JEE MAIN | IIT JEE

Center of Mass

Concept Revision & Examples

REVISION in 25 Min

Mohit Sir, IIT Kharagpur

EDUNITI

Topics to covered

1. Understanding of COM
2. COM of Multiparticle System
3. COM of Two Particle System
4. COM of Continuous Mass Distribution
5. COM of Standard Bodies (*uniform mass density*)
6. COM of Bodies (*Non-uniform mass density*)
7. COM of Combined Structure
8. COM in Cavity Problems (*2 Methods*)
9. Displacement of COM
10. Velocity & Acceleration of COM

JOIN

Eduniti for Physics

EDUNITI

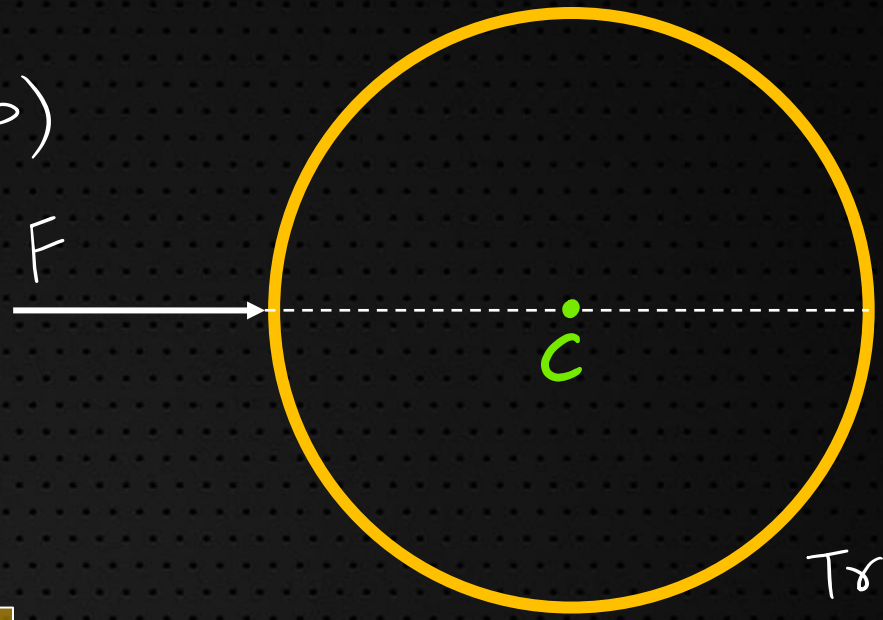
Chapter	Formulae_Concept VIDEO LINK
Unit & Dimensions	https://youtu.be/wdd-wlZF4Hk
Errors and Vectors	
Vernier Calliper	https://youtu.be/pVoN045dV8I
Screw Gauge	https://youtu.be/gYd2PtmZ0mw
Kinematics_Motion in 1d	https://youtu.be/U4NNxFaFliE
Kinematics_Motion in 2d	https://youtu.be/4_Zo5WhMf7w
Laws of Motion	https://youtu.be/7JlR8gNRQIs
Friction	https://youtu.be/Rn1bLst7eGk
Work Energy Power	https://youtu.be/kjrXoE-kDI8
Circular Motion	https://youtu.be/KnFymKHlkT0
Centre of Mass	
Collision	
Rotational Motion_Moment of Inertia	https://youtu.be/9ckZdOhy3z0
Gravitation	https://youtu.be/rAj2huLVaEk
Properties of Solids	https://youtu.be/gSXxjk89l_c
Fluids Statics (Part 1)	https://youtu.be/RFKx9B9yo3M
Fluid Dynamics (Part 2)	https://youtu.be/Y717vQpUEJQ
Fluid Properties (Part 3)	https://youtu.be/V8xUWWK2oT0
Simple Harmonic Motion	https://youtu.be/RIb7ofNG09I
Thermal Properties	
KTG	https://youtu.be/XO1tvFhIa0I
Thermodynamics	https://youtu.be/iz_kf1jRDRw
Wave Motion -Organ Pipes and Resonance Tube	https://youtu.be/fB7pfJ77za8
Wave Motion - Doppler's Effect	https://youtu.be/9-BxOaamnwg
Electrostatics	https://youtu.be/3stXbGRMcrk
Capacitors	https://youtu.be/EXEiickNUkY
Current Electricity	https://youtu.be/gm8FUfjrX18
Moving Charges and Magnetic Effect of Current	https://youtu.be/ULD2Ok1CGJk
Earth's Magnetism	https://youtu.be/a4CT5uVwAK4
Magnetic Properties	https://youtu.be/63cwdYXNlYE
EMI	https://youtu.be/puVavm_GFRM
Alternating Current	https://youtu.be/74dTY-pzM_o
Ray Optics	https://youtu.be/BhnyTWzIlBA
Wave Optics Part 1_Interference	https://youtu.be/LG5nlE8XTeI
Wave Optics Part 2_Diffraction_Polarization	https://youtu.be/ymMyyJGGqnY
Optical Instruments	https://youtu.be/OQssbDH0A4I
Electromagnetic Waves	https://youtu.be/bcVXgEkyQZY
Semiconductors_Basics + Zener Diode	https://youtu.be/_A2JomQ7-50
Semiconductors_Transistors	https://youtu.be/psDwl84Nzb0
Semiconductors_Logic Gates	https://youtu.be/pZdQAzLbFTo
Communication Systems	https://youtu.be/8NgMqK9X79Y
Modern Physics_Part 1_Atomic Physics	https://youtu.be/9VKUnE3mpHk
Modern Physics_Part 2_Photoelectric Effect	https://youtu.be/24oTQp84jrk
Modern Physics_Part 3_Dual Nature of Light	https://youtu.be/0zoR_saMAQY
Modern Physics_Part 4_Radioactivity	https://youtu.be/AdX3YBhQyog
Modern Physics_Part 5_Nuclear Physics	https://youtu.be/VDWqVahGixc
Modern Physics_Part 6_X Rays	https://youtu.be/dSHXdzX7NX0

1. Understanding of COM

(a)

Balance

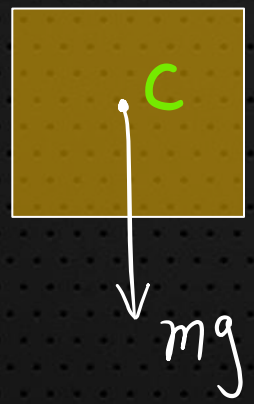
(b)



