A Real Option Analysis of an Oil Refinery Project

Junichi Imai and Mutsumi Nakajima

This paper evaluates an oil refinery project when the prices of the output products are uncertain and management has some flexibility to switch operating process units. We develop a multinomial lattice model and provide numerical examples that are based on an actual case study. The results of the case study show that our lattice-based real option approach is useful in practice. We evaluate the project including the value of flexibility, and specify the value of each process unit. Furthermore, we discuss the optimal construction and switching strategy and demonstrate it under a sample price fluctuation. Our main conclusion is that the flexibility of the project is so profitable that management should never ignore the value of flexibility in evaluating a project. [JEL: G13, G31, C61]

■In this paper, we evaluate an oil refinery project when the prices of the output products are uncertain and management has the flexibility to switch operating process units. To evaluate the project we first develop a lattice framework for valuing a switching option when there are several sources of uncertainty. Next, we apply this framework to an actual oil refinery project and evaluate the managerial flexibility of the project.

First, we review multinomial lattice models that converge to multidimensional geometric Brownian motions. Boyle (1988) develops a lattice framework with two state variables. Boyle, Evnine, and Gibbs (1989) develop a 2^n-jump lattice framework by equating the first two moments of lognormal distribution to those of the approximating distribution. Kamrad and Ritchken (1991) suggest a (2^n+1)-jump lattice model and show that their model converges to the corresponding continuous-time model more quickly. Nelson and Rawaswamy (1990) discuss general convergence problems. He (1990) and Cheyette (1988) develop (n+1)-nomial lattice models that converge to the corresponding continuous-time process. He, in particular, discusses a general theory of convergence and develops a more efficient procedure from a computational viewpoint. Therefore, this paper constructs a multidimensional lattice procedure based on He's approach.

Second, we describe a valuation model of a switching option when the underlying assets follow a multinomial process. A switching option is defined as an option that has exercise rights to switch from one "stage" to another. It is important not only for theorists but also for practitioners to understand the concept of the switching option. This is a very useful concept and most real options as well as some financial options can be regarded as switching options. Options to defer, expand, contract, and abandon are all switching options with two stages. Each decision that management makes corresponds to an exercise of a switching option. Kogut and Kulatilaka (1994) propose the idea of this option to evaluate operating flexibility when an exchange rate is fluctuating.

We apply the method to an oil refinery project. An oil refinery project is a large and complex project that

Junichi Imai is a Post Doctoral Fellow at the Centre for Advanced Studies in Finance, University of Waterloo, Canada, and a Lecturer at the Faculty of Policy Studies, Iwate Prefectural University, Iwate, Japan. Mutsumi Nakajima is in Industrial Systems Department, Project Systems Division, JGC Corporation, Kanagawa, Japan.

The authors gratefully acknowledge three anonymous referees who helped to improve this paper and Raj Aggarwal, the Editor, for helpful comments. The authors are also grateful to Koichi Furukawa, Phelim Boyle for helpful suggestions. The authors also thank Nobuaki Ishii and Akinori Ogawa for giving us the data for the case study, and Tomonori Kameoka for his cooperation. J. Imai acknowledges Japan Society for Promotion of Science (JSPS) for research support. We thank JGC Corporation for giving us relevant data.

consists of a sizeable number of process units, and it produces various kinds of products and byproducts. It is not easy to evaluate the entire project from the beginning. However, the project can be divided into a set of sub-projects and management can evaluate the entire project by integrating these sub-projects. We focus on one of these sub-projects in this paper and evaluate it. The project here is considered to have managerial flexibility, where the manager of the sub-project can change to different process units during its' lifetime.

The traditional approach to evaluate the project in capital budgeting is based on the net present value (NPV) rule. Although this rule derives from sound theoretical foundations, it is now well recognized that it underestimates the project because it does not capture managerial flexibility under uncertainty. In other words, we ignore the possibility of future managerial actions with the NPV rule. To overcome this deficiency, a number of recent studies have proposed the real option approach to explore various applications to the capital budgeting area (See Ingersoll and Ross, 1992, Aggarwal, 1993, Dixit and Pindyck, 1993, Trigeorgis, 1995, 1996, and Sick, 1995). In the real option approach, we obtain an expected value of a project by using risk neutral probability. As Mason and Merton (1985) mention, the real option approach estimates the market value of the project as if it were traded.

A project for a natural resource development is suitable for valuation by the real option approach, because the price of the natural resource is fluctuating, and the real option approach can evaluate managerial flexibility under the price uncertainty. Brennan and Schwartz (1985) evaluate the operating flexibility of a natural resource project. Ekern (1988) shows that the real option approach is effective in evaluating a petroleum project. However, most studies, so far, focus on proposing a theoretical framework and few papers provide empirical implications because of data unavailability. Paddock, Siegel and Smith (1988) apply the real option approach to value offshore petroleum leases. Quigg (1993) examines the option premium to wait to invest by using market prices of land. Nichols (1994) reports that Merck, a pharmaceutical company, applies the real option approach to project valuation. Amram and Kulatilaka (1999), and Luehrman (1998) discuss the real options from practical viewpoints.

We apply the real option approach to a sub-project of the oil refinery and examine the value of flexibility in this sub-project. The real option approach is superior to a traditional sensitivity analysis in the sense that we can estimate interactive effects of different variables because we incorporate the correlation structure directly into the model. The approach is also superior to a traditional simulation analysis because we can find the optimal switching strategy as well as estimate the project value.

It is appropriate to evaluate the sub-project by the real option approach for the following two reasons. First, we can regard products produced in an oil refinery as the underlying assets of the real option because their prices are uncertain. Second, the manager of the sub-project has the flexibility of changing the type of process units. This flexibility can be valued as a switching option.

We also discuss the optimal strategy. Sick (1995) insists that the determination of the optimal exercise strategy is one of the central problems in a real option problem. Dixit (1989), Kogut and Kulatilaka (1994), and Kulatilaka and Kogut (1996) examine a hurdle price of the switching, and show that the optimal decision at each exercise date depends upon prior decisions when switching is costly (they call it hysteresis).

The results of the case study show that the real option approach developed here provides a practical and powerful method. We evaluate the project value under product price uncertainty, and specify the value of each process unit. In addition, we discuss the optimal construction and switching strategy and illustrate the solution under a sample price change. We conclude that the value of the managerial flexibility in the sub-project is significant and should not be ignored. This implies that a static analysis, such as the traditional NPV, fails to capture the value of flexibility and can significantly underestimate the project value. In other words, estimating a project without uncertainty may result in underestimating it, because the value of managerial flexibility is not captured correctly.

