by Pradeep Kshetrapal

Phvsical World

- **Science** means organized knowledge.

It is human nature to observe things and happenings around in the nature and then to relate them. This knowledge is organized so that it become well connected and logical. Then it is known as Science. It is a systematic attempt to understand natural phenomenon and use this knowledge to predict, modify and control phenomena.

Scientific Method

Scientific methods are used to observe things and natural phenomena. It includes several steps :

- Observations
- Controlled experiments,
- Qualitative and quantitative reasoning,
- Mathematical modeling,
- Prediction and
- Verification or falsification of theories.

There is no 'final' theory in science and no unquestioned authority in science.

- Observations and experiments need theories to support them. Sometimes the existing theory is unable to explain the new observations, hence either new theories are formed or modification is done in the existing theories.
- For example to explain different phenomena in light, theories are changed. To explain bending of light a new Wave-theory was formed, and then to explain photoelectric effect help of quantum mechanics was taken.

Natural Sciences can be broadly divided in three branches namely Physics, Chemistry and biology

- **Physics** is a study of basic laws of nature and their manifestation in different phenomenas.

Principal thrusts in Physics

- There are two principal thrusts in Physics;
- 1.Unification 2. reduction

Unification

- Efforts are made to explain different phenomena in nature on the basis of one or minimum laws. This is principle of Unification.

Example: Phenomena of apple falling to ground, moon revolving around earth and weightlessness in the rocket, all these phenomena are explained with help of *one* Law that is, Newtons Law of Gravitation.

Reductionism

- To understand or to derive the properties of a bigger or more complex system the properties of its simpler constituents are taken into account. This approach is called reductionism.

It is supposed to be the heart of Physics.
For example a complex thermo dynamical system can be understood by the properties of its constituent like kinetic energy of molecules and atoms.

- ***The scope of Physics*** can be divided in to two domains; Macroscopic and Microscopic.
- Macroscopic domain includes phenomena at the level of Laboratory, terrestrial and astronomical scales.
- Microscopic domain I ncludes atomic, molecular and nuclear phenomena.
- Recently third domain in between is also thought of with name Mesoscopic Physics. This deals with group of Hundreds of atoms
  - Scope of physics is very wide and exciting because it deals with objects of size as large as Universe (10^{25}m) and as small as 10^{-14} m, the size of a nucleus.

The excitement of Physics is experienced in many fields Like:

- Live transmissions through television.
- Computers with high speed and memory,
- Use of Robots,
- Lasers and their applications

Physics in relation to other branches of Science

Physics in relation to Chemistry.

- Chemical bonding, atomic number and complex structure can be explained by physics phenomena of Electrostatic forces,
- taking help of X-ray diffraction.

Physics in relation to other Science

- Physics in relation to Biological Sciences: Physics helps in study of Biology through its inventions. Optical microscope helps to study bio-samples, electron microscope helps to study biological cells. X-rays have many applications in biological sciences. Radio isotopes are used in cancer.
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Physics in relation with Astronomy:

- Giant astronomical telescope developed in physics are used for observing planets. Radio telescopes have enabled astronomers to observe distant limits of universe.
- Physics related to other sciences: Laws of Physics are used to study different phenomenas in other sciences like Biophysics, oceanography, seismology etc.

<u>Fundamental Forces in Nature</u>

There is a large number of forces experienced or applied. These may be macroscopic forces like gravitation, friction, contact forces and microscopic forces like electromagnetic and inter-atomic forces. But all these forces arise from some basic forces called Fundamental Forces.

Fundamental Forces in Nature..

1. Gravitational force.

- *It is due to Mass of the two bodies.*
- *It is always attractive.*
- *It operates in all objects of universe.*
- *Its range is infinite*

It's a weak force. 10^{-38} times compared to strong Nuclear force

2.Electromagnetic Forces:

- It's due to stationery or moving Electrical charge
- It may be attractive or repulsive.
- It operates on charged particles
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- Its stronger 10^{36} times than gravitational force but 10^{-2} times of strong Nuclear force.

3. Strong nuclear force:

- Operate between Nucleons
- It may be attractive or repulsive
- Its range is very short, within nuclear size (10^{-15} m).
- Its strongest force in nature

4.Weak Nuclear force:

- Operate within nucleons I.e. elementary particles like electron and neutrino.
- It appears during radioactive b decay.
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<u>Conservation Laws</u>

- In any physical phenomenon governed by different forces, several quantities do not change with time. These special quantities are conserved quantities of nature.

1.For motion under conservative force, the total mechanical Energy of a body is constant.

2.Total energy of a system is conserved, and it is valid across all domains of nature from microscopic to macroscopic. Total energy of the universe is believed to be constant.

3.Conservation of Mass was considered another conservation law, till advent of Einstein. Then it was converted to law of conservation of mass plus energy. Because mass is converted into energy and vice-versa according to equation E = mc^2 The examples are annihilation and pair production.

4.Momentum is another quantity which is preserved. Similar is angular momentum of an isolated system.

5.Conservation of Electric charge is a fundamental law of nature.

6. Later there was development of law of conservation of attributes called baryon number, lepton number and so on.

The laws of nature do not change with change of space and time. This is known as symmetry of space and time. This and some other symmetries play a central role in modern physics. Conservation laws are connected to this.

Laws of Physics related to technology :

Principal of Physics	***Technology***
Electromagnetic Induction	Electricity Generation
Laws of Thermodynamics	Steam, petrol, or diesel Engine
Electromagnetic Waves propagation	Radio, TV, Phones
Nuclear chain reaction	Nuclear reactor for power
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Bernoulli's theorem	Aero planes
Population inversion	Lasers
X-rays	Medical Diagnosis
Ultra high magnetic fields	Superconductors
Digital electronics	Computers and calculators
Electromagnetic Induction	Electricity Generation

Physicist and their contributions

Name	***Contribution***	***country***
Isaac Newton	Law of Gravitation, Laws of Motion, Reflecting telescope	U.K.
Galileo Galilei	Law of Inertia	Italy
Archimedes	Principle of Buoyancy, Principle of Lever	Greece
James Clerk Maxwell	Electromagnetic theory, light is an e/m wave.	U.K.
W.K.Roentgen	X-rays	Germany
Marie S. Curie	Discovery of Radium, Polonium, study of Radioactivity	Poland
Albert Einstein	Law of Photo electricity, Theory of Relativity	Germany
S.N.Bose	Quantum Statistics	India
James Chadwick	Neutron	U.K.
Niels Bohr	Quantum model of Hydrogen atom	Denmark
Earnest Rutherford	Nuclear model of Atom	New Zealand
C.V.Raman	Inelastic Scattering of light by molecules	India
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R.A.Milikan	Measurement of Electronic Charge	U.S.A.
E.O.Lawrence	Cyclotron	U.S.A.

Wolfgong Pauli	Quantum Exclusion principle	Austria
Louis de Broglie	Wave nature of matter	France
J.J.Thomson	Electron	U.K.
S.Chandrashekhar	Chandrashekhar limit, structure of stars	India
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Notes and exercises
by

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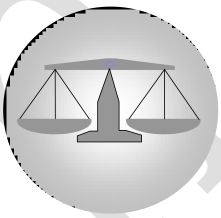

Physical World, Units, Dimensions and Measurements

Physical World

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Archimedes	Principle of Buoyancy, Principle of Lever	Greece
James Clerk Maxwell	Electromagnetic theory, light is an e/m wave.	U.K.
W.K.Roentgen	X-rays	Germany
Marie S. Curie	Discovery of Radium, Polonium, study of Radioactivity	Poland
Albert Einstein	Law of Photo electricity, Theory of Relativity	Germany
S.N.Bose	Quantum Statistics	India
James Chadwick	Neutron	U.K.
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R.A.Milikan	Measurement of Electronic Charge	U.S.A.
E.O.Lawrence	Cyclotron	U.S.A.
Wolfgong Pauli	Quantum Exclusion principle	Austria
Louis de Broglie	Wave nature of matter	France
J.J.Thomson	Electron	U.K.
S.Chandrashekhar	Chandrashekhar limit, structure of stars	India
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1.1 Physical Quantity

A quantity which can be measured and by which various physical happenings can be explained and expressed in form of laws is called a physical quantity. For example length, mass, time, force *etc.*

On the other hand various happenings in life *e.g.*, happiness, sorrow *etc.* are not physical quantities because these can not be measured.

Measurement is necessary to determine magnitude of a physical quantity, to compare two similar physical quantities and to prove physical laws or equations.

A physical quantity is represented completely by its magnitude and unit. For example, 10 *metre* means a length which is ten times the unit of length 1 *kg*. Here 10 represents the numerical value of the given quantity and *metre* represents the unit of quantity under consideration. Thus in expressing a physical quantity we choose a unit and then find that how many times that unit is contained in the given physical quantity, *i.e.*

$$\text{Physical quantity } (Q) = \text{Magnitude} \times \text{Unit } = n \times u$$

Where, n represents the numerical value and u represents the unit. Thus while expressing definite amount of physical quantity, it is clear that as the unit(u) changes, the magnitude(n) will also change but product 'nu' will remain same.

i.e. $n\,u = \text{constant}$, or $n_1u_1 = n_2u_2 = \text{constant}$; $\therefore\ n \propto \frac{1}{u}$

i.e. magnitude of a physical quantity and units are inversely proportional to each other .Larger the unit, smaller will be the magnitude.

1.2 Types of Physical Quantity

(1) **Ratio (numerical value only) :** When a physical quantity is a ratio of two similar quantities, it has no unit.

e.g. Relative density = Density of object/Density of water at 4°*C*

Refractive index = Velocity of light in air/Velocity of light in medium

Strain = Change in dimension/Original dimension

Note : ❑ Angle is exceptional physical quantity, which though is a ratio of two similar physical quantities (angle = arc / radius) but still requires a unit (degrees or radians) to specify it along with its numerical value.

(2) **Scalar (Magnitude only) :** These quantities do not have any direction *e.g.* Length, time, work, energy *etc.*

Magnitude of a physical quantity can be negative. In that case negative sign indicates that the numerical value of the quantity under consideration is negative. It does not specify the direction.

Scalar quantities can be added or subtracted with the help of following ordinary laws of addition or subtraction.

(3) **Vector (magnitude and direction) :** *e.g.* displacement, velocity, acceleration, force *etc.*

Vector physical quantities can be added or subtracted according to vector laws of addition. These laws are different from laws of ordinary addition.

Note : ❑ There are certain physical quantities which behave neither as scalar nor as vector. For example, moment of inertia is not a vector as by changing the sense of rotation its value is not changed. It is also not a scalar as it has different values in different directions (*i.e.* about different axes). Such physical quantities are called Tensors.

1.3 Fundamental and Derived Quantities

(1) **Fundamental quantities :** Out of large number of physical quantities which exist in nature, there are only few quantities which are independent of all other quantities and do not require the help of any other physical quantity for their definition, therefore these are called absolute quantities. These quantities are also called fundamental or base quantities, as all other quantities are based upon and can be expressed in terms of these quantities.

(2) **Derived quantities :** All other physical quantities can be derived by suitable multiplication or division of different powers of fundamental quantities. These are therefore called derived quantities.

If length is defined as a fundamental quantity then area and volume are derived from length and are expressed in term of length with power 2 and 3 over the term of length.

Note : ❑ In mechanics Length, Mass and time are arbitrarily chosen as fundamental quantities. However this set of fundamental quantities is not a unique choice. In fact any three quantities in mechanics can be termed as fundamental as all other quantities in mechanics can be expressed in terms of these. *e.g.* if speed and time are taken as fundamental quantities, length will become a derived quantity because then length will be expressed as Speed $\times$ Time. and if force and acceleration are taken as fundamental quantities, then mass will be defined as Force / acceleration and will be termed as a derived quantity.

1.4 Fundamental and Derived Units

Normally each physical quantity requires a unit or standard for its specification so it appears that there must be as many units as there are physical quantities. However, it is not so. It has been found that if in *mechanics* we choose arbitrarily units of any *three* physical quantities we can express the units of all other physical quantities in mechanics in terms of these. Arbitrarily the physical quantities *mass*, *length* and *time* are choosen for this purpose. *So any unit of mass, length and time in mechanics is called a* **fundamental, absolute or base unit.** *Other units which can be expressed in terms of fundamental units, are called derived units*. For example light year or *km* is a fundamental units as it is a unit of length while s^{-1}, m^2 or kg/m are derived units as these are derived from units of time, mass and length respectively.

System of units : A complete set of units, both fundamental and derived for all kinds of physical quantities is called system of units. The common systems are given below –

(1) **CGS system :** The system is also called Gaussian system of units. In it length, mass and time have been chosen as the fundamental quantities and corresponding fundamental units are centimetre (*cm*), gram (*g*) and second (*s*) respectively.

(2) **MKS system :** The system is also called Giorgi system. In this system also length, mass and time have been taken as fundamental quantities, and the corresponding fundamental units are *metre*, kilogram and second.

(3) **FPS system :** In this system foot, pound and second are used respectively for measurements of length, mass and time. In this system force is a derived quantity with unit poundal.

(4) **S. I. system :** It is known as International system of units, and is infact extended system of units applied to whole physics. There are seven fundamental quantities in this system. These quantities and their units are given in the following table

Quantity	**Name of Unit**	**Symbol**
Length	metre	*m*
Mass	kilogram	*kg*
Time	second	*s*
Electric Current	ampere	*A*
Temperature	Kelvin	*K*
Amount of Substance	mole	*mol*
Luminous Intensity	candela	*cd*

Besides the above seven fundamental units two supplementary units are also defined –

Radian (*rad*) for plane angle and Steradian (*sr*) for solid angle.

Note : ❑ Apart from fundamental and derived units we also use very frequently practical units. These may be fundamental or derived units

e.g., light year is a practical unit (fundamental) of distance while horse power is a practical unit (derived) of power.

❑ Practical units may or may not belong to a system but can be expressed in any system of units

e.g., 1 mile = 1.6 *km* = 1.6×10^3 *m*.

1.5 S.I. Prefixes

In physics we have to deal from very small (*micro*) to very large (*macro*) magnitudes as one side we talk about the atom while on the other side of universe, *e.g.*, the mass of an electron is 9.1×10^{-31} *kg* while that of the sun is 2×10^{30} *kg*. To express such large or small magnitudes simultaneously we use the following prefixes :

Power of 10	**Prefix**	**Symbol**
10^{18}	exa	*E*
10^{15}	peta	*P*
10^{12}	tera	*T*
10^{9}	giga	*G*
10^{6}	mega	*M*
10^{3}	kilo	*k*
10^{2}	hecto	*h*
10^{1}	deca	*da*
10^{-1}	deci	*d*
10^{-1}	centi	*c*
10^{-3}	milli	*m*
10^{-6}	micro	μ

10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1.6 Standards of Length, Mass and Time

(1) **Length :** Standard metre is defined in terms of wavelength of light and is called atomic standard of length.

The metre is the distance containing 1650763.73 wavelength in vacuum of the radiation corresponding to orange red light emitted by an atom of krypton-86.

Now a days metre is defined as length of the path travelled by light in vacuum in 1/299,7792, 458 part of a second.

(2) **Mass :** The mass of a cylinder made of platinum-iridium alloy kept at International Bureau of Weights and Measures is defined as 1 *kg*.

On atomic scale, 1 *kilogram* is equivalent to the mass of 5.0188×10^{25} atoms of ${}_6C^{12}$ (an isotope of carbon).

(3) **Time :** 1 *second* is defined as the time interval of 9192631770 vibrations of radiation in *Cs*-133 atom. This radiation corresponds to the transition between two hyperfine level of the ground state of *Cs*-133.

1.7 Practical Units

(1) **Length :**

(i) 1 fermi = 1 $fm = 10^{-15}\ m$

(ii) 1 *X*-ray unit = 1$XU = 10^{-13}\ m$

(iii) 1 angstrom = 1Å = $10^{-10}\ m = 10^{-8}\ cm = 10^{-7}\ mm = 0.1\ \mu mm$

(iv) 1 micron = $\mu m = 10^{-6}\ m$

(v) 1 astronomical unit = 1 *A.U.* = $1.49 \times 10^{11}\ m \approx 1.5 \times 10^{11}\ m \approx 10^8\ km$

(vi) 1 Light year = 1 $ly = 9.46 \times 10^{15}\ m$

(vii) 1 *Parsec* = 1*pc* = 3.26 light year

(2) **Mass :**

(i) Chandra Shekhar unit : 1 *CSU* = 1.4 times the mass of sun = $2.8 \times 10^{30}\ kg$

(ii) Metric tonne : 1 Metric tonne = 1000 *kg*

(iii) Quintal : 1 Quintal = 100 *kg*

(iv) Atomic mass unit (*amu*) : $amu = 1.67 \times 10^{-27}\ kg$ mass of proton or neutron is of the order of 1 *amu*

(3) **Time :**

(i) Year : It is the time taken by earth to complete 1 revolution around the sun in its orbit.

(ii) Lunar month : It is the time taken by moon to complete 1 revolution around the earth in its orbit.

1 *L.M.* = 27.3 days

(iii) Solar day : It is the time taken by earth to complete one rotation about its axis with respect to sun. Since this time varies from day to day, average solar day is calculated by taking average of the duration of all the days in a year and this is called Average Solar day.

1 Solar year = 365.25 average solar day

or average solar day $= \frac{1}{365.25}$ the part of solar year

(iv) Sedrial day : It is the time taken by earth to complete one rotation about its axis with respect to a distant star.

1 Solar year = 366.25 Sedrial day = 365.25 average solar day

Thus 1 Sedrial day is less than 1 solar day.

(v) Shake : It is an obsolete and practical unit of time.

1 Shake = 10^{-8} *sec*

1.8 Dimensions of a Physical Quantity

When a derived quantity is expressed in terms of fundamental quantities, it is written as a product of different powers of the fundamental quantities. The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its dimensions.

To make it more clear, consider the physical quantity force

Force = mass × acceleration $= \frac{\text{mass} \times \text{velocity}}{\text{time}} = \frac{\text{mass} \times \text{length/time}}{\text{time}}$ = mass × length × $(\text{time})^{-2}$ .... (i)

Thus, the dimensions of force are 1 in mass, 1 in length and – 2 in time.

Here the physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to indicate that the equation is among the dimensions and not among the magnitudes.

Thus equation (i) can be written as [force] = $[MLT^{-2}]$.

Such an expression for a physical quantity in terms of the fundamental quantities is called the dimensional equation. If we consider only the R.H.S. of the equation, the expression is termed as dimensional formula.

Thus, dimensional formula for force is, $[MLT^{-2}]$.

1.9 Important Dimensions of Complete Physics

Mechanics

S. N.	Quantity	Unit	Dimension
(1)	Velocity or speed (v)	m/s	$[M^0L^1T^{-1}]$
(2)	Acceleration (a)	m/s^2	$[M^0LT^{-2}]$
(3)	Momentum (P)	$kg\text{-}m/s$	$[M^1L^1T^{-1}]$
(4)	Impulse (I)	*Newton-sec* or $kg\text{-}m/s$	$[M^1L^1T^{-1}]$
(5)	Force (F)	*Newton*	$[M^1L^1T^{-2}]$
(6)	Pressure (P)	*Pascal*	$[M^1L^{-1}T^{-2}]$
(7)	Kinetic energy (E_K)	*Joule*	$[M^1L^2T^{-2}]$
(8)	Power (P)	*Watt* or *Joule/s*	$[M^1L^2T^{-3}]$
(9)	Density (d)	kg/m^3	$[M^1L^{-3}T^0]$
(10)	Angular displacement (θ)	*Radian* (*rad.*)	$[M^0L^0T^0]$
(11)	Angular velocity (ω)	*Radian*/sec	$[M^0L^0T^{-1}]$
(12)	Angular acceleration (α)	*Radian/sec*^{2}	$[M^0L^0T^{-2}]$
(13)	Moment of inertia (I)	$kg\text{-}m^2$	$[M^1L^2T^0]$

S. N.	Quantity	Unit	Dimension
(14)	Torque (τ)	*Newton-meter*	$[M^1L^2T^{-2}]$
(15)	Angular momentum (L)	*Joule-sec*	$[M^1L^2T^{-1}]$
(16)	Force constant or spring constant (k)	*Newton/m*	$[M^1L^0T^{-2}]$
(17)	Gravitational constant (G)	*N-m²/kg²*	$[M^{-1}L^3T^{-2}]$
(18)	Intensity of gravitational field (E_g)	*N/kg*	$[M^0L^1T^{-2}]$
(19)	Gravitational potential (V_g)	*Joule/kg*	$[M^0L^2T^{-2}]$
(20)	Surface tension (T)	*N/m* or *Joule/m²*	$[M^1L^0T^{-2}]$
(21)	Velocity gradient (V_g)	*Second*$^{-1}$	$[M^0L^0T^{-1}]$
(22)	Coefficient of viscosity (η)	*kg/m-s*	$[M^1L^{-1}T^{-1}]$
(23)	Stress	*N/m²*	$[M^1L^{-1}T^{-2}]$
(24)	Strain	No unit	$[M^0L^0T^0]$
(25)	Modulus of elasticity (E)	*N/m²*	$[M^1L^{-1}T^{-2}]$
(26)	Poisson Ratio (σ)	No unit	$[M^0L^0T^0]$
(27)	Time period (T)	*Second*	$[M^0L^0T^1]$
(28)	Frequency (n)	*Hz*	$[M^0L^0T^{-1}]$

Heat

S. N.	Quantity	Unit	Dimension
(1)	Temperature (T)	*Kelvin*	$[M^0L^0T^0\theta^1]$
(2)	Heat (Q)	*Joule*	$[ML^2T^{-2}]$
(3)	Specific Heat (c)	*Joule/kg-K*	$[M^0L^2T^{-2}\theta^{-1}]$
(4)	Thermal capacity	*Joule/K*	$[M^1L^2T^{-2}\theta^{-1}]$
(5)	Latent heat (L)	*Joule/kg*	$[M^0L^2T^{-2}]$
(6)	Gas constant (R)	*Joule/mol-K*	$[M^1L^2T^{-2}\theta^{-1}]$
(7)	Boltzmann constant (k)	*Joule/K*	$[M^1L^2T^{-2}\theta^{-1}]$
(8)	Coefficient of thermal conductivity (K)	*Joule/m-s-K*	$[M^1L^1T^{-3}\theta^{-1}]$
(9)	Stefan's constant (σ)	*Watt/m²-K⁴*	$[M^1L^0T^{-3}\theta^{-4}]$
(10)	Wien's constant (b)	*Meter-K*	$[M^0L^1T^0\theta^1]$
(11)	Planck's constant (h)	*Joule-s*	$[M^1L^2T^{-1}]$
(12)	Coefficient of Linear Expansion (α)	*Kelvin*$^{-1}$	$[M^0L^0T^0\theta^{-1}]$
(13)	Mechanical eq. of Heat (J)	*Joule/Calorie*	$[M^0L^0T^0]$
(14)	Vander wall's constant (a)	*Newton-m*^{4}	$[ML^5T^{-2}]$
(15)	Vander wall's constant (b)	m^3	$[M^0L^3T^0]$

Electricity

S. N.	Quantity	Unit	Dimension
(1)	Electric charge (q)	*Coulomb*	$[M^0L^0T^1A^1]$
(2)	Electric current (I)	*Ampere*	$[M^0L^0T^0A^1]$
(3)	Capacitance (C)	*Coulomb/volt or Farad*	$[M^{-1}L^{-2}T^4A^2]$
(4)	Electric potential (V)	*Joule/coulomb*	$M^1L^2T^{-3}A^{-1}$

S. N.	Quantity	Unit	Dimension
(5)	Permittivity of free space (ε_0)	$\frac{Coulomb^2}{Newton-meter^2}$	$[M^{-1}L^{-3}T^4A^2]$
(6)	Dielectric constant (K)	Unitless	$[M^0L^0T^0]$
(7)	Resistance (R)	*Volt/Ampere* or *ohm*	$[M^1L^2T^{-3}A^{-2}]$
(8)	Resistivity or Specific resistance (ρ)	*Ohm-meter*	$[M^1L^3T^{-3}A^{-2}]$
(9)	Coefficient of Self-induction (L)	$\frac{volt-second}{ampere}$ or *henery or ohm-second*	$[M^1L^2T^{-2}A^{-2}]$
(10)	Magnetic flux (ϕ)	*Volt-second* or *weber*	$[M^1L^2T^{-2}A^{-1}]$
(11)	Magnetic induction (B)	$\frac{newton}{ampere-meter}$ $\frac{Joule}{ampere-meter^2}$ $\frac{volt-second}{meter^2}$ or *Tesla*	$[M^1L^0T^{-2}A^{-1}]$
(12)	Magnetic Intensity (H)	*Ampere/meter*	$[M^0L^{-1}T^0A^1]$
(13)	Magnetic Dipole Moment (M)	*Ampere-meter*^{2}	$[M^0L^2T^0A^1]$
(14)	Permeability of Free Space (μ_0)	$\frac{Newton}{ampere^2}$ or $\frac{Joule}{ampere^2-meter}$ or $\frac{Volt-second}{ampere-meter}$ or $\frac{Ohm-second}{meter}$ or $\frac{henery}{meter}$	$[M^1L^1T^{-2}A^{-2}]$
(15)	Surface charge density (σ)	$Coulomb\ metre^{-2}$	$[M^0L^{-2}T^1A^1]$
(16)	Electric dipole moment (p)	$Coulomb-meter$	$[M^0L^1T^1A^1]$
(17)	Conductance (G) ($1/R$)	ohm^{-1}	$[M^{-1}L^{-2}T^3A^2]$
(18)	Conductivity (σ) ($1/\rho$)	$ohm^{-1}meter^{-1}$	$[M^{-1}L^{-3}T^3A^2]$
(19)	Current density (J)	*Ampere/m*^{2}	$M^0L^{-2}T^0A^1$
(20)	Intensity of electric field (E)	*Volt/meter*, *Newton/coulomb*	$M^1L^1T^{-3}A^{-1}$
(21)	Rydberg constant (R)	m^{-1}	$M^0L^{-1}T^0$

1.10 Quantities Having Same Dimensions

S. N.	Dimension	Quantity
(1)	$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient and decay constant
(2)	$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque, moment of force
(3)	$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
(4)	$[M^1L^1T^{-1}]$	Momentum, impulse
(5)	$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity
(6)	$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
(7)	$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
(8)	$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area)
(9)	$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.
(10)	$[M^0L^2T^{-2}]$	Latent heat and gravitational potential
(11)	$[M^0L^2T^{-2}\theta^{-1}]$	Thermal capacity, gas constant, Boltzmann constant and entropy
(12)	$[M^0L^0T^1]$	$\sqrt{l/g}, \sqrt{m/k}, \sqrt{R/g}$, where l = length g = acceleration due to gravity, m = mass, k = spring constant
(13)	$[M^0L^0T^1]$	L/R, $\sqrt{LC}$, RC where L = inductance, R = resistance, C = capacitance

(14)	$[ML^2T^{-2}]$	$I^2Rt, \frac{V^2}{R}t, VIt, qV, LI^2, \frac{q^2}{C}, CV^2$ where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance

1.11 Application of Dimensional Analysis

(1) **To find the unit of a physical quantity in a given system of units** : Writing the definition or formula for the physical quantity we find its dimensions. Now in the dimensional formula replacing M, L and T by the fundamental units of the required system we get the unit of physical quantity. However, sometimes to this unit we further assign a specific name, *e.g.*, Work = Force × Displacement

So $[W] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

So its units in C.G.S. system will be $g\ cm^2/s^2$ which is called *erg* while in M.K.S. system will be $kg\ m^2/s^2$ which is called *joule*.

Sample problems based on unit finding

Problem 1. The equation $\left(P + \frac{a}{V^2}\right)(V - b)$ = constant. The units of a is **[MNR 1995; AFMC 1995]**

(a) $Dyne \times cm^5$ (b) $Dyne \times cm^4$ (c) $Dyne / cm^3$ (d) $Dyne / cm^2$

Solution : (b) According to the principle of dimensional homogenity $[P] = \left[\frac{a}{V^2}\right]$

$\Rightarrow [a] = [P]\,[V^2] = [ML^{-1}T^{-2}][L^6] = [ML^5T^{-2}]$

or unit of $a = gm \times cm^5 \times sec^{-2}$ = Dyne × cm^4

Problem 2. If $x = at + bt^2$, where x is the distance travelled by the body in *kilometre* while t the time in seconds, then the units of b are **[CBSE 1993]**

(a) km/s (b) $km\text{-}s$ (c) km/s^2 (d) $km\text{-}s^2$

Solution : (c) From the principle of dimensional homogenity $[x] = [bt^2] \Rightarrow [b] = \left[\frac{x}{t^2}\right]$ $\therefore$ Unit of $b = km/s^2$.

Problem 3. The unit of absolute permittivity is **[EAMCET (Med.) 1995; Pb. PMT 2001]**

(a) *Farad - meter* (b) *Farad / meter* (c) $Farad/meter^2$ (d) *Farad*

Solution : (b) From the formula $C = 4\pi\varepsilon_0 R$ $\therefore \varepsilon_0 = \frac{C}{4\pi R}$

By substituting the unit of capacitance and radius : unit of ε_0 = *Farad/ meter*.

Problem 4. Unit of Stefan's constant is

(a) Js^{-1} (b) $Jm^{-2}s^{-1}K^{-4}$ (c) Jm^{-2} (d) Js

Solution : (b) Stefan's formula $\frac{Q}{At} = \sigma T^4$ $\therefore \sigma = \frac{Q}{AtT^4}$ $\therefore$ Unit of $\sigma = \frac{\text{Joule}}{m^2 \times \sec \times K^4} = Jm^{-2}s^{-1}K^{-4}$

Problem 5. The unit of surface tension in SI system is

[MP PMT 1984; AFMC 1986; CPMT 1985, 87; CBSE 1993; Karnataka CET (Engg/Med.) 1999; DCE 2000, 01]

(a) $Dyne / cm^2$ (b) *Newton/m* (c) *Dyne/cm* (d) $Newton/m^2$

Solution : (b) From the formula of surface tension $T = \frac{F}{l}$

By substituting the S.I. units of force and length, we will get the unit of surface tension = *Newton/m*

Problem 6. A suitable unit for gravitational constant is **[MNR 1988]**

(a) $kg\ \ metre\ \sec^{-1}$ (b) $Newton\ metre^{-1}\sec$ (c) $Newton\ \ metre^2 kg^{-2}$ (d) $kg\ metre\ \sec^{-1}$

Solution : (c) As $F = \frac{Gm_1 m_2}{r^2}$ $\therefore\ G = \frac{Fr^2}{m_1 m_2}$

Substituting the unit of above quantities unit of G = $Newton\ \ metre^2 kg^{-2}$.

Problem 7. The SI unit of universal gas constant (R) is

[MP Board 1988; JIPMER 1993; AFMC 1996; MP PMT 1987, 94; CPMT 1984, 87; UPSEAT 1999]

(a) $Watt\ K^{-1} mol^{-1}$ (b) $Newton\ K^{-1} mol^{-1}$ (c) $Joule\ K^{-1} mol^{-1}$ (d) $Erg\ K^{-1} mol^{-1}$

Solution : (c) Ideal gas equation $PV = nRT$ $\therefore [R] = \frac{[P][V]}{[nT]} = \frac{[ML^{-1}T^{-2}][L^3]}{[mole][K]} = \frac{[ML^2T^{-2}]}{[mole]\times[K]}$

So the unit will be $Joule\ K^{-1} mol^{-1}$.

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(2) **To find dimensions of physical constant or coefficients** : As dimensions of a physical quantity are unique, we write any formula or equation incorporating the given constant and then by substituting the dimensional formulae of all other quantities, we can find the dimensions of the required constant or coefficient.

(i) Gravitational constant : According to Newton's law of gravitation $F = G\frac{m_1 m_2}{r^2}$ or $G = \frac{Fr^2}{m_1 m_2}$

Substituting the dimensions of all physical quantities $[G] = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$

(ii) Plank constant : According to Planck $E = h\nu$ or $h = \frac{E}{\nu}$

Substituting the dimensions of all physical quantities $[h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

(iii) Coefficient of viscosity : According to Poiseuille's formula $\frac{dV}{dt} = \frac{\pi pr^4}{8\eta l}$ or $\eta = \frac{\pi pr^4}{8l(dV/dt)}$

Substituting the dimensions of all physical quantities $[\eta] = \frac{[ML^{-1}T^{-2}][L^4]}{[L][L^3/T]} = [ML^{-1}T^{-1}]$

Sample problems based on dimension finding

Problem 8. $X = 3YZ^2$ find dimension of Y in (MKSA) system, if X and Z are the dimension of capacity and magnetic

field respectively **[MP PMT 2003]**

(a) $M^{-3}L^{-2}T^{-4}A^{-1}$ (b) ML^{-2} (c) $M^{-3}L^{-2}T^4A^4$ (d) $M^{-3}L^{-2}T^8A^4$

Solution : (d) $X = 3YZ^2$ $\therefore\ [Y] = \frac{[X]}{[Z^2]} = \frac{[M^{-1}L^{-2}T^4A^2]}{[MT^{-2}A^{-1}]^2} = [M^{-3}L^{-2}T^8A^4]$.

Problem 9. Dimensions of $\frac{1}{\mu_0\varepsilon_0}$, where symbols have their usual meaning, are **[AIEEE 2003]**

(a) $[LT^{-1}]$ (b) $[L^{-1}T]$ (c) $[L^{-2}T^2]$ (d) $[L^2T^{-2}]$

Solution : (d) We know that velocity of light $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ $\therefore \frac{1}{\mu_0\varepsilon_0} = C^2$

$\therefore$ So $\left[\frac{1}{\mu_0 \varepsilon_0}\right] = [LT^{-1}]^2 = [L^2T^{-2}]$.

Problem 10. If L, C and R denote the inductance, capacitance and resistance respectively, the dimensional formula for C^2LR is **[UPSEAT 2002]**

(a) $[ML^{-2}T^{-1}I^0]$ (b) $[M^0L^0T^3I^0]$ (c) $[M^{-1}L^{-2}T^6I^2]$ (d) $[M^0L^0T^2I^0]$

Solution : (b) $[C^2LR] = \left[C^2L^2\frac{R}{L}\right] = \left[(LC)^2\left(\frac{R}{L}\right)\right]$

and we know that frequency of LC circuits is given by $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$ *i.e.*, the dimension of LC is equal to $[T^2]$

and $\left[\frac{L}{R}\right]$ gives the time constant of $L-R$ circuit so the dimension of $\frac{L}{R}$ is equal to $[T]$.

By substituting the above dimensions in the given formula $\left[(LC)^2\left(\frac{R}{L}\right)\right] = [T^2]^2[T^{-1}] = [T^3]$.

Problem 11. A force F is given by $F = at + bt^2$, where t is time. What are the dimensions of a and b **[BHU 1998; AFMC 2001]**

(a) MLT^{-3} and ML^2T^{-4} (b) MLT^{-3} and MLT^{-4} (c) MLT^{-1} and MLT^0 (d) MLT^{-4} and MLT^1

Solution : (b) From the principle of dimensional homogenity $[F] = [at]$ $\therefore [a] = \left[\frac{F}{t}\right] = \left[\frac{MLT^{-2}}{T}\right] = [MLT^{-3}]$

Similarly $[F] = [bt^2]$ $\therefore [b] = \left[\frac{F}{t^2}\right] = \left[\frac{MLT^{-2}}{T^2}\right] = [MLT^{-4}]$.

Problem 12. The position of a particle at time t is given by the relation $x(t) = \left(\frac{v_0}{\alpha}\right)(1 - c^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$. The dimensions of v_0 and α are respectively

(a) $M^0L^1T^{-1}$ and T^{-1} (b) $M^0L^1T^0$ and T^{-1} (c) $M^0L^1T^{-1}$ and LT^{-2} (d) $M^0L^1T^{-1}$ and T

Solution : (a) From the principle of dimensional homogeneity $[\alpha\, t]$ = dimensionless $\therefore [\alpha] = \left[\frac{1}{t}\right] = [T^{-1}]$

Similarly $[x] = \frac{[v_0]}{[\alpha]}$ $\therefore [v_0] = [x][\alpha] = [L][T^{-1}] = [LT^{-1}]$.

Problem 13. The dimensions of physical quantity X in the equation Force $= \frac{X}{\text{Density}}$ is given by

(a) $M^1L^4T^{-2}$ (b) $M^2L^{-2}T^{-1}$ (c) $M^2L^{-2}T^{-2}$ (d) $M^1L^{-2}T^{-1}$

Solution : (c) $[X]$ = [Force] × [Density] = $[MLT^{-2}] \times [ML^{-3}] = [M^2L^{-2}T^{-2}]$.

Problem 14. Number of particles is given by $n = -D\frac{n_2 - n_1}{x_2 - x_1}$ crossing a unit area perpendicular to X- axis in unit time, where n_1 and n_2 are number of particles per unit volume for the value of x meant to x_2 and x_1. Find dimensions of D called as diffusion constant **[CPMT 1979]**

(a) M^0LT^2 (b) $M^0L^2T^{-4}$ (c) M^0LT^{-3} (d) $M^0L^2T^{-1}$

Solution : (d) (n) = Number of particle passing from unit area in unit time $= \frac{\text{No. of particle}}{A \times t} = \frac{[M^0L^0T^0]}{[L^2]\,[T]} = [L^{-2}T^{-1}]$

$[n_1] = [n_2] =$ No. of particle in unit volume $= [L^{-3}]$

Now from the given formula $[D] = \dfrac{[n][x_2 - x_1]}{[n_2 - n_1]} = \dfrac{[L^{-2}T^{-1}][L]}{[L^{-3}]} = [L^2T^{-1}]$.

Problem 15. E, m, l and G denote energy, mass, angular momentum and gravitational constant respectively, then the dimension of $\dfrac{El^2}{m^5G^2}$ are

(a) Angle (b) Length (c) Mass (d) Time

Solution : (a) $[E]$ = energy = $[ML^2T^{-2}]$, $[m]$ = mass = $[M]$, $[l]$ = Angular momentum = $[ML^2T^{-1}]$

$[G]$ = Gravitational constant = $[M^{-1}L^3T^{-2}]$

Now substituting dimensions of above quantities in $\dfrac{El^2}{m^5G^2} = \dfrac{[ML^2T^{-2}]\times[ML^2T^{-1}]^2}{[M^5]\times[M^{-1}L^3T^{-2}]^2} = [M^0L^0T^0]$

i.e., the quantity should be angle.

Problem 16. The equation of a wave is given by $Y = A \sin \omega\left(\dfrac{x}{v} - k\right)$ where ω is the angular velocity and v is the linear velocity. The dimension of k is

(a) LT (b) T (c) T^{-1} (d) T^2

Solution : (b) According to principle of dimensional homogeneity $[k] = \left[\dfrac{x}{v}\right] = \left[\dfrac{L}{LT^{-1}}\right] = [T]$.

Problem 17. The potential energy of a particle varies with distance x from a fixed origin as $U = \dfrac{A\sqrt{x}}{x^2 + B}$, where A and B are dimensional constants then dimensional formula for AB is

(a) $ML^{7/2}T^{-2}$ (b) $ML^{11/2}T^{-2}$ (c) $M^2L^{9/2}T^{-2}$ (d) $ML^{13/2}T^{-3}$

Solution : (b) From the dimensional homogeneity $[x^2] = [B]$ $\therefore [B] = [L^2]$

As well as $[U] = \dfrac{[A][x^{1/2}]}{[x^2] + [B]} \Rightarrow [ML^2T^{-2}] = \dfrac{[A][L^{1/2}]}{[L^2]}$ $\therefore [A] = [ML^{7/2}T^{-2}]$

Now $[AB] = [ML^{7/2}T^{-2}] \times [L^2] = [ML^{11/2}T^{-2}]$

Problem 18. The dimensions of $\dfrac{1}{2}\varepsilon_0 E^2$ (ε_0 = permittivity of free space ; E = electric field) is

(a) MLT^{-1} (b) ML^2T^{-2} (c) $ML^{-1}T^{-2}$ (d) ML^2T^{-1}

Solution : (c) Energy density = $\dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{\text{Energy}}{\text{Volume}} = \left[\dfrac{ML^2T^{-2}}{L^3}\right] = [ML^{-1}T^{-2}]$

Problem 19. You may not know integration. But using dimensional analysis you can check on some results. In the integral $\displaystyle\int \frac{dx}{(2ax - x^2)^{1/2}} = a^n \sin^{-1}\left(\frac{x}{a} - 1\right)$ the value of n is

(a) 1 (b) -1 (c) 0 (d) $\dfrac{1}{2}$

Solution : (c) Let x = length $\therefore [X] = [L]$ and $[dx] = [L]$

By principle of dimensional homogeneity $\left[\dfrac{x}{a}\right]$ = dimensionless $\therefore [a] = [x] = [L]$

By substituting dimension of each quantity in both sides: $\dfrac{[L]}{[L^2 - L^2]^{1/2}} = [L^n]$ $\therefore n = 0$

***Problem* 20.** A physical quantity $P = \frac{B^2 l^2}{m}$ where B= magnetic induction, l= length and m = mass. The dimension of P is

(a) MLT^{-3} (b) $ML^2T^{-4}I^{-2}$ (c) $M^2L^2T^{-4}I$ (d) $MLT^{-2}I^{-2}$

Solution : (b) $F = BIL$ $\therefore$ Dimension of $[B] = \frac{[F]}{[I][L]} = \frac{[MLT^{-2}]}{[I][L]} = [MT^{-2}I^{-1}]$

Now dimension of $[P] = \frac{B^2 l^2}{m} = \frac{[MT^{-2}I^{-1}]^2 \times [L^2]}{[M]} = [ML^2T^{-4}I^{-2}]$

***Problem* 21.** The equation of the stationary wave is $y = 2a \sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$, which of the following statements is wrong

(a) The unit of ct is same as that of λ (b) The unit of x is same as that of λ

(c) The unit of $2\pi c/\lambda$ is same as that of $2\pi x/\lambda t$ (d) The unit of c/λ is same as that of x/λ

Solution : (d) Here, $\frac{2\pi ct}{\lambda}$ as well as $\frac{2\pi x}{\lambda}$ are dimensionless (angle) *i.e.* $\left[\frac{2\pi ct}{\lambda}\right] = \left[\frac{2\pi x}{\lambda}\right] = M^0L^0T^0$

So (i) unit of $c\,t$ is same as that of λ (ii) unit of x is same as that of λ (iii) $\left[\frac{2\pi c}{\lambda}\right] = \left[\frac{2\pi x}{\lambda t}\right]$

and (iv) $\frac{x}{\lambda}$ is unit less. It is not the case with $\frac{c}{\lambda}$.

(3) **To convert a physical quantity from one system to the other** : The measure of a physical quantity is nu = constant

If a physical quantity X has dimensional formula $[M^aL^bT^c]$ and if (derived) units of that physical quantity in two systems are $[M_1^aL_1^bT_1^c]$ and $[M_2^aL_2^bT_2^c]$ respectively and n_1 and n_2 be the numerical values in the two systems respectively, then $n_1[u_1] = n_2[u_2]$

$$\Rightarrow n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$$

$$\Rightarrow n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

where M_1, L_1 and T_1 are fundamental units of mass, length and time in the first (known) system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the second (unknown) system. Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

Example : (1) conversion of *Newton* into *Dyne*.

The Newton is the S.I. unit of force and has dimensional formula $[MLT^{-2}]$.

So 1 N = 1 $kg\text{-}m/sec^2$

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 1\left[\frac{kg}{gm}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 1\left[\frac{10^3\,gm}{gm}\right]^1\left[\frac{10^2\,cm}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 10^5$

$\therefore$ 1 $N = 10^5$ *Dyne*

(2) Conversion of gravitational constant (G) from C.G.S. to M.K.S. system

The value of G in C.G.S. system is 6.67×10^{-8} C.G.S. units while its dimensional formula is $[M^{-1}L^3T^{-2}]$

So $G = 6.67 \times 10^{-8}\ cm^3/g\ s^2$

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 6.67\times10^{-8}\left[\frac{gm}{kg}\right]^{-1}\left[\frac{cm}{m}\right]^3\left[\frac{sec}{sec}\right]^{-2}$

$$= 6.67\times10^{-8}\left[\frac{gm}{10^3\,gm}\right]^{-1}\left[\frac{cm}{10^2\,cm}\right]^3\left[\frac{sec}{sec}\right]^{-2} = 6.67\times10^{-11}$$

$\therefore$ $G = 6.67\times10^{-11}$ M.K.S. units

Sample problems based on conversion

***Problem* 22.** A physical quantity is measured and its value is found to be nu where $n =$ numerical value and $u =$ unit.
Then which of the following relations is true **[RPET 2003]**

(a) $n \propto u^2$ (b) $n \propto u$ (c) $n \propto \sqrt{u}$ (d) $n \propto \frac{1}{u}$

Solution : (d) We know $P = nu =$ constant $\therefore n_1u_1 = n_2u_2$ or $n \propto \frac{1}{u}$.

***Problem* 23.** In C.G.S. system the magnitude of the force is 100 *dynes*. In another system where the fundamental physical quantities are *kilogram*, *metre* and *minute*, the magnitude of the force is

(a) 0.036 (b) 0.36 (c) 3.6 (d) 36

Solution : (c) $n_1 = 100$, $M_1 = g$, $L_1 = cm$, $T_1 = \sec$ and $M_2 = kg$, $L_2 = meter$, $T_2 = \text{minute}$, $x = 1$, $y = 1$, $z = -2$

By substituting these values in the following conversion formula $n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^2$

$$n_2 = 100\left[\frac{gm}{kg}\right]^1\left[\frac{cm}{meter}\right]^1\left[\frac{\sec}{\text{minute}}\right]^{-2}$$

$$n_2 = 100\left[\frac{gm}{10^3\,gm}\right]^1\left[\frac{cm}{10^2\,cm}\right]^1\left[\frac{\sec}{60\sec}\right]^{-2} = 3.6$$

***Problem* 24.** The temperature of a body on Kelvin scale is found to be X K. When it is measured by a Fahrenheit thermometer, it is found to be X F. Then X is **[UPSEAT 200]**

(a) 301.25 (b) 574.25 (c) 313 (d) 40

Solution : (c) Relation between centigrade and Fahrenheit $\frac{K-273}{5} = \frac{F-32}{9}$

According to problem $\frac{X-273}{5} = \frac{X-32}{9}$ $\therefore X = 313$.

***Problem* 25.** Which relation is wrong **[RPMT 1997]**

(a) 1 *Calorie* = 4.18 *Joules* (b) 1Å $=10^{-10}\,m$

(c) 1 *MeV* = 1.6×10^{-13} *Joules* (d) 1 *Newton* $=10^{-5}$ *Dynes*

Solution : (d) Because 1 *Newton* = 10^5 *Dyne.*

***Problem* 26.** To determine the Young's modulus of a wire, the formula is $Y = \frac{F}{A}.\frac{L}{\Delta l}$; where L= length, A= area of cross- section of the wire, $\Delta L =$ Change in length of the wire when stretched with a force F. The conversion factor to change it from CGS to MKS system is

(a) 1 (b) 10 (c) 0.1 (d) 0.01

Solution : (c) We know that the dimension of young's modulus is $[ML^{-1}T^{-2}]$

C.G.S. unit : $gm\ cm^{-1}\sec^{-2}$ and M.K.S. unit : $kg.\ m^{-1}\ sec^{-2}$.

By using the conversion formula: $n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^{-1}\left[\frac{T_1}{T_2}\right]^{-2} = \left[\frac{gm}{kg}\right]^1\left[\frac{cm}{meter}\right]^{-1}\left[\frac{\sec}{\sec}\right]^{-2}$

$\therefore$ Conversion factor $\frac{n_2}{n_1} = \left[\frac{gm}{10^3\,gm}\right]^1\left[\frac{cm}{10^2\,cm}\right]^{-1}\left[\frac{\sec}{\sec}\right]^{-2} = \frac{1}{10} = 0.1$

***Problem* 27.** Conversion of 1 MW power on a new system having basic units of mass, length and time as 10*kg*, 1*dm* and 1 *minute* respectively is

(a) $2.16\times10^{12}\,unit$ (b) $1.26\times10^{12}\,unit$ (c) $2.16\times10^{10}\,unit$ (d) $2\times10^{14}\,unit$

Solution : (a) $[P]=[ML^2T^{-3}]$

Using the relation $n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z = 1\times10^6\left[\frac{1\,kg}{10\,kg}\right]^1\left[\frac{1m}{1\,dm}\right]^2\left[\frac{1s}{1\min}\right]^{-3}$ [As $1MW = 10^6 W$]

$$= 10^6\left[\frac{1kg}{10kg}\right]\left[\frac{10\,dm}{1\,dm}\right]^2\left[\frac{1\sec}{60\sec}\right]^{-3} = 2.16\times10^{12}\,unit$$

***Problem* 28.** In two systems of relations among velocity, acceleration and force are respectively $v_2 = \frac{\alpha^2}{\beta}v_1$, $a_2 = \alpha\beta a_1$ and $F_2 = \frac{F_1}{\alpha\beta}$. If α and β are constants then relations among mass, length and time in two systems are

(a) $M_2 = \frac{\alpha}{\beta}M_1, L_2 = \frac{\alpha^2}{\beta^2}L_1, T_2 = \frac{\alpha^3 T_1}{\beta}$ (b) $M_2 = \frac{1}{\alpha^2\beta^2}M_1, L_2 = \frac{\alpha^3}{\beta^3}L_1, T_2 = T_1\frac{\alpha}{\beta^2}$

(c) $M_2 = \frac{\alpha^3}{\beta^3}M_1, L_2 = \frac{\alpha^2}{\beta^2}L_1, T_2 = \frac{\alpha}{\beta}T_1$ (d) $M_2 = \frac{\alpha^2}{\beta^2}M_1, L_2 = \frac{\alpha}{\beta^2}L_1, T_2 = \frac{\alpha^3}{\beta^3}T_1$

Solution : (b) $v_2 = v_1\frac{\alpha^2}{\beta} \Rightarrow [L_2T_2^{-1}]=[L_1T_1^{-1}]\frac{\alpha^2}{\beta}$ ......(i)

$a_2 = a_1\alpha\beta \Rightarrow [L_2T_2^{-2}]=[L_1T_1^{-2}]\alpha\beta$ ......(ii)

and $F_2 = \frac{F_1}{\alpha\beta} \Rightarrow [M_2L_2T_2^{-2}]=[M_1L_1T_1^{-2}]\times\frac{1}{\alpha\beta}$ ......(iii)

Dividing equation (iii) by equation (ii) we get $M_2 = \frac{M_1}{(\alpha\beta)\alpha\beta} = \frac{M_1}{\alpha^2B^2}$

Squaring equation (i) and dividing by equation (ii) we get $L_2 = L_1\frac{\alpha^3}{\beta^3}$

Dividing equation (i) by equation (ii) we get $T_2 = T_1\frac{\alpha}{\beta^2}$

***Problem* 29.** If the present units of length, time and mass (*m*, *s*, *kg*) are changed to 100*m*, 100*s*, and $\frac{1}{10}$ *kg* then

(a) The new unit of velocity is increased 10 times (b) The new unit of force is decreased $\frac{1}{1000}$ times

(c) The new unit of energy is increased 10 times (d) The new unit of pressure is increased 1000 times

Solution : (b) Unit of velocity = *m/sec* ; in new system = $\frac{100m}{100\sec} = \frac{m}{\sec}$ (same)

Unit of force $= \dfrac{kg \times m}{\sec^2}$; in new system $= \dfrac{1}{10} kg \times \dfrac{100m}{100\sec \times 100\sec} = \dfrac{1}{1000} \dfrac{kg \times m}{\sec^2}$

Unit of energy $= \dfrac{kg \times m^2}{\sec^2}$; in new system $= \dfrac{1}{10} kg \times \dfrac{100m \times 100m}{100\sec \times 100\sec} = \dfrac{1}{10} \dfrac{kg \times m^2}{\sec^2}$

Unit of pressure $= \dfrac{kg}{m \times \sec^2}$; in new system $= \dfrac{1}{10} kg \times \dfrac{1}{100} m \times \dfrac{1}{100\,sec \times 100\,sec} = 10^{-7} \dfrac{kg}{m \times sec^2}$

***Problem* 30.** Suppose we employ a system in which the unit of mass equals 100 *kg*, the unit of length equals 1 *km* and the unit of time 100 *s* and call the unit of energy *eluoj* (*joule* written in reverse order), then

(a) 1 *eluoj* = 10^4 *joule* (b) 1 *eluoj* = 10^{-3} *joule* (c) 1 *eluoj* = 10^{-4} *joule* (d) 1 *joule* = 10^3 *eluoj*

Solution : (a) $[E] = [ML^2T^{-2}]$

1 *eluoj* $= [100kg] \times [1km]^2 \times [100\sec]^{-2} = 100kg \times 10^6 m^2 \times 10^{-4} \sec^{-2} = 10^4\, kg\, m^2 \times \sec^{-2} = 10^4$ *Joule*

***Problem* 31.** If 1*gm cms*$^{-1}$ = *x Ns*, then number *x* is equivalent to

(a) 1×10^{-1} (b) 3×10^{-2} (c) 6×10^{-4} (d) 1×10^{-5}

Solution : (d) $gm\text{ - }cm\, s^{-1} = 10^{-3} kg \times 10^{-2} m \times s^{-1} = 10^{-5} kg \times m \times s^{-1}$ = 10^{-5} *Ns*

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(4) **To check the dimensional correctness of a given physical relation :** This is based on the '*principle of homogeneity*'. According to this principle the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$,

then according to principle of homogeneity $[X] = [A] = [(BC)^2] = [\sqrt{DEF}]$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. A dimensionally correct equation may or may not be physically correct.

Example : (1) $F = mv^2 / r^2$

By substituting dimension of the physical quantities in the above relation –

$$[MLT^{-2}] = [M][LT^{-1}]^2 / [L]^2$$

i.e. $[MLT^{-2}] = [MT^{-2}]$

As in the above equation dimensions of both sides are not same; this formula is not correct dimensionally, so can never be physically.

(2) $s = ut - (1/2)at^2$

By substituting dimension of the physical quantities in the above relation –

$$[L] = [LT^{-1}][T] - [LT^{-2}][T^2]$$

i.e. $[L] = [L] - [L]$

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct. However, from equations of motion we know that $s = ut + (1/2)at^2$

Sample problems based on formulae checking

***Problem* 32.** From the dimensional consideration, which of the following equation is correct

(a) $T = 2\pi\sqrt{\dfrac{R^3}{GM}}$ (b) $T = 2\pi\sqrt{\dfrac{GM}{R^3}}$ (c) $T = 2\pi\sqrt{\dfrac{GM}{R^2}}$ (d) $T = 2\pi\sqrt{\dfrac{R^2}{GM}}$

Solution : (a) $T = 2\pi\sqrt{\dfrac{R^3}{GM}} = 2\pi\sqrt{\dfrac{R^3}{gR^2}} = 2\pi\sqrt{\dfrac{R}{g}}$ [As $GM = gR^2$]

Now by substituting the dimension of each quantity in both sides.

$$[T] = \left[\frac{L}{LT^{-2}}\right]^{1/2} = [T]$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

***Problem* 33.** A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn block A executes small oscillations. The time period of which is given by

(a) $2\pi\sqrt{\dfrac{M\eta}{L}}$ (b) $2\pi\sqrt{\dfrac{L}{M\eta}}$ (c) $2\pi\sqrt{\dfrac{ML}{\eta}}$ (d) $2\pi\sqrt{\dfrac{M}{\eta L}}$

Solution : (d) Given m = mass = $[M]$, η = coefficient of rigidity = $[ML^{-1}T^{-2}]$, L = length = $[L]$

By substituting the dimension of these quantity we can check the accuracy of the given formulae

$$[T] = 2\pi\left(\frac{[M]}{[\eta][L]}\right)^{1/2} = \left[\frac{M}{ML^{-1}T^{-2}L}\right]^{1/2} = [T].$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

***Problem* 34.** A small steel ball of radius r is allowed to fall under gravity through a column of a viscous liquid of coefficient of viscosity. After some time the velocity of the ball attains a constant value known as terminal velocity v_T. The terminal velocity depends on (i) the mass of the ball. (ii) η (iii) r and (iv) acceleration due to gravity g. which of the following relations is dimensionally correct

(a) $v_T \propto \dfrac{mg}{\eta r}$ (b) $v_T \propto \dfrac{\eta r}{mg}$ (c) $v_T \propto \eta rmg$ (d) $v_T \propto \dfrac{mgr}{\eta}$

Solution : (a) Given v_T = terminal velocity = $[LT^{-1}]$, m = Mass = $[M]$, g = Acceleration due to gravity = $[LT^{-2}]$

r = Radius = $[L]$, η = Coefficient of viscosity = $[\eta]$

By substituting the dimension of each quantity we can check the accuracy of given formula $v_T \propto \dfrac{mg}{\eta r}$

$$\therefore\ [LT^{-1}] = \frac{[M][LT^{-2}]}{[ML^{-1}T^{-1}][L]} = [LT^{-1}]$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

***Problem* 35.** A dimensionally consistent relation for the volume V of a liquid of coefficient of viscosity η flowing per second through a tube of radius r and length l and having a pressure difference p across its end, is

(a) $V = \dfrac{\pi pr^4}{8\eta l}$ (b) $V = \dfrac{\pi\eta l}{8pr^4}$ (c) $V = \dfrac{8p\eta l}{\pi r^4}$ (d) $V = \dfrac{\pi p\eta}{8lr^4}$

Solution : (a) Given V = Rate of flow = $\dfrac{\text{Volume}}{\text{sec}} = [L^3T^{-1}]$, P = Pressure = $[ML^{-1}T^{-2}]$, r = Radius = $[L]$

η = Coefficient of viscosity = $[ML^{-1}T^{-1}]$, l = Length = $[L]$

By substituting the dimension of each quantity we can check the accuracy of the formula $V = \dfrac{\pi P r^4}{8\eta l}$

$$\therefore\ [L^3T^{-1}] = \frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L]} = [L^3T^{-1}]$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

Problem 36. With the usual notations, the following equation $S_t = u + \frac{1}{2}a(2t-1)$ is

(a) Only numerically correct (b) Only dimensionally correct

(c) Both numerically and dimensionally correct (d) Neither numerically nor dimensionally correct

Solution : (c) Given S_t = distance travelled by the body in t^{th} *sec.*= $[LT^{-1}]$, a = Acceleration = $[LT^{-2}]$,

v = velocity = $[LT^{-1}]$, t = time = $[T]$

By substituting the dimension of each quantity we can check the accuracy of the formula

$$S_t = u + \frac{1}{2}a(2t-1)$$

$$\therefore\ [LT^{-1}] = [LT^{-1}] + [LT^{-2}]\,[T] \Rightarrow [LT^{-1}] = [LT^{-1}] + [LT^{-1}]$$

Since the dimension of each terms are equal therefore this equation is dimensionally correct. And after deriving this equation from Kinematics we can also proof that this equation is correct numerically also.

Problem 37. If velocity v, acceleration A and force F are chosen as fundamental quantities, then the dimensional formula of angular momentum in terms of v, A and F would be

(a) $FA^{-1}v$ (b) Fv^3A^{-2} (c) Fv^2A^{-1} (d) $F^2v^2A^{-1}$

Solution : (b) Given, v = velocity = $[LT^{-1}]$, A = Acceleration = $[LT^{-2}]$, F = force = $[MLT^{-2}]$

By substituting, the dimension of each quantity we can check the accuracy of the formula

[Angular momentum] = Fv^3A^{-2}

$$[ML^2T^{-1}] = [MLT^{-2}]\,[LT^{-1}]^3[LT^{-2}]^{-2}$$

$$= [ML^2T^{-1}]$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

Problem 38. The largest mass (m) that can be moved by a flowing river depends on velocity (v), density (ρ) of river water and acceleration due to gravity (g). The correct relation is

(a) $m \propto \frac{\rho^2 v^4}{g^2}$ (b) $m \propto \frac{\rho v^6}{g^2}$ (c) $m \propto \frac{\rho v^4}{g^3}$ (d) $m \propto \frac{\rho v^6}{g^3}$

Solution : (d) Given, m = mass = $[M]$, v = velocity = $[LT^{-1}]$, ρ = density = $[ML^{-3}]$, g = acceleration due to gravity = $[LT^{-2}]$

By substituting, the dimension of each quantity we can check the accuracy of the formula

$$m = K\frac{\rho v^6}{g^3}$$

$$\Rightarrow [M] = \frac{[ML^{-3}][LT^{-1}]^6}{[LT^{-2}]^3}$$

$$= [M]$$

L.H.S. = R.H.S. *i.e.,* the above formula is Correct.

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(5) **As a research tool to derive new relations :** If one knows the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, relation between the quantities can be derived.

Example : (i) Time period of a simple pendulum.

Let time period of a simple pendulum is a function of mass of the bob (m), effective length (l), acceleration due to gravity (g) then assuming the function to be product of power function of m, l and g

i.e., $T = Km^x l^y g^z$; where K = dimensionless constant

If the above relation is dimensionally correct then by substituting the dimensions of quantities –

$$[T] = [M]^x [L]^y [LT^{-2}]^z$$

or $[M^0L^0T^1] = [M^xL^{y+z}T^{-2z}]$

Equating the exponents of similar quantities $x = 0$, $y = 1/2$ and $z = -1/2$

So the required physical relation becomes $T = K\sqrt{\frac{l}{g}}$

The value of dimensionless constant is found (2π) through experiments so $T = 2\pi\sqrt{\frac{l}{g}}$

(ii) Stoke's law : When a small sphere moves at low speed through a fluid, the viscous force F, opposing the motion, is found experimentally to depend on the radius r, the velocity of the sphere v and the viscosity η of the fluid.

So $F = f(\eta, r, v)$

If the function is product of power functions of η, r and v, $F = K\eta^x r^y v^z$; where K is dimensionless constant.

If the above relation is dimensionally correct $[MLT^{-2}] = [ML^{-1}T^{-1}]^x [L]^y [LT^{-1}]^z$

or $[MLT^{-2}] = [M^x L^{-x+y+z} T^{-x-z}]$

Equating the exponents of similar quantities $x = 1$; $-x + y + z = 1$ and $-x - z = -2$

Solving these for x, y and z, we get $x = y = z = 1$

So eq^n (i) becomes $F = K\eta rv$

On experimental grounds, $K = 6\pi$; so $F = 6\pi\eta rv$

This is the famous Stoke's law.

Sample problem based on formulae derivation

Problem 39. If the velocity of light (c), gravitational constant (G) and Planck's constant (h) are chosen as fundamental units, then the dimensions of mass in new system is

(a) $c^{1/2}G^{1/2}h^{1/2}$ (b) $c^{1/2}G^{1/2}h^{-1/2}$ (c) $c^{1/2}G^{-1/2}h^{1/2}$ (d) $c^{-1/2}G^{1/2}h^{1/2}$

Solution : (c) Let $m \propto c^x G^y h^z$ or $m = K c^x G^y h^z$

By substituting the dimension of each quantity in both sides

$[M^1L^0T^0] = K[LT^{-1}]^x[M^{-1}L^3T^{-2}]^y[ML^2T^{-1}]^z = [M^{-y+z}L^{x+3y+2z}T^{-x-2y-z}]$

By equating the power of M, L and T in both sides : $-y + z = 1$, $x + 3y + 2z = 0$, $-x - 2y - z = 0$

By solving above three equations $x = 1/2$, $y = -1/2$ and $z = 1/2$.

$\therefore \; m \propto c^{1/2}G^{-1/2}h^{1/2}$

Problem 40. If the time period (T) of vibration of a liquid drop depends on surface tension (S), radius (r) of the drop and density (ρ) of the liquid, then the expression of T is

(a) $T = K\sqrt{\rho r^3/S}$ (b) $T = K\sqrt{\rho^{1/2}r^3/S}$ (c) $T = K\sqrt{\rho r^3/S^{1/2}}$ (d) None of these

Solution : (a) Let $T \propto S^x r^y \rho^z$ or $T = K S^x r^y \rho^z$

By substituting the dimension of each quantity in both sides

$[M^0L^0T^1] = K\,[MT^{-2}]^x[L]^y[ML^{-3}]^z = [M^{x+z}L^{y-3z}T^{-2x}]$

By equating the power of M, L and T in both sides $x+z=0$, $y-3z=0$, $-2x=1$

By solving above three equations $\therefore$ $x=-1/2$, $y=3/2$, $z=1/2$

So the time period can be given as, $T = K S^{-1/2} r^{3/2} \rho^{1/2} = K\sqrt{\frac{\rho r^3}{S}}$.

***Problem* 41.** If P represents radiation pressure, C represents speed of light and Q represents radiation energy striking a unit area per second, then non-zero integers x, y and z such that $P^x Q^y C^z$ is dimensionless, are

[AFMC 1991; CBSE 1992; CPMT 1981, 92; MP PMT 1992]

(a) $x=1, y=1, z=-1$ (b) $x=1, y=-1, z=1$ (c) $x=-1, y=1, z=1$ (d) $x=1, y=1, z=1$

Solution : (b) $[P^x Q^y C^z] = M^0 L^0 T^0$

By substituting the dimension of each quantity in the given expression

$[ML^{-1}T^{-2}]^x [MT^{-3}]^y [LT^{-1}]^z = [M^{x+y} L^{-x+z} T^{-2x-3y-z}] = M^0 L^0 T^0$

by equating the power of M, L and T in both sides: $x+y=0$, $-x+z=0$ and $-2x-3y-z=0$

by solving we get $x=1, y=-1, z=1$.

***Problem* 42.** The volume V of water passing through a point of a uniform tube during t seconds is related to the cross-sectional area A of the tube and velocity u of water by the relation $V \propto A^\alpha u^\beta t^\gamma$, which one of the following will be true

(a) $\alpha = \beta = \gamma$ (b) $\alpha \neq \beta = \gamma$ (c) $\alpha = \beta \neq \gamma$ (d) $\alpha \neq \beta \neq \gamma$

Solution : (b) Writing dimensions of both sides $[L^3] = [L^2]^\alpha [LT^{-1}]^\beta [T]^\gamma \Rightarrow [L^3 T^0] = [L^{2\alpha+\beta} T^{\gamma-\beta}]$

By comparing powers of both sides $2\alpha + \beta = 3$ and $\gamma - \beta = 0$

Which give $\beta = \gamma$ and $\alpha = \frac{1}{2}(3-\beta)$ i.e. $\alpha \neq \beta = \gamma$.

***Problem* 43.** If velocity (V), force (F) and energy (E) are taken as fundamental units, then dimensional formula for mass will be

(a) $V^{-2}F^0E$ (b) V^0FE^2 (c) $VF^{-2}E^0$ (d) $V^{-2}F^0E$

Solution : (d) Let $M = V^a F^b E^c$

Putting dimensions of each quantities in both side $[M]=[LT^{-1}]^a [MLT^{-2}]^b [ML^2T^{-2}]^c$

Equating powers of dimensions. We have $b+c=1$, $a+b+2c=0$ and $-a-2b-2c=0$

Solving these equations, $a=-2$, $b=0$ and $c=1$

So $M = [V^{-2}F^0E]$

***Problem* 44.** Given that the amplitude A of scattered light is :

(i) Directly proportional to the amplitude (A_0) of incident light.

(ii) Directly proportional to the volume (V) of the scattering particle

(iii) Inversely proportional to the distance (r) from the scattered particle

(iv) Depend upon the wavelength (λ) of the scattered light. then:

(a) $A \propto \frac{1}{\lambda}$ (b) $A \propto \frac{1}{\lambda^2}$ (c) $A \propto \frac{1}{\lambda^3}$ (d) $A \propto \frac{1}{\lambda^4}$

Solution : (b) Let $A = \frac{KA_0 V \lambda^x}{r}$

By substituting the dimension of each quantity in both sides

$\Rightarrow [L] = \frac{[L].[L^3][L^x]}{[L]}$

$\therefore\ [L] = [L^{3+x}]$; $\Rightarrow\ 3 + x = 1$ or $x = -2$

$\therefore\ A \propto \lambda^{-2}$

1.12 Limitations of Dimensional Analysis

Although dimensional analysis is very useful it cannot lead us too far as,

(1) If dimensions are given, physical quantity may not be unique as many physical quantities have same dimensions. For example if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may be work or energy or torque.

(2) Numerical constant having no dimensions [*K*] such as (1/2), 1 or 2π *etc.* cannot be deduced by the methods of dimensions.

(3) The method of dimensions can not be used to derive relations other than product of power functions. For example,

$$s = ut + (1/2)at^2 \qquad \text{or} \qquad y = a\sin\omega t$$

cannot be derived by using this theory (try if you can). However, the dimensional correctness of these can be checked.

(4) The method of dimensions cannot be applied to derive formula if in mechanics a physical quantity depends on more than 3 physical quantities as then there will be less number (= 3) of equations than the unknowns (>3). However still we can check correctness of the given equation dimensionally. For example $T = 2\pi\sqrt{1/mgl}$ can not be derived by theory of dimensions but its dimensional correctness can be checked.

(5) Even if a physical quantity depends on 3 physical quantities, out of which two have same dimensions, the formula cannot be derived by theory of dimensions, *e.g.*, formula for the frequency of a tuning fork $f = (d/L^2)v$ cannot be derived by theory of dimensions but can be checked.

1.13 Significant Figures

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

(1) All non-zero digits are significant.

Example : 42.3 has three significant figures.

243.4 has four significant figures.

24.123 has five significant figures.

(2) A zero becomes significant figure if it appears between to non-zero digits.

Example : 5.03 has three significant figures.

5.604 has four significant figures.

4.004 has four significant figures.

(3) Leading zeros or the zeros placed to the left of the number are never significant.

Example : 0.543 has three significant figures.

0.045 has two significant figures.

0.006 has one significant figures.

(4) Trailing zeros or the zeros placed to the right of the number are significant.

Example : 4.330 has four significant figures.

433.00 has five significant figures.

343.000 has six significant figures.

(5) In exponential notation, the numerical portion gives the number of significant figures.

Example : 1.32×10^{-2} has three significant figures.

1.32×10^{4} has three significant figures.

1.14 Rounding Off

While rounding off measurements, we use the following rules by convention:

(1) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example : $x = 7.82$ is rounded off to 7.8, again $x = 3.94$ is rounded off to 3.9.

(2) If the digit to be dropped is more than 5, then the preceding digit is raised by one.

Example : x = 6.87 is rounded off to 6.9, again x = *12.78* is rounded off to 12.8.

(3) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.

Example : x = 16.351 is rounded off to 16.4, again x = 6.758 is rounded off to 6.8.

(4) If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.

Example : x = 3.250 becomes 3.2 on rounding off, again x = 12.650 becomes 12.6 on rounding off.

(5) If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.

Example : x = 3.750 is rounded off to 3.8, again x = 16.150 is rounded off to 16.2.

1.15 Significant Figures in Calculation

In most of the experiments, the observations of various measurements are to be combined mathematically, *i.e.*, added, subtracted, multiplied or divided as to achieve the final result. Since, all the observations in measurements do not have the same precision, it is natural that the final result cannot be more precise than the least precise measurement. The following two rules should be followed to obtain the proper number of significant figures in any calculation.

(1) The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as are present in the number having the least number of decimal places. The rule is illustrated by the following examples :

(i)
$$\begin{array}{rl} 33.3 & \leftarrow \text{(has only one decimal place)} \\ 3.11 & \\ +\ 0.313 & \\ \hline \underline{36.723} & \leftarrow \text{(answer should be reported to one decimal place)} \end{array}$$

Answer = 36.7

(ii)
$$\begin{array}{rl} 3.1421 & \\ 0.241 & \\ +\ 0.09 & \leftarrow \text{(has 2 decimal places)} \\ \hline \underline{3.4731} & \leftarrow \text{(answer should be reported to 2 decimal places)} \end{array}$$

Answer = 3.47

(iii)
$$\begin{array}{rl} 62.831 & \leftarrow \text{(has 3 decimal places)} \\ -\ 24.5492 & \end{array}$$

38.2818 ← (answer should be reported to 3 decimal places after rounding off)

Answer = 38.282

(2) The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation. The rule is illustrated by the following examples :

(i) 142.06

$\times$ 0.23 ← (two significant figures)

32.6738 ← (answer should have two significant figures)

Answer = 33

(ii) 51.028

$\times$ 1.31 ← (three significant figures)

66.84668

Answer = 66.8

(iii) $\frac{0.90}{4.26} = 0.2112676$

Answer = 0.21

1.16 Order of Magnitude

In scientific notation the numbers are expressed as, Number $= M \times 10^x$. Where M is a number lies between 1 and 10 and x is integer. Order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one. For example,

(1) Speed of light in vacuum $= 3 \times 10^8 ms^{-1} \approx 10^8 m/s$ (ignoring 3 < 5)

(2) Mass of electron $= 9.1 \times 10^{-31} kg \approx 10^{-30} kg$ (as 9.1 > 5).

Sample problems based on significant figures

***Problem* 45.** Each side a cube is measured to be 7.203 *m*. The volume of the cube up to appropriate significant figures is

(a) 373.714 (b) 373.71 (c) 373.7 (d) 373

Solution : (c) Volume $= a^3 = (7.023)^3 = 373.715\, m^3$

In significant figures volume of cube will be $373.7\, m^3$ because its side has four significant figures.

***Problem* 46.** The number of significant figures in 0.007 m^2 is

(a) 1 (b) 2 (c) 3 (d) 4

Solution : (a)

***Problem* 47.** The length, breadth and thickness of a block are measured as 125.5 *cm*, 5.0 *cm* and 0.32 *cm* respectively. Which one of the following measurements is most accurate

(a) Length (b) Breadth (c) Thickness (d) Height

Solution : (a) Relative error in measurement of length is minimum, so this measurement is most accurate.

***Problem* 48.** The mass of a box is 2.3 kg. Two marbles of masses 2.15 *g* and 12.39 *g* are added to it. The total mass of the box to the correct number of significant figures is

(a) 2.340 *kg* (b) 2.3145 *kg*. (c) 2.3 *kg* (d) 2.31 *kg*

Solution : (c) Total mass $= 2.3 + 0.00215 + 0.01239 = 2.31\, kg$

Total mass in appropriate significant figures be 2.3 *kg*.

***Problem* 49.** The length of a rectangular sheet is 1.5 cm and breadth is 1.203 cm. The area of the face of rectangular sheet to the correct no. of significant figures is :

(a) $1.8045\ cm^2$ (b) $1.804\ cm^2$ (c) $1.805\ cm^2$ (d) $1.8\ cm^2$

Solution : (d) Area $= 1.5 \times 1.203 = 1.8045\ cm^2 = 1.8\ cm^2$ (Upto correct number of significant figure).

***Problem* 50.** Each side of a cube is measured to be 5.402 cm. The total surface area and the volume of the cube in appropriate significant figures are :

(a) $175.1\ cm^2, 157\ cm^2$ (b) $175.1\ cm^2, 157.6\ cm^3$

(c) $175\ cm^2, 157\ cm^2$ (d) $175.08\ cm^2, 157.639\ cm^3$

Solution : (b) Total surface area $= 6 \times (5.402)^2 = 175.09\ cm^2 = 175.1\ cm^2$ (Upto correct number of significant figure)

Total volume $= (5.402)^3 = 175.64\ cm^3 = 175.6\ cm^3$ (Upto correct number of significant figure).

***Problem* 51.** Taking into account the significant figures, what is the value of 9.99 *m* + 0.0099 *m*

(a) 10.00 *m* (b) 10 *m* (c) 9.9999 *m* (d) 10.0 *m*

Solution : (a) $9.99\ m + 0.0099\ m = 9.999\ m = 10.00\ m$ (In proper significant figures).

***Problem* 52.** The value of the multiplication 3.124 × 4.576 correct to three significant figures is

(a) 14.295 (b) 14.3 (c) 14.295424 (d) 14.305

Solution : (b) $3.124 \times 4.576 = 14.295$ =14.3 (Correct to three significant figures).

***Problem* 53.** The number of the significant figures in $11.118 \times 10^{-6}\ V$ is

(a) 3 (b) 4 (c) 5 (d) 6

Solution : (c) The number of significant figure is 5 as 10^{-6} does not affect this number.

***Problem* 54.** If the value of resistance is 10.845 *ohms* and the value of current is 3.23 *amperes*, the potential difference is 35.02935 *volts*. Its value in significant number would be

(a) 35 *V* (b) 35.0 *V* (c) 35.03 *V* (d) 35.025 *V*

Solution : (b) Value of current (3.23 *A*) has minimum significant figure (3) so the value of potential difference $V(= IR)$ have only 3 significant figure. Hence its value be 35.0 *V*.

1.17 Errors of Measurement

The measuring process is essentially a process of comparison. Inspite of our best efforts, the measured value of a quantity is always somewhat different from its actual value, or true value. This difference in the true value of a quantity is called error of measurement.

(1) **Absolute error :** Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured *n* times. Let the measured value be $a_1, a_2, a_3, \ldots\ a_n$. The arithmetic mean of these value is $a_m = \dfrac{a_1 + a_2 + \ldots . a_n}{n}$

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise.

By definition, absolute errors in the measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

....

$$\Delta a_n = a_m - a_n$$

The absolute errors may be positive in certain cases and negative in certain other cases.

(2) **Mean absolute error :** It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by $\overline{\Delta a}$. Thus

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ... |\Delta a_n|}{n}$$

Hence the final result of measurement may be written as $a = a_m \pm \overline{\Delta a}$

This implies that any measurement of the quantity is likely to lie between $(a_m + \overline{\Delta a})$ and $(a_m - \overline{\Delta a})$.

(3) **Relative error or Fractional error :** The relative error or fractional error of measurement is defined as the ratio of mean absolute error to the mean value of the quantity measured. Thus

$$\text{Relative error or Fractional error } = \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\overline{\Delta a}}{a_m}$$

(4) **Percentage error :** When the relative/fractional error is expressed in percentage, we call it percentage error. Thus

$$\text{Percentage error } = \frac{\overline{\Delta a}}{a_m} \times 100\,\%$$

1.18 Propagation of Errors

(1) **Error in sum of the quantities :** Suppose $x = a + b$

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x *i.e.* sum of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of $x = \frac{(\Delta a + \Delta b)}{a + b} \times 100\,\%$

(2) **Error in difference of the quantities :** Suppose $x = a - b$

Let Δa = absolute error in measurement of a,

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x *i.e.* difference of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of $x = \frac{(\Delta a + \Delta b)}{a - b} \times 100\,\%$

(3) **Error in product of quantities :** Suppose $x = a \times b$

Let Δa = absolute error in measurement of a,

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x *i.e.* product of a and b.

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

(4) **Error in division of quantities :** Suppose $x = \frac{a}{b}$

Let Δa = absolute error in measurement of a,

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x *i.e.* division of a and b.

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

(5) **Error in quantity raised to some power :** Suppose $x = \frac{a^n}{b^m}$

Let Δa = absolute error in measurement of a,

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$

Percentage error in the value of x = n (Percentage error in value of a) + m (Percentage error in value of b)

Note : ❑ The quantity which have maximum power must be measured carefully because it's contribution to error is maximum.

Sample problems based on errors of measurement

***Problem* 55.** A physical parameter a can be determined by measuring the parameters b, c, d and e using the relation $a = b^{\alpha}c^{\beta}/d^{\gamma}e^{\delta}$. If the maximum errors in the measurement of b, c, d and e are $b_1\%$, $c_1\%$, $d_1\%$ and $e_1\%$, then the maximum error in the value of a determined by the experiment is **[CPMT 19**

(a) $(b_1 + c_1 + d_1 + e_1)\%$ (b) $(b_1 + c_1 - d_1 - e_1)\%$

(c) $(\alpha b_1 + \beta c_1 - \gamma d_1 - \delta e_1)\%$ (d) $(\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$

Solution : (d) $a = b^{\alpha}c^{\beta}/d^{\gamma}e^{\delta}$

So maximum error in a is given by

$$\left(\frac{\Delta a}{a}\times 100\right)_{\max} = \alpha.\frac{\Delta b}{b}\times 100 + \beta.\frac{\Delta c}{c}\times 100 + \gamma.\frac{\Delta d}{d}\times 100 + \delta.\frac{\Delta e}{e}\times 100$$

$$= (\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$$

***Problem* 56.** The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in the measurement of force and length are respectively 4% and 2%, The maximum error in the measurement of pressure is

(a) 1% (b) 2% (c) 6% (d) 8%

Solution : (d) $P = \frac{F}{A} = \frac{F}{l^2}$, so maximum error in pressure (P)

$$\left(\frac{\Delta P}{P}\times 100\right)_{\max} = \frac{\Delta F}{F}\times 100 + 2\frac{\Delta l}{l}\times 100 = 4\% + 2 \times 2\% = 8\%$$

***Problem* 57.** The relative density of material of a body is found by weighing it first in air and then in water. If the weight in air is (5.00 ± 0.05) *Newton* and weight in water is (4.00 ± 0.05) *Newton*. Then the relative density along with the maximum permissible percentage error is

(a) 5.0 ± 11% (b) 5.0 ± 1% (c) 5.0 ± 6% (d) 1.25 ± 5%

Solution : (a) Weight in air $= (5.00 \pm 0.05)N$

Weight in water $= (4.00 \pm 0.05)\, N$

Loss of weight in water $= (1.00 \pm 0.1)\, N$

Now relative density $= \dfrac{\text{weight in air}}{\text{weight loss in water}}$ *i.e.* $R\,.\,D = \dfrac{5.00 \pm 0.05}{1.00 \pm 0.1}$

Now relative density with max permissible error $= \dfrac{5.00}{1.00} \pm \left(\dfrac{0.05}{5.00} + \dfrac{0.1}{1.00}\right) \times 100 = 5.0 \pm (1 + 10)\%$

$= 5.0 \pm 11\%$

***Problem* 58.** The resistance $R = \dfrac{V}{i}$ where $V = 100 \pm 5$ volts and $i = 10 \pm 0.2$ *amperes*. What is the total error in R

(a) 5% (b) 7% (c) 5.2% (d) $\dfrac{5}{2}\%$

Solution : (b) $R = \dfrac{V}{I}$ $\therefore \left(\dfrac{\Delta R}{R} \times 100\right)_{\max} = \dfrac{\Delta V}{V} \times 100 + \dfrac{\Delta I}{I} \times 100 = \dfrac{5}{100} \times 100 + \dfrac{0.2}{10} \times 100 = (5 + 2)\% = 7\%$

***Problem* 59.** The period of oscillation of a simple pendulum in the experiment is recorded as 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s respectively. The average absolute error is

(a) 0.1 s (b) 0.11 s (c) 0.01 s (d) 1.0 s

Solution : (b) Average value $= \dfrac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5} = 2.62$ sec

Now $|\Delta T_1| = 2.63 - 2.62 = 0.01$

$|\Delta T_2| = 2.62 - 2.56 = 0.06$

$|\Delta T_3| = 2.62 - 2.42 = 0.20$

$|\Delta T_4| = 2.71 - 2.62 = 0.09$

$|\Delta T_5| = 2.80 - 2.62 = 0.18$

Mean absolute error $\Delta T = \dfrac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4| + |\Delta T_5|}{5} = \dfrac{0.54}{5} = 0.108 = 0.11\,sec$

***Problem* 60.** The length of a cylinder is measured with a meter rod having least count 0.1 *cm*. Its diameter is measured with venier calipers having least count 0.01 *cm*. Given that length is 5.0 *cm*. and radius is 2.0 *cm*. The percentage error in the calculated value of the volume will be

(a) 1% (b) 2% (c) 3% (d) 4%

Solution : (c) Volume of cylinder $V = \pi r^2 l$

Percentage error in volume $\dfrac{\Delta V}{V} \times 100 = \dfrac{2\Delta r}{r} \times 100 + \dfrac{\Delta l}{l} \times 100$

$= \left(2 \times \dfrac{0.01}{2.0} \times 100 + \dfrac{0.1}{5.0} \times 100\right) = (1 + 2)\% = 3\%$

***Problem* 61.** In an experiment, the following observation's were recorded : L = 2.820 *m*, M = 3.00 *kg*, l = 0.087 *cm*, Diameter D = 0.041 *cm* Taking $g = 9.81\ m/s^2$ using the formula , $Y = \dfrac{4Mg}{\pi D^2 l}$, the maximum permissible error in Y is

(a) 7.96% (b) 4.56% (c) 6.50% (d) 8.42%

Solution : (c) $Y = \dfrac{4MgL}{\pi D^2 l}$ so maximum permissible error in $Y = \dfrac{\Delta Y}{Y} \times 100 = \left(\dfrac{\Delta M}{M} + \dfrac{\Delta g}{g} + \dfrac{\Delta L}{L} + \dfrac{2\Delta D}{D} + \dfrac{\Delta l}{l}\right) \times 100$

$$= \left(\frac{1}{300} + \frac{1}{9.81} + \frac{1}{9820} + 2 \times \frac{1}{41} + \frac{1}{87}\right) \times 100 \quad = 0.065 \times 100 = 6.5\%$$

Problem 62. According to Joule's law of heating, heat produced $H = I^2 Rt$, where I is current, R is resistance and t is time. If the errors in the measurement of I, R and t are 3%, 4% and 6% respectively then error in the measurement of H is

(a) ± 17% (b) ± 16% (c) ± 19% (d) ± 25%

Solution : (b) $H = I^2 R t$

$$\therefore \frac{\Delta H}{H} \times 100 = \left(\frac{2\Delta I}{I} + \frac{\Delta R}{R} + \frac{\Delta t}{t}\right) \times 100 \quad = (2 \times 3 + 4 + 6)\% \quad = 16\%$$

Problem 63. If there is a positive error of 50% in the measurement of velocity of a body, then the error in the measurement of kinetic energy is

(a) 25% (b) 50% (c) 100% (d) 125%

Solution : (c) Kinetic energy $E = \frac{1}{2} m v^2$

$$\therefore \frac{\Delta E}{E} \times 100 = \left(\frac{\Delta m}{m} + \frac{2\Delta v}{v}\right) \times 100$$

Here $\Delta m = 0$ and $\frac{\Delta v}{v} \times 100 = 50\%$

$$\therefore \frac{\Delta E}{E} \times 100 = 2 \times 50 = 100\%$$

Problem 64. A physical quantity P is given by $P = \dfrac{A^3 B^{\frac{1}{2}}}{C^{-4} D^{\frac{3}{2}}}$. The quantity which brings in the maximum percentage error in P is

(a) A (b) B (c) C (d) D

Solution : (c) Quantity C has maximum power. So it brings maximum error in P.

Problems based on units and dimensions

1. Number of base SI units is **[MP PET 2003]**

(a) 4 (b) 7 (c) 3 (d) 5

2. The unit of Planck's constant is **[RPMT 1999; MP PET 2003]**

(a) *Joule* (b) *Joule/s* (c) *Joule/m* (d) *Joule- s*

3. The unit of reactance is **[MP PET 2003]**

(a) *Ohm* (b) *Volt* (c) *Mho* (d) *Newton*

4. The dimension of $\frac{R}{L}$ are **[MP PET 2003]**

(a) T^2 (b) T (c) T^{-1} (d) T^{-2}

5. Dimensions of potential energy are **[MP PET 2003]**

(a) MLT^{-1} (b) ML^2T^{-2} (c) $ML^{-1}T^{-2}$ (d) $ML^{-1}T^{-1}$

6. The dimensions of electric potential are **[UPSEAT 2003]**

(a) $[ML^2T^{-2}Q^{-1}]$ (b) $[MLT^{-2}Q^{-1}]$ (c) $ML^2T^{-1}Q$ (d) $ML^2T^{-2}Q$

7. The physical quantities not having same dimensions are **[AIEEE 2003]**

(a) Speed and $(\mu_0\varepsilon_0)^{-1/2}$ (b) Torque and work

(c) Momentum and Planck's constant (d) Stress and Young's modulus

8. The dimensional formula for Boltzmann's constant is **[MP PET 2002]**

(a) $[ML^2T^{-2}\theta^{-1}]$ (b) $[ML^2T^{-2}]$ (c) $[ML^0T^{-2}\theta^{-1}]$ (d) $[ML^{-2}T^{-1}\theta^{-1}]$

9. Which of the following quantities is dimensionless **[MP PET 2002]**

(a) Gravitational constant (b) Planck's constant (c) Power of a convex lens (d) None of these

10. Which of the two have same dimensions **[AIEEE 2002]**

(a) Force and strain (b) Force and stress

(c) Angular velocity and frequency (d) Energy and strain

11. The dimensions of pressure is equal to **[AIEEE 2002]**

(a) Force per unit volume (b) Energy per unit volume (c) Force (d) Energy

12. Identify the pair whose dimensions are equal **[AIEEE 2002]**

(a) Torque and work (b) Stress and energy (c) Force and stress (d) Force and work

13. A physical quantity x depends on quantities y and z as follows: $x = Ay + B\tan Cz$, where A, B and C are constants. Which of the following do not have the same dimensions **[AMU (Eng.) 2001]**

(a) x and B (b) C and z^{-1} (c) y and B/A (d) x and A

14. $ML^3T^{-1}Q^{-2}$ is dimension of **[RPET 2000]**

(a) Resistivity (b) Conductivity (c) Resistance (d) None of these

15. Two quantities A and B have different dimensions. Which mathematical operation given below is physically meaningful **[CPMT 1997]**

(a) A/B (b) $A+B$ (c) $A-B$ (d) None of these

16. Let $[\varepsilon_0]$ denotes the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current, then

(a) $[\varepsilon_0] = M^{-1}L^{-3}T^2I$ (b) $[\varepsilon_0] = M^{-1}L^{-3}T^4I^2$ (c) $[\mu_0] = MLT^{-2}I^{-2}$ (d) $[\mu_0] = ML^2T^{-1}I$

17. The dimension of quantity (L/RCV) is **[Roorkee 1994]**

(a) $[A]$ (b) $[A]^2$ (c) $[A^{-1}]$ (d) None of these

18. The quantity $X = \frac{\varepsilon_0 LV}{t}$; here ε_0 is the permittivity of free space, L is length, V is potential difference and t is time. The dimensions of X are same as that of

(a) Resistance (b) Charge (c) Voltage (d) Current

19. The unit of permittivity of free space ε_0 is **[MP PET 1993; MP PMT 2003]**

(a) *Coulomb/Newton-metre* (b) *Newton-metre²/Coulomb²*

(c) *Coulomb²/(Newton-metre)²* (d) *Coulomb²/Newton-metre²*

20. Dimensional formula of capacitance is **[CPMT 1978; MP PMT 1979; IIT-JEE 1983]**

(a) $M^{-1}L^{-2}T^4A^2$ (b) $ML^2T^4A^{-2}$ (c) $MLT^{-4}A^2$ (d) $M^{-1}L^{-2}T^{-4}A^{-2}$

21. The dimensional formula for impulse is **[EAMCET 1981; CBSE PMT 1991; CPMT 1978; AFMC 1998; BCECE 2003]**

(a) MLT^{-2} (b) MLT^{-1} (c) ML^2T^{-1} (d) M^2LT^{-1}

22. The dimensions of universal gravitational constant are **[MP PMT 1984, 87, 97, 2000; CBSE PMT 1988, 92, 2004; MP PET 1984, 96, 99; MNR 1992; DPMT 1984; CPMT 1978, 84, 89, 90, 92, 96; AFMC 1999; NCERT 1975; DPET 1993; AIIMS 2002; RPET 2001; Pb. PMT 2002; UPSEAT 1999; BCECE 2003]**

(a) $M^{-2}L^2T^{-2}$ (b) $M^{-1}L^3T^{-2}$ (c) $ML^{-1}T^{-2}$ (d) ML^2T^{-2}

23. How many wavelength of Kr^{86} are there in one *metre* **[MNR 1985; UPSEAT 2000]**

(a) 1553164.13 (b) 1650763.73 (c) 652189.63 (d) 2348123.73

24. Light year is a unit of **[MP PMT 1989; AFMC 1991; CPMT 1991]**

(a) Time (b) mass (c) Distance (d) Energy

25. L, C and R represent physical quantities inductance, capacitance and resistance respectively. The combination which has the dimensions of frequency is **[IIT-JEE 1984]**

(a) $1/RC$ and R/L (b) $1/\sqrt{RC}$ and $\sqrt{R/L}$ (c) $1/\sqrt{LC}$ (d) C/L

26. In the relation $P = \frac{\alpha}{\beta} e^{-\frac{\alpha z}{k\theta}}$, P is pressure, z is distance, k is Boltzmann constant and θ is temperature. The dimensional formula of β will be **[IIT-JEE (Screening) 2004]**

(a) $[M^0L^2T^0]$ (b) $[M^1L^2T^1]$ (c) $[M^1L^0T^0]$ (d) $[M^0L^2T^1]$

27. If the acceleration due to gravity be taken as the unit of acceleration and the velocity generated in a falling body in one second as the unit of velocity then

(a) The new unit of length is g *metre*

(b) The new unit of length is 1 *metre*

(c) The new unit of length is g^2 *metre*

(d) The new unit of time is $\frac{1}{g}$ *second*

28. The famous Stefan's law of radiation states that the rate of emission of thermal radiation per unit by a black body is proportional to area and fourth power of its absolute temperature that is $Q = \sigma AT^4$ where $A =$ area, $T =$ temperature and σ is a universal constant. In the 'energy- length- time temperature' (E-L-T-K) system the dimension of σ is

(a) $E^2T^2L^{-2}K^{-2}$ (b) $E^{-1}T^{-2}L^{-2}K^{-1}$ (c) $ET^{-1}L^{-3}K^{-4}$ (d) $ET^{-1}L^{-2}K^{-4}$ 2.

29. The resistive force acting on a body moving with a velocity V through a fluid at rest is given by $F = C_DV^2A\rho$ where, C_D = coefficient of drag, A = area of cross-section perpendicular to the direction of motion. The dimensions of C_D are

(a) ML^3T^{-2} (b) $M^{-1}L^{-1}T^2$ (c) $M^{-1}L^{-1}T^{-2}$ (d) $M^0L^0T^0$

30. The dimensions of (angular momentum)/(magnetic moment) are :

(a) $[M^3LT^{-2}A^2]$ (b) $[MA^{-1}T^{-1}]$ (c) $[ML^2A^{-2}T]$ (d) $[M^2L^{-3}AT^2]$

31. The frequency n of vibrations of uniform string of length l and stretched with a force F is given by $n = \frac{P}{2l}\sqrt{\frac{F}{m}}$ where p is the number of segments of the vibrating string and m is a constant of the string. What are the dimensions of m

(a) $ML^{-1}T^{-1}$ (b) $ML^{-3}T^0$ (c) $ML^{-2}T^0$ (d) $ML^{-1}T^0$

32. Choose the wrong statement(s)

(a) A dimensionally correct equation may be correct

(b) A dimensionally correct equation may be incorrect

(c) A dimensionally incorrect equation may be incorrect

(d) A dimensionally incorrect equation may be incorrect

33. A certain body of mass M moves under the action of a conservative force with potential energy V given by $V = \frac{Kr}{x^2 + a^{2'}}$ where x is the displacement and a is the amplitude. The units of K are

(a) *Watt* (b) *Joule* (c) *Joule-metre* (d) None of these.

34. The Richardson equation is given by $I = AT^2e^{-B/kT}$. The dimensional formula for AB^2 is same as that for

(a) IT^2 (b) kT (c) IK^2 (d) IK^2/T

35. If the units of force, energy and velocity are 10 N, 100 J and 5 ms^{-1}, the units of length, mass and time will be

(a) 10*m*, 5*kg*, 1*s* (b) 10*m*, 4*kg*, 2*s* (c) 10*m*, 4*kg*, 0.5*s* (d) 20*m*, 5*kg*, 2*s*.

Problems based on error of measurement

36. The period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ where l is about 100 *cm* and is known to 1*mm* accuracy. The period is about 2*s*. The time of 100 oscillations is measured by a stop watch of least count 0.1 s. The percentage error in g is

(a) 0.1% (b) 1% (c) 0.2% (d) 0.8%

37. The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimation of the kinetic energy obtained by measuring mass and speed **[NCERT 1990; Orissa JEE 1990]**

(a) 11% (b) 8% (c) 5% (d) 1%

38. While measuring the acceleration due to gravity by a simple pendulum, a student makes a positive error of 1% in the length of the pendulum and a negative error of 3% in the value of time period. His percentage error in the measurement of g by the relation $g = 4\pi^2 (l/T^2)$ will be

(a) 2% (b) 4% (c) 7% (d) 10%

39. The random error in the arithmetic mean of 100 observations is *x*; then random error in the arithmetic mean of 400 observations would be

(a) $4x$ (b) $\frac{1}{4}x$ (c) $2x$ (d) $\frac{1}{2}x$

40. What is the number of significant figures in 0.310×10^3

(a) 2 (b) 3 (c) 4 (d) 6

41. Error in the measurement of radius of a sphere is 1%. The error in the calculated value of its volume is

(a) 1% (b) 3% (c) 5% (d) 7%

42. The mean time period of second's pendulum is 2.00*s* and mean absolute error on the time period is 0.05*s*. To express maximum estimate of error, the time period should be written as

(a) $(2.00 \pm 0.01)\ s$ (b) $(2.00 + 0.025)\ s$ (c) $(2.00 \pm 0.05)\ s$ (d) $(2.00 \pm 0.10)\ s$

43. A body travels uniformly a distance of (13.8 ±0.2) m in a time (4.0 ± 0.3) s. The velocity of the body within error limits is

(a) $(3.45 \pm 0.2)\ ms^{-1}$ (b) $(3.45 \pm 0.3)\ ms^{-1}$ (c) $(3.45 \pm 0.4)\ ms^{-1}$ (d) $(3.45 \pm 0.5)\ ms^{-1}$

44. The percentage error in the above problem is

(a) 7% (b) 5.95% (c) 8.95% (d) 9.85%

45. The unit of percentage error is

(a) Same as that of physical quantity

(b) Different from that of physical quantity

(c) Percentage error is unit less

(d) Errors have got their own units which are different from that of physical quantity measured

46. The decimal equivalent of 1/20 upto three significant figures is

(a) 0.0500 (b) 0.05000 (c) 0.0050 (d) 5.0×10^{-2}

47. If 97.52 is divided by 2.54, the correct result in terms of significant figures is

(a) 38.4 (b) 38.3937 (c) 38.394 (d) 38.39

48. Accuracy of measurement is determined by

(a) Absolute error (b) Percentage error (c) Both (d) None of these

49. The radius of a sphere is (5.3 ± 0.1) *cm*. The percentage error in its volume is

(a) $\frac{0.1}{5.3} \times 100$ (b) $3 \times \frac{0.1}{5.3} \times 100$ (c) $\frac{0.1 \times 100}{3.53}$ (d) $3 + \frac{0.1}{5.3} \times 100$

50. A thin copper wire of length *l* metre increases in length by 2% when heated through 10°C. What is the percentage increase in area when a square copper sheet of length *l* metre is heated through 10°*C*

(a) 4% (b) 8% (c) 16% (d) None of the above.

51. In the context of accuracy of measurement and significant figures in expressing results of experiment, which of the following is/are correct

(1) Out of the two measurements 50.14 *cm* and 0.00025 *ampere*, the first one has greater accuracy

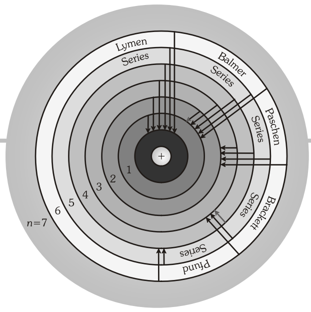
(2) If one travels 478 *km* by rail and 397 *m*. by road, the total distance travelled is 478 *km*.

(a) Only (1) is correct (b) Only (2) is correct (c) Both are correct (d) None of them is correct.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.

b	d	a	c	b	a	c	a	d	c
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
b	a	d	a	a	c	c	d	d	a
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
b	b	b	c	a	a	a	d	d	b
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
d	c	c	c	b	c	b	c	d	b
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
b	c	b	c	c	a	a	b	b	a
51.									

4



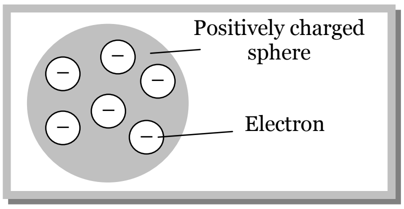
Atomic Structure

Important Atomic Models

(1) **Thomson's model**

J.J. Thomson gave the first idea regarding structure of atom. According to this model.

(i) An atom is a solid sphere in which entire and positive charge and it's mass is uniformly distributed and in which negative charge (*i.e.* electron) are embedded like seeds in watermelon.



Success and failure

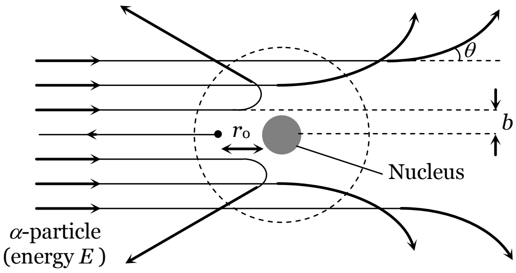
Explained successfully the phenomenon of thermionic emission, photoelectric emission and ionization.

The model fail to explain the scattering of α- particles and it cannot explain the origin of spectral lines observed in the spectrum of hydrogen and other atoms.

