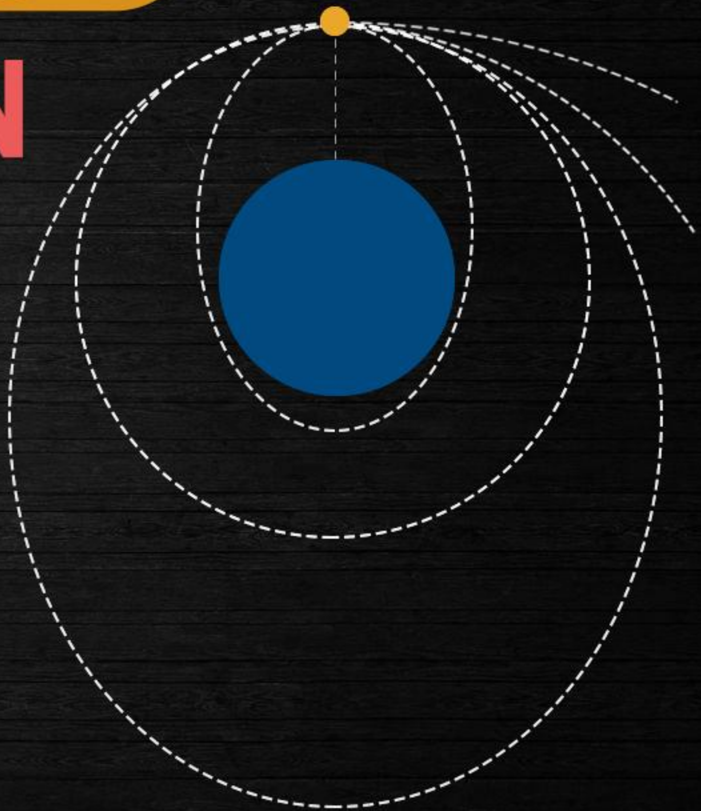
EDUNITI

GRAVITATION

ONE SHOT

REVISION

CLASS XI



NEET
JEE MAIN
JEE ADVANCED
OLYMPIADS

MOHIT GOENKA
IIT KHARAGPUR

LAW OF GRAVITATION

$$F \propto m_1 m_2$$
$$F \propto \frac{1}{r^2}$$
$$\Rightarrow F \propto \frac{m_1 m_2}{r^2} \Rightarrow \boxed{F = \frac{G m_1 m_2}{r^2}}$$

G : Universal Gravitational constant

$6.67 \times 10^{-11}\ N m^2 / kg^2$

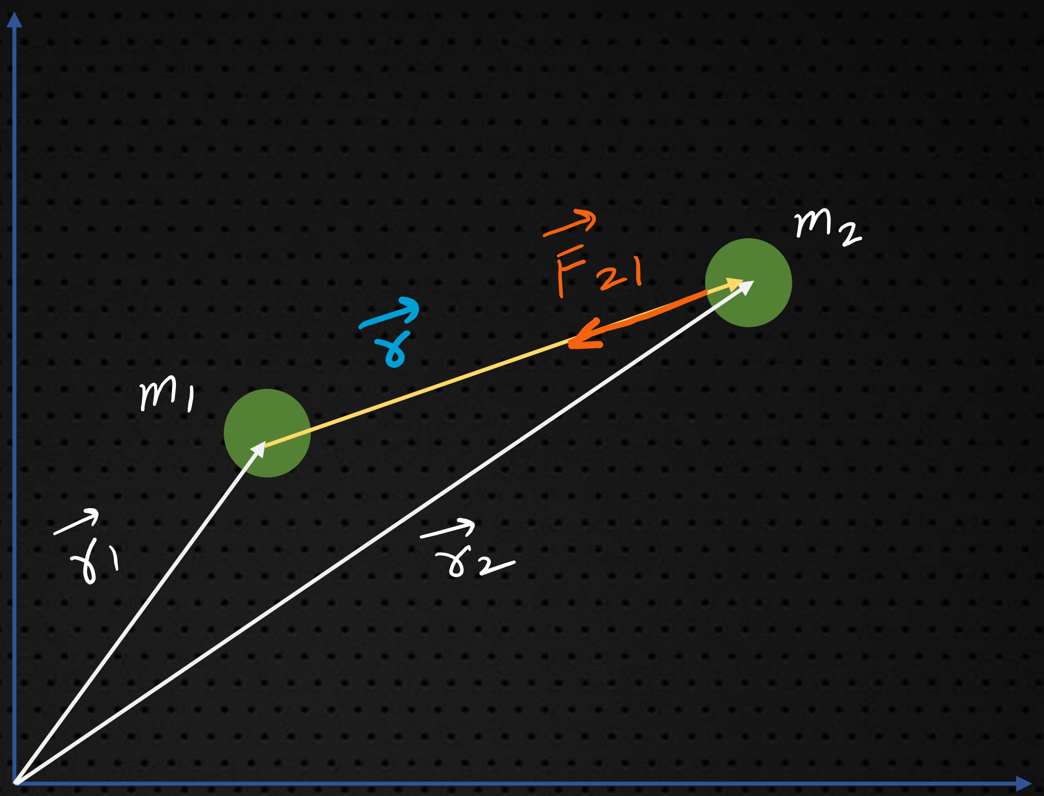
NOTE:
1. F acts along line joining masses.
2. Here m_1 and m_2 are point masses.

IN VECTOR FORM

$\vec{F}_{21}$: force on m_2 due to m_1.

$$\vec{F}_{21} = \frac{G m_1 m_2}{r^2}(-\hat{r})$$

$\{-\hat{r}$ as $\vec{F}_{21}$ is in opposite direction of $\vec{r}\}$



$\hat{r}$ being unit vector,

$$\Rightarrow \hat{r} = \frac{\vec{r}}{r}$$

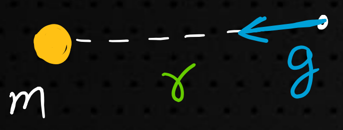
thus, $\vec{F}_{21} = -\frac{G m_1 m_2 \vec{r}}{r^3}$ $\{\vec{r} = \vec{r}_2 - \vec{r}_1\}$

$$\Rightarrow \boxed{\vec{F}_{21} = -\frac{G m_1 m_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)}$$

GRAVITATIONAL FIELD

↳ A way of expressing influence of a mass.

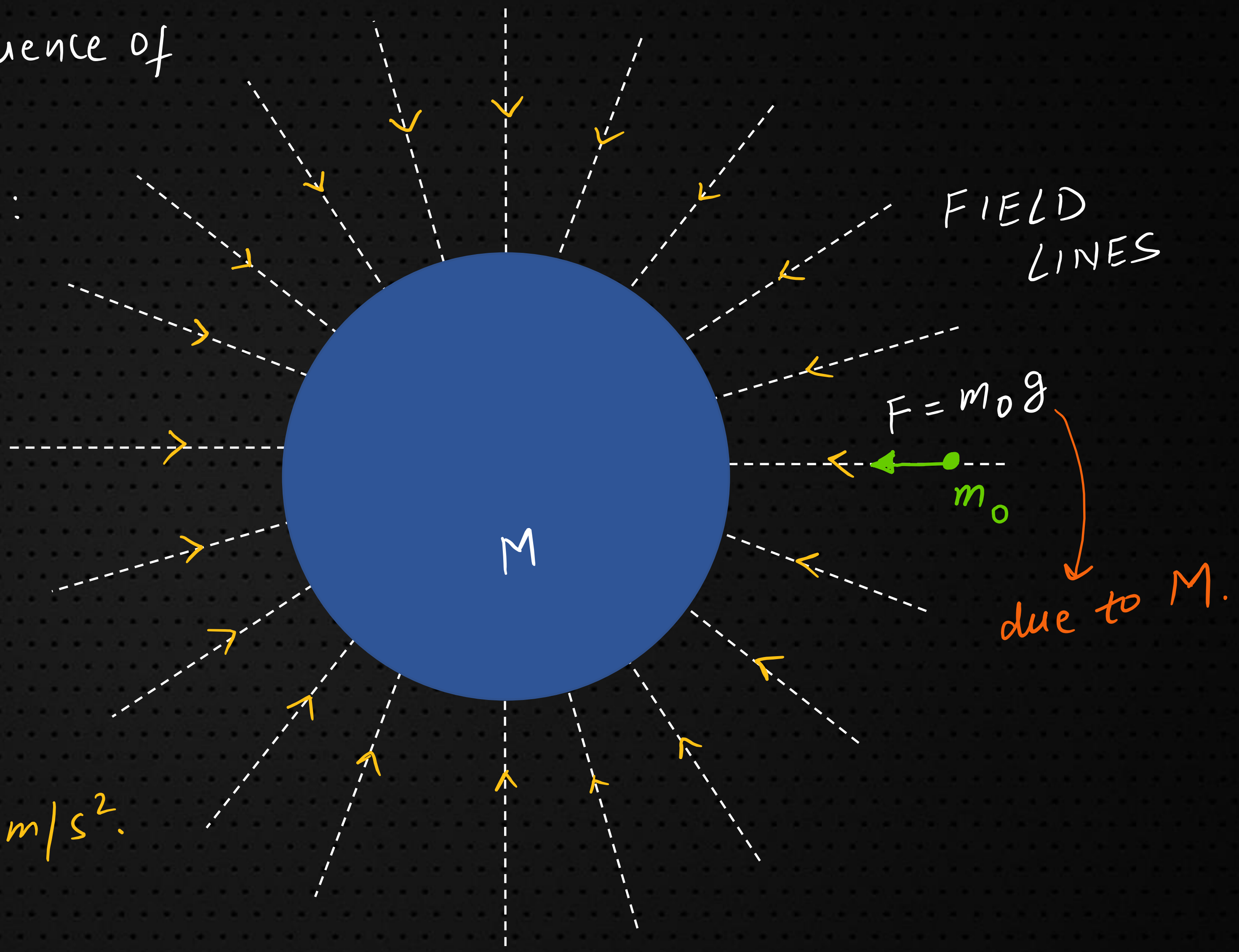
Gravitational Field strength (g):



$$g = \frac{Gm}{r^2}$$

NOTE:

1. g is also called "acceleration due to gravity"
2. At earth's surface its close to $g_0 = 9.8\ m/s^2$.



