

Particles in Fields

DEMOCRITUS SPECULATED that everything is made of small, indivisible units he called “atoms.” He was correct, but the project of identifying and classifying these indivisible units got off to a slow start. Atoms were initially thought to be like tiny rocks whose shapes determined their properties. For example, a substance that stings when it is touched might be made of atoms covered in sharp spikes. These models provided interesting explanations, but no testable predictions about how substances interact.

Real progress began in the late eighteenth century when chemists, notably Antoine-Laurent de Lavoisier, began taking careful measurements of reactants and products to test theories about the identities and properties of elements. Lavoisier listed thirty-three elements in his 1789 *Elements of Chemistry*. By the late nineteenth century, chemists had identified several dozen elements and could predict the quantities of elements required to make certain compounds.

In 1871, Russian chemist Dmitri Mendeleev organized the sixty-six known elements into a repeating pattern according to their chemical properties. He used this pattern to create the first draft of the modern periodic table of the elements. The beginning of Mendeleev’s table (table 1.1) shows several familiar elements in their correct positions.

Mendeleev identified several gaps in his table and predicted new elements to fill these gaps – and he predicted the new elements’ properties. The discoveries of Gallium (1875), Scandium (1879), and Germanium (1886), each with the predicted properties, convinced most scientists that Mendeleev’s table was correct and useful.

The periodic table of the elements, expanded by the discovery of many more elements, is the foundation of chemistry to this day. We now define “atom” as the smallest unit of an element having the chemical properties of that element. Although we will discover that there are smaller, more fundamental particles, the periodic table will always be the definitive list of atoms.

ELECTRIC SPARKS TO ELECTRON BEAMS

While chemists discovered and studied more new elements, physicists turned their attention to electricity, especially electric sparks. Sparks are exciting and easy to make, so physicists and curious children have always studied them with great enthusiasm. Physicists of the nineteenth century knew that sparks carry electrical charge from one object to another. However, sparks happen so quickly that it is not obvious which way the charge is moving.

We now know that sparks are streams of electrons. Electrons pick up speed very quickly in a spark. They move so quickly that when they collide with something, like an air molecule, the collision

By convention sweet and by convention bitter, by convention hot, by convention cold, by convention color; but in reality atoms and void.

DEMOCRITUS
c.460 – c.370 BCE

H						
Li	Be	B	C	N	O	F
Na	Mg	Al	Si	P	S	Cl
K	Ca

TABLE 1.1 The first periodic table of the elements, 1871.

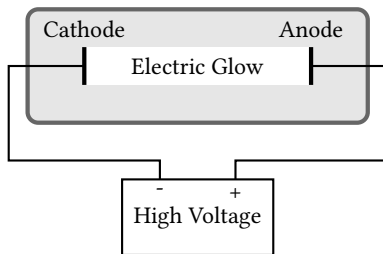


FIGURE 1.1 An evacuated tube for creating a visible beam of current. Electrons traveling from the cathode to the anode create a glow as they collide with gas molecules.

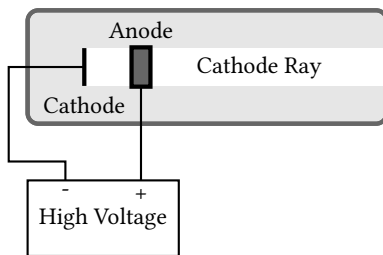


FIGURE 1.2 In a cathode-ray tube the anode is a hoop that electrons pass through. The electrons continue in a beam until reaching the end of the tube. Since the beam starts on the cathode, it is called a cathode ray.

produces a small but visible flash. The glowing path of a spark beam is a multitude of these flashes. This is the case with small sparks from static electricity and, more spectacularly, with lightning.

Sparks are much easier to produce in the absence, or near absence, of air. Physicists produced sealed glass “tubes” (fig. 1.1) for their experiments which had nearly all air removed, allowing the electrons to travel farther between each collision. Inside the sealed tube are two metal plates held by wires going through the tube’s glass wall. These wires are connected to a high voltage power supply. The metal plate connected to the power supply’s negative terminal is called the cathode, while the plate connected to the positive terminal is called the anode. The power supply drives an excess of electrons to the cathode, making the cathode negatively charged. The power supply pulls electrons from the anode, leaving the anode positively charged.

