ASSIGNMENT-14

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Download all python codes from

https://github.com/Gayathri1729/SRFP/tree/main/Assignment14

and latex-tikz codes from

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1 OPTIMIZATION 2.2

Find the maximum profit that a company can make, if the profit function is given by $p(x) = 41 - 72x - 18x^2$

2 SOLUTION

Lemma 2.1. *A function f(x) is said to be concave if following inequality is true for* $\lambda \in [0, 1]$:

$$\lambda f(x_1) + (1-\lambda) f(x_2) \leq f(\lambda x_1 + (1-\lambda)x_2) \quad (2.0.1)$$

Given the profit function of the company is

$$p(x) = 41 - 72x - 18x^2 \quad (2.0.2)$$

Checking convexity of $p(x)$:

$$\lambda\left(41 - 72x_1 - 18{x_1}^2\right) + (1-\lambda)\left(41 - 72x_2 - 18{x_2}^2\right) \leq \left(41 - 72(\lambda x_1 + (1-\lambda)x_2) - 18\left(\lambda x_1 + (1-\lambda)x_2\right)^2\right) \quad (2.0.3)$$

resulting in

$$18\lambda(\lambda - 1)(x_1 - x_2)^2 \leq 0 \quad (2.0.4)$$

$$\implies \lambda(\lambda - 1) \leq 0 \quad (2.0.5)$$

is true .

$\implies$ The function is concave.

Using gradient ascent method we can find its maxima,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \quad (2.0.6)$$

$$\implies x_{n+1} = x_n + \alpha\left(-36x_n - 72\right) \quad (2.0.7)$$

Taking $x_0 = 2, \alpha = 0.001$ and precision= 0.00000001,values obtained using python are:

$$\boxed{\text{Maxima} = 112.99999999999876 \approx 113} \quad (2.0.8)$$

$$\boxed{\text{Maxima Point} = -1.9999997364868565 \approx -2} \quad (2.0.9)$$

We can verify this by the derivative test. Since $p(x)$ is a concave function it has a maxima.

$$\frac{dp(x)}{dx} = -36x - 72 \quad (2.0.10)$$

Critical point :

$$\frac{dp(x)}{dx} = 0 \quad (2.0.11)$$

$$-36x - 72 = 0 \quad (2.0.12)$$

$$x = -2 \quad (2.0.13)$$

is a critical point.And since $p(x)$ is a concave function there will be a maxima at x=-2. And the maxima is

$$p(-2) = 113 \quad (2.0.14)$$

Again Fig.2.1 verifies this.

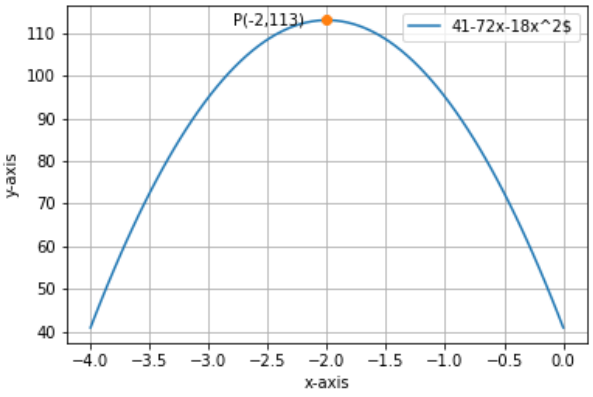

Fig. 2.1: $p(x) = 41 - 72x - 18x^2$