Pohl pendulum

Principle

If an oscillating system is allowed to swing freely it is observed that the decrease of successive maximum amplitudes is highly dependent on the damping.
If the oscillating system is stimulated to swing by an external periodic torque, we observe that in the steady state the amplitude is a function of the frequency and the amplitude of the external periodic torque and of the damping.
The characteristic frequencies of the free oscillation as well as the resonance curves of the forced oscillation for different damping values are to be determined.

Set-up

The DC motor need to be powered by constant voltage. First, put the current knob to maximum. Then set the voltage to $20V$. Finally connect the power supply to the motor controller like shown on the figure.

The DC output of the power supply unit is connected to the eddy current brake. FIRST, put all voltage knobs to zero, SECOND put current knob to maximum. Then you can adjust the eddy current brake I_B by rising the voltage with the FINE VOLTAGE KNOB.

ATTENTION DO NOT EXCEED 1A for I_B !

Setup of wiring

Procedure

Free oscillations

1. *Determine the oscillating period and the characteristic frequency of the undamped case.*
   To determine the characteristic frequency ω_0 of the torsion pendulum without damping ($I_B = 0$),

   - deflect the pendulum completely to one side,
   - measure the time for several oscillations (minimum 20 if possible),
   - divide this time by the number of oscillations to get an average of period T of the oscillations

   The measurement is to be repeated several times and the average value $\overline{T}$ of the characteristic period is to be calculated.

2. *Determine the oscillating periods and the corresponding characteristic frequencies for different damping values.*
   For that, proceed the same way you got the caracteristic frequency in the first task. But each time increase the eddy current brake. For that

- GENTLY turn the FINE voltage knob of the coil power supply, (if problem to attain some values, turn it back to zero and increase slightly the coarse voltage knob, then adjust current with fine voltage knob again)
- list of mesured current should be $I_B = 0.25/0.40/0.55/0.9A$

To determine the damping values for the above mentioned cases measure unidirectional maximum amplitudes as follows :

- initially it has to be ensured that the pendulum pointer at rest coincides with the zero-position of the graduations. This can be achieved by turning the eccentric disc of the motor,
- deflect the pendulum completely to one side,
- observe and insert in a spreadsheet the magnitude of successive amplitudes on the other side.
- get the average damping ratio K, damping constant δ and logarithmic decrement Λ .

Forced oscillations

To stimulate the torsion pendulum, the rod connecting the motor and the spring is fixed to the upper third of the groove of the lever arm.

1. *The main goal of thins section is to determine the resonance curves and represent them graphically for different values of the damping coefficient.*
   In order to do that, we will relate the voltage supplied by the controller to the motor to its rotation frequency. Then we will relate the amplitude of the oscillations to the voltage in order to have a better precision of the frequency.

   - open a spreadsheet and make two columns, one for voltage the other for frequency,
   - increase progressively voltage supplied to the motor with controller knob ; start from $0V$ to $20V$ by steps of $2V$,
   - using the stopwatch, measure the frequency of the stimulation by counting 20 periods each time and store the values in the table,
   - realize a graph with the different values and fit the data with an equation of the form $f(V) = aV^2 + bV$ (second order polynomial with intersect zero)

   From now on, you will change the stimulation period using voltage and you will be able to get the corresponding frequency by using $f(V)$.
   The goal is now to plot the amplitude of the oscillations for different frequencie and different damping currents.

   - set the eddy current to one of the following values $I_B = 0.25/0.5/0.9A$
   - open a spreadsheet and establish different columns for V, f(V), amplitude for $I_B = 0.25/0.5/0.9A$ (one column for each current),
   - vary the stimulating voltage of the motor controller by large steps below and after resonance and small steps around resonance,
   - store the values of measured amplitudes when steady regime is established after each increase of voltage (wait at least $2.5/\delta(I_B)$ between increase and measure),
   - plot the amplitude of oscillations in fuction of frequency for different break currents.

2. *Observe the phase shifting between the torsion pendulum and the stimulating lever for a small damping value for different stimulating frequencies*
   In each case, readings should only be taken after a stable pendulum amplitude has been established.

   - chose a small damping value,
   - stimulate the pendulum according to the following frequencies : one far below the resonance, one at resonance, one far after resonance,
   - observe the corresponding phase shifts between the torsion pendulum and the external torque and compare to the expected theorical values.

Theory

Free oscillations case

In case of free and damped torsional vibration torques τ_1 (spiral spring) and τ_2 (eddy current brake) act on the pendulum. We have

$$\tau_1 = -D\phi, \tau_2 = -C\dot{\phi}$$

with ϕ the angle of rotation, $\dot{\phi}$ the angular velocity, D the torque per unit angle and C a constant that relates damping and the current I_B.

The resultant torque

$$\tau_{tot} = I\ddot{\phi} = \tau_1 + \tau_2 = -D\phi - C\dot{\phi}$$

with I the pendulum moment of intertia and $\ddot{\phi}$ the angular acceleration.
This leads us to the following equation

$$I\ddot{\phi} + C\dot{\phi} + D\phi = 0$$

By dividing by I and setting $\delta = C/2I$ and ${\omega_0}^2 = D/I$ we get

$$\ddot{\phi} + 2\delta\dot{\phi} + {\omega_0}^2\phi = 0$$

where δ is the damping constant and $\omega_0 = \sqrt{I/D}$ is the caracteristic frequency of the undamped system.

The temporal solution of this equation is

$$\phi(t) = \phi_0 \mathrm{e}^{-\delta t}\cos(\omega t)$$

with $\omega = \sqrt{{\omega_0}^2 - \delta^2}$.

The damping ratio K, damping constant δ and logarithmic decrement Λ are defied by the following formulas

$$\frac{\phi_{n+1}}{\phi_n} = K = \exp^{\delta T}, \Lambda = \ln K = \delta T = ln\frac{\phi_{n+1}}{\phi_n}$$

Forced oscillations case

If the pendulum is acted on by a periodic torque $\tau_a = \tau_0\cos(\omega_a t)$ the mouvement equation becomes

$$\ddot{\phi} + 2\delta\dot{\phi} + {\omega_0}^2\phi = F_0\cos(\omega_a t)$$

where $F_0 = \tau_0/I$.

In the steady state the solution becomes

$$\phi(t) = \phi_a\cos(\omega t + \alpha)$$

where

$$\phi_a = \frac{F_0/{\omega_0}^2}{\sqrt{\left(1 - \left(\frac{\omega_a}{\omega_0}\right)^2\right)^2 + \left(\frac{2\delta\omega_a}{{\omega_0}^2}\right)^2}}$$

Furthermore

$$\alpha = -\arctan\frac{2\delta\omega_a}{{\omega_0}^2 - {\omega_a}^2}$$