This paper is organized as follows. Section I provides a valuation model of a switching option when the underlying assets follow multidimensional geometric Brownian motions. Section II describes an oil refinery project and shows that it can be evaluated with the valuation model of a switching option. In Section III we analyze the project. Section IV provides some concluding remarks.

I. A Valuation Model

In this section, we first develop a multidimensional lattice model that can evaluate options when there are several underlying assets. It derives from a standard option pricing theory. Next, we describe a switching option and provide an example of it to be understood clearly.

A. A Multidimensional Lattice Model

In this paper we assume that there are M sources of uncertainty and that they follow geometric Brownian motions such that:

$$dS_i/S_i=\mu_i dt + \sigma_i dW_i\ ;\ i=1,...,M \tag{1}$$

where S_i is the asset price which represents the *i*th source of uncertainty, μ_i and σ_i are the drift and the volatility of each asset and Wi is the standard Wiener process. We assume that the pairwise correlation between *i*th and *j*th Brownian motions is $\rho_{i,j}$ so that

$$E[dW_i dW_j]=\rho_{i,j} dt \tag{2}$$

where $E[\cdot]$ represents an expectation. The risk free rate is assumed constant and is e]qual to r.

We set up the lattice procedure as developed by He (1990).[1] In his approach, the M-dimensional diffusion process for the underlying assets can be approximated by a (M+1)- nomial lattice process. He shows that the lattice process converges to the continuous time process.

B. A Valuation Model of A Switching Option

In this subsection, we provide a valuation model of a switching option when the underlying assets follow a multinomial lattice process. As mentioned earlier, the concept of a switching option is important because the idea can be applied to many kinds of options to evaluate them. A switching option is defined as an option that has exercise rights to switch from one "stage" to another. A holder of a switching option has rights to change a stage at any time during the life of the option contract. The cash flow, which the option holder receives, is dependent on the stage selected by the holder as well as prices of the underlying assets. The holder of a switching option can change stages at different times, but switching costs are often incurred when the holder exercises the right. It could be regarded as a series of American options in the sense that exercising an option produces another option.

We provide an example of a switching option contract to clarify the model.[2] Exhibit 1 illustrates a four-jump lattice process over two periods when there are two underlying assets. We assume that each underlying asset follows a binomial process. The rate of return on the first asset over each period can have two possible values. Similarly, the second asset has two possible rates. Suppose that the initial values of the assets at time zero are 100 and 110, respectively. There are four possible states at time one, and nine possible states at time two. For simplicity, the risk-free rate is assumed to be zero. Let p1 through p4 denote the risk neutral probability to each state.

Consider a switching option written on these two underlying assets. We assume that the switching option has two stages, and a holder of the option can select *Stage 1* or *Stage2* at time zero. The holder can receive payments at both time one and time two. The payment at time one is determined according to the stage selected at time zero as well as the assets prices at time one. Let S1(1) and S2(1) denote assets prices at time one, respectively. The cash flow received from the switching option is defined as follows. When the holder selects *Stage1* at time zero, the cash flow at time one denoted by CF_stage1(1), is defined as:

$$\text{CF\_stage1}(1)=\max(S1(1)-150,0)+\max(S2(1)-100,0) \tag{3}$$

On the other hand, when the holder selects *Stage2* at time zero, the holder receives a payment which is equal to

$$\text{CF\_stage2}(1)=\max(S1(1)-120,0)+\max(S2(1)-120,0) \tag{4}$$

at time one.

The option holder can change the stage at time one by paying a switching cost, which is illustrated in Exhibit 2. Suppose that the switching cost from *Stage 1* to *Stage 2* denoted by SW12 is five, and that from *Stage 2* to *Stage 1* denoted by SW21, is three. The selection of the stage at time one affects the cash payment received at time two. At maturity (i.e., t=2), the option holder simply receives a payment according to assets prices at time two and the stage selected at time one.

We will value the switching option by dynamic programming. Let V1(t) denote the value of the switching option at time t if a holder selects the *Stage 1* at time t - 1. We can define V2(t) in the same way.

The value of the switching option at maturity is equal to the payment received at time two.

$$V1(2)=\text{CF\_stage1}(2) \tag{5}$$
$$V2(2)=\text{CF\_stage2}(2) \tag{6}$$

The option holder on *Stage 1* has two alternatives. The holder, 1) does not change *Stage 1* until time two, 2) changes from *Stage 1* to *Stage 2* by paying the switching cost of SW12. In either case, the holder receives the amount of CF_stage1(1) at time one. When the holder selects the first alternative, the value of the switching option is equal to

$$\text{CF\_stage1}(1)+E[\,V1(2)\,] \tag{7}$$

where $E[\cdot]$ represents the expectation under the risk

[1]For details of the construction of the lattice see Boyle (1990) as well as He (1990).

[2]Because this example is illustrated for explaining a switching option contract, we do not use the He's model for simplicity.

Exhibit 1. Two-Dimensional Lattice Model (Two Periods)

This exhibit illustrates a two-dimensional lattice process over two periods. This lattice is not built by the He's model because it is made only for explaining a switching option. Each underlying asset follows a binomial process, respectively. The initial price of the first asset is 100 and the rate of return over one period can have two possible values (1.5 or 1/1.5). The initial price of the second asset is 110 and the rate of return over one period can have two possible values (1.2 or 1/1.2). Consequently, in this exhibit there are four possible states from each node. A branch that connects two nodes indicates possible transition. There are four possible states at time one and nine possible states at time two. Let p1 through p4 denote the risk neutral probability of transition to each node. We assume that they are 0.182, 0.218, 0.273, 0.327 in order.

