Cross-Sectional Household Heterogeneity in Responses to Macroeconomic Shocks

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Abstract

*Replication code available at https://github.com/GavinEngelstad/HANK-Honors.

1 Introduction

2 Literature Review

3 Model

I model a discrete time, one-asset HANK with incomplete markets stemming from uninsurable, idiosyncratic income risks and nominal rigidities. The economy is composed of households, unions, firms and a government. Within the model, there are shocks to total factor productivity (TFP) A_t, price markups ψ_t, wage markups ψ_t^W, government spending g_t, transfers to households η_t, tax progressivity τ_t^P, and monetary policy ξ_t.

The household sector features a continuum of dynamically optimizing heterogeneous households that choose to consume and save. Households earn income from their wages, firm profits, and government transfers. Household productivity levels evolve idiosyncratically over time, which they self-insure against by investing in a risk-free government bond.

The union sector includes a labor packer and a continuum of unions. The labor packer aggregates the labor provided by the unions, which choose a homogenous level of labor to be supplied by households to maximize aggregate utility. Unions are subject to quadratic wage adjustment costs paid in utils following Auclert, Bardóczy, and Rognlie (2023).

The firm sector comprises a representative perfectly competitive final goods firm and a continuum of monopolistically competitive intermediate goods firms. The final goods firm aggregates production from the intermediate goods firms, who produce differentiated goods using labor supplied by unions. Following Rotemberg (1982), intermediate goods firms face quadratic price adjustment costs, creating pricing frictions in the economy.

The government acts as the fiscal and monetary authority. As the fiscal authority, the government supplies a risk-free bond to households, spends endogenously, pays a lump-sum transfer amount to households, and imposes a progressive tax scheme to balance the budget. As the monetary authority, the government sets the interest rate according to a Taylor rule based on the levels of inflation and output.

In this section, I give the assumptions and key equations in the model. For a derivation of the equations and characterization of the model, see Appendix B.

3.1 Households

The model is populated by a unit continuum of infinitely lived households indexed $i \in [0,1]$. Each period, households provide the amount of labor $\ell_{i,t}$ decided by the union and choose to consume $c_{i,t}$ and hold $b_{i,t}$ of a risk-free government bond which has gross real returns R_t to maximize expected discounted utility. Households have constant relative risk aversion (CRRA) preferences given by

$$\max_{\{c_{i,t},\ell_{i,t},b_{i,t}\}_{t=0}^{\infty}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{c_{i,t}^{1-\gamma}}{1-\gamma}-\phi\frac{\ell_{i,t}^{1+\chi}}{1+\chi}\right]$$

where β, γ, ϕ, and χ represent the intertemporal discount rate, risk aversion level, relative disutility of labor, and inverse Frisch elasticity of labor supply.

Household productivity $z_{i,t}$ evolves stochastically over time subject to the log-AR(1) process

$$\log z_{i,t} = \rho_z \log z_{i,t-1} + \epsilon_{z,i,t}, \quad \epsilon_{z,i,t} \sim \mathcal{N}(0,\sigma_z^2)$$

where ρ_z and σ_z^2 represent the persistence and variance of individual productivity shocks. Based on their productivity, labor supply, and the real wage W_t, households generate pre-tax labor income $W_t z_{i,t}\ell_{i,t}$. Additionally, dividends D_t and transfers η_t are evenly distributed across households from the profits of intermediate goods firms and exogenously by the government.

Following McKay, Nakamura, and Steinsson (2016), the government imposes a progressive tax on productivity. Since productivity is exogenous, this acts like a lump sum tax and does not distort household decisions. The tax scheme is given by $\tau_t^L z_{i,t}^{\tau_t^P}$ where τ_t^L and τ_t^P measure the level and progressivity of the tax scheme respectively. Therefore, $\tau_t^P < 1$ creates a regressive tax scheme, $\tau_t^P = 1$ creates a proportional tax scheme, and $\tau_t^P > 1$ creates a progressive tax scheme.

Combined, this gets the household budget constraint

$$b_{i,t} + c_{i,t} = R_t b_{i,t-1} + W_t z_{i,t}\ell_{i,t} + D_t + \eta_t - \tau_t^L z_{i,t}^{\tau_t^P}.$$

Households are also subject to the borrowing constraint $b_{i,t} \geq \underline{b}$ which enforces a no-Ponzi condition for all households.

Because productivity $z_{i,t}$ follows and exogenous law of motion that is time invariant, the distribution of household productivity $\Gamma_t^z(z)$ is also fully exogenous and follows a time invariant process. Assuming the initial distribution for $\Gamma_t^z(z)$ is equal to the ergodic distribution of the AR-1 process, the overall distribution stays constant over time, even as individual households change state within it.

Each period, household's choices depend on their states $z_{i,t}$ and $b_{i,t-1}$ entering the period. Given these states, households follow the decision rules

$$b_t(b_{i,t-1}, z_{i,t}) = b_{i,t}$$

$$c_t(b_{i,t-1}, z_{i,t}) = c_{i,t}$$

Therefore, the distribution of household states $\Gamma_t(b, z)$ evolves according to

$$\Gamma_{t+1}(b', z') = \int_{\{(b,z):b_t(b,z)=b'\}} \Pr(z'|z) d\Gamma_t(b, z)$$

which says the density of households with savings b' and productivity z' is equal to the density of households that choose to save b' times the probability that their productivity ends up z'.

3.2 Unions

Labor is provided by a single labor packer that aggregates labor from a unit continuum of unions indexed $k \in [0, 1]$.

The labor packer aggregates labor supplied by each union $n_{k,t}$ into aggregate labot N_t according to the Dixit-Stiglitz aggregator

$$N_t = \left(\int_0^1 n_{k,t}^{\frac{1}{\psi_t^W}} \right)^{\psi_t^W}$$

where $\dfrac{\psi_t^W}{\psi_t^W - 1}$ represents the elasticity of substitution for labor provided by each union. Profit maximization for the union gets the demand for labor provided by each union k

$$n_{k,t} = N_t \left(\frac{w_{k,t}}{W_t} \right)^{\frac{\psi_t^W}{1-\psi_t^W}}$$

where $w_{k,t}$ is the real wage demanded by union k.

Unions choose a level of labor to demand uniformly from households $\ell_{k,t}$ and aggregates it according to

$$n_{k,t} = \int z \ell_{k,t} d\Gamma_t^z(z).$$

The uniform labor demand assumption follows Auclert, Bardóczy, and Rognlie (2023), and suggests that households supply the same level of labor to the union regardless of their productivity and wealth differences. This ignores household differences in willingness to work and does require that some households are required to work more than they would choose to (Gerke et al. 2024). Alternative approaches would allow unions to vary the quantity of labor demanded or wage for different households, but add substantial mathematical and computational complexity to the model (Gerke et al. 2024).

The union chooses $\ell_{k,t}$ to maximize household utility subject to quadratic adjustment $m_{k,t}^W$ costs

$$m_{k,t}^W = \frac{\psi_t^W}{\psi_t^W - 1} \frac{1}{2\kappa^W} \log \left(\frac{w_{k,t}}{\overline{\pi}^W w_{k,t-1}} \right)^2$$

which is paid in utils where κ^W denotes the responsiveness of wages to economic changes, $\pi_t^W = \dfrac{W_t}{W_{t-1}}$ is wage inflation, and the overline over a variable represents its steady state value. The aggregate utility maximization problem gets the wage Philips curve

$$\log \left(\frac{\pi_t^W}{\overline{\pi}^W} \right) = \kappa^W \left(\phi L_t^{1+\chi} - \frac{1}{\psi_t^W} W_t L_t \int z c_t(b,z)^{-\gamma} d\Gamma_t(b,z) \right) + \beta \log \left(\frac{\pi_{t+1}^W}{\overline{\pi}^W} \right)$$

where L_t is the amount of labor demanded from each household.

3.3 Firms

The model is populated by a representative, competitive final goods firm and a unit continuum of intermediate goods firms indexed $j \in [0, 1]$.

Like the labor packer, the final goods firm aggregates intermediate goods $y_{j,t}$ into output Y_t according to the Dixit-Stiglitz aggregator

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{1}{\psi_t}} \, dj \right)^{\psi_t}$$

where $\dfrac{\psi_t}{\psi_t - 1}$ represents the elasticity of substitution for intermediate goods. Profit maximization for the final goods firm gets the demand for intermediate good j

$$y_{j,t} = Y_t \left(\frac{p_{j,t}}{P_t} \right)^{\frac{\psi}{\psi-1}}$$

where $p_{j,t}$ is the price of intermediate good j and P_t is the overall price level of the economy given by

$$P_t = \left(\int_0^1 p_{j,t}^{\frac{1}{1-\psi_t}} \, dj \right)^{1-\psi_t} .$$

Intermediate goods firms use productive units of labor $n_{j,t}$ to produce their intermediate good according to

$$y_{j,t} = A_t n_{j,t}$$

where A_t represents the overall productivity level of the economy.

Intermediate goods firms also choose prices subject to quadratic adjustment costs $m_{j,t}$ à la Rotemberg (1982) given by

$$m_{j,t} = \frac{\psi_t}{\psi_t - 1} \frac{1}{2\kappa} \log \left(\frac{p_{j,t}}{\overline{\pi} p_{j,t-1}} \right)^2 Y_t$$

where $\pi = \dfrac{P_t}{P_{t-1}}$ is inflation, κ determines the responsiveness of inflation to changes in output, and an overline denotes the steady state of a variable. Compared to the alternative Calvo (1983) rule, the Rotemberg price frictions have a couple advantages. First, price frictions under a Rotemberg rule are more consistent with real data (Richter and Throckmorton 2016). Additionally, a Rotemberg rule has an analytically solvable Philips curve, which makes the model easier to solve. The Philips curve is

$$\log\left(\frac{\pi_t}{\overline{\pi}}\right) = \kappa\left(\frac{W_t}{A_t} - \frac{1}{\psi_t}\right) + R_{t+1}\frac{Y_{t+1}}{Y_t}\log\left(\frac{\pi_{t+1}}{\overline{\pi}}\right).$$

Finally, since intermediate goods firms are monopolistically competitive, they can make a profit. Profits will be paid out in the form of real dividends $d_{j,t}$ such that

$$d_{j,t} = \frac{p_{j,t}}{P_t}y_{i,t} - W_t n_{j,t} - m_{j,t}$$

where firms earn real revenue $\frac{p_{j,t}}{P_t}y_{i,t}$ and pay labor costs $W_t n_{j,t}$ and price adjustment costs. Aggregate dividends D_t are

$$D_t = \int_0^1 d_{j,t}dj.$$

3.4 Government

In the economy, the government acts as both the fiscal and monetary authority.

As the fiscal authority, the government spends an exogenous fraction g_t of output so that government spending G_t follows

$$G_t = g_t Y_t.$$

The government also offers the risk-free bond B_t and pays out transfers to households subject to the law of motion for bonds

$$B_t = \overline{B} + \rho_B\left(R_t B_{t-1} - \overline{RB} + G_t - \overline{G} + \eta_t - \overline{\eta}\right)$$

following Auclert, Rognlie, and Straub (2024) where ρ_B represents how quickly the government pays back non-steady state levels of debt. In the steady state, this means the government holds a constant stock of debt which it pays all the interest on every period. However, increases in transfers η_t, the interest rate R_t, or government spending G_t will be financed by taking on more debt and paying it back over time. To balance the budget, the government sets the tax level τ_L so that

$$R_t B_{t-1} + G_t + \eta_t = \tau_t^L \int z^{\tau_t^P} d\Gamma_t^Z(z) + B_t.$$

As the monetary authority, the government sets the interest rate I_t according to the Taylor Rule

$$I_t = \overline{I} \hat{\pi}_t^{\omega_\pi} \hat{Y}_t^{\omega_Y} \xi_t$$

where ω_π and ω_Y represent the relative importance of inflation and output stabilization and ξ_t is the monetary policy shock. The Fisher relation means

$$R_t = \frac{I_{t-1}}{\pi_t}.$$

3.5 Equilibrium

For the economy to be in equilibrium, the labor, bond, and goods markets all need to clear. Labor market clearing requires unions to provide as much labor as firms demand so that

$$N_t = \int_0^1 n_{j,t} dj.$$

Bond market clearing requires the supply of bonds by the government to equal household savings

$$B_t = \int b_t(b, z) d\Gamma_t(b, z).$$

Finally, goods market clearing requires consumption, government spending, and price adjustment costs to equal output

$$Y_t = \int c_t(b, z) d\Gamma_t(b, z) + M_t + G_t$$

where $M_t = \displaystyle\int_0^1 m_{j,t} dj$.

Therefore, a solution to the model consists of sequences for prices $\{\pi_t, W_t, \pi_t^W, M_t, D_t, R_t, I_t, \tau_t^L\}_{t=0}^{\infty}$, household decision rules $\{b_t, c_t\}_{t=0}^{\infty}$ that solve the household utility maximization problem, the distribution of household states $\{\Gamma_t\}_{t=0}^{\infty}$ that evolves following the policy rules, and macroeconomic aggregates $\{Y_t, N_t, L_t, B_t, G_t\}_{t=0}^{\infty}$ all so that the labor, bond, and goods markets clear subject to exogenous, AR(1) processes for $\{A_t, \psi_t, \psi_t^W, g_t, \xi_t, \tau_t^P, \eta_t\}_{t=0}^{\infty}$.

3.6 Computational Methods

I solve the model in the sequence-space following Auclert et al. (2021). This method has significant computational advantages over standard state-space methods like Reiter (2009) or even dimensionality-reduced state-space methods like Bayer and Luetticke (2018) since it removes household states, of which there can be thousands, from the system used to solve the model.

The first step to solve the model is to find the steady state. I discretize the household asset and productivity levels into a grid. Household transitions between productivity levels are modeled using a Rouwenhorst process (Kopecky and Suen 2010). Following Reiter (2009), I add more asset gridpoints closer to the borrowing constraint $\underline{b}$ to address the nonlinearities in the decision rules near that point. I solve for household decision rules using the endogenous grid method (Carroll 2006). Then, following Young (2010), the distribution Γ_t is represented as a histogram at each of the asset-productivity gridpoints, which households travel between based on the savings decision rule.

Shocks are modeled as linear perturbations around the steady state in the sequence space (Auclert et al. 2021). I use the Python automatic differentiation library Jax to solve for derivatives of the aggregate conditions and the Fake News Algorithm with two-sided numerical differentiation to solve for derivatives of the heterogeneous agent block aggregates (Auclert et al. 2021). To model the effect of shocks on individual policy rules, I use the disaggregated Fake News derivative and aggregate economic conditions to solve for the linearized effect of the shock on households.

The grid dimensions and sequence space truncation horizon are outlined in Table 3.1. In Appendix C, I test the effect of the computational choices on my results, including using a high-dimensional model with substantially more gridpoints and different truncation horizons.

Table 3.1: Computational Parameters

Parameter	**Value**	**Description**
n_b	501	Number of asset gridpoints
$\underline{b}$	0	Borrowing constraint
$\overline{b}$	50	Maximum asset gridpoint
n_z	7	Number of productivity gridpoints
T	300	Sequence space perturbation time horizon

4 Parameterization

I use a two-step procedure to parameterize the model at a quarterly frequency. First, I calibrate the micro-parameters and frictions within the model. Then, I use a Bayesian estimation of the shocks to decompose business cycle dynamics into each different shock channel. This "calibrate then estimate" approach is common in HANK literature since the method reuses the perturbation matrix instead of having to recompute it each draw, which is the most computationally difficult part of the solution process (Winberry 2018; Auclert, Rognlie, and Straub 2020; Auclert et al. 2021; Bayer, Born, and Luetticke 2024). Alternative approaches use parallelized estimation strategies and still can take days to estimate the full set of parameters in the model (Acharya et al. 2023).

4.1 Calibration

The calibrated model parameters are listed in Table 4.1. Risk aversion is set to 4, which is standard in HANK literature (Kaplan, Moll, and Violante 2018). I take Frisch elasticity of 0.5 from (Chetty 2012). Household productivity transitions are based on Storesletten, Telmer, and Yaron (2004) to have persistence 0.963 and cross-sectional standard deviation of 0.5. The slope of the Philips Curve and the Taylor coefficients for inflation and output are set based on standard values in the literature. The government inflation target ensures a 0% inflation steady state. The values for TFP and monetary policy shocks have no effect in the steady state. The price and wage markups are set to give intermediate goods firms and unions a 20% markup in the steady state. Tax progressivity of 1.18 creates a progressive taxation scheme for the economy (Heathcote, Storesletten, and Violante 2017). The government debt persistence parameter is set to match Auclert, Rognlie, and Straub

Table 4.1: Model Parameters

Parameter	Value	Description	Target
Preferences			
β	0.945	Discount rate	2% annual interest rate
γ	4	Risk aversion	Kaplan, Moll, and Violante (2018)
$1/\chi$	1/2	Frisch elasticity	Chetty (2012)
ϕ	3.16	Disutility of labor	$\overline{N} = 1$
$\underline{b}$	0	Borrowing constraint	
Productivity			
ρ_z	0.963	Productivity persistence	Storesletten, Telmer, and Yaron (2004)
σ_z	0.134	Productivity STD	Cross-sectional STD of 0.5
Unions			
κ_W	0.1	Wage Philips Curve	
Firms			
κ	0.1	Philips Curve	
Government			
ρ_B	0.93	Debt persistence	Auclert, Rognlie, and Straub (2024)
$\overline{B}$	0.577	Govt. debt target	57.7% debt to GDP steady state
ω_π	1.5	Taylor inflation	
ω_Y	0	Taylor output	
$\overline{\pi}$	1	Inflation target	0% inflation steady state
Shock SS			
$\overline{A}$	1	TFP	
$\overline{\psi}$	1.2	Markup	20% markup
$\overline{\psi}^W$	1.2	Wage markup	20% markup
$\overline{g}$	0.202	Govt. spending	20.1% govt. spending
$\overline{\eta}$	0.081	Transfers	8.1% transfers
$\overline{\tau}^P$	1.18	Tax progressivity	Heathcote, Storesletten, and Violante (2017)
$\overline{\xi}$	1	Monetary shock	

(2024).

The government debt target, government spending rate, and transfers are calibrated to match historical US averages for debt to GDP, government spending to GDP, and household transfers to GDP between 1966 and 2019. This process is explained in Appendix A.1. The values for the discount rate and the disutility of labor are calibrated within the model to match a 2% annual (0.5% quarterly) interest rate and full employment in the steady state ($\overline{N} = 1$).

4.2 Estimation Strategy

To estimate the shocks to the model, I use the Bayesian estimation procedure from Auclert et al. (2021). This method matches the covariances of endogenous variables with different time offsets in the impulse response functions (IRFs) in the sequence space of the model to their covariances in real data. Similar to other estimations of HANKs, I use a standard random-walk Metropolis-Hastings (RWMH) algorithm with 250,000 draws and a 50,000 draw burn-in (Auclert et al. 2021; Bayer, Born, and Luetticke 2024).

I estimate the persistence and standard deviation for each of the seven shocks on quarterly macroeconomic time-series for GDP, inflation, the federal funds rate, hours worked, consumption, government debt, and wages from 1966 to 2019. For inflation and the interest rate, I estimate on the difference from the mean. For GDP, employment, consumption, debt, and wages, I estimate on the difference from log-linear trend over time. The data series and detrending process are explained further in Appendix A.2. I do not include any microdata in the estimation process, which is a limitation of the paper. However, fitting to distributional microdata generally has a negligible effect on the overall estimates (Bayer, Born, and Luetticke 2024).[1]

I assume weak prior distributions for each of the estimated parameters. The prior for the persistence of each shock is assumed to be a beta distribution with mean 0.5 and standard deviation 0.15. The prior for the standard deviation of each shock is assumed to be an inverse gamma distribution with mean 0.1 and standard deviation 2%.

4.3 Estimation Results

The estimation results are presented in Table 4.2. Impulse response functions for the estimated shocks can be found in Appendix E. I find wage markup shocks are the most persistent, with a ρ of 0.997. Price markup shocks have a ρ of 0.983, making them also very persistent. Shocks to TFP, tax progressivity, household transfers, and government spending are also found to be fairly persistent, with ρ estimates of 0.952, 0.905, 0.851, and 0.856 respectively. Shocks to monetary policy are the least persistent, with a ρ estimate of 0.627. The estimated standard deviation σ is highest for

1. Iao and Selvakumar (2024) finds a smaller error band for estimates using microdata, but the parameter estimates themselves are very similar.

Table 4.2: Estimation Results

Parameter		**Prior**			**Posterior**			
Shock	Statistic	Distribution	Mean	Std. Dev.	Mode	Mean	5%	95%
TFP	ρ	Beta	0.50	0.15	0.952	0.952	0.934	0.969
	σ	Inv. Gamma	0.20	2.00	0.152	0.154	0.142	0.166
Markup	ρ	Beta	0.50	0.15	0.987	0.983	0.970	0.991
	σ	Inv. Gamma	0.20	2.00	0.549	0.558	0.511	0.611
Wage Markup	ρ	Beta	0.50	0.15	0.997	0.997	0.996	0.997
	σ	Inv. Gamma	0.20	2.00	1.761	1.765	1.621	1.921
Govt. Spend	ρ	Beta	0.50	0.15	0.850	0.856	0.807	0.906
	σ	Inv. Gamma	0.20	2.00	0.648	0.856	0.807	0.906
Mon. Pol.	ρ	Beta	0.50	0.15	0.634	0.627	0.574	0.678
	σ	Inv. Gamma	0.20	2.00	0.440	0.444	0.409	0.481
Tax Prog.	ρ	Beta	0.50	0.15	0.914	0.905	0.874	0.934
	σ	Inv. Gamma	0.20	2.00	1.707	1.852	1.512	2.242
Transfers	ρ	Beta	0.50	0.15	0.834	0.851	0.791	0.918
	σ	Inv. Gamma	0.20	2.00	2.460	2.409	2.050	2.784

transfer shocks (2.409%), tax progresivity shocks (1.707%), and wage markup shocks (1.761%). Comparatively, the standard deviations of shocks to government spending, price markups, and monetary policy are found to be small with values of 0.856%, 0.558%, and 0.444%, and 0.132%. TFP has the smallest average shock size with an estimated standard deviation of 0.154%.

These estimates generally line up with both representative agent and HANK literature. My estimated TFP persistence and standard deviation is nearly identical to Bayer, Born, and Luetticke (2024). Similarly, the estimates for government spending and the interest rate mostly line up with Bayer, Born, and Luetticke (2024) and Smets and Wouters (2007). I estimate a similarly sized but slightly more persistent price markup shock than Bayer, Born, and Luetticke (2024). The estimated wage markup shock is both bigger and more persistent than Smets and Wouters (2007) and Bayer, Born, and Luetticke (2024). The differences in markup shocks could be explained by recent trends of increasing markups within the later estimation window I use (De Loecker, Eeckhout, and Unger 2020). My estimated tax progressivity shock is more persistent and larger than that of Bayer, Born, and Luetticke (2024), although they use a different taxation scheme that should expect a different parameter estimate. An estimation of a household transfer shock is, to my knowledge, novel.

The credible intervals for the estimates are high compared to other literature (Smets and Wouters 2007; Bayer, Born, and Luetticke 2024). This is common when estimating a one-asset, as opposed to two-asset, model (Auclert et al. 2021). This does add uncertainty to my analysis, however the parameters are all well identified with means of the RWMH process near the posterior modes and credible intervals that, despite being larger than those in other liturature, are still reasonably narrow. Appendix D features plots of the recursive means (Figure D.1), posterior distributions (Figure D.2), and posterior covariances (Figure D.3) which all suggest good convergence.

