Multi-Agent Optimization

Tutorial at AAMAS'23

Filippo Bistaffa   Gauthier Picard   Roie Zivan

Some contents adapted from previous tutorials (http://https//www2.isye.gatech.edu/~fferdinando3/cfp/AAMAS19/)

Introduction and Motivations

Who are we?

Filippo Bistaffa, PhD

IIIA-CSIC, Barcelona
Expertises: coalition formation, parallel computing, shared mobility

Gauthier Picard, PhD, Hab.

ONERA, the French Aerospace Lab
Expertises: DCOPs, self-organization, resource allocation

Roie Zivan, PhD

Ben Gurion University of the Negev
Expertises: DCOPs, partial cooperation

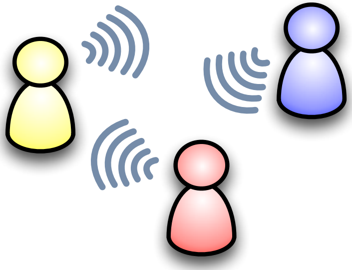
Introduction and Motivations

Multiagent Systems

- Agent: An entity that behaves autonomously in the pursuit of goals
- Multi-agent system: A system of multiple interacting agents

An agent is...

- Autonomous: Is of full control of itself
- Interactive: May communicate with other agents
- Reactive: Responds to changes in the environment or requests by other agents
- Proactive:Takes initiatives to achieve its goals



Introduction and Motivations

Research questions addressed during this tutorial

- How to make collective optimal decisions?
  - How to model the collective decision?
  - Which protocols to implement these decisions?
- How to form groups *wrt* to some utility criteria?
  - How to model the utility of each group?
  - How to express which groups are feasible or not?

Today's Menu

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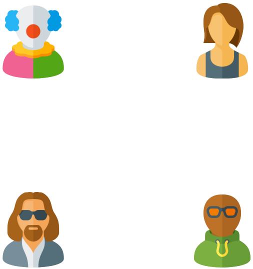
Characteristic Function Games (CFGs)

[CHALKIADAKIS et al., 2011]



Set of Agents A

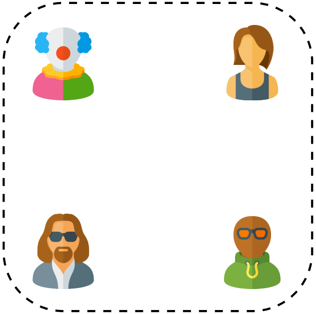
$A = \{\ ,\ ,\ ,\ \}$

Characteristic Function $v(\cdot)$

- $v(\{\ ,\ \}) = 0$
- $v(\{\ ,\ ,\ \}) = -7$
- $v(\{\ ,\ \}) = 3$
- $\dots$

Characteristic Function Games (CFGs)

[CHALKIADAKIS et al., 2011]



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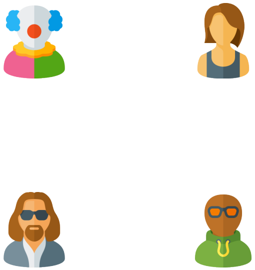

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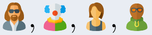
$A = \{$  $\}$

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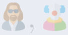
- $v(\{$  $\}) = 3$
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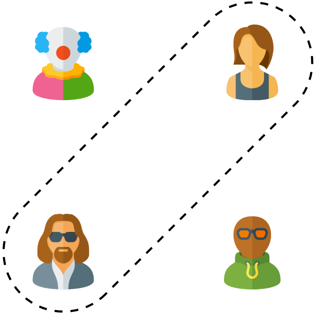

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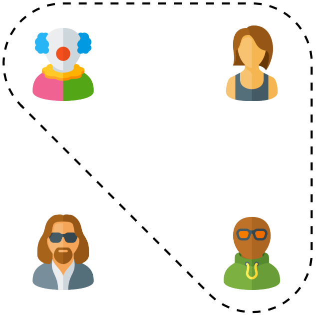

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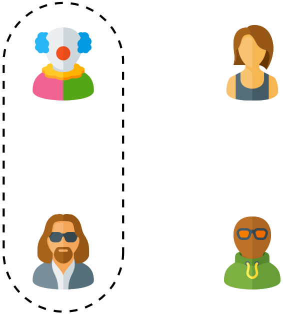

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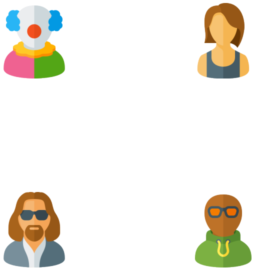

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Characteristic Function

[CHALKIADAKIS et al., 2011]

Characteristic Function

The function $v : \mathcal{P}(A) \rightarrow \mathbb{R}$ associates a value to *every coalition* (i.e., subset) of A

Exponential Complexity

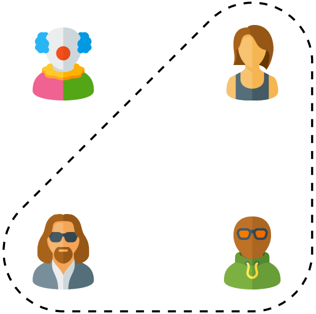
Representing $v(\cdot)$ as a *table* requires an *exponential* number of steps (i.e., $2^{|A|}$)

Mitigate this Complexity

(1) *Restrict* the set of coalitions or (2) consider $v(\cdot)$ with a specific *structure*

Cardinality-Restricted CFGs

[SHEHORY and KRAUS, 1998]



Maximum Cardinality k

E.g., only coalitions of size ≤ 3 are feasible

Polynomial Number of Coalitions

Total number of coalitions is $\sum_{i=1}^{k} \binom{|A|}{i} = \mathcal{O}(|A|^k)$, i.e., *polynomial wrt* $|A|$

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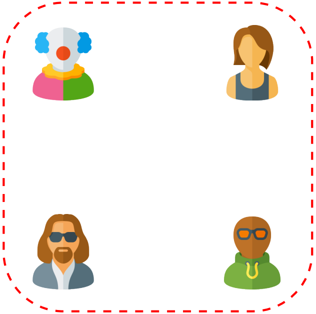

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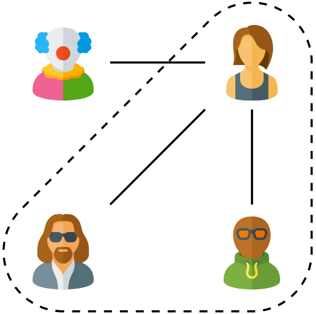
Graph-Restricted CFGs

[Myerson, 1977], [Demange, 2004]



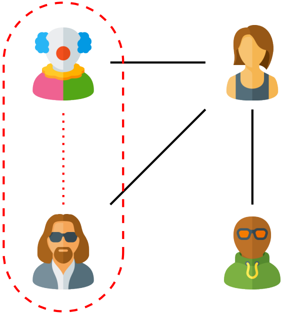
Graph G among Agents

Connected Subgraphs

A coalition is *feasible* only if it induces a *connected* subgraph of G

Graph-Restricted CFGs

[MYERSON, 1977], [DEMANGE, 2004]



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Real-World Example: Social Ridesharing

[Bistaffa et al., 2017b]

Social Ridesharing

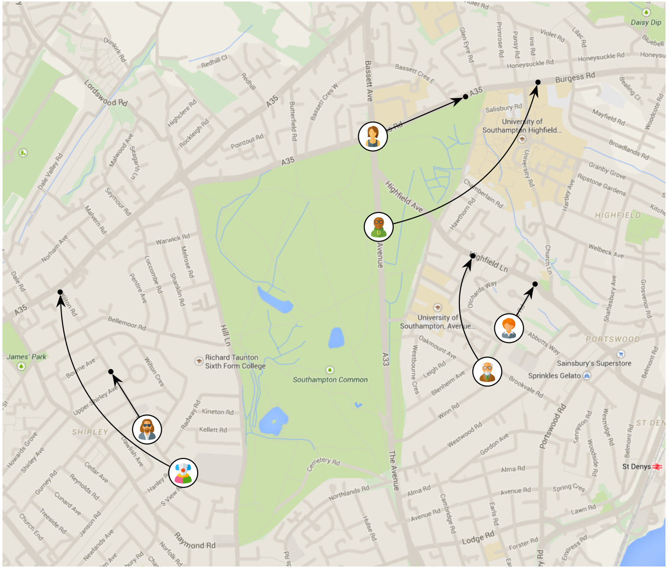
Arrange *cost-effective* shared cars among agents connected by a *social network*

Cardinality-Based Constraints

Cars (i.e., coalitions) can contain *up to 5 passengers*

Graph-Based Constraints

We only form coalitions among "*friends of friends*" (connected subgraph)


Daisy Dip
Burgess Rd
A35
Bassett Ave
Lordswood Rd
Redhill Cl
Dunkirk Rd
Highclere Rd
Rockleigh Rd
Pointout Rd
Butterfield Rd
Bassett Cres W
Bassett Cres E
Salisbury Rd
University of Southampton Highfield...
Glen Eyre Rd
Primrose Rd
Pansy Rd
Iris Rd
Violet Rd
Lilac Rd
Honeysuckle Rd
Bealing Cl
Woodcote Rd
Mayfield Rd
Broadlands Rd
Granby Grove
Ripstone Gardens
HIGHFIELD
Highfield Ave
Chamberlain Rd
Hawthorn Rd
University Rd
Hartley Ave
Welbeck Ave
Church Ln
Highfield Ln
Orchards Way
Shaftesbury Ave
Grosvenor Rd
PORTSWOOD
University of Southampton, Avenue...
Oakmount Ave
Westbourne Cres
Leigh Rd
Blenheim Ave
Abbotts Way
Sainsbury's Superstore
Sprinkles Gelato
Brookvale Rd
Belmont Rd
Winn Rd
Westwood Rd
Gordon Ave
Portswood Rd
Tennyson Rd
Westridge Rd
Woodside Rd
St Denys
Alma Rd
Avenue Rd
Spring Cres
Lawn Rd
Cambridge Rd
Earls Rd
Lodge Rd
Forster Rd
Empress Rd
Northlands Rd
Hulse Rd
Cemetery Rd
The Avenue
A33
Avenue
Southampton Common
Richard Taunton Sixth Form College
Hill Ln
Kineton Rd
Kellett Rd
Radway Rd
Raymond Rd
Norfolk Rd
Dale Valley Rd
Malwood Ave
Seagarth Ln
Seymour Rd
Malvern Rd
Warwick Rd
Melrose Rd
Luccombe Rd
Pentire Ave
Shanklin Rd
Norham Ave
Dale Rd
Wilton Rd
Bellemoor Rd
Wilton Cres
James' Park
Bourne Ave
Upper Shirley Ave
SHIRLEY
Dawlish Ave
Hanley Rd
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Shirley Ave
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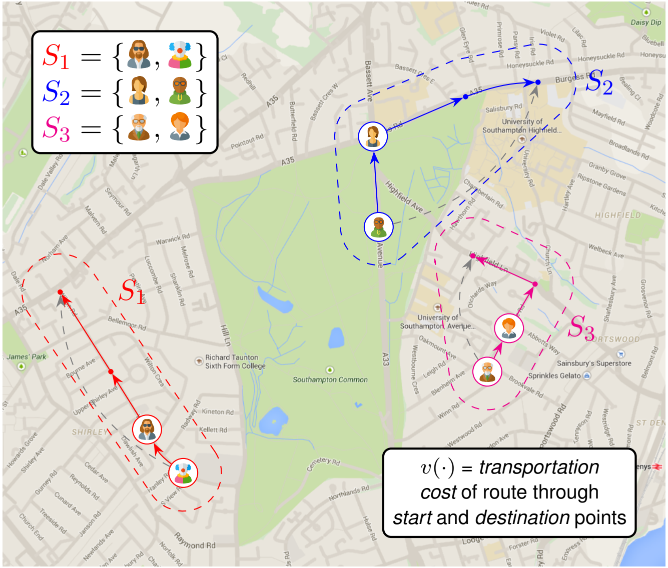
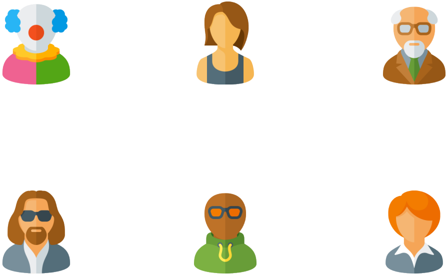
Gurney Rd
Cunard Ave
Treeside Rd
Janson Rd
Church End
Newlands Ave


$S_1 = \{ , \}$
$S_2 = \{ , \}$
$S_3 = \{ , \}$
S_1
S_2
S_3
$v(\cdot)$ = *transportation cost* of route through *start* and *destination* points

Today's Menu

Coalition Structure Generation (CSG)

[RAHWAN et al., 2015]



Solving the Coalition Structure Generation (CSG) Problem

Compute the partition $\mathcal{S}$ of A into *feasible* coalitions that *maximizes* the sum $\sum_{S \in \mathcal{S}} v(S)$

Coalition Structure Generation (CSG)

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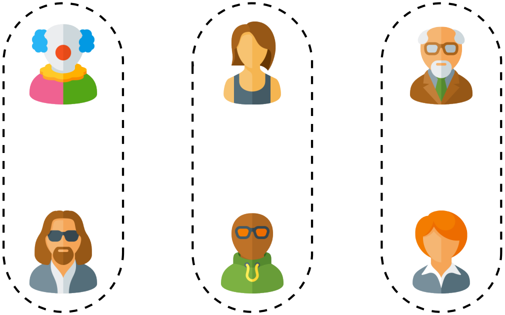

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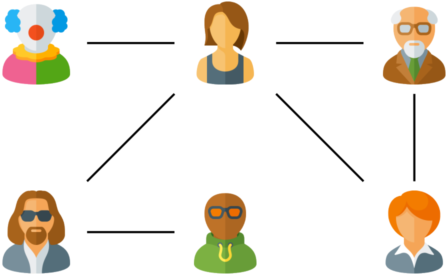

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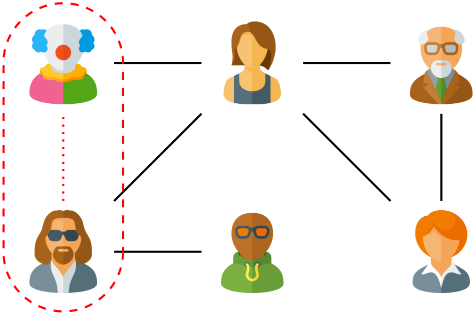

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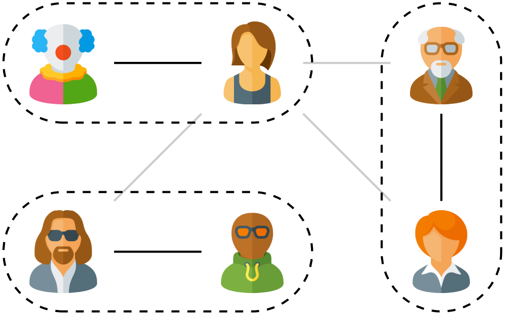

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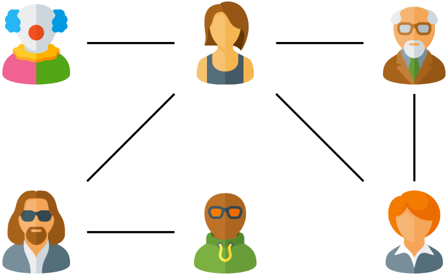
CSG Approaches based on Search

[Bistaffa et al., 2017a]



Edge Contraction Operation

Contraction of edge $(S_i, S_j) \rightarrow$ form coalition $S_i \cup S_j$

CSG Approaches based on Search

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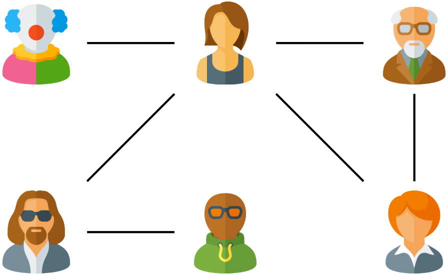

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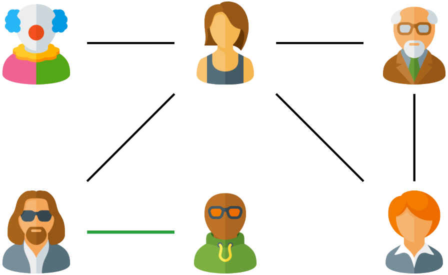

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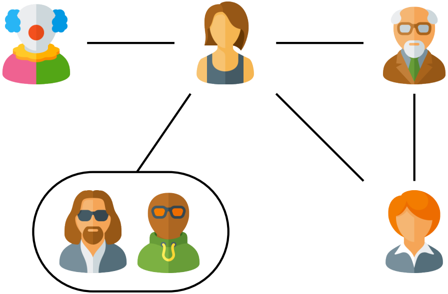

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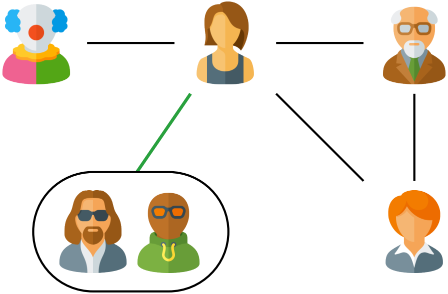

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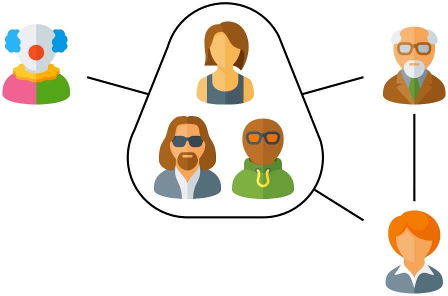

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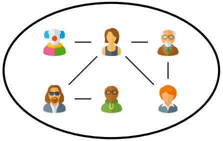

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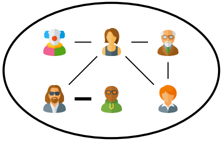

