# 分析力学的拉格朗日体系

## 本节导读

- 拉格朗日方程
- 泛函简介
- 哈密顿原理
- 循环积分与能量积分、诺特定理

# 基本形式的拉格朗日方程

$$\sum_i (\vec{F}_i - m_i \ddot{\vec{r}}_i) \cdot \delta \vec{r}_i = 0$$ 达朗贝尔原理

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T}{\partial \dot{q}_\alpha} - \frac{\partial T}{\partial q_\alpha} = Q_\alpha \qquad (\alpha = 1, 2, \cdots, s)$$

<u>——基本形式的拉格朗日方程</u>

# 保守系的拉格朗日方程

$$Q_\alpha = -\frac{\partial V}{\partial q_\alpha}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T}{\partial \dot{q}_\alpha} - \frac{\partial T}{\partial q_\alpha} = -\frac{\partial V}{\partial q_\alpha} \quad (\alpha = 1, 2, \cdots, s)$$

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_\alpha} - \frac{\partial L}{\partial q_\alpha} = 0 \quad (\alpha = 1, 2, \cdots, s)}$$

其中L=T-V，叫做拉格朗日函数

# 拉格朗日方程

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_\alpha}-\frac{\partial L}{\partial q_\alpha}=0 \quad (\alpha=1,2,\cdots,s)$$

——拉格朗日方程

其中，$L(q_\alpha,\dot{q}_\alpha,t)$叫做拉格朗日函数或拉氏量

$P_\alpha \equiv \dfrac{\partial L}{\partial \dot{q}_\alpha}$，　为广义坐标$q_\alpha$对应的广义动量

# 几种系统的拉格朗日函数举例

## 在电磁场中运动的带电粒子

$$L = \frac{1}{2}mv^2 - Q(\varphi - \vec{v} \cdot \vec{A})$$

$\varphi$和$\vec{A}$为标势和矢势，满足：

$$\boxed{\vec{E} = \vec{\nabla}\varphi}$$

$$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$$

# 几种系统的拉格朗日函数举例

**电磁场**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{c}J_\mu A^\mu$$

四维矢势 $A^\mu = (\phi/c, \mathbf{A})$

四维流密度 $J^\mu = (c\rho, \mathbf{j})$

电磁张量
$$F_{\alpha\beta} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

# 几种系统的拉格朗日函数举例

引力场

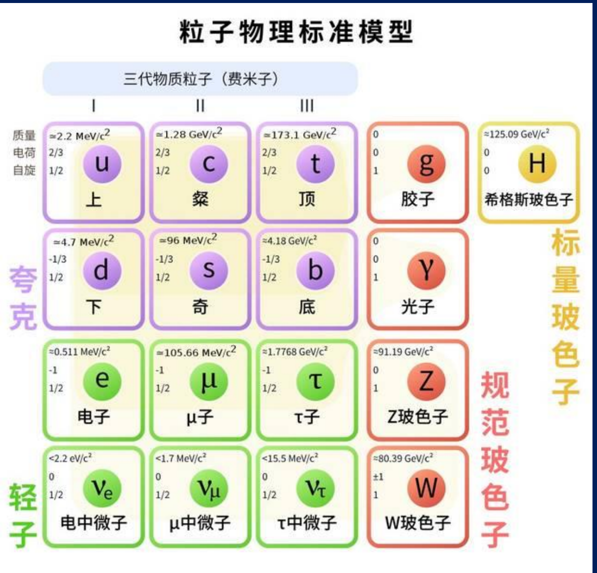
$$\mathcal{L} = \frac{1}{4\pi G} R\sqrt{-g}$$

