Actuarial Pricing of Equity-Linked Insurance via Simulation

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Group Member Introduction

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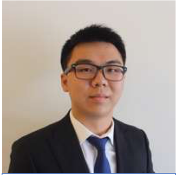

Wayne Tao
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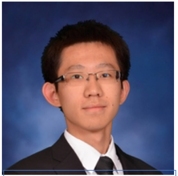

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Figure 1: Group Members

Agenda

- Variable Annuity Introduction
- Models
    - Decrement
    - Equity investment
    - (Risk-free) Discount Rate
- Methods Used
- R Shiny Demo
- Conclusion

Variable Annuity Introduction

There are a wide range of variable annuity products on the market, this project is focused on the **Guaranteed Minimum Death Benefit(GMDB)**.

- Traditional Annuity
  - An annuity is periodically payable upon survival
- Traditional Whole Life
  - A pre-determined single payment at death
- Guaranteed Minimum Death Benefit
  - the higher of ...
  - pre-determined monetary amount G
  - premiums invested into stock value (i.e. equity-linked)

Variable Annuity Introduction (Cont.)

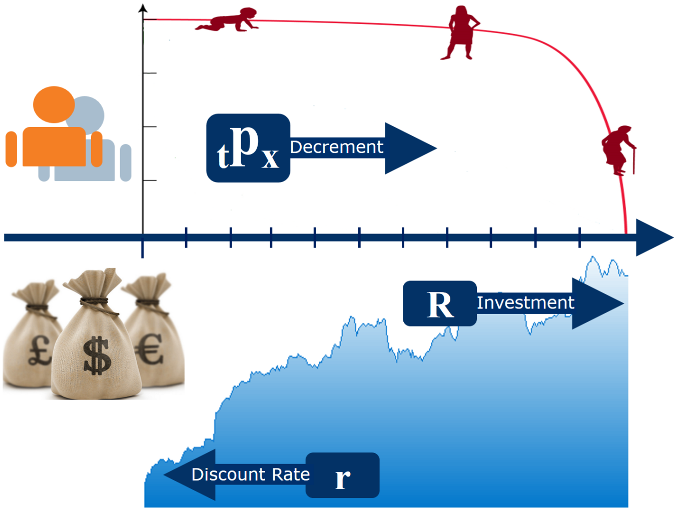

Figure 2: Variable Annuity Introduction

Models: Mortality Decrement

- $T_{(x)}$: Remaining life time random variable of (x), a life of age x

$$
\begin{aligned}
{}_t p_x &:= \text{Probability that a life (x) survive an extra t years} \\
&= \mathbb{P}(T_{(x)} > t)
\end{aligned}
$$

- Life table (Discretized continuous model)

x	l_x	d_x
0	100000	637
1	99363	45
2	99318	28

- Estimator for integral ages $\widehat{{}_t p_x} = \frac{l_{x+t}}{l_x}$
- Apply linear interpolation for fractional ages, aka UDD assumption

Models: Equity Investment Return

Geometric Brownian Motion Model A continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion (also called a Wiener process) with drift. [from Wiki]

Model Explanation

Let S_0 denote the last stock closing price of the training data
Let S_t denote the predicted stock closing price after t periods
Assume Stochastic Differential Equation (SDE):

$$d[lnS_t] = \frac{dS_t}{S_t} = \mu t + \sigma dW_t$$

Solution:

$$\frac{S_t}{S_0} \sim lgnorm(\mu - \frac{1}{2}\sigma^2, \sigma)$$

Models: (Risk-free) Discount Rate

Vasicek Short Rate Model A mathematical model describing the evolution of interest rates. It is a type of one-factor short rate model as it describes interest rate movements as driven by only one source of market risk. [from Wiki]

Model Explanation

Stochastic Differential Equation (SDE):

$$dr_t = a[b - r_t]dt + \sigma dW_t$$

- a : speed of reversion
- b : long term mean level
- σ: instantaneous volatility

Solution: $\mathbb{E}[r_t] = r_0 e^{-at} + b(1 - e^{-at})$, $Var[r_t] = \frac{\sigma^2}{2a}(1 - e^{-2at})$

STAT 428 Methods Used (Group 1)

- Random Number Generator
  - Methods: inverse CDF, Accept-Rejection, Metropolis- Hasting
  - Use case: simulation of three sources of risks based on parameters estimated

STAT 428 Methods Used (Group 2)

- Bootstrap
  - Method: regular bootstrap, jackknife
  - Use case: errors of estimators, accuracy of fit
- Optimization:
  - Methods: Newton-Raphson, BFGS (quasi-Newton)
  - Use case: MLE parameter estimates

R Shiny Demonstration

- Presenter: Haoen Cui (hcui10)

Conclusion