Boolean Algebraic Expression Simplification

SCS 1205
Computer Systems

Axioms

- $0.0 = 0$
- $0.1 = 0$
- $1.0 = 0$
- $1.1 = 1$

- $0 + 0 = 0$
- $0 + 1 = 1$
- $1 + 0 = 1$
- $1 + 1 = 1$

- $0' = 1$
- $1' = 0$

- Boolean algebra has its own axioms and laws which is foundation of boolean algebra.
- Those axioms and rules can be used to simplify the Boolean expressions.
- We assume these axioms are true, we do not need to prove them.

Annulment Law

- A.0 = 0
- A + 1 = 1
- Result is fixed irrespective of whether A is 1 or 0

A.0 = 0

A	0	A.0	0
1	0	0	0
0	0	0	0

A+1 = 1

A	1	A+1	1
1	1	1	1
0	1	1	1

Identity Law

- A.1 = A
- A + 0 = A
- Result is identical to A

A.1 = A

A	1	A.1	A
1	1	1	1
0	1	0	0

A+0 = A

A	0	A+0	A
1	0	1	1
0	0	0	0

Idempotent Law

- A.A=A
- A+A = A
- Result is identical to A

A	A	A.A	A
1	1	1	1
0	0	0	0

A	A	A+A	A
1	1	1	1
0	0	0	0

Complement Law

- A.A′ = 0
- A + A′ = 1
- Result is fixed irrespective of whether A is 1 or 0

A.A′ = 0

A	A’	A.A’	0
1	0	0	0
0	1	0	0

A+A′ = 1

A	A’	A+A’	1
1	0	1	1
0	1	1	1

Commutative Law

- A.B = B.A
- A + B = B + A
- Can change terms without affecting the result

A	B	A.B	B.A
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

A.B = B.A

Commutative Law contd.

A	B	A+B	B+A
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

$$A+B = B+A$$

Double Negation Law

A	A’	A”	A
1	0	1	1
0	1	0	0

$$A'' = A$$

Distributive Law

- A(B + C) = (A.B) + (A.C)
- A + (B.C) = (A + B) .(A + C)
- Can expand terms

A(B + C) = (A.B)+ (A.C)

A	B	C	B+C	A(B+C)	A.B	A.C	(A.B) + (A.C)
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Distributive Law contd.

$$A + (B.C) = (A + B) .(A + C)$$

A	B	C	B.C	A + (B.C)	A+B	A+C	(A+B).(A+C)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Redundancy Law (Absorption Law)

- $A + A.B = A$
- $A(A + B) = A$

A + A.B = A

A	B	A.B	A+A.B	A
0	0	0	0	0
0	1	0	0	0
1	0	0	1	1
1	1	1	1	1

Redundancy Law (Absorption Law)

$$A(A + B) = A$$

A	B	A+B	A(A+B)	A
0	0	0	0	0
0	1	1	0	0
1	0	1	1	1
1	1	1	1	1

De Morgan’s Law (De Morgan’s Theorem)

- (A.B)′ = A′ + B′
- (A+B)′ = A′.B′

(A.B)′ = A′ + B′

A	B	A.B	(A.B)’	A’	B’	A’+B’
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

De Morgan's Law (De Morgan's Theorem) contd.

A	B	A+B	(A+B)'	A'	B'	A'.B'
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

(A+B)' = A'.B'

Associative Law

- (A+B) + C = A + (B+C)
- (AB)C=A(BC)

(A+B) + C = A + (B+C)

A	B	C	A+B	(A+B)+C	B+C	A+(B+C)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Associative Law contd.

(A.B).C=A.(B.C)

A	B	C	A.B	(AB)C	B.C	A(BC)
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

Simplification Examples

1. **C + (BC)'**

   = C + B' + C' (DeMorgan's Law)

   = C + C' + B' (Commutative Law)

   = 1 + B' (Complement Law)

   = 1 (Annulment Law)

2. $(AB)'(A' + B)(B' + B)$

$= (AB)'(A' + B)(1)$ (Complement Law)

$= (AB)'(A' + B)$ (Identity Law)

$= (A' + B')(A' + B)$ (De Morgan's Law)

$= A'.A' + A'B + B'A' + B'B$ (Distributive Law)

$= A' + A'B + B'A' + B'B$ (Idempotent Law)

$= A' + A'B + B'A' + 0$ (Complement Law)

$= A' + A'B + B'A'$ (Identity Law)

$=A' + A'(B+B')$ (Distributive Law)

$= A' + A'(1)$ (Complement Law)

$= A' + A'$ (Identity Law)

$= A'$ (Idempotent Law)

3. $(A+C)(AD+AD')+AC+C$

$= (A+C)A(D+D') + AC + C$ (Distributive Law)

$= (A+C)A(1) + AC + C$ (Complement Law)

$= (A+C)A + AC + C$ (Identity Law)

$= A((A+C)+C)+C$ (Distributive Law)

$= A(A + (C+C)) + C$ (Associate Law)

$= A(A+C) + C$ (Identity Law)

$= AA + AC + C$ (Distributive Law)

$= A+AC+C$ (Idempotent Law)

$= A + C(A+1)$ (Distributive Law)

$= A+C(1)$ (Annulment Law)

$= A+C$ (Identity Law)