Translate ✓

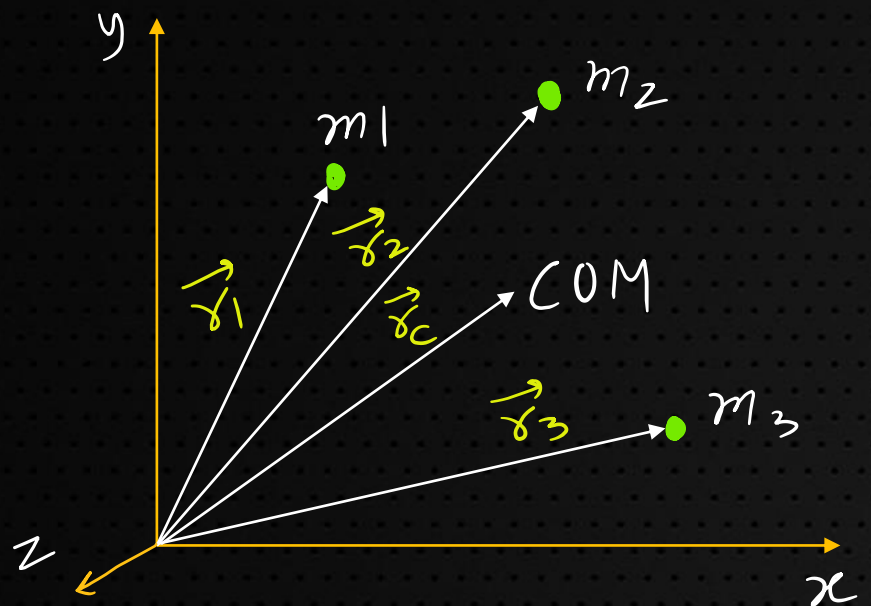
Rotate ✗

(C)



body lies in uniform gravitational field

2. COM of Multiparticle System



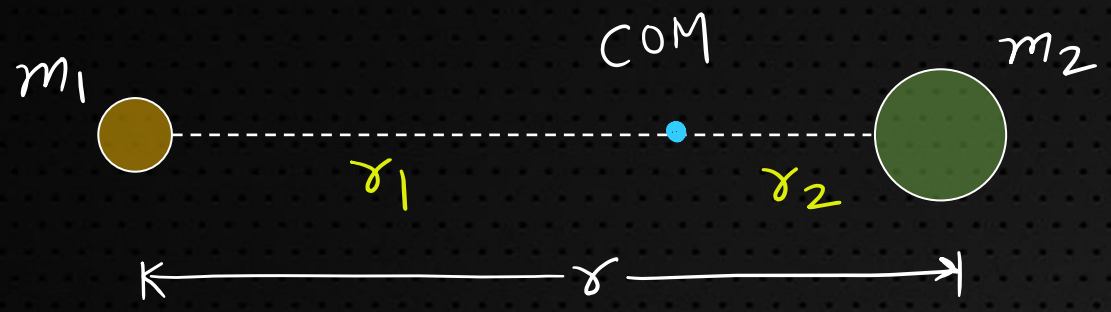
$$\vec{r}_c = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

$$x_c = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$y_c = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$z_c = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3}{m_1 + m_2 + m_3}$$

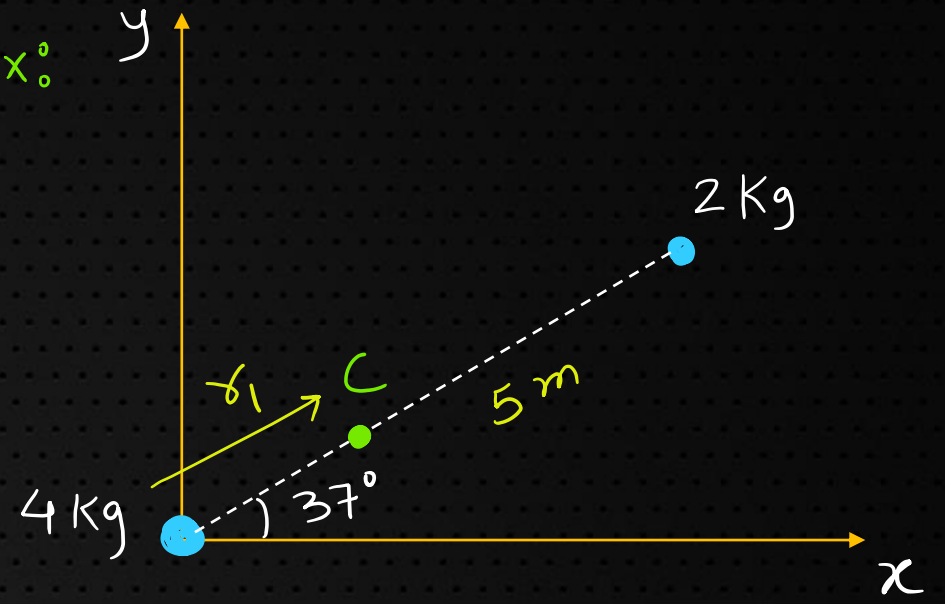
3. COM of 2-Particle System



(i) $r_1 = \frac{m_2 r}{m_1 + m_2}$, $r_2 = \frac{m_1 r}{m_1 + m_2}$

(ii) $m_1 r_1 = m_2 r_2$

Ex:



Find COM co-ordinates ?

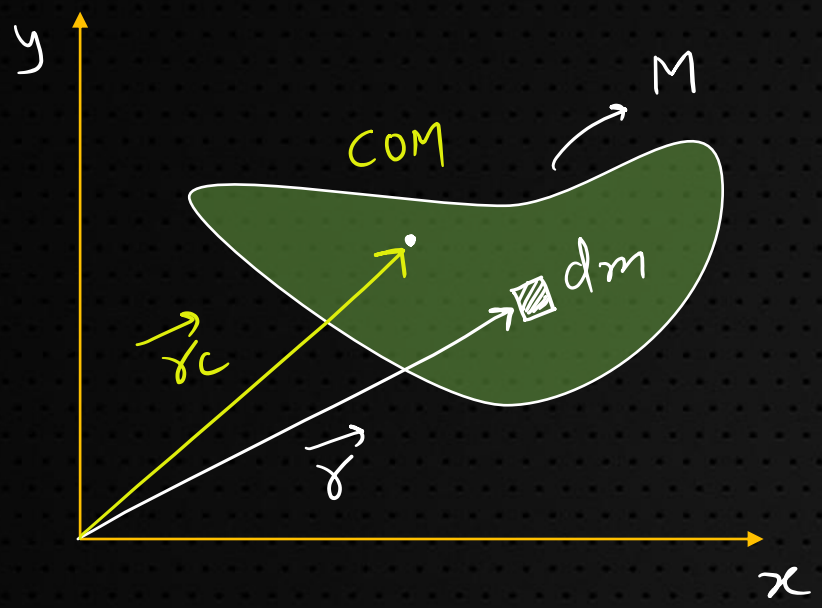
Soln: $r_1 = \frac{2 \times 5}{2+4} = \frac{10}{6} = \frac{5}{3}$ m

