Computing Nash Equilibria for Strategic Games

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Dominance: For two strategies x_i, x_i', we say x_i (weakly) dominates x_i' if for all x_{-i}:

$$U_i(x_{-i}; x_i) \geq U_i(x_{-i}; x_i')$$

We say x_i strictly dominates x_i' if for all x_{-i}:

$$U_i(x_{-i}; x_i) > U_i(x_{-i}; x_i')$$

An even stronger proposition is that x_i dominates x_i' if for all pure counter profiles π_{-1}:

$$U_i(\pi_{-i}; x_i) \geq U_i(\pi_{-i}; x_i')$$

A strategy is **dominant** if it dominates every other strategy. A strategy is **dominated** if there exists at least one other strategy that dominates it.

A strictly dominant strategy must be played in an NE, because the player should always want to switch to it.

A strictly dominated strategy is clearly bad, and it does not make sense for a rational player to play such strategy. If we assume every play is rational and that each of them knows everyone else is rational: we can eliminate such strictly dominated strategy without eliminate any NE. Note that:

- NEs might be eliminated if weakly dominated strategies are eliminated
- A strategy can be dominated by mixed strategies

We can find the NEs of the game by iteratively eliminating strictly dominated strategies, and then find the NEs in the residual game.

Proposition: In an n-player game, strategy profile x^* is an NE iff there exists $w_1, \ldots, w_n \in \mathbb{R}$ (payoffs for every player) such that:

- $\forall$ player i, $\forall \pi_{i,j} \in \text{support}(x_i^*)$, $U_i(x_{-i}^*; \pi_{i,j}) = w_i$
- $\forall$ player i, $\forall \pi_{i,j} \notin \text{support}(x_i^*)$, $U_i(x_{-i}^*; \pi_{i,j}) \leq w_i$

In a two-person game, we can enumerate all possible support sets, and solve LPs according to the above constraints. This enumeration is clearly in exponential time. Or use the Lemke-Howson algorithm to find 1 NE, omitted here.