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Volatility cones and volatility arbitrage strategies – empirical study based on SSE ETF option

Hong Yu Xin Pan and Jun Song
Fudan University, Shanghai, China

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Abstract

Purpose – Using volatility cones as the estimate of actual volatility instead of GARCH models, the purpose of this paper is to explore whether volatility arbitrage strategy can provide positive profits and how the transaction costs existed in the real market affect the effectiveness of volatility arbitrage strategy.

Design/methodology/approach – A number of hedging approaches proposed to improve the hedging results and final returns of Black-Scholes model are analyzed and compared.

Findings – The general finding is that volatility arbitrage strategy can provide satisfactory returns based on the samples in Chinese market. Regarding transaction costs, the variable bandwidth delta and delta tolerance approach showed better results. Besides, choosing futures together with ETFs as hedging underlying can increase the VaR for better risk management.

Practical implications – This paper offers a new method for volatility arbitrage in Chinese financial market.

Originality/value – This paper researches the profitability of the volatility arbitrage strategy on ETF 50 options using volatility cones method for the first time. This method has advantage over the point-wise estimation such as GARCH model and stochastic volatility model.

Keywords ETF options, Transaction costs, Option hedging, Volatility arbitrage, Volatility cones

Paper type Research paper

1. Introduction

As an indispensable risk management tool in the investment field, equity-linked options have covered the capital markets of the major developed and developing countries and have been listed in more than 20 countries and regions. Based on market demand, the transactions of the Chinese Shanghai Stock Exchange (SSE) 50 ETF options, which was listed in February 2015 (Table I), show that the total volume of this option has increased significantly since February. Its average monthly volume rose by 47.52 percent, indicating that investors have an ever-increasing demand of the options market.

With the introduction of 50 ETF options, option-involved portfolios and a number of trading strategies have expanded to practical use from the previous theoretical analysis, and the option arbitrage is a strategy with small risk and good return. The core of option price is the volatility. So volatility arbitrage is a trading strategy which can make use of the option mispricing, attracting wide attention of option traders and institutional investors.

Academics have done considerable research on volatility arbitrage products. Guo (2006) was the first scholar who provided the idea of designing volatility-trading products in China and proposed that the product can be designed by constructing the portfolio of options and underlying assets with continuous delta hedging. Liao (2009) used the GARCH model and Monte Carlo simulation to analyze the value of a FX volatility-trading product through case study. Zhang (2009) designed the volatility swap-based arbitrage strategy for volatility products in US market, and tested the effectiveness of the corresponding strategies in the Chinese market. Gao (2014) introduced volatility trading to deal with residual risk exposure of delta hedging and found that the introduction of volatility trading can improve the return of delta hedging. Wang and Liu (2016) use the VAR-structural GARCH model to analyze the

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spillover effect in Asian financial markets. Yet, Chinese scholars have completed relatively few researches on arbitrage products, and mainly for bank products. This is because the Chinese stock market did not have the conditions to vigorously develop the volatility-trading products. Besides, exchange-traded stock options have not been listed, and OTC options trading is also not active. However, with the introduction of Shanghai 50 ETF options, volatility arbitrage products and volatility arbitrage-related strategy research will gain more widespread attention.

In addition, domestic and foreign academics have also carried out theoretical research on the key issue of volatility arbitrage – hedging. After the Black-Scholes pricing model was published in 1973, some unreasonable hypotheses such as continuous hedging and non-transaction costs had been studied and perfected. Wilmott (1994) proposed discrete delta strategy to adjust the hedging ratio at fixed intervals. This strategy unleashes the assumption of continuous hedging of the B-S model. Leland (1985) was the first to initiate the research on transaction costs within hedging strategy. Hodges and Anthony (1989) pioneered the utility-based option hedging strategy and the "non-trading interval" concept, which was considered to be the optimal option hedging strategy with transaction costs and was widely used in subsequent research and empirical analysis. But such solutions based on utility model are not closed solutions and can only be obtained by numerical computation, which significantly increases difficulty of calculation. Whalley and Wilmott (1997) conducted the asymptotic analysis on Hodges and Neuberger (1989) models to solve the defect of non-closed solutions. Under the assumption of small transaction costs, Whalley and Wilmott (1997) derive an optimal form of Black-Scholes delta-centered hedging bandwidth. Among the existing domestic researches, Tang (2009), Liao and Zhang (2010), Pan (2013), and Su (2014) have done comparative analysis of all kinds of discrete hedging strategies. Pan (2012) divided the volatility into the upper part and the lower part and studied the asymmetry behaviors of the volatility. Tang (2009) found that the stochastic volatility model and the GARCH-family model behave better in pricing rather than in hedging. In a nutshell, although domestic and foreign scholars have researched on volatility arbitrage and dynamic hedging of options, the data used in analysis and simulation are based on developed market, and the previous relevant researches are mostly based on assumption of known volatility or based on GARCH model and stochastic volatility model to estimate the volatility parameters needed for volatility arbitrage. However, the point-wise estimation has questionable significance to the interval volatility and has no advantage in risk management. Furthermore, the negative basis phenomenon in current Chinese future market will also exert a certain impact on volatility arbitrage strategy while there are few papers about using futures for delta hedging.

The paper will examine the validity and profitability of the arbitrage strategy by using the volatility cone as the interval volatility forecast instead of the traditional GARCH

Table I. Transactions of Chinese SSE 50 ETF options (units)

Months	Total turnover	Turnover for calls	Turnover for puts	Total positions	Call positions	Put positions
February 2015	232,508	123,727	108,781	40,665	21,910	18,755
March 2015	550,699	304,034	246,665	56,410	31,864	24,546
April 2015	690,136	389,899	300,237	108,900	60,234	48,666
May 2015	1,136,752	676,628	460,124	170,358	103,956	66,402
June 2015	1,886,856	1,040,645	846,211	230,732	173,172	57,560
July 2015	2,858,200	1,746,338	1,111,862	271,866	178,599	93,267
August 2015	3,046,438	1,726,557	1,319,881	373,755	251,418	122,337
September 2015	2,144,420	1,164,212	980,208	291,158	174,254	116,904
October 2015	1,957,154	1,064,484	892,670	341,682	202,913	138,769
November 2015	4,210,044	2,354,994	1,855,050	543,503	330,349	213,154

Source: Wind

volatility point estimation on account of real market analysis of the listed 50 ETF options in Shanghai Security Exchange. Considering transaction costs, different hedging strategies are analyzed to find the optimal dynamic hedging strategy. Besides, the impact of using futures for delta hedging is also studied under the background of negative future-spot basis.

2. Volatility arbitrage strategy design

2.1 Volatility arbitrage strategy

Volatility arbitrage strategy is to build an asset portfolio with option and its underlying assets, whose returns depend on the difference between the actual volatility and implied volatility. Actual volatility determines the theoretical price of option, normally obtained by forecast. Volatility arbitrage can be seen as trading Vega. When the implied volatility of option is quite high, which means that the option value is overpriced, this strategy requires to short options, and replicate the option returns by dynamic delta hedging so as to earn the returns resulting from the difference between implied volatility and actual volatility. When the implied volatility is quite low, then the option value is underestimated, we need to keep a long position on options and hedge the delta risk. Whether the volatility arbitrage can earn profits depends on two factors. The first is delta hedging accuracy, where the more accurate the hedging is, the lower the cost is, the more profitable the strategy is; the second is to the deviation between predicted volatility and the actual volatility, that is, the more precise the predicted volatility is, the greater the profit is.

In the framework of the Black-Scholes model, we can replicate the change of returns on options by holding Δ units of underlying asset and dynamically adjusts the position of underlying assets. Specifically, when we long a call option, we need to sell Δ units of underlying asset while a long position on put options means that we need to buy Δ units of underlying asset. When the cost of buying options in the market differs from the cost of dynamic replication, the arbitrage opportunity exists.

2.2 Choice of volatility for delta hedging

The hedging position delta can be calculated by the following formula:

$$\Delta(t)_{call} = \frac{\partial C(t,S)}{\partial S} = N(d_1) \tag{1}$$

$$\Delta(t)_{put} = \frac{\partial P(t,S)}{\partial S} = N(d_1)-1$$

Δ is the delta of option, C the value of call option, P the value of put option, S the price of underlying asset. With other parameters all known, the main issue of delta calculation lies on the choice of volatility. Considering we could use actual volatility and implied volatility to calculate the delta hedging position, the following analysis will be divided into two cases. Here σ is used to denote actual volatility while $\tilde{\sigma}$ represents implied volatility, both are assumed constant in following analysis.

2.2.1 Hedge with actual volatility. Suppose we hold a long position in option, we can replicate the P&L of option by selling Δ units of underlying assets (here we only consider call options) and put the remaining cash in the bank with interest rates r. The portfolio values are as follows: V is the option value, D the continuous dividend yield, the subscript "a" denotes actual and i represents implied (Tables II and III).

