# SCS 1302 - Discrete Mathematics

## Tutorial 1

1. What is meant by a "proposition" in logic?
2. Are the following sentences propositions?
   a. Toronto is the capital of Canada.
   b. Read this carefully.
   c. $1 + 2 = 3$
   d. $x + 1 = 2$
   e. What time is it?
3. Let $p$, $q$, $r$ be propositions. Which of the following are well formed propositional formulas?
   a. $\vee\, pq$
   b. $(\neg(p \to (q \wedge p)))$
   c. $(\neg(p \to (q = p)))$
   d. $(\neg(\blacklozenge(q \vee p)))$
   e. $(p \wedge \neg q) \vee (q \to r)$
   f. $p\neg r$
4. Let $p$, $q$, $r$ be propositions. Let's consider the interpretation $v$, where $v(p) = F$, $v(q) = T$, $v(r) = T$. Does $v$ satisfy the following propositional formulas? (For a compound proposition $P$, an interpretation is a possible assignment of truth values to its variables. As you know, because there are three propositions, there are eight such assignments(or combinations); $TTT$, $TTF$, … and so on. The interpretation $v$ satisfies the compound proposition $P$ if $P$ is true for $v$.)
   a. $(p \to \neg q) \vee \neg(r \wedge q)$
   b. $(\neg p \vee \neg q) \to (p \vee \neg r)$
   c. $\neg(\neg p \to \neg q) \wedge r$
   d. $\neg(\neg p \to q \wedge \neg r)$

5. Compute the truth table of $(p \vee q) \wedge \neg(p \wedge q)$, where $p$ and $q$ are propositions.

6. Let $p$, $q$, $r$ be propositions. Use the truth tables method to determine whether $(\neg p \vee q) \wedge (q \rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$ is satisfiable.

7. Let $p$, $q$, $r$ be propositions. Use the truth table method to verify whether the following formulas are tautologies, satisfiable or unsatisfiable:

a. $(p \rightarrow q) \wedge \neg q \rightarrow \neg p$

b. $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

c. $(p \vee q \rightarrow r) \vee p \vee q$

d. $(p \vee q) \wedge (p \rightarrow r \wedge q) \wedge (q \rightarrow \neg r \wedge p)$

e. $(p \rightarrow q) \rightarrow (p \rightarrow \neg q)$

f. $(\neg p \rightarrow q) \vee ((p \wedge \neg r) \leftrightarrow q)$

g. $(p \vee q) \wedge (\neg q \wedge \neg p)$

8. Let $p$, $q$, $r$ be propositions. Use the truth table method to verify whether the following logical consequences and equivalences are correct (a compound proposition $Q$ is said to be a logical consequence of the compound proposition $P$, denoted by $P \models Q$, if $P \rightarrow Q$ is a tautology):

a. $(p \rightarrow q) \models \neg p \rightarrow \neg q$

b. $p \rightarrow q \wedge r \models (p \rightarrow q) \rightarrow r$

c. $(p \rightarrow q) \wedge \neg q \models \neg p$

d. $p \vee (\neg q \wedge r) \models q \vee \neg r \rightarrow p$

e. $(p \vee q) \wedge (\neg p \rightarrow \neg q) \ \Xi\ p$

f. $(p \wedge q) \vee r \ \Xi\ (p \rightarrow \neg q) \rightarrow r$

g. $(p \vee q) \wedge (\neg p \rightarrow \neg q) \ \Xi\ q$

h. $((p \rightarrow q) \rightarrow q) \rightarrow q \ \Xi\ p \rightarrow q$

9. Let's consider a propositional language where,
    - $p$ means "Paola is happy",
    - $q$ means "Paola paints a picture",
    - $r$ means "Renzo is happy".

Formalize the following sentences (that is, translate each of these English sentences into expressions involving propositional variables and logical connectives):

a. "if Paola is happy and paints a picture then Renzo isn't happy"
b. "if Paola is happy, then she paints a picture"
c. "Paola is happy only if she paints a picture"

10. Let $p =$"Aldo is Italian" and $q =$"Bob is English". Formalize the following sentences:

a. "Aldo isn't Italian"
b. "Aldo is Italian while Bob is English"
c. "If Aldo is Italian then Bob is not English"
d. "Aldo is Italian or if Aldo isn't Italian then Bob is English"
e. "Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English"

11. Angelo, Bruno and Carlo are three students that took the Logic exam. Let's consider a propositional language where,
    - $p =$"Aldo passed the exam",
    - $q =$"Bruno passed the exam",
    - $r =$"Carlo passed the exam".

Formalize the following sentences:

a. "Carlo is the only one passing the exam"

b. “Aldo is the only one not passing the exam”
c. “Only one, among Aldo, Bruno and Carlo, passed the exam”
d. “At least one among Aldo, Bruno and Carlo passed”
e. “At least two among Aldo, Bruno and Carlo passed the exam”
f. “At most two among Aldo, Bruno and Carlo passed the exam”
g. “Exactly two, among Aldo, Bruno and Carlo passed the exam”

12. Let’s consider a propositional language where
    - $p =$“Angelo comes to the party”,
    - $q =$“Bruno comes to the party”,
    - $r =$“Carlo comes to the party”,
    - $s =$“Davide comes to the party”.

Formalize the following sentences:

a. “If Davide comes to the party then Bruno and Carlo come too”
b. “Carlo comes to the party only if Angelo and Bruno do not come”
c. “Davide comes to the party if and only if Carlo comes and Angelo doesn’t come”
d. “If Davide comes to the party, then, if Carlo doesn’t come then Angelo comes”
e. “Carlo comes to the party provided that Davide doesn’t come, but, if Davide comes, then Bruno doesn’t come”
f. “A necessary condition for Angelo coming to the party, is that, if Bruno and Carlo aren’t coming, Davide comes”
g. “Angelo, Bruno and Carlo come to the party if and only if Davide doesn’t come, but, if neither Angelo nor Bruno come, then Davide comes only if Carlo comes”

13. What is meant by Disjunctive and Conjunctive normal form? Explain it by using two examples.

14. For each of the following compound propositions, find a compound proposition in normal form which is equivalent to the original compound proposition. Answers should be in the format below:

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\begin{aligned}
(p \to q) \to (\neg r \wedge q) &\equiv (p \wedge \neg q) \vee (\neg r \wedge q) & \\
&\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee (\neg r \wedge q)) & \text{[distributive]} \\
&\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee \neg r) \wedge (\neg q \vee q) & \text{[distributive]} \\
&\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee \neg r) \wedge T & \text{[negation]} \\
&\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee \neg r) & \text{[identity]} \\
&\equiv (p \vee \neg r) \wedge (p \vee q) \wedge (\neg q \vee \neg r) & \text{[distributive]}
\end{aligned}
$$

a. $\neg[(\neg p \to q) \to r]$

b. $p \to (q \wedge r)$

c. $(p \wedge r)$ iff $(\neg q \vee r)$

d. $\neg((\neg p \to \neg q) \wedge \neg r)$

e. $(p \to q) \to (\neg r \wedge q)$

f. $(p \to (\neg q \wedge r)) \wedge (p \to \neg q)$

g. $\neg((p \to q) \to ((q \to r) \to (p \to r)))$

h. $(p \to (\neg q \to r)) \wedge (p \to \neg q) \to (p \to r)$

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