JEE MAIN | IIT JEE

Motion in 2-D

Concept + 10 Numerical

REVISION in 42 Min

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EDUNITI

Motion in Two Dimensions

- 2D Motion
- Projectile Questions
  - Ground to Ground
  - From tower/building
  - Inclined Plane
  - Moving trolley/car

10 Numerical Examples

JOIN

Eduniti for Physics

EDUNITI

Chapter	Formulae_Concept VIDEO LINK
Unit & Dimensions	https://youtu.be/wdd-wlZF4Hk
Errors and Vectors	
Vernier Calliper	https://youtu.be/pVoN045dV8I
Screw Gauge	https://youtu.be/gYd2PtmZ0mw
Kinematics_Motion in 1d	https://youtu.be/U4NNxFaFliE
Kinematics_Motion in 2d	
Laws of Motion	
Work Energy Power	https://youtu.be/kjrXoE-kDI8
Centre of Mass	
Centre of Mass of Standard Bodies	https://youtu.be/oCeACfryB-U
Collision	
Rotational Motion_Moment of Inertia	https://youtu.be/9ckZdOhy3z0
Gravitation	https://youtu.be/rAj2huLVaEk
Properties of Solids	https://youtu.be/gSXxjk89I_c
Fluids Statics (Part 1)	https://youtu.be/RFKx9B9yo3M
Fluid Dynamics (Part 2)	https://youtu.be/Y717vQpUEJQ
Fluid Properties (Part 3)	https://youtu.be/V8xUWWK2oT0
Simple Harmonic Motion	https://youtu.be/RIb7ofNG09I
Thermal Properties	
KTG	https://youtu.be/XO1tvFhIa0I
Thermodynamics	https://youtu.be/iz_kf1jRDRw
Wave Motion -Organ Pipes and Resonance Tube	https://youtu.be/fB7pfJ77za8
Wave Motion - Doppler's Effect	https://youtu.be/9-BxOaamnwg
Electrostatics	https://youtu.be/3stXbGRMcrk
Capacitors	https://youtu.be/EXEiickNUkY
Current Electricity	https://youtu.be/gm8FUfjrX18
Moving Charges and Magnetic Effect of Current	https://youtu.be/ULD2Ok1CGJk
Earth's Magnetism	https://youtu.be/a4CT5uVwAK4
Magnetic Properties	https://youtu.be/63cwdYXNlYE
EMI	https://youtu.be/puVavm_GFRM
Alternating Current	https://youtu.be/74dTY-pzM_o
Ray Optics	https://youtu.be/BhnyTWzIlBA
Wave Optics Part 1_Interference	https://youtu.be/LG5nlE8XTeI
Wave Optics Part 2_Diffraction_Polarization	https://youtu.be/ymMyyJGGqnY
Optical Instruments	https://youtu.be/OQssbDH0A4I
Electromagnetic Waves	https://youtu.be/bcVXgEkyQZY
Semiconductors_Basics + Zener Diode	https://youtu.be/_A2JomQ7-50
Semiconductors_Transistors	https://youtu.be/psDwl84Nzb0
Semiconductors_Logic Gates	https://youtu.be/pZdQAzLbFTo
Communication Systems	https://youtu.be/8NgMqK9X79Y
Modern Physics_Part 1_Atomic Physics	https://youtu.be/9VKUnE3mpHk
Modern Physics_Part 2_Photoelectric Effect	https://youtu.be/24oTQp84jrk
Modern Physics_Part 3_Dual Nature of Light	https://youtu.be/0zoR_saMAQY
Modern Physics_Part 4_Radioactivity	https://youtu.be/AdX3YBhQyog
Modern Physics_Part 5_Nuclear Physics	https://youtu.be/VDWqVahGixc
Modern Physics_Part 6_X Rays	https://youtu.be/dSHXdzX7NX0

1. Motion in a Plane

↳ 2D motion is Vector Sum of two independent 1D motion (along x & y)

If $\vec{a}$ is constant:

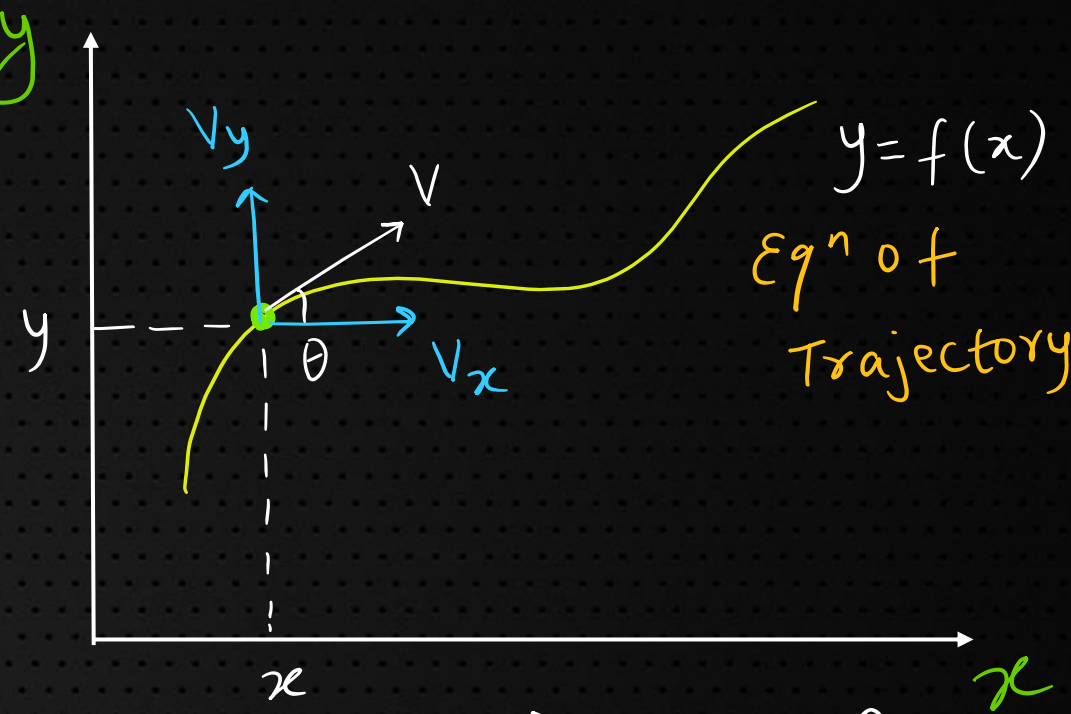
↳ $v_x = u_x + a_x t$

$v_y = u_y + a_y t$

$x = u_x t + \frac{1}{2} a_x t^2$

$y = u_y t + \frac{1}{2} a_y t^2$

$\vec{r} = x\hat{i} + y\hat{j}$



$\vec{v} = v_x\hat{i} + v_y\hat{j}$

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

... Continued

Ex 1. A particle starts from the origin at $t = 0$ sec with a velocity of $10\,m/s$ along y axis and moves in $X - Y$ plane with a constant acceleration of $(8\hat{i} + 2\hat{j})\ m/s^2$. At what time is the x coordinate of the particle $16\,m$? What is y coordinate of the particle at that time and what is velocity of the particle at that time?

