Risk Management Institute

MSc in Financial Engineering

Mixing vs. Integrating: An Empirical Study on Factor and Alpha Signal Generation in Systematic Investing

FE5110 (2240) - Financial Engineering Project

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Appendix 1 – Sample Cover Page

Please add the following information to your cover page of the final report. Project Title	*Mixing vs. Integrating: An Empirical Study on Factor and Alpha Signal Generation in Systematic Investing*
Name	*HUI JIA SHUN*
Student ID	*A0250406R*
Itemized workload	
Data collection	*4 hours (wasted around 20 hours on a topic of systematic fixed income when there are insufficient data points to conduct the study effectively)*
Literature review	*40 hours running through research papers and reading books*
Programming	*30 hours coding, 20 hours running results*
Any other workload	*Analysis and Report writing - 30 hours*
Total workload	*124 hours (excluding time wasted on original topic of systematic fixed income)*
Original contribution	*This project has the following original contributions:* - Project inspiration comes from S2E8 of the Flirting with Models Podcast (Liqian Ren – In Search of Modern Alpha) - Data was extracted by me from MSCI's Barra Portfolio Manager platform. Descriptor data is generated by Barra. - Built full stack quant library containing data cleaning pipeline, portfolio construction library, portfolio analytics library and back testing/performance library on python from scratch. - Results are mainly generated from the dataset by me. - The analysis and insights presented were generated independently by me.

Abstract

This paper presents an empirical examination of two portfolio construction strategies for factor-based investing, namely the "integrated" and the "mixing" approach. Using extensive financial data, we ran a comparative analysis for the performance of these two methods across several dimensions, including returns, risk-adjusted returns, intended factor tilts and potential collinearity. The study found that the mixing approach yielded a higher return along with higher volatility and increased risk of collinearity. In contrast, the integrated approach led to higher risk-adjusted returns largely due to lower volatility. These findings lead us to believe that the optimal strategy will largely be dependent on the constraints and requirements of the portfolio manager. Our findings revealed that the mixing approach may lead to unintended factor exposure, inhibiting the manager’s ability to accurately express his market conviction. Also, the integrated approach may offer potential benefits in managing turnover. This work provides insights for systematic managers both in the asset management and the hedge fund space to optimize their factor or alpha based strategies.

Acknowledgments

I would like to extend my heartfelt appreciation to Vanessa for her unwavering support throughout this study. My appreciation also goes to my fellow team members at MSCI, who not only make my work enjoyable but also helped me discover my passion and my life's purpose. Special thanks to Ming Fang and Jing Woo for the innumerable sacrifices you've made on my behalf. Your selflessness has not gone unnoticed.

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Introduction

Introduction to Quantitative Finance and its Primary Domain

The field of quantitative finance, a discipline dedicated to applying mathematical and statistical concepts to financial markets has become an integral part of the global markets.

The discipline can be deconstructed into two primary domains, the p-quant, and the q-quant space. For a p-quant, the focus will primarily be modelling with the intent to describe real-world probabilities. These types of quants will build models to capture risk and design trading strategies.

On the other hand, the q-quant is focused on “risk-neutral probabilities”. A theoretical concept that is used to calculate the expected future values of financial instruments. Such mathematical techniques used are largely applied to the derivative space, under the assumption that all investors are ambivalent to risk.

In this research paper, our primary emphasis will be on the domain of p-quant. Interest in quantitative strategies within the investment realm has dramatically escalated over the last ten years. These strategies are particularly enticing due to their capacity to generate diversified return streams, potentially offer compelling risk-adjusted returns, and, most recently, navigate drawdowns with a focus on tail hedging. Such attributes are highly valued by allocators, including pension funds and asset owners. The primary reason is that these stakeholders prefer to evade correlated return streams from their external managers. Moreover, they are persistently in pursuit of robust risk-adjusted returns.

Key Market Participants and Factor-Based Strategies

When it comes to quantitative strategies, the buy-side, mainly hedge funds and asset managers have traditionally been the key market participants, and more recently, asset owners have also started rolling out systematic mandates. One of the frameworks widely adopted in quantitative strategies on the buy-side is the concept of factors. factors are attributes that explain the differences in returns of different securities. Factors such as company size, value, momentum, volatility, and quality are widely recognized as systematic sources of risk that have historically commanded risk premia over the long term. Understanding these factors and harnessing their potential can be a powerful tool for portfolio construction, allowing investors to create portfolios that deliver strong risk-adjusted returns. Each factor carries the potential to yield considerable returns over an extended period, but there's no assurance that they will always be successful. Chow, Li and Shim [2017] saw that during and in the aftermath of the global financial crisis, for example, the U.S. value factor saw a total loss of 12.0% in 2007 and 2008, and the U.S. momentum factor experienced a 52.6% downturn in 2009. Given this, many investors have started diversifying their active risk positions by pursuing strategies that capitalize on multiple factors.

Hedge funds primarily research and develop their own factors that are not covered by the traditional known factors. They would call these alpha signals. On the other hand, asset managers would have mandates in running a strategy to harvest long-term risk premia from these traditional known factors. These are called smart beta strategies.

Process of Implementing Quantitative Strategies

The process of implementing quantitative strategies usually initiates with the phase of idea generation. During this stage, a hypothesis or concept is developed, often aimed at capitalizing on market anomalies. These anomalies can originate from informational, behavioural, analytical, or structural advantages that an investment manager may possess. Following this, the workflow proceeds to the data collection phase, which involves gathering pertinent data. This data can encompass a variety of types, including price, economic, fundamental, and alternative data.

A sequence of data manipulations is then carried out, involving the management of missing data and outliers, and transforming them into a signal. This is followed by backtesting to get an understanding of how the investment strategy has performed historically, thus building confidence in its effectiveness. Once these tests yield satisfactory results, the signal is moved into production to run a live portfolio. The portfolio manager will continuously supervise this strategy while simultaneously conducting evaluations and making necessary adjustments.

When it comes to integrating a signal into a live portfolio, there are typically two methods to achieve this. The first approach is based on heuristics, in which a predefined set of rules is applied to the portfolio to establish the weights of its components. Various design choices made by the user could impact this, including the selection of the universe, the weighting decision (whether equal or market capitalization-weighted), the constraints, and the objectives.

The second method involves utilizing an optimizer to build the portfolio. The idea here is to define an objective function that aims to amplify the signal while mitigating risk. This results in optimized weights that shape the ideal portfolio. By using a more sophisticated optimizer that employs convex optimization, users can define limitations and multiple objectives to arrive at a viable solution.

In this report, due to limitations on having access to a reliable optimizer, we will mainly leverage the heuristic approach in constructing our portfolios.

Role of Signals in Portfolio Performance

The signals employed in portfolio construction significantly influence the portfolio's actual performance. A dependable signal usually comprises an amalgamation of several uncorrelated signals to enhance confidence in the indicator. A basic rule of thumb is that these combined signals should be economically logical and exhibit low correlation with each other. From the standpoint of a hedge fund manager, they might leverage the combination of multiple signals to create an alpha signal that detects market irregularities or trends. Conversely, an asset manager might construct a multi-factor portfolio by combining various factor exposures into a single portfolio.

Approaches to Combining Signals and Constructing Portfolios

There are really two schools of thought for the approach to combining signals and subsequently constructing the portfolio. The first approach is the mixing approach. This approach is done by defining factors individually and implementing individual portfolios on each factor. This would be a signal that will be implemented on different sleeves of the portfolio. An example when a portfolio manager is looking to construct a value and momentum portfolio, he would primarily construct the first sleeve of the portfolio using value indicators and the second portfolio sleeve using a momentum-based indicator. Then the manager will go through a selection heuristically to determine the rules to incorporate into the portfolio construction (i.e., weighting scheme). The second method employed is the integrated approach, wherein we assign subjective weights to the emphasized signals and combine them accordingly while adhering to the constraint:

$$\Sigma W_i = 1$$

where:

Σ denotes the sum, W_i represents each individual weight for the signals, and the sum of all these weights (W_i) equals 1.

An additional illustration of the integrated approach, when applied to the combination of value and momentum signals using equal weights, can be seen as follows:

$$Combined\ Signal = 0.5 * Value + 0.5 * Momentum$$

Through this research, the study provides valuable insights into the dynamics of factor investing and signal generation, offering quantitative managers, asset managers, and hedge funds a robust understanding of the relative strengths and weaknesses of the mixing and integrating approaches. Given the increased prevalence of systematic and factor-based investing strategies, these findings hold critical implications for investment outcomes and risk management.

