Multi-Agent Optimization

Tutorial at AAMAS'23

Filippo Bistaffa   Gauthier Picard   Roie Zivan

Some contents adapted from previous tutorials (`http://https//www2.isye.gatech.edu/~fferdinando3/cfp/AAMAS19/`)

AAMAS 2023 LONDON

IIIA Institut d'Investigació en Intel·ligència Artificial

ONERA THE FRENCH AEROSPACE LAB

Ben-Gurion University of the Negev

Introduction and Motivations

Who are we?

Filippo Bistaffa, PhD

IIIA-CSIC, Barcelona
Expertises: coalition formation, parallel computing, shared mobility

Gauthier Picard, PhD, Hab.

ONERA, the French Aerospace Lab
Expertises: DCOPs, self-organization, resource allocation

Roie Zivan, PhD

Ben Gurion University of the Negev
Expertises: DCOPs, partial cooperation

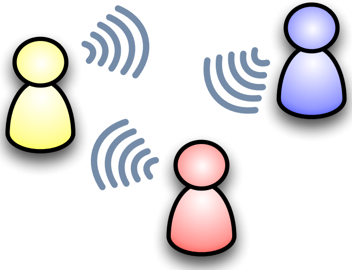
Introduction and Motivations

Multiagent Systems

- Agent: An entity that behaves autonomously in the pursuit of goals
- Multi-agent system: A system of multiple interacting agents

An agent is...

- Autonomous: Is of full control of itself
- Interactive: May communicate with other agents
- Reactive: Responds to changes in the environment or requests by other agents
- Proactive:Takes initiatives to achieve its goals



Introduction and Motivations

Research questions addressed during this tutorial

- How to make collective optimal decisions?
  - How to model the collective decision?
  - Which protocols to implement these decisions?
- How to form groups *wrt* to some utility criteria?
  - How to model the utility of each group?
  - How to express which groups are feasible or not?

Today's Menu

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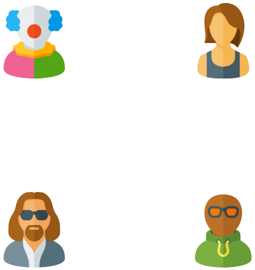
Characteristic Function Games (CFGs)

[CHALKIADAKIS et al., 2011]



Set of Agents A

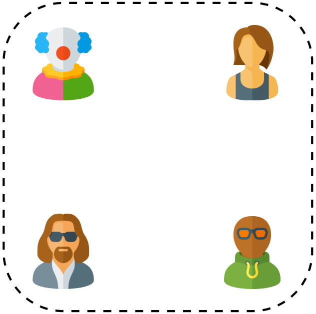
$A = \{$, , , $\}$

Characteristic Function $v(\cdot)$

- $v(\{$, $\}) = 0$
- $v(\{$, , $\}) = -7$
- $v(\{$, $\}) = 3$
- $\ldots$

Characteristic Function Games (CFGs)

[CHALKIADAKIS et al., 2011]



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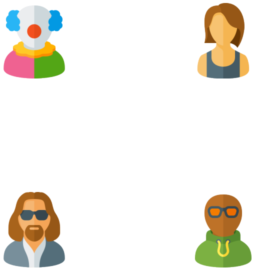

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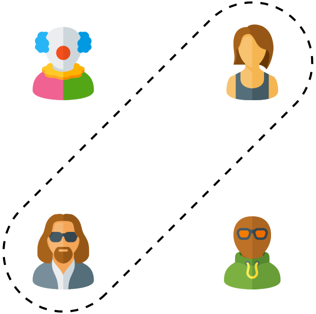

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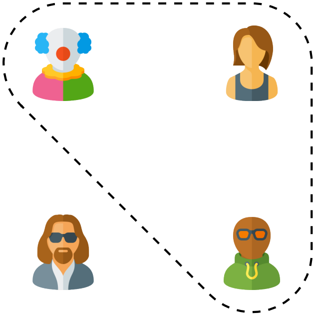

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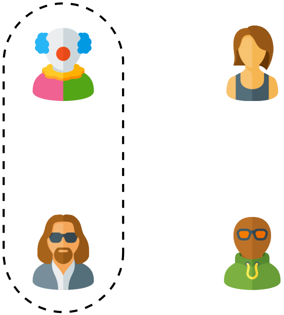

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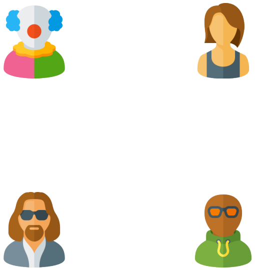

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- …

Characteristic Function

[CHALKIADAKIS et al., 2011]

Characteristic Function

The function $v : \mathcal{P}(A) \rightarrow \mathbb{R}$ associates a value to *every coalition* (i.e., subset) of A

Exponential Complexity

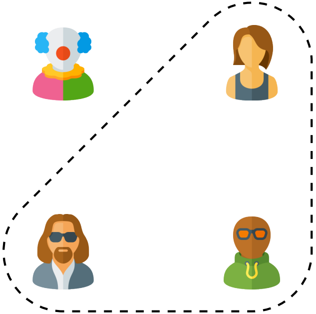
Representing $v(\cdot)$ as a *table* requires an *exponential* number of steps (i.e., $2^{|A|}$)

Mitigate this Complexity

(1) *Restrict* the set of coalitions or (2) consider $v(\cdot)$ with a specific *structure*

Cardinality-Restricted CFGs

[SHEHORY and KRAUS, 1998]



Maximum Cardinality k

E.g., only coalitions of size ≤ 3 are feasible

Polynomial Number of Coalitions

Total number of coalitions is $\sum_{i=1}^{k} \binom{|A|}{i} = \mathcal{O}(|A|^k)$,
i.e., *polynomial wrt* $|A|$

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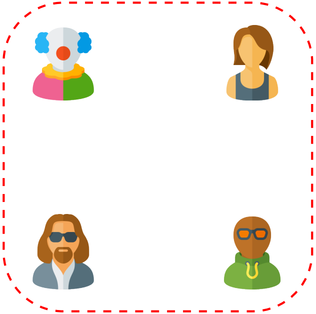

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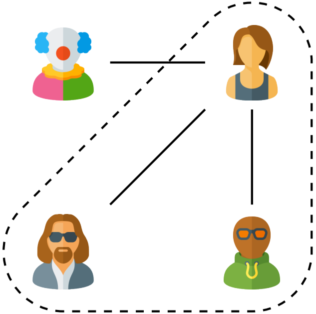
Graph-Restricted CFGs

[MYERSON, 1977], [DEMANGE, 2004]



Graph G among Agents

Connected Subgraphs

A coalition is *feasible* only if it induces a *connected* subgraph of G

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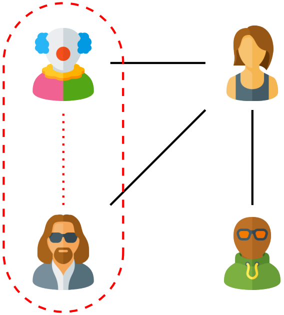
[MYERSON, 1977], [DEMANGE, 2004]



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Real-World Example: Social Ridesharing

[Bistaffa et al., 2017b]

Social Ridesharing

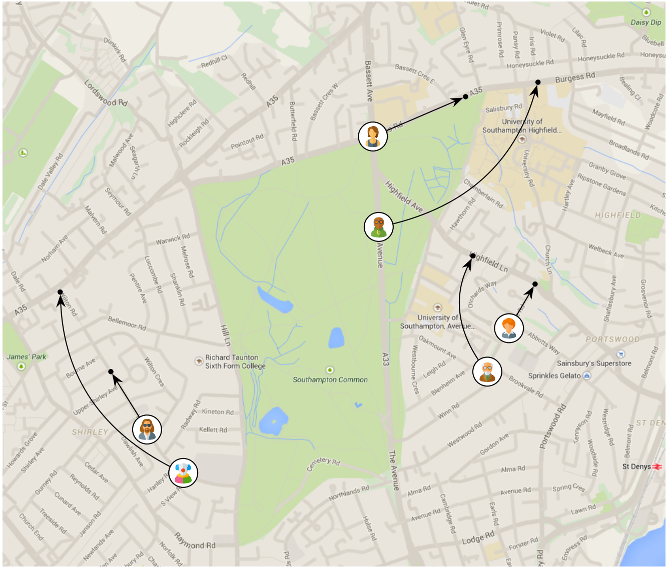
Arrange *cost-effective* shared cars among agents connected by a *social network*

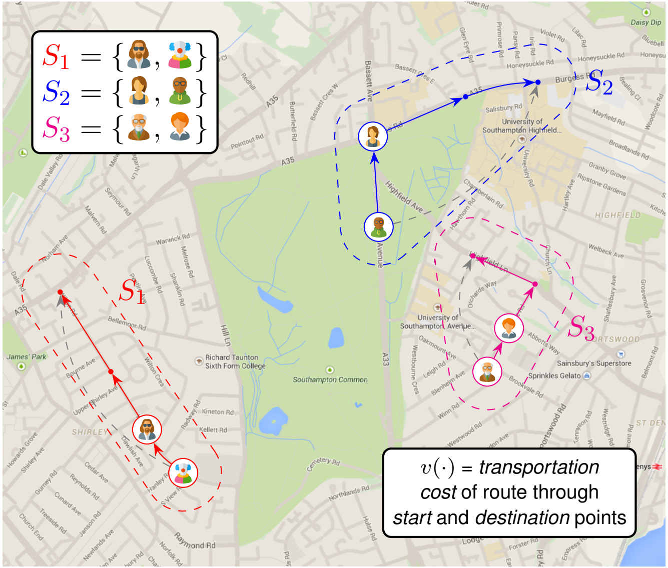
Cardinality-Based Constraints

Cars (i.e., coalitions) can contain *up to 5 passengers*

Graph-Based Constraints

We only form coalitions among "*friends of friends*" (connected subgraph)




$S_1 = \{ , \}$
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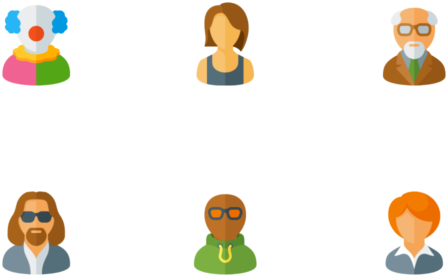
S_1
S_2
S_3
$v(\cdot)$ = *transportation cost* of route through *start* and *destination* points
Southampton Common
University of Southampton Highfield
University of Southampton, Avenue
Richard Taunton Sixth Form College
Sainsbury's Superstore
Sprinkles Gelato

Today's Menu

Coalition Structure Generation (CSG)

[RAHWAN et al., 2015]



Solving the Coalition Structure Generation (CSG) Problem

Compute the partition $\mathcal{S}$ of A into *feasible* coalitions that *maximizes* the sum $\sum_{S \in \mathcal{S}} v(S)$

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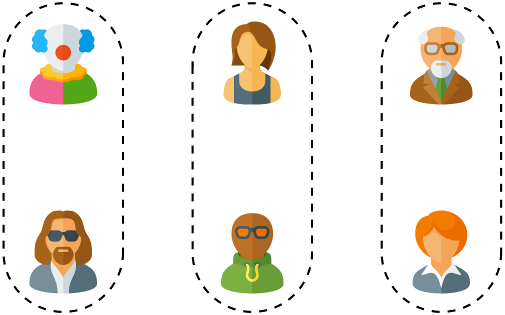

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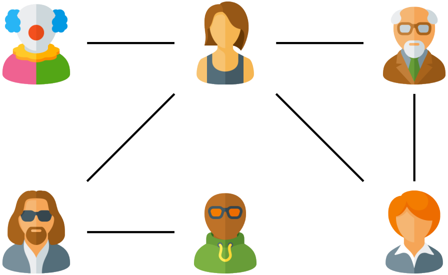

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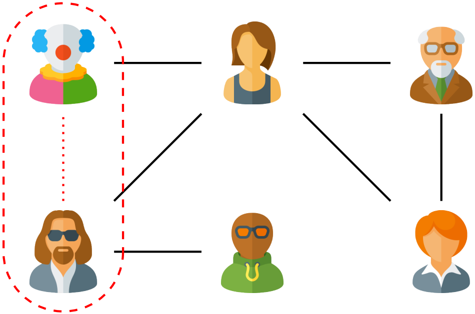

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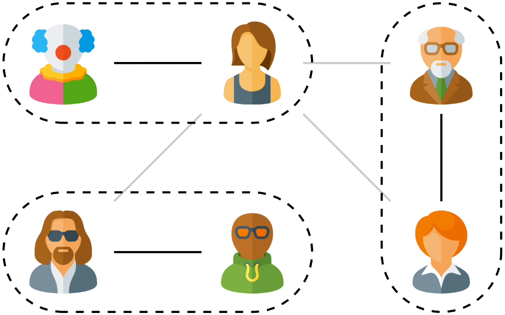

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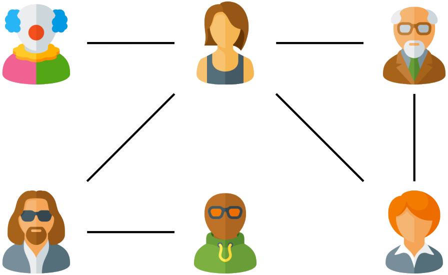
CSG Approaches based on Search

[BISTAFFA et al., 2017a]



Edge Contraction Operation

Contraction of edge $(S_i, S_j) \rightarrow$ form coalition $S_i \cup S_j$

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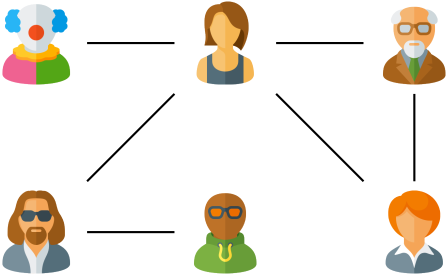

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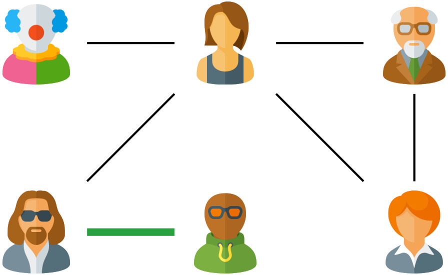

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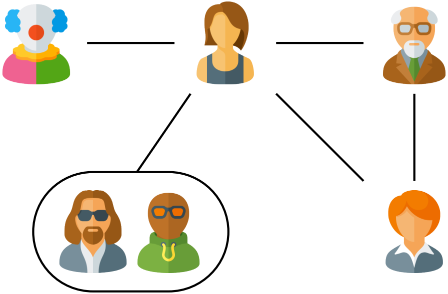

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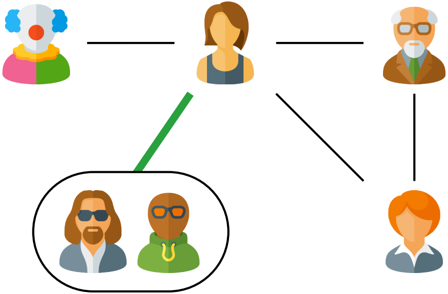

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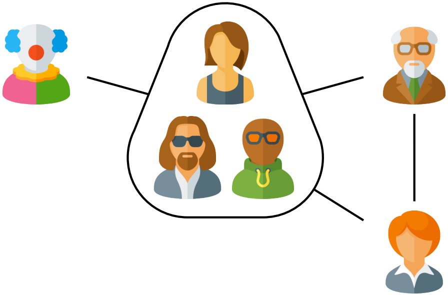

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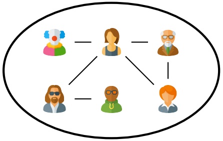

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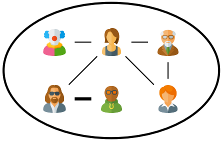

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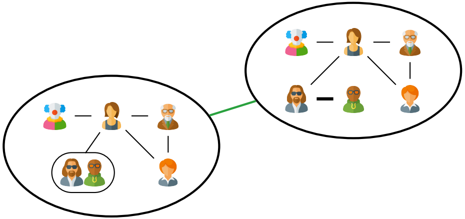

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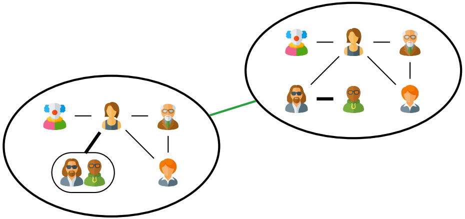

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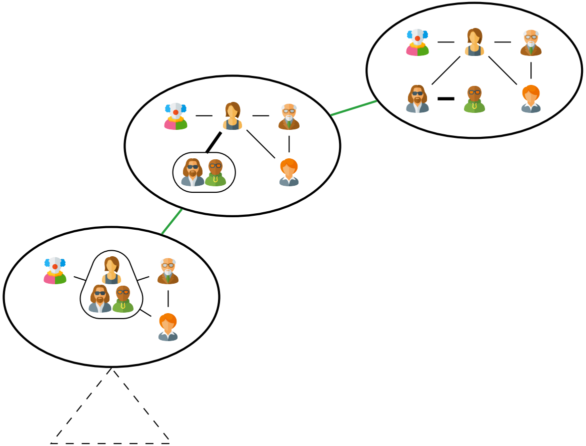

