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Elastic buckling behavior of rectangular plates with holes subjected to partial edge loading

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ABSTRACT

Cut-outs or openings are inevitable part of steel structural systems since they are required as access ports for mechanical and electrical systems or to reduce the amount of material that is used. When such perforated plates experience compression loads, they may buckle or show instability due to axial compression. In this study, the buckling of perforated square and rectangular plates subjected to in-plane compressive edge loading is investigated using the finite element method. To analyze the behavior of the plates the following four edge loading cases were applied; concentrated edge loading, asymmetric partial loading at the opposite edges, partial loading at the center of the opposite edges and partial loading at the two ends of the opposite edges. The plate aspect ratio, the length and location of the edge loading and diameter of the circular hole are taken as the variables that have a buckling effect on the behavior of the plate. The results show that the square plates are highly sensitive to buckling when the loading at the center of the plates.

1. Introduction

In the literature there are a number of studies reporting the linear buckling behavior of rectangular plates, however, limited in depth work has been published on perforated plates under in-plane uniform loading [1–8] and linearly varying in-plane loading [9, 10]. The studies concerning perforated plates show that the presence of holes change the buckling mode shape and may reduce the elastic buckling load capacity of perforated steel plates.

Deolasi and Datta [11] investigated the elastic buckling and vibration behavior of rectangular plates without holes using the finite element method based on ordinary first order shear deformation theory (FSDT). They concluded that the plates are less susceptible to buckling when partial edge loading is located near the supported edges. Srivastava et al. [12] used the finite element method to investigate the buckling and vibration characteristics of stiffened plates subjected to in-plane partial and concentrated loading at the opposite edges. They stated that the stiffened plate is less prone to buckling when the loading is located near the supported edges and near the stiffeners. Mariorana et al. [13–15] conducted a series of investigations which generally were concerned with the elastic buckling of behaviors of perforated plates under axial compression and bending moment and localized symmetrical loads. The localized symmetrical loads were applied at the short edge of the plate with different ratios of load length to short edge length. Ikhenazen et al. [16] investigated the linear buckling of simply supported thin plates subjected to patch compression using the finite element method. They determined the buckling coefficient for two different load cases applied to a range of plate with various edge ratios ($a/b = 1, 2,.., 10$). Recently, Singh et al. [17] undertook a buckling analysis of thin rectangular plates with cutouts subjected to partial edge compression using the finite element method. In their work, the diameter of circular hole numerically set to 0.2 of the short edge of plates is taken as the constant in all kinds of plates, They showed that square plates are significantly affected by partial edge loading in comparison with rectangular plates having aspect ratios $a/b = 1, 2, 3, 4$.

Since there are few studies that are available in the literature related to the linear buckling behavior of perforated plates subjected to partial edge loading cases the present study aims to fill that gap. The buckling behavior of perforated plates under in-plane different partial edge loading was investigated and in addition the effects of the hole size on buckling load was analyzed. The plate aspect ratio, the length and location of the edge loading and diameter of the circular hole are considered to be the variables that affect the behavior of the perforated buckling plate. The finite element method was used to determine the plate buckling load of the perforated plates. The aim of this study is to determine the variation of elastic buckling load of rectangular plates subjected to partially edge loading as the plate aspect ratios and the diameter of the circular holes were changed.

2. Description of the problem

Fig. 1 shows a sample rectangular plate with a circular perforation having a width b, length a, thickness t and a circular cut-out with diameter d located at the center of plate. All the four plate edges are simply supported in the out-of-plane direction.

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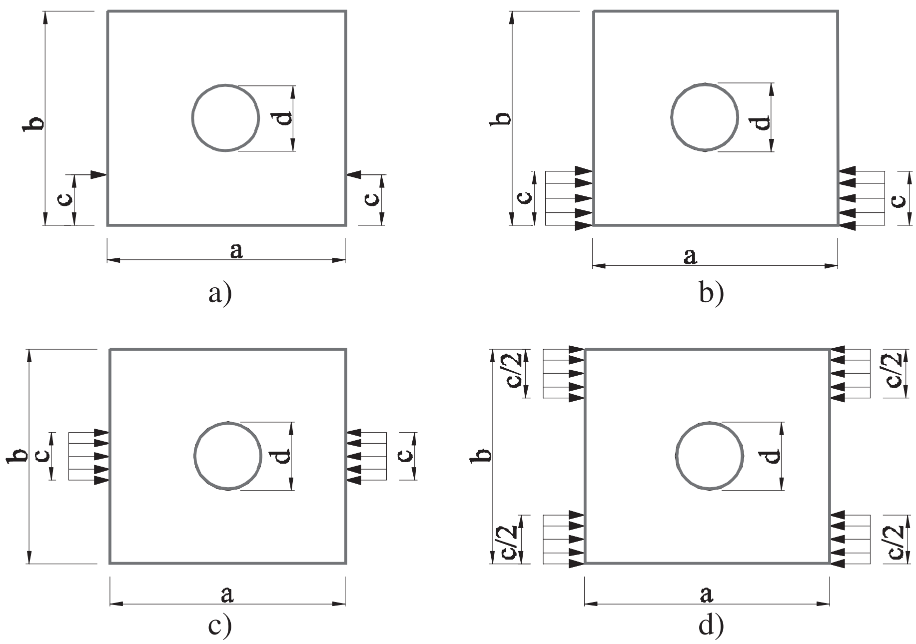

