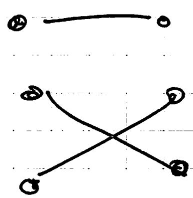
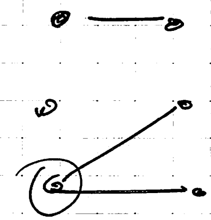
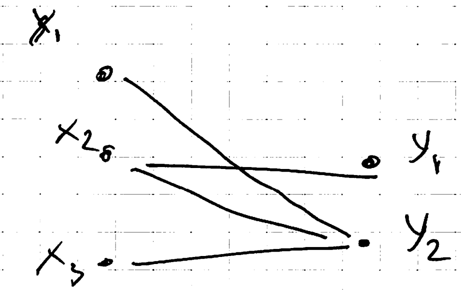


Matching



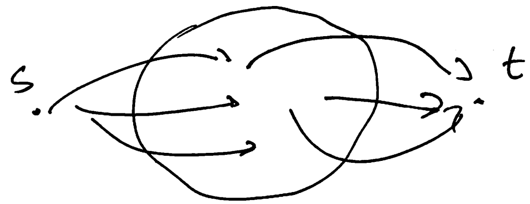
Not a matching

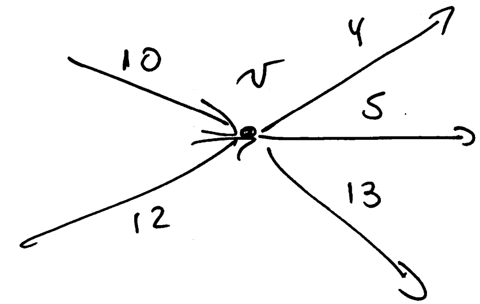


$$(X, Y, E)$$

$$\forall S \subseteq X \;:\; |S| \leq |N(S)|$$

$$\forall S \subseteq Y \;:\; |S| \leq |N(S)|$$



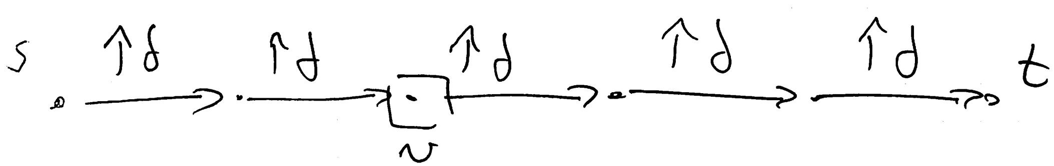


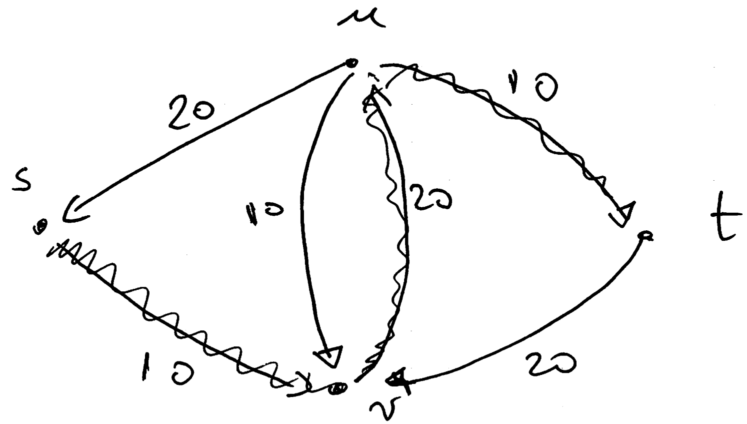
$f^{in}(v) = 22$

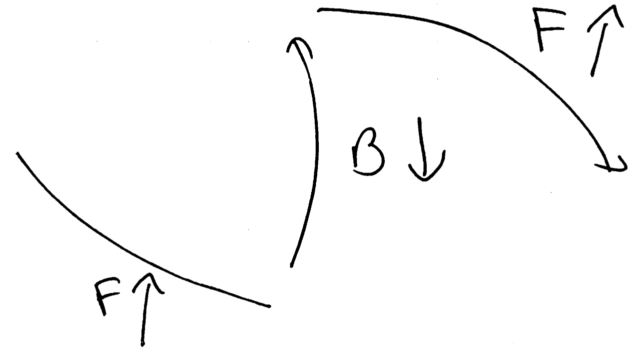
$f^{out}(v) = 22$

Def: $\quad f^{in}(v) = \sum_{e \in \delta^{in}(v)} f(e) \qquad f^{out}(v) = \sum_{e \in \delta^{out}(v)} f(e)$

