STATS 260 Class 20

Gavin Jaeger-Freeborn

1. Review

small sample cs $n < 40$

$$\bar{x} \pm t_{n-1,\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$$

NOTE: d is still everything to the right of ± therefor

$$d = t_{n-1,\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$$

and

$$n = \left(\frac{z_{\frac{\alpha}{2}} s}{d} \right)^2$$

NOTE: the resulting n must be $n < 40$ also round up

2. set 24

3. Population Proportion Estimation

if we have a **binomial distribution** with $n < 40$. we can estimate the value of the true proportion of success

if n is the number of observations and x is the number of successes, then $\hat{p} = x/n$

p is the **sample proportion**

$$\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \approx N(0, 1)$$

this can be rewriten as

$$\frac{\hat{p} - p}{\frac{\sqrt{p(1-p)}}{\sqrt{n}}} \approx N(0, 1)$$

$$X \sim Bin(n, p)$$

NOTE: X is the total number of successes where $X_1, X_2, \ldots, X_n \sim Bin(n = 1, p)$ each one is either a success or a failure.

$$E(x_i) = P = \mu$$

$$V(X_i) = E({X^2}_i) - \mu^2$$

$$= p(1-p)$$

$$\sigma_{x_i} = \sqrt{p(1-p)}$$

$$\frac{\hat{p} - p}{\frac{\sqrt{p(1-p)}}{\sqrt{n}}} \approx N(0, 1) = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Because of central limit Theorem

Standard error of $\hat{p}$ is just $\sqrt{p(1-p)/n}$ since the value of p is unknown

We cannot use the standard error in out confidence interval.

Instead, we use the **estimated standard error**

estimated standard error of $\hat{p}$ is

$$\sqrt{p(1-p)/n}$$

Using the same formula as before

$$(\text{estimate}) \pm (\text{ critical value }) \cdot (\text{ estimated standard error })$$

We get

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

NOTE: there must be at least 5 success and 5 failures

Example

A sample of 1380 randomly selected books produced by a publishing company finds that 25 have bookbinding errors. Find a 95% confidence interval for p, the proportion of books with bookbinding errors.

$$n = 1380$$

$$x = 25$$

$$cl = 95\%$$

$$\alpha = 1 - 0.95 = 0.05$$

$$\frac{\alpha}{2} = 0.025$$

Looking up z value for p = 1-0.025

$$z_{0.025} = 1.96$$

$$\hat{p} = \frac{x}{n} = \frac{25}{1380}$$

$$\frac{25}{1380} \pm 1.96 \sqrt{\frac{\frac{25}{1380}\left(1 - \frac{25}{1380}\right)}{1389}}$$

$$0.018116 \pm 0.007036831$$

$$(0.011079, 0.02515)$$

We are 95% confident that the true proportion of books with errors is between 1.11% and 2.52%

If we are given d (margin of error) we can estimate the sample size using

$$n = \frac{(z_{\alpha/2})^2 \hat{p}(1-\hat{p})}{d^2}$$

Option 1

Sometimes we use a previous study to estimate for $\hat{p}$

Option 2

$$\text{use } \hat{p} = \frac{1}{2} \text{ (based on calculus)}$$

This gives you

$$n = \frac{(z_{a/2})^2}{4d^2} \Leftarrow \hat{p}(1-\hat{p}) = \frac{1}{4}$$

Example

In an earlier study, it was found that 1.4% of all microchips made by a particular manufacturer were defective. Using this as a pilot study, estimate the sample size needed to create a 99% confidence interval for p, the true proportion of defective microchips, with a margin of error of 0.005.

$$n = \frac{(z_{\alpha/2})^2 \hat{p}(1-\hat{p})}{d^2}$$

$$\frac{2.575^2(0.014)(1-0.014)}{0.005^2}$$

$$n = 3661.166$$

Remember to round up

$$n = 3662$$

Example

We wish to carry out a telephone survey to estimate p, the proportion of island residents who want a bridge to the mainland. How many people must we call in order to estimate p with 98% confidence, to within 0.01?

Margin of error

$$d = 0.01$$

$$cl = 98\%$$

$$\alpha = 1 - .98, \frac{\alpha}{2} = 0.01$$

$$z_{0.01} = 2.326348$$

$$\hat{p} = 1/2$$

$$n = \frac{(z_{a/2})^2}{4d^2}$$

$$n = \frac{(2.326348)^2}{4(0.01)^2}$$

$$n = \frac{(5.4289)}{(4e-04)}$$

$$n = 13572.25$$

$$n = 13573$$