So the mark-to-market portfolio value change is as follows:

$$dV^i - \Delta^a dS - r\left(V^i - \Delta^a S\right)dt - \Delta^a DSdt \tag{2}$$

According to Ahmad and Wilmott (2005), when implied volatility equals actual volatility, the option is correctly priced. Considering No Arbitrage Principle, the return of delta hedging strategy should be 0:

$$dV^a - \Delta^a dS - r(V^a - \Delta^a S)dt - \Delta^a DSdt = 0 \tag{3}$$

After deducting the formula above, the mark-to-market value change becomes following:

$$dV^i - dV^a + r(V^a - \Delta^a S)dt - r\left(V^i - \Delta^a S\right)dt$$

$$dV^i - dV^a - r\left(V^i - V^a\right)dt$$

$$= e^{rt} d\left(e^{-rt}\left(V^i - V^a\right)\right) \tag{4}$$

Then, the present value of this profit at time t_0 is as follows:

$$e^{r(t-t_0)} e^{rt} d\left(e^{-rt}\left(V^i - V^a\right)\right)$$

$$e^{rt_0} d\left(e^{-rt}\left(V^i - V^a\right)\right)$$

So the total profit from t_0 to expiration is as follows:

$$e^{rt_0} \int_{t_0}^{T} d\left(e^{-rt}\left(V^i - V^a\right)\right) = V^a - V^i \tag{5}$$

So the total profit of the portfolio is guaranteed and constant, that is, $V^a - V^i$.

Table II.
Portfolio values at time t of hedging case with actual volatility

Component	Value
Option	V^i
Stock	$-\Delta^a S$
Cash	$-V^i + \Delta^a S$
Total	0

Table III.
Portfolio values at time $t + dt$ of hedging case with actual volatility

Component	Value
Option	$V^i + dV^i$
Stock	$-\Delta^a S - \Delta^a Ds$
Cash	$(-V^i + \Delta^a S)\ (1 + rdt) - \Delta^a DSdt$
Total	$dV^i - \Delta^a dS - r(V^i - \Delta^a S)dt - \Delta^a DSdt$

Using Ito's lemma, the mark-to-market profit can be rewritten as follows:

$$dV^i - \Delta^a dS - r\left(V^i - \Delta^a S\right)dt - \Delta^a DSdt$$

$$= \theta^i dt + \Delta^i dS + \frac{1}{2}\sigma^2 S^2 \Gamma^i dt - \Delta^a S - r\left(V^i - \Delta^a S\right)dt - \Delta^a DSdt$$

$$= \theta^i dt + \mu S\left(\Delta^i - \Delta^a\right)dt + \frac{1}{2}\sigma^2 S^2 \Gamma^i dt - r\left(V^i - V^a\right)dt + \left(\Delta^i - \Delta^a\right)\sigma SdW - \Delta^a DSdt$$

$$= \left(\Delta^i - \Delta^a\right)\sigma SdW + (\mu + D)S\left(\Delta^i - \Delta^a\right)dt + \frac{1}{2}\left(\sigma^2 - \tilde{\sigma}^2\right)S^2 \Gamma^i dt$$

$$= \frac{1}{2}\left(\sigma^2 - \tilde{\sigma}^2\right)S^2 \Gamma^i dt + \left(\Delta^i - \Delta^a\right)((\mu - r + D)Sdt + \sigma SdW) \tag{6}$$

Although the final return of the portfolio is guaranteed, but the daily value changes have a random item dW, which indicates the portfolio P&L may be negative at a certain date.

Here, we conducted several Monte Carlo simulations of delta hedging with actual volatility. The option is a European call with maturity of one year, strike of 100 and at the money initially. The actual volatility is 40 percent, the implied volatility is 20 percent[1], the drift and dividend yield are 0 and the risk-free rate is 5 percent. The simulation results are shown in Figure 1. Note that the final profits here are not exactly the same, but the errors are small. It is mainly due to the discrete delta hedge instead of continuous hedge and the limited number of hedging times (1,000 times).

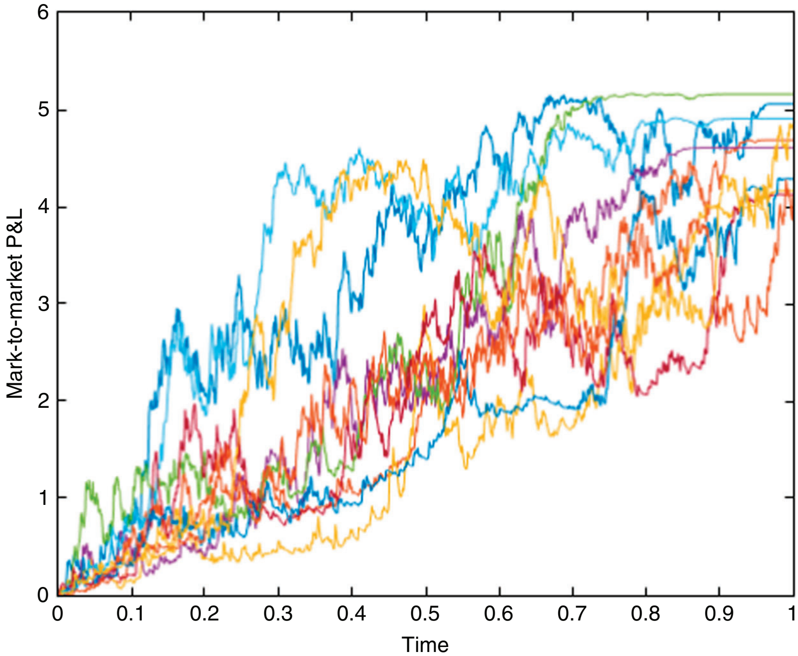


Figure 1.
P&L for a delta-hedged option hedged with actual volatility on mark-to-market basis

2.2.2 Hedge with implied volatility. Suppose you hold a call long, replicate the profit and loss of the option by selling the underlying asset, and deposit the remaining cash into the bank at the interest rate r. The value of the portfolio changes as shown in Tables IV and V.

The mark-to-market P&L of delta-hedged portfolio with implied volatility is as follows:

$$dV^i - \Delta^i dS - r\left(V^i - \Delta^i S\right)dt - \Delta^i DSdt \tag{7}$$

To rewrite the mark-to-market P&L using Ito's lemma similar to the previous case, we can get the following:

$$\begin{aligned} &dV^i - \Delta^i dS - r\left(V^i - \Delta^i S\right)dt - \Delta^i DSdt \\ &= \theta^i dt + \frac{1}{2}\sigma^2 S^2 \Gamma^i dt - r\left(V^i - \Delta^i S\right)dt - \Delta^i DSdt \\ &= \frac{1}{2}\left(\sigma^2 - \tilde{\sigma}^2\right) S^2 \Gamma^i dt \end{aligned} \tag{8}$$

After discounting the formula above, we can have the present value of portfolio P&L:

$$\frac{1}{2}\left(\sigma^2 - \tilde{\sigma}^2\right) \int_{t_0}^{T} e^{-er(t-t_0)} S^2 \Gamma^i dt \tag{9}$$

Different from the first case, final P&L here is not guaranteed, instead it is highly path dependent (shown in Figure 2). However, it is deterministic because the stochastic term does not exist here, which is better for risk management perspective. During the time period before expiration, we only need to assure the actual volatility is always greater than the implied volatility for long position of options or always less for short positions to make a profit.

Carr (2005) shows the more general result that if the option is delta-hedged based on a volatility σ_h, then the present value of the total P&L is given by the following:

$$V(S,t;\sigma_h) - V(S,t;\sigma) + \frac{1}{2}\left(\sigma^2 - \sigma_h^2\right) \int_{t_0}^{T} e^{-r(t-t_0)} S^2 \Gamma^h dt \tag{10}$$

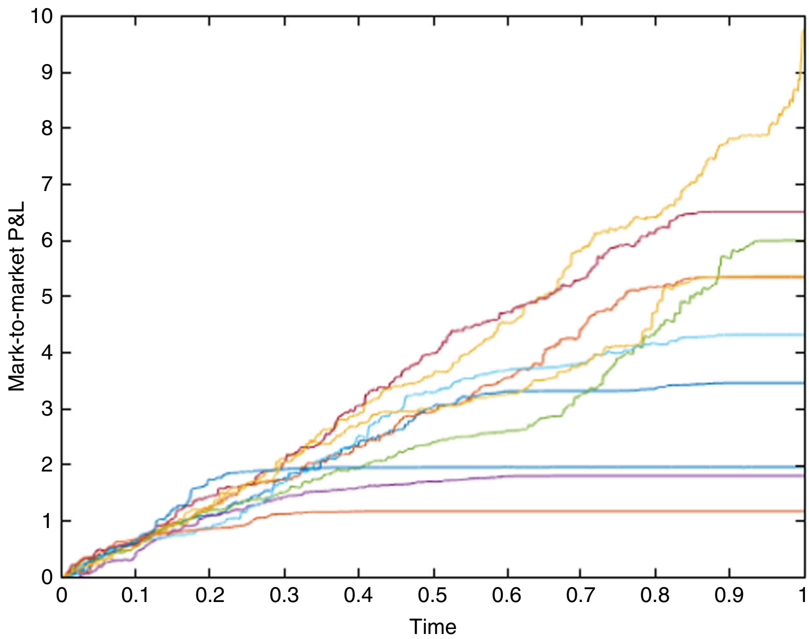
where σ is the actual volatility, and Γ^h the Gamma of option calculated by σ_h.

