lecture 7

scale space

(Slides, courtesy Prof. Langer)

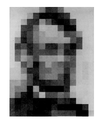
Overview of today's lecture

- Motivation (eye candy)
- Gaussian scale space (definition)
- Gaussian pyramid
  - Application: image registration (Lucas-Kanade – to be discussed soon in a future lecture)
- Features in scale space
  - edge, box

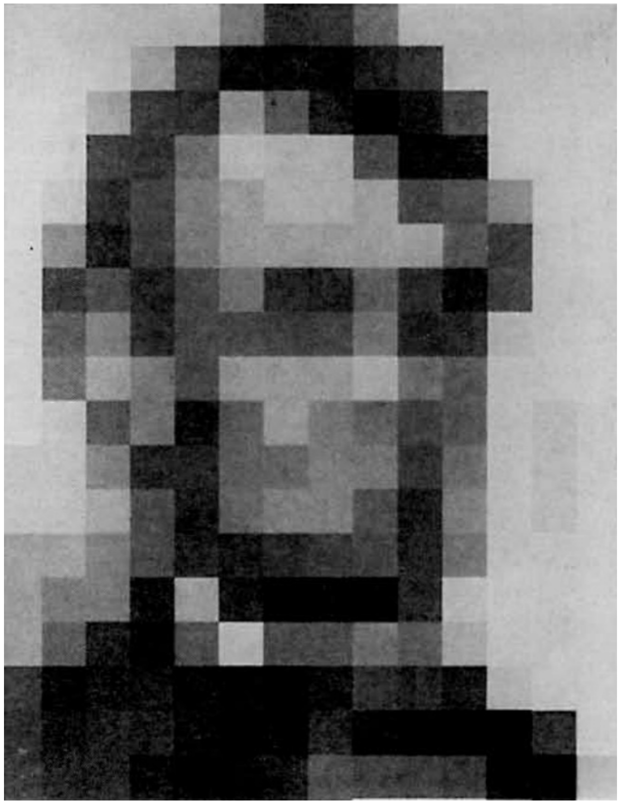
The image size of an object depends on the object's 3D size and its distance from the camera.

We need to analyze images at different " scales".

Today we will introduce some basic ideas.



14 x 18 pixel
image of
famous face

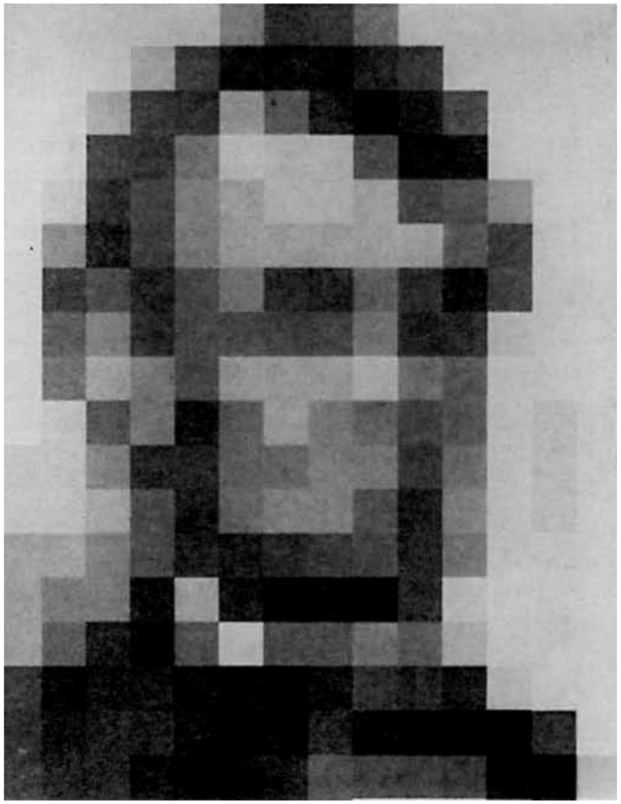


14 x 18 pixel
image

Here we have expanded the image.

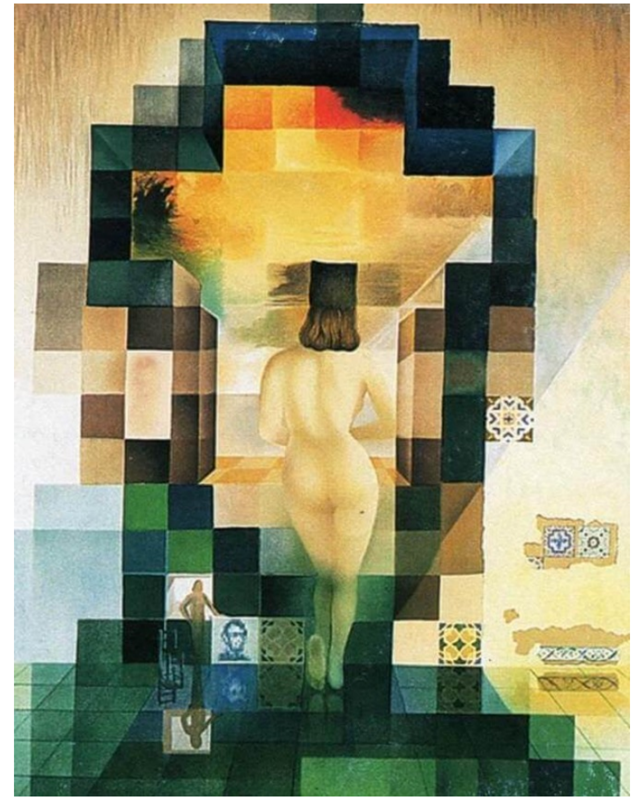
Defocus your eyes on this image
or take off your glasses,
or view it from a distance.

The edges of the expanded pixels will blur out
and you may see the recognizable face.

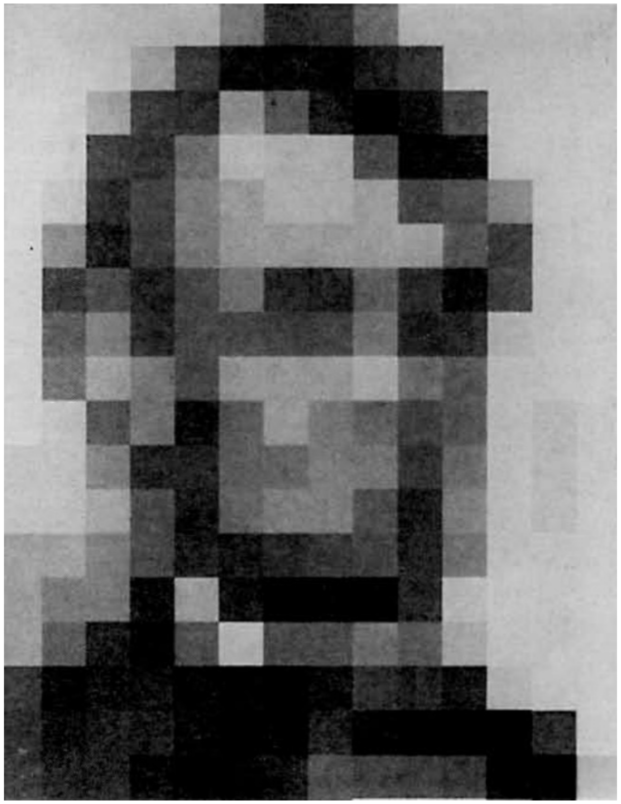


14 x 18 pixel
image of
Abraham Lincoln
by Leon Harmon 1976,
Scientific American

(first reported in 1973)

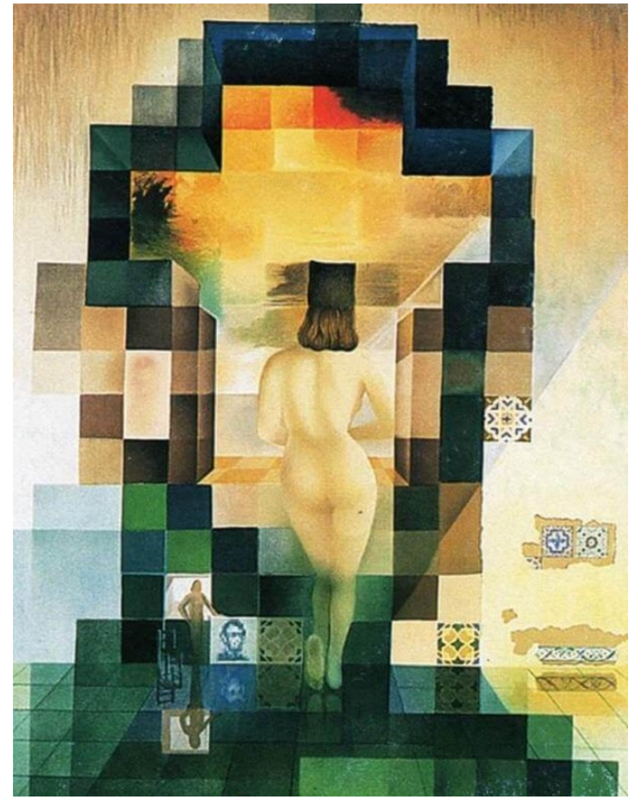


Salvador Dali painting (1974):
2.5 x 2 m
“Gala Contemplating the Mediterranean Sea Which at Twenty Meters Becomes the Portrait of Abraham Lincoln”



