Pre-tutorial questions

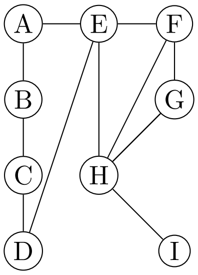
Do you know the basic concepts of this week's lecture content? These questions are only to test yourself. They will not be explicitly discussed in the tutorial, and no solutions will be given to them.

1. Graph terminologi
   (a) What is a graph $G(V, E)$?
   (b) Graphs are usually represented either as an adjacency matrix or as an adjacency list. Can you explain the two representations?
   (c) What are the advantages/disadvantages between the two different representations?
   (d) What is a simple path in a graph?
   (e) What is a cycle in a graph?
   (f) What is a tree? If a tree has n vertices, how many edges does it have?
   (g) What's the difference between a rooted tree and an unrooted tree?
   (h) What is a bipartite graph?
2. Graph traversals
   (a) What's the difference between BFS and DFS?
   (b) Explain the two search algorithms BFS and DFS.

Tutorial

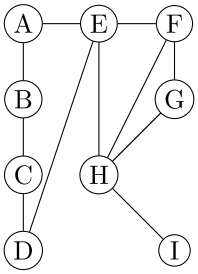
Problem 1

Run a Breadth First Traversal for graph G starting at A. Write the order the nodes are explored. Assuming breaking ties lexicographically (i.e., if more than one child pick the one that comes first in lexicographic order).



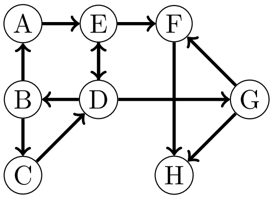
Problem 2

Run a Depth First Traversal for graph G starting at A. Write the order the nodes are explored. Assuming breaking ties lexicographically.



Problem 3

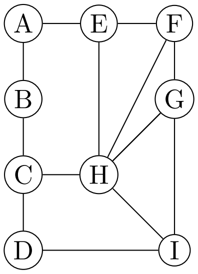
Consider the following directed graph G



1. Give the adjacency matrix and the adjacency list representation of this graph
2. Is this graph strongly connected?
3. This graph has a few directed cycles. Find the minimum number of edges that you need to delete from this graph in order to make it a directed acyclic graph (DAG).
4. Write down the transitive closure of the graph.
5. [**Advanced**] Write down a topological ordering of the resulting DAG.

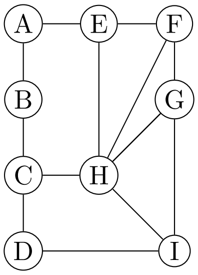
Problem 4

Run a Breadth First Search for graph G starting at A and searching for I. Write out the path that is explored. Assuming breaking ties lexicographically.



Problem 5

Run a Depth First Search for graph G starting at A and searching for I. Write out the path that is explored. Assuming breaking ties lexicographically.



Problem 6

An undirected graph $G = (V, E)$ is said to be bipartite if its vertex set V can be partition into two sets A and B such that $E \subseteq A \times B$. Design an $O(n + m)$ algorithm to test if a given input graph is bipartite using the following guide:

1. Suppose we run BFS from some vertex $s \in V$ and obtain layers $L_1, \ldots, L_k$. Let (u, v) be some edge in E. Show that if $u \in L_i$ and $v \in L_j$ then $|i - j| \leq 1$.
2. Suppose we run BFS on G. Show that if there is an edge (u, v) such that u and v belong to the same layer then the graph is not bipartite
3. Suppose G is connected and we run BFS. Show that if there are no intra-layer edges then the graph is bipartite
4. Put together all the above to design an $O(n + m)$ time algorithm for testing bipartiness.

Problem 7

Give an $O(n)$ time algorithm to detect whether a given undirected graph contains a cycle. If the answer is yes, the algorithm should produce a cycle. (Assume adjacency list representation.)

Problem 8

[Advanced] In class we sketched a proof that a Directed Acyclic Graph always has a topological ordering. Write out a formal proof using induction.