(2) **Rutherford's model**

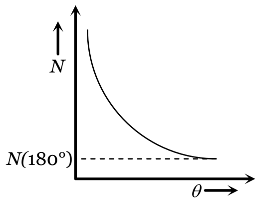
Rutherford's α-particle scattering experiment

Rutherford performed experiments on the scattering of alpha particles by extremely thin gold foils and made the following observations



Number of scattered particles :

$$N \propto \frac{1}{\sin^4(\theta/2)}$$



(i) Most of the α-particles pass through the foil straight away undeflected.

(ii) Some of them are deflected through small angles.

(iii) A few α-particles (1 in 1000) are deflected through the angle more than 90°.

(iv) A few α -particles (very few) returned back *i.e.* deflected by 180°.

(v) Distance of closest approach (Nuclear dimension)

The minimum distance from the nucleus up to which the α-particle approach, is called the distance of closest approach (r_0). From figure $r_0 = \frac{1}{4\pi\varepsilon_0}.\frac{Ze^2}{E}$; $E = \frac{1}{2}mv^2 =$ K.E. of α-particle

(vi) Impact parameter (b) : The perpendicular distance of the velocity vector ($\vec{v}$) of the α-particle from the centre of the nucleus when it is far away from the nucleus is known as impact parameter. It is given as

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)} \Rightarrow b \propto \cot(\theta/2)$$

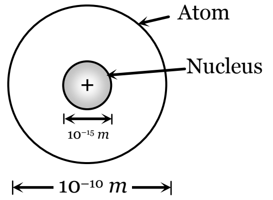
Note : ❑ If t is the thickness of the foil and N is the number of α-particles scattered in a particular direction (θ = constant), it was observed that $\frac{N}{t}$ = constant $\Rightarrow \frac{N_1}{N_2} = \frac{t_1}{t_2}$.

After Rutherford's scattering of α-particles experiment, following conclusions were made as regard as atomic structure :

(a) Most of the mass and all of the charge of an atom concentrated in a very small region is called atomic nucleus.

(b) Nucleus is positively charged and it's size is of the order of $10^{-15}\ m \approx 1$ *Fermi*.

(c) In an atom there is maximum empty space and the electrons revolve around the nucleus in the same way as the planets revolve around the sun.



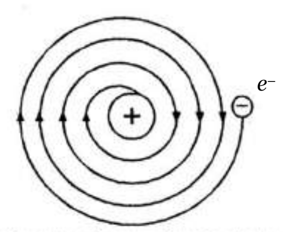
Size of the nucleus = 1 Fermi = $10^{-15}\ m$
Size of the atom 1 Å = $10^{-10}\ m$

Draw backs

(i) Stability of atom : It could not explain stability of atom because according to classical electrodynamic theory an accelerated charged particle should continuously radiate energy. Thus an electron moving in an circular path around the nucleus should also radiate energy and thus move into smaller and smaller orbits of gradually decreasing radius and it should ultimately fall into nucleus.

(ii) According to this model the spectrum of atom must be continuous where as practically it is a line spectrum.

(iii) It did not explain the distribution of electrons outside the nucleus.



Instability of atom

(3) **Bohr's model**

Bohr proposed a model for hydrogen atom which is also applicable for some lighter atoms in which a single electron revolves around a stationary nucleus of positive charge Ze (called hydrogen like atom)

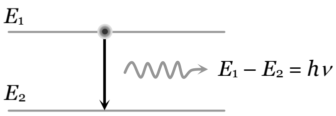
Bohr's model is based on the following postulates.

(i) The electron can revolve only in certain discrete non-radiating orbits, called stationary orbits, for which total angular momentum of the revolving electrons is an integral multiple of $\frac{h}{2\pi}\ (=\hbar)$

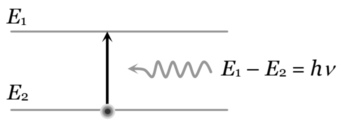
i.e. $L = n\left(\frac{h}{2\pi}\right) = mvr$; where n = 1, 2, 3, ...= Principal quantum number

(ii) The radiation of energy occurs only when an electron jumps from one permitted orbit to another.

When electron jumps from higher energy orbit (E_1) to lower energy orbit (E_2) then difference of energies of these orbits *i.e.* $E_1 - E_2$ emits in the form of photon. But if electron goes from E_2 to E_1 it absorbs the same amount of energy.



Emission



Absorption

Note : ❑ According to Bohr theory the momentum of an e^- revolving in second orbit of H_2 atom will be $\frac{h}{\pi}$

❑ For an electron in the n^{th} orbit of hydrogen atom in Bohr model, circumference of orbit $= n\lambda$; where λ = de-Broglie wavelength.

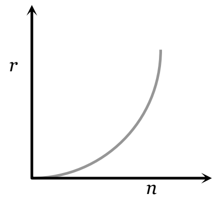
Bohr's Orbits (For Hydrogen and H_2-Like Atoms)

(1) **Radius of orbit**

For an electron around a stationary nucleus the electrostatics force of attraction provides the necessary centripetal force

i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ .... (i) also $mvr = \frac{nh}{2\pi}$ ....(ii)

From equation (i) and (ii) radius of n^{th} orbit



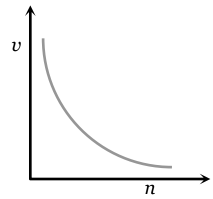
$$r_n = \frac{n^2h^2}{4\pi^2 kZme^2} = \frac{n^2h^2\varepsilon_0}{\pi mZe^2} = 0.53\frac{n^2}{Z}\text{Å} \qquad \left[\text{where } k = \frac{1}{4\pi\varepsilon_0}\right]$$

$\Rightarrow$ $\boldsymbol{r_n \propto \frac{n^2}{Z}}$

Note : ❑ The radius of the innermost orbit (n = 1) hydrogen atom (z = 1) is called Bohr's radius a_0 i.e. $a_0 = 0.53$ Å .

(2) **Speed of electron**

From the above relations, speed of electron in n^{th} orbit can be calculated as



$$v_n = \frac{2\pi kZe^2}{nh} = \frac{Ze^2}{2\varepsilon_0 nh} = \left(\frac{c}{137}\right).\frac{Z}{n} = 2.2\times10^6\frac{Z}{n} m/sec$$

where (c = speed of light 3×10^8 m/s)

Note : ❑ The ratio of speed of an electron in ground state in Bohr's first orbit of hydrogen atom to velocity of light in air is equal to $\frac{e^2}{2\varepsilon_0 ch} = \frac{1}{137}$ (where c = speed of light in air)

(3) **Some other quantities**

For the revolution of electron in n^{th} orbit, some other quantities are given in the following table

Quantity	Formula	Dependency on *n* and *Z*
(1) Angular speed	$\omega_n = \frac{v_n}{r_n} = \frac{\pi m z^2 e^4}{2\varepsilon_0^2 n^3 h^3}$	$\omega_n \propto \frac{Z^2}{n^3}$
(2) Frequency	$\nu_n = \frac{\omega_n}{2\pi} = \frac{mz^2e^4}{4\varepsilon_0^2 n^3 h^3}$	$\nu_n \propto \frac{Z^2}{n^3}$
(3) Time period	$T_n = \frac{1}{\nu_n} = \frac{4\varepsilon_0^2 n^3 h^3}{mz^2e^4}$	$T_n \propto \frac{n^3}{Z^2}$
(4) Angular momentum	$L_n = mv_n r_n = n\left(\frac{h}{2\pi}\right)$	$L_n \propto n$
(5) Corresponding current	$i_n = e\nu_n = \frac{mz^2e^5}{4\varepsilon_0^2 n^3 h^3}$	$i_n \propto \frac{Z^2}{n^3}$
(6) Magnetic moment	$M_n = i_n A = i_n(\pi r_n^2)$	$M_n \propto n$

	(where $\mu_0 = \frac{eh}{4\pi m}$ = Bohr magneton)	
(7) Magnetic field	$B = \frac{\mu_0 i_n}{2r_n} = \frac{\pi m^2 z^3 e^7 \mu_0}{8\varepsilon_0^3 n^5 h^5}$	$B \propto \frac{Z^3}{n^5}$

(4) **Energy**

(i) **Potential energy :** An electron possesses some potential energy because it is found in the field of nucleus potential energy of electron in n^{th} orbit of radius r_n is given by $U = k.\frac{(Ze)(-e)}{r_n} = -\frac{kZe^2}{r_n}$

(ii) **Kinetic energy :** Electron posses kinetic energy because of it's motion. Closer orbits have greater kinetic energy than outer ones.

As we know $\frac{mv^2}{r_n} = \frac{k.(Ze)(e)}{r_n^2}$ $\Rightarrow$ Kinetic energy $K = \frac{kZe^2}{2r_n} = \frac{|U|}{2}$

(iii) **Total energy :** Total energy (E) is the sum of potential energy and kinetic energy *i.e.* $E = K + U$

$$\Rightarrow E = -\frac{kZe^2}{2r_n} \text{ also } r_n = \frac{n^2h^2\varepsilon_0}{\pi mze^2}. \text{ Hence } E = -\left(\frac{me^4}{8\varepsilon_0^2h^2}\right).\frac{z^2}{n^2} = -\left(\frac{me^4}{8\varepsilon_0^2ch^3}\right)ch\frac{z^2}{n^2} = -Rch\frac{Z^2}{n^2} = -13.6\frac{Z^2}{n^2}eV$$

where $R = \frac{me^4}{8\varepsilon_0^2ch^3}$ = Rydberg's constant = 1.09×10^7 *per metre*

Note : ❑ Each Bohr orbit has a definite energy

❑ For hydrogen atom ($Z = 1$) $\Rightarrow E_n = -\frac{13.6}{n^2}eV$

❑ The state with $n = 1$ has the lowest (most negative) energy. For hydrogen atom it is $E_1 = -13.6$ eV.

❑ Rch = Rydberg's energy $\simeq 2.17 \times 10^{-18} J \simeq 31.6\, eV$.

❑ $E = -K = \frac{U}{2}$.

(iv) **Ionisation energy and potential :** The energy required to ionise an atom is called ionisation energy. It is the energy required to make the electron jump from the present orbit to the infinite orbit.

Hence $E_{ionisation} = E_\infty - E_n = 0 - \left(-13.6\frac{Z^2}{n^2}\right) = +\frac{13.6Z^2}{n^2}eV$

For H_2-atom in the ground state $E_{ionisation} = \frac{+13.6(1)^2}{n^2} = 13.6\,eV$

The potential through which an electron need to be accelerated so that it acquires energy equal to the ionisation energy is called ionisation potential. $V_{ionisation} = \frac{E_{ionisation}}{e}$

(v) **Excitation energy and potential :** When the electron is given energy from external source, it jumps to higher energy level. This phenomenon is called excitation.

The minimum energy required to excite an atom is called excitation energy of the particular excited state and corresponding potential is called exciting potential.

$$E_{Excitation} = E_{Final} - E_{Initial} \text{ and } V_{Excitation} = \frac{E_{excitation}}{e}$$

(vi) **Binding energy (B.E.) :** Binding energy of a system is defined as the energy released when it's constituents are brought from infinity to form the system. It may also be defined as the energy needed to separate it's constituents to large distances. If an electron and a proton are initially at rest and brought from large distances to form a hydrogen atom, 13.6 *eV* energy will be released. The binding energy of a hydrogen atom is therefore 13.6 *eV*.

Note : ❑ For hydrogen atom principle quantum number $n = \sqrt{\frac{13.6}{(\text{B.E.})}}$.

(5) **Energy level diagram**

The diagrammatic description of the energy of the electron in different orbits around the nucleus is called energy level diagram.

Energy level diagram of hydrogen/hydrogen like atom

	Principle quantum number	Orbit	Excited state	Energy for H_2 – atom	Energy for H_2 – like atom	Ionisation energy from this level (for H_2 – atom)
– – – – – –	$n = \infty$	Infinite	Infinite	$E_\infty = 0\ eV$	$0\ eV$	$0\ eV$
———	$n = 4$	Fourth	Third	$E_4 = -0.85\ eV$	$-0.85\ Z^2$	$+0.85\ eV$
———	$n = 3$	Third	Second	$E_3 = -1.51\ eV$	$-1.51\ Z^2$	$+1.51\ eV$
———	$n = 2$	Second	First	$E_2 = -3.4\ eV$	$-3.4\ Z^2$	$+3.4\ eV$
———	$n = 1$	First	Ground	$E_1 = -13.6\ eV$	$-13.6\ Z^2$	$+13.6\ eV$

Note : ❑ In hydrogen atom excitation energy to excite electron from ground state to first excited state will be $-3.4-(-13.6)=10.2\ eV$.

and from ground state to second excited state it is $[-1.51-(-13.6)=12.09\ eV]$.

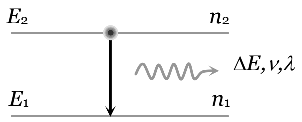
❑ In an H_2 atom when e^- makes a transition from an excited state to the ground state it's kinetic energy increases while potential and total energy decreases.

(6) **Transition of electron**

When an electron makes transition from higher energy level having energy $E_2(n_2)$ to a lower energy level having energy E_1 (n_1) then a photon of frequency ν is emitted

(i) **Energy of emitted radiation**

$$\Delta E = E_2 - E_1 = \frac{-Rc\,h\,Z^2}{n_2^2} - \left(-\frac{Rch\,Z^2}{n_1^2}\right) = 13.6 Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$



Emission

(ii) **Frequency of emitted radiation**

$$\Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h} = Rc\,Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

(iii) **Wave number/wavelength**

Wave number is the number of waves in unit length $\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c}$

$$\Rightarrow \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) = \frac{13.6Z^2}{hc}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

(iv) **Number of spectral lines** : If an electron jumps from higher energy orbit to lower energy orbit it emits raidations with various spectral lines.

If electron falls from orbit n_2 to n_1 then the number of spectral lines emitted is given by

$$N_E = \frac{(n_2 - n_1 + 1)(n_2 - n_1)}{2}$$

If electron falls from n^{th} orbit to ground state (*i.e.* $n_2 = n$ and $n_1 = 1$) then number of spectral lines emitted $N_E = \frac{n(n-1)}{2}$

Note : ❑ Absorption spectrum is obtained only for the transition from lowest energy level to higher energy levels. Hence the number of absorption spectral lines will be $(n-1)$.

(v) **Recoiling of an atom** : Due to the transition of electron, photon is emitted and the atom is recoiled

Recoil momentum of atom = momentum of photon $= \frac{h}{\lambda} = hRZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Also recoil energy of atom $= \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$ (where m = mass of recoil atom)

(7) **Drawbacks of Bohr's atomic model**

(i) It is valid only for one electron atoms, *e.g.* : H, He^+, Li^{+2}, Na^{+1} *etc.*

(ii) Orbits were taken as circular but according to Sommerfield these are elliptical.

(iii) Intensity of spectral lines could not be explained.

(iv) Nucleus was taken as stationary but it also rotates on its own axis.

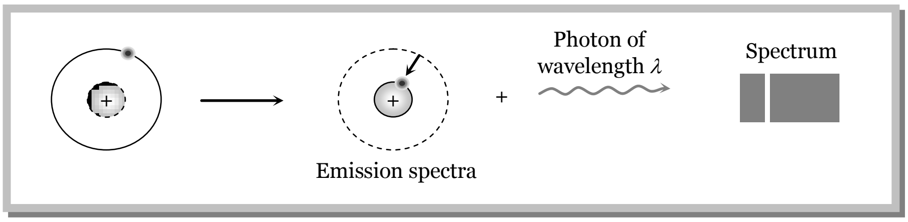
(v) It could not be explained the minute structure in spectrum line.

(vi) This does not explain the Zeeman effect (splitting up of spectral lines in magnetic field) and Stark effect (splitting up in electric field)

(vii) This does not explain the doublets in the spectrum of some of the atoms like sodium (5890$Å$ & 5896$Å$)

Hydrogen Spectrum and Spectral Series

When hydrogen atom is excited, it returns to its normal unexcited (or ground state) state by emitting the energy it had absorbed earlier. This energy is given out by the atom in the form of radiations of different wavelengths as the electron jumps down from a higher to a lower orbit. Transition from different orbits cause different wavelengths, these constitute spectral series which are characteristic of the atom emitting them. When observed through a spectroscope, these radiations are imaged as sharp and straight vertical lines of a single colour.



Spectral series

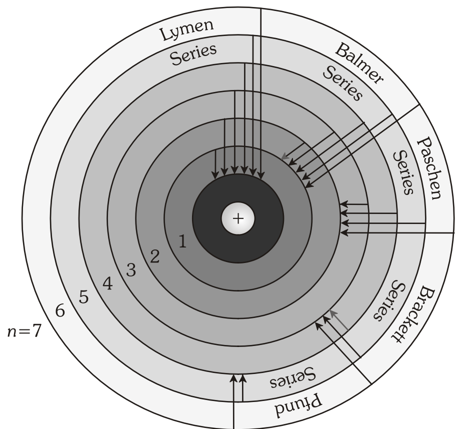
The spectral lines arising from the transition of electron forms a spectra series.

(i) Mainly there are five series and each series is named after it's discover as Lymen series, Balmer series, Paschen series, Bracket series and Pfund series.

(ii) According to the Bohr's theory the wavelength of the radiations emitted from hydrogen atom is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where n_2 = outer orbit (electron jumps from this orbit), n_1 = inner orbit (electron falls in this orbit)



(iii) First line of the series is called first member, for this line wavelength is maximum (λ_{max})

(iv) Last line of the series ($n_2 = \infty$) is called series limit, for this line wavelength is minimum (λ_{min})

Spectral series	**Transition**	**Wavelength (λ)** $= \frac{n_1^2 n_2^2}{(n_2^2 - n_1^2)R} = \frac{n_1^2}{\left(1 - \frac{n_1^2}{n_2^2}\right)R}$		$\frac{\lambda_{max}}{\lambda_{min}} = \frac{(n+1)^2}{(2n+1)}$	**Region**
		Maximum wavelength $(n_1 = n$ **and** $n_2 = n+1)$ $\lambda_{max} = \frac{n^2(n+1)^2}{(2n+1)R}$	**Minimum wavelength** $(n_2 = \infty, n_1 = n)$ $\lambda_{min} = \frac{n^2}{R}$		
1. Lymen series	$n_2 = 2, 3, 4 \ldots\infty$ $n_1 = 1$	$\lambda_{max} = \frac{(1)^2 (1+1)^2}{(2\times 1+1)R} = \frac{4}{3R}$	$n_1 = n = 1$ $\lambda_{min} = \frac{1}{R}$	$\frac{4}{3}$	Ultraviolet region
2.Balmer series	$n_2 = 3, 4, 5 \ldots\infty$ $n_1 = 2$	$n_1 = n = 2,\ n_2 = 2 + 1 = 3$ $\lambda_{max} = \frac{36}{5R}$	$\lambda_{min} = \frac{4}{R}$	$\frac{9}{5}$	Visible region
3. Paschen series	$n_2 = 4, 5, 6 \ldots\infty$ $n_1 = 3$	$n_1 = n = 3, n_2 = 3 + 1 = 4$ $\lambda_{max} = \frac{144}{7R}$	$n_1 = n = 3$ $\lambda_{min} = \frac{9}{R}$	$\frac{16}{7}$	Infrared region
4. Bracket series	$n_2 = 5, 6, 7 \ldots \infty$ $n_1 = 4$	$n_1 = n = 4, n_2 = 4 + 1 = 5$ $\lambda_{max} = \frac{400}{9R}$	$n_1 = n = 4$ $\lambda_{min} = \frac{16}{R}$	$\frac{25}{9}$	Infrared region
5. Pfund series	$n_2 = 6, 7, 8 \ldots \infty$ $n_1 = 5$	$n_1 = \lambda = 5, n_2 = 5 + 1 = 6$ $\lambda_{max} = \frac{900}{11R}$	$\lambda_{min} = \frac{25}{R}$	$\frac{36}{11}$	Infrared region

Quantum Numbers

An atom contains large number of shells and subshells. These are distinguished from one another on the basis of their size, shape and orientation (direction) in space. The parameters are expressed in terms of different numbers called quantum number.

Quantum numbers may be defined as a set of four number with the help of which we can get complete information about all the electrons in an atom. It tells us the address of the electron *i.e.* location, energy, the type of orbital occupied and orientation of that orbital.

(1) **Principal Quantum number (*n*) :** This quantum number determines the main energy level or shell in which the electron is present. The average distance of the electron from the nucleus and the energy of the electron depends on it. $E_n \propto \frac{1}{n^2}$ and $r_n \propto n^2$ (in *H*-atom)

The principal quantum number takes whole number values, $n = 1, 2, 3, 4, \ldots\ldots \infty$

(2) **Orbital quantum number (*l*) or azimuthal quantum number (*l*)**

This represents the number of subshells present in the main shell. These subsidiary orbits within a shell will be denoted as 1, 2, 3, 4 … or *s*, *p*, *d*, *f* … This tells the shape of the subshells.

The orbital angular momentum of the electron is given as $L = \sqrt{l(l+1)}\frac{h}{2\pi}$ (for a particular value of *n*).

For a given value of *n* the possible values of *l* are $l = 0, 1, 2, \ldots\ldots$ upto $(n - 1)$

(3) **Magnetic quantum number (m_l) :** An electron due to it's angular motion around the nucleus generates an electric field. This electric field is expected to produce a magnetic field. Under the influence of external magnetic field, the electrons of a subshell can orient themselves in certain preferred regions of space around the nucleus called orbitals.

The magnetic quantum number determines the number of preferred orientations of the electron present in a subshell.

The angular momentum quantum number m can assume all integral value between $-l$ to $+l$ including zero. Thus m_l can be $-1, 0, +1$ for $l = 1$. Total values of m_l associated with a particular value of l is given by $(2l + 1)$.

(4) **Spin (magnetic) quantum number (m_s) :** An electron in atom not only revolves around the nucleus but also spins about its own axis. Since an electron can spin either in clockwise direction or in anticlockwise direction. Therefore for any particular value of magnetic quantum number, spin quantum number can have two values, *i.e.* $m_s = \frac{1}{2}$ (Spin up) or $m_s = -\frac{1}{2}$ (Spin down)

This quantum number helps to explain the magnetic properties of the substance.

Electronic Configurations of Atoms

The distribution of electrons in different orbitals of an atom is called the electronic configuration of the atom. The filling of electrons in orbitals is governed by the following rules.

(1) **Pauli's exclusion principle**

"It states that no two electrons in an atom can have all the four quantum number (n, l, m_l and m_s) the same."

It means each quantum state of an electron must have a different set of quantum numbers n, l, m_l and m_s. This principle sets an upper limit on the number of electrons that can occupy a shell.

N_{max} in one shell = $2n^2$; Thus N_{max} in K, L, M, N .... shells are 2, 8, 18, 32,

Note : ❑ The maximum number of electrons in a subshell with orbital quantum number l is $2(2l + 1)$.

(2) **Aufbau principle**

Electrons enter the orbitals of lowest energy first.

As a general rule, a new electron enters an empty orbital for which $(n + l)$ is minimum. In case the value $(n + l)$ is equal for two orbitals, the one with lower value of n is filled first.

Thus the electrons are filled in subshells in the following order (memorize)

1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, 3*d*, 4*p*, 5*s*, 4*d*, 5*p*, 6*s*, 4*f*, 5*d*, 6*p*, 7*s*, 5*f*, 6*d*, 7*p*, ...

(3) **Hund's Rule**

When electrons are added to a subshell where more than one orbital of the same energy is available, their spins remain parallel. They occupy different orbitals until each one of them has at least one electron. Pairing starts only when all orbitals are filled up.

Pairing takes place only after filling 3, 5 and 7 electrons in *p*, *d* and *f* orbitals, respectively.

Concepts

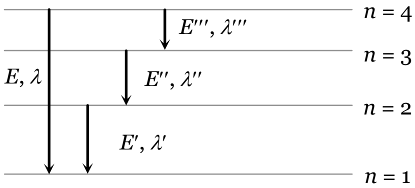
☞ *With the increase in principal quantum number the energy difference between the two successive energy level decreases, while wavelength of spectral line increases.*

$E' > E'' > E'''$

$\lambda' < \lambda'' < \lambda'''$

$E = E' + E'' + E'''$

$$\frac{1}{\lambda} = \frac{1}{\lambda'} + \frac{1}{\lambda''} + \frac{1}{\lambda'''}$$



☞ ***Rydberg constant is different for different elements***

$R (= 1.09 \times 10^7\ m^{-1})$ is the value of Rydberg constant when the nucleus is considered to be infinitely massive as compared to the revolving electron. In other words, the nucleus is considered to be stationary.

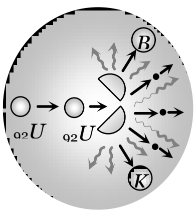
In case, the nucleus is not infinitely massive or stationary, then the value of Rydberg constant is given as $R' = \dfrac{R}{1 + \dfrac{m}{M}}$

where m is the mass of electron and M is the mass of nucleus.

☞ ***Atomic spectrum is a line spectrum***

Each atom has it's own characteristic allowed orbits depending upon the electronic configuration. Therefore photons emitted during transition of electrons from one allowed orbit to inner allowed orbit are of some definite energy only. They do not have a continuous graduation of energy. Therefore the spectrum of the emitted light has only some definite lines and therefore atomic spectrum is line spectrum.

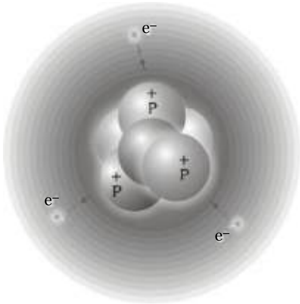
☞ *Just as dots of light of only three colours combine to form almost every conceivable colour on T.V. screen, only about 100 distinct kinds of atoms combine to form all the materials in the universe.*



Nuclear Physics & Radioactivity

Rutherford's α-scattering experiment established that the mass of atom is concentrated with small positively charged region at the centre which is called 'nucleus'.

Nuclei are made up of proton and neutron. The number of protons in a nucleus (called the atomic number or proton number) is represented by the symbol *Z*. The number of neutrons (neutron number) is represented by *N*. The total number of neutrons and protons in a nucleus is called it's mass number *A* so $A = Z + N$.



Neutrons and proton, when described collectively are called ***nucleons.***

Nucleus contains two types of particles : Protons and neutrons

Nuclides are represented as $_Z X^A$; where *X* denotes the chemical symbol of the element.

Neutron

Neutron is a fundamental particle which is essential constituent of all nuclei except that of hydrogen atom. It was discovered by Chadwick.

(1) The charge of neutron : It is neutral

(2) The mass of neutron : $1.6750 \times 10^{-27}\ kg$

(3) It's spin angular momentum : $\frac{1}{2} \times \left(\frac{h}{2\pi}\right) J\text{-}s$

(4) It's magnetic moment : $9.57 \times 10^{-27}\ J/Tesla$

(5) It's half life : 12 minutes

(6) Penetration power : High

(7) Types : Neutrons are of two types slow neutron and fast neutron, both are fully capable of penetrating a nucleus and causing artificial disintegration.

A free neutron outside the nucleus is unstable and decays into proton and electron.

$$_0n^1 \rightarrow \underset{\text{Proton}}{_1H^1} + \underset{\text{Electron}}{_{-1}\beta^0} + \underset{\text{Antinutrino}}{\bar{\nu}}$$

Thermal neutrons

Fast neutrons can be converted into slow neutrons by certain materials called moderator's (Paraffin wax, heavy water, graphite) when fast moving neutrons pass through a moderator, they collide with the molecules of the moderator, as a result of this, the energy of moving neutron decreases while that of the molecules of the moderator increases. After sometime they both attains same energy. The neutrons are then in thermal equilibrium with the molecules of the moderator and are called thermal neutrons.

Note : ❑ Energy of thermal neutron is about 0.025 *eV* and speed is about 2.2 *km/s*.

Nucleus

(1) **Different types of nuclei**

The nuclei have been classified on the basis of the number of protons (atomic number) or the total number of nucleons (mass number) as follows

(i) **Isotopes :** The atoms of element having same atomic number but different mass number are called isotopes. All isotopes have the same chemical properties. The isotopes of some elements are the following

$$_1H^1,\ _1H^2,\ _1H^3 \quad _8O^{16},\ _8O^{17},\ _8O^{18} \quad _2He^3,\ _2He^4 \quad _{17}Cl^{35},\ _{17}Cl^{37} \quad _{92}U^{235},\ _{92}U^{238}$$

(ii) **Isobars :** The nuclei which have the same mass number (*A*) but different atomic number (*Z*) are called isobars. Isobars occupy different positions in periodic table so all isobars have different chemical properties. Some of the examples of isobars are

$$_1H^3 \text{ and } _2He^3, \quad _6C^{14} \text{ and } _7N^{14},\ _8O^{17} \text{ and } _9F^{17}$$

(iii) **Isotones :** The nuclei having equal number of neutrons are called isotones. For them both the atomic number (*Z*) and mass number (*A*) are different, but the value of $(A - Z)$ is same. Some examples are

$$_4Be^9 \text{ and } _5B^{10},\ _6C^{13} \text{ and } _7N^{14},\ _8O^{18} \text{ and } _9F^{19},\ _3Li^7 \text{ and } _4Be^8,\ _1H^3 \text{ and } _2He^4$$

(iv) **Mirror nuclei :** Nuclei having the same mass number A but with the proton number (Z) and neutron number ($A - Z$) interchanged (or whose atomic number differ by 1) are called mirror nuclei for example.

$$_1H^3 \text{ and } _2He^3,\ _3Li^7 \text{ and } _4Be^7$$

(2) **Size of nucleus**

(i) Nuclear radius : Experimental results indicates that the nuclear radius is proportional to $A^{1/3}$, where A is the mass number of nucleus *i.e.* $R \propto A^{1/3} \Rightarrow R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}\ m = 1.2\ fm$.

Note : ❑ Heavier nuclei are bigger in size than lighter nuclei.

(ii) Nuclear volume : The volume of nucleus is given by $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \Rightarrow V \propto A$

(iii) Nuclear density : Mass per unit volume of a nucleus is called nuclear density.

$$\text{Nuclear density}(\rho) = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi(R_0 A^{1/3})^3}$$

where m = Average of mass of a nucleon (= mass of proton + mass of neutron = $1.66 \times 10^{-27}\ kg$)

and mA = Mass of nucleus

$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3} = 2.38 \times 10^{17}\ kg/m^3$$

Note : ❑ ρ is independent of A, it means ρ is same of all atoms.

❑ Density of a nucleus is maximum at it's centre and decreases as we move outwards from the nucleus.

(3) **Nuclear force**

Forces that keep the nucleons bound in the nucleus are called nuclear forces.

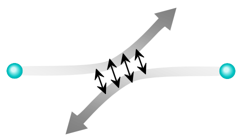
(i) Nuclear forces are short range forces. These do not exist at large distances greater than $10^{-15}\ m$.

(ii) Nuclear forces are the strongest forces in nature.

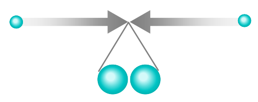
(iii) These are attractive force and causes stability of the nucleus.

(iv) These forces are charge independent.

(v) Nuclear forces are non-central force.



At low speeds, electromagnetic repulsion prevents the collision of nuclei



At high speeds, nuclei come close enough for the strong force to bind them together.

Nuclear forces are exchange forces

According to scientist Yukawa the nuclear force between the two nucleons is the result of the exchange of particles called mesons between the nucleons.

π - mesons are of three types – Positive π meson (π^+), negative π meson (π^-), neutral π *meson* (π^0)

The force between neutron and proton is due to exchange of charged meson between them *i.e.*

$$p \to \pi^+ + n, \quad n \to p + \pi^-$$

The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^0) between them *i.e.* $p \to p' + \pi^0$ and $n \to n' + \pi^0$

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the nuclear forces.

Dog-Bone analogy

The above interactions can be explained with the dog bone analogy according to which we consider the two interacting nucleons to be two dogs having a common bone clenched in between their teeth very firmly. Each one of these dogs wants to take the bone and hence they cannot be separated easily. They seem to be bound to each other with a strong attractive force (which is the bone) though the dogs themselves are strong enemies. The meson plays the same role of the common bone in between two nucleons.

(4) **Atomic mass unit (*amu*)**

The unit in which atomic and nuclear masses are measured is called atomic mass unit (*amu*)

$$1\ amu\ (\text{or } 1u) = \frac{1}{12}th \text{ of mass of } {}_6C^{12} \text{ atom} = 1.66 \times 10^{-27}\ kg$$

Masses of electron, proton and neutrons

Mass of electron (m_e) = $9.1 \times 10^{-31}\ kg = 0.0005486\ amu$, Mass of proton ($m_p$) = $1.6726 \times 10^{-27}\ kg = 1.007276\ amu$

Mass of neutron (m_n) = $1.6750 \times 10^{-27}\ kg = 1.00865\ amu$, Mass of hydrogen atom ($m_e + m_p$) = $1.6729 \times 10^{-27}\ kg = 1.0078\ amu$

Mass-energy equivalence

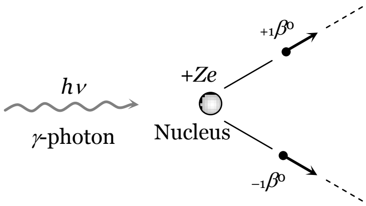
According to Einstein, mass and energy are inter convertible. The Einstein's mass energy relationship is given by $E = mc^2$

If $m = 1\ amu$, $c = 3 \times 10^8\ m/sec$ then $E = 931\ MeV$ *i.e.* 1 *amu* is equivalent to 931 *MeV* or **1 *amu* (or 1 *u*) = 931 *MeV***

(5) **Pair production and pair-annihilation**

When an energetic γ-ray photon falls on a heavy substance. It is absorbed by some nucleus of the substance and an electron and a positron are produced. This phenomenon is called pair production and may be represented by the following equation

$$\underset{(\gamma-\text{photon})}{h\nu} = \underset{(\text{Positron})}{{}_{1}\beta^0} + \underset{(\text{Electron})}{{}_{-1}\beta^0}$$



The rest-mass energy of each of positron and electron is

$$E_0 = m_0c^2 = (9.1 \times 10^{-31}\ kg) \times (3.0 \times 10^8\ m/s)^2$$

$$= 8.2 \times 10^{-14}\ J = \mathbf{0.51\ MeV}$$

Hence, for pair-production it is essential that the energy of γ-photon must be at least $2 \times 0.51 = 1.02\ MeV$. If the energy of γ-photon is less than this, it would cause photo-electric effect or Compton effect on striking the matter.

The converse phenomenon pair-annihilation is also possible. Whenever an electron and a positron come very close to each other, they annihilate each other by combining together and two γ-photons (energy) are produced. This phenomenon is called pair annihilation and is represented by the following equation.

$$\underset{(\text{Positron})}{{}_{+1}\beta^0} + \underset{(\text{Electron})}{{}_{-1}\beta^0} = \underset{(\gamma\text{-photon})}{h\nu} + \underset{(\gamma\text{-photon})}{h\nu}$$

(6) **Nuclear stability**

Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α, β-particles and γ - EM waves. (This process is called radioactivity). The stability of nucleus is determined by many factors. Few such factors are given below :

(i) Neutron-proton ratio $\left(\frac{N}{Z}\text{ Ratio}\right)$

The chemical properties of an atom are governed entirely by the number of protons (Z) in the nucleus, the stability of an atom appears to depend on both the number of protons and the number of neutrons.

For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal ($N \approx Z$) *i.e.* $\frac{N}{Z} = 1$

Heavy nuclei are stable only when they have more neutrons than protons. Thus heavy nuclei are neutron rich compared to lighter nuclei (for heavy nuclei, more is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore more neutrons are added to provide the strong attractive forces necessary to keep the nucleus stable.)

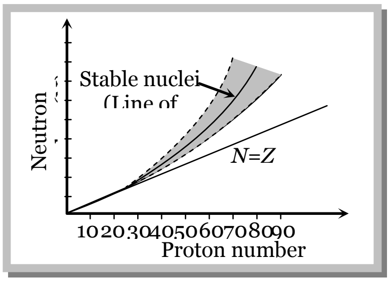
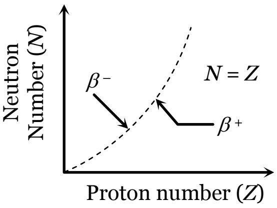


Figure shows a plot of *N* verses *Z* for the stable nuclei. For mass number upto about *A* = 40. For larger value of *Z* the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At *Bi* (*Z* = 83, *A* = 209), the neutron excess in *N* – *Z* = 43. There are no stable nuclides with *Z* > 83.

Note : ❑ The nuclide $_{83}Bi^{209}$ is the heaviest stable nucleus.

❑ A nuclide above the line of stability *i.e.* having excess neutrons, decay through β^- emission (neutron changes into proton). Thus increasing atomic number *Z* and decreasing neutron number *N*. In β^- emission, $\frac{N}{Z}$ ratio decreases.

A nuclide below the line of stability have excess number of protons. It decays by β^+ emission, results in decreasing *Z* and increasing *N*. In β^+ emission, the $\frac{N}{Z}$ ratio increases.



(ii) Even or odd numbers of *Z* or *N* : The stability of a nuclide is also determined by the consideration whether it contains an even or odd number of protons and neutrons.

It is found that an even-even nucleus (even *Z* and even *N*) is more stable (60% of stable nuclide have even *Z* and even *N*).

An even-odd nucleus (even *Z* and odd *N*) or odd-even nuclide (odd *Z* and even *N*) is found to be lesser sable while the odd-odd nucleus is found to be less stable.

Only five stable odd-odd nuclides are known : $_1H^2$, $_3Li^6$, $_5Be^{10}$, $_7N^{14}$ and $_{75}Ta^{180}$

(iii) Binding energy per nucleon : The stability of a nucleus is determined by value of it's binding energy per nucleon. In general higher the value of binding energy per nucleon, more stable the nucleus is

Mass Defect and Binding Energy.

(1) **Mass defect (Δm)**

It is found that the mass of a nucleus is always less than the sum of masses of it's constituent nucleons in free state. This difference in masses is called mass defect. Hence mass defect

Δm = Sum of masses of nucleons – Mass of nucleus

$$= \{Zm_p + (A-Z)m_n\} - M = \{Zm_p + Zm_e + (A-Z)m_z\} - M'$$

where m_p = Mass of proton, m_n = Mass of each neutron, m_e = Mass of each electron

M = Mass of nucleus, Z = Atomic number, A = Mass number, M' = Mass of atom as a whole.

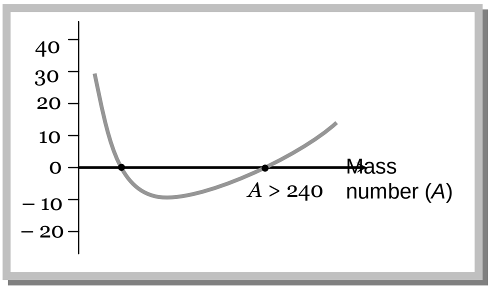
Note : ❑ The mass of a typical nucleus is about 1% less than the sum of masses of nucleons.

(2) **Packing fraction**

Mass defect per nucleon is called packing fraction

Packing fraction $(f) = \dfrac{\Delta m}{A} = \dfrac{M-A}{A}$ where M = Mass of nucleus, A = Mass number

Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, larger is the stability of the nucleus.



(i) Packing fraction may be of positive, negative or zero value.

(iii) At $A = 16, f \to$ Zero

(3) **Binding energy (B.E.)**

The neutrons and protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them infinitely apart (or the same energy is released during the formation of the nucleus). This energy is called the binding energy of the nucleus.

or

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect then according to Einstein's mass energy relation

Binding energy = $\Delta m \cdot c^2 = [\{m_pZ + m_n(A-Z)\} - M] \cdot c^2$

(This binding energy is expressed in *joule*, because Δm is measured in *kg*)

If Δm is measured in *amu* then binding energy = Δm *amu* = $[\{m_pZ + m_n(A-Z)\} - M]$ *amu* = $\Delta m \times 931$ *MeV*

(4) **Binding energy per nucleon**

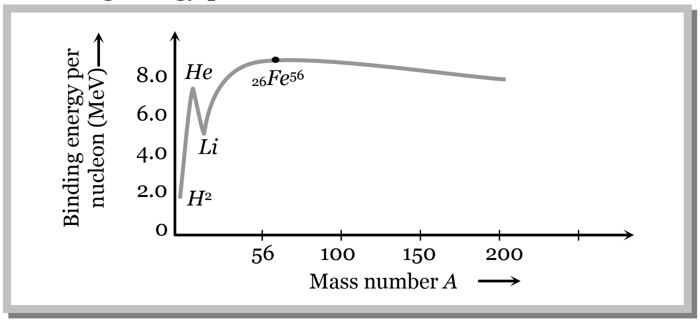
The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

$$\text{Binding energy per nucleon} = \frac{\text{Total binding energy}}{\text{Mass number (i.e. total number of nucleons)}} = \frac{\Delta m \times 931}{A}\,\frac{MeV}{Nucleon}$$

Binding energy per nucleon $\propto$ Stability of nucleus

Binding Energy Curve

It is the graph between binding energy per nucleon and total number of nucleons (*i.e.* mass number A)



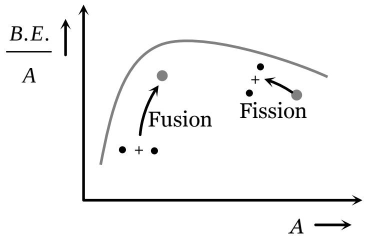
(1) Some nuclei with mass number $A < 20$ have large binding energy per nucleon than their neighbour nuclei. For example $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$. These nuclei are more stable than their neighbours.

(2) The binding energy per nucleon is maximum for nuclei of mass number A = 56 ($_{26}Fe^{56}$). It's value is 8.8 MeV per nucleon.

(3) For nuclei having A > 56, binding energy per nucleon gradually decreases for uranium (A = 238), the value of binding energy per nucleon drops to 7.5 MeV.

Note : ❑ When a heavy nucleus splits up into lighter nuclei, then binding energy per nucleon of lighter nuclei is more than that of the original heavy nucleus. Thus a large amount of energy is liberated in this process (nuclear fission).

❑ When two very light nuclei combines to form a relatively heavy nucleus, then binding energy per nucleon increases. Thus, energy is released in this process (nuclear fusion).



Nuclear Reactions

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction. The general expression for the nuclear reaction is as follows.

$$\underset{\text{(Parent nucleus)}}{X} + \underset{\text{(Incident particle)}}{a} \longrightarrow \underset{\text{(Compound nucleus)}}{C} \longrightarrow \underset{\text{(Compound nucleus)}}{Y} + \underset{\text{(Product particles)}}{b} + \underset{\text{(Energy)}}{Q}$$

Here X and a are known as reactants and Y and b are known as products. This reaction is known as (a, b) reaction and can be represented as $X(a, b)\ Y$

(1) ***Q* value or energy of nuclear reaction**

The energy absorbed or released during nuclear reaction is known as Q-value of nuclear reaction.

Q-value = (Mass of reactants – mass of products)c^2 *Joules*

= (Mass of reactants – mass of products) *amu*

If Q < 0, The nuclear reaction is known as endothermic. (The energy is absorbed in the reaction)

If Q > 0, The nuclear reaction is known as exothermic (The energy is released in the reaction)

(2) **Law of conservation in nuclear reactions**

(i) Conservation of mass number and charge number : In the following nuclear reaction

$$_2He^4 + {_7N^{14}} \to {_8O^{17}} + {_1H^1}$$

	Before the reaction	After the reaction
Mass number (A) →	4 +14 = 18	17 + 1 = 18
Charge number (Z) →	2 + 7 = 9	8 + 1 = 9

(ii) Conservation of momentum : Linear momentum/angular momentum of particles before the reaction is equal to the linear/angular momentum of the particles after the reaction. That is $\Sigma p = 0$

(iii) Conservation of energy : Total energy before the reaction is equal to total energy after the reaction. Term Q is added to balance the total energy of the reaction.

(3) **Common nuclear reactions**

The nuclear reactions lead to artificial transmutation of nuclei. Rutherford was the first to carry out artificial transmutation of nitrogen to oxygen in the year 1919.

$$_2He^4 + {_7N^{14}} \to {_9F^{18}} \to {_8O^{17}} + {_1H^1}$$

It is called (α, p) reaction. Some other nuclear reactions are given as follows.

(p, n) reaction $\Rightarrow$ ${}_1H^1 + {}_5B^{11} \rightarrow {}_6C^{12} \rightarrow {}_6C^{11} + {}_0n^1$

(p, α) reaction $\Rightarrow$ ${}_1H^1 + {}_3Li^{11} \rightarrow {}_4Be^8 \rightarrow {}_2He^4 + {}_2He^4$

(p, γ) reaction $\Rightarrow$ ${}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} \rightarrow {}_7N^{13} + \gamma$

(n, p) reaction $\Rightarrow$ ${}_0n^1 + {}_7N^{14} \rightarrow {}_7N^{15} \rightarrow {}_6C^{14} + {}_1H^1$

(γ, n) reaction $\Rightarrow$ $\gamma + {}_1H^2 \rightarrow {}_1H^1 + {}_0n^1$

Nuclear Fission and Fusion

Nuclear fission

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy is called nuclear fission.

The phenomenon of nuclear fission was discovered by scientist Ottohann and F. Strassman and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.

(1) **Fission reaction of U^{235}**

(i) Nuclear reaction :

$$ {}_{92}U^{235} + {}_0n^1 \rightarrow \underset{\text{(unstable nucleus)}}{{}_{92}U^{236}} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q $$

(ii) The energy released in U^{235} fission is about 200 MeV or 0.8 MeV per nucleon.

(iii) By fission of ${}_{92}U^{235}$, on an average 2.5 neutrons are liberated. These neutrons are called fast neutrons and their energy is about 2 MeV (for each). These fast neutrons can escape from the reaction so as to proceed the chain reaction they are need to slow down.

(iv) Fission of U^{235} occurs by slow neutrons only (of energy about 1eV) or even by thermal neutrons (of energy about 0.025 eV).

(v) 50 kg of U^{235} on fission will release $\approx 4 \times 10^{15}$ J of energy. This is equivalence to 20,000 tones of TNT explosion. The nuclear bomb dropped at Hiroshima had this much explosion power.

(vi) The mass of the compound nucleus must be greater than the sum of masses of fission products.

(vii) The $\dfrac{\text{Binding energy}}{\text{A}}$ of compound nucleus must be less than that of the fission products.

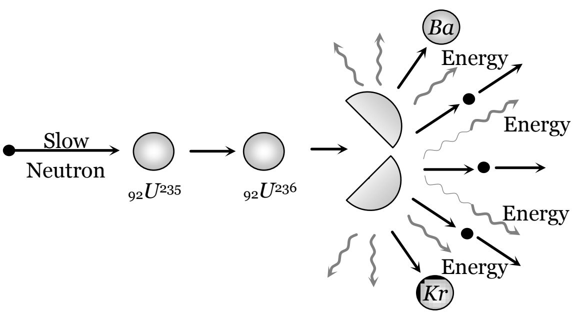
(viii) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments ${}_{56}Ba$ and ${}_{36}Kr$ are formed but they may be any stable isotopes of middle weight atoms.

Same other U^{235} fission reactions are

$$ {}_{92}U^{235} + {}_0n^1 \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1 $$
$$ \rightarrow {}_{57}La^{148} + {}_{35}Br^{85} + 3\,{}_0n^1 $$
$$ \rightarrow \text{Many more} $$

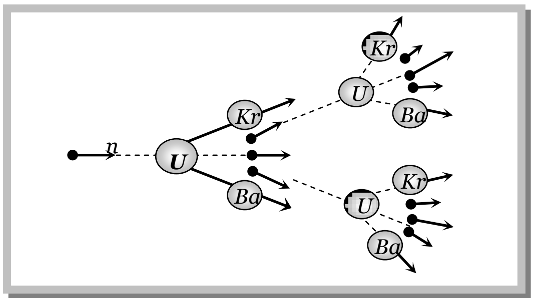
(ix) The neutrons released during the fission process are called prompt neutrons.

(x) Most of energy released appears in the form of kinetic energy of fission fragments.



(2) **Chain reaction**

In nuclear fission, three neutrons are produced along with the release of large energy. Under favourable conditions, these neutrons can cause further fission of other nuclei, producing large number of neutrons. Thus a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.



In the chain reaction, the number of nuclei undergoing fission increases very fast. So, the energy produced takes a tremendous magnitude very soon.

Difficulties in chain reaction

(i) Absorption of neutrons by U^{238}, the major part in natural uranium is the isotope U^{238} (99.3%), the isotope U^{235} is very little (0.7%). It is found that U^{238} is fissionable with fast neutrons, whereas U^{235} is fissionable with slow neutrons. Due to the large percentage of U^{238}, there is more possibility of collision of neutrons with U^{238}. It is found that the neutrons get slowed on coliding with U^{238}, as a result of it further fission of U^{238} is not possible (Because they are slow and they are absorbed by U^{238}). This stops the chain reaction.

Removal : (i) To sustain chain reaction $_{92}U^{235}$ is separated from the ordinary uranium. Uranium so obtained $\left({}_{92}U^{235}\right)$ is known as enriched uranium, which is fissionable with the fast and slow neutrons and hence chain reaction can be sustained.

(ii) If neutrons are slowed down by any method to an energy of about 0.3 eV, then the probability of their absorption by U^{238} becomes very low, while the probability of their fissioning U^{235} becomes high. This job is done by moderators. Which reduce the speed of neutron rapidly graphite and heavy water are the example of moderators.

(iii) Critical size : The neutrons emitted during fission are very fast and they travel a large distance before being slowed down. If the size of the fissionable material is small, the neutrons emitted will escape the fissionable material before they are slowed down. Hence chain reaction cannot be sustained.

Removal : The size of the fissionable material should be large than a critical size.

The chain reaction once started will remain steady, accelerate or retard depending upon, a factor called neutron reproduction factor (k). It is defined as follows.

$$k = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

→ If $k = 1$, the chain reaction will be steady. The size of the fissionable material used is said to be the critical size and it's mass, the critical mass.

→ If $k > 1$, the chain reaction accelerates, resulting in an explosion. The size of the material in this case is super critical. (Atom bomb)

→ If $k < 1$, the chain reaction gradually comes to a halt. The size of the material used us said to be sub-critical.

Types of chain reaction : Chain reactions are of following two types

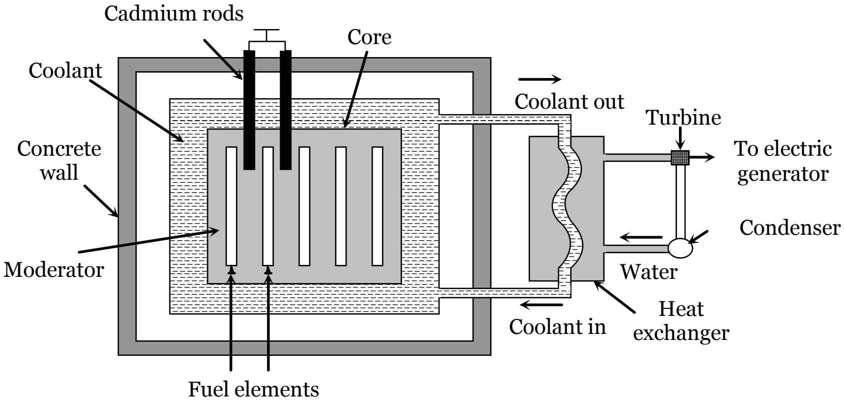
Controlled chain reaction	Uncontrolled chain reaction
Controlled by artificial method	No control over this type of nuclear reaction
All neurons are absorbed except one	More than one neutron takes part into reaction
It's rate is slow	Fast rate

Reproduction factor $k = 1$	Reproduction factor $k > 1$
Energy liberated in this type of reaction is always less than explosive energy	A large amount of energy is liberated in this type of reaction
Chain reaction is the principle of nuclear reactors	Uncontrolled chain reaction is the principle of atom bomb.

Note : ❑ The energy released in the explosion of an atom bomb is equal to the energy released by 2000 tonn of TNT and the temperature at the place of explosion is of the order of 10^7 ^oC.

Nuclear Reactor

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction. It is also called an atomic pile. It is thus a source of controlled energy which is utilised for many useful purposes.



(1) **Parts of nuclear reactor**

(i) **Fissionable material (Fuel) :** The fissionable material used in the reactor is called the fuel of the reactor. Uranium isotope (U^{235}) Thorium isotope (Th^{232}) and Plutonium isotopes (Pu^{239}, Pu^{240} and Pu^{241}) are the most commonly used fuels in the reactor.

(ii) **Moderator :** Moderator is used to slow down the fast moving neutrons. Most commonly used moderators are graphite and heavy water (D_2O).

(iii) **Control Material :** Control material is used to control the chain reaction and to maintain a stable rate of reaction. This material controls the number of neutrons available for the fission. For example, cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rods in or out of the core of the reactor.

(iv) **Coolant :** Coolant is a cooling material which removes the heat generated due to fission in the reactor. Commonly used coolants are water, CO_2 nitrogen *etc.*

(v) **Protective shield :** A protective shield in the form a concrete thick wall surrounds the core of the reactor to save the persons working around the reactor from the hazardous radiations.

Note : ❑ It may be noted that Plutonium is the best fuel as compared to other fissionable material. It is because fission in Plutonium can be initiated by both slow and fast neutrons. Moreover it can be obtained from U^{238}.

❑ Nuclear reactor is firstly devised by fermi. ❑ Apsara was the first Indian nuclear reactor.

(2) **Uses of nuclear reactor**

(i) In electric power generation.

(ii) To produce radioactive isotopes for their use in medical science, agriculture and industry.

(iii) In manufacturing of PU^{239} which is used in atom bomb.

(iv) They are used to produce neutron beam of high intensity which is used in the treatment of cancer and nuclear research.

Note : ❑ A type of reactor that can produce more fissile fuel than it consumes is the breeder reactor.

Nuclear fusion

In nuclear fusion two or more than two lighter nuclei combine to form a single heavy nucleus. The mass of single nucleus so formed is less than the sum of the masses of parent nuclei. This difference in mass results in the release of tremendous amount of energy

$$_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4\,MeV$$

$$_1H^3 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1 + 17.6\,MeV$$

or $$_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + 24\,MeV$$

For fusion high pressure ($\approx 10^6$ *atm*) and high temperature (of the order of 10^7 *K* to 10^8 *K*) is required and so the reaction is called thermonuclear reaction.

Fusion energy is greater then fission energy fission of one uranium atom releases about 200 *MeV* of energy. But the fusion of a deutron ($_1H^2$) and triton ($_1H^3$) releases about 17.6 *MeV* of energy. However the energy released per nucleon in fission is about 0.85 *MeV* but that in fusion is 4.4 *MeV*. So for the same mass of the fuel, the energy released in fusion is much larger than in fission.

Plasma : The temperature of the order of 10^8 *K* required for thermonuclear reactions leads to the complete ionisation of the atom of light elements. The combination of base nuclei and electron cloud is called plasma. The enormous gravitational field of the sun confines the plasma in the interior of the sun.

The main problem to carryout nuclear fusion in the laboratory is to contain the plasma at a temperature of $10^8 K$. No solid container can tolerate this much temperature. If this problem of containing plasma is solved, then the large quantity of deuterium present in sea water would be able to serve as in-exhaustible source of energy.

Note : ❑ To achieve fusion in laboratory a device is used to confine the plasma, called **Tokamak**.

Stellar Energy

Stellar energy is the energy obtained continuously from the sun and the stars. Sun radiates energy at the rate of about 10^{26} *joules* per *second*.

Scientist Hans Bethe suggested that the fusion of hydrogen to form helium (thermo nuclear reaction) is continuously taking place in the sun (or in the other stars) and it is the source of sun's (star's) energy.

The stellar energy is explained by two cycles

Proton-proton cycle	Carbon-nitrogen cycle
$_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_1e^0 + Q_1$	$_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} + Q_1$
$_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + Q_2$	$_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e^0$
$_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2{}_1H^1 + Q_3$	$_1H^1 + {}_6C^{13} \rightarrow {}_7N^{14} + Q_2$
$4{}_1H^1 \rightarrow {}_2He^4 + 2{}_{+1}e^0 + 2\gamma + 26.7\,MeV$	$_1H^1 + {}_7N^{14} \rightarrow {}_8O^{15} + Q_3$
	$_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 + Q_4$
	$_1H^1 + {}_7N^{15} \rightarrow {}_6C^{12} + {}_2He^4$
	$4{}_1H^1 \rightarrow {}_2He^4 + 2{}_1e^0 + 24.7\,MeV$

About 90% of the mass of the sun consists of hydrogen and helium.

Nuclear Bomb

Based on uncontrolled nuclear reactions.

Atom bomb	Hydrogen bomb
Based on fission process it involves the fission of U^{235}	Based on fusion process. Mixture of deutron and tritium is used in it
In this critical size is important	There is no limit to critical size
Explosion is possible at normal temperature and pressure	High temperature and pressure are required
Less energy is released compared to hydrogen bomb	More energy is released as compared to atom bomb so it is more dangerous than atom bomb

Concepts

☞ *A test tube full of base nuclei will weight heavier than the earth.*

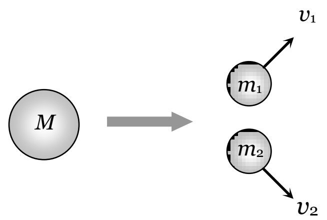
- *The nucleus of hydrogen contains only one proton. Therefore we may say that the proton is the nucleus of hydrogen atom.*
- *If the relative abundance of isotopes in an element has a ratio $n_1 : n_2$ whose atomic masses are m_1 and m_2 then atomic mass of the element is* $M = \dfrac{n_1 m_1 + n_2 m_2}{n_1 + n_2}$

Examples

Example: 1 A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio 8 : 1. The ratio of radii of the fragments is

(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 2 : 1

Solution : (a)



By conservation of momentum $m_1 v_1 = m_2 v_2$

$\Rightarrow \dfrac{v_1}{v_2} = \dfrac{8}{1} = \dfrac{m_2}{m_1}$ ...... (i)

Also from $r \propto A^{1/3} \Rightarrow \dfrac{r_1}{r_2} = \left(\dfrac{A_1}{A_2}\right)^{1/3} = \left(\dfrac{1}{8}\right)^{1/3} = \dfrac{1}{2}$

Example: 2 The ratio of radii of nuclei $^{27}_{13}Al$ and $^{125}_{52}Te$ is approximately **[J & K CET 2000]**

(a) 6 : 10 (b) 13 : 52 (c) 40 : 177 (d) 14 : 7

Solution : (a) By using $r \propto A^{1/3} \Rightarrow \dfrac{r_1}{r_2} = \left(\dfrac{A_1}{A_2}\right)^{1/3} = \left(\dfrac{27}{125}\right)^{1/3} = \dfrac{8}{5} = \dfrac{6}{10}$

Example: 3 If Avogadro's number is 6×10^{23} then the number of protons, neutrons and electrons in 14 *g* of $_6C^{14}$ are respectively

(a) $36 \times 10^{23}, 48 \times 10^{23}, 36 \times 10^{23}$ (b) $36 \times 10^{23}, 36 \times 10^{23}, 36 \times 10^{21}$

(c) $48 \times 10^{23}, 36 \times 10^{23}, 48 \times 10^{21}$ (d) $48 \times 10^{23}, 48 \times 10^{23}, 36 \times 10^{21}$

Solution : (a) Since the number of protons, neutrons and electrons in an atom of $_6C^{14}$ are 6, 8 and 6 respectively. As 14 *gm* of $_6C^{14}$ contains 6×10^{23} atoms, therefore the numbers of protons, neutrons and electrons in 14 *gm* of $_6C^{14}$ are $6 \times 6 \times 10^{23} = 36 \times 10^{23}$, $8 \times 6 \times 10^{23} = 48 \times 10^{23}$, $6 \times 6 \times 10^{23} = 36 \times 10^{23}$.

Example: 4 Two Cu^{64} nuclei touch each other. The electrostatics repulsive energy of the system will be

(a) 0.788 *MeV* (b) 7.88 *MeV* (c) 126.15 *MeV* (d) 788 *MeV*

Solution : (c) Radius of each nucleus $R = R_0 (A)^{1/3} = 1.2\,(64)^{1/3} = 4.8\ fm$

Distance between two nuclei (r) = 2R

So potential energy $U = \dfrac{k \cdot q^2}{r} = \dfrac{9 \times 10^9 \times (1.6 \times 10^{-19} \times 29)^2}{2 \times 4.8 \times 10^{-15} \times 1.6 \times 10^{-19}} = 126.15\ MeV$.

Example: 5 When $_{92}U^{235}$ undergoes fission. 0.1% of its original mass is changed into energy. How much energy is released if 1 *kg* of $_{92}U^{235}$ undergoes fission **[MP PET 1994; MP PMT/PET 1998; BHU 2001; BVP 2003]**

(a) $9 \times 10^{10}\ J$ (b) $9 \times 10^{11}\ J$ (c) $9 \times 10^{12}\ J$ (d) $9 \times 10^{13}\ J$

Solution : (d) By using $E = \Delta m \cdot c^2 \Rightarrow E = \left(\dfrac{0.1}{100} \times 1\right)(3 \times 10^8)^2 = 9 \times 10^{13}\ J$

Example: 6 1 *g* of hydrogen is converted into 0.993 *g* of helium in a thermonuclear reaction. The energy released is **[EAMCET (Med.) 1995; CPMT 1999]**

(a) $63 \times 10^7\ J$ (b) $63 \times 10^{10}\ J$ (c) $63 \times 10^{14}\ J$ (d) $63 \times 10^{20}\ J$

Solution : (b) $\Delta m = 1 - 0.993 = 0.007\ gm$

$\therefore E = \Delta mc^2 = 0.007 \times 10^{-3} \times (3 \times 10^8)^2 = 63 \times 10^{10}\ J$

Example: 7 The binding energy per nucleon of deuteron $({}_1^2H)$ and helium nucleus $({}_2^4He)$ is 1.1 *MeV* and 7 *MeV* respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is **[MP PMT 1992; Roorkee 1994; IIT-JEE 1996; AIIMS 1997; Haryana PMT 2000; Pb PMT 2001; CPMT 2001; AIEEE 2004]**

(a) 13.9 *MeV* (b) 26.9 *MeV* (c) 23.6 *MeV* (d) 19.2 *MeV*

Solution : (c) ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + Q$

Total binding energy of helium nucleus = 4 × 7 = 28 *MeV*

Total binding energy of each deutron = 2 × 1.1 = 2.2 *MeV*

Hence energy released = 28 – 2 × 2.2 = 23.6 *MeV*

Example: 8 The masses of neutron and proton are 1.0087 *amu* and 1.0073 *amu* respectively. If the neutrons and protons combine to form a helium nucleus (alpha particles) of mass 4.0015 *amu*. The binding energy of the helium nucleus will be [1 *amu*= 931 *MeV*] **[CPMT 1986; MP PMT 1995; CBSE 2003]**

(a) 28.4 *MeV* (b) 20.8 *MeV* (c) 27.3 *MeV* (d) 14.2 *MeV*

Solution : (a) Helium nucleus consist of two neutrons and two protons.

So binding energy $E = \Delta m$ *amu* $= \Delta m \times 931$ *MeV*

$\Rightarrow E = (2 \times m_p + 2m_n - M) \times 931$ *MeV* $= (2 \times 1.0073 + 2 \times 1.0087 - 4.0015) \times 931 = 28.4$ *MeV*

Example: 9 A atomic power reactor furnace can deliver 300 *MW*. The energy released due to fission of each of uranium atom U^{238} is 170 *MeV*. The number of uranium atoms fissioned per hour will be

(a) 5×10^{15} (b) 10×10^{20} (c) 40×10^{21} (d) 30×10^{25}

Solution : (c) By using $P = \frac{W}{t} = \frac{n \times E}{t}$ where n = Number of uranium atom fissioned and E = Energy released due to each fission so $300 \times 10^6 = \frac{n \times 170 \times 10^6 \times 1.6 \times 10^{-19}}{3600}$ $\Rightarrow n = 40 \times 10^{21}$

Example: 10 The binding energy per nucleon of O^{16} is 7.97 *MeV* and that of O^{17} is 7.75 *MeV*. The energy (in *MeV*) required to remove a neutron from O^{17} is **[IIT-JEE 1995]**

(a) 3.52 (b) 3.64 (c) 4.23 (d) 7.86

Solution : (c) $O^{17} \rightarrow O^{16} + {}_0n^1$

$\therefore$ Energy required = Binding of O^{17} – binding energy of O^{16} = 17 × 7.75 – 16 × 7.97 = 4.23 *MeV*

Example: 11 A gamma ray photon creates an electron-positron pair. If the rest mass energy of an electron is 0.5 *MeV* and the total kinetic energy of the electron-positron pair is 0.78 *MeV*, then the energy of the gamma ray photon must be **[MP PMT 1991]**

(a) 0.78 *MeV* (b) 1.78 *MeV* (c) 1.28 *MeV* (d) 0.28 *MeV*

Solution : (b) Energy of γ-rays photon = 0.5 + 0.5 +0.78 = 1.78 *MeV*

Example: 12 What is the mass of one Curie of U^{234} **[MNR 1985]**

(a) 3.7×10^{10} *gm* (b) 2.348×10^{23} *gm* (c) 1.48×10^{-11} *gm* (d) 6.25×10^{-34} *gm*

Solution : (c) 1 *curie* = 3.71×10^{10} disintegration/*sec* and mass of 6.02×10^{23} atoms of U^{234} = 234 *gm*

$\therefore$ Mass of 3.71×10^{10} atoms $= \frac{234 \times 3.71 \times 10^{10}}{6.02 \times 10^{23}} = 1.48 \times 10^{-11}$ *gm*

Example: 13 In the nuclear fusion reaction ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$, given that the repulsive potential energy between the two nuclei is $-7.7 \times 10^{-14}\ J$, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}\ J/K$] **[AIEEE 2003]**

(a) $10^9\ K$ (b) $10^7\ K$ (c) $10^5\ K$ (d) $10^3\ K$

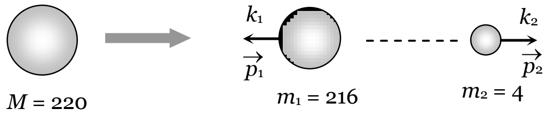
Solution : (a) Kinetic energy of molecules of a gas at a temperature T is 3/2 kT

$\therefore$ To initiate the reaction $\frac{3}{2}kT = 7.7 \times 10^{-14}\ J$ $\Rightarrow T = 3.7 \times 10^9\ K$.

Example: 14 A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 *MeV*. Calculate the kinetic energy of the α-particle **[IIT-JEE (Screening) 2003]**

(a) 4.4 *MeV* (b) 5.4 *MeV* (c) 5.6 *MeV* (d) 6.5 *MeV*

Solution : (b)



Q-value of the reaction is 5.5 *eV* *i.e.* $k_1 + k_2 = 5.5\ MeV$ ......(i)

By conservation of linear momentum $p_1 = p_2 \Rightarrow \sqrt{2(216)k_1} = \sqrt{2(4)k_2} \Rightarrow k_2 = 54\,k_1$ ......(ii)

On solving equation (i) and (ii) we get k_2 = 5.4 *MeV*.

***Example*: 15** Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a ${}^{20}_{10}Ne$ nucleus and M_2 the mass of a ${}^{40}_{20}Ca$ nucleus. Then **[IIT 1998; DPMT 2000]**

(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$ (c) $M_2 < 2M_1$ (d) $M_1 < 10(m_n + m_p)$

Solution : (c, d) Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less then the sum of masses of it's constituent particles ${}^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons. Therefore, mass of ${}^{20}_{10}Ne$ nucleus $M_1 < 10\,(m_p + m_n)$

Also heavier the nucleus, more is he mass defect thus $20\,(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

or $10\,(m_p + m_n) > M_2 - M_1$

$\Rightarrow$ $M_2 < M_1 + 10\,(m_p + m_n) \Rightarrow M_2 < M_1 + M_1 \Rightarrow M_2 < 2M_1$

Tricky example: 1

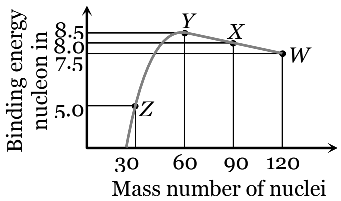
Binding energy per nucleon *vs* mass number curve for nuclei is shown in the figure. *W*, *X*, *Y* and *Z* are four nuclei indicated on the curve. The process that would release energy is **[IIT-JEE 1999]**

(a) $Y \to 2Z$

(b) $W \to X + Z$

(c) $W \to 2Y$

(d) $X \to Y + Z$



Solution : (c) Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleon) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (c) this happens.

Given $W \to 2Y$

Binding energy of reactants = 120 × 7.5 = 900 *MeV*

and binding energy of products = 2 (60 × 8.5) = 1020 *MeV* > 900 *MeV*

Radioactivity

The phenomenon of spontaneous emission of radiatons by heavy elements is called radioactivity. The elements which shows this phenomenon are called radioactive elements.

(1) Radioactivity was discovered by Henery Becquerel in uranium salt in the year 1896.

(2) After the discovery of radioactivity in uranium, Piere Curie and Madame Curie discovered a new radioactive element called radium (which is 10^6 times more radioactive than uranium)

(3) Some examples of radio active substances are : Uranium, Radium, Thorium, Polonium, Neptunium *etc.*

(4) Radioactivity of a sample cannot be controlled by any physical (pressure, temperature, electric or magnetic field) or chemical changes.

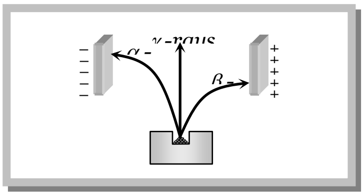
(5) All the elements with atomic number (Z) > 82 are naturally radioactive.

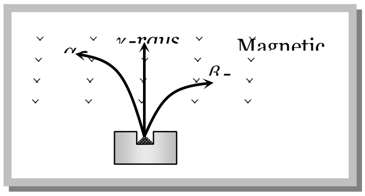
(6) The conversion of lighter elements into radioactive elements by the bombardment of fast moving particles is called artificial or induced radioactivity.

(7) Radioactivity is a nuclear event and not atomic. Hence electronic configuration of atom don't have any relationship with radioactivity.

Nuclear radiatons

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only. When the radiation enters into the external electric field, they splits into three parts





(i) Radiations which deflects towards negative plate are called α-rays (stream of positively charged particles)

(ii) Radiations which deflects towards positive plate are called β particles (stream of negatively charged particles)

(iii) Radiations which are undeflected called γ-*rays*. (E.M. waves or photons)

Note : ❑ Exactly same results were obtained when these radiations were subjected to magnetic field.

❑ No radioactive substance emits both α and β particles simultaneously. Also γ-rays are emitted after the emission of α or β-particles.

❑ β-particles are not orbital electrons they come from nucleus. The neutron in the nucleus decays into proton and an electron. This electron is emitted out of the nucleus in the form of β-rays.

Properties of α, β and γ-rays

Features	α- particles	β - particles	γ - rays
1. Identity	Helium nucleus or doubly ionised helium atom ($_2He^4$)	Fast moving electron $(-\beta^0$ or $\beta^-)$	Photons (E.M. waves)
2. Charge	+ 2*e*	− *e*	Zero
3. Mass 4 m_p (m_p = mass of proton = 1.87×10^{-27}	4 m_p	m_e	Massless
4. Speed	$\approx 10^7\ m/s$	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 *MeV* to 9 *MeV*	All possible values between a minimum certain value to 1.2 *MeV*	Between a minimum value to 2.23 *MeV*
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 *cm* of iron (or *Pb*) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete
10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	${}_Z X^A \xrightarrow{\alpha-decay}$ ${}_{Z-2}Y^{A-4} + {}_2He^4$	${}_Z X^A \to {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$ ${}_Z X^A \xrightarrow{n_\beta} {}_{Z'}X^A$	${}_Z X^A \to {}_Z X^a + \gamma$

	${}_{Z}X^{A} \xrightarrow{n_\alpha} {}_{Z'}Y^{A'}$ $\Rightarrow \boldsymbol{n_\alpha = \frac{A'-A}{4}}$	$\Rightarrow \boldsymbol{n_\beta = (2n_\alpha - Z + Z')}$	

Radioactive Disintegration

(1) **Law of radioactive disintegration**

According to Rutherford and Soddy law for radioactive decay is as follows.

"At any instant the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant"

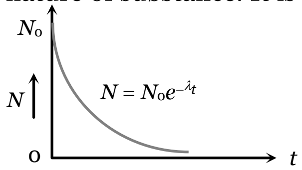
i.e. $-\frac{dN}{dt} \propto N \quad \Rightarrow \frac{dN}{dt} = -\lambda N$. It can be proved that $\boldsymbol{N = N_0 e^{-\lambda t}}$

This equation can also be written in terms of mass *i.e.* $M = M_0 e^{-\lambda t}$

where N = Number of atoms remains undecayed after time t, N_0 = Number of atoms present initially (*i.e.* at t = 0), M = Mass of radioactive nuclei at time t, M_0 = Mass of radioactive nuclei at time t = 0, $N_0 - N$ = Number of disintegrated nucleus in time t

$\frac{dN}{dt}$ = rate of decay, λ = Decay constant or disintegration constant or radioactivity constant or Rutherford Soddy's constant or the probability of decay per unit time of a nucleus.

Note : ❑ λ depends only on the nature of substance. It is independent of time and any physical or chemical changes.



(2) **Activity**

It is defined as the rate of disintegration (or count rate) of the substance (or the number of atoms of any material decaying per second) *i.e.* $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$

where A_0 = Activity of t = 0, A = Activity after time t

Units of activity (Radioactivity)

It's units are *Becqueral* (*Bq*), Curie (*Ci*) and Rutherford (*Rd*)

1 *Becquerel* = 1 *disintegration/sec*, 1 *Rutherford* = 10^6 *dis/sec*, 1 *Curie* = 3.7×10^{11} *dis/sec*

Note : ❑ Activity per *gm* of a substance is known as specific activity. The specific activity of 1 *gm* of radium – 226 is 1 *Curie*.

❑ 1 *millicurie* = 37 *Rutherford*

❑ The activity of a radioactive substance decreases as the number of undecayed nuclei decreases with time.

❑ Activity $\propto \frac{1}{\text{Half life}}$

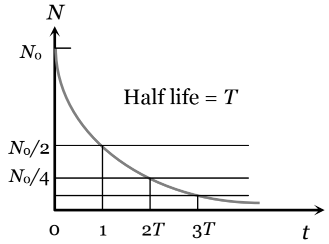
(3) **Half life ($T_{1/2}$)**

Time interval in which the mass of a radioactive substance or the number of it's atom reduces to half of it's initial value is called the half life of the substance.

i.e. if $N = \frac{N_0}{2}$ then $t = T_{1/2}$

Hence from $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda (T_{1/2})} \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$



Time (t)	**Number of undecayed atoms (N) (N_0 = Number of initial atoms)**	**Remaining fraction of active atoms (N/N_0) probability of survival**	**Fraction of atoms decayed ($N_0 - N$) /N_0 probability of decay**
t = 0	N_0	1 (100%)	0
$t = T_{1/2}$	$\frac{N_0}{2}$	$\frac{1}{2}$ (50%)	$\frac{1}{2}$ (50%)

$t = 2(T_{1/2})$	$\frac{1}{2}\times\frac{N_0}{2}=\frac{N_0}{(2)^2}$	$\frac{1}{4}$ (25%)	$\frac{3}{4}$ (75%)
$t = 3(T_{1/2})$	$\frac{1}{2}\times\frac{N_0}{(2)}=\frac{N_0}{(2)^3}$	$\frac{1}{8}$ (12.5%)	$\frac{7}{8}$ (87.5%)
$t = 10\,(T_{1/2})$	$\frac{N_0}{(2)^{10}}$	$\left(\frac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = n\,(N_{1/2})$	$\frac{N}{(2)^2}$	$\left(\frac{1}{2}\right)^{n}$	$\left\{1-\left(\frac{1}{2}\right)^{n}\right\}$

Useful relation

After n half-lives, number of undecayed atoms $N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

(4) **Mean (or average) life (τ)**

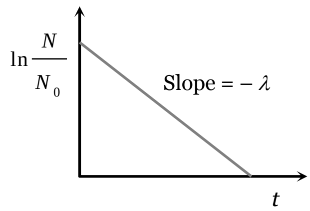
The time for which a radioactive material remains active is defined as mean (average) life of that material.

Other definitions

(i) It is defined as the sum of lives of all atoms divided by the total number of atoms

$$i.e.\ \tau = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}} = \frac{1}{\lambda}$$

(ii) From $N = N_0 e^{-\lambda t} \Rightarrow \dfrac{\ln\frac{N}{N_0}}{t} = -\lambda$ slope of the line shown in the graph



i.e. the magnitude of inverse of slope of $\ln\frac{N}{N_0}$ *vs* t curve is known as mean life (τ).

(iii) From $N = N_0 e^{-\lambda t}$

If $t = \frac{1}{\lambda} = \tau \Rightarrow N = N_0 e^{-1} = N_0\left(\frac{1}{e}\right) = 0.37\,N_0 = 37\%$ of N_0.

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\frac{1}{e}$ times or 0.37 times or 37% of original number of atoms. or

It is the time in which number of decayed atoms ($N_0 - N$) becomes $\left(1-\frac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(iv) From $T_{1/2} = \frac{0.693}{\lambda} \Rightarrow \frac{1}{\lambda} = \tau = \frac{1}{0.693}.(t_{1/2}) = 1.44\,(T_{1/2})$

i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$

Note : ❑ Half life and mean life of a substance doesn't change with time or with pressure, temperature *etc.*

Radioactive Series

If the isotope that results from a radioactive decay is itself radioactive then it will also decay and so on.

The sequence of decays is known as radioactive decay series. Most of the radio-nuclides found in nature are members of four radioactive series. These are as follows

Mass number	Series (Nature)	Parent	Stable and product	Integer n	Number of lost particles
$4n$	Thorium (natural)	$_{90}Th^{232}$	$_{82}Pb^{208}$	52	$\alpha = 6, \beta = 4$
$4n + 1$	Neptunium (Artificial)	$_{93}Np^{237}$	$_{83}Bi^{209}$	52	$\alpha = 8, \beta = 5$

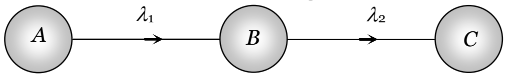
$4n+2$	Uranium (Natural)	$_{92}U^{238}$	$_{82}Pb^{206}$	51	$\alpha = 8, \beta = 6$
$4n+3$	Actinium (Natural)	$_{89}Ac^{227}$	$_{82}Pb^{207}$	51	$\alpha = 7, \beta = 4$

Note : ❑ The $4n + 1$ series starts from $_{94}PU^{241}$ but commonly known as neptunium series because neptunium is the longest lived member of the series.

❑ The $4n + 3$ series actually starts from $_{92}U^{235}$.

Successive Disintegration and Radioactive Equilibrium

Suppose a radioactive element *A* disintegrates to form another radioactive element *B* which intern disintegrates to still another element *C*; such decays are called successive disintegration.



Rate of disintegration of $A = \frac{dN_1}{dt} = -\lambda_1 N_1$ (which is also the rate of formation of *B*)

Rate of disintegration of $B = \frac{dN_2}{dt} = -\lambda_2 N_2$

$\therefore$ Net rate of formation of *B* = Rate of disintegration of *A* – Rate of disintegration of *B*
$= \lambda_1 N_1 - \lambda_2 N_2$

Equilibrium

In radioactive equilibrium, the rate of decay of any radioactive product is just equal to it's rate of production from the previous member.

i.e. $\lambda_1 N_1 = \lambda_2 N_2$ $\Rightarrow$ $\frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_2} = \frac{\tau_2}{\tau_1} = \frac{(T_{1/2})}{(T_{1/2})_1}$

Note : ❑ In successive disintegration if N_0 is the initial number of nuclei of *A* at $t = 0$ then number of nuclei of product *B* at time *t* is given by $N_2 = \frac{\lambda_1 N_0}{(\lambda_2 - \lambda_1)}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$ where $\lambda_1 \lambda_2$ – decay constant of *A* and *B*.

Uses of radioactive isotopes

(1) **In medicine**

(i) For testing blood-chromium - 51 (ii) For testing blood circu

(iii) For detecting brain tumor- Radio mercury - 203

(iv) For detecting fault in thyroid gland - Radio iodine - 131

(v) For cancer - cobalt – 60 (vi) For blood - Gold - 189

(vii) For skin diseases - Phospohorous - 31

(2) **In Archaeology**

(i) For determining age of archaeological sample (carbon dating) C^{14}

(ii) For determining age of meteorites - K^{40} (iii) For determining age of earth-Lead isotopes

(3) **In agriculture**

(i) For protecting potato crop from earthworm- CO^{60} (ii) For artificial rains - AgI (iii) As fertilizers - P^{32}

(4) **As tracers** - (Tracer) : Very small quantity of radioisotopes present in a mixture is known as tracer

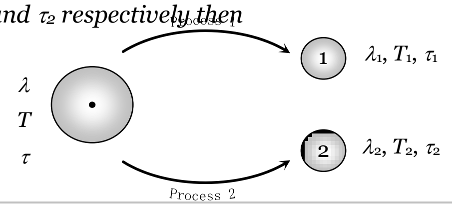
(i) Tracer technique is used for studying biochemical reaction in tracer and animals.

(5) **In industries**

(i) For detecting leakage in oil or water pipe lines (ii) For determining the age of planets.

Concept

☞ *If a nuclide can decay simultaneously by two different process which have decay constant λ_1 and λ_2, half life T_1 and T_2 and mean lives τ_1 and τ_2 respectively then*



$\Rightarrow \lambda = \lambda_1 + \lambda_2$

$\Rightarrow T = \frac{T_1 T_2}{T_1 + T_2}$

$\Rightarrow \tau = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$

***Example*: 16** When $_{90}Th^{228}$ transforms to $_{83}Bi^{212}$, then the number of the emitted α–and β–particles is, respectively **[MP PET 2002]**

(a) $8\alpha, 7\beta$ (b) $4\alpha, 7\beta$ (c) $4\alpha, 4\beta$ (d) $4\alpha, 1\beta$

Solution : (d) $_{Z=90}Th^{A=228} \rightarrow {}_{Z'=83}Bi^{A'=212}$

Number of α-particles emitted $n_\alpha = \frac{A-A'}{4} = \frac{228-212}{4} = 4$

Number of β-particles emitted $n_\beta = 2n_\alpha - Z + Z' = 2\times4 - 90 + 83 = 1.$

***Example*: 17** A radioactive substance decays to 1/16th of its initial activity in 40 days. The half-life of the radioactive substance expressed in days is

(a) 2.5 (b) 5 (c) 10 (d) 20

Solution : (c) By using $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{N}{N_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^{40/T_{1/2}} \Rightarrow T_{1/2} = 10$ days.

***Example*: 18** A sample of radioactive element has a mass of 10 *gm* at an instant *t* = 0. The approximate mass of this element in the sample after two mean lives is **[CBSE PMT 2003]**

(a) 2.50 *gm* (b) 3.70 *gm* (c) 6.30 *gm* (d) 1.35 *gm*

Solution : (d) By using $M = M_0e^{-\lambda t} \Rightarrow M = 10e^{-\lambda(2\tau)} = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 = 1.359$ *gm*

***Example*: 19** The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is **[IIT-JEE (Screening) 2002]**

(a) 400 μs (b) 6.3 μs (c) 40 μs (d) 300 μs

Solution : (a) By using $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{t/100} \Rightarrow t = 400\ \mu sec.$

***Example*: 20** The mean lives of a radioactive substance for α and β emissions are 1620 years and 405 years respectively. After how much time will the activity be reduced to one fourth **[RPET 1999]**

(a) 405 year (b) 1620 year (c) 449 year (d) None of these

Solution : (c) $\lambda_\alpha = \frac{1}{1620}$ *per year* and $\lambda_\beta = \frac{1}{405}$ *per year* and it is given that the fraction of the remained activity $\frac{A}{A_0} = \frac{1}{4}$

Total decay constant $\lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ *per year*

We know that $A = A_0e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda}\log_e\frac{A_0}{A} \Rightarrow t = \frac{1}{\lambda}\log_e 4 = \frac{2}{\lambda}\log_e 2 = 324\times2\times0.693 = 449$ *years*.

***Example*: 21** At any instant the ratio of the amount of radioactive substances is 2 : 1. If their half lives be respectively 12 and 16 hours, then after two days, what will be the ratio of the substances

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

Solution : (a) By using $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2}\times\frac{(1/2)^{n_1}}{(1/2)^{n_2}} = \frac{2}{1}\times\frac{\left(\frac{1}{2}\right)^{\frac{2\times24}{12}}}{\left(\frac{1}{2}\right)^{\frac{2\times24}{16}}} = \frac{1}{1}$

***Example*: 22** From a newly formed radioactive substance (Half-life 2 *hours*), the intensity of radiation is 64 times the permissible safe level. The minimum time after which work can be done safely from this source is **[IIT 1983; SCRA 1996]**

(a) 6 *hours* (b) 12 *hours* (c) 24 *hours* (d) 128 *hours*

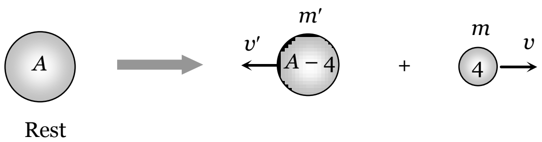
Solution : (b) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow \frac{A}{A_0} = \frac{1}{64} = \left(\frac{1}{2}\right)^0 = \left(\frac{1}{2}\right)^n \Rightarrow n = 6$

$\Rightarrow \frac{t}{T_{1/2}} = 6 \Rightarrow t = 6\times2 = 12$ *hours.*

***Example*: 23** nucleus of mass number *A*, originally at rest, emits an α-particle with speed *v*. The daughter nucleus recoils with a speed **[DCE 2000; AIIMS 2004]**

(a) $2v/(A+4)$ (b) $4v/(A+4)$ (c) $4v/(A-4)$ (d) $2v/(A-4)$

Solution : (c)



According to conservation of momentum $4v = (A-4)v' \Rightarrow v' = \frac{4v}{A-4}$.

Example: 24 The counting rate observed from a radioactive source at t = 0 second was 1600 counts per second and at t = 8 seconds it was 100 counts per second. The counting rate observed as counts per second at t = 6 seconds will be **[MP PET 1996; UPSEAT 2000]**

(a) 400 (b) 300 (c) 200 (d) 150

Solution : (c) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow 100 = 1600\left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow T_{1/2} = 2\,sec$

Again by using the same relation the count rate at t = 6 *sec* will be $A = 1600\left(\frac{1}{2}\right)^{6/2} = 200$.

Example: 25 The kinetic energy of a neutron beam is 0.0837 *eV*. The half-life of neutrons is 693*s* and the mass of neutrons is 1.675×10^{-27} *kg*. The fraction of decay in travelling a distance of 40*m* will be

(a) 10^{-3} (b) 10^{-4} (c) 10^{-5} (d) 10^{-6}

Solution : (c) $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times0.0837\times1.6\times10^{-19}}{1.675\times10^{-27}}} = 4\times10^3\ m/sec$

$\therefore$ Time taken by neutrons to travel a distance of 40 *m* $\Delta t' = \frac{40}{4\times10^3} = 10^{-2}\,sec$

$\therefore \frac{dN}{dt} = \lambda N \Rightarrow \frac{dN}{N} = \lambda\, dt$

$\therefore$ Fraction of neutrons decayed in Δt *sec* in $\frac{\Delta N}{N} = \lambda\,\Delta t = \frac{0.693}{T}\Delta t = \frac{0.693}{693}\times10^{-2} = 10^{-5}$

Example: 26 The fraction of atoms of radioactive element that decays in 6 days is 7/8. The fraction that decays in 10 days will be

(a) 77/80 (b) 71/80 (c) 31/32 (d) 15/16

Solution : (c) By using $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow t = \frac{T_{1/2}\log_e\left(\frac{N_0}{N}\right)}{\log_e(2)} \Rightarrow t \propto \log_e\frac{N_0}{N} \Rightarrow \frac{t_1}{t_2} = \frac{\left(\log_e\frac{N_0}{N}\right)_1}{\left(\log_e\frac{N_0}{N}\right)_2}$

Hence $\frac{6}{10} = \frac{\log_e(8/1)}{\log_e(N_0/N)} \Rightarrow \log_e\frac{N_0}{N} = \frac{10}{6}\log_e(8) = \log_e 32 \Rightarrow \frac{N_0}{N} = 32$.

So fraction that decays $= 1 - \frac{1}{32} = \frac{31}{32}$.

Tricky example: 2

Half-life of a substance is 20 minutes. What is the time between 33% decay and 67% decay **[AIIMS 200**

(a) 40 minutes (b) 20 minutes (c) 30 minutes (d) 25 minutes

Solution : (b) Let N_0 be the number of nuclei at beginning

$\therefore$ Number of undecayed nuclei after 33% decay = 0.67 N_0

and number of undecayed nuclei after 67% of decay = 0.33 N_0

$\because$ 0.33 $N_0 \simeq \frac{0.67 N_0}{2}$ and in the half-life time the number of undecayed nuclei becomes half.

Example

Example: 1 The ratio of areas within the electron orbits for the first excited state to the ground state for hydrogen atom is

(a) 16 : 1 (b) 18 : 1 (c) 4 : 1 (d) 2 : 1

Solution : (a) For a hydrogen atom

Radius $r \propto n^2 \Rightarrow \frac{r_1^2}{r_2^2} = \frac{n_1^4}{n_2^4} \Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{n_1^4}{n_2^4} \Rightarrow \frac{A_1}{A_2} = \frac{n_1^4}{n_2^4} = \frac{2^4}{1^4} = 16 \Rightarrow \frac{A_1}{A_2} = \frac{16}{1}$

Example: 2 The electric potential between a proton and an electron is given by $V = V_0 \ln \frac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with *n*, *n* being the principal quantum number

[IIT-JEE (Screening) 2003]

(a) $r_n \propto n$ (b) $r_n \propto 1/n$ (c) $r_n \propto n^2$ (d) $r_n \propto 1/n^2$

Solution : (a) Potential energy $U = eV = eV_0 \ln \frac{r}{r_0}$

$\therefore$ Force $F = -\left|\frac{dU}{dr}\right| = \frac{eV_0}{r}$. The force will provide the necessary centripetal force. Hence

$\frac{mv^2}{r} = \frac{eV_0}{r} \Rightarrow v = \sqrt{\frac{eV_0}{m}}$ .....(i) and $mvr = \frac{nh}{2\pi}$ .....(ii)

Dividing equation (ii) by (i) we have $mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}}$ or $r \propto n$

Example: 3 The innermost orbit of the hydrogen atom has a diameter 1.06 $Å$. The diameter of tenth orbit is

[UPSEAT 2002]

(a) 5.3 $Å$ (b) 10.6 $Å$ (c) 53 $Å$ (d) 106 $Å$

Solution : (d) Using $r \propto n^2 \Rightarrow \frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2$ or $\frac{d_2}{d_1} = \left(\frac{n_2}{n_1}\right)^2 \Rightarrow \frac{d_2}{1.06} = \left(\frac{10}{1}\right)^2 \Rightarrow d = 106\ Å$

Example: 4 Energy of the electron in n^{th} orbit of hydrogen atom is given by $E_n = -\frac{13.6}{n^2} eV$. The amount of energy needed to transfer electron from first orbit to third orbit is

(a) 13.6 *eV* (b) 3.4 *eV* (c) 12.09 *eV* (d) 1.51 *eV*

Solution : (c) Using $E = -\frac{13.6}{n^2} eV$

For $n = 1$, $E_1 = \frac{-13.6}{1^2} = -13.6\, eV$ and for $n = 3$ $E_3 = -\frac{13.6}{3^2} = -1.51\, eV$

So required energy $= E_3 - E_1 = -1.51 - (-13.6) = 12.09\, eV$

Example: 5 If the binding energy of the electron in a hydrogen atom is 13.6 *eV*, the energy required to remove the electron from the first excited state of Li^{++} is **[AIEEE 2003]**

(a) 122.4 *eV* (b) 30.6 *eV* (c) 13.6 *eV* (d) 3.4 *eV*

Solution : (b) Using $E_n = -\frac{13.6 \times Z^2}{n^2} eV$

For first excited state *n* = 2 and for Li^{++}, $Z = 3$

$\therefore E = -\frac{13.6}{2^2} \times 3^2 = -\frac{13.6 \times 9}{4} = -30.6\, eV$. Hence, remove the electron from the first excited state of Li^{++} be 30.6 *eV*

Example: 6 The ratio of the wavelengths for 2 → 1 transition in Li^{++}, He^{+} and *H* is **[UPSEAT 2003]**

(a) 1 : 2 : 3 (b) 1 : 4 : 9 (c) 4 : 9 : 36 (d) 3 : 2 : 1

Solution : (c) Using $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \lambda \propto \frac{1}{Z^2} \Rightarrow \lambda_{Li} : \lambda_{He^+} : \lambda_H = \frac{1}{9} : \frac{1}{4} : \frac{1}{1} = 4 : 9 : 36$

Example: 7 Energy *E* of a hydrogen atom with principal quantum number *n* is given by $E = \frac{-13.6}{n^2} eV$. The energy of a photon ejected when the electron jumps *n* = 3 state to *n* = 2 state of hydrogen is approximately

[CBSE PMT/PDT Screening 2004]

(a) 1.9 *eV* (b) 1.5 *eV* (c) 0.85 *eV* (d) 3.4 *eV*

Solution : (a) $\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{36} = 1.9\, eV$

***Example*: 8** In the Bohr model of the hydrogen atom, let *R*, *v* and *E* represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which of the following quantity is proportional to the quantum number *n* **[KCET 2002]**

(a) *R*/*E* (b) *E*/*v* (c) *RE* (d) *vR*

Solution : (d) Rydberg constant $R = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2}$

Velocity $v = \frac{Ze^2}{2\varepsilon_0 nh}$ and energy $E = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2}$

Now, it is clear from above expressions $R.v \propto n$

***Example*: 9** The energy of hydrogen atom in *n*th orbit is E_n, then the energy in *n*th orbit of singly ionised helium atom will be

(a) $4E_n$ (b) $E_n/4$ (c) $2E_n$ (d) $E_n/2$

Solution : (a) By using $E = -\frac{13.6\, Z^2}{n^2} \Rightarrow \frac{E_H}{E_{He}} = \left(\frac{Z_H}{Z_{He}}\right)^2 = \left(\frac{1}{2}\right)^2 \Rightarrow E_{He} = 4E_n$.

***Example*: 10** The wavelength of radiation emitted is λ_0 when an electron jumps from the third to the second orbit of hydrogen atom. For the electron jump from the fourth to the second orbit of the hydrogen atom, the wavelength of radiation emitted will be **[SCRA 1998; MP PET 2001]**

(a) $\frac{16}{25}\lambda_0$ (b) $\frac{20}{27}\lambda_0$ (c) $\frac{27}{20}\lambda_0$ (d) $\frac{25}{16}\lambda_0$

Solution : (b) Wavelength of radiation in hydrogen atom is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \frac{1}{\lambda_0} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5}{36}R \qquad .....(i)$$

and $$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = R\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3R}{16} \qquad .....(ii)$$

From equation (i) and (ii) $\frac{\lambda'}{\lambda} = \frac{5R}{36} \times \frac{16}{3R} = \frac{20}{27} \Rightarrow \lambda' = \frac{20}{27}\lambda_0$

***Example*: 11** If scattering particles are 56 for 90^o angle then this will be at 60^o angle **[RPMT 2000]**

(a) 224 (b) 256 (c) 98 (d) 108

Solution : (a) Using Scattering formula

$$N \propto \frac{1}{\sin^4(\theta/2)} \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_2}{2}\right)}\right]^4 \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin\left(\frac{90^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}\right]^4 = \left[\frac{\sin 45^\circ}{\sin 30^\circ}\right]^4 = 4 \Rightarrow N_2 = 4N_1 = 4 \times 56 = 224$$

***Example*: 12** When an electron in hydrogen atom is excited, from its 4[th] to 5[th] stationary orbit, the change in angular momentum of electron is (Planck's constant: $h = 6.6 \times 10^{-34}$ *J–s*) **[AFMC 2000]**

(a) 4.16×10^{-34} *J- s* (b) 3.32×10^{-34} *J-s* (c) 1.05×10^{-34} *J-s* (d) 2.08×10^{-34} *J-s*

Solution : (c) Change in angular momentum

$$\Delta L = L_2 - L_1 = \frac{n_2 h}{2\pi} - \frac{n_1 h}{2\pi} \Rightarrow \Delta L = \frac{h}{2\pi}(n_2 - n_1) = \frac{6.6 \times 10^{-34}}{2 \times 3.14}(5-4) = 1.05 \times 10^{-34}\, J\text{-}s$$

***Example*: 13** In hydrogen atom, if the difference in the energy of the electron in *n* = 2 and *n* = 3 orbits is *E*, the ionization energy of hydrogen atom is

(a) 13.2 *E* (b) 7.2 *E* (c) 5.6 *E* (d) 3.2 *E*

Solution : (b) Energy difference between n = 2 and n = 3; $E = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = K\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}K$ .....(i)

Ionization energy of hydrogen atom $n_1 = 1$ and $n_2 = \infty$; $E' = K\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = K$ .....(ii)

From equation (i) and (ii) $E' = \frac{36}{5}E = 7.2\,E$

***Example*: 14** In Bohr model of hydrogen atom, the ratio of periods of revolution of an electron in n = 2 and n = 1 orbits is **[EAMCET (Engg.) 2000]**

(a) 2 : 1 (b) 4 : 1 (c) 8 : 1 (d) 16 : 1

Solution : (c) According to Bohr model time period of electron $T \propto n^3 \Rightarrow \frac{T_2}{T_1} = \frac{n_2^3}{n_1^3} = \frac{2^3}{1^3} = \frac{8}{1} \Rightarrow T_2 = 8T_1$.

***Example*: 15** A double charged lithium atom is equivalent to hydrogen whose atomic number is 3. The wavelength of required radiation for emitting electron from first to third Bohr orbit in Li^{++} will be (Ionisation energy of hydrogen atom is 13.6 eV)

(a) 182.51 $Å$ (b) 177.17 $Å$ (c) 142.25 $Å$ (d) 113.74 $Å$

Solution : (d) Energy of a electron in nth orbit of a hydrogen like atom is given by

$E_n = -13.6\frac{Z^2}{n^2}eV$, and Z = 3 for Li

Required energy for said transition

$$\Delta E = E_3 - E_1 = 13.6Z^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 13.6 \times 3^2\left[\frac{8}{9}\right] = 108.8\,eV = 108.8 \times 1.6 \times 10^{-19}\,J$$

Now using $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} \Rightarrow \lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{108.8 \times 1.6 \times 10^{-19}} = 0.11374 \times 10^{-7}\,m \Rightarrow \lambda = 113.74\,Å$

***Example*: 16** The absorption transition between two energy states of hydrogen atom are 3. The emission transitions between these states will be

(a) 3 (b) 4 (c) 5 (d) 6

Solution : (d) Number of absorption lines = (n – 1) $\Rightarrow$ 3 = (n – 1) $\Rightarrow n$ = 4

Hence number of emitted lines $= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

***Example*: 17** The energy levels of a certain atom for 1st, 2nd and 3rd levels are E, $4E/3$ and $2E$ respectively. A photon of wavelength λ is emitted for a transition 3 $\rightarrow$ 1. What will be the wavelength of emissions for transition 2 $\rightarrow$ 1 **[CPMT 1996]**

(a) $\lambda/3$ (b) $4\lambda/3$ (c) $3\lambda/4$ (d) 3λ

Solution : (d) For transition 3 $\rightarrow$ 1 $\Delta E = 2E - E = \frac{hc}{\lambda} \Rightarrow E = \frac{hc}{\lambda}$ .....(i)

For transition 2 $\rightarrow$ 1 $\frac{4E}{3} - E = \frac{hc}{\lambda'} \Rightarrow E = \frac{3hc}{\lambda'}$ .....(ii)

From equation (i) and (ii) $\lambda' = 3\lambda$

***Example*: 18** Hydrogen atom emits blue light when it changes from n = 4 energy level to n = 2 level. Which colour of light would the atom emit when it changes from n = 5 level to n = 2 level **[KCET 1993]**

(a) Red (b) Yellow (c) Green (d) Violet

Solution : (d) In the transition from orbits $5 \rightarrow 2$ more energy will be liberated as compared to transition from 4 $\rightarrow$ 2. So emitted photon would be of violet light.

***Example*: 19** A single electron orbits a stationary nucleus of charge $+Ze$, where Z is a constant. It requires 47.2 eV to excited electron from second Bohr orbit to third Bohr orbit. Find the value of Z **[IIT-JEE 1981]**

(a) 2 (b) 5 (c) 3 (d) 4

Solution : (b) Excitation energy of hydrogen like atom for $n_2 \to n_1$

$$\Delta E = 13.6 Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) eV \Rightarrow 47.2 = 13.6 Z^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{36} Z^2 \Rightarrow Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 24.98 \simeq 25$$

$\Rightarrow Z = 5$

***Example*: 20** The first member of the Paschen series in hydrogen spectrum is of wavelength 18,800 Å. The short wavelength limit of Paschen series is **[EAMCET (Med.) 2000]**

(a) 1215 Å (b) 6560 Å (c) 8225 Å (d) 12850 Å

Solution : (c) First member of Paschen series mean it's $\lambda_{max} = \frac{144}{7R}$

Short wavelength of Paschen series means $\lambda_{min} = \frac{9}{R}$

Hence $\frac{\lambda_{max}}{\lambda_{min}} = \frac{16}{7} \Rightarrow \lambda_{min} = \frac{7}{16} \times \lambda_{max} = \frac{7}{16} \times 18{,}800 = 8225\,Å$.

