SCS 1307
Probability & Statistics

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What is a continuous random variable?

- Consider a random variable which can take any value in an interval. Such a random variable is not a discrete random variable because the values in the interval cannot be placed in one-to-one corresponding with counting numbers.

Definition

A random variable which can take any value in an interval is called a **continuous** random variable.

Probability Density Function

The probability properties of a continuous random variable X are specified by its probability density function *f(x)*

This function has the properties that

(1) $f(x) \geq 0$ for every $x \in R$

(2) $$\int_{-\infty}^{\infty} f(x)dx = 1$$

The probability of the event { a<X≤ b} is found from the density function *f* as follows

$$P(a < X \le b) = \int_a^b f(x)dx$$

Note that the density function of a continuous random variable is such that probabilities are given by areas under its graph.

Example

Suppose that the random variable T has a density function given by ,

$f_t(T) = 0$ when t<0 and

$f_t(T) = \lambda e^{-\lambda t}$ when t >=0

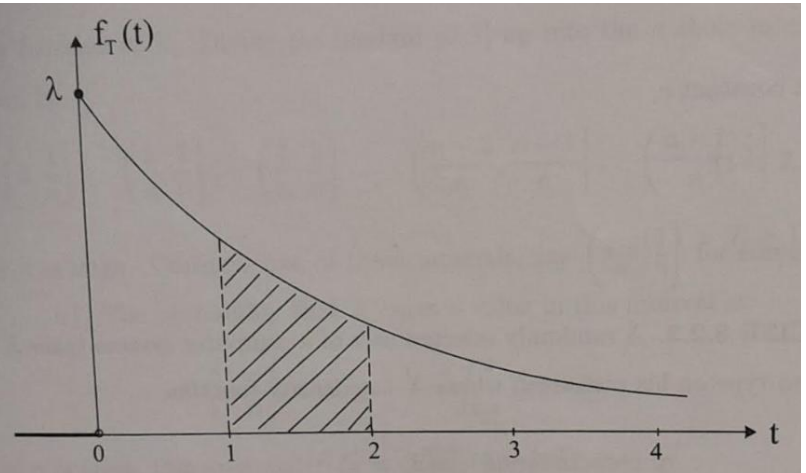
Show the area corresponding to the P(1 < T =< 2) in a graph.

Solution

$f_t(T) = 0$ when t<0 and

$f_t(T) = \lambda e^{-\lambda t}$ when t >=0

P(1 < T =< 2) is shown below



The area corresponding to

$$P(1 < T \le 2) = \int_1^2 \lambda e^{-\lambda t} dt = e^{-\lambda} - e^{-2\lambda}$$

has been shaded in the above diagram.

Exercise

A continuous random variable has p.d.f f(x) where

$$f(x)=kx \;,\; 0=<x=<4$$

(a) Find the value of the constant k

(b) sketch y = $f(x)$

(c) find P(1 =<X=<2.5)

Solution

A continuous random variable has p.d.f f(x) where

f(x)=kx , 0=<x=<4

(a) Find the value of the constant k

Since X is r.v $\int_0^4 f(x)dx = 1$

$$\int_0^4 kx\,dx = 1$$

$$\left[\frac{kx^2}{2}\right]_0^4 = 1$$

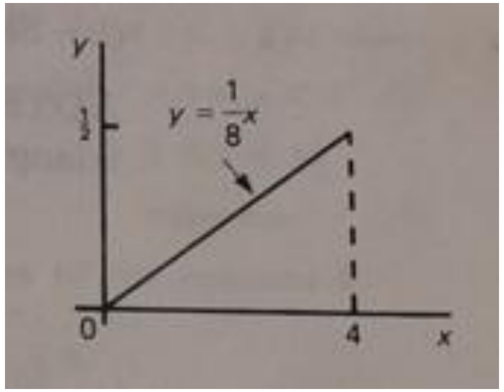
8k =1 → k=1/8

Solution

A continuous random variable has p.d.f f(x) where

f(x)=kx , 0=<x=<4

(b) sketch y = *f*(x) = x/8

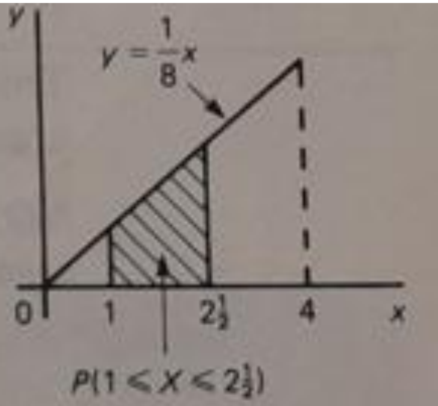


(c) find P(1 =<X=<2.5)

$$P(1 \leqslant X \leqslant 2\tfrac{1}{2}) = \int_1^{2\frac{1}{2}} \frac{1}{8}x\,\mathrm{d}x$$

$$= \left[\frac{x^2}{16}\right]_1^{2\frac{1}{2}}$$

$$= 0.328 \quad (3\text{ S.F.})$$



Exercise

The continuous r.v X has pdf $f(x)$ where $f(x)$ =k (4-x), 1=<X=<3.

(a) Find the value of constant k

(b) Sketch y=$f(x)$

(c)Find P(1.2 =<X=<2.4).

Expectation and Variance

If X is a continuous random variable with p.d.f f(x), then the expectation of X is E(X) where

$$E(X) = \int x\, f(x) dx$$

E(X) is often denoted by μ and referred to as the mean of X.

The variance of X is Var(X) where

$$\text{Var}(X) = E(X-\mu)^2 = E(X^2) - \mu^2 \text{ where } \mu = E(X)$$

$$= \int_{-\infty}^{\infty} x^2\, f(x) dx - \mu^2$$

Example

If X is a continuous r.v. with p.d.f $f(x) = \frac{3x^2}{64}$, 0=<x=<4, find E(X) and Var(X).

Solution

If X is a continuous r.v. with p.d.f $f(x) = \frac{3x^2}{64}$, 0=<x=<4,

E(X) = $\int x\, f(x) dx \;\; = \int_0^4 x\, \frac{3x^2}{64} dx$

Var(X) $= E(X-\mu)^2 = E(x^2) - \mu^2 = \int_0^4 x^2\, \frac{3x^2}{64} dx$ - μ^2