Computer Systems

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Introduction to Boolean Algebra & Logic Operators

History – Boolean Algebra

Boolean Algebra – George Boole

(1854, A British mathematician)

The branch of algebra in which the values of the variables are the truth values true and false.

Boolean Algbra

- There are only two states
  - State 0
  - State 1
- Usually, a variable is considered in state 1 unless it is specifically designated with a negation.
  - x– variable x is assumed to be in state 1
  - $\bar{F}$ - it is assumed that $\bar{F}$ denotes state 0

Basic Operations: Boolean Algebra

$A.B$	$A+B$	$\bar{A}$
AND	OR	NOT

Boolean Function

- A Boolean function is an expression that consists of binary variables and boolean logic operations.

$$f(x, y) = x.\bar{y} + \bar{x}.(x + y)$$

- A Boolean function can be evaluated for all possible values of the variables.
- Finite number of possible value combinations for all the variables.

Truth Table

- A boolean function can be expressed in a truth table.
- The number of rows will be 2^n where n is the number of variables in the function.

$$F = x + y.\bar{z}$$

x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Truth Table (Cont.)

- Truth table can be elaborated with additional columns for each term

$$F = x + y.\bar{z}$$

x	y	z	$y.\bar{z}$	F
0	0	0	0	0
0	0	1	0	0
0	1	0	1	1
0	1	1	0	0
1	0	0	0	1
1	0	1	0	1
1	1	0	1	1
1	1	1	0	1

Algebra to Logic Circuit

The operational behavior of a Boolean algebraic expression can be represented with a circuit diagram composed of logic gates.

A logic gate is a piece of hardware which emulates Boolean algebraic operation(s) and operates on Boolean inputs.

Inside of a Logic Gate

- Transistors
- Resistors
- Diodes

Transistor AND Gate

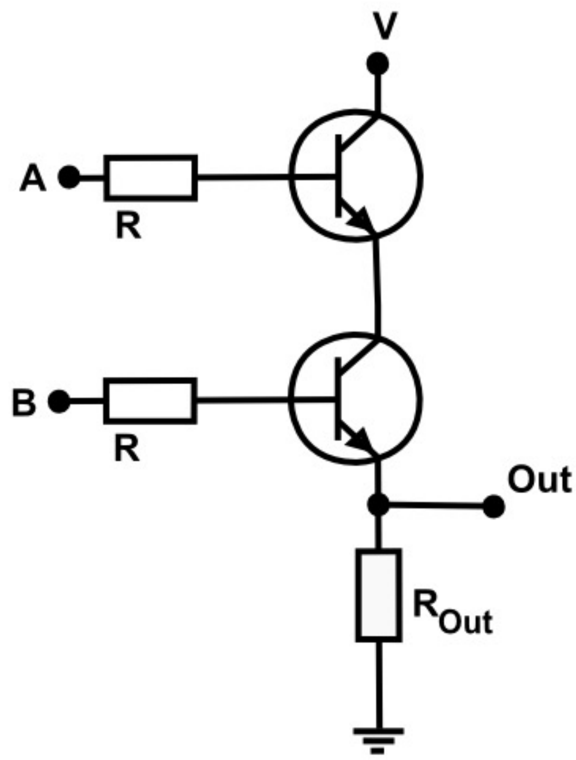
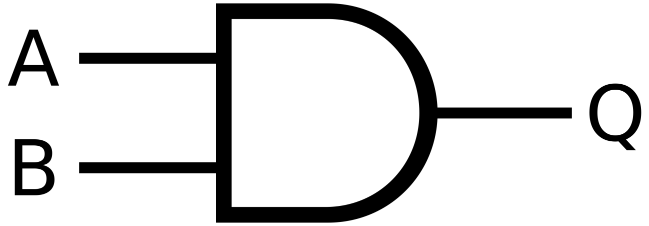


