Probability

① $P = \boxed{\frac{\binom{15}{7}}{15^8}}$

② $P(1 \text{ event}) = \frac{5\times4\times[\text{crossed out}]\times210}{100000} = 0.042$

$P = \boxed{(0.042)^5 \cdot (1-0.042)^3 \cdot \binom{8}{5}}$

③ $\mathbb{P}(A) = \left(\frac{3}{6}\right)^3 \cdot \left[\binom{3}{3} + \binom{3}{2}\right] = \frac{1}{2}$

$\mathbb{P}(B) = \frac{6}{6^3} = \frac{1}{36}$

$\mathbb{P}(AB) = \frac{3}{6^3} = \frac{1}{72}$

$\mathbb{P}(AB) = \mathbb{P}(A) \cdot \mathbb{P}(B) \Rightarrow \boxed{\text{independent}}$

④ $\mathbb{P}(\text{flush}) = \frac{\binom{13}{5}\times4}{\binom{52}{5}} = \frac{1287\times4}{2598960} = 0.000495\,2\times4 = 0.00198$

$E(\text{flush}) = \frac{1}{0.00198} = \boxed{505}$

⑤ Bayes

$P(W|P) = 0.7$ $\quad P(W|\bar{P}) = 0.5$ $\quad P(W_4|P) = (0.7)^4 \cdot 0.3 \cdot \binom{5}{4} \approx 0.360$

$P(P) = 0.75$ $\quad P(W_4|\bar{P}) \approx (0.5)^5 \cdot \binom{5}{4} = 0.156$

$P(P|W_4) = \frac{P(W_4|P)\cdot P(P)}{P(W_4)} = \frac{0.36\times0.75}{0.36\times0.75 + 0.156\times0.25}$

$\approx \boxed{0.874}$