CA PREPARATION

LIMIT FUNGSI TRIGONOMETRI

Tentukan nilai limit fungsi trigonometri berikut

1. $\lim\limits_{x\to 0} \dfrac{3x}{\tan 5x}$

2. $\lim\limits_{x\to 0} \dfrac{sin^3\, 3x}{6x^3}$

3. $\lim\limits_{t\to 0} \dfrac{6t^2+5t}{\sin 2t}$

4. $\lim\limits_{t\to 0} \dfrac{\cos 2t\, .\tan 3t+\sin 6t}{8t}$

5. $\lim\limits_{t\to 1} \dfrac{(t^3-4t^2-t+4)\sin(t-1)}{t^3-6t^2+9t-4}$

6. $\lim\limits_{x\to a} \dfrac{2x-2a}{\tan(x-a)+3x-3a}$

7. $\lim\limits_{x\to 2} \dfrac{4-4\cos(x-2)}{x^2-4x+4}$

8. $\lim\limits_{x\to 0} \dfrac{\cos 6x-1}{1-\cos 5x}$

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9. $\lim_{x \to y} \frac{\cos 4x - \cos 4y}{x - y}$

10. Tentukan nilai dari $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ untuk $f(x) = 2\sin 5x$

11. $\lim_{x \to 0} \frac{x \tan 5x}{\cos 2x - \cos 7x}$

12. $\lim_{x \to 0} \frac{sin^2 9x}{4x^2 \tan\left(x + \frac{\pi}{6}\right)}$

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13. $\lim_{x \to \frac{\pi}{2}} \frac{1 - sin^2 x}{\left(\sin\frac{1}{2}x - \cos\frac{1}{2}x\right)^2}$

14. $\lim_{x \to \pi} \frac{1 + \cos x}{(x - \pi)^2}$

15. $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\cos x - sin\ x}$

1.) $\lim_{x \to 0} \frac{3x}{\tan 5x} = \frac{3}{5}$

2.) $\lim_{x \to 0} \frac{\sin^3 3x}{6x^3} = \lim_{x \to 0} \frac{\sin 3x \cdot \sin 3x \cdot \sin 3x}{6x \cdot x \cdot x} = \frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2}$

3.) $\lim_{t \to 0} \frac{6t^2 + 5t}{\sin 2t} = \lim_{t \to 0} \frac{t(6t+5)}{\sin 2t} = \frac{1}{2} \cdot (6 \cdot 0 + 5) = \frac{5}{2}$

4.) $\lim_{t \to 0} \frac{\cos 2t \cdot \tan 3t + \sin 6t}{8t} = \cos(2 \cdot 0) \cdot \frac{3}{8} + \frac{6}{8} = \frac{9}{8}$

5.) $$\lim_{t \to 1} \frac{(t^3 - 4t^2 - t + 4)\sin(t-1)}{t^3 - 6t^2 + 9t - 4} = \lim_{(t-1)\to 0} \frac{\cancel{(t-4)}(t+1)\cancel{(t-1)}\sin(t-1)}{\cancel{(t-4)}\cancel{(t-1)}(t-1)}$$

$$= (1+1) \cdot 1 = 2$$

$$t=1 \left| \begin{array}{cccc} 1 & -4 & -1 & 4 \\ & 1 & -3 & -4 \\ \hline 1 & -3 & -4 & |0 \end{array} \right. +$$

$(t-4)(t+1)$

$$t=1 \left| \begin{array}{cccc} 1 & -6 & 9 & -4 \\ & 1 & -5 & 4 \\ \hline 1 & -5 & 4 & |0 \end{array} \right. +$$

$(t-4)(t-1)$

6.) $$\lim_{x \to a} \frac{2x - 2a}{\tan(x-a) + 3x - 3a} \cdot \frac{\frac{1}{x-a}}{\frac{1}{x-a}} = \lim_{(x-a)\to 0} \frac{\frac{2(x-a)}{(x-a)}}{\frac{\tan(x-a)}{(x-a)} + \frac{3(x-a)}{(x-a)}}$$

$$= \frac{2}{1+3} = \frac{1}{2}$$

7.) $\lim\limits_{x\to 2} \dfrac{4-4\cos(x-2)}{x^2-4x+4} = \lim\limits_{(x-2)\to 0} \dfrac{4(1-\cos(x-2))}{(x-2)^2}$

$= \lim\limits_{(x-2)\to 0} \dfrac{4(1-(1-2\sin^2\frac{1}{2}(x-2)))}{(x-2)^2}$

$= \lim\limits_{(x-2)\to 0} \dfrac{4 \cdot 2 \sin\frac{1}{2}(x-2) \sin\frac{1}{2}(x-2)}{(x-2)\quad(x-2)}$