***Example*: 21** Ratio of the wavelengths of first line of Lyman series and first line of Balmer series is

[EAMCET (Engg.) 1995; MP PMT 1997]

(a) 1 : 3 (b) 27 : 5 (c) 5 : 27 (d) 4 : 9

Solution : (c) For Lyman series $\frac{1}{\lambda_{L_1}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$ .....(i)

For Balmer series $\frac{1}{\lambda_{B_1}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$ .....(ii)

From equation (i) and (ii) $\frac{\lambda_{L_1}}{\lambda_{B_1}} = \frac{5}{27}$.

***Example*: 22** The third line of Balmer series of an ion equivalent to hydrogen atom has wavelength of 108.5 nm. The ground state energy of an electron of this ion will be **[RPET 1997]**

(a) 3.4 eV (b) 13.6 eV (c) 54.4 eV (d) 122.4 eV

Solution : (c) Using $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{108.5 \times 10^{-9}} = 1.1 \times 10^7 \times Z^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$

$$\Rightarrow \frac{1}{108.5 \times 10^{-9}} = 1.1 \times 10^7 \times Z^2 \times \frac{21}{100} \Rightarrow Z^2 = \frac{100}{108.5 \times 10^{-9} \times 1.1 \times 10^{-7} \times 21} = 4 \Rightarrow Z = 2$$

Now Energy in ground state $E = -13.6 Z^2\, eV = -13.6 \times 2^2\, eV = -54.4\, eV$

***Example*: 23** Hydrogen (H), deuterium (D), singly ionized helium (He^{+}) and doubly ionized lithium (Li^{++}) all have one electron around the nucleus. Consider $n = 2$ to $n = 1$ transition. The wavelengths of emitted radiations are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively. Then approximately **[KCET 1994]**

(a) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (b) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (c) $\lambda_1 = 2\lambda_2 = 2\sqrt{2}\lambda_3 = 3\sqrt{2}\lambda_4$ (d) $\lambda_1 = \lambda_2 = 2\lambda_3 = 3\lambda_4$

Solution : (a) Using $\Delta E \propto Z^2$ ($\because n_1$ and n_2 are same)

$$\Rightarrow \frac{hc}{\lambda} \propto Z^2 \Rightarrow \lambda Z^2 = \text{constant} \Rightarrow \lambda_1 Z_1^2 = \lambda_2 Z_2^2 = \lambda_3 Z_3^2 = \lambda_4 Z^4 \Rightarrow \lambda_1 \times 1 = \lambda_2 \times 1^2 = \lambda_3 \times 2^2 = \lambda_4 \times 3^3$$

$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$.

***Example*: 24** Hydrogen atom in its ground state is excited by radiation of wavelength 975 $Å$. How many lines will be there in the emission spectrum

(a) 2 (b) 4 (c) 6 (d) 8

Solution : (c) Using $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \frac{1}{975 \times 10^{-10}} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow n = 4$

Now number of spectral lines $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$.

***Example*: 25** A photon of energy 12.4 *eV* is completely absorbed by a hydrogen atom initially in the ground state so that it is excited. The quantum number of the excited state is

(a) $n = 1$ (b) $n = 3$ (c) $n = 4$ (d) $n = \infty$

Solution : (c) Let electron absorbing the photon energy reaches to the excited state *n*. Then using energy conservation

$\Rightarrow -\frac{13.6}{n^2} = -13.6 + 12.4 \Rightarrow -\frac{13.6}{n^2} = -1.2 \Rightarrow n^2 = \frac{13.6}{1.2} = 12 \Rightarrow n = 3.46 \simeq 4$

***Example*: 26** The wave number of the energy emitted when electron comes from fourth orbit to second orbit in hydrogen is 20,397 cm^{-1}. The wave number of the energy for the same transition in He^+ is

(a) 5,099 cm^{-1} (b) 20,497 cm^{-1} (c) 40,994 cm^{-1} (d) 81,998 cm^{-1}

Solution : (d) Using $\frac{1}{\lambda} = \bar{\nu} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \bar{\nu} \propto Z^2 \Rightarrow \frac{\bar{\nu}_2}{\bar{\nu}_1} = \left(\frac{Z_2}{Z_1}\right)^2 = \left(\frac{Z}{1}\right)^2 = 4 \Rightarrow \bar{\nu}_2 = \bar{\nu} \times 4 = 81588\ cm^{-1}$.

***Example*: 27** In an atom, the two electrons move round the nucleus in circular orbits of radii *R* and 4*R*. the ratio of the time taken by them to complete one revolution is

(a) 1/4 (b) 4/1 (c) 8/1 (d) 1/8

Solution : (d) Time period $T \propto \frac{n^3}{Z^2}$

For a given atom (Z = constant) So $T \propto n^3$ .....(i) and radius $R \propto n^2$ .....(ii)

$\therefore$ From equation (i) and (ii) $T \propto R^{3/2} \Rightarrow \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2} = \frac{1}{8}$.

***Example*: 28** Ionisation energy for hydrogen atom in the ground state is *E*. What is the ionisation energy of Li^{++} atom in the 2nd excited state

(a) *E* (b) 3*E* (c) 6*E* (d) 9*E*

Solution : (a) Ionisation energy of atom in *n*th state $E_n = \frac{Z^2}{n^2}$

For hydrogen atom in ground state (n = 1) and Z = 1 $\Rightarrow E = E_0$ .....(i)

For Li^{++} atom in 2nd excited state n = 3 and Z = 3, hence $E' = \frac{E_0}{3^2} \times 3^2 = E_0$ .....(ii)

From equation (i) and (ii) $E' = E$.

***Example*: 29** An electron jumps from n = 4 to n = 1 state in *H*-atom. The recoil momentum of *H*-atom (in *eV*/*C*) is

(a) 12.75 (b) 6.75 (c) 14.45 (d) 0.85

Solution : (a) The *H*-atom before the transition was at rest. Therefore from conservation of momentum

Photon momentum = Recoil momentum of *H*-atom or $P_{recoil} = \frac{h\nu}{c} = \frac{E_4 - E_1}{c} = \frac{-0.85eV - (-13.6\,eV)}{c} = 12.75\frac{eV}{c}$

***Example*: 30** If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be

[IIT-JEE 1983; CBSE PMT 1991, 93; MP PET 1999; RPET 1993, 2001; RPMT 1999, 2003; J & K CET 2004]

(a) 60 (b) 32 (c) 4 (d) 64

Solution : (a) Maximum value of n = 4

So possible (maximum) no. of elements

$$N = 2\times1^2 + 2\times2^2 + 2\times3^2 + 2\times4^2 = 2+8+18+32 = 60 .$$

Tricky example: 1

If the atom ${}_{100}Fm^{257}$ follows the Bohr model and the radius of ${}_{100}Fm^{257}$ is n times the Bohr radius, then find n

[IIT-JEE (Screening) 2003]

(a) 100 (b) 200 (c) 4 (d) 1/4

Solution : (d) $(r_m) = \left(\frac{m^2}{Z}\right)(0.53\,\mathring{A}) = (n\times0.53\,\mathring{A}) \Rightarrow \frac{m^2}{Z} = n$

$m = 5$ for ${}_{100}Fm^{257}$ (the outermost shell) and $z = 100$

$\therefore \quad n = \frac{(5)^2}{100} = \frac{1}{4}$

Tricky example: 2

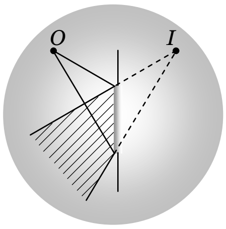
An energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy (in eV) required to remove both the electrons from a neutral helium atom is

(a) 79.0 (b) 51.8 (c) 49.2 (d) 38.2

Solution : (a) After the removal of first electron remaining atom will be hydrogen like atom.

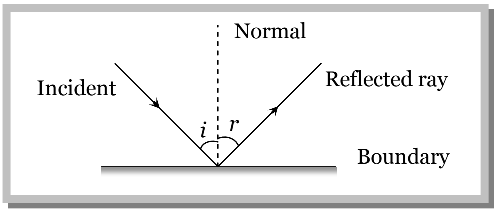
So energy required to remove second electron from the atom $E = 13.6\times\frac{2^2}{1} = 54.4\,eV$

$\therefore$ Total energy required = 24.6 + 54.4 = 79 eV



Reflection of Light

When a ray of light after incidenting on a boundary separating two media comes back into the same media, then this phenomenon, is called reflection of light.



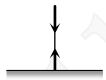
⇒ $\angle i = \angle r$

⇒ After reflection, velocity, wave length and frequency of light remains same but intensity decreases

⇒ There is a phase change of π if reflection takes place from denser medium

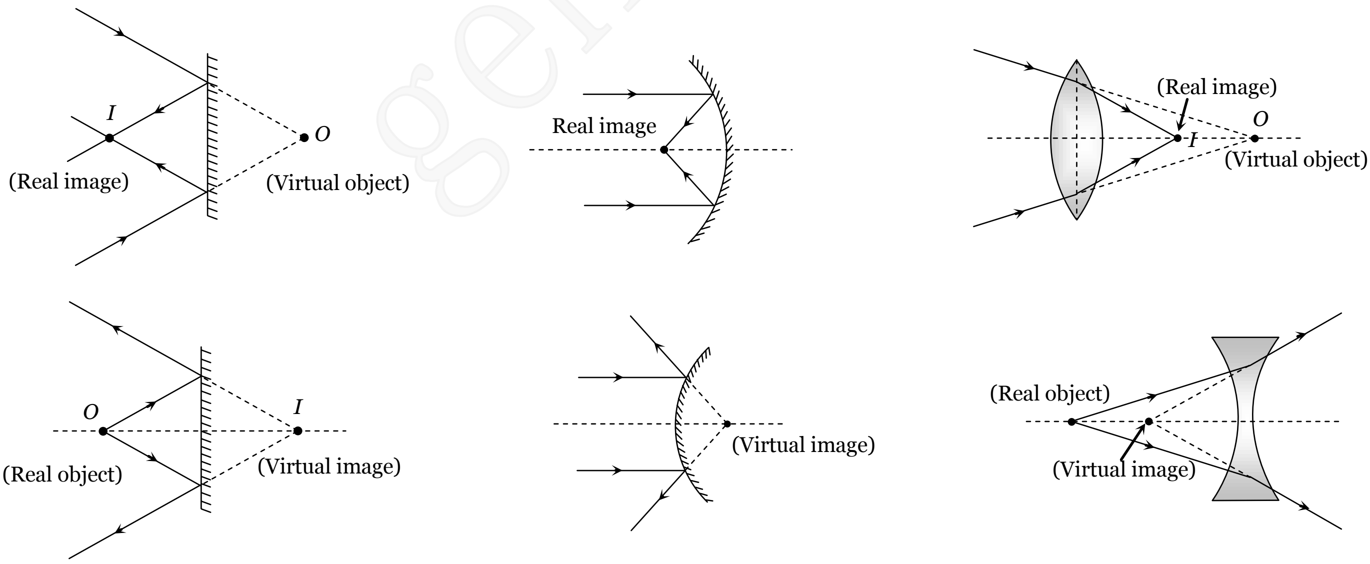
Note : ❑ After reflection velocity, wavelength and frequency of light remains same but intensity decreases.

❑ If light ray incident normally on a surface, after reflection it retraces the path.



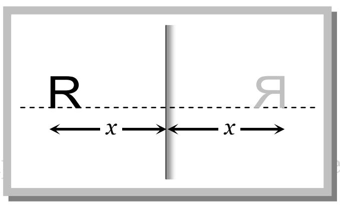
Real and virtual images

If light rays, after reflection or refraction, actually meets at a point then real image is formed and if they appears to meet virtual image is formed.

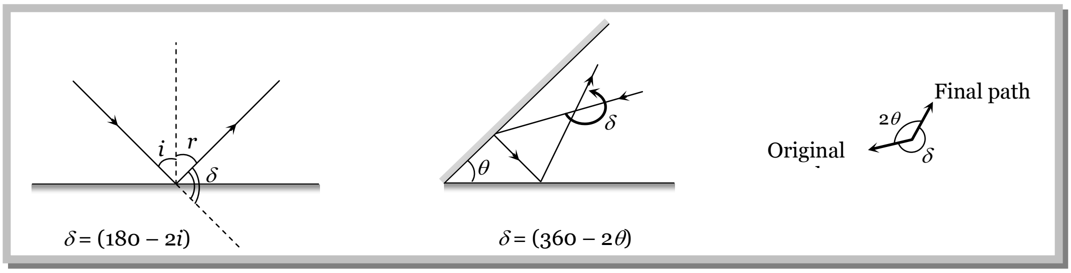


Plane Mirror

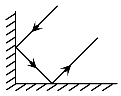
The image formed by a plane mirror is virtual, erect, laterally inverted, equal in size that of the object and at a distance equal to the distance of the object in front of the mirror.



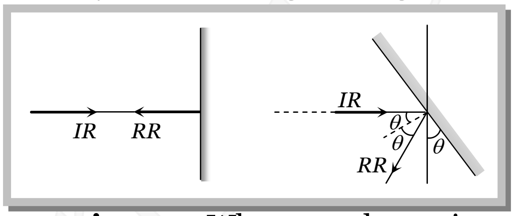
(1) **Deviation :** Deviation produced by a plane mirror and by two inclined plane mirrors.



Note : ❑ If two plane mirrors are inclined to each other at 90°, the emergent ray is anti-parallel to incident ray, if it suffers one reflection from each. Whatever be the angle to incidence.



(2) **Rotation :** If a plane mirror is rotated in the plane of incidence through angle θ, by keeping the incident ray fixed, the reflected ray turned through an angle 2θ.



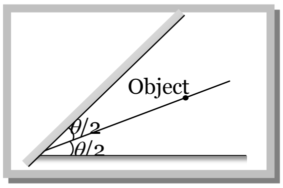
(3) **Images by two inclined plane mirrors :** When two plane mirrors are inclined to each other at an angle θ, then number of images (n) formed of an object which is kept between them.

(i) $n = \left(\frac{360}{\theta} - 1\right)$; If $\frac{360}{\theta}$ = even integer

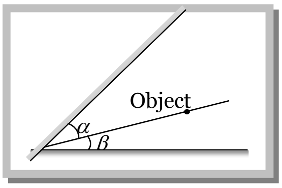
(ii) If $\frac{360}{\theta}$ = odd integer then there are two possibilities

(a) Object is placed symmetrically (b) Object is placed asymmetrically

$n = \left(\frac{360}{\theta} - 1\right)$



$n = \frac{360}{\theta}$



Note : ❑ If $\theta = 0°$ *i.e.* mirrors are parallel to each other so $n = \infty$ *i.e.* infinite images will be formed.

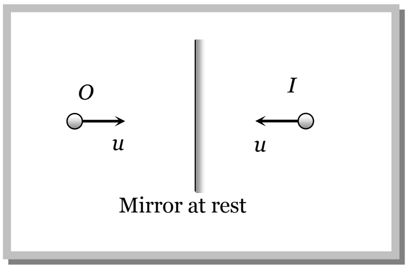
❑ If $\theta = 90°$, $n = \frac{360}{90} - 1 = 3$

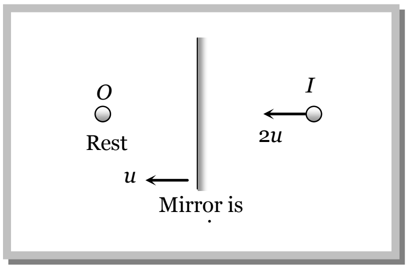
❑ If $\theta = 72°$, $n = \frac{360}{72} - 1 = 4$ (If nothing is said object is supposed to be symmetrically placed).

(4) **Other important informations**

(i) When the object moves with speed u towards (or away) from the plane mirror then image also moves toward (or away) with speed u. But relative speed of image $w.r.t.$ object is $2u$.

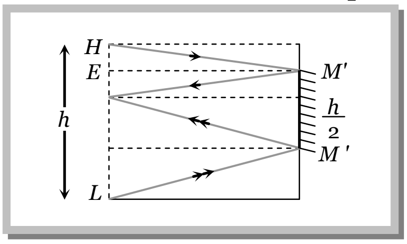
(ii) When mirror moves towards the stationary object with speed u, the image will move with speed $2u$.

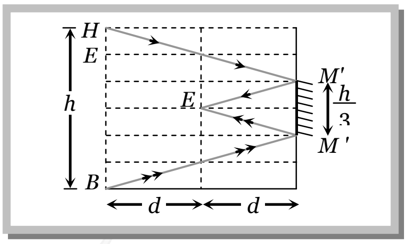




(iii) A man of height h requires a mirror of length at least equal to $h/2$, to see his own complete image.

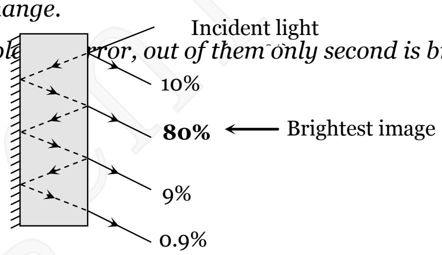
(iv) To see complete wall behind himself a person requires a plane mirror of at least one third the height of wall. It should be noted that person is standing in the middle of the room.



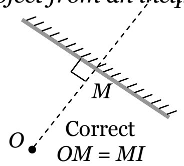


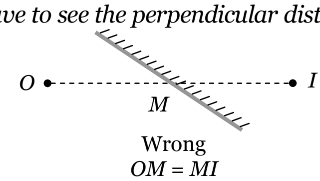
Example ***Concepts***

- *The reflection from a denser medium causes an additional phase change of π or path change of $\lambda/2$ while reflection from rarer medium doesn't cause any phase change.*
- *We observe number of images in a thick plane mirror, out of them only second is brightest.*



- *To find the location of an object from an inclined plane mirror, you have to see the perpendicular distance of the object from the mirror.*

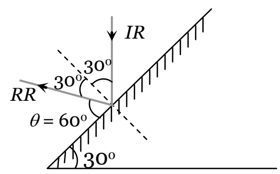




Example: 1 A plane mirror makes an angle of 30° with horizontal. If a vertical ray strikes the mirror, find the angle between mirror and reflected ray

(a) 30° (b) 45° (c) 60° (d) 90°

Solution : (c) Since angle between mirror and normal is 90° and reflected ray (RR) makes an angle of 30° with the normal so required angle will be $\theta = 60°$.



Example: 2 Two vertical plane mirrors are inclined at an angle of 60° with each other. A ray of light travelling horizontally is reflected first from one mirror and then from the other. The resultant deviation is

(a) 60° (b) 120° (c) 180° (d) 240°

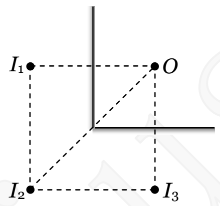
Solution : (d) By using $\delta = (360 - 2\theta)$ $\Rightarrow$ $\delta = 360 - 2 \times 60 = 240^o$

Example: 3 A person is in a room whose ceiling and two adjacent walls are mirrors. How many images are formed

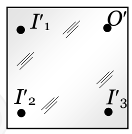
[AFMC 2002]

(a) 5 (b) 6 (c) 7 (d) 8

Solution : (c) The walls will act as two mirrors inclined to each other at 90° and so sill form $\frac{360}{90} - 1 = 3$ images of the person. Now these images with object (Person) will act as objects for the ceiling mirror and so ceiling will form 4 images as shown. Therefore total number of images formed = 3 + 4 = 7



Three images by walls



Four images by

Note : ❑ The person will see only six images of himself (I_1, I_2, I_3, I_1', I_2', I_3')

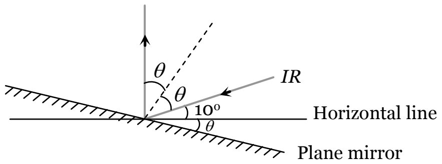
Example: 4 A ray of light makes an angle of 10° with the horizontal above it and strikes a plane mirror which is inclined at an angle θ to the horizontal. The angle θ for which the reflected ray becomes vertical is

(a) 40° (b) 50° (c) 80° (d) 100°

Solution : (a) From figure

$\theta + \theta + 10 = 90$

$\Rightarrow$ $\theta = 40^o$



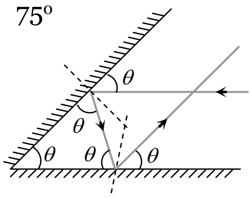
Example: 5 A ray of light incident on the first mirror parallel to the second and is reflected from the second mirror parallel to first mirror. The angle between two mirrors is

(a) 30° (b) 60° (c) 75° (d) 90°

Solution : (b) From geometry of figure

$\theta + \theta + \theta = 180^o$

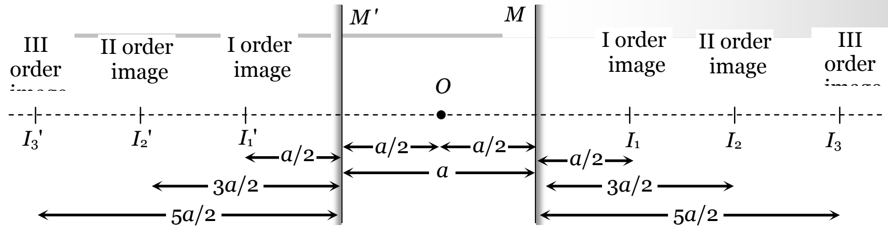
$\Rightarrow$ $\theta = 60^o$



Example: 6 A point object is placed mid-way between two plane mirrors distance 'a' apart. The plane mirror forms an infinite number of images due to multiple reflection. The distance between the nth order image formed in the two mirrors is

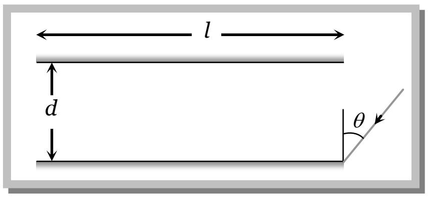
(a) na (b) $2na$ (c) $na/2$ (d) $n^2 a$

Solution : (b)



From above figure it can be proved that seperation between nth order image formed in the two mirrors = $2na$

Example: 7 Two plane mirrors P and Q are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of θ at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is



(a) $\dfrac{l}{d\tan\theta}$

(b) $\dfrac{d}{l\tan\theta}$

(c) $ld\tan\theta$

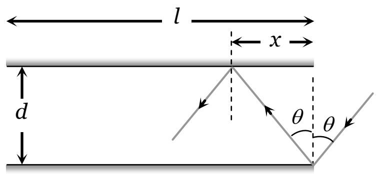
(d) None of these

Solution : (a) Suppose n = Total number of reflection light ray undergoes before exist out.

x = Horizontal distance travelled by light ray in one reflection.

So $nx = l$ also $\tan\theta = \dfrac{x}{d}$

$\Rightarrow n = \dfrac{l}{d\tan\theta}$



Example: 8 A plane mirror and a person are moving towards each other with same velocity v. Then the velocity of the image is

(a) v (b) $2v$ (c) $3v$ (d) $4v$

Solution : (c) If mirror would be at rest, then velocity of image should be $2v$. but due to the motion of mirror, velocity of image will be $2v + v = 3v$.

Example: 9 A ray reflected successively from two plane mirrors inclined at a certain angle undergoes a deviation of 300^o. The number of images observable are

(a) 10 (b) 11 (c) 12 (d) 13

Solution : (b) By using $\delta = (360 - 2\theta) \Rightarrow 300 = 360 - 2\theta$

$\Rightarrow \theta = 30^o$. Hence number of images $= \dfrac{360}{30} - 1 = 11$

Tricky example: 1

A small plane mirror placed at the centre of a spherical screen of radius R. A beam of light is falling on the mirror. If the mirror makes n revolution. per second, the speed of light on the screen after reflection from the mirror will be

(a) $4\pi nR$ (b) $2\pi nR$ (c) $\dfrac{nR}{2\pi}$ (d) $\dfrac{nR}{4\pi}$

Solution : (a) When plane mirror rotates through an angle θ, the reflected ray rotates through an angle 2θ. So spot on the screen will make $2n$ revolution per second

$\therefore$ Speed of light on screen $v = \omega R = 2\pi(2n)R = 4\pi nR$

Tricky example: 2

A watch shows time as 3 : 25 when seen through a mirror, time appeared will be **[RPMT 1997; JIPMER 2001, 2002]**

(a) 8 : 35 (b) 9 : 35 (c) 7 : 35 (d) 8 : 25

Solution : (a) For solving this type of problems remember

Actual time = 11 : 60 – given time

So here Actual time = 11 : 60 – 3 : 25 = 8 : 35

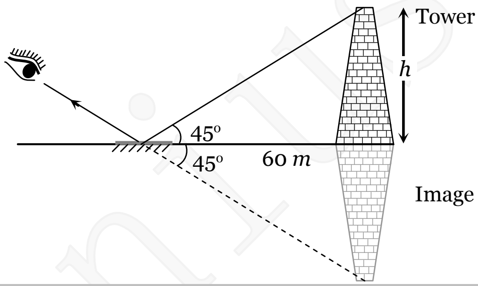
Tricky example: 3

When a plane mirror is placed horizontally on a level ground at a distance of 60 *m* from the foot of a tower, the top of the tower and its image in the mirror subtend an angle of 90° at the eye. The height of the tower will be **[CPMT 1984]**

(a) 30 *m* (b) 60 *m* (c) 90 *m* (d) 120 *m*

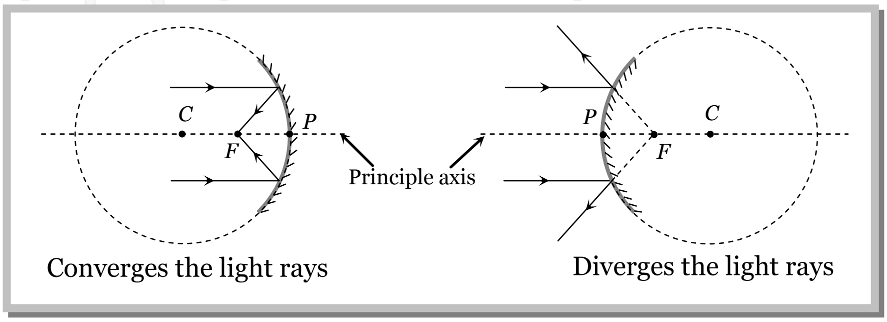
Solution : (b) Form the figure it is clear that $\frac{h}{60} = \tan 45^o$

$\Rightarrow h = 60\ m$



Curved Mirror

It is a part of a transparent hollow sphere whose one surface is polished.



(1) **Some definitions :**

(i) **Pole (P)** : Mid point of the mirror

(ii) Centre of curvature (C) : Centre of the sphere of which the mirror is a part.

(iii) Radius of curvature (R) : Distance between pole and centre of curvature.

($R_{concave} = -ve$, $R_{convex} = +ve$, $R_{plane} = \infty$)

(iv) Principle axis : A line passing through *P* and *C*.

(v) Focus (F) : An image point on principle axis for which object is at ∞

(vi) Focal length (f) : Distance between *P* and *F*.

(vii) Relation between f and R : $f = \frac{R}{2}$ ($f_{concare} = -ve$, $f_{convex} = +ve$, $f_{plane} = \infty$)

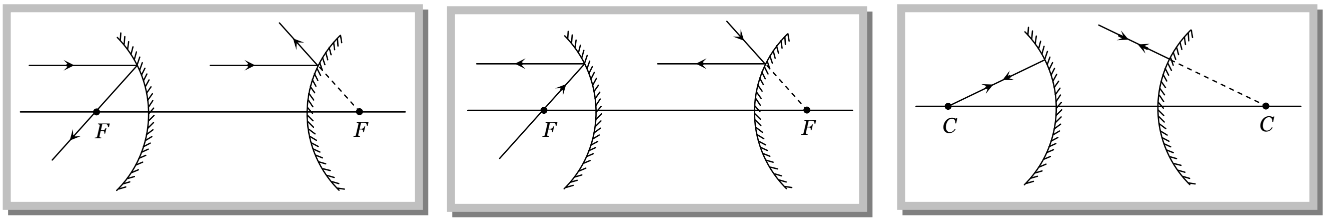
(viii) Power : The converging or diverging ability of mirror

(ix) Aperture : Effective diameter of light reflecting area.
Intensity of image $\propto$ Area $\propto$ (Aperture)^2

(x) Focal plane : A plane passing from focus and perpendicular to principle axis.

(2) **Rules of image formation and sign convention** :

Rule (i) Rule (ii) Rule (iii)

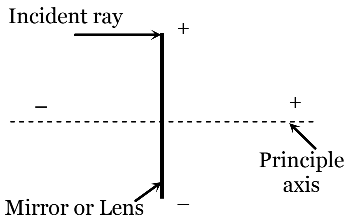


(3) **Sign conventions :**

(i) All distances are measured from the pole.

(ii) Distances measured in the direction of incident rays are taken as positive while in the direction opposite of incident rays are taken negative.

(iii) Distances above the principle axis are taken positive and below the principle axis are taken negative.

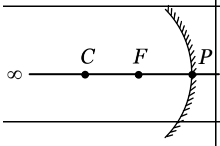
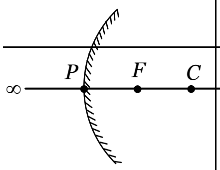


Note : ❑ Same sign convention are also valid for lenses.

Use following sign while solving the problem :

	Concave mirror		Convex mirror
	Real image (u ≥ f)	**Virtual image (u< f)**	
Distance of object	$u \to -$	$u \to -$	$u \to -$
Distance of image	$v \to -$	$v \to +$	$v \to +$
Focal length	$f \to -$	$f \to -$	$f \to +$
Height of object	$O \to +$	$O \to +$	$O \to +$
Height of image	$I \to -$	$I \to +$	$I \to +$
Radius of curvature	$R \to -$	$R \to -$	$R \to +$
Magnification	$m \to -$	$m \to +$	$m \to +$

(4) **Position, size and nature of image formed by the spherical mirror**

Mirror	Location of the object	Location of the image	Magnification, Size of the image	Nature	
				Real virtual	**Erect inverted**
(a) Concave	At infinity i.e. $u = \infty$	At focus i.e. $v = f$	$m << 1$, diminished	Real	inverted
	Away from centre of curvature ($u > 2f$)	Between f and $2f$ i.e. $f < v < 2f$	$m < 1$, diminished	Real	inverted
	At centre of curvature $u = 2f$	At centre of curvature i.e. $v = 2f$	$m = 1$, same size as that of the object	Real	inverted
	Between centre of curvature and focus : $F < u < 2f$	Away from the centre of curvature $v > 2f$	$m > 1$, magnified	Real	inverted
	At focus i.e. $u = f$	At infinity i.e. $v = \infty$	$m = \infty$, magnified	Real	inverted
	Between pole and focus $u < f$	$v > u$	$m > 1$ magnified	Virtual	erect
(b) Convex	At infinity i.e. $u = \infty$	At focus i.e., $v = f$	$m < 1$, diminished	Virtual	erect
	Anywhere between infinity and pole	Between pole and focus	$m < 1$, diminished	Virtual	erect

Note : ❑ In case of convex mirrors, as the object moves away from the mirror, the image becomes smaller and moves closer to the focus.

❑ Images formed by mirrors do not show chromatic aberration.

❑ For convex mirror maximum image distance is it's focal length.

❑ In concave mirror, minimum distance between a real object and it's real image is zero.
(i.e. when $u = v = 2f$)

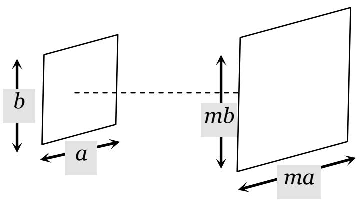
Mirror formula and magnification

For a spherical mirror if u = Distance of object from pole, v = distance of image from pole, f = Focal length, R = Radius of curvature, O = Size of object, I = size of image, m = magnification (or linear magnification), m_s = Areal magnification, A_o = Area of object, A_i = Area of image

(1) **Mirror formula :** $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$; (use sign convention while solving the problems).

Note : ❑ **Newton's formula :** If object distance (x_1) and image distance (x_2) are measured from focus instead of pole then $f^2 = x_1 x_2$

(2) **Magnification :** m = $\frac{\text{Size of object}}{\text{Size of image}}$

Linear magnification		**Areal magnification**
Transverse	**Longitudinal**	
When a object is placed perpendicular to the principle axis, then linear magnification is called lateral or transverse magnification. It is given by $m = \frac{I}{O} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$ (* Always use sign convention while solving the problems)	When object lies along the principle axis then its longitudinal magnification $m = \frac{I}{O} = \frac{-(v_2 - v_1)}{(u_2 - u_1)}$ If object is small; $m = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2$ Also Length of image = $\left(\frac{v}{u}\right)^2 \times \text{Length of object } (L_0)$ $(L_i) = \left(\frac{f}{u-f}\right)^2 .L_o$	 If a 2*D*-object is placed with it's plane perpendicular to principle axis It's Areal magnification $M_s = \frac{\text{Area of image } (A_i)}{\text{Area of object } (A_o)}$ $= \frac{ma \times mb}{ab} = m^2$ $\Rightarrow m_s = m^2 = \frac{A_i}{A_o}$

Note : ❑ Don't put the sign of quantity which is to be determined.

❑ If a spherical mirror produces an image '*m*' times the size of the object (*m* = magnification) then *u*, *v* and *f* are given by the followings

$$u = \left(\frac{m-1}{m}\right)f, \quad v = -(m-1)f \quad \text{and} \quad f = \left(\frac{m}{m-1)}\right)u \quad \text{(use sign convention)}$$

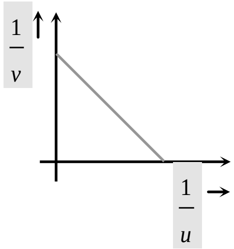
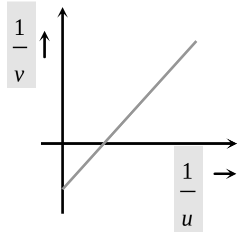
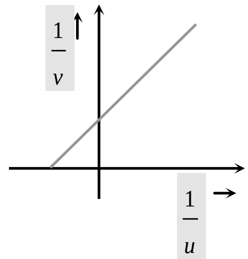
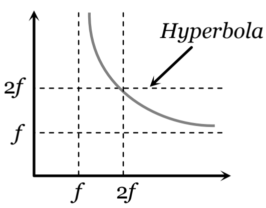
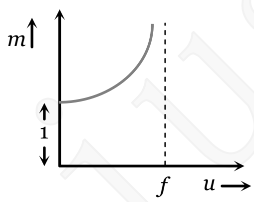
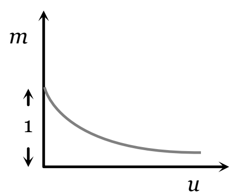
(3) **Uses of mirrors**

(i) **Concave mirror :** Used as a shaving mirror, In search light, in cinema projector, in telescope, by E.N.T. specialists etc.

(ii) **Convex mirror :** In road lamps, side mirror in vehicles *etc.*

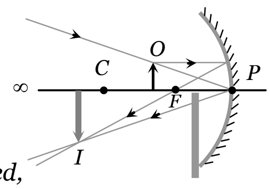
Note : ❑ Field of view of convex mirror is more than that of concave mirror.

Different graphs

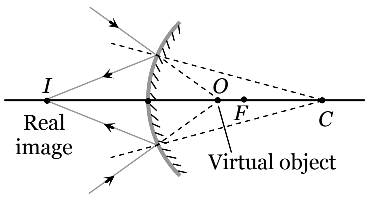
Graph between $\frac{1}{v}$ and $\frac{1}{u}$		
(a) Real image formed by concave mirror 	(b) Virtual image formed by concave mirror 	(c) Virtual image formed by convex mirror 
Graph between *u* and *v* for real image of concave mirror	**Graph between *u* and *m* for virtual image by concave mirror**	**Graph between *u* and *m* for virtual image by convex mirror.**
		

Concepts

- *Focal length of a mirror is independent of material of mirror, medium in which it is placed, wavelength of incident light*
- *Divergence or Convergence power of a mirror does not change with the change in medium.*
- *If an object is moving at a speed v_o towards a spherical mirror along it's axis then speed of image away from mirror is* $v_i = -\left(\frac{f}{u-f}\right)^2 . v_o$ *(use sign convention)*



- *When object is moved from focus to infinity at constant speed, the image will move faster in the beginning and slower later on, towards the mirror.*
- *As every part of mirror forms a complete image, if a part of the mirror is obstructed, full image will be formed but intensity will be reduced.*
- *Can a convex mirror form real images?*
  yes if (distance of virtual object) u < f (focal length)



Example

***Example*: 10** A convex mirror of focal length f forms an image which is $1/n$ times the object. The distance of the object from the mirror is

(a) $(n-1)f$ (b) $\left(\frac{n-1}{n}\right)f$ (c) $\left(\frac{n+1}{n}\right)f$ (d) $(n+1)f$

Solution : (a) By using $m = \frac{f}{f-u}$

Here $m = +\frac{1}{n}$, $f \to +f$ So, $+\frac{1}{n} = \frac{+f}{+f-u} \Rightarrow u = -(n-1)f$

***Example*: 11** An object 5 *cm* tall is placed 1 *m* from a concave spherical mirror which has a radius of curvature of 20 *cm*. The size of the image is

(a) 0.11 *cm* (b) 0.50 *cm* (c) 0.55 *cm* (d) 0.60 *cm*

Solution : (c) By using $\frac{I}{O} = \frac{f}{f-u}$

Here $O + 5\,cm$, $f = -\frac{R}{2} = -10\,cm$, $u = -1\,m = -100\,cm$

So, $\frac{I}{+5} = \frac{-10}{-10-(-100)} \Rightarrow I = -0.55\,cm.$

***Example*: 12** An object of length 2.5 *cm* is placed at a distance of 1.5 f from a concave mirror where f is the magnitude of the focal length of the mirror. The length of the object is perpendicular to the principle axis. The length of the image is

(a) 5 *cm*, erect (b) 10 *cm*, erect (c) 15 *cm*, erect (d) 5 *cm*, inverted

Solution : (d) By using $\frac{I}{O} = \frac{f}{f-u}$; where $I = ?$, $O = +2.5\,cm.$ $f \to -f, u = -1.5f$

$\therefore \frac{I}{+2.5} = \frac{-f}{-f-(-1.5f)} \Rightarrow I = -5\,cm.$ (Negative sign indicates that image is inverted.)

***Example*: 13** A convex mirror has a focal length f. A real object is placed at a distance f in front of it from the pole produces an image at

(a) Infinity (b) f (c) $f/2$ (d) $2f$

Solution : (c) By using $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{+f} = \frac{1}{v} + \frac{1}{(-f)} \Rightarrow v = \frac{f}{2}$

***Example*: 14** Two objects A and B when placed one after another infront of a concave mirror of focal length 10 *cm* from images of same size. Size of object A is four times that of B. If object A is placed at a distance of 50 *cm* from the mirror, what should be the distance of B from the mirror

(a) 10 *cm* (b) 20 *cm* (c) 30 *cm* (d) 40 *cm*

Solution : (b) By using $\frac{I}{O} = \frac{f}{f-u} \Rightarrow \frac{I_A}{I_B} \times \frac{O_B}{O_A} = \frac{f-u_B}{f-u_A} \Rightarrow \frac{1}{1} \times \frac{1}{4} = \frac{-10-u_B}{-10-(-50)} \Rightarrow u_B = -20\,cm.$

***Example*: 15** A square of side 3 *cm* is placed at a distance of 25 *cm* from a concave mirror of focal length 10 *cm*. The centre of the square is at the axis of the mirror and the plane is normal to the axis. The area enclosed by the image of the wire is

(a) 4 cm^2 (b) 6 cm^2 (c) 16 cm^2 (d) 36 cm^2

Solution : (a) By using $m^2 = \frac{A_i}{A_o}$; where $m = \frac{f}{f-u}$

Hence from given values $m = \frac{-10}{-10-(-25)} = \frac{-2}{3}$ and $A_o = 9\,cm^2$ $\therefore$ $A_i = \left(\frac{-2}{3}\right)^2 \times 9 = 4\,cm^2$

***Example*: 16** A convex mirror of focal length 10 *cm* is placed in water. The refractive index of water is 4/3. What will be the focal length of the mirror in water

(a) 10 *cm* (b) 40/3 *cm* (c) 30/4 *cm* (d) None of these

Solution : (a) No change in focal length, because *f* depends only upon radius of curvature *R*.

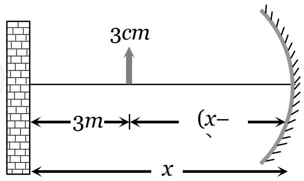
***Example*: 17** A candle flame 3 *cm* is placed at distance of 3 *m* from a wall. How far from wall must a concave mirror be placed in order that it may form an image of flame 9 *cm* high on the wall

(a) 225 *cm* (b) 300 *cm* (c) 450 *cm* (d) 650 *cm*

Solution : (c) Let the mirror be placed at a distance *x* from wall

By using

$$\frac{I}{O} = \frac{-v}{u} \Rightarrow \frac{-9}{+3} = \frac{-(-x)}{-(x-3)} \qquad \Rightarrow x = -4.5m = -450cm.$$



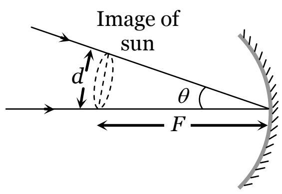
***Example*: 18** A concave mirror of focal length 100 *cm* is used to obtain the image of the sun which subtends an angle of 30'. The diameter of the image of the sun will be

(a) 1.74 *cm* (b) 0.87 *cm* (c) 0.435 *cm* (d) 100 *cm*

Solution : (b) Diameter of image of sun $d = f\theta$

$$\Rightarrow d = 100 \times \left(\frac{30}{60}\right) \times \frac{\pi}{180}$$

$$\Rightarrow \quad d = 0.87\,cm.$$



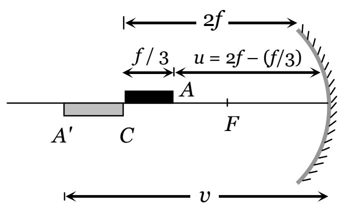
***Example*: 19** A thin rod of length $f/3$ lies along the axis of a concave mirror of focal length *f*. One end of its magnified image touches an end of the rod. The length of the image is **[MP PET 1995]**

(a) f (b) $\frac{1}{2}f$ (c) $2f$ (d) $\frac{1}{4}f$

Solution : (b) If end *A* of rod acts an object for mirror then it's image will be *A*' and if $u = 2f - \frac{f}{3} = \frac{5f}{3}$

So by using $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-f} = \frac{1}{v} + \frac{1}{-\frac{5f}{3}} \Rightarrow v = -\frac{5}{2}f$

$\therefore$ Length of image $= \frac{5}{2}f - 2f = \frac{f}{2}$

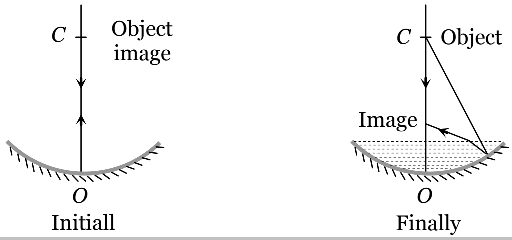


***Example*: 20** A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let *O* be the pole of the mirror and *C* its centre of curvature. A point object is placed at *C*. It has a real image, also located at *C*. If the mirror is now filled with water, the image will be

(a) Real, and will remain at C (b) Real, and located at a point between C and ∞

(c) Virtual and located at a point between C and O (d) Real, and located at a point between C and O

Solution : (d)



Tricky example: 4

An object is placed infront of a convex mirror at a distance of 50 *cm*. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and plane mirror is 30 *cm,* it is found that there is no parallel between the images formed by two mirrors. Radius of curvature of mirror will be

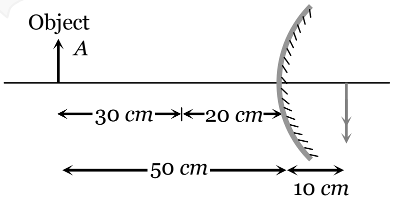
(a) 12.5 *cm* (b) 25 *cm* (c) $\frac{50}{3}$ *cm* (d) 18 *cm*

Solution : (b) Since there is no parallel, it means that both images (By plane mirror and convex mirror) coinciding each other.

According to property of plane mirror it will form image at a distance of 30 *cm* behind it. Hence for convex mirror $u = -50\ cm$, $v = +10\ cm$

By using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\Rightarrow \frac{1}{f} = \frac{1}{+10} + \frac{1}{-50} = \frac{4}{50}$

$\Rightarrow f = \frac{25}{2} cm$ $\Rightarrow R = 2f = 25 cm.$

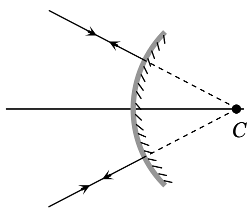


Tricky example: 5

A convergent beam of light is incident on a convex mirror so as to converge to a distance 12 *cm* from the pole of the mirror. An inverted image of the same size is formed coincident with the virtual object. What is the focal length of the mirror

(a) 24 *cm* (b) 12 *cm* (c) 6 *cm* (d) 3 *cm*

Solution : (c) Here object and image are at the same position so this position must be centre of curvature



$\therefore R = 12\ cm \Rightarrow f = \frac{R}{2}$

Practice Questions Basic Level

1. A light bulb is placed between two mirrors (plane) inclined at an angle of 60°. Number of images formed are

[NCERT 1980; CPMT 1996, 97; SCRA 1994; AIIMS 1997; RPMT 1999; AIEEE 2002; Orissa JEE 2003; MP PET 2004]

(a) 2 (b) 4 (c) 5 (d) 6

2. Two plane mirrors are inclined at an angle of 72^o. The number of images of a point object placed between them will be **[KCET (Engg. & Med.)1999; BCECE 2003]**

(a) 2 (b) 3 (c) 4 (d) 5

3. To get three images of a single object, one should have two plane mirrors at an angle of **[AIEEE 2003]**

(a) 30^o (b) 60^o (c) 90^o (d) 120^o

4. A man of length h requires a mirror of length at least equal to, to see his own complete image **[MP PET 2003]**

(a) $\frac{h}{4}$ (b) $\frac{h}{3}$ (c) $\frac{h}{2}$ (d) h

5. Two plane mirrors are at 45° to each other. If an object is placed between them then the number of images will be **[MP PMT 200**

(a) 5 (b) 9 (c) 7 (d) 8

6. An object is at a distance of 0.5 m in front of a plane mirror. Distance between the object and image is **[CPMT 2002]**

(a) 0.5 m (b) 1 m (c) 0.25 m (d) 1.5 m

7. A man runs towards a mirror at a speed 15 m/s. The speed of the image relative to the man is **[RPMT 1999; Kerala PET 2002]**

(a) 15 ms^{-1} (b) 30 ms^{-1} (c) 35 ms^{-1} (d) 20 ms^{-1}

8. The light reflected by a plane mirror may form a real image **[KCET (Engg. & Med.) 2002]**

(a) If the rays incident on the mirror are diverging (b) If the rays incident on the mirror are converging

(c) If the object is placed very close to the mirror (d) Under no circumstances

9. A man is 180 cm tall and his eyes are 10 cm below the top of his head. In order to see his entire height right from toe to head, he uses a plane mirror kept at a distance of 1 m from him. The minimum length of the plane mirror required is **[MP PMT 1993; DPMT 2001]**

(a) 180 cm (b) 90 cm (c) 85 cm (d) 170 cm

10. A small object is placed 10 cm infront of a plane mirror. If you stand behind the object 30 cm from the object and look at its image, the distance focused for your eye will be

(a) 60 cm (b) 20 cm (c) 40 cm (d) 80 cm

11. Two plane mirrors are at right angles to each other. A man stands between them and combs his hair with his right hand. In how many of the images will he be seen using his right hand

(a) None (b) 1 (c) 2 (d) 3

12. A man runs towards mirror at a speed of 15 m/s. What is the speed of his image **[CBSE PMT 2000]**

(a) 7.5 m/s (b) 15 m/s (c) 30 m/s (d) 45 m/s

13. A ray of light is incidenting normally on a plane mirror. The angle of reflection will be **[MP PET 2000]**

(a) 0° (b) 90° (c) Will not be reflected (d) None of these

14. A plane mirror produces a magnification of **[MP PMT/PET 1997]**

(a) – 1 (b) + 1 (c) Zero (d) Between 0 and + ∞

15. When a plane mirror is rotated through an angle θ, then the reflected ray turns through the angle 2θ, then the size of the image **[MP PAT 1996]**

(a) Is doubled (b) Is halved (c) Remains the same (d) Becomes infinite

16. What should be the angle between two plane mirrors so that whatever be the angle of incidence, the incident ray and the reflected ray from the two mirrors be parallel to each other

(a) 60° (b) 90° (c) 120° (d) 175°

17. Ray optics is valid, when characteristic dimensions are **[CBSE PMT 1994]**

(a) Of the same order as the wavelength of light (b) Much smaller than the wavelength of light

(c) Of the order of one millimeter (d) Much larger than the wavelength of light

18. It is desired to photograph the image of an object placed at a distance of 3 m from the plane mirror. The camera which is at a distance of 4.5 m from the mirror should be focussed for a distance of

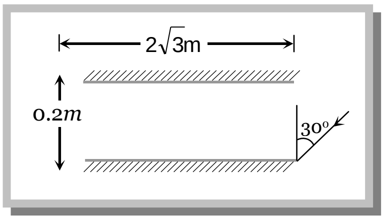
(a) 3 m (b) 4.5 m (c) 6 m (d) 7.5 m

19. Two plane mirrors are parallel to each other an spaced 20 *cm* apart. An object is kept in between them at 15 *cm* from *A*. Out of the following at which point an image is not formed in mirror *A* (distance measured from mirror *A*)

(a) 15 *cm* (b) 25 *cm* (c) 45 *cm* (d) 55 *cm*

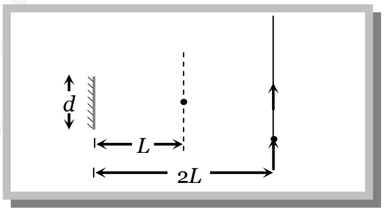
Advance Level

20. Two plane mirrors *A* and *B* are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of 30° at a point just inside one end of *A*. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is



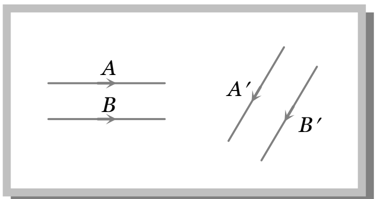
(a) 28
(b) 30
(c) 32
(d) 34

21. A point source of light *B* is placed at a distance *L* in front of the centre of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2*L* from it as shown. The greatest distance over which he can see the image of the light source in the mirror is



(a) $d/2$
(b) d
(c) $2d$
(d) $3d$

22. The figure shows two rays *A* and *B* being reflected by a mirror and going as *A*′ and *B*′. The mirror is



(a) Plane (b) Concave
(c) Convex (d) May be any spherical mirror

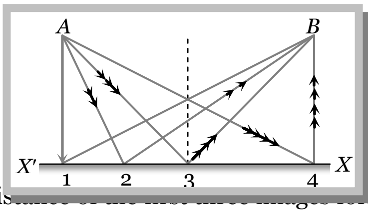
23. An object is initially at a distance of 100 *cm* from a plane mirror. If the mirror approaches the object at a speed of 5 *cm/s*, then after 6 *s* the distance between the object and its image will be

(a) 60 *cm* (b) 140 *cm* (c) 170 *cm* (d) 150 *cm*

24. An object placed in front of a plane mirror is displaced by 0.4 *m* along a straight line at an angle of 30° to mirror plane. The change in the distance between the object and its image is

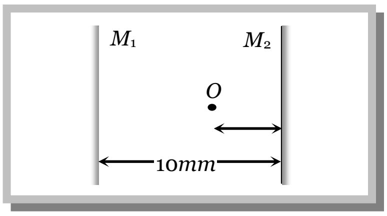
(a) 0.20 *m* (b) 0.40 *m* (c) 0.25 *m* (d) 0.80 *m*

25. A ray of light travels from *A* to *B* with uniform speed. On its way it is reflected by the surface *XX*′. The path followed by the ray to take least time is



(a) 1 (b) 2
(c) 3 (d) 4

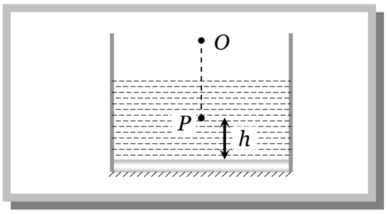
26. A point object *O* is placed between two plan mirrors as shown is fig. The distance of the first three images formed by mirror M_2 from it are



(a) 2 *mm*, 8 *mm*, 18 *mm*
(b) 2 *mm*, 18 *mm*, 28 *mm*
(c) 2 *mm*, 18 *mm*, 22 *mm*

(d) 2 *mm*, 18 *mm*, 58 *mm*

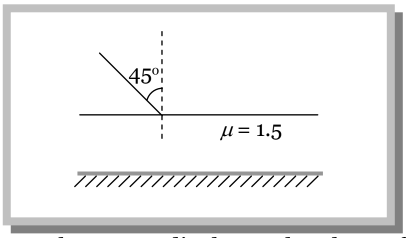
27. A plane mirror is placed at the bottom of the tank containing *a* liquid of refractive index μ . *P* is a small object at a height *h* above the mirror. An observer *O*-vertically above *P* outside the liquid see *P* and its image in the mirror. The apparent distance between these two will be



(a) $2\mu h$ (b) $\frac{2h}{\mu}$

(c) $\frac{2h}{\mu - 1}$ (d) $h\left(1+\frac{1}{\mu}\right)$

28. One side of a glass slab is silvered as shown. A ray of light is incident on the other side at angle of incidence $i = 45^o$. Refractive index of glass is given as 1.5. The deviation of the ray of light from its initial path when it comes out of the slab is



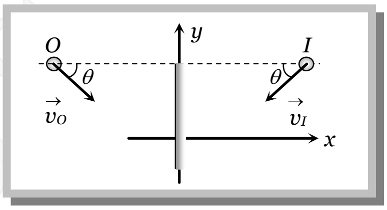
(a) 90°

(b) 180°

(c) 120°

(d) 45°

29. If an object moves towards a plane mirror with a speed *v* at an angle θ to the perpendicular to the plane of the mirror, find the relative velocity between the object and the image



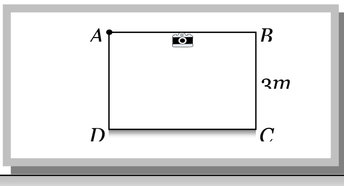
(a) *v*

(b) 2*v*

(c) $2v\cos\theta$

(d) $2v\sin\theta$

30. Figure shows a cubical room *ABCD* will the wall *CD* as a plane mirror. Each side of the room is 3*m*. We place a camera at the midpoint of the wall *AB*. At what distance should the camera be focussed to photograph an object placed at *A*



(a) 1.5 *m* (b) 3 *m*

(c) 6 *m* (d) More than 6 *m*

Reflection of light at spherical surface

Basic Level

31. A man having height 6 *m*, want to see full height in mirror. They observe image of 2*m* height erect, then used mirror is **[J & K CET 2004]**

(a) Concave (b) Convex (c) Plane (d) None of these

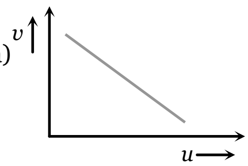
32. An object of length 6*cm* is placed on the principal axis of a concave mirror of focal length *f* at a distance of 4 *f*. The length of the image will be **[MP PET 2003]**

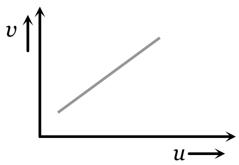
(a) 2 *cm* (b) 12 *cm* (c) 4 *cm* (d) 1.2 *cm*

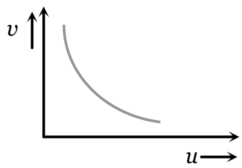
33. Convergence of concave mirror can be decreased by dipping in **[AFMC 2003]**

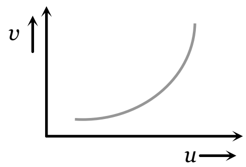
(a) Water (b) Oil (c) Both (d) None of these

34. In an experiment of find the focal length of a concave mirror a graph is drawn between the magnitudes of *u* and *v*. The graph looks like

(a) 

(b) 

(c) 

(d) 

35. An object 2.5 *cm* high is placed at a distance of 10 *cm* from a concave mirror of radius of curvature 30 *cm* The size of the image is **[BVP 2003]**
(a) 9.2 *cm* (b) 10.5 *cm* (c) 5.6 *cm* (d) 7.5 *cm*

36. A diminished virtual image can be formed only in **[MP PMT 2002]**
(a) Plane mirror (b) A concave mirror (c) A convex mirror (d) Concave-parabolic mirror

37. A point object is placed at a distance of 30 *cm* from a convex mirror of focal length 30*cm*. The image will form at **[JIPMER 2002**
(a) Infinity (b) Focus (c) Pole (d) 15 *cm* behind the mirror

38. The focal length of a convex mirror is 20 *cm* its radius of curvature will be **[MP PMT 2001]**
(a) 10 *cm* (b) 20 *cm* (c) 30 *cm* (d) 40 *cm*

39. A concave mirror of focal length 15 *cm* forms an image having twice the linear dimensions of the object. The position of the object when the image is virtual will be
(a) 22.5 *cm* (b) 7.5 *cm* (c) 30 *cm* (d) 45 *cm*

40. Under which of the following conditions will a convex mirror of focal length *f* produce an image that is erect, diminished and virtual **[AMU (Engg.) 2001]**
(a) Only when $2f > u > f$ (b) Only when $u = f$ (c) Only when $u < f$ (d) Always

41. A concave mirror gives an image three times as large as the object placed at a distance of 20 *cm* from it. For the image to be real, the focal length should be **[SCRA 1998; JIPMER 2**
(a) 10 *cm* (b) 15 *cm* (c) 20 *cm* (d) 30 *cm*

42. A point object is placed at a distance of 10 *cm* and its real image is formed at a distance of 20*cm* from a concave mirror. If the object is moved by 0.1*cm* towards the mirror, the image will shift by about
(a) 0.4 *cm* away from the mirror (b) 0.4 *cm* towards the mirror
(c) 0.8 *cm* away from the mirror (d) 0.8 *cm* towards the mirror

43. The minimum distance between the object and its real image for concave mirror is **[RPMT 1999]**
(a) *f* (b) 2*f* (c) 4*f* (d) Zero

44. An object is placed at 20 *cm* from a convex mirror of focal length 10 *cm*. The image formed by the mirror is **[JIPMER 1999]**
(a) Real and at 20 *cm* from the mirror (b) Virtual and at 20 *cm* from the mirror
(c) Virtual and at 20/3 *cm* from the mirror (d) Real and at 20/3 *cm* from the mirror

45. An object is placed 40 *cm* from a concave mirror of focal length 20 *cm*. The image formed is **[MP PET 1986; MP PMT/PET 19**
(a) Real, inverted and same in size (b) Real, inverted and smaller
(c) Virtual, erect and larger (d) Virtual, erect and smaller

46. Match List I with List II and select the correct answer using the codes given below the lists **[SCRA 1998]**

List I (Position of the object)	List II (Magnification)
(I) An object is placed at focus before a convex mirror	(A) Magnification is $-\infty$
(II) An object is placed at centre of curvature before a concave mirror	(B) Magnification is 0.5
(III) An object is placed at focus before a concave mirror	(C) Magnification is + 1
(IV) An object is placed at centre of curvature before a convex mirror	(D) Magnification is – 1
	(E) Magnification is 0.33

Codes :
(a) I-B, II-D, III-A, IV-E (b) I-A, II-D, III-C, IV-B (c) I-C, II-B, III-A, IV-E (d) I-B, II-E, III-D, IV-C

47. In a concave mirror experiment, an object is placed at a distance x_1 from the focus and the image is formed at a distance x_2 from the focus. The focal length of the mirror would be
(a) x_1x_2 (b) $\sqrt{x_1x_2}$ (c) $\frac{x_1+x_2}{2}$ (d) $\sqrt{\frac{x_1}{x_2}}$

48. Which of the following forms a virtual and erect image for all positions of the object **[IIT-JEE 1996]**
(a) Convex lens (b) Concave lens (c) Convex mirror (d) Concave mirror

49. A convex mirror has a focal length *f*. A real object is placed at a distance *f* in front of it from the pole produces an image at **[MP PAT 1996]**

(a) Infinity (b) f (c) $f/2$ (d) $2f$

50. Radius of curvature of concave mirror is 40 *cm* and the size of image is twice as that of object, then the object distance is **[AFMC 1995]**

(a) 60 *cm* (b) 20 *cm* (c) 40 *cm* (d) 30 *cm*

51. All of the following statements are correct except **[Manipal MEE 1995]**

(a) The magnification produced by a convex mirror is always less than one

(b) A virtual, erect, same-sized image can be obtained using a plane mirror

(c) A virtual, erect, magnified image can be formed using a concave mirror

(d) A real, inverted, same-sized image can be formed using a convex mirror

52. Radius of curvature of convex mirror is 40 *cm* and the size of object is twice as that of image, then the image distance is **[AFMC 1995]**

(a) 10 *cm* (b) 20 *cm* (c) 40 *cm* (d) 30 *cm*

53. If an object is placed 10 *cm* in front of a concave mirror of focal length 20 *cm*, the image will be **[MP PMT 1995]**

(a) Diminished, upright, virtual (b) Enlarged, upright, virtual (c) Diminished, inverted, real (d) Enlarged, upright, real

54. An object 1 *cm* tall is placed 4 *cm* in front of a mirror. In order to produce an upright image of 3 *cm* height one needs a **[SCRA 1994]**

(a) Convex mirror of radius of curvature 12 *cm* (b) Concave mirror of radius of curvature 12 *cm*

(c) Concave mirror of radius of curvature 4 *cm* (d) Plane mirror of height 12 *cm*

55. The image formed by a convex mirror of a real object is larger than the object **[CPMT 1994]**

(a) When $u < 2f$ (b) When $u > 2f$ (c) For all values of u (d) For no value of u

56. An object 5 *cm* tall is placed 1 *m* from a concave spherical mirror which has a radius of curvature of 20 *cm*. The size of the image is **[MP PET 1993]**

(a) 0.11 *cm* (b) 0.50 *cm* (c) 0.55 *cm* (d) 0.60 *cm*

57. A virtual image three times the size of the object is obtained with a concave mirror of radius of curvature 36 *cm*. The distance of the object from the mirror is

(a) 5 *cm* (b) 12 *cm* (c) 10 *cm* (d) 20 *cm*

58. Given a point source of light, which of the following can produce a parallel beam of light **[CPMT 1974]**

(a) Convex mirror (b) Concave mirror

(c) Concave lens (d) Two plane mirrors inclined at an angle of 90^o

59. A convex mirror is used to form the image of an object. Then which of the following statements is wrong

(a) The images lies between the pole and the focus (b) The image is diminished in size

(c) The images is erect (d) The image is real

60. A boy stands straight infront of a mirror at a distance of 30 *cm* away from it. He sees his erect image whose height is $\frac{1}{5}$ th of his real height. The mirror he is using is

(a) Plane mirror (b) Convex mirror (c) Concave mirror (d) Plano-convex mirror

61. For the largest distance of the image from a concave mirror of focal length 10*cm*, the object should be kept at

(a) 10 *cm* (b) Infinite (c) 40 *cm* (d) 60 *cm*

62. A dentist uses a small mirror that gives a magnification of 4 when it is held 0.60 *cm* from a tooth. The radius of curvature of the mirror is

(a) 1.60 *cm* (convex) (b) 0.8 *cm* (concave) (c) 1.60 *cm* (concave) (d) 0.8 *cm* (convex)

63. A dice is placed with its one edge parallel to the principal axis between the principal focus and the centre of the curvature of a concave mirror. Then the image has the shape of

(a) Cube (b) Cuboid (c) Barrel shaped (d) Spherical

Advance Level

64. A short linear object of length l lies along the axis of a concave mirror of focal length f at a distance u form the pole of the mirror. The size of the image is approximately equal to **[IIT 1988; BHU 2003]**

(a) $l\left(\frac{u-f}{f}\right)^{1/2}$ (b) $l\left(\frac{u-f}{f}\right)^{2}$ (c) $l\left(\frac{f}{u-f}\right)^{1/2}$ (d) $l\left(\frac{f}{u-f}\right)^{2}$

65. A point object is moving on the principal axis of a concave mirror of focal length 24 *cm* towards the mirror. When it is at a distance of 60 *cm* from the mirror, its velocity is 9 *cm/sec*. What is the velocity of the image at that instant

(a) 5 *cm/sec* towards the mirror (b) 4 *cm/sec* towards the mirror

(c) 4 *cm/sec* away from the mirror (d) 9 *cm/sec* away from the mirror

66. A convex mirror of focal length 10 *cm* forms an image which is half of the size of the object. The distance of the object from the mirror is

(a) 10 *cm* (b) 20 *cm* (c) 5 *cm* (d) 15 *cm*

67. A concave mirror is used to focus the image of a flower on a nearby well 120 *cm* from the flower. If a lateral magnification of 16 is desired, the distance of the flower from the mirror should be

(a) 8 *cm* (b) 12 *cm* (c) 80 *cm* (d) 120 *cm*

68. A thin rod of 5 *cm* length is kept along the axis of a concave mirror of 10 *cm* focal length such that its image is real and magnified and one end touches the rod. Its magnification will be

(a) 1 (b) 2 (c) 3 (d) 4

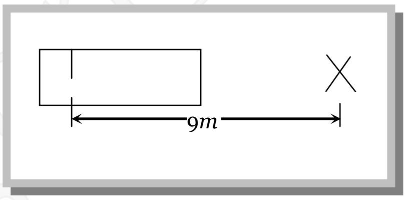
69. A luminous object is placed 20 *cm* from surface of a convex mirror and a plane mirror is set so that virtual images formed in two mirrors coincide. If plane mirror is at a distance of 12 *cm* from object, then focal length of convex mirror, is

(a) 5 *cm* (b) 10 *cm* (c) 20 *cm* (d) 40 *cm*

70. A rear mirror of a vehicle is cylindrical having radius of curvature 10 *cm*. The length of arc of curved surface is also 10 *cm*. If the eye of driver is assumed to be at large distance, from the mirror, then the field of view in radian is

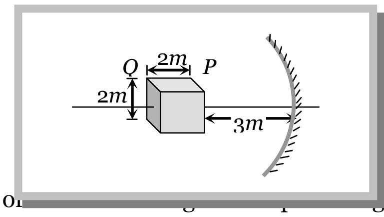
(a) 0.5 (b) 1 (c) 2 (d) 4

71. A vehicle has a driving mirror of focal length 30 *cm*. Another vehicle of dimension $2 \times 4 \times 1.75\, m^3$ is 9 *m* away from the mirror of first vehicle. Position of the second vehicle as seen in the mirror of first vehicle is



(a) 30 *cm*
(b) 60 *cm*
(c) 90 *cm*
(d) 9 *cm*

72. A cube of side 2 *m* is placed in front of a concave mirror focal length 1*m* with its face *P* at a distance of 3 *m* and face *Q* at a distance of 5 *m* from the mirror. The distance between the images of face *P* and *Q* and height of images of *P* and *Q* are



(a) 1 *m*, 0.5 *m*, 0.25 *m*
(b) 0.5 *m*, 1 *m*, 0.25 *m*
(c) 0.5 *m*, 0.25 *m*, 1*m*
(d) 0.25 *m*, 1*m*, 0.5 *m*

73. A concave mirror of radius of curvature 60 *cm* is placed at the bottom o ht of 20 *cm*. The mirror faces upwards with its axis vertical. Solar light falls normally on the surface of water and the image of the sun is formed. If $_a\mu_w = \frac{4}{3}$ then with the observer in air, the distance of the image from the surface of water is

(a) 30 *cm* (b) 10 *cm* (c) 7.5 *cm* above (d) 7.5 *cm* below

74. A concave mirror forms an image of the sun at a distance of 12 *cm* from it

(a) The radius of curvature of this mirror is 6 *cm*
(b) To use it as a shaving mirror, it must be held at a distance of 8-10 *cm* from the face
(c) If an object is kept at a distance of 12 *cm* from it, the image formed will be of the same size as the object
(d) All the above a alternatives are correct

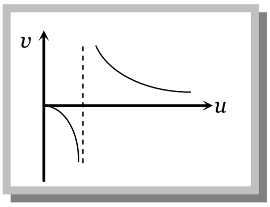
75. A small piece of wire bent into an L shape with upright and horizontal portions of equal lengths, is placed with the horizontal portion along the axis of the concave mirror whose radius of curvature is 10 *cm*. If the bend is 20 *cm* from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portions of the wire is

(a) 1 : 2 (b) 3 : 1 (c) 1 : 3 (d) 2 : 1

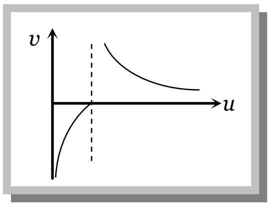
76. As the position of an object (u) reflected from a concave mirror is varied, the position of the image (v) also varies. By letting the u changes from 0 to $+\infty$ the graph between v versus u will be

77.

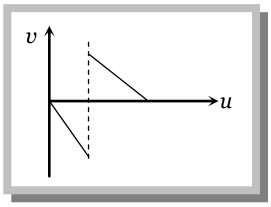
(a)



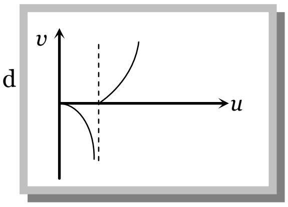
b



c



d



78. A concave mirror has a focal length 20 *cm*. The distance between the two positions of the object for which the image size is double of the object size is

(a) 20 *cm* (b) 40 *cm* (c) 30 *cm* (d) 60 *cm*

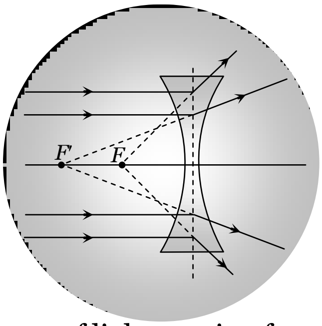
79. A concave mirror of focal length 10 *cm* and a convex mirror of focal length 15 *cm* are placed facing each other 40 *cm* apart. A point object is placed between the mirrors, on their common axis and 15 *cm* from the concave mirror. Find the position and nature of the image produced by the successive reflections, first at concave mirror and then at convex mirror

(a) 2 *cm* (b) 4 *cm* (c) 6 *cm* (d) 8 *cm*

Answer Sheet

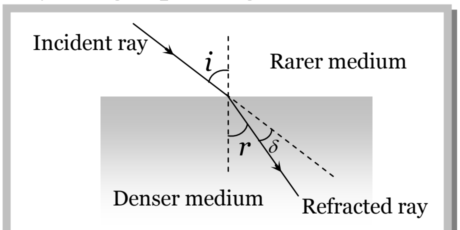
Assignments

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	c	c	c	c	b	b	b	b	c	b	b	a	b	c	b	d	d	c	b
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
d	a	b	b	c	c	b	a	c	d	b	a	d	c	d	c	d	d	b	d
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
b	a	d	c	a	a	b	b, c	c	d	d	a	b	b	d	c	b	b	d	b
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78		
a	c	b	d	c	a	a	a	a	b	a	d	c	b	b	a	a	c		

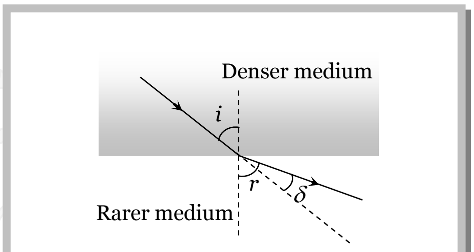


Refraction of Light

The bending of the ray of light passing from one medium to the other medium is called refraction.



Deviation $\delta = (i - r)$



Deviation $\delta = (r - i)$

Snell's law

The ratio of sine of the angle of incidence to the angle of refraction (r) is a constant called refractive index

i.e. $\frac{\sin i}{\sin r} = \mu$ (a constant). For two media, Snell's law can be written as ${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$

$\Rightarrow \mu_1 \times \sin i = \mu_2 \times \sin r$ *i.e.* $\mu \sin\theta =$ constant

Also in vector form : $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{n}} = \boldsymbol{\mu}(\hat{\boldsymbol{r}} \times \hat{\boldsymbol{n}})$

Refractive Index

Refractive index of a medium is that characteristic which decides speed of light in it. It is a scalar, unit less and dimensionless quantity

.(1) **Types :** It is of following two types

Absolute refractive index	Relative refractive index
(i) When light travels from air to any transparent medium then R.I. of medium *w.r.t.* air is called it's absolute R.I. *i.e.* ${}_{\text{air}}\mu_{\text{medium}} = \frac{c}{v}$	(i) When light travels from medium (1) to medium (2) then R.I. of medium (2) *w.r.t.* medium (1) is called it's relative R.I. *i.e.* ${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$ (where v_1 and v_2 are the speed of light in medium 1 and 2 respectively).
(ii) Some absolute R.I.	(ii) Some relative R.I. (a) When light enters from water to glass :

$_a\mu_{glass} = \frac{3}{2} = 1.5$, $_a\mu_{water} = \frac{4}{3} = 1.33$

$_a\mu_{diamond} = 2.4$, $_a\mu_{Cs_2} = 1.62$

$_a\mu_{crown} = 1.52$, $\mu_{vacuum} = 1$, $\mu_{air} = 1.0003 \approx 1$

$_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$

(b) When light enters from glass to diamond :

$_g\mu_D = \frac{\mu_D}{\mu_g} = \frac{2.4}{1.5} = \frac{8}{5}$

Note : ❑ Cauchy's equation : $\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ...$ $(\lambda_{Red} > \lambda_{violet}$ so $\mu_{Red} < \mu_{violet})$

❑ If a light ray travels from medium (1) to medium (2), then $_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$

$\mu \propto \frac{1}{\lambda}$

$\mu \propto \frac{1}{v}$

$v \propto \lambda$

(2) **Dependence of Refractive index**

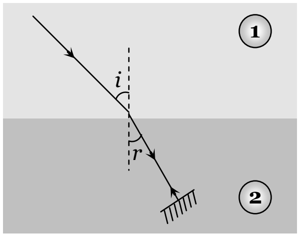
(i) Nature of the media of incidence and refraction.

(ii) Colour of light or wavelength of light.

(iii) Temperature of the media : Refractive index decreases with the increase in temperature.

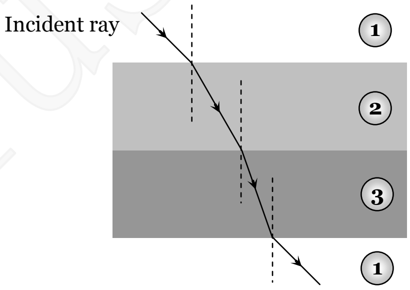
(3) **Principle of reversibility of light and refraction through several media :**

Principle of reversibility



$_1\mu_2 = \frac{1}{_2\mu_1}$

Refraction through several media

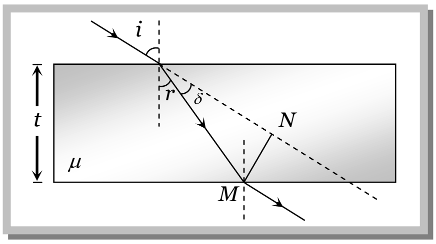


$_1\mu_2 \times {_2\mu_3} \times {_3\mu_1} = 1$

Refraction Through a Glass Slab and Optical Path

(1) **Lateral shift**

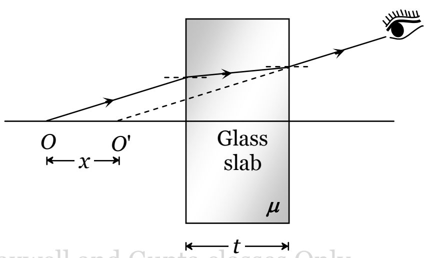
The refracting surfaces of a glass slab are parallel to each other. When a light ray passes through a glass slab it is refracted twice at the two parallel faces and finally emerges out parallel to it's incident direction *i.e.* the ray undergoes no deviation δ = 0. The angle of emergence (e) is equal to the angle of incidence (i)



The Lateral shift of the ray is the perpendicular distance between the incident and the emergent ray, and it is given by ***MN* = *t* sec *r* sin (*i* – *r*)**

Normal shift

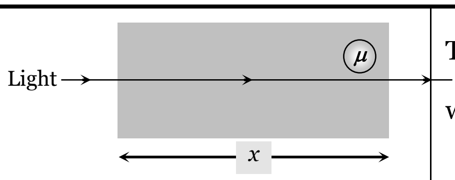
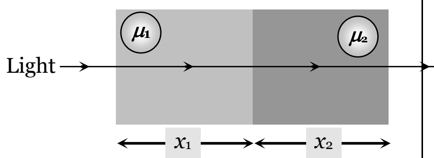
Normal shift $OO' = x = \left(1 - \frac{1}{\mu}\right)t$



Or the object appears to be shifted towards the slab by the distance x

(2) **Optical path :**

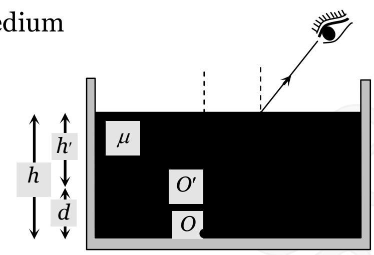
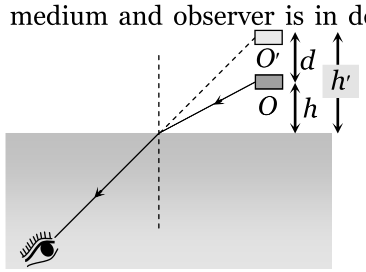
It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium.

	Time taken by light ray to pass through the medium $= \frac{\mu x}{c}$; where x = geometrical path and μx = optical path
	For two medium in contact optical path = $\mu_1 x_1 + \mu_2 x_2$

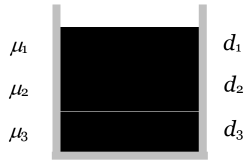
Note : ❑ Since for all media $\mu > 1$, so optical path length (μx) is always greater than the geometrical path length (x).

Real and Apparent Depth

If object and observer are situated in different medium then due to refraction, object appears to be displaced from it's real position. There are two possible conditions.

(1) When object is in denser medium and observer is in rarer medium 	(1) Object is in rarer medium and observer is in denser medium. 
(2) $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{h}{h'}$ Real depth >Apparent depth that's why *a* coin at the bottom of bucket (full of water) appears to be raised)	(2) $\mu = \frac{h'}{h}$ Real depth < Apparent depth that's why high flying aeroplane appears to be higher than it's actual height.
(3) Shift $d = h - h' = \left(1 - \frac{1}{\mu}\right)h$	(3) $d = (\mu - 1)h$
(4) For water $\mu = \frac{4}{3} \Rightarrow d = \frac{h}{4}$ For glass $\mu = \frac{3}{2} \Rightarrow d = \frac{h}{3}$	(4) Shift for water $d_w = \frac{h}{3}$ Shift for glass $d_g = \frac{h}{2}$

Note : ❑ If a beaker contains various immisible liquids as shown then



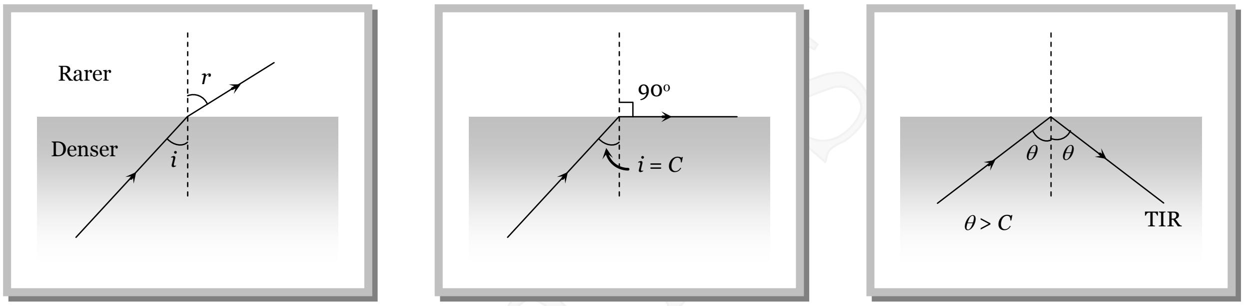
Apparent depth of bottom $= \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} + ...$

$\mu_{\text{combination}} = \frac{d_{AC}}{d_{App.}} = \frac{d_1 + d_2 + ...}{\frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + ...}$ (In case of two liquids if $d_1 = d_2$ than $\mu = \frac{2\mu_1\mu_2}{\mu_1 + \mu_2}$)

Total Internal Reflection

When a ray of light goes from denser to rarer medium it bends away from the normal and as the angle of incidence in denser medium increases, the angle of refraction in rarer medium also increases and at a certain angle, angle of refraction becomes 90°, this angle of incidence is called critical angle (C).

When Angle of incidence exceeds the critical angle than light ray comes back in to the same medium after reflection from interface. This phenomenon is called Total internal reflection (TIR).

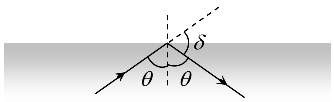


Important formula $\mu = \frac{1}{\sin C} = \text{cosec } C$; where $\mu \rightarrow {}_{\text{Rerer}}\mu_{\text{Denser}}$

Note : ❑ When a light ray travels from denser to rarer medium, then deviation of the ray is

$\delta = \pi - 2\theta \Rightarrow \delta \rightarrow \text{max. when } \theta \rightarrow \text{min.} = C$

i.e. $\delta_{\max} = (\pi - 2C)$; $C \rightarrow$ critical angle



(1) **Dependence of critical angle**

(i) Colour of light (or wavelength of light) : Critical angle depends upon wavelength as $\lambda \propto \frac{1}{\mu} \propto \sin C$

(a) $\lambda_R > \lambda_V \Rightarrow C_R > C_V$

(b) Sin $C = \frac{1}{{}_R\mu_D} = \frac{\mu_R}{\mu_D} = \frac{\lambda_D}{\lambda_R} = \frac{v_D}{v_R}$ (for two media) (c) For TIR from boundary of two media $i > \sin^{-1}\frac{\mu_R}{\mu_D}$

(ii) Nature of the pair of media : Greater the refractive index lesser will be the critical angle.

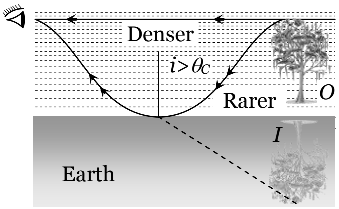
(a) For (glass- air) pair $\rightarrow C_{\text{glass}} = 42^o$ (b) For (water-air) pair $\rightarrow C_{\text{water}} = 49^o$

(c) For (diamond-air) pair $\rightarrow C_{\text{diamond}} = 24^o$

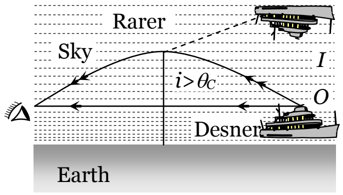
(iii) Temperature : With temperature rise refractive index of the material decreases therefore critical angle increases.

(2) **Examples of total internal reflection (TIR)**

(i)



Mirage : An optical illusion in deserts



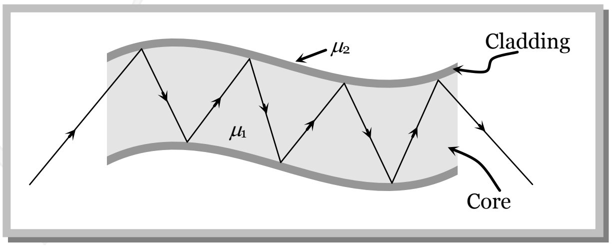
Looming : An optical illusion in cold countries

(ii) **Brilliance of diamond :** Due to repeated internal reflections diamond sparkles.

(iii) **Optical fibre :** Optical fibres consist of many long high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core (μ_1) is higher than that of the cladding (μ_2).

When the light is incident on one end of the fibre at a small angle, the light passes inside, undergoes repeated total internal reflections along the fibre and finally comes out. The angle of incidence is always larger than the critical angle of the core material with respect to its cladding. Even if the fibre is bent, the light can easily travel through along the fibre

A bundle of optical fibres can be used as a 'light pipe' in medical and optical examination. It can also be used for optical signal transmission. Optical fibres have also been used for transmitting and receiving electrical signals which are converted to light by suitable transducers.

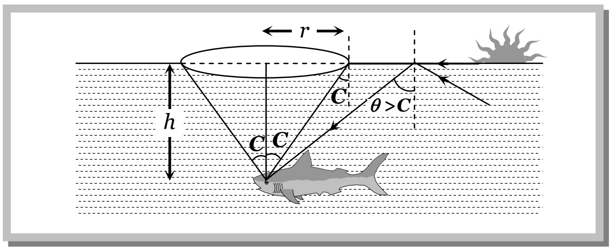


(iv) **Field of vision of fish (or swimmer) :** A fish (diver) inside the water can see the whole world through a cone with.

(a) Apex angle $= 2C = 98^o$

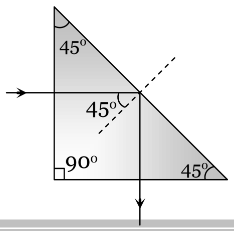
(b) Radius of base $r = h\tan C = \dfrac{h}{\sqrt{\mu^2 - 1}}$

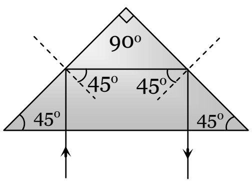
(c) Area of base $A = \dfrac{\pi h^2}{(\mu^2 - 1)}$

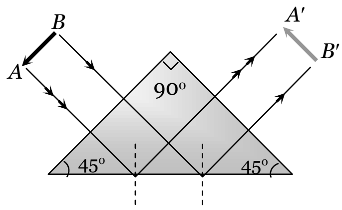


Note : ❑ For water $\mu = \dfrac{4}{3}$ so $r = \dfrac{3h}{\sqrt{7}}$ and $A = \dfrac{9\pi h^2}{7}$.

(v) **Porro prism :** A right angled isosceles prism, which is used in periscopes or binoculars. It is used to deviate light rays through 90^o and 180^o and also to erect the image.







Example

Example: 1 A beam of monochromatic blue light of wavelength 4200 $\mathring{A}$ in air travels in water $(\mu = 4/3)$. Its wavelength in water will be

(a) 2800 $\mathring{A}$ (b) 5600 $\mathring{A}$ (c) 3150 $\mathring{A}$ (d) 4000 $\mathring{A}$

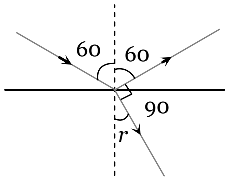
Solution: (c) $\mu \propto \frac{1}{\lambda} \Rightarrow \frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{1}{4/3} = \frac{\lambda_2}{4200} \Rightarrow \lambda_2 = 3150\ \mathring{A}$

Example: 2 On a glass plate a light wave is incident at an angle of 60°. If the reflected and the refracted waves are mutually perpendicular, the refractive index of material is **[MP PMT 1994; Haryana CEE 1996]**

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) $\frac{3}{2}$ (d) $\frac{1}{\sqrt{3}}$

Solution: (b) From figure $r = 30^o$

$\therefore \quad \mu = \frac{\sin i}{\sin r} = \frac{\sin 60^o}{\sin 30^o} = \sqrt{3}$



Example: 3 Velocity of light in glass whose refractive index with respect to air is 1.5 is 2 × $10^8 m/s$ and in certain liquid the velocity of light found to be $2.50 \times 10^8 m/s$. The refractive index of the liquid with respect to air is **[CPMT 1978; MP PET/PMT 1988]**

(a) 0.64 (b) 0.80 (c) 1.20 (d) 1.44

Solution: (c) $\mu \propto \frac{1}{v} \Rightarrow \frac{\mu_{li}}{\mu_g} = \frac{v_g}{v_l} \Rightarrow \frac{\mu_l}{1.5} = \frac{2 \times 10^8}{2.5 \times 10^8} \Rightarrow \mu_l = 1.2$

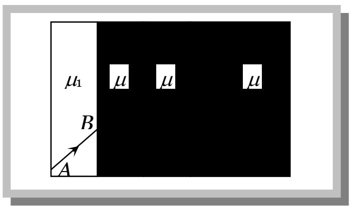
Example: 4 A ray of light passes through four transparent media with refractive indices $\mu_1 . \mu_2, \mu_3$, and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray *CD* is parallel to the incident ray *AB*, we must have

(a) $\mu_1 = \mu_2$

(b) $\mu_2 = \mu_3$

(c) $\mu_3 = \mu_4$

(d) $\mu_4 = \mu_1$



Solution: (d) For successive refraction through difference media $\mu \sin\theta =$ constant.

Here as θ is same in the two extreme media. Hence $\mu_1 = \mu_4$

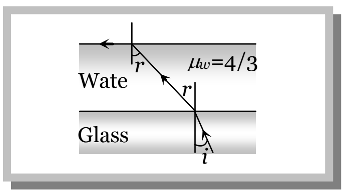
Example: 5 A ray of light is incident at the glass–water interface at an angle *i*, it emerges finally parallel to the surface of water, then the value of μ_g would be

(a) (4/3) sin *i*

(b) 1/ sin *i*

(c) 4/ 3

(d) 1



Solution: (b) For glass water interface $_g\mu_\omega = \frac{\sin i}{\sin r}$ ......(i) and For water-air interface $_\omega\mu_a = \frac{\sin r}{\sin 90}$ .....(ii)

$\therefore \quad {}_g\mu_\omega \times_\omega \mu_a = \sin i \qquad \Rightarrow \quad \mu_g = \frac{1}{\sin i}$

Example: 6 The ratio of thickness of plates of two transparent mediums A and B is 6 : 4. If light takes equal time in passing through them, then refractive index of B with respect to A will be

(a) 1.4 (b) 1.5 (c) 1.75 (d) 1.33

Solution: (b) By using $t = \frac{\mu x}{c}$

$\Rightarrow \frac{\mu_B}{\mu_A} = \frac{x_A}{x_B} = \frac{6}{4} \Rightarrow {}_A\mu_B = \frac{3}{2} = 1.5$

Example: 7 A ray of light passes from vacuum into a medium of refractive index μ, the angle of incidence is found to be twice the angle of refraction. Then the angle of incidence is

(a) $\cos^{-1}(\mu/2)$ (b) $2\cos^{-1}(\mu/2)$ (c) $2\sin^{-1}(\mu)$ (d) $2\sin^{-1}(\mu/2)$

Solution: (b) By using $\mu = \frac{\sin i}{\sin r} \Rightarrow \mu = \frac{\sin 2r}{\sin r} = \frac{2\sin r\cos r}{\sin r}$ $(\sin 2\theta = 2\sin\theta\cos\theta)$

$\Rightarrow r = \cos^{-1}\left(\frac{\mu}{2}\right)$. So, $i = 2r = 2\cos^{-1}\left(\frac{\mu}{2}\right)$.

Example: 8 A ray of light falls on the surface of a spherical glass paper weight making an angle α with the normal and is refracted in the medium at an angle β. The angle of deviation of the emergent ray from the direction of the incident ray is

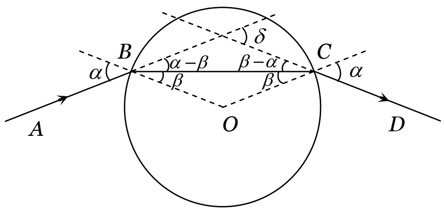
(a) $(\alpha-\beta)$ (b) $2(\alpha-\beta)$ (c) $(\alpha-\beta)/2$ (d) $(\alpha+\beta)$

Solution: (b) From figure it is clear that ΔOBC is an isosceles triangle,

Hence $\angle OCB = \beta$ and emergent angle is α

Also sum of two in terior angles = exterior angle

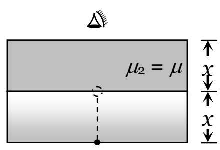
$\therefore \quad \delta = (\alpha-\beta)+(\alpha-\beta) = 2(\alpha-\beta)$



Example: 9 A rectangular slab of refractive index μ is placed over another slab of refractive index 3, both slabs being identical in dimensions. If a coin is placed below the lower slab, for what value of μ will the coin appear to be placed at the interface between the slabs when viewed from the top

(a) 1.8 (b) 2 (c) 1.5 (d) 2.5

Solution: (c) Apparent depth of coin as seen from top $= \frac{x}{\mu_1} + \frac{x}{\mu_2} = x$



$\Rightarrow \frac{1}{\mu_1} + \frac{1}{\mu_2} = 1 \quad \Rightarrow \frac{1}{3} + \frac{1}{\mu} = 1 \quad \Rightarrow \mu = 1.5$

Example: 10 A coin is kept at bottom of an empty beaker. A travelling microscope is focussed on the coin from top, now water is poured in beaker up to a height of 10 *cm*. By what distance and in which direction should the microscope be moved to bring the coin again in focus

(a) 10 *cm* up ward (b) 10 *cm* down ward (c) 2.5 *cm* up wards (d) 2.5 *cm* down wards

Solution: (c) When water is poured in the beaker. Coin appears to shift by a distance $d = \frac{h}{4} = \frac{10}{4} = 2.5 cm$

Hence to bring the coil again in focus, the microscope should be moved by 2.5 *cm* in upward direction.

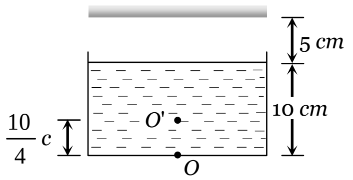
***Example*: 11** Consider the situation shown in figure. Water $\left(\mu_w = \frac{4}{3}\right)$ is filled in a breaker upto a height of 10 *cm*. A plane mirror fixed at a height of 5 *cm* from the surface of water. Distance of image from the mirror after reflection from it of an object *O* at the bottom of the beaker is

(a) 15 *cm* (b) 12.5 *cm* (c) 7.5 *cm* (d) 10 *cm*

Solution: (b) From figure it is clear that object appears to be raised by $\frac{10}{4} cm\,(2.5\, cm)$

Hence distance between mirror and $O' = 5 + 7.5 = 12.5\, cm$

So final image will be formed at 12.5 *cm* behind the plane mirror.



***Example*: 12** The wavelength of light in two liquids '*x*' and '*y*' is 3500 Å and 7000 Å, then the critical angle of *x* relative to *y* will be

(a) 60° (b) 45° (c) 30° (d) 15°

Solution: (c) $\sin C = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{3500}{7000} = \frac{1}{2} \Rightarrow C = 30^o$

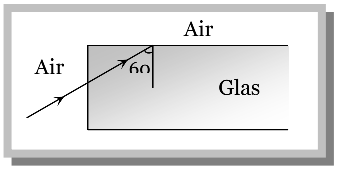
***Example*: 13** A light ray from air is incident (as shown in figure) at one end of a glass fiber (refractive index $\mu = 1.5$) making an incidence angle of 60° on the lateral surface, so that it undergoes a total internal reflection. How much time would it take to traverse the straight fiber of length 1 *km* **[Orissa JEE 2002]**

(a) 3.33 μ *sec*

(b) 6.67 μ *sec*

(c) 5.77 μ *sec*

(d) 3.85 μ *sec*



Solution: (d) When total internal reflection just takes place from lateral surface then $i = C$ *i.e.* $C = 60°$

From $\mu = \frac{1}{\sin C} \Rightarrow \mu = \frac{1}{\sin 60} = \frac{2}{\sqrt{3}}$

Hence time taken by light traverse some distance in medium $t = \frac{\mu x}{C}$

$$\Rightarrow t = \frac{\frac{2}{\sqrt{3}} \times (1 \times 10^3)}{3 \times 10^8} = 3.85\, \mu\, sec.$$

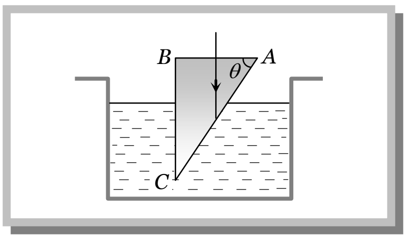
***Example*: 14** A glass prism of refractive index 1.5 is immersed in water $(\mu = 4/3)$. A light beam incident normally on the face *AB* is totally reflected to reach the face *BC* if

(a) $\sin\theta > 8/9$

(b) $2/3 < \sin\theta < 8/9$

(c) $\sin\theta \le 2/3$

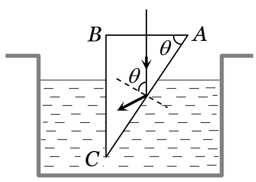
(d) $\cos\theta \ge 8/9$



Solution: (a) From figure it is clear that

Total internal reflection takes place at *AC*, only if $\theta > C$

$\Rightarrow \sin\theta > \sin C$ $\quad \Rightarrow \sin\theta > \frac{1}{{}_{\omega}\mu_g}$



$\Rightarrow \sin\theta > \dfrac{1}{9/8}$ $\qquad \Rightarrow \sin\theta > \dfrac{8}{9}$

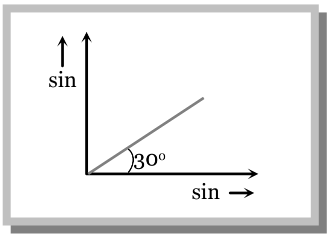
***Example*: 15** When light is incident on a medium at angle i and refracted into a second medium at an angle r, the graph of sin i *vs* sin r is as shown in the graph. From this, one can conclude that

(a) Velocity of light in the second medium is 1.73 times the velocity of light in the I medium

(b) Velocity of light in the I medium is 1.73 times the velocity in the II medium

(c) The critical angle for the two media is given by $\sin i_c = \dfrac{1}{\sqrt{3}}$

(d) $\sin i_c = \dfrac{1}{2}$



Solution: (b, c) From graph $\tan 30^o = \dfrac{\sin r}{\sin i} = \dfrac{1}{{}_1\mu_2} \Rightarrow {}_1\mu_2 = \sqrt{3} \Rightarrow \dfrac{\mu_2}{\mu_1} = \dfrac{v_1}{v_2} = 1.73 \Rightarrow v_1 = 1.75\, v_2$

Also from $\mu = \dfrac{1}{\sin C} \Rightarrow \sin C = \dfrac{1}{{}_{Rarer}\mu_{Denser}} \Rightarrow \sin C = \dfrac{1}{{}_1\mu_2} = \dfrac{1}{\sqrt{3}}$.

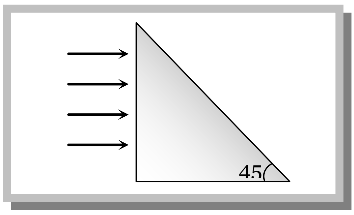
***Example*: 16** A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive indices of the material of the prism for the above red, green and blue wavelength are 1.39, 1.44 and 1.47 respectively. The prism will

(a) Separate part of red colour from the green and the blue colours

(b) Separate part of the blue colour from the red and green colours

(c) Separate all the colours from one another

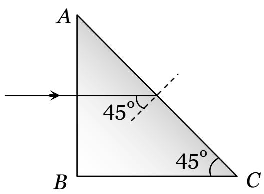
(d) Not separate even partially any colour from the other two colours



Solution: (a) At face AB, i = 0 so r = 0, *i.e.*, no refraction will take place. So light will be incident on face AC at an angle of incidence of 45°. The face AC will not transmit the light for which $i > \theta_C$, *i.e.*, $\sin i > \sin\theta_C$

or $\sin 45^o > (1/\mu)$ *i.e.*, $\mu > \sqrt{2}$ $(= 1.41)$

Now as $\mu_R < \mu$ while μ_G and $\mu_B > \mu$, so red will be transmitted through the face AC while green and blue will be reflected. So the prism will separate red colour from green and blue.



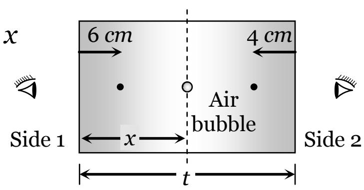
***Example*: 17** An air bubble in a glass slab $(\mu = 1.5)$ is 6 *cm* deep when viewed from one face and 4 *cm* deep when viewed from the opposite face. The thickness of the glass plate is

(a) 10 *cm* (b) 6.67 *cm* (c) 15 *cm* (d) None of these

Solution: (c) Let thickness of slab be t and distance of air bubble from one side is x

When viewed from side (1) : $1.5 = \dfrac{x}{6} \Rightarrow x = 9cm$

When viewed from side (2) : $1.5 = \dfrac{(t-x)}{4} \Rightarrow 1.5 = \dfrac{(t-9)}{4} \Rightarrow t = 15cm$



Tricky example: 1

One face of a rectangular glass plate 6 *cm* thick is silvered. An object held 8 *cm* in front of the first face, forms an image 12 *cm* behind the silvered face. The refractive index of the glass is

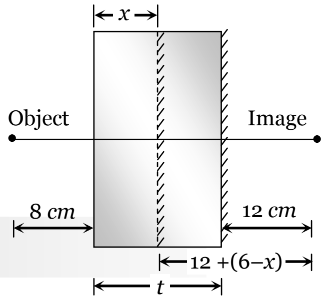
(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.6

Solution : (c) From figure thickness of glass plate $t = 6\ cm$.