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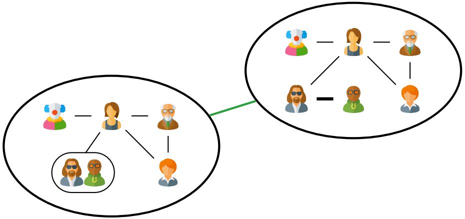

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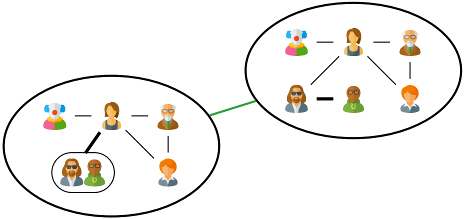

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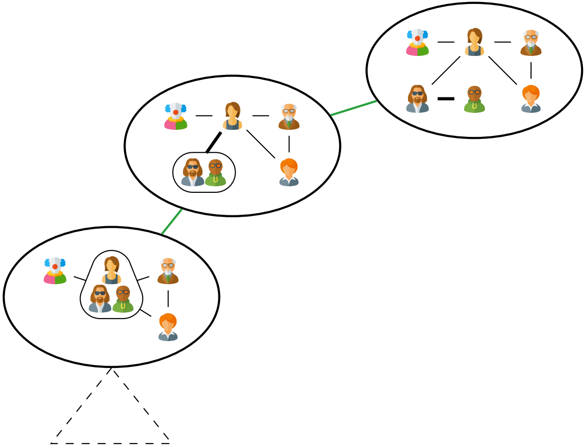

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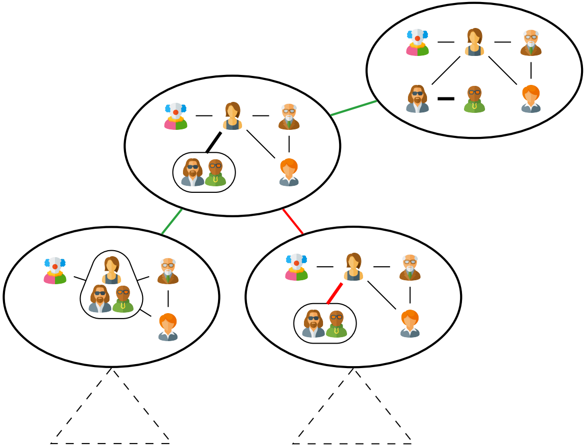

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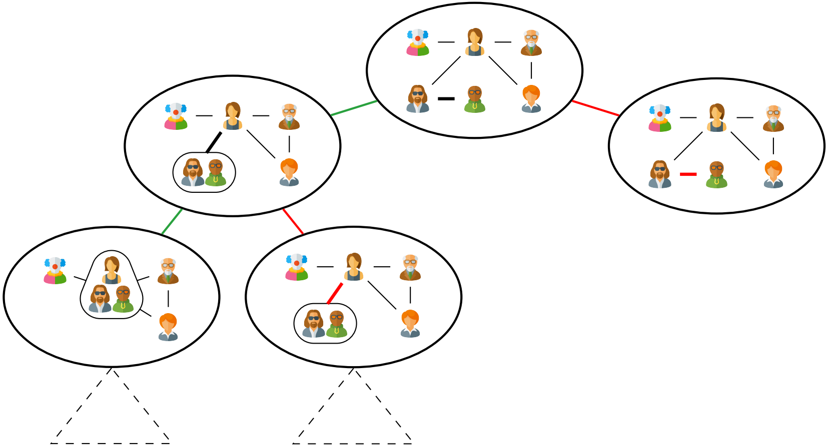

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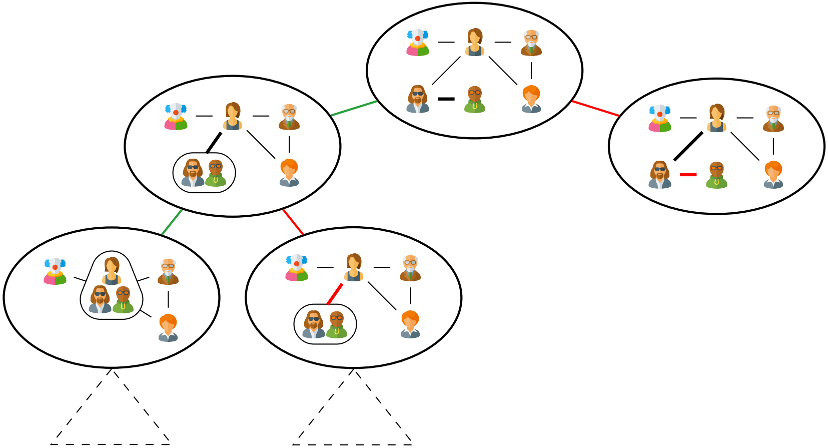

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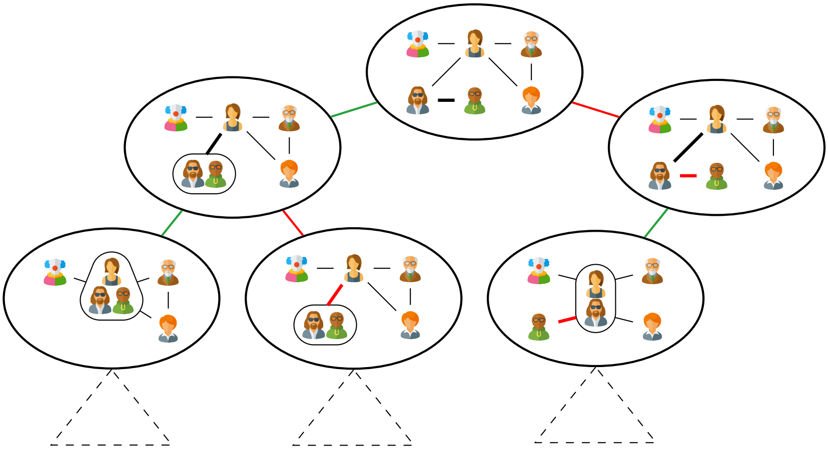

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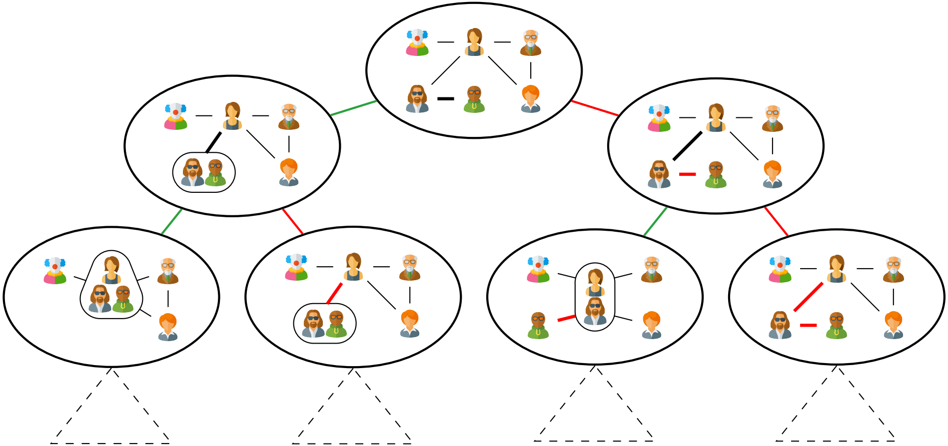

CSG Approaches based on Search

[BISTAFFA et al., 2017a]

CFSS Algorithm

- Builds a *Binary Decision Diagram* (BDD) by contracting (or not) an edge at each step
- Each coalition structure (i.e., partition of A) is represented *only once* in the BDD
- The optimal coalition structure is computed by doing a *depth-first* traversal of the BDD

Pros

Approximate algorithm with quality guarantees if used in conjunction with *Branch-and-Bound*

Cons

Performance depends on the assumption that $v(\cdot)$ can be expressed in *closed-form*

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CSG Approaches based on Integer Linear Programming

Background on Integer Linear Programming

Weighted Knapsack Problem

We want to fill our knapsack (capacity = c) with the goal of maximizing the total value

What is the Optimal Subset of Object for $c = 5$?

A Pick 🍎 (weight = 1) $\to 1$	$w($🍎$) = 1, v($🍎$) = 1$
B Pick 💎 (weight = 2) $\to 4$	$w($💎$) = 2, v($💎$) = 4$
C Pick 💍 (weight = 4) $\to 3$	$w($💍$) = 4, v($💍$) = 3$
D Pick 👑 (weight = 5) $\to 9$	$w($👑$) = 6, v($👑$) = 1$
E Pick (weight = 3) $\to 6$	$w() = 3, v() = 6$

CSG Approaches based on Integer Linear Programming

Background on Integer Linear Programming

Our Ingredients

- Let x_A, x_B, x_C, x_D, x_E be binary decision variables (either pick the object or not)
- Objective function: maximize the value of selected objects
- Constraint: do not exceed the knapsack capacity

Integer *Linear* Programming (ILP) Formulation

$$
\begin{aligned}
\text{maximize} \quad & 1 \cdot x_A + 4 \cdot x_B + 3 \cdot x_C + 9 \cdot x_D + 6 \cdot x_E && \text{(Values of selected objects)} \\
\text{subject to} \quad & 1 \cdot x_A + 2 \cdot x_B + 4 \cdot x_C + 5 \cdot x_D + 3 \cdot x_E \leq 5 && \text{(Capacity constraint)} \\
& x_A, x_B, x_C, x_D, x_E \in \{0, 1\} && \text{(Binary decision variables)}
\end{aligned}
$$

CSG Approaches based on Integer Linear Programming

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Our Ingredients

- Let x_A, x_B, x_C, x_D, x_E be binary decision variables (either pick the object or not)
- Objective function: maximize the value of selected objects
- Constraint: do not exceed the knapsack capacity

Integer *Linear* Programming (ILP) Formulation

maximize	$1 \cdot x_A + 4 \cdot x_B + 3 \cdot x_C + 9 \cdot x_D + 6 \cdot x_E$	(Values of selected objects)
subject to	$1 \cdot x_A + 2 \cdot x_B + 4 \cdot x_C + 5 \cdot x_D + 3 \cdot x_E \leq 5$	(Capacity constraint)
	$x_A, x_B, x_C, x_D, x_E \in \{0, 1\}$	(Binary decision variables)

CSG Approaches based on Integer Linear Programming

Background on Integer Linear Programming

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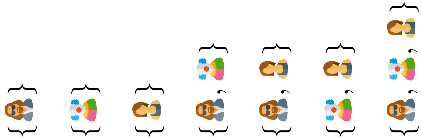
Integer *Linear* Programming (ILP) Formulation

$$
\begin{aligned}
\text{maximize} \quad & 1 \cdot x_A + 4 \cdot x_B + 3 \cdot x_C + 9 \cdot x_D + 6 \cdot x_E && \text{(Values of selected objects)} \\
\text{subject to} \quad & 1 \cdot x_A + 2 \cdot x_B + 4 \cdot x_C + 5 \cdot x_D + 3 \cdot x_E \leq 5 && \text{(Capacity constraint)} \\
& x_A, x_B, x_C, x_D, x_E \in \{0, 1\} && \text{(Binary decision variables)}
\end{aligned}
$$

CSG Approaches based on Integer Linear Programming

[Rahwan et al., 2015]

- Given A and a set $\mathcal{S}$ of *coalitions* (i.e., subsets) of A, let M be a $|A| \times |\mathcal{S}|$ matrix
- $M_{iS} = 1$ if and only if agent $a \in A$ is part of coalition $S \in \mathcal{S}$, $M_{iS} = 0$ otherwise



$$M = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

CSG Approaches based on Integer Linear Programming

[Rahwan et al., 2015]

Objective of Coalition Structure Generation

Compute the *partition* of A that *maximizes* the sum of the corresponding values

ILP Formulation for Coalition Structure Generation

$$\begin{array}{lll} \text{maximize} & \sum_{S \in \mathcal{S}} v(S) \cdot x_S & \text{(Value of each selected coalition)} \\ \text{subject to} & \sum_{S \in \mathcal{S}} M_{iS} \cdot x_S = 1 \quad \forall i \in A & \text{(Each agent exactly in } \textit{one} \text{ coalition)} \end{array}$$

CSG Approaches based on Integer Linear Programming

[Rahwan et al., 2015]

Solving Integer Linear Programs

ILPs can be solved with state-of-the-art solvers like CPLEX (very mature technology)

Pros

Does not require any assumption on $v(\cdot)$ (very general approach)

Cons

- Memory requirements can become unmanageable for more than 20–30 agents
- Difficult to directly exploit the structure of the problem (i.e., graph)

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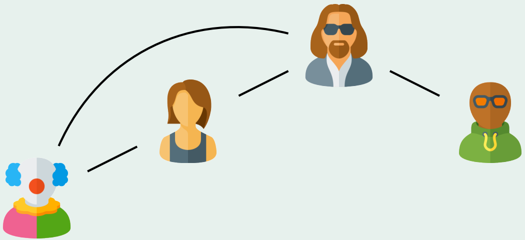
CSG as a COP

[BISTAFFA and FARINELLI, 2018]

Graph-Restricted CFG Example



Pseudotree among Agents (Hierarchy)

Adjacent agents in the graph fall in the same branch of the tree (e.g., and)

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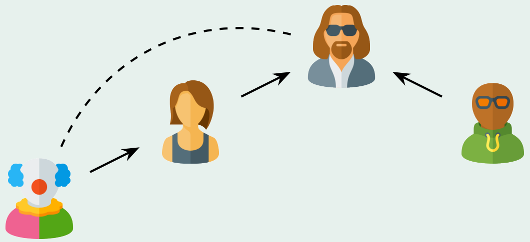

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[Bistaffa and Farinelli, 2018]

Challenge

How can we exploit the structure (i.e., hierarchy among agents)?

Main Idea

- Each coalition (i.e., decision variable) is "controlled" by the highest agent
- "Delegate" the formation of coalitions to descendants by means of *required* variables

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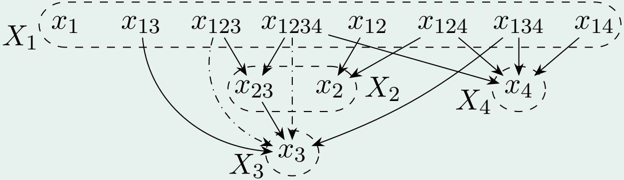

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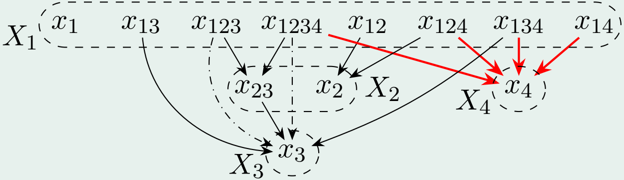

CSG as a COP

[BISTAFFA and FARINELLI, 2018]

Required Variables

- Any two variables that require the same variable *cannot* be enabled simultaneously
- As a result *no overlapping variables* are activated *at the same time*

Number of Constraints

- Naive COP: $\binom{\#\text{ coalitions}}{2}$
- This approach: linear *wrt* the number of agents

Open Question

Can we make this COP a Distributed COP (DCOP)?

CSG as a COP

[BISTAFFA and FARINELLI, 2018]

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Today’s Menu

Characteristic Function

[CHALKIADAKIS et al., 2011]

Characteristic Function

The function $v : \mathcal{P}(A) \rightarrow \mathbb{R}$ associates a value to *every coalition* (i.e., subset) of A

Exponential Complexity

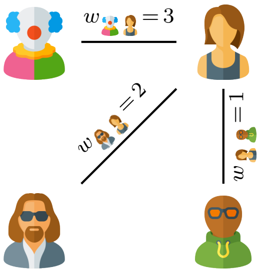
Representing $v(\cdot)$ as a *table* requires an *exponential* number of steps (i.e., $2^{|A|}$)

Mitigate this Complexity

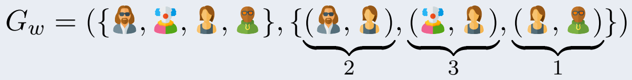
(1) *Restrict* the set of coalitions or (2) consider $v(\cdot)$ with a specific *structure*

Induced Subgraph Games (ISGs)

[DENG and PAPADIMITRIOU, 1994]



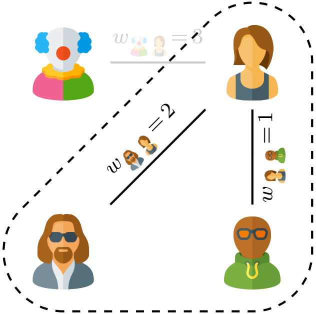
Weighted Graph G among Agents



Value is the Sum of Induced Edges

Induced Subgraph Games (ISGs)

[DENG and PAPADIMITRIOU, 1994]



Weighted Graph G among Agents

$$G_w = (\{\ ,\ ,\ ,\ \}, \{\underbrace{(\ ,\ )}_{2}, \underbrace{(\ ,\ )}_{3}, \underbrace{(\ ,\ )}_{1}\})$$

Value is the Sum of Induced Edges

$$v(\{\ ,\ ,\ \}) = 2 + 1 = 3$$

Induced Subgraph Games (ISGs)

[CHALKIADAKIS et al., 2011]

Succinct Game Representation

The characteristic function is *entirely* represented by the weighted graph G_w

Computational Advantages

CSG on ISGs can be treated as a *graph clustering* problem (“easier” than CSG)

Limited Representation Power

Not every characteristic function game can be *perfectly* represented as an ISG

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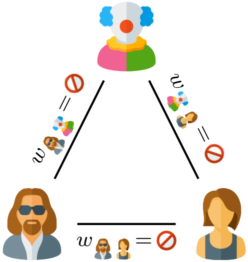


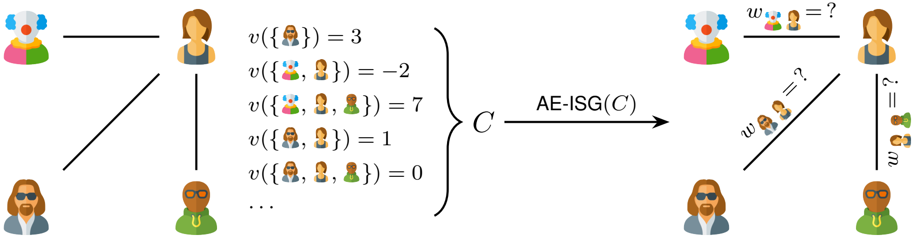
$$v(S) = \begin{cases} 0, & \text{if } |S| = 1, \\ 1, & \text{if } |S| = 2, \\ 6, & \text{if } |S| = 3. \end{cases}$$

Can We Approximate a CFG as an ISG?