Table IV. Portfolio values at time t of hedging case with implied volatility

Component	Value
Option	V^i
Stock	$-\Delta^i S$
Cash	$-V^i + \Delta^i S$
Total	0

Table V. Portfolio values at time $t+dt$ of hedging case with implied volatility

Component	Value
Option	$V^i + dV^i$
Stock	$-\Delta^i_S - \Delta^i dS$
Cash	$(-V^i + \Delta^i S)(1 + rdt) - \Delta^i DSdt$
Total	$dV^i - \Delta^i dS - r(V^i - \Delta^i S)dt - \Delta^i DSdt$



Notes: $T = 1$ year, $S = K = 100$, $\sigma = 0\%$, $\tilde{\sigma} = 20\%$, $\mu = q = 0$, $r = 5\%$

Figure 2. Monte Carlo simulation using implied volatility hedges

Delta hedging with actual volatility can foresee the final profit before expiration, but the key point of this strategy is the estimation of actual volatility. There still exist some risks if we cannot accurately predict the actual volatility. Meanwhile, the strategy is not that satisfactory for risk management as the fluctuation of mark-to-market profits is quite large.

It is possible to guarantee a positive daily return by delta hedging with implied volatility. Moreover, the implied volatility can be calculated from market price of options which ensures the accuracy of hedge positions. However, we cannot foresee the final profit using this strategy but can only guarantee a positive final P&L.

2.2.3 Volatility cones. Because the expected return depends on the difference between actual volatility and implied volatility, the prediction of actual volatility would be particularly important. There are a lot of volatility predicting models, including extrapolation, moving average model and GARCH. The GARCH model is the most widely used model in theory and practice. The GARCH (p, q) model is usually defined as follows:

$$r_t = \mu + \varepsilon_t$$

$$\varepsilon_t = h_t z_t, \quad z_t \sim i.i.d(0, 1)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i h_{t-j}^2 \tag{11}$$

where r_t, μ, ε_t, h_t represent the daily log returns, the mean returns, the disturbance terms and the conditional variance of the assets, respectively, and p and q represent the lagged items of the disturbance term and the conditional variance, respectively.

However, the GARCH model also has many flaws. First, the model cannot explain the inconsistency between the direction of the volatility change and the direction of its asset return. This is mainly because the model design does not take into account the impact of residual symbols on volatility. Second, the GARCH model only obtains the term structure which converges to the long-term mean in an exponential form but cannot get the volatility term structure with a peak. Third, the existing analysis also points out that the GARCH model is worse than the implied volatility method and worse than the historical volatility method in short predictive period case (Zheng and Huang, 2010; Wang, 2012). In addition, the predictable models mentioned above can only provide the point estimates of volatility, which is not that usable for prediction of realized volatility within a specific period. To solve this problem, we use volatility cones (Burghardt and Lane, 1990) here. The concept of volatility cones is based on the range of historical volatility within different time periods. With volatility cones, we could have a general idea of the distribution of volatility during a specific time period in the future. The method shares same merits of quantile regression with Wang *et al.* (2014).

To construct volatility cones, we can first divide the original return sequence into $n = T-h+1$ subsequences if we set the length of asset return sequence as T and time period for measurement as h days. Then volatility (or variance) distribution can be obtained, and the different quantiles of this distribution form the volatility cones.

In order to get more information from a given return sequence, generally we need to use data repeatedly, resulting in data overlapping. This will obviously create correlations within the results, leading to some bias of estimates. In order to reduce the estimation error, Hodges and Tompkins (2002) introduced a corresponding adjustment factor for the volatility estimates-based overlapping sequences of asset returns:

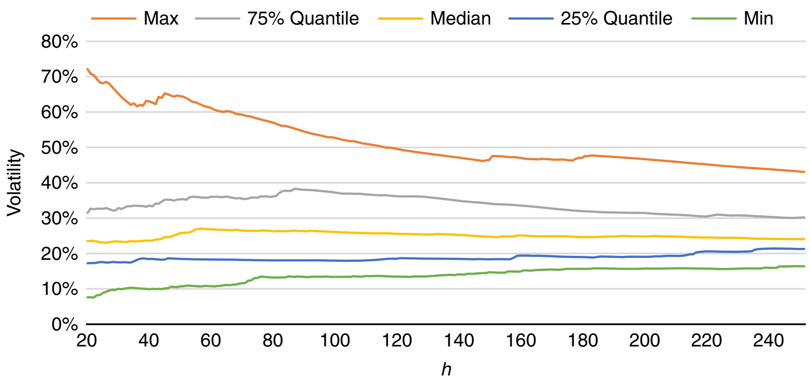
$$m = \frac{1}{1-\frac{h}{n}+\frac{h^2-1}{3n^2}} \tag{12}$$

where h means the length of subsequences, and $n = (T-h)+1$ the number of non-overlapping subsequence (Table VI).

The purpose of the volatility cone is to clarify how the volatility interval changes over different trading ranges. As can be seen from Figure 3, short-term volatility has greater amplitude than long term, indicating that large fluctuations in the longer period of time will be averaged. The volatility cone can reflect the volatility differences of different time intervals. In addition, for stocks and futures, the information effect is very large in predicting the actual volatility. Since the volatility cone can place the current market information in the historical context, it is wiser to construct a trading strategy by comparing the implied volatility and the distribution of historical volatility given by volatility cone. Furthermore, for position traders, market-adjusted volatility cone can help them monitor volatility changes without trading all the time, and they can selectively trade until volatility is large and then build positions.

Table VI. Volatility cones of 50 ETF

	20 days	40 days	60 days	120 days
Max.	0.7428	0.6681	0.6700	0.6084
75% quantile	0.3237	0.3560	0.3949	0.4449
Median	0.2417	0.2506	0.2944	0.3138
25% quantile	0.1774	0.1954	0.2004	0.2271
Min.	0.0786	0.1049	0.1180	0.1650



Note: Database: three-year data till December 1, 2015
Source: Data: Wind

Figure 3. Volatility cones of 50 ETF

3. The arbitrage analysis of volatility without transaction costs

3.1 Hedging strategy

3.1.1 Black-Scholes model. Black and Scholes (1973) proposed the following assumptions when deriving the option pricing formula:

(1) The rate of return on the riskless asset is known and constant.

(2) The stock price follows a geometric Brownian Motion.

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{13}$$

(3) The stock does not pay a dividend.

(4) Only examine European options.

(5) Stock and option transactions do not incur any costs and fees.

(6) It is possible to lend and borrow any amount of cash at the riskless rate.

(7) There is no penalty for short-selling.

According to the above assumptions, Black and Scholes derive the Black and Scholes option pricing formula:

$$C_t = S_t N(d_1) - K e^{-r(T-t)} N(d_2)$$

$$P_t = K e^{-r(T-t)} N(-d_2) - S_t N(-d_1) \tag{14}$$

Where N is the cumulative normal distribution function $\Phi(0, 1)$:

$$d_1 = \frac{\ln \frac{S_t}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sqrt{\sigma(T-t)}}$$

$$d_2 = d_1 - \sqrt{\sigma(T-t)}$$

3.1.2 Discrete hedging within fixed interval. The Black-Scholes Hedging at fixed regular intervals relaxes the assumption of continuous hedging and considers to modify the hedging delta and re-hedge in the underlying stock at fixed regular intervals. With the volatility modification, Wilmott (1994) proposed an optimal hedge ratio for discrete delta hedging to reduce the standard deviation of hedging error. He used an adjusted volatility for the model amendment:

$$\sigma' = \sigma \times \left\{1+\frac{\Delta_t}{2\sigma^2}(\mu-r)\left[3(\mu-r)+\sigma^2\right]\right\} \tag{15}$$

where Δt is the time interval of delta hedging, σ' the adjusted volatility, and σ the original volatility.

The adjusted hedge ratio, delta, is as follows:

$$\text{Delta} = \frac{\partial V(S,t,r,K,\sigma')}{\partial S}+\Delta_t\left[\left(\mu-r+\frac{\sigma^2}{2}\right)S\frac{\partial V^2(S,t,r,K,\sigma')}{\partial S^2}\right] \tag{16}$$

3.2 Empirical implementation of volatility arbitrage

In this empirical strategy implementation, we tested volatility arbitrage strategy on 440 options including existing and expired options in Chinese market within the time period of February 9-November 30, 2015. The data used in this paper are all from Wind database, and the data are analyzed and processed by MATLAB.

As mentioned before, the return of volatility arbitrage strategy comes from the difference between actual volatility and implied volatility. Similar to the VIX index of S&P 500, IVIX index also shows the implied volatility of China 50 ETF in 30 days from now. China IVIX index, released by the SSE, is based on the principle of variance swaps and calculated by China 50 ETF prices traded on SSE. On the other hand, realized volatility is an estimate for actual volatility. Figures 4 and 5 show the IVIX and realized volatility of China 50 ETF and their differences since the launch of China 50 ETF options in February 2015. During February to mid-March and July to mid-October, the IVIX index is lower than the realized volatility, indicating an arbitrage opportunity to long volatility. While during mid-March to June and from November, short-selling volatility might be a better strategy for volatility traders.

