FLUID STATICS

REVISION in 35 Min

Now that's how you REVISE

-Mohit Goenka, IIT Kharagpur

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1. PYQs Video Solution Topic Wise:
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EDUNITI

Eduniti for Physics

FLUID MECHANICS

FLUID STATICS

Fluid is at rest with respect to Vessel

(Part 1)

FLUID DYNAMICS

Fluid is in motion w.r.t vessel / Pipe

(part 2)

FLUID PROPERTIES

Surface tension and viscosity

(part 3)

Revision Series Playlist Link https://bit.ly/3eBbib9

TOPICS COVERED

1. Introductory Points *(density, pressure)*
2. Pressure Variation *(vessel at rest)*
3. Free Surface *(vessel at rest, vessel accelerated linearly, vessel rotating)*
4. Pressure Variation *(vessel linearly accelerated , rotating)*
5. Pascal’s Law *(hydraulic lift)*
6. Barometer
7. Force on side walls due to liquid
8. Torque on side walls
9. Archimedes' Principle *(Floatation, Apparent weight, center of buoyancy)*

1. Introductory points

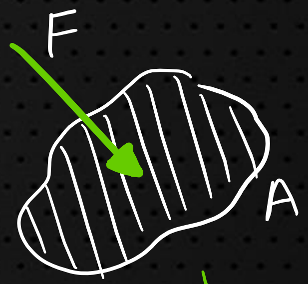
(a) Density, $\rho = m/V$ (kg/m^3 or g/cm^3)

(b) $\rho_{mix} = \dfrac{\text{total mass}}{\text{total volume}}$

(c) Relative density or specific gravity $= \rho/\rho_w$ at $4^\circ C$

Ex: Rel density of Hg = 13.6 $\Rightarrow \rho_{Hg} = 13.6 \times 1000$
$= 13600\ kg/m^3$
(or $13.6\ g/cc$)

(d) Pressure $= F_\perp / A$



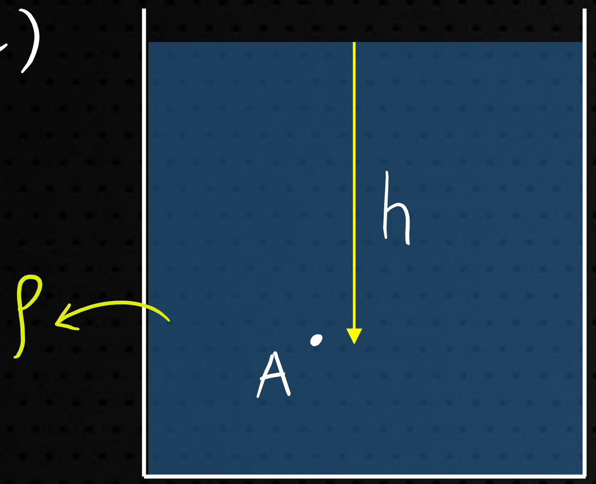
→ Pascal or Pa (N/m^2)

→ $P_0 = 1\ atm = 1.013 \times 10^5\ Pa$
$\approx 10^5\ Pa$

Pressure acts ⊥ to surface

2. Pressure Variation (vessel at rest)

(a)

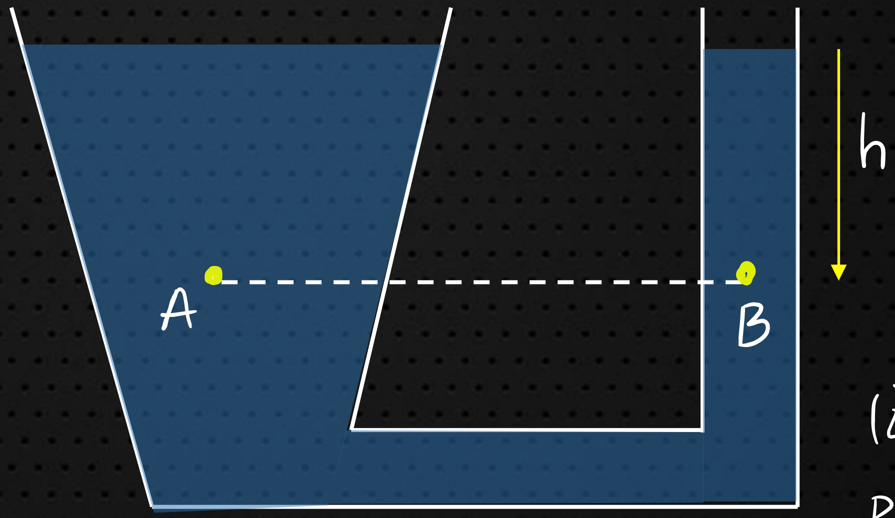


$P_A = \rho g h + P_0$

P_A → Absolute Pressure

$\rho g h$ → Gauge pressure (P_g)