14 x 18 pixel
image of
Abraham Lincoln
by Leon Harmon 1976,
[*Scientific American*]

Original portrait



Salvador Dali painting (1974):
2.5 x 2 m
“Gala Contemplating the Mediterranean Sea Which at Twenty Meters Becomes the Portrait of Abraham Lincoln”

Painting by Chuck Close (1986)

Actual detail of right eye

IMAGINE
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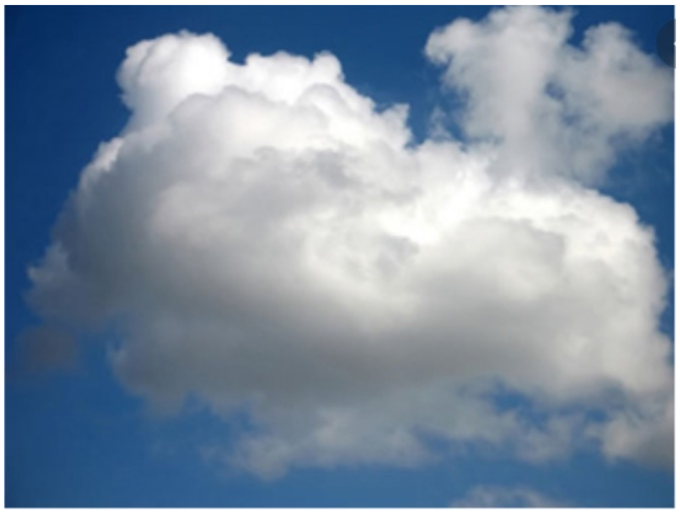
THE IMMERSIVE EXHIBITION

https://imagine-vangogh.ca/en/

Another example of image structure at many scales.

Fractals

Many natural phenomena produce similar geometric structure at multiple scales. The branch of mathematics that describes these geometries is called Fractals.



Today we will examine *basic* ideas for how to analyze images at different scales.

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- Motivation (eye candy)
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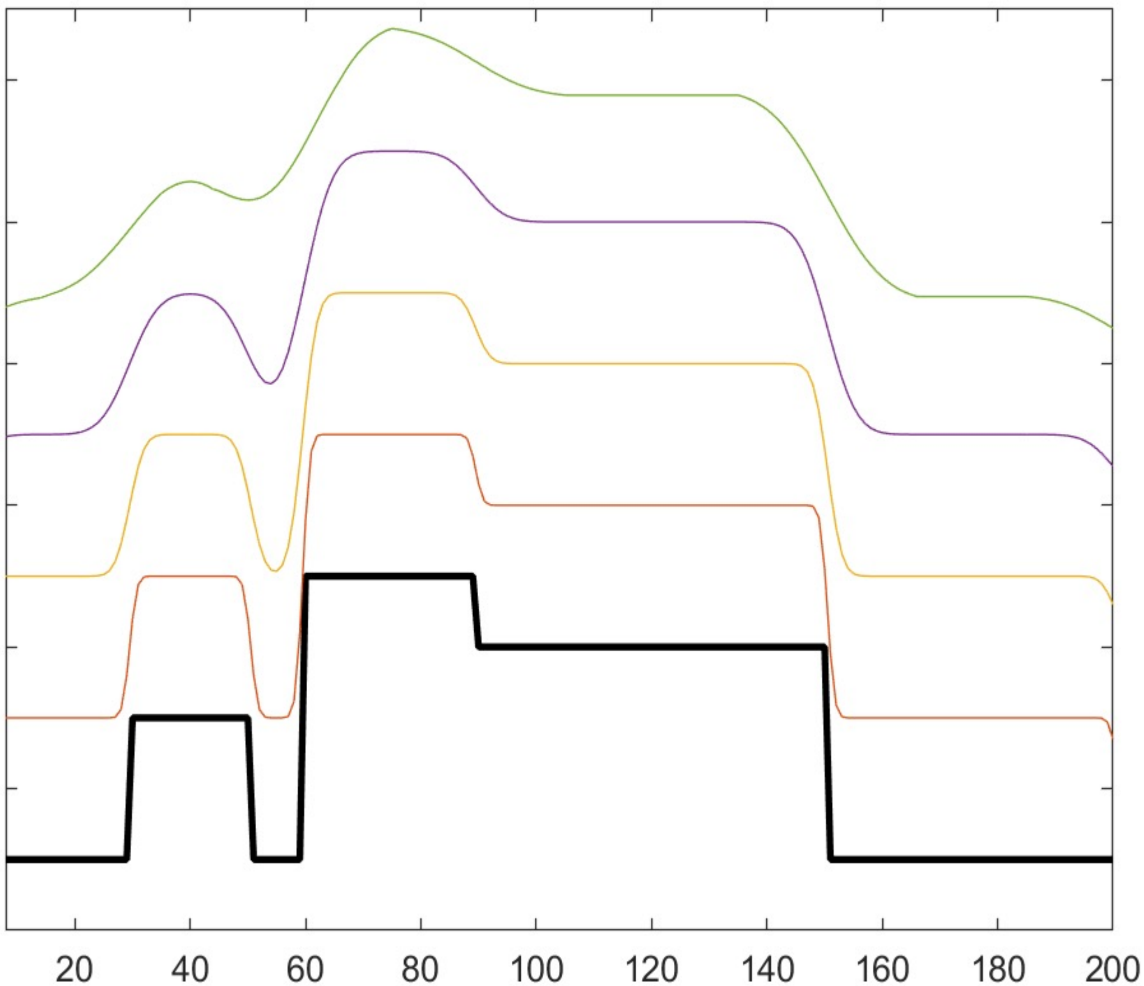
Gaussian Scale Space (1D image)

$$I(x, \sigma) = I(x) * G(x; \sigma)$$

Each curve is shifted upwards for easier comparison.



$I(x, 8)$

$I(x, 4)$

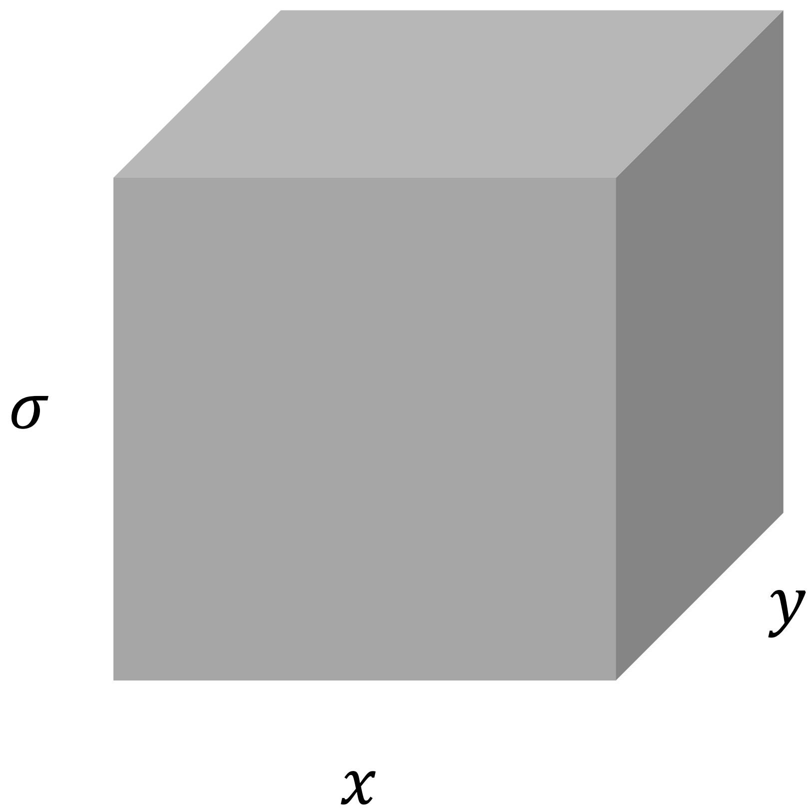
$I(x, 2)$

$I(x, 1)$

$I(x)$

Gaussian Scale Space (2D image)

$$I(x, y, \sigma) = I(x, y) * G(x, y; \sigma)$$



The next slides will show examples σ slices for different σ.

