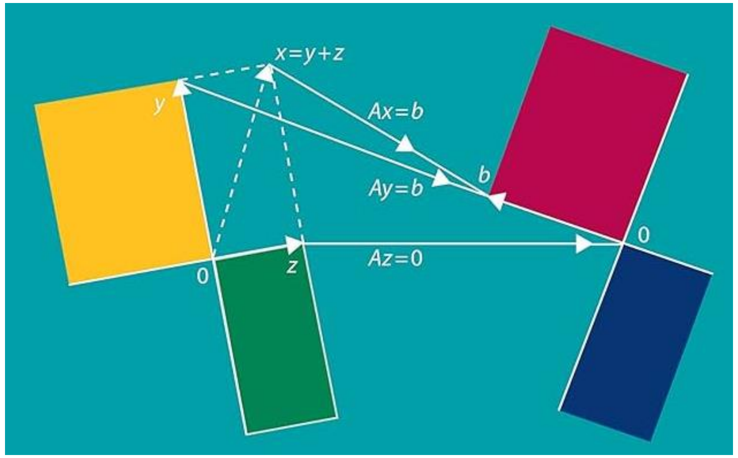
Introduction to Linear Algebra

(Linear Algebra)

Randil Pushpananda, PhD

rpn@ucsc.cmb.ac.lk



Who am I…

- *BSc in Engineering Physics* from the University of Colombo, Sri Lanka in 2006.
- *Bachelor of Information Technology* (BIT) (External), UCSC in 2009.
- *Research Assistant*, Language Technology Research Laboratory (LTRL) (2008 – 2018)
- *PhD in Computer Science* from the University of Colombo, Sri Lanka, 2017.
- *Principal Investigator*, Language Technology Research Laboratory (LTRL), UCSC.

Research Interests:

- *Natural Language Processing/Human Language Technology - Machine Translation, Speech Processing (ASR and TTS) and Machine Learning, Conversational Agents (Chatbots), Large Language Models (LLMs).*

Language Technology Research Laboratory

LTRL

- Addresses the need for local language computing in Sri Lanka through research and development in Localization and Language Processing
- Corpus Creation (Dataset Creation) - *Sinhala and Tamil*
- Voice (Speech) to Text System (ASR) – *Sinhala*
- Text to Voice (Speech) System (TTS) – *Sinhala and Tamil*
- Machine Translation System – *Sinhala – Tamil Language Pair*
- AI Chatbot Systems – *English and Sinhala (with Bank of Ceylon)*
- Spell Checking Application – *Sinhala*
- Image to Text Conversion System (OCR) – *Sinhala and Tamil*
- Grammar Checker– *Sinhala*

Course Organization

Lecturer: Dr. Randil Pushpananda

Assistant Staff: Mr. S. K. A. Kavinda
Ms. N. P. Keragala

Lectures – Friday 08.00 AM – 10.00 AM
Practicals – Wednesday 01.00 PM – 03.00 PM

Course Organization

Recommended Reading:

- Strang, G., 2022. Introduction to linear algebra
- Friedberg, S.H., Insel, A.J. and Spence, L.E., 2018. *Linear algebra*
- Jim Hefferon, Linear Algebra

Evaluation Criteria :

80% paper, 20% Assignments

Assignments:

- 2 In-Class Assignments

Course Outline

Number of Credits	**2L + 1P (3 Credits)**	
Core / Optional	Core	
Evaluation Criteria	Assignments:	20%
	Final Exam:	80%

		Per Week	Total
Method of Delivery	Lectures	2 Hours	30 Hours
	Tutorials	2 Hours	30 Hours
	Lab Work	-	-
	Group Work	-	-

Course Contents

1. Linear Systems
2. Vector spaces and subspaces
3. Orthogonality
4. Determinants
5. Eigenvalues and Eigenvectors
6. Singular Value Decomposition (SVD)
7. Linear Transformations
8. Applications of Linear Algebra in Various Disciplines