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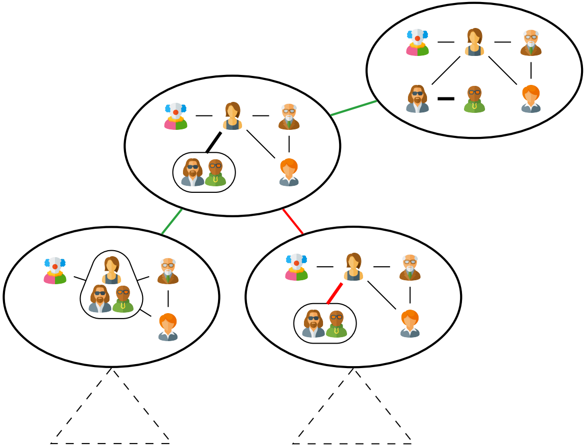

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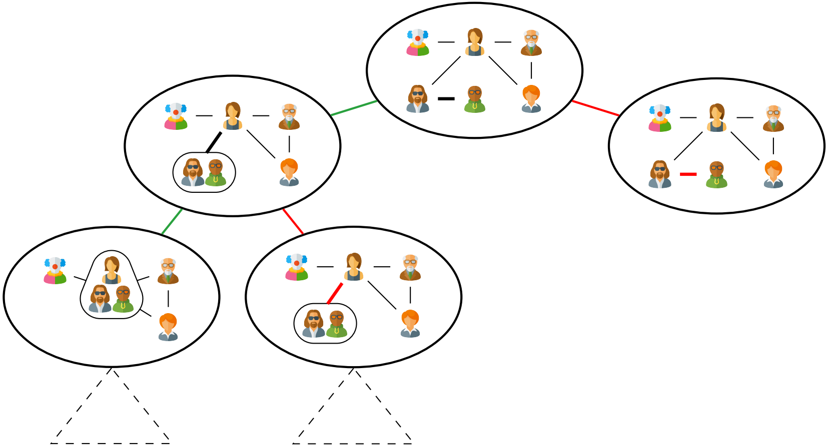

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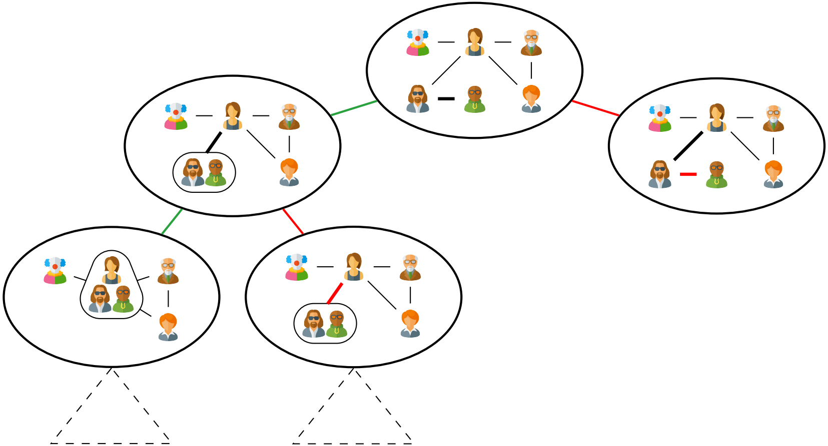

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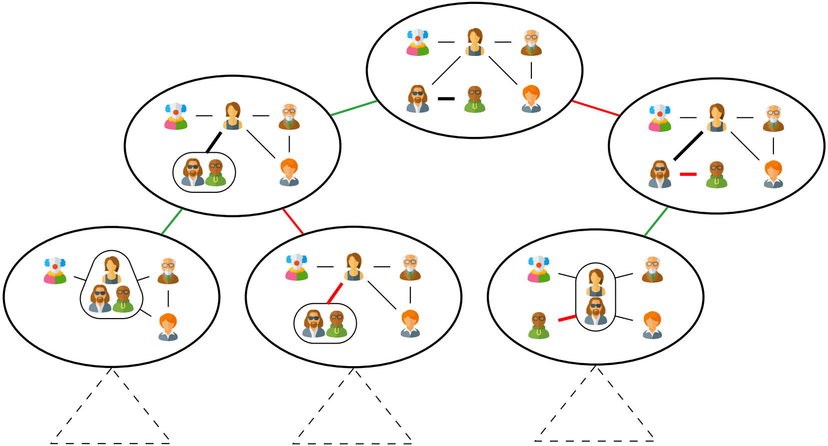

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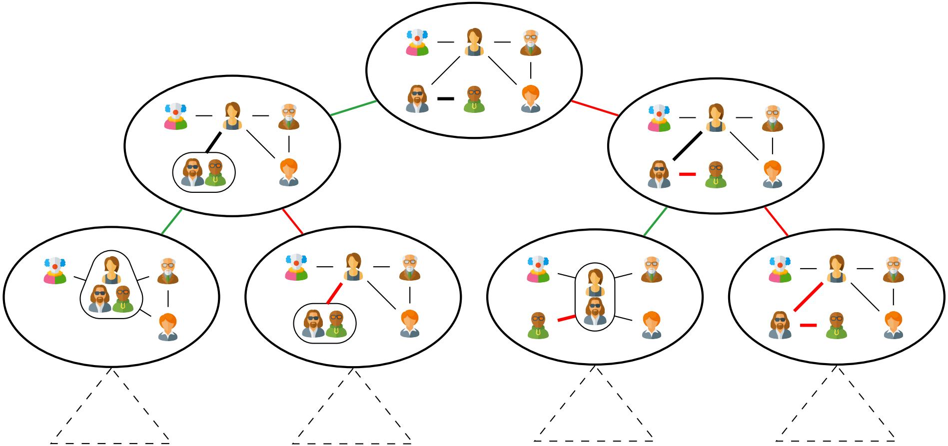

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CFSS Algorithm

- Builds a *Binary Decision Diagram* (BDD) by contracting (or not) an edge at each step
- Each coalition structure (i.e., partition of A) is represented *only once* in the BDD
- The optimal coalition structure is computed by doing a *depth-first* traversal of the BDD

Pros

Approximate algorithm with quality guarantees if used in conjunction with *Branch-and-Bound*

Cons

Performance depends on the assumption that $v(\cdot)$ can be expressed in *closed-form*

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CSG Approaches based on Integer Linear Programming

Background on Integer Linear Programming

Weighted Knapsack Problem

We want to fill our knapsack (capacity = c) with the goal of maximizing the total value

What is the Optimal Subset of Object for $c = 5$?

A Pick 🍎 (weight = 1) $\rightarrow 1$ — $w(🍎) = 1, v(🍎) = 1$

B Pick 💎 (weight = 2) $\rightarrow 4$ — $w(💎) = 2, v(💎) = 4$

C Pick 💍 (weight = 4) $\rightarrow 3$ — $w(💍) = 4, v(💍) = 3$

D Pick 👑 (weight = 5) $\rightarrow 9$ — $w(👑) = 6, v(👑) = 1$

E Pick 📟 (weight = 3) $\rightarrow 6$ — $w(📟) = 3, v(📟) = 6$

CSG Approaches based on Integer Linear Programming

Background on Integer Linear Programming

Our Ingredients

- Let x_A, x_B, x_C, x_D, x_E be binary decision variables (either pick the object or not)
- Objective function: maximize the value of selected objects
- Constraint: do not exceed the knapsack capacity

Integer *Linear* Programming (ILP) Formulation

$$
\begin{array}{lll}
\text{maximize} & 1 \cdot x_A + 4 \cdot x_B + 3 \cdot x_C + 9 \cdot x_D + 6 \cdot x_E & \text{(Values of selected objects)} \\
\text{subject to} & 1 \cdot x_A + 2 \cdot x_B + 4 \cdot x_C + 5 \cdot x_D + 3 \cdot x_E \leq 5 & \text{(Capacity constraint)} \\
 & x_A, x_B, x_C, x_D, x_E \in \{0, 1\} & \text{(Binary decision variables)}
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subject to	$1 \cdot x_A + 2 \cdot x_B + 4 \cdot x_C + 5 \cdot x_D + 3 \cdot x_E \leq 5$	(Capacity constraint)
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CSG Approaches based on Integer Linear Programming

Background on Integer Linear Programming

Our Ingredients

- Let x_A, x_B, x_C, x_D, x_E be binary decision variables (either pick the object or not)
- Objective function: maximize the value of selected objects
- Constraint: do not exceed the knapsack capacity

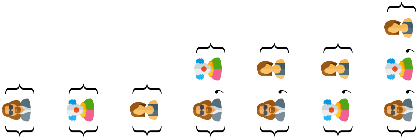
Integer *Linear* Programming (ILP) Formulation

$$
\begin{aligned}
\text{maximize} \quad & 1 \cdot x_A + 4 \cdot x_B + 3 \cdot x_C + 9 \cdot x_D + 6 \cdot x_E && \text{(Values of selected objects)} \\
\text{subject to} \quad & 1 \cdot x_A + 2 \cdot x_B + 4 \cdot x_C + 5 \cdot x_D + 3 \cdot x_E \leq 5 && \text{(Capacity constraint)} \\
& x_A, x_B, x_C, x_D, x_E \in \{0, 1\} && \text{(Binary decision variables)}
\end{aligned}
$$

CSG Approaches based on Integer Linear Programming

[RAHWAN et al., 2015]

- Given A and a set $\mathcal{S}$ of *coalitions* (i.e., subsets) of A, let M be a $|A| \times |\mathcal{S}|$ matrix
- $M_{iS} = 1$ if and only if agent $a \in A$ is part of coalition $S \in \mathcal{S}$, $M_{iS} = 0$ otherwise



$$M = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

CSG Approaches based on Integer Linear Programming

[RAHWAN et al., 2015]

Objective of Coalition Structure Generation

Compute the *partition* of A that *maximizes* the sum of the corresponding values

ILP Formulation for Coalition Structure Generation

maximize	$\sum_{S \in \mathcal{S}} v(S) \cdot x_S$	(Value of each selected coalition)
subject to	$\sum_{S \in \mathcal{S}} M_{iS} \cdot x_S = 1 \quad \forall i \in A$	(Each agent exactly in *one* coalition)

CSG Approaches based on Integer Linear Programming

[RAHWAN et al., 2015]

Solving Integer Linear Programs

ILPs can be solved with state-of-the-art solvers like CPLEX (very mature technology)

Pros

Does not require any assumption on $v(\cdot)$ (very general approach)

Cons

- Memory requirements can become unmanageable for more than 20–30 agents
- Difficult to directly exploit the structure of the problem (i.e., graph)

CSG Approaches based on Integer Linear Programming

[Rahwan et al., 2015]

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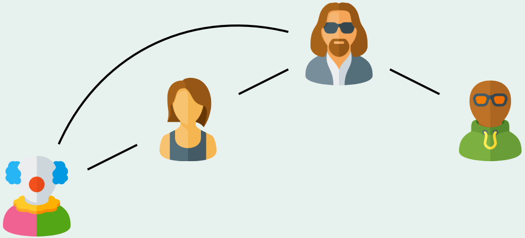
CSG as a COP

[BISTAFFA and FARINELLI, 2018]

Graph-Restricted CFG Example



Pseudotree among Agents (Hierarchy)

Adjacent agents in the graph fall in the same branch of the tree (e.g., and)

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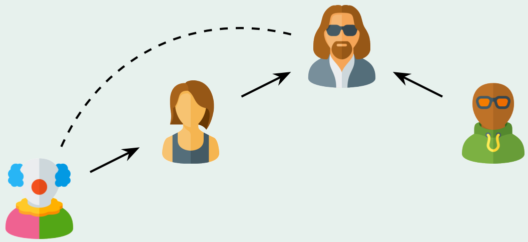

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Adjacent agents in the graph fall in the same branch of the tree (e.g.,  and )

CSG as a COP

[Bistaffa and Farinelli, 2018]

Challenge

How can we exploit the structure (i.e., hierarchy among agents)?

Main Idea

- Each coalition (i.e., decision variable) is "controlled" by the highest agent
- "Delegate" the formation of coalitions to descendants by means of *required* variables

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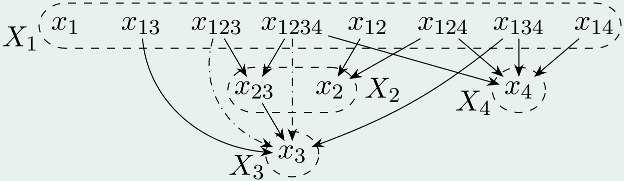

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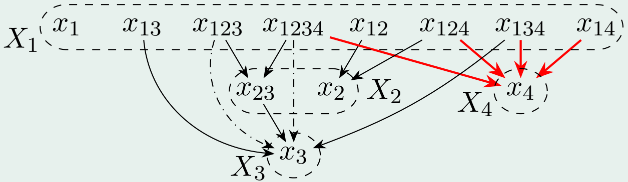

CSG as a COP

[Bistaffa and Farinelli, 2018]

Required Variables

- Any two variables that require the same variable *cannot* be enabled simultaneously
- As a result *no overlapping variables* are activated *at the same time*

Number of Constraints

- Naive COP: $\binom{\#\text{ coalitions}}{2}$
- This approach: linear *wrt* the number of agents

Open Question

Can we make this COP a Distributed COP (DCOP)?

CSG as a COP

[Bistaffa and Farinelli, 2018]

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Today's Menu

Characteristic Function

[CHALKIADAKIS et al., 2011]

Characteristic Function

The function $v : \mathcal{P}(A) \rightarrow \mathbb{R}$ associates a value to *every coalition* (i.e., subset) of A

Exponential Complexity

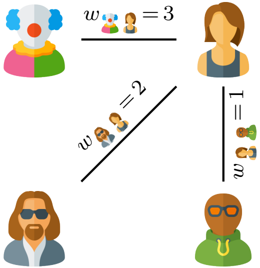
Representing $v(\cdot)$ as a *table* requires an *exponential* number of steps (i.e., $2^{|A|}$)

Mitigate this Complexity

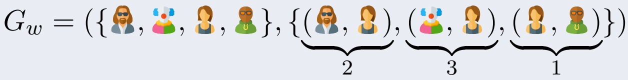
(1) *Restrict* the set of coalitions or (2) consider $v(\cdot)$ with a specific *structure*

Induced Subgraph Games (ISGs)

[DENG and PAPADIMITRIOU, 1994]



Weighted Graph G among Agents



Value is the Sum of Induced Edges

Induced Subgraph Games (ISGs)

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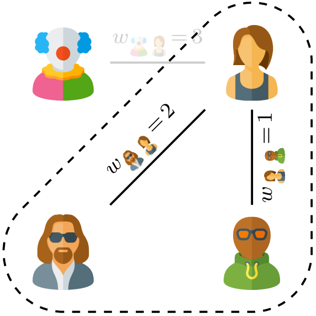

Weighted Graph G among Agents

$$G_w = (\{\ ,\ ,\ ,\ \}, \{\underbrace{(\ ,\ )}_{2}, \underbrace{(\ ,\ )}_{3}, \underbrace{(\ ,\ )}_{1}\})$$

Value is the Sum of Induced Edges

$$v(\{\ ,\ ,\ \}) = 2 + 1 = 3$$

Induced Subgraph Games (ISGs)

[CHALKIADAKIS et al., 2011]

Succinct Game Representation

The characteristic function is *entirely* represented by the weighted graph G_w

Computational Advantages

CSG on ISGs can be treated as a *graph clustering* problem (“easier” than CSG)

Limited Representation Power

Not every characteristic function game can be *perfectly* represented as an ISG

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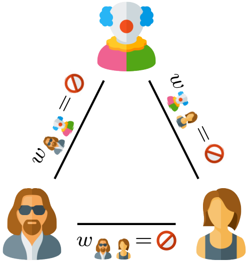


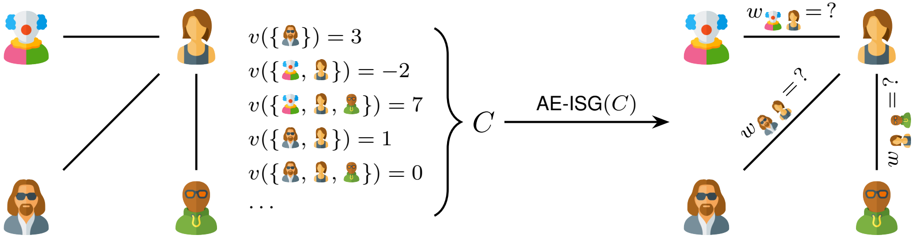
$$v(S) = \begin{cases} 0, & \text{if } |S| = 1, \\ 1, & \text{if } |S| = 2, \\ 6, & \text{if } |S| = 3. \end{cases}$$

Can We Approximate a CFG as an ISG?

