STATS 260 Class 6

Gavin Jaeger-Freeborn

Thu 21 May 2020 11:20:51 PM

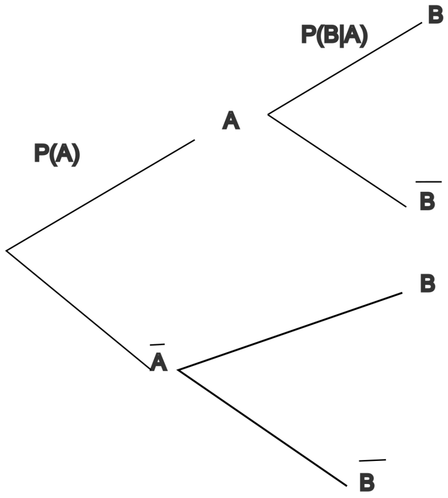
1. Multiplication Rule

$$P(B \bigcap A) = P(A)P(B|A)$$

This is from

$$P(B|A) = \frac{P(A \bigcap B)}{P(A)}$$

NOTE: This is useful for tree diagrams



$$P(\cap B) = P(B \cap A) = P(A)P(B|A)$$

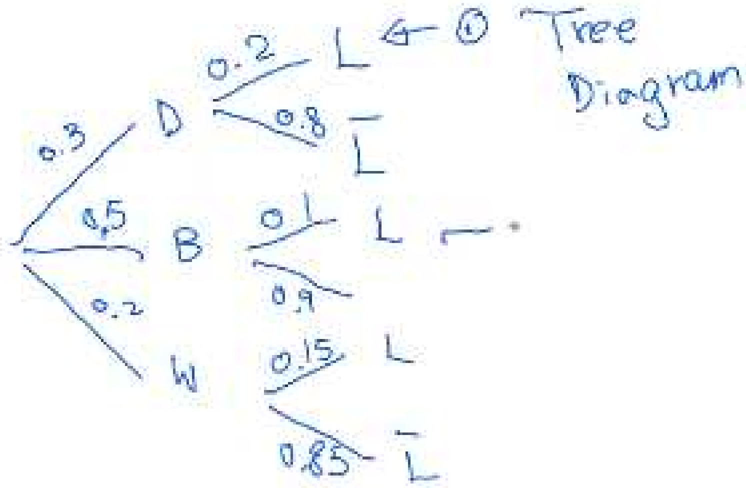
$$P(\bar{A} \cap B) = P(\bar{A})P(B|\bar{A})$$

$$P(B) = P(A \cap B) + P(\bar{A} \cap B)$$

$$A \cup \bar{A} = SA \cap \bar{A} = \phi$$

Example

Suppose that 30% of all students drive to school, 50% take the bus, and 20% walk. Of those who drive, 20% are usually late for their first class of the day. Of those who take the bus, 10% are usually late for their first class of the day. Of those who walk, 15% are usually late for their first class of the day. What is the probability that a randomly selected student is regularly late for their first class?



