Optimization Techniques for Agent Coordination

Tutorial at AAMAS'26

Filippo Bistaffa Gauthier Picard

2026 AAMAS PAPHOS • CYPRUS 25 YEARS

IIIA Institut d'Investigació en Intel·ligència Artificial

ONERA THE FRENCH AEROSPACE LAB

Introduction and Motivations

Who are we?

Filippo Bistaffa, PhD

IIIA-CSIC, Barcelona
Expertises: coalition formation, parallel computing, shared mobility

Gauthier Picard, PhD, Hab.

ONERA, the French Aerospace Lab
Expertises: DCOPs, self-organization, resource allocation

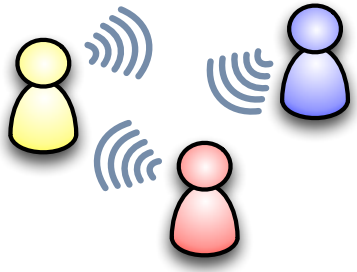
Introduction and Motivations

Multiagent Systems

- Agent: An entity that behaves autonomously in the pursuit of goals
- Multi-agent system: A system of multiple interacting agents

An agent is...

- Autonomous: Is of full control of itself
- Interactive: May communicate with other agents
- Reactive: Responds to changes in the environment or requests by other agents
- Proactive:Takes initiatives to achieve its goals



Introduction and Motivations

Research questions addressed during this tutorial

- How to make collective optimal decisions?
  - How to model the collective decision?
  - Which protocols to implement these decisions?
- How to form groups *wrt* to some utility criteria?
  - How to model the utility of each group?
  - How to express which groups are feasible or not?

Today's Menu

- Distributed Constraint Optimization
  - Preliminaries
  - DCOP Model
  - DCOP Algorithms
  - Extensions
- Auctions
  - Combinatorial Auctions
  - Relaxations
  - Decentralization
- Coalition Formation on MAS
  - Characteristic Function Games
  - Coalition Structure Generation
- Real-World Applications
  - Observation Scheduling in Multi-Owner Constellations
  - Shared Mobility
  - Multi-Agent Pick-up and Delivery
- Discussion and Wrap-up

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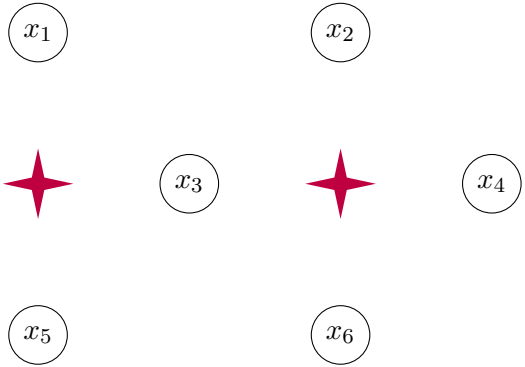
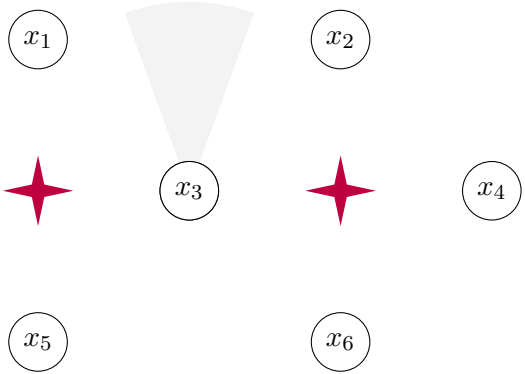
Motivating example

Sensor networks



Motivating example

Sensor networks



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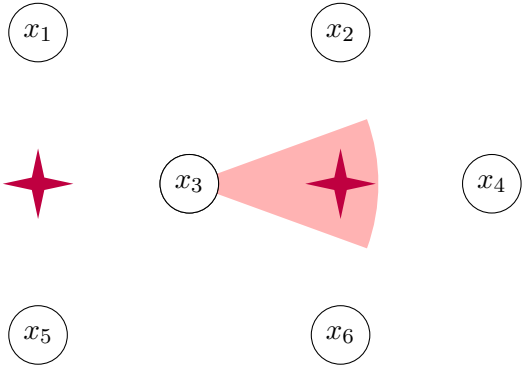

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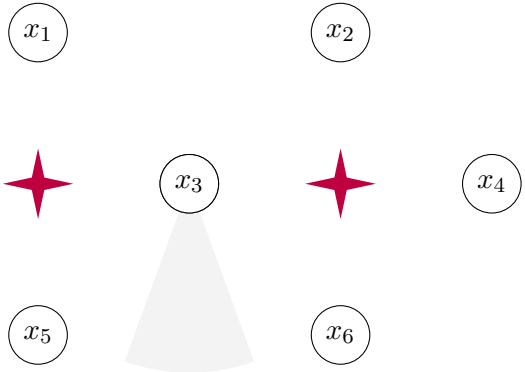

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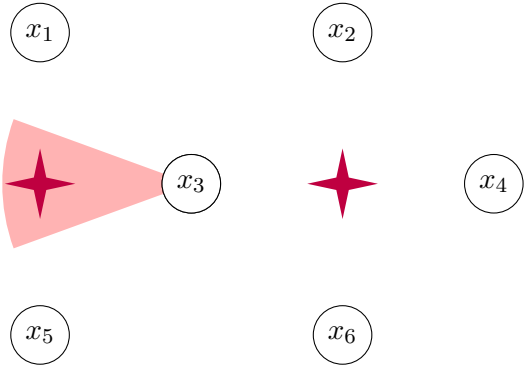

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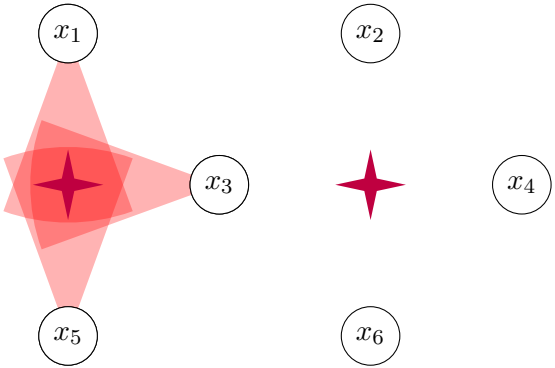

x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem as a CSP!

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CSP

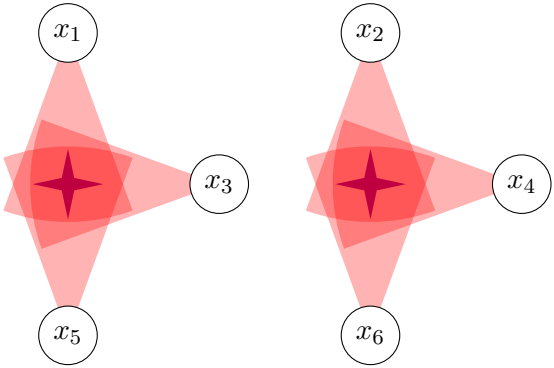
Constraint Satisfaction

- Variables $X = \{x_1, \ldots, x_n\}$
- Domains $D = \{D_1, \ldots, D_n\}$
- Constraints $C\{c_1, \ldots, c_m\}$
  where a constraint $c_i \subseteq D_{i_1} \times D_{i_2} \times \ldots \times D_{i_n}$ denotes the possible valid joint assignments for the variables $x_{i_1}, x_{i_1}, \ldots, x_{i_n}$ it involves

- **Goal**: Find an assignment to all variables that satisfies all the constraints

CSP

Constraint Satisfaction

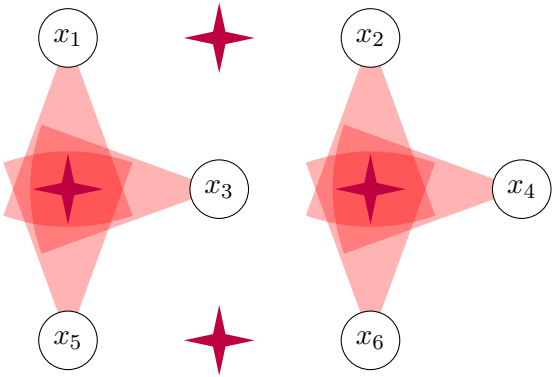


x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem as a CSP!

Max-CSP

Max Constraint Satisfaction



x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem as a Max-CSP!

Max-CSP

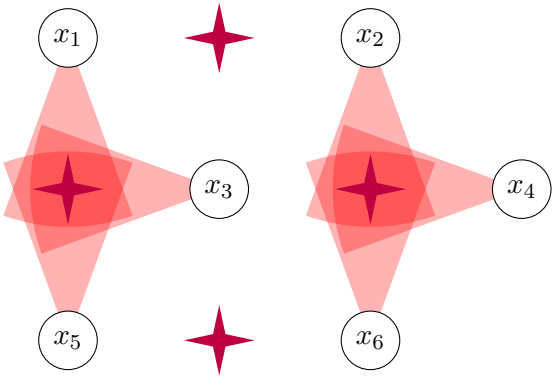
Max Constraint Satisfaction

- Variables $X = \{x_1, \ldots, x_n\}$
- Domains $D = \{D_1, \ldots, D_n\}$
- Constraints $C\{c_1, \ldots, c_m\}$
  where a constraint $c_i \subseteq D_{i_1} \times D_{i_2} \times \ldots \times D_{i_n}$ denotes the possible valid joint assignments for the variables $x_{i_1}, x_{i_1}, \ldots, x_{i_n}$ it involves

- **Goal**: Find an assignment to all variables that satisfies a maximum number of constraints

Max-CSP

Max Constraint Satisfaction

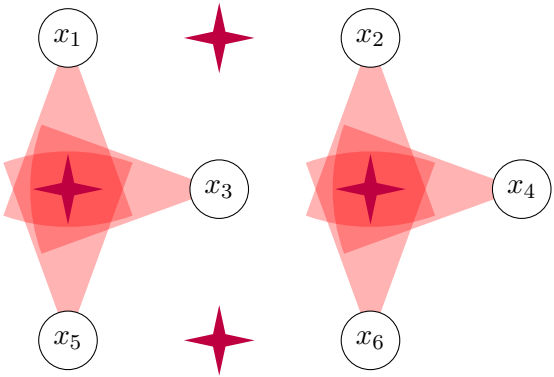


x_1	x_3	x_5	Sat?
N	N	N	✗
N	N	E	✗
	...		✗
S	W	N	✓
	...		✗
W	W	W	✗

Model the problem as a Max-CSP!

WCSP (or COP)

Constraint Optimization



x_1	x_3	x_5	Cost
N	N	N	∞
N	N	E	∞
	...		∞
S	W	N	10
	...		∞
W	W	W	∞

Model the problem as a COP!

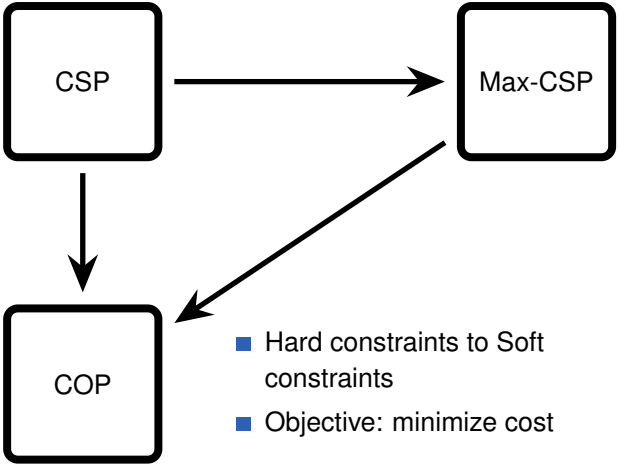
WCSP (or COP)

Constraint Optimization

- Variables $X = \{x_1, \ldots, x_n\}$
- Domains $D = \{D_1, \ldots, D_n\}$
- Constraints $C\{c_1, \ldots, c_m\}$
  where a constraint $c_i : D_{i_1} \times D_{i_2} \times \ldots \times D_{i_n} \rightarrow \mathbb{R}_+ \cup \{\infty\}$ expresses the degree of constraint violation

- **Goal**: Find an assignment to all variables that minimizes the sum of all the constraints

Constraint Reasoning

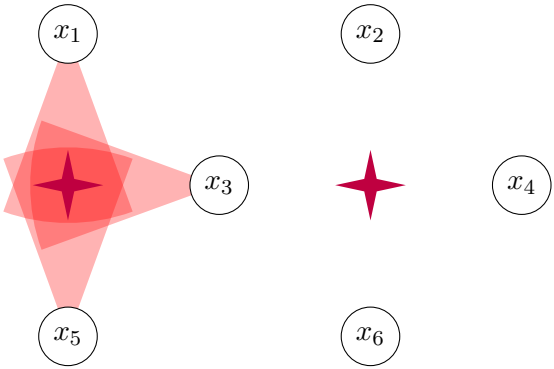


- Objective: maximize #constraints satisfied

- Hard constraints to Soft constraints
- Objective: minimize cost

WCSP (or COP)

Constraint Optimization



Imagine that each sensor is an autonomous agent

How should this problem be modeled and solved in a decentralized manner?

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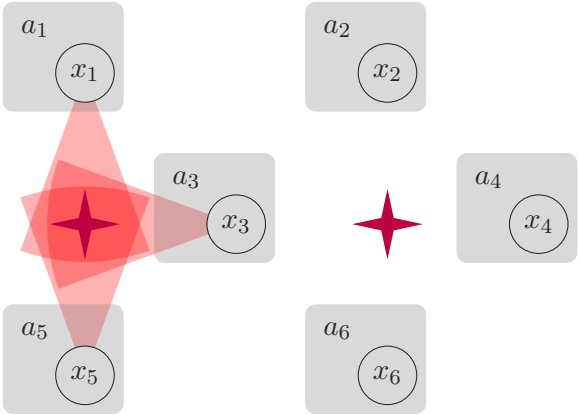
DCOP

Distributed Constraint Optimization [Modi et al., 2005]



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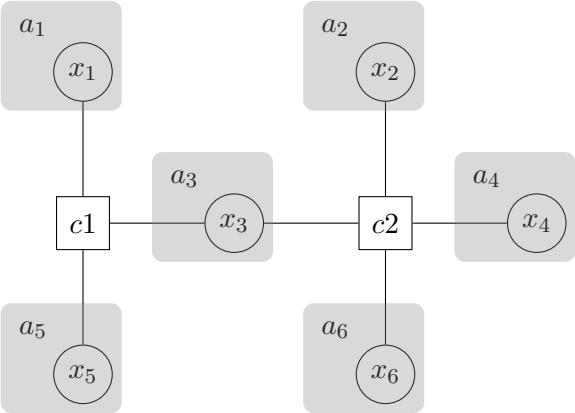


DCOP

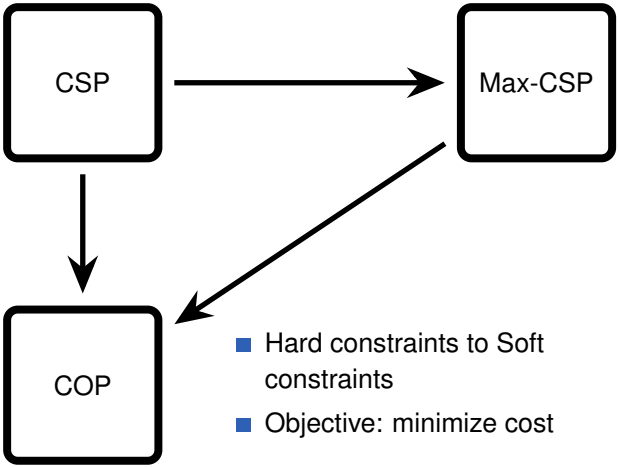
Distributed Constraint Optimization [MODI et al., 2005]

- Agents $X = \{a_1, \ldots, a_l\}$
- Variables $X = \{x_1, \ldots, x_n\}$
- Domains $D = \{D_1, \ldots, D_n\}$
- Constraints $C\{c_1, \ldots, c_m\}$
- Mapping of variables to agents

- **Goal**: Find an assignment to all variables that minimizes the sum of all the constraints

DCOP

Distributed Constraint Optimization [Modi et al., 2005]



- Objective: maximize #constraints satisfied

- Hard constraints to Soft constraints
- Objective: minimize cost

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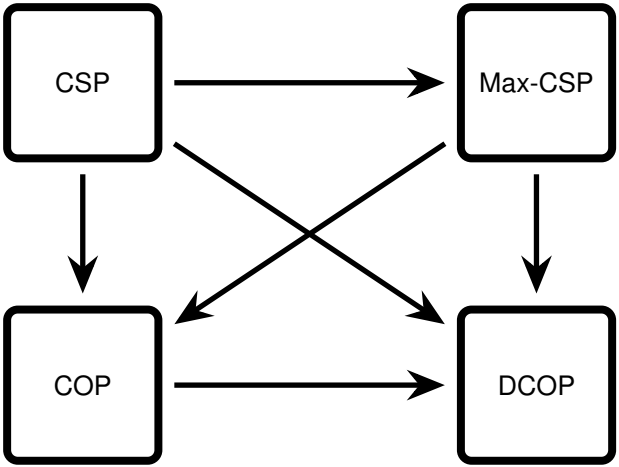

- Variables are controlled by agents
- Communication model
- Local knowledge

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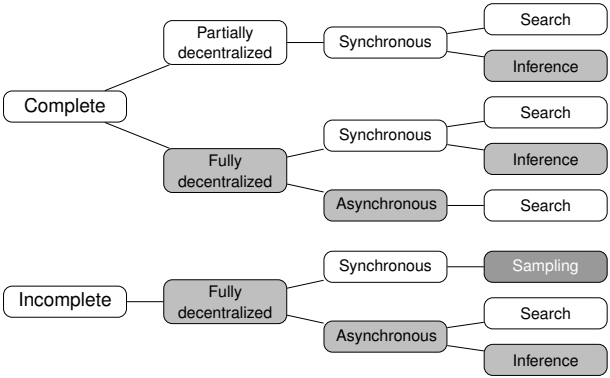
DCOP Algorithms

See [FIORETTO et al., 2018]



DCOP Algorithms

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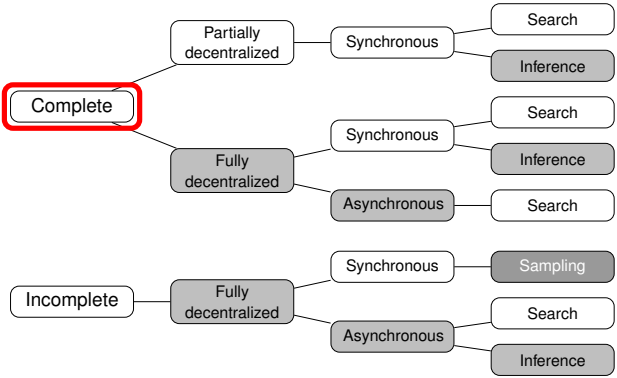
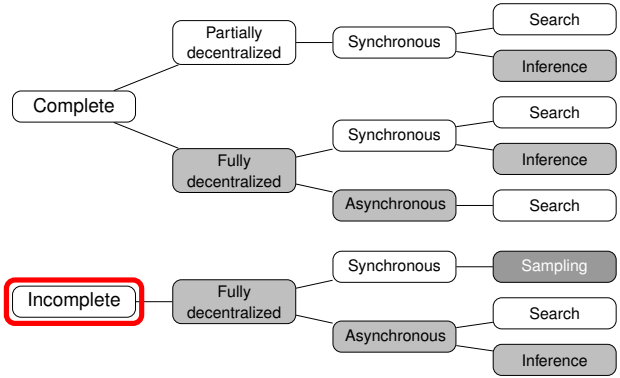


Important metrics

- Agent complexity
- Network loads
- Message size

DCOP Algorithms

See [FIORETTO et al., 2018]



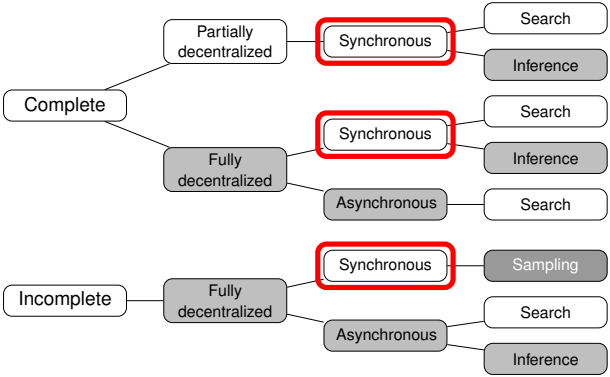
Important metrics

- Agent complexity
- Network loads
- Message size

- Anytime
- Quality guarantees
- Execution time vs. solution quality

DCOP Algorithms

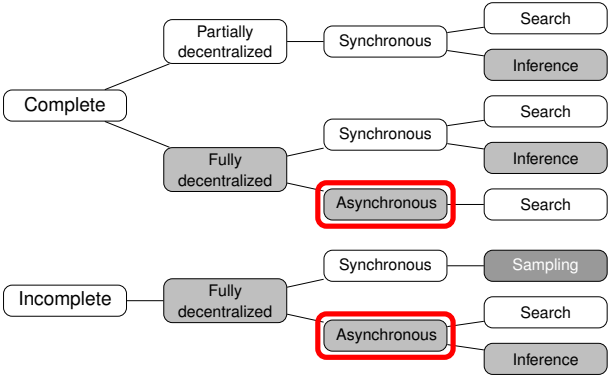
See [FIORETTO et al., 2018]



- Systematic process, divided in steps
- Each agent waits for particular messages before acting
- Consistent view of the search process
- Typically, increases idle-time

DCOP Algorithms

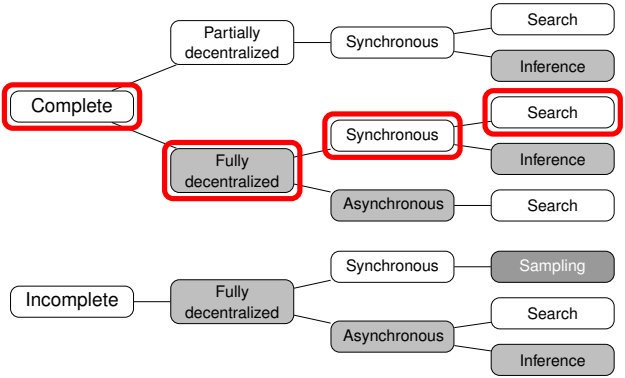
See [FIORETTO et al., 2018]



- Decision based on agents' local state
- Agents' actions do not depend on sequence of received messages
- Minimizes idle-time
- No guarantees on validity of local views

DCOP Algorithms

See [FIORETTO et al., 2018]



Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]

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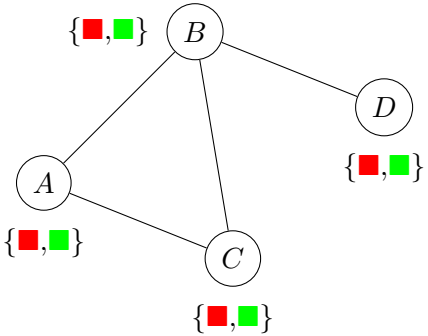
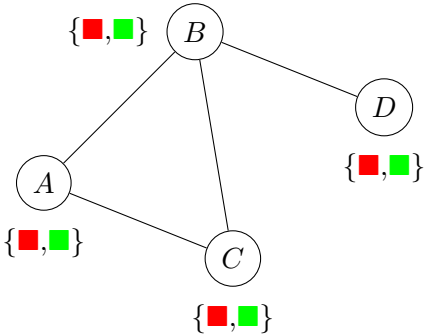


x_i	x_j	(A,B)	(A,C)	(B,C)	(B,C)
■	■	5	5	5	3
■	■	8	10	4	8
■	■	20	20	3	10
■	■	3	3	3	3

How do we solve this distributedly?

