APPENDIX A

A. Preliminaries

We first introduce a general recursion as a minor modification of [19, eq. (83)-(85)]. Consider an $I \times J$ random matrix $\mathbf{A} \in \mathbb{C}^{I \times J}$ with i.i.d. entries $a_{ij} \sim \mathcal{CN}(a_{ij}; 0, \zeta_a/I)$. Define a set $\{\boldsymbol{\theta}_{i_n}(l) \in \mathbb{C}^{k_n} | n = 1, ..., n_I.\ i_n = \sum_{k=0}^{n-1} I_k + 1, ..., \sum_{k=0}^{n} I_k.\ l = 0, ..., L-1.\}$ with $I_0 = 0$ and $\sum_{n=1}^{n_I} I_n = I$. Similarly, define a set $\{\boldsymbol{\varphi}_{j_n}(l) \in \mathbb{C}^{q_n} | n = 1, ..., n_J.\ j_n = \sum_{k=0}^{n-1} J_k + 1, ..., \sum_{k=0}^{n} J_k.\ l = 0, ..., L-1.\}$ with $J_0 = 0$ and $\sum_{n=1}^{n_J} J_n = J$. The recursion below involves the updates of $\mathbf{q}(l), \mathbf{m}_1(l), \mathbf{m}_2(l) \in \mathbb{C}^J$ and $\mathbf{e}_1(l), \mathbf{e}_2(l), \mathbf{v}(l) \in \mathbb{C}^I$. Specifically, given $\{\boldsymbol{\theta}_{i_n}(l)\}$ and $\{\boldsymbol{\varphi}_{j_n}(l)\}$, we have

$$\mathbf{q}(l+1) = \mathbf{A}^H \mathbf{v}(l) - (\mathbf{m}_1(l), \mathbf{m}_2(l))\, \boldsymbol{\xi}(l)$$
$$v_{i_n}(l) = g_{l,n}\left(e_{1i_n}(l), e_{2i_n}(l), \boldsymbol{\theta}_{i_n}(l)\right) \tag{55a}$$
$$(\mathbf{e}_1(l), \mathbf{e}_2(l)) = \mathbf{A}\,(\mathbf{m}_1(l), \mathbf{m}_2(l)) - \mathbf{v}(l-1)\boldsymbol{\gamma}(l)^T$$
$$(m_{1j_n}(l), m_{2j_n}(l)) = f_{l,n}\left(q_{j_n}(l), \boldsymbol{\varphi}_{j_n}(l)\right) \tag{55b}$$

where $\boldsymbol{\xi}(l) = (\frac{\zeta_a}{I}\sum_{n,i_n} \frac{\partial v_{i_n}(l)}{\partial e_{1i_n}(l)}, \frac{\zeta_a}{I}\sum_{i}\frac{\partial v_{i_n}(l)}{\partial e_{2i_n}(l)})^T \in \mathbb{R}^2$, $\boldsymbol{\gamma}(l) = (\frac{\zeta_a}{I}\sum_{n,j_n}\frac{\partial m_{1j_n}(l)}{\partial q_{j_n}(l)}, \frac{\zeta_a}{I}\sum_{i}\frac{\partial m_{2j_n}(l)}{\partial q_{j_n}(l)})^T \in \mathbb{R}^2$, and $\mathbf{v}(-1) = \mathbf{0}$. (55) reduces to [19, eq. (83)-(85)] by letting $n_I = n_J = 1$, and letting $\boldsymbol{\theta}_{i_n}(l)$ and $\boldsymbol{\varphi}_{j_n}(l)$ invariant to recursion number l. We note that in each iteration, both $\mathbf{q}(l)$ and $(\mathbf{e}_1(l), \mathbf{e}_2(l))$ are updated through a linear mixing of $\mathbf{v}(l)$ (or $(\mathbf{m}_1(l), \mathbf{m}_2(l))$) by random Gaussian matrix $\mathbf{A}$ (or $\mathbf{A}^H$) together with a point-wise subtraction. The linear mixing makes $\mathbf{q}(l)$ (or $(\mathbf{e}_1(l), \mathbf{e}_2(l))$) distributed as Gaussian vectors (or matrix) in the large system limit and the point-wise subtraction removes the correlation of the components of $\mathbf{q}(l)$ (or the rows of $(\mathbf{e}_1(l), \mathbf{e}_2(l))$). Furthermore, due to symmetry, the components of $\mathbf{q}(l)$ (or the rows of $(\mathbf{e}_1(l), \mathbf{e}_2(l))$) have the same distribution.