For 2D images, today we will work with the following example.

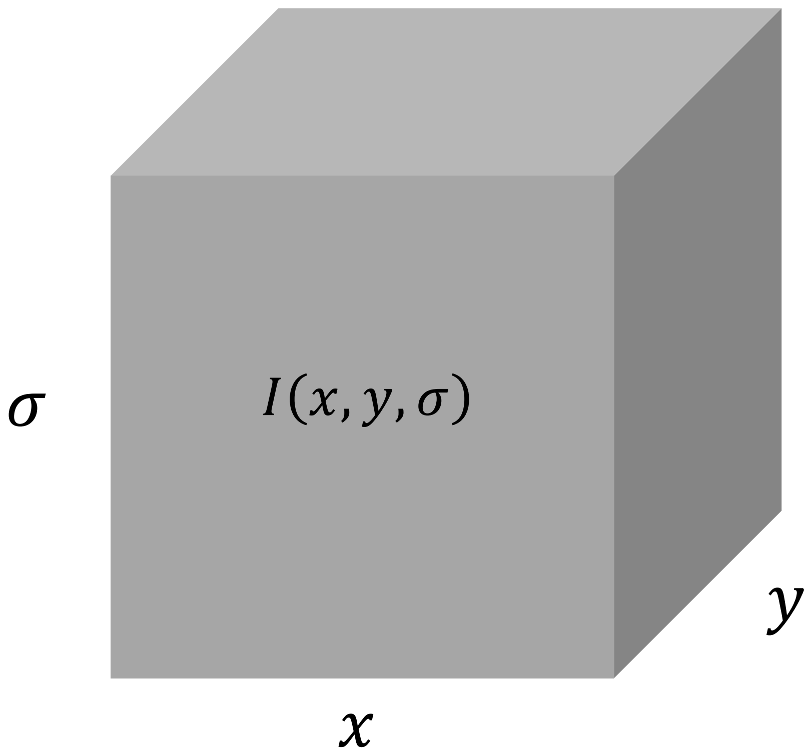
$I(x, y)$

512 x 512 pixels

[Habitat 67 (Montreal Old Port)]

Gaussian Scale Space (2D image)

$$I\left(x, y, \sigma = 2^k\right) = I(x, y) * G(x, y; \sigma = 2^k)$$



For our example, we will sample σ slices using $\sigma = 2^k$, starting at k=0.
This choice is arbitrary.

$$I(x, y, \sigma = 1) = I(x, y) * G(x, y; \sigma = 1)$$

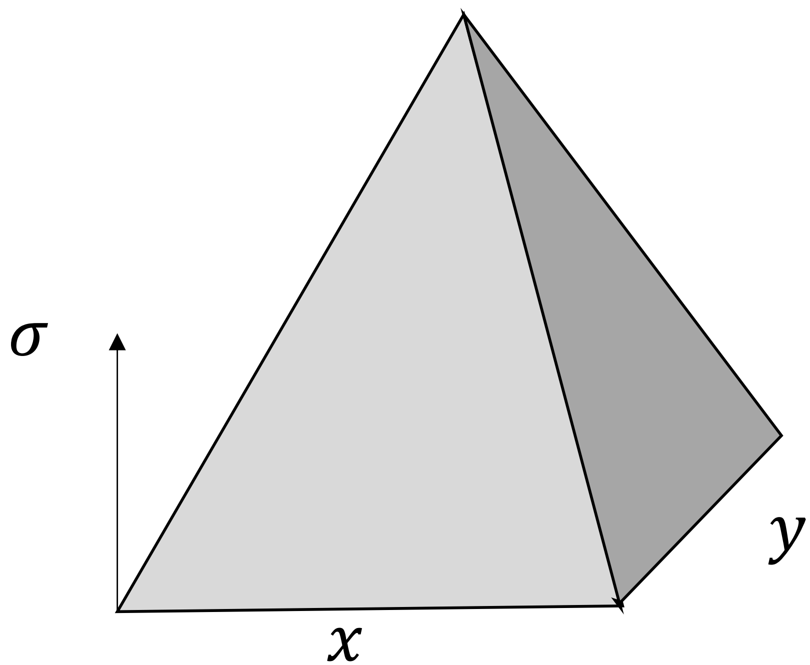
$$I(x, y, \sigma = 2) = I(x, y) * G(x, y; \sigma = 2)$$

$$I(x, y, \sigma = 4) = I(x, y) * G(x, y; \sigma = 4)$$

$$I(x, y, \sigma = 8) = I(x, y) * G(x, y; \sigma = 8)$$

Gaussian Pyramid

Create image at layer k by *subsampling* the blurred version of the image.



Gaussian Pyramid

Create image at layer k by *subsampling* $I\left(x, y, \sigma = 2^{k-1}\right)$ with step size ("stride") 2^k.

This results in each successive image being a factor 2 smaller in both x and y.

σ

$$
\begin{aligned}
I(x,y) &= I(x,y,0) \\
I_1(x,y) &= I(2x,2y,1) \\
I_2(x,y) &= I(4x,4y,2) \\
I_3(x,y) &= I(8x,8y,4) \\
&= \ldots \\
I_k(x,y) &= I(2^k x, 2^k y, 2^{k-1})
\end{aligned}
$$

Gaussian Pyramid (*in practice*)

On the previous slide, we subsampled from an image that is the same size as the original.

In practice, one forms a Gaussian pyramid instead by: iteratively blur (using $\sigma = 1$) and subsample (using stride 2).
This gives a slightly different result.

$I_0(x, y) \leftarrow I(x, y)$

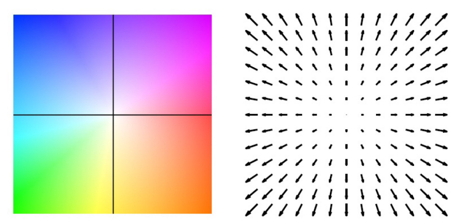
for k = 0 to k_{max} {

$I_{k+1}(x, y) \leftarrow$ subsample $I_k(x, y) * G(x, y; \sigma = 1)$
using a stride of 2

}

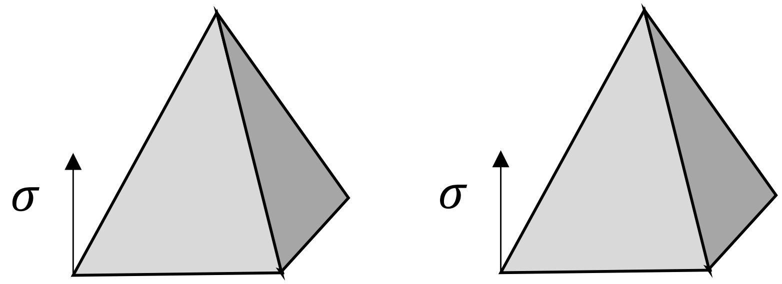
Application of Gaussian pyramids: Multiscale Image Registration (Lucas-Kanade)

We will come back to this in some detail in a future lecture.



Multiscale Image Registration
(Lucas-Kanade)

Given images $I(x, y)$ and $J(x, y)$, compute their Gaussian pyramids:



For each σ from largest to smallest {

If σ is largest scale
initialize estimates of $(h_x, \; h_y)$ at each pixel to be (0,0)
else {
initialize estimates of $(h_x, \; h_y)$ at each pixel by interpolating
the final estimates at the next larger scale
}
run iterative Lucas-Kanade algorithm using these initial estimates
}

Multiscale Image Registration
(Lucas-Kanade)

“coarse to fine”




Note that a motion of one pixel at the scale $\sigma = 2^k$ corresponds to a motion of 2^k pixels in the original image.

What are the advantages ?
What are the disadvantages ?