Def: Value of flow f is $\quad v(f) = f^{out}(s) = f^{in}(t)$



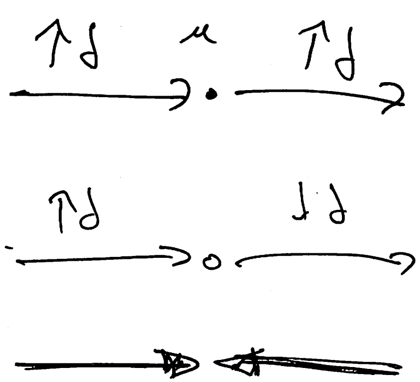




Feasibility of output

① ~~Flow~~ Capacity constraint

② Flow cons constraint



$\in G_f$

If (u,v) is forward

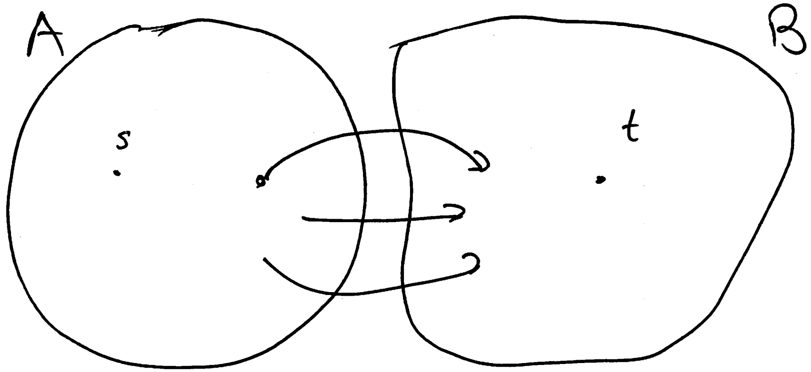
$\delta \leq$ res. cap of (u,v)

$= c(u,v) - f(u,v)$

If (u,v) is backward

$\delta \leq$ res. cap of (u,v)

$= f(v,u)$



$s \in A$

$t \in B$

$\cancel{A \cap B \neq}$

$A \cap B = \emptyset$

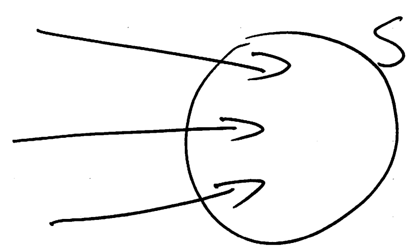
$A \cup B = V$

$$\underline{\text{Def}} = \quad \text{cap}(A,B) = \sum_{\substack{(u,v)\in E \\ u \in A,\ v \in B}} c(u,v) = c(A,B)$$

For $S \subseteq V$

$$f^{out}(S) = \sum_{\substack{(u,v)\in E \\ u \in S,\ v \notin S}} f(u,v) \quad \text{and} \quad f^{in}(S) = \sum_{\substack{(u,v)\in E \\ u \notin S,\ v \in S}} f(u,v)$$

S



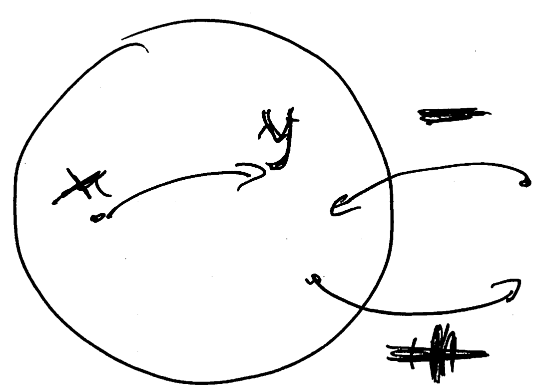
Claim let f be an s-t flow and (A, B) be an s-t cut

$$v(f) = f^{out}(A) - f^{in}(A)$$

proof

$$\sum_{u \in A} f^{out}(u) - f^{in}(u) = f^{out}(s) = v(f)$$

$$= f^{out}(A) - f^{in}(A)$$



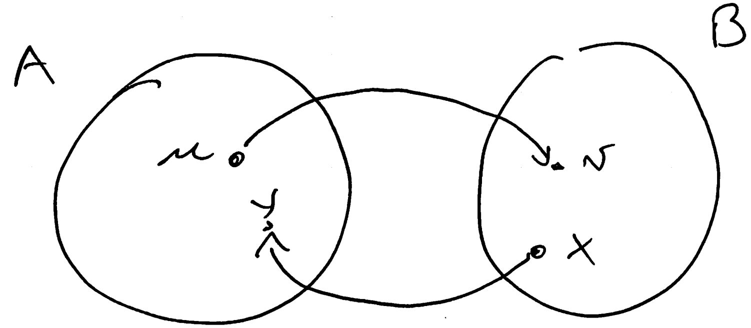
Coro: $v(f) \leq c(A, B)$

Let f be output of FF algo.

