Key Words: Buckling, buckling factor, shear loading

References

[1] RAS, Data Sheet 02.03.01, 1963

1 General

This HSB sheet provides buckling factors for rectangular plates under shear loading for various support conditions and length/width ratios. The buckling factors are applicable only to plates of isotropic material.

2 List of Symbols

Symbol	Unit	Description
a	mm	length of plate
b	mm	width of plate
k	–	buckling factor
k^*	–	modified buckling factor
t	mm	thickness of plate
E	MPa	Young's modulus
ν	–	Poisson's ratio
τ_{cr}	MPa	critical buckling stress

3 Analyses

The critical buckling stress is given by:

$$\tau_{cr} = k \cdot \frac{\pi^2 \cdot E}{12 \cdot (1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 \tag{3-1}$$

With the modified buckling factor $k^* = k \cdot \frac{\pi^2}{12 \cdot (1-\nu^2)}$ it follows:

$$\tau_{cr} = k^* \cdot E \cdot \left(\frac{t}{b}\right)^2 \tag{3-2}$$

For $\nu = 0.3$ the modified buckling factor is calculated as $k^* = k \cdot \frac{\pi^2}{12 \cdot (1-0.3^2)} = 0.904 \cdot k$. It should be noted, however, that for most of the aluminum alloys used in airframe design $\nu = 0.33$, resulting in a slightly different critical buckling stress. The curves provided in Fig. 1 are valid for $\nu = 0.3$.

