SCS 1306 Linear Algebra

Tutorial 09

Eigenvalues and Eigenvectors

Select the most suitable answer for question 1-5.

1. The eigenvalues of $\begin{bmatrix} 5 & 6 & 17 \\ 0 & -19 & 23 \\ 0 & 0 & 37 \end{bmatrix}$ are

(A) -19 , 5, 37
(B) 19, -5, -37
(C) 2, -3, 7
(D) 3, -5, 37

2. If $\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -4 \end{bmatrix}$, the eigenvalue corresponding to the eigenvector is

(A) 1
(B) 4
(C) -4.5
(D) 6

3. The eigenvalue of the following matrix $\begin{bmatrix} 3 & 2 & 9 \\ 7 & 5 & 13 \\ 6 & 17 & 19 \end{bmatrix}$ are given by solving the cubic equation

(A) $\lambda^3 - 27\lambda^2 + 167\lambda - 285$
(B) $\lambda^3 - 27\lambda^2 - 122\lambda - 313$
(C) $\lambda^3 + 27\lambda^2 + 167\lambda + 285$
(D) $\lambda^3 + 23.23\lambda^2 - 158.3\lambda + 313$

4. The eigenvalues of a 4 × 4 matrix [A] are given as 2, -3, 13, and 7. Then the |det(A)| is

(A) 546
(B) 19
(C) 25
(D) Cannot be determined

5. Given the matrix $[A] = \begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -3 \end{bmatrix}$ has an eigenvalue value of 4 with the corresponding eigenvector of $[X]= \begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$, then $[A]^5[X]$ =

(A) $\begin{bmatrix} -18 \\ -16 \\ 4 \end{bmatrix}$

(B) $\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$

(C) $\begin{bmatrix} -4608 \\ -4096 \\ 1024 \end{bmatrix}$

(D) $\begin{bmatrix} -0.004395 \\ -0.003906 \\ 0.0009766 \end{bmatrix}$

6. Find all the eigenvalues and corresponding eigenvectors, and say whether the matrix A can or cannot be diagonalized. If the matrix can be diagonalized, give a matrix P such that $P^{-1}AP= D$ is diagonal.

(A) $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

(B) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

(C) $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

(D) $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

7. Verify if the matrix A is orthogonal hence find its inverse.

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$

8. Verify whether the following matrix is orthogonal or not? If not, can it convert into an orthogonal matrix. It yes, how?

$$A = \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$

9. (A) Prove if matrix A is orthogonal, then its determinants must be either 1 or -1.

   (B) Prove if matrix A and B are both orthogonal, then AB is also orthogonal.

10. Suppose that $n \times n$ matrix A can be computed as QBQ^{-1} where Q is an $n \times n$ orthogonal matrix, and B is an $n \times n$ diagonal matrix.