$= 4 \cdot 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = 2$

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8.) $\lim_{x \to 0} \frac{\cos 6x - 1}{1 - \cos 5x} = \lim_{x \to 0} \frac{1 - 2\sin^2 3x - 1}{1 - (1 - 2\sin^2 \frac{5}{2}x)}$

$= \lim_{x \to 0} \frac{-\cancel{2} \sin 3x \cdot \sin 3x}{\cancel{2} \sin \frac{5}{2}x \cdot \sin \frac{5}{2}x} = -1 \cdot \frac{3}{5/2} \cdot \frac{3}{5/2} = -\frac{36}{25}$

9.) $\lim_{x \to y} \frac{\cos 4x - \cos 4y}{x - y} = \lim_{(x-y) \to 0} \frac{-2 \sin 2(x+y) \sin 2(x-y)}{(x-y)}$

$= -2 \cdot 2 \sin(y+y) \cdot 2 = -8 \sin 2y$

10.) $f(x) = 2\sin 5x$

$f(x+h) = 2\sin 5(x+h) = 2\sin(5x+5h)$

$$= \lim_{h\to 0} \frac{2\sin(5x+5h) - 2\sin 5x}{h}$$

$$= \lim_{h\to 0} \frac{2\left(2\cos\left(5x+\frac{5}{2}h\right)\sin\left(\frac{5}{2}h\right)\right)}{h}$$

$$= 2 \cdot 2\cos\left(5x+\frac{5}{2}\cdot 0\right)\cdot\frac{5}{2} = 10\cos 5x$$

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11.) $\lim_{x \to 0} \dfrac{x \tan 5x}{\cos 2x - \cos 7x} = \lim_{x \to 0} \dfrac{x . \tan 5x}{-2 \sin \frac{9}{2}x \sin\left(-\frac{5}{2}x\right)} = -\dfrac{1}{2} \cdot \dfrac{1}{9/2} \cdot \dfrac{5}{-5/2}$

$= -\dfrac{1}{2} \cdot \dfrac{2}{9} \cdot 5 \cdot \left(-\dfrac{2}{5}\right)$

$= \dfrac{2}{9}$

12.) $\lim_{x \to 0} \dfrac{\sin^2 9x}{4x^2 . \tan\left(x + \frac{\pi}{6}\right)} = \lim_{x \to 0} \dfrac{\sin 9x . \sin 9x}{4x \cdot x .} \cdot \dfrac{1}{\tan\left(x + \frac{\pi}{6}\right)}$

$= \dfrac{9}{4} , 9 \cdot \dfrac{1}{\tan\left(0 + \frac{\pi}{6}\right)} = \dfrac{81}{4} \cdot \dfrac{1}{\sqrt{3}/3}$

$= \dfrac{81}{4}\sqrt{3}$

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13.) $$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin^2 x}{(\sin\frac{1}{2}x-\cos\frac{1}{2}x)^2} = \lim_{x\to\frac{\pi}{2}} \frac{1-\sin^2 x}{\sin^2\frac{1}{2}x - \underbrace{2\sin\frac{1}{2}x\cos\frac{1}{2}x}_{2\sin a\cos a = \sin 2a} + \cos^2\frac{1}{2}x}$$

$\sin^2 a + \cos^2 a = 1$

$$= \lim_{x\to\frac{\pi}{2}} \frac{(1-\cancel{\sin x})(1+\sin x)}{1-\cancel{\sin x}}$$

$$= 1+\sin\frac{\pi}{2} = 1+1 = \boxed{2}$$

14.) $$\lim_{x \to \pi} \frac{1 + \cos x}{(x-\pi)^2} = \lim_{(x-\pi)\to 0} \frac{1 - \cos(\pi - x)}{(x-\pi)^2}$$

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$\cos(\pi - x) = -\cos x$

$-\cos(\pi - x) = \cos x$

$$= \lim_{(x-\pi)\to 0} \frac{1 - \left(1 - 2\sin^2 \frac{1}{2}(\pi - x)\right)}{(x-\pi)^2}$$

$$= \lim_{(x-\pi)\to 0} \frac{2\sin\frac{1}{2}(\pi - x)}{(x-\pi)} \cdot \frac{\sin\frac{1}{2}(\pi - x)}{(x-\pi)}$$

$$= 2 \cdot -\frac{1}{2} \cdot -\frac{1}{2} = \frac{1}{2}$$

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15. $$\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\cos x - \sin x} = \lim_{x \to \frac{\pi}{4}} \frac{1 - \frac{\sin x}{\cos x}}{\cos x - \sin x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\frac{\cancel{\cos x - \sin x}}{\cos x}}{\cancel{\cos x - \sin x}}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{1}{\cos x} = \frac{1}{\cos \frac{\pi}{4}}$$

$$= \sqrt{2}$$