(b)



(i) $P_A = P_B = \rho g h + P_0$

(ii) Pressure at depth h is independent of shape of vessel

(iii) At same level, pressure is same (same liquid)

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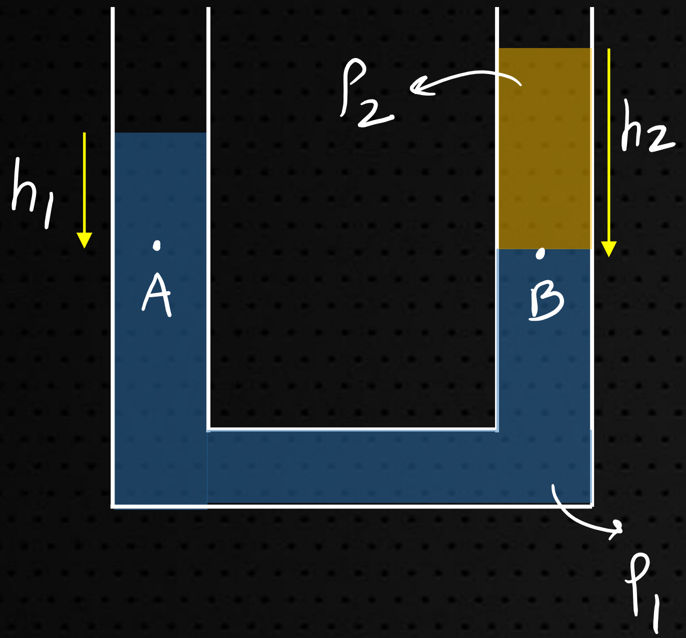
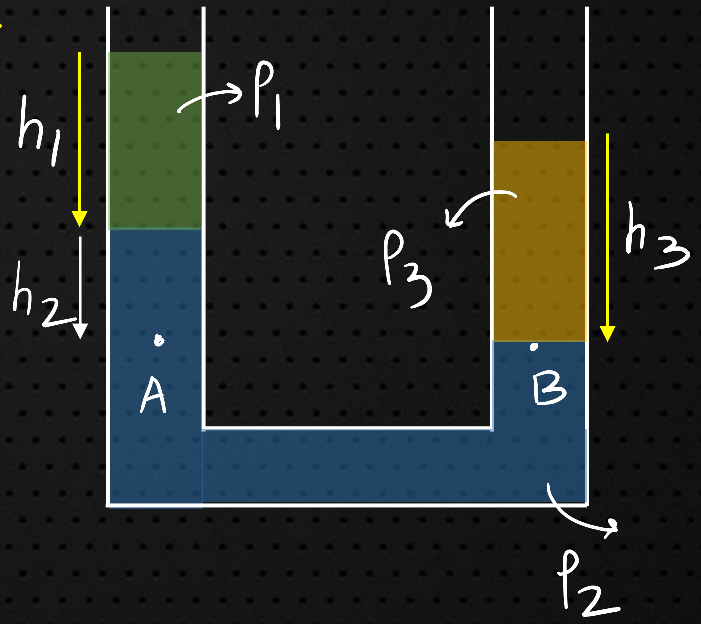
Ex 1.



$$P_A = P_B$$

$$\Rightarrow \rho_1 g h_1 + P_0 = \rho_2 g h_2 + P_0$$

Ex 2.



