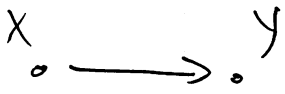
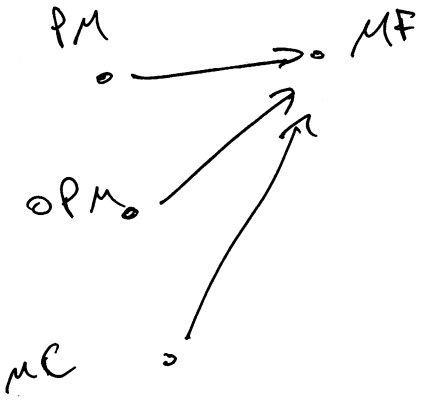
$X \leq_p Y$

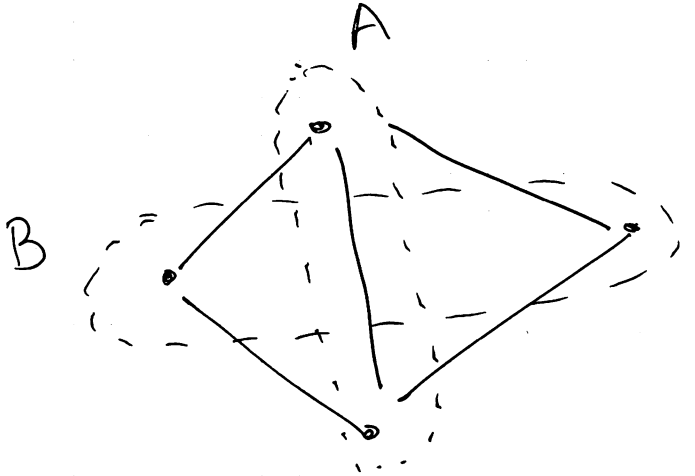




Want to show

i) $IS \leq_p VC$

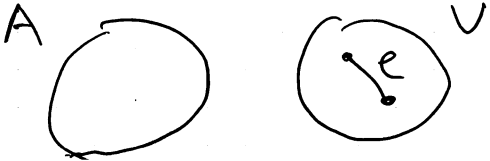
ii) $VC \leq_p IS$



A is a vertex cover

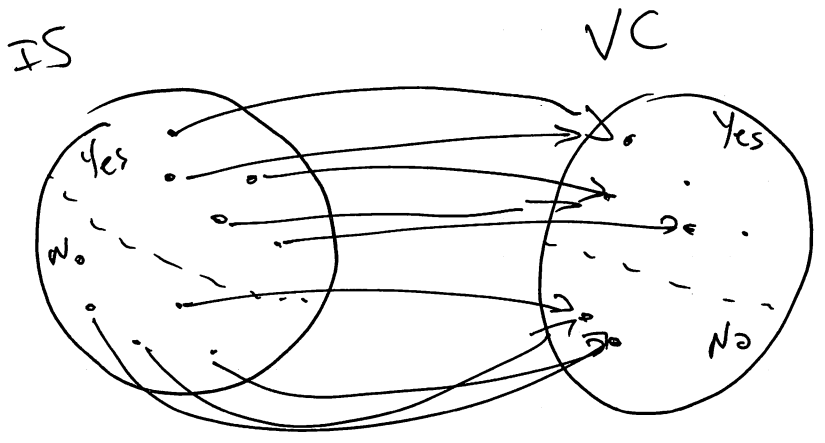
B is an indep set

Claim: ~~If~~ A is a vertex cover iff ~~then~~ $V \setminus A$ is an indep set



$\Rightarrow$ e cannot exist

IS $\leq_p$ VC



Mapping:

$G' = G$

$k = |V| - t$

(G, t) instance of IS

(G', k) instance of VC

Need G has an indep set of size $\geq t$

iff G' has a vertex cover of size $\leq k$

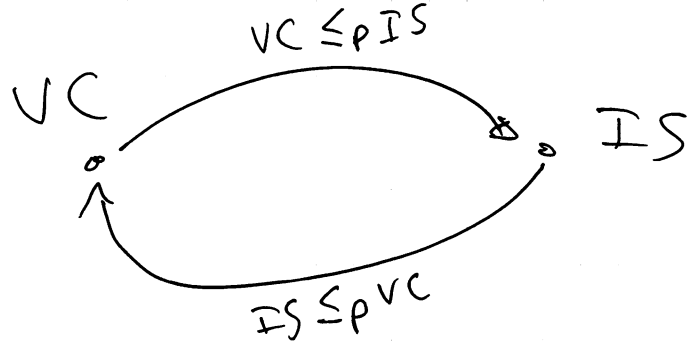
$VC \leq_p IS$

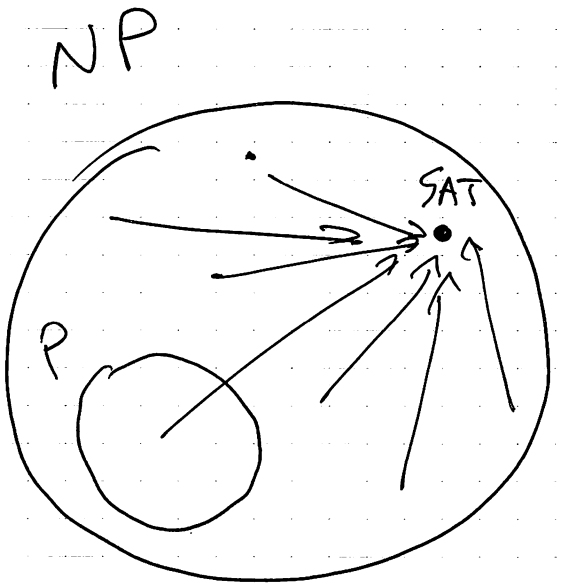
Instance of vertex cover : (G, k)

Instance of independent set : (G', t)