Let x be the apparent position of the silvered surface.

According to property of plane mirror

$x + 8 = 12 + 6 - x \quad \Rightarrow \quad x = 5\ cm$



Tricky example: 2

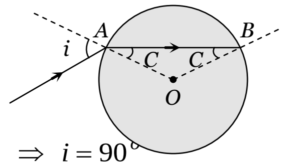
A ray of light is incident on a glass sphere of refractive index 3/2. What should be the angle of incidence so that the ray which enters the sphere doesn't come out of the sphere

(a) $\tan^{-1}\left(\frac{2}{3}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$ (c) 90° (d) $\cos^{-1}\left(\frac{1}{3}\right)$

Solution : (c) Ray doesn't come out from the sphere means TIR takes place.

Hence from figure $\angle ABO = \angle OAB = C$

$\therefore \quad \mu = \frac{1}{\sin C} \Rightarrow \sin C = \frac{1}{\mu} = \frac{2}{3}$



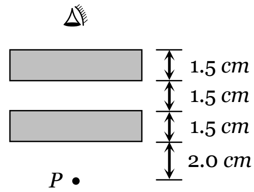
Applying Snell's Law at A $\quad \frac{\sin i}{\sin C} = \frac{3}{2} \Rightarrow \sin i = \frac{3}{2}\sin C = \frac{3}{2}\times\frac{2}{3} = 1 \quad \Rightarrow \quad i = 90^o$

Tricky example: 3

The image of point P when viewed from top of the slabs will be

(a) 2.0 *cm* above P (b) 1.5 *cm* above P (c) 2.0 *cm* below P (d) 1 *cm* above P

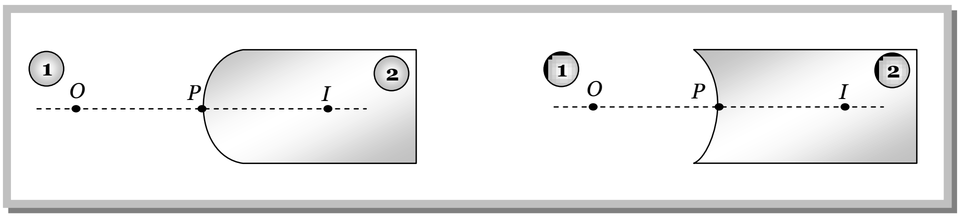
Solution: (d) The two slabs will shift the image a distance



$$d = 2\left(1 - \frac{1}{\mu}\right)t = 2\left(1 - \frac{1}{1.5}\right)(1.5) = 1\ cm$$

Therefore, final image will be 1 *cm* above point P.

Refraction From Curved Surface



μ_1 = Refractive index of the medium from which light rays are coming (from object).

μ_2 = Refractive index of the medium in which light rays are entering.

u = Distance of object, v = Distance of image, R = Radius of curvature

Refraction formula : $\frac{\mu_2 - \mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$ (use sign convention while solving the problem)

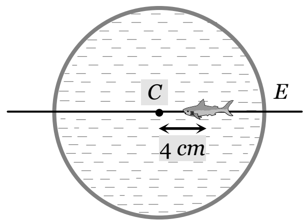
Note : ❑ Real image forms on the side of a refracting surface that is opposite to the object, and virtual image forms on the same side as the object.

❑ Lateral (Transverse) magnification $m = \frac{I}{O} = \frac{\mu_1 v}{\mu_2 u}$.

Specific Example

In a thin spherical fish bowl of radius 10 *cm* filled with water of refractive index 4/3 there is a small fish at a distance of 4 *cm* from the centre *C* as shown in figure. Where will the image of fish appears, if seen from *E*

(a) 5.2 *cm* (b) 7.2 *cm* (c) 4.2 *cm* (d) 3.2 *cm*



Solution : (a) By using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

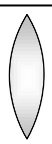
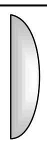
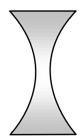
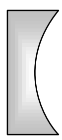
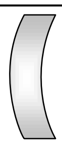
where $\mu_1 = \frac{4}{3}$, $\mu_2 = 1$, $u = -6\,cm$, $v = ?$

On putting values $v = -5.2\,cm$

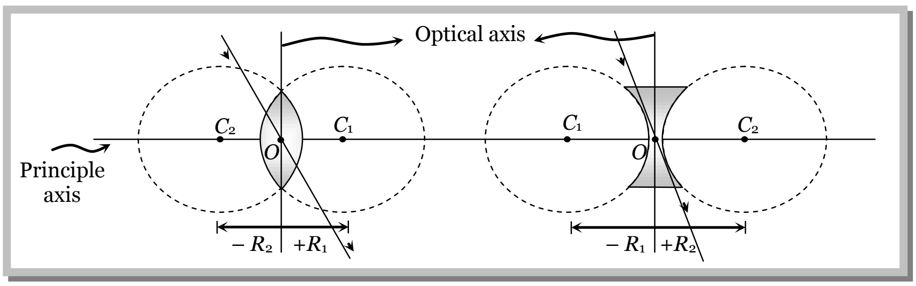
Lens

Lens is a transparent medium bounded by two refracting surfaces, such that at least one surface is spherical.

(1) Type of lenses

Convex lens (Converges the light rays)			Concave lens (Diverges the light rays)		
					
Double convex	Plano convex	Concavo convex	Double concave	Plane concave	Convexo concave
Thick at middle			Thin at middle		
It forms real and virtual images both			It forms only virtual images		

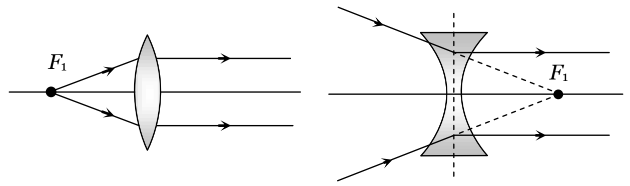
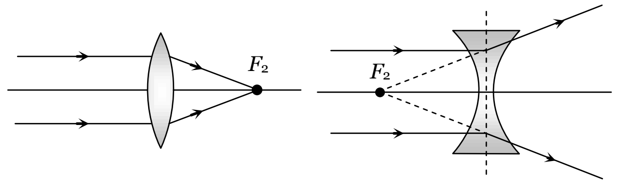
(2) Some definitions



C_1, C_2 – Centre of curvature, R_1, R_2 – Radii of curvature

(i) **Optical centre (O) :** A point for a given lens through which light ray passes undeviated (Light ray passes undeviated through optical centre).

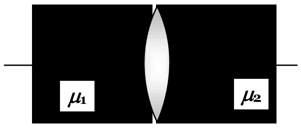
(ii) **Principle focus**

First principle focus	**Second principle focus**
	

Note : ❑ Second principle focus is the principle focus of the lens.

❑ When medium on two sides of lens is same then $|F_1| = |F_2|$.

❑ If medium on two sides of lens are not same then the ratio of two focal lengths $\frac{f_1}{f_2} = \frac{\mu_1}{\mu_2}$



(iii) **Focal length (*f*) :** Distance of second principle focus from optical centre is called focal length

$$f_{\text{convex}} \to \text{positive}, \quad f_{\text{concave}} \to \text{negative}, \quad f_{\text{plane}} \to \infty$$

(iv) **Aperture :** Effective diameter of light transmitting area is called aperture. Intensity of image $\propto (\text{Aperture})^2$

(v) **Power of lens (*P*) :** Means the ability of a lens to converge the light rays. Unit of power is Diopter (*D*).

$$P = \frac{1}{f(m)} = \frac{100}{f(cm)}; \quad P_{\text{convex}} \to \text{positive}, \quad P_{\text{concave}} \to \text{negative}, \quad P_{\text{plane}} \to \text{zero}.$$

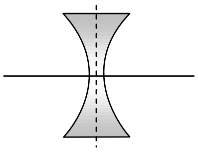
Note : ❑ Thick lens Thin lens

$P\uparrow f\downarrow R\downarrow$ $P\downarrow f\uparrow R\uparrow$

(2) **Image formation by lens**

Lens	Location of the object	Location of the image	Nature of image		
			Magnification	Real / virtual	Erect / inverted
Convex	At infinity i.e. $u = \infty$	At focus i.e. $v = f$	$m < 1$ diminished	Real	Inverted
	Away from 2*f* i.e. $(u > 2f)$	Between *f* and 2*f* i.e. $f < v < 2f$	$m < 1$ diminished	Real	Inverted

2f f f 2f

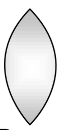
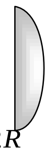
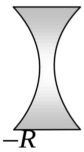
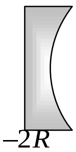
	At $2f$ or $(u = 2f)$	At $2f$ *i.e.* $(v = 2f)$	$m = 1$ same size	Real	Inverted
	Between f and $2f$ *i.e.* $f < u < 2f$	Away from $2f$ *i.e.* $(v > 2f)$	$m > 1$ magnified	Real	Inverted
	At focus *i.e.* $u = f$	At infinity *i.e.* $v = \infty$	$m = \infty$ magnified	Real	Inverted
	Between optical centre and focus, $u < f$	At a distance greater than that of object $v > u$	$m > 1$ magnified	Virtual	Erect
Concave 	At infinity *i.e.* $u = \infty$	At focus *i.e.* $v = f$	$m < 1$ diminished	Virtual	Erect
	Anywhere between infinity and optical centre	Between optical centre and focus	$m < 1$ diminished	Virtual	Erect

Note : ❑ Minimum distance between an object and it's real image formed by a convex lens is $4f$.

❑ Maximum image distance for concave lens is it's focal length.

(4) **Lens maker's formula**

The relation between f, μ, R_1 and R_2 is known as lens maker's formula and it is $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Equiconvex lens	**Plano convex lens**	**Equi concave lens**	**Plano concave lens**
$R_1 = R$ and $R_2 = -R$ $f = \frac{R}{2(\mu - 1)}$  for $\mu = 1.5$, $f = R$	$R_1 = \infty$, $R_2 = -R$ $f = \frac{R}{(\mu - 1)}$  for $\mu = 1.5$, $f = 2R$	$R_1 = -R$, $R_2 = +R$ $f = -\frac{R}{2(\mu - 1)}$  for $\mu = 1.5$ $f = -R$	$R_1 = \infty$, $R_2 = R$ $f = \frac{R}{2(\mu - 1)}$  for $\mu = 1.5$, $f = -2R$

(5) **Lens in a liquid**

Focal length of a lens in a liquid (f_l) can be determined by the following formula

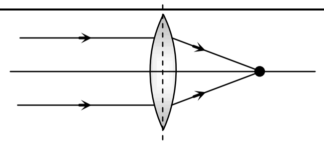
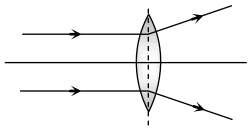
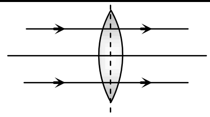
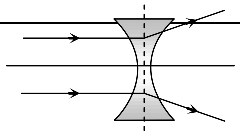
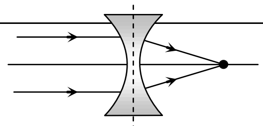
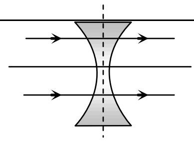
$$\frac{f_l}{f_a} = \frac{({}_a\mu_g - 1)}{({}_l\mu_g - 1)}$$ (Lens is supposed to be made of glass).

Note : ❑ Focal length of a glass lens $(\mu = 1.5)$ is f in air then inside the water it's focal length is $4f$.

❑ In liquids focal length of lens increases (↑) and it's power decreases (↓).

(6) **Opposite behaviour of a lens**

In general refractive index of lens $(\mu_L) >$ refractive index of medium surrounding it (μ_M).

$\mu_L > \mu_M$	$\mu_L < \mu_M$	$\mu_L = \mu_M$
		
		

(7) **Lens formula and magnification of lens**

(i) Lens formula : $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$; (use sign convention)

(ii) Magnification : The ratio of the size of the image to the size of object is called magnification.

(a) Transverse magnification : $m=\frac{I}{O}=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f}$ (use sign convention while solving the problem)

(b) Longitudinal magnification : $m=\frac{I}{O}=\frac{v_2-v_1}{u_2-u_1}$. For very small object $m=\frac{dv}{du}=\left(\frac{v}{u}\right)^2=\left(\frac{f}{f+u}\right)^2=\left(\frac{f-v}{f}\right)^2$

(c) Areal magnification : $m_s=\frac{A_i}{A_o}=m^2=\left(\frac{f}{f+u}\right)^2$, ($A_i$ = Area of image, A_o = Area of object)

(8) **Relation between object and image speed**

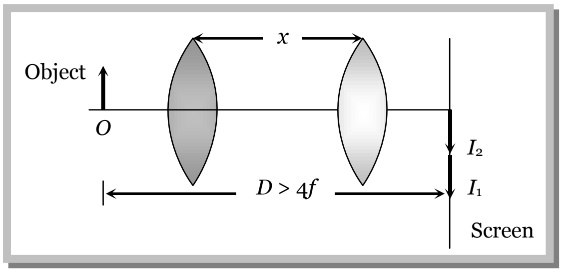
If an object move with constant speed (V_o) towards a convex lens from infinity to focus, the image will move slower in the beginning and then faster. Also $V_i=\left(\frac{f}{f+u}\right)^2.V_o$

(9) **Focal length of convex lens by displacement method**

(i) For two different positions of lens two images (I_1 and I_2) of an object is formed at the same location.

(ii) Focal length of the lens $f=\frac{D^2-x^2}{4D}=\frac{x}{m_1-m_2}$

where $m_1=\frac{I_1}{O}$ and $m_2=\frac{I_2}{O}$

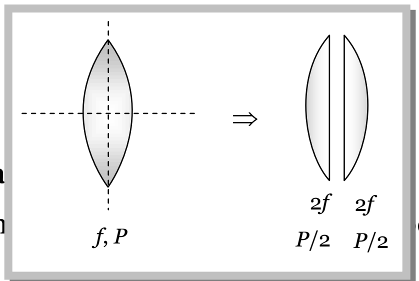


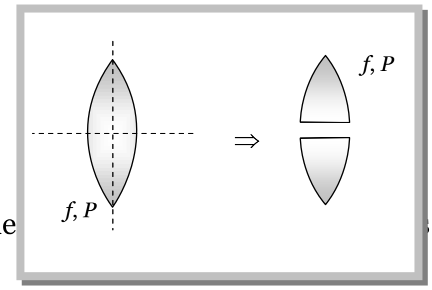
(iii) Size of object $O=\sqrt{I_1.I_2}$

(10) **Cutting of lens**

(i) A symmetric lens is cut along optical axis in two equal parts. Intensity of image formed by each part will be same as that of complete lens.

(ii) A symmetric lens is cut along principle axis in two equal parts. Intensity of image formed by each part will be less compared as that of complete lens.(aperture of each part is $\frac{1}{\sqrt{2}}$ times that of complete lens)





(11) **Combina**

(i) For a systen cal le follows :

$P = P_1 + P_2 + P_3 ....$, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...$,

$m = m_1 \times m_2 \times m_3 \times ....$ ..

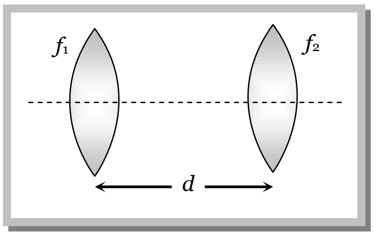
(ii) In case when two thin lens are in contact : Combination will behave as a lens, which have more power or lesser focal length.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow F = \frac{f_1 f_2}{f_1 + f_2} \quad \text{and} \quad P = P_1 + P_2$$

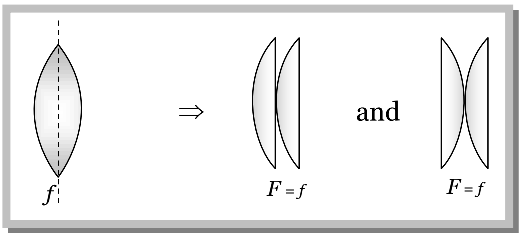
(iii) If two lens of equal focal length but of opposite nature are in contact then combination will behave as a plane glass plate and $F_{\text{combination}} = \infty$

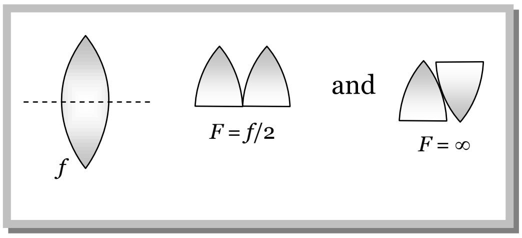
(iv) When two lenses are placed co-axially at a distance d from each other then equivalent focal length (F).

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad \text{and} \quad P = P_1 + P_2 - dP_1 P_2$$



(v) Combination of parts of a lens :



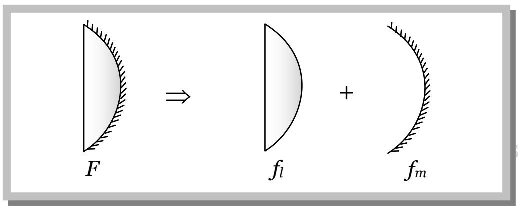


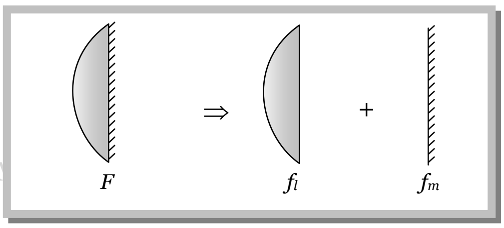
(12) **Silvering of lens**

On silvering the surface of the lens it behaves as a mirror. The focal length of the silvered lens is $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$ where f_l = focal length of lens from which refraction takes place (twice)

f_m = focal length of mirror from which reflection takes place.

(i) Plano convex is silvered



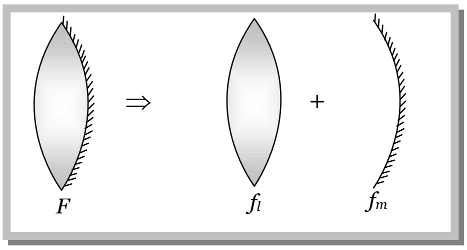


$f_m = \frac{R}{2}$, $f_l = \frac{R}{(\mu-1)}$ so $F = \frac{R}{2\mu}$ $\qquad$ $f_m = \infty$, $f_l = \frac{R}{(\mu-1)}$ so $F = \frac{R}{2(\mu-1)}$

(ii) Double convex lens is silvered

Since $f_l = \frac{R}{2(\mu-1)}$, $f_m = \frac{R}{2}$

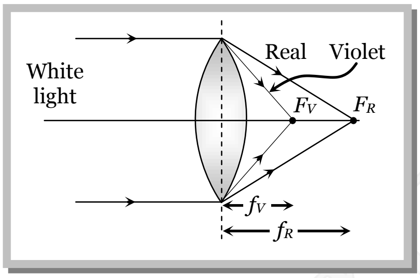
So $F = \frac{R}{2(2\mu-1)}$



Note : ❑ Similar results can be obtained for concave lenses.

(13) **Defects in lens**

(i) **Chromatic aberration :** Image of a white object is coloured and blurred because μ (hence f) of lens is different for different colours. This defect is called chromatic aberration.



$\mu_V > \mu_R$ so $f_R > f_V$

Mathematically chromatic aberration $= f_R - f_V = \omega f_y$

ω = Dispersion power of lens.

f_y = Focal length for mean colour $= \sqrt{f_R f_V}$

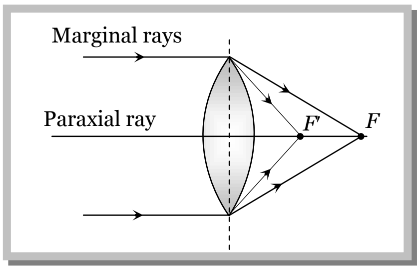
Removal : To remove this defect *i.e.* for Achromatism we use two or more lenses in contact in place of single lens.

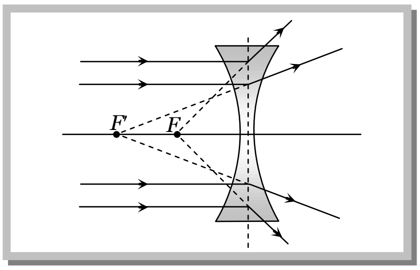
Mathematically condition of Achromatism is : $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ or $\omega_1 f_2 = -\omega_2 f_1$

Note : ❑ Component lenses of an achromatic doublet cemented by canada blasam because it is transparent and has a refractive index almost equal to the refractive of the glass.

(ii) **Spherical aberration :** Inability of a lens to form the point image of a point object on the axis is called Spherical aberration.

In this defect all the rays passing through a lens are not focussed at a single point and the image of a point object on the axis is blurred.





Removal : A simple method to reduce spherical aberration is to use a stop before and infront of the lens. (but this method reduces the intensity of the image as most of the light is cut off). Also by using plano-convex lens, using two lenses separated by distance $d = F - F'$, using crossed lens.

Note : ❑ Marginal rays : The rays farthest from the principal axis.

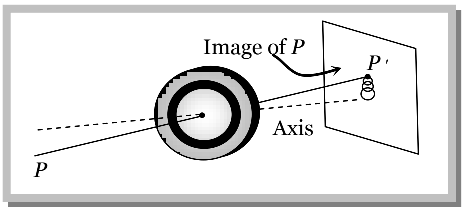
Paraxial rays : The rays close to the principal axis.

❑ Spherical aberration can be reduced by either stopping paraxial rays or marginal rays, which can be done by using a circular annular mask over the lens.

❑ Parabolic mirrors are free from spherical aberration.

(iii) **Coma :** When the point object is placed away from the principle axis and the image is received on a screen perpendicular to the axis, the shape of the image is like a comet. This defect is called Coma.

It refers to spreading of a point object in a plane ⊥ to principle axis.

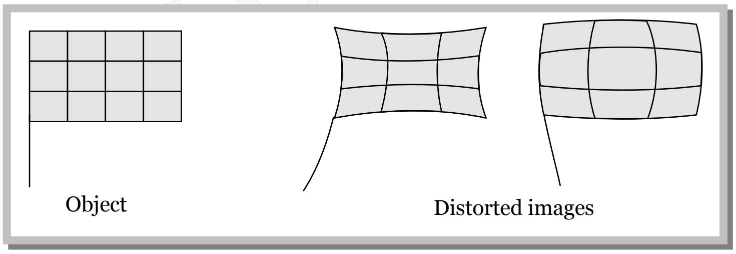


Removal : It can be reduced by properly designing radii of curvature of the lens surfaces. It can also be reduced by appropriate stops placed at appropriate distances from the lens.

(iv) **Curvature :** For a point object placed off the axis, the image is spread both along and perpendicular to the principal axis. The best image is, in general, obtained not on a plane but on a curved surface. This defect is known as Curvature.

Removal : Astigmatism or the curvature may be reduced by using proper stops placed at proper locations along the axis.

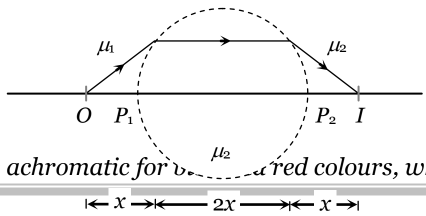
(v) **Distortion :** When extended objects are imaged, different portions of the object are in general at different distances from the axis. The magnification is not the same for all portions of the extended object. As a result a line object is not imaged into a line but into a curve.



(vi) **Astigmatism :** The spreading of image (of a point object placed away from the principal axis) along the principal axis is called Astigmatism.

Concepts

☞ *If a sphere of radius R made of material of refractive index μ_2 is placed in a medium of refractive index μ_1, Then if the object is placed at a distance $\left(\frac{\mu_1}{\mu_2-\mu_1}\right)R$ from the pole, the real image formed is equidistant from the sphere.*



☞ *The lens doublets used in telescope are achromatic for blue and red colours, while these used in camera are achromatic for*

μ_1

violet and green colours. The reason for this is that our eye is most sensitive between blue and red colours, while the photographic plates are most sensitive between violet and green colours.

☞ ***Position of optical centre***

Equiconvex and equiconcave	*Exactly at centre of lens*
Convexo-concave and concavo-convex	*Outside the glass position*
Plano convex and plano concave	*On the pole of curved surface*

☞ ***Composite lens :*** *If a lens is made of several materials then*

Number of images formed = Number of materials used

Here no. of images = 5

Example

Example: 18 A thin lens focal length f_1 and its aperture has diameter d. It forms an image of intensity I. Now the central part of the aperture upto diameter $d/2$ is blocked by an opaque paper. The focal length and image intensity will change to

(a) $\frac{f}{2}$ and $\frac{I}{2}$ (b) f and $\frac{I}{4}$ (c) $\frac{3f}{4}$ and $\frac{I}{2}$ (d) f and $\frac{3I}{4}$

Solution: (d) Centre part of the aperture up to diameter $\frac{d}{2}$ is blocked *i.e.* $\frac{1}{4}$*th* area is blocked $\left(A=\frac{\pi d^2}{4}\right)$. Hence remaining area $A' = \frac{3}{4}A$. Also, we know that intensity $\propto$ Area $\Rightarrow$ $\frac{I'}{I}=\frac{A'}{A}=\frac{3}{4} \Rightarrow I'=\frac{3}{4}I$.

Focal length doesn't depend upon aperture.

Example: 19 The power of a thin convex lens $({}_a\mu_g = 1.5)$ is + 5.0 D. When it is placed in a liquid of refractive index ${}_a\mu_l$, then it behaves as a concave lens of local length 100 *cm*. The refractive index of the liquid ${}_a\mu_l$ will be

(a) 5 / 3 (b) 4 / 3 (c) $\sqrt{3}$ (d) 5 / 4

Solution: (a) By using $\frac{f_l}{f_a}=\frac{{}_a\mu_g-1}{{}_l\mu_g-1}$; where ${}_l\mu_g=\frac{\mu_g}{\mu_l}=\frac{1.5}{\mu_l}$ and $f_a=\frac{1}{P}=\frac{1}{5}m=20\,cm$

$$\Rightarrow \frac{-100}{20}=\frac{1.5-1}{\frac{1.5}{\mu_l}-1} \Rightarrow \mu_l = 5/3$$

Example: 20 A double convex lens made of a material of refractive index 1.5 and having a focal length of 10 *cm* is immersed in liquid of refractive index 3.0. The lens will behave as **[NCERT 1973]**

(a) Diverging lens of focal length 10 *cm* (b) Diverging lens of focal length 10 / 3 *cm*

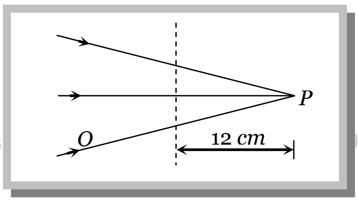
(c) Converging lens of focal length 10 / 3 *cm* (d) Converging lens of focal length 30 *cm*

Solution: (a) By using $\frac{f_l}{f_a}=\frac{{}_a\mu_g-1}{{}_l\mu_g-1} \Rightarrow \frac{f_l}{+10}=\frac{1.5-1}{\frac{1.5}{3}-1} \Rightarrow f_l=-10\,cm$ (*i.e.* diverging lens)

Example: 21 Figure given below shows a beam of light converging at point P. When a concave lens of focal length 16 *cm* is introduced in the path of the beam at a place O shown by dotted line, the beam converges at a distance x from the lens. The value x will be equal to

(a) 12 *cm*

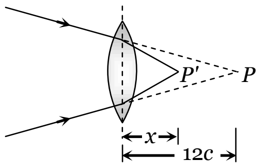
(b) 24 *cm*



(c) 36 *cm*

(d) 48 *cm*

Solution: (d) From the figure shown it is clear that



For lens : $u = 12\ cm$ and $v = x = ?$

By using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\Rightarrow \frac{1}{+16} = \frac{1}{x} - \frac{1}{+12}$ $\Rightarrow x = 48\ cm$.

***Example*: 22** A convex lens of focal length 40 *cm* is an contact with a concave lens of focal length 25 *cm*. The power of combination is

(a) – 1.5 *D* (b) – 6.5 *D* (c) + 6.5 *D* (d) + 6.67 *D*

Solution: (a) By using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ $\Rightarrow \frac{1}{F} = \frac{1}{+40} + \frac{1}{-25}$

$\Rightarrow F = -\frac{200}{3} cm$, hence $P = \frac{100}{f(cm)} = \frac{100}{-200/3} = -1.5\ D$

***Example*: 23** A combination of two thin lenses with focal lengths f_1 and f_2 respectively forms an image of distant object at distance 60 *cm* when lenses are in contact. The position of this image shifts by 30 *cm* towards the combination when two lenses are separated by 10 *cm*. The corresponding values of f_1 and f_2 are **[AIIMS 1995]**

(a) 30 *cm*, – 60 *cm* (b) 20 *cm*, – 30 *cm* (c) 15 *cm*, – 20 *cm* (d) 12 *cm*, – 15 *cm*

Solution: (b) Initially $F = 60\ cm$ (Focal length of combination)

Hence by using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{60}$ $\Rightarrow \frac{f_1 f_2}{f_1 + f_2}$ ......(i)

Finally by using $\frac{1}{F'} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ where $F' = 30\ cm$ and $d = 10\ cm$ $\Rightarrow$

$\frac{1}{30} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{10}{f_1 f_2}$ ......(ii)

From equations (i) and (ii) $f_1 f_2 = -600$.

From equation (i) $f_1 + f_2 = -10$ .....(iii)

Also,difference of focal lengths can written as $f_1 - f_2 = \sqrt{(f_1 + f_2)^2 - 4 f_1 f_2}$ $\Rightarrow f_1 - f_2 = 50$.(iv)

From (iii) × (iv) $f_1 = 20$ and $f_2 = -30$

***Example*: 24** A thin double convex lens has radii of curvature each of magnitude 40 *cm* and is made of glass with refractive index 1.65. Its focal length is nearly

(a) 20 *cm* (b) 31 *cm* (c) 35 *cm* (d) 50 *cm*

Solution: (b) By using $f = \frac{R}{2(\mu - 1)}$ $\Rightarrow$ $f = \frac{40}{2(1.65 - 1)} = 30.7\ cm \approx 31\ cm$.

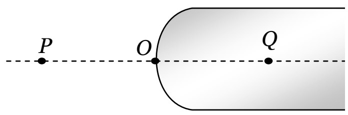
***Example*: 25** A spherical surface of radius of curvature *R* separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object *P* placed in air is found to have a real image *Q* in the glass. The line *PQ* cuts the surface at a point *O* and $PO = OQ$. The distance *PO* is equal to

[MP PMT 1994; Haryana CEE 1996]

(a) $5R$ (b) $3R$ (c) $2R$ (d) $1.5R$

Solution: (a) By using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Where $\mu_1 = 1$, $\mu_2 = 1.5$, $u = -OP$, $v = OQ$



Hence $\frac{1.5}{OQ} - \frac{1}{-OP} = \frac{1.5-1}{(+R)} \Rightarrow \frac{1.5}{OP} + \frac{1}{OP} = \frac{0.5}{R}$

$\Rightarrow OP = 5R$

***Example*: 26** The distance between an object and the screen is 100 *cm*. A lens produces an image on the screen when placed at either of the positions 40 *cm* apart. The power of the lens is

(a) $3D$ (b) $5D$ (c) $7D$ (d) $9D$

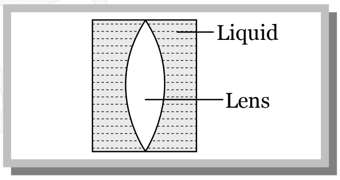
Solution: (b) By using $f = \frac{D^2 - x^2}{4D} \Rightarrow f = \frac{100^2 - 40^2}{4 \times 100} = 21\,cm$

Hence power $P = \frac{100}{F(cm)} = \frac{100}{21} \approx +5D$

***Example*: 27** Shown in figure here is a convergent lens placed inside a cell filled with a liquid. The lens has focal length +20 *cm* when in air and its material has refractive index 1.50. If the liquid has refractive index 1.60, the focal length of the system is

(a) + 80 *cm* (b) – 80 *cm*

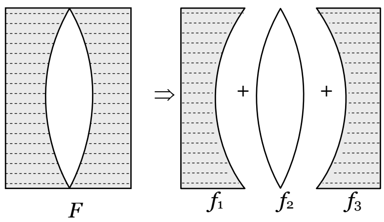
(c) – 24 *cm* (d) – 100 *cm*



Solution: (d) Here $\frac{1}{f_1} = (1.6-1)\left(\frac{1}{\infty} - \frac{1}{20}\right) = \frac{-3}{100}$ ....(i)

$\frac{1}{f_2} = (1.5-1)\left(\frac{1}{20} - \frac{1}{-20}\right) = \frac{1}{20}$ ....(ii)

$\frac{1}{f_3} = (1.6-1)\left(\frac{1}{-20} - \frac{1}{\infty}\right) = \frac{-3}{100}$ ....(iii)



By using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \Rightarrow \frac{1}{F} = \frac{-3}{100} + \frac{1}{20} - \frac{3}{100} \Rightarrow F = -100\,cm$

***Example*: 28** A concave lens of focal length 20 *cm* placed in contact with a plane mirror acts as a **RA 1998]**

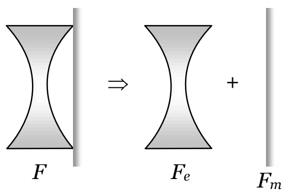
(a) Convex mirror of focal length 10 *cm* (b) Concave mirror of focal length 40 *cm*

(c) Concave mirror of focal length 60 *cm* (d) Concave mirror of focal length 10 *cm*

Solution: (a) By using $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$

Since $f_m = \infty \Rightarrow F = \frac{f_l}{2} = \frac{20}{2} = 10\,cm$

(After silvering concave lens behave as convex mirror)



***Example*: 29** A candle placed 25 *cm* from a lens, forms an image on a screen placed 75 *cm* on the other end of the lens. The focal length and type of the lens should be

(a) + 18.75 *cm* and convex lens (b) – 18.75 *cm* and concave lens

(c) + 20.25 *cm* and convex lens (d) – 20.25 *cm* and concave lens

Solution: (a) In concave lens, image is always formed on the same side of the object. Hence the given lens is a convex lens for which $u = -25\,cm$, $v = 75\,cm$.

By using $\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{f}=\frac{1}{(+75)}-\frac{1}{(-25)} \Rightarrow f=+18.75\ cm.$

Example: 30 A convex lens forms a real image of an object for its two different positions on a screen. If height of the image in both the cases be 8 *cm* and 2 *cm*, then height of the object is **[KCET (Engg./Med.) 2**

(a) 16 *cm* (b) 8 *cm* (c) 4 *cm* (d) 2 *cm*

Solution: (c) By using $O=\sqrt{I_1 I_2} \Rightarrow O=\sqrt{8\times 2}=4\ cm$

Example: 31 A convex lens produces a real image *m* times the size of the object. What will be the distance of the object from the lens **[JIPMER 2002]**

(a) $\left(\frac{m+1}{m}\right)f$ (b) $(m-1)f$ (c) $\left(\frac{m-1}{m}\right)f$ (d) $\frac{m+1}{f}$

Solution: (a) By using $m=\frac{f}{f+u}$ here $-m=\frac{(+f)}{(+f)+u} \Rightarrow -\frac{1}{m}=\frac{f+u}{f}=1+\frac{u}{f} \Rightarrow u=-\left(\frac{m+1}{m}\right).f$

Example: 32 An air bubble in a glass sphere having 4 *cm* diameter appears 1 *cm* from surface nearest to eye when looked along diameter. If ${}_a\mu_g=1.5$, the distance of bubble from refracting surface is **[CPMT 2002]**

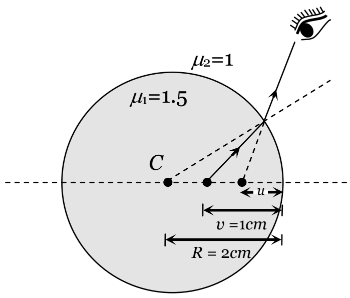
(a) 1.2 *cm* (b) 3.2 *cm* (c) 2.8 *cm* (d) 1.6 *cm*

Solution: (a) By using

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

where $u=?,\ v=-1\ cm,\ \mu_1=1.5,\ \mu_2=1,\ R=-2\ cm.$

$$\frac{1}{-1}-\frac{1.5}{u}=\frac{1-1.5}{(-2)} \Rightarrow u=-\frac{6}{5}=-1.2\ cm.$$

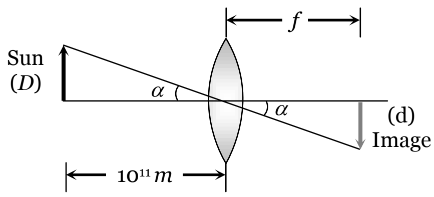


Example: 33 The sun's diameter is $1.4\times 10^{9}\,m$ and its distance from the earth is $10^{11}\,m$. The diameter of its image, formed by a convex lens of focal length 2*m* will be

(a) 0.7 *cm* (b) 1.4 *cm* (c) 2.8 *cm* (d) Zero (*i.e.* point image)

Solution: (c) From figure

$$\frac{D}{d}=\frac{10^{11}}{2} \Rightarrow d=\frac{2\times 1.4\times 10^{9}}{10^{11}}=2.8\ cm.$$



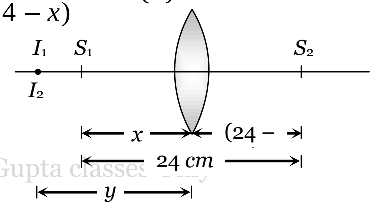
Example: 34 Two point light sources are 24 *cm* apart. Where should a convex lens of focal length 9 *cm* be put in between them from one source so that the images of both the sources are formed at the same place

(a) 6 *cm* (b) 9 *cm* (c) 12 *cm* (d) 15 *cm*

Solution: (a) The given condition will be satisfied only if one source (S_1) placed on one side such that $u<f$ (*i.e.* it lies under the focus). The other source (S_2) is placed on the other side of the lens such that $u>f$ (*i.e.* it lies beyond the focus).

If S_1 is the object for lens then $\frac{1}{f}=\frac{1}{-y}-\frac{1}{-x} \Rightarrow \frac{1}{y}=\frac{1}{x}-\frac{1}{f}$ ....(i)

If S_2 is the object for lens then $\frac{1}{f}=\frac{1}{+y}-\frac{1}{-(24-x)} \Rightarrow \frac{1}{y}=\frac{1}{f}-\frac{1}{(24-x)}$ ....(ii)



From equation (i) and (ii)

$$\frac{1}{x}-\frac{1}{f}=\frac{1}{f}-\frac{1}{(24-x)} \Rightarrow \frac{1}{x}+\frac{1}{(24-x)}=\frac{2}{f}=\frac{2}{9} \Rightarrow x^2-24x+108=0$$

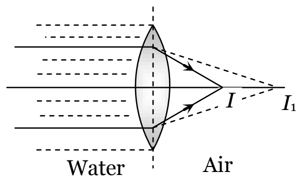
On solving the equation $x = 18\ cm$, $6\ cm$

Example: 35 There is an equiconvex glass lens with radius of each face as R and $_a\mu_g = 3/2$ and $_a\mu_w = 4/3$. If there is water in object space and air in image space, then the focal length is

(a) $2R$ (b) R (c) $3R/2$ (d) R^2

Solution: (c) Consider the refraction of the first surface *i.e.* refraction from rarer medium to denser medium

$$\frac{\mu_2-\mu_1}{R}=\frac{\mu_1}{-u}+\frac{\mu_2}{v_1} \Rightarrow \frac{\left(\frac{3}{2}\right)-\left(\frac{4}{3}\right)}{R}=\frac{\frac{4}{3}}{\infty}+\frac{\frac{3}{2}}{v_1} \Rightarrow v_1 = 9R$$



Now consider the refraction at the second surface of the lens *i.e.* refraction from denser medium to rarer medium

$$\frac{1-\frac{3}{2}}{-R}=-\frac{\frac{3}{2}}{9R}+\frac{1}{v_2} \Rightarrow v_2=\left(\frac{3}{2}\right)R$$

The image will be formed at a distance do $\frac{3}{2}R$. This is equal to the focal length of the lens.

Tricky example: 4

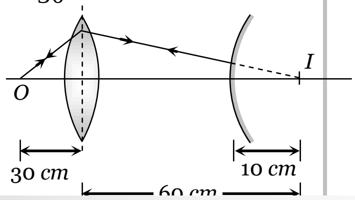
A luminous object is placed at a distance of 30 *cm* from the convex lens of focal length 20 *cm*. On the other side of the lens. At what distance from the lens a convex mirror of radius of curvature 10 *cm* be placed in order to have an upright image of the object coincident with it

[CBSE PMT 1998; JIPMER 2001, 2002]

(a) 12 *cm* (b) 30 *cm* (c) 50 *cm* (d) 60 *cm*

Solution : (c) For lens $u = 30\ cm$, $f = 20\ cm$, hence by using $\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{+20}=\frac{1}{v}-\frac{1}{-30} \Rightarrow v = 60\ cm$

The final image will coincide the object, if light ray falls normally on convex mirror as shown. From figure it is seen clear that reparation between lens and mirror is 60 – 10 = 50 *cm*.



Tricky example: 5

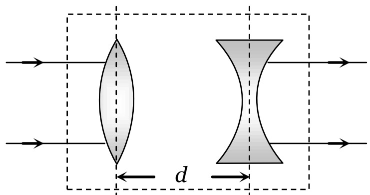
A convex lens of local length 30 *cm* and a concave lens of 10 *cm* focal length are placed so as to have the same axis. If a parallel beam of light falling on convex lens leaves concave lens as a parallel beam, then the distance between two lenses will be

(a) 40 *cm* (b) 30 *cm* (c) 20 *cm* (d) 10 *cm*

Solution : (c) According to figure the combination behaves as plane glass plate (*i.e.*, $F = \infty$)

Hence by using $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}$

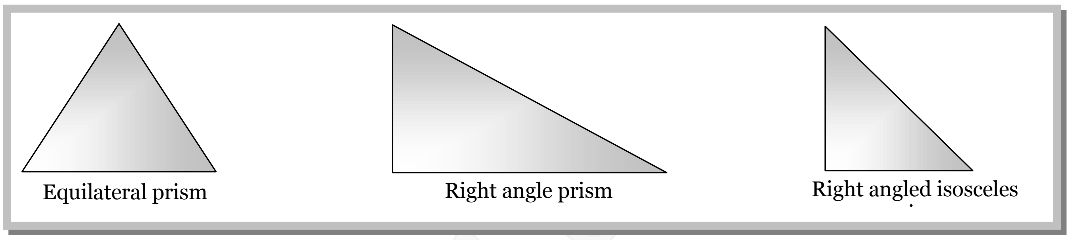
$$\Rightarrow \frac{1}{\infty}=\frac{1}{+30}+\frac{1}{-10}-\frac{d}{(30)(-10)} \Rightarrow d = 20\ cm$$



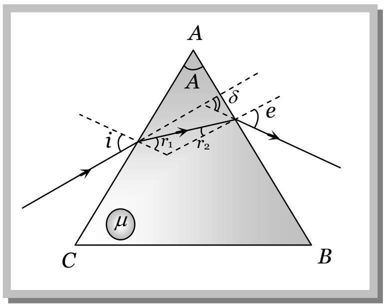
Prism

Prism is a transparent medium bounded by refracting surfaces, such that the incident surface (on which light ray is incidenting) and emergent surface (from which light rays emerges) are plane and non parallel.

Commonly used prism :



(1) **Refraction through a prism**



$A = r_1 + r_2$ and $i + e = A + \delta$

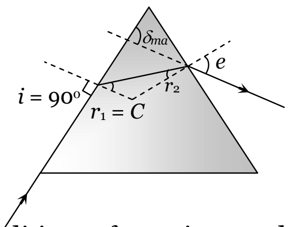
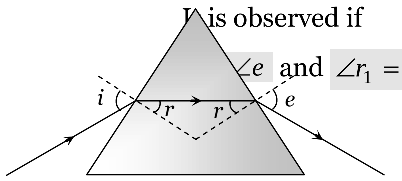
For surface AC $\mu = \dfrac{\sin i}{\sin r_1}$;

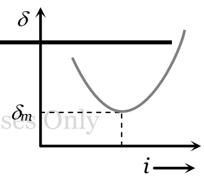
i – Angle of incidence, e – Angle of emergence, A – Angle of prism or refracting angle of prism, r_1 and r_2 – Angle of refraction, δ – Angle of

For surface AB $\mu = \dfrac{\sin r_2}{\sin e}$

(2) **Deviation through a prism**

For thin prism $\delta = (\mu - 1)A$. Also deviation is different for different colour light *e.g.* $\mu_R < \mu_V$ so $\delta_R < \delta_V$. And $\mu_{\text{Flint}} > \mu_{\text{Crown}}$ so $\delta_F > \delta_C$

Maximum deviation	**Minimum deviation**
 In this condition of maximum deviation $\angle i = 90^o$, $r_1 = C$, $r_2 = A - C$ and from Snell's law on emergent surface	 It is observed if $\angle i = \angle e$ and $\angle r_1 = \angle r_2 = r$ (i) Refracted ray inside the prism is parallel to the base of the prism

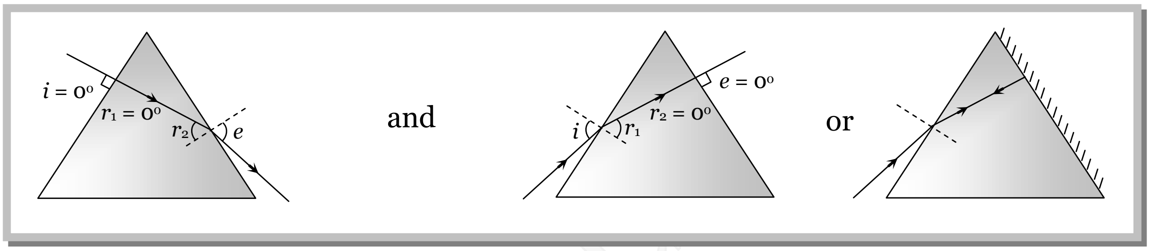


$e = \sin^{-1}\left[\dfrac{\sin(A-C)}{\sin C}\right]$	(ii) $r = \dfrac{A}{2}$ and $i = \dfrac{A+\delta_m}{2}$ (iii) $\mu = \dfrac{\sin i}{\sin A/2}$ or $\mu = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin A/2}$

Note : ❑ If $\delta_m = A$ then $\mu = 2\cos A/2$

(3) **Normal incidence on a prism**

If light ray incident normally on any surface of prism as shown

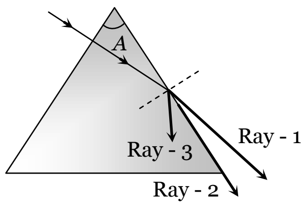


In any of the above case use $\mu = \dfrac{\sin i}{\sin A}$ and $\delta = i - A$

(4) **Grazing emergence and TIR through a prism**

When a light ray falls on one surface of prism, it is not necessary that it will exit out from the prism. It may or may not be exit out as shown below

Normal incidence



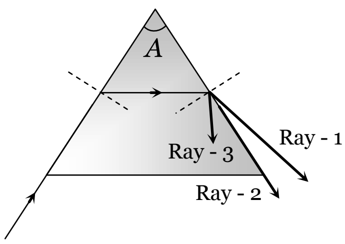
Ray –1 : General emergence
$A < C$ and
$\mu < \operatorname{cosec} A$

Ray – 2: Grazing emergence
$A = C$ and
$\mu = \operatorname{cosec} A$

Ray – 3: TIR
$A > C$ and
$\mu > \operatorname{cosec} A$

A = angle of prism and C = Critical angle for the material of the prism

Grazing incidence



Ray –1 : General emergence
$A < 2C$ and
$\mu < \operatorname{cosec}(A/2)$

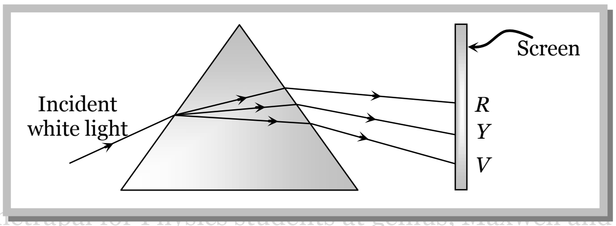
Ray – 2: Grazing emergence
$A = 2C$ and
$\mu = \operatorname{cosec}(A/2)$

Ray – 3: TIR
$A > 2C$ and
$\mu > \operatorname{cosec}(A/2)$

Note : ❑ For the condition of grazing emergence. Minimum angle of incidence $i_{min} = \sin^{-1}\left[\sqrt{\mu^2 - 1}\,\sin A - \cos A\right]$.

(5) **Dispersion through a prism**

The splitting of white light into it's constituent colours is called dispersion of light.



(i) Angular dispersion (θ) : Angular separation between extreme colours *i.e.* $\boldsymbol{\theta = \delta_V - \delta_R = (\mu_V - \mu_R)A}$. It depends upon μ and A.

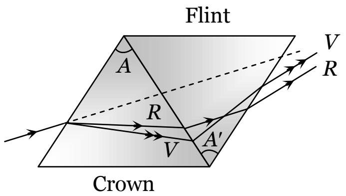
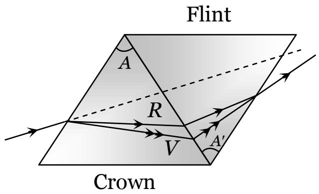
(ii) Dispersive power (ω) : $\omega = \dfrac{\theta}{\delta_y} = \dfrac{\mu_V - \mu_R}{\mu_y - 1}$ wher e $\left\{\mu_y = \dfrac{\mu_V + \mu_R}{2}\right\}$

$\Rightarrow$ It depends only upon the material of the prism *i.e.* μ and it doesn't depends upon angle of prism A

Note : ❑ Remember $\omega_{\text{Flint}} > \omega_{\text{Crown}}$.

(6) **Combination of prisms**

Two prisms (made of crown and flint material) are combined to get either dispersion only or deviation only.

<table>
<tr><td>Dispersion without deviation (chromatic combination)

</td><td>Deviation without dispersion (Achromatic combination)

</td></tr>
<tr><td>(i) $\dfrac{A'}{A} = -\dfrac{(\mu_y - 1)}{(\mu'_y - 1)}$</td><td>(i) $\dfrac{A'}{A} = -\dfrac{(\mu_V - \mu_R)}{(\mu'_V - \mu'_R)}$</td></tr>
<tr><td>(ii) $\theta_{\text{net}} = \theta\left(1 - \dfrac{\omega'}{\omega}\right) = (\omega\delta - \omega'\delta')$</td><td>(ii) $\delta_{\text{net}} = \delta\left(1 - \dfrac{\omega}{\omega'}\right)$</td></tr>
</table>

Scattering of Light

Molecules of a medium after absorbing incoming light radiations, emits them in all direction. This phenomenon is called Scattering.

(1) **According to scientist Rayleigh :** Intensity of scattered light $\propto \dfrac{1}{\lambda^4}$

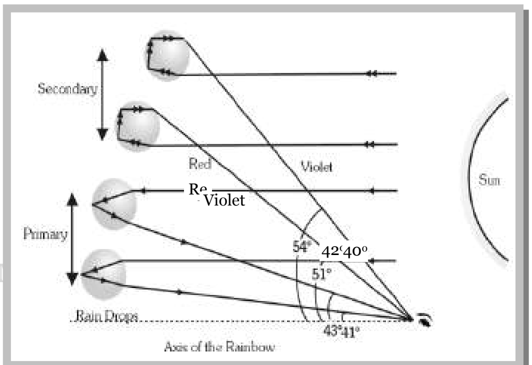
(2) **Some phenomenon based on scattering :** (i) Sky looks blue due to scattering.
(ii) At the time of sunrise or sunset it looks reddish. (iii) Danger signals are made from red.

(3) **Elastic scattering :** When the wavelength of radiation remains unchanged, the scattering is called elastic.

(4) **Inelastic scattering (Raman's effect) :** Under specific condition, light can also suffer inelastic scattering from molecules in which it's wavelength changes.

Rainbow

Rainbow is formed due to the dispersion of light suffering refraction and TIR in the droplets present in the atmosphere.



(1) **Primary rainbow :** (i) Two refraction and one TIR. (ii) Innermost arc is violet and outermost is red. (iii) Subtends an angle of 42° at the eye of the observer. (iv) More bright

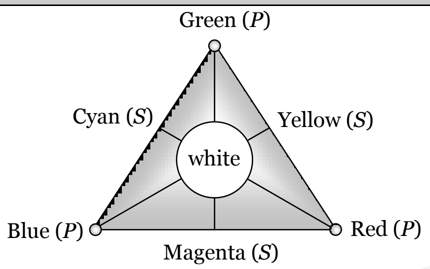
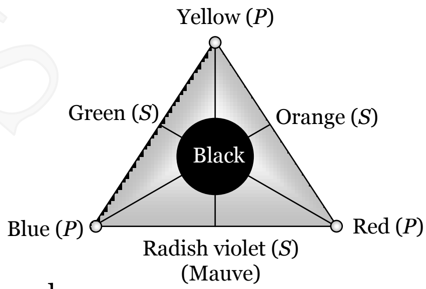
(2) **Secondary rainbow :** (i) Two refraction and two TIR. (ii) Innermost arc is red and outermost is violet.

(iii) It subtends an angle of 52.5° at the eye. (iv) Comparatively less bright.

Colours

Colour is defined as the sensation received by the eye (cone cells of the eye) due to light coming from object.

(1) **Types of colours**

Spectral colours	**Colours of pigment and dyes**
 	
(i) Complementary colours : Green and magenta Blue and yellow Red and cyan (ii) Combination : Green + red + blue = White Blue + yellow = White Red + cyan = White Green + magenta = White	(i) Complementary colours : yellow and mauve Red and green Blue and orange (ii) Combination : Yellow + red + blue = Black Blue + orange = Black Red + green = Black Yellow + mauve = Black

(2) **Colours of object :** The perception of a colour by eye depends on the nature of object and the light incident on it.

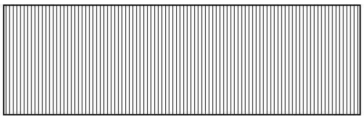
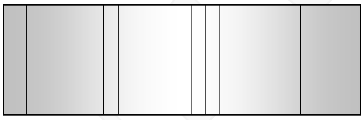
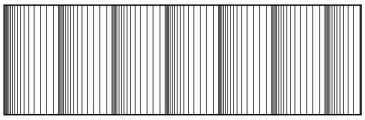
Colours of opaque object	**Colours of transparent object**
(i) Due to selective reflection.	(i) Due to selective transmission.
(ii) A rose appears red in white light because it reflects red colour and absorbs all remaining colours.	(ii) A red glass appears red because it absorbs all colours, except red which it transmits.
(iii) When yellow light falls on a bunch of flowers, then yellow and white flowers looks yellow. Other flowers looks black.	(iii) When we look on objects through a green glass or green filter then green and white objects will appear green while other black.

Note : A hot object will emit light of that colour only which it has observed when it was heated.

Spectrum

The ordered arrangements of radiations according to wavelengths or frequencies is called Spectrum. Spectrum can be divided in two parts (I) Emission spectrum and (II) Absorption spectrum.

(1) **Emission spectrum :** When light emitted by a self luminous object is dispersed by a prism to get the spectrum, the spectrum is called emission spectra.

Continuous emission spectrum	Line emission spectrum	Band emission spectrum
(i) It consists of continuously varying wavelengths in a definite wavelength range.	(i) It consist of distinct bright lines.	(iii) It consist of district bright bands.
(ii) It is produced by solids, liquids and highly compressed gases heated to high temperature.	(ii) It is produced by an excited source in atomic state.	(ii) It is produced by an excited source in molecular state.
(iii) *e.g.* Light from the sun, filament of incandescent bulb, candle flame *etc.*	(iii) *e.g.* Spectrum of excited helium, mercury vapours, sodium vapours or atomic hydrogen.	(iii) *e.g.* Spectra of molecular H_2, CO, NH_3 etc.
		

(2) **Absorption spectrum :** When white light passes through a semi-transparent solid, or liquid or gas, it's spectrum contains certain dark lines or bands, such spectrum is called absorption spectrum (of the substance through which light is passed).

(i) Substances in atomic state produces line absorption spectra. Polyatomic substances such as H_2, CO_2 and $KMnO_4$ produces band absorption spectrum.

(ii) Absorption spectra of sodium vapour have two (yellow lines) wavelengths $D_1 (5890\ \mathring{A})$ and $D_2 (5896\ \mathring{A})$

Note : ❑ If a substance emits spectral lines at high temperature then it absorbs the same lines at low temperature. This is Kirchoff's law.

(3) **Fraunhoffer's lines :** The central part (photosphere) of the sun is very hot and emits all possible wavelengths of the visible light. However, the outer part (chromosphere) consists of vapours of different elements. When the light emitted from the photosphere passes through the chromosphere, certain wavelengths are absorbed. Hence, in the spectrum of sunlight a large number of dark lines are seen called Fraunhoffer lines.

(i) The prominent lines in the yellow part of the visible spectrum were labelled as *D*-lines, those in blue part as *F*-lines and in red part as *C*-line.

(ii) From the study of Fraunhoffer's lines the presence of various elements in the sun's atmosphere can be identified *e.g.* abundance of hydrogen and helium.

(4) **Spectrometer :** A spectrometer is used for obtaining pure spectrum of a source in laboratory and calculation of μ of material of prism and μ of a transparent liquid.

It consists of three parts : Collimator which provides a parallel beam of light; Prism Table for holding the prism and Telescope for observing the spectrum and making measurements on it.

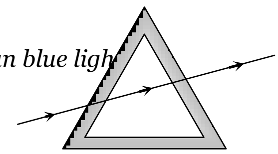
The telescope is first set for parallel rays and then collimator is set for parallel rays. When prism is set in minimum deviation position, the spectrum seen is pure spectrum. Angle of prism (A) and angle of minimum deviation (δ_m) are measured and μ of material of prism is calculated using prism formula. For μ of a transparent liquid, we take a hollow prism with thin glass sides. Fill it with the liquid and measure (δ_m) and A of liquid prism. μ of liquid is calculated using prism formula.

(5) **Direct vision spectroscope :** It is an instrument used to observe pure spectrum. It produces dispersion without deviation with the help of n crown prisms and $(n-1)$ flint prisms alternately arranged in a tabular structure.

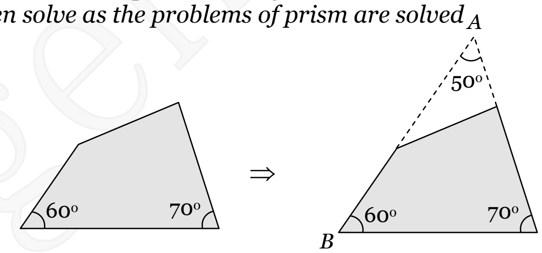
For no deviation $n(\mu-1)A = (n-1)(\mu'-1)A'$.

Concepts

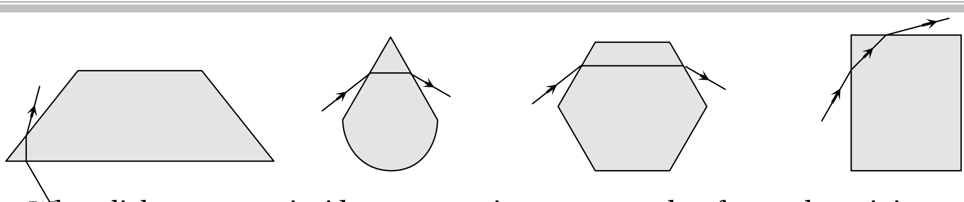
- *When a ray of white light passes through a glass prism red light is deviated less than blue light*
- *For a hollow prism $A \neq 0$ but $\delta = 0$*



- *If an opaque coloured object or crystal is crushed to fine powder it will appear white (in sun light) as it will lose it's property of selective reflection.*
- *Our eye is most sensitive to that part at the spectrum which lies between the F line (sky green) one the C-line (red) of hydrogen equal to the refractive index for the D line (yellow) of sodium. Hence for the dispersive power, the following formula is internationally accepted* $\omega = \dfrac{\mu_F - \mu_C}{\mu_D - 1}$
- *Sometimes a part of prism is given and we keep on thinking whether how should we proceed ? To solve such problems first complete the prism then solve as the problems of prism are solved*



- *Some other types of prism*



***Example:* 36** When light rays are incident on a prism at an angle of 45°, the minimum deviation is obtained. If refractive index of the material of prism is $\sqrt{2}$, then the angle of prism will be

(a) 30° (b) 40° (c) 50° (d) 60°

Solution: (d) $\mu = \dfrac{\sin i}{\sin\dfrac{A}{2}} \Rightarrow \sqrt{2} = \dfrac{\sin 45}{\sin\dfrac{A}{2}} \Rightarrow \sin\dfrac{A}{2} = \dfrac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \dfrac{1}{2} \Rightarrow \dfrac{A}{2} = 30^o \Rightarrow A = 60^o$

***Example:* 37** Angle of minimum deviation for a prism of refractive index 1.5 is equal to the angle of prism. The angle of prism is $(\cos 41^o = 0.75)$

(a) 62° (b) 41° (c) 82° (d) 31°

Solution: (c) Given $\delta_m = A$, then by using $\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} \Rightarrow \mu = \frac{\sin\frac{A+A}{2}}{\sin\frac{A}{2}} = \frac{\sin A}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$

$\left\{\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}\right\}$

$\Rightarrow 1.5 = 2\cos\frac{A}{2} \Rightarrow 0.75 = \cos\frac{A}{2} \Rightarrow 41^o = \frac{A}{2} \Rightarrow A = 82^o$.

***Example:* 38** Angle of glass prism is 60° and refractive index of the material of the prism is 1.414,then what will be the angle of incidence, so that ray should pass symmetrically through prism

(a) 38° 61' (b) 35° 35' (c) 45° (d) 53° 8'

Solution: (c) incident ray and emergent ray are symmetrical in the cure, when prism is in minimum deviation position.

Hence in this condition

$\mu = \frac{\sin i}{\sin\frac{A}{2}} \Rightarrow \sin i = \mu \sin\left(\frac{A}{2}\right) \Rightarrow \sin i = 1.414 \times \sin 30^o = \frac{1}{\sqrt{2}} \Rightarrow i = 45^o$

***Example:* 39** A prism $(\mu = 1.5)$ has the refracting angle of 30°. The deviation of a monochromatic ray incident normally on its one surface will be $(\sin 48^o 36' = 0.75)$

(a) 18° 36' (b) 20° 30' (c) 18° (d) 22° 1'

Solution: (a) By using $\mu = \frac{\sin i}{\sin A} \Rightarrow 1.5 = \frac{\sin i}{\sin 30} \Rightarrow \sin i = 0.75 \Rightarrow i = 48^o 36'$

Also from $\delta = i - A \Rightarrow \delta = 48^o 36' - 30^o = 18^o 36'$

***Example:* 40** Angle of a prism is 30° and its refractive index is $\sqrt{2}$ and one of the surface is silvered. At what angle of incidence, a ray should be incident on one surface so that after reflection from the silvered surface, it retraces its path

(a) 30° (b) 60° (c) 45° (d) $\sin^{-1}\sqrt{1.5}$

Solution: (c) This is the case when light ray is falling normally an second surface.

Hence by using $\mu = \frac{\sin i}{\sin A} \Rightarrow \sqrt{2} = \frac{\sin i}{\sin 30^o} \Rightarrow \sin i = \sqrt{2} \times \frac{1}{2} \Rightarrow i = 45^o$

***Example:* 41** The refracting angle of prism is A and refractive index of material of prism is $\cot\frac{A}{2}$. The angle of minimum deviation is

(a) $180^o - 3A$ (b) $180^o + 2A$ (c) $90^o - A$ (d) $180^o - 2A$

Solution: (d) By using $\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} \Rightarrow \cot\frac{A}{2} = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} \Rightarrow \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$

$\Rightarrow \sin\left(90 - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right) \Rightarrow 90 - \frac{A}{2} = \frac{A+\delta_m}{2} \Rightarrow \delta_m = 180 - 2A$

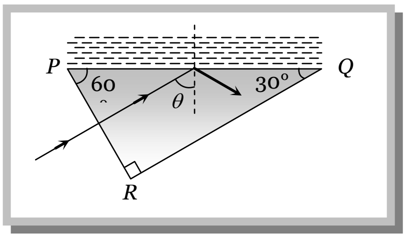
***Example:* 42** A ray of light passes through an equilateral glass prism in such a manner that the angle of incidence is equal to the angle of emergence and each of these angles is equal to 3/4 of the angle of the prism. The angle of deviation is

(a) 45° (b) 39° (c) 20° (d) 30°

Solution: (d) Given that $A = 60^o$ and $i = e = \frac{3}{4}A = \frac{3}{4} \times 60 = 45^o$

By using $i + e = A + \delta \Rightarrow 45 + 45 = 60 + \delta \Rightarrow \delta = 30^o$

***Example:* 43** *PQR* is a right angled prism with other angles as 60° and 30°. Refractive index of prism is 1.5. *PQ* has a thin layer of liquid. Light falls normally on the face *PR*. For total internal reflection, maximum refractive index of liquid is



(a) 1.4

(b) 1.3

(c) 1.2

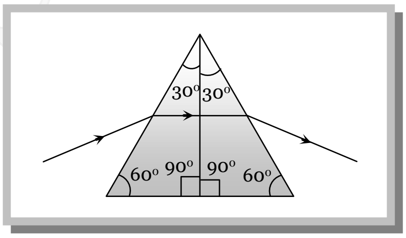
(d) 1.6

Solution: (c) For *TIR* at *PQ* $\theta < C$

From geometry of figure $\theta = 60$ *i.e.* $60 > C \Rightarrow \sin 60 > \sin C$

$$\Rightarrow \frac{\sqrt{3}}{2} > \frac{\mu_{Liquid}}{\mu_{Prism}} \Rightarrow \mu_{Liquid} < \frac{\sqrt{3}}{2} \times \mu_{Prism} \Rightarrow \mu_{Liquid} < \frac{\sqrt{3}}{2} \times 1.5 \Rightarrow \mu_{Liquid} < 1.3\,.$$

***Example:* 44** Two identical prisms 1 and 2, each will angles of 30°, 60° and 90° are placed in contact as shown in figure. A ray of light passed through the combination in the position of minimum deviation and suffers a deviation of 30°. If the prism 2 is removed, then the angle of deviation of the same ray is **[PMT (Andhra) 1995]**



(a) Equal to 15°

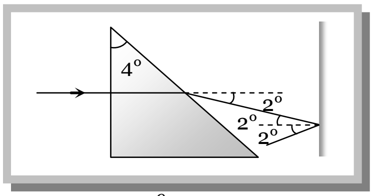
(b) Smaller than 30°

(c) More than 15°

(d) Equal to 30°

Solution: (a) $\delta = (\mu - 1)A$ as *A* is halved, so δ is also halves

***Example:* 45** A prism having an apex angle 4° and refraction index 1.5 is located in front of a vertical plane mirror as shown in figure. Through what total angle is the ray deviated after reflection from the mirror



(a) 176° (b) 4°

(c) 178° (d) 2°

Solution: (c) $\delta_{Prism} = (\mu - 1)A = (1.5 - 1)4^o = 2^o$

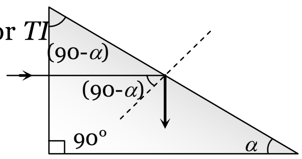
$$\therefore \delta_{Total} = \delta_{Prism} + \delta_{Mirror} = (\mu - 1)A + (180 - 2i) = 2^o + (180 - 2 \times 2) = 178^o$$

***Example:* 46** A ray of light is incident to the hypotenuse of a right-angled prism after travelling parallel to the base inside the prism. If μ is the refractive index of the material of the prism, the maximum value of the base angle for which light is totally reflected from the hypotenuse is **[EAMCET 2003]**

(a) $\sin^{-1}\left(\frac{1}{\mu}\right)$ (b) $\tan^{-1}\left(\frac{1}{\mu}\right)$ (c) $\sin^{-1}\left(\frac{\mu - 1}{\mu}\right)$ (d) $\cos^{-1}\left(\frac{1}{\mu}\right)$

Solution: (d) If α = maximum value of vase angle for which light is totally reflected from hypotenuse.

$(90 - \alpha) = C$ = minimum value of angle of incidence an hypotenuse for TIR



$$\sin(90 - \alpha) = \sin C = \frac{1}{\mu} \Rightarrow \alpha = \cos^{-1}\left(\frac{1}{\mu}\right)$$

***Example:* 47** If the refractive indices of crown glass for red, yellow and violet colours are 1.5140, 1.5170 and 1.5318 respectively and for flint glass these are 1.6434, 1.6499 and 1.6852 respectively, then the dispersive powers for crown and flint glass are respectively

(a) 0.034 and 0.064 (b) 0.064 and 0.034 (c) 1.00 and 0.064 (d) 0.034 and 1.0

Solution: (a) $\omega_{\text{Crown}} = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.5318 - 1.5140}{(1.5170 - 1)} = 0.034$ and

$$\omega_{\text{Flint}} = \frac{\mu_v' - \mu_r'}{\mu_y' - 1} = \frac{1.6852 - 1.6434}{1.6499 - 1} = 0.064$$

***Example*: 48** Flint glass prism is joined by a crown glass prism to produce dispersion without deviation. The refractive indices of these for mean rays are 1.602 and 1.500 respectively. Angle of prism of flint prism is 10°, then the angle of prism for crown prism will be

(a) $12^o 2.4'$ (b) $12^o 4'$ (c) 1.24^o (d) 12°

Solution: (a) For dispersion without deviation

$$\frac{A_C}{A_F} = \frac{(\mu_F - 1)}{(\mu_C - 1)} \Rightarrow \frac{A}{10} = \frac{(1.602 - 1)}{(1.500 - 1)} \Rightarrow A = 12.04^o = 12^o 2.4'$$

Tricky example: 6

An achromatic prism is made by crown glass prism $(A_C = 19^o)$ and flint glass prism $(A_F = 6^o)$. If ${}^C\mu_v = 1.5$ and ${}^F\mu_v = 1.66$, then resultant deviation for red coloured ray will be

(a) 1.04° (b) 5° (c) 0.96° (d) 13.5°

Solution : (d) For achromatic combination $w_C = -w_F \Rightarrow [(\mu_v - \mu_r)A]_C = -[(\mu_v - \mu_r)A]_F$

$\Rightarrow [\mu_r A]_C + [\mu_r A]_F = [\mu_v A]_C + [\mu_v A]_F = 1.5 \times 19 + 6 \times 1.66 = 38.5$

Resultant deviation $\delta = [(\mu_r - 1)A]_C + [(\mu_r - 1)A]_F$

$$= [\mu_r A]_C + [\mu_r A]_F - (A_C + A_F) = 38.5 - (19 + 6) = 13.5^o$$

Tricky example: 7

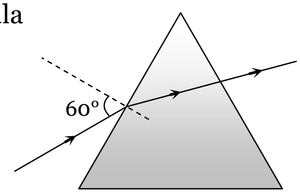
The light is incident at an angle of 60° on a prism of which the refracting angle of prism is 30°. The refractive index of material of prism will be

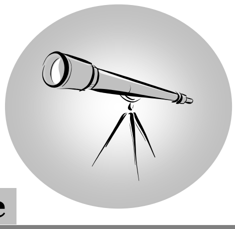
(a) $\sqrt{2}$ (b) $2\sqrt{3}$ (c) 2 (d) $\sqrt{3}$

Solution : (d) By using $i + e = A + \delta \Rightarrow 60 + e = 30 + 30 \Rightarrow e = 0$.

Hence ray will emerge out normally so by using the formula

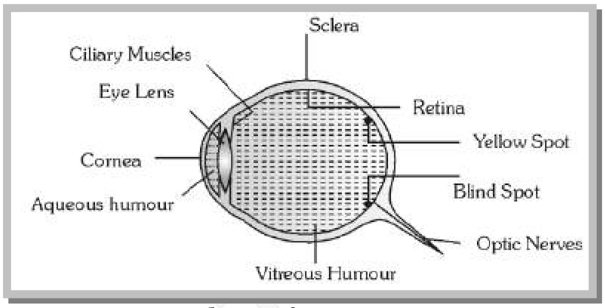
$$\mu = \frac{\sin i}{\sin A} = \frac{\sin 60}{\sin 30} = \sqrt{3}$$





Optical Instruments

Human Eye



(1) **Eye lens :** Over all behaves as a convex lens of $\mu = 1.437$

(2) **Retina :** Real and inverted image of an object, obtained at retina, brain sense it erect.

(3) **Yellow spot :** It is the most sensitive part, the image formed at yellow spot is brightest.

(4) **Blind spot :** Optic nerves goes to brain through blind spot. It is not sensitive for light.

(5) **Ciliary muscles –** Eye lens is fixed between these muscles. It's both radius of curvature can be changed by applying pressure on it through ciliary muscles.

(6) **Power of accomodation :** The ability of eye to see near objects as well as far objects is called power of accomodation.

Note : ❑ When we look distant objects, the eye is relaxed and it's focal length is largest.

(7) **Range of vision :** For healthy eye it is 25 *cm* (near point) to ∞ (far point).

A normal eye can see the objects clearly, only if they are at a distance greater than 25 *cm*. This distance is called Least distance of distinct vision and is represented by *D*.

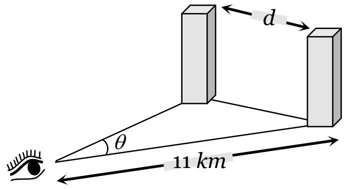
(8) **Persistence of vision :** Is 1/10 *sec*. *i.e.* if time interval between two consecutive light pulses is lesser than 0.1 *sec*., eye cannot distinguish them separately.

(9) **Binocular vision :** The seeing with two eyes is called binocular vision.

(10) **Resolving limit :** The minimum angular displacement between two objects, so that they are just resolved is called resolving limit. For eye it is $1' = \left(\frac{1}{60}\right)^o$.

Specific Example

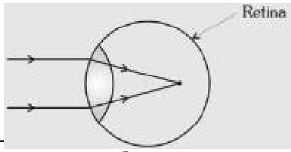
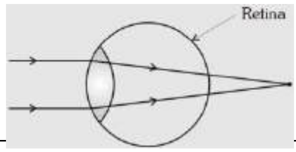
A person wishes to distinguish between two pillars located at a distances of 11 *Km*. What should be the minimum distance between the pillars.



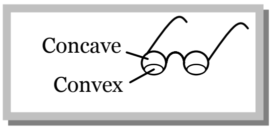
Solution : As the limit of resolution of eye is $\left(\frac{1}{60}\right)^o$

So $\theta > \left(\frac{1}{60}\right)^o$ $\Rightarrow \frac{d}{11\times 10^3} > \left(\frac{1}{60}\right)\times\frac{\pi}{180} \Rightarrow d > 3.2\,m$

(11) **Defects in eye**

Myopia (short sightness)	Hypermetropia (long sightness)
(i) Distant objects are not seen clearly but nearer objects are clearly visible.	(i) Distant objects are seen clearly but nearer object are not clearly visible.
(ii) Image formed before the retina.  	(ii) Image formed behind the retina. 
(iii) Far point comes closer.	(iii) Near point moves away
(iv) Reasons : (a) Focal length or radii of curvature of lens reduced or power of lens increases. (b) Distance between eye lens and retina increases.	(iv) Reasons : (a) Focal length or radii of curvature of lens increases or power of lens decreases. (b) Distance between eye lens and retina decreases.
(v) Removal : By using a concave lens of suitable focal length.	(v) Removal : By using a convex lens.
(vi) Focal length : (a) A person can see upto distance $\to x$ wants to see $\to \infty$, so focal length of used lens $f = -x = -$ (defected far point) (b) A person can see upto distance $\to x$ wants to see distance $\to y$ $(y > x)$ so $f = \frac{xy}{x-y}$	(vi) Focal length : (a) A person cannot see before distance $\to d$ wants to see the object place at distance $\to D$ so $f = \frac{dD}{d-D}$

Presbyopia : In this defect both near and far objects are not clearly visible. It is an old age disease and it is due to the loosing power of accommodation. It can be removed by using bifocal lens.



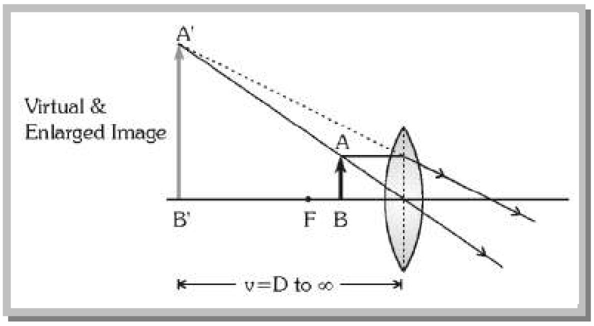
Astigmatism : In this defect eye cannot see horizontal and vertical lines clearly, simultaneously. It is due to imperfect spherical nature of eye lens. This defect can be removed by using cylindrical lens (Torric lenses).

Microscope

It is an optical instrument used to see very small objects. It's magnifying power is given by

$$m = \frac{\text{Visual angle with instrument } (\beta)}{\text{Visual angle when object is placed at least distance of distinct vision } (\alpha)}$$

(1) **Simple miscroscope**



(i) It is a single convex lens of lesser focal length.

(ii) Also called magnifying glass or reading lens.

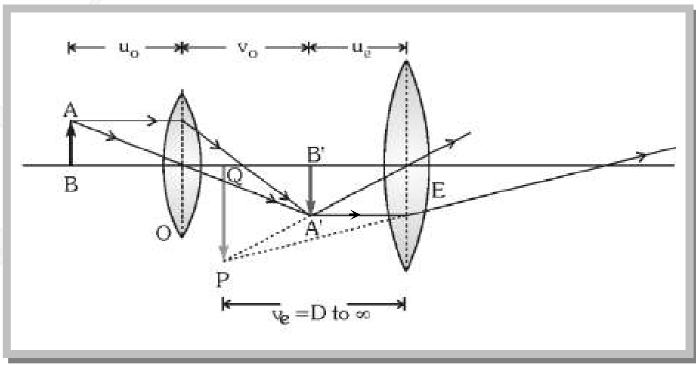
(iii) Magnification's, when final image is formed at D and ∞ (*i.e.* m_D and m_∞)

$$m_D = \left(1+\frac{D}{f}\right)_{\max} \quad \text{and } m_\infty = \left(\frac{D}{f}\right)_{\min}$$

Note : ❑ $m_{\max.} - m_{\min.} = 1$

❑ If lens is kept at a distance a from the eye then $m_D = 1 + \frac{D-a}{f}$ and $m_\infty = \frac{D-a}{f}$

(2) **Compound microscope**



(i) Consist of two converging lenses called objective and eye lens.

(ii) $f_{\text{eye lens}} > f_{\text{objective}}$ and

$(\text{diameter})_{\text{eye lens}} > (\text{diameter})_{\text{objective}}$

(iii) Final image is magnified, virtual and inverted.

(iv) u_0 = Distance of object from objective (o), v_0 = Distance of image ($A'B'$) formed by objective from objective, u_e = Distance of $A'B'$ from eye lens, v_e = Distance of final image from eye lens, f_0 = Focal length of objective, f_e = Focal length of eye lens.

Magnification : $m_D = -\frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right) = -\frac{f_0}{(u_0 - f_0)}\left(1+\frac{D}{f_e}\right) = -\frac{(v_0 - f_0)}{f_0}\left(1+\frac{D}{f_e}\right)$

$$m_\infty = -\frac{v_0}{u_0}.\frac{D}{F_e} = \frac{-f_0}{(u_0 - f_0)}\left(\frac{D}{f_e}\right) = -\frac{(v_0 - f_0)}{f_0}.\frac{D}{F_e}$$

Length of the tube (*i.e.* distance between two lenses)

When final image is formed at D ; $L_D = v_0 + u_e = \frac{u_0 f_0}{u_0 - f_0} + \frac{f_e D}{f_e + D}$

When final images is formed at ∞ ; $L_\infty = v_0 + f_e = \frac{u_0 f_0}{u_0 - f_0} + f_e$

(Do not use sign convention while solving the problems)

Note : ❑ $m_\infty = \dfrac{(L_\infty - f_0 - f_e)D}{f_0 f_e}$

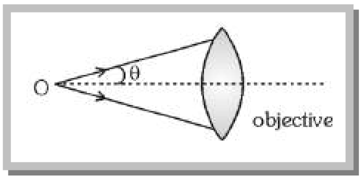
❑ For maximum magnification both f_0 and f_e must be less.

❑ $m = m_{\text{objective}} \times m_{\text{eyelens}}$

❑ If objective and eye lens are interchanged, practically there is no change in magnification.

(3) **Resolving limit and resolving power :** In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (*RL*) and it's reciprocal is called Resolving power (*RP*)

$$R.L. = \frac{\lambda}{2\mu \sin\theta} \text{ and } R.P. = \frac{2\mu \sin\theta}{\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$



λ = Wavelength of light used to illuminate the object,

μ = Refractive index of the medium between object and objective,

θ = Half angle of the cone of light from the point object, $\mu \sin\theta$ = Numerical aperture.

Note : ❑ Electron microscope : electron beam $(\lambda \approx 1 Å)$ is used in it so it's *R.P.* is approx 5000 times more than that of ordinary microscope $(\lambda \approx 5000\ Å)$

Telescope

By telescope distant objects are seen.

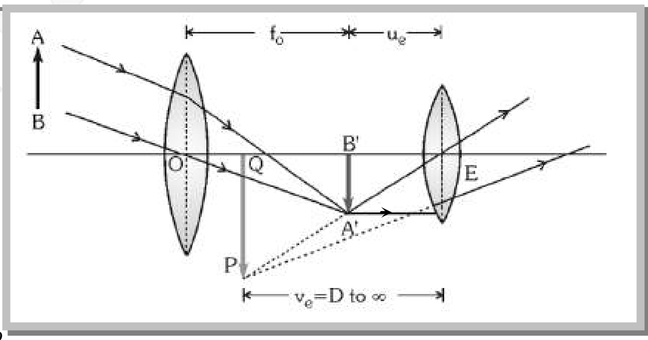
(1) **Astronomical telescope**

(i) Used to see heavenly bodies.

(ii) $f_{\text{objective}} > f_{\text{eyelens}}$ and $d_{\text{objective}} > d_{\text{eyelens}}$.

(iii) Intermediate image is real, inverted and small.

(iv) Final image is virtual, inverted and small.



(v) Magnification : $m_D = -\dfrac{f_0}{f_e}\left(1 + \dfrac{f_e}{D}\right)$ and $m_\infty = -\dfrac{f_0}{f_e}$

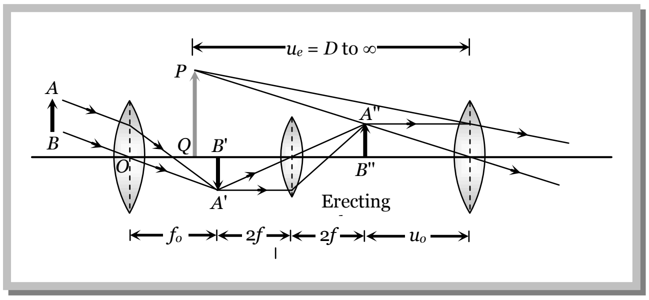
(vi) Length : $L_D = f_0 + u_e = f_0 + \dfrac{f_e D}{f_e + D}$ and $L_\infty = f_0 + f_e$

(2) **Terrestrial telescope**

(i) Used to see far off object on the earth.

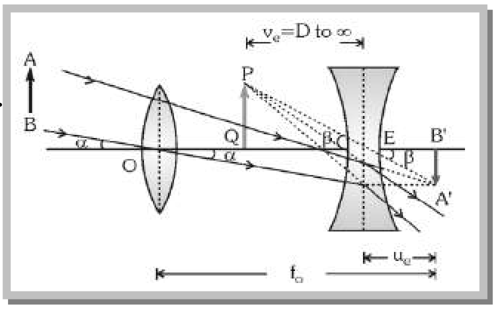
(ii) It consists of three converging lens : objective, eye lens and erecting lens.

(iii) It's final image is virtual erect and smaller.



(iv) Magnification : $m_D = \dfrac{f_0}{f_e}\left(1 + \dfrac{f_e}{D}\right)$ and $m_\infty = \dfrac{f_0}{f_e}$

(v) Length : $L_D = f_0 + 4f + u_e = f_0 + 4f + \dfrac{f_e D}{f_e + D}$ and $L_\infty = f_0 + 4f + f_e$



(3) **Galilean telescope**

(i) It is also a terrestrial telescope but of much smaller field of view.

(ii) Objective is a converging lens while eye lens is diverging lens.

(iii) Magnification : $m_D = \dfrac{f_0}{f_e}\left(1 - \dfrac{f_e}{D}\right)$ and $m_\infty = \dfrac{f_0}{f_e}$

(iv) Length : $L_D = f_0 - u_e$ and $L_\infty = f_0 - f_e$

(4) **Resolving limit and resolving power**

Smallest angular separations ($d\theta$) between two distant objects, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \dfrac{1.22\lambda}{a}$

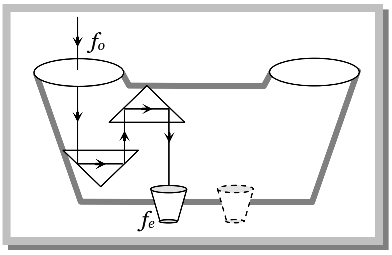
and resolving power $(RP) = \dfrac{1}{d\theta} = \dfrac{a}{1.22\lambda} \Rightarrow R.P. \propto \dfrac{1}{\lambda}$ where a = aperture of objective.

Note :Minimum separation (d) between objects, so they can just resolved by a telescope is –

$d = \dfrac{r}{R.P.}$ where r = distance of objects from telescope.

(5) **Binocular**

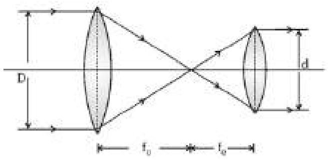
If two telescopes are mounted parallel to each other so that an object can be seen by both the eyes simultaneously, the arrangement is called 'binocular'. In a binocular, the length of each tube is reduced by using a set of totally reflecting prisms which provided intense, erect image free from lateral inversion. Through a binocular we get two images of the same object from different angles at same time. Their superposition gives the perception of depth also along with length and breadth, *i.e.*, binocular vision gives proper three-dimensional (3*D*) image.



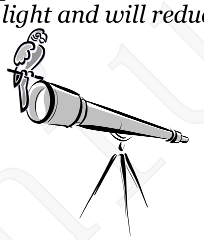
Concepts

- *As magnifying power is negative, the image seen in astronomical telescope is truly inverted, i.e., left is turned right with upside down simultaneously. However, as most of the astronomical objects are symmetrical this inversion does not affect the observations.*
- *Objective and eye lens of a telescope are interchanged, it will not behave as a microscope but object appears very small.*
- *In a telescope, if field and eye lenses are interchanged magnification will change from (f_o / f_e) to (f_e / f_o), i.e., it will change from m to (1/m), i.e., will become (1/m²) times of its initial value.*
- *As magnification for normal setting as (f_o / f_e), so to have large magnification, f_o must be as large as practically possible and f_e small. This is why in a telescope, objective is of large focal length while eye piece of small.*
- *In a telescope, aperture of the field lens is made as large as practically possible to increase its resolving power as resolving power of a telescope $\propto (D/\lambda)^*$. Large aperture of objective also helps in improving the brightness of image by gathering more light from distant object. However, it increases aberrations particularly spherical.*
- *For a telescope with increase in length of the tube, magnification decreases.*
- *In case of a telescope if object and final image are at infinity then :*

$$m = \frac{f_o}{f_e} = \frac{D}{d}$$



- *If we are given four convex lenses having focal lengths $f_1 > f_2 > f_3 > f_4$. For making a good telescope and microscope. We choose the following lenses respectively. Telescope $f_1(o), f_4(e)$ Microscope $f_4(o), f_3(e)$*
- *If a parrot is sitting on the objective of a large telescope and we look towards (or take a photograph)of distant astronomical object (say moon) through it, the parrot will not be seen but the intensity of the image will be slightly reduced as the parrot will act as obstruction to light and will reduce the aperture of the objective.*



Example

Example: 1 A man can see the objects upto a distance of one metre from his eyes. For correcting his eye sight so that he can see an object at infinity, he requires a lens whose power is

or

A man can see upto 100 *cm* of the distant object. The power of the lens required to see far objects will be **[MP PMT 1993, 2003]**

(a) +0.5 *D* (b) +1.0 *D* (c) +2.0 *D* (d) –1.0 *D*

Solution: (d) f = –(defected far point) = – 100 *cm*. So power of the lens $P = \frac{100}{f} = \frac{100}{-100} = -1D$

Example: 2 A man can see clearly up to 3 *metres*. Prescribe a lens for his spectacles so that he can see clearly up to 12 *metres* **[DPMT 2002]**

(a) – 3/4 *D* (b) 3 *D* (c) – 1/4 *D* (d) – 4 *D*

Solution: (c) By using $f = \frac{xy}{x-y} \Rightarrow f = \frac{3\times 12}{3-12} = -4m$. Hence power $P = \frac{1}{f} = -\frac{1}{4}D$

Example: 3 The diameter of the eye-ball of a normal eye is about 2.5 *cm*. The power of the eye lens varies from

(a) 2 *D* to 10 *D* (b) 40 *D* to 32 *D* (c) 9 *D* to 8 *D* (d) 44 *D* to 40 *D*

Solution: (d) An eye sees distant objects with full relaxation so $\frac{1}{2.5\times10^{-2}}-\frac{1}{-\infty}=\frac{1}{f}$ or

$$P=\frac{1}{f}=\frac{1}{25\times10^{-2}}=40D$$

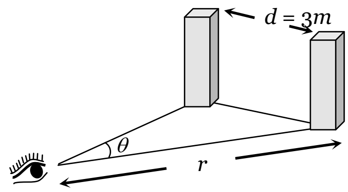
An eye sees an object at 25 *cm* with strain so $\frac{1}{2.5\times10^{-2}}-\frac{1}{-25\times10^{-2}}=\frac{1}{f}$ or

$$P=\frac{1}{f}=40+4=44D$$

***Example*: 4** The resolution limit of eye is 1 *minute*. At a distance of *r* from the eye, two persons stand with a lateral separation of 3 *metre*. For the two persons to be just resolved by the naked eye, *r* should be

(a) 10 *km* (b) 15 *km* (c) 20 *km* (d) 30 *km*

Solution: (a) From figure $\theta=\frac{d}{r}$; where $\theta=1'=\left(\frac{1}{60}\right)^{o}=\left(\frac{1}{60}\right)\times\frac{\pi}{180}\,rad$



$$\Rightarrow 1\times\frac{1}{60}\times\frac{\pi}{180}=\frac{3}{r}\Rightarrow r=10\ km$$

***Example*: 5** Two points separated by a distance of 0.1 *mm* can just be resolved in a microscope when a light of wavelength 6000 *Å* is used. If the light of wavelength 4800 *Å* is used this limit of resolution becomes

[UPSEAT 2002]

(a) 0.08 *mm* (b) 0.10 *mm* (c) 0.12 *mm* (d) 0.06 *mm*

Solution: (a) By using resolving limit (R.L.) $\propto\lambda$ $\Rightarrow\frac{(R.L.)_1}{(R.L.)_2}=\frac{\lambda_1}{\lambda_2}\Rightarrow\frac{0.1}{(R.L.)_2}=\frac{6000}{4800}\Rightarrow(R.L.)_2=0.08\,mm$.

***Example*: 6** In a compound microscope, the focal lengths of two lenses are 1.5 *cm* and 6.25 *cm* an object is placed at 2 *cm* form objective and the final image is formed at 25 *cm* from eye lens. The distance between the two lenses is

[EAMCET (Med.) 2000]

(a) 6.00 *cm* (b) 7.75 *cm* (c) 9.25 *cm* (d) 11.00 *cm*

Solution: (d) It is given that $f_o=1.5\ cm$, $f_e=6.25\ cm$, $u_o=2\ cm$

When final image is formed at least distance of distinct vision, length of the tube

$$L_D=\frac{u_of_o}{u_o-f_o}+\frac{f_eD}{f_e+D}$$

$$\Rightarrow L_D=\frac{2\times1.5}{(2-1.5)}+\frac{6.25\times25}{(6.25+25)}=11\,cm\,.$$

***Example*: 7** The focal lengths of the objective and the eye-piece of a compound microscope are 2.0 *cm* and 3.0 *cm* respectively. The distance between the objective and the eye-piece is 15.0 *cm*. The final image formed by the eye-piece is at infinity. The two lenses are thin. The distances in *cm* of the object and the image produced by the objective measured from the objective lens are respectively **[IIT-JEE 1995]**

(a) 2.4 and 12.0 (b) 2.4 and 15.0 (c) 2.3 and 12.0 (d) 2.3 and 3.0

Solution: (a) Given that $f_o=2\,cm$, $f_e=3\,cm$, $L_\infty=15\,cm$

By using $L_\infty=v_o+f_e\Rightarrow15=v_o+3\Rightarrow v_o=12\,cm$. Also $\frac{v_o}{u_o}=\frac{v_o-f_o}{f_o}\Rightarrow\frac{12}{u_o}=\frac{12-2}{2}\Rightarrow$ $u_o=2.4\,cm$.

***Example*: 8** The focal lengths of the objective and eye-lens of a microscope are 1 *cm* and 5 *cm* respectively. If the magnifying power for the relaxed eye is 45, then the length of the tube is

(a) 30 *cm* (b) 25 *cm* (c) 15 *cm* (d) 12 *cm*

Solution: (c) Given that $f_o = 1\,cm$, $f_e = 5\,cm$, $m_\infty = 45$

By using $m_\infty = \frac{(L_\infty - f_o - f_e)}{f_o f_e} \Rightarrow 45 = \frac{(L_\infty - 1 - 5)\times 25}{1\times 5} \Rightarrow L_\infty = 15\,cm$

***Example*: 9** If the focal lengths of objective and eye lens of a microscope are 1.2 *cm* and 3 *cm* respectively and the object is put 1.25 *cm* away from the objective lens and the final image is formed at infinity, then magnifying power of the microscope is

(a) 150 (b) 200 (c) 250 (d) 400

Solution: (b) Given that $f_o = 1.2\,cm$, $f_e = 3\,cm$, $u_o = 1.25\,cm$

By using $m_\infty = -\frac{f_o}{(u_o - f_o)}\cdot\frac{D}{f_e} \Rightarrow m_\infty = -\frac{1.2}{(1.25-1.2)}\times\frac{25}{3} = -200$.

***Example*: 10** The magnifying power of an astronomical telescope is 8 and the distance between the two lenses is 54*cm*. The focal length of eye lens and objective lens will be respectively **[MP PMT 1991; CPMT 1991**

(a) 6 *cm* and 48 *cm* (b) 48 *cm* and 6 *cm* (c) 8 *cm* and 64 *cm* (d) 64 *cm* and 8 *cm*

Solution: (a) Given that $m_\infty = 8$ and $L_\infty = 54$

By using $|m_\infty| = \frac{f_o}{f_e}$ and $L_\infty = f_o + f_e$ we get $f_o = 6\,cm$ and $f_e = 48\,cm$.

***Example*: 11** If an object subtend angle of 2° at eye when seen through telescope having objective and eyepiece of focal length $f_o = 60\,cm$ and $f_e = 5\,cm$ respectively than angle subtend by image at eye piece will be **[UPSEAT 2001]**

(a) 16° (b) 50° (c) 24° (d) 10°

Solution: (c) By using $\frac{\beta}{\alpha} = \frac{f_o}{f_e} \Rightarrow \frac{\beta}{20} = \frac{60}{5} \Rightarrow \beta = 24°$

***Example*: 12** The focal lengths of the lenses of an astronomical telescope are 50 *cm* and 5 *cm*. The length of the telescope when the image is formed at the least distance of distinct vision is

(a) 45 *cm* (b) 55 *cm* (c) $\frac{275}{6}$ *cm* (d) $\frac{325}{6}$ *cm*

Solution: (d) By using $L_D = f_o + u_e = f_o + \frac{f_e D}{f_e + D} = 50 + \frac{5\times 25}{(5+25)} = \frac{325}{6}\,cm$

***Example*: 13** The diameter of moon is $3.5\times 10^3\,km$ and its distance from the earth is $3.8\times 10^5\,km$. If it is seen through a telescope whose focal length for objective and eye lens are 4 *m* and 10 *cm* respectively, then the angle subtended by the moon on the eye will be approximately

(a) 15° (b) 20° (c) 30° (d) 35°

Solution: (b) The angle subtended by the moon on the objective of telescope $\alpha = \frac{3.5\times 10^3}{3.8\times 10^5} = \frac{3.5}{3.8}\times 10^{-2}\,rad$

Also $m = \frac{f_o}{f_e} = \frac{\beta}{\alpha} \Rightarrow \frac{400}{10} = \frac{\beta}{\alpha} \Rightarrow \beta = 40\,\alpha \Rightarrow \beta = 40\times\frac{3.5\times 10^3}{3.8\times 10^5}\times\frac{180}{\pi} = 20°$

***Example*: 14** A telescope has an objective lens of 10 *cm* diameter and is situated at a distance one *kilometre* from two objects. The minimum distance between these two objects, which can be resolved by the telescope, when the mean wavelength of light is 5000 *Å*, is of the order of

(a) 0.5 *m* (b) 5 *m* (c) 5 *mm* (d) 5*cm*

Solution: (b) Suppose minimum distance between objects is *x* and their distance from telescope is *r*

So Resolving limit $d\theta = \frac{1.22\lambda}{a} = \frac{x}{r} \Rightarrow x = \frac{1.22\lambda\times r}{a} = \frac{1.22\times(5000\times 10^{-10})\times(1\times 10^3)}{(0-1)} = 6.1\times 10^{-3}m = 6.1\,mm$

Hence, It's order is ≈ 5 *mm*.

Example: 15 A compound microscope has a magnifying power 30. The focal length of its eye-piece is 5 *cm*. Assuming the final image to be at the least distance of distinct vision. The magnification produced by the objective will be

(a) +5 (b) – 5 (c) +6 (d) – 6

Solution (b) Magnification produced by compound microscope $m = m_o \times m_e$

where m_o = ? and $m_e = \left(1 + \frac{D}{f_e}\right) = 1 + \frac{25}{5} = 6 \Rightarrow 30 = -m_o \times 6 \Rightarrow m_o = -5$.

Tricky Example 1 : A man is looking at a small object placed at his least distance of distinct vision. Without changing his position and that of the object he puts a simple microscope of magnifying power 10 *X* and just sees the clear image again. The angular magnification obtained is

(a) 2.5 (b) 10.0 (c) 5.0 (d) 1.0

Solution : (d) Angular magnification $= \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{I/D}{O/D} = \frac{I}{O}$

Since image and object are at the same position, $\frac{I}{O} = \frac{v}{u} = 1 \Rightarrow$ Angular magnification = 1

Tricky Example 2: A compound microscope is used to enlarge an object kept at a distance 0.03*m* from it's objective which consists of several convex lenses in contact and has focal length 0.02*m*. If a lens of focal length 0.1*m* is removed from the objective, then by what distance the eye-piece of the microscope must be moved to refocus the image

(a) 2.5 *cm* (b) 6 *cm* (c) 15 *cm* (d) 9 *cm*

Solution : (d) If initially the objective (focal length F_o) forms the image at distance v_o then $v_o = \frac{u_o f_o}{u_o - f_o} = \frac{3 \times 2}{3 - 2} = 6\ cm$

Now as in case of lenses in contact $\frac{1}{F_o} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ... = \frac{1}{f_1} + \frac{1}{F'_o}$ $\left\{\text{where } \frac{1}{F'_o} = \frac{1}{f_2} + \frac{1}{f_3} + ...\right\}$

So if one of the lens is removed, the focal length of the remaining lens system

$$\frac{1}{F'_o} = \frac{1}{F_0} - \frac{1}{f_1} = \frac{1}{2} - \frac{1}{10} \Rightarrow F'_o = 2.5\ cm$$

This lens will form the image of same object at a distance v'_o such that $v'_o = \frac{u_o F'_o}{u_o - F'_o} = \frac{3 \times 2.5}{(3 - 2.5)} = 15\ cm$

So to refocus the image, eye-piece must be moved by the same distance through which the image formed by the objective has shifted *i.e.* 15 – 6 = 9 *cm*.