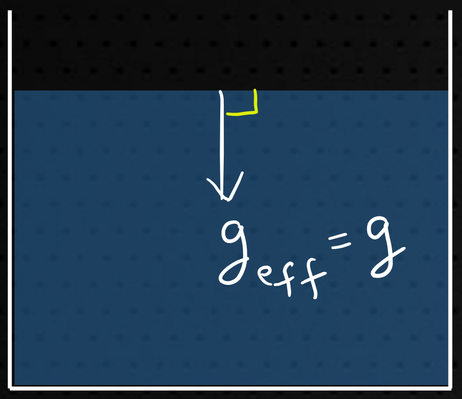
$$P_A = P_B$$

$$\Rightarrow P_0 + \rho_1 g h_1 + \rho_2 g h_2 = P_0 + \rho_3 g h_3$$

3. Free Surface

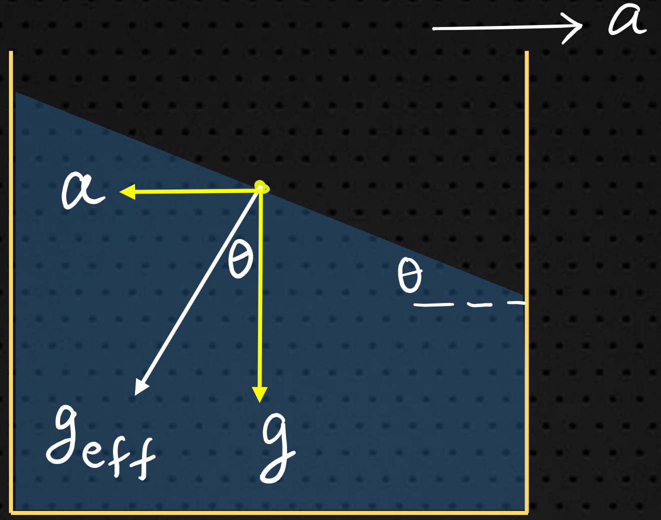
↳ free surface is always $\perp$ to g_{eff}

(a)



At rest

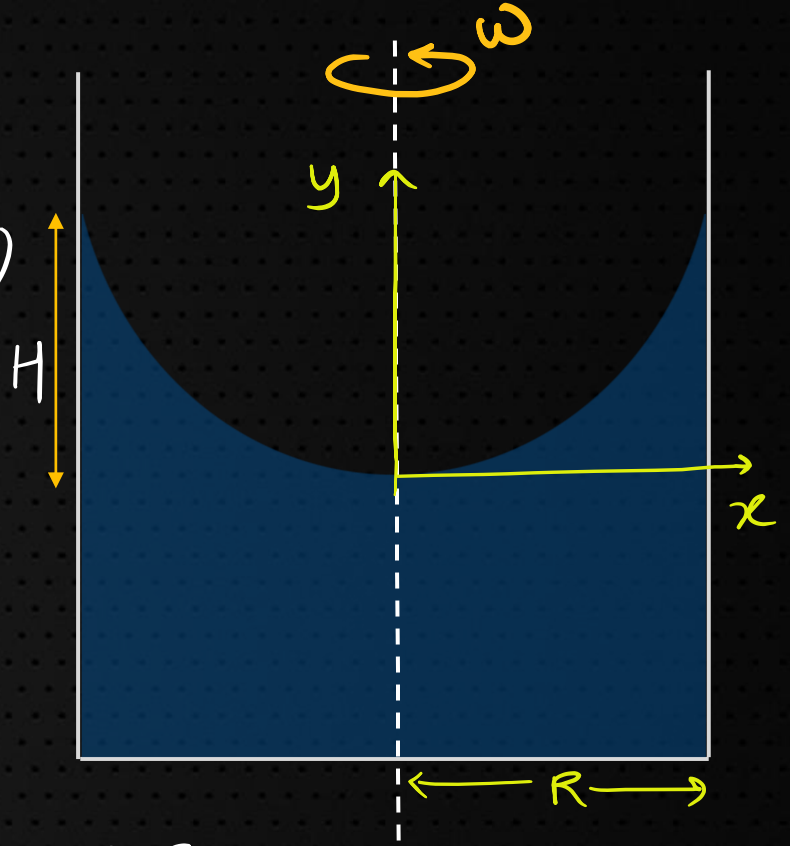
(b)



$$g_{eff} = \sqrt{a^2 + g^2}$$

$$\tan\theta = \frac{a}{g}$$

(c)