FOR A RING g

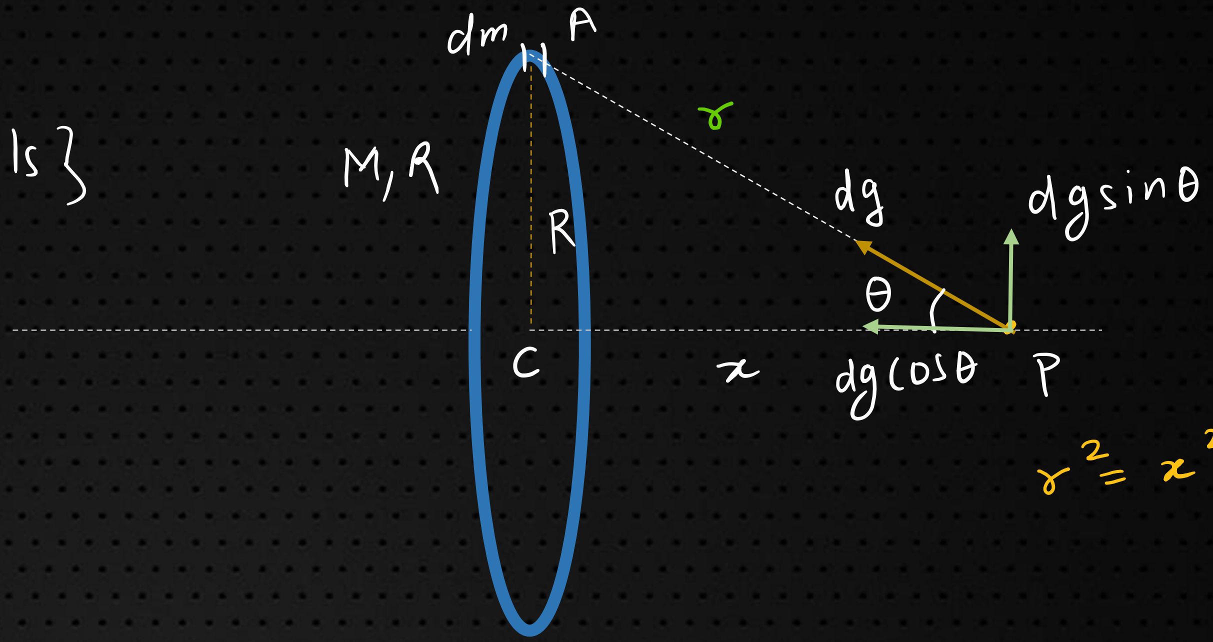
$\int dg\sin\theta = 0$ {vertical comp. cancels}

$g(x) = \int dg\cos\theta$

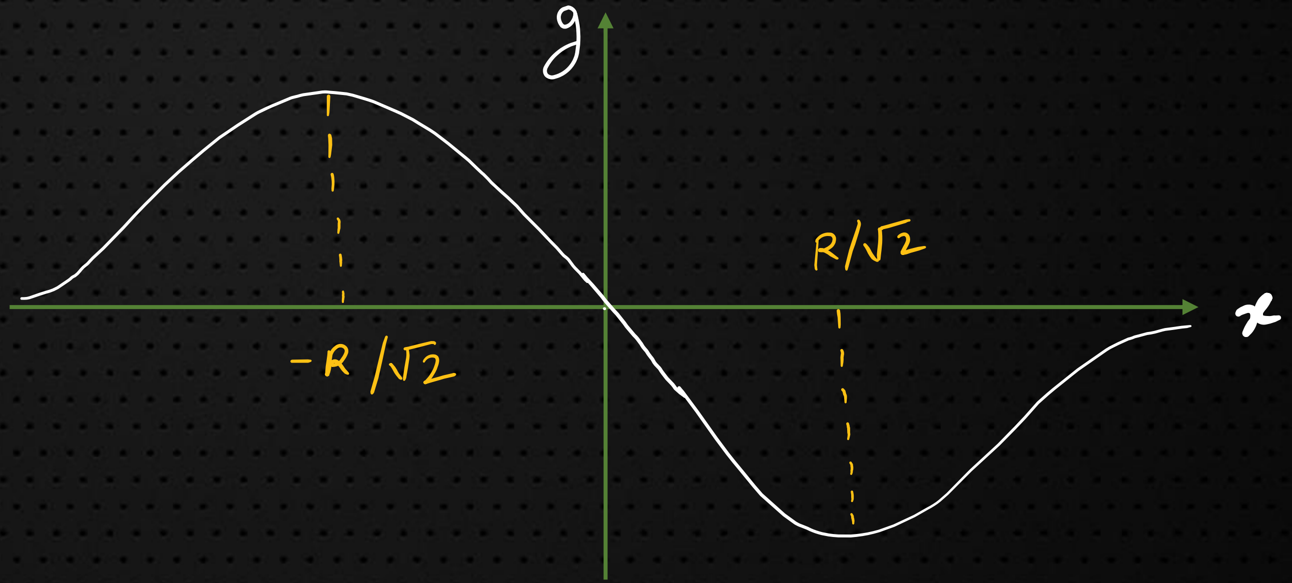
$= \int \frac{G\,dm}{r^2}\cos\theta = \int \frac{G\,dm}{r^2} \times \frac{x}{r}$

