Examples and Intuitions II

The $\Theta^{(1)}$ matrices for AND, NOR, and OR are:

$$AND:$$
$$\Theta^{(1)} = \begin{bmatrix} -30 & 20 & 20 \end{bmatrix}$$
$$NOR:$$
$$\Theta^{(1)} = \begin{bmatrix} 10 & -20 & -20 \end{bmatrix}$$
$$OR:$$
$$\Theta^{(1)} = \begin{bmatrix} -10 & 20 & 20 \end{bmatrix}$$

We can combine these to get the XNOR logical operator (which gives 1 if x_1 and x_2 are both 0 or both 1).

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \rightarrow \begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \end{bmatrix} \rightarrow \begin{bmatrix} a^{(3)} \end{bmatrix} \rightarrow h_\Theta(x)$$

For the transition between the first and second layer, we'll use a $\Theta^{(1)}$ matrix that combines the values for AND and NOR:

$$\Theta^{(1)} = \begin{bmatrix} -30 & 20 & 20 \\ 10 & -20 & -20 \end{bmatrix}$$

For the transition between the second and third layer, we'll use a $\Theta^{(2)}$ matrix that uses the value for OR:

$$\Theta^{(2)} = \begin{bmatrix} -10 & 20 & 20 \end{bmatrix}$$

Let's write out the values for all our nodes:

$$a^{(2)} = g(\Theta^{(1)} \cdot x)$$
$$a^{(3)} = g(\Theta^{(2)} \cdot a^{(2)})$$
$$h_\Theta(x) = a^{(3)}$$

And there we have the XNOR operator using a hidden layer with two nodes! The following summarizes the above algorithm:

Putting it together: x_1 XNOR x_2

$\rightarrow$ x_1 AND x_2

$\rightarrow$ (NOT x_1) AND (NOT x_2)

$\rightarrow$ x_1 OR x_2

$h_\Theta(x)$

x_1	x_2	$a_1^{(2)}$	$a_2^{(2)}$	$h_\Theta(x)$
0	0	0	1	1
0	1	0	0	0
1	0	0	0	0
1	1	1	0	1