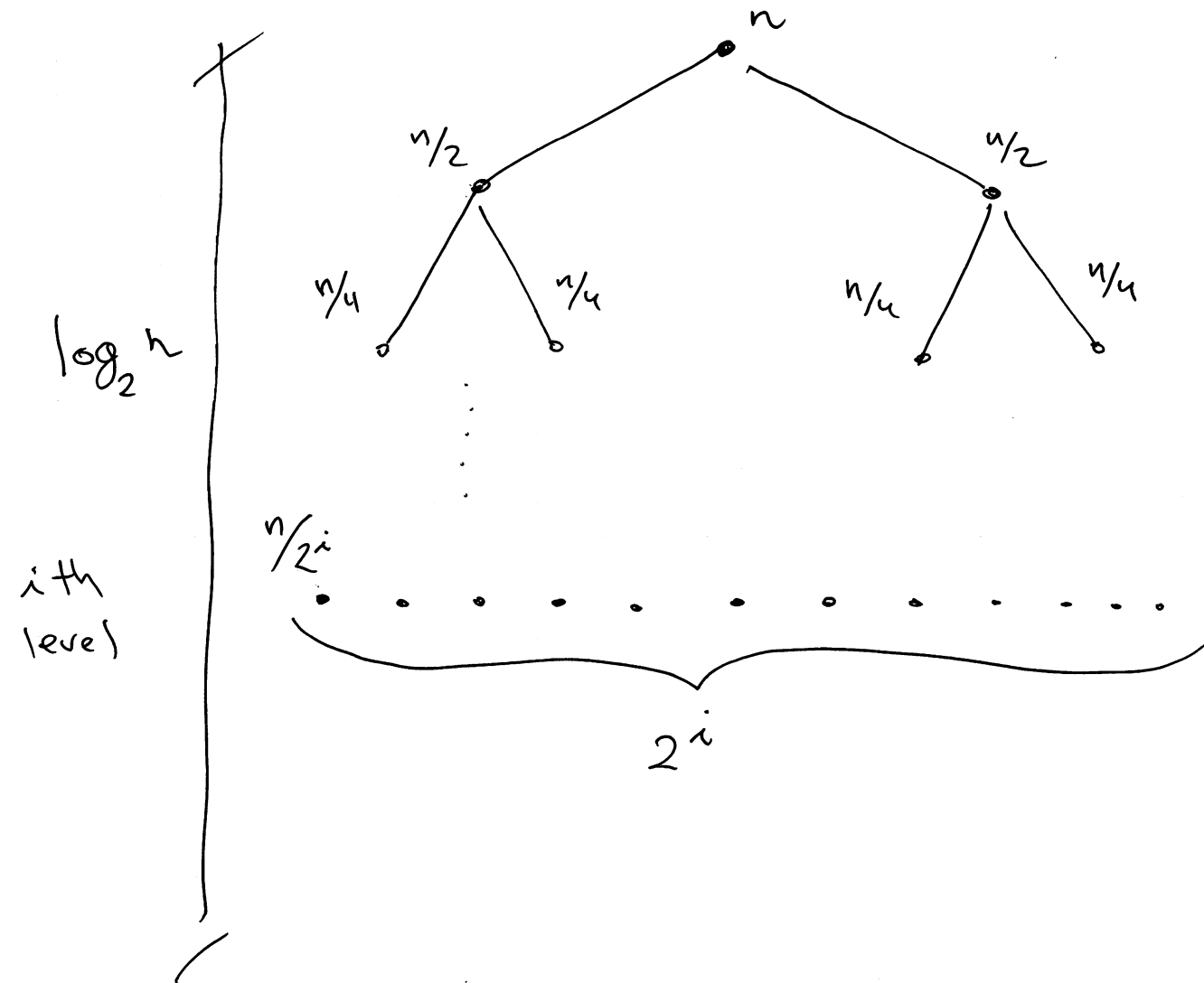


$$T(n) = 2T(n/2) + cn \implies T(n) = O(n \log n)$$



cn time

$$2 \cdot c \frac{n}{2} = cn$$

$$2^i \cdot c \frac{n}{2^i} = cn$$