$= \frac{Gx}{(x^2+R^2)^{3/2}} \int dm$

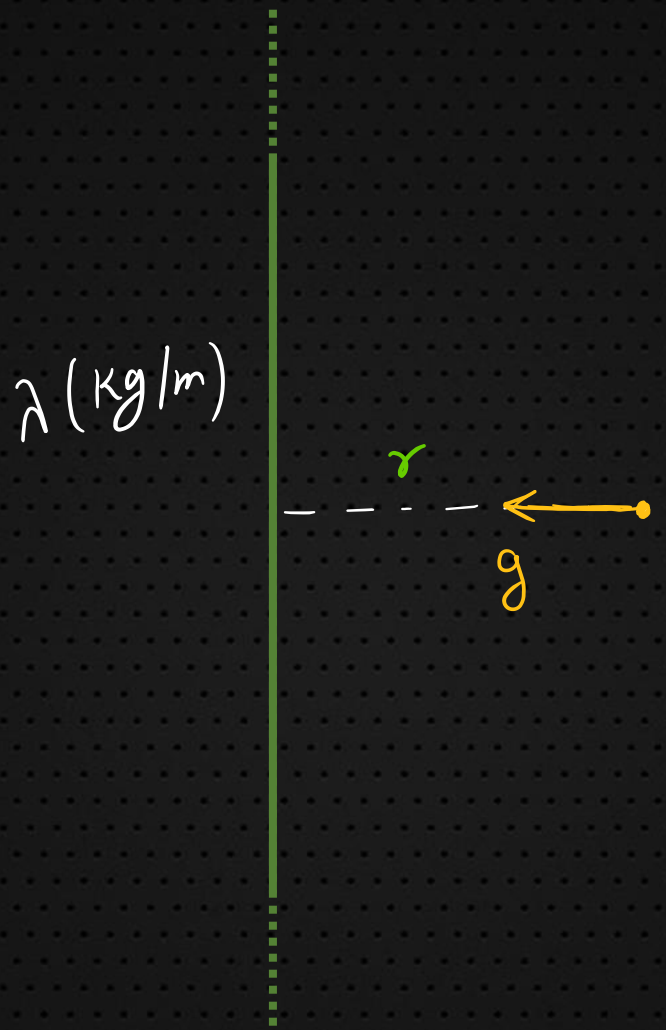
$= \boxed{\frac{GMx}{(x^2+R^2)^{3/2}}}$



$r^2 = x^2 + R^2$

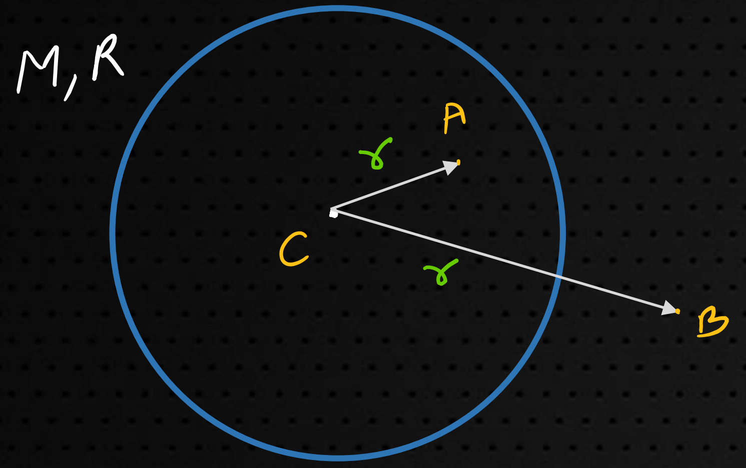


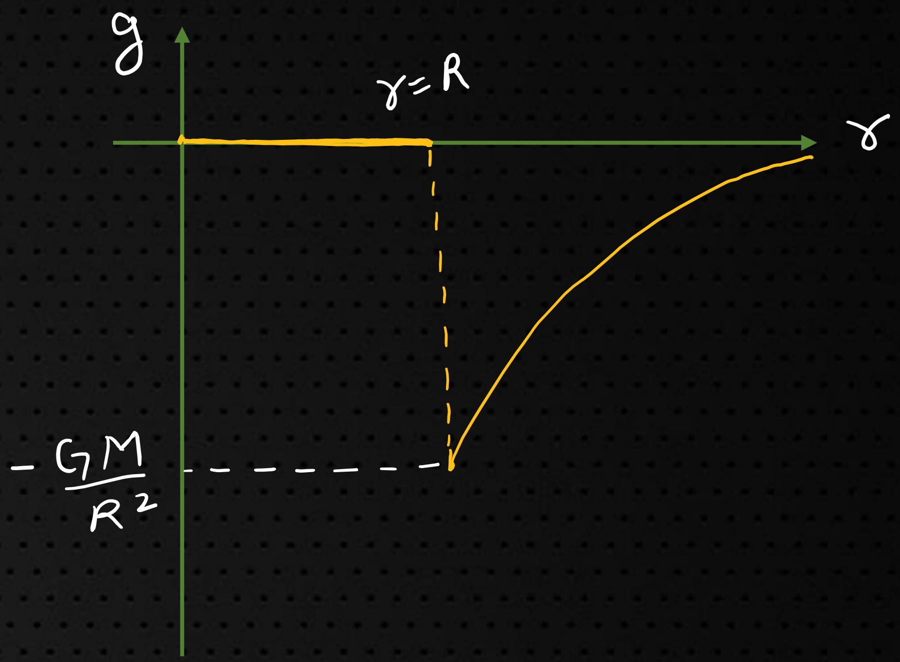
FOR A LONG WIRE g



$$\boxed{g = \frac{2G\lambda}{r}}$$

FOR THIN SPHERICAL SHELL g



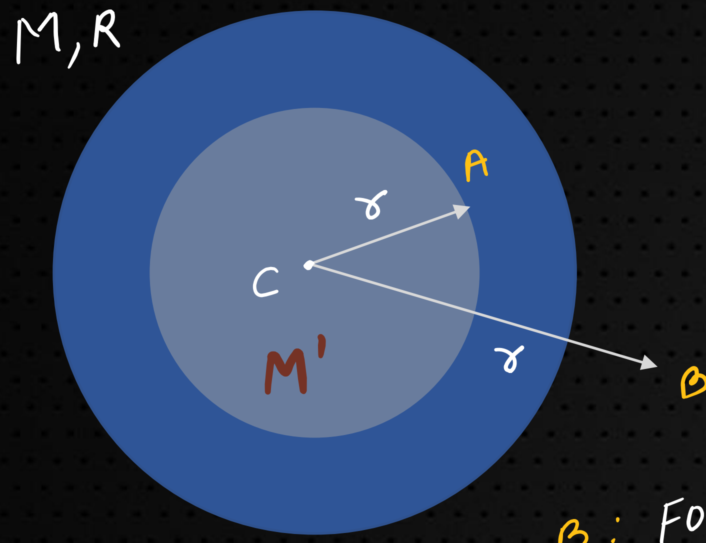


A : For $r < R$, $g = 0$

B : for $r \geq R$, $g = -\frac{GM}{r^2}$

NOTE : Taking radially inwards as -VE direction

FOR SOLID SPHERE g



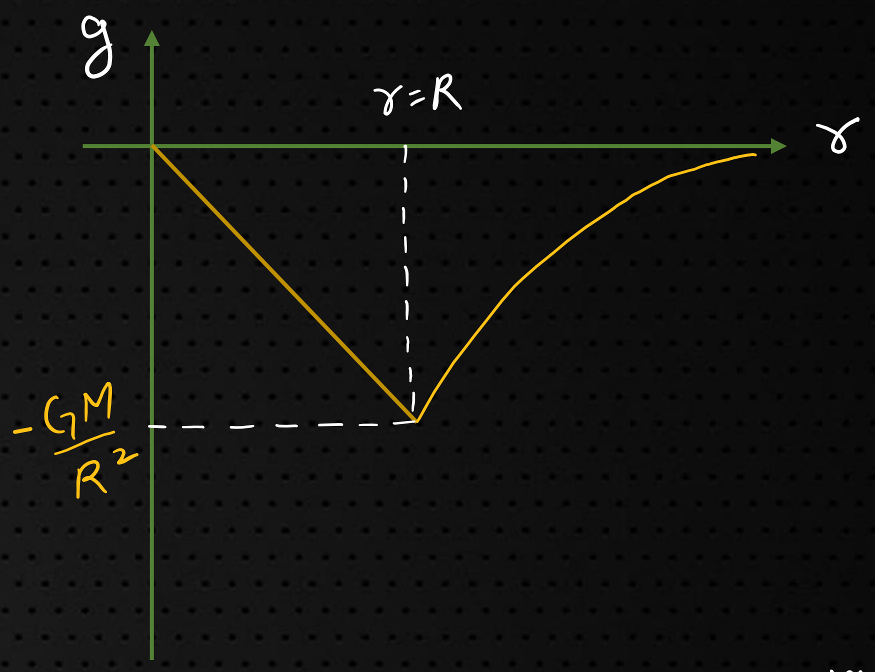
A: For $r < R$:

$$M' = Mr^3/R^3$$

$$\Rightarrow g = -\frac{GM'}{r^2} = -\frac{GMr}{R^3}$$

B: FOR $r \geq R$:

$$g = -\frac{GM}{r^2}$$



NOTE: Taking radially inwards as -VE direction

POTENTIAL ENERGY

U : Work done to bring m_2 very slowly from ∞ to a separation r from m_1.

$$W = \int_{\infty}^{r} \frac{Gm_1m_2}{r^2}\, dr = \boxed{-\frac{Gm_1m_2}{r}}$$

NOTE: At infinity, $U=0$

Also, $W = \Delta U \Rightarrow W = U_f - U_i$

$$\Rightarrow -\frac{Gm_1m_2}{r} = U_f - 0$$

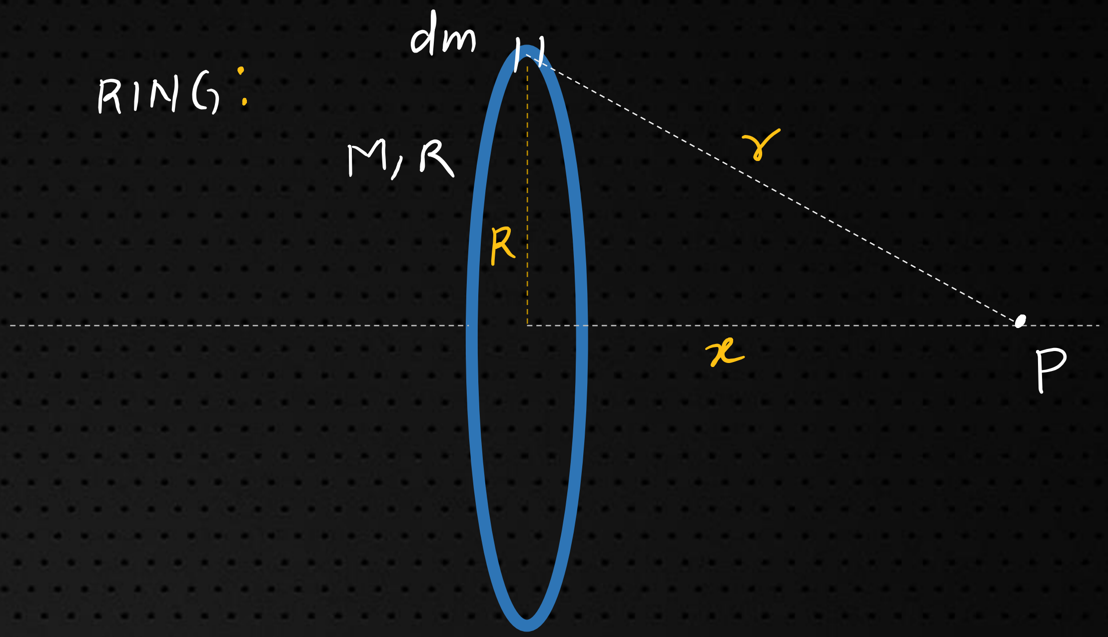
$$\therefore U_f = -\frac{Gm_1m_2}{r}$$

GRAVITATIONAL POTENTIAL V

POINT Mass:

$$V = -\frac{Gm}{r}$$

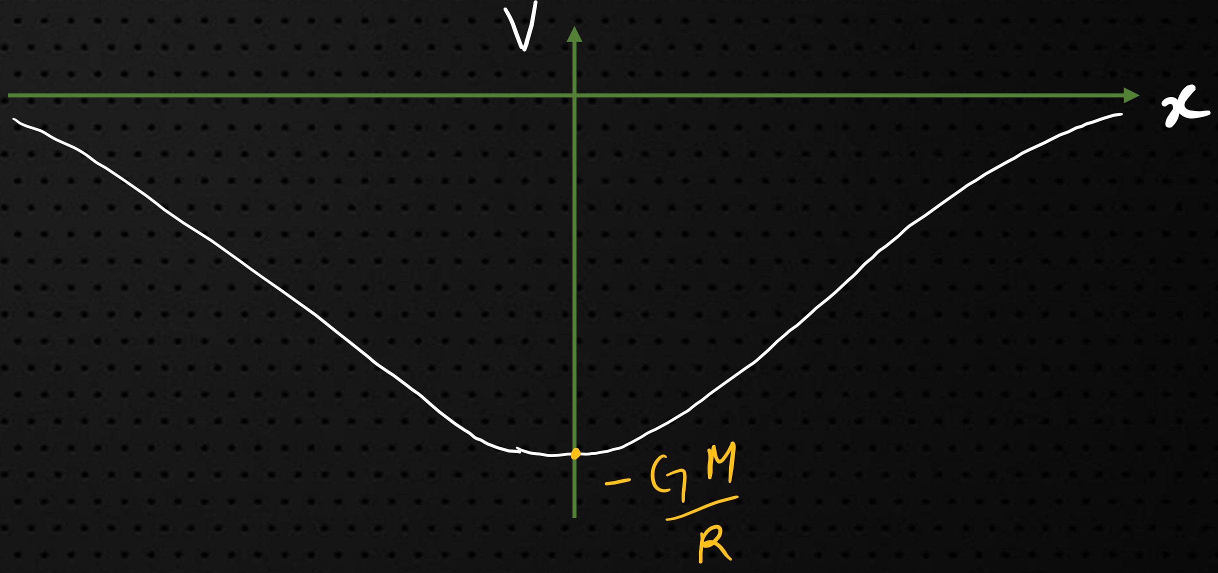
RING:



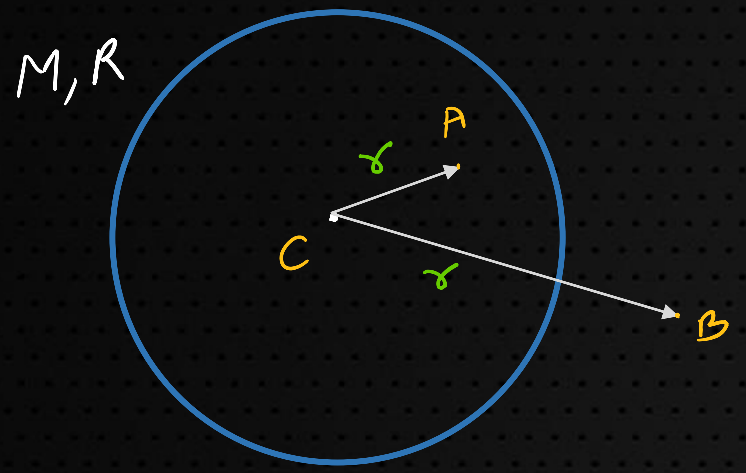
RING: $dV = -\frac{G\,dm}{r}$

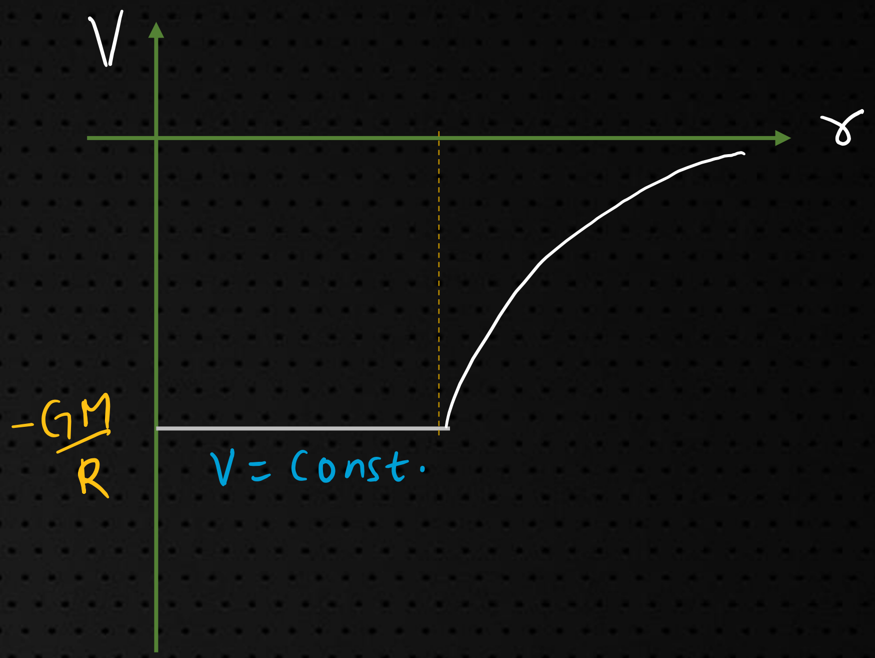
$$\Rightarrow V = -\frac{G}{r}\int dm$$

$$\boxed{V = \frac{-GM}{(R^2+x^2)^{1/2}}}$$



V DUE TO SPHERICAL SHELL



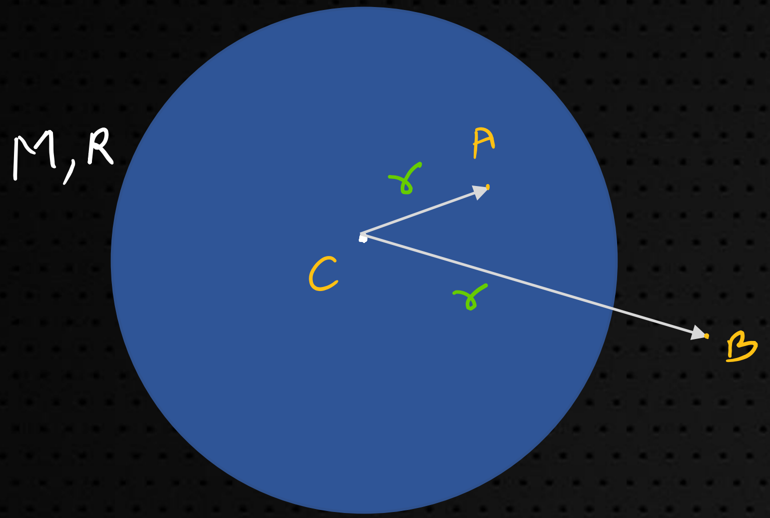


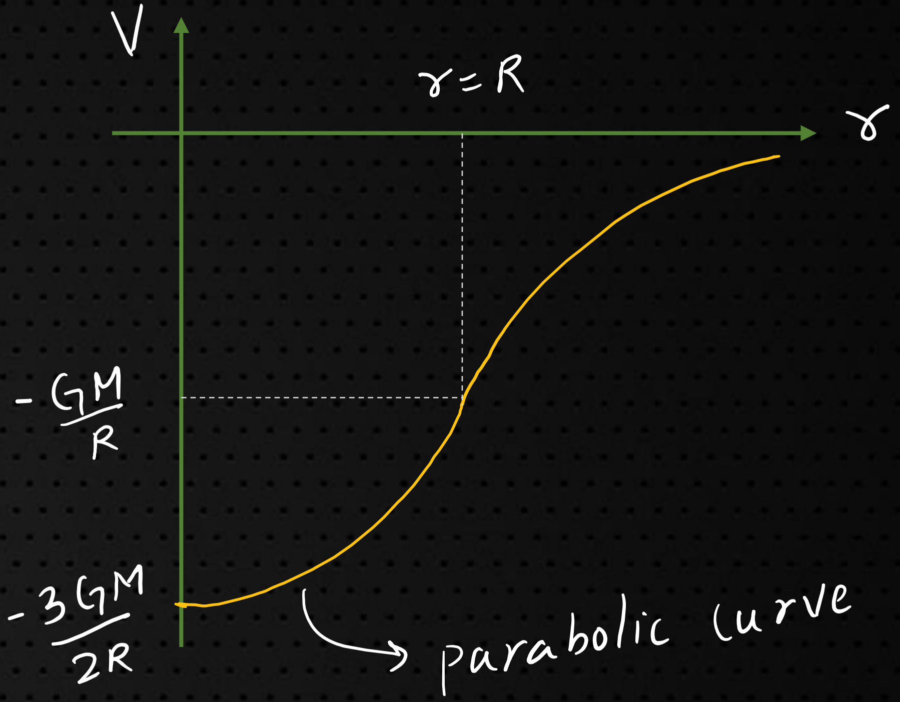
A: For $r < R$, $V = -GM/R$

B: For $r \geq R$, $V = -\frac{GM}{r}$

→ **NOTE:** For an outside point, assume all mass to situated at centre.

V DUE TO SOLID SPHERE





A : For $r < R$, $V = -\frac{GM}{2R^3}(3R^2 - r^2)$

B : For $r \geq R$, $V = -GM/r$

→ **NOTE :**
For an outside point, assume all mass to situated at centre.

