Assign 1. 20230232 변가윤

#1

$3 \to +a$

$6 \to -b$

$2,4,5 \to x$

$1 \to \text{stop}$

한번 던질때의 expect

$$\Rightarrow E_x = \frac{1}{6}\cdot 0 + \frac{1}{6}\cdot 0 + \frac{1}{6}\cdot a + \frac{1}{6}\cdot 0 + \frac{1}{6}\cdot 0 + \frac{1}{6}\cdot(-b)$$

$$= \frac{a-b}{6}$$

게임이 끝날때 까지의 평균 던지는 횟수 $\Rightarrow \frac{1}{\frac{1}{6}} = 6$ (1이 나올 확률이 $\frac{1}{6}$ 이어서)

- answer $= \frac{a-b}{6} \cdot 6 = a-b$

#2

$$l_p = 6\ln p + \ln(1-p)$$

$\to \frac{d}{dp} l(p) = \frac{6}{p} - \frac{1}{1-p}$ $\to$ $\frac{d}{dp} l(p) = 0$ 인 지점을 구해 보자

$$\frac{6}{p} - \frac{1}{1-p} = 0 \Rightarrow \frac{6}{p} = \frac{1}{1-p} \Rightarrow 6 - 6p = p \Rightarrow p = \frac{6}{7}$$

$\frac{d^2}{dp^2} l(p)$의 부호를 확인 하여 $p = \frac{6}{7}$ 일때가 최댓값인지 확인 하자.

$\frac{d^2}{dp^2} l(p) = -\frac{6}{p^2} - \frac{1}{(1-p)^2}$ $\to$ $p = \frac{6}{7}$일때 $\frac{d^2}{dp^2} l(p)$의 값은 $-\frac{49}{6} - 49 < 0$

따라서 $p = \frac{6}{7}$ 일때 최댓값이다.

- answer : $p = \frac{6}{7}$

#3

$$P(A|B) = P(B|A) \Rightarrow \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{P(A)} \Rightarrow P(A) = P(B)$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{1}{3} = 2P(A) - P(A\cap B)$$

$$\Downarrow$$

이때 $P(A\cap B) > 0$ 이므로 $\Rightarrow 2P(A) > \frac{1}{3} \Rightarrow \underline{P(A) > \frac{1}{6}}$

#4

$$f(w) = \sum_{i=1}^{m}\sum_{j=1}^{n} (a_i^T w - b_j^T w)^2 + \frac{\lambda}{2}\|w\|_2^2$$

$$\frac{\partial}{\partial w}(a_i^T w - b_j^T w)^2 = 2(a_i - b_j)(a_i^T w - b_j^T w)$$

$$\Rightarrow \nabla_w \sum_{i=1}^{m}\sum_{j=1}^{n} (a_i^T w - b_j^T w)^2 = 2\sum_{i=1}^{m}\sum_{j=1}^{n}(a_i^T w - b_j^T w)(a_i - b_j)$$

$$\nabla_w \frac{\lambda}{2}\|w\|_2^2 = \lambda w$$

$$\Downarrow$$

$$\nabla_w f(w) = 2\sum_{i=1}^{m}\sum_{j=1}^{n}(a_i^T w - b_j^T w)(a_i - b_j) + \lambda w$$