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Optimization Algorithms

Understanding
exponentially
weighted averages

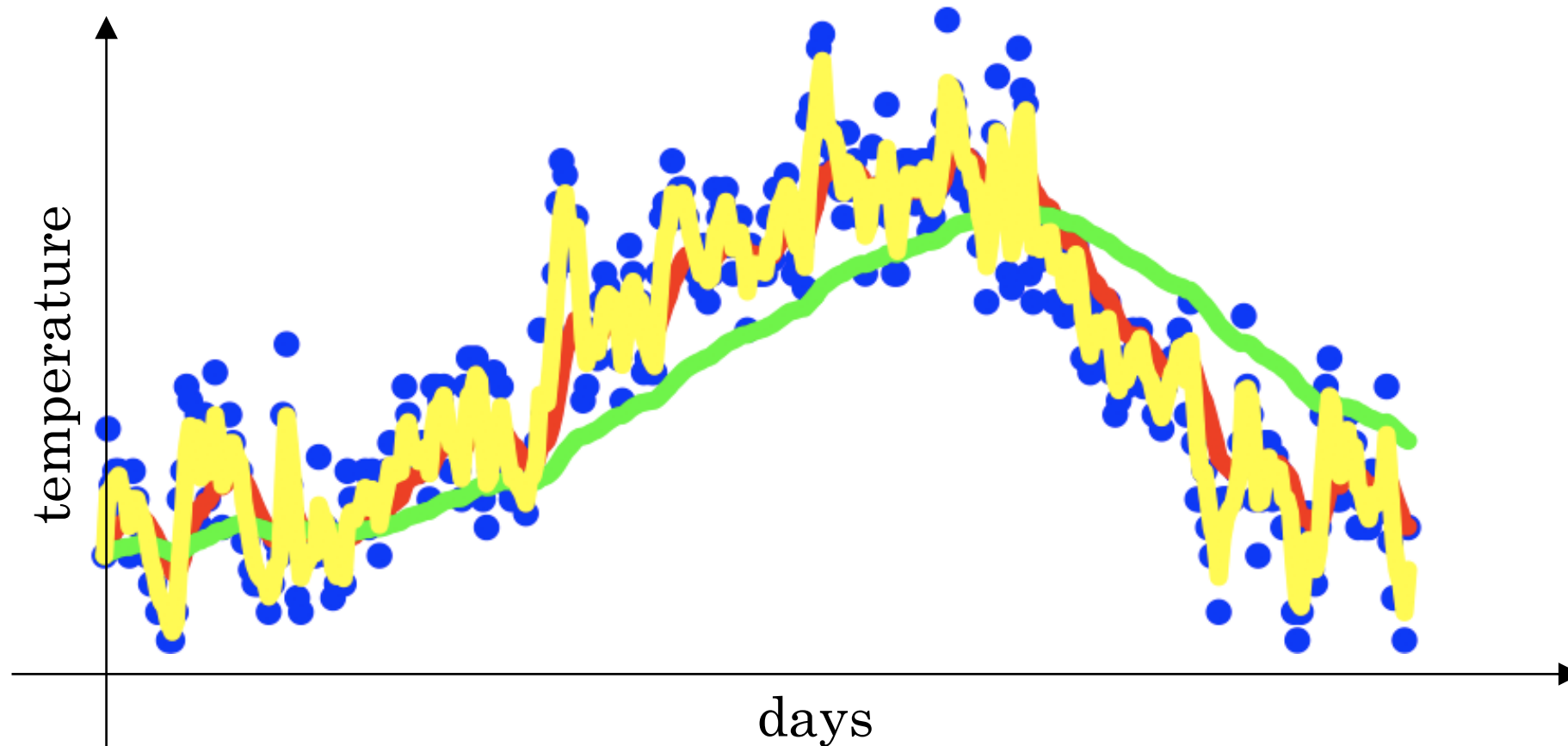
Exponentially weighted averages

$$v_t = \beta v_{t-1} + (1 - \beta) \theta_t$$

$$\beta = 0.9$$

$$0.98$$

$$0.5$$



Exponentially weighted averages

$$v_t = \beta v_{t-1} + (1 - \beta) \theta_t$$

$$v_{100} = 0.9 v_{99} + 0.1 \theta_{100}$$

$$v_{99} = 0.9 v_{98} + 0.1 \theta_{99}$$

$$v_{98} = 0.9 v_{97} + 0.1 \theta_{98}$$

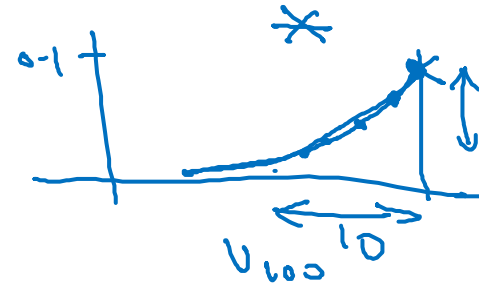
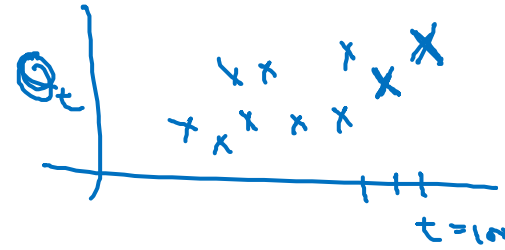
...

$$\begin{aligned} \rightarrow v_{100} &= 0.1 \theta_{100} + 0.9 \cancel{v_{99}} (0.1 \theta_{99} + 0.9 \cancel{v_{98}}) \\ &= \underbrace{0.1 \theta_{100}} + \underbrace{0.1 \times 0.9 \cdot \theta_{99}} + \underbrace{0.1 (0.9)^2 \theta_{98}} + \underbrace{0.1 (0.9)^3 \theta_{97}} + \underbrace{0.1 (0.9)^4 \theta_{96}} + \dots \end{aligned}$$

$$\underbrace{0.9^{10}} \approx \underbrace{0.35} \approx \frac{1}{e}$$

$$\frac{(1-\epsilon)^{1/\epsilon}}{0.9} \approx \frac{1}{e}$$

$$\epsilon = 0.02 \rightarrow \underbrace{0.98^{50}} \approx \frac{1}{e}$$



$$\approx \frac{1}{1-\beta}$$

$$\epsilon = 1 - \beta$$

$$0.1 \theta_{98} + 0.9 v_{97}$$

Implementing exponentially weighted averages

$$v_0 = 0$$

$$v_1 = \beta v_0 + (1 - \beta) \theta_1$$

$$v_2 = \beta v_1 + (1 - \beta) \theta_2$$

$$v_3 = \beta v_2 + (1 - \beta) \theta_3$$

...

An implementation which
saves memory.

$$V_\theta := 0$$

$$V_\theta := \beta v + (1 - \beta) \theta_1$$

$$V_\theta := \beta v + (1 - \beta) \theta_2$$

⋮

$$\rightarrow V_\theta = 0$$

Repeat {

Get next θ_t

$$\underline{V_\theta := \beta V_\theta + (1 - \beta) \theta_t} \leftarrow$$

}