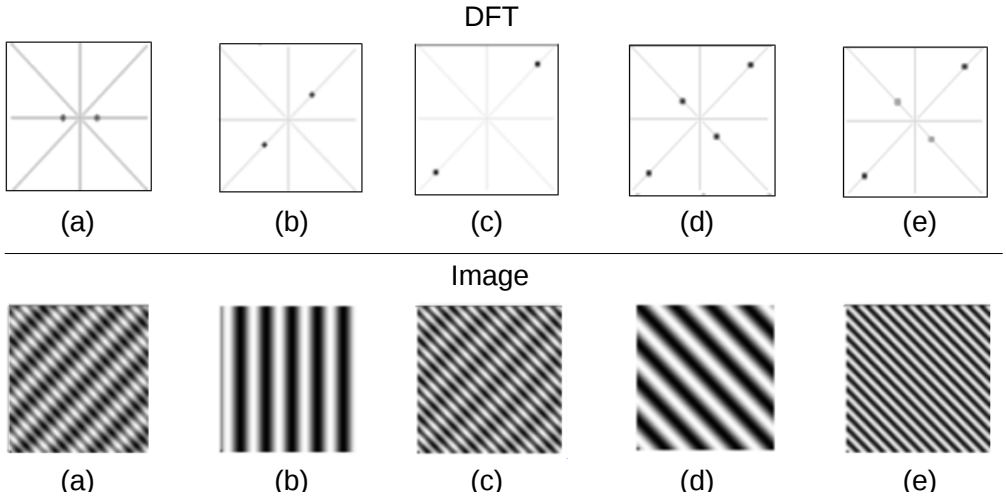
Image Signal Processing

Tutorial – 4: (Image Transforms)

Q 1. In the given figure, match DFT spectra and corresponding images:



Sol: (a:b); (b:d); (c-e); (d-a); (e-c)

Q 2. Write down the SVD decomposition $U\Sigma V^T$ of the following matrices:

1. Orthogonal matrix O
2. Diagonal matrix D
3. Symmetric matrix A

Sol: 1. $U = O, \Sigma = I, V = I$. 2. $U = I, \Sigma = D, V = I$. 3. $U = R, V = R$.

Q 3. Which of the following statements about the Singular Value Decomposition are true?

1. Every real matrix has an SVD.
2. The SVD reveals the rank of a matrix.
3. A matrix is not invertible if it has a singular value 0.

Sol: All three.

Q 4. Assume that the two-dimensional Fourier spectrum of an image with size 640×480 and a spatial resolution of 72 dpi shows a dominant peak at position $\pm(100, 100)$. Determine the orientation and effective frequency (in cycles per cm) of the corresponding image pattern.

Sol: The image contains a periodic pattern with effective frequency $\hat{f} = \frac{1}{\tau}$ and orientation ϕ, The corresponding frequency coefficient for this pattern is found at position (m, n) in the Fourier spectrum.

The orientation is given by

$$\phi = \arctan \frac{n \cdot M}{m \cdot N} = 53^o$$

The effective frequency will be $f = f_s \cdot \sqrt{(\frac{m}{M})^2 + (\frac{n}{N})^2}$ where $f_s = \frac{72}{2.54}$ cycles/cm.
Hence $f \approx 7.5$ cycles/cm.

Q 5. The unitary 1D Discrete Sine Transform (DST) of an N-point sequence $u(n)$ is given by

$$v(k) = \sqrt{\frac{2}{N+1}} \sum_{n=0}^{N-1} u(n) \sin\left(\frac{\pi(k+1)(n+1)}{(N+1)}\right), \quad 0 \leq k \leq N-1$$

Determine the 2D DST matrix for N=3.

Sol:

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 0.707 & 1 & 0.707 \\ 1 & 0 & -1 \\ 0.707 & -1 & 0.707 \end{bmatrix}$$