Subset Sum Problem

Input:

- Set $A = \{a_1, a_2, \ldots, a_n\}$
- target T

Question:

Is there $S \subseteq A$ that adds up to exactly T?

Knapsack

Input: v_i, w_i for $i = 1, \ldots, n$

capacity W

Objective:

Find $S \subseteq$ of items

s.t. $w(S) \leq W$

maximize $v(S)$

Subset Sum is a special case of knapsack

$v_i = w_i = a_i$

$W = T$

Subset sum is NP-hard

There is an $O(nW)$ time algo

$VC \leq_p SS$

$((V,E), k) \to (A, T)$

		e_1	e_2	e_3	...	e_m
v_1	1	1	0	0		1
v_2	1	0	0	0		0
v_3	1	1	0	0	...	0
⋮	1	0	1	0		0
	⋮	⋮	⋮	⋮		⋮
						0
v_n	1	0	0	0		1
d_1		1	0	0	...	0
d_2		0	1	0	...	0
d_3		0	0	1	...	0
⋮						
d_m		0	0	0	...	1
T		"k" 2	2	2	...	2

say $e_1 = (v_1, v_3)$

Subset of vertices $\longleftrightarrow$ Subset of rows

subset of vertices is a vertex cover $\longleftrightarrow$ subset of rows add upto 1/2 vector

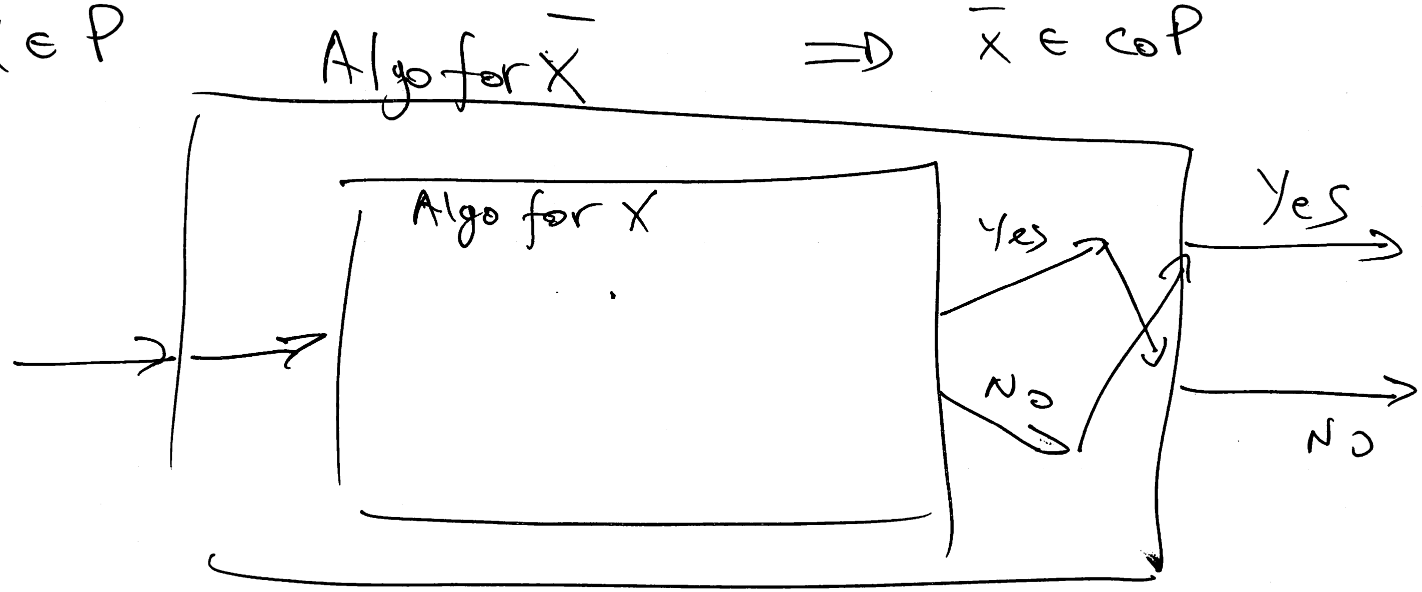
$\exists$ subset of vertices is a vertex cover of size k $\longleftrightarrow$ $\exists$ subset of rows that adds up to T

