ASSIGNMENT 11

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Download all python codes from

https://github.com/Gayathri1729/SRFP/tree/main/Assignment11

and latex-tikz codes from

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1 OPTIMIZATION 2.17

A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?

2 SOLUTION

- All the data can be tabularised as:

	Grinding machine	Sprayer	Profit
Pedestal lamps	2	3	5
Wooden shades	1	2	3
Max Hours	≤ 12	≤ 20	

TABLE 2.1
TIME NEEDED AND PROFIT FOR EACH OBJECT

- Let the number of pieces of pedestal lamp manufactured be x and the number of pieces of wooden shades manufactured be y such that :

$$x \geq 0 \tag{1}$$
$$y \geq 0 \tag{2}$$

- From the data given we have:

$$2x + y \leq 12 \tag{3}$$

and,

$$3x + 2y \leq 20 \tag{4}$$

$\therefore$ The maximizing function is:

$$\max Z = \begin{pmatrix} 5 & 3 \end{pmatrix} \mathbf{x} \tag{5}$$
$$s.t. \quad \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 12 \\ 20 \end{pmatrix} \tag{6}$$
$$-\mathbf{x} \leq \mathbf{0} \tag{7}$$

- The Lagrangian function can be given as:

$$\begin{aligned} &L(\mathbf{x}, \boldsymbol{\lambda}) \\ &= \begin{pmatrix} 5 & 3 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} - 12 \right] \right. \\ &+ \left[\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} - 20 \right] \\ &+ \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{8}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} \tag{9}$$

- Now, we have

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 5 + \begin{pmatrix} 2 & 3 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 3 + \begin{pmatrix} 1 & 2 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} - 12 \\ \begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} - 20 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{10}$$

$\therefore$ The Lagrangian matrix is given by:-

$$\begin{pmatrix} 0 & 0 & 2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 & -1 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -5 \\ -3 \\ 12 \\ 20 \\ 0 \\ 0 \end{pmatrix} \tag{11}$$

- Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -5 \\ -3 \\ 12 \\ 20 \end{pmatrix} \tag{12}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -5 \\ -3 \\ 12 \\ 20 \end{pmatrix} \tag{13}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & -1 \\ 0 & 0 & -3 & 2 \\ 2 & -3 & 0 & 0 \\ -1 & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} -5 \\ -3 \\ 12 \\ 20 \end{pmatrix} \tag{14}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 4 \\ 24 \\ -1 \\ -1 \end{pmatrix} \tag{15}$$

$$\because \boldsymbol{\lambda} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \prec \mathbf{0}$$

- The Optimal solution is given by:

$$\mathbf{x} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{16}$$

$$Z = \begin{pmatrix} 5 & 3 \end{pmatrix} \mathbf{x} \tag{17}$$

$$Z = \begin{pmatrix} 5 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{18}$$

$$Z = \text{Rs.}32 \tag{19}$$

- So, to maximise profit
  No. of pedestal lamps manufactured is $\boxed{x = 4}$
  and
  No. of wooden shades manufactured is $\boxed{y = 4}$.
- The maximum daily profit is $\boxed{Z = \text{Rs.}32}$.

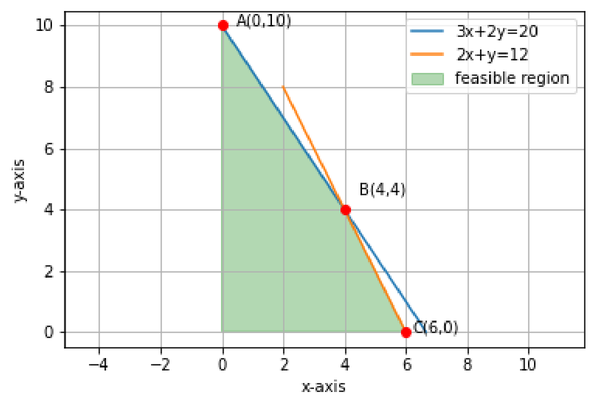

Fig. 2.1. Graphical Representataion