5 Business Cycles

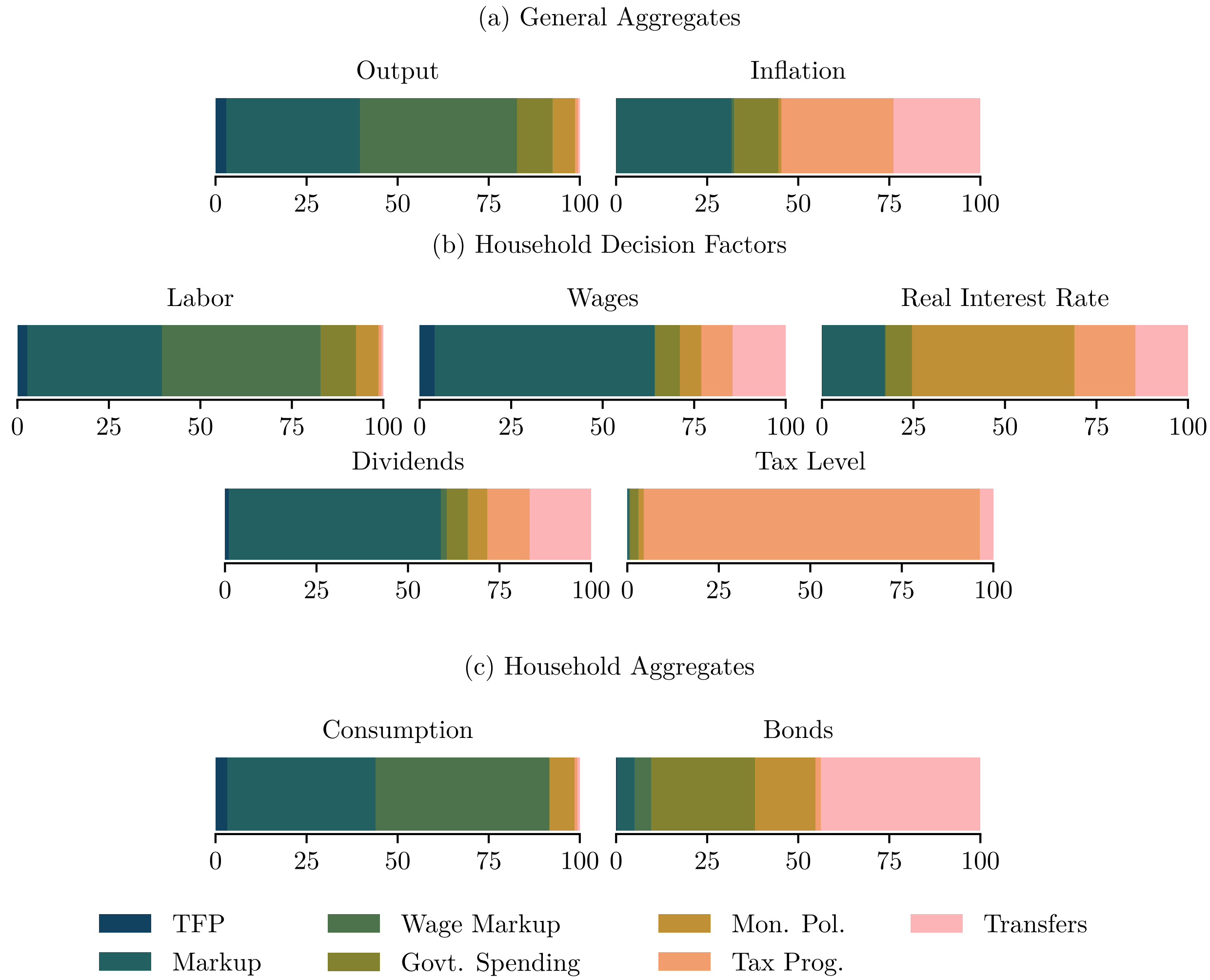
5.1 Aggregate Outcomes

Using the estimated shock parameters, I examine the role of each shock within business cycles. Figure 5.1 features a forecast error variance decomposition for key aggregates in the model.[2] Panel 5.1a describes how output and inflation are affected by different business cycle shocks. Compared to other literature, I find TFP plays a smaller role in my model, especially in explaining output variance (Smets and Wouters 2007; Bayer, Born, and Luetticke 2024). I find that price and wage markups are important business cycle drivers in the model, and play a much larger role explaining output than seen in Bayer, Born, and Luetticke (2024). This difference is likely explained by the lack of a capital sector in the economy causing changes in firm behavior to be explained by price, not production, factors. Consistent with other estimates, supply-side factors (TFP and price-markups) account for about 80% of output volatility, suggesting my estimated TFP, price markup, and wage markup shocks act as general supply side shocks, including the investment shock seen in Bayer, Born, and Luetticke (2024). I find that government spending, tax progressivity, and transfers have an important effect on prices, explaining the majority of inflation, but not production, explaining only a small portion of output.

Panel 5.1b explores how business cycles affect the factors that directly impact household deci-

2. See Appendix F for how the forecast error variance decomposition was calculated.

Figure 5.1: Variance Decomposition: Aggregates



Notes: Forecast error variance decomposition calculated at a 4 quarter time horizon.

sions. Variance in the amount of labor supplied by each household is mostly explained by price and wage markups. Price markups also play an important role determining the wages and dividends paid out to households. Monetary policy shocks explain the largest share of interest rate variance, and tax progressivity shocks explain almost all variance in the tax level. Altogether, TFP and government spending shocks have a relatively small effect on the factors that impact households. Price markups affect households income and labor supply while wage markups only significantly impact the amount of labor households supply. Monetary policy primarily affects the interest rate and tax progressivity primarily affects the tax level. Government transfers have a small but non-negligible effect across the board.

A variance decomposition of household aggregates is presented in Panel 5.1c. Price and wage markups have the largest impacts on consumption, suggesting the union's labor supply choice, wages, and dividends, are important factors to the household's consumption decisions. In contrast, household bond holdings are mostly affected by government spending, monetary policy, and transfers. Since government spending, the interest rate, and transfers play into the government bond law of motion, this suggests household reactions to the supply side of the market is most important explaining variation in aggregate household savings.

5.2 Decision Rules

Next, I examine the business cycle factors that drive changes in household behavior at different points along the income and wealth distribution. I focus on the 10th, 50th, and 90th percentiles on the productivity distribution. Since labor is supplied homogeneously across households, these represent low, middle, and high income households. On the wealth distribution, I look at the 0th, 50th, 90th, and 99th percentiles. Over 40% of the households in the model hold 0 savings, so the decision rules for the 0th percentile apply to a significant number of households. The 50th, 90th and 99th percentiles represent households that have small, medium, and large savings. In my analysis, I analyze decision rules at fixed points on the wealth distribution. This means the analysis does not apply to specific individuals in the model, who can move along the distribution, or account shifts in the distribution changing the threshold for wealth percentiles.

Table 5.1 gives information about the decision rules and income sources at these productivity and wealth levels in the steady state. Both consumption and savings, as expected, are increasing with both productivity and wealth. Consumption increases more between income than wealth shares while savings increase more between wealth than income shares. This suggests higher wealth households exhibit significant consumption smoothing over time attempting to save more to be able to maintain slightly higher levels of consumption, however higher income households count on having a more sustained income to be able to consumption smooth. Despite this, within the model there exists precautionary saving from high income households to protect against idiosyncratic

Table 5.1: Household Steady State Behavior

	Low Income (10%)				Middle Income (50%)				High Income (90%)			
	0%	50%	90%	99%	0%	50%	90%	99%	0%	50%	90%	99%
States												
Productivity	0.444	0.444	0.444	0.444	1.000	1.000	1.000	1.000	2.252	2.252	2.252	2.252
Assets	0.000	0.040	1.874	5.896	0.000	0.040	1.874	5.896	0.000	0.040	1.874	5.896
Decisions												
Consumption	0.483	0.507	0.696	0.896	0.744	0.754	0.898	1.079	1.109	1.111	1.200	1.352
Savings	0.000	0.016	1.670	5.512	0.000	0.030	1.730	5.591	0.173	0.211	1.965	5.855
Income												
Wages	0.327	0.327	0.327	0.327	0.737	0.737	0.737	0.737	1.660	1.660	1.660	1.660
	(67.79)	(62.60)	(13.83)	(5.11)	(99.00)	(93.95)	(28.05)	(11.05)	(129.54)	(125.62)	(52.45)	(23.03)
Interest	0.000	0.040	1.883	5.926	0.000	0.040	1.883	5.926	0.000	0.040	1.883	5.926
	(0.00)	(7.66)	(79.60)	(92.47)	(0.00)	(5.10)	(71.67)	(88.83)	(0.00)	(3.03)	(59.51)	(82.22)
Transfers	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248	0.248
	(51.30)	(47.37)	(10.47)	(3.86)	(33.27)	(31.57)	(9.42)	(3.71)	(19.33)	(18.74)	(7.83)	(3.44)
Taxes	-0.092	-0.092	-0.092	-0.092	-0.240	-0.240	-0.240	-0.240	-0.626	-0.626	-0.626	-0.626
	(19.09)	(17.63)	(3.89)	(1.44)	(32.27)	(30.62)	(9.14)	(3.60)	(48.86)	(47.38)	(19.79)	(8.69)
Total	0.483	0.523	2.366	6.408	0.744	0.785	2.628	6.670	1.282	1.326	3.165	7.207
	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)	(100.00)

Notes: After-tax income share in parenthesis. Column percentiles correspond to the 0th, 50th, 90th, and 99th wealth percentiles.

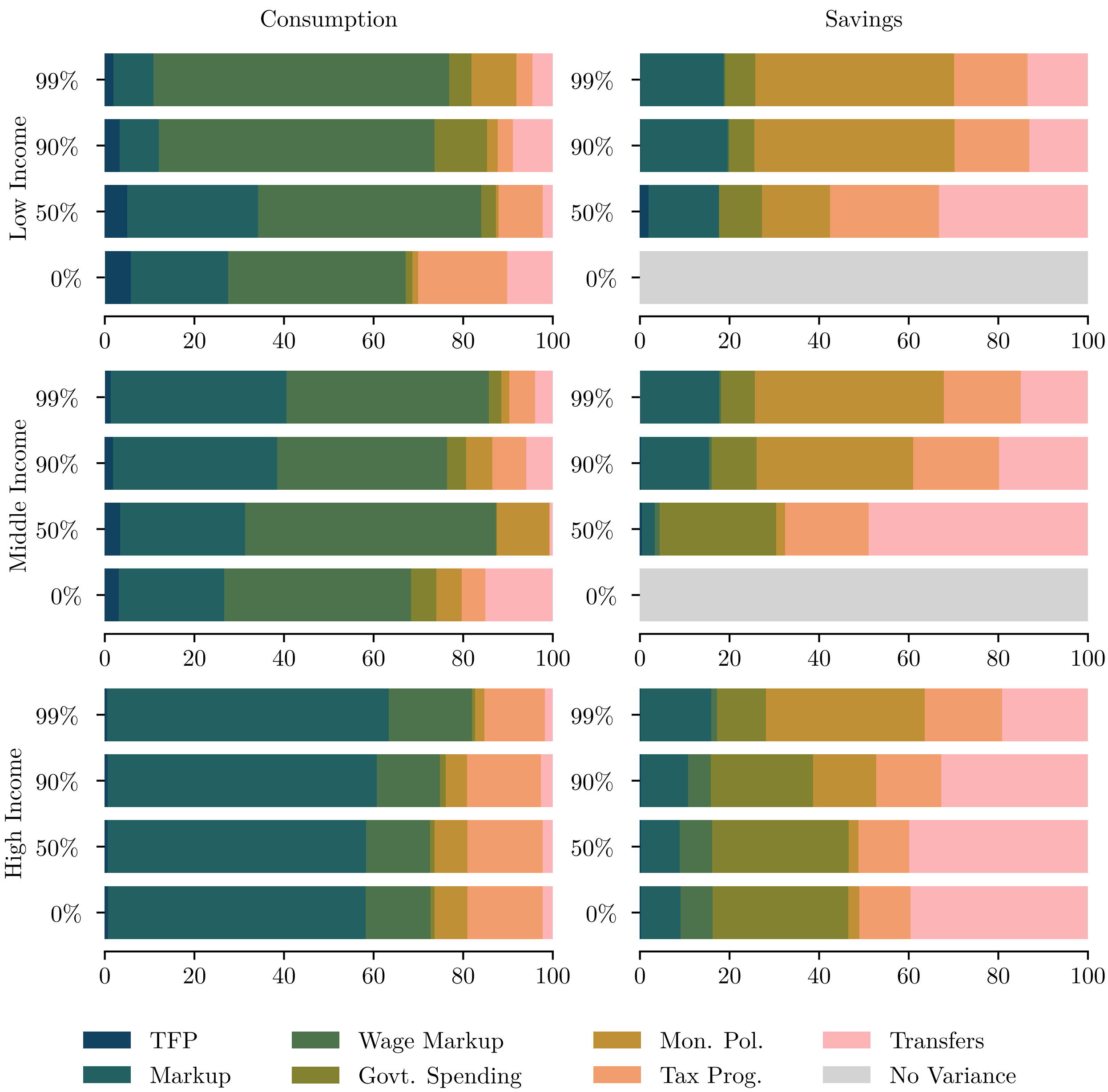
income shocks since, unlike low and middle income households, high income households at all but the highest wealth levels are net savers and end the period with more saved than they start with.

Households at different points along the income and wealth distribution get their budget each period from different sources. For low income households, transfers and, at higher wealth levels, interest rates contribute significantly to their budget. In contrast, higher income households get a significantly larger share from their wages. Because of the progressive tax structure, higher income households contribute a significantly higher share of their income to taxes. For a more detailed breakdown, see Appendix G for surface plots of household decision rules (Figure G.1) and income sources (Figure G.2) at all wealth and productivity levels.

Within business cycles, the shock determinants of household behavior exhibit significant variability across the wealth and income distribution. Figure 5.2 gives a forecast error variance decomposition for household decision rules for consumption and savings at different productivity and wealth levels. Appendix G also features IRFs for each decision rule in response to each shock at all income and wealth levels.

In Figure 5.2, changes in consumption for low income households is best explained by wage markup shocks while changes in consumption for high income households are best explained by

Figure 5.2: Variance Decomposition: Household Decision Rules



Notes: Forecast error variance decomposition calculated at a 4 quarter time horizon. Subplot y-axis is the household position on the wealth distribution.

price markup shocks. Given the decompositions for household decision determinants in Figure 5.1c, low income households are highly responsive to changes in the labor supply decided by the unions while higher income households respond more to changes in the level of wages and dividends. Interestingly, tax progressivity and transfers have more significant effects on household consumption at all wealth and income levels than on aggregate consumption. This suggests the consumption effects of tax progressivity and transfers for different households cancel each other out, making the shocks less important in aggregate. This emphasizes the heterogeneous effects that macroeconomic shocks can have on different households and the importance of understanding the disaggregated effects of macroeconomic events.

The variance decompositions of household savings decisions also vary across the wealth and income distributions. Low and middle income households that hold no wealth never choose to save, so they have no variance explained within the model. Monetary policy shocks effect already high wealth households far more than any other group, suggesting the effect of interest rate hikes can be explained more through income effects, which only affect households that already hold wealth, than households choosing to save because of the higher potential gains, which would affect all households. Government spending and transfer shocks explain the most variance for high income households, suggesting increases in government spending and transfers, which the government funds through offering more bonds to households, are primarily funded by higher income households, though transfers are also important to median wealth, middle and low income households. Again, tax progressivity shocks play a more significant role explaining individual savings than aggregate savings, meaning households responses to the shocks must cancel each other out.

Similar to the steady states in Table 5.1, variance decompositions of household consumption vary more between income levels than wealth levels while variance decompositions for household savings vary more between wealth levels than income levels. This means heterogeneity in household decisions depends on both the distribution of households states and the decision being made, suggesting more complex models where households make more decisions could exhibit more complex types of heterogeneity.

6 Endogenous Effects

Except for tax progressivity and government transfers, household behavior within the model is not a direct response to shocks, but rather a response to the macroeconomic consequences of the shock. In this section, I decompose the variance in household decisions into the different direct channels that affect household decisions. Within the framework of my estimated HANK, this analysis pinpoints the most important macroeconomic factors for different households.

6.1 Direct Effects Decomposition

I expand the direct-indirect decomposition for monetary policy shocks from Kaplan, Moll, and Violante (2018) to the full set of shocks and direct household effects within my model. Household conditions depend on the union's labor supply choice L, wages W, the interest rate R, dividends D, household transfers η, the tax level τ^L, and tax progressivity τ^P. Therefore, I can decompose the vector $d\mathbf{C}$ representing the linearized impulse response function (IRF) for consumption as

$$d\mathbf{C} = \frac{\partial \mathbf{C}}{\partial \mathbf{L}} d\mathbf{L} + \frac{\partial \mathbf{C}}{\partial \mathbf{W}} d\mathbf{W} + \frac{\partial \mathbf{C}}{\partial \mathbf{R}} d\mathbf{R} + \frac{\partial \mathbf{C}}{\partial \mathbf{D}} d\mathbf{D} + \frac{\partial \mathbf{C}}{\partial \boldsymbol{\eta}} d\boldsymbol{\eta} + \frac{\partial \mathbf{C}}{\partial \boldsymbol{\tau}^P} d\boldsymbol{\tau}^P + \frac{\partial \mathbf{C}}{\partial \boldsymbol{\tau}^L} d\boldsymbol{\tau}^L$$

where $\dfrac{\partial \mathbf{C}}{\partial \mathbf{X}}$ is the direct effect of X on consumption and $d\mathbf{X}$ is the IRF for X. For my analysis, I combine dividends and direct transfers since both are evenly distributed transfers to all households. I also combine the tax level and tax progressivity since tax level variation is almost entirely explained by changes in tax progressivity (Figure 5.1). Denoting transfers T and taxes τ, this means

$$d\mathbf{C} = \underbrace{\frac{\partial \mathbf{C}}{\partial \mathbf{L}} d\mathbf{L}}_{\text{Labor effects}} + \underbrace{\frac{\partial \mathbf{C}}{\partial \mathbf{W}} d\mathbf{W}}_{\text{Wage effects}} + \underbrace{\frac{\partial \mathbf{C}}{\partial \mathbf{R}} d\mathbf{R}}_{\text{Interest effects}} + \underbrace{\frac{\partial \mathbf{C}}{\partial \mathbf{T}} d\mathbf{T}}_{\text{Transfer effects}} + \underbrace{\frac{\partial \mathbf{C}}{\partial \boldsymbol{\tau}} d\boldsymbol{\tau}}_{\text{Tax effects}} .$$

Then, denoting $\dfrac{\partial \mathbf{C}}{\partial \mathbf{X}} d\mathbf{X}$ as $d\mathbf{C}_X$, variance in consumption within the model can be explained as

$$\begin{aligned}
\mathrm{Var}\left(d\mathbf{C}\right) = {} & \mathrm{Var}\left(d\mathbf{C}_L\right) + \mathrm{Var}\left(d\mathbf{C}_W\right) + \mathrm{Var}\left(d\mathbf{C}_R\right) + \mathrm{Var}\left(d\mathbf{C}_T\right) + \mathrm{Var}\left(d\mathbf{C}_\tau\right) \\
& + 2\mathrm{Cov}\left(d\mathbf{C}_L, d\mathbf{C}_W\right) + 2\mathrm{Cov}\left(d\mathbf{C}_L, d\mathbf{C}_R\right) + 2\mathrm{Cov}\left(d\mathbf{C}_L, d\mathbf{C}_T\right) + 2\mathrm{Cov}\left(d\mathbf{C}_L, d\mathbf{C}_\tau\right) \\
& + 2\mathrm{Cov}\left(d\mathbf{C}_W, d\mathbf{C}_R\right) + 2\mathrm{Cov}\left(d\mathbf{C}_W, d\mathbf{C}_T\right) + 2\mathrm{Cov}\left(d\mathbf{C}_W, d\mathbf{C}_\tau\right) \\
& + 2\mathrm{Cov}\left(d\mathbf{C}_R, d\mathbf{C}_T\right) + 2\mathrm{Cov}\left(d\mathbf{C}_R, d\mathbf{C}_\tau\right) \\
& + 2\mathrm{Cov}\left(d\mathbf{C}_T, d\mathbf{C}_\tau\right).
\end{aligned}$$

Unlike the variance decompositions in Section 5, this features covariance terms. This is because the shocks to the model decomposed in Section 5 are assumed to be orthogonal to each other, while direct effects within the model are not. Positive covariance within the decomposition implies comovement between the decomposed effects that complement each other. Negative covariance implies comovement between decomposed effects that, in part, cancel each other out. Substituting in any other household variable for consumption, including the policy rules at specific states, results in an identical decomposition.

6.2 Decomposition Results

I perform this decomposition on aggregates and decision rules for consumption and savings. Like in section 5.2, I decompose decision rules at the 10th, 50th, and 90th productivity percentiles and the 0th, 50th, 90th, and 99th wealth percentiles. Appendix G decomposes the IRFs for household behavior into each direct channel.

Table 6.1 presents the decomposition results for consumption. For low income households, changes in direct transfers and labor supply are extremely impactful on their consumption decisions, while for higher income households these factors, especially transfers, are unimportant. Instead, interest rates are, compared to other factors, more important for higher income households than lower and middle income households. The variance decomposition for aggregate consumption is very similar to that of middle income, middle wealth households, suggesting that median households tend to act similarly to aggregate consumption.

There are large, negative covariances for many factors affecting the consumption of low income, and to a lesser extent middle income, households. This means that the macroeconomic effects of business cycle shocks push households in conflicting ways. In contrast, higher income households have negligible negative covariances, meaning business cycle induced macroeconomic movements homogeneously push these households to either consume more or less.

The decomposition for household savings decisions is presented in Table 6.2. Consistent with the business cycle decomposition in Figure 5.2, low and middle income households at the 0th wealth percentile have no variance in their savings decisions. For all other households and in aggregate, the interest rate is by far the most important factor in determining household savings. Other

Table 6.1: Direct Effects Decomposition: Consumption

	Total	**Low Income**				**Middle Income**				**High Income**			
		0th	50th	90th	99th	0th	50th	90th	99th	0th	50th	90th	99th
Variance Components $\times 100$													
Var(L)	0.57	0.14	0.13	0.14	0.17	0.71	0.52	0.43	0.45	1.25	1.24	1.15	1.12
	(70.8)	(130.0)	(1,048.1)	(66.8)	(135.2)	(912.6)	(77.4)	(24.8)	(39.2)	(13.5)	(13.4)	(13.8)	(19.6)
Var(W)	0.00	0.00	0.00	0.00	0.02	0.01	0.00	0.01	0.03	0.02	0.02	0.05	0.07
	(0.0)	(2.6)	(22.8)	(1.7)	(12.4)	(18.2)	(0.7)	(0.8)	(3.1)	(0.3)	(0.3)	(0.6)	(1.3)
Var(R)	0.25	0.00	0.02	0.20	0.10	0.00	0.24	0.75	0.35	2.62	2.63	2.30	1.13
	(31.6)	(0.0)	(150.7)	(94.3)	(78.6)	(0.0)	(36.2)	(43.0)	(30.5)	(28.3)	(28.4)	(27.6)	(19.7)
Var(T)	0.09	0.08	0.06	0.09	0.13	0.08	0.06	0.11	0.14	0.12	0.12	0.15	0.15
	(11.3)	(77.6)	(460.0)	(41.9)	(98.2)	(107.3)	(8.2)	(6.3)	(12.2)	(1.3)	(1.3)	(1.7)	(2.7)
Var(τ)	0.00	0.13	0.10	0.01	0.00	0.04	0.01	0.00	0.00	0.26	0.26	0.22	0.19
	(0.6)	(120.7)	(769.9)	(6.0)	(2.7)	(54.7)	(1.7)	(0.1)	(0.3)	(2.9)	(2.8)	(2.7)	(3.2)
Covariance Components $\times 100$													
Cov(L, W)	0.00	-0.02	-0.02	0.02	0.05	-0.10	-0.05	0.08	0.12	0.17	0.17	0.24	0.29
	(0.1)	(-17.5)	(-153.8)	(10.2)	(40.8)	(-123.0)	(-6.8)	(4.3)	(10.9)	(1.8)	(1.9)	(2.8)	(5.0)
Cov(L, R)	0.36	0.00	0.05	0.15	0.03	0.00	0.35	0.55	0.33	1.80	1.80	1.61	1.06
	(44.9)	(0.0)	(382.5)	(71.5)	(25.1)	(0.0)	(52.3)	(31.6)	(28.8)	(19.4)	(19.4)	(19.2)	(18.6)
Cov(L, T)	-0.23	-0.11	-0.08	-0.11	-0.15	-0.24	-0.17	-0.22	-0.25	-0.39	-0.39	-0.41	-0.42
	(-28.2)	(-98.3)	(-686.4)	(-52.9)	(-115.2)	(-306.4)	(-25.2)	(-12.5)	(-21.9)	(-4.2)	(-4.2)	(-4.9)	(-7.3)
Cov(L, τ)	-0.03	-0.13	-0.11	-0.04	-0.02	-0.16	-0.07	0.02	0.03	0.57	0.57	0.51	0.46
	(-3.8)	(-124.5)	(-890.2)	(-19.5)	(-18.4)	(-204.8)	(-9.7)	(1.1)	(3.0)	(6.2)	(6.2)	(6.0)	(8.0)
Cov(W, R)	0.00	0.00	-0.01	0.02	0.01	0.00	-0.03	0.09	0.09	0.24	0.24	0.32	0.26
	(0.1)	(0.0)	(-55.1)	(9.9)	(5.8)	(0.0)	(-4.8)	(5.3)	(7.6)	(2.5)	(2.6)	(3.9)	(4.6)
Cov(W, T)	-0.00	0.01	0.01	-0.02	-0.04	0.03	0.02	-0.04	-0.07	-0.05	-0.05	-0.08	-0.11
	(-0.1)	(12.4)	(99.9)	(-8.1)	(-34.8)	(38.7)	(2.2)	(-2.2)	(-6.1)	(-0.6)	(-0.6)	(-1.0)	(-1.9)
Cov(W, τ)	-0.00	0.02	0.02	-0.01	-0.01	0.02	0.01	0.00	0.01	0.08	0.08	0.10	0.12
	(-0.0)	(16.3)	(130.2)	(-2.8)	(-5.4)	(24.1)	(1.0)	(0.2)	(0.9)	(0.8)	(0.9)	(1.2)	(2.0)
Cov(R, T)	-0.15	0.00	-0.03	-0.12	-0.03	0.00	-0.11	-0.28	-0.18	-0.56	-0.56	-0.57	-0.39
	(-18.4)	(0.0)	(-261.1)	(-56.2)	(-20.9)	(0.0)	(-17.2)	(-15.9)	(-16.0)	(-6.1)	(-6.1)	(-6.8)	(-6.9)
Cov(R, τ)	-0.03	0.00	-0.04	-0.05	-0.01	0.00	-0.05	0.02	0.02	0.82	0.82	0.70	0.43
	(-3.5)	(0.0)	(-336.2)	(-22.7)	(-6.2)	(0.0)	(-7.2)	(1.1)	(1.9)	(8.9)	(8.9)	(8.4)	(7.6)
Cov(T, τ)	0.01	0.10	0.07	0.03	0.02	0.06	0.02	-0.01	-0.02	-0.18	-0.18	-0.18	-0.17
	(1.7)	(96.2)	(594.4)	(15.3)	(15.7)	(74.9)	(3.3)	(-0.6)	(-1.7)	(-1.9)	(-1.9)	(-2.1)	(-3.0)
Total $\times 100$													
Var(c)	0.80	0.11	0.01	0.21	0.13	0.08	0.67	1.74	1.14	9.27	9.27	8.35	5.72
	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)

Notes: Forecast error variance decomposition calculated at a 4 quarter time horizon. Variance components presented in the table are multiplied by 100. Variance percent share in parentheses. Column percentiles correspond to the 0th, 50th, 90th, and 99th wealth percentiles.

factors, like the labor supply and taxes, are also important only to high income households. Unlike consumption, the variance components of aggregate savings are very different from that of any individual household, suggesting macroeconomic movements in aggregate savings tell us very little about any individual household.