Impact on Research Findings on Hedge Funds and Asset Managers

1. **Hedge Funds:** For quantitative managers in hedge funds who design and implement algorithms and models to aid in the investment decision-making process, the study's findings offer a roadmap for refining strategies. It provides a nuanced understanding of how to best generate factor signals, which could significantly improve the accuracy and efficacy of their quantitative models. This, in turn, could lead to improved trading decisions and ultimately, enhanced factor/alpha capture. Furthermore, the study's insights could also enable hedge funds to manage systematic risk more efficiently, thereby providing a competitive edge in the marketplace.

2. **Asset Managers:** Asset managers, who make investment decisions across a variety of securities and asset classes, can use the insights from this study to optimize their portfolio construction and management processes. The findings can guide them in balancing their factor exposures more effectively, managing risk more prudently, and delivering more consistent returns to their clients.

Overall, the study provides an empirical framework for investment professionals to navigate the complexities of factor/alpha signal generation in systematic investing. This new understanding can ultimately lead to potentially improved performance, risk management, and turnover awareness.

The study is structured in the following way: initially, we navigate through a literature review, scrutinizing previous studies from both an academic and a practitioner's standpoint. Subsequently, we delve into the methodology devised for this empirical investigation, highlighting key areas like the source of data, how missing data and outliers are handled, and the standardization (z-score) of factor signals. Afterward, we clarify the empirical framework utilized in this study before moving on to the interpretation of the results and comparison. A subsequent discussion explores the potential implications of these results. The study concludes with a final summation of our findings.

Literature Review

Fama and French [1993] introduced the three-factor model consisting of size, value, and the market factor. This is an expansion from the CAPM model, and the three-factor model seeks to explain the higher risk-adjusted returns coming from the three factors. Where small companies typically outperform large companies and lower-valuation companies outperform those that are highly valued. Subsequently, Fama and French [2015] further expanded the factor model to capture size, value, profitability and investment in average stock returns. The five-factor provided a more satisfactory description of average returns than the three-factor model. However, the research acknowledges that the result might be specific to the sample used and warrants further exploration.

Gupta, Lodh, and Barman [2020] delved into an optimal framework that revolves around a heuristic approach for constructing single-factor portfolios. According to the study, building a factor portfolio using multiple descriptors offers numerous potential advantages. These include the ability to better define a factor and mitigate the risks associated with events, industry shifts, and market cycles that could arise when a single descriptor is employed. The study further argues that an enhanced and robust metric can be achieved by averaging across multiple fundamental and technical measures. For instance, when trying to capture a value factor, reliance on both fundamental and technical descriptors can lead to a more calibrated approach. Moreover, the authors assert that while augmenting factor exposures often comes with increased concentration risk, constructing a multi-factor portfolio can alleviate the impact of such portfolio concentration.

Menchero and Lee [2014] dived into several approaches for capturing the return premium for alpha factors. Mainly using simple factor portfolios where the portfolio is constructed directly by taking long positions in stocks with high exposure to the factor and short positions in stocks with negative exposure. This approach does not account for the degree of collinearity among factors and will create inadvertent tilts as well. Another approach is a pure factor portfolio where a factor portfolio will have a unit exposure to the factor in question and zero exposure to all factors. A pure factor portfolio will disentangle the confounding effects of collinearity. The final construction style is a minimum volatility factor portfolio where there is a unit exposure to the factor but at the same time, conduct mean-variance optimization using the forecasted risk to account for the interaction effects amongst securities. The paper concluded that the optimal method for combining multiple sources of alpha is through a weighted combination of minimum volatility factor portfolios for each alpha signal or in the context of this paper, a mixing approach but each sleeve is constructed using a mean-variance optimization.

Bender and Wang [2016] discuss two approaches to multi-factor portfolio construction: the combination (we use the term integrated for this report) approach and the bottom-up approach. It also explores how factors like value and momentum naturally diversify each other due to their contrasting behaviour (high prices are often linked to momentum and low prices are linked to value). The article argues that combination approach is theoretically superior as it considers the interaction effects among factors at the security level. Such interactions may be overlooked when combining single-factor portfolios. However, the portfolio employing the mixing approach can at times outperform the integrating approach especially during periods of poor performance by the underlying investment styles or factors.

Chow, Li, and Shim [2017] conducted comparative analysis for the integrating approach and the mixing approach and found that integrating strategy tends to yield higher returns. However, it is also associated to higher trading costs and more idiosyncratic risk with higher concentration in securities and lower diversification benefits. On the other hand, the mixing approach yields lower returns but is cheaper to implement and provide more efficient factor exposure and it is easily attributable to the factor in focus. The paper concludes that the mixing approach will be more effective in smart beta indexes while the integrated approach will be recommended for systematic active strategies that can better manage concentration risk and trading costs. Overall, the paper emphasizes the importance of considering trading costs, risk factors and implementation issues when choosing between these two strategies for constructing multi-factor portfolios and we should not overlook practical implications and their limitations.

All in all, these industry discussions shine light on the importance of weighing performance outcomes, trading costs, risk factors and practical implementation issues in deciding the optimal approach for constructing multi-factor portfolios. Each approach has its strength and limitations, and the choice of a portfolio managers depends on the specific objectives, risk tolerance and mandate constraints.

Methodology

Data

The data is primarily sourced from MSCI's Barra. The historical data ranges for June 1998 to May 2023. It consists of monthly snapshot of the universe along with its variables. The universe selected for this study is the MSCI World Index. The MSCI World Index is a broad global equity benchmark that represents large and mid-cap equity performance across 23 developed markets countries. It covers approximately 85% of the free float-adjusted market capitalization in each country and MSCI World Index does not offer exposure to emerging markets. As for descriptors extracted from Barra, which is used in defining each factor:

Table 1: The descriptor definition and methodology

Factors	**Descriptor**	**Description**
Value	Book-to-Price	Computed by dividing the most recently reported book value of common equity by the current market capitalization.
Volatility	Historical Beta	Computed as the slope coefficient from a time-series regression of stock excess returns against the market cap-weighted excess returns of the market over a trailing window of 504 trading days, with a 252-day half-life.
Momentum	Relative Strength	The non-lagged Relative Strength is first computed as the exponentially weighted sum of the log excess returns of the stock relative to the market over a trailing 252-day window, with a 126-day half-life. The final RSTR descriptor is then computed as the equal-weighted average of the non-lagged values over an 11-day window lagged by 11 days.
Quality	Gross Profitability	Computed as $$GP = \frac{Sales - COGS}{TA}$$ Where COGS is cost of goods sold and TA is total assets from the last fiscal year.
Yield	Dividend Yield	Computed by dividing the trailing 12-month dividend per share by the price at the last month end.
Growth	Earnings per Share Growth Rate	Computed by dividing the slope coefficient from the regression of the annual earnings per share from the last five fiscal years against time, by the average annual earnings per share.
Size	Logarithm of Market Capitalization	Computed as the natural logarithm of the market capitalization of the firm. Applying the logarithm in the market capitalization will produce a linear effect in relation to returns.

The data we retrieved from Barra encompasses elements such as market capitalization, index weighting, risk-free rate, industry, and country classification. These aspects will be crucial in the assembly of our portfolio and in the subsequent calculation of performance ratios. These calculations will help in comparing portfolios that use both the mixing and integrated approaches. To convert the Barra-extracted data into a practical format, some pre-processing steps are necessary. We needed to execute three iterations to gather historical results dating back to 1998. We then merged these three separate extractions to create a unified, comprehensive master data file.

Factor Interpretation

In this portion, we will briefly run through the factor's definition. We will dive deep into the details of the factor definition.

For the value factor, it captures the extent to which a company is miss-priced using the Book-to-Price ratio. This factor captures the outperformance of cheaper companies with reference to its book value. High value stocks are expected to outperform the market in the long term.

The volatility factor is associated with the standard deviation of the security. Stocks with higher volatility are consider riskier and hold a negative risk premia. They are empirically tested to underperform the market in the long run. We flip the value of the descriptor by multiplying -1 to it. This is to indicate that a higher value of exposure to the volatility factor will command a higher expected return in the long run.

