



Machine Learning

# Large scale machine learning

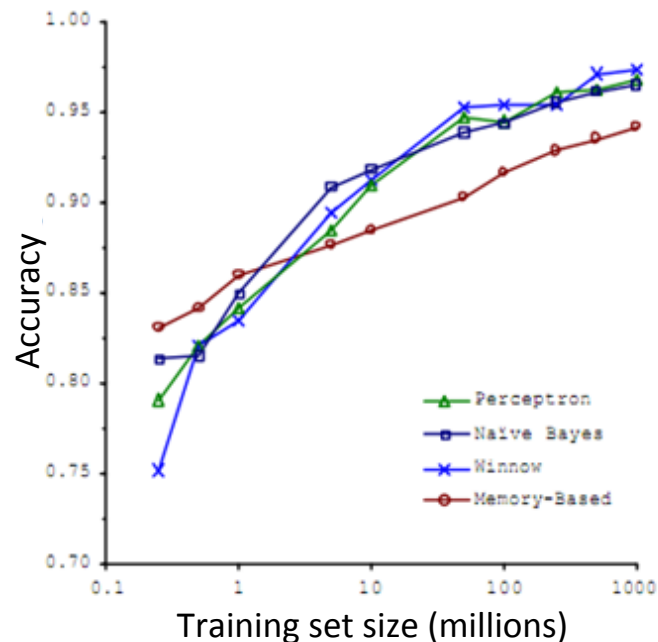
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## Learning with large datasets

# Machine learning and data

Classify between confusable words.  
E.g., {to, two, too}, {then, than}.

For breakfast I ate two eggs.



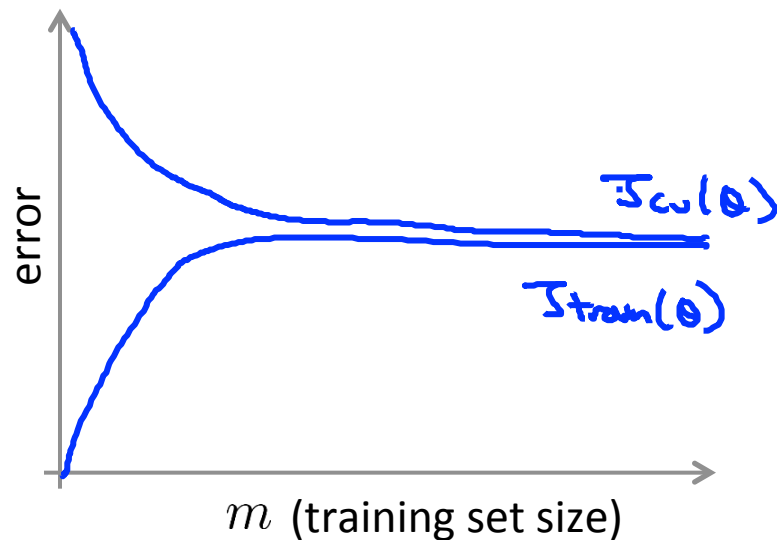
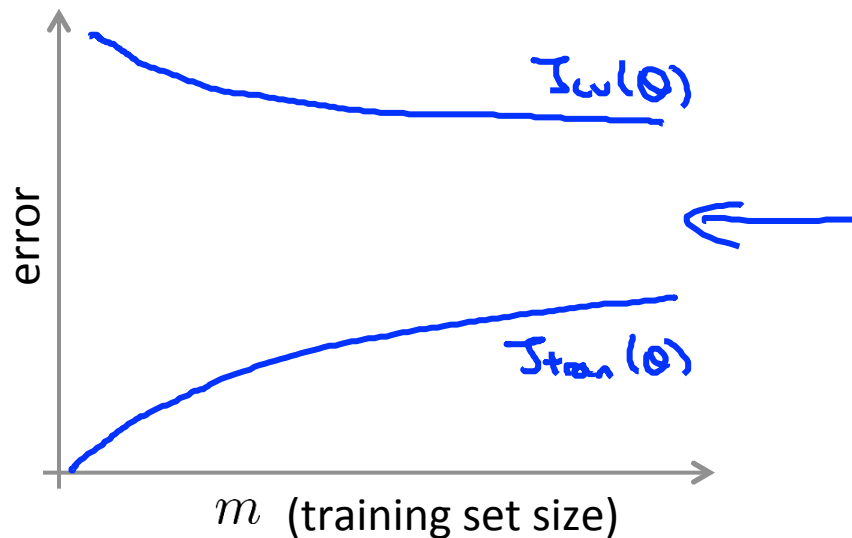
“It’s not who has the best algorithm that wins.  
It’s who has the most data.”

