# ASSIGNMENT 7

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Download all python codes from

https://github.com/Gayathri1729/SRFP/tree/main/Assignment7

and latex-tikz codes from

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## 1 Quadratic Forms-2.39

Find the equation of the hyperbola with vertices $\begin{pmatrix} 0 \\ \pm\frac{\sqrt{11}}{2} \end{pmatrix}$, foci $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$

## 2 Solution

The standard equation of hyperbola is

$$\frac{\mathbf{x}^\top \mathbf{V}\mathbf{x}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{1}$$

Since focus $\mathbf{F}=\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$, the vertices lie in the y-axis.Thus,

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{u} = \mathbf{0} \tag{3}$$

$$f = 1 \tag{4}$$

$$\mathbf{V} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{5}$$

where $\lambda_1 > 0$ and $\lambda_2 < 0$. Vertices is given by,

$$\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} = \frac{\sqrt{11}}{2} \tag{6}$$

$$\sqrt{\frac{-1}{\lambda_2}} = \frac{\sqrt{11}}{2} \tag{7}$$

$$\lambda_2 = -\frac{4}{11} \tag{8}$$

$$\mathbf{F} = \pm\left(\sqrt{\frac{(\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f)(\lambda_1 - \lambda_2)}{\lambda_1\lambda_2}}\right)\mathbf{p_2} \tag{9}$$

where $\mathbf{p_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\pm\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \pm\left(\sqrt{\frac{(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{10}$$

$$3 = \sqrt{\frac{(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}} \tag{11}$$

$$3 = \sqrt{\frac{(-4 - 11\lambda_1)}{-4\lambda_1}} \tag{12}$$

$$\lambda_1 = \frac{4}{25} \tag{13}$$

The equation of the given hyperbola is

$$\mathbf{x}^\top \begin{pmatrix} \frac{4}{25} & 0 \\ 0 & -\frac{4}{11} \end{pmatrix}\mathbf{x} = -1 \tag{14}$$

The plot of the hyperbola is given in the fig 2.1

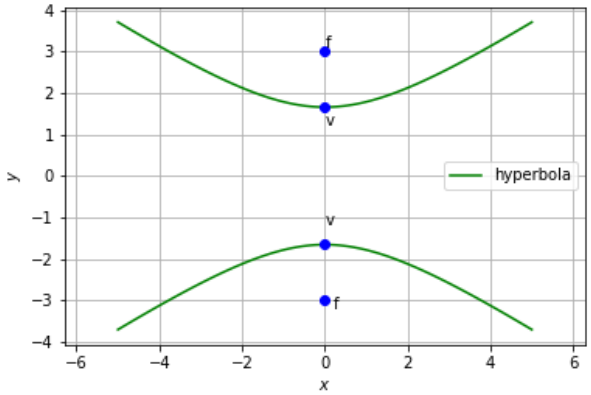


Fig. 2.1. Hyperbola