The covariance terms between factors are very small for low and middle income households, and very large for high income ones. This is opposite what was observed affecting household consumption decisions, and suggests that the factors affecting savings decisions after each shock

Table 6.2: Direct Effects Decomposition: Savings

	Total	Low Income				Middle Income				High Income			
		0th	50th	90th	99th	0th	50th	90th	99th	0th	50th	90th	99th
Variance Components $\times 100$													
Var(L)	0.34	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.03	0.61	0.61	0.68	0.71
	(12.2)	(0.0)	(0.4)	(0.0)	(0.0)	(0.0)	(4.6)	(0.4)	(0.0)	(837.3)	(772.3)	(10.8)	(1.2)
Var(W)	0.12	0.00	0.00	0.01	0.03	0.00	0.00	0.06	0.09	0.18	0.18	0.24	0.29
	(4.4)	(0.0)	(0.3)	(0.2)	(0.0)	(0.0)	(1.3)	(0.6)	(0.1)	(243.5)	(226.6)	(3.8)	(0.5)
Var(R)	5.05	0.00	0.02	7.16	64.35	0.00	0.26	8.69	67.51	2.62	2.70	11.72	71.98
	(182.6)	(0.0)	(43.7)	(87.9)	(95.8)	(0.0)	(76.9)	(92.0)	(97.3)	(3,599.0)	(3,398.3)	(185.9)	(118.0)
Var(T)	0.00	0.00	0.00	0.01	0.01	0.00	0.01	0.01	0.02	0.01	0.01	0.02	0.02
	(0.1)	(0.0)	(5.2)	(0.1)	(0.0)	(0.0)	(1.6)	(0.1)	(0.0)	(16.5)	(15.4)	(0.3)	(0.0)
Var(τ)	0.12	0.00	0.00	0.06	0.09	0.00	0.01	0.05	0.06	0.89	0.89	0.97	1.05
	(4.4)	(0.0)	(4.6)	(0.8)	(0.1)	(0.0)	(2.9)	(0.5)	(0.1)	(1,215.6)	(1,118.6)	(15.4)	(1.7)
Covariance Components $\times 100$													
Cov(L,W)	-0.20	0.00	0.00	0.00	0.01	0.00	-0.00	-0.04	-0.05	-0.31	-0.31	-0.38	-0.43
	(-7.2)	(0.0)	(0.2)	(0.0)	(0.0)	(0.0)	(-1.3)	(-0.4)	(-0.1)	(-419.6)	(-389.0)	(-6.0)	(-0.7)
Cov(L,R)	-1.24	0.00	-0.00	0.05	0.32	0.00	-0.06	-0.28	-0.41	-1.26	-1.29	-2.00	-3.28
	(-45.0)	(0.0)	(-3.5)	(0.6)	(0.5)	(0.0)	(-18.3)	(-3.0)	(-0.6)	(-1,735.0)	(-1,616.9)	(-31.7)	(-5.4)
Cov(L,T)	0.01	0.00	-0.00	-0.00	-0.01	0.00	-0.01	0.01	0.01	0.05	0.05	0.07	0.09
	(0.5)	(0.0)	(-1.5)	(-0.0)	(-0.0)	(0.0)	(-2.2)	(0.1)	(0.0)	(63.1)	(59.5)	(1.2)	(0.1)
Cov(L,τ)	0.20	0.00	-0.00	0.00	0.01	0.00	-0.01	-0.04	-0.04	0.72	0.72	0.79	0.84
	(7.2)	(0.0)	(-1.3)	(0.0)	(0.0)	(0.0)	(-3.7)	(-0.4)	(-0.1)	(985.0)	(907.5)	(12.6)	(1.4)
Cov(W,R)	0.77	0.00	-0.00	0.22	0.82	0.00	0.02	0.55	1.56	0.64	0.66	1.46	3.08
	(28.0)	(0.0)	(-0.5)	(2.7)	(1.2)	(0.0)	(7.1)	(5.9)	(2.2)	(879.8)	(832.5)	(23.2)	(5.1)
Cov(W,T)	-0.01	0.00	-0.00	-0.00	-0.02	0.00	-0.00	-0.02	-0.03	-0.04	-0.04	-0.06	-0.07
	(-0.4)	(0.0)	(-0.8)	(-0.1)	(-0.0)	(0.0)	(-0.1)	(-0.2)	(-0.0)	(-51.3)	(-48.1)	(-0.9)	(-0.1)
Cov(W,τ)	-0.12	0.00	-0.00	0.03	0.05	0.00	0.00	0.05	0.07	-0.39	-0.40	-0.48	-0.55
	(-4.4)	(0.0)	(-0.3)	(0.3)	(0.1)	(0.0)	(1.0)	(0.5)	(0.1)	(-537.0)	(-497.1)	(-7.6)	(-0.9)
Cov(R,T)	-0.08	0.00	0.01	-0.17	-0.76	0.00	0.03	-0.24	-0.86	-0.10	-0.11	-0.41	-1.00
	(-3.0)	(0.0)	(12.3)	(-2.1)	(-1.1)	(0.0)	(7.4)	(-2.6)	(-1.2)	(-136.6)	(-134.6)	(-6.5)	(-1.6)
Cov(R,τ)	-0.77	0.00	0.01	0.34	0.92	0.00	0.05	0.33	0.60	-1.50	-1.53	-2.77	-5.32
	(-28.0)	(0.0)	(14.0)	(4.1)	(1.4)	(0.0)	(14.5)	(3.5)	(0.9)	(-2,054.4)	(-1,922.9)	(-43.9)	(-8.7)
Cov(T,τ)	0.01	0.00	0.00	-0.00	-0.02	0.00	0.01	-0.01	-0.02	0.07	0.07	0.11	0.12
	(0.4)	(0.0)	(4.4)	(-0.0)	(-0.0)	(0.0)	(1.8)	(-0.1)	(-0.0)	(100.0)	(93.7)	(1.7)	(0.2)
Total $\times 100$													
Var(a)	2.76	0.00	0.06	8.15	67.17	0.00	0.34	9.45	69.36	0.07	0.08	6.31	61.02
	(100.0)	(0.0)	(100.0)	(100.0)	(100.0)	(0.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)	(100.0)

Notes: Forecast error variance decomposition calculated at a 4 quarter time horizon. Variance components presented in the table are multiplied by 100. Variance percent share in parentheses. Column percentiles correspond to the 0th, 50th, 90th, and 99th wealth percentiles.

for low and middle income households have homogenous effects that all either push households to save more or save less. In contrast, shocks cause high income households to face diverging forces that, to some degree, cancel each other out.

7 Historical Decomposition

The variance shares of the model give a good general idea of which factors contribute to changes in macroeconomic aggregates and household decisions, however they do not give any specific view of which shocks and factors have been important over time. To analyze this, I perform a historical decomposition of the shocks to the model using the same macroeconomic series in the Bayesian estimation.

7.1 Decomposition Strategy

To perform the historical decomposition, I use the process in Auclert and Rognlie (2023). Using the deviation from trend in the observed data $d\mathbf{X}^{\text{data}}$ and the IRFs of the model $d\mathbf{X}$, I solve for a matrix of shocks $\boldsymbol{\epsilon}$ that create simulated paths for macroeconomic series $d\tilde{\mathbf{X}}$ to solve

$$\begin{aligned} \min_{\boldsymbol{\epsilon}} \quad & \sum_{t=0}^{T_{\text{obs}}} \left\| d\mathbf{X}_t^{\text{data}} - d\tilde{\mathbf{X}}_t \right\|^2 \\ \text{subject to} \quad & d\tilde{\mathbf{X}}_t = \sum_{s=0}^{T-1} d\mathbf{X}_s \boldsymbol{\epsilon}_{t-s}. \end{aligned}$$

Since I use seven data series to fit seven shocks that all have linearly independent IRFs, the sequences for shocks $\boldsymbol{\epsilon}$ when simulated $d\tilde{\mathbf{X}}_t$ perfectly match the data. Using the sequences for shocks in $\boldsymbol{\epsilon}$, the simulated $d\tilde{\mathbf{X}}_t$ can be decomposed as the sum of the effects from each individual shock. Figure H.1 shows the decompositions for each of the fitted data series.

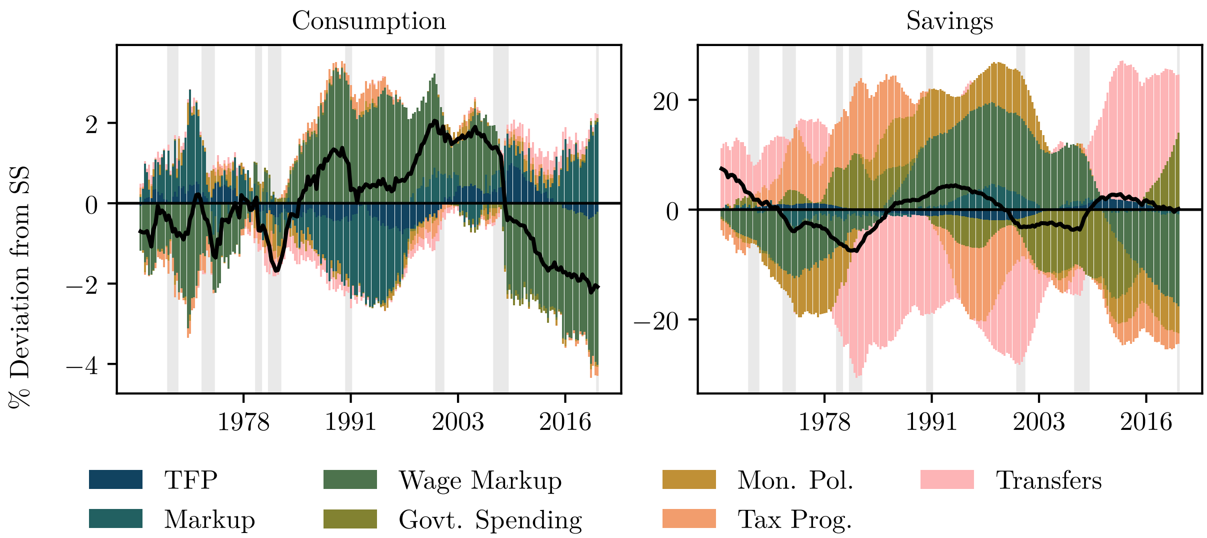
Then, I use the sequences of shocks to simulate the behavior of different households over time within the model. These series are pure simulation, and not fit to any microdata, so they should not be taken as true paths for the consumption and savings decisions for actual households. Therefore, I interpret these sequences more weakly to get insight into the factors affecting household decisions. In addition, I apply a moving average to better get at general trends and reduce noise.

7.2 Historical Decompositions

The historical decompositions for aggregate consumption and savings are presented in Figure 7.1. Like in the variance decompositions, price and wage markup shocks have been the most important determinants of consumption. In contrast, many factors, including wage markups, monetary policy, and transfers, impact aggregate savings.

Looking at specific time periods, price markups increase consumption initially while wage

Figure 7.1: Historical Decomposition: Household Aggregates



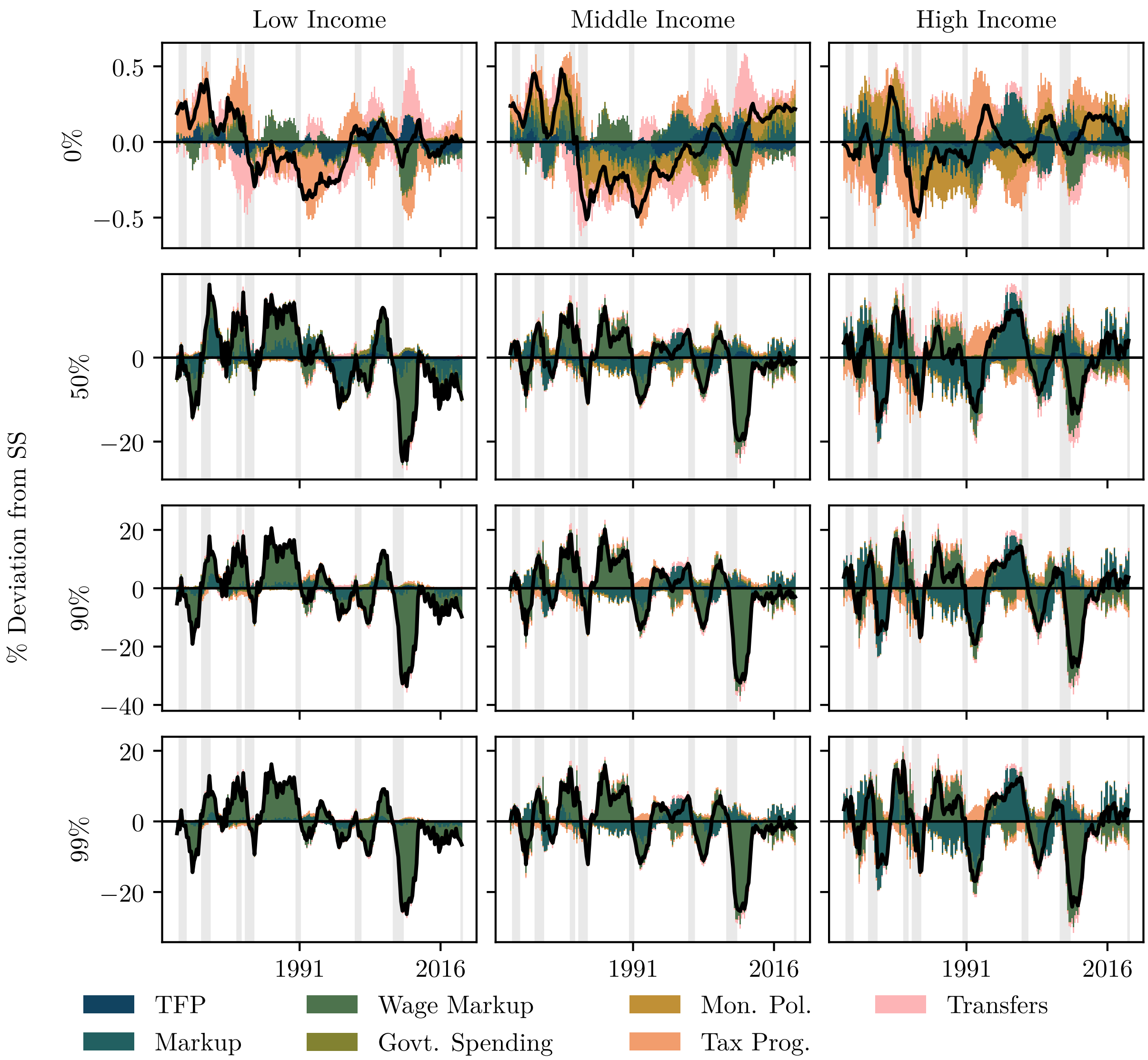
Notes: NBER-dated recessions highlighted in gray.

markups decrease consumption. Then, in the 80s the two effects flip-flop and price markups start to decrease consumption while wage markups increase it. Finally, around the time of the Great Recession the two effects switch again until the end of the estimation window. Across almost all periods, the effects from wage markups are slightly stronger than those of price markups, consistent with the variance decomposition in Figure 5.1c. Instead of price markups, transfers and monetary policy have significant impacts on savings, but the direction of the effects again flips in the early 1980s and late 2000s.

Figure 7.2 has historical decompositions for consumption decisions for households across the income and wealth distribution. Like in Section 5, I focus on low, middle, and high income households at the 0th, 50th, 90th, and 99th wealth percentiles. Simulated paths for consumption are very different for households at the 0th wealth percentile than other points along the wealth distribution, which all look very similar. Specifically, low wealth households are effected by a more diverse array of shocks, while the consumption patterns for higher wealth households are almost entirely explained by price and wage markups. Wage markup shocks impact low income households more than price markups, though for high income households price markup shocks are more important.

Tax progressivity shocks have opposite effects on low and high income households — when

Figure 7.2: Historical Decomposition: Household Consumption



Notes: Row labeled by wealth percentile. 12 quarter moving average applied. NBER-dated recessions highlighted in gray.

tax progressivity shocks cause low income households to consume more they cause high income households to consume less. Despite the decomposition only being fit on aggregate data, the effects of the Reagan-era tax cuts for higher income households are clear within the decomposition (Prasad 2012). Before the 80s when the top marginal tax rate in the US was highest, the level of tax progressivity makes higher income households consume less, and it makes lower and middle income households consume more. After the 80s, this flips and tax progressivity has positive effects on the consumption of high income households and negative effects on the consumption of low and middle income households. This lends credence to the use of shock paths fitted on aggregate data to gain understanding of individual-level phenomena within the model.

A similar decomposition for savings decisions is shown in Figure 7.3. Low and middle income households at the 0th wealth percentile never save, and therefore the decomposition is constant over time. Low and middle income households at other wealth levels do have some variability in their savings decisions, though substantially less than higher income households. This contrasts the consumption decomposition in Figure 7.2, where within a wealth band the consumption has a similar variability at all income levels. Price markups and tax progressivity are most important for low income households' savings decisions. Middle income household saving is similarly affected by price markups and tax progressivity though also face significant wage markup effects. Changes in savings decisions for high income households, especially at higher wealth levels, are almost entirely caused by wage markup shocks.

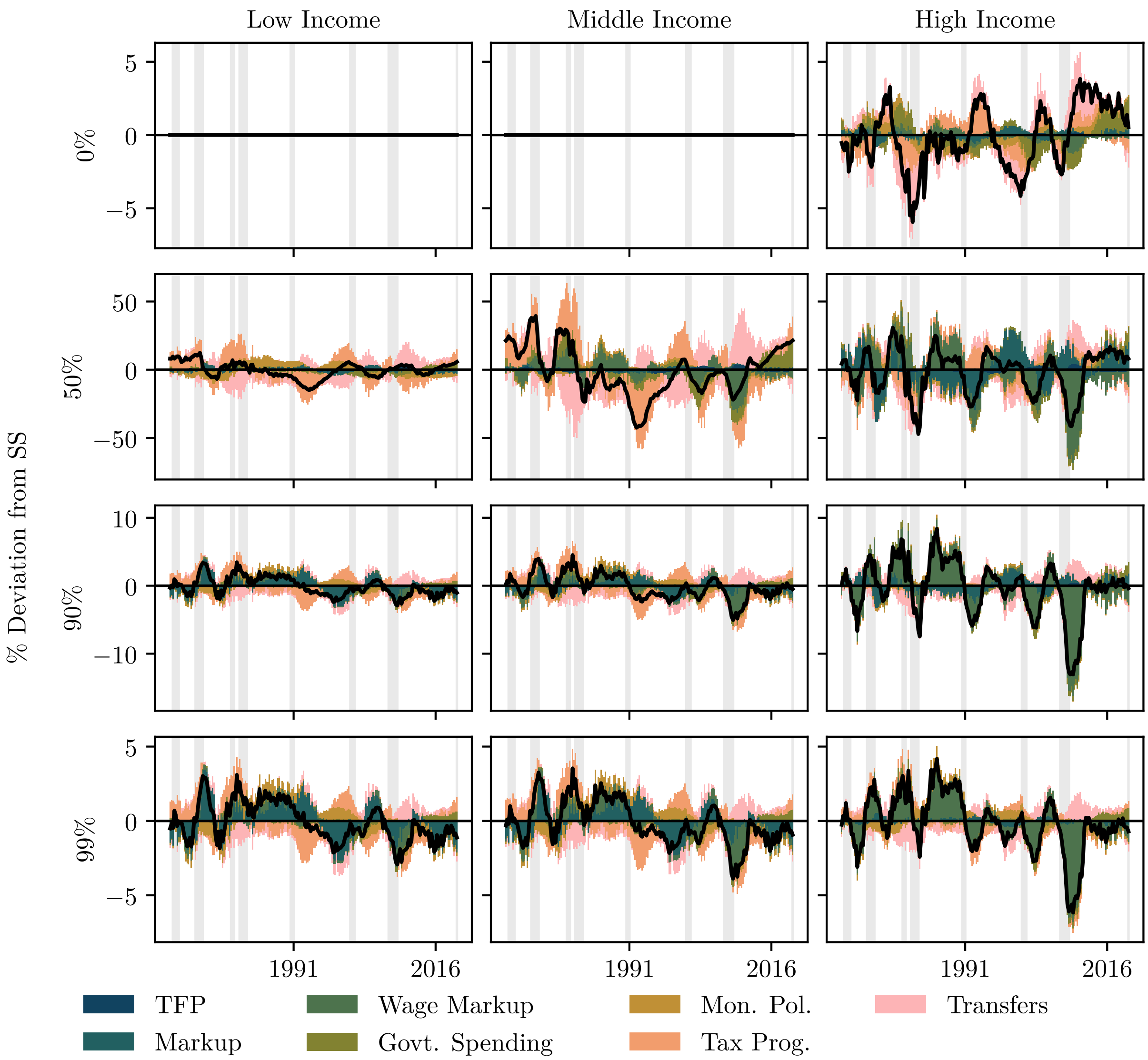
7.3 Historical Decomposition of Endogenous Effects

To get a more precise view of what affects households, not just the overall macroeconomic shocks, I separate the decomposed paths into the direct factors that play into household decisions. Like outlined in Section 6, shocks affect households through labor supply, wages, interest rates, transfers, and taxes. Therefore, paths for household and aggregate consumption and saving can be explained as the sum of the effects from each of these sources.

Figure 7.4 shows the effect of each of these channels on aggregate consumption and saving. The specific paths for both series are identical to those in Figure 7.1. Wages and transfers, which move inverse to each other, are the most significant factors affecting consumption, through the labor supply decided by the union also plays an important role. In fact, the transfer and wage effects almost perfectly cancel each other out, so the overall series very nearly follows the path for the labor supply effects. Interest rates and taxes have minimal effects on aggregate consumption within the window.

Wages, interest rates, and taxes are the most important determinants of aggregate savings.

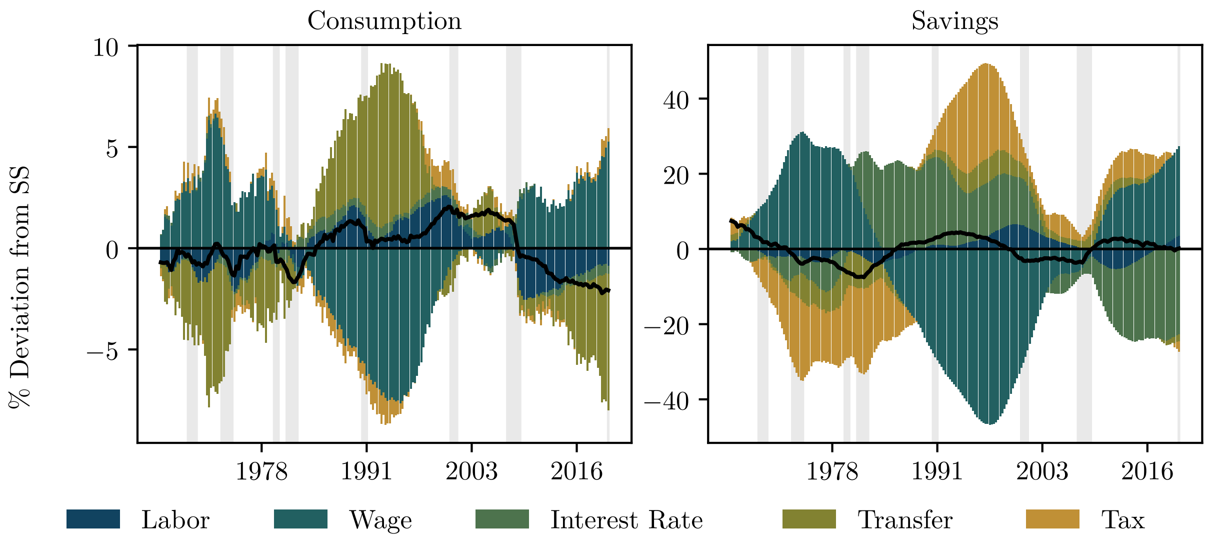
Figure 7.3: Historical Decomposition: Household Savings



Notes: Row labeled by wealth percentile. 12 quarter moving average applied. NBER-dated recessions highlighted in gray.