Assignment

Human eye

80. Near and far points of human eye are **[EAMCET (Med.) 1995; MP PET 2001; Bihar CECE 2004]**

(a) 25 *cm* and infinite (b) 50 *cm* and 100 *cm* (c) 25 *cm* and 50 *cm* (d) 0 *cm* and 25 *cm*

81. A defective eye cannot see close objects clearly because their image is formed **[MP PET 2003]**

(a) On the eye lens (b) Between eye lens and retina

(c) On the retina (d) Beyond retina

82. Retina of eye acts like ..... of camera **[AFMC 2003]**

(a) Shutter (b) Film (c) Lens (d) None of these

83. A person who can see things most clearly at a distance of 10 *cm*. Requires spectacles to enable him to see clearly things at a distance of 30 *cm*. What should be the focal length of the spectacles **[BHU 2003]**

(a) 15 *cm* (concave) (b) 15 *cm* (convex) (c) 10 *cm* (d) 0

84. An astronaut is looking down on earth's surface from a space shuttle at an altitude of $400\,km$. Assuming that the astronaut's pupil diameter is $5\,mm$ and the wavelength of visible light is $500\,nm$. The astronaut will be able to resolve linear object of the size of about **[AIIMS 2003]**

(a) 0.5 *m* (b) 5 *m* (c) 50 *m* (d) 500 *m*

85. A person uses a lens of power + 3*D* to normalise vision. Near point of hypermetropic eye is **[CPMT 2002]**

(a) 1 *m* (b) 1.66 *m* (c) 2 *m* (d) 0.66 *m*

86. The separation between two microscopic particles is measured P_A and P_B by two different lights of wavelength 2000 Å and 3000 Å respectively, then **[AIEEE 2002]**

(a) $P_A > P_B$ (b) $P_A < P_B$ (c) $P_A < 3/2P_B$ (d) $P_A = P_B$

87. To remove myopia (short sightedness) a lens of power 0.66 *D* is required. The distant point of the eye is approximately **[MP PMT 2001]**

(a) 100 *cm* (b) 150 *cm* (c) 50 *cm* (d) 25 *cm*

88. A person suffering from 'presbyopia' should use **[MP PET 2001]**

(a) A concave lens (b) A convex lens

(c) A bifocal lens whose lower portion is convex (d) A bifocal lens whose upper portion is convex

89. The resolving limit of healthy eye is about **[MP PET 1999; RPMT 1999; AIIMS 2001]**

(a) 1′ (b) 1″ (c) 1° (d) $\frac{1}{60}''$

90. A person uses spectacles of power + 2*D*. He is suffering from **[MP PET 2000]**

(a) Short sightedness or myopia (b) Long sightedness or hypermetropia

(c) Presbyopia (d) Astigmatism

91. The hyper metropia is a **[CBSE PMT 2000]**

(a) Short-side defect (b) Long-side defect

(c) Bad vision due to old age (d) None of these

92. A man cannot see clearly the objects beyond a distance of 20 *cm* from his eyes. To see distant objects clearly he must use which kind of lenses and of what focal length **[MP PMT 2000]**

(a) 100 *cm* convex (b) 100 *cm* concave (c) 20 *cm* convex (d) 20 *cm* concave

93. An eye specialist prescribes spectacles having a combination of convex lens of focal length 40 *cm* in contact with a concave lens of focal length 25 *cm*. The power of this lens combination in diopters is **[IIT 1997 Cancelled; DPMT 2000]**

(a) + 1.5 (b) –1.5 (c) +6.67 (d) –6.67

94. Two parallel pillars are 11 *km* away from an observer. The minimum distance between the pillars so that they can be seen separately will be **[RPET 1997; RPMT 2000]**

(a) 3.2 *m* (b) 20.8 *m* (c) 91.5 *m* (d) 183 *m*

95. A person cannot see objects clearly beyond 2.0 *m*. The power of lens required to correct his vision will be **[MP PMT/PET 1998; JIPMER 2000; KCET (Engg./Med.) 2000]**

(a) + 2.0 *D* (b) – 1.0 *D* (c) + 1.0 *D* (d) – 0.5 *D*

96. When objects at different distances are seen by the eye, which of the following remains constant **[MP PMT 1999]**

(a) The focal length of the eye lens (b) The object distance from the eye lens

(c) The radii of curvature of the eye lens (d) The image distance from the eye lens

97. A person wears glasses of power $-2.0\ D$. The defect of the eye and the far point of the person without the glasses will be **[MP PMT 1999]**

(a) Nearsighted, 50 *cm* (b) Farsighted, 50 *cm* (c) Nearsighted, 250 *cm* (d) Astigmatism, 50 *cm*

98. A person is suffering from the defect astigmatism. Its main reason is **[MP PMT 1997]**

(a) Distance of the eye lens from retina is increased (b) Distance of the eye lens from retina is decreased

(c) The cornea is not spherical (d) Power of accommodation of the eye is decreased

99. Myopia is due to **[AFMC 1996]**

(a) Elongation of eye ball (b) Irregular change in focal length

(c) Shortening of eye ball (d) Older age

100. Human eye is most sensitive to visible light of the wavelength **[CPMT 1996]**

(a) 6050 $Å$ (b) 5500 $Å$ (c) 4500 $Å$ (d) 7500 $Å$

101. Match the List I with the List II from the combinations shown **[ISM Dhanbad 1994]**

(I) Presbiopia (A) Sphero-cylindrical lens

(II) Hypermetropia (B) Convex lens of proper power may be used close to the eye

(III) Astigmatism (C) Concave lens of suitable focal length

(IV) Myopia (D) Convex spectacle lens of suitable focal length

(a) I-A; II-C; III-B; IV-D (b) I-B; II-D; III-C; IV-A (c) I-D; II-B; III-A; IV-C (d) I-D; II-A; III-C; IV-B

102. The human eye has a lens which has a **[MP PET 1994]**

(a) Soft portion at its centre (b) Hard surface

(c) Varying refractive index (d) Constant refractive index

103. A man with defective eyes cannot see distinctly object at the distance more than 60 *cm* from his eyes. The power of the lens to be used will be **[MP PMT 1994]**

(a) $+60D$ (b) $-60D$ (c) $-1.66D$ (d) $\frac{1}{1.66}D$

104. A person's near point is 50 *cm* and his far point is 3 *m*. Power of the lenses he requires for

(i) Reading and (ii) For seeing distant stars are **[MP PMT 1994]**

(a) $-2D$ and $0.33D$ (b) $2D$ and $-0.33D$ (c) $-2D$ and $3D$ (d) $2D$ and $-3D$

105. The focal length of a simple convex lens used as a magnifier is 10 *cm*. For the image to be formed at a distance of distinct vision ($D = 25\ cm$), the object must be placed away from the lens at a distance of **[CPMT 1991]**

(a) 5 *cm* (b) 7.14 *cm* (c) 7.20 *cm* (d) 16.16 *cm*

106. A person is suffering from myopic defect. He is able to see clear objects placed at 15 *cm*. What type and of what focal length of lens he should use to see clearly the object placed 60 *cm* away **[MP PMT 1991]**

(a) Concave lens of 20 *cm* focal length (b) Convex lens of 20 *cm* focal length

(c) Concave lens of 12 *cm* focal length (d) Convex lens of 12 *cm* focal length

107. A person can see a thing clearly when it is at a distance of 1 *metre* only. If he wishes to see a distance star, he needs a lens of focal length **[MP PET 1990]**

(a) +100 *cm* (b) – 100 *cm* (c) +50 *cm* (d) –50 *cm*

108. A man suffering from myopia can read a book placed at 10 *cm* distance. For reading the book at a distance of 60 *cm* with relaxed vision, focal length of the lens required will be **[MP PMT 1989]**

(a) 45 *cm* (b) – 20 *cm* (c) –12 *cm* (d) 30 *cm*

109. A person can see clearly objects at 100 *cm* distance. If he wants to see objects at 40 *cm* distance, then the power of the lens he shall require is **[MP PET 1989]**

(a) +1.5 *D* (b) – 1.5 *D* (c) +3.0 *D* (d) –3.0 *D*

110. If the distance of the far point for a myopia patient is doubled, the focal length of the lens required to cure it will become **[MP PET 1989]**

(a) Half (b) Double
(c) The same but a convex lens (d) The same but a concave lens

111. Image is formed for the short sighted person at **[AFMC 1988]**

(a) Retina (b) Before retina (c) Behind the retina (d) Image is not formed at all

112. A man who cannot see clearly beyond 5 *m* wants to see stars clearly. He should use a lens of focal length **[MP PET/PMT 1988]**

(a) – 100 *metre* (b) + 5 *metre* (c) – 5 *metre* (d) Very large

113. Far point of myopic eye is 250 *cm*, then the focal length of the lens to be used will be **[CPMT 1986; DPMT 2002]**

(a) + 250 *cm* (b) – 250 *cm* (c) + 250/9 *cm* (d) – 250/9 *cm*

114. One can take pictures of objects which are completely invisible to the eye using camera film which are invisible to **[MNR 1985]**

(a) Ultra-violet rays (b) Sodium light (c) Visible light (d) Infra-red rays

115. In human eye the focussing is done by **[CPMT 1983]**

(a) To and fro movement of eye lens (b) To and fro movement of the retina
(c) Change in the convexity of the lens surface (d) Change in the refractive index of the eye fluids

116. The minimum light intensity that can be perceived by the eye is about $10^{-10}\ watt/metre^2$. The number of photons of wavelength $5.6\times10^{-7}\ metre$ that must enter per second the pupil of area $10^{-4}\ metre^2$ for vision, is approximately equal to $(h = 6.6\times10^{-34}\ joule-\text{sec})$ **[NCERT 1982]**

(a) 3×10^2 *photons* (b) 3×10^6 *photons* (c) 3×10^4 *photons* (d) 3×10^5 *photons*

117. A far sighted man who has lost his spectacles, reads a book by looking through a small hole (3-4 *mm*) in a sheet of paper. The reason will be **[CPMT 1977]**

(a) Because the hole produces an image of the letters at a longer distance
(b) Because in doing so, the focal length of the eye lens is effectively increased
(c) Because in doing so, the focal length of the eye lens is effectively decreased
(d) None of these

118. The maximum focal length of the eye-lens of a person is greater than its distance from the retina. The eye is

(a) Always strained in looking at an object (b) Strained for objects at large distances only
(c) Strained for objects at short distances only (d) Unstrained for all distances

119. The focal length of a normal eye-lens is about

(a) 1 *mm* (b) 2 *cm* (c) 25 *cm* (d) 1

120. The distance of the eye-lens from the retina is *x*. For normal eye, the maximum focal length of the eye-lens is

(a) = *x* (b) < *x* (c) > *x* (d) = 2*x*

121. A man wearing glasses of focal length +1*m* can clearly see beyond 1*m*

(a) If he is farsighted (b) If he is nearsighted (c) If his vision is normal (d) In each of these cases

122. The near point of a person is 50 *cm* and the far point is 1.5 *m*. The spectacles required for reading purpose and for seeing distance are respectively

(a) $+2D, -\left(\frac{2}{3}\right)D$ (b) $+\left(\frac{2}{3}\right)D-2D$ (c) $-2D, +\left(\frac{2}{3}\right)D$ (d) $-\left(\frac{2}{3}\right)D+2D$

123. A man, wearing glasses of power $+2D$ can read clearly a book placed at a distance of 40 *cm* from the eye. The power of the lens required so that he can read at 25 *cm* from the eye is

(a) $+4.5\ D$ (b) $+4.0\ D$ (c) $+3.5\ D$ (d) $+3.0\ D$

124. A person can see clearly between 1 *m* and 2*m*. His corrective lenses should be

(a) Bifocals with power $-0.5D$ and additional $+3.5D$ (b) Bifocals with power $-1.0D$ and additional $+3.0\ D$

(c) Concave with power 1.0 *D* (d) Convex with power 0.5 *D*

125. While reading the book a man keeps the page at a distance of 2.5 *cm* from his eye. He wants to read the book by holding the page at 25 *cm*. What is the nature of spectacles one should advice him to use to completely cure his eye sight

(a) Convex lens of focal length 25 *cm* (b) Concave lens of focal length 25 *cm*

(c) Convex lens of focal length 2.5 *cm* (d) Concave lens of focal length 2.5 *cm*

126. The blades of a rotating fan can not be distinguished from each other due to

(a) Parallex (b) Power of accommodation (c) Persistence of vision (d) Binocular vision

127. Aperture of the human eye is 2 *mm*. Assuming the mean wavelength of light to be 5000 $\AA$, the angular resolution limit of the eye is nearly

(a) 2 *minutes* (b) 1 *minute* (c) 0.5 *minute* (d) 1.5 *minutes*

128. If there had been one eye of the man, then

(a) Image of the object would have been inverted (b) Visible region would have decreased

(c) Image would have not been seen three dimensional (d) (b) and (c) both

129. A man can see the object between 15*cm* and 30*cm*. He uses the lens to see the far objects. Then due to the lens used, the near point will be at

(a) $\frac{10}{3}\ cm$ (b) 30 *cm* (c) 15 *cm* (d) $\frac{100}{3}\ cm$

130. A presbyopic patient has near point as 30 *cm* and far point as 40 *cm*. The dioptric power for the corrective lens for seeing distant objects is

(a) 40 *D* (b) 4 *D* (c) 2.5 *D* (d) 0.25 *D*

131. A man swimming under clear water is unable to see clearly because

(a) The size of the aperture decreases (b) The size of the aperture increases

(c) The focal length of eye lens increases (d) The focal length of eye lens decreases

132. The distance between retina and eye-lens in a normal eye is 2.0 *cm*. The accommodated power of eye lens range from

(a) 45 *D* to 50 *D* (b) 50 *D* to 54 *D* (c) 10 *D* to 16 *D* (d) 5 *D* to 8 *D*

133. If the eye is taken as a spherical ball of radius 1 *cm*, the range of accommodated focal length of eye-lens is

(a) 1.85 *cm* to 2.0 *cm* (b) 1.0 *cm* to 2.8 *cm* (c) 1.56 *cm* to 2.5 *cm* (d) 1.6 *cm* to 2.0 *cm*

134. A person cannot read printed matter within 100 *cm* from his eye. The power of the correcting lens required to read at 20 *cm* from his eye if the distance between the eye lens and the correcting lens is 2 *cm* is

(a) 4.8 *D* (b) 1.25 *D* (c) 4.25 *D* (d) 4.55 *D*

135. A student having $-1.5\ D$ spectacles uses a lens of focal length 5 *cm* as a simple microscope to read minute scale divisions in the laboratory. The least distance of distinct vision without glasses is 20 *cm* for the student. The maximum magnifying power he gets with spectacles on is

(a) 6 (b) 9 (c) 5 (d) 4

Microscope

136. In a compound microscope the object of f_o and eyepiece of f_e are placed at distance L such that L equals **[Kerala PMT 2004]**

(a) $f_o + f_e$ (b) $f_o - f_e$

(c) Much greater than f_o or f_e (d) Need not depend either value of focal lengths

137. In a simple microscope, if the final image is located at infinity then its magnifying power is **[CPMT 1985; MP PMT 2004]**

(a) $\frac{25}{f}$ (b) $\frac{D}{25}$ (c) $\frac{f}{25}$ (d) $\frac{f}{D^{+1}}$

138. In a simple microscope, if the final image is located at 25 *cm* from the eye placed close to the lens, then the magnifying power is **[BVP 2003]**

(a) $\frac{25}{f}$ (b) $1+\frac{25}{f}$ (c) $\frac{f}{25}$ (d) $\frac{f}{25}+1$

139. The maximum magnification that can be obtained with a convex lens of focal length 2.5 *cm* is (the least distance of distinct vision is 25 *cm*) **[MP PET 2003]**

(a) 10 (b) 0.1 (c) 62.5 (d) 11

140. In a compound microscope, the intermediate image is **[IIT-JEE (Screening) 2000; AIEEE 2003]**

(a) Virtual, erect and magnified (b) Real, erect and magnified

(c) Real, inverted and magnified (d) Virtual, erect and reduced

141. A compound microscope has two lenses. The magnifying power of one is 5 and the combined magnifying power is 100. The magnifying power of the other lens is **[Kerala PMT 2002]**

(a) 10 (b) 20 (c) 50 (d) 25

142. Wavelength of light used in an optical instrument are $\lambda_1 = 4000\ Å$ and $\lambda_2 = 5000\ Å$, then ratio of their respective resolving power (corresponding to λ_1 and λ_2) is **[AIEEE 2002]**

(a) 16 : 25 (b) 9 : 1 (c) 4 : 5 (d) 5 : 4

143. The angular magnification of a simple microscope can be increased by increasing **[Orissa JEE 2002]**

(a) Focal length of lens (b) Size of object (c) Aperture of lens (d) Power of lens

144. The magnification produced by the objective lens and the eye lens of a compound microscope are 25 and 6 respectively. The magnifying power of this microscope is **[Manipal MEE 1995; DPMT 2002]**

(a) 19 (b) 31 (c) 150 (d) $\sqrt{150}$

145. The length of the compound microscope is 14 *cm*. The magnifying power for relaxed eye is 25. If the focal length of eye lens is 5 *cm*, then the object distance for objective lens will be **[Pb. PMT 2002]**

(a) 1.8 *cm* (b) 1.5 *cm* (c) 2.1 *cm* (d) 2.4 *cm*

146. The magnifying power of a simple microscope is 6. The focal length of its lens in *metres* will be, if least distance of distinct vision is 25 *cm* **[MP PMT 2001]**

(a) 0.05 (b) 0.06 (c) 0.25 (d) 0.12

147. Relative difference of focal lengths of objective and eye lens in the microscope and telescope is given as **[MH CET (Med.) 2001]**

(a) It is equal in both (b) It is more in telescope (c) It is more in microscope (d) It may be more in any one

148. Three objective focal lengths (f_o) and two eye piece focal lengths (f_e) are available for a compound microscope. By combining these two, the magnification of microscope will be maximum when **[RPMT 2001]**

(a) $f_o = f_e$ (b) $f_o >> f_e$ (c) f_o and f_e both are small (d) $f_o >> f_e$

149. If the red light is replaced by blue light illuminating the object in a microscope the resolving power of the microscope **[DCE 2001]**

(a) Decreases (b) Increases (c) Gets halved (d) Remains unchanged

150. In case of a simple microscope, the object is placed at **[UPSEAT 2000]**

(a) Focus f of the convex lens (b) A position between f and $2f$ (c) Beyond $2f$ (d) Between the lens and f

151. In a compound microscope cross-wires are fixed at the point **[EAMCET (Engg.) 2000]**

(a) Where the image is formed by the objective (b) Where the image is formed by the eye-piece

(c) Where the focal point of the objective lies (d) Where the focal point of the eye-piece lies

152. The length of the tube of a microscope is 10 *cm*. The focal lengths of the objective and eye lenses are 0.5 *cm* and 1.0 *cm*. The magnifying power of the microscope is about **[MP PMT 2000]**

(a) 5 (b) 23 (c) 166 (d) 500

153. Least distance of distinct vision is 25 *cm*. Magnifying power of simple microscope of focal length 5 *cm* is **[EAMCET (Engg.) 1995; Pb. PMT 1999]**

(a) 1/5 (b) 5 (c) 1/6 (d) 6

154. The objective of a compound microscope is essentially **[SCRA 1998]**

(a) A concave lens of small focal length and small aperture (b) Convex lens of small focal length and large aperture

(c) Convex lens of large focal length and large aperture (d) Convex lens of small focal length and small aperture

155. For relaxed eye, the magnifying power of a microscope is **[CBSE PMT 1998]**

(a) $-\frac{v_o}{u_o}\times\frac{D}{f_e}$ (b) $-\frac{v_o}{u_o}\times\frac{f_e}{D}$ (c) $\frac{u_o}{v_o}\times\frac{D}{f_e}$ (d) $\frac{u_o}{v_o}\times\left(-\frac{D}{f_e}\right)$

156. A person using a lens as a simple microscope sees an **[AIIMS 1998]**

(a) Inverted virtual image (b) Inverted real magnified image

(c) Upright virtual image (d) Upright real magnified image

157. The focal length of the objective lens of a compound microscope is **[CPMT 1985; MNR 1986; MP PET 1997]**

(a) Equal to the focal length of its eye piece (b) Less than the focal length of eye piece

(c) Greater than the focal length of eye piece (d) Any of the above three

158. To produce magnified erect image of a far object, we will be required along with a convex lens, is **[MNR 1983; MP PAT 1996]**

(a) Another convex lens (b) Concave lens (c) A plane mirror (d) A concave mirror

159. An object placed 10 *cm* in front of a lens has an image 20 *cm* behind the lens. What is the power of the lens (in *dioptres*) **[MP PMT 1995]**

(a) 1.5 (b) 3.0 (c) – 15.0 (d) +15.0

160. Resolving power of a microscope depends upon **[MP PET 1995]**

(a) The focal length and aperture of the eye lens (b) The focal lengths of the objective and the eye lens

(c) The apertures of the objective and the eye lens (d) The wavelength of light illuminating the object

161. If the focal length of the objective lens is increased then **[MP PMT 1994]**

(a) Magnifying power of microscope will increase but that of telescope will decrease

(b) Magnifying power of microscope and telescope both will increase

(c) Magnifying power of microscope and telescope both will decrease

(d) Magnifying power of microscope will decrease but that of telescope will increase

162. If in compound microscope m_1 and m_2 be the linear magnification of the objective lens and eye lens respectively, then magnifying power of the compound microscope will be **[CPMT 1985; KCET 1994]**

(a) $m_1 - m_2$ (b) $\sqrt{m_1 + m_2}$ (c) $(m_1 + m_2)/2$ (d) $m_1 \times m_2$

163. The magnifying power of a microscope with an objective of 5 *mm* focal length is 400. The length of its tube is 20 *cm*. Then the focal length of the eye-piece is **[MP PMT 1991]**

(a) 200 *cm* (b) 160 *cm* (c) 2.5 *cm* (d) 0.1 *cm*

164. In a compound microscope, if the objective produces an image I_o and the eye piece produces an image I_e, then **[MP PET 1990]**

(a) I_o is virtual but I_e is real (b) I_o is real but I_e is virtual (c) I_o and I_e are both real (d) I_o and I_e are both virtual

165. In an electron microscope if the potential is increased from 20 *kV* to 80 *kV*, the resolving power of the microscope will change from *R* to **[CPMT 1988, 89]**

(a) $R/4$ (b) $4R$ (c) $2R$ (d) $R/2$

166. When the length of a microscope tube increases, its magnifying power **[MNR 1986]**

(a) Decreases (b) Increases (c) Does not change (d) May decrease or increase

167. An electron microscope is superior to an optical microscope in **[CPMT 1984]**

(a) Having better resolving power (b) Being easy to handle

(c) Low cost (d) Quickness of observation

168. In a compound microscope magnification will be large, if the focal length of the eye piece is **[CPMT 1984]**

(a) Large (b) Smaller (c) Equal to that of objective (d) Less than that of objective

169. An electron microscope gives better resolution than optical microscope because **[CPMT 1982]**

(a) Electrons are abundant (b) Electrons can be focused nicely

(c) Effective wavelength of electron is small (d) None of these

170. A man is looking at a small object placed at his near point. Without altering the position of his eye or the object, he puts a simple microscope of magnifying power 5*X* before his eyes. The angular magnification achieved is

(a) 5 (b) 2.5 (c) 1 (d) 0.2

171. The focal length of the objective of a compound microscope is f_0 and its distance from the eyepiece is *L*. The object is placed at a distance *u* from the objective. For proper working of the instrument

(a) $L < u$ (b) $L > u$ (c) $f_0 < L < 2f_0$ (d) $L > 2f_0$

172. Find the maximum magnifying power of a compound microscope having a 25 *diopter* lens as the objective, a 5 *diopter* lens as the eyepiece and the separation 30 *cm* between the two lenses. The least distance for clear vision is 25 *cm*

(a) 8.4 (b) 7.4 (c) 9.4 (d) 10.4

173. The focal length of the objective and the eye-piece of a microscope are 2 *cm* and 5 *cm* respectively and the distance between them is 30 *cm*. If the image seen by the eye is 25 *cm* from the eye-piece, the distance of the object from the objective is

(a) 0.8 *cm* (b) 2.3 *cm* (c) 0.4 *cm* (d) 1.2 *cm*

174. The focal length of objective and eye-piece of a microscope are 1 *cm* and 5 *cm* respectively. If the magnifying power for relaxed eye is 45, then length of the tube is

(a) 6 *cm* (b) 9 *cm* (c) 12 *cm* (d) 15 *cm*

175. A microscope has an objective of focal length 1.5 *cm* and an eye-piece of focal length 2.5 *cm*. If the distance between objective and eye-piece is 25 *cm*. What is the approximate value of magnification produced for relaxed eye is

(a) 75 (b) 110 (c) 140 (d) 25

176. The magnifying power of a microscope is generally marked as 10*X*, 100 *X*, etc. These markings are for a normal relaxed eye. A microscope marked 10*X* is used by an old man having his near point at 40 *cm*. The magnifying power of the microscope for the old man with his eyes completely relaxed is

(a) 10 (b) 18 (c) 12 (d) 16

177. If the focal length of objective and eye lens are 1.2 *cm* and 3 *cm* respectively and the object is put 1.25 *cm* away from the objective lens and the final image is formed at infinity. The magnifying power of the microscope is

(a) 150 (b) 200 (c) 250 (d) 400

178. A compound microscope is adjusted for viewing the distant image of an object, the distance of the object from the object glass is now slightly increased, what re-adjustment of the instrument would be necessary for obtaining a distant image again

(a) Objective should be moved away from the eye-piece (b) Eye-piece should be moved towards the objective

(c) Both should be moved towards each other (d) Both should be moved away from each other

179. When the object is self-luminous, the resolving power of a microscope is given by the expression

(a) $\frac{2\mu\sin\theta}{\lambda}$ (b) $\frac{\mu\sin\theta}{\lambda}$ (c) $\frac{2\mu\cos\theta}{\lambda}$ (d) $\frac{2\mu}{\lambda}$

180. In a compound microscope, maximum magnification is obtained when the final image

(a) Is formed at infinity (b) Is formed at the least of distinct vision

(c) Coincides with the object (d) Coincides with the objective lens

181. How should people wearing spectacles work with a microscope

(a) They should keep on wearing their spectacles

(b) They should take off their spectacles

(c) They may keep on wearing or take off their spectacles, It makes no difference

(d) They cannot use a microscope at all

Telescope

182. The focal length of the objective and eyepiece of an astronomical telescope for normal adjustments are 50 *cm* and 5 *cm*. The length of the telescope should be **[MP PMT 2004]**

(a) 50 *cm* (b) 55 *cm* (c) 60 *cm* (d) 45 *cm*

183. The resolving power of an astronomical telescope is 0.2 *seconds*. If the central half portion of the objective lens is covered, the resolving power will be **[MP PMT 2004]**

(a) 0.1 *sec* (b) 0.2 *sec* (c) 1.0 *sec* (d) 0.6 *sec*

184. If F_o and F_e are the focal length of the objective and eye-piece respectively of a telescope, then its magnifying power will be

[CPMT 1977, 82, 97, 99, 2003; SCRA 1994; KCET (Engg./Med.) 1999; Pb. PMT 2000; BHU 2001; BCECE 2003, 2004]

(a) $F_o + F_e$ (b) $F_o \times F_e$ (c) F_o / F_e (d) $\frac{1}{2}(F_o + F_e)$

185. The length of an astronomical telescope for normal vision (relaxed eye) (f_o = focal length of objective lens and f_e = focal length of eye lens) is **[EAMCET (Med.) 1995; MP PAT 1996; CPMT 1999; BVP 2003]**

(a) $f_o \times f_e$ (b) $\frac{f_o}{f_e}$ (c) $f_o + f_e$ (d) $f_o - f_e$

186. A telescope of diameter 2*m* uses light of wavelength 5000 *Å* for viewing stars. The minimum angular separation between two stars whose image is just resolved by this telescope is **[MP PET 2003]**

(a) 4×10^{-4} *rad* (b) 0.25×10^{-6} *rad* (c) 0.31×10^{-6} *rad* (d) 5.0×10^{-3} *rad*

187. The aperture of the objective lens of a telescope is made large so as to **[AIEEE 2003; KCET 2003]**

(a) Increase the magnifying power of the telescope (b) Increase the resolving power of the telescope

(c) Make image aberration less (d) Focus on distant objects

188. The distance of the moon from earth is 3.8×10^5 *km*. The eye is most sensitive to light of wavelength 5500 *Å*. The separation of two points on the moon that can be resolved by a 500 *cm* telescope will be **[AMU (Med.) 2002]**

(a) 51 *m* (b) 60 *m* (c) 70 *m* (d) All of the above

189. To increase both the resolving power and magnifying power of a telescope **[Kerala PET 2002; KCET (Engg.) 2002]**

(a) Both the focal length and aperture of the objective has to be increased

(b) The focal length of the objective has to be increased

(c) The aperture of the objective has to be increased

(d) The wavelength of light has to be decreased

190. The focal lengths of the objective and eye lenses of a telescope are respectively 200*cm* and 5*cm*. The maximum magnifying power of the telescope will be **[MP PMT/PET 1998; JIPMER 2001, 2002]**

(a) – 40 (b) – 48 (c) – 60 (d) – 100

191. A telescope has an objective of focal length 50 *cm* and an eye piece of focal length 5 *cm*. The least distance of distinct vision is 25 *cm*. The telescope is focussed for distinct vision on a scale 200 *cm* away. The separation between the objective and the eye-piece is **[Kerala PET 2002]**

(a) 75 *cm* (b) 60 *cm* (c) 71 *cm* (d) 74 *cm*

192. In a laboratory four convex lenses L_1, L_2, L_3 and L_4 of focal lengths 2, 4, 6 and 8*cm* respectively are available. Two of these lenses form a telescope of length 10*cm* and magnifying power 4. The objective and eye lenses are **[MP PMT 2001]**

(a) L_2, L_3 (b) L_1, L_4 (c) L_3, L_2 (d) L_4, L_1

193. Four lenses of focal length + 15 *cm*, + 20 *cm*, + 150 *cm* and + 250 *cm* are available for making an astronomical telescope. To produce the largest magnification, the focal length of the eye-piece should be **[CPMT 2001; AIIMS 2001]**

(a) + 15 *cm* (b) + 20 *cm* (c) + 150 *cm* (d) + 250 *cm*

194. In a terrestrial telescope, the focal length of objective is 90 *cm*, of inverting lens is 5 *cm* and of eye lens is 6 *cm*. If the final image is at 30 *cm*, then the magnification will be **[DPMT 2001]**

(a) 21 (b) 12 (c) 18 (d) 15

195. The focal lengths of the objective and the eyepiece of an astronomical telescope are 20 *cm* and 5 *cm* respectively. If the final image is formed at a distance of 30 *cm* from the eye piece, find the separation between the lenses for distinct vision **[BHU (Med.) 2000]**

(a) 32.4 *cm* (b) 42.3 *cm* (c) 24.3 *cm* (d) 30.24 *cm*

196. Resolving power of reflecting type telescope increases with **[DPMT 2000]**

(a) Decrease in wavelength of incident light (b) Increase in wavelength of incident light

(c) Increase in diameter of objective lens (d) None of these

197. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 *m* and an eye-piece of focal length 2*cm* **[IIT-JEE 1992; Roorkee 2000]**

(a) The distance between the objective and the eye-piece is 16.02 *m*

(b) The angular magnification of the planet is 800

(c) The image of the planet is inverted

(d) All of the above

198. The astronomical telescope consists of objective and eye-piece. The focal length of the objective is **[AIIMS 1998; BHU 2000]**

(a) Equal to that of the eye-piece (b) Greater than that of the eye-piece

(c) Shorter than that of the eye-piece (d) Five times shorter than that of the eye-piece

199. The diameter of the objective of a telescope is a, the magnifying power is m and wavelength of light is λ. The resolving power of the telescope is **[MP PMT 2000]**

(a) $(1.22\lambda)/a$ (b) $(1.22a)/\lambda$ (c) $\lambda m/(1.22a)$ (d) $a/(1.22\lambda m)$

200. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 *cm* and final image is formed at infinity. The focal lengths of the objective and eyepiece are respectively

[IIT-JEE 1989; MP PET 1995; JIPMER 2000]

(a) 20 *cm*, 16 *cm* (b) 50 *cm*, 10 *cm* (c) 30 *cm*, 6 *cm* (d) 45 *cm*, –9 *cm*

201. A photograph of the moon was taken with telescope. Later on, it was found that a housefly was sitting on the objective lens of the telescope. In photograph **[NCERT 1970; MP PET 1999]**

(a) The image of housefly will be reduced (b) There is a reduction in the intensity of the image

(c) There is an increase in the intensity of the image (d) The image of the housefly will be enlarged

202. The magnifying power of a telescope is M. If the focal length of eye piece is doubled, then the magnifying power will become

[Haryana CEET 1998]

(a) $2M$ (b) $M/2$ (c) $\sqrt{2}M$ (d) $3M$

203. The minimum magnifying power of a telescope is M. If the focal length of its eyelens is halved, the magnifying power will become

[MP PMT/PET 1998]

(a) $M/2$ (b) $2M$ (c) $3M$ (d) $4M$

204. The final image in an astronomical telescope is **[EAMCET (Engg.) 1998]**

(a) Real and errect (b) Virtual and inverted (c) Real and inverted (d) Virtual and errect

205. The astronomical telescope has two lenses of focal powers 0.5 *D* and 20 *D*. Its magnifying power will be **[CPMT 1997]**

(a) 40 (b) 10 (c) 100 (d) 35

206. An astronomical telescope of ten-fold angular magnification has a length of 44 *cm*. The focal length of the objective is **[CBSE PMT 19**

(a) 4 *cm* (b) 40 *cm* (c) 44 *cm* (d) 440 *cm*

207. A telescope consisting of an objective of focal length 100 *cm* and a single eyes lens of focal length 10 *cm* is focussed on a distant object in such a way that parallel rays emerge from the eye lens. If the object subtends an angle of 2° at the objective, the angular width of the image is **[JIPMER 1997]**

(a) 20° (b) 1/6° (c) 10° (d) 24°

208. When diameter of the aperture of the objective of an astronomical telescope is increased, its **[MP PMT 1997]**

(a) Magnifying power is increased and resolving power is decreased

(b) Magnifying power and resolving power both are increased

(c) Magnifying power remains the same but resolving power is increased

(d) Magnifying power and resolving power both are decreased

209. The focal length of objective and eye-piece of a telescope are 100 *cm* and 5 *cm* respectively. Final image is formed at least distance of distinct vision. The magnification of telescope is **[RPET 1997]**

(a) 20 (b) 24 (c) 30 (d) 36

210. A simple telescope, consisting of an objective of focal length 60 *cm* and single eye lens of focal length 5 *cm* is focussed on a distant object in such a way that parallel rays comes out from the eye lens. If the object subtends an angle 2° at the objective, the angular width of the image **[CPMT 1979; NCERT 1980; MP PET 1992; JIPMER 1997]**

(a) 10° (b) 24° (c) 50° (d) 1/6°

211. The diameter of the objective of the telescope is 0.1 *metre* and wavelength of light is 6000 Å. Its resolving power would be approximately **[MP PET 1997]**

(a) 7.32×10^{-6} *radian* (b) 1.36×10^{6} *radian* (c) 7.32×10^{-5} *radian* (d) 1.36×10^{5} *radian*

212. A Gallilean telescope has objective and eye-piece of focal lengths 200 *cm* and 2 *cm* respectively. The magnifying power of the telescope for normal vision is **[MP PMT 1996]**

(a) 90 (b) 100 (c) 108 (d) 198

213. All of the following statements are correct except **[Manipal MEE 1995]**

(a) The total focal length of an astronomical telescope is the sum of the focal lengths of its two lenses

(b) The image formed by the astronomical telescope is always erect because the effect of the combination of the two lenses its divergent

(c) The magnification of an astronomical telescope can be increased by decreasing the focal length of the eye-piece

(d) The magnifying power of the refracting type of astronomical telescope is the ratio of the focal length of the objective to that of the eye-piece

214. The length of a telescope is 36 *cm*. The focal length of its lenses can be **[Bihar MEE 1995]**

(a) 30 *cm*, 6 *cm* (b) – 30 *cm*, – 6 *cm* (c) – 30 *cm*, – 6 *cm* (d) – 30 *cm*, 6 *cm*

215. The diameter of the objective lens of telescope is 5.0 *m* and wavelength of light is 6000 Å. The limit of resolution of this telescope will be **[MP PMT 1994]**

(a) 0.03 *sec* (b) 3.03 *sec* (c) 0.06 *sec* (d) 0.15 *sec*

216. If tube length of astronomical telescope is 105 *cm* and magnifying power is 20 for normal setting, calculate the focal length of objective **[AFMC 1994]**

(a) 100 *cm* (b) 10 *cm* (c) 20 *cm* (d) 25 *cm*

217. Radio telescope is used to see **[AFMC 1994]**

(a) Distant start and planets (b) Sun and to measure its temperature

(c) Stars and to measures diameters (d) None of these

218. Four lenses with focal lens ± 15 *cm* and ± 150 *cm* are being placed for used as a telescopic objective. The focal length of the lens which produces the largest magnification with a given eye-piece is **[CBSE PMT 1994]**

(a) –15 *cm* (b) +150 *cm* (c) –150 *cm* (d) +15 *cm*

219. The image of a star (effectively a point source) is made by convergent lens of focal length 50 *cm* and diameter of aperture 5.0 *cm*. If the lens is ideal, and the effective wavelength in image formation is taken as 5×10^{-5} *cm*, the diameter of the image formed will be nearest to **[NSEP 1994]**

(a) Zero (b) 10^{-6} *cm* (c) 10^{-5} *cm* (d) 10^{-3} *cm*

220. To increase the magnifying power of telescope (f_o = focal length of the objective and f_e = focal length of the eye lens) **[MP PET/PMT 1988; MP PMT 1992, 94]**

(a) f_o should be large and f_e should be small (b) f_o should be small and f_e should be large

(c) f_o and f_e both should be large (d) f_o and f_e both should be small

221. The limit of resolution of a 100 *cm* telescope $(\lambda = 5.5\times10^{-7}\ m)$ is **[BHU 1993]**

(a) 0.14" (b) 0.3" (c) 1′ (d) 1"

222. In a reflecting astronomical telescope, if the objective (a spherical mirror) is replaced by a parabolic mirror of the same focal length and aperture, then **[IIT-JEE 1993]**

(a) The final image will be erect (b) The larger image will be obtained

(c) The telescope will gather more light (d) Spherical aberration will be absent

223. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 *m* and an eyepiece of focal length 2 *cm* **[IIT-JEE 1993]**

(a) The distance between the objective and the eyepiece is 16.02 *m*

(b) The angular magnification of the planet is 800

(c) The image of the planet is inverted

(d) The objective is larger than the eyepiece

224. The average distance between the earth and moon is 38.6×10^4 *km*. The minimum separation between the two points on the surface of the moon that can be resolved by a telescope whose objective lens has a diameter of 5 *m* with $\lambda = 6000\ Å$ is **[MP PMT 1993]**

(a) 5.65 *m* (b) 28.25 *m* (c) 11.30 *m* (d) 56.51 *m*

225. The focal length of the objective and eye piece of a telescope are respectively 60 *cm* and 10 *cm*. The magnitude of the magnifying power when the image is formed at infinity is **[MP PET 1991]**

(a) 50 (b) 6 (c) 70 (d) 5

226. The focal length of an objective of a telescope is 3 *metre* and diameter 15 *cm*. Assuming for a normal eye, the diameter of the pupil is 3 *mm* for its complete use, the focal length of eye piece must be **[MP PET 1989]**

(a) 6 *cm* (b) 6.3 *cm* (c) 20 *cm* (d) 60 *cm*

227. An opera glass (Gallilean telescope) measures 9 *cm* from the objective to the eyepiece. The focal length of the objective is 15 *cm*. Its magnifying power is **[DPMT 1988]**

(a) 2.5 (b) 2/5 (c) 5/3 (d) 0.4

228. The focal length of objective and eye lens of a astronomical telescope are respectively 2 *m* and 5 *cm*. Final image is formed at (i) least distance of distinct vision (ii) infinity. The magnifying power in both cases will be **[MP PMT/PET 1988]**

(a) – 48, – 40 (b) – 40, – 48 (c) – 40, 48 (d) – 48, 40

229. An optical device that enables an observer to see over or around opaque objects, is called **[CPMT 1986]**

(a) Microscope (b) Telescope (c) Periscope (d) Hydrometer

230. The magnifying power of a telescope can be increased by **[CPMT 1979]**

(a) Increasing focal length of the system (b) Fitting eye piece of high power

(c) Fitting eye piece of low power (d) Increasing the distance of objects

231. An achromatic telescope objective is to be made by combining the lenses of flint and crown glasses. This proper choice is **[CPMT 1977]**

(a) Convergent of crown and divergent of flint (b) Divergent of crown and convergent of flint

(c) Both divergent (d) Both convergent

232. An observer looks at a tree of height 15 *m* with a telescope of magnifying power 10. To him, the tree appears **[CPMT 1975]**

(a) 10 times taller (b) 15 times taller (c) 10 times nearer (d) 15 times nearer

233. The magnification produced by an astronomical telescope for normal adjustment is 10 and the length of the telescope is 1.1 *m*. The magnification when the image is formed at least distance of distinct vision (D = 25 *cm*) is

(a) 14 (b) 6 (c) 16 (d) 18

234. The objective of a telescope has a focal length of 1.2 *m*. it is used to view a 10.0 *m* tall tower 2 *km* away. What is the height of the image of the tower formed by the objective

(a) 2 *mm* (b) 4 *mm* (c) 6 *mm* (d) 8 *mm*

235. A giant telescope in an observatory has an objective of focal length 19 *m* and an eye-piece of focal length 1.0 *cm*. In normal adjustment, the telescope is used to view the moon. What is the diameter of the image of the moon formed by the objective? The diameter of the moon is 3.5×10^6 *m* and the radius of the lunar orbit round the earth is 3.8×10^8 *m*

(a) 10 *cm* (b) 12.5 *cm* (c) 15 *cm* (d) 17.5 *cm*

236. The aperture of the largest telescope in the world is ≈ 5 *metre*. If the separation between the moon and the earth is $\approx 4 \times 10^5$ *km* and the wavelength of the visible light is $\cong 5000\ Å$, then the minimum separation between the objects on the surface of the moon which can be just resolved is

(a) 1 *metre* approximately (b) 10 *metre* approximately (c) 50 *metre* approximately (d) 200 *metre* approximately

237. In Galileo's telescope, magnifying power for normal vision is 20 and power of eye-piece is –20 *D*. Distance between the objective and eye-piece should be

(a) 90 *cm* (b) 95 *cm* (c) 100 *cm* (d) 105 *cm*

238. The least resolve angle by a telescope using objective of aperture 5 *m* and light of wavelength = 4000 *A.U.* is nearly

(a) $\frac{1}{50}°$ (b) $\frac{1}{50}$ *sec* (c) $\frac{1}{50}$ *minute* (d) $\frac{1}{500}$ *sec*

239. The limit of resolution of a 10 *cm* telescope for visible light of wavelength 6000 Å is approximately

(a) 0.1 *s* or arc (b) 30° (c) $\left(\frac{1}{6}\right)^o$ (d) None of these

240. An eye-piece of a telescope with a magnification of 100 has a power of 20 diopters. The object of this telescope has a power of

(a) 2 *diopters* (b) 0.2 *diopters* (c) 2000 *diopters* (d) 20 *diopters*

241. The Yerkes Observatory telescope has a large telescope with objective of diameter of about 1 *m*. Assuming wavelength of light to be $6\times10^{-7}m$, the angular distance θ between two stars which can just be resolved is

(a) $(7.3\times10^{-7})°$ (b) 7.3×10^{-7} *rad* (c) $\frac{1}{40}$ of a second (d) None of these

242. A Galilean telescope measures 9 *cm* from the objective to the eye-piece. The focal length of the objective is 15 *cm*. Its magnifying power is

(a) 2.5 (b) 2/5 (c) 5/3 (d) 0.4

243. For seeing a cricket match, we prefer binoculars to the terrestrial telescope, because

(a) Binoculars give three-dimensional view (b) Terrestrial telescope gives inverted image

(c) To avoid chromatic aberration (d) To have larger magnification

244. A simple two lens telescope has an objective of focal length 50 *cm* and an eye-piece of 2.5 *cm*. The telescope is pointed at an object at a very large distance which subtends at an angle of 1 *milliradian* on the naked eye. The eye piece is adjusted so that the final virtual image is formed at infinity. The size of the real image formed by the objective is

(a) 5 *mm* (b) 1 *mm* (c) 0.5 *mm* (d) 0.1 *mm*

245. The objective of a telescope, after focussing for infinity is taken out and a slit of length *L* is placed in its position. A sharp image of the slit is formed by the eye-piece at a certain distance from it on the other side. The length of this image is *l*, then magnification of telescope is

(a) $\frac{l}{2L}$ (b) $\frac{2L}{l}$ (c) $\frac{l}{L}$ (d) $\frac{L}{l}$

246. An astronomical telescope in normal adjustment receives light from a distant source *S*. The tube length is now decreased slightly

(a) A virtual image of *S* will be formed at a finite distance

(b) No image will be formed

(c) A small, real image of *S* will be formed behind the eye-piece, close to it

(d) A large, real image of *S* will be formed behind the eye-piece, far away from it

247. A telescope consisting of object glass of power + 2 *D* and eye-glass of power + 20 *D* is focussed on an object 1*m* from the object glass. The final image is seen with completely relaxed eye. The magnifying power of the telescope is

(a) 20 (b) 41 (c) 24 (d) 49.2

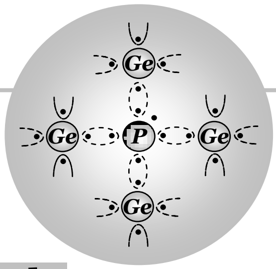
248. An astronomical telescope and a Galilean telescope use identical objective lenses. They have the same magnification, when both are in normal adjustment. The eye-piece of the astronomical telescope has a focal length *f*

(a) The tube lengths of the two telescopes differ by *f* (b) The tube lengths of the two telescopes differ by 2*f*

(c) The Galilean telescope has a shorter tube length (d) The Galilean telescope has a longer tube length

79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98

a	d	b	a	c	a	b	b	c	a	b	b	d	b	a	d	d	a	c	a
99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118
b	c	c	c	b	b	c	b	c	a	b	b	c	b	d	c	c	a	a	b
119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138
a	d	a	c	a	d	c	b	d	b	c	c	b	a	d	a	c	a	b	d
139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158
c	b	d	d	c	a	a	b	c	b	d	a	d	d	d	a	c	b	b	d
159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178
d	d	d	c	b	c	a	a	b	c	c	b,d	a	b	d	c	d	b	b	a
179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198
b	b	b	c	c	c	c	b	a	a	b	c	d	a	c	c	a, c	d	b	d
199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218
c	b	b	b	b	a	b	a	c	b	b	d	b	b	a	a	a	a	b	d
219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238
a	a	d	a	d	b	a	a	a	c	b	a	c	a	c	d	c	b	b	a
239	240	241	242	243	244	245	246	247											
b	b	a	a	c	d	a	b	b, c											



Semi-conductor Devices

Energy Bands

In isolated atom the valence electrons can exist only in one of the allowed orbitals each of a sharply defined energy called energy levels. But when two atoms are brought nearer to each other, there are alterations in energy levels and they spread in the form of bands.

Energy bands are of following types

(1) **Valence band**

The energy band formed by a series of energy levels containing valence electrons is known as valence band. At 0 *K*, the electrons fills the energy levels in valence band starting from lowest one.

(i) This band is always fulfill by electron.

(ii) This is the band of maximum energy.

(iii) Electrons are not capable of gaining energy from external electric field.

(iv) No flow of current due to such electrons.

(v) The highest energy level which can be occupied by an electron in valence band at 0 *K* is called fermi level.

(2) **Conduction band**

The higher energy level band is called the conduction band.

(i) It is also called empty band of minimum energy.

(ii) This band is partially filled by the electrons.

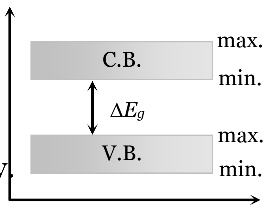
(iii) In this band the electrons can gain energy from external electric field.

(iv) The electrons in the conduction band are called the free electrons. They are able to move any where within the volume of the solid.

(v) Current flows due to such electrons.

(3) **Forbidden energy gap (ΔE_g)**

Energy gap between conduction band and valence band $\Delta E_g = (C.B.)_{\min} - (V.B.)_{\max}$



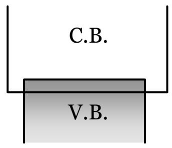
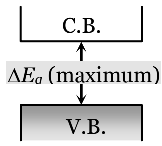
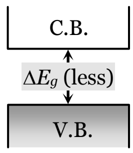
(i) No free electron present in forbidden energy gap.

(ii) Width of forbidden energy gap upon the nature of substance.

(iii) As temperature increases (↑), forbidden energy gap decreases (↓) very slightly.

Types of Solids

On the basis of band structure of crystals, solids are divided in three categories.

S.No.	Properties	Conductors	Insulators	Semiconductors
(1)	Electrical conductivity	10^2 to 10^8 $\mho/m$	10^{-8} $\mho/m$	10^{-5} to 10^0 $\mho/m$
(2)	Resistivity	10^{-2} to 10^{-8} Ω-m (negligible)	10^8 Ω-m	10^5 to 10^0 Ω-m
(3)	Band structure			
(4)	Energy gap	Zero or very small	Very large; for diamond it is 6 eV	For Ge E_g = 0.7 eV for Si E_g = 1.1 eV
(5)	Current carries	Free electrons	––	Free electrons and holes
(6)	Condition of V.B. and C.B. at ordinary temperature	V.B. and C.B. are completely filled or C.B. is some what empty	V.B. – completely filled C.B. – completely unfilled	V.B. – somewhat empty C.B. – somewhat filled
(7)	Temperature co-efficient of resistance (α)	Positive	Zero	Negative
(8)	Effect of temperature on conductivity	Decreases	—	Increases
(9)	Effect of temperature on resistance	Increases	—	Decreases
(11)	Examples	Cu, Ag, Au, Na, Pt, Hg etc.	Wood, plastic, mica, diamond, glass etc.	Ge, Si, Ga, As etc.
(12)	Electron density	$10^{29}/m^3$	—	$Ge \sim 10^{19}$ $/m^3$ $Si \sim 10^{16}$ $/m^3$

Holes in semiconductors

At absolute zero temperature (0 K) conduction band of semiconductor is completely empty and the semiconductor behaves as an insulator.

When temperature increases the valence electrons acquires thermal energy to jump to the conduction band (Due to the braking of covalent bond). If they jumps to C.B. they leaves behind the deficiency of electrons in the valence band. This deficiency of electron is known as ***hole*** or cotter. A hole is considered as a seat of positive charge, having magnitude of charge equal to that of an electron.

(1) Holes acts as virtual charge, although there is no physical charge on it.

(2) Effective mass of hole is more than electron.

(3) Mobility of hole is less than electron.

Types of Semiconductors

(1) **Intrinsic semiconductor**

A pure semiconductor is called intrinsic semiconductor. It has thermally generated current carriers

(i) They have four electrons in the outermost orbit of atom and atoms are held together by covalent bond

(ii) Free electrons and holes both are charge carriers and n_e (in C.B.) $= n_h$ (in V.B.)

(iii) The drift velocity of electrons (v_e) is greater than that of holes (v_h)

(iv) For them fermi energy level lies at the centre of the C.B. and V.B.

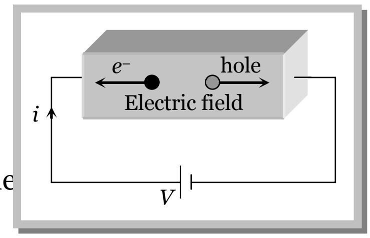
(v) In pure semiconductor, impurity must be less than 1 in 10^8 parts of semiconductor.

(vi) In intrinsic semiconductor $n_e^{(o)} = n_h^{(o)} = n_i = AT^{3/2} e^{-\Delta E_g / 2KT}$; where $n_e^{(o)}$ = Electron density in conduction band, $n_h^{(o)}$ = Hole density in V.B., n_i = Density of intrinsic carriers.

(vii) Because of less number of charge carriers at room temperature, intrinsic semiconductors have low conductivity so they have no practical use.

Net current and conductivity

When some potential difference is applied across a piece of intrinsic semiconductor current flows in it due to both electron and holes *i.e.* $i = i_e + i_h \Rightarrow i = n_e eAv_e \quad i = eA[n_e v_e + n_h v_h]$



Hence conductivity of semiconductor $\sigma = e[n_e \mu_e + n_h \mu_h]$

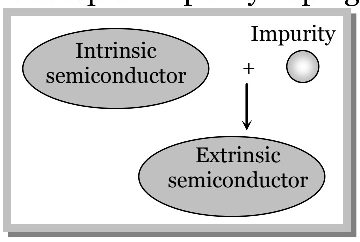
where v_e = drift velocity of electron, v_h = drift velocity of holes,

E = Applied electric field $\mu_e = \frac{v_e}{E}$ = mobility of e^- and $\mu_n = \frac{v_h}{E}$ = mobility of hole

Note : ❑ $(ni)_{Ge} \simeq 2.4 \times 10^{19} / m^3$ and $(ni)_{Si} \simeq 1.5 \times 10^{16} / m^3$

- ❑ At room temperature $\sigma_{Ge} > \sigma_{Si}$
- ❑ $\mu_e > \mu_h$
- ❑ Conductivity of semiconductor increases with temperature because number density of charge carriers increases.
- ❑ In a doped semiconductor, the number density of electrons and holes is not equal. But it can be established that $\boldsymbol{n_e n_h = n_i^2}$; where n_e, n_h are the number density of electrons and holes respectively and n_i is the number density of intrinsic curries (*i.e.* electrons or holes) in a pure semiconductor. This product is independent of donor and acceptor impurity doping.

(2) **Extrinsic semiconductor**



(i) It is also called impure semiconductor.

(ii) The process of adding impurity is called Doping.

(iii) Impurities are of two types :

Pentavalent impurity	**Trivalent impurity**
The elements whose atom has five valance impurities e.g. As, P, Sb etc. These are also called donor impurities. These impurities are also called donor impurities because they donates extra free electron.	*The elements whose each atom has three valance electrons are called trivalent impurities e.g. In, Ga, Al, B, etc. These impurities are also called acceptor impurities as they accept electron.*

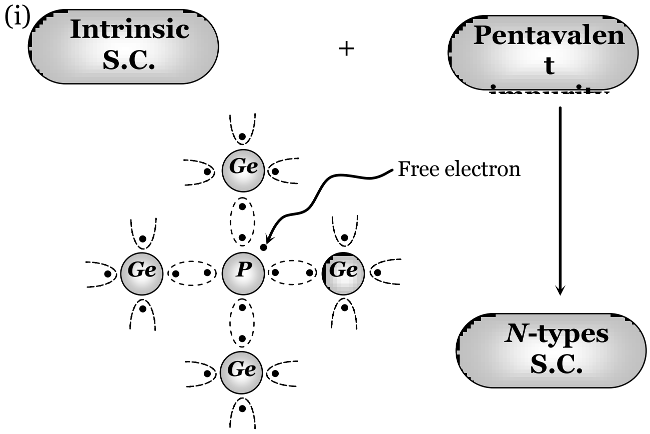
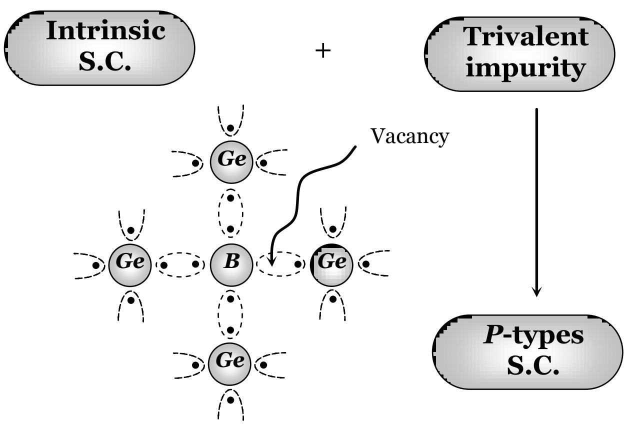
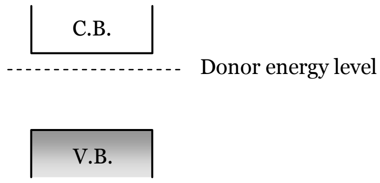
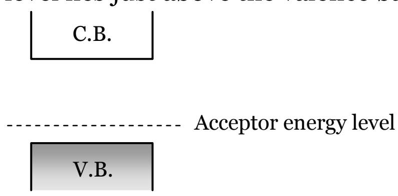
(iv) The number of atoms of impurity element is about 1 in 10^8 atoms of the semiconductor.

(v) $n_e \neq n_h$

(vi) In these fermi level shifts towards valence or conduction energy bands.

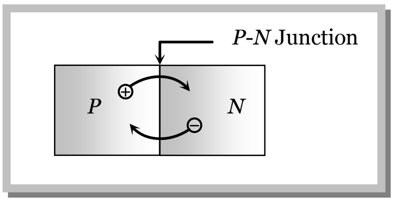
(vii) Their conductivity is high and they are practically used.

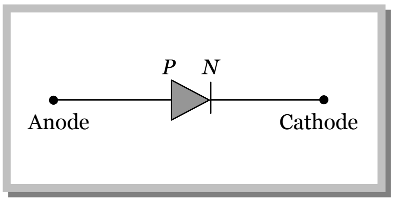
(3) **Types of extrinsic semiconductor**

***N*-type semiconductor**	***P*-type semiconductor**
(i) 	
(ii) Majority charge carriers – electrons Minority charge carriers – holes	Majority charge carriers – holes Minority charge carriers – electrons
(iii) $n_e >> n_h$; $i_e >> i_h$	$n_h >> n_e$; $i_h >> i_e$
(iv) Conductivity $\sigma \approx n_e \mu_e e$	Conductivity $\sigma \approx n_h \mu_h e$
(iv) *N*-type semiconductor is electrically neutral (not negatively charged)	*P*-type semiconductor is also electrically neutral (not positively charged)
(v) Impurity is called Donar impurity because one impurity atom generate one e^- .	Impurity is called Acceptor impurity.
(vi) Donor energy level lies just below the conduction band. 	Acceptor energy level lies just above the valence band. 

P-N Junction Diode

When a *P*-type semiconductor is suitably joined to an *N*-type semiconductor, then resulting arrangement is called *P-N* junction or *P-N* junction diode

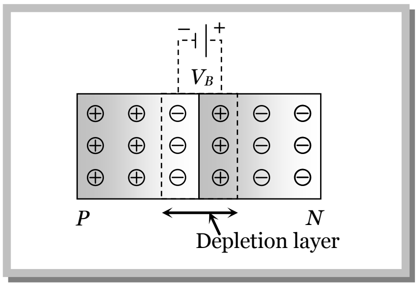




(1) **Depletion region**

On account of difference in concentration of charge carrier in the two sections of *P-N* junction, the electrons from *N*-region diffuse through the junction into *P*-region and the hole from *P* region diffuse into *N*-region.

Due to diffusion, neutrality of both *N* and *P*-type semiconductor is disturbed, a layer of negative charged ions appear near the junction in the *P*-crystal and a layer of positive ions appears near the junction in *N*-crystal. This layer is called depletion layer



(i) The thickness of depletion layer is 1 *micron* = 10^{-6} *m*.

(ii) Width of depletion layer $\propto \frac{1}{\text{Dopping}}$

(iii) Depletion is directly proportional to temperature.

(iv) The *P-N* junction diode is equivalent to capacitor in which the depletion layer acts as a dielectric.

(2) **Potential barrier**

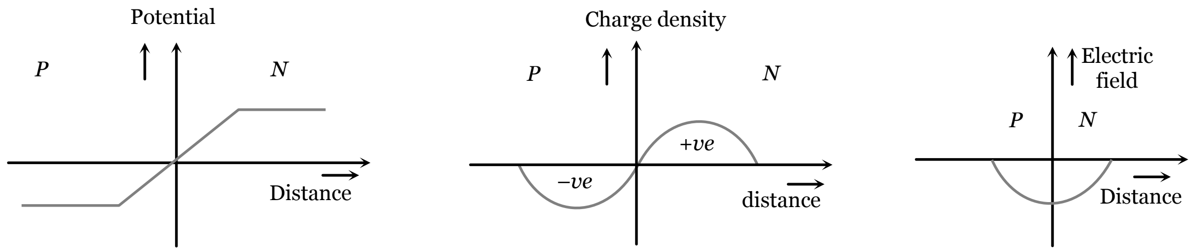
The potential difference created across the *P-N* junction due to the diffusion of electron and holes is called potential barrier.

For *Ge* $V_B = 0.3V$ and for silicon $V_B = 0.7V$

On the average the potential barrier in *P-N* junction is ~ 0.5 *V* and the width of depletion region ~ 10^{-6}.

So the barrier electric field $E = \frac{V}{d} = \frac{0.5}{10^{-6}} = 5 \times 10^5\ V/m$

Some important graphs



(3) **Diffusion and drift current**

Because of concentration difference holes/electron try to diffuse from their side to other side. Only these holes/electrons crosses the junction, having high kinetic energy. This diffusion results is an electric current from the *P*-side to the *N*-side known as diffusion current (i_{df})

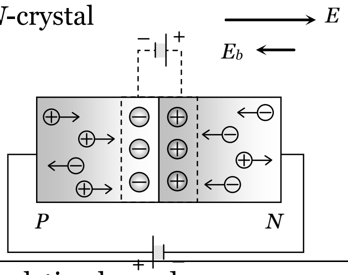
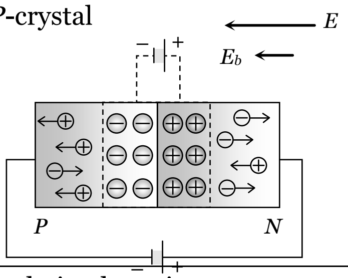
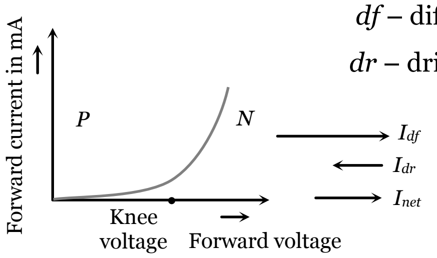
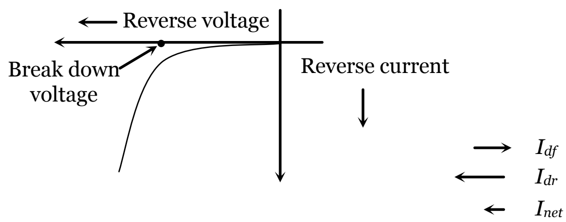
As electron hole pair (because of thermal collisions) are continuously created in the depletion region. These is a regular flow of electrons towards the *N*-side and of holes towards the *P*-side. This makes a current from the *N*-side to the *P*-side. This current is called the drift current (i_{dr}).

Note : ❑ In steady state $i_{df} = i_{dr}$ so $i_{net} = 0$

❑ When no external source is connected, diode is called unbiased.

(4) **Biasing**

Means the way of connecting *emf* source to *P-N* junction diode

Forward biasing	**Reverse biasing**
(i) Positive terminal of the battery is connected to the *P*-crystal and negative terminal of the battery is connected to *N*-crystal 	(i) Positive terminal of the battery is connected to the *N*-crystal and negative terminal of the battery is connected to *P*-crystal 
(ii) Width of depletion layer decreases	(ii) Width of depletion layer increases
(iii) $R_{Forward} \approx 10\Omega - 25\Omega$	(iii) $R_{Reverse} \approx 10^5\Omega$
(iv) Forward bias opposes the potential barrier and for $V > V_B$ a forward current is set up across the junction.	(iv) Reverse bias supports the potential barrier and no current flows across the junction due to the diffusion of the majority carriers. (A very small reverse currents may exist in the circuit due to the drifting of minority carriers across the junction)
(v) Cut-in (Knee) voltage : The voltage at which the current starts to increase. For *Ge* it is 0.3 *V* and for *Si* it is 0.7 *V*.	(v) Break down voltage : Reverse voltage at which break down of semiconductor occurs. For *Ge* it is 25 *V* and for *Si* it is 35 *V*.
(vi) *df* – diffusion *dr* – drift 	(vi) 

Reverse Breakdown and Special Purpose Diodes

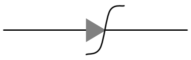
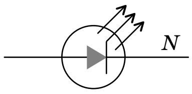
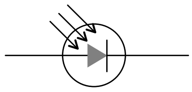
(1) **Zener breakdown**

When reverse bias is increased the electric field at the junction also increases. At some stage the electric field becomes so high that it breaks the covalent bonds creating electron, hole pairs. Thus a large number of carriers are generated. This causes a large current to flow. This mechanism is known as **Zener breakdown.**

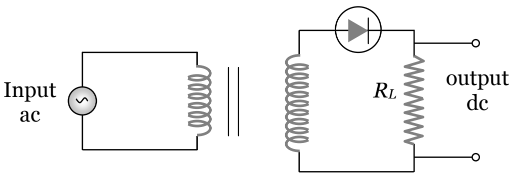
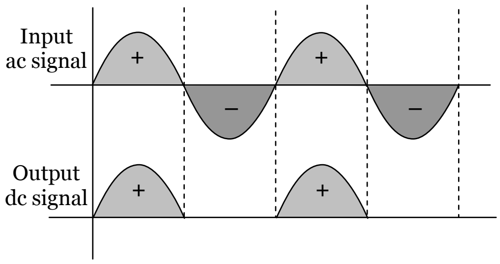
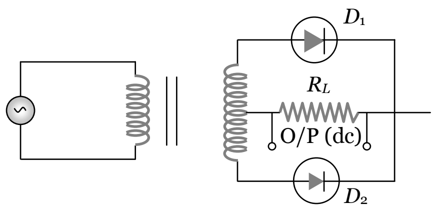
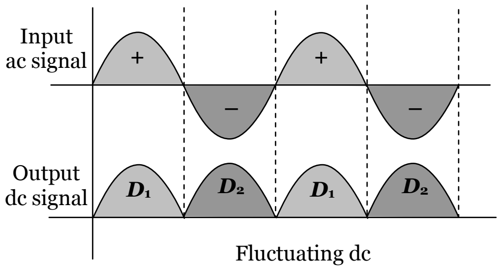
(2) **Avalanche breakdown**

At high reverse voltage, due to high electric field, the miniority charge carriers, while crossing the junction acquires very high velocities. These by collision breaks down the covalent bonds, generating more carriers. A chain reaction is established, giving rise to high current. This mechanism is called **avalanche breakdown.**

(3) **Special purpose diodes**

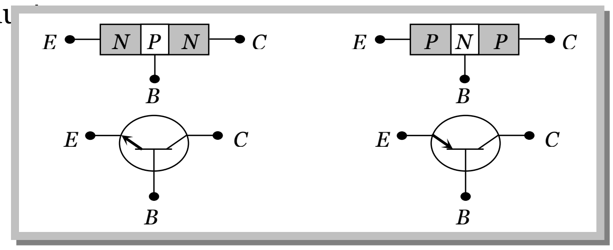
Zener diode	**Light emitting diode (LED)**	**Photo diode**	**Solar cells**
 It is a highly doped *p-n* junction which is not damaged by high reverse current. The breakdown voltage is made very sharp. In the forward bias, the zener diode acts as ordinary diode. It can be used as voltage regulator	 Specially designed diodes, which give out light radiations when forward biases. LED'S are made of *GaAsp*, *Gap* etc.	 In these diodes electron and hole pairs are created by junction photoelectric effect. That is the covalent bonds are broken by the EM radiations absorbed by the electron in the V.B. These are used for detecting light signals.	It is based on the photovoltaic effect. One of the semiconductor region is made so thin that the light incident on it reaches the *p-n* junction and gets absorbed. It converts solar energy into electrical energy.

P-N Junction Diode as a Rectifier

Half wave rectifier	**Full wave rectifier**
 	  Fluctuating dc
During positive half cycle Diode ⟶ forward biased Output signal ⟶ obtained During negative half cycle Diode ⟶ reverse biased Output signal ⟶ not obtained	During positive half cycle Diode : D_1 ⟶ forward biased D_2 ⟶ reverse biased Output signal ⟶ obtained due to D_1 only During negative half cycle Diode : D_1 ⟶ reverse biased D_2 ⟶ forward biased Output signal ⟶ obtained due to D_2 only Note : ❑ Fluctuating dc ⟶ Filter ⟶ constant dc.

Transistor

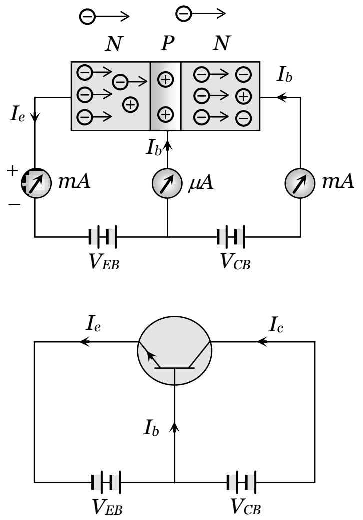
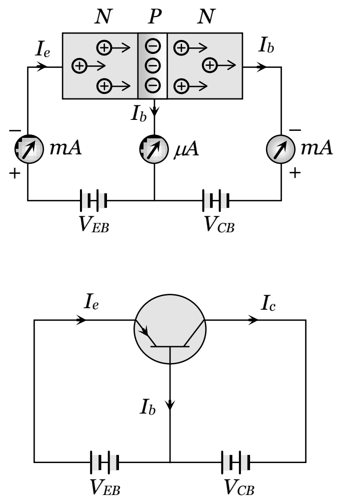
A junction transistor is formed by sandwiching a thin layer of *P*-type semiconductor between two *N*-type semiconductors or by sandwiching a thin layer of *n*-type semiconductor between two *P*-type semicondu



E – Emitter (emits majority charge carriers)
C – Collects majority charge carriers
B – Base (provide proper interaction between E and *C*)

Note : In normal operation base-emitter is forward biased and collector base junction is reverse biased.

(1) **Working of Transistor :** In both transistor emitter - base junction is forward biased and collector – base junction is reverse biased.

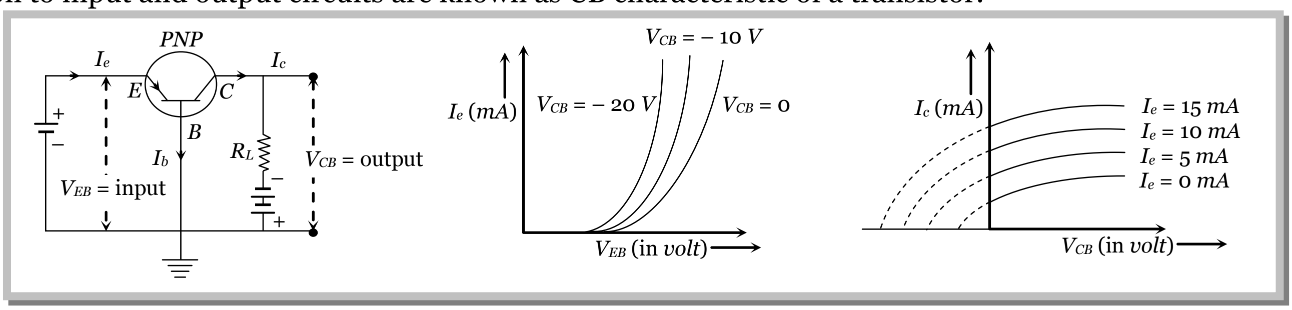
***NPN* – transistor**	***PNP* – transistor**
	
5% emitter electron combine with the holes in the base region resulting in small base current. Remaining 95% electrons enter the collector region.	5% emitter holes combine with the electrons in the base region resulting in small base current. Remaining 95% holes enter the collector region.
$I_e > I_c$, and $I_c = I_b + I_c$	$I_e > I_c$, and $I_c = I_b + I_c$

Note : ❑ In a transistor circuit the reverse bias is high as compared to the forward bias. So that it may exert a large attractive force on the charge carriers to enter the collector region.

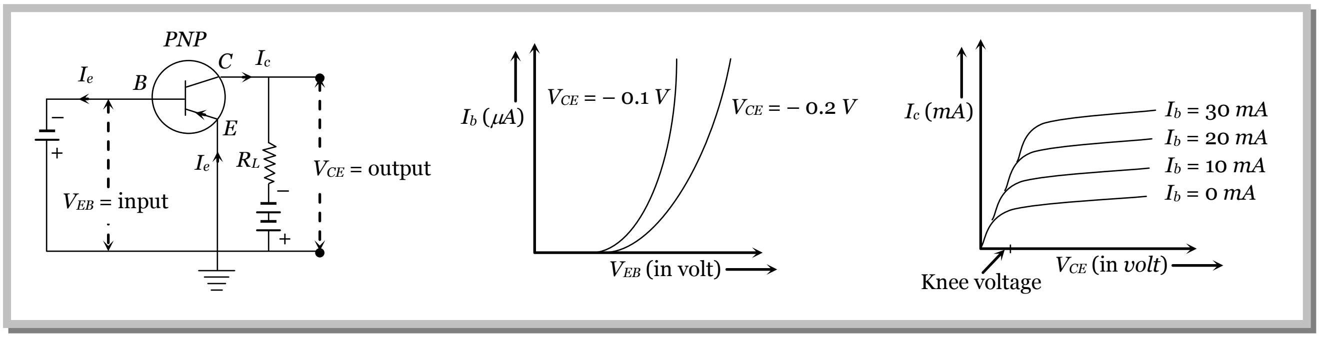
(2) **Characteristics of transistors :** A transistor can be connected in a circuit in the following three different configurations.

(i) Common base (CB) (ii) Common emitter (CE) (iii) Common collector (CC)

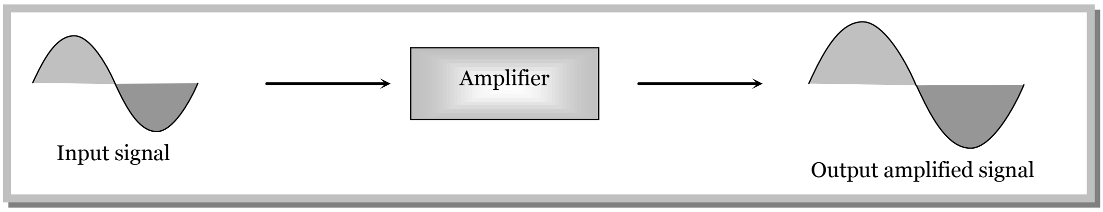
(i) CB characteristics : The graphs between voltages and currents when base of a transistor is common to input and output circuits are known as CB characteristic of a transistor.



(ii) CE characteristics : The graphs between voltages an d currents when emitter of a transistor is common to input and output circuits are known as CE characteristics of a transistor.

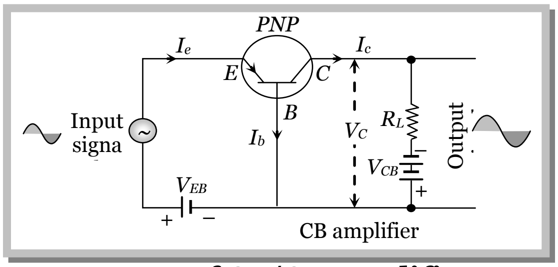


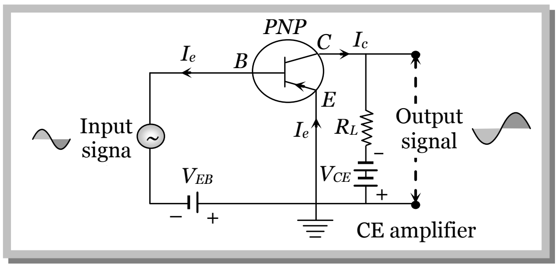
(3) **Transistor as an amplifier :** A device which increases the amplitude of the input signal is called amplifier.



The transistor can be used as an amplifier in the following three configuration

(i) CB amplifier (ii) CE amplifier (iii) CC amplifier





(4) **Parameters of CE/CB amplifiers**

Transistor as C.E. amplifier	**Transistor as C.B. amplifier**
(i) Current gain (α) (a) $\alpha_{ac} = \dfrac{\text{Small change in collector current } (\Delta i_c)}{\text{Small change in collector current } (\Delta i_e)}$; V_B (constant) (b) $\alpha_{dc}\,(\text{or}\,\alpha) = \dfrac{\text{Collector current}(i_c)}{\text{Emitter current } (i_e)}$ valve of α_{dc} lies between 0.95 to 0.99	(i) Current gain (β) (a) $\beta_{ac} = \left(\dfrac{\Delta i_c}{\Delta i_b}\right)$ V_{CE} = constant (b) $\beta_{dc} = \dfrac{i_c}{i_b}$ valve of β_{ac} lies between 15 and 20
(ii) Voltage gain $A_v = \dfrac{\text{Change in output voltage}(\Delta V_o)}{\text{Change in input voltage } (\Delta V_i)}$ $\Rightarrow A_v = \alpha_{ac} \times$ Resistance gain	(ii) Voltage gain $A_v = \dfrac{\Delta V_o}{\Delta V_i} = \beta_{ac} \times \text{Resistance gain}$
(iii) Power gain $= \dfrac{\text{Change in output power}(\Delta P_o)}{\text{Change in input power}(\Delta P_c)}$ $\Rightarrow$ Power gain $= \alpha_{ac}^2 \times$ Resistance gain	(iii) Power gain $= \dfrac{\Delta P_o}{\Delta P_i} = \beta_{ac}^2 \times \text{Resistance gain}$

Note : ❑ **Trans conductance (*gm*) :** The ratio of the change in collector current to the change in emitter base voltage is called trans conductance. *i.e.* $g_m = \frac{\Delta i_c}{\Delta V_{EB}}$. Also $g_m = \frac{A_V}{R_L}$; R_L = Load resistance

(5) **Relation between α and β :** $\beta = \frac{\alpha}{1-\alpha}$ or $\alpha = \frac{\beta}{1+\beta}$

(6) **Comparison between *CB*, *CE* and *CC* amplifier**

S.No.	Characteristic	Amplifier		
		CB	*CE*	*CC*
(i)	Input resistance (R_i)	≈ 50 to 200 Ω low	≈ 1 to 2 *k*Ω medium	≈ 150 – 800 *k*Ω high
(ii)	Output resistance (R_o)	≈ 1 – 2 *k*Ω high	≈ 50 *k*Ω medium	≈ *k*Ω low
(iii)	Current gain	0.8 – 0.9 low	20 – 200 high	20 – 200 high
(iv)	Voltage gain	Medium	High	Low
(v)	Power gain	Medium	High	Low
(vi)	Phase difference between input and output voltages	Zero	180°	Zero
(vii)	Used as amplifier for	current	Power	Voltage

Example

Example: 2 A *Ge* specimen is doped with *Al*. The concentration of acceptor atoms is ~10^{21} *atoms/m*^{3}. Given that the intrinsic concentration of electron hole pairs is ~ $10^{19}/m^3$, the concentration of electrons in the specimen is **[AIIMS 2004]**

(a) $10^{17}/m^3$ (b) $10^{15}/m^3$ (c) $10^4/m^3$ (d) $10^2/m^3$

Solution : (a) $n_i^2 = n_h n_e \Rightarrow (10^{19})^2 = 10^{21} \times n_e \Rightarrow n_e = 10^{17}/m^3$.

Example: 3 A silicon specimen is made into a *P*-type semi-conductor by doping, on an average, one Indium atom per 5×10^7 silicon atoms. If the number density of atoms in the silicon specimen is 5×10^{28} *atoms/m*^{3}, then the number of acceptor atoms in silicon will be

(a) 2.5×10^{30} *atoms/cm*^{3} (b) 1.0×10^{13} *atoms/cm*^{3} (c) 1.0×10^{15} *atoms/cm*^{3} (d) 2.5×10^{36} *atoms/cm*^{3}

Solution : (c) Number density of atoms in silicon specimen = 5×10^{28} *atom/m*^{3} = 5×10^{22} *atom/cm*^{3}

Since one atom of indium is doped in 5×10^7 *Si* atom. So number of indium atoms doped per *cm*$^{-3}$ of silicon.

$$n = \frac{5\times10^{22}}{5\times10^7} = 1\times10^{15}\ atom/cm^3.$$

Example: 4 A *P*-type semiconductor has acceptor levels 57 *meV* above the valence band. The maximum wavelength of light required to create a hole is (Planck's constant $h = 6.6\times10^{-34}$ *J-s*)

(a) 57 Å (b) 57×10^{-3} Å (c) 217100 Å (d) 11.61×10^{-33} Å

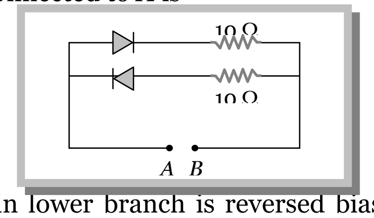
Solution : (c) $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.6\times10^{-34}\times3\times10^8}{57\times10^{-3}\times1.6\times10^{-19}} = 217100$ Å.

Example: 5 A potential barrier of 0.50*V* exists across a *P-N* junction. If the depletion region is $5.0\times10^{-7}\ m$ wide, the intensity of the electric field in this region is **[UPSEAT 2002]**

(a) $1.0\times10^6\ V/m$ (b) $1.0\times10^5\ V/m$ (c) $2.0\times10^5\ V/m$ (d) $2.0\times10^6\ V/m$

Solution : (a) $E = \frac{V}{d} = \frac{0.50}{5 \times 10^{-7}} = 1 \times 10^6\ V/m.$

***Example*: 6** A 2V battery is connected across the points A and B as shown in the figure given below. Assuming that the resistance of each diode is zero in forward bias and infinity in reverse bias, the current supplied by the battery when its positive terminal is connected to A is

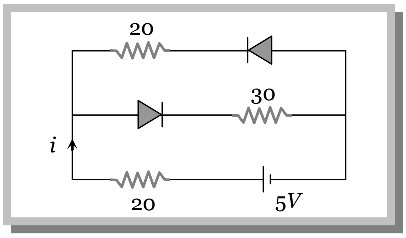


(a) 0.2 A
(b) 0.4 A
(c) Zero
(d) 0.1 A

Solution : (a) Since diode in upper branch is forward biased and in lower branch is reversed biased. So current through circuit $i = \frac{V}{R + r_d}$; here r_d = diode resistance in forward biasing = 0

So $i = \frac{V}{R} = \frac{2}{10} = 0.2 A$.

***Example*: 7** Current in the circuit will be

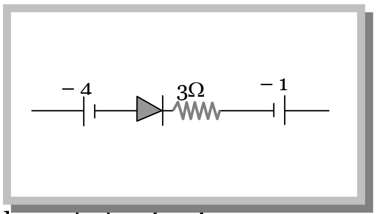


(a) $\frac{5}{40} A$
(b) $\frac{5}{50} A$
(c) $\frac{5}{10} A$
(d) $\frac{5}{20} A$

Solution : (b) The diode in lower branch is forward biased and diode in upper branch is reverse biased

$\therefore\ i = \frac{5}{20 + 30} = \frac{5}{50} A$

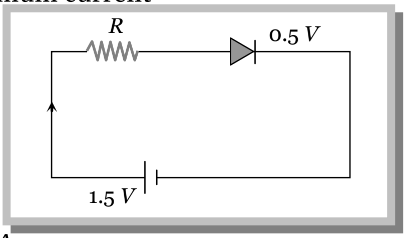
***Example*: 8** Find the magnitude of current in the following circuit **[RPMT 2001]**



(a) 0
(b) 1 *amp*
(c) 0.1 *amp*
(d) 0.2 *amp*

Solution : (a) Diode is reverse biased. Therefore no current will flow through the circuit.

***Example*: 9** The diode used in the circuit shown in the figure has a constant voltage drop of 0.5 V at all currents and a maximum power rating of 100 *milliwatts*. What should be the value of the resistor R, connected in series with the diode for obtaining maximum current



(a) 1.5 Ω
(b) 5 Ω
(c) 6.67 Ω
(d) 200 Ω

Solution : (b) The current through circuit $i = \frac{P}{V} = \frac{100 \times 10^{-3}}{0.5} = 0.2 A$

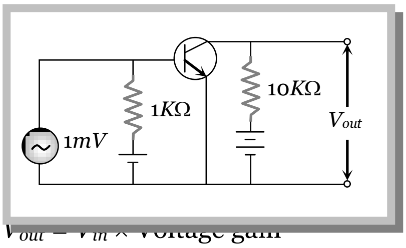
$\therefore$ voltage drop across resistance = 1.5 – 0.5 = 1 V $\Rightarrow R = \frac{1}{0.2} = 5\,\Omega$

***Example*: 10** For a transistor amplifier in common emitter configuration for load impedance of 1 $k\Omega$ (h_{fe} = 50 and h_{oe} = 25) the current gain is

(a) – 5.2 (b) – 15.7 (c) – 24.8 (d) – 48.78

Solution : (d) In common emitter configuration current gain $A_i = \frac{-h_{fe}}{1 + h_{oe} R_L} = \frac{-50}{1 + 25 \times 10^{-6} \times 10^3} = -48.78.$

***Example*: 11** In the following common emitter configuration an *NPN* transistor with current gain $\beta = 100$ is used. The output voltage of the amplifier will be **[AIIMS 2003]**



(a) 10 *mV*
(b) 0.1 *V*
(c) 1.0 *V*
(d) 10 *V*

Solution : (c) Voltage gain $= \frac{\text{Output voltage}}{\text{Input voltage}}$ $\Rightarrow V_{out} = V_{in} \times$ Voltage gain

$\Rightarrow V_{out} = V_{in} \times$ Current gain $\times$ Resistance gain $= V_{in} \times \beta \times \frac{R_L}{R_{BE}} = 10^{-3} \times 100 \times \frac{10}{1} = 1V.$

***Example*: 12** While a collector to emitter voltage is constant in a transistor, the collector current changes by 8.2 *mA* when the emitter current changes by 8.3 *mA*. The value of forward current ratio h_{fe} is

(a) 82 (b) 83 (c) 8.2 (d) 8.3

Solution : (a) $h_{fe} = \left(\frac{\Delta i_c}{\Delta i_b}\right)_{V_{ce}} = \frac{8.2}{8.3 - 8.2} = 82$

***Example*: 13** The transfer ratio of a transistor is 50. The input resistance of the transistor when used in the common-emitter configuration is 1 *K*Ω. The peak value for an ac input voltage of 0.01 *V* peak is

(a) 100 μA (b) 0.01 *mA* (c) 0.25 *mA* (d) 500 μA

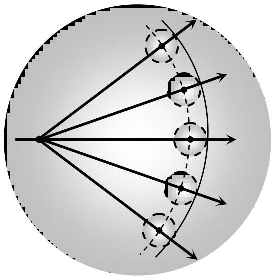
Solution : (d) $i_c = \beta i_b = \beta \times \frac{V_i}{R_i} = 50 \times \frac{0.01}{1000} = 500 \times 10^{-6} A = 500 \mu A$

***Example*: 14** In a common base amplifier circuit, calculate the change in base current if that in the emitter current is 2 *mA* and $\alpha = 0.98$ **[BHU 1995]**

(a) 0.04 *mA* (b) 1.96 *mA* (c) 0.98 *mA* (d) 2 *mA*

Solution : (a) $\Delta i_c = \alpha \Delta i_e = 0.98 \times 2 = 196\, mA$

$\therefore \Delta i_b = \Delta i_e - \Delta i_c = 2 - 1.96 = 0.04\, mA.$



Wave Optics

Light Propagation

Light is a form of energy which generally gives the sensation of sight.

(1) **Different theories**

(2) **Optical phenomena explained (√) or not explained (×) by the different theories of light**

(3) **Wave front**

(i) Suggested by Huygens

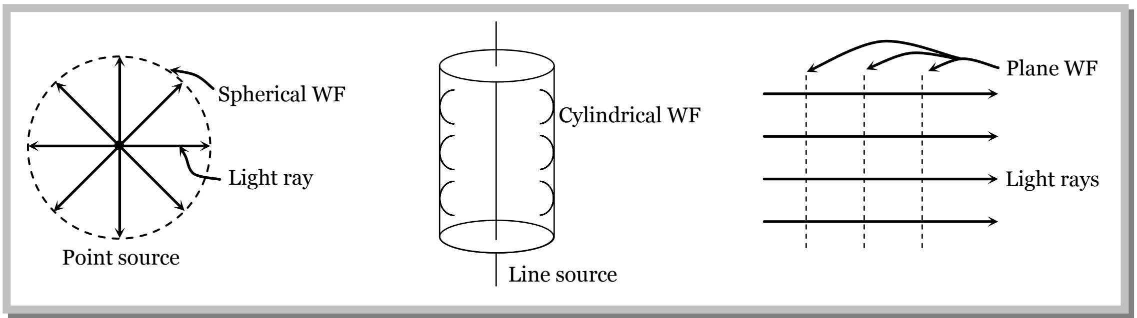
(ii) The locus of all particles in a medium, vibrating in the same phase is called Wave Front (WF)

(iii) The direction of propagation of light (ray of light) is perpendicular to the WF.

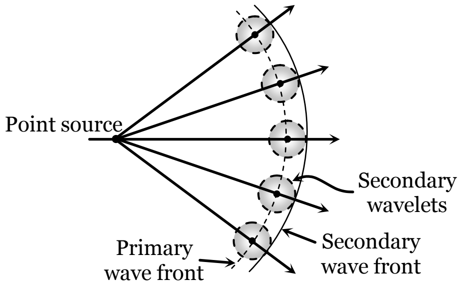
Newtons corpuscular theory	Huygen's wave theory	Maxwell's EM wave theory	Einstein's quantum theory	de-Broglie's dual theory of light
(i) Based on Rectilinear propagation of light	(i) Light travels in a hypothetical medium ether (high elasticity very low density) as waves	(i) Light travels in the form of EM waves with speed in free space $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	(i) Light is produced, absorbed and propagated as packets of energy called photons	(i) Light propagates both as particles as well as waves
(ii) Light propagates in the form of tiny particles called Corpuscles. Colour of light is due to different size of corpuscles	(ii) He proposed that light waves are of longitudinal nature. Later on it was found that they are transverse	(ii) EM waves consists of electric and magnetic field oscillation and they do not require material medium to travel	(ii) Energy associated with each photon $E = h\nu = \frac{hc}{\lambda}$ h = planks constant $= 6.6 \times 10^{-34}\ J\text{-sec}$ ν = frequency λ = wavelength	(ii) Wave nature of light dominates when light interacts with light. The particle nature of light dominates when the light interacts with matter (micro-scopic particles)

S. No.	Phenomena	Theory				
		Corpuscular	Wave	E.M. wave	Quantum	Dual
(i)	Rectilinear Propagation	√	√	√	√	√
(ii)	Reflection	√	√	√	√	√
(iii)	Refraction	√	√	√	√	√
(iv)	Dispersion	×	√	√	×	√
(v)	Interference	×	√	√	×	√
(vi)	Diffraction	×	√	√	×	√
(vii)	Polarisation	×	√	√	×	√
(viii)	Double refraction	×	√	√	×	√
(ix)	Doppler's effect	×	√	√	×	√
(x)	Photoelectric effect	×	×	×	√	√

(iv) **Types of wave front**



(v) Every point on the given wave front acts as a source of new disturbance called secondary wavelets. Which travel in all directions with the velocity of light in the medium.



A surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new wave front at that instant. This is called secondary wave front

Note : ❑ Wave front always travels in the forward direction of the medium.

❑ Light rays is always normal to the wave front.

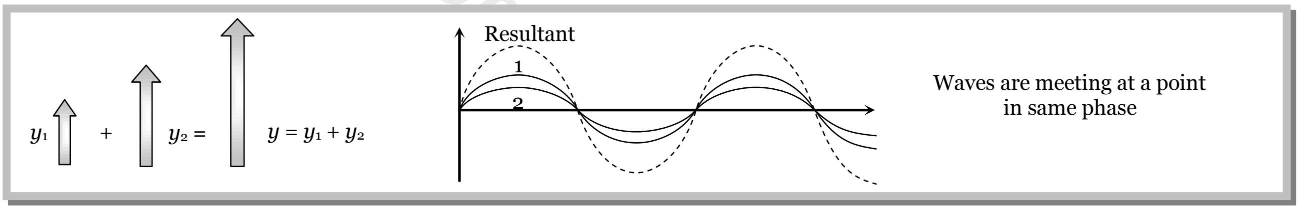
❑ The phase difference between various particles on the wave front is zero.

Principle of Super Position

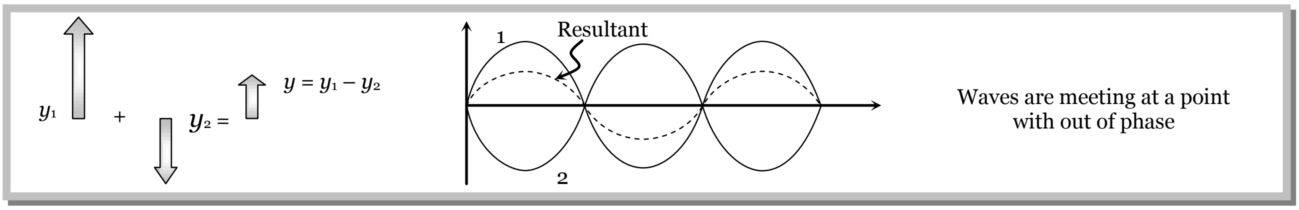
When two or more than two waves superimpose over each other at a common particle of the medium then the resultant displacement (y) of the particle is equal to the vector sum of the displacements (y_1 and y_2) produced by individual waves. *i.e.* $\vec{y} = \vec{y}_1 + \vec{y}_2$

(1) **Graphical view :**

(i)



(ii)

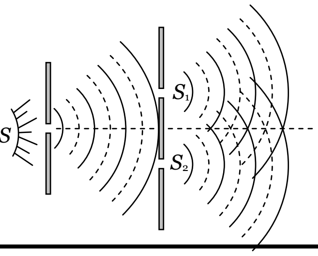
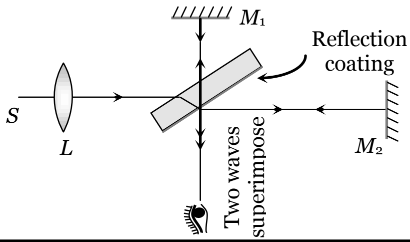


(2) **Phase / Phase difference / Path difference / Time difference**

(i) Phase : The argument of sine or cosine in the expression for displacement of a wave is defined as the phase. For displacement $y = a \sin \omega t$; term ωt = phase or instantaneous phase

(ii) Phase difference (ϕ) : The difference between the phases of two waves at a point is called phase difference *i.e.* if $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$ so phase difference = ϕ

(iii) Path difference (Δ) : The difference in path length's of two waves meeting at a point is called path difference between the waves at that point. Also $\Delta = \frac{\lambda}{2\pi} \times \phi$

Division of wave front	Division of amplitude
The light source is narrow	Light sources is extended. Light wave partly reflected (50%) and partly transmitted (50%)
The wave front emitted by a narrow source is divided in two parts by reflection of refraction.	The amplitude of wave emitted by an extend source of light is divided in two parts by partial reflection and partial refraction.
The coherent sources obtained are imaginary *e.g.* Fresnel's biprism, Llyod's mirror Youngs' double slit *etc.*	The coherent sources obtained are real *e.g.* Newtons rings, Michelson's interferrometer colours in thin films
	

(iv) Time difference (*T.D.*) : Time difference between the waves meeting at a point is $\boldsymbol{T.D.} = \dfrac{\boldsymbol{T}}{\boldsymbol{2\pi}} \times \boldsymbol{\phi}$

(3) **Resultant amplitude and intensity**

If suppose we have two waves $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$; where a_1, a_2 = Individual amplitudes, ϕ = Phase difference between the waves at an instant when they are meeting a point. I_1, I_2 = Intensities of individual waves

Resultant amplitude : After superimposition of the given waves resultant amplitude (or the amplitude of resultant wave) is given by $\boldsymbol{A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}}$

For the interfering waves $y_1 = a_1 \sin\omega t$ and $y_2 = a_2 \cos\omega t$, Phase difference between them is 90°. So resultant amplitude $A = \sqrt{a_1^2 + a_2^2}$

Resultant intensity : As we know intensity ∝ (Amplitude)² $\Rightarrow I_1 = ka_1^2, I_2 = ka_2^2$ and $I = kA^2$ (*k* is a proportionality constant). Hence from the formula of resultant amplitude, we get the following formula of resultant intensity $\boldsymbol{I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi}$

Note : ❑ The term $2\sqrt{I_1I_2}\cos\phi$ is called interference term. For incoherent interference this term is zero so resultant intensity $I = I_1 + I_2$

(4) **Coherent sources**

The sources of light which emits continuous light waves of the same wavelength, same frequency and in same phase or having a constant phase difference are called coherent sources.

Two coherent sources are produced from a single source of light by adopting any one of the following two methods

Note : ❑ Laser light is highly coherent and monochromatic.

❑ Two sources of light, whose frequencies are not same and phase difference between the waves emitted by them does not remain constant *w.r.t.* time are called non-coherent.

❑ The light emitted by two independent sources (candles, bulbs *etc.*) is non-coherent and interference phenomenon cannot be produced by such two sources.

❑ The average time interval in which a photon or a wave packet is emitted from an atom is defined as the **time of coherence**. It is $\tau_c = \dfrac{L}{c} = \dfrac{\text{Distance of coherence}}{\text{Velocity of light}}$, it's value is of the order of 10^{-10} *sec*.

Interference of Light

When two waves of exactly same frequency (coming from two coherent sources) travels in a medium, in the same direction simultaneously then due to their superposition, at some points intensity of light is maximum while at some other points intensity is minimum. This phenomenon is called Interference of light.

(1) **Types :** It is of following two types

Constructive interference	Destructive interference
(i) When the waves meets a point with same phase, constructive interference is obtained at that point (*i.e.* maximum light)	(i) When the wave meets a point with opposite phase, destructive interference is obtained at that point (*i.e.* minimum light)
(ii) Phase difference between the waves at the point of observation $\phi = 0^o$ or $2n\pi$	(ii) $\phi = 180^o$ or $(2n-1)\pi$; $n = 1, 2, ...$ or $(2n+1)\pi$; $n = 0,1,2.....$
(iii) Path difference between the waves at the point of observation $\Delta = n\lambda$ (*i.e.* even multiple of $\lambda/2$)	(iii) $\Delta = (2n-1)\frac{\lambda}{2}$ (*i.e.* odd multiple of $\lambda/2$)
(iv) Resultant amplitude at the point of observation will be maximum $a_1 = a_2 \Rightarrow A_{min} = 0$ If $a_1 = a_2 = a_0 \Rightarrow A_{max} = 2a_0$	(iv) Resultant amplitude at the point of observation will be minimum $A_{min} = a_1 - a_2$ If $a_1 = a_2 \Rightarrow A_{min} = 0$
(v) Resultant intensity at the point of observation will be maximum $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{max} = 2I_0$	(v) Resultant intensity at the point of observation will be minimum $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{min} = 0$

(2) **Resultant intensity due to two identical waves :**

For two coherent sources the resultant intensity is given by $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

For identical source $I_1 = I_2 = I_0 \Rightarrow I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi = 4I_0\cos^2\frac{\phi}{2}$ $\quad$ $[1 + \cos\theta = 2\cos^2\frac{\theta}{2}]$

Note : ❑ In interference redistribution of energy takes place in the form of maxima and minima.

❑ Average intensity : $I_{av} = \frac{I_{max} + I_{min}}{2} = I_1 + I_2 = a_1^2 + a_2^2$

❑ Ratio of maximum and minimum intensities :

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2}+1}{\sqrt{I_1/I_2}-1}\right)^2 = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2 = \left(\frac{a_1/a_2+1}{a_1/a_2-1}\right)^2$$ also

$$\sqrt{\frac{I_1}{I_2}} = \frac{a_1}{a_2} = \left(\frac{\sqrt{\frac{I_{max}}{I_{min}}}+1}{\sqrt{\frac{I_{max}}{I_{min}}}-1}\right)$$

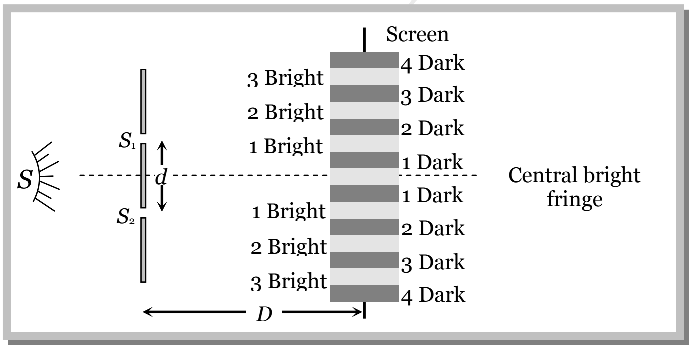
- If two waves having equal intensity ($I_1 = I_2 = I_0$) meets at two locations P and Q with path difference Δ_1 and Δ_2 respectively then the ratio of resultant intensity at point P and Q will be

$$\frac{I_P}{I_Q} = \frac{\cos^2\frac{\phi_1}{2}}{\cos^2\frac{\phi_2}{2}} = \frac{\cos^2\left(\frac{\pi\Delta_1}{\lambda}\right)}{\cos^2\left(\frac{\pi\Delta_2}{\lambda}\right)}$$

Young's Double Slit Experiment (YDSE)

Monochromatic light (single wavelength) falls on two narrow slits S_1 and S_2 which are very close together acts as two coherent sources, when waves coming from two coherent sources (S_1, S_2) superimposes on each other, an interference pattern is obtained on the screen. In YDSE alternate bright and dark bands obtained on the screen. These bands are called Fringes.

d = Distance between slits
D = Distance between slits and screen
λ = Wavelength of monochromatic light emitted from source



(1) Central fringe is always bright, because at central position $\phi = 0^o$ or $\Delta = 0$

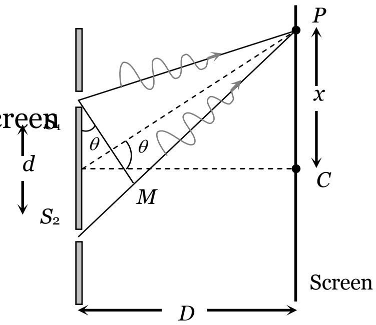
(2) The fringe pattern obtained due to a slit is more bright than that due to a point.

