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MA1102R
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00. FUNCTIONS & SETS

sets

$$A = \{x \mid \; properties \; of x\}$$

- $A \subseteq B$: A is a subset of B
- $A \nsubseteq B$: A is not a subset of B
- $A = B \leftrightarrow A \subseteq B \wedge B \subseteq A$

operations on sets

- union: $A \cup B = \{x \mid x \in A \vee x \in B\}$
- intersection: $A \cap B = \{x \mid x \in A \wedge x \in B\}$
- difference: $A \backslash B = \{x \mid x \in A \wedge x \notin B\}$

notations of sets

- $\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{N}$
- $\mathbb{N} = \mathbb{Z}^+$
- $\emptyset$: empty set

notations of intervals

- closed interval (inclusive): $[a, b] = \{x \mid a \leq x \leq b\}$
- open interval (exclusive): $(a, b) = \{x \mid a < x < b\}$
  - $(a, \infty) = \{x \mid a < x\}$

functions

- **existence**: $\forall a \in A, f(a) \in B$
- **uniqueness**: $\forall a \in A$ has only one image in B.
- for $f : A \to B$
  - domain: A
  - codomain: B
  - range: $\{f(x) \mid x \in A\}$
- for this mod:
  - $A, B \subseteq \mathbb{R}$
  - if A is not stated, the domain of f is the largest possible set for which f is defined
  - if B is not stated, $B = \mathbb{R}$

graphs of functions

The graph of f is the set
$$G(f) := \{(x, f(x)) \mid x \in A\}$$

- if $A, B \subseteq R$ then $G(f) \subseteq A \times B \subseteq \mathbb{R} \times \mathbb{R}$
- each element is a point on the Cartesian plane $\mathbb{R}^2$

algebra of functions

function	domain
$(f+g)(x) := f(x) + g(x)$	$A \cap B$
$(f-g)(x) := f(x) - g(x)$	$A \cap B$
$(fg)(x) := f(x)g(x)$	$A \cap B$
$(f/g)(x) := f(x)/g(x)$	$\{x \in A \cap B \mid g(x) \neq 0\}$

types of functions

- **rational function**: $R(x) = \frac{P(x)}{Q(x)}$, where P, Q are polynomials and $Q(x) \neq 0$
  - every polynomial is a rational function ($Q(x) = 1$)
- **algebraic function**: constructed from polynomials using algebraic operations
- a function f is **increasing** on a set I if $x_q < x_2 \Rightarrow f(x_1) < f(x_2)$ for any $x_1, x_2 \in I$.
- a function f is **decreasing** on a set I if $x_q < x_2 \Rightarrow f(x_1) > f(x_2)$ for any $x_1, x_2 \in I$.
- even/odd:
  - **even function**: $\forall x, f(-x) = f(x)$
    - ⋆ symmetric about the y-axis
  - **odd function**: $\forall x, f(-x) = -f(x)$
    - ⋆ symmetric about the origin O
  - any function defined on $\mathbb{R}$ can be decomposed *uniquely* into the sum of an even function and an odd function
- **power function**: x^n

$$x^n \text{ is } \begin{cases} \text{an odd function,} & \text{if } n \text{ is odd} \\ \text{an even function,} & \text{if } n \text{ is even} \end{cases}$$