

MA1521 Cheatsheet

AY20/21 Sem 1 | Chapter 1-3

01. FUNCTIONS & LIMITS

Rules of Limits

1. $\lim_{x \rightarrow a} (f \pm g)(x) = L \pm L'$
2. $\lim_{x \rightarrow a} (fg)(x) = LL'$
3. $\lim_{x \rightarrow a} \frac{f}{g}(x) = \frac{L}{L'}$, provided $L' \neq 0$
4. $\lim_{x \rightarrow a} kf(x) = kL$ for any real number k .

Estimation

first order estimate: $y' \approx y + \Delta x \times \frac{dy}{dx} \Big|_{x=2}$

second order estimate:
 $y' \approx \text{1st estimate} + \left(\frac{(\Delta x)^2}{2} \times \frac{d^2y}{dx^2} \Big|_{x=2} \right)$

Stats

pop. variance: $\sigma^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n}$

pop. covariance: $\text{cov}(x, y) = \frac{\sum xy^2 - \frac{\sum x \sum y}{n}}{n}$

pop. correlation: $\frac{\text{cov}(x, y)}{\sigma_x \times \sigma_y}$

02. DIFFERENTIATION

extreme values:

- $f'(x) = 0$
- $f'(x)$ does not exist
- end points of the domain of f

parametric differentiaton: $\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$

Differentiation Techniques

$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\csc x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$
$a^{f(x)}$	$\ln a \cdot f'(x) a^{f(x)}$
$\log_a f(x)$	$\log_a e \cdot \frac{f'(x)}{f(x)}$
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}, \quad f(x) < 1$
$\cos^{-1} f(x)$	$-\frac{f'(x)}{\sqrt{1-[f(x)]^2}}, \quad f(x) < 1$
$\tan^{-1} f(x)$	$\frac{f'(x)}{1+[f(x)]^2}$
$\cot^{-1} f(x)$	$-\frac{f'(x)}{1+[f(x)]^2}$
$\sec^{-1} f(x)$	$\frac{f'(x)}{ f(x) \sqrt{[f(x)]^2-1}}$
$\csc^{-1} f(x)$	$-\frac{f'(x)}{ f(x) \sqrt{[f(x)]^2-1}}$

L'Hopital's Rule

$$\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow x_0} \frac{f'(x)}{g'(x)}$$

- for indeterminate forms ($\frac{0}{0}$ or $\frac{\infty}{\infty}$), cannot directly substitute $x = a$.
- for other forms: convert to ($\frac{0}{0}$ or $\frac{\infty}{\infty}$) then apply L'Hopital's rule
- for exponents: use \ln , then sub into $e^{f(x)}$

03. INTEGRATION

Integration Techniques

$f(x)$	$\int f(x)$
$\tan x$	$\ln(\sec x), \quad x < \frac{\pi}{2}$
$\cot x$	$\ln(\sin x), \quad 0 < x < \pi$
$\csc x$	$-\ln(\csc x + \cot x), \quad 0 < x < \pi$
$\sec x$	$\ln(\sec x + \tan x), \quad x < \frac{\pi}{2}$
$\frac{1}{x^2+a^2}$	$\frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1} \left(\frac{x}{a} \right), \quad x < a$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a} \ln \left(\frac{x-a}{x+a} \right), \quad x > a$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a} \ln \left(\frac{x+a}{x-a} \right), \quad x < a$
a^x	$\frac{a^x}{\ln a}$

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

- indefinite integral** — the integral of the function without any limits
- antiderivative** — any function whose derivative will be the same as the original function

substitution: $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

by parts: $\int uv' dx = uv - \int u'v dx$

volume of revolution

about x-axis:

- (with hollow area) $V = \pi \int_a^b [f(x)]^2 - [g(x)]^2 dx$
- (about line $y = k$) $V = \pi \int_a^b [f(x) - k]^2 dx$

Logistic Models

$$N = \frac{N_{t=\infty}}{1 + \left(\frac{N_{t=\infty}}{N_{t=0}} - 1 \right) e^{-Bt}}$$

- N - number
- B - birth rate
- t - time