The momentum factor involves investing in securities that have higher returns over a pre-defined period and expecting those returns to continue produce returns. We primarily look at relative strength in defining momentum as opposed to the traditional definition. This differs from the traditional definition, which looks at a period of returns. However, we encounter an issue while running performance calculation. The results were too perfect. Upon investigation, I realized that we encountered look-ahead bias. Essentially, we are using today's price data and including that into the model when we have yet to receive the information. We account for look-ahead bias for the descriptor by lagging the descriptor by one time period.

Quality involves investing companies that have stable earnings. Quality stocks are expected to perform better than lower quality stocks. Quality stocks are typically defined by multiple dimensions like low debt, ROE and corporate governance. In this study, we use profitability as a measure.

The yield factor refers to the income return on an investment. Such as interest or dividend received from hold a particular security. We note that we are not able to compute income returns, only price returns. The returns from exposure coming from the yield factor could potentially understanded given the outperformance primarily comes from income returns as opposed to price returns.

The growth factor involves investing in companies that are expected to grow at an above average rate compared to other companies. We look at EPS growth rate for this factor.

Finally, the size factor represents that smaller companies with lower market capitalization will tend to outperform large companies over time. Like the volatility factor, we invert the sign of the descriptor by multiplying the value by -1. This is to essentially capture the effect of being long small capitalization when represented by a higher value of the descriptor.

For the simplicity of the study, we define a factor using only a single descriptor. We are aware that this is not a robust methodology in defining a factor where we could subject the factor to be mis specified.

Missing Data

After running through a short sniff test on the data, there are certain variables that are missing for some securities. A few methods are evaluated to determine the best way to treat the missing data. We only fill missing data for descriptors only.

Table 2: Number of missing variables for data source

Variable	Number of Missing Values
Book-to-Price	1249
Historical Beta	154
Relative Strength	1184
Gross profitability	1603
Dividend yield	251
Earnings per share Growth rate	10308
Logarithm of Market Capitalization	7

We considered the methods:

1. Imputation: replacing missing values with imputed values like mean, median or mode. We believe that this will cause the security to dilute its idiosyncratic feature that is isolated to industry. We drop this selection.
2. Deletion: removal of the data completely. This will require the rescaling of weights for the universe and will might introduce a bias in benchmark calculations later. We drop this selection.
3. Interpolation: filling missing data points by using adjacent data points. This will allow the replacement value to be highly subjected to outlier adjacent values and exposed to look ahead bias. We drop this selection.
4. Replacement using industry average: filling the missing data with the industry mean. This method assumes that companies in the same industry tend to have similar characteristics. We believe that is the most appropriate approach as the universe to determine the industry mean is large enough (at estimated 1500 securities per day). The mean will then be a reasonable representative. We also note that, this will disregard the idiosyncratic characteristic of the security.

Treatment of Outliers

Outliers can emerge as a result of data inaccuracies, or they could indicate novel phenomena. In practical terms, however, distinguishing between a data error and a novel phenomenon may be challenging. Being aware of the effects of outliers is imperative as extreme values are used to calculate the mean and the standard deviation when we Z-score the descriptors later. This could potentially affect the centrality and the dispersion of the distribution.

The process of identifying outliers involves two steps. The initial step is to tag the outliers. In our context, outliers are defined as values that are 2.5 standard deviations away from the mean of the raw descriptors for our factors. The subsequent step is to decide how to handle these observations. We could either completely eliminate these observations, an approach known as trimming, or we could substitute these extreme values with less extreme ones, a process referred to as winsorization.

The raw data from all six descriptors undergo a process of winsorization at a limit of three standard deviations. For every descriptor and corresponding date, if a raw descriptor value exceeds 2.5 standard deviations, we replace it with its value at 2.5 standard deviations. This means we cap values greater than 2.5 at 2.5 and elevate those less than negative 2.5 to negative 2.5. We chose winsorization over trimming or transformation due to its effectiveness in reducing the influence of extreme values while preserving the overall data structure. It should be noted that while winsorization alters the data, it does so in a way that minimizes the potential distortion of the original information. To understand the effect of winsorization on our data, we will employ before-and-after visualizations, such as box plots or histograms, demonstrating the impact of this treatment on our data's distribution.

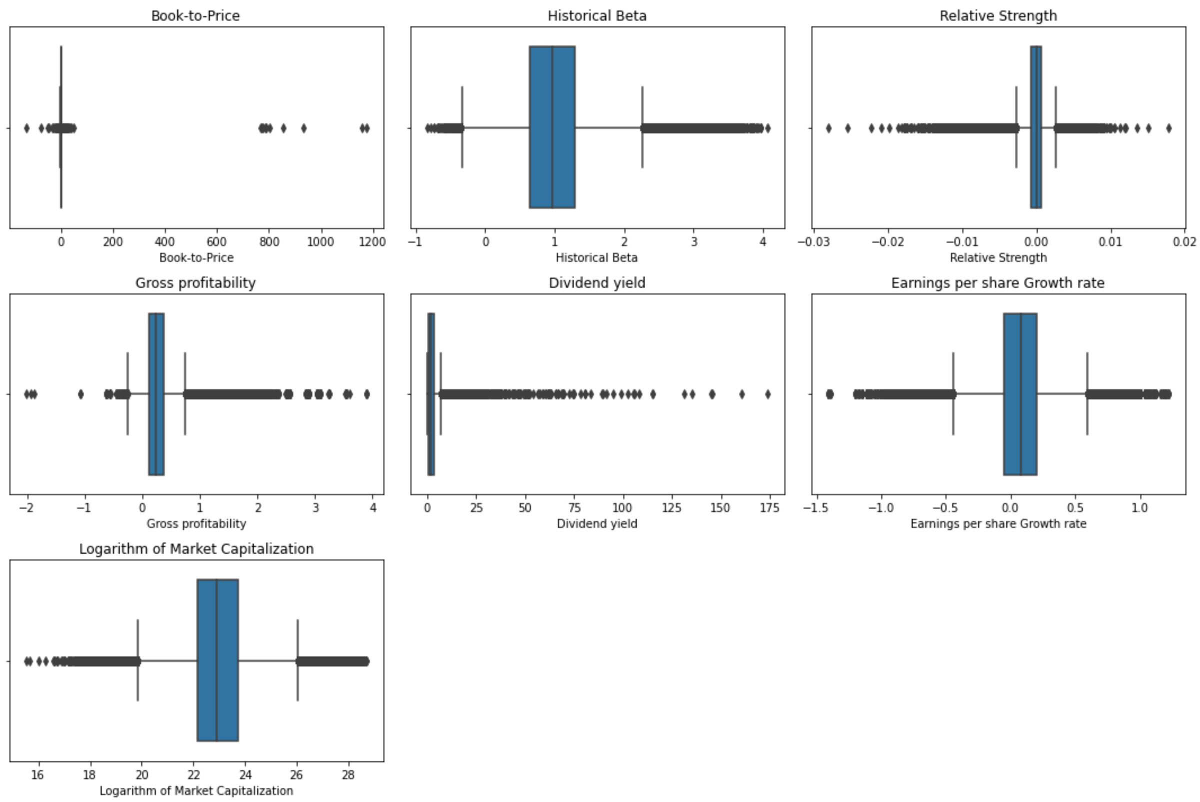

Figure 1: Boxplot of pre-winsorized descriptors

On first look, it is noted that there are some outliers, especially for Book-to-Price. We believe that this is a data error and will proceed winsorize the variables.

