ASSIGNMENT 6

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Download all python codes from

https://github.com/Gayathri1729/SRFP/tree/main/Assignment6

and latex-tikz codes from

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1 Quadratic Forms-2.74 j

In each of the following, find the equation for the ellipse that satisfies the given conditions:

Foci $\begin{pmatrix} 0 \\ \pm\sqrt{10} \end{pmatrix}$, passing through $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

Lemma 1.1. *For ellipse,*

Property:

$$|\mathbf{V}| > 0 \tag{1}$$

$$\lambda_1 > 0, \lambda_2 < 0 \tag{2}$$

Standard Form:

$$\frac{\mathbf{x}^T\mathbf{D}\mathbf{x}}{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{3}$$

Centre:

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} \tag{4}$$

Axes:

$$\begin{cases} \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_1}} \\ \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u}-f}{\lambda_2}} \end{cases} \tag{5}$$

Focus:

$$\mathbf{F} = \sqrt{\frac{(\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}} \tag{6}$$

Focal Length:

$$\beta = \frac{1}{2}\left|\frac{(\mathbf{c}^T\mathbf{V} + \mathbf{u}^T)\mathbf{p}_1}{(\lambda_1 + \lambda_2)}\right| \tag{7}$$

Latus Rectum:

$$(\mathbf{V}\mathbf{c} + \mathbf{u})^T(\mathbf{x} - \beta) + \mathbf{u}^T\mathbf{c} + \mathbf{f} = 0 \tag{8}$$

End points of latus rectum:

$$\mathbf{u}^T\kappa = -\frac{(\kappa^T\mathbf{V}\kappa + f)}{2} \tag{9}$$

Length of latus rectum:

$$l = \left\|\beta(\mathbf{V}\mathbf{c} + \mathbf{u})^T\right\| \tag{10}$$

2 Solution

The equation for the given ellipse is

$$\mathbf{x}^\top\mathbf{V}\mathbf{x} = 1 \tag{11}$$

where

$$\mathbf{V} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{12}$$

Then from (3),

$$\mathbf{u} = \mathbf{0} \tag{13}$$

$$f = -1 \tag{14}$$

Given the ellipse is passing through $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

$$\implies \begin{pmatrix} 2 & 3 \end{pmatrix}\mathbf{V}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 1 \tag{15}$$

$$\begin{pmatrix} 2 & 3 \end{pmatrix}\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 1 \tag{16}$$

$$\implies 4\lambda_1 + 9\lambda_2 = 1 \tag{17}$$

Note that $\lambda_1 > \lambda_2$.

Given the focus is $\mathbf{F} = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ \pm\sqrt{10} \end{pmatrix}$.

Thus, the major axis of the ellipse is y-axis. We know that for the ellipse whose major axis in y-axis,

$$\mathbf{F} = \sqrt{\frac{(\mathbf{u}^\top\mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}} \tag{18}$$

$$\mathbf{F} = \sqrt{\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}} \tag{19}$$

$$10 = \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \tag{20}$$

From (17)

$$\lambda_2 = \frac{1 - 4\lambda_1}{9} \tag{21}$$

From (20)

$$\lambda_2 = \frac{\lambda_1}{10\lambda_1 + 1} \tag{22}$$

$$\implies \frac{1 - 4\lambda_1}{9} = \frac{\lambda_1}{10\lambda_1 + 1} \tag{23}$$

$$\lambda_1 = \frac{1}{8} \tag{24}$$

From (17),

$$\lambda_2 = \frac{1}{18} \tag{25}$$

Thus the equation of the given ellipse is

$$\mathbf{x}^\top \begin{pmatrix} \frac{1}{8} & 0 \\ 0 & \frac{1}{18} \end{pmatrix} \mathbf{x} = 1 \tag{26}$$

Fig 2.1 represents the given ellipse.

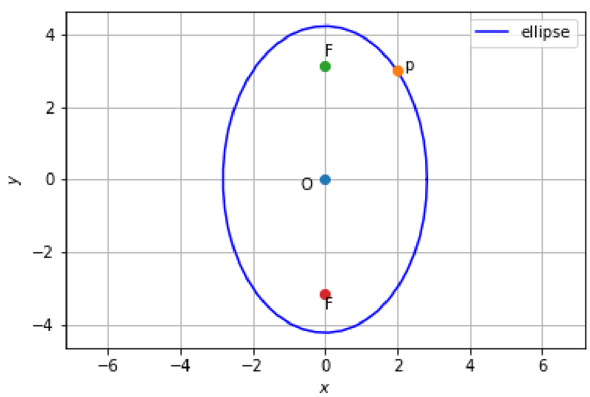

Fig. 2.1. Ellipse