Fig. 1. Partial edge loading patterns of perforated plates: (a) concentrated loading at the opposite edges, (b) partial loading at one end of the opposite edges, (c) partial loading at the center of the opposite edges (d) partial loading at the two ends of the opposite edges.

In order to investigate the effect of partial loading on the perforated plates, they are subjected to localized edge loading of four different types. The external loads are placed on the short edge of plates as shown in Fig. 1. For the concentrated edge loading case in Fig. 1a, the load is moved along the short edge and c denotes the distance between the load and the lower left corner of plate. In the other cases shown in Fig. 1b to d, c denotes the length of the partial load.

This study reports on the investigation of the elastic buckling behavior of perforated plates subjected to partial edge loading. Two different aspect ratios $a/b = 1$ and $a/b = 2$, different normalized partial loading ($c/d = 0.0, 0.05, 0.1, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50$) and a different normalized hole size ($d/b = 0, 0.1, 0.2, 0.3, 0.4, 0.5$) were selected.

3. The finite element model and buckling analysis

The maximum load carrying capacity of a structure can be obtained by performing either nonlinear analysis or buckling analysis. In nonlinear analysis, the externally applied load is divided into smaller load steps, then these smaller loads are applied at each increment and an equilibrium state is found through iteration. Hence, the maximum load carrying capacity or instability point(s) of the structure is determined. The second method is the buckling analysis in which not only is the critical loads obtained but also the corresponding deformation shapes of the modeled structure. The linear buckling analysis (or eigenvalue analysis) consists of two stages. First, a unit load or stress which has same load pattern as a given external load is applied to determine the internal stresses in structure due to externally applied loads. In the second stage, based on the initial stress, the geometric stiffness matrix is obtained in order to perform the eigenvalue analysis. In the linear buckling analysis, it is assumed that the deformation is small and the material obeys the Hook's Law. When one of these assumptions is violated, the nonlinear buckling analysis must be performed. In the present study, the material behavior was assumed to be linear elastic and the deformations compared with the overall dimensions of plate were assumed to be small. Based on these assumptions, a linear buckling analysis was carried out to investigate the buckling behavior of the perforated plates.

3.1. Finite element modeling

All the necessary computations of the plate buckling were performed using ANSYS; a commercial finite element analysis program [18]. The SHELL63 element used for modeling and analyzing the perforated plates has four nodes and six degrees of freedom at each node since the flexural behavior and membrane action can be taken into consideration using this element. The material of the plates was assumed to be homogeneous, isotropic and elastic. The material properties for Young's modulus $E = 210$ GPa and Poision's ratio $= 0.3$ were selected. The dimensions of the square plate were set to 100×100 and thickness of plates was taken as 1 mm.

The typical meshed plates and boundary conditions are given in Fig. 2. The maximum size of the shell elements was selected $b/20$. The shell element size along the hole perimeter was set to the smaller of $b/50$ or $d/40$ [5, 9, 17]. All the edges of the plates were modeled as simply supporting the out-of-plane direction. Three roller supports along the edge at $y = -b/2$ were used in order to prevent the plate from exhibiting a rigid body motion.

After concentrated or partial edge loads were applied to the plates as shown in Fig. 1, the linear buckling analysis was carried out to investigate the buckling behavior of the perforated plates. The lowest critical buckling load parameters for distributed and concentrated in-plane loading were calculated according to the following dimensionless form:

$$N^*_{cr} = \frac{N_{cr} b^2}{D} \text{ for distributed loading}$$

$$N^*_{cr} = \frac{P_{cr} b}{D} \text{ for concentrated loading} \tag{1}$$

where N_{cr} is the intensity of the buckling load per unit length and P_{cr} is the concentrated buckling load. D is the flexural rigidity of the plate defined by:

$$D = \frac{Et^3}{12(1-\upsilon^2)}. \tag{2}$$

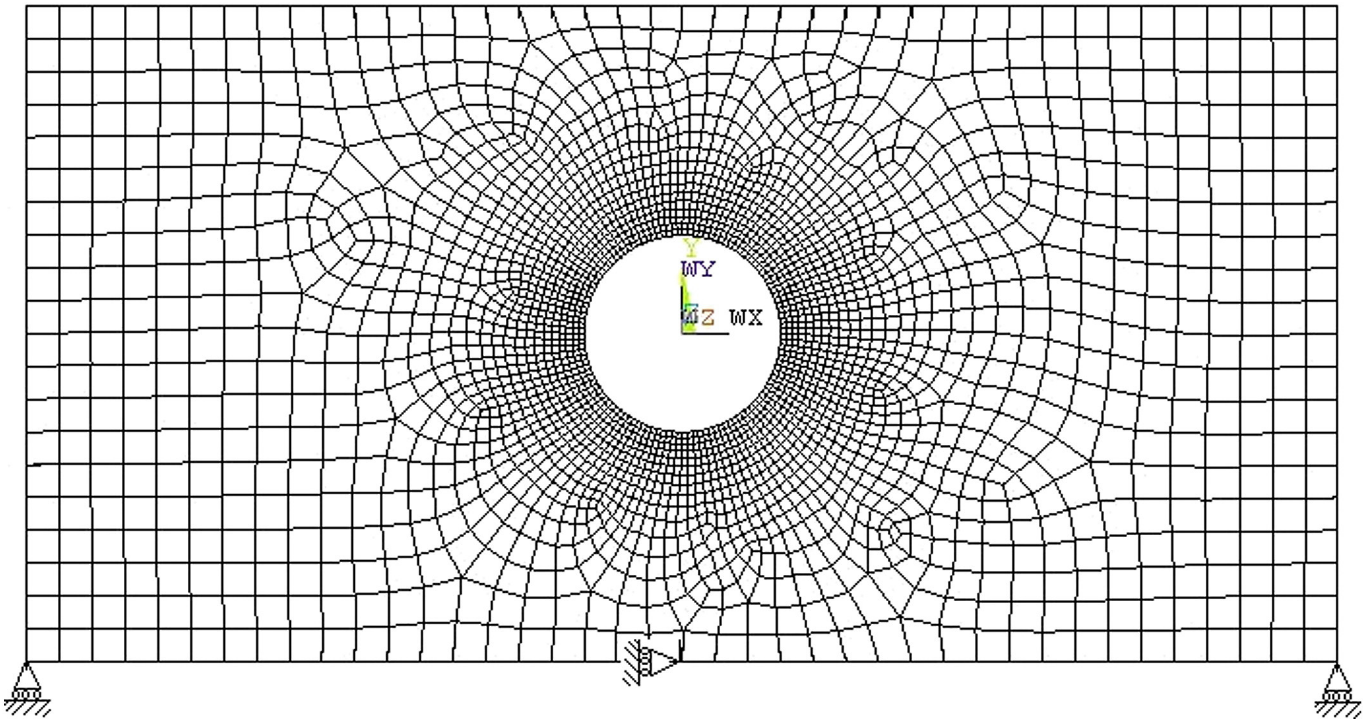

Fig. 2. A typical meshed plate and boundary condition.

3.2. Validation of finite element modeling

In the finite element analysis, complex physical systems having infinite degrees of freedom are modeled as a set of a piecewise continuous functions. Each continuous function is valid only in a finite element. By connecting all of these finite elements are assumed to approximate the behavior of continuous system. During the modeling, it is possible that two different kinds of errors emerge, namely; i) discretization error due to the size of the finite elements and ii) a modeling error due to the element type selection. If improperly sized and inappropriate finite element types are chosen, discretization and modeling errors may arise. In order to eradicate these types of errors, a validation analysis must be performed to determine that the mesh size and type of finite element are acceptable. In order to demonstrate that the selected mesh size and finite element were valid for this study, a series of analyses were performed then results obtained from this work were compared with the results of other research in the literature.

In order to show that the selected mesh sizes was adequate for the elastic buckling analysis, a mesh convergence study was performed for the plate with/without a hole under uniformly distributed load at the opposite edges and a concentrated load at the center of opposite edges. The elastic buckling load of square plates subjected to uniformly distributed loading without holes was calculated analytically $N_{cr}^* = 4 \times \pi^2 = 39.478$ by Timeshenko and Gere [19]. When same plate was subjected to a concentrated load at the centre of opposite edges, the value of N_{cr}^* was calculated 25.700 by Jana and Bhaskar [20], 25.5097 by Brown [21] and 25.814 by Lesissa and Ayoub [22]. Jana and Bhaskar [20] used a superposition of Airy's stress functions and approximately using extended Kantorovich method to obtain the elasticity buckling of a square plate subjected to concentrated loading cases. Leissa and Ayoub [22] determined the elastic buckling loads using the classical Ritz Method. On the other hand, Brown [21] used the conjugate load/displacement method of elastic stability analysis. A measure of convergence as a function of the maximum mesh size and convergence error with comparing to Timoshenko's value and Jana and Bhaskar's value were given in Table 1. When a maximum mesh size of $b/20$ for distributed loading case was selected, the percentage error in critical buckling load parameter was found to be less than 0.15 % of the analytical solution. And the convergence error based on the result of Jana and Braskar for concentrated loading case is about 0.3 %. Therefore, the maximum mesh size was set to $b/20$ along the shorter edges of plates in this study.