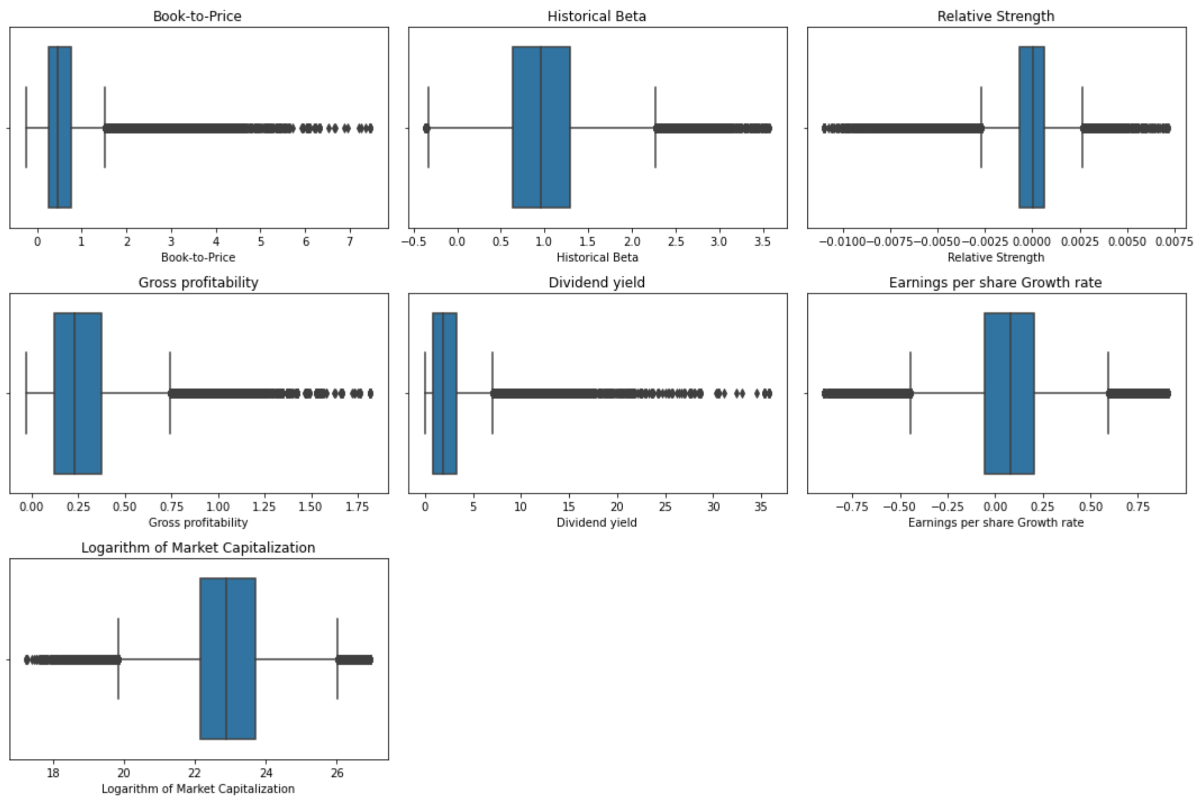

Figure 2:Boxplot of post-winsorized descriptors

After the process of winsorization, the values of the descriptors are less extreme. Specifically, in the case of the Book-to-Price ratio, we observe a significant reduction in the maximum value from an estimated 1200 down to a more plausible estimate of 8. This transformation helps to bring about a more reasonable data distribution.

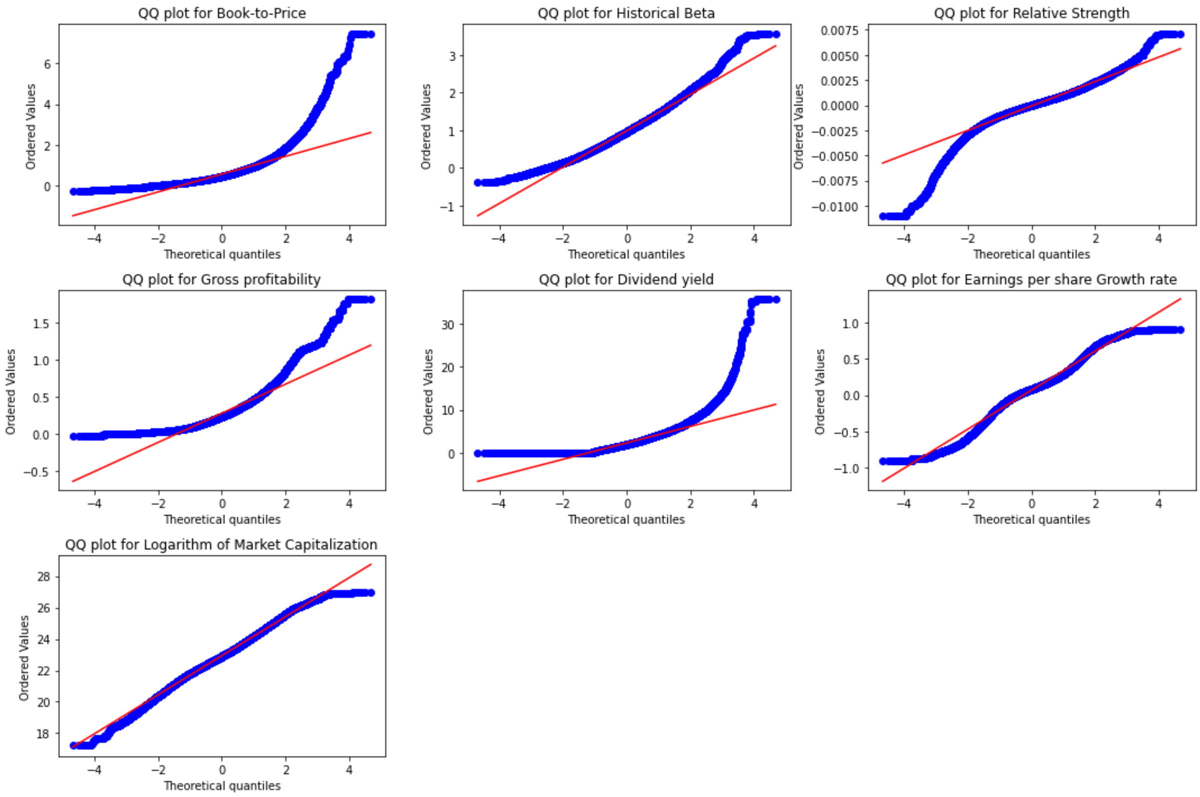

Figure 3: QQ Plot for Descriptors

In terms of normality, we see that most descriptors exhibit non-normal characteristics. They mostly exhibit bimodal distribution other than EPS growth rate and logarithm of market capitalization which are lightly tailed. This is common in financial data set and will proceed without any further modifications. However, we note this could potentially distort statistical tests that assumes normality and are sensitive to outliers (effects are managed post-winsorization).

