SCS 1307
Probability & Statistics

by

Dr Dilshani Tissera

Department of Statistics

University of Colombo

Uniform Distribution

- Sample space $S = \{1,2,3,\ldots.k\}$
- Probability measure : equal assignment to all outcomes. i.e all outcomes are equally likely
- Random variable X defined by $X(i)=i$, where $i = 1,2,3,\ldots..k$
- Distribution: $P(X=x)=1/k$

Moments of Uniform Distribution

Mean = μ = $\left(\frac{(1+2+\cdots..+k}{k}\right) = \frac{\frac{1}{2}k(k+1)}{k} = \frac{k+1}{2}$

Variance = $\sigma^2 = \frac{k^2-1}{12}$

Geometric Distribution

Consider a sequence of iid Bernoulli trials with P({S})=p

Variable of interest: Number of trails that has to be performed until the first success

- Random variable: X is the number of trials on which the first success occurs

- Distribution: For X=x there must be a run of (x-1) failures followed by a success

So P(X=x)= $p(1-p)^{x-1}$, x=1,2,3……; 0<p<1

Moments of Geometric Distribution

Mean = $\mu = \frac{1}{p}$

Variance = $\sigma^2 = \frac{(1-p)}{p^2}$

Example

Suppose that the probability of having a male or a female child is equal, find the probability that family's fourth child is their first son.

$$P= \frac{1}{2}^3 * \frac{1}{2} = 0.0625$$

Negative Binomial Distribution

This is a generalisation of the geometric distribution

- Random variable: X is the number of trials on which the kth success occurs where k is a positive integer

- Distribution: $P(X=x)=\binom{x-1}{k-1}p^k(1-p)^{x-k}$, x=k, k+1, …..,0<p<1

  We say that X has negative binomial (k,p) distribution

Geometric distribution is special case of this random variable when k=1

Moments of Negative Binomial Distribution

Mean = $\mu = \frac{k}{p}$

Variance = $\sigma^2 = \frac{k(1-p)}{p^2}$

Example

An oil company conducts a geological study that indicates that an exploratory oil well should have a 20% chance of striking oil. What is the probability that the third strike comes on the seventh well drilled?

Solution

X=7, k=3, p=0.2

P(X=x)=$\binom{x-1}{k-1}p^k(1-p)^{x-k}$

P(X=7)=$\binom{7-1}{3-1}0.2^3(1-0.2)^{7-3}$= 0.049

Hypergeometric Distribution

This is the finite population equivalent of the binomial distribution, in the following sense.

Suppose objects are selected at random, one after the another, without replacement, from a finite population consisting of k successes and N-k failures. The trials are not independent.

- Random variable: X is the number of ‘successes’ in a sample size n from a population of size N that has k ‘successes’ and (N-k) failures.
- Distribution: $P(X=x)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$, x= 0,1,2,3,…..k, 0<p<1

Moments of Hypergeometric Distribution

Mean = μ = $\frac{nk}{N}$

Variance = $\sigma^2 = \frac{nk(N-k)(N-n)}{p^2}$

Example

A deck of cards contains 20 cards: 6 red cards and 14 black cards. 5 cards are drawn randomly without replacement. What is the probability that exactly 4 red cards are drawn?

Solution:

The probability of choosing exactly 4 red cards is :

P(4 red cards)= $\dfrac{\text{\# samples with 4 red cards and 1 black card}}{\text{\# of possible 4 card samples}}$

$$P(X=4)=\frac{\binom{6}{4}\binom{20-6}{5-4}}{\binom{20}{5}} = 0.0135$$

Revision Exercises

1. Suppose that 1% of the items made by a certain machine are defective. To keep a check on the quality of the output a batch of ten items is inspected occasionally. What is the probability that the next ten items inspected include more than one defective?

Solution

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X: no of defectives in a sample of 10

$X \sim Binomial(10,0.01)$

$P(X>1)=1- P(X=0)-P(x=1) =$

Revision Exercises

2. Pat is required to sell candy bars to raise money for the 6th grade field trip. There is a 40% chance of him selling a candy bar at each house. He has to sell 5 candy bars in all. What is the probability he sells his last candy bar at the 11th house?

Solution

Need to find the 5th success in 11th trial

X=11, k=5, p=0.4

P(X=x)=$\binom{x-1}{k-1}p^k(1-p)^{x-k}$

The probability that he sells his last candy bar at the 11th house =

P(X=11)=$\binom{10}{4}0.4^5(1-0.4)^6$

Revision Exercises

3. Products produced by a machine has a 3% defective rate. What is the probability that the first defective occurs in the fifth item inspected?

Solution

3. Products produced by a machine has a 3% defective rate. What is the probability that the first defective occurs in the fifth item inspected?

X- the number of trials until the first success

P(X=5)= $0.03(1-0.03)^4$ = 0.072

Revision Exercises

4. An ordinary die is thrown seven times. Find the probability of obtaining exactly three sixes.

Solution

4. An ordinary die is thrown seven times. Find the probability of obtaining exactly three sixes.

X~ No: of sixes obtained

X~Binomial(7,1/6) = 0.07814

Revision Exercises

5. A call center receives calls at a mean rate of 3 per minute. Find the probability that during a randomly selected 3 minutes there will be no calls.

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X~No: of calls per 3 minutes

X~Poisson(9)

$$P(X=0) = \frac{e^{-9}9^{0}}{0!} = 0.000123$$