



Program Complexity Analysis

Question 01

```
1 for (i = 0; i < n; i++):  
2     for(j = 0; j < n; j++):  
3         a = b + 1;
```

Question 02 - Runtime of Selection Sort

```
1 void selection_sort(item_type s[], int n) {  
2     int i, j; /* counters */  
3     int min; /* index of minimum */  
4     for (i = 0; i < n; i++) {  
5         min = i;  
6         for (j = i + 1; j < n; j++) {  
7             if (s[j] < s[min]) {  
8                 min = j;  
9             }  
10        }  
11        swap(&s[i], &s[min]);  
12    }  
13 }
```

Question 03 - Matrix Multiplication

```
1 /* Input: Two matrices [A (x by y), B (y by z)]  
2    Output: C (x by z) where C[i][j] = dot(A_row_i, B_col_j)  
3 */  
4 for (i = 1; i <= a->rows; i++) {  
5     for (j = 1; j <= b->columns; j++) {  
6         c->m[i][j] = 0;  
7         for (k = 1; k <= b->rows; k++) {  
8             c->m[i][j] += a->m[i][k] * b->m[k][j];  
9         }  
10    }  
11 }
```

Question 04

```
1 r := 0
2 for i := 1 to n-1 do
3     for j := i+1 to n do
4         for k := 1 to j do
5             r := r + 1
6 return(r)
```

Question 05

```
1 r := 0
2 for i := 1 to n do
3     for j := 1 to i do
4         for k := j to i+j do
5             r := r + 1
6 return(r)
```

Question 06

```
1 count := 0
2 i := n
3 while i > 1 do
4     count := count + 1
5     i := i / 2
6 return count
```

Question 07

```
1 sum := 0
2 for i := 1 to n do
3     i := i * 2
4     sum := sum + 1
5 return sum
```

Question 08

```
1 int i, j, k = 0;
2 for (i = n / 2; i <= n; i++) {
3     for (j = 2; j <= n; j = j * 2) {
4         k = k + n / 2;
5     }
6 }
```

Question 09

```
1 r := 0
2 for i := 1 to n do
3     for j := 1 to i do
4         for k := j to i+j do
5             for l := 1 to i+j-k do
6                 r := r + 1
7 return(r)
```

Question 10

```
1 r = 0
2 for i = 1 to n do
3     for j = i + 1 to n do
4         for k = i + j - 1 to n do
5             r = r + 1
6 return(r)
```

A Summation formulas

Basic counting

$$\sum_{j=a}^b 1 = b - a + 1 \quad (\text{number of integers from } a \text{ to } b, \text{ inclusive})$$

Arithmetic sums (polynomial sums)

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2} \right)^2$$

$$\sum_{j=i+1}^n j = \frac{n(n+1)}{2} - \frac{i(i+1)}{2}$$