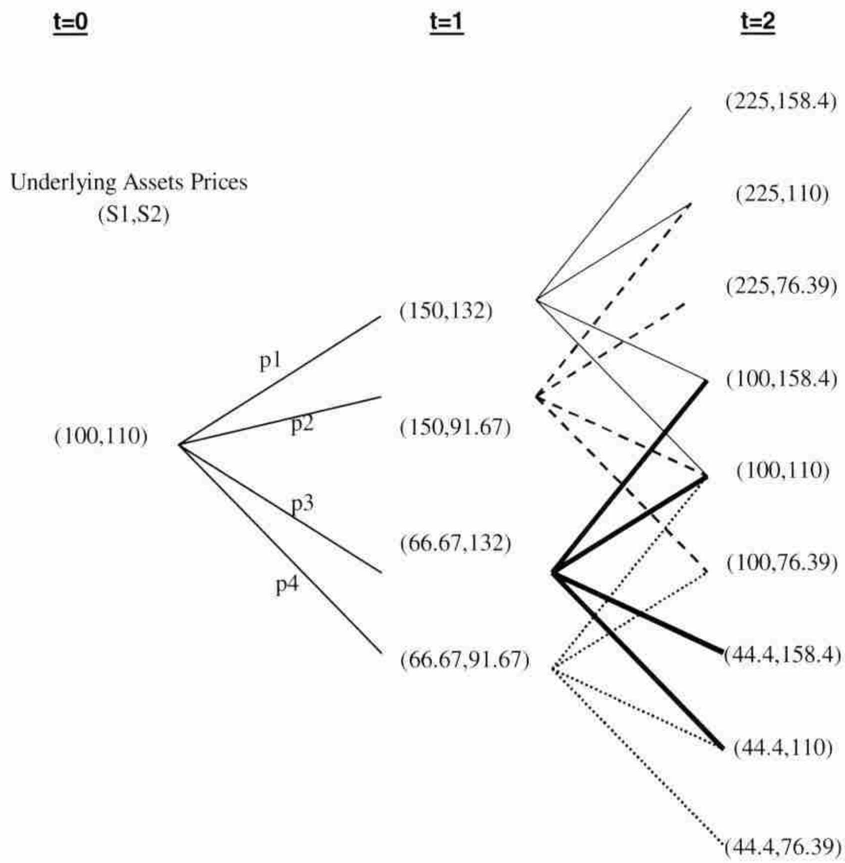
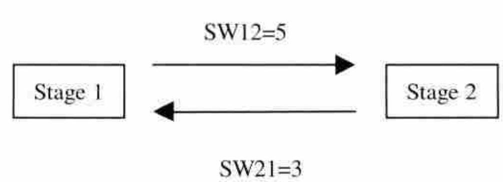


Exhibit 2. Transition Between Two Stages

This exhibit illustrates possible transition between two stages in the switching option and switching costs from one stage to the other. A switching option holder must pay five to switch from *Stage 1* to *Stage 2*, while pay three from *Stage 2* to *Stage 1*.



neutral probability. On the other hand, when the holder selects the second alternative, it is equal to

$$CF\_stage1(1) + E[\,V2(2)\,] - SW12 \tag{8}$$

The holder of the switching option can select the better alternative. Therefore, the value of the option at time one on *Stage 1* is obtained by

$$V1(1)=CF\_stage1(1)+\max\{E[\,V1(2)\,],\ E[V2(2)]-SW12\} \tag{9}$$

Exhibit 3 illustrates the value process of this switching option. The value of the switching option at time zero is 45.8 when the holder selects *Stage 2*. The exhibit also shows that it is optimal to switch from *Stage 2* to *Stage 1* at time one when the price of the first asset goes down to 66.67.

We can formulate a valuation model of a general switching option by extending the idea indicated in this section.

II. An Application to an Oil Refinery Project

In this section we first describe the entire oil refinery in brief and specify the sub-project that we focus on in this paper. Next, we show that the sub-project corresponds to a switching option.

A. A Project Description

This application is based on an actual case study. An oil refinery project is large and complex. A refinery processes crude oil and produces various petroleum products, such as liquid petroleum gas (LPG), gasoline, jet/kerosene, gas oil, and fuel oil. It consists of a large number of process units. The refinery process configuration is illustrated in Exhibit 4. A manufacture unit of Fluid Catalytic Cracking (FCC) complex is built to maximize gasoline production. The process configuration of the FCC complex is shown in Exhibit 5.

This paper focuses on the sub-project that includes the unsaturated LPG process in the FCC complex and will evaluate the value of flexibility of this sub-project. Since the sub-project that we focus on is a part of the entire oil refinery, its' value does not represent the profitability of the oil refinery. The project manager must examine the true profitability of the refinery by integrating all sub-projects. It is useful to examine the profitability of each sub-project before integration.

Exhibit 6 illustrates the product flow scheme of the sub-project. There are four alternative process units. These are unit of methyl tertiary butyl ether (MTBE), Alkylation, Polymerization, and a combination of MTBE and Alkylation. Thus, we have five alternative cases in total including no processing as the *Base Case*. These five cases are actually all exclusive alternatives because other alternatives are evidently inefficient from a practical viewpoint. In Exhibit 6, each process unit (i.e., each *Case*) produces different kinds of products of LPG, MTBE, Alkylate, polymerization gasoline (Poly-Gasoline).

The market prices of the products are fluctuating and uncontrollable.[3] Thus, the cash flow that the manager receives from this sub-project depends on both the case and the prices of the products. Let CFt denote the cash flow (Unit: US dollars) such as:[4]

$$CF_t(\text{Base Case})=0 \tag{10}$$

$$CF_t(\text{Case A})=\{6900P_t(LPG)+1400P_t(MTBE)-8000P_t(LPG-9300\}\times 10^{-6} \tag{11}$$

$$CF_t(\text{Case B})=\{4300P_t(LPG)+8300P_t(Alkylate)-8000P_t(LPG)-99370\}\times 10^{-6} \tag{12}$$

$$CF_t(\text{Case C})=\{4000P_t(LPG)+3300P_t(Gasoline)-8000P_t(LPG)-3300\}\times 10^{-6} \tag{13}$$

$$CF_t(\text{Case D})=\{1900P_t(LPG)+1400P_t(MTBE)+6100P_t(Alkylate)-8000P_t(LPG)-54305\}\times 10^{-6} \tag{14}$$

where $P_t(X)$ is a price of the product X at time t. Since all cases are exclusive the manager must select a case in order to maximize the sub-project value. We assume that one year is equal to 333 days.

The construction cost of each case is 10 (*Case A*), 49 (*Case B*), 11 (*Case C*), and 55 (*Case D*) million dollars, respectively. The manager has the flexibility of changing a case during the project period. For example, if the price of gasoline rises dramatically, the manager can build a Polymerization process unit and switch to *Case C*. Furthermore, the manager can stop the operating process unit temporarily and resume it. It is costly to stop a process unit temporarily and to resume it, but these costs are much less than building a new process unit. The possibility of switching between cases is illustrated in Exhibit 7. Consequently, the value of the project is path-dependent because the cash flows depend on both the prices of the products and the selected case. We assume that the planning period of the project is 20 years. Namely, the manager stops this project after 20 years regardless of the useful life of the process unit.

