

分析力学的拉格朗日体系

本节导读

- 拉格朗日方程
- 泛函简介
- 哈密顿原理
- 循环积分与能量积分、诺特定理

基本形式的拉格朗日方程

$$\sum_i (\vec{F}_i - m_i \ddot{\vec{r}}_i) \cdot \delta \vec{r}_i = 0 \quad \text{达朗贝尔原理}$$



$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_\alpha} - \frac{\partial T}{\partial q_\alpha} = Q_\alpha \quad (\alpha = 1, 2, \dots, s)$$

——基本形式的拉格朗日方程

保守系的拉格朗日方程

$$Q_{\alpha} = -\frac{\partial V}{\partial q_{\alpha}}$$



$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_{\alpha}} - \frac{\partial T}{\partial q_{\alpha}} = -\frac{\partial V}{\partial q_{\alpha}} \quad (\alpha = 1, 2, \dots, s)$$



$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_{\alpha}} - \frac{\partial L}{\partial q_{\alpha}} = 0 \quad (\alpha = 1, 2, \dots, s)$$

其中 $L = T - V$, 叫做拉格朗日函数

拉格朗日方程

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\alpha} - \frac{\partial L}{\partial q_\alpha} = 0 \quad (\alpha = 1, 2, \dots, s)$$

——拉格朗日方程

其中, $L(q_\alpha, \dot{q}_\alpha, t)$ 叫做拉格朗日函数或拉氏量

$P_\alpha \equiv \frac{\partial L}{\partial \dot{q}_\alpha}$, 为广义坐标 q_α 对应的广义动量

几种系统的拉格朗日函数举例

在电磁场中运动的带电粒子

$$L = \frac{1}{2}mv^2 - Q(\varphi - \vec{v} \cdot \vec{A})$$

φ 和 \vec{A} 为标势和矢势，满足：

$$\vec{E} = -\vec{\nabla}\varphi$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

几种系统的拉格朗日函数举例

电磁场

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{c}J_{\mu}A^{\mu}$$

四维矢势

$$A^{\mu} = (\phi/c, \mathbf{A})$$

四维流密度

$$J^{\mu} = (c\rho, \mathbf{j})$$

电磁张量

$$F_{\alpha\beta} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

几种系统的拉格朗日函数举例

引力场

$$\mathcal{L} = \frac{1}{4\pi G} R \sqrt{-g}$$

时空度规的行列式

$$g = \frac{1}{c^2} \det(g_{\alpha\beta})$$

里奇标量

R

几种系统的拉格朗日函数举例

粒子物理标准模型

粒子物理标准模型					
三代物质粒子 (费米子)					
	I	II	III		
质量	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 125.09 \text{ GeV}/c^2$
电荷	$2/3$	$2/3$	$2/3$	0	0
自旋	$1/2$	$1/2$	$1/2$	1	0
	u 上	c 粲	t 顶	g 胶子	H 希格斯玻色子
夸克	$\approx 4.7 \text{ MeV}/c^2$	$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	$-1/3$	$-1/3$	$-1/3$	0	
	d 下	s 奇	b 底	γ 光子	
轻子	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 91.19 \text{ GeV}/c^2$	
	-1	-1	-1	0	
	e 电子	μ μ子	τ τ子	Z Z玻色子	
	$< 2.2 \text{ eV}/c^2$	$< 1.7 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$	
	$1/2$	$1/2$	$1/2$	± 1	
	ν_e 电中微子	ν_μ μ中微子	ν_τ τ中微子	W W玻色子	

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}\text{tr}(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}\text{tr}(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) \\
 & + (\bar{\nu}_L, \bar{e}_L) \tilde{\sigma}^\mu i D_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \bar{e}_R \sigma^\mu i D_\mu e_R + \bar{\nu}_R \sigma^\mu i D_\mu \nu_R + (\text{h.c.}) \\
 & - \frac{\sqrt{2}}{v} \left[(\bar{\nu}_L, \bar{e}_L) \phi M^e e_R + \bar{e}_R \tilde{M}^e \tilde{\phi} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \right] \\
 & - \frac{\sqrt{2}}{v} \left[(-\bar{e}_L, \bar{\nu}_L) \phi^* M^\nu \nu_R + \bar{\nu}_R \tilde{M}^\nu \phi^T \begin{pmatrix} -e_L \\ \nu_L \end{pmatrix} \right] \\
 & + (\bar{u}_L, \bar{d}_L) \tilde{\sigma}^\mu i D_\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix} + \bar{u}_R \sigma^\mu i D_\mu u_R + \bar{d}_R \sigma^\mu i D_\mu d_R + (\text{h.c.}) \\
 & - \frac{\sqrt{2}}{v} \left[(\bar{u}_L, \bar{d}_L) \phi M^d d_R + \bar{d}_R \tilde{M}^d \tilde{\phi} \begin{pmatrix} u_L \\ d_L \end{pmatrix} \right] \\
 & - \frac{\sqrt{2}}{v} \left[(-\bar{d}_L, \bar{u}_L) \phi^* M^u u_R + \bar{u}_R \tilde{M}^u \phi^T \begin{pmatrix} -d_L \\ u_L \end{pmatrix} \right] \\
 & + \overline{(D_\mu \phi)} D^\mu \phi - m_h^2 [\bar{\phi} \phi - v^2/2]^2 / 2v^2.
 \end{aligned}$$

