SCS 1307
Probability & Statistics

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A discrete r.v. X having pdf of the form

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x\,!} \quad \text{for } x=0,1,2,3\ldots\text{to infinity}$$

Where λ can take any positive value, is said to follow the Poisson distribution.

Expectation and Variance

If $X \sim Po(\lambda)$ then it can be shown that $E(X) = \lambda$ and $Var(X) = \lambda$

Uses of the Poisson Distribution

If an event is randomly scattered in time/space and has mean number of occurrence λ in a given time/space and if X is the r.v. ‘the number of occurrences in the given interval, then $X \sim Po(\lambda)$

Examples…….

Number of

- Car accidents on a particular stretch of a road in one day
- Accidents in a factory in one week
- Telephone calls made to a switch board in a given minute
- Insurance claims made to a company in a given time

Example

(1) If $X \sim Po(2)$, find

(a) $P(X=4)$

(b) $P(X>=3)$

Solution

(1) If X~ Po(2), find P(X=4) and P(X>=3)

Solution

(a) $P(X=4) = \dfrac{e^{-2}\,2^4}{4\,!} = 0.0902$

Solution

(1) If X~ Po(2), find P(X=4) and P(X>=3)

Solution

(b) P(X>=3) =1- [P(X=0)+P(X=1)+P(X=2)]

$$=1 - \left[\frac{e^{-2} 2^0}{0\,!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} \right]$$

=0.323

Examples

(1) The mean number of bacteria per milliliter of a liquid is known to be 4. Assuming that the number of bacteria follows a Poisson distribution, find the probability that, in 1ml of liquid, there will be

(a)No bacteria (b) 4 bacteria (c) less than 3 bacteria

Solution

$X \sim Po(4)$,

(a)No bacteria

$$P(X=0) = \frac{e^{-4}\,4^{0}}{0\,!} = 0.0183$$

(b) 4 bacteria

$$P(X=4) = \frac{e^{-4}\,4^{4}}{4\,!} = 0.195$$

Solution

$X \sim Po(4)$,

(c) less than 3 bacteria

$$P(X<3) = P(X=0)+P(X=1)+P(X=2)$$

$$= \frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} = 0.238$$

Example

(2). Using the data of previous exercise find the probability that in 3ml of liquid there will be less than 2 bacteria.

Solution

(2). Using the data of previous exercise find the probability that in 3ml of liquid there will be less than 2 bacteria.

Let Y be the number of bacteria in 3ml of liquid

$Y \sim Po(12)$

$P(Y<2) = P(Y=0) + P(Y=1)$

$$= \frac{e^{-12}\, 2^{0}}{0\,!} + \frac{e^{-12}\, 12^{1}}{1!} = 7.99 \times 10^{-5}$$

The Normal Approximation to the Poisson Distribution

If $X \sim Po(\lambda)$ then $E(X) = \lambda$ and $Var(X) = \lambda$.

Now for large λ, $X \sim N(\lambda, \lambda)$ approximately.

Example

A radioactive disintegration gives counts that follow a Poisson distribution with mean count per second of 25. Find the probability that in 1 second the count is between 23 and 27 inclusive,

(a) using Poisson distribution

(b) using the Normal approximation to the Poisson distribution

Solution

(a) using Poisson distribution

$X \sim Po(25)$

$P(23 =< X =< 27)$

$= P(X=23)+P(X=24)+P(X=25)+P(X=26)+PX=27)$

$=0.076342$

Solution

(b) using the Normal approximation to the Poisson distribution

using the normal approximation to the Poisson distribution.

$$X \sim N(25,25)$$

with the continuity correction we need to find $P(22.5<X<27.5) = P(-0.5<Z<0.5)$

$$= 0.383$$