[3]We assume that prices of other feedstock such as Methanol and Isobutane are constant. This assumption is not crucial from an economical viewpoint.

[4]These data were provided by JGC Corporation.

Exhibit 3. The Value Process of the Switching Option

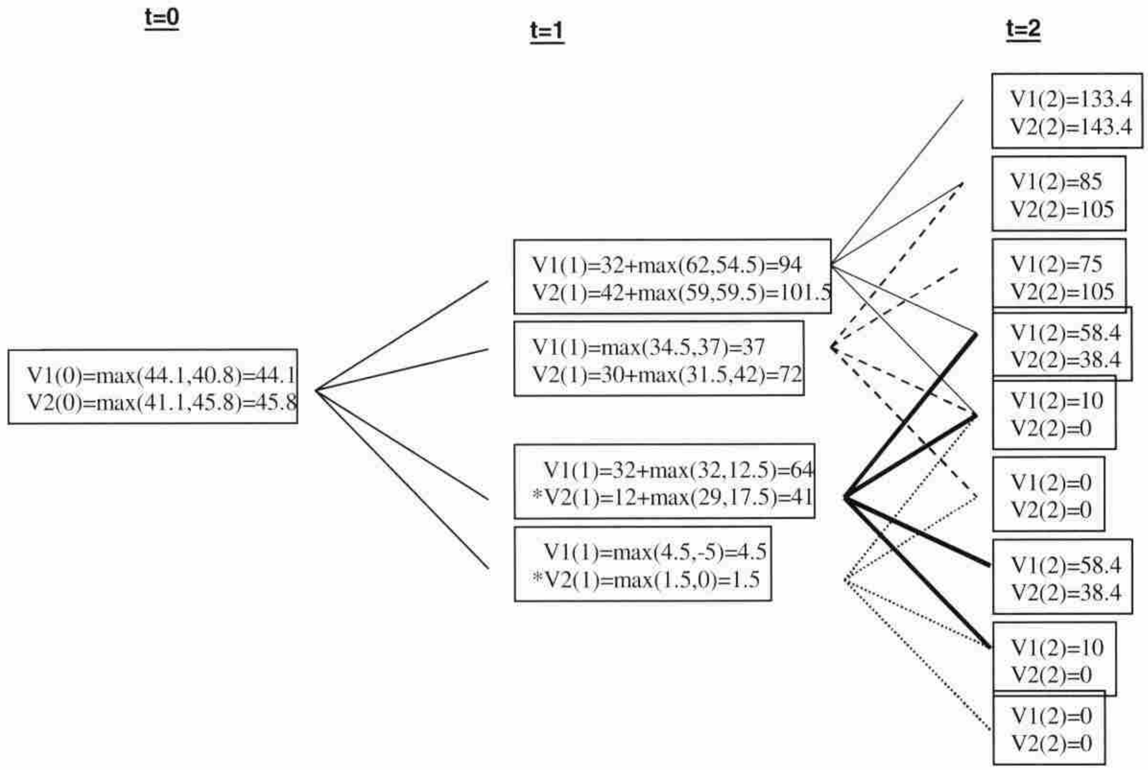
This exhibit illustrates the value process of the switching option when the underlying assets follow the multinomial process shown in Exhibit 2. The option expires at time two. For simplicity, we assume that the risk-free rate is zero. V1(t); t=0, 1, 2 is the value of the option on *Stage 1* before switching at time t. V2(t) is defined in the similar way. As a computation example, we pick up a node where the assets prices are 150 and 132, respectively at time one. We obtain the value of V1(1) according to the following computation.

$32 = CF\_stage1(1) = \max(150 - 150, 0) + \max(132 - 100, 0),$

$62 = p1*133.4 + p2*85 + p3*58.4 + p4*10,$

$54.5 = p1*143.4 + p2*105 + p3*38.4 + p4*0 - SW12,$

where p1 through p4 are the risk neutral probabilities. Thus, the holder should keep *Stage 1* instead of switching to *Stage 2* in this case. A mark "*" in this exhibit indicates that the option holder should change the stage from *Stage 2* to *Stage1* at time one. In the similar way at time zero, we can obtain both values of V1(0) and V2(0). By comparing V1(0) with V2(0) we can conclude that the fair price of the switching option is 45.8, and that the option holder should select *Stage 2* at time zero.



B. The Project Value as a Switching Option

This sub-project can be evaluated using the valuation model of the switching option developed in the previous section because a change of a case can be regarded as an exercise of a switching option. It is evident that this project has four underlying assets (i.e., LPG, MTBE, Alkylate, and Gasoline).

We must figure out the flexibility of the sub-project as a switching option. If the switching cost is constant, it is possible to regard each case as the stage of the option. However, the sub-project here cannot be viewed in this way because the switching cost is not constant. For example, consider the switching cost when the manager changes a process unit from *Base Case* to *Case B*. If a process unit of Alkylation has not been built yet, switching cost is 49 million dollars, which is equal to the construction cost of the Alkylation unit. On the other hand, if an Alkylation unit has been already built and is not temporarily operating, switching cost from *Base Case* to *Case B* is 0.1 million dollars which is equal to the cost to resume the Alkylation unit. Consequently, the switching cost depends on whether the process unit has been already constructed or not.

Therefore, to include these features we set up 22

Exhibit 4. The Entire Process of a Typical Oil Refinery

This exhibit illustrates the entire process of a typical oil refinery. The refinery processes crude oil and produces various petroleum products, such as LPG, gasoline, jet/kerosene, gas oil, and fuel oil. It consists of a number of process units. We focus on Fluid Catalytic Cracking (FCC) complex as a sub-project in this paper. The FCC complex mainly converts low value heavy oil into high value gasoline, and therefore contributes to improving refinery margin.