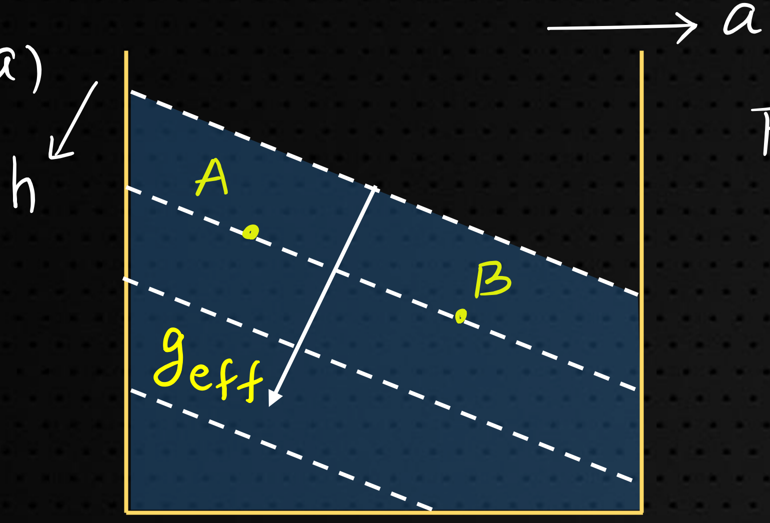
$$y = \frac{\omega^2 x^2}{2g}$$

↳ If $x = R$, $y = H$

$$\therefore H = \frac{\omega^2 R^2}{2g}$$

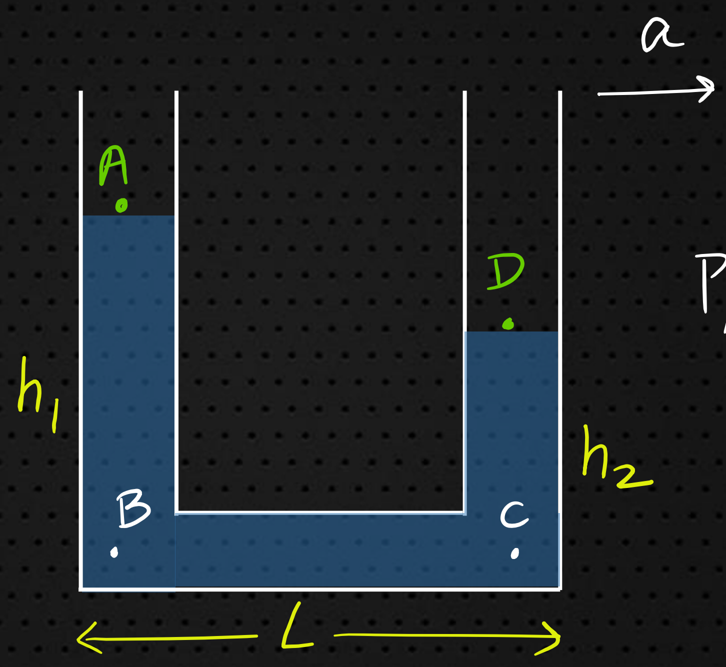
4. Pressure Variation (accelerated system)

(a)



$$P_A = P_B = P_0 + \rho g_{eff} h$$

(b)



↳ tube dia very small

Write eqn from A → D :

$$P_A + \rho g h_1 - \rho a L - \rho g h_2 = P_D$$

$$P_A = P_D = P_0$$

$$\Rightarrow \boxed{h_1 - h_2 = \frac{aL}{g}}$$

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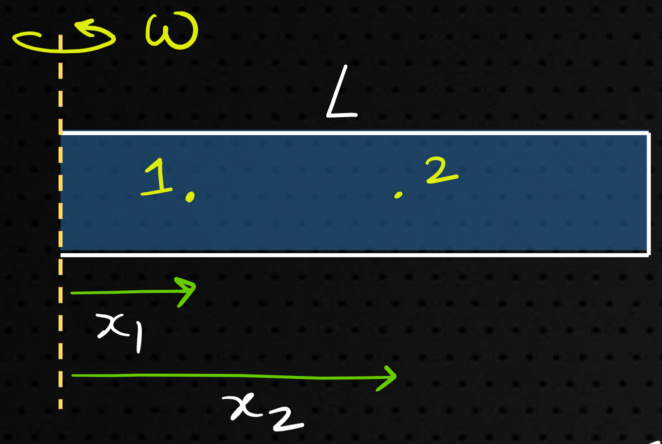
(c)



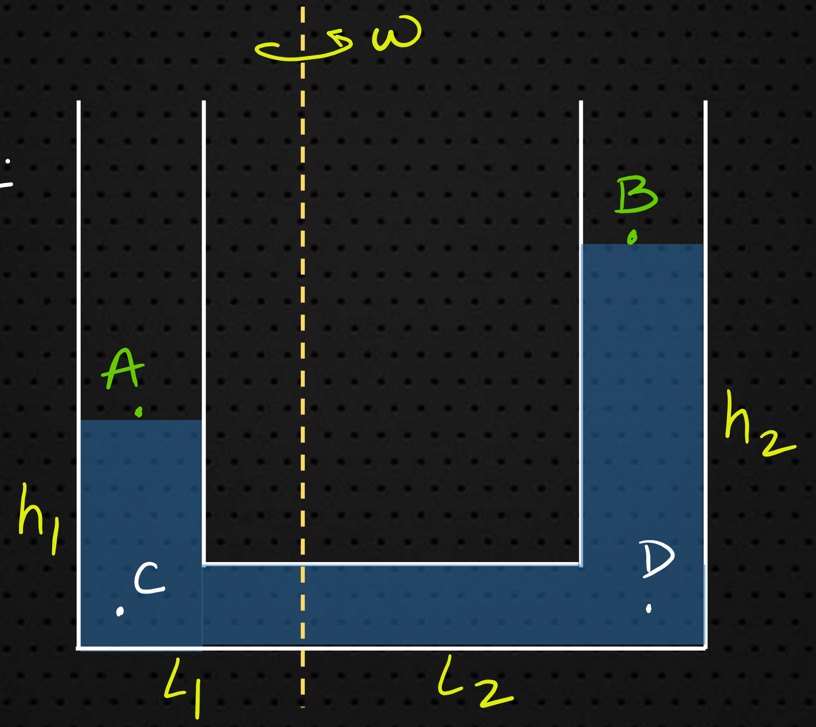
$$P_2 - P_1 = \frac{1}{2}\rho\omega^2\left(x_2^2 - x_1^2\right)$$