So in the empirical back-test, we would adjust our volatility-trading directions based on the real-time data of actual volatility and implied volatility. As mentioned before, volatility cones are better choices for interval estimate of actual volatility, so we use the rolling volatility cones based on one-year historical return data and time period of corresponding tenor of each option as entry and exit signals of volatility-trading strategy. Specifically,

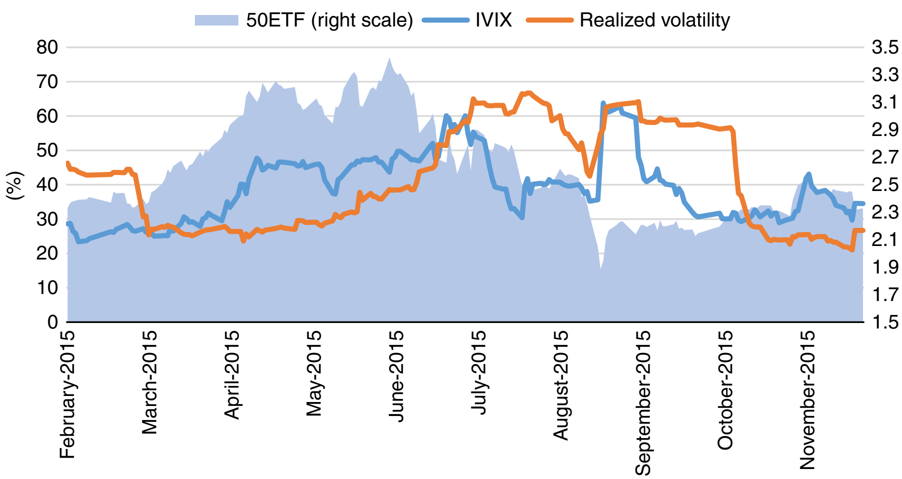


Figure 4.
Settle price of 50 ETF, IVIX and realized volatility

we will open a long position on options when real-time implied volatility is below the 25 percent quantile of historical volatility distribution and open a short position when real-time implied volatility exceeds the 75 percent quantile of volatility distribution. As for the exit signal, if the real-time implied volatility crosses the median line of historical volatility, the option position is closed to stop the gains. Besides, this strategy is on a rolling-basis which means it is possible to re-open an option position if the entry signal reoccurs after position close-out. Moreover, here we did not consider the impact of position volume we hold and only assume that we will buy or sell one option contract when entry signal appears.

After opening the positions on volatility trading, the most important thing is to hedge our portfolio for risk management. Comparing the pros and cons of choices of different hedging volatilities as mentioned before, we choose implied volatility here as hedging factors because it is easy to calculate the hedging positions on underlying asset, and the volatility traders need positive returns and less retracement during the trading period rather than deterministic returns.

We replicate the returns of options based on the analysis of Tables IV and V with underlying assets and cash, and adjust the hedge position daily. Meanwhile, we use the yield of ten-year government bond as the risk-free rate, and the transaction costs are assumed to be 0 (this assumption will be released later).

3.2.1 Original Black-Scholes hedging approach. Within the 440 options, we can get positive returns on 307 options, which means the winning percentage is 70.23 percent. Moreover, there are ten cases where we never opened a position on options. So the volatility arbitrage strategy is quite effective because the winning percentage is relatively high. We can get average P&L of 0.0607 and Sharpe Ratio of 0.1213 (risk-free rate is 3.39 percent). Figure 6 shows the scatter diagram of final P&Ls, indicating there are only few cases that get negative profit. Figures 7 and 8 show that the majority of back-test results has final P&L around 0-0.2 and yield around 0-25 percent. The existence of negative results might be due to the estimation errors of actual volatility. Because the volatility cones within a specific time period may not completely represent the actual volatility in the future, we also back-tested other strategies based on different volatility cones.

Table VII shows that strategy based on one-year volatility cones performs much better than the strategies based on three- year and five-year volatility. This volatility arbitrage strategy can achieve an average yield of 4.68 percent with the VaR (95 percent) of yield of −8.66 percent and Sharpe ratio of 0.1213. However, the strategy has more obvious extreme situation than the other two strategies; the maximum retracement reached 130.28 percent[2].

Considering the retracement of previous strategies, we modified our trading strategies by adding a stop-loss signal. If the accumulated yield hit the −20 percent line, we should close out our position on options and underlying assets. Table VIII shows the result with stop-loss signals.

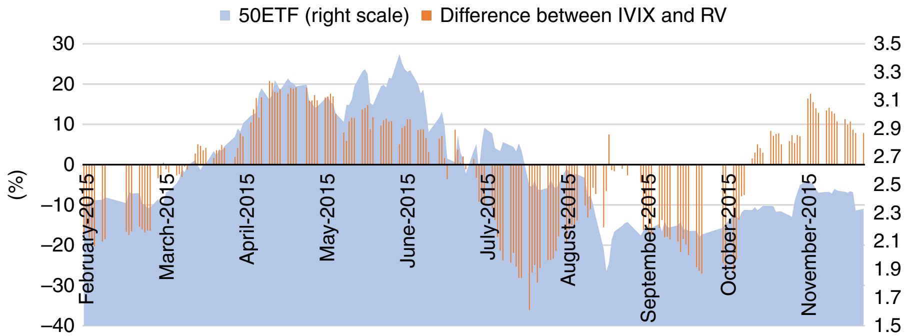


Figure 5.
Settle price of 50 ETF and difference between IVIX and realized volatility

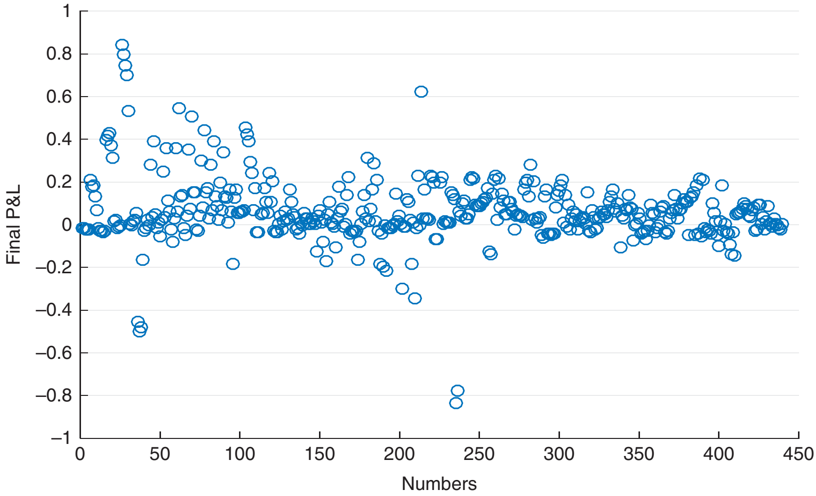


Figure 6.
Final P&Ls of 440 options based on one-year volatility cones

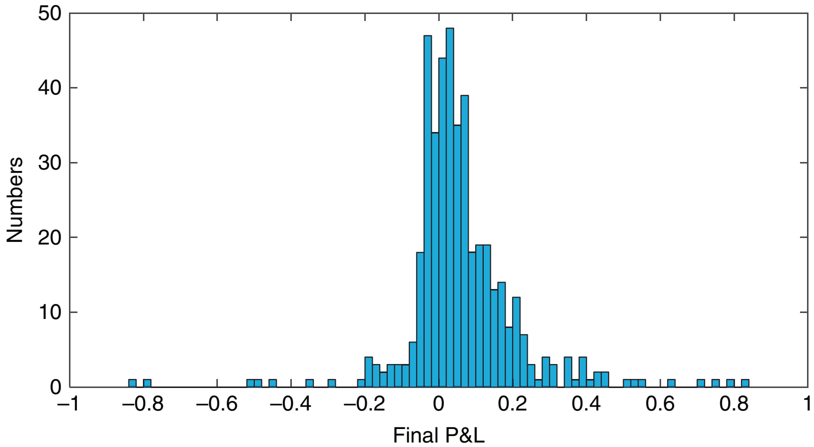


Figure 7.
P&L distribution of 440 options based on one-year volatility cones

We can see the retracement has been greatly improved, and the maximum retracement is about 30 percent. The VaR value and the average return also increased, but the winning percentage, average yield, Sharpe ratio, earnings the median yield, and 75 percent quantile points have declined. However, the overall situation of extreme gains and losses have been greatly improved.

3.2.2 Discrete hedging approach within fixed interval. For discrete hedging within fixed interval, we assume stock yield μ as 19.43 percent, calculated by the expected return of China 50 ETF within three year, to calculate the adjusted volatility and delta for hedging. Considering the results above, we chose one-year volatility cones as the estimates of actual volatility distribution. Besides, we brought in the stop-loss signal of 20 percent.

Figures 9 and 10 show the P&L and yield of discrete hedging approach are similar to Black-Scholes approach. Most final returns lie on 0-0.2 and nearly 49.31 percent of final yields are between 0 and 10 percent. Table IX shows that by discrete hedging within fixed interval, apart from the decrease in standard deviation, most indicators remain similar to Black-Scholes approach or even slightly worse.