... Continued

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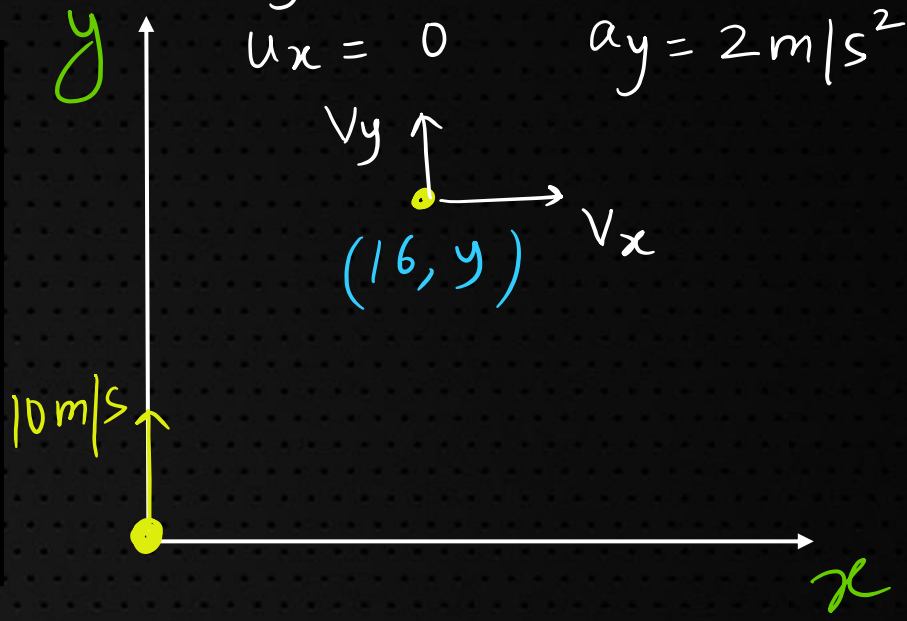
Soln:

$u_y = 10\,m/s$, $a_x = 8\,m/s^2$

$u_x = 0$, $a_y = 2\,m/s^2$



(i) $x = u_x t + \frac{1}{2} a_x t^2$

$\Rightarrow 16 = 0 + \frac{1}{2} \times 8 \times t^2 \Rightarrow$ **$t = 2\,s$**

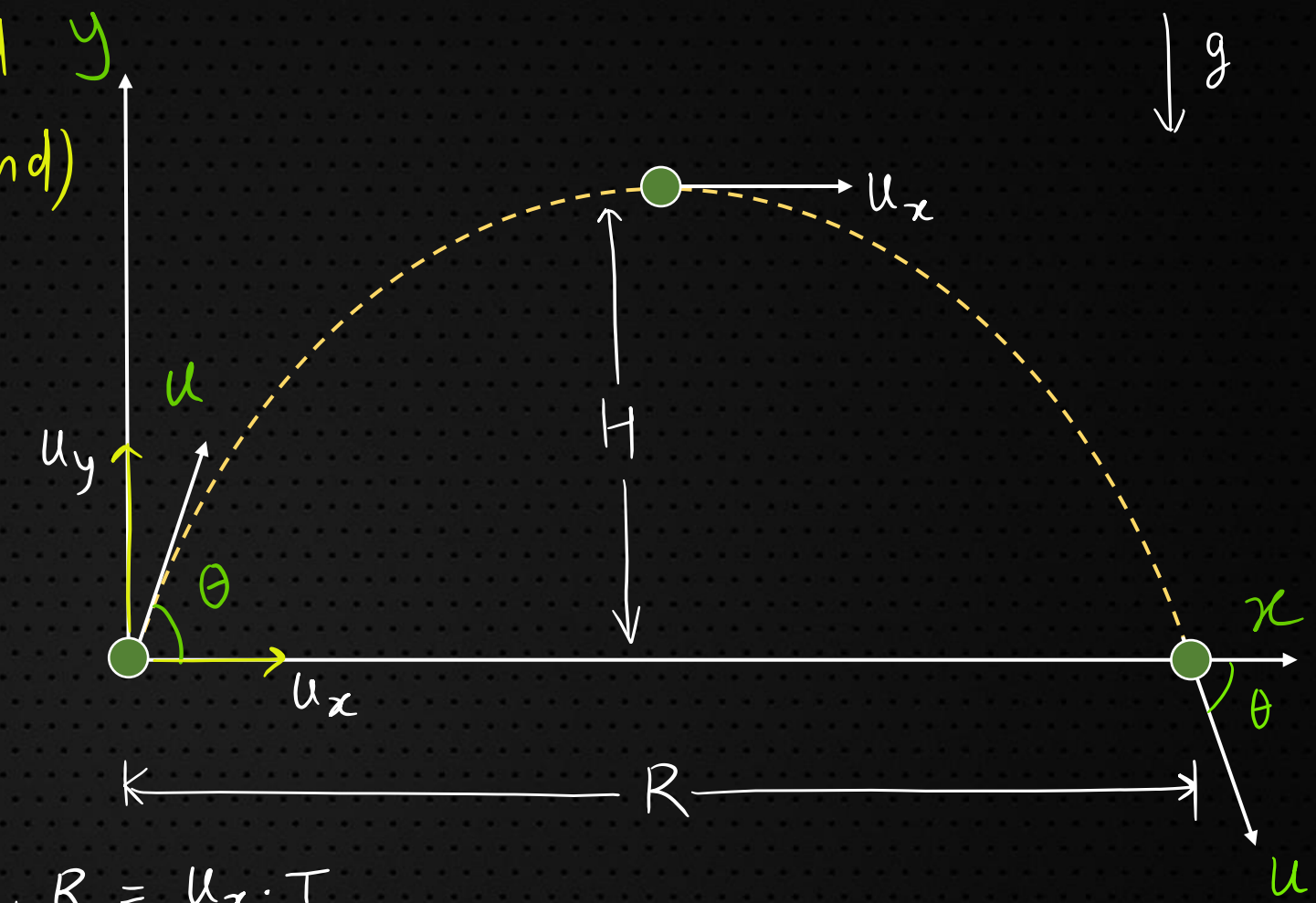
(ii) $y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow y = 10 \times 2 + \frac{1}{2} \times 2 \times 4 =$ **$24\,m$**

(iii) $V_y = 10 + 2 \times 2 = 14\,m/s$, $V_x = 0 + 8 \times 2 = 16\,m/s$

$\therefore V = \sqrt{V_y^2 + V_x^2} \approx$ **$21.3\,m/s$**

2. Projectile Motion (Ground to Ground)

2a. Standard eqn :



(i) Time of Flight

$$T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$$

(ii) $$H = \frac{u^2_y}{2g} = \frac{u^2\sin^2\theta}{2g}$$

(iii) Range, $R = u_x \cdot T$

$$= \frac{2u_x u_y}{g} = \frac{u^2\sin 2\theta}{g}$$

(iv) $\because a_x = 0 \Rightarrow u_x = u\cos\theta$ is const.

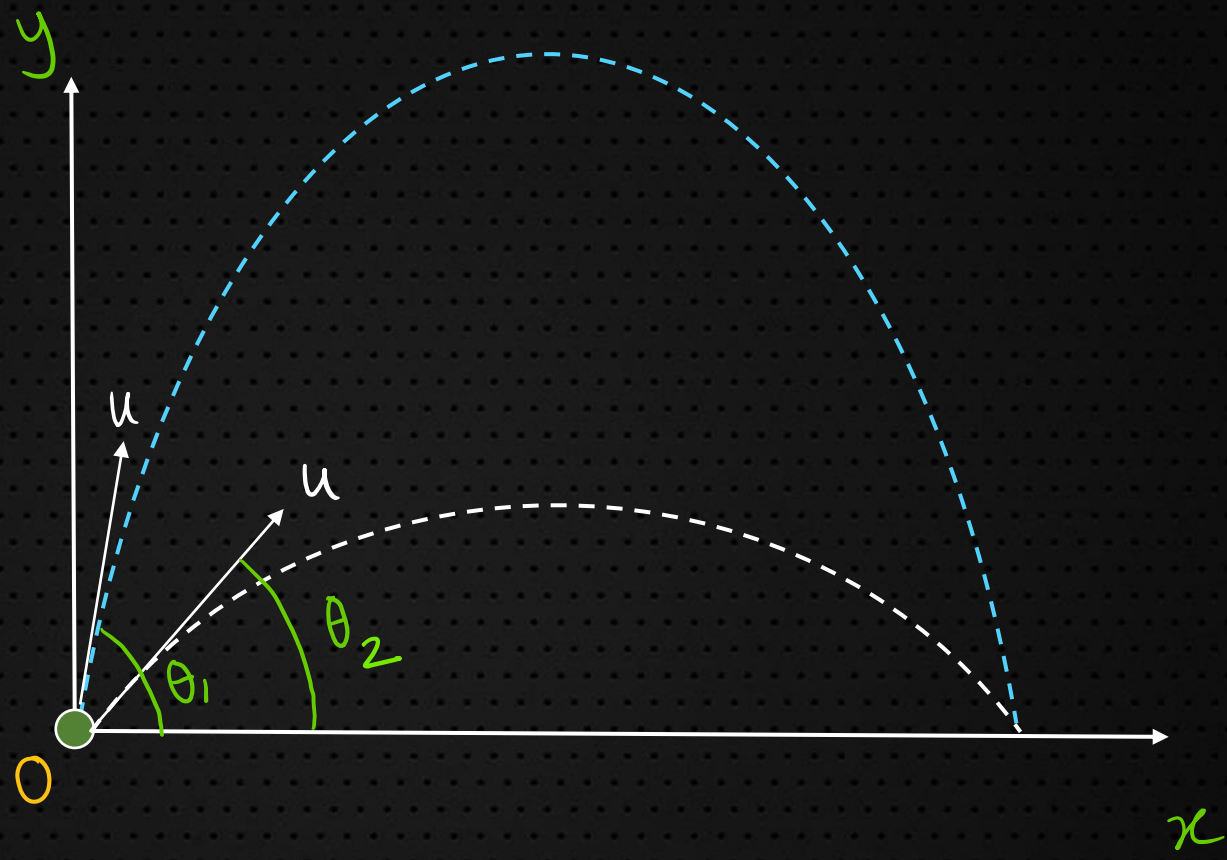
2b. Condition for R_{max} & Same Range :