[Bistaffa et al., 2022]

Approximately Equivalent ISG (AE-ISG)

Given a CFG C, compute the ISG that *best approximates* C, namely AE-ISG(C)

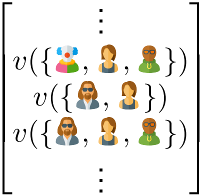


AE-ISG as Norm Approximation (ℓ_p Linear Regression)

[BISTAFFA et al., 2022]

minimise $\underbrace{\|Mw - v\|_p}_{\text{residuals}}$

$$M = \begin{array}{c} \begin{array}{ccc} w & w & w \end{array} \\ \begin{bmatrix} \vdots & \vdots & \vdots \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ \vdots & \vdots & \vdots \end{bmatrix} \end{array}$$

$v =$ 

AE-ISG as Norm Approximation (ℓ_p Linear Regression)

[BISTAFFA et al., 2022]

$$\text{minimise} \quad \|\underbrace{Mw - v}_{\text{residuals}}\|_p$$

Residual Vector

The *residual vector* $r = Mw - v$ is the vector of *differences* between *approximated* coalitional values (i.e., Mw) and *original* coalitional values (i.e., v)

Constrained Norm Approximation

Some coalitions (singletons) can be represented *exactly* via additional constraints

AE-ISG as Norm Approximation (ℓ_p Linear Regression)

[BISTAFFA et al., 2022]

Size of AE-ISG Model

Building M and v requires to go through the set of coalitional values (obviously)

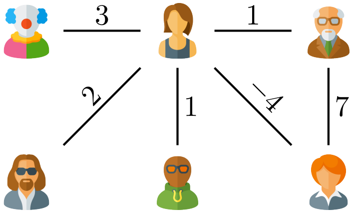
Computational Complexity

If the set of feasible coalitions is *polynomial* (e.g., ridesharing), computing AE-ISG(C) has a *manageable* complexity, depending on the norm ℓ_p:

- $\ell_1/\ell_\infty \rightarrow$ Linear Programming (exact, CPU)
- $\ell_2 \rightarrow$ Least Squares (exact/analytical, GPU)
- $\ell_{>2} \rightarrow$ Iteratively Reweighted Least Squares (numerical)

CSG on ISGs as Graph Clustering

[Bistaffa et al., 2022]



CSG on ISGs Optimisation Objective

Maximise sum of of *clusters' internal weights* (namely, *coverage* measure)

CSG on ISGs as Graph Clustering

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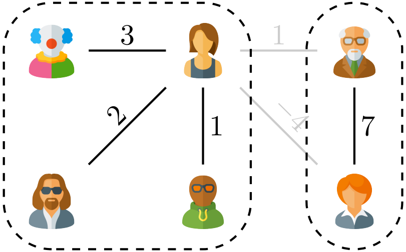


CSG on ISGs Optimisation Objective

Maximise sum of of *clusters' internal weights* (namely, *coverage* measure)

ILP for Optimal Graph Clustering (KGC)

[BISTAFFA et al., 2022]

$X_{ij} = 1 \rightarrow$ edge $\{i, j\}$ is "activated" (i and j are in the same cluster)

maximise $\sum_{i,j \in A} w_{ij} \cdot X_{ij}$ (Coverage objective function)

subject to $\forall i, j, z \in A : \begin{cases} X_{ij} + X_{jz} - 2 \cdot X_{iz} \leq 1 \\ X_{iz} + X_{ij} - 2 \cdot X_{jz} \leq 1 \\ X_{jz} + X_{iz} - 2 \cdot X_{ij} \leq 1 \end{cases}$ (Transitivity)

$\forall i \in A : \sum_{j \in A} X_{ij} \leq k$ (Cardinality constraint)

Theoretical Quality Guarantees

[BISTAFFA et al., 2022]

KGC computes the value of the optimal CSG solution
for the *approximated game AE-ISG*(C), namely $V(CS^*_{AE})$

What about the value of the optimal CSG solution
for the *original game* C, namely $V(CS^*_C)$?

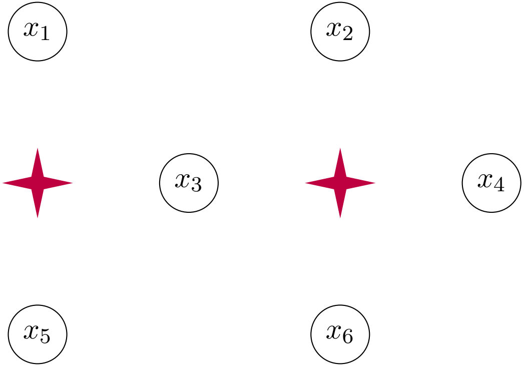
We proposed a way to compute B_r
such that $V(CS^*_C) - V(CS^*_{AE}) \leq B_r$

Today's Menu

Today's Menu

Motivating example

Sensor networks



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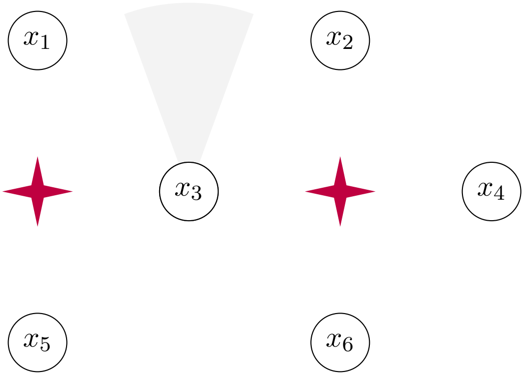

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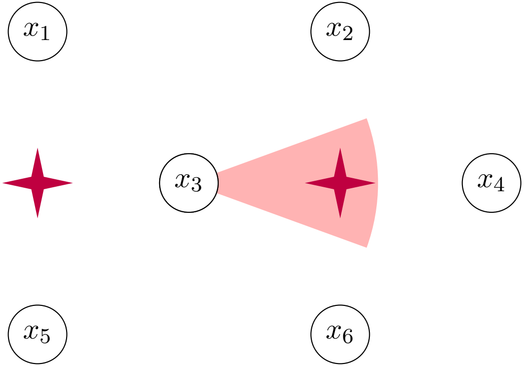

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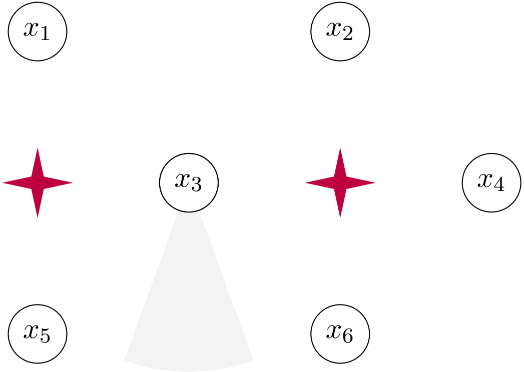

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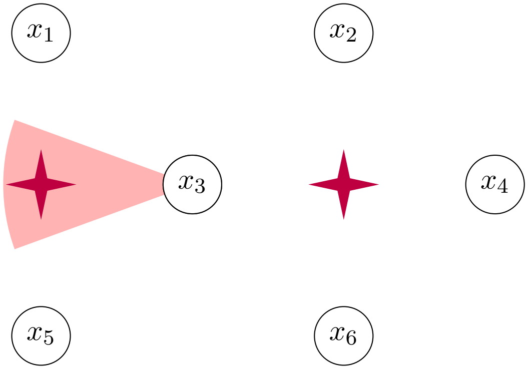

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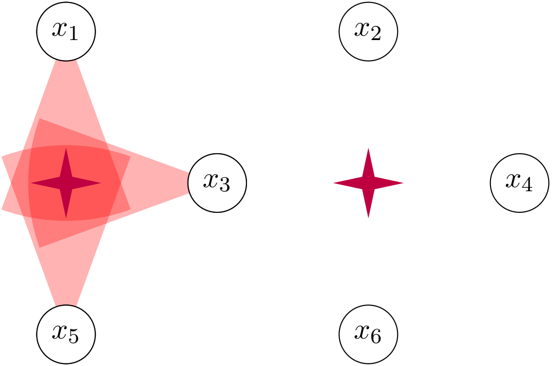


x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem as a CSP!

Today's Menu

CSP

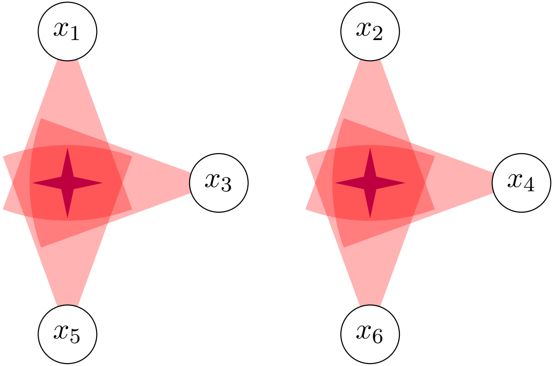
Constraint Satisfaction

- Variables $X = \{x_1, \ldots, x_n\}$
- Domains $D = \{D_1, \ldots, D_n\}$
- Constraints $C\{c_1, \ldots, c_m\}$
  where a constraint $c_i \subseteq D_{i_1} \times D_{i_2} \times \ldots \times D_{i_n}$ denotes the possible valid joint assignments for the variables $x_{i_1}, x_{i_1}, \ldots, x_{i_n}$ it involves

- **Goal**: Find an assignment to all variables that satisfies all the constraints

CSP

Constraint Satisfaction

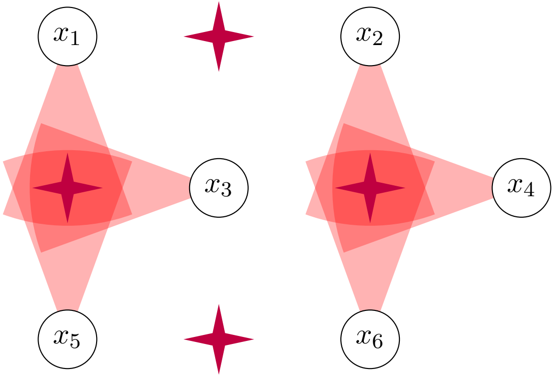


x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem as a CSP!

Max-CSP

Max Constraint Satisfaction



x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

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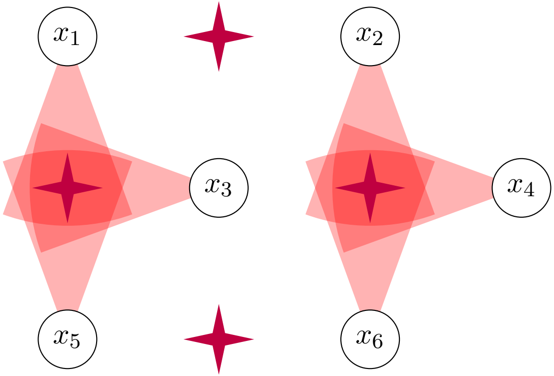
Max-CSP

Max Constraint Satisfaction

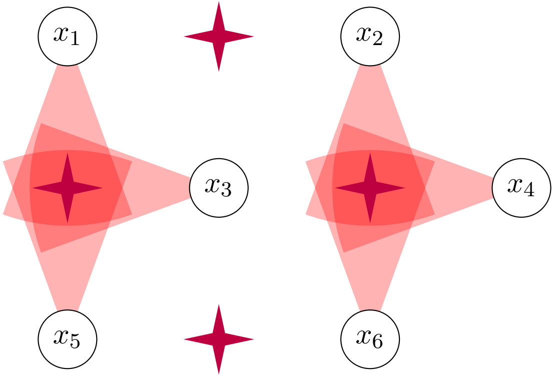


x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem
as a Max-CSP!

WCSP (or COP)

Constraint Optimization



x_1	x_3	x_5	Cost
N	N	N	∞
N	N	E	∞
...			∞
S	W	N	10
...			∞
W	W	W	∞

Model the problem as a COP!

WCSP (or COP)

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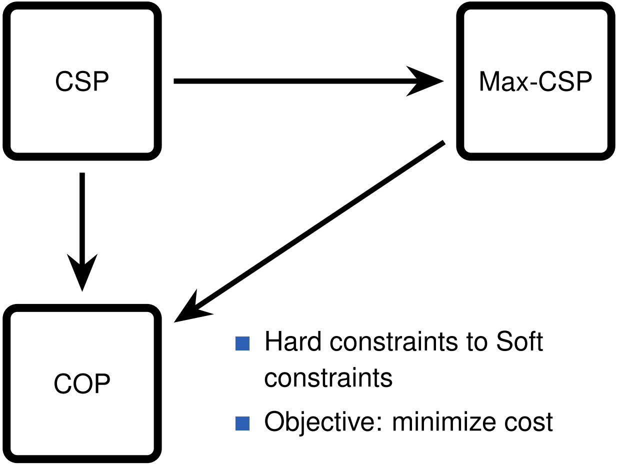
  where a constraint $c_i : D_{i_1} \times D_{i_2} \times \ldots \times D_{i_n} \to \mathbb{R}_+ \cup \{\infty\}$ expresses the degree of constraint violation

- **Goal**: Find an assignment to all variables that minimizes the sum of all the constraints

Constraint Reasoning

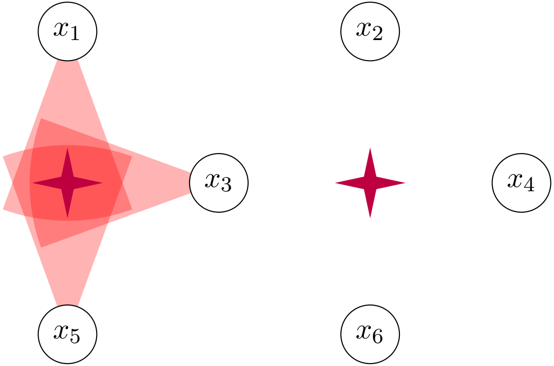


- Objective: maximize #constraints satisfied

- Hard constraints to Soft constraints
- Objective: minimize cost

WCSP (or COP)

Constraint Optimization



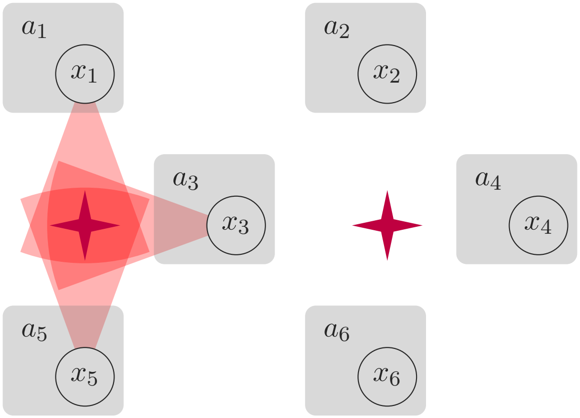
Imagine that each sensor is an autonomous agent

How should this problem be modeled and solved in a decentralized manner?

Today's Menu

DCOP

Distributed Constraint Optimization [Modi et al., 2005]



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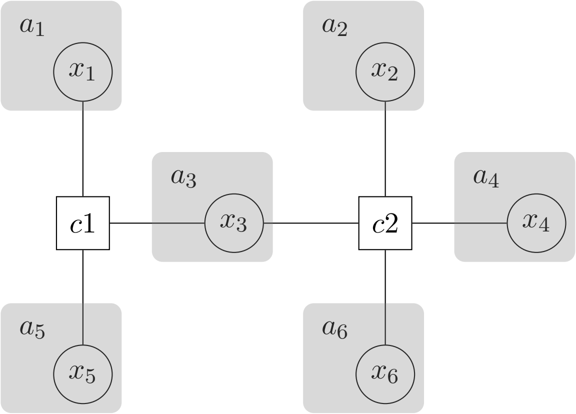

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- Mapping of variables to agents

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DCOP

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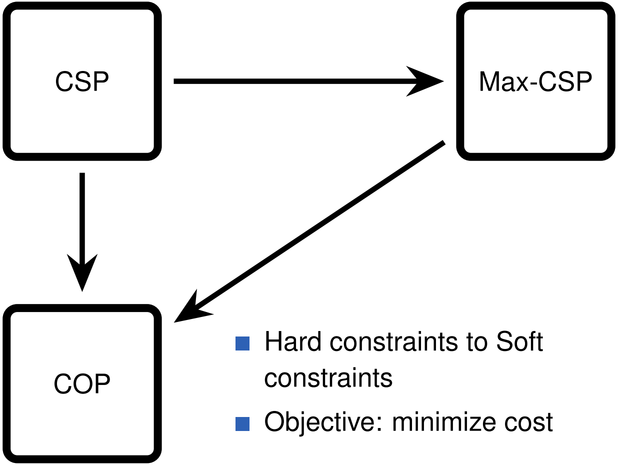

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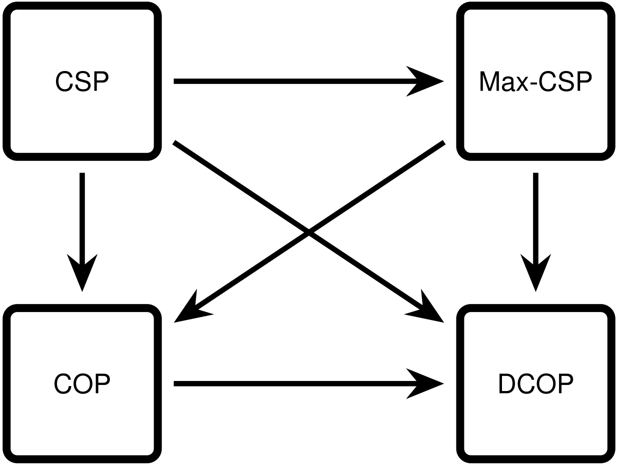
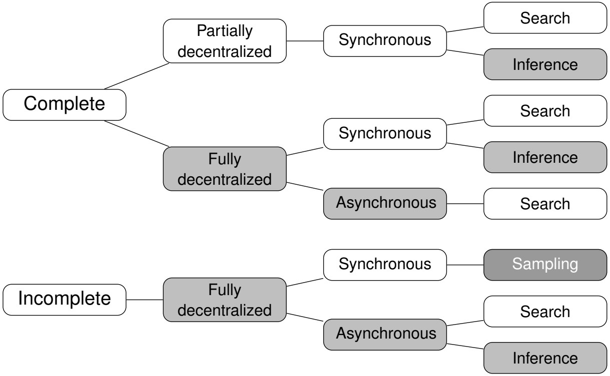


