

1.To show that  $\not\sim_L$  is an equivalence relation I must prove it is

Reflexive:  $aRa$  for all  $a \in X$

Symmetric:  $aRb$  implies  $bRa$  for all  $a,b \in X$

Transitive:  $aRb$  and  $bRc$  imply  $aRc$  for all  $a,b,c \in X$

$a \not\sim_L a$

If  $az \in L$  iff  $az \in L$  is true because  $az$  cannot both be in the language and not

$a \not\sim_L b$  and  $b \not\sim_L a$

If  $az \in L$  iff  $bz \in L$  then  $bz \in L$  iff  $az \in L$ , this is true because in the first case if there is no string  $z$  that can be concatenated to make  $a=b$  then there would still not be any sting  $z$  that can make  $b=a$

$a \not\sim_L b$  and  $b \not\sim_L c$  implies  $a \not\sim_L c$

If  $az_1 \in L$  iff  $bz_1 \in L$  and  $bz_2 \in L$  iff  $cz_2 \in L$ , because there is no string  $z_1$  that can make  $bz_1 \notin L$ , and no  $z_2$   $bz_2 \notin L$  than there is no string  $z_3$  that can make  $az_3 \in NL$  and  $cz_3 \in L$ , so  $a \not\sim_L c$

Example: is we have a Language  $L$  of 0,1 that only accepts strings that start with a 1.

If we have string  $a = 101$  then  $a \not\sim_L a$  is reflexive

If we have string  $b = 1$  then  $a \not\sim_L b$  implies  $b \not\sim_L a$  is true because there is no string  $z$  to make  $az$  and  $bz$  not start with 1.

If we have string  $c = 1111$   $a \not\sim_L b$  and  $b \not\sim_L c$  implies  $a \not\sim_L c$  because if there is no string  $z$  to make  $a$  and  $b$  not in language  $L$ , and no sting  $z$  to make  $b$  and  $c$  not in language  $L$ . Then there can't be any  $z$  to make  $a$  and  $c$  not in language  $L$ .

2.If there is a finite number of strings  $k$ , then there must be a limit to a length of a string I will call that  $H$ .

Let  $x$  and  $y$  be strings  $\leq H$ , and let  $L$  be any language of strings  $\leq H$ . We say that  $x$  and  $y$  are differentiable by  $L$  if some string  $z \leq H$ , exists whereby exactly one of the strings  $xz$  and  $yz$  is a member of  $L$ . Otherwise, for every string  $z \leq H$ ,  $xz \notin L$  iff  $yz \notin L$  and we say that  $x$  and  $y$  are in differentiable by  $L$ .

# HW1

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