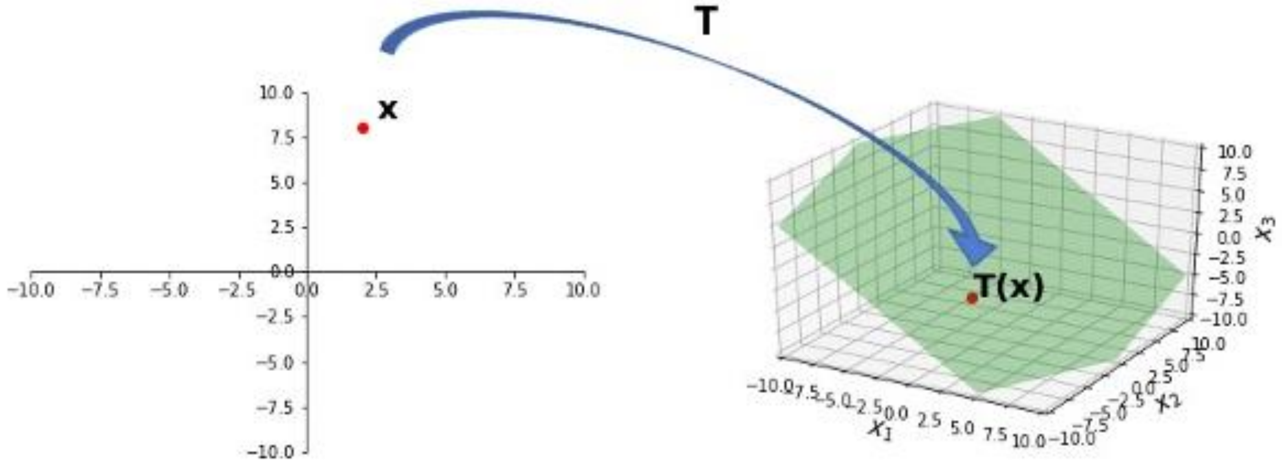
What is Linear Algebra?

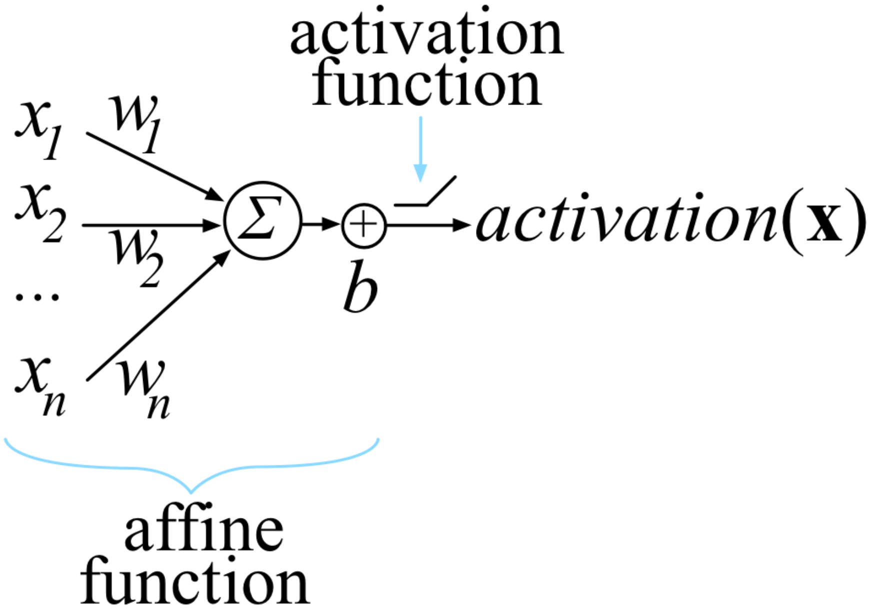
- Linear algebra is a branch of mathematics that deals with vector spaces and linear mappings between these spaces.
- Includes various concepts such as vectors, matrices, systems of linear equations, determinants, eigenvalues, and eigenvectors.
- Linear algebra finds extensive applications in diverse fields including physics, engineering, computer science, economics, and statistics.

