JEE MAIN | IIT JEE

ERROR ANALYSIS

CONCEPT + PYQs

REVISION in 20 Min

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TOPICS TO BE DISCUSSED

1. Absolute Error, Relative Error & Percentage Error
2. Combination of Errors
3. Questions on Errors

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Chapter	Formulae_Concept VIDEO LINK
Unit & Dimensions	https://youtu.be/wdd-wlZF4Hk
Error Analysis	https://youtu.be/AEMsHUUAo5s
Vernier Calliper	https://youtu.be/pVoN045dV8I
Screw Gauge	https://youtu.be/gYd2PtmZ0mw
Kinematics_Motion in 1d	https://youtu.be/U4NNxFaFliE
Kinematics_Motion in 2d	https://youtu.be/4_Zo5WhMf7w
Laws of Motion	https://youtu.be/7JlR8gNRQIs
Friction	https://youtu.be/Rn1bLst7eGk
Work Energy Power	https://youtu.be/kjrXoE-kDI8
Circular Motion	https://youtu.be/KnFymKHlkT0
Centre of Mass	https://youtu.be/ads35RKD618
Cons of Momentum & Collision	https://youtu.be/3f0u4L-lyyw
Rotational Motion – Part 1	https://youtu.be/O6j1mLp06XI
Rotational Motion – Part 2	https://youtu.be/OHni1DRdfAQ
Rotational Motion_Part 3	https://youtu.be/qugIqfYRCrk
Gravitation	https://youtu.be/rAj2huLVaEk
Properties of Solids	https://youtu.be/gSXxjk89l_c
Fluids Statics (Part 1)	https://youtu.be/RFKx9B9yo3M
Fluid Dynamics (Part 2)	https://youtu.be/Y717vQpUEJQ
Fluid Properties (Part 3)	https://youtu.be/V8xUWWK2oT0
Simple Harmonic Motion	https://youtu.be/RIb7ofNG09I
Thermal Properties	https://youtu.be/OYjjyPIzddE
Heat Transfer	https://youtu.be/PyNboHgtYzM
KTG	https://youtu.be/XO1tvFhIa0I
Thermodynamics	https://youtu.be/iz_kf1jRDRw
String Waves (Part-1)	https://youtu.be/mZWNmH19wDQ
String Waves (Part-2)	https://youtu.be/PDGq4d3xA6c
Wave Motion -Organ Pipes and Resonance Tube	https://youtu.be/fB7pfJ77za8
Wave Motion - Doppler's Effect	https://youtu.be/9-BxOaamnwg
Electrostatics	https://youtu.be/3stXbGRMcrk
Capacitors	https://youtu.be/EXEiickNUkY
Current Electricity	https://youtu.be/gm8FUfjrX18
Moving Charges and Magnetic Effect of Current	https://youtu.be/ULD2Ok1CGJk
Earth's Magnetism	https://youtu.be/a4CT5uVwAK4
Magnetic Properties	https://youtu.be/63cwdYXNlYE
EMI	https://youtu.be/puVavm_GFRM
Alternating Current	https://youtu.be/74dTY-pzM_o
Ray Optics	https://youtu.be/BhnyTWzIlBA
Wave Optics Part 1_Interference	https://youtu.be/LG5nlE8XTeI
Wave Optics Part 2_Diffraction_Polarization	https://youtu.be/ymMyyJGGqnY
Optical Instruments	https://youtu.be/OQssbDH0A4I
Electromagnetic Waves	https://youtu.be/bcVXgEkyQZY
Semiconductors_Basics + Zener Diode	https://youtu.be/_A2JomQ7-50
Semiconductors_Transistors	https://youtu.be/psDwl84Nzb0
Semiconductors_Logic Gates	https://youtu.be/pZdQAzLbFTo
Communication Systems	https://youtu.be/8NgMqK9X79Y
Modern Physics_Part 1_Atomic Physics	https://youtu.be/9VKUnE3mpHk
Modern Physics_Part 2_Photoelectric Effect	https://youtu.be/24oTQp84jrk
Modern Physics_Part 3_Dual Nature of Light	https://youtu.be/0zoR_saMAQY
Modern Physics_Part 4_Radioactivity	https://youtu.be/AdX3YBhQyog
Modern Physics_Part 5_Nuclear Physics	https://youtu.be/VDWqVahGixc
Modern Physics_Part 6_X Rays	https://youtu.be/dSHXdzX7NX0