Overview of today's lecture

- Gaussian scale space (definition)
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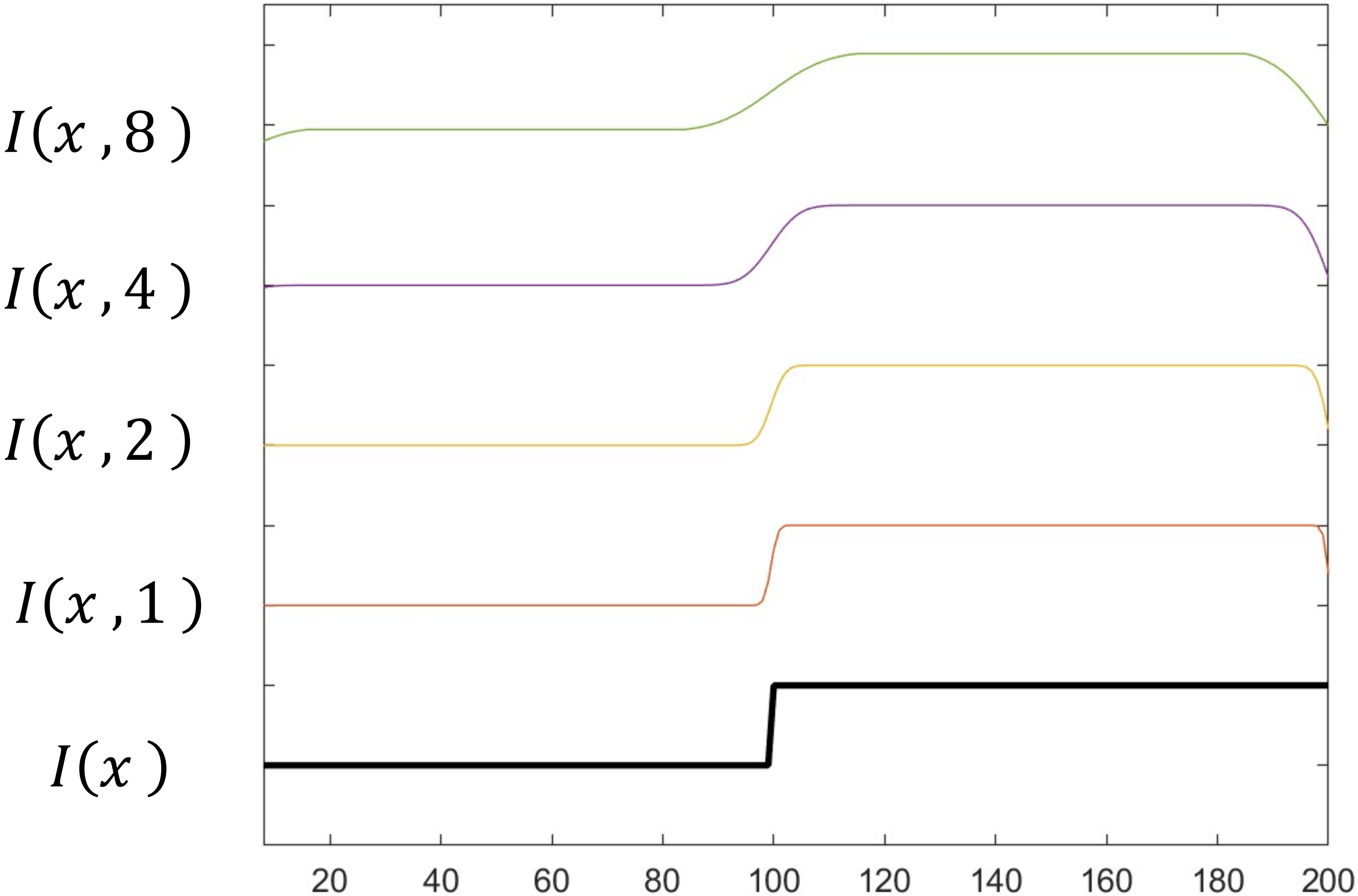
  - Application: image registration (Lucas-Kanade)
- Features in scale space
  - edge, box

Gaussian scale space

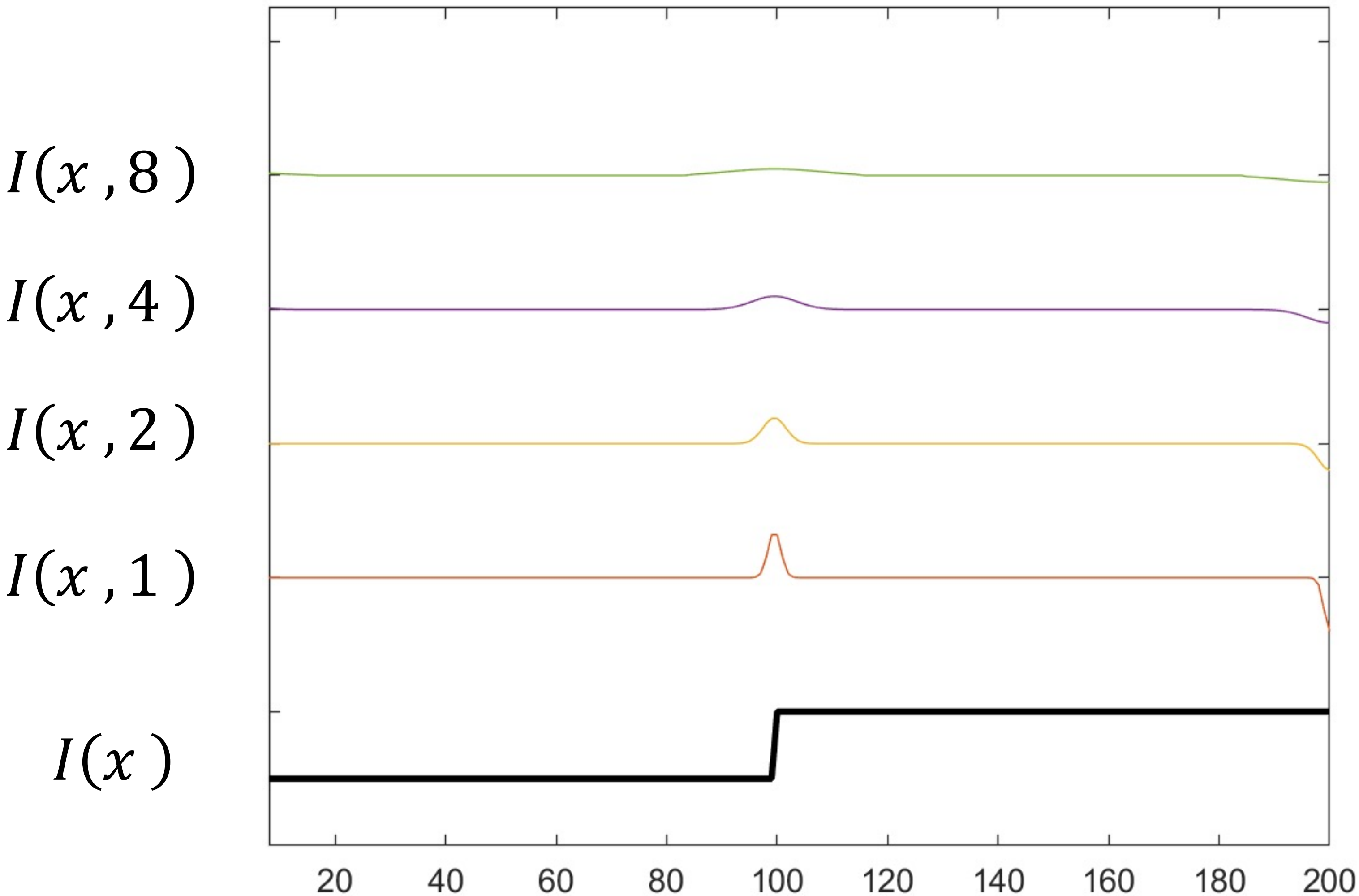
$$I(x, \sigma) = I(x) * G(x; \sigma)$$



Increasing σ smooths the step edge.

