STATS 260 Class 13

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Let p be a number between 0 and 1. The 100p th percentile of a continuous random variable is the value α such that $F(\alpha) = p$.

Example

For the random variable from the previous example, find the 90 th percentile.

$$F(\alpha) = \frac{\alpha}{1+\alpha} = 0.9$$

$$\alpha = 0.9 + 0.9\alpha$$

$$0.1\alpha = 0.9\alpha = 1$$

1. Mean and Variance of an Interval

The **expected value** or **mean** of a continuous random variable X with pdf f (x) is:

$$E(X) = \mu = \int_{-\infty}^{\infty} x\, f(x) dx$$

similar to

$$E(X) = \mu = \sum_{x} x\, f(x)$$

(provided this integral converges)

The **variance** of a continuous random variable X with pdf f (x) is:

$$V(X) = \sigma \; = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$$

similar to

$$V(X) = \sigma \; = \sum_{x} (x - \mu)^2 f(x)$$

(provided this integral converges) and the standard deviation, $\sigma \; = \sqrt{\sigma^2}$.

As with discrete random variables, we have the following:

- $V(X) = E(X^2) - \mu^2$
- $E(aX + b) = aE(X) + b$
- $V(aX + b) = a^2 V(X)$

Example

Suppose the random variable X has pdf

$$f(x) = \begin{cases} 2e^{-2x} & 0 \le x < \infty \\ 0 & otherwise \end{cases}$$

Find the median of the distribution.

NOTE: The median, $\tilde{\mu}$ of a continuous random variable is the 50^{th} percentile.

$$\mu = E(x) = \int_0^\infty x2e^{-2x}\,dx$$

$$= \lim_{b\to\infty} \int_0^b x \cdot 2e^{-2x}\,dx$$

$$= \lim_{b\to\infty} \left[(-be^{-2b} - \frac{e^{-2b}}{2}) - (0 - \tfrac{1}{2}) \right]$$

$$= \lim_{b\to\infty} (-be^{-2b} - \frac{e^{-2b}}{2}) - \lim_{b\to\infty} (0 - \tfrac{1}{2})$$

$$= \tfrac{1}{2}$$

$$E(X^2) = \int {}^{\infty}_{0} x^2 2e^{-2x}\,dx$$

$$V(X) = \frac{1}{2} - (\frac{1}{2})^2 = \frac{1}{4}$$