- Variables are controlled by agents
- Communication model
- Local knowledge

Today's Menu

DCOP Algorithms

See [FIORETTO et al., 2018]



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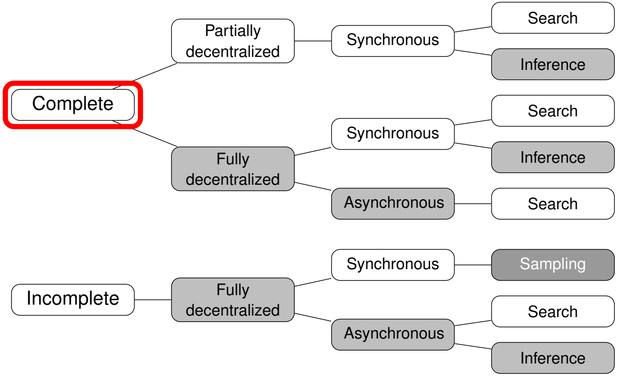
See [FIORETTO et al., 2018]

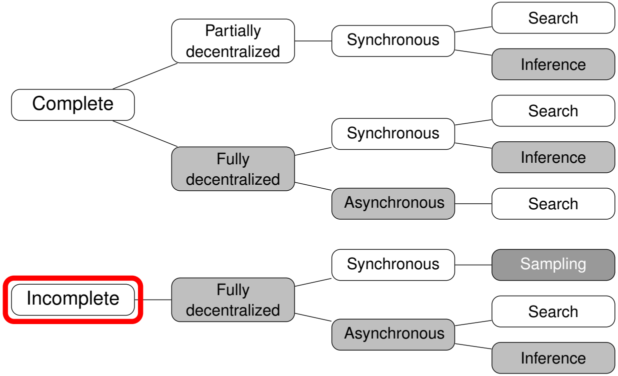


Important metrics

- Agent complexity
- Network loads
- Message size

DCOP Algorithms

See [FIORETTO et al., 2018]



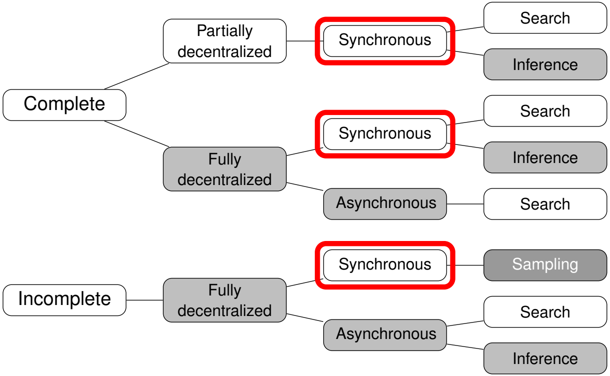
Important metrics

- Agent complexity
- Network loads
- Message size

- Anytime
- Quality guarantees
- Execution time vs. solution quality

DCOP Algorithms

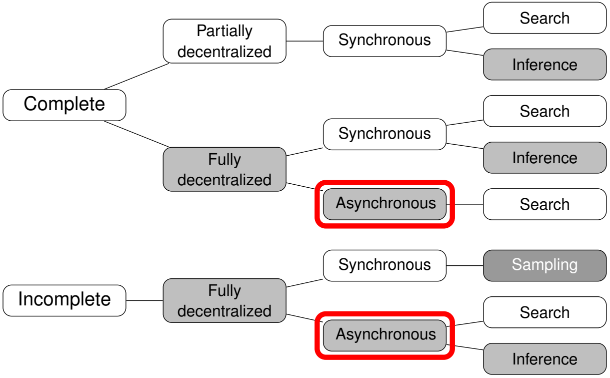
See [FIORETTO et al., 2018]



- Systematic process, divided in steps
- Each agent waits for particular messages before acting
- Consistent view of the search process
- Typically, increases idle-time

DCOP Algorithms

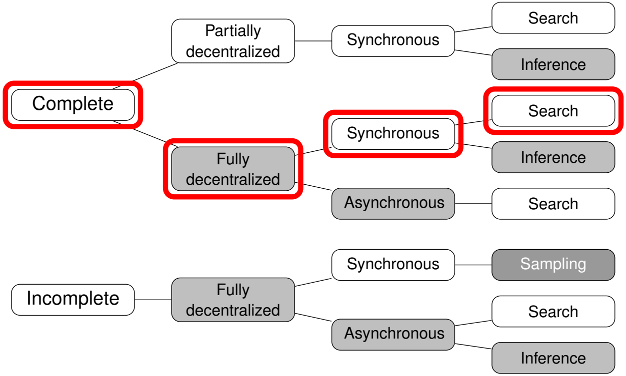
See [FIORETTO et al., 2018]



- Decision based on agents' local state
- Agents' actions do not depend on sequence of received messages
- Minimizes idle-time
- No guarantees on validity of local views

DCOP Algorithms

See [FIORETTO et al., 2018]



Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]

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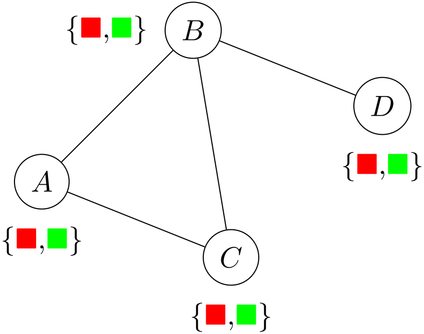
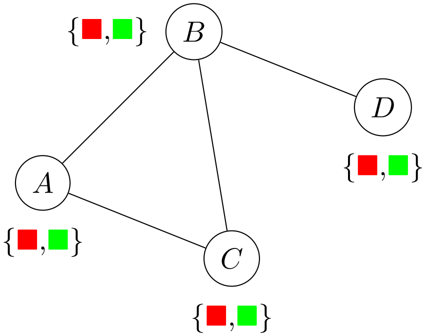


x_i	x_j	(A, B)	(A, C)	(B, C)	(B, C)
		5	5	5	3
		8	10	4	8
		20	20	3	10
		3	3	3	3

How do we solve this distributedly?

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]



x_i	x_j	(A,B)	(A,C)	(B,C)	(B,C)
		5	5	5	3
		8	10	4	8
		20	20	3	10
		3	3	3	3

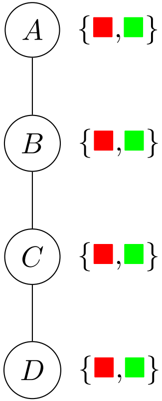
How do we solve this distributedly?

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]

- Agents operate on a complete ordering
- Agents exchange CPA messages containing partial assignments
- When a solution is found, its solution cost as an UB is broadcasted to all agents
- The UB is used for branch pruning

Complete ordering



Synchronous Branch-and-Bound (SBB)

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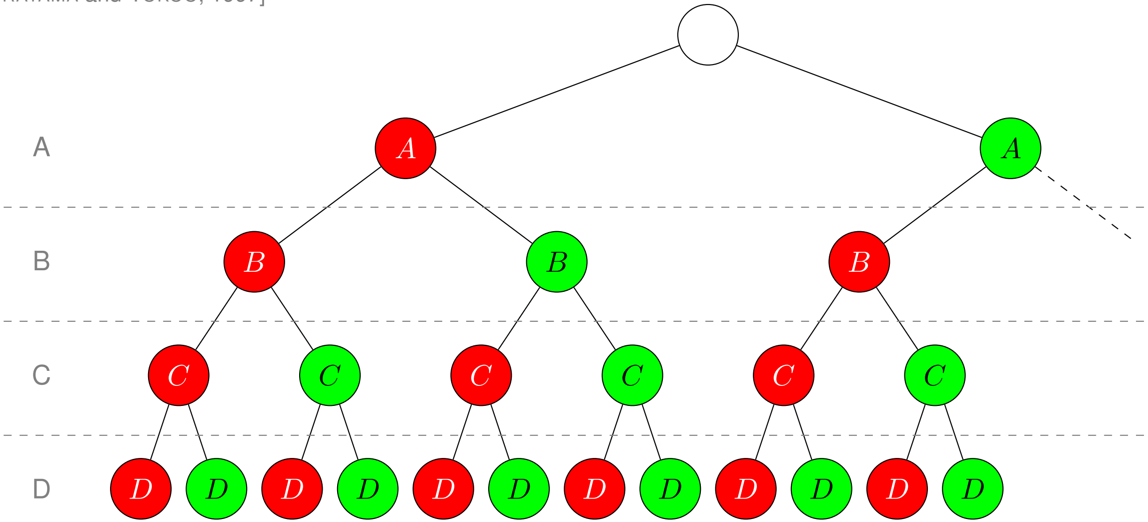

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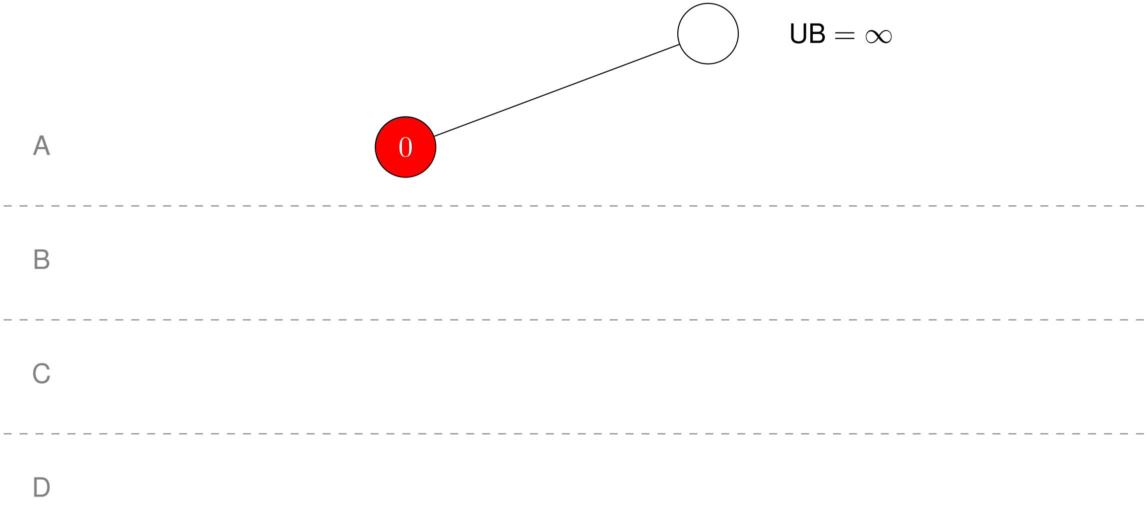

Synchronous Branch-and-Bound (SBB)

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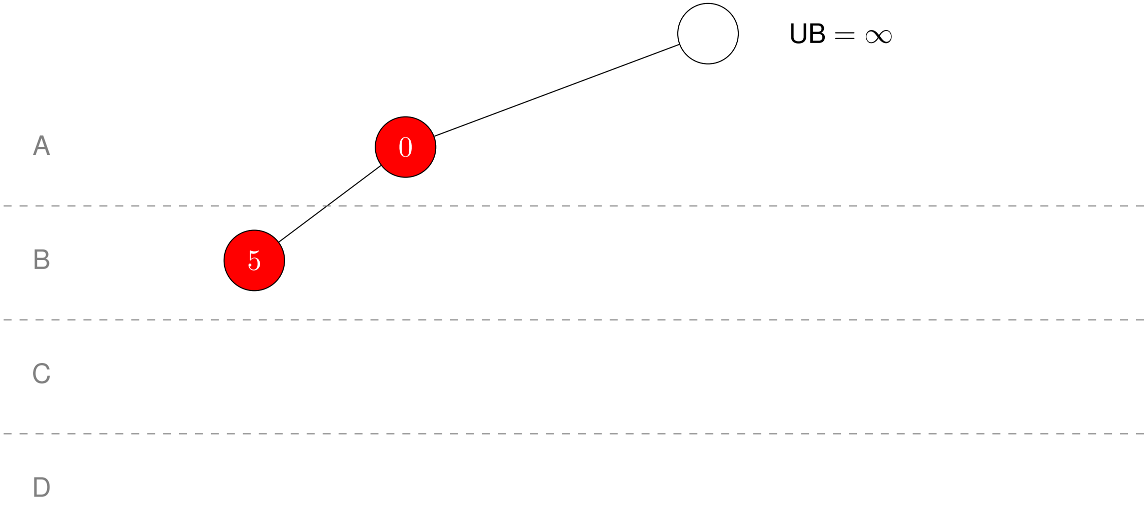

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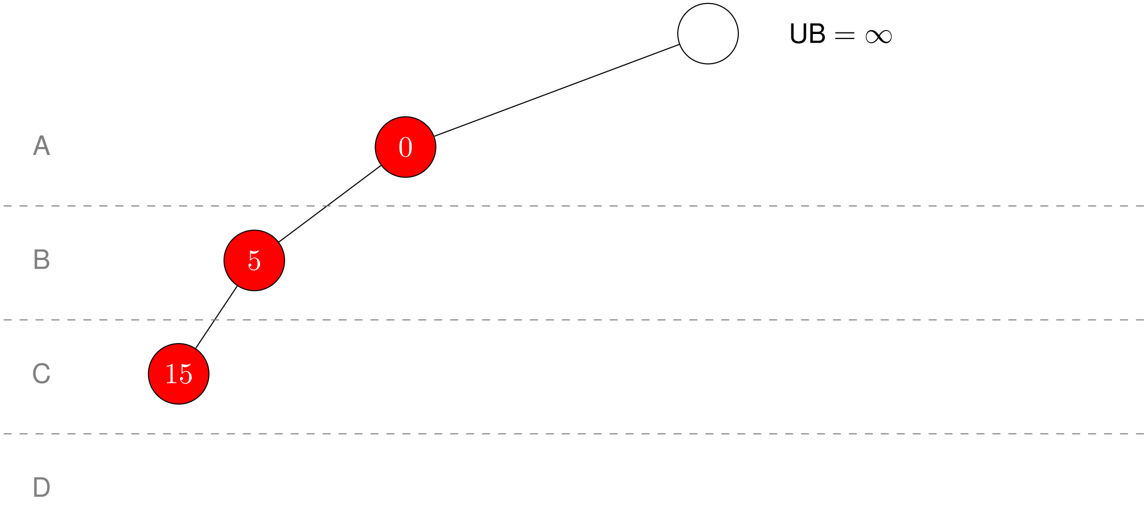

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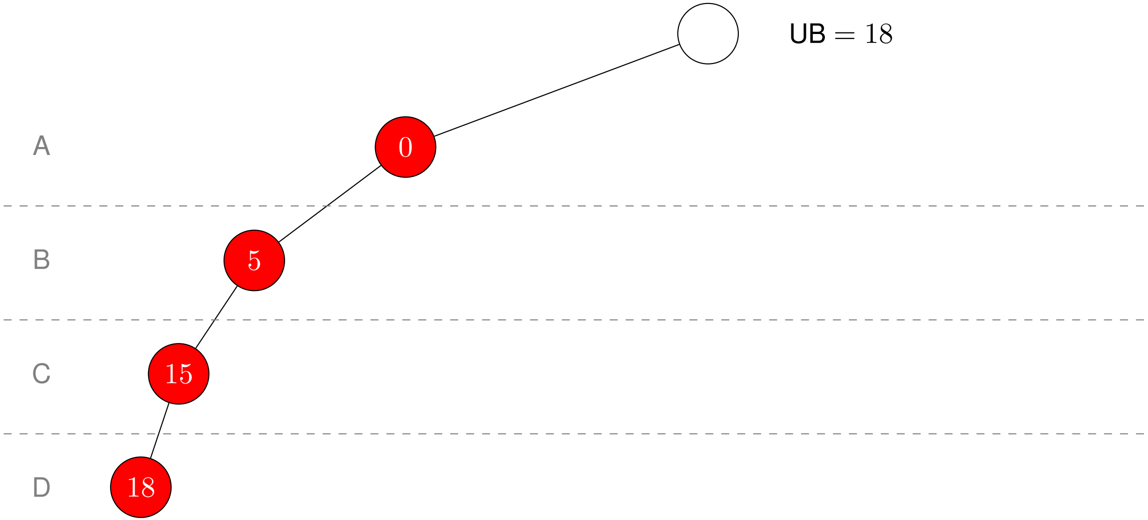

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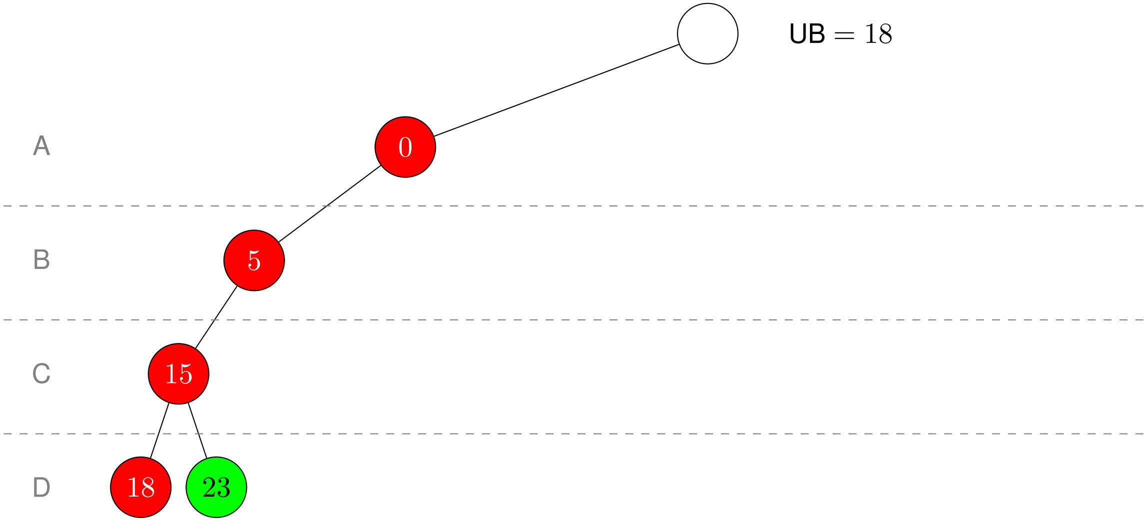