## Learning with large datasets

$m = 100,000,000$

$m = 1,000?$

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$





Machine Learning

# Large scale machine learning

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Stochastic  
gradient descent

# Linear regression with gradient descent

→  $h_{\theta}(x) = \sum_{j=0}^n \theta_j x_j$  **Linear Regression Hypothesis**  
**Linear Regression Cost Function**

→  $J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$

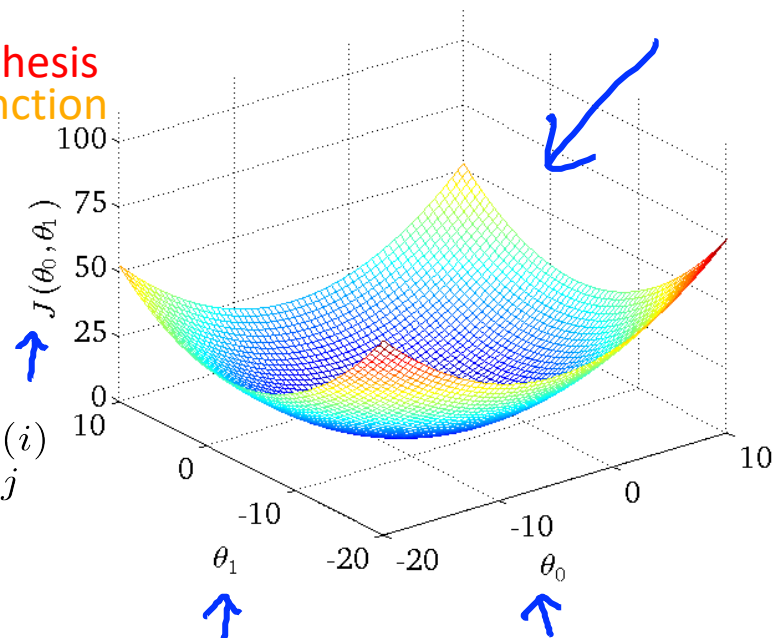
Repeat {

→  $\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$

(for every  $j = 0, \dots, n$ )

}

Linear Regression with Batch Gradient Descent



# Linear regression with gradient descent

$$h_{\theta}(x) = \sum_{j=0}^n \theta_j x_j$$

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Repeat {

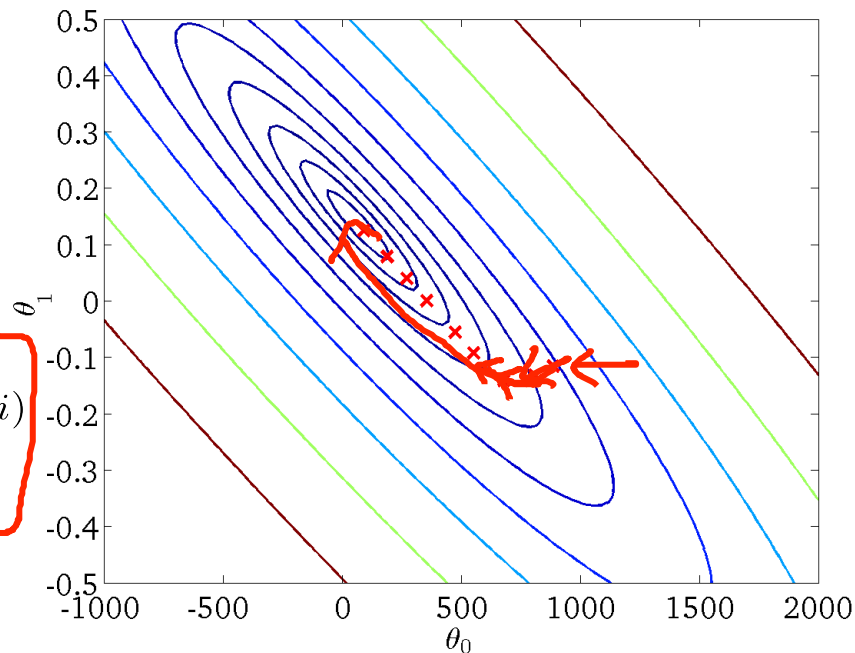
$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(for every  $j = 0, \dots, n$ )