时空度规的行列式 $$g = \frac{1}{c^2} \det\left(g_{\alpha\beta}\right)$$

里奇标量 $R$

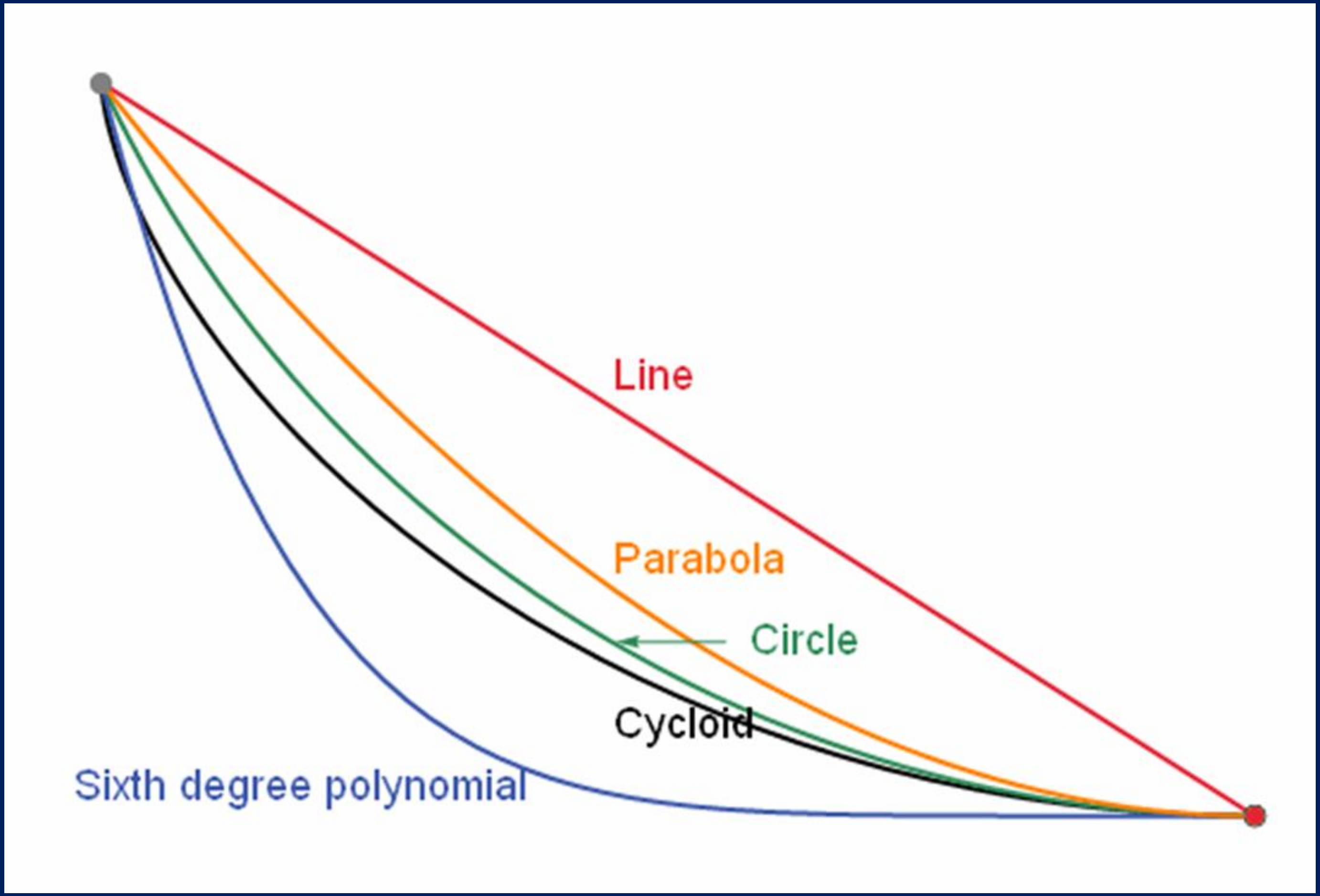
# 几种系统的拉格朗日函数举例

## 粒子物理标准模型




$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) && \text{(U(1), SU(2) and SU(3) gauge terms)} \\
& +(\bar{\nu}_L, \bar{e}_L)\tilde{\sigma}^\mu iD_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \bar{e}_R\sigma^\mu iD_\mu e_R + \bar{\nu}_R\sigma^\mu iD_\mu \nu_R + \text{(h.c.)} && \text{(lepton dynamical term)} \\
& -\frac{\sqrt{2}}{v}\left[(\bar{\nu}_L, \bar{e}_L)\phi M^e e_R + \bar{e}_R \bar{M}^e \bar{\phi}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}\right] && \text{(electron, muon, tauon mass term)} \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{e}_L, \bar{\nu}_L)\phi^* M^\nu \nu_R + \bar{\nu}_R \bar{M}^\nu \phi^T\begin{pmatrix} -e_L \\ \nu_L \end{pmatrix}\right] && \text{(neutrino mass term)} \\
& +(\bar{u}_L, \bar{d}_L)\tilde{\sigma}^\mu iD_\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix} + \bar{u}_R\sigma^\mu iD_\mu u_R + \bar{d}_R\sigma^\mu iD_\mu d_R + \text{(h.c.)} && \text{(quark dynamical term)} \\
& -\frac{\sqrt{2}}{v}\left[(\bar{u}_L, \bar{d}_L)\phi M^d d_R + \bar{d}_R \bar{M}^d \bar{\phi}\begin{pmatrix} u_L \\ d_L \end{pmatrix}\right] && \text{(down, strange, bottom mass term)} \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{d}_L, \bar{u}_L)\phi^* M^u u_R + \bar{u}_R \bar{M}^u \phi^T\begin{pmatrix} -d_L \\ u_L \end{pmatrix}\right] && \text{(up, charmed, top mass term)} \\
& +\overline{(D_\mu\phi)}D^\mu\phi - m_h^2[\bar{\phi}\phi - v^2/2]^2/2v^2. && \text{(Higgs dynamical and mass term)} \quad (1)
\end{aligned}
$$

# 捷线问题



# 捷线问题