$$\text{total time} = \sum_{i=0}^{\log_2 n} cn = O(n \log n)$$

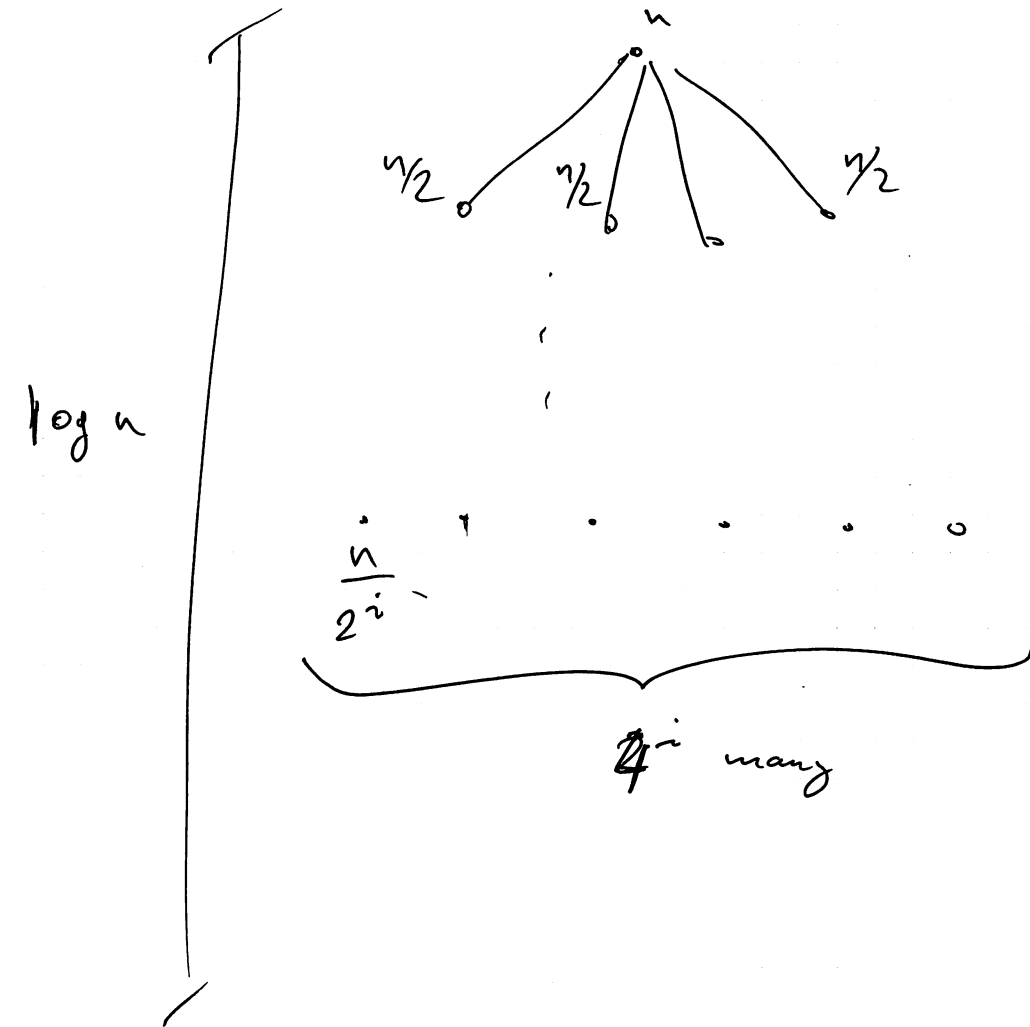
$$T(n) = 4 T\left(\frac{n}{2}\right) + cn \Rightarrow T(n) = O(n^2)$$

