STATS 260 Class 9

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1. Set 10

2. Variance V(X)

The variance of X is written as σ^2

REMEMBER this is related to the population not a sample

$$\sigma^2 = V(X) = E[(X - \mu)]$$

The **standard deviation of X_1 written σ_1 is $\sigma = \sqrt{\sigma^2}$**

We can interpret V(X) in a similar way to E(X): If we were to carry out the experiment many times, and each time keep track of the observed value of X, then the variance of these observed values would approach V(X), as the number of repetitions of the experiment approaches infinity.

2.1. Variance V(X)

$$\sigma^2 = V(X) = E(X^2) - \mu^2$$

Laptop Example

$$E(X) = 1.2$$

$$E(X^2) = 2.16$$

$$V(X) = E(X^2) - [E(X)]^2$$

$$= 2.16 - 1.2^2$$

$$V(X) = 0.72$$

NOTE: standard deviation is just the $\sqrt{V(X)} = \sigma$

NOTE: $\sigma^2, \sigma \geq 0$

2.2. Laws of Variance: (a, b are constants)
1. $V(b) = 0$
2. $V(X + b) = V(X)$
3. $V(aX) = (a)^2 V(X)$

Example
If the random variable X has $V(X) = 2$, then $V(3X + 1) = (3)^2 V(X) = 9(2) = 18$.

Notation:
We may write the variance of aX + b as either V (aX + b) or 2 as σ^2_{aX+b}.

We would write the standard deviation of aX + b as σ_{aX+b}.

Important:
These laws apply to variance, and **not** to standard deviation.

Example:
If the random variable X has $\sigma = 5$, find σ_{-2X+1}.

Example:

Suppose the random variable X has E(X) = 1.9 and V (X) = 0.5.

Find E(3X + 2).

Select the closest to your unrounded answer:

$$E(3X + 2) = 3E(X) + 2 = 3(1.9) + 2 = 7.7$$

(A) 2

(B) 4

(C) 6

(D) 8

Find V (−4X + 8).

Select the closest to your unrounded answer:

$$(-4)^2 V(X) = 16V(X) = 16 \times .5 = 8$$

(A) − 8

(B) 0

(C) 8

(D) 16