SCS 1307
Probability & Statistics

by

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Q1

A telephone operator receives calls independently of one another. If the probability of receiving no calls in one hour is 0.0325, find the probability that the company will receive exactly 2 calls in an hour.

Solution

A telephone operator receives calls independently of one another. If the probability of receiving no calls in one hour is 0.0325, find the probability that the company will receive exactly 2 calls in an hour.

Let X be the numb er of calls receive in an hour

X~Poisson(λ) P(X=x) = $\frac{e^{-\lambda}\lambda^{x}}{x!}$ given that P(X=0) = $\frac{e^{-\lambda}\lambda^{0}}{0!}$ = $e^{-\lambda}$ = 0.0325 → λ= 3.42

P(X=2) = $\frac{e^{-3.42}3.42^{2}}{2!}$ = 0.1912

Q2

Let $\bar{X}$ be the mean of a random sample with size 30 taken from a N(106,150) distribution and $\bar{Y}$ be the mean of a random sample of size 50 taken from a N(103, 200) distribution.

a) Write down the sampling distribution of $\bar{X}$ and $\bar{Y}$

b) If X and Y are independent what does the distribution that ($\bar{X}$ - $\bar{Y}$) follow?

c) Find the probability that $\bar{X}$ exceeding $\bar{Y}$ by at least 1.2

Solution

Let $\bar{X}$ be the mean of a random sample with size 30 taken from a N(106,150) distribution and $\bar{Y}$ be the mean of a random sample of size 50 taken from a N(103, 200) distribution.

a) Write down the sampling distribution of $\bar{X}$ and $\bar{Y}$

$\bar{X}$ ~ N(106, 150/30) and $\bar{Y}$ ~ N(103, 200/50)

b) If X and Y are independent what does the distribution that ($\bar{X}$ - $\bar{Y}$) follow?

($\bar{X}$ - $\bar{Y}$) ~ N(3, 5+4) = N(3,9)

c) Find the probability that $\bar{X}$ exceeding $\bar{Y}$ by at least 1.2

P(($\bar{X}$ - $\bar{Y}$) >1.2) = P(Z > $\frac{1.2 - 3}{3}$) = 1 - P(Z< -0.6)

Q3

The probability that a birdwatcher see an eagle on any given day is 1/8. It is assumed that this probability is unaffected by whether he has seen an eagle on any other day. What is the probability that the birdwatcher first sees an eagle on the third day?

Solution

The probability that a birdwatcher see an eagle on any given day is 1/8. It is assumed that this probability is unaffected by whether he has seen an eagle on any other day. What is the probability that the birdwatcher first sees an eagle on the third day?

p= 1/8 and q = 7/8

X – the day he sees an eagle for the first time: X~ Geometric(1/8)

$P(X=3) = (\frac{7}{8})^2 * \frac{1}{8}$

Q4

Three coins are thrown. If one head turns up , Rs1.00 is paid. If two heads turn up , Rs 3.00 is paid , and if three heads turn up Rs 5.00 is paid If the game is to be considered as fair what should be the penalty if no heads turn up?

Solution

Three coins are thrown. If one head turns up , Rs1.00 is paid. If two heads turn up , Rs 3.00 is paid , and if three heads turn up Rs 5.00 is paid If the game is to be considered as fair what should be the penalty if no heads turn up?

To determine the penalty that makes the game **fair**, we need to calculate the **expected payout** and then set the penalty equal to that amount so the average net gain is zero.

Let x be the penalty for getting no heads.

$$E = \left(\frac{1}{8}\right)(-x) + \left(\frac{3}{8}\right)(1) + \left(\frac{3}{8}\right)(3) + \left(\frac{1}{8}\right)(5)$$

$$E = -\frac{x}{8} + \frac{3}{8} + \frac{9}{8} + \frac{5}{8} = -\frac{x}{8} + \frac{17}{8}$$

For the game to be fair, expected value must be zero:

$$-\frac{x}{8} + \frac{17}{8} = 0 \Rightarrow x = 17$$

Q5

Five identically shaped discs are in a bag; two of them are black and the rest is white. Discs are drawn at random from the bag in turn and not replaced. Let X be the number of discs drawn up to and including the first black one.

(i) list the values of X and the associated theoretical probabilities.

(ii) Calculate the mean value of X.

Solution

5 discs total: 2 black, 3 white

Discs are drawn **without replacement**

X is the number of draws **up to and including** the first black disc

X=1,2,3,4

X	P(X=x)
1	2/5
2	3/5*2/4
3	3/5*2/4*2/3
4	3/5*2/4*1/3*2/2