[BISTAFFA et al., 2022]

Approximately Equivalent ISG (AE-ISG)

Given a CFG C, compute the ISG that *best approximates* C, namely AE-ISG(C)



AE-ISG as Norm Approximation (ℓ_p Linear Regression)

[BISTAFFA et al., 2022]

minimise $\|\underbrace{Mw - v}_{\text{residuals}}\|_p$

$$M = \begin{array}{ccc} w & w & w \\ \end{array}\begin{bmatrix} \vdots & \vdots & \vdots \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ \vdots & \vdots & \vdots \end{bmatrix} \qquad v = \begin{bmatrix} \vdots \\ v(\{\ ,\ ,\ \}) \\ v(\{\ ,\ \}) \\ v(\{\ ,\ ,\ \}) \\ \vdots \end{bmatrix}$$

AE-ISG as Norm Approximation (ℓ_p Linear Regression)

[BISTAFFA et al., 2022]

$$\text{minimise } \; \|\underbrace{Mw - v}_{\text{residuals}}\|_p$$

Residual Vector

The *residual vector* $r = Mw - v$ is the vector of *differences* between *approximated* coalitional values (i.e., Mw) and *original* coalitional values (i.e., v)

Constrained Norm Approximation

Some coalitions (singletons) can be represented *exactly* via additional constraints

AE-ISG as Norm Approximation (ℓ_p Linear Regression)

[Bistaffa et al., 2022]

Size of AE-ISG Model

Building M and v requires to go through the set of coalitional values (obviously)

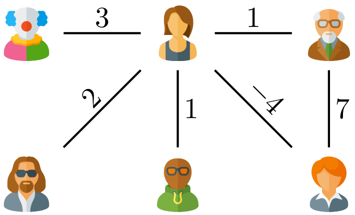
Computational Complexity

If the set of feasible coalitions is *polynomial* (e.g., ridesharing), computing AE-ISG(C) has a *manageable* complexity, depending on the norm ℓ_p:

- $\ell_1/\ell_\infty \rightarrow$ Linear Programming (exact, CPU)
- $\ell_2 \rightarrow$ Least Squares (exact/analytical, GPU)
- $\ell_{>2} \rightarrow$ Iteratively Reweighted Least Squares (numerical)

CSG on ISGs as Graph Clustering

[BISTAFFA et al., 2022]



CSG on ISGs Optimisation Objective

Maximise sum of of *clusters' internal weights* (namely, *coverage* measure)

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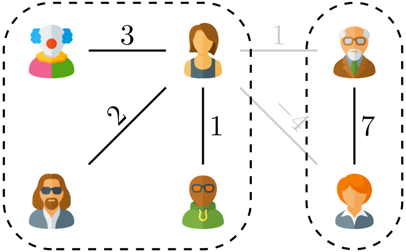


CSG on ISGs Optimisation Objective

Maximise sum of of *clusters' internal weights* (namely, *coverage* measure)

ILP for Optimal Graph Clustering (KGC)

[BISTAFFA et al., 2022]

$X_{ij} = 1 \rightarrow$ edge $\{i, j\}$ is "activated" (i and j are in the same cluster)

maximise $\sum_{i,j \in A} w_{ij} \cdot X_{ij}$ (Coverage objective function)

subject to $\forall i, j, z \in A : \begin{cases} X_{ij} + X_{jz} - 2 \cdot X_{iz} \leq 1 \\ X_{iz} + X_{ij} - 2 \cdot X_{jz} \leq 1 \\ X_{jz} + X_{iz} - 2 \cdot X_{ij} \leq 1 \end{cases}$ (Transitivity)

$\forall i \in A : \sum_{j \in A} X_{ij} \leq k$ (Cardinality constraint)

Theoretical Quality Guarantees

[Bistaffa et al., 2022]

KGC computes the value of the optimal CSG solution
for the *approximated game AE-ISG*(C), namely $V(CS^*_{AE})$

What about the value of the optimal CSG solution
for the *original game* C, namely $V(CS^*_C)$?

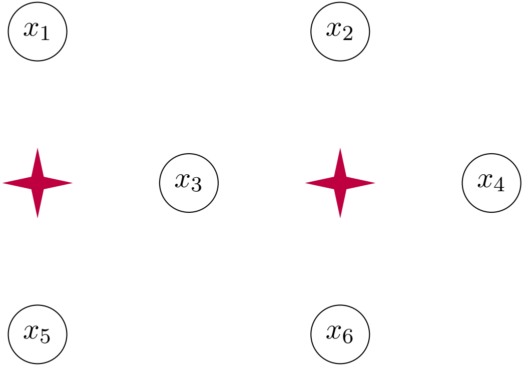
We proposed a way to compute B_r
such that $V(CS^*_C) - V(CS^*_{AE}) \leq B_r$

Today's Menu

Today's Menu

Motivating example

Sensor networks



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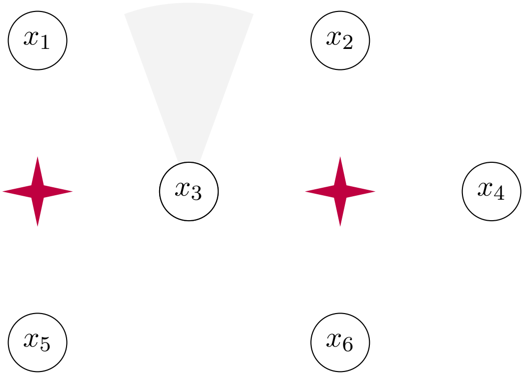
Sensor networks



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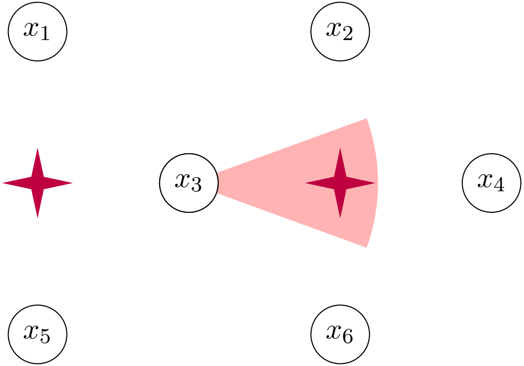

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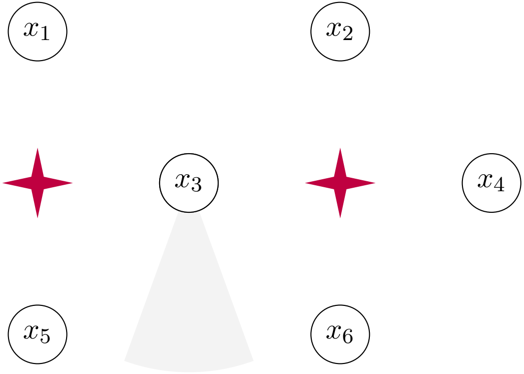

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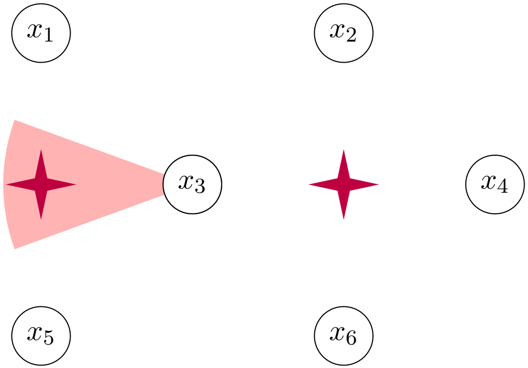

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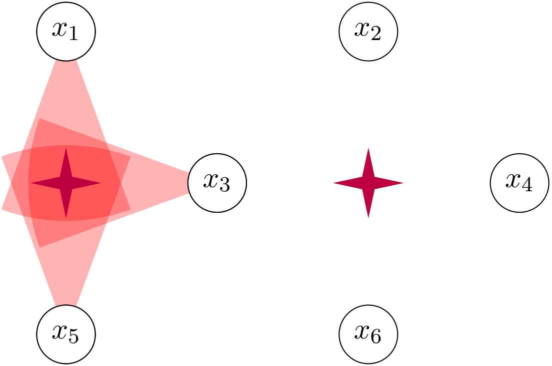


x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem as a CSP!

Today's Menu

CSP

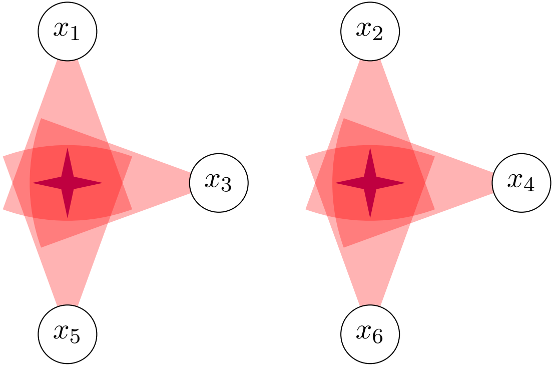
Constraint Satisfaction

- Variables $X = \{x_1, \ldots, x_n\}$
- Domains $D = \{D_1, \ldots, D_n\}$
- Constraints $C\{c_1, \ldots, c_m\}$
  where a constraint $c_i \subseteq D_{i_1} \times D_{i_2} \times \ldots \times D_{i_n}$ denotes the possible valid joint assignments for the variables $x_{i_1}, x_{i_1}, \ldots, x_{i_n}$ it involves

- **Goal**: Find an assignment to all variables that satisfies all the constraints

CSP

Constraint Satisfaction

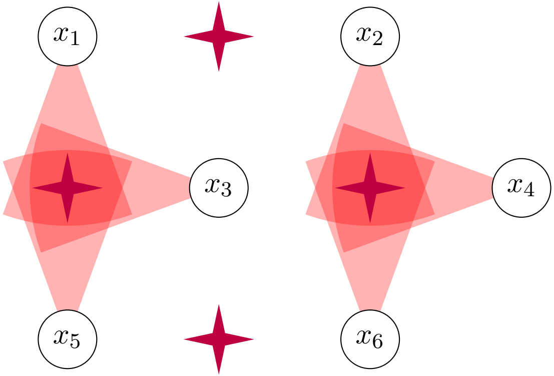


x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	…		✗
S	W	N	✓
	…		✗
W	W	W	✗

Model the problem as a CSP!

Max-CSP

Max Constraint Satisfaction



x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem as a Max-CSP!

Max-CSP

Max Constraint Satisfaction

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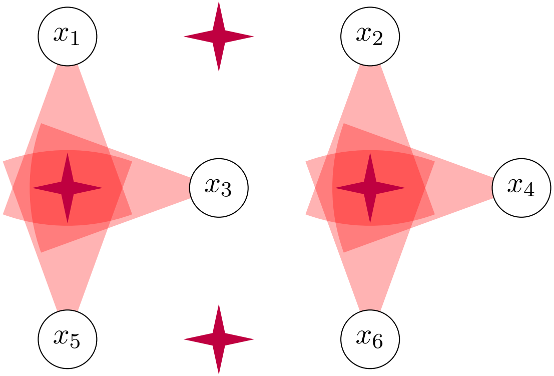
Max-CSP

Max Constraint Satisfaction

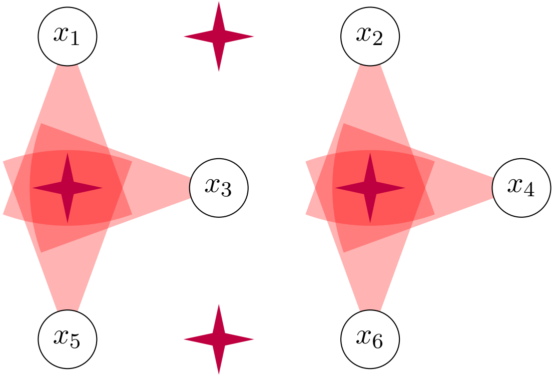


x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem
as a Max-CSP!

WCSP (or COP)

Constraint Optimization



x_1	x_3	x_5	Cost
N	N	N	∞
N	N	E	∞
	...		∞
S	W	N	10
	...		∞
W	W	W	∞

Model the problem as a COP!

WCSP (or COP)

Constraint Optimization

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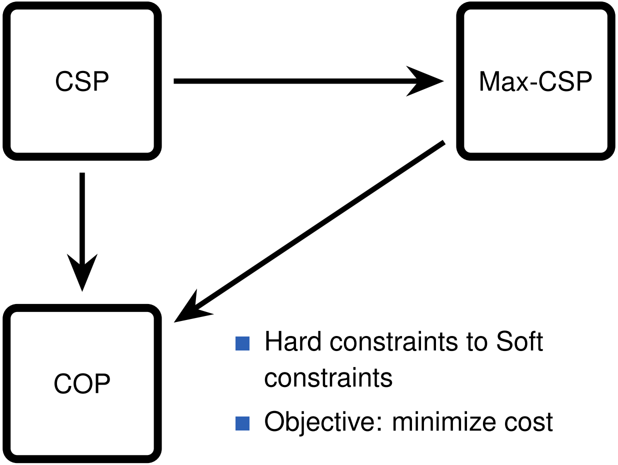
- Domains $D = \{D_1, \ldots, D_n\}$
- Constraints $C\{c_1, \ldots, c_m\}$
  where a constraint $c_i : D_{i_1} \times D_{i_2} \times \ldots \times D_{i_n} \rightarrow \mathbb{R}_+ \cup \{\infty\}$ expresses the degree of constraint violation

- **Goal**: Find an assignment to all variables that minimizes the sum of all the constraints

Constraint Reasoning

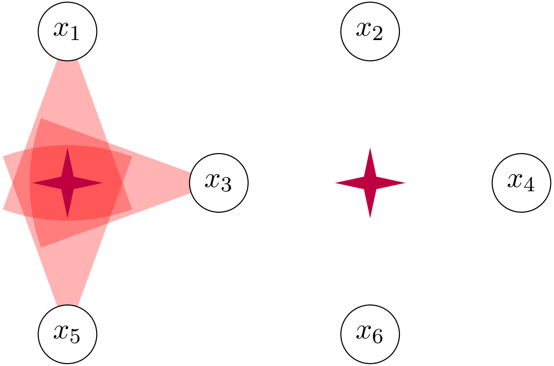


- Objective: maximize #constraints satisfied

- Hard constraints to Soft constraints
- Objective: minimize cost

WCSP (or COP)

Constraint Optimization



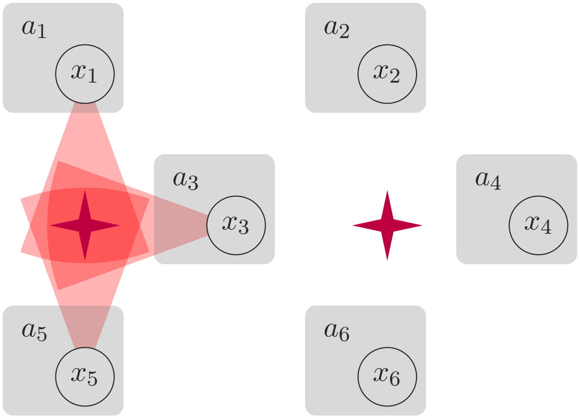
Imagine that each sensor is an autonomous agent

How should this problem be modeled and solved in a decentralized manner?