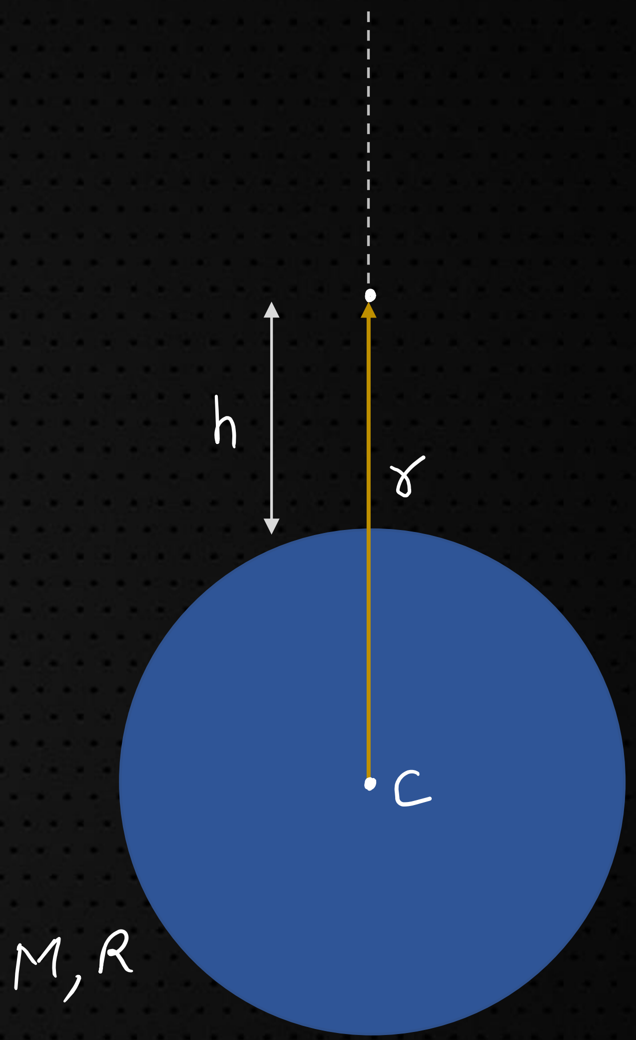
VARIATION OF g WITH HEIGHT

At height "h" :

$$g = \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\left(1+\frac{h}{R}\right)^{-2}, \quad g_0 = \frac{GM}{R^2}$$

If $\frac{h}{R} << 1$ or $h << R$

$$\Rightarrow \boxed{g = g_0\left(1-\frac{2h}{R}\right)}$$



NOTE :

1. only if $h << R$ (for earth till $h \sim 400$ Km)
   use $g = g_0\left(1-\frac{2h}{R}\right)$

2. If h is comparable with R, use

$$g = \frac{GM}{(R+h)^2} = \frac{g_0 R^2}{(R+h)^2}$$

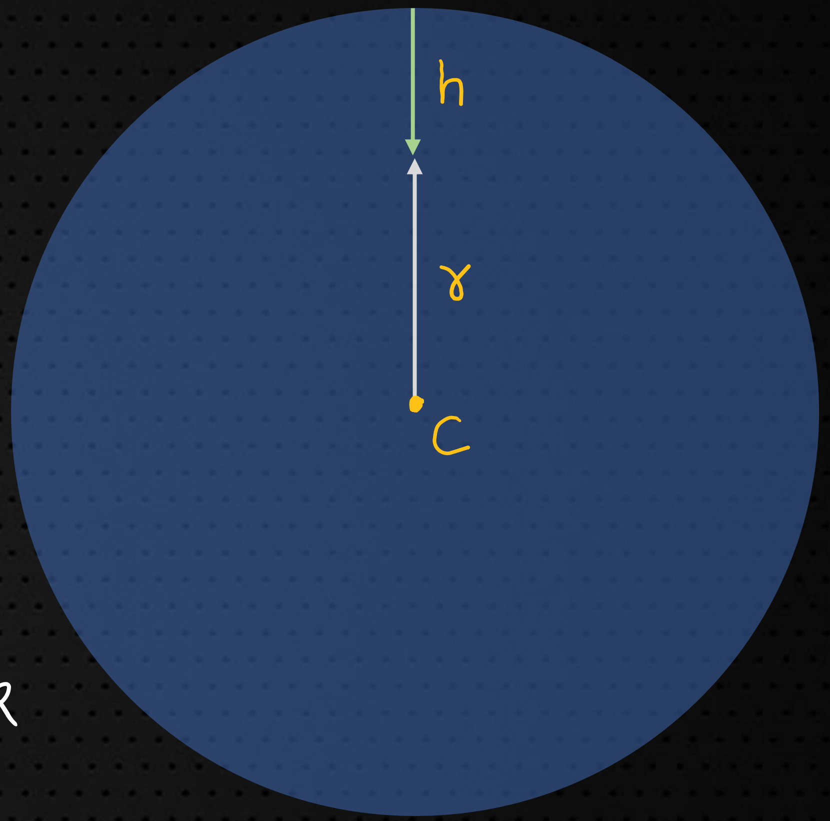
VARIATION OF g WITH DEPTH

At a depth "h" :

$$g = \frac{GMr}{R^3} \quad , \quad r = R-h$$

$$\Rightarrow g = \frac{GM(R-h)}{R^3} = \frac{GM}{R^2}\left(\frac{R-h}{R}\right)$$

$$\therefore \boxed{g = g_0\left(1-\frac{h}{R}\right)}$$



M, R

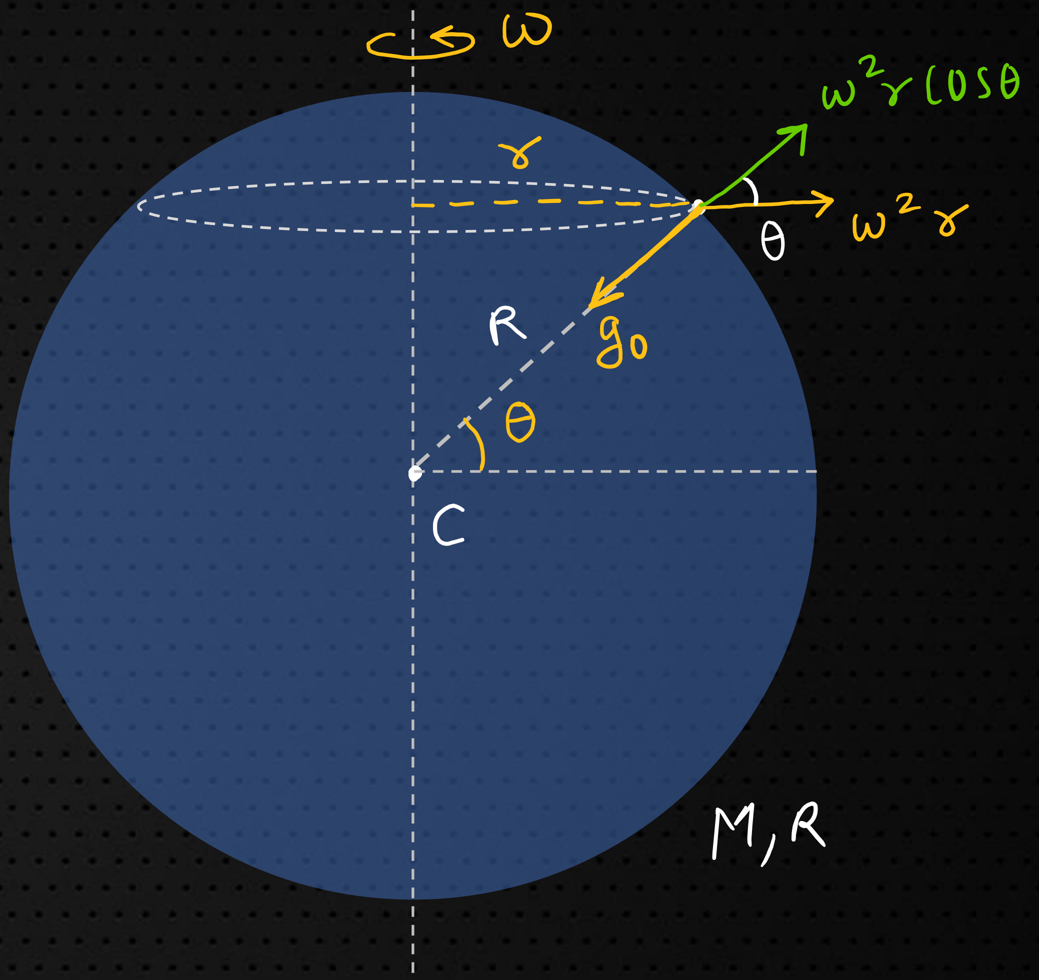
VARIATION OF g WITH ROTATION

$r = R\cos\theta$

$g_{effective}$ towards centre is

$$g_{eff} = g_0 - \omega^2 r\cos\theta$$

$$= \boxed{g_0 - \omega^2 R\cos^2\theta}$$



NOTE:

1. $\omega^2 R = 0.034$ rad/s (for Earth)
2. At equator, $g_{eff} = g_0 - \omega^2 R$, $\theta = 0°$
3. At poles, $g_{eff} = g_0$, $\theta = 90°$.

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ESCAPE VELOCITY

V_e

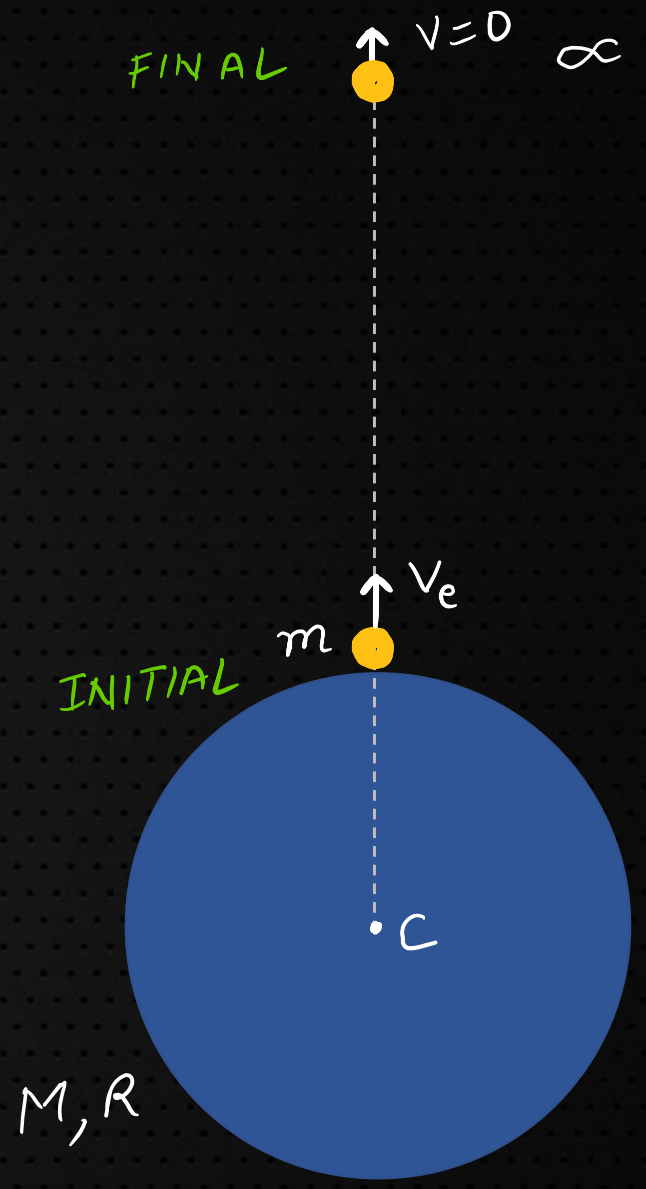
Minimum velocity V_e, so that particle escapes planet's gravitational pull.