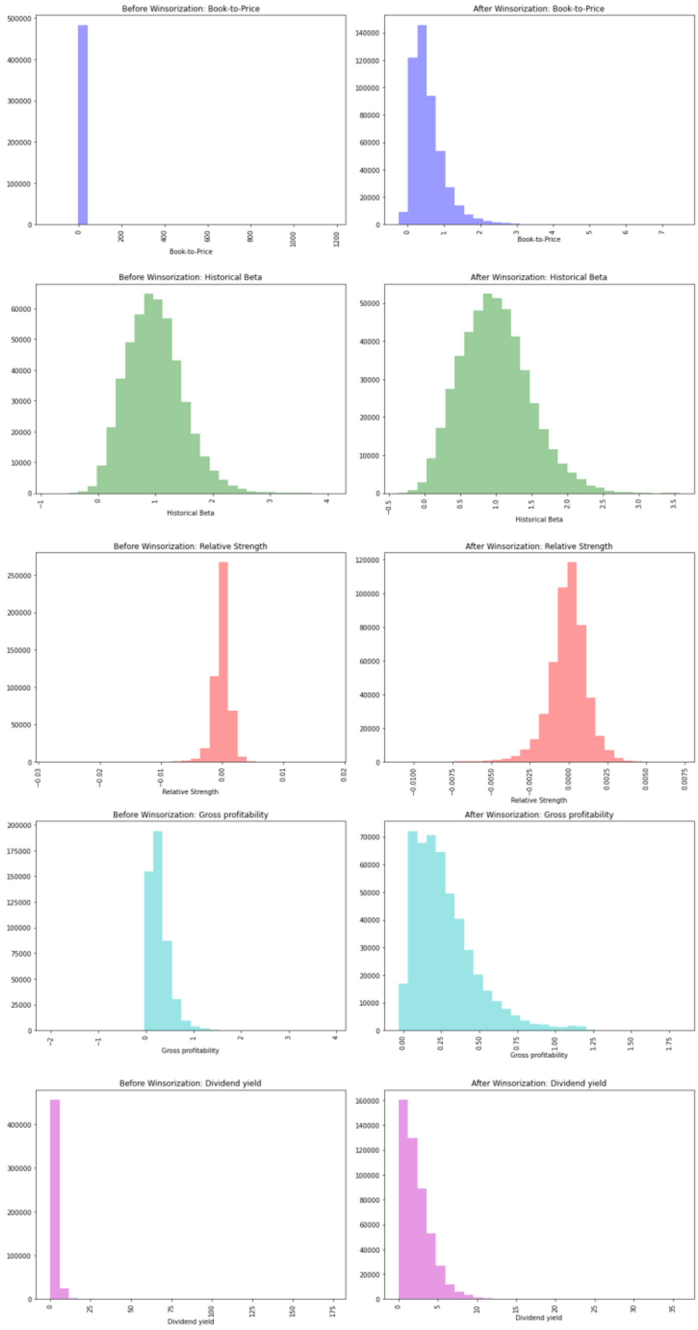

Figure 4: Histogram for Descriptors

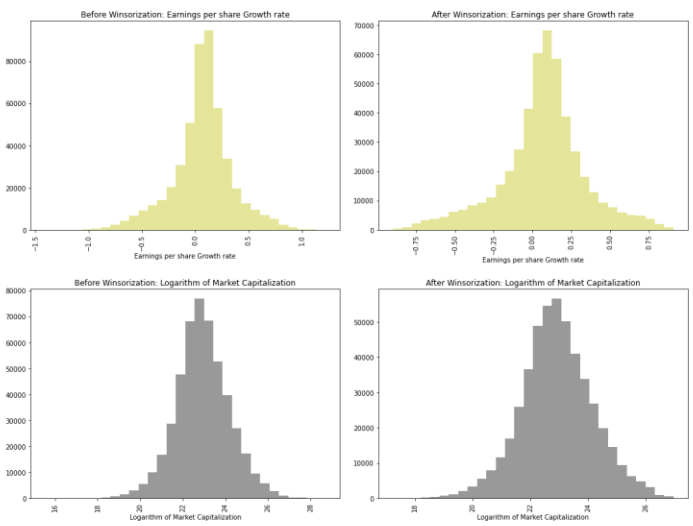

Figure 5:Histogram for descriptors

In this chart, we see an improvement in results post-winsorization. We conduct the winsorization for returns as well as there are discrepancies and data error for price which translates to spurious returns for certain securities.

Standardization (Z-scoring)

We proceed to z-score the values to define the factors. Z-scoring allows us to compare the values of two different factors. This portion is relevant to the integration approach given that we are aggregating two factors with a different scale and z-scoring the factors will make it additive. In certain situations, standardization is truly normalization. If the original cross-sectional distribution of the stocks' factor exposures is a normal distribution, we can then make clear statements about how probable an observation is or how rare it is. The Z-score concept is still works if the underlying cross-sectional distribution is not normal. It gives a sense on how far a variable is from the mean.

The standardization process is as follows:

$$z = \frac{(x - \mu)}{\sigma}$$

where:

- x is the value of the descriptor,
- μ is the market capitalization weighted mean of the descriptor,
- σ is the equal weighted standard deviation of the descriptor.

The process of standardization is applied to all seven factors. For each respective time period, we calculate the mean and standard deviation. This procedure is repeated across all dates, resulting in each date having a unique set of standardized cross-sectional data.

The use of market capitalization-weighted mean will allow us to have a bias to large caps in determining the mean. We proceed to use equal-weighted standard deviation to give each observation in the sample an equal weight, regardless of market capitalization. This reduces the influence of large caps in the de-meaning process and alleviate the potential bias. Once we obtain our z-scores, we will then have factor exposures. From here on, I will refer the z-score of the descriptors as factor exposures.

Empirical Framework

Portfolio Construction

In an empirical study like this, it is crucial to limit the variability between the parameters as much as possible. This ensures that the results we observe are not the product of unrelated discrepancies, but rather, are indicative of the unique characteristics of each approach. With this principle in mind, we seek to identify and control any potential nuances that are specific to each approach, such as different methods of portfolio construction or the application of different risk models. These nuances, though perhaps small, could potentially introduce variability into our results, thereby masking the true differences between the two approaches.

However, it is worth acknowledging that our experiment will not be flawless. Despite our best efforts to mitigate inconsistencies, it is inevitable that some deviations may arise during the portfolio construction process. This is simply an inherent challenge associated with empirical research. To put our hypotheses to the test, we construct two portfolios using identical inputs. The aim here is to ensure that any differences we observe in performance and characteristics are due to the distinct methods of factor and alpha signal generation between the two approaches rather than any extraneous differences in the portfolio inputs.

Once the portfolios are constructed, we measure and compare their performances, examining the return, risk, information ratio, Sharpe ratio, and other pertinent metrics. This comparison will shed light on which approach (mixing or integrating) proves to be more effective in systematic investing, based on the criteria that we have defined.

We begin by specifying the parameters. This includes the top N stocks, or a certain percentile of the stock universe selected based on the factor score. We will vary the parameter to conduct sensitivity analysis at a later stage.

Mixing Approach

For the portfolio assembled via the mixing approach, we rank the securities based on their factor exposures for a given date. Post ranking, we select the highest N securities, which in our scenario means choosing the top N percent of stocks. These chosen securities are then equally weighted to formulate the portfolio for that particular time frame. We repeat this for the number of factors specified to construct other factor portfolios. This is rebalanced end of the month for each factor portfolio. We then blend the factor portfolios together into a final portfolio. The aggregation of the final portfolio is equal-weighted for each factor. As mentioned earlier in the report the choice of weighting contributes implicit bias to the characteristic of the portfolio. The main advantage of equal weighting is that it avoids the concentration in a few large companies that dominate the returns while providing exposure to smaller companies. However, one main pitfall for equal-weighted portfolios is that they can incur higher transaction costs primarily due to rebalancing back to target weights as opposed to market cap weighting where there is an element of self-adjustment. Being equal weighted also might invest in more companies that are riskier and weaker fundamentals.

We present a diagram showcasing how the portfolio is constructed using the mixing approach.

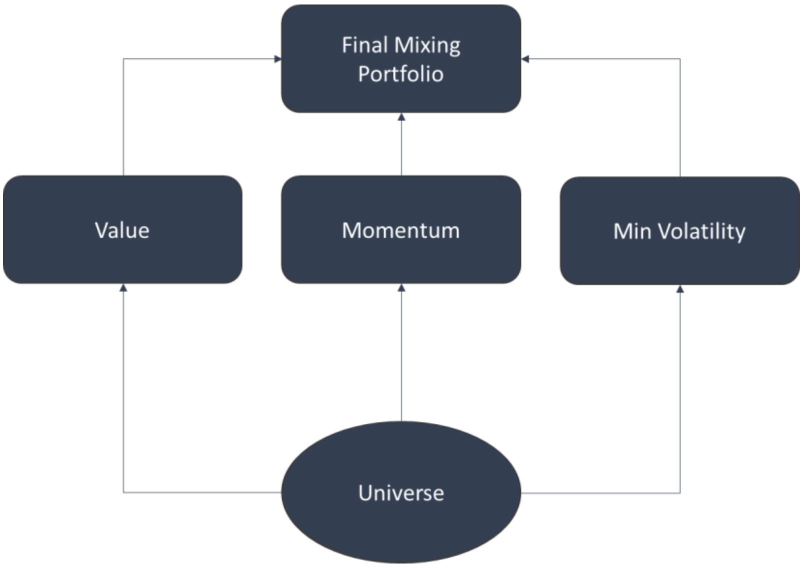

Figure 6: Mixing workflow

Integrated Approach

In the integrated approach to portfolio construction, we initially aggregate the highlighted factors into a consolidated score. For instance, if we select value, momentum, and minimum volatility as the primary factors for the model, the integrated method would aggregate each factor's exposure, allocating equal weight to each signal. This aggregated signal is subsequently used to rank the securities, selecting the top N percent from the total securities in the universe. The chosen securities are then equally weighted and placed into the final portfolio. This procedure is reiterated for each date, with a rebalancing conducted at the month's end. We offer a diagram that illustrates the portfolio construction using this integrated methodology.

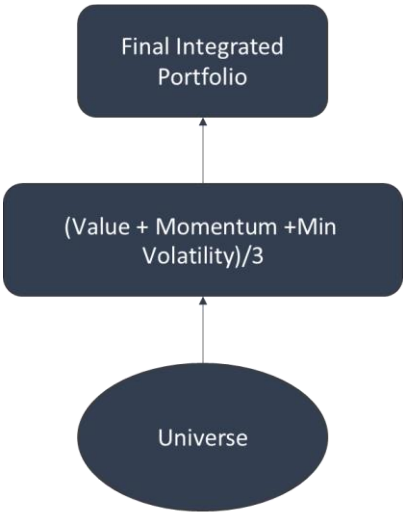

Figure 7:Integrated workflow

To gain a deeper comprehension of the differences between the portfolios, the visualization below presents a scatterplot organized by the factors in focus. For the mixing approach, it's observed that stocks with the highest exposure to each factor are chosen from each dimension. This could potentially overlook certain stocks that have substantial exposures to both factors. In contrast, the integrated approach primarily captures a vast majority of stocks that exhibit exposure to both factors and possess the desired characteristics. However, a potential downside of the integrated approach is that it might inadvertently include stocks that have moderate exposure to all factors, thereby possibly diluting the factor exposures and potentially reducing the potency of any individual factor's contribution to the portfolio's performance.

