ps

STATS 260 Class 10

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1. Recap up to set 11

Capital letters	X, Y, ...	Random Variables (r.v)
small letters	x, y, ...	numerical values
discrete r.v	$P(X = x) = f(x)$	pmf
	$P(X \leq x) = F(x)$	cdf

Parameters quantities about population

$E(X) = \mu$	mean, or expected clue
$V(X) = \sigma^2$	variance
$SD(X) = \sigma$	standard deviation

2. Counting

2.1. Permutations

2.1.1. n factorial

Permutations n distinct items is n!,

$$n! = n(n-1)(n-2)\ldots(2)(1)$$

NOTE: We define 0! to be equal to 1.

Example

The number of different ways to arrange 4 people for a photograph is 4! = 24.

$$4 \times 3 \times 2 \times 1$$

The number of arrangements of r items taken from a collection of n distinct items is:

$$P(n, r) =_n P_r = n^{(r)} = n \frac{!}{(n-r)!}$$

Example

Suppose I have a class of 20 students. The number of ways I can select 4 of these students and arrange them for a photograph is:

$$20P4 = \frac{20!}{16!} = 116280$$

$$= \frac{20 \times 19 \times 18 \times 16!}{16!} = \frac{20!}{16!}$$

2.2. Combinations

The number of combinations (selections) of r items taken from a collection of n distinct items is:

$$C(n,r) = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Example

Suppose I have a class of 20 students. The number of ways I can select (but not arrange) 4 of these students is:

$$\binom{20}{4} = \frac{20!}{4!16!} = 4845$$

$$20C4 = 4845$$

Example

Suppose I have a box containing slips of paper, numbered 1, 2, . . . 30. If I select three of the thirty slips at random, what is the probability that all three slips show a number which is 9 or less?

$$n = 30$$

Select 3 means

$$= r = 3$$

$$\text{Prob} = \frac{n(A)}{n(S)}$$

$$= \frac{9C3}{30C3}$$

3. Set 11

4. Bernoulli Process

An experiment consisting of one or more trials, each having the following properties.

1. Each trial has exactly two outcomes, which we call success and failure.
2. The trials are independent of each other.
3. For all trials the probability of success, p, is a constant.

A **binomial experiment** is a Bernoulli process where n , the number of trials, is fixed in advance.

Let *X* count the number of successes in a binomial experiment Then *X* is a binomial random variable, and we write *X~ Bin(n, p)* , where n is the number of trials, and p is the probability of successes. For a binomial random variable, n and p are its parameters.

~ means X follows n,p parameters

Example

In a manufacturing process, each item has a probability of 0.05 of being defective, independent of all other items. Suppose 12 items are selected at random, and we let W denote the number of defective items.

NOTE: Defective is success

$$P = P(success) = 0.05$$

$$n = 12$$

$$W \sim Bin(12, 0.05)$$

$$w = 0, 1, \ldots, 12$$

0 to all defective

5. Binomial Probability Distribution

$$f(x) =$$

$$pmf \ =$$

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x = 0, 1, 2, \ldots, n$$

where x = success

Example

On a multiple choice test, there are 10 questions, each with 8 possible responses. I will complete the test by randomly selecting answers. What is the probability that I will get 1 question correct?

Assume Independence

Exactly

Let X = # of correct answers

$$X = Bin(10, \frac{1}{8})$$

$$P(X - 1) = \binom{10}{1}\left(\frac{1}{8}\right)^1\left(1 - \frac{1}{8}\right)^{10-1}$$

$$= \binom{10}{1}\left(\frac{1}{8}\right)^1\left(\frac{7}{8}\right)^9$$

$$= 0.375$$

Example

In the manufacture of lithium batteries, is is found that 7% of all batteries are defective. Suppose that we test 6 randomly selected batteries. What is the probability that at least two batteries are defective?

X = # of defective batteries

$$p = 0.07, n = 6$$

$$X \sim Bin(6, 0.07)$$

$$P(x \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$$

$$= 1 - P(X < 2)$$

$$= 1 - \binom{6}{0} 0.07^0 (0.93)^6 - \binom{6}{1} 0.07^1 (0.93)^3$$

$$= 0.0608$$

5.1. Expected Value and Variance

If X ~ Bin(n, p), then:

$$E(X) = \sum xf(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$E(X) = np \text{ and } V(X) = np(1-p)$$

Example

What is the expected number of defective lithium batteries per batch of 6? What is the variance?

$$n = 6, \ p = 0.07$$

$$E(X) = 6 \times 0.07 = 0.42$$

$$V(X) = 6 \times 0.07 \times 0.93 = 0.3906$$

5.2. Cumulative Distribution Tables F(X)

These tables give $P(X \leq x)$ for "nice" values of n and p

Example

It is known that 20% of all tablet computers will need the touch-screen repaired within the first two years of use. Suppose we select 15 tablet computers at random.