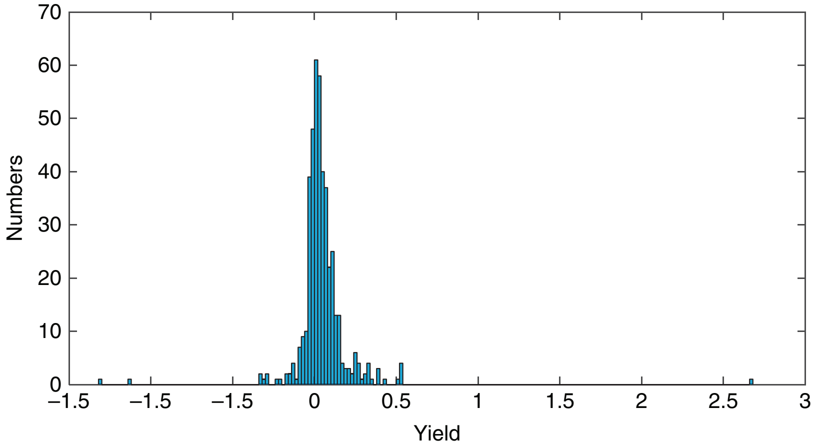


Figure 8.
Yield distribution of 440 options based on one-year volatility cones

Strategy	1 year	3 year	5 year
Winning percentage	70.23	68.64	67.27
Average P&L	0.0607	0.0064	0.0023
SD of P&L	0.1558	0.1430	0.1931
Average yield (%)	4.68	0.89	0.14
Sharp ratio	0.1213	−0.1306	−0.2173
VaR (95%) of yield	−8.66%	−42.27%	−27.98%
Minimum yield (%)	−130.28	−91.16	−97.48
25% quantile of yield	−1.04%	−2.66%	−2.38%
Median of yield (%)	2.90	3.31	2.30
75% quantile of yield	8.11%	8.11%	6.95%
Maximum yield (%)	267.18	44.07	34.60

Table VII.
P&L based on volatility cones within different time period

Strategy	1 year	3 year	5 year
Winning percentage	69.77	66.59	63.64
Average P&L	0.0617	0.0058	−0.0013
SD of P&L	0.1444	0.1695	0.1812
Average yield (%)	4.47	0.56	−0.54
Sharp ratio	0.1015	−0.2387	−0.3102
VaR (95%) of yield	−7.84%	−22.91%	−24.65%
Minimum yield (%)	−26.58	−32.88	−32.88
25% quantile of yield	0%	−2.3%	−4.08%
Median of yield (%)	2.28	2.29	2.11
75% quantile of yield	7.47%	6.29%	6.12%
Maximum yield (%)	53.62	33.59	34.60

Table VIII.
P&L based on different volatility cones with stop-loss signal

3.2.3 Comparison of volatility cone and GARCH point estimates. The interval estimate of volatility using the volatility cone is superior to the point estimate of the GARCH model, and the advantage of the volatility cone is verified by the empirical analysis of the two estimation methods. We choose the simplest GARCH (1, 1) model to analyze the time-varying volatility of different options in the remaining period and calculate the volatility

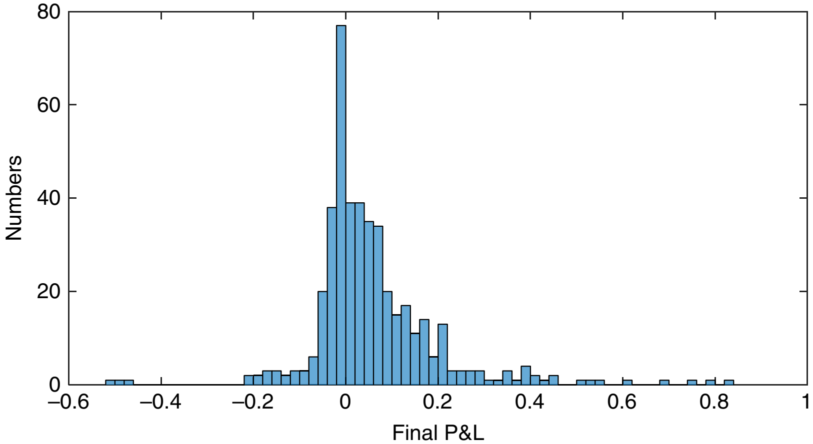


Figure 9.
P&L distribution of discrete hedging within fixed intervals based on one-year volatility cones

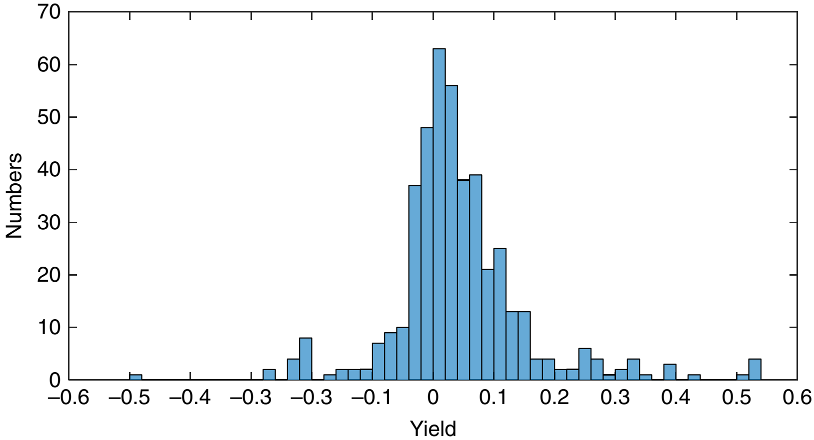


Figure 10.
Yield distribution of discrete hedging within fixed intervals

Table IX.
Returns based on different hedging strategy

Strategy	Black-Scholes approach	Discrete hedging approach within fixed interval
Winning percentage	69.77	69.77
Average P&L	0.0617	0.0617
SD of P&L	0.1444	0.1443
Average yield (%)	4.47	4.31
Sharpe ratio	0.1015	0.0807
VaR (95%) of yield	–7.84%	–9.84%
Minimum yield (%)	–26.58	–48.19
25% quantile of yield	0%	–1.07%
Median of yield (%)	2.28	2.81
75% quantile of yield	7.47%	8.17%
Maximum yield (%)	53.62	53.55

using the one-year daily return rate data. In the framework of the GARCH model, the arbitrage strategy is as follows: when the real-time implied volatility is less than 75 percent of the predicted volatility, long the option; when the real-time implied volatility exceeds 125 percent of the expected volatility, short the option; and in the back-test period, if the implied volatility return to the predicted volatility after opening the positions, the options and positions are closed in order to get the profit. The stop signal on the strategy is considered here. As can be seen from Table X, although using GARCH model can get a relatively high winning percentage and lower standard deviation of returns, the average P&L, average yield, Sharpe ratio and VaR values are significantly lower than volatility cone. Therefore, it is more reasonable to use the volatility cone as the actual volatility estimates.

4. Hedging with transaction costs

4.1 Hedging approaches

4.1.1 Leland model. Leland (1985) proposed hedging strategies with transaction costs based on the Black-Scholes formula, by adjusting the volatility. Adjusted volatility is given by the following:

$$\sigma'^2 = \sigma^2\left[1+\sqrt{\frac{8}{\pi}\frac{k}{\sigma\sqrt{\Delta_t}}}\right] \tag{17}$$

Here k represents the proportional transaction cost, measured as a fractional of trading volumes. The adjusted delta is given by the following:

$$\text{Delta} = \frac{\partial V\left(S,t,r,K,\sigma'\right)}{\partial S} \tag{18}$$

4.1.2 Delta tolerance. Delta tolerance strategies mean to re-hedge the position of underlying asset only when Delta exceeds the bound of predetermined range. There are two ways of re-hedging: first, re-hedge to the optimal Black-Scholes delta and second, keep a minimum position adjustment only to keep Delta within the predetermined tolerance range. Whalley and Wilmott (1993) used the first method to define H as the deviation of the optimal delta, and Δ is the underlying stock positions we hold:

$$\Delta = \frac{\partial V}{\partial S} \pm H \tag{19}$$

Strategy	One-year volatility cone	GARCH (1, 1)	Discrete hedging approach within fixed interval (volatility cone)	Discrete hedging approach within fixed interval (GARCH)
Winning percentage	69.77	70.23	69.77	70.23
Average P&L	0.0617	0.0341	0.0617	0.0346
SD of P&L	0.1444	0.1390	0.1443	0.1385
Average yield (%)	4.47	2.02	4.31	2.04
Sharpe ratio	0.1015	−0.1230	0.0807	−0.1353
VaR (95%) of yield	−7.84%	−21.44%	−9.84%	−21.84%
Minimum yield (%)	−26.58	−35.93	−48.19	35.78
25% quantile of yield	0%	−0.69%	−1.07%	−0.70%
Median of yield (%)	2.28	2.53	2.81	2.52
75% quantile of yield	7.47%	7.15%	8.17%	7.10%
Maximum yield (%)	53.62	46.01	53.55	44.78

Table X. Profit and loss based on different volatility estimation methods

When H exceeds the predetermined value, we should re-adjust the hedging position to assure H equals to 0. The value of option follows the equation:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=\frac{k}{H_0}\left(\frac{\partial^2 V}{\partial S^2}\right)^2 \tag{20}$$

where H_0 is the pre-setting value of H.

