University of Colombo School of Computing
SCS 1308 - Foundations of Algorithms
Take-home 03

Instructions

- **Try the following questions and upload your answer script as a zip file to the given link in the UGVLE on/before 15th December at 6 pm.**
- **Note: Rename your zip file with your index number and name. (i.e: indexNo_Name.zip).**

01. Construct a graph for the following 3-SAT formula and determine the clique size:
    1. $(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_3 \vee x_4) \wedge (x_2 \vee \neg x_3 \vee \neg x_4)$
    ii. Represent literals as vertices.
    iii. Add edges between non-conflicting literals.
    iv. Find a clique of size 3.

02. The Traveling Salesman Problem (TSP) is a well-known optimization problem. Answer the following questions to explore how it can be transformed into a decision problem and analyzed for NP-completeness:
    a. Explain how TSP (Decision Version) is related to the Hamiltonian Cycle problem. Discuss:
        i. What constraints make TSP equivalent to finding a Hamiltonian cycle?
        ii. Why this transformation shows that TSP is NP-complete.

03. Explain why the linear search algorithm has a time complexity of O(n).
    a. What would be the worst-case time complexity for a list of size n=10 and n=100?
    b. Which algorithm ($O(n^3)$ or ($O(2^n)$) scales better as n increases? Justify your answer with a comparison of their growth rates.

04. Given the 3-SAT formula:

$(x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee \neg x_3) \wedge (x_1 \vee x_2 \vee \neg x_4) \wedge (\neg x, \vee \neg x_3 \vee x_4)$,

perform the following tasks:

i. Represent each literal in a clause as a vertex
ii. Add edges between vertices of different clauses if their literals are non-conflicting.
iii. Draw the graph to visualize the relationships.
iv. Determine the size of the largest clique in the graph.
v. If the graph contains a clique of size equal to the number of clauses, explain what this implies about the satisfiability of the formula.
vi. If no such clique exists, explain why the formula is unsatisfiable.