Synchronous Branch-and-Bound (SBB)

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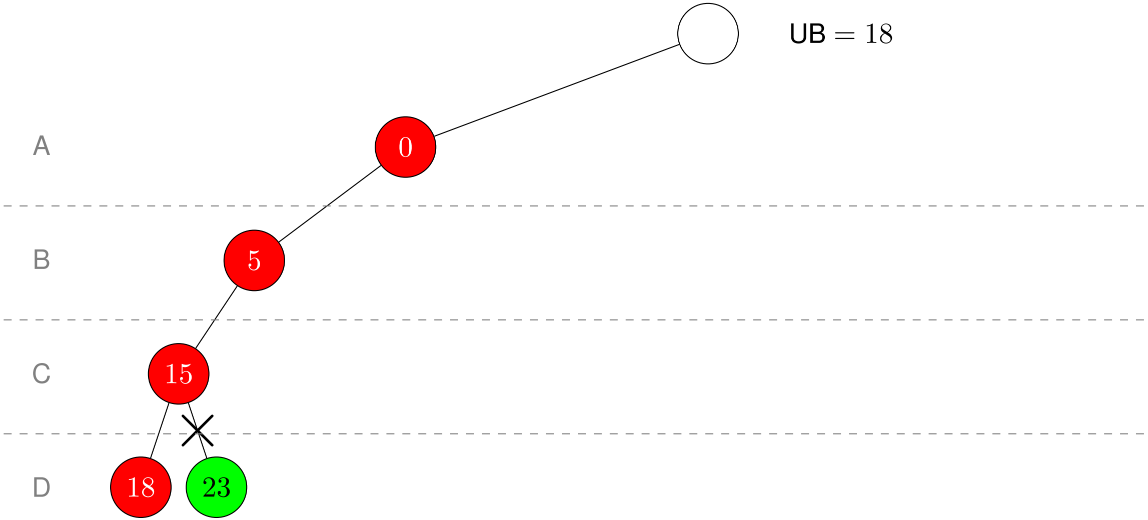

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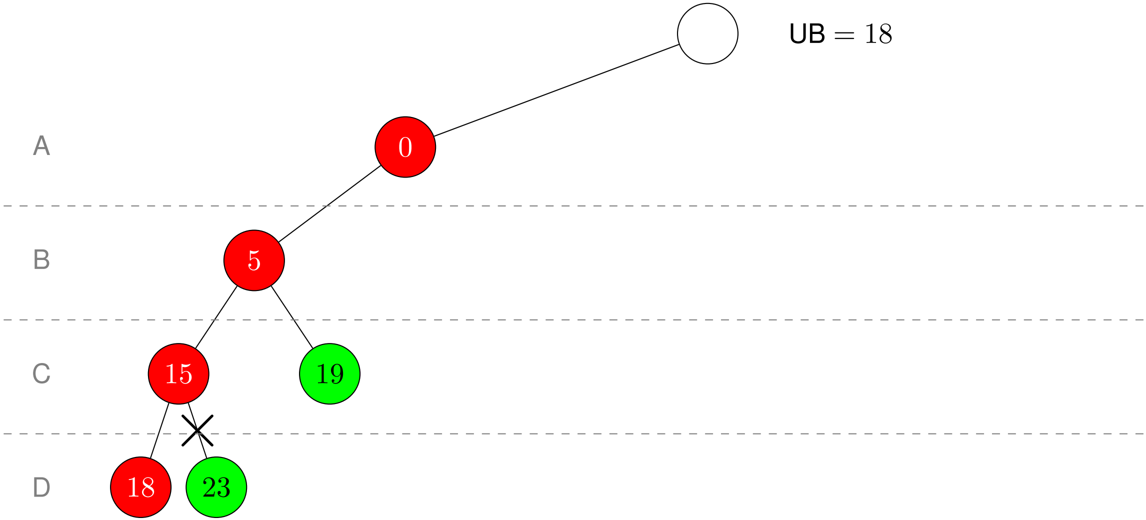

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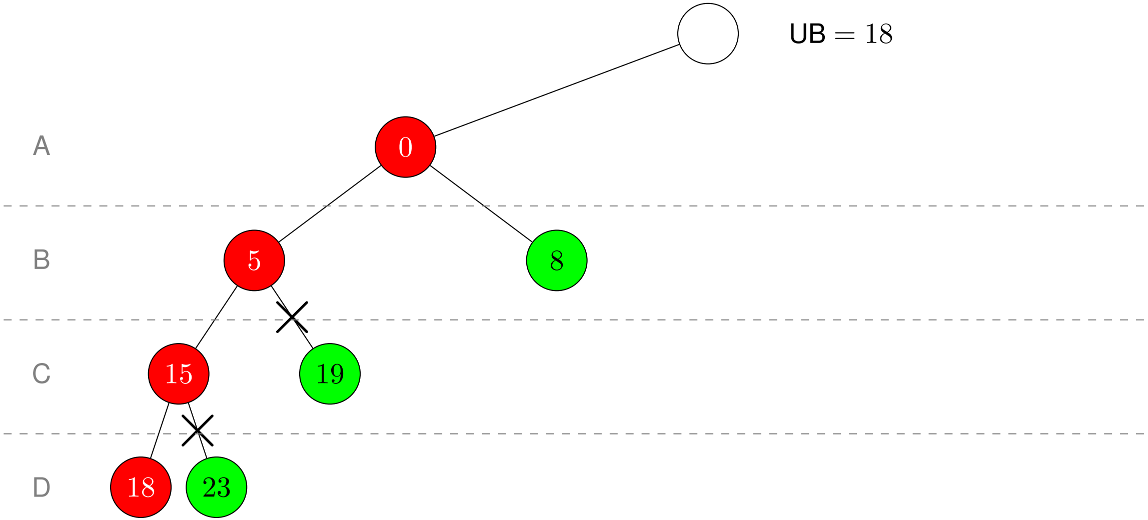

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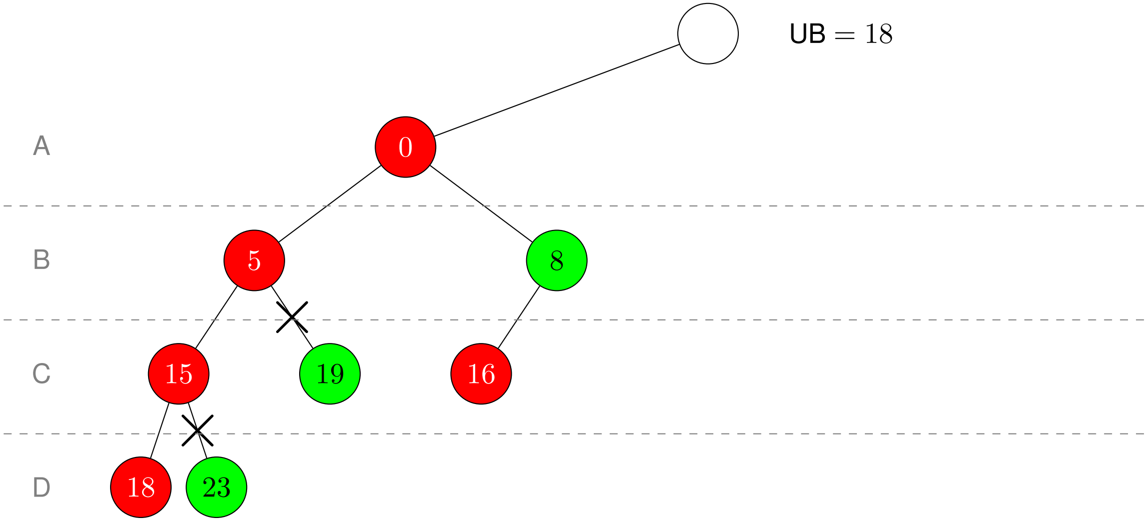

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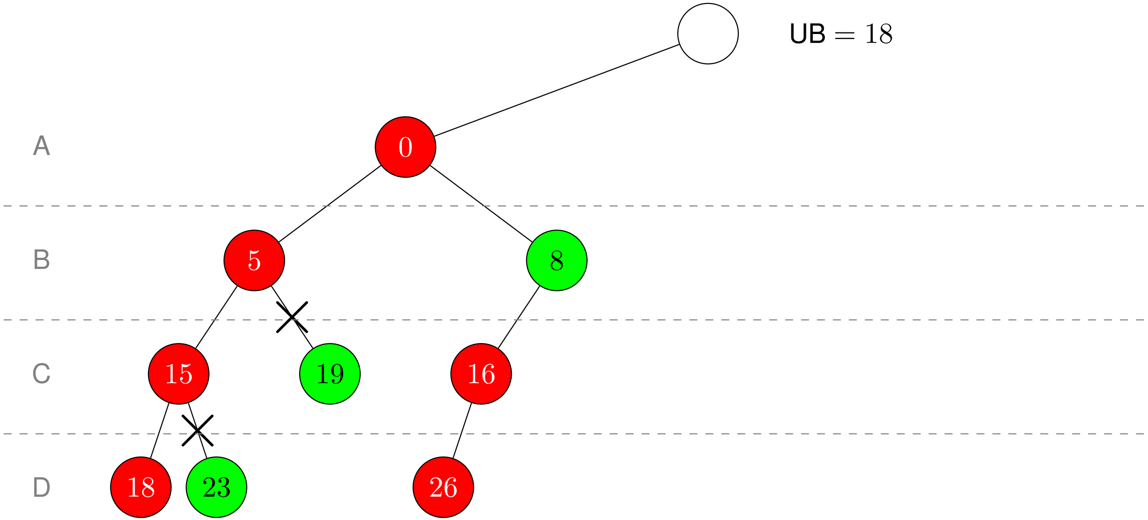

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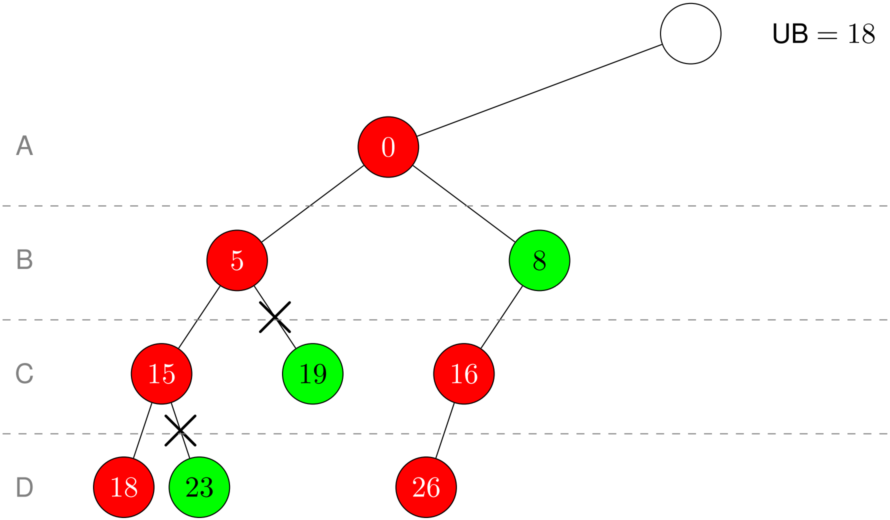

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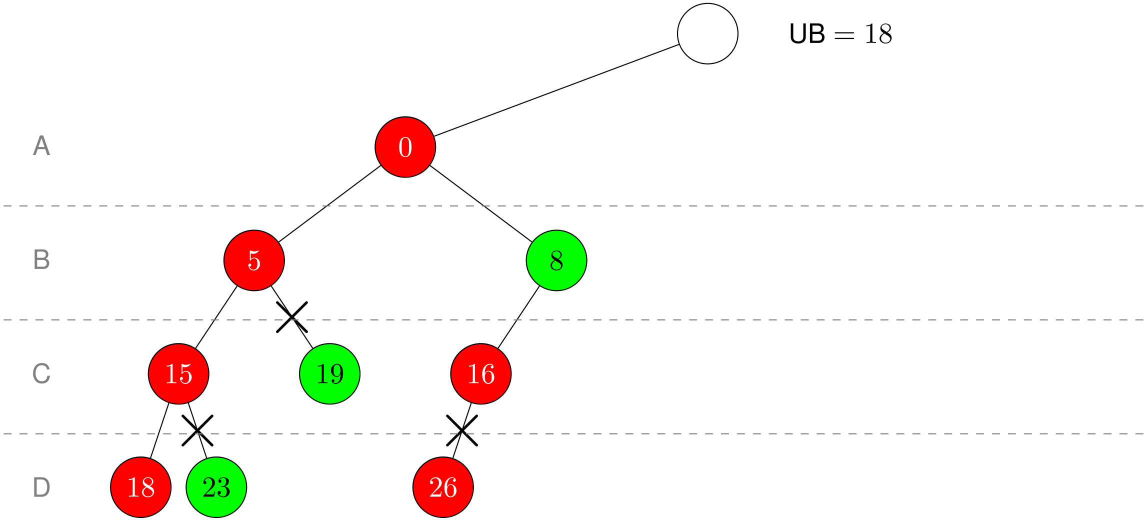


Synchronous Branch-and-Bound (SBB)

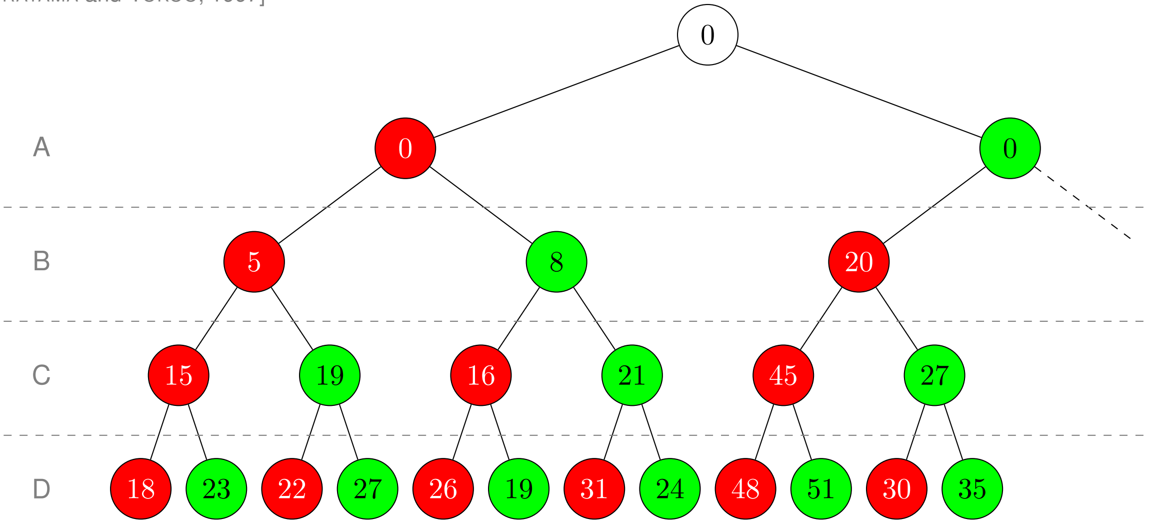
[HIRAYAMA and YOKOO, 1997]

	SBB
Correct the solution it finds is optimal	Yes
Complete it terminates	Yes
Message complexity max size of messages	$\mathcal{O}(d)$
Network load max number of messages	$\mathcal{O}(b^d)$
Runtime how long it takes	$\mathcal{O}(b^d)$

branching factor = b
num variables = d

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]



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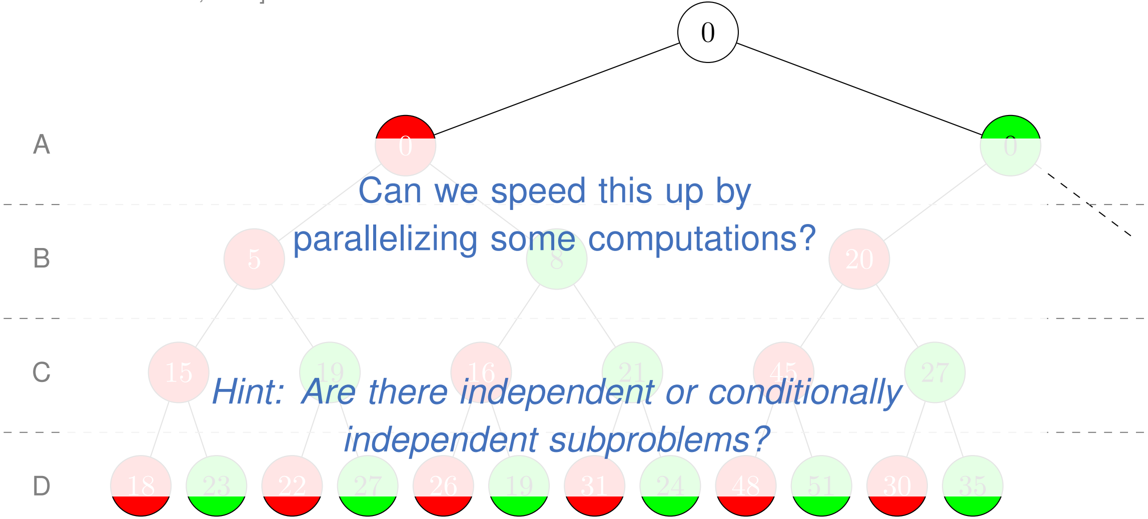
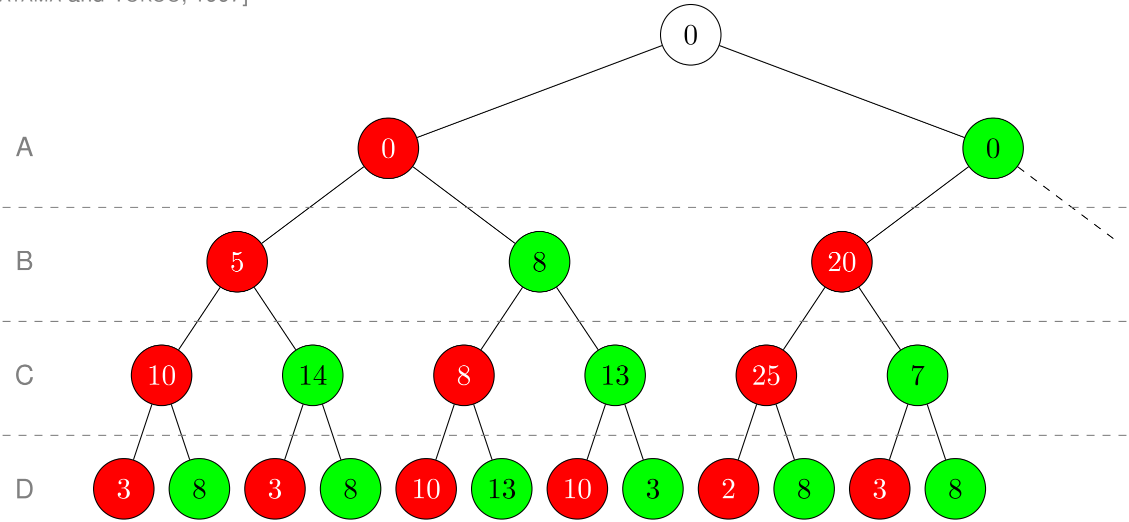


Can we speed this up by parallelizing some computations?

