# STATS 260 Class 21

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**1. Sets 25 to 27 Hypothesis Testing**

**2. Single Sample Hypothesis**

**Example**

A factory has a machine that dispenses 80ml of fluid in a bottle, an employee believes the average amount of fluid is not 80ml. Using 40 samples, he measures the average amount dispensed by the machine to be 78ml with a standard deviation of 2.5.

(a) state the null and alternative Hypothesis.

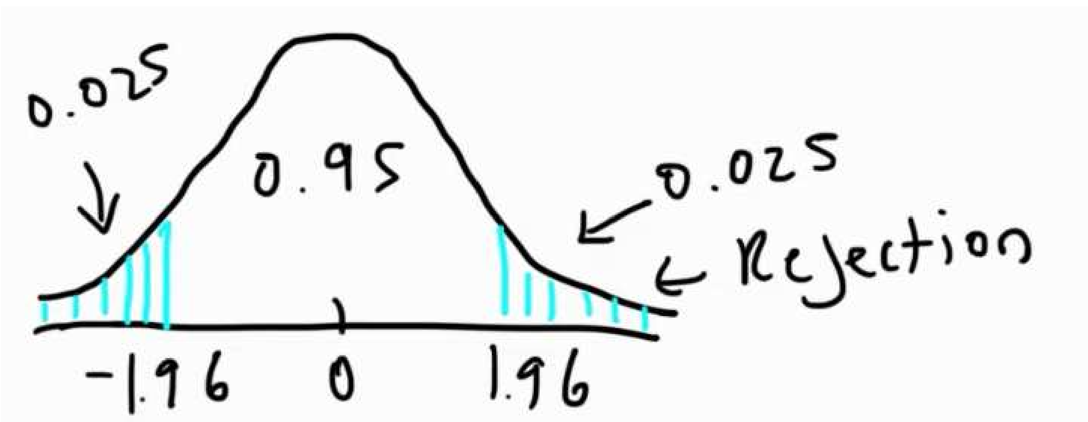
(b) at a 95% confidence level, is there enough evidence to support the idea that the machine is not working properly.

$$H_0\colon \mu \neq 80$$

Alternatively Hypothesis

$$H_a\colon \mu \neq 80$$

$$\bar{x} = 78,\ s = 2.5,\ n = 40 > 30$$



Since n > 30 we can use

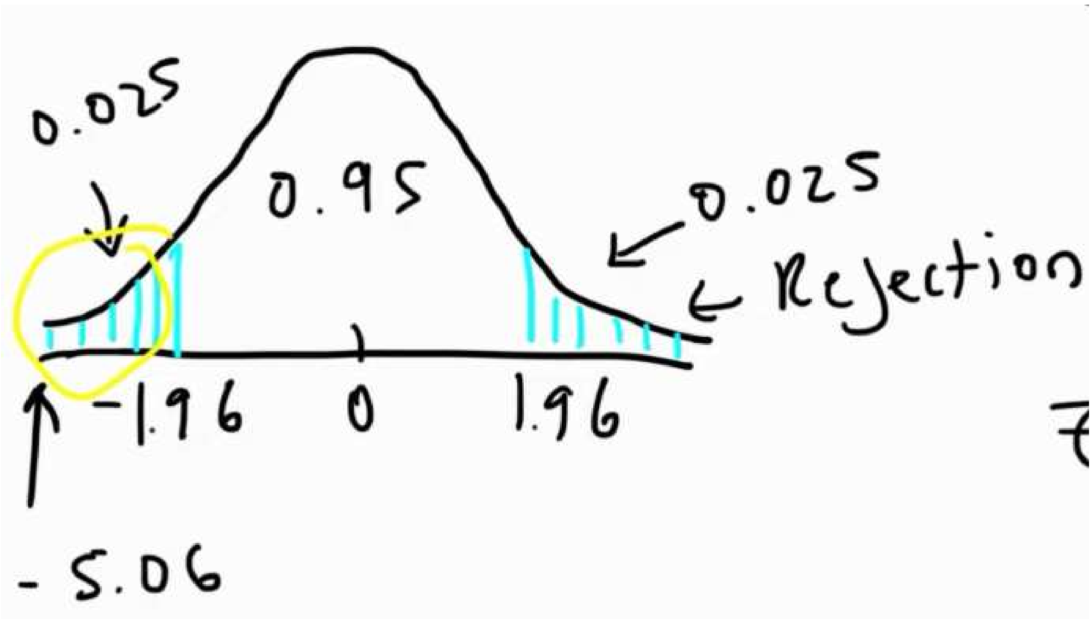
$$Z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{78 - 80}{2.5/\sqrt{40}}$$

$$= \frac{-2}{0.39528}$$

Calculated z value

$$z_c \approx -5.06$$




With a 95% confidence we can say that the machine is not working properly. Since our z value falls outside of out confidence interval.