}

$M = 300,000,000$

Batch gradient descent



## Batch gradient descent

$$\rightarrow J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Repeat {

$$\rightarrow \theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

$$\frac{\partial}{\partial \theta_j} J_{train}(\theta)$$

(for every  $j = 0, \dots, n$ )

}

$m = 300,000,000$

## Stochastic gradient descent

$$\rightarrow \text{cost}(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2} (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$\rightarrow J_{train}(\theta) = \frac{1}{m} \sum_{i=1}^m \text{cost}(\theta, (x^{(i)}, y^{(i)}))$$

1. Randomly shuffle dataset. ←

2. Repeat {

for  $i = 1, \dots, m$  {

$$\theta_j := \theta_j - \alpha (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(for  $j = 0, \dots, n$ )

$$\rightarrow \frac{\partial}{\partial \theta_j} \text{cost}(\theta, (x^{(i)}, y^{(i)}))$$

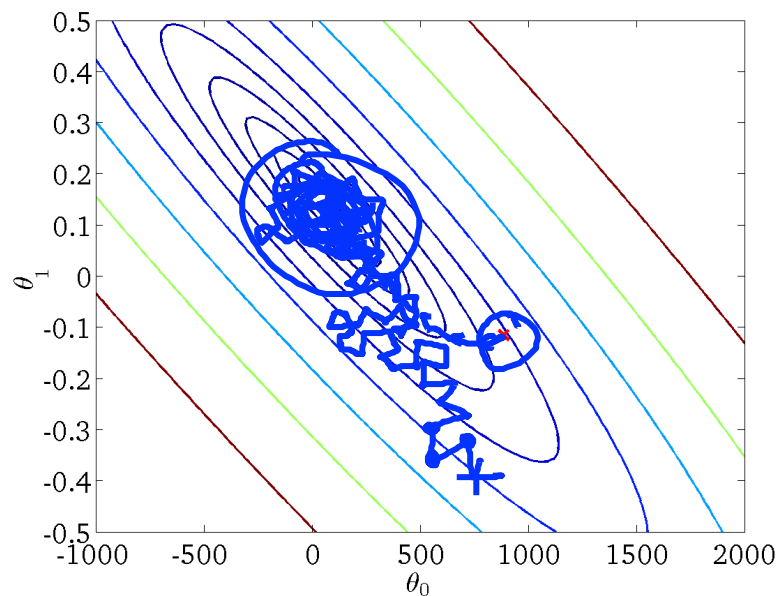
$$\rightarrow (x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), (x^{(3)}, y^{(3)}), \dots$$

# Stochastic gradient descent

→ 1. Randomly shuffle (reorder) training examples

→ 2. Repeat { 1-10x  
    for  $i := 1, \dots, m$  {  
        →  $\theta_j := \theta_j - \alpha(h_{\theta}(x^{(i)}) - y^{(i)})x_j^{(i)}$   
            (for  $j = 0, \dots, n$ )  
    }  
    every }  
}

→  $m = 300,000,000$



Gradually goes toward the minimum





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Mini-batch  
gradient descent

## Mini-batch gradient descent

→ Batch gradient descent: Use all  $m$  examples in each iteration

→ Stochastic gradient descent: Use 1 example in each iteration

Mini-batch gradient descent: Use  $b$  examples in each iteration

$b$  = mini-batch size.