1. Absolute Error, Relative Error & Percentage Error

(Error = True Value − Measured Value)

STEP 1 : $R_{avg} = \dfrac{R_1 + R_2 + \dots + R_n}{n}$ (we take R_{avg} as True Value)

STEP 2 : Absolute Error $\Delta R_1 = R_1 - R_{avg}$

$\Delta R_2 = R_2 - R_{avg}$

. . . . . .

. . . . . .

$\Delta R_n = R_n - R_{avg}$

(take only magnitude)

STEP 3 : Mean Absolute Error

$$\Delta R_{avg} = \frac{|\Delta R_1| + |\Delta R_2| + \dots + |\Delta R_n|}{n}$$

Final Reading = $R_{avg} \pm \Delta R_{avg}$

Rel Error $\dfrac{\Delta R_{avg}}{R_{avg}}$

% Error $\dfrac{\Delta R_{avg}}{R_{avg}} \times 100$

... Continued

Example 2.7 We measure the period of oscillation of a simple pendulum. In successive measurements, the readings turn out to be 2.63 s, 2.56 s, 2.42 s, 2.71s and 2.80 s. Calculate the absolute errors, relative error or percentage error.

Answer The mean period of oscillation of the pendulum

$$T = \frac{(2.63 + 2.56 + 2.42 + 2.71 + 2.80)\text{s}}{5}$$

$$= \frac{13.12}{5}\ \text{s}$$

$$= 2.624\ \text{s}$$

$$= 2.62\ \text{s}$$

As the periods are measured to a resolution of 0.01 s, all times are to the second decimal; it is proper to put this mean period also to the second decimal.

The errors in the measurements are

2.63 s – 2.62 s = 0.01 s
2.56 s – 2.62 s = – 0.06 s
2.42 s – 2.62 s = – 0.20 s
2.71 s – 2.62 s = 0.09 s
2.80 s – 2.62 s = 0.18 s

Note that the errors have the same units as the quantity to be measured.

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is

$$\Delta T_{mean} = [(0.01 + 0.06 + 0.20 + 0.09 + 0.18)\text{s}]/5$$
$$= 0.54\ \text{s}/5$$
$$= 0.11\ \text{s}$$

That means, the period of oscillation of the simple pendulum is (2.62 ± 0.11) s i.e. it lies between (2.62 + 0.11) s and (2.62 – 0.11) s or between 2.73 s and 2.51 s.

2. Combination of Errors

↳ error in measurement is very small

(a) Sum or difference

$l = 4.1 \pm 0.1$ cm, $b = 3.3 \pm 0.1$ cm

$S = l + b = 7.4 \pm 0.2$ cm

$S = l - b = 0.8 \pm 0.2$ cm

(b) Product or Division

$P = d\dfrac{x^a y^b}{z^c}$

{ Error is Δx, Δy & Δz. Find Error in P

$\Rightarrow \ln P = a\ln x + b\ln y - c\ln z + \ln d$

$\Rightarrow \dfrac{dP}{P} = a\dfrac{dx}{x} + b\dfrac{dy}{y} - c\dfrac{dz}{z} + 0$

To find Max error in P

$$\frac{\Delta P}{P} = a\frac{\Delta x}{x} + b\frac{\Delta y}{y} + c\frac{\Delta z}{z}$$