Gaussian Derivative Scale Space

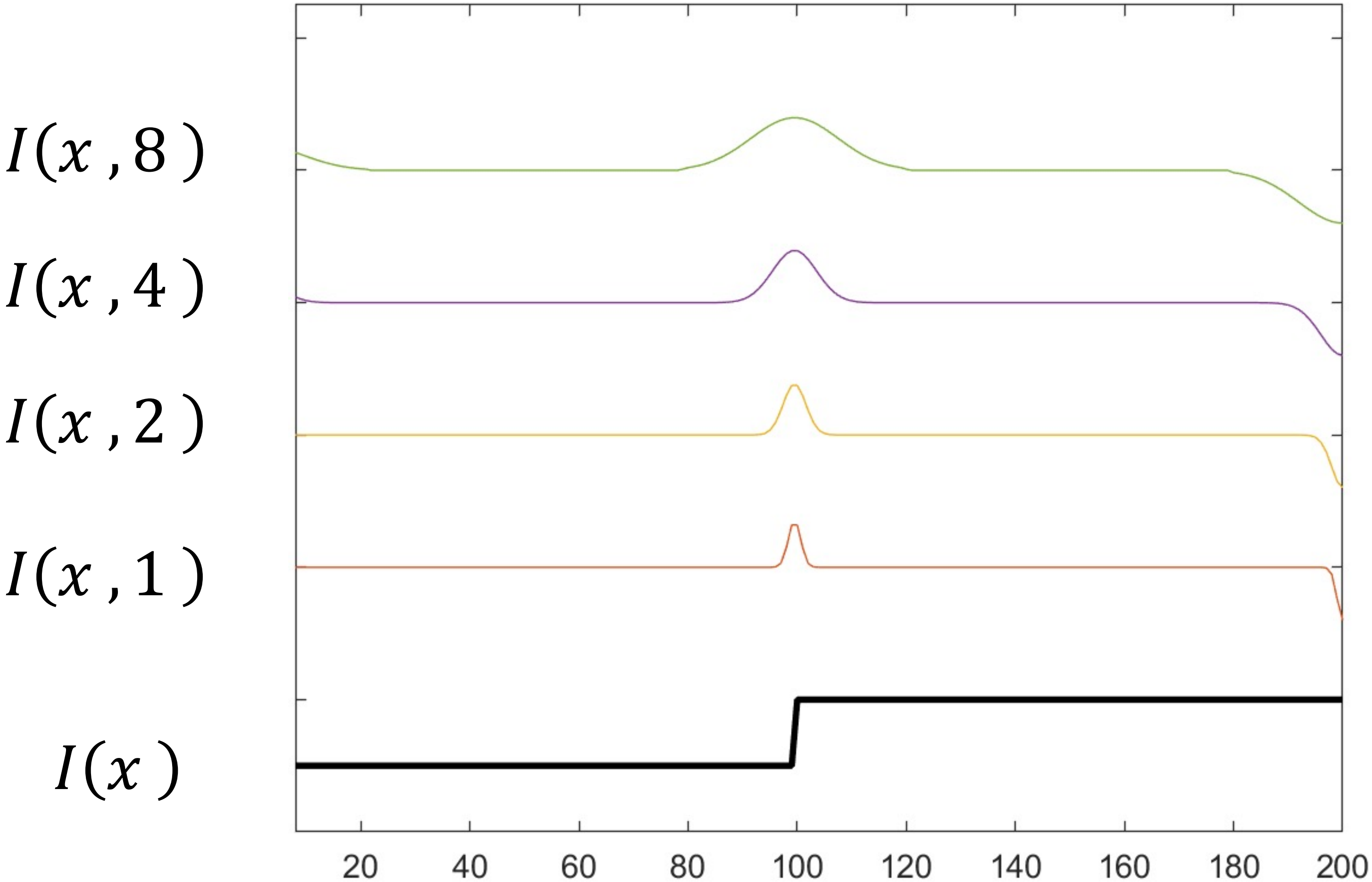
$$I(x, \sigma) = I(x) * \frac{dG(x;\, \sigma)}{dx}$$



The magnitude of the gradient is reduced for larger σ.

Normalized Gaussian Derivative Scale Space

$$I(x, \sigma) = I(x) * \boldsymbol{\sigma} \frac{dG(x;\, \sigma)}{dx}$$



If we multiply the Gaussian derivative filter by the scale σ, then the peak magnitude of the response at the edge is the same for all σ.

(See lecture notes.)