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]



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■	■	20	20	3	10
■	■	3	3	3	3

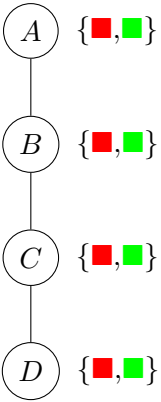
How do we solve this distributedly?

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]

- Agents operate on a complete ordering
- Agents exchange CPA messages containing partial assignments
- When a solution is found, its solution cost as an UB is broadcasted to all agents
- The UB is used for branch pruning

Complete ordering



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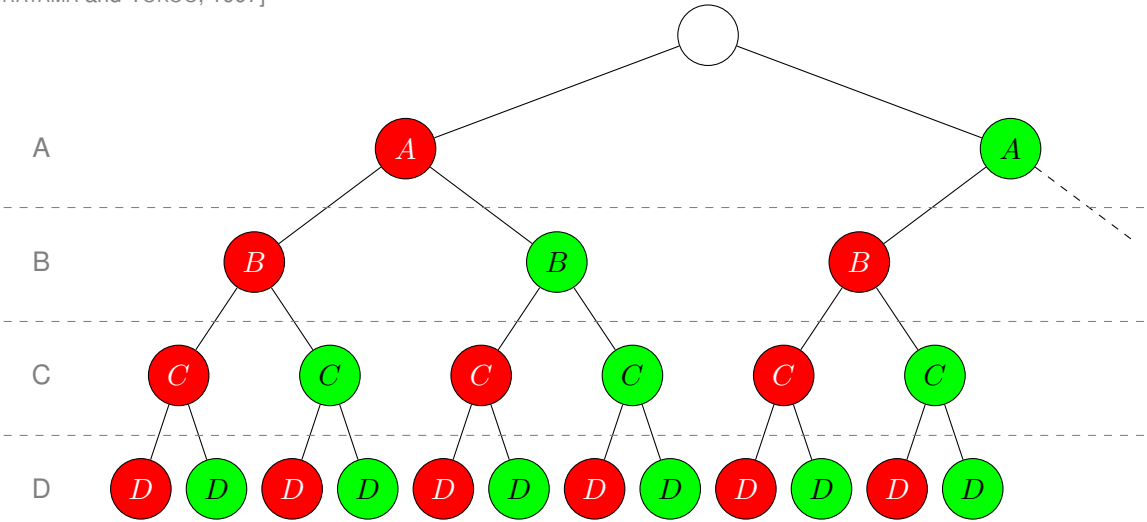

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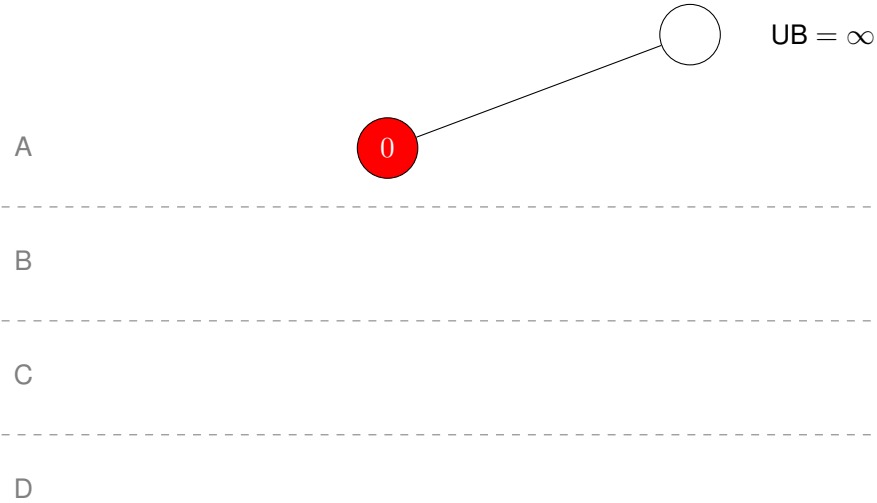

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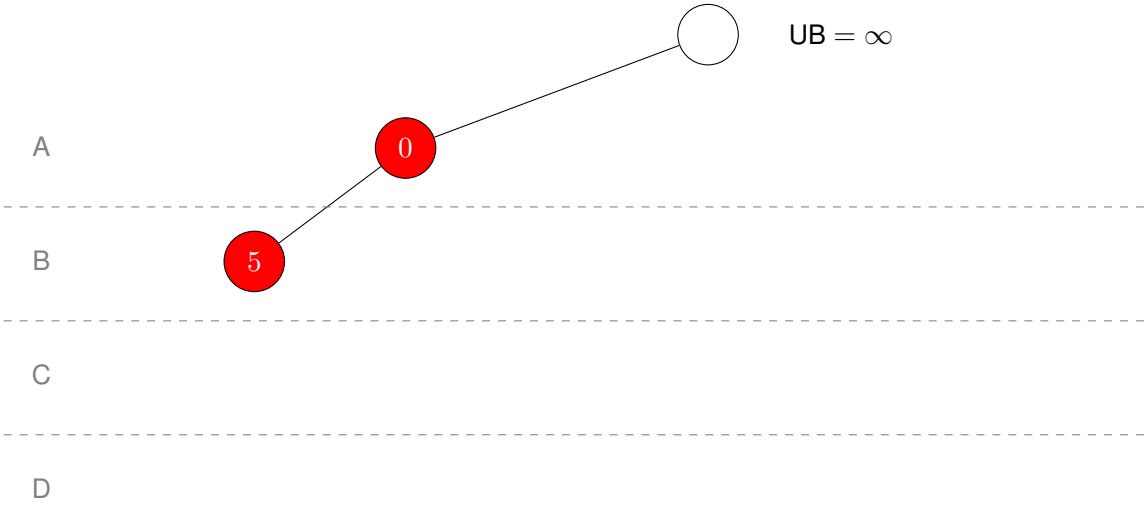

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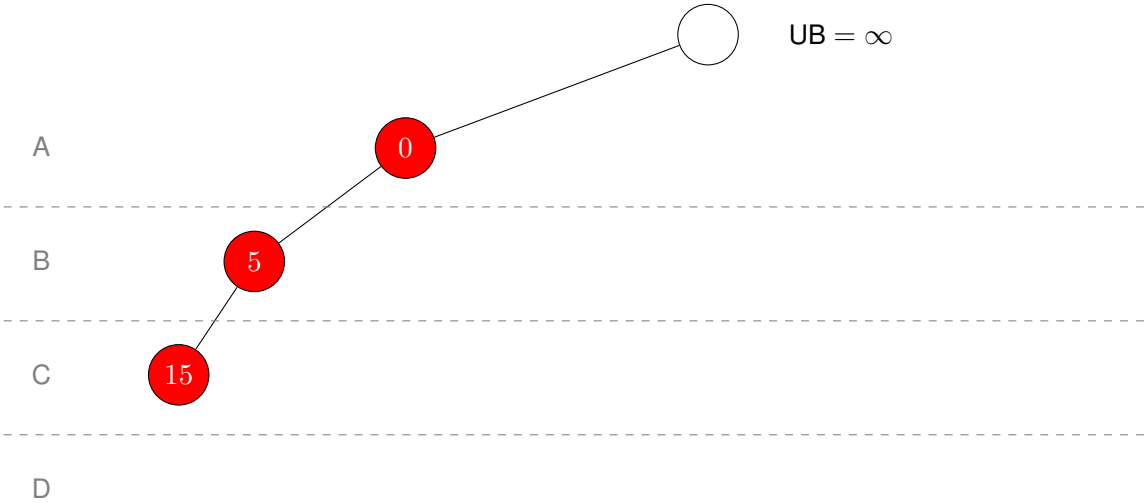

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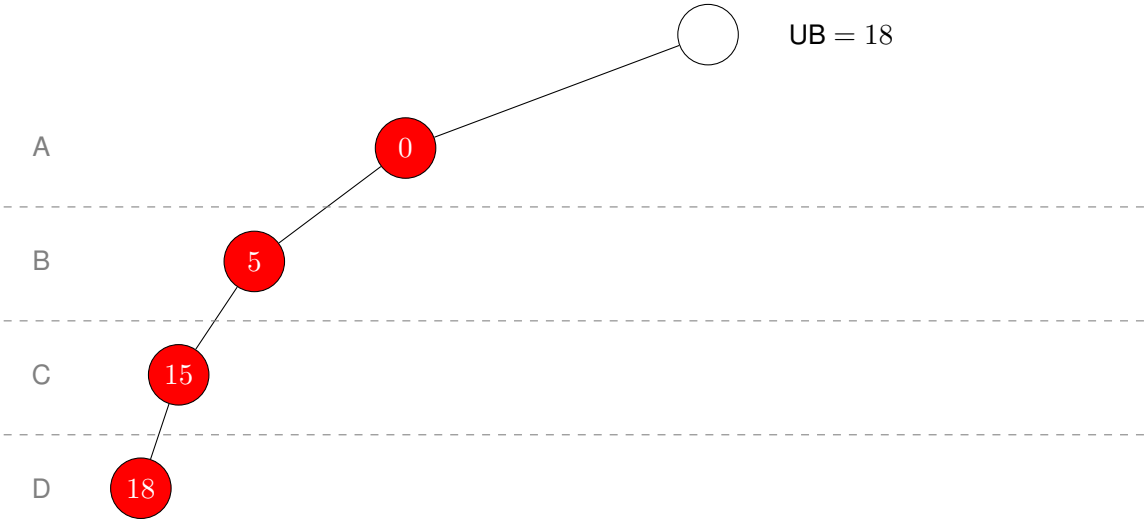

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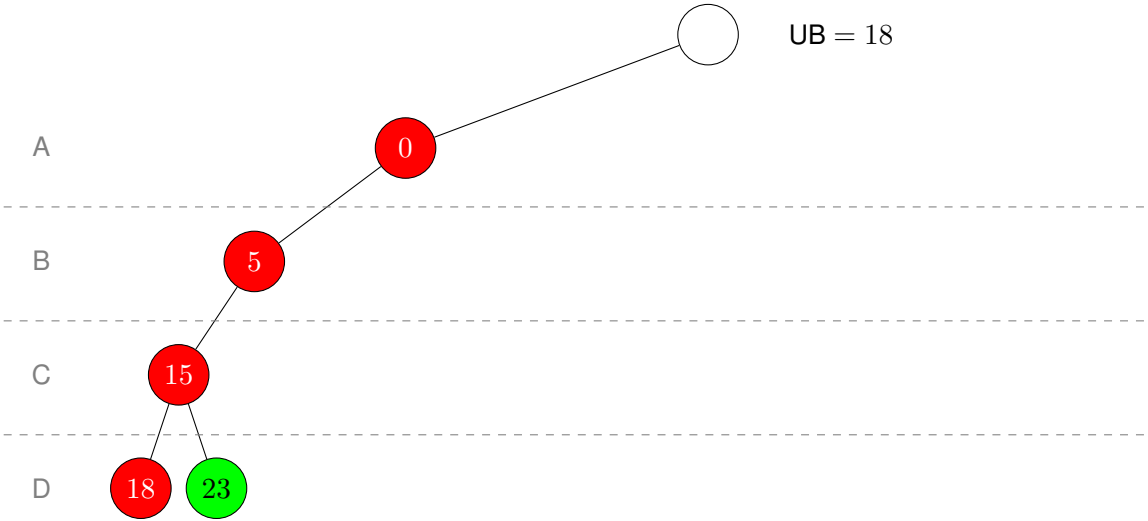

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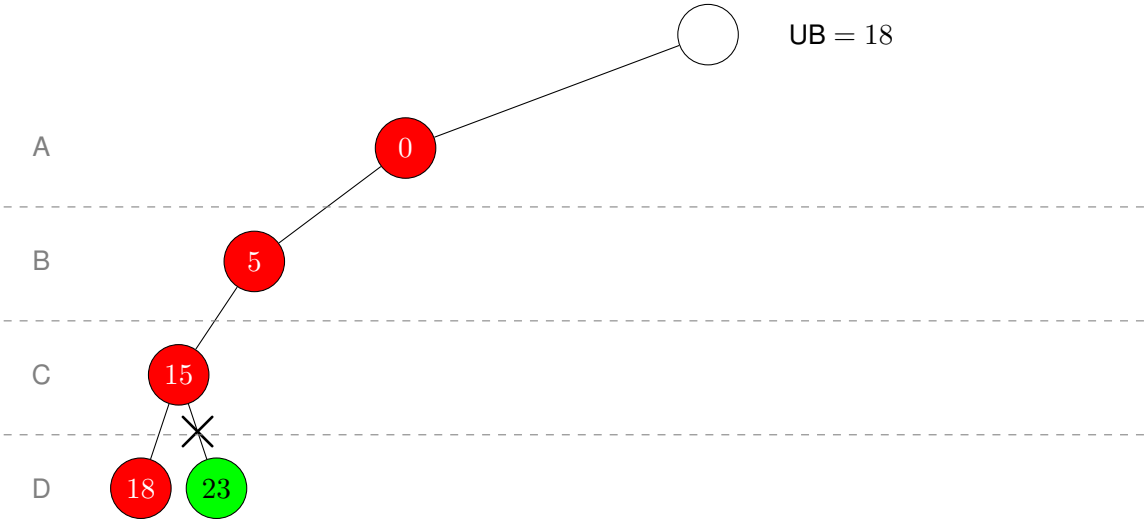

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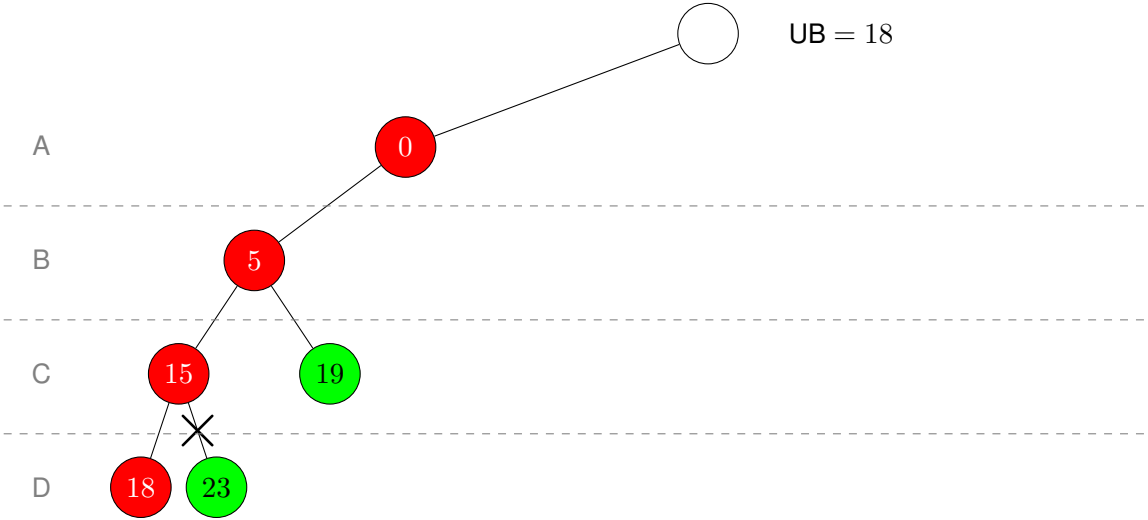

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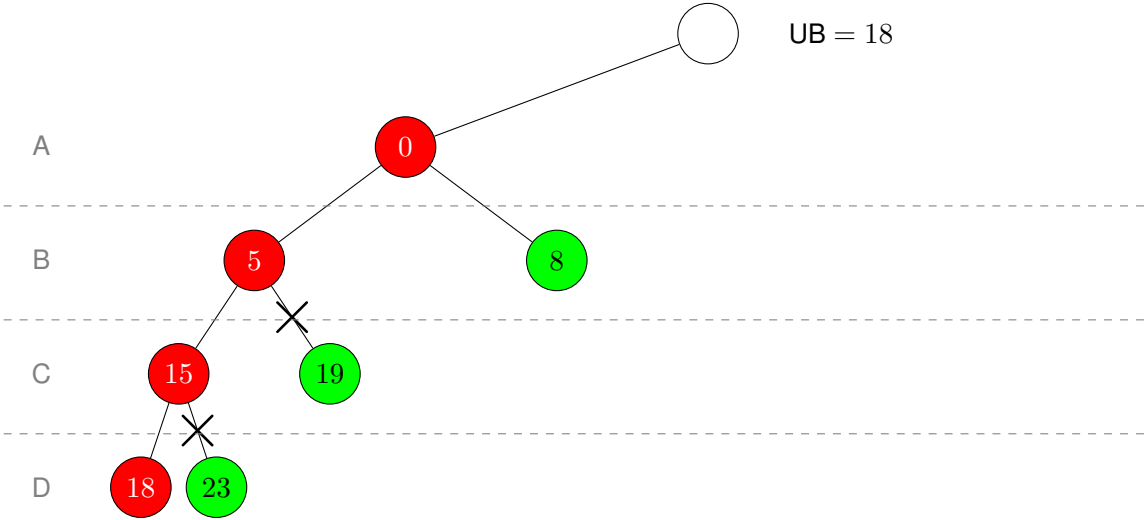