Let $A = \{ u \in V : \exists$ an s-u path in $G_f \}$

$B = V \setminus A$

$\Rightarrow (A, B)$ is an s-t cut

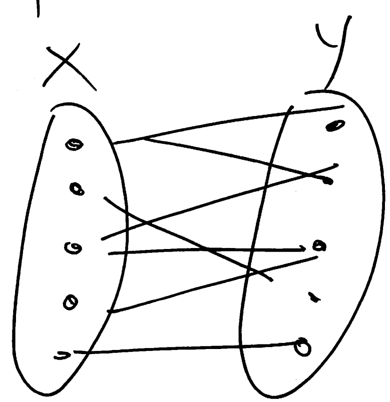


$(u,v) \in E$
$(u,v) \notin G_f$ $\Rightarrow f(u,v) = C(u,v)$

$(x,y) \in E$
$(y,x) \notin G_f$ $\Rightarrow f(x,y) = 0$

$$\Rightarrow v(f) = f^{out}(A) - f^{in}(A)$$
$$= C(A,B) - 0$$

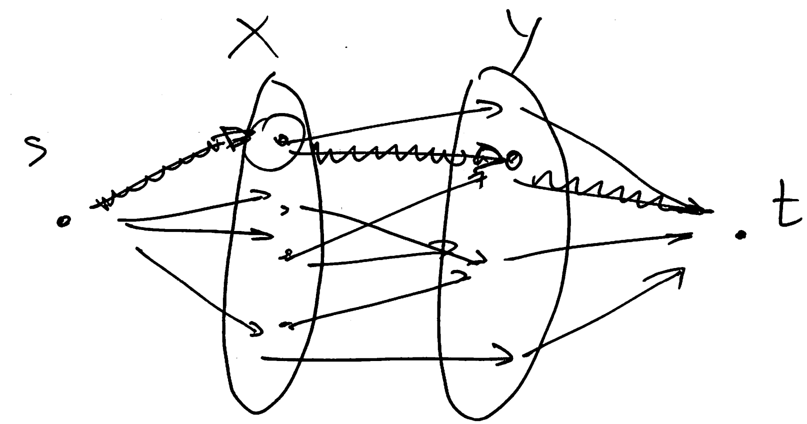
Bipartite Graph



Objective is to find matching M

max. $|M|$

Max flow problem



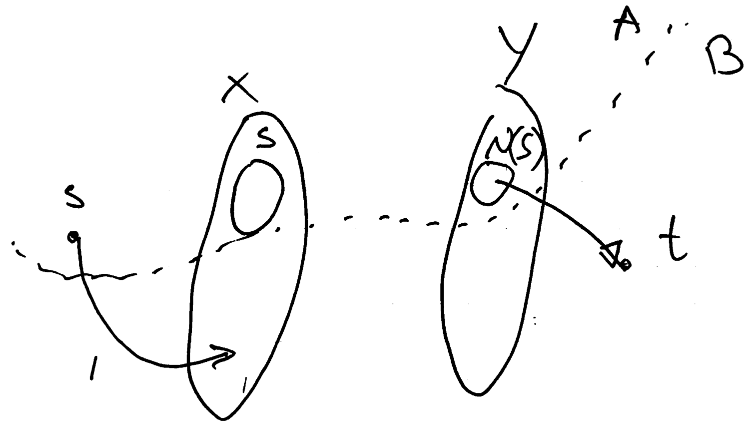
$c(e)=1 \quad \forall e$

Objective is to find an s-t flow

max $v(f)$

Assume $|X| = |Y| = n$

(X, Y, E) has a perfect matching $\Longleftrightarrow$ min cut in H has capacity n



$$cap(A, B) = |X \setminus S| + |N(S)|$$
$$= n - |S| + |N(S)|$$

If $|N(S)| < |S| \Rightarrow c(A, B) < n$

But $c(A, B) \geq n \Rightarrow |N(S)| \geq |S|$