$\therefore \frac{5}{3}\cos 37°, \frac{5}{3}\sin 37°$

or

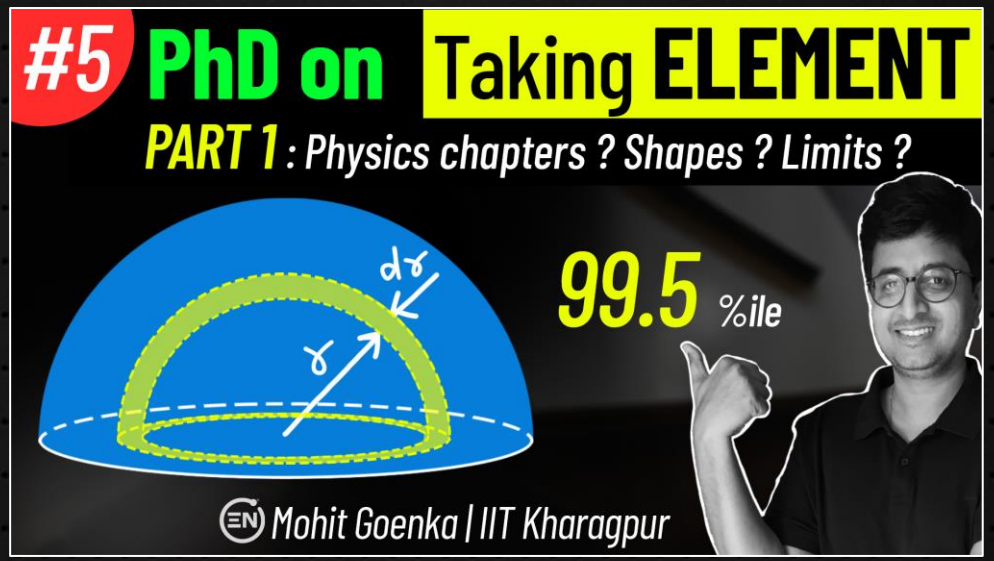
$\boxed{\frac{4}{3}, 1}$

4. COM of a Continuous Mass Distribution



$$\vec{r}_c = \frac{\int \vec{r}\, dm}{\int dm} = \frac{1}{M}\int \vec{r}\, dm$$

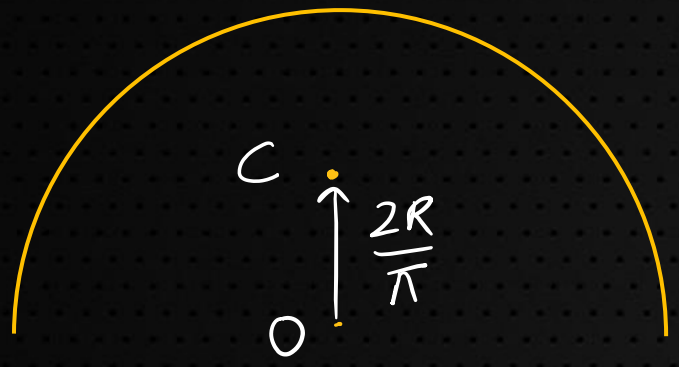
$$\begin{cases} x_c = \dfrac{\int x\, dm}{\int dm} \\ y_c = \dfrac{\int y\, dm}{\int dm} \\ z_c = \dfrac{\int z\, dm}{\int dm} \end{cases}$$