(U(1), SU(2) and SU(3) gauge terms)

(lepton dynamical term)

(electron, muon, tauon mass term)

(neutrino mass term)

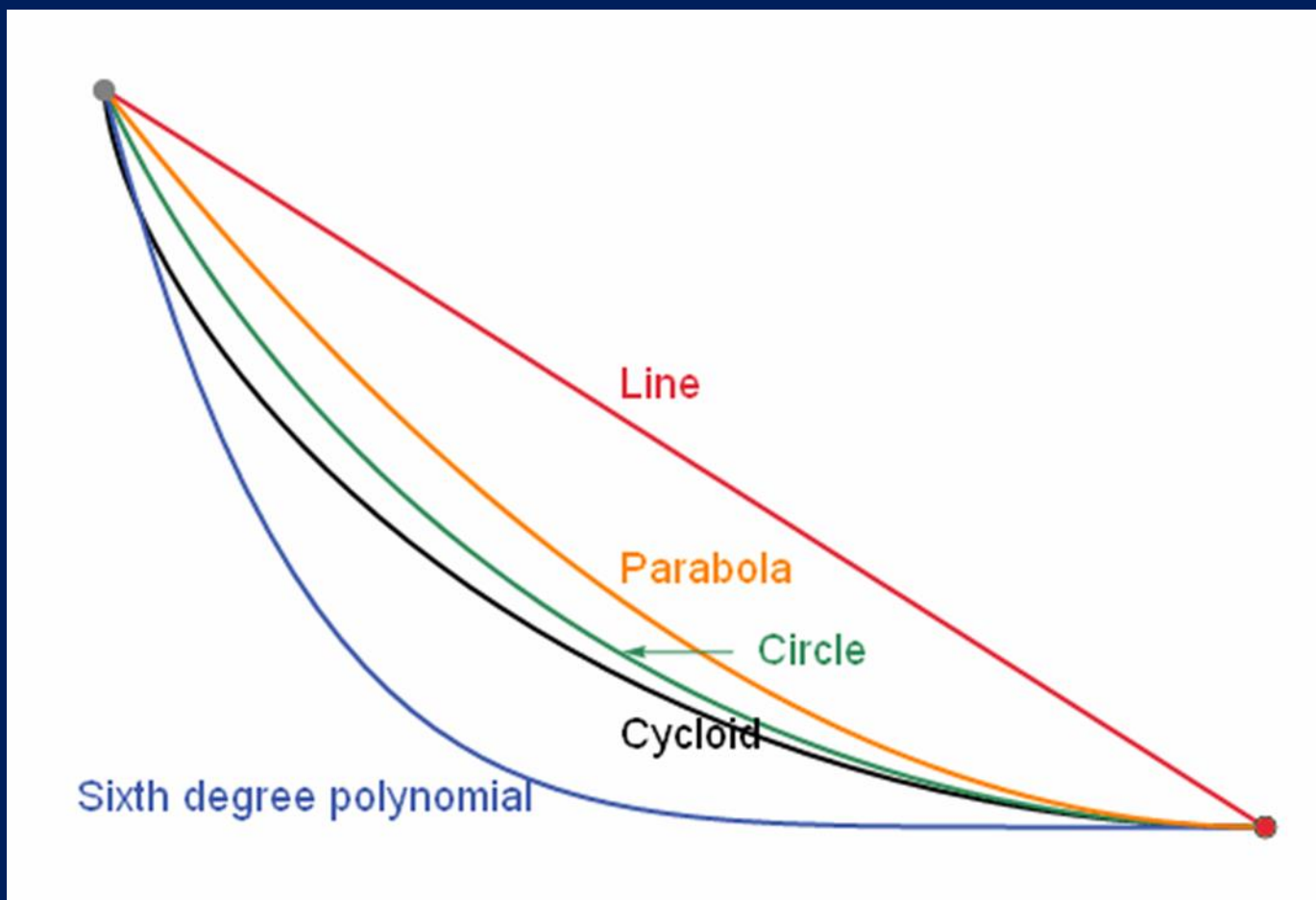
(quark dynamical term)

(down, strange, bottom mass term)

(up, charmed, top mass term)

(Higgs dynamical and mass term) (1)

捷线问题



捷线问题

