#11

PHD ON CONCENTRIC SHELLS

ELECTRIC POTENTIAL

EDUNITI

TOPICS TO BE DISCUSSED

1. Electric Field and Potential in Charged Shell
2. How to write Electric Field & Potential in Concentric Shells
3. Questions on finding Potential (*charge density*)
4. Question involving Earthing
5. Question involving connecting shells

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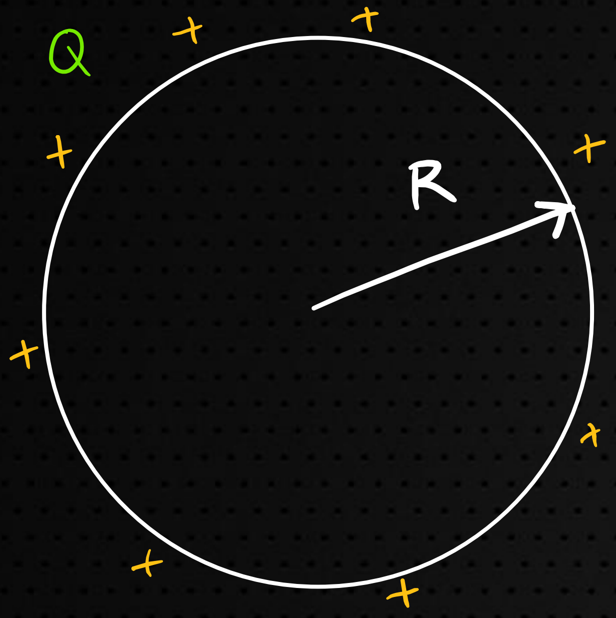
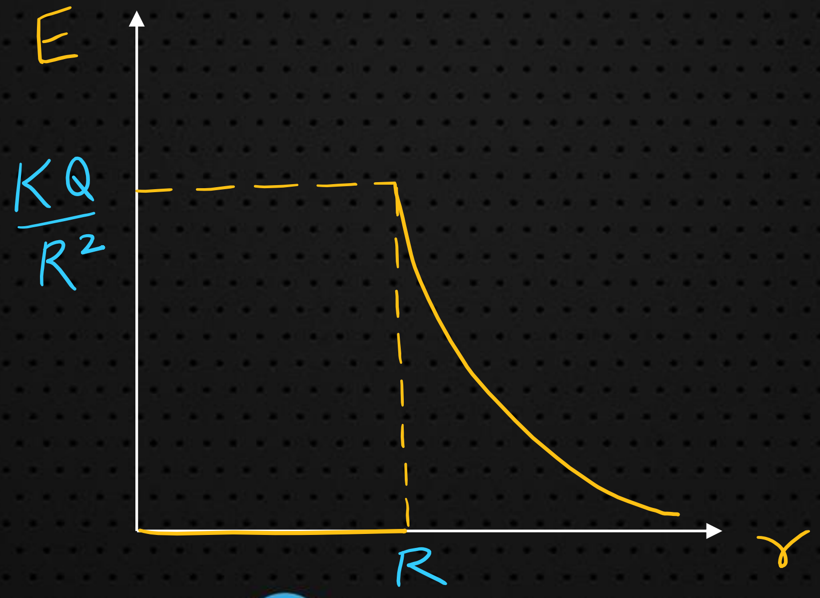
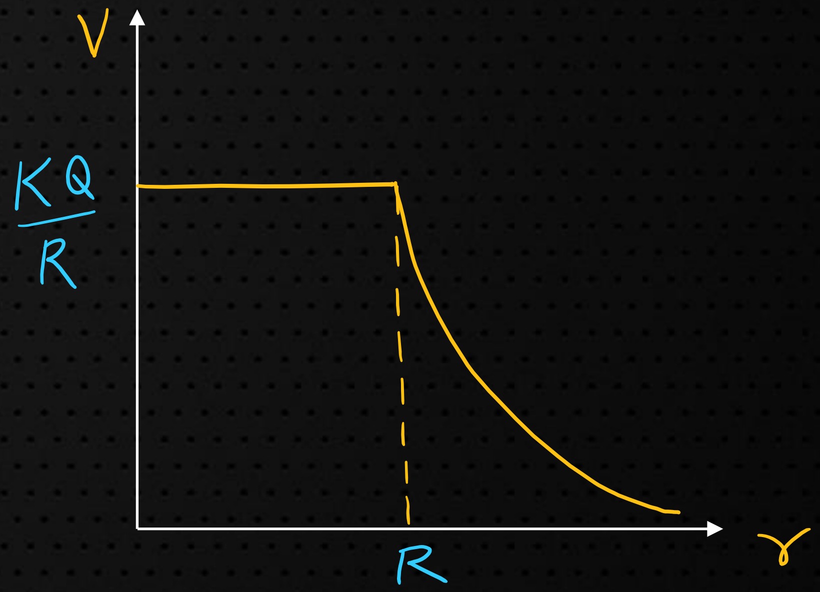
Eduniti for Physics