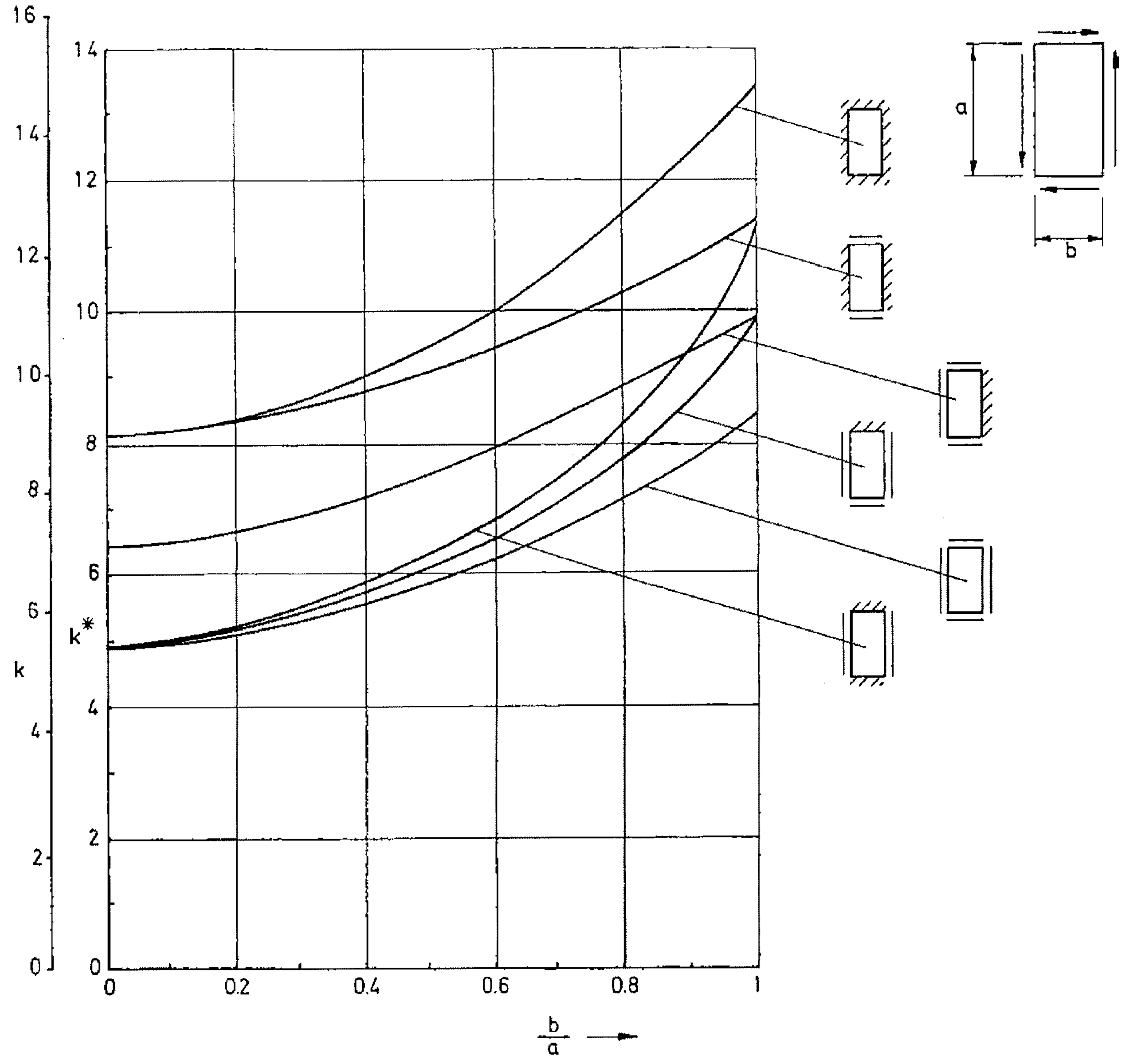


Figure 1: Buckling factor k and modified buckling factor k^* vs aspect ratio b/a

4 Application

The data provided for the critical buckling stress are valid for linear elastic conditions. The Young's modulus E is to be reduced if the plastic regime is entered.

Key Words: Buckling, buckling factor, orthotropic plate, shear loading

References

[1] Davenport, O.B. and Bert, C.W.: Buckling of Orthotropic, Curved, Sandwich Panels Subjected of Edge Shear Loads, J. of Aircraft VOL. 9, No. 7, July 1971

[2] Seydel, E.: Ausbeul-Schublast rechteckiger Platten, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 24. Jahrg. 1933 Nr. 3

1 General

This HSB sheet provides buckling factors for rectangular orthotropic laminated plates under shear loading. The plate is simply supported at all four edges.

2 List of Symbols

Symbol	Unit	Description
a	mm	length of plate
b	mm	width of plate
k	–	buckling factor
q_{cr}	N/mm	critical buckling section shear force
t	mm	thickness of plate
$x,\ y$	mm	coordinates
D_{ij}	$\mathrm{N \cdot mm}$	bending stiffness of laminate
$E_x,\ E_y$	MPa	Young's modulus (in x- and y-direction)
G_{xy}	MPa	shear modulus (in the xy-plane)
$\bar{\alpha}$	–	effective aspect ratio
β	–	Seydel's orthotropy parameter
$\nu_{xy},\ \nu_{yx}$	–	Poisson's ratios of the laminate

Subscripts	
cr	critical buckling
x, y	coordinate directions
i, j	stiffness matrix subscripts
$1, 2, 3$	orthotropic axes

3 Analyses

3.1 Critical buckling section shear force

The critical buckling section shear force is calculated as

$$q_{cr} = k \cdot \left(\frac{\pi}{b}\right)^2 \cdot \sqrt[4]{D_{11} \cdot D_{22}^3} \tag{3-1}$$

In Fig. 1, the buckling factor k is given as a function of the effective aspect ratio

