CS3223 AY23/24 SEM 2

Notation	Meaning
r	relational algebra expression
$\|\|r\|\|$	number of tuples in output of r
$\|r\|$	number of pages in output of r
b_d	number of data records that can fit on a page
b_i	number of data entries that can fit on a page
F	average fanout of B$^+$-tree index (i.e., number of pointers to child nodes)
h	height of B$^+$-tree index (i.e., number of levels of internal nodes) $h = \lceil \log_F(\lceil \frac{\|\|R\|\|}{b_i} \rceil) \rceil$ if format-2 index on table R
B	number of available buffer pages

04.1 SORTING

- **clustered index** $\rightarrow$ order of data entries $\approx$ data records
  - ≥ 1 per relation; format 1 is always clustered

External Merge Sort

- **sorted run** $\rightarrow$ sorted data records written to a file on disk
  1. create temporary file R_i for each B pages of R sorted
  2. merge: use $B-1$ pages for input, 1 page for output
- total I/O = $2N(\lceil \log_{B-1}(N_0) \rceil + 1)$
  - $2N$ to create $\lceil N/B \rceil$ sorted runs of B pages each
  - merging sorted runs: $2N \times \lceil \log_{B-1} N_0 \rceil$

optimisation with blocked I/O

- sequential I/O - read/write in *buffer blocks* of b pages
- one block (b pages) for output, remaining blocks for input
  - number of runs merged per pass, $F = \lfloor \frac{B}{b} \rfloor - 1$
  - number of passes = $\lceil \log_F(N_0) \rceil + 1$

Sorting with B$^+$-trees

- when *sort key is a prefix of the index key* of the B$^+$-tree
- sequentially scan leaf pages of B$^+$-tree
  - for Format-2/3, use RID to retrieve data records

04.2 SELECTION: $\sigma_p(R)$

- $\sigma_p(R)$ selects rows from relation R satisfying predicate p
- **selectivity** of an access path $\rightarrow$ number of index & data pages retrieved (more selective = fewer pages retrieved)
- **covering index** I for $Q \rightarrow$ if all attributes referenced in Q are part of the key of I (**index-only plan**: no RID lookup)

Matching Predicates

- **term** $\rightarrow$ of form $R.A$ op c or $R.A_i$ op $R.A_j$
- **conjunct** $\rightarrow \geq 1$ terms connected by $\vee$ (**disjunctive**: > 1)
- **CNF predicate** $\rightarrow$ one or more conjuncts connected by $\wedge$

$$\overbrace{(\underbrace{\text{rating} \geq 8}_{\text{term/conjunct}} \vee \underbrace{\text{director} = \text{"Coen"}}_{\text{term/conjunct}})}^{\text{disjunctive conjunct}} \wedge \underbrace{(\text{year} > 2003)}_{\text{term/conjunct}} \wedge \underbrace{(\text{language} = \text{"English"})}_{\text{term/conjunct}}$$

B$^+$-tree matching predicates

- for index $I = (K_1, K_2, \dots, K_n)$ and non-disjunctive CNF predicate p, I matches p if p is of the form

$$\underbrace{(K_1 = c_1) \wedge \cdots \wedge (K_{i-1} = c_{i-1})}_{\text{zero or more equality predicates}} \wedge (K_i\ op_i\ c_i),\ i \in [1, n]$$

- matching index: matching records are in contiguous pages

Hash index matching predicates

- hash index I matches p if p is of form $(K_1 = c_1) \wedge (K_2 = c_2) \wedge \cdots \wedge (K_n = C_n)$

Primary/Covered Conjuncts

- **primary conjuncts** $\rightarrow$ subset of conjuncts that I matches
  - e.g. p = (A $\geq$ 18) $\wedge$ (A $\leq$ 20) $\wedge$ (W=65) for I = (A,W,H)
- **covered conjuncts** $\rightarrow$ attribute appears in the key of I
  - primary conjuncts $\subseteq$ covered conjuncts

