Tutorial on Type, Set, and Venn Diagram

Preparation

It is recommended that you read the following chapters before doing this tutorial:

- Type
- Set
- Venn Diagram and Propositional Logic

You should also be able to:

- Navigate around Linux
- Use GHCi, especially loading source files in GHCi.

Set Operations

We have two sets A and B where:

$$\begin{aligned}
A &= \{77, 89, 78, 65, 77, 69, 73, 83, 71, 65, 86, 73, 78\} \\
B &= \{76, 79, 71, 73, 67, 73, 83, 70, 85, 78\} \\
C &= \{1, 2, 3\}
\end{aligned}$$

For each of the sets below, list its elements and find its cardinality.

1. $A \cap B$

> **Solution:**
>
> $$\begin{aligned} A \cap B &= \{71, 73, 78, 83\} \\ |A \cap B| &= 4 \end{aligned}$$

2. $A \cup B$

> **Solution:**
>
> $$\begin{aligned} A \cup B &= \{65, 67, 69, 70, 71, 73, 76, 77, 78, 79, 83, 85, 86, 89\} \\ |A \cup B| &= 14 \end{aligned}$$
>
> Note that $|A \cup B| = |A| + |B| - |A \cap B|$ where $|A| = 9$, $|B| = 9$, $|A \cap B| = 4$.

3. $A - B$

> **Solution:**
>
> $$\begin{aligned} A \cap B &= \{65, 69, 77, 86, 89\} \\ |A \cap B| &= 5 \end{aligned}$$

4. $\wp(C)$

Solution:

$$\begin{aligned}&\wp(C) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}\\&|\wp(C)| = 8\end{aligned}$$

Note that for any set S, $|\wp(S)| = 2^{|S|}$.

5. $\{(x, 22) \mid x \in C\}$

Solution:

$$\begin{aligned}&\{(x, 22) \mid x \in C\} = \{(1, 22), (2, 22), (3, 22)\}\\&|\{(x, 22) \mid x \in C\}| = 3\end{aligned}$$

6. $\{p \mid p \in B, p < 70\}$

Solution:

$$\begin{aligned}&\{p \mid p \in B, p < 70\} = \{67\}\\&|\{p \mid p \in B, p < 70\}| = 1\end{aligned}$$

7. $\{(x, y) \mid x \in A, y \in B\}$, you only need to find this set's cardinality

Solution:

$$|\{(x, y) \mid x \in A, y \in B\}| = |A| \times |B| = 9 \times 9 = 81$$

$\{(x, y) \mid x \in A, y \in B\}$ is called the Cartesian product of A and B, often written as $A \times B$.

8. $\wp(\varnothing)$

Solution:

$$\begin{aligned}&\wp(\varnothing) = \{\varnothing\}\\&|\wp(\varnothing)| = 1\end{aligned}$$

- The empty set $\varnothing$ is a subset of all sets, including itself.
- $|\varnothing| = 0$ and $|\wp(\varnothing)| = 2^{|\varnothing|} = 1$

“Set” Operations in Haskell

Download template.hs[1] and load it in GHCi. `setA`, `setB`, `setC`, and a function `powerset` are already defined in this file and you can use them to verify your solution to the previous exercise. **Note that we are simulating sets using Haskell lists.** Unlike mathematical sets, elements are ordered and there can be duplications in lists. Use the commands below to verify your solution:

1. `intersect setA setB`

Solution:

```
GHCi> :l template.hs
[1 of 1] Compiling Main             ( template.hs, interpreted )
Ok, modules loaded: Main.
GHCi> intersect setA setB
[78,73,83,71]
GHCi> length (intersect setA setB)
4
```

2. `union setA setB`
3. `setA \\ setB`
4. `powerset setC`
5. `[(x, 22) | x <- setC]`
6. `[p | p <- setB, p < 70]`
7. `[(x,y) | x <- setA, y <- setB]`
8. `powerset []`

Make sure to take some time and understand every command. Click here[2] for the documentation.