Image Source - https://en.wikipedia.org/wiki/AND_gate

AND Operation

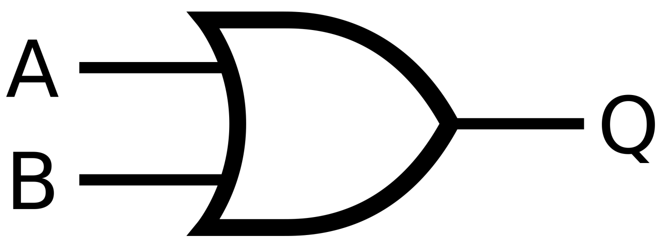
- Operator is symbolically denoted as follows:
  - A . B
  - A ∧ B



A	B	Q = A.B
0	0	0
0	1	0
1	0	0
1	1	1

OR Operation

- Operator is symbolically denoted as follows:
  - A + B
  - A V B

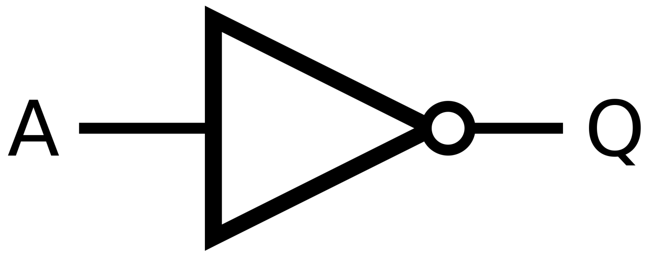


A	B	Q = A+B
0	0	0
0	1	1
1	0	1
1	1	1

NOT Operation

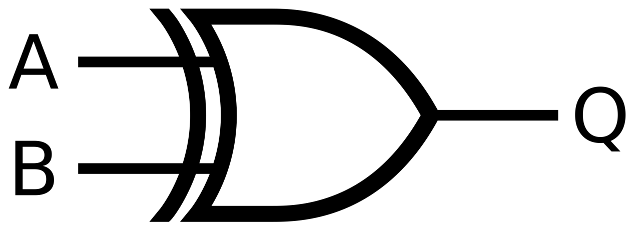
- Operator is symbolically denoted as follows:
  - $\overline{A}$
  - ¬A
  - ∼A
  - A’

A	Q = $\overline{A}$
0	1
1	0



XOR Operation

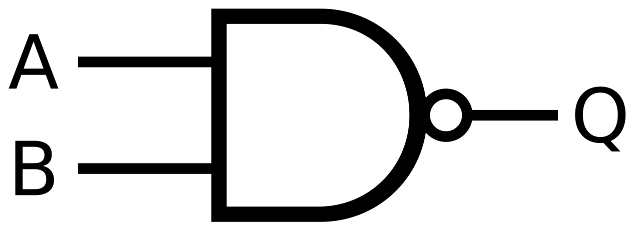
- Operator is symbolically denoted as follows:
  - A ⊕ B



A	B	Q = A⊕B
0	0	0
0	1	1
1	0	1
1	1	0

NAND Operation

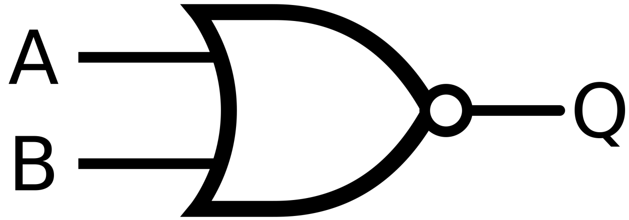
- Operator is symbolically denoted as follows:
  - $\overline{(A\,.\,B)}$
  - $\overline{(A \wedge B)}$
  - (A . B)’
  - ¬(A ∧ B)