(3) If the slit widths are unequal, the minima will not be complete dark. For very large width uniform illumination occurs.

(4) If one slit is illuminated with red light and the other slit is illuminated with blue light, no interference pattern is observed on the screen.

(5) If the two coherent sources consist of object and it's reflected image, the central fringe is dark instead of bright one.

(6) **Path difference**



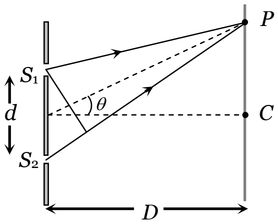
Path difference between the interfering waves meeting at a point P on the screen is given by $\Delta = \frac{xd}{D} = d\sin\theta$

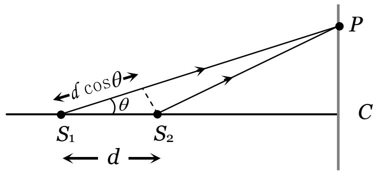
where x is the position of point P from central maxima.

For maxima at P: $\Delta = n\lambda$; where n = 0, ± 1, ± 2, ...

and For minima at P: $\Delta = \frac{(2n-1)\lambda}{2}$; where $n = \pm 1, \pm 2, ...$

Note : ❑ If the slits are vertical, the path difference (Δ) is $d\sin\theta$, so as θ increases, Δ also increases. But if slits are horizontal path difference is $d\cos\theta$, so as θ increases, Δ decreases.





(7) **More about fringe**

(i) All fringes are of

equal width. Width of each fringe is $\boldsymbol{\beta = \frac{\lambda D}{d}}$ and angular fringe width $\boldsymbol{\theta = \frac{\lambda}{d} = \frac{\beta}{D}}$

(ii) If the whole YDSE set up is taken in another medium then λ changes so β changes

e.g. in water $\lambda_w = \frac{\lambda_a}{\mu_w} \Rightarrow \beta_w = \frac{\beta_a}{\mu_w} = \frac{3}{4}\beta_a$

(iii) Fringe width $\beta \propto \frac{1}{d}$ *i.e.* with increase in separation between the sources, β decreases.

(iv) Position of n^{th} bright fringe from central maxima $\boldsymbol{x_n = \frac{n\lambda D}{d} = n\beta}$; $n = 0, 1, 2 ....$

(v) Position of n^{th} dark fringe from central maxima $\boldsymbol{x_n = \frac{(2n-1)\lambda D}{2d} = \frac{(2n-1)\beta}{2}}$; $n = 1, 2, 3 ....$

(vi) In YDSE, if n_1 fringes are visible in a field of view with light of wavelength λ_1, while n_2 with light of wavelength λ_2 in the same field, then $\boldsymbol{n_1\lambda_1 = n_2\lambda_2}$.

(vii) Separation (Δx) between fringes

Between n^{th} bright and m^{th} bright fringes $(n > m)$	Between n^{th} bright and m^{th} dark fringe
$\Delta x = (n-m)\beta$	(a) If $n > m$ then $\Delta x = \left(n - m + \frac{1}{2}\right)\beta$ (b) If $n < m$ then $\Delta x = \left(m - n - \frac{1}{2}\right)\beta$

(8) **Identification of central bright fringe**

To identify central bright fringe, monochromatic light is replaced by white light. Due to overlapping central maxima will be white with red edges. On the other side of it we shall get a few coloured band and then uniform illumination.

(9) **Condition for observing sustained interference**

(i) The initial phase difference between the interfering waves must remain constant : Otherwise the interference will not be sustained.

(ii) The frequency and wavelengths of two waves should be equal : If not the phase difference will not remain constant and so the interference will not be sustained.

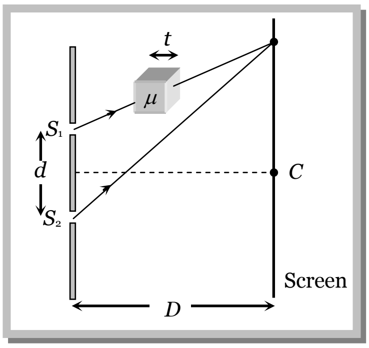
(iii) The light must be monochromatic : This eliminates overlapping of patterns as each wavelength corresponds to one interference pattern.

(iv) The amplitudes of the waves must be equal : This improves contrast with $I_{max} = 4I_0$ and $I_{min} = 0$.

(v) The sources must be close to each other : Otherwise due to small fringe width $\left(\beta \propto \frac{1}{d}\right)$ the eye can not resolve fringes resulting in uniform illumination.

(10) **Shifting of fringe pattern in YDSE**

If a transparent thin film of mica or glass is put in the path of one of the waves, then the whole fringe pattern gets shifted.



If film is put in the path of upper wave, fringe pattern shifts upward and if film is placed in the path of lower wave, pattern shift downward.

$$\text{Fringe shift} = \frac{D}{d}(\mu - 1)t = \frac{\beta}{\lambda}(\mu - 1)t$$

⇒ Additional path difference $= (\mu - 1)t$

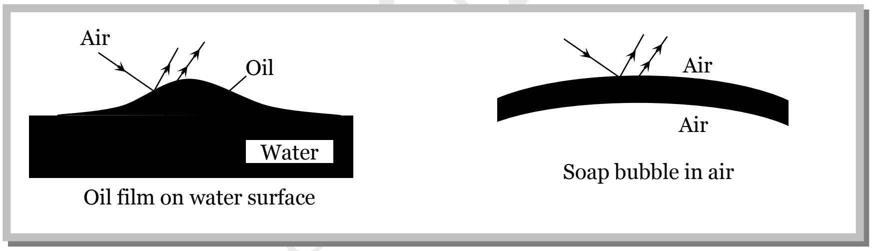
⇒ If shift is equivalent to n fringes then $n = \frac{(\mu - 1)t}{\lambda}$ or $t = \frac{n\lambda}{(\mu - 1)}$

⇒ Shift is independent of the order of fringe (*i.e.* shift of zero order maxima = shift of n^{th} order maxima.

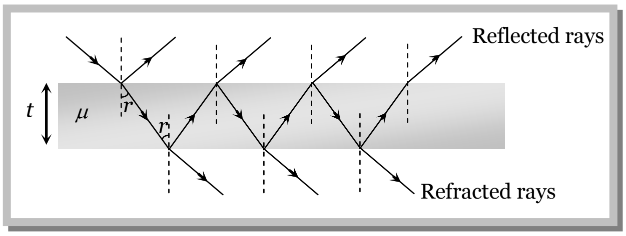
⇒ Shift is independent of wavelength.

Illustrations of Interference

Interference effects are commonly observed in thin films when their thickness is comparable to wavelength of incident light (If it is too thin as compared to wavelength of light it appears dark and if it is too thick, this will result in uniform illumination of film). Thin layer of oil on water surface and soap bubbles shows various colours in white light due to interference of waves reflected from the two surfaces of the film.



(1) **Thin films :** In thin films interference takes place between the waves reflected from it's two surfaces and waves refracted through it.



Interference in reflected light	Interference in refracted light
Condition of constructive interference (maximum intensity)	Condition of constructive interference (maximum intensity)
$\Delta = 2\mu\ t\cos r = (2n \pm 1)\frac{\lambda}{2}$	$\Delta = 2\mu t \cos r = (2n)\frac{\lambda}{2}$
For normal incidence $r = 0$	For normal incidence
so $2\mu t = (2n \pm 1)\frac{\lambda}{2}$	$2\mu t = n\lambda$

Condition of destructive interference (minimum intensity) $\Delta = 2\mu t \cos r = (2n)\frac{\lambda}{2}$ For normal incidence $2\mu t = n\lambda$	Condition of destructive interference (minimum intensity) $\Delta = 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$ For normal incidence $2\mu t = (2n \pm 1)\frac{\lambda}{2}$

Doppler's Effect in Light

The phenomenon of apparent change in frequency (or wavelength) of the light due to relative motion between the source of light and the observer is called Doppler's effect.

If $\nu =$ actual frequency, $\nu' =$ Apparent frequency, v = speed of source *w.r.t* stationary observer, c = speed of light

Source of light moves towards the stationary observer ($v << c$)	**Source of light moves away from the stationary observer ($v << c$)**
(i) Apparent frequency $\nu' = \nu\left(1 + \frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1 - \frac{v}{c}\right)$	(i) Apparent frequency $\nu' = \nu\left(1 - \frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1 + \frac{v}{c}\right)$
(ii) Doppler's shift : Apparent wavelength < actual wavelength, So spectrum of the radiation from the source of light shifts towards the red end of spectrum. This is called Red shift Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$	(ii) Doppler's shift : Apparent wavelength > actual wavelength, So spectrum of the radiation from the source of light shifts towards the violet end of spectrum. This is called Violet shift Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$

Note : ❑ Doppler's shift $(\Delta\lambda)$ and time period of rotation (T) of a star relates as $\Delta\lambda = \frac{\lambda}{c} \times \frac{2\pi r}{T}$; r = radius of star.

Applications of Doppler effect

(i) Determination of speed of moving bodies (aeroplane, submarine etc) in RADAR and SONAR.

(ii) Determination of the velocities of stars and galaxies by spectral shift.

(iii) Determination of rotational motion of sun.

(iv) Explanation of width of spectral lines.

(v) Tracking of satellites. (vi) In medical sciences in echo cardiogram, sonography *etc.*

Concepts

☞ *The angular thickness of fringe width is defined as* $\delta = \frac{\beta}{D} = \frac{\lambda}{d}$, *which is independent of the screen distance D.*

☞ *Central maxima means the maxima formed with zero optical path difference. It may be formed anywhere on the screen.*

☞ *All the wavelengths produce their central maxima at the same position.*

☞ *The wave with smaller wavelength from its maxima before the wave with longer wavelength.*

☞ *The first maxima of violet colour is closest and that for the red colour is farthest.*

- *Fringes with blue light are thicker than those for red light.*
- *In an interference pattern, whatever energy disappears at the minimum, appears at the maximum.*
- *In YDSE, the nth maxima always comes before the nth minima.*
- *In YDSE, the ratio* $\frac{I_{max}}{I_{min}}$ *is maximum when both the sources have same intensity.*
- *For two interfering waves if initial phase difference between them is* ϕ_0 *and phase difference due to path difference between them is* ϕ'. *Then total phase difference will be* $\phi = \phi_0 + \phi' = \phi_0 + \frac{2\pi}{\lambda}\Delta$.
- *Sometimes maximm number of maximas or minimas are asked in the question which can be obtained on the screen. For this we use the fact that value of sin* θ *(or cos* θ*) can't be greater than 1. For example in the first case when the slits are vertical*

$$\sin\theta = \frac{n\lambda}{d} \qquad \text{(for maximum intensity)}$$

$$\because \quad \sin\theta \not> 1 \qquad \therefore \quad \frac{n\lambda}{d} \not> 1 \quad \text{or} \quad n \not> \frac{d}{\lambda}$$

Suppose in some question d/λ *comes out say 4.6, then total number of maximuas on the screen will be 9. Corresponding to* $n = 0, \pm 1, \pm 2, \pm 3$ *and* ± 4.

- ***Shape of wave front***

If rays are parallel, wave front is plane. If rays are converging wave front is spherical of decreasing radius. If rays are diverging wave front is spherical of increasing radius.

Example

Example: 1 If two light waves having same frequency have intensity ratio 4 : 1 and they interfere, the ratio of maximum to minimum intensity in the pattern will be

(a) 9 : 1 (b) 3 : 1 (c) 25 : 9 (d) 16 : 25

Solution: (a) By using $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right)^2 = \left(\frac{\sqrt{\frac{4}{1}}+1}{\sqrt{\frac{4}{1}}-1}\right)^2 = \frac{9}{1}$.

Example: 2 In Young's double slit experiment using sodium light (λ = 5898Å), 92 fringes are seen. If given colour (λ = 5461Å) is used, how many fringes will be seen

(a) 62 (b) 67 (c) 85 (d) 99

Solution: (d) By using $n_1\lambda_1 = n_2\lambda_2 \Rightarrow 92 \times 5898 = n_2 \times 5461 \Rightarrow n_2 = 99$

Example: 3 Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point A and π at point B. Then the difference between the resultant intensities at A and B is

(a) $2I$ (b) $4I$ (c) $5I$ (d) $7I$

Solution: (b) By using $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

At point A : Resultant intensity $I_A = I + 4I + 2\sqrt{I \times 4I}\cos\frac{\pi}{2} = 5I$

At point B : Resultant intensity $I_B = I + 4I + 2\sqrt{I \times 4I}\cos\pi = I$. Hence the difference $= I_A - I_B = 4I$

***Example*: 4** If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point, the amplitude of the resulting wave will be about **[MP PMT 2000]**

(a) 7 (b) 6 (c) 5 (d) 3.

Solution: (b) By using $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$ $\Rightarrow$ $A = \sqrt{(4)^2 + (3)^2 + 2\times 4\times 3\cos\frac{\pi}{3}} = \sqrt{37} \approx 6$.

***Example*: 5** Two waves being produced by two sources S_1 and S_2. Both sources have zero phase difference and have wavelength λ. The destructive interference of both the waves will occur of point P if $(S_1P - S_2P)$ has the value **[MP PET 1987]**

(a) 5λ (b) $\frac{3}{4}\lambda$ (c) 2λ (d) $\frac{11}{2}\lambda$

Solution: (d) For destructive interference, path difference the waves meeting at P (*i.e.* $S_1P - S_2P$) must be odd multiple of $\lambda/2$. Hence option (d) is correct.

***Example*: 6** Two interfering wave (having intensities are 9*I* and 4*I*) path difference between them is 11 λ. The resultant intensity at this point will be

(a) I (b) $9I$ (c) $4I$ (d) $25I$

Solution: (d) Path difference $\Delta = \frac{\lambda}{2\pi}\times\phi$ $\Rightarrow$ $\frac{2\pi}{\lambda}\times 11\lambda = 22\pi$ *i.e.* constructive interference obtained at the same point

So, resultant intensity $I_R = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{9I} + \sqrt{4I})^2 = 25I$.

***Example*: 7** In interference if $\frac{I_{\max}}{I_{\min}} = \frac{144}{81}$ then what will be the ratio of amplitudes of the interfering wave

(a) $\frac{144}{81}$ (b) $\frac{7}{1}$ (c) $\frac{1}{7}$ (d) $\frac{12}{9}$

Solution: (b) By using $\frac{a_1}{a_2} = \left(\frac{\sqrt{\frac{I_{\max}}{I_{\min}}} + 1}{\sqrt{\frac{I_{\max}}{I_{\min}}} - 1}\right) = \left(\frac{\sqrt{\frac{144}{81}} + 1}{\sqrt{\frac{144}{81}} - 1}\right) = \left(\frac{\frac{12}{9} + 1}{\frac{12}{5} - 1}\right) = \frac{7}{1}$

***Example*: 8** Two interfering waves having intensities x and y meets a point with time difference $3T/2$. What will be the resultant intensity at that point

(a) $(\sqrt{x} + \sqrt{y})$ (b) $(\sqrt{x} + \sqrt{y} + \sqrt{xy})$ (c) $x + y + 2\sqrt{xy}$ (d) $\frac{x+y}{2xy}$

Solution: (c) Time difference T.D. $= \frac{T}{2\pi}\times\phi \Rightarrow \frac{3T}{2} = \frac{T}{2\pi}\times\phi \Rightarrow \phi = 3\pi$; This is the condition of constructive interference.

So resultant intensity $I_R = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}$.

***Example*: 9** In Young's double-slit experiment, an interference pattern is obtained on a screen by a light of wavelength 6000 $\mathring{A}$, coming from the coherent sources S_1 and S_2. At certain point P on the screen third dark fringe is formed. Then the path difference $S_1P - S_2P$ in microns is **[EAMCET 2003]**

(a) 0.75 (b) 1.5 (c) 3.0 (d) 4.5

Solution: (b) For dark fringe path difference $\Delta = (2n-1)\frac{\lambda}{2}$; here $n = 3$ and $\lambda = 6000\times 10^{-10}\ m$

So $\Delta = (2 \times 3 - 1) \times \frac{6 \times 10^{-7}}{2} = 15 \times 10^{-7} m = 1.5\, microns.$

***Example*: 10** In a Young's double slit experiment, the slit separation is 1 *mm* and the screen is 1 *m* from the slit. For a monochromatic light of wavelength 500 *nm*, the distance of 3rd minima from the central maxima is

(a) 0.50 *mm* (b) 1.25 *mm* (c) 1.50 *mm* (d) 1.75 *mm*

Solution: (b) Distance of n^{th} minima from central maxima is given as $x = \frac{(2n-1)\lambda D}{2d}$

So here $x = \frac{(2 \times 3 - 1) \times 500 \times 10^{-9} \times 1}{2 \times 10^{-3}} = 1.25\, mm$

***Example*: 11** The two slits at a distance of 1 *mm* are illuminated by the light of wavelength 6.5×10^{-7} *m*. The interference fringes are observed on a screen placed at a distance of 1 *m*. The distance between third dark fringe and fifth bright fringe will be

[NCERT 1982; MP PET 1995; BVP 2003]

(a) 0.65 *mm* (b) 1.63 *mm* (c) 3.25 *mm* (d) 4.88 *mm*

Solution: (b) Distance between n^{th} bright and m^{th} dark fringe ($n > m$) is given as $x = \left(n - m + \frac{1}{2}\right)\beta = \left(n - m + \frac{1}{2}\right)\frac{\lambda D}{d}$

$\Rightarrow x = \left(5 - 3 + \frac{1}{2}\right) \times \frac{6.5 \times 10^{-7} \times 1}{1 \times 10^{-3}} = 1.63\, mm$.

***Example*: 12** The slits in a Young's double slit experiment have equal widths and the source is placed symmetrically relative to the slits. The intensity at the central fringes is I_0. If one of the slits is closed, the intensity at this point will be**[MP PMT 1999**

(a) I_0 (b) $I_0/4$ (c) $I_0/2$ (d) $4I_0$

Solution: (b) By using $I_R = 4I\cos^2\frac{\phi}{2}$ {where I = Intensity of each wave}

At central position $\phi = 0^o$, hence initially $I_0 = 4I$.

If one slit is closed, no interference takes place so intensity at the same location will be I only *i.e.* intensity become s $\frac{1}{4}$ *th* or $\frac{I_0}{4}$.

***Example*: 13** In double slit experiment, the angular width of the fringes is 0.20° for the sodium light (λ = 5890 $\mathring{A}$). In order to increase the angular width of the fringes by 10%, the necessary change in the wavelength is

(a) Increase of 589 $\mathring{A}$ (b) Decrease of 589 $\mathring{A}$ (c) Increase of 6479 $\mathring{A}$ (d) Zero

Solution: (a) By using $\theta = \frac{\lambda}{d} \Rightarrow \frac{\theta_1}{\theta_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{0.20^o}{(0.20^o + 10\%\text{ of } 0.20)} = \frac{5890}{\lambda_2} \Rightarrow \frac{0.20}{0.22} = \frac{5890}{\lambda_2} \Rightarrow \lambda_2 = 6479$

So increase in wavelength = 6479 – 5890 = 589 Å.

***Example*: 14** In Young's experiment, light of wavelength 4000 $\mathring{A}$ is used, and fringes are formed at 2 *metre* distance and has a fringe width of 0.6 *mm*. If whole of the experiment is performed in a liquid of refractive index 1.5, then width of fringe will be

[MP PMT 1994, 97]

(a) 0.2 *mm* (b) 0.3 *mm* (c) 0.4 *mm* (d) 1.2 *mm*

Solution: (c) $\beta_{medium} = \frac{\beta_{air}}{\mu} \Rightarrow \beta_{medium} = \frac{0.6}{1.5} = 0.4 mm$.

***Example*: 15** Two identical sources emitted waves which produces intensity of *k* unit at a point on screen where path difference is λ. What will be intensity at a point on screen at which path difference is $\lambda/4$ **[RPET 1996]**

(a) $\frac{k}{4}$ (b) $\frac{k}{2}$ (c) k (d) Zero

Solution: (b) By using phase difference $\phi = \frac{2\pi}{\lambda}(\Delta)$

For path difference λ, phase difference $\phi_1 = 2\pi$ and for path difference $\lambda/4$, phase difference $\phi_2 = \pi/2$.

Also by using $I = 4I_0 \cos^2\frac{\phi}{2} \Rightarrow \frac{I_1}{I_2} = \frac{\cos^2(\phi_1/2)}{\cos^2(\phi_2/2)} \Rightarrow \frac{k}{I_2} = \frac{\cos^2(2\pi/2)}{\cos^2\left(\frac{\pi/2}{2}\right)} = \frac{1}{1/2} \Rightarrow I_2 = \frac{k}{2}$.

***Example*: 16** A thin mica sheet of thickness $2\times10^{-6}\,m$ and refractive index (μ = 1.5) is introduced in the path of the first wave. The wavelength of the wave used is 5000Å. The central bright maximum will shift **[CPMT 1999]**

(a) 2 fringes upward (b) 2 fringes downward (c) 10 fringes upward (d) None of these

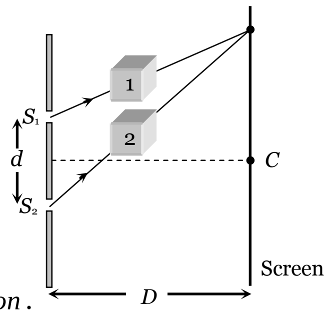
Solution: (a) By using shift $\Delta x = \frac{\beta}{\lambda}(\mu-1)t \Rightarrow \Delta x = \frac{\beta}{5000\times10^{-10}}(1.5-1)\times2\times10^{-6} = 2\beta$

Since the sheet is placed in the path of the first wave, so shift will be 2 fringes upward.

***Example*: 17** In a *YDSE* fringes are observed by using light of wavelength 4800 Å, if a glass plate (μ = 1.5) is introduced in the path of one of the wave and another plates is introduced in the path of the (μ = 1.8) other wave. The central fringe takes the position of fifth bright fringe. The thickness of plate will be

(a) 8 *micron* (b) 80 *micron* (c) 0.8 *micron* (d) None of these

Solution: (a) Shift due to the first plate $x_1 = \frac{\beta}{\lambda}(\mu_1-1)t$ (Upward)



and shift due to the second $x_2 = \frac{\beta}{\lambda}(\mu_2-1)t$ (Downward)

Hence net shift = $x_2 - x_1 = \frac{\beta}{\lambda}(\mu_2-\mu_1)t$

$\Rightarrow 5p = \frac{\beta}{\lambda}(1.8-1.5)t \Rightarrow t = \frac{5\lambda}{0.3} = \frac{5\times4800\times10^{-10}}{0.3} = 8\times10^{-6}\,m = 8\,micron$.

***Example*: 18** In young double slit experiment $\frac{d}{D} = 10^{-4}$ (d = distance between slits, D = distance of screen from the slits). At a point P on the screen resulting intensity is equal to the intensity due to individual slit I_0. Then the distance of point P from the central maxima is (λ = 6000 Å)

(a) 2 *mm* (b) 1 *mm* (c) 0.5 *mm* (d) 4 *mm*

Solution: (a) By using shift $I = 4I_0\cos^2(\phi/2) \Rightarrow I_0 = 4I_0\cos^2(\phi/2) \Rightarrow \cos(\phi/2) = \frac{1}{2}$ or $\frac{\phi}{2} = \frac{\pi}{3} \Rightarrow \phi = \frac{2\pi}{3}$

Also path difference $\Delta = \frac{xd}{D} = \frac{\lambda}{2\pi}\times\phi \Rightarrow x\times\left(\frac{d}{D}\right) = \frac{6000\times10^{-10}}{2\pi}\times\frac{2\pi}{3} \Rightarrow x = 2\times10^{-3}\,m = 2mm.$

***Example*: 19** Two identical radiators have a separation of $d = \lambda/4$, where λ is the wavelength of the waves emitted by either source. The initial phase difference between the sources is $\pi/4$. Then the intensity on the screen at a distance point situated at an angle $\theta = 30^o$ from the radiators is (here I_0 is the intensity at that point due to one radiator)

(a) I_0 (b) $2I_0$ (c) $3I_0$ (d) $4I_0$

Solution: (a) Initial phase difference $\phi_0 = \frac{\pi}{4}$; Phase difference due to path difference $\phi' = \frac{2\pi}{\lambda}(\Delta)$

where $\Delta = d\sin\theta \Rightarrow \phi' = \frac{2\pi}{\lambda}(d\sin\theta) = \frac{2\pi}{\lambda}\times\frac{\lambda}{4}(\sin30^o) = \frac{\pi}{4}$

Hence total phase difference $\phi = \phi_0 + \phi' = \frac{\phi}{4}$. By using $I = 4I_0 \cos^2(\phi/2) = 4I_0 \cos^2\left(\frac{\pi/2}{2}\right) = 2I_0$.

Example: 20 In *YDSE* a source of wavelength 6000 Å is used. The screen is placed 1 *m* from the slits. Fringes formed on the screen, are observed by a student sitting close to the slits. The student's eye can distinguish two neighbouring fringes. If they subtend an angle more than 1 minute of arc. What will be the maximum distance between the slits so that the fringes are clearly visible

(a) 2.06 *mm* (b) 2.06 *cm* (c) 2.06×10^{-3} *mm* (d) None of these

Solution: (a) According to given problem angular fringe width $\theta = \frac{\lambda}{d} \geq \frac{\pi}{180 \times 60}$ [As $1' = \frac{\pi}{180 \times 60} rad$]

i.e. $d < \frac{6 \times 10^{-7} \times 180 \times 60}{\pi}$ *i.e.* $d < 2.06 \times 10^{-3} m \Rightarrow d_{\max} = 2.06\, mm$

Example: 21 the maximum intensity in case of interference of *n* identical waves, each of intensity I_0, if the interference is (i) coherent and (ii) incoherent respectively are

(a) $n^2 I_0, nI_0$ (b) $nI_0, n^2 I_0$ (c) nI_0, I_0 (d) $n^2 I_0, (n-1)I_0$

Solution: (a) In case of interference of two wave $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$

(i) In case of coherent interference ϕ does not vary with time and so I will be maximum when $\cos\phi = \max = 1$

i.e. $(I_{\max})_{co} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

So for *n* identical waves each of intensity I_0 $(I_{\max})_{co} = (\sqrt{I_0} + \sqrt{I_0} + ...)^2 = (n\sqrt{I_0})^2 = n^2 I_0$

(ii)In case of incoherent interference at a given point, ϕ varies randomly with time, so $(\cos\phi)_{av} = 0$ and hence $(I_R)_{Inco} = I_1 + I_2$

So in case of *n* identical waves $(I_R)_{Inco} = I_0 + I_0 + ... = nI_0$

Example: 22 The width of one of the two slits in a Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit width. The ratio of the maximum to the minimum intensity in interference pattern will be

(a) $\frac{1}{a}$ (b) $\frac{9}{1}$ (c) $\frac{2}{1}$ (d) $\frac{1}{2}$

Solution: (b) $A_{\max} = 2A + A = 3A$ and $A_{\min} = 2A - A = A$. Also $\frac{I_{\max}}{I_{\min}} = \left(\frac{A_{\max}}{A_{\min}}\right)^2 = \left(\frac{3A}{A}\right)^2 = \frac{9}{1}$

Example: 23 A star is moving towards the earth with a speed of $4.5 \times 10^6 m/s$. If the true wavelength of a certain line in the spectrum received from the star is 5890 Å, its apparent wavelength will be about $[c = 3 \times 10^8 m/s]$

[MP PMT 1999]

(a) 5890 Å (b) 5978 Å (c) 5802 Å (d) 5896 Å

Solution: (c) By using $\lambda' = \lambda\left(1 - \frac{v}{c}\right) \Rightarrow \lambda' = 5890\left(1 - \frac{4.5 \times 10^6}{3 \times 10^8}\right) = 5802\, Å$.

Example: 24 Light coming from a star is observed to have a wavelength of 3737 Å, while its real wavelength is 3700 Å. The speed of the star relative to the earth is [Speed of light $= 3 \times 10^8 m/s$] **[MP PET 1997]**

(a) $3 \times 10^5 m/s$ (b) $3 \times 10^6 m/s$ (c) $3.7 \times 10^7 m/s$ (d) $3.7 \times 10^6 m/s$

Solution: (b) By using $\Delta\lambda = \lambda\frac{v}{c} \Rightarrow$ (3737-3700)= $3700 \times \frac{v}{3 \times 10^8} \Rightarrow v = 3 \times 10^6\, m/s$.

***Example*: 25** Light from the constellation Virgo is observed to increase in wavelength by 0.4%. With respect to Earth the constellation is **[MP PMT 1994, 97; MP PET 2003]**

(a) Moving away with velocity $1.2 \times 10^6 m/s$
(b) Coming closer with velocity $1.2 \times 10^6 m/s$
(c) Moving away with velocity $4 \times 10^6 m/s$
(d) Coming closer with velocity $4 \times 10^6 m/s$

Solution: (a) By using $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$; where $\frac{\Delta\lambda}{\lambda} = \frac{0.4}{100}$ and $c = 3 \times 10^8$ *m/s* $\Rightarrow \frac{0.4}{100} = \frac{v}{3 \times 10^8} \Rightarrow v = 1.2 \times 10^6$ *m/s*

Since wavelength is increasing *i.e.* it is moving away.

Tricky example: 1

In *YDSE*, distance between the slits is 2×10^{-3} *m*, slits are illuminated by a light of wavelength 2000Å –9000 Å. In the field of view at a distance of 10^{-3} *m* from the central position which wavelength will be observe. Given distance between slits and screen is 2.5 *m*

(a) 40000 Å (b) 4500 Å (c) 5000 Å (d) 5500 Å

Solution : (b) $x = \frac{n\lambda D}{d} \Rightarrow \lambda = \frac{xd}{nD} = \frac{10^{-3} \times 2 \times 10^{-3}}{n \times 2.5} \Rightarrow \frac{8 \times 10^{-7}}{n} m = \frac{8000}{n} Å$

For n = 1, 2, 3……. λ = 8000 Å, 4000 Å, $\frac{8000}{3}$ Å………

Hence only option (a) is correct.

Tricky example: 2

I is the intensity due to a source of light at any point *P* on the screen. If light reaches the point *P* via two different paths (a) direct (b) after reflection from a plane mirror then path difference between two paths is $3\lambda/2$, the intensity at *P* is

(a) *I* (b) Zero (c) 2*I* (d) 4*I*

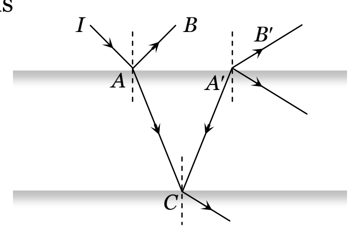
Solution : (d) Reflection of light from plane mirror gives additional path difference of $\lambda/2$ between two waves

$\therefore$ Total path difference $= \frac{3\lambda}{2} + \frac{\lambda}{2} = 2\lambda$

Which satisfies the condition of maxima. Resultant intensity $= (\sqrt{I} + \sqrt{I})^2 = 4I$.

Tricky example: 3

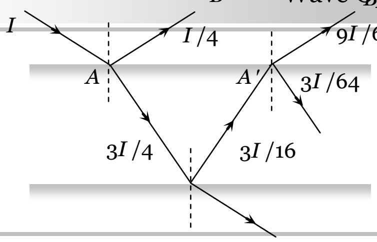
A ray of light of intensity *I* is incident on a parallel glass-slab at a point *A* as shown in figure. It undergoes partial reflection and refraction. At each reflection 25% of incident energy is reflected. The rays *AB* and *A′B′* undergo interference. The ratio I_{max}/I_{min} is



(a) 4 : 1
(b) 8 : 1
(c) 7 : 1
(d) 49 : 1

Solution : (d) From figure $I_1 = \frac{I}{4}$ and $I_2 = \frac{9I}{64} \Rightarrow \frac{I_2}{I_1} = \frac{9}{16}$

By using $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1}\right) = \left(\frac{\sqrt{\frac{9}{16}}+1}{\sqrt{\frac{9}{16}}-1}\right) = \frac{49}{1}$



Fresnel's Biprism

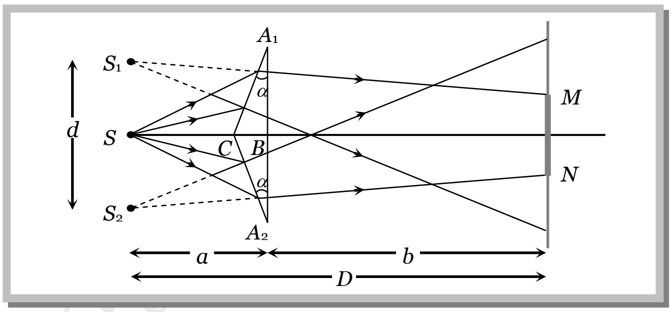
(1) It is an optical device of producing interference of light Fresnel's biprism is made by joining base to base two thin prism (A_1BC and A_2BC as shown in the figure) of very small angle or by grinding a thick glass plate.

(2) Acute angle of prism is about 1/2° and obtuse angle of prism is about 179°.

(3) When a monochromatic light source is kept in front of biprism two coherent virtual source S_1 and S_2 are produced.

(4) Interference fringes are found on the screen (in the *MN* region) placed behind the biprism interference fringes are formed in the limited region which can be observed with the help eye piece.

(5) Fringe width is measured by a micrometer attached to the eye piece. Fringes are of equal width and its value is $\beta = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{\beta d}{D}$

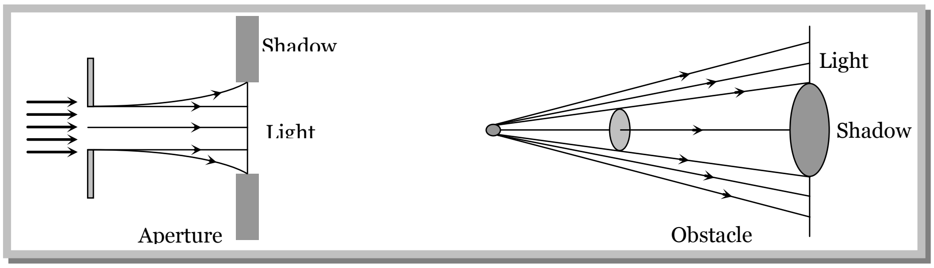


Let the separation between S_1 and S_2 be d and the distance of slits and the screen from the biprism be a and b respectively *i.e.* $D = (a + b)$. If angle of prism is α and refractive index is μ then $d = 2a(\mu-1)\alpha$

$$\therefore \qquad \lambda = \frac{\beta[2a(\mu-1)\alpha]}{(a+b)} \Rightarrow \beta = \frac{(a+b)\lambda}{2a(\mu-1)\alpha}$$

Diffraction of Light

It is the phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wavelength of light.



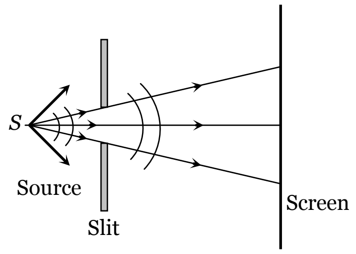
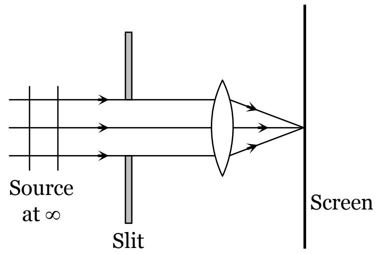
Note : ❑ Diffraction is the characteristic of all types of waves.

❑ Greater the wavelength of wave, higher will be it's degree of diffraction.

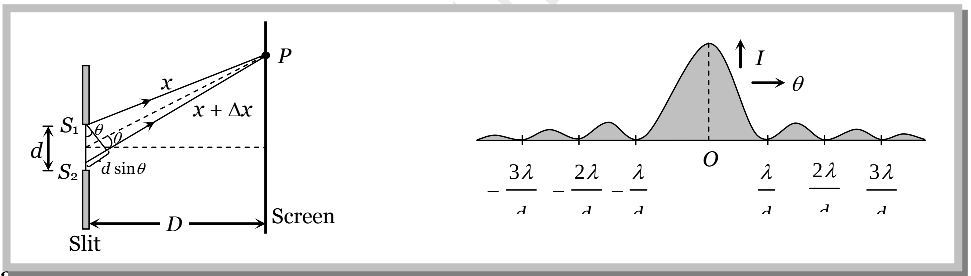
❑ Experimental study of diffraction was extended by Newton as well as Young. Most systematic study carried out by Huygens on the basis of wave theory.

- The minimum distance at which the observer should be from the obstacle to observe the diffraction of light of wavelength λ around the obstacle of size d is given by $x = \frac{d^2}{4\lambda}$.

(1) **Types of diffraction :** The diffraction phenomenon is divided into two types

Fresnel diffraction	Fraunhofer diffraction
(i) If either source or screen or both are at finite distance from the diffracting device (obstacle or aperture), the diffraction is called Fresnel type.	(i) In this case both source and screen are effectively at infinite distance from the diffracting device.
(ii) Common examples : Diffraction at a straight edge, narrow wire or small opaque disc etc.	(ii) Common examples : Diffraction at single slit, double slit and diffraction grating.
	

(2) **Diffraction of light at a single slit :** In case of diffraction at a single slit, we get a central bright band with alternate bright (maxima) and dark (minima) bands of decreasing intensity as shown



(i) Width of central maxima $\beta_0 = \frac{2D\lambda}{d}$, and angular width $= \frac{2\lambda}{d}$

(ii) Minima occurs at a point on either side of the central maxima, such that the path difference between the waves from the two ends of the aperture is given by $\Delta = n\lambda$; where $n = 1, 2, 3 ....$

i.e. $d\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d}$

(iii) The secondary maxima occurs, where the path difference between the waves from the two ends of the aperture is given by $\Delta = (2n+1)\frac{\lambda}{2}$; where $n = 1, 2, 3 ....$

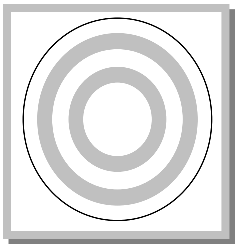
i.e. $d\sin\theta = (2n+1)\frac{\lambda}{2} \Rightarrow \sin\theta = \frac{(2n+1)\lambda}{2d}$

(3) **Comparison between interference and diffraction**

Interference	Diffraction
Results due to the superposition of waves from two coherent sources.	Results due to the superposition of wavelets from different parts of same wave front. (single coherent source)

All fringes are of same width $\beta = \frac{\lambda D}{d}$	All secondary fringes are of same width but the central maximum is of double the width $\beta_0 = 2\beta = 2\frac{\lambda D}{d}$
All fringes are of same intensity	Intensity decreases as the order of maximum increases.
Intensity of all minimum may be zero	Intensity of minima is not zero.
Positions of nth maxima and minima $x_{n(\text{Bright})} = \frac{n\lambda D}{d}$, $x_{n(\text{Dark})} = (2n-1)\frac{\lambda D}{d}$	Positions of nth secondary maxima and minima $x_{n(\text{Bright})} = (2n+1)\frac{\lambda D}{d}$, $x_{n(\text{Dark})} = \frac{n\lambda D}{d}$
Path difference for nth maxima $\Delta = n\lambda$	for nth secondary maxima $\Delta = (2n+1)\frac{\lambda}{2}$
Path difference for nth minima $\Delta = (2n-1)\lambda$	Path difference for nth minima $\Delta = n\lambda$

(4) **Diffraction and optical instruments :** The objective lens of optical instrument like telescope or microscope etc. acts like a circular aperture. Due to diffraction of light at a circular aperture, a converging lens cannot form a point image of an object rather it produces a brighter disc known as Airy disc surrounded by alternate dark and bright concentric rings.

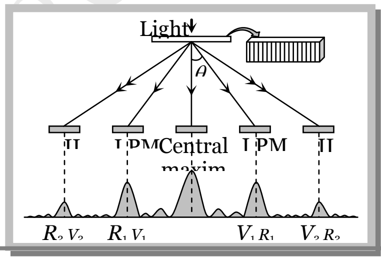


The angular half width of Airy disc $= \theta = \frac{1.22\lambda}{D}$ (where D = aperture of lens)

The lateral width of the image $= f\theta$ (where f = focal length of the lens)

Note : ❑ Diffraction of light limits the ability of optical instruments to form clear images of objects when they are close to each other.

(5) **Diffraction grating :** Consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the diffraction of principle maxima (PM) is given by $d\sin\theta = n\lambda$; where d = distance between two consecutive slits and is called grating element.

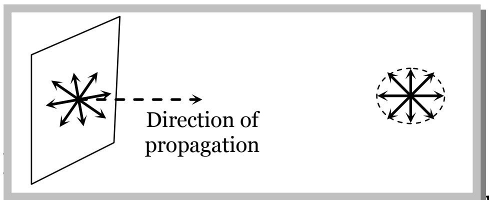


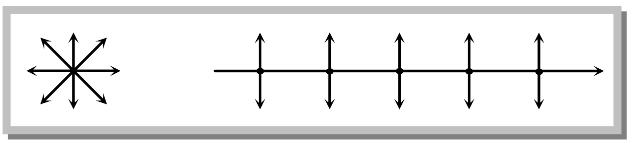
Polarisation of Light

Light propagates as transverse EM waves. The magnitude of electric field is much larger as compared to magnitude of magnetic field. We generally prefer to describe light as electric field oscillations.

(1) **Unpolarised light**

The light having electric field oscillations in all directions in the plane perpendicular to the direction of propagation is called Unpolarised light. The oscillation may be resolved into horizontal and vertical component.





(2)

→ Direction of ↕ Vertical oscillation • Horizontal

The light having oscillations only in one plane is called Polarised or plane polarised light.

(i) The plane in which oscillation occurs in the polarised light is called plane of oscillation.

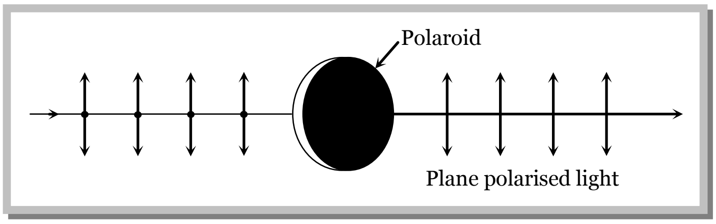
(ii) The plane perpendicular to the plane of oscillation is called plane of polarisation.

(iii) Light can be polarised by transmitting through certain crystals such as tourmaline or polaroids.

(3) **Polaroids**

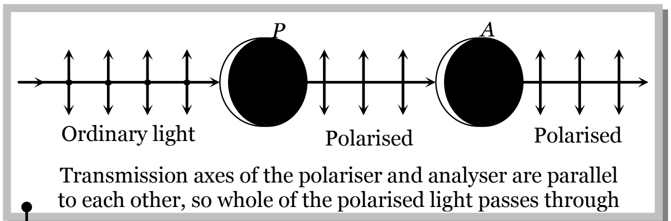
It is a device used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal. or

It is a thin film of ultramicroscopic crystals of quinine idosulphate with their optic axis parallel to each other.

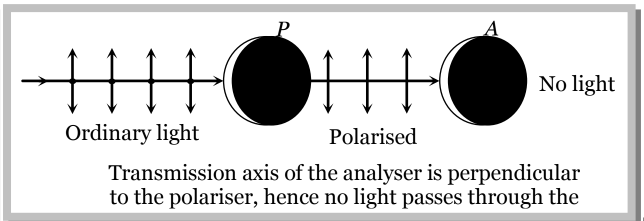


(i) Polaroids allow the light oscillations parallel to the transmission axis pass through them.

(ii) The crystal or polaroid on which unpolarised light is incident is called polariser. Crystal or polaroid on which polarised light is incident is called analyser.



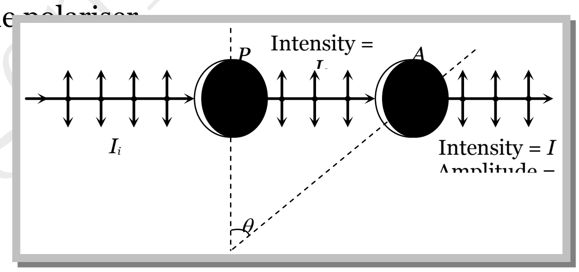
Transmission axes of the polariser and analyser are parallel to each other, so whole of the polarised light passes through



Transmission axis of the analyser is perpendicular to the polariser, hence no light passes through the

Note : ❑ When unpolarised light is incident on the polariser, the intensity of the transmitted polarised light is half the intensity of unpolarised light.

(4) **Malus law** This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.



(i) $I = I_0 \cos^2\theta$ and $A^2 = A_0^2 \cos^2\theta \Rightarrow A = A_0 \cos\theta$

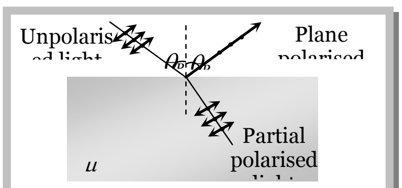
If $\theta = 0^o$, $I = I_0$, $A = A_0$, If $\theta = 45^o$, $I = \frac{I_0}{2}$, $A = \frac{A_0}{\sqrt{2}}$, If $\theta = 90^o$, $I = 0$, $A = 0$

(ii) If $I_i =$ Intensity of unpolarised light.

So $I_0 = \frac{I_i}{2}$ *i.e.* if an unpolarised light is converted into plane polarised light (say by passing it through a plaroid or a Nicol-prism), its intensity becomes half. and $I = \frac{I_i}{2}\cos^2\theta$

Note : ❑ Percentage of polarisation $= \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} \times 100$

(5) **Brewster's law :** Brewster discovered that when a beam of unpolarised light is reflected from a transparent medium (refractive index $=\mu$), the reflected light is completely plane polarised at a certain angle of incidence (called the angle of polarisation θ_p).



Also $\mu = \tan\theta_p$ Brewster's law

(i) For $i < \theta_P$ or $i > \theta_P$

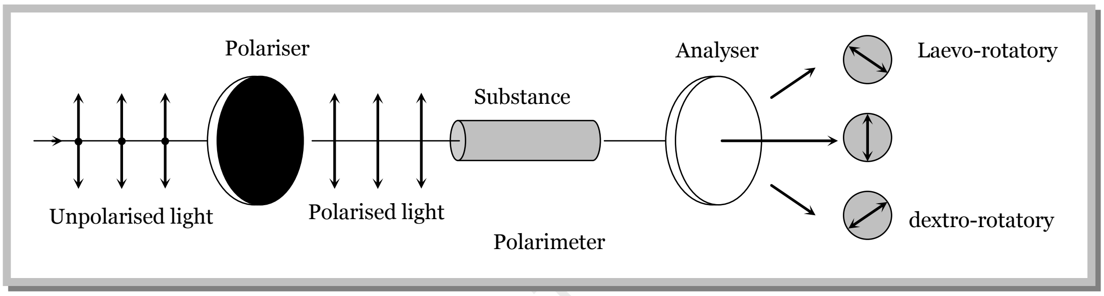
Both reflected and refracted rays becomes partially polarised

(ii) For glass $\theta_P \approx 57^o$, for water $\theta_P \approx 53^o$

(6) **Optical activity and specific rotation**

When plane polarised light passes through certain substances, the plane of polarisation of the light is rotated about the direction of propagation of light through a certain angle. This phenomenon is called optical activity or optical rotation and the substances optically active.

If the optically active substance rotates the plane of polarisation clockwise (looking against the direction of light), it is said to be *dextro-rotatory* or *right-handed*. However, if the substance rotates the plane of polarisation anti-clockwise, it is called *laevo-rotatory* or *left-handed*.



The optical activity of a substance is related to the asymmetry of the molecule or crystal as a whole, *e.g.*, a solution of cane-sugar is dextro-rotatory due to asymmetrical molecular structure while crystals of quartz are dextro or laevo-rotatory due to structural asymmetry which vanishes when quartz is fused.

Optical activity of a substance is measured with help of polarimeter in terms of 'specific rotation' which is defined as the rotation produced by a solution of length 10 *cm* (1 *dm*) and of unit concentration (*i.e.* 1 *g*/cc) for a given wavelength of light at a given temperature. *i.e.* $[\alpha]_{t^oC}^{\lambda} = \frac{\theta}{L \times C}$ where θ is the rotation in length L at concentration C.

(7) **Applications and uses of polarisation**

(i) By determining the polarising angle and using Brewster's law, *i.e.* $\mu = \tan\theta_P$, refractive index of dark transparent substance can be determined.

(ii) It is used to reduce glare.

(iii) In calculators and watches, numbers and letters are formed by liquid crystals through polarisation of light called liquid crystal display **(LCD)**.

(iv) In CD player polarised laser beam acts as needle for producing sound from compact disc which is an encoded digital format.

(v) It has also been used in recording and reproducing three-dimensional pictures.

(vi) Polarisation of scattered sunlight is used for navigation in solar-compass in polar regions.

(vii) Polarised light is used in optical stress analysis known as 'photoelasticity'.

(viii) Polarisation is also used to study asymmetries in molecules and crystals through the phenomenon of 'optical activity'.

Assignment

Nature of light and interference of light

1. The dual nature of light is exhibited by **[KCET 1999; AIIMS 2001; BHU 2001; Bihar CEE 2004]**

(a) Diffraction and photoelectric effect (b) Diffraction and reflection
(c) Refraction and interference (d) Photoelectric effect

2. Huygen wave theory allows us to know **[AFMC 2004]**

(a) The wavelength of the wave (b) The velocity of the wave
(c) The amplitude of the wave (d) The propagation of wave fronts

3. When a beam of light is used to determine the position of an object, the maximum accuracy is achieved if the light is **[AIIMS 2003]**

(a) Polarised (b) Of longer wavelength (c) Of shorter wavelength (d) Of high intensity

4. Which of the following phenomenon does not show the wave nature of light **[RPET 2003; MP PMT 2003]**

(a) Diffraction (b) Interference (c) Refraction (d) Photoelectric effect

5. As a result of interference of two coherent sources of light, energy is **[MP PMT 2002; KCET 2003]**

(a) Increased
(b) Redistributed and the distribution does not vary with time
(c) Decreased
(d) Redistributed and the distribution changes with time

6. To demonstrate the phenomenon of interference, we require two sources which emit radiation **[AIEEE 2003]**

(a) Of the same frequency and having a definite phase relationship
(b) Of nearly the same frequency
(c) Of the same frequency
(d) Of different wavelengths

7. Consider the following statements

Assertion (A): Thin films such as soap bubble or a thin layer of oil on water show beautiful colours, when illuminated by white light.

Reason (R) : It happens due to the interference of light reflected from the upper surface of the thin film.

Of these statements **[AIIMS 2002]**

(a) Both A and R are true but R is a correct explanation of A (b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false (d) A is false but R is true
(e) Both A and R are false

8. When light passes from one medium into another medium, then the physical property which does not change is

[CPMT 1990; MNR 1995; AMU 1995; UPSEAT 1999, 2000; MP PET 2002; RPET 1996, 2003; AFMC 1993, 98, 2003]

(a) Velocity (b) Wavelength (c) Frequency (d) Refractive index

9. The frequency of light ray having the wavelength 3000$\mathring{A}$ is **[DPMT 2002]**

(a) 9×10^{13} *cycles/sec* (b) 10^{15} *cycles/sec* (c) 90 *cycles/sec* (d) 3000 *cycles/sec*

10. Two coherent sources of different intensities send waves which interfere. The ratio of maximum intensity to the minimum intensity is 25. The intensities of the sources are in the ratio **[RPMT 1989; UPSEAT 2002]**

(a) 25 : 1 (b) 5 : 1 (c) 9 : 4 (d) 25 : 16

11. What is the path difference of destructive interference **[AIIMS 2002]**

(a) $n\lambda$ (b) $n(\lambda+1)$ (c) $\frac{(n+1)\lambda}{2}$ (d) $\frac{(2n+1)\lambda}{2}$

12. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are

[IIT-JEE 1988; AIIMS 1997; MP PMT 1997; MP PET 1999; KCET (Engg./Med.) 2000; MP PET 2002]

(a) $5I$ and I (b) $5I$ and $3I$ (c) $9I$ and I (d) $9I$ and $3I$

13. Laser beams are used to measure long distance because **[DCE 2001]**

(a) They are monochromatic (b) They are highly polarised

(c) They are coherent (d) They have high degree of parallelism

14. Wave nature of light is verified by **[RPET 2001]**

(a) Interference (b) Photoelectric effect (c) Reflection (d) Refraction

15. If the wavelength of light in vacuum be λ, the wavelength in a medium of refractive index n will be **[UPSEAT 2001; MP PET 2001]**

(a) $n\lambda$ (b) $\frac{\lambda}{n}$ (c) $\frac{\lambda}{n^2}$ (d) $n^2\lambda$

16. Newton postulated his corpuscular theory on the basis of **[UPSEAT 2001; KCET 2001]**

(a) Newton's rings (b) Colours of thin films

(c) Rectilinear propagation of light (d) Dispersion of white light

17. Two coherent sources of intensities. I_1 and I_2 produce an interference pattern. The maximum intensity in the interference pattern will be **[UPSEAT 2001; MP PET 2001]**

(a) $I_1 + I_2$ (b) $I_1^2 + I_2^2$ (c) $(I_1 + I_2)^2$ (d) $(\sqrt{I_1} + \sqrt{I_2})^2$

18. Which one among the following shows particle nature of light **[CBSE PM/PD 2001]**

(a) Photo electric effect (b) Interference (c) Refraction (d) Polarization

19. For constructive interference to take place between two monochromatic light waves of wavelength λ, the path difference should be

[MNR 1992; UPSEAT 2001]

(a) $(2n-1)\frac{\lambda}{4}$ (b) $(2n-1)\frac{\lambda}{2}$ (c) $n\lambda$ (d) $(2n+1)\frac{\lambda}{2}$

20. In a wave, the path difference corresponding to a phase difference of ϕ is **[MP PET 2000]**

(a) $\frac{\pi}{2\lambda}\phi$ (b) $\frac{\pi}{\lambda}\phi$ (c) $\frac{\lambda}{2\pi}\phi$ (d) $\frac{\lambda}{\pi}\phi$

21. A beam of monochromatic blue light of wavelength $4200 Å$ in air travels in water, its wavelength in water will be **[UPSEAT 2000]**

(a) $2800 Å$ (b) $5600 Å$ (c) $3150 Å$ (d) $4000 Å$

22. Wave front originating from a point source is **[RPET 2000]**

(a) Cylindrical (b) Spherical (c) Plane (d) Cubical

23. Waves that can not be polarised are **[KCET 2000]**

(a) Transverse waves (b) Longitudinal waves (c) Light waves (d) Electromagnetic waves

24. According to Huygen's wave theory, point on any wave front may be regarded as **[J & K CET 2000]**

(a) A photon (b) An electron (c) A new source of wave (d) Neutron

25. The light produced by a laser is all the following except **[JIPMER 2000]**

(a) Incoherent (b) Monochromatic

(c) In the form of a narrow beam (d) Electromagnetic

26. The phenomena of interference is shown by **[MNR 1994; MP PMT 1997; AIIMS 1999, 2000; JIPMER 2000; UPSEAT 1994, 2000]**

(a) Longitudinal mechanical waves only (b) Transverse mechanical waves only

(c) Electromagnetic waves only (d) All the above types of waves

27. If the ratio of amplitude of two waves is 4 : 3, then the ratio of maximum and minimum intensity is

[MP PMT 1996; AFMC 1997; RPET 2000]

(a) 16 : 18 (b) 18 : 16 (c) 49 : 1 (d) 94 : 1

28. If the distance between a point source and screen is doubled, then intensity of light on the screen will become

[RPET 1997; RPMT 1999]

(a) Four times (b) Double (c) Half (d) One-fourth

29. Soap bubble appears coloured due to the phenomenon of

[CPMT 1972, 83, 86; AFMC 1995, 97; RPET 1997; CBSE PMT 1997; AFMC 1997]

(a) Interference (b) Diffraction (c) Dispersion (d) Reflection

30. Two waves are known to be coherent if they have **[RPMT 1994, 95, 97; MP PMT 1996; MNR 1995]**

(a) Same amplitude (b) Same wavelength

(c) Same amplitude and wavelength (d) Constant phase difference and same wavelength

31. An oil flowing on water seems coloured due to interference. For observing this effect, the approximate thickness of the oil film should be **[DPMT 1987; JIPMER 1997]**

(a) 100 $\mathring{A}$ (b) 10000 $\mathring{A}$ (c) 1 *mm* (d) 1 *cm*

32. If L is the coherence length and c the velocity of light, the coherent time is **[MP PMT 1996]**

(a) cL (b) $\frac{L}{c}$ (c) $\frac{c}{L}$ (d) $\frac{1}{Lc}$

33. By a monochromatic wave, we mean **[AFMC 1995]**

(a) A single ray (b) A single ray of a single colour

(c) Wave having a single wavelength (d) Many rays of a single colour

34. Two coherent sources of light produce destructive interference when phase difference between them is **[MP PMT 1994; CPMT 1995]**

(a) 2π (b) π (c) $\pi/2$ (d) 0

35. Which one of the following statements is correct **[KCET 1994]**

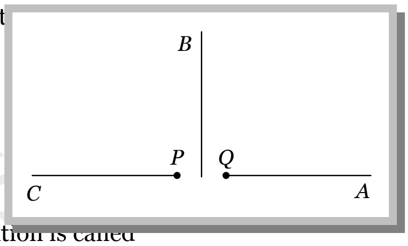
(a) In vacuum, the speed of light depends upon frequency

(b) In vacuum, the speed of light does not depend upon frequency

(c) In vacuum, the speed of light is independent of frequency and wavelength

(d) In vacuum, the speed of light depends upon wavelength

36. Figure here shows P and Q as two equally intense coherent sources emitting radiations of wavelength 20 *m*. The separation PQ is 5.0 *m* and phase of P is ahead of the phase of Q by 90°. A, B and C are three distant points of observation equidistant from the mid-point of PQ. The intensity of radiations at A, B, C will bear t **[NSEP 1994]**



(a) 0 : 1 : 4

(b) 4 : 1 : 0

(c) 0 : 1 : 2

(d) 2 : 1 : 0

37. In Huygen's wave theory, the locus of all points in the same state of vibration is called **[CBSE PMT 1993]**

(a) A half period zone (b) Vibrator (c) A wavefront (d) A ray

38. The idea of the quantum nature of light has emerged in an attempt to explain **[CPMT 1990]**

(a) Interference (b) Diffraction

(c) Radiation spectrum of a black body (d) Polarisation

39. The necessary condition for an interference by two source of light is that the **[RPMT 1988; CPMT 1989]**

(a) Two monochromatic sources should be of same amplitude but with a constant phase

(b) Two sources should be of same amplitude

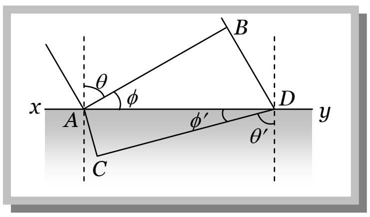
(c) Two point sources should have phase difference varying with time

(d) Two sources should be of same wavelength

40. If the intensity of the waves observed by two coherent sources is I. Then the intensity of resultant waves in constructive interference will be **[RPET 1988]**

(a) $2I$ (b) $4I$ (c) I (d) None of these

41. In figure, a wavefront AB moving in air is incident on a plane glass surface xy. Its position CD after refraction through a glass slab is shown also along with normals drawn at A and D. the refractive index of glass with respect to air will be equal to **[CPMT 19**



(a) $\frac{\sin\theta}{\sin\theta'}$

(b) $\frac{\sin\theta}{\sin\phi'}$

(c) (BD/AC)

(d) (AB/CD)

42. Four independent waves are expressed as

(i) $y_1 = a_1 \sin\omega t$ (ii) $y_2 = a_2 \sin 2\omega t$ (iii) $y_3 = a_3 \cos\omega t$ (iv) $y_4 = a_4 \sin(\omega t + \pi/3)$

The interference is possible between **[CPMT 1986]**

(a) (i) and (ii) (b) (i) and (iv) (c) (iii) and (iv) (d) Not possible at all

43. Colour of light is known by its **[MP PMT 1984]**

(a) Velocity (b) Amplitude (c) Frequency (d) Polarisation

44. Laser light is considered to be coherent because it consists of **[CPMT 1972]**

(a) Many wavelengths (b) Uncoordinated wavelengths
(c) Coordinated waves of exactly the same wavelength (d) Divergent beams

45. A laser beam may be used to measure very large distances because **[CPMT 1972]**
(a) It is unidirectional (b) It is coherent (c) It is monochromatic (d) It is not absorbed

46. Interference patterns are not observed in thick films, because
(a) Most of the incident light intensity is observed within the film
(b) A thick film has a high coefficient of reflection
(c) The maxima of interference patterns are far from the minima
(d) There is too much overlapping of colours washing out the interference pattern

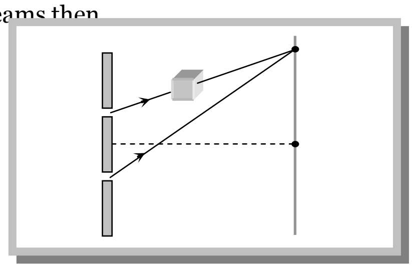
47. Phenomenon of interference is not observed by two sodium lamps of same power. It is because both waves have
(a) Not constant phase difference (b) Zero phase difference
(c) Different intensity (d) Different frequencies

Young's double slit experiment

Basic Level

48. In a Young's double slit experiment, the separation between the two slits is 0.9 *mm* and the fringes are observed one *metre* away. If it produces the second dark fringe at a distance of 1 *mm* from the central fringe, the wavelength of monochromatic source of light used is **[KCET 2004]**
(a) 500 *nm* (b) 600 *nm* (c) 450 *nm* (d) 400 *nm*

49. A monochromatic beams of light is used for the formation of fringes on the screen by illuminating the two slits in the Young's double slit mica is interposed in the path of one of the interfering beams then **[AIIMS 2004]**



(a) The fringe width increases
(b) The fringe width decreases
(c) The fringe width remains the same but the pattern shifts
(d) The fringe pattern disappears

50. In a Young's double-slit experiment the fringe width is 0.2 *mm*. If the wavelength of light used is increased by 10% and the separation between the slits is also increased by 10%, the fringe width will be **[MP PMT 2004]**
(a) 0.20 *mm* (b) 0.401 *mm* (c) 0.242 *mm* (d) 0.165 *mm*

51. In Young's experiment, the distance between the slits is reduced to half and the distance between the slit and screen is doubled, then the fringe width **[IIT 1981; MP PMT 1994; RPMT 1997; KCET (Engg./Med.) 2000; UPSEAT 2000; AMU (Engg.) 2000;**
(a) Will not change (b) Will become half (c) Will be doubled (d) Will become four times

52. In an interference experiment, third bright fringe is obtained at a point on the screen with a light of 700 *nm*. What should be the wavelength of the light source in order obtain 5th bright fringe at the same point **[KCET 2003]**
(a) 500 *nm* (b) 630 *nm* (c) 750 *nm* (d) 420 *nm*

53. In Young's double-slit experiment the fringe width is β. If entire arrangement is placed in a liquid of refractive index n, the fringe width becomes **[KCET 2003]**
(a) $\frac{\beta}{n+1}$ (b) $n\beta$ (c) β / n (d) $\beta / n-1$

54. If the separation between slits in Young's double slit experiment is reduced to $\frac{1}{3}rd$, the fringe width becomes n times. The value of n is **[MP PET 2003]**
(a) 3 (b) $\frac{1}{3}$ (c) 9 (d) $\frac{1}{9}$

55. When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the two interfering waves of light, then the path difference changes by **[MP PMT 2002]**
(a) $(\mu+1)t$ (b) $(\mu-1)t$ (c) $\frac{(\mu+1)}{t}$ (d) $\frac{(\mu-1)}{t}$

56. In a Young's double slit experiment, the source illuminating the slits is changed from blue to violet. The width of the fringes **[Kerala CET (Med.) 2002]**

(a) Increases (b) Decreases (c) Becomes unequal (d) Remains constant

57. In Young's double slit experiment, the intensity of light coming from the first slit is double the intensity from the second slit. The ratio of the maximum intensity to the minimum intensity on the interference fringe pattern observed is **[KCET (Med.) 2002]**

(a) 34 (b) 40 (c) 25 (d) 38

58. In Young's double slit experiment the wavelength of light was changed from 7000$Å$ to 3500$Å$. While doubling the separation between the slits which of the following is not true for this experiment **[Orissa JEE 2002]**

(a) The width of the fringes changes

(b) The colour of bright fringes changes

(c) The separation between successive bright fringes changes

(d) The separation between successive dark fringes remains unchanged

59. In Young's double slit experiment, the central bright fringe can be identified **[KCET (Engg.) 2002]**

(a) By using white light instead of monochromatic light (b) As it is narrower than other bright fringes

(c) As it is wider than other bright fringes (d) As it has a greater intensity than the other bright fringes

60. Interference was observed in interference chamber when air was present, now the chamber is evacuated and if the same light is used, a careful observer will see **[CBSE PMT 1993; DPMT 2000; BHU 2002]**

(a) No interference

(b) Interference with bright bands

(c) Interference with dark bands

(d) Interference in which width of the fringe will be slightly increased

61. A slit of width *a* is illuminated by white light. For red light $(\lambda = 6500\ Å)$. The first minima is obtained at $\theta = 30^o$. Then the value of *a* will be **[MP PMT 1987; CPMT 2002]**

(a) 3250 $Å$ (b) $6.5\times10^{-4}\ mm$ (c) 1.24 *microns* (d) $2.6\times10^{-4}\ cm$

62. In the Young's double slit experiment with sodium light. The slits are 0.589 *m* apart. The angular separation of the third maximum from the central maximum will be (given $\lambda = 589\ mm$) **[Pb. PMT 2002]**

(a) $\sin^{-1}(0.33\times10^{8})$ (b) $\sin^{-1}(0.33\times10^{-6})$ (c) $\sin^{-1}(3\times10^{-8})$ (d) $\sin^{-1}(3\times10^{-6})$

63. In the Young's double slit experiment for which colour the fringe width is least **[MP PMT 1994; UPSEAT 2001; MP PET 2001]**

(a) Red (b) Green (c) Blue (d) Yellow

64. In a Young's double slit experiment, the separation of the two slits is doubled. To keep the same spacing of fringes, the distance *D* of the screen from the slits should be made **[AMU (Engg.) 2001]**

(a) $\frac{D}{2}$ (b) $\frac{D}{\sqrt{2}}$ (c) $2D$ (d) $4D$

65. Consider the following statements

Assertion (*A*): In Young's experiment, the fringe width for dark fringes is different from that for bright fringes.

Reason (*R*) : In Young's double slit experiment performed with a source of white light, only black and bright fringes are observed

Of these statements **[AIIMS 2001]**

(a) Both *A* and *R* are true and *R* is a correct explanation of *A* (b) Both *A* and *R* are true but *R* is not a correct explanation of *A*

(c) Both *A* and *R* are false (d) *A* is false but *R* is true

(e) *A* is true but *R* is false

66. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 *nm* is used. If the wavelength of light is changed to 400 *nm*, number of fringes observed in the same segment of the screen is given by **[IIT-JEE (Screening) 2001]**

(a) 12 (b) 18 (c) 24 (d) 30

67. In Young's double slit experiment, a mica slit of thickness *t* and refractive index μ is introduced in the ray from the first source S_1. By how much distance the fringes pattern will be displaced **[RPMT 1996, 97; JIPMER 2000]**

(a) $\frac{d}{D}(\mu-1)t$ (b) $\frac{D}{d}(\mu-1)t$ (c) $\frac{d}{(\mu-1)D}$ (d) $\frac{D}{d}(\mu-1)$

68. Young's double slit experiment is performed with light of wavelength 550 *nm*. The separation between the slits is 1.10 *mm* and screen is placed at distance of 1 *m*. What is the distance between the consecutive bright or dark fringes **[Pb. PMT 2000]**

(a) 1.5 *mm* (b) 1.0 *m* (c) 0.5 *mm* (d) None of these

69. In interference obtained by two coherent sources, the fringe width (β) has the following relation with wavelength (λ) **[CPMT 1997; MP PMT 2000]**

(a) $\beta \propto \lambda^2$ (b) $\beta \propto \lambda$ (c) $\beta \propto 1/\lambda$ (d) $\beta \propto \lambda^{-2}$

70. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then in the interference pattern **[IIT-JEE (Screening) 2000]**

(a) The intensities of both the maxima and the minima increase
(b) The intensity of maxima increases and the minima has zero intensity
(c) The intensity of maxima decreases and that of the minima increases
(d) The intensity of maxima decreases and the minima has zero intensity

71. In Young's double slit experiment with a source of light of wavelength 6320$\mathring{A}$, the first maxima will occur when **[Roorkee 1999]**

(a) Path difference is 9480 $\mathring{A}$ (b) Phase difference is 2π radian
(c) Path difference is 6320 $\mathring{A}$ (d) Phase difference is π radian

72. If a transparent medium of refractive index μ = 1.5 and thickness $t = 2.5 \times 10^{-5}\, m$ is inserted in front of one of the slits of Young's double slit experiment, how much will be the shift in the interference pattern? The distance between the slits is 0.5 *mm* and that between slits and screen is 100 *cm* **[AIIMS 1999]**

(a) 5 *cm* (b) 2.5 *cm* (c) 0.25 *cm* (d) 0.1 *cm*

73. If a torch is used in place of monochromatic light in Young's experiment what will happens **[MH CET (Med.) 1999; KCET (Med.) 1999]**

(a) Fringe will appear for a moment then it will disappear (b) Fringes will occur as from monochromatic light
(c) Only bright fringes will appear (d) No fringes will appear

74. When a thin metal plate is placed in the path of one of the interfering beams of light **[KCET (Engg./Med.) 1999]**

(a) Fringe width increases (b) Fringes disappear (c) Fringes become brighter (d) Fringes become blurred

75. What happens by the use of white light in Young's double slit experiment **[Similar to (AIIMS 2001; Kerala 2000); IIT-JEE 1987; RPMT 1993; MP PMT 1996; RPET 1998; UPSEAT 1999]**

(a) Bright fringes are obtained
(b) Only bright and dark fringes are obtained
(c) Central fringe is bright and two or three coloured and dark fringes are observed
(d) None of these

76. Young's experiment is performed in air and then performed in water, the fringe width **[CPMT 1990; MP PMT 1994; RPMT 1997]**

(a) Will remain same (b) Will decrease (c) Will increase (d) Will be infinite

77. In Young's experiment, one slit is covered with a blue filter and the other (slit) with a yellow filter. Then the interference pattern **[MP PET 1997]**

(a) Will be blue (b) Will be yellow (c) Will be green (d) Will not be formed

78. Two sources give interference pattern which is observed on a screen. *D* distance apart from the sources. The fringe width is 2*w*. If the distance *D* is now doubled, the fringe width will **[MP PET 1997]**

(a) Become $w/2$ (b) Remain the same (c) Become *w* (d) Become 4*w*

79. In Young's double slit experiment, angular width of fringes is 0.20° for sodium light of wavelength 5890 $\mathring{A}$. If complete system is dipped in water, then angular width of fringes becomes **[RPET 1997]**

(a) 0.11° (b) 0.15° (c) 0.22° (d) 0.30°

80. In two separate set-ups of the Young's double slit experiment, fringes of equal width are observed when lights of wavelengths in the ratio 1 : 2 are used. If the ratio of the slit separation in the two cases is 2 : 1, the ratio of the distances between the plane of the slits and the screen in the two set-ups is **[Kurukshetra CEE 1996]**

(a) 4 : 1 (b) 1 : 1 (c) 1 : 4 (d) 2 : 1

81. In a Young's double slit experiment, the central point on the screen is **[MP PMT 1996]**

(a) Bright (b) Dark (c) First bright and then dark (d) First dark and then bright

82. In Young's double slit experiment, the distance between sources is 1 *mm* and distance between the screen and source is 1*m*. If the fringe width on the screen is 0.06 *cm*, then λ = **[CPMT 1996]**

(a) 6000 $\mathring{A}$ (b) 4000 $\mathring{A}$ (c) 1200 $\mathring{A}$ (d) 2400 $\mathring{A}$

83. In a Young's double slit experiment, the distance between two coherent sources is 0.1 *mm* and the distance between the slits and the screen is 20 *cm*. If the wavelength of light is 5460 $\mathring{A}$ then the distance between two consecutive maxima is **[RPMT 1995]**

(a) 0.5 *mm* (b) 1.1 *mm* (c) 1.5 *mm* (d) 2.2 *mm*

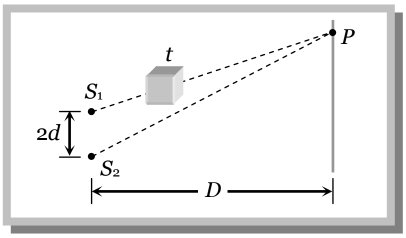
84. If a thin mica sheet of thickness t and refractive index $\mu = (5/3)$ is placed in the path of one of the interfering beams as shown in figure, then the displacement of the fringe system is **[CPMT 1995]**

(a) $\frac{Dt}{3d}$

(b) $\frac{Dt}{5d}$

(c) $\frac{Dt}{4d}$

(d) $\frac{2Dt}{5d}$



85. In a double slit experiment, the first minimum on either side of the central maximum occurs where the path difference between the two paths is **[CPMT 1995]**

(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$ (c) λ (d) 2λ

86. In Young's double slit experiment, the phase difference between the light waves reaching third bright fringe from the central fringe will be (λ = 6000 $Å$) **[MP PMT 1994]**

(a) Zero (b) 2π (c) 4π (d) 6π

87. Sodium light $(\lambda = 6\times10^{-7} m)$ is used to produce interference pattern. The observed fringe width is 0.12 *mm*. The angle between the two interfering wave trains is **[CPMT 1993]**

(a) 5×10^{-1} *rad* (b) 5×10^{-3} *rad* (c) 1×10^{-2} *rad* (d) 1×10^{-3} *rad*

88. The contrast in the fringes in any interference pattern depends on **[Roorkee 1992]**

(a) Fringe width (b) Intensity ratio of the sources

(c) Distance between the slits (d) Wavelength

89. In Young's double slit experiment, carried out with light of wavelength λ = 5000 $Å$, the distance between the slits is 0.2 *mm* and the screen is at 200 *cm* from the slits. The central maximum is at x = 0. The third maximum (taking the central maximum as zeroth maximum) will be at x equal to **[CBSE PMT 1992]**

(a) 1.67 *cm* (b) 1.5 *cm* (c) 0.5 *cm* (d) 5.0 *cm*

90. In a Young's experiment, two coherent sources are placed 0.90 *mm* apart and the fringes are observed one *metre* away. If it produces the second dark fringe at a distance of 1 *mm* from the central fringe, the wavelength of monochromatic light used would be

[CBSE PMT 1992]

(a) 60×10^{-4} *cm* (b) 10×10^{-4} *cm* (c) 10×10^{-5} *cm* (d) 60×10^{-5} *cm*

91. In Fresnel's biprism, coherent sources are obtained by **[RPET 1991]**

(a) Division of wavefront (b) Division of amplitude (c) Division of wavelength (d) None of these

92. In Young's experiment, the ratio of maximum and minimum intensities in the fringe system is 9 : 1. The ratio of amplitudes of coherent sources is **[NCERT 1990]**

(a) 9 : 1 (b) 3 : 1 (c) 2 : 1 (d) 1 : 1

93. In a certain double slit experimental arrangement interference fringes of width 1.0 *mm* each are observed when light of wavelength 5000 $Å$ is used. Keeping the set up unaltered, if the source is replaced by another source of wavelength 6000 $Å$, the fringe width will be **[CPMT 1988]**

(a) 0.5 *mm* (b) 1.0 *mm* (c) 1.2 *mm* (d) 1.5 *mm*

94. In Young's double slit experiment, if the slit widths are in the ratio 1 : 9, then the ratio of the intensity at minima to that at maxima will be **[MP PET 1987]**

(a) 1 (b) 1/9 (c) 1/4 (d) 1/3

95. The Young's experiment is performed with the lights of blue (λ = 4360 $Å$) and green colour (λ = 5460 $Å$). If the distance of the 4th fringe from the centre is x, then **[CPMT 1987]**

(a) $x(\text{Blue}) = x(\text{Green})$ (b) $x(\text{Blue}) > x(\text{Green})$ (c) $x(\text{Blue}) < x(\text{Green})$ (d) $\frac{x(\text{Blue})}{x(\text{Green})} = \frac{5460}{4360}$

96. In Young's experiment, keeping the distance of the slit from screen constant if the slit width is reduced to half, then **[CPMT 1986]**

(a) The fringe width will be doubled (b) The fringe width will reduce to half

(c) The fringe width will not change (d) The fringe width will become $\sqrt{2}$ times

97. In Young's experiment, if the distance between screen and the slit aperture is increased the fringe width will **[RPET 1986]**

(a) Decrease (b) Increases but intensity will decrease

(c) Increase but intensity remains unchanged (d) Remains unchanged but intensity decreases

98. In Fresnel's biprism experiment, the two coherent sources are **[RPET 1985]**

(a) Real (b) Imaginary

(c) One is real and the other is imaginary (d) None of these

99. In Fresnel's experiment, the width of the fringe depends upon the distance **[RPET 1985]**

(a) Between the prism and the slit aperture

(b) Of the prism from the screen

(c) Of screen from the imaginary light sources

(d) Of the screen from the prism and the distance from the imaginary sources

100. In the Young's double slit experiment, the ratio of intensities of bright and dark fringes is 9. This means that **[IIT-JEE 1982]**

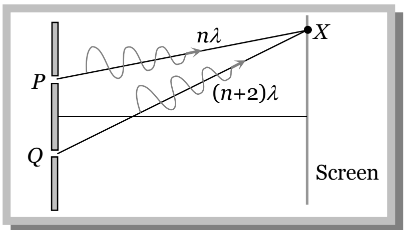
(a) The intensities of individual sources are 5 and 4 units respectively

(b) The intensities of individual sources are 4 and 1 units respectively

(c) The ratio of their amplitudes is 3

(d) The ratio of their amplitudes is 2

101. The figure below shows a double slit experiment. P and Q are the slits. The path lengths PX and QX are $n\lambda$ and $(n + 2)\lambda$ respectively where n is a whole number and λ is the wavelength. Taking the central bright fringe as zero, what is formed at X



(a) First bright

(b) First dark

(c) Second bright

(d) Second dark

102. A plate of thickness t made of a material of refractive index μ is placed in front of one of the slits in a double slit experiment. What should be the minimum thickness t which will make the intensity at the centre of the fringe pattern zero

(a) $(\mu-1)\frac{\lambda}{2}$ (b) $(\mu-1)\lambda$ (c) $\frac{\lambda}{2(\mu-1)}$ (d) $\frac{\lambda}{(\mu-1)}$

103. The thickness of a plate (refractive index μ for light of wavelength λ) which will introduce a path difference of $\frac{3\lambda}{4}$ is

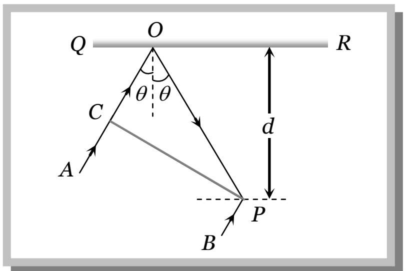
(a) $\frac{3\lambda}{4(\mu-1)}$ (b) $\frac{3\lambda}{2(\mu-1)}$ (c) $\frac{\lambda}{2(\mu-1)}$ (d) $\frac{3\lambda}{4\mu}$

Advance Level

104. In the Young's double slit experiment, if the phase difference between the two waves interfering at a point is ϕ, the intensity at that point can be expressed by the expression (where $A + B$ depends upon the amplitude of the two waves) **[MP PMT/PET 1998; MP PMT 2003]**

(a) $I=\sqrt{A^2+B^2\cos^2\phi}$ (b) $I=\frac{A}{B}\cos\phi$ (c) $I=A+B\cos\phi/2$ (d) $I=A+B\cos\phi$

105. In the adjacent diagram CP represents wavefronts and AO and BP the corresponding two rays. Find the condition on θ for constructive interference at P between the ray BP and reflected ray OP **[IIT-JEE (Screening) 2003]**



(a) $\cos\theta=3\lambda/2d$

(b) $\cos\theta=\lambda/4d$

(c) $\sec\theta-\cos\theta=\lambda/d$

(d) $\sec\theta - \cos\theta = 4\lambda / d$

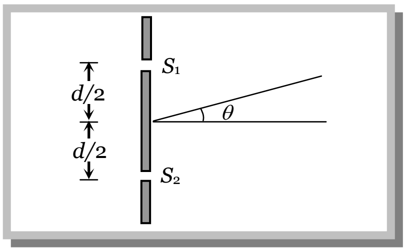
106. When one of the slits of Young's experiment is covered with a transparent sheet of thickness 4.8 *mm*, the central fringe shifts to a position originally occupied by the 30th bright fringe. What should be the thickness of the sheet if the central fringe has to shift to the position occupied by 20th bright fringe **[KCET (Engg.) 2002]**

(a) 3.8 *mm* (b) 1.6 *mm* (c) 7.6 *mm* (d) 3.2 *mm*

107. In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness *t* is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is **[IIT-JEE (Screening) 2002)]**

(a) 2λ (b) $\frac{2\lambda}{3}$ (c) $\frac{\lambda}{3}$ (d) λ

108. In an interference arrangement similar to Young's double slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources each of frequency 10^6 *Hz*. The sources are synchronized to have zero phase difference. The slits are separated by distance *d* = 150 *m*. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is maximum intensity, then $I(\theta)$ for $0 \le \theta \le 90°$ is given by **[IIT-JEE 1995]**



(a) $I(\theta) = I_0$ for $\theta = 90°$

(b) $I(\theta) = I_0 / 2$ for $\theta = 30°$

(c) $I(\theta) = I_0 / 4$ for $\theta = 90°$

(d) $I(\theta)$ is constant for all values of θ

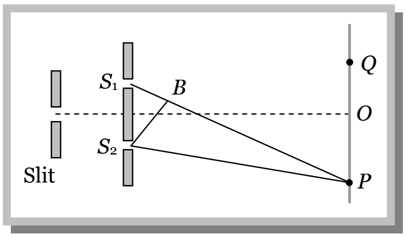
109. In Young's double slit experiment, white light is used. The separation between the slits is *b*. the screen is at a distance $d(d >> b)$ from the slits. Some wavelengths are missing exactly in front of one slit. These wavelengths are **[IIT-JEE 1984; AIIMS 1995]**

(a) $\lambda = \frac{b^2}{d}$ (b) $\lambda = \frac{2b^2}{d}$ (c) $\lambda = \frac{b^2}{3d}$ (d) $\lambda = \frac{2b^2}{3d}$

110. In a two slit experiment with monochromatic light fringes are obtained on a screen placed at some distance from the sits. If the screen is moved by $5 \times 10^{-2} m$ towards the slits, the change in fringe width is $3 \times 10^{-5} m$. If separation between the slits is $10^{-3} m$, the wavelength of light used is **[Roorkee 1992]**

(a) 6000 *Å* (b) 5000 *Å* (c) 3000 *Å* (d) 4500 *Å*

111. In the figure is shown Young's double slit experiment. *Q* is the position of the first bright fringe on the right side of *O*. *P* is the 11th fringe on the other side, as measured from *Q*. If the wavelength of the light used is $6000 \times 10^{-10} m$, then S_1B will be equal to **[CPMT 1986, 92]**



(a) $6 \times 10^{-6} m$

(b) $6.6 \times 10^{-6} m$

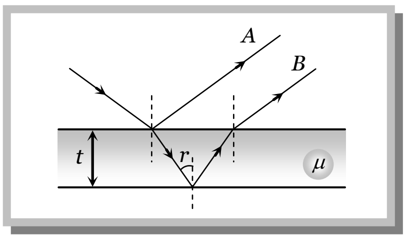
(c) $3.138 \times 10^{-7} m$

(d) $3.144 \times 10^{-7} m$

112. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude *A* and wavelength λ. In another experiment with the same set up the two slits are of equal amplitude *A* and wavelength λ but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is **[IIT-JJE 1986]**

(a) 1 : 2 (b) 2 : 1 (c) 4 : 1 (d) 1 : 1

113. When light of wavelength λ falls on a thin film of thickness *t* and refractive index *n*, the essential condition for the production of constructive interference fringes by the rays *A* and *B* are (*m* = 1, 2, 3, ...)



(a) $2nt \cos r = \left(m - \frac{1}{2}\right)\lambda$

(b) $2nt \cos r = m\lambda$

(c) $nt \cos r = m\lambda$

(d) $nt \cos r = (m-1)\lambda$

114. Four light waves are represented by

(i) $y = a_1 \sin\omega t$ (ii) $y = a_2 \sin(\omega t + \phi)$ (iii) $y = a_1 \sin 2\omega t$ (iv) $y = a_2 \sin 2(\omega t + \phi)$

Interference fringes may be observed due to superposition of

(a) (i) and (ii) (b) (i) and (iii) (c) (ii) and (iv) (d) (iii) and (iv)

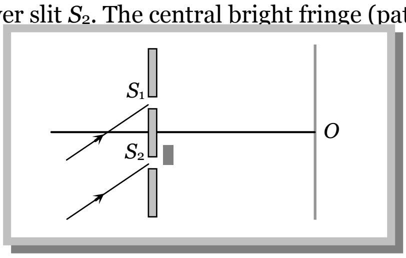
115. In Young's double slit experiment the y-coordinates of central maxima and 10^{th} maxima are 2 cm and 5 cm respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5 the corresponding y-coordinates will be

(a) 2 cm, 7.5 cm (b) 3 cm, 6 cm (c) 2 cm, 4cm (d) 4/3 cm, 10/3 cm

116. The maximum intensity in Young's double slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 10\, d$

(a) $\frac{I_0}{2}$ (b) $\frac{3}{4}I_0$ (c) I_0 (d) $\frac{I_0}{4}$

117. A monochromatic beam of light falls on YDSE apparatus at some angle (say θ) as shown in figure. A thin sheet of glass is inserted in front of the lower slit S_2. The central bright fringe (path difference = 0) will be obtained



(a) At O

(b) Above O

(c) Below O

(d) Anywhere depending on angle θ, thickness of plate t and refractive index of glass μ

118. In Young's double slit experiment how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringe if $\lambda = 2000\ Å$ and $d = 7000\ Å$

(a) 12 (b) 7 (c) 18 (d) 4

119. Young's double slit experiment is made in a liquid. The 10^{th} bright fringe in liquid lies where 6^{th} dark fringe lies in vacuum. The refractive index of the liquid is approximately

(a) 1.8 (b) 1.54 (c) 1.67 (d) 1.2

120. Light of wavelength λ_0 in air enters a medium of refractive index n. If two points A and B in this medium lie along the path of this light at a distance x, then phase difference ϕ_0 between these two points is

(a) $\phi_0 = \frac{1}{n}\left(\frac{2\pi}{\lambda_0}\right)x$ (b) $\phi_0 = n\left(\frac{2\pi}{\lambda_0}\right)x$ (c) $\phi_0 = (n-1)\left(\frac{2\pi}{\lambda_0}\right)x$ (d) $\phi_0 = \frac{1}{(n-1)}\left(\frac{2\pi}{\lambda_0}\right)x$

121. In a Young's double slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelength $\lambda_0 = 750\, nm$ and $\lambda = 900\, nm$. The minimum distance from the common central bright fringe on a screen 2m from the slits where a bright fringe from one interference pattern coincides with a bright fringe from the other is

(a) 1.5 mm (b) 3 mm (c) 4.5 mm (d) 6 mm

122. In the ideal double slit experiment, when a glass plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is

(a) 2λ (b) $\frac{2\lambda}{3}$ (c) $\frac{\lambda}{3}$ (d) λ

123. Two wavelengths of light λ_1 and λ_2 are sent through a Young's double slit apparatus simultaneously. If the third order λ_1 bright fringe coincides with the fourth order λ_2 bright fringe then

(a) $\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$ (b) $\frac{\lambda_1}{\lambda_2} = \frac{3}{4}$ (c) $\frac{\lambda_1}{\lambda_2} = \frac{5}{4}$ (d) $\frac{\lambda_1}{\lambda_2} = \frac{4}{5}$

124. A flake of glass (refractive index 1.5) is placed over one of the openings of a double slit apparatus. The interference pattern displaces itself through seven successive maxima towards the side where the flake is placed. if wavelength of the diffracted light is $\lambda = 600\, nm$, then the thickness of the flake is

(a) 2100 nm (b) 4200 nm (c) 8400 nm (d) None of these

125. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then in the interference pattern

(a) The intensitites of both the maxima and the minima increase

(b) The intensity of the maxima increases and minima has zero intensity

(c) The intensity of the maxima decreases and that of minima increases

(d) The intensity of the maxima decreases and the minima has zero intensity

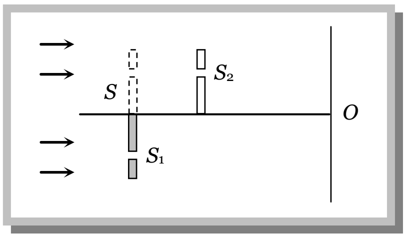
126. In Young's experiment the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of n for which the $(n+1)th$ blue bright band coincides with the n^{th} red band is

(a) 4 (b) 3 (c) 2 (d) 1

127. In a double slit experiment if 5^{th} dark fringe is formed opposite to one of the slits, the wavelength of light is

(a) $\frac{d^2}{6D}$ (b) $\frac{d^2}{5D}$ (c) $\frac{d^2}{15D}$ (d) $\frac{d^2}{9D}$

128. In a Young's double slit experiment one of the slits is advanced towards the screen by a distance $d/2$ and $d = n\lambda$ where n is an odd integer and d is the initial distance between the slits. If I_0 is the intensity of each wave from the slits, the intensity at O is



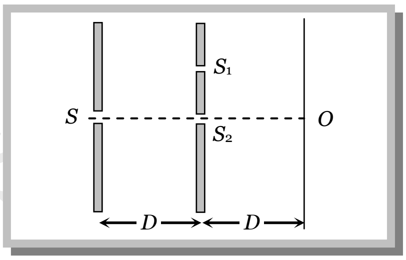
(a) I_0

(b) $\frac{I_0}{4}$

(c) 0

(d) $2I_0$

129. Two ideal slits S_1 and S_2 are at a distance d apart, and illuminated by light of wavelength λ passing through an ideal source slit S placed on the line through S_2 as shown. The distance between the planes of slits and the source slit is D. A screen is held at a distance D from the plane of the slits. The minimum value of d for which there is darkness at O is



(a) $\sqrt{\frac{3\lambda D}{2}}$

(b) $\sqrt{\lambda D}$

(c) $\sqrt{\frac{\lambda D}{2}}$

(d) $\sqrt{3\lambda D}$

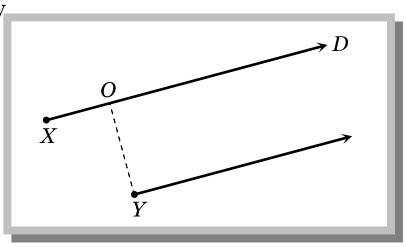
130. In a double slit experiment interference is obtained from electron waves produced in an electron gun supplied with voltage V. if λ is the wavelength of the beam, D is the distance of screen, d is the spacing between coherent source, h is Planck's constant, e is charge on electron and m is mass of electron then fringe width is given as

(a) $\frac{hD}{\sqrt{2meV}\,d}$ (b) $\frac{2hD}{\sqrt{meV}\,d}$ (c) $\frac{hd}{\sqrt{2meV}\,D}$ (d) $\frac{2hd}{\sqrt{meV}\,D}$

131. In a double slit arrangement fringes are produced using light of wavelength 4800 Å. One slit is covered by a thin plate of glass of refractive index 1.4 and the other with another glass plate of same thickness but of refractive index 1.7. By doing so the central bright shifts to original fifth bright fringe from centre. Thickness of glass plate is

(a) 8 μm (b) 6 μm (c) 4 μm (d) 10 μm

132. Two point sources X and Y emit waves of same frequency and speed but Y lags in phase behind X by $2\pi l$ radian. If there is a maximum in direction D the distance XO using n as an integer is given by



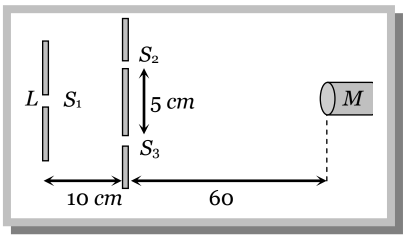
(a) $\frac{\lambda}{2}(n-l)$

(b) $\lambda(n+l)$

(c) $\frac{\lambda}{2}(n+l)$

(d) $\lambda(n-l)$

133. A student is asked to measure the wavelength of monochromatic light. He sets up the apparatus sketched below. S_1, S_2, S_3 are narrow parallel slits, L is a sodium lamp and M is a micrometer eye-piece. The student fails to observe interference fringes. You would advise him to



(a) Increase the width of S_1

(b) Decrease the distance between S_2 and S_3

(c) Replace L with a white light source

(d) Replace M with a telescope

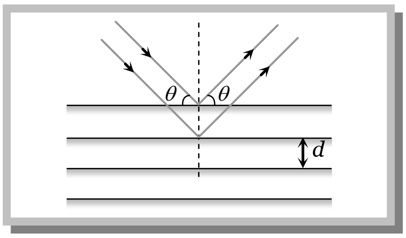
134. A beam with wavelength λ falls on a stack of partially reflecting planes with separation d. The angle θ that the beam should make with the planes so that the beams reflected from successive planes may interfere constructively is (where n =1, 2, ...)

(a) $\sin^{-1}\left(\frac{n\lambda}{d}\right)$

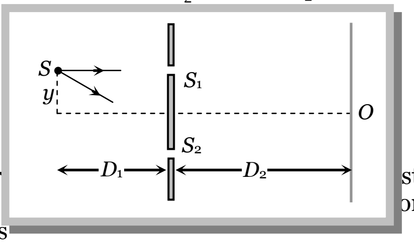
(b) $\tan^{-1}\left(\frac{n\lambda}{d}\right)$

(c) $\sin^{-1}\left(\frac{n\lambda}{2d}\right)$

(d) $\cos^{-1}\left(\frac{n\lambda}{2d}\right)$



135. In a double slit experiment the source slit S is at a distance D_1 and the screen at a distance D_2 from the plane of ideal slit cuts S_1 and S_2 as shown. If the source slit is shifted to by parallel to S_1S_2, the central bright fringe will be shifted by



(a) y (b) $-y$ (c) $\frac{D_2}{D_1}y$ (d) $-\frac{D_2}{D_1}y$

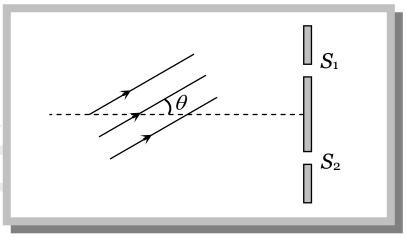
136. A parallel beam of monochromatic light is used in a Young's double slit exper... stance d and the screen is placed parallel to the plane of the slits. The angle which the ... ormal to the plane of the slits to produce darkness at the position of central brightness is

(a) $\cos^{-1}\frac{\lambda}{d}$

(b) $\cos^{-1}\frac{2\lambda}{d}$

(c) $\sin^{-1}\frac{\lambda}{d}$

(d) $\sin^{-1}\frac{\lambda}{2d}$



137. In a Young's double slit experiment, let β be the fringe width, and let I_0 be the intensity at the central bright fringe. At a distance x from the central bright fringe, the intensity will be

(a) $I_0\cos\left(\frac{x}{\beta}\right)$ (b) $I_0\cos^2\left(\frac{x}{\beta}\right)$ (c) $I_0\cos^2\left(\frac{\pi x}{\beta}\right)$ (d) $\left(\frac{I_0}{4}\right)\cos^2\left(\frac{\pi x}{\beta}\right)$

138. In Young's double slit experiment the distance d between the slits S_1 and S_2 is 1 *mm*. What should be the width of each slit be so as to obtain 10 maxima of the two slit interference pattern with in the central maximum of the single slit diffraction pattern

(a) 0.1 *mm* (b) 0.2 *mm* (c) 0.3 *mm* (d) 0.4 *mm*

Diffraction of light

139. When light is incident on a diffraction grating the zero order principal maximum will be **[KCET 2004]**

(a) One of the component colours (b) Absent

(c) Spectrum of the colours (d) White

140. A beam of light of wavelength 600 *nm* from a distant source falls on a single slit 1 *mm* wide and the resulting diffraction pattern is observed on a screen 2 *m* away. The distance between the first dark fringes on either side of the central bright fringe is

[IIT-JEE 1994; KCET 2004]

(a) 1.2 *mm* (b) 1.2 *cm* (c) 2.4 *cm* (d) 2.4 *mm*

141. Consider the following statements

Assertion (A): When a tiny circular obstacle is placed in the path of light from some distance, a bright spot is seen at the centre of the shadow of the obstacle.

Reason (R) : Destructive interference occurs at the centre of the shadow.

Of these statements **[AIIMS 2002]**

(a) Both A and R are true and R is a correct explanation of A (b) Both A and R are true but R is not a correct explanation of A

(c) A is true but R is false (d) A is false but R is true

(e) Both A and R are false

142. The light of wavelength 6328 $\mathring{A}$ is incident on a slit of width 0.2 *mm* perpendicularly situated at a distance of 9 *m* and the central maxima between two minima, the angular is approximately **[MP PMT 1987; Pb. PMT 2002]**

(a) 0.36° (b) 0.18° (c) 0.72° (d) 0.08°

143. A diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light **[KCET (Eng./Med.) 2000; BHU 2001]**

(a) No change (b) diffraction bands become narrower and crowded together

(c) Bands become broader and farther apart (d) Bands disappear

144. Angular width (β) of central maximum of a diffraction pattern on a single slit does not depend upon **[DCE 2000, 2001]**

(a) Distance between slit and source (b) Wavelength of light used

(c) Width of the slit (d) Frequency of light used

145. In order to see diffraction the thickness of the film is **[J&K CEE 2001]**

(a) 100 Å (b) 10,000 Å (c) 1 *mm* (d) 1 *cm*

146. What will be the angle of diffracting for the first minimum due to Fraunhoffer diffraction with sources of light of wave length 550 *nm* and slit of width 0.55 *mm* **[Pb. PMT 2001]**

(a) 0.001 *rad* (b) 0.01 *rad* (c) 1 *rad* (d) 0.1 *rad*

147. The bending of beam of light around corners of obstacles is called **[NCERT 1990; AFMC 1995; RPET 1997; CPMT 1999; JIPMER 2000]**

(a) Reflection (b) Diffraction (c) Refraction (d) Interference

148. Diffraction effects are easier to notice in the case of sound waves than in the case of light waves because **[RPET 1978; KCET 2000]**

(a) Sound waves are longitudinal (b) Sound is perceived by the ear

(c) Sound waves are mechanical waves (d) Sound waves are of longer wavelength

149. Direction of the first secondary maximum in the Fraunhofer diffraction pattern at a single slit is given by (*a* is the width of the slit) **[KCET 1999]**

(a) $a\sin\theta = \frac{\lambda}{2}$ (b) $a\cos\theta = \frac{3\lambda}{2}$ (c) $a\sin\theta = \lambda$ (d) $a\sin\theta = \frac{3\lambda}{2}$

150. A slit of size 0.15 *cm* is placed at 2.1 *m* from a screen. On illuminated it by a light of wavelength $5\times10^{-5}\,cm$. The width of diffraction pattern will be **[RPET 1999]**

(a) 70 *mm* (b) 0.14 *mm* (c) 1.4 *cm* (d) 0.14 *cm*

151. Yellow light is used in a single slit diffraction experiment with a slit of 0.6 *mm*. If yellow light is replaced by *x*-rays, than the observed pattern will reveal **[IIT-JEE 1999]**

(a) That the central maxima is narrower (b) More number of fringes

(c) Less number of fringes (d) No diffraction pattern

152. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of incident beam. At the first maximum of the diffraction pattern the phase difference between the rays coming from the edges of the slit is **[IIT–JEE 1995, 98]**

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

153. Diffraction and interference of light suggest **[CPMT 1995; RPMT 1998]**

(a) Nature of light is electro-magnetic (b) Wave nature

(c) Nature is quantum (d) Nature of light is transverse

154. A light wave is incident normally over a slit of width $24\times10^{-5}\,cm$. The angular position of second dark fringe from the central maxima is 30°. What is the wavelength of light **[RPET 1995]**

(a) 6000 Å (b) 5000 Å (c) 3000 Å (d) 1500 Å

155. A beam of light of wavelength 600 *nm* from a distant source falls on a single slit 1.00 *nm* wide and the resulting diffraction pattern is observed on a screen 2 *m* away. The distance between the first dark fringes on either side of the central bright fringe is **[IIT-JEE 1994]**

(a) 1.2 *cm* (b) 1.2 *mm* (c) 2.4 *cm* (d) 2.4 *mm*

156. A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 *mm*. The light is focused by a convex lens on a screen placed on the focal plane. The first minimum will be formed for the angle of diffraction equal to **[CBSE PMT 1993]**

(a) 0° (b) 15° (c) 30° (d) 60°

157. Light appears to travel in straight lines since **[RPMT 1997; AIIMS 1998; CPMT 1987, 89, 90, 2001; KCET (Engg.) 2002; BHU 2002]**

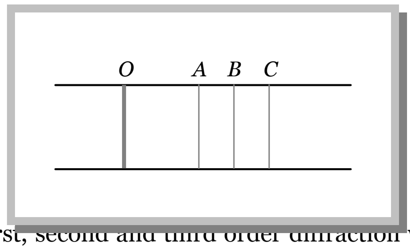
(a) It is not absorbed by the atmosphere (b) It is reflected by the atmosphere

(c) It's wavelength is very small (d) It's velocity is very large

158. The condition for observing Fraunhofer diffraction from a single slit is that the light wavefront incident on the slit should be **[MP PMT 1987]**

(a) Spherical (b) Cylindrical (c) Plane (d) Elliptical

159. The position of the direct image obtained at O, when a monochromatic beam of light is passed through a plane transmission grating at normal incidence is shown in fig.