To formally describe the asymptotic properties of $\mathbf{q}(l+1)$ and $(\mathbf{e}_1(l), \mathbf{e}_2(l))$, we introduce some definitions by following [20]. We say that a function $\phi(\cdot) : \mathbb{C}^m \to \mathbb{C}$ is *pseudo-Lipschitz* of order 2, if there exists a constant $c > 0$ such that, for any $\mathbf{x}, \mathbf{y} \in \mathbb{C}^m$: $|\phi(\mathbf{x}) - \phi(\mathbf{y})| \leq c\,(1 + \|\mathbf{x}\| + \|\mathbf{y}\|)\,\|\mathbf{x} - \mathbf{y}\|$. We say that the empirical distribution of vector sequences $\mathbf{x}_i, i = 1, ..., N$ (denoted by $\hat{p}_\mathbf{x}$) converges weakly to a probability density function $p(\tilde{\mathbf{x}})$ if $\lim_{N \to \infty} \mathbb{E}_{\hat{p}_\mathbf{x}}[\psi(x)] = E_{p_{\tilde{\mathbf{x}}}}[\psi(x)]$ for any bounded continuous function $\psi(\cdot)$. Our goal is to characterize the distribution of $\mathbf{q}(l+1)$ and $(\mathbf{e}_1(l), \mathbf{e}_2(l))$ conditioned on the quantities previously calculated and used in (55). To this end, define $\mathfrak{S}_{l_1,l_2}$ as the probability space of $\{\mathbf{q}(l)\}_{l=0}^{l_1-1}, \{\mathbf{v}(l)\}_{l=0}^{l_1-1}, \{\boldsymbol{\theta}_{i_n}(l)\}_{l=0}^{l_1-1}$, $\{\boldsymbol{\varphi}_{j_n}(l)\}_{l=1}^{l_2}, \{\mathbf{e}_1(l), \mathbf{e}_2(l)\}_{l=1}^{l_2}$, and $\{\mathbf{m}_1(l), \mathbf{m}_2(l)\}_{l=0}^{l_2}$. Define matrix $\mathbf{V}_l = [\mathbf{v}(0), ..., \mathbf{v}(l-1)]$ and $\mathbf{M}_l = [\mathbf{m}_1(0), \mathbf{m}_2(0)..., \mathbf{m}_1(l-1), \mathbf{m}_2(l-1)]$. Then we express $\mathbf{v}(l)$ as $\mathbf{v}(l) = \mathbf{v}_\|(l) + \mathbf{v}_\perp(l)$, where $\mathbf{v}_\|(l)$ is the orthogonal projection of $\mathbf{v}(l)$ onto the column space of $\mathbf{V}_l$, and $\mathbf{v}_\perp(l)$ is a vector in the orthogonal complementary space of the column space of $\mathbf{V}_l$. Furthermore, $\mathbf{v}_\|(l)$ can be expressed as $\mathbf{v}_\|(l) = \sum_{i=1}^{l-1}\alpha_i \mathbf{v}(i)$ with α_i representing the i-th projection coefficient. Analogously to the expression of $\mathbf{v}(l)$, let $(\mathbf{m}_1(l), \mathbf{m}_2(l)) = (\mathbf{m}_{1,\|}(l), \mathbf{m}_{2,\|}(l)) + (\mathbf{m}_{1,\perp}(l), \mathbf{m}_{2,\perp}(l))$

where $(\mathbf{m}_{1,\|}(l), \mathbf{m}_{2,\|}(l)) = \sum_{i=0}^{l-1}(\mathbf{m}_1(i), \mathbf{m}_2(i))\boldsymbol{\beta}_i$ with $\boldsymbol{\beta}_i \in \mathbb{R}^{2 \times 2}$.

The state variables of the recursion (55) are $\tau_q(l) \in \mathbb{R}$ and $\boldsymbol{\Sigma}(l) \in \mathbb{C}^{2 \times 2}$ given by

$$\tau_q(l) = \zeta_a \sum_{n=1}^{n_I} \frac{I_n}{I}\mathbb{E}\left[\left|g_{l,n}\left(\sqrt{\boldsymbol{\Sigma}(l)}\mathbf{n}, \tilde{\boldsymbol{\theta}}_n(l)\right)\right|^2\right] \tag{56a}$$

$$\boldsymbol{\Sigma}(l) = \zeta_a \sum_{n=1}^{n_J} \frac{J_n}{I}\mathbb{E}\Big[f_{l,n}(\sqrt{\tau_q(l)}w_q, \tilde{\boldsymbol{\varphi}}_n(l))^H \times$$
$$f_{l,n}(\sqrt{\tau_q(l)}w_q, \tilde{\boldsymbol{\varphi}}_n(l))\Big] \tag{56b}$$

where $\mathbf{n} \sim \mathcal{CN}(\mathbf{n}; 0, \mathbf{I})$, $w_q \sim \mathcal{CN}(w_q; 0, 1)$, and $\boldsymbol{\Sigma}(0) = \lim_{J \to \infty}\frac{1}{J}(\mathbf{m}_1(0), \mathbf{m}_2(0))^H(\mathbf{m}_1(0), \mathbf{m}_2(0))$. The expectation $\mathbb{E}[\cdot]$ is taken over $\mathbf{n}$ and $\tilde{\boldsymbol{\theta}}_n(l)$ in (56a), and w_q and $\tilde{\boldsymbol{\varphi}}_n(l)$ in (56b).