Likely due to the ρ_B parameter in the bond law of motion, the decomposed factors appear much smoother for savings than consumption, but the general shape of the savings decomposition is similar to that of the consumption decomposition, especially for the wage effects. Interest rates and taxes are far more important for savings than for consumption. The difference between the effects of taxes on consumption and savings suggests households in the model that were given tax breaks within the estimation window chose to save the extra money, not spend it. This could be

Figure 7.4: Historical Endogenous Decomposition: Household Aggregates



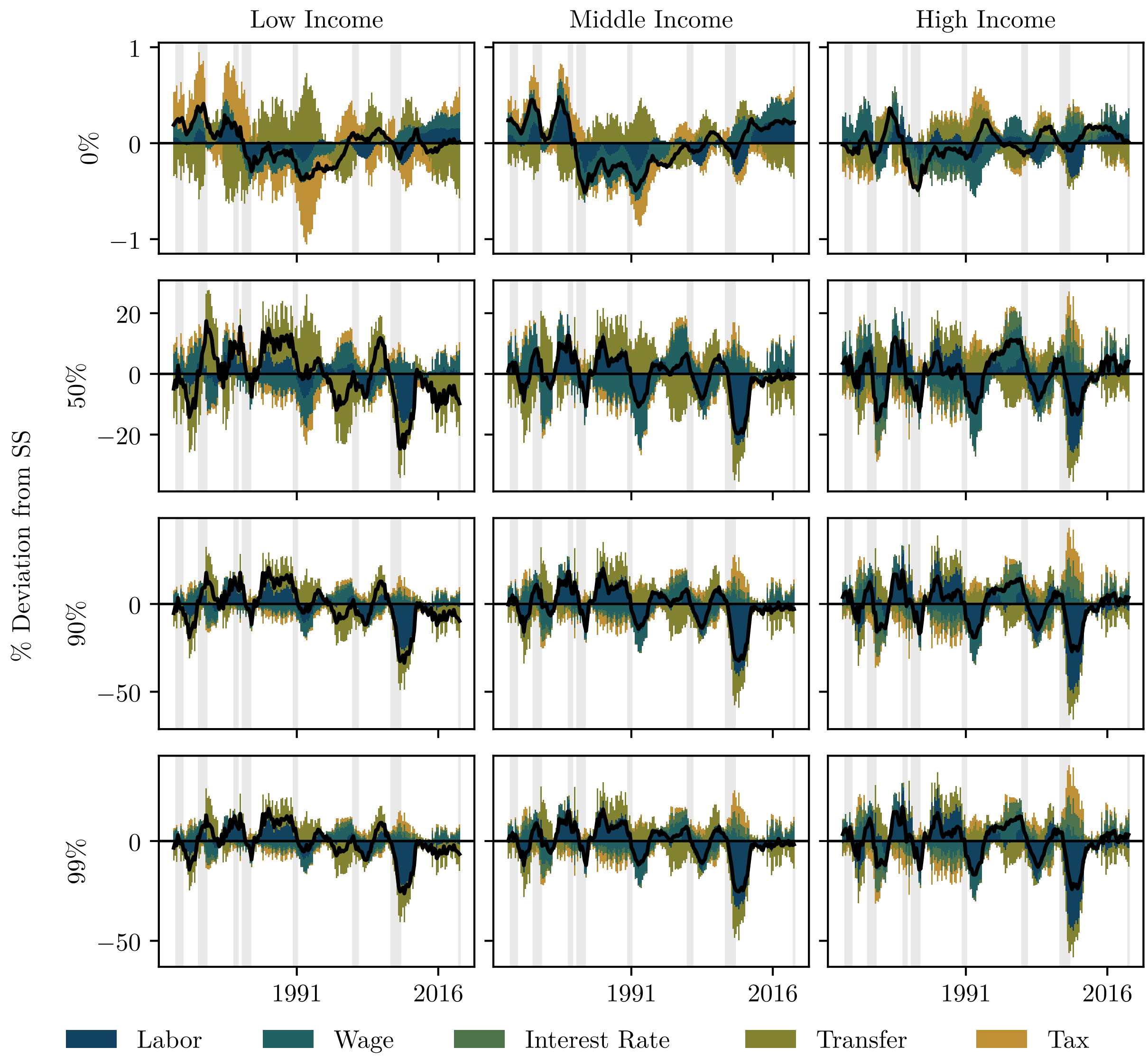
Notes: NBER-dated recessions highlighted in gray.

because the largest tax breaks in the estimation window were given to higher wealth households, which have lower MPCs (Auclert, Bardóczy, and Rognlie 2023).

The decomposition separated by income and wealth level is presented in Figure 7.5. The labor supply choice is, in general, the most important factor affecting household consumption decisions, especially for higher income or higher wealth households. For lower income and lower wealth households, direct household transfers and taxes are also important. Wages and interest rates are moderately important to all households except low and middle income 0th percentile households that are never impacted by the interest rate.

Figure 7.6 presents the same decomposition for household savings decisions. Again, low and middle income households at the 0th wealth percentile never choose to save. Interest rates are moderately important across all households, but most important to higher wealth households. In fact, interest rates determine almost all movement in savings decisions for 99th percentile low and middle income households. Households with either lower levels of wealth or higher income also face significant wage, labor, and tax effects. This suggests households choose to save more to consumption smooth after changes in income rather than to increase their future earnings when interest rates are higher.

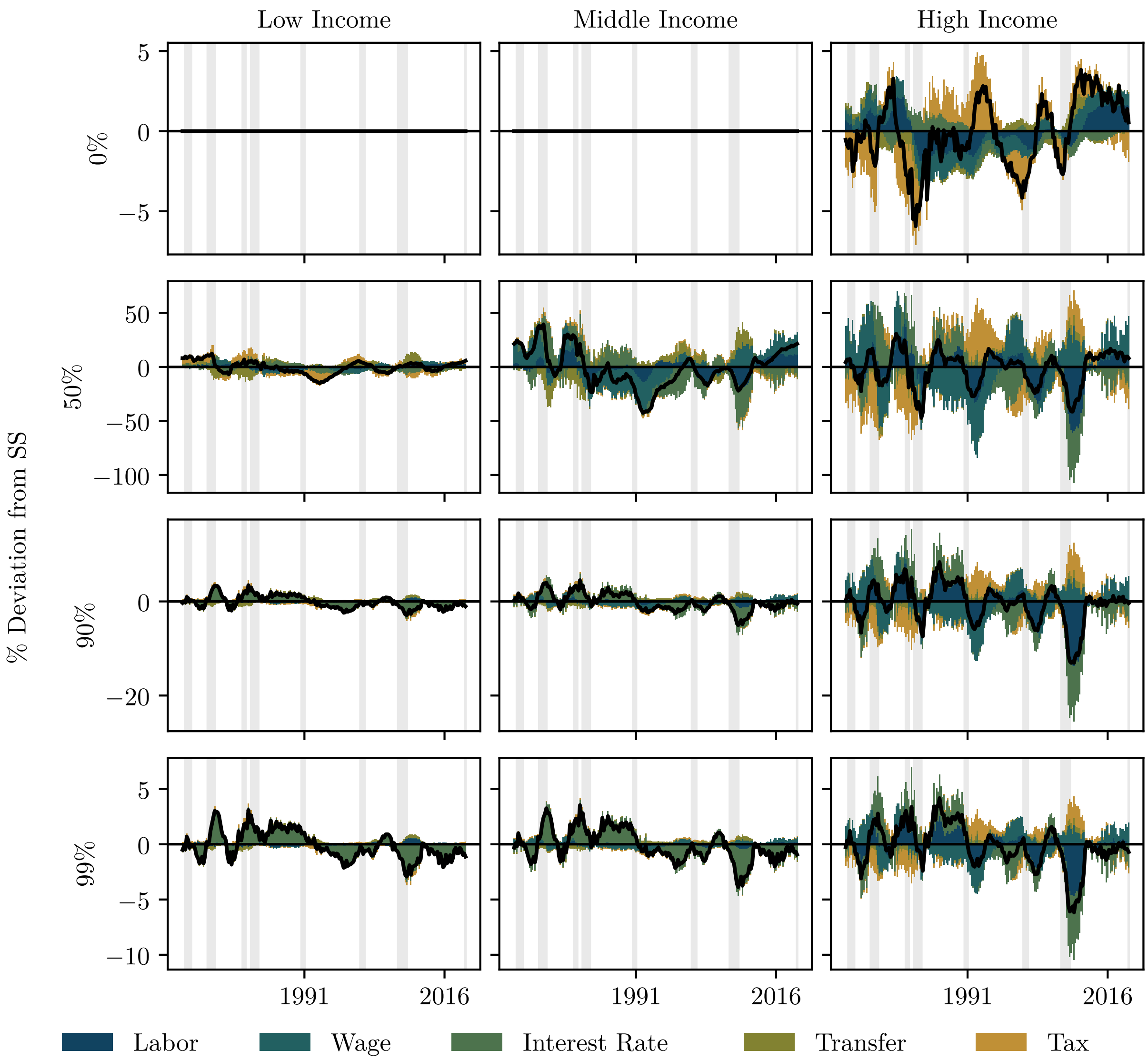
Figure 7.5: Historical Endogenous Decomposition: Household Consumption



Notes: Row labeled by wealth percentile. 12 quarter moving average applied. NBER-dated recessions highlighted in gray.

8 Conclusion

Figure 7.6: Historical Endogenous Decomposition: Household Savings



Notes: Row labeled by wealth percentile. 12 quarter moving average applied. NBER-dated recessions highlighted in gray.

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A Data

A.1 Calibration Data

I calibrate parameters in the model to match historical US averages relative to GDP. To match the estimation window, all data is quarterly from 1966 to 2019. Since the calibration target for $\overline{N}$ implies $\overline{Y} = 1$, I calibrate both levels ($\overline{B}$ and $\overline{\eta}$) and rates ($\overline{g}$) to their average fraction of GDP. The data is all from FRED (FRED codes in parentheses).

Debt Target. I target the steady state level of debt to match the mean US debt to GDP ratio. To calculate this ratio, I divide the historical nominal debt level (GFDEBTN) by the historical nominal GDP level (GDP). To account for differences in units, I divide this ratio by 1,000. Taking the mean gets $\overline{B} = 0.577$.

Government Spending. I target the steady state rate of government spending to match the mean fraction of GDP spent by the government To calculate this, I divide nominal government spending (GCE) by nominal GDP (GDP). Taking the mean gets $\overline{g} = 0.202$.

Transfers. I target the steady state government transfers to households to match the ratio of government transfers to households to GDP. I divide nominal social benefits transfers to households (B087RC1Q027SBEA) by nominal GDP (GDP). Taking the mean gets $\overline{\eta} = 0.081$.

A.2 Estimation Data

I estimate Y_t, π_t, I_t, N_t, C_t, B_t, and W_t against US aggregate data for GDP, inflation, the Federal Funds Rate, hours worked, consumption, government debt, and wages. I get the data from FRED (FRED codes in parentheses) at a quarterly frequency from 1966 to 2019. Since the model works in levels instead of percent deviation, the series are all multiplied by the steady state variable in the model before estimation.

GDP. To represent Y_t in the model, I use nominal GDP (GDP). I divide by the GDP deflator (GDPDEF) to get real GDP and by population (POPTHM) to make it per-capita. Then, I use the difference from the log-linear trend to estimate off of. Finally, I divide by 4 to make it quarterly and multiply by 100 to make it a percent.

Inflation. To represent π_t in the model, I use the log quarter to quarter difference in the GDP deflator (GDPDEF). I then subtract out the mean to make it into the difference from trend and multiply by 100 to make it a percent.

Federal Funds Rate. To represent I_t in the model, I use the Federal Funds Rate (FEDFUNDS). I subtract out the mean to make it into the difference from trend and divide by 4 to make it quarterly.

Hours Worked. To represent N_t in the model, I use total hours worked (HOANBS). I divide by population (POPTHM) to make it per capita. Then, I take the difference from log-linear trend to estimate off of. Finally, I divide by 4 to make it quarterly and multiply by 100 to make it a percent.

Consumption. To represent C_t in the model, I use personal consumption expenditure (PCE). I divide by the GDP deflator (GDPDEF) to get real consumption and by population (POPTHM) to make it per capita. Then, I take the difference from the log-linear trend, divide by 4 to make it quarterly, and multiply by 100 to make it a percent.

Government Debt. To represent B_t in the model, I use the level of government debt (GFDEBTN). I divide by the GDP deflator (GDPDEF) to get real debt and by population (POPTHM) to make it per capita. Then, I take the difference from the log-linear trend. Finally, I divide by 4 to make it quarterly and multiply by 100 to make it a percent.

Wages. To represent W_t in the model, I use the average hourly earnings of production and nonsupervisory employees (AHETPI). I divide by the GDP deflator (GDPDEF) to get real wages. Then, I take the difference from the log-linear trend, divide by 4 to make it quarterly, and multiply by 100 to make it a percent.

B Additional Model Details

B.1 Household Decision Rules

From the household budget and preferences in Section 3.1, households solve

$$\begin{aligned}
\max_{\{c_{i,t}, b_{i,t}\}_{t=0}^{\infty}} \quad & \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{c_{i,t}^{1-\gamma}}{1-\gamma} - \phi\frac{\ell_{i,t}^{1+\chi}}{1+\chi}\right] \\
\text{subject to} \quad & b_{i,t} + c_{i,t} = R_t b_{i,t-1} + W_t z_{i,t}\ell_{i,t} + D_t + \eta_t - \tau_t^L z_{i,t}^{\tau_t^P} \\
& b_{i,t} \geq \underline{b}.
\end{aligned}$$

This gets the Lagrangian

$$\begin{aligned}
\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\Bigg[\frac{c_{i,t}^{1-\gamma}}{1-\gamma} - \phi\frac{\ell_{i,t}^{1+\chi}}{1+\chi} \\
&+ \lambda_{i,t}\left(R_t b_{i,t-1} + W_t z_{i,t}\ell_{i,t} + D_t + \eta_t - \tau_t^L z_{i,t}^{\tau_t^P} - b_{i,t} - c_{i,t}\right) \\
&+ \mu_{i,t}\left(b_{i,t} - \underline{b}\right)\Bigg]
\end{aligned}$$

which has the FOCs

$$\lambda_{i,t} = c_{i,t}^{-\gamma} \tag{$c_{i,t}$}$$

$$\lambda_{i,t} = \mathbb{E}\beta R_{t+1}\lambda_{i,t+1} + \mu_{i,t} \tag{$b_{i,t}$}$$

for consumption and bonds respectively. Combining the FOCs for consumption and bonds gets

$$c_{i,t}^{-\gamma} = \beta R_{t+1} c_{i,t+1}^{-\gamma} + \mu_{i,t}.$$

Since $\mu_{i,t} \geq 0$, this becomes the Euler Equation

$$c_{i,t}^{-\gamma} \geq \beta R_{t+1} c_{i,t+1}^{-\gamma}$$

which holds with equality whenever the borrowing constraint is not binding and $b_{i,t} > \underline{b}$.

B.2 Labor Packer Demand Function

Since the labor packer earns revenue $W_t N_t$ and has costs given by $\int_0^1 w_{k,t} n_{k,t} dk$, the profit maximization condition is

$$\max_{\{n_{k,t}\}_{k\in[0,1]}} \quad W_t N_t - \int_0^1 w_{k,t} n_{k,t} dk.$$

Plugging in the aggregator, this becomes

$$\max_{\{n_{k,t}\}_{k\in[0,1]}} \quad W_t \left(\int_0^1 n_{k,t}^{\frac{1}{\psi_t^W}} dk \right)^{\psi_t^W} - \int_0^1 w_{k,t} n_{k,t} dk.$$

This has the FOC

$$w_{k,t} = W_t \left(\int_0^1 n_{k,t}^{\frac{1}{\psi_t^W}} dk \right)^{\psi_t^W - 1} n_{k,t}^{\frac{1-\psi_t^W}{\psi_t^W}}$$

which, rearranged, becomes the demand function

$$n_{k,t} = \left(\frac{w_{k,t}}{W_t} \right)^{\frac{\psi_t^W}{1-\psi_t^W}} \left(\int_0^1 n_{k,t}^{\frac{1}{\psi_t^W}} dk \right)^{\psi_t^W} = N_t \left(\frac{w_{k,t}}{W_t} \right)^{\frac{\psi_t^W}{1-\psi_t^W}}.$$

B.3 Wage Philips Curve

At time t, unions decide ℓ_k by solving

$$\begin{aligned}
\max_{\{w_{k,s},\ell_{k,s}\}_{s=t}^{\infty}} \quad & \mathbb{E} \sum_{s=t}^{\infty} \beta^{s-t} \left[\int \frac{c_s(b,z)^{1-\gamma}}{1-\gamma} d\Gamma_s(b,z) - \phi \int_0^1 \frac{\ell_{k,s}^{1+\chi}}{1+\chi} dk - m_{k,s}^W \right] \\
\text{subject to} \quad & c_s(b,z) + b_s(b,z) = R_s b + z \int_0^1 w_{k,s} \ell_{k,s} dk + D_s + \eta_s - \tau_s^L z^{\tau_s^P}, \quad (b,z) \in \Gamma_t(b,z) \\
& m_{k,s}^W = \frac{\psi_s^W}{\psi_s^W - 1} \frac{1}{2\kappa^W} \log \left(\frac{w_{k,s}}{\overline{\pi}^W w_{k,s-1}} \right)^2 \\
& n_{k,s} = \ell_{k,s} \int z d\Gamma_t^z(z) \\
& n_{k,s} = N_s \left(\frac{w_{k,s}}{W_s} \right)^{\frac{\psi_s^W}{1-\psi_s^W}}.
\end{aligned}$$

Letting

$$h_s(b,z) = R_s b + D_s + \eta_s - \tau_s^L z^{\tau_s^P} - b_s(b,z)$$

this becomes

$$\max_{\{w_{k,s},\ell_{k,s}\}_{s=t}^{\infty}} \quad \mathbb{E}\sum_{s=t}^{\infty}\beta^{s-t}\left[\int \frac{\left(h_s(b,z)+z\int_0^1 w_{k,s}\ell_{k,s}dk\right)^{1-\gamma}}{1-\gamma}d\Gamma_s(b,z)-\phi\int_0^1\frac{\ell_{k,s}^{1+\chi}}{1+\chi}dk\right.$$
$$\left.-\frac{\psi_s^W}{\psi_s^W-1}\frac{1}{2\kappa^W}\log\left(\frac{w_{k,s}}{\overline{\pi}^W w_{k,s-1}}\right)^2\right]$$
$$\text{subject to} \quad N_s\left(\frac{w_{k,s}}{W_s}\right)^{\frac{\psi_s^W}{1-\psi_s^W}}=\ell_{k,s}\int z d\Gamma_t^z(z).$$

Therefore, we have the Lagrangian

$$\mathcal{L}=\mathbb{E}\sum_{s=t}^{\infty}\beta^{s-t}\left[\int \frac{\left(h_s(b,z)+z\int_0^1 w_{k,s}\ell_{k,s}dk\right)^{1-\gamma}}{1-\gamma}d\Gamma_s(b,z)-\phi\int_0^1\frac{\ell_{k,s}^{1+\chi}}{1+\chi}dk\right.$$
$$-\frac{\psi_s^W}{\psi_s^W-1}\frac{1}{2\kappa^W}\log\left(\frac{w_{k,s}}{\overline{\pi}^W w_{k,s-1}}\right)^2$$
$$\left.+\lambda_{k,s}\left(N_s\left(\frac{w_{k,s}}{W_s}\right)^{\frac{\psi_s^W}{1-\psi_s^W}}-\ell_{k,s}\int z d\Gamma_t^z(z)\right)\right].$$

This has the FOCs

$$z w_{k,s} c_s(z,b)^{-\gamma}=\phi\ell_{k,s}^{\chi}+\lambda_{k,s}z \tag{$\ell_{k,s}$}$$

$$\frac{\psi_s^W}{\psi_s^W-1}\frac{1}{w_{k,s}\kappa^W}\log\left(\frac{w_{k,s}}{\overline{\pi}^W w_{k,s-1}}\right)=\ell_{k,s}\int z c_s(b,z)^{-\gamma}d\Gamma_s(b,z) \tag{$w_{k,s}$}$$
$$-\frac{\psi_s^W}{\psi_s^W-1}\lambda_{k,s}N_s\left(\frac{w_{k,s}}{W_s}\right)^{\frac{\psi_s^W}{1-\psi_s^W}}w_{k,s}^{-1}$$
$$+\frac{\psi_s^W}{\psi_s^W-1}\frac{1}{w_{k,s}\kappa^W}\log\left(\frac{w_{k,s+1}}{\overline{\pi}^W w_{k,s}}\right).$$

Since the conditions for unions are all identical, we can plug in $w_{k,s}=W_s$, $\ell_{k,s}=L_s$, and $\lambda_{k,s}=\Lambda_s$. Integrating and rearranging the FOC for $\ell_{k,s}$ gets

$$\Lambda_s=\frac{W_sL_s\int z c_s(b,z)^{-\gamma}d\Gamma_s(b,z)-\phi L_s^{1+\chi}}{L_s\int z d\Gamma_s^z(z)}=\frac{W_sL_s\int z c_s(b,z)^{-\gamma}d\Gamma_s(b,z)-\phi L_s^{1+\chi}}{N_s}.$$

Plugging this into the FOC for $w_{k,s}$ and multiplying by $\kappa^W W_s \frac{\psi_s^W - 1}{\psi_s^W}$ gets the Philips curve

$$\log\left(\frac{\pi_s^W}{\overline{\pi}^W}\right) = \kappa^W \left(\phi L_s^{1+\chi} - \frac{1}{\psi_s^W} W_s L_s \int z c_s(b,z) d\Gamma_s(b,z)\right) + \beta \log\left(\frac{\pi_{s+1}^W}{\overline{\pi}^W}\right).$$

B.4 Final Goods Firm Conditions

Final goods firms earn revenue $P_t Y_t$ and have costs $\int_0^1 y_{j,t} p_{j,t} dj$. Therefore, the profit maximization condition for firms is

$$\max_{\{y_{j,t}\}_{j\in[0,1]}} \quad P_t Y_t - \int_0^1 p_{j,t} y_{j,t} dj.$$

Plugging in the aggregator, this becomes

$$\max_{\{y_{j,t}\}_{j\in[0,1]}} \quad P_t \left(\int_0^1 y_{j,t}^{\frac{1}{\psi_t}} dj\right)^{\psi_t} - \int_0^1 p_{j,t} y_{j,t} dj$$

which has the FOC

$$p_{j,t} = P_t \left(\int_0^1 y_{j,t}^{\frac{1}{\psi_t}} dj\right)^{\psi_t - 1} y_{j,t}^{\frac{1-\psi_t}{\psi_t}}.$$

Rearranging this gets

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{\frac{\psi_t}{1-\psi_t}} \left(\int_0^1 y_{j,t}^{\frac{1}{\psi_t}} dj\right)^{\psi_t} = Y_t \left(\frac{p_{j,t}}{P_t}\right)^{\frac{\psi_t}{1-\psi_t}}$$

which is the demand for intermediate good j. Plugging this back into the aggregator means

$$\begin{aligned} Y_t &= \left(\int_0^1 \left(Y_t \left(\frac{p_{j,t}}{P_t}\right)^{\frac{\psi_t}{1-\psi_t}}\right)^{\frac{1}{\psi_t}} dj\right)^{\psi_t} \\ &= Y_t \frac{1}{P_t^{\frac{\psi_t}{1-\psi_t}}} \left(\int_0^1 p_{j,t}^{\frac{1}{1-\psi_t}} dj\right)^{\psi_t}. \end{aligned}$$

Rearranging this gets the price aggregator

$$P_t = \left(\int_0^1 p_{j,t}^{\frac{1}{1-\psi_t}} dj\right)^{1-\psi_t}.$$

B.5 Philips Curve

Intermediate goods firms pick prices to maximize expected discounted real profits solving

$$\begin{aligned}
\max_{\{p_{j,s}\}_{s=t}^{\infty}} \quad & \mathbb{E}\sum_{s=t}^{\infty} R_{t,s}^{-1}\left[\frac{p_{j,s}}{P_s}y_{j,s} - W_s n_{j,s} - m_{j,s}\right] \\
\text{subject to} \quad & m_{j,s} = \frac{\psi_s}{\psi_s - 1}\frac{1}{2\kappa}\log\left(\frac{p_{j,s}}{\overline{\pi}p_{j,s-1}}\right)^2 Y_s \\
& y_{j,s} = A_s n_{j,s} \\
& y_{j,s} = Y_s\left(\frac{p_{j,s}}{P_s}\right)^{\frac{\psi_s}{1-\psi_s}}
\end{aligned}$$

where $R_{t,s} = \prod_{q=t+1}^{s} R_q$ represents the real gross return of bonds from period t to s. Plugging in the demand function, production function, and adjustment costs gets

$$\max_{\{p_{j,s}\}_{s=t}^{\infty}} \quad \mathbb{E}\sum_{s=t}^{\infty} R_{t,s}^{-1}Y_s\left[\left(\frac{p_{j,s}}{P_s}\right)^{\frac{1}{1-\psi_s}} - \frac{W_s}{A_s}\left(\frac{p_{j,s}}{P_s}\right)^{\frac{\psi_s}{1-\psi_s}} - \frac{\psi_s}{\psi_s-1}\frac{1}{2\kappa}\left(\frac{p_{j,s}}{\overline{\pi}p_{j,s-1}} - 1\right)^2\right]$$