In order to assess the effects of mesh size along the hole perimeter on the elastic buckling load, a series of convergence studies were performed on the perforated plates under uniformly distributed load at the opposite edges and a concentrated load at the center of opposite edges for different hole size ($d/b = 0.1$, 0.3 and 0.5). The plates were analyzed with the proposed element using mesh size of $b/20$ along the edges in all cases and the value of buckling load parameters obtained for both loading cases were presented in Table 2. From the table, it can be noticed that the buckling load parameters decreased with decreasing the mesh size along the hole perimeter for all hole

Table 1
Examining convergence error for a solid plate.

Max size	Concentrated loading case		Uniformly distributed loading case	
	FEM	Error (%)	FEM	Error (%)
$b/6$	25.381	1.24	38.887	1.496
$b/10$	25.508	0.75	39.252	0.572
$b/16$	25.597	0.40	39.388	0.228
$b/20$	**25.621**	**0.31**	**39.420**	**0.147**
$b/30$	25.648	0.20	39.452	0.065
$b/40$	25.657	0.17	39.464	0.036
$b/50$	25.662	0.15	39.469	0.023
$b/60$	25.664	0.14	39.472	0.015
$b/70$	25.666	0.13	39.474	0.011
Reference	25.7000 [20]	0.00	39.4780 [19]	0.000

Table 2
Examining convergence error for perforated plate subjected to concentrated loading and uniformly distributed loading cases.

Mesh size along hole perimeter	Concentrated loading case			Uniformly distributed loading case		
	d/*b*			*d*/*b*		
	0.1	0.3	0.5	0.1	0.3	0.5
π ∗ d/20	24.859	–	–	37.937	–	–
π ∗ d/30	24.837	20.927	–	37.895	31.914	–
π ∗ d/40	**24.823**	20.913	–	**37.886**	31.915	–
π ∗ d/47(≈b/50)	–	**20.858**	–	–	**31.886**	–
π ∗ d/50	24.810	20.914	17.890	37.879	31.892	28.705
π ∗ d/60	24.827	20.795	17.889	37.875	31.885	28.709
π ∗ d/70	24.831	20.862	17.816	37.874	31.872	28.699
π ∗ d/78(≈b/50)	–	–	**17.825**	–	–	**28.691**
π ∗ d/80	24.852	20.917	17.825	37.869	31.861	28.691
π ∗ d/90	24.796	20.883	17.753	37.860	31.857	28.686
π ∗ d/100	24.837	20.853	17.823	37.864	31.858	28.689
Relative Error (%)	0.059	0.026	0.012	0.057	0.086	0.008

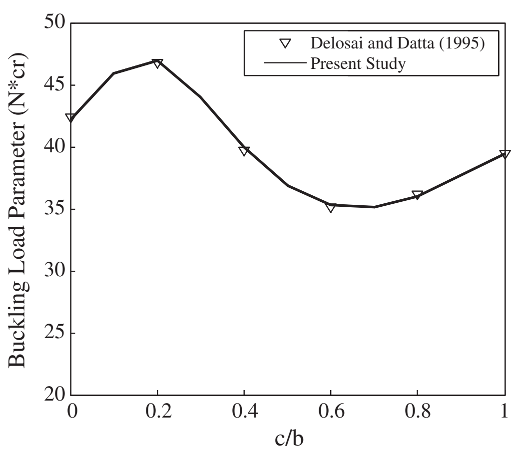

Fig. 3. Comparison of buckling load parameters of solid plates for partial loading at one end of the opposite edges.

sizes. Relative errors between the buckling load parameter obtained when the maximum mesh size along the hole perimeter is $\pi \times d/100$ and that when maximum mesh size is the smallest of $b/50$ or $\pi \times d/40$ (bold fonts in Table 2) were less than 0.1 %. It means that buckling load parameters did not differ from that in the analysis of plates higher mesh numbers. The mesh size along the perimeter was set to the smallest of $b/50$ or $\pi \times d/40$ in the present study. The results obtained by the finite element method using the parameters given above are in agreement with the other results. It shows that the chosen mesh size and element type adequately represent real cases for concentrated load case.

In order to compare the results for the partial edge loading cases; partial loading at one end of the opposite edges was selected. The comparison of the result of the current study with the result of Delosia and Datta [23] who used the finite element method gave a good agreement as shown in Fig. 3.

4. Results and discussion

Results were obtained concerning the effects of a circular hole size on the behavior of square and rectangular plates which are subjected to different edge loading. This was carried out for two different aspect ratios ($a/b = 1$ and $a/b = 2$) and for four different edge loading patterns.