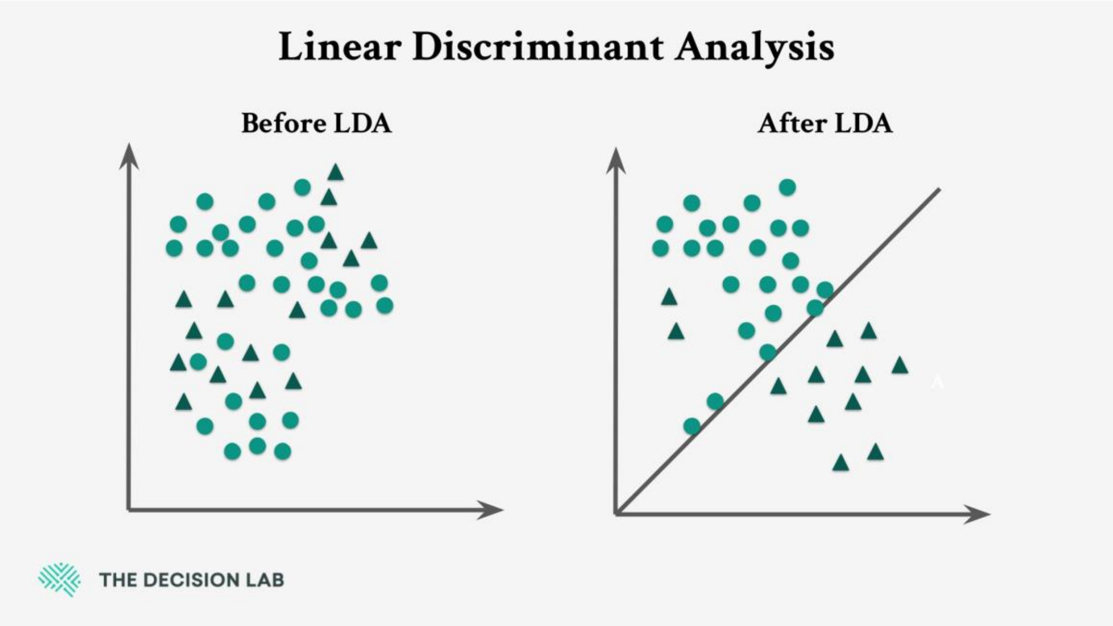
Why?

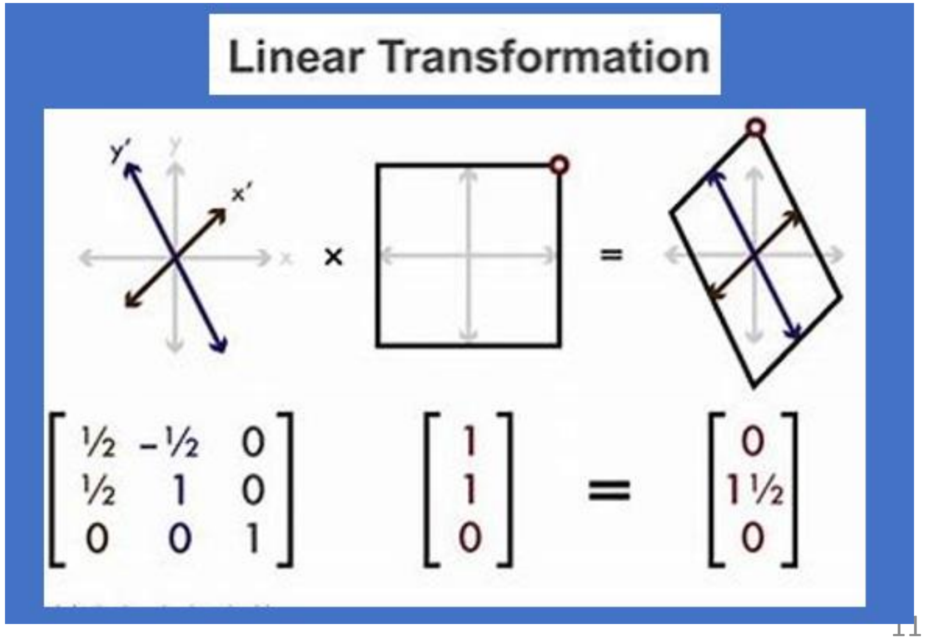
Why?