$b = 10$

$\frac{2-100}{10}$

Get  $b = 10$  examples  $(x^{(i)}, y^{(i)}) \dots (x^{(i+9)}, y^{(i+9)})$

$$\Theta_j := \Theta_j - \alpha \frac{1}{10} \sum_{k=i}^{i+9} (h_{\Theta}(x^{(k)}) - y^{(k)}) \cdot x_j^{(k)}$$

$i := i + 10$

# Mini-batch gradient descent

Say  $b = 10$ ,  $m = 1000$ .

Repeat {

→ for  $i = 1, 11, 21, 31, \dots, 991$  {

→  $\theta_j := \theta_j - \alpha \frac{1}{10} \sum_{k=i}^{i+9} (h_{\theta}(x^{(k)}) - y^{(k)}) x_j^{(k)}$

(for every  $j = 0, \dots, n$ )

}

}

$m = 300, 600, 900$

↑

→  $b$  examples

→ 1 example

Vectorization

Use  $b$   
examples  
for each  
iteration

where  $b$  is  
the mini-  
batch size

$b = 10$   
↑



Machine Learning

# Large scale machine learning

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Stochastic  
gradient descent  
convergence

## Checking for convergence

By plotting the cost function

→ Batch gradient descent:

→ Plot  $J_{train}(\theta)$  as a function of the number of iterations of gradient descent.

$$J_{train}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$M = 300,000,000$$

→ Stochastic gradient descent:

$$\rightarrow cost(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2} (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

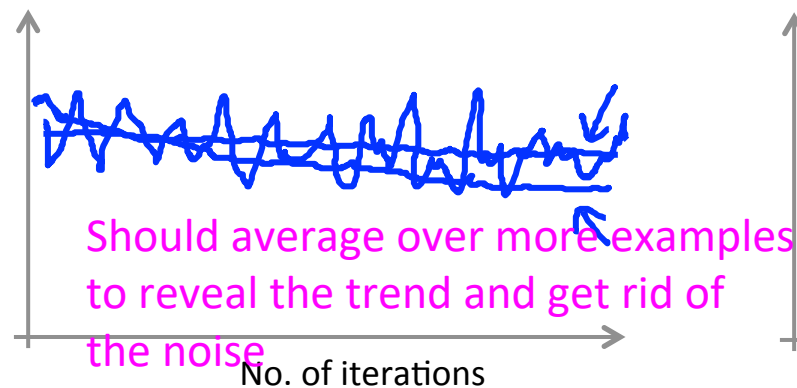
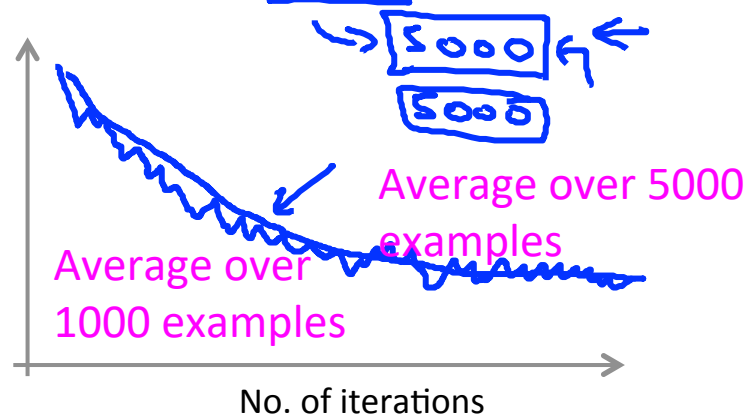
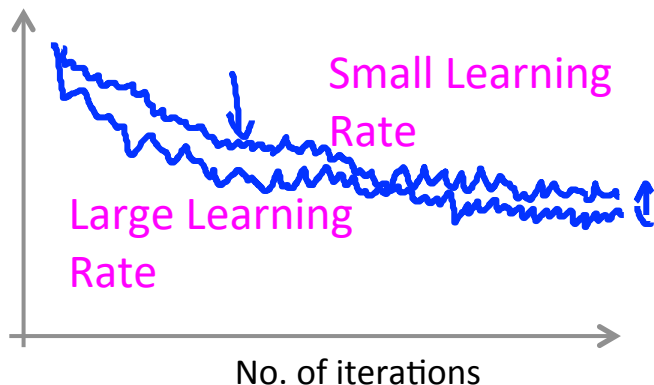
→ During learning, compute  $cost(\theta, (x^{(i)}, y^{(i)}))$  before updating  $\theta$  using  $(x^{(i)}, y^{(i)})$ .

