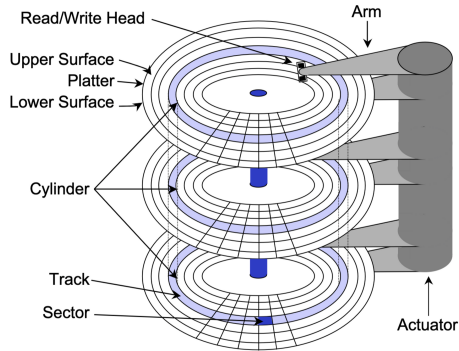


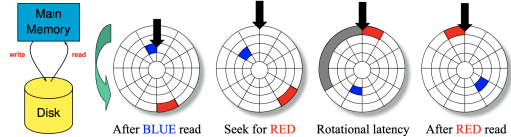
01. DBMS STORAGE

- store data on non-volatile disk
- process data in main memory (RAM) (*volatile storage*)

Magnetic HDD



- disk access time** =
 - seek time** → move arms to position disk head on track
 - rotational delay** → wait for block to rotate under head
 - average rotational delay = time for $\frac{1}{2}$ revolutions
 - transfer time** → move data to/from disk surface
 - time for 1 revolution $\times \frac{\text{\# of requested sectors on the same track}}{\text{\# of sectors in track}}$
- response time** for disk access = queuing delay + access time



- command processing time: interpreting access command by disk controller (part of access time, considered negligible)
- small requests are dominated by seek time; large requests dominated by transfer time
- access order**:
 - contiguous blocks within the same track (same surface)
 - cylinder tracks within the same cylinder
 - next cylinder

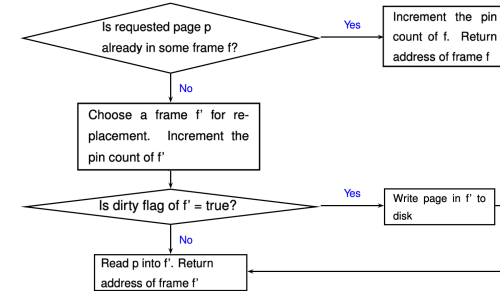
SSD (Solid-State Drive)

- no mechanical moving parts
- advantages: ✓ significantly faster than HDD
✓ higher data transfer rate ✓ lower power consumption
- disadvantages: ✗ update to a page requires erasure of multiple pages before overwriting page
✗ limited number of times a page can be erased

Buffer Manager

- data is stored & retrieved in **disk blocks** (pages)
 - each block = sequence of ≥ 1 contiguous sectors
- buffer pool**: main memory allocated for DBMS
 - partitioned into **frames** (block-sized pages)

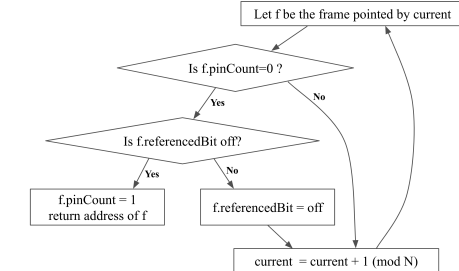
- pin count**: number of clients using page (initialised 0)
 - $\neq 0 \Rightarrow$ page is utilised by some transaction; don't replace
- dirty flag**: initialised false
 - dirty** → page is modified & not updated on the disk
 - dirty page must be written back to the disk if the transaction has committed



! unpinning: update dirty flag to true if page is dirty

replacement policies

- decide which unpinned (pinCount==0) page to replace
- LRU** uses a queue of pointers to frames with pinCount==0
- clock**: cheaper than LRU, used in postgres
 - referenced bit - turns on when pinCount==0
 - replace page with referenced bit off && pinCount==0

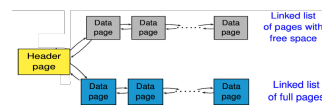


File abstraction

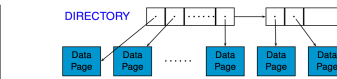
- each relation is a file of records
- each record has a unique record identifier, **RID**
- heap file** → unordered file
 - vs sorted/hashed file: records are ordered/hashed

heap file implementations

- linked list** implementation
 - header page: metadata about the file

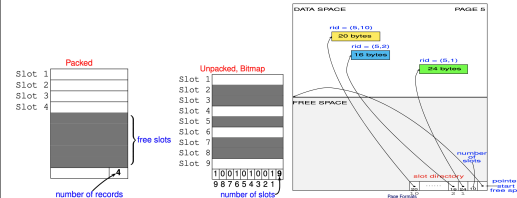


- page directory** implementation: more efficient
 - maintain directory structure with one entry per page
 - stores address of and amount of free space on page
 - insertion: scan directory to find page with enough space to store the new record
 - insertion worst case: scan number of pages + data page itself (vs LL worst case: entire list)



