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CS 510 Automata Theory
Homework 4

1. Consider the following grammar for a fragment of a programming language:

$S \rightarrow A|I|E$
$I \rightarrow if\ condition\ then\ S$
$E \rightarrow if\ condition\ then\ S\ else\ S$
$A \rightarrow assignment$

Where S represents a general statement, A an assignment statement, I an if-then statement and E and if-then-else statement. Anything that is not capitalized ("if", "condition", "then", "else", "assignment") is a single terminal symbol.

(a) Show that this grammar is ambiguous.

If we take a look at the string:

$if\ condition\ then\ if\ condition\ then\ S\ else\ S$

This string can be interpreted in two ways, aka is can be made with two parse trees. It can be made by:

$if\ condition\ then\ \{if\ condition\ then\ S\ else\ S\}$
$if\ condition\ then\ \{if\ condition\ then\ S\}\ else\ S$

This by definition is ambiguous

(b) Give a new unambiguous grammar for the same language and explain why your grammar is unambiguous.

$S \rightarrow I|E$
$I \rightarrow if\ condition\ then\ S \mid if\ condition\ then\ E\ else\ I$
$E \rightarrow if\ condition\ then\ E\ else\ E|A$
$A \rightarrow assignment$

Now whenever there is an *if then else* statement, there is no way to have an *if then* statement nested "inside", the only possibility is to have a *if then else* nested inside. This eliminates the second parse tree that was generated for part a, while including the first parse tree for part a. With this restriction we are still able to generate the same strings as before.

2. Let $\Sigma = \{0, 1, \#\}$. Let $L = \{x\#y | x, y \in \{0,1\}^*$ and $x \neq y$ Prove that L is context free by describing a PDA that accepts L.

Whe need to check that $x_k \neq y_k$ where $k \leq$ x.

q_o, it transitions to q_1 while pushing the bottom of the stack symbol.

q_1 it loops while reading a character of the input string and pushing a onto the stack, while reading nothing off the stack(a is just a counter). Nondeterministically it reads a character of the input string, and pushes that character (e.g 0 or 1), while still reading nothing off the stack, this transitions the machine to q_2.

q_2 it continues to read the rest of string x while not pushing or popping anything on/off the stack(so 0 or 1 is on top of the stack). When it reads #, it transitions to q_3, while not pushing or popping anything on/off the stack.

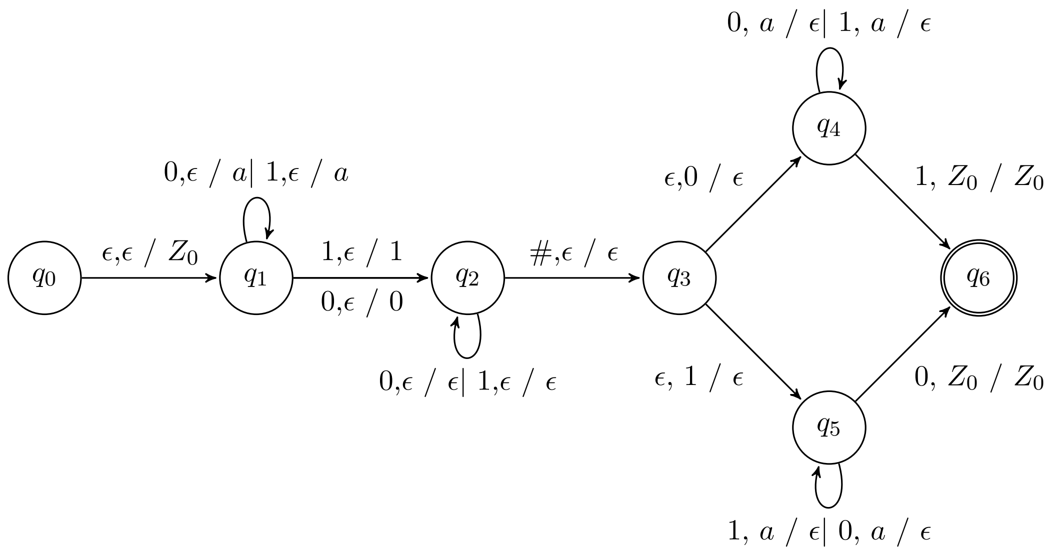
q_3 it then reads ϵ and pops, depending on a 0 or 1 it goes to state q_4 or q_5 accordingly.

q_4 or q_5 it reads a character of string and pops a off the stack. If the string becomes empty before the stack, go to q_6. Otherwise when you pop the Z_0 symbol, depending on what state your in either go to accepting state q_6 or stay in current state never able to accept.

q_6 accept.

P $= (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$
$\Sigma = \{0, 1, \#\}$
$\Gamma = \{0, 1, a, Z_0\}$
$\delta =$ the behavior of the machine bellow
$q_0 = q_0$
$Z_0 = Z_0$
$F = \{q_6\}$


0, a / ϵ| 1, a / ϵ
q_4
0,ϵ / a| 1,ϵ / a
ϵ,0 / ϵ
1, Z_0 / Z_0
ϵ,ϵ / Z_0
1,ϵ / 1
#,ϵ / ϵ
q_0
q_1
q_2
q_3
q_6
0,ϵ / 0
0,ϵ / ϵ| 1,ϵ / ϵ
ϵ, 1 / ϵ
0, Z_0 / Z_0
q_5
1, a / ϵ| 0, a / ϵ