(i) $R = \dfrac{u^2 \sin 2\theta}{g}$

$\therefore$ for $\theta = 45^\circ$, $R_{max} = \dfrac{u^2}{g}$

(ii) If $\theta_1 + \theta_2 = 90^\circ$

$\Rightarrow R_1 = R_2$



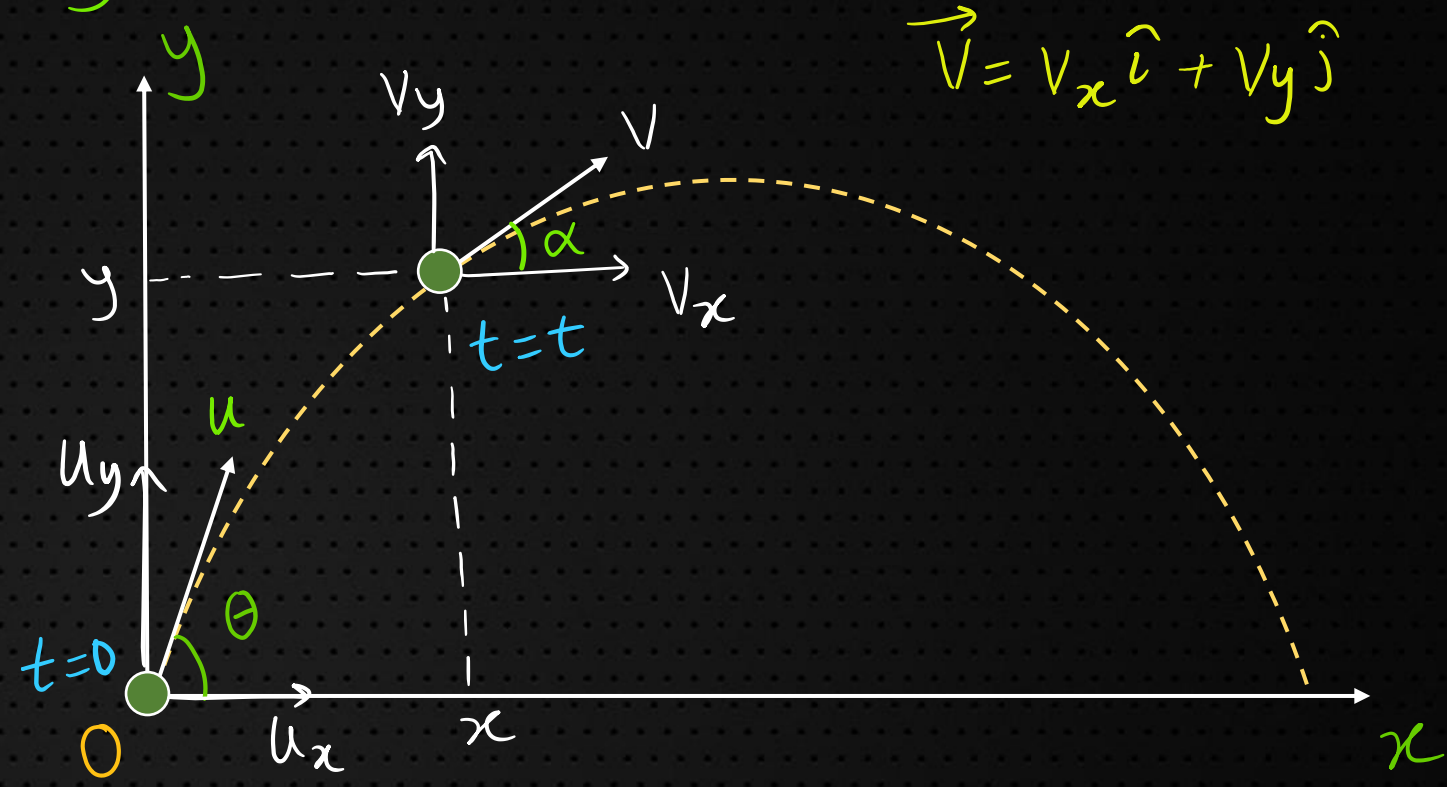
2C. Analysis of Projectile at any time t

(i) $V_x = u_x$, $V_y = u_y - gt$

$V = \sqrt{V_x^2 + V_y^2}$, $\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

(ii) $x = u_x t$, $y = u_y t - \frac{1}{2} g t^2$

$\vec{r} = x\hat{i} + y\hat{j}$, $r = \sqrt{x^2 + y^2}$



$\vec{V} = V_x\hat{i} + V_y\hat{j}$

$u_x = u\cos\theta$, $u_y = u\sin\theta$

$a_x = 0$, $a_y = -g$ } Given

2d. Eqⁿ of Trajectory

$u_x = u\cos\theta$, $u_y = u\sin\theta$

$a_x = 0$, $a_y = -g$

Using $x = u\cos\theta \cdot t$ and

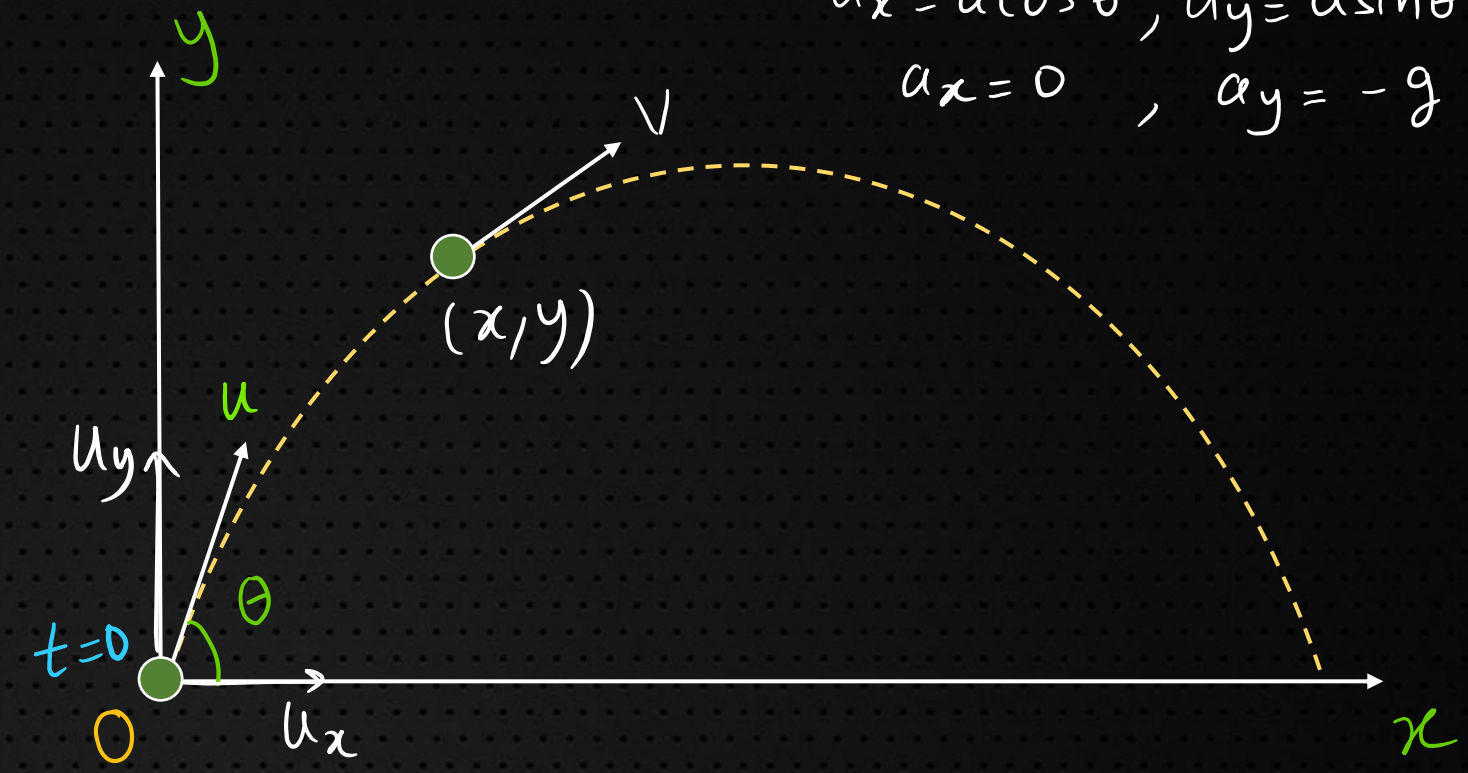
$y = u\sin\theta \cdot t - \frac{1}{2}gt^2$

