SCS 1307
Probability & Statistics

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Sum and difference of two independent normal variables

If X and Y are two independent normal variables such that $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$

then $X+Y \sim N(\mu_1+\mu_2, \sigma_1^2 + \sigma_2^2)$

and $X-Y \sim N(\mu_1-\mu_2, \sigma_1^2 + \sigma_2^2)$

Example

If $X \sim N(60,16)$ and $Y \sim N(70,9)$, find

(a) $P(X+Y<140)$

(b) $P(120<X+Y<135)$

(c) $P(Y-X>7)$

Solution

If $X \sim N(60,16)$ and $Y \sim N(70,9)$, find

(a) $P(X+Y<140)$

$(X+Y) \sim N(60+70, 16+9)$

i.e $R = X+Y \sim N(130,25)$

We require $P(R<140) = P(Z<2)$

$= 0.9772$

Solution

(b) $(X+Y) \sim N(60+70, 16+9)$

i.e $R = X+Y \sim N(130, 25)$

We require $P(120 < R < 135) = P(-2 < Z < 1)$

$= 0.8185$

Solution

(c) $(Y-X) \sim N(70-60, 9+16)$

i.e $T = Y-X \sim N(10, 25)$

We require $P(Y-X>7) = P(T>7)$

$= P(Z>-0.6)$

$= 0.7257$

Multiples of Normal Variables

If X is a normal variable such that $X \sim N(\mu, \sigma^2)$ then

$$aX \sim N(a\mu, a^2\sigma^2)$$

Example: If $X \sim N(50,25)$, find $P(3X>160)$

Solution

If $X \sim N(50,25)$, find $P(3X>160)$

$3X \sim N(150,225)$

We require $P(3X>160) = P(Z>0.667)$

$= 0.2523$

Exercise

Each weekday Mr Jones walks to the local library to read the newspapers. The time he takes to walk to and from the library is a normal variable with mean 15 minutes and standard deviation 2 minutes. The time he spends in the library is a normal variable with mean 25 minutes and standard deviation $\sqrt{12}$ minutes. Find the probability that, on a particular day,

(i) Mr Jones is away from the house for more than 45 minutes

(ii) Mr Jones spends more time travelling than in the library

Exercise

Let X be the time he takes to walk to and from the library

$X \sim N(15,4)$

Let Y be the time he spends in the library

$Y \sim N(25,12)$

(i) Mr Jones is away from the house for more than 45 minutes

$X+Y \sim N(40, 16)$

$P(X+Y>45)$

(ii) Mr Jones spends more time travelling than in the library

$X-Y \sim N(-10, 16)$

$P(X>Y) = P(X-Y>0)$

The Distribution of the Sample Mean

If $X_1, X_2, \ldots\ldots X_m$ is a random sample of size n taken from a normal distribution with mean μ and variance σ^2 such that $X \sim N(\mu, \sigma^2)$, then the distribution of $\overline{X}$ is also normal with mean μ and variance σ^2/n.

Example: A random sample of size 15 is taken from a normal distribution with mean 60 and standard deviation 4. Find the probability that the mean of the sample is less than 58. $\overline{X}$

Solution

A random sample of size 15 is taken from a normal distribution with mean 60 and standard deviation 4. Find the probability that the mean of the sample is less than 58.

$X \sim N(60,16)$

Sample mean of size 15, $\overline{X} \sim N(60, 16/15)$

We require $P(\overline{X}<58) = P(Z<-1.936) = 0.0264$

The Central Limit Theorem

The Distribution of the sample mean from any population

If X1, X2,…….Xn is a random sample of size n taken from **any** distribution with mean μ and variance σ^2 , **for large n**, then the distribution of sample mean is approximately normal with mean μ and variance σ^2/n.

Note: the approximation gets better as n gets larger

Example

If a random sample of size 30 is taken from the Binomial(9,0.5) distribution, find the probability that the sample mean exceeds 5.

Solution

If a random sample of size 30 is taken from the Binomial(9,0.5) distribution, find the probability that the sample mean exceeds 5.

X~Bin(9,0.5) np=4.5 npq=2.25

By CLT $\overline{X}$ ~N(4.5, 2.25/30)

P($\overline{x}$ >5) =P(Z>1.826) = 0.0340