Student ID: _____

Number of additional sheets: _____

AUT UNIVERSITY COMPUTING + MATHEMATICAL SCIENCES

716181 – Algorithm Design and Analysis

Semester 2, 2013

Final Examination

November 2013

Time Allowed Two hours (plus reading time).

Instructions

- This is a closed-book exam.
- Please answer all questions in the space provided.
- Calculator is allowed for this exam.

Question	Marks given	Marks possible
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
Total		80

1. Answer the following questions:

   (a) Consider the following statements. Which ones are true? (6 *marks*)

   (i) If $f(n) = \log n$, $g(n) = \log^2 n + \log n$, then $f(n)$ is $O(g(n))$. **[T/F]**

   (ii) If $T(n) = 2T\left(\frac{n}{2}\right) + n$ and $T(0) = 0$, then $T(n)$ is $O(n^2)$. **[T/F]**

   (iii) The average-case time complexity of the insertion operation on a binomial heap is $O(1)$. **[T/F]**

   (iv) Kosaraju-Sharir algorithm computes a linearization of a given directed graph if it exists. **[T/F]**

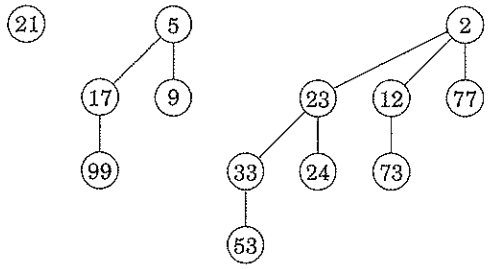
   (v) Bellman-Ford algorithm is a greedy algorithm for solving the all-pair shortest path problem in $O(n^3)$ time. **[T/F]**

   (vi) RSA is an algorithm for public-key cryptography that is based on the presumed difficulty of factoring large integers. Therefore a user of RSA normally publishes a large prime number p along with an auxliary value e that is coprime with $p - 1$ as the public key. **[T/F]**

   (b) Write down the pseudocode for the DFS-Explore algorithm starting from a given node s in a directed graph. (4 *marks*)

2. Answer the following questions:

(a) Perform *deleteMin* operation on the following binomial heap. Show the steps. (6 *marks*)



(b) Describe a situation where the priority queue data structure is used in the implementation of an algorithm. (4 *marks*)

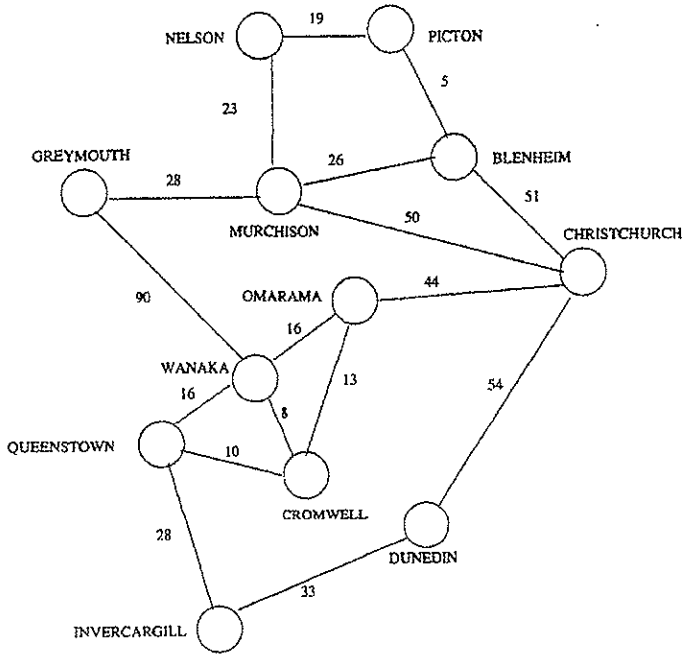
3. Answer the following questions:

   (a) What is a walk in a directed graph? (2 *marks*)

   (b) What is a linearization of a directed graph? (2 *marks*)

   (c) Describe an algorithm that runs in time $O(m+n)$ and computes all strongly connected components of a given directed graph, where m and n are respectively the size and order of the graph. (6 *marks*)

4. **[Laying Cables]** A cable TV company is planning to introduce a new type of cable TV to South Island of New Zealand. The graph below shows the distances between various South Island towns. As a consultant for the company, you are asked to design the most cost-efficient way to lay cable so that it connects all towns on the map.

