

- if $\ell = d$: directory doubles; $d++$
- else $\ell < d$: redistribute and increment ℓ
- deletion: if bucket B_i becomes empty or B_i and B_j can merge,
 - deallocate B_i and decrement $\ell--$ for split image B_j
 - if each pair of corresponding entries point to the same bucket, the directory can be halved
- **performance**: at most 2 disk I/Os (for equality query)
- collisions: when 2 data entries have the same hashed value
 - use **overflow pages** if # collisions exceeds page capacity

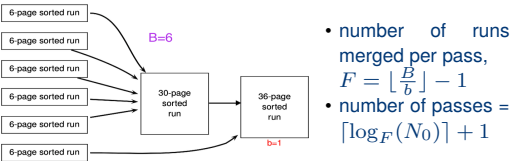
04.1 SORTING

External Merge Sort

- **sorted run** \rightarrow sorted data records written to a file on disk
- divide and conquer
 1. create temporary file R_i for each B pages of R sorted
 2. merge: use $B - 1$ pages for input, 1 page for output
- total I/O = $2N(\lceil \log_{B-1}(N_0) \rceil + 1)$
 - $2N$ to create $\lceil N/B \rceil$ sorted runs of B pages each
 - merging sorted runs: $2N \times \lceil \log_{B-1} N_0 \rceil$

optimisation with blocked I/O

- sequential I/O - read/write in *buffer blocks* of b pages
- one block (b pages) for output, remaining blocks for input



Sorting with B⁺-trees

- when *sort key* is a *prefix of the index key* of the B⁺-tree
- sequentially scan leaf pages of B⁺-tree
 - for Format-2/3, use RID to retrieve data records

04.2 SELECTION: $\sigma_p(R)$

- $\sigma_p(R)$: selects rows from relation R satisfying predicate p
- **access path**: a way of accessing data records/entries
 - **table scan** \rightarrow scan all data pages
 - **index scan** \rightarrow scan index pages
 - **index intersection** \rightarrow combine results from index scans
- **selectivity** of an access path \rightarrow number of index & data pages retrieved to access data records/entries
 - more selective = fewer pages retrieved
- index I is a **covering index** for query $Q \rightarrow$ if all attributes referenced in Q are part of the key of or include columns of I
 - Q can be evaluated using I without any RID lookup (**index-only** plan)

Matching Predicates

- **term** \rightarrow of form $R.A \text{ op } c$ or $R.A_i \text{ op } R.A_j$
- **conjunct** \rightarrow one or more terms connected by \vee
 - **disjunctive** conjunct \rightarrow contains \vee
- conjunctive normal form, **CNF predicate** \rightarrow comprises one or more conjuncts connected by \wedge

$$(\text{rating} \geq 8 \vee \text{director} = \text{"Coen"}) \wedge (\text{year} > 2003) \wedge (\text{language} = \text{"English"})$$

term/conjunct
term/conjunct
term/conjunct
term/conjunct

B⁺-tree matching predicates

- for index $I = (K_1, K_2, \dots, K_n)$ and non-disjunctive CNF predicate p , I matches p if p is of the form

$$\underbrace{(K_1 = c_1) \wedge \dots \wedge (K_{i-1} = c_{i-1})}_{\text{zero or more equality predicates}} \wedge (K_i \text{ op}_i c_i), i \in [1, n]$$
 - *at most one* non-equality comparison operator which must be on the last attribute of the prefix (K_i)
- matching index: matching records are in contiguous pages
 - non-matching index: not contiguous \Rightarrow less efficient

Hash index matching predicates

- for hash index $I = (K_1, K_2, \dots, K_n)$ and non-disjunctive CNF predicate p , I matches p if p is of form

$$(K_1 = c_1) \wedge (K_2 = c_2) \wedge \dots \wedge (K_n = c_n)$$

Primary/Covered Conjuncts

- **primary conjuncts** \rightarrow subset of conjuncts that I matches
 - e.g. $p = (\text{age} \geq 18) \wedge (\text{age} \leq 20) \wedge (\text{weight} = 65)$ for $I = (\text{age}, \text{weight}, \text{height})$
- **covered conjuncts** \rightarrow subset of conjuncts covered by I
 - each attribute in covered conjuncts appears in key of I
- primary conjuncts \subseteq covered conjuncts

