ASSIGNMENT 2

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1 Linear forms 2.11

Which of the following pairs of linear equations has a unique solution,no solution,or infinitely many solutions?

1) $\begin{pmatrix} 3 & -5 \end{pmatrix} \mathbf{x} = 20$ and $\begin{pmatrix} 6 & -10 \end{pmatrix} \mathbf{x} = 40$
2) $\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 7$ and $\begin{pmatrix} 3 & -3 \end{pmatrix} \mathbf{x} = 15$

2 Solution

1) Given $\begin{pmatrix} 3 & -5 \end{pmatrix} \mathbf{x} = 20$ and $\begin{pmatrix} 6 & -10 \end{pmatrix} \mathbf{x} = 40$.The above equations can be expressed as a matrix equation.

$$\begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 20 \\ 40 \end{pmatrix} \tag{1}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 3 & -5 & 20 \\ 6 & -10 & 40 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 3 & -5 & 20 \\ 0 & 0 & 0 \end{pmatrix} \tag{2}$$

$$\therefore rank \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = rank \begin{pmatrix} 3 & -5 & 20 \\ 6 & -10 & 40 \end{pmatrix} \tag{3}$$

$$= 1 < dim \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = 2 \tag{4}$$

$\implies$ 1 has infinitely many solutions. Fig2.1 shows that the lines are the same.

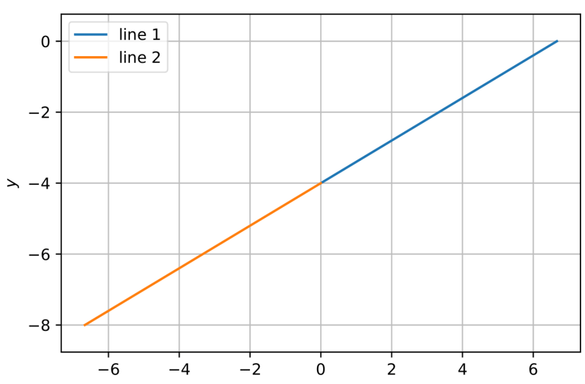

Fig. 2.1. Lines coincide:infinitely many solutions

2) Given $\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 7$ and $\begin{pmatrix} 3 & -3 \end{pmatrix} \mathbf{x} = 15$. The above equations can be expressed as a matrix equation.

$$\begin{pmatrix} 1 & -3 \\ 3 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 7 \\ 15 \end{pmatrix} \tag{5}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & -3 & 7 \\ 3 & -3 & 15 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & -3 & 7 \\ 2 & 0 & 8 \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} 1 & -3 & 7 \\ 2 & 0 & 8 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & -3 & 7 \\ 0 & 6 & -6 \end{pmatrix} \tag{7}$$

$$\begin{pmatrix} 1 & -3 & 7 \\ 0 & 6 & -6 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 + \frac{R_2}{2}} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 6 & -6 \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} 1 & 0 & 4 \\ 0 & 6 & -6 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{6}} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & -1 \end{pmatrix} \tag{9}$$

$$\implies \mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \tag{10}$$

is a solution of 5.

$$\because rank \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = rank \begin{pmatrix} 3 & -5 & 20 \\ 6 & -10 & 40 \end{pmatrix} \tag{11}$$

$$= dim \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = 2 \tag{12}$$

$\implies$ 5 has a unique solution, $\mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ Fig2.2 shows that the lines intersect only at one point.

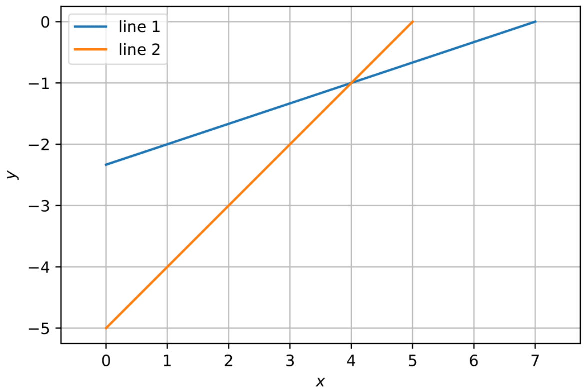

Fig. 2.2. Lines intersecting only at one point:unique solution