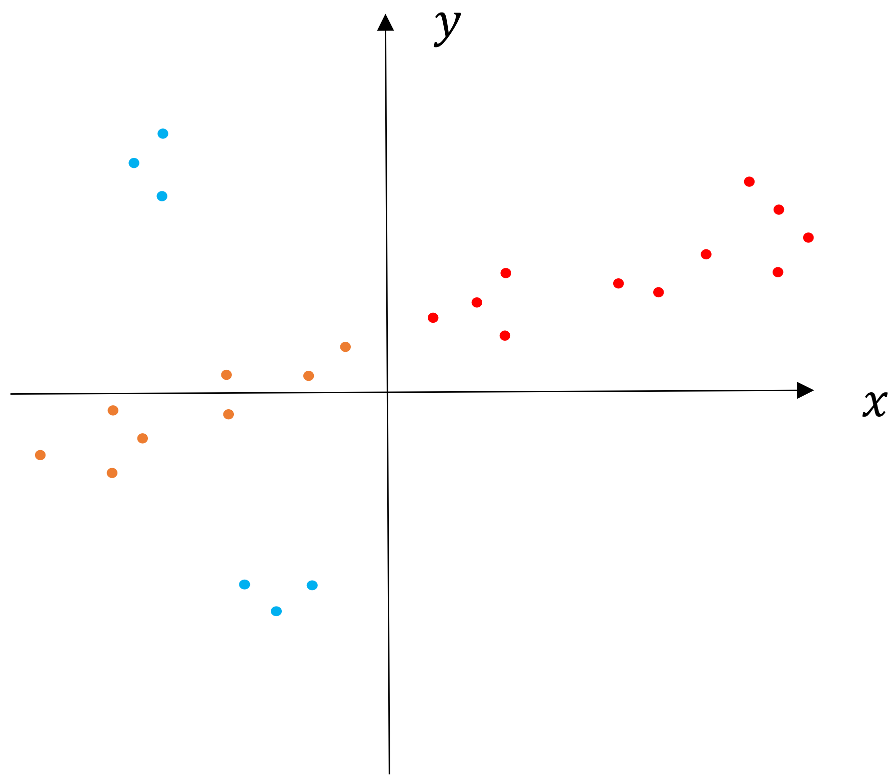
Lecture 6

Robust Estimation:
Hough transform & RANSAC

Limitation of least squares methods

fitting a line to a set of points



Inliers : point location is explained by the line model + measurement noise

Outliers : point location is not explained by the line model

How to reduce *sensitivity to outliers*?

(1) “Robust statistics”

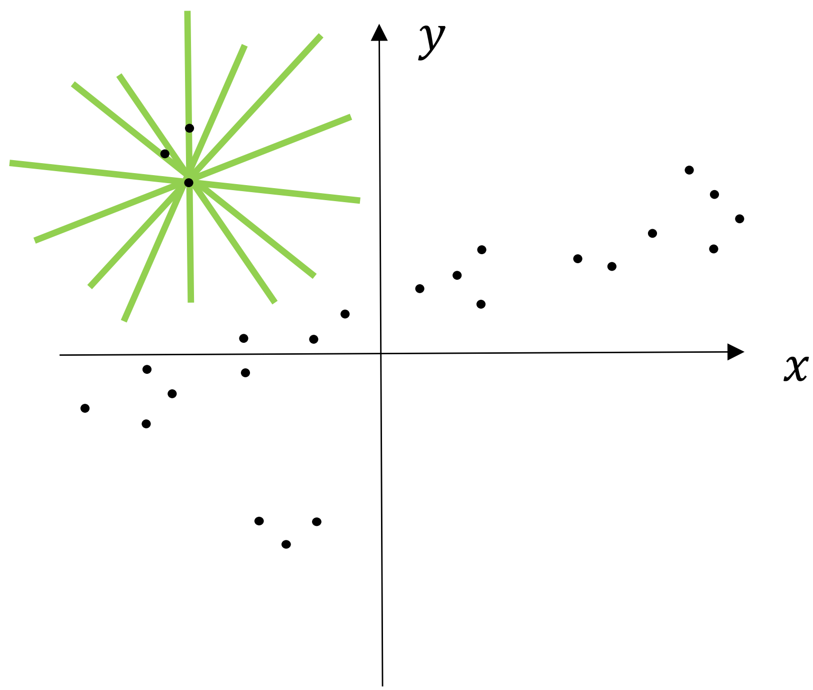
e.g. Don’t penalize so heavily for large distances

- Use distance rather than distance squared (L1 rather than L2)
- Beyond a certain distance, make the error penalty *constant*.
- …..

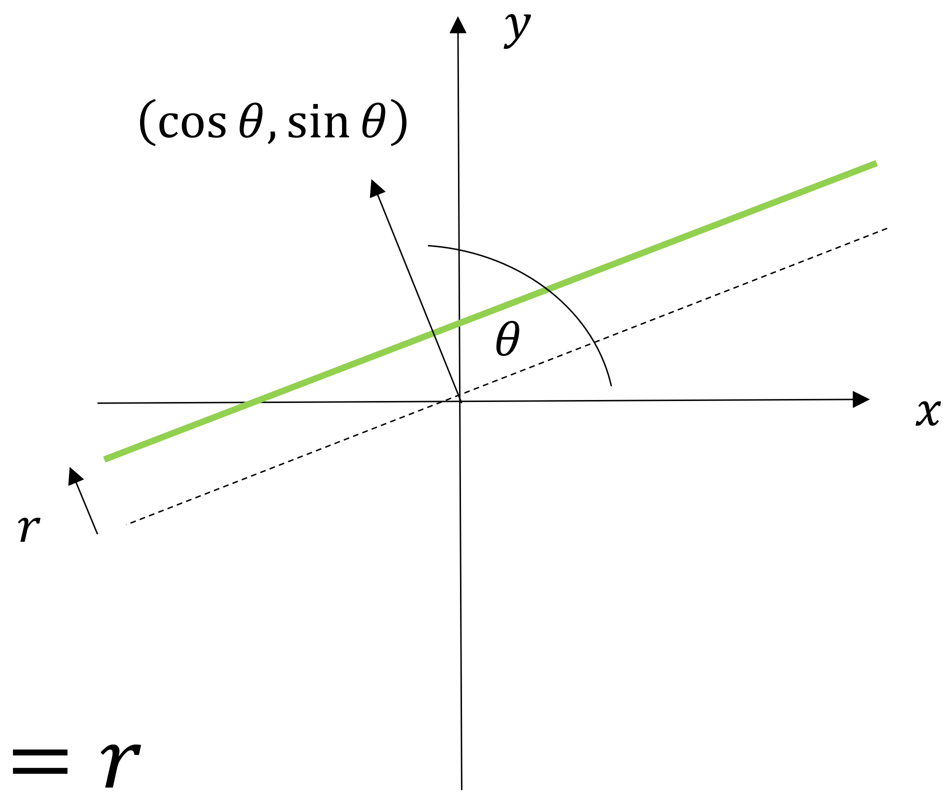
How to reduce *sensitivity to outliers*?

(2) Hough Transform (1962)

For each point, *vote* for all the lines that pass near the point.