A	B	$\overline{(A\,.\,B)}$
0	0	1
0	1	1
1	0	1
1	1	0

NOR Operation

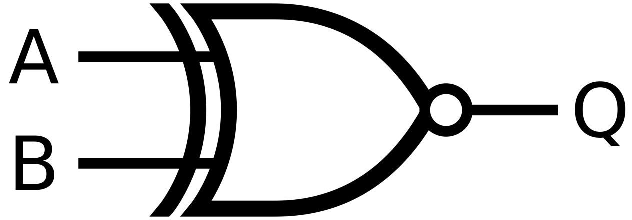
- Operator is symbolically denoted as follows:
  - $\overline{(A + B)}$
  - $\overline{(A \vee B)}$
  - (A + B)’
  - ¬(A ∨ B)



A	B	$\overline{(A + B)}$
0	0	1
0	1	0
1	0	0
1	1	0

XNOR Operation

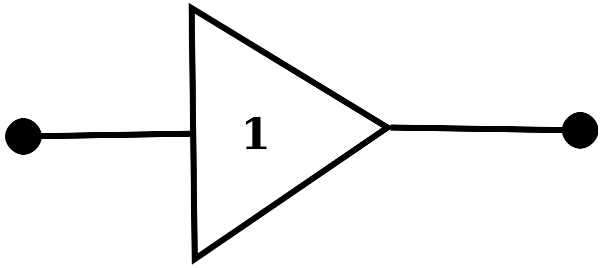
- Operator is symbolically denoted as follows:
  - $\overline{A \oplus B}$



A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

Buffer

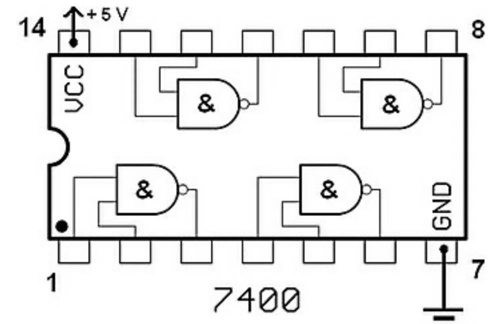
- A basic logic gate that passes its input, unchanged, to its output.
- To increase the propagation delay of circuits



A	Q
0	0
1	1

Logic Gates Manufacturing

- Gates are built into IC form
- Single IC may have several gates
- Typically one type of gates in an IC
- Multi input single gate ICs available



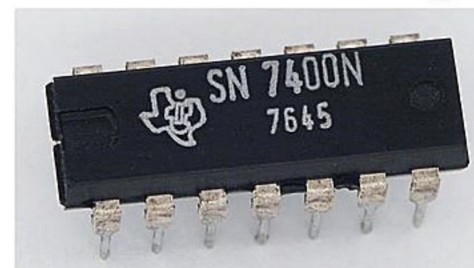
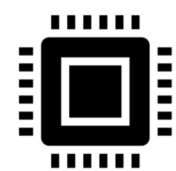


