INDEX NUMBER:

UNIVERSITY OF COLOMBO, SRI LANKA
FACULTY OF SCIENCE

LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) – 2022

ST 1008 – PROBABILITY AND DISTRIBUTIONS

(Two Hours)

Answer all questions

No. of questions: 04
No. of pages: 09

Important Instructions to the Candidates:

- If a page or a part of this question paper is not printed, please inform the supervisor immediately.
- Enter your index number on all pages of the answer script/question paper.
- MULTIPLE CHOICE QUESTIONS: Question (1) and (2) consist of 10 Multiple Choice Questions (MCQ) of each. Each of the MCQs will have 5 choices with only one correct answer. **Encircle the correct choice** in the tables given on the question paper.
- SEMI-STRUCTURED TYPE: Write the answers to questions (3) and (4) on the papers provided.
- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.
- **Attach the exam paper to the answer sheets.**
- Statistical tables are attached to the paper.
- **You are not permitted to remove any part of the question paper except the page that the statistical tables are given from the Examination Hall.**

For Examiner's use only						
Question No.	Q1	Q2	Q3	Q4	Total	%
Marks						

INDEX NUMBER:

Question Number 01.

This question consists of 10 multiple choice questions. Select the most suitable answer and encircle the correct choice in Table 01.

(Question No. 01. – 10 × 10 Marks)

Table 01: Answer Table of Question Number 01.

Question Number	Answers					Question Number	Answers				
1.	a	b	c	d	e	**6.**	a	b	c	d	e
2.	a	b	c	d	e	**7.**	a	b	c	d	e
3.	a	b	c	d	e	**8.**	a	b	c	d	e
4.	a	b	c	d	e	**9.**	a	b	c	d	e
5.	a	b	c	d	e	**10.**	a	b	c	d	e

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Number of Correct Answers:……... **Marks:**…….

Question Number 02.

This question consists of 10 multiple choice questions. Select the most suitable answer and encircle the correct choice in Table 2.

(Question No. 02. – 10 × 10 Marks)

Table 2: Answer Table of Question Number 02.

Question Number	Answers					Question Number	Answers				
1.	a	b	c	d	e	**6.**	a	b	c	d	e
2.	a	b	c	d	e	**7.**	a	b	c	d	e
3.	a	b	c	d	e	**8.**	a	b	c	d	e
4.	a	b	c	d	e	**9.**	a	b	c	d	e
5.	a	b	c	d	e	**10.**	a	b	c	d	e

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Number of Correct Answers:……... **Marks:**…….

INDEX NUMBER:

Question Number 03.

(a) Following table shows the probability distribution for a random variable X.

Table 3: Probability distribution of X

x	0	1	2	3
$P(X = x)$	0.2	a	0.3	b

The expected value of the random variable X is given as 1.5.

Calculate,

(i) the values of a and b. **(10 marks)**

(ii) $E[(X-1)^2]$. **(15 marks)**

(iii) $Var(2X-1)$. **(10 marks)**

(b) The continuous random variable X has probability density function (pdf)

$$f_X(x;\theta) = \frac{1}{2\theta}, \qquad -\theta \le x \le \theta.$$

(i) Show that the cumulative distribution function of X is

$$F_X(x) = \frac{\theta + x}{2\theta}, \qquad -\theta \le x \le \theta. \ (\textbf{10 marks})$$

(ii) Deduce an expression for $P(X > x)$. **(05 marks)**

(iii) The random variable Y has the same distribution as X and is **independent** of X. The random variable Z is defined as $max(X,Y)$.

Considering the condition that $(Z \le z) \rightarrow (X \le z)\ and\ (Y \le z)$, Show that

$$P(Z \le z) = \left(\frac{\theta + z}{2\theta}\right)^2, \qquad -\theta \le z \le \theta. \ (\textbf{10 marks})$$

(iv) Deduce the pdf of Z and hence find $E(Z)$. **(15 marks)**

(v) The random variable W is defined as $min(X,Y)$. Considering a suitable condition as in (ii), show that

$$P(W > w) = \left(\frac{\theta - w}{2\theta}\right)^2, \qquad -\theta \le z \le \theta. \ (\textbf{15 marks})$$

(vi) Deduce the pdf of W. **(10 marks)**

(Question No. 03 – 100 Marks)

INDEX NUMBER:

Question Number 04.

(a) (i) The random variable X has the Poisson distribution with mean λ, so that

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad x = 0, 1, 2, \ldots.$$

Show that the Moment Generating Function of X is

$$M_X(t) = e^{\lambda(e^t - 1)}. \quad \textbf{(10 marks)}$$

$$\left[Hint: \sum_{x=0}^{\infty} \frac{a^x}{x!} = e^a\right]$$

(ii) Hence, derive $E(X)$ and $Var(X)$. **(20 marks)**

(b) Assume that, for any $t > 0$, the number N of incoming telephone calls to a 24-hour international call centre in a time t minutes follows a Poisson distribution with mean $3t$.

(i) Find the probability that exactly 2 calls arrive in the next minute. **(10 marks)**

(ii) Find the probability that no calls arrive in the next $1\frac{1}{2}$ minutes. **(10 marks)**

(iii) State the exact distribution of N in the case $t = 60$. Staffing levels at the call centre are based on the assumption that not more than 200 calls will be made in an hour. Use a suitable approximation to the distribution of N to calculate the approximate probability that this assumption is violated in a given hour. (Use a continuity correction if required.) **(20 marks)**

(c) Suppose a call center operator starts his work at 7.30 a.m. at the international call centre described in (b). It is known that the time T minutes to wait until the first call is received by the operator follows exponential distribution with probability density function

$$f_T(t; \lambda) = \lambda e^{-\lambda t}, \qquad \lambda > 0, \qquad t \geq 0.$$

(i) Give the value of the exponential parameter λ. **(05 marks)**

(ii) Find the probability that the operator waits not more than 1 minute to receive the first call. **(10 marks)**

(iii) **Derive** and calculate the expected waiting time for the operator to receive the first call. **(15 marks)**

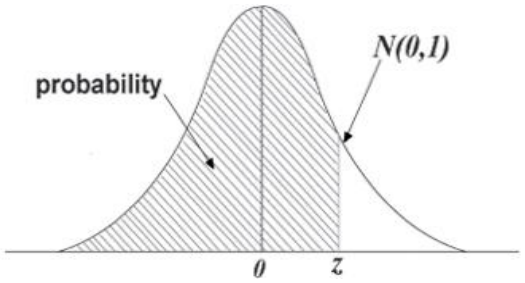
(Question No. 04 – 100 Marks)

********** END **********

INDEX NUMBER:

(You can separate this sheet from the Examination Paper)

Table 03. The Standardized Normal Distribution Probabilities



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of ***Z***, the standardized normal deviate, (the proportion ***P***, of the distribution less than ***Z***) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value ***X*** is obtained by calculating $Z = (X - \mu)/\sigma$ and reading the proportion corresponding to this value of ***Z***.

Z	*P*	*Z*	*P*	*Z*	*P*
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997