(the distance where g of planet is zero is taken is ∞)

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \frac{1}{2} m V_e^2 - \frac{GMm}{R} = 0 + 0$$

$$\therefore \boxed{V_e = \sqrt{\frac{2GM}{R}}} \approx 11.2 \text{ Km/s at EARTH's surface.}$$



ORBITAL VELOCITY

$V_0, T, K, U, T.E., B.E.$

(a) $F = mV_0^2/r \Rightarrow \frac{GMm}{r^2} = \frac{mV_0^2}{r} \Rightarrow V_0 = \sqrt{\frac{GM}{r}}$

(b) $T = \frac{2\pi r}{V_0} = \frac{2\pi r}{\sqrt{GM/r}} = \frac{2\pi}{\sqrt{GM}} r^{3/2}$

(c) $K = \frac{1}{2} m V_0^2 = GMm/2r$

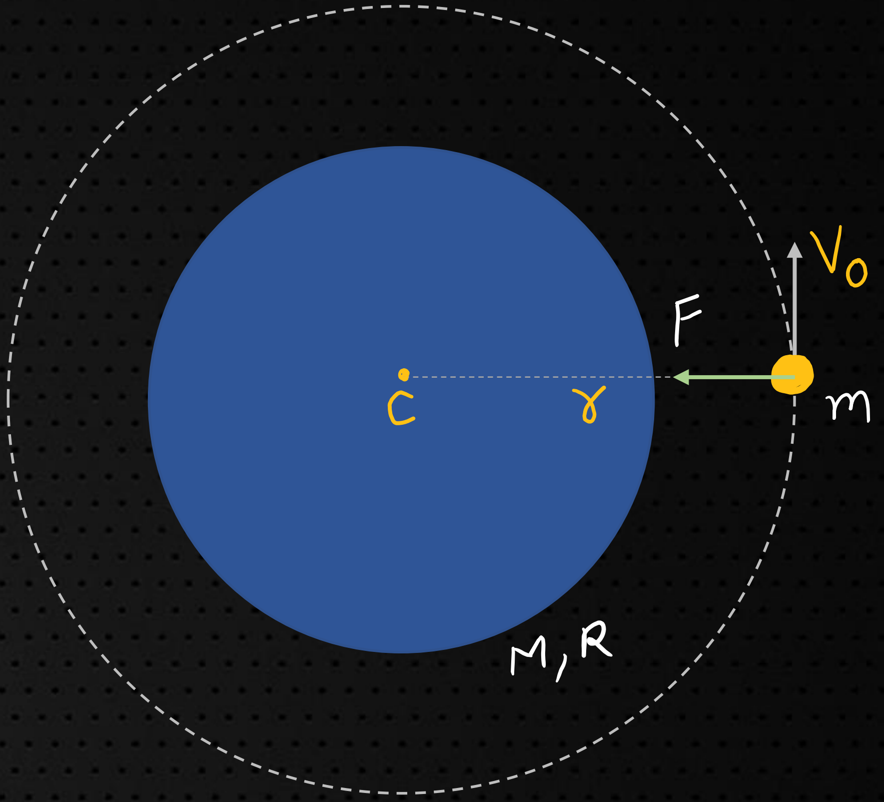
$U = -GMm/r$, $T.E = K + U$

$= -\frac{GMm}{2r}$

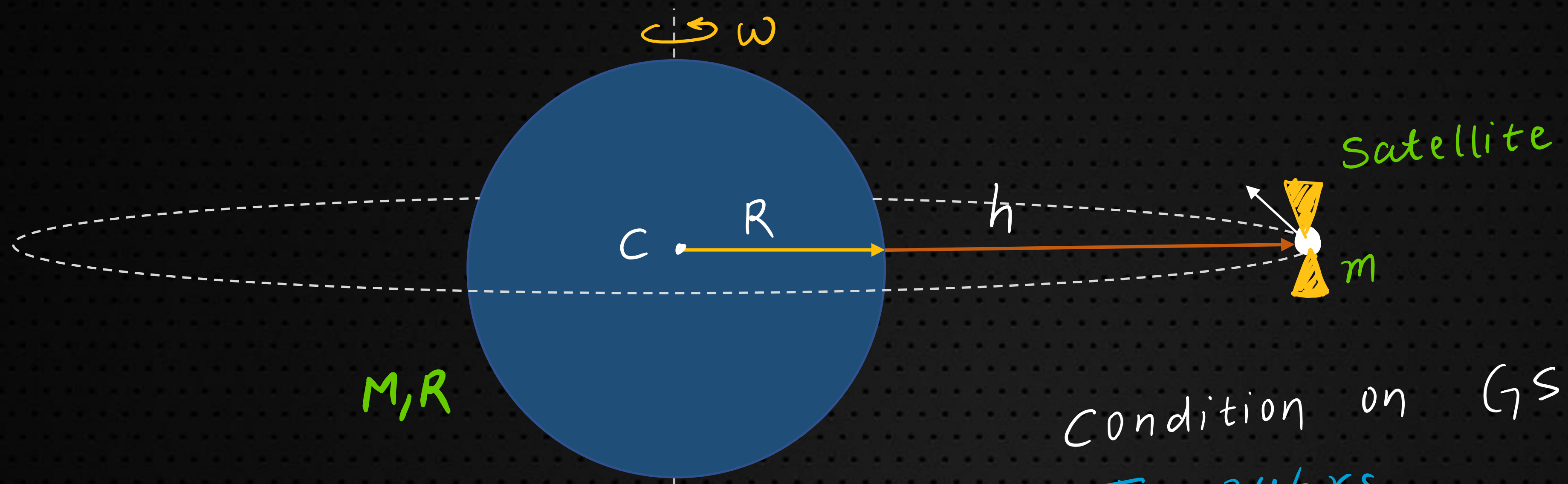
$\therefore K = \left|\frac{U}{2}\right| = |T.E|$

(d) Binding Energy, $\boxed{B.E = -T.E} = \frac{GMm}{2r}$

↳ Energy required to be given so that "m" escapes.



GEOSTATIONARY SATELLITE



$$T = \frac{2\pi}{\sqrt{GM}} r^{3/2} \quad , \quad r = R + h$$

$T = 24\,hrs$, $R = 6400\ Km$,

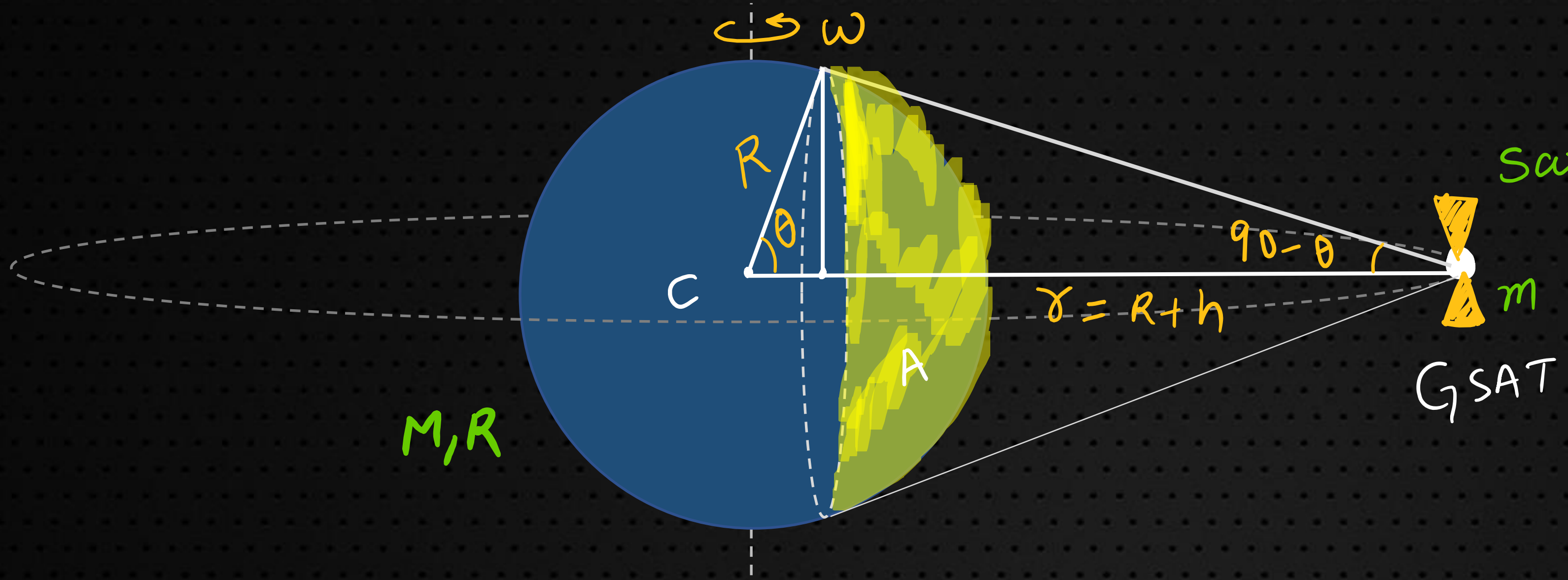
$M = 6 \times 10^{24}\ Kg$

We get $\boxed{h \approx 36000\ Km}$

Condition on GSAT:

1. $T = 24\,hrs$
2. Must lie on equatorial plane
3. must revolve along the direction of rotation of earth.