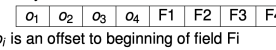
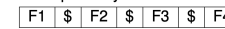
Page Formats

- RID** = (page ID, slot number)
- fixed-length** records
 - packed organisation: inefficient deletion (transferring last record to deleted record changes RID of record)
- variable-length** records: **slotted page organisation**



Record formats

- fixed-length** records: store consecutively
- variable-length** records:
 - Delimit fields with special symbols
 - Use an array of field offsets



Data entry formats

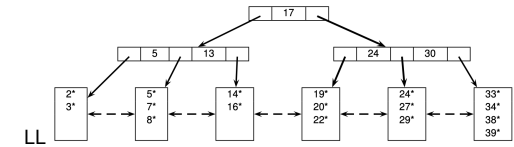
- k^* is an actual **data record** (with search key k)
- k^* is of the form (k, RID) - fixed length (k, \bullet)
- k^* is of the form $(k, RID\text{-list})$ - e.g. $(k, \{RID11, RID12\})$

02. TREE-BASED INDEXING

- search key** → sequence of k data attributes, $k \geq 1$
 - composite search key** → if $k > 1$
- unique index** → search key is a candidate key
- clustered index** → order of data entries \approx order of records
 - Format-1** is always clustered
 - at most one clustered index for each relation
- dense index** → there is an index record for every search key value in the data. *unclustered index* must be dense

B⁺-tree Index

- leaf nodes: sorted data entries (k^* is of form (k, RID))
- internal nodes: stores index entries $(p_0, k_1, p_1, \dots, p_n)$ for $k_1 < k_2 < \dots < k_n$ where p_i is the page disk address
 - each (k_i, p_i) is an **index entry**
 - for k^* in index subtree T_i rooted at p_i , $k \in [k_i, k_{i+1}]$
- order** of index tree, $d \in \mathbb{Z}^+$
 - each non-root node contains m entries, $m \in [d, 2d]$
 - root node contains $[1, 2d]$ entries
- equality search**: at each internal node N , find the largest k_i s.t. $k \geq k_i$. search subtree at p_i if k_i exists, else p_0
- range search**: find first matching record; traverse doubly



LL

Insertion: splitting

- splitting leaf node: distribute $d + 1$ entries to a new leaf node
- if parent overflows: push the middle $(d+1)$ key up to parent
- root node overflows: create new root (parent of current root)

insertion: redistribution (of leaf nodes only)

- try right sibling first, then left sibling, else use splitting
- sibling** → two nodes at the *same level* & *same parent* node

deletion: redistribution - try right sibling, then left, else merge

deletion: merging (siblings have d entries) - try right first

- if leaf underflows: delete parent key, combine with sibling
- if internal node underflows: pull down its index entry in parent, combine with sibling, push a key back up
 - becomes the new root if parent is root & becomes empty

Bulk Loading a B⁺-tree

- sort data entries by search key and store sequentially
- construct leaf pages with $2d$ entries
- construct internal pages by attempting to insert leaf pages into rightmost parent page

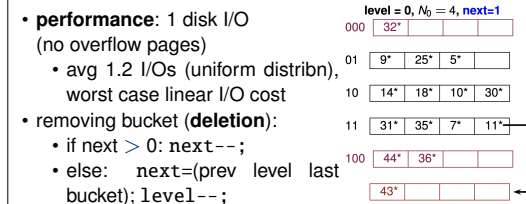
03. HASH-BASED INDEXING

Static Hashing

- hash record to $B_i \in B_0, \dots, B_{N-1}$ with $i = h(k) \bmod N$
- when full, reconstruct hash table with more buckets
 - each bucket:
 - 1 primary data page
 - ≥ 0 overflow data pages

Linear Hashing (Dynamic)

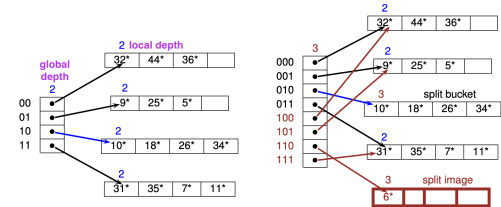
- grows **linearly**: split when some **bucket overflows**
- how to split bucket B_i :
 - add a new bucket $B_j = B_{i+N_i}$ (split image of B_i)
 - redistribute entries in B_i between B_i and B_j
 - next++; if next== N_{level} : level++; next=0
- file size at the beginning of round i , $N_i = 2^i N_0$
- at round i , hash $x = B_x$ has been split $\rightarrow h_i(k) : h_{i+1}(k)$