Cost of Evaluation

let p' = primary conjuncts of p, p_c = covered conjuncts of p

B$^+$-tree index evaluation of p

1. navigate internal nodes to find first leaf page
   $\text{cost}_{\text{internal}} = \lceil \log_F(\lceil \frac{||R||}{b_{d \text{ or } i}} \rceil) \rceil$ for format-1/otherwise
2. scan leaf pages to access all qualifying data entries
   $\text{cost}_{\text{leaf}} = \lceil \frac{||\sigma_{p'}(R)||}{b_{d \text{ or } i}} \rceil$ for format-1/otherwise
3. retrieve qualified data records via RID lookups
   $\text{cost}_{\text{RID}} = ||\sigma_{p_c}(R)||$ or 0 if I is covering or format-1
   - reduce cost with **clustered** data records (sort RIDs):
     $\lceil \frac{||\sigma_{p_c}(R)||}{b_d} \rceil \leq \text{cost}_{RID} \leq \min\{||\sigma_{p_c}(R)||, |R|\}$

hash index evaluation of p

- **format-1**: cost to retrieve data records $\geq \lceil \frac{||\sigma_{p'}(R)||}{b_d} \rceil$
- **format-2**: cost to retrieve data entries $\geq \lceil \frac{||\sigma_{p'}(R)||}{b_i} \rceil$
  cost to retrieve data records =
  $\begin{cases} 0 & \text{if I is a covering index,} \\ ||\sigma_{p_c}(R)|| & \text{otherwise} \end{cases}$

05.1 PROJECTION $\pi_{A_1,\dots,A_m}(R)$

- $\pi_L(R)$ eliminates duplicates, $\pi^*_L(R)$ preserves duplicates
- can **index scan** if index contains the attributes *as a prefix*

Sort-based approach

cost analysis

1. extract attributes: $|R|$ scan + $|\pi^*_L(R)|$ output temp result
2. sort records: $2|\pi^*_L(R)|(\log_m(N_0) + 1)$
3. remove duplicates: $|\pi^*_L(R)|$ to scan records

optimised sort-based approach

1. create sorted runs with projected attributes only
2. merge sorted runs and remove duplicates

- if $B > \sqrt{|\pi^*_L(R)|}$, same I/O cost as hash-based approach
  - $N_0 = \lfloor \frac{|R|}{B} \rfloor \approx \sqrt{|\pi^*_L(R)|}$ initial sorted runs
  - $\log_{B-1}(N_0) \approx 1$ merge passes

Hash-based approach

1. **partitioning phase**: hash each tuple $t \in R$ to some R_i
   - one buffer for input, $(B-1)$ buffers for output
   - for each t: project attributes to form t', hash $h(t')$ to one output buffer, flush output buffer to disk when full
2. **duplicate elimination** from each $\pi^*_L(R_i)$
   - for each R_i: initialise in-mem hash table, hash each $t \in R_i$ to bucket B_j with $h' \neq h$, insert if $t \notin B_j$
   - write tuples in hash table to results

- **I/O cost** (no partition overflow): $|R| + 2|\pi^*_L(R)|$
  - partitioning cost: $|R| + |\pi^*_L(R)|$
  - duplicate elimination cost: $|\pi^*_L(R)|$
- partition overflow: recursively apply partitioning
  - to avoid, $B >$ size of hash table for R_i = $\frac{|\pi^*_L(R)|}{B-1} \times f$
    - approximately $B > \sqrt{f \times |\pi^*_L(R)|}$

05.2 JOIN $R \bowtie_\theta S$

R = outer relation (smaller relation); S = inner relation

! for **format-2** index, add cost of retrieving record

- **tuple-based** nested loop join: $|R| + ||R|| \times |S|$
- **page-based** nested loop join: $|R| + |R| \times |S|$
- **block nested loop join**: $|R| + (\lceil \frac{|R|}{B-2} \rceil \times |S|)$, $|R| \leq |S|$
  - 1 page output, 1 page input, $(B-2)$ pages to read R
  - for each $(B-2)$ pages of R: for each P_S of S: check r,s
- **index nested loop join**: for joining $R.A_i = S.B_j$
  $|R| + ||R|| \times \left(\log_F(\lceil \frac{||S||}{b_d} \rceil) + \lceil \frac{||S||}{b_d ||\pi_{B_j}(S)||} \rceil + c \right)$

sort-merge join

- sort R & S: $2|R|(\log_m(N_R) + 1) + 2|S|(\log_m(N_S) + 1)$
- merge cost: $|R| + |S|$ (worst case $|R| + ||R|| \times |S|$)

optimised sort-merge join

- merge sorted runs until $B > N(R, i) + N(S, j)$; then join
- $3(|R| + |S|) = 2 + 1$ (for initial sorted runs + merging)
  - if $B > \sqrt{2|S|}$, one pass to merge initial sorted runs