If $x_1 = 0$ i.e at axis

$$P_2 - P_1 = \frac{1}{2}\rho\omega^2 x_2^2$$

→ As you from axis to location 2, Press. increases by $\frac{1}{2}\rho\omega^2 x_2^2$

Ex.



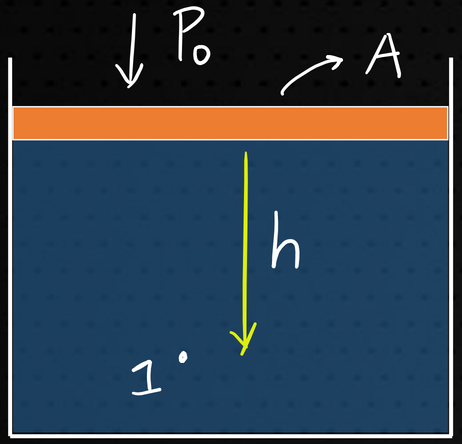
$$P_A + \rho g h_1 - \frac{1}{2}\rho\omega^2 L_1^2 + \frac{1}{2}\rho\omega^2 L_2^2 - \rho g h_2 = P_B$$

$$(P_A = P_B = P_0)$$

$$h_2 - h_1 = \frac{\omega^2}{2g}\left(L_2^2 - L_1^2\right)$$

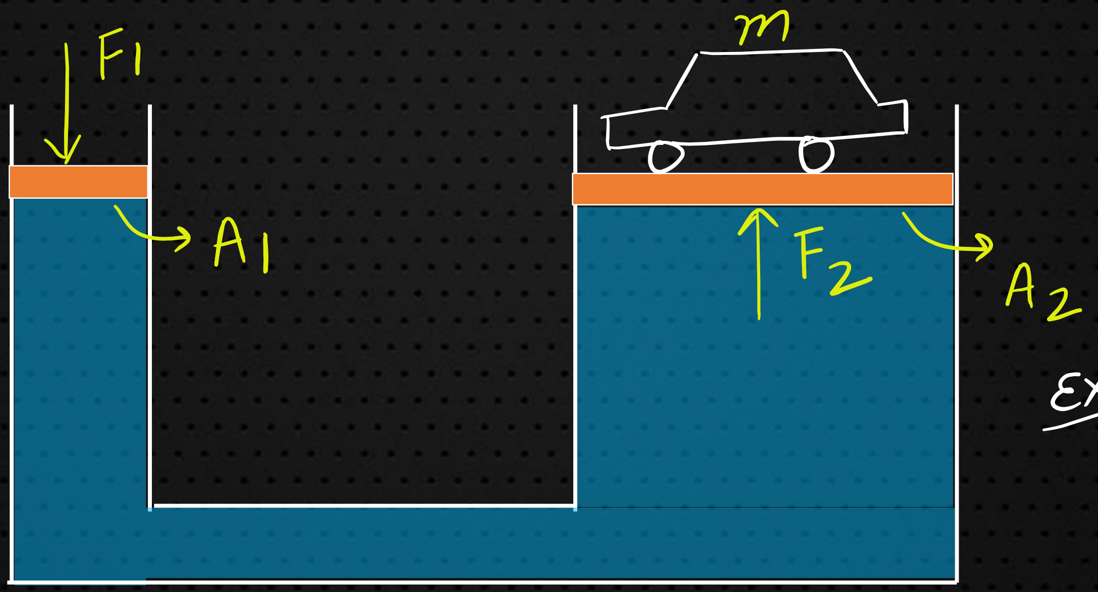
5. Pascal's Law

↳ External Pressure gets distributed evenly in all direction



$$P_1 = \rho g h + \frac{mg}{A} + P_0$$

Hydraulic Lift



$$F_2 = P A_2$$
$$= \frac{F_1}{A_1} \cdot A_2$$
$$\Rightarrow F_2 = F_1 \left(\frac{A_2}{A_1} \right)$$