Gaussian 2nd Derivative Scale Space

$$I(x, \sigma) = I(x) * \frac{d^2 G(x\,;\,\sigma)}{d^2 x}$$

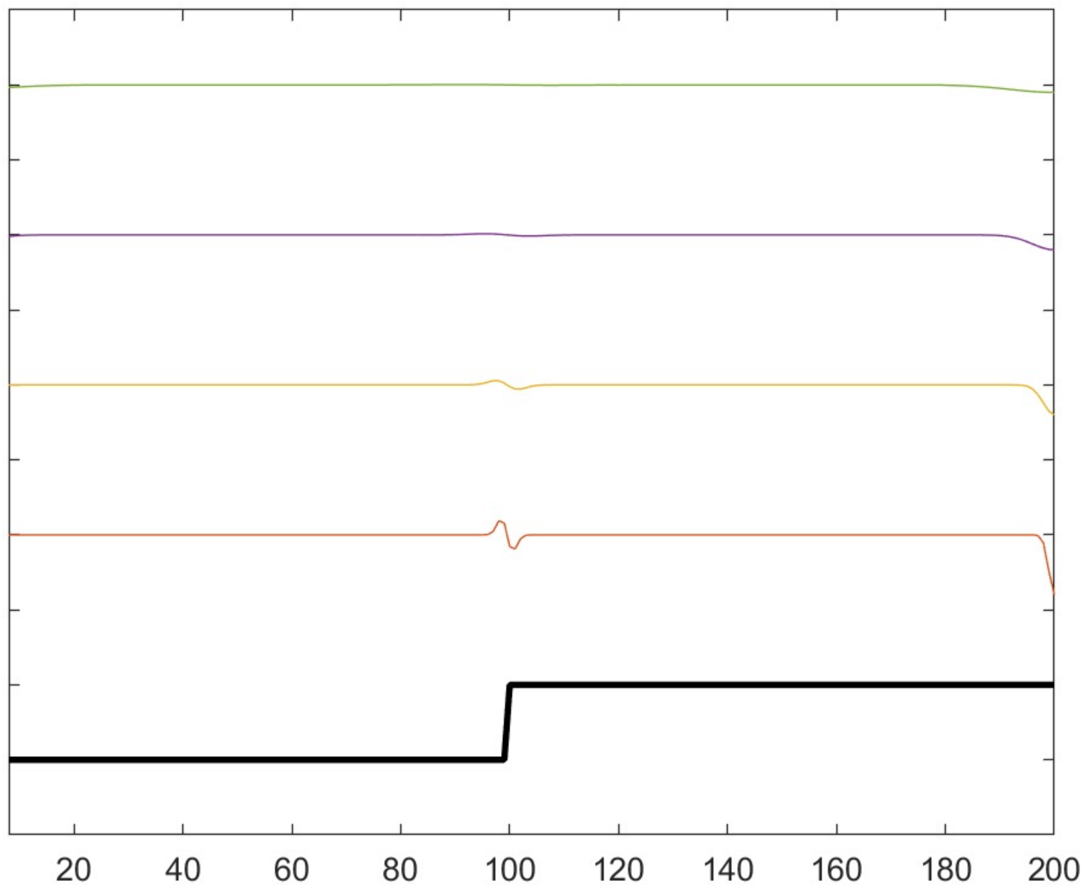
$I(x, 8)$

$I(x, 4)$

$I(x, 2)$

$I(x, 1)$

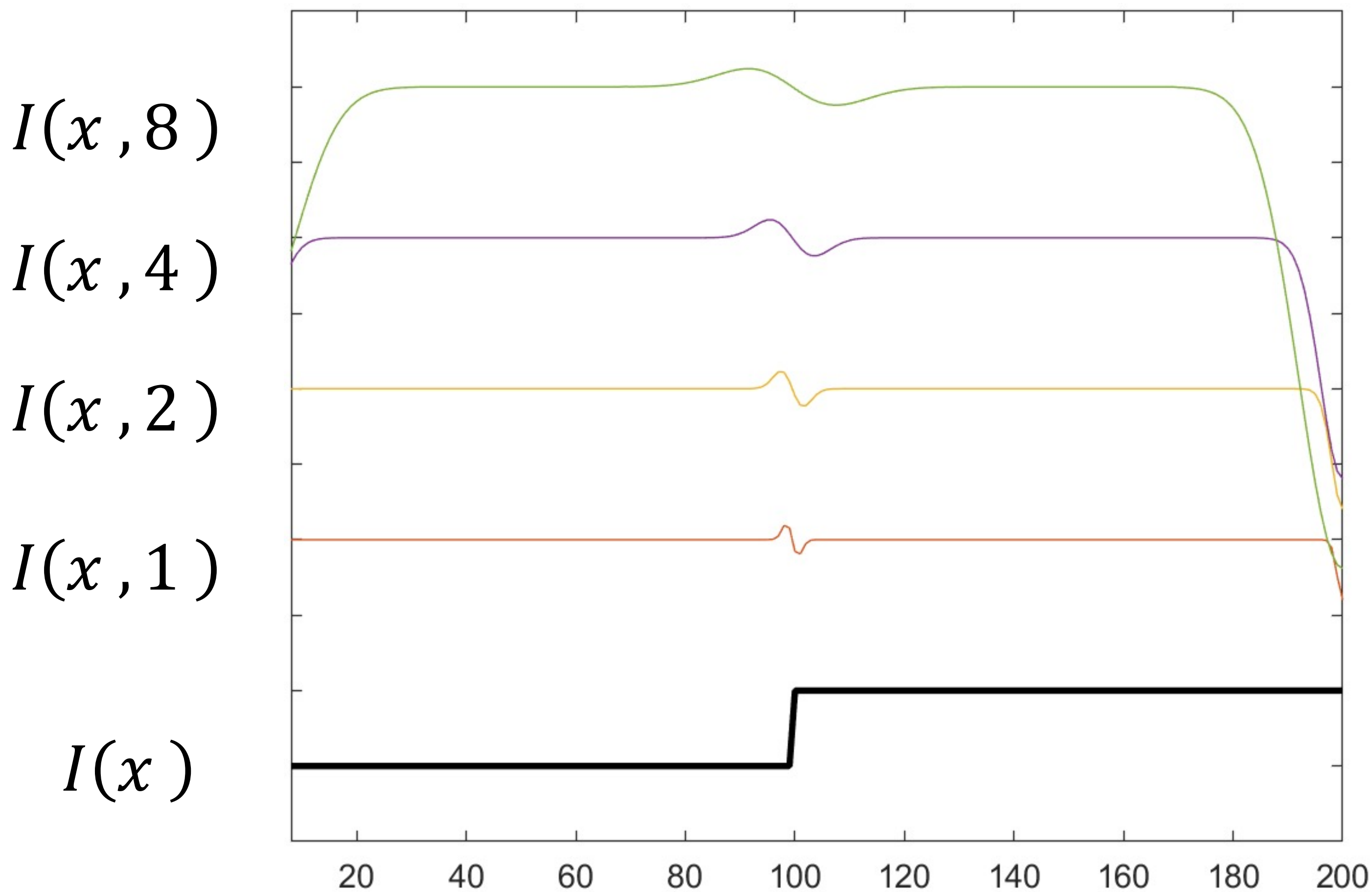
$I(x)$



The magnitude of the step edge blurred with a second derivative of Gaussian is greatly reduced for larger σ.

Normalized Gaussian 2nd Derivative Scale Space

$$I(x,\sigma) = I(x) * \sigma^2 \frac{d^2 G(x\,;\,\sigma)}{d^2 x}$$

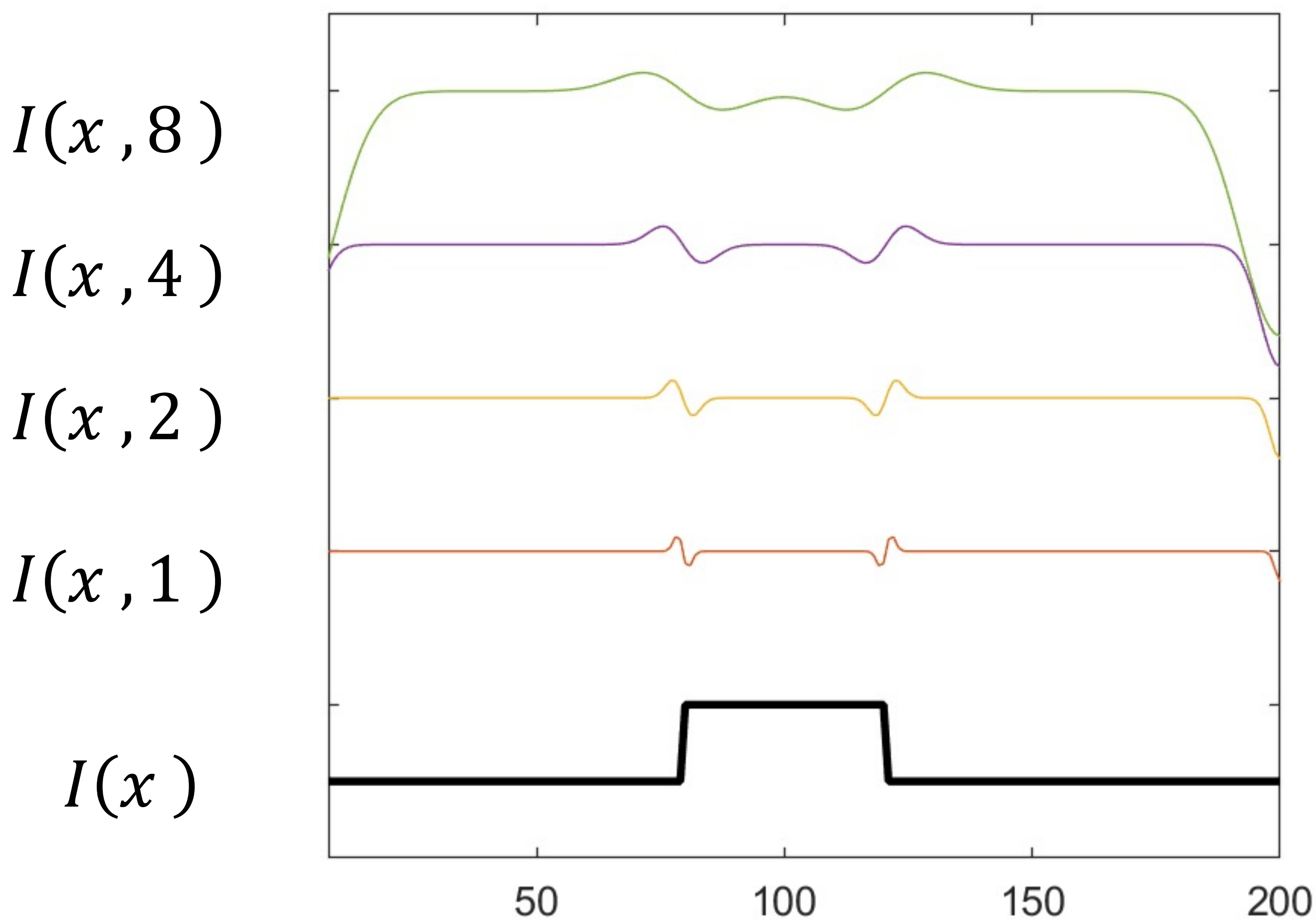


If we multiply the 2nd Gaussian derivative filter by σ^2 then the peak magnitude of the response at the edge is the same for all σ.

(See lecture notes.)

Normalized Gaussian 2nd Derivative Scale Space for “**box**” image

$$I(x,\sigma) = I(x) * \sigma^2 \frac{d^2G(x\,;\,\sigma)}{d^2x}$$



The “box” image in an interesting case because the box has a natural scale (width).

Note that the up edge and down edge of the box give responses that are flipped relative to each other.