- Foundational Understanding:
  - Key mathematical concepts such as vectors, matrices, and linear transformations.
  - Many algorithms and techniques used in computer science are based on mathematics.


T
x
T(x)
x_1
x_2
x_3

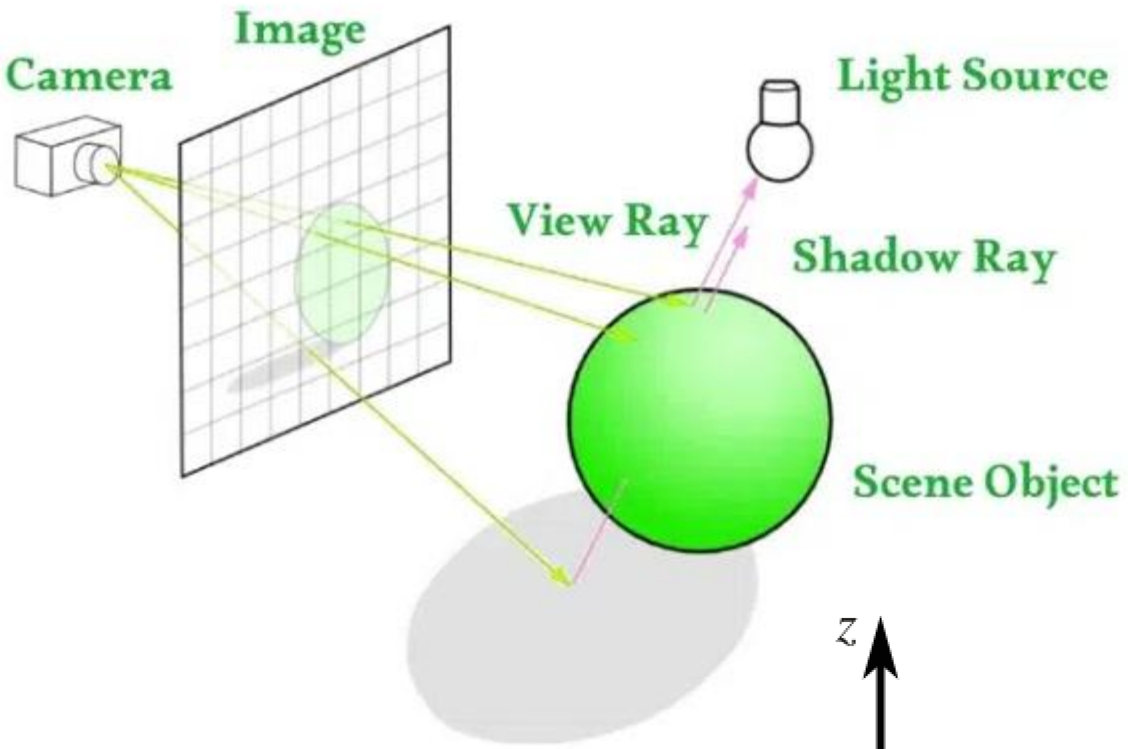

activation function
x_1 w_1
x_2 w_2
...
x_n w_n
Σ
b
$activation(\mathbf{x})$
affine function

