ASSIGNMENT 2

Gayathri S

1 Linear forms 2.11

Which of the following pairs of linear equations has a unique solution,no solution,or infinitely many solutions?

1) $\begin{pmatrix}3, -5\end{pmatrix}\mathbf{x} = 20$ and $\begin{pmatrix}6, -10\end{pmatrix}\mathbf{x} = 40$
2) $\begin{pmatrix}1, -3\end{pmatrix}\mathbf{x} = 7$ and $\begin{pmatrix}3, -3\end{pmatrix}\mathbf{x} = 15$

2 Solution

1) Given $\begin{pmatrix}3, -5\end{pmatrix}\mathbf{x} = 20$ and $\begin{pmatrix}6, -10\end{pmatrix}\mathbf{x} = 40$.The above equations can be expressed as a matrix equation.

$$\begin{pmatrix}3 & -5\\6 & -10\end{pmatrix}\mathbf{x} = \begin{pmatrix}20\\40\end{pmatrix} \tag{1}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}3 & -5 & 20\\6 & -10 & 40\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix}3 & -5 & 20\\0 & 0 & 0\end{pmatrix} \tag{2}$$

The rank of

$$\begin{pmatrix}3 & -5\\6 & -10\end{pmatrix} = 1 \tag{3}$$

and the rank of

$$\begin{pmatrix}3 & -5 & 20\\6 & -10 & 40\end{pmatrix} = 1 \tag{4}$$

from 2 .

$$\therefore rank\begin{pmatrix}3 & -5\\6 & -10\end{pmatrix} = rank\begin{pmatrix}3 & -5 & 20\\6 & -10 & 40\end{pmatrix} \tag{5}$$

$$= 1 < dim\begin{pmatrix}3 & -5\\6 & -10\end{pmatrix} = 2 \tag{6}$$

$\implies$ 1 has infinitely many solutions. Fig2.1 shows that the lines are the same.

2) Given $\begin{pmatrix}1, -3\end{pmatrix}\mathbf{x} = 7$ and $\begin{pmatrix}3, -3\end{pmatrix}\mathbf{x} = 15$. The above equations can be expressed as a matrix equation.

$$\begin{pmatrix}1 & -3\\3 & -3\end{pmatrix}\mathbf{x} = \begin{pmatrix}7\\15\end{pmatrix} \tag{7}$$

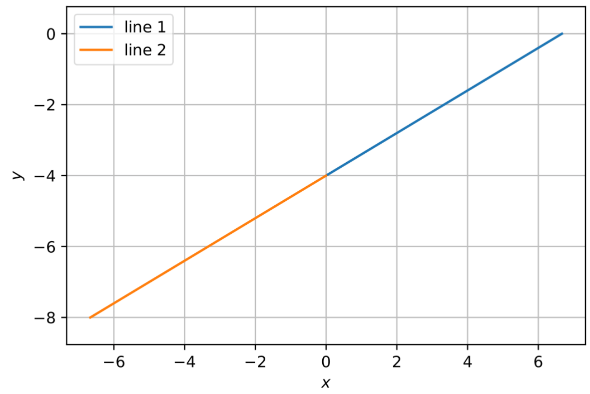

Fig. 2.1. Lines coincide:infinitely many solutions

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}1 & -3 & 7\\3 & -3 & 15\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix}1 & -3 & 7\\2 & 0 & 8\end{pmatrix} \tag{8}$$

$$\begin{pmatrix}1 & -3 & 7\\2 & 0 & 8\end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix}1 & -3 & 7\\0 & 6 & -6\end{pmatrix} \tag{9}$$

$$\begin{pmatrix}1 & -3 & 7\\0 & 6 & -6\end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 + \frac{R_2}{2}} \begin{pmatrix}1 & 0 & 4\\0 & 6 & -6\end{pmatrix} \tag{10}$$

$$\begin{pmatrix}1 & 0 & 4\\0 & 6 & -6\end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{6}} \begin{pmatrix}1 & 0 & 4\\0 & 1 & -1\end{pmatrix} \tag{11}$$

$$\implies \mathbf{x} = \begin{pmatrix}4\\-1\end{pmatrix} \tag{12}$$

is a solution of 7. The rank of

$$\begin{pmatrix}1 & -3\\3 & -3\end{pmatrix} = 2 \tag{13}$$

and the rank of

$$\begin{pmatrix}1 & -3 & 7\\3 & -3 & 15\end{pmatrix} = 2 \tag{14}$$

from 11 .

$$\therefore\ rank\begin{pmatrix}3 & -5\\ 6 & -10\end{pmatrix} = rank\begin{pmatrix}3 & -5 & 20\\ 6 & -10 & 40\end{pmatrix} \quad (15)$$

$$= dim\begin{pmatrix}3 & -5\\ 6 & -10\end{pmatrix} = 2 \quad (16)$$

$\implies$ 7 has a unique solution, $\mathbf{x} = \begin{pmatrix}4\\ -1\end{pmatrix}$ Fig2.2 shows that the lines intersect only at one point.

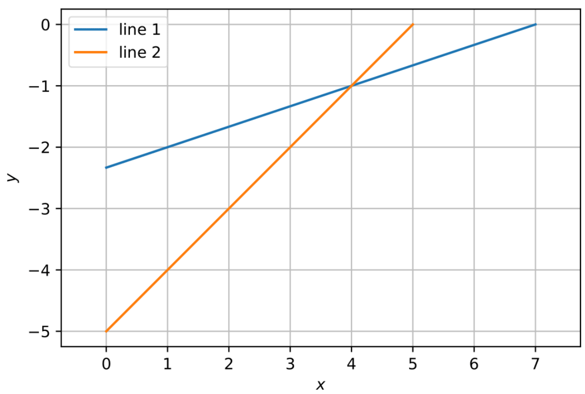

Fig. 2.2. Lines intersecting only at one point:unique solution