SCS 1302 – Discrete Mathematics

Tutorial 2

1. Let $P(x)$ be the propositional function (or the predicate) "the word x contains the letter a", where the domain of discourse is the set of all English words. What are these truth values?
   a. $P(\text{orange})$
   b. $P(\text{lemon})$
   c. $P(\text{true})$
   d. $P(\text{false})$
2. State the value of x after the statement if $P\ (x)$, then $x := 1$ is executed, where $P\ (x)$ is the propositional function (or the predicate) "$x > 1$," if the value of x when this statement is reached is
   a. $x = 0$
   b. $x = 1$
   c. $x = 2$
3.
   a. Let $N(x)$ be the propositional function (or the predicate) "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.
      i. $\exists x N(x)$
      ii. $\forall x N(x)$
      iii. $\neg\exists x N(x)$
      iv. $\exists x \neg N(x)$
      v. $\neg\forall x N(x)$
      vi. $\forall x \neg N(x)$
   b. Translate these predicates into English, where $R(x)$ is the propositional function "x is a rabbit" and $H(x)$ is the propositional function "x hops" and the domain consists of all animals.
      i. $\forall x(R(x) \rightarrow H(x))$

ii. $\forall x(R(x) \land H(x))$

iii. $\exists x(R(x) \rightarrow H(x))$

iv. $\exists x(R(x) \land H(x))$

4. Let $C(x)$ be the propositional function "x has a cat," let $D(x)$ be the propositional function "x has a dog," and let $F(x)$ be the propositional function "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

a. A student in your class has a cat, a dog, and a ferret.

b. All students in your class have a cat, a dog, or a ferret.

c. Some student in your class has a cat and a ferret, but not a dog.

d. No student in your class has a cat, a dog, and a ferret.

e. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

5.

a. Suppose that the domain of the propositional function $P(x)$ consists of the integers $-2, -1, 0, 1$, and 2. Write out each of these propositions using disjunctions, conjunctions, and negations.

i. $\exists x P(x)$

ii. $\forall x P(x)$

iii. $\exists x \neg P(x)$

iv. $\forall x \neg P(x)$

v. $\neg \exists x P(x)$

vi. $\neg \forall x P(x)$

b. Suppose that the domain of the propositional function $P(x)$ consists of $-5, -3, -1$, 1, 3, and 5. Express these statements (propositions) without using quantifiers, instead using only negations, disjunctions, and conjunctions.

i. $\exists x \neg P(x)$

ii. $\forall x P(x)$

iii. $\forall x((x \neq 1) \rightarrow P(x))$

iv. $\exists x((x \geq 0) \land P(x))$

v. $\exists x(\neg P(x)) \wedge \forall x((x < 0) \rightarrow P(x))$

6. For each of these statements find a domain for which the statement is true and a domain for which the statement is false.
   a. Everyone speaks Hindi.
   b. There is someone older than 21 years.
   c. Every two people have the same first name.
   d. Someone knows more than two other people.
7. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.
   a. Everyone in your class has a cellular phone.
   b. Somebody in your class has seen a foreign movie.
   c. There is a person in your class who cannot swim.
   d. All students in your class can solve quadratic equations.
   e. Some student in your class does not want to be rich.
8. Express each of these statements using logical operators, predicates, and quantifiers.
   a. Some propositions are tautologies.
   b. The negation of a contradiction is a tautology.
   c. The disjunction of two contingencies can be a tautology.
   d. The conjunction of two tautologies is a tautology.
9. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.
   a. $\exists x\, P(x, 3)$
   b. $\forall y\, P(1, y)$
   c. $\exists y \neg P(2, y)$
   d. $\forall x \neg P(x, 2)$
10.

a. Determine whether $\forall x(P\ (x) \rightarrow Q(x))$ and $\forall x P\ (x) \rightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

b. Determine whether $\forall x(P\ (x) \leftrightarrow Q(x))$ and $\forall x\ P\ (x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

c. Show that $\exists x(P\ (x) \lor Q(x))$ and $\exists x P\ (x) \lor \exists x Q(x)$ are logically equivalent.

d. Show that $\forall x P\ (x) \lor \forall x Q(x)$ and $\forall x(P\ (x) \lor Q(x))$ are not logically equivalent.

e. Show that $\exists x P\ (x) \land \exists x Q(x)$ and $\exists x(P\ (x) \land Q(x))$ are not logically equivalent.