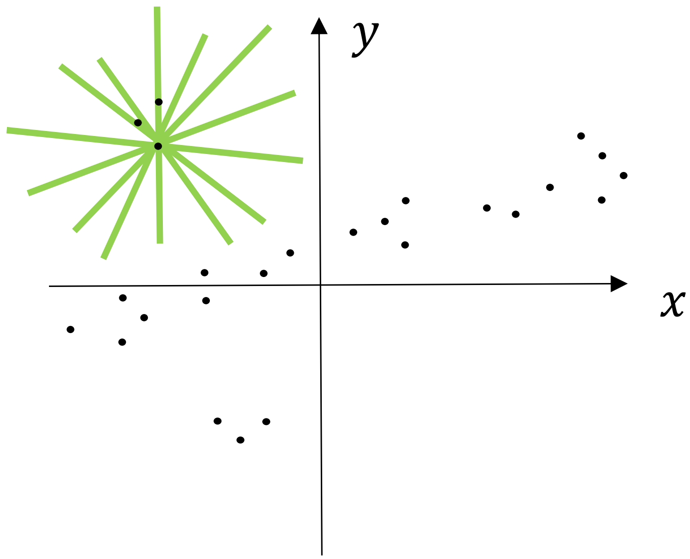


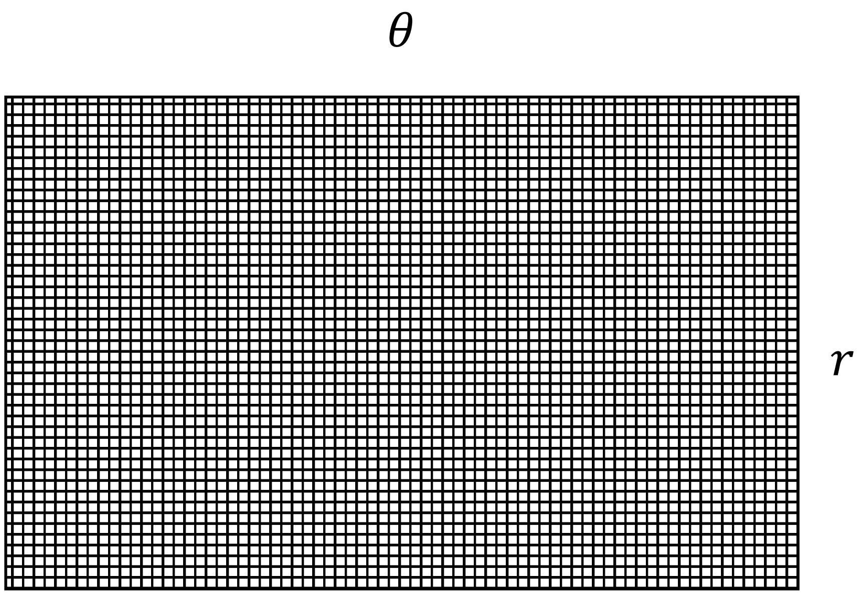
As we did last lecture, represent a line using an angle θ in [0, 360) degrees and a non-negative perpendicular distance r of the line from the origin.



$$x \cos\theta + y \sin\theta = r$$

Define a *discrete* set of candidates lines (r, θ).

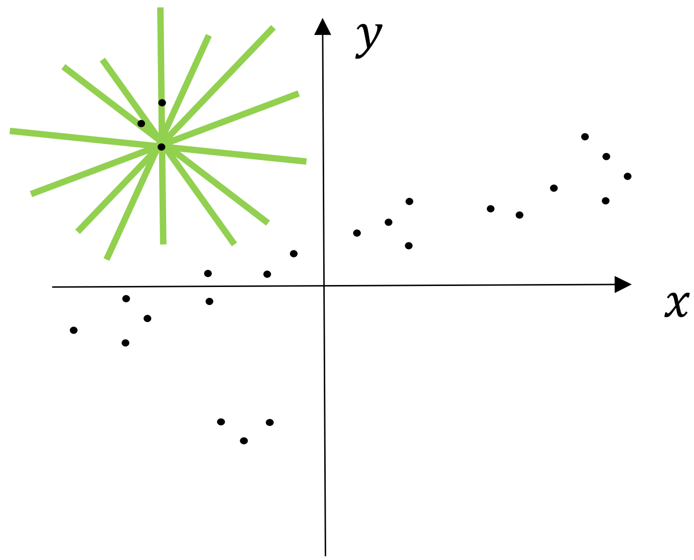


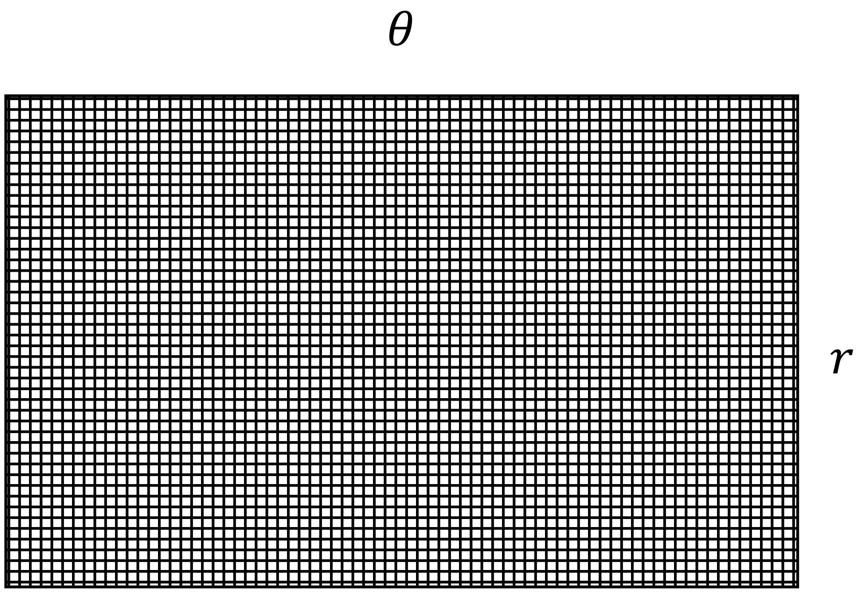


Hough Transform:

For each point, *vote* for all the lines that pass near the point.

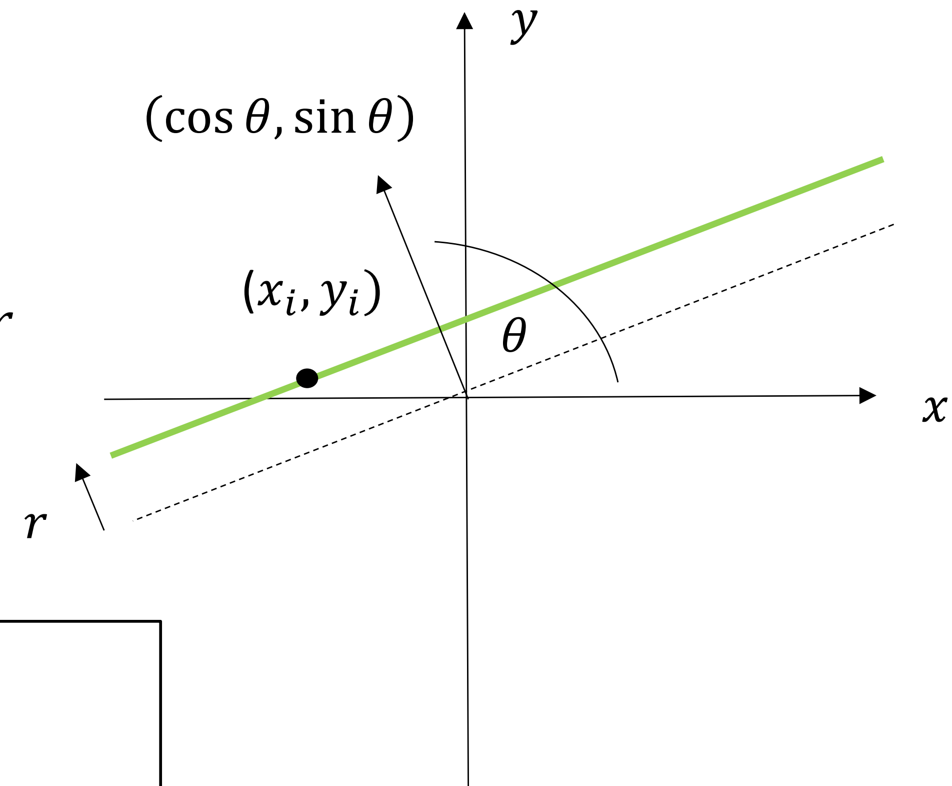
Hough transform





for each sample point (x_i, y_i)
    for each line *near* that point, parameterized $(r\ ,\theta)$
        cast one vote for this line
return the line $(r\ ,\theta)$ that has the maximum count