This has the FOC of

$$\begin{aligned}
\frac{\psi_s}{\psi_s-1}\frac{1}{\kappa}Y_s\log\left(\frac{p_{j,s}}{\overline{\pi}p_{j,s-1}}\right)p_{j,s}^{-1} =& \frac{\psi_s}{\psi_s-1}Y_s\frac{W_s}{A_s}\left(\frac{p_{j,s}}{P_s}\right)^{\frac{\psi_s}{1-\psi_s}}p_{j,s}^{-1} - \frac{1}{\psi_s-1}Y_s\left(\frac{p_{j,s}}{P_s}\right)^{\frac{1}{1-\psi_s}}p_{j,s}^{-1} \\
&+ \frac{\psi_s}{\psi_s-1}\frac{1}{\kappa}R_{s+1}^{-1}Y_{s+1}\log\left(\frac{p_{j,s+1}}{\overline{\pi}p_{j,s}}\right)p_{j,s}^{-1}.
\end{aligned}$$

Since firm conditions are identical, we can assume price symmetry across firms so $p_{j,s} = p_{j',s}$ for $j \neq j'$. Using the price aggregator, this gets

$$P_s = \left(\int_0^1 p_{j,s}^{1-\psi_s}dj\right)^{\frac{1}{1-\psi_s}} = \left(p_{j,s}^{1-\psi_s}\right)^{\frac{1}{1-\psi_s}} = p_j.$$

Plugging this in and rearranging the system yields the Philips Curve

$$\log\left(\frac{\pi_t}{\overline{\pi}}\right) = \kappa\left(\frac{W_t}{A_t} - \frac{1}{\psi_t}\right) + R_{t+1}\frac{Y_{t+1}}{Y_t}\log\left(\frac{\pi_{t+1}}{\overline{\pi}}\right).$$

B.6 Aggregation

In the model, firm conditions are perfectly symmetrical. Therefore, for $j \neq j'$ we can assume

$$y_{j,t} = y_{j',t}$$

$$n_{j,t} = n_{j',t}$$

$$d_{j,t} = d_{j',t}$$

$$m_{j,t} = m_{j',t}$$

Using the aggregators for each variable this gets

$$\begin{aligned}
Y_t &= \left(\int_0^1 y_{j,t}^{\frac{\psi_t-1}{\psi_t}} dj\right)^{\frac{\psi_t}{\psi_t-1}} = \left(y_{j,t}^{\frac{\psi_t-1}{\psi_t}}\right)^{\frac{\psi_t}{\psi_t-1}} = y_{j,t} \\
N_t &= \int_0^1 n_{j,t} dj = n_{j,t} \\
D_t &= \int_0^1 d_{j,t} dj = d_{j,t} \\
M_t &= \int_0^1 m_{j,t} dj = m_{j,t}.
\end{aligned}$$

Then, integrating across the production function gets

$$Y_t = \int_0^1 y_{j,t} dj = \int_0^1 A_t n_{j,t} dj = A_t N_t,$$

integrating across the dividend expression gets

$$D_t = \int_0^1 d_{j,t} dj = \int_0^1 \left(\frac{p_{j,t}}{P_t} y_{j,t} - W_t n_{j,t} - m_{j,t}\right) dj = Y_t - W_t N_t - M_t,$$

and integrating across the adjustment cost expression gets

$$M_t = \int_0^1 m_{j,t} dj = \int_0^1 \frac{\psi_t}{\psi_t - 1} \frac{1}{2\kappa} \left(\frac{p_{j,t}}{\overline{\pi} p_{j,t-1}} - 1\right)^2 Y_t dj = \frac{\psi_t}{\psi_t - 1} \frac{1}{2\kappa} \left(\frac{\pi_t}{\overline{\pi}} - 1\right)^2 Y_t.$$

B.7 Characterization

The model is characterized by the household decision rules

$$
\begin{aligned}
b_t(b_{i,t-1}, z_{i,t}) + c_t(b_{i,t-1}, z_{i,t}) &= R_t b_{i,t-1} + W_t z_{i,t} L_t + D_t + \eta_t - \tau_t^L z_{i,t}^{\tau_t^P} \\
c_t(b_{i,t-1}, z_{i,t})^{-\gamma} &= \beta \mathbb{E} R_{t+1} c_{t+1}(b_{i,t-1}, z_{i,t}),
\end{aligned}
$$

distributional movement condition

$$
\Gamma_{t+1}(b', z') = \int_{\{(b,z): b_t(b,z)=b'\}} \Pr(z'|z) d\Gamma_t(b, z),
$$

aggregate equations

$$
\begin{aligned}
\pi_t^W &= \frac{W_t}{W_{t-1}} \\
L_t &= \frac{N_t}{\int z d\Gamma_t^z(z)} \\
\log\left(\frac{\pi_t^W}{\overline{\pi}^W}\right) &= \kappa^W \left(\phi L_t^{1+\chi} - \frac{1}{\psi_t^W} W_t L_t \int z c_t(b,z) d\Gamma_s(b,z)\right) + \beta \log\left(\frac{\pi_{t+1}^W}{\overline{\pi}^W}\right) \\
Y_t &= A_t N_t \\
M_t &= \frac{\psi_t}{\psi_t - 1} \frac{1}{2\kappa} \left(\frac{\pi_t}{\overline{\pi}} - 1\right)^2 Y_t \\
\log\left(\frac{\pi_t}{\overline{\pi}}\right) &= \kappa \left(\frac{W_t}{A_t} - \frac{1}{\psi_t}\right) + R_{t+1} \frac{Y_{t+1}}{Y_t} \log\left(\frac{\pi_{t+1}}{\overline{\pi}}\right) \\
D_t &= Y_t - W_t N_t - M_t \\
G_t &= g_t Y_t \\
B_t &= \overline{B} + \rho_B \left(R_t B_{t-1} - \overline{RB} + G_t - \overline{G} + \eta_t - \overline{\eta}\right) \\
R_t B_{t-1} + G_t + \eta_t &= \tau_t^L \int z d\Gamma_t^Z(z) + B_t \\
I_t &= \overline{I} \left(\frac{\pi_t}{\overline{\pi}}\right)^{\omega_\pi} \left(\frac{Y_t}{\overline{\overline{Y}}}\right)^{\omega_Y} \xi_t \\
R_t &= \frac{I_{t-1}}{\pi_t},
\end{aligned}
$$

and market clearing condition

$$B_t = \int b_t(b,z) d\Gamma_t(b,z)$$

where the goods market clears by Walras's Law.

C Computational Error

TBD

D Estimation Results

Figure D.1: Recursive Means

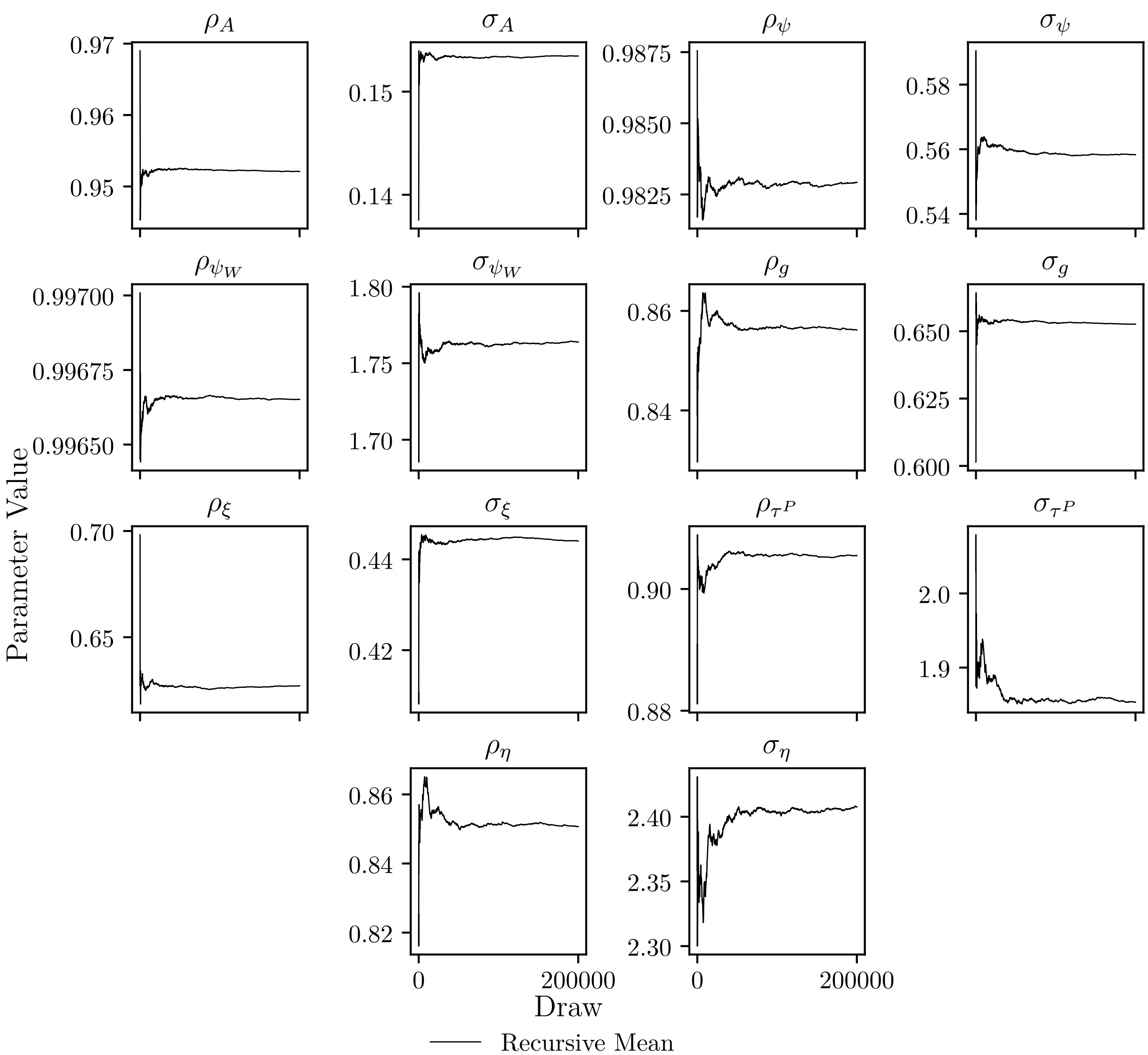


Figure D.2: Posterior Distributions

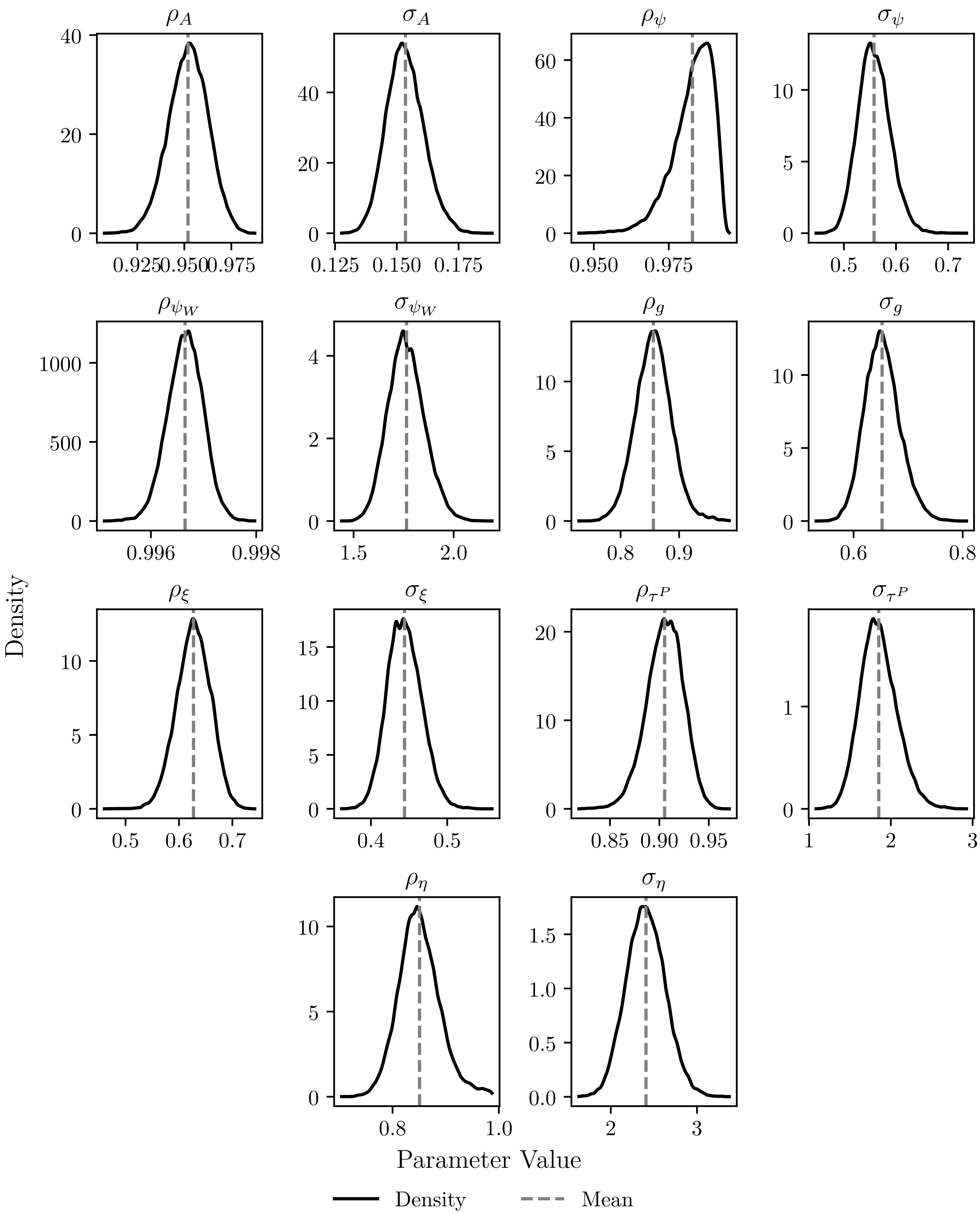
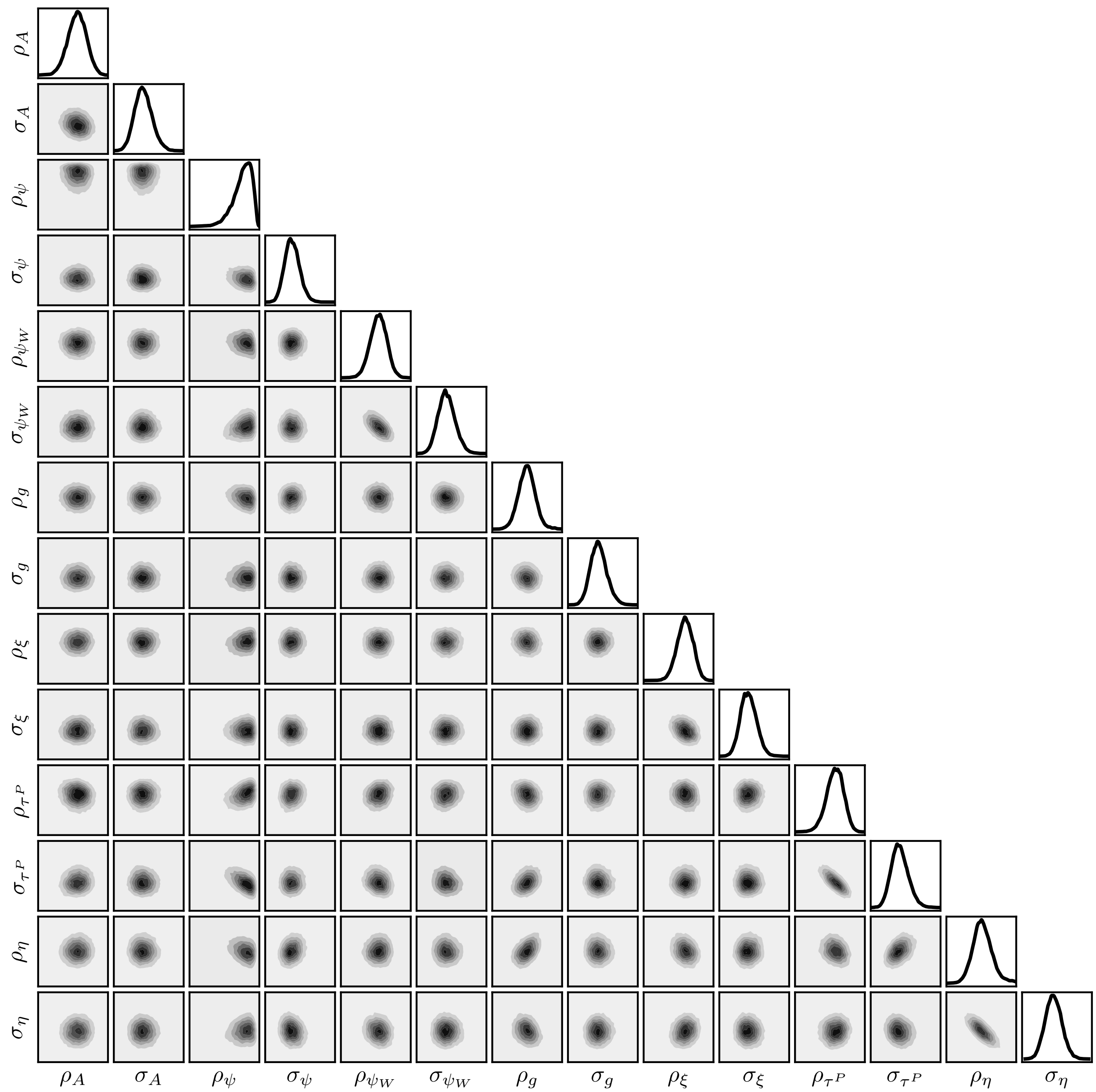