Grace hash join

for *build relation* R and *probe relation* S,

1. **partition** R and S into k partitions each, $k = B - 1$
   - $\pi_A(R_i) \cap \pi_B(S_j) = \emptyset \quad \forall R_i, S_j, i \neq j$
   - $R = R_1 \cup R_2 \cup \cdots \cup R_k, \quad t \in R_i \iff h(t.A) = i$
2. **probing phase**: hash $r \in R_i$ with $h'(r.A)$ to table T; $\forall s \in S_i, r \in$ bucket $h'(s.B)$: output (r, s) if match
   - $R \bowtie_{R.A=S.B} S = (R_1 \bowtie S_1) \cup \cdots \cup (R_k \bowtie S_k)$

- **partition overflow** if R_i cannot fit in memory: recurse
- I/O cost: $\#\text{Partition Phases} \times 2(|R| + |S|) + |R| + |S|$
- $B > \frac{f \times |R|}{B-1} + 2$ (input & output buffer) $\approx B > \sqrt{f \times |R|}$
  - during probing, $B >$ size of each partition $+2$

adapting join algorithms

- **multiple equality-join** conditions: $(R.A{=}S.A) \wedge (R.B{=}S.B)$
  - index nested loop join: use index on some/all join attribs
  - sort-merge join: sort on *combination* of attributes
- **inequality-join** conditions: $(R.A < S.A)$
  - index nested loop join: requires B$^+$-tree index
  - not applicable: sort-merge join, hash-based joins
- **set operations**
  - intersection: $R(A,B) \cap S(A,B) = \pi_{R.A,R.B}(R \bowtie_p S)$
  - cross product: $R \times S = R \bowtie_{true} S$
  - union/difference: duplicate elimination/slightly modified

06. QUERY EVALUATION

- **aggregation**: maintain running information while table scan
  - index scan if there is a covering index for the query
- **group-by**: sort/hash to group by attributes then aggregate
  - if group-by attributes are a B+tree prefix, just aggregate

materialised evaluation

- evaluates bottom-up; materialise intermediate results to disk
- $\times$ incurs I/O $\checkmark$ simple implementation $\checkmark$ less memory

pipelined evaluation (top-down, demand-driven)

- interleaved execution of operators - pass output directly to parent operator - can switch execution to where it is needed
- *blocking operator:* can't produce output until all input tuples received (grace hash & sort-merge join, external mergesort)

hybrid: pipelined evaluation + partial materialisation

materialise if repeatedly scanned (e.g. nested loop join)

query plans

query: ≥ 1 logical plans: implemented by ≥ 1 physical plans

query plan trees

- **linear** $\geq$ 1 operand per join operation is a base relation (else **bushy**)
- **left-deep** every right join operand is a base relation

query optimisation

- binary operators ($\bowtie$, $\times$) are **commutative & associative**
  - push selection and projection to operands first
- DP query plan enumeration: use all optimal sub-plans to build overall plan (single-relation $\Rightarrow$ two-relation $\Rightarrow \dots$)

System R Optimiser

- enumerate only left-deep query plans; avoid cross-product query plans; consider early selections and projections
- DP + **sort order** o_i of query plan output: $optPlan(S_i, o_i)$

cost estimation

- estimation assumptions
  1. **uniformity** - of distribn of attr values
  2. **independence** - for distribn of values in different attrs
  3. **inclusion** - for $R \bowtie_{R.A=S.B} S$, if $||\pi_A(R)|| \leq ||\pi_B(S)||$, then $\pi_A(R) \subseteq \pi_B(S)$ $\Rightarrow$ every R tuple joins with some S tuple
- **size estimation** for query $q = \sigma_p(e)$, $p = t_1 \wedge \cdots \wedge t_n$
  - **selectivity factor** $\rightarrow$ fraction of tuples satisfying term
    - aka *reduction factor*, $rf(t_i) = \frac{||\sigma_t(e)||}{||e||}$
  - $||q|| \approx ||e|| \times \prod_{i=1}^{n} rf(t_i)$
  - join selectivity estimation:
    $rf(R.A = S.B) \approx \frac{1}{\max\{||\pi_A(R)||, ||\pi_B(S)||\}}$
  - given $\pi_A(R) \leq \pi_B(S)$:
    $||Q|| = ||R|| \times \frac{||S||}{\pi_B(S)}$
- **histogram estimation**
  - **equiwidth** $\rightarrow \approx$equal number of *values* per bucket
  - **equidepth** $\rightarrow \approx$equal number of *tuples* per bucket
  - with **MCV**: keep a k/v pair of value/#tuples

