STATS 260 Class 7

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1. Probability Modeling

1.1. Random Variable

a function which maps each outcome of an experiment to a number

$$events \rightarrow \#'s$$

Example

The number of defective items could be 0, 1,..., 10. Thus, X can take on the values 0, 1,..., 10.

$$X = \{0, 1, \ldots, 10\}$$

Probability one item is defective is P(X=1)

Probability at least 2 items are defective is P(X≥1)

Example

I randomly select a student and ask if they have taken Math 122. For this experiment, I have the random variable Y , which takes on two values: 0 and 1. The random variable Y will take a value of 1, if the answer is "Yes", and will take on a value of 0 if the answer is "No".

$$P(X = 0) \rightarrow NO, P(X = 1) \rightarrow YES, X\{0, 1, \}$$

1.2. Support

possible values it can take. In the last example question.

$$X = \{0, 1\}$$

1.2.1. Continuous

Support is real numbers

1.2.2. Discrete

Support is non real numbers

2. Probability Mass Function or Probability Distribution f(X)

$$f(x) = P(X = x)$$

2.1. Probability Distribution Table

x	**0**	**1**	**...**	**10**
f(x)	**0.1**	**0.03**	**...**	**0.005**

Example

At a small taco shop, it has been noted that 80% of customers order beef tacos, and the other 20% of customers order veggie tacos. **Three customers** enter the store, and each customer independently orders one taco. Construct the probability distribution table for the random variable X, where **X is number of veggie tacos ordered** by the three customers.

Outcomes {BBB,VBB,BVB,BBV,VVB,VBV,BVV,VVV}

$X = 0 \rightarrow BBB$

$X = 1 \rightarrow VBB, BVB, BBV$

$X = 2 \rightarrow VVB, VBV, BVV$

$x = 3 \rightarrow VVV$

R.V. X Support of X = $\{0, 1, 2, 3\}$

$$f(0) = P(X = D) = P(BBB) = 0.8 \times 0.8 \times 0.8 = 0.512$$

$$f(1) = P(\{VBB, BVB, BBV\})$$

$$= (0.2)(0.8)(0.8) + (0.8)(0.2)(0.8) + (0.8)(0.8)(0.2) = 0.384$$

$$f(2) = P(\{VVB, VBV, BVV\})$$

$$= 3 \times (0.2)(0.2)(0.8) = 0.096$$

$$f(2) = P(\{VVV\})$$

$$= 0.2^3 = 0.003$$

x	0	1	2	3
f(x)	0.512	0.384	0.096	0.008

NOTE:

$\sum_x f(x) = 1$

What is the probability that exactly one veggie taco will be ordered?

$$P(x = 1) = f(1) = 0.384$$

What is the probability that at least two veggie tacos will be ordered?

$$P(X \geq 2) = P(X = 2) + P(X = 3)$$

$$= f(2) + f(3)$$

$$= 0.96 + 0.008 = 0.104$$

Suppose we know that at **least one veggie taco** is ordered. What is the probability that **exactly two veggie tacos** will be ordered?

Conditional Probability

$$P(X = 2 | X \geq 1)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(X = 2 \cap X \geq 1)}{P(\geq 1)} = \frac{P(X = 2)}{P(X \geq 1)}$$

x	0	1	2	3
f(x)	0.512	0.384	0.096	0.008

$$\frac{0.096}{0.384 + 0.096 + 0.008} = \frac{0.096}{0.488} = \frac{12}{61}$$

3. Cumulative Distrobution Function F(X) cdf

$F(X) = P(X \leq x)$

Example

Suppose the random variable X has the following probability distribution:

x	1	2	3	4	5
f(x)	0.3	0.15	0.05	0.2	0.3

Find the cdf for this random variable

$$F(1) = P(X \leq 1) = P(X = 1) = 0.3$$

$$F(2) = P(X \leq 2) = P(X = 1) + P(X = 2) = f(1) + f(2) = 0.3 + 0.15 = 0.45$$

$$F(3) = P(X \leq 3) = Pf(1) + f(2) + f(3) = 0.5$$

$$F(4) = 0.7$$

$$F(5) = 1$$

x	1	2	3	4	5
F(x)	0.3	0.45	0.5	0.7	1

The easier way is to just add them

x	1	2	3	4	5
f(x)	0.3	0.15	0.05	0.2	0.3
F(x)	0.3	0.45	0.5	0.7	1

f(x) $\rightarrow$ F(X)

3.1. Properties of a cdf

- F (x) is monotone increasing.
- $\lim_{x\to\infty} F(x) = 0$ and $\lim_{x\to\infty} F(x) = 1$.

