



University of Colombo School of Computing

SCS 1304 - Problem Solving Strategies and Computation Approaches

Lab Sheet 09

Brute Force & Divide-and-Conquer

Answer the following questions and you are allowed to use non programmable calculators.

- A. An image processing task where you need to perform matrix multiplications frequently. Given the two matrices below, representing pixel values in grayscale images

Matrix A (2x2): $\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix}$

Matrix B (2x2): $\begin{bmatrix} 5 & 7 \\ 6 & 2 \end{bmatrix}$

Use Strassen's matrix multiplication algorithm to compute the product of matrices A and B. Show all intermediate steps, including the computation of the seven Strassen products.

- B. In a signal processing application, you are required to multiply two large matrices frequently. Consider the following matrices

Matrix C (4x4): $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

Matrix D (4x4): $\begin{bmatrix} 16 & 15 & 14 & 13 \\ 12 & 11 & 10 & 9 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}$

Apply Strassen's algorithm to multiply matrices C and D. Start by dividing each matrix into four 2x2 submatrices. Then, compute the Strassen products and use them to find the final matrix product.

C. You are a software engineer working on an e-commerce platform that handles a large volume of transactions daily. Your task is to optimize the processing of these transactions by sorting them based on the time of occurrence. Given that the dataset contains 10 thousand transaction records, you decide to implement the Merge Sort algorithm, which is based on the Divide and Conquer strategy.

1. Describe how the Divide and Conquer approach is utilized in Merge Sort to sort the transaction records. Provide a step-by-step explanation using a smaller dataset example [50, 23, 9, 18, 61, 32].
2. Calculate the total number of comparisons that Merge Sort would require to sort a dataset of 10 thousand elements. Use the formula $T(n) = n * \log_2(n)$ to estimate the complexity.
3. Suppose the transaction records are spread across different servers. How would you modify the Merge Sort algorithm to work efficiently in a distributed environment?

D. A logistics company is using Quick Sort to organize the departure times of 2,000 trucks to ensure timely delivery. Meanwhile, the company also needs to solve a routing problem using a Divide and Conquer strategy to break down the large geographic area into manageable zones for delivery optimization.

1. Explain the role of the pivot in the Quick Sort algorithm and how it determines the partitioning of the array.
2. Considering the worst-case scenario, calculate the number of comparisons required to sort the list of 2,000 delivery times.
3. Describe how the Divide and Conquer approach can be applied to optimize the delivery routes. Provide an example of how the region might be divided and the benefits of this approach.
4. If the region is divided into 4 zones, each requiring 500 comparisons to optimize, calculate the total number of comparisons needed for the entire region.

Quiz

01. What is the key idea behind the Divide and Conquer strategy?

- a) To solve problems by directly finding the solution
- b) To break the problem into smaller subproblems, solve each subproblem, and combine their solutions
- c) To iterate through all possible solutions
- d) To guess and check until a solution is found

02. In which case does the binary search algorithm perform the best?

- a) When the list is already sorted
- b) When the list is unsorted
- c) When the list is partially sorted
- d) When the list is very small

03. Which of the following algorithms is based on the Divide and Conquer principle?

- a) Merge Sort
- b) Bubble Sort
- c) Linear Search
- d) Insertion Sort

04. What is the main advantage of the Merge Sort algorithm?

- a) It uses less memory
- b) It guarantees $O(n \log n)$ time complexity
- c) It is easier to implement
- d) It works well with small datasets

- 05.** Which of the following is an essential condition for applying the Divide and Conquer strategy?
- a) The problem can be broken down into smaller independent subproblems
 - b) The problem must be solved as a whole
 - c) The problem must have a linear solution
 - d) The problem must be complex and unsolvable by other methods
- 06.** In Merge Sort, what happens after dividing the array into smaller subarrays?
- a) The array is discarded
 - b) The subarrays are recursively sorted and merged
 - c) The smallest elements are directly sorted
 - d) The subarrays are compared with each other
- 07.** Which of the following scenarios best fits the application of the Divide and Conquer approach?
- a) Searching for an item in an unordered list
 - b) Sorting a large dataset efficiently
 - c) Iterating over a small array
 - d) Calculating the factorial of a number
- 08.** How does Divide and Conquer differ from Dynamic Programming?
- a) Divide and Conquer is iterative, while Dynamic Programming is recursive
 - b) Divide and Conquer solves subproblems independently, while Dynamic Programming solves overlapping subproblems
 - c) Divide and Conquer is more efficient than Dynamic Programming
 - d) Dynamic Programming does not use subproblems

- 09.** You are tasked with finding a specific word in a dictionary. Which algorithm would be most appropriate for this task?
- a) Linear Search b) Bubble Sort
 - c) Binary Search d) Quick Sort
- 10.** When designing a new distributed file system, how can the Divide and Conquer strategy be applied to enhance performance?
- a) By handling each file independently
 - b) By dividing the data across multiple nodes and processing them in parallel
 - c) By using a single node for all processing
 - d) By using a brute-force approach to data processing
- 11.** You are given a sorted array of 1,000,000 elements. How many comparisons would the Binary Search algorithm need, in the worst case, to find an element?
- a) 10 b) 20 c) 50 d) 100
- 12.** In Merge Sort, how many comparisons are needed to sort an array of 32 elements?
- a) 32 b) 160 c) 128 d) 64
- 13.** If Quick Sort is applied to a sorted array of 100 elements, what is the worst-case number of comparisons?
- a) 4950 b) 1000 c) 5050 d) 1500
- 14.** You need to find the 50th smallest element in an unsorted array of 100 elements using a Divide and Conquer approach (like Quickselect). What is the expected time complexity?
- a) $O(n)$ b) $O(n \log n)$
 - c) $O(\log n)$ d) $O(n^2)$

15. If you apply Merge Sort to a dataset of size 64, how many merge operations will occur?

- a) 32 b) 63 c) 64 d) 128

16. You are using Divide and Conquer to solve a matrix multiplication problem using Strassen's algorithm. If the matrices are of size 128x128, how many subproblems will Strassen's algorithm create?

- a) 7 b) 49 c) 128 d) 14

17. When using Binary Search to find an element in an array of 1,048,576 elements, how many levels deep would the search go in the worst case?

- a) 10 b) 20 c) 30 d) 40

18. If you use the Divide and Conquer approach to compute the n th Fibonacci number using matrix exponentiation, what will be the time complexity?

- a) $O(n)$ b) $O(\log n)$
c) $O(n \log n)$ d) $O(n^2)$

19. Suppose you are sorting a list of 128 numbers using a Divide and Conquer algorithm (Merge Sort). How many times will the list be divided before starting to merge?

- a) 7 b) 6 c) 8 d) 128

20. If a Divide and Conquer algorithm reduces a problem of size n into 4 subproblems each of size $n/2$, what is the time complexity of the algorithm?

- a) $O(n^2)$ b) $O(n \log n)$
c) $O(n^3)$ d) $O(n^4)$