Def: Take a computational problem X, define $\bar{X}$ it's complement

Def: ~~For~~ $X \in coP$ if $\bar{X} \in P$

Obs: $P = coP$

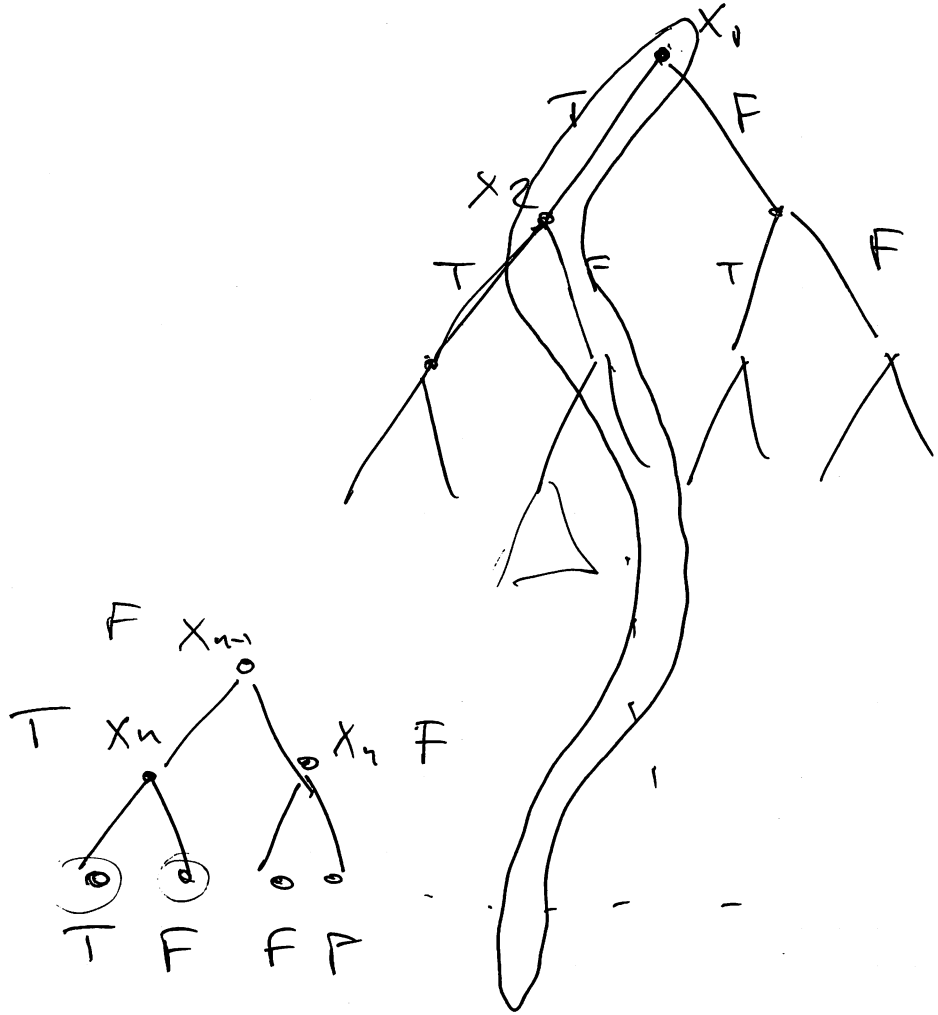
If $X \in P$ $\Rightarrow$ $\bar{X} \in coP$



$\Rightarrow P \subseteq coP$

QSAT $\in$ PSPACE

$$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \cdots \exists x_n \cdot \varphi(x_1, x_2, \ldots, x_n)$$



what is the size of the tree?

$\simeq 2^n$

If we re-use space then only use $O(n)$ space

Key idea in showing $QSAT \leq_p GG$

View QSAT as a game between two player

Player 1 : set variables $x_1 \; x_3 \; x_5 \ldots$ $(\exists)$

Playe 2 : set variables $x_2 \; x_4 \; x_6 \ldots$ $(\forall)$

Player 1 wins if formula maps to T

" 2 " " " " " F

$QSAT \leq_P GG \quad \exists x_1 \forall x_2 \cdots \underline{\exists_n} \cdot \varphi = C_1 \wedge C_2 \wedge \cdots \wedge C_m$

$C_1 = x_1 \vee \neg x_2 \vee x_{10}$