$$n = 15, \; p = 0.2$$

$$X \sim Bin(15, 0.2)$$

What is the probability that <u>no more than 6</u> tablets will need repairs to the touch-screen within the first two years of use?

$$P(X \leq 6)$$

		p									
n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
14	0	0.2288	0.0440	0.0178	0.0068	0.0008	0.0001	0.0000			
	1	0.5846	0.1979	0.1010	0.0475	0.0081	0.0009	0.0001			
	2	0.8416	0.4481	0.2811	0.1608	0.0398	0.0065	0.0006	0.0000		
	3	0.9559	0.6982	0.5213	0.3552	0.1243	0.0287	0.0039	0.0002		
	4	0.9908	0.8702	0.7415	0.5842	0.2793	0.0898	0.0175	0.0017	0.0000	
	5	0.9985	0.9561	0.8883	0.7805	0.4859	0.2120	0.0583	0.0083	0.0004	
	6	0.9998	0.9884	0.9617	0.9067	0.6925	0.3953	0.1501	0.0315	0.0024	0.0000
	7	1.0000	0.9976	0.9897	0.9685	0.8499	0.6047	0.3075	0.0933	0.0116	0.0002
	8		0.9996	0.9978	0.9917	0.9417	0.7880	0.5141	0.2195	0.0439	0.0015
	9		1.0000	0.9997	0.9983	0.9825	0.9102	0.7207	0.4158	0.1298	0.0092
	10			1.0000	0.9998	0.9961	0.9713	0.8757	0.6448	0.3018	0.0441
	11				1.0000	0.9994	0.9935	0.9602	0.8392	0.5519	0.1584
	12					0.9999	0.9991	0.9919	0.9525	0.8021	0.4154
	13					1.0000	0.9999	0.9992	0.9932	0.9560	0.7712
	14						1.0000	1.0000	1.0000	1.0000	1.0000
15	0	0.2059	0.0352	0.0134	0.0047	0.0005	0.0000				
	1	0.5490	0.1671	0.0802	0.0353	0.0052	0.0005	0.0000			
	2	0.8159	0.3980	0.2361	0.1268	0.0271	0.0037	0.0003	0.0000		
	3	0.9444	0.6482	0.4613	0.2969	0.0905	0.0176	0.0019	0.0001		
	4	0.9873	0.8358	0.6865	0.5155	0.2173	0.0592	0.0093	0.0007	0.0000	
	5	0.9978	0.9389	0.8516	0.7216	0.4032	0.1509	0.0338	0.0037	0.0001	
	6	0.9997	0.9819	0.9434	0.8689	0.6098	0.3036	0.0950	0.0152	0.0008	
	7	1.0000	0.9958	0.9827	0.9500	0.7869	0.5000	0.2131	0.0500	0.0042	0.0000
	8		0.9992	0.9958	0.9848	0.9050	0.6964	0.3902	0.1311	0.0181	0.0003
	9		0.9999	0.9992	0.9963	0.9662	0.8491	0.5968	0.2784	0.0611	0.0022
	10		1.0000	0.9999	0.9993	0.9907	0.9408	0.7827	0.4845	0.1642	0.0127
	11			1.0000	0.9999	0.9981	0.9824	0.9095	0.7031	0.3518	0.0556
	12				1.0000	0.9997	0.9963	0.9729	0.8732	0.6020	0.1841
	13					1.0000	0.9995	0.9948	0.9647	0.8329	0.4510
	14						1.0000	0.9995	0.9953	0.9648	0.7941
	15							1.0000	1.0000	1.0000	1.0000

$$P(X \leq 6) = 0.9819$$

Example

What is the probability that exactly 5 tablets will need touch-

$$P(X = 5) = \binom{15}{5} 0.02^5 (0.8)^{10}$$

$$= P(X = 5) - P(X \leq 4)$$

$$= 0.9389 - 0.8358 = 0.1031$$

Example

What is the probability that at least 2 tablets will need touch- screen repairs?

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X \leq 1)$$

$$= 1 - 0.1671$$

$$= 0.8329$$

Example

It is known that 30% of all laptops of a certain brand experience hard-drive failure within 3 years of purchase. Suppose that 20 laptops are selected at random. Let the random variable X denote the number of laptops which have experienced hard-drive failure within 3 years of purchase. If it is known that at least 3 laptops experience hard-drive failure , what is the probability that no more than 6 laptops will experience hard-drive failure?

$$X \sim Bin(20, 0.3)$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(X \le 6 | X \ge 3) = \frac{P(X \le 6 \cap X \ge 3)}{P(X \ge 3)}$$

$$= \frac{P(3 \le X \le 6)}{P(X \ge 3)}$$

don’t forget to convert to F(X) aka cdf

$$= \frac{P(X \le 6) - P(X \le 2)}{1 - P(X \le 2)}$$

look up in table

$$= \frac{0.6080 - 0.0355}{1 - 0.0355}$$

$$= 0.5936$$