Figure D.3: Posterior Covariences



E Aggregate IRFs

Figure E.1: TFP (A) Shock Impulse Response Functions

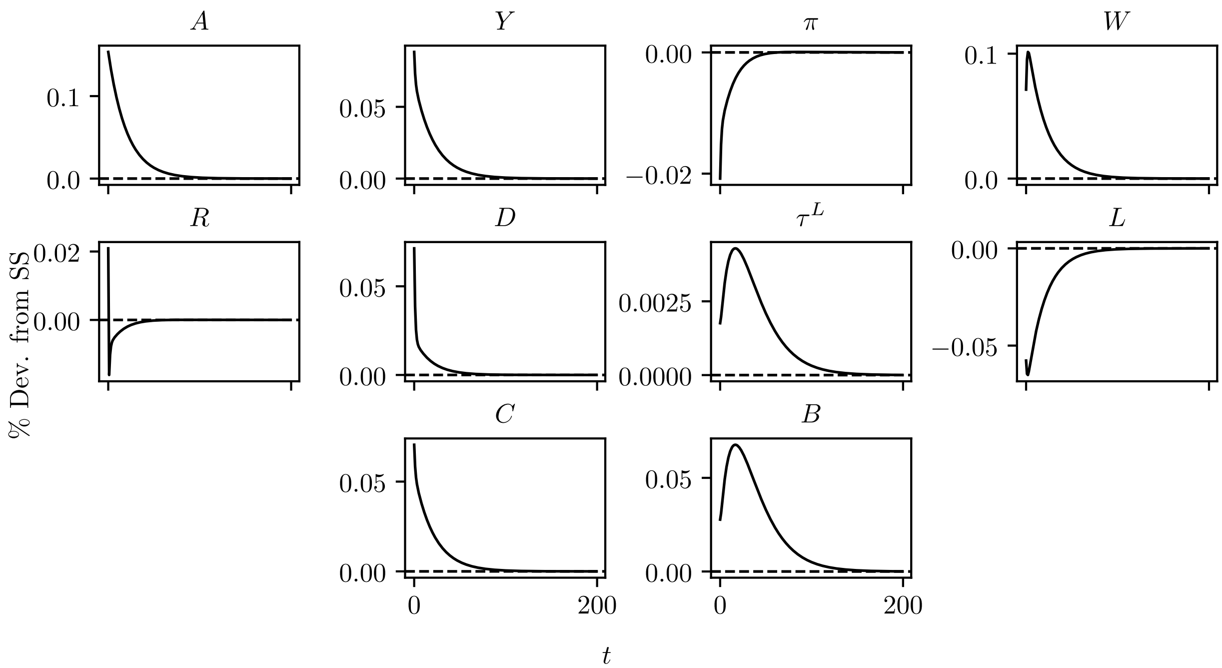


Figure E.2: Price Markup (ψ) Shock Impulse Response Functions

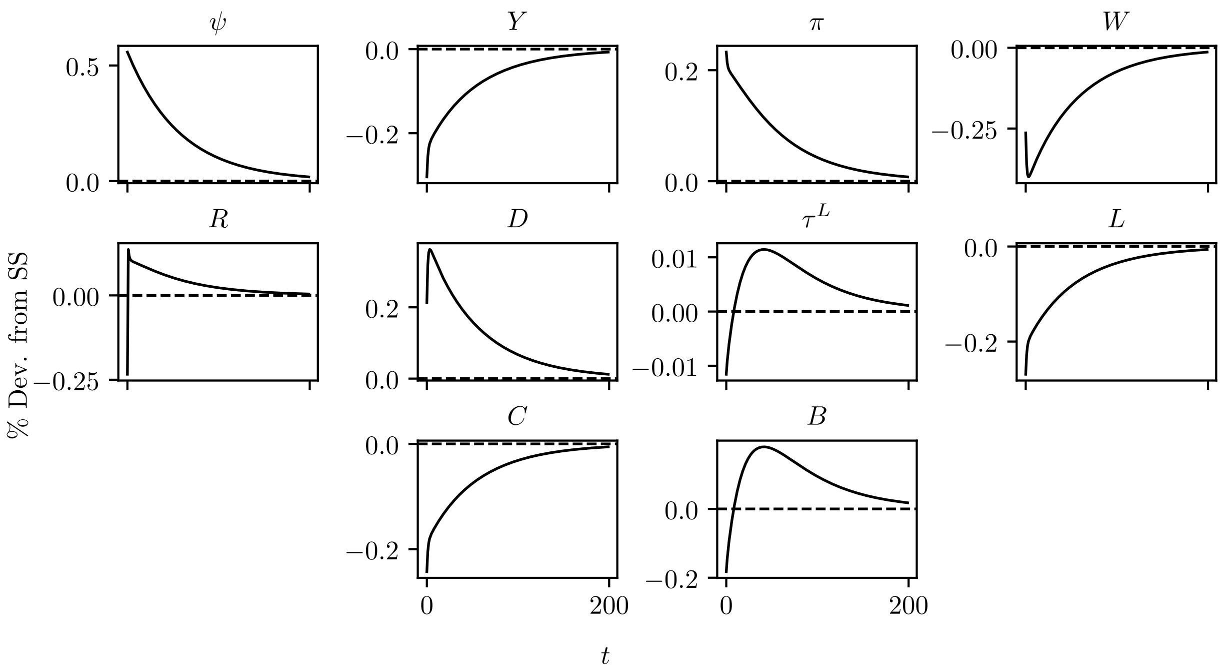


Figure E.3: Wage Markup (ψ_W) Shock Impulse Response Functions

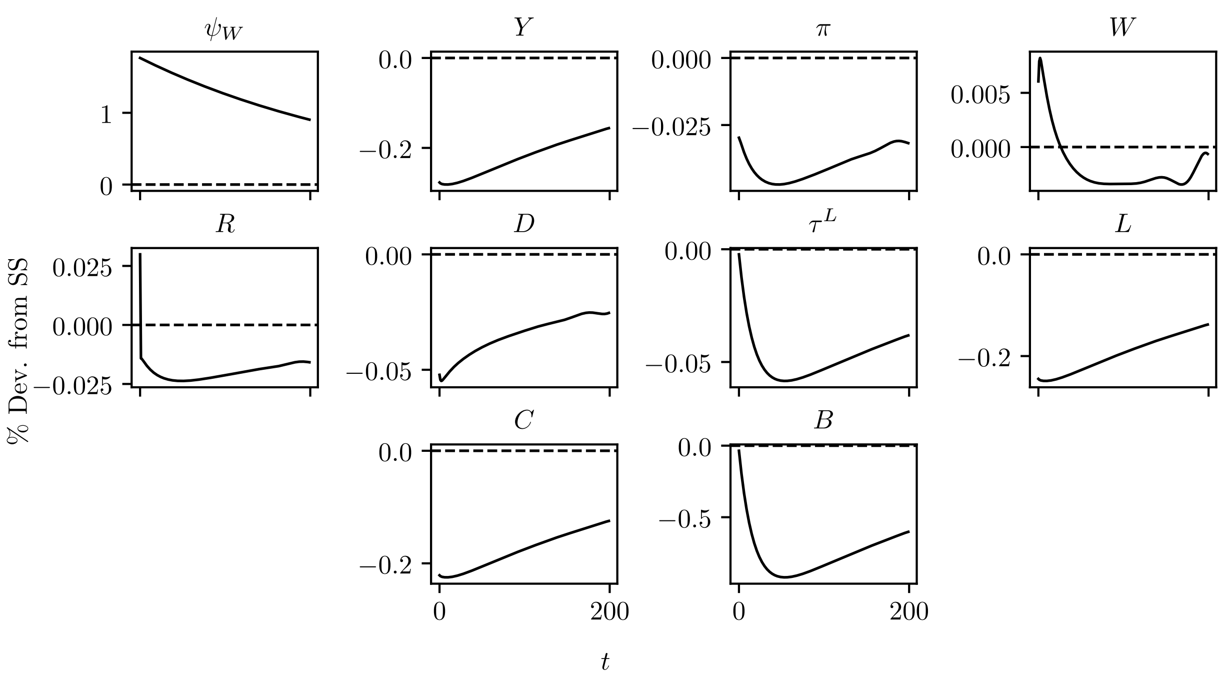


Figure E.4: Govt. Spending (g) Shock Impulse Response Functions

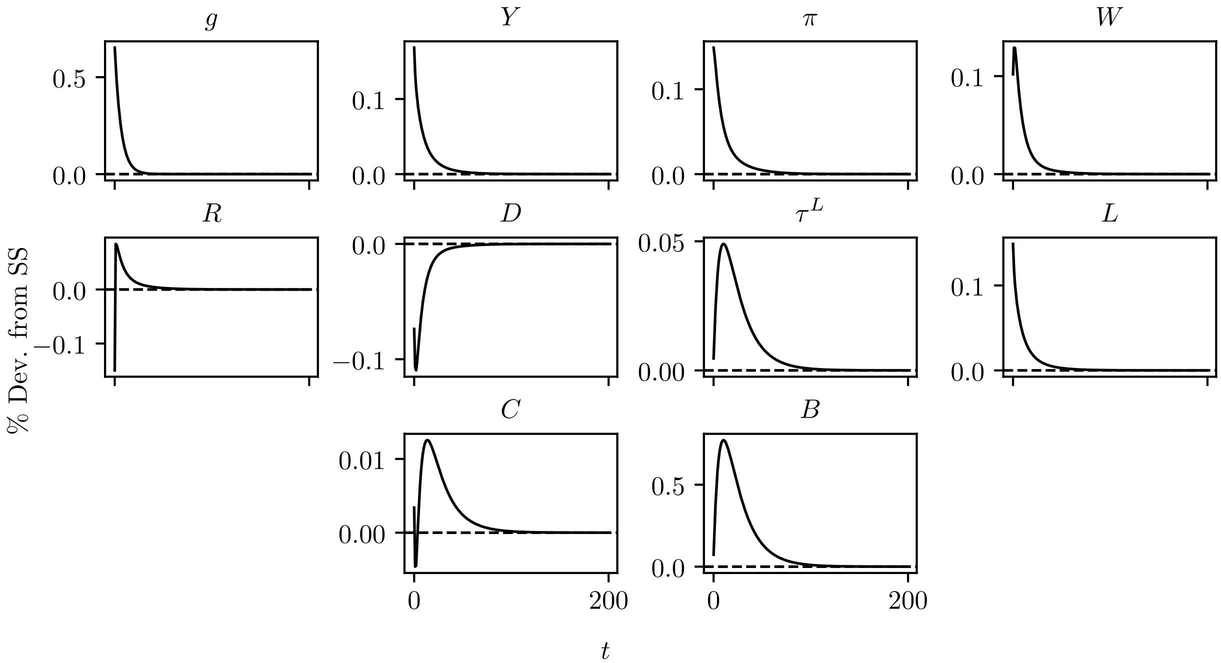


Figure E.5: Monetary Policy (ξ) Shock Impulse Response Functions

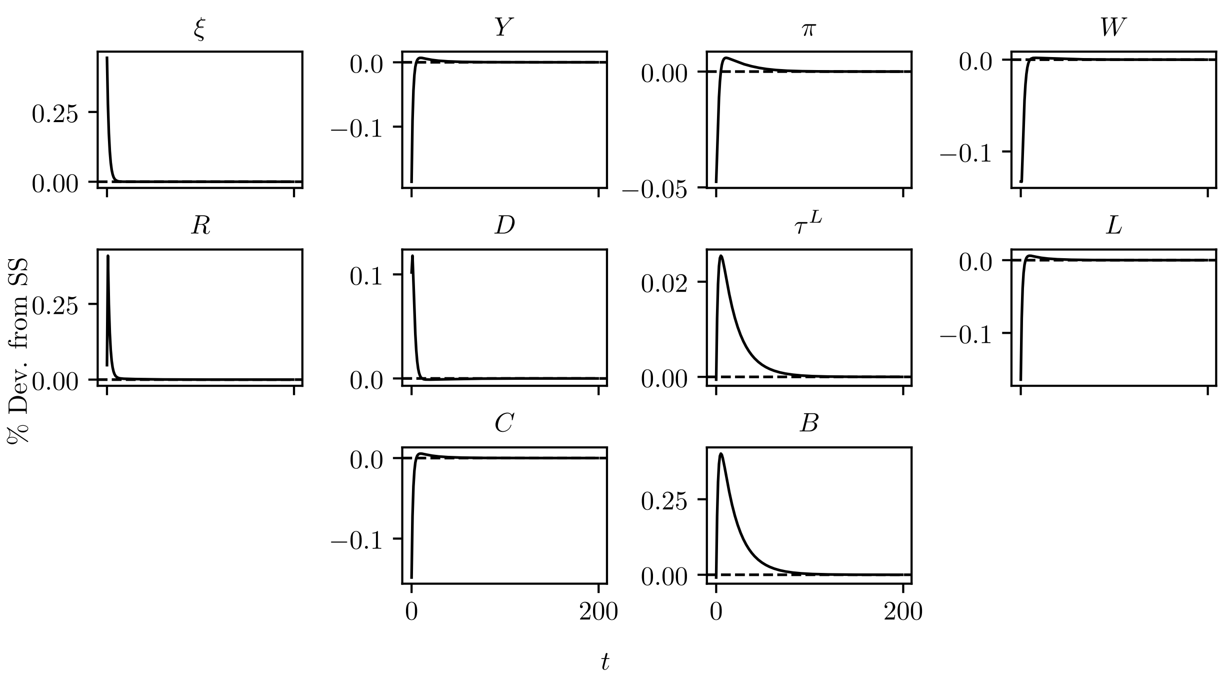


Figure E.6: Tax Progressivity (τ^P) Shock Impulse Response Functions

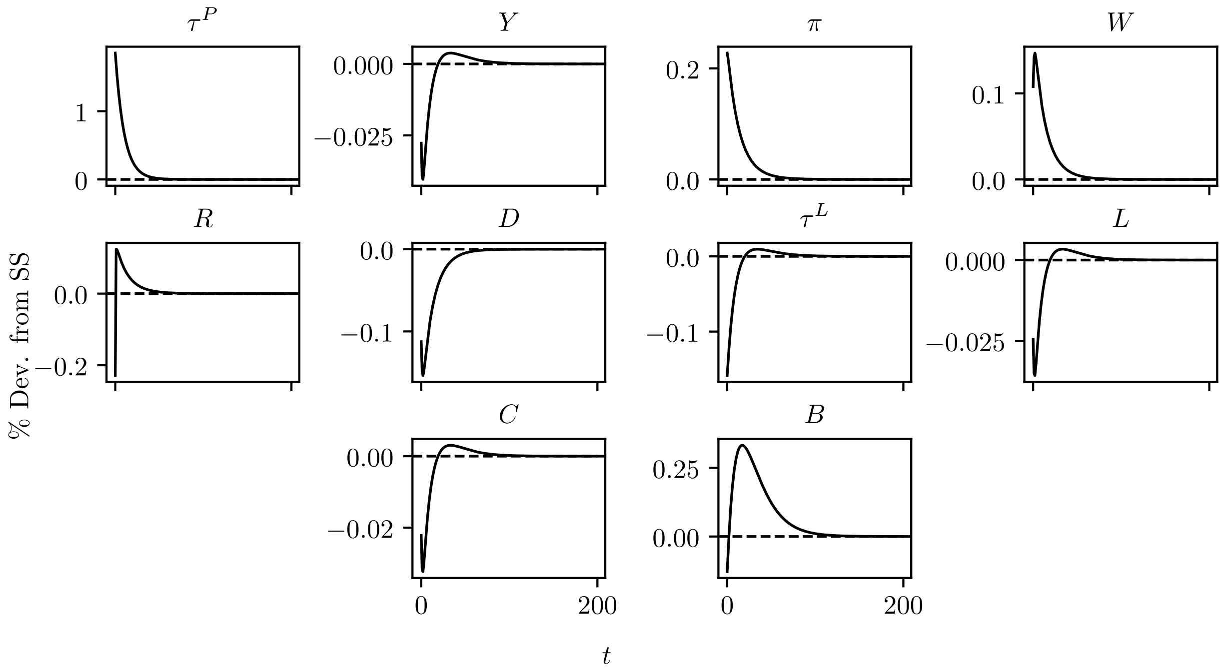
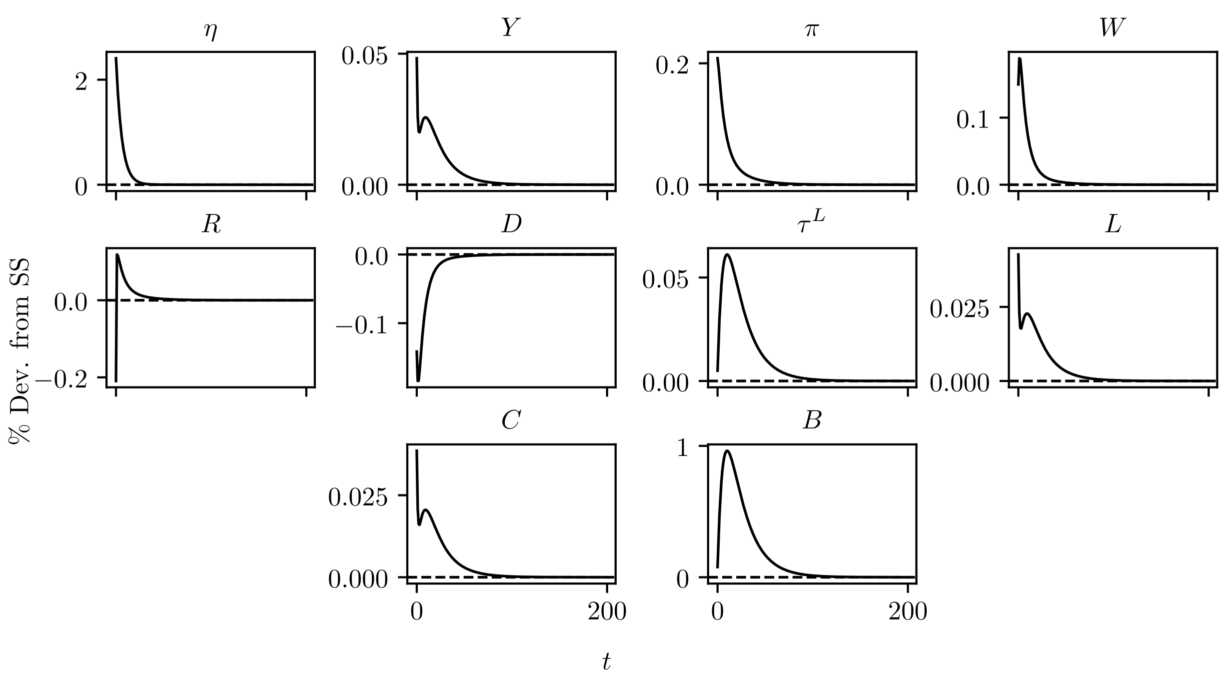


Figure E.7: Household Transfer (η) Shock Impulse Response Functions



F Forecast Error Variance Decomposition Calculation

To calculate the forecast error variance decomposition (FEVD) in the sequence space, I start with the moving average process from Auclert et al. (2021)

$$d\tilde{\mathbf{X}}_t = \sum_{s=0}^{\infty} d\mathbf{X}_s \epsilon_{t-s}$$

where $d\tilde{\mathbf{X}}_t$ is a vector of outcome differences from trend $d\tilde{x}_{j,t}$, $d\mathbf{X}_s$ is a matrix of impulse responses where the i-jth element represents the change in outcome j s periods after a shock to ϵ_i, $\dfrac{dx_{j,s}}{d\epsilon_{j,s}}$, and ϵ_t is a vector of iid shocks $\epsilon_{i,t}$ with diagonal variance-covariance matrix Σ. Assuming no effects from shocks at time t, we know

$$d\tilde{\mathbf{X}}_{t+h} - d\tilde{\mathbf{X}}_t = \sum_{s=0}^{h-1} d\mathbf{X}_s \epsilon_{t+h-s}.$$

This gets Mean Squared Error for output j of

$$\begin{aligned}
\mathrm{MSE}\left(d\tilde{x}_{j,t+h}\right) &= \mathbb{E}\left[\left(\sum_{s=0}^{h-1} dx_{j,s}\epsilon_{t+h-s}\right)^2\right] \\
&= \mathbb{E}\left[\sum_{s=0}^{h-1}\sum_{r=0}^{h-1} dx_{j,s}\epsilon_{t+h-s}\epsilon_{t+r-s}^{\top} dx_{j,r}^{\top}\right] \\
&= \sum_{s=0}^{h-1} dx_{j,s}\Sigma dx_{j,s}^{\top} \\
&= \mathrm{Var}\left(d\tilde{x}_{j,t+h}\right)
\end{aligned}$$

where the part of the variance coming from by shock i is

$$\sum_{s=0}^{h-1} dx_{ij,s}\sigma_i^2 dx_{ij,s}^{\top}.$$

Therefore, the FEVD is

$$\mathrm{FEVD}_{ij} = \frac{\sum_{s=0}^{h-1} dx_{ij,s}\sigma_i^2 dx_{ij,s}^{\top}}{\sum_{s=0}^{h-1} dx_{j,s}\Sigma dx_{j,s}^{\top}}.$$