Today's Menu

DCOP

Distributed Constraint Optimization [Modi et al., 2005]



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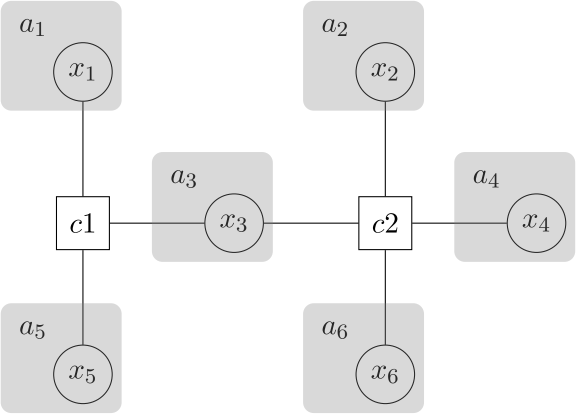


DCOP

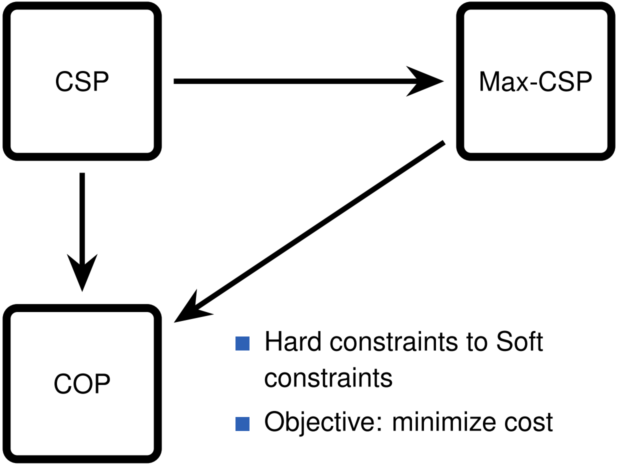
Distributed Constraint Optimization [Modi et al., 2005]

- Agents $X = \{a_1, \ldots, a_l\}$
- Variables $X = \{x_1, \ldots, x_n\}$
- Domains $D = \{D_1, \ldots, D_n\}$
- Constraints $C\{c_1, \ldots, c_m\}$
- Mapping of variables to agents

- **Goal**: Find an assignment to all variables that minimizes the sum of all the constraints

DCOP

Distributed Constraint Optimization [Modi et al., 2005]

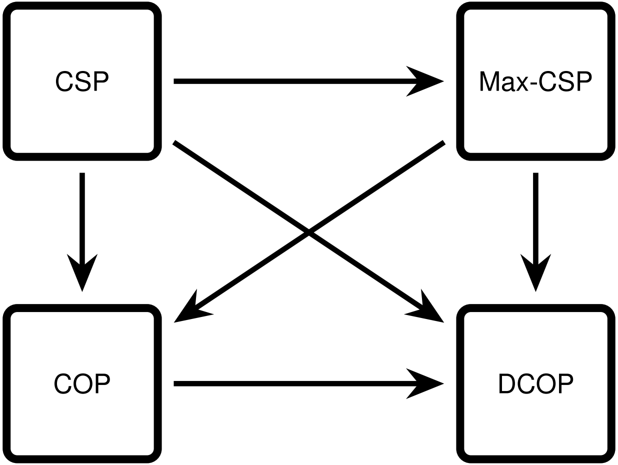


- Objective: maximize #constraints satisfied

- Hard constraints to Soft constraints
- Objective: minimize cost

DCOP

Distributed Constraint Optimization [Modi et al., 2005]

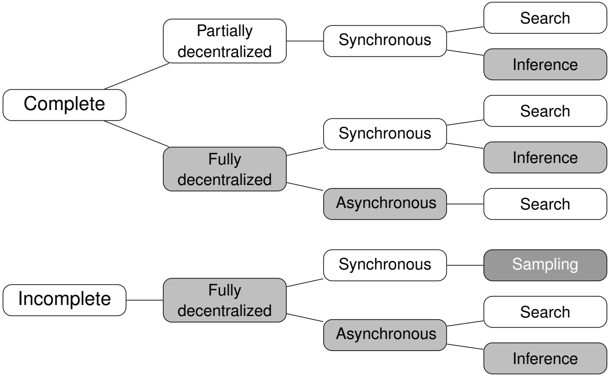


- Variables are controlled by agents
- Communication model
- Local knowledge

Today's Menu

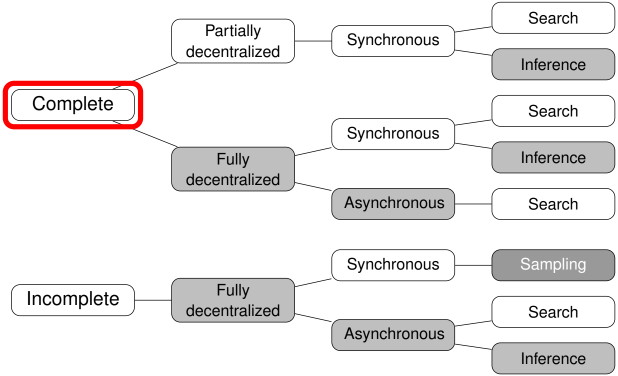
DCOP Algorithms

See [FIORETTO et al., 2018]



DCOP Algorithms

See [FIORETTO et al., 2018]

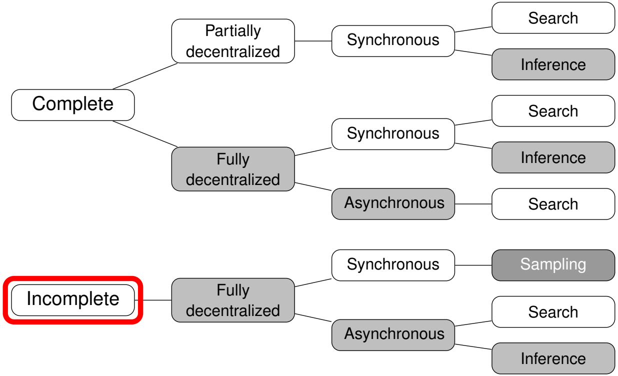


Important metrics

- Agent complexity
- Network loads
- Message size

DCOP Algorithms

See [Fioretto et al., 2018]



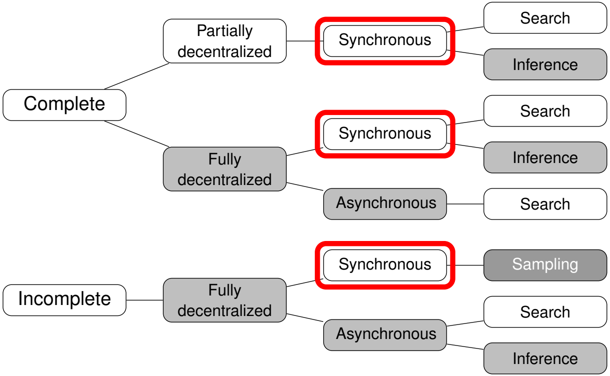
Important metrics

- Agent complexity
- Network loads
- Message size

- Anytime
- Quality guarantees
- Execution time vs. solution quality

DCOP Algorithms

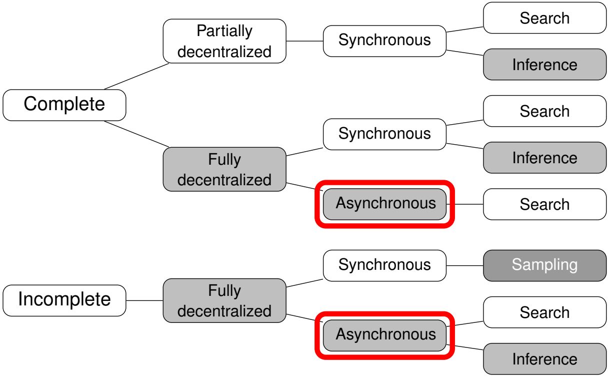
See [FIORETTO et al., 2018]



- Systematic process, divided in steps
- Each agent waits for particular messages before acting
- Consistent view of the search process
- Typically, increases idle-time

DCOP Algorithms

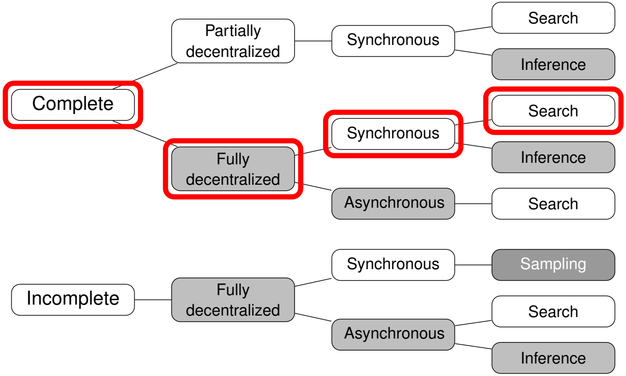
See [FIORETTO et al., 2018]



- Decision based on agents' local state
- Agents' actions do not depend on sequence of received messages
- Minimizes idle-time
- No guarantees on validity of local views

DCOP Algorithms

See [FIORETTO et al., 2018]



Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]

Synchronous Branch-and-Bound (SBB)

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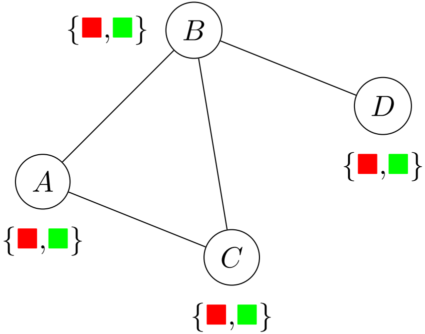
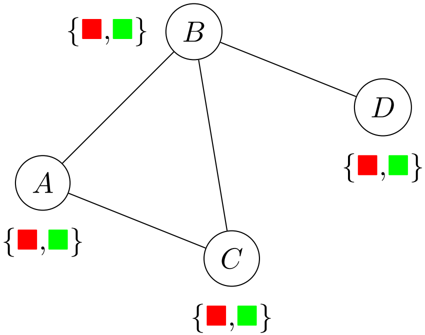


x_i	x_j	(A, B)	(A, C)	(B, C)	(B, C)
		5	5	5	3
		8	10	4	8
		20	20	3	10
		3	3	3	3

How do we solve this distributedly?

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]



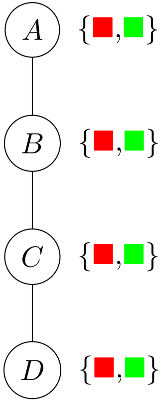
x_i	x_j	(A, B)	(A, C)	(B, C)	(B, C)
		5	5	5	3
		8	10	4	8
		20	20	3	10
		3	3	3	3

How do we solve this distributedly?

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]

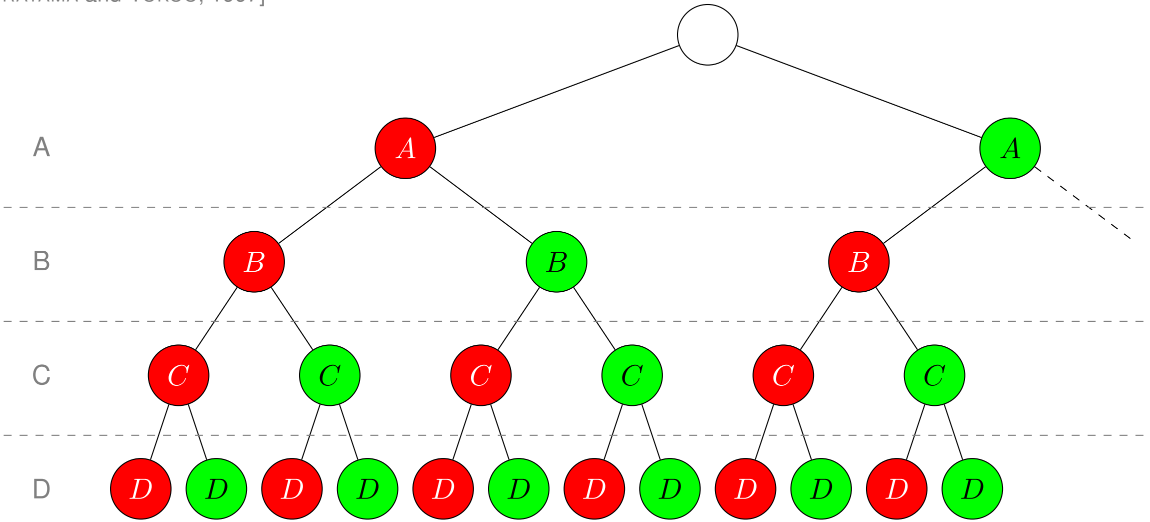
Complete ordering



- Agents operate on a complete ordering
- Agents exchange CPA messages containing partial assignments
- When a solution is found, its solution cost as an UB is broadcasted to all agents
- The UB is used for branch pruning

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]



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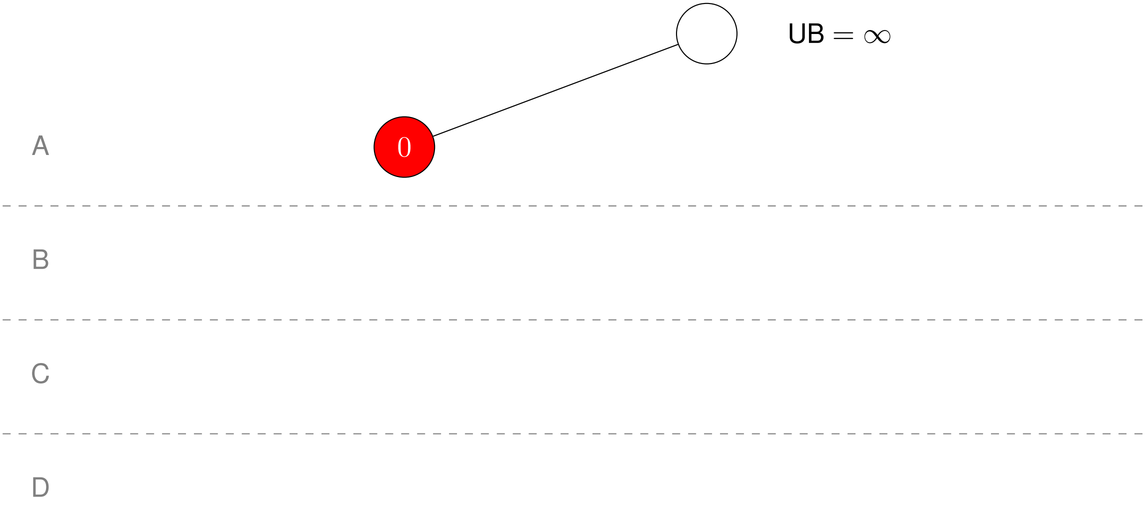

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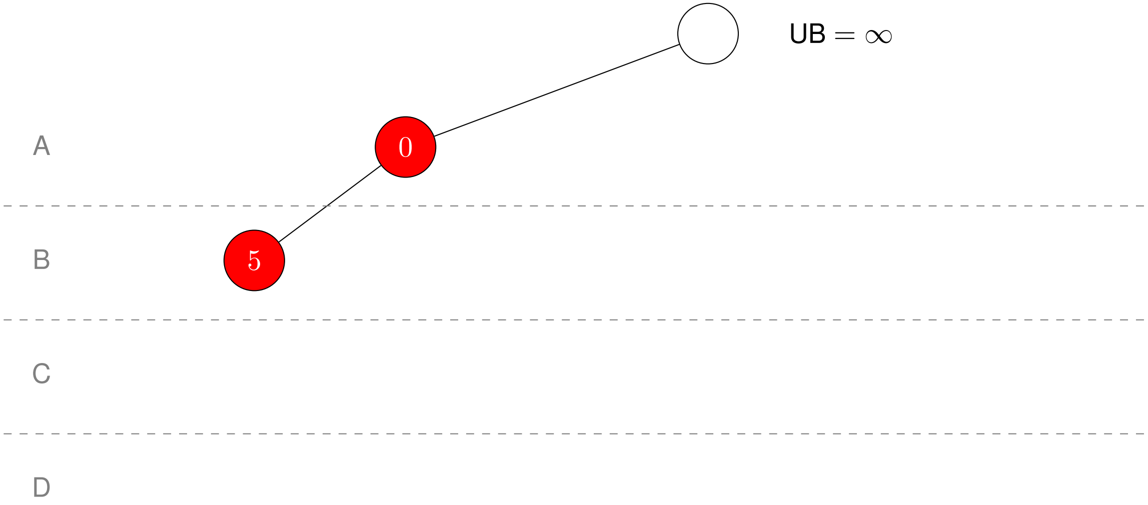

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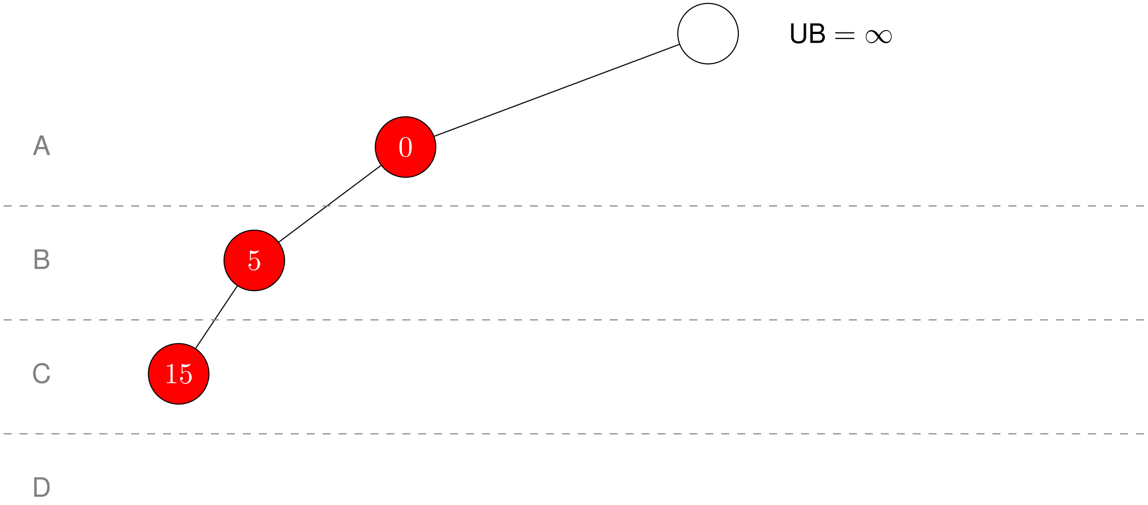

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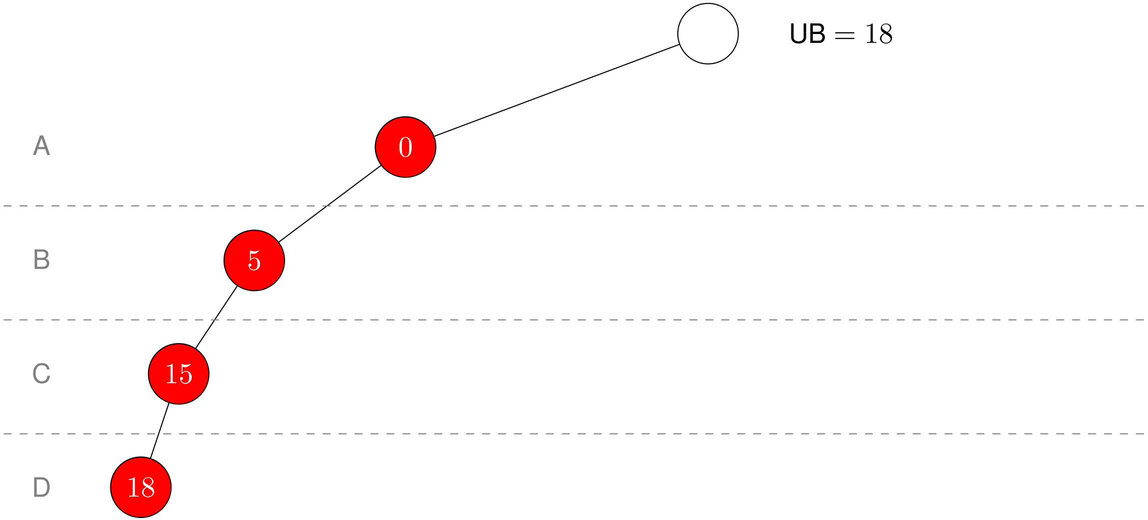