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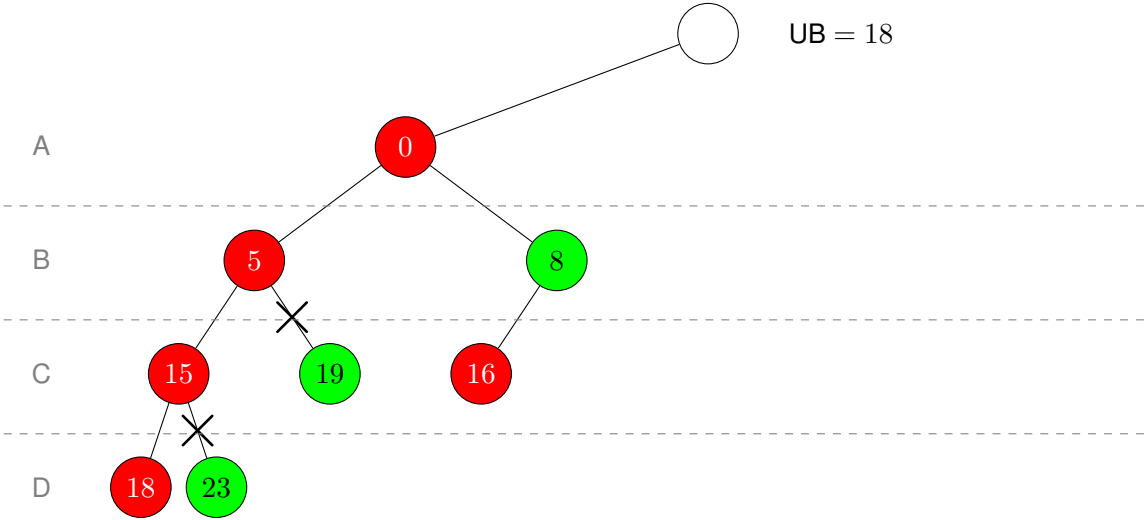

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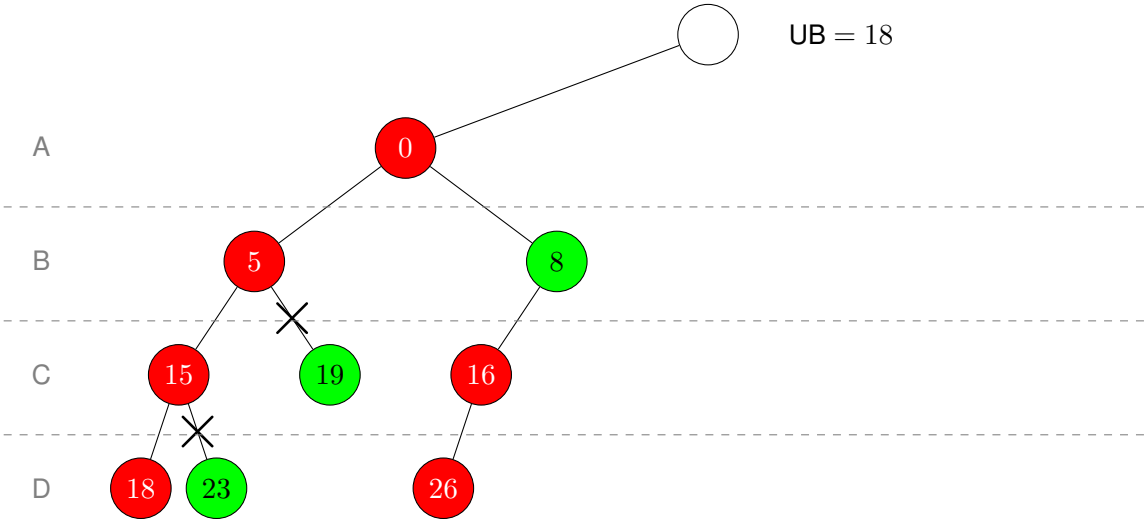

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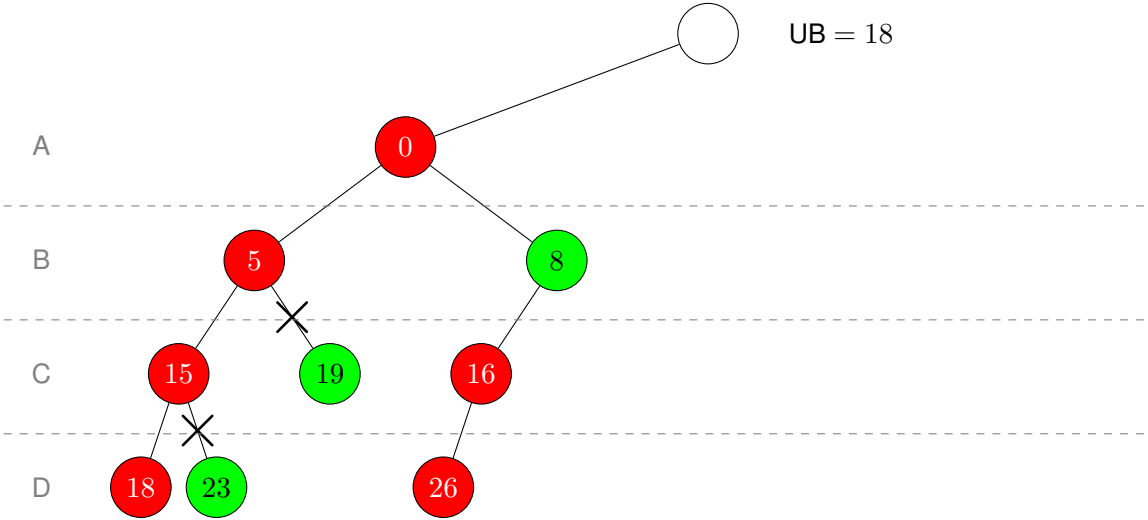

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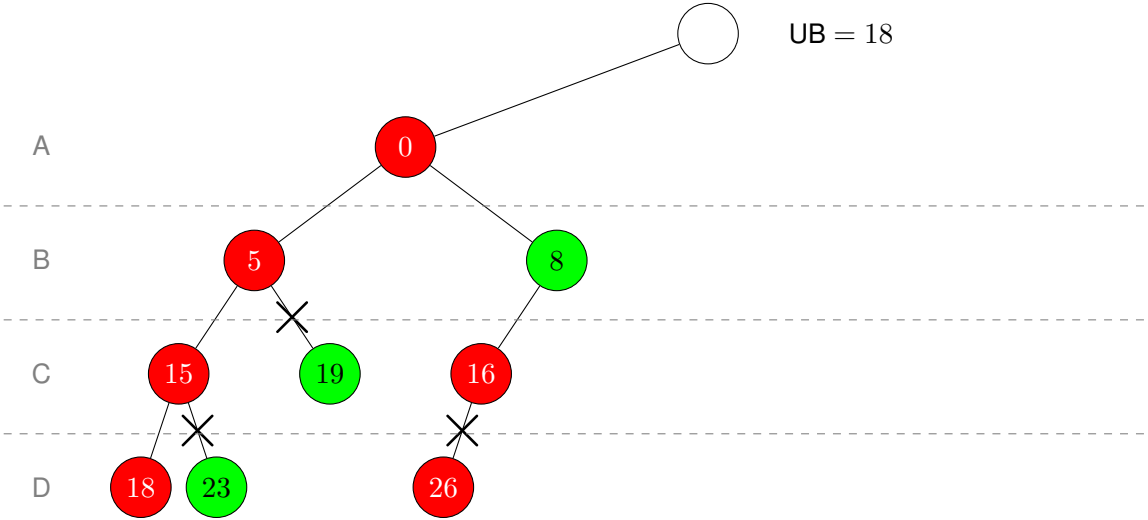

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]

	SBB
Correct the solution it finds is optimal	Yes
Complete it terminates	Yes
Message complexity max size of messages	$\mathcal{O}(d)$
Network load max number of messages	$\mathcal{O}(b^d)$
Runtime how long it takes	$\mathcal{O}(b^d)$

branching factor = b
num variables = d

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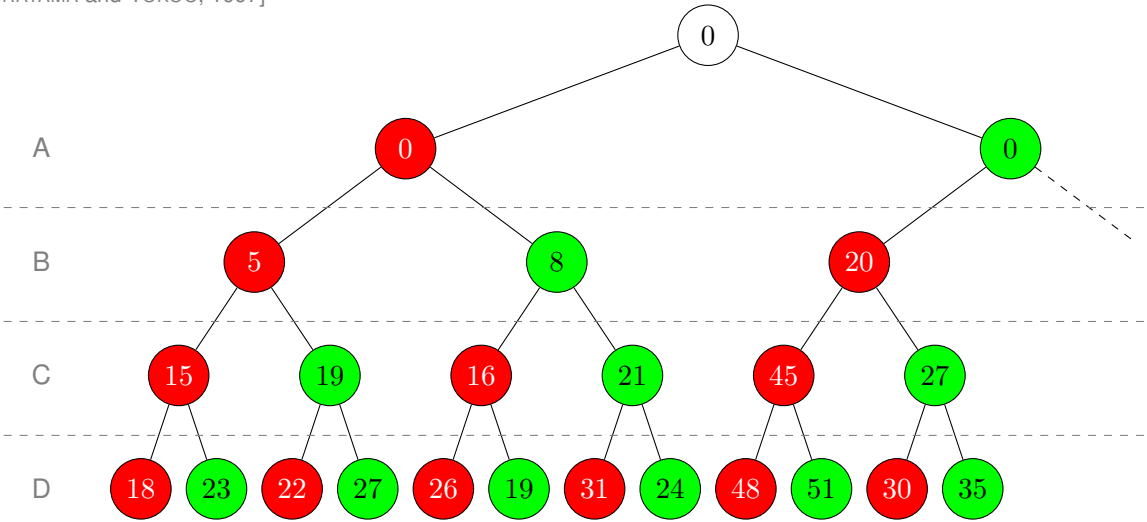

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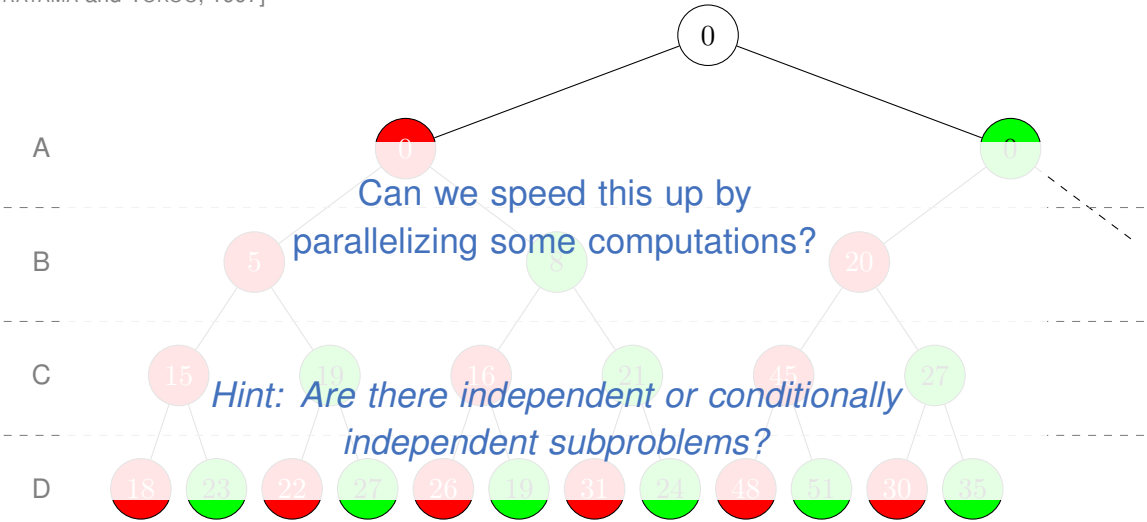
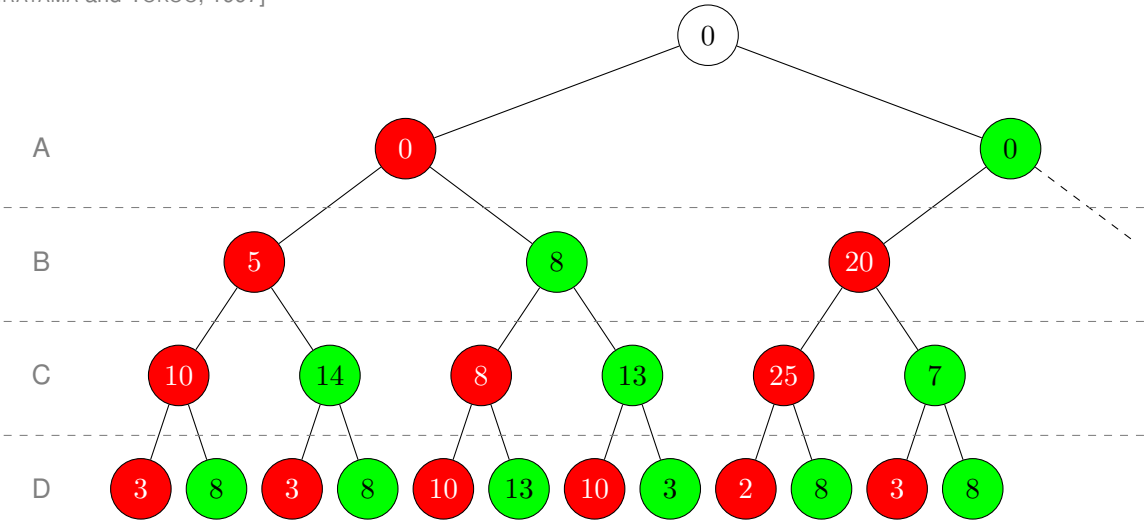


Can we speed this up by parallelizing some computations?

Hint: Are there independent or conditionally independent subproblems?

Synchronous Branch-and-Bound (SBB)

[HIRAYAMA and YOKOO, 1997]



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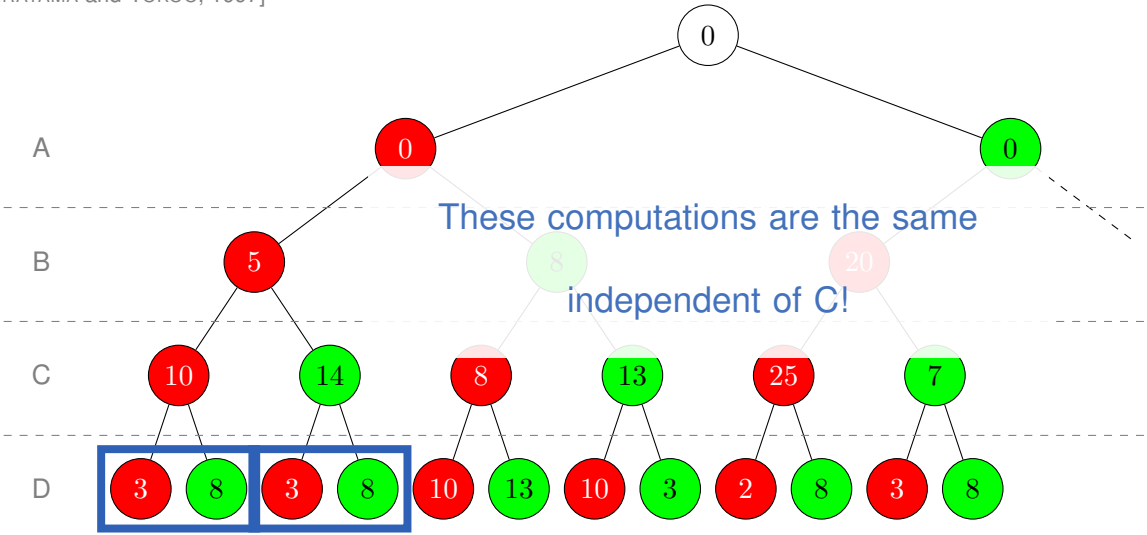
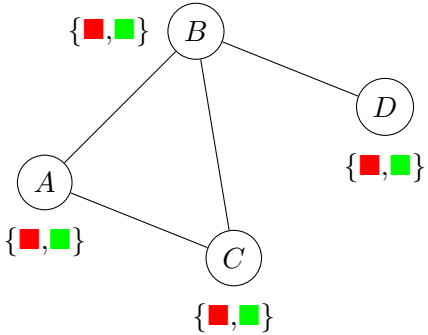
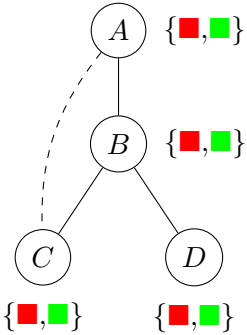


Pseudo-Tree

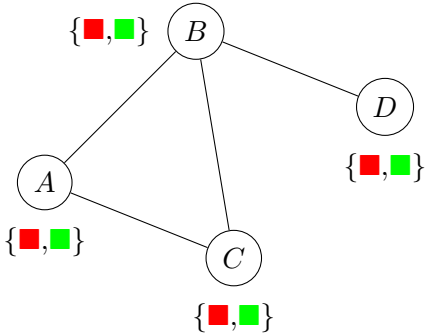


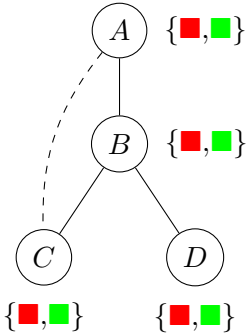


Definition (Pseudo-Tree)

A spanning tree of the constraint graph such that no two nodes in sibling subtrees share a constraint in the constraint graph

Pseudo-Tree





Definition (Pseudo-Tree)

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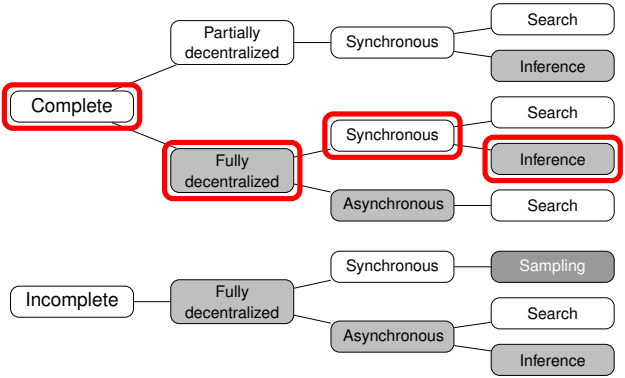
DCOP Algorithms

See [FIORETTO et al., 2018]



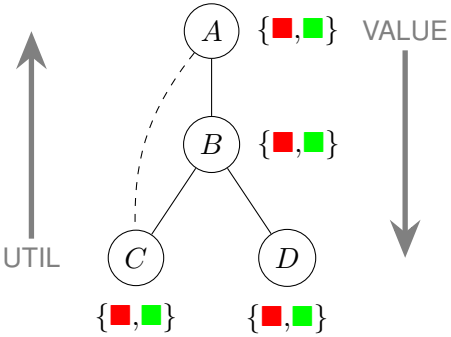
Distributed Pseudotree Optimization Procedure (DPOP)

[PETCU and FALTINGS, 2005b]

DPOP

[PETCU and FALTINGS, 2005b]

- Extension of the Bucket Elimination (BE)
- Agents operate on a pseudo-tree ordering
- UTIL phase: Leaves to root
- VALUE phase: Root to leaves



DPOP

[PETCU and FALTINGS, 2005b]

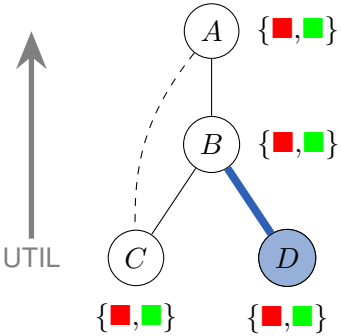
B	D	(B, D)
r	r	3
r	g	8
g	r	10
g	g	3

$\min\{3, 8\} = 3$

$\min\{10, 3\} = 3$

Message to B

B	cost
r	3
g	3



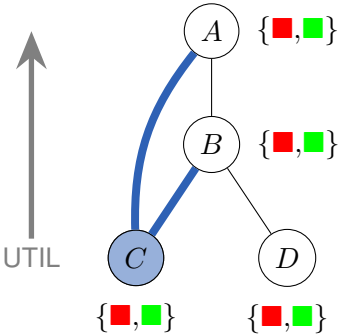
DPOP

[PETCU and FALTINGS, 2005b]

A	B	C	(B,C)	(A,C)	cost
r	r	r	5	5	10
r	r	g	4	8	12
r	g	r	3	5	8
r	g	g	3	8	11
g	r	r	5	10	15
g	r	g	4	3	7
g	g	r	3	10	13
g	g	g	3	3	6

Message to B

A	B	cost
r	r	10
r	g	8
g	r	7
g	g	6



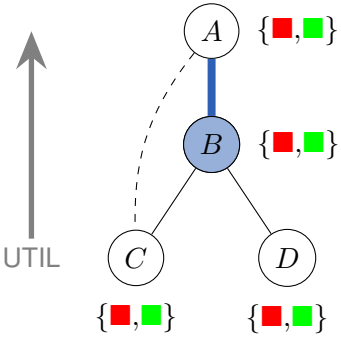
DPOP

[PETCU and FALTINGS, 2005b]

A	B	(A, B)	Util C	Util D	cost
r	r	5	10	53	18
r	g	8	8	3	19
g	r	20	7	3	30
g	g	3	6	3	12

