MSc in Artificial Intelligence

Batch 17

IT 5845 - Mathematics for Artificial Intelligence

Tutorial 1

1. Find three vectors $\boldsymbol{u}$, $\boldsymbol{v}$, and $\boldsymbol{w}$ in $\mathbb{R}^3$ such that $\boldsymbol{w} = 3\boldsymbol{u}$, $\boldsymbol{v} = 2\boldsymbol{u}$ and $2\boldsymbol{u} + 3\boldsymbol{v} + 4\boldsymbol{w} = \begin{bmatrix} 20 \\ 10 \\ -25 \end{bmatrix}$.

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2. Let $\boldsymbol{u} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$, $\boldsymbol{x} = \begin{bmatrix} -7 \\ 1 \\ 2 \end{bmatrix}$, and $\boldsymbol{y} = \begin{bmatrix} 0 \\ 8 \\ 3 \end{bmatrix}$. Compute the following.

a) $(\boldsymbol{u} + \boldsymbol{v}) \cdot (\boldsymbol{x} + \boldsymbol{y})$

b) $((3\boldsymbol{u} \cdot \boldsymbol{x})\boldsymbol{v}) \cdot \boldsymbol{y}$

c) $\boldsymbol{u} \cdot \boldsymbol{x} - 4\boldsymbol{v} \cdot \boldsymbol{y}$

d) $\boldsymbol{u} \cdot \boldsymbol{x} + 3\boldsymbol{u} \cdot \boldsymbol{y} - \boldsymbol{v} \cdot \boldsymbol{y}$

e) $(2(\boldsymbol{v} + \boldsymbol{u}) \cdot \boldsymbol{u}) - 5\boldsymbol{u} \cdot \boldsymbol{y}$

f) $4\boldsymbol{u} \cdot \boldsymbol{x} + 6\,[\boldsymbol{v} \cdot (3\boldsymbol{x} - \boldsymbol{y})]$

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3. Calculate $\|\boldsymbol{u}\|$ for each of the following vectors.

a) $\boldsymbol{u} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$ b) $\boldsymbol{u} = \begin{bmatrix} 1 \\ 0 \\ -7 \end{bmatrix}$ c)$\boldsymbol{u} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$ d) $\boldsymbol{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \\ 3 \end{bmatrix}$ e) $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$

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4. For each pair of vectors $\boldsymbol{u}$, $\boldsymbol{v}$, calculate the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.

a) $\boldsymbol{u} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ b) $\boldsymbol{u} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ c) $\boldsymbol{u} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} 5 \\ -4 \\ 2 \end{bmatrix}$

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5. The angle between the vectors $\begin{bmatrix} 1 \\ 7 \\ b \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$ is given by $\arccos(1/3)$. Find b.

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6. For what values of a are the vectors $\begin{bmatrix} -6 \\ a \\ 2 \end{bmatrix}$ and $\begin{bmatrix} a \\ a^2 \\ a \end{bmatrix}$ orthogonal?

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7. Given the vectors $\boldsymbol{u} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ and $\boldsymbol{v} = \begin{bmatrix} 7 \\ -5 \\ 1 \end{bmatrix}$.

a) Find the unit vector in the direction of $\boldsymbol{u}$.

b) Find a unit vector perpendicular to both $\boldsymbol{u}$ and $\boldsymbol{v}$.

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8. Find the orthogonal projection of the vector $\boldsymbol{u} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ on the vector $\boldsymbol{v} = \begin{bmatrix} 4 \\ -4 \\ 7 \end{bmatrix}$.

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