07. TRANSACTION MANAGEMENT

- to ensure **ACID properties** of transactions $\rightarrow$
  1. **atomicity** - either all or none of the actions happen
  2. **consistency** - if each txn is consistent, and the DB starts consistent, then the DB ends up consistent
  3. **isolation** - execution of one txn is isolated from other txn
  4. **durability** - if txn commits, its effects persist
- **view equivalent** $\rightarrow$ same reads-from and final write
- **view serialisable** $\rightarrow$ view equiv to some serial schedule
- **conflict** $\rightarrow$ at least 1 write + different txns + same object
- **conflict equivalent** $\rightarrow$ all pairs of conflicting actions are ordered in the same way
- **conflict serialisable** $\rightarrow$ conflict equivalent to a serial sch
  1. acyclic conflict serialisability graph (node: committed txn, edge: precedes and conflicts with any action)
  2. conflict serialisable $\Rightarrow$ view serialisable

3. view serialisable + no blind writes $\Rightarrow$ conflict serialisable
    - **blind write** $\rightarrow$ did not read before write

anomalies arise due to conflicting actions

- **dirty read** - due to WR conflicts
- **unrepeatable read** - due to RW conflict (R_1, W_2, R_1)
- **lost update** - due to WW conflict
- **phantom read** - re-executing a query on a search condition gives different results (prevent by predicate/index locking)

recovery

- **cascading abort** $\rightarrow$ if T_1 reads from T_2, T_1 must abort when T_2 aborts (for correctness)
- **recoverable** $\rightarrow$ if T reads from T', then T commits after T'
    - guarantees that committed txns will not be aborted
- **cascadeless** $\rightarrow$ whenever T_i reads from T_j, $Commit_j$ must precede this action
    - all values read are produced by a committed transaction
- **before-images**: log before action & restore (must be strict)
- **strict** $\rightarrow$ for every $W_i(O)$ in S, O is not read/written by another txn until T_i either aborts or commits
- strict schedule $\Rightarrow$ cascadeless $\Rightarrow$ recoverable

08. CONCURRENCY CONTROL

Lock-based Concurrency Control

2PL (Two Phase Locking)

- may release locks any time
- once a txn releases a lock, it cannot request any more locks
    - **growing/shrinking phase**: before/after releasing 1^{st} lock
- prevents all anomalies, including phantom read

Strict 2PL

- **Strict 2PL** $\rightarrow$ txn must hold locks until it commits/aborts
- 2PL $\Rightarrow$ conflict serialisable
- strict 2PL $\Rightarrow$ strict & conflict serialisable

Lock Management

deadlocks

- **deadlock detection:** waits-for graph (WFG)
    - nodes represent active txns
    - edge $T_i \rightarrow T_j$ if T_i is waiting for T_j to release a lock
    - WFG has a cycle $\Rightarrow$ deadlock
        - abort one transaction and its edges from WFG
- **deadlock prevention:** older = higher priority
    - **wait-die** policy $\rightarrow$ lower-priority aborts instead of waiting
        - less aggressive; younger txns may keep aborting
    - **wound-wait** policy $\rightarrow$ (preemptive) higher- aborts lower-
        - preemptive - can abort another txn

Prevention Policy	T_i has higher priority	T_i has lower priority
Wait-die	T_i waits for T_j	T_i aborts
Wound-wait	T_j aborts	T_i waits for T_j

- restarted txn uses original timestamp to avoid starvation

lock conversion

- increases concurrency; only in the growing phase
- **lock upgrade**, $UG_i(A)$: allowed if no other txn is holding a shared lock on A and T_i has not yet released any lock
    - ensures serialisable schedule
- **lock downgrade**, $DG_i(A)$: allowed if T_i has not modified A and has not released any lock

ANSI SQL Isolation Levels

Isolation Level	Dirty Read	Unrepeatable Read	Phantom Read
READ UNCOMMITTED	possible	possible	possible
READ COMMITTED	not possible	possible	possible
REPEATABLE READ	not possible	not possible	possible
SERIALIZABLE	not possible	not possible	not possible

Degree	Isolation level	Write Locks	Read Locks	Predicate Locking
0	Read Uncommitted	long duration	none	none
1	Read Committed	long duration	short duration	none
2	Repeatable Read	long duration	long duration	none
3	Serializable	long duration	long duration	yes