Lemma 1. *Consider the recursion (55). Assume that the empirical distributions of $\boldsymbol{\theta}_{i_n}(l)$, $\boldsymbol{\varphi}_{j_n}(l)$, and $(m_{1j_n}(0), m_{2j_n}(0))^T$ respectively converge weakly to the probability distributions of random variables $\tilde{\boldsymbol{\theta}}_n(l)$, $\tilde{\boldsymbol{\varphi}}_n(l)$, and $(\tilde{m}_{1n}(0), \tilde{m}_{2n}(0))^T$ with bounded second moments. Further assume that the empirical second moments of those vectors respectively converge to the second moments of corresponding random variables. Assume that $g_{l,n}(\cdot)$ and $f_{l,n}(\cdot)$ are Lipschitz continuous and continuously differentiable almost everywhere with bounded derivatives. We have*

$$\mathbf{q}(l+1)|_{\mathfrak{S}_{l+1,l}}$$
$$\stackrel{\mathrm{d}}{=} \sum_{i=0}^{l-1}\alpha_i \mathbf{q}(i+1) + \tilde{\mathbf{A}}^H \mathbf{v}_\perp(l) + \tilde{\mathbf{M}}_{l+1}\mathbf{o}_{l+1} \tag{57a}$$
$$(\mathbf{e}_1(l), \mathbf{e}_2(l))|_{\mathfrak{S}_{l,l}}$$
$$\stackrel{\mathrm{d}}{=} \sum_{i=0}^{l-1}(\mathbf{e}_1(i), \mathbf{e}_2(i))\boldsymbol{\beta}_i + \tilde{\mathbf{A}}(\mathbf{m}_{1,\perp}(l), \mathbf{m}_{2,\perp}(l)) + \tilde{\mathbf{V}}_l(\mathbf{o}_l, \mathbf{o}_l) \tag{57b}$$

where $\tilde{\mathbf{A}}$ is an independent copy of $\mathbf{A}$; the columns of $\tilde{\mathbf{M}}_l$ (or $\tilde{\mathbf{V}}_l$) form an orthogonal basis of the column space of $\mathbf{M}_l$ (or $\mathbf{V}_l$) with $\tilde{\mathbf{M}}_l^H \tilde{\mathbf{M}}_l = N\mathbf{I}_{l \times l}$ (or $\tilde{\mathbf{V}}_l^H \tilde{\mathbf{V}}_l = J\mathbf{I}_{l \times l}$; $\mathbf{o}_l$ is a vector of length l whose elements converge to zero almost surely as $I, J \to \infty$. For any pseudo-Lipschitz functions $\phi_q(\cdot)$ and $\phi_e(\cdot)$ of order 2, we have

$$\lim_{J_n \to \infty} \frac{1}{J_n}\sum_{j_n}\phi_q\left(q_{j_n}(l), \boldsymbol{\varphi}_{j_n}(l)\right)$$
$$\stackrel{a.s.}{=} \mathbb{E}\left[\phi_q\left(\sqrt{\tau_q(l-1)}w_q, \tilde{\boldsymbol{\varphi}}_n(l)\right)\right], n = 1, ..., n_J \tag{58a}$$

$$\lim_{I_n \to \infty} \frac{1}{I_n}\sum_{i_n}\phi_e\left(e_{1i_n}(l), e_{2i_n}(l), \boldsymbol{\theta}_{i_n}(l)\right)$$
$$\stackrel{a.s.}{=} \mathbb{E}\left[\phi_e\left(\sqrt{\boldsymbol{\Sigma}(l)}\mathbf{n}, \tilde{\boldsymbol{\theta}}_n(l)\right)\right], n = 1, ..., n_I \tag{58b}$$

where $w_q \sim \mathcal{CN}(w_q; 0, 1)$ is independent of $\tilde{\boldsymbol{\varphi}}_n(l)$ and $\mathbf{n} \sim \mathcal{CN}(\mathbf{n}; 0, \mathbf{I})$ is independent of $\tilde{\boldsymbol{\theta}}_n(l)$; $a \stackrel{a.s.}{=} b$ represent a equals b almost surely.

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