UNIVERSITY OF COLOMBO, SRI LANKA

FACULTY OF SCIENCE

LEVEL I EXAMINATION IN SCIENCE (SEMESTER I) – 2021

ST 1008 – PROBABILITY AND DISTRIBUTIONS

(Two Hours)

Answer all questions

No. of questions: 04
No. of pages: 09

Important Instructions to the Candidates:

- If a page or a part of this question paper is not printed, please inform the supervisor immediately.

- Enter your index number on all pages of the answer script/question paper.

- MULTIPLE CHOICE QUESTIONS: Question (1) and (2) consist of 10 Multiple Choice Questions (MCQ) of each. Each of the MCQs will have 5 choices with only one correct answer. **Encircle the correct choice** in the tables given on the question paper.

- SEMI-STRUCTURED TYPE: Write the answers to questions (3) and (4) on the papers provided.

- Electronic devices capable of storing and retrieving text, including electronic dictionaries and mobile phones are not allowed.

- **Attach the exam paper to the answer sheets.**

- Statistical tables are attached to the paper.

- **You are not permitted to remove any part of the question paper except the page that the statistical tables are given from the Examination Hall.**

For Examiner's use only						
Question No.	Q1	Q2	Q3	Q4	Total	%
Marks						

Question Number 03.

(a) Suppose the rules of a carnival game are as follows:

1. The player initially pays Rs.10 to play the game.
2. The player then flips a fair coin. If the player gets a head, the game organizer gives the player Rs.20 and the player stops playing.
3. If the player gets a tail on the coin, the player rolls a fair six-sided die. If the player gets number six on the die, the game organizer gives the player Rs.10 and the game is over.
4. If the player does not get number six on the die, the game is over and the game organizer gives nothing to the player.

Let X denote the player's **(net)earnings** (= winning amount – initial pay) for this game.

(i) Draw a tree diagram for all possible outcomes of the game. **(10 marks)**

(ii) Find the probability distribution of X. **(15 marks)**

(iii) Suppose, **in another carnival game**, probability distribution of player's net earnings X is given as follows.

X	-10	0	10
$P(X)$	1/3	1/6	1/2

Find the player's **expected** and **variance** (net)earnings of this game. Give your answers to 3 decimal places. **(20 marks)**

(iv) If a player plays the game in (iii) 5 times, find the **expected value** and **variance** of the player's total net earnings. Give your answers to 3 decimal places. **(10 marks)**

(b) The lifetime, X, **in tens of hours**, of a battery has a cumulative distribution function $F(x)$ given by

$$F(x) = \begin{cases} 0 & , \quad x < 1 \\ \dfrac{4}{9}(x^2 + 2x - 3) & , \quad 1 \le x \le 1.5 \\ 1 & , \quad x > 1.5 \end{cases}$$

(i) Find the median of X, giving your answer to 3 decimal places. **(15 marks)**

$$\left[\text{Hint: Solutions for the equation } ay^2 + by + c = 0 \text{ can be obtain from} \\ y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$

(ii) Find the probability density function of the random variable X. **(10 marks)**

(iii) Using the cumulative distribution function or otherwise find $P(X \ge 1.2)$. **(10 marks)**

(iv) A radio runs on 4 of these batteries, all of which must be working. Four new batteries are put into the radio. Find the probability that the radio will still be working after 12 hours. **(10 marks)**

(Question No. 03 – 100 Marks)

Question Number 04.

(a) (i) State the conditions under which the binomial distribution may be used for the calculations of probabilities of an event. **(10 marks)**

(ii) The probability that a boy chosen at random has a weekend birthday in 2014 is 0.25. Among a group of 10 boys chosen at random, calculate the probability of more than two have a weekend birthday in 2014. Give your answer to 3 decimal places. **(10 marks)**

(iii) 100 such groups of ten boys have been randomly selected for a new experiment. Consider the distribution of the number of groups having more than two boys with weekend birthday in 2014.

Among the 100 groups, how many groups would you **expect** to contain more than two boys with weekend birthday in 2014? (Refer the answers obtained in (ii) if needed) **(10 marks)**

(iv) Considering normal approximation to the distribution in (iii) find the probability that more than 50 groups are having more than two boys with weekend birthday in 2014. **(15 marks)**

(b) The random variable X has the geometric distribution with probability mass function (pmf)
$$P(X = x) = q^x p \qquad x = 0, 1, 2, 3, \ldots \quad 0 < p < 1,$$
where $q = 1 - p$.

(i) Show that $P(X \le x) = 1 - q^{x+1}$. **(10 marks)**

$$\left[\begin{array}{l} Hint \\ \sum_{y=0}^{n-1} r^y = \dfrac{1 - r^n}{1 - r} \quad for\ r \ne 1 \end{array}\right]$$

(ii) Show that $P[X\ is\ odd\ ] = q \times P[X\ is\ even]$.

here $P[X\ is\ odd] = P[X = 1\ or\ X = 3\ or\ X = 5\ or \ldots]$ and

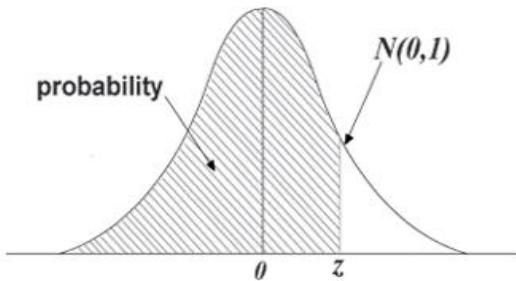$P[X\ is\ even] = P[X = 0\ or\ X = 2\ or\ X = 4\ or \ldots]$. **(15 marks)**

(iii) Show that
$$P(X\ is\ odd) = \frac{q}{1+q}. \textbf{ (10 marks)}$$

(iv) Find $P(X\ is\ odd | X \le 3)$ as a function of q. **(20 marks)**

********** END **********

(You can separate this sheet from the Examination Paper)

Table 03. The Standardized Normal Distribution Probabilities

_N(0,1)_

probability

_0_   _z_

The distribution tabulated is that of the normal distribution with mean **zero** and standard deviation **1**. For each value of Z, the standardized normal deviate, (the proportion P, of the distribution less than Z) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value X is obtained by calculating $Z = (X - \mu)/ \sigma$ and reading the proportion corresponding to this value of Z.

Z	P	Z	P	Z	P
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997