($\frac{\Delta x}{x}$ is rel error in x

3. Questions on Errors

Ex 1. The density of a solid metal sphere is determined by measuring its mass and its diameter. The maximum error in the density of the sphere is $(x/100)\%$. If the relative errors in measuring the mass and the diameter are 6.0% and 1.5% respectively, the value of x is

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Soln: $\frac{\Delta m}{m} \times 100 = 6\%$

$\frac{\Delta d}{d} \times 100 = 1.5\%$

$\rho = \frac{m}{V} \Rightarrow \rho = \frac{m}{\frac{4}{3}\pi\frac{d^3}{8}} \Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta d}{d}$

$\frac{\Delta\rho}{\rho} \times 100 = 6 + 3 \times 1.5 = 10.5\%$

$\therefore 10.5 = \frac{x}{100} \Rightarrow \boxed{x = 1050}$

Ans

... Continued

Ex2. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1% , the maximum error in determining the density is **(2018 Main)**

(a) 6% (b) 2.5% (c) 3.5% (d) 4.5%

... Continued

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(a) 6% (b) 2.5% (c) 3.5% ✓(d) 4.5%

Soln:

$\frac{\Delta m}{m} \times 100 = 1.5\%$

$\frac{\Delta L}{L} \times 100 = 1\%$

$$\rho = \frac{m}{L^3} \Rightarrow \frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta L}{L}$$

$$\Rightarrow \frac{\Delta \rho}{\rho} \times 100 = 1.5 + 3 \times 1$$

$$= \boxed{4.5\%}$$

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Ex 3. In a simple pendulum, experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained 55.0 cm. The percentage error in the determination of g is close to

(2019 Main, 8 April II)

(a) 0.7% (b) 6.8% (c) 3.5% (d) 0.2%

... Continued

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Soln: Least Count is taken as Error.

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

$(\Delta L = 1mm = 0.1\,cm,\ L = 55\,cm,\ \Delta T = 1s,\ T = 30s)$

$$\Rightarrow \frac{\Delta g}{g} = \frac{0.1}{55} + 2\times\frac{1}{30} \Rightarrow \%\ Error = \frac{\Delta g}{g}\times 100$$

$$= \left(\frac{10}{55} + \frac{20}{3}\right)\%$$

$$= \boxed{6.8\%}$$

... Continued

Ex 4. A physical quantity z depends on four observables a, b, c and d, as $z = \dfrac{a^2 b^{2/3}}{\sqrt{c}d^3}$. The percentage of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is

(a) 12.25 %
(b) 14.5 %
(c) 16.5 %
(d) 13.5 %

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... Continued

Ex 4. A physical quantity z depends on four observables a, b, c and d, as $z = \dfrac{a^2 b^{2/3}}{\sqrt{c}\, d^3}$. The percentage of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is

(a) 12.25 %
(b) 14.5 %
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(d) 13.5 %

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Soln:

$$z = \frac{a^2 b^{2/3}}{\sqrt{c}\, d^3}$$

$$\Rightarrow \frac{\Delta z}{z} = 2\frac{\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + 3\frac{\Delta d}{d}$$

$$\therefore \frac{\Delta z}{z} \times 100 = (2\times 2) + \left(\frac{2}{3}\times 1.5\right) + \left(\frac{1}{2}\times 4\right) + (3\times 2.5)$$

$$= 4 + 1 + 2 + 7.5$$

$$= \boxed{14.5\ \%}$$

... Continued

Ex 5. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)\,\text{mA}$, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? **(2014 Main)**

