Title TBD

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Gavin Engelstad*

gengelst@macalester.edu.

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Abstract

Research into the effects of business cycles often focuses on aggregate macroeconomic consequences. This paper explores the heterogeneous effects of business cycles on households at different wealth and earnings potential levels. I perform a Bayesian estimation on a HANK to identify business cycle forces. Then, I decompose the business cycle shocks and transmission channels driving changes in household decision rules. I find that low and middle income households are most affected by price markup shocks while high income households are most affected by tax progressivity shocks. Within the model, interest rate changes impact high earning and wealthier households through income and precautionary saving channels. Middle income households are most responsive to changes in wages while low income households are most responsive to changes in transfers. This suggests macroeconomic-level substitution effects are driven by household decisions only above a certain earnings threshold.

1 Introduction

The significant cross-sectional differences between households in the United States suggests business cycle fluctuations should have heterogeneous effects across households. At the same time, most

*Replication code available at https://github.com/GavinEngelstad/HANK-Honors macroeconomic research, both within representative agent and heterogeneous models, is interested in explaining changes in aggregates (Smets and Wouters 2007; Krusell and Smith 1998; Kaplan, Moll, and Violante 2018; Auclert 2019; McKay, Nakamura, and Steinsson 2016). I examine how business cycles affect households at different wealth and income levels. I also examine the important transmission channels for changes in behavior for different households. I find that wealthier and higher earning households are most affected by changes in the interest through precautionary saving mechanisms. Wages affect higher earning households through strong substitution effects, while direct transfers affect lower earning households through income effects.

I analyze these effects within the framework of an estimated Heterogeneous Agent New Keynesian (HANK) model. HANKs add household heterogeneity to standard New Keynesian models that feature realistic price and market frictions (Kaplan, Moll, and Violante 2018). Idiosyncratic changes in household states mean the model features incomplete markets and uninsurable risks that give households a strong precautionary motive that plays an important role in the economy (McKay, Nakamura, and Steinsson 2016; Bayer et al. 2019). My model follows the standard within HANK literature and features an idiosyncratic productivity process for households that determines their income potential (Kaplan, Moll, and Violante 2018; McKay, Nakamura, and Steinsson 2016). Within the model, households decide how much to save, consume, and work, which determines their movements along the wealth distribution.

To understand the effect of business cycles on the model, I perform a Bayesian estimation against US data from 1966 to 2019 of six macroeconomic shocks to the model: total factor productivity (TFP), price markups, government spending, monetary policy, government transfers to households, and tax progressivity. The first four shocks are chosen from representative agent literature as structural shocks to aggregates in the model (Smets and Wouters 2007). The tax progressivity shock is unique to heterogeneous models and applies non-uniformly across the distribution of households (Bayer, Born, and Luetticke 2024). The household transfer shock does the opposite, applying a uniform shock to all households. My estimations suggest price markups and tax progressivity play the most important role explaining the changes in aggregate outcomes during business cycles.

Given my estimates, I then examine the effect of business cycles on household decisions at different productivity and wealth levels. This goes one step farther than other Bayesian estimates of HANKs in the literature that typically focus on aggregates (Auclert, Rognlie, and Straub 2020; Acharya et al. 2023) or macro-level movements of the wealth distribution (Bayer, Born, and Luetticke 2024). I find that business cycle driven changes in household decisions are caused by a combination of price markups, tax progressivity, and household transfers. Price markups are especially important for low productivity and low wealth households while tax progressivity and transfers have larger effects on high productivity and high wealth households.

Finally, I expand the direct-indirect effects decomposition from Kaplan, Moll, and Violante (2018) to all factors that directly affect household decisions in my model: wages, interest rates, transfers, and taxes. Using this decomposition, I analyze which macroeconomic factors that directly play into household decisions are the most important for households at different locations along the wealth and productivity distributions. I find that interest rates are most important to higher income, net-saver households, which I attribute to precautionary saving factors. Wages are the most important for middle income households that display less precautionary saving but still have significant substitution effects as wages change. For low income households, I find that income effects dominate and transfers are the most important determinant of household decisions. Across all groups, there is comovement of aggregate variables that together cause opposite changes in household behavior that dampen each other's effects.

2 Literature Review

This paper adds to a growing body of work that adds household heterogeneity and market incompleteness to workhorse New Keynesian models that have been used to inform governmental policy for decades (Woodford and Walsh 2005; Smets and Wouters 2007). Specifically, it contributes to work examining transmission channels and business cycle dynamics within these models.

HANK models have developed our understanding of the forces that affect the macroeconomy. Idiosyncratic household income risks give an additional motive for precautionary savings beyond the aggregate forces within representative agent models (McKay, Nakamura, and Steinsson 2016; Auclert, Rognlie, and Straub 2020; Acharya, Challe, and Dogra 2023). Heightened uncertainty from these risks explains parts of specific business cycle events, including the Great Recession (Bayer et al. 2019). Borrowing constrained households are more responsive to macroeconomic conditions, so their presence can exacerbate or dampen the consequences of macroeconomic shocks (Bilbiie 2020). After a monetary policy shock when heterogeneity is present, indirect, as opposed to direct, effects cause the aggregate household response to shocks (Kaplan, Moll, and Violante 2018). In this paper, I extend the direct-indirect decomposition from Kaplan, Moll, and Violante (2018) to all macroeconomic factors that directly effect household decisions.

Cross-sectional variation in marginal propensity to consume plays an important role in HANK models. Transfers have a trickle-up effect since poorer households have a smaller marginal propensity to consume (Auclert, Rognlie, and Straub 2023). Wealthy households are self-insured against macroeconomic shocks, so the household response to the shock varies across the wealth distribution (Gornemann, Kuester, and Nakajima 2016). Shocks have unequal effects on households since earnings, balance sheet, and interest rate exposure are not evenly distributed (Auclert 2019). Heterogeneous changes in savings behavior creates a "redistribution channel" that affects aggregates (Auclert 2019). My analysis examines the transmission channels for changes in decisions for households across the wealth distribution. I focus on the heterogeneous household outcomes that are driven by the different exposure channels.

Bayesian estimates for business cycles are similar for HANKs and representative agent models (Smets and Wouters 2007; Bayer, Born, and Luetticke 2024). Investment, markups, and technology channels play the most important role in estimated business cycles (Auclert, Rognlie, and Straub 2020; Bayer, Born, and Luetticke 2024). Heterogeneous MPCs and precautionary motives drive economic outcomes in response to the estimates shocks (Auclert, Rognlie, and Straub 2020). The main obstacle to estimation is that the number of potential household states makes estimation slow. The estimation process in Bayer, Born, and Luetticke (2024) uses dimensionality reduction and parallelization to speed up the process and still takes days. Newer sequence-space methods make estimation much faster (Auclert et al. 2021). Therefore, I use a sequence-space method for the estimation in this paper.

3 Model

I model a discrete time, one-asset HANK with incomplete markets stemming from uninsurable, idiosyncratic income risks and nominal rigidities. The economy is composed of a household sector, firm sector, and government sector. Within the model, there are shocks to total factor productivity (TFP) A_t, price markups ψ_t, government spending g_t, transfers to households η_t, tax progressivity τ_t^P, and monetary policy ξ_t.

The household sector features a block of dynamically optimizing heterogeneous households that choose to supply labor, consume, and save. Households earn income from their wages, firm profits, and government transfers. Household productivity levels evolve idiosyncratically over time, which they self-insure against by investing in a risk-free government bond.

The firm sector comprises a representative perfectly competitive final goods firm and a block of monopolistically competitive intermediate goods firms. The final goods firm aggregates production from the intermediate goods firms, who produce differentiated goods using labor supplied by households. Following Rotemberg (1982), intermediate goods firms face quadratic price adjustment costs, creating pricing frictions in the economy.

The government acts as the fiscal and monetary authority. As the fiscal authority, the government supplies a risk-free bond to households, spends endogenously, pays a lump-sum transfer amount to households, and imposes a progressive tax scheme to balance the budget. As the monetary authority, the government sets the interest rate according to a Taylor rule based on the levels of inflation and output.

In this section, I give the assumptions and key equations in the model. For a derivation of the equations and characterization of the model, see Appendix B.

3.1 Households

The model is populated by a unit continuum of infinitely lived households indexed $i \in [0, 1]$. Each period, households choose to consume $c_{i,t}$, provide labor $\ell_{i,t}$, and hold $b_{i,t}$ of a risk-free government bond which has gross real returns R_t to maximize expected discounted utility. Households have constant relative risk aversion (CRRA) preferences given by

$$\max_{\{c_{i,t},\ell_{i,t},b_{i,t}\}_{t=0}^{\infty}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{c_{i,t}^{1-\gamma}}{1-\gamma}-\phi\frac{\ell_{i,t}^{1+\chi}}{1+\chi}\right]$$

where β, γ, ϕ, and χ represent the intertemporal discount rate, risk aversion level, relative disutility of labor, and inverse Frisch elasticity of labor supply.

Household productivity $z_{i,t}$ evolves stochastically over time subject to the log-AR(1) process

$$\log z_{i,t} = \rho_z \log z_{i,t-1} + \epsilon_{z,i,t}, \quad \epsilon_{z,i,t} \sim \mathcal{N}(0,\sigma_z^2)$$

where ρ_z and σ_z^2 represent the persistence and variance of individual productivity shocks. Based on their productivity, labor supply, and the real wage W_t, households generate pre-tax labor income $W_t z_{i,t}\ell_{i,t}$. Additionally, dividends D_t and transfers η_t are evenly distributed across households from the profits of intermediate goods firms and exogenously by the government.

Following McKay, Nakamura, and Steinsson (2016), the government imposes a progressive tax on productivity. Since productivity is exogenous, this acts like a lump sum tax and does not distort household decisions. The tax scheme is given by $\tau_t^L z_{i,t}^{\tau_t^P}$ where τ_t^L and τ_t^P measure the level and progressivity of the tax scheme respectively. Therefore, $\tau_t^P < 1$ creates a regressive tax scheme, $\tau_t^P = 1$ creates a proportional tax scheme, and $\tau_t^P > 1$ creates a progressive tax scheme.

Combined, this gets the household budget constraint

$$b_{i,t} + c_{i,t} = R_t b_{i,t-1} + W_t z_{i,t}\ell_{i,t} + D_t + \eta_t - \tau_t^L z_{i,t}^{\tau_t^P}.$$

Households are also subject to the borrowing constraint $b_{i,t} \geq \underline{b}$ which enforces a no-Ponzi condition for all households.

3.2 Distribution of Households

The distribution of household productivity $\Gamma_t^z(z)$ is fully exogenous and follows a time invariant process. Assuming the initial distribution for $\Gamma_t^z(z)$ is equal to the ergodic distribution of the AR-1 process, the overall distribution stays constant over time.

Each period, households choices depend on their states $z_{i,t}$ and $b_{i,t-1}$. Given their states, households will follow the decision rules

$$b_t(b_{i,t-1}, z_{i,t}) = b_{i,t}$$
$$c_t(b_{i,t-1}, z_{i,t}) = c_{i,t}$$
$$\ell_t(b_{i,t-1}, z_{i,t}) = \ell_{i,t}.$$

Therefore, the distribution of household states $\Gamma_t(b, z)$ evolves according to

$$\Gamma_{t+1}(b', z') = \int_{\{(b,z):b_t(b,z)=b'\}} \Pr(z'|z) d\Gamma_t(b, z).$$

3.3 Firms

The model is populated by a representative, competitive final goods firm and a continuum of intermediate goods firms indexed $j \in [0, 1]$.

The final goods firm aggregates intermediate goods $y_{j,t}$ into output Y_t according to the Dixit-Stiglitz aggregator

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{1}{\psi_t}} \, dj \right)^{\psi_t}$$

where $\dfrac{\psi_t}{\psi_t - 1}$ represents the elasticity of substitution for intermediate goods. Profit maximization for the final goods firm gets the demand for intermediate good j

$$y_{j,t} = Y_t \left(\frac{p_{j,t}}{P_t} \right)^{\frac{\psi}{\psi-1}}$$

where $p_{j,t}$ is the price of intermediate good j and P_t is the overall price level of the economy given by

$$P_t = \left(\int_0^1 p_{j,t}^{\frac{1}{1-\psi_t}} \, dj \right)^{1-\psi_t}.$$

Intermediate goods firms use productive units of labor $n_{j,t}$ to produce their intermediate good according to

$$y_{j,t} = A_t n_{j,t}$$

where A_t represents the overall productivity level of the economy.

Intermediate goods firms also choose prices subject to quadratic adjustment costs $m_{j,t}$ à la Rotemberg (1982) given by

$$m_{j,t} = \frac{\psi_t}{\psi_t - 1} \frac{1}{2\kappa} \left(\frac{p_{j,t}}{\overline{\pi} p_{j,t-1}} - 1 \right)^2 y_{j,t}$$

where $\pi = \dfrac{P_t}{P_{t-1}}$ is inflation, κ determines the responsiveness of inflation to changes in output, and an overline denotes the steady state of a variable. Compared to the alternative Calvo (1983) rule, the Rotemberg price frictions have a couple advantages. First, price frictions under a Rotemberg rule are more realistic (Richter and Throckmorton 2016). Additionally, a Rotemberg rule has an analytically solvable Philips curve, which reduces the size of the system. The linearized Philips curve is

$$(\hat{\pi}_t - 1) = \kappa \left(\frac{W_t}{A_t} - \frac{1}{\psi_t} \right) + \mathbb{E} R_{t+1}^{-1} \frac{Y_{t+1}}{Y_t} (\hat{\pi}_{t+1} - 1)$$

where the circumflex denotes the geometric deviation from the steady state so that $\hat{\pi}_t = \dfrac{\pi_t}{\overline{\pi}}$.

Finally, since intermediate goods firms are monopolistically competitive, they can make a profit. Profits will be paid out in the form of real dividends $d_{j,t}$ such that

$$d_{j,t} = \frac{p_{j,t}}{P_t} y_{i,t} - W_t n_{j,t} - m_{j,t}$$

where firms earn real revenue $\dfrac{p_{j,t}}{P_t} y_{i,t}$ and pay labor costs $W_t n_{j,t}$ and price adjustment costs. Aggregate dividends D_t are

$$D_t = \int_0^1 d_{j,t} dj.$$

3.4 Government

In the economy, the government acts as both the fiscal and monetary authority.

As the fiscal authority, the government spends an exogenous fraction g_t of output so that government spending G_t follows

$$G_t = g_t Y_t.$$

The government also offers the risk-free bond B_t and pays out transfers to households so that

$$B_t = \overline{B} + \rho_B \left(R_t B_{t-1} - \overline{RB} + G_t - \overline{G} + \eta_t - \overline{\eta} \right)$$

where ρ_B represents how quickly the government pays back non-steady state levels of debt. In the steady state, this means the government holds a constant stock of debt which it pays all the interest on every period. However, increases in transfers η_t, the interest rate R_t, or government spending G_t will be financed by taking on more debt and paying it back over time. To balance the budget, the government sets the tax level τ_L so that

$$R_t B_{t-1} + G_t + \eta_t = \tau_t^L \int z^{\tau_t^P} d\Gamma_t^Z(z) + B_t.$$

As the monetary authority, the government sets the interest rate I_t according to the Taylor Rule

$$I_t = \overline{I} \hat{\pi}_t^{\omega_\pi} \hat{Y}_t^{\omega_Y} \xi_t$$

where ω_π and ω_Y represent the relative importance of inflation and output stabilization and ξ_t is the monetary policy shock. The Fisher relation suggests

$$R_t = \frac{I_{t-1}}{\pi_t}.$$

3.5 Equilibrium

For the economy to be in equilibrium, the labor, bond, and goods markets all need to clear. Labor market clearing requires aggregate productive labor demand from firms to equal to aggregate productive labor supply by households

$$N_t = \int z \ell_t(b, z) d\Gamma_t(b, z)$$

where $N_t = \int_0^1 n_{j,t} dj$. Bond market clearing requires the supply of bonds by the government to equal household savings

$$B_t = \int b_t(b,z) d\Gamma_t(b,z).$$

Finally, goods market clearing requires consumption, government spending, and price adjustment costs to equal output

$$Y_t = \int c_t(b,z) d\Gamma_t(b,z) + M_t + G_t$$

where $M_t = \int_0^1 m_{j,t} dj$.

Therefore, a solution to the model consists of sequences for prices $\{\pi_t, W_t, M_t, D_t, R_t, I_t, \tau_t^L\}_{t=0}^{\infty}$, household decision rules $\{b_t, c_t, \ell_t\}_{t=0}^{\infty}$ that solve the household utility maximization problem, the distribution of household states $\{\Gamma_t\}_{t=0}^{\infty}$ that evolves following the policy rules, and macroeconomic aggregates $\{Y_t, N_t, B_t, G_t\}_{t=0}^{\infty}$ all so that the labor, bond, and goods markets clear subject to exogenous, AR(1) processes for $\{A_t, g_t, \xi_t, \psi_t, \tau_t^P, \eta_t\}_{t=0}^{\infty}$.

3.6 Computational Methods

I solve the model in the sequence-space following Auclert et al. (2021). This method has significant computational advantages over standard state-space methods like Reiter (2009) or even dimensionality-reduced state-space methods like Bayer and Luetticke (2018) since it removes household states, of which there can be thousands, from the system used to solve the model.

The first step to solve the model is to find the steady state. I discretize the household asset and productivity levels into a grid. Household transitions between productivity levels are modeled using a Rouwenhorst process (Kopecky and Suen 2010). Following Reiter (2009), I add more asset gridpoints closer to the borrowing constraint $\underline{b}$ to address the nonlinearities in the decision rules near that point. I solve for household decision rules using the endogenous grid method (Carroll 2006). Then, following Young (2010), the distribution Γ_t is represented as a histogram at each of the asset-productivity gridpoints, which households travel between based on the savings decision rule.

Shocks are modeled as linear perturbations around the steady state in the sequence space

Table 3.1: Computational Parameters

Parameter	**Value**	**Description**
n_b	501	Number of asset gridpoints
$\underline{b}$	0	Borrowing constraint
$\overline{b}$	50	Maximum asset gridpoint
n_z	7	Number of productivity gridpoints
T	300	Sequence space perturbation time horizon

(Auclert et al. 2021). I use the Python automatic differentiation library Jax to solve for derivatives of the aggregate conditions and the Fake News Algorithm with two-sided numerical differentiation to solve for derivatives of the heterogeneous agent block aggregates (Auclert et al. 2021). To model the effect of shocks on individual policy rules, I use the disaggregated Fake News derivative and aggregate economic conditions to solve for the linearized effect of the shock on households.

The grid dimensions and sequence space truncation horizon are outlined in Table 3.1. In Appendix TBD, I test the effect of the computational choices on my results, including using a high-dimensional model with substantially more gridpoints and different truncation horizons.

4 Parameterization

I use a two-step procedure to parameterize the model at a quarterly frequency. First, I calibrate the micro-parameters and frictions within the model. Then, I use a Bayesian estimation of the shock effects to decompose business cycle dynamics into each different shock channel. This "calibrate then estimate" approach is common in HANK literature since the method reuses the perturbation matrix instead of having to recompute it each draw, which is the most computationally difficult part of the solution process (Winberry 2018; Auclert, Rognlie, and Straub 2020; Auclert et al. 2021; Bayer, Born, and Luetticke 2024). Alternative approaches use parallelized estimation strategies and still can take days to estimate the full set of parameters in the model (Acharya et al. 2023).

Table 4.1: Model Parameters

Parameter	Value	Description	Target
Preferences			
β	0.965	Discount rate	2% annual interest rate
γ	4	Risk aversion	Kaplan, Moll, and Violante (2018)
$1/\chi$	1/2	Frisch elasticity	Chetty (2012)
ϕ	2.55	Disutility of labor	$\overline{N} = 1$
$\underline{b}$	0	Borrowing constraint	
Productivity			
ρ_z	0.963	Productivity persistence	Storesletten, Telmer, and Yaron (2004)
σ_z	0.134	Productivity STD	Cross-sectional STD of 0.5
Firms			
κ	0.1	Philips Curve slope	
Government			
ρ_B	0.95	Debt persistence	1 year half-life
$\overline{B}$	0.577	Govt. debt target	57.7% debt to GDP steady state
ω_π	1.5	Taylor inflation	
ω_Y	0	Taylor output	
$\overline{\pi}$	1	Inflation target	0% inflation steady state
Shock SS			
$\overline{A}$	1	TFP	
$\overline{\psi}$	1.2	Markup	20% markup
$\overline{g}$	0.202	Govt. spending	20.1% govt. spending
$\overline{\eta}$	0.081	Transfers	8.1% transfers
$\overline{\tau^P}$	1.18	Tax progressivity	Heathcote, Storesletten, and Violante (2017)
$\overline{\xi}$	1	Monetary shock	

4.1 Calibration

The calibrated model parameters are listed in Table 4.1. Risk aversion is set to 4, which is standard in HANK literature (Kaplan, Moll, and Violante 2018). I take Frisch elasticity of 0.5 from (Chetty 2012). Household productivity transitions are based on Storesletten, Telmer, and Yaron (2004) to have persistence 0.963 and cross-sectional standard deviation of 0.5. The slope of the Philips Curve and the Taylor coefficients for inflation and output are set based on standard values in the literature. The government inflation target ensures a 0% inflation steady state. The values for TFP and monetary policy shocks have no effect in the steady state. The markup is set to give intermediate goods firms a 20% markup when setting prices. Tax progressivity of 1.18 creates a progressive taxation scheme for the economy (Heathcote, Storesletten, and Violante 2017).

