Assignment 1

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Download all python codes from

https://github.com/Gayathri1729/SRFP/tree/main/Assignment3

and latex-tikz codes from

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1 CONSTR-2.33

Construct LIFT such that $LI = 4, IF = 3, TL = 2.5, LF = 4.5, IT = 4$.

2 Explanation

1) Assume vertices of the given quadrilateral:-
Let the vertices of the quadrilateral $LIFT$ be **L**,**I**,**F** and **T** .
2) List out given data in form of vectors:-
Given:
$LI = 4, IF = 3, TL = 2.5, LF = 4.5, IT = 4$.
In vector form,

$$\|\mathbf{L} - \mathbf{I}\| = 4 \tag{2.0.1}$$
$$\|\mathbf{I} - \mathbf{F}\| = 3 \tag{2.0.2}$$
$$\|\mathbf{T} - \mathbf{L}\| = 2.5 \tag{2.0.3}$$
$$\|\mathbf{L} - \mathbf{F}\| = 4.5 \tag{2.0.4}$$
$$\|\mathbf{I} - \mathbf{T}\| = 4 \tag{2.0.5}$$

3) Find out two triangles of given quadrilateral having same base:
Quadrilateral $LIFT$ is made up of two triangles $\triangle LIF$ and $\triangle LIT$ placed on base LI .
4) Verify that construction of both triangles,is possible or not by using the fact that "sum of any two sides of a triangle is greater than the third side":-

(a) Consider $\triangle LIF$,

$$\|L - I\| + \|I - F\| = 7 > \|L - F\| \tag{2.0.6}$$
$$\|I - F\| + \|L - F\| = 7.5 > \|L - I\| \tag{2.0.7}$$
$$\|L - I\| + \|L - F\| = 8.5 > \|I - F\| \tag{2.0.8}$$

Sum of any two sides is greater than the third side in $\triangle LIF$.
$\therefore$ Construction of $\triangle LIF$ is possible.

(b) Similarly in $\triangle LIT$,

$$\|L - I\| + \|I - T\| = 8 > \|L - T\| \tag{2.0.9}$$
$$\|L - T\| + \|I - T\| = 6.5 > \|L - I\| \tag{2.0.10}$$
$$\|L - I\| + \|L - T\| = 6.5 > \|I - T\| \tag{2.0.11}$$

Sum of any two sides is greater than the third side in $\triangle LIT$.
$\therefore$ Construction of $\triangle LIT$ is possible.

5) Conclude that construction of quadrilateral is possible if both triangles can be constructed otherwise not possible:-
$\because$ both the triangles can be constructed,we can construct the quadrilateral with the given sides.
6) To find the coordinates of the vertices of the given quadrilateral:
Let the sides of the triangles be denoted by $LI = f, IF = l, TL = t, LF = i, IT = g$ Then,

$$f = 4, l = 3, t = 2.5, i = 4.5, g = 4 \tag{2.0.12}$$

Suppose $\angle FLI$=N and $\angle TLI$=M Now,let

$$\mathbf{L} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.13}$$
$$\mathbf{I} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{2.0.14}$$
$$\mathbf{F} = \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} i\cos N \\ i\sin N \end{pmatrix} \tag{2.0.15}$$
$$\mathbf{T} = \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} t\cos M \\ t\sin M \end{pmatrix} \tag{2.0.16}$$

Then we know that,

$$\cos N = \frac{f^2 + i^2 - l^2}{2fi} \tag{2.0.17}$$

$$p = i\cos N = \frac{f^2 + i^2 - l^2}{2f} \tag{2.0.18}$$

$$= \frac{4^2 + 4.5^2 - 3^2}{2 \times 4} = 3.406 \tag{2.0.19}$$

$$\sin N = \pm\sqrt{1 - \cos^2 N} \tag{2.0.20}$$

$$q = i\sin N = \pm\sqrt{i^2 - i^2\cos^2 N} \tag{2.0.21}$$

$$= \pm\sqrt{4.5^2 - 3.406^2} = \pm 2.94 \tag{2.0.22}$$

$$\cos M = \frac{t^2 + f^2 - g^2}{2ft} \tag{2.0.23}$$

$$r = t\cos M = \frac{t^2 + f^2 - g^2}{2f} \tag{2.0.24}$$

$$= \frac{2.5^2 + 4^2 - 4^2}{2 \times 4} = 0.781 \tag{2.0.25}$$

$$\sin M = \pm\sqrt{1 - \cos^2 M} \tag{2.0.26}$$

$$s = t\sin M = \pm\sqrt{t^2 - t^2\cos^2 M} \tag{2.0.27}$$

$$= \pm\sqrt{2.5^2 - 0.781^2} = \pm 2.374 \tag{2.0.28}$$

Consider q and s to be positive.Then the coordinates of the quadrilateral can be obtained from 2.0.13, 2.0.14, 2.0.15 and 2.0.16.

$$\mathbf{L} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 3.406 \\ 2.94 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 0.781 \\ 2.374 \end{pmatrix} \tag{2.0.29}$$

7) Knowing all the coordinates, now we can construct the quadrilateral.

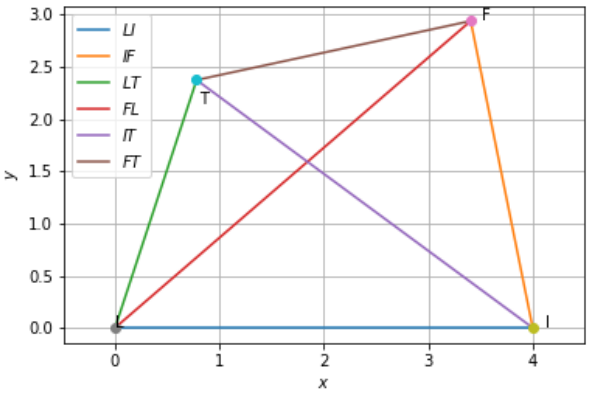

Fig. 2.1: Quadrilateral *LIFT*