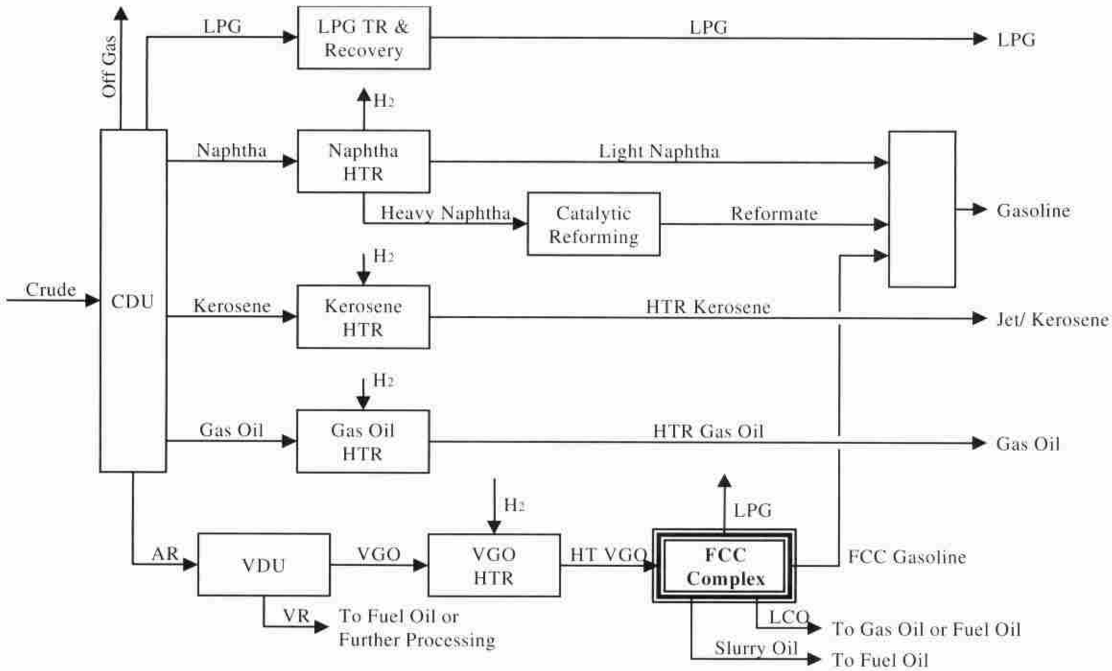


Exhibit 5. A Process Configuration of the FCC Complex

This exhibit illustrates the product flow of the FCC complex. In addition to FCC gasoline, the FCC produces unsaturated liquefied Petroleum Gas (LPG) which is separated into C3 LPG and C4 LPG. Unsaturated LPG can be further processed to convert it into high-octane gasoline components such as Methyl Tertiary Butyl Ether (MTBE), Alkylate, and Polymerization gasoline (Poly-gasoline).

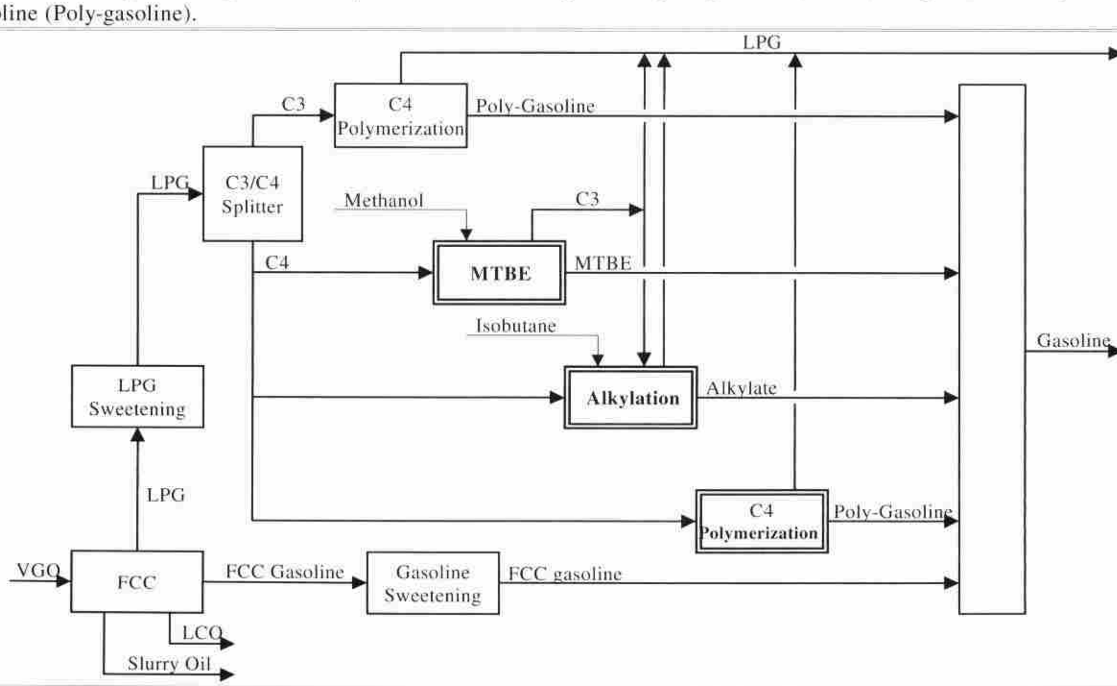
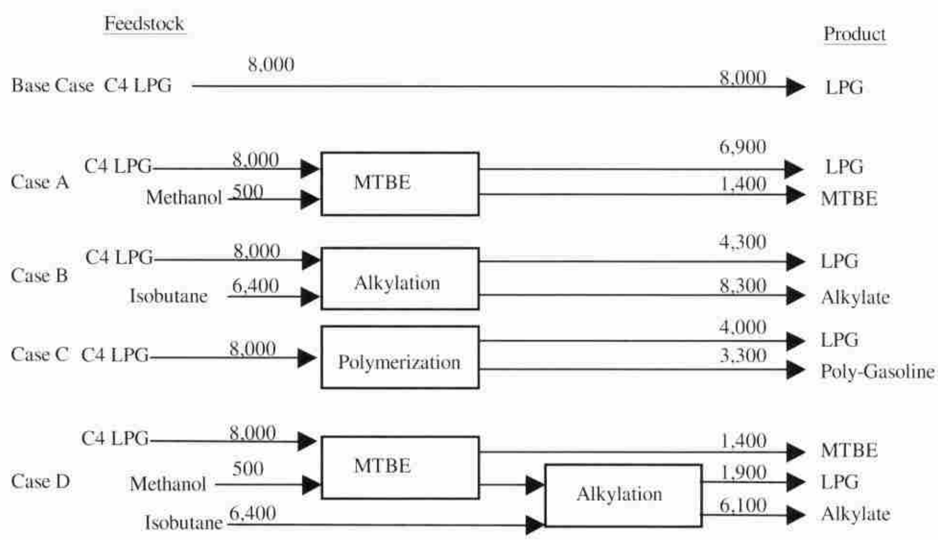