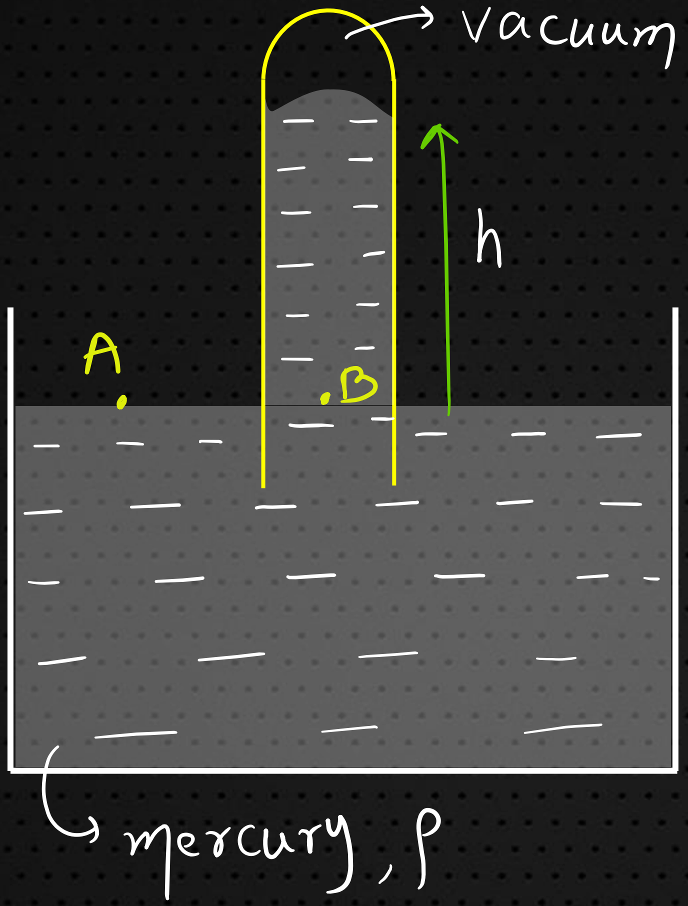
$\because A_2 >> A_1 \Rightarrow F_2 >> F_1$

Ex. $mg = F_2 = F_1 \frac{A_2}{A_1}$

$$\Rightarrow F_1 = \frac{A_1}{A_2} \times mg$$

6. Barometer

↳ setup to measure atmospheric pressure



$$P_A = P_B$$

$$\Rightarrow P_0 = \rho g h$$

If $P_0 = 1.013 \times 10^5$ Pa

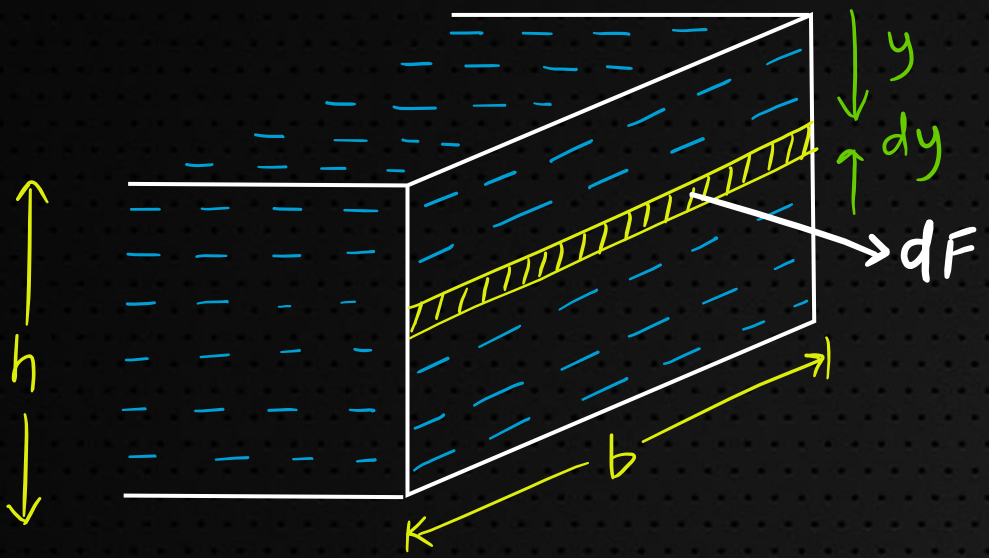
$$h = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5}{13600 \times 9.8}$$

$$= 0.76 \text{ m or } 760 \text{ mm}$$

$\therefore$ 1 atm is also written as 760 mm of Hg.