$$\rightarrow (x^{(i)}, y^{(i)}), (x^{(i+1)}, y^{(i+1)}), \dots$$

→ Every 1000 iterations (say), plot  $cost(\theta, (x^{(i)}, y^{(i)}))$  averaged over the last 1000 examples processed by algorithm.

## Checking for convergence

Plot  $cost(\theta, (x^{(i)}, y^{(i)}))$ , averaged over the last 1000 (say) examples

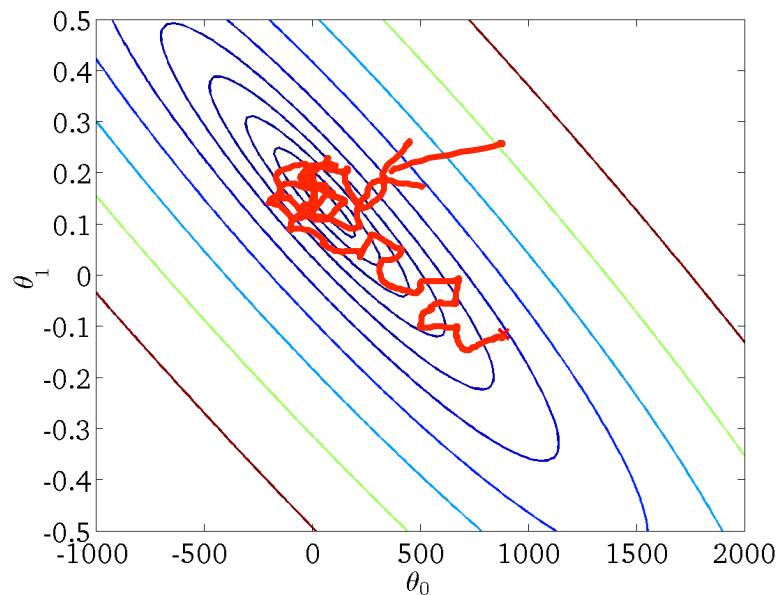


## Stochastic gradient descent

$$\text{cost}(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2}(h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$J_{\text{train}}(\theta) = \frac{1}{2m} \sum_{i=1}^m \text{cost}(\theta, (x^{(i)}, y^{(i)}))$$

1. Randomly shuffle dataset.
2. Repeat {  
    for  $i := 1, \dots, m$  {  
         $\theta_j := \theta_j - \alpha(h_{\theta}(x^{(i)}) - y^{(i)})x_j^{(i)}$   
        (for  $j = 0, \dots, n$ )  
    }  
}



Learning rate  $\alpha$  is typically held constant. Can slowly decrease  $\alpha$  over time if we want  $\theta$  to converge. (E.g.  $\alpha = \frac{\text{const1}}{\text{iterationNumber} + \text{const2}}$ )

# Stochastic gradient descent

$$\text{cost}(\theta, (x^{(i)}, y^{(i)})) = \frac{1}{2}(h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$J_{\text{train}}(\theta) = \frac{1}{2m} \sum_{i=1}^m \text{cost}(\theta, (x^{(i)}, y^{(i)}))$$