Hint: Are there independent or conditionally independent subproblems?

Synchronous Branch-and-Bound (SBB)

[Hirayama and Yokoo, 1997]



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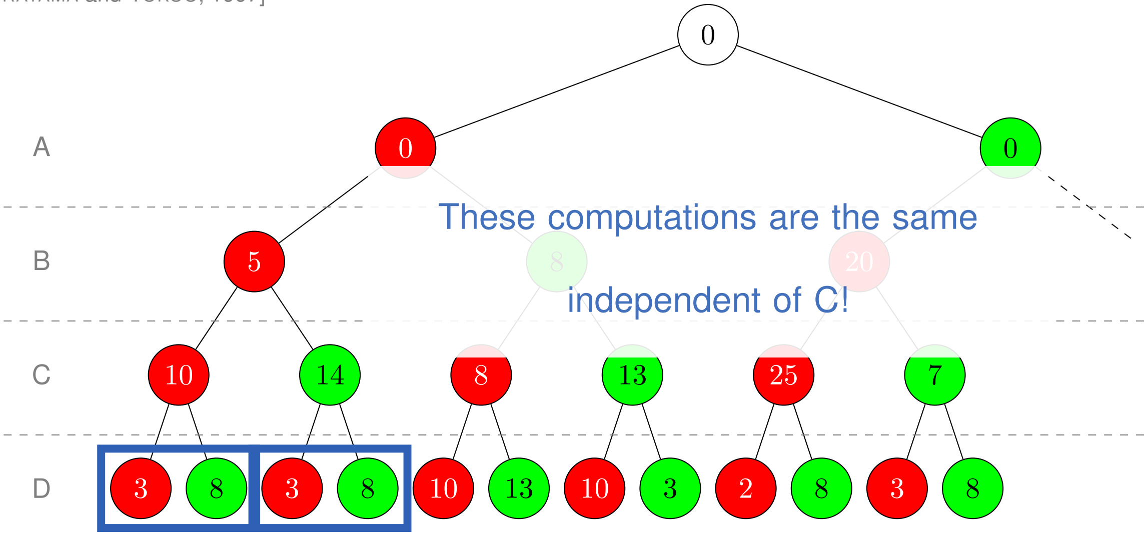
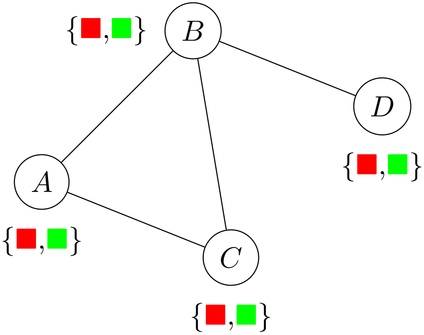
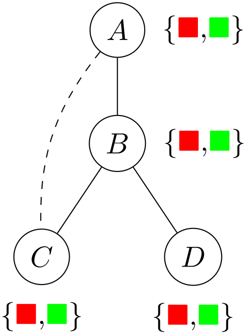


Pseudo-Tree

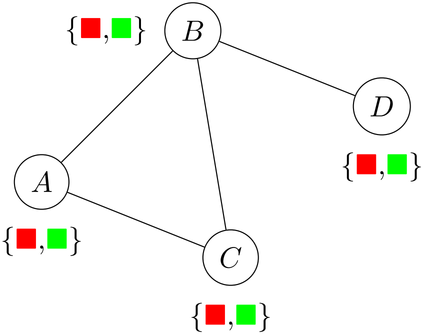


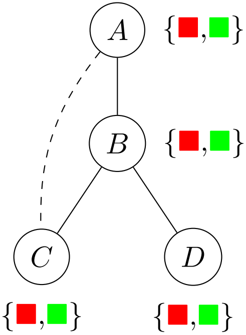


Definition (Pseudo-Tree)

A spanning tree of the constraint graph such that no two nodes in sibling subtrees share a constraint in the constraint graph

Pseudo-Tree





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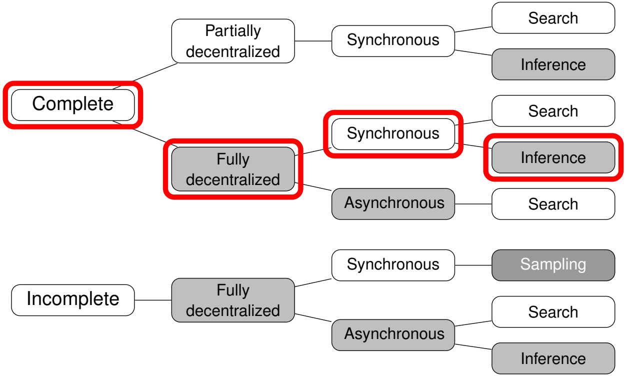
DCOP Algorithms

See [FIORETTO et al., 2018]



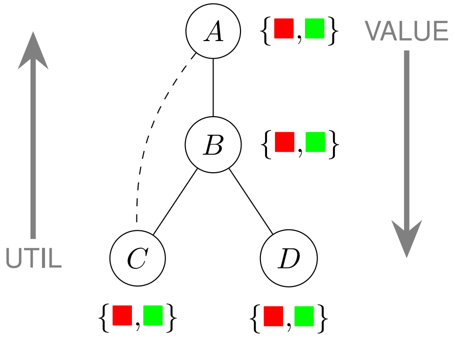
Distributed Pseudotree Optimization Procedure (DPOP)

[PETCU and FALTINGS, 2005b]

DPOP

[PETCU and FALTINGS, 2005b]

- Extension of the Bucket Elimination (BE)
- Agents operate on a pseudo-tree ordering
- UTIL phase: Leaves to root
- VALUE phase: Root to leaves



DPOP

[PETCU and FALTINGS, 2005b]

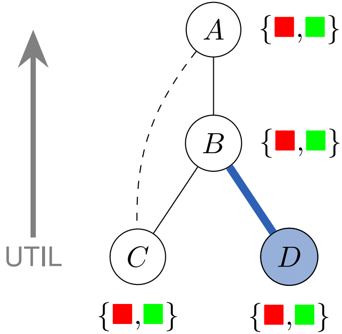
B	D	(B, D)
r	r	3
r	g	8
g	r	10
g	g	3

$\min\{3, 8\} = 3$

$\min\{10, 3\} = 3$

Message to B

B	cost
r	3
g	3



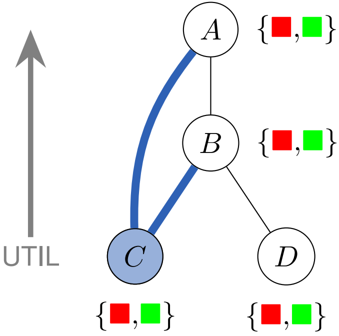
DPOP

[PETCU and FALTINGS, 2005b]

A	B	C	(B,C)	(A,C)	cost
r	r	r	5	5	10
r	r	g	4	8	12
r	g	r	3	5	8
r	g	g	3	8	11
g	r	r	5	10	15
g	r	g	4	3	7
g	g	r	3	10	13
g	g	g	3	3	6

Message to B

A	B	cost
r	r	10
r	g	8
g	r	7
g	g	6



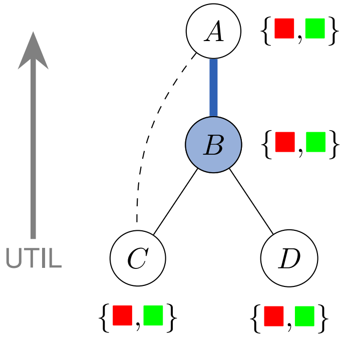
DPOP

[PETCU and FALTINGS, 2005b]

A	B	(A, B)	Util C	Util D	cost
r	r	5	10	53	18
r	g	8	8	3	19
g	r	20	7	3	30
g	g	3	6	3	12

Message to A

A	cost
r	18
g	12

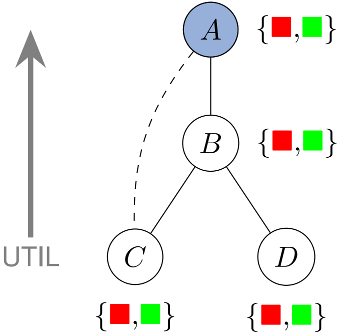


DPOP

[PETCU and FALTINGS, 2005b]

A	cost
r	18
g	12

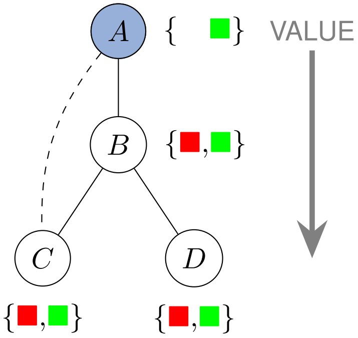
optimal cost $= 12$



DPOP

[PETCU and FALTINGS, 2005b]

A	cost
r	18
g	12

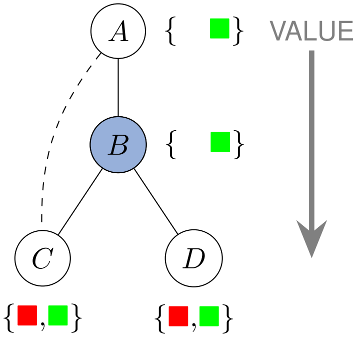


- Select value for $A = g$
- Send MSG "$A = g$" to agents B and C

DPOP

[PETCU and FALTINGS, 2005b]

A	B	(A, B)	Util C	Util D	cost
r	r	5	10	53	18
r	g	8	8	3	19
g	r	20	7	3	30
g	g	3	6	3	12



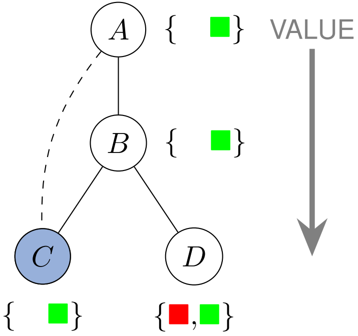
- Select value for $B = g$
- Send MSG "$B = g$" to agents C and D

DPOP

[PETCU and FALTINGS, 2005b]

A	B	C	(B,C)	(A,C)	cost
r	r	r	5	5	10
r	r	g	4	8	12
r	g	r	3	5	8
r	g	g	3	8	11
g	r	r	5	10	15
g	r	g	4	3	7
g	**g**	r	3	10	13
g	**g**	g	3	3	6

- Select value for $C = g$



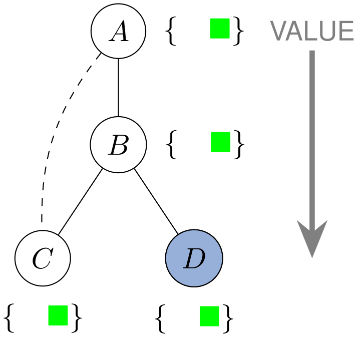
DPOP

[PETCU and FALTINGS, 2005b]

B	D	(B, D)
r	r	3
r	g	8
g	r	10
g	g	3

$\min\{3, 8\} = 3$

$\min\{10, 3\} = 3$



- Select value for $D = g$

DPOP

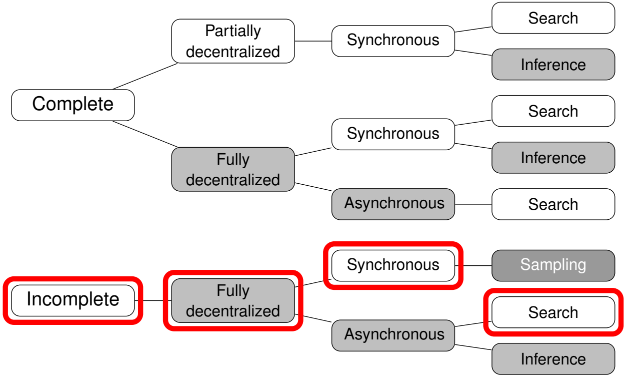
[PETCU and FALTINGS, 2005b]

	SBB	DPOP
Correct the solution it finds is optimal	Yes	Yes
Complete it terminates	Yes	Yes
Message complexity max size of messages	$\mathcal{O}(d)$	$\mathcal{O}(b^d)$
Network load max number of messages	$\mathcal{O}(b^d)$	$\mathcal{O}(d)$
Runtime how long it takes	$\mathcal{O}(b^d)$	$\mathcal{O}(b^d)$

branching factor = b
num variables = d

DCOP Algorithms

See [FIORETTO et al., 2018]



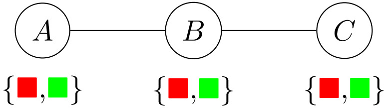
Distributed Local Search

[MAHESWARAN et al., 2004; ZHANG et al., 2003]

Local Search Algorithms

- DSA: Distributed Stochastic Search [Zhang et al., 2005]
- MGM: Maximum Gain Messages Algorithm [Maheswaran et al., 2004]
- Note: we now maximize utilities
- Every agent individually decides whether to change its value or not
- Decision involves
  - knowing neighbors' values
  - calculation of utility gain by changing values
  - probabilities

x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8



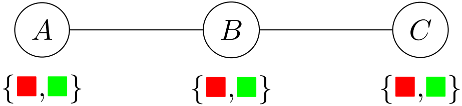
DSA Algorithm

[ZHANG et al., 2005]

- All agents execute the following
  - Randomly choose a value
  - while (termination is not met)
    - if (a new value is assigned): send the new value to neighbors
    - collect neighbors' new values if any
    - select and assign the next value based on assignment rule

DSA Algorithm

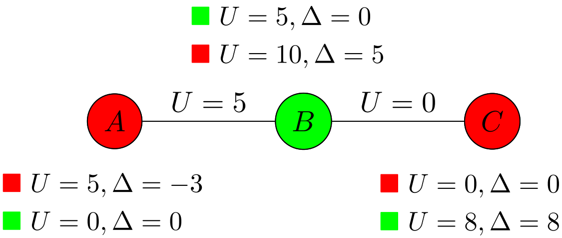
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

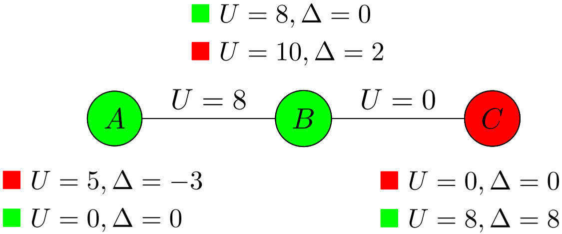
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

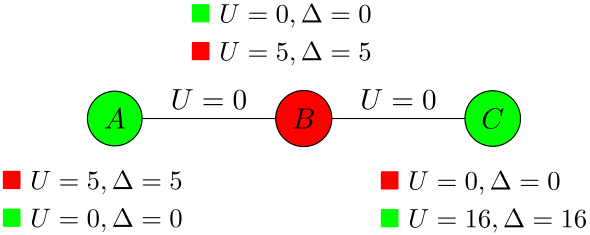
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

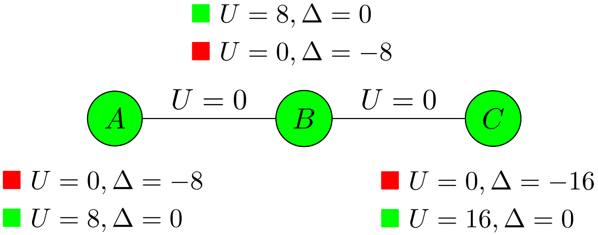
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

MGM Algorithm

[MAHESWARAN et al., 2004]

- All agents execute the following
  - Randomly choose a value
  - while (termination is not met)
    - if (a new value is assigned): send the new value to neighbors
    - collect neighbors' new values if any
    - calculate gain and send it to neighbors
    - collect neighbors' gains
    - if (it has the highest gain among all neighbors): change value to the value that maximizes gain

Large Great if you need an anytime algorithm!