NOTE: We don't use H_2O because $h \approx 10$ m (not practicall)

7. Force on side walls (due to liquid)



$$dF = P(y) \times dA = \rho g y \times b\,dy$$

$$\Rightarrow F = \rho g b \int_0^h y\,dy = \rho g b \times \frac{h^2}{2} = \frac{\rho g h}{2} \times bh$$

$$\therefore F = P_{av} \times \text{Area of wall}$$

$$P_{av} = \rho g \frac{h}{2} \text{ (Pressure at center)}$$

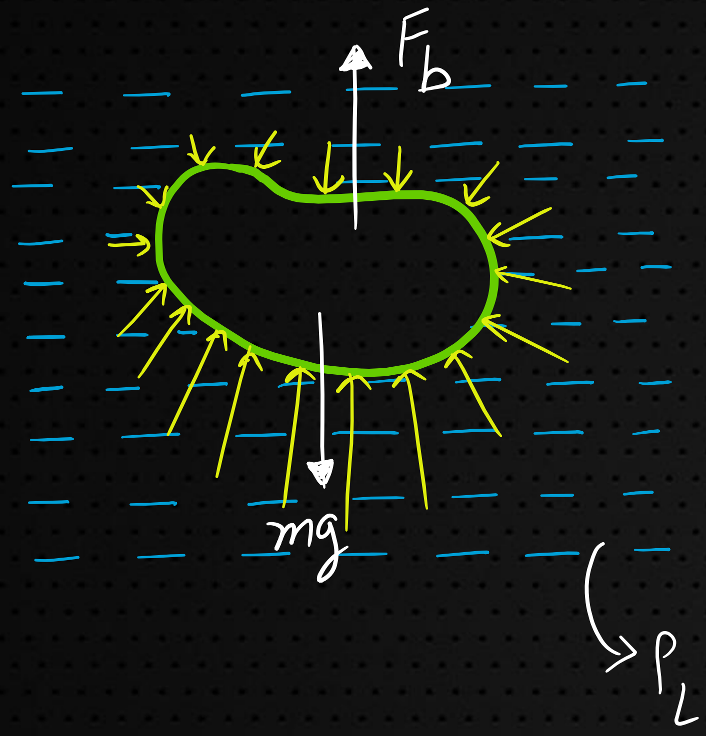
8. Torque on side walls

$$d\tau = dF \times (h-y) \Rightarrow d\tau = \rho g y \cdot b\,dy \cdot (h-y)$$

$$\Rightarrow \tau = \rho g b \int_0^h y(h-y)\,dy = \boxed{\frac{\rho g b h^3}{6}}$$

9. Archimedes' Principle / Floatation

(a)



Buoyant force, $F_b = \rho_L V g$ { V is volume of displaced liquid

Body is solid (not hollow from inside) :

(i) Body sinks : $\rho_s > \rho_L$

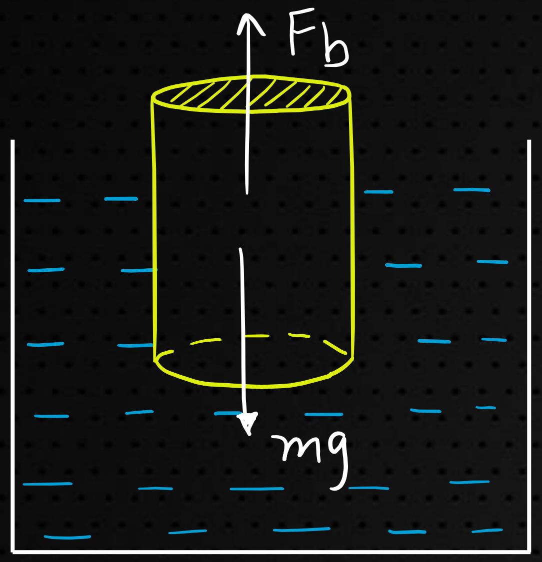
(ii) Body Floats : $\rho_s \leq \rho_L$

$\rho_s < \rho_L$ — Some Portion is outside

$\rho_s = \rho_L$ — Completely Submerged

... Continued

(b)