Synchronous Branch-and-Bound (SBB)

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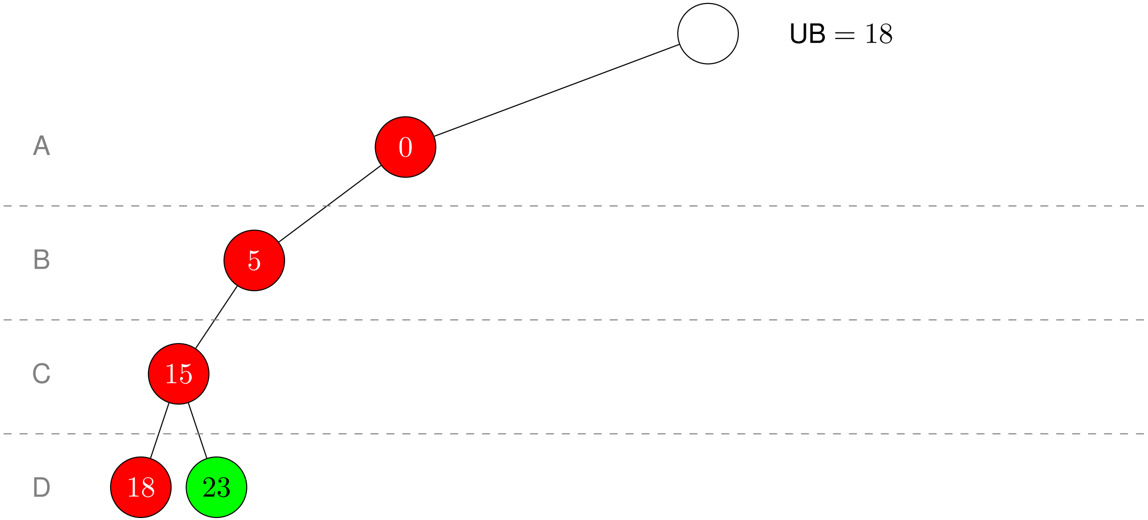

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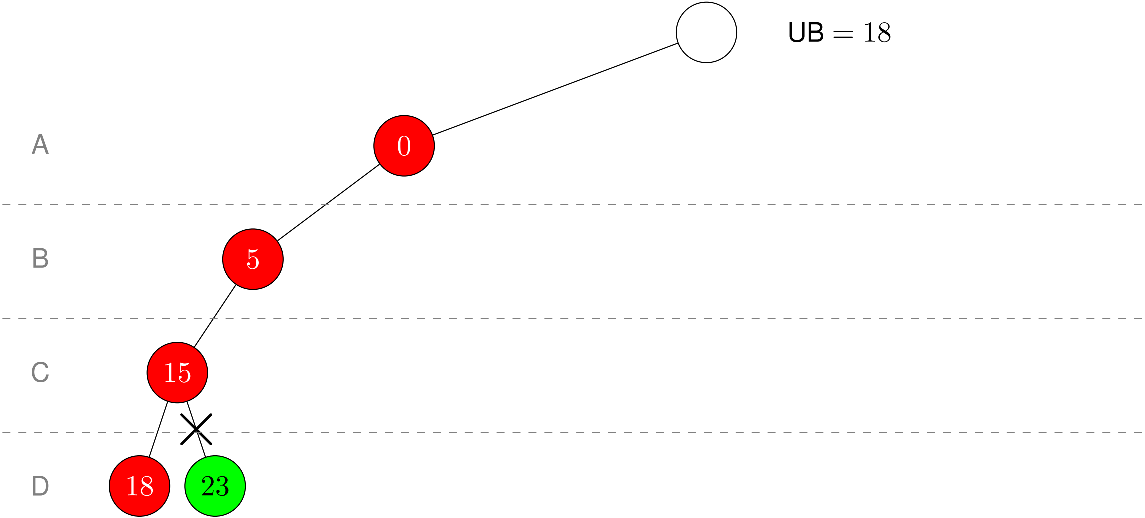

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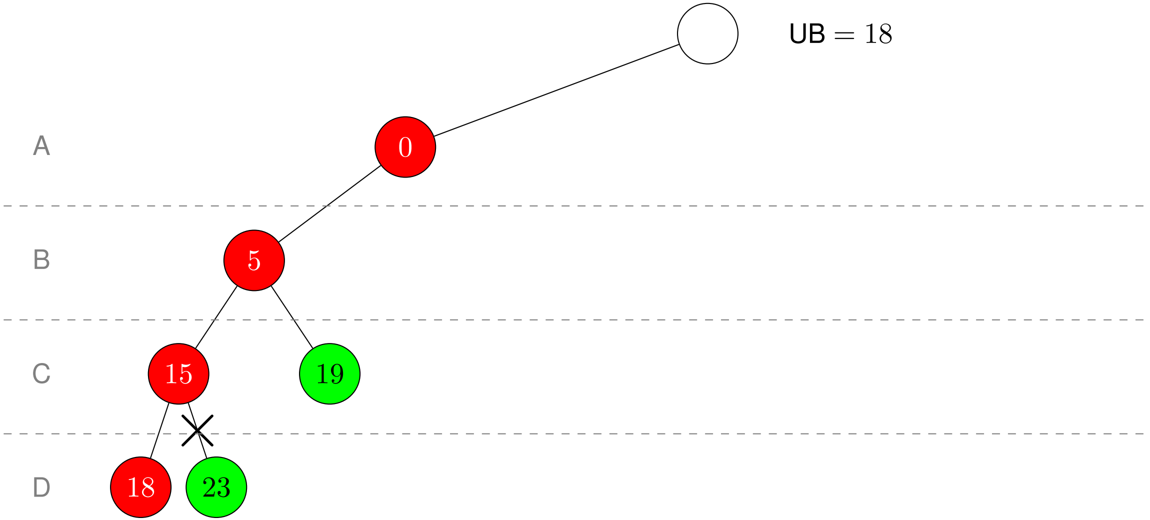

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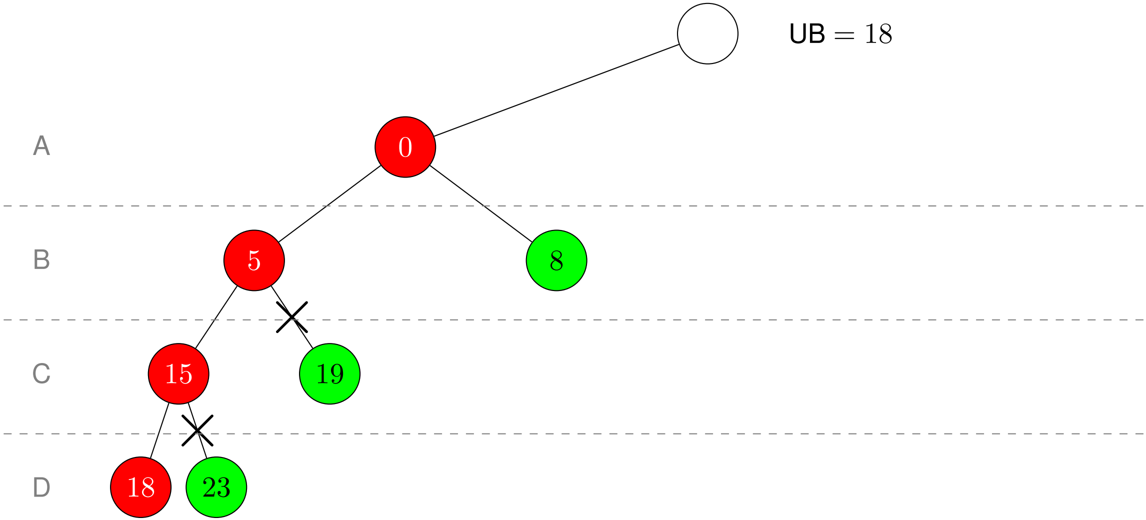

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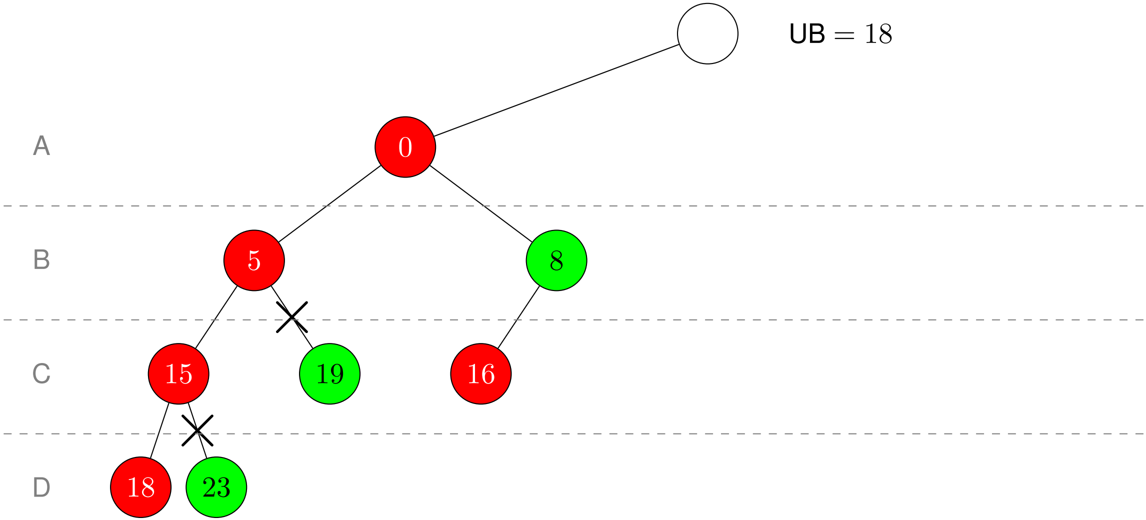

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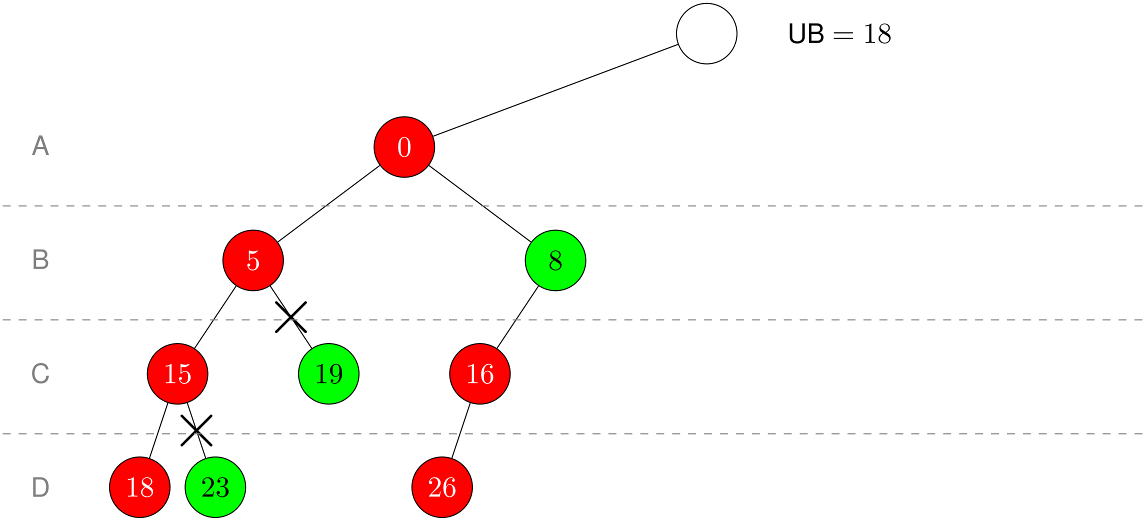

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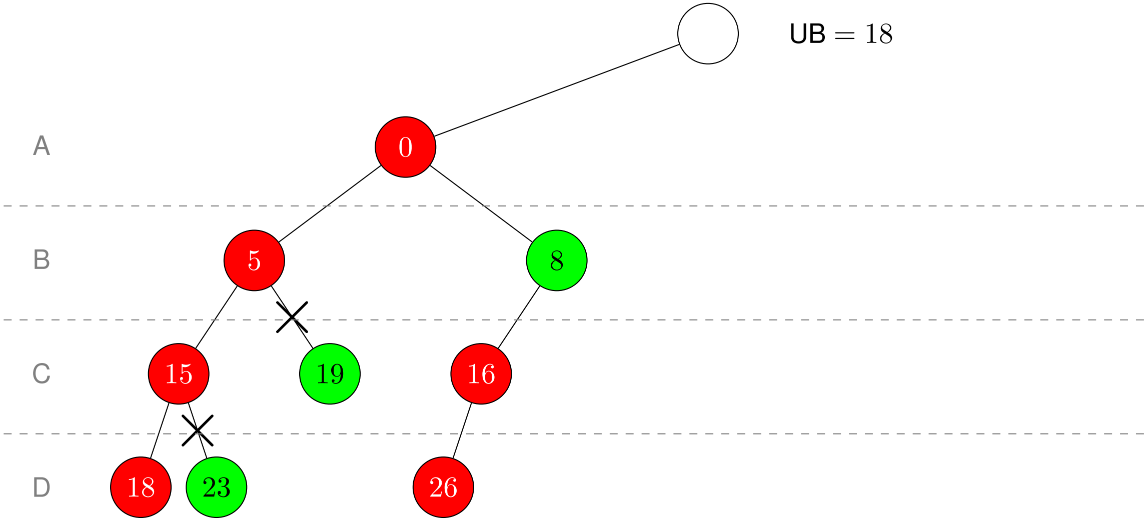

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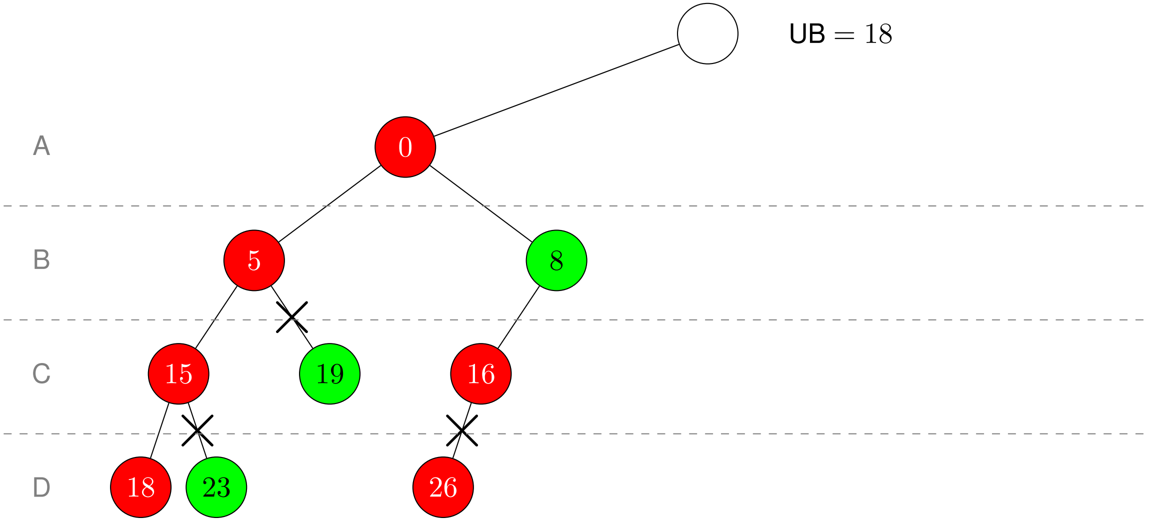


Synchronous Branch-and-Bound (SBB)

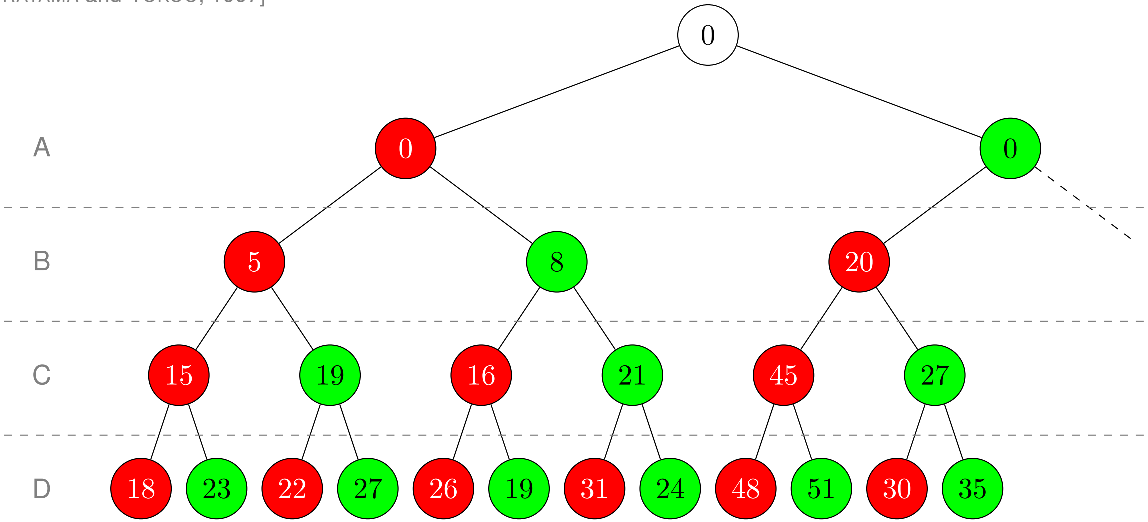
[HIRAYAMA and YOKOO, 1997]

	SBB
Correct the solution it finds is optimal	Yes
Complete it terminates	Yes
Message complexity max size of messages	$\mathcal{O}(d)$
Network load max number of messages	$\mathcal{O}(b^d)$
Runtime how long it takes	$\mathcal{O}(b^d)$

branching factor = b
num variables = d

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]



