

# ASSIGNMENT 2

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## 1 LINEAR FORMS 2.11

Which of the following pairs of linear equations has a unique solution, no solution, or infinitely many solutions?

- 1)  $\begin{pmatrix} 3 \\ -5 \end{pmatrix} \mathbf{x} = 20$  and  $\begin{pmatrix} 6 \\ -10 \end{pmatrix} \mathbf{x} = 40$
- 2)  $\begin{pmatrix} 1 \\ -3 \end{pmatrix} \mathbf{x} = 7$  and  $\begin{pmatrix} 3 \\ -3 \end{pmatrix} \mathbf{x} = 15$

## 2 SOLUTION

- 1) Given  $\begin{pmatrix} 3 \\ -5 \end{pmatrix} \mathbf{x} = 20$  and  $\begin{pmatrix} 6 \\ -10 \end{pmatrix} \mathbf{x} = 40$ . The above equations can be expressed as a matrix equation.

$$\begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 20 \\ 40 \end{pmatrix} \quad (1)$$

The augmented matrix for the above equation is row reduced as follows

$$\left( \begin{array}{cc|c} 3 & -5 & 20 \\ 6 & -10 & 40 \end{array} \right) \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left( \begin{array}{cc|c} 3 & -5 & 20 \\ 0 & 0 & 0 \end{array} \right) \quad (2)$$

The rank of

$$\begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = 1 \quad (3)$$

and the rank of

$$\begin{pmatrix} 3 & -5 & 20 \\ 6 & -10 & 40 \end{pmatrix} = 1 \quad (4)$$

from 2 .

$$\therefore \text{rank} \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = \text{rank} \begin{pmatrix} 3 & -5 & 20 \\ 6 & -10 & 40 \end{pmatrix} \quad (5)$$

$$= 1 < \dim \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = 2 \quad (6)$$

$\Rightarrow$  1 has infinitely many solutions. Fig 2.1 shows that the lines are the same.

- 2) Given  $\begin{pmatrix} 1 \\ -3 \end{pmatrix} \mathbf{x} = 7$  and  $\begin{pmatrix} 3 \\ -3 \end{pmatrix} \mathbf{x} = 15$ . The above equations can be expressed as a matrix equation.

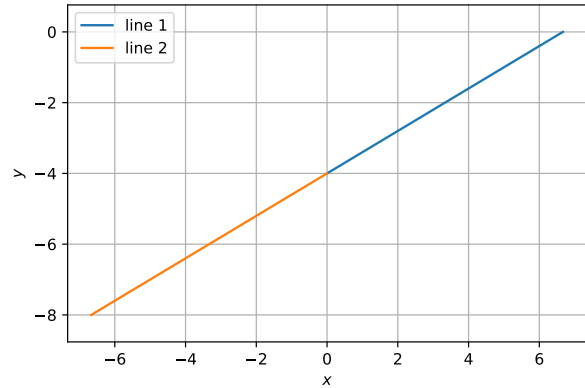


Fig. 2.1. Lines coincide: infinitely many solutions

tion.

$$\begin{pmatrix} 1 & -3 \\ 3 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 7 \\ 15 \end{pmatrix} \quad (7)$$

The augmented matrix for the above equation is row reduced as follows

$$\left( \begin{array}{cc|c} 1 & -3 & 7 \\ 3 & -3 & 15 \end{array} \right) \xrightarrow{R_2 \leftarrow R_2 - R_1} \left( \begin{array}{cc|c} 1 & -3 & 7 \\ 2 & 0 & 8 \end{array} \right) \quad (8)$$

$$\left( \begin{array}{cc|c} 1 & -3 & 7 \\ 2 & 0 & 8 \end{array} \right) \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left( \begin{array}{cc|c} 1 & -3 & 7 \\ 0 & 6 & -6 \end{array} \right) \quad (9)$$

$$\left( \begin{array}{cc|c} 1 & -3 & 7 \\ 0 & 6 & -6 \end{array} \right) \xrightarrow{R_1 \leftarrow R_1 + \frac{R_2}{2}} \left( \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 6 & -6 \end{array} \right) \quad (10)$$

$$\left( \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 6 & -6 \end{array} \right) \xrightarrow{R_2 \leftarrow \frac{R_2}{6}} \left( \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array} \right) \quad (11)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \quad (12)$$

is a solution of 7. The rank of

$$\begin{pmatrix} 1 & -3 \\ 3 & -3 \end{pmatrix} = 2 \quad (13)$$

and the rank of

$$\begin{pmatrix} 1 & -3 & 7 \\ 3 & -3 & 15 \end{pmatrix} = 2 \quad (14)$$

from 11 .

$$\therefore \text{rank} \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = \text{rank} \begin{pmatrix} 3 & -5 & 20 \\ 6 & -10 & 40 \end{pmatrix} \quad (15)$$

$$= \dim \begin{pmatrix} 3 & -5 \\ 6 & -10 \end{pmatrix} = 2 \quad (16)$$

$\Rightarrow$  7 has a unique solution,  $\mathbf{x} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$  Fig2.2 shows that the lines intersect only at one point.

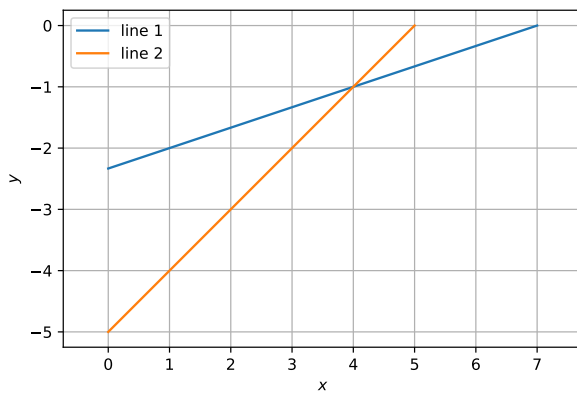


Fig. 2.2. Lines intersecting only at one point:unique solution