The diffracted images A, B and C correspond to the first, second and third order diffraction when the source is replaced by an another source of shorter wavelength **[CPMT 1986]**

(a) All the four shift in the direction C to O
(b) All the four will shift in the direction O to C
(c) The images C, B and A will shift toward O
(d) The images C, B and A will shift away from O

160. To observe diffraction the size of an obstacle **[CPMT 1982]**

(a) Should be of the same order as wavelength
(b) Should be much larger than the wavelength
(c) Have no relation to wavelength
(d) Should be exactly $\frac{\lambda}{2}$

161. The first diffraction minima due to a single slit diffraction is at $\theta = 30^o$ for a light of wavelength 5000 Å. The width of the slit is **[CPMT 1985]**

(a) $5\times10^{-5}\,cm$ (b) $1.0\times10^{-4}\,cm$ (c) $2.5\times10^{-5}\,cm$ (d) $1.25\times10^{-5}\,cm$

162. Radio waves diffract pronoucedly around buildings while light waves which are also electromagnetic waves do not because **[PPE 1978]**

(a) Wavelength of the radio waves is not comparable with the size of the obstacle
(b) Wavelength of radio waves is of the order of 200-500 m hence they bend more than the light waves whose wavelength is very small
(c) Light waves are transverse whereas radio waves are longitudinal
(d) None of the above

163. One cannot obtain diffraction from a wide slit illuminated by a monochromatic light because **[PPE 1978]**

(a) The half period elements contained in a wide slit are very large so the resultant effect is general illumination
(b) The half period elements contained in a wide slit are small so the resultant effect is general illumination
(c) Diffraction patterns are superimposed by interference pattern and hence the result is general illumination
(d) None of these

164. In the far field diffraction pattern of a single slit under polychromatic illumination, the first minimum with the wavelength λ_1 is found to be coincident with the third maximum at λ_2. So

(a) $3\lambda_1 = 0.3\lambda_2$ (b) $3\lambda_1 = \lambda_2$ (c) $\lambda_1 = 3.5\lambda_2$ (d) $0.3\lambda_1 = 3\lambda_2$

165. In case of Fresnel diffraction

(a) Both source and screen are at finite distance from diffracting device
(b) Source is at finite distance while screen at infinity from diffraction device
(c) Screen is at finite distance while source at infinity from diffracting device
(d) Both source and screen are effectively at infinity from diffracting device

166. Light of wavelength λ = 5000 $\mathring{A}$ falls normally on a narrow slit. A screen placed at a distance of 1 m from the slit and perpendicular to the direction of light. The first minima of the diffraction pattern is situated at 5 mm from the centre of central maximum. The width of the slit is

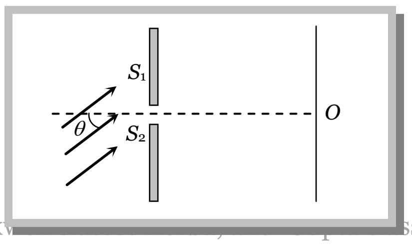
(a) 0.1 mm (b) 1.0 mm (c) 0.5 mm (d) 0.2 mm

167. Light falls normally on a slit of width 0.3 mm. A lens of focal length 40 cm collects the rays at its focal plane. The distance of the first dark band from the direct one is 0.8 mm. The wavelength of light is

(a) 4800 Å (b) 5000 Å (c) 6000 Å (d) 5896 Å

168. A parallel monochromatic beam of light is incident at an angle θ to the normal of a slit of width e. The central point O of the screen will be dark if

(a) $e\sin\theta = n\lambda$ where n = 1, 3, 5 ...
(b) $e\sin\theta = n\lambda$ where n = 1, 2, 3 ...
(c) $e\sin\theta = (2n-1)\lambda/2$ where n = 1, 2, 3 ......



(d) $e\cos\theta = n\lambda$ where n = 1, 2, 3, 4 ....

Polarization of Light

169. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refraction index n) is **[AIEEE 2004]**

(a) $\sin^{-1}(n)$ (b) $\sin^{-1}\left(\frac{1}{n}\right)$ (c) $\tan^{-1}\left(\frac{1}{n}\right)$ (d) $\tan^{-1}(n)$

170. Through which character we can distinguish the light waves from sound waves **[CBSE PMT 1990; RPET 2002]**

(a) Interference (b) Refraction (c) Polarisation (d) Reflection

171. Which of following can not be polarised **[Kerala PMT 2001]**

(a) Radio waves (b) Ultraviolet rays (c) Infrared rays (d) Ultrasonic waves

172. A polaroid is placed at 45° to an incoming light of intensity I_0. Now the intensity of light passing through polaroid after polarisation would be **[CPMT 1995]**

(a) I_0 (b) $I_0/2$ (c) $I_0/4$ (d) Zero

173. Plane polarised light is passed through a polaroid. On viewing through the polaroid we find that when the polariod is given one complete rotation about the direction of the light, one of the following is observed **[MNR 1993]**

(a) The intensity of light gradually decreases to zero and remains at zero
(b) The intensity of light gradually increases to a maximum and remains at maximum
(c) There is no change in intensity
(d) The intensity of light is twice maximum and twice zero

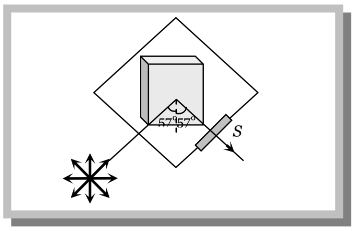
174. Out of the following statements which is not correct **[CPMT 1991]**

(a) When unpolarised light passes through a Nicol's prism, the emergent light is elliptically polarised
(b) Nicol's prism works on the principle of double refraction and total internal reflection
(c) Nicol's prism can be used to produce and analyse polarised light
(d) Calcite and Quartz are both doubly refracting crystals

175. A ray of light is incident on the surface of a glass plate at an angle of incidence equal to Brewster's angle ϕ. If μ represents the refractive index of glass with respect to air, then the angle between reflected and refracted rays is **[CPMT 1989]**

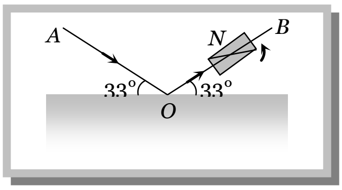
(a) $90+\phi$ (b) $\sin^{-1}(\mu\cos\phi)$ (c) 90° (d) $90^{o}-\sin^{-1}(\sin\phi/\mu)$

176. Figure represents a glass plate placed vertically on a horizontal table with a beam of unpolarised light falling on its surface at the polarising angle of 57° with the normal. The electric vector in the reflected light on screen S will vibrate with respect to the plane of incidence in a **[CPMT 1988]**



(a) Vertical plane
(b) Horizontal plane
(c) Plane making an angle of 45° with the vertical
(d) Plane making an angle of 57° with the horizontal

177. A beam of light AO is incident on a glass slab $(\mu = 1.54)$ in a direction as shown in figure. The reflected ray OB is passed through a Nicol prism on viewing through a Nicole prism, we find on rotating the prism that **[CPMT 1986]**



(a) The intensity is reduced down to zero and remains zero
(b) The intensity reduces down some what and rises again
(c) There is no change in intensity
(d) The intensity gradually reduces to zero and then again increases

178. Polarised glass is used in sun glasses because **[CPMT 1981]**

(a) It reduces the light intensity to half an account of polarisation (b) It is fashionable
(c) It has good colour (d) It is cheaper

179. In the propagation of electromagnetic waves the angle between the direction of propagation and plane of polarisation is **[CPMT 1978]**

(a) 0° (b) 45° (c) 90° (d) 180°

180. The transverse nature of light is shown by

[CPMT 1972, 74, 78; RPMT 1999; MP PMT 2000; AFMC 2001; AIEEE 2002; MP PET 2004]

(a) Interference of light (b) Refraction of light (c) Polarisation of light (d) Dispersion of light

181. A calcite crystal is placed over a dot on a piece of paper and rotated, on seeing through the calcite one will be see **[CPMT 1971]**

(a) One dot (b) Two stationary dots

(c) Two rotating dots (d) One dot rotating about the other

182. In a doubly refracting crystal, optic axis is a direction along which

(a) A plane polarised beam does not suffer deviation

(b) Any beam of light does not suffer any deviation

(c) Double refraction does not take place

(d) Ordinary and extraordinary rays undergo maximum deviation

183. Which is incorrect with reference to polarisation by reflection

(a) The degree of polarisation varies with the angle of incidence

(b) Percentage of the polarising light in the reflected beam is the greatest at the angle of polarisation

(c) Reflected light is plane polarised in the plane of incidence

(d) Reflected light is plane polarised in the plane perpendicular to plane of incidence

184. Two polarising plates have polarising directions parallel so as to transmit maximum intensity of light. Through what angle must either plate be turned if the intensities of the transmitted beam is to drop by one-third

(a) 55°18' (b) 144°22' (c) Both of these (d) None of these

185. The polaroid is

(a) Celluloid film (b) Big crystal

(c) Cluster of small crystals arranged in a regular way (d) Cluster of small crystals arranged in a haphazard way

186. Light from the cloudless sky is

(a) Fully polarised (b) Partially polarised (c) Unpolarised (d) Can not be said

Doppler's Effect of Light

187. The observed wavelength of light coming from a distant galaxy is found to be increased by 0.5% as compared with that comparing from a terrestrial source. The galaxy is **[MP PMT 1993, 2003]**

(a) Stationary with respect to the earth

(b) Approaching the earth with velocity of light

(c) Receding from the earth with the velocity of light

(d) Receding from the earth with a velocity equal to $1.5\times10^{6} m/s$

188. In hydrogen spectrum the wavelength of H_α line is 656 *nm* whereas in the spectrum of a distant galaxy. H_α line wavelength is 706*nm*. Estimated speed of the galaxy with respect to earth is **[IIT-JEE 1999; UPSEAT 2003]**

(a) $2\times10^{8} m/s$ (b) $2\times10^{7} m/s$ (c) $2\times10^{6} m/s$ (d) $2\times10^{5} m/s$

189. A star emits light of 5500 Å wavelength. Its appears blue to an observer on the earth, it means **[DPMT 2002]**

(a) Star is going away from the earth (b) Star is stationary

(c) Star is coming towards earth (d) None of the above

190. The 6563 Å line emitted by hydrogen atom in a star is found to be red shifted by 5 Å. The speed with which the star is receding from the earth is **[Pb. PMT 2002]**

(a) $17.29\times10^{9} m/s$ (b) $4.29\times10^{7} m/s$ (c) $3.39\times10^{5} m/s$ (d) $2.29\times10^{5} m/s$

191. Three observers *A*, *B* and *C* measure the speed of light coming from a source to be v_A, v_B and v_C. The observer *A* moves towards the source, the observer *C* moves away from the source with the same speed. The observer *B* stays stationary. The surrounding space is vacuum very where. Then **[Kerala CET (Med.) 2002]**

(a) $v_A > v_B > v_C$ (b) $v_A < v_B < v_C$ (c) $v_A = v_B = v_C$ (d) $v_A = v_B > v_C$

192. A star emitting light of wavelength 5896 Å is moving away from the earth with a speed of 3600 *km/sec*. The wavelength of light observed on earth will ($c = 3\times10^{8} m/\sec$ is the speed of light) **[MP PET 1995, 2002]**

(a) Decrease by 5825.25 Å (b) Increase by 5966.75 Å (c) Decrease by 70.75 Å (d) Increase by 70.75 Å

193. The periodic time of rotation of a certain star is 22 days and its radius is $7\times10^{8} m$. If the wavelength of light emitted by its surface be 4320 Å, the Doppler shift will be (1 day = 86400 *sec*) **[MP PET 2001]**

(a) 0.033 $Å$ (b) 0.33 $Å$ (c) 3.3 $Å$ (d) 33 $Å$

194. A heavenly body is receding from earth such that the fractional change in λ is 1, then its velocity is **[DCE 2000]**

(a) C (b) $\frac{3C}{5}$ (c) $\frac{C}{5}$ (d) $\frac{2C}{5}$

195. A star is going away from the earth. An observer on the earth will see the wavelength of light coming from the star **[MP PMT 1999]**

(a) Decreased

(b) Increased

(c) Neither decreased nor increased

(d) Decreased or increased depending upon the velocity of the star

196. If the shift of wavelength of light emitted by a star is towards violet, then this shows that star is **[RPET 1996; RPMT 1999]**

(a) Stationary (b) Moving towards earth (c) Moving away from earth (d) Information is incomplete

197. When the wavelength of light coming from a distant star is measured it is found shifted towards red. Then the conclusion is **[JIPMER 1999]**

(a) The star is approaching the observer (b) The star recedes away from earth

(c) There is gravitational effect on the light (d) The star remains stationary

198. In the spectrum of light of a luminous heavenly body the wavelength of a spectral line is measured to be 4747 Å while actual wavelength of the line is 4700 Å. The relative velocity of the heavenly body with respect to earth will be (velocity of light is $3\times10^8 m/s$) **[MP PET 1997; MP PMT/PET 1998]**

(a) $3\times10^5 m/s$ moving towards the earth (b) $3\times10^5 m/s$ moving away from the earth

(c) $3\times10^6 m/s$ moving towards the earth (d) $3\times10^6 m/s$ moving away from the earth

199. The wavelength of light observed on the earth, from a moving star is found to decrease by 0.05%. Relative to the earth the star is **[MP PMT/PET 1998]**

(a) Moving away with a velocity of $1.5\times10^5 m/s$ (b) Coming closer with a velocity of $1.5\times10^5 m/s$

(c) Moving away with a velocity of $1.5\times10^4 m/s$ (d) Coming closer with a velocity of $1.5\times10^4 m/s$

200. Due to Doppler's effect, the shift in wavelength observed is 0.1 Å for a star producing wavelength 6000 Å. Velocity of recession of the star will be **[KCET 1998]**

(a) 2.5 *km/s* (b) 10 *km/s* (c) 5 *km/s* (d) 20 *km/s*

201. A rocket is going away from the earth at a speed of $10^6 m/s$. If the wavelength of the light wave emitted by it be 5700 Å, what will be its Doppler's shift **[MP PMT 1990, 94; RPMT 1996]**

(a) 200 Å (b) 19 Å (c) 20 Å (d) 0.2 Å

202. A rocket is going away from the earth at a speed 0.2 *c*, where *c* = speed of light, it emits a signal of frequency $4\times10^7 Hz$. What will be the frequency observed by an observer on the earth **[RPMT 1996]**

(a) $4\times10^6 Hz$ (b) $3.3\times10^7 Hz$ (c) $3\times10^6 Hz$ (d) $5\times10^7 Hz$

203. A star moves away from earth at speed 0.8 *c* while emitting light of frequency $6\times10^{14} Hz$. What frequency will be observed on the earth (in units of $10^{14} Hz$) (*c* = speed of light) **[MP PMT 1995]**

(a) 0.24 (b) 1.2 (c) 30 (d) 3.3

204. The sun is rotating about its own axis. The spectral lines emitted from the two ends of its equator, for an observer on the earth, will show **[MP PMT 1994]**

(a) Shift towards red end

(b) Shift towards violet end

(c) Shift towards red end by one line and towards violet end by other

(d) No shift

205. The time period of rotation of the sun is 25 days and its radius is $7\times10^8 m$. The Doppler shift for the light of wavelength 6000 Å emitted from the surface of the sun will be **[MP PMT 1994]**

(a) 0.04 $Å$ (b) 0.40 $Å$ (c) 4.00 $Å$ (d) 40.0 $Å$

206. The apparent wavelength of the light from a star moving away from the earth is 0.01 % more than its real wavelength. Then the velocity of star is **[CPMT 1979]**

(a) 60 *km/sec* (b) 15 *km/sec* (c) 150 *km/sec* (d) 30 *km/sec*

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
a	**d**	**c**	**d**	**b**	**a**	**c**	**c**	**b**	**c**	**d**	**c**	**d**	**a**	**b**	**c**	**d**	**a**	**c**	**c**
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
c	**b**	**b**	**c**	**a**	**d**	**c**	**d**	**a**	**d**	**b**	**b**	**c**	**b**	**c**	**d**	**c**	**c**	**a**	**b**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
c	**d**	**c**	**c**	**a**	**d**	**a**	**b**	**c**	**a**	**d**	**d**	**c**	**a**	**b**	**b**	**a**	**d**	**a**	**d**
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
c	**d**	**c**	**c**	**c**	**b**	**b**	**c**	**b**	**a**	**b, c**	**b**	**d**	**b**	**c**	**b**	**d**	**d**	**b**	**a**
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
a	**a**	**b**	**a**	**b**	**d**	**b**	**b**	**b**	**d**	**a**	**c**	**c**	**c**	**c**	**a**	**b**	**b**	**d**	**b,d**
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
c	**c**	**a**	**d**	**b**	**d**	**a**	**a,b**	**a,c**	**a**	**a**	**b**	**a**	**a,d**	**c**	**a**	**d**	**b**	**a**	**b**
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
c	**a**	**a**	**c**	**a**	**c**	**d**	**c**	**c**	**a**	**a**	**b**	**b**	**c**	**d**	**d**	**c**	**b**	**d**	**d**
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
c	**a**	**b**	**a**	**b**	**a**	**b**	**d**	**d**	**b**	**a**	**c**	**b**	**a**	**d**	**c**	**c**	**c**	**c**	**a**
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b	**b**	**a**	**c**	**a**	**a**	**c**	**b**	**d**	**c**	**d**	**b**	**d**	**a**	**c**	**a**	**d**	**a**	**a**	**c**
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
d	**c**	**c**	**c**	**c**	**d**	**d**	**b**	**c**	**d**	**c**	**d**	**a**	**a**	**b**	**b**	**b**	**d**	**b**	**c**
201	**202**	**203**	**204**	**205**	**206**														
b	**b**	**b**	**c**	**a**	**d**														

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Revision Notes

1. Magnetic flux. Magnetic flus if denoted by (ϕ)

$$\phi = \vec{A} . \vec{B}$$

$$\phi = AB \cos \theta$$

Unit. Wb. or Tesla m^2. Thus 1 T = 1 W b m^2.

1. Faraday's laws of electromagnetic induction. On the basis of his experiments, Faraday gave the following laws :

(*i*) *Whenever magnetic flux linked with a circuit changes, induced e.m.f. is produced.*

(*ii*) *The induced e.m.f. lasts as long as the change in the magnetic flux continues.*

(*iii*) *The magnitude of induced e.m.f. is directly proportional to the rate of change of magnetic flux.*

The magnitude of induced e.m.f. is given by

$$e = \frac{\phi_2 - \phi_1}{dt}.$$

where ϕ_1 and ϕ_2 are magnetic flux linked with the coil initially and after time t. The negative sign shows that induced e.m.f. opposes the change taking plane in magnetic flux/

In differential notation,

$$e = -\frac{d\phi}{dt}.$$

In CGS system, e is measured in *e.m.u.* and ϕ in maxwell while in SI system, e is measured in volt and ϕ in weber.

Note. It may be remembered that

$$1 \text{ volt} = 10^8 \text{ e.m.u. of potential,}$$

$$1 \text{ ampere} = \frac{1}{10} \, e.m.u. \text{ of current,}$$

$$1 \text{ coulomb} = \frac{1}{10} \, e.m.u. \text{ of charge,}$$

and $$1 \text{ ohm} = 10^9 \text{ e.m.u. of resistance.}$$

> **Key Points**
>
> The magnetic flux linked with a loop does not change with time when
>
> (*i*) When both loop and magnet move in same direction with same velocity.
>
> (*ii*) magnet is rotated about its axis keeping its position from loop unchanged.
>
> (*iii*) loop is rotated in a uniform magnet field keeping it fully within the field.

(*a*) **Induced current.** If the coil is a closed circuit and has a resistance R, then induced current

$$i = \frac{e}{R} = \frac{N}{R}\frac{d\phi}{dt} (N = \text{no. of turns in the coil})$$

(*b*) **Induced charge.** Incuded charge is given by :

$$q = i \times t = \frac{N\phi}{R} \text{ where } \phi = \text{change of flux.}$$

Also $$q = -\frac{N(\phi_2 - \phi_1)}{Rt} \times t$$

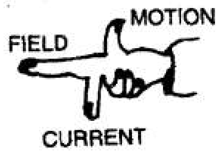
$$q = -\frac{N(\phi_2 - \phi_1)}{R}$$

$$q = \frac{e}{R} \times t \qquad q = \frac{N(\phi_2 - \phi_1)\,t}{R}$$

This shows that induced change is independent of **time interval.**

(*c*) **Len's law.** The direction of induced *e.m.f.* due to electromagnetic induction is such that its effect opposes the cause which has produced it.

(*d*) **Fleming's right hand rile.** *If thumb, fore–finger and the middle finger are spread perpendicular to one another (in two different⊥ planes) such that the fore–finger denotes the direction oif magnetic field and thumb, the direction of motion then the middle finger denotes the direction of induced e.m.f.*



3. E.M.F. induced in a moving conductor. If a straight conductor of length l is moving perpendicular to a uniform magnetic field of flux density B with a velocity v, then

Infuced *e.m.f.* $e = Blv$

If R is the resistance of the conductor, then

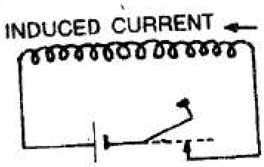
Induced current $i = \frac{e}{R} = \frac{Blv}{R}$

4. Self inductance or coefficient of self induction. The magnetic flux linked with a coil through which current I is flowing is given by

$$\phi = LI$$

where L *is called self inductance or coefficient of self induction of the coil.*

The coefficient of self induction of a coil is numerically equal to the magnetic flux linked with it, when unit current flowes through it.



The instantaneous induced e.m.f. produced in the coil is given by

$$e = -L\frac{dI}{dt}$$

where $\frac{dI}{dt}$ is rate of change of current at that instant,

The coefficient of self induction of a coil is also numerically equal to the induced e.m.f set up in the coil, when the rate of change of current in the coil is unity.

The unit of self inductance is e.m.u. in CGS system and henry in SI.

Self inductance of a solenoid. Self inductance of a solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

$L = \mu_r\mu_0 n^2 lA$ where m_r is relative permeability of material of core used.

where μ_0 = permeability of free space.

N = Total no. of turns and n = no. of turns per unit length.

l = length of the coil.

A = area of cross-section of coil.

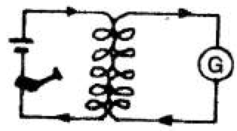
Also $$L = \frac{\mu_0 n^2 l^2 A}{l} = \mu_0 n^2 lA.$$

5. Mutual inductance or coefficient of mutual induction. The magnetic flux linked with one coil when a current I flows through a neighbouring coil is given by

$$\phi = MI$$

where M is called *mutual inductance or coefficient of mutual induction between the two coils.*

The coefficient of mutual induction between two coils is numerically equal to the magnetic flux linked with one coil, when unit current flows through the neighbouring coil.



The instantaneous induced *e.m.f.* produced in one coil is given by

$$e = -M\frac{dI}{dt}$$

where $\frac{dI}{dt}$ is rate to change of current in the neighbouring coil at that instant.

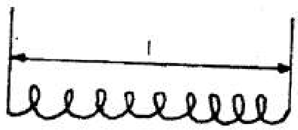
The coefficient of mutual induction between two coils is also numerically equal to the induced e.m.f. set up in one coil, when the rate of change of current in the neighbouring coil is unity.

In CGS system, the unit of mutual inductance is *e.m.u.* and in S.I., the unit is *henry*.

1 henry = 10^9 *e.m.u.* of mutual inductance.

Self induction is called inertia of electricity.

Mutual inductance between two coils. The mutual inductance between two coils of area A, no. of turns N_1 and N_2 with length of secondary or primary as l is given by :



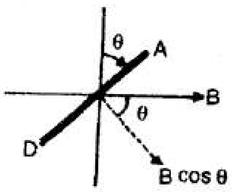
$$M = -\mu_0\frac{N_1N_2A}{l}$$

If n_1 and n_2 are the no. of turns per unit length in the two coils,

then $M = \mu_0 n_1 n_2 A l$. $(\because N_1 = n_1 l \text{ and } N_2 = n_2 l)$

Note. In CGS system I, e, ϕ and M are measured in *e.m.u.* of current, *e.m.u.* of *e.m.f.*, maxwell and e.m.u. of mutual induction respectively, whele in SI, they are respectively measured in ampere, volt, weber and henry.

6. Induced emf produced in a coil rotating inside a magnetic field (a.c generator). Consider coil of area A, number of induction turns N and rotating inside a magnetic field of induction B with angular velocity ω. At $t = 0$, the coil is vertical. At any time t, the plane of coil will make an angle θ equal to ωt with the vertical.



The induced *e.m.f.* produced in the coil at time t given by

$$e = NAB\omega \sin \omega t$$

where N is frequency and T is time period of rotation.

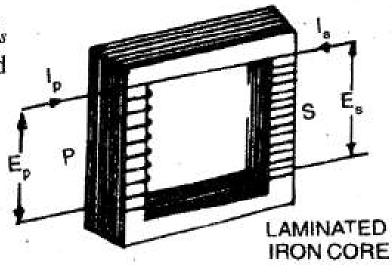
$$e_{max} = NAB\omega$$

Thus $$e = e_{max} \sin \omega t.$$

7. Transformer. *A transformer is a device of changing a low voltage alternating current into a high voltage alternating current or vice-versa.*

A transformer which increases the voltage (current will decrease), is called **step-up transformer** while another while which decreases the voltage (current will increases) is **step-down transformer.**

Suppose a transformer consists of a primary of n_p turns and the secondary coil of n_s turns [Fig.]. Let E_p and E_s be the values of *e.m.f.* across primary and secondary coil and I_p and I_s be the respective values of the current.



Then, $\frac{E_s}{E_p} = \frac{n_s}{n_p} = \frac{I_p}{I_s} = k$ where k is known as transformation ratio.

For a step-up transformer, $k > 1$ and for a step down transformer, $k > 1$

For a 100% efficient (ideal) transformer, Input power = Output power *i.e.*,

$$E_pI_p = E_sI_s.$$

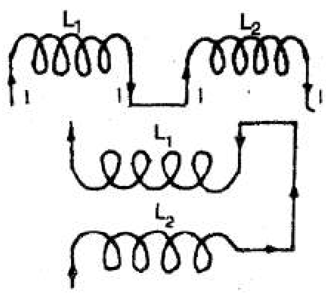
8. (*i*) An electric field produced by time varying magnetic field, which has non-vanishing closed line integral is called as non-conservative field.Here $\oint \vec{E}\,d\vec{l} \neq 0$,

(*ii*) In conservative fields $\oint E\,.\,dl = 0$.

(*iii*) The direction of induced current is given by Fleming's right hand rule.

(*iv*) The circulating currents induced in metal sheets, blocks when the magnetic flux linked with them changes are called eddy currents or Focault current

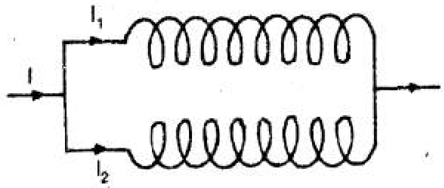
(*v*) If coils are in series as shown in (a) the



$$L = L_1 + L_2 + 2M$$

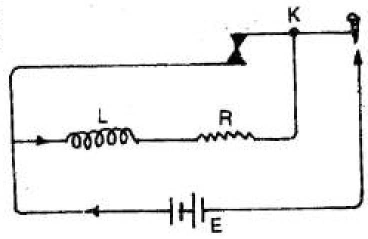
If the coils are as shown in (b), then

$$L = L_1 + L_2 - 2M.$$



$$\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}.$$

9. (*i*) Growth and decay of current in L–R circuit. During growth of current in L–R circuit, let R be the inductanceless resistance and L the resistanceless inductance. If I is the instantaneous value of the current at any time t, then



The maximum value of current $I_m=\frac{E}{R}$

The current I at any instant is given by

$$I=I_m\left(1-e^{-\frac{R}{L}t}\right)$$

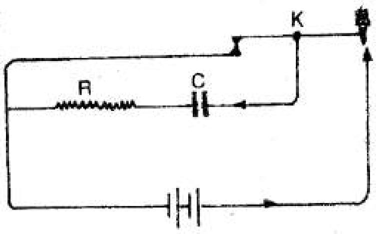
The expression L/R is called the time constant and is measured in seconds.

(*ii*) Decay of current. On switching off the circuit without introducing and additional resistance, the current takes some time to decay from maximum to zero value. The current at any instant is given by

$$I=I_m\,e^{-\frac{R}{L}t}$$

The value of current I after L/2, 2L/R, 3L/R ...... is given by 0.3679 I_m, 0.1357 I_m, 0.0498 I_m ...... etc.

10. Charging and discharging of a condenser. (*i*) When a circuit containing capacitance and resistance in series with a battery is switched on the charge grows from zero to maximum value through the capacitor in a certain time. If q is the instantaneous value of charge and q_m the maximum value of charge.



Maximum value of charge q_m = EC

The instantaneous value of charge q is given by

$$q=q_m\left(1-e^{\frac{-t}{RC}}\right)$$

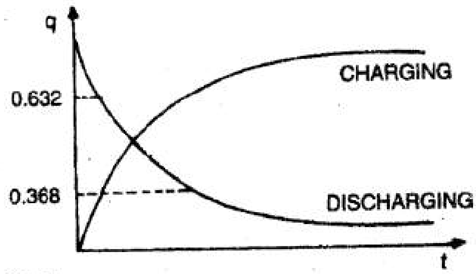
(*ii*) Similarly when the circuit is switched off without introducing any additional resistance, then the charge takes some time to decay from maximum to zero value. The value of charge at any instant is given by

$$q=q_m\,.\,e^{\frac{-t}{RC}}$$

In both these cases RC is called the **time constant** as it has the dimensions of time.

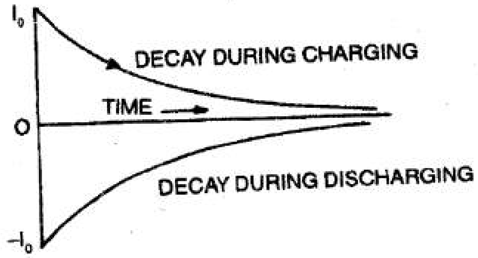
(*iii*) After t = 5RC, the capacitor gets almost fully charged.

(*iv*) Discharging of a capacitor $q=q_0e^{-t/RC}$ is graphically shows as



RC is time constant = 0.368 q_0

(v) Decay of current during charging and discharging is shown in Fig. below



11. Energy stored in an inductance coil. When the current grows in an inductance, work has to be done in establishing the current in it. This work is stored into the inductance as magnetic energy. When the circuit is broken, this energy is liberated.The energy stored at the make and liberated at the break is given by

$$\text{Energy} = \frac{1}{2} LI_m^2$$

$$\text{Energy at any instant} = \frac{1}{2} LI^2$$

If L is in henry and I in amperes, then energy is in Joules.

12. Circuit containing inductance and capacitance. When a condenser is allowed to discharge through an inductance, the discharge is oscillatory and is according to S.H.M. equation.

$$q = q_m \cos \omega t$$

where $$\omega = \frac{1}{\sqrt{LC}}$$

L being the inductance and C the capacitance of the circuit. The period of oscillation is given by

$$T = 2\pi \sqrt{LC}$$

and the frequency $n = \dfrac{1}{T} = \dfrac{1}{2\pi\sqrt{LC}}$ and is called the natural frequency of L-C circuit.

13. Maximum or peak value of alternating voltage and alternating current. *The maximum value of e.m.f. in either direction is called the* **peak value** *of alternating e.m.f. It is given by*

$$E = E_0 \sin \omega t$$

where E is the instantaneous value and $\omega = 2\pi n$ in which n is the frequency of A.C.

Maximum value of the current I_0 *in either direction is called the* **peak value** *of the alternating current.*

$$I = I_0 \sin \omega t$$

where I is the instantaneous value of current.

14. Mean or average value of A.C. voltage and current. *The average or mean value of current is that steady current which sends the same charge through a circuit in the same time as the alternating current does in half the time period.* If I_m denotes the mean value, then

$$I_m = \frac{2I_0}{\pi} = 0.637\, I_0$$

Similarly the mean or average voltage E_m is given by

$$E_m = \frac{2E_0}{\pi} = 0.637\, E_m$$

15. Root mean square value or virtual value. *It is that steady current or voltage which produces the same heating effect in a resistance in a given time as the A.C. does in the same resistance in the same time.*

D.C. ammeter and volt-meter are not capable of measuring A.C. currents of voltages. They will give zero reading when used in a.c. circuit. It is due to reason that mean value of A.C. current and voltages is zero over complete cycle.

If I_p denotes the virtual value of current, then

$$I_p = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

Similarly $$E_p = \frac{E_0}{\sqrt{2}} = 0.707\, E_0$$

16. Relation between virtual and mean values. $I_v = I_m \frac{\pi}{2\sqrt{2}}$ and $E_v = E_m \frac{\pi}{2\sqrt{2}}$

17. Impedance and reactance. The ratio of the applied voltage to the current is called the impedance (Z) of the A.C. circuit if all the three elements (R, L, C) are present in general.

When only inductance or only capacitance is present in the circuit, then the ratio of E_{rms} and i_{rms} is called reactance of inductance or of capacitance respectively (represented by X_L and X_C).

Alternating currents/voltage a always measured by a.c. ammete and voltmeters. They alwa record their virtual values only

18. Different types of alternating circuits : (*i*) Circuits containing only resistance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin \omega t$$

Instantaneous current and voltage are in phase always.

(*ii*) Circuit containing only inductance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin (\omega t - \pi/2)$$

The current lags behind the voltage by phase $\pi/2$. The inductive reactance $X_L = \omega L = 2\pi n L$

(*iii*) Circuit containing only capacitance :

$$E = E_0 \sin \omega t \text{ then } i = i_0 \sin (\omega t + \pi/2)$$

The current leads the voltage by a phase angle $\pi/2$ radian.

The capacitance reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi n C}$

(*iv*) Circuit containing resistance and inductance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin (\omega t - \phi)$$

where $\tan \phi = \frac{\omega L}{R}$ and $i_0 = \frac{E_0}{Z}$ and $I_v = \frac{E_v}{Z}$

The current lags behind the voltage by phase angle ϕ radian. The impedance

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{R^2 + X_L^2}$$

(*v*) Circuit containing resistance and capacitance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin (\omega t - \phi)$$

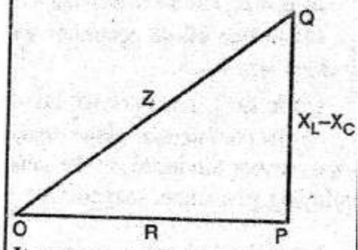
where $\tan \phi = \frac{I/\omega C}{R}$

$$i_0 = \frac{E_0}{Z} \text{ and } I_v = \frac{E_v}{Z}$$

The current leads the voltage by phase angle ϕ. The impedance

$$Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2} = \sqrt{R^2 + X_C{}^2}$$

The impedance triangle is given as below :



It measures the total effective resistance offered by LCR-circuit.

(*vi*) Circuit containing resistance, inductance and capacitance in series (LCR circuit) :

The impedance $$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

or $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

When $E = E_0 \sin \omega t$, then $i = i_0 \sin(\omega t - \phi)$

where $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{\omega L - 1/\omega C}{R}$

and $i_0 = \dfrac{E_0}{Z}$ and $I_v = \dfrac{E_v}{Z}$

At resonance : $n = \dfrac{1}{2\pi\sqrt{LC}}$ and $X_L = X_C$.

$Z = R, \phi = 0$ and I_0 is maximum.

(*vii*) **Coefficient of coupling of two coils :**

$$k = \sqrt{\frac{M}{L_1 L_2}}\text{, } k \text{ is always less than one}$$

(*viii*) **In case of L-C circuit :**

$$\frac{d^2q}{dt^2} + \omega^2 q = 0$$

here $\omega^2 = \dfrac{1}{LC}$.

Power of an A.C. circuit. If $E = E_0 \sin \omega t$ is the applied *e.m.f.* and $I = I_0 \sin(\omega t + \phi)$ is the corresponding value of the current, then

Power of circuit $P = E_v \,.\, I_v \cos\phi$

$\cos\phi$ is called the **power factor** of the circuit and is given by the ratio $\dfrac{R}{Z}$ where Z is the impedance of the circuit.

(*i*) For pure resistance $\phi = 0$ and $\cos\phi = 1$

$\therefore$ Power $P = E_v I_v$

(*ii*) For pure inductance and capacitance circuit $\phi = \pi/2$ and $\cos\phi = 0$. Such circuits are called **wattless circuit for which P = 0.**

(*iii*) For L–R circuit $\cos\phi = \dfrac{R}{\sqrt{R^2 + L^2\omega^2}}$.

19. (1) A genegator or a dynamo is a machine used for generating electric current by mechanical means. Here mechanical energy is converted into electrical energy.

(2) The frequency of A.C. in India is 50 Hz. In certain other countries it is 30 Hz, 50 Hz or 60 Hz.

(3) EMF is A.C. dynamo, $E = NBA\,\omega \sin\theta$ where N is the number of turns in the armature and θ is the angle between B and A (A is the area)

or $E = E_0 \sin \omega t$ where $E_0 = NBA\omega$

(4) In two phase generator we use two armatures and in three phase generator we use three armatures.

(5) In D.C. generator we use commutator. Here $E = E_0 \sin \omega t$.

(6) In commercial generators, we make use of electromagnets which are energised by the current produced by the generator itself. These are called dynamos whereas those employing permanent magnets are called magnetos.

(7) In two phase generator $E_1 = E_0 \sin \omega t$ and $E_2 = E_0 \sin\left(\omega t \pm \dfrac{\pi}{2}\right)$. In three phase generator $E_1 = E_0 \sin \omega t$, $E_2 = E_0 \sin\left(\omega t \pm \dfrac{2\pi}{3}\right)$, $E_3 = E_0 \sin\left(\omega t \pm \dfrac{4\pi}{3}\right)$.

(8) The flux at time t in the case of a generator is given by

$$\phi = NBA \cos \omega t$$

Key Points

(*i*) The reciprocal of reactance X_L of a coil is called susceptance.
(*ii*) The reciprocal of impedance Z of a.c. circuit is called admittance.

Key Points

Q factor of a resonant LCR circuit is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It is voltage multiplication factor of a a.c. circuit.

(9) The choke coil does a wonderful job in A.C. It finds extensive use in battery eliminators, ratio and T.V. sets, mercury, fluorescent lamps etc.

(10) Capacitor can also do the job done by an inductor but it is inferior to choke.

(11) Induction coil is an apparatus for obtaining high potential difference from a low potential difference supply (D.C.). It is based on the principal of mutual induction.

(12) An induction coil is used in laboratory as high voltage supply for studying discharge through gases and as a high tension supply for the spark plugs in car engines.

(13) An electric motor is a machine for converting electrical energy into mechanical energy.

(14) Efficiency of a D.C. motor

$$\eta = \frac{\text{output mechanical power}}{\text{input electrical power}} = \frac{\text{back e.m.f.}}{\text{applied e.m.f.}}$$

For efficiency to be maximum, the back e.m.f. should be half of the applied e.m.f.

(15) Since $I_0 = \frac{E_0}{\omega L}$ so for low frequency AC, choke coild with laminated soft iron cores are used and for reducing high frequency A.C., air core chokes are used.

(16) Impedance triangle is a right angled triangle whose base is ohmic resistance R, normal $(X_L - X_C)$ and hypotenuse is impedance Z.

(17) When LCR are connected in parallel

$$\frac{1}{|Z|} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

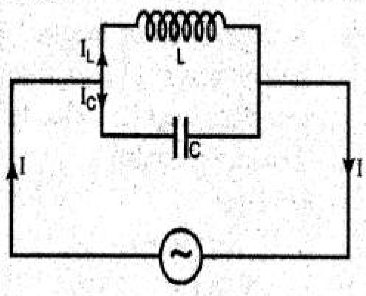
At $\omega = \omega_r = \frac{1}{\sqrt{LC}}, \frac{1}{|Z|}$ is minimum or $|Z|$ is maximum.

(18) For C-R circuit $\cos\phi = \frac{R}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}$.

(19) For L-C-R circuit $\cos\phi = \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$

Remember

Parallel resonant circuit is shown below



Formulas

2. $E = -\dfrac{N(\phi_2 - \phi_1)}{t} = -\dfrac{N\,d\phi}{dt}$
3. $E = Blv$
4. $E = E_0 \sin \omega t,\ E_0 = BAN\omega$
5. $E = -L\dfrac{dI}{dt},\ \phi = LI$
6. $E = -M\dfrac{dI}{dt},\ \phi = MI$
7. $\dfrac{E_1}{E_2} = \dfrac{I_2}{I_1} = \dfrac{N_1}{N_2}$
8. $\eta = \dfrac{\text{Output power}}{\text{Input power}}$
9. $L = \dfrac{\mu_0 N^2 A}{l}$
10. $M = \dfrac{\mu_0 N_1 N_2 A}{l}$
11. $f = \dfrac{1}{2\pi\sqrt{LC}}$
12. $I = I_0(1 - e^{-R/L\,t})$

    $I = I_0 e^{-R/L\,t},\ \tau = \dfrac{L}{R}$
13. $Q = Q_0(1 - e^{-t/CR}),\ Q = Q_0 e^{-t/CR}$

    $\tau = CR$
14. $I = I_0 \sin \omega t$
15. $I_{rms} = \dfrac{I_0}{\sqrt{2}}$
16. $X_L = \omega L$
17. $X_C = \dfrac{1}{\omega C}$
18. LR circuit, $I = I_0 \sin(\omega t - \theta)$

$$= \frac{E_0}{\sqrt{R^2 + X_L^2}} \sin(\omega t - \theta)$$

$$\theta = \frac{\tan^{-1}\omega L}{R}$$

$$Z = \sqrt{R^2 + X_L^2}$$

19. CR-circuit, $I = I_0 \sin(\omega t + \theta)$

$$= \frac{E_0}{\sqrt{R^2 + X_C^2}} \sin(\omega t + \theta)$$

$$\theta = \tan^{-1}\frac{1}{\omega CR}$$

$$Z = \sqrt{R^2 + X_C^2}$$

20. **L-C-R circuit**

    $I = I_0 \sin(\omega t - \theta)$

$$\Rightarrow \quad I = \frac{E_0}{\sqrt{R^2 + (X_L - X_C^2)}} \sin(\omega t - \theta)$$

$$\theta = \tan^{-1}\frac{X_L - X_C}{R}$$

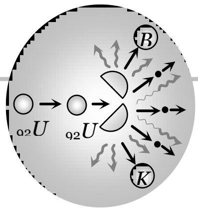
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

21. $P = 1/2\, E_0 I_0$

    $= E_{rms}.I_{rms}$ (Non-inductive circuit)
22. $P = 1/2\, E_0 I_0 \cos\theta$

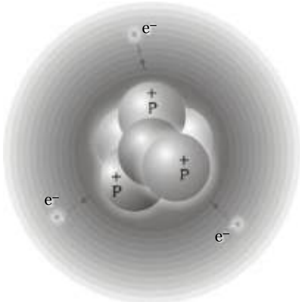
    $= E_{rms}.I_{rms} \cos\theta$ (inductive circuit)
23. $\cos\theta = \dfrac{\text{True power}}{\text{Apparent power}} = \dfrac{R}{Z}$
24. $Q = \dfrac{\omega_r L}{R} = \dfrac{2\pi f_r L}{R}$

    = Quality factor



Nuclear Physics & Radioactivity

Rutherford's α-scattering experiment established that the mass of atom is concentrated with small positively charged region at the centre which is called 'nucleus'.



Nuclei are made up of proton and neutron. The number of protons in a nucleus (called the atomic number or proton number) is represented by the symbol Z. The number of neutrons (neutron number) is represented by N. The total number of neutrons and protons in a nucleus is called it's mass number A so $A = Z + N$.

Neutrons and proton, when described collectively are called ***nucleons.***

Nucleus contains two types of particles : Protons and neutrons

Nuclides are represented as $_ZX^A$; where X denotes the chemical symbol of the element.

Neutron

Neutron is a fundamental particle which is essential constituent of all nuclei except that of hydrogen atom. It was discovered by Chadwick.

(1) The charge of neutron : It is neutral

(2) The mass of neutron : $1.6750 \times 10^{-27}\ kg$

(3) It's spin angular momentum : $\frac{1}{2} \times \left(\frac{h}{2\pi}\right) J\text{ - }s$

(4) It's magnetic moment : $9.57 \times 10^{-27}\ J/Tesla$

(5) It's half life : 12 minutes

(6) Penetration power : High

(7) Types : Neutrons are of two types slow neutron and fast neutron, both are fully capable of penetrating a nucleus and causing artificial disintegration.

> *A free neutron outside the nucleus is unstable and decays into proton and electron.*
>
> $$_0n^1 \rightarrow \underset{\text{Proton}}{_1H^1} + \underset{\text{Electron}}{_{-1}\beta^0} + \underset{\text{Antinutrino}}{\bar{\nu}}$$

Thermal neutrons

Fast neutrons can be converted into slow neutrons by certain materials called moderator's (Paraffin wax, heavy water, graphite) when fast moving neutrons pass through a moderator, they collide with the molecules of the moderator, as a result of this, the energy of moving neutron decreases while that of the molecules of the moderator increases. After sometime they both attains same energy. The neutrons are then in thermal equilibrium with the molecules of the moderator and are called thermal neutrons.

Note : ❑ Energy of thermal neutron is about 0.025 eV and speed is about 2.2 km/s.

Nucleus

(1) **Different types of nuclei**

The nuclei have been classified on the basis of the number of protons (atomic number) or the total number of nucleons (mass number) as follows

(i) **Isotopes :** The atoms of element having same atomic number but different mass number are called isotopes. All isotopes have the same chemical properties. The isotopes of some elements are the following

$$_1H^1,\ _1H^2,\ _1H^3 \quad _8O^{16},\ _8O^{17},\ _8O^{18} \quad _2He^3,\ _2He^4 \quad _{17}Cl^{35},\ _{17}Cl^{37} \quad _{92}U^{235},\ _{92}U^{238}$$

(ii) **Isobars :** The nuclei which have the same mass number (*A*) but different atomic number (*Z*) are called isobars. Isobars occupy different positions in periodic table so all isobars have different chemical properties. Some of the examples of isobars are

$${}_1H^3 \text{ and } {}_2He^3, \quad {}_6C^{14} \text{ and } {}_7N^{14}, \; {}_8O^{17} \text{ and } {}_9F^{17}$$

(iii) **Isotones :** The nuclei having equal number of neutrons are called isotones. For them both the atomic number (*Z*) and mass number (*A*) are different, but the value of (*A* – *Z*) is same. Some examples are

$${}_4Be^9 \text{ and } {}_5B^{10}, {}_6C^{13} \text{ and } {}_7N^{14}, {}_8O^{18} \text{ and } {}_9F^{19}, {}_3Li^7 \text{ and } {}_4Be^8, {}_1H^3 \text{ and } {}_2He^4$$

(iv) **Mirror nuclei :** Nuclei having the same mass number *A* but with the proton number (*Z*) and neutron number (*A* – *Z*) interchanged (or whose atomic number differ by 1) are called mirror nuclei for example.

$${}_1H^3 \text{ and } {}_2He^3, \; {}_3Li^7 \text{ and } {}_4Be^7$$

(2) **Size of nucleus**

(i) Nuclear radius : Experimental results indicates that the nuclear radius is proportional to $A^{1/3}$, where *A* is the mass number of nucleus *i.e.* $R \propto A^{1/3} \Rightarrow R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}\, m = 1.2\, fm$.

Note : ❑ Heavier nuclei are bigger in size than lighter nuclei.

(ii) Nuclear volume : The volume of nucleus is given by $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \Rightarrow V \propto A$

(iii) Nuclear density : Mass per unit volume of a nucleus is called nuclear density.

$$\text{Nuclear density}(\rho) = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi (R_0 A^{1/3})^3}$$

where m = Average of mass of a nucleon (= mass of proton + mass of neutron = $1.66 \times 10^{-27}\, kg$)

and mA = Mass of nucleus

$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3} = 2.38 \times 10^{17}\, kg/m^3$$

Note : ❑ ρ is independent of *A*, it means ρ is same of all atoms.

❑ Density of a nucleus is maximum at it's centre and decreases as we move outwards from the nucleus.

(3) **Nuclear force**

Forces that keep the nucleons bound in the nucleus are called nuclear forces.

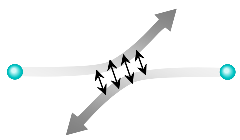
(i) Nuclear forces are short range forces. These do not exist at large distances greater than $10^{-15}\, m$.

(ii) Nuclear forces are the strongest forces in nature.

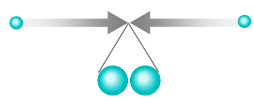
(iii) These are attractive force and causes stability of the nucleus.

(iv) These forces are charge independent.

(v) Nuclear forces are non-central force.



At low speeds, electromagnetic repulsion prevents the collision of nuclei



At high speeds, nuclei come close enough for the strong force to bind them together.

Nuclear forces are exchange forces

According to scientist Yukawa the nuclear force between the two nucleons is the result of the exchange of particles called mesons between the nucleons.

π - mesons are of three types – Positive π meson (π^+), negative π meson (π^-), neutral π *meson* (π^0)

The force between neutron and proton is due to exchange of charged meson between them *i.e.*

$$p \to \pi^+ + n, \quad n \to p + \pi^-$$

The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^0) between them *i.e.* $p \to p' + \pi^0$ and $n \to n' + \pi^0$

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the nuclear forces.

Dog-Bone analogy

The above interactions can be explained with the dog bone analogy according to which we consider the two interacting nucleons to be two dogs having a common bone clenched in between their teeth very firmly. Each one of these dogs wants to take the bone and hence they cannot be separated easily. They seem to be bound to each other with a strong attractive force (which is the bone) though the dogs themselves are strong enemies. The meson plays the same role of the common bone in between two nucleons.

(4) **Atomic mass unit (*amu*)**

The unit in which atomic and nuclear masses are measured is called atomic mass unit (*amu*)

1 *amu* (or 1*u*) $= \frac{1}{12} th$ of mass of ${}_6C^{12}$ atom = 1.66×10^{-27} *kg*

Masses of electron, proton and neutrons

Mass of electron (m_e) = 9.1×10^{-31} *kg* = 0.0005486 *amu*, Mass of proton (m_p) = 1.6726×10^{-27} *kg* = 1.007276 *amu*

Mass of neutron (m_n) = 1.6750×10^{-27} *kg* = 1.00865 *amu*, Mass of hydrogen atom ($m_e + m_p$) = 1.6729×10^{-27} *kg* = 1.0078 *amu*

Mass-energy equivalence

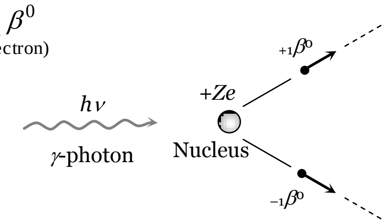
According to Einstein, mass and energy are inter convertible. The Einstein's mass energy relationship is given by $E = mc^2$

If $m = 1$ *amu*, $c = 3 \times 10^8$ *m/sec* then $E = 931$ *MeV* *i.e.* 1 *amu* is equivalent to 931 *MeV* or **1 *amu* (or 1 *u*) = 931 *MeV***

(5) **Pair production and pair-annihilation**

When an energetic γ-ray photon falls on a heavy substance. It is absorbed by some nucleus of the substance and an electron and a positron are produced. This phenomenon is called pair production and may be represented by the following equation

$$\underset{(\gamma-\text{photon})}{h\nu} = \underset{(\text{Positron})}{{}_{1}\beta^0} + \underset{(\text{Electron})}{{}_{-1}\beta^0}$$



The rest-mass energy of each of positron and electron is

$$E_0 = m_0c^2 = (9.1 \times 10^{-31} \, kg) \times (3.0 \times 10^8 \, m/s)^2$$

$$= 8.2 \times 10^{-14} \, J = \mathbf{0.51 \, MeV}$$

Hence, for pair-production it is essential that the energy of γ-photon must be at least $2 \times 0.51 = 1.02$ *MeV*. If the energy of γ-photon is less than this, it would cause photo-electric effect or Compton effect on striking the matter.

The converse phenomenon pair-annihilation is also possible. Whenever an electron and a positron come very close to each other, they annihilate each other by combining together and two γ-photons (energy) are produced. This phenomenon is called pair annihilation and is represented by the following equation.

$$\underset{\text{(Positron)}}{{}_{+1}\beta^0} + \underset{(Electron)}{{}_{-1}\beta^0} = \underset{(\gamma\text{-photon})}{h\nu} + \underset{(\gamma\text{-photon})}{h\nu}$$

(6) **Nuclear stability**

Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α, β-particles and γ- EM waves. (This process is called radioactivity). The stability of nucleus is determined by many factors. Few such factors are given below :

(i) Neutron-proton ratio $\left(\frac{N}{Z}\text{ Ratio}\right)$

The chemical properties of an atom are governed entirely by the number of protons (Z) in the nucleus, the stability of an atom appears to depend on both the number of protons and the number of neutrons.

For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal ($N \approx Z$) *i.e.* $\frac{N}{Z} = 1$

Heavy nuclei are stable only when they have more neutrons than protons. Thus heavy nuclei are neutron rich compared to lighter nuclei (for heavy nuclei, more is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore more neutrons are added to provide the strong attractive forces necessary to keep the nucleus stable.)

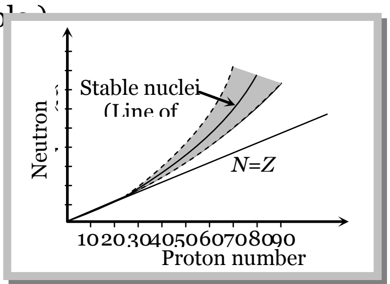
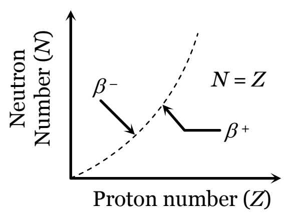


Figure shows a plot of N verses Z for the stable nuclei. For mass number upto about A = 40. For larger value of Z the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At Bi (Z = 83, A = 209), the neutron excess in $N - Z$ = 43. There are no stable nuclides with $Z > 83$.

Note : ❑ The nuclide ${}_{83}Bi^{209}$ is the heaviest stable nucleus.

❑ A nuclide above the line of stability *i.e.* having excess neutrons, decay through β^- emission (neutron changes into proton). Thus increasing atomic number Z and decreasing neutron number N. In β^- emission, $\frac{N}{Z}$ ratio decreases.

A nuclide below the line of stability have excess number of protons. It decays by β^+ emission, results in decreasing Z and increasing N. In β^+ emission, the $\frac{N}{Z}$ ratio increases.



(ii) Even or odd numbers of Z or N : The stability of a nuclide is also determined by the consideration whether it contains an even or odd number of protons and neutrons.

It is found that an even-even nucleus (even Z and even N) is more stable (60% of stable nuclide have even Z and even N).

An even-odd nucleus (even Z and odd N) or odd-even nuclide (odd Z and even N) is found to be lesser sable while the odd-odd nucleus is found to be less stable.

Only five stable odd-odd nuclides are known : ${}_1H^2$, ${}_3Li^6$, ${}_5Be^{10}$, ${}_7N^{14}$ and ${}_{75}Ta^{180}$

(iii) Binding energy per nucleon : The stability of a nucleus is determined by value of it's binding energy per nucleon. In general higher the value of binding energy per nucleon, more stable the nucleus is

Mass Defect and Binding Energy

(1) **Mass defect (Δm)**

It is found that the mass of a nucleus is always less than the sum of masses of it's constituent nucleons in free state. This difference in masses is called mass defect. Hence mass defect

$$\Delta m = \text{Sum of masses of nucleons} - \text{Mass of nucleus}$$

$$= \{Zm_p + (A-Z)m_n\} - M = \{Zm_p + Zm_e + (A-Z)m_z\} - M'$$

where m_p = Mass of proton, m_n = Mass of each neutron, m_e = Mass of each electron

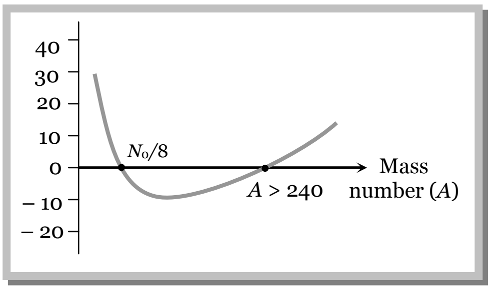
M = Mass of nucleus, Z = Atomic number, A = Mass number, M' = Mass of atom as a whole.

Note : ❑ The mass of a typical nucleus is about 1% less than the sum of masses of nucleons.

(2) **Packing fraction**

Mass defect per nucleon is called packing fraction

Packing fraction $(f) = \dfrac{\Delta m}{A} = \dfrac{M-A}{A}$ where M = Mass of nucleus, A = Mass number



Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, larger is the stability of the nucleus.

(i) Packing fraction may be of positive, negative or zero value.

(iii) At $A = 16, f \to$ Zero

(3) **Binding energy (B.E.)**

The neutrons and protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them infinitely apart (or the same energy is released during the formation of the nucleus). This energy is called the binding energy of the nucleus.

or

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect then according to Einstein's mass energy relation

Binding energy $= \Delta m \cdot c^2 = [\{m_pZ + m_n(A-Z)\} - M] \cdot c^2$

(This binding energy is expressed in *joule*, because Δm is measured in *kg*)

If Δm is measured in *amu* then binding energy = Δm *amu* = $[\{m_p Z + m_n(A - Z)\} - M]$ *amu* = $\Delta m \times 931$ *MeV*

(4) **Binding energy per nucleon**

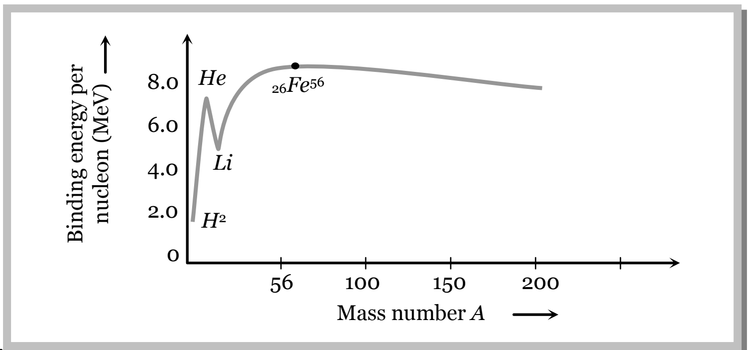
The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

$$\text{Binding energy per nucleon} = \frac{\text{Total binding energy}}{\text{Mass number } (i.e.\text{ total number of nucleons})} = \frac{\Delta m \times 931}{A}\frac{MeV}{Nucleon}$$

Binding energy per nucleon $\propto$ Stability of nucleus

Binding Energy Curve

It is the graph between binding energy per nucleon and total number of nucleons (*i.e.* mass number *A*)

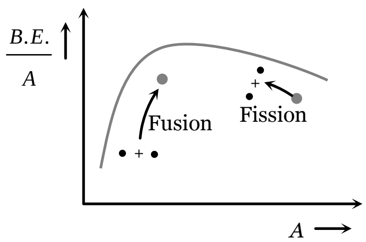


(1) Some nuclei with mass number $A < 20$ have large binding energy per nucleon than their neighbour nuclei. For example $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$. These nuclei are more stable than their neighbours.

(2) The binding energy per nucleon is maximum for nuclei of mass number $A = 56$ ($_{26}Fe^{56}$). It's value is 8.8 *MeV* per nucleon.

(3) For nuclei having $A > 56$, binding energy per nucleon gradually decreases for uranium ($A = 238$), the value of binding energy per nucleon drops to 7.5 *MeV*.

Note :
- ❑ When a heavy nucleus splits up into lighter nuclei, then binding energy per nucleon of lighter nuclei is more than that of the original heavy nucleus. Thus a large amount of energy is liberated in this process (nuclear fission).
- ❑ When two very light nuclei combines to form a relatively heavy nucleus, then binding energy per nucleon increases. Thus, energy is released in this process (nuclear fusion).



Nuclear Reactions

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction. The general expression for the nuclear reaction is as follows.

$$\underset{\text{(Parent nucleus)}}{X} + \underset{\text{(Incident particle)}}{a} \longrightarrow \underset{\text{(Compound nucleus)}}{C} \longrightarrow \underset{\text{(Compound nucleus)}}{Y} + \underset{\text{(Product particles)}}{b} + \underset{\text{(Energy)}}{Q}$$

Here *X* and *a* are known as reactants and *Y* and *b* are known as products. This reaction is known as (*a*, *b*) reaction and can be represented as *X*(*a*, *b*) *Y*

(1) ***Q* value or energy of nuclear reaction**

The energy absorbed or released during nuclear reaction is known as *Q*-value of nuclear reaction.

Q-value = (Mass of reactants – mass of products)c^2 *Joules*

= (Mass of reactants – mass of products) *amu*

If $Q < 0$, The nuclear reaction is known as endothermic. (The energy is absorbed in the reaction)

If $Q > 0$, The nuclear reaction is known as exothermic (The energy is released in the reaction)

(2) **Law of conservation in nuclear reactions**

(i) Conservation of mass number and charge number : In the following nuclear reaction

$$_2He^4 + _7N^{14} \rightarrow _8O^{17} + _1H^1$$

	Before the reaction	After the reaction
Mass number (A) →	4 +14 = 18	17 + 1 = 18
Charge number (Z) →	2 + 7 = 9	8 + 1 = 9

(ii) Conservation of momentum : Linear momentum/angular momentum of particles before the reaction is equal to the linear/angular momentum of the particles after the reaction. That is $\Sigma p = 0$

(iii) Conservation of energy : Total energy before the reaction is equal to total energy after the reaction. Term Q is added to balance the total energy of the reaction.

(3) **Common nuclear reactions**

The nuclear reactions lead to artificial transmutation of nuclei. Rutherford was the first to carry out artificial transmutation of nitrogen to oxygen in the year 1919.

$$_2He^4 + _7N^{14} \rightarrow _9F^{18} \rightarrow _8O^{17} + _1H^1$$

It is called (α, p) reaction. Some other nuclear reactions are given as follows.

(p, n) reaction $\Rightarrow$ $_1H^1 + _5B^{11} \rightarrow _6C^{12} \rightarrow _6C^{11} + _0n^1$

(p, α) reaction $\Rightarrow$ $_1H^1 + _3Li^{11} \rightarrow _4Be^8 \rightarrow _2He^4 + _2He^4$

(p, γ) reaction $\Rightarrow$ $_1H^1 + _6C^{12} \rightarrow _7N^{13} \rightarrow _7N^{13} + \gamma$

(n, p) reaction $\Rightarrow$ $_0n^1 + _7N^{14} \rightarrow _7N^{15} \rightarrow _6C^{14} + _1H^1$

(γ, n) reaction $\Rightarrow$ $\gamma + _1H^2 \rightarrow _1H^1 + _0n^1$

Nuclear Fission and Fusion

Nuclear fission

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy is called nuclear fission.

The phenomenon of nuclear fission was discovered by scientist Ottohann and F. Strassman and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.

(1) **Fission reaction of U^{235}**

(i) Nuclear reaction :

$$_{92}U^{235} + {_0n^1} \rightarrow \underset{\text{(unstable nucleus)}}{_{92}U^{236}} \rightarrow {_{56}Ba^{141}} + {_{36}Kr^{92}} + 3{_0n^1} + Q$$

(ii) The energy released in U^{235} fission is about 200 *MeV* or 0.8 *MeV* per nucleon.

(iii) By fission of $_{92}U^{235}$, on an average 2.5 neutrons are liberated. These neutrons are called fast neutrons and their energy is about 2 *MeV* (for each). These fast neutrons can escape from the reaction so as to proceed the chain reaction they are need to slow down.

(iv) Fission of U^{235} occurs by slow neutrons only (of energy about 1*eV*) or even by thermal neutrons (of energy about 0.025 *eV*).

(v) 50 *kg* of U^{235} on fission will release $\approx 4 \times 10^{15}$ *J* of energy. This is equivalence to 20,000 tones of *TNT* explosion. The nuclear bomb dropped at Hiroshima had this much explosion power.

(vi) The mass of the compound nucleus must be greater than the sum of masses of fission products.

(vii) The $\frac{\text{Binding energy}}{\text{A}}$ of compound nucleus must be less than that of the fission products.

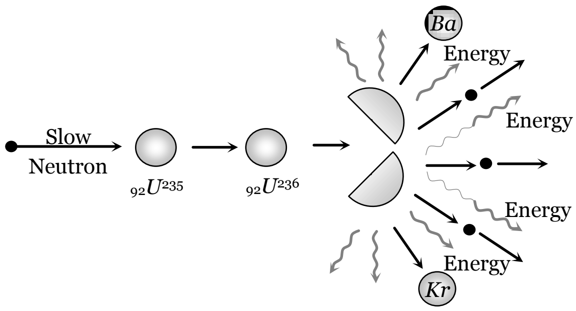
(viii) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments $_{56}Ba$ and $_{36}Kr$ are formed but they may be any stable isotopes of middle weight atoms.

Same other U^{235} fission reactions are

$$_{92}U^{235} + {_0n^1} \rightarrow {_{54}Xe^{140}} + {_{38}Sr^{94}} + 2{_0n^1}$$
$$\rightarrow {_{57}La^{148}} + {_{35}Br^{85}} + 3{_0n^1}$$
$$\rightarrow \text{Many more}$$

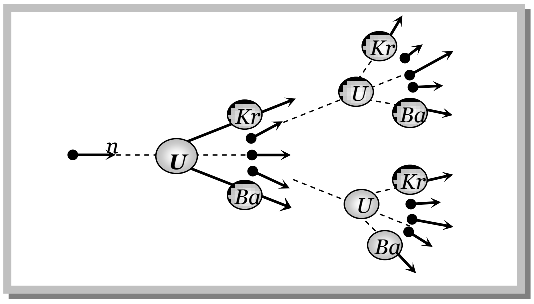
(ix) The neutrons released during the fission process are called prompt neutrons.

(x) Most of energy released appears in the form of kinetic energy of fission fragments.



(2) **Chain reaction**

In nuclear fission, three neutrons are produced along with the release of large energy. Under favourable conditions, these neutrons can cause further fission of other nuclei, producing large number of neutrons. Thus a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.



In the chain reaction, the number of nuclei undergoing fission increases very fast. So, the energy produced takes a tremendous magnitude very soon.

Difficulties in chain reaction

(i) Absorption of neutrons by U^{238}, the major part in natural uranium is the isotope U^{238} (99.3%), the isotope U^{235} is very little (0.7%). It is found that U^{238} is fissionable with fast neutrons, whereas U^{235} is fissionable with slow neutrons. Due to the large percentage of U^{238}, there is more possibility of collision of neutrons with U^{238}. It is found that the neutrons get slowed on coliding with U^{238}, as a result of it further fission of U^{238} is not possible (Because they are slow and they are absorbed by U^{238}). This stops the chain reaction.

Removal : (i) To sustain chain reaction ${}_{92}U^{235}$ is separated from the ordinary uranium. Uranium so obtained $\left({}_{92}U^{235}\right)$ is known as enriched uranium, which is fissionable with the fast and slow neutrons and hence chain reaction can be sustained.

(ii) If neutrons are slowed down by any method to an energy of about 0.3 *eV*, then the probability of their absorption by U^{238} becomes very low, while the probability of their fissioning U^{235} becomes high. This job is done by moderators. Which reduce the speed of neutron rapidly graphite and heavy water are the example of moderators.

(iii) Critical size : The neutrons emitted during fission are very fast and they travel a large distance before being slowed down. If the size of the fissionable material is small, the neutrons emitted will escape the fissionable material before they are slowed down. Hence chain reaction cannot be sustained.

Removal : The size of the fissionable material should be large than a critical size.

The chain reaction once started will remain steady, accelerate or retard depending upon, a factor called neutron reproduction factor (k). It is defined as follows.

$$k = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

→ If $k = 1$, the chain reaction will be steady. The size of the fissionable material used is said to be the critical size and it's mass, the critical mass.

→ If $k > 1$, the chain reaction accelerates, resulting in an explosion. The size of the material in this case is super critical. (Atom bomb)

→ If $k < 1$, the chain reaction gradually comes to a halt. The size of the material used us said to be sub-critical.

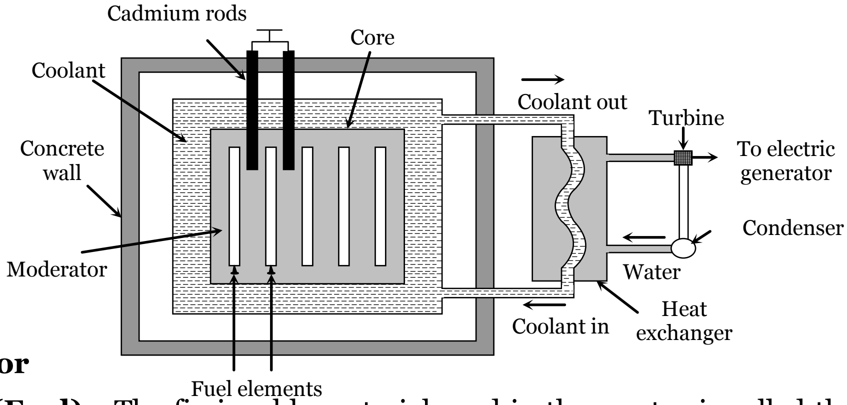
Types of chain reaction : Chain reactions are of following two types

Controlled chain reaction	**Uncontrolled chain reaction**
Controlled by artificial method	No control over this type of nuclear reaction
All neurons are absorbed except one	More than one neutron takes part into reaction
It's rate is slow	Fast rate
Reproduction factor $k = 1$	Reproduction factor $k > 1$
Energy liberated in this type of reaction is always less than explosive energy	A large amount of energy is liberated in this type of reaction
Chain reaction is the principle of nuclear reactors	Uncontrolled chain reaction is the principle of atom bomb.

Note : ❑ The energy released in the explosion of an atom bomb is equal to the energy released by 2000 tonn of TNT and the temperature at the place of explosion is of the order of 10^7 °C.

Nuclear Reactor

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction. It is also called an atomic pile. It is thus a source of controlled energy which is utilised for many useful purposes.



(1) **Parts of nuclear reactor**

(i) **Fissionable material (Fuel) :** The fissionable material used in the reactor is called the fuel of the reactor. Uranium isotope (U^{235}) Thorium isotope (Th^{232}) and Plutonium isotopes (Pu^{239}, Pu^{240} and Pu^{241}) are the most commonly used fuels in the reactor.

(ii) **Moderator :** Moderator is used to slow down the fast moving neutrons. Most commonly used moderators are graphite and heavy water (D_2O).

(iii) **Control Material :** Control material is used to control the chain reaction and to maintain a stable rate of reaction. This material controls the number of neutrons available for the fission. For example, cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rods in or out of the core of the reactor.

(iv) **Coolant :** Coolant is a cooling material which removes the heat generated due to fission in the reactor. Commonly used coolants are water, CO_2 nitrogen *etc.*

(v) **Protective shield :** A protective shield in the form a concrete thick wall surrounds the core of the reactor to save the persons working around the reactor from the hazardous radiations.

Note : ❑ It may be noted that Plutonium is the best fuel as compared to other fissionable material. It is because fission in Plutonium can be initiated by both slow and fast neutrons. Moreover it can be obtained from U^{238}.

❑ Nuclear reactor is firstly devised by fermi. ❑ Apsara was the first Indian nuclear reactor.

(2) **Uses of nuclear reactor**

(i) In electric power generation.

(ii) To produce radioactive isotopes for their use in medical science, agriculture and industry.

(iii) In manufacturing of PU^{239} which is used in atom bomb.

(iv) They are used to produce neutron beam of high intensity which is used in the treatment of cancer and nuclear research.

Note : ❑ A type of reactor that can produce more fissile fuel than it consumes is the breeder reactor.

Nuclear fusion

In nuclear fusion two or more than two lighter nuclei combine to form a single heavy nucleus. The mass of single nucleus so formed is less than the sum of the masses of parent nuclei. This difference in mass results in the release of tremendous amount of energy

$$_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4\,MeV$$

$$_1H^3 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1 + 17.6\,MeV$$

or

$$_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + 24\,MeV$$

For fusion high pressure ($\approx 10^6$ *atm*) and high temperature (of the order of 10^7 *K* to 10^8 *K*) is required and so the reaction is called thermonuclear reaction.

Fusion energy is greater then fission energy fission of one uranium atom releases about 200 *MeV* of energy. But the fusion of a deutron $({}_1H^2)$ and triton $({}_1H^3)$ releases about 17.6 *MeV* of energy. However the energy released per nucleon in fission is about 0.85 *MeV* but that in fusion is 4.4 *MeV*. So for the same mass of the fuel, the energy released in fusion is much larger than in fission.

Plasma : The temperature of the order of 10^8 *K* required for thermonuclear reactions leads to the complete ionisation of the atom of light elements. The combination of base nuclei and electron cloud is called plasma. The enormous gravitational field of the sun confines the plasma in the interior of the sun.

The main problem to carryout nuclear fusion in the laboratory is to contain the plasma at a temperature of $10^8 K$. No solid container can tolerate this much temperature. If this problem of containing plasma is solved, then the large quantity of deuterium present in sea water would be able to serve as in-exhaustible source of energy.

Note : ❑ To achieve fusion in laboratory a device is used to confine the plasma, called **Tokamak**.

Stellar Energy

Stellar energy is the energy obtained continuously from the sun and the stars. Sun radiates energy at the rate of about 10^{26} *joules* per *second*.

Scientist Hans Bethe suggested that the fusion of hydrogen to form helium (thermo nuclear reaction) is continuously taking place in the sun (or in the other stars) and it is the source of sun's (star's) energy.

The stellar energy is explained by two cycles

Proton-proton cycle	Carbon-nitrogen cycle
${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_1e^0 + Q_1$	${}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} + Q_1$
${}_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + Q_2$	${}_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e^0$
${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2{}_1H^1 + Q_3$	${}_1H^1 + {}_6C^{13} \rightarrow {}_7N^{14} + Q_2$
$\cancel{4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\gamma + 26.7\,MeV}$	${}_1H^1 + {}_7N^{14} \rightarrow {}_8O^{15} + Q_3$
	${}_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 + Q_4$
	${}_1H^1 + {}_7N^{15} \rightarrow {}_6C^{12} + {}_2He^4$
	$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_1e^0 + 24.7\,MeV$

About 90% of the mass of the sun consists of hydrogen and helium.

Nuclear Bomb

Based on uncontrolled nuclear reactions.

Atom bomb	Hydrogen bomb
Based on fission process it involves the fission of U^{235}	Based on fusion process. Mixture of deutron and tritium is used in it
In this critical size is important	There is no limit to critical size
Explosion is possible at normal temperature and pressure	High temperature and pressure are required
Less energy is released compared to hydrogen bomb	More energy is released as compared to atom bomb so it is more dangerous than atom bomb

Concepts

☞ *A test tube full of base nuclei will weight heavier than the earth.*

☞ *The nucleus of hydrogen contains only one proton. Therefore we may say that the proton is the nucleus of hydrogen atom.*

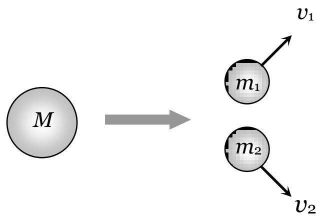
☞ *If the relative abundance of isotopes in an element has a ratio $n_1 : n_2$ whose atomic masses are m_1 and m_2 then atomic mass of the element is* $M = \dfrac{n_1 m_1 + n_2 m_2}{n_1 + n_2}$

Example

Example: 1 A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio 8 : 1. The ratio of radii of the fragments is

(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 2 : 1

Solution : (a)



By conservation of momentum $m_1v_1 = m_2v_2$

$\Rightarrow \frac{v_1}{v_2} = \frac{8}{1} = \frac{m_2}{m_1}$ ...... (i)

Also from $r \propto A^{1/3} \Rightarrow \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$

Example: 2 The ratio of radii of nuclei $^{27}_{13}Al$ and $^{125}_{52}Te$ is approximately **[J & K CET 2000]**

(a) 6 : 10 (b) 13 : 52 (c) 40 : 177 (d) 14 : 7

Solution : (a) By using $r \propto A^{1/3} \Rightarrow \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{8}{5} = \frac{6}{10}$

Example: 3 If Avogadro's number is 6×10^{23} then the number of protons, neutrons and electrons in 14 *g* of $_6C^{14}$ are respectively

(a) 36×10^{23}, 48×10^{23}, 36×10^{23} (b) 36×10^{23}, 36×10^{23}, 36×10^{21}

(c) 48×10^{23}, 36×10^{23}, 48×10^{21} (d) 48×10^{23}, 48×10^{23}, 36×10^{21}

Solution : (a) Since the number of protons, neutrons and electrons in an atom of $_6C^{14}$ are 6, 8 and 6 respectively. As 14 *gm* of $_6C^{14}$ contains 6×10^{23} atoms, therefore the numbers of protons, neutrons and electrons in 14 *gm* of $_6C^{14}$ are $6\times6\times10^{23} = 36\times10^{23}$, $8\times6\times10^{23} = 48\times10^{23}$, $6\times6\times10^{23} = 36\times10^{23}$.

Example: 4 Two Cu^{64} nuclei touch each other. The electrostatics repulsive energy of the system will be

(a) 0.788 *MeV* (b) 7.88 *MeV* (c) 126.15 *MeV* (d) 788 *MeV*

Solution : (c) Radius of each nucleus $R = R_0(A)^{1/3} = 1.2\,(64)^{1/3} = 4.8\ fm$

Distance between two nuclei (r) = $2R$

So potential energy $U = \frac{k\cdot q^2}{r} = \frac{9\times10^9 \times (1.6\times10^{-19}\times 29)^2}{2\times4.8\times10^{-15}\times1.6\times10^{-19}} = 126.15\ MeV$.

Example: 5 When $_{92}U^{235}$ undergoes fission. 0.1% of its original mass is changed into energy. How much energy is released if 1 *kg* of $_{92}U^{235}$ undergoes fission **[MP PET 1994; MP PMT/PET 1998; BHU 2001; BVP 2003]**

(a) 9×10^{10} *J* (b) 9×10^{11} *J* (c) 9×10^{12} *J* (d) 9×10^{13} *J*

Solution : (d) By using $E = \Delta m\cdot c^2 \Rightarrow E = \left(\frac{0.1}{100}\times1\right)(3\times10^8)^2 = 9\times10^{13}\ J$

Example: 6 1 *g* of hydrogen is converted into 0.993 *g* of helium in a thermonuclear reaction. The energy released is **[EAMCET (Med.) 1995; CPMT 1999]**

(a) 63×10^{7} *J* (b) 63×10^{10} *J* (c) 63×10^{14} *J* (d) 63×10^{20} *J*

Solution : (b) $\Delta m = 1 - 0.993 = 0.007\ gm$

$\therefore E = \Delta mc^2 = 0.007 \times 10^{-3} \times (3 \times 10^8)^2 = 63 \times 10^{10}\ J$

Example: 7 The binding energy per nucleon of deuteron $\left(^2_1H\right)$ and helium nucleus $\left(^4_2He\right)$ is 1.1 *MeV* and 7 *MeV* respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is

[MP PMT 1992; Roorkee 1994; IIT-JEE 1996; AIIMS 1997; Haryana PMT 2000; Pb PMT 2001; CPMT 2001; AIEEE 2004]

(a) 13.9 *MeV* (b) 26.9 *MeV* (c) 23.6 *MeV* (d) 19.2 *MeV*

Solution : (c) $_1H^2 + {_1H^2} \rightarrow {_2He^4} + Q$

Total binding energy of helium nucleus = 4 × 7 = 28 *MeV*

Total binding energy of each deutron = 2 × 1.1 = 2.2 *MeV*

Hence energy released = 28 – 2 × 2.2 = 23.6 *MeV*

Example: 8 The masses of neutron and proton are 1.0087 *amu* and 1.0073 *amu* respectively. If the neutrons and protons combine to form a helium nucleus (alpha particles) of mass 4.0015 *amu*. The binding energy of the helium nucleus will be [1 *amu*= 931 *MeV*] **[CPMT 1986; MP PMT 1995; CBSE 2003]**

(a) 28.4 *MeV* (b) 20.8 *MeV* (c) 27.3 *MeV* (d) 14.2 *MeV*

Solution : (a) Helium nucleus consist of two neutrons and two protons.

So binding energy $E = \Delta m\ amu = \Delta m \times 931\ MeV$

$\Rightarrow E = (2 \times m_p + 2m_n - M) \times 931\ MeV = (2 \times 1.0073 + 2 \times 1.0087 - 4.0015) \times 931 = 28.4\ MeV$

Example: 9 A atomic power reactor furnace can deliver 300 *MW*. The energy released due to fission of each of uranium atom U^{238} is 170 *MeV*. The number of uranium atoms fissioned per hour will be

(a) 5×10^{15} (b) 10×10^{20} (c) 40×10^{21} (d) 30×10^{25}

Solution : (c) By using $P = \frac{W}{t} = \frac{n \times E}{t}$ where n = Number of uranium atom fissioned and E = Energy released due to each fission so $300 \times 10^6 = \frac{n \times 170 \times 10^6 \times 1.6 \times 10^{-19}}{3600}$ $\Rightarrow n = 40 \times 10^{21}$

Example: 10 The binding energy per nucleon of O^{16} is 7.97 *MeV* and that of O^{17} is 7.75 *MeV*. The energy (in *MeV*) required to remove a neutron from O^{17} is **[IIT-JEE 1995]**

(a) 3.52 (b) 3.64 (c) 4.23 (d) 7.86

Solution : (c) $O^{17} \rightarrow O^{16} + {}_0n^1$

$\therefore$ Energy required = Binding of O^{17} – binding energy of O^{16} $= 17 \times 7.75 - 16 \times 7.97 = 4.23\ MeV$

Example: 11 A gamma ray photon creates an electron-positron pair. If the rest mass energy of an electron is 0.5 *MeV* and the total kinetic energy of the electron-positron pair is 0.78 *MeV*, then the energy of the gamma ray photon must be **[MP PMT 1991]**

(a) 0.78 *MeV* (b) 1.78 *MeV* (c) 1.28 *MeV* (d) 0.28 *MeV*

Solution : (b) Energy of γ-rays photon = 0.5 + 0.5 +0.78 = 1.78 *MeV*

Example: 12 What is the mass of one Curie of U^{234} **[MNR 1985]**

(a) $3.7 \times 10^{10}\ gm$ (b) $2.348 \times 10^{23}\ gm$ (c) $1.48 \times 10^{-11}\ gm$ (d) $6.25 \times 10^{-34}\ gm$

Solution : (c) 1 *curie* = 3.71×10^{10} disintegration/*sec* and mass of 6.02×10^{23} atoms of $U^{234} = 234\ gm$

$\therefore$ Mass of 3.71×10^{10} atoms $= \frac{234 \times 3.71 \times 10^{10}}{6.02 \times 10^{23}} = 1.48 \times 10^{-11}\ gm$

Example: 13 In the nuclear fusion reaction ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$, given that the repulsive potential energy between the two nuclei is $-7.7 \times 10^{-14}\ J$, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}\ J/K$] **[AIEEE 2003]**

(a) $10^9\ K$ (b) $10^7\ K$ (c) $10^5\ K$ (d) $10^3\ K$

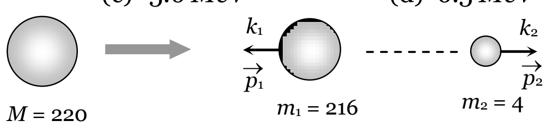
Solution : (a) Kinetic energy of molecules of a gas at a temperature *T* is 3/2 *kT*

$\therefore$ To initiate the reaction $\frac{3}{2}kT = 7.7 \times 10^{-14}\ J$ $\Rightarrow T = 3.7 \times 10^9\ K$.

Example: 14 A nucleus with mass number 220 initially at rest emits an α-particle. If the *Q* value of the reaction is 5.5 *MeV*. Calculate the kinetic energy of the α-particle **[IIT-JEE (Screening) 2003]**

(a) 4.4 *MeV* (b) 5.4 *MeV* (c) 5.6 *MeV* (d) 6.5 *MeV*

Solution : (b)



Q-value of the reaction is 5.5 *eV i.e.* $k_1 + k_2 = 5.5\ MeV$ ......(i)

By conservation of linear momentum $p_1 = p_2 \Rightarrow \sqrt{2(216)k_1} = \sqrt{2(4)k_2}$ $\Rightarrow k_2 = 54\ k_1$ ......(ii)

On solving equation (i) and (ii) we get $k_2 = 5.4\ MeV$.

Example: 15 Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a ${}_{10}^{20}Ne$ nucleus and M_2 the mass of a ${}_{20}^{40}Ca$ nucleus. Then **[IIT 1998; DPMT 2000]**

(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$ (c) $M_2 < 2M_1$ (d) $M_1 < 10(m_n + m_p)$

Solution : (c, d) Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less then the sum of masses of it's constituent particles $^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons. Therefore, mass of $^{20}_{10}Ne$ nucleus $M_1 < 10(m_p + m_n)$

Also heavier the nucleus, more is he mass defect thus $20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

or $10(m_p + m_n) > M_2 - M_1$

$\Rightarrow M_2 < M_1 + 10(m_p + m_n) \Rightarrow M_2 < M_1 + M_1 \Rightarrow M_2 < 2M_1$

Tricky example: 1

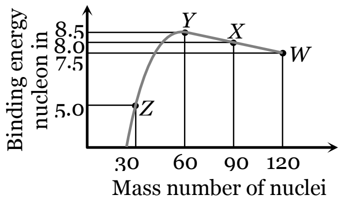
Binding energy per nucleon *vs* mass number curve for nuclei is shown in the figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is **[IIT-JEE 1999]**

(a) $Y \rightarrow 2Z$

(b) $W \rightarrow X + Z$

(c) $W \rightarrow 2Y$

(d) $X \rightarrow Y + Z$



Solution : (c) Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleon) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (c) this happens.

Given $W \rightarrow 2Y$

Binding energy of reactants = 120 × 7.5 = 900 MeV

and binding energy of products = 2 (60 × 8.5) = 1020 MeV > 900 MeV

Radioactivity

The phenomenon of spontaneous emission of radiatons by heavy elements is called radioactivity. The elements which shows this phenomenon are called radioactive elements.

(1) Radioactivity was discovered by Henery Becquerel in uranium salt in the year 1896.

(2) After the discovery of radioactivity in uranium, Piere Curie and Madame Curie discovered a new radioactive element called radium (which is 10^6 times more radioactive than uranium)

(3) Some examples of radio active substances are : Uranium, Radium, Thorium, Polonium, Neptunium *etc.*

(4) Radioactivity of a sample cannot be controlled by any physical (pressure, temperature, electric or magnetic field) or chemical changes.

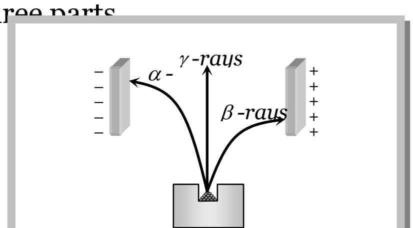
(5) All the elements with atomic number (Z) > 82 are naturally radioactive.

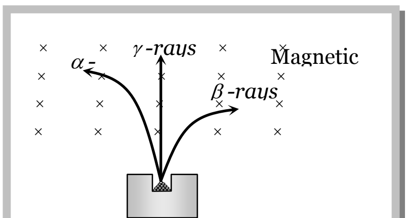
(6) The conversion of lighter elements into radioactive elements by the bombardment of fast moving particles is called artificial or induced radioactivity.

(7) Radioactivity is a nuclear event and not atomic. Hence electronic configuration of atom don't have any relationship with radioactivity.

Nuclear radiatons

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only. When the radiation enters into the external electric field, they splits into three parts





(i) Radiations which deflects towards negative plate are called α-rays (stream of positively charged particles)

(ii) Radiations which deflects towards positive plate are called β particles (stream of negatively charged particles)

(iii) Radiations which are undeflected called γ-*rays*. (E.M. waves or photons)

Note :
- Exactly same results were obtained when these radiations were subjected to magnetic field.
- No radioactive substance emits both α and β particles simultaneously. Also γ-rays are emitted after the emission of α or β-particles.
- β-particles are not orbital electrons they come from nucleus. The neutron in the nucleus decays into proton and an electron. This electron is emitted out of the nucleus in the form of β-rays.

Properties of α, β and γ-rays

Features	α- particles	β - particles	γ - rays
1. Identity	Helium nucleus or doubly ionised helium atom ($_2He^4$)	Fast moving electron $(-\beta^0$ or $\beta^-)$	Photons (E.M. waves)
2. Charge	+ 2e	– *e*	Zero
3. Mass 4 m_p (m_p = mass of proton = 1.87×10^{-27}	4 m_p	m_e	Massless
4. Speed	$\approx 10^7$ m/s	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 *MeV* to 9 *MeV*	All possible values between a minimum certain value to 1.2 *MeV*	Between a minimum value to 2.23 *MeV*
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 *cm* of iron (or *Pb*) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete
10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	$_ZX^A \xrightarrow{\alpha-decay}$ $_{Z-2}Y^{A-4} + {_2He^4}$ $_ZX^A \xrightarrow{n_\alpha} {_{Z'}Y^{A'}}$	$_ZX^A \to {_{Z+1}Y^A} + {_{-1}e^0} + \bar{\nu}$ $_ZX^A \xrightarrow{n_\beta} {_{Z'}X^A}$ $\Rightarrow \boldsymbol{n_\beta = (2n_\alpha - Z + Z')}$	$_ZX^A \to {_ZX^a} + \gamma$

	$\Rightarrow n_\alpha = \frac{A' - A}{4}$		

Radioactive Disintegration

(1) **Law of radioactive disintegration**

According to Rutherford and Soddy law for radioactive decay is as follows.

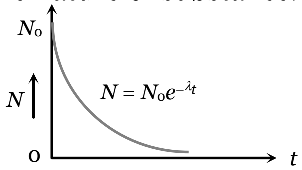
"At any instant the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant" *i.e.* $-\frac{dN}{dt} \propto N \Rightarrow \frac{dN}{dt} = -\lambda N$. It can be proved that $\boldsymbol{N = N_0 e^{-\lambda t}}$

This equation can also be written in terms of mass *i.e.* $M = M_0 e^{-\lambda t}$

where N = Number of atoms remains undecayed after time t, N_0 = Number of atoms present initially (*i.e.* at t = 0), M = Mass of radioactive nuclei at time t, M_0 = Mass of radioactive nuclei at time t = 0, $N_0 - N$ = Number of disintegrated nucleus in time t

$\frac{dN}{dt}$ = rate of decay, λ = Decay constant or disintegration constant or radioactivity constant or Rutherford Soddy's constant or the probability of decay per unit time of a nucleus.

Note : ❑ λ depends only on the nature of substance. It is independent of time and any physical or chemical changes.



(2) **Activity**

It is defined as the rate of disintegration (or count rate) of the substance (or the number of atoms of any material decaying per second) *i.e.* $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$

where A_0 = Activity of t = 0, A = Activity after time t

Units of activity (Radioactivity)

It's units are *Becqueral* (*Bq*), Curie (*Ci*) and Rutherford (*Rd*)

1 *Becquerel* = 1 *disintegration/sec*, 1 *Rutherford* = 10^6 *dis/sec*, 1 *Curie* = 3.7×10^{11} *dis/sec*

Note : ❑ Activity per *gm* of a substance is known as specific activity. The specific activity of 1 *gm* of radium – 226 is 1 *Curie*.

❑ 1 *millicurie* = 37 *Rutherford*

❑ The activity of a radioactive substance decreases as the number of undecayed nuclei decreases with time.

❑ Activity $\propto \frac{1}{\text{Half life}}$

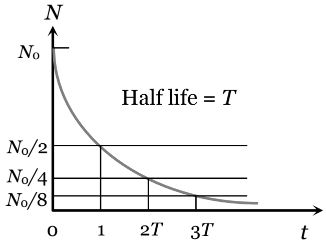
(3) **Half life ($T_{1/2}$)**

Time interval in which the mass of a radioactive substance or the number of it's atom reduces to half of it's initial value is called the half life of the substance.

i.e. if $N = \frac{N_0}{2}$ then $t = T_{1/2}$

Hence from $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda (T_{1/2})} \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$



Time (t)	Number of undecayed atoms (N) (N_0 = Number of initial atoms)	Remaining fraction of active atoms (N/N_0) probability of survival	Fraction of atoms decayed ($N_0 - N$) $/N_0$ probability of decay
t = 0	N_0	1 (100%)	0

$t = T_{1/2}$	$\frac{N_0}{2}$	$\frac{1}{2}$ (50%)	$\frac{1}{2}$ (50%)
$t = 2(T_{1/2})$	$\frac{1}{2}\times\frac{N_0}{2}=\frac{N_0}{(2)^2}$	$\frac{1}{4}$ (25%)	$\frac{3}{4}$ (75%)
$t = 3(T_{1/2})$	$\frac{1}{2}\times\frac{N_0}{(2)}=\frac{N_0}{(2)^3}$	$\frac{1}{8}$ (12.5%)	$\frac{7}{8}$ (87.5%)
$t = 10\ (T_{1/2})$	$\frac{N_0}{(2)^{10}}$	$\left(\frac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = n\ (N_{1/2})$	$\frac{N}{(2)^2}$	$\left(\frac{1}{2}\right)^{n}$	$\left\{1-\left(\frac{1}{2}\right)^{n}\right\}$

Useful relation

After n half-lives, number of undecayed atoms $N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

(4) **Mean (or average) life (τ)**

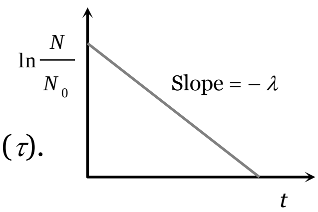
The time for which a radioactive material remains active is defined as mean (average) life of that material.

Other definitions

(i) It is defined as the sum of lives of all atoms divided by the total number of atoms

$$i.e.\ \tau = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}} = \frac{1}{\lambda}$$

(ii) From $N = N_0 e^{-\lambda t} \Rightarrow \frac{\ln \frac{N}{N_0}}{t} = -\lambda$ slope of the line shown in the graph



i.e. the magnitude of inverse of slope of $\ln\frac{N}{N_0}$ *vs* t curve is known as mean life (τ).

(iii) From $N = N_0 e^{-\lambda t}$

If $t = \frac{1}{\lambda} = \tau \Rightarrow N = N_0 e^{-1} = N_0\left(\frac{1}{e}\right) = 0.37\,N_0 = 37\%$ of N_0.

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\frac{1}{e}$ times or 0.37 times or 37% of original number of atoms. or

It is the time in which number of decayed atoms ($N_0 - N$) becomes $\left(1-\frac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(iv) From $T_{1/2} = \frac{0.693}{\lambda} \Rightarrow \frac{1}{\lambda} = \tau = \frac{1}{0.693}.(t_{1/2}) = 1.44\,(T_{1/2})$

i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$

Note : ❑ Half life and mean life of a substance doesn't change with time or with pressure, temperature *etc.*

Radioactive Series

If the isotope that results from a radioactive decay is itself radioactive then it will also decay and so on.

The sequence of decays is known as radioactive decay series. Most of the radio-nuclides found in nature are members of four radioactive series. These are as follows

Mass number	Series (Nature)	Parent	Stable and product	Integer n	Number of lost particles
$4n$	Thorium (natural)	$_{90}Th^{232}$	$_{82}Pb^{208}$	52	$\alpha = 6, \beta = 4$
$4n + 1$	Neptunium (Artificial)	$_{93}Np^{237}$	$_{83}Bi^{209}$	52	$\alpha = 8, \beta = 5$

$4n + 2$	Uranium (Natural)	${}_{92}U^{238}$	${}_{82}Pb^{206}$	51	$\alpha = 8, \beta = 6$
$4n + 3$	Actinium (Natural)	${}_{89}Ac^{227}$	${}_{82}Pb^{207}$	51	$\alpha = 7, \beta = 4$

Note :
- The $4n + 1$ series starts from ${}_{94}PU^{241}$ but commonly known as neptunium series because neptunium is the longest lived member of the series.
- The $4n + 3$ series actually starts from ${}_{92}U^{235}$.

Successive Disintegration and Radioactive Equilibrium

Suppose a radioactive element A disintegrates to form another radioactive element B which intern disintegrates to still another element C; such decays are called successive disintegration.

$$A \xrightarrow{\lambda_1} B \xrightarrow{\lambda_2} C$$

Rate of disintegration of $A = \frac{dN_1}{dt} = -\lambda_1 N_1$ (which is also the rate of formation of B)

Rate of disintegration of $B = \frac{dN_2}{dt} = -\lambda_2 N_2$

$\therefore$ Net rate of formation of B = Rate of disintegration of A – Rate of disintegration of B

$= \lambda_1 N_1 - \lambda_2 N_2$

Equilibrium

In radioactive equilibrium, the rate of decay of any radioactive product is just equal to it's rate of production from the previous member.

i.e. $\lambda_1 N_1 = \lambda_2 N_2 \quad \Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_2} = \frac{\tau_2}{\tau_1} = \frac{(T_{1/2})}{(T_{1/2})_1}$

Note :
- In successive disintegration if N_0 is the initial number of nuclei of A at $t = 0$ then number of nuclei of product B at time t is given by $N_2 = \frac{\lambda_1 N_0}{(\lambda_2 - \lambda_1)}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$ where $\lambda_1 \lambda_2$ – decay constant of A and B.

Uses of radioactive isotopes

(1) **In medicine**

(i) For testing blood-chromium - 51

(ii) For testing blood circulation - Na - 24

(iii) For detecting brain tumor- Radio mercury - 203

(iv) For detecting fault in thyroid gland - Radio iodine - 131

(v) For cancer - cobalt - 60

(vi) For blood - Gold - 189

(vii) For skin diseases - Phospohorous - 31

(2) **In Archaeology**

(i) For determining age of archaeological sample (carbon dating) C^{14}

(ii) For determining age of meteorites - K^{40}

(iii) For determining age of earth-Lead isotopes

(3) **In agriculture**

(i) For protecting potato crop from earthworm- CO^{60} (ii) For art As fertilizers - P^{32}

(4) **As tracers** - (Tracer) : Very small quantity of radioisotopes present in a mixture is known as tracer

(i) Tracer technique is used for studying biochemical reaction in tracer and animals.

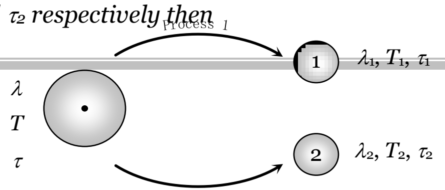
(5) **In industries**

(i) For detecting leakage in oil or water pipe lines (ii) For determining the age of planets.

Concept

- *If a nuclide can decay simultaneously by two different process which have decay constant λ_1 and λ_2, half life T_1 and T_2 and mean lives τ_1 and τ_2 respectively then*

$\Rightarrow \lambda = \lambda_1 + \lambda_2$



$$\Rightarrow \quad T = \frac{T_1 T_2}{T_1 + T_2}$$

$$\Rightarrow \quad \tau = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$$

***Example*: 16** When ${}_{90}Th^{228}$ transforms to ${}_{83}Bi^{212}$, then the number of the emitted α–and β–particles is, respectively **[MP PET 2002]**

(a) $8\alpha, 7\beta$ (b) $4\alpha, 7\beta$ (c) $4\alpha, 4\beta$ (d) $4\alpha, 1\beta$

Solution : (d) ${}_{Z=90}Th^{A=228} \rightarrow {}_{Z'=83}Bi^{A'=212}$

Number of α-particles emitted $n_\alpha = \frac{A - A'}{4} = \frac{228 - 212}{4} = 4$

Number of β-particles emitted $n_\beta = 2n_\alpha - Z + Z' = 2 \times 4 - 90 + 83 = 1$.

***Example*: 17** A radioactive substance decays to 1/16th of its initial activity in 40 days. The half-life of the radioactive substance expressed in days is

(a) 2.5 (b) 5 (c) 10 (d) 20

Solution : (c) By using $N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{N}{N_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^{40/T_{1/2}} \Rightarrow T_{1/2} = 10$ days.