4.1. Concentrated loading case

The relation between buckling load parameter (N_{cr}^*) and normalized load positions (c/b) for perforated plates subjected to a concentrated loading at the opposite edges is given in Fig. 4. The critical buckling mode shapes for square and rectangular plates having the largest hole ($d/b = 0.5$) and the smallest hole ($d/b = 0.1$) are also given in Fig. 5. The values of c/b were limited to 0.5 due to the symmetry of loading. It can be seen in Fig. 4a that the buckling load parameter of a square plate decreases as the concentrated edge load approaches to the middle of the plates and the size of cut-outs are increased. Fig. 4a also shows that the buckling load parameters for $c/b = 0.1$ are about two times higher than those for $c/b = 0.5$. Therefore, the size of the cut-outs has a lower effect on the variation of elastic buckling load capacity for

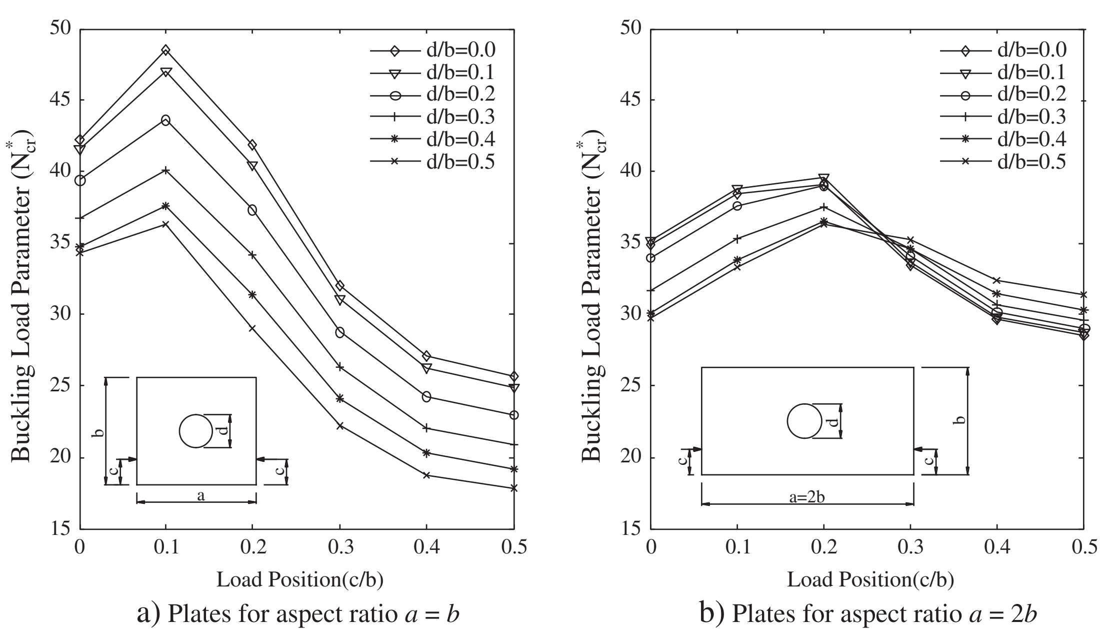

Fig. 4. Buckling load parameter versus load position for perforated plates subject to concentrated loading at the opposite edges.

