Tutorial 05

1) Elementary Row Operations

   a) Add a scalar multiple of one row to another row, and replace the latter row with that sum

   b) Multiply one row by a nonzero scalar

   c) Swap the position of two rows

Do the following operations on **matrix A** = $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 8 & 0 \\ 0 & -3 & 4 \end{pmatrix}$

   a) Add 2 times the first row to the second row, and replace the latter row with that sum and obtain **matrix** B

   b) Multiply the 3rd row of the **matrix B** by (- 1/3) and hence obtain **matrix C**

   c) Swap the position of 1st and 2nd rows of the **matrix C** and hence obtain **matrix D**

2) Elementary matrices

   a) $E_{(2\times1+2)}$ = $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Find P = $E_{(2\times1+2)} \times A$

   b) $E_{(-1/3,3)}$ = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$ Find Q = $E_{(-1/3,3)} \times P$

   c) P^{12} = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Find R = $P^{12} \times Q$

3) Gaussian elimination → Row echelon form

   a) Let K = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 1 & 2 & 1 \\ 4 & 3 & 2 & 1 \end{pmatrix}$, Find M such M is in row echelon form

   b) Let A = $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$. Write A as a product of elementary matrices.

4) Gauss-Jordan elimination → reduced row echelon form.

   a) Let A = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 0 \end{pmatrix}$, Find B, the reduced row-echelon form of A, and write it in the form B=UA.

b) Let K = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 1 & 2 & 1 \\ 4 & 3 & 2 & 1 \end{pmatrix}$, Find M such M is in reduced row echelon form. And find U such that UK = M.

5) Find the rank of each of the following

a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 4 & 5 & 6 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ e) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$