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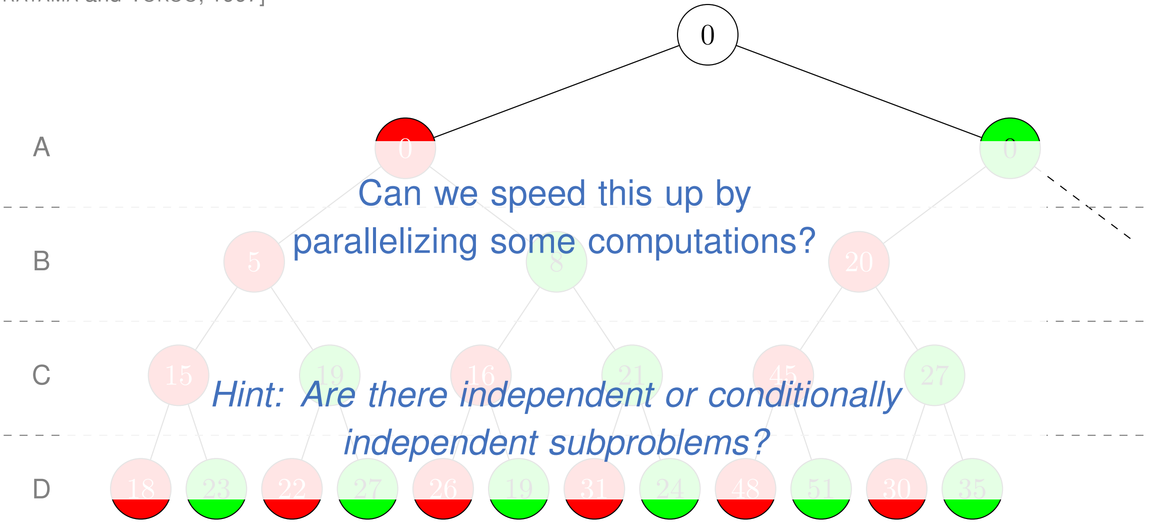
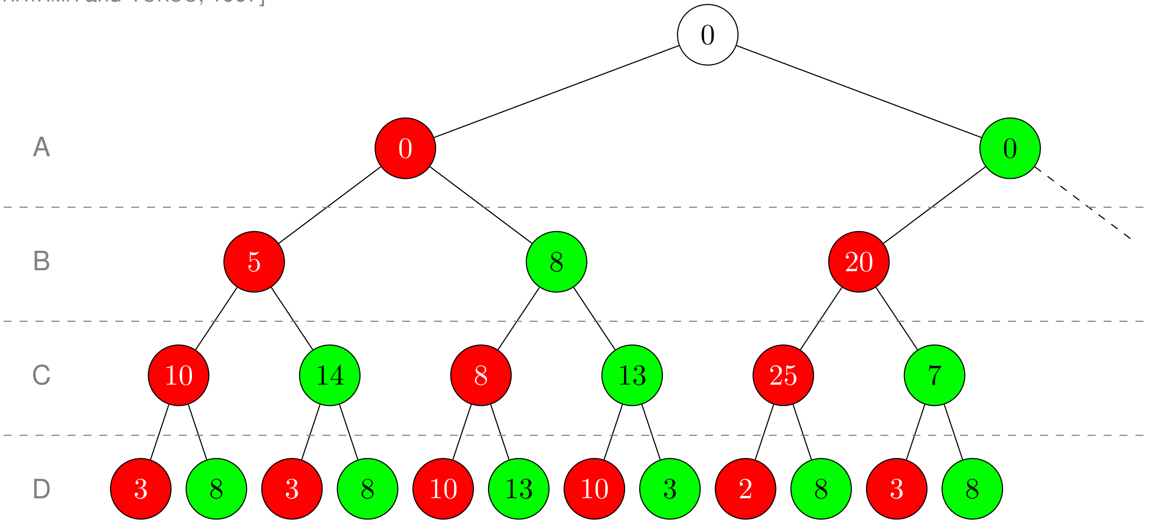


Can we speed this up by parallelizing some computations?

Hint: Are there independent or conditionally independent subproblems?

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]



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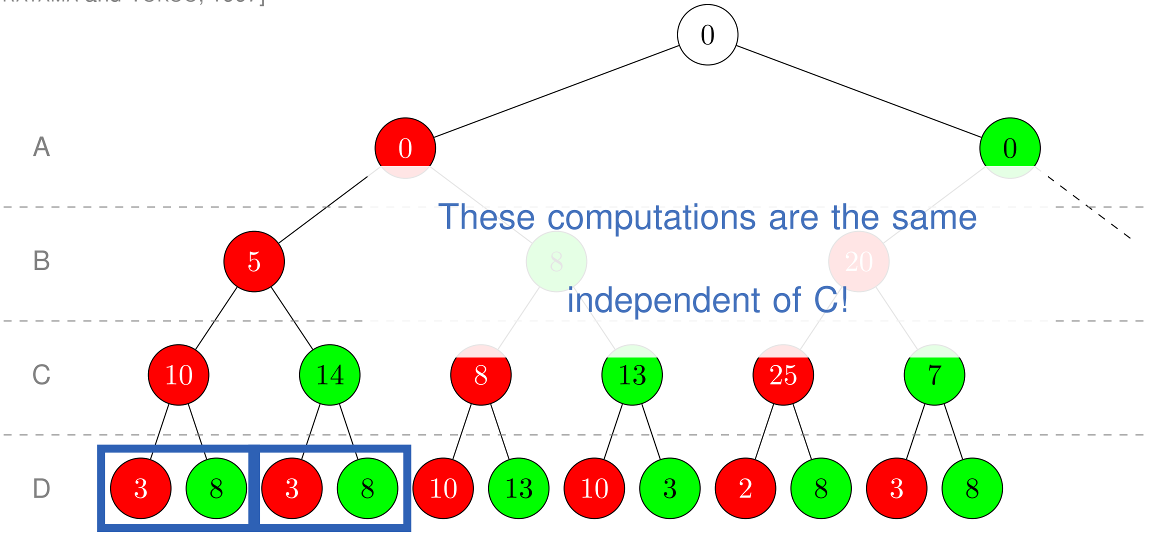
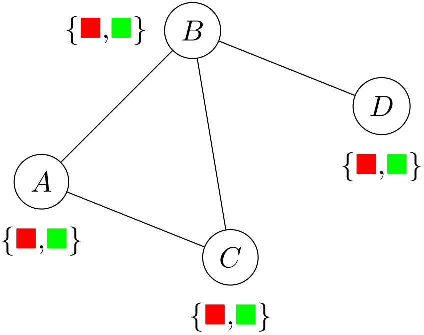
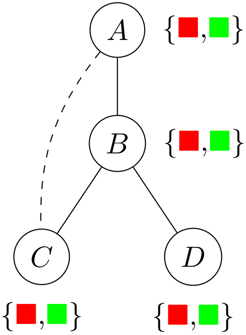


Pseudo-Tree

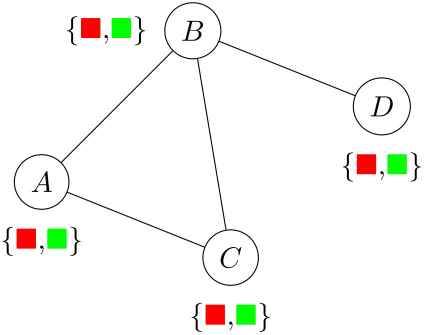


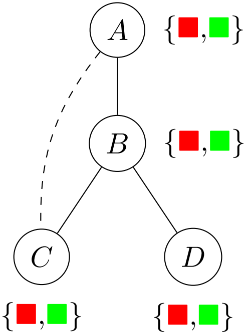


Definition (Pseudo-Tree)

A spanning tree of the constraint graph such that no two nodes in sibling subtrees share a constraint in the constraint graph

Pseudo-Tree



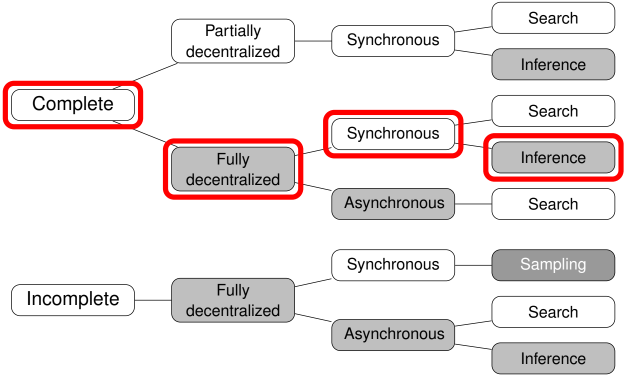


Definition (Pseudo-Tree)

A spanning tree of the constraint graph such that no two nodes in sibling subtrees share a constraint in the constraint graph

DCOP Algorithms

See [Fioretto et al., 2018]



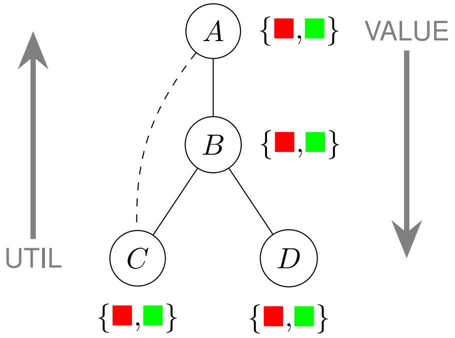
Distributed Pseudotree Optimization Procedure (DPOP)

[Petcu and Faltings, 2005b]

DPOP

[PETCU and FALTINGS, 2005b]

- Extension of the Bucket Elimination (BE)
- Agents operate on a pseudo-tree ordering
- UTIL phase: Leaves to root
- VALUE phase: Root to leaves



DPOP

[PETCU and FALTINGS, 2005b]

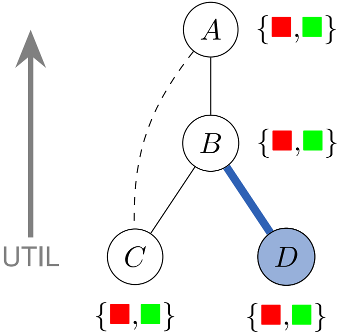
B	D	(B, D)
r	r	3
r	g	8
g	r	10
g	g	3

$\min\{3, 8\} = 3$

$\min\{10, 3\} = 3$

Message to B

B	cost
r	3
g	3



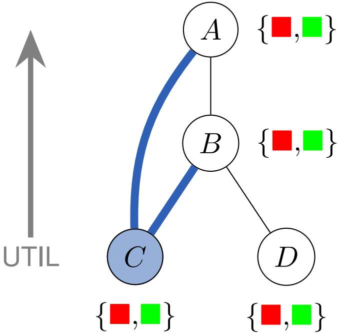
DPOP

[PETCU and FALTINGS, 2005b]

A	B	C	(B, C)	(A, C)	cost
r	r	r	5	5	10
r	r	g	4	8	12
r	g	r	3	5	8
r	g	g	3	8	11
g	r	r	5	10	15
g	r	g	4	3	7
g	g	r	3	10	13
g	g	g	3	3	6

Message to B

A	B	cost
r	r	10
r	g	8
g	r	7
g	g	6



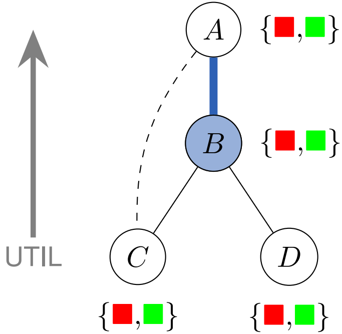
DPOP

[PETCU and FALTINGS, 2005b]

A	B	(A, B)	Util C	Util D	cost
r	r	5	10	53	18
r	g	8	8	3	19
g	r	20	7	3	30
g	g	3	6	3	12

Message to A

A	cost
r	18
g	12

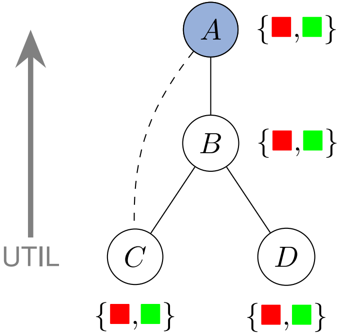


DPOP

[PETCU and FALTINGS, 2005b]

A	cost
r	18
g	12

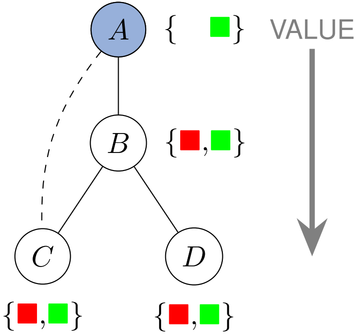
optimal cost $= 12$



DPOP

[PETCU and FALTINGS, 2005b]

A	cost
r	18
g	12



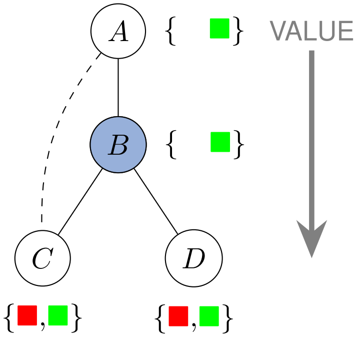
- Select value for $A = g$
- Send MSG "$A = g$" to agents B and C

DPOP

[PETCU and FALTINGS, 2005b]

A	B	(A,B)	Util C	Util D	cost
r	r	5	10	53	18
r	g	8	8	3	19
g	r	20	7	3	30
g	g	3	6	3	12

- Select value for $B = g$
- Send MSG "$B = g$" to agents C and D

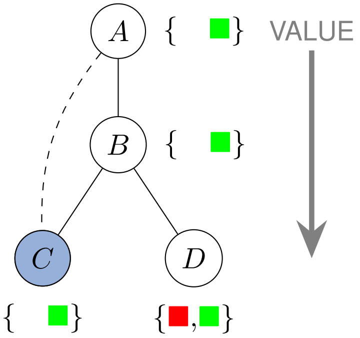


DPOP

[PETCU and FALTINGS, 2005b]

A	B	C	(B, C)	(A, C)	cost
r	r	r	5	5	10
r	r	g	4	8	12
r	g	r	3	5	8
r	g	g	3	8	11
g	r	r	5	10	15
g	r	g	4	3	7
g	**g**	r	3	10	13
g	**g**	g	3	3	6

- Select value for $C = g$



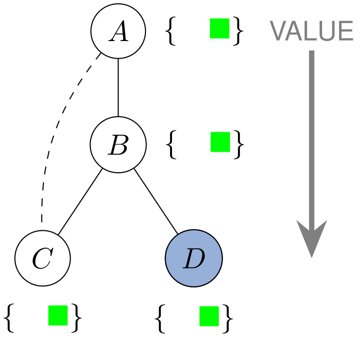
DPOP

[PETCU and FALTINGS, 2005b]

B	D	(B, D)
r	r	3
r	g	8
g	r	10
g	g	3

$\min\{3, 8\} = 3$

$\min\{10, 3\} = 3$



- Select value for $D = g$

DPOP

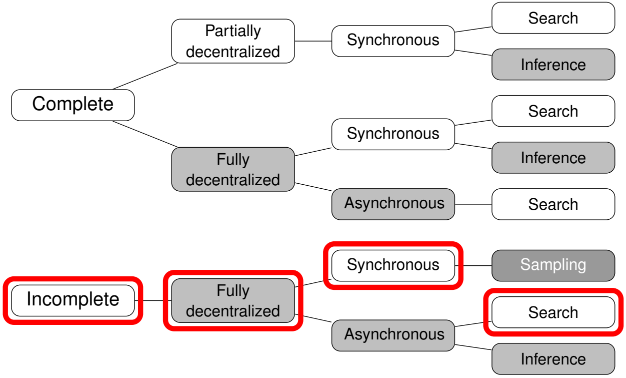
[PETCU and FALTINGS, 2005b]

	SBB	DPOP
Correct the solution it finds is optimal	Yes	Yes
Complete it terminates	Yes	Yes
Message complexity max size of messages	$\mathcal{O}(d)$	$\mathcal{O}(b^d)$
Network load max number of messages	$\mathcal{O}(b^d)$	$\mathcal{O}(d)$
Runtime how long it takes	$\mathcal{O}(b^d)$	$\mathcal{O}(b^d)$

branching factor = b
num variables = d

DCOP Algorithms

See [FIORETTO et al., 2018]



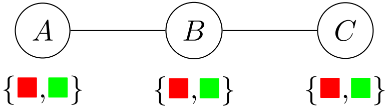
Distributed Local Search

[MAHESWARAN et al., 2004; ZHANG et al., 2003]

Local Search Algorithms

- DSA: Distributed Stochastic Search [Zhang et al., 2005]
- MGM: Maximum Gain Messages Algorithm [Maheswaran et al., 2004]
- Note: we now maximize utilities
- Every agent individually decides whether to change its value or not
- Decision involves
  - knowing neighbors' values
  - calculation of utility gain by changing values
  - probabilities

x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8



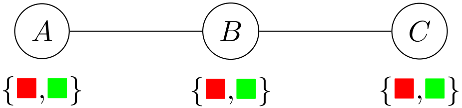
DSA Algorithm

[ZHANG et al., 2005]

- All agents execute the following
  - Randomly choose a value
  - while (termination is not met)
    - if (a new value is assigned): send the new value to neighbors
    - collect neighbors' new values if any
    - select and assign the next value based on assignment rule

DSA Algorithm

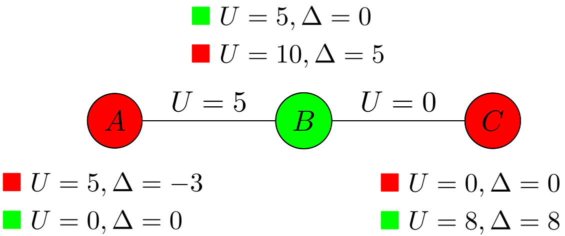
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

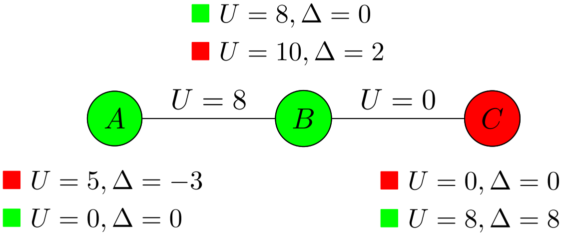
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
red	red	5	5
red	green	5	0
green	red	0	0
green	green	8	8

DSA Algorithm

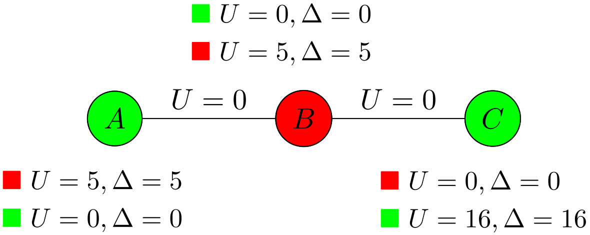
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

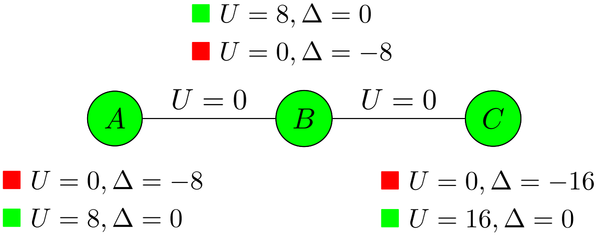
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

MGM Algorithm

[MAHESWARAN et al., 2004]

- All agents execute the following
  - Randomly choose a value
  - while (termination is not met)
    - if (a new value is assigned): send the new value to neighbors
    - collect neighbors' new values if any
    - calculate gain and send it to neighbors
    - collect neighbors' gains
    - if (it has the highest gain among all neighbors): change value to the value that maximizes gain

Large Great if you need an anytime algorithm!