$$cn$$

$$4 cn = 4cn$$

$$4^2 cn = 2^2 cn$$

$$4^i c \frac{n}{2^i} = 2^i cn$$



$$\text{total} = \sum_{i=0}^{\log n} 2^i cn$$

$$\leq 2^{\log n + 1} cn$$

$$= 2cn^2$$

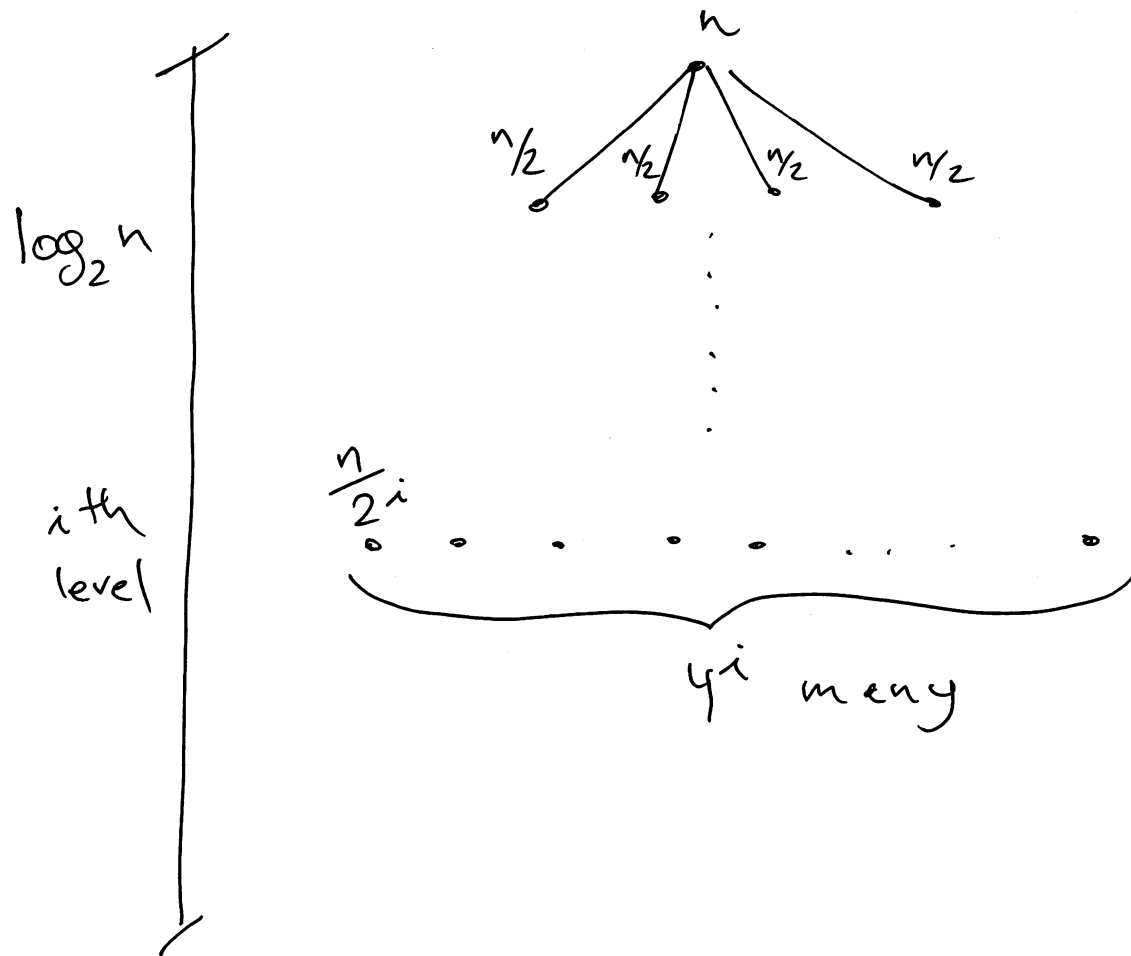
$$T(n) = 2T(n/2) + cn$$

$$= 2 \left(2T\left(\frac{n}{4}\right) + c \frac{n}{2} \right) + cn$$

$$= 4T\left(\frac{n}{4}\right) + cn + \textcircled{cn}$$

$$\begin{array}{cccc}
 & 1 & 3 & \\
 & 2 & 5 & 3 \quad 7 \\
 \lambda & & 1 & 5 \quad 1 \\
 \hline
 & 2 & 5 & 3 \quad 7 \quad \leftarrow O(n) \\
 & \dots & 8 & 5 \quad \leftarrow O(n) \\
 & & & \vdots \\
 & & & \leftarrow O(n)
 \end{array}
 \left. \vphantom{\begin{array}{c} O(n) \\ O(n) \\ \vdots \\ O(n) \end{array}} \right\} n \text{ many}$$