1. Randomly shuffle dataset.
2. Repeat {
 

for  $i := 1, \dots, m$  {
 

$\theta_j := \theta_j - \alpha(h_{\theta}(x^{(i)}) - y^{(i)})x_j^{(i)}$   
 (for  $j = 0, \dots, n$ )

 }

 }



Learning rate  $\alpha$  is typically held constant. Can slowly decrease  $\alpha$  over time if we want  $\theta$  to converge. (E.g.  $\alpha = \frac{\text{const1}}{\text{iterationNumber} + \text{const2}}$ )  $\alpha \rightarrow 0$





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Online learning

## Online learning

Shipping service website where user comes, specifies origin and destination, you offer to ship their package for some asking price, and users sometimes choose to use your shipping service ( $y = 1$ ), sometimes not ( $y = 0$ ).

Features  $x$  capture properties of user, of origin/destination and asking price. We want to learn  $p(y = 1|x; \theta)$  to optimize price.

Repeat forever {  
  Get  $(x, y)$  corresponding to user.  
  Update  $\theta$  using  $(x, y)$ :  
     $\rightarrow \theta_j := \theta_j - \alpha (h_\theta(x) - y) \cdot x_j \quad (j=0, \dots, n)$   
  }  
   $\rightarrow$  Can adapt to changing user preference.

price      logistic regression

~~$(x, y)$   $(f(x))$~~

## Other online learning example:

Product search (learning to search)

User searches for "Android phone 1080p camera" ←

Have 100 phones in store. Will return 10 results.

→  $x =$  features of phone, how many words in user query match name of phone, how many words in query match description of phone, etc.

→  $y = 1$  if user clicks on link.  $y = 0$  otherwise.  $(x, y)$  ←

→ Learn  $p(y = 1|x; \theta)$ . ← predicted CTR

→ Use to show user the 10 phones they're most likely to click on.

Other examples: Choosing special offers to show user; customized selection of news articles; product recommendation; ...



Machine Learning

# Large scale machine learning

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Map-reduce and  
data parallelism

## Map-reduce

Batch gradient descent:

$$\theta_j := \theta_j - \alpha \frac{1}{400} \sum_{i=1}^{400} (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

$$m = 400 \leftarrow$$

$$m = 400,000,000$$

Machine 1: Use  $(x^{(1)}, y^{(1)}), \dots, (x^{(100)}, y^{(100)})$ .

$$\text{temp}_j^{(1)} = \sum_{i=1}^{100} (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

Machine 2: Use  $(x^{(101)}, y^{(101)}), \dots, (x^{(200)}, y^{(200)})$ .

$$\text{temp}_j^{(2)} = \sum_{i=101}^{200} (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

Machine 3: Use  $(x^{(201)}, y^{(201)}), \dots, (x^{(300)}, y^{(300)})$ .

$$\text{temp}_j^{(3)} = \sum_{i=201}^{300} (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

Machine 4: Use  $(x^{(301)}, y^{(301)}), \dots, (x^{(400)}, y^{(400)})$ .