Message to A

A	cost
r	18
g	12

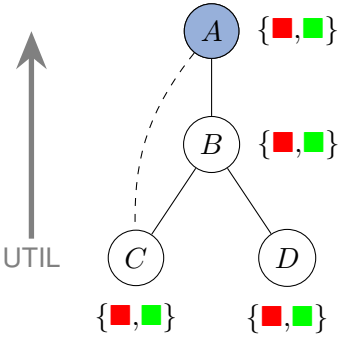


DPOP

[PETCU and FALTINGS, 2005b]

A	cost
r	18
g	12

optimal cost $= 12$



DPOP

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A	cost
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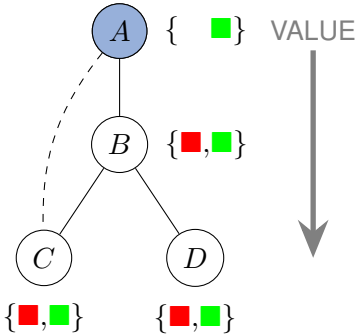
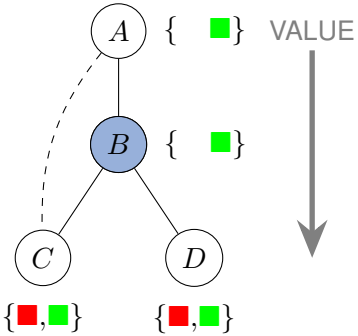


- Select value for $A = g$
- Send MSG "$A = g$" to agents B and C

DPOP

[PETCU and FALTINGS, 2005b]

A	B	(A, B)	Util C	Util D	cost
r	r	5	10	53	18
r	g	8	8	3	19
g	r	20	7	3	30
g	g	3	6	3	12



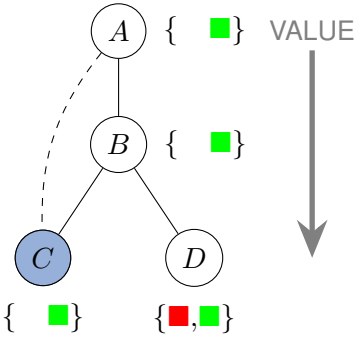
- Select value for $B = g$
- Send MSG "$B = g$" to agents C and D

DPOP

[PETCU and FALTINGS, 2005b]

A	B	C	(B,C)	(A,C)	cost
r	r	r	5	5	10
r	r	g	4	8	12
r	g	r	3	5	8
r	g	g	3	8	11
g	r	r	5	10	15
g	r	g	4	3	7
g	**g**	r	3	10	13
g	**g**	g	3	3	6

- Select value for $C = g$



DPOP

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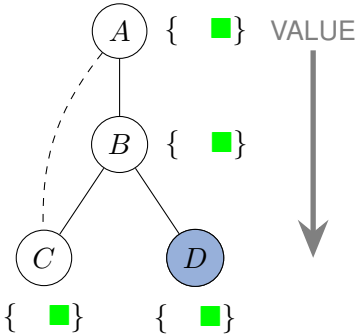

- Select value for $D = g$

DPOP

[PETCU and FALTINGS, 2005b]

	SBB	DPOP
Correct the solution it finds is optimal	Yes	Yes
Complete it terminates	Yes	Yes
Message complexity max size of messages	$\mathcal{O}(d)$	$\mathcal{O}(b^d)$
Network load max number of messages	$\mathcal{O}(b^d)$	$\mathcal{O}(d)$
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branching factor = b
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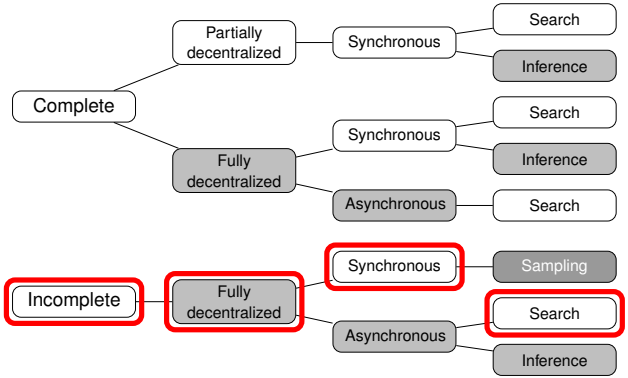
DCOP Algorithms

See [FIORETTO et al., 2018]



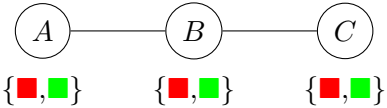
Distributed Local Search

[MAHESWARAN et al., 2004; ZHANG et al., 2003]

Local Search Algorithms

- DSA: Distributed Stochastic Search [ZHANG et al., 2005]
- MGM: Maximum Gain Messages Algorithm [MAHESWARAN et al., 2004]
- Note: we now maximize utilities
- Every agent individually decides whether to change its value or not
- Decision involves
  - knowing neighbors' values
  - calculation of utility gain by changing values
  - probabilities

x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8



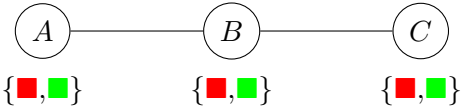
DSA Algorithm

[ZHANG et al., 2005]

- All agents execute the following
  - Randomly choose a value
  - while (termination is not met)
    - if (a new value is assigned): send the new value to neighbors
    - collect neighbors' new values if any
    - select and assign the next value based on assignment rule

DSA Algorithm

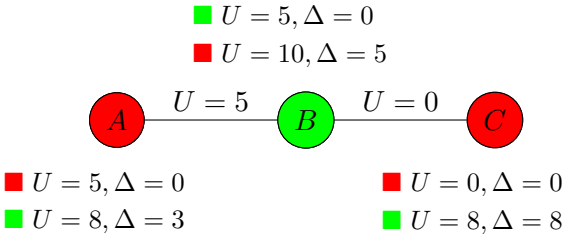
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
■	■	5	5
■	■	5	0
■	■	0	0
■	■	8	8

DSA Algorithm

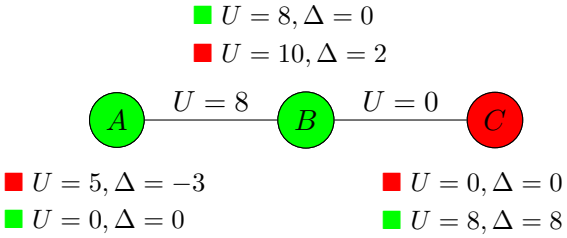
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

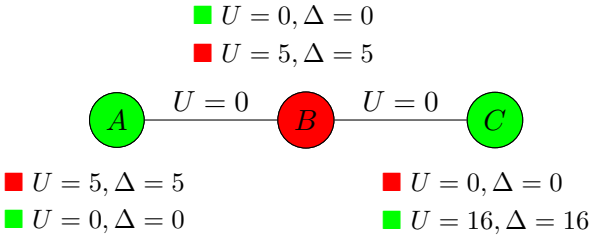
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

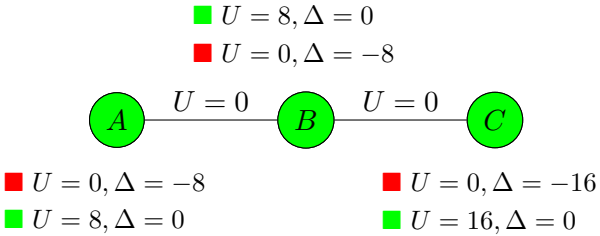
[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

DSA Algorithm

[ZHANG et al., 2005]



x_i	x_j	(A, B)	(B, C)
		5	5
		5	0
		0	0
		8	8

MGM Algorithm

[MAHESWARAN et al., 2004]

- All agents execute the following
  - Randomly choose a value
  - while (termination is not met)
    - if (a new value is assigned): send the new value to neighbors
    - collect neighbors' new values if any
    - calculate gain and send it to neighbors
    - collect neighbors' gains
    - if (it has the highest gain among all neighbors): change value to the value that maximizes gain

Large Great if you need an anytime algorithm!

MGM Algorithm

[MAHESWARAN et al., 2004]

- All agents execute the following
  - Randomly choose a value
  - while (termination is not met)
    - if (a new value is assigned): send the new value to neighbors
    - collect neighbors' new values if any
    - calculate gain and send it to neighbors
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Large Great if you need an anytime algorithm!

MGM vs DSA

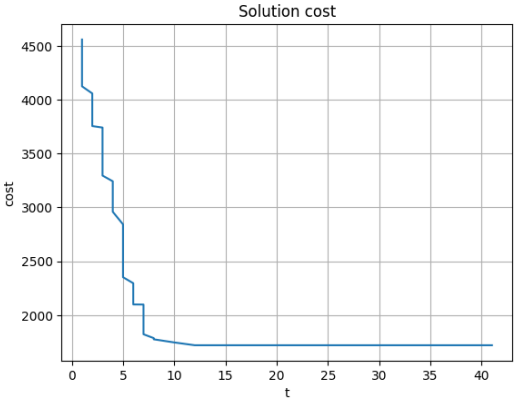


Figure: MGM

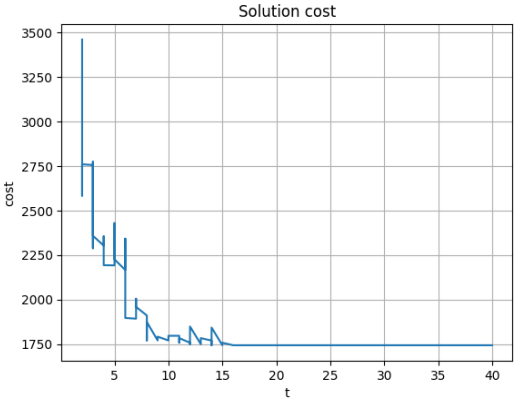


Figure: DSA

Today's Menu

Distributed Constraint Optimization
- Preliminaries
- DCOP Model
- DCOP Algorithms
- Extensions

Auctions

Coalition Formation on MAS

Real-World Applications

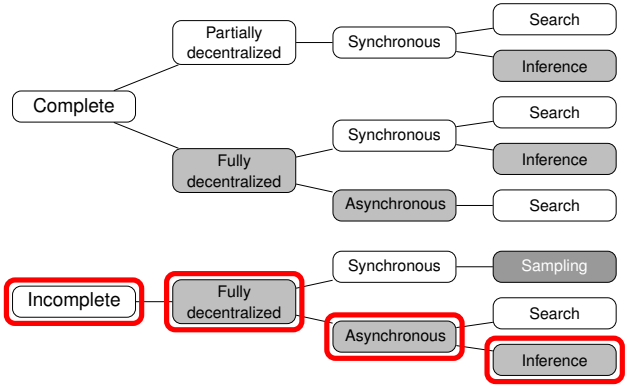
Discussion and Wrap-up

Extensions to the DCOP Framework

- Dynamic DCOPs
  - SDPOP [Petcu and Faltings, 2005a], I-ADOPT and I-BnB-ADOPT [Yeoh et al., 2011], FMS [Ramchurn et al., 2010]
- Multi-Objective DCOPs
  - MO-SBB [Medi et al., 2014], Pseudo-tree Based Algorithm [Matsui et al., 2012], B-MOMS [Delle Fave et al., 2011], DP-AOF [Okimoto et al., 2013]
- Asymetric DCOPs
  - SyncABB-2ph, SyncABB-1ph, ACLS, MCS-MGM [Grinshpoun et al., 2013]
- Probabilistic DCOPs
  - $\mathbb{E}$[DPOP] and SD-DPOP [Léauté and Faltings, 2011; Nguyen et al., 2012], U-GDL [Stranders et al., 2011]
- Continuous DCOPs
  - CMS [Stranders et al., 2009], HCMS [Voice et al., 2010], PFD [Choudhury et al., 2020], EC-DPOP, AC-DPOP, CAC-DPOP, C-DSA [Hoang et al., 2020], C-CoCoA [Sarker et al., 2021]
- ...

Deeper Focus on Max-Sum

See [FIORETTO et al., 2018]



Distributed Inference, Max-Sum

[FARINELLI et al., 2008]

Today's Menu

Distributed Constraint Optimization

Auctions
- Combinatorial Auctions
- Relaxations
- Decentralization

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Discussion and Wrap-up

Today's Menu

Distributed Constraint Optimization

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Discussion and Wrap-up

What is an Auction?

[LEYTON-BROWN et al., 2009]

"An *auction* is a protocol that allows agents to indicate their *interests* in one or more *resources* and that uses these indications of interest to determine both an *allocation* of resources and a set of *payments* by the agents."

Where are Auctions used Nowadays?

[LEYTON-BROWN et al., 2009]

- Resource allocation
  - Treasury auctions
  - Right to drill oil, off-shore oil lease
  - Use the EM spectrum
  - Private and public goods and services acquisition
  - Internet auctions
- Market based computing
  - Production control
  - Robot navigation
  - Sensor networks

Single-Item Auctions

Single-Item Auction Protocols

English Auction

The auctioneer announces a suggested opening bid, accepting increasingly higher bids from bidders interested in the item

Japanese Auction

An initial price is displayed, and all interested bidders enter the auction arena; a bidder exits the arena when no longer interested

Dutch Auction

The auctioneer begins with a high asking price, and lowers it until some participant accepts the price, or it reaches a predetermined reserve price

Winner Determination Problem (WDP)

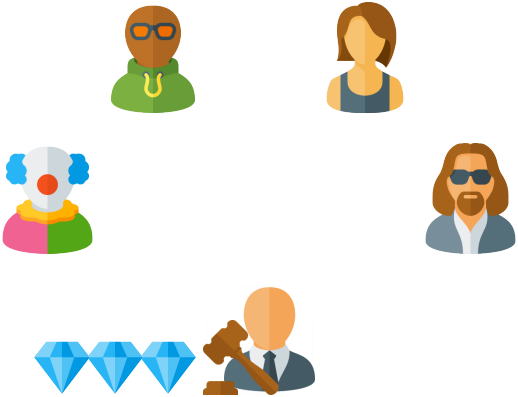
Objective

Given a set of bids, allocate the good to the bidder whose bid *maximizes* the auctioneer's revenue

WDPs for Single-Item Auctions are Easy

- English: last bid wins
- Japanese: last remaining bidder wins
- Dutch: first bid wins

Multi-Unit Auctions



WDP for Multi-Unit Auctions

Example of a Multi-Unit Auction

We want to sell 15 apples maximising the revenue

What it the Optimal Allocation with these Bids?

- A: buy 12 apples for 4€ $(V_A(\{🍎, 🍎, 🍎, 🍎, 🍎, 🍎, 🍎, 🍎, 🍎, 🍎, 🍎, 🍎\}) = 4€)$
- B: buy 2 apples for 2€ $(V_B(\{🍎, 🍎\}) = 2€)$
- C: buy 1 apple for 2€ $(V_C(\{🍎\}) = 2€)$
- D: buy 1 apple for 1€ $(V_D(\{🍎\}) = 1€)$
- E: buy 4 apples for 10€ $(V_E(\{🍎, 🍎, 🍎, 🍎\}) = 10€)$

WDP as a Weighted Knapsack Problem (WKP)

- Goal of WKP: maximize the revenue obtained by "filling the knapsack"
- Let x_A, x_B, x_C, x_D, x_E be decision variables (One binary variable for each bid)

Integer *Linear* Programming (ILP) Formulation

$$
\begin{aligned}
\text{maximize} \quad & 4 \cdot x_A + 2 \cdot x_B + 2 \cdot x_C + x_D + 10 \cdot x_E && \text{(Values of accepted bids)} \\
\text{subject to} \quad & 12 \cdot x_A + 2 \cdot x_B + x_C + x_D + 4 \cdot x_E \leq 15 && \text{("Capacity" constraint)} \\
& x_A, x_B, x_C, x_D, x_E \in \{0, 1\} && \text{(Binary decision variables)}
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$$

WDP as a Weighted Knapsack Problem (WKP)

- Goal of WKP: maximize the revenue obtained by “filling the knapsack”
- Let x_A, x_B, x_C, x_D, x_E be decision variables (One binary variable for each bid)

Integer *Linear* Programming (ILP) Formulation

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Multi-Item (Combinatorial) Auctions

[LEYTON-BROWN et al., 2009]

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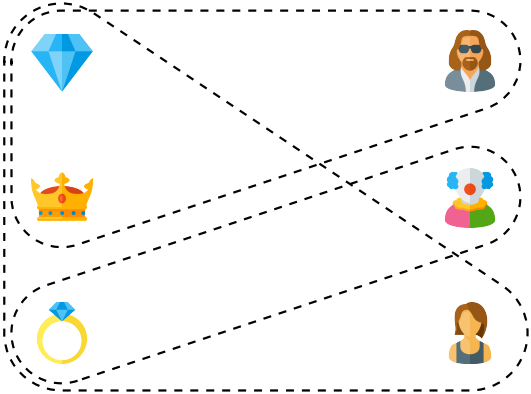

Multi-item bids

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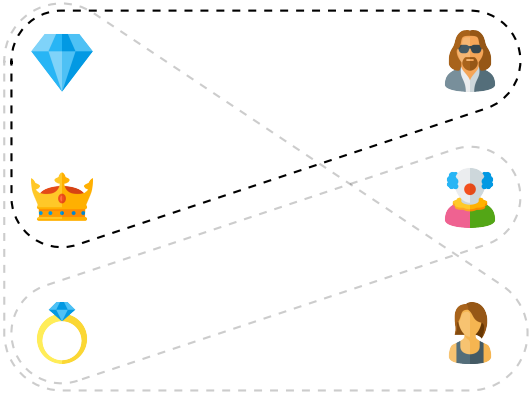

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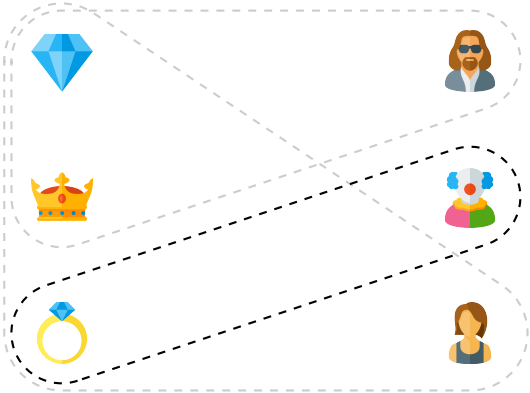

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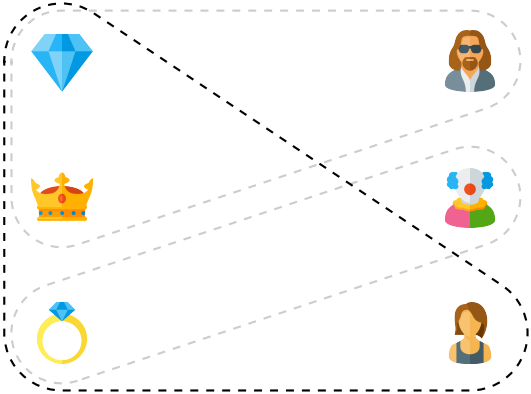

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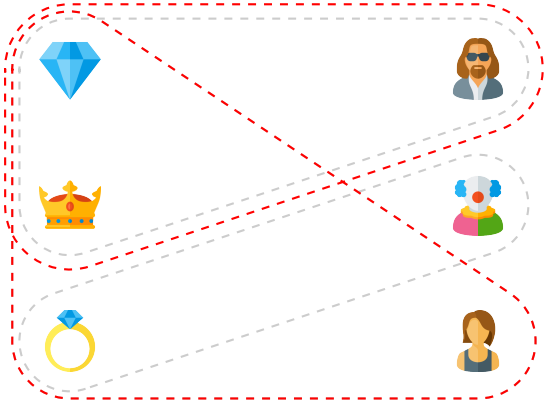

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Weighted Set Packing (WSP) Problem

[LEYTON-BROWN et al., 2009]

Our Goal when Solving WSP as an Optimization Problem

Maximize value of accepted bids *without selecting conflicting* ones

ILP Formulation for WSP

$$\text{maximize} \quad x_1 \cdot V(\{💎, 👑\}) + x_2 \cdot V(\{💍\}) + x_3 \cdot V(\{💎, 💍, 👑\})$$

$$\text{subject to} \quad x_1 \cdot 💎 \overset{?}{\in} \{💎, 👑\} + x_2 \cdot 💎 \overset{?}{\in} \{💍\} + x_3 \cdot 💎 \overset{?}{\in} \{💎, 💍, 👑\} \leq 1 \qquad (💎)$$

$$x_1 \cdot 👑 \overset{?}{\in} \{💎, 👑\} + x_2 \cdot 👑 \overset{?}{\in} \{💍\} + x_3 \cdot 👑 \overset{?}{\in} \{💎, 💍, 👑\} \leq 1 \qquad (👑)$$

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\text{subject to} & x_1 \cdot & 1 & + x_2 \cdot & 0 & + x_3 \cdot & 1 & \leq 1 & (💎) \\
 & x_1 \cdot & 1 & + x_2 \cdot & 0 & + x_3 \cdot & 1 & \leq 1 & (👑) \\
 & x_1 \cdot & 0 & + x_2 \cdot & 1 & + x_3 \cdot & 1 & \leq 1 & (💍)
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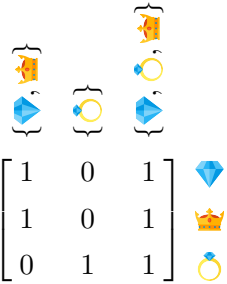
$$