***Example*: 18** A sample of radioactive element has a mass of 10 *gm* at an instant *t* = 0. The approximate mass of this element in the sample after two mean lives is **[CBSE PMT 2003]**

(a) 2.50 *gm* (b) 3.70 *gm* (c) 6.30 *gm* (d) 1.35 *gm*

Solution : (d) By using $M = M_0 e^{-\lambda t} \Rightarrow M = 10e^{-\lambda(2\tau)} = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 = 1.359\ gm$

***Example*: 19** The half-life of ${}^{215}At$ is 100 μs. The time taken for the radioactivity of a sample of ${}^{215}At$ to decay to 1/16th of its initial value is **[IIT-JEE (Screening) 2002]**

(a) 400 μs (b) 6.3 μs (c) 40 μs (d) 300 μs

Solution : (a) By using $N = N_0 \left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{t/100} \Rightarrow t = 400\ \mu sec.$

***Example*: 20** The mean lives of a radioactive substance for α and β emissions are 1620 years and 405 years respectively. After how much time will the activity be reduced to one fourth **[RPET 1999]**

(a) 405 year (b) 1620 year (c) 449 year (d) None of these

Solution : (c) $\lambda_\alpha = \frac{1}{1620}$ *per year* and $\lambda_\beta = \frac{1}{405}$ *per year* and it is given that the fraction of the remained activity $\frac{A}{A_0} = \frac{1}{4}$

Total decay constant $\lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ *per year*

We know that $A = A_0 e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda}\log_e \frac{A_0}{A} \Rightarrow t = \frac{1}{\lambda}\log_e 4 = \frac{2}{\lambda}\log_e 2 = 324 \times 2 \times 0.693 = 449$ *years.*

***Example*: 21** At any instant the ratio of the amount of radioactive substances is 2 : 1. If their half lives be respectively 12 and 16 hours, then after two days, what will be the ratio of the substances

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

Solution : (a) By using $N = N_0 \left(\frac{1}{2}\right)^n \Rightarrow \frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2} \times \frac{(1/2)^{n_1}}{(1/2)^{n_2}} = \frac{2}{1} \times \frac{\left(\frac{1}{2}\right)^{\frac{2\times 24}{12}}}{\left(\frac{1}{2}\right)^{\frac{2\times 24}{16}}} = \frac{1}{1}$

***Example*: 22** From a newly formed radioactive substance (Half-life 2 *hours*), the intensity of radiation is 64 times the permissible safe level. The minimum time after which work can be done safely from this source is **[IIT 1983; SCRA 1996]**

(a) 6 *hours* (b) 12 *hours* (c) 24 *hours* (d) 128 *hours*

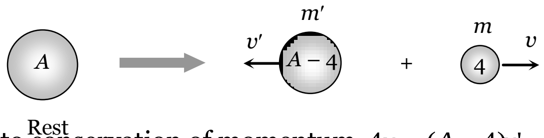
Solution : (b) By using $A = A_0 \left(\frac{1}{2}\right)^n \Rightarrow \frac{A}{A_0} = \frac{1}{64} = \left(\frac{1}{2}\right)^0 = \left(\frac{1}{2}\right)^n \Rightarrow n = 6$

$\Rightarrow \frac{t}{T_{1/2}} = 6 \Rightarrow t = 6 \times 2 = 12$ *hours.*

***Example*: 23** nucleus of mass number A, originally at rest, emits an α-particle with speed v. The daughter nucleus recoils with a speed **[DCE 2000; AIIMS 2004]**

(a) $2v/(A+4)$ (b) $4v/(A+4)$ (c) $4v/(A-4)$ (d) $2v/(A-4)$

Solution : (c)



According to conservation of momentum $4v = (A-4)v' \Rightarrow v' = \frac{4v}{A-4}$.

***Example*: 24** The counting rate observed from a radioactive source at t = 0 second was 1600 counts per second and at t = 8 seconds it was 100 counts per second. The counting rate observed as counts per second at t = 6 seconds will be **[MP PET 1996; UPSEAT 2000]**

(a) 400 (b) 300 (c) 200 (d) 150

Solution : (c) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow 100 = 1600\left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow T_{1/2} = 2\ sec$

Again by using the same relation the count rate at t = 6 *sec* will be $A = 1600\left(\frac{1}{2}\right)^{6/2} = 200$.

***Example*: 25** The kinetic energy of a neutron beam is 0.0837 eV. The half-life of neutrons is 693s and the mass of neutrons is 1.675×10^{-27} kg. The fraction of decay in travelling a distance of 40m will be

(a) 10^{-3} (b) 10^{-4} (c) 10^{-5} (d) 10^{-6}

Solution : (c) $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 0.0837 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}} = 4 \times 10^3\ m/sec$

$\therefore$ Time taken by neutrons to travel a distance of 40 m $\Delta t' = \frac{40}{4 \times 10^3} = 10^{-2}\ sec$

$\therefore \frac{dN}{dt} = \lambda N \Rightarrow \frac{dN}{N} = \lambda\, dt$

$\therefore$ Fraction of neutrons decayed in Δt *sec* in $\frac{\Delta N}{N} = \lambda \Delta t = \frac{0.693}{T}\Delta t = \frac{0.693}{693} \times 10^{-2} = 10^{-5}$

***Example*: 26** The fraction of atoms of radioactive element that decays in 6 days is 7/8. The fraction that decays in 10 days will be

(a) 77/80 (b) 71/80 (c) 31/32 (d) 15/16

Solution : (c) By using $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow t = \frac{T_{1/2}\log_e\left(\frac{N_0}{N}\right)}{\log_e(2)} \Rightarrow t \propto \log_e\frac{N_0}{N} \Rightarrow \frac{t_1}{t_2} = \frac{\left(\log_e\frac{N_0}{N}\right)_1}{\left(\log_e\frac{N_0}{N}\right)_2}$

Hence $\frac{6}{10} = \frac{\log_e(8/1)}{\log_e(N_0/N)} \Rightarrow \log_e\frac{N_0}{N} = \frac{10}{6}\log_e(8) = \log_e 32 \Rightarrow \frac{N_0}{N} = 32$.

So fraction that decays $= 1 - \frac{1}{32} = \frac{31}{32}$.

Tricky example: 2

Half-life of a substance is 20 minutes. What is the time between 33% decay and 67% decay **[AIIMS 200**

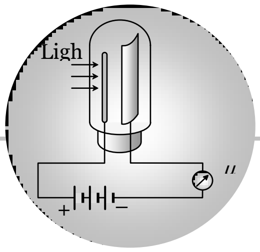
(a) 40 minutes (b) 20 minutes (c) 30 minutes (d) 25 minutes

Solution : (b) Let N_0 be the number of nuclei at beginning

$\therefore$ Number of undecayed nuclei after 33% decay = 0.67 N_0

and number of undecayed nuclei after 67% of decay = 0.33 N_0

$\because$ 0.33 $N_0 \simeq \frac{0.67 N_0}{2}$ and in the half-life time the number of undecayed nuclei becomes half.



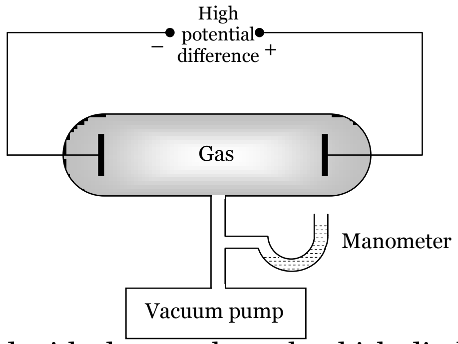
Electron, Photon, Photoelectric Effect and X-rays

Electric Discharge Through Gases

At normal atmospheric pressure, the gases are poor conductor of electricity. If we establish a potential difference (of the order of 30 *kV*) between two electrodes placed in air at a distance of few *cm* from each other, electric conduction starts in the form of sparks.

The passage of electric current through air is called electric discharge through the air.

The discharge of electricity through gases can be systematically studied with the help of discharge tube shown below



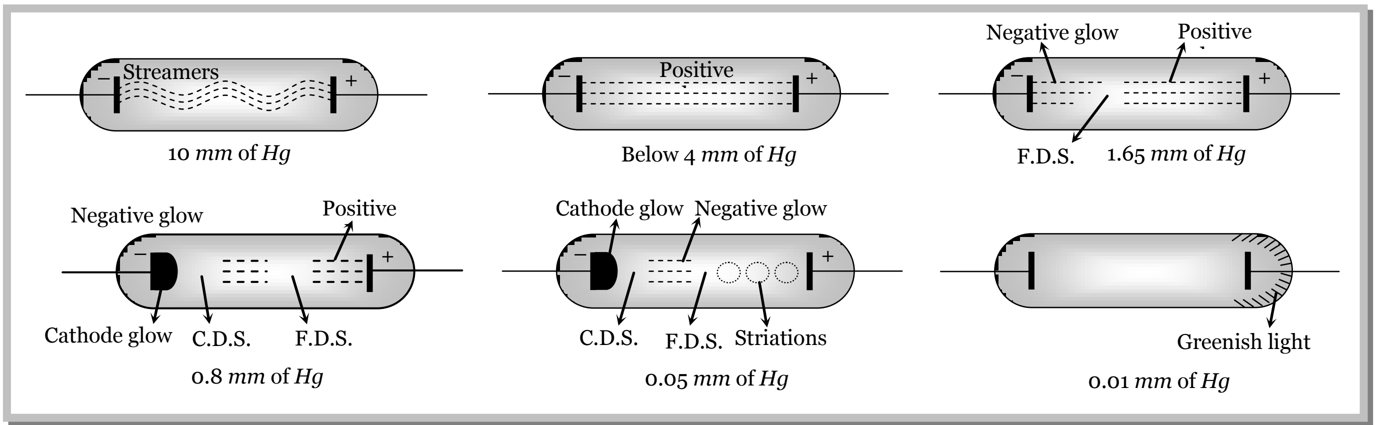
Length of discharge tube ≈ 30 to 40 *cm*

Diameter of the tube ≈ 4*cm*

The discharge tube is filled with the gas through which discharge is to be studied. The pressure of the enclosed gas can be reduced with the help of a vacuum pump and it's value is read by manometer.

Sequence of phenomenon

As the pressure inside the discharge tube is gradually reduced, the following is the sequence of phenomenon that are observed.



(1) At normal pressure no discharge takes place.

(2) At the pressure 10 *mm* of *Hg*, a zig-zag thin red spark runs from one electrode to other and cracking sound is heard.

(3) At the pressure 4 *mm*. of *Hg*, an illumination is observed at the electrodes and the rest of the tube appears dark. This type of discharge is called dark discharge.

(4) When the pressure falls below 4 *mm* of *Hg* then the whole tube is filled with bright light called positive column and colour of light depends upon the nature of gas in the tube as shown in the following table.

Gas	Colour
Air	Purple red

H_2	Blue
N_2	Red
Cl_2	Green
CO_2	Bluish white
Na	Yellow
Neon	Dark red

(5) At a pressure of 1.65 *mm* of *Hg* :

(i) Sky colour light is produced at the cathode it is called as negative glow.

(ii) Positive column shrinks towards the anode and the dark space between positive column and negative glow is called Faradays dark space (FDS)

(6) At a pressure of 0.8 *mm Hg* : At this pressure, negative glow is detached from the cathode and moves towards the anode. The dark space created between cathode and negative glow is called as Crook's dark space length of positive column further reduced. A glow appear at cathode called cathode glow.

(7) At a pressure of 0.05 *mm* of *Hg* : The positive column splits into dark and bright disc of light called striations.

(8) At the pressure of 0.01 or 10^{-2} *mm* of *Hg* some invisible particle move from cathode which on striking with the glass tube of the opposite side of cathode cause the tube to glow. These invisible rays emerging from cathode are called cathode rays.

(9) Finally when pressure drops to nearly 10^{-4} *mm* of *Hg*, there is no discharge in tube.

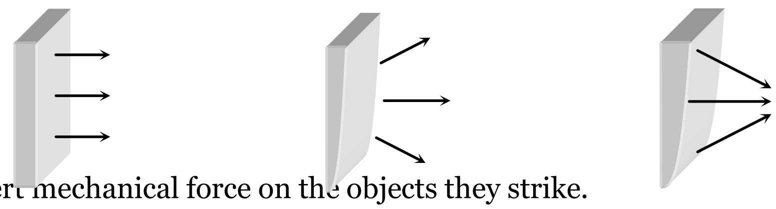
Cathode Rays

Cathode rays, discovered by sir Willium Crooke are the stream of electrons. They can be produced by using a discharge tube containing gas at a low pressure of the order of 10^{-2} *mm* of *Hg*. At this pressure the gas molecules ionise and the emitted electrons travel towards positive potential of anode. The positive ions hit the cathode to cause emission of electrons from cathode. These electrons also move towards anode. Thus the cathode rays in the discharge tube are the electrons produced due to ionisation of gas and that emitted by cathode due to collision of positive ions.

(1) **Properties of cathode rays**

(i) Cathode rays travel in straight lines (cast shadows of objects placed in their path)

(ii) Cathode rays emit normally from the cathode surface. Their direction is independent of the position of the anode.



(iii) Cathode rays exert mechanical force on the objects they strike.

(iv) Cathode rays produce heat when they strikes a material surface.

(v) Cathode rays produce fluorescence.

(vi) When cathode rays strike a solid object, specially a metal of high atomic weight and high melting point *X*-rays are emitted from the objects.

(vii) Cathode rays are deflected by an electric field and also by a magnetic field.

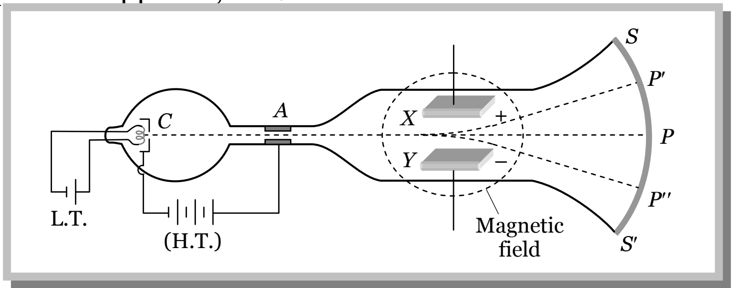
(viii) Cathode rays ionise the gases through which they are passed.

(ix) Cathode rays can penetrate through thin foils of metal.

(x) Cathode rays are found to have velocity ranging $\frac{1}{30}th$ to $\frac{1}{10}th$ of velocity of light.

(2) **J.J. Thomson's method to determine specific charge of electron**

It's working is based on the fact that if a beam of electron is subjected to the crossed electric field $\vec{E}$ and magnetic field $\vec{B}$, it experiences a force due to each field. In case the forces on the electrons in the electron beam due to these fields are equal and opposite, the beam remains undeflected.



C = Cathode, A = Anode, F = Filament, LT = Battery to heat the filament, V = potential difference to accelerate the electrons, SS' = ZnS coated screen, XY = metallic plates (Electric field produced between them)

(i) When no field is applied, the electron beam produces illuminations at point P.

(ii) In the presence of any field (electric and magnetic) electron beam deflected up or down (illumination at P' or P'')

(iii) If both the fields are applied simultaneously and adjusted such that electron beam passes undeflected and produces illumination at point P.

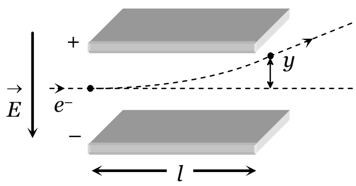
In this case; Electric force = Magnetic force $\Rightarrow eE = evB \Rightarrow v = \frac{E}{B}$; v = velocity of electron

As electron beam accelerated from cathode to anode its potential energy at the cathode appears as gain in the K.E. at the anode. If suppose V is the potential difference between cathode and anode then, potential energy $= eV$

And gain in kinetic energy at anode will be K.E. $= \frac{1}{2}mv^2$ i.e. $eV = \frac{1}{2}mv^2 \Rightarrow \frac{e}{m} = \frac{v^2}{2V} \Rightarrow \frac{e}{m} = \frac{E^2}{2VB^2}$

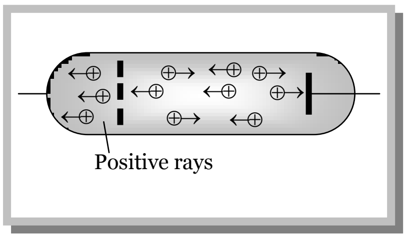
Thomson found, $\frac{e}{m} = 1.77 \times 10^{11}\ C/kg$.

Note : ❑ The deflection of an electron in a purely electric field is given by $y = \frac{1}{2}\left(\frac{eE}{m}\right)\cdot\frac{l^2}{v^2}$; where l length of each plate, y = deflection of electron in the field region, v = speed of the electron.



Positive Rays

Positive rays are sometimes known as the canal rays. These were discovered by Goldstein. If the cathode of a discharge tube has holes in it and the pressure of the gas is around 10^{-3} *mm* of *Hg* then faint luminous glow comes out from each hole on the backside of the cathode. It is said positive rays which are coming out from the holes.



(1) **Origin of positive rays**

When potential difference is applied across the electrodes, electrons are emitted from the cathode. As they move towards anode, they gain energy. These energetic electrons when collide with the atoms of the gas in the discharge tube, they ionize the atoms. The positive ions so formed at various places between cathode and anode, travel towards the cathode. Since during their motion, the positive ions when reach the cathode, some pass through the holes in the cathode. These streams are the positive rays.

(2) **Properties of positive rays**

(i) These are positive ions having same mass if the experimental gas does not have isotopes. However if the gas has isotopes then positive rays are group of positive ions having different masses.

(ii) They travels in straight lines and cast shadows of objects placed in their path. But the speed of the positive rays is much smaller than that of cathode rays.

(iii) They are deflected by electric and magnetic fields but the deflections are small as compared to that for cathode rays.

(iv) They show a spectrum of velocities. Different positive ions move with different velocities. Being heavy, their velocity is much less than that of cathode rays.

(v) q/m ratio of these rays depends on the nature of the gas in the tube (while in case of the cathode rays q/m is constant and doesn't depend on the gas in the tube). q/m for hydrogen is maximum.

(vi) They carry energy and momentum. The kinetic energy of positive rays is more than that of cathode rays.

(vii) The value of charge on positive rays is an integral multiple of electronic charge.

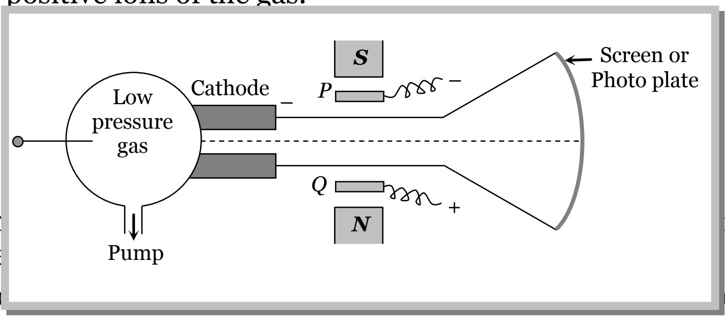
(viii) They cause ionisation (which is much more than that produced by cathode rays).

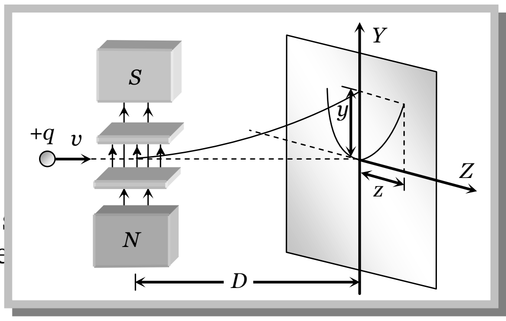
Mass Spectrograph

It is a device used to determine the mass or (q/m) of positive ions.

(1) **Thomson mass spectrograph**

It is used to measure atomic masses of various isotopes in gas. This is done by measuring q/m of singly ionised positive ions of the gas.





Th … hand … rds
cathode … in the … s is
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$\vec{E}$ or $\vec{B} \perp \vec{v}$).

If the initial motion of the ions is in $+x$ direction and electric and magnetic fields are applied along $+y$ axis then force due to electric field is in the direction of y-axis and due to magnetic field it is along z-direction.

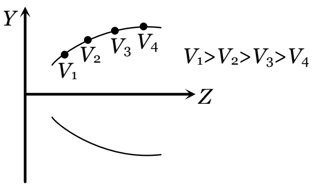
The deflection due to electric field alone $y = \dfrac{qELD}{mv^2}$ ....(i)

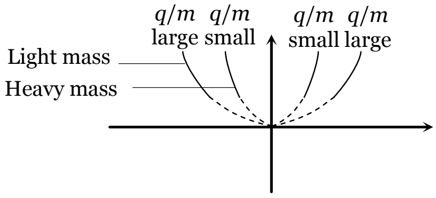
The deflection due to magnetic field alone $z = \dfrac{qBLD}{mv}$ ....(ii)

From equation (i) and (ii)

$z^2 = k\left(\dfrac{q}{m}\right)y$, where $k = \dfrac{B^2LD}{E}$; This is the equation of parabola. It means all the charged particles moving with different velocities but of same q/m value will strike the screen placed in yz plane on a parabolic track as shown in the above figure.

Note : ❑ All the positive ions of same. q/m moving with different velocity lie on the same parabola. Higher is the velocity lower is the value of y and z. The ions of different specific charge will lie on different parabola.





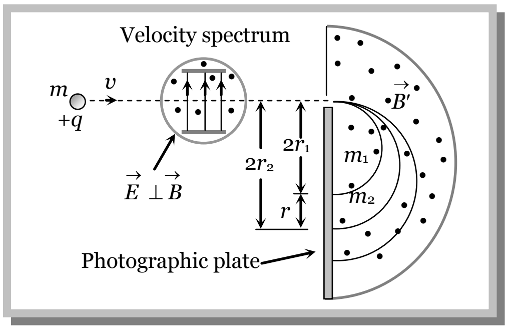
❑ The number of parabola tells the number of isotopes present in the given ionic beam.

(2) **Bainbridge mass spectrograph**

In Bainbridge mass spectrograph, field particles of same velocity are selected by using a velocity selector and then they are subjected to a uniform magnetic field perpendicular to the velocity of the particles. The particles corresponding to different isotopes follow different circular paths as shown in the figure.

(i) **Velocity selector :** The positive ions having a certain velocity v gets isolated from all other velocity particles. In this chamber the electric and magnetic fields are so balanced that the particle moves undeflected. For this the necessary condition is $v = \frac{E}{B}$.

(ii) **Analysing chamber :** In this chamber magnetic field B is applied perpendicular to the direction of motion of the particle. As a result the particles move along a circular path of radius



$$r = \frac{mE}{qBB'} \Rightarrow \frac{q}{m} = \frac{E}{BB'r} \text{ also } \frac{r_1}{r_2} = \frac{m_1}{m_2}$$

In this way the particles of different masses gets deflected on circles of different radii and reach on different points on the photo plate.

Note : ❑ Separation between two traces $= d = 2r_2 - 2r_1 \Rightarrow d = \frac{2v(m_2 - m_1)}{qB'}$.

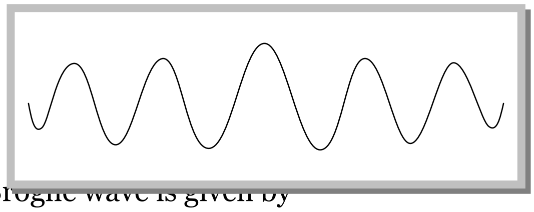
Matter waves (de-Broglie Waves)

According to de-Broglie a moving material particle sometimes acts as a wave and sometimes as a particle.

or

A wave is associated with moving material particle which control the particle in every respect.

The wave associated with moving particle is called matter wave or de-Broglie wave and it propagates in the form of wave packets with group velocity.



(1) **de-Broglie wavelength**

According to de-Broglie theory, the wavelength of de-Broglie wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \qquad \Rightarrow \lambda \propto \frac{1}{p} \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}}$$

Where h = Plank's constant, m = Mass of the particle, v = Speed of the particle, E = Energy of the particle.

The smallest wavelength whose measurement is possible is that of γ-rays.

The wavelength of matter waves associated with the microscopic particles like electron, proton, neutron, α-particle *etc.* is of the order of 10^{-10} m.

(i) **de-Broglie wavelength associated with the charged particles.**

The energy of a charged particle accelerated through potential difference V is $E = \frac{1}{2}mv^2 = qV$

Hence de-Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

$$\lambda_{electron} = \frac{12.27}{\sqrt{V}}Å, \quad \lambda_{proton} = \frac{0.286}{\sqrt{V}}Å, \quad \lambda_{deutron} = \frac{0.202 \times 10^{-10}}{\sqrt{V}}Å, \quad \lambda_{\alpha-particle} = \frac{0.101}{\sqrt{V}}Å$$

(ii) **de-Broglie wavelength associated with uncharged particles.**

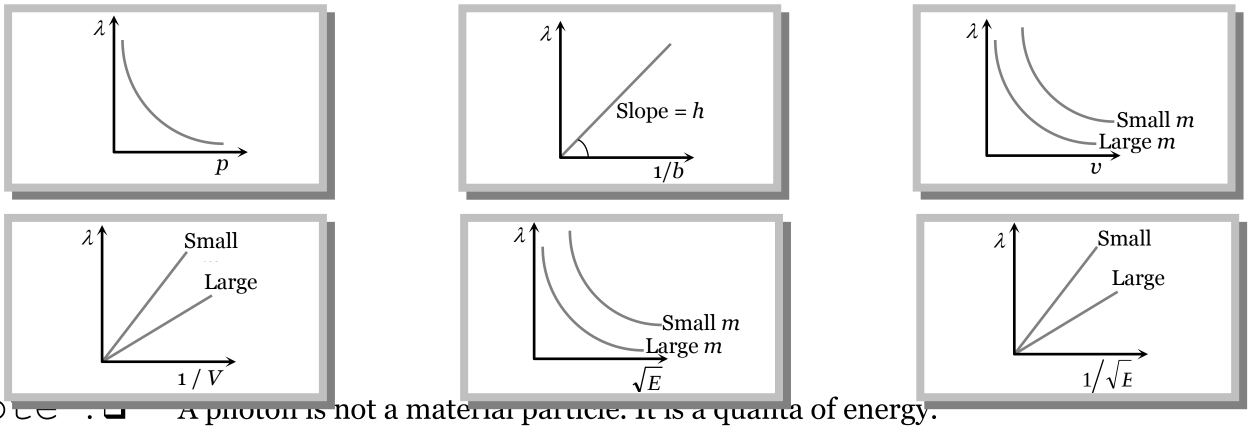
For *Neutron* de-Broglie wavelength is given as $\lambda_{Neutron} = \frac{0.286 \times 10^{-10}}{\sqrt{E(\text{in } eV)}} m = \frac{0.286}{\sqrt{E(\text{in } eV)}}Å$

Energy of thermal neutrons at ordinary temperature

$\because\ E = kT \Rightarrow \lambda = \dfrac{h}{\sqrt{2mkT}}$; where k = Boltzman's constant = 1.38×10^{-23} *Joules/kelvin*, T = Absolute temp.

So $\lambda_{\text{Thermal Neutron}} = \dfrac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.07 \times 10^{-17} \times 1.38 \times 10^{-23}\, T}} = \dfrac{30.83}{\sqrt{T}}\,\mathring{A}$

(2) **Some graphs**



Note : ❑ A photon is not a material particle. It is a quanta of energy.

❑ When a particle exhibits wave nature, it is associated with a wave packet, rather then a wave.

(3) **Characteristics of matter waves**

(i) Matter wave represents the probability of finding a particle in space.

(ii) Matter waves are not electromagnetic in nature.

(iii) de-Brogile or matter wave is independent of the charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).

(iv) Practical observation of matter waves is possible only when the de-Broglie wavelength is of the order of the size of the particles is nature.

(v) Electron microscope works on the basis of de-Broglie waves.

(vi) The electric charge has no effect on the matter waves or their wavelength.

(vii) The phase velocity of the matter waves can be greater than the speed of the light.

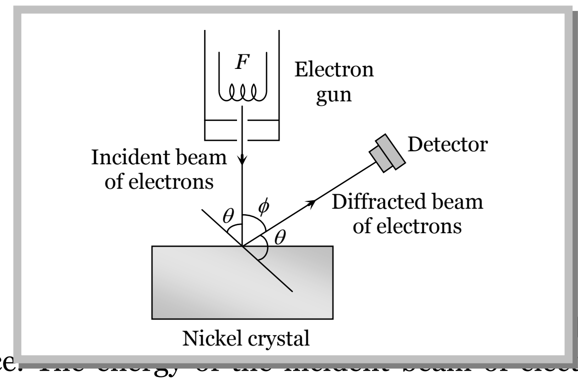
(viii) Matter waves can propagate in vacuum, hence they are not mechanical waves.

(ix) The number of de-Broglie waves associated with n^{th} orbital electron is n.

(x) Only those circular orbits around the nucleus are stable whose circumference is integral multiple of de-Broglie wavelength associated with the orbital electron.

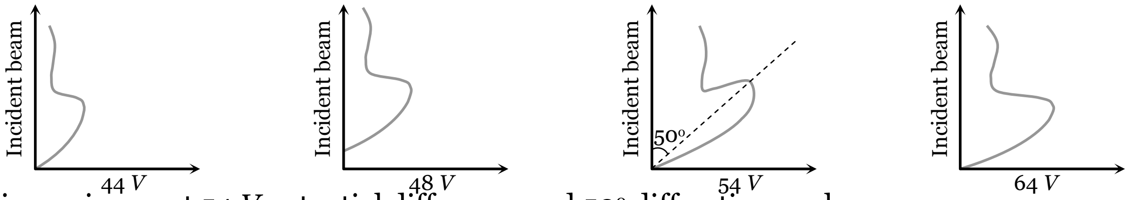
(4) **Davision and Germer experiment**

It is used to study the scattering of electron from a solid or to verify the wave nature of electron. A beam of electrons emitted by electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle. *Ni* crystal behaves like a three dimensional diffraction grating and it diffracts the electron beam obtained from electron gun.



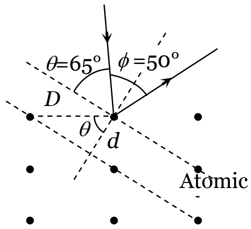
The diffracted beam of electrons ... positioned at any angle by rotating it about the point of incidence. The energy of the incident beam of electrons can also be varied by changing the applied voltage to the electron gun.

According to classical physics, the intensity of scattered beam of electrons at all scattering angle will be same but Davisson and Germer, found that the intensity of scattered beam of electrons was not the same but different at different angles of scattering.



Intensity is maximum at 54 *V* potential difference and 50° diffraction angle.

If the de-Broglie waves exist for electrons then these should be diffracted as *X*-rays. Using the Bragg's formula $2d\sin\theta = n\lambda$, we can determine the wavelength of these waves.



Where *d* = distance between diffracting planes, $\theta = \frac{(180-\phi)}{2}$ = glancing angle for incident beam = Bragg's angle.

The distance between diffraction planes in *Ni*-crystal for this experiment is *d* = 0.91Å and the Bragg's angle = 65°. This gives for *n* = 1, $\lambda = 2\times 0.91\times 10^{-10}\sin 65^{o} = 1.65\,Å$

Now the de-Broglie wavelength can also be determined by using the formula $\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} = 1.67\,Å$.

Thus the de-Broglie hypothesis is verified.

Heisenberg Uncertainty Principle

According to Heisenberg's uncertainty principle, it is impossible to measure simultaneously both the position and the momentum of the particle.

Let Δx and Δp be the uncertainty in the simultaneous measurement of the position and momentum of the particle, then $\Delta x\Delta p = \hbar$; where $\hbar = \frac{h}{2\pi}$ and $h = 6.63\times 10^{-34}$ *J-s* is the Planck's constant.

If $\Delta x = 0$ then $\Delta p = \infty$

and if $\Delta p = 0$ then $\Delta x = \infty$ *i.e.*, if we are able to measure the exact position of the particle (say an electron) then the uncertainty in the measurement of the linear momentum of the particle is infinite. Similarly, if we are able to measure the exact linear momentum of the particle *i.e.*, $\Delta p = 0$, then we can not measure the exact position of the particle at that time.

Photon

According to Eienstein's quantum theory light propagates in the bundles (packets or quanta) of energy, each bundle being called a photon and possessing energy.

(1) **Energy of photon**

Energy of each photon is given by $E = h\nu = \frac{hc}{\lambda}$; where *c* = Speed of light, *h* = Plank's constant = 6.6 × 10^{-34} *J-sec*, ν = Frequency in *Hz*, λ = Wavelength of light

Energy of photon in electron volt $E(eV) = \frac{hc}{e\lambda} = \frac{12375}{\lambda(Å)} \approx \frac{12400}{\lambda(Å)}$

(2) **Mass of photon**

Actually rest mass of the photon is zero. But it's effective mass is given as

$E = mc^2 = h\nu \Rightarrow m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{c\lambda}$. This mass is also known as kinetic mass of the photon

(3) **Momentum of the photon**

Momentum $p = m\times c = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

(4) **Number of emitted photons**

The number of photons emitted per second from a source of monochromatic radiation of wavelength λ and power P is given as $(n) = \frac{P}{E} = \frac{P}{h\nu} = \frac{P\lambda}{hc}$; where E = energy of each photon

(5) **Intensity of light (I)**

Energy crossing per unit area normally per second is called intensity or energy flux

i.e. $I = \frac{E}{At} = \frac{P}{A}$ $\left(\frac{E}{t} = P = \text{radiation power}\right)$

At a distance r from a point source of power P intensity is given by $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

Concepts

- *Discovery of positive rays helps in discovering of isotopes.*
- *The de-Broglie wavelength of electrons in first Bohr orbit of an atom is equal to circumference of orbit.*
- *A particle having zero rest mass and non zero energy and momentum must travels with a speed equal to speed of light.*
- ***de-Broglie wave length associates with gas molecules** is given as $\lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3\,mkT}}$ (Energy of gas molecules at temperature T is $E = \frac{3}{2}kT$)*

Example: 1 The ratio of specific charge of an α-particle to that of a proton is **[BCECE 2003]**

(a) 2 : 1 (b) 1 : 1 (c) 1 : 2 (d) 1 : 3

Solution : (c) Specific charge $= \frac{q}{m}$; Ratio $= \frac{(q/m)_\alpha}{(q/m)_p} = \frac{q_\alpha}{q_p} \times \frac{m_p}{m_\alpha} = \frac{1}{2}$.

Example: 2 The speed of an electron having a wavelength of $10^{-10}\,m$ is **[AIIMS 2002]**

(a) $7.25 \times 10^6\,m/s$ (b) $6.26 \times 10^6\,m/s$ (c) $5.25 \times 10^6\,m/s$ (d) $4.24 \times 10^6\,m/s$

Solution : (a) By using $\lambda_{electron} = \frac{h}{m_e v} \Rightarrow v = \frac{h}{m_e \lambda_e} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}} = 7.25 \times 10^6\,m/s.$

Example: 3 In Thomson experiment of finding e/m for electrons, beam of electron is replaced by that of muons (particle with same charge as of electrons but mass 208 times that of electrons). No deflection condition in this case satisfied if **[Orissa (Engg.) 2002]**

(a) B is increased 208 times (b) E is increased 208 times

(c) B is increased 14.4 times (d) None of these

Solution : (c) In the condition of no deflection $\frac{e}{m} = \frac{E^2}{2VB^2}$. If m is increased to 208 times then B should be increased by $\sqrt{208} = 14.4$ times.

Example: 4 In a Thomson set-up for the determination of e/m, electrons accelerated by 2.5 kV enter the region of crossed electric and magnetic fields of strengths $3.6 \times 10^4\,Vm^{-1}$ and $1.2 \times 10^{-3}\,T$ respectively and go through undeflected. The measured value of e/m of the electron is equal to **[AMU 2002]**

(a) 1.0×10^{11} C-kg^{-1} (b) 1.76×10^{11} C-kg^{-1} (c) 1.80×10^{11} C-kg^{-1} (d) 1.85×10^{11} C-kg^{-1}

Solution : (c) By using $\frac{e}{m} = \frac{E^2}{2VB^2} \Rightarrow \frac{e}{m} = \frac{(3.6 \times 10^4)^2}{2 \times 2.5 \times 10^3 \times (1.2 \times 10^{-3})^2} = 1.8 \times 10^{11}\,C/kg.$

Example: 5 In Bainbridge mass spectrograph a potential difference of 1000 V is applied between two plates distant 1 cm apart and magnetic field in B = 1T. The velocity of undeflected positive ions in m/s from the velocity selector is

(a) $10^7\,m/s$ (b) $10^4\,m/s$ (c) $10^5\,m/s$ (d) $10^2\,m/s$

Solution : (c) By using $v = \frac{E}{B}$; where $E = \frac{V}{d} = \frac{1000}{1 \times 10^{-2}} = 10^5\,V/m \Rightarrow v = \frac{10^5}{1} = 10^5\,m/s$.

Example: 6 An electron and a photon have same wavelength. It p is the momentum of electron and E the energy of photon. The magnitude of p/E in S.I. unit is

(a) 3.0×10^8 (b) 3.33×10^{-9} (c) 9.1×10^{-31} (d) 6.64×10^{-34}

Solution : (b) $\lambda = \frac{h}{p}$ (for electron) or $p = \frac{h}{\lambda}$ and $E = \frac{hc}{\lambda}$ (for photon)

$\therefore \frac{p}{E} = \frac{1}{c} = \frac{1}{3 \times 10^8 \, m/s} = 3.33 \times 10^{-9} \, s/m$

***Example*: 7** The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is E. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio λ_1/λ_2 is proportional to **[UPSEAT 2003; IIT-JEE (Screening) 2004]**

(a) E^0 (b) $E^{1/2}$ (c) E^{-1} (d) E^{-2}

Solution : (b) For photon $\lambda_2 = \frac{hc}{E}$ .... (i) and For proton $\lambda_1 = \frac{h}{\sqrt{2mE}}$ ....(ii)

Therefore $\frac{\lambda_1}{\lambda_2} = \frac{E^{1/2}}{\sqrt{2m}\, c} \Rightarrow \frac{\lambda_1}{\lambda_2} \propto E^{1/2}$.

***Example*: 8** The de-Broglie wavelength of an electron having 80eV of energy is nearly ($1eV = 1.6 \times 10^{-19} J$, Mass of electron $9 \times 10^{-31} kg$ and Plank's constant 6.6×10^{-34} *J-sec*)

(a) 140 $Å$ (b) 0.14 $Å$ (c) 14 $Å$ (d) 1.4 $Å$

Solution : (d) By using $\lambda = \frac{h}{\sqrt{2mE}} = \frac{12.27}{\sqrt{V}}$. If energy is 80 eV then accelerating potential difference will be 80 V. So $\lambda = \frac{12.27}{\sqrt{80}} = 1.37 \approx 1.4 \, Å$.

***Example*: 9** The kinetic energy of electron and proton is $10^{-32} J$. Then the relation between their de-Broglie wavelengths is

(a) $\lambda_p < \lambda_e$ (b) $\lambda_p > \lambda_e$ (c) $\lambda_p = \lambda_e$ (d) $\lambda_p = 2\lambda_e$

Solution : (a) By using $\lambda = \frac{h}{\sqrt{2mE}}$ $E = 10^{-32} J$ = Constant for both particles. Hence $\lambda \propto \frac{1}{\sqrt{m}}$

Since $m_p > m_e$ so $\lambda_p < \lambda_e$.

***Example*: 10** The energy of a proton and an α particle is the same. Then the ratio of the de-Broglie wavelengths of the proton and the α is **[RPET 1991]**

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

Solution : (b) By using $\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$ (E – same) $\Rightarrow \frac{\lambda_{proton}}{\lambda_{\alpha-particle}} = \sqrt{\frac{m_\alpha}{m_p}} = \frac{2}{1}$.

***Example*: 11** The de-Broglie wavelength of a particle accelerated with 150 *volt* potential is 10^{-10} *m*. If it is accelerated by 600 *volts* p.d., its wavelength will be **[RPET 1988]**

(a) 0.25 $Å$ (b) 0.5 $Å$ (c) 1.5 $Å$ (d) 2 $Å$

Solution : (b) By using $\lambda \propto \frac{1}{\sqrt{V}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}} \Rightarrow \frac{10^{-10}}{\lambda_2} = \sqrt{\frac{600}{150}} = 2 \Rightarrow \lambda_2 = 0.5 \, Å$.

***Example*: 12** The de-Broglie wavelength of an electron in an orbit of circumference $2\pi r$ is **[MP PET 1987]**

(a) $2\pi r$ (b) πr (c) $1/2\pi r$ (d) $1/4\pi r$

Solution : (a) According to Bohr's theory $mvr = n\frac{h}{2\pi} \Rightarrow 2\pi r = n\left(\frac{h}{mv}\right) = n\lambda$

For $n = 1$ $\lambda = 2\pi r$

***Example*: 13** The number of photons of wavelength 540 *nm* emitted per second by an electric bulb of power 100W is (taking $h = 6 \times 10^{-34}$ *J-sec*) **[Kerala (Engg.) 2002]**

(a) 100 (b) 1000 (c) 3×10^{20} (d) 3×10^{18}

Solution : (c) By using $n = \frac{P\lambda}{hc} = \frac{100 \times 540 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 3 \times 10^{20}$

***Example*: 14** A steel ball of mass 1*kg* is moving with a velocity 1 *m/s*. Then its de-Broglie waves length is equal to

(a) h (b) $h/2$ (c) Zero (d) $1/h$

Solution : (a) By using $\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{\lambda}{1 \times 1} = h$.

***Example*: 15** The de-Broglie wavelength associated with a hydrogen atom moving with a thermal velocity of 3 *km/s* will be

(a) 1 $\mathring{A}$ (b) 0.66 $\mathring{A}$ (c) 6.6 $\mathring{A}$ (d) 66 $\mathring{A}$

Solution : (b) By using $\lambda = \frac{h}{mv_{rms}} \Rightarrow \lambda = \frac{6.6\times 10^{-34}}{2\times 1.67\times 10^{-27}\times 3\times 10^{3}} = 0.66\mathring{A}$

***Example*: 16** When the momentum of a proton is changed by an amount P_0, the corresponding change in the de-Broglie wavelength is found to be 0.25%. Then, the original momentum of the proton was **[CPMT 2002]**

(a) p_0 (b) 100 p_0 (c) 400 p_0 (d) 4 p_0

Solution : (c) $\lambda \propto \frac{1}{p} \Rightarrow \frac{\Delta p}{p} = -\frac{\Delta\lambda}{\lambda} \Rightarrow \left|\frac{\Delta p}{p}\right| = \left|\frac{\Delta\lambda}{\lambda}\right| \Rightarrow \frac{p_0}{p} = \frac{0.25}{100} = \frac{1}{400} \Rightarrow p = 400\, p_0$.

***Example*: 17** If the electron has same momentum as that of a photon of wavelength 5200Å, then the velocity of electron in m/sec is given by

(a) 10^3 (b) 1.4×10^3 (c) 7×10^{-5} (d) 7.2×10^6

Solution : (b) $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda} = \frac{6.6\times 10^{-34}}{9.1\times 10^{-31}\times 5200\times 10^{-10}} \Rightarrow v = 1.4\times 10^3\ m/s.$

***Example*: 18** The de-Broglie wavelength of a neutron at 27°*C* is λ. What will be its wavelength at 927°*C*

(a) $\lambda/2$ (b) $\lambda/3$ (c) $\lambda/4$ (d) $\lambda/9$

Solution : (a) $\lambda_{neutron} \propto \frac{1}{\sqrt{T}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{\lambda}{\lambda_2} = \sqrt{\frac{(273+927)}{(273+27)}} = \sqrt{\frac{1200}{300}} = 2 \Rightarrow \lambda_2 = \frac{\lambda}{2}$.

***Example*: 19** The de-Broglie wavelength of a vehicle is λ. Its load is changed such that its velocity and energy both are doubled. Its new wavelength will be

(a) λ (b) $\frac{\lambda}{2}$ (c) $\frac{\lambda}{4}$ (d) 2λ

Solution : (a) $\lambda = \frac{h}{mv}$ and $E = \frac{1}{2}mv^2 \Rightarrow \lambda = \frac{hv}{2E}$ when v and E both are doubled, λ remains unchanged *i.e.* $\lambda' = \lambda$.

***Example*: 20** In Thomson mass spectrograph when only electric field of strength 20 *kV/m* is applied, then the displacement of the beam on the screen is 2 *cm*. If length of plates = 5 *cm*, distance from centre of plate to the screen = 20 *cm* and velocity of ions = 10^6 *m/s*, then *q/m* of the ions is

(a) 10^6 *C/kg* (b) 10^7 *C/Kg* (c) 10^8 *C/kg* (d) 10^{11} *C/kg*

Solution : (c) By using $y = \frac{qELD}{mv^2}$; where y = deflection on screen due to electric field only

$\Rightarrow \frac{q}{m} = \frac{yv^2}{ELD} = \frac{2\times 10^{-2}\times (10^6)^2}{20\times 10^3\times 5\times 10^{-2}\times 0.2} = 10^8\ C/kg.$

***Example*: 21** The minimum intensity of light to be detected by human eye is $10^{-10} W/m^2$. The number of photons of wavelength $5.6\times 10^{-7} m$ entering the eye, with pupil area $10^{-6} m^2$, per second for vision will be nearly

(a) 100 (b) 200 (c) 300 (d) 400

Solution : (c) By using $I = \frac{P}{A}$; where P = radiation power

$\Rightarrow P = I\times A \Rightarrow \frac{nhc}{t\lambda} = IA \Rightarrow \frac{n}{t} = \frac{IA\lambda}{hc}$

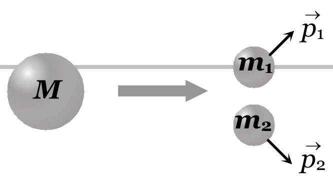
Hence number of photons entering per sec the eye $\left(\frac{n}{t}\right) = \frac{10^{-10}\times 10^{-6}\times 5.6\times 10^{-7}}{6.6\times 10^{-34}\times 3\times 10^{8}} = 300.$

Example 1. A particle of mass *M* at rest decays into two particles of masses m_1 and m_2, having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles, λ_1/λ_2 is **[IIT-JEE 1999]**

(a) m_1/m_2 (b) m_2/m_1 (c) 1.0 (d) $\sqrt{m_1}/\sqrt{m_1}$

Solution : (c) According to conservation of momentum *i.e.* $p_1 = p_2$

Hence from $\lambda = \frac{h}{p} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{p_1}{p_2} = \frac{1}{1}$



The curve drawn between velocity and frequency of photon in vacuum will be a **[MP PET 2000]**

(a) Straight line parallel to frequency axis
(b) Straight line parallel to velocity axis
(c) Straight line passing through origin and making an angle of 45^o with frequency axis
(d) Hyperbola

Solution : (a) Velocity of photon (*i.e.* light) doesn't depend upon frequency. Hence the graph between velocity of photon and frequency will be as follows.

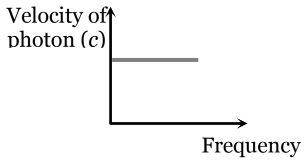


Photo-electric Effect

It is the phenomenon of emission of electrons from the surface of metals, when light radiations (Electromagnetic radiations) of suitable frequency fall on them. The emitted electrons are called photoelectrons and the current so produced is called photoelectric current.

This effect is based on the principle of conservation of energy.

(1) **Terms related to photoelectric effect**

(i) **Work function (or threshold energy) (W_o) :** The minimum energy of incident radiation, required to eject the electrons from metallic surface is defined as work function of that surface.

$$W_0 = h\nu_0 = \frac{hc}{\lambda_0}\ Joules\ ; \quad \nu_0 = \text{Threshold frequency};\ \lambda_0 = \text{Threshold wavelength}$$

Work function in electron volt $W_o(eV) = \dfrac{hc}{e\lambda_0} = \dfrac{12375}{\lambda_0(\mathring{A})}$

Note : ❑ By coating the metal surface with a layer of barium oxide or strontium oxide it's work function is lowered.

(ii) **Threshold frequency (ν_o) :** The minimum frequency of incident radiations required to eject the electron from metal surface is defined as threshold frequency.

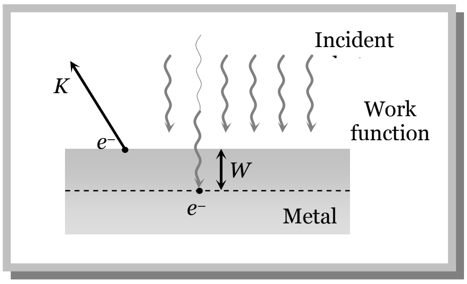
If incident frequency $\nu < \nu_0 \Rightarrow$ No photoelectron emission

(iii) **Threshold wavelength (λ_o) :** The maximum wavelength of incident radiations required to eject the electrons from a metallic surface is defined as threshold wavelength.

If incident wavelength $\lambda > \lambda_0 \Rightarrow$ No photoelectron emission

(2) **Einstein's photoelectric equation**

According to Einstein, photoelectric effect is the result of one to one inelastic collision between photon and electron in which photon is completely absorbed. So if an electron in a metal absorbs a photon of energy E (= $h\nu$), it uses the energy in three following ways.



(i) Some energy (say W) is used in shifting the electron from interior to the surface of the metal.

(ii) Some energy (say W_o) is used in making the surface electron free from the metal.

(iii) Rest energy will appear as kinetic energy (K) of the emitted photoelectrons.

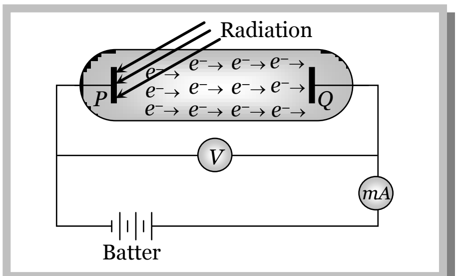
Hence $E = W + W_o + K$

For the electrons emitting from surface W = 0 so kinetic energy of emitted electron will be max.

Hence $\boldsymbol{E = W_o + K_{max}}$; This is the Einstein's photoelectric equation

(3) **Experimental arrangement to observe photoelectric effect**

When light radiations of suitable frequency (or suitable wavelength and suitable energy) falls on plate P, photoelectrons are emitted from P.



(i) If plate Q is at zero potential *w.r.t.* P, very small current flows in the circuit because of some electrons of high kinetic energy are reaching to plate Q, but this current has no practical utility.

(ii) If plate Q is kept at positive potential *w.r.t.* P current starts flowing through the circuit because more electrons are able to reach upto plate Q.

(iii) As the positive potential of plate Q increases, current through the circuit increases but after some time constant current flows through the circuit even positive potential of plate Q is still increasing, because at this condition all the electrons emitted from plate P are already reached up to plate Q. This constant current is called **saturation current.**

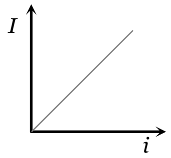
(iv) To increase the photoelectric current further we will have to increase the intensity of incident light.

Photoelectric current (i) depends upon

(a) Potential difference between electrodes (till saturation)

(b) Intensity of incident light (I)

(c) Nature of surface of metal



(v) To decrease the photoelectric current plate Q is maintained at negative potential *w.r.t.* P, as the anode Q is made more and more negative, fewer and fewer electrons will reach the cathode and the photoelectric current decreases.

(vi) At a particular negative potential of plate Q no electron will reach the plate Q and the current will become zero, this negative potential is called **stopping potential** denoted by V_0.

(vii) If we increase further the energy of incident light, kinetic energy of photoelectrons increases and more negative potential should be applied to stop the electrons to reach upto plate Q. Hence $\boldsymbol{eV_0 = K_{max}}$.

Note : ❑ Stopping potential depends only upon frequency or wavelength or energy of incident radiation. It doesn't depend upon intensity of light.

We must remember that intensity of incident light radiation is inversely proportional to the square of distance between source of light and photosensitive plate P *i.e.*, $I \propto \frac{1}{d^2}$ so $I \propto i \propto \frac{1}{d^2}$)

Important formulae

$\Rightarrow h\nu = h\nu_0 + K_{\max}$

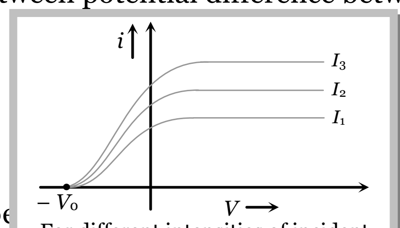
$$\Rightarrow K_{\max} = eV_0 = h(\nu - \nu_0) \quad \Rightarrow \frac{1}{2} m v_{\max}^2 = h(\nu - \nu_0) \quad \Rightarrow v_{\max} = \sqrt{\frac{2h(\nu - \nu_0)}{m}}$$

$$\Rightarrow K_{\max} = \frac{1}{2} m v_{\max}^2 = eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = hc\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right) \Rightarrow v_{\max} = \sqrt{\frac{2hc}{m}\frac{(\lambda - \lambda_0)}{\lambda\lambda_0}}$$

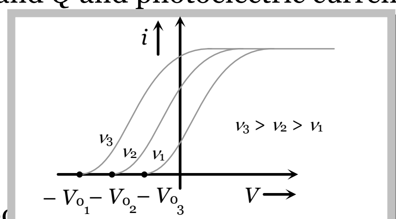
$$\Rightarrow V_0 = \frac{h}{e}(\nu - \nu_0) = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = 12375\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

(4) **Different graphs**

(i) Graph between potential difference between the plates P and Q and photoelectric current

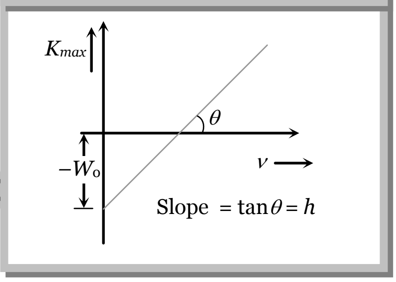


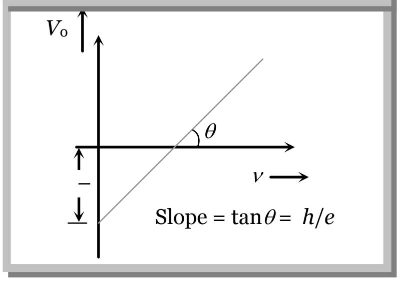
For different intensities of incident light



For different Frequencies of incident light

(ii) Graph be… rgy / stopping p… d frequency of incident light

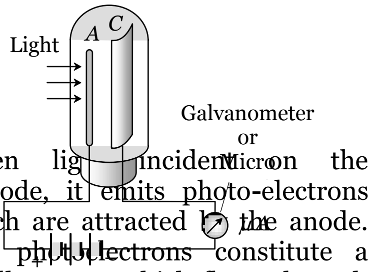
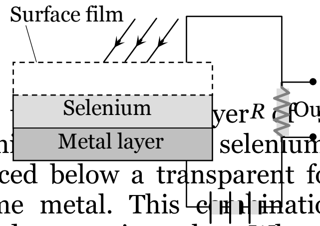
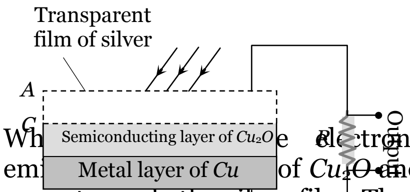




Photoelectric C

A device which converts light energy into electrical energy is called photoelectric cell. It is also known as photocell or electric eye.

Photoelectric cell are mainly of three types

Photo-emissive cell	Photo-conductive cell	Photo-voltaic cell
It consists of an evacuated glass or quartz bulb containing anode *A* and cathode *C*. The cathode is semi-cylindrical metal on which a layer of photo-sensitive material is coated.  When light [illegible] incident on the cathode, it emits photo-electrons which are attracted [illegible] the anode. The photoelectrons constitute a small current which flows through the external circuit.	It is based on the principle that conductivity of a semiconductor increases with increase in the intensity of incident light.  In [illegible] layer of some semi[illegible] selenium) is placed below a transparent foil of some metal. This [illegible] combination is fixed over an iron plate. When light is incident on the transparent foil, the electrical resistance of the semiconductor layer is reduced. Hence a current starts flowing in the battery circuit connected.	It consists of a *Cu* plate coated with a thin layer of cuprous oxide (Cu_2O). On this plate is laid a semi transparent thin film of silver.  Wh[illegible]e electrons emi[illegible] of Cu_2O and move towards the silver film. Then the silver film becomes negatively charged and copper plate becomes positively charged. A potential difference is set up between these two and current is set up in the external resistance.

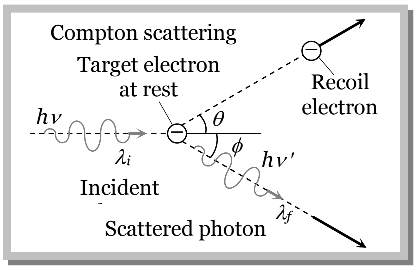
Note : ❑ The photoelectric current can be increased by filling some inert gas like Argon into the bulb. The photoelectrons emitted by cathode ionise the gas by collision and hence the current is increased.

Compton effect

The scattering of a photon by an electron is called Compton effect. The energy and momentum is conserved. Scattered photon will have less energy (more wavelength) as compare to incident photon (less wavelength). The energy lost by the photon is taken by electron as kinetic energy.

The change in wavelength due to Compton effect is called Compton shift. Compton shift

$$\lambda_f - \lambda_i = \frac{h}{m_0 c}(1 - \cos\theta)$$



Note : ❑ Compton effect shows

X-rays

X-rays was discovered by scientist Rontgen that's why they are also called Rontgen rays.

Rontgen discovered that when pressure inside a discharge tube kept 10^{-3} *mm* of *Hg* and potential difference is 25 *kV* then some unknown radiations (X-rays) are emitted by anode.

(1) **Production of X-rays**

There are three essential requirements for the production of X-rays

(i) A source of electron

(ii) An arrangement to accelerate the electrons

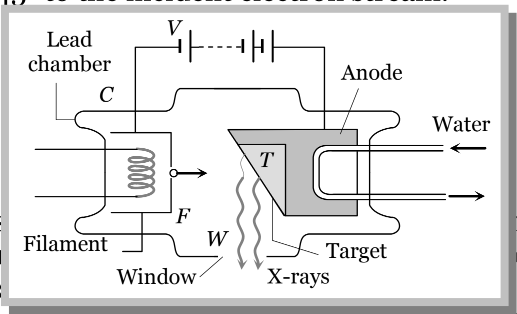
(iii) A target of suitable material of high atomic weight and high melting point on which these high speed electrons strike.

(2) **Coolidge X-ray tube**

It consists of a highly evacuated glass tube containing cathode and target. The cathode consist of a tungsten filament. The filament is coated with oxides of barium or strontium to have an emission of electrons even at low temperature. The filament is surrounded by a molybdenum cylinder kept at negative potential *w.r.t.* the target.

The target (it's material of high atomic weight, high melting point and high thermal conductivity) made of tungsten or molybdenum is embedded in a copper block.

The face of the target is set at 45° to the incident electron stream.



The filament is heated by pass ntial difference (≈ 10 kV to 80 kV) is applied between the target and cath e emitted by filament. The stream of highly energetic electrons are focus

Most of the energy of the electrons is converted into heat (above 98%) and only a fraction of the energy of the electrons (about 2%) is used to produce X-rays.

During the operation of the tube, a huge quantity of heat is produced in this target, this heat is conducted through the copper anode to the cooling fins from where it is dissipated by radiation and convection.

(i) **Control of intensity of X-rays** : Intensity implies the number of X-ray photons produced from the target. The intensity of X-rays emitted is directly proportional to the electrons emitted per second from the filament and this can be increased by increasing the filament current. So *intensity of X-rays* $\propto$ *Filament current*

(ii) **Control of quality or penetration power of *X*-rays :** Quality of X-rays implies the penetrating power of X-rays, which can be controlled by varying the potential difference between the cathode and the target.

For large potential difference, energy of bombarding electrons will be large and hence larger is the penetration power of X-rays.

Depending upon the penetration power, X-rays are of two types

Hard X-rays	Soft X-rays
More penetration power	Less penetration power
More frequency of the order of ≈ 10^{19} Hz	Less frequency of the order of ≈ 10^{16} Hz
Lesser wavelength range (0.1Å – 4Å)	More wavelength range (4Å – 100Å)

Note : ❑ Production of X-ray is the reverse phenomenon of photoelectric effect.

(3) **Properties of X-rays**

(i) X-rays are electromagnetic waves with wavelength range 0.1Å – 100Å.

(ii) The wavelength of X-rays is very small in comparison to the wavelength of light. Hence they carry much more energy (This is the only difference between X-rays and light)

(iii) X-rays are invisible.

(iv) They travel in a straight line with speed of light.

(v) X-rays are measured in Rontgen (measure of ionization power).

(vi) X-rays carry no charge so they are not deflected in magnetic field and electric field.

(vii) $\lambda_{Gama\ rays} < \lambda_{X\text{-}rays} < \lambda_{UV\ rays}$

(viii) They used in the study of crystal structure.

(ix) They ionise the gases

(x) X-rays do not pass through heavy metals and bones.

(xi) They affect photographic plates.

(xii) Long exposure to X-rays is injurious for human body.

(xiii) Lead is the best absorber of X-rays.

(xiv) For X-ray photography of human body parts, $BaSO_4$ is the best absorber.

(xv) They produce photoelectric effect and Compton effect

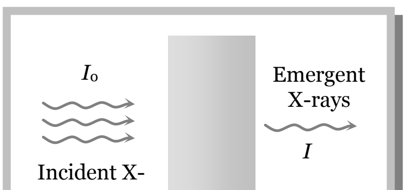
(xvi) X-rays are not emitted by hydrogen atom.

(xvii) These cannot be used in Radar because they are not reflected by the target.

(xviii) They show all the important properties of light rays like; reflection, refraction, interference, diffraction and polarization *etc*.

(4) **Absorption of X-rays**

X-rays are absorbed when they incident on substance.



Intensity of emergent X-rays $I = I_0 e^{-\mu x}$

So intensity of absorbed X-rays $I' = I_0 - I = I_0(1 - e^{-\mu x})$

where x = thickness of absorbing medium, μ = absorption coefficient

Note : ❑ The thickness of medium at which intensity of emergent X-rays becomes half *i.e.* $I' = \frac{I_0}{2}$ is called half value thickness ($x_{1/2}$) and it is given as $x_{1/2} = \frac{0.693}{\mu}$.

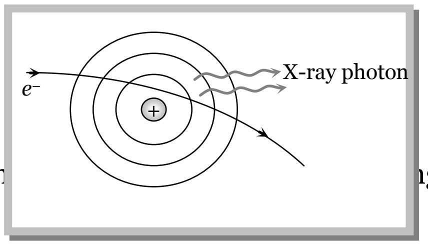
Classification of X-rays

In X-ray tube, when high speed electrons strikes the target, they penetrate the target. They loses their kinetic energy and comes to rest inside the metal. The electron before finally being stopped makes several collisions with the atoms in the target. At each collision one of the following two types of X-rays may get form.

(1) **Continuous X-rays**

As an electron passes close to the positive nucleus of atom, the electron is deflected from it's path as shown in figure. This results in deceleration of the electron. The loss in energy of the electron during deceleration is emitted in the form of X-rays.

The X-ray photons emitted so form the continuous X-ray spectrum.



Note : ❑ Continuos X-rays are produced due to the phen ng". It means slowing down or braking radiation.

Minimum wavelength

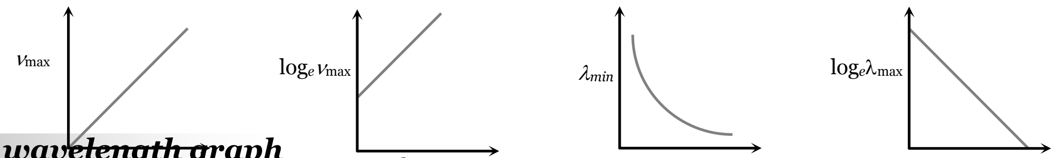
When the electron looses whole of it's energy in a single collision with the atom, an X-ray photon of maximum energy $h\nu_{max}$ is emitted *i.e.* $\frac{1}{2}mv^2 = eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$

where v = velocity of electron before collision with target atom, V = potential difference through which electron is accelerated, c = speed of light = 3×10^8 *m/s*

Maximum frequency of radiations (X-rays) $\nu_{max} = \frac{eV}{h}$

Minimum wave length = cut off wavelength of X-ray $\lambda_{min} = \frac{hc}{eV} = \frac{12375}{V}$ Å

Note : ❑ Wavelength of continuous X-ray photon ranges from certain minimum (λ_{min}) to infinity.

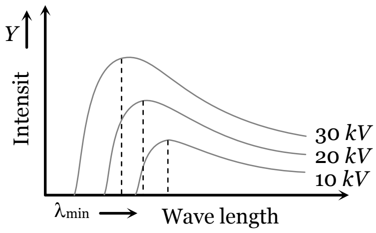


Intensity wavelength graph

The continuous X-ray spectra consist of all the wavelengths over a given range. These wavelength are of different intensities. Following figure shows the intensity variation of different wavelengths for various accelerating voltages applied to X-ray tube.

For each voltage, the intensity curve starts at a particular minimum wavelength (λ_{min}). Rises rapidly to a maximum and then drops gradually.

The wavelength at which the intensity is maximum depends on the accelerating voltage, being shorter for higher voltage and vice-versa.



(2) **Characteristic X-rays**

Few of the fast moving electrons having high velocity penetrate the surface atoms of the target material and knock out the tightly bound electrons even from the inner most shells of the atom. Now when the electron is knocked out, a vacancy is created at that place. To fill this vacancy electrons from higher shells jump to fill the created vacancies, we know that when an electron jumps from a higher energy orbit E_1 to lower energy orbit E_2, it radiates energy ($E_1 - E_2$). Thus this energy difference is radiated in the form of X-rays of very small but definite wavelength which depends upon the target material. The X-ray spectrum consist of sharp lines and is called characteristic X-ray spectrum.

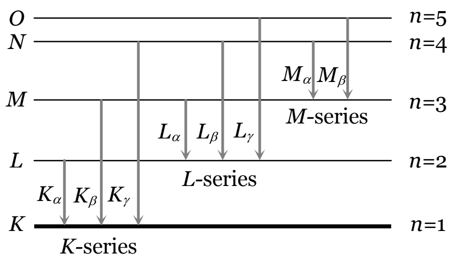
e⁻

X-ray photon

e⁻

e⁻

K, L, M, ... series



If the electron striking the target eject an electron from the *K*-shell of the atom, a vacancy is crated in the *K*-shell. Immediately an electron from one of the outer shell, say *L*-shell jumps to the *K*-shell, emitting an X-ray photon of energy equal to the energy difference between the two shells. Similarly, if an electron from the *M*-shell jumps to the *K*-shell, X-ray photon of higher energy is emitted. The X-ray photons emitted due to the jump of electron from the *L*, *M*, *N* shells to the K-shells gives K_α, K_β, K_γ lines of the *K*-series of the spectrum.

If the electron striking the target ejects an electron from the *L*-shell of the target atom, an electron from the *M*, *N* ..... shells jumps to the *L*-shell so that X-rays photons of lesser energy are emitted. These photons form the lesser energy emission. These photons form the *L*-series of the spectrum. In a similar way the formation of *M* series, *N* series *etc.* may be explained.

Energy and wavelength of different lines

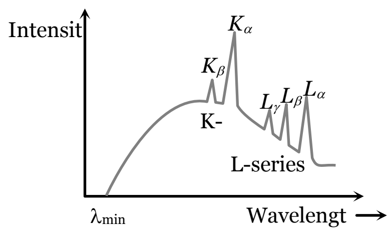
Series	Transition	Energy	Wavelength
K_α	$\underset{(2)}{L} \rightarrow \underset{(1)}{K}$	$E_L - E_K = h\nu_{K\alpha}$	$\lambda_{K\alpha} = \dfrac{hc}{E_L - E_K} = \dfrac{12375}{(E_L - E_K)eV}$ Å
K_β	$\underset{(3)}{M} \rightarrow \underset{(1)}{K}$	$E_M - E_K = h\nu_{K\beta}$	$\lambda_{K\beta} = \dfrac{hc}{E_M - E_K} = \dfrac{12375}{(E_M - E_K)eV}$ Å
L_α	$\underset{(3)}{M} \rightarrow \underset{(2)}{L}$	$E_M - E_L = h\nu_{L\alpha}$	$\lambda_{L\alpha} = \dfrac{hc}{E_M - E_L} = \dfrac{12375}{(E_M - E_L)eV}$ Å

Note : ❑ The wavelength of characteristic X-ray doesn't depend on accelerating voltage. It depends on the atomic number (*Z*) of the target material.

❑ $\lambda_{K\alpha} < \lambda_{L\alpha} < \lambda_{M\alpha}$ and $\nu_{K\alpha} > \nu_{L\alpha} > \nu_{M\alpha}$

❑ $\lambda_{K\alpha} > \lambda_{L\beta} < \lambda_{K\gamma}$

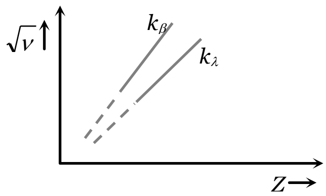
Intensity-wavelength graph



At certain sharply defined wavelengths, the intensity of X-rays is very large

as marked K_α, K_β .... As shown in figure. These X-rays are known as characteristic X-rays. At other wavelengths the intensity varies gradually and these X-rays are called continuous X-rays.

Mosley's law



Mosley studied the characteristic X-ray spectrum of a number of a heavy elements and concluded that the spectra of different elements are very similar and with increasing atomic number, the spectral lines merely shift towards higher frequencies.

He also gave the following relation $\sqrt{\nu} = a(Z - b)$

where ν = Frequency of emitted line, Z = Atomic number of target, a = Proportionality constant, b = Screening constant.

Note : ❑ a and b doesn't depend on the nature of target. Different values of b are as follows

$b = 1$ for K-series

$b = 7.4$ for L-series

$b = 19.2$ for M-series

- $(Z - b)$ is called effective atomic number.

More about Mosley's law

(i) It supported Bohr's theory

(ii) It experimentally determined the atomic number (Z) of elements.

(iii) This law established the importance of ordering of elements in periodic table by atomic number and not by atomic weight.

(iv) Gaps in Moseley's data for A = 43, 61, 72, 75 suggested existence of new elements which were later discovered.

(v) The atomic numbers of Cu, Ag and Pt were established to be 29, 47 and 78 respectively.

(vi) When a vacancy occurs in the K-shell, there is still one electron remaining in the K-shell. An electron in the L-shell will feel an effective charge of $(Z - 1)e$ due to $+ Ze$ from the nucleus and $- e$ from the remaining K-shell electron, because L-shell orbit is well outside the K-shell orbit.

(vii) Wave length of characteristic spectrum $\frac{1}{\lambda} = R(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ and energy of X-ray radiations.

$$\Delta E = h\nu = \frac{hc}{\lambda} = Rhc(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(viii) If transition takes place from $n_2 = 2$ to $n_1 = 1$ (K_α - line)

(a) $a = \sqrt{\frac{3RC}{4}} = 2.47 \times 10^{15}\ Hz$

(b) $\nu_{K\alpha} = RC(Z-1)^2\left(1 - \frac{1}{2^2}\right) = \frac{3RC}{4}(Z-1)^2 = 2.47 \times 10^{15}(Z-1)^2\ Hz$

(c) In general the wavelength of all the K-lines are given by $\frac{1}{\lambda_K} = R(Z-1)^2\left(1 - \frac{1}{n^2}\right)$ where n = 2, 3, 4, ...

While for K_α line $\lambda_{K\alpha} = \frac{1216}{(Z-1)}\ Å$

(d) $E_{K\alpha} = 10.2(Z-1)^2\ eV$

Uses of X-rays

(i) In study of crystal structure : Structure of DNA was also determined using X-ray diffraction.

(ii) In medical science. (iii) In radiograph

(iv) In radio therapy (v) In engineering

(vi) In laboratories (vii) In detective department

(viii) In art the change occurring in old oil paintings can be examined by X-rays.

Concepts

- *Nearly all metals emits photoelectrons when exposed to UV light. But alkali metals like lithium, sodium, potassium, rubidium and cesium emit photoelectrons even when exposed to visible light.*
- *Oxide coated filament in vacuum tubes is used to emit electrons at relatively lower temperature.*
- *Conduction of electricity in gases at low pressure takes because colliding electrons acquire higher kinetic energy due to increase in mean free path.*
- *Kinetic energy of cathode rays depends on both voltage and work function of cathode.*
- *Photoelectric effect is due to the particle nature of light.*
- *Hydrogen atom does not emit X-rays because it's energy levels are too close to each other.*
- *The essential difference between X-rays and of γ-rays is that, γ-rays emits from nucleus while X-rays from outer part of atom.*
- *There is no time delay between emission of electron and incidence of photon i.e. the electrons are emitted out as soon as the light falls on metal surface.*
- *If light were wave (not photons) it will take about an year take about an year to eject a photoelectron out of the metal surface.*
- *Doze of X-ray are measured in terms of produced ions or free energy via ionisaiton.*
- *Safe doze for human body per week is one Rontgen (One Rontgon is the amount of X-rays which emits 2.5×10^4 J free rough ionization of 1 gm air at NTP*

Example

Example: 22 The work function of a substance is 4.0 *eV*. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **[AIEEE 2004]**

(a) 540 *nm* (b) 400 *nm* (c) 310 *nm* (d) 220 *nm*

Solution : (c) By using $\lambda_0 = \frac{12375}{W_0(eV)} \Rightarrow \lambda_0 = \frac{12375}{4} = 3093.7\,Å \simeq 310\,nm$

Example: 23 Photo-energy 6 *eV* are incident on a surface of work function 2.1 *eV*. What are the stopping potential **[MP PMT 2004]**

(a) $-5V$ (b) $-1.9\ V$ (c) $-3.9\ V$ (d) $-8.1\ V$

Solution : (c) By using Einstein's equation $E = W_0 + K_{max} \Rightarrow 6 = 2.1 + K_{\max} \Rightarrow K_{\max} = 3.9\,eV$

Also $V_0 = -\frac{K_{\max}}{\rho} = -3.9\ V.$

Example: 24 When radiation of wavelength λ is incident on a metallic surface the stopping potential is 4.8 *volts*. If the same surface is illuminated with radiation of double the wavelength, then the stopping potential becomes 1.6 *volts*. Then the threshold wavelength for the surface is

(a) 2λ (b) 4λ (c) 6λ (d) 8λ

Solution : (b) By using $V_0 = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$

$$4.8 = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \quad ......(i) \quad \text{and} \quad 1.6 = \frac{hc}{e}\left[\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right] \quad ......(ii)$$

From equation (i) and (ii) $\lambda_0 = 4\lambda$.

Example: 25 When radiation is incident on a photoelectron emitter, the stopping potential is found to be 9 volts. If e/m for the electron is $1.8 \times 10^{11}\,Ckg^{-1}$ the maximum velocity of the ejected electrons is

(a) $6 \times 10^5 ms^{-1}$ (b) $8 \times 10^5 ms^{-1}$ (c) $1.8 \times 10^6 ms^{-1}$ (d) $1.8 \times 10^5 ms^{-1}$

Solution : (c) $\frac{1}{2} m v_{\max}^2 = eV_0 \Rightarrow v_{\max} = \sqrt{2\left(\frac{e}{m}\right).V_0} = \sqrt{2 \times 1.8 \times 10^{11} \times 9} = 1.8 \times 10^6\ m/s.$

Example: 26 The lowest frequency of light that will cause the emission of photoelectrons from the surface of a metal (for which work function is 1.65 *eV*) will be **[JIPMER 2002]**

(a) $4 \times 10^{10}\,Hz$ (b) $4 \times 10^{11}\,Hz$ (c) $4 \times 10^{14}\,Hz$ (d) $4 \times 10^{-10}\,Hz$

Solution : (c) Threshold wavelength $\lambda_0 = \frac{12375}{W_0(eV)} = \frac{12375}{1.65} = 7500\ Å.$

$\therefore$ so minimum frequency $\nu_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{7500 \times 10^{-10}} = 4 \times 10^{14}\ Hz.$

***Example*: 27** Light of two different frequencies whose photons have energies 1 *eV* and 2.5 *eV* respectively, successively illuminates a metal of work function 0.5 *eV*. The ratio of maximum kinetic energy of the emitted electron will be

(a) 1 : 5 (b) 1 : 4 (c) 1 : 2 (d) 1 : 1

Solution : (b) By using $K_{\max} = E - W_0 \Rightarrow \frac{(K_{\max})_1}{(K_{\max})_2} = \frac{1-0.5}{2.5-0.5} = \frac{0.5}{2} = \frac{1}{4}$.

***Example*: 28** Photoelectric emission is observed from a metallic surface for frequencies ν_1 and ν_2 of the incident light rays $(\nu_1 > \nu_2)$. If the maximum values of kinetic energy of the photoelectrons emitted in the two cases are in the ratio of 1 : *k*, then the threshold frequency of the metallic surface is **[EAMCET (Engg.) 2001]**

(a) $\frac{\nu_1 - \nu_2}{k-1}$ (b) $\frac{k\nu_1 - \nu_2}{k-1}$ (c) $\frac{k\nu_2 - \nu_1}{k-1}$ (d) $\frac{\nu_2 - \nu_1}{k-1}$

Solution : (b) By using $h\nu - h\nu_0 = k_{\max} \Rightarrow h(\nu_1 - \nu_0) = k_1$ and $h(\nu_1 - \nu_0) = k_2$

Hence $\frac{\nu_1 - \nu_0}{\nu_2 - \nu_0} = \frac{k_1}{k_2} = \frac{1}{k} \Rightarrow \nu_0 = \frac{k\nu_1 - \nu_2}{k-1}$

***Example*: 29** Light of frequency $8 \times 10^{15}\,Hz$ is incident on a substance of photoelectric work function 6.125 *eV*. The maximum kinetic energy of the emitted photoelectrons is **[AFMC 2001]**

(a) 17 *eV* (b) 22 *eV* (c) 27 *eV* (d) 37 *eV*

Solution : (c) Energy of incident photon $E = h\nu = 6.6 \times 10^{-34} \times 8 \times 10^{15} = 5.28 \times 10^{-18}\,J = 33\,eV$.

From $E = W_0 + K_{\max} \Rightarrow K_{\max} = E - W_0 = 33 - 6.125 = 26.87\,eV \approx 27\,eV$.

***Example*: 30** A photo cell is receiving light from a source placed at a distance of 1 *m*. If the same source is to be placed at a distance of 2 *m*, then the ejected electron **[MNR 1986; UPSEAT 2000, 2001]**

(a) Moves with one-fourth energy as that of the initial energy
(b) Moves with one fourth of momentum as that of the initial momentum
(c) Will be half in number
(d) Will be one-fourth in number

Solution : (d) Number of photons $\propto$ Intensity $\propto \frac{1}{(\text{distance})^2}$

$\Rightarrow \frac{N_1}{N_2} = \left(\frac{d_2}{d_1}\right)^2 \Rightarrow \frac{N_1}{N_2} - \left(\frac{2}{1}\right)^2 \Rightarrow N_2 = \frac{N_1}{4}$.

***Example*: 31** When yellow light incident on a surface no electrons are emitted while green light can emit. If red light is incident on the surface then **[MNR 1998; MH CET 2000; MP PET 2000]**

(a) No electrons are emitted (b) Photons are emitted
(c) Electrons of higher energy are emitted (d) Electrons of lower energy are emitted

Solution : (a) $\lambda_{\text{Green}} < \lambda_{\text{Yellow}} < \lambda_{\text{Red}}$

According to the question λ_{Green} is the maximum wavelength for which photoelectric emission takes place. Hence no emission takes place with red light.

***Example*: 32** When a metal surface is illuminated by light of wavelengths 400 *nm* and 250 *nm* the maximum velocities of the photoelectrons ejected are *v* and 2*v* respectively. The work function of the metal is (*h* = Planck's constant, *c* = velocity of light in air) **[EMCET (Engg.) 2000]**

(a) $2hc \times 10^6\,J$ (b) $1.5hc \times 10^6\,J$ (c) $hc \times 10^6\,J$ (d) $0.5hc \times 10^6\,J$

Solution : (a) By using $E = W_0 + K_{\max} \Rightarrow \frac{hc}{\lambda} = W_0 + \frac{1}{2}mv^2$

$\frac{hc}{400 \times 10^{-9}} = W_0 + \frac{1}{2}mv^2$ ......(i) and $\frac{hc}{250 \times 10^{-9}} = W_0 + \frac{1}{2}m(2v)^2$ ......(ii)

From equation (i) and (ii) $W_0 = 2hc \times 10^6\,J$.

***Example*: 33** The work functions of metals *A* and *B* are in the ratio 1 : 2. If light of frequencies *f* and 2*f* are incident on the surfaces of *A* and *B* respectively, the ratio of the maximum kinetic energies of photoelectrons emitted is (*f* is greater than threshold frequency of *A*, 2*f* is greater than threshold frequency of *B*) **[EAMCET (Med.) 2000]**

(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

Solution : (b) By using $E = W_0 + K_{\max} \Rightarrow E_A = hf = W_A + K_A$ and $E_B = h(2f) = W_B + K_B$

So, $\frac{1}{2} = \frac{W_A + K_A}{W_B + K_B}$ ......(i) also it is given that $\frac{W_A}{W_B} = \frac{1}{2}$ ....(ii)

From equation (i) and (ii) we get $\frac{K_A}{K_B} = \frac{1}{2}$.

***Example*: 34** When a point source of monochromatic light is at a distance of 0.2*m* from a photoelectric cell, the cut-off voltage and the saturation current are 0.6 *volt* and 18 *mA* respectively. If the same source is placed 0.6 *m* away from the photoelectric cell, then **[IIT-JEE 1992; MP PMT 1999]**

(a) The stopping potential will be 0.2 V
(b) The stopping potential will be 0.6 V
(c) The saturation current will be 6 mA
(d) The saturation current will be 18 mA

Solution : (b) Photoelectric current (i) $\propto$ Intensity $\propto \frac{1}{(\text{distance})^2}$. If distance becomes 0.6 m (*i.e.* three times) so current becomes $\frac{1}{9}$ times *i.e.* 2mA.

Also stopping potential is independent of intensity *i.e.* it remains 0.6 V.

***Example*: 35** In a photoemissive cell with exciting wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be **[CBSE 1998]**

(a) $v(3/4)^{1/2}$ (b) $v(4/3)^{1/2}$ (c) Less then $v(4/3)^{1/2}$ (d) Greater then $v(4/3)^{1/2}$

Solution : (d) From $E = W_0 + \frac{1}{2}mv_{\max}^2 \Rightarrow v_{\max} = \sqrt{\frac{2E}{m} - \frac{2W_0}{m}}$ (where $E = \frac{hc}{\lambda}$)

If wavelength of incident light charges from λ to $\frac{3\lambda}{4}$ (decreases)

Let energy of incident light charges from E to E' and speed of fastest electron changes from v to v' then

$$v = \sqrt{\frac{2E}{m} - \frac{2W_0}{m}} \quad .....(i) \quad \text{and} \quad v' = \sqrt{\frac{2E'}{m} - \frac{2W_0}{m}} \quad ....(ii)$$

$$\text{As } E \propto \frac{1}{\lambda} \Rightarrow E' = \frac{4}{3}E \text{ hence } v' = \sqrt{\frac{2\left(\frac{4}{3}E\right)}{m} - \frac{2W_0}{m}} \Rightarrow v' = \left(\frac{4}{3}\right)^{1/2}\sqrt{\frac{2E}{m} - \frac{2W_0}{m\left(\frac{4}{3}\right)^{1/2}}}$$

$$\Rightarrow v' = \left(\frac{4}{3}\right)^{1/2} \quad X = \sqrt{\frac{2E}{m} - \frac{2W_0}{m\left(\frac{4}{3}\right)^{1/2}}} > v \text{ so } v' > \left(\frac{4}{3}\right)^{1/2} v.$$

***Example*: 36** The minimum wavelength of X-rays produced in a coolidge tube operated at potential difference of 40 kV is **[BCECE 2003]**

(a) 0.31Å (b) 3.1Å (c) 31Å (d) 311Å

Solution : (a) $\lambda_{\min} = \frac{12375}{40 \times 10^3} = 0.309\,Å \approx 0.31\,Å$

***Example*: 37** The X-ray wavelength of L_α line of platinum (Z = 78) is $1.30Å$. The X-ray wavelength of L_a line of Molybdenum (Z = 42) is **[EAMCET (Engg.) 2000]**

(a) 5.41Å (b) 4.20Å (c) 2.70Å (d) 1.35 Å

Solution : (a) The wave length of L_α line is given by $\frac{1}{\lambda} = R(z-7.4)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda \propto \frac{1}{(z-7.4)^2}$

Tricky example: 3

***Example*: 38** The cut off wavelength of continuous X-ray from two coolidge tubes operating at 30 kV but using different target materials (molybdenum Z= 42 and tungsten Z = 74) are

(a) 1Å, 3Å (b) 0.3 Å, 0.2 Å (c) 0.414 Å, 0.8 Å (d) 0.414 Å, 0.414 Å

Solution : (d) Cut off wavelength of continuous X-rays depends solely on the voltage applied and does not depend on the material of the target. Hence the two tubes will have the same cut off wavelength.

$$Ve = h\nu = \frac{hc}{\lambda} \quad \text{or} \quad \lambda = \frac{hc}{Ve} = \frac{6.627 \times 10^{-34} \times 3 \times 10^8}{30 \times 10^3 \times 1.6 \times 10^{-19}} m = 414 \times 10^{-10} m = 0.414\,Å.$$

Tricky example: 4

Two photons, each of energy $2.5 eV$ are simultaneously incident on the metal surface. If the work function of the metal is $4.5\ eV$, then from the surface of metal

(a) Two electrons will be emitted
(b) Not even a single electron will be emitted
(c) One electron will be emitted
(d) More than two electrons will be emitted

Solution : (b) Photoelectric effect is the phenomenon of one to one elastic collision between incident photon and an electron. Here in this question one electron absorbs one photon and gets energy $2.5\ eV$ which is lesser than $4.5\ eV$. Hence no photoelectron emission takes place.

Tricky example: 5

In X-ray tube when the accelerating voltage V is halved, the difference between the wavelength of K_α line and minimum wavelength of continuous X-ray spectrum

(a) Remains constant
(b) Becomes more than two times
(c) Becomes half
(d) Becomes less than two times

Solution : (c) $\Delta\lambda = \lambda_{K_\alpha} - \lambda_{\min}$ when V is halved $\lambda_{\min}$ becomes two times but λ_{K_α} remains the same.

$\therefore \quad \Delta\lambda' = \lambda_{K_\alpha} - 2\lambda_{\min} = 2(\Delta\lambda) - \lambda_{K_a}$

$\Delta\lambda'$ [illegible] $2(\Delta\lambda)$

Tricky example: 6

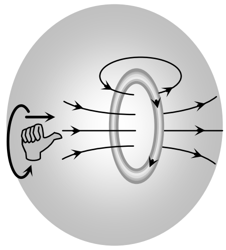
Molybdenum emits K_α-photons of energy $18.5\ keV$ and iron emits K_α photons of energy $34.7\ keV$. The times taken by a molybdenum K_α photon and an iron K_α photon to travel 300 m are

(a) (3 μs, 15 μs)
(b) (15 μs, 3μs)
(c) (1 μs, 1 μs)
(d) (1 μs, 5μs)

Solution : (c) Photon have the same speed whatever be their energy, frequency, wavelength, and origin.

$\therefore$ time of travel of either photon $= \dfrac{300}{3\times 10^{8}} = 10^{-6} s = 1 \mu s$

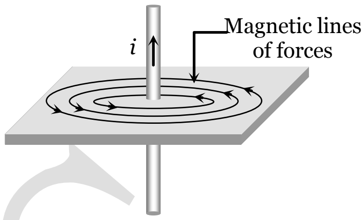
Magnetic Effect of Current



Oersted found that a magnetic field is established around a current carrying conductor.

Magnetic field exists as long as there is current in the wire.

The direction of magnetic field was found to be changed when direction of current was reversed.



Note : ❐ A moving charge produces magnetic as well as electric field, unlike a stationary charge which only produces electric field.

Biot Savart's Law

Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductors.

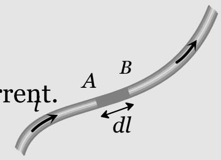
This law is although for infinitesimally small conductors yet it can be used for long conductors. In order to understand the Biot-Savart's law, we need to understand the term current-element.

Current element

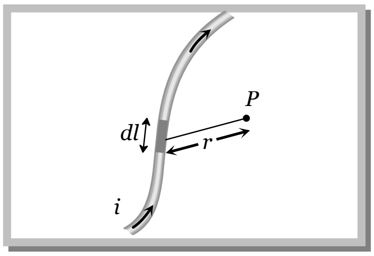
It is the product of current and length of infinitesimal segment of current carrying wire.

The current element is taken as a vector quantity. Its direction is same as the direction of current.

Current element $AB = i\vec{dl}$



In the figure shown below, there is a segment of current carrying wire and P is a point where magnetic field is to be calculated. $i\vec{dl}$ is a current element and r is the distance of the point 'P' with respect to the current element $i\vec{dl}$. According to Biot-Savart Law, magnetic field at point 'P' due to the current element $i\vec{dl}$ is given by the expression, $\boldsymbol{dB = k\dfrac{i\,dl\sin\theta}{r^2}}$ also $B = \int dB = \dfrac{\mu_0 i}{4\pi}.\int\dfrac{dl\sin\theta}{r^2}$



In C.G.S. : $k = 1 \Rightarrow dB = \dfrac{idl\sin\theta}{r^2}$ *Gauss*

In S.I. : $k = \dfrac{\mu_0}{4\pi} \Rightarrow dB = \dfrac{\mu_0}{4\pi}.\dfrac{idl\sin\theta}{r^2}$ *Tesla*

where μ_0 = Absolute permeability of air or vacuum $= 4\pi\times10^{-7}\dfrac{Wb}{Amp-metre}$. It's other units are $\dfrac{Henry}{metre}$ or $\dfrac{N}{Amp^2}$ or $\dfrac{Tesla-metre}{Ampere}$

(1) **Different forms of Biot-Savarts law**

Vector form	Biot-Savarts law in terms of current density	Biot-savarts law in terms of charge and it's velocity
Vectorially, $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\vec{l} \times \hat{r})}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\vec{l} \times \vec{r})}{r^3}$ $\Rightarrow$ Direction of $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$. This is given by right hand screw rule.	In terms of current density $d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \vec{r}}{r^3} dV$ where $j = \frac{i}{A} = \frac{idl}{Adl} = \frac{idl}{dV}$ = current density at any point of the element, dV = volume of element	In terms of charge and it's velocity, $d\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\vec{v} \times \vec{r})}{r^3}$ $\because \; id\vec{l} = \frac{q}{dt} d\vec{l} = q\frac{d\vec{l}}{dt} = q\vec{v}$

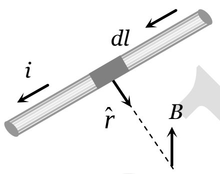
(2) **Similarities and differences between Biot-Savart law and Coulomb's Law**

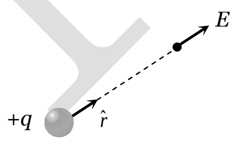
(i) The current element produces a magnetic field, whereas a point charge produces an electric field.

(ii) The magnitude of magnetic field varies as the inverse square of the distance from the current element, as does the electric field due to a point charge.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{l} \times \hat{r}}{r^2} \quad \text{Biot-Savart Law} \qquad \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \quad \text{Coulomb's Law}$$

(iii) The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element $d\vec{l}$ and the unit vector $\hat{r}$.

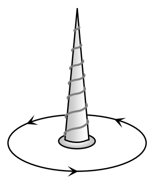




Direction of Magnetic Field

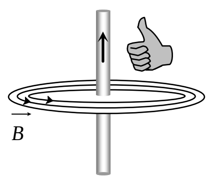
The direction of magnetic field is determined with the help of the following simple laws :

(1) **Maxwell's cork screw rule**



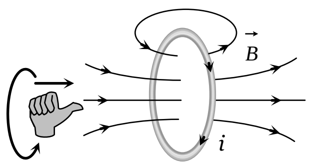
According to this rule, if we imagine a right handed screw placed along the current carrying linear conductor, be rotated such that the screw moves in the direction of flow of current, then the direction of rotation of the thumb gives the direction of magnetic lines of force.

(2) **Right hand thumb rule**



According to this rule if a current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of folding fingers will represent the direction of magnetic lines of force.

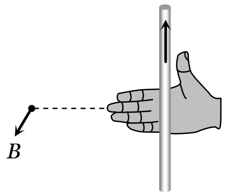
(3) **Right hand thumb rule of circular currents**



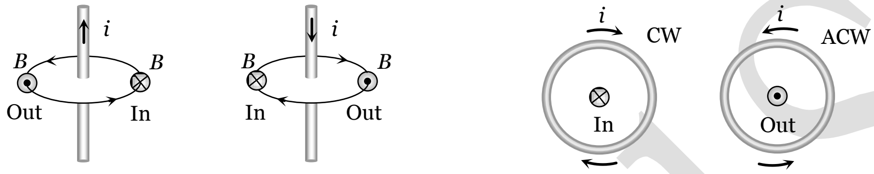
According to this rule if the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb.

(4) **Right hand palm rule**

If we stretch our right hand such that fingers point towards the point. At which magnetic field is required while thumb is in the direction of current then normal to the palm will show the direction of magnetic field.



Note : ❑ If magnetic field is directed perpendicular and into the plane of the paper it is represented by ⊗ (cross) while if magnetic field is directed perpendicular and out of the plane of the paper it is represented by ⊙ (dot)



In : Magnetic field is away from the observer or perpendicular inwards.

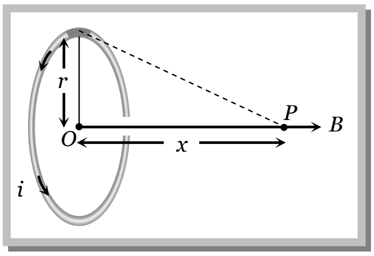
Out : Magnetic field is towards the observer or perpendicular outwards.

Application of Biot-Savarts Law

(1) **Magnetic field due to a circular current**

If a coil of radius r, carrying current i then magnetic field on it's axis at a distance x from its centre given by

$$B_{axis} = \frac{\mu_0}{4\pi}\cdot\frac{2\pi N i r^2}{(x^2+r^2)^{3/2}}$$; where N = number of turns in coil.



Different cases

***Case* 1 : Magnetic field at the centre of the coil**

(i) At centre $x = 0 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi}\cdot\frac{2\pi N i}{r} = \frac{\mu_0 N i}{2r} = B_{max}$

(ii) For single turn coil $N = 1 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{r} = \frac{\mu_0 i}{2r}$ (iii) In C.G.S. $\frac{\mu_0}{4\pi} = 1 \Rightarrow B_{centre} = \frac{2\pi i}{r}$

Note : ❑ $B_{centre} \propto N$ (i, r constant), $B_{centre} \propto i$ (N, r constant), $B_{centre} \propto \frac{1}{r}$ (N, i constant)

***Case* 2 : Ratio of B_{centre} and B_{axis}**

The ratio of magnetic field at the centre of circular coil and on it's axis is given by $\frac{B_{centre}}{B_{axis}} = \left(1+\frac{x^2}{r^2}\right)^{3/2}$

(i) If $x = \pm a,\ B_c = 2\sqrt{2}\, B_a$ $\quad x = \pm\frac{a}{2},\ B_c = \frac{5\sqrt{5}}{8} B_a$ $\quad x = \pm\frac{a}{\sqrt{2}},\ B_c = \left(\frac{3}{2}\right)^{3/2} B_a$

(ii) If $B_a = \frac{B_c}{n}$ then $x = \pm r\sqrt{(n^{2/3}-1)}$ and if $B_a = \frac{B_c}{\sqrt{n}}$ then $x = \pm r\sqrt{(n^{1/3}-1)}$

***Case* 3 : Magnetic field at very large/very small distance from the centre**

(i) If $x >> r$ (very large distance) $\Rightarrow B_{axis} = \frac{\mu_0}{4\pi}.\frac{2\pi Nir^2}{x^3} = \frac{\mu_0}{4\pi}.\frac{2NiA}{x^3}$ where $A = \pi r^2$ = Area of each turn of the coil.

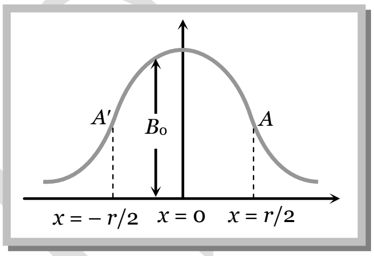
(ii) If $x << r$ (very small distance) $\Rightarrow B_{axis} \neq B_{centre}$, but by using binomial theorem and neglecting higher power of $\frac{x^2}{r^2}$; $B_{axis} = B_{centre}\left(1-\frac{3}{2}\frac{x^2}{r^2}\right)$

***Case 4* : B-x curve**

The variation of magnetic field due to a circular coil as the distance x varies as shown in the figure.

B varies non-linearly with distance x as shown in figure and is maximum when $x^2 = \min = 0$, *i.e.*, the point is at the centre of the coil and it is zero at $x = \pm\infty$.

Point of inflection (A and A') **:** Also known as points of curvature change or pints of zero curvature.



(i) At these points B varies linearly with $x \Rightarrow \frac{dB}{dx}$ = constant $\Rightarrow \frac{d^2B}{dx^2} = 0$.

(ii) They locates at $x = \pm\frac{r}{2}$ from the centre of the coil.

(iii) Separation between point of inflextion is equal to radius of coil (r)

(iv) Application of points of inflextion is "Hamholtz coils" arrangement.

Note : ❑ The magnetic field at $x = \frac{r}{2}$ is $B = \frac{4\mu_0 Ni}{5\sqrt{5}\,r}$

(2) **Helmholtz coils**

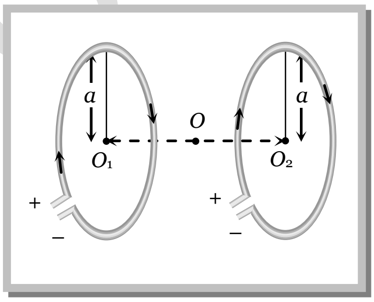
(i) This is the set-up of two coaxial coils of same radius such that distance between their centres is equal to their radius.

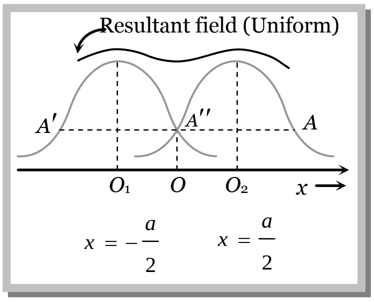
(ii) These coils are used to obtain uniform magnetic field of short range which is obtained between the coils.

(iii) At axial mid point O, magnetic field is given by $B = \frac{8\mu_0 Ni}{5\sqrt{5}R} = 0.716\frac{\mu_0 Ni}{R} = 1.432\,B$, where $B = \frac{\mu_0 Ni}{2R}$

(iv) Current direction is same in both coils otherwise this arrangement is not called Helmholtz's coil arrangement.

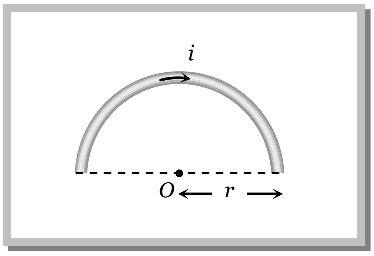
(v) Number of points of inflextion $\Rightarrow$ Three (A, A', A'')



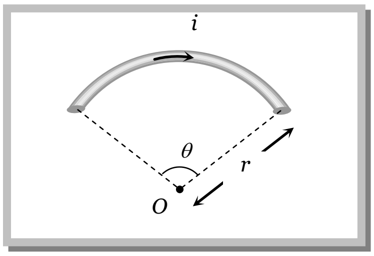


Note : ❑ The device whose working principle based on this arrangement and in which uniform magnetic field is used called as "Halmholtz galvanometer".

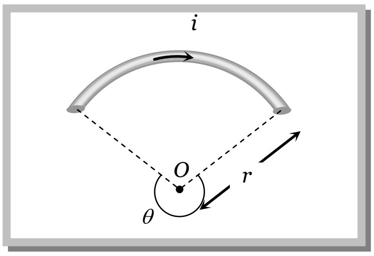
(3) **Magnetic field due to current carrying circular arc :** Magnetic field at centre O



$B = \frac{\mu_0}{4\pi}.\frac{\pi i}{r} = \frac{\mu_0 i}{4r}$



$B = \frac{\mu_0}{4\pi}.\frac{\theta i}{r}$



$B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \theta)i}{r}$

Special results

If magnetic field at the centre of circular coil is denoted by $B_0 \left(= \frac{\mu_0}{4\pi}.\frac{2\pi i}{r}\right)$

Magnetic field at the centre of arc which is making an angle θ at the centre is

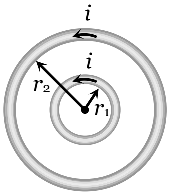
$B_{arc} = \left(\frac{B_0}{2\pi}\right).\theta$

Angle at centre	Magnetic field at centre in term of B_0
360° (2π)	B_0
180° (π)	$B_0 / 2$
120° ($2\pi/3$)	$B_0 / 3$
90° ($\pi/2$)	$B_0 / 4$
60° ($\pi/3$)	$B_0 / 6$
30° ($\pi/6$)	$B_0 / 12$

(4) **Concentric circular loops** ($N = 1$)

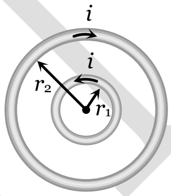
(i) Coplanar and concentric : It means both coils are in same plane with common centre

(a) Current in same direction



$B_1 = \frac{\mu_0}{4\pi} 2\pi i\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

(