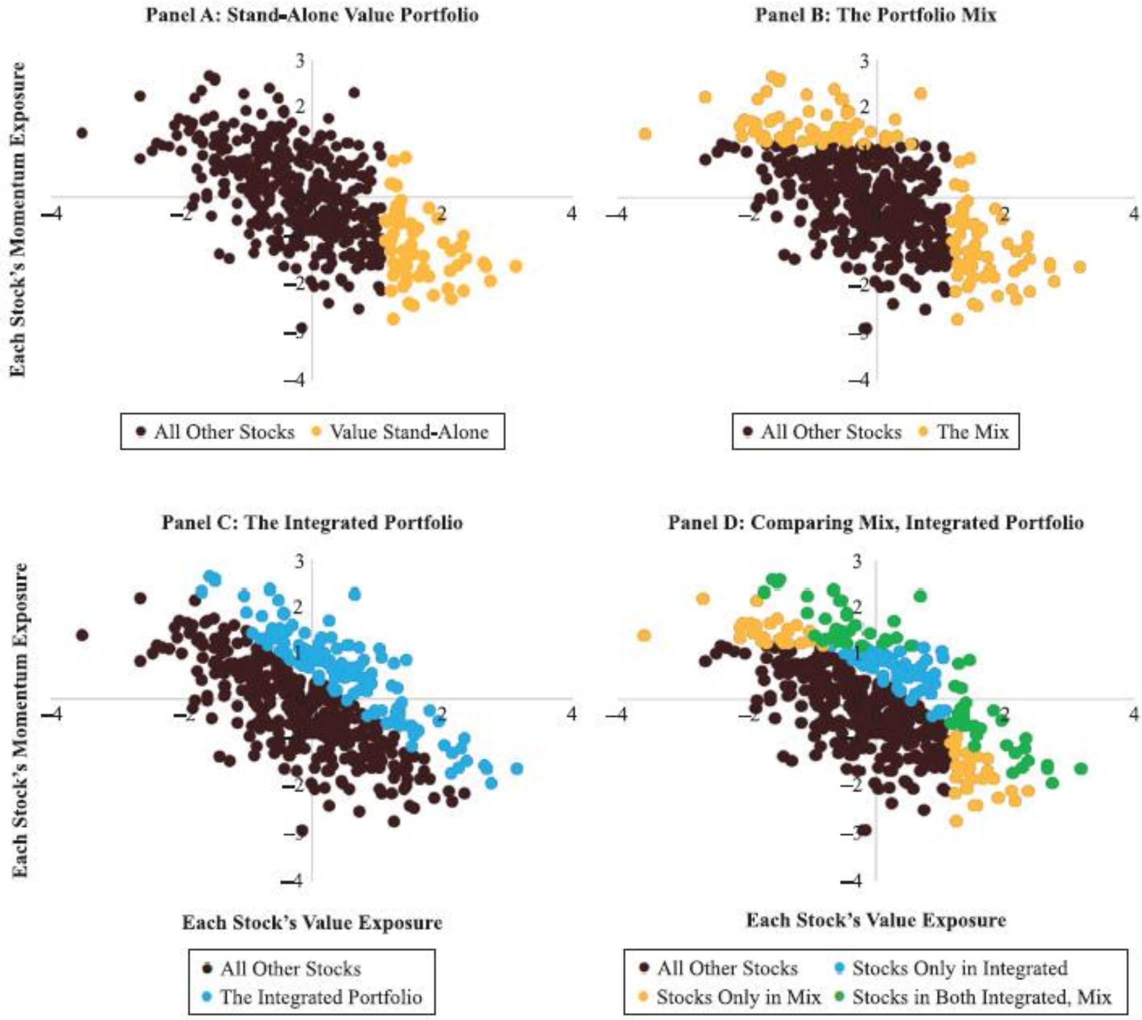

Figure 8: Scatterplot for mixing and integrated strategies

Results

The primary aim of this study is to evaluate the efficiency of portfolio building through a combined and comprehensive strategy. The study covers a span from June 1998 to May 2023. It's crucial to acknowledge that the performance may be influenced by the specific timeframe and could potentially face path dependency risk. However, in this analysis, we operate under the presumption that we are indifferent to the effects of path dependency. Also note that the results are overstated as the returns are gross of transaction fees.

The metrics used for comparison include annualized returns, excess returns, various factor exposures, cumulative returns, annualized portfolio volatility, annualized Sharpe ratio, and the annualized information ratio, among others. We vary the proportion of stock holdings from 5% to 35%. It should be noted that the benchmark return remains constant for both approaches.

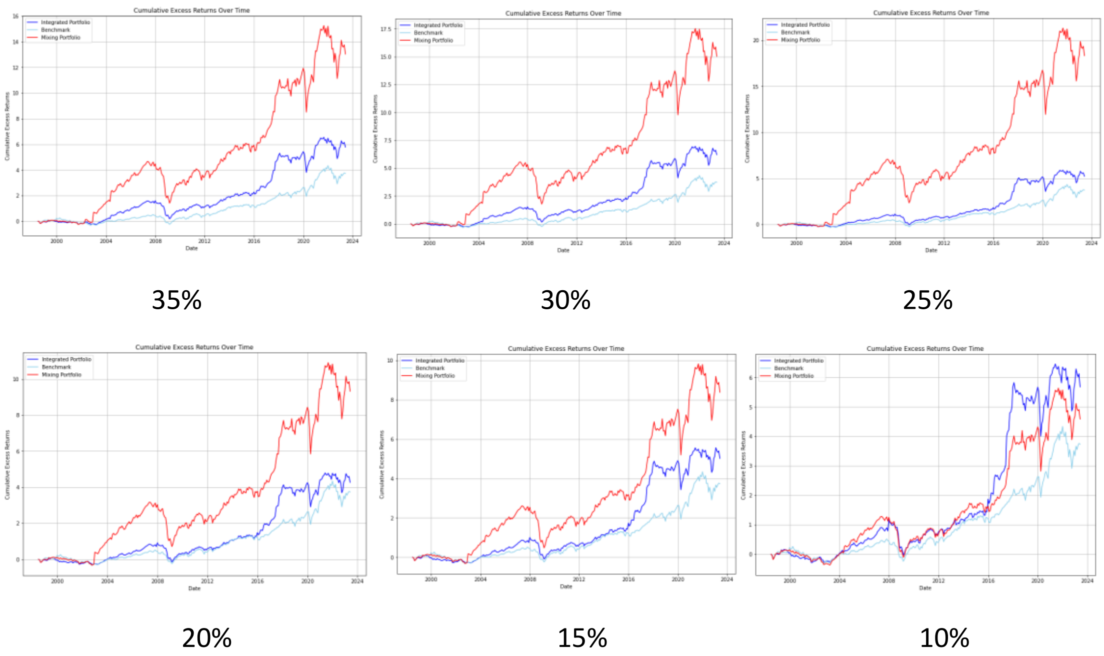

Figure 9: Cumulative excess returns

In terms of cumulative returns, the mixed approach shows significantly higher portfolio and excess cumulative returns across higher proportion of stock holdings (from 15 to 35%). However, as the percentage of stock holdings decrease, we see that the returns converge between the mixed and integrated portfolios. At 10% stock holding, the integrated approach outperforms the mixing approach in cumulative terms.

	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing
Percentage of Stocks	0.35		0.3		0.25		0.2		0.15		0.1		0.05	
Portfolio Annualized Return	9.56%	12.82%	9.85%	13.42%	9.23%	14.28%	8.52%	11.45%	9.14%	11.04%	9.66%	8.79%	7.14%	8.72%
Benchmark Annualized Return	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%
Portfolio Annualized Excess Return	7.96%	11.15%	8.22%	11.74%	7.59%	12.58%	6.87%	9.79%	7.44%	9.37%	7.89%	7.13%	5.31%	7.02%
Benchmark Annualized Excess Return	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%

Table 3: Table of returns

The study has also shown that both the mixing and the integrated approach outperformed the benchmark. However, it is noted that the mixing approach consistently generated higher annualized returns than the integrated approach. The highest return comes from the 25% stock selection at 14.28% compared to 9.85% for the integrated approach. As for the annualized excess return, the mixing approach outperformed the integrated approach with 12.58% compared against 7.59%.