- **short-duration** lock $\rightarrow$ can be released before commit/abort
- **long-duration** lock $\rightarrow$ held until txn commits/aborts

Locking Granularity

- (coarsest/most granular) database $>$ relation $>$ page $>$ tuple
- **multi-granular** lock $\rightarrow$ can request different granularity
    - if T holds lock mode M on data granule D, then T implicitly holds M on data granules finer than D

I-lock (intention)

- before acquiring any S-/X-lock on G, must acquire I-locks on granules coarser than G in a *top-down* manner
- can be shared with other I-locks
- $\times$ limited concurrency: S-lock is incompatible with I-lock

IS- and IX-lock (intention shared/exclusive)

- acquire locks *top-down*, release locks *bottom-up*
    - to obtain **S or IS lock**: must hold **IS or IX** lock on parent
    - to obtain **X or IX lock**: must hold **IX** lock on parent

Lock compatability matrix

Lock Requested	Lock Held: -	IS	IX	S	X
IS	$\surd$	$\surd$	$\surd$	$\surd$	$\times$
IX	$\surd$	$\surd$	$\surd$	$\times$	$\times$
S	$\surd$	$\surd$	$\times$	$\surd$	$\times$
X	$\surd$	$\times$	$\times$	$\times$	$\times$

Lock compatability matrix

Lock Requested	Lock Held: -	I	S	X
I	$\surd$	$\surd$	$\times$	$\times$
S	$\surd$	$\times$	$\surd$	$\times$
X	$\surd$	$\times$	$\times$	$\times$

09. MULTIVERSION CONCURRENCY CONTROL (MVCC)

- maintain multiple versions of each object
    - $W_i(O)$ creates new version, $R_i(O)$ reads some version
- $\checkmark$ read-only txns not blocked by update txns $\checkmark$ update txns not blocked by read-only txns $\checkmark$ read-only txns never aborted
- **multi-version** schedule $\rightarrow$ read can return *any* version
- **mono-version** $\rightarrow$ always reads most recent version
- **multi-version view equivalent**, $S \equiv_{mv} S'$ $\rightarrow$ same set of read-from relationships; $R_i(x_j) \in S \iff R_i(x_j) \in S'$
    - final write doesn't matter (concept in monoversion only)
- **multi-version view serialisable** (MVSS) $\rightarrow$ exists a *serial mono-version schedule* that is multi-version view equivalent
    - mono-version view serialisable $\Rightarrow$ MVSS
    - VSS $\subseteq$ MVSS; VSS $\Rightarrow$ MVSS; MVSS $\not\Rightarrow$ VSS

Snapshot Isolation

- each txn T sees a snapshot of the DB comprising updates by transactions that committed before T starts
- **concurrent** txns $\rightarrow$ overlap, defined by start(T)/commit(T)
- **protocol**: O_i is more recent if commit(T_i) is later
    - $W_i(O)$ creates version i of O
    - $R_i(O)$ reads either its latest $W_i(O)$ or the latest version of O created by a txn that committed before start(T_i)
- **concurrent update property**: if multiple concurrent txn update the same object, only 1 commits (ensure serialisable)
    - **FCW** (first committer wins): commit $\iff$ no committed concurrent txn on updated object
    - **FUW** (first updater wins): acquire X-lock to update
        - T proceeds iff all concurrent T' (previously holding the X-lock) aborts and O has not been updated by any concurrent txn. / else abort T
- **garbage collection**: if not read by any (active/future) txn
    - delete O_i if there exists a newer O_j (commit(T_i) $<$ commit(T_j)) such that for every active txn T_k that started after commit(T_i), we have commit(T_j) $<$ start(T_k)
- **performance**: $\checkmark$ similar to READ_COMMITTED but without *lost update* or *unrepeatable read* anomalies
    - $\times$ $\not\Rightarrow$ serialisability (some non-serialisable executions)
        - **write skew** anomaly: $R_1(x_0), R_2(y_0), W_1(y_1), W_2(x_2)$
        - **read-only txn** anomaly: $T_3 \overset{rw}{\dashrightarrow} T_2 \overset{rw}{\dashrightarrow} T_1 \xrightarrow{wr} T_3$
    - $\times$ does not guarantee serialisability

Serialisable Snapshot Isolation (SSI)

