

SCS 1307 Probability & Statistics

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Q1

A soft-drink vending machine is set so that the amount of drink dispensed is a random variable with a mean of 200ml and standard deviation of 15ml. what is the probability that the average amount dispensed in a random sample of size 36 is at least 204 ml?

Q1- Solution

Let X be the amount of drink dispensed

$$X \sim (\text{mean} = 200\text{ml}, \text{sd} = 15\text{ml})$$

Probability that the mean in a random sample of size 36 is at least 204 ml?

By the CLT

$$\bar{X} \sim N(200, 225/36)$$

$$P(\bar{X} > 204) = P(Z > \frac{204 - 200}{15/\sqrt{36}}) = P(Z > 1.6) = 1 - P(Z < 1.6)$$

In the Growmore Market Garden plants are inspected for the presence of the deadly red angus leaf bug. The number of bugs per leaf is known to follow a Poisson distribution with mean one.

- (a) What is the probability that any one leaf on a given plant will have been attacked (at least one bug is found on it)?
- (b) A random sample of twelve plants are taken. For each plant 10 leaves are selected at random and inspected for these bugs. If more than eight leaves on any particular plant have been attacked, then the plant is destroyed. What is the probability that exactly two of these twelve plants are destroyed?

Q2 - Solution

(a) Let X be the r.v the number of bugs per leaf

$$X \sim Po(1)$$

We require $P(X \geq 1) = 1 - P(X=0)$

$$\begin{aligned} &= 1 - e(-1) = 1 - 0.368 \\ &= 0.632 \end{aligned}$$

The probability that any one leaf on a given plants will have been attacked is 0.632.

Let Y – the number of leaves that have been attacked on a plant

$$Y \sim \text{Bin}(10, 0.632)$$

$$P(Y > 8) = P(Y=9) + P(Y=10) = 0.069$$

The probability of any one plant is destroyed 0.069

Let R – The number of plants that are destroyed

$$R \sim \text{Bin}(12, 0.069)$$

$$P(R=2) = 0.154$$

The heights of a particular species of plant follow a normal distribution with mean 21cm and standard deviation $\sqrt{90}$ cm. A random sample of 10 plants is taken and the mean height calculated. Find the probability that this sample mean lies 18cm and 27cm.

Q3 - Solution

Let X be the r.v. the height in cm of a plant.

Then $X \sim N(21, 90)$

Now $n=10$, so $\bar{X} \sim N(21, 90/10) = N(21, 9)$

$$P(18 < \bar{X} < 27) = P\left(\frac{18-21}{3} < Z < \frac{27-21}{3}\right) = P(-1 < Z < 2) = 0.8185$$

Q4

If a large number of samples of size n are taken from $\text{Po}(2.5)$ and approximately 5% of the sample means are less than 2.025, estimate n .

Q4 - Solution

If $X \sim Po(2.5)$

$$E(X) = 2.5 \text{ and } V(X)=2.5$$

So, by the CLT, $\bar{X} \sim N(2.5, 2.5/n)$

$$\text{Now } P(\bar{X} < 2.025) = 0.05$$

$$P\left(Z < \frac{2.025 - 2.5}{\sqrt{2.5/n}}\right) = 0.05$$

$$P\left(Z < \frac{-0.475}{\sqrt{2.5/n}}\right) = 0.05 \quad \text{from the tables} \quad P(Z < -1.645) = 0.05$$

$$\text{So } \frac{0.475}{\sqrt{2.5/n}} = 1.645 \rightarrow n = 29.98 \approx 30$$

Q5

Out of 60 applicants to a private university 40 are male students. If 20 applicants are selected at random find the probability that 10 are males.

Q5 - solution

Hypergeometric distribution with

N=60 n = 20 k= 40 x=10

$$P(X=10) = \frac{\binom{40}{10} \binom{60-40}{20-10}}{\binom{60}{20}}$$