G Household Decision Rules

Figure G.1: Household Decision Rules

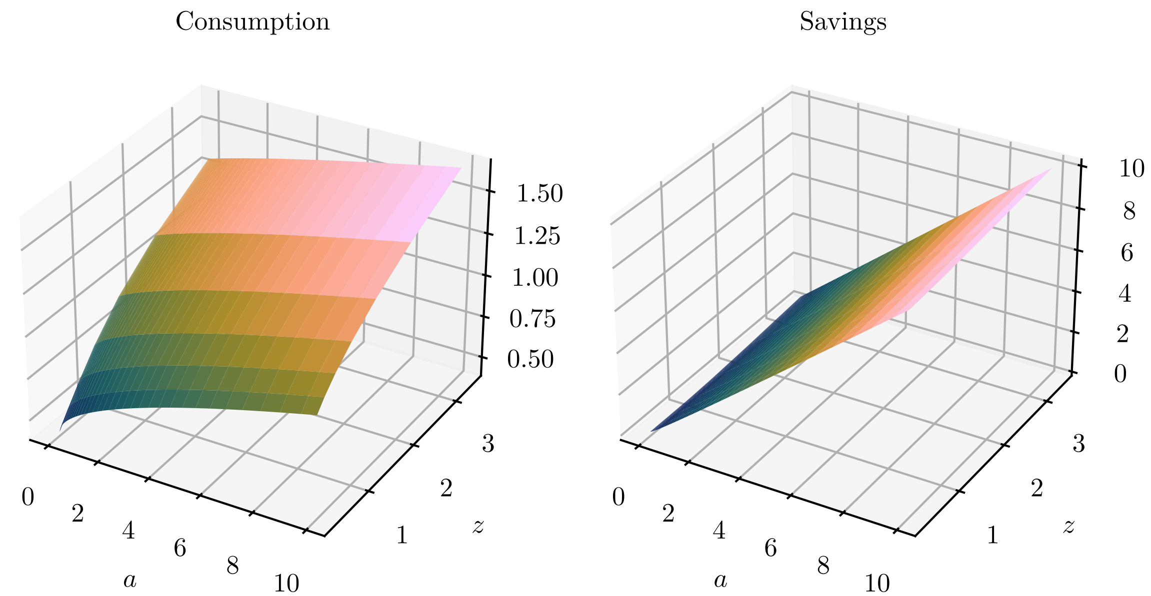


Figure G.2: Household Income Shares

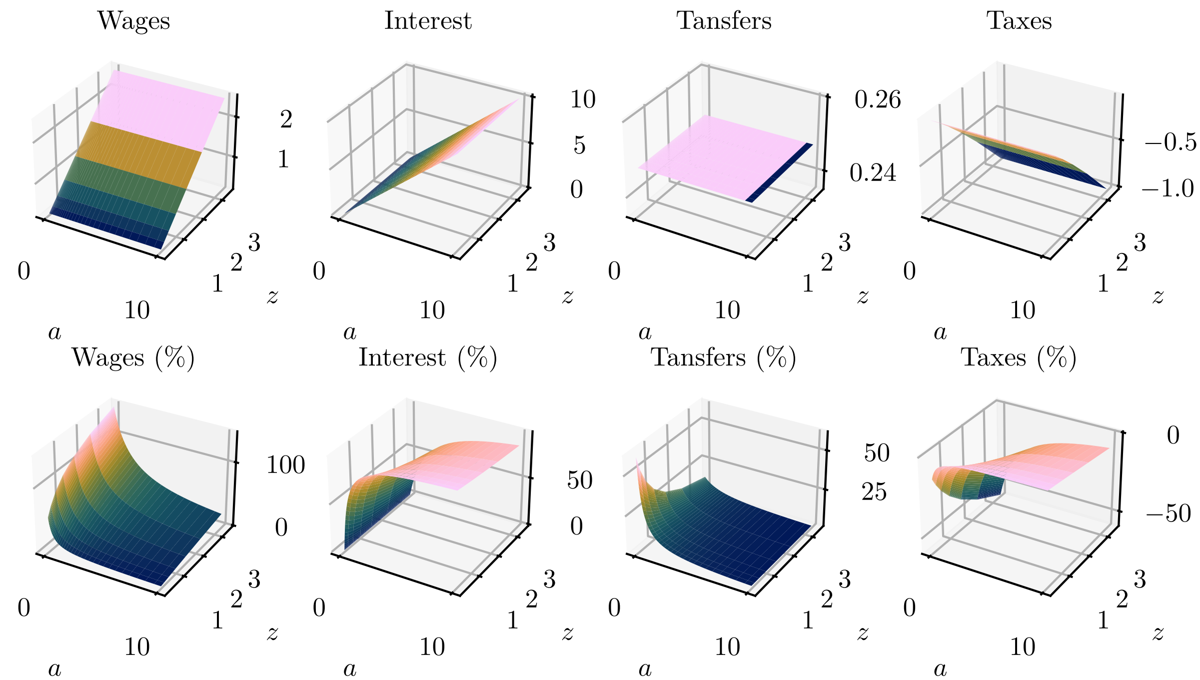


Figure G.3: Consumption Response to a TFP Shock

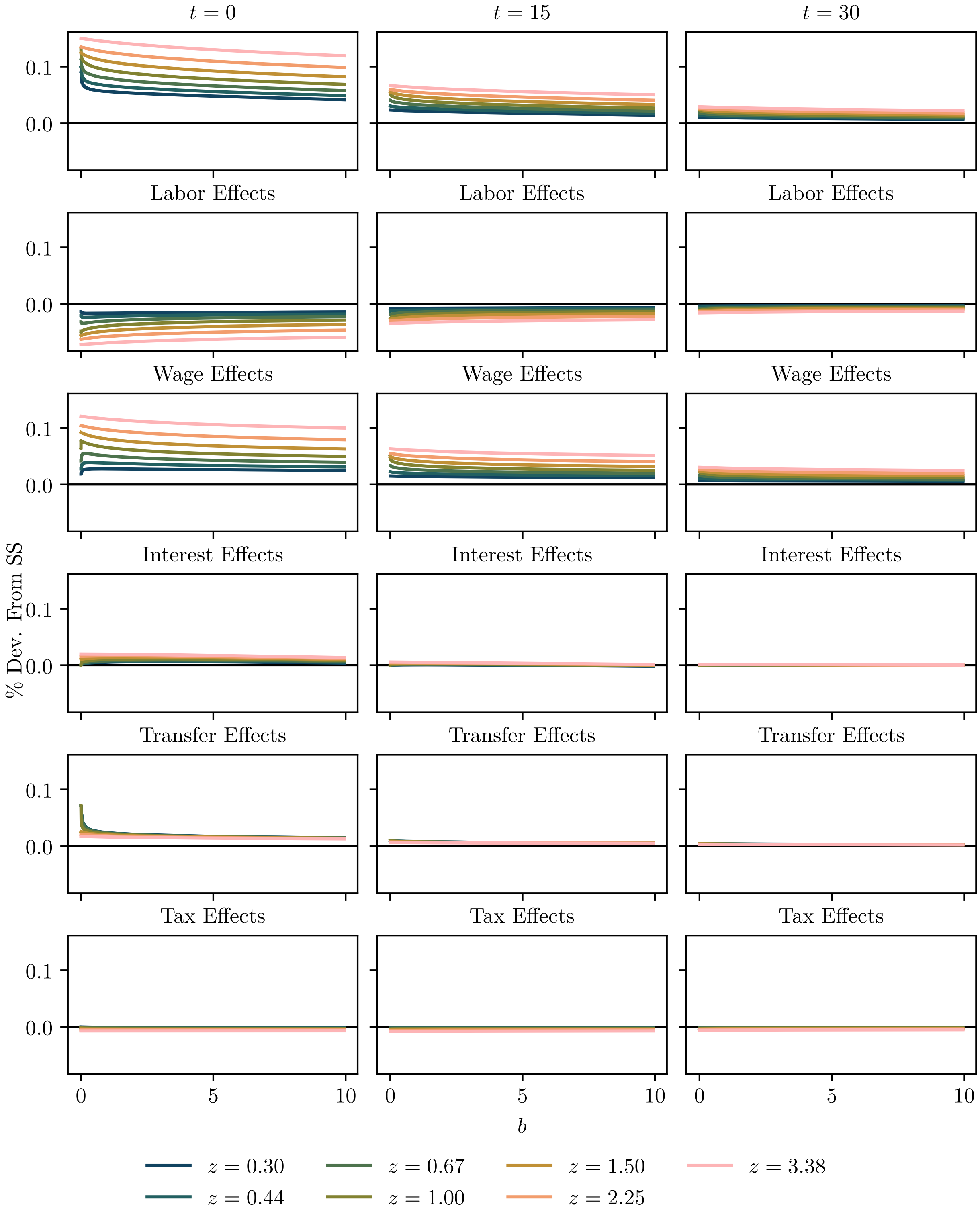


Figure G.4: Savings Response to a TFP Shock

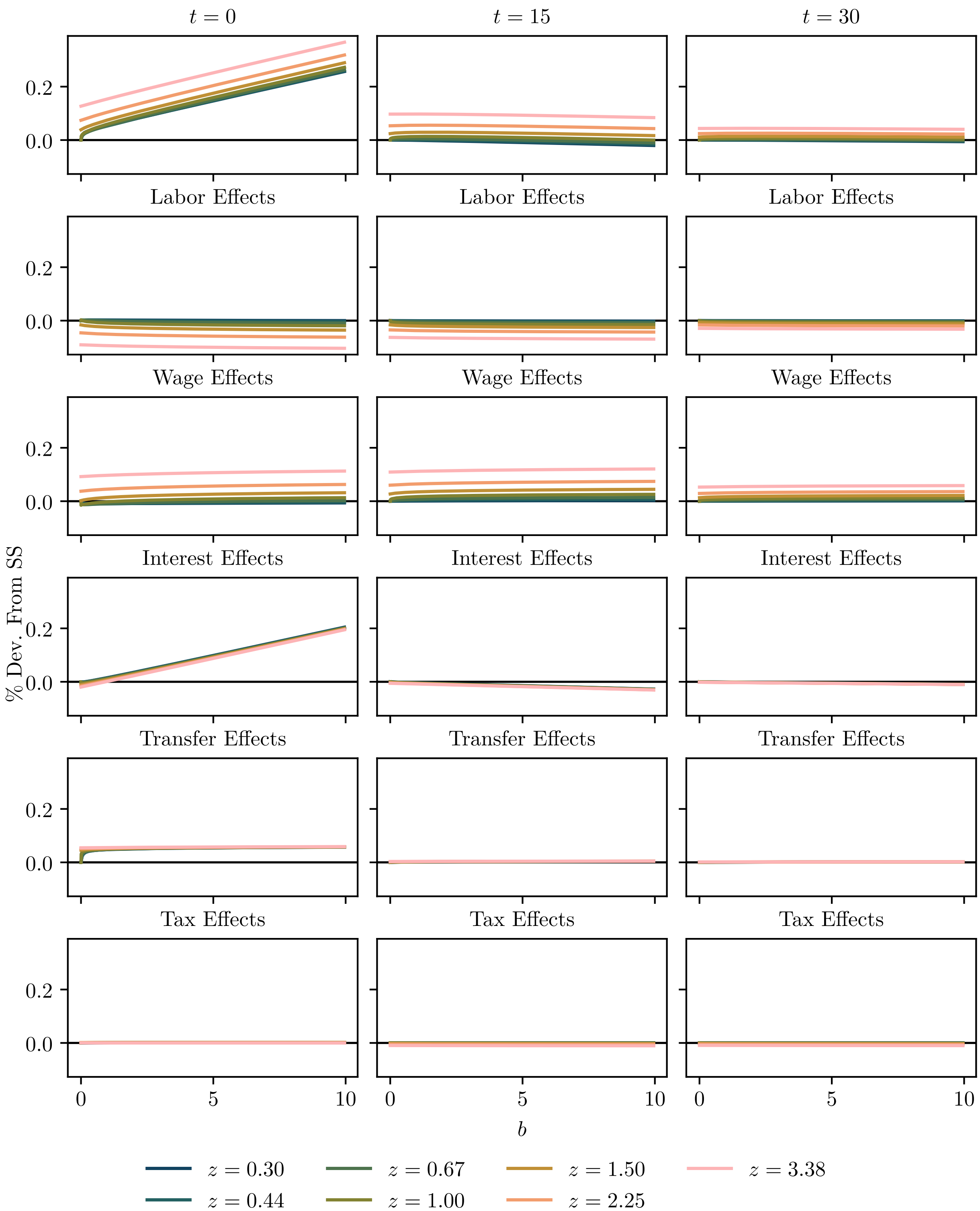


Figure G.5: Consumption Response to a Markup Shock

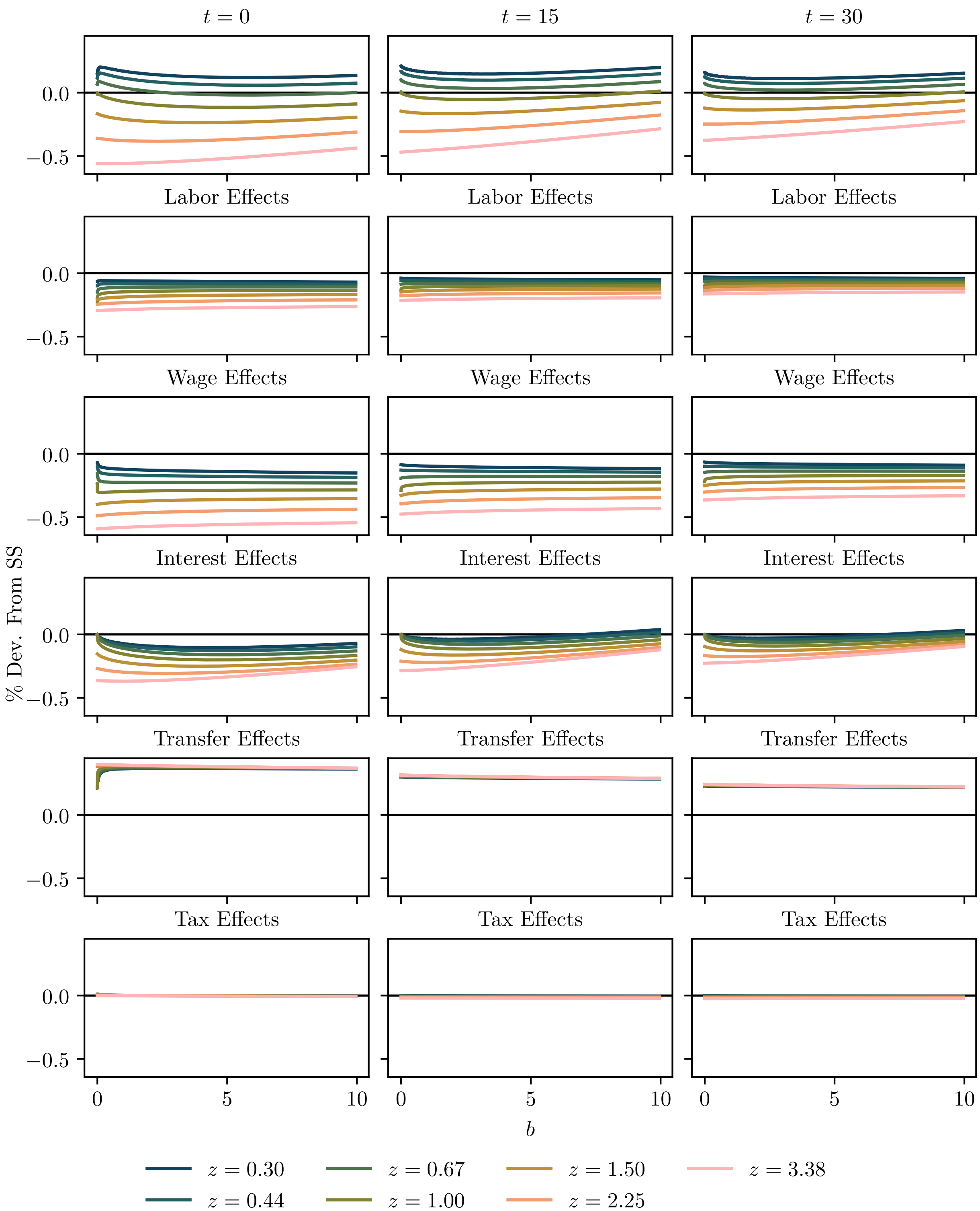


Figure G.6: Savings Response to a Markup Shock

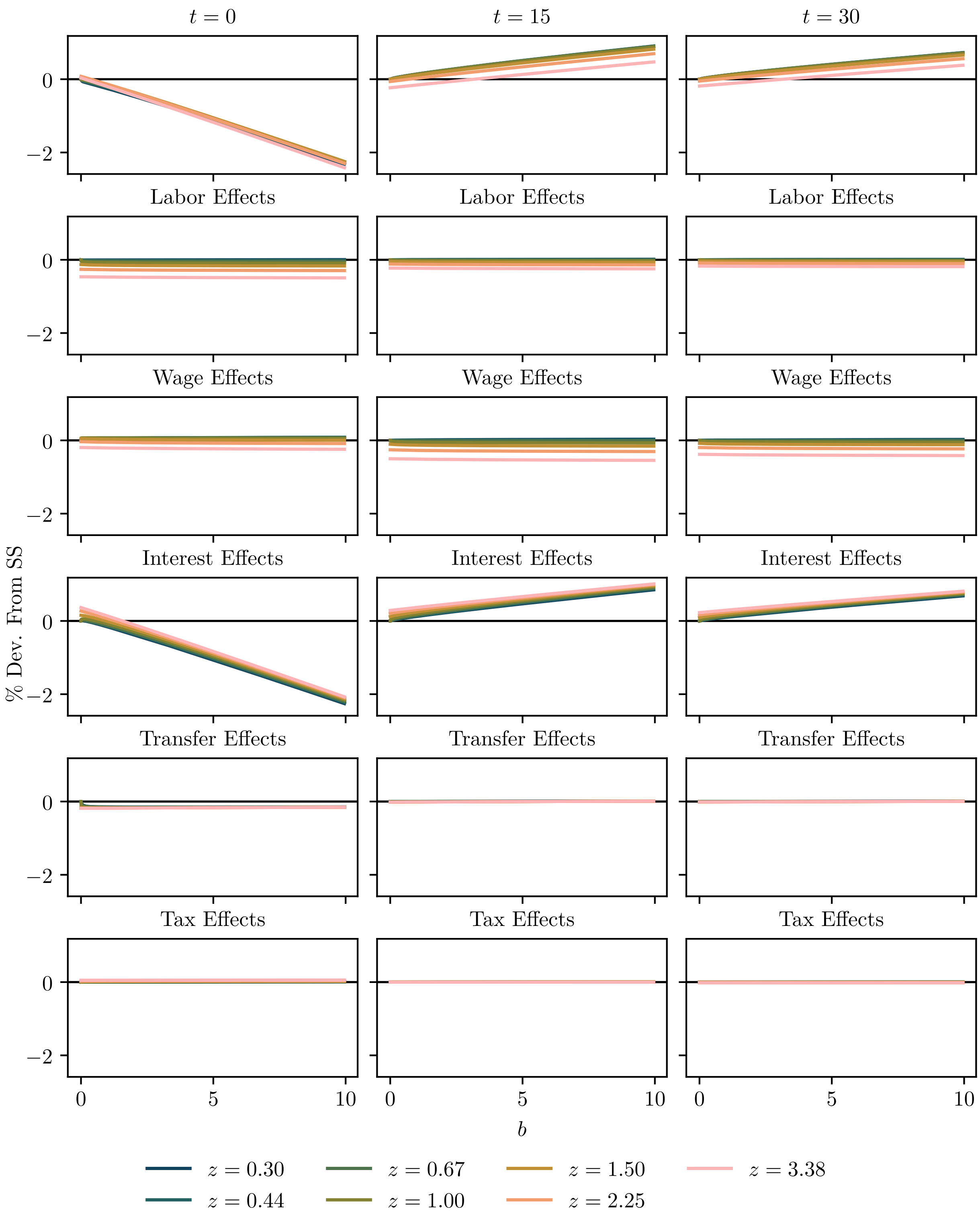


Figure G.7: Consumption Response to a Wage Markup Shock

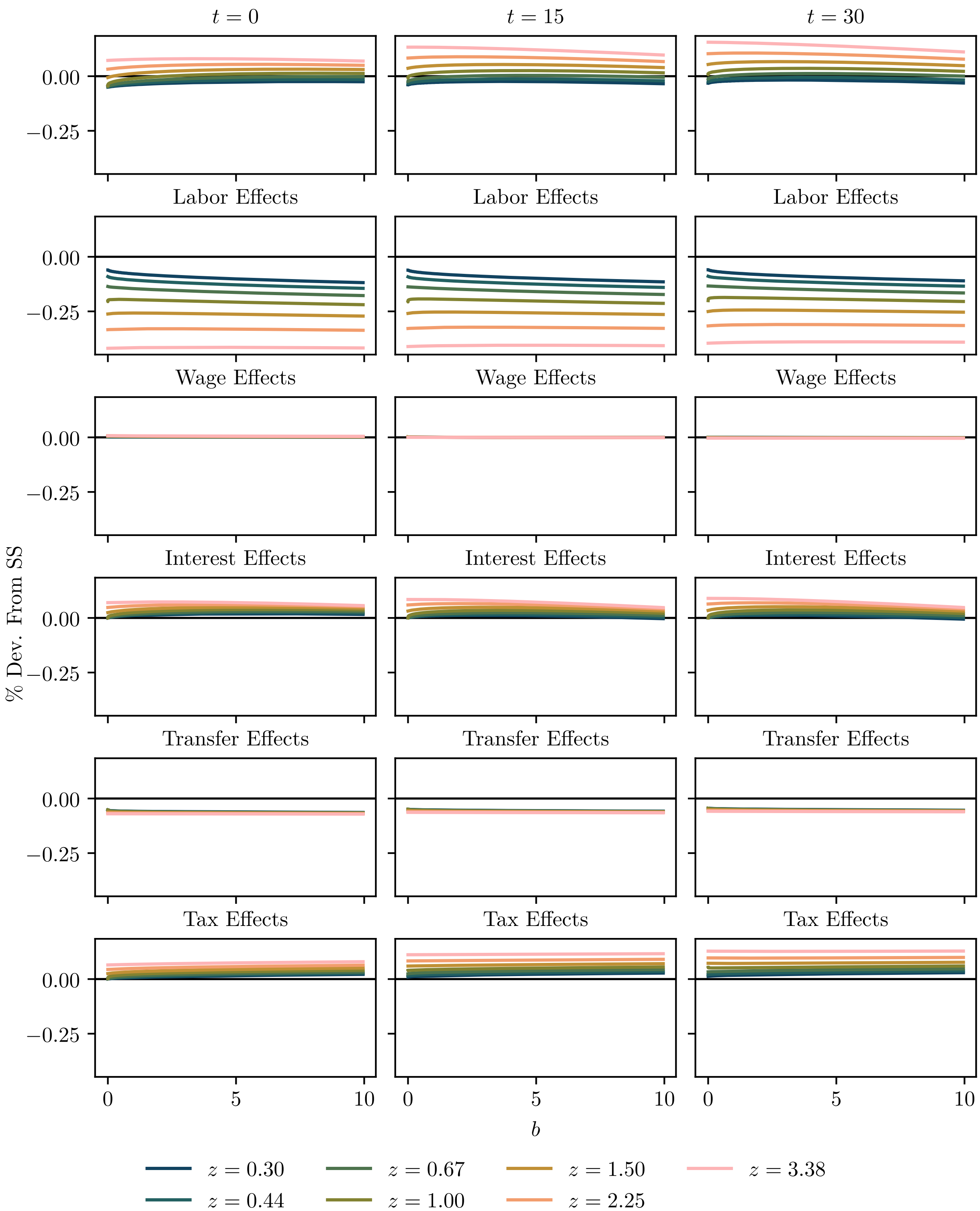


Figure G.8: Savings Response to a Wage Markup Shock

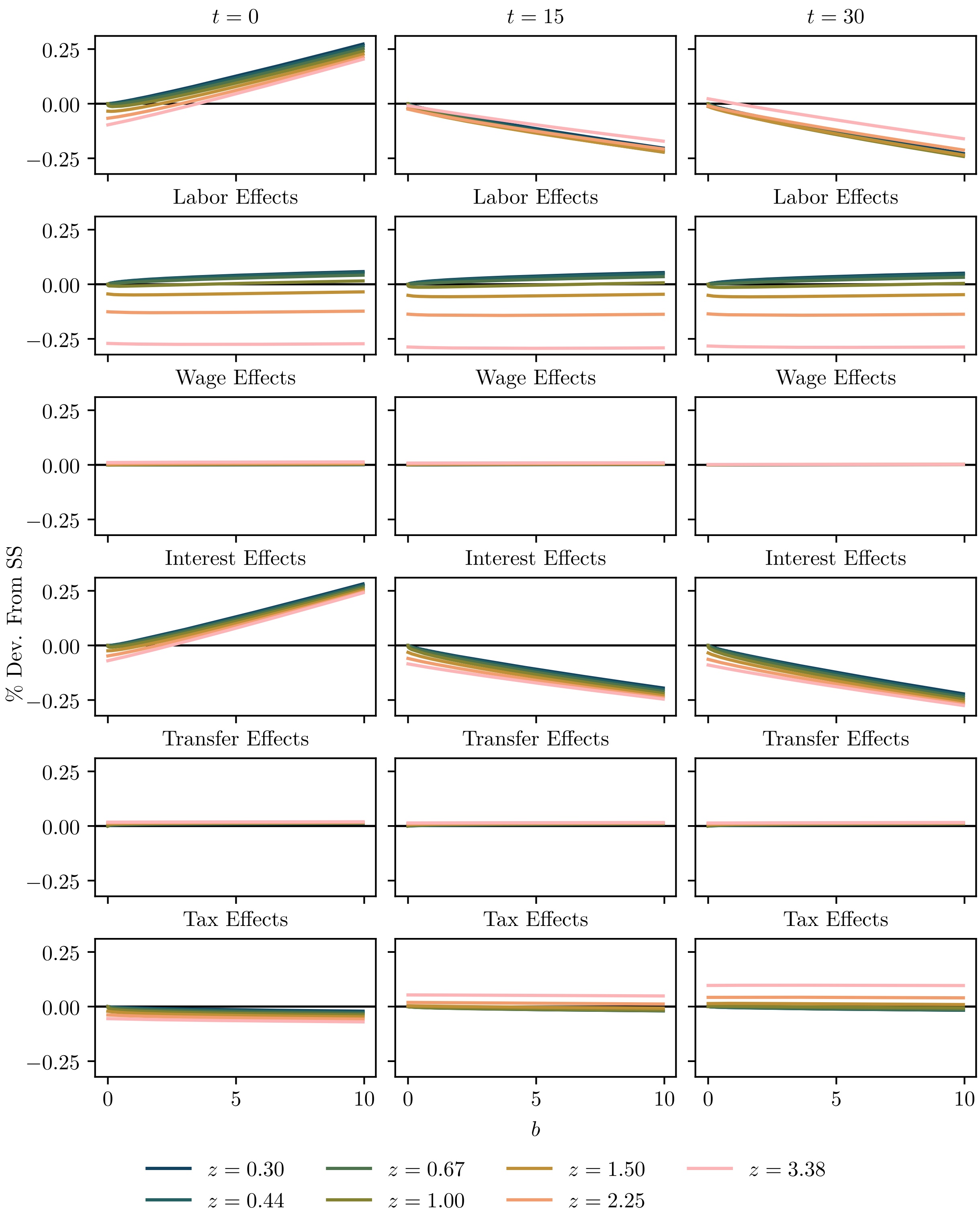


Figure G.9: Consumption Response to a Government Spending Shock

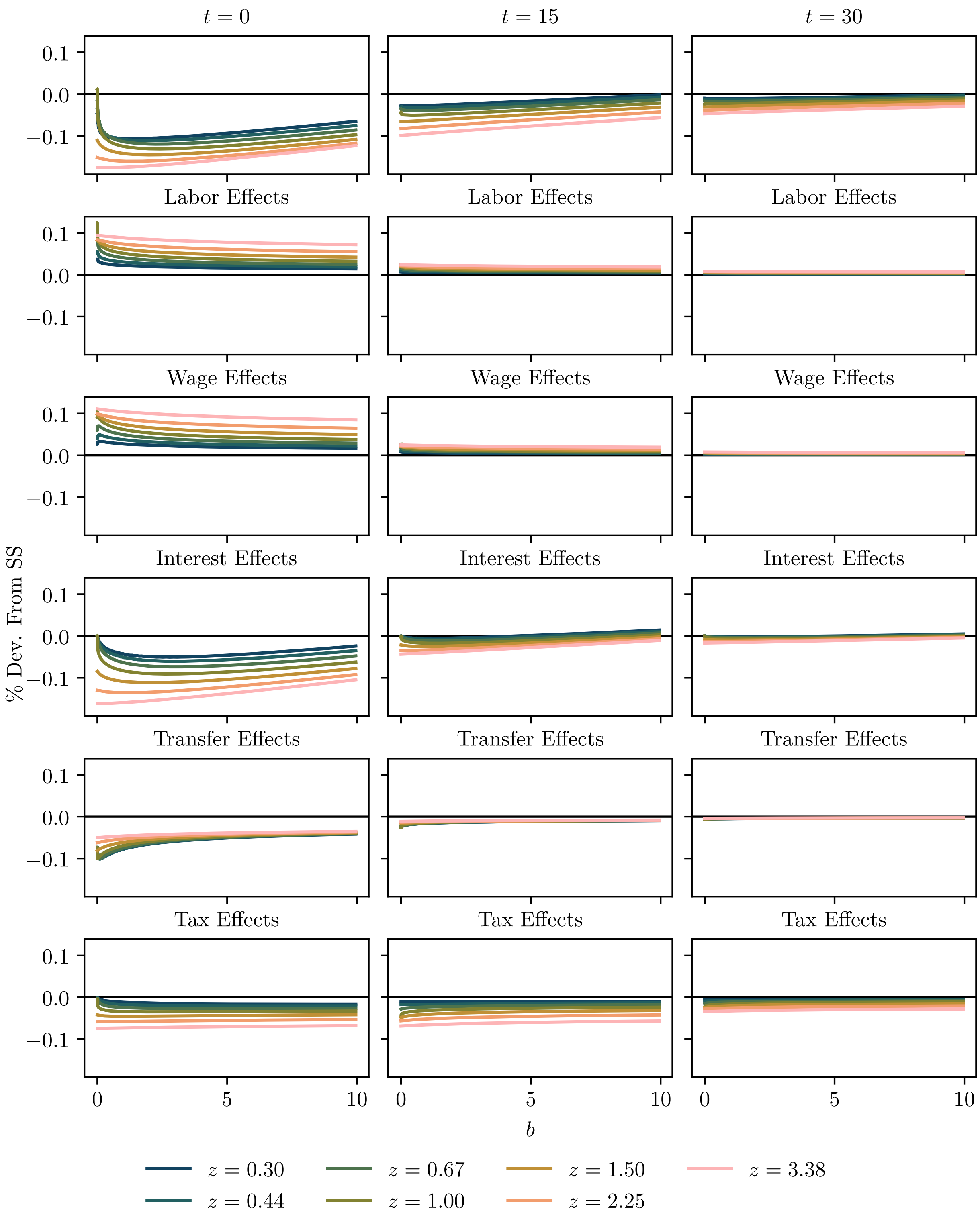


