INDEX NUMBER:

UNIVERSITY OF COLOMBO, SRI LANKA
FACULTY OF SCIENCE

LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) – 2024

ST 1008 – PROBABILITY AND DISTRIBUTIONS

(Two Hours)

Answer all questions

No. of questions: 04
No. of pages: 08

Important Instructions to the Candidates:

- If a page or a part of this question paper is not printed, please inform the supervisor immediately.
- Enter your index number on all pages of the answer script/question paper.
- MULTIPLE CHOICE QUESTIONS: Question (1) and (2) consist of 10 Multiple Choice Questions (MCQ) of each. Each of the MCQs will have 5 choices with only one correct answer. **Encircle the correct choice** in the tables given on the question paper.
- SEMI-STRUCTURED TYPE: Write the answers to questions (3) and (4) on the papers provided.
- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.
- **Attach the exam paper to the answer sheets.**
- Statistical tables are attached to the paper.
- **You are not permitted to remove any part of the question paper except the page that the statistical tables are given from the Examination Hall.**

For Examiner's use only						
Question No.	Q1	Q2	Q3	Q4	Total	%
Marks						

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Question Number 01.

This question consists of 10 multiple choice questions. Select the most suitable answer and encircle the correct choice in Table 01.

(Question No. 01. – 10 × 10 Marks)

Table 01: Answer Table of Question Number 01.

Question Number	Answers					Question Number	Answers				
1.	a	b	c	d	e	**6.**	a	b	c	d	e
2.	a	b	c	d	e	**7.**	a	b	c	d	e
3.	a	b	c	d	e	**8.**	a	b	c	d	e
4.	a	b	c	d	e	**9.**	a	b	c	d	e
5.	a	b	c	d	e	**10.**	a	b	c	d	e

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Number of Correct Answers:……... **Marks:**…….

Question Number 02.

This question consists of 10 multiple choice questions. Select the most suitable answer and encircle the correct choice in Table 2.

(Question No. 02. – 10 × 10 Marks)

Table 2: Answer Table of Question Number 02.

Question Number	Answers					Question Number	Answers				
1.	a	b	c	d	e	**6.**	a	b	c	d	e
2.	a	b	c	d	e	**7.**	a	b	c	d	e
3.	a	b	c	d	e	**8.**	a	b	c	d	e
4.	a	b	c	d	e	**9.**	a	b	c	d	e
5.	a	b	c	d	e	**10.**	a	b	c	d	e

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Number of Correct Answers:……... **Marks:**…….

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Question Number 03.

(a) Events A and B are such that $P(A) = \frac{1}{3}$, $P(B) = \frac{2}{5}$, and $P(B|\overline{A}) = \frac{11}{20}$.

(i) Highlight the events $(A \cap \overline{B})$, $(B \cap \overline{A})$ and $(A \cap B)$ in a Venn diagram? **(10 marks)**

(ii) Find $P(A \cap B)$ **(10 marks)**

(iii) Find $P(A \cup B)$. **(10 marks)**

(iv) Find $P(\overline{A}|B)$. **(10 marks)**

(v) Find $P(\overline{B}|A)$. **(10 marks)**

(b) The continuous random variable X has probability density function

$$f(x) = \begin{cases} cx, & 0 \leq x < 1 \\ c, & 1 \leq x < 2 \\ 0, & otherwise \end{cases}$$

where c is a constant

(i) Show that $c = \frac{2}{3}$. **(10 marks)**

(ii) Find $E(X)$. **(10 marks)**

(iii) Show that the median m of X is 1.25. **(10 marks)**

(iv) Show that $P\left(|X - m| < \frac{1}{2}\right) = 0.65$. **(10 marks)**

(v) Three independent observations of X are taken. Find the probability of one of the observations is satisfying the condition $|X - m| < \frac{1}{2}$ and other two are not satisfying it. **(10 marks)**

(Question No. 03 – 100 Marks)

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Question Number 04.

(a) A darts player practices throwing a dart at the bull's eye on a dart board. Independently for each throw, her probability of hitting the bull's eye is 0.2. Let X be the number of failures before her first success hit on the bull's eye and X takes geometric distribution with the probability function

$$P(X = x) = 0.8^x 0.2, \qquad x = 0, 1, 2, \ldots$$

(i) Find the probability that she is successful for the first time on her 3rd throw. **(05 marks)**

(ii) Find the probability that she will have at least 3 failures before her first success. **(10 marks)**

(iii) Show that the mean value of X is 4. **(15 marks)**

Hint: $\sum_{x=1}^{\infty} rq^{r-1} = \frac{1}{(1-q)^2}$ $when$ $|q| < 1$

(iv) On another occasion the player throws the dart at the bull's eye until she has 2 successes. Let Y be number of failures before her 2nd success hit on the bull's eye. Write down the probability distribution function of Y, and give the name of this distribution. **(10 marks)**

(b) A machine is producing components whose lengths are normally distributed with a mean of 6.50 cm. As a policy to reject components an upper tolerance limit of 6.45 cm for the length of the components has been adopted and, when the machine is correctly set, 1 in 20 components is rejected as exceeding this limit. After servicing the machine, it is found that 1 in 15 components is rejected for exceeding this limit.

[Note: Use the interpolation technique to estimate the values of unknown data points that fall in between existing, known data points.]

(i) Show that the standard deviation of the lengths when the machine is correctly set is 0.0243. **(15 marks)**

(ii) Assuming that the mean has not changed but that the production has become more variable after the service, show that the new standard deviation is 0.0266. **(15 marks)**

(iii) Assuming that the standard deviation has not changed but that the mean has moved after the service, estimate the new mean. **(15 marks)**

(iv) If 1000 components are produced in a shift, how many of them may be expected to have lengths in the rage 4.48 to 6.53 cm if the machine is set as in (ii). **(15 marks)**

Hint: $P(Z < -0.752) = 0.2260$ and $P(Z < 1.128) = 0.8703$ where Z is a standard normal random variable.

(Question No. 04 – 100 Marks)

********** END **********

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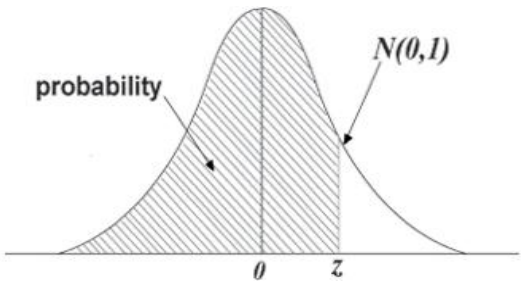
Table 3. The Standardized Normal Distribution Probabilities



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of ***Z***, the standardized normal deviate, (the proportion ***P***, of the distribution less than ***Z***) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value ***X*** is obtained by calculating $Z = (X - \mu)/\sigma$ and reading the proportion corresponding to this value of ***Z***.

Z	*P*	*Z*	*P*	*Z*	*P*
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997