**3. p-value**

The **smaller the** the p-value the **stronger** the evedence against $H_0$

The **larger** the p-value is, the **weaker** the evidence we have **against** $H_0$

**4. The p-value approach**

1. Define the parameters to be tested. eg μ or P
2. Define H 0 and H 1 .

3. Specify the test statistic and the distribution under $H_0$ . (assume $H_0$ is true)
4. Find the observed value of the test statistic.
5. Find the p-value.
6. Report the strength of evidence against $H_0$ :
   - Very strong if $p \leq 0.01$
   - Strong if $0.01 < p \leq 0.05$
   - Moderate if $0.05 < p \leq 0.1$
   - Little or none if $0.1 < p$
7. Answer any other questions given (i.e. report the value of the estimate, report the value of the estimated standard error, etc.)

| | |
|---|---|
| single tail | $\mu >$ *or* $\mu <$ *or* $\mu \geq$ *or* $\mu \leq$ |
| 2 tail test | $\mu =$ *or* $\mu \neq$ |

**Example**

A certain medication is supposed to contain 350 mg of the active ingredient per pill. It is known from previous work that this content is normally distributed with a standard deviation of 3.5 mg. Suppose a random sample of 5 pills are taken, and the average content is 346.4 mg.

Is the mean pill content not 350 mg?

1. let μ = true mean active ingredients per pill (mg)

2. $H_{0:}\mu = 350$

   $H_{1:}\mu$ != 350 (therefore its 2 tail)

3 test statistic ad distribution

$\sigma = 3.5, n = 5$

$$Z_{obs} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

4

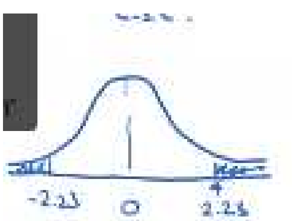
$$Z_{obs} = \frac{346.4 - 350}{3.5/\sqrt{5}} = \frac{-3.6}{1.5652} = -2.23$$

NOTE: the 1.5652 is the estimated standard error (e.s.e)

5. $p - value = 2 \times P(Z > 2.23) = 2 \times p(z < -2.23)$

$= 2 \times 0.0129$

$= 0.0258$



6. there is **strong** evidence against $H_0$

7. Estimate = 346.4 , e.s.e = 1.5652

**5. Relationship between Hypothesis and pvalue**

| Alternate Hypothesis | p-value |
|---|---|
| $h_1: \mu > \mu_0$ | $p(z > z_{observed})$ |
| $h_1: \mu < \mu_0$ | $p(z < z_{observed})$ |
| $h_1: \mu \neq \mu_0$ | $2P(z < -\lvert z_{observed}\rvert) or 2P(z > \lvert z_{observed}\rvert)$ |

**6. Errors and Hypothesis Tests**

Two types of errors are possible in a hypothesis test.

**Type I error**

(Rejection Error) is made when we reject the null hypothesis when it is true.

**Type II error**

(Acceptance Error) is made when we do not reject the null hypothesis when it is false.

**6.1. Possibility of each type of error**

$\alpha$ is the probability of making a Type 1 error

$\beta$ is the probability of making a Type 2 error

| | $H_0$ true | $H_0$ false |
|---|---|---|
| Reject $H_0$ | Type I | ✓ |
| do not reject $H_0$ | ✓ | Type II |

| | |
|---|---|
| $\downarrow \alpha$ | $\uparrow \beta$ |
| $\uparrow \alpha$ | $\downarrow \beta$ |

### 6.2. When do we reject $H_0$?

We are asked to test $H_0$ at some significance level $\alpha$. We carry out the hypothesis test in much the same way: defining parameters, calculating the value of the test statistic, finding the p–value. Rather than giving the level of strength against $H_0$ , as in the p–value

Approach, we instead either reject or don’t reject $H_0$ by the following rule:

- If $p \le \alpha$, then reject the null hypothesis.
- If $p > \alpha$, then do not reject the null hypothesis. (Some will phrase this as ”maintain the null hypothesis” or ”fail to reject the null hypothesis”)

**Example**

For the pill example, if we were asked to test our hypotheses at the level $\alpha = 0.01$, what would our conclusion be?

$$p - value = 0.0258 > \alpha = 0.01$$

Conclusion: maintian $H_0$ at $\alpha$

Example: What if we were testing at the level $\alpha = 0.05$?

$$p - value < \alpha = 0.05$$

conclusion = request $H_0$

IMPORTANT: It is dishonest to set your value of $\alpha$ after the data has been collected and examined; the value of $\alpha$ should be made by taking into account the consequences of Type I and II errors before the study is carried out.

