Probability Theory Graded Assignment Week 2
Problem 1

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First let's find the PMF of the random variable X

Case	X	$P(X)$
Head and win	$100 - 10 = 90$	$0.5 * 0.1 = 0.05$
Head and lose	-10	$0.5 * 0.9 = 0.45$
Tail and win \$50	$50 - 20 = 30$	$0.5 * 0.1 = 0.05$
Tail and win \$500	$500 - 20 = 480$	$0.5 * 0.01 = 0.005$
Tail and lose	-20	$0.5 * 0.89 = 0.445$

$$E(X) = 0.5 \times [0.1 \times 90 + 0.9 \times (-10)] + 0.5 \times [0.1 \times 30 + 0.01 \times 480 + 0.89 \times (-20)] \quad (1)$$
$$= 0.5 \times [9 - 9] + 0.5 \times [3 + 4.8 - 17.8] \quad (2)$$
$$= -5 \quad (3)$$

We can see that $E(X)$ is the mean of the net payout of each lottery because the probability for each lottery is 0.5. And the net payout of each lottery does not change this fact.

Let X_1 denote the payout for the first lottery and X_2 for the second.

Proof:

$$E(X) = 0.5 \times E(X_1) + 0.5E(X_2) \quad (4)$$
$$= \frac{E(X_1) + E(X_2)}{2} \quad (5)$$

By definition, this is the mean.