## Written Assignment 1

1. Consider $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

   (a) Find rank of matrix A

   (b) Find eigen values and eigen vectors of A

   (c) Compute the trace and determinant of the matrix A using eigen values and cross check with respect to the ones computed from the matrix elements

   (d) Form an eigen vector matrix $\phi = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}$ where e_1, e_2, e_3 being normalized column eigen vectors of A

   (e) Given $\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = diag \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}$ is a diagonal matrix
   with respective eigen values as elements.
   Find $\phi \Lambda \phi^T$. Give your inference by comparing the result with matrix A

2. Consider a unit length square region in which another square is inscribed as shown
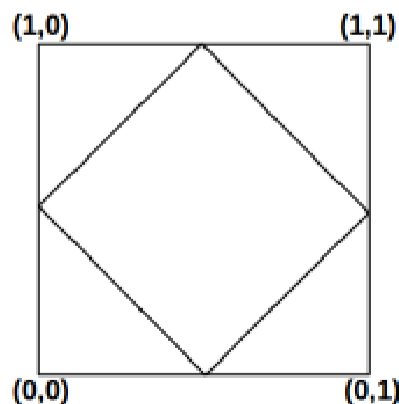
(a) Give the Probability Density Function of the same

   [Hint : Write pdf in functional form]

(b) What is the probability of a point falling in the region covered by the inner space

3. Given $x \in \{0, 1\}$ and $P\{x = 1\} = p$ i.e., probability of x being 1 is p

   (a) Compute the mean (μ) and variance (σ^2) of the event space

   (b) Compute mean and variance if $p = 0.5$

   (c) Compute mean and variance if $p = 0.3$

   (d) Try to describe verbally what does this mean ?

   (e) Sketch Probability Mass Function (PMF)

4. Given $x \in \{-1, 0, 1\}$ and $2P(x = -1) = 2P(x = 1) = P(x = 0)$

   (a) Sketch Probability Mass Function (PMF)

   (b) Compute the mean of the random variable x

   (c) Compute standard deviation (or) variance

5. Probability Density Function of a continuous random varibale x is given below
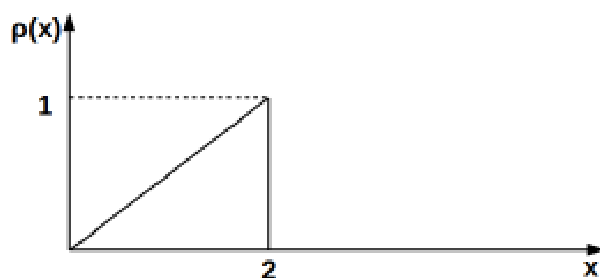
   (a) Compute the mean value for the random variable x

   (b) Give some intutive explanation to (a) regarding the position of the mean

   (c) Compute variance (or) standard deviation