MGM Algorithm

[MAHESWARAN et al., 2004]

- All agents execute the following
  - Randomly choose a value
  - while (termination is not met)
    - if (a new value is assigned): send the new value to neighbors
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    - calculate gain and send it to neighbors
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MGM vs DSA

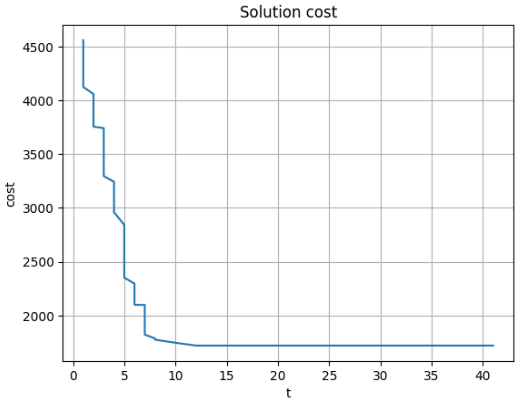


Figure: MGM

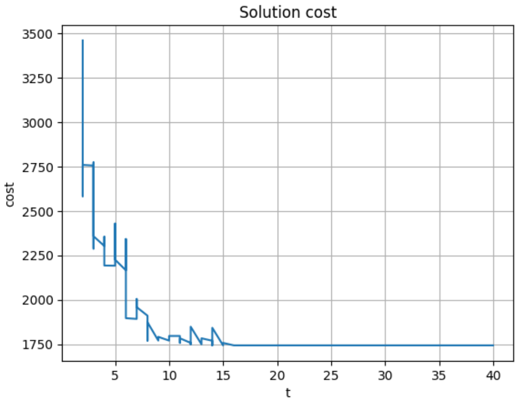


Figure: DSA

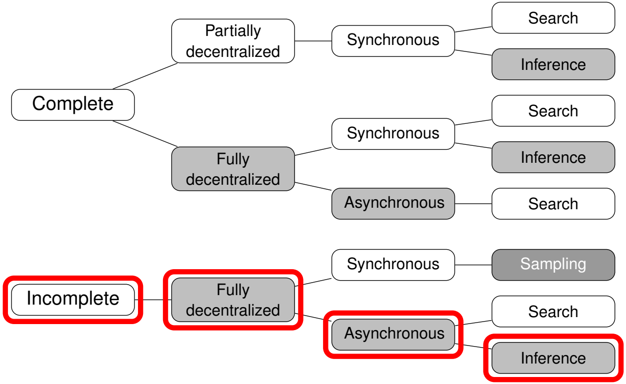
Today's Menu

Extensions to the DCOP Framework

- Dynamic DCOPs
  - SDPOP [Petcu and Faltings, 2005a], I-ADOPT and I-BnB-ADOPT [Yeoh et al., 2011], FMS [Ramchurn et al., 2010]
- Multi-Objective DCOPs
  - MO-SBB [Medi et al., 2014], Pseudo-tree Based Algorithm [Matsui et al., 2012], B-MOMS [Delle Fave et al., 2011], DP-AOF [Okimoto et al., 2013]
- Asymetric DCOPs
  - SyncABB-2ph, SyncABB-1ph, ACLS, MCS-MGM [Grinshpoun et al., 2013]
- Probabilistic DCOPs
  - $\mathbb{E}[\text{DPOP}]$ and SD-DPOP [Léauté and Faltings, 2011; Nguyen et al., 2012], U-GDL [Stranders et al., 2011]
- Continuous DCOPs
  - CMS [Stranders et al., 2009], HCMS [Voice et al., 2010], PFD [Choudhury et al., 2020], EC-DPOP, AC-DPOP, CAC-DPOP, C-DSA [Hoang et al., 2020], C-CoCoA [Sarker et al., 2021]
- ...

Deeper Focus on Max-Sum

See [FIORETTO et al., 2018]



Distributed Inference, Max-Sum

[FARINELLI et al., 2008]

Roie Zivan's Talk

Today's Menu

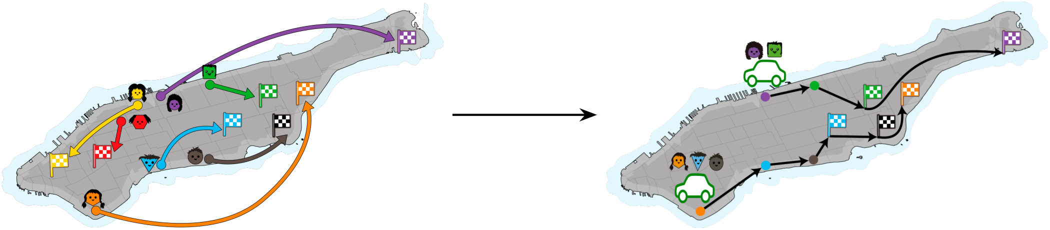
Today's Menu

Shared Mobility as (Online) Coalition Structure Generation

[BISTAFFA et al., 2019]

What is Shared Mobility for Us?

Arrange *shared rides* (coalitions) among users that submit *real-time* requests, with the objective of *maximizing* a given *objective function*



Shared Mobility as (Online) Coalition Structure Generation

[ibid.]

Our Task

At each time step, arrange a (possibly empty) set of non-overlapping feasible cars among the requests currently active in the system

Our Objective Function

Maximize environmental benefits 🌳 and quality of service 🕑

Our Case Study [Bistaffa et al., 2019]

Densely populated areas (e.g., Manhattan) with request rate of 400 reqs/minute

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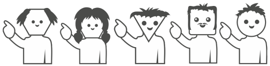
Input of the Online CSG Problem

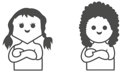
[BISTAFFA et al., 2019]

Incoming Requests

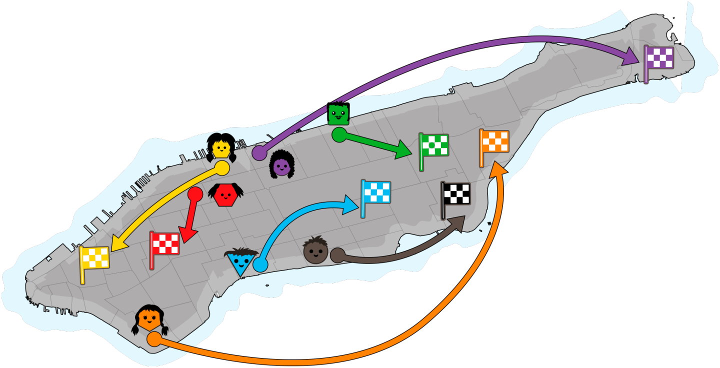


“I just issued a trip request”

Waiting Trip Requests



“I am waiting to share my ride”



Input of the Online CSG Problem

[BISTAFFA et al., 2019]

Example of a Shared Mobility Request

"I want to go from point i to point j, and I am willing to wait δ minutes to be picked up by somebody ($d =$ false) / before I leave with *my own car* ($d =$ true)"

- $r = \langle i, j, d, \delta \rangle$ (A request is a tuple r)
- $r \in R_t$ (The system receives a set R_t of requests at each time step t)
- $\langle R_1, \ldots, R_t, \ldots, R_h \rangle$ (Sequence of inputs over a time horizon h)
- The input sequence is *not known a priori* (Online optimization problem)

Input of the Online CSG Problem

[Bistaffa et al., 2019]

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Value $v(S)$ of a Coalition S

[BISTAFFA et al., 2019]

- The *value* (utility) of a coalition S is defined as:

$$v(S) = \overbrace{\rho_{\text{CO}_2} \cdot E_{\text{CO}_2}(S) + \rho_{\text{noise}} \cdot E_{\text{noise}}(S) + \rho_{\text{traffic}} \cdot E_{\text{traffic}}(S)}^{\text{environmental benefits}} + \overbrace{\rho_{\text{QoS}} \cdot Q(S)}^{\text{quality of service}}$$

- $|S| \leq k$ (Maximum cardinality constraint)

$$F(S) = |S| \leq k \,\wedge\, \ldots$$

- $\mathcal{F}(R) = \{S \in 2^R \mid F(S)\}$ (Set of feasible coalitions from a set R of requests)

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Curse of Dimensionality

[BISTAFFA et al., 2019]

- Recall that $\mathcal{F}(R) = \{S \in 2^R \mid F(S)\}$
- With $|S| \leq k$, $|\mathcal{F}(R)| \leq \sum_{i=1}^{k} \binom{|R|}{i}$, i.e., $\mathcal{O}(|R|^k)$ (Polynomial complexity)
- In practice, $|R_t|$ can be as high as 400 (Request rate in NY taxi dataset)

Scalability Problem

Enumerating all coalitions in $\mathcal{F}(R)$ is impractical, especially in realistic application scenarios with *very limited time budget* for the solution

Our Solution

Consider a restricted set $\hat{\mathcal{F}}(R)$ of *good candidate coalitions* instead of $\mathcal{F}(R)$

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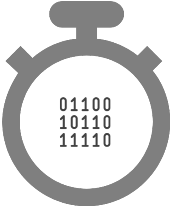
Generation of Good Candidate Coalitions (Step 1)

[BISTAFFA et al., 2019]

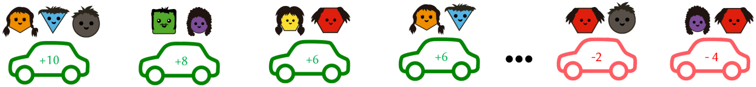
- CO_2 emissions
- Acoustic pollution
- Traffic congestion
- Quality of service

20 seconds



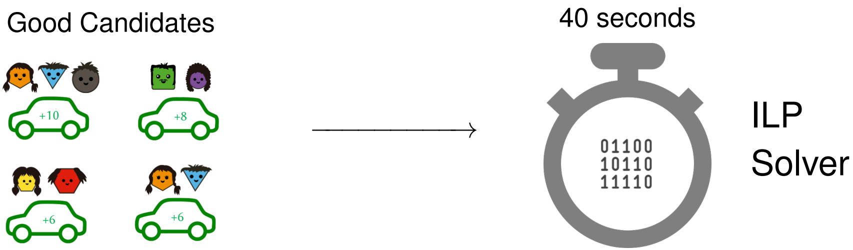
Probabilistic Greedy Algorithm

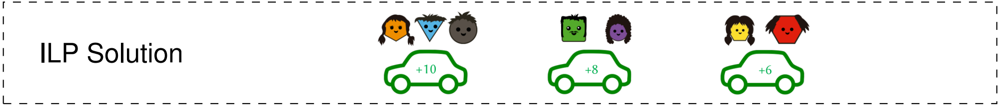
Candidate Cars



ILP Optimization (Step 2)

[BISTAFFA et al., 2019]





Approximated ILP Formulation

[BISTAFFA et al., 2019]

$$\text{maximize} \quad \sum_{S \in \hat{\mathcal{F}}(\text{Pool})} v(S) \cdot x_S \qquad \text{(Only good candidates)}$$

$$\text{such that} \quad x_S + x_{S'} \leq 1 \quad \forall\, \hat{\mathcal{F}}(\text{Pool}) : S \cap S' \neq \varnothing$$

Computational Advantage

Approximated ILP has a number of variables that is $< 0.01\%$ of the optimal ILP

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Today's Menu

Collective Energy Purchasing

[Farinelli et al., 2013]

Collective Energy Purchasing Scenario

- Each agent has an energy consumption profile
- Customers form coalitions to buy energy at reduced tariffs from two different markets:
  - *Spot market:* a short-term market intended for smaller amounts of energy
  - *Forward market:* a long-term market to buy more energy at cheaper prices

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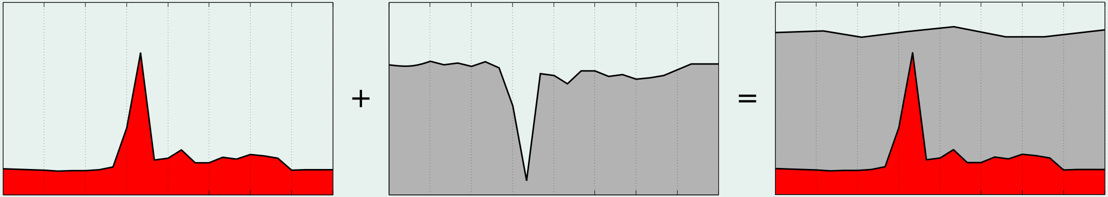
Collective Energy Purchasing

[FARINELLI et al., 2013]

Profile Merging

- Peaks in energy profiles require the use of *expensive*, *carbon-intensive*, peaking plant generators, resulting in higher consumers electricity bill
- A *flattened* profile results in a more efficient grid, with *lower carbon emissions* and *lower prices* for consumers

Example



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- $q_S^t(S)$: energy purchased from spot market at time t
- $q_F(S)$: total energy purchased from forward market
- p_S: spot market energy price
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$m + a$ Characteristic Functions

[BISTAFFA et al., 2017a]

$m + a$ Characteristic Function

- $m + a =$ Superadditive function $+$ subadditive function
  - Superadditive: $v(S_1 \cup S_2) > v(S_1) + (S_2)$
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Open Question

Is the characteristic function of shared mobility $m + a$?

$m + a$ Characteristic Functions

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Branch-and-Bound for $m + a$ Characteristic Functions

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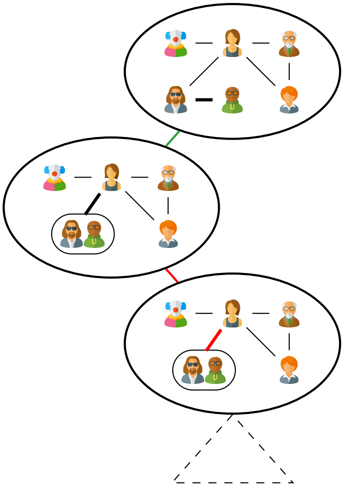


Can we Find an Upper Bound on $v(\cdot)$ in this Subtree?

Upper Bound M for $m + a$ Functions

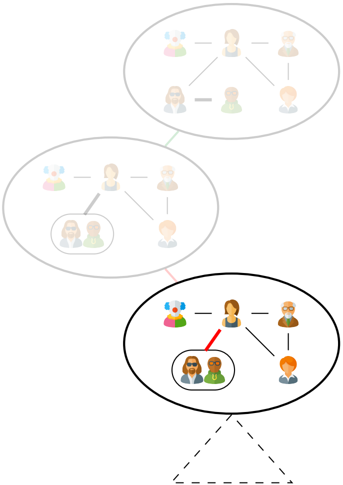
$$M = v^+(\textit{contract all edges}) + v^-(\textit{contract no edge})$$

Branch-and-Bound Algorithm

If M is $<$ than current best solution, do not visit this subtree

Branch-and-Bound for $m + a$ Characteristic Functions

[Bistaffa et al., 2017a]



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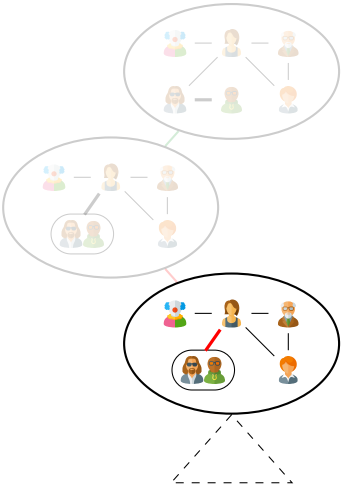

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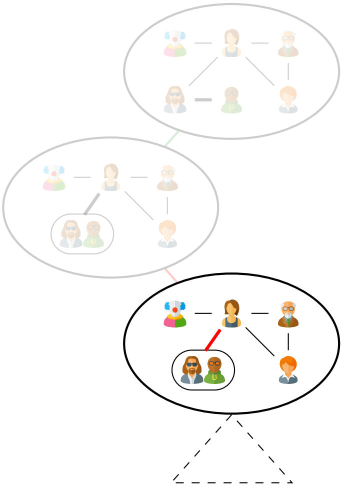

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Today's Menu

SECP model

Smart Environment Configuration Problem [RUST et al., 2016]

- Example of applying DCOPs to a "real" problem
- Coordinate objects in the building
- Model
  - objects
  - relations between objects and environment
  - user objectives and requirements
- Formulate the problem as an optimization problem

SECP model

Smart Environment Configuration Problem [RUST et al., 2016]

Focus on smart lighting use cases

- **Objects:** anything that can produce light: light bulbs, windows with rolling shutter, etc.
- **User preferences:** having a predefined luminosity level in a room, under some conditions
- **Energy efficiency**

Linking objects and user preferences:

- How to model the luminosity in a room ? **variable**
- How to model the dependency between the light sources and the luminosity ? **function / constraint**