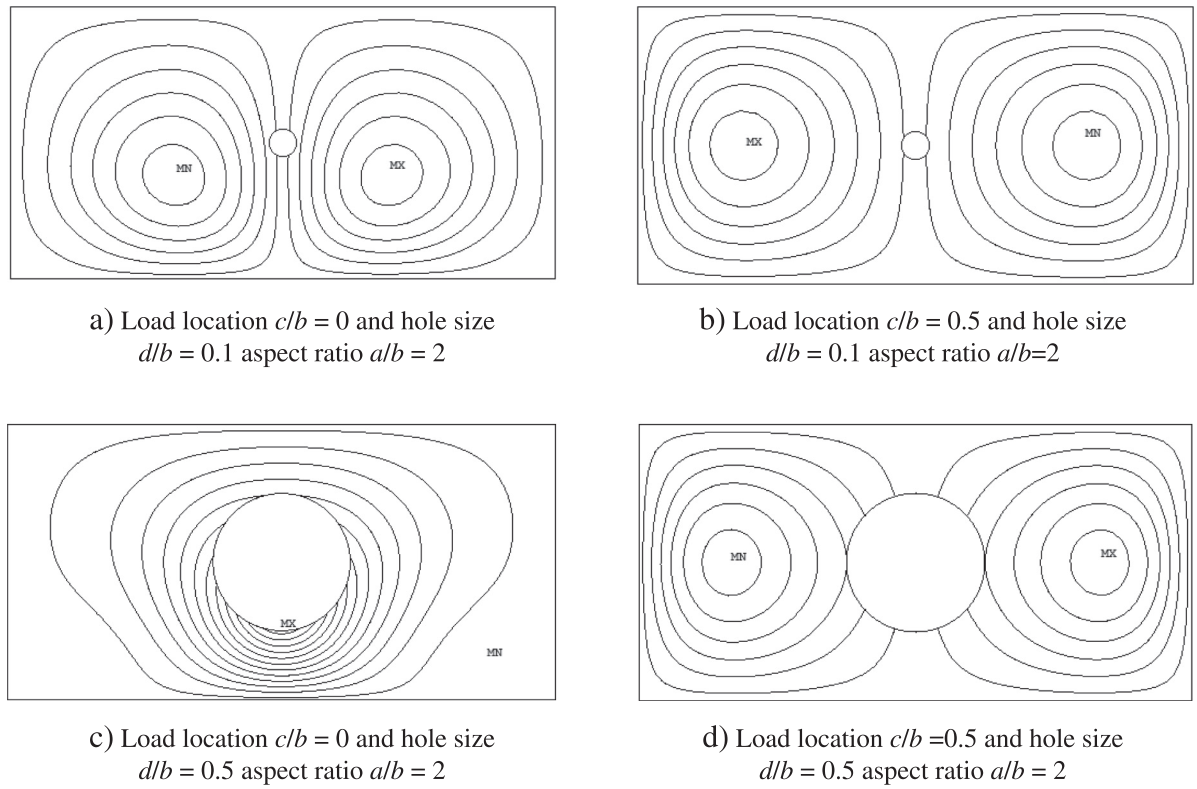

Fig. 5. Critial buckling mode shape of rectangular plates subjected to concentrated loading at the opposite edges.

rectangular plates (Fig. 4b). The variations of the load parameters were less than 30 % and 12 % for the small and large cut-outs, respectively. While the location of concentrated load was changed along the edge of square plate, the buckling load parameter decreased gradually based on the size of hole. The buckling load parameters for the plates with a small hole were always larger than the plates having large cut-outs. In contrast, for the rectangular plates the value of buckling load parameter was not directly related to the size of cut-outs. It is interesting to note that as the load location c/b is bigger than 0.3 this means that as the concentrated load approaches the center of the hole, the buckling

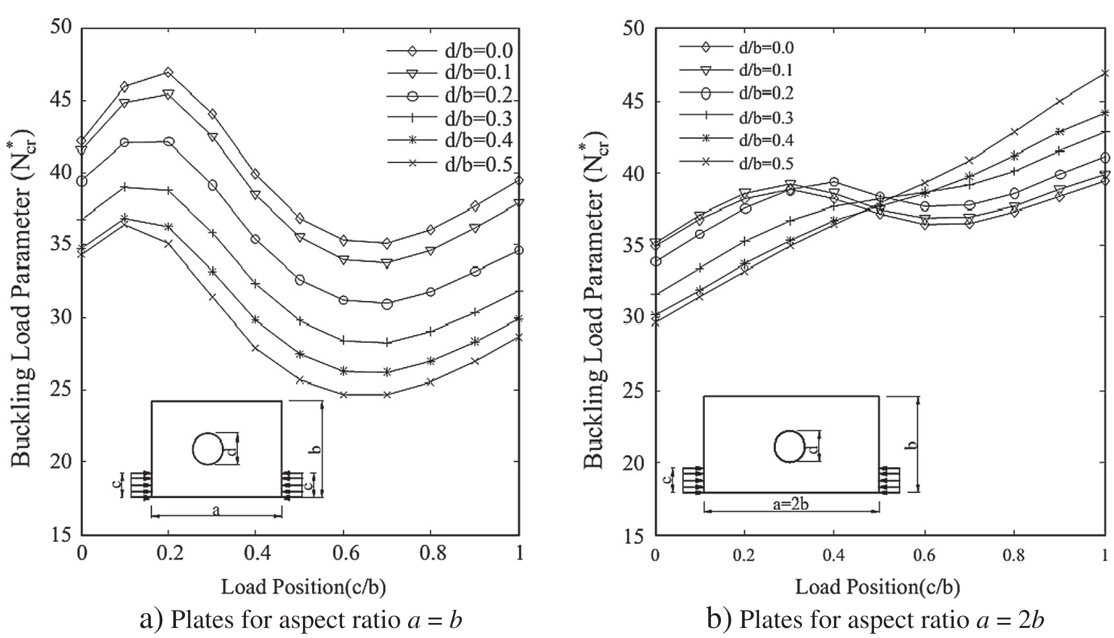

Fig. 6. Buckling load parameter versus load position for plates subject to partial load at one end of opposite edges.