$$\bar{\alpha} = \frac{b}{a} \cdot \sqrt[4]{\frac{D_{11}}{D_{22}}} \tag{3-2}$$

and Seydel's orthotropy parameter

$$\beta = \frac{D_{12} + 2 \cdot D_{33}}{\sqrt{D_{11} \cdot D_{22}}} \tag{3-3}$$

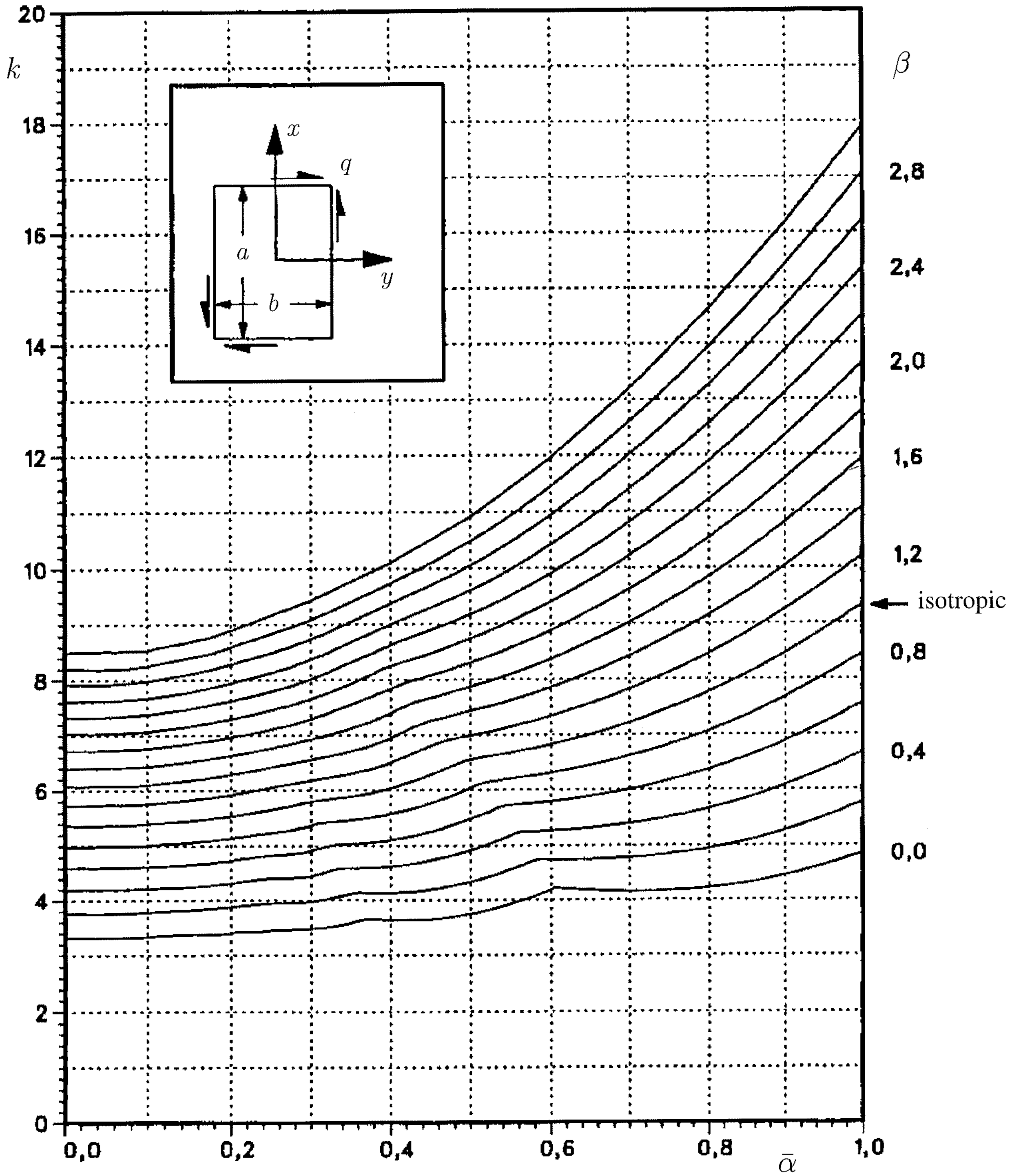


Figure 1: Buckling factor k vs effective aspect ratio $\bar{\alpha}$

3.2 Procedure

For fibre composite laminates, the plate stiffness values D depend on the percentage share of the different fibre directions as well as on their distribution over the plate thickness.