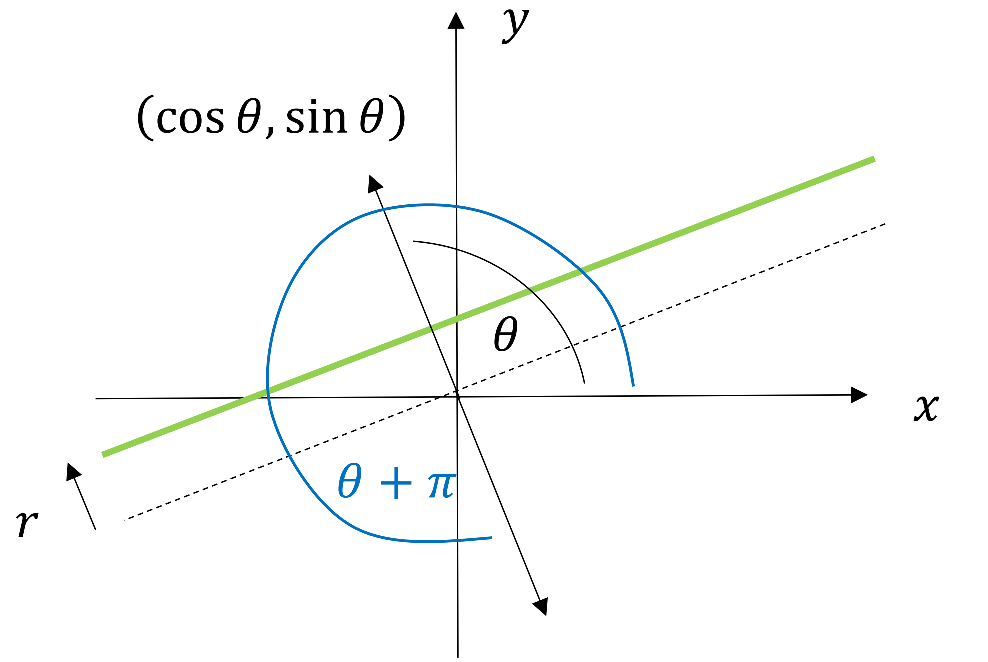
Hough Transform



$$\color{green}{x}\ \cos\theta + \color{green}{y}\ \sin\theta\ = r$$

for each sample point (x_i, y_i)
    for each θ bin in [0, 2π) {
        $\boldsymbol{r} = round(\ x_i \cos\theta + y_i \sin\theta\ )$
        vote for this (r, θ) // increment a counter
    }
return the $(r\ , \theta)$ that has the maximum count

Exercise: The r value computed above can be negative.   How ?



$$\cos\theta = -\cos(\theta+\pi)$$
$$\sin\theta = -\sin(\theta+\pi)$$

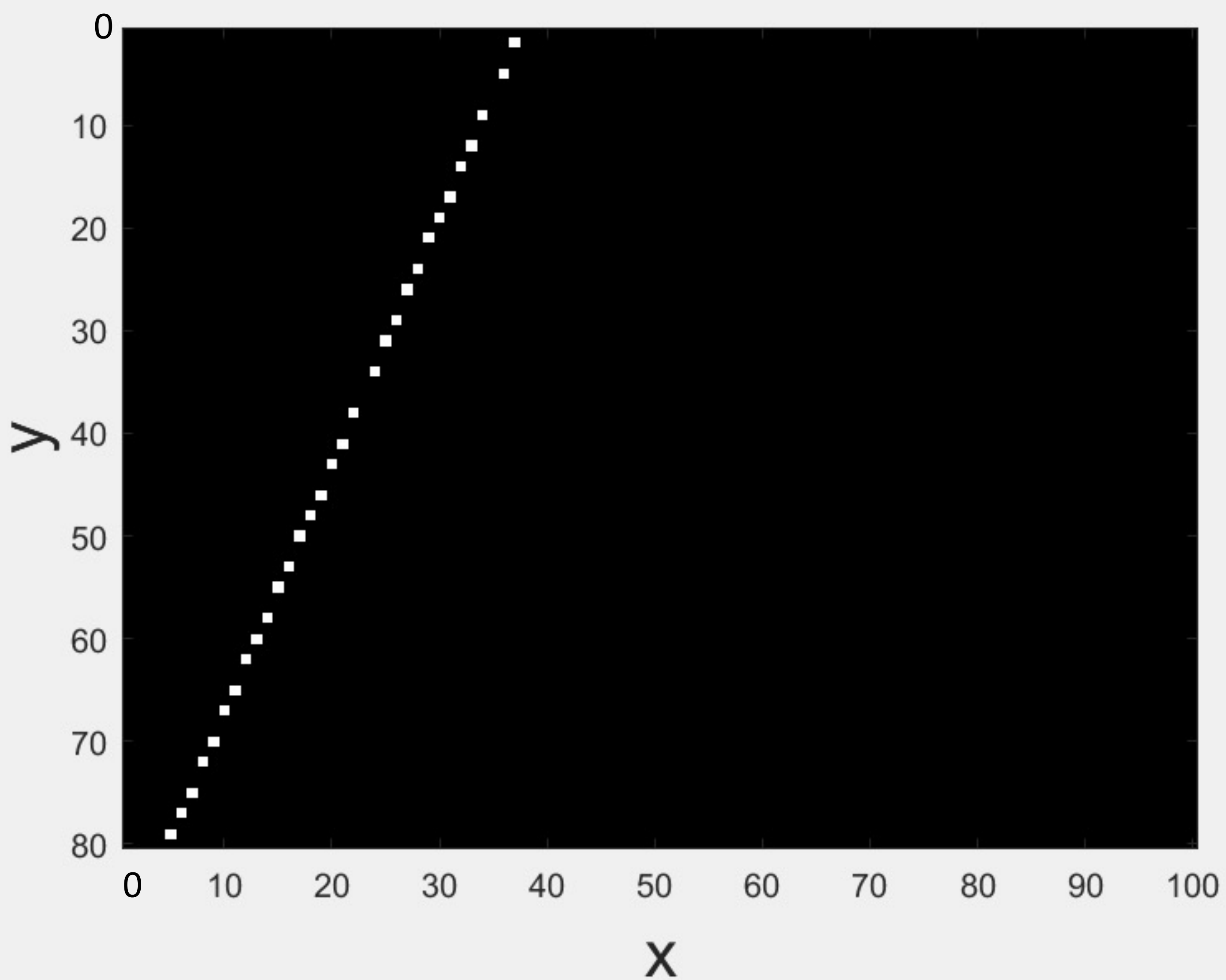
For any line $\quad r = x\cos\theta + y\sin\theta$