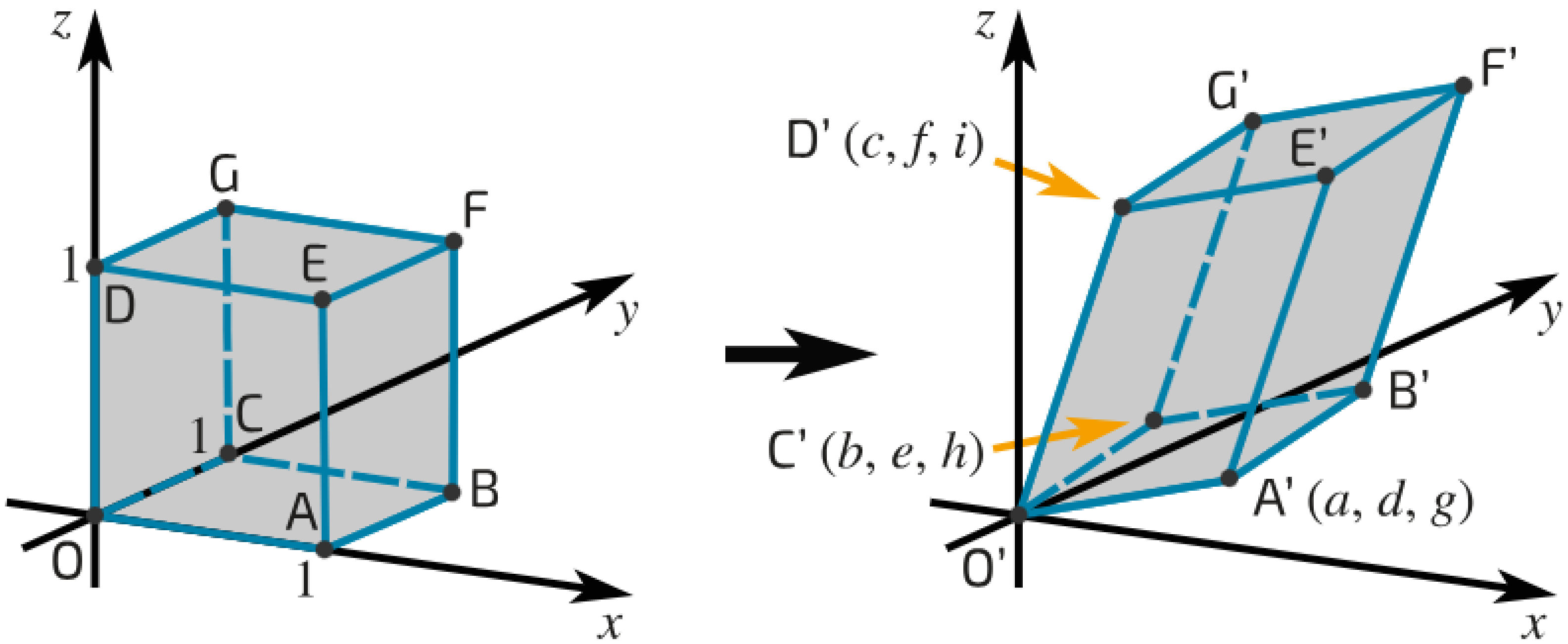

Linear Discriminant Analysis
Before LDA
After LDA
THE DECISION LAB


Linear Transformation
$\begin{bmatrix} ½ & -½ & 0 \\ ½ & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1½ \\ 0 \end{bmatrix}$

Why?

- Graphics and Computer Vision:
  - Linear algebra is crucial in graphics programming for tasks such as rendering 3D scenes, transformations, and lighting calculations.
  - In computer vision, techniques like image processing, feature extraction, and object recognition heavily rely on linear algebra.


Camera
Image
Light Source
View Ray
Shadow Ray
Scene Object


z
y
x
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D
C
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O
1
1
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z
y
x
G'
F'
E'
D' (c, f, i)
C' (b, e, h)
B'
A' (a, d, g)
O'

(a) (b)


AIR
COMBAT


Score: 00000
X
Pause
Y
Sound On
SPEED
ALT


18/10
Ali Sheikh
Hook


HD ENVIRONMENT
20
HEALTH
05/05
AUTO FIRE

Why?

- Machine Learning and Data Science:
    - Backbone of many machine learning algorithms, including regression, dimensionality reduction techniques like neural networks.
    - Understanding concepts like matrices, vectors, and matrix operations is essential for implementing and understanding these algorithms.
- Data Representation and Manipulation:
    - Data in computer science is often represented and manipulated using matrices and vectors.
    - Linear algebra provides tools for organizing, transforming, and analyzing data efficiently, making it indispensable in fields like databases, computer graphics, and cryptography.

