SCS 1307
Probability & Statistics

by

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Binomial Distribution

Consider an experiment which has two possible outcomes, one which is considered as the ‘success’ and the other is ‘failure’.

A Binomial situation arises when ***n*** independent trials of the experiment are conducted, for example

- Toss a coin 6 times, considering obtaining a head on a single trial as the success
- Roll a die 10 times, considering obtaining an even number as the ‘success’

Let p be the probability that an event will happen in any single trial.

The probability that the event will fail to happen in any single trial is 1-p.

The probability that the event will happen exactly $\boldsymbol{k}$ times in $\boldsymbol{n}$ independent trials is given by the probability function.

$$Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

When the random variable X denote the number of successes in $\boldsymbol{n}$ independent trials, the above given function is often called the Binomial Distribution since for x = 0,1,………,$\boldsymbol{n}$, it corresponds to successive terms in the binomial expansion.

Example: The probability of getting exactly 2 heads in 6 tosses of a fair coin is,

$$X \sim Bin(6,1/2)$$

$$P(X=2) = {}^{6}C_{2}\,(1/2)^{2}\,(1/2)^{4} = 15/64$$

Exercise

1) Find the probability that in tossing a fair coin three times there will appear

   (a) 3 heads

   (b) 2 tails and 1 head

   (c) at least 1 head

   (d) not more than 1 tail

Solution

1) $X \sim Bin(3,1/2)$. Probability of getting

(a) 3 heads

$P(X=3) = {}^{3}C_{3}\,(1/2)^{3}\,(1/2)^{0} = 1/8$

(b) 2 tails and 1 head ; Let Y be the number of tails : $Y \sim Bin(3,1/2)$

$P(Y=2) = {}^{3}C_{2}\,(1/2)^{2}\,(1/2)^{1} = 3/8$

Solution

1) Probability of getting

(c) at least 1 head

$P(X>=1) = 1 - P(X=0) = 1- {}^{3}C_{0}\ (1/2)^{0}\ (1/2)^{3} = 1-1/8 = 7/8$

(d) not more than 1 tail (3 Heads or 2 Heads)

$P(X>=2) = P(X=2) + P(X=3) = 1/8 + 3/8 = 4/8$

Exercise

Find the probability that in 5 tosses of a fair die a 3 appears (a) twice (b) at most once (c) at least two times.

Solution

The probability that in 5 tosses of a fair die a 3 appears

X~Bin(5,1/6)

(a) twice = $P(X=2) = {}^{5}C_{2}\,(1/6)^{2}\,(5/6)^{3}$

(b) at most once $=P(X=0)+P(X=1)$

$$= {}^{5}C_{0}\,(1/6)^{0}\,(5/6)^{5} + {}^{5}C_{1}\,(1/6)^{1}\,(5/6)^{4}$$

(c) at least two times $=1- [P(0)+P(1)]$

Exercise

Find the probability that in a family of 4 children there will be

(a) at least 1 boy,

(b) at least 1 boy and at least 1 girl.

Assume that the probability of a male birth is 1/2.

Solution

The probability that in a family of 4 children there will be

Let X be a child being a boy: X~Bin(4,1/2)

(a) at least 1 boy = 1- $^4C_0\ (1/2)^0\ (1/2)^4$

(b) at least 1 boy and at least 1 girl = 1-P(X=4)-P(X=0)

Some properties of the Binomial Distribution

Mean = np

Variance =npq where q=1-p

Example

In 100 tosses of a fair coin what is the expected number of Heads ?

$E(X) = np = 100 * 0.5 = 50$

Exercise

If the probability of a defective bolt is 0.1, find

(a) the mean

(b) the standard deviation

for the number of defective bolts in a total of 400 bolts.

Solution

If the probability of a defective bolt is 0.1 and total number of bolts available is 400,

(a) the mean = np = 400 * 0.1

(a) the standard deviation = 400 * 0.1 * 0.9

Exercise

The random variable X has a binomial distribution with mean 5.76 and standard deviation 1.92. Find P(X=6)

Exercise

The random variable X has a binomial distribution with mean 5.76 and standard deviation 1.92. Find P(X=6)

$X \sim Bin(n,p)$

$np = 5.76$ $np(1-p)=1.92 * 1.92 = 3.686$ $\rightarrow$ $1-p = 0.6399$ $\rightarrow$ $p= 0.36$

$n = 5.76/0.36 = 16$