Mapping: $G' = G$

$t = n - k$




NP
SAT
P

3-SAT $\leq_p$ IS

Istance of 3-SAT

$$\phi = (x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$$

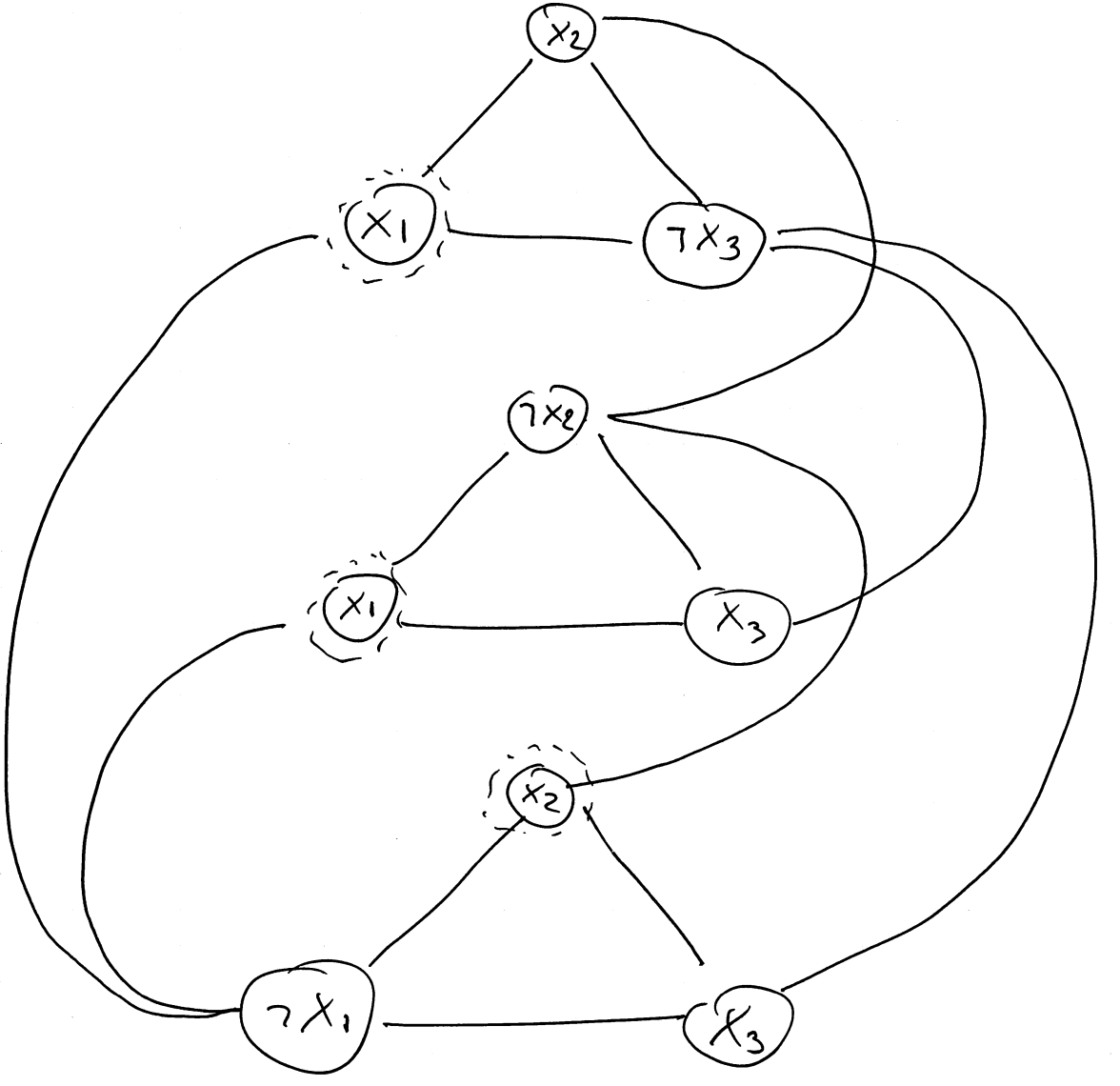
$\Downarrow$

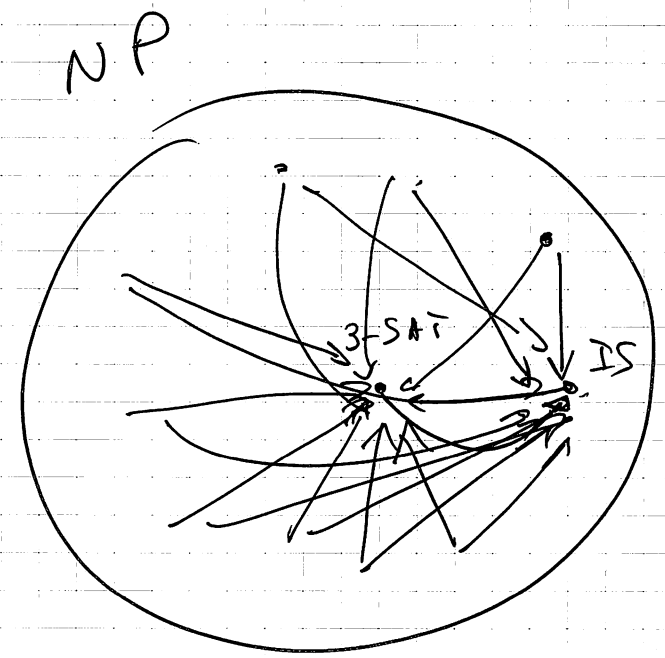
Instance of IS

(G, t)

$x_1 = T \quad x_2 = T \quad x_3 = F$

$t = 3$




NP
3-SAT
IS

3-SAT $\leq_p$ 3-COLORING

Instance of 3-SAT

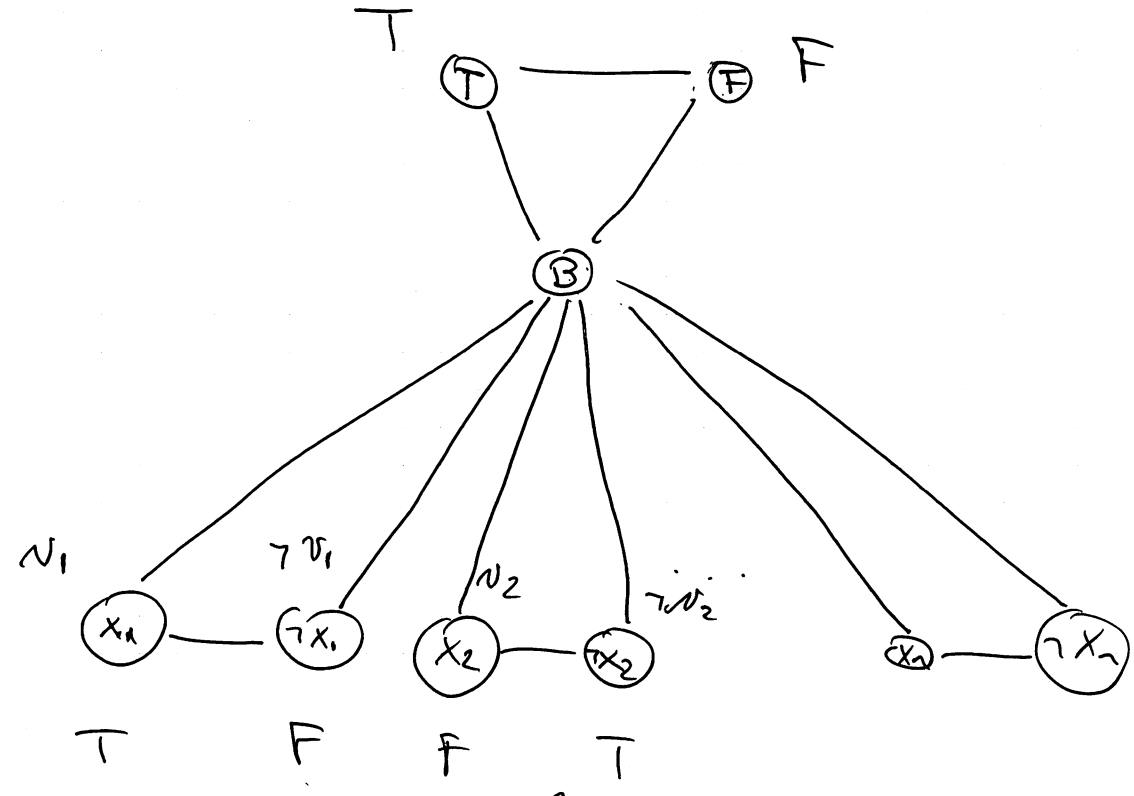
$\phi = C_1 \wedge C_2 \wedge \dots \wedge C_m$

and variables $x_1, x_2, \dots x_n$

Instance of 3-COLORING

G

3 colors: $\{B, T, F\}$



interpret this as setting $x_1 = T$

interpret this as setting $x_2 = F$

Clause gadget $C_j = (\overset{F}{x_1} \vee \overset{F}{\neg x_2} \vee \overset{T}{x_3})$