During the conceptual design of structural items, the lay-up of the laminate is normally not yet exactly decided on, with the implication that the stiffness values are not exactly known, either. But to allow for a prediction of the critical buckling section shear forces, it may be assumed that the fibre layers are equally distributed over the thickness, leading to a homogeneous model of the plate. Based on this simplification, the stiffness values can be directly obtained from the elastic constants of the laminate and the plate thickness:

$$\begin{aligned} D_{11} &= \frac{E_x \cdot t^3}{12 \cdot (1 - \nu_{xy} \cdot \nu_{yx})} \\ D_{22} &= \frac{E_y \cdot t^3}{12 \cdot (1 - \nu_{xy} \cdot \nu_{yx})} \\ D_{12} &= \nu_{xy} \cdot D_{22} = \nu_{yx} \cdot D_{11} \\ D_{33} &= \frac{G_{xy} \cdot t^3}{12} \end{aligned} \tag{3-4}$$

The critical buckling section shear force predicted with this simplification provides a typical value. The exact critical buckling section shear force is higher if the layers in the direction of the short edges generally lie nearer to the outer surfaces of the plate than those in the direction of the long edges. Therefore, in the latter case, it is strongly recommended to use the actual stiffness values for design verification.

4 Application

The following prerequisites are assumed:

- The orthotropic axes are parallel to the edges of the rectangular plate.
- The laminate is symmetric to its center plane.
- The predicted critical buckling section shear force lies within the regime where the stresses still remain linear elastic.

Key Words: Buckling, buckling factor, corrugated web, shear loading

References

[1] Seydel, E.: Beitrag zur Frage des Ausbeulens von versteiften Platten bei Schubbeanspruchung. DVL-Jahrbuch 1930, S. 235-254

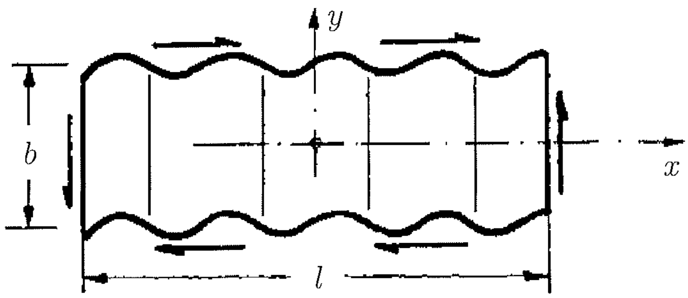
[2] Schapitz, E.: Festigkeitslehre für den Leichtbau. VDI-Verlag, 1963

[3] Hertel, H.: Leichtbau. Springer Verlag, 1960

1 General

This HSB sheet provides the critical buckling shear stress of a corrugated web. The corrugated web is treated as a whole item. It can be modelled as an geometrically orthotropic plate with two different bending stiffness values in directions perpendicular to each other (see [3]). All four edges are simply supported. The external section shear force acts in the center plane. If the sheet is very thin, local buckling may occur, which needs to be checked additionally.

The configuration is shown in the following sketch:



2 List of Symbols

Symbol	Unit	Description
a	mm	characteristic length in a trapezium-shaped profile
b	mm	width of the corrugated web
c_a	–	buckling coefficient
k^*	–	buckling factor
l	mm	length of the corrugated web
s	mm	overall height of the corrugated web
t	mm	thickness of the sheet
$x,\ y$	mm	coordinates
E	MPa	Young's modulus
$D_x,\ D_y$	N · mm	average stiffness of the orthotropic plate in the x- and y-direction
D_{xy}	N · mm	average twisting stiffness of the orthotropic plate
$\bar{\alpha}$	–	effective aspect ratio
β	–	orthotropy parameter
ν	–	Poisson's ratio
τ_{cr}	MPa	critical buckling shear stress

Subscript	
$_{cr}$	critical buckling
$_{x,y}$	coordinate directions

3 Analyses

The critical buckling shear stress of an orthotropic plate in the linear elastic regime is given by (see [1, 2]):

$$\tau_{cr} = c_a \cdot \frac{4}{b^2 \cdot t} \cdot \sqrt[4]{D_x \cdot D_y^3} \tag{3-1}$$

