STATS 260 Class 8

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1. Population mean E(X) (expected value) mu

Let X be a discrete random variable. The **expected value**, or **mean**, of X, denoted by μ, or by E(X) is

$$E(X) = \sum_{\text{all}\, x} x \cdot f(x)$$

f(x) is the pmf of X probability mass function (pmf) or probability distribution

Example

Suppose that X has the following distribution.

x	5	15	100
f (x)	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{5}{12}$

Find E(X) (center of distribution)

$$= 5(1/3) + 15(1/4) + 100(5/12)$$

$$= \frac{20}{12} + \frac{45}{12} + \frac{500}{12} = \frac{565}{12}$$

Example

Approximately 40% of all laptops of a particular brand will need a battery replacement within 3 years of purchase. **Three laptops** of this brand are selected at random. What is the expected number of laptops (in each group of three laptops) which will need a battery replacement within 3 years of purchase?

R = replacement

N = no replacement

let X = # of laptops needing replacement

E(X) = ?

Guess : 3 laptops, 40% replacement

$3 \times 0.4 = 1.2$

$$P(X = 0) = P(NNN) = (0.6)^3 = 0.216 = f(0)$$

$$P(X = 1) = P(RNN, NRN, NNR) = 3 \times (0.6)^2(0.4) = 0.432 = f(1)$$

$$P(X = 2) = P(RRN, RNR, NRR) = 3 \times (0.6)(0.4)2 = 0.288 = f(2)$$

$$P(X = 3) = P(RRR) = 0.4^3 = 0.064$$

$$E(X) = 0(0.216) + 1(0.432) + 2(0.288) + 3(0.064)$$

$\boxed{= 1.2}$ NOTE: this is exactly $3 \times 0.4 = 1.2$

If X is a random variable., and Y - g(X) then:

$$E(Y) - E(g(X)) = \sum g(x)P(X = x) = \sum_x g(x)f(x) \equiv \mu_y \equiv \mu_{g(x)}$$

Example

Using the pmf from the previous example, find E(X + 2), and E(X 2).

x	0	1	2	3
$g(x) = x + 2$	2	3	4	5
f(x)	0.216	0.432	0.288	0.064
X^2	0	1	4	9

$$E(X+2) = 2 \cdot (0.216) + 3 \cdot (0.432) + 4 \cdot (0.288) + 5 \cdot (0.064)$$

$$= 3.2$$

$$[1.2 + 2]$$

$$E(X^2) = 0(0.216) + 1 \cdot (0.432) + 4 \cdot (0.288) + 9 \cdot (0.064)$$

$$= 2.16$$

$$[E(X)]^2 = 1.2^2 = 1.44$$

$$2.16 \neq 1.44$$

NOTE: In this example, and in most cases, E(X 2) is not the same thing as [E(X)] 2 .

$E(x^2) \neq [E(X)]^2$

In general

$$E[g(x)] \neq g(E(X))$$

When is E[g(x) = g[E(X)] ?

When g(x) is linear y = ax + b

2. Laws of Expected Value: (a, b are constants)

1) $E(b) = b$
2) $E(X + b) = E(X) + b$
3) $E(aX) = aE(X)$

2.1. Notation

We may also express $E(aX + b)$ as μ_{aX+b}.

2.2. Proof of 2

$$E(X + b) = \sum(x + b)f(x)$$

$$= \sum xf(x) + \sum bf(x)$$

$$= E(x) + b\sum f(x)$$

$$E(X) + B$$

Example:

If the random variable X is known to have expected value 3.8, find E(7X + 3).

$$E(X) = 3.8$$

$$= 7E(X) + 3$$

$$7(3.8) + 3$$

$$= 29.6$$

Example:

For the laptop experiment, the cost for a replacement battery is \$30 per laptop. What is the expected cost for each group of three laptops? (Assume that each laptop will need at most one replacement battery.)

Let y = cost of each group of 3 laptops.

$$y = 30X$$

$$E(Y) = E(30x) = 30E(X)$$

$$= 30 \times 1.3 = \$36$$

Example

Suppose a random variable X has the following cdf:

x	1	2	3
F(x)	0.3	0.8	1

- Find $E(X)$.

  Select the closest to your unrounded answer:

  must convert to f(x)

x	1	2	3
f(x)	0.3	0.5	0.2

$$\sum x \cdot f(x) = 1.9$$

$\boxed{\text{(A) 2}}$

(B) 3

(C) 4

(D) 5

- Find $E(X^2)$.

  Select the closest to your unrounded answer:

$$E(X^2) = \sum x^2 f(x) = 4.1$$

  (A) 3.5

  $\boxed{\text{(B) 4}}$

  (C) 4.5

  (D) 5

3. Set 10

4. Variance V(X)

The variance of X is written as σ^2

REMEMBER this is related to the population not a sample

$$\sigma^2 = V(X) = E[(X - \mu)]$$

The **standard deviation of** X_1 **written** σ_1 **is** $\sigma = \sqrt{\sigma^2}$

We can interpret V(X) in a similar way to E(X): If we were to carry out the experiment many times, and each time keep track of the observed value of X, then the variance of these observed values would approach V(X), as the number of repetitions of the experiment approaches infinity.

4.1. Computational Formula for Variance

$$\sigma^2 = V(X) = E(X^2) - \mu^2$$

Laptop Example

$$E(X) = 1.2$$

$$E(X^2) = 2.16$$

$$V(X) = E(X^2) - [E(X)]^2$$

$$= 2.16 - 1.2^2$$

$$V(X) = 0.72$$