1. Electric Field & Potential due to charged shell

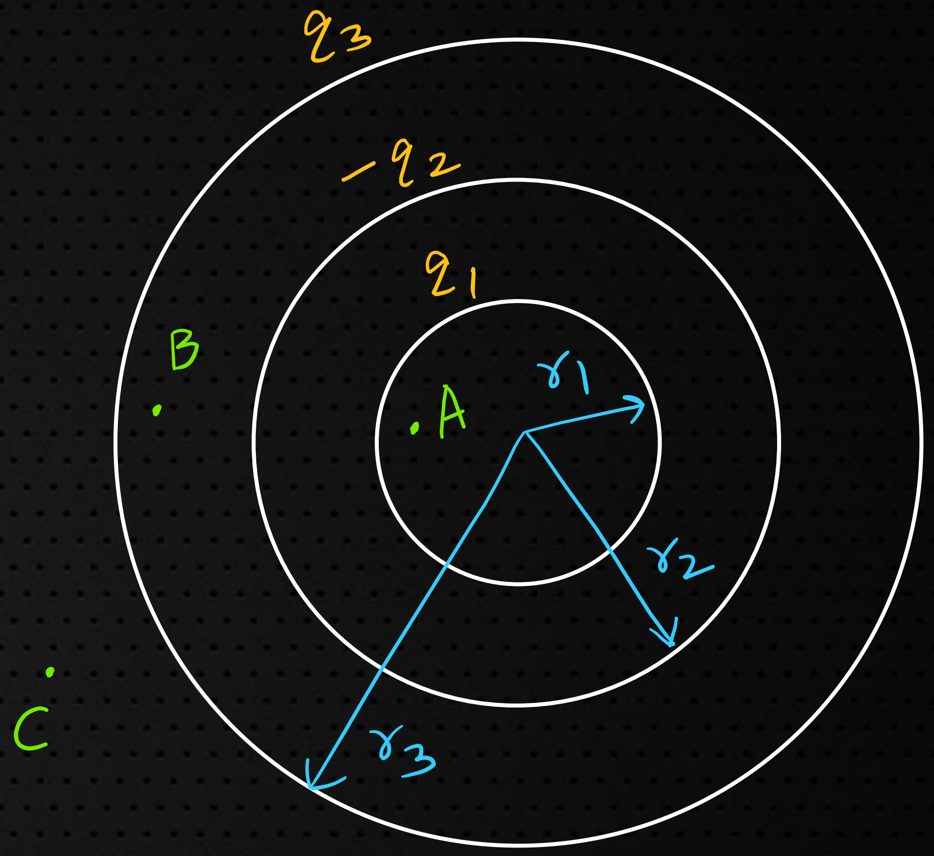


- for $r < R$, $E = 0$, $V = \frac{KQ}{R}$ (constant)
- for $r \geq R$, $E = \frac{KQ}{r^2}$, $V = \frac{KQ}{r}$