Exhibit 6. Product Flow Scheme of the Sub-Project

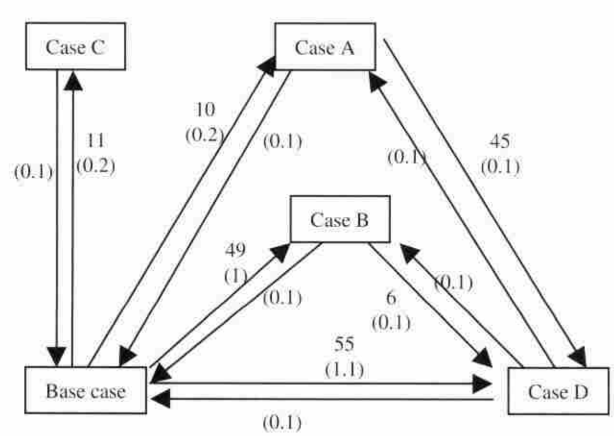
This exhibit represents the product flow of the sub-project. In this paper, we estimate this sub-project value as an independent project. The arrows on the figure indicate the product flow, and rectangles (MTBE, Alkylation Polymerization) indicate process units in the process. A figure that is attached to an arrow on the exhibit indicates quantity of each product per day. The project has five exclusive alternatives that are called "Case". *Base Case* has no process unit in this process, which we use as a benchmark. In *Case A*, the process unit "MTBE" produces 6900 barrels of LPG and 1400 barrels of MTBE from 8000 barrels of LPG and 500 barrels of Methanol. Similarly, LPG and Alkylate are produced in *Case B*, LPG and Gasoline are produced in *Case C*. In *Case D*, there are two process units (i.e., MTBE and Alkylation), and they produce MTBE, LPG, and Alkylate.



Unit: BPD (Barrels per day)

Exhibit 7. The Possibility of Switching Between Cases

This exhibit illustrates the possibility of switching between cases. An arrow on this exhibit indicates a possible switching. It is possible to switch to all cases from *Base Case*. The figure without parentheses indicates the construction cost of a new process unit. On the other hand, the figure in parentheses indicates the cost to resume the non-operating process units that were built before. For example, consider a switch from *Base Case* to *Case B*. When a process unit "Alkylation" has not been built yet, we need 49 million dollars to construct the unit. However, when the unit has been already constructed and is temporarily non-operating, we need only one million dollars to resume it. Conversely, the switching cost from *Case B* to *Base Case* is always 0.1 million dollars.



Unit: million dollars

stages according to not only the operating process units but also temporarily non-operating process units. Exhibit 8 summarizes the stage.

III. Empirical Results

In this section we apply the valuation model of a switching option to the sub-project described in the previous section and estimate the value of its flexibility. We assume that the four underlying assets of the project follow geometric Brownian motions. Parameter values of the underlying assets are estimated using daily data from 1986 to 1995. We give the estimated parameter values in Exhibit 9.

The instantaneous risk-free rate is assumed constant and is equal to 5%. Before discussing the project valuation, we examine the convergence of the lattice procedure. Exhibit 10 illustrates the convergence of the project value in *Situation 3*. This exhibit shows that our model is practical enough to compute the sub-project value of the oil refinery. We can confirm that the value of the sub-project converges as the number of time steps increases.

Exhibit11 shows the value of flexibility in several situations. By comparing these situations, we can estimate flexibility of each process unit. As mentioned in the introduction, this paper does not attempt to estimate the project value of the entire oil refinery but attempts to estimate the value of flexibility that the sub-project could have. Therefore, we use the *Base Case* in *Situation 1* as the benchmark of the sub-project and assume that its value is equal to zero.

According to *Situation 1*, when there is no flexibility of switching from the initial case, it is optimal to start this sub-project with *Case B*. This implies that a process unit of Alkylation is profitable. It is ascertained that the value of the sub-project in *Situation 3* is the largest among all situations because sub-projects in *Situation 3* have full flexibility of switching. According to Exhibit 11, we can conclude that it is optimal to start a sub-project with *Case D* (i.e., both MTBE and Alkylation units) at time zero. The difference between the values in *Situation 1* and *Situation 3* represents the value of full flexibility. This indicates that the value of flexibility in the sub-project is so significant that it should not be ignored by the manager. For example, if the manager ignores the flexibility, the sub-project value is approximately 517 million dollars with *Case B*. On the other hand, if the manager takes advantage of the flexibility, the sub-project is worth 678 million dollars. This implies that the value of the flexibility is 161 million dollars.

In order to analyze the value of flexibility in each case, we compute values from *Situation 4* to *Situation 7*. It is possible to estimate the value contribution of each process unit for the sub-project. We confirm that a process unit of Alkylation is more valuable, and a unit of MTBE is less valuable.

Next, we change the values of parameters and examine value changes of the sub-project. Exhibit 12 summarizes the values with different volatilities, initial prices of the underlying assets, and construction costs when the manager has full flexibility of switching.

In the left panel, we compute the value of the sub-project when the volatilities of all the underlying assets are 0.1, 0.2, 0.3, respectively. It is meaningful to analyze this case because volatility estimation is difficult. It is possible to confirm that the value of the sub-project is an increasing function of the volatility, and the optimal strategy is to start the sub-project with *Case D* regardless of the value of the volatility. In the middle panel, the sub-project values are computed when all initial prices are 10, 20, 30, respectively. It is optimal to start the sub-project with the *Base Case*. The right panel provides the values under the assumption that all construction costs are one-tenth, twice, and ten times the original costs. The switching costs directly affect the optimal strategy. Only when the construction costs are ten times the original costs, the manager starts the sub-project with the *Base Case*, because it is too costly to build process units.

Finally, we examine the optimal switching strategy. We repeatedly apply the valuation model every year. To show the validity of this approach we generate a sample path of output product prices, and illustrate the optimal strategy if this path is realized. Exhibit 13 indicates a sample of the optimal strategy.

In Exhibit 13, we exercise the switching option four times. Consequently, a process unit of Polymerization is not constructed in this example. Since the price change is not predictable, the manager cannot determine the optimal switching strategy in advance. However, the manager can decide the optimal switching according to the actual price change.

IV. Concluding Remarks

This paper evaluates an oil refinery project by using a real option approach. We first develop a model to evaluate a switching option in a multidimensional lattice framework. We adopt the lattice procedure of He (1990) and formulate a valuation model of a switching option based on his lattice model. The model enables us to evaluate switching options when there are several sources of uncertainty and management has flexibility. Next, we apply the valuation model to a part of an oil refinery project and estimate the value of this sub-project.