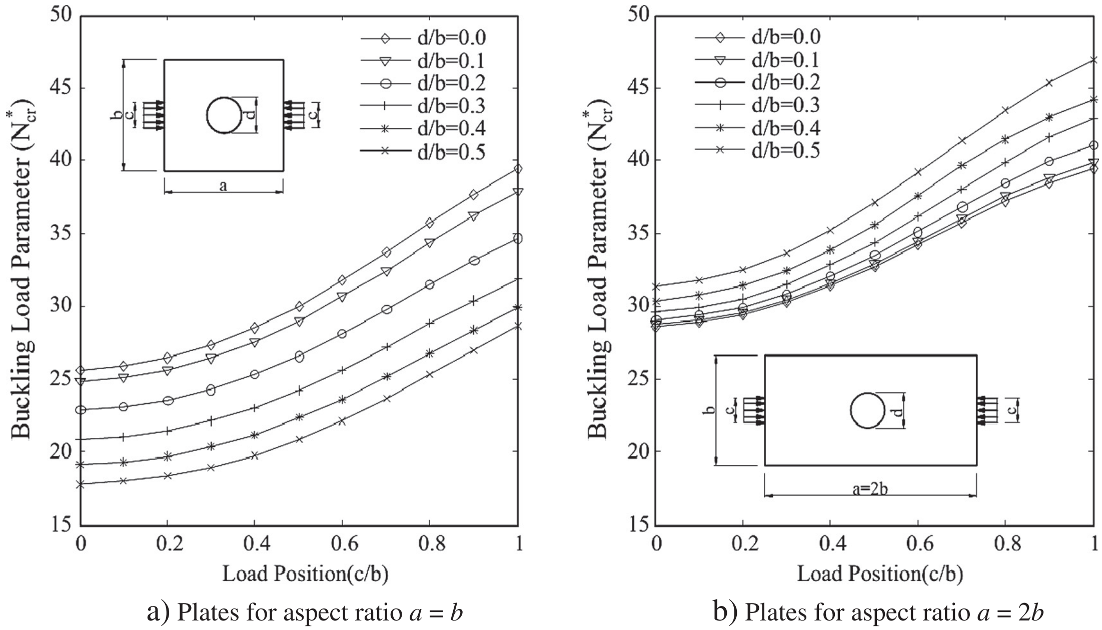

Fig. 7. Buckling load parameter versus load position for plates subject to partial loading at center of opposite edges.

load parameter increase even if the diameter of cut-out diameter becomes larger (see Fig. 4b). This may be due to the critical buckling shape of the rectangular plates.

4.2. Partial loading at one end of opposite edges case

Fig. 6 shows the variation of the buckling load parameters with non-dimensional edge loading at one end. This edge loading is an asymmetric; therefore, the buckling load values given in Fig. 6 are asymmetric. The minimum load parameters were obtained while c/b ratio was between 0.5 and 0.7. For the square plates subjected to edge loading at one end, the diameter of cut-out increased as the buckling load decreases. For the rectangular plate shown in Fig. 6b, the buckling load parameter increased as c/b were increased. This phenomenon is explained by the buckling behavior of plate. When the ratio $c/b = 1$ this means that the plate has been subjected to full compression, the plates show a symmetric buckling shape comprising a full sinus wave. In such a case the buckling parameter takes the maximum value. On the other hand, when the rectangular plate is subjected to an asymmetric edge loading, the plates show an asymmetric buckling shape so the buckling load shape becomes smaller.

4.3. Central partial edge loading case

Fig. 7 represents values of non-dimensional buckling load parameters (N^*_{cr}) for two different aspect ratios ($a/b = 1$ and $a/b = 2$) and for partial edge loading from center of the plates. It is interesting to note that the values of buckling parameter increase while as ratios c/b increase and the diameter ratios of circular cut-outs are decreased. As an example when the diameters of cut-outs were small such as $d/b = 0.1$, the rectangular plates showed a full sinus wave shape; on the other hand, the rectangular plates gave a one and a half sinus wave as the diameter of cut-outs was increased as shown in Fig. 8. If the loading parameters c/b were to be set to 1, the critical buckling mode would become more significant and then the buckling load would increase.

4.4. Partial edge loading at two ends of the opposite edges case

For the perforated square plates subjected to edge loading from both ends, the maximum load parameters were obtained when the load was applied at the corners of square plates. For a given hole size the buckling load decreased as the load ratio (c/b) increased. On the other hand, the effect of location of edge loading load had no or a little effect on the elastic buckling loads for the rectangular plate (Fig. 9).