Explanation

x → ∞

P(X ≤ x)

X≤x → Sample Space

Remember

P(S) = 1

When S is sample space

x → -∞

ϕ is the empty set

P(ϕ) = 0

- F (x) is right-continuous (continuous at each point x = k where x approaches k from the right)

NOTE: In the previous example, the support for the pmf was x = 1, 2, 3, 4, 5. As we've discussed previously, for any x which is not part of the support (i.e. impossible outcomes), the probability of that value of being observed is zero.

Example

In the previous example, the event X = 3.5 is an impossible event. Therefore,

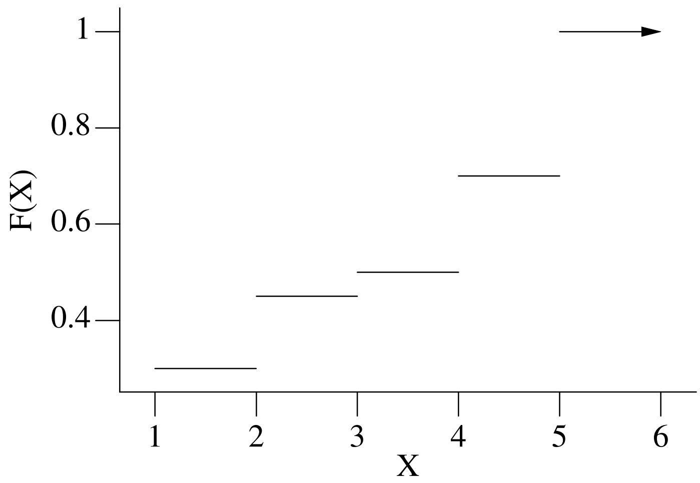
f (3.5) = P (X = 3.5) = 0.

However, **this does not mean the cdf also has a value of zero** :

Example

F (3.5) = P (X ≤ 3.5)

x	1	2	3	4	5
F(X)	0.3	0.45	0.5	0.7	1



$$\lim_{x \to k^+} F(X) = F(k)$$

Example

Let the discrete random variable X count the number of classes a randomly selected UVic student is currently taking. The cdf for X is the following.

x	1	2	3	4	5	6	7
F(x)	0.15	0.25	0.4	0.6	0.75	0.90	1

Remember F(X) = P(X<=x)

- What is the probability that the student is taking no more than 4 classes?

$$P(X \leq 4) = F(4) = 0.6$$

- Calculate F(4.5).

$$F(4.5) = F(4) = 0.6$$

- What is the probability that the student is taking at least 3 classes?

$$P(X \geq 3)$$

we can then use the complement of $F(3)$ since $F(3) = P(x \leq 3)$

$$P(X \geq 3) = 1 - P(x < 3)$$

$$= 1 - P(x \leq 2) = 1 - F(2)$$

$$= 1 - 0.25$$

$$\boxed{= 0.75}$$

- What is the probability that the student is taking exactly 3 classes?

$$P(x \leq 3) - P(x \leq 2) = F(3) - F(2) = 0.4 - 0.25$$

$$= 0.15$$

- What is the probability that the student is taking at **least 2 but no more than 5 classes**?

$$P(x \geq 2) \cap P(x \leq 5) = P(2 \leq x \leq 5)$$

$$F(5) = \{1, 2, 3, 4, 5\}, \text{and } F(1) = \{1\}$$

$$F(5) - F(1) = 0.75 - 0.15 = 0.6$$