Exploring Monotonicity in Ranked Choice Voting

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What is Ranked Choice Voting

Ranked Choice Voting (RCV) is a newly popular method of voting. The primary idea behind it is that it takes your preference of candidate instead of your top choice which is seen in usual voting methods. Unlike conventional voting methods, where you would choose your favorite candidate, you would rank the candidates in order of preference. The winner is selected by going through rounds of elimination. In round 1, if a candidate has a majority (50% of the votes) that candidate is the winner. If no one has the majority, the candidate with the least number of first place votes gets eliminated, we go into round 2 and so on.

What is Monotonicity?

There are a few properties that each Voting System is desired to fulfill. For example, Anonymity is a property where the voting system has to make sure that all voters are treated equally; that is if one is switch ballots with another voter, the result is to remain the same. In this research, we mainly focused on Monotonicity:

- **Monotonicity**
  A ranked voting system is monotonic if it is neither possible to prevent the election of a candidate by ranking them higher on some of the ballots, nor possible to elect an otherwise unelected candidate by ranking them lower on some of the ballots (while nothing else is altered on any ballot).

Why is this a Problem?

We do know that RCV fails Monotonicity, and so do other voting systems. No voting system is perfect; hence, these properties are 'desired' and not 'required'. We are more interested in how often does RCV fail Monotonicity. And that's where our Literature Search began...

Literature Search

Public Choice (2014) 161:1–9
DOI 10.1007/s11127-013-0118-2

Frequency of monotonicity failure under Instant Runoff Voting: estimates based on a spatial model of elections

Joseph T. Ornstein · Robert Z. Norman

After a comprehensive literature search, we found a couple of interesting papers that deal with the frequency of monotonicity failure in a 3-candidate election. The first paper from Dr. Crispin Allard (1996) totally rules out the possibility of monotonicity failure. The second paper, by Joe Ornstein and Robert Norman, discuss if and only if conditions for upward monotonicity failure (candidate goes from winning to losing the election with an increase in number of votes) which decides if a voting profile (collection of ballots) is Monotonic or not.
We do have to keep in mind that both papers didn't have real world data with them and the model used by them was based on empirical data.

Implementing Real World Data

RCV Results Report

Customer Name:	Alameda County	Election Date:	11/8/2016
Election Name:	ALA_20161108_E	Run Date:	11/18/2016
Contest:	**Member, City Council, Dist. 2 - Berkeley (RCV)**	Load Type:	Complete

Run Id: 104 - Pass 0 **Pass Number: 0**

Final State

Candidate		Votes	% Vote
CHERYL DAVILA		2215	31.03%
NANCI IRA ARMSTRONG-TEMPLE		2086	29.22%
DARRYL MOORE		2837	39.75%
Write-In		0	0.00%
	Continuing Ballots	**7138**	
Exhausted by Over Votes		17	
Under Votes		968	
Exhausted Ballots		0	
	Total Ballots	**8123**	

Figure 1:Berkley RCV Election 2016

Since the Ornstein-Norman paper was based on non-empirical data, we wanted to apply these conditions to real world data. After implementing data from the Bay Area 2016 elections, we found one profile from Berkeley - District 2 that failed Monotonicity. This profile met the required conditions but we would require ballot to ballot data to come up with a different profile where a different candidate is the RCV winner.

Conditions and Results

Let A, B, and C be three candidates in the RCV election. Let a, b and c be the corresponding number of votes the candidates get respectively; V is the total number of votes. Assume A is not the majority winner, B gets eliminated in round 1 and C is the RCV winner The if and only if statements are:

$$c > \frac{V+2}{4} \tag{1}$$

and

$$c + b_2 > \frac{V}{2} \tag{2}$$

where b_2 is a subset of the total votes where the voter prefer candidate B over C over A.
Using empirical data, the frequency of upward monotoncity failure was found to be somewhere between 0.7% and 51%.

Combinatorial Count

A more theoretical approach to the Monotonicity problem in RCV would be to an Exhaustible Combinatorial Count. If we were to consider 100 votes in a 3 candidate RCV election, the total number of rank orderings would be 6 (3!). Now, if we look at the all the possible combinations between the 100 votes and the 6 rank orderings, we would get:

$$C(105, 5) = 96,560,646 \text{ combinations} \tag{3}$$

To restrict the number of possibilities we deal with, we can assume that all 6 rank orderings will get at least 2% of the votes. Now we can venture on how many of these possibilities will fit the conditions given by the paper. This would give us a lower bound on the Monotonicity Failure from a theoretical point of view.

Future Ventures

There are definitely avenues to venture on when it comes to RCV and Monotonicity. There is certainly more data out there to test out the conditions from the Ornstein-Norman paper. Here are some more areas to venture on:

1. Continue with the Exhaustible Combinatorial Count of all Possibilities
2. Comparing Real World Data to the Model
3. Finding Conditions for 4 or more candidate elections
4. Adjusting the Ornstein-Norman model to incorporate real world voter distribution patterns.

References

[1] Joseph T. Ornstein, Robert Z. Norman *Frequency of monotonicity failure under Instant Runoff Voting: estimates based on a spatial model of elections.*
Published online: 17 October 2013 © Springer Science+Business Media New York 2013

[2] Crispin Allard
Estimating the Probability of Monotonicity Failure in a UK General Election Voting matters - Issue 5, January 1996

[3] RCV Data
`https://www.acvote.org/election-information/archived-elections`

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