Cost of Evaluation

let p' = primary conjuncts of p , p_c = covered conjuncts of p

B⁺-tree index evaluation of p

1. navigate internal nodes to find first leaf page

$$\text{cost}_{\text{internal}} = \begin{cases} \lceil \log_F(\lceil \frac{|R|}{b_d} \rceil) \rceil & \text{if } I \text{ is a format-1 index} \\ \lceil \log_F(\lceil \frac{|R|}{b_i} \rceil) \rceil & \text{otherwise} \end{cases}$$
2. scan leaf pages to access all qualifying data entries

$$\text{cost}_{\text{leaf}} = \begin{cases} \lceil \frac{||\sigma_{p'}(R)||}{b_d} \rceil & \text{if } I \text{ is a format-1 index} \\ \lceil \frac{||\sigma_{p'}(R)||}{b_i} \rceil & \text{otherwise} \end{cases}$$
3. retrieve qualified data records via RID lookups

$$\text{cost}_{\text{RID}} = \begin{cases} 0 & \text{if } I \text{ is a covering format-1 index,} \\ ||\sigma_{p_c}(R)|| & \text{otherwise} \end{cases}$$
 - reduce cost with **clustered** data records (sort RIDs):

$$\lceil \frac{||\sigma_{p_c}(R)||}{b_d} \rceil \leq \text{cost}_{\text{RID}} \leq \min\{||\sigma_{p_c}(R)||, |R|\}$$

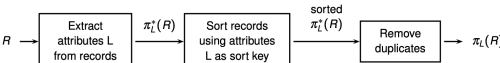
hash index evaluation of p

- **format-1**: cost to retrieve data records $\geq \lceil \frac{||\sigma_{p'}(R)||}{b_d} \rceil$
 - **format-2**: cost to retrieve data entries $\geq \lceil \frac{||\sigma_{p'}(R)||}{b_i} \rceil$
- $$\text{cost to retrieve data records} = \begin{cases} 0 & \text{if } I \text{ is a covering index,} \\ ||\sigma_{p'}(R)|| & \text{otherwise} \end{cases}$$

05.1 PROJECTION $\pi_{A_1, \dots, A_m}(R)$

- $\pi_L(R)$ eliminates duplicates, $\pi_L^*(R)$ preserves duplicates

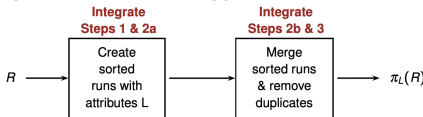
Sort-based approach



cost analysis

1. extract attributes: $|R|$ scan + $|\pi_L^*(R)|$ output temp result
2. sort records: $2|\pi_L^*(R)|(\log_m(N_0) + 1)$
3. remove duplicates: $|\pi_L^*(R)|$ to scan records

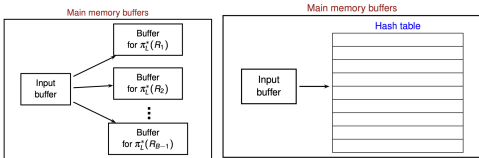
optimised sort-based approach



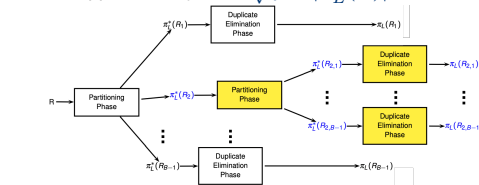
- if $B > \sqrt{\lceil \pi_L^*(R) \rceil}$, same I/O cost as hash-based approach
 - $N_0 = \lfloor \frac{|R|}{B} \rfloor \approx \sqrt{\lceil \pi_L^*(R) \rceil}$ initial sorted runs
 - $\log_{B-1}(N_0) \approx 1$ merge passes

Hash-based approach

1. **partitioning phase**: hash each tuple $t \in R$
 - $R = R_1 \cup R_2 \cup \dots \cup R_{B-1}$
 - for each R_i & $R_j, i \neq j, \pi_L^*(R_i) \cap \pi_L^*(R_j) = \emptyset$
 - for each t : project attributes to form t' , hash $h(t')$ to one output buffer, flush output buffer to disk when full
 - one buffer for input, $(B - 1)$ buffers for output
2. **duplicate elimination** from each $\pi_L^*(R_i)$
 - for each R_i : initialise in-mem hash table, hash each $t \in R_i$ to bucket B_j with $h' \neq h$, insert if $t \notin B_j$
 - write tuples in hash table to results