Since like charges repel, the electron excess on the cathode can become great enough to accelerate electrons away from the cathode toward the anode, producing a visible beam of glowing current in the tube. Within these evacuated tubes current can travel much farther than most sparks in open air, and the current can be maintained to produce a continuous glowing beam. The beam of electrons is visible in sealed tubes with most, but not all, of the air removed. When there is no air in the tube the beam is invisible because the electrons do not run into anything along their path.

If the anode is shaped like a hoop it can mostly avoid getting hit by the electrons, as shown in figure 1.2. Electrons are attracted by the anode, but as they approach they are going so fast that they shoot right through the center of the hoop and continue until they run into the end of the tube. These continuing electrons revealed to physicists that the beam began at the cathode, and the beams became known as *cathode rays*. Tubes that are designed so that the cathode rays miss the anode and strike the tube’s wall are called *cathode ray tubes*.

Twentieth century televisions are direct descendants of the nineteenth century cathode ray tubes. At the back of the television is an electron gun that produces a cathode ray pointed at the back of the screen. The screen is covered with phosphorus or a similar material that glows brightly when hit by electrons. The cathode ray is bent by electric and magnetic fields so that it moves rapidly all over the screen. The beam’s intensity changes as it moves, drawing a picture on the screen. Televisions and computer monitors of this type are often called CRTs: cathode ray tubes.

Cathode rays provided a new way for physicists to study extremely small pieces of matter. Similar beams with ever increasing energy have become the primary tool in the search for the fundamental

pieces of matter. Understanding these beams requires a little knowledge about electrical energy and one of the units used to measure energy, the *electron-volt* or eV.

The concept behind the electron-volt is quite simple. Batteries store electrical energy which can be used to run electrical devices. The energy is retrieved by allowing electrons to flow through a circuit from the negative end of the battery, through the device, and back to the battery's positive end. The voltage of the battery is a measure of how much energy is provided by each electron flowing through the circuit. A nine-volt battery, for example, provides nine electron volts (9 eV) of energy for every electron that makes the journey. An electron volt is a very small amount of energy, about the amount of energy released by a single molecule in the battery. Electron volts are related to our other unit of energy, the joule.

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

A typical nine-volt battery has about 10^{22} electrons stored that can make the journey through a circuit, so the battery can provide about 9×10^{22} eV of energy, which is a significant amount of energy of energy.

$$9 \times 10^{22} \times \left(\frac{1.60 \times 10^{-19} \text{ J}}{1} \right) = 14\,000 \text{ J}$$

Individual chemical reaction typically release a few electron volts of energy or less, making the volt a convenient unit for items powered by chemical reactions, like batteries.

While the electron volt is a small amount of energy, it is enough to get an electron moving quite quickly. By the mid nineteenth century physicists were not using a few volts to make their sparks, but several kilovolts (1 kV = 1000 V).

EXAMPLE 1.1 A cathode ray tube is powered by a 5.0 kV power supply. If an electron makes the trip from cathode to anode without running into anything, what will its kinetic energy be when it passes the anode? Give your answer in both electron volts and joules.

A 5.0 kV power supply delivers 5.0 keV of energy to each electron. If the electron does not lose any energy in collisions, its kinetic energy will be 5.0 keV, or

$$K = 5.0 \times 10^3 \times \left(\frac{1.60 \times 10^{-19} \text{ J}}{1} \right) = 8.0 \times 10^{-16} \text{ J}$$

Once the electron passes the anode it no longer feels much force and will continue with 5.0 keV of kinetic energy until it runs into something.

EXAMPLE 1.2 You decide to make a cathode ray whose electrons are going 5.0% of the speed of light ($c = 3.00 \times 10^8$ m/s). You build a tube that has no air in it, so that the electrons will not be slowed by collisions with air molecules. What voltage do you need from your power supply?

Five percent of the speed of light is

$$v = 0.05c = 0.05(3.00 \times 10^8 \text{ m/s}) = 1.50 \times 10^7 \text{ m/s}.$$

This is very fast, but still slow enough that we can use the usual kinetic energy formula, $K = \frac{1}{2}mv^2$, where the mass of an electron is $m = 9.11 \times 10^{-31}$ kg. The electron's kinetic energy must be

$$K = \frac{1}{2}mv^2 = (9.11 \times 10^{-31} \text{ kg})(1.50 \times 10^7 \text{ m/s})^2 = 1.02 \times 10^{-16} \text{ J}$$

Convert this to electron volts.

$$K = 4.10 \times 10^{-14} \cancel{\text{J}} \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \cancel{\text{J}}} \right) = 640 \text{ eV}$$

All of the energy provided by the power supply is going to the electron's kinetic energy, so the power supply must provide 640 eV to each electron. The power supply's voltage, therefore, must be 640 V. Kilovolt power supplies easily get electrons up to a significant fraction of the speed of light!

