University of Colombo School of Computing

SCS1308 - Foundations of Algorithms

Tutorial 03

Iteration Method

1. $T(n) = 2T(n/2) + n^4$ with the base case $T(1) = \Theta(1)$
2. $T(n) = 16T(n/4) + n^2$ with the base case $T(1) = \Theta(1)$
3. $T(n) = T(n/4) + \sqrt{n}$ with the base case $T(1) = \Theta(1)$
4. $T(n) = T(n/2) + n \log_2 n$ with the base case $T(1) = 0$, assume $n = 2^k for k \geq 0$
5. $T(n) = 2T(n-1) + 2^n$ with the base case $T(1) = 2$
6. $T(n) = T(n - \sqrt{n}) + \sqrt{n}$, with the base case $T(1) = 1$
7. $T(n) = T(n/2) + \log n$ with the base case $T(1) = 0$ assume that $n = 2^k$
8. $T(n) = T(n-3) + 2n$ with the base cases, $T(1) = 1, \quad T(2) = 2, \quad and \quad T(3) = 3$ (assume n is divisible by 3 for simplicity)

Substitution Method

1. Prove the solution of $T(n) = T(n-1) + n$ is $O(n^2)$.
2. Prove that $T(n) = 2T(n/2) + n/\log_2 n$ with the base case $T(1) = 1$ is $O(n \log n)$
3. $T(n) = 3T(\sqrt{n}) + \log n$, with the base case $T(2) = 1$
4. $T(n) = T(\sqrt{n}) + 1$, with the base case $T(2) = 0$

Masters Method

1. $T(n) = 9T(n/3) + n$
2. $T(n) = T(n/3) + 1$
3. $T(n) = 7T(n/2) + \Theta(n^2)$
4. $T(n) = 2T(n/4) + \sqrt{n}$
5. $T(n) = \sqrt{2}T(n/2) + n$
6. Use the master method to show that the solution to the binary-search recurrence $T(n) = T(n/2) + \Theta(1) \ is \ T(n) = \Theta(\log_2 n)$.
7. $T(n) = 5T(n/2) + n^{2.3}, \quad \log_2 5 \approx 2.32$

A Summation formulas

Basic counting

$$\sum_{j=a}^{b} 1 = b - a + 1 \qquad \text{(number of integers from } a \text{ to } b\text{, inclusive)}$$

Arithmetic sums (polynomial sums)

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{j=i+1}^{n} j = \frac{n(n+1)}{2} - \frac{i(i+1)}{2}$$

Geometric

$$\sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r} \quad (r \neq 1)$$

$$\sum_{i=1}^{n} r^i = \frac{r(1 - r^n)}{1 - r} \quad (r \neq 1)$$

$$\sum_{i=0}^{\infty} r^i = \frac{1}{1 - r} \quad (|r| < 1)$$

$$\sum_{i=1}^{\infty} r^i = \frac{r}{1 - r} \quad (|r| < 1)$$

$$\sum_{i=0}^{n} ar^i = a \cdot \frac{1 - r^{n+1}}{1 - r} \quad (r \neq 1)$$

$$\sum_{i=1}^{n} ar^{i-1} = a \cdot \frac{1 - r^n}{1 - r} \quad (r \neq 1)$$

Logarithmic

$$\sum_{k=1}^{n} \log k = \log(n!)$$

$$\log(n!) = n \log n - n + \frac{1}{2} \log(2\pi n) + O\left(\frac{1}{n}\right)$$

$$\sum_{k=1}^{n} \frac{\log k}{k} = \frac{1}{2}(\log n)^2 + O(1)$$