Figure G.10: Savings Response to a Government Spending Shock

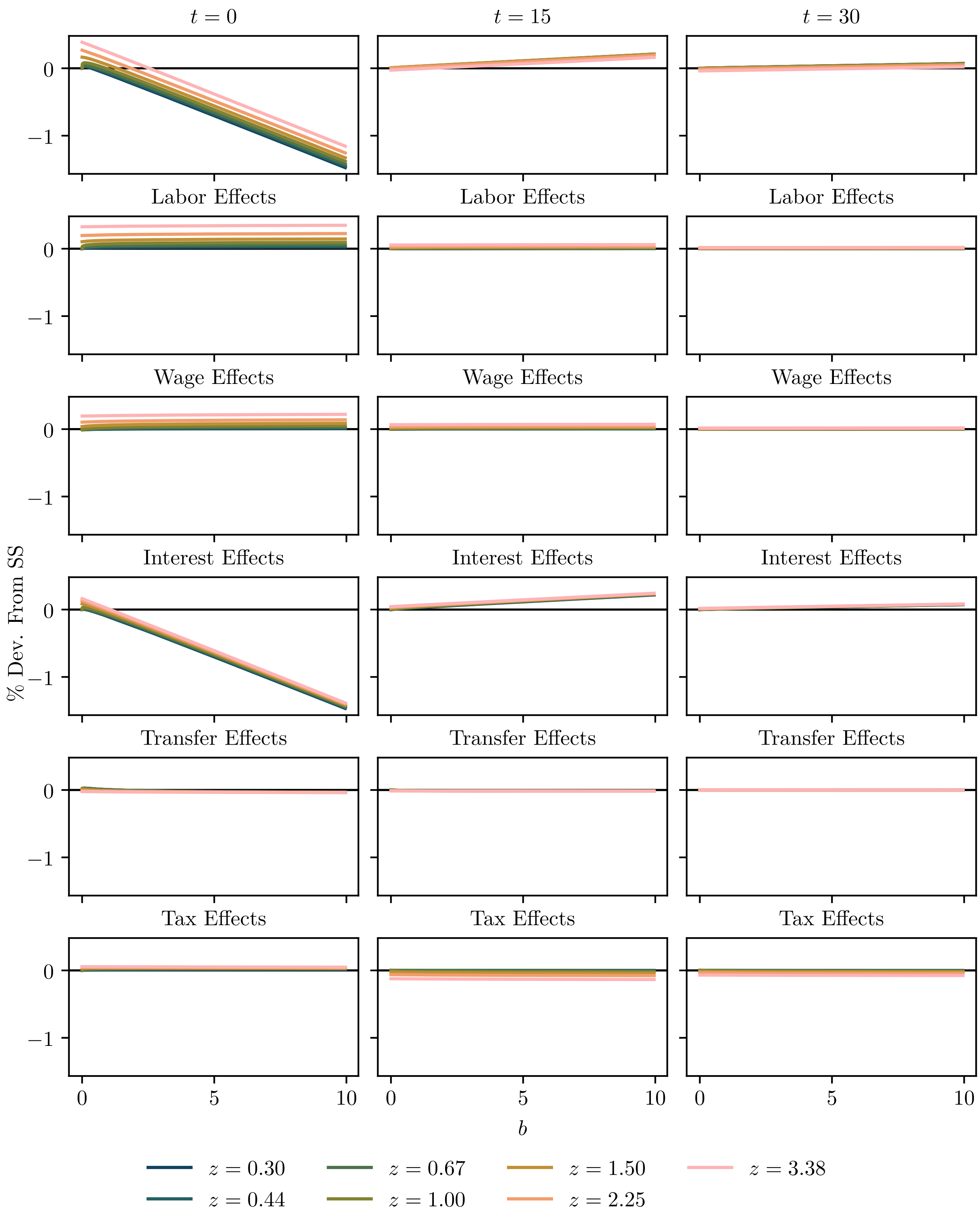


Figure G.11: Consumption Response to a Monetary Policy Shock

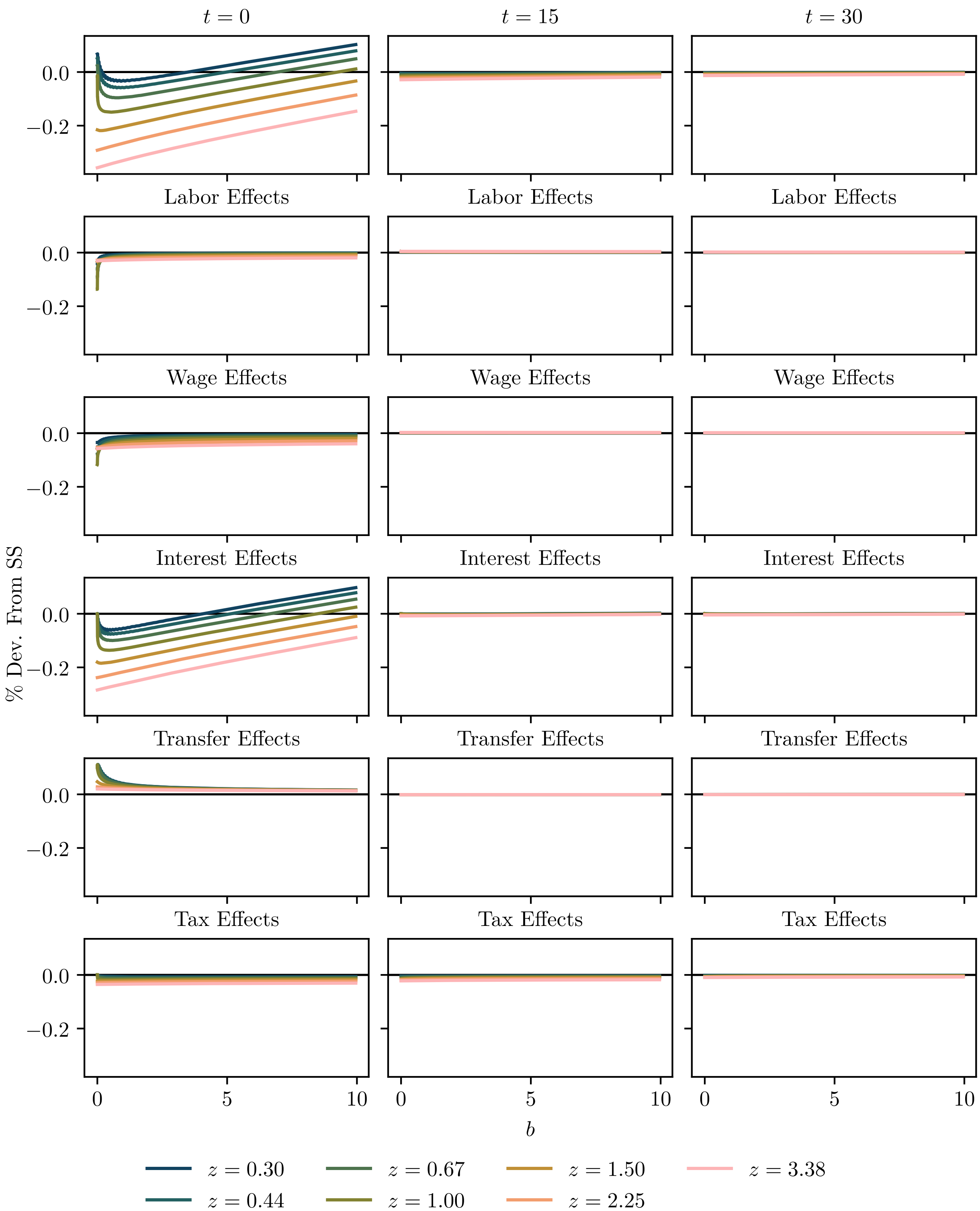


Figure G.12: Savings Response to a Monetary Policy Shock

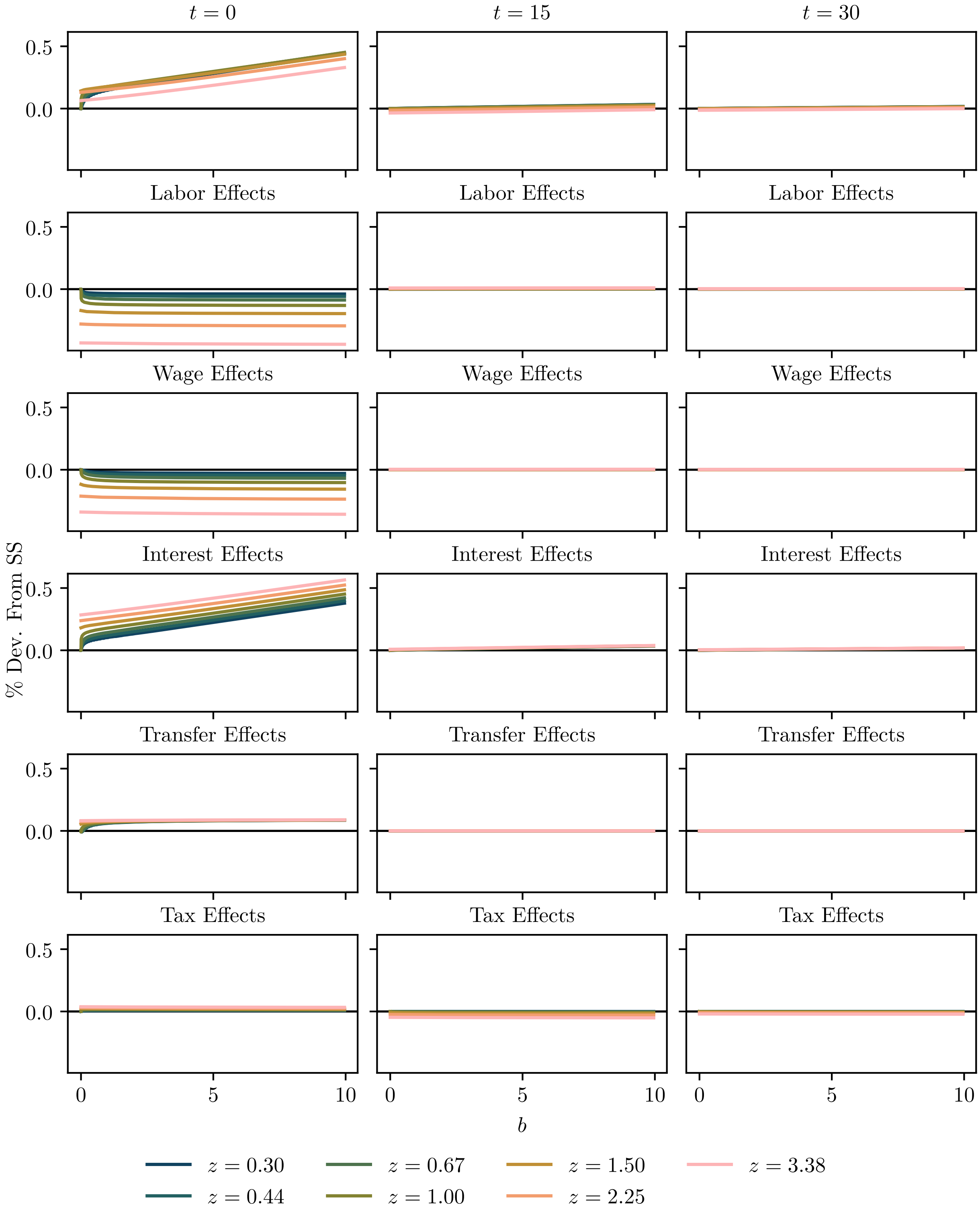


Figure G.13: Consumption Response to a Government Transfer Shock

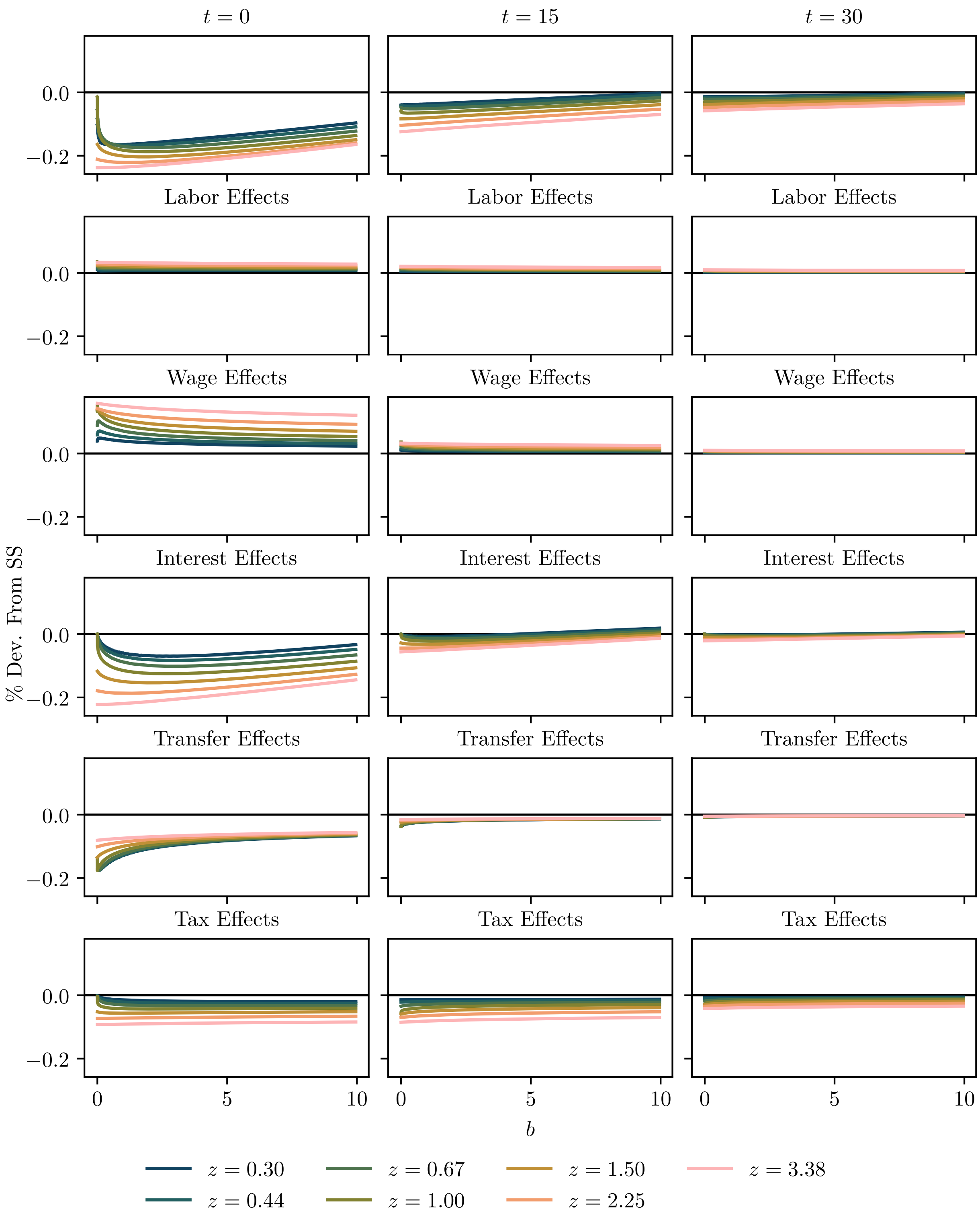


Figure G.14: Savings Response to a Government Transfer Shock

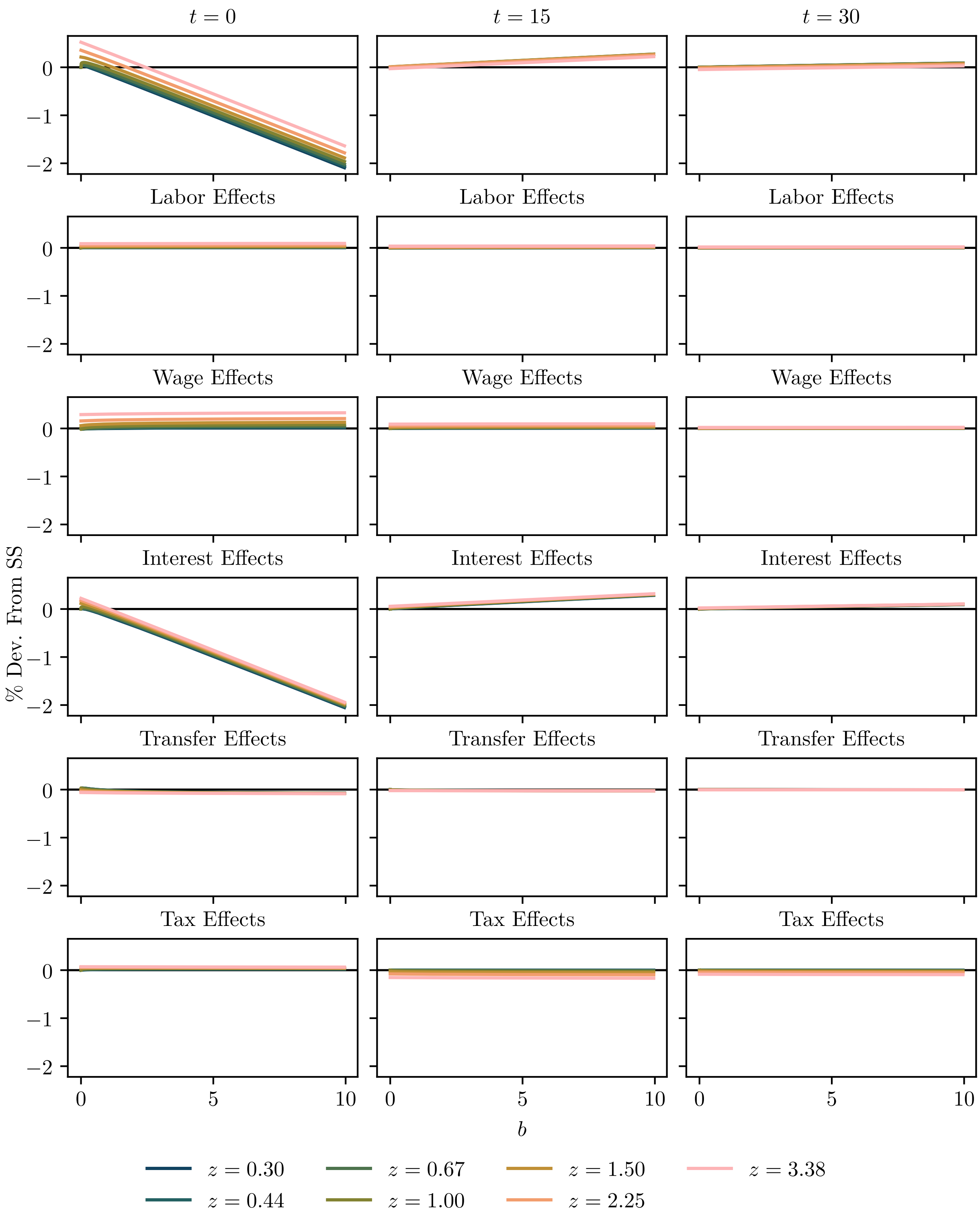


Figure G.15: Consumption Response to a Tax Progressivity Shock

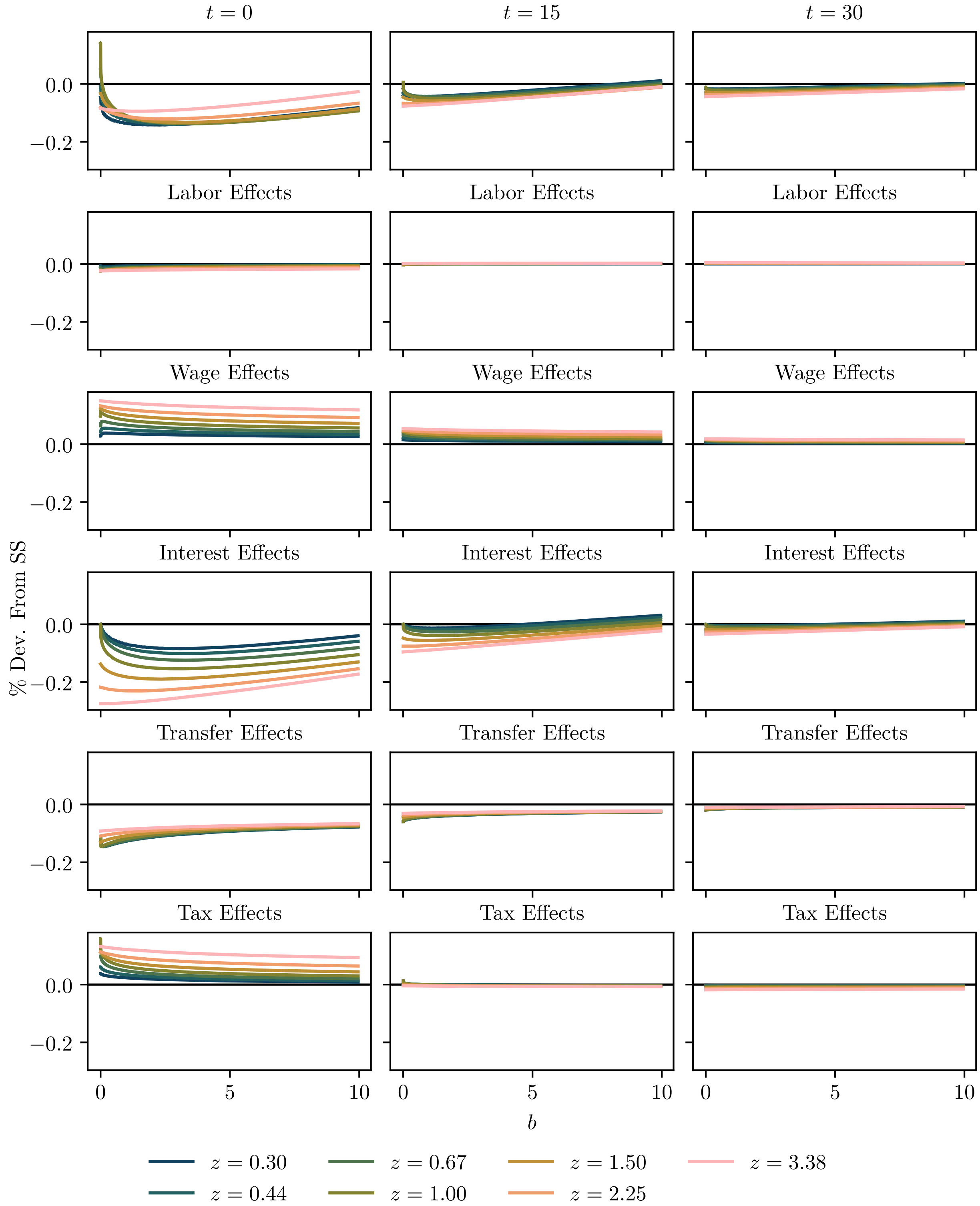
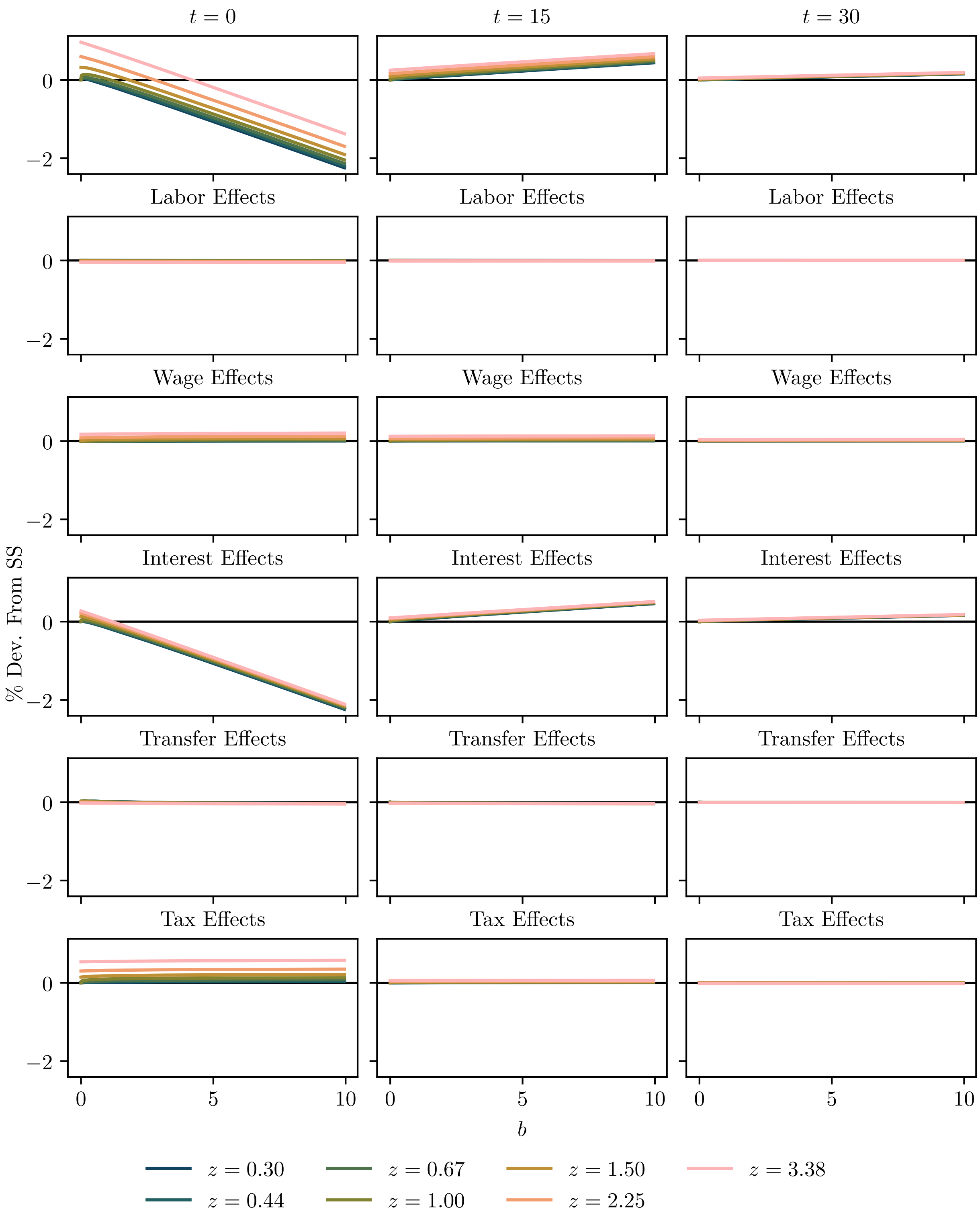
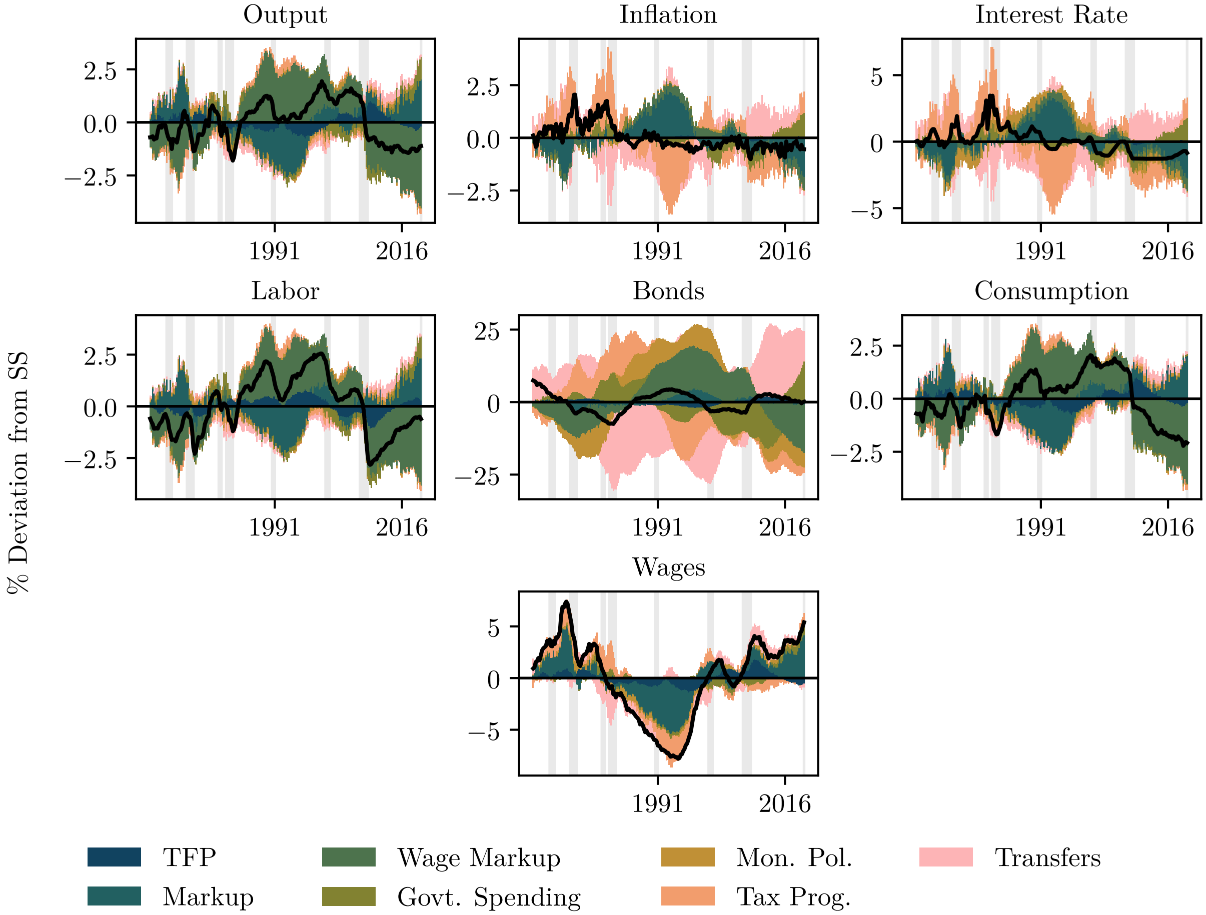


Figure G.16: Savings Response to a Tax Progressivity Shock



H Additional Historical Decompostions

Figure H.1: Fitted Historical Decompositions



Notes: NBER-dated recessions highlighted in gray.