Extendible Hashing (Dynamic)

- add a new bucket whenever existing bucket overflows
 - no overflow pages unless # collisions \geq page capacity
- directory of pointers to buckets - 2^d entries $(b_d b_{d-1} \dots b_1)$
 - d = **global depth** of hashed file
- corresponding** directory entries differ only in the d^{th} bit

- entries in a bucket of **local depth** $\ell \in [0, d]$: same last ℓ bits
 - a split bucket & its image have the *same local depth*
- number of directory entries pointing to a bucket = $2^{d-\ell}$



- splitting bucket: $\ell++$ (repeat until no more overflow)
 - if $\ell = d$: directory doubles; $d++$
 - else $\ell < d$: redistribute and increment ℓ
- deletion: if bucket B_i becomes empty or B_i and B_j can merge,
 - deallocate B_i and decrement $\ell--$ for split image B_j
 - if each pair of corresponding entries point to the same bucket, the directory can be halved
- performance**: at most 2 disk I/Os (for equality query)
- collisions: when 2 data entries have the same hashed value
 - use **overflow pages** if # collisions exceeds page capacity

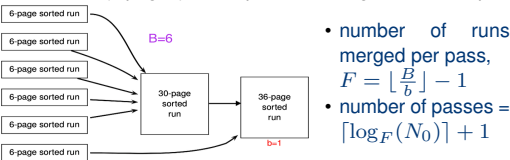
04.1 SORTING

External Merge Sort

- sorted run** \rightarrow sorted data records written to a file on disk
- divide and conquer
 - create temporary file R_i for each B pages of R sorted
 - merge: use $B-1$ pages for input, 1 page for output
- total I/O = $2N(\lceil \log_{B-1}(N_0) \rceil + 1)$
 - $2N$ to create $\lceil N/B \rceil$ sorted runs of B pages each
 - merging sorted runs: $2N \times \lceil \log_{B-1} N_0 \rceil$

optimisation with blocked I/O

- sequential I/O - read/write in *buffer blocks* of b pages
- one block (b pages) for output, remaining blocks for input



Sorting with B⁺-trees

- when *sort key is a prefix of the index key* of the B⁺-tree
- sequentially scan leaf pages of B⁺-tree
 - for Format-2/3, use RID to retrieve data records

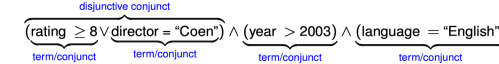
04.2 SELECTION: $\sigma_p(R)$

- $\sigma_p(R)$: selects rows from relation R satisfying predicate p
- access path**: a way of accessing data records/entries
 - table scan** \rightarrow scan all data pages
 - index scan** \rightarrow scan index pages
 - index intersection** \rightarrow combine results from index scans
- selectivity** of an access path \rightarrow number of index & data pages retrieved to access data records/entries

- more selective = fewer pages retrieved
- index I is a **covering index** for query $Q \rightarrow$ if all attributes referenced in Q are part of the key of or include columns of I
 - Q can be evaluated using I without any RID lookup (**index-only plan**)

Matching Predicates

- term** \rightarrow of form $R.A$ op c or $R.A_i$ op $R.A_j$
- conjunct** \rightarrow one or more terms connected by \vee
 - disjunctive conjunct** \rightarrow contains \vee
- conjunctive normal form, **CNF predicate** \rightarrow comprises one or more conjuncts connected by \wedge



B⁺-tree matching predicates

- for index $I = (K_1, K_2, \dots, K_n)$ and non-disjunctive CNF predicate p , I matches p if p is of the form

$$(K_1 = c_1) \wedge \dots \wedge (K_{i-1} = c_{i-1}) \wedge (K_i \text{ op}_i c_i), \quad i \in [1, n]$$
 zero or more equality predicates
 - at *most one* non-equality comparison operator which must be on the last attribute of the prefix (K_i)
- matching index: matching records are in contiguous pages
 - non-matching index: not contiguous \Rightarrow less efficient