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[MAHESWARAN et al., 2004]

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MGM vs DSA

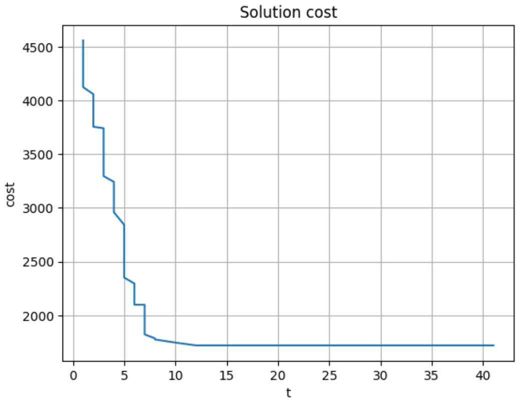


Figure: MGM

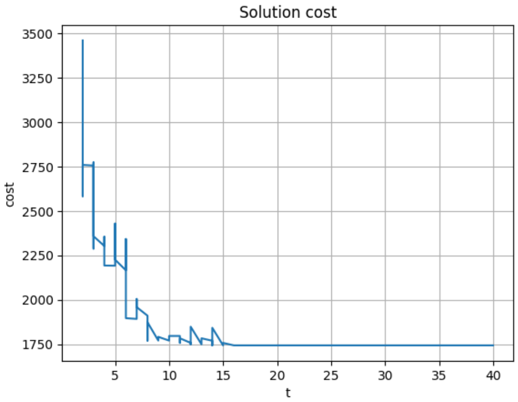


Figure: DSA

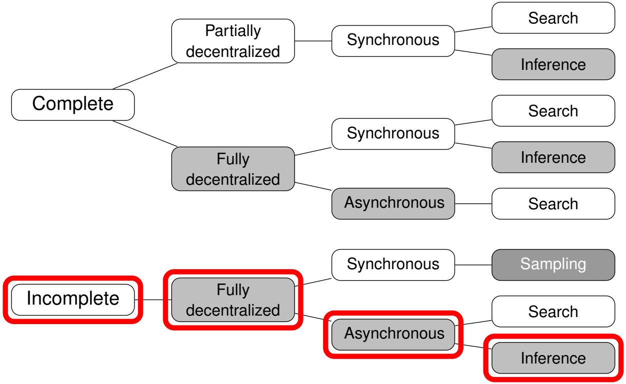
Today's Menu

Extensions to the DCOP Framework

- Dynamic DCOPs
  - SDPOP [Petcu and Faltings, 2005a], I-ADOPT and I-BnB-ADOPT [Yeoh et al., 2011], FMS [Ramchurn et al., 2010]
- Multi-Objective DCOPs
  - MO-SBB [Medi et al., 2014], Pseudo-tree Based Algorithm [Matsui et al., 2012], B-MOMS [Delle Fave et al., 2011], DP-AOF [Okimoto et al., 2013]
- Asymetric DCOPs
  - SyncABB-2ph, SyncABB-1ph, ACLS, MCS-MGM [Grinshpoun et al., 2013]
- Probabilistic DCOPs
  - $\mathbb{E}$[DPOP] and SD-DPOP [Léauté and Faltings, 2011; Nguyen et al., 2012], U-GDL [Stranders et al., 2011]
- Continuous DCOPs
  - CMS [Stranders et al., 2009], HCMS [Voice et al., 2010], PFD [Choudhury et al., 2020], EC-DPOP, AC-DPOP, CAC-DPOP, C-DSA [Hoang et al., 2020], C-CoCoA [Sarker et al., 2021]
- ...

Deeper Focus on Max-Sum

See [FIORETTO et al., 2018]



Distributed Inference, Max-Sum

[FARINELLI et al., 2008]

Roie Zivan's Talk

Today's Menu

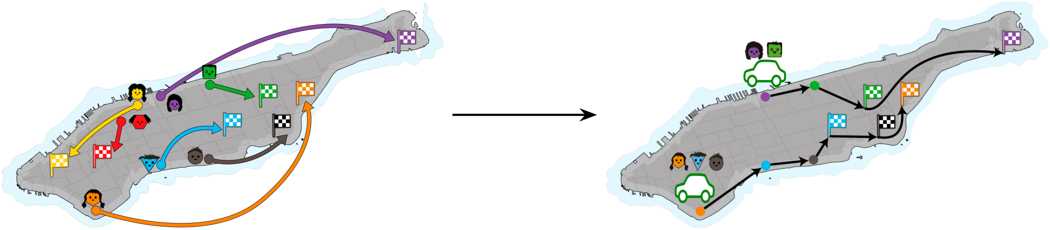
Today's Menu

Shared Mobility as (Online) Coalition Structure Generation

[BISTAFFA et al., 2019]

What is Shared Mobility for Us?

Arrange *shared rides* (coalitions) among users that submit *real-time* requests, with the objective of *maximizing* a given *objective function*



Shared Mobility as (Online) Coalition Structure Generation

[ibid.]

Our Task

At each time step, arrange a (possibly empty) set of non-overlapping feasible cars among the requests currently active in the system

Our Objective Function

Maximize environmental benefits 🌳 and quality of service 🕑

Our Case Study [Bistaffa et al., 2019]

Densely populated areas (e.g., Manhattan) with request rate of 400 reqs/minute

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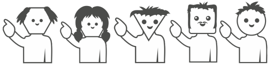
Input of the Online CSG Problem

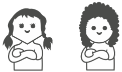
[BISTAFFA et al., 2019]

Incoming Requests

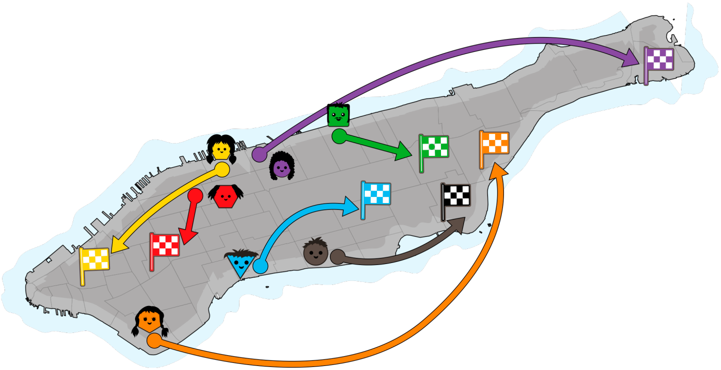


"I just issued a trip request"

Waiting Trip Requests



"I am waiting to share my ride"



Input of the Online CSG Problem

[BISTAFFA et al., 2019]

Example of a Shared Mobility Request

"I want to go from point i to point j, and I am willing to wait δ minutes to be picked up by somebody ($d =$ false) / before I leave with *my own car* ($d =$ true)"

- $r = \langle i, j, d, \delta \rangle$ (A request is a tuple r)
- $r \in R_t$ (The system receives a set R_t of requests at each time step t)
- $\langle R_1, \ldots, R_t, \ldots, R_h \rangle$ (Sequence of inputs over a time horizon h)
- The input sequence is *not known a priori* (Online optimization problem)

Input of the Online CSG Problem

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Value $v(S)$ of a Coalition S

[BISTAFFA et al., 2019]

- The *value* (utility) of a coalition S is defined as:

$$v(S) = \overbrace{\rho_{\text{CO}_2} \cdot E_{\text{CO}_2}(S) + \rho_{\text{noise}} \cdot E_{\text{noise}}(S) + \rho_{\text{traffic}} \cdot E_{\text{traffic}}(S)}^{\text{environmental benefits}} + \overbrace{\rho_{\text{QoS}} \cdot Q(S)}^{\text{quality of service}}$$

- $|S| \leq k$ (Maximum cardinality constraint)

$$F(S) = |S| \leq k \ \land \ \ldots$$

- $\mathcal{F}(R) = \{S \in 2^R \mid F(S)\}$ (Set of feasible coalitions from a set R of requests)

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Curse of Dimensionality

[Bistaffa et al., 2019]

- Recall that $\mathcal{F}(R) = \{S \in 2^R \mid F(S)\}$
- With $|S| \leq k$, $|\mathcal{F}(R)| \leq \sum_{i=1}^{k} \binom{|R|}{i}$, i.e., $\mathcal{O}(|R|^k)$ (Polynomial complexity)
- In practice, $|R_t|$ can be as high as 400 (Request rate in NY taxi dataset)

Scalability Problem

Enumerating all coalitions in $\mathcal{F}(R)$ is impractical, especially in realistic application scenarios with *very limited time budget* for the solution

Our Solution

Consider a restricted set $\hat{\mathcal{F}}(R)$ of *good candidate coalitions* instead of $\mathcal{F}(R)$

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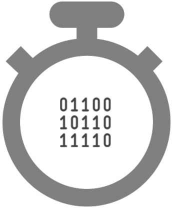
Generation of Good Candidate Coalitions (Step 1)

[BISTAFFA et al., 2019]

- CO_2 emissions
- Acoustic pollution
- Traffic congestion
- Quality of service

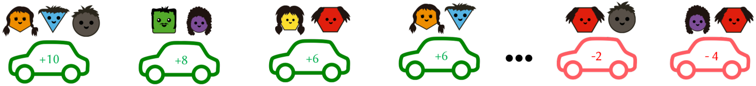
→

20 seconds



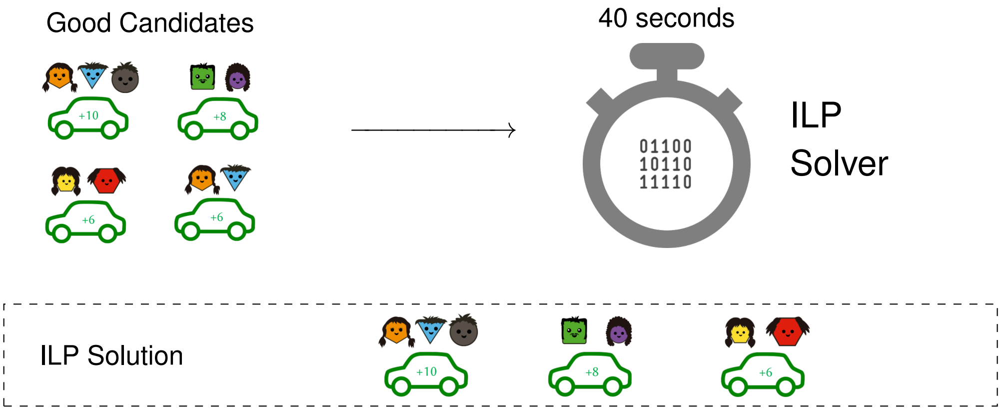
Probabilistic Greedy Algorithm

Candidate Cars



ILP Optimization (Step 2)

[BISTAFFA et al., 2019]



Approximated ILP Formulation

[BISTAFFA et al., 2019]

$$\text{maximize} \quad \sum_{S \in \hat{\mathcal{F}}(\text{Pool})} v(S) \cdot x_S$$

(Only good candidates)

$$\text{such that} \quad x_S + x_{S'} \leq 1 \quad \forall\, \hat{\mathcal{F}}(\text{Pool}) : S \cap S' \neq \varnothing$$

Computational Advantage

Approximated ILP has a number of variables that is $< 0.01\%$ of the optimal ILP

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Today's Menu

Collective Energy Purchasing

[FARINELLI et al., 2013]

Collective Energy Purchasing Scenario

- Each agent has an energy consumption profile
- Customers form coalitions to buy energy at reduced tariffs from two different markets:
  - *Spot market:* a short-term market intended for smaller amounts of energy
  - *Forward market:* a long-term market to buy more energy at cheaper prices

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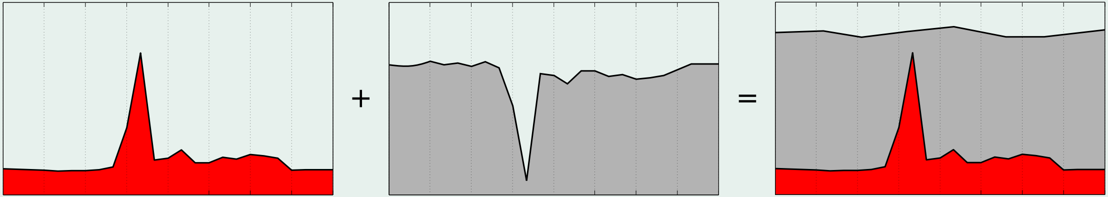
Collective Energy Purchasing

[FARINELLI et al., 2013]

Profile Merging

- Peaks in energy profiles require the use of *expensive*, *carbon-intensive*, peaking plant generators, resulting in higher consumers electricity bill
- A *flattened* profile results in a more efficient grid, with *lower carbon emissions* and *lower prices* for consumers

Example



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- $q_S^t(S)$: energy purchased from spot market at time t
- $q_F(S)$: total energy purchased from forward market
- p_S: spot market energy price
- p_F: forward market energy price

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$m + a$ Characteristic Functions

[BISTAFFA et al., 2017a]

$m + a$ Characteristic Function

- $m + a =$ Superadditive function $+$ subadditive function
  - Superadditive: $v(S_1 \cup S_2) > v(S_1) + (S_2)$
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Open Question

Is the characteristic function of shared mobility $m + a$?

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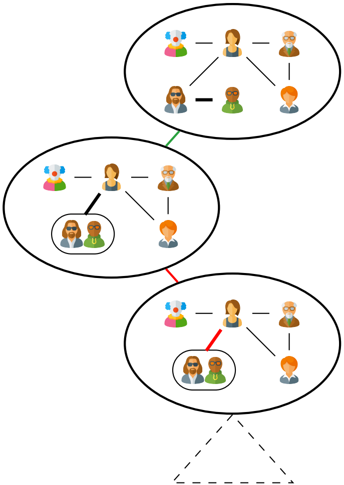
Branch-and-Bound for $m + a$ Characteristic Functions

[BISTAFFA et al., 2017a]



Can we Find an Upper Bound on $v(\cdot)$ in this Subtree?