EXAMPLE 1.3 While carefully connecting a 640 V power supply to your cathode ray tube, you wonder what the average force is on each electron as it is accelerated up to 5.0% of the speed of light. The distance from the cathode to the anode is 3.0 cm. What is the average force?

We have not learned anything about how the power supply gives energy to the electrons (that is the next section) so we will not include the power supply or any potential energy in the system. The system will just be a single electron's kinetic energy, which is increased by work done on the electron by the mysterious power supply to the kinetic energy we found in example 1.2 above. Start with conservation of energy.

$$H_i + W + \cancel{U} = H_f$$

$$\cancel{U_i} + F \Delta x = K_f$$

$$F = \frac{K_f}{\Delta x} = \frac{1.02 \times 10^{-16} \text{ J}}{3.0 \times 10^{-2} \text{ m}} = 3.4 \times 10^{-15} \text{ N}$$

We used the kinetic energy in joules to get a force in newtons. What if we had used the kinetic energy in electron volts for the last step?

$$F = \frac{K_f}{\Delta x} = \frac{640 \text{ eV}}{3.0 \times 10^{-2} \text{ m}} = 2.1 \times 10^4 \text{ eV/m}$$

This is the same force, $2.1 \times 10^4 \text{ eV/m} = 3.4 \times 10^{-15} \text{ N}$. Sometimes measuring force in eV/m will be the more convenient, especially when dealing with tiny particles. Sometimes joules are more convenient. Both are acceptable units of force.

ELECTRICAL POTENTIAL AND ENERGY

The power supply does not give kinetic energy directly to the electrons. Instead, it gives the electrons potential energy. Since like charges repel, negatively charged electrons are pushed away from the negatively charged cathode. Every time the power supply pushes an electron onto the cathode it stores potential energy, much like compressing a spring. Since opposite charges attract, electrons are pulled towards the positively charged anode. Every time the power supply pulls an electron away from the anode it stores potential energy, much like stretching a spring.

This potential energy is stored in the *electrical potential*. The electrical potential is a field that exists everywhere. At any location the electrical potential can be positive, negative, or zero. It is mostly zero, but electric charges alter the electrical potential. Positive charges produce a region of positive potential, while negative charges create a region of negative potential.

The top graph in figure 1.3 shows the electrical potential produced by the charged cathode and anode in a cathode ray tube. Near the negatively charged cathode the electrical potential is negative. Near the positively charged anode the electrical potential is positive. In the middle the effects cancel and the electrical potential is zero.

Electrical potential is measured in volts. In figure 1.3, negative charge has been added to the cathode until its electrical potential is -320 V . Negative charge has been removed from the anode until its electrical potential is 320 V . The potential difference is therefore 640 V , just like examples 1.2 and 1.3 above.

The symbol for electrical potential is V , which looks very similar to the symbol V for volts. Be careful not to confuse them! In figure 1.3 the cathode's potential is $V = -320 \text{ V}$, the anode's is $V = 320 \text{ V}$, and the potential difference is $\Delta V = 640 \text{ V}$.

Electrical potential energy is energy stored in the electrical potential. Notice that electrical potential and electrical potential energy are two distinct things. When a charge q is placed at a location where the potential is V , the electrical potential energy is

