1

ASSIGNMENT 4

Gayathri S

Download all python codes from

https://github.com/Gayathri1729/SRFP/tree/main/ Assignment4

and latex-tikz codes from

https://github.com/Gayathri1729/SRFP/tree/main/ Assignment4

1 Linear Forms-2.43 B

Determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angle between them.

$$(2 \ 1 \ 3) \mathbf{x} = 2 \text{ and } (1 \ -2 \ 5) \mathbf{x} = 0$$

2 EXPLANATION

Given the planes,

$$P_1: (2 \ 1 \ 3) \mathbf{x} = 2 \tag{1}$$

$$P_2: (1 -2 5) \mathbf{x} = 0 \tag{2}$$

The normal vector of P_1 and P_2 are

$$n_1 = \begin{pmatrix} 2 & 1 & 3 \end{pmatrix} \tag{3}$$

and

$$n_2 = \begin{pmatrix} 1 & -2 & 5 \end{pmatrix}, \tag{4}$$

respectively.

The angle between two planes is same as the angle between their normal vectors. Let θ be the angle between n_1 and n_2 . Then

$$\cos \theta = \frac{\mathbf{n_1} \cdot \mathbf{n_2}}{\|\mathbf{n_1}\| \|\mathbf{n_2}\|} \tag{5}$$

$$\|\mathbf{n_1}\| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$$
 (6)

$$\|\mathbf{n_2}\| = \sqrt{1^2 + (-2)^2 + 5^2} = \sqrt{30}$$
 (7)

$$\mathbf{n_1} \cdot \mathbf{n_2} = 2 \times 1 + 1 \times -2 + 3 \times 5 = 15$$
 (8)

Then,

$$\cos \theta = \frac{15}{\sqrt{14}\sqrt{30}} = \sqrt{\frac{15}{28}} \tag{9}$$

 \therefore the planes P_1 and P_2 are neither parallel nor perpendicular.

From 2, the angle between the planes is

$$\theta = \cos^{-1}\left(\sqrt{\frac{15}{28}}\right) \tag{10}$$

$$= \cos^{-1}(0.7319) = 42.95^{\circ} \tag{11}$$

Fig 2.1 shows the planes are neither parallel nor perpendicular.

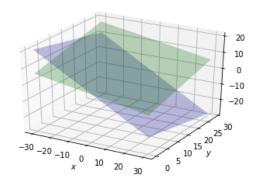


Fig. 2.1. Planes P_1 and P_2