Normalized Gaussian 2nd Derivative Scale Space for "box" image

$$I(x\,,\sigma\,) = I(x\,) * \sigma^2 \, \frac{d^2 G(x\,;\,\sigma)}{d^2 x}$$

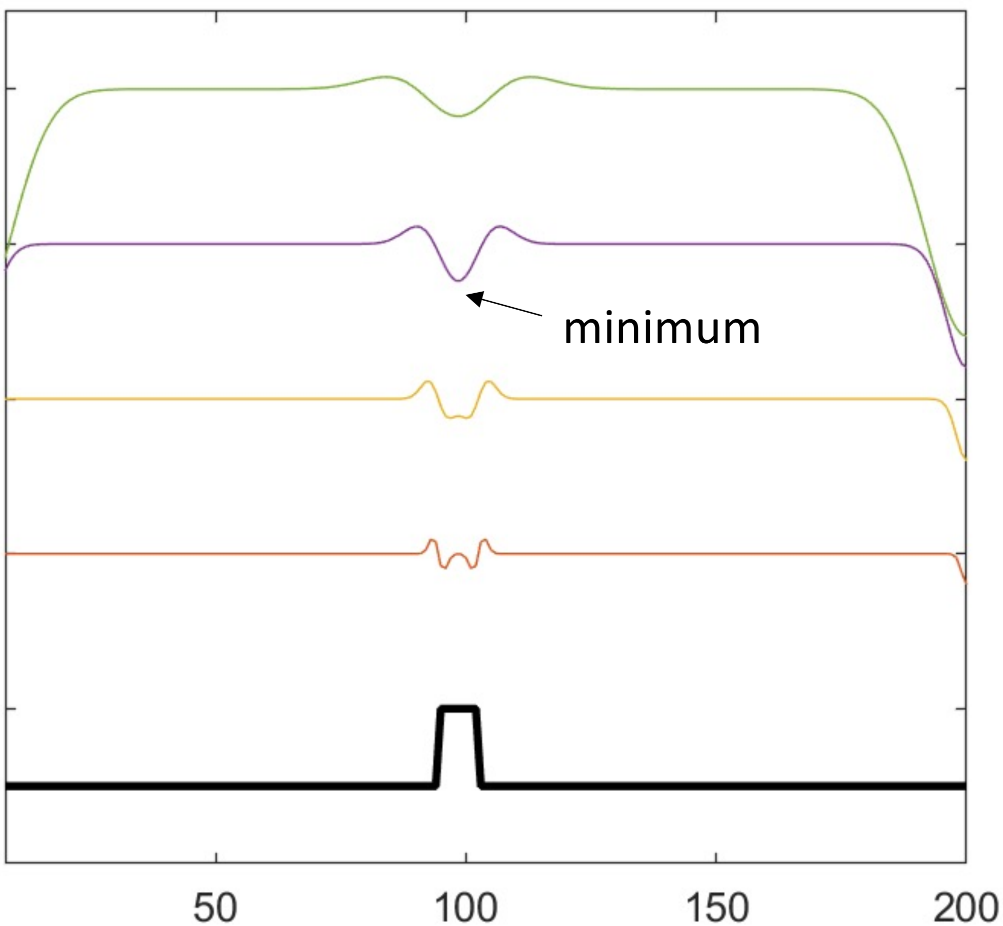
$I(x\,,8\,)$

$I(x\,,4\,)$

$I(x\,,2\,)$

$I(x\,,1\,)$

$I(x\,)$



The response to the box has a minimum in 2D scale space (x, σ) with x at the center of the box and scale σ that is the half width of the box.

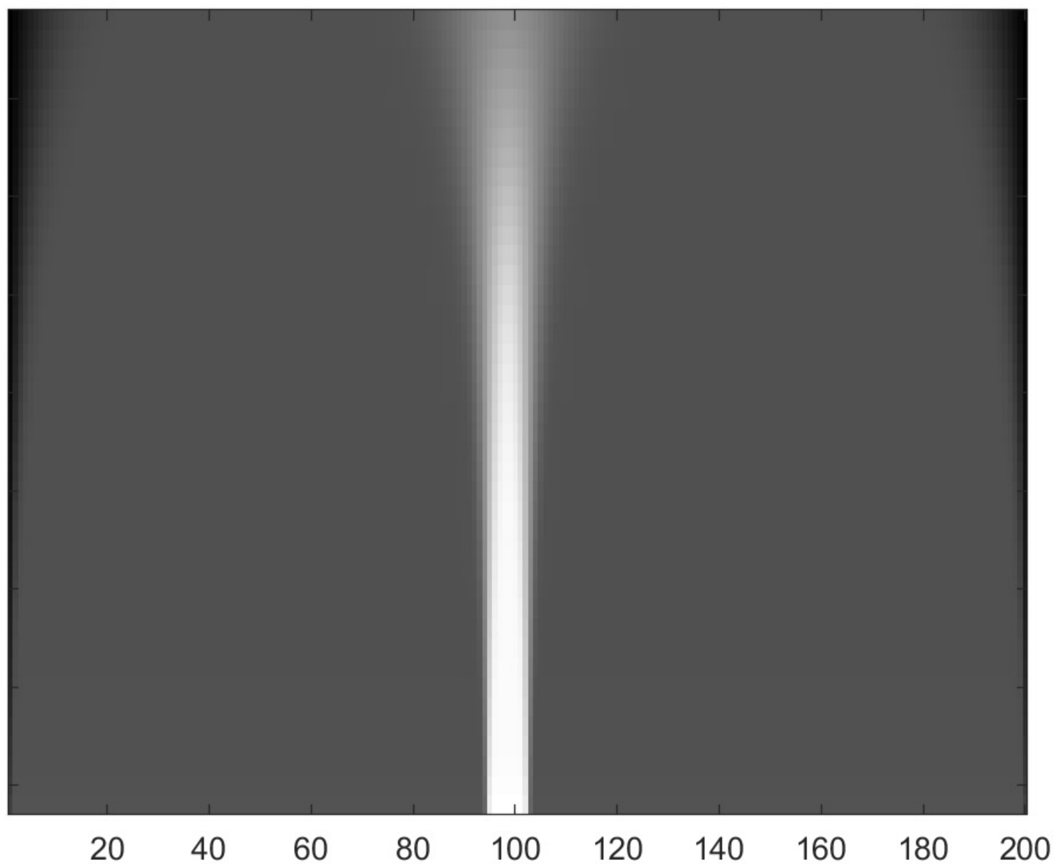
For this example, the box has width 8 and the minimum occurs at σ=4.

(See lecture notes.)

Gaussian scale space for 1D box image

$$I(x, \sigma) = I(x) * G(x; \sigma)$$

$I(x, 8)$

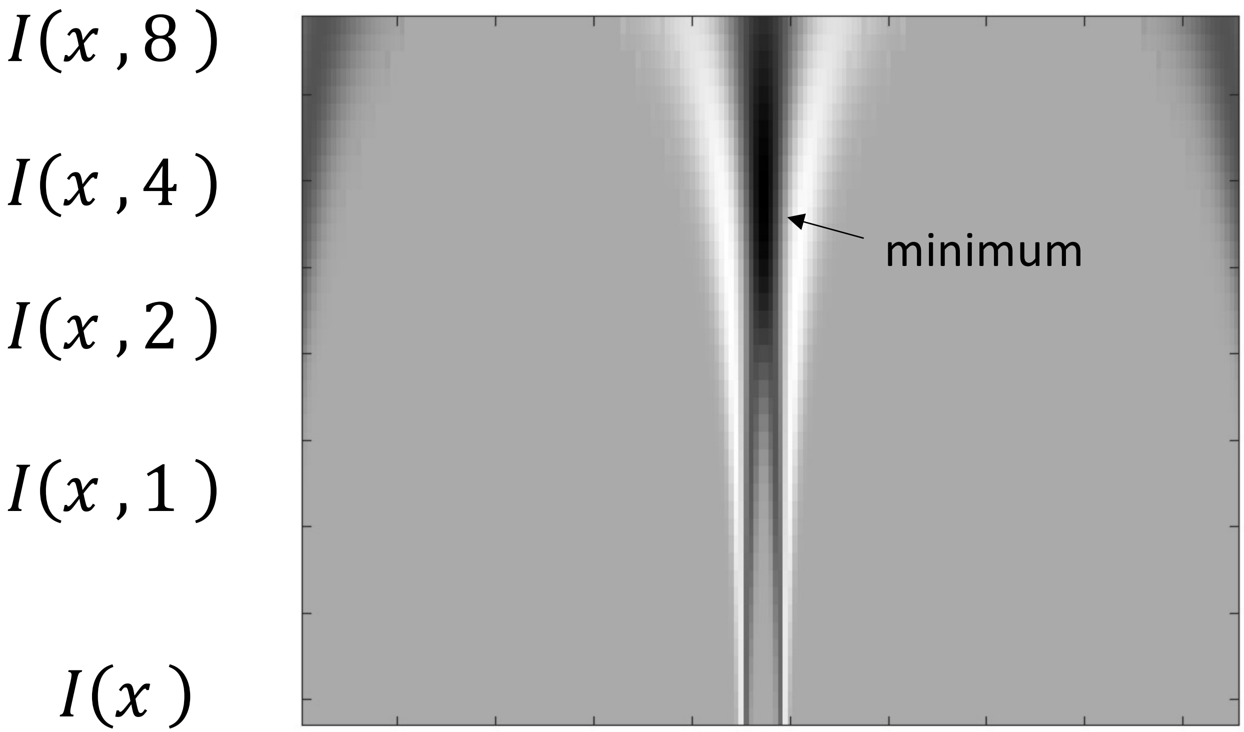


$I(x)$

Increasing σ smooths the step edge.

