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# MA1101R
AY20/21 sem 2
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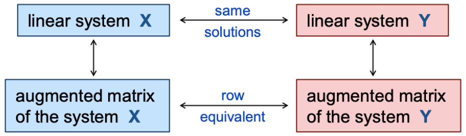
## 01. LINEAR SYSTEMS

- **zero equation** $\rightarrow$ coefficients are all zero
  - either 0 or infinitely many solutions
- **inconsistent** $\rightarrow$ has no solutions
- **solution set** $\rightarrow$ *set* of all solutions to the equation
  - $\{(1+s, 2s, s) \mid s \in \mathbb{R}\}$
- **general solution** $\rightarrow$ *expression* that gives us all solutions to the equation
  - $\begin{cases} x = t \\ y = 2t + 1 \end{cases}$




### elementary row operations

1. $cR_i, c \neq 0$ - multiply by a non-zero constant
2. $R_i \leftrightarrow R_j$ - interchange 2 equations
3. $R_i + cR_j, c \in \mathbb{R}$ - add a multiple of one equation to another equation

### (reduced) row echelon forms

- # of pivot columns = # of leading entries = # of nonzero rows
- every matrix has a **unique** RREF but can have multiple REF.

### homogenous linear systems

- **homogenous** $\rightarrow$ rightmost column is all zeros
- either:
  - one solution - **trivial solution**
  - infinitely many solutions AND the trivial solution

## 02. MATRICES

### types of matrices

- **row/column** matrix $\rightarrow$ only one row/column
  - 1x1 matrix is both row & column matrix
- **square** $\rightarrow$ same number of rows & columns
- **diagonal** $\rightarrow$ all non-diagonal entries are zero
  - **scalar** $\rightarrow$ all diagonal entries are the same
  - **identity**, $I_n$ $\rightarrow$ all diagonal entries are 1
- **zero** $\rightarrow$ all entries are equal to zero
- **symmetric** $\rightarrow a_{ij} = a_{ji} \forall i, j$
  - symmetric $\leftrightarrow A^T = A$
- **triangular**
  - **upper** $\rightarrow a_{ij} = 0$ whenever $i > j$
  - **lower** $\rightarrow a_{ij} = 0$ whenever $i < j$
  - for any upper triangular matrix $D$ where all entries are 1, $D^n = 0$.

### useful notation

<fill in lol>

### transpose

- $(A^T)^T = A$
- $(A + B)^T = A^T + B^T$
- if $c$ is a scalar, then $(cA)^T = cA^T$
- $(AB)^T = B^T A^T$

### inverse

- **uniqueness of inverses** $\rightarrow$ if $B$ and $C$ are inverses of $A$, then $B = C$.
- **cancellation laws** - only hold if $A$ is invertible.
  - if $B_1$ and $B_2$ are $m \times n$ matrices such that $AB_1 = AB_2$, then $B_1 = B_2$.
  - if $C_1$ and $C_2$ are $m \times n$ matrices such that $C_1 A = C_2 A$, then $C_1 = C_2$.

inverse of 2x2 matrix:
$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

### properties of inverses

if $A, B$ are invertible matrices and $c$ is a nonzero scalar,
- $cA$ is invertible: $(cA)^{-1} = \frac{1}{c} A^{-1}$
- $A^T$ is invertible: $(A^T)^{-1} = (A^{-1})^T$
- $A^{-1}$ is invertible: $(A^{-1})^{-1} = A$
- $AB$ is invertible: $(AB)^{-1} = B^{-1} A^{-1}$

if $A, B$ are square matrices of the same size and $AB = I$, then
- $A$ and $B$ are invertible
- $A^{-1} = B; \quad B^{-1} = A$
- $BA = I$

negative powers of square matrices
- $A^{-n} = (A^{-1})^n = A^{-1} A^{-1} \cdots A^{-1}$
- if $A$ is invertible, $(A^n)^{-1} = (A^{-1})^n$