BROADCASTING AREA



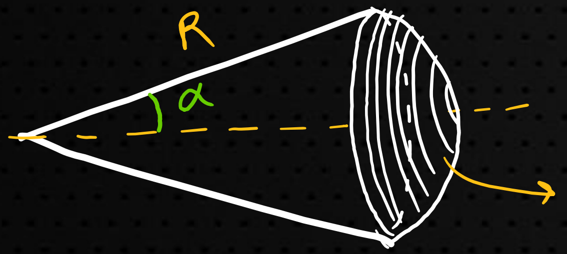
$$\sin(90-\theta) = R/r$$

$$\Rightarrow \cos\theta = \frac{R}{R+h}$$

$$A = 2\pi R^2(1-\cos\theta)$$

$$\Rightarrow A = 2\pi R^2\left(1-\frac{R}{R+h}\right)$$

$$\Rightarrow \boxed{A = \frac{2\pi h R^2}{R+h}}$$



$$A = 2\pi R^2(1-\cos\alpha)$$

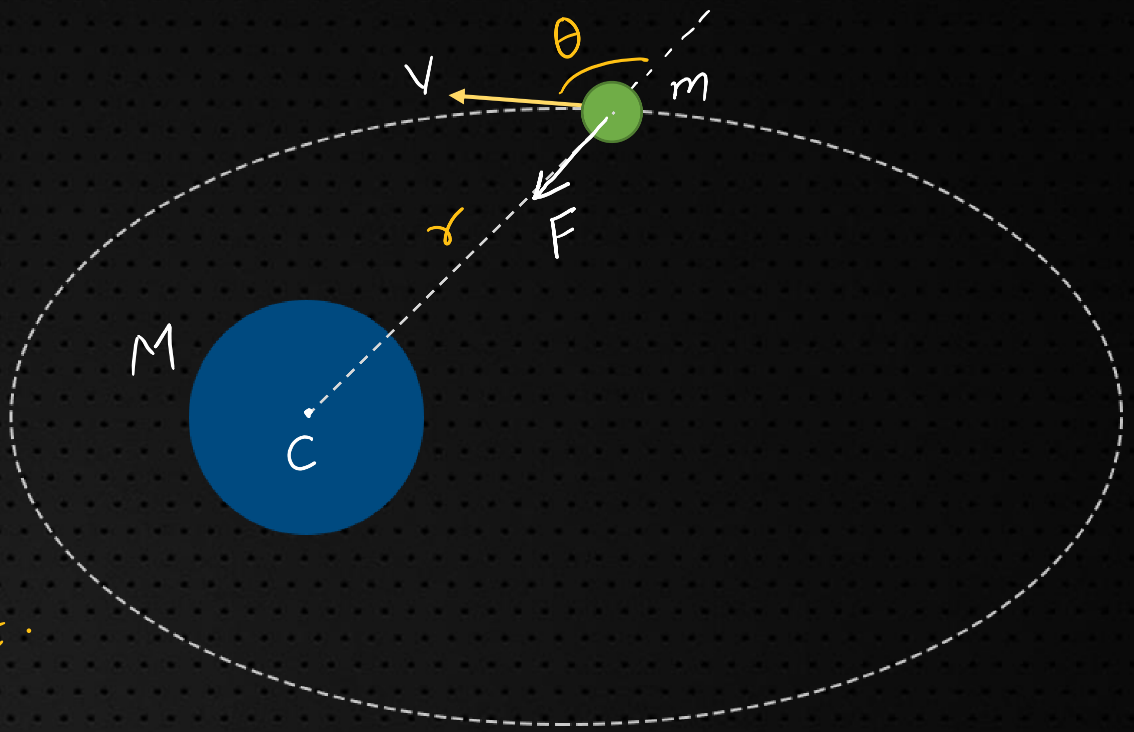
DISCUSSION ON ELLIPTICAL PATH

About τ, L, $U+K$

(1.) As "m" revolves, F always passes through C. Thus torque τ about C is zero.

$\because \tau = 0 \Rightarrow L$ is constant.

or $mvr\sin\theta = \text{const.}$



(2.) $U + K = \text{constant}$

$$\Rightarrow -\frac{GMm}{r} + \frac{1}{2}mv^2 = \text{const.}$$

SPEED AT PERIGEE/APOGEE

$$L_P = L_A \Rightarrow m v_P a(1-e) = m v_A a(1+e) \quad \text{---(1)}$$

$$U_P + K_P = U_A + K_A$$

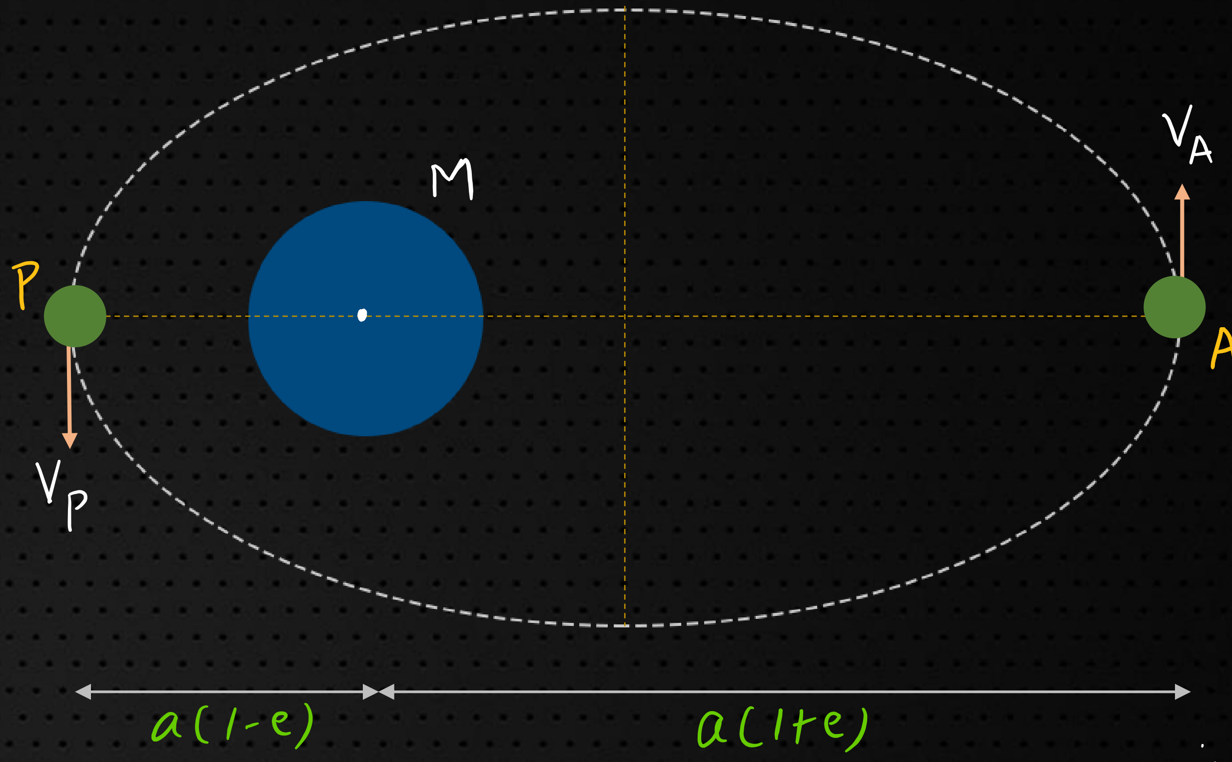
$$\Rightarrow -\frac{GMm}{a(1-e)} + \frac{1}{2} m v_P^2 = -\frac{GMm}{a(1+e)} + \frac{1}{2} m v_A^2 \quad \text{---(2)}$$

solving (1) and (2), we get :

$$V_P = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}, \quad V_A = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)}$$

NOTE: Total energy of system is constant

i.e. $-\frac{GMm}{2a}$.



a : semi-major axis

e: eccentricity

PROJECTED VELOCITY V/S PATH

If :

