

# Hypothesis Representation

We could approach the classification problem ignoring the fact that  $y$  is discrete-valued, and use our old linear regression algorithm to try to predict  $y$  given  $x$ . However, it is easy to construct examples where this method performs very poorly. Intuitively, it also doesn't make sense for  $h_\theta(x)$  to take values larger than 1 or smaller than 0 when we know that  $y \in \{0, 1\}$ . To fix this, let's change the form for our hypotheses  $h_\theta(x)$  to satisfy  $0 \leq h_\theta(x) \leq 1$ . This is accomplished by plugging  $\theta^T x$  into the Logistic Function.

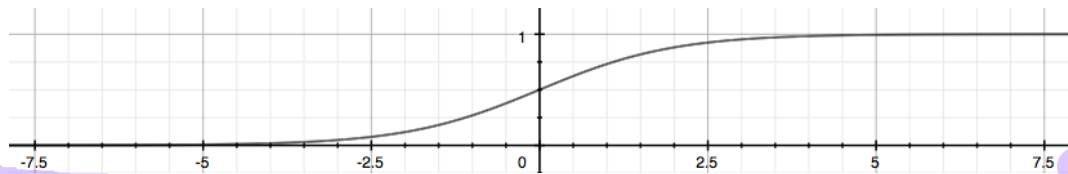
Our new form uses the "Sigmoid Function," also called the "Logistic Function":

$$h_\theta(x) = g(\theta^T x)$$

$$z = \theta^T x$$

$$g(z) = \frac{1}{1 + e^{-z}}$$

The following image shows us what the sigmoid function looks like:



The function  $g(z)$ , shown here, maps any real number to the  $(0, 1)$  interval, making it useful for transforming an arbitrary-valued function into a function better suited for classification.

$h_\theta(x)$  will give us the **probability** that our output is 1. For example,  $h_\theta(x) = 0.7$  gives us a probability of 70% that our output is 1. Our probability that our prediction is 0 is just the complement of our probability that it is 1 (e.g. if probability that it is 1 is 70%, then the probability that it is 0 is 30%).

$$h_\theta(x) = P(y = 1|x; \theta) = 1 - P(y = 0|x; \theta)$$
$$P(y = 0|x; \theta) + P(y = 1|x; \theta) = 1$$