WDP as Weighted Set Packing (WSP) Problem

[LEYTON-BROWN et al., 2009]

- Given a set N of items and a set $\mathcal{S}$ of bids, let M be a $|N| \times |\mathcal{S}|$ matrix
- $M_{iS} = 1$ if and only if item $i \in N$ is part of bid $S \in \mathcal{S}$, $M_{iS} = 0$ otherwise



$$M = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Weighted Set Packing (WSP) Problem

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ILP Formulation for WSP

maximize	$\sum_{S \in \mathcal{S}} x_S \cdot V(S)$	(Value of each active bid)
subject to	$\sum_{S \in \mathcal{S}} M_{iS} \cdot x_S = 1 \quad \forall i \in N$	(All items must be sold)
	$\sum_{S \in \mathcal{S}} M_{iS} \cdot x_S \leq 1 \quad \forall i \in N$	(Items can remain unsold)

Today's Menu

Distributed Constraint Optimization

Auctions
- Combinatorial Auctions
- Relaxations
- Decentralization

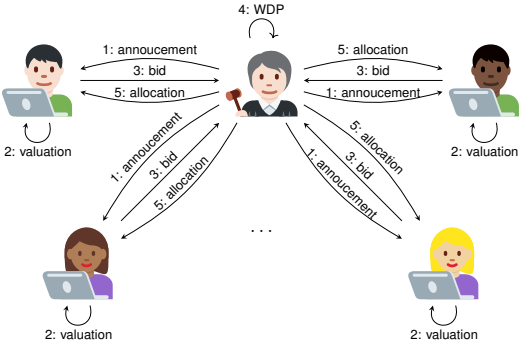
Coalition Formation on MAS

Real-World Applications

Discussion and Wrap-up

Auction-based Coordination

Many auction schemes [PARSONS et al., 2011]



- **Combinatorial Auctions** (CA)
  [CRAMTON et al., 2010]
- **Parallel Single Item Auctions** (PSI)
  [KOENIG et al., 2006]
  - Each agent bids on the *whole set of items* in parallel
- **Sequential Single Item Auctions** (SSI)
  [LAGOUDAKIS et al., 2005]
  - Each agent *sequentially* bids on a *single item* wrt to the already allocated items
- **Consensus-based Bundle Auction** (CBBA)
  [CHOI et al., 2009]
  - WDP decentralized as a *consensus* on bundles

Auction-based Coordination

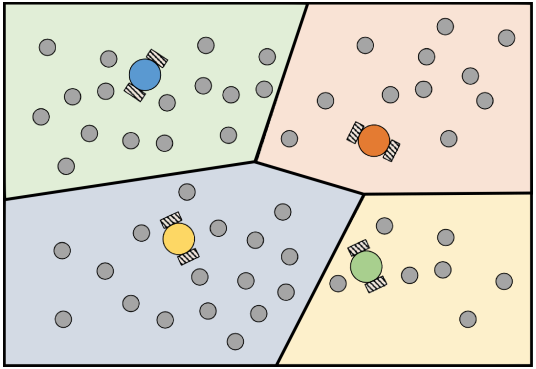
Focus on Task Allocation

Many application fields make use of auction-based approaches

- A set of **agents** (robots, satellites, etc.), $R = \{r_1, \ldots, r_{|R|}\}$ with capabilities
- A set of **tasks**, $T = \{t_1, \ldots, t_{|T|}\}$, with time-related and operation constraints and requirements
- Find an assignment of tasks to agents, wrt. some consistency **constraints**
  - e.g. capabilities, dependencies between tasks, resource capacity, plan consistency

  whilst optimizing some specific **objective**
  - e.g. completion time, energy

$\approx$ **multi-item allocation**: each agent is allocated several tasks (bundle)



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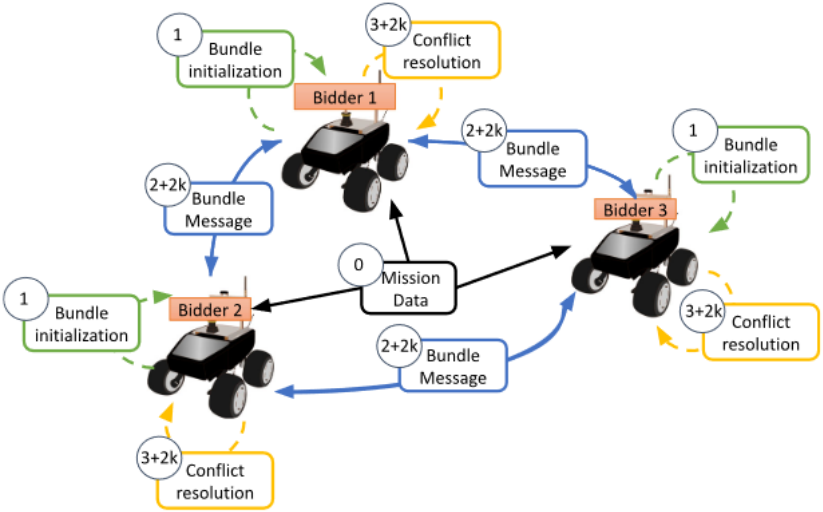

CBBA Algorithm

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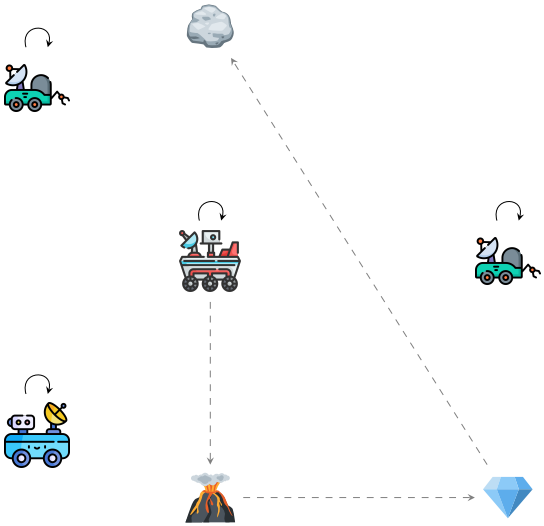

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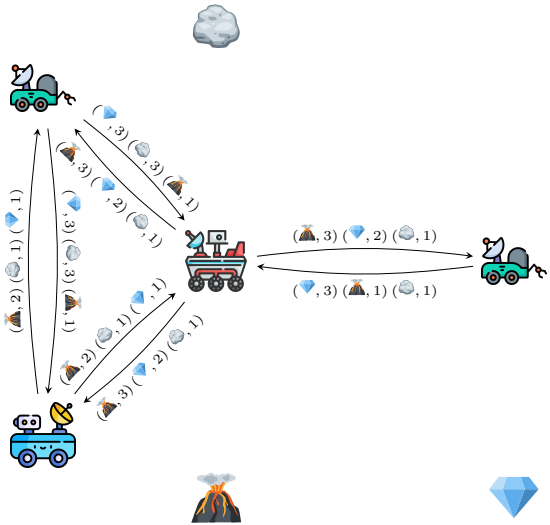

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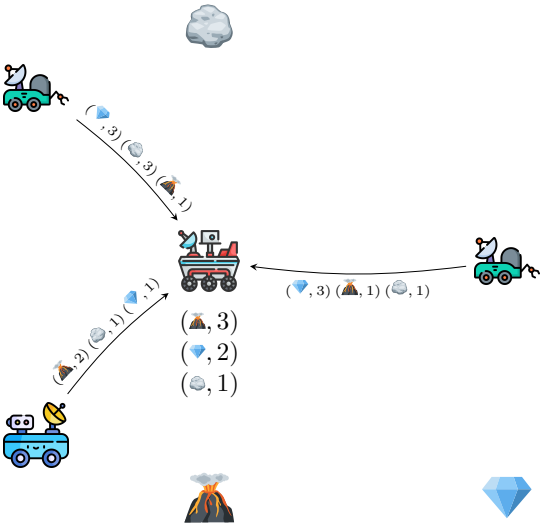

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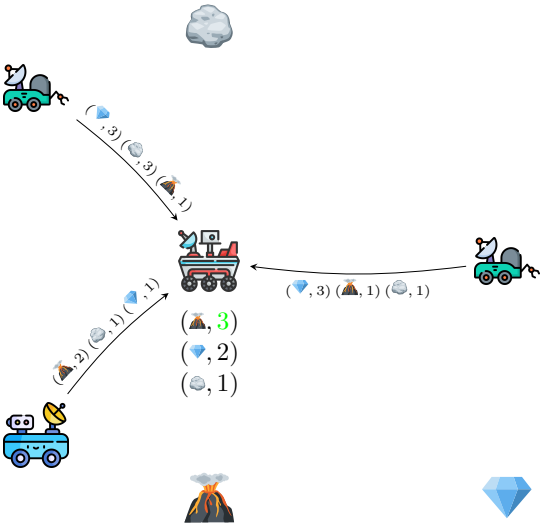

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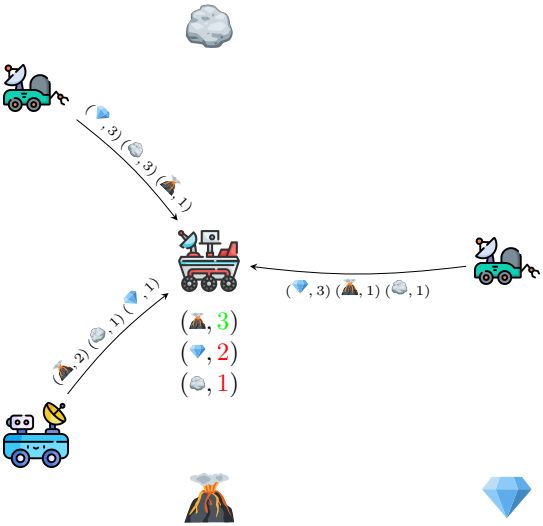

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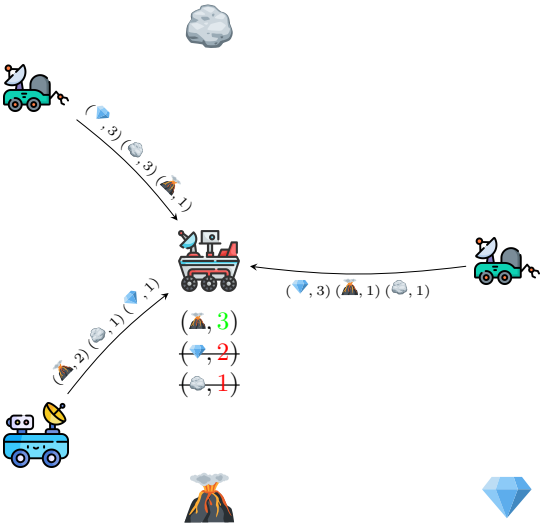

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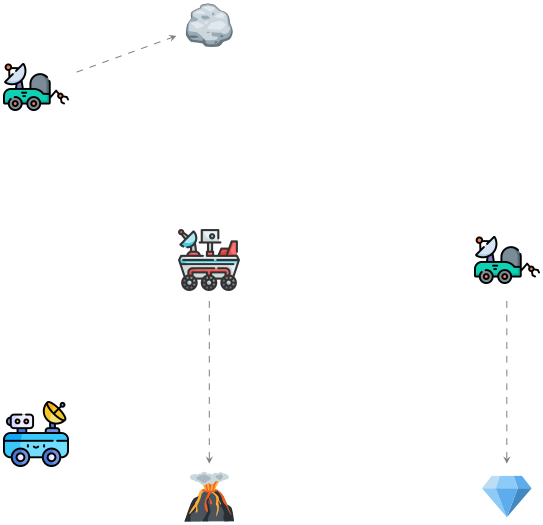
- **Allocation**
  - Winning bundles are allocated
  - Agents execute the tasks in their



Consensus-based Bundle Auction (CBBA)

Advantages of CBBA

- **Decentralized:** No central authority required, enabling robust operation in dynamic environments
- **Scalable:** Efficiently handles large numbers of agents and tasks
- **Flexible:** Can be adapted to different task allocation problems and objective functions

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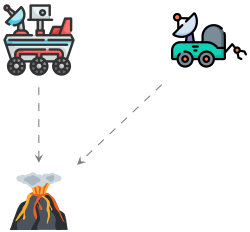

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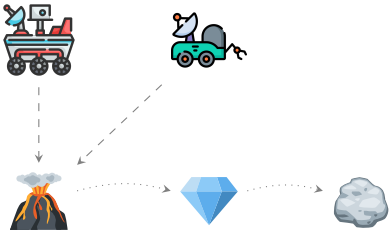

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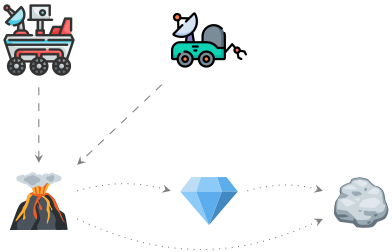
- How to handle composite/sequenced tasks?
- How to handle alternative sequences/modes?



Today's Menu

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Characteristic Function Games (CFGs)

[CHALKIADAKIS et al., 2011]

Set of Agents A

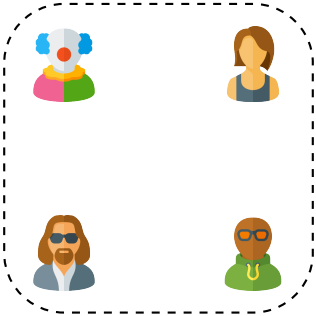
$A = \{$, , , $\}$

Characteristic Function $v(\cdot)$

- $v(\{$, $\}) = 0$
- $v(\{$, , $\}) = -7$
- $v(\{$, $\}) = 3$
- $\ldots$

Characteristic Function Games (CFGs)

[CHALKIADAKIS et al., 2011]



Set of Agents A

$A = \{$, , , $\}$

Characteristic Function $v(\cdot)$

- $v(\{$, $\}) = 0$
- $v(\{$, , $\}) = -7$
- $v(\{$, $\}) = 3$
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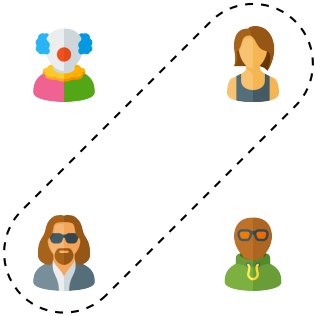

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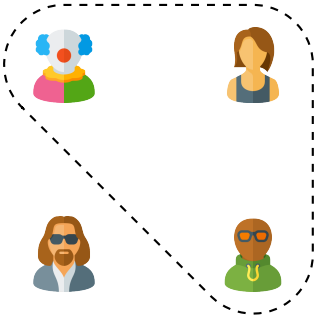

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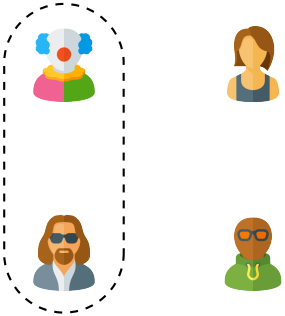

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Characteristic Function

[CHALKIADAKIS et al., 2011]

Characteristic Function

The function $v : \mathcal{P}(A) \rightarrow \mathbb{R}$ associates a value to *every coalition* (i.e., subset) of A

Exponential Complexity

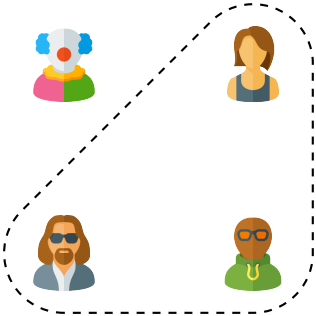
Representing $v(\cdot)$ as a *table* requires an *exponential* number of steps (i.e., $2^{|A|}$)

Mitigate this Complexity

(1) *Restrict* the set of coalitions or (2) consider $v(\cdot)$ with a specific *structure*

Cardinality-Restricted CFGs

[SHEHORY and KRAUS, 1998]



Maximum Cardinality k

E.g., only coalitions of size ≤ 3 are feasible

Polynomial Number of Coalitions

Total number of coalitions is $\sum_{i=1}^{k} \binom{|A|}{i} = \mathcal{O}(|A|^k)$, i.e., *polynomial wrt* $|A|$

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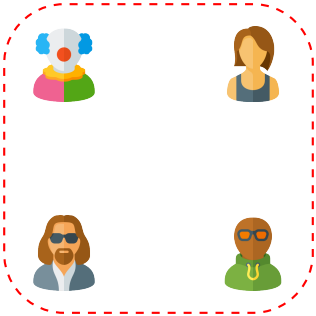

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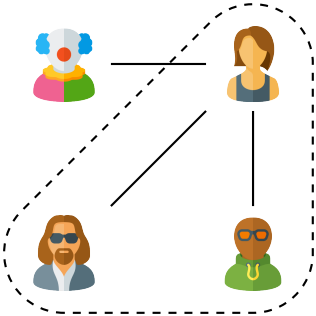
Graph-Restricted CFGs

[MYERSON, 1977], [DEMANGE, 2004]



Graph G among Agents

$G = (\{$🧛‍♂️, 🤹‍♀️, 👩‍🦰, 🧟‍♂️$\}, \{($🧛‍♂️, 👩‍🦰$), ($🤹‍♀️, 👩‍🦰$), ($👩‍🦰, 🧟‍♂️$)\})$

Connected Subgraphs

A coalition is *feasible* only if it induces a *connected* subgraph of G

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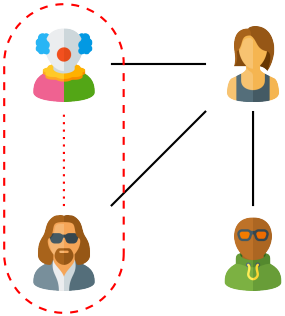

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Today's Menu

Distributed Constraint Optimization

Auctions

Coalition Formation on MAS

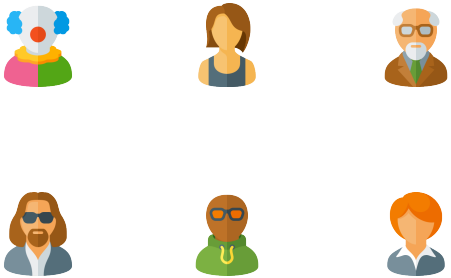
- Characteristic Function Games
- Coalition Structure Generation

Real-World Applications

Discussion and Wrap-up

Coalition Structure Generation (CSG)

[RAHWAN et al., 2015]



Solving the Coalition Structure Generation (CSG) Problem

Compute the partition $\mathcal{S}$ of A into *feasible* coalitions that *maximizes* the sum $\sum_{S \in \mathcal{S}} v(S)$

Coalition Structure Generation (CSG)

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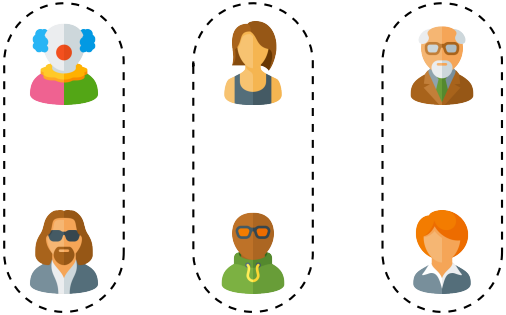

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Coalition Structure Generation as Integer Linear Programming

[RAHWAN et al., 2015]

ILP Formulation for (Approximate) Coalition Structure Generation

$$
\begin{aligned}
\text{maximize} \quad & x_1 \cdot V(\{\text{👩}, \text{🧟}\}) + x_2 \cdot V(\{\text{🧟}, \text{🧑‍🦰}\}) + x_3 \cdot V(\{\text{🧟}, \text{👩}\}) \\
\text{subject to} \quad & x_1 \cdot \text{🧑‍🦰} \overset{?}{\in} \{\text{👩}, \text{🧟}\} + x_2 \cdot \text{🧑‍🦰} \overset{?}{\in} \{\text{🧟}, \text{🧑‍🦰}\} + x_3 \cdot \text{🧑‍🦰} \overset{?}{\in} \{\text{🧟}, \text{👩}\} = 1 \qquad (\text{🧑‍🦰}) \\
& x_1 \cdot \text{🧟} \overset{?}{\in} \{\text{👩}, \text{🧟}\} + x_2 \cdot \text{🧟} \overset{?}{\in} \{\text{🧟}, \text{🧑‍🦰}\} + x_3 \cdot \text{🧟} \overset{?}{\in} \{\text{🧟}, \text{👩}\} = 1 \qquad (\text{🧟}) \\
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\end{aligned}
$$