2. How Write E.F. & Potential in Concentric shells



(a) Potential at A, B & C

$$V_A = \frac{Kq_1}{r_1} - \frac{Kq_2}{r_2} + \frac{Kq_3}{r_3}$$

$$V_B = \frac{Kq_1}{r_B} - \frac{Kq_2}{r_B} + \frac{Kq_3}{r_3}$$

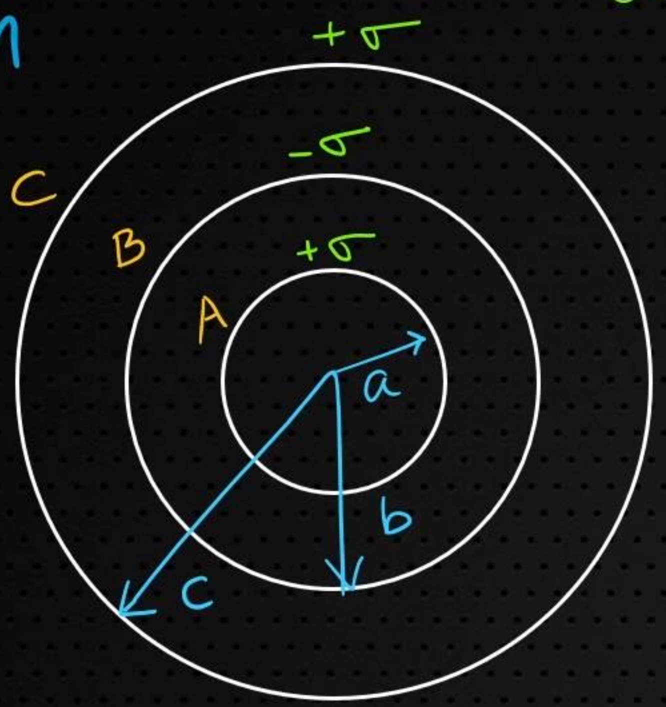
$$V_C = \frac{Kq_1}{r_C} - \frac{Kq_2}{r_C} + \frac{Kq_3}{r_C}$$

(b) E.F at A, B & C

$$E_A = 0, \quad E_B = \frac{K}{r_B^2}(q_1 - q_2), \quad E_C = \frac{K}{r_C^2}(q_1 - q_2 + q_3)$$

3. Question

$q = \sigma 4\pi R^2$



Ex 1. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma, -\sigma$ and $+\sigma$ respectively. The potential of shell B is

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(a) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$ (b) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$

✓(c) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$ (d) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$

Sol^n:

$$V_B = \frac{Kq_A}{b} + \frac{Kq_B}{b} + \frac{Kq_C}{c} = \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[\frac{a^2}{b} - \frac{b^2}{b} + c\right] = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b} + c\right]$$

... continued

Ex2. A charge Q is distributed over three concentric spherical shells of radii a, b, c ($a < b < c$) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where $r < a$, would be

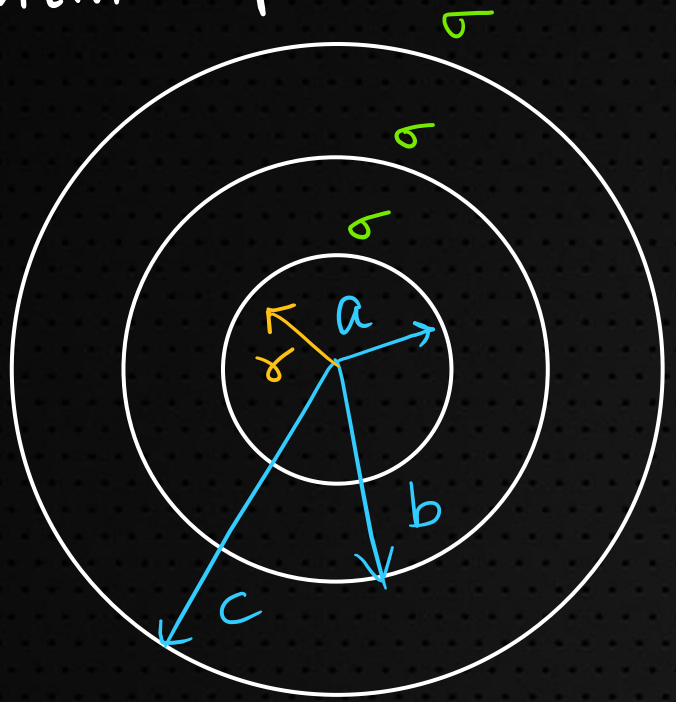
(a) $\frac{Q}{12\pi\varepsilon_0}\frac{ab+bc+ca}{abc}$

(b) $\frac{Q(a^2+b^2+c^2)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$

(c) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$

(d) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

... Continued



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(a) $\dfrac{Q}{12\pi\varepsilon_0}\dfrac{ab+bc+ca}{abc}$ (b) $\dfrac{Q(a^2+b^2+c^2)}{4\pi\varepsilon_0(a^3+b^3+c^3)}$

(c) $\dfrac{Q}{4\pi\varepsilon_0(a+b+c)}$ ✓ (d) $\dfrac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

