Discrete Mathematics - Kuppi 04
Set Theory & Functions

1 Answers:

1.
Usual method:

$$
\begin{aligned}
\text{Let } x \in\ & \overline{(A \cap B)} \\
& \equiv x \notin (A \cap B) \\
& \equiv \neg(x \in (A \cap B)) \\
& \equiv \neg(x \in A \wedge x \in B) \\
& \equiv (x \notin A) \vee (x \notin B) \\
& \equiv x \in \overline{A} \vee x \in \overline{B} \\
& \in \overline{A} \vee \overline{B} \\
& \therefore \overline{(A \cap B)} \subseteq \overline{A} \cup \overline{B}
\end{aligned}
$$

$$
\begin{aligned}
\text{Let } x \in\ & \overline{A} \cup \overline{B} \\
& \in \overline{A} \vee \overline{B} \\
& \equiv x \in \overline{A} \vee x \in \overline{B} \\
& \equiv \neg(x \in A \wedge x \in B) \\
& \equiv (x \notin A) \vee (x \notin B) \\
& \equiv \neg(x \in (A \cap B)) \\
& \equiv x \notin (A \cap B) \\
& \therefore \overline{(A \cap B)} \subseteq \overline{A} \cup \overline{B}
\end{aligned}
$$

Therefore $\overline{(A \cap B)} \subseteq \overline{A} \cup \overline{B}$

Using set builder method:

$$\begin{aligned}
\overline{(A \cup B)} &\equiv \{x \mid x \notin (A \cup B)\} \\
&\equiv \{x \mid \neg x \in (A \cup B)\} \\
&\equiv \{x \mid \neg(x \in A \vee x \in B)\} \\
&\equiv \{x \mid \neg(x \in A) \wedge \neg(x \in B)\} \\
&\equiv \{x \mid x \in \overline{A} \wedge x \in overline B\} \\
&\equiv \{x \mid x \in (\overline{A} \cap \overline{B})\}
\end{aligned}$$

2. **Proof:** We want to show that

$$A \oplus B = (A - B) \cup (B - A).$$

$$\begin{aligned}
x \in A \oplus B &\iff x \in (A - B) \vee x \in (B - A) \\
&\iff (x \in A \wedge x \notin B) \ \vee \ (x \in B \wedge x \notin A) \\
&\iff x \in (A - B) \cup (B - A)
\end{aligned}$$

Hence,

$$A \oplus B = (A - B) \cup (B - A).$$

3. **Proof:** We want to show that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

$$\begin{aligned}
x \in A \cap (B \cup C) &\iff x \in A \wedge x \in (B \cup C) \\
&\iff x \in A \wedge (x \in B \vee x \in C) \\
&\iff (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C) \\
&\iff x \in (A \cap B) \vee x \in (A \cap C) \\
&\iff x \in (A \cap B) \cup (A \cap C)
\end{aligned}$$

Hence,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

4. **Proof:** We want to show that

$$A - (B \cup C) = (A - B) \cap (A - C).$$

$$\begin{aligned} x \in A - (B \cup C) &\iff x \in A \wedge x \notin (B \cup C) \\ &\iff x \in A \wedge (x \notin B \text{ and } x \notin C) \\ &\iff (x \in A \wedge x \notin B) \text{ and } (x \in A \text{ and } x \notin C) \\ &\iff x \in (A - B) \cap (A - C) \end{aligned}$$

$\bigcap_{k=1}^{n} A_k = A_1 \cap A_2 \cap \cdots \cap A_n$