$\rho_s < \rho_L$

$F_b = mg \Rightarrow \rho_L V_1 g = \rho_s V g$

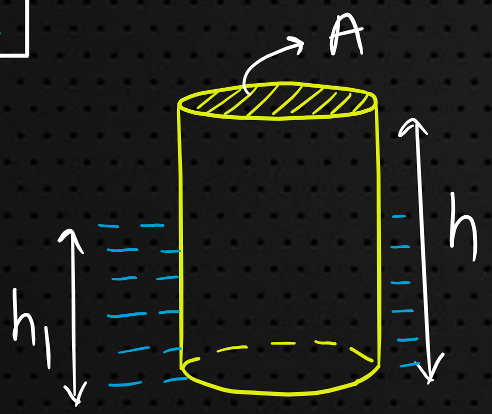
$\left\{ \begin{array}{l} V_1 : \text{displaced liquid vol.} \\ V : \text{Vol of body} \end{array} \right.$

$$\Rightarrow \boxed{\frac{V_1}{V} = \frac{\rho_s}{\rho_L}} \longrightarrow$$

Ex : Ice floating in water

$$\frac{V_1}{V} = \frac{900}{1000} \Rightarrow \frac{V_1}{V} = 0.9$$

$\therefore$ 90% of ice is inside water



$$\frac{Ah_1}{Ah} = \frac{\rho_s}{\rho_L} \Rightarrow \boxed{\frac{h_1}{h} = \frac{\rho_s}{\rho_L}}$$

$\hookrightarrow$ valid for uniform cross-section.

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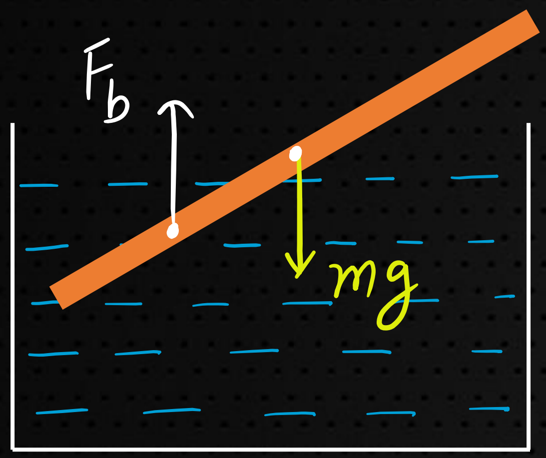
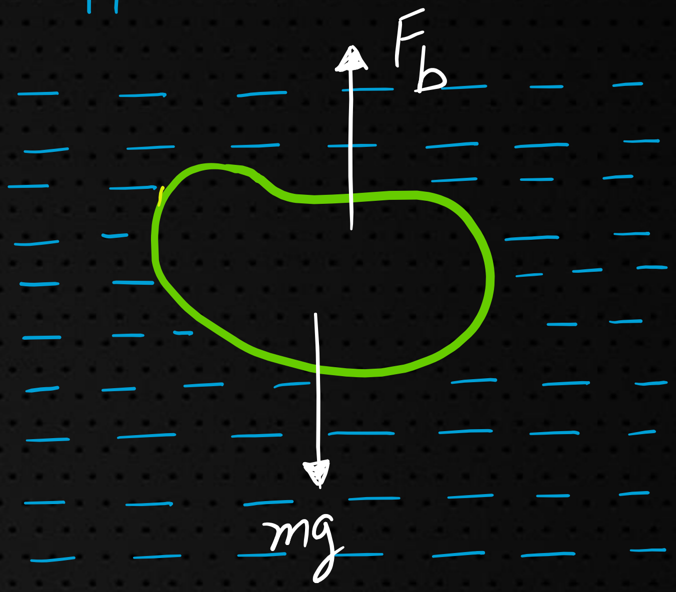
(C) Center of Buoyancy



(i) C.o.b is c.o.m of displaced Liquid

(ii) F_b passes through c.o.b

(d) Apparent weight



(i) $W_{app} = mg - F_b$

$$\Rightarrow W_{app} = mg - \rho_L V g$$

$$= mg - \rho_L \cdot \frac{m}{\rho_s} g = mg\left(1 - \frac{\rho_L}{\rho_s}\right)$$

$\left\{ V = \frac{m}{\rho_s} \right.$

$$\therefore \text{Reading} = m\left(1 - \frac{\rho_L}{\rho_s}\right)$$

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JEE Main PYQs Link https://bit.ly/2S54jzh Chapter wise 2021, 2020, 2018

GoldMine Link https://bit.ly/2VhOGFF

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