SCS 1302 - Discrete Mathematics

Take Home Assignment 01

Instructions:

- Handwritten or printed answers are allowed.
- Write your index number on each page. Please use your new index number.
- All the answer scripts must be in PDF format.
- Rename your PDF as `Index_A1`.
  Example: `240000_A1`
- Late submissions will not be marked. Submit on or before the deadline: **28th July 2025, 5.00 PM**.

Question 01

(a) Show that the compound propositions $(p \wedge \neg q) \rightarrow r$ and $(p \rightarrow q) \rightarrow r$ are logically equivalent by using a truth table (10 marks).

Is the compound proposition $(p \wedge \neg q) \rightarrow r$ valid?

Is the compound proposition $(p \wedge \neg q) \rightarrow r$ a tautology?

(b) State the converse, contrapositive, and inverse of the conditional statement: "If Yomal is concerned about his cholesterol levels, then he will walk 2 km every day." (05 marks)

(c) Show that the propositions $(p \vee q) \wedge p$ and p are logically equivalent by developing a series of logical equivalences (05 marks).

(d) Show that the propositions $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences (05 marks).

Question 02

(a) Let $p(x)$ and $q(x)$ be two predicates of variable x defined by $p(x) : x < -1$ and $q(x) : x \geq 1$, where $x \in \mathbb{R}$. Determine the truth values of the following propositions and justify your answers (10 marks).

(i) $\neg\forall x\big(\neg p(x) \wedge \neg q(x)\big)$

(ii) $\neg\exists x\big(\neg p(x) \vee \neg q(x)\big)$

(iii) $\neg\forall x \neg p(x) \vee \neg\exists x q(x)$

(iv) $\neg\exists x \neg p(x) \wedge \neg\exists x \neg q(x)$

(b) Let $N(x)$ be the predicate "x is a non-negative integer," let $E(x)$ be the predicate "x is even," let $O(x)$ be the predicate "x is odd," and let $P(x)$ be the predicate "x is prime," where the domain for x is the set of integers. Express each of the following statements using $N(x)$, $E(x)$, $O(x)$, $P(x)$, quantifiers, and logical connectives (20 marks).

(i) There exists an odd prime integer.

(ii) If an integer is not even, then it is odd.

(iii) The only even prime is 2.

(iv) The addition of two prime numbers need not be a prime number.

(v) The product of any two prime numbers is not a prime number.

Question 03

(a) Determine whether the following argument is valid. If valid, prove it formally using rules of inference and equivalence. If invalid, provide a truth assignment that shows the invalidity: (10 marks)

$$(m \wedge n) \to (p \vee \neg q), \quad m \to (r \wedge \neg p), \quad q \to \neg r. \quad \text{Therefore, } \neg n.$$

(b) Consider the following scenario: If AI systems are ethical, then either programmers are unbiased or regulations are strict. If regulations are not strict and programmers are biased, then AI systems are not ethical (20 marks).

Define the following propositions:

- e: AI systems are ethical
- u: programmers are unbiased
- s: regulations are strict

Answer the following:

(i) Translate the argument into symbolic form.

(ii) Express the premises and conclusion as a single compound proposition.

(iii) Use a truth table to evaluate the validity of the argument.

(iv) Is the conclusion logically valid? Explain.

(c) Determine whether the following argument is valid. If the argument is valid, state the rule of inference being used (15 marks).

> "If n is a real number with $n > 4$, then $n^2 > 16$. Suppose that $n^2 \leq 16$. Then $n \leq 4$."

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