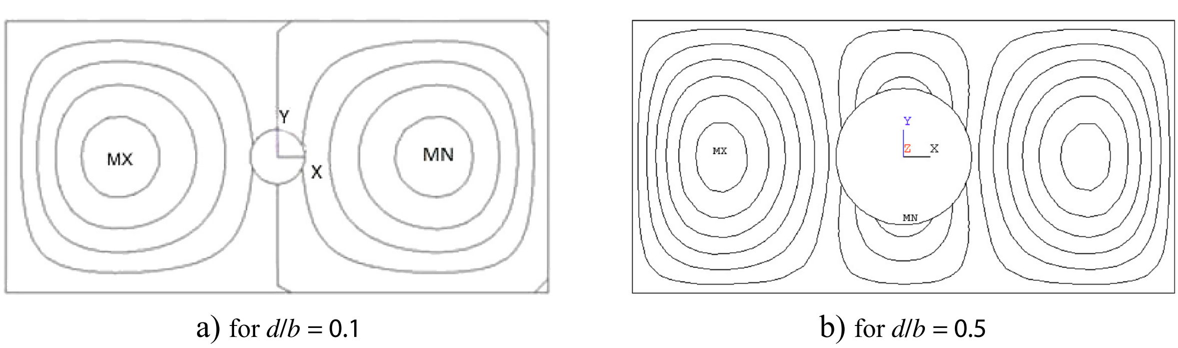

Fig. 8. Critical buckling mode shapes of rectangular plates subjected to edge loading at center for (a) small diameter ($d/b = 0.1$) and (b) large diameter ($d/b = 0.5$) while $c/b = 0.6$.

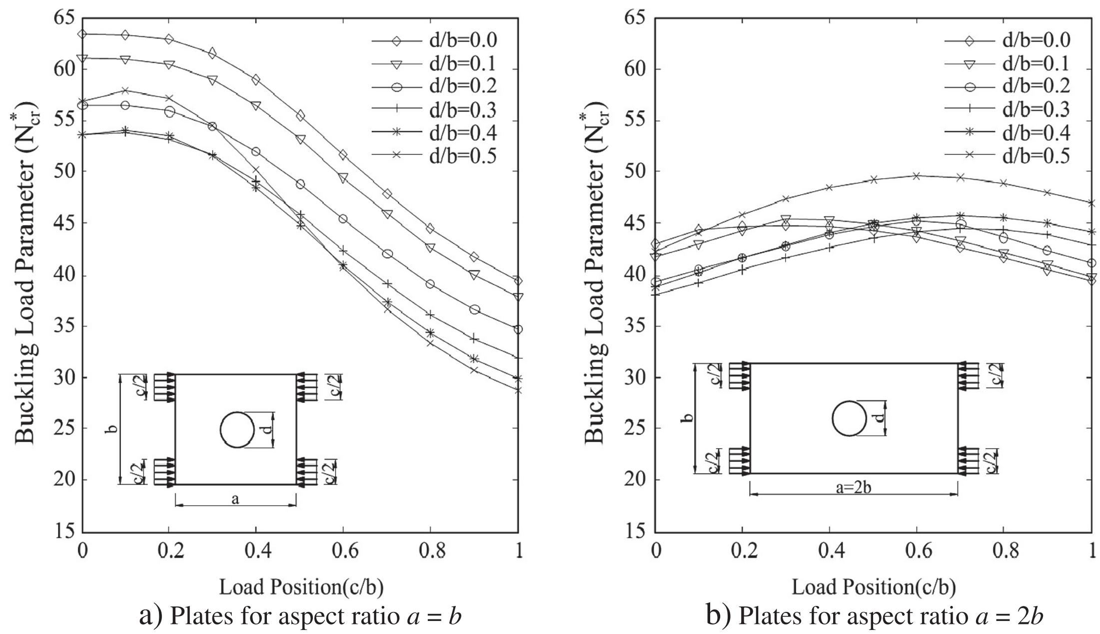

Fig. 9. Buckling load parameter versus load position for plates subject to partial loading at two ends of opposite edges.

5. Conclusions

The buckling behavior of perforated square and rectangular plates subjected to different edge loading was investigated using the finite element method. This was carried out for two different aspect ratios ($a/b = 1$ and $a/b = 2$) and four different edge loading patterns. Based on the findings, the results from this study can be summarized as follows:

- The comparison of buckling analysis of the square plates shows that the buckling load parameters becomes higher when the loading is applied near the ends of the edges. The main reason for this behavior is that the restraining effects of the supported edges which undergo small out of plane deformation. The presence of holes reduces the buckling load parameter in all cases based on their size. Increasing the size of the hole decreases the buckling load parameter for all square plates. While the value of the load position (c/b) approached 1 for partial edge load cases, the value of buckling load parameters became same for all cases. On the other hand, the concentrated loading case, the maximum loading parameter is observed when the load position (c/b) is about 0.2.
- The value of the buckling load parameters for rectangular plates with circular holes increases with the increase of the size of holes for distributed loading cases because the buckling mode shapes change from two half sinus waves to three half sinus waves. On the contrary, the size of the hole causes little change in the buckling load parameter for concentrated loading cases.
- For the rectangular plates the presence of a circular hole of different ratios (d/b) did not decrease the elastic buckling load of plates for all partial and concentrated loading cases.
- For the square plates, the value of buckling load is sensitive to the location of the load. When the applied load was near to center of edge, the buckling values dramatically decreased. Therefore, an extra measure must be taken to avoid the elastic buckling of square plates when partial loading is applied at the center and concentrated loading is applied to the middle of the edge.

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