Hash index matching predicates

- for hash index $I = (K_1, K_2, \dots, K_n)$ and non-disjunctive CNF predicate p , I matches p if p is of form

$$(K_1 = c_1) \wedge (K_2 = c_2) \wedge \dots \wedge (K_n = c_n)$$

Primary/Covered Conjuncts

- primary conjuncts** \rightarrow subset of conjuncts that I matches
 - e.g. $p = (\text{age} \geq 18) \wedge (\text{age} \leq 20) \wedge (\text{weight}=65)$ for $I = (\text{age}, \text{weight}, \text{height})$
- covered conjuncts** \rightarrow subset of conjuncts covered by I
 - each attribute in covered conjuncts appears in key of I
- primary conjuncts \subseteq covered conjuncts

Cost of Evaluation

let p' = primary conjuncts of p , p_c = covered conjuncts of p

B⁺-tree index evaluation of p

- navigate internal nodes to find first leaf page

$$\text{cost}_{\text{internal}} = \begin{cases} \lceil \log_F(\lceil \frac{|R|}{b_d} \rceil) \rceil & \text{if } I \text{ is a format-1 index} \\ \lceil \log_F(\lceil \frac{|R|}{b_i} \rceil) \rceil & \text{otherwise} \end{cases}$$
- scan leaf pages to access all qualifying data entries

$$\text{cost}_{\text{leaf}} = \begin{cases} \lceil \frac{||\sigma_{p'}(R)||}{b_d} \rceil & \text{if } I \text{ is a format-1 index} \\ \lceil \frac{||\sigma_{p'}(R)||}{b_i} \rceil & \text{otherwise} \end{cases}$$
- retrieve qualified data records via RID lookups

$$\text{cost}_{\text{RID}} = \begin{cases} 0 & \text{if } I \text{ is a covering format-1 index,} \\ ||\sigma_{p_c}(R)|| & \text{otherwise} \end{cases}$$
 - reduce cost with **clustered** data records (sort RIDs):

$$\lceil \frac{||\sigma_{p_c}(R)||}{b_d} \rceil \leq \text{cost}_{\text{RID}} \leq \min\{||\sigma_{p_c}(R)||, |R|\}$$

hash index evaluation of p

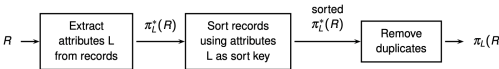
- format-1**: cost to retrieve data records $\geq \lceil \frac{||\sigma_{p'}(R)||}{b_d} \rceil$
- format-2**: cost to retrieve data entries $\geq \lceil \frac{||\sigma_{p'}(R)||}{b_i} \rceil$
- cost to retrieve data records =

$$\begin{cases} 0 & \text{if } I \text{ is a covering index,} \\ ||\sigma_{p_c}(R)|| & \text{otherwise} \end{cases}$$

05.1 PROJECTION $\pi_{A_1, \dots, A_m}(R)$

- $\pi_L(R)$ eliminates duplicates, $\pi_L^*(R)$ preserves duplicates

Sort-based approach



cost analysis

- extract attributes: $|R|$ scan + $|\pi_L^*(R)|$ output temp result
- sort records: $2|\pi_L^*(R)|(\log_m(N_0) + 1)$
- remove duplicates: $|\pi_L^*(R)|$ to scan records

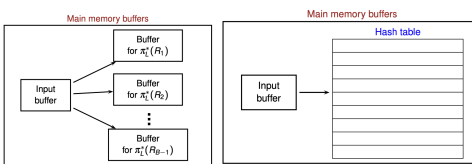
optimised sort-based approach



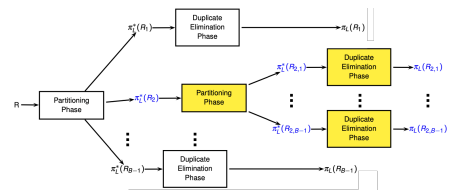
- if $B > \sqrt{|\pi_L^*(R)|}$, same I/O cost as hash-based approach
 - $N_0 = \lceil \frac{|R|}{B} \rceil \approx \sqrt{|\pi_L^*(R)|}$ initial sorted runs
 - $\log_{B-1}(N_0) \approx 1$ merge passes

Hash-based approach

- partitioning phase**: hash each tuple $t \in R$
 - $R = R_1 \cup R_2 \cup \dots \cup R_{B-1}$
 - for each R_i & R_j , $i \neq j$, $\pi_L^*(R_i) \cap \pi_L^*(R_j) = \emptyset$
 - for each t : project attributes to form t' , hash $h(t')$ to one output buffer, flush output buffer to disk when full
 - one buffer for input, $(B-1)$ buffers for output
- duplicate elimination** from each $\pi_L^*(R_i)$
 - for each R_i : initialise in-mem hash table, hash each $t \in R_i$ to bucket B_j with $h' \neq h$, insert if $t \notin B_j$
 - write tuples in hash table to results