The government debt target, government spending rate, and transfers are calibrated to match historical US averages for debt to GDP, government spending to GDP, and household transfers to GDP between 1966 and 2019. This process is explained in Appendix A.1. The values for the discount rate and the disutility of labor are calibrated within the model to match a 2% annual (0.5% quarterly) interest rate and full employment in the steady state ($\overline{N} = 1$).

4.2 Estimation Strategy

To estimate the shocks to the model, I use the Bayesian estimation procedure from Auclert et al. (2021). This method matches the covariances of endogenous variables with different time offsets in the impulse response functions (IRFs) in the sequence space of the model to their covariances in real data. Similar to other estimations of HANKs, I use a standard random-walk Metropolis-Hastings (RWMH) algorithm with 250,000 draws and a 50,000 draw burn-in (Auclert et al. 2021; Bayer, Born, and Luetticke 2024).

I estimate the persistence and standard deviation for each of the six shocks on quarterly macroeconomic time-series for GDP, inflation, the federal funds rate, hours worked, consumption, and government debt from 1966 to 2019. For inflation and the interest rate, I estimate on the difference from the mean. For GDP, employment, and consumption, I estimate on the difference from log-linear trend over time. The data series and detrending process are explained further in Appendix A.2. I do not include any microdata in the estimation process, which is a limitation of the paper. However, fitting to distributional microdata generally has a negligible effect on the overall estimates (Bayer, Born, and Luetticke 2024).[1]

I assume weak prior distributions for each of the estimated parameters. The prior for the persistence of each shock is assumed to be a beta distribution with mean 0.5 and standard deviation 0.15. The prior for the standard deviation of each shock is assumed to be an inverse gamma distribution with mean 0.2 and standard deviation 2.

1. Iao and Selvakumar (2024) finds a smaller error band for estimates using microdata, but the parameter estimates themselves are very similar.

Table 4.2: Estimation Results

Parameter		Prior			Posterior			
Shock	Statistic	Distribution	Mean	Std. Dev.	Mode	Mean	5%	95%
TFP	ρ	Beta	0.50	0.15	0.952	0.951	0.934	0.968
	σ	Inv. Gamma	0.20	2.00	0.153	0.154	0.143	0.167
Markup	ρ	Beta	0.50	0.15	0.995	0.995	0.994	0.996
	σ	Inv. Gamma	0.20	2.00	1.524	1.535	1.414	1.665
Govt. Spend	ρ	Beta	0.50	0.15	0.901	0.893	0.845	0.936
	σ	Inv. Gamma	0.20	2.00	0.131	0.132	0.122	0.143
Mon. Pol.	ρ	Beta	0.50	0.15	0.609	0.607	0.567	0.647
	σ	Inv. Gamma	0.20	2.00	0.444	0.447	0.413	0.485
Tax Prog.	ρ	Beta	0.50	0.15	0.929	0.926	0.895	0.960
	σ	Inv. Gamma	0.20	2.00	5.847	5.943	5.418	6.512
Transfers	ρ	Beta	0.50	0.15	0.926	0.914	0.854	0.960
	σ	Inv. Gamma	0.20	2.00	0.254	0.264	0.226	0.305

4.3 Estimation Results

The estimation results are presented in Table 4.2. Impulse response functions for the estimated shocks can be found in Appendix D. I find price markup shocks are the most persistent, with a ρ of 0.995. Shocks to TFP, tax progressivity, household transfers, and government spending are also found to be very persistent, with ρ estimates of 0.951, 0.926, 0.914, and 0.893 respectively. Shocks to monetary policy are the least persistent, with a ρ estimate of 0.607. The estimated standard deviation σ is highest for tax progressivity shocks (5.943) and price markup shocks (1.535). Comparatively, the standard deviations of shocks to monetary policy, household transfers, TFP, and government spending are found to be small with values of 0.447, 0.264, 0.154, and 0.132.

These estimates generally line up with both representative agent and HANK literature. The estimates for shocks to TFP, government spending, and the interest rate mostly line up with Smets and Wouters (2007) and Bayer, Born, and Luetticke (2024). My estimate for the persistence of price markup shocks is slightly higher than theirs, but I exclude the wage markup shock which has a very similar estimate, especially in Smets and Wouters (2007). Alternatively, this difference could be explained by recent trends of increasing markups within the later estimation window I use (De Loecker, Eeckhout, and Unger 2020). My estimated tax progressivity shock decays slightly faster than that of Bayer, Born, and Luetticke (2024). An estimation of a household transfer shock is, to my knowledge, novel.

The credible intervals for the estimates are high compared to other literature (Smets and Wouters 2007; Bayer, Born, and Luetticke 2024). This is common when estimating a one-asset, as opposed to two-asset, model (Auclert et al. 2021). This does add uncertainty to my analysis, however the parameters are all well identified with means of the RWMH process near the posterior modes and still narrow credible intervals. Appendix C features plots of the recursive means (Figure C.1), posterior distributions (Figure C.2), and posterior covariances (Figure C.3) which all suggest good convergence.

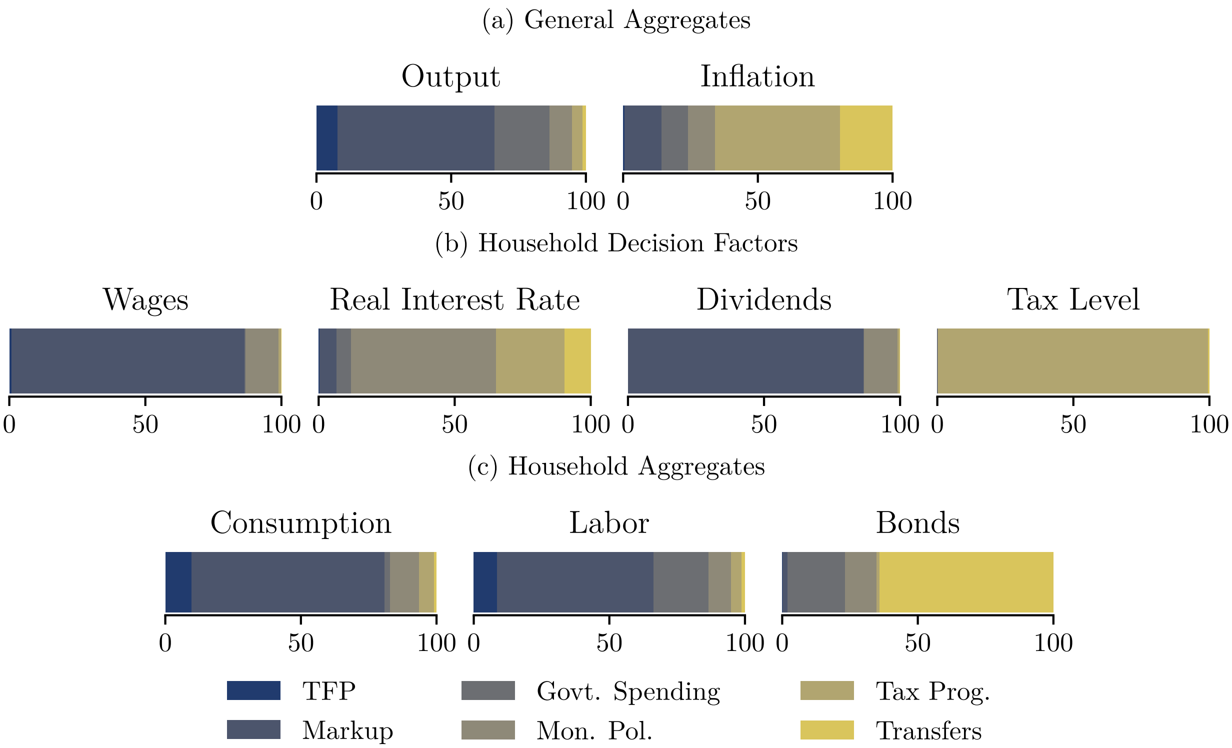
5 Business Cycles

5.1 Aggregate Outcomes

Using the estimated shock parameters, I examine the role each shock plays in business cycles. Figure 5.1 features a variance decomposition for key aggregates in my model calculated using the process in Appendix E. Panel (a) describes the business cycle effect on output and inflation. Compared to other literature, I find a larger effect of price markups and a smaller effect of TFP, especially on output (Smets and Wouters 2007; Bayer, Born, and Luetticke 2024). This difference is likely explained by the lack of a capital sector in the economy causing changes in firm behavior to be explained by price, not production, factors. Consistent with other estimates, aggregate changes in supply-side factors (TFP and price-markups) account for about 65% of output volatility, suggesting my estimated price-markup shock plays the role of general supply side shocks, like investment and wage markups, in other estimates (Bayer, Born, and Luetticke 2024). Inflation, I find, is most affected by tax progressivity, which explains approximately 50% of the variance.

Panels (b) and (c) in Figure 5.1 explore the effect of business cycles on households. Panel (b) shows how factors that directly affect household decisions are impacted by business cycles. The estimates suggest price markups explain most of the volatility in the wages and dividends paid out

Figure 5.1: Variance Decomposition: Aggregates



Notes: Forecast error variance decomposition calculated at a 4 quarter time horizon.

to households. Changes in the tax level is almost entirely explained by changes in tax progressivity. Real interest rate variation is mostly explained by monetary policy, though tax progressivity also plays a significant role. Panel (c) breaks down the effect on household aggregates. Consumption and labor variation is mostly explained by supply side factors, especially price markups, while savings variation is explained primarily by shocks to government transfers.

5.2 Decision Rules

Next, I examine the business cycle factors that drive changes in household behavior at different points along the wealth distribution. I focus on the 10th, 50th, and 90th percentiles on the productivity distribution to represent low, middle, and high earning households. On the wealth distribution, I look at the 50th, 90th, and 99th percentiles. More than half of households in the model hold 0 savings, so the decision rules for the 50th percentile apply to the entire bottom 50%. The 90th and 99th percentiles represent households that get a significant portion of their income from their savings. In my analysis, I analyze decision rules at fixed points on the wealth distribution. This means my analysis does not apply to specific individuals in the model, who can move along the distribution, or account shifts in the distribution changing the threshold for wealth percentiles.

Table 5.1 gives the decision rules and income shares for households at these productivity and wealth levels in the steady state. Household consumption is most closely associated with productivity. The labor decision rules exhibit cross-sectional income and substitution effects. At the same productivity level, higher savings is always associated with lower labor supply (income effects). However, at low savings levels, higher productivity households work less (income effects dominate) while at high savings levels, higher productivity households work more (substitution effects dominate). Ignoring borrowing constrained households, at low and medium productivity levels, households save less than they start the period with. At high productivity levels, 50th and 90th wealth percentile households end the period with higher than initial savings, but 99th percentile households are net-negative savers. Wages make up a larger fraction of the incomes for higher productivity households and interest makes up a larger fraction of the incomes for higher wealth households. Transfers are most important for households with low productivity and low savings. The tax share for households follows the progressive taxation scheme. For a more detailed breakdown, see Appendix F for surface plots for household decision rules (Figure F.1) and income shares (Figure F.2) for all income and productivity levels.

Within business cycles, the shock determinants of household behavior exhibit significant variability across the wealth and productivity distributions. Figure 5.2 gives a variance decomposition of household decision rules for consumption, labor supply, and savings at different productivity and wealth levels. Figure Appendix F features IRFs for how decision rules change in response to each shock. Variation in household consumption is explained primarily by price markup shocks across almost all productivity and wealth levels. The importance of price markups suggests wages and dividends, both of which are mostly explained by the shock (Figure 5.1), are the most important determinants of consumption. Especially for low wealth, low productivity households and high productivity households, tax progressivity also plays a very significant role. Relative to other productivity levels, middle productivity households are effected the most by monetary policy. This

Table 5.1: Household Steady State Behavior

Productivity		Wealth		Decision Rules			Income (%)			
Percentile	Value	Percentile	Value	c	ℓ	b	W	R	T	τ
10th	0.44	50th	0.00	0.58	1.14	0.00	63.0	0.0	37.0	13.8
		90th	1.94	0.70	0.78	1.70	11.5	78.5	10.0	3.7
		99th	6.31	0.82	0.57	5.89	3.1	93.3	3.6	1.4
50th	1.00	50th	0.00	0.78	0.93	0.00	75.8	0.0	24.2	23.5
		90th	1.94	0.85	0.79	1.76	22.9	68.4	8.7	8.4
		99th	6.31	0.94	0.65	5.95	7.6	88.9	3.5	3.4
90th	2.25	50th	0.00	1.00	0.85	0.21	86.6	0.0	13.4	34.0
		90th	1.94	1.03	0.80	2.05	40.7	52.6	6.7	16.9
		99th	6.31	1.09	0.73	6.24	17.1	79.7	3.1	7.9

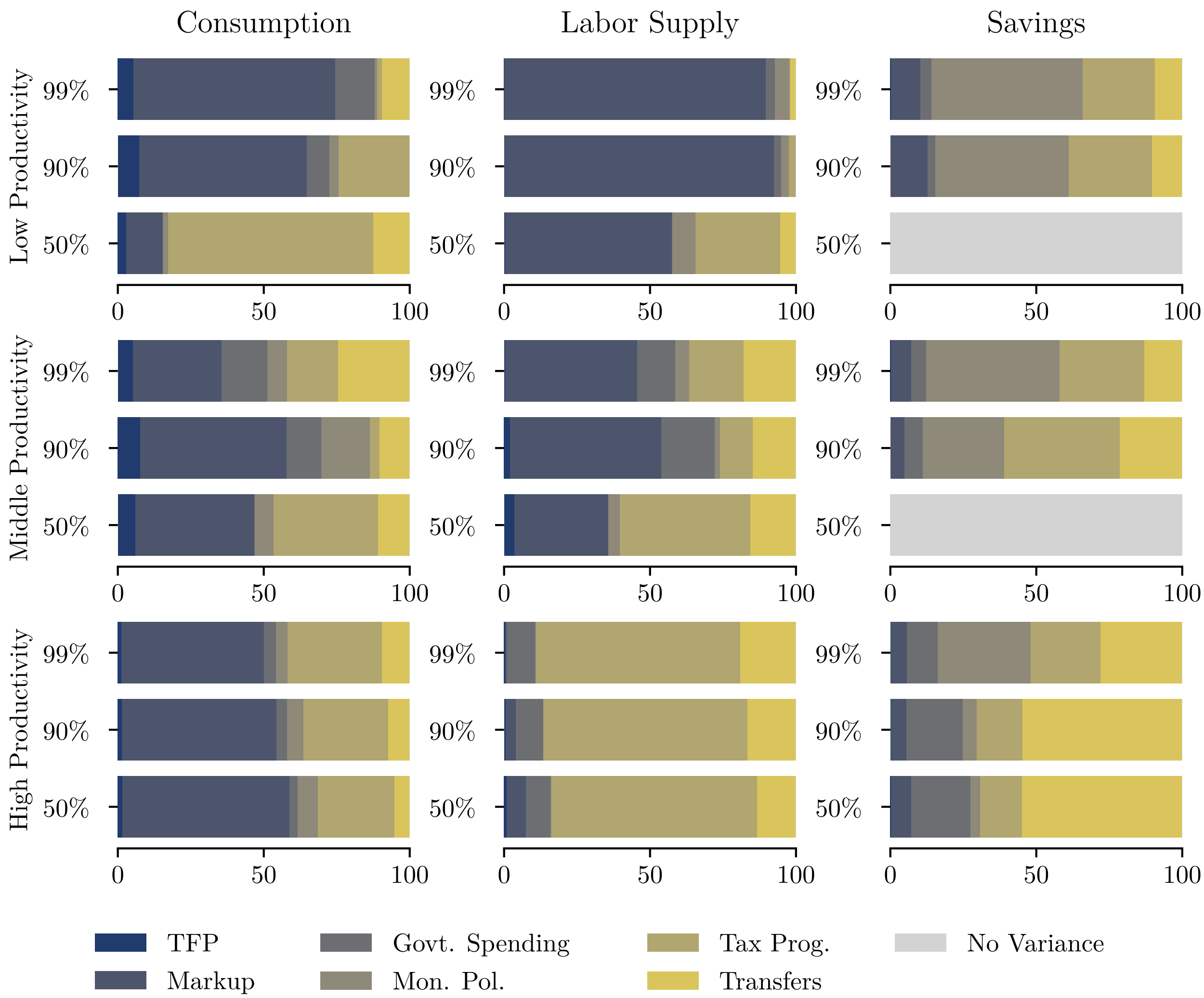
Notes: c: Consumption, ℓ: Labor, b: Savings, W: Wage, R: Interest Rate, T: Transfers (Dividends + Govt. Transfers), τ: Taxes. Income is a percentage of pre-tax budget.

could be driven by a combination of consumption smoothing by low productivity households (since expected future incomes without savings are the lowest for them) and the smaller share of income interest rates have for higher productivity households (Table 5.1).

Labor supply decisions are mostly determined by price markup shocks for low productivity households and tax progressivity shocks for high productivity households. Since price markups play a significant role determining the wage, the importance of price markups on low productivity households suggests substitution effects dominate within this group. In contrast, income effects appear to dominate for high productivity households since the taxes in my model are non-distorting.

Monetary policy shocks explain a significant portion of the variation in savings decisions, especially for low and middle productivity households. Tax progressivity also plays a significant role for this group, suggesting the decreased tax burden on lower-income households when a more progressive tax scheme is imposed is primarily funneled into increased saving by this group. Low and middle productivity borrowing constrained households that do not save in the steady state also do not save at other points in the business cycle.

Figure 5.2: Variance Decomposition: Household Decision Rules



Notes: Forecast error variance decomposition calculated at a 4 quarter time horizon. Subplot y-axis is the household position on the wealth distribution.

6 Endogenous Effects

Except for tax progressivity and government transfers, household behavior within the model is not a direct response to shocks, but rather a response to the macroeconomic consequences of the shock. In this section, I decompose the variance in household decisions into the different direct channels that affect household decisions. Within the framework of my estimated HANK, this analysis pinpoints the most important macroeconomic factors for different households.