Pay special attention to the type declaration of `powerset`:

```
powerset :: [a] -> [[a]]
```

`a` is a type variable which acts like a place-holder. Haskell figures out what `a` is automatically when `powerset` is called and this allows for polymorphism. For example:

`a` can be a number.

```
GHCi> powerset [1]
[[1],[]]
```

`a` can be a list of numbers.

```
GHCi> powerset [[1],[]]
[[[1],[]],[[1]],[[]],[]]
```

`a` can be a character.

```
GHCi> powerset ['x','y','z']
["xyz","xy","xz","x","yz","y","z",""]
```

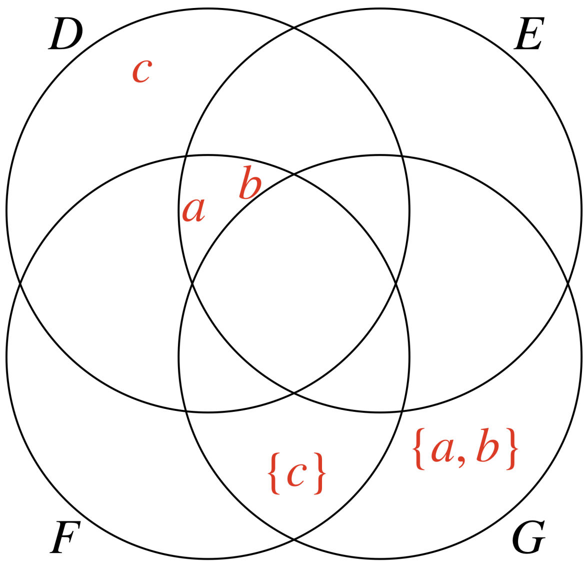
[1] http://homepages.inf.ed.ac.uk/s1757135/type_set_venn/template.hs
[2] http://hackage.haskell.org/package/base-4.12.0.0/docs/Data-List.html#g:20

Sets and Venn Diagrams

Given the following sets:

$$D = \{a, b, c\} \qquad E = \{a, b\}$$
$$F = \{a, b, \{c\}\} \qquad G = \{\{a, b\}, \{c\}\}$$

Fill in the Venn Diagram correspondingly:



Difference Between $\in$ and $\subseteq$

Determine if the statements below are true of false:

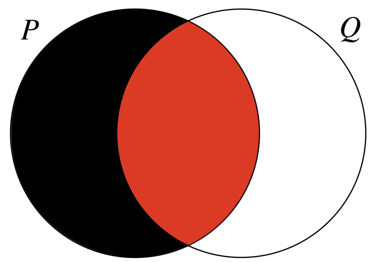
9. $c \in D$ $\surd$ **True** ◯ False
10. $c \in F$ ◯ True $\surd$ **False**
11. $E \subseteq D$ $\surd$ **True** ◯ False
12. $E \in G$ $\surd$ **True** ◯ False
13. $E \subseteq G$ ◯ True $\surd$ **False**
14. $\{\{c\}\} \subseteq F$ $\surd$ **True** ◯ False

Tutorial Activity - Logical Arguments and Venn Diagrams

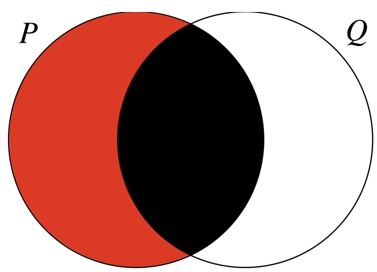
Given two non-empty sets P and Q, for each of the four arguments below:

- Shade the definitely **non-empty** region in the Venn diagram. (Colour red in the solution)
- Fill the definitely **empty** region in the Venn diagram. (Colour black in the solution)

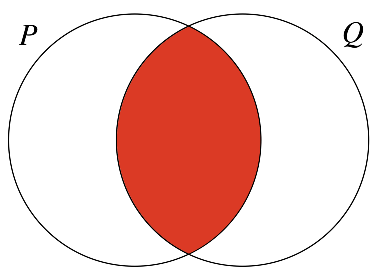
15. **Example:** All P are Q.



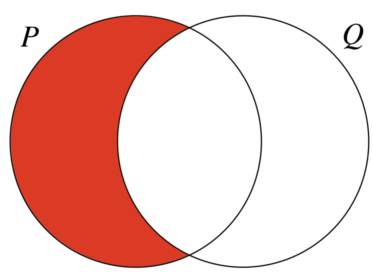
16. No P is Q.