$$T(n) = 4 T\left(\frac{n}{2}\right) + cn \implies T(n) = O(n^2)$$



$$cn$$

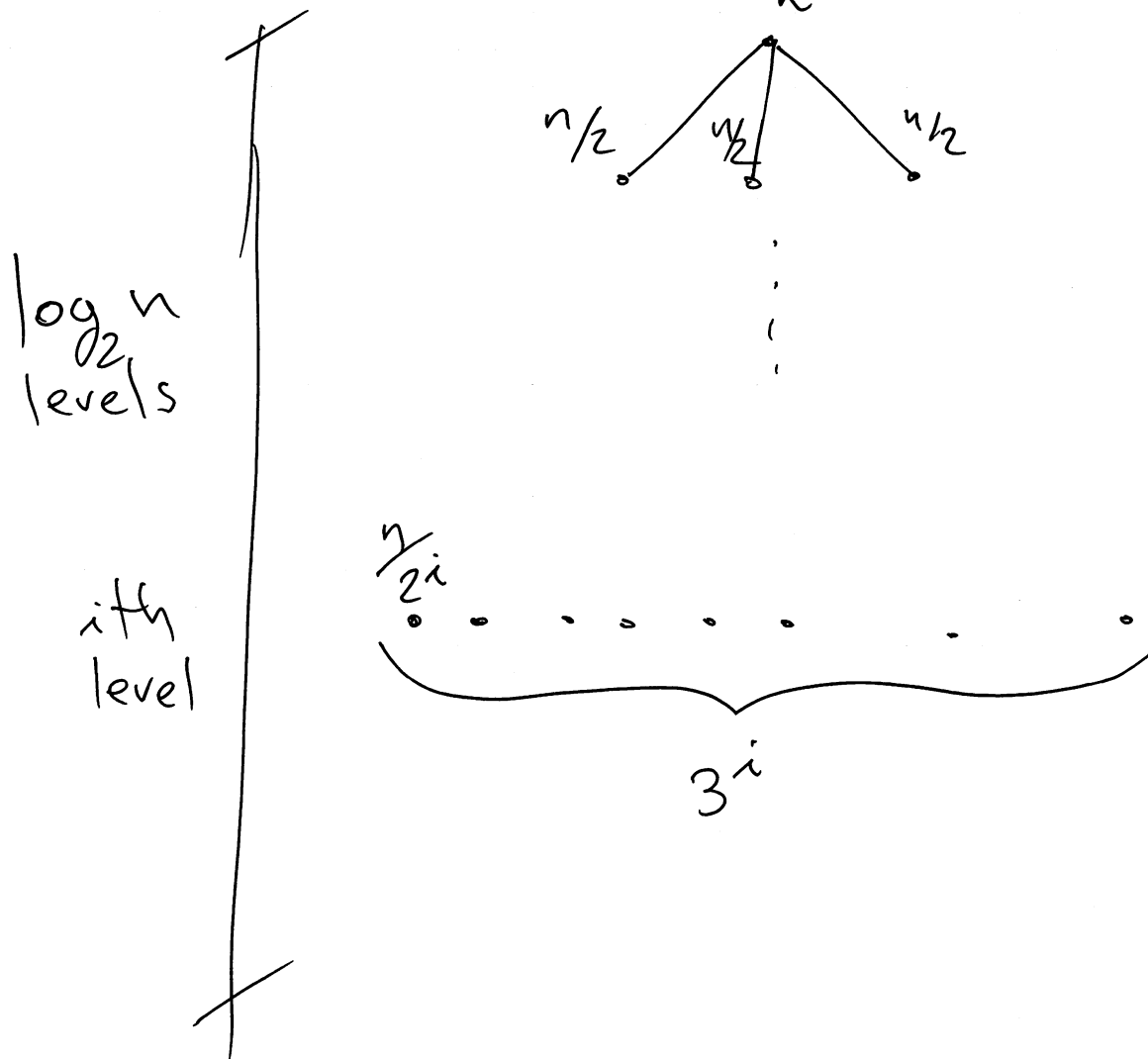
$$4 \times \frac{cn}{2} = 2cn$$

...