6.1 Direct Effects Decomposition

I expand the direct-indirect decomposition for monetary policy shocks from Kaplan, Moll, and Violante (2018) to the full set of shocks and direct household effects within my model. Household conditions depend on wages W, the interest rate R, dividends D, household transfers η, the tax level τ^L, and tax progressivity τ^P. Therefore, I can decompose the vector $d\mathbf{C}$ representing the linearized impulse response function (IRF) for consumption as

$$d\mathbf{C} = \frac{\partial \mathbf{C}}{\partial \mathbf{W}} d\mathbf{W} + \frac{\partial \mathbf{C}}{\partial \mathbf{R}} d\mathbf{R} + \frac{\partial \mathbf{C}}{\partial \mathbf{D}} d\mathbf{D} + \frac{\partial \eta}{\partial \eta} d\eta + \frac{\partial \mathbf{C}}{\partial \tau^P} d\tau^P + \frac{\partial \mathbf{C}}{\partial \tau^L} d\tau^L$$

where $\dfrac{\partial \mathbf{C}}{\partial \mathbf{X}}$ is the direct effect of X on consumption and $d\mathbf{X}$ is the IRF for X. For my analysis, I combine dividends and direct transfers since both are evenly distributed transfers to all households. I also combine the tax level and tax progressivity since tax level variation is almost entirely explained by changes in tax progressivity (Figure 5.1). Denoting transfers T and taxes τ, this means

$$d\mathbf{C} = \underbrace{\frac{\partial \mathbf{C}}{\partial \mathbf{W}} d\mathbf{W}}_{\text{Wage effects}} + \underbrace{\frac{\partial \mathbf{C}}{\partial \mathbf{R}} d\mathbf{R}}_{\text{Interest effects}} + \underbrace{\frac{\partial \mathbf{C}}{\partial \mathbf{T}} d\mathbf{T}}_{\text{Transfer effects}} + \underbrace{\frac{\partial \mathbf{C}}{\partial \tau} d\tau}_{\text{Tax effects}} .$$

Then, denoting $\dfrac{\partial \mathbf{C}}{\partial \mathbf{X}} d\mathbf{X}$ as $d\mathbf{C}_X$, variance in consumption within the model can be explained as

$$\begin{aligned} \mathrm{Var}\left(d\mathbf{C}\right) = {} & \mathrm{Var}\left(d\mathbf{C}_W\right) + \mathrm{Var}\left(d\mathbf{C}_R\right) + \mathrm{Var}\left(d\mathbf{C}_T\right) + \mathrm{Var}\left(d\mathbf{C}_\tau\right) \\ & + 2\mathrm{Cov}\left(d\mathbf{C}_W, d\mathbf{C}_R\right) + 2\mathrm{Cov}\left(d\mathbf{C}_W, d\mathbf{C}_T\right) + 2\mathrm{Cov}\left(d\mathbf{C}_W, d\mathbf{C}_\tau\right) \\ & + 2\mathrm{Cov}\left(d\mathbf{C}_R, d\mathbf{C}_T\right) + 2\mathrm{Cov}\left(d\mathbf{C}_R, d\mathbf{C}_\tau\right) \\ & + 2\mathrm{Cov}\left(d\mathbf{C}_T, d\mathbf{C}_\tau\right). \end{aligned}$$

Unlike the variance decompositions in Section 5, this features covariance terms. This is because the shocks to the model decomposed in Section 5 are assumed to be orthogonal to each other, while direct effects within the model are not. Positive covariance within the decomposition implies comovement between the decomposed effects that complement each other. Negative covariance

Table 6.1: Direct Effects Decomposition, Consumption

	Aggregate	**Low Productivity**			**Middle Productivity**			**High Productivity**		
		50th	90th	99th	50th	90th	99th	50th	90th	99th
Variances										
Var(W)	1,607.7	55.0	272.6	1,030.4	4,382.9	156.2	106.2	30.3	27.7	28.2
Var(R)	289.5	0.0	205.6	1,151.5	0.0	174.8	166.7	42.7	48.3	52.4
Var(T)	1,707.1	164.7	1,032.4	3,940.4	3,884.9	198.5	158.5	14.0	14.6	17.9
Var(τ)	33.0	21.0	69.7	109.9	190.8	2.7	0.4	2.9	2.6	2.8
Covariances										
Cov(W, R)	672.3	0.0	232.5	1,060.9	0.0	164.1	131.8	35.9	36.4	38.3
Cov(W, T)	-1,656.6	-95.1	-530.5	-2,014.7	-4,125.5	-176.0	-129.7	-20.6	-20.1	-22.5
Cov(W, τ)	-694.1	0.0	-453.6	-2,081.2	0.0	-185.2	-161.1	-24.4	-26.5	-30.5
Cov(R, T)	-226.8	-33.9	-137.3	-334.3	-914.0	-20.0	-6.4	9.4	8.5	9.0
Cov(R, τ)	-97.7	0.0	-118.8	-351.8	0.0	-21.5	-8.2	11.1	11.3	12.2
Cov(T, τ)	234.1	58.7	267.5	654.9	860.2	22.6	7.8	-6.4	-6.2	-7.1

Notes: Forecast error variance decomposition calculated at a 4 quarter time horizon with total variance normalized to 100%. Percentiles correspond to the 50th, 90th, and 99th wealth percentiles.

implies comovement between decomposed effects that, in part, cancel each other out. Substituting in any other household variable for consumption, including the policy rules at specific states, results in an identical decomposition.

6.2 Decomposition Results

I perform this decomposition on aggregates and decision rules for consumption, labor supply, and savings. Like in section 5.2, I decompose decision rules at the 10th, 50th, and 90th productivity percentiles and the 50th, 90th, and 99th wealth percentiles. Appendix F decomposes the IRFs for household behavior into each direct channel.

Table 6.1 presents the decomposition results for consumption. Across all productivity levels, the variance explained by the interest rate relative to the other variances is higher for households at the 99th wealth percentile. Wages and transfers explain more variance in consumption for households with lower wealth levels. For low productivity households, transfers are more important and for high productivity households, wages are more important. Interestingly, relative to other factors, changes in the interest rate explain more of the variance in consumption for low wealth, high productivity households than high wealth, low productivity households. This suggests the effects

Table 6.2: Direct Effects Decomposition, Labor Supply

	Aggregate	**Low Productivity**			**Middle Productivity**			**High Productivity**		
		50th	90th	99th	50th	90th	99th	50th	90th	99th
Variances										
Var(W)	1,158.7	5.3	2.1	2.0	58.5	1,168.6	166.5	10.4	8.4	7.6
Var(R)	1,809.4	0.0	37.1	82.0	0.0	14,061.0	4,016.7	99.4	111.9	130.8
Var(T)	6,267.1	82.1	186.1	280.9	208.7	15,972.1	3,818.5	32.6	33.9	44.7
Var(τ)	49.0	10.5	12.6	7.8	10.3	214.3	10.2	6.8	6.1	7.1
Covariances										
Cov(W, R)	1,405.5	0.0	6.4	4.6	0.0	3,670.8	676.0	30.9	29.0	29.1
Cov(W, T)	-2,672.7	-20.8	-16.3	-12.6	-110.5	-4,084.5	-710.1	-18.0	-16.4	-17.6
Cov(W, τ)	-3,345.3	0.0	-81.8	-148.3	0.0	-14,898.6	-3,882.3	-56.7	-61.4	-76.1
Cov(R, T)	-221.5	-7.4	-4.0	-1.8	-24.5	-437.3	-29.5	8.2	6.9	7.0
Cov(R, τ)	-294.9	0.0	-21.4	-25.1	0.0	-1,729.4	-198.8	25.9	26.1	30.3
Cov(T, τ)	536.8	29.3	48.2	46.7	46.2	1,821.0	188.6	-14.9	-14.4	-17.8

Notes: Forecast error variance decomposition calculated at a 4 quarter time horizon with total variance normalized to 100%. Percentiles correspond to the 50th, 90th, and 99th wealth percentiles.

of the interest rate comes from precautionary channels, which would impact the high-productivity net savers more, rather than income channels, which would affect higher wealth households more. Negative comovements between the effects of wages and transfers cancel each other out, especially for low productivity households.

The results for the decomposition of labor supply is presented in Table 6.2. Across all wealth and productivity levels, the amount of variation explained by transfers is larger than the variation explained by wages. This suggests that income effects from direct transfers dominate within the model, though the negative covariance between the two factors suggests substitution effects dampen income effects. Like with consumption, interest rates explain more of the variance for higher wealth and higher productivity households relative to the amount of variance explained by the other factors. Similar to consumption, likely due to precautionary motives, interest rates explain the majority of variation in labor supply for high productivity households. Middle productivity households face wage, tax, and transfer effects with massive negative covariances suggesting the consumption effects of aggregate trends for this group cancel each other out.

The decomposition for savings is presented in Table 6.3. Like in Figure 5.2, borrowing constrained 50th wealth percentile households in the low and middle productivity groups have no

Table 6.3: Direct Effects Decomposition, Savings

	Aggregate	Low Productivity			Middle Productivity			High Productivity		
		50th	90th	99th	50th	90th	99th	50th	90th	99th
Variances										
Var(W)	41.4	0.0	0.3	0.0	0.0	3.3	0.5	491.3	64.9	9.6
Var(R)	371.2	0.0	44.7	67.4	0.0	79.4	82.9	3,219.7	667.1	231.6
Var(T)	2.6	0.0	8.2	2.5	0.0	3.6	1.3	13.2	1.9	0.3
Var(τ)	20.3	0.0	2.4	1.0	0.0	1.1	0.5	636.6	94.6	17.6
Covariances										
Cov(W, R)	-120.8	0.0	-0.6	1.3	0.0	-10.6	-2.0	-1,185.7	-178.3	-31.5
Cov(W, T)	-10.2	0.0	-1.4	0.1	0.0	-3.4	-0.7	-5.2	-1.5	-0.6
Cov(W, τ)	28.1	0.0	12.3	6.4	0.0	12.1	5.3	-56.2	-13.3	-3.4
Cov(R, T)	28.5	0.0	-0.6	0.1	0.0	-1.7	-0.4	516.7	72.4	12.0
Cov(R, τ)	-86.7	0.0	8.2	5.2	0.0	8.1	4.5	-1,429.7	-246.9	-56.1
Cov(T, τ)	-6.7	0.0	4.3	1.6	0.0	1.8	0.8	29.5	3.4	0.1

Notes: Forecast error variance decomposition calculated at a 4 quarter time horizon with total variance normalized to 100%. Percentiles correspond to the 50th, 90th, and 99th wealth percentiles.

variation in savings during business cycles. Variation in savings for 90th and 99th wealth percentile households in these groups is almost entirely explained by the interest rate. For high productivity households, the interest rate still explains most of the variance in savings, though significant portions are also explained by wages and taxes. These likely play a larger role for high productivity households compared to low productivity ones since earnings from wages and taxes paid increase with productivity, especially given the progressive taxation scheme. High productivity households also face significant negative covariance between interest rates and wages and interest rates and taxation, suggesting these explanatory factors move against each other.

7 Conclusion

This paper explores how cross-sectional household heterogeneity is associated with different household behavior during business cycles. I find that the business cycle determinants differ substantially at different points along the distribution for wealth and earnings potential (measured via productivity). In general, decisions for lower earning households are most affected by price markup shocks. In contrast, variation in higher earning household decisions is most explained through changes in transfers and tax progressivity.

I also examine the effects of different transmission channels for these shocks during business cycles. Looking at the relative importance of each aggregate economic factor that directly affect households decisions, I find that interest rates are most important for high-earning net-saver households. Median income households are most affected by wages, suggesting moderate earnings potential is high enough that substitution effects can dominate in response to macroeconomic changes. Transfers explain most of the variation in low earning households, meaning income effects dominate.

This analysis focuses on changes to household decision rules at specific points on the steady-state wealth and income distribution. The results, therefore, should be interpreted as generalizations of household behavior at different points on the wealth and income distributions, not a specific household's response to business cycle shocks. This is especially important since components within the model (especially utility functions) do not perfectly represent any specific household.

My research suggests household responses to business cycles are very heterogeneous. Future work should explore these differences in behavior as a source of inequality. This is especially important given the important role business cycles play determining the levels of inequality (Bayer, Born, and Luetticke 2024).

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A Data

A.1 Calibration Data

I calibrate parameters in the model to match historical US averages relative to GDP. To match the estimation window, all data is quarterly from 1966 to 2019. Since the calibration target for $\overline{N}$ implies $\overline{Y} = 1$, I calibrate both levels ($\overline{B}$ and $\overline{\eta}$) and rates ($\overline{g}$) to their average fraction of GDP. The data is all from FRED (FRED codes in parentheses).

Debt Target. I target the steady state level of debt to match the mean US debt to GDP ratio. To calculate this ratio, I divide the historical nominal debt level (GFDEBTN) by the historical nominal GDP level (GDP). To account for differences in units, I divide this ratio by 1,000. Taking the mean gets $\overline{B} = 0.577$.

Government Spending. I target the steady state rate of government spending to match the mean fraction of GDP spent by the government To calculate this, I divide nominal government spending (GCE) by nominal GDP (GDP). Taking the mean gets $\overline{g} = 0.202$.

Transfers. I target the steady state government transfers to households to match the ratio of government transfers to households to GDP. I divide nominal social benefits transfers to households (B087RC1Q027SBEA) by nominal GDP (GDP). Taking the mean gets $\overline{\eta} = 0.081$.

A.2 Estimation Data

I estimate Y_t, π_t, I_t, N_t, C_t, B_t against US aggregate data for GDP, inflation, the Federal Funds Rate, hours worked, consumption, and government debt. I get the data from FRED (FRED codes in parentheses) at a quarterly frequency from 1966 to 2019. Since the model works in levels instead of percent deviation, the series are all multiplied by the steady state variable in the model before estimation.

GDP. To represent Y_t in the model, I use nominal GDP (GDP). I divide by the GDP deflator (GDPDEF) to get real GDP and by population (POPTHM) to make it per-capita. Then, I use the difference from the log-linear trend to estimate off of. Finally, I divide by 4 to make it quarterly and multiply by 100 to make it a percent.

Inflation. To represent π_t in the model, I use the log quarter to quarter difference in the GDP deflator (GDPDEF). I then subtract out the mean to make it into the difference from trend and multiply by 100 to make it a percent.

Federal Funds Rate. To represent I_t in the model, I use the Federal Funds Rate (FEDFUNDS). I subtract out the mean to make it into the difference from trend and divide by 4 to make it quarterly.

Hours Worked. To represent N_t in the model, I use total hours worked (HOANBS). I divide by population (POPTHM) to make it per capita. Then, I take the difference from log-linear trend to estimate off of. Finally, I divide by 4 to make it quarterly and multiply by 100 to make it a percent.

Consumption. To represent C_t in the model, I use personal consumption expenditure (PCE). I divide by the GDP deflator (GDPDEF) to get real consumption and by population (POPTHM) to make it per capita. Then, I take the difference from the log-linear trend, divide by 4 to make it quarterly, and multiply by 100 to make it a percent.

Government Debt. To represent B_t in the model, I use the level of government debt (GFDEBTN). I divide by the GDP deflator (GDPDEF) to get real debt and by population (POPTHM) to make it per capita. Then, I take the difference from the log-linear trend. Finally, I divide by 4 to make it quarterly and multiply by 100 to make it a percent.

B Additional Model Details

B.1 Household Decision Rules

From the household budget and preferences in Section 3.1, households solve

$$
\begin{aligned}
\max_{\{c_{i,t},\ell_{i,t},b_{i,t}\}_{t=0}^{\infty}} \quad & \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{c_{i,t}^{1-\gamma}}{1-\gamma}-\phi\frac{\ell_{i,t}^{1+\chi}}{1+\chi}\right] \\
\text{subject to} \quad & b_{i,t}+c_{i,t}=R_t b_{i,t-1}+W_t z_{i,t}\ell_{i,t}+D_t+\eta_t-\tau_t^L z_{i,t}^{\tau_t^P} \\
& b_{i,t}\geq \underline{b}.
\end{aligned}
$$

This gets the Lagrangian

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \Bigg[\frac{c_{i,t}^{1-\gamma}}{1-\gamma} - \phi\frac{\ell_{i,t}^{1+\chi}}{1+\chi} \\
&+ \lambda_{i,t}\left(R_t b_{i,t-1} + W_t z_{i,t}\ell_{i,t} + D_t + \eta_t - \tau_t^L z_{i,t}^{\tau_t^P} - b_{i,t} - c_{i,t}\right) \\
&+ \mu_{i,t}\left(b_{i,t} - \underline{b}\right)\Bigg]
\end{aligned}
$$

which has the FOCs

$$
\begin{aligned}
\lambda_{i,t} &= c_{i,t}^{-\gamma} \\
\lambda_{i,t} &= \frac{\phi}{W_t z_{i,t}}\ell_{i,t}^{\chi} \\
\lambda_{i,t} &= \mathbb{E}\beta R_{t+1}\lambda_{i,t+1} + \mu_{i,t}
\end{aligned}
$$

for consumption, labor, and bonds respectively. Combining the FOCs for consumption and labor gets the intratemporal constraint

$$
c_{i,t}^{-\gamma} = \frac{\phi}{W_t z_{i,t}}\ell_{i,t}^{\chi},
$$

and combining the FOCs for consumption and bonds gets

$$
c_{i,t}^{-\gamma} = \beta R_{t+1} c_{i,t+1}^{-\gamma} + \mu_{i,t}.
$$

Since $\mu_{i,t} \geq 0$, this becomes the Euler Equation

$$
c_{i,t}^{-\gamma} \geq \beta R_{t+1} c_{i,t+1}^{-\gamma}
$$

which holds with equality whenever the borrowing constraint is not binding and $b_{i,t} > \underline{b}$.

B.2 Final Goods Firm Conditions

Final goods firms earn revenue P_tY_t and have costs $\int_0^1 y_{j,t}p_{j,t}dj$. Therefore, the profit maximization condition for firms is

$$\max_{y_{j,t}} \quad P_tY_t - \int_0^1 p_{j,t}y_{j,t}dj.$$

Plugging in the aggregator, this becomes

$$\max_{y_{j,t}} \quad P_t \left(\int_0^1 y_{j,t}^{\frac{1}{\psi_t}} \, dj \right)^{\psi_t} - \int_0^1 p_{j,t}y_{j,t}dj$$

which has the FOC

$$p_{j,t} = P_t \left(\int_0^1 y_{j,t}^{\frac{1}{\psi_t}} \, dj \right)^{\psi_t - 1} y_{j,t}^{\frac{1-\psi_t}{\psi_t}}.$$

Rearranging this gets

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{\frac{\psi_t}{1-\psi_t}} \left(\int_0^1 y_{j,t}^{\frac{1}{\psi_t}} \, dj \right)^{\psi_t} = Y_t \left(\frac{p_{j,t}}{P_t} \right)^{\frac{\psi_t}{1-\psi_t}}$$

which is the demand for intermediate good j. Plugging this back into the aggregator means

$$\begin{aligned} Y_t &= \left(\int_0^1 \left(Y_t \left(\frac{p_{j,t}}{P_t} \right)^{\frac{\psi_t}{1-\psi_t}} \right)^{\frac{1}{\psi_t}} dj \right)^{\psi_t} \\ &= Y_t \frac{1}{P_t^{\frac{\psi_t}{1-\psi_t}}} \left(\int_0^1 p_{j,t}^{\frac{1}{1-\psi_t}} \, dj \right)^{\psi_t}. \end{aligned}$$

Rearranging this gets the price aggregator

$$P_t = \left(\int_0^1 p_{j,t}^{\frac{1}{1-\psi_t}} \, dj \right)^{1-\psi_t}.$$

B.3 Philips Curve Derivation

Intermediate goods firms pick prices to maximize expected discounted real profits solving

$$\max_{\{p_{j,t}\}_{t=0}^{\infty}} \quad \mathbb{E}\sum_{s=t}^{\infty} R_{t,s}^{-1}\left[\frac{p_{j,s}}{P_s}y_{j,s} - W_s n_{j,s} - m_{j,s}\right]$$

where $R_{t,s} = \prod_{q=t+1}^{s} R_q$ represents the real gross return of bonds from period t to s. Plugging in the demand function, production function, and adjustment costs gets

$$\max_{\{p_{j,t}\}_{t=0}^{\infty}} \quad \mathbb{E}\sum_{s=t}^{\infty} R_{t,s}^{-1}Y_s\left(\frac{p_{j,s}}{P_s}\right)^{\frac{1}{1-\psi_t}}\left[\frac{p_{j,s}}{P_s} - \frac{W_s}{A_s} - \frac{\psi_s}{\psi_s-1}\frac{1}{2\kappa}\left(\frac{p_{j,s}}{\pi p_{j,s-1}}-1\right)^2\right]$$

This has the FOC with respect to $p_{j,t}$ of

$$\begin{aligned}
\frac{1}{2\kappa}Y_t\left(\frac{p_{j,t}}{P_t}\right)^{\frac{\psi_t}{1-\psi_t}} p_{j,t}^{-1}\left(\frac{p_{j,t}}{\pi p_{j,t-1}}-1\right)^2 + \frac{\psi_t}{\psi_t-1}\frac{1}{\kappa}Y_t\left(\frac{p_{j,t}}{P_t}\right)^{\frac{\psi_t}{1-\psi_t}}\frac{1}{\pi p_{j,t-1}}\left(\frac{p_{j,t}}{\pi p_{j,t-1}}-1\right)& \\
= \frac{\psi_t}{\psi_t-1}\frac{1}{\kappa}R_{t+1}^{-1}Y_{t+1}\left(\frac{p_{j,t}}{P_t}\right)^{\frac{\psi_t}{1-\psi_t}}\frac{p_{j,t+1}}{\pi p_{j,t}}p_{j,t}^{-1}\left(\frac{p_{j,t+1}}{\pi p_{j,t}}-1\right)& \\
+\frac{\psi_t}{\psi_t-1}\left(\frac{p_{j,t}}{P_t}\right)^{\frac{\psi_t}{1-\psi_t}} p_{j,t}^{-1}\frac{W_t}{A_t} - \frac{1}{\psi_t-1}Y_t\left(\frac{p_{j,t}}{P_t}\right)^{\frac{\psi_t}{1-\psi_t}} p_{j,t}^{-1}.&
\end{aligned}$$