4.1.3 Whalley-Wilmott utility maximization strategy. Hodges and Neuberger (1989) used exponential utility function and stochastic control methods to maximize the utility and obtain the option price and a non-trading area. When hedge position is below the lower boundary of non-trading area, we should buy the underlying stock to make the position equal to the lower limit; when hedge position exceeds the upper boundary of non-trading area, we should sell the underlying stock to make the position equal to the upper limit; and no transactions when the hedge position is within the non-trading area.

The utility function is usually assumed as the formula below, which means that the hedger has the negative exponential utility function:

$$U(z)=-e^{-\lambda z},\lambda>0 \tag{21}$$

where λ is a measure of the hedger's (constant) absolute risk aversion.

Whalley and Wilmott used the global-in-time method. The asymptotic analysis results in a relatively simple formula used for calculating the non-trading area. They got a Black-Scholes delta bandwidth:

$$\left[\tfrac{\partial V}{\partial S}-B_t,\tfrac{\partial V}{\partial S}+B_t\right]$$

$$B_t=\left(\frac{3kS_t e^{-r(T-t)}\Gamma^2}{2\lambda}\right)^{\frac{1}{3}} \tag{22}$$

So the hedging strategy is given by the following:

$$y_t=\begin{cases}\Delta_t-B_t, & if\ \ y_{t-1}<\Delta_t-B_t\\ \Delta_t+B_t, & if\ \ y_{t-1}>\Delta_t+B_t\\ y_{t-1}, & if\ \ \Delta_t-B_t<y_{t-1}<\Delta_t+B_t\end{cases} \tag{23}$$

where y_t is the hedge position at time t, and λ the risk aversion coefficient of investors.

4.2 Empirical results on hedging approaches with transaction cost

The transaction cost to buy or sell China 50 ETF in Chinese market is 3.65 BP proportional to the trading amount and the cost to long or short a 50 ETF option is 2.3 BP.

4.2.1 Black-Scholes approach. Based on the Black-Scholes approach, we use the delta same as the no-cost case of original Black-Scholes model for hedging without any adjustment. However, the transaction cost to open an option position is also taken into account for the calculation of initial payoff, and the proportional cost parameter is included in the P&L calculation for every re-hedging step, which will reduce the final P&L definitely. Based on Black-Scholes approach, the winning percentage is 63.41 percent, the average yield is 4.46 percent, the Sharpe ratio is 0.0644 and the average P&L is 0.0600, with a standard deviation of 0.1405. Moreover, the VaR (95 percent) of yield is −10.50 percent. Besides, Figure 11 shows the accumulated costs of 440 options with Black-Scholes approach. Most accumulated costs are higher than −0.003, and the average cost is −0.0017. Although the performance is worse due to the existence of transaction costs, the strategy still brings satisfactory returns.

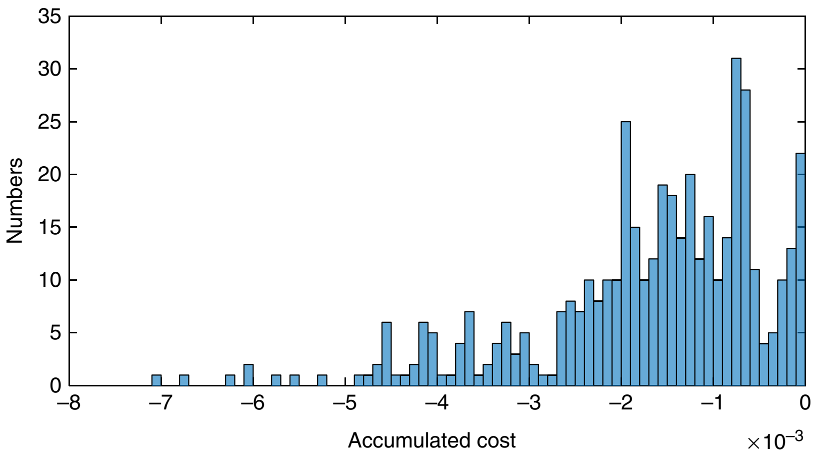


Figure 11.
Accumulated costs with Black-Scholes approach

In addition, the GARCH (1, 1) model is also used to predict the actual volatility here for comparison. Compared to the volatility cone method (Table XI), it is found that, while the use of GARCH model can reduce the accumulated costs and extreme losses and the winning percentage rises slightly, the average P&L, the Sharpe ratio and the VaR are all significantly worse than those based on the one-year volatility cone. Therefore, it is advisable to use the one-year volatility cone instead of the GARCH point estimate to predict volatility when transaction costs are taken into account.

4.2.2 Leland approach. Comparing with the Black-Scholes approach (Table VII), the 75 percent quantile and maximum yield got slightly improved while other indicators remain similar. In a whole, Leland approach performs quite similar with the Black-Scholes approach (Table XII).

Figure 12 shows the implied volatility and the adjusted volatility based on the same option contract (10000297.SH) with Figure 9. We could see from the figure that the difference between two volatilities is quite small, which results in the similar final returns of two different approaches.

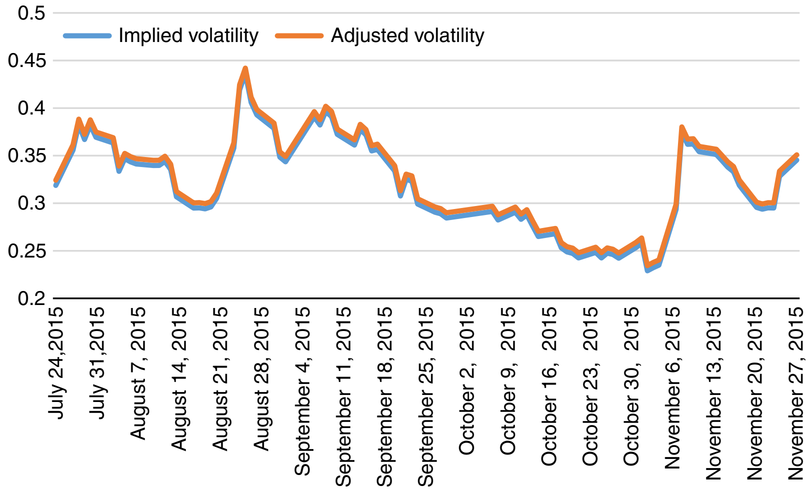
4.2.3 Delta tolerance. Figure 13 and Table XIII show the final results using delta tolerance approach with different predetermined H. When H increases, the frequency of

Strategy	Black-Scholes (GARCH)	Black-Scholes (one-year volatility cone)
Accumulated costs	−0.0013	−0.0017
Winning percentage	69.54	68.86
Average P&L	0.0328	0.0592
SD of P&L	0.1390	0.1451
Average yield (%)	1.92	4.13
Sharpe ratio	−0.1471	0.0644
VaR (95%) of yield	−21.61%	−10.50%
Minimum yield (%)	−35.94	−49.30
25% quantile of yield	−0.82%	−1.17%
Median of yield (%)	2.48	2.57%
75% quantile of yield	7.00%	8.09%
Maximum yield (%)	45.95	53.50

Table XI.
Returns based on different hedging strategy with transaction cost

Table XII. Returns based on different hedging strategy with transaction costs

Strategy	Black-Scholes approach	Leland approach
Accumulated costs	−0.0017	−0.0017
Winning percentage	68.86	69.09
Average P&L	0.0592	0.0600
SD of P&L	0.1451	0.1441
Average yield (%)	4.13	4.20
Sharpe ratio	0.0644	0.0714
VaR (95%) of yield	−10.50%	−10.02%
Minimum yield (%)	−49.30	−46.96
25% quantile of yield	−1.17%	−1.09%
Median of yield (%)	2.57	2.66
75% quantile of yield	8.09%	8.07%
Maximum yield (%)	53.50	53.33



Note: Based on a 50 ETF call option with a strike price of 2.9 in March 2016

Figure 12. Volatilities with Leland approach

delta hedging decreases, resulting in a decrease in accumulated cost and an increase in hedging errors. When H equals to 0.135, we can get the highest VaR of yield, which means the largest loss will not exceed 8.51 percent with the probability of 95 percent.

According to the above analysis, we choose $H = 0.135$ as the deviation to the optimal delta. Except the maximum yield, all other indicators perform better than the Black-Scholes approach (Table XIV), and because of the existence of delta tolerance range, the average cost drops down to −0.0001.

4.2.4 Whalley-Wilmott utility maximization strategy. From Figure 14, we can see when we choose λ equals to 2.31, we can get the largest VaR of yield, which is −9.13 percent, and the average yield is equal to 4.41 percent. With a small risk aversion coefficient, the delta hedging is less frequent so the average cost is only −0.0011.

According to the above analysis, we choose $\lambda = 2.31$ as the risk aversion coefficient. Table XV shows the result compared with the result with Black-Scholes approach. Except the maximum yield, all other indicators performs better than Black-Scholes approach with the average cost decreases to −0.0011.