$$4^i \times \frac{cn}{2^i} = 2^i cn$$

$$\begin{aligned} \text{total time} &= \sum_{i=0}^{\log_2 n} 2^i cn \\ &= cn \cdot \sum_{i=0}^{\log_2 n} 2^i \leq 2cn^2 \end{aligned}$$

$$T(n) = 3 T(n/2) + cn \Rightarrow T(n) = O(n^{\log_2 3})$$



$$cn$$

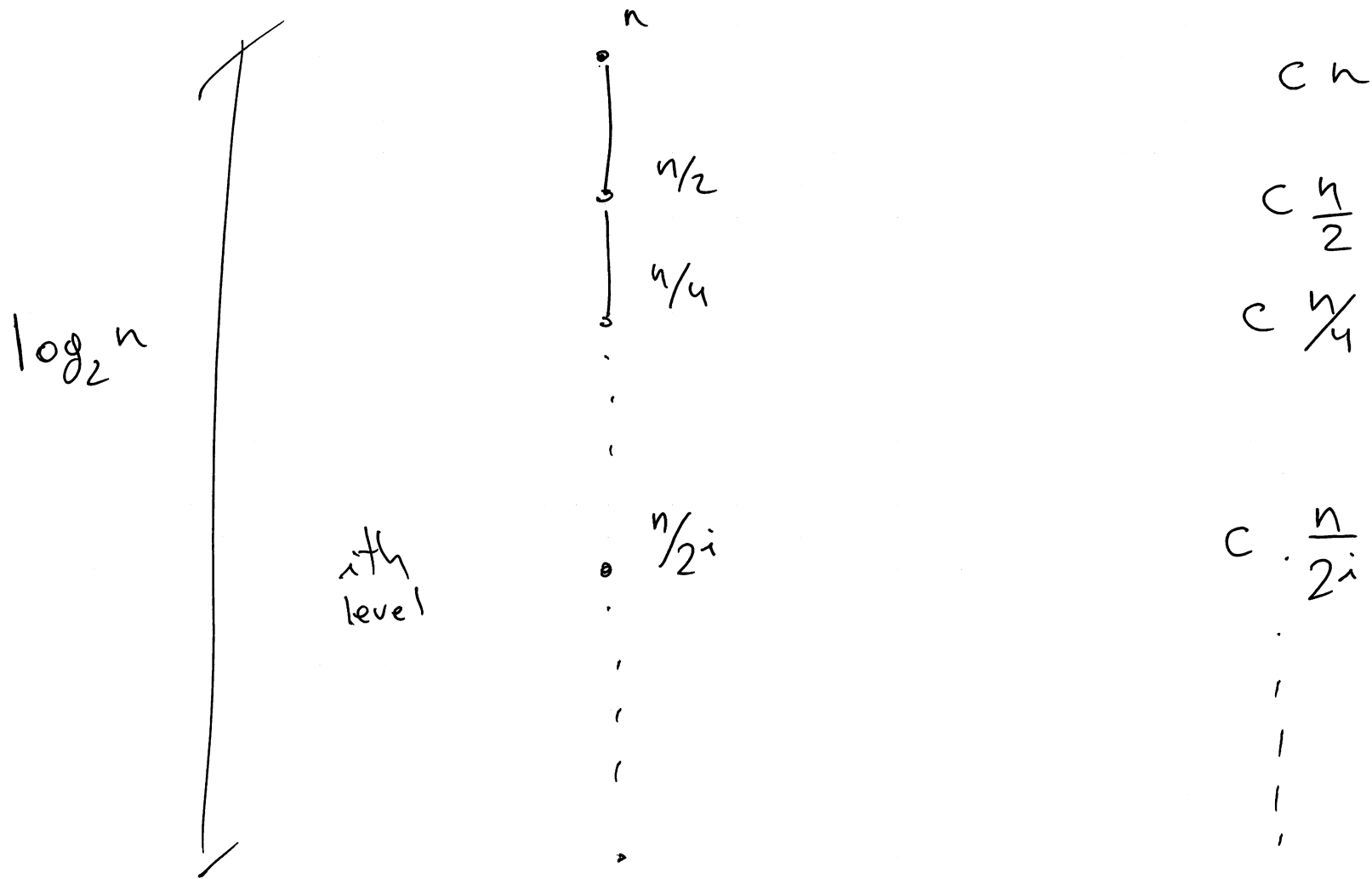
$$3 \times c \frac{n}{2} = \frac{3}{2} cn$$

$$3^i \times c \frac{n}{2^i} = \left(\frac{3}{2}\right)^i cn$$

⋮

$$\begin{aligned} \text{total time} &= \sum_{i=0}^{\log_2 n} \left(\frac{3}{2}\right)^i cn \\ &\leq \frac{3}{2} c \left(\frac{3}{2}\right)^{\log_2 n} n \end{aligned}$$