Since firm conditions are identical, we can assume price symmetry across firms so $p_{j,t} = p_{j',t}$ for $j \neq j'$. Using the price aggregator, this gets

$$P_t = \left(\int_0^1 p_{j,t}^{1-\psi_t}dj\right)^{\frac{1}{1-\psi_t}} = \left(p_{j,t}^{1-\psi_t}\right)^{\frac{1}{1-\psi_t}} = p_j.$$

Additionally, since our perturbation method linearizes the system around the steady state, we can ignore the part that's zero in a first order approximation. Finally, rearranging the system yields the Philips Curve

$$\left(\frac{\pi_t}{\pi}-1\right) = \kappa\left(\frac{W_t}{A_t} - \frac{1}{\psi_t}\right) + R_{t+1}\frac{Y_{t+1}}{Y_t}\left(\frac{\pi_{t+1}}{\pi}-1\right)$$

B.4 Aggregation

In the model, firm conditions are perfectly symmetrical. Therefore, for $j \neq j'$ we can assume

$$y_{j,t} = y_{j',t}$$

$$n_{j,t} = n_{j',t}$$

$$d_{j,t} = d_{j',t}$$

$$m_{j,t} = m_{j',t}$$

Using the aggregators for each variable this gets

$$\begin{aligned}
Y_t &= \left(\int_0^1 y_{j,t}^{\frac{\psi_t - 1}{\psi_t}} dj \right)^{\frac{\psi_t}{\psi_t - 1}} = \left(y_{j,t}^{\frac{\psi_t - 1}{\psi_t}} \right)^{\frac{\psi_t}{\psi_t - 1}} = y_{j,t} \\
N_t &= \int_0^1 n_{j,t} dj = n_{j,t} \\
D_t &= \int_0^1 d_{j,t} dj = d_{j,t} \\
M_t &= \int_0^1 m_{j,t} dj = m_{j,t}.
\end{aligned}$$

Then, integrating across the production function gets

$$Y_t = \int_0^1 y_{j,t} dj = \int_0^1 A_t n_{j,t} dj = A_t N_t,$$

integrating across the dividend expression gets

$$D_t = \int_0^1 d_{j,t} dj = \int_0^1 \left(\frac{p_{j,t}}{P_t} y_{j,t} - W_t n_{j,t} - m_{j,t} \right) dj = Y_t - W_t N_t - M_t,$$

and integrating across the adjustment cost expression gets

$$M_t = \int_0^1 m_{j,t} dj = \int_0^1 \frac{\psi_t}{\psi_t - 1} \frac{1}{2\kappa} \left(\frac{p_{j,t}}{\overline{\pi} p_{j,t-1}} - 1 \right)^2 y_{j,t} dj = \frac{\psi_t}{\psi_t - 1} \frac{1}{2\kappa} \left(\frac{\pi_t}{\overline{\pi}} - 1 \right)^2 Y_t.$$

B.5 Characterization

The model is characterized by the household decision rules

$$
\begin{aligned}
b_t(b_{i,t-1}, z_{i,t}) + c_t(b_{i,t-1}, z_{i,t}) &= R_t b_{i,t-1} + W_t z_{i,t} \ell_t(b_{i,t-1}, z_{i,t}) + D_t + \eta_t - \tau_t^L z_{i,t}^{\tau_t^P} \\
c_t(b_{i,t-1}, z_{i,t})^{-\gamma} &= \beta \mathbb{E} R_{t+1} c_{t+1}(b_{i,t-1}, z_{i,t}) \\
c_t(b_{i,t-1}, z_{i,t})^{-\gamma} &= \frac{\phi}{W_t z_{i,t}} \ell_t(b_{i,t-1}, z_{i,t}),
\end{aligned}
$$

distributional movement condition

$$
\Gamma_{t+1}(b', z') = \int_{\{(b,z): b_t(b,z) = b'\}} \Pr(z'|z) d\Gamma_t(b, z),
$$

aggregate equations

$$
\begin{aligned}
Y_t &= A_t N_t \\
M_t &= \frac{\psi_t}{\psi_t - 1} \frac{1}{2\kappa} \left(\frac{\pi_t}{\overline{\pi}} - 1 \right)^2 Y_t \\
\kappa \frac{\pi_t}{\overline{\pi}} \left(\frac{\pi_t}{\overline{\pi}} - 1 \right) &= 1 - \psi_t + \frac{W_t}{A_t} \psi_t + \kappa \mathbb{E} R_{t+1}^{-1} \frac{Y_{t+1}}{Y_t} \frac{\pi_{t+1}}{\overline{\pi}} \left(\frac{\pi_{t+1}}{\overline{\pi}} - 1 \right) \\
D_t &= Y_t - W_t N_t - M_t \\
G_t &= g_t Y_t \\
B_t &= (\rho_B + R_t - 1) B_{t-1} + (2 - \rho_B - \overline{R}) \overline{B} + \eta_t - \overline{\eta} + G_t - \overline{G} \\
R_t B_{t-1} + G_t + \eta_t &= \tau_t^L \int z d\Gamma_t^Z(z) + B_t \\
I_t &= \overline{I} \left(\frac{\pi_t}{\overline{\pi}} \right)^{\omega_\pi} \left(\frac{Y_t}{\overline{Y}} \right)^{\omega_Y} \xi_t \\
R_t &= \frac{I_{t-1}}{\pi_t},
\end{aligned}
$$

and market clearing conditions

$$B_t = \int b_t(b,z) d\Gamma_t(b,z)$$
$$N_t = \int z\ell_z(b,z) d\Gamma_t(b,z)$$

where the goods market clears by Walras's Law.

C Estimation Results

Figure C.1: Recursive Means

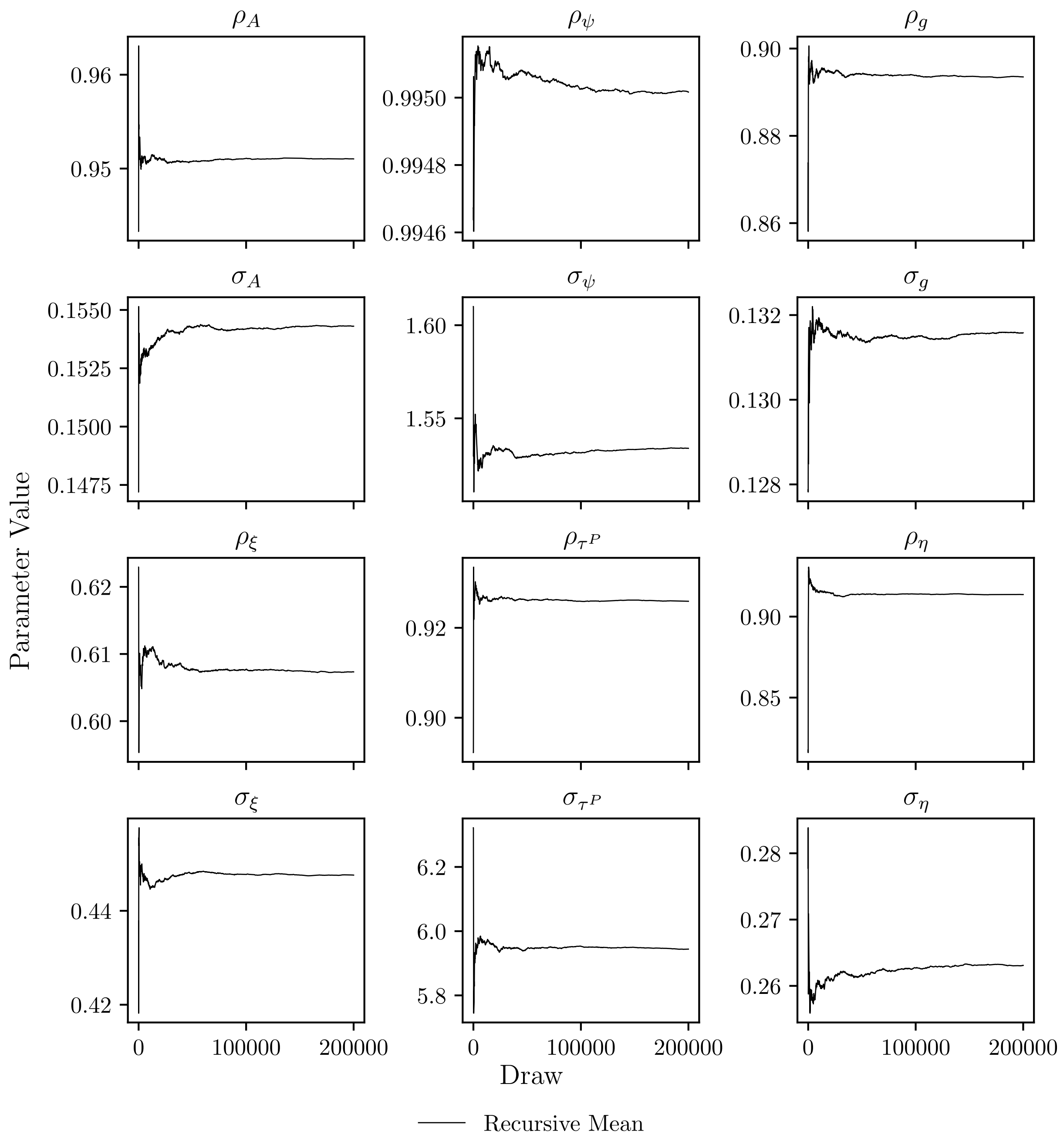


Figure C.2: Posterior Distributions

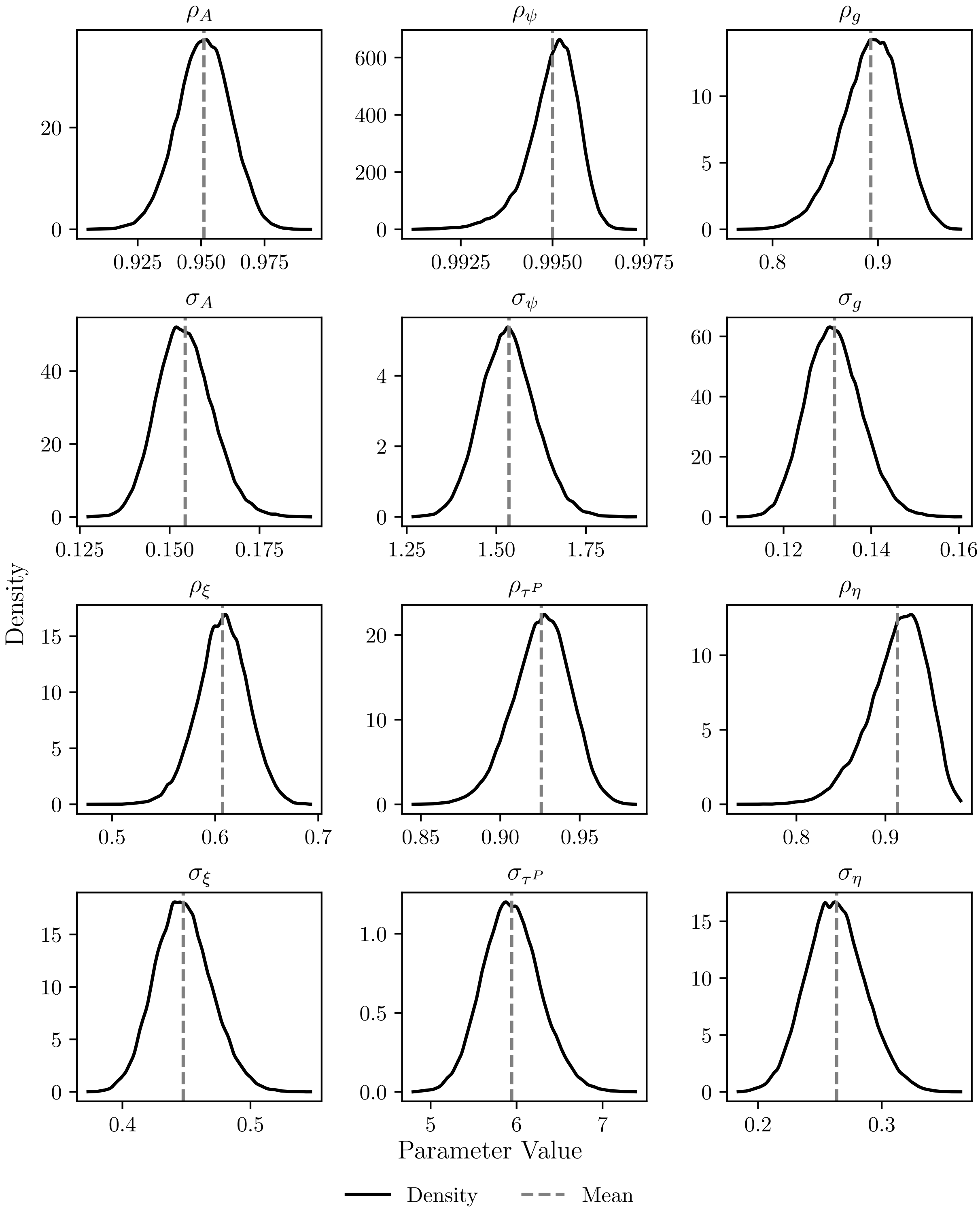
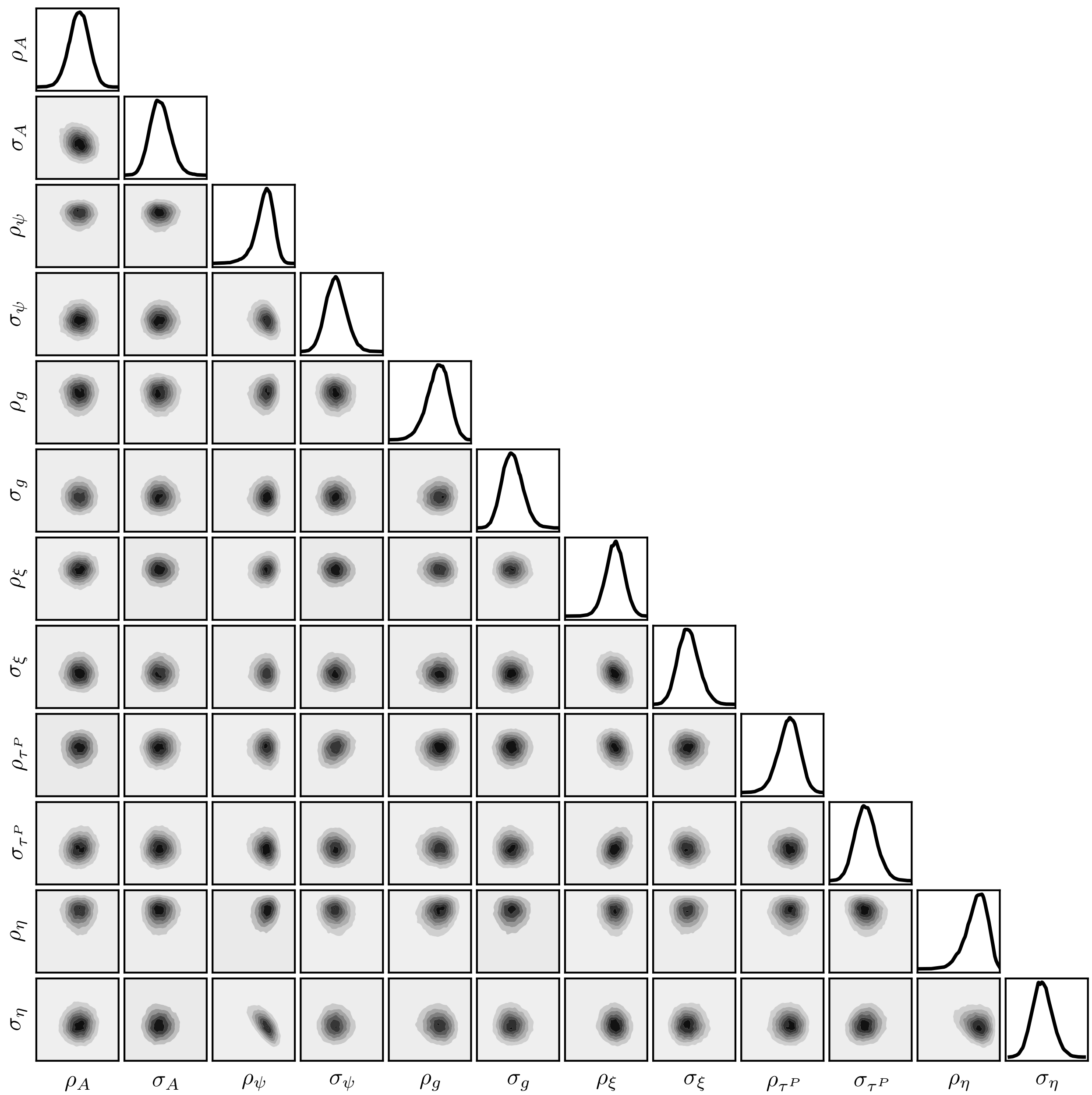