- ensures MVSS
- detect $T_i \overset{rw}{\dashrightarrow} T_j \overset{rw}{\dashrightarrow} T_k$ and abort one of T_i, T_j, T_k
    - keeps track of **rw dependencies**; possible false positives

transactional dependencies: `ww`, `wr`, `rw`

- **immediate successor** $\rightarrow$ no W(x) commits betw commits
- **dependency serialisation graph**, DSG
    - nodes: (committed) transactions
    - edges: transactional dependencies, e.g. $T_i \xrightarrow{wr} T_j$
        - $\dashrightarrow$/$\rightarrow$ for concurrent/non-concurrent
- if S is a SI schedule that is not MVSS, then
    - there is *at least one* cycle in $DSG(S)$
    - for each cycle in $DSG(S)$, $\exists T_i, T_j, T_k$ such that
        - $T_i \overset{rw}{\dashrightarrow} T_j \overset{rw}{\dashrightarrow} T_k$ exists
        - T_i and T_k may be same txn (eg. write-skew anomaly)

10. CRASH RECOVERY

- **recovery manager** guarantees atomicity and durability
    - undo: preserve atomicity (remove effects of aborts)
    - redo: durability (re-install effects of commits)
- **steal policy** $\rightarrow$ can write dirty page to disk before commit
- **force policy** $\rightarrow$ must write all dirty pages to disk at commit
    - no-steal: may run out of buffer pages
    - force: incur random I/O

	Force	No-force
Steal	undo & no redo	undo & redo
No-steal	no undo & no redo	no undo & redo

ARIES Recovery Algorithm

- steal; no-force; assumes strict 2PL for concurrency control
- **transaction table (TT)** - 1 entry for each active txn
    - (txn ID, last LSN, C/U status)
- **dirty page table (DPT)** - (pageID, recLSN) = earliest LR that dirtied page
- **log records**: (type, txn ID, prevLSN, undoNextLSN)
- **write-ahead logging (WAL)**: do not flush uncommitted update to DB until log record containing before-image is flushed to log **(abort)**
- **force-at-commit**: do not commit txn until the after-images of all its updated records are in stable storage **(commit)**
- (**!** redoable) = update LR and compensation LR
- All types of log records must have type, txnID, prevLSN

implementing restart

1. **analysis phase** - TT (active txns) & DPT (superset of dirty)
    1.1. initialise TT & DPT (retrieve ECPLR from BCPLR)
    1.2. for each r in log file in forward direction/chronological
        - if *end LR*, remove T from TT; continue
        - if *redoable LR* for P and P not in DPT:
            - create P's entry in DPT with recLSN=r.LSN
        - add or update entry for T in TT: lastLSN = r.LSN
            - if *commit LR*: update status=C
2. **redo phase** - restore DB to state at time of crash
    2.1. start from **redoLSN** = smallest recLSN in DPT
    2.2. scan in forward direction for all *redoable LR*
        i. if *not redoable* or NOT optimisation cond: continue
        ii. if P.pageLSN $<$ r.LSN: (r has not been installed)
            - reapply logged action in r to P
            - update P.pageLSN = r.LSN
        iii. (optimisation) else: recLSN $\leq$ r.LSN $\leq$ P.pageLSN
            - update P in DPT: recLSN=P.pageLSN+1
    2.3. create *end LR* for all status=C in TT; remove entry
        - optimisation cond: (P $\notin$ DPT) *or* (DPT P.recLSN $>$ r.LSN)
            - update of r has already been applied to P
3. **undo phase** - abort **loser** txns (active at crash) in *reverse*
    3.1. initialise L = set of lastLSN (status=U) from TT
        - `update-L-and-TT(LSN)` := if LSN is not null, add to L; else create *end LR* for T and remove T from TT
    3.2. while $L \neq \emptyset$:
        i. r = largest lastLSN in L; delete r from L
        ii. if r is *update LR* for T on P:
            - create *CLR* r_2 with r_2.undoNextLSN=r.prevLSN
            - update TT: T.lastLSN=r_2.LSN
            - create DPT entry for P with recLSN=r_2.LSN if P not in DPT
            - undo logged action and update P.pageLSN=r_2.LSN
            - `update-L-and-TT`(r.prevLSN)
        iii. else if r is *CLR*: `update-L-and-TT`(r.undoNextLSN)
        iv. else r is *abort LR*: `update-L-and-TT`(r.prevLSN)