(i) $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$(y = ax - bx^2) \rightarrow$ Quadratic Eqⁿ

NOTE

Range, $R = \frac{a}{b}$



(ii) $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$\rightarrow$ Range

Ex. 2

A ball is projected with speed v_0 at an angle θ from a point on the ground. If after $t = 3\sqrt{2}$ s the velocity becomes perpendicular to the initial direction of projection, find the angle of projection. Assume air resistance is negligible and take $v_0 = 30$ m/s and $g = 10$ m/s^2.

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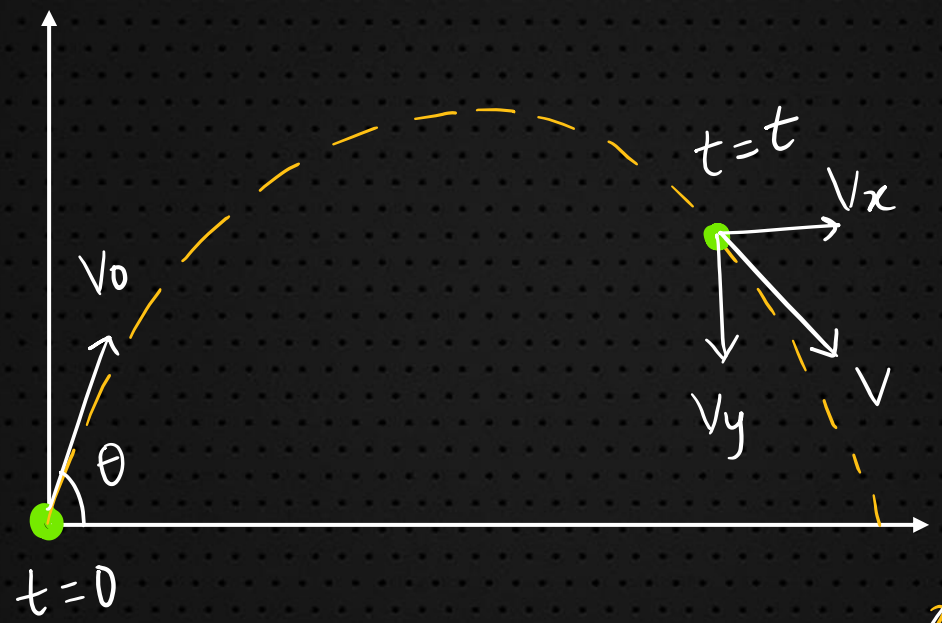
Soln:



$\vec{V_0} = V_0\cos\theta\,\hat{i} + V_0\sin\theta\,\hat{j}$

$\vec{V} = V_x\hat{i} + V_y\hat{j}$

$= V_0\cos\theta\,\hat{i} + (V_0\sin\theta - gt)\hat{j}$

$\Rightarrow \vec{V_0}\cdot\vec{V} = 0$

$\Rightarrow V_0^2\cos^2\theta + V_0^2\sin^2\theta - V_0\sin\theta\cdot gt = 0$

$\Rightarrow V_0^2 = V_0\sin\theta\cdot gt$

$\Rightarrow \sin\theta = \frac{30}{10\times 3\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\therefore \boxed{\theta = 45^\circ}$

Ex 3. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})\ ms^{-1}$, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\ ms^{-2}$, the equation of its trajectory is

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

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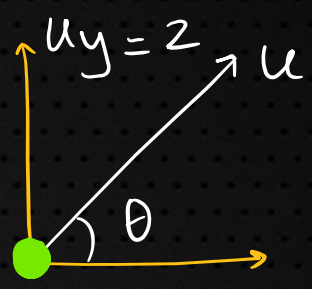
$\vec{u} = \hat{i} + 2\hat{j}$



$\tan\theta = \frac{u_y}{u_x} = 2$

and $u = \sqrt{1^2 + 2^2} = \sqrt{5}$

Soln:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\Rightarrow y = x \cdot 2 - \frac{10x^2}{2 \cdot 1^2}$$

$$\Rightarrow \boxed{y = 2x - 5x^2}$$

Ex 4.

A particle moves in the y-z plane with a constant acceleration a directed along the negative z- axis. The equation of motion of the particle has the form $z = py - qy^2$, where p and q are positive constants. What is the velocity (v) of the particle at the origin?

A $\sqrt{\frac{a(p+1)}{2q}}$

B $\sqrt{\frac{a(p^2-1)}{q}}$

C $\sqrt{\frac{a(p^2+1)}{2q}}$

D $\sqrt{\frac{a(p^2+1)}{q^2}}$

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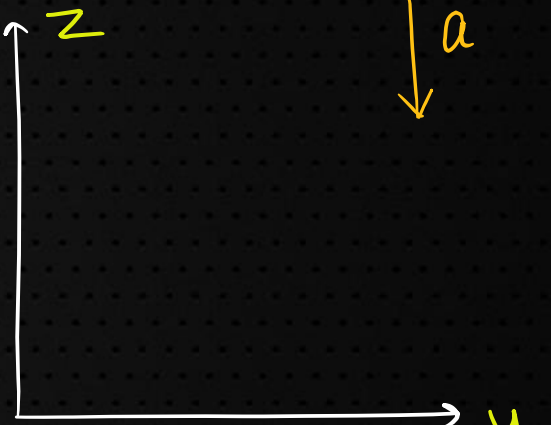
A $\sqrt{\frac{a(p+1)}{2q}}$ B $\sqrt{\frac{a(p^2-1)}{q}}$ C $\sqrt{\frac{a(p^2+1)}{2q}}$ D $\sqrt{\frac{a(p^2+1)}{q^2}}$

Soln: $z = y\tan\theta - \frac{ay^2}{2u^2\cos^2\theta}$ $\Rightarrow P = \tan\theta$, $Q = \frac{a}{2u^2\cos^2\theta}$

$\Rightarrow Q = \frac{a}{2u^2}(1+\tan^2\theta) = \frac{a}{2u^2}(1+P^2)$

$\therefore u = \sqrt{\frac{a(P^2+1)}{2Q}}$ Ans.