- I/O cost** (no partition overflow): $|R| + 2|\pi_L^*(R)|$
 - partitioning cost: $|R| + |\pi_L^*(R)|$
 - duplicate elimination cost: $|\pi_L^*(R)|$
- partition overflow: recursively apply partitioning
 - to avoid, $B >$ size of hash table for $R_i = \frac{|\pi_L^*(R)|}{B-1} \times f$
 - approximately $B > \sqrt{f \times |\pi_L^*(R)|}$



Projection using Indexes

- if index search key contains all wanted attributes *as a prefix*
 - index scan** data entries in order & eliminate duplicates

05.2 JOIN $R \bowtie_{\theta} S$

- R = outer relation (smaller relation); S = inner relation
- ! for **format-2** index, add cost of retrieving record

nested loop joins

- tuple-based** nested loop join: $|R| + ||R|| \times |S|$
- page-based** nested loop join: $|R| + |R| \times |S|$
- block nested loop join**: $|R| + (\lceil \frac{|R|}{B-2} \rceil \times |S|)$, $|R| \leq |S|$
 - 1 page output, 1 page input, $(B-2)$ pages to read R
 - for each $(B-2)$ pages of R : for each P_S of S : check r, s
- index nested loop join**:

$$|R| + ||R|| \times \left(\log_F(\lceil \frac{|S|}{b_d} \rceil) + \lceil \frac{|S|}{b_d ||\pi_{B_j}(S)||} \rceil \right)$$
 - joining $R(A, B) \bowtie_A S(A, C)$ with B+tree index on $S.A$
 - for each tuple $r \in R$, use r to probe S 's index for match

sort-merge join

- sort R & S : $2|R|(\log_m(N_R) + 1) + 2|S|(\log_m(N_S) + 1)$
- merge cost: $|R| + |S|$ (worst case $|R| + ||R|| \times |S|$)
- optimised sort-merge join**
 - merge sorted runs until $B > N(R, i) + N(S, j)$; then do merge and join at the same time
- I/O cost: $3 \times (|R| + |S|)$
 - if $B > \sqrt{2|S|}$, one pass to merge initial sorted runs
 - $2(|R| + |S|)$ for initial sorted runs, $|R| + |S|$ for merging

hash join

- partition R and S into k partitions on join column
 - $\pi_A(R_i) \cap \pi_B(S_j) = \emptyset \quad \forall R_i, S_j, i \neq j$
 - $R = R_1 \cup R_2 \cup \dots \cup R_k, \quad t \in R_i \iff h(t.A) = i$
 - $S = S_1 \cup S_2 \cup \dots \cup S_k, \quad t \in S_i \iff h(t.B) = i$
- join corresponding partitions:

$$R \bowtie_{R.A=S.B} S = (R_1 \bowtie S_1) \cup \dots \cup (R_k \bowtie S_k)$$

Grace hash join

for *build relation* R and *probe relation* S ,

- partition** R and S into k partitions each, $k = B-1$
- probing phase**: hash $r \in R_i$ with $h'(r.A)$ to table T
 - $\forall s \in S_i, r \in \text{bucket } h'(s.B)$: output (r, s) if match
- I/O cost: $3(|R| + |S|)$ (no partition overflow)
- $B > \frac{f \times |R|}{B-1} + 2$ (input & output buffer) $\approx B > \sqrt{f \times |R|}$
 - during probing, $B >$ size of each partition +2
- partition overflow** if R_i cannot fit in memory
 - recursively apply partitioning to overflow partition

General join conditions

- multiple equality-join conditions:
 $(R.A = S.A) \wedge (R.B = S.B)$
 - index nested loop join: use index on some/all join attribs
 - sort-merge join: sort on *combination* of attributes
 - other algos: no change
- inequality-join conditions: $(R.A < S.A)$
 - index nested loop join: requires B⁺-tree index
 - not applicable: sort-merge join (too much rewinding), hash-based joins
 - other algos: no change

NOTATION

Notation	Meaning
r	relational algebra expression
$ r $	number of tuples in output of r
$ r $	number of pages in output of r
b_d	number of data records that can fit on a page
b_l	number of data entries that can fit on a page
F	average fanout of B ⁺ -tree index (i.e., number of pointers to child nodes)
h	height of B ⁺ -tree index (i.e., number of levels of internal nodes) $h = \lceil \log_r \{ \lceil \frac{ R }{b_l} \rceil \} \rceil$ if format-2 index on table R
B	number of available buffer pages