University of Colombo School of Computing

SCS 1308 - Foundations of Algorithms

Tutorial - 03

01. Professor Amongus has shown that a decision problem L is polynomial-time reducible to an NP-complete problem M. Moreover, after 80 pages of dense mathematics, he has also just proven that L can be solved in polynomial time. Has he just proven that P=NP ? Why or why not?

02. What is the difference between problems which are proven to be NP-hard and NP-complete?

03. Find a satisfying assignment to the following 3SAT formula, or argue that one doesn't exist.

$$(w \lor x \lor \sim z) \land (x \lor \sim y \lor z) \land (w \lor \sim x \lor z) \land (\sim w \lor \sim x \lor y)$$

04. The problem CLIQUE takes as input a graph G = (V, E) and integer k > 0 and returns True if there is a set C subset V of size k such that every pair of vertices in C are connected with an edge from E.

I. What does it mean to say that the Boolean Satisfiability problem SAT is polynomially reducible to CLIQUE

II. Given the following input formula

$$F = (\sim P \lor Q \lor \sim R) \land (P \lor Q) \land (\sim Q \lor R)$$

construct a graph Gr such that F is satisfiable if and only if G_F has a clique of size k = 3.

III. Explain why the existence of a clique of size k = 3 guarantees that there is a satisfying assignment for F?

05. Explain the Knapsack algorithm. Is it a NP-complete problem?

Given the values and weights of four items as below:

Item	1	2	3	4
Value	10	20	30	40
Weight	30	10	40	20

The capacity of the sack is given as, W = 40

What is the maximum value achieved, illustrate the Knapsack algorithm.

06. A set of nodes "V" is a vertex cover. The removal of V from the graph destroys every edge. This is the VERTEX_COVER problem. In the graph shown below {A, C, D, F} is a vertex cover. Given the input as the graph G and an integer k,

A. Does there exist a vertex cover of G at most k nodes? Explain

B. Then, how you can use 3SAT (using two connected nodes for each variable and three connected nodes for each clause) for showing that vertex cover problem is NP complete. You can be creative in your explanation