Image Source
https://en.wikipedia.org/wiki/Logic_gate



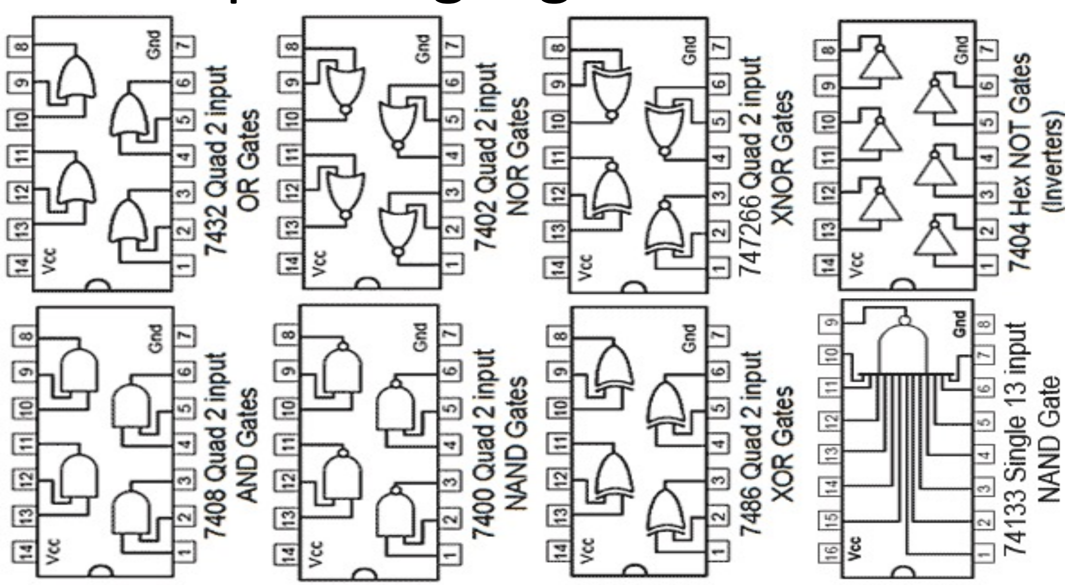
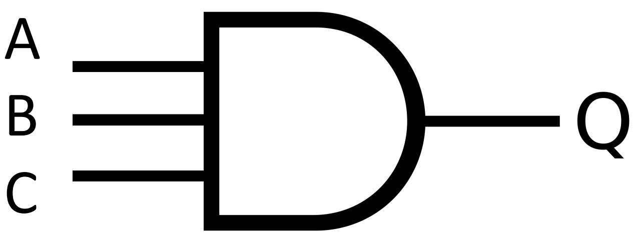
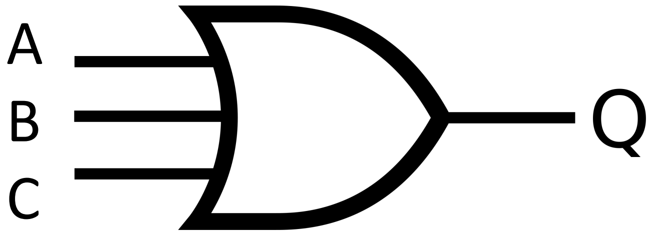


Image Source https://learnabout-electronics.org/Digital/dig21.php

Multi Input Logic Gates

- Gates with more than two inputs are available
- AND Gate with three inputs (A.B.C)
- OR Gate with ~~four~~ three inputs (A+B+C)





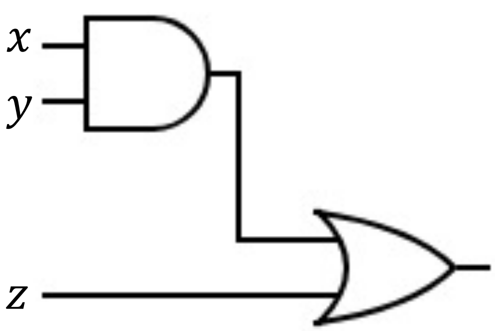
Multi Input Logic Gates (Cont.)

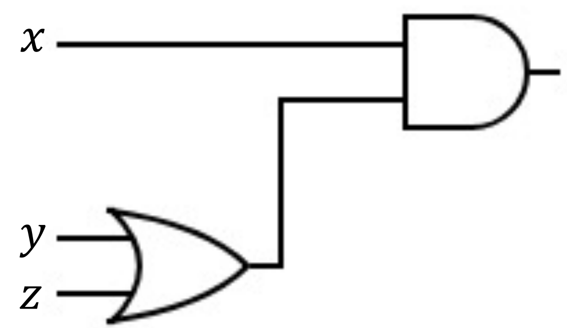
- Gates with more than two inputs are available
- AND Gate with three inputs
  - $F = x \wedge y \wedge z$
- OR Gate with three inputs
  - $F = x \vee y \vee z$
- XOR Gate with three inputs
  - $F = x \oplus y \oplus z$

Drawing Logic Circuits

- How to draw the logic circuit for a given boolean expression?