As stiffness input for the corrugated web one can use:

- for the bending stiffness of the sheet

$$D_x = \frac{E \cdot t^3}{12 \cdot (1 - \nu^2)} \tag{3-2}$$

- for the special case of a 60 ° sinusoidal profile shape

$$D_y \approx \frac{E \cdot s^2 \cdot t}{6} \tag{3-3}$$

(for other profile shapes, D_y is to be determined individually)

- for the twisting stiffness, it is

$$D_{xy} \approx D_x \tag{3-4}$$

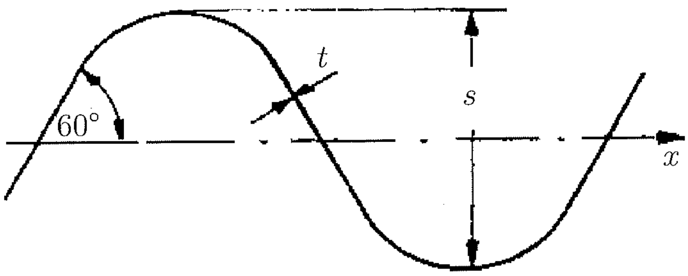
The buckling coefficient c_a is a function of the effective aspect ratio

$$\bar{\alpha} = \frac{b}{l} \cdot \sqrt[4]{\frac{D_x}{D_y}} \tag{3-5}$$

and the orthotropy parameter

$$\beta = \frac{D_{xy}}{\sqrt{D_x \cdot D_y}} \tag{3-6}$$

For a long sinusoidally shaped corrugated web according to the following sketch

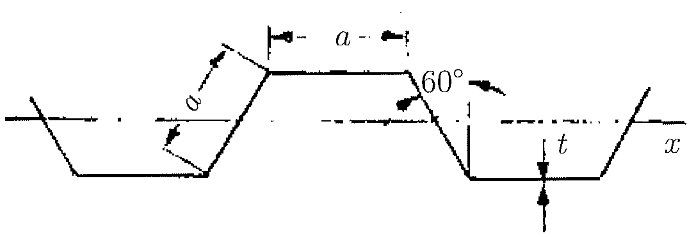


it is $b \ll l$ and $t \ll s$, resulting in $D_x \ll D_y$ and $D_{xy} \ll D_y$, which leads to $\bar{\alpha} \approx 0$ and $\beta \approx 0$.

For an orthotropic plate with $\bar{\alpha} = 0$ and $\beta = 0$, the buckling coefficient is $c_a = 8.125$ (see [1]) and Eq. (3-1) can be approximately transformed (for the special value $\nu = 0.3$) into

$$\tau_{cr} \approx 4.7 \cdot E \cdot \left(\frac{s}{b}\right)^2 \cdot \sqrt{\frac{t}{s}} \tag{3-7}$$

For a trapezium-shaped profile according to the following sketch



one obtains

$$\tau_{cr} \approx 4.48 \cdot E \cdot \left(\frac{a}{b}\right)^2 \cdot \sqrt{\frac{t}{a}} \tag{3-8}$$

In analogy to the buckling formula for isotropic plates, the critical buckling shear stress given in Eq. (3-7) can be written as

$$\tau_{cr} = k^* \cdot E \cdot \left(\frac{s}{b}\right)^2 \tag{3-9}$$

with the buckling factor $k^* = 4.7 \cdot \sqrt{t/s}$; the corresponding diagram of k^* is shown in Fig. 1.