	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing
Percentage of Stocks	0.35		0.3		0.25		0.2		0.15		0.1		0.05	
Portfolio Volatility	15.67%	24.31%	15.87%	26.67%	15.56%	30.13%	15.46%	25.07%	15.93%	21.21%	17.44%	18.24%	18.31%	20.74%
Tracking Error	9.68%	20.12%	10.32%	23.05%	10.01%	27.14%	10.31%	20.83%	11.23%	15.44%	13.08%	9.09%	14.07%	12.28%

Table 4: Table of risk

From a risk perspective, we see that the portfolio constructed using the mixing approach has consistently exhibited higher risk both on a standalone and active basis. One interesting finding is that at 10% stock holding, the integrated approach has a lower total volatility despite higher annualized return.

	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing
Percentage of Stocks	0.35		0.3		0.25		0.2		0.15		0.1		0.05	
Portfolio Sharpe Ratio	0.665	0.602	0.674	0.582	0.648	0.558	0.610	0.536	0.632	0.593	0.618	0.556	0.469	0.507
Portfolio Information Ratio	0.124	0.269	0.144	0.274	0.087	0.281	0.020	0.203	0.076	0.219	0.119	0.102	-0.045	0.106

Table 5: Table for risk-adjusted returns

According to the results, the integrated approach generally has a higher Sharpe ratio compared to the mixed approach. This suggests that, on a risk-adjusted basis, the integrated approach delivered better performance relative to the Mixing approach. On the other hand, the mixing approach has a higher information ratio across the board, compared to the integrated approach. This is primarily driven by the returns as opposed to a lower tracking error as the mixing approach has consistently deliver a higher tracking error as compared to the integrated approach.

	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing
Percentage of Stocks	0.35		0.3		0.25		0.2		0.15		0.1		0.05	
Average of Value Exp	0.599	0.412	0.637	0.436	0.683	0.465	0.738	0.505	0.804	0.556	0.889	0.631	1.035	0.759
Average of Min Vol Exp	0.586	0.231	0.626	0.240	0.669	0.248	0.710	0.255	0.759	0.262	0.814	0.263	0.888	0.249
Average of Momentum Exp	0.031	-0.100	0.040	-0.102	0.050	-0.103	0.059	-0.107	0.065	-0.113	0.068	-0.123	0.066	-0.155
Average of Quality Exp	0.111	-0.060	0.137	-0.048	0.168	-0.035	0.204	-0.019	0.252	0.005	0.315	0.039	0.420	0.110
Average of Yield Exp	0.471	0.181	0.527	0.201	0.596	0.225	0.680	0.253	0.789	0.286	0.955	0.325	1.253	0.384
Average of Growth Exp	0.206	0.005	0.230	0.014	0.256	0.024	0.288	0.035	0.329	0.051	0.387	0.074	0.471	0.107
Average of Low Size	1.913	1.567	1.959	1.585	2.004	1.610	2.057	1.642	2.116	1.674	2.185	1.718	2.286	1.795

Table 6:Table for portfolio factor exposures

Regarding portfolio exposure, our primary interest lies in a portfolio that effectively manifests the intended exposures. We have noticed that the mixing approach seems to struggle in reflecting the intentions behind portfolio construction. The objective of this construction is to attain exposure across all six factors, but it appears that there are instances of underexposure (indicated by negative values) to certain factors for most periods. This underexposure is most noticeable with the momentum and quality factors.

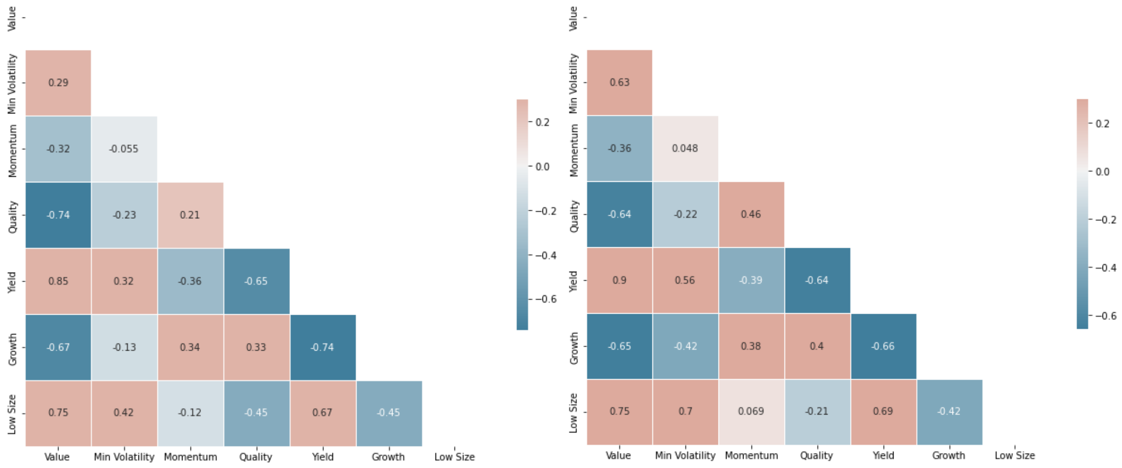

Figure 10: Correlation Matrix for integrated(left) and mixed(right) approach

In the realm of factor investing, a prevalent challenge encountered is collinearity. This phenomenon transpires when exposure to a specific factor inadvertently leads to exposure to an unanticipated factor as well. This overlap can blur the distinct impacts of individual factors, creating ambiguity in deciphering the specific contributions of each factor to the portfolio performance. To gain insights into the collinearity issue in our study, we executed a correlation test for each factor for both the integrated and the mixed approach. Upon analysing the results, we observed that the correlation amongst factors are higher when using the mixing approach. This suggests that the mixed approach could potentially heighten the risk of collinearity and complicate the task of isolating and assessing the impact of individual factors. This coincides with our findings in the factor exposure results.

Discussion

Based on the observed data, we can extract important insights and implications for quantitative investment strategies. There are perks and pitfalls for both approaches and it will be mainly dependent on the portfolio managers' requirements to leverage these approaches effectively.

First and foremost, it's important to note that the mixing strategy consistently yielded higher returns when a greater proportion of stocks were held, compared to the integrated strategy. Consequently, portfolio managers need to consider their constraints before selecting a strategy. For instance, if a manager's investment strategy leans towards a more concentrated portfolio or the focus is on a smaller universe of stocks, the integrated approach could potentially serve as a more suitable choice for portfolio construction. Conversely, if a manager can operate within a broader universe of stocks and is equipped to manage the increased turnover or transaction costs associated with a more diversified portfolio, the mixing approach might be the preferred strategy. There are also many innovations to execute the mixing approach. One example is smart beta products. The process of constructing these factor portfolios can be outsourced to smart beta asset managers with an attractive management fee. In essence, the choice between an integrated or a mixing approach depends on the specific constraints and capabilities of the portfolio manager.

Secondly, the analysis revealed that the integrated approach has a higher risk-adjusted return (Sharpe) primarily driven by the lower portfolio volatility. This characteristic is relevant for portfolio managers with a mandate to stick within a certain risk tolerance. A manager with a higher risk tolerance might use the mixing approach while a more risk-averse manager might prefer an integrated approach to achieve their objectives.

Furthermore, our analysis uncovers an intriguing aspect of the mixing approach - the portfolio doesn't necessarily align with the intended factor exposures. This could suggest that the superior returns may be a consequence of unintended factor exposures, implying that these high returns could be more a product of luck rather than skill. It's important for a portfolio manager to fully comprehend the risk factors involved, as any misunderstanding or misrepresentation could skew how a specific market view is actualized within the portfolio. Such misalignments could potentially restrict the portfolio's capacity to seize market opportunities to the fullest, consequently affecting the overall performance. In simple terms, while the mixing approach may yield higher returns, it's essential to recognize that these returns might not be the result of strategic investment decisions. Instead, they could be lucky outcomes of unintended factor exposures. This realization underscores the importance of properly understanding and managing risk factors for optimal portfolio performance.

