MA1521 Cheatsheet

AY20/21 Sem 1 | Chapter 1-3

Differentiation Techniques

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\csc x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$a^{f(x)}$	$\ln a \cdot f'(x) a^{f(x)}$
$\log_a f(x)$	$\log_a e \cdot \frac{f'(x)}{f(x)}$
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}, \quad \lvert f(x)\rvert<1$
$\cos^{-1} f(x)$	$-\frac{f'(x)}{\sqrt{1-[f(x)]^2}}, \quad \lvert f(x)\rvert<1$
$\tan^{-1} f(x)$	$\frac{f'(x)}{1+[f(x)]^2}$
$\cot^{-1} f(x)$	$-\frac{f'(x)}{1+[f(x)]^2}$
$\sec^{-1} f(x)$	$\frac{f'(x)}{\lvert f(x)\rvert\sqrt{[f(x)]^2-1}}$
$\csc^{-1} f(x)$	$-\frac{f'(x)}{\lvert f(x)\rvert\sqrt{[f(x)]^2-1}}$

Integration Techniques

$f(x)$	$\int f(x)$
$\tan x$	$\ln(\sec x), \ \lvert x\rvert < \frac{\pi}{2}$
$\cot x$	$\ln(\sin x), \ 0 < x < \pi$
$\csc x$	$-\ln(\csc x + \cot x), \ 0 < x < \pi$
$\sec x$	$\ln(\sec x + \tan x), \ \lvert x\rvert < \frac{\pi}{2}$
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}(\frac{x}{a})$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}(\frac{x}{a}), \ \lvert x\rvert < a$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln(\frac{x-a}{x+a}), \ x > a$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln(\frac{x+a}{x-a}), \ x < a$
a^x	$\frac{a^x}{\ln a}$

Estimation

first order estimate: $y' \approx y + \Delta x \times \frac{dy}{dx}\Big|_{x=2}$

second order estimate:
$y' \approx$ 1st estimate $+ (\frac{(\triangle x)^2}{2} \times \frac{d^2y}{dx^2}\Big|_{x=2})$

Stats

pop. variance: $\sigma^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n}$

pop. covariance: $\text{cov}(x, y) = \frac{\sum xy^2 - \frac{\sum x \sum y}{n}}{n}$

pop. correlation: $\frac{\text{cov}(x,y)}{\sigma_x \times \sigma_y}$

Differentiation

extreme values:

- $f'(x) = 0$
- $f'(x)$ does not exist
- end points of the domain of f

parametric differentiaton: $\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx}) = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$

L'Hospital's Rule

- for indeterminate forms ($\frac{0}{0}$ or $\frac{\infty}{\infty}$), cannot directly substitute $x = a$.
- for other forms: convert to ($\frac{0}{0}$ or $\frac{\infty}{\infty}$) then apply L'Hospital's rule
- for exponents: use $\ln$, then sub into $e^{f(x)}$

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)}$$

Rules of Limits

1. $\lim_{x\to a}(f \pm g)(x) = L \pm L'$
2. $\lim_{x\to a}(fg)(x) = LL'$
3. $\lim_{x\to a} \frac{f}{g}(x) = \frac{L}{L'}$, provided $L' \neq 0$
4. $\lim_{x\to a} kf(x) = kL$ for any real number k.

Integration

$\frac{d}{dx}\int_a^x f(t)dt = f(x)$

- **indefinite integral** — the integral of the function without any limits
- **antiderivative** — any function whose derivative will be the same as the original function

substitution: $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

by parts: $\int uv' \, dx = uv - \int u'v \, dx$

volume of revolution

about x-axis

(with hollow area) $V = \pi\int_a^b [f(x)]^2 - [g(x)]^2 dx$

(about line $y = k$) $V = \pi\int_a^b [f(x) - k]^2 dx$

Logistic Models

$$N = \frac{N_{t=\infty}}{1+(\frac{N_{t=\infty}}{N_{t=0}}-1)e^{-Bt}}$$

- N - number
- B - birth rate
- t - time