$$P(L \cap D) = 0.3 * 0.2 = 0.06$$

$$P(B \cap L) = 0.5 * .1 = .05$$

$$P(W \cap L) = 0.2 * 0.15 = 0.03$$

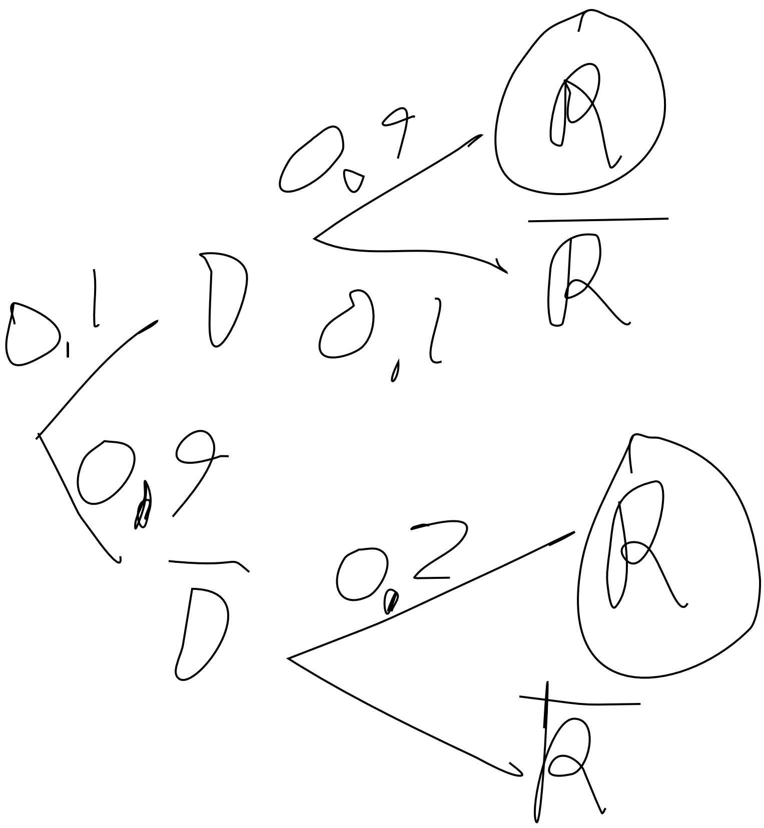
$$P(L) = P(L \cap D) + P(B \cap L) + P(W \cap L)$$

$$= 0.06 + 0.05 + 0.03 = 0.14$$

Example

The probability of an item on a certain production line being defective is 0.1. If an item is defective, the probability that the inspector will remove it from the line is 0.9. If an item is not defective, the probability that the inspector will remove it from the line is 0.2.

What is the probability that a randomly selected item will be removed from the production line?



$$P(R) = (0.1)(0.9) + (0.9)(0.2) = 0.27$$

2. Law of Total Probability

if $A_1, A_2, \ldots, A_k$ are a collection of mutually exclusive and exhaustive events, then for any event B we have:

$$P(B) = P(B \cap A_1) + P(B \cap A_2) + \ldots P(B \cap A_k)$$

$$= P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \cdots + P(B|A_k)P(A_k)$$

3. Bayes Theorem

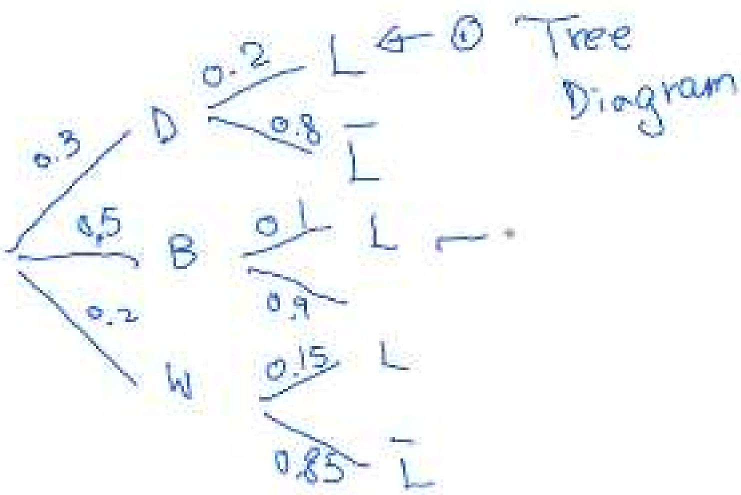
If $A_1, A_2, \ldots, A_k$ are a collection of mutually exclusive and exhaustive events, then for any event B (where $P(B)6 \neq 0$) we have the following, for $1 \leq i \leq k$:

$$P(Ai|B) = \frac{P(A_i \cap B)}{P(B)}$$

$$= \frac{P(B|Ai)P(Ai)}{P(B|A1)P(A1) + P(B|A2)P(A2) + \cdots + P(B|Ak)P(Ak)}$$

Example

using the previous tree calculate P(Late)



