

Decision Boundary

In order to get our discrete 0 or 1 classification, we can translate the output of the hypothesis function as follows:

$$h_{\theta}(x) \geq 0.5 \rightarrow y = 1$$

$$h_{\theta}(x) < 0.5 \rightarrow y = 0$$

The way our logistic function g behaves is that when its input is greater than or equal to zero, its output is greater than or equal to 0.5:

$$g(z) \geq 0.5$$

$$\text{when } z \geq 0$$

Remember.

$$z = 0, e^0 = 1 \Rightarrow g(z) = 1/2$$

$$z \rightarrow \infty, e^{-\infty} \rightarrow 0 \Rightarrow g(z) = 1$$

$$z \rightarrow -\infty, e^{\infty} \rightarrow \infty \Rightarrow g(z) = 0$$

So if our input to g is $\theta^T X$, then that means:

$$h_{\theta}(x) = g(\theta^T x) \geq 0.5$$

$$\text{when } \theta^T x \geq 0$$

From these statements we can now say:

$$\theta^T x \geq 0 \Rightarrow y = 1$$

$$\theta^T x < 0 \Rightarrow y = 0$$

The **decision boundary** is the line that separates the area where $y = 0$ and where $y = 1$. It is created by our hypothesis function.

Example :

$$\theta = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}$$

$$y = 1 \text{ if } 5 + (-1)x_1 + 0x_2 \geq 0$$

$$5 - x_1 \geq 0$$

$$-x_1 \geq -5$$

$$x_1 \leq 5$$

In this case, our decision boundary is a straight vertical line placed on the graph where $x_1 = 5$, and everything to the left of that denotes $y = 1$, while everything to the right denotes $y = 0$.

Again, the input to the sigmoid function $g(z)$ (e.g. $\theta^T X$) doesn't need to be linear, and could be a function that describes a circle (e.g. $z = \theta_0 + \theta_1 x_1^2 + \theta_2 x_2^2$) or any shape to fit our data.