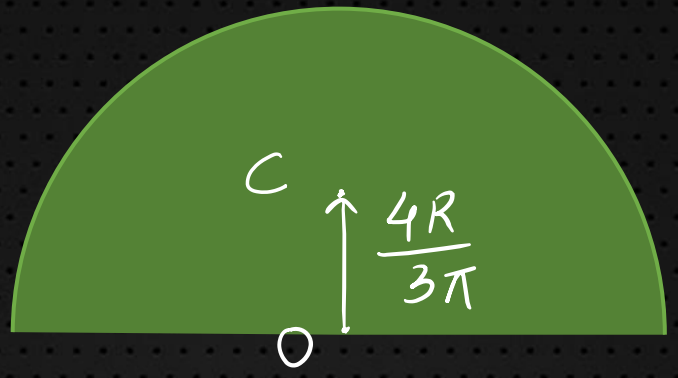
https://youtu.be/J0mV6kR9uRo

5. COM of Standard bodies (uniform mass distribution)

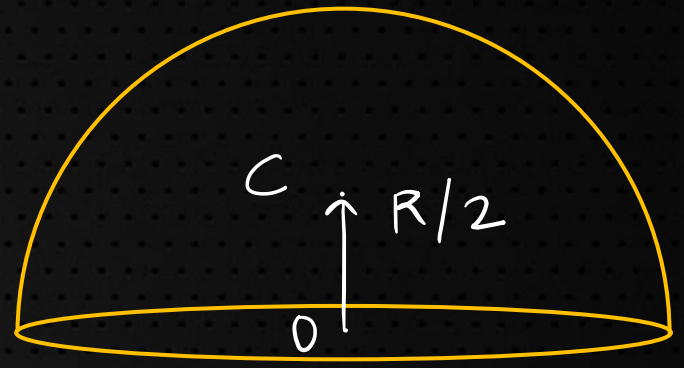
1. Semicircular Ring



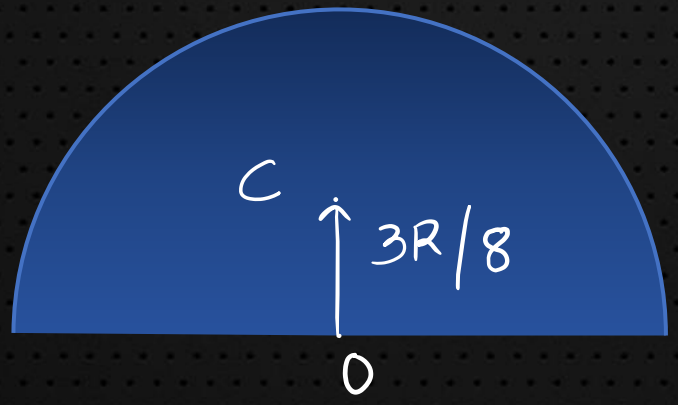
2. Semicircular Disc



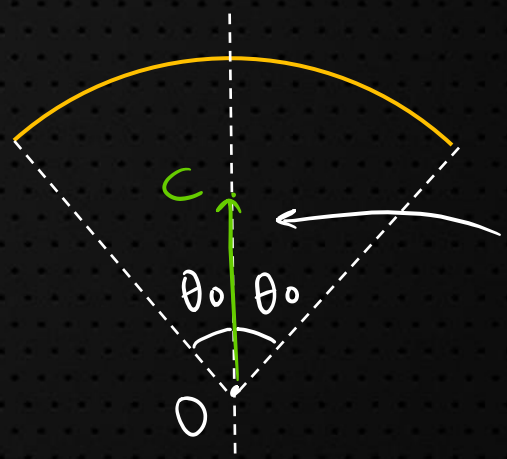
3. Hollow Hemisphere



4. Solid Hemisphere



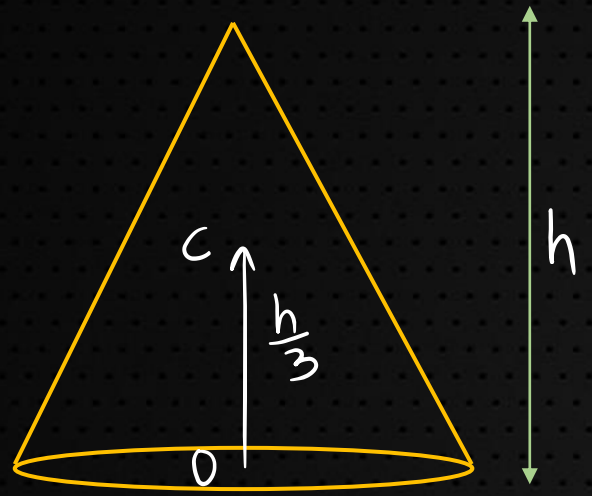
5. Arc of Ring



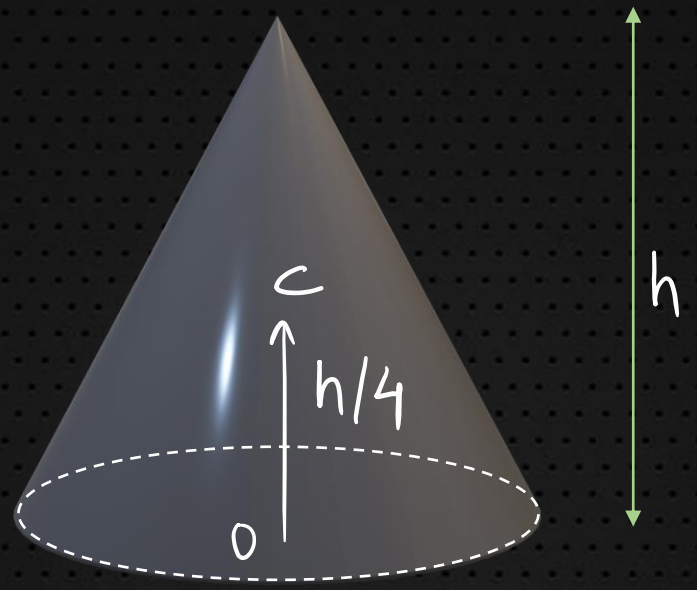
$$OC = \frac{R\sin\theta_0}{\theta_0}$$

... Continued

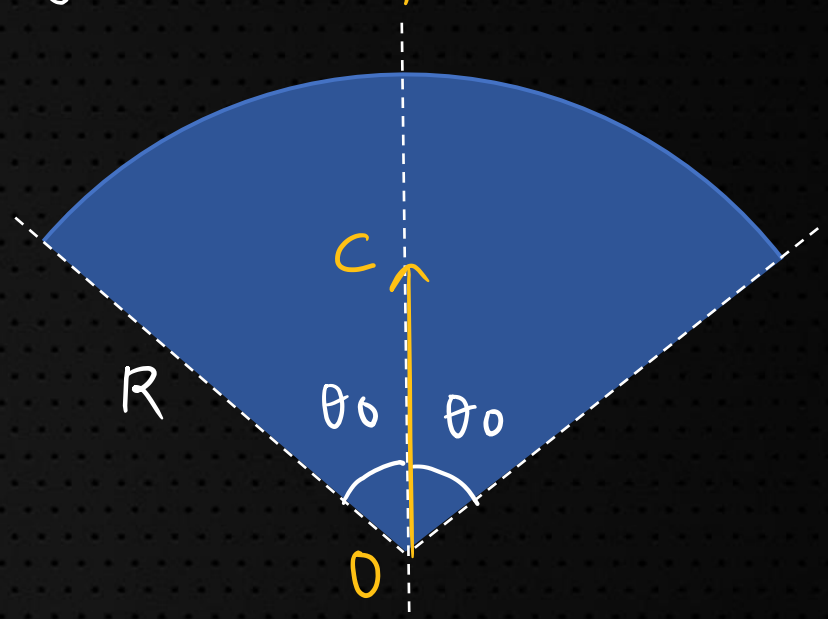
6. Hollow Cone



7. Solid Cone



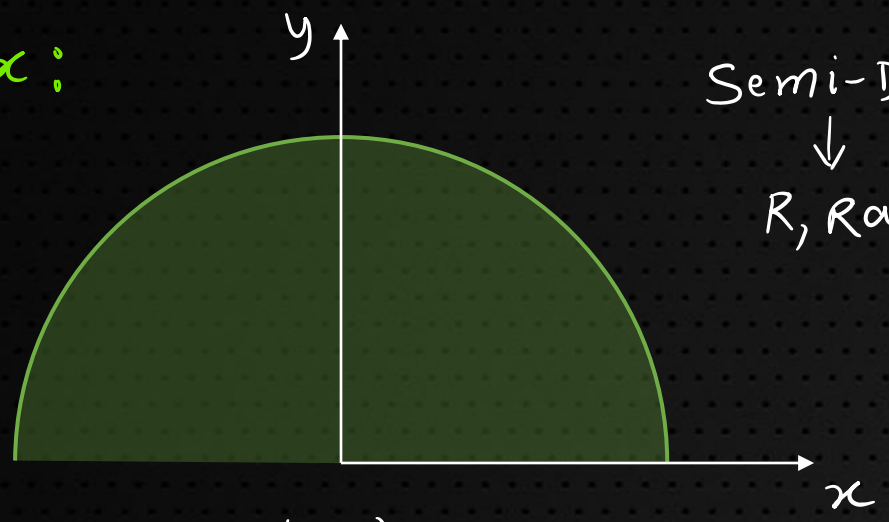
8. Sector of a Disc



$$OC = \frac{2R}{3}\,\frac{\sin\theta_0}{\theta_0}$$

6. COM of body (non-uniform mass distribution)

ex:

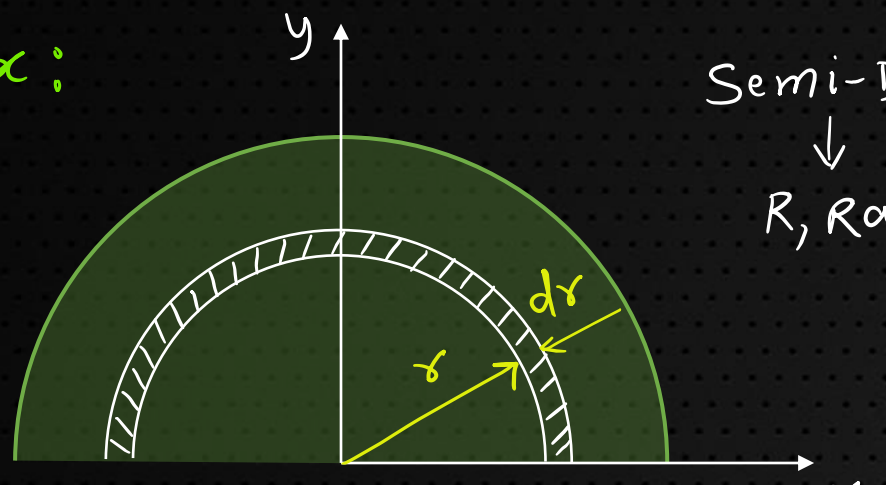


Semi-Disc
