SCS 1307
Probability & Statistics

by

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Normal Approximation to the Binomial Distribution

Under certain circumstances the normal distribution can be used as an approximation to the binomial distribution.

- If X~ Bin(n,p) then E(X)=np and Var(X)=npq.

- Now for large n and p not too small or too large

  Then X~ N(np,npq)

  (Hint: When n>10 and p close to ½ or n>30 and p moving away from 1/2)

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Example

Find the probability of obtaining between 4 and 7 heads inclusive with 12 tosses of a fair coin,

(a) using the binomial distribution,

(b) using the normal approximation to the binomial distribution.

Solution

Find the probability of obtaining between 4 and 7 heads inclusive with 12 tosses of a fair coin,

(a) using the binomial distribution

Let X be the r.v "the number of heads obtained"

$$X \sim Bin(12, 0.5)$$

$$P(4 =< X =< 7) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$$

$$= 0.121 + 0.193 + 0.226 + 0.193 = 0.733$$

Solution

(b) using the normal approximation to the binomial distribution.

$n=12$ & $p=0.5$ so $np=6$, $npq=3$

$X \sim N(6,3)$

with the continuity correction we need to find

$P(3.5<X<7.5) = P(-1.443<Z<0.866)$

$= 0.732$

Exercise

It is known that in a sack of mixed grass seeds 35% are ryegrass. Use the normal approximation to the binomial distribution to find the probability that in a sample of 400 seeds there are

(a) less than 120 ryegrass seeds

(b) between 120 and 150 ryegrass seeds (inclusive)

(c) more than 160 ryegrass seeds

Solution

 (a) less than 120 ryegrass seeds

Let X be the number of ryegrass seeds.

Then X~Bin(400,0.35)

np=400*0.35=140 and npq=91

X~N(140,91)

We require P(X<119.5)=P(Z<-2.149)=0.0158

Solution

X~N(140,91)

(b) between 120 and 150 ryegrass seeds (inclusive)

We require P(120=<X=<150) =P(119.5<X<150.5)

$$=P(-2.149<Z<1.101)$$

$$= 0.8487$$

Solution

X~N(140,91)

(c) more than 160 ryegrass seeds

We require P(X>160) =P(X>160.5)

$$=P(Z>2.149)$$

$$= 0.0158$$