Exhibit 8. Switching Costs Among 22 Stages

This exhibit shows switching costs among twenty-two stages. The exhibit can be provided according to Exhibit 6 and 7. The five cases are divided into 22 stages according to both operating and non-operating process units. The first panel indicates the stage number. The second panel indicates an operating case. The next two panels indicate operating and non-operating process units, respectively. ALKY means an Alkylation unit and POLY means a Polymerization unit. "Nothing" in the forth panel means that there are no non-operating process units.

Base Case is divided into eight stages from *Stage 1* to *Stage 8*. *Stage 1* has no non-operating process units. *Stage 2* through *Stage 4* has a non-operating process unit, and *Stage 5* through *Stage 7* has two non-operating units. In *Stage 8* there are three non-operating process units such as MTBE, Alkylation and Polymerization. In the same way, *Case A* through *Case C* is divided into four stages and *Case D* is divided into two stages. The fifth panel shows switching costs among the stages. A mark "-" indicates impossibility of switching. We pick up *Stage 11* as an example. In *Stage 11*, a process unit MTBE is operating and a process unit Polymerization is non-operating. The manager has three alternatives for the next period. The manager can, 1) switch to *Stage 6* by paying shutdown cost of MTBE, 2) continue in *Stage 11*, 3) switch to *Stage 22* by paying construction cost of a process unit of Alkylation. The value of the oil refinery project can be estimated as a switching option with 22 stages.

Stage		Operating	Non-Operating	Base Case								Case A				Case B				Case C				Case D	
No.	Case	Process Unit	Process Unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	Base	-	Nothing	0	-	-	-	-	-	-	-	10	-	-	-	49	-	-	-	11	-	-	-	55	-
2	Base	-	MTBE	-	0	-	-	-	-	-	-	0.2	-	-	-	-	49	-	-	-	11	-	-	45	-
3	Base	-	ALKY	-	-	0	-	-	-	-	-	-	10	-	-	1	-	-	-	-	-	11	-	6	-
4	Base	-	POLY	-	-	-	0	-	-	-	-	-	-	10	-	-	-	49	-	0.2	-	-	-	-	55
5	Base	-	MTBE, ALKY	-	-	-	-	0	-	-	-	-	0.2	-	-	-	1	-	-	-	-	-	11	1.1	-
6	Base	-	MTBE, POLY	-	-	-	-	-	0	-	-	-	-	0.2	-	-	-	-	49	-	0.2	-	-	-	45
7	Base	-	ALKY, POLY	-	-	-	-	-	-	0	-	-	-	-	10	-	-	1	-	-	-	0.2	-	-	6
8	Base	-	MTBE, ALKY, POLY	-	-	-	-	-	-	-	0	-	-	-	0.2	-	-	-	1	-	-	-	0.2	-	1.1
9	A	MTBE	Nothing	-	0.1	-	-	-	-	-	-	0	-	-	-	-	-	-	-	-	-	-	-	45	-
10	A	MTBE	ALKY	-	-	-	-	0.1	-	-	-	-	0	-	-	-	-	-	-	-	-	-	-	0.9	-
11	A	MTBE	POLY	-	-	-	-	-	0.1	-	-	-	-	0	-	-	-	-	-	-	-	-	-	-	45
12	A	MTBE	ALKY, POLY	-	-	-	-	-	-	-	0.1	-	-	-	0	-	-	-	-	-	-	-	-	-	0.9
13	B	ALKY	Nothing	-	-	0.1	-	-	-	-	-	-	-	-	-	0	-	-	-	-	-	-	-	6	-
14	B	ALKY	MTBE	-	-	-	-	0.1	-	-	-	-	-	-	-	-	0	-	-	-	-	-	-	0.1	-
15	B	ALKY	POLY	-	-	-	-	-	-	0.1	-	-	-	-	-	-	-	0	-	-	-	-	-	-	6
16	B	ALKY	MTBE, POLY	-	-	-	-	-	-	-	0.1	-	-	-	-	-	-	-	0	-	-	-	-	-	0.1
17	C	POLY	Nothing	-	-	-	0.1	-	-	-	-	-	-	-	-	-	-	-	-	0	-	-	-	-	-
18	C	POLY	MTBE	-	-	-	-	-	0.1	-	-	-	-	-	-	-	-	-	-	-	0	-	-	-	-
19	C	POLY	ALKY	-	-	-	-	-	-	0.1	-	-	-	-	-	-	-	-	-	-	-	0	-	-	-
20	C	POLY	MTBE, ALKY	-	-	-	-	-	-	-	0.1	-	-	-	-	-	-	-	-	-	-	-	0	-	-
21	D	MTBE, ALKY	Nothing	-	-	-	-	0.1	-	-	-	-	0.1	-	-	-	0.1	-	-	-	-	-	-	0	-
22	D	MTBE, ALKY	POLY	-	-	-	-	-	-	-	0.1	-	-	-	0.1	-	-	-	0.1	-	-	-	-	-	0

Exhibit 9. Parameter Values of the Underlying Assets

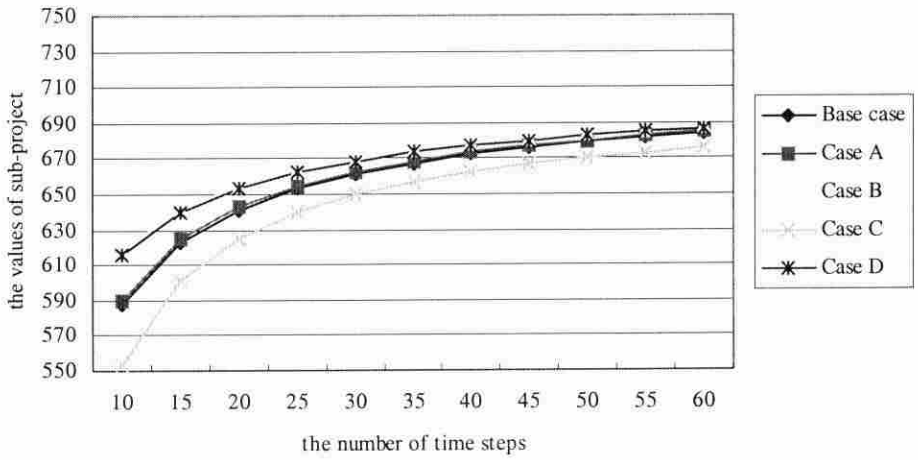
This exhibit summarizes parameter values of the four underlying assets in the sub-project. They are estimated by price data in the US market from 1986 to 1995. The data used in this paper is on a daily basis. The daily price is computed as a mean of the maximum and the minimum price in the day. When a product is not priced, we removed the day from our data set. We estimate both volatility and correlation coefficients under the assumption that each underlying asset follows a geometric Brownian motion. Moreover, we slightly modify some values of parameters according to an expert's forecast in this industry.