Coalition Structure Generation as Integer Linear Programming

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$$\text{maximize} \quad x_1 \cdot V(\{\text{👩}, \text{🧟}\}) + x_2 \cdot V(\{\text{🧓}, \text{🧑‍🦰}\}) + x_3 \cdot V(\{\text{🧓}, \text{👩}\})$$

$$\text{subject to} \quad x_1 \cdot \text{🧑‍🦰} \notin \{\text{👩}, \text{🧟}\} + x_2 \cdot \text{🧑‍🦰} \in \{\text{🧓}, \text{🧑‍🦰}\} + x_3 \cdot \text{🧑‍🦰} \notin \{\text{🧓}, \text{👩}\} = 1 \qquad (\text{🧑‍🦰})$$

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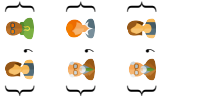
Coalition Structure Generation as Integer Linear Programming

[RAHWAN et al., 2015]

- Given A and a set $\mathcal{S}$ of coalitions, let M be a $|A| \times |\mathcal{S}|$ binary matrix
- $M_{aS} = 1$ if and only if agent $a \in A$ is part of coalition $S \in \mathcal{S}$



$$M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Coalition Structure Generation as Integer Linear Programming

[RAHWAN et al., 2015]

Optimization Objective of Coalition Structure Generation

Compute the *partition* of A that *maximizes* the sum of the utility values

ILP Formulation for (Approximate) Coalition Structure Generation

maximize	$\sum_{S \in \mathcal{S}} x_S \cdot V(S)$	(Value of each selected coalition)
subject to	$\sum_{S \in \mathcal{S}} M_{aS} \cdot x_S = 1 \quad \forall a \in A$	(Each agent in one coalition)

CSG as Integer Linear Programming $\approx$ WDP for CFGs

[RAHWAN et al., 2015]

Solving Integer Linear Programs

ILPs can be solved with state-of-the-art solvers like CPLEX (very mature technology)

Pros

Does not require any assumption on $v(\cdot)$ (very general approach)

Cons

- Memory requirements can become unmanageable for more than 20–30 agents
- Difficult to directly exploit the structure of the problem (i.e., graph)

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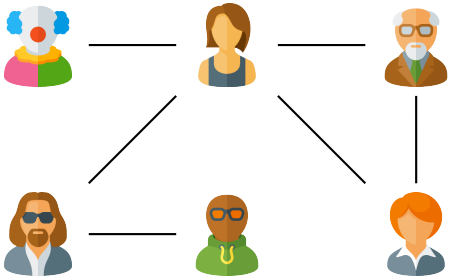

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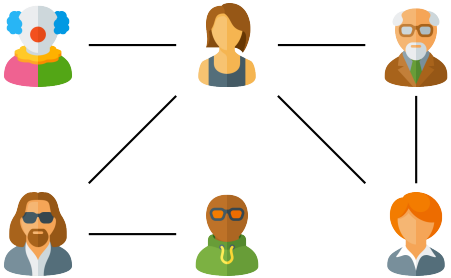
CSG Approaches based on Search

[BISTAFFA et al., 2017a]



Edge Contraction Operation

Contraction of edge $(S_i, S_j) \rightarrow$ form coalition $S_i \cup S_j$

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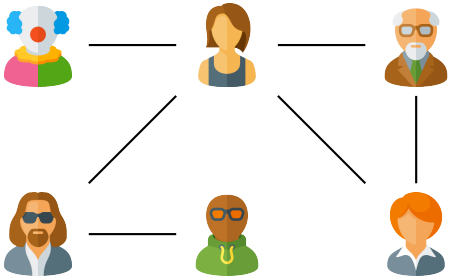

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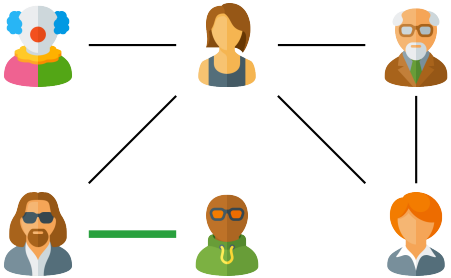

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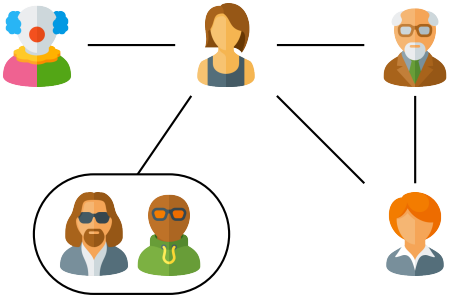

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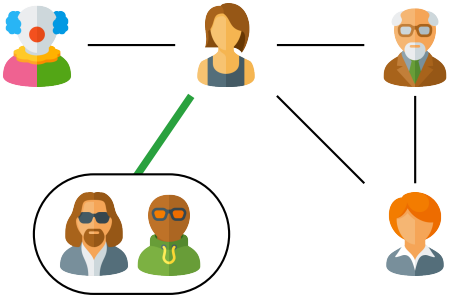

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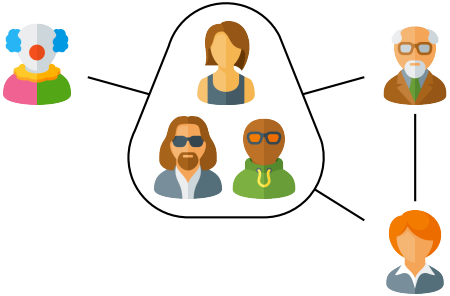

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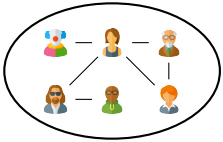

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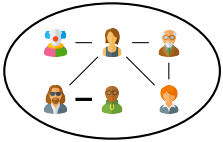

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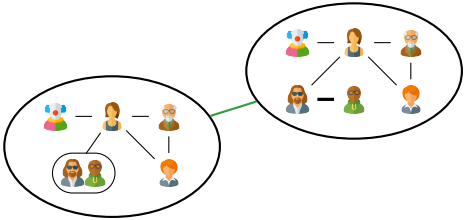

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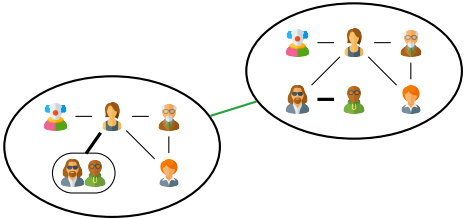

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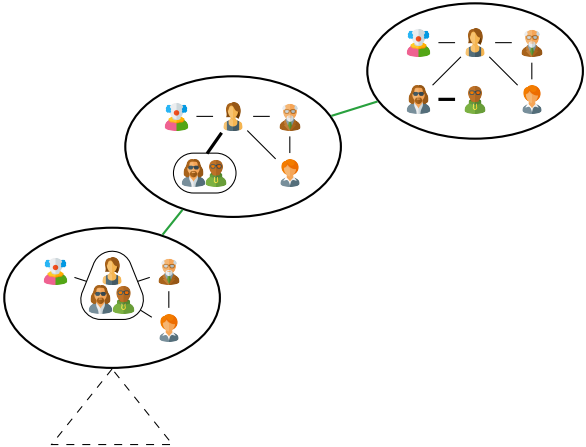

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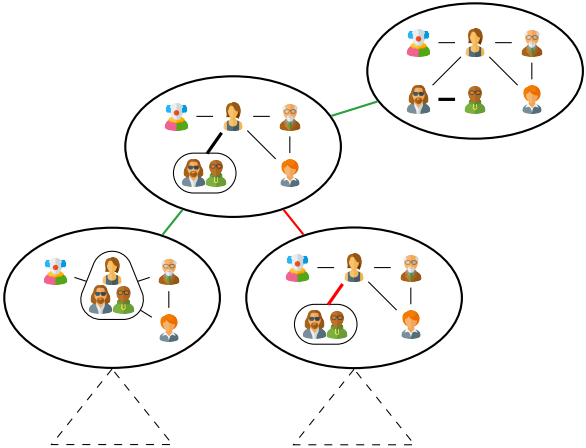

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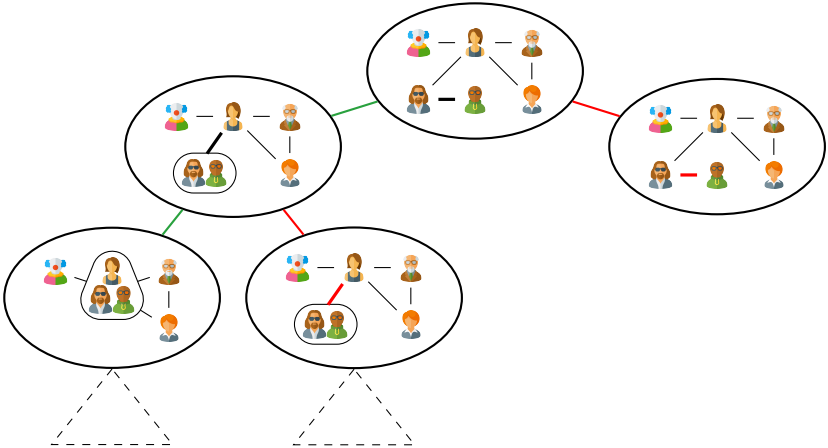

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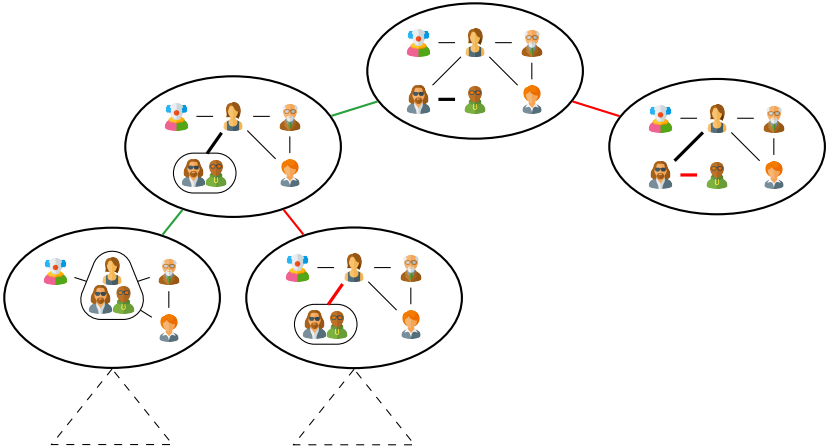

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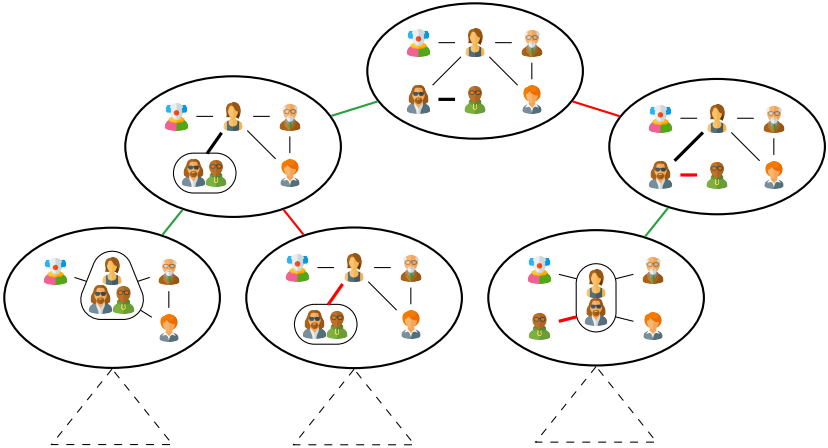

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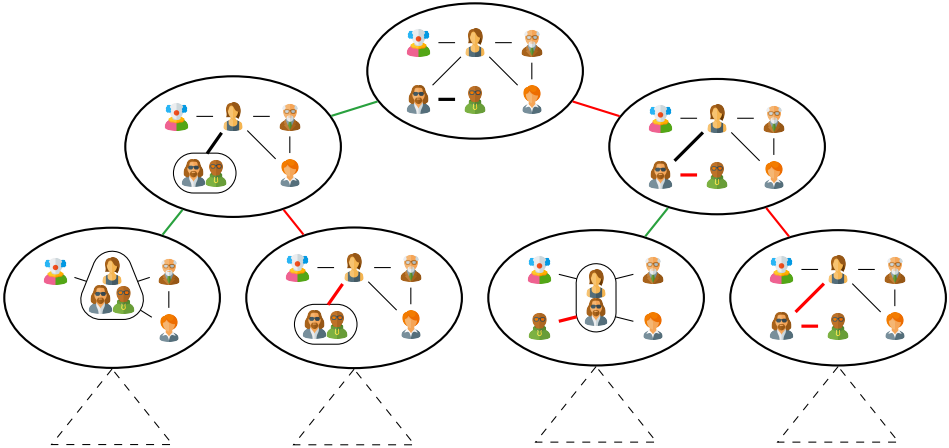

CSG Approaches based on Search

[BISTAFFA et al., 2017a]

CFSS Algorithm

- Builds a *binary search tree* by contracting (or not) an edge at each step
- Each coalition structure (i.e., partition of A) is represented *only once* in the tree
- The optimal coalition structure is computed by doing a *depth-first* traversal of the tree

Pros

Approximate algorithm with quality guarantees if used in conjunction with *Branch-and-Bound*

Cons

Performance depends on the assumption that $v(\cdot)$ can be expressed in *closed-form*

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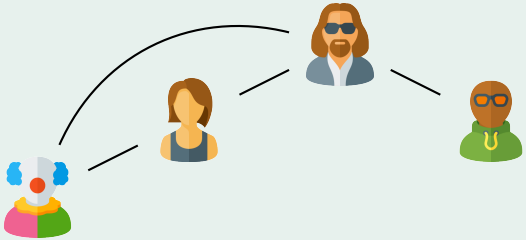
CSG as a COP

[BISTAFFA and FARINELLI, 2018]

Graph-Restricted CFG Example



Pseudotree among Agents (Hierarchy)

Adjacent agents in the graph fall in the same branch of the tree (e.g., and)

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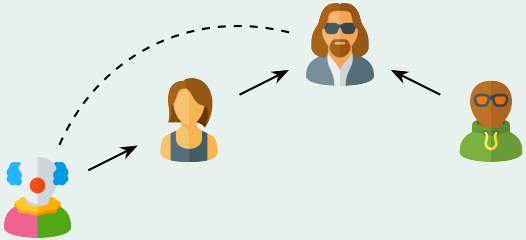

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How can we exploit the structure (i.e., hierarchy among agents)?

Main Idea

- Each coalition (i.e., decision variable) is "controlled" by the highest agent
- "Delegate" the formation of coalitions to descendants by means of *required* variables

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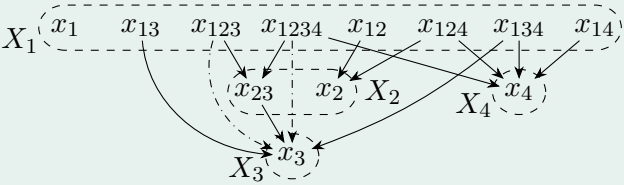

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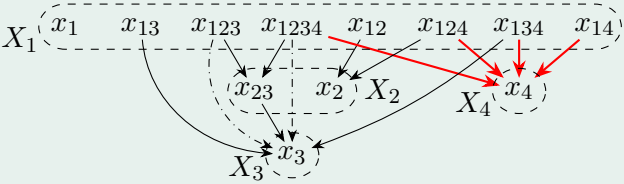

CSG as a COP

[BISTAFFA and FARINELLI, 2018]

Required Variables

- Any two variables that require the same variable *cannot* be enabled simultaneously
- As a result *no overlapping variables* are activated *at the same time*

Number of Constraints

- Naive COP: $\binom{\#\text{ coalitions}}{2}$
- This approach: linear *wrt* the number of agents

Open Question

Can we make this COP a Distributed COP (DCOP)?

CSG as a COP

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Observation Scheduling in Multi-Owner Constellations [PICARD, 2022]

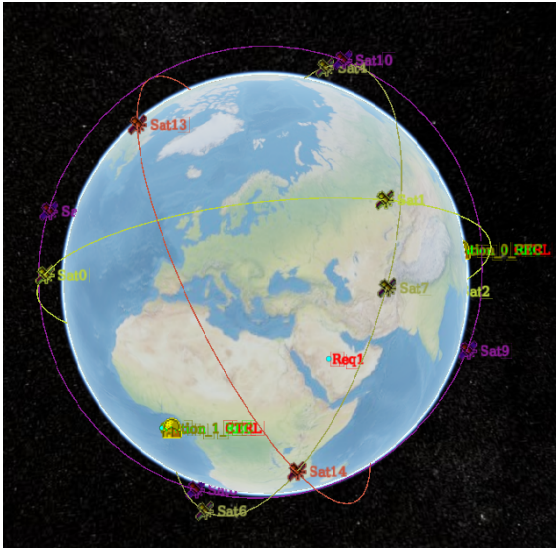
- Increasing size of deployed EOS constellations

⇒ Observe any point on Earth at higher frequency, e.g. Planet constellation

- **but**, requires to improve coordination and cooperation between assets and stakeholders

- We focus here on collective observation scheduling on a constellation where some users have exclusive access to some orbit portions

⇒ Answer to strong user expectations to benefit both from a shared system (to reduce costs) and a proprietary system (total control and confidentiality)



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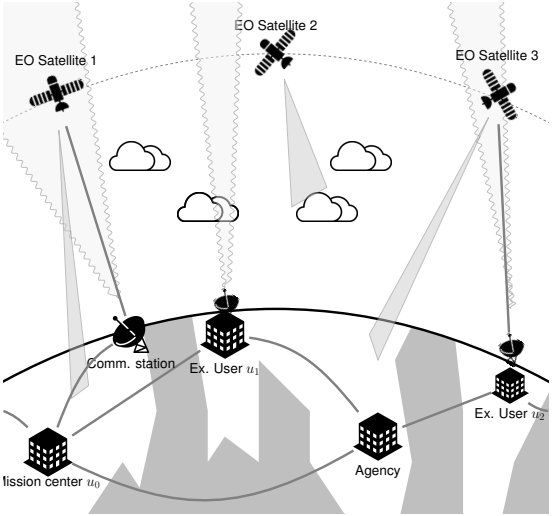
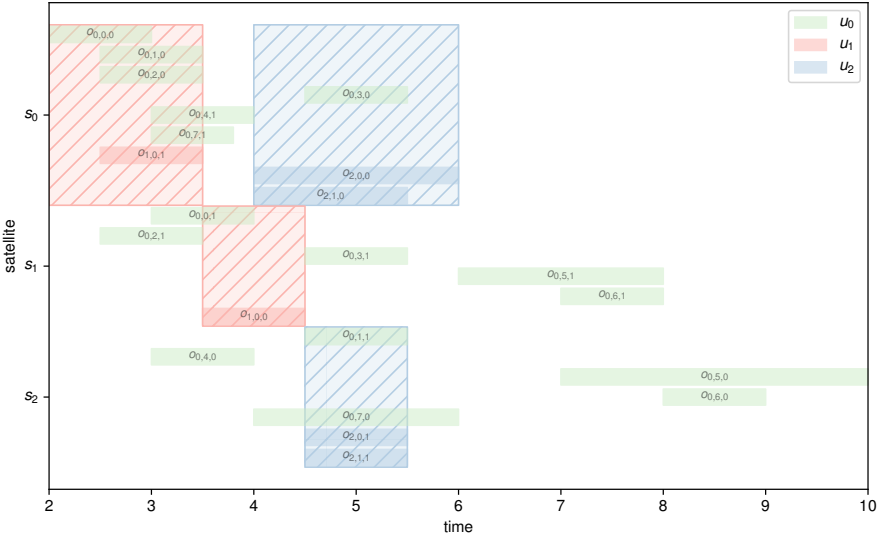


Scheduling Observations with Multiple Exclusive Orbit Portions

Illustrative Example



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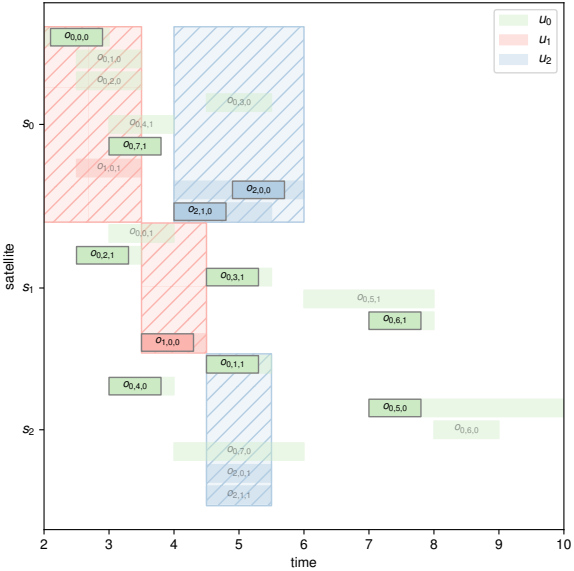

DCOP Model

A DCOP $\langle \mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{C}, \mu \rangle$ is defined for a given request r, and a current scheduling

- The agents are the exclusive users which can potentially schedule r:

$$\mathcal{A} = \{u \in \mathcal{U}^{\text{ex}} | \exists (s, (t_u^{\text{start}}, t_u^{\text{end}})) \in e_u, \exists o \in \theta_r \text{ s.t. } s_o = s, [t_u^{\text{start}}, t_u^{\text{end}}] \cap [t_o^{\text{start}}, t_o^{\text{end}}] \neq \varnothing\} \tag{1}$$

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$$\mathcal{X} = \{x_{e,o} | e \in \bigcup_{u \in \mathcal{A}} e_u, o \in \mathcal{O}[u]^r\} \tag{2}$$

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DCOP Model (cont.)