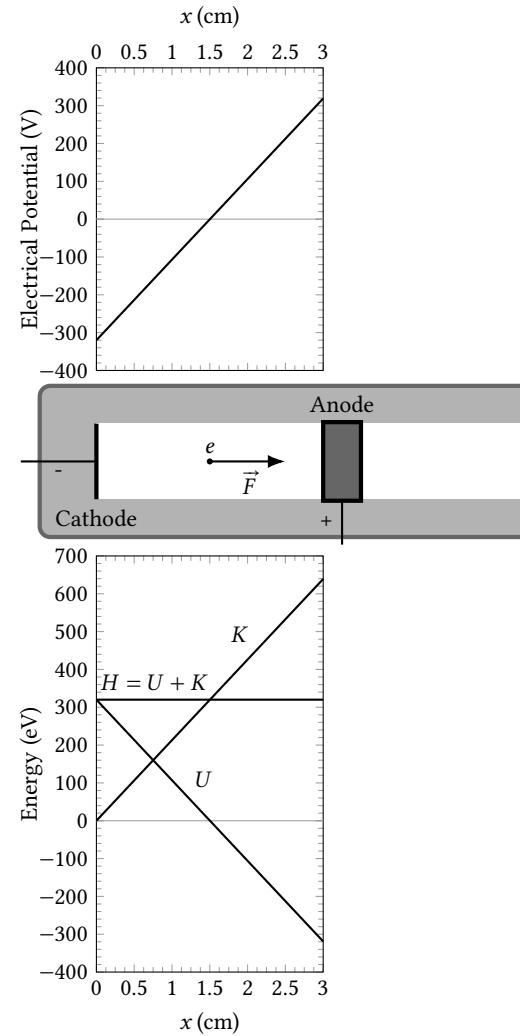


FIGURE 1.3 Above the cathode ray tube is the electrical potential in the 3 cm gap between the cathode and anode when they are attached to a 640 V power supply. Below is the energy graph for an electron accelerating from the cathode towards the anode.

$$U = qV$$

An electron's charge is $q = -1\text{ e}$, where an *elementary unit of charge* is represented by the symbol e . This symbol can be a little confusing because it looks like the symbol for an electron, which is e . Be careful not to confuse them! The charge on an e is -1 e .

The energy graph in figure 1.3 shows the potential energy of an electron anywhere between the cathode and the anode. At the cathode, where the electron beam starts, an electron's electrical potential energy is

$$U = qV = (-1\text{ e})(-320\text{ V}) = 320\text{ eV}.$$

At the anode the electron's potential energy is

$$U = qV = (-1\text{ e})(320\text{ V}) = -320\text{ eV}.$$

Since an electron's charge is negative, its electrical potential energy U is highest where the electrical potential V is most negative. As an electron accelerates from the cathode its potential energy U decreases and its kinetic energy K increases. If the electron does not lose any energy in collisions, all of its lost potential energy will become kinetic energy. The energy graph in figure 1.3 shows the kinetic energy increasing from $K = 0\text{ eV}$ at the cathode up to $K = 640\text{ eV}$ at the anode, with the total energy $H = U + K = 320\text{ eV}$ staying constant for the entire journey.

With the formula for electrical potential energy, we can include potential energy when using energy conservation to solve problems. Including the potential energy in the system is almost always the best choice.

EXAMPLE 1.4 A cathode ray tube is powered by a 5.0 kV power supply which raises the potential of the anode to 2.5 kV and lowers the potential of the cathode to -2.5 kV . If an electron makes the trip from cathode to anode without running into anything, what will its kinetic energy be when it passes the anode?

Start with conservation of energy, including the electron's electrical potential energy.

$$H_i + \cancel{U_i} + \cancel{K_i} = H_f$$

$$U_i + \cancel{U_f} = U_f + K_f$$

$$qV_i + \cancel{qV_f} = qV_f + K_f$$

$$K_f = q(V_i - V_f)$$

The final kinetic energy depends on the electron's charge and the electrical potential difference provided by the power supply.

$$K_f = (-1\text{ e})(-2.5\text{ kV} - 2.5\text{ kV}) = (-1\text{ e})(-5.0\text{ kV}) = 5.0\text{ keV}$$

The electron passes the anode with kinetic energy $K = 5.0 \text{ keV}$.

Knowing the electron's electrical potential energy allows us to calculate the force on the electron using Hamilton's equation. The force shown in figure 1.3 is

$$F = -\frac{\Delta U}{\Delta x} = -\frac{-320 \text{ eV} - 320 \text{ eV}}{3.0 \text{ cm}} = -\frac{-640 \text{ eV}}{3.0 \times 10^{-2} \text{ m}} = 2.1 \times 10^4 \text{ eV/m},$$

which agrees with the result in example 1.3, where the force was found using work. Using Hamilton's equation is usually a better choice.

