STATS 260 Class 4

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Guaranteed event	S	will always happen
Impossible/null event	$\varnothing$	will never happen

S is called a **guaranteed** or **certain event**, because it will always occur.

The event $\varnothing$, which consists of no outcomes, is called the **impossible event** or **null event**, because it never occurs.

If for events A and B, we have A $\cap$ B = $\varnothing$, then we say that A and B are disjoint or mutually exclusive events.

We can often use tree diagrams to help us find all possible outcomes.

Example

Suppose that a box contains red, blue, and green marbles (several of each color). Two marbles are selected one at a time from the box, and the sequence of colors is noted. What is the sample space?

1. Probability ($Pr(A)$ or $P(A)$)

Likelihood that some event will or will not occur.

We measure probability on a scale from 0 to 1

$0 \rightarrow$ impossible for the event to occur

$1 \rightarrow$ event is guaranteed to occur.

1.1. Approaches

Experimentally

- repeat an experiment n times
- count f, the number of time s the event in question occurs.
- then $P(A) = f/n$

Classical (the one we will use)

Theoretically

1.2. Probability Axioms

1. $P(S) = 1 \leftarrow$ Guaranteed
2. $P(A) \geq 0)$ for any event A
3. $P(A_2 \bigcup A_2 \bigcup \cdots) = \sum P(A_i)$ for all **infinite** collection of **mutually exclusive** events.
   $\therefore A_i \bigcap A_j = \phi$

From these axioms, we can derive other properties of probability, including:

- P (∅) = 0
- $P(A_1 \bigcup A_2 \bigcup \cdots \bigcup_k) = \sum_{i=1}^{k} P(A_i)$. (where the events are all mutually exclusive)
- $P(A) = 1 - P(\bar{A})$ for any event A. $\leftarrow$ or $P(\bar{A}) = 1 - P(A)$
- $P(A) \leq 1$ for any event A
- P (A ∪ B) = P (A) + P (B) - P (A ∩ B) for any events A and B.
- P (A ∪ B ∪ C) = P (A) + P (B) + P (C) - P (A ∩ B) - P (A ∩ C) - P (B ∩ C) + P (A ∩ B ∩ C) for any events A, B, and C.

P (A ∪ B) = P (A) + P (B) - P (A ∩ B)

If we just did P(A) + P(B) we would over count so we - P (A ∩ B)

Example

NOTE: End of first quiz

2. Uniform Sample Space

Each sample is equivalently likely to be picked

Example

Since every element of S appears the same amount of times they are all equivalently likely to be picked.

$$S = \{1, 2, 3, 4, 5, 6,\}, P(\{1\}) = \frac{1}{6}$$

$$n(S) = 6$$

$n(S)$ = size of the sample space

$n(A)$ = size of event A

$$A = \{2, 4, 6\}$$

$$n(A) = 3$$

$n(S)$ sample events must have the same probability, and those probabilities must add to 1.

The probability of each event must be $1/n(S)$

The **probability** of **any event A** in a **uniform, finite sample space S** is

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$\frac{3}{6} = \frac{1}{2}$$

Example

There are 80 students in a classroom. I will select one of the 80 students at random to answer a question. Of the 80 students, 7 are sitting in the front row. What is the probability that I select a student who is sitting in the front row?

$$n(S) = 80, n(A) = 7$$

$$P(A) = \frac{7}{80}$$

Example

The 2001 Census found that in Tofino, there were 790 residents who traveled to work. Here are the results of this census question

Mode of Transportation	**Total Numbers**
Car/truck/van	435
Walk/bicycle	250
Other method	105

Suppose a Tofino resident who travels to work is selected at random. What is the probability that this resident walks or bikes to work?

$$435 + 250 + 105 = 790$$

Example

Consider the results of the following survey of 250 single-crop farms:

	Wheat	Corn	Soy
Alberta	69	15	16
Saskatchewan	61	65	24

If we select one farm at random, what is the probability that the **farm grows wheat, or is in Saskatchewan**?

$$\text{Prob} = \frac{69 + 61 + 65 + 24}{250}$$

3. P(B|A)

$P(B|A)$ = probability that B will occur if A occurs.

$$P(B|A) = \frac{n(B \cap A)}{n(A)} = \frac{P(B \cap A)}{P(A)}$$

Example

Consider the results of the following survey of 250 single-crop farms:

	Wheat	Corn	Soy
Alberta	69	15	16
Saskatchewan	61	65	24

Suppose that a single-crop farm is selected at random. If the farm is in Alberta, what is the probability the farm grows soy?

$$P(Soy|Alberta) = \frac{16}{69+15+16}$$

Example 2

If a farm which **grows soy** is selected, what is the probability that the farm is **in Alberta**?

$$P(Alberta|Soy) = \frac{16}{16+24}$$

NOTE: $P(A|B) \neq P(B|A)$ - in general

Example

Suppose 80% **(A)** of all Canadians exercise one or more days a week, and also, that 20% **(B)** of all Canadians exercise at five or more days a week. If we randomly select a Canadian who exercises at least one day a week, what is the probability that this Canadian exercises five or more days a week?

$$B \subseteq A$$

$$B \bigcap A = B$$

$$P(B|A) = \frac{P(A \bigcap B)}{P(A)}$$

$$= \frac{P(B)}{P(A)} = \frac{0.2}{0.8}$$

$$\boxed{= 0.25}$$

Example

Suppose we would like to know the probability that someone orders **chocolate ice cream in a waffle cone**.

- We want P(Chocolate $\cap$ Waffle)

Example

Suppose we would like to know the probability that someone **who wants a waffle cone will order chocolate ice cream**. Which of the following are we trying to find:

- We want P(Chocolate|Waffle)

4. Multiplication Rule

$$P(B \cap A) = P(A)P(B|A)$$

This is from

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$