Case Study

1) Write an algorithm that suits to solve the problem?

Step 1 :- Divide the pile of socks into two roughly equal parts

Step 2 :- Each group of campers independently sorts their respective piles of socks using any efficient sorting algorithm

Step 3 :- Merge the 2 sorted piles into single sorted pile, Combine the sorted piles in a way that maintains the sorted order, Use a merge algorithm, similar to the one used in MergeSort.

Step 4 :- If there are more than 1 pile, continue step 1, 2, 3 again.

Step 5 :- The final result is a completely sorted list of socks.

2) Determine the no: of phases needed for PLP to arrange the socks for faster wash.

Sol The no: of phases needed for PLP is determined by the no: of recursive calls in the algorithm. Each recursive call represents a phase. In this case, since the algorithm divides the piles into 2 halves at each level, the no: of phases is $\log_2(n)$, where n is the no: of socks.

3) Write a pseudocode algorithm outlining the sorting process, considering individual sorting & merging phase.

Sol

```
function PLP_Sort(socks):
    if length(socks) <= 1:
        return socks

    mid = length(socks) / 2
    left_pile = socks [0 to mid-1]
    right_pile = socks [mid to end]
```

```
    left_pile = PLP_Sort(left_pile)
    right_pile = PLP_Sort(right_pile)
    result = Merge(left_pile, right_pile)
    return result

function Merge(left, right):
    result = []
    left_index = 0
    right_index = 0
    while left_index < length(left) & right_index < length(right):
        if left[left_index] < right[right_index]:
            result.append(left[left_index])
            left_index++
        else:
            result.append(right[right_index])
            right_index++
    result.extend(left[left_index:])
    result.extend(right[right_index:])
    return result
```

4) Derive the recurrence relation of above algorithm?

Sol) The recurrence relation for the given algorithm is:

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + O(n)$$

This recurrence relation describes the time complexity of the PLP sorting algorithm. The $2 \cdot T\left(\frac{n}{2}\right)$ term represents the time to sort & merge 2 sub-piles, & $O(n)$ represents the time to merge the 2 sorted sub-piles.