17. Some P are Q.

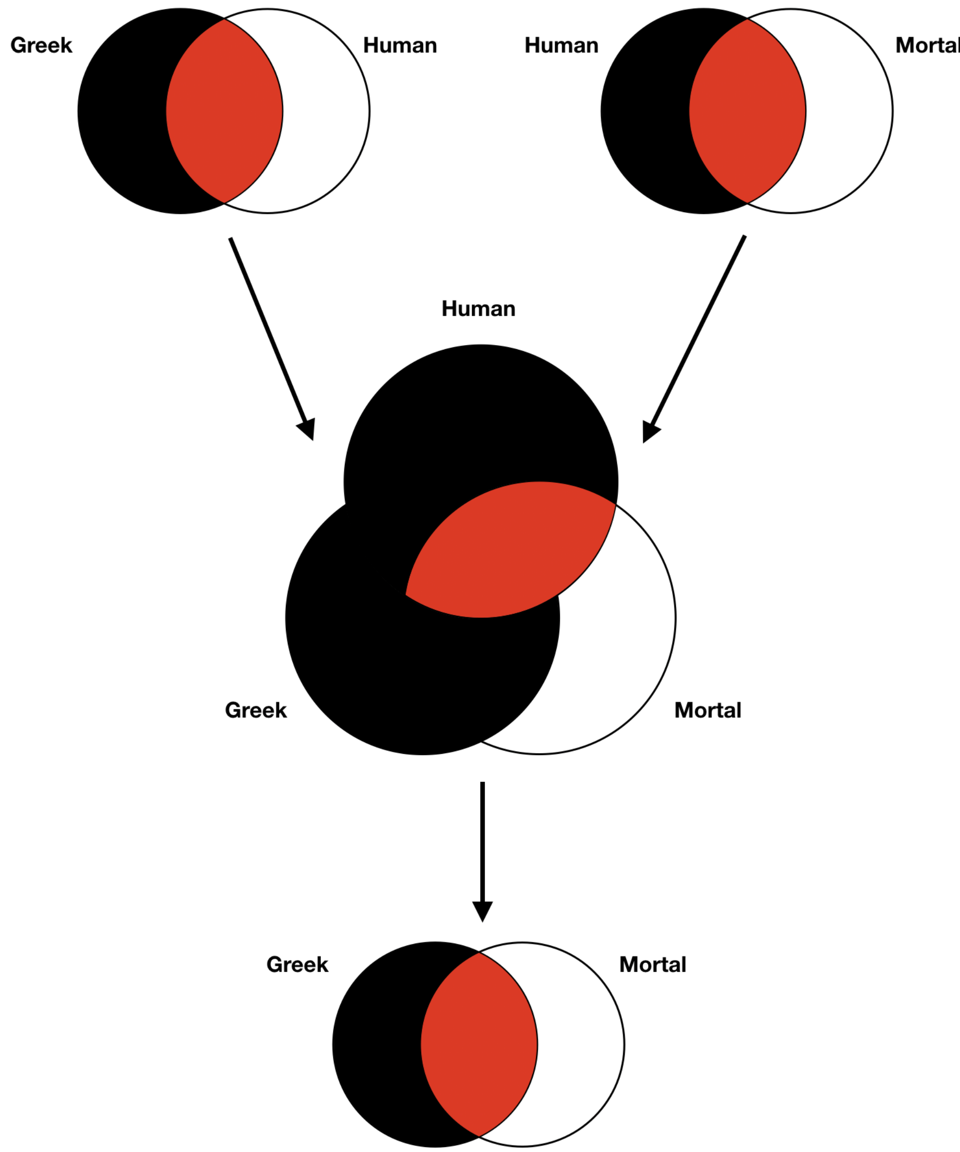


18. Some P are not Q.



We can use the Venn diagrams constructed to visualize syllogism like this one:

```
Premise:    All humans are mortal.
Premise:    All Greeks are humans.
Conclusion: All Greeks are mortal.
```



Collaborate with your group members, draw a similar diagram for this syllogism:

```
Premise:    All logic tutorials are fun.
Premise:    Some tutorials are not fun.
Conclusion: Some tutorials are not logic tutorials.
```