

ASSIGNMENT 2

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1 LINEAR FORMS 2.11

Which of the following pairs of linear equations has a unique solution, no solution, or infinitely many solutions?

- 1) $\begin{pmatrix} 3 & -5 \end{pmatrix} \mathbf{x} = 20$ and $\begin{pmatrix} 6 & -10 \end{pmatrix} \mathbf{x} = 40$
- 2) $\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 7$ and $\begin{pmatrix} 3 & -3 \end{pmatrix} \mathbf{x} = 15$

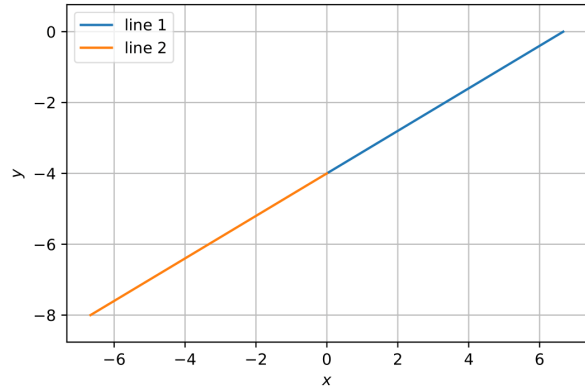


Fig. 2.1. Lines coincide: infinitely many solutions

2 SOLUTION

- 1) Given $\begin{pmatrix} 3 & -5 \end{pmatrix} \mathbf{x} = 20$ and $\begin{pmatrix} 6 & -10 \end{pmatrix} \mathbf{x} = 40$. The above equations can be expressed as a matrix equation.

$$\begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 20 \\ 40 \end{pmatrix} \quad (1)$$

The augmented matrix for the above equation is row reduced as follows

$$\left(\begin{array}{cc|c} 3 & -5 & 20 \\ 6 & -10 & 40 \end{array} \right) \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{cc|c} 3 & -5 & 20 \\ 0 & 0 & 0 \end{array} \right) \quad (2)$$

The rank of

$$\begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = 1 \quad (3)$$

and the rank of

$$\left(\begin{array}{cc|c} 3 & -5 & 20 \\ 6 & -10 & 40 \end{array} \right) = 1 \quad (4)$$

from 2 .

$$\therefore \text{rank} \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = \text{rank} \left(\begin{array}{cc|c} 3 & -5 & 20 \\ 6 & -10 & 40 \end{array} \right) \quad (5)$$

$$= 1 < \dim \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = 2 \quad (6)$$

\Rightarrow 1 has infinitely many solutions. Fig2.1 shows that the lines are the same.

- 2) Given $\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 7$ and $\begin{pmatrix} 3 & -3 \end{pmatrix} \mathbf{x} = 15$. The above equations can be expressed as a matrix equation.

$$\begin{pmatrix} 1 & -3 \\ 3 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 7 \\ 15 \end{pmatrix} \quad (7)$$

The augmented matrix for the above equation is row reduced as follows

$$\left(\begin{array}{cc|c} 1 & -3 & 7 \\ 3 & -3 & 15 \end{array} \right) \xrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{cc|c} 1 & -3 & 7 \\ 2 & 0 & 8 \end{array} \right) \quad (8)$$

$$\left(\begin{array}{cc|c} 1 & -3 & 7 \\ 2 & 0 & 8 \end{array} \right) \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & -3 & 7 \\ 0 & 6 & -6 \end{array} \right) \quad (9)$$

$$\left(\begin{array}{cc|c} 1 & -3 & 7 \\ 0 & 6 & -6 \end{array} \right) \xrightarrow{R_1 \leftarrow R_1 + \frac{R_2}{2}} \left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 6 & -6 \end{array} \right) \quad (10)$$

$$\left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 6 & -6 \end{array} \right) \xrightarrow{R_2 \leftarrow \frac{R_2}{6}} \left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array} \right) \quad (11)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \quad (12)$$

is a solution of 7. The rank of

$$\begin{pmatrix} 1 & -3 \\ 3 & -3 \end{pmatrix} = 2 \quad (13)$$

and the rank of

$$\begin{pmatrix} 1 & -3 & 7 \\ 3 & -3 & 15 \end{pmatrix} = 2 \quad (14)$$

from 11 .

$$\therefore \text{rank} \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = \text{rank} \begin{pmatrix} 3 & -5 & 20 \\ 6 & -10 & 40 \end{pmatrix} \quad (15)$$

$$= \dim \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = 2 \quad (16)$$

\Rightarrow 7 has a unique solution, $\mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ Fig2.2 shows that the lines intersect only at one point.

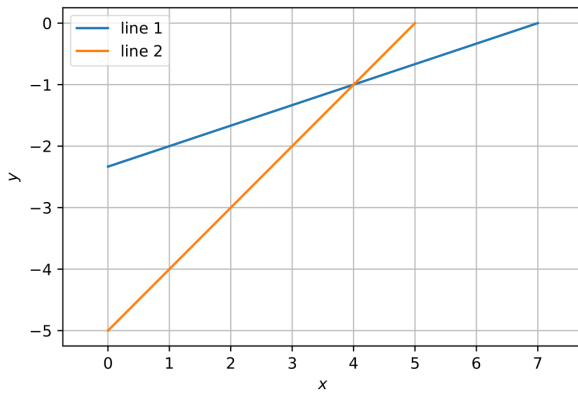


Fig. 2.2. Lines intersecting only at one point:unique solution