SECP model

Example application to ambient intelligence scenario

- **Actuators**
  - Connected light bulbs, TV, Rolling shutters, ...
- **Sensors**
  - Presence detector, Luminosity Sensor, etc.
- **Physical Dependency Models**
  - E.g. Living-room light model
- **User Preferences**
  - Expressed as rules :

IF	presence_living_room	$=$	1
AND	light_sensor_living_room	$<$	60
THEN	light_level_living_room	$\leftarrow$	60
AND	shutter_living_room	$\leftarrow$	0

SECP model

Example application to ambient intelligence scenario

- **Actuators**
  - *Decision* variable x_i, domain $\mathcal{D}_{x_i}$
  - Cost function $c_i : \mathcal{D}_{x_i} \rightarrow \mathbb{R}$
- **Sensors**
  - *Read-only* variable s_l, domain $\mathcal{D}_{s_l}$
- **Physical Dependency Models** $\langle y_j, \phi_j \rangle$
  - Give the expected state of the environment from a set of actuator-variables influencing this model
  - Variable y_j representing the *expected* state of the environment
  - Function $\phi_j : \prod_{\varsigma \in \sigma(\phi_j)} \mathcal{D}_\varsigma \rightarrow \mathcal{D}_{y_j}$
- **User Preferences**
  - Utility function u_k
  - Distance from the current expected state to the target state of the environment

Formulating SECP as a DCOP

Multi-objective optimization problem

$$\min_{x_i \in \nu(\mathfrak{A})} \sum_{i \in \mathfrak{A}} c_i \quad \text{and} \quad \max_{\substack{x_i \in \nu(\mathfrak{A}) \\ y_j \in \nu(\Phi)}} \sum_{k \in \mathfrak{R}} u_k$$

$$\textbf{s.t. } \phi_j(x_j^1, \ldots, x_j^{\overline{\phi_j}}) = y_j \quad \forall y_j \in \nu(\Phi)$$

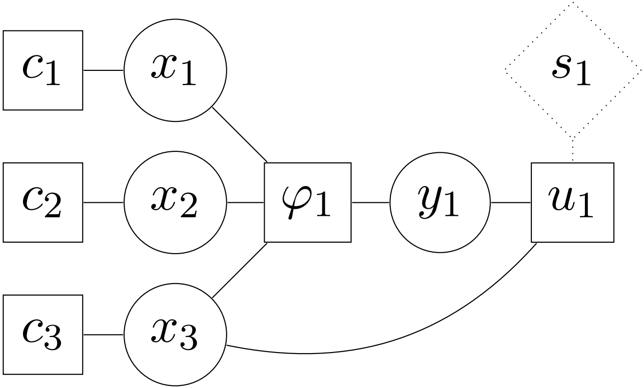
Mono-objective DCOP formulation

$$\max_{\substack{x_i \in \nu(\mathfrak{A}) \\ y_j \in \nu(\Phi)}} \quad \omega_u \sum_{k \in \mathfrak{R}} u_k - \omega_c \sum_{i \in \mathfrak{A}} c_i + \sum_{\varphi_j \in \boldsymbol{\Phi}} \varphi_j$$

$$\varphi_j(x_j^1, \ldots, x_j^{|\sigma(\phi_j)|}, y_j) = \begin{cases} 0 & \text{if } \phi_j(x_j^1, \ldots, x_j^{|\sigma(\phi_j)|}) = y_j \\ -\infty & \text{otherwise} \end{cases}$$

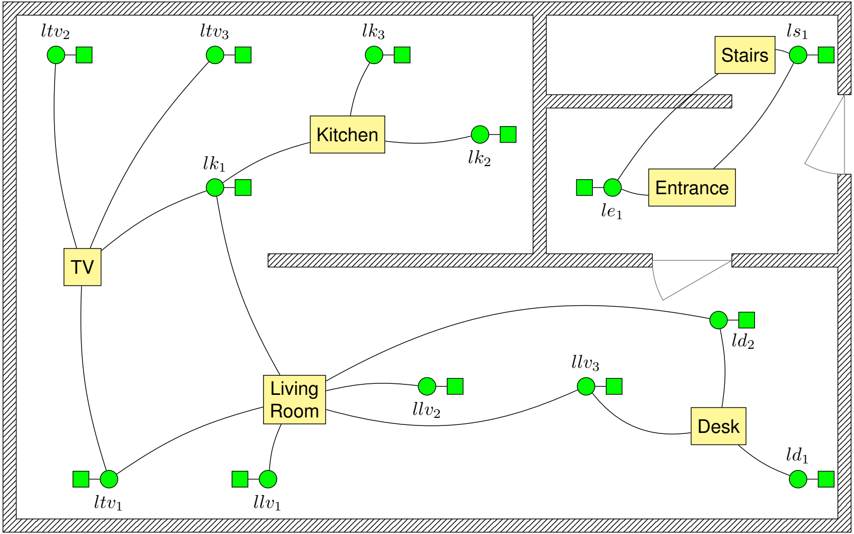
Formulating SECP as a DCOP

Representing a DCOP as a factor graph

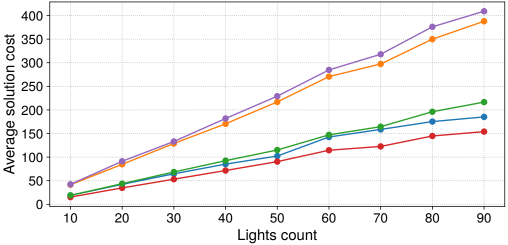


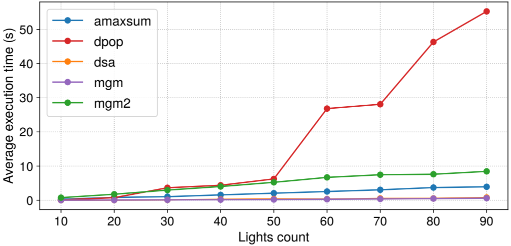
SECP Factor Graph

in a house (without rules)



Algorithms’ performances





- Best solutions: DPOP, MaxSum, MGM2
- Worst runtime: DPOP
- Best compromise: MaxSum, MGM2

SECP: further readings

- Experiments with various algorithms [Rust et al., 2016, 2022]
- How to deploy DCOPs [Rust et al., 2017, 2022]
- How to adapt deployment at runtime [Rust et al., 2018, 2020, 2022]

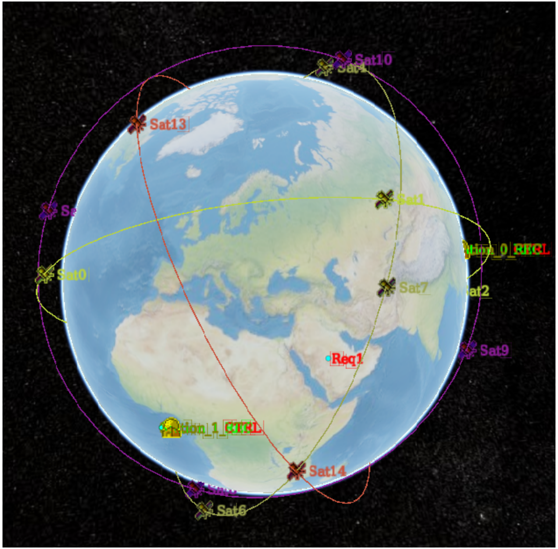
Today's Menu

Observation Scheduling in Multi-Owner Constellations [Picard, 2022]

- Increasing size of deployed EOS constellations

⇒ Observe any point on Earth at higher frequency, e.g. Planet constellation

- **but**, requires to improve coordination and cooperation between assets and stakeholders



- We focus here on collective observation scheduling on a constellation where some users have exclusive access to some orbit portions

⇒ Answer to strong user expectations to benefit both from a shared system (to reduce costs) and a proprietary system (total control and confidentiality)

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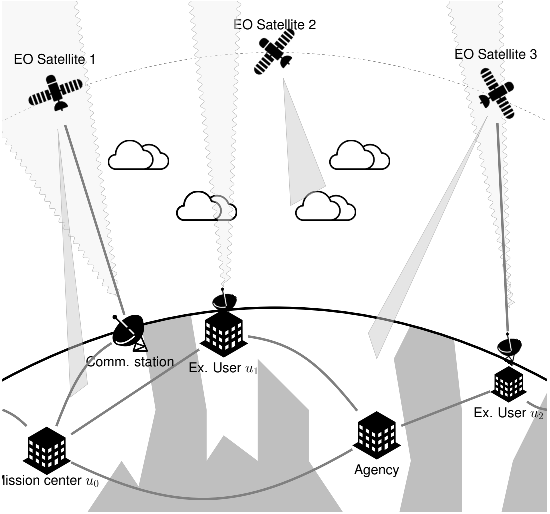
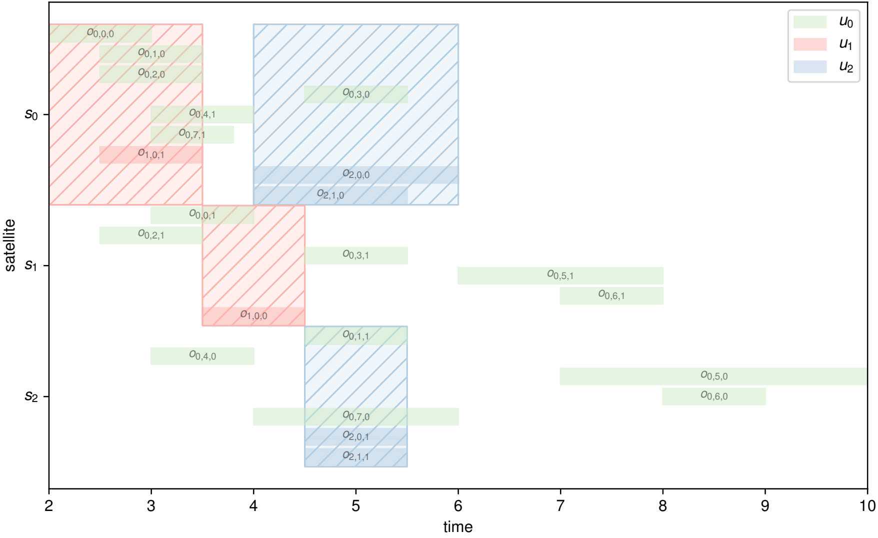


Scheduling Observations with Multiple Exclusive Orbit Portions

Illustrative Example



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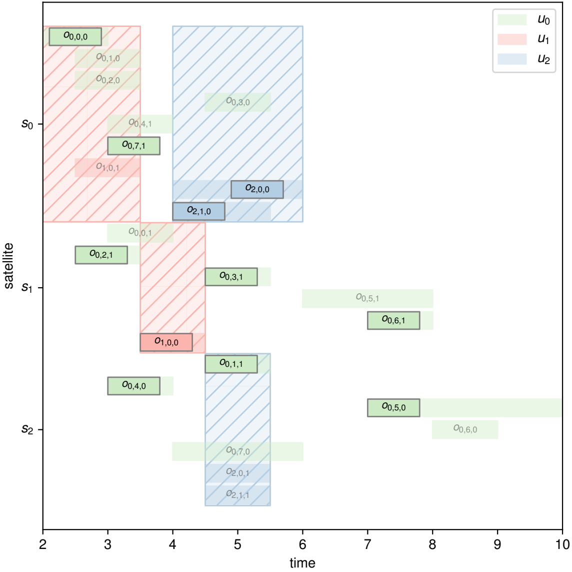

DCOP Model

A DCOP $\langle \mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{C}, \mu \rangle$ is defined for a given request r, and a current scheduling

- The agents are the exclusive users which can potentially schedule r:

$$\mathcal{A} = \{u \in \mathcal{U}^{\text{ex}} | \exists (s, (t_u^{\text{start}}, t_u^{\text{end}})) \in e_u, \exists o \in \theta_r \text{ s.t. } s_o = s, [t_u^{\text{start}}, t_u^{\text{end}}] \cap [t_o^{\text{start}}, t_o^{\text{end}}] \neq \varnothing\} \tag{1}$$

- Each agent u owns binary decision variables, one for each observation $o \in \mathcal{O}[u]^r$ and exclusive e in its exclusives e_u, stating whether it schedules o in e or not:

$$\mathcal{X} = \{x_{e,o} | e \in \bigcup_{u \in \mathcal{A}} e_u, o \in \mathcal{O}[u]^r\} \tag{2}$$

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- μ associates each variable $x_{e,o}$ to e's owner

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DCOP Model (cont.)

- Constraints should check that at most one observation is scheduled per request (4), that satellites are not overloaded (5), that at most one agent serves the same observation (6)

$$\sum_{e \in \bigcup_{u \in \mathcal{A}} e_u} x_{e,o} \leq 1, \quad \forall u \in \mathcal{X}, \forall o \in \mathcal{O}[u]^r \tag{4}$$

$$\sum_{o \in \{o \in \mathcal{O}[u]^r | u \in \mathcal{A}, s_o = s\}, e \in \bigcup_{u \in \mathcal{A}} e_u} x_{e,o} \leq \kappa_s^*, \ \forall s \in \mathcal{S} \tag{5}$$

$$\sum_{e \in \bigcup_{u \in \mathcal{A}} e_u} x_{e,o} \leq 1, \quad \forall o \in \mathcal{O} \tag{6}$$

- The cost to integrate an observation in the current user's schedule should be assessed to guide the optimization process

$$c(x_{e,o}) = \pi(o, \mathcal{M}_{u_o}), \quad \forall x_{e,o} \in \mathcal{X} \tag{7}$$

where π evaluates the best cost obtained when scheduling o and any combination of observations from $\mathcal{M}_{u_o}$, as to consider all possible revisions of u_o's current schedule

$$\mathcal{C} = \{(4), (5), (6), (7)\} \tag{8}$$

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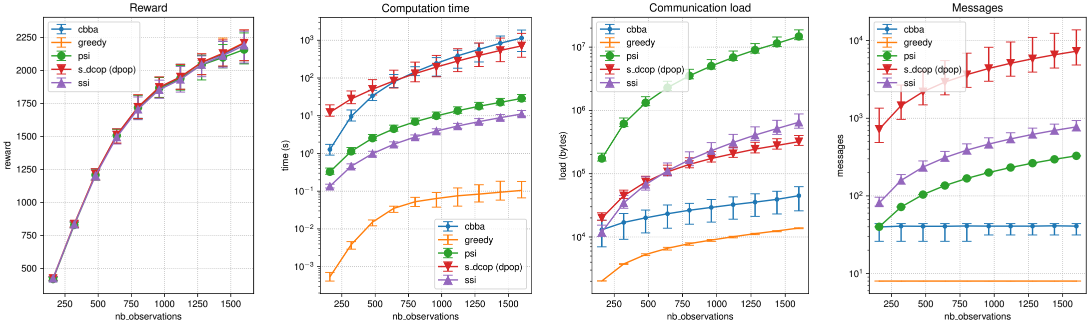
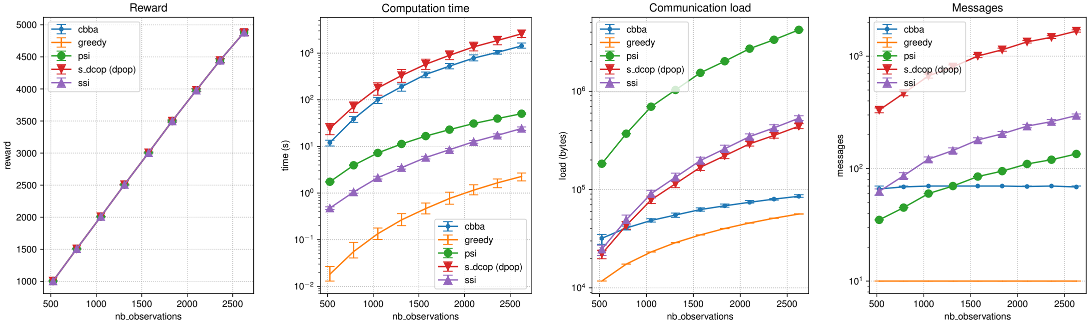
Highly conflicting randomly generated problems

5-min horizon with overlapping requests and limited capacity



Realistic randomly generated problems

6-hour horizon with numerous requests and large capacity



Today's Menu

Conclusion and Wrap-up

What We've Seen Today

- 2 major multi-agent constraint optimization frameworks: **DCOP**, **CF**
  - DCOP: how to **collectively solve** constraint optimization problems
  - CF: how to **form coalitions/groups** with respect to some criteria and constraints
- Various **techniques and algorithms** to attach these problems
- Examples of **applications** in the transportation, IoT, space and energy domain

Conclusion and Wrap-up

Open questions

Distributed constraint optimization

- How to **decompose or regroup** as to reduce **interactions**?
- How to **structure** the system as to improve **parallelism**?
- How to deploy and make systems robust and resilient in **dynamic environments**?

Coalition formation

- Which other **realistic** scenarios can we model as $m + a$?
- Can we exploit some other **properties** for scenarios that are not $m + a$ (e.g., shared mobility)?
- More in general, how can we **improve** the **scalability** of CF approaches?

Common questions

- How to use DCOPs in CF and vice versa?
- Maintaining libraries and data sets

Special Thanks

Special thanks to all previous contributors to tutorials on multi-agent optimization and related topics, notably

Ferdinando Fioretto, Long Tran-Thanh, Pierre Rust, Enrico Pontelli, William Yeoh, Jesus Cerquides, Juan Antonio Rodriguez Aguilar, Alessandro Farinelli, Pedro Meseguer, Sarvapali Ramchurn, Amnon Meisels

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