↓
R, Radius

$\sigma = 2r \ (Kg/m^2)$

6. COM of body (non-uniform mass distribution)

Ex:



Semi-Disc
↓
R, Radius

$\sigma = 2r \ (Kg/m^2)$

Soln. COM lies on y axis

$$y_{cm} = \frac{\int y\,dm}{\int dm}$$

$$dm = \sigma(r) \times dA$$
$$= 2r \times \pi r\,dr$$
$$= 2\pi r^2\,dr$$

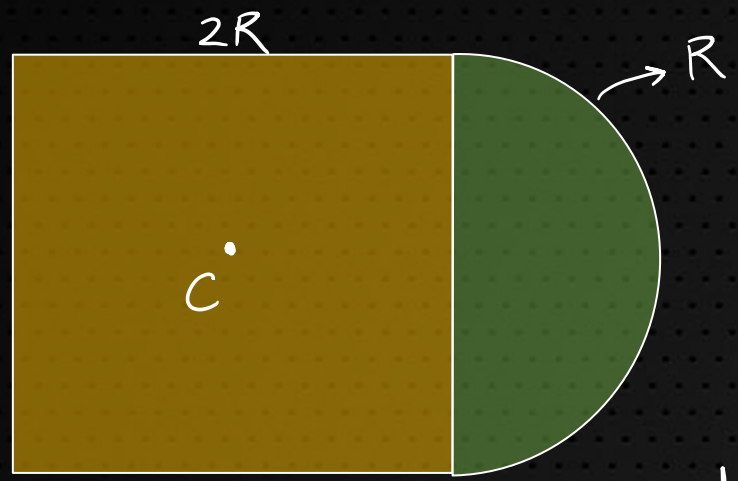
$$y = \frac{2r}{\pi} \ (\text{com of } dm)$$

$$\Rightarrow y_{cm} = \frac{\int_0^R \frac{2r}{\pi} \times 2\pi r^2\,dr}{\int_0^R 2\pi r^2\,dr}$$

$$= \frac{4r^4/4 \Big|_0^R}{2\pi \frac{r^3}{3} \Big|_0^R} = \boxed{\frac{3R}{2\pi}}$$

7. COM of Combined Structure

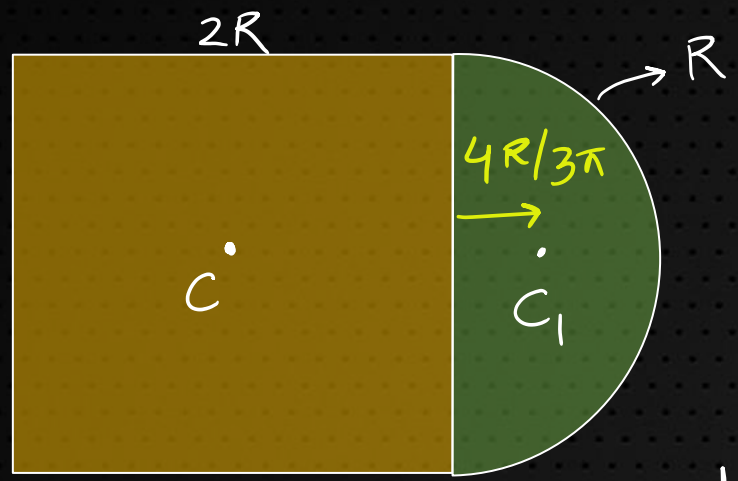
Ex:



Both are same material. Find COM from C.

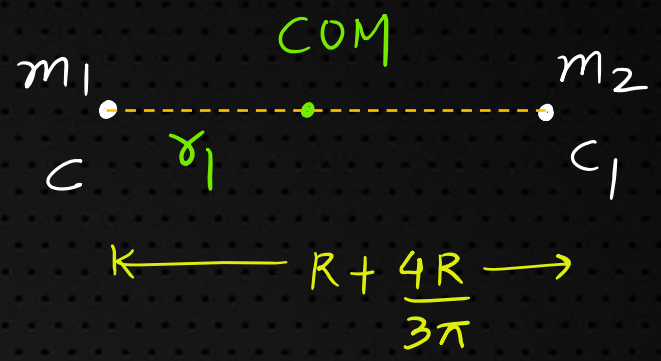
7. COM of Combined Structure

Ex:



Both are same material. Find COM from C.