Upper Bound M for $m + a$ Functions

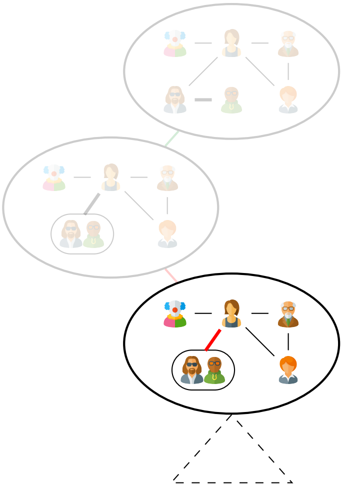
$$M = v^{+}(\textit{contract all edges}) + v^{-}(\textit{contract no edge})$$

Branch-and-Bound Algorithm

If M is $<$ than current best solution, do not visit this subtree

Branch-and-Bound for $m+a$ Characteristic Functions

[BISTAFFA et al., 2017a]



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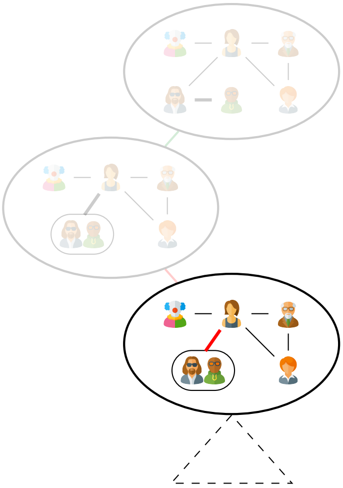

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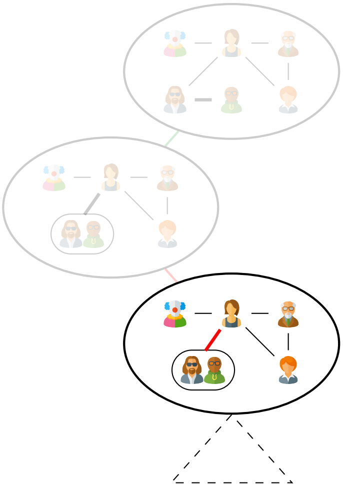

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Today's Menu

SECP model

Smart Environment Configuration Problem [RUST et al., 2016]

- Example of applying DCOPs to a "real" problem
- Coordinate objects in the building
- Model
  - objects
  - relations between objects and environment
  - user objectives and requirements
- Formulate the problem as an optimization problem

SECP model

Smart Environment Configuration Problem [RUST et al., 2016]

Focus on smart lighting use cases

- **Objects:** anything that can produce light: light bulbs, windows with rolling shutter, etc.
- **User preferences:** having a predefined luminosity level in a room, under some conditions
- **Energy efficiency**

Linking objects and user preferences:

- How to model the luminosity in a room ? **variable**
- How to model the dependency between the light sources and the luminosity ? **function / constraint**

SECP model

Example application to ambient intelligence scenario

- **Actuators**
  - Connected light bulbs, TV, Rolling shutters, ...
- **Sensors**
  - Presence detector, Luminosity Sensor, etc.
- **Physical Dependency Models**
  - E.g. Living-room light model
- **User Preferences**
  - Expressed as rules :

IF	presence_living_room	$=$	1
AND	light_sensor_living_room	$<$	60
THEN	light_level_living_room	$\leftarrow$	60
AND	shutter_living_room	$\leftarrow$	0

SECP model

Example application to ambient intelligence scenario

- **Actuators**
  - *Decision* variable x_i, domain $\mathcal{D}_{x_i}$
  - Cost function $c_i : \mathcal{D}_{x_i} \to \mathbb{R}$
- **Sensors**
  - *Read-only* variable s_l, domain $\mathcal{D}_{s_l}$
- **Physical Dependency Models** $\langle y_j, \phi_j \rangle$
  - Give the expected state of the environment from a set of actuator-variables influencing this model
  - Variable y_j representing the *expected* state of the environment
  - Function $\phi_j : \prod_{\varsigma \in \sigma(\phi_j)} \mathcal{D}_\varsigma \to \mathcal{D}_{y_j}$
- **User Preferences**
  - Utility function u_k
  - Distance from the current expected state to the target state of the environment

Formulating SECP as a DCOP

Multi-objective optimization problem

$$\min_{x_i \in \nu(\mathfrak{A})} \sum_{i \in \mathfrak{A}} c_i \quad \text{and} \quad \max_{\substack{x_i \in \nu(\mathfrak{A}) \\ y_j \in \nu(\Phi)}} \sum_{k \in \mathfrak{R}} u_k$$

$$\textbf{s.t. } \phi_j(x_j^1, \ldots, x_j^{\overline{\phi_j}}) = y_j \quad \forall y_j \in \nu(\Phi)$$

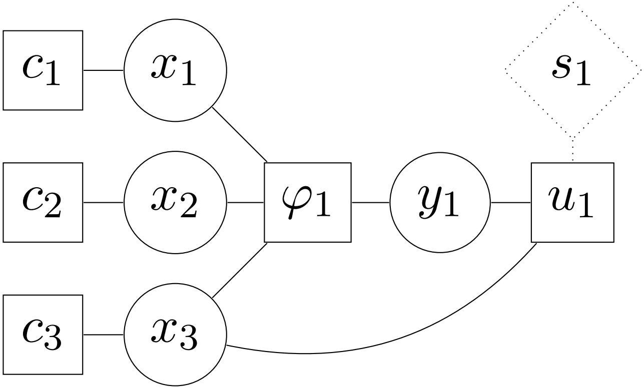
Mono-objective DCOP formulation

$$\max_{\substack{x_i \in \nu(\mathfrak{A}) \\ y_j \in \nu(\Phi)}} \quad \omega_u \sum_{k \in \mathfrak{R}} u_k - \omega_c \sum_{i \in \mathfrak{A}} c_i + \sum_{\varphi_j \in \boldsymbol{\Phi}} \varphi_j$$

$$\varphi_j(x_j^1, \ldots, x_j^{|\sigma(\phi_j)|}, y_j) = \begin{cases} 0 & \text{if } \phi_j(x_j^1, \ldots, x_j^{|\sigma(\phi_j)|}) = y_j \\ -\infty & \text{otherwise} \end{cases}$$

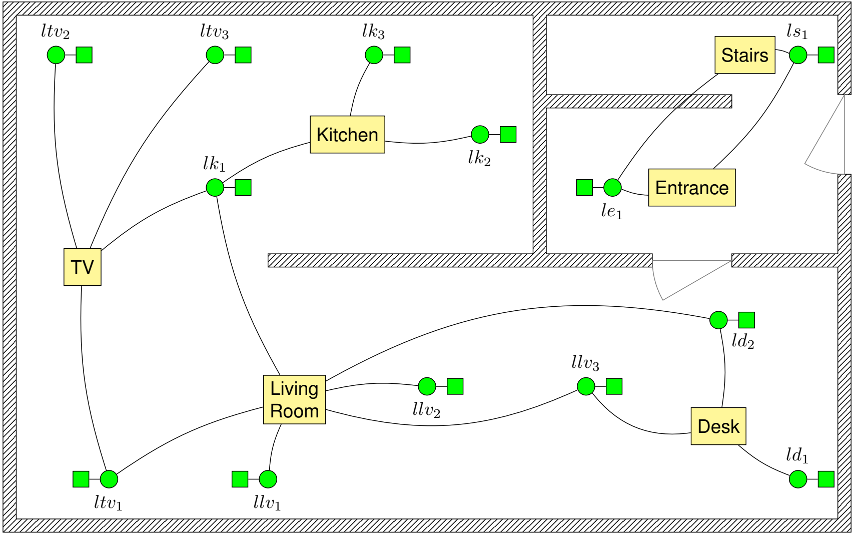
Formulating SECP as a DCOP

Representing a DCOP as a factor graph

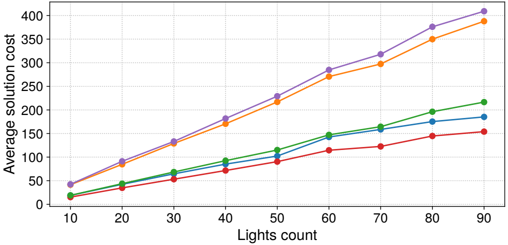


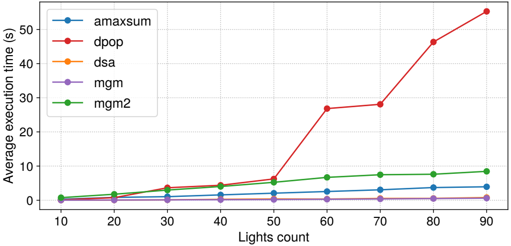
SECP Factor Graph

in a house (without rules)



Algorithms' performances





- Best solutions: DPOP, MaxSum, MGM2
- Worst runtime: DPOP
- Best compromise: MaxSum, MGM2

SECP: further readings

- Experiments with various algorithms [Rust et al., 2016, 2022]
- How to deploy DCOPs [Rust et al., 2017, 2022]
- How to adapt deployment at runtime [Rust et al., 2018, 2020, 2022]

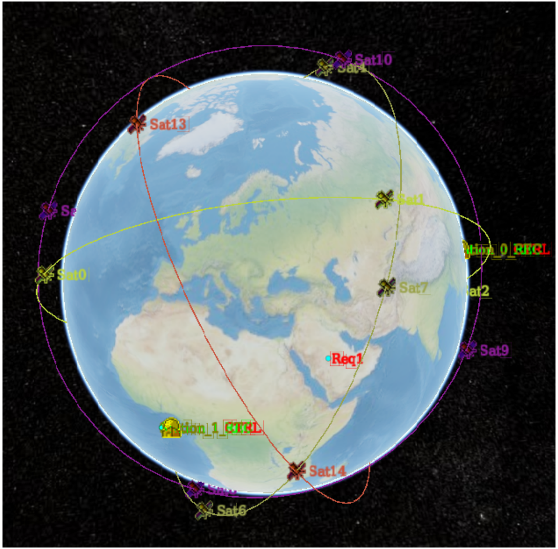
Today's Menu

Observation Scheduling in Multi-Owner Constellations [Picard, 2022]

- Increasing size of deployed EOS constellations

⇒ Observe any point on Earth at higher frequency, e.g. Planet constellation

- **but**, requires to improve coordination and cooperation between assets and stakeholders



- We focus here on collective observation scheduling on a constellation where some users have exclusive access to some orbit portions

⇒ Answer to strong user expectations to benefit both from a shared system (to reduce costs) and a proprietary system (total control and confidentiality)

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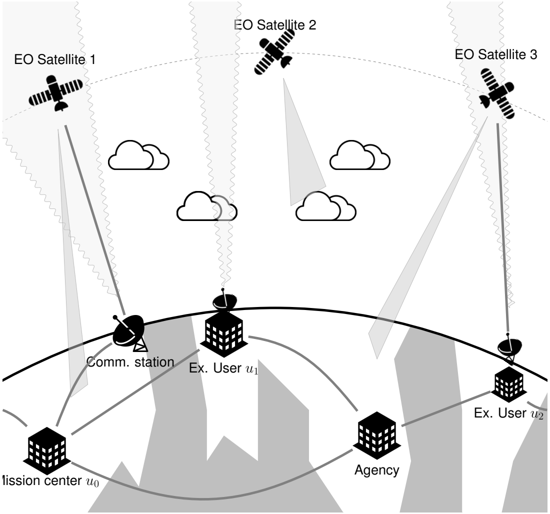
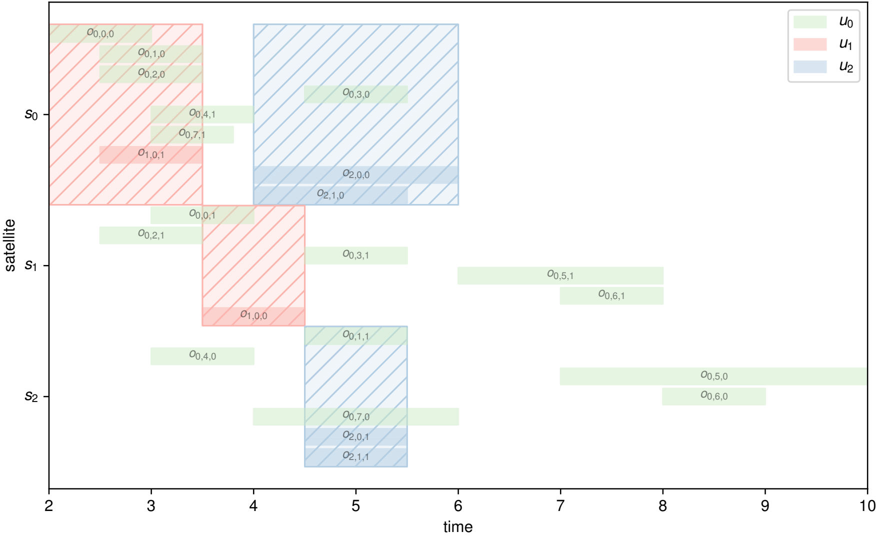


Scheduling Observations with Multiple Exclusive Orbit Portions

Illustrative Example



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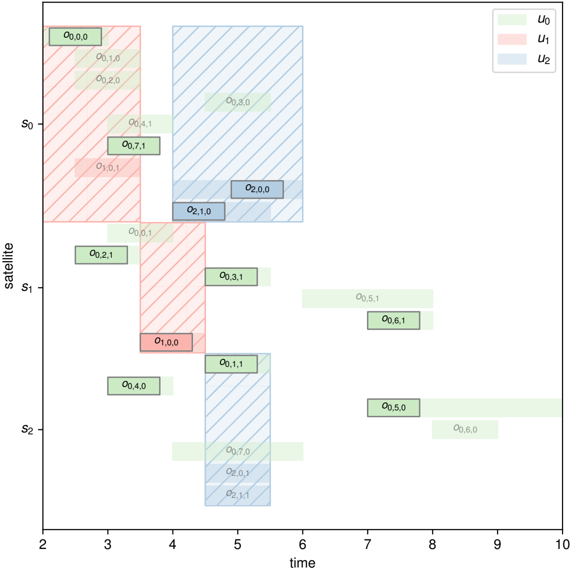

DCOP Model

A DCOP $\langle \mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{C}, \mu \rangle$ is defined for a given request r, and a current scheduling

- The agents are the exclusive users which can potentially schedule r:

$$\mathcal{A} = \{u \in \mathcal{U}^{\text{ex}} | \exists (s, (t_u^{\text{start}}, t_u^{\text{end}})) \in e_u, \exists o \in \theta_r \text{ s.t. } s_o = s, [t_u^{\text{start}}, t_u^{\text{end}}] \cap [t_o^{\text{start}}, t_o^{\text{end}}] \neq \varnothing\} \quad (1)$$

- Each agent u owns binary decision variables, one for each observation $o \in \mathcal{O}[u]^r$ and exclusive e in its exclusives e_u, stating whether it schedules o in e or not:

$$\mathcal{X} = \{x_{e,o} | e \in \textstyle\bigcup_{u \in \mathcal{A}} e_u, o \in \mathcal{O}[u]^r\} \quad (2)$$

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- μ associates each variable $x_{e,o}$ to e's owner

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DCOP Model (cont.)

- Constraints should check that at most one observation is scheduled per request (4), that satellites are not overloaded (5), that at most one agent serves the same observation (6)

$$\sum_{e \in \bigcup_{u \in \mathcal{A}} e_u} x_{e,o} \leq 1, \quad \forall u \in \mathcal{X}, \forall o \in \mathcal{O}[u]^r \tag{4}$$

$$\sum_{o \in \{o \in \mathcal{O}[u]^r | u \in \mathcal{A}, s_o = s\}, e \in \bigcup_{u \in \mathcal{A}} e_u} x_{e,o} \leq \kappa_s^*, \ \forall s \in \mathcal{S} \tag{5}$$

$$\sum_{e \in \bigcup_{u \in \mathcal{A}} e_u} x_{e,o} \leq 1, \quad \forall o \in \mathcal{O} \tag{6}$$

- The cost to integrate an observation in the current user's schedule should be assessed to guide the optimization process

$$c(x_{e,o}) = \pi(o, \mathcal{M}_{u_o}), \quad \forall x_{e,o} \in \mathcal{X} \tag{7}$$

where π evaluates the best cost obtained when scheduling o and any combination of observations from $\mathcal{M}_{u_o}$, as to consider all possible revisions of u_o's current schedule

$$\mathcal{C} = \{(4), (5), (6), (7)\} \tag{8}$$

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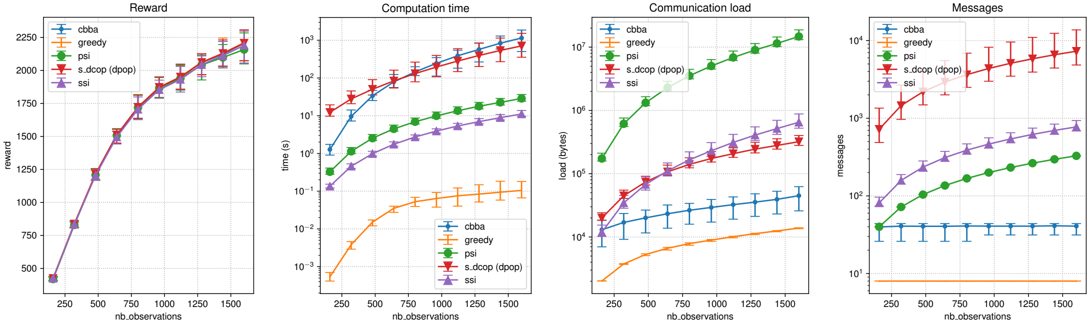
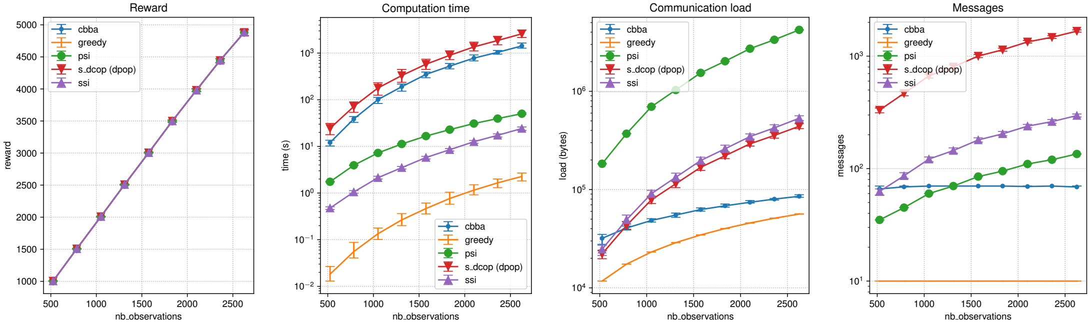
Highly conflicting randomly generated problems

5-min horizon with overlapping requests and limited capacity



Realistic randomly generated problems

6-hour horizon with numerous requests and large capacity



Today's Menu

Conclusion and Wrap-up

What We've Seen Today

- 2 major multi-agent constraint optimization frameworks: **DCOP**, **CF**
    - DCOP: how to **collectively solve** constraint optimization problems
    - CF: how to **form coalitions/groups** with respect to some criteria and constraints
- Various **techniques and algorithms** to attach these problems
- Examples of **applications** in the transportation, IoT, space and energy domain

Conclusion and Wrap-up

Open questions

Distributed constraint optimization

- How to **decompose or regroup** as to reduce **interactions**?
- How to **structure** the system as to improve **parallelism**?
- How to deploy and make systems robust and resilient in **dynamic environments**?

Coalition formation

- Which other **realistic** scenarios can we model as $m + a$?
- Can we exploit some other **properties** for scenarios that are not $m + a$ (e.g., shared mobility)?
- More in general, how can we **improve** the **scalability** of CF approaches?

Common questions

- How to use DCOPs in CF and vice versa?
- Maintaining libraries and data sets

Special Thanks

Special thanks to all previous contributors to tutorials on multi-agent optimization and related topics, notably

Ferdinando Fioretto, Long Tran-Thanh, Pierre Rust, Enrico Pontelli, William Yeoh, Jesus Cerquides, Juan Antonio Rodriguez Aguilar, Alessandro Farinelli, Pedro Meseguer, Sarvapali Ramchurn, Amnon Meisels

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