

# Statistics Basics Week 5 SGA

Yumen Cao

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Proof: Standard decision-making procedure of t-test is equivalent to the following: reject null hypothesis if and only if  $\mu_0$  does not belong to  $I$ . If  $\mu_0$  does not lie in  $I$ , either  $\mu_0$  is larger than the right endpoint of  $I$  or  $\mu_0$  is smaller than the left endpoint of  $I$ . Let's assume one sample null hypothesis  $\mathbb{E}X = \mu_0$  holds. Without loss of generality, we prove the latter which is

$$\mu_0 < \bar{x} - 1.96 \cdot \frac{SD(x)}{\sqrt{n}}$$

This can be reorganised as

$$\frac{\bar{x} - \mu_0}{SD(x)} \cdot \sqrt{n} > 1.96$$

We picked the magic number 1.96 because it represents a confidence interval,  $I$ , of 95% in standard normal distribution. Hence the probability of  $\mu_0$  smaller than the left endpoint of  $I$  is 2.5%. And  $\mu_0$  does not lie in  $I$  is 5%. The probability that we falsely reject null hypothesis provided that it is true is also 5%. We will reject this assumption only when  $\mu_0$  falls out of the confidence interval  $I$  which is equivalent to the rule used in ordinary two-sided one-sample t-test.