Soln:



$\sigma = m/A$

$m = \sigma A$

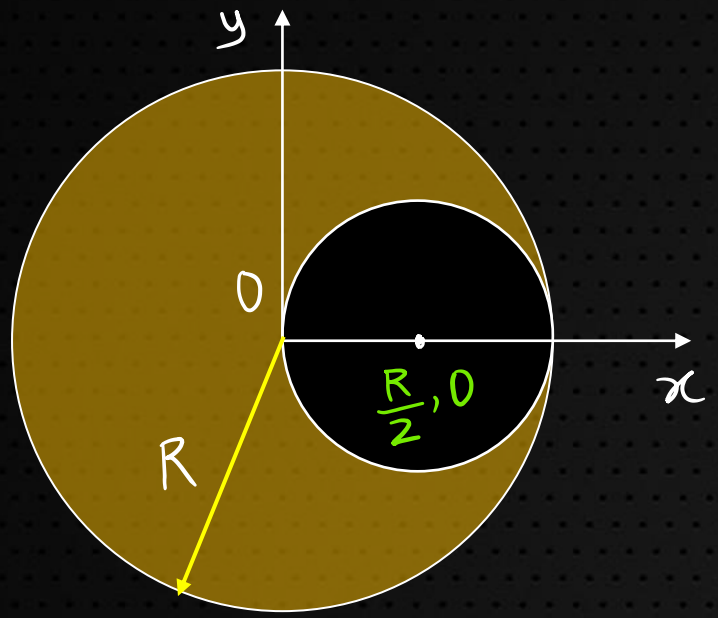
$$\gamma_1 = \frac{m_2\left(R+\frac{4R}{3\pi}\right)}{m_1+m_2} = \frac{\sigma\frac{\pi R^2}{2}\left(R+\frac{4R}{3\pi}\right)}{\sigma 4R^2+\sigma\frac{\pi R^2}{2}}$$

$$= \frac{R(3\pi+4)}{3(8+\pi)}$$ Ans.

8. CoM in Cavity Problems

Ex:

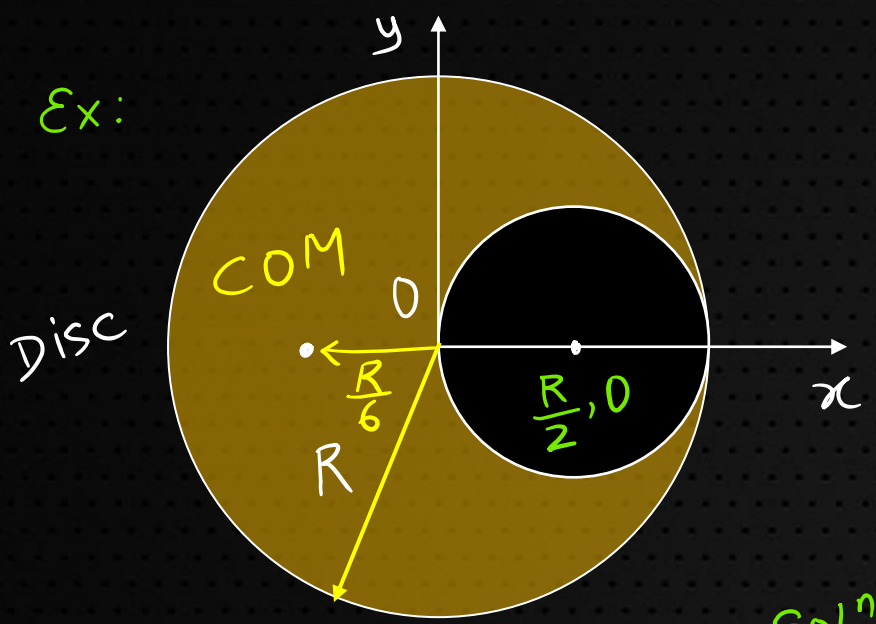
Disc



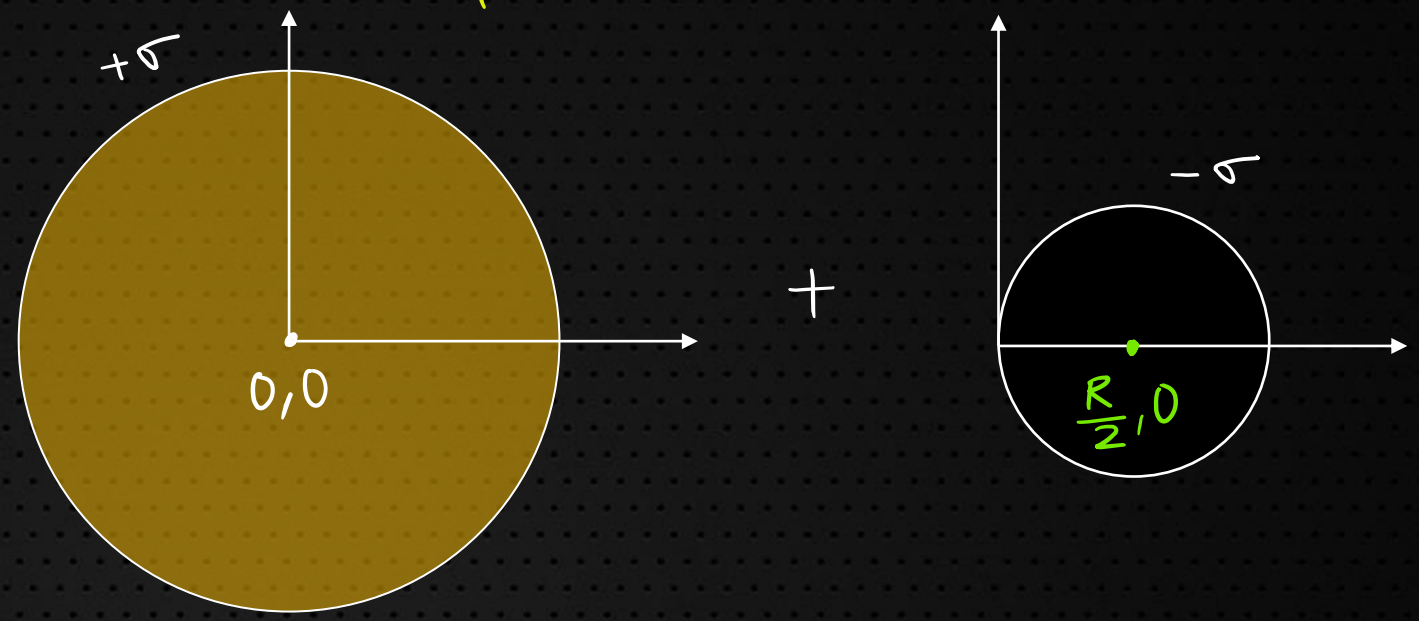
8. CoM in Cavity Problems

$m = \sigma \times A$

Ex:



1st_Method



Soln:

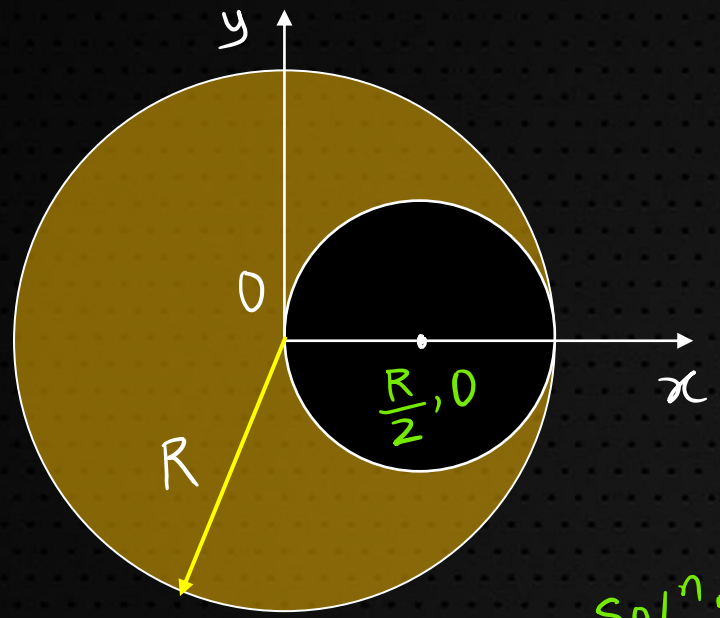
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(\sigma \pi R^2 \times 0) + (-\sigma \pi \frac{R^2}{4} \times \frac{R}{2})}{\sigma \pi R^2 - \sigma \pi \frac{R^2}{4}}$$

$$= -\frac{R}{6}$$

8. CoM in Cavity Problems

2nd_Method

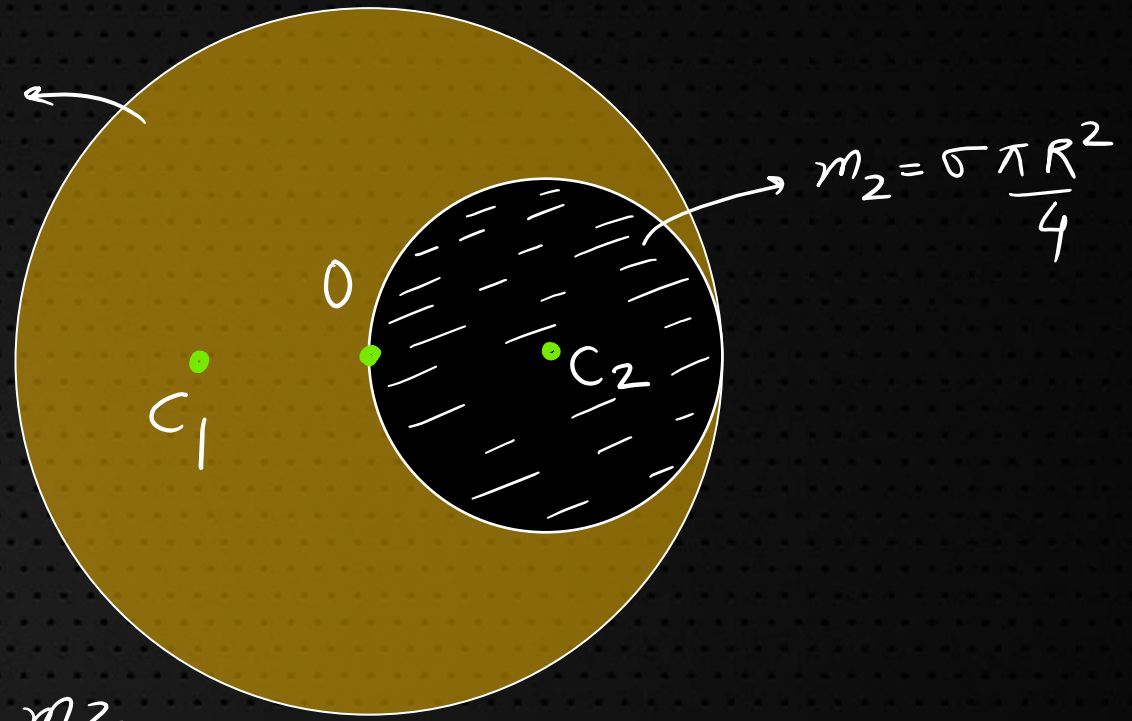
Ex:



Disc

$\sigma(\pi R^2 - \frac{\pi R^2}{4}) = m_1$

$\sigma\frac{3\pi R^2}{4}$



Soln:

$m_1 x = m_2 \frac{R}{2} \Rightarrow \sigma\frac{3\pi R^2}{4} \cdot x = \sigma\frac{\pi R^2}{4} \times \frac{R}{2}$

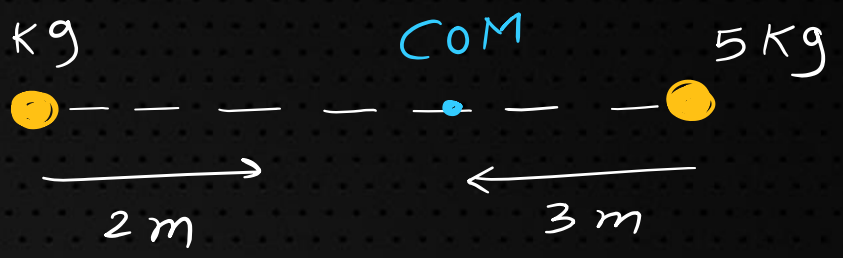
$\Rightarrow \boxed{x = \frac{R}{6}}$

9. Displacement of COM

$$\vec{r}_C = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$\Rightarrow \Delta\vec{r}_C = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

Ex:



Find displacement of COM.