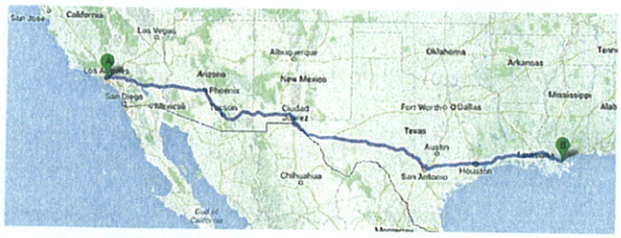


(a) Re-formulate this problem in graph-theoretical terms: (2 *marks*)

(b) Describe an efficient algorithm to solve this problem: (4 *marks*)

(c) Find and draw an optimal solution on the graph. (4 *marks*)

5. **[Easy Riders]** Wyatt and Billy wants to drive from Los Angeles to New Orleans. Their fuel tank, when full, holds enough fuel to travel m km, and they have a map that gives distances between towns with petrol stations along the route. Let $p_1 < p_2 < \cdots < p_n$ be the locations of all the petrol stations along the route where p_i is the distance from Los Angeles to the petrol station. You can assume that the distance between neighbouring petrol stations is at most m km. To save time,



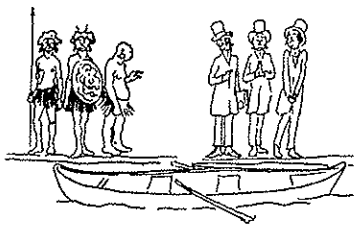
they want to make as few petrol stops as possible along the way. Give an $O(n)$ time algorithm to determine the sequence of petrol stations they should stop.

(a) Someone suggested them to use a greedy algorithm to solve this problem. Provide a greedy choice criterion that they may use here. (3 *marks*)

(b) State the greedy choice property for the above criteria that is needed to guarantee an optimal solution. (3 *marks*)

(c) Describe your algorithm and analyze its time complexity. (4 *marks*)

6. **[Missionaries and Cannibals]** Three missionaries and three cannibals must cross a river using a boat which can carry at most two people, under the constraint that, for both banks, if there are missionaries present on the bank, they cannot be out-numbered by cannibals (if they were, the cannibals would eat the missionaries.) The boat cannot cross the river by itself with no people on board. Your task is to decide if there is a way for all the missionaries and cannibals to cross the river. Answer the following questions:



(a) Model this situation as a directed graph. State clearly what the nodes and edges represent in the context of the question. Then rewrite this as a graph-theoretical question. (5 *marks*)

(b) Suppose the problem is generalized to allow n missionaries and n cannibals, while the boat could carry k people, design an algorithm to solve the above problem. Furthermore, your algorithm should provide a solution that includes the smallest number of boat crosses, if a solution exists at all. (5 *marks*)

7. **[King Arthur's Wizard]** You wake up one day finding yourself turned into Merlin the Wizard, the advisor of King Arthur. King Arthur's court had n knights. He ruled over m counties. Each knight i had a quota q_i of the number of counties he could oversee. Each county j, in turn, produced a set S_j of the knights that it would be willing to be overseen by. King Arthur sets upon you the task of computing an assignment of counties to the knights so that no knight would exceed his quota, while every county j is overseen by a knight from its set S_j.

(a) Model this situation using a flow network. (5 *marks*)

- What are the nodes in the flow network?

- What are the edges?

What are the weights on the edges?

(b) Show how you can employ the Max-flow algorithm to compute the assignment. (5 *marks*)

8. **[Want to Pass the Exam?]** In an Algorithm Design and Analysis exam your lecturer gives you n questions, each takes certain number of minutes to finish. However you have only K minutes in total. Suppose the ith question takes t_i number of minutes to finish, and has a value of m_i marks. Suppose also that there is no dependency among the questions so that you can answer them in any order. Furthermore, your lecturer is so mean that he wouldn't give any mark to any partial solutions (therefore there is no point for you to complete only part of a question). Your task is to find out the best selection of questions to finish so that you can get the highest marks (assuming you will answer all questions correctly). Act quickly, as time is running out.

   You think of using dynamic programming to solve this problem. Let $B(i, w)$ be the maximum marks that you can get out of the questions 1 to i in w minutes. The question now is to compute the value of $B(n, K)$.

   (a) What is the value of $B(0, w)$ for any number $w \geq 0$? (2 *marks*)

   (b) Write a recurrence expressing the value of $B(i+1, w)$ for $i \geq 0$ and $w \geq 0$. (4 *marks*)

   Hint: Consider the two cases when $w < t_{i+1}$ and when $w \geq t_{i+1}$ separately.

   (c) Describe your algorithm for computing the solution. What is the running time of your algorithm? (4 *marks*)