$\uparrow$ set $\alpha$ before the study is completed

### 7. "Relationship between hypothesis testing and confidence intervals"

Suppose we construct a $(1 - \alpha)100\%$ confidence interval for μ.

It is true that for any number k in this interval, that if we were to test H 0 : μ = k, H 1 : μ 6 = k, we'd have a p-value greater than α.

This means that if we were testing $H0\colon \mu = k, H1\colon \mu 6 = k$ at the level of α, we would reject the null hypothesis if and only if k were not inside the $(1–\alpha)100\%$ confidence interval for μ.

**Example**

Using our pill data, we can find that a 95% confidence interval for μ is (343.77, 349.03).

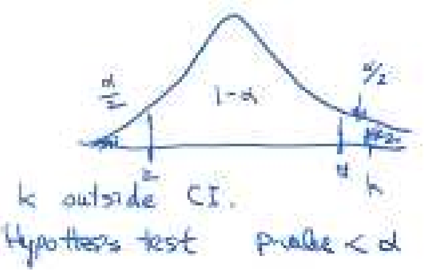
What would our conclusion be if we test H 0 : μ = 344, H 1 : μ 6 = 344 at the level α = 0.05?

$$\alpha = 1 - \textit{confidence interval} = 1 - 95 = 0.05$$

344 is inside the CI

$$p - value > 0.05$$

$$\boxed{\therefore \text{ Retain } H_0}$$




**Example**

What would our conclusion be if we test $H_0\colon \mu = 342, H_1\colon \mu 6 = 342$ at the level $\alpha = 0.05$?

$$(343.77, 349.03)$$

342 is outside the CI

$\therefore$ reject $H_0$ because p-value < 0.05

**Example**

The lengths of mourning doves (from beak to tail) are known to be normally distributed. Suppose that 5 mourning doves are selected at random, and it is found that the average length of the mourning doves is 32.4 cm, with a standard deviation of 2.9 cm.

Let μ denote the true mean length of mourning doves. Test the hypotheses $H_0$ : μ = 30, H a : μ > 30 at the level α = 0.1.

3. Test statistic and distribution:

Population is normal

$n = 5, s = 2.9$

Therefore we sue $t_{n-1}$

$$t_{obs} = \frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_{n-1} = t_4$$

$$s = 2.9cm$$

$$\bar{x} = 32.4cm$$

$$n = 5 < 30$$

$$H_0: \mu = 30$$

$$H_a: \mu > 30$$

$$\alpha = 0.1$$

4. $t_{obs} \frac{\bar{X} - \mu}{so/\sqrt{n}}$

$$t_{obs} = \frac{32.4 - 30}{2.9/\sqrt{5}}$$

$$\frac{2.4}{1.296919}$$

$$t_{observed} = 1.85054$$

Now find p-value using t table

$$p - value = P(t_4 > 1.8505)$$

NOTE: the reason for using > is becouse the alternative is >

In R

```
> 1 - pt(1.8505, 4)
[1] 0.06895478
```

Therefore the p-value is $0.06895478$

$$0.05 < p - value < 0.1$$

6. Moderate evidence against $H_0$

7. estimate = 32.4

   e.s.e = 1.2969

   p-value > 0.05

   $\therefore$ retain $H_0$

**Example**

In a sample of 46 people, we find the average blood glucose level upon waking up is 5.3 mmol/L with a standard deviation of 1.2 mmol/L. Is there reason to believe that the true mean blood glucose level upon waking for people is not 5 mmol/L?

let μ denote true mean blood glucose level of people upon waking up

2. $h_0 : \mu = 5$ $h_1 : \mu \neq 5$

   2 tail

3. test statistic , distrubution

$$z_o = \frac{\bar{x} - \mu}{s/\sqrt{n}} \sim N(0, 1)$$

4. z sub o = {5.3 - 5 } over {1.2 / sqrt 46} = 0.3 over 0.1764 = 1.6956

5. $p - value = 2P(z > 1.6956)$

$= 2P(z < -1.6956)$

$= 0.0892$

6. There is moderate evidence against $H_0$

$\bar{x} = 5.3mol/L$, $ese = 0.1769mmol/L$