Sol^n: $V = \dfrac{Kq_A}{a} + \dfrac{Kq_B}{b} + \dfrac{Kq_C}{c} \Rightarrow V = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{\sigma 4\pi a^2}{a} + \dfrac{\sigma 4\pi b^2}{b} + \dfrac{\sigma 4\pi c^2}{c}\right)$

$\Rightarrow V = \dfrac{1}{4\pi\varepsilon_0} \times \sigma 4\pi(a+b+c)$ — (1)

$\sigma 4\pi a^2 + \sigma 4\pi b^2 + \sigma 4\pi c^2 = Q \Rightarrow \sigma 4\pi = \dfrac{Q}{a^2+b^2+c^2}$ — (2)

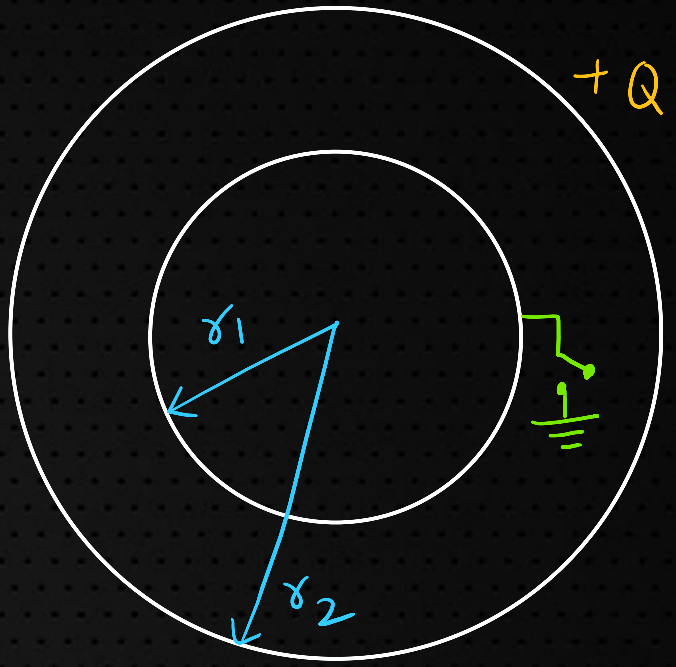
Put (2) in (1) $\Rightarrow$ option (d)

4. Question involving Earthing

Ex 3. In the figure shown, if the outer shell has charge $+Q$ and the inner shell is earthed, then

(a) determine the charge on the inner shell, and

(b) find the potential of the outer shell



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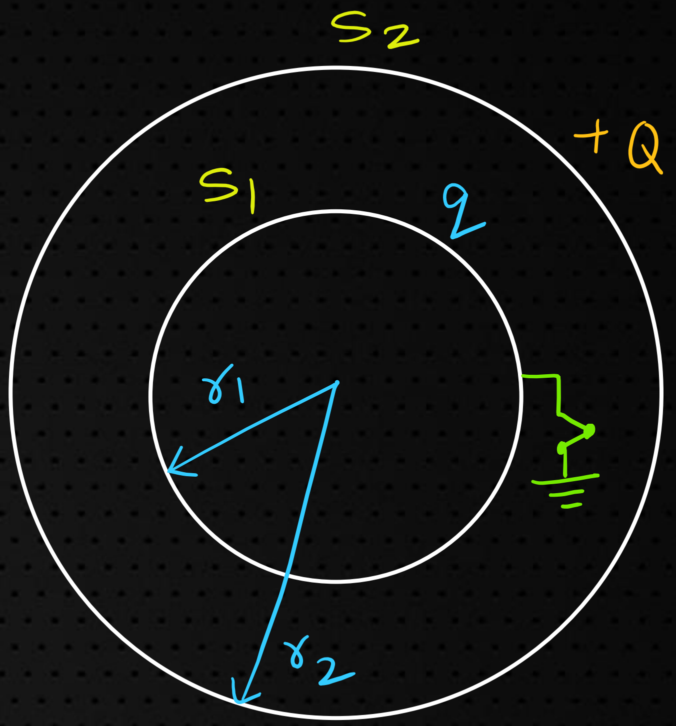


