ASSIGNMENT 4

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Download all python codes from

https://github.com/Gayathri1729/SRFP/tree/main/Assignment4

and latex-tikz codes from

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1 Linear Forms-2.43 B

Determine whether the given planes are parallel or perpendicular, and in case they are neither,find the angle between them.

$\begin{pmatrix}2 & 1 & 3\end{pmatrix}\mathbf{x}=2$ and $\begin{pmatrix}1 & -2 & 5\end{pmatrix}\mathbf{x}=0$

2 Explanation

Given the planes,

$$P_1 : \begin{pmatrix}2 & 1 & 3\end{pmatrix}\mathbf{x} = 2 \tag{1}$$

$$P_2 : \begin{pmatrix}1 & -2 & 5\end{pmatrix}\mathbf{x} = 0 \tag{2}$$

The normal vector of P_1 and P_2 are

$$\mathbf{n}_1 = \begin{pmatrix}2\\1\\3\end{pmatrix} \tag{3}$$

and

$$\mathbf{n}_2 = \begin{pmatrix}1\\-2\\5\end{pmatrix}, \tag{4}$$

respectively.

The angle between two planes is same as the angle between their normal vectors.Let θ be the angle between $\mathbf{n}_1$ and $\mathbf{n}_2$.Then

$$\cos\theta = \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{5}$$

$$\|\mathbf{n_1}\| = \sqrt{2^2+1^2+3^2} = \sqrt{14} \tag{6}$$

$$\|\mathbf{n_2}\| = \sqrt{1^2+(-2)^2+5^2} = \sqrt{30} \tag{7}$$

$$\mathbf{n_1}^\top \mathbf{n_2} = 2\times 1 + 1\times -2 + 3\times 5 = 15 \tag{8}$$

Then,

$$\cos\theta = \frac{15}{\sqrt{14}\sqrt{30}} = \sqrt{\frac{15}{28}} \tag{9}$$

$\therefore$ the planes P_1 and P_2 are neither parallel nor perpendicular.

From 9, the angle between the planes is

$$\theta = \cos^{-1}\left(\sqrt{\frac{15}{28}}\right) \tag{10}$$

$$= \cos^{-1}(0.7319) = 42.95^\circ \tag{11}$$

Fig 2.1 shows the planes are neither parallel nor perpendicular.

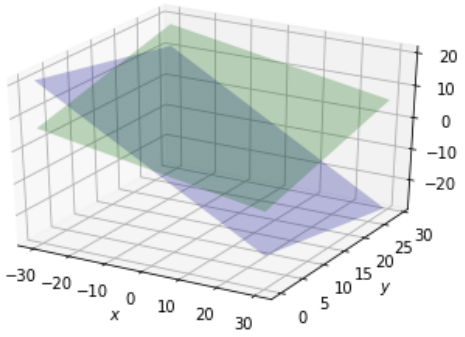

Fig. 2.1. Planes P_1 and P_2