Armed with the formula for electrical potential energy, we have all of the tools we need to explain the well known facts that *opposite charges attract* while *like charges repel*. First, consider two positive charges. Each creates a small region of positive potential. When you try to put the charges close together, each positive charge enters the positive potential of the other. This gives them positive potential energy which increases as they get closer. Hamilton taught us that the force is always towards lower potential energy, so there will be a force pushing the two positive charges apart. Two positive charges repel.

Two negative charges each produce a small region of negative potential. Putting these negative charges into each others' negative potential gives them positive potential energy which increases as they get closer. They will feel a force pushing them apart, towards lower potential energy. Two negative charges repel.

If the charges are opposite, then putting them close together puts the positive charge in the other's negative potential and the negative charge in the other's positive potential. The resulting potential energy is negative, getting more negative as they get closer together. Opposite charges will feel a force towards this more negative, lower potential energy. Opposite charges attract.

These forces between charges are responsible for all of the contact forces you experience in your everyday life. The forces between your feet and the floor, between a book and the table it sits on, and the force of wind on your face are all due to the electrical potentials of electrons and protons interacting. Gravitation is the only force you encounter which is not electrical.

ELECTRIC FIELD AND FORCE

The electrical potential only exerts a force on charges if the potential has a slope. This slope is called the *electric field*. The electric field provides a useful short-cut for calculating electrical forces. Here is how it works.

Consider a charge q in a potential, like the electron in between the cathode and anode shown in figure 1.4. The force on the charge

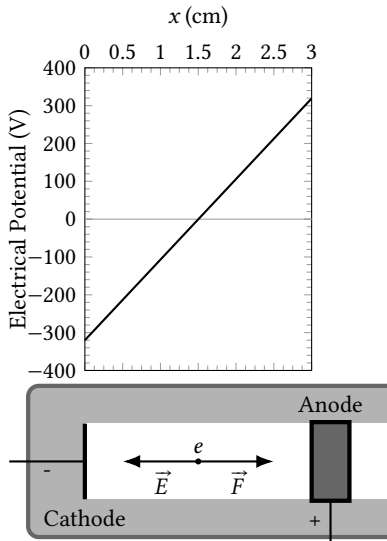


FIGURE 1.4 Above the cathode ray tube is the electrical potential in the 3 cm gap between the cathode and anode when they are attached to a 640 V power supply. The electric field \vec{E} in the gap is a vector pointing in the direction of decreasing potential, with a magnitude equal to the potential's slope. This force on the electron is $\vec{F} = q\vec{E}$.

is given by Hamilton's equation, which can be separated into two factors: the charge q and the slope of the electric potential.

$$F = -\frac{\Delta U}{\Delta x} = -\frac{qV_f - qV_i}{\Delta x} = q\left(-\frac{V_f - V_i}{\Delta x}\right) = q\left(-\frac{\Delta V}{\Delta x}\right)$$

The last factor on the right is the electric field.

$$E = -\frac{\Delta V}{\Delta x}$$

The electric field always points in the direction of decreasing electrical potential. Finding the electric field from the electrical potential can be difficult, but often the field is given. Once you have the electric field, electric force formula is a simple multiplication.

$$\vec{F} = q\vec{E}$$

In many situations calculating the electric field eliminates the need for an energy graph or energy calculations.

EXAMPLE 1.5 A 640 V power supply is connected to a cathode ray tube, producing the potential plotted on the graph in figure 1.4. Find the electric field between the cathode and anode, and the force exerted on an electron by this field.

First, find the electric field from the slope of the electrical potential. This calculation is almost identical to the force calculation at the end of the last section (p. 7), but take care with signs.

$$E = -\frac{\Delta V}{\Delta x} = -\frac{320 \text{ V} - (-320 \text{ V})}{3.0 \text{ cm}} = -2.1 \times 10^4 \text{ V/m},$$

The electric field is negative, which means it points to the left in the diagram. To the left is the direction of decreasing electrical potential.

Now find the force on the electron, which has charge $q = -1 \text{ e}$.

$$\vec{F} = q\vec{E} = (-1 \text{ e})(-2.1 \times 10^4 \text{ V/m}) = 2.1 \times 10^4 \text{ eV/m}.$$

This force is positive, which means it points to the right in the diagram, opposite the electric field.

This force agrees with the results in example 1.3 and at on p. 7.