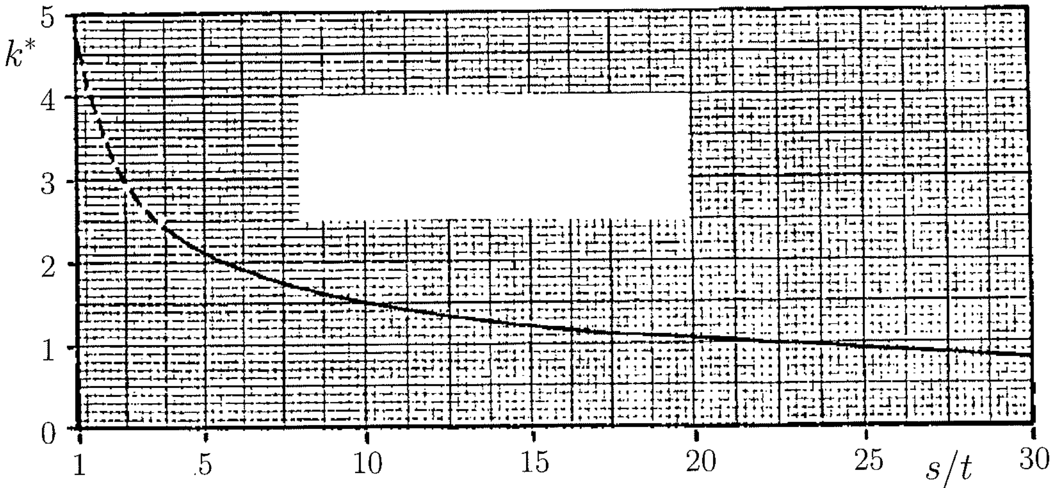


Figure 1: Buckling factor k^* vs s/t for sinusoidal, 60°-shaped profile

The following diagrams (Figs. 2, 3) show the critical buckling shear stresses for corrugated webs with sinusoidal sheet profile for aluminum alloys with $E = 70\,600$ MPa and titanium alloys with $E = 108\,000$ MPa, respectively.