- Constraints should check that at most one observation is scheduled per request (4), that satellites are not overloaded (5), that at most one agent serves the same observation (6)

$$\sum_{e \in \bigcup_{u \in \mathcal{A}} e_u} x_{e,o} \leq 1, \quad \forall u \in \mathcal{X}, \forall o \in \mathcal{O}[u]^r \tag{4}$$

$$\sum_{o \in \{o \in \mathcal{O}[u]^r | u \in \mathcal{A}, s_o = s\}, e \in \bigcup_{u \in \mathcal{A}} e_u} x_{e,o} \leq \kappa_s^*, \ \forall s \in \mathcal{S} \tag{5}$$

$$\sum_{e \in \bigcup_{u \in \mathcal{A}} e_u} x_{e,o} \leq 1, \quad \forall o \in \mathcal{O} \tag{6}$$

- The cost to integrate an observation in the current user's schedule should be assessed to guide the optimization process

$$c(x_{e,o}) = \pi(o, \mathcal{M}_{u_o}), \quad \forall x_{e,o} \in \mathcal{X} \tag{7}$$

where π evaluates the best cost obtained when scheduling o and any combination of observations from $\mathcal{M}_{u_o}$, as to consider all possible revisions of u_o's current schedule

$$\mathcal{C} = \{(4), (5), (6), (7)\} \tag{8}$$

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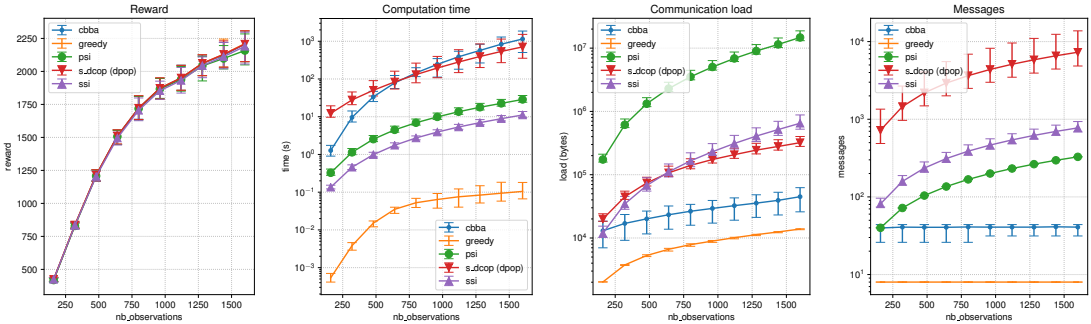
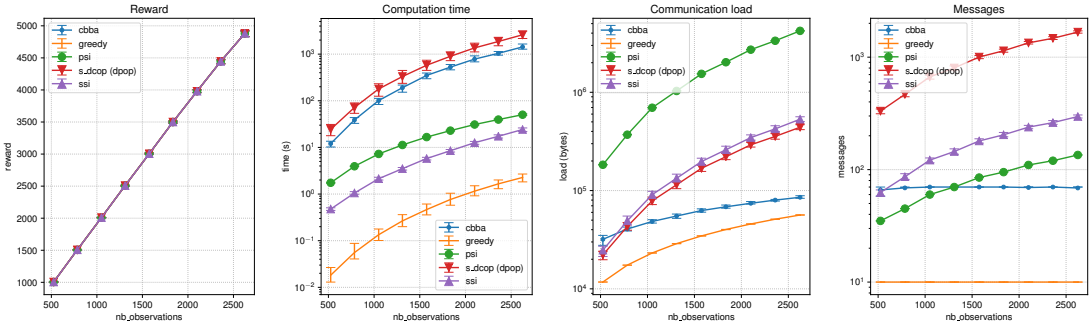
Highly conflicting randomly generated problems

5-min horizon with overlapping requests and limited capacity



Realistic randomly generated problems

6-hour horizon with numerous requests and large capacity



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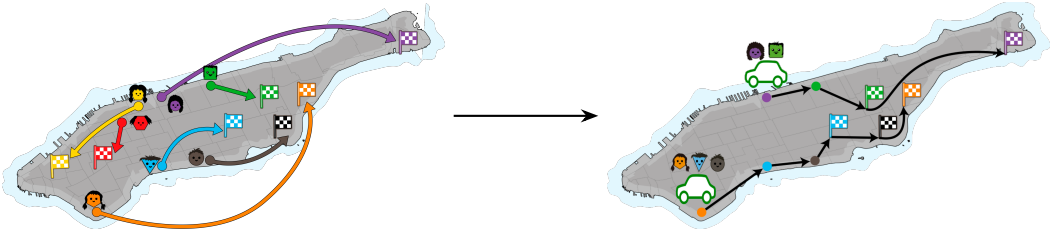
Shared Mobility as (Online) Coalition Structure Generation

[BISTAFFA et al., 2019]

What is Shared Mobility for Us?

Arrange *shared rides* (coalitions) among users that submit *real-time* requests, with the objective of *maximizing* a given *objective function*



Shared Mobility as (Online) Coalition Structure Generation

[BISTAFFA et al., 2019]

Our Task

At each time step, arrange a (possibly empty) set of non-overlapping feasible cars among the requests currently active in the system

Our Objective Function

Maximize environmental benefits and quality of service

Our Case Study [BISTAFFA et al., 2019]

Densely populated areas (e.g., Manhattan) with request rate of 400 reqs/minute

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Input of the Online CSG Problem

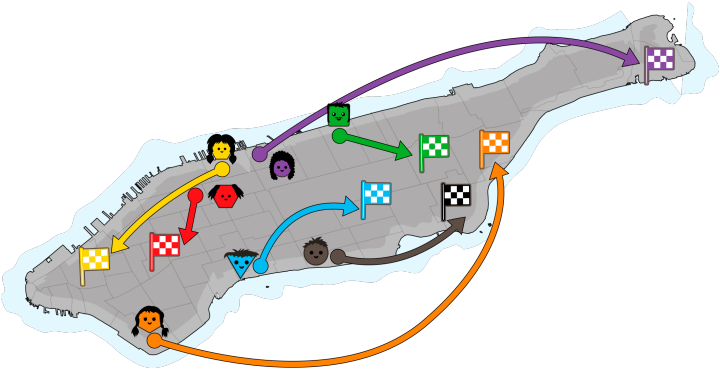
[BISTAFFA et al., 2019]

Incoming Requests

"I just issued a trip request"

Waiting Trip Requests

"I am waiting to share my ride"



Input of the Online CSG Problem

[BISTAFFA et al., 2019]

Example of a Shared Mobility Request

"I want to go from point i to point j, and I am willing to wait δ minutes to be picked up by somebody ($d =$ false) / before I leave with *my own car* ($d =$ true)"

- $r = \langle i, j, d, \delta \rangle$ (A request is a tuple r)
- $r \in R_t$ (The system receives a set R_t of requests at each time step t)
- $\langle R_1, \ldots, R_t, \ldots, R_h \rangle$ (Sequence of inputs over a time horizon h)
- The input sequence is *not known a priori* (Online optimization problem)

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Value $v(S)$ of a Coalition S

[BISTAFFA et al., 2019]

- The *value* (utility) of a coalition S is defined as:

$$v(S) = \overbrace{\rho_{\text{CO}_2} \cdot E_{\text{CO}_2}(S) + \rho_{\text{noise}} \cdot E_{\text{noise}}(S) + \rho_{\text{traffic}} \cdot E_{\text{traffic}}(S)}^{\text{environmental benefits}} + \overbrace{\rho_{\text{QoS}} \cdot Q(S)}^{\text{quality of service}}$$

- $|S| \leq k$ (Maximum cardinality constraint)

$$F(S) = |S| \leq k \ \wedge \ \ldots$$

- $\mathcal{F}(R) = \left\{ S \in 2^R \mid F(S) \right\}$ (Set of feasible coalitions from a set R of requests)

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Curse of Dimensionality

[BISTAFFA et al., 2019]

- Recall that $\mathcal{F}(R) = \{S \in 2^R \mid F(S)\}$
- With $|S| \leq k$, $|\mathcal{F}(R)| \leq \sum_{i=1}^{k} \binom{|R|}{i}$, i.e., $\mathcal{O}(|R|^k)$ (Polynomial complexity)
- In practice, $|R_t|$ can be as high as 400 (Request rate in NY taxi dataset)

Scalability Problem

Enumerating all coalitions in $\mathcal{F}(R)$ is impractical, especially in realistic application scenarios with *very limited time budget* for the solution

Our Solution

Consider a restricted set $\hat{\mathcal{F}}(R)$ of *good candidate coalitions* instead of $\mathcal{F}(R)$

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Generation of Good Candidate Coalitions (Step 1)

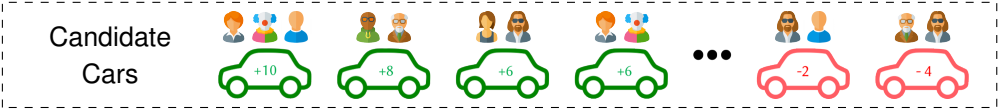
[BISTAFFA et al., 2019]

- CO_2 emissions
- Acoustic pollution
- Traffic congestion
- Quality of service

→

20 seconds

Probabilistic Greedy Algorithm



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[FENOY et al., 2024b]

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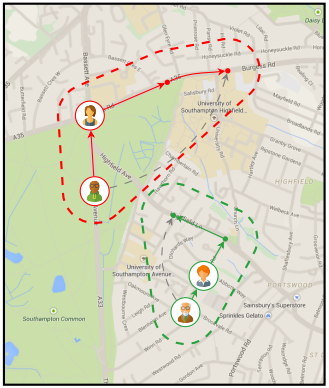
Machine Learning Model

Candidate Cars

Limitations of State-of-the-Art



General "Complete" Algorithms

Impossible to use in practice (limited to 20–25 humans), require to enumerate *all* 2^n *subsets*

Ad Hoc Heuristic Algorithms

Exploit *domain-specific knowledge*, require *expensive ad hoc* algorithm development

Towards a General and Scalable Approach

Can we somehow *learn* this *domain-specific knowledge* in an *automated way*?

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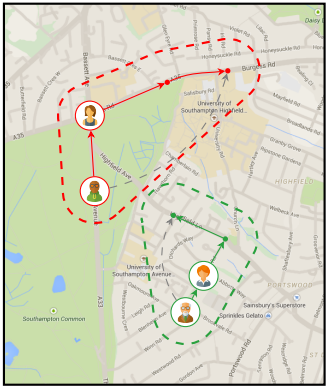

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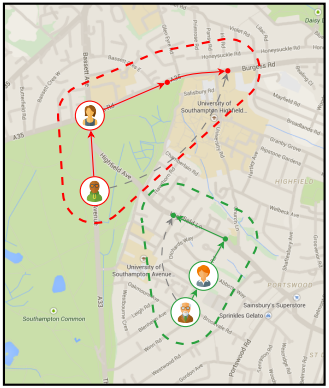

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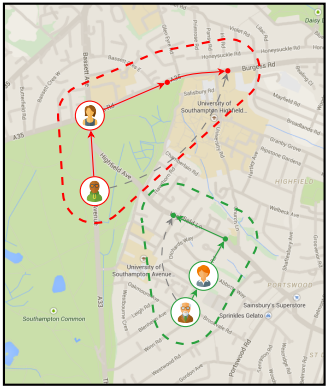

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Combine Optimization and Machine Learning

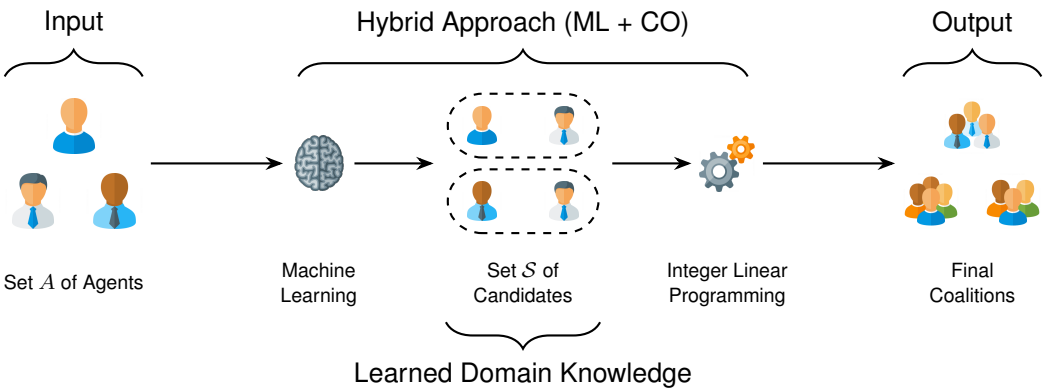
“There is interest in using machine learning as a modeling tool for optimization or to extract intuition and knowledge about algorithms.”

Yoshua Bengio, 2018 Turing Award Laureate

“For conceptual and engineering breakthroughs that have made deep neural networks a critical component of computing”

General Coalition Formation Approach

[FENOY et al., 2024b]



Our Journey towards General Coalition Formation

[FENOY et al., 2024b]

What We Tried

- Generative adversarial networks (👎 mode collapse)
- Pointer networks (👎 difficult to employ)
- Monte Carlo tree search (👎 collectives of poor quality)

What Worked in the End

- *Attention*-based model
- Entropy to enhance *diversity* of generated candidates during training
- We are comparable with ad hoc SOTA, but we are *more general*

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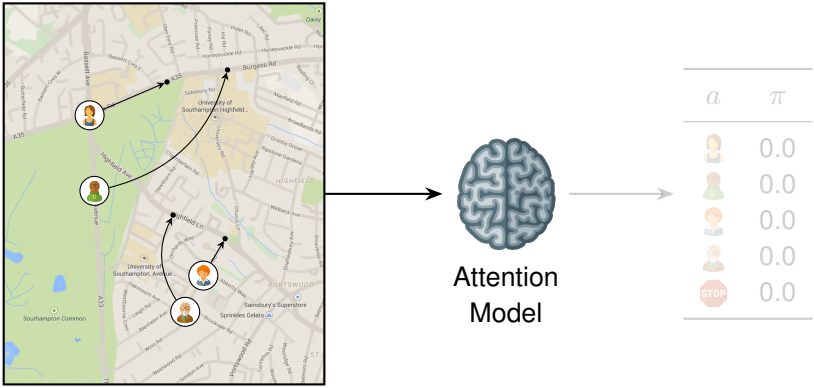
Proposing Coalitions with Our Attention Model

[FENOY et al., 2024b]



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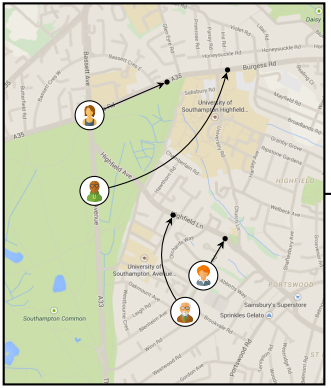
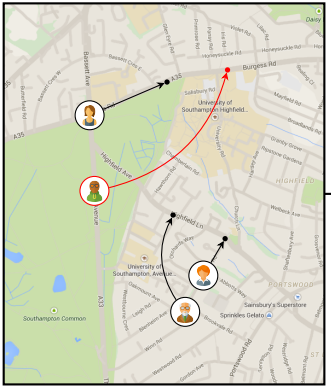


Attention Model

a	π
	0.1
	0.5
	0.2
	0.1
STOP	0.1

Proposing Coalitions with Our Attention Model

[FENOY et al., 2024b]



Attention Model

a	π
	0.0
	–
	0.0
	0.0
STOP	0.0

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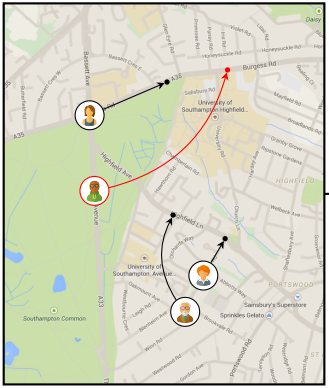
[FENOY et al., 2024b]



Attention Model

a	π
	0.6
	–
	0.3
	0.1
STOP	0.0

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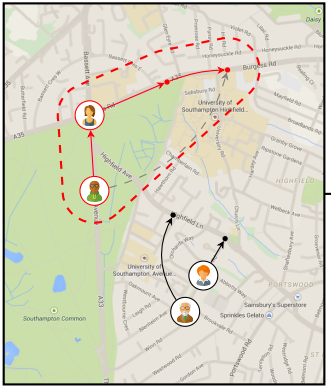

Attention Model

a	π
	–
	–
	0.0
	0.0
STOP	0.0

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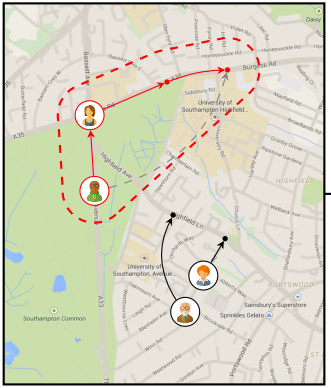
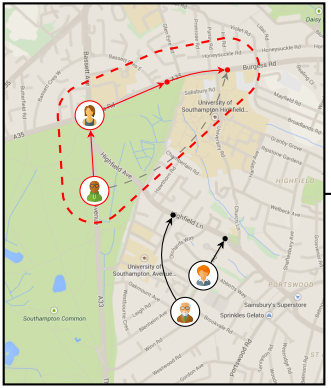


Attention Model

a	π
	–
	–
	0.3
	0.1
STOP	0.6

Proposing Coalitions with Our Attention Model

[FENOY et al., 2024b]



Attention Model

a	π
	–
	–
	0.1
	0.8
STOP	0.1

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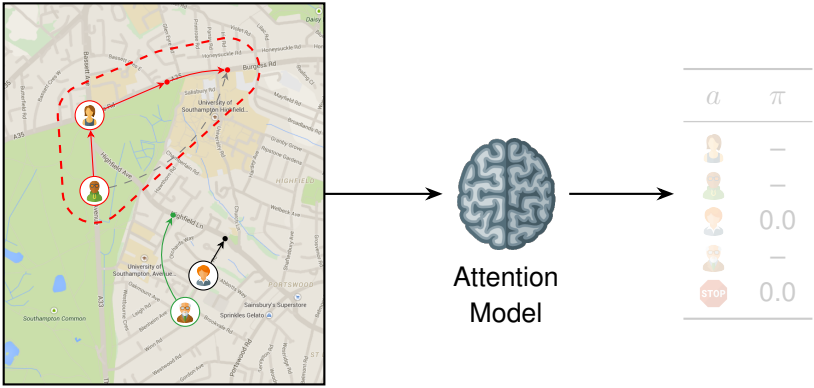
[FENOY et al., 2024b]



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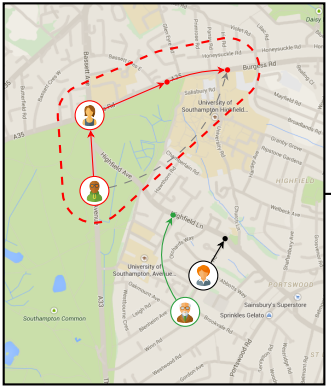
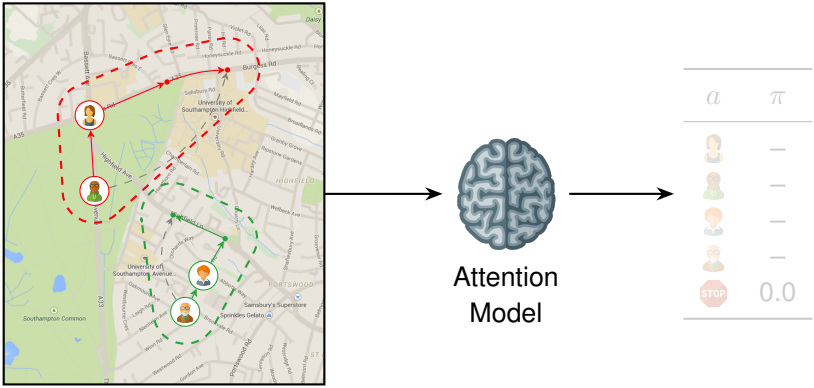


