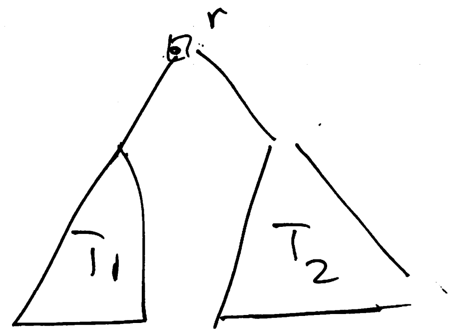
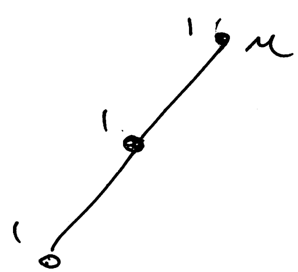


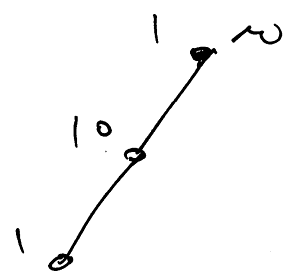
$r \in OPT \rightarrow w(r) + w(VC(T_1)) + w(VC(T_2))$

$r \notin OPT \rightarrow w(VC(T_1)$ that uses root$) + w(VC(T_2)$ that uses root$)$



$L^{in}[u] = 2$

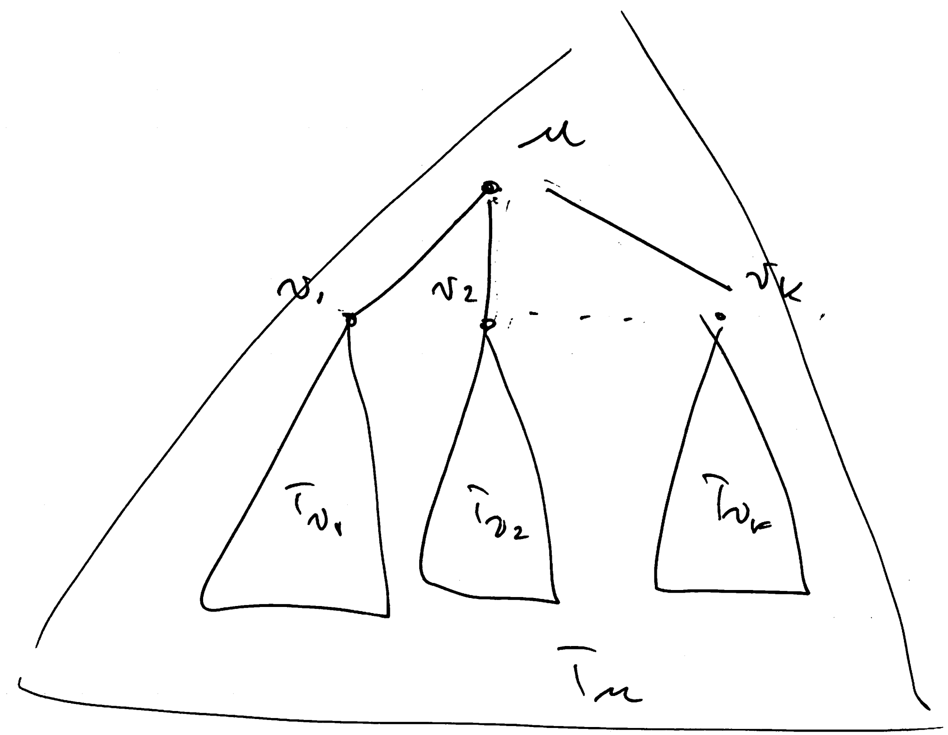
$L^{out}[u] = 1$



$L^{in}[u] = 2$

$L^{out}[u] = 10$

Let T_u rooted at u



$$L^{out}[u] = \sum_{v:\text{ child of } u} L^{in}[v]$$

$$L^{in}[u] = w(u) + \sum_{v:\text{child of } u} \min\left(L^{out}[v]; L^{in}[v]\right)$$

\# DP states = $2n$

each takes = $O(n)$

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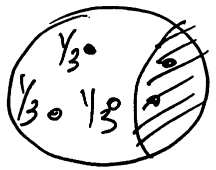
total takes = $O(n^2)$ time

A



A is chosen by Greedy

If u is now covered by A

$$\text{bill}(u) = \frac{1}{\#\text{ newly covered elements}}$$

Obs: $\displaystyle\sum_{u \in U} \text{bill}(u) =$ cost of Greedy

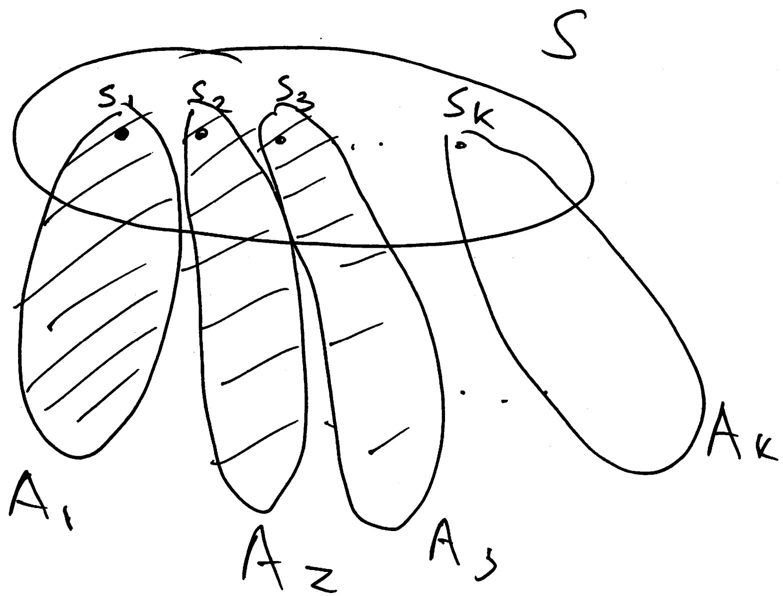
---

Claim: $\displaystyle\sum_{u \in S} \text{bill}(u) \le 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{|S|} \quad \forall S \in OPT$

$$\text{cost of greedy} = \sum_{u \in U} \text{bill}(u) \le \sum_{S \in OPT} \sum_{u \in S} \text{bill}(u) \le \sum_{S \in OPT} H_{|S|}$$

$$\le \sum_{S \in OPT} H_{|U|} = H_{|U|} \sum_{S \in OPT} 1 = H_{|U|}\, |OPT|$$

Claim: $\sum_{u \in S} \text{bill}(u) \leq 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{|S|}$ $\quad \forall S \in OPT$



$\text{bill}(s_1) \leq \frac{1}{k}$

$\text{bill}(s_2) \leq \frac{1}{k-1}$

$\text{bill}(s_3) \leq \frac{1}{k-2}$

$\vdots$

$\text{bill}(s_k) \leq 1$

Write IP for Vertex Cover Problem

Variables: for each vertex u

create a variable $x_u \in \{0, 1\}$

Constraints: for each edge (u, v)

$$x_u + x_v \geq 1$$

Objective: $\min \sum_{u \in V} x_u$

TITLE: DATE / NO.

Write IP for knapsack

Variables : for each item i

$$x_i \in \{0,1\}$$

Constraint :
$$\sum_{\text{item } i} w_i x_i \leq W$$

Objective :
$$\max \sum_{\text{item } i} v_i x_i$$