There is an equivalent line

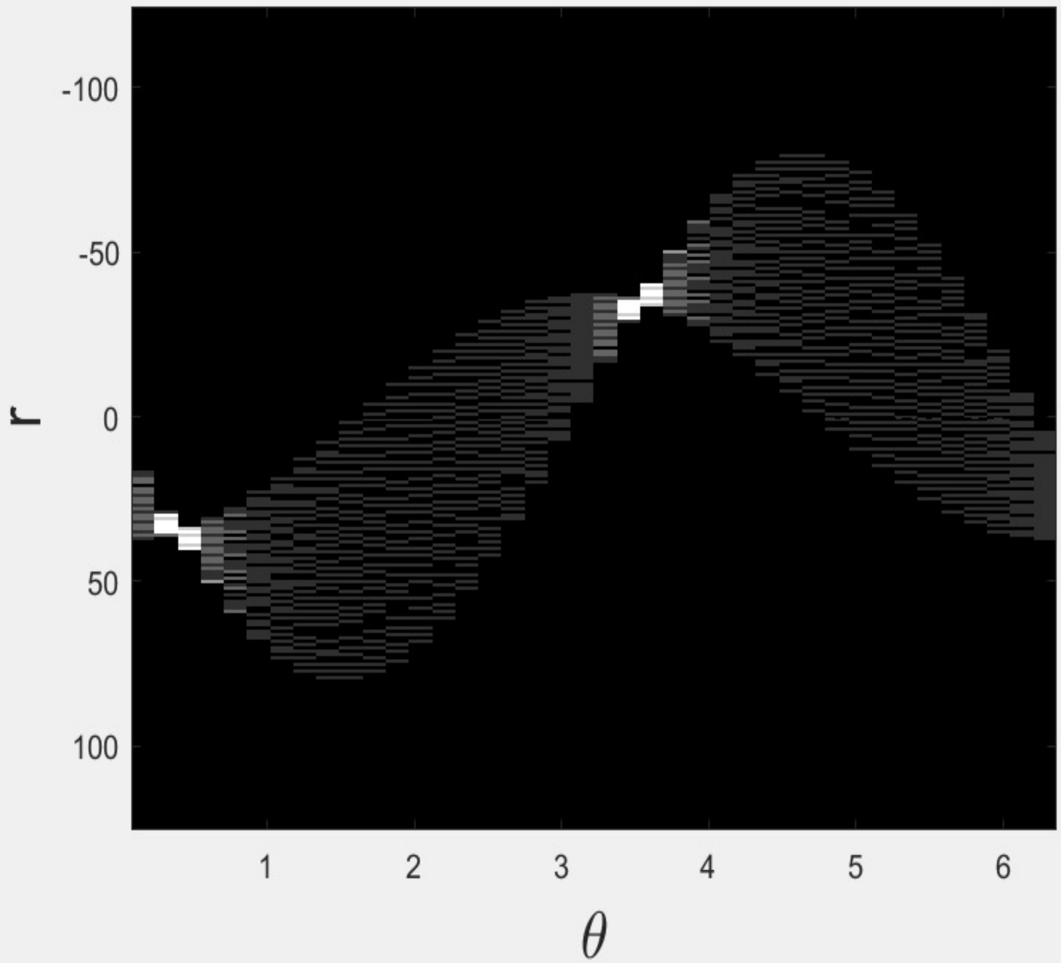
$$-r = x\cos(\theta+\pi) + y\sin(\theta+\pi)$$

Example 1: points on line

Matlab puts image origin (0,0) at top left.



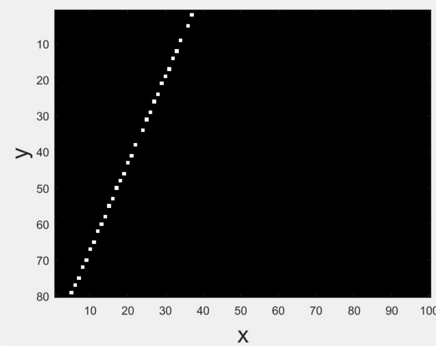
for each sample point (x_i, y_i)
    for each θ in [0, 2π) {
        $\boldsymbol{r} = round(\, x_i \cos\theta + y_i \sin\theta \,)$
        vote for this (r, θ) // increment a counter
    }
return the $(r\,,\theta)$ that has the maximum count

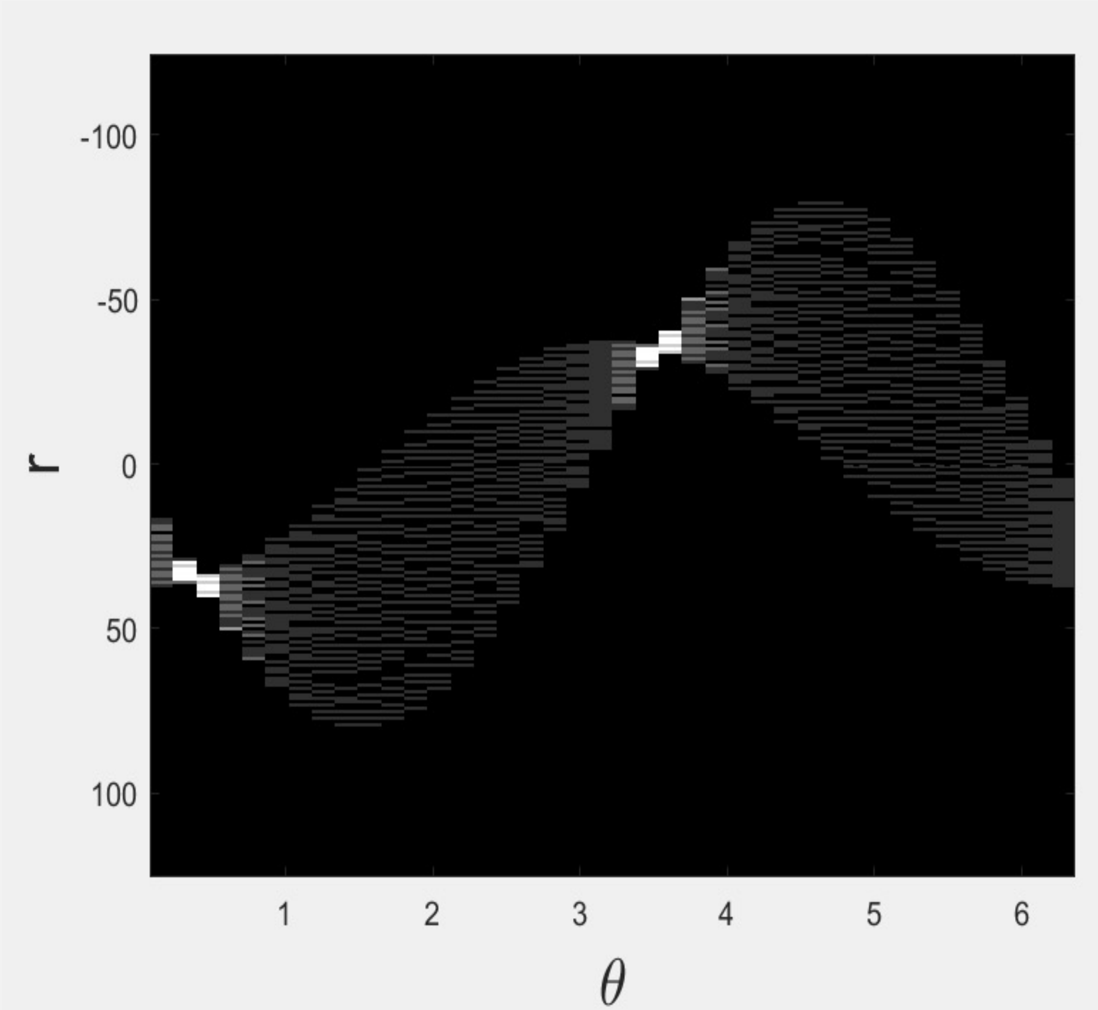


(θ goes from 0 to 2π)

For each (x_i, y_i) , we get a cosine function in Hough space. Why?

A set of points that fall on a line gives two H peaks, since $H(r, \theta) = H(-r, \theta + \pi)$ where H is vote count.





(θ goes from 0 to 2π)