Introduction to Linear Algebra

- Questions:
  - $4x = 8$, Find x?

  - Saman has bought two ice creams and two drinks for Rs. 300.00. How much did john pay for each item?

Introduction to Linear Algebra

- Questions:
  - Saman has bought two ice creams and two drinks for Rs. 300.00. Kamala bought two ice creams and one drink for Rs. 200.00. How much did they pay for each item?

To extract information from multiple linear equations, we need linear algebra

Complex scientific, or engineering problem can be solved by using linear algebra on linear equations

Linear Equation

- A linear equation is one where all the variables such as x, y, z have index (power) of 1 or 0 only

$$x + 2y + z = 5$$

$$x = 3$$

$$3x + y + z + w = -8$$

Fundamental Problem of Linear Algebra

Solving Systems of Linear Equations

Questions

Which of the following equations are Linear Equations

(a) $x - y - z = 3$

(c) $\cos(x) + \sin(y) = 1$

(b) $\sqrt{x} + y + z = 6$

(d) $e^{x+y+z} = 1$

(f) $x = -3y$

(e) $x - 2y + 5z = \sqrt{3}$

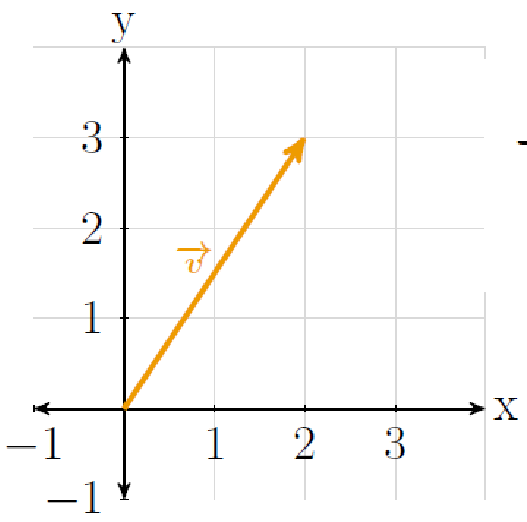
Vectors and Matrices

Vectors

- Vector is a quantity that has size (magnitude) and direction
  - walk due north for 5 kilometers
  - Velocity, acceleration, force and displacement are all vector quantities
- Scalar is a number that measures the size of a particular quantity.
  - Length, area, volume, mass and temperature are all scalar quantities

In a 2-dimensional space, a vector can be written as $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$.

In a 3-dimensional space, a vector can be written as $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$.



$$\vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Vector Addition

$$\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \mathbf{2} \\ \mathbf{4} \end{bmatrix}$$

$$\boldsymbol{v} + \boldsymbol{w} =$$

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \mathbf{1} \\ \mathbf{3} \end{bmatrix}$$

Scalar Multiplication

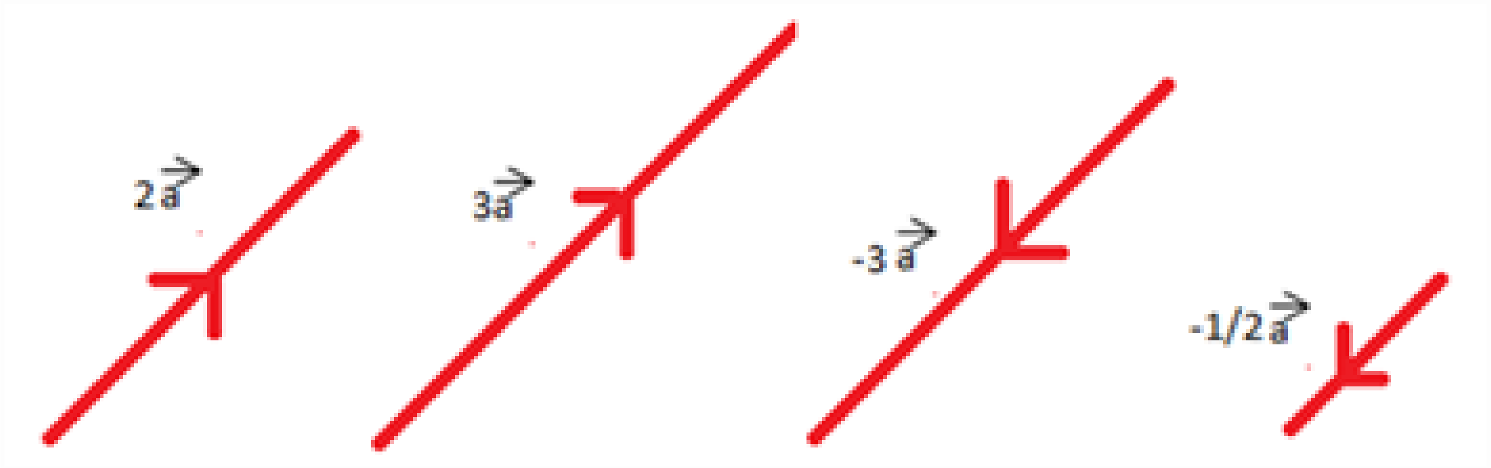
- Scalar can be a real number. $\lambda \in \mathbb{R}$

