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CS 510 Automata Theory
Homework 3

1. Prove the following language, $L_1$, is not regular using the pumping lemma. Assume $\Sigma = \{0, 1\}$. $L_1 = \{1^k y \mid y \in \Sigma^*$ and $y$ contains at most $k$ 1's for $k \geq 1$ $\}$

We have some number n
$w \in L_1$ $w = 1^n 0^n 1^n$
To prove $L_1$ is regular, these 3 steps must hold true while breaking $w$ into $xyz$
- $y \neq \epsilon$
- $|xy| \leq n$
- for each $k \leq 0$, $xy^k z$ is in $L$

This string $w$, forces y to be $1^i$ where $1 \leq$ i $\leq$ n, since $|xy| \leq n$
We can then not pump at all, but make k = 0
So then we are left with $1^{n-i} 0^n 1^n$
This is not in the Language $L_1$ because "$y$ contains at most $k$ 1's for $k \geq 1$" is not true
So $L_1$ is not regular

2. Prove the following language, $L_2$, is not regular. You may use the pumping lemma or closure properties.
$L_2 = \{w \mid w = x_1 0 x_2 0 ... x_{k-1} 0 x_k$ for $k \geq 0$ and each $x_i \in 1^*$ and $x_i \neq x_j$ for $i \neq j\}$

We have some number n
$w \in L_2$ $w = 1^n 0 1^{n+1} 0 ... 0^{2n-1} 0 1^{2n}$
To prove $L_2$ is regular, these 3 steps must hold true while breaking $w$ into $xyz$
- $y \neq \epsilon$
- $|xy| \leq n$
- for each $k \leq 0$, $xy^k z$ is in $L$

This string $w$, forces y to be $1^i$ where $1 \leq$ i $\leq$ n, since $|xy| \leq n$
So we are left with $xy1^a 0 1^{n+1} 0 ... 0 1^{2n-1} 0 1^{2n}$, where $0 \leq a \leq n-1$, because it is possible $x = \epsilon$ and $y = 1$
Because all possible stings of $y$ contain only 1's we can then pump $y^k$ to the point where $1^n < xy \leq 1^{2n}$, this makes $x_1$ equal to some $x_i$ from $x_2$ to $x_k$
This is not in the Language $L_2$ because "$x_i \neq x_j$ for $i \neq j$" is not true
So $L_2$ is not regular