The electric field is a vector field which exists everywhere in space. (The electric field vector could be the zero vector if there are no charges around.) A charge placed where the electric field is not zero will feel a force. A positive charge feels a force in the direction of the electric field, while a negative charge feels a force in the opposite direction. A neutral particle does not feel any force from electric fields.

The electric field vector's individual components are the potential's slope in the coordinate directions.

$$E_x = -\frac{\Delta V}{\Delta x} \quad E_y = -\frac{\Delta V}{\Delta y} \quad E_z = -\frac{\Delta V}{\Delta z}$$

Since the electric potential is measured in volts, the electric field is in volts per meter.

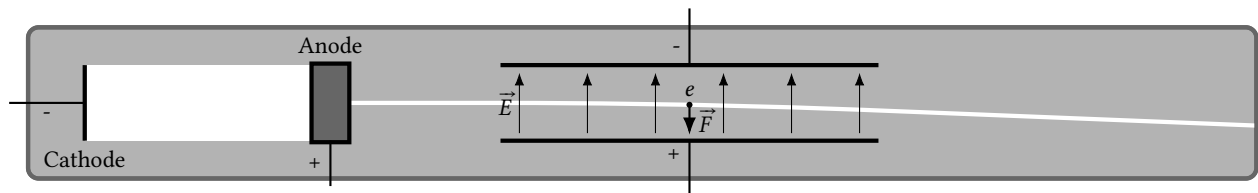
The cathode ray experiment in figure 1.5 shows a common example of how electric force is used to deflect a cathode ray. The plates near the center are charged so that the top plate has a lower potential while the bottom plate has a higher potential. This produces an electric field between the plates directed upwards – towards lower potential. The upwards electric field produces a downwards electric force on the negatively charged electrons, deflecting the beam downwards.

The direction of the electric field is always towards lower potential. A positive charge creates a region of positive potential, with the highest potential close to the charge, so *the electric field points away from a positive charge*, toward the lower potential far away. A negative charge creates a region of negative potential, with the most negative potential close to the charge, so *the electric field points toward a negative charge*, toward the nearby lower potential. In both cases the electric field has the greatest magnitude close to the charge, where the potential is changing most rapidly with distance. Far from the charge, where the potential is inching towards zero, the magnitude of the electric field is small.

The electric field gives another explanation for why like charges repel and opposite charges attract. For example, a positive charge produces an electric field that points away from it. Another positive charge will feel a force in the direction of this field, so the second positive charge is repelled by the first. If the second charge is negative, it will feel a force opposite the electric field and be attracted to the positive charge. Take a moment to determine the direction of the force that a negative particle would exert on another charge, either positive or negative.

Historically, the electrical potential, electric field, and electric force were discovered in reverse order. The force law between charges was discovered by Charles Coulomb in 1785, and is called Coulomb's law.

FIGURE 1.5 A cathode ray deflected by an electric field between two charged plates. The anode has a small hole so the cathode ray makes a thin beam. The beam is deflected by the electric field between the two charged plates



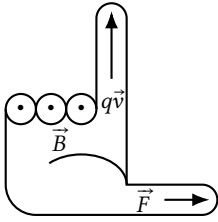


FIGURE 1.6 The right-hand rule for a magnetic field \vec{B} coming out of the page (dots). The magnetic force \vec{F} (thumb) is perpendicular to the electrical current $q\vec{v}$ (pointer finger) and the magnetic field \vec{B} (little fingers)

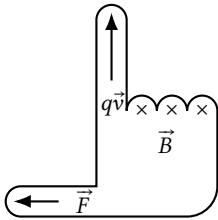


FIGURE 1.7 The right-hand rule for a magnetic field \vec{B} going into the page (crosses). The magnetic force \vec{F} (thumb) is perpendicular to the electrical current $q\vec{v}$ (pointer finger) and the magnetic field \vec{B} (little fingers)

The electric field was described by Michael Faraday in his *Experimental Researches in Electricity*, begun in 1831. The derivation of the electrical field from the electric potential was given by James Clerk Maxwell in *A Dynamical Theory of the Electromagnetic Field*, published in 1865.

MAGNETIC FIELD AND FORCE

The magnetic field, like the electric field, is a vector field that exists everywhere. The value of the magnetic field can be zero, but this is actually rare. Earth is not electrically charged, so it does not produce an electric field, but Earth does produce a huge magnetic field which extends far out into space. This magnetic field is produced by electric currents deep in Earth's molten core, powered by convection currents and Earth's rotation. This magnetic field protects Earth from the dangerous solar wind, a wind of fast charged particles coming at us constantly from the Sun.