Sol^n: $\because$ Earthed, $V_{S_1} = 0$

(a) $\dfrac{Kq}{r_1} + \dfrac{KQ}{r_2} = 0 \Rightarrow q = -\dfrac{Qr_1}{r_2}$

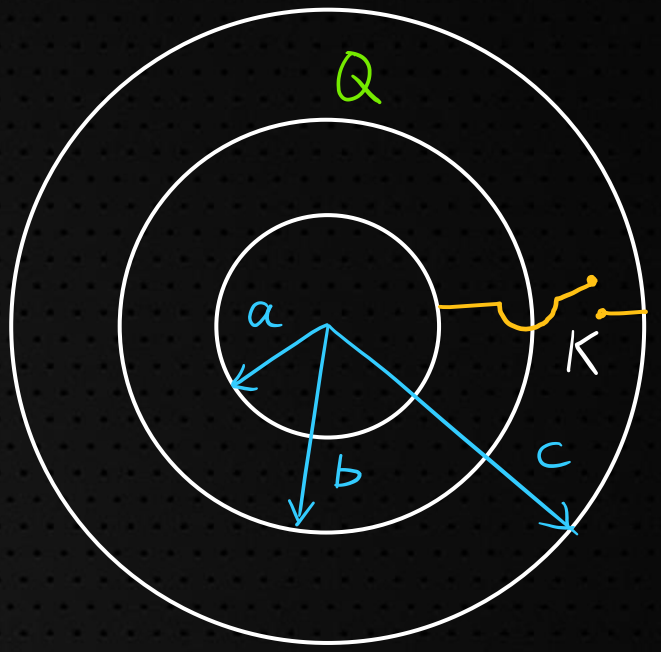
(b) $V_{S_2} = \dfrac{Kq}{r_2} + \dfrac{KQ}{r_2} = \dfrac{K}{r_2}\left(-\dfrac{Qr_1}{r_2}\right) + \dfrac{KQ}{r_2}$

$= \boxed{\dfrac{KQ}{r_2}\left(1 - \dfrac{r_1}{r_2}\right)}$

5. Question involving Connecting shells

Ex 4. Three concentric conducting shells of radii a, b, and c are shown in fig. Charge on the shell of radius b is Q. If key K is closed, find the charges on the innermost and outermost shells.

Given that $a : b : c = 1 : 2 : 3$.



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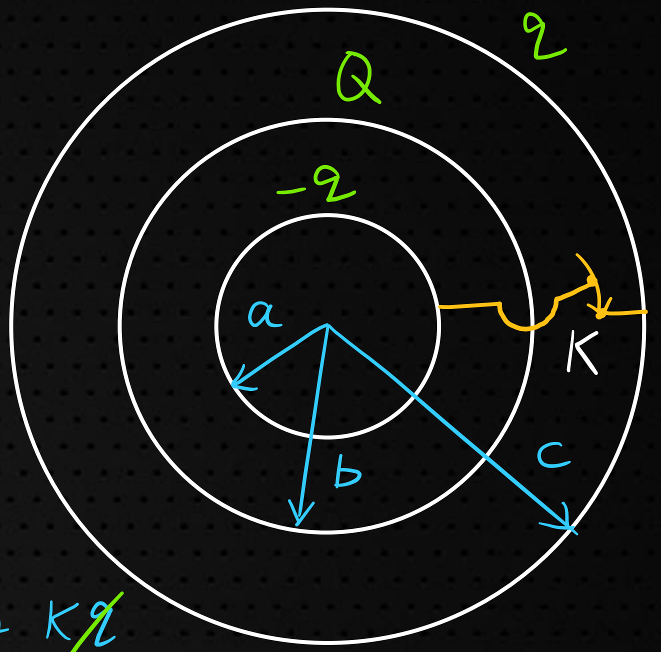
Ex 4. Three concentric conducting shells of radii a, b, and c are shown in fig. Charge on the shell of radius b is Q. If key K is closed, find the charges on the innermost and outermost shells.

$-q$ q

Given that $a : b : c = 1 : 2 : 3. \longrightarrow b = 2a, \; c = 3a$



Sol^n: $V_{\text{inner shell}} = V_{\text{outer shell}}$

$$\Rightarrow -\frac{Kq}{a} + \frac{KQ}{b} + \cancel{\frac{Kq}{c}} = \frac{-Kq}{c} + \frac{KQ}{c} + \cancel{\frac{Kq}{c}}$$

$$\Rightarrow -\frac{Kq}{a} + \frac{KQ}{2a} = -\frac{Kq}{3a} + \frac{KQ}{3a} \Rightarrow -q + \frac{Q}{2} = -\frac{q}{3} + \frac{Q}{3}$$

$$\therefore \frac{Q}{6} = \frac{2q}{3} \Rightarrow \boxed{q = \frac{Q}{4}}$$

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