$$P(L \cap D) = 0.3 \cdot 0.2 = 0.06$$

$$P(B \cap L) = 0.5 \cdot 0.1 = 0.05$$

$$P(W \cap L) = 0.2 \cdot 0.15 = 0.03$$

$$P(L) = P(L \cap D) + P(B \cap L) + P(W \cap L)$$

$$P(L) = 0.06 + 0.05 + 0.03 = 0.14$$

Example

Suppose that 30% of all students drive to school, 50% take the bus, and 20% walk. Of those who drive, 20% are usually late for their first class of the day. Of those who take the bus, 10% are usually late for their first class of the day. Of those who walk, 15% are usually late for their first class of the day. **Suppose that a student is late for class. What is the probability that this student walks to school?**

$$P(W|L) = \frac{P(W \cap L)}{P(L)}$$

$$P(W|L) = \frac{0.03}{0.14} = \frac{3}{14}$$

4. Set 7

5. Independant events

If A occured but dose not change the likelihood of B occuring. then A and B are Independant events.

If Independant then

$$\boxed{\begin{array}{c} P(B|A) = P(B) \\ P(B \cap A) = P(A)P(B) \end{array}}$$

6. Mutually Exclusive

The probability of A and B arw mutually exclusive if and only if

$$\boxed{P(A \cap B) = 0}$$

Example

to check if a probability is independent or mutually exclusive just check

If $P(A \cap B) = 0$ then its **Mutually Exclusive**.

If $P(B \cap A) = P(A)P(B)$ then it is **Independant**

7. Pairwise

if $P(A_i \cap A_j) = P(A_i)P(A_j)$ for all i, j.)

These events A, B, C

Pairwise

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cap C) = P(A)P(C)$$

$$P(B \cap C) = P(B)P(C)$$

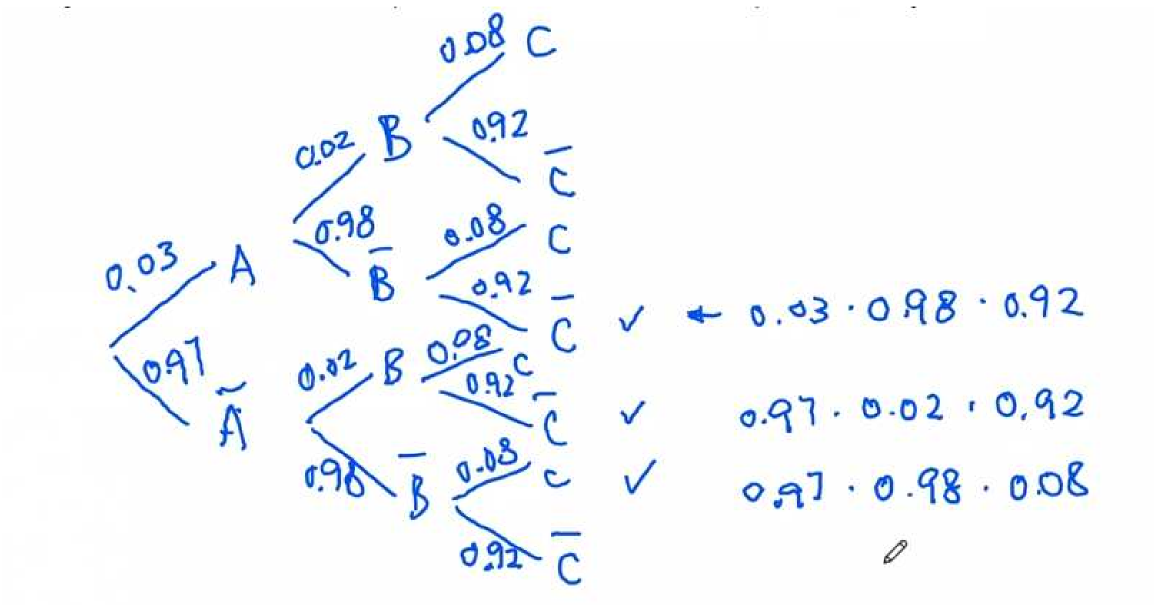
if Pairwise and

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$

then it is just independent

Example

A machine is made of three components (A,B,C) which function independently. The probability that components A,B,C will need to be repaired today is 0.03, 0.02, 0.08 (respectively). What is the probability **exactly one** of the three components will need to be repaired today?



$$P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$$