Examples and Intuitions II

The $\Theta^{(1)}$ matrices for AND, NOR, and OR are:

$$\begin{aligned}&AND:\\&\Theta^{(1)} = \begin{bmatrix}-30 & 20 & 20\end{bmatrix}\\&NOR:\\&\Theta^{(1)} = \begin{bmatrix}10 & -20 & -20\end{bmatrix}\\&OR:\\&\Theta^{(1)} = \begin{bmatrix}-10 & 20 & 20\end{bmatrix}\end{aligned}$$

We can combine these to get the XNOR logical operator (which gives 1 if x_1 and x_2 are both 0 or both 1).

$$\begin{bmatrix}x_0 \\ x_1 \\ x_2\end{bmatrix} \rightarrow \begin{bmatrix}a_1^{(2)} \\ a_2^{(2)}\end{bmatrix} \rightarrow \begin{bmatrix}a^{(3)}\end{bmatrix} \rightarrow h_\Theta(x)$$

For the transition between the first and second layer, we'll use a $\Theta^{(1)}$ matrix that combines the values for AND and NOR:

$$\Theta^{(1)} = \begin{bmatrix}-30 & 20 & 20 \\ 10 & -20 & -20\end{bmatrix}$$

For the transition between the second and third layer, we'll use a $\Theta^{(2)}$ matrix that uses the value for OR:

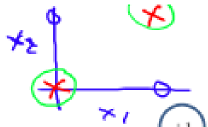
$$\Theta^{(2)} = \begin{bmatrix}-10 & 20 & 20\end{bmatrix}$$

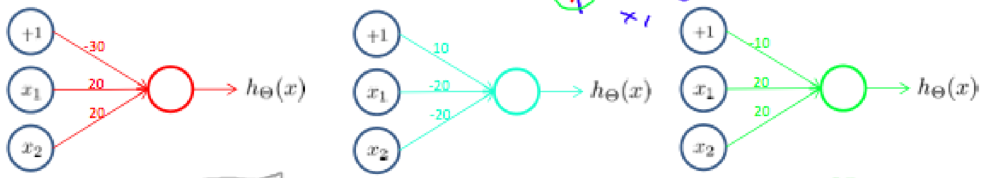
Let's write out the values for all our nodes:

$$\begin{aligned}&a^{(2)} = g(\Theta^{(1)} \cdot x)\\&a^{(3)} = g(\Theta^{(2)} \cdot a^{(2)})\\&h_\Theta(x) = a^{(3)}\end{aligned}$$

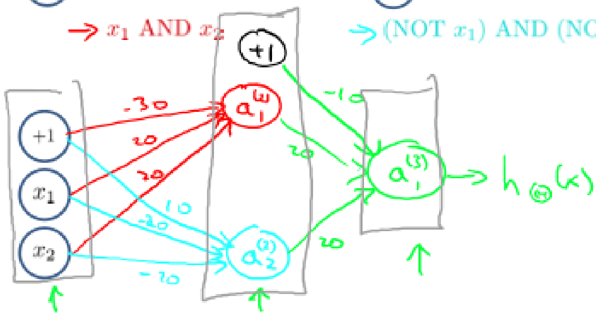
And there we have the XNOR operator using a hidden layer with two nodes! The following summarizes the above algorithm:

Putting it together: $x_1 \text{ XNOR } x_2$





$\rightarrow x_1 \text{ AND } x_2$ $\rightarrow (\text{NOT } x_1) \text{ AND } (\text{NOT } x_2)$ $\rightarrow x_1 \text{ OR } x_2$



x_1	x_2	$a_1^{(2)}$	$a_2^{(2)}$	$h_\Theta(x)$
0	0	0	1	1
0	1	0	0	0
1	0	0	0	0
1	1	1	0	1