Mars, being somewhat smaller than Earth, cooled more quickly after the formation of the solar system, so it no longer has a molten core and therefore has lost its magnetic field, exposing it to the solar wind. The solar wind gradually blew Mars's atmosphere away, making it a rather inhospitable place. Magnetic fields are very important!

The magnetic field exerts a force on charged particles, but only on charged particles that are moving. The direction of the magnetic force is *perpendicular* to the direction of the field and also *perpendicular* to the particle's direction of motion. This is described by a vector cross product.

$$\vec{F} = q\vec{v} \times \vec{B}$$

In this book we will only consider situations where the magnetic field is perpendicular to the charged particle's velocity. In this case the magnitude of the magnetic force is simply

$$F = qvB.$$

The direction of the magnetic force is determined by the right-hand rule, shown in figures 1.6 for magnetic fields out of the page and in figure figure 1.7 for magnetic fields into the page. The three vectors in the magnetic force formula are assigned to different fingers on your right-hand. The vector $q\vec{v}$ is assigned to your index finger, as if you are firing the charge like an imaginary bullet out of your straight, extended finger. However, if q is negative then you need to point your index finger in the direction *opposite* the velocity vector \vec{v} , keeping it extended and straight while reversing your entire hand. The direction of $q\vec{v}$ is the direction of the *electrical current* due to the charge's movement.

The magnetic field vectors are assigned to your remaining fingers – lots of fingers for the lots of magnetic field vectors. These fingers

are not extended and should bend naturally in the direction of the magnetic field.

If the index finger and the remaining fingers produce some angle between 0° and 180° , then the thumb, the most forceful of fingers, shows the direction of the force. The thumb should be extended and straight, perpendicular to both the extended index finger and the remaining bent fingers.

The right-hand rule can be used when the velocity and magnetic field are pointing in any direction, not just the directions I have described. For more complicated situations, the right-hand rule is a fully three dimensional exercise. You should train with an experienced right-hand rule practitioner to avoid sign errors and injury.

Gravitational forces are responsible for the orbits of celestial objects – moons orbit planets, planets orbit stars, and stars orbit galaxies. Electrical forces are responsible for the orbits of electrons around the nucleus of an atom. In both cases the field, gravitational or electrical, is produced by an object at the center which causes an attractive force holding the smaller object in orbit.

Magnetic orbits are totally different because they have *nothing* at the center. This is possible because the magnetic force is perpendicular to the particle's motion, changing the particle's direction, but not its speed. As the particle's path bends, the direction of the force also changes, staying always perpendicular to the particle's velocity. This changing force will curve the particle's path around into a circle (or a helix in more complicated situations).

Circular magnetic orbits are used to keep particles orbiting in circular particle colliders. Charged particles in the solar wind follow tight helical orbits that follow Earth's magnetic field to the poles, where the particles crash into the atmosphere producing northern and southern lights.

The electric potential and the electric field are both very useful. The electric potential is especially useful when you want to understand the energy in the system, while the electric field is especially convenient when you want to understand the electric forces. We have learned about the magnetic field, which is convenient when we want to understand the magnetic forces. (The magnetic field is not quite as convenient as the electric field because of the cross product and the right-hand rule, but it works.) Surely, you are now anxious to learn about the magnetic potential for situations where you want to study magnetic potential energy.

Unfortunately, I have bad news. The magnetic potential is quite complicated. It is actually a *vector* potential. This is related to the fact the magnetic force involves the velocity, which is also a vector. The magnetic potential energy involves the charge, the magnetic vector potential, and the velocity vector, which makes it complicated. The real killer, however, is the magnetic vector potential's contribution

to a magnetic potential momentum! The formula is not complicated, but working with the potential momentum requires extreme mental discipline.

Fortunately, I have good news, too. Because the magnetic force is perpendicular to the charge's motion, magnetic forces never do any work! Magnetic potential energy only interacts with the kinetic energy of charges and currents through electric forces, which can do work. While I recommend learning vector calculus so you can contemplate the supreme beauty of the magnetic vector potential, you won't actually need the magnetic vector potential for any problems in introductory physics. Use the *electric* force, the magnetic force will not work!¹

¹ Kenobi, Obi-Wan. Private communication.