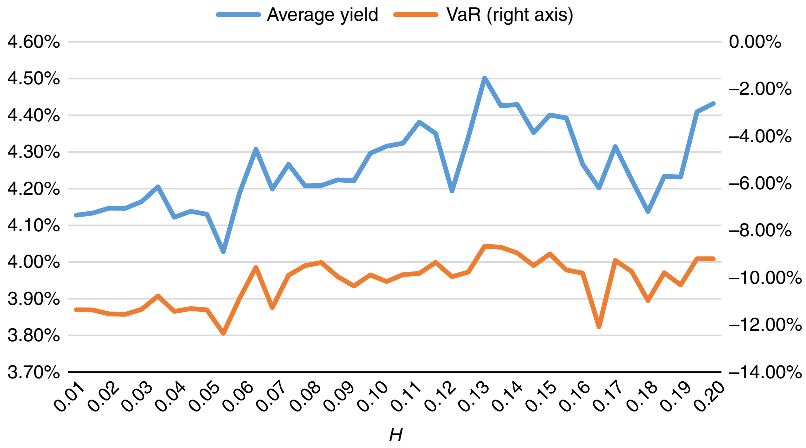


Figure 13. Average yield and VaR based on different *H*

Table XIII. Results by delta tolerance approach based on different *H*

H	Average P&L	SD of P&L	Average yield %	SD of yield	VaR (95%) of yield	Median of yield (%)
0.1000	0.0614	0.1384	4.30	0.1098	−9.88	3.08
0.1050	0.0619	0.1381	4.32	0.1097	−10.16	3.20
0.1100	0.0621	0.1387	4.32	0.1099	−9.87	3.35
0.1150	0.0627	0.1367	4.38	0.1084	−9.82	3.33
0.1200	0.0623	0.1368	4.35	0.1084	−9.35	3.39
0.1250	0.0604	0.1399	4.19	0.1113	−9.96	3.36
0.1300	0.0627	0.1386	4.34	0.1108	−9.76	3.29
0.1350	0.0647	0.1367	4.50	0.1094	−8.51	3.31
0.1400	0.0637	0.1368	4.43	0.1092	−8.71	3.31
0.1450	0.0636	0.1370	4.43	0.1092	−8.95	3.19
0.1500	0.0624	0.1364	4.35	0.1089	−9.48	3.19
0.1550	0.0629	0.1360	4.40	0.1084	−8.99	3.28
0.1600	0.0631	0.1354	4.39	0.1083	−9.67	3.31
0.1650	0.0611	0.1346	4.27	0.1078	−9.80	3.23
0.1700	0.0600	0.1358	4.20	0.1083	−12.08	3.29
0.1750	0.0618	0.1343	4.31	0.1072	−9.26	3.35
0.1800	0.0606	0.1341	4.22	0.1071	−9.73	3.39
0.1850	0.0593	0.1354	4.14	0.1079	−10.97	3.35
0.1900	0.0606	0.1350	4.23	0.1075	−9.79	3.35
0.1950	0.0607	0.1353	4.23	0.1077	−10.30	3.35

4.3 Comparison of different approaches with transaction costs

Table XVI shows the final results of four approaches regarding the transaction costs. Using GARCH (1, 1) as the volatility estimate is significantly worse than the volatility cone. Delta tolerance approach has the lowest average costs which means the least frequency of delta hedging. Moreover, this approach also has the highest winning percentage, average P&L and the lowest standard deviation. For investors who are quite risk-averse, the delta tolerance approach also has the highest VaR of yield and Sharpe ratio. For the Variable Bandwidth delta which is based on utility maximization theory, it has the highest medium yield, 75 percent Quantile of yield and lowest standard deviation. Besides, the results of Leland approach are slightly improved comparing with

Table XIV.
Returns based on Black-Scholes approach and delta tolerance

Strategy	Black-Scholes approach	Delta tolerance ($H = 0.135$)
Accumulated costs	−0.0017	−0.0001
Winning percentage	68.86	71.59
Average P&L	0.0592	0.0647
SD of P&L	0.1451	0.1367
Average yield (%)	4.13	4.50
Sharpe ratio	0.0644	0.1014
VaR (95%) of yield	−10.50%	−8.51%
Minimum yield (%)	−49.30	−49.29
25% quantile of yield	−1.17%	−0.81%
Median of yield (%)	2.57	3.31
75% quantile of yield	8.09%	8.15%
Maximum yield (%)	53.50	51.29

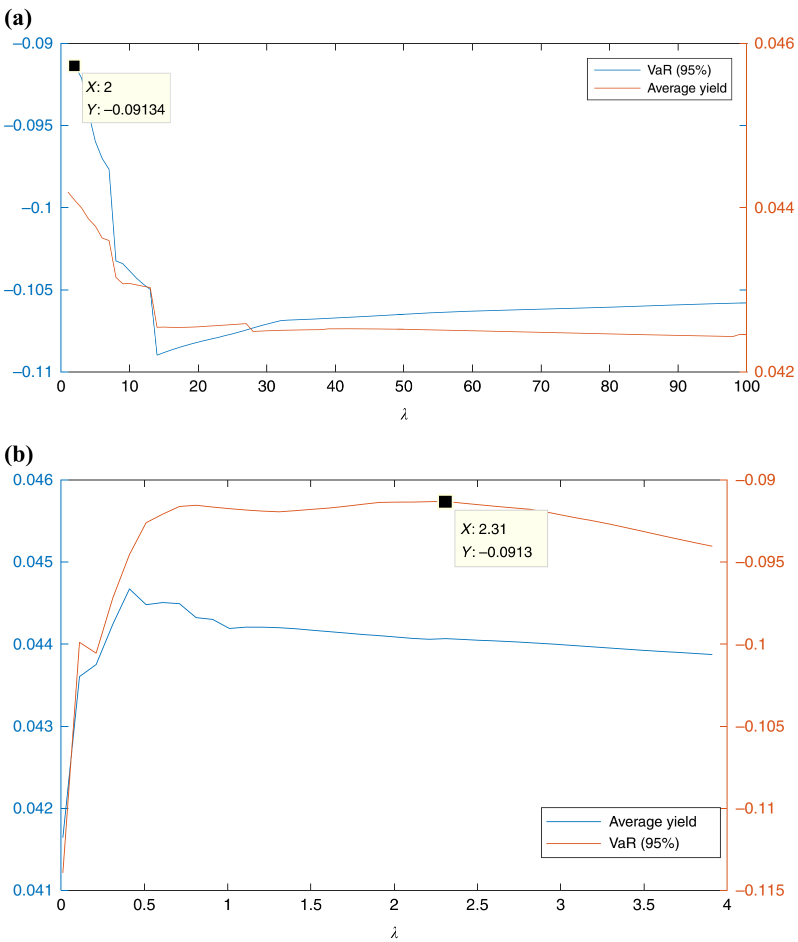


Figure 14.
Average yield and VaR with different λ

Strategy	Black-Scholes approach	Whalley-Wilmott utility maximization strategy ($\lambda = 2.31$)
Accumulated costs	−0.0017	−0.0011
Winning percentage	68.86	70.23
Average P&L	0.0592	0.0631
SD of P&L	0.1451	0.1369
Average yield (%)	4.13	4.41
Sharpe ratio	0.0644	0.0937
VaR (95%) of yield	−10.50%	−9.13%
Minimum yield (%)	−49.30	−49.29
25% quantile of yield	−1.17%	−1.03%
Median of yield (%)	2.57	3.56
75% quantile of yield	8.09%	8.18%
Maximum yield (%)	53.50	52.03

Table XV. Returns by Black-Scholes approach and utility maximization strategy

Strategy	Black-Scholes		Leland		Delta tolerance ($H = 0.135$)		Whalley-Wilmott ($\lambda = 2.31$)	
	Volatility cone	GARCH	Volatility cone	GARCH	Volatility cone	GARCH	Volatility cone	GARCH
Accumulated costs	−0.0017	−0.0013	−0.0017	−0.0013	−0.0001	−0.0007	−0.0011	−0.0007
Winning percentage	68.86	69.54	69.09	69.77	71.59	73.64	70.23	72.27
Average P&L	0.0592	0.0328	0.0600	0.0330	0.0647	0.0369	0.0631	0.0383
SD of P&L	0.1451	0.1390	0.1441	0.1386	0.1367	0.1331	0.1369	0.1349
Average yield (%)	4.13	1.92	4.20	1.93	4.50	2.37	4.41	2.47
Sharpe ratio	0.0644	−0.1471	0.0714	−0.1461	0.1014	−0.1050	0.0937	−0.0682
VaR (95%) of yield	−10.50%	−21.61%	−10.02%	21.34%	−8.51%	−21.80%	−9.13%	−21.29%
Minimum yield (%)	−49.30	−35.94	−46.96	−35.46	−49.29	−34.46	−49.29	−35.06
25% quantile of yield	−1.17%	−0.82%	−1.09%	−0.78%	−0.81%	−0.11%	−1.03%	−0.29%
Median of yield (%)	2.57	2.48	2.66	2.50	3.31	3.09	3.56	2.93
75% quantile of yield	8.09%	7.00%	8.07%	7.04%	8.15%	7.43%	8.18%	7.63%
Maximum yield (%)	53.50	45.95	53.33	42.98	51.29	48.10	52.03	48.10

Table XVI. Returns based on different hedging approaches with transaction costs

the Black-Scholes approach, and this strategy has the largest minimum yield. As Black-Scholes approach provides the largest maximum yield and the lowest minimum yields, this approach seems to show more extreme cases of returns comparing to other hedging strategies.

Concerning all the characteristics of four hedging approaches with transaction costs, for investors who are more risk-averse or cost-conscious, it is better to choose delta tolerance approach as it can provide higher VaR, lower costs and relatively higher profit.