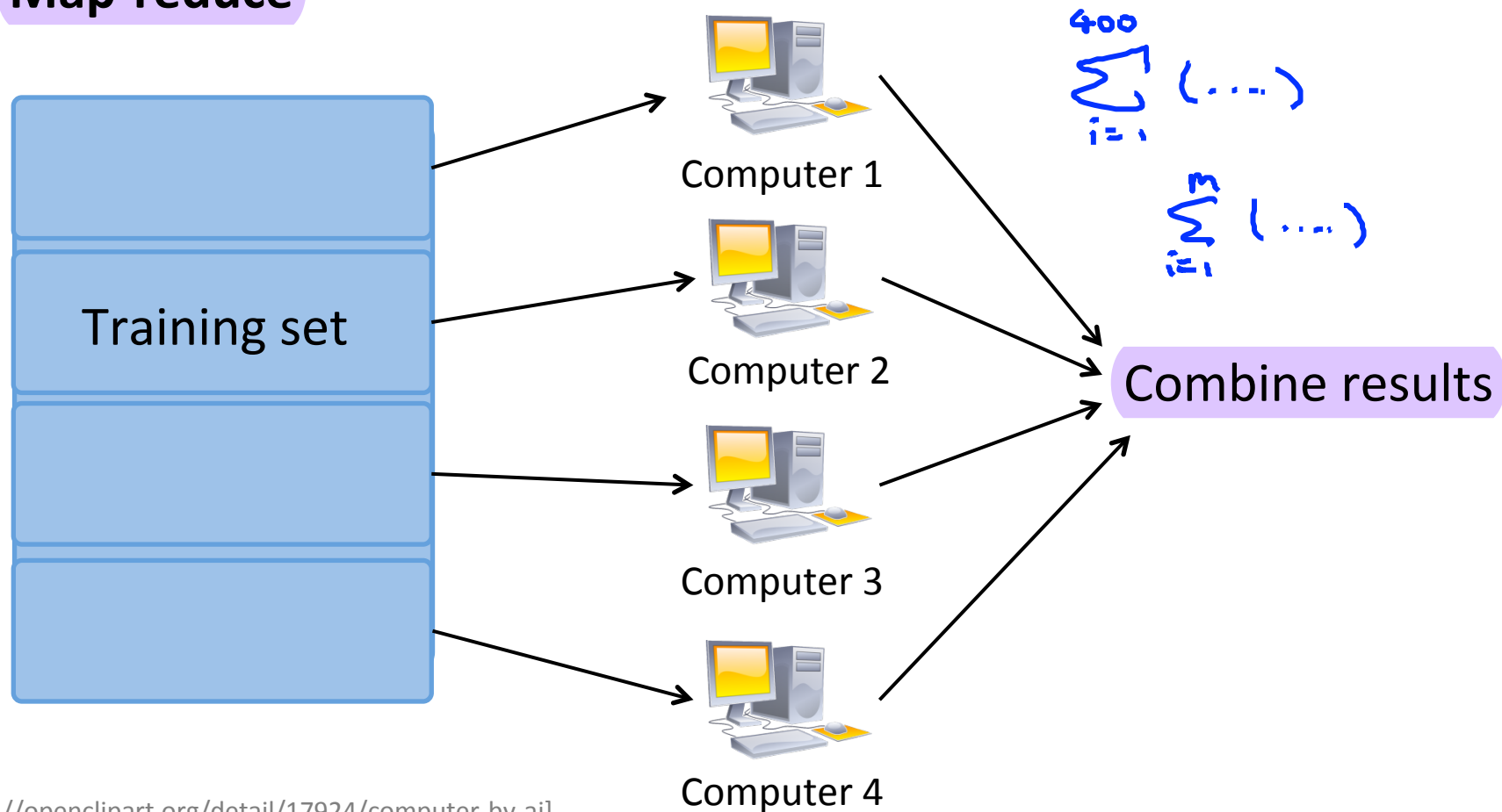
$$\text{temp}_j^{(4)} = \sum_{i=301}^{400} (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

Combine:

$$\begin{aligned} \theta_j &:= \theta_j \\ &- \alpha \frac{1}{400} ( \\ &\quad \text{temp}_j^{(1)} + \text{temp}_j^{(2)} \\ &\quad + \text{temp}_j^{(3)} + \text{temp}_j^{(4)} ) \end{aligned}$$

$$(j = 0, \dots, n)$$


# Map-reduce



## Map-reduce and summation over the training set

Many learning algorithms can be expressed as computing sums of functions over the training set.

E.g. for advanced optimization, with logistic regression, need:

$$\rightarrow \underline{J_{train}(\theta)} = -\frac{1}{m} \sum_{i=1}^m \underline{y^{(i)} \log h_{\theta}(x^{(i)}) - (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)}))}$$


$$\rightarrow \underline{\frac{\partial}{\partial \theta_j} J_{train}(\theta)} = \frac{1}{m} \sum_{i=1}^m \underline{(h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}}$$

$temp^{(i)}$

$temp_j^{(i)} \leftarrow$

## Multi-core machines