Ex 5. ***A ball is thrown from ground level so as to just clear a wall 4 m high at a distance of 4 m and falls at a distance of 14 m from the wall. Find the*** (projection θ) ***direction of the velocity.***

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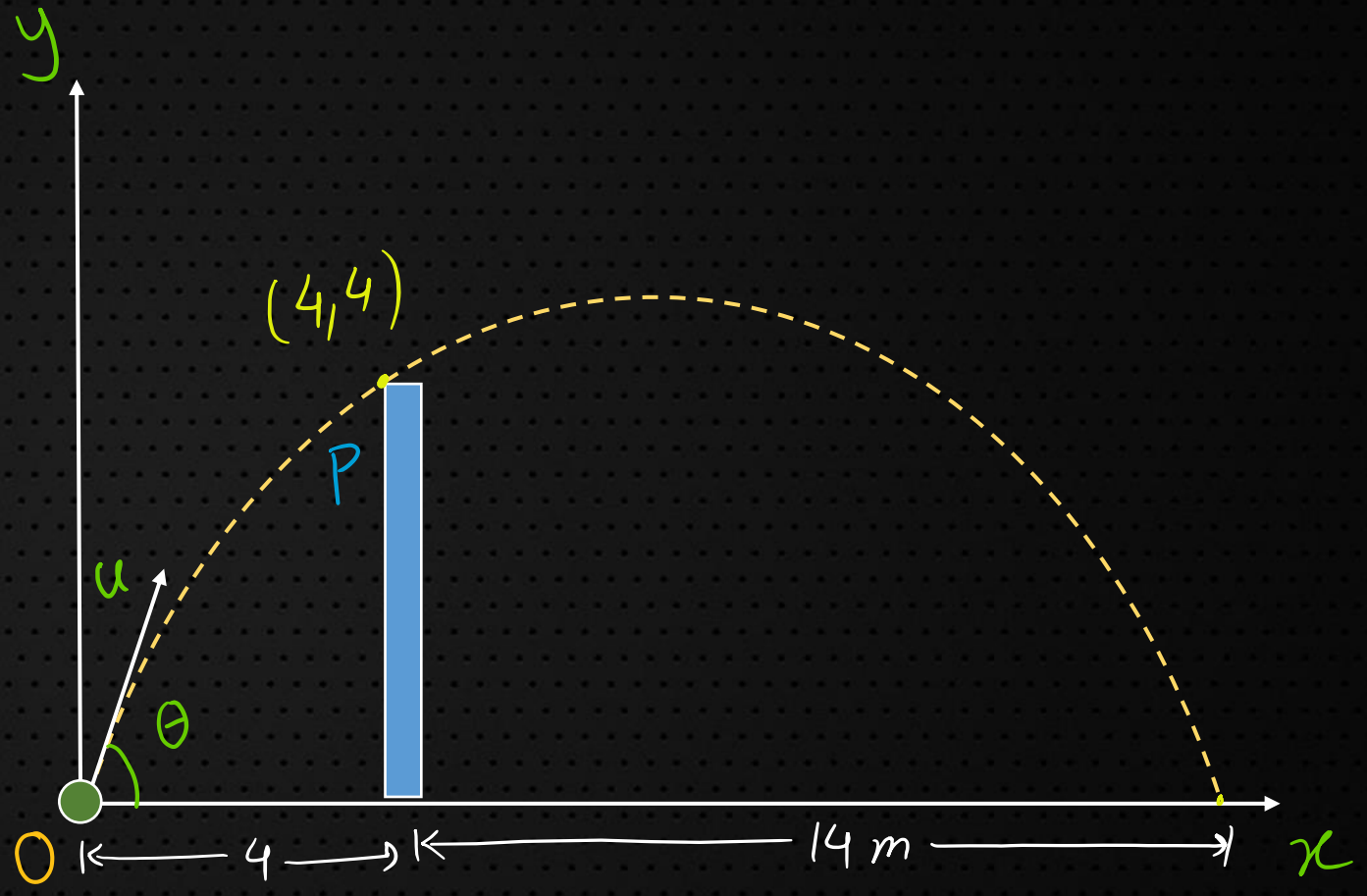
Soln:

(i) $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

$\Rightarrow 4 = 4\tan\theta\left(1 - \frac{4}{18}\right)$

$\therefore \tan\theta = \frac{9}{7}$ or $\boxed{\theta = \tan^{-1}\frac{9}{7}}$



$R = 4 + 14 = 18\,m$

3. Projectile Fired from a Tower

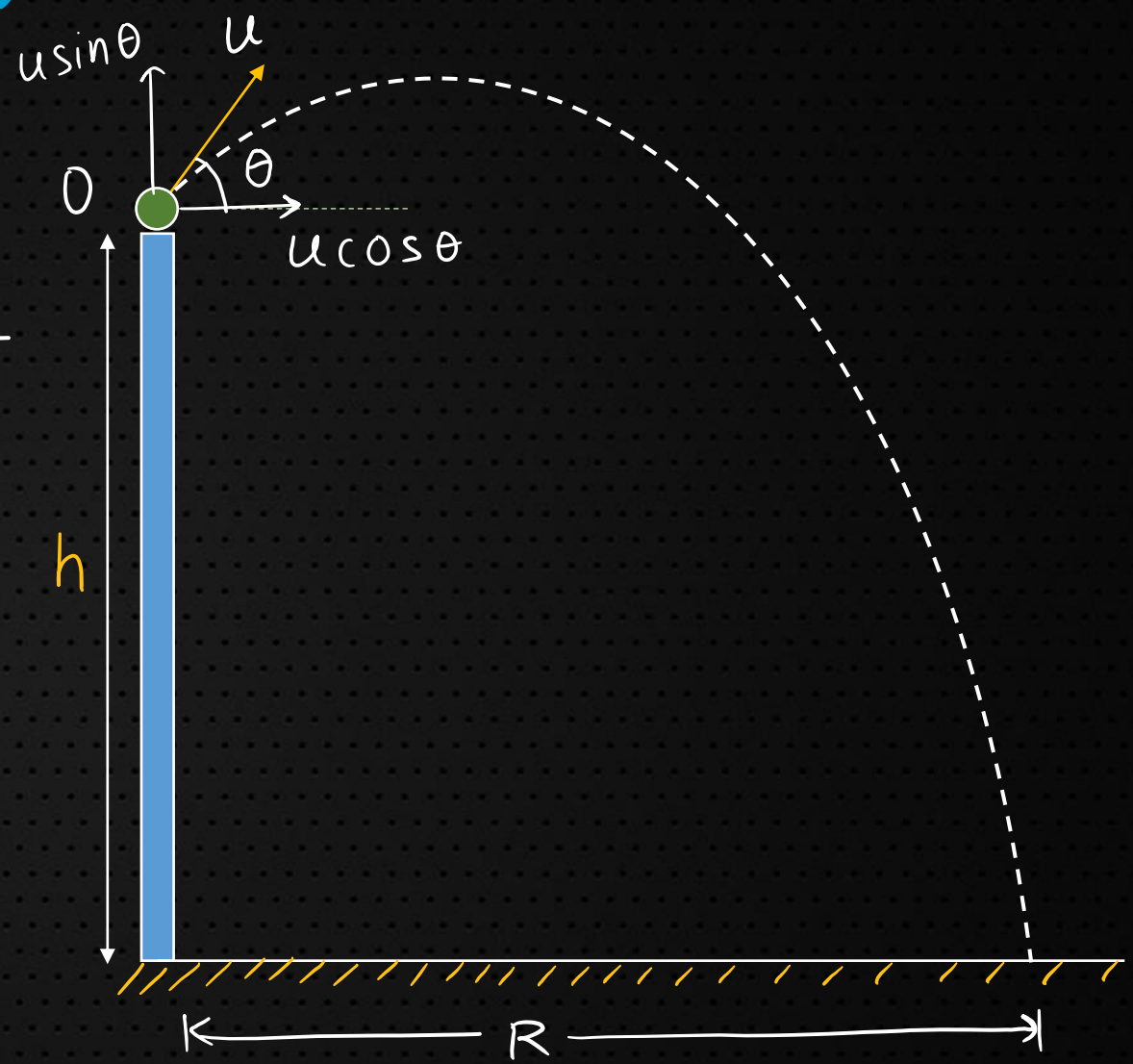
3a. At Some angle

(i) To find T (write eqn along y)

$$s_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow -h = u\sin\theta \cdot T - \frac{1}{2} g T^2$$

