

Searching Algorithms

* Linear Search

- ↳ unsorted search list data set
- ↳ small dataset
- ↳ low memory ^{restrictions} (algorithm has low space complexity $[O(1)]$)
- implemented using a simple for loop

* Binary Search

- ↳ most popular search
- ↳ need sorted array as input
- ↳ not suitable for linked lists (slow middle access)
- ↳ $T(n) = O(\log_2 n)$

```
def bSearchIterative(arr, target):
```

```
    left = 0
```

```
    right = len(arr) - 1
```

```
    while left <= right:
```

```
        mid = (left + right) // 2    ← floor division
```

```
        if arr[mid] == target:
```

```
            return mid
```

```
        elif arr[mid] > target:
```

```
            right = mid - 1
```

```
        else:
```

```
            left = mid + 1
```

```
    return -1
```


* Jump Search

↳ for sorted arrays

↳ $T(n) = O(\sqrt{n})$

↳ Skips ahead by fixed intervals.

↳ Works better than bSearch for linked lists

↳ bSearch may cause many cache misses, when accessing high latency memory.

↳ jump search is more cache friendly

↳ can be modified for partially sorted arrays

↳ optimum jump size is \sqrt{n} (geeksforgeeks has calculation)

$n = \text{len(arr)}$

$\text{step} = \sqrt{n}$

$\text{prev} = 0$

while $\text{arr}[\text{min}(\text{step}, n) - 1] < \text{target}$:

$\text{prev} = \text{step}$

$\text{step} += \sqrt{n}$

if $\text{prev} \geq n$:

return -1

for i in $\text{range}(\text{prev}, \text{min}(\text{step}, n))$:

if $\text{arr}[i] == \text{target}$:

return i

return -1

* Interpolation Search

↳ $T_n = O(\log(\log(n)))$

↳ effective for uniformly distributed arrays

↳ improves upon binary search by estimating probable position of target

$$pos = low + \left(\frac{target - arr[low]}{arr[high] - arr[low]} \times (high - low) \right)$$

interpolationSearch(arr, low, high, target)

if (low <= high AND target >= arr[low] AND target <= arr[high])

pos = low + ((target - arr[low]) / (arr[high] - arr[low])) * (high - low)

if arr[pos] == target

return pos

elif arr[pos] < target

return interpolationSearch(arr, pos + 1, hi, target)

else

return interpolationSearch(arr, lo, pos - 1, target)

Sorting Algorithms

* Stable - maintains relative order of equal elements

* Insertion Sort

↳ split array into 2

↳ assume left half is sorted and add unsorted items to correct position in sorted array

↳ start with 2nd element (take 1st element alone to be sorted part)

↳ compare 1 and 2 and swap if needed

↳ move to 3rd element and compare with 1 and 2 and put in correct position

for $i = 1$ to n ← start with 1 [$n = \overset{\text{number of elements}}{\text{size of array}}$]
 element → $\text{key} = \text{arr}[i]$ since 0 is already sorted
 that we are "inserting" $j = i - 1$

while ($j \geq 0$ && $\text{arr}[j] > \text{key}$)

$\text{arr}[j+1] = \text{arr}[j]$

$j = j - 1$

$\text{arr}[j+1] = \text{key}$

* look for position of key by moving each compared element to the right along the sorted part of the array till we find the correct place

* Shell Sort

↳ 1st algo to break quadratic time barrier (i.e $T(n) \leq O(n^2)$)

↳ AKA diminishing increment sort

↳ Best $\rightarrow O(n \log n)$ Avg $\rightarrow O(n \log n^2)$ - Worst $O(n^2)$

```
for (int interval = n/2; interval > 0; interval /= 2) {
    for (int i = interval; i < n; i++) {
        int temp = arr[i];
        int j;
        for (j = i; j >= interval && arr[j - interval] > temp; j -= interval)
            arr[j] = arr[j - interval];
        arr[j] = temp;
    }
}
```

* A worst case, this devolves to insertion sort

* Radix Sort

↳ considers the structure of keys

↳ groups numbers by their individual digits (radix)

↳ using radix as key, uses counting sort to sort them

↳ is a stable sort (will not work otherwise)

↳ usually use for a fixed range

↳ has high auxiliary space

↳ $T(n) = O(d \cdot (n + k))$

n - number of elements

d - max digits in a number

b - base of counting (10 for decimal etc)

```

int getMax(int arr[], int n) {
    int mx = arr[0];
    for (int i = 1; i < n; i++) {
        if (arr[i] > mx) {
            mx = arr[i];
        }
    }
    return mx;
}

