SCS 1204 – Discrete Mathematics
Tutorial 4

1. Prove that there is no positive integer n such that $n^2 + n^3 = 100$.

2. **a.** Prove that for each positive integer n such that $n \leq 4$, $(n+1)^3 \geq 3^n$.

b. Prove that for each positive integer n such that $1 \leq n \leq 4$, $n^2+1 \geq 2^n$.

3. Prove that for all $x \in \mathbb{R}$, $|x + 3| - x > 2$.

4. Prove that for any two integers a and b, if ab is even, then at least one of a and b is even.
(Hint: Use a proof by contraposition.)

5. Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

6. Prove or disprove the following statements:

(i) If n is an integer and $n^3 + 5$ is odd, then n is even.

(ii) If x is a non-zero real number, then $x^2 + \frac{1}{x^2} \geq 2$.

(iii) If x is a real number, then x^2 is a positive real number.

(iv) Every positive integer is the sum of the squares of three integers.

(v) Show that if x and y are integers and both xy and $x + y$ are even, then both x and y are even. (Hint: use a proof by cases.)