$$T(n) = T(n/2) + cn \implies T(n) = O(n \log n)$$

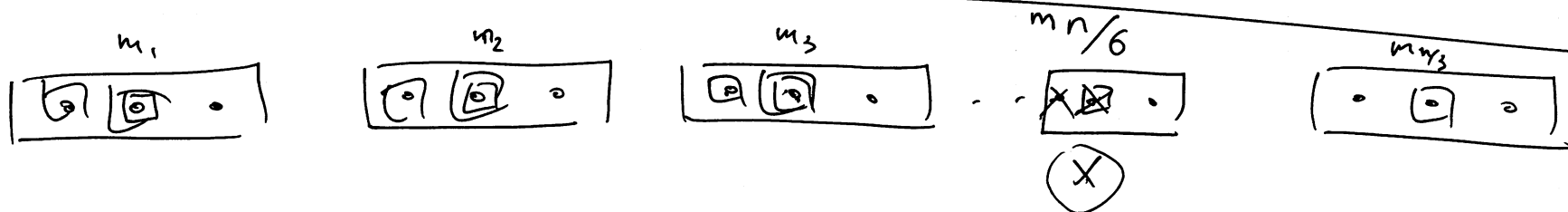


$$\text{total time} = \sum_{i=0}^{\log_2 n} c \frac{n}{2^i} = cn \sum_{i=0}^{\log_2 n} \frac{1}{2^i}$$

$\underbrace{2 \quad 1 \quad 5}_2 \quad \underbrace{10 \quad 7 \quad 8}_8 \quad \underbrace{11 \quad 20 \quad 4}_{11}$

\Downarrow

8

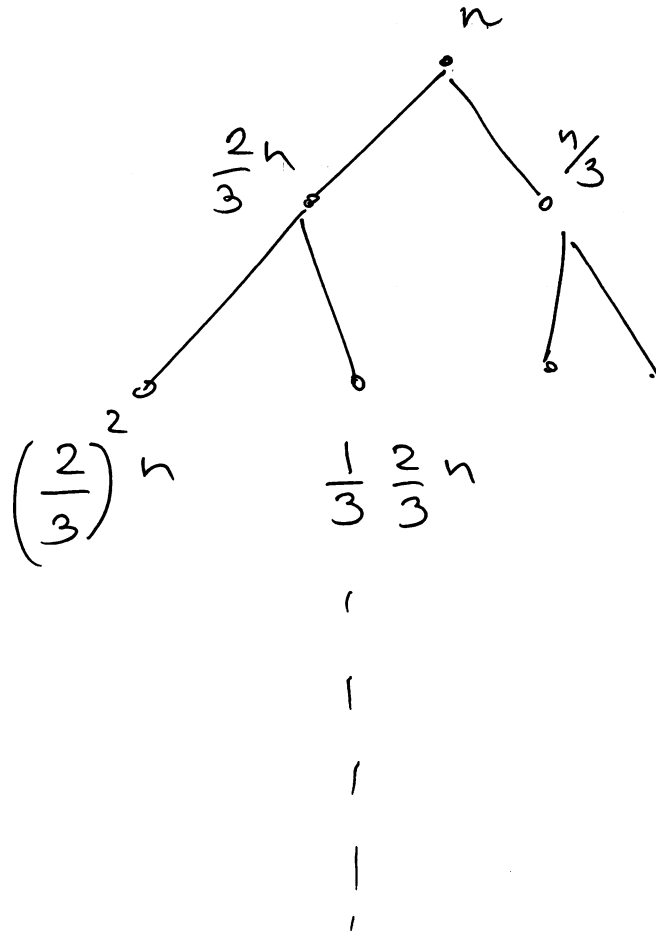


$$\frac{1}{2} \frac{n}{3} \times 2 = \frac{n}{3} \Rightarrow \text{rank}(x, A) \geq \frac{n}{3}$$

$$\frac{1}{2} \frac{n}{3} \times 2 = \frac{n}{3} \Rightarrow \text{rank}(x, A) \leq \frac{2}{3} n$$

$$T(n) = T\left(\frac{2}{3}n\right) + T\left(\frac{n}{3}\right) + cn \Rightarrow O(n \log n)$$

$\log_{3/2} n$



cn

$$c \frac{2}{3}n + c \frac{1}{3}n = cn$$

$$= cn$$

total time $\leq \sum_{i=0}^{\log_{3/2} n} cn = O(n \log n)$