TRIGONOMETRY FORMULA LIST

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common values

θ / °	0°	30°	45°	60°	90°
θ / rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

reciprocal

$$\csc A = \frac{1}{\sin A}$$
$$\sec A = \frac{1}{\cos A}$$
$$\cot A = \frac{1}{\tan A}$$

periodicity

$$\sin(A \pm 2\pi) = \sin A$$
$$\cos(A \pm 2\pi) = \cos A$$
$$\tan(A \pm \pi) = \tan A$$

co-function

$$\sin(\tfrac{\pi}{2} - A) = \cos A$$
$$\cos(\tfrac{\pi}{2} - A) = \sin A$$
$$\tan(\tfrac{\pi}{2} - A) = \cot A$$

sum & difference

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

pythagorean identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A - \tan^2 A = 1$$
$$\csc^2 A - \cot^2 A = 1$$

double angle

$$\begin{aligned}\sin(2A) &= 2\sin A\cos A\\ \cos(2A) &= \cos^2 A - \sin^2 A\\ &= 1 - 2\sin^2 A\\ &= 2\cos^2 A - 1\\ &= \frac{1-\tan^2 A}{1+\tan^2 A}\\ \tan(2A) &= \frac{2\tan A}{1-\tan^2 A}\end{aligned}$$

product to sum

$$\sin A\sin B = \frac{1}{2}\Big[\cos(A-B) - \cos(A+B)\Big]$$
$$\cos A\cos B = \frac{1}{2}\Big[\cos(A-B) + \cos(A+B)\Big]$$
$$\sin A\cos B = \frac{1}{2}\Big[\sin(A+B) + \sin(A-B)\Big]$$
$$\cos A\sin B = \frac{1}{2}\Big[\sin(A+B) - \sin(A-B)\Big]$$

sum to product

$$\sin A \pm \sin B = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right)$$
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

geometry

sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$$

area of triangle

$$\tfrac{1}{2}ab\sin C$$

Heron's formula

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$\text{where } s = \tfrac{1}{2}(a+b+c)$$

differentiation

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\csc x$	$-\csc x\cot x$
$\sec x$	$\sec x\tan x$
$\cot x$	$-\csc^2 x$
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$, $\lvert f(x)\rvert<1$
$\cos^{-1} f(x)$	$-\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$, $\lvert f(x)\rvert<1$
$\tan^{-1} f(x)$	$\frac{f'(x)}{1+[f(x)]^2}$
$\cot^{-1} f(x)$	$-\frac{f'(x)}{1+[f(x)]^2}$
$\sec^{-1} f(x)$	$\frac{f'(x)}{\lvert f(x)\rvert\sqrt{[f(x)]^2-1}}$
$\csc^{-1} f(x)$	$-\frac{f'(x)}{\lvert f(x)\rvert\sqrt{[f(x)]^2-1}}$

integration

$f(x)$	$\int f(x)$
$\tan x$	$\ln(\sec x)$, $\lvert x\rvert < \frac{\pi}{2}$
$\cot x$	$\ln(\sin x)$, $0 < x < \pi$
$\csc x$	$-\ln(\csc x + \cot x)$, $0 < x < \pi$
$\sec x$	$\ln(\sec x + \tan x)$, $\lvert x\rvert < \frac{\pi}{2}$