Figure C.3: Posterior Covariences



D Aggregate IRFs

Figure D.1: TFP (A) Shock Impulse Response Functions

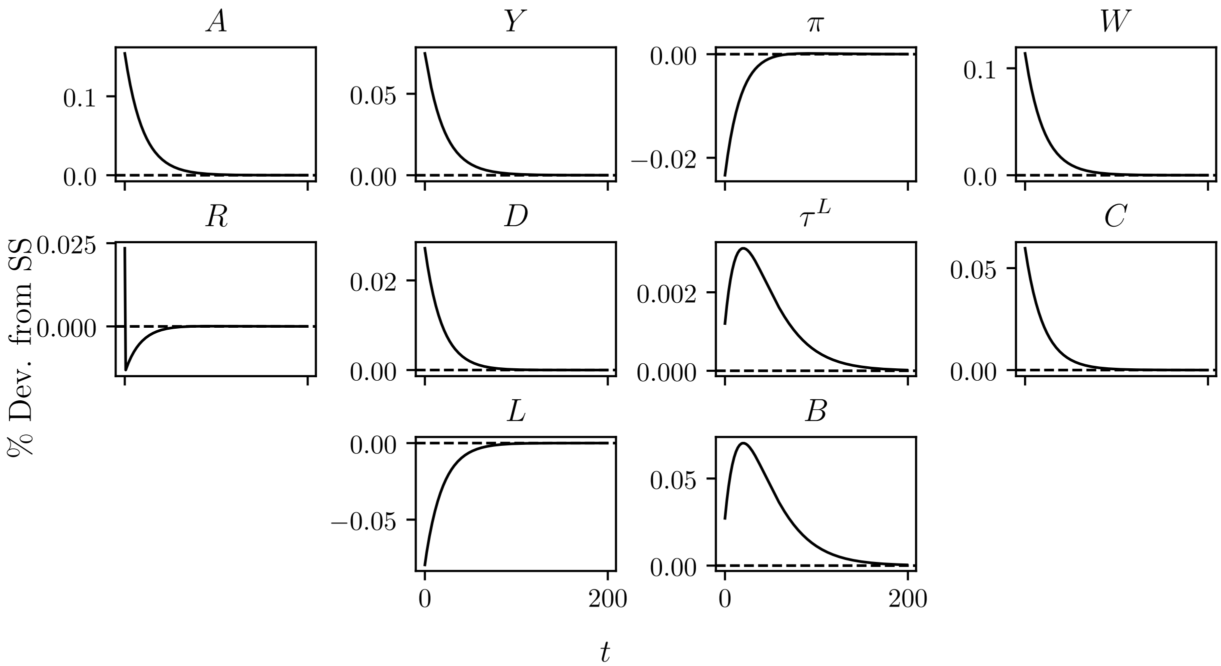


Figure D.2: Price Markup (ψ) Shock Impulse Response Functions

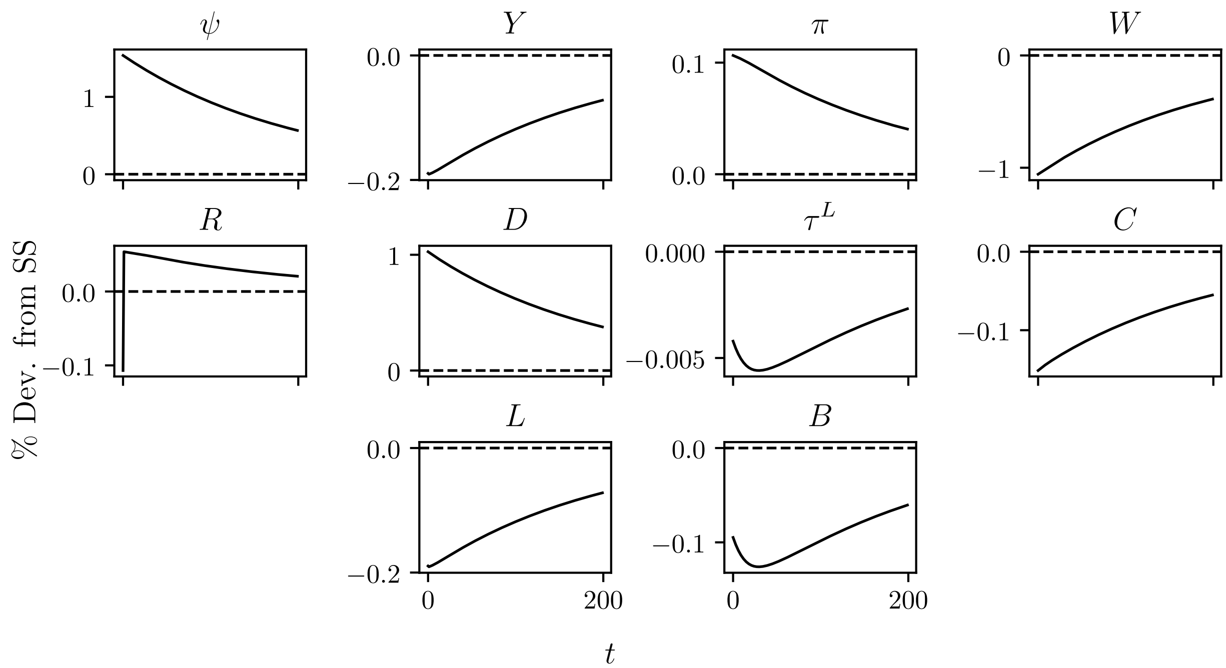


Figure D.3: Govt. Spending (g) Shock Impulse Response Functions

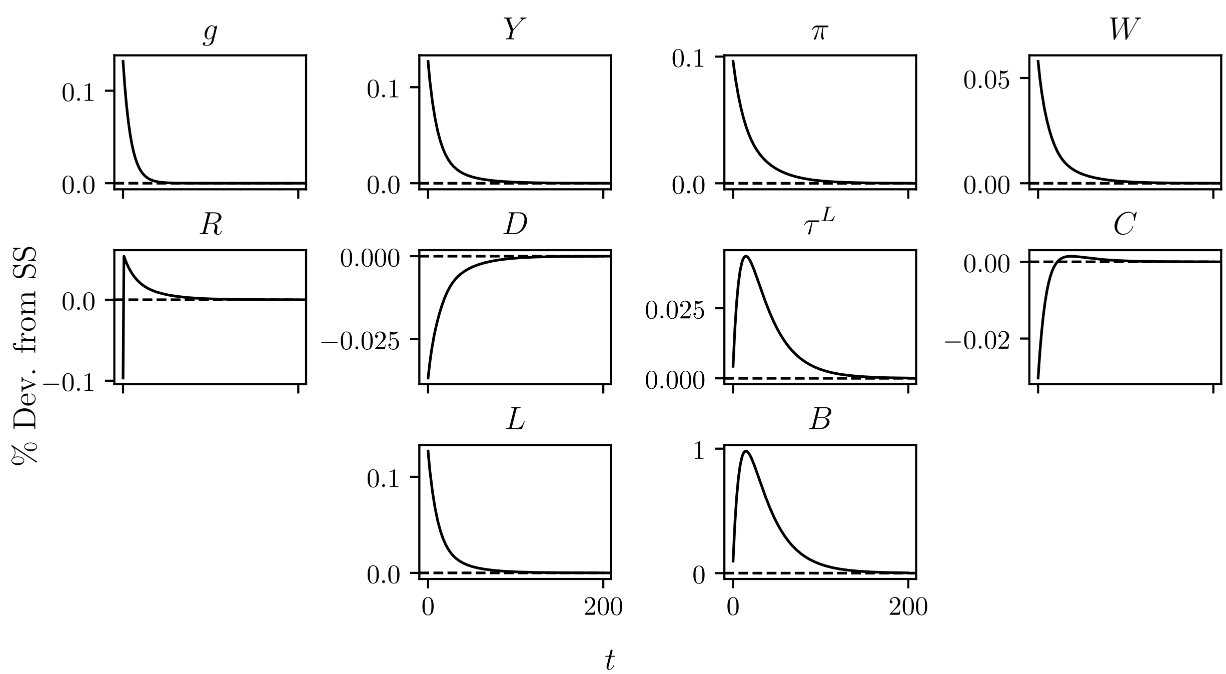


Figure D.4: Monetary Policy (ξ) Shock Impulse Response Functions

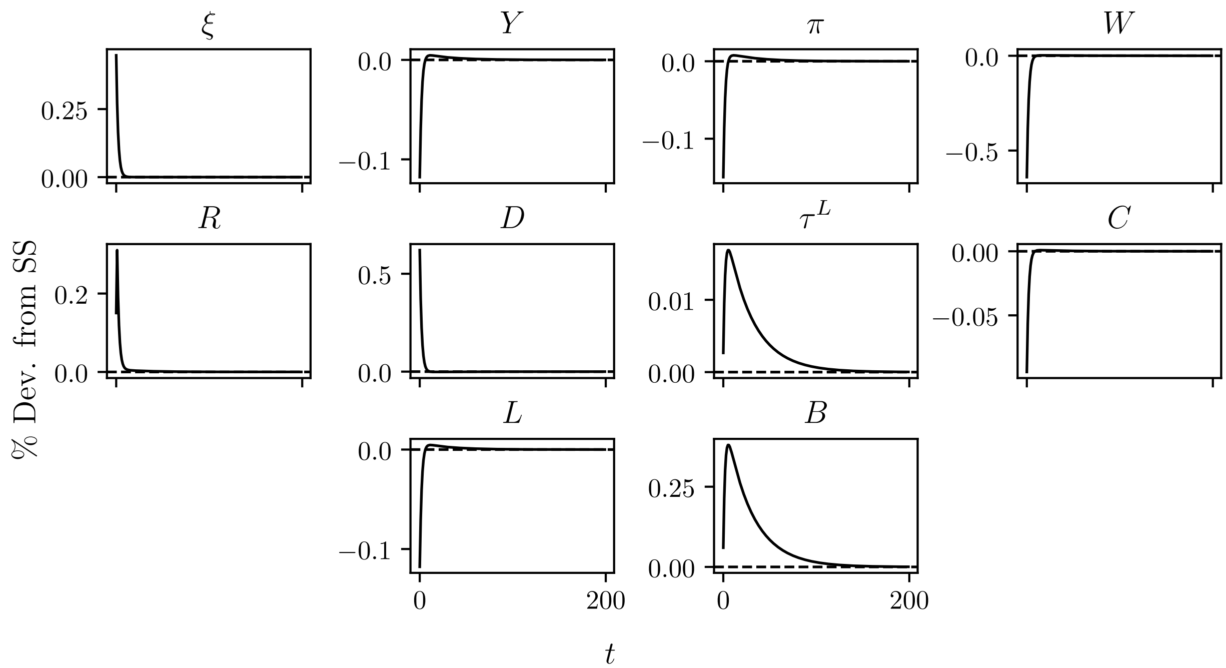


Figure D.5: Tax Progressivity (τ^P) Shock Impulse Response Functions

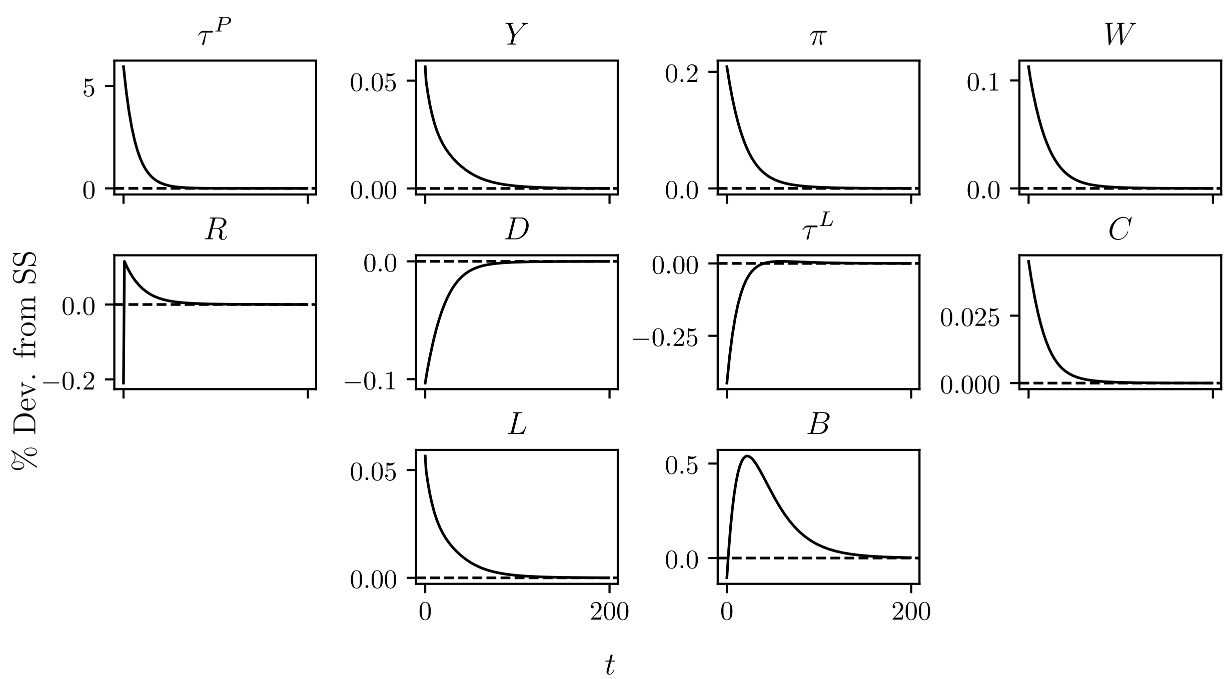
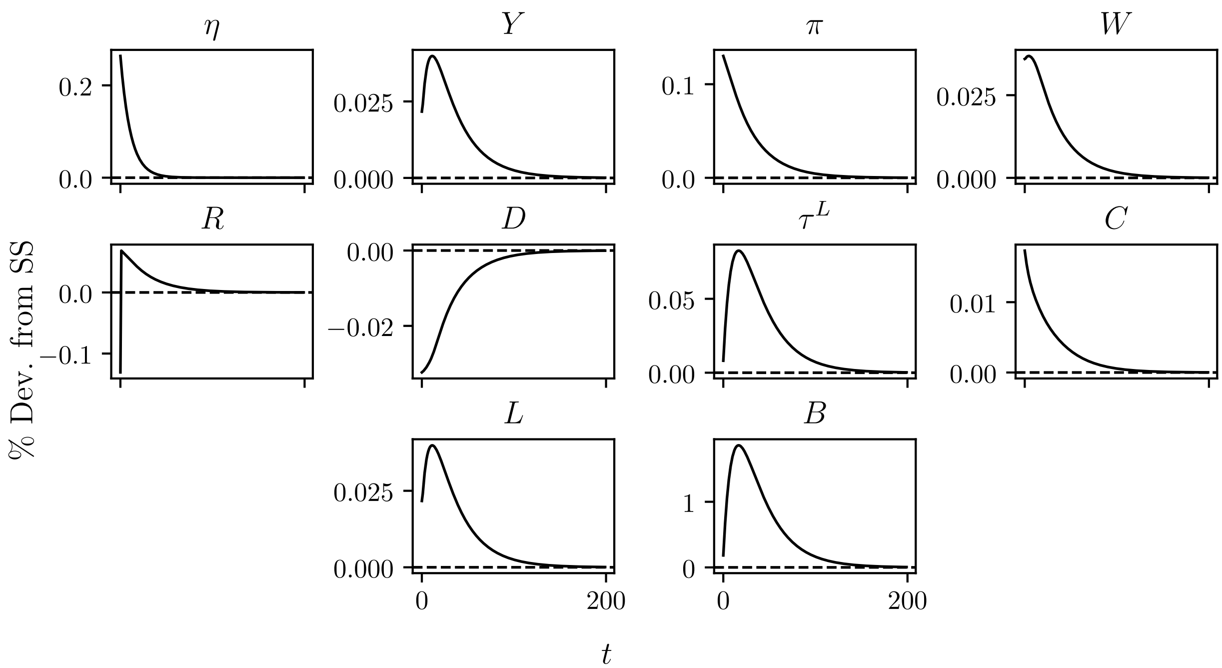


Figure D.6: Household Transfer (η) Shock Impulse Response Functions



E Forecast Error Variance Decomposition Calculation

To calculate the forecast error variance decomposition (FEVD) in the sequence space, I start with the moving process from Auclert et al. (2021)

$$d\tilde{\mathbf{X}}_t = \sum_{s=0}^{\infty} d\mathbf{X}_s \epsilon_{t-s}$$

where $d\tilde{\mathbf{X}}_t$ is a vector of outcome differences from trend $d\tilde{x}_{j,t}$, $d\mathbf{X}_s$ is a matrix of impulse responses where the i-jth element represents the change in outcome j s periods after a shock to ϵ_i, $\frac{dx_{j,s}}{d\epsilon_{j,s}}$, and ϵ_t is a vector of iid shocks $\epsilon_{i,t}$ with diagonal variance-covariance matrix Σ. Assuming no effects from shocks at time t, we know

$$d\tilde{\mathbf{X}}_{t+h} - d\tilde{\mathbf{X}}_t = \sum_{s=0}^{h-1} d\mathbf{X}_s \epsilon_{t+h-s}.$$

This gets Mean Squared Error for output j of

$$\begin{aligned}
\text{MSE}\left(d\tilde{x}_{j,t+h}\right) &= \mathbb{E}\left[\left(\sum_{s=0}^{h-1} dx_{j,s}\epsilon_{t+h-s}\right)^2\right] \\
&= \mathbb{E}\left[\sum_{s=0}^{h-1}\sum_{r=0}^{h-1} dx_{j,s}\epsilon_{t+h-s}\epsilon_{t+r-s}^{\top} dx_{j,r}^{\top}\right] \\
&= \sum_{s=0}^{h-1} dx_{j,s}\Sigma dx_{j,s}^{\top} \\
&= \text{Var}\left(d\tilde{x}_{j,t+h}\right)
\end{aligned}$$

where the part of the variance coming from by shock i is

$$\sum_{s=0}^{h-1} dx_{ij,s}\sigma_i^2 dx_{ij,s}^{\top}.$$

Therefore, the FEVD is

$$\text{FEVD}_{ij} = \frac{\sum_{s=0}^{h-1} dx_{ij,s}\sigma_i^2 dx_{ij,s}^{\top}}{\sum_{s=0}^{h-1} dx_{j,s}\Sigma dx_{j,s}^{\top}}.$$