- If $\lambda > 1$ the vector will keep the same direction but stretch.
- If $1 > \lambda > 0$ the vector will keep the same direction but shrink.

- If $\lambda < -1$ the vector will change direction and stretch.
- If $0 > \lambda > -1$ the vector will change direction and shrink.

Scalar Multiplication

$$3v = 3\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$



Linear Combination

- The heart of Linear Algebra is in two operations: Addition and Multiplication
- Combining those two operations give Linear Combination

$$\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad \boldsymbol{v} + \boldsymbol{w} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

The linear combinations of $\boldsymbol{v}$ and $\boldsymbol{w}$ are the vectors $c\boldsymbol{v} + d\boldsymbol{w}$ for all numbers c and d:

The linear combinations $$c\begin{bmatrix} 2 \\ 4 \end{bmatrix} + d\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2c + 1d \\ 4c + 3d \end{bmatrix}$$ **fill the xy plane**

Linear Combination

All combinations $c\begin{bmatrix}4\\1\end{bmatrix}+d\begin{bmatrix}2\\-1\end{bmatrix}$ fill the xy plane. They produce every $\begin{bmatrix}x\\y\end{bmatrix}$

The vectors $c\begin{bmatrix}1\\1\\2\end{bmatrix}+d\begin{bmatrix}0\\0\\4\end{bmatrix}$ fill a **plane** in xyz space.

Problem

- Linear algebra is not limited to 2 vectors in 2-dimensional and 3-dimentional space.
- As long as we stay linear, the problems can get more dimensions and more vectors.
- vectors ---> m components instead of 2 components.
- Can have n vectors $v_1, v_2, \ldots, v_n$, instead of 2 vectors.
- Finally, n vectors in m-dimensional space will go into the columns of an m by n matrix A:

m rows
n columns
m by n matrix

$$A = \begin{bmatrix} \\ v_1 & v_2 & \cdots & v_n \\ \\ \end{bmatrix}$$

Problem

m **rows**
n **columns**
m **by** n **matrix**

$$A = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}$$

1 **Describe all the combinations** $Ax = x_1v_1 + x_2v_2 + \cdots + x_nv_n$ **of the columns**

2 **Find the numbers** x_1 **to** x_n **that produce a desired output vector** $Ax = b$

Solving Two Equations

Solve $\quad c\begin{bmatrix} 2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$

This means $\quad \begin{aligned} 2c + 2d &= 8 \\ c - d &= 2 \end{aligned}$

Column Way, Row Way, Matrix Way

Column way
Linear combination

$$c\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + d\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Row way
Two equations for c and d

$$\begin{aligned} v_1c + w_1d &= b_1 \\ v_2c + w_2d &= b_2 \end{aligned}$$

Matrix way
2 by 2 matrix

$$\begin{bmatrix} v_1 & w_1 \\ v_2 & w_2 \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$