ASSIGNMENT 6

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Download all python codes from

```
https://github.com/Gayathri1729/SRFP/tree/main/
    Assignment6
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and latex-tikz codes from

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    Assignment6
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1 Quadratic Forms-2.74 j

In each of the following, find the equation for the ellipse that satisfies the given conditions:

Foci $\begin{pmatrix} 0 \\ \pm\sqrt{10} \end{pmatrix}$, passing through $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

2 Solution

The standard equation of conic is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 1 \tag{1}$$

For ellipse, the equation becomes

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} = 1 \tag{2}$$

where

$$\mathbf{V} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{3}$$

$$\mathbf{u} = \mathbf{0} \tag{4}$$

$$f = -1 \tag{5}$$

Given the ellipse is passing through $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

$$\implies \begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{V} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 1 \tag{6}$$

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 1 \tag{7}$$

$$\implies 4\lambda_1 + 9\lambda_2 = 1 \tag{8}$$

Note that $\lambda_1 > \lambda_2$.

Given the focus is $F = \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ \pm\sqrt{10} \end{pmatrix}$.

Thus, the major axis of the ellipse is y-axis. We know that for the ellipse whose major axis in y-axis,

$$F = \sqrt{\frac{(\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}} \tag{9}$$

$$F = \sqrt{\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}} \tag{10}$$

$$10 = \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \tag{11}$$

From (8)

$$\lambda_2 = \frac{1 - 4\lambda_1}{9} \tag{12}$$

From (11)

$$\lambda_2 = \frac{\lambda_1}{10\lambda_1 + 1} \tag{13}$$

$$\implies \frac{1 - 4\lambda_1}{9} = \frac{\lambda_1}{10\lambda_1 + 1} \tag{14}$$

$$\lambda_1 = \frac{1}{8} \tag{15}$$

From (8),

$$\lambda_2 = \frac{1}{18} \tag{16}$$

Thus the equation of the given ellipse is

$$\mathbf{x}^\top \begin{pmatrix} \frac{1}{8} & 0 \\ 0 & \frac{1}{18} \end{pmatrix} \mathbf{x} = 1 \tag{17}$$

Fig 2.1 represents the given ellipse.

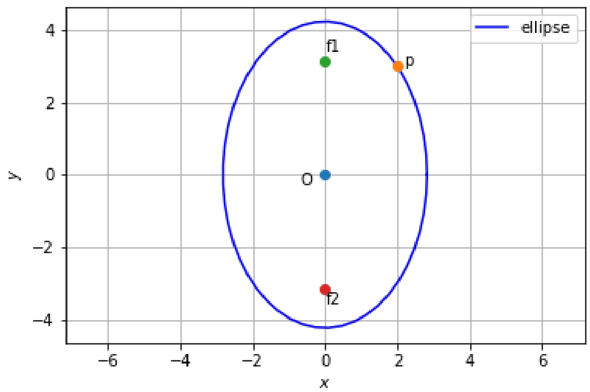

Fig. 2.1. Ellipse