Steiner Tree

Input:

- $G = (V, E)$ graph
- $\ell : E \to \mathbb{R}_+$ lengths
- $X \subseteq V$ terminals

Output:

- $Z \subseteq V \setminus X$ (steiner nodes)
- T a MST of $G[X \cup Z]$

Objective:

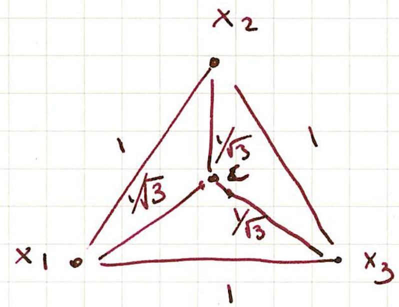
minimize $\ell(T)$

Trivial algorithm:

guess Z $\longleftarrow$ 2^n choices!

compute MST in $G[X \cup Z]$

return best solution



$MST(\{x_1, x_2, x_3\}) = 2$

$MST(\{x_1, x_2, x_3, c\}) = \sqrt{3}$

Even if $G[X]$ is connected, extra Steiner nodes can be useful!

Can we get something that is exponential only on k?

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① For $A \subseteq X$, $r \in V$

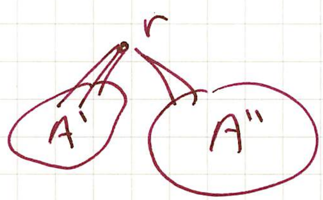
$M[A, r]$ = cost of best solution rooted at r and spanning A

$= \min \ell(T) : \exists Z \subseteq V \setminus A$ and T is ST of $G[Z \cup A \cup \{r\}]$

② Suppose r has one child u in OPT solution

$$M[A,r] = \begin{cases} M[A-r, u] + \ell(u,r) & \text{if } r \in A \\ M[A, u] + \ell(u,r) & \text{if } r \notin A \end{cases}$$

Suppose r has two or more children



$$M[A,r] = M[A', r] + M[A'', r]$$

$$\Rightarrow M[A,r] = \min\Big(\min_{u \in V} \big(M[A-r, u] + \ell(u,r)\big), \min_{A' \subseteq A} \big(M[A', r] + M[A \setminus A', r]\big) \Big)$$

(3) # DP states $= 2^k \times n$

each takes $= O(n + 2^k)$ time

total time $= O(2^k n^2 + 4^k n)$

if $k = \Omega(\log n) \Rightarrow O(4^k n)$ time

Better analysis

$$\sum_{A \subseteq X} 2^{|A|} = \sum_{\ell=0}^{k} \binom{k}{\ell} 2^\ell = 3^k$$

strings of length k over $\{0,1,2\}$

choose ℓ positions out ~~of~~ of k

write binary string there

write 2's elsewhere

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