- **I/O cost** (no partition overflow): $|R| + 2|\pi_L^*(R)|$
 - partitioning cost: $|R| + |\pi_L^*(R)|$
 - duplicate elimination cost: $|\pi_L^*(R)|$
- partition overflow: recursively apply partitioning
 - to avoid, $B >$ size of hash table for $R_i = \frac{|\pi_L^*(R)|}{B_1} \times f$
 - approximately $B > \sqrt{f \times |\pi_L^*(R)|}$



Projection using Indexes

- if index search key contains all wanted attributes *as a prefix*
 - **index scan** data entries in order & eliminate duplicates

05.2 JOIN $R \bowtie_{\theta} S$

R = outer relation (smaller relation); S = inner relation

! for **format-2** index, add cost of retrieving record

nested loop joins

- **tuple-based** nested loop join: $|R| + ||R|| \times |S|$
- **page-based** nested loop join: $|R| + |R| \times |S|$
- **block nested loop join**: $|R| + (\lceil \frac{|R|}{B-2} \rceil \times |S|)$, $|R| \leq |S|$
 - 1 page output, 1 page input, $(B - 2)$ pages to read R
 - for each $(B - 2)$ pages of R : for each P_S of S : check r, s
- **index nested loop join**:

$$|R| + ||R|| \times \left(\log_F(\lceil \frac{||S||}{b_d} \rceil) + \lceil \frac{||S||}{b_d ||\pi_{B_j}(S)||} \rceil \right)$$

- joining $R(A, B) \bowtie_A S(A, C)$ with B-tree index on $S.A$

- for each tuple $r \in R$, use r to probe S 's index for match

sort-merge join

- sort R & S : $2|R|(\log_m(N_R) + 1) + 2|S|(\log_m(N_S) + 1)$
- merge cost: $|R| + |S|$ (worst case $|R| + ||R|| \times |S|$)
- **optimised sort-merge join**
 - merge sorted runs until $B > N(R, i) + N(S, j)$; then do merge and join at the same time
- I/O cost: $3 \times (|R| + |S|)$
 - if $B > \sqrt{2|S|}$, one pass to merge initial sorted runs
 - $2(|R| + |S|)$ for initial sorted runs, $|R| + |S|$ for merging

hash join

1. partition R and S into k partitions on join column
 - $\pi_A(R_i) \cap \pi_B(S_j) = \emptyset \quad \forall R_i, S_j, i \neq j$
 - $R = R_1 \cup R_2 \cup \dots \cup R_k, t \in R_i \iff h(t.A) = i$
 - $S = S_1 \cup S_2 \cup \dots \cup S_k, t \in S_i \iff h(t.B) = i$
2. join corresponding partitions:

$$R \bowtie_{R.A=S.B} S = (R_1 \bowtie S_1) \cup \dots \cup (R_k \bowtie S_k)$$

Grace hash join

for *build relation* R and *probe relation* S ,

1. **partition** R and S into k partitions each, $k = B - 1$
 2. **probing phase**: hash $r \in R_i$ with $h'(r.A)$ to table T
 - 2.1. $\forall s \in S_i, r \in \text{bucket } h'(s.B)$: output (r, s) if match
- I/O cost: $3(|R| + |S|)$ (no partition overflow)
 - $B > \frac{f \times |R|}{B-1} + 2$ (input & output buffer) $\approx B > \sqrt{f \times |R|}$
 - during probing, $B >$ size of each partition + 2
 - **partition overflow** if R_i cannot fit in memory
 - recursively apply partitioning to overflow partition

General join conditions

- **multiple equality-join** conditions: $(R.A = S.A) \wedge (R.B = S.B)$
 - index nested loop join: use index on some/all join attribs
 - sort-merge join: sort on *combination* of attributes
 - other algos: no change
- **inequality-join** conditions: $(R.A < S.A)$
 - index nested loop join: requires B⁺-tree index
 - not applicable: sort-merge join (too much rewinding), hash-based joins
 - other algos: no change

NOTATION

Notation	Meaning
r	relational algebra expression
$ r $	number of tuples in output of r
$ r $	number of pages in output of r
b_d	number of data records that can fit on a page
b_i	number of data entries that can fit on a page
F	average fanout of B ⁺ -tree index (i.e., number of pointers to child nodes)
h	height of B ⁺ -tree index (i.e., number of levels of internal nodes)
$h = \lceil \log_F(\lceil \frac{ r }{b_i} \rceil) \rceil$	if format-2 index on table R
B	number of available buffer pages