Middle Model

Potential and field of a point charge

The potential in the vicinity of a single point charge q_c is

$$V = k_C \frac{q_c}{r}$$

where r is the distance from the central point charge q_c. The constant in front is *Coulomb's constant*.

$$k_C = 1.44 \times 10^{-9} \, \text{V·m/e}$$

This equation will get you started finding energies and forces between charged particles.

The first route for finding the force on a charge q, which you already learned, is to start with the electric potential, multiply by q to get the potential energy, and then find the slope to get the force. The second method is to start with the potential, find the slope to get the electric field, and then multiply by q to get the force. Either method gives the same force.

The hard step in either method is finding the slope, which can require calculus or drawing tangent lines on graphs. Using the electric field gets that step out of the way. You have already seen the formula for the potential due to a point charge. With some calculus the charge's electric field can be found. The resulting magnitude of the the electric field is

$$|\vec{E}| = k_C \frac{|q_c|}{r^2}$$

The calculus is done! The remaining steps – finding the field's direction and multiplying by the other charge – do not require sophisticated mathematics.

The direction of the electric field is always towards lower potential. A positive charge creates a region of positive potential, with the highest potential close to the charge, so *the electric field points away from a positive charge,* toward the lower potential far away. A negative charge creates a region of negative potential, with the most negative potential close to the charge, so the *the electric field points toward a negative charge,* toward the nearby lower potential. In both cases the electric field has the greatest magnitude close to the charge, where the potential is changing most rapidly with distance. Far from the charge, where the potential is inching towards zero, the magnitude of the electric field is small.

The electric field gives another explanation for why like charges repel and opposite charges attract. For example, a positive charge

produces an electric field that points away from it. Another positive charge will feel a force in the direction of this field, so the second positive charge is repelled by the first. If the second charge is negative, it will feel a force opposite the electric field and be attracted to the positive charge. Take a moment to determine the direction of the force that a negative particle would exert on another charge, either positive or negative.

The electric field, like the electric potential, obeys the superposition principle, but the electric field vectors must be added *as vectors* which makes the addition more complicated than it is for the electric potential. A charge does not feel its own electric field, only the total field produced by all other charges.

Historically, all of the steps above were discovered in reverse order. The force law between charges was discovered by Charles Coulomb in 1785, and is called Coulomb's law. The electric field was described by Michael Faraday in his *Experimental Researches in Electricity,* begun in 1831. The derivation of the electrical field from the electric potential was given by James Clerk Maxwell in *A Dynamical Theory of the Electromagnetic Field,* published in 1865.

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