F Household Decision Rules

Figure F.1: Household Decision Rules

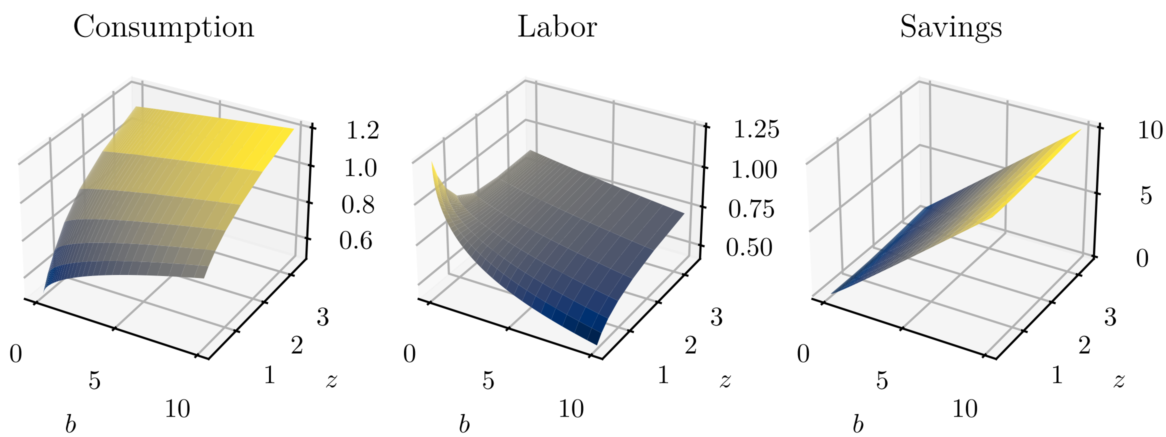


Figure F.2: Household Income Shares

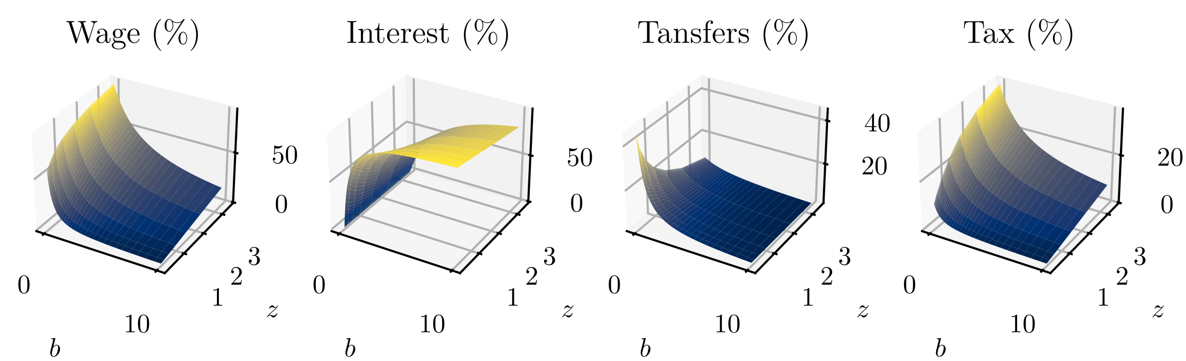


Figure F.3: Consumption Response to a TFP Shock

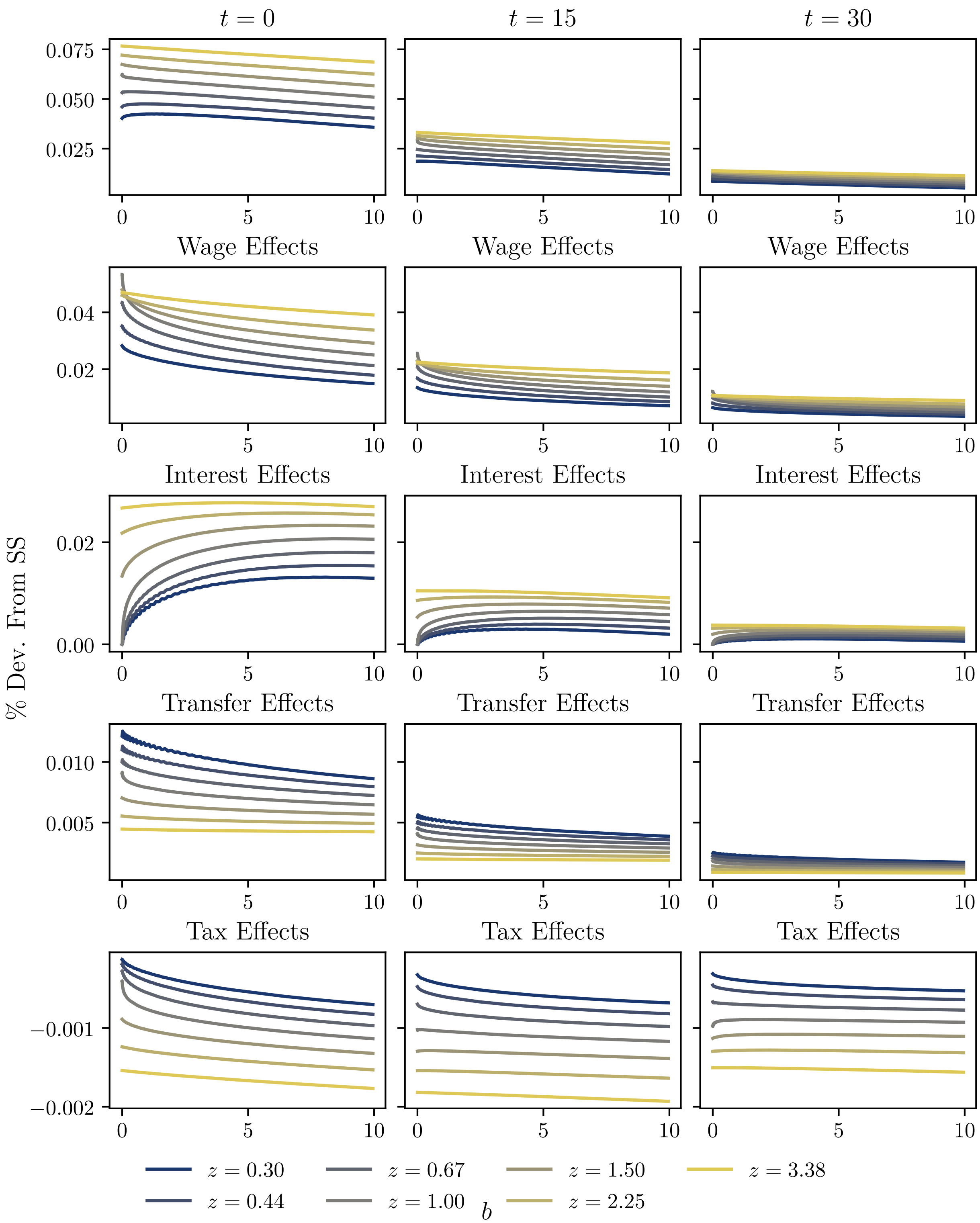


Figure F.4: Labor Supply Response to a TFP Shock

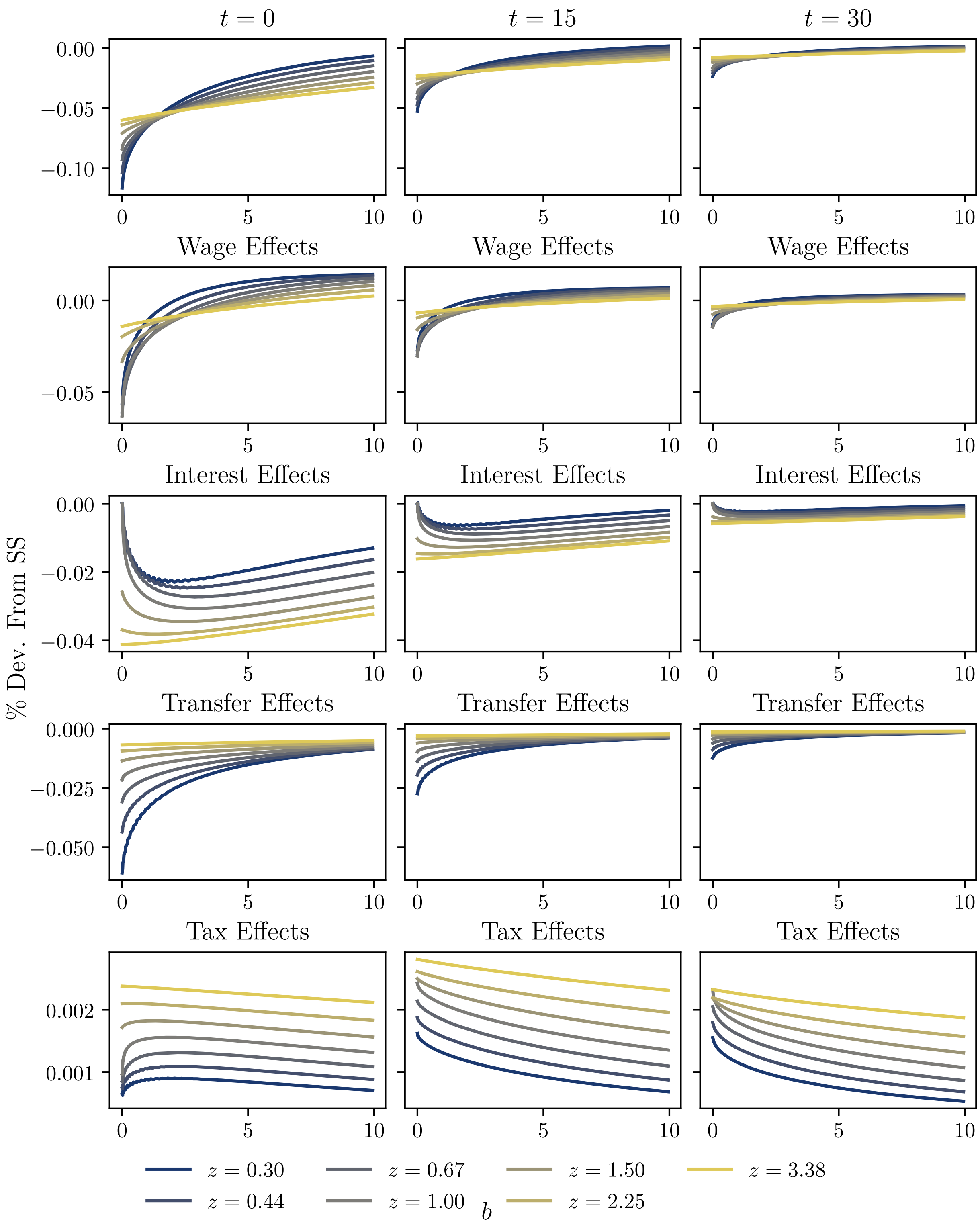


Figure F.5: Savings Response to a TFP Shock

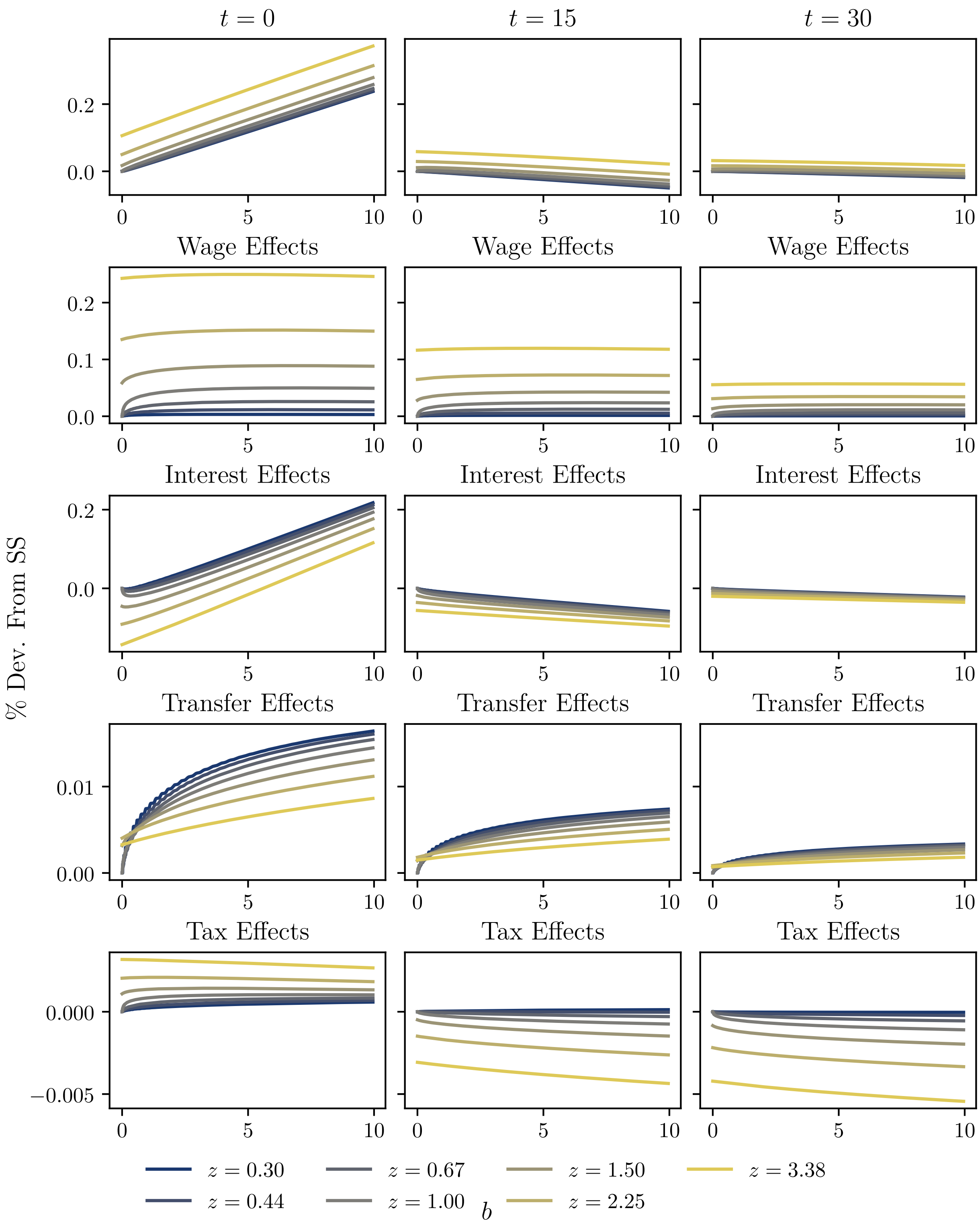


Figure F.6: Consumption Response to a Markup Shock

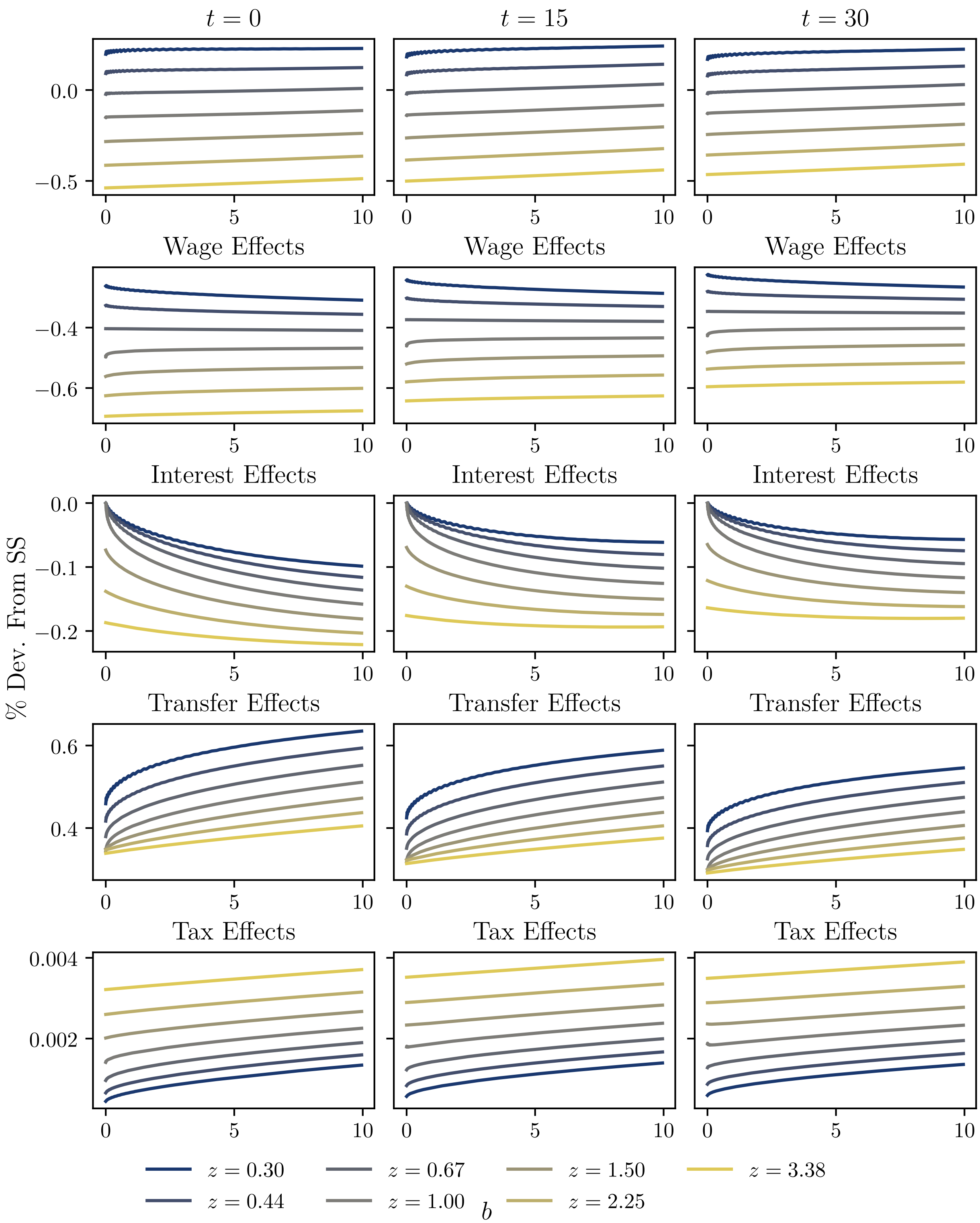


Figure F.7: Labor Supply Response to a Markup Shock

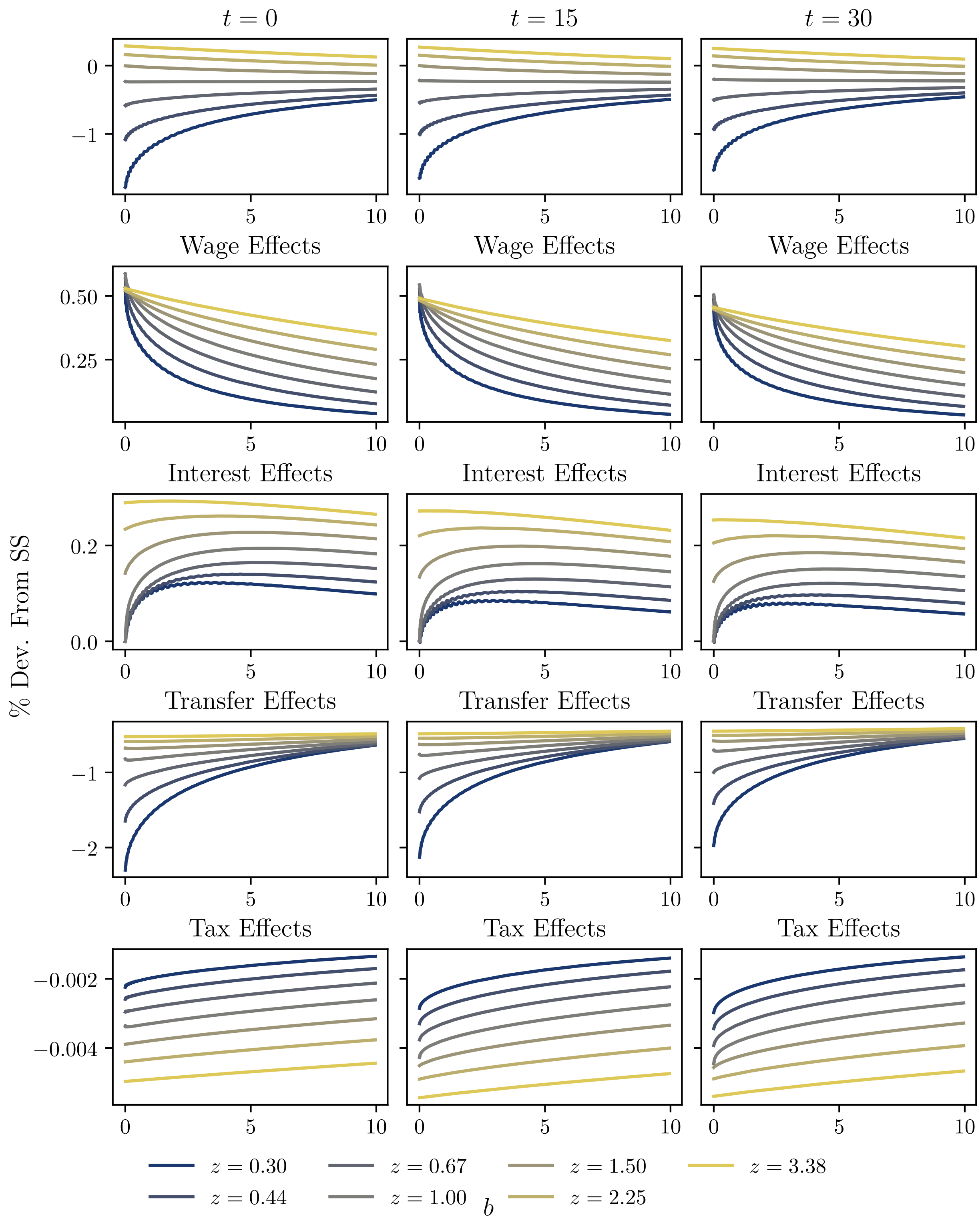