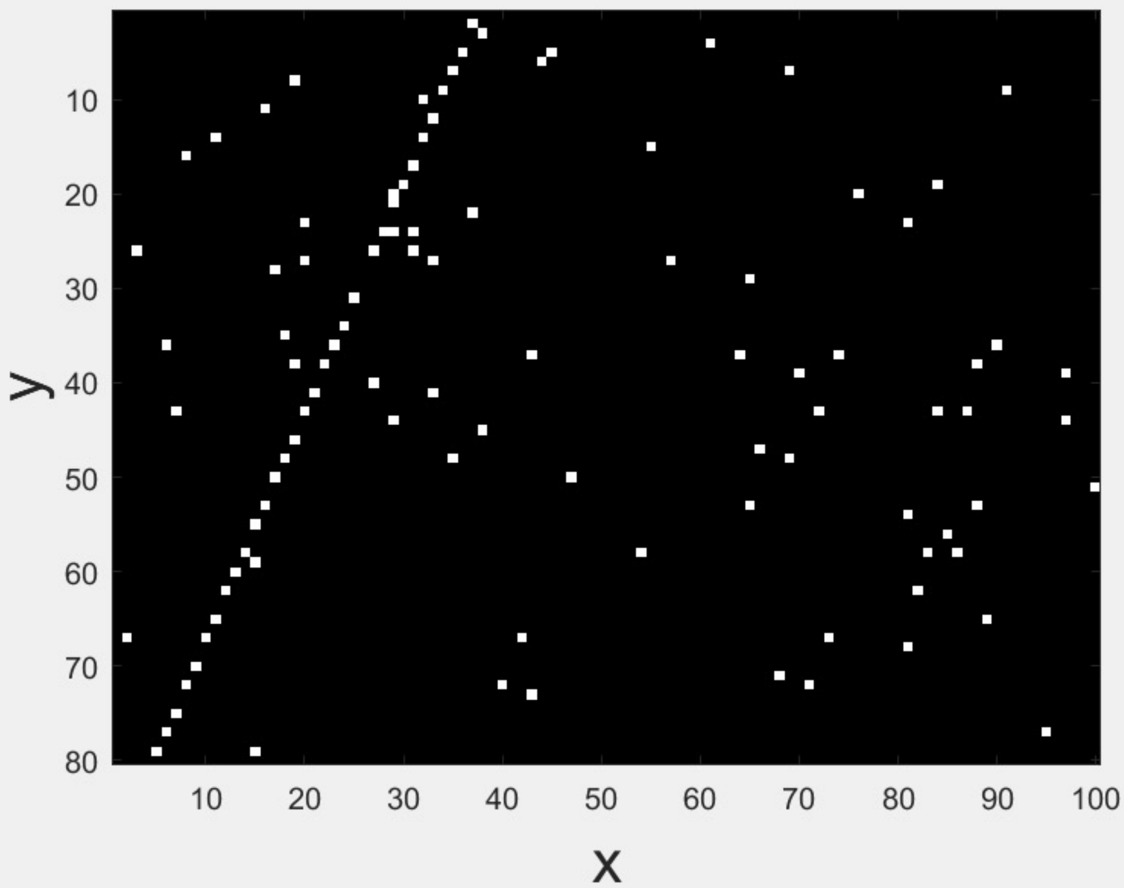
r can be negative or positive.

We can ignore the negative ones if we want.

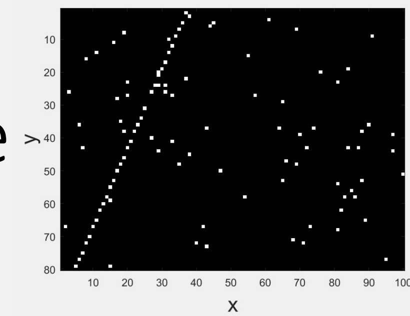
Alternatively, we could let θ go up to π only, and allow both positive and negative r.

Example 2: points on line + points not on line

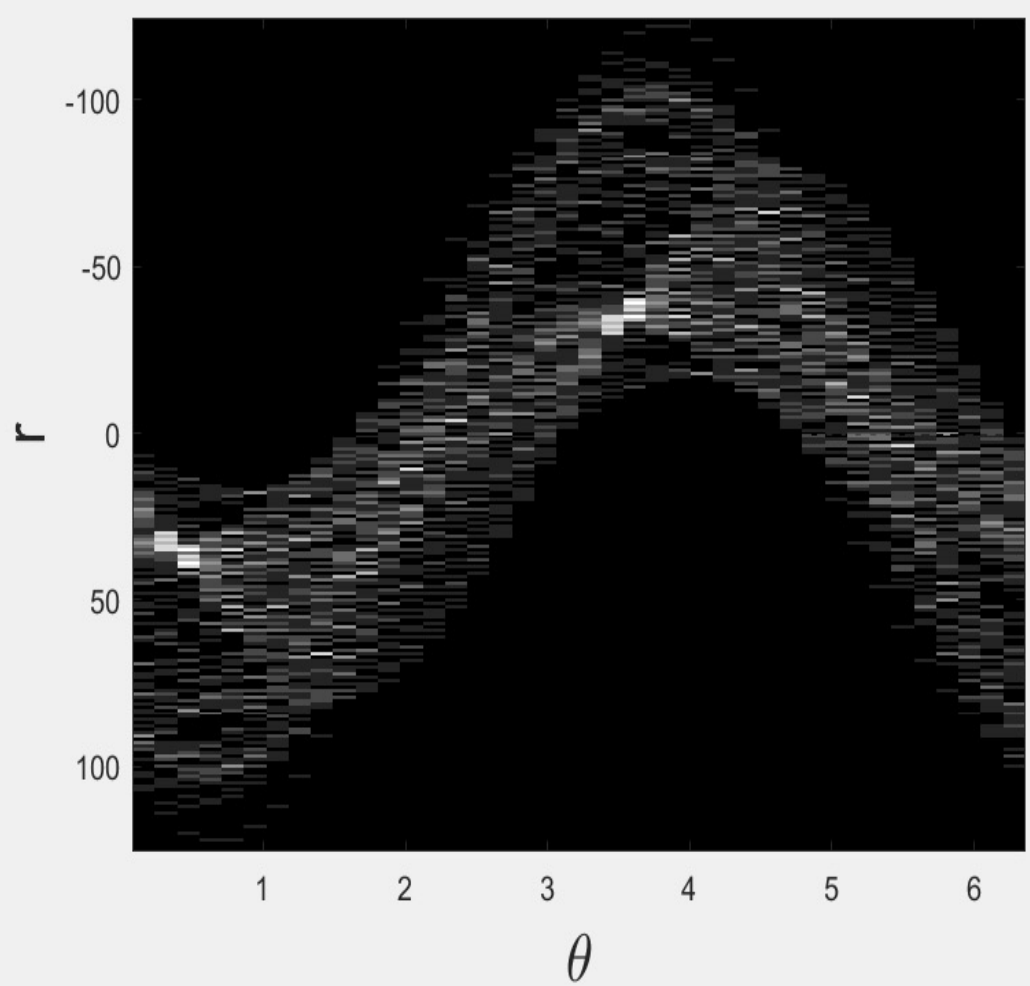
What do you think the Hough transform $H(r, \theta)$ will look like?