Normalized Gaussian 2nd Derivative Scale Space for "box" image

$$I(x\,,\sigma\,) = \; I(x\,) * \sigma^2 \; \frac{d^2G(x\,;\,\sigma)}{d^2x}$$

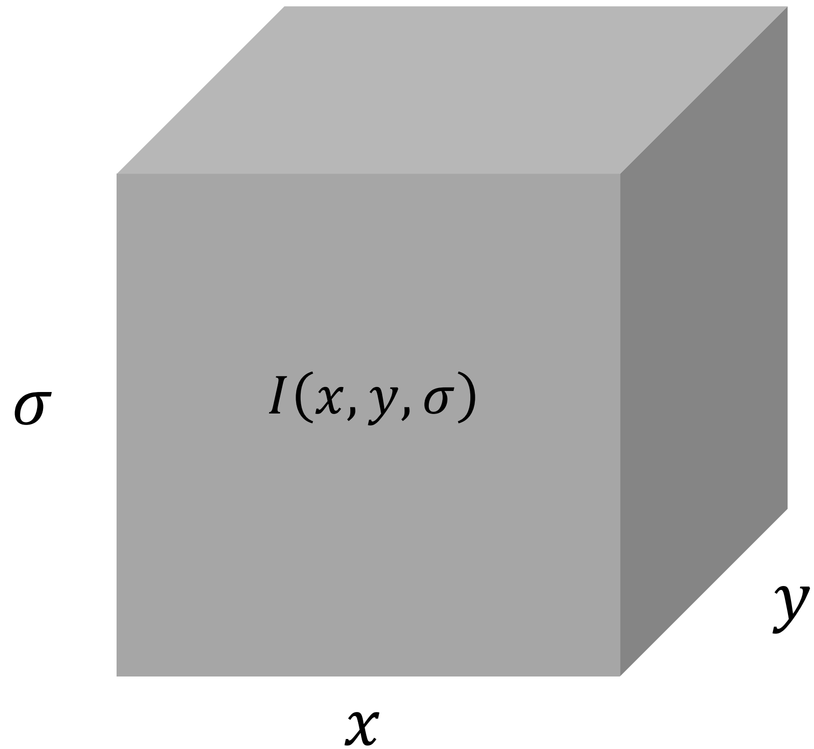


As mentioned two slides ago, the response to the box has a minimum in scale space (x, σ) with x at the center of the box and scale σ that is the half width of the box.

For this example, the minimum occurs at σ=4.

Gaussian Scale Space (2D image)

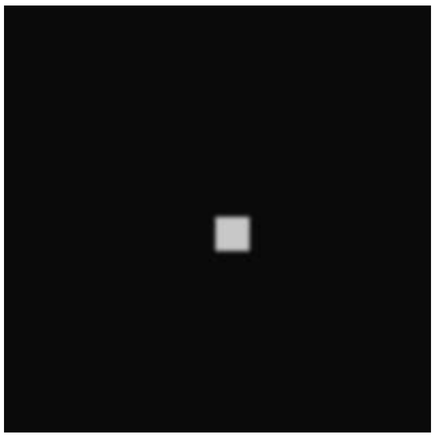
$$I(x,y,\sigma) = I(x,y) * G(x,y;\sigma)$$



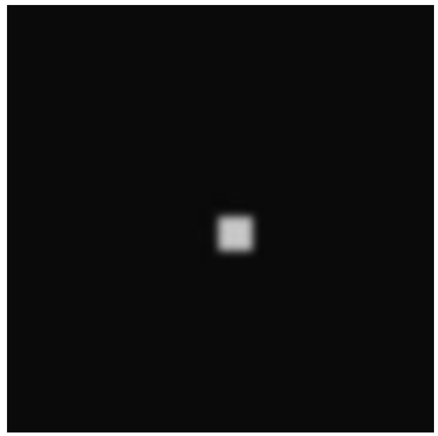
What is a "box" in a 2D image ?

Gaussian Scale Space (2D image)

$$I\left(x, y, \sigma = 2^k\right) = I(x, y) * G(x, y; \sigma = 2^k)$$

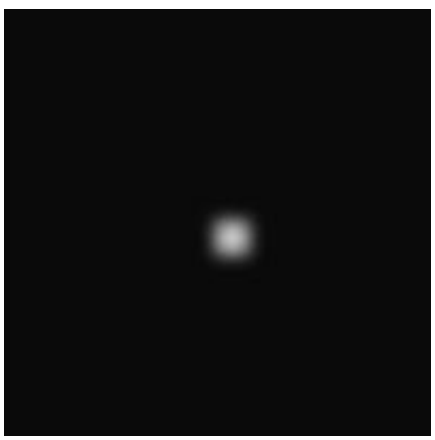


$\sigma = 1$

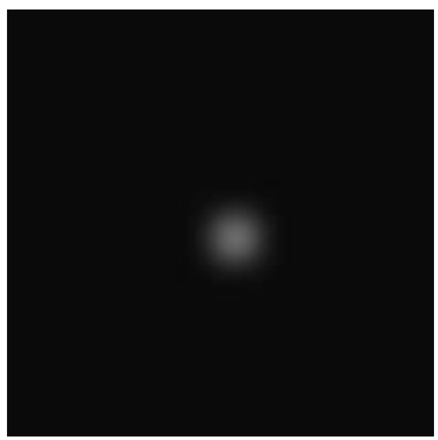


$\sigma = 2$

In this example, the box width and height is 16 pixels.



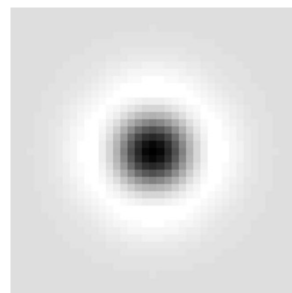
$\sigma = 4$



$\sigma = 8$

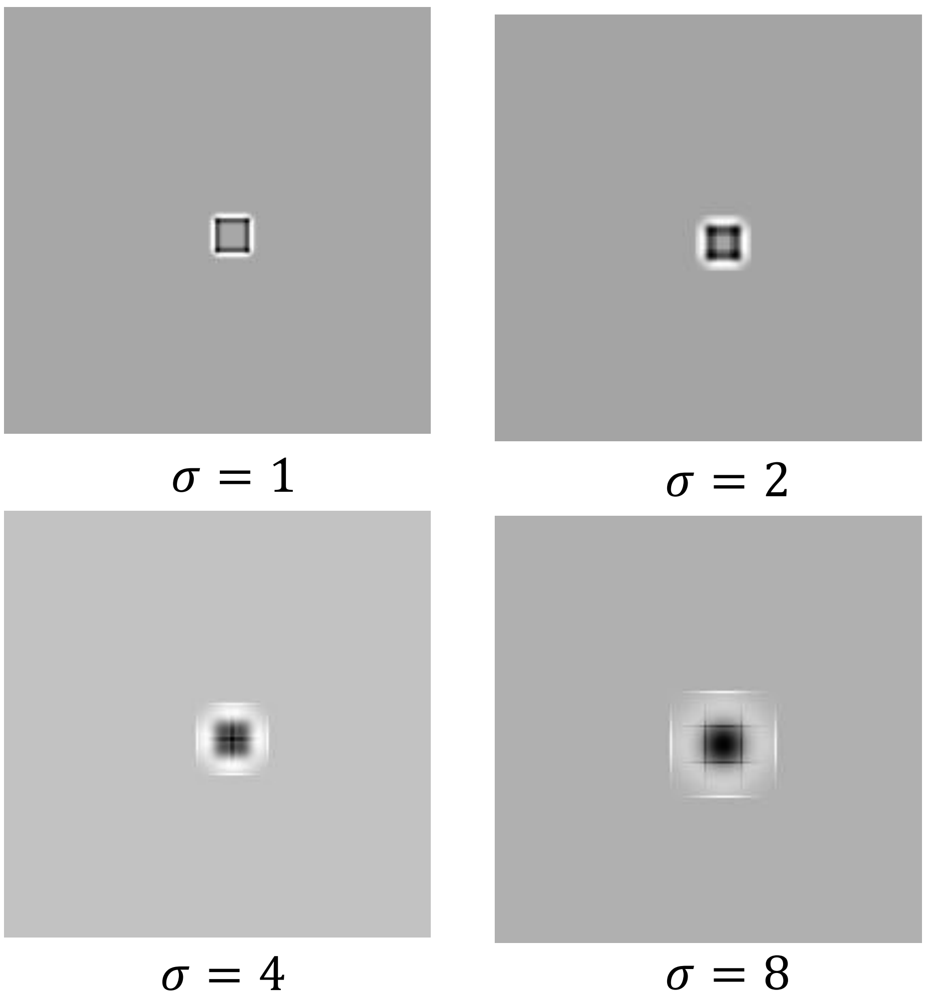
Recall: Laplacian of a Gaussian

$$\nabla^2 G(x, y, \sigma) \equiv \frac{\partial^2 G(x, y, \sigma)}{\partial x^2} + \frac{\partial^2 G(x, y, \sigma)}{\partial y^2}$$



Normalized Laplacian of Gaussian
Scale Space for "box" image

$$I\left(x,y,\sigma=2^k\right)= I(x,y)*\sigma^2\ \nabla^2 G(x,y;\sigma=2^k)$$



As in the 1D case, the response to the box has a minimum in scale space (x,σ) with x at the center of the box and scale σ that is the half width of the box.

For this example, the box width is 16 and so the minimum occurs at σ=8. (Larger σ not shown.)

Box (blob) detector in scale space

Filter with Laplacian of a Gaussian and look for local maxima or minima in (x, y, σ) scale space.

- The idea of a SIFT feature, which we will discuss in a future lecture, builds on this construction.