```

0 1 2 3 4 5 6 7
 170 45 75 90 800 24 2 66

0 1 2 3 4 5 6 7 8 9
 1 2 3 4 4 7 8 8 8 1

1 1 1 2 4 1 66

```

void countS(int arr[], int n, int place) {
    int output[n];
    int count[10] = {0};

    for (int i = 0; i < n; i++) {
        count[(arr[i] / place) % 10]++;
    }

    for (int i = n - 1; i >= 0; i--) {
        output[count[(arr[i] / place) % 10] - 1] = arr[i];
        count[(arr[i] / place) % 10]--;
    }

    for (int i = 0; i < n; i++) {
        arr[i] = output[i];
    }
}

void radixS(int arr[], int n) {
    int m = getMax(arr, n);

    for (int place = 1; m / place > 0; place *= 10) {
        countS(arr, n, place);
    }
}

```


Hashing

- * Have average case time complexity = $O(1)$
Worst case is still = $O(n)$

- * 2 main types

- ↳ Static hashing
- ↳ Dynamic hashing

- * Hashing has 3 main parts

- 1) Key: input given to the hash function
- 2) Hash function: takes key and returns index of an element in hash table called hash index
- 3) Hash table: data structure that maps keys to values.

- * Hash table is usually an array of structs that includes a char array to store the key followed by any other required data types

- * Various implementations of hashing

- ↳ hash list
- ↳ hash trees

- * Ideal Hashing

- ↳ Each key is converted into a separate index and stored in its home bucket
- ↳ No collisions

- * But collisions do occur so we consider 2 things.

- 1) Choose a good hash function to minimize collisions
- 2) Handle overflow efficiently

RICHARD

- * Clustering - hash function groups multiple key value pairs together
 - not in same bucket but many nearby buckets are filled
- * Collisions - hash function evaluates completely different keys and returns the same hash index there by storing them in the same bucket.
 - ↳ pigeonhole principle
- * Good hash functions avoids above 2 conditions as much as possible
- * Why can't we use n -sized hash table?
 - ↳ high memory usage
 - ↳ need a perfect hash function that maps each unique key to a unique hash index
 - ↳ very hard to do for dynamic data (data is stored and deleted)

* Characteristics of hash functions

- Efficiency
- Uniform distribution (avoid clustering)
- Determinism (same input key always gives same hash index)
- Minimized collisions

Hash Functions

1) Middle Square Hash Function

- * Take input key x
- * Square the key (x^2)
- * Choose a specific number of digits from the middle of x^2
- * Use $\%m$ on extracted digits to fit to table (m = table size)
- Used for small hash tables with integer keys.

2) Multiplicative Method

$$* h(x) = \text{floor}(\text{table size} \times (\text{key} \times A \% 1))$$
$$= L(m \cdot (k \cdot A \% 1))$$

* A is a constant (usually 0.618 is chosen) $[0 < A < 1]$.
Knuth's recommendation

- * Better distribution than division method.
- * Slower than division

4) Folding Hash Function

1) Separate key into smaller fixed-size parts

2) Combine parts

↳ XOR, ~~AND~~ or Add together

3) Mod by table size

5) Digit Analysis

- * Assumes that distribution of keys is known in advance
 - ↳ Analyse digits in key that have skewed distribution
 - ↳ Eliminate skewed digits
 - ↳ Use remaining digits

6) Division Method

- * Simplest and most widely used
- * $h(x) = x \% m$ ($m = \text{table size}$)
- * need to choose good m
- * Generally: a prime number that is not close to a power of 2.
- * for a hash table of size b , a given bucket will have approximately $2^{32}/b$ integers (when trying to hash all integers)
- * If divisor is even, even ints go into even buckets and odd into odd-buckets.
- * If divisor is odd, ~~even~~, odd both will hash into any bucket.

- + Biased distribution occurs if we use a divisor that is a multiple of prime numbers.
- + Ideally choose a large prime for divisor
- + Or choose divisor such that it has no prime factors smaller than 20.

+ Criterion of a Hash Table

↳ Key density = n/T (n - number of keys in table, T - number of distinct possible keys)

↳ loading density / factor

$$\alpha = n / (sb) \quad (s - \text{number of slots}) \\ (b - \text{number of buckets})$$

Overflow Handling

- + When a collision occurs we get an overflow
- + 2 Solutions

1) Search for an empty bucket (open addressing)

↳ linear probing (linear open addressing)

↳ Quadratic Probing

↳ Random Probing

2) Each bucket has a list of all keys indexed to it

↳ Array ~~linear~~ linear list

↳ Chaining

⊛ Open addressing - all elements stored directly into hash table

- + Quadratic probing $\Rightarrow h'(k, i) = (h(k) + i^2) \% m$ } i = number of collisions for given k .
- + Random probing $\Rightarrow h'(k) = (h(k) + r[i]) \% m$ }