5. Volatility arbitrage analysis of dynamic hedging using futures

5.1 Introduction of approach

Similar to hedging with spot ETF, we can also use futures for delta hedging. When the price fluctuates in a small range, the P&L of the future is approximately equal to the P&L of the option. When the price fluctuates largely, we can re-adjust the future positions based on the currently calculated delta value. Based on the Black-Scholes formula, we know the following:

$$C_t = S_t N(d_1) - K e^{-r(T-t)} N(d_2) \tag{24}$$

Thus, the delta of call option is $\Delta(t)_{call} = \frac{\partial C(t,S)}{\partial S} = N(d_1)$, while the relationship between spot and future is $F = Se^{r(T-t)}$. So the delta of call option for future is as follows:

$$\Delta(t)_{F/call} = \frac{\partial C}{\partial F} = \frac{\partial C}{\partial S} \cdot \frac{\partial S}{\partial F} = e^{-r(T-t)} N(d_1) \tag{25}$$

Similarly, the delta of put option for future is as follows:

$$\Delta(t)_{F/put} = \frac{\partial P}{\partial F} = \frac{\partial P}{\partial S} \cdot \frac{\partial S}{\partial F} = -e^{-r(T-t)} N(-d_1) \tag{26}$$

The method is actually equivalent to using futures to adjust the position of risky assets, and the transaction costs of trading futures is much lower, compared with spot ETF. Transaction fee for the 50 futures is 0.023 percent of transaction amount. The filing fee is $1 each, while the total transaction cost for institutional investors to buy and sell 50 ETF is 3.65 BP proportional.

In addition, comparing Shanghai 50 Index with its future (Figure 15), over the past year, the future-spot basis is always negative. Futures contracts are substantially discounted, and the discount rate of most distant future contract of CSI 500 was close to 20 percent. While the negative basis makes the costs of using spot and future to hedge different. When the portfolio is required to hold the long position of underlying assets, hedging with futures can reduce the cost. We can also distinguish between long and short positions of options to choose the underlying for hedging in order to obtain a higher return. However, there exists some disadvantages of this strategy. First of all, the strategy can only guarantee Delta neutral, but cannot achieve Gamma neutral; moreover, the relationship between future and spot is simplified as $F = Se^{r(T-t)}$, which may lead to basis risks.

5.2 Back-test result

In the empirical analysis, this paper discusses two hedging strategies using futures respectively: the first is to use futures instead of spot, while the second is to use both spot and futures but to distinguish between long and short positions of options to hedge differently. Specifically, when the delta hedging requires holding the long positions of

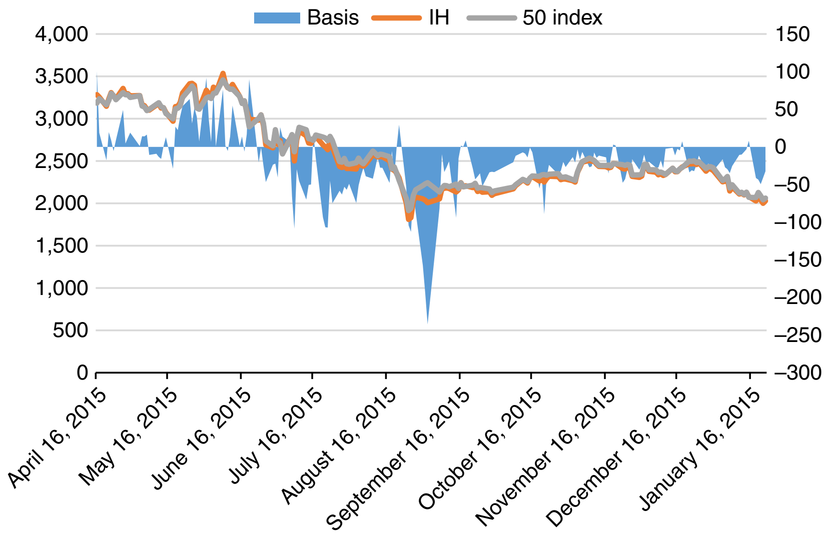


Figure 15.
SSE 50 index and IH dominant contract

underlying assets, we choose the futures to hedge, and when the delta hedging requires holding the short positions, we use the spot to hedge in order to earn part of the basis return. Since the SSE 50 futures are only launched by the China Financial Futures Exchange on April 16, 2015, and the research object of this paper includes the options listed before that date, here this paper only considers the options listed after April 16, 2015 for hedging with futures, and options listed before are selected for hedging with 50 ETF, and stop-loss signal of 20 percent is also taken into account here. In addition, since the delta tolerance hedging strategy can lead to the best results with transaction costs, which is studied above, this strategy is also selected as the complement for the Black-Scholes strategy with the tolerable delta error value H of 0.135.

From Table XVII, it is easy to find that although the median yield and 75 percent quantile of hedging using futures are slightly increased, this strategy does not improve the back-test results of the original strategy. However, notably the cumulative transaction cost decreased from −0.0017 to −0.0001 due to the low transaction cost advantage of future trading. If we use spot and futures to hedge respectively, although the winning percentage is lower than original strategy, the average P&L, average yield, Sharpe ratio, VaR, median and other indicators have improved, and the standard deviation also decreased slightly. So when holding long positions, using futures instead of the spot to hedge can improve the original hedging strategy. In addition, with the introduction of delta tolerance, the winning percentage, average P&L, average yield, Sharpe ratio and median of the strategy have a slight increase, and transaction costs fall to 0.0003. However, considering the low transaction cost characteristic of futures itself, the improvement of the strategy is not as significant as the spot hedging.

6. Conclusion

The results of volatility arbitrage strategies show that there are volatility-trading opportunities in Chinese market on China 50 ETF options. With volatility cones as the entry and exit signals, Black-Scholes dynamic hedging can achieve a winning percentage over 75 percent and average yield of 4.68 percent regardless of transaction costs. Moreover, after examination of volatility cones based on calculation within different time period, the best choice for the time period is one year. Besides, due to the large extreme retracement, the introduction of stop-loss signal can improve the performance of the strategy and the retracement results. It is also found that the volatility cone is better than GARCH model as the volatility estimate. However, Discrete Hedging within Fixed Intervals proposed by Wilmott in 1994 does not improve the final returns of volatility arbitrage while this approach is theoretically better than the original Black-Scholes hedging methods.

Strategy	Black-Scholes approach	Futures hedging	Spot hedging	Spot hedging with delta tolerance ($H = 0.135$)
Accumulated costs	−0.0017	−0.0001	−0.0006	−0.0003
Winning percentage	68.86	65.45	67.04	67.05
Average P&L	0.0592	0.0571	0.0593	0.0596
SD of P&L	0.1451	0.1492	0.1385	0.1384
Average yield (%)	4.13	4.04	4.21	4.24
Sharpe ratio	0.0644	0.0550	0.0745	0.0773
VaR (95%) of yield	−10.50%	−11.32%	−9.95%	−9.95%
Minimum yield (%)	−49.30	−47.48	−49.30	−49.29
25% quantile of yield	−1.17%	−1.36%	−1.20%	−1.20%
Median of yield (%)	2.57	3.35	2.72	2.76
75% quantile of yield	8.09%	8.43%	8.12%	8.12%
Maximum yield (%)	53.50	53.50	51.29	51.30

Table XVII. Futures strategy and Black-Scholes approach

Regarding the existence of transaction costs, final results of volatility arbitrage is still satisfactory because all the approaches can achieve winning percentages over 65 percent and average yield similar to the cases without transaction costs, which means the costs have little impact on the efficiency of volatility-trading strategies. The variable bandwidth delta strategy is theoretically the most favorable strategies studied. However, the empirical study shows the delta tolerance approach can generate best results. The Leland approach did not provide improved results brought by Black-Scholes as expected because the adjustment on volatility is quite small. In addition, Black-Scholes compared to other strategies has easier access to extreme profit and loss results. On the other hand, regarding cases with transaction costs, volatility cones are also better than GARCH models as the volatility estimates.

The complete replacement of hedging underlying from spot to future has poor effects, with all indicators declining slightly. However, hedging with spot and future respectively can improve the Black-Scholes strategy back-test results. The VaR increases significantly, which shows its great contribution on risk management. The introduction of delta tolerance has little contribution. The main reason is that the transaction cost of the futures is already quite small, and the reduction of hedging frequencies has little influence on total transaction costs.

It is also noticeable that all the approaches for volatility arbitrage cannot guarantee a positive profit mainly because of the following: since it is common for Gamma values to become larger at the expiration of an option, it is difficult to replicate delta, and the prediction of the actual volatility is difficult. So some hedge funds tend to use more complex strategies mixing several different derivatives to guarantee smaller losses or take more risks to generate larger profits potentially.

Notes

1. The analysis here considers only the case where the actual volatility is less than the implied volatility. If the implied volatility is considered to be higher than the actual volatility, the call option should be shorted for the corresponding analysis.
2. In Tables VII-XI, the Sharpe ratio is sometimes negative while the average returns are positive, mainly because the risk-free rate of return makes it possible for some strategies to have a negative Sharpe ratio at lower average gains and losses.

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Further reading

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Corresponding author

Jun Song can be contacted at: songjun@fudan.edu.cn