(a) 0.2 mA (b) 0.02 mA (c) 0.5 mA (d) 0.05 mA

... Continued

Ex 5. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? **(2014 Main)**

(a) 0.2 mA (b) 0.02 mA (c) 0.5 mA (d) 0.05 mA

Soln: We want dI

Given: $dV = 0.01$, $I = 5$ mA

$T = 300$ K

$I = e^{1000V/T} - 1$

$$\Rightarrow \frac{dI}{dV} = \frac{1000}{T} e^{1000V/T} \Rightarrow \frac{dI}{dV} = \frac{1000}{T}(I+1)$$

$$\Rightarrow dI = \frac{1000}{T}(I+1)dV$$

$$\therefore dI = \frac{1000}{300}(5+1) \times 0.01 = \boxed{0.2\ mA}$$

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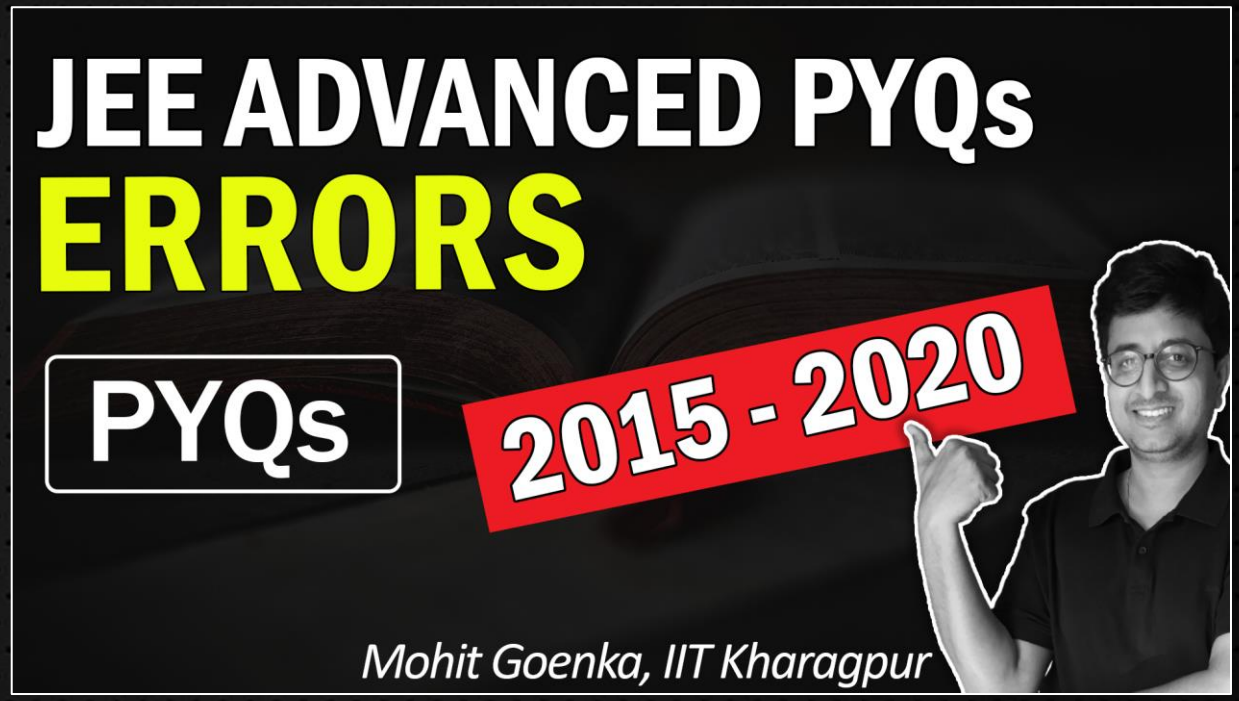
PYQs LINKS (JEE MAIN)

2021 Feb	2021 March	2021 July	2021 August
https://youtu.be/irjQwBIrEoM	https://youtu.be/7za14uUHkps	https://youtu.be/OFQ8bUdFafc	https://youtu.be/0SHRzwUopXo



https://youtu.be/ri69BwOlh3A

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Revision Series Playlist Link https://bit.ly/3eBbib9

JEE Main PYQs Link https://bit.ly/2S54jzh *Chapter wise 2021, 2020, 2018*

GoldMine Link https://bit.ly/2VhOGFF