Soln:

$$\Delta x_{cm} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2}$$

$$= \frac{(2\times 2) + (5\times -3)}{2+5}$$

$$= \frac{4 - 15}{7} = \boxed{\frac{-11}{7}\,m}$$

$\frac{11}{7}$ m COM

10. Velocity & Acceleration of COM

V_{cm} a_{cm}

$$\vec{r}_c = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots} \xrightarrow{d\vec{r}_c/dt} \vec{V}_{cm} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2 + \cdots}{\underbrace{m_1 + m_2 + \cdots}_{M}} \Rightarrow M\vec{V}_{cm} = \vec{P}_{net}$$

$$\downarrow d\vec{V}_{cm}/dt$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots}{\underbrace{m_1 + m_2 + \cdots}_{M}} \Rightarrow M\vec{a}_{cm} = \vec{F}_{net}$$

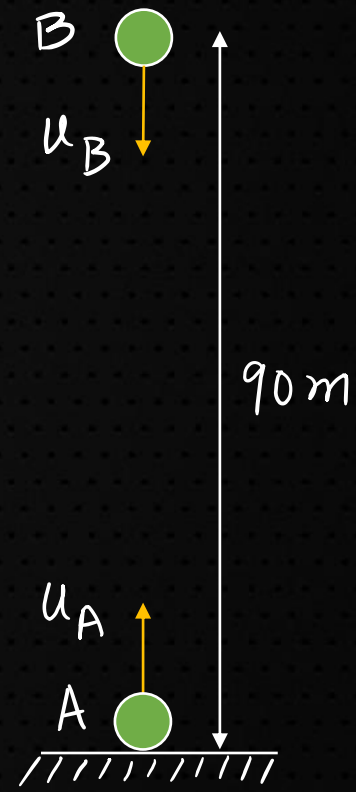
NOTE:

(i) $\vec{F}_{net} = \frac{d\vec{P}_{net}}{dt}$ (ii) $\vec{F}_{net} = 0 \Rightarrow \vec{P}_{net} = \text{Const.}$

($a_{cm} = 0$) ($\vec{V}_{cm} = \text{Const.}$)

... Continued

Ex: ***Two particles A and B of mass 1 kg and 2 kg respectively are projected in the directions shown in figure with speeds u_A = 200 m/s and u_B = 50 m/s. Initially they were 90 m apart. Find the maximum height attained by the centre of mass of the particles. Assume acceleration due to gravity to be constant. (g = 10 m/s²)***



... Continued

Ex: ***Two particles A and B of mass 1 kg and 2 kg respectively are projected in the directions shown in figure with speeds u_A = 200 m/s and u_B = 50 m/s. Initially they were 90 m apart. Find the maximum height attained by the centre of mass of the particles. Assume acceleration due to gravity to be constant. (g = 10 m/s²)***

Soln: $y_1 = \frac{m_B \times 90}{m_A + m_B} = \frac{180}{3} = 60\ m$

$u_C = \frac{m_A u_A + m_B u_B}{m_A + m_B} = \frac{200 - 100}{3} = \frac{100}{3}\ m/s$

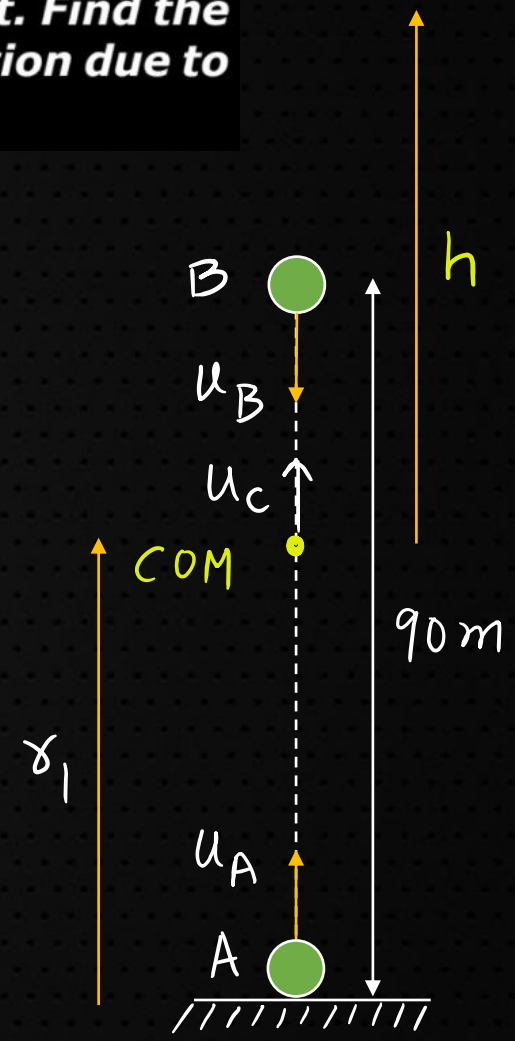
$a_C = \frac{m_A a_A + m_B a_B}{m_A + m_B} = \frac{-g(m_A + m_B)}{m_A + m_B} = -g$

$v_C^2 = u_C^2 + 2 a_C h$

$\Rightarrow 0 = \left(\frac{100}{3}\right)^2 - 2 \times 10 \times h \Rightarrow h = 55.55\ m$

$\therefore H = y_1 + h$

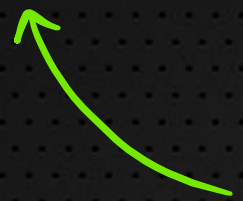
$= 115.55\ m$ Ans



EDUNITI

PYQs LINKS (JEE MAIN)

2021 Feb	2021 March	2021 July	2021 August	2020
https://youtu.be/JBndilWvPwA	https://youtu.be/ocJJa_lXm64	https://youtu.be/5P1QCbU-t84	https://youtu.be/LIKVg9txoas	https://youtu.be/ZeEzCS5uCmk



CLICK

(Practice these Questions)

EDUNITI

Revision Series Playlist Link https://bit.ly/3eBbib9

JEE Main PYQs Link https://bit.ly/2S54jzh *Chapter wise 2021, 2020, 2018*

GoldMine Link https://bit.ly/2VhOGFF