Figure F.8: Savings Response to a Markup Shock

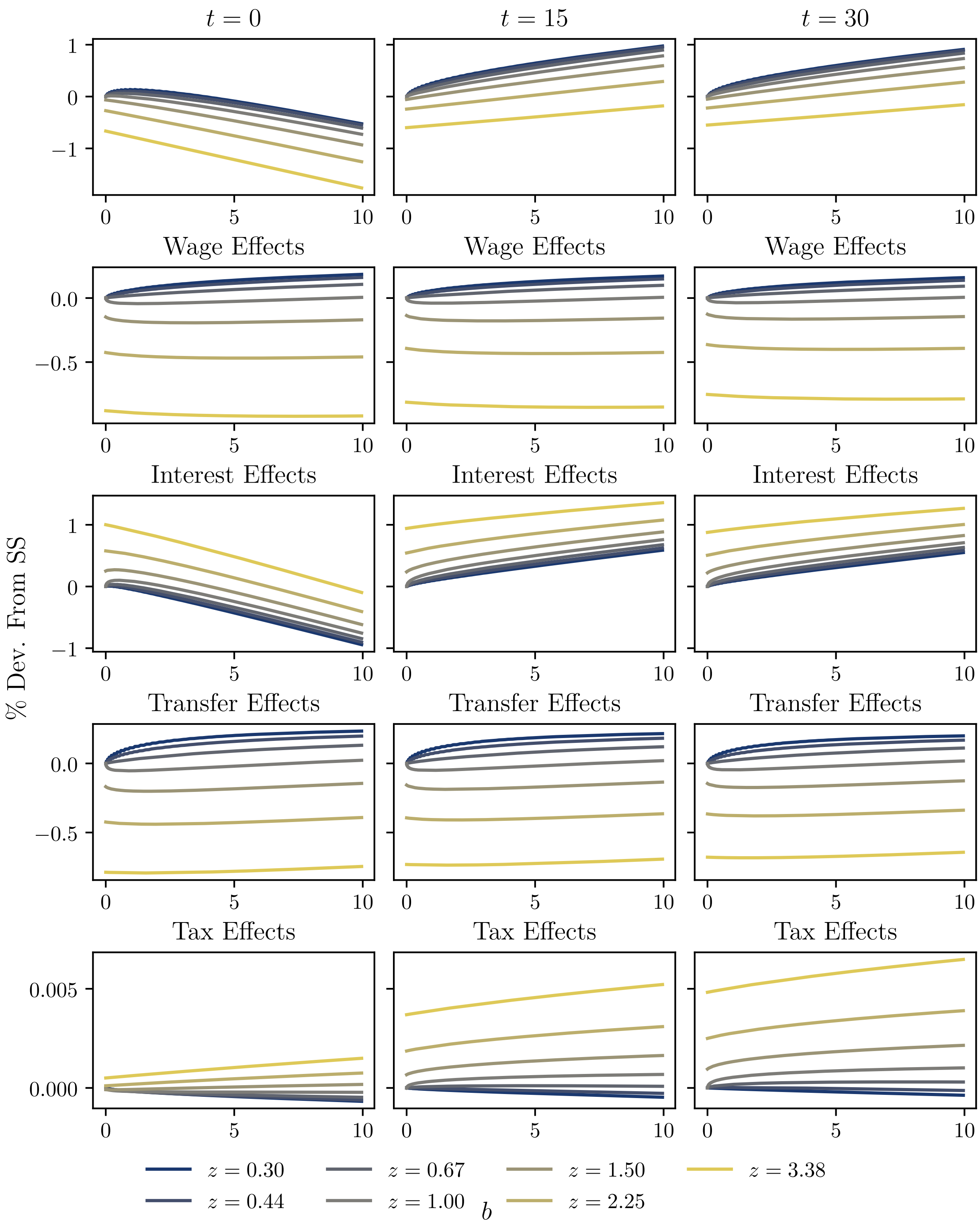


Figure F.9: Consumption Response to a Government Spending Shock

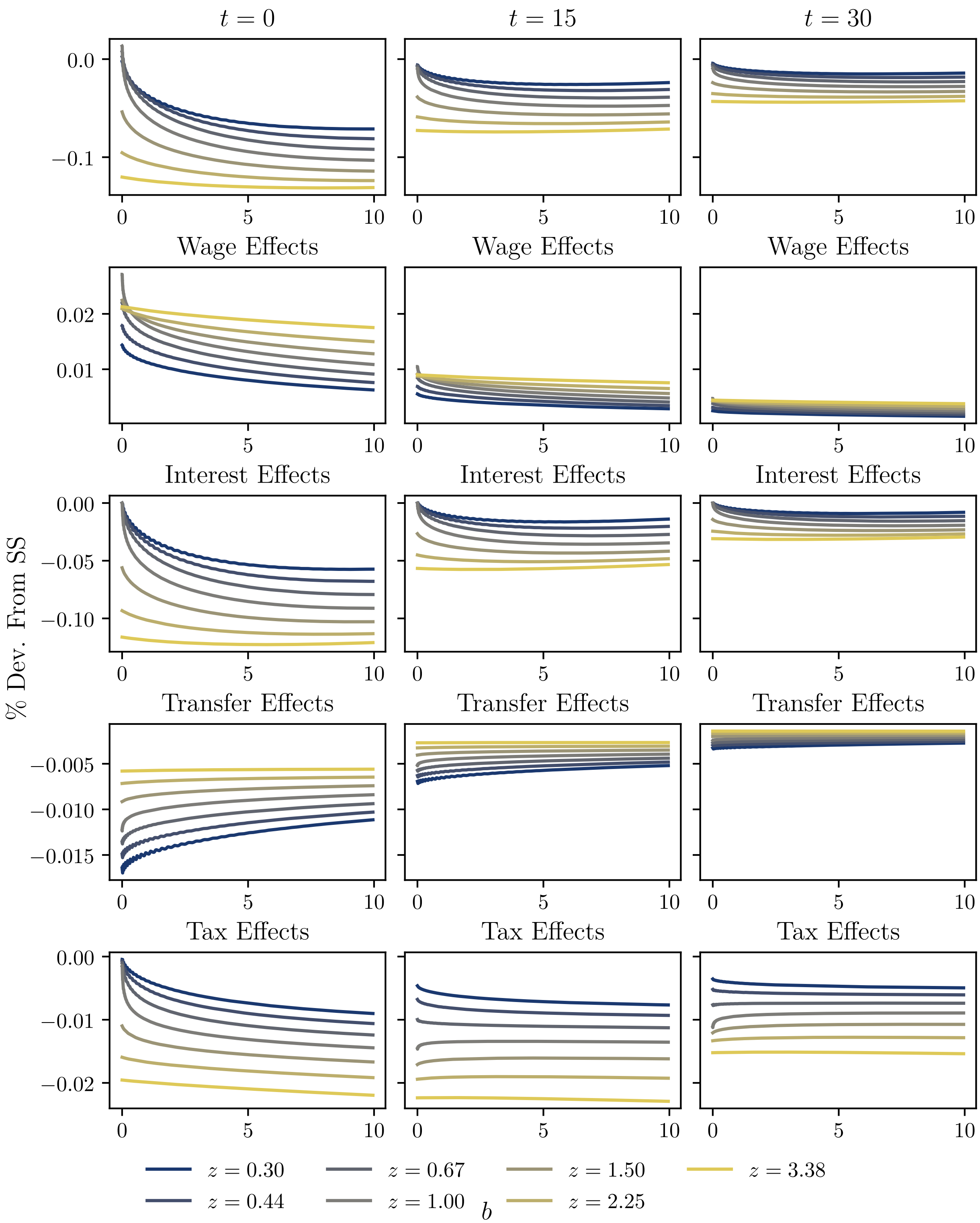


Figure F.10: Labor Supply Response to a Government Spending Shock

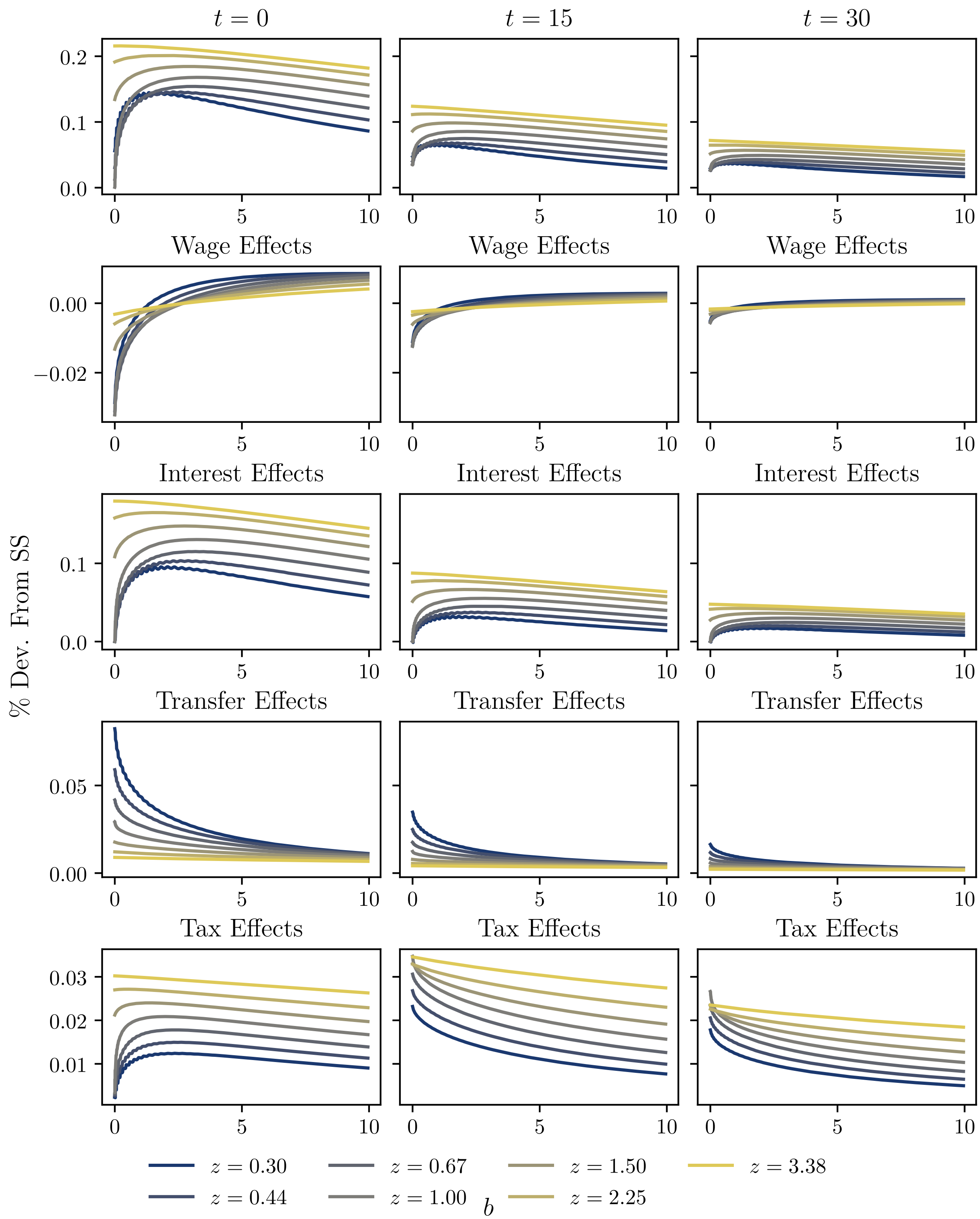


Figure F.11: Savings Response to a Government Spending Shock

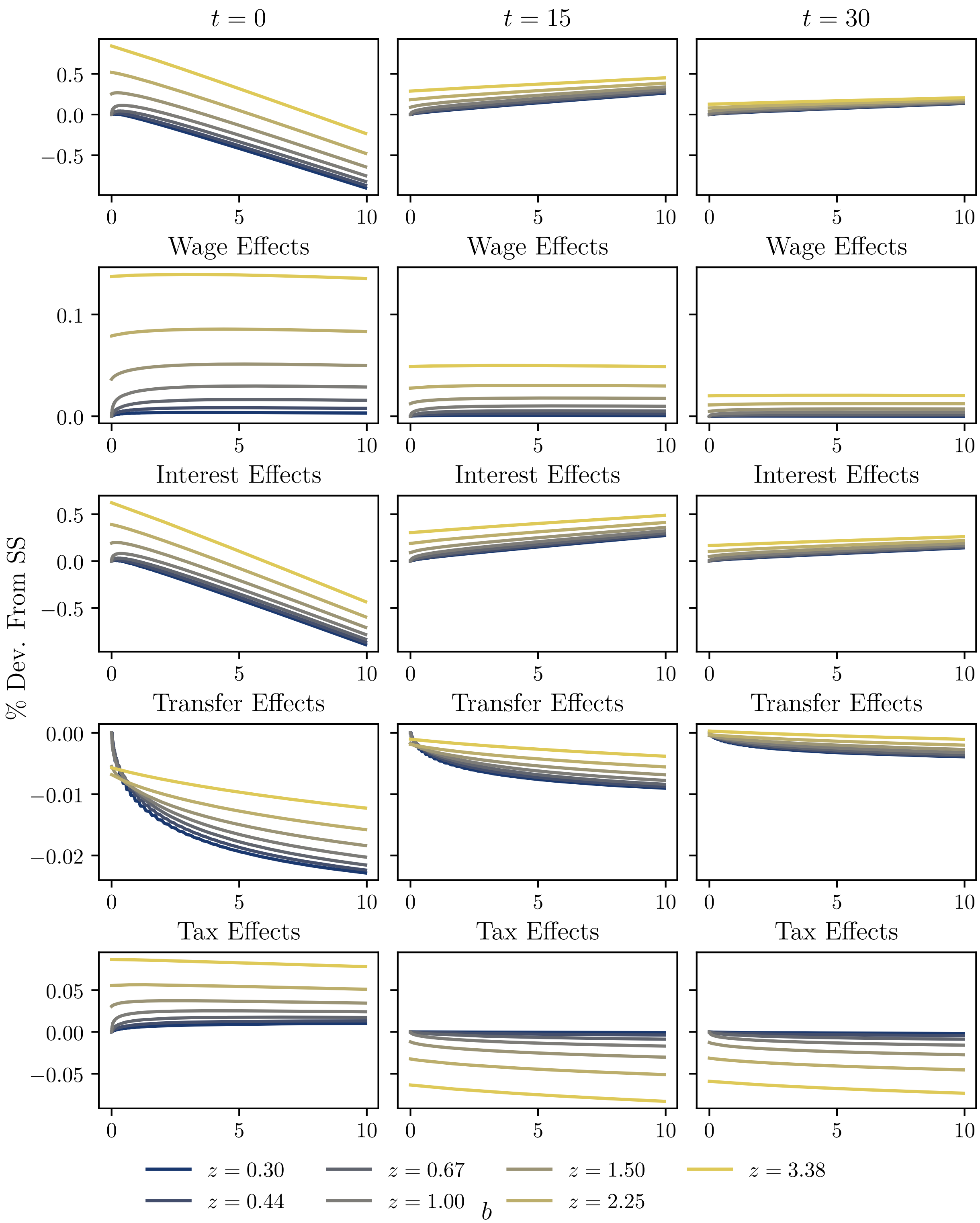


Figure F.12: Consumption Response to a Monetary Policy Shock

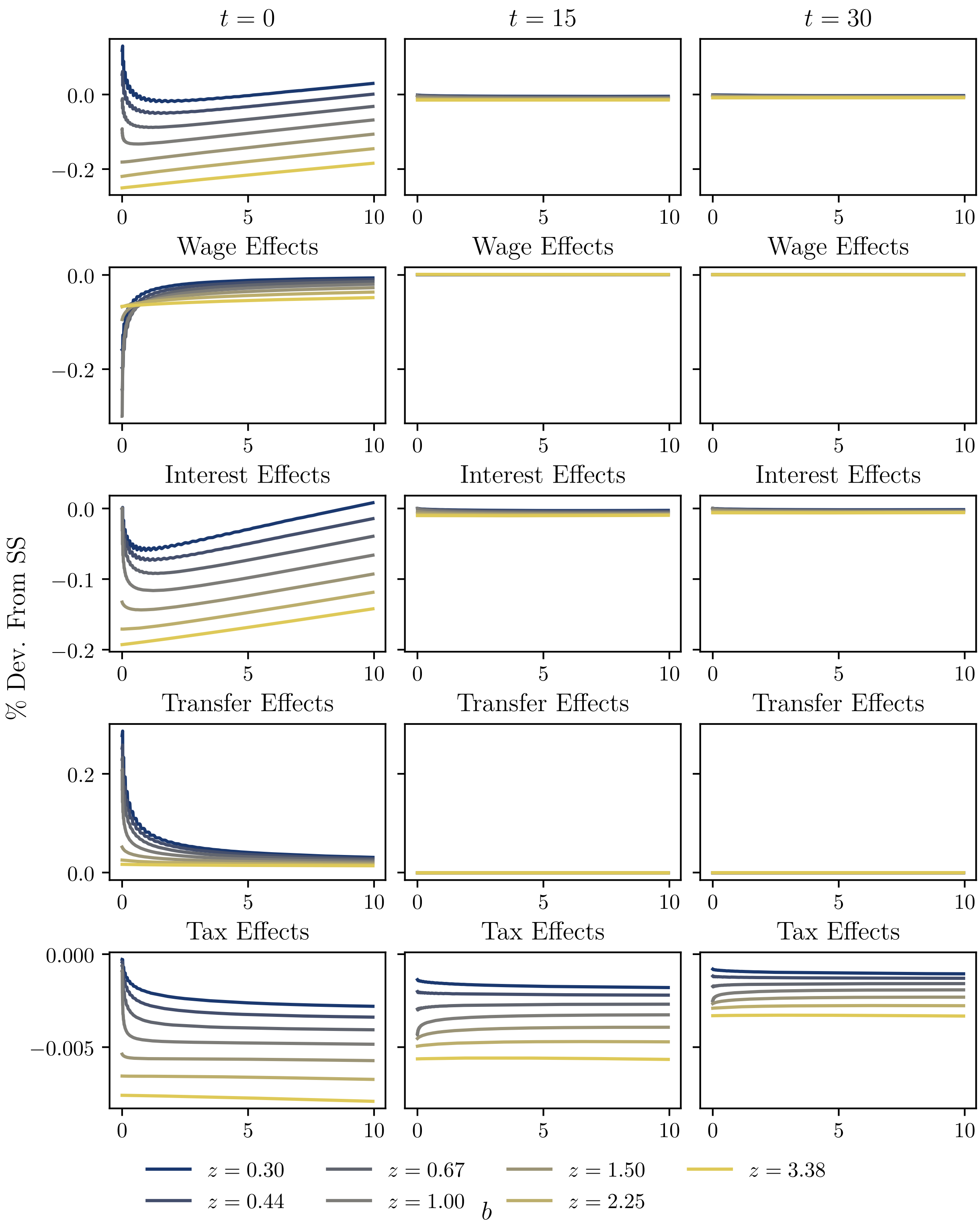


Figure F.13: Labor Supply Response to a Monetary Policy Shock

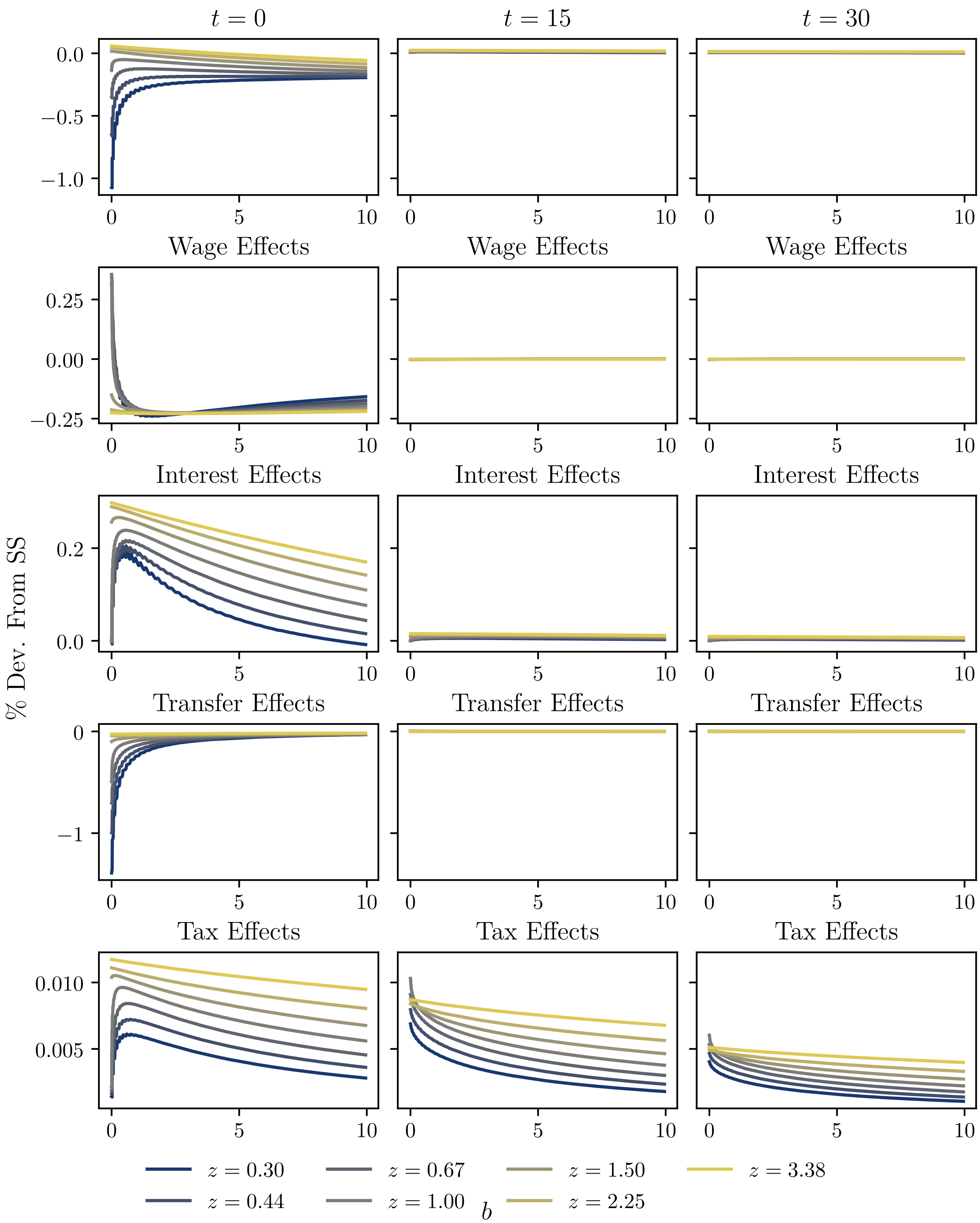


Figure F.14: Savings Response to a Monetary Policy Shock

