SCS 1307
Probability & Statistics

by

Dr Dilshani Tissera

Department of Statistics

University of Colombo

Experiment and Outcome

- **A Random Experiment**

  an experiment in which there is some uncertainty about the outcome that will be observed

- **Outcome**

  The results of an observation of the experiment

- **Outcome space (universal set / sample space)**

  Set of all possible outcomes of the experiment (S)

- **Equally Likely Outcome**

  If any outcome of random experiment is as likely to occur as any other outcome then they are called equally likely outcomes.

- **Example for a Random Experiment**

  Roll a die (outcomes: 1,2,3,4,5,6)

Events

- **An Event**

  A sub set of outcome space

- **Equally Likely Events**

  If any event is as likely to occur as any other event then they are called equally likely events.

- **Mutually Exclusive Events**

  Two events A and B are said to be mutually exclusive (or disjoint) if $A \cap B = \varnothing$.

- **Exhaustive events**

  If two events A and B are such that P(A U B) =S and P(A U B) = 1

- **An Elementary event**

  An event with just one outcome

Example

- A drawing pin is dropped twice and it is observed, each time, whether the pin is pointing upwards or downwards. Possible outcomes are:

    * outcome ‘pin points upwards on both occasions’

    * outcome ‘pin points downwards on both occasions’

    * outcome ‘pin points upwards on first drop and downwards on second drop’

    * outcome ‘pin points downwards on first drop and upwards on second drop

Examples

- A random experiment consists of tossing two coins and observing whether each falls Heads or tails.

- A random experiment consists of tossing three coins and observing whether each falls Heads or tails.

Probability function

- Probability is a function , which we denote by P. Like all functions , P has a domain which we denote by G. Since P assigns values to events, G is a set of events. A complete specification of a probability model requires us to specify
- The outcome space Ω corresponding to the random experiment
- The domain G of the probability function
- The probability function P.

Probability function cntd….

- A probability function P, defined on domain G , assigns to each event A $\in$ G a unique real number P(A) in a way which satisfies the following axioms

    Axiom 1: P(A) >= 0 for any A $\in$ G .

    Axiom 2: P(Ω) =1.

    Axiom 3: P(A U B) = P(A) + P(B) whenever

  P($A \cap$ B) = $\emptyset$, for any A, B $\in$ G.

Some properties of P

- Property 1

$$P(A') = 1 - P(A)$$

- Property 2

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Exercise 1

A random experiment consists of tossing a coin and observing whether Heads or tails results. The outcome space is $\Omega = \{h, t\}$. Complete the following table in a way that consistent with the axioms of probability.

A	$\varnothing$	{h}	{t}	{h,t}
P(A)		0.4		

Exercise 2

- A random experiment consists of rolling a six-sided die and observing the number of dots on the upper face. Complete the following table in a way that is consistent with the axioms of probability.

A	$\varnothing$	{1,2}	{3}	{1,2,3}	{4,5,6}	{3,4,5,6}	{1,2,4,5,6}	Ω
P(A)				0.5			0.8	

Symmetry considerations

- Physical symmetry sometimes enables us to easily assign probability values to certain events.
- Example: roll a uniform six-sided die and observe the number of dots.

Here $\Omega = \{1,2,3,4,5,6\}$. Clearly each of the six elementary events is equally likely provided the die is properly constructed.

- Thus $P(A) = n(A)/n(\Omega)$

Example

- Toss a fair coin twice and observe the sequence of Heads and Tails.
- Here Ω ={hh, ht, th, tt}. Then the probability of each elementary event is ¼.

Probability values by relative frequency

- Suppose a random experiment can be repeated and one event which can be observed is A. Then if we denote $f_r(A)$ the number of times that A occurs in r repetitions of the experiment we have the following. $f_r(A)$ is called the *frequency of occurrence* of A in r repetitions.
- We often assign the value $f_r(A)/r$ to P(A) if r is large.

Example

- We drop a drawing pin 1000 times and observe that it comes to rest with the pin pointing downwards 721 times. We therefore assign P(D)=0.721, where D = event 'pin points downwards when the pin is dropped.'

Exercise 3

- Two marbles are drawn simultaneously from a bag containing three red marbles and two blue marble. Their colours are observed. Apart from colour the marbles are indistinguishable.

i. Specify the outcome space

ii. Find the probability of getting 1 red and 1 blue ball

iii. Find the probability of getting at least 1 blue ball

Exercise 4

- A card is drawn at random from an ordinary card pack of 52 playing cards. Find the probability that the card is;

  a) a seven

  b) not a seven

Exercise 5

- A coin and a die are thrown together. Draw a sample space diagram and find probability of obtaining

  a)a head

  b)a number greater than 4

  c)a head and a number greater than 4

  d)a head or a number greater than 4

Exercise 6

- In a survey of 1500 country homes there were 1200 which had an air-conditioning unit, 900 which had a solar hot-water, while 700 had both.

A country home is selected at random, from the same area, and it is observed whether it has an air-conditioning unit and whether it has a solar hot-water unit. Give a suitable outcome space and find the probability of having

a. only a solar hot water

b. A solar hot water unit or an air conditioning unit

Exercise 7

A card is drawn at random from a pack of cards. Find the probability that

1. A heart card is drawn
2. A card, which is not a face card, is drawn
3. The number on the drawn card is divisible by 3