

# Counting

①  $\frac{7!}{1!} + \frac{5!}{2!} \cdot \binom{4}{1} + \frac{5!}{3!} \cdot \binom{4}{2} = 480$  different strings  
 $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} = 11$  unique subsets

②  $\binom{13}{2} \binom{4}{2} \binom{4}{2} \cdot (52 - 8) = 123,552$

③  $\binom{15+6-1}{6-1} + \binom{15+7-1}{6-1} = 35853$

④  $\begin{matrix} 3 \\ \swarrow \\ 9 \end{matrix}$  left child of 3 can only be 1 or 2 2

$\begin{matrix} 9 \\ \swarrow \\ 10 \end{matrix}$  right child of 9 : 10-12 5

$\begin{matrix} 10 \\ \swarrow \searrow \\ 11 \quad 12 \end{matrix}$ 
 $\begin{matrix} 10 \\ \swarrow \searrow \\ 11 \quad 12 \end{matrix}$ 
 $\begin{matrix} 11 \\ \swarrow \searrow \\ 10 \quad 12 \end{matrix}$ 
 $\begin{matrix} 12 \\ \swarrow \searrow \\ 11 \quad 10 \end{matrix}$ 
 $\begin{matrix} 12 \\ \swarrow \searrow \\ 10 \quad 11 \end{matrix}$

$\begin{matrix} 9 \\ \swarrow \\ 1 \end{matrix}$  left child of 9 : 4-8

2 children : 2 42  
 3 children : 5  
 4 children : 14  
 5 children : 42

total :  $2 \times 5 \times 42 = 420$

⑤ case 1 : 3 nurses :

$(1,1,8) (2,3,5)$   
 $(1,2,7) (2,4,4)$   
 $(1,3,6) (3,3,4)$   
 $(1,4,5)$   
 $(2,2,6)$

case 2 : 4 nurses

$(1,1,1,7) (1,2,2,5) (2,2,3,3)$   
 $(1,1,2,6) (1,2,3,4)$   
 $(1,1,3,5) (1,3,3,3)$   
 $(1,1,4,4) (2,2,2,4)$

assume all get vaccines

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