			Correlation			
Product	**Initial Price**	**Volatility**	**LPG**	**MTBE**	**Alkylate**	**Poly-Gasoline**
LPG	13.2	0.309	1	0.365	0.314	0.314
MTBE	30.0	0.252	-	1	0.537	0.537
Alkylate	23.7	0.249	-	-	1	0.537
Poly-Gasoline	22.0	0.299	-	-	-	1

Unit: US dollars/Barrel

Exhibit 10. Convergence of the Sub-Project Values

This exhibit illustrates the convergence of the sub-project values when the manager has full flexibility to change between cases. It corresponds to the values of *Situation 3* in Exhibit 11. It is shown that these values quickly tend to converge as the number of time steps.



The results of the case study show that our real option valuation model is useful in practice. They show that the value of flexibility in the sub-project is so large that the manager should take it into consideration. Because the value of flexibility is significantly large, managers who always conduct their analysis on the traditional NPV rule and do not use the real option approach, are in great danger of seriously underestimating the value of their project.■

Exhibit 11. Values of Flexibility

This exhibit summarizes values of flexibility of the sub-project if the manager starts this sub-project with the initial case at time zero. We use 40 as the number of time steps. We evaluate project values under seven situations. The manager has no flexibility to change the *Case* in *Situation 1*. We use the *Base Case* in *Situation 1* as the benchmark of the sub-project and assume that its' value is equal to zero. For example, if the project starts from *Case A* in *Situation 1*, the manager never changes *Case A* during the lifetime of the project, and the value of *Case A* is 133 million dollars compared with the *Base Case*. The sub-project becomes the most profitable when it is started with *Case B* in *Situation 1*. The manager can switch only to *Base Case* and switch back to the initial case in *Situation 2*. Namely, the manager can switch between *Base Case* and the initial case in *Situation 2*. In *Situation 3*, the manager has full flexibility illustrated in Exhibit 7. In order to estimate the value of the flexibility of switching to each *Case*, *Situation 4* through *Situation 7* is examined. In *Situation 4*, the manager has full flexibility of switching except for *Case A*. Namely, we assume that the manager can not switch to *Case A*. In other words, the difference of values between *Situation 3* and *Situation 4* represents the value of flexibility of switching to *Case A*. Similarly, the manager can not switch to *Case B* in *Situation 6*, and can not switch to *Case D* in *Situation 7*. All programs are written with C++ language. We compute on a PC with Pentium II (366MHz) and a memory of 128Mbytes. Computation time is shown in the last line of the table. It takes about an hour to compute in any Situations.

Initial Case at Time 0	Situation 1	Situation 2	Situation 3	Situation 4	Situation 5	Situation 6	Situation 7
Base Case	0	-	672	661	566	638	644
Case A	133	144	673	-	569	639	637
Case B	517	559	678	668	-	645	648
Case C	106	199	662	651	557	-	637
Case D	422	491	677	667	569	644	-
Computation Time (sec.)	3114	3550	3200	3571	3827	3639	4039

Unit: million US dollars

Exhibit 12. The Project Values with Different Parameter Values

This exhibit summarizes values of the sub-project with different parameter values when the manager has full flexibility of switching, namely, in *Situation 3*. The left panel shows the values with three volatilities. We compute values of the project when the volatilities of all underlying assets are 0.1, 0.2, 0.3, respectively. The middle panel represents the values with three initial prices of the underlying assets. We assume that the initial prices of all underlying assets are 10, 20, 30, respectively. The right panel indicates the values under the assumption that all construction costs are one-tenth, twice, and ten times the original costs.

	Volatility			Initial Price			Construction Cost		
	20%	30%	40%	10	20	30	X 0.1	X 2	X 10
Base Case	597	717	853	141	402	711	720	628	392
Case A	598	718	853	132	394	703	722	627	354
Case B	604	721	851	98	383	710	729	623	200
Case C	585	708	844	133	390	695	712	613	340
Case D	603	719	850	97	377	697	729	620	170

Unit: million US dollars

Exhibit 13. The Optimal Switching Strategy

This exhibit shows how the optimal switching occurs in response to a sample path of the underlying assets process. A sample path is generated under the assumption that the four underlying assets follow geometric Brownian motions. Parameter values are used according to Exhibit 9. The left panel indicates time of the project. The middle shows a set of prices of the underlying assets. The next panel shows the stage number of the switching option. The right panel shows the switching that the manager decides. In the last two lines, mean and std represent the *ex post* mean and the standard deviation of the underlying assets in the sample path.

	Product Price				Stage	Switching
Year	LPG	MTBE	ALKY	C4p-gas	Number	Case
0	13.2	30.0	23.7	22.0	21	Build D
1	13.6	32.3	20.6	24.8	21	
2	13.2	36.0	18.3	25.1	21	
3	18.0	39.1	22.4	26.7	21	
4	15.2	33.5	18.7	20.1	10	Resume A
5	15.6	35.0	23.1	22.6	21	Resume D
6	14.0	40.2	25.8	26.4	21	
7	17.9	41.6	29.1	21.2	14	Resume B
8	17.2	31.9	26.2	17.1	14	
9	13.6	32.0	27.6	17.4	14	
10	135	36.3	33.8	24.6	14	
11	17.5	35.0	27.1	18.7	14	
12	13.1	24.5	26.4	17.2	14	
13	17.1	28.7	24.7	20.4	14	
14	15.3	23.6	25.5	19.3	14	
15	15.5	28.5	26.4	22.4	14	
16	17.0	24.1	25.4	24.3	14	
17	11.1	24.0	23.2	16.8	14	
18	10.4	29.1	21.9	16.8	21	Resume D
19	10.4	31.2	23.8	19.6	21	
20	12.9	29.9	23.6	22.4	21	
Mean	14.5	31.7	24.6	21.2		
Std	2.4	5.3	3.5	3.3		

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