Attention Model

a	π
	–
	–
	0.6
	–
STOP	0.4

Proposing Coalitions with Our Attention Model

[FENOY et al., 2024b]



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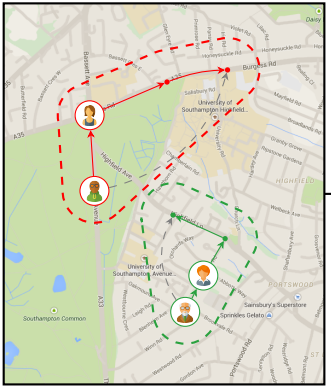

Attention Model

a	π
	–
	–
	–
	–
STOP	1.0

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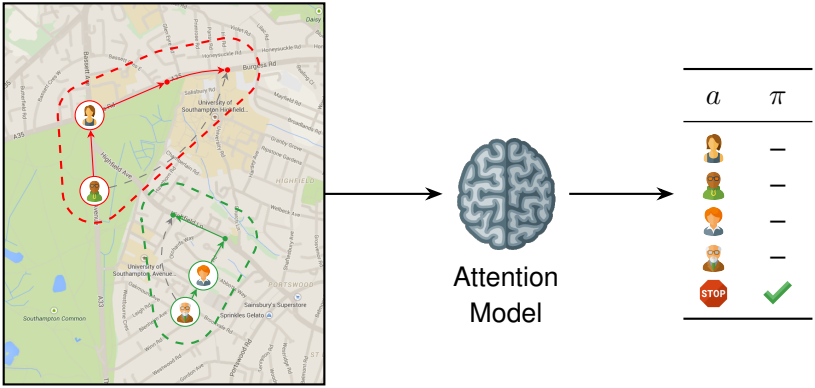
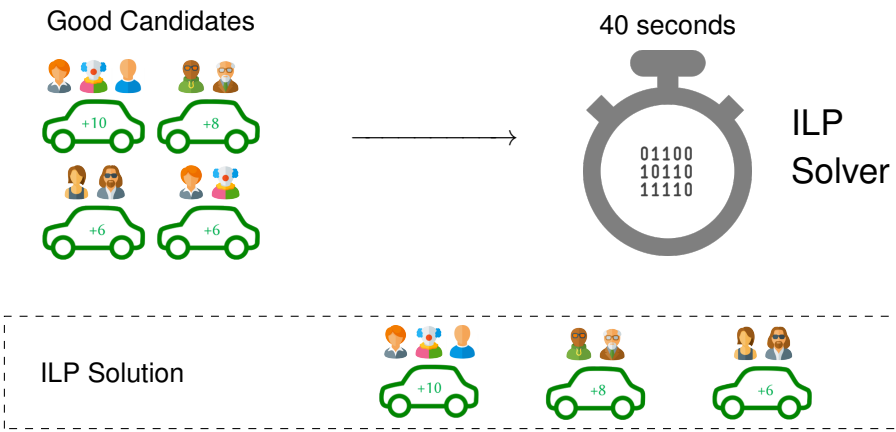


ILP Optimization (Step 2)

[BISTAFFA et al., 2019; FENOY et al., 2024b]



Quality Comparison with Ad Hoc SOTA

[FENOY et al., 2024b]

n	*Attention Model*	MCTS[1]	P-Greedy[2]
50	0.96	0.92	0.98
100	0.90	0.89	0.98
200*	0.87	0.01	1.00

[1] Wu and Ramchurn, Proceedings of IJCAI, 2020 (10.24963/ijcai.2020/57).

[2] Bistaffa *et al.*, Trans. on Intelligent Transport. Systems, 2021 (10.1109/TITS.2019.2954982).

*Optimality ratio computed *wrt* P-Greedy (true optimal is too costly to compute).

Today's Menu

Distributed Constraint Optimization

Auctions

Coalition Formation on MAS

Real-World Applications

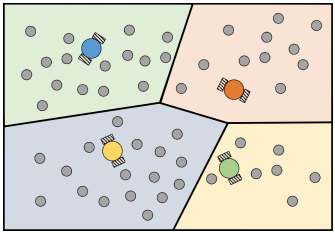
- Observation Scheduling in Multi-Owner Constellations
- Shared Mobility
- Multi-Agent Pick-up and Delivery

Discussion and Wrap-up

Multi-Agent Pickup and Delivery (MAPD)

Multi-Agent Pickup and Delivery (MAPD)

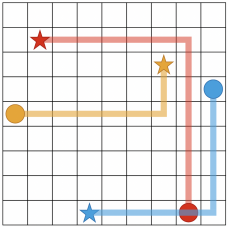
Multi-Robot Task Allocation (MRTA)



Assign tasks to robots

+

Multi-Agent Path Finding (MAPF)



Compute conflict-free paths

Approaches to Solve MAPD

Two-Step Approaches (Task Assignment + Path Planning)

- CENTRAL [MA et al., 2017]
- TA-Prioritized, TA-Hybrid [LIU et al., 2019]

Integrated Approaches (Better Solution Quality)

- TP, TPTS [MA et al.,2017]
- RMCA [CHEN et al., 2021]

Approaches to Solve MAPD

Two-Step Approaches (Task Assignment + Path Planning)

- CENTRAL [Ma et al., 2017]
- TA-Prioritized, TA-Hybrid [Liu et al., 2019]

Integrated Approaches (Better Solution Quality)

- TP, TPTS [Ma et al.,2017]
- RMCA [Chen et al., 2021]

Regret-Based Marginal Cost Assignment (RMCA)

Main Features of RMCA

- Employs a *priority heap* of potential assignments based on their *marginal cost*
- Agents receive tasks and update paths and priorities until all tasks are allocated

Limitations of RMCA

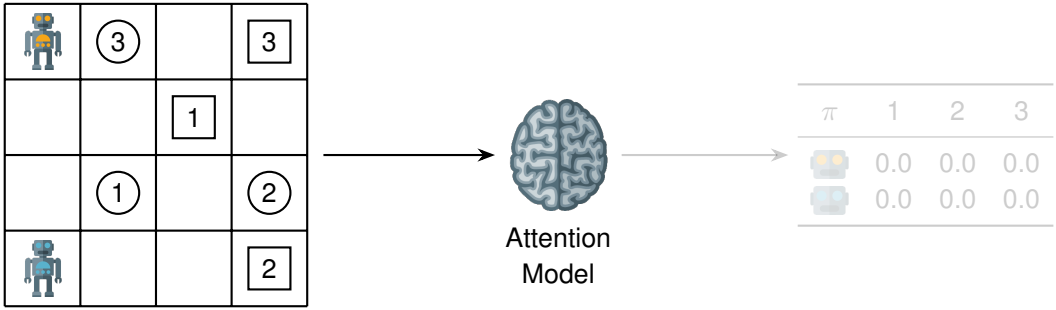
- The assignment is performed with an incremental greedy heuristic
- The impact of the current decision on future ones is disregarded

Our Proposal

Learn the task assignment policy with an *attention model*

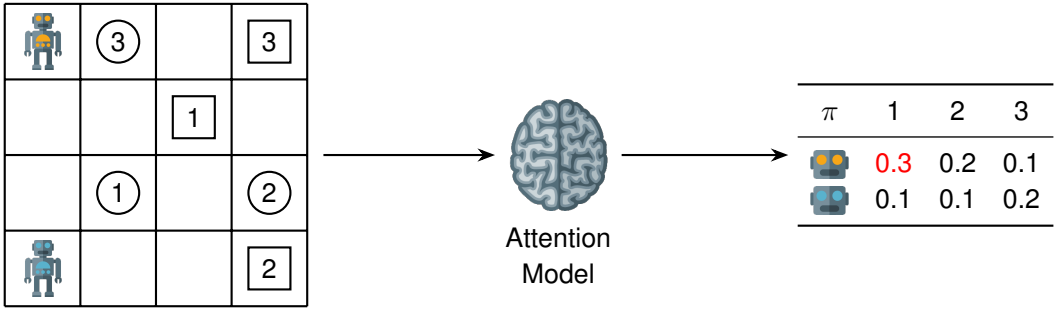
Task Assignment with Our Attention Model

[FENOY et al., 2024a]



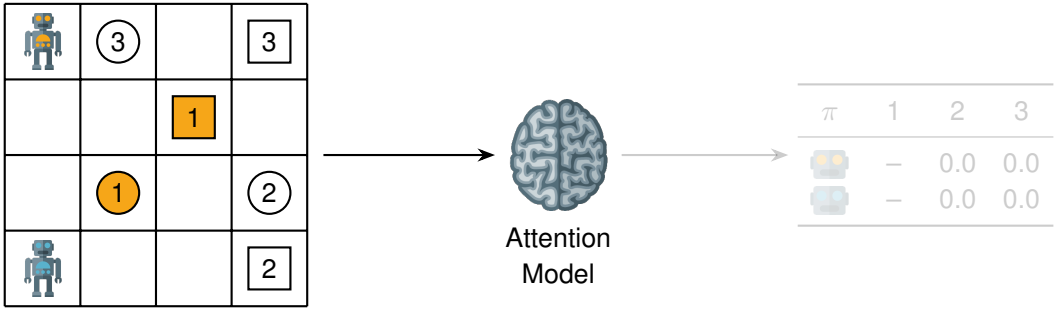
Task Assignment with Our Attention Model

[FENOY et al., 2024a]



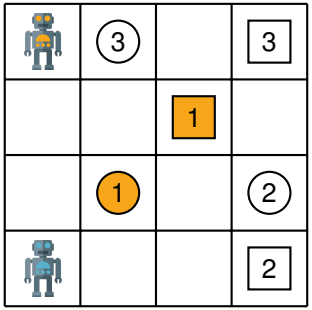
Task Assignment with Our Attention Model

[FENOY et al., 2024a]



Task Assignment with Our Attention Model

[FENOY et al., 2024a]

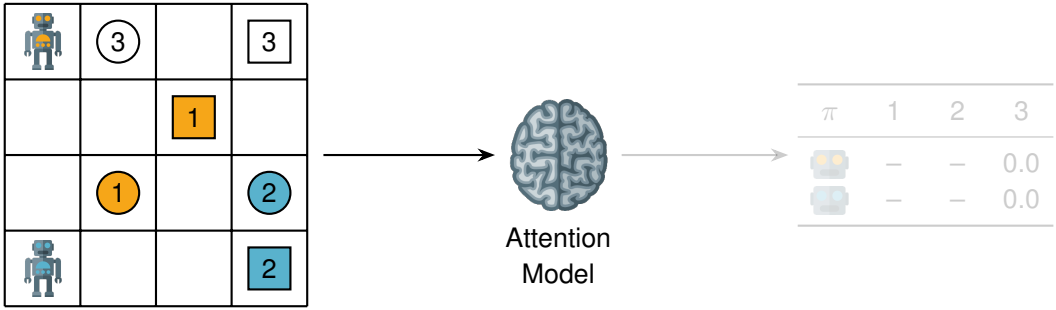


Attention Model

π	1	2	3
	–	0.1	0.3
	–	0.5	0.1

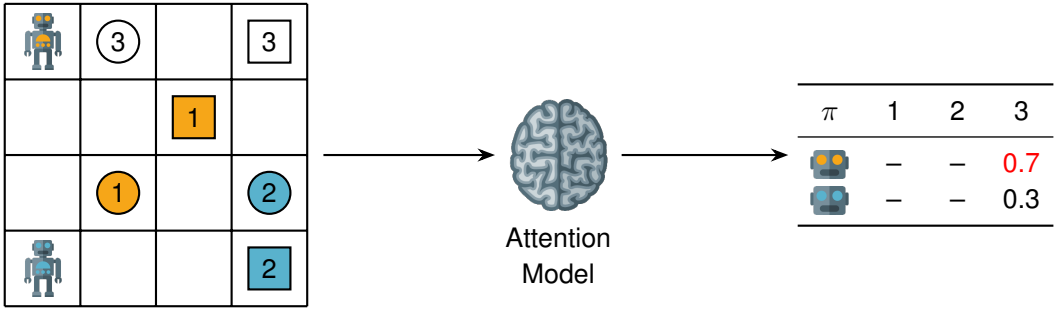
Task Assignment with Our Attention Model

[FENOY et al., 2024a]



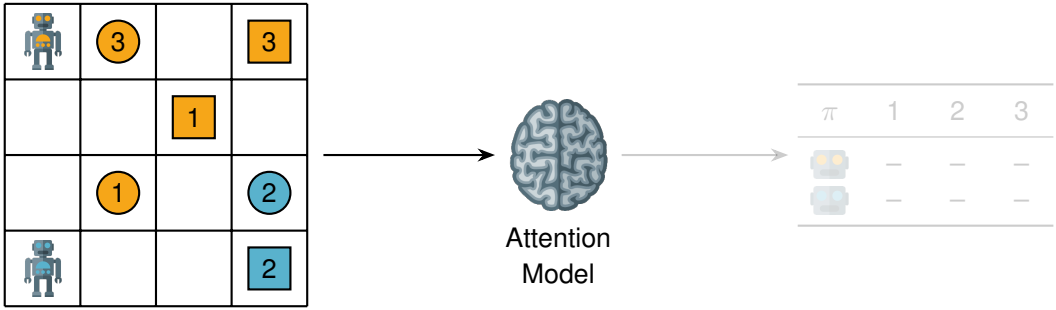
Task Assignment with Our Attention Model

[FENOY et al., 2024a]



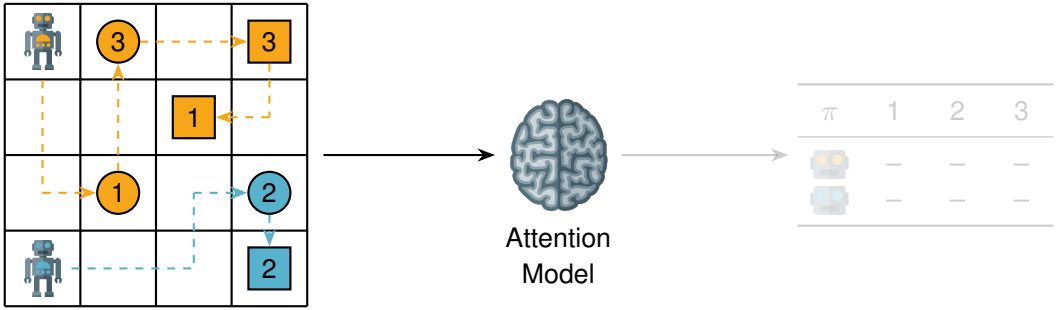
Task Assignment with Our Attention Model

[FENOY et al., 2024a]



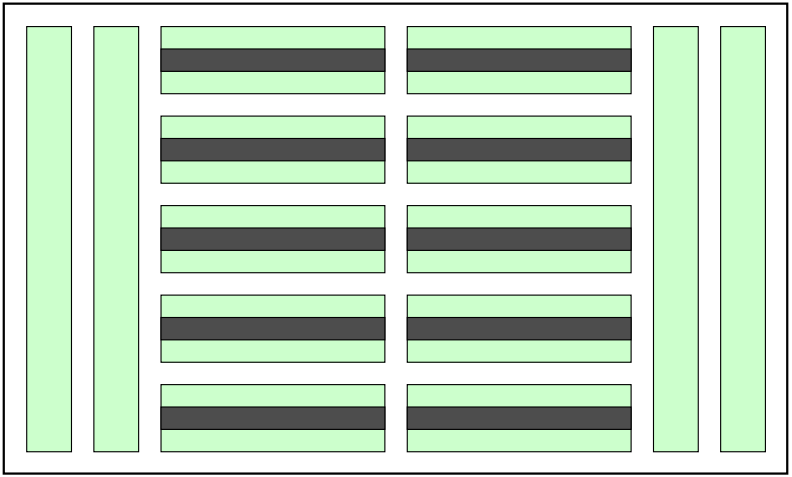
Compute Conflict-Free Paths for Pick-Up and Delivery

[FENOY et al., 2024a]



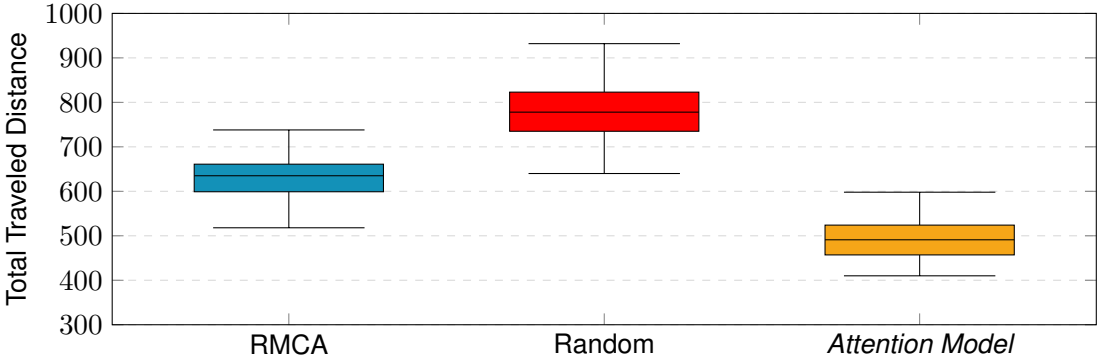
Experiments on Well-Formed Infrastructure

[FENOY et al., 2024a]



Experimental Results (10 Agents, 20 Tasks)

[FENOY et al., 2024a]



Today's Menu

Distributed Constraint Optimization

Auctions

Coalition Formation on MAS

Real-World Applications

Discussion and Wrap-up

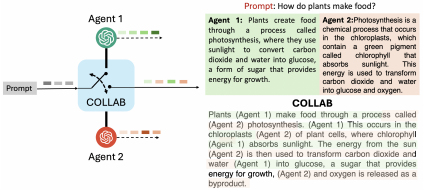
Installing Coordination with LLMs

[CHAKRABORTY et al., 2025; MAHMUD et al., 2025a,b]

DCOPs for LLMs

Coordinating Multi-LLM Systems

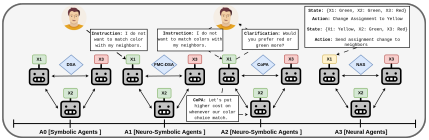
Using DCOP frameworks to manage the output of multiple LLM agents to optimize joint responses and alignment



LLMs for DCOPs

LLMs as Optimizers

Leveraging reasoning capabilities to formulate constraints from natural language (e.g. VL-DCOPs) or to act as heuristic solvers for complex optimization problems



LLMs in Auctions

[FILIPPAS et al., 2024; SHAH et al., 2025]

LLMs as Economic Agents

- **Strategic Bidding:** LLMs can interpret auction context and formulate strategies in natural language games
- **Emergent Collusion:** LLM agents spontaneously learn to collude to maximize collective reward, even without explicit instruction
- **Mechanism Design:** This poses a new challenge for designing "LLM-proof" auctions that are robust to such tactit collusion

Challenges & Opportunities

Synergizing Classical Multi-Agent Models with Generative Agency

Contemporary discourse on Agentic AI frequently suffers from historical amnesia, rediscovering established Multi-Agent Systems principles while simultaneously producing ad-hoc, over-engineered architectures that are tailored to the quirks of current models rather than sound engineering

- **Reinterpreting** Agent and Multi-Agent contributions in the age of large models: look at 50 years of active research!
- Grounded models and algorithms are as many opportunities to **improve the use of large models**
- Large models are powerful bricks to **augment agents**

Discussion and Wrap-up

What We've Seen Today

- Three major multi-agent optimization frameworks: **DCOP**, **Auctions**, and **CF**
  - DCOP: how to **collectively solve** constraint optimization problems
  - Aucitons : how to **allocate tasks/resources** to agents
  - CF: how to **form coalitions/groups** with respect to some criteria and constraints
- Various **techniques and algorithms** to attach these problems
- Examples of **applications** in the transportation and space robotics domains

Discussion and Wrap-up

Open questions

Coalition formation

- How can we **improve** the **scalability** of CF approaches?
- How can we **improve** the **generality** of CF approaches?
- Can **Machine Learning** help with these challenges?

Distributed constraint optimization

- How to **decompose or regroup** as to reduce **interactions**?
- How to **structure** the system as to improve **parallelism**?
- How to deploy robust and resilient systems in **dynamic environments**?

Common questions

- How to use DCOPs in CF and vice versa?
- How different are Auctions and DCOPs?
- Maintaining libraries and data sets

Special Thanks

Special thanks to all previous contributors to tutorials on multi-agent optimization and related topics, notably

Ferdinando Fioretto, Long Tran-Thanh, Pierre Rust, Enrico Pontelli, William Yeoh, Jesus Cerquides, Juan Antonio Rodriguez Aguilar, Alessandro Farinelli, Pedro Meseguer, Sarvapali Ramchurn, Amnon Meisels

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