The third point is further supported by the correlation numbers for both approaches. There is a higher susceptibility of collinearity during the portfolio construction for the mixing approach. This could introduce additional obstacles in constructing a portfolio with the manager's intended tilts. A common approach to effectively navigate this problem is to rely on an optimizer with prescribed constraints but that will depend on the sophistication of the manager himself.

Finally, turnover is a key consideration in systematic investing as the processes are mostly scaled to accommodate a larger trading volume. A higher turnover will lead a fee paid for transacting in and out of multiple positions. Intuitively, the integrated approach will offer a lower turnover. This stems from certain factors being negatively correlated like value and momentum. If a security has a higher price, will typically have a higher exposure to momentum and when the price decreases, the security will cycle into a value stock. With these considerations, an integrated approach of 50/50 to value and momentum, the stock will likely still be included in the portfolio and will not trigger a rebalance. For the mixing approach, a stock moving out of factor will trigger a rebalance in the factor portfolio without the total portfolio in mind, incurring a higher transaction cost.

Limitations of the Study

Beginning with the data integrity in this study, it's crucial to recognize that it exhibits certain inconsistencies. Missing data and outliers are frequent challenges encountered in financial data analysis, potentially skewing the findings away from an accurate representation of reality. Despite these difficulties, we endeavour to navigate these obstacles using carefully calibrated measures designed to manage the data. Our primary objective is to minimize the introduction of biases, thus ensuring that the results derived are as objective and reliable as possible. These steps help to ensure that, despite inherent limitations in the data, our analysis can yield meaningful and applicable insights.

Another issue with the study is the performance calculations. There are extreme and spurious returns for certain securities where we simply set it as 0. This might possibly understate/overstate returns. The returns are also calculated from price, and we ignore dividend returns. This introduces another understatement of returns.

Our study does not consider transaction costs associated with portfolio rebalancing. In the real world, the frequency of trades can significantly impact net returns, especially for the mixing approach which might require more frequent rebalancing due to a larger number of held assets.

The study also uses simple measures of risk which typically does not capture tail events. A multidimensional approach in investigating measures like maximum drawdown or Value at Risk could offer different perspectives.

Going Forward

The findings of this empirical study on the mixing versus integrated approach to systematic investing provide rich insights, however, the research field is always evolving. Going forward, there are several paths this study could take.

Varying the number of factors and if there is an optimal number of factors for each approach. A sensitivity analysis can be explored on the number of factors and to establish any meaningful relationships. Quantitative managers typically have alpha signals relying on either a sole raw signal, a combination, or a thematic capture. Understanding the benefits and pitfalls for both approaches will provide incremental value in the portfolio construction process.

Time-variant factor weights in portfolio construction. As factor will cycle in and out of favour along with economic cycles, instead of relying on static weights for different factors, time-variant factor weights allow the portfolio's composition to adapt over time. These dynamic weights could respond to observable shifts in market sentiment, economic data, or business cycle indicators. For example, a recessionary environment will typically reward factors like value, minimum volatility, and quality. A study leveraging both mixing and integrated approach on dynamic factor weights will generate valuable insights that is useful.

The integrated and mixing approach could be tested using a commercial optimizer to see if there is an incremental gain in risk-adjusted performance. An optimizer could potentially construct a portfolio with a precise exposure to target factors and minimize realized volatility of the portfolio by considering the interaction effects of different factors. It is an efficient implementation and most quantitative managers rely on an optimizer to construct their portfolio.

Finally, while this study focuses on equity portfolios, future research could extend this analysis to other asset classes like bonds, commodities, or currencies. Different asset classes might respond differently to systematic factors, and these responses could change over time. Hence, an empirical study exploring the effectiveness of the mixing and integrated approaches across different asset classes and over different time periods could be useful.

Conclusion

In conclusion, this empirical study dives into the comparative performance and implications of the mixing and integrated approach for systematic investing. The findings could potentially provide insights for quantitative strategies and portfolio construction both in the asset management and the hedge fund space. The study demonstrated that both approaches have their unique strength and downsides. The mixing approach delivers a higher return but at an expense of higher risk and potential misalignment with intended factor exposures. Conversely, the integrated approach provides superior risk-adjusted returns and less prone to collinearity at the cost of lower returns.

The choice ultimately sits on the portfolio manager. Considerations such as asset constraints, risk tolerance, transaction costs, and desired factor exposures all come into play in selecting the optimal approach. A more concentrated or risk-averse strategy will lean towards an integrated approach while a diversified and aggressive strategy will favour the mixing approach.

Future research can delve deeper into the study's limitations like the disregard of transaction costs and data inconsistencies. Additional avenues of exploration could include other asset classes, varying number of factors and time-variant factor weights.

The area of systematic investing is a dynamic and evolving field. As the landscape continues to change and become more complex, the development of robust and adaptable portfolio construction methods becomes increasingly important. This research seeks to contribute a better understanding and provide a launchpad for further innovations.

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Appendices

	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing	Integrated	Mixing
Percentage of Stocks	0.35		0.3		0.25		0.2		0.15		0.1		0.05	
Portfolio Annualized Return	9.56%	12.82%	9.85%	13.42%	9.23%	14.28%	8.52%	11.45%	9.14%	11.04%	9.66%	8.79%	7.14%	8.72%
Benchmark Annualized Return	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%	8.29%
Portfolio Annualized Excess Return	7.96%	11.15%	8.22%	11.74%	7.59%	12.58%	6.87%	9.79%	7.44%	9.37%	7.89%	7.13%	5.31%	7.02%
Benchmark Annualized Excess Return	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%	6.42%
Average of Value Exp	0.599	0.412	0.637	0.436	0.683	0.465	0.738	0.505	0.804	0.556	0.889	0.631	1.035	0.759
Average of Min Vol Exp	0.586	0.231	0.626	0.240	0.669	0.248	0.710	0.255	0.759	0.262	0.814	0.263	0.888	0.249
Average of Momentum Exp	0.031	-0.100	0.040	-0.102	0.050	-0.103	0.059	-0.107	0.065	-0.113	0.068	-0.123	0.066	-0.155
Average of Quality Exp	0.111	-0.060	0.137	-0.048	0.168	-0.035	0.204	-0.019	0.252	0.005	0.315	0.039	0.420	0.110
Average of Yield Exp	0.471	0.181	0.527	0.201	0.596	0.225	0.680	0.253	0.789	0.286	0.955	0.325	1.253	0.384
Average of Growth Exp	0.206	0.005	0.230	0.014	0.256	0.024	0.288	0.035	0.329	0.051	0.387	0.074	0.471	0.107
Average of Low Size	1.913	1.567	1.959	1.585	2.004	1.610	2.057	1.642	2.116	1.674	2.185	1.718	2.286	1.795
Benchmark Cumulative Return	631.55%	631.55%	631.55%	631.55%	631.55%	631.55%	631.55%	631.55%	631.55%	631.55%	631.55%	631.55%	631.55%	631.55%
Portfolio Cumulative Return	880.71%	1938.67%	946.12%	2229.78%	807.95%	2710.58%	672.32%	1403.90%	790.79%	1269.97%	902.94%	721.00%	460.16%	709.10%
Benchmark Excess Cumulative Return	373.59%	373.59%	373.59%	373.59%	373.59%	373.59%	373.59%	373.59%	373.59%	373.59%	373.59%	373.59%	373.59%	373.59%
Portfolio Excess Cumulative Return	578.28%	1304.25%	621.26%	1503.72%	523.37%	1833.01%	426.79%	932.04%	502.06%	837.85%	568.37%	459.20%	264.47%	444.90%
Average Number of Assets	564	1552	483	1500	402	1414	322	1284	241	1093	160	830	80	473
Portfolio Volatility	15.67%	24.31%	15.87%	26.67%	15.56%	30.13%	15.46%	25.07%	15.93%	21.21%	17.44%	18.24%	18.31%	20.74%
Tracking Error	9.68%	20.12%	10.32%	23.05%	10.01%	27.14%	10.31%	20.83%	11.23%	15.44%	13.08%	9.09%	14.07%	12.28%
Portfolio Sharpe Ratio	0.665	0.602	0.674	0.582	0.648	0.558	0.610	0.536	0.632	0.593	0.618	0.556	0.469	0.507
Portfolio Information Ratio	0.124	0.269	0.144	0.274	0.087	0.281	0.020	0.203	0.076	0.219	0.119	0.102	-0.045	0.106