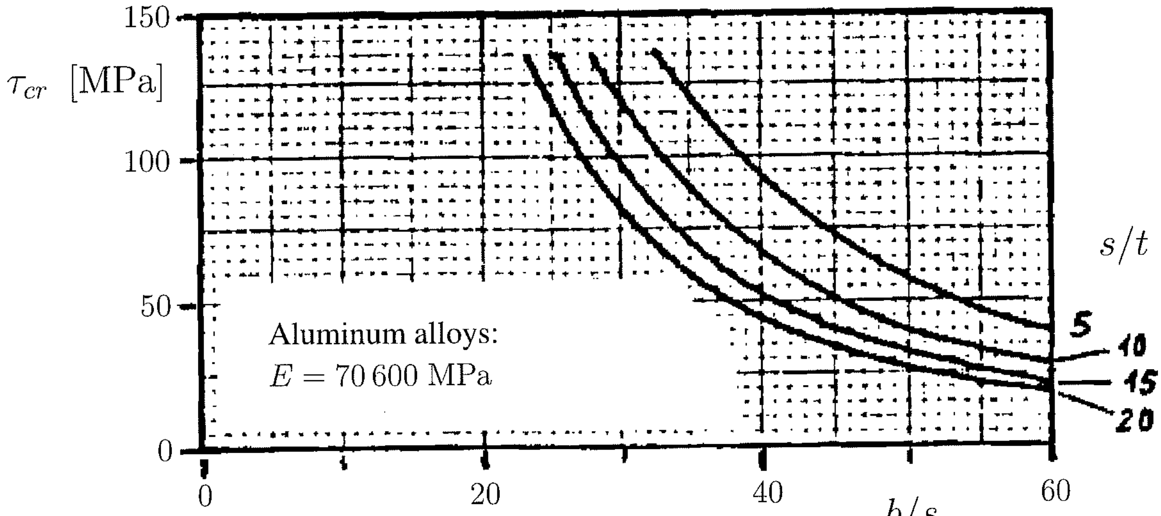


Figure 2: Critical buckling shear stress for aluminum alloys

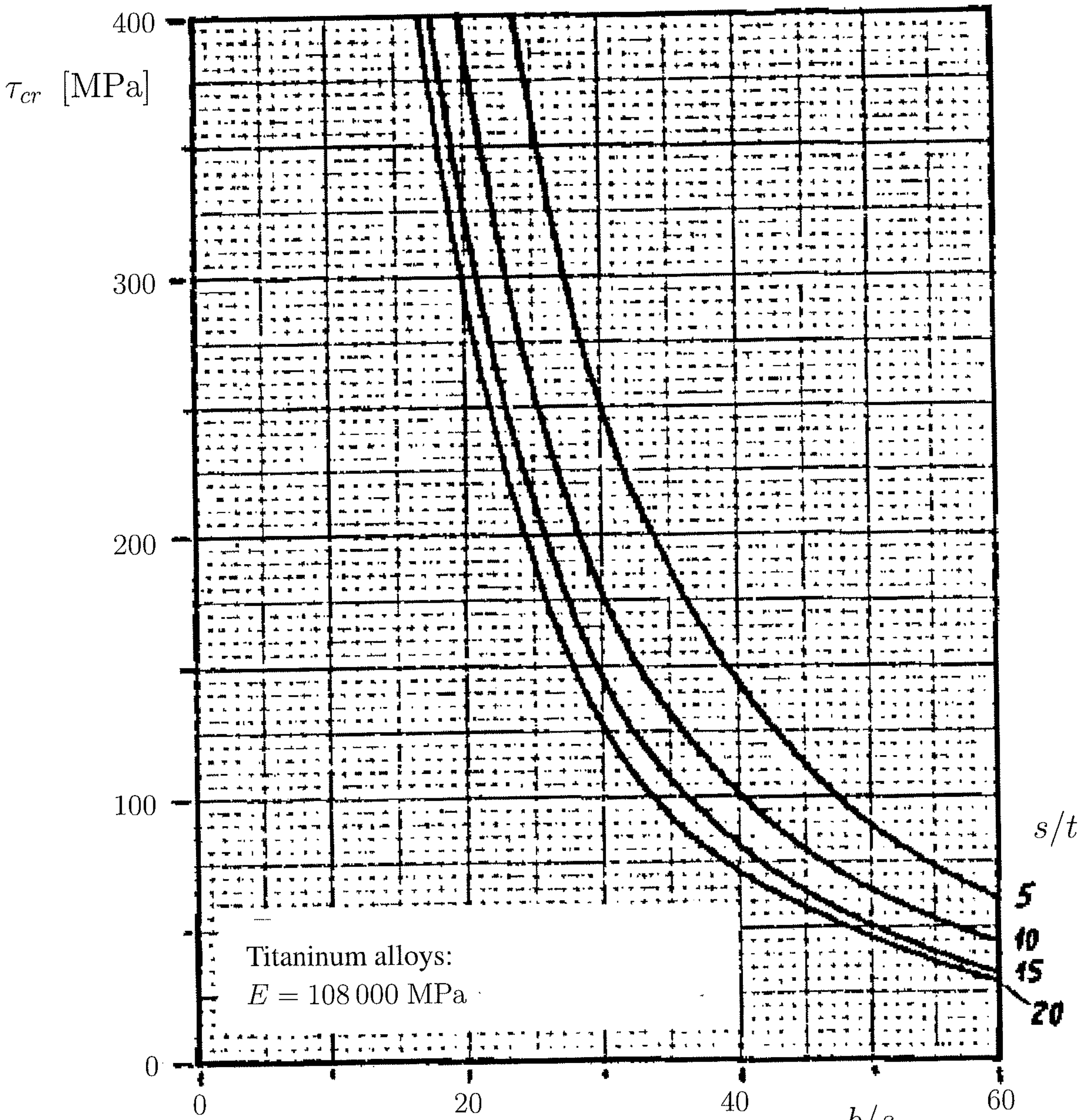


Figure 3: Critical buckling shear stress for titanium alloys

4 Application

The data provided for the critical buckling shear stress are valid for linear elastic conditions. The Young's modulus E is to be reduced if the plastic regime is entered.