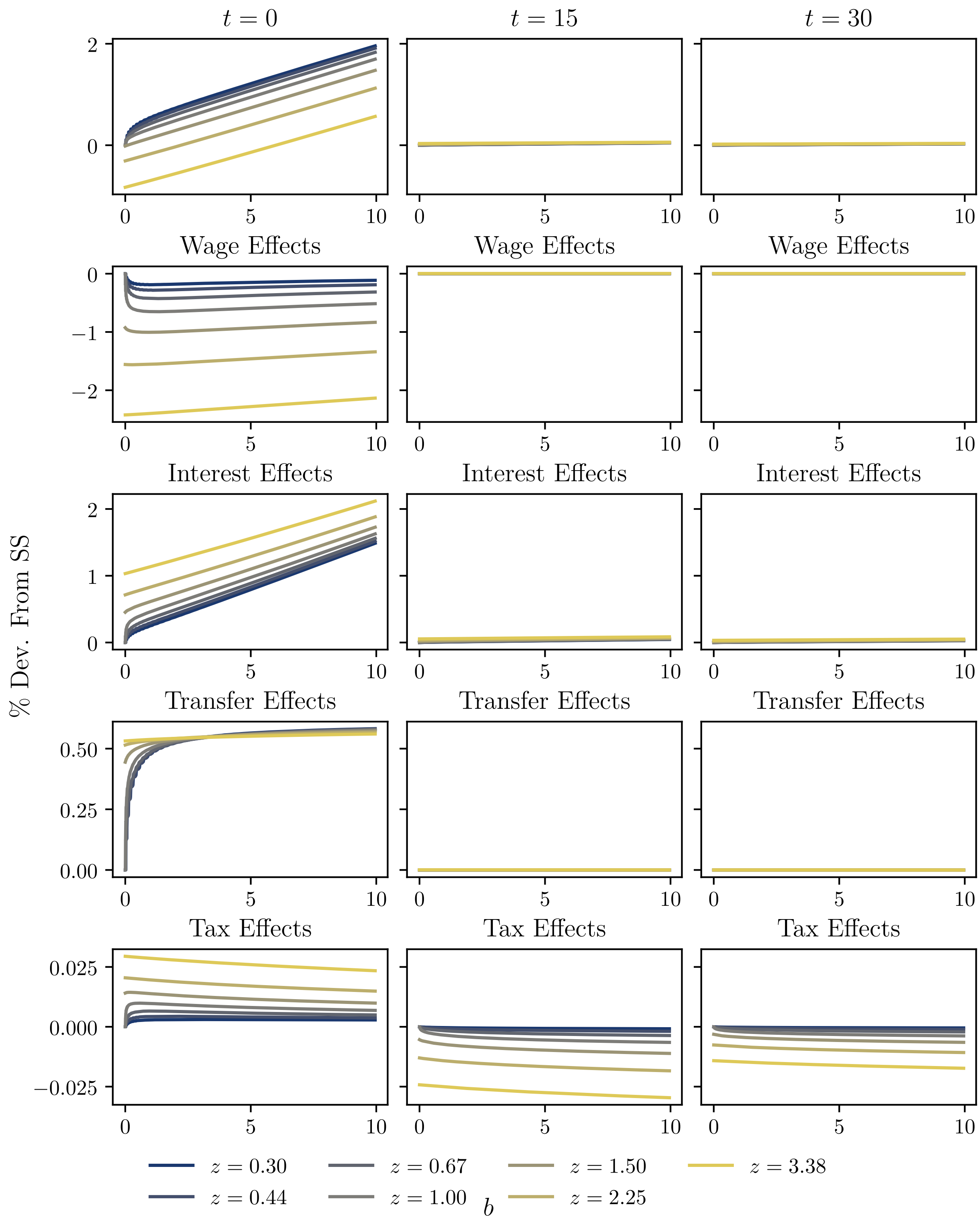


Figure F.15: Consumption Response to a Government Transfer Shock

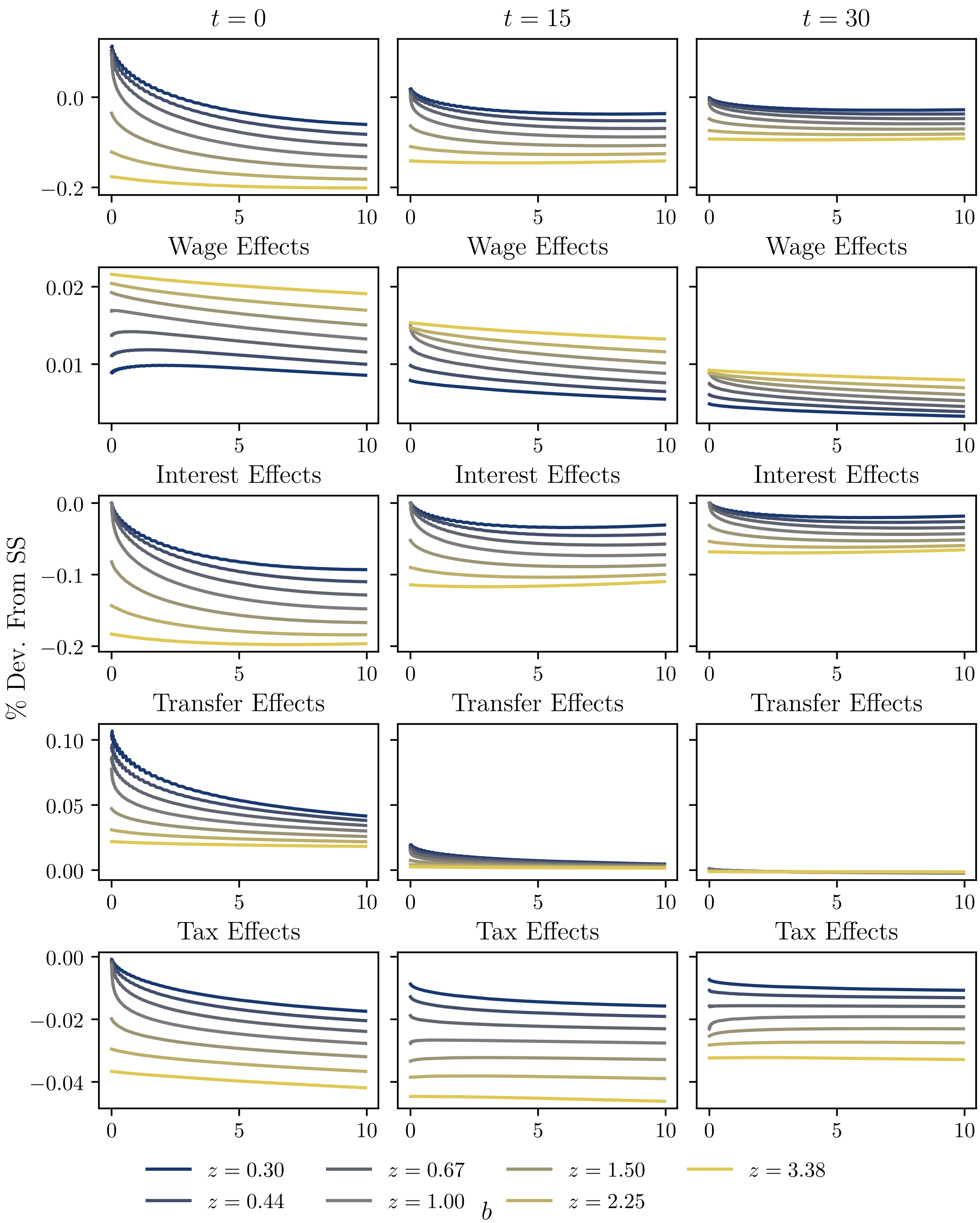


Figure F.16: Labor Supply Response to a Government Transfer Shock

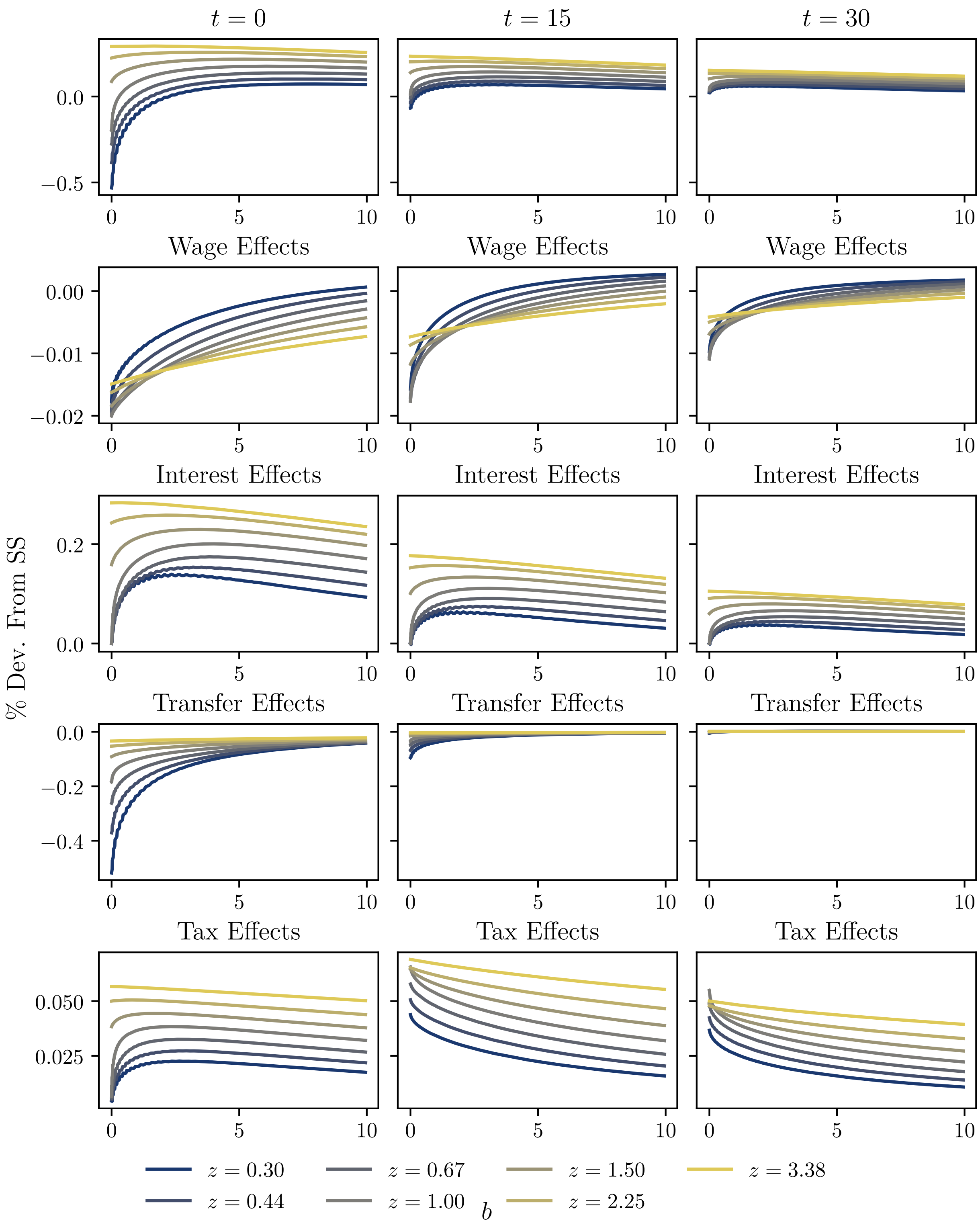


Figure F.17: Savings Response to a Government Transfer Shock

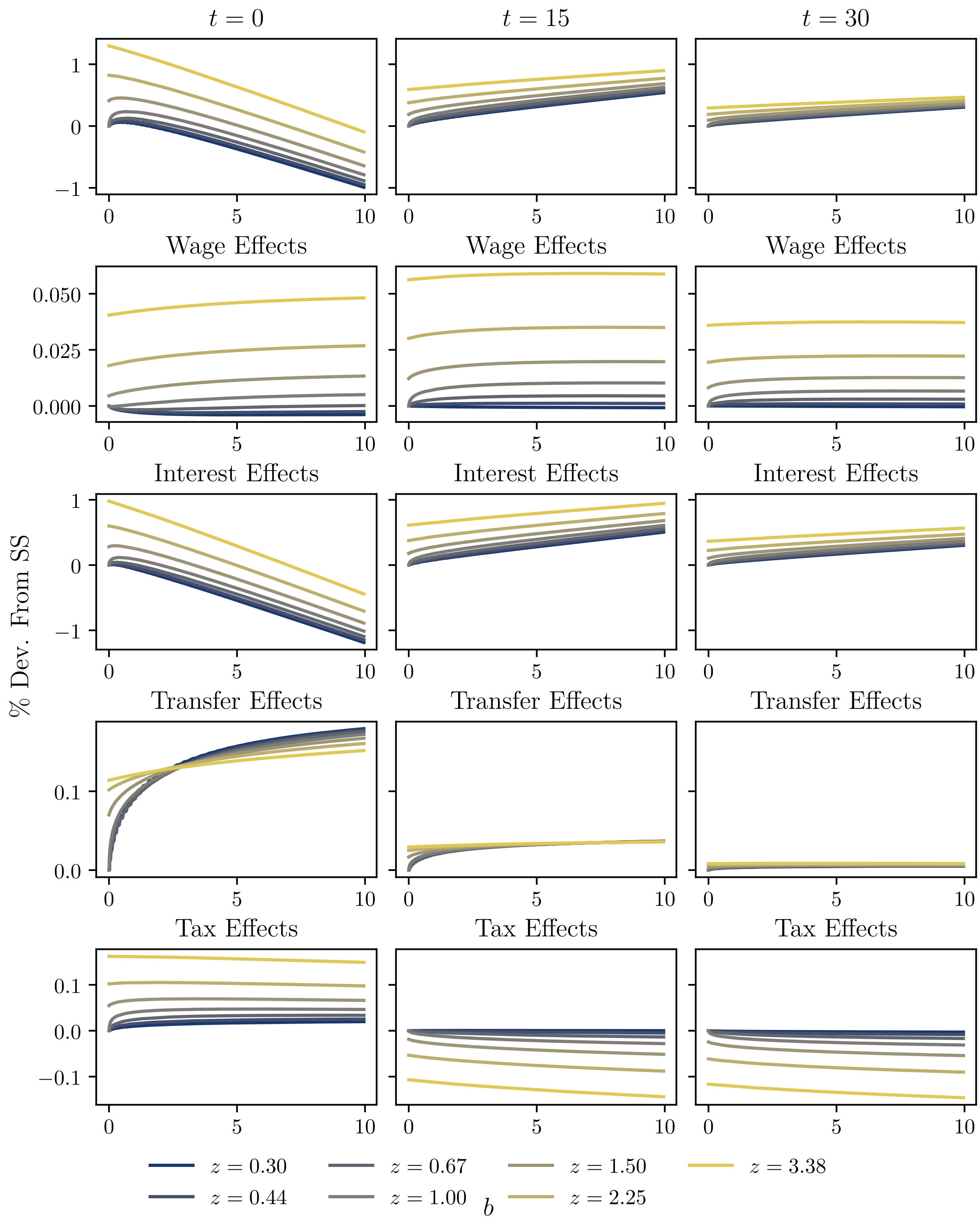


Figure F.18: Consumption Response to a Tax Progressivity Shock

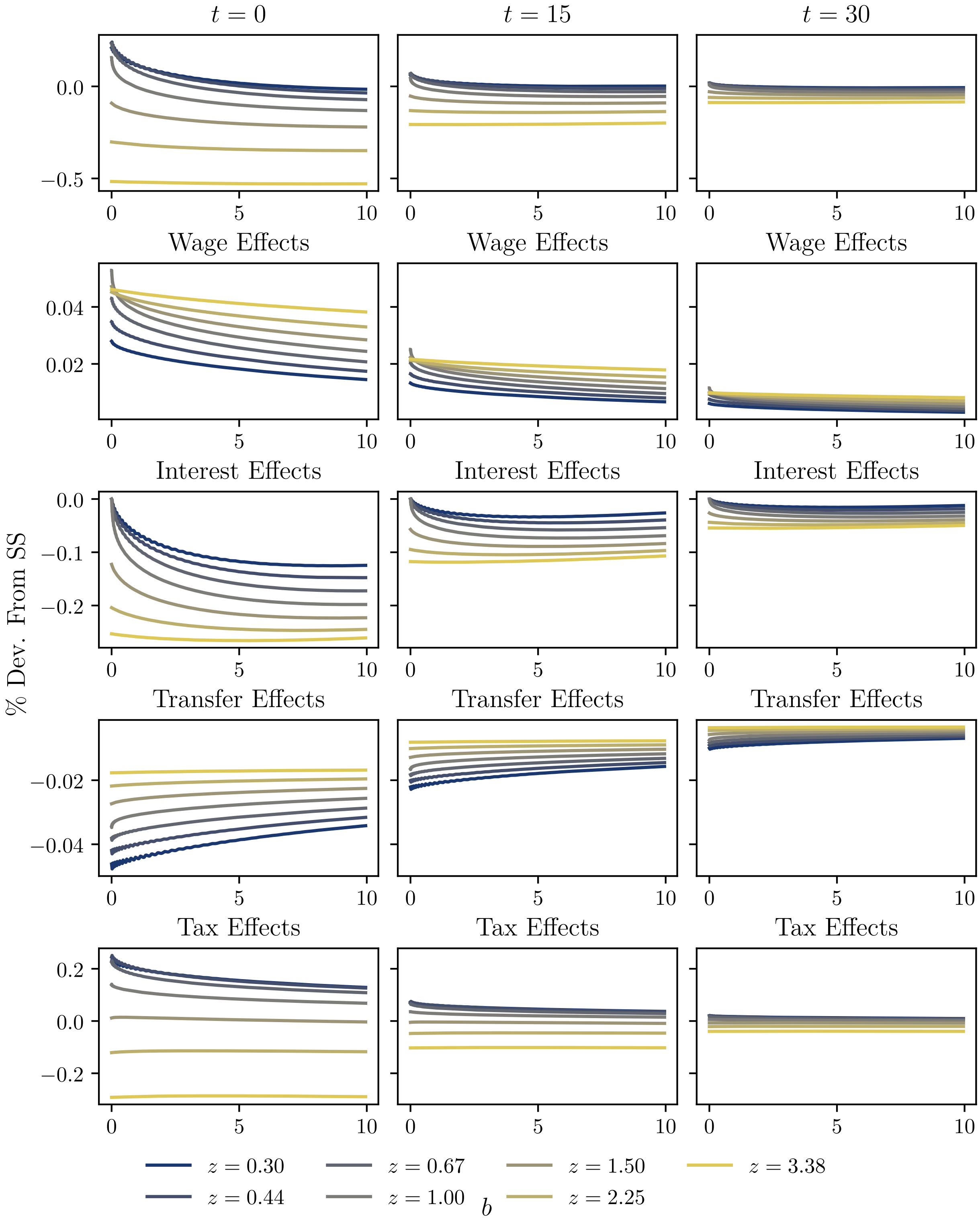


Figure F.19: Labor Supply Response to a Tax Progressivity Shock

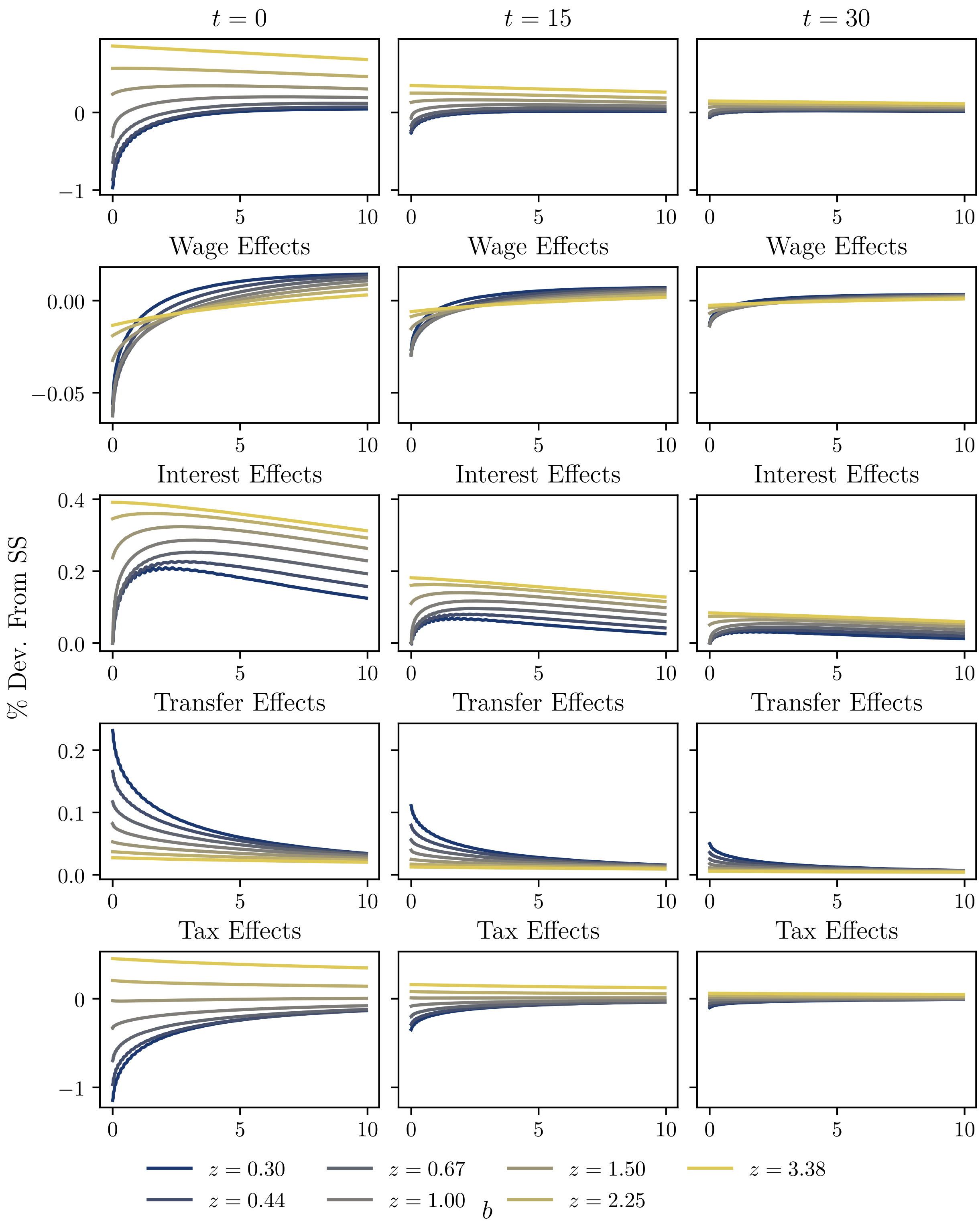


Figure F.20: Savings Response to a Tax Progressivity Shock