#Q	Mark
1	
2	
3	
4	
5	
6	
Total	

SCS 1308 – Foundation of Algorithms

In-Class Assignment 1/ (2 hours)

Answer all the questions.

Index no : ……

Consider the following equations when considering masters theorem.

T(n) is a monotonically increasing function as follows:

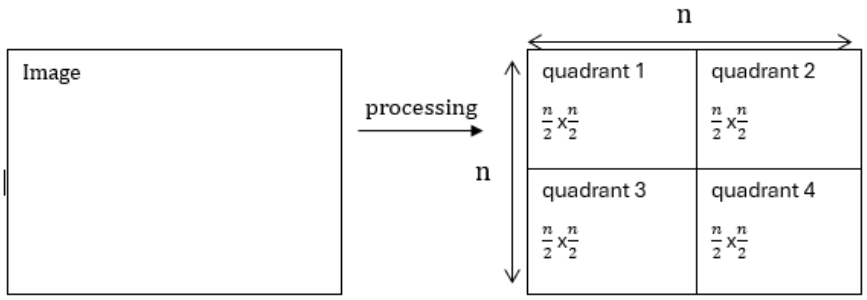
$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$$T(1) = c$$

Where $a \geq 1, b \geq 2, c > 0, if\ f(n) is\ \Theta(n^d) where\ d \geq 0\ then,$

$$T(n) = \begin{cases} \Theta(n^d)\ if\ a < b^d \\ \Theta(n^d log\ n)\ if\ a = b^d \\ \Theta(n^{log_b^a})\ if\ a > b^d \end{cases}$$

1. High resolution image is divided into 4 quadrants for processing. Each quadrant is processed recursively, and merging takes n^2 time due to pixel blending. Consider the dimensions of an image .



A. Write the recurrence equation for the above scenario considering recursive and non-recursive terms. Your final answer should be given in T(n) terms.

[5 marks]

ANSWER IN THIS BOX

B. Solve the recurrence equation using the iteration method. Ensure that your solution includes a detailed tree structure with the following components to achieve full marks: the root, depth, and leaf formations at each level.

[10 marks]

ANSWER IN THIS BOX

C. Verify solution using substitution method.

[5 marks]

ANSWER IN THIS BOX

2. A network splits data packet routing into 3 smaller subproblems. Processing each subproblem takes n time, and merging takes n^2 time.

    A. Write the recurrence equation for the above scenario considering recursive and non-recursive terms. Your final answer should be given in T(n) terms.

    [5 marks]

ANSWER IN THIS BOX

B. Solve the recurrence equation using the iteration method. Ensure that your solution includes a detailed tree structure with the following components to achieve full marks: the root, depth, and leaf formations at each level.

[10 marks]

ANSWER IN THIS BOX

C. Verify solution using substitution method.

[5 marks]

ANSWER IN THIS BOX

3. A deep learning model splits its dataset into two halves for training. Each half is trained recursively and combining results (using gradient merging) takes nlogn time.

   A. Write the recurrence equation for the above scenario considering recursive and non-recursive terms. Your final answer should be given in T(n) terms.

[5 marks]

ANSWER IN THIS BOX

   B. Solve the recurrence equation using the iteration method. Ensure that your solution includes a detailed tree structure with the following components to achieve full marks: the root, depth, and leaf formations at each level.

[10 marks]

ANSWER IN THIS BOX

C. Verify solution using substitution method. [5 marks]

ANSWER IN THIS BOX

4. Consider the following program that reverse an array of integers in place. Prove the correctness of the loop invariant for initialization, maintenance and termination phases. Provide a brief explanation for each phases using 1-3 sentences **only**.

```
function reverseArray(arr, n):
    left ← 0
    right ← n - 1

    while left < right:
        // Swap the elements at 'left' and 'right'
        temp ← arr[left]
        arr[left] ← arr[right]
        arr[right] ← temp

        // Move the pointers closer to the center
        left ← left + 1
        right ← right - 1

    return arr
```

a. What is the loop invariant of the above code ? [3 marks]

ANSWER IN THIS BOX

b. Show that loop invariant hold for initialization, maintenance and termination phases. [7marks]