$$F = x.y + z$$





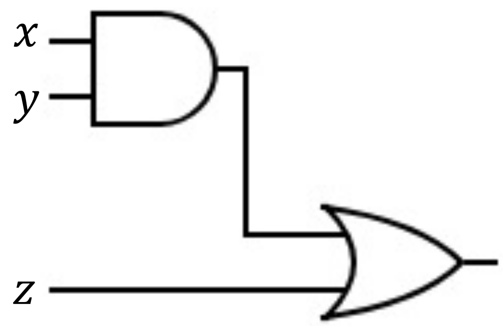
Operator Precedence

( )	Parentheses
~	NOT
∧	AND
∨	OR

Drawing Logic Circuits

- The logic circuit for a given boolean expression can be drawn using appropriate gates.

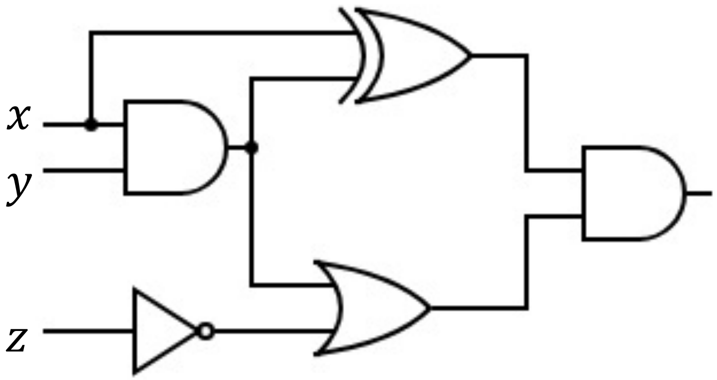
$$F = x.y + z$$



Drawing Logic Circuits

- Complex expressions require complex gate arrangements.

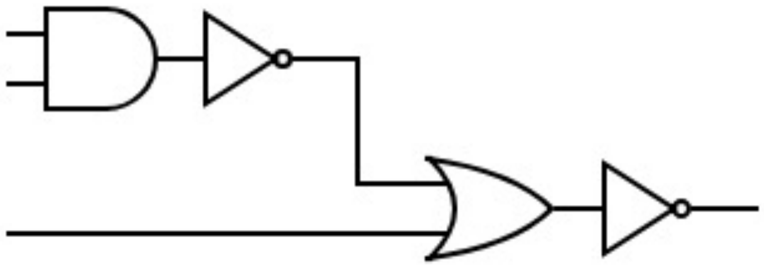
$$F = (x \oplus (x.y)).(x.y + \bar{z})$$



Drawing Logic Circuits (Cont.)

- Mulitiple circuits for the same expression

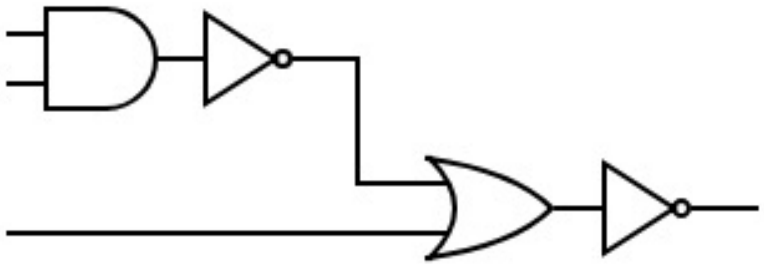
$$F = \overline{\overline{x.y} + z}$$

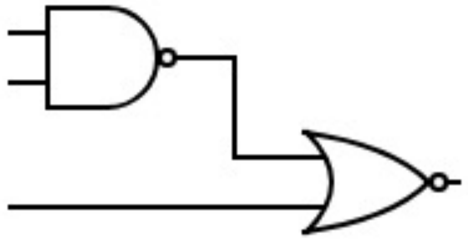


Drawing Logic Circuits (Cont.)

- Mulitiple circuits for the same expression

$$F = \overline{\overline{x.y} + z}$$





Circuit drawing - https://www.circuit-diagram.org/editor/

Thank You..!