(a) $V = 0 \Rightarrow$ path 1, straight line

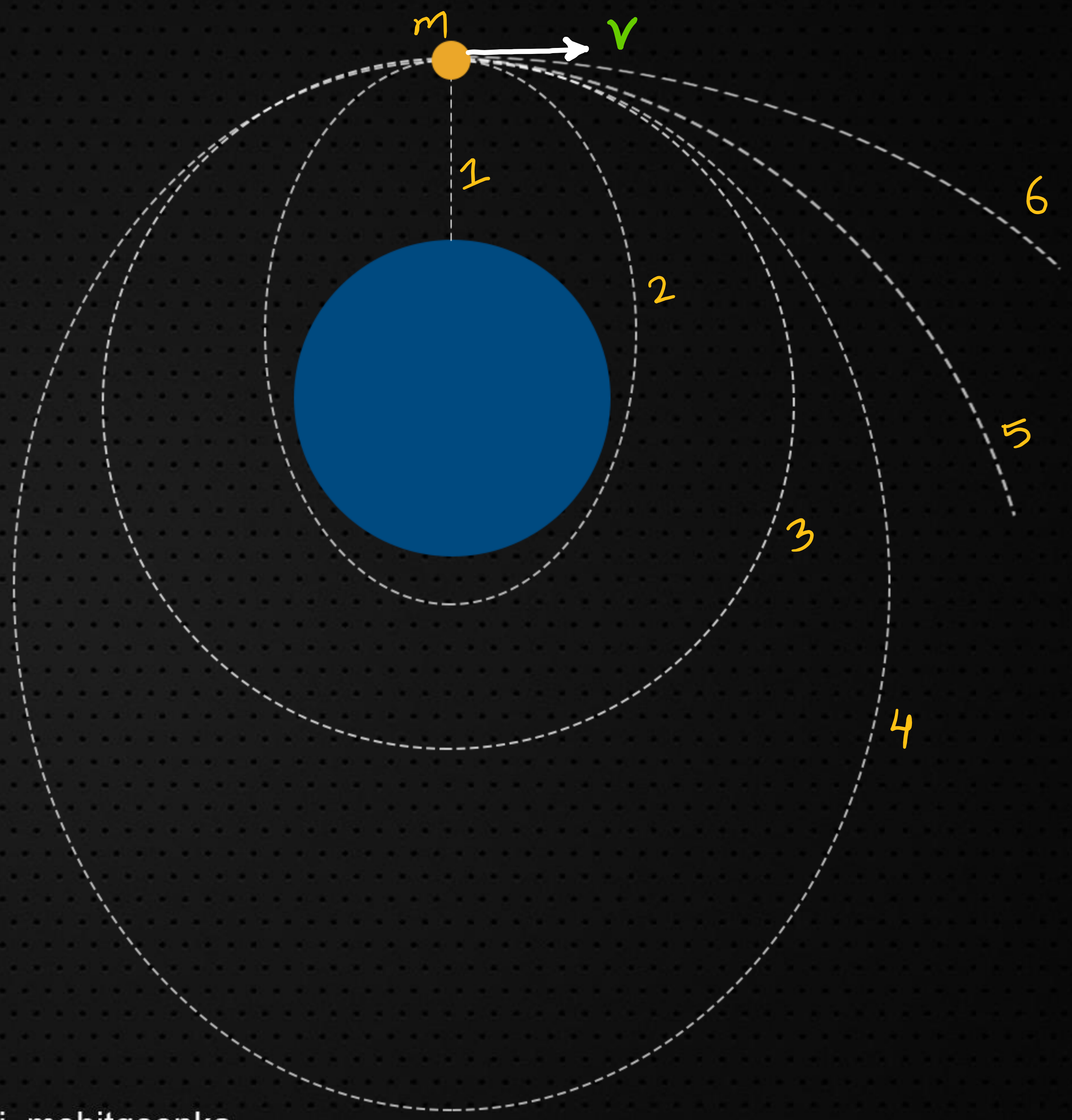
(b) $0 < V < V_o \Rightarrow$ path 2, Elliptical

(c) $V = V_o \Rightarrow$ path 3, Circular

(d) $V_o < V < V_e \Rightarrow$ path 4, Elliptical

(e) $V = V_e \Rightarrow$ path 5, Parabola

(f) $V > V_e \Rightarrow$ path 6, Hyperbola

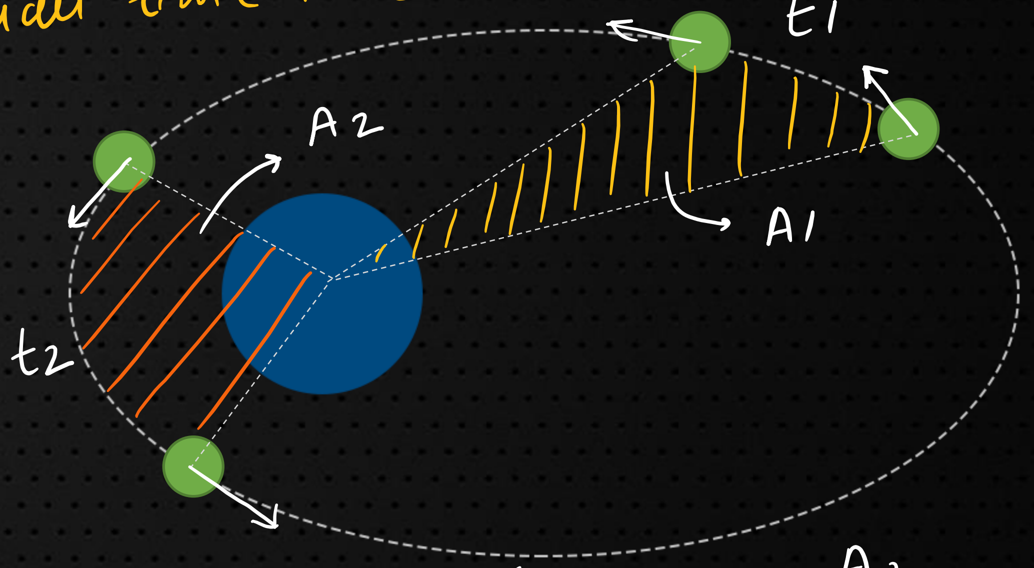


KEPLER'S LAW

1. Law of Orbit : Planet revolves around sun in an elliptical path with sun at one of the foci.

2. Law of Area : Line joining planet and sun sweeps equall Area in equall time interval.

$$\boxed{\frac{dA}{dt} = \frac{L}{2m}} = \text{Constant.}$$



$$\frac{A1}{t1} = \frac{A2}{t2}$$

3. Law of Periods : $\boxed{T^2 \propto a^3}$

DOES SUN REVOLVE ?

Both EARTH and SUN pull upon each.

1. Why do we only talk about EARTH's REVOLUTION ?

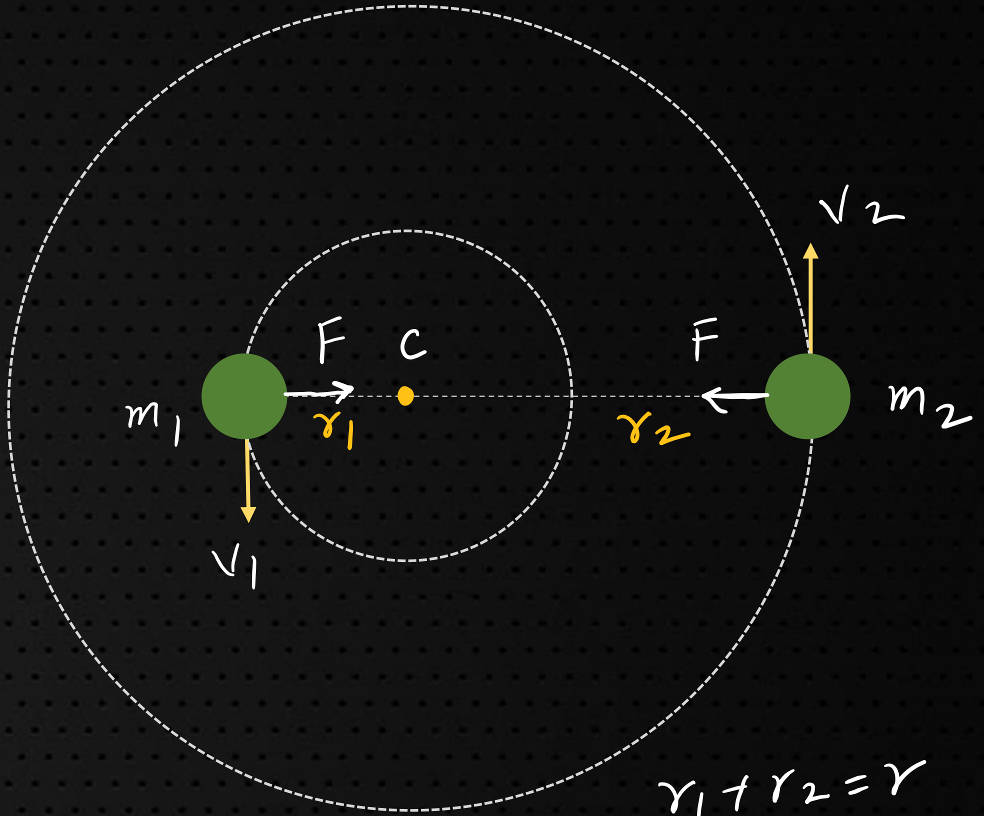
2. Does SUN REVOLVE ?

If yes about which point ?

What's that point called ?

Double Mass System

- Considering 2 Mass System, they revolve about their common centre of mass, "C".
- That point is called "BARYCENTER"



$$r_1 + r_2 = r$$

$$r_1 = \frac{m_2 r}{m_1 + m_2}, \quad r_2 = \frac{m_1 r}{m_1 + m_2}$$

NOTE:

1. Both m_1 and m_2 has same Time Period T and ω, $\omega = \sqrt{\frac{G(m_1+m_2)}{r^3}}$

2. $\frac{V_1}{V_2} = \frac{\omega r_1}{\omega r_2} \Rightarrow \frac{V_1}{V_2} = \frac{r_1}{r_2}$