ANSWER IN THIS BOX

5. The following program shows whether a given number n is prime.

```
function isPrime(n):
    if n ≤ 1:
        return false  // Numbers less than or equal to 1 are not prime

    for i from 2 to √n:
        if n % i == 0:
            return false  // n is divisible by i, so it's not prime

    return true  // n is prime if no divisors are found
```

a. What is the loop invariant of the above code ? [3 marks]

ANSWER IN THIS BOX

b. Show that loop invariant hold for initialization, maintenance and termination phases. [7 marks]

ANSWER IN THIS BOX

6. Multiple-choice questions: Please note that some questions may have multiple correct answers. Each incorrect answer will incur a penalty of -0.25 points. Minimum mark for question 6 is 0. Mark your answer in the provided boxes in page 10. [20 marks = (2 marks x 10]]

1) A file of size n MB is split into two parts and transmitted over separate channels. The first part is $2/3$ of the file size, and the second is $1/3$. Each transmission takes time proportional to the file size, and splitting takes O(n). The recurrence relation is:

$$T(n) = T(\frac{2n}{3}) + T(\frac{n}{3}) + cn$$

What is the asymptotic time complexity?
A. O(nlogn)
B. $O(n\log_2 n)$
C. $O(n^2)$
D. O(n)

2) You need to merge n sorted lists, where the size of each list decreases by half at every step. The merging takes O(k) time for two lists of size k. What is the recurrence and solution?

A. Recurrence: T(n)=2T(n/2)+cn, Complexity: O(nlogn)
B. Recurrence: T(n)=T(n/2)+cn, Complexity: O(n)
C. Recurrence: T(n)=T(n/2)+n, Complexity: O(nlogn)
D. Recurrence: $T(n)=T(n/2)+cn^2$, Complexity: $O(n^2)$

3) In parallel matrix multiplication, a n x n matrix is divided into 8 submatrices, and each multilication takes O(n2). The recurrence is $T(n) = 8T(n/2) + O(n^2)$. What is the complexity ?

A. $O(n^3)$
B. $O(n^2)$
C. $O(n^{log_2^8})$
D. $O(n^{log_2^7})$

4) To merge k sorted files of size n, splitting and merging each takes O(nlogk). The recurrence is:
T(n)=kT(n/k)+O(nlogk) What is the complexity?

A. O(nlogn)
B. $(nlog^2n)$
C. $O(n^2)$
D.O(n log n log k)

5) In a cryptographic hash function, a loop processes chunks of data to compute the hash. What invariants ensure correctness?

A. Each chunk contributes uniquely to the hash.
B. Chunks are processed in the same order for the same input.
C. The final hash size is fixed regardless of input size.
D. All chunks must be equal in size.

6) A loop iterates through tasks to schedule them in a time slot. The invariant ensures no overlap between tasks. What additional conditions might ensure correctness?

A. Tasks are scheduled in the order of their deadlines.
B. A task is only scheduled if it fits within the time slot.
C. Unscheduled tasks are moved to the next time slot.
D. The algorithm terminates when all tasks are considered.

7) You are tasked with writing a loop to sort an array A[1…n] in ascending order. Which of the following could be valid loop invariants?

A. The subarray A[1…i] is sorted at the i-th iteration.
B. The largest element in A[i…n] is always at A[i].
C. No element in A[1…i] is greater than any element in A[i+1…n].
D. All elements are sorted when the loop exits.

8) A loop checks if a string of parentheses is balanced. What invariants hold?

A. The count of open parentheses is non-negative at each step.
B. The total count of open and closed parentheses matches.
C. The string is balanced at any intermediate step.
D. The algorithm terminates with a count of zero.

9) A loop calculates the n-th Fibonacci number iteratively. What invariants ensure correctness?

A. At step i, the variable fib1 stores F(i−1).
B. At step i, the variable fib2 stores F(i).
C. The variables fib1 and fib2 always hold consecutive Fibonacci numbers.
D. The algorithm terminates after calculating F(n)

10) When iterating through an array to find the maximum element, which invariants ensure correctness?

A. The variable max_so_far is greater than or equal to any element in A[1…i].
B. The variable max_so_far is updated whenever a larger element is found.
C. After the loop exits, max_so_far is the maximum element in A[1…n].
D. No element before iii is greater than max_so_far.

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