INDEX NUMBER:

UNIVERSITY OF COLOMBO, SRI LANKA
FACULTY OF SCIENCE

LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) – 2023

ST 1008 – PROBABILITY AND DISTRIBUTIONS

(Two Hours)

Answer all questions

No. of questions: 04
No. of pages: 08

Important Instructions to the Candidates:

- If a page or a part of this question paper is not printed, please inform the supervisor immediately.
- Enter your index number on all pages of the answer script/question paper.
- MULTIPLE CHOICE QUESTIONS: Question (1) and (2) consist of 10 Multiple Choice Questions (MCQ) of each. Each of the MCQs will have 5 choices with only one correct answer. **Encircle the correct choice** in the tables given on the question paper.
- SEMI-STRUCTURED TYPE: Write the answers to questions (3) and (4) on the papers provided.
- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.
- **Attach the exam paper to the answer sheets.**
- Statistical tables are attached to the paper.
- **You are not permitted to remove any part of the question paper except the page that the statistical tables are given from the Examination Hall.**

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Question No.	Q1	Q2	Q3	Q4	Total	%
Marks						

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Question Number 01.

This question consists of 10 multiple choice questions. Select the most suitable answer and encircle the correct choice in Table 01.

Table 01: Answer Table of Question Number 01.

Question Number	**Answers**					**Question Number**	**Answers**				
1.	a	b	c	d	e	**6.**	a	b	c	d	e
2.	a	b	c	d	e	**7.**	a	b	c	d	e
3.	a	b	c	d	e	**8.**	a	b	c	d	e
4.	a	b	c	d	e	**9.**	a	b	c	d	e
5.	a	b	c	d	e	**10.**	a	b	c	d	e

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Number of Correct Answers:……….. **Marks:**……….

Question Number 02.

This question consists of 10 multiple choice questions. Select the most suitable answer and encircle the correct choice in Table 2.

Table 2: Answer Table of Question Number 02.

Question Number	**Answers**					**Question Number**	**Answers**				
1.	a	b	c	d	e	**6.**	a	b	c	d	e
2.	a	b	c	d	e	**7.**	a	b	c	d	e
3.	a	b	c	d	e	**8.**	a	b	c	d	e
4.	a	b	c	d	e	**9.**	a	b	c	d	e
5.	a	b	c	d	e	**10.**	a	b	c	d	e

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Number of Correct Answers:……….. **Marks:**……….

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Question Number 03.

(a) A standard pack of playing cards consists of 4 suits (Clubs, Diamonds, Hearts and Spades), each of 13 cards numbered in ascending order as 2, 3, 4, …, 10, Jack, Queen, King, Ace. A player receives 13 cards from a well shuffled pack.

When answering the following questions, you may leave your answers in terms of factorials unless you are asked to indicate any other value.

(i) In how many ways can 13 cards be selected from the pack of playing cards? **(05 marks)**

(ii) How many possible combinations of cards are there in which the player receives no Spades? **(05 marks)**

(iii) Show the probability that the player receives no Spades is 0.01279. **(05 marks)**

(v) Find the probability that the player receives exactly 6 Hearts among the 13 cards. **(15 marks)**

(b) The discrete random variable X can take only the values 0, 1, 2, 3, 4, 5. The probability distribution of X is given by the following:
$P(X = 0) = P(X = 1) = P(X = 2) = a$
$P(X = 3) = P(X = 4) = P(X = 5) = b$
$P(X \geq 2) = 3P(X < 2)$
where a and b are constants.

(i) Show that $a = \frac{1}{8}$ and $b = \frac{5}{24}$. **(10 marks)**

(ii) Show that expectation of X is $\frac{23}{8}$ and hence find the expectation of $(2X - 5)$. **(10 marks)**

(iii) Determine the variance of X. **(10 marks)**

(iv) Determine the mode and the median of X. **(15 marks)**

(v) Let X_1 and X_2 be two independent observations from this distribution. List down all the possible pairs of (X_1, X_2) which satisfy the condition that the sum of X_1 and X_2 exceeds 7. **(10 marks)**

(vi) Determine the probability that the sum of X_1 and X_2 exceeds 7. **(15 marks)**

(Question No. 03 – 100 Marks)

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Question Number 04.

(a) The random variable X has the binomial distribution with probability mass function

$$P(X = x) = \frac{n!}{x!\,(n-x)!} p^x q^{n-x}, \qquad x = 0, 1, 2, \dots, n.$$

(i) Show that the Moment Generating Function of X is

$$M_X(t) = (pe^t + q)^n.$$ **(15 marks)**

Hint: $\sum_{x=0}^{n} {}^{n}C_x a^x b^{n-x} = (a+b)^n$

(ii) Hence, derive $E(X)$ and $Var(X)$. **(20 marks)**

(b) A manufacturer produces components of two quality grades:
High quality components, with lifetimes(H) distributed as $N(2500{,}15625)$, i.e. Normally with mean 2500 hours and standard deviation $\sqrt{15625}$ = 125 hours.
Standard quality components, with lifetimes(S) distributed as $N(2000{,}90000)$.

(i) Show that the probability of a randomly chosen high quality component lasts at least 2300 hours is 0.9452 and find the corresponding probability for a standard component. **(20 marks)**

(ii) Suppose a box contains randomly packed 10 high quality components. Suggesting a distribution for the number of components(Y) in the box that has a lifetime of at least 2300 hours, find the probability that all 10 components in the box have lifetime of at least 2300 hours. **(15 marks)**

(iii) Find the probability that a randomly chosen standard component has a lifetime longer than a randomly chosen high quality component. **(15 marks)**

(c) Consider the scenario given in (b) above. A large batch of components is produced, of which 40% are high quality and 60% are standard. However, due to a machine malfunction, these components are unlabelled and indistinguishable in appearance. A single component is chosen at random from this batch.

Find the probability that it has a lifetime of at least 2300 hours. **(15 marks)**

[Hint: Use the concept behind total probability theorem to answer this question.]

(Question No. 04 – 100 Marks)

********** END **********

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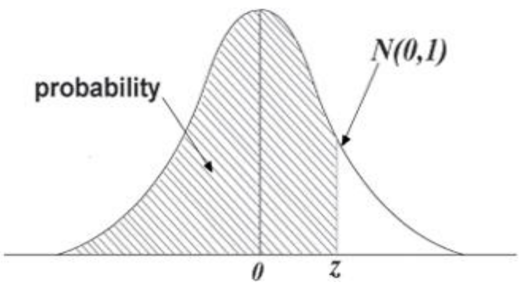
Table 3. The Standardized Normal Distribution Probabilities



The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of ***Z***, the standardized normal deviate, (the proportion ***P***, of the distribution less than ***Z***) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value ***X*** is obtained by calculating $Z = (X - \mu)/\sigma$ and reading the proportion corresponding to this value of ***Z***.

Z	*P*	*Z*	*P*	*Z*	*P*
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997