↳ Find T

(ii) $R = u\cos\theta \cdot T$



3b. Projected horizontally

(i) Time of Flight, $T = \sqrt{\frac{2h}{g}}$

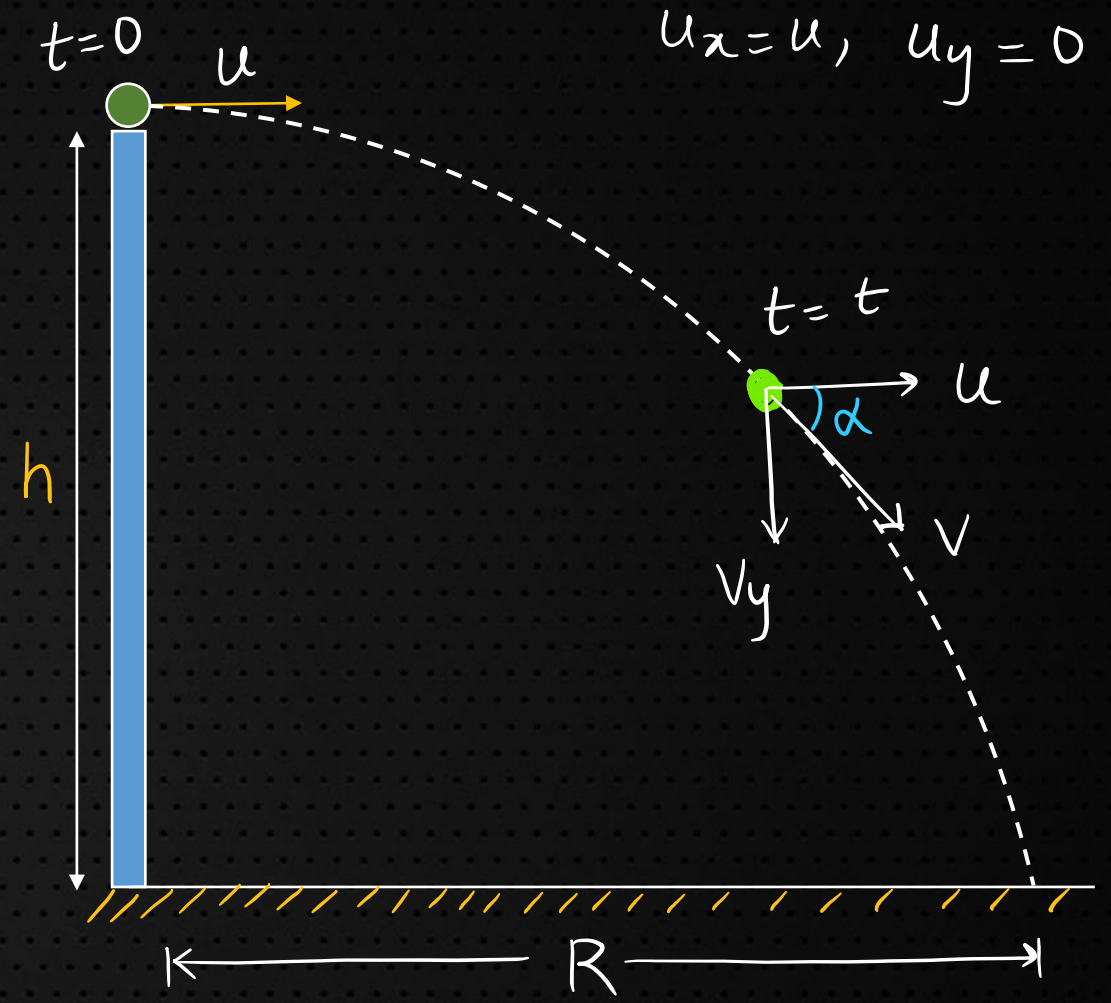
(ii) Range, $R = u \cdot T = u\sqrt{\frac{2h}{g}}$

(iii) $V_y = gt$

$\therefore \alpha = \tan^{-1}\left(\frac{V_y}{u}\right)$

and

$V = \sqrt{u^2 + V_y^2}$



Ex 6. A helicopter on flood relief mission, flying horizontally with a speed u at an altitude H, has to drop a food packet for a victim standing on the ground. At what distance from the victim should the packet be dropped ? The victim stands in the vertical plane of the helicopter's motion.

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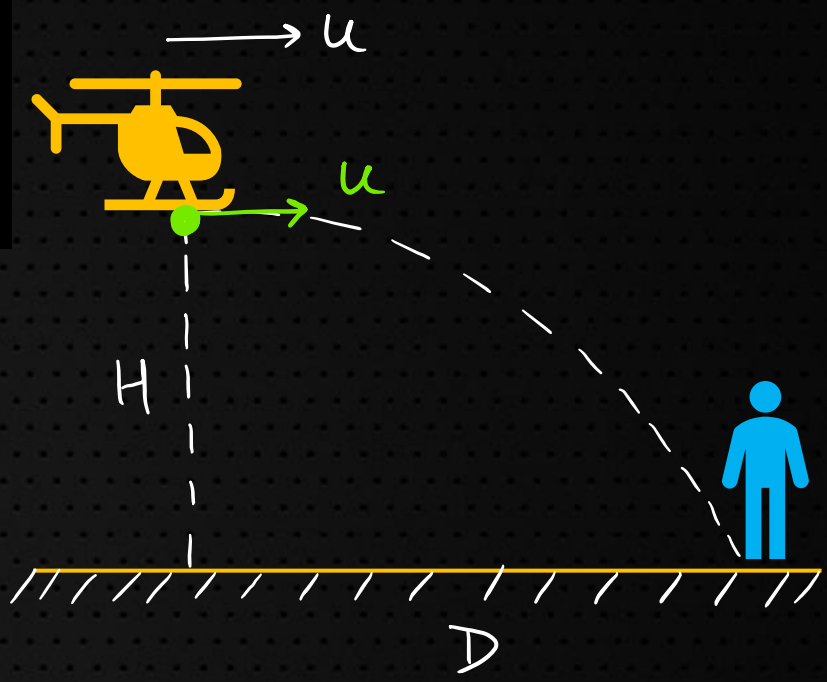

Soln:

$$t = \sqrt{\frac{2H}{g}} \Rightarrow D = u\sqrt{\frac{2H}{g}}$$

∴ Distance between Victim & Packet at time of drop $= \sqrt{H^2 + D^2}$

$$= \sqrt{H^2 + \frac{2u^2H}{g}}$$ Ans

Ex 7.

A ball rolls off the edge of a horizontal table top 4 m high. If it strikes the floor at a point 5 m horizontally away from the edge of the table, what was its speed at the instant it left the table?

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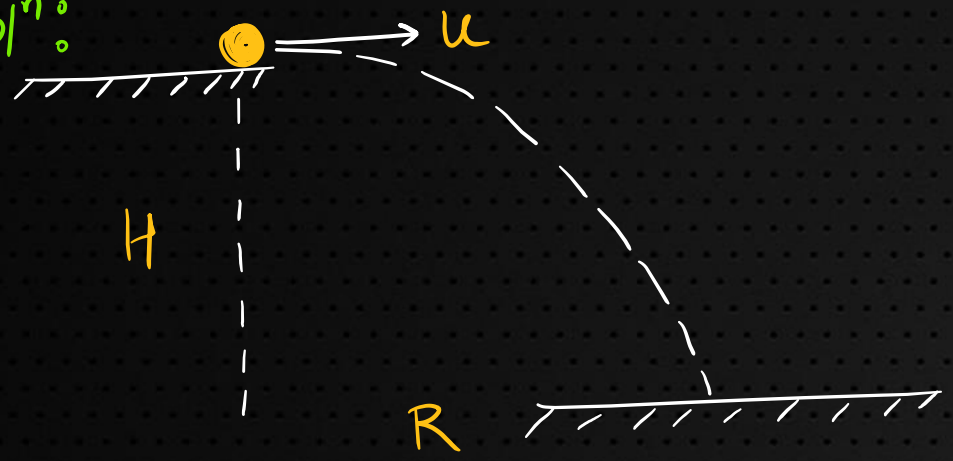


$$R = u \cdot T = u\sqrt{\frac{2H}{g}}$$

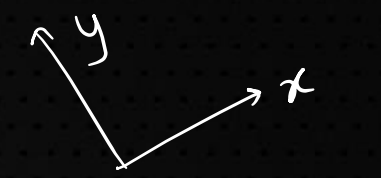
$$\Rightarrow u = R\sqrt{\frac{g}{2H}} = 5\sqrt{\frac{9.8 \times 4.9}{2 \times 4}}$$