Example 2:   points on line + points not on line



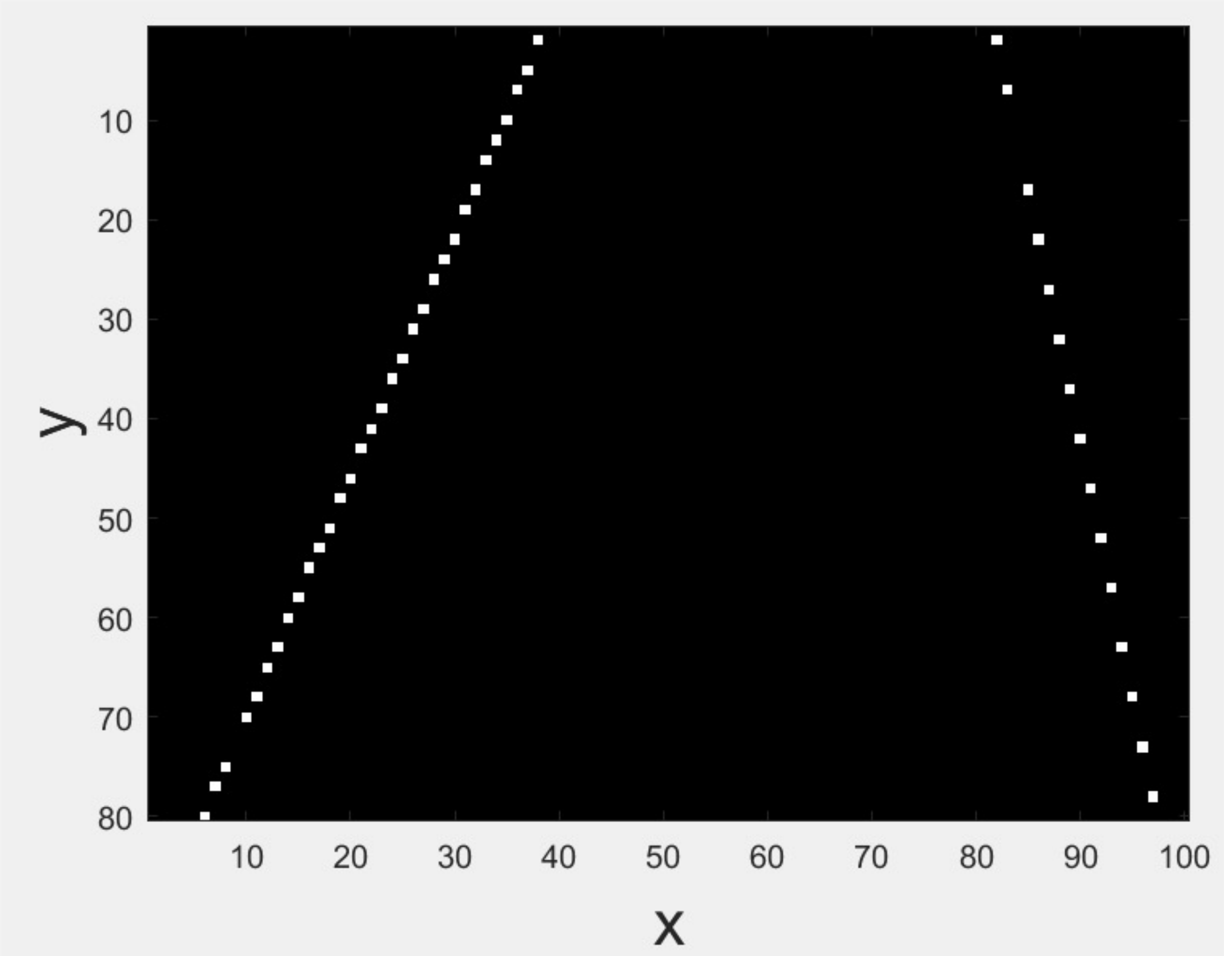
$H(r, \theta)$



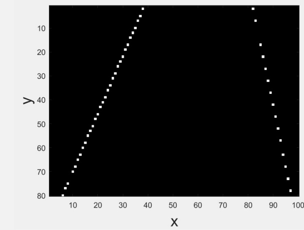
(θ goes from 0 to 2π)

Example 3:   points on two lines

What do you think the Hough transform will look like?

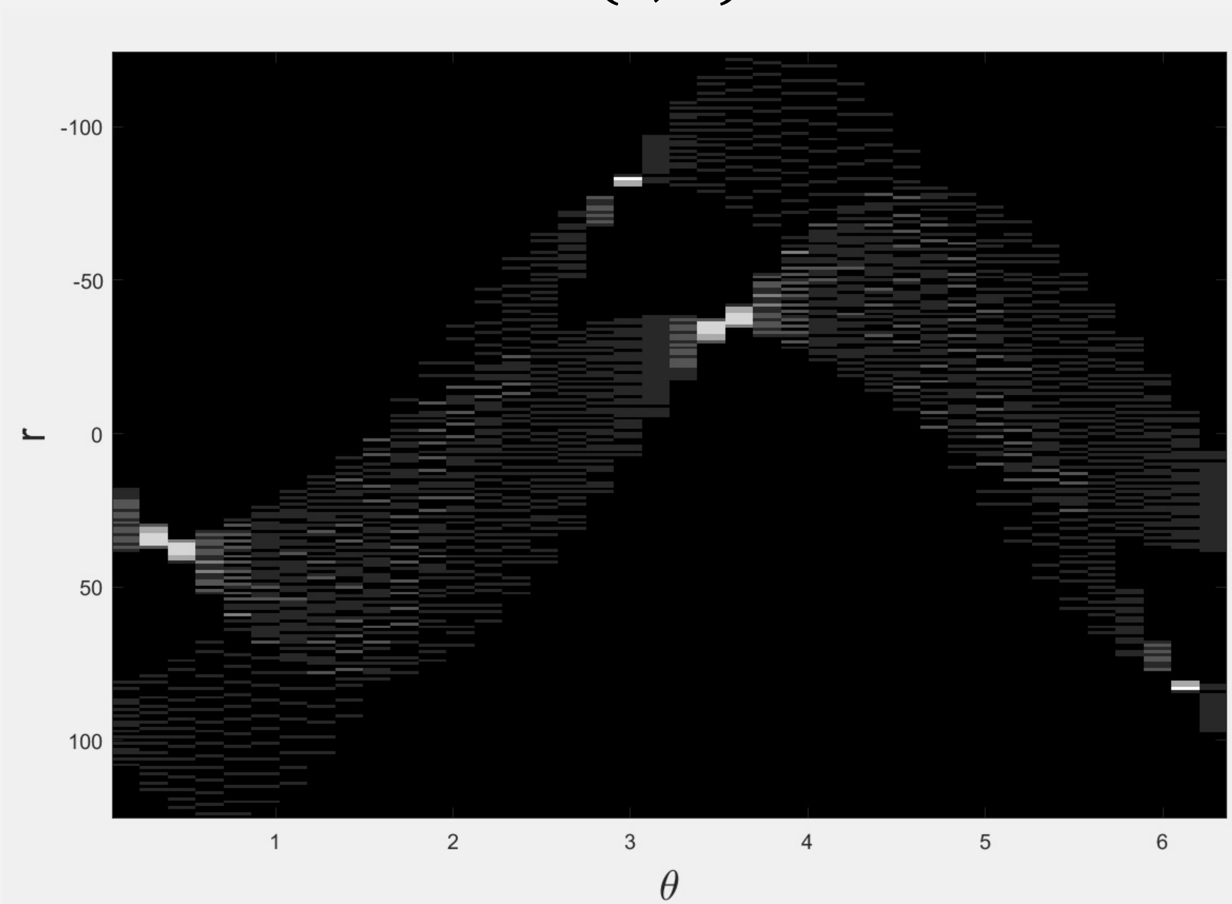


Example 3: points on two lines



The second line produces a second pair of peaks.   The relative heights of the two pairs of peaks depend on the number of points on each line.

$$H(r, \theta)$$



Hough transform : discussion

- When many lines are present, it is difficult to detect the peak.
  You could *smooth the voting map* to remove this "noise".

- How to choose the bin size for the radius and angle of the lines ?
  → Speed/accuracy tradeoff

Exercise: Can you think of another way to vote for lines, given a set of points in the image ?

Exercise: Can you think of another way to vote for lines, given a set of points in the image ?

Answer: Take pairs of points. Each pair defines a line, namely the unique line passing through those points. Vote for the line. So the total number of votes will be the number of pairs of points.

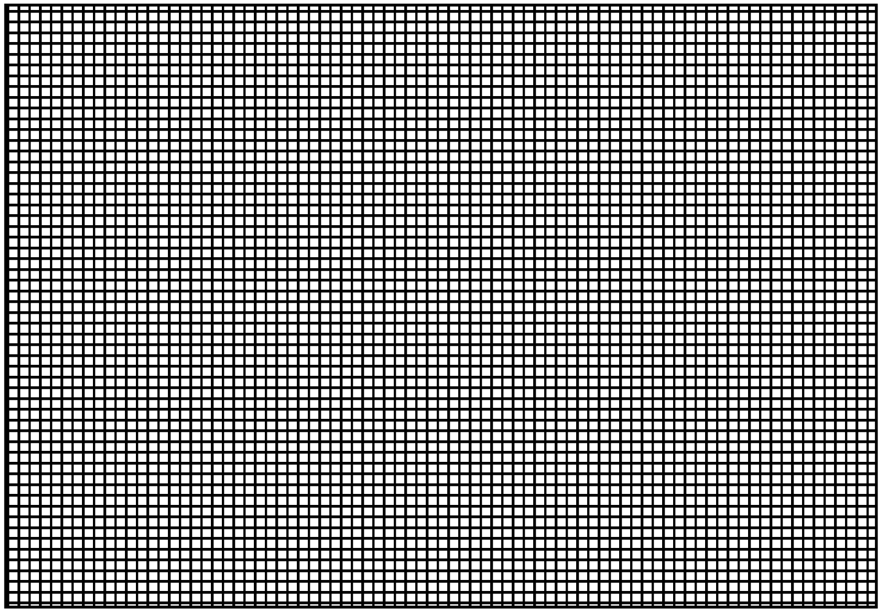
Vanishing point *detection*

(output of Canny edge detector)

Suppose we have a set of edge estimates $(x_i, y_i,\ \theta_i)$.
We would like to estimate a vanishing point (x_v, y_v).
Last lecture we saw how to do this using least squares.

Hough transform for vanishing point detection

(output of Canny edge detector)



?

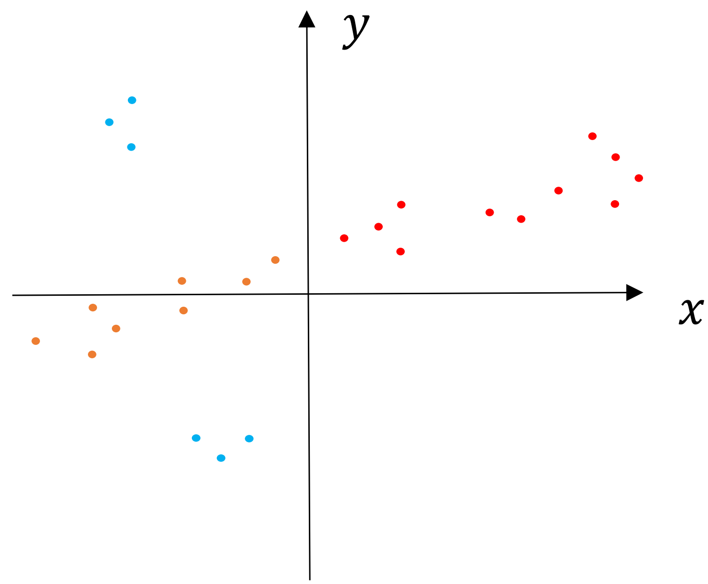
Suppose we have a set of edge estimates $(x_i, y_i,\ \theta_i)$.
We would like to estimate a vanishing point (x_v, y_v).

Exercise: How would you formulate this problem using Hough transform ? (Assignment 1)

How to reduce sensitivity to outliers?

(3) RANSAC (Random Sample Consensus)

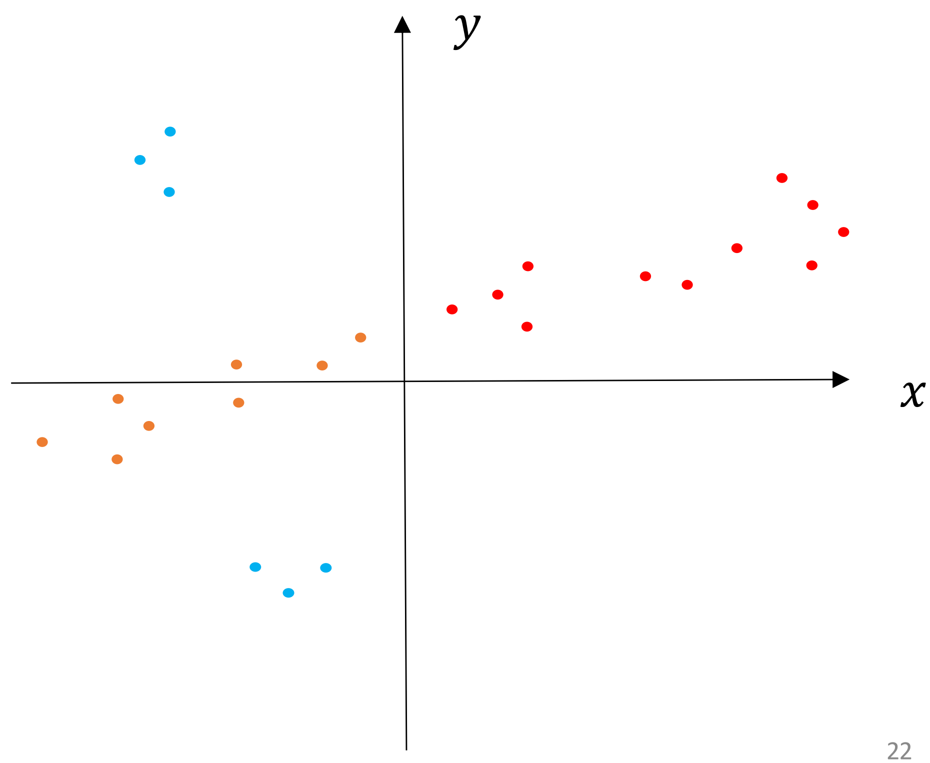
Decide which points are inliers and which are outliers, and fit the model to the inliers only.



Chicken and egg problem !

RANSAC idea: randomly pick two different points, and fit a line model to these two points.

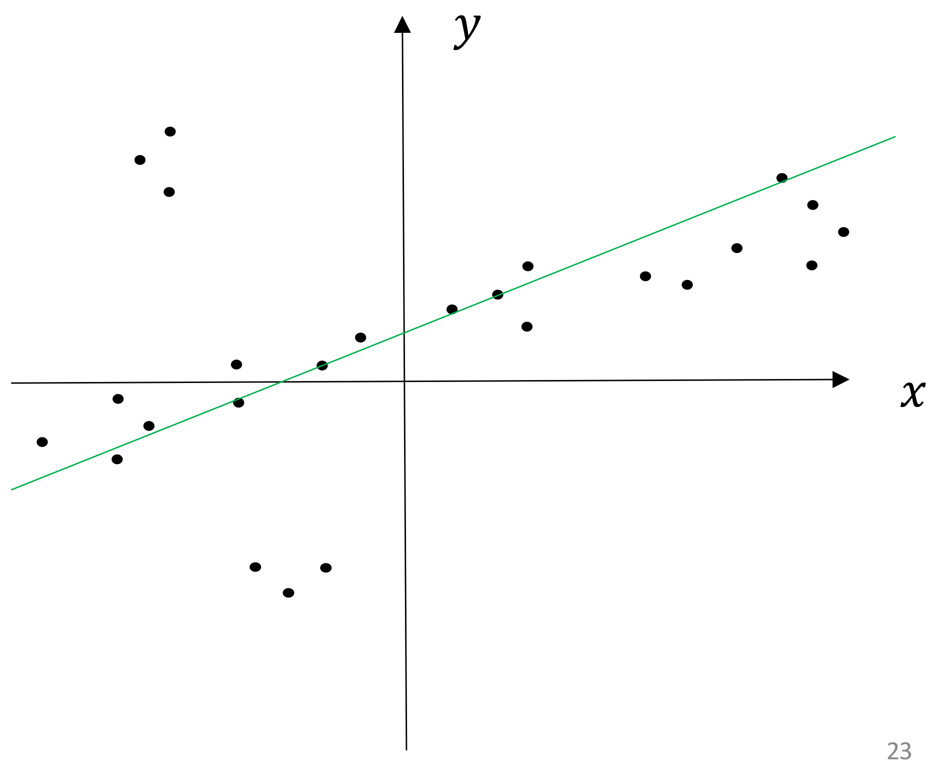
Q: When can we say that we have a good line model ?



RANSAC idea: randomly pick two different points, and fit a line model to these two points.

Q: When can we say that we have a good line model ?

A: It is a good model if many other points lie near the line.



RANSAC (Random Sample Consensus)

[Fischler and Bolles, 1981]

https://www.sri.com/sites/default/files/publications/ransac-publication.pdf

Example: fitting a line model through two points

Repeat many times {

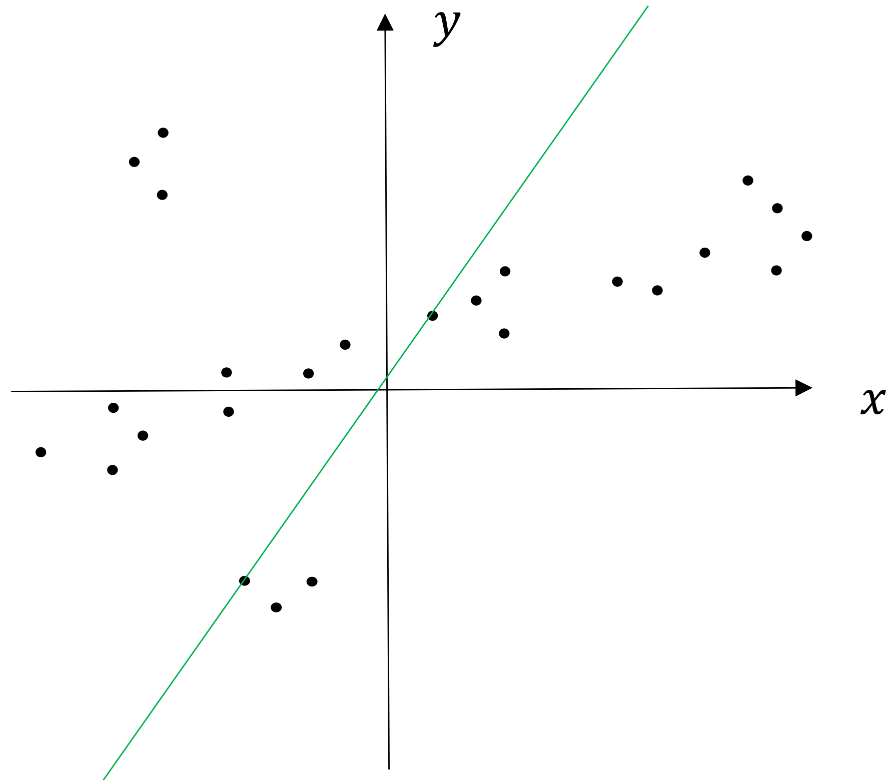
- randomly sample 2 data points and fit a line to them
- examine all remaining points $N-2$ data point; count how many C of them are within a threshold distance τ from the line ("consensus set")
- if C is sufficiently large (greater than some threshold), then refit the model using all points in the consensus set e.g using least squares. If the model fit is the best so far, then save it.

[I changed this condition slightly from what I presented during the lecture.]

}

Q: Why use the *minimal* number of points of points to fit a model ?

It seems like a crazy strategy for the case of fitting a line to a set of (x,y) points with noisy positions.



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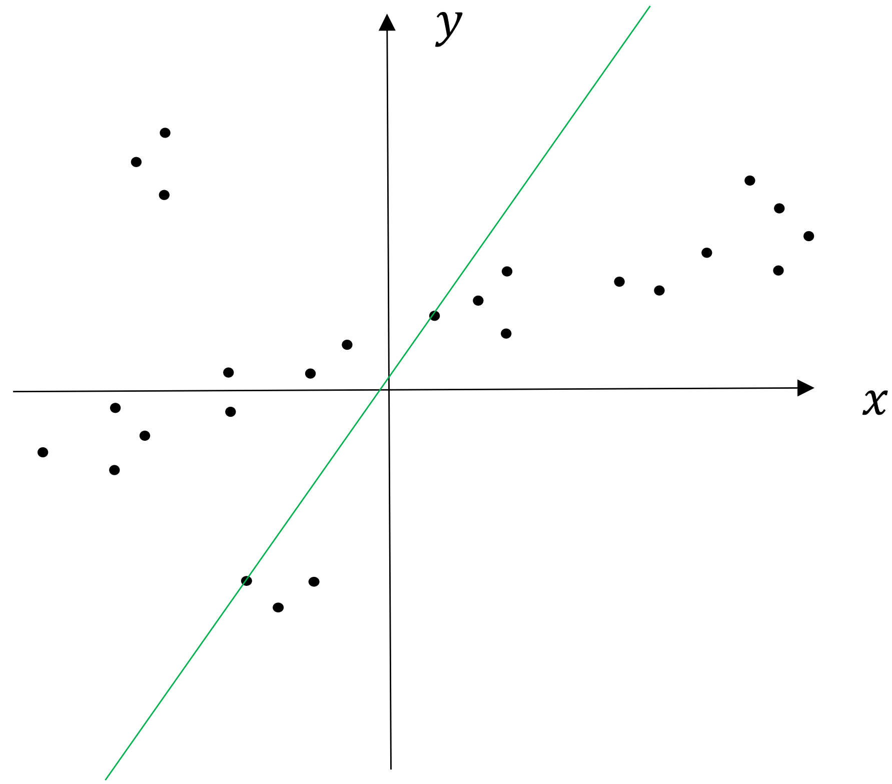

A: There is a tradeoff here.
Using more randomly chosen data points to fit a model will potentially give a better model, but only if the extra points are inliers. The more points you use, the greater the chance that one point will be an outlier and if that happens then your model will be a poor fit.

RANSAC for vanishing point detection

(output of Canny edge detector)

Exercise: How would you formulate the problem? You are given a set of edges $(x_i\,, y_i\,, \theta_i)$. (Hint: What is the model you are trying to fit? How many samples do you need to fit the model?)

RANSAC (Random Sample Consensus)

[Fischler and Bolles, 1981]

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More general description:

Repeat many times{

- randomly sample n data "points" and fit a model where n is the *minimum* number of "points" needed for an *exact* model fit;
- examine all remaining "points" $(N - n)$ and count how many C are within some threshold distance τ from the model ("consensus set")
- if C is sufficiently large (greater than some threshold), then refit the model using all points in the consensus set e.g using least squares. If the model fit is the best so far, then save it.

}

RANSAC is a probabilistic model

Let w be the fraction of points that are inliers, that is, the probability that a randomly chosen point is an inlier.

(We assume that there is one correct model and “points” are either inliers or outliers.)

If we randomly sample n data points (to fit a model), then the probability that all n samples will be inliers is ?

Thus, the probability that at least 1 of the n samples will be an outlier is ?

Suppose we sample and fit a model k times. The probability that we will find a model with all inliers is ?

RANSAC is a probabilistic model

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?

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RANSAC is a probabilistic model

Let w be the fraction of points that are inliers, that is, the probability that a randomly chosen point is an inlier.

(We assume that there is one correct model and "points" are either inliers or outliers.)

If we randomly sample n data points (to fit a model), then the probability that all n samples will be inliers is w^n.

Thus, the probability that at least 1 of the n samples will be an outlier is $1 - w^n$. This is close to 1 when w is small and n is large. Thus, we will need to sample more times to get a good model.

Suppose we sample and fit a model k times. The probability that we will find a model with all inliers is ?

RANSAC is a probabilistic model

Let w be the fraction of points that are inliers, that is, the probability that a randomly chosen point is an inlier.

(We assume that there is one correct model and "points" are either inliers or outliers.)

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Suppose we sample and fit a model k times. The probability that we will find a model with all inliers is $1 - (1 - w^n)^k$.

RANSAC algorithm parameters (“secret sauce”)

- How close does a “point” need to be to the model to count it in the consensus set?

- How many points need to be in the consensus set for us to consider the model (and refit using the consensus set) ?
  [ADDED after lecture]

- How many times do we sample (fit a model) ?

[ASIDE: RANSAC algorithm variations]

- Rather than counting the number of elements in the consensus set, use a more general measure of how well the consensus set fits the model.

- What if there are multiple good models (e.g. multiple vanishing points) ? Different models would define different inlier sets. Solutions: (1) once we have found a good model, remove inliers and find other models. (2) Keep track of multiple models.

- …

COMP 558 Overview

Part 1 : 2D Vision

RGB

Image filtering

Edge detection

Least Squares Estimation

Robust Estimation: Hough transform & RANSAC

Features 1: corners

Image Registration: the Lucas-Kanade method

Scale spaces (Gaussian and Laplacian)

Histogram-based Tracking:

Features 2: SIFT, HOG

Features 3: CNN's

Object classification and detection

Segmentation

Part 2 : 3D Vision

Linear perspective, camera translation

Vanishing points, camera rotation

Homogeneous coordinates, camera intrinsics

Least Squares methods (eigenspaces, SVD)

Camera Calibration

Homographies & rectification

Stereo and Epipolar Geometry

Stereo correspondence

Cameras and Photography

RGBD Cameras

RANSAC