

## Day 19

### Assignment 2: Traveling Salesman Problem

Create a function `int FindMinCost(int[,] graph)`

that takes a 2D array representing the graph where `graph[i][j]` is the cost to travel from city `i` to city `j`. The function should return the minimum cost to visit all cities and return to the starting city. Use dynamic programming for this solution.

A)

*Java implementation of the Traveling Salesman Problem (TSP) using dynamic programming. This approach utilizes bitmasking to represent subsets of cities and memoization to store already computed states, making it efficient for solving the problem.*

**Java code for the FindMinCost function:**

```
import java.util.Arrays;
public class TravelingSalesman {

    // Function to find the minimum cost to visit all cities and return to the starting city
    public static int FindMinCost(int[][] graph) {
        int n = graph.length;
        int[][] dp = new int[n][1 << n];
        for (int[] row : dp) {
            Arrays.fill(row, -1);
        }
        return tsp(0, 1, graph, dp);
    }

    // Helper function for the TSP using dynamic programming and bitmasking
    private static int tsp(int pos, int mask, int[][] graph, int[][] dp) {
        int n = graph.length;

        // Base case: if all cities have been visited
        if (mask == (1 << n) - 1) {
            return graph[pos][0]; // Return to the starting city
        }

        // Check if the result is already computed
        if (dp[pos][mask] != -1) {
            return dp[pos][mask];
        }

        int minCost = Integer.MAX_VALUE;

        // Try to go to an unvisited city
        for (int city = 0; city < n; city++) {
            if ((mask & (1 << city)) == 0) {
```

```

        int newCost = graph[pos][city] + tsp(city, mask | (1 << city), graph, dp);
        minCost = Math.min(minCost, newCost);
    }
}

// Save the result in dp array and return
dp[pos][mask] = minCost;
return minCost;
}

public static void main(String[] args) {
    // Example usage
    int[][] graph = {
        {0, 10, 15, 20},
        {10, 0, 35, 25},
        {15, 35, 0, 30},
        {20, 25, 30, 0}
    };

    System.out.println("The minimum cost to visit all cities and return to the starting city is: " +
FindMinCost(graph));
}
}

```

### Explanation:

#### 1. Initialization:

- *dp* is a 2D array where *dp[i][mask]* represents the minimum cost to visit all cities in the subset represented by *mask* starting from city *i*.
- *mask* is a bitmask where the bit at position *j* is set if city *j* is visited.

#### 2. Recursive Function *tsp*:

- *pos* is the current city.
- *mask* is the current set of visited cities.
- If all cities are visited ( $mask == (1 \ll n) - 1$ ), return the cost to return to the starting city.
- If the result for this state is already computed, return the saved result.
- Iterate over all cities and recursively compute the cost for the unvisited cities, updating the minimum cost.

#### 3. Main Function *FindMinCost*:

- Initializes the *dp* array and calls the *tsp* function starting from city 0 with the initial mask 1 (indicating that the first city is visited).

#### *4. Example Usage:*

*- The main method provides an example graph and prints the minimum cost to visit all cities and return to the starting city.*

*This approach ensures that each state is computed only once, making it efficient for solving the Traveling Salesman Problem for a moderate number of cities.*