SCS 1307
Probability & Statistics

by

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Normal Distribution

Normal distribution is the most important continuous distribution in statistics. Many measured quantities in the natural sciences follow a normal distribution.

eg. heights, masses, ages, examination results.

p.d.f of Normal Variable

A continuous random variable X having p.d.f

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

is said to have a normal distribution with mean μ and variance σ^2.

μ and σ^2 are the parameters of the distribution.

The distribution is bell shaped and symmetrical about $x = \mu$.

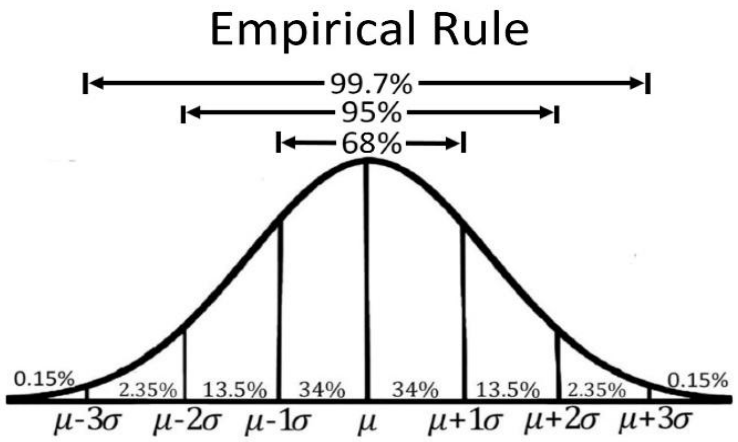
Expectation and Variance

If $X \sim N(\mu, \sigma^2)$ then

$$E(X) = \mu \text{ and}$$

$$Var(X) = \sigma^2$$

Empirical Rule



Area under the total curve is 1

Approximately 68% of the distribution lies within $\pm$1 σ of the mean.

Approximately 95% of the distribution lies within $\pm$2 σ of the mean.

Approximately 99.8% of the distribution lies within $\pm$3 σ of the mean.

The Standard Normal Distribution

Consider the random variable Z where $Z = \frac{(X-\mu)}{\sigma}$

Now if the random variable X has a normal distribution with mean μ and variance σ^2, then the random variable Z has a standard normal distribution with mean 0 and variance 1.

The p.d.f for Z, Ø(X)

The p.d.f. of the standard normal variable X is denoted by Ø(X) where

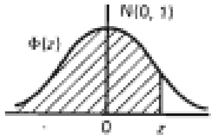
$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}.$$

The cumulative distribution function for X

This integral is difficult to evaluate, so we refer to tables.

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$$

THE DISTRIBUTION FUNCTION $\Phi(z)$ OF THE NORMAL DISTRIBUTION N(0, 1)



z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	36
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	7	11	15	19	22	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7954	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	20	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	7	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	5	8	11	14	16	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	19	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	18
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	17
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9635	.9645	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	4	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898								0	1	1	1	1	2	2	2	2
				.9901	.99036	.99061	.99086				3	5	8	10	13	15	18	20	23
								.99111	.99134	.99158	2	5	7	9	12	14	16	18	21
2.4	.99180	.99202	.99224	.99245	.99266						2	4	6	8	11	13	15	17	19
						.99286	.99305	.99324	.99343	.99361	2	4	6	7	9	11	13	15	17
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520	2	3	5	6	8	9	11	12	14
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643	1	2	3	5	6	7	8	9	10
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736	1	2	3	4	5	6	7	8	9
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807	1	1	2	3	4	4	5	6	6
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861	0	1	1	2	2	3	3	4	4
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900	0	1	1	2	2	2	3	3	4
3.1	$.9^{3}032$	$.9^{3}065$	$.9^{3}096$								3	6	9	13	16	19	22	25	28
				$.9^{3}126$	$.9^{3}155$	$.9^{3}184$	$.9^{3}211$				3	6	8	11	14	17	20	22	25
								$.9^{3}238$	$.9^{3}264$	$.9^{3}289$	2	5	7	10	12	15	17	20	22
3.2	$.9^{3}313$	$.9^{3}336$	$.9^{3}359$	$.9^{3}381$	$.9^{3}402$						2	4	7	9	11	13	15	18	20
						$.9^{3}423$	$.9^{3}443$	$.9^{3}462$	$.9^{3}481$	$.9^{3}499$	2	4	6	8	9	11	13	15	17
3.3	$.9^{3}517$	$.9^{3}534$	$.9^{3}550$	$.9^{3}566$	$.9^{3}581$						2	3	5	6	8	10	11	13	14
						$.9^{3}596$	$.9^{3}610$	$.9^{3}624$	$.9^{3}638$	$.9^{3}651$	1	3	4	5	7	8	9	10	12
3.4	$.9^{3}663$	$.9^{3}675$	$.9^{3}687$	$.9^{3}698$	$.9^{3}709$	$.9^{3}720$	$.9^{3}730$	$.9^{3}740$	$.9^{3}749$	$.9^{3}758$	1	2	3	4	5	6	7	8	9
3.5	$.9^{3}767$	$.9^{3}776$	$.9^{3}784$	$.9^{3}792$	$.9^{3}800$	$.9^{3}807$	$.9^{3}815$	$.9^{3}822$	$.9^{3}828$	$.9^{3}835$	1	1	2	3	4	4	5	6	7
3.6	$.9^{3}841$	$.9^{3}847$	$.9^{3}853$	$.9^{3}858$	$.9^{3}864$	$.9^{3}869$	$.9^{3}874$	$.9^{3}879$	$.9^{3}883$	$.9^{3}888$	0	1	1	2	2	3	3	4	5
3.7	$.9^{3}892$	$.9^{3}896$	$.9^{3}90$	$.9^{4}04$	$.9^{4}08$	$.9^{4}12$	$.9^{4}15$	$.9^{4}18$	$.9^{4}22$	$.9^{4}250$									
3.8	$.9^{4}28$	$.9^{4}31$	$.9^{4}33$	$.9^{4}36$	$.9^{4}38$	$.9^{4}41$	$.9^{4}43$	$.9^{4}46$	$.9^{4}48$	$.9^{4}500$									
3.9	$.9^{4}52$	$.9^{4}54$	$.9^{4}56$	$.9^{4}58$	$.9^{4}59$	$.9^{4}61$	$.9^{4}63$	$.9^{4}64$	$.9^{4}66$	$.9^{4}670$									

For negative values of z use $\Phi(z) = 1 - \Phi(-z)$