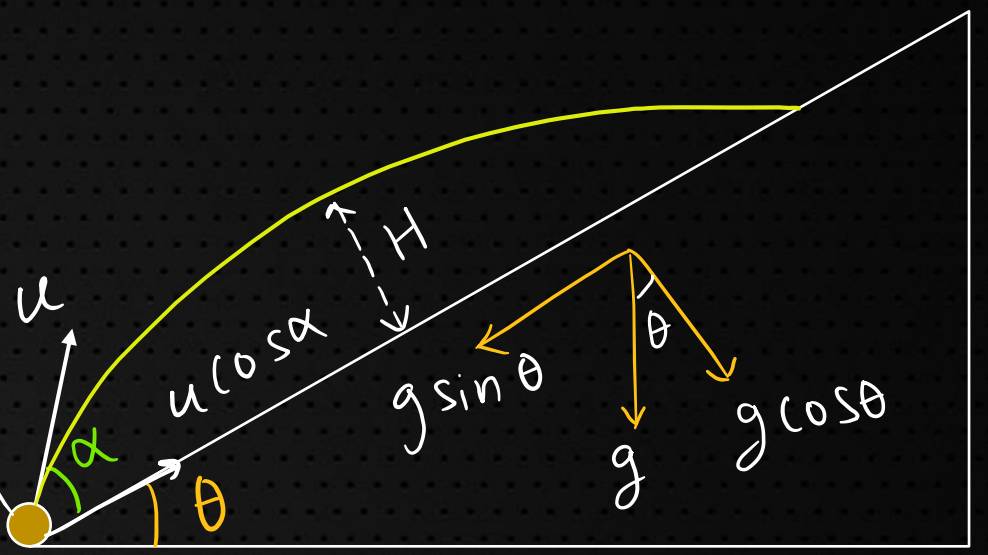
$$= \frac{5}{2} \times 7 \times \frac{1}{\sqrt{10}}$$

$$= \frac{17.5}{\sqrt{10}} \text{ m/s}$$ Ans

4. Projectile Motion on Inclined Plane

4a. Up the Incline





(i) $T = \frac{2u_y}{a_y} = \frac{2u\sin\alpha}{g\cos\theta}$

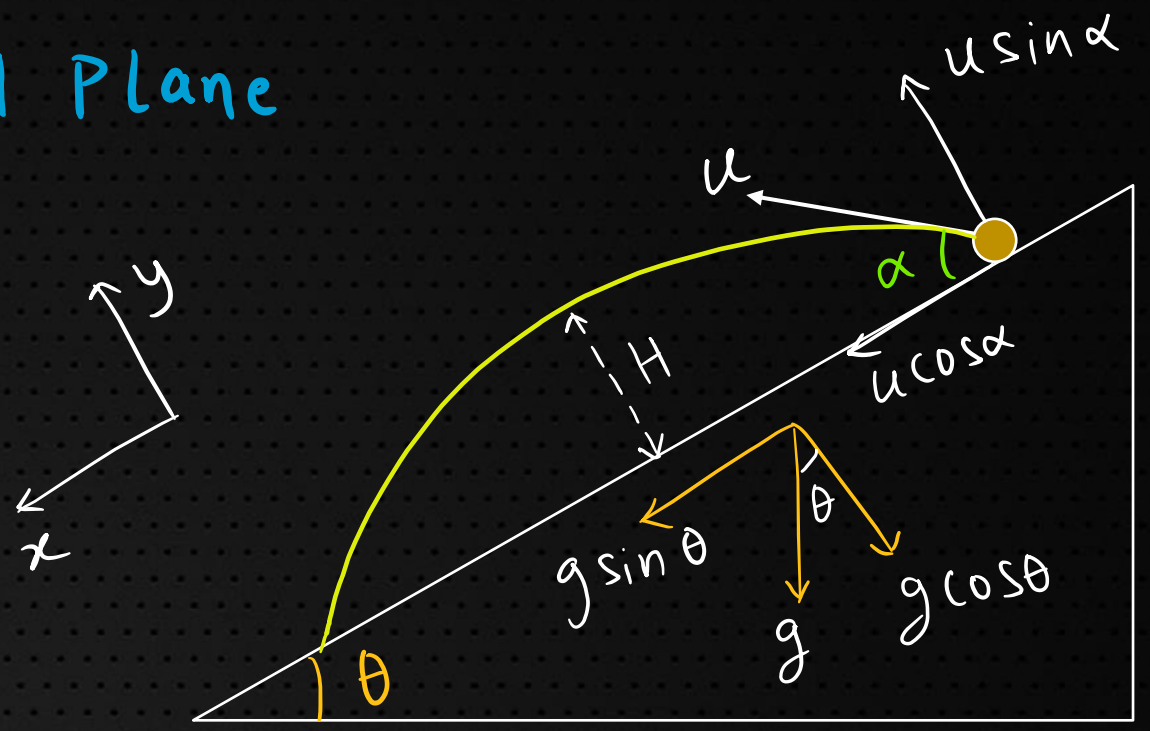
(ii) $H = \frac{u^2_y}{2a_y} = \frac{u^2\sin^2\alpha}{2g\cos\theta}$

(iii) For Range, $x = u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow R = u\cos\alpha \cdot T - \frac{1}{2}g\sin\theta \cdot T^2$

4. Projectile Motion on Inclined Plane

4b. Down the Incline



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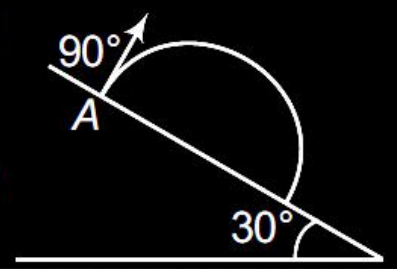
(ii) $H = \frac{u^2_y}{2a_y} = \frac{u^2\sin^2\alpha}{2g\cos\theta}$

(iii) For Range, $x = u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow R = u\cos\alpha \cdot T + \frac{1}{2}g\sin\theta \cdot T^2$

Ex 8 : A ball is projected from point A with velocity $10\ ms^{-1}$ perpendicular to the inclined plane as shown in figure. Range of the ball on the inclined plane is

(a) $\frac{40}{3}$ m (b) $\frac{20}{3}$ m (c) $\frac{12}{3}$ m (d) $\frac{60}{3}$ m



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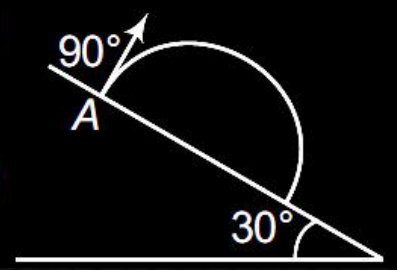
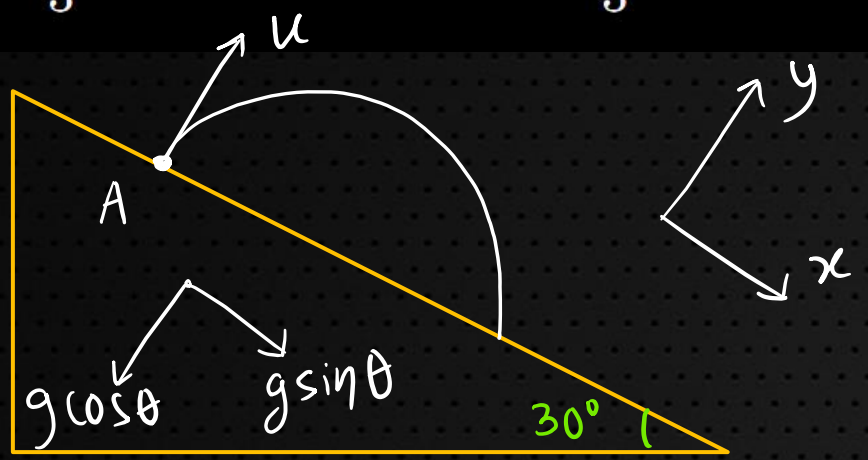


Soln:



$$T = \frac{2u_y}{a_y} = \frac{2u}{g\cos\theta} = \frac{2\times10\times2}{10\times\sqrt{3}} = \frac{4}{\sqrt{3}}\ s$$

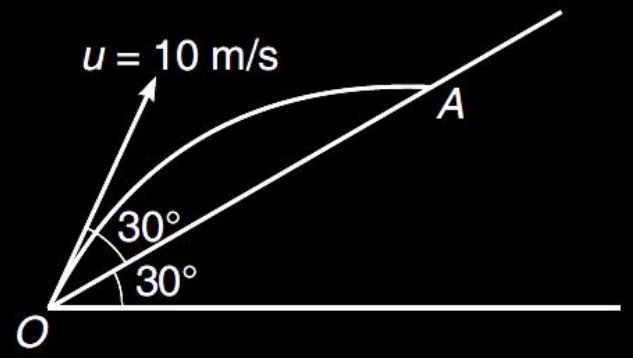
$$\therefore R = u_x T + \frac{1}{2}a_x T^2$$

$$= 0 + \frac{1}{2}\times g\sin\theta \times T^2$$

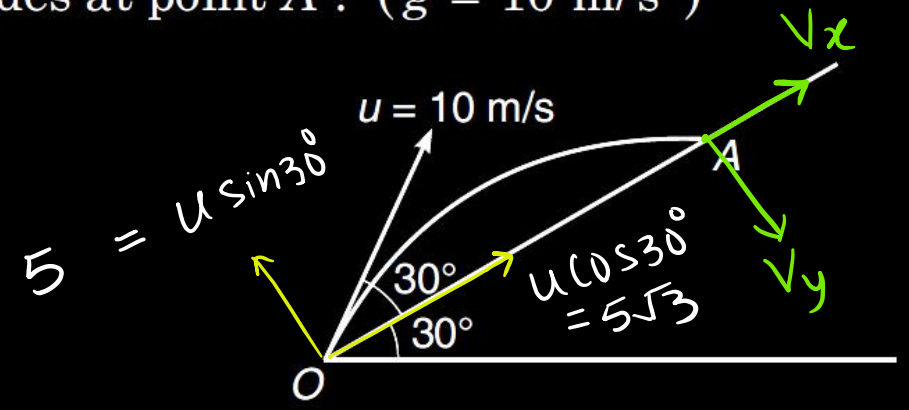
$$= \frac{1}{2}\times10\times\frac{1}{2}\times\frac{16}{3}$$

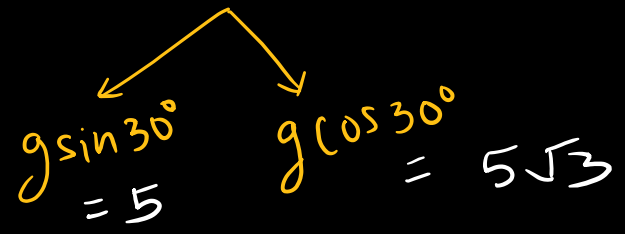
$$= \frac{40}{3}\ m \quad \text{Ans}$$

Ex 9. A particle is projected along an inclined plane as shown in figure. What is the speed of the particle when it collides at point A ? ($g = 10\ \text{m/s}^2$)



Ex 9. A particle is projected along an inclined plane as shown in figure. What is the speed of the particle when it collides at point A ? ($g = 10\ m/s^2$)





Sol^{n}: $T = \dfrac{2u_y}{a_y} = \dfrac{2u\sin 30°}{g\cos 30°} = \dfrac{2\times 5}{5\sqrt{3}} = 2/\sqrt{3}$

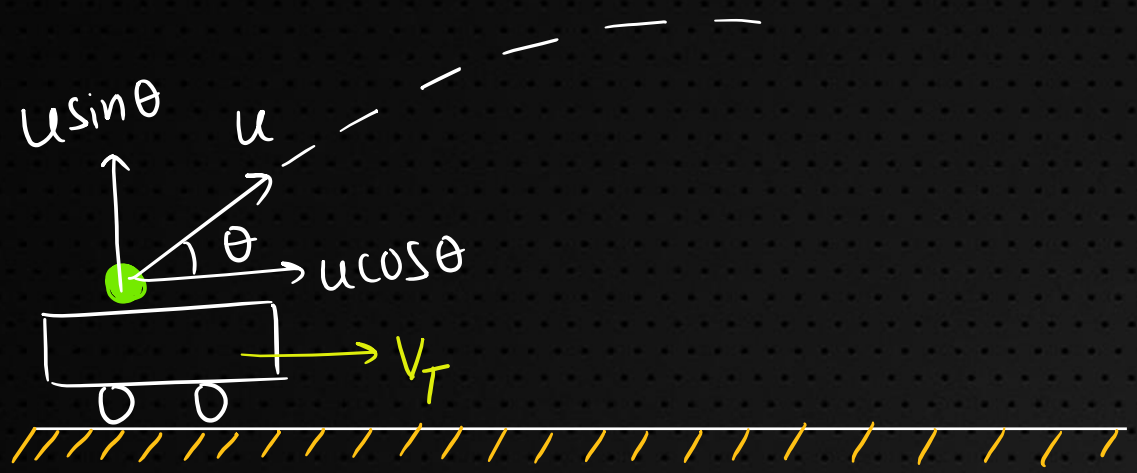
$V_x = u_x + a_x t = 5\sqrt{3} - 5\times\dfrac{2}{\sqrt{3}} = \dfrac{15-10}{\sqrt{3}} = 5/\sqrt{3}$ m/s

$V_y = u_y + a_y t = 5 - 5\sqrt{3}\times\dfrac{2}{\sqrt{3}} = -5$ m/s

$\therefore V = \sqrt{V_x^2 + V_y^2} = \sqrt{\dfrac{25}{3} + 25} = \dfrac{10}{\sqrt{3}}$ m/s

Ans

5. Projectile from moving Trolley



Concept: W.r.t ground

$u_x = u\cos\theta + V_T$

$u_y = u\sin\theta$

(i) $T = \frac{2u\sin\theta}{g}$

(ii) $H = \frac{u^2\sin^2\theta}{2g}$

(iii) $R = u_x \cdot T = (u\cos\theta + V_T) \cdot \frac{2u\sin\theta}{g}$

↳ To find R_{max}, $\frac{dR}{d\theta} = 0$

Ex 10. A boy of height 1.5 m, making move on a skateboard due east with velocity 4 m s^{-1} , throws a coin vertically up with a velocity of 3 m s^{-1} relative to himself.

(a) Find the total displacement of the coin relative to ground till it comes to the hand of the boy.

(b) What is the maximum height attained by the coin w.r.t to ground? (Take $g = 10 m/s^2$)

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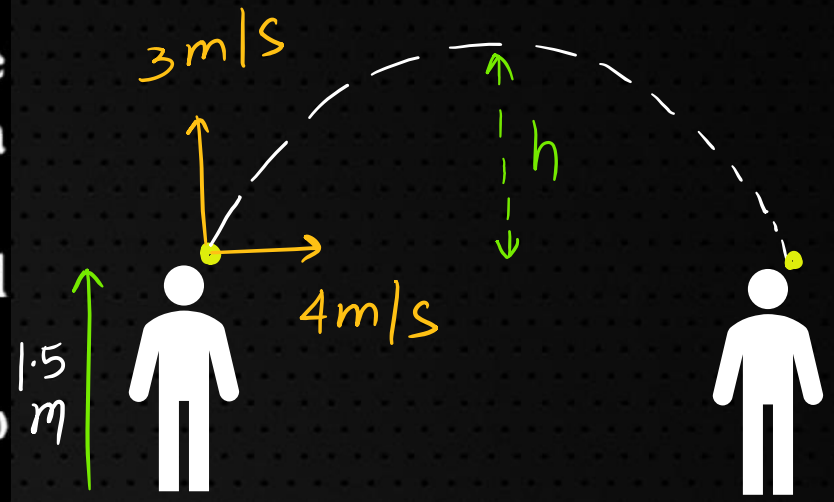

Soln: (a) $R = \frac{2u_x u_y}{g} = \frac{2 \times 4 \times 3}{10} = 2.4\,m$ Ans

(b) H (w.r.t ground) $= 1.5 + \frac{u_y^2}{2g} = 1.5 + \frac{9}{20}$ ← h

$= 1.5 + 0.45$

$= 1.95\,m$ Ans

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KINEMATICS PYQs LINKS (JEE MAIN)

2021 Feb	2021 March	2021 July	2021 August	2020
https://youtu.be/A47na5ifAMc	https://youtu.be/ek7EgrGh3fg	https://youtu.be/mQSeejQAVwU	https://youtu.be/ao2TFneTnVM	https://youtu.be/OLKrZuYe150

CLICK
(Practice these Questions)

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Revision Series Playlist Link https://bit.ly/3eBbib9

JEE Main PYQs Link https://bit.ly/2S54jzh *Chapter wise 2021, 2020, 2018*

GoldMine Link https://bit.ly/2VhOGFF