VV 285
RC 5

SUN YAN

Content

- Functions and Derivatives

- Exercise

Integrals

- 2.2.25. Definition. A ***step function*** with respect to a partition $P=(a_0,\dots,a_n)$with elements $y_i \in V, f(t)=y_i$ whenever $a_{i-1} < t < a_i, i = 1,\dots,n$.

- 2.2.29. Theorem. Let $f : [a, b] \to V$ be a step function with respect to some partition P. Then the ***integral*** of f is $I_P(f):=(a_1 - a_0)y_1 + \cdots + (a_n - a_{n-1})y_n \in V$ and is independent of the choice of P.

$$\left\| \int_a^b f(x)\mathrm{d}x \right\|_V \leq \int_a^b \|f(x)\|_V \mathrm{d}x \leq |b-a| \cdot \sup_{x\in[a,b]} \|f(x)\|_V$$

Mean Value Theorem

2.2.30. Mean Value Theorem. X, V are finite-dimensional vector spaces, $\Omega \subset X$ is open and $f \in C(\Omega, V)$. $x, y \in \Omega$ and the line segment $x + ty, 0 \le t \le 1$ is wholly contained in Ω. Then

$$f(x+y) - f(x) = \int_0^1 Df|_{x+ty} y \mathrm{d}t = \left(\int_0^1 Df_{x+ty} \mathrm{d}t \right) y$$

Differentiating Under an Integral

- 2.2.33. Theorem.
- 1. X, V are finite-dimensional vector spaces.
- 2. $I=[a,b]\subset \mathrm{R}, \Omega\subset X$ an open set.
- 3. $f: I\times\Omega\to V$, $Df(t,\cdot)$ exists and is continuous for every $t\in I$.
- Then
- $g(x) = \int_a^b f(t,x)\mathrm{d}t$, $\quad Dg(x) = \int_a^b Df(t,\cdot)|_x \, \mathrm{d}t$

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Exercise 1

In each of the following cases, find $D_1\psi(x, y)$ and $D_2\psi(x, y)$, by evaluating the integrals.

1. $\psi(x, y) = \int_1^x e^{ty}\, dt$
2. $\psi(x, y) = \int_0^x \cos(ty)\, dt$
3. $\psi(x, y) = \int_1^x (y + t)^2\, dt$
4. $\psi(x, y) = \int_1^x e^{y+t}\, dt$

Exercise 2

Prove the Euler's integral formula for $n!$.

$$\int_0^{\infty} x^n e^{-x} \mathrm{d}x = n!$$

Exercise 3- Review of Product Rule and Chain Rule

Suppose f: $R^2 \to R^3$ and g : $R^2 \to R^2$ is defined as

$$f(u,v)=\begin{pmatrix} u+v \\ u^2 \\ uv \end{pmatrix}$$

$$g(x,y)=\begin{pmatrix} xy \\ x^y \end{pmatrix}$$

Calculate the derivative of $f \circ g$.

Exercise 3- Review of Product Rule and Chain Rule

Suppose $f: \mathbb{R}^2 \to \mathbb{R}^3$ and $g: \mathbb{R}^2 \to \mathbb{R}^2$ is defined as

$$f(u,v)=\begin{pmatrix} u+v \\ u^2 \\ uv \end{pmatrix}$$

$$g(u,v)=\begin{pmatrix} u^2 \\ uv \\ u^v \end{pmatrix}$$

Calculate the derivative of $f \cdot g$ *in two ways..*

Reference

Hohberger, Horst. “VV285_main.pdf”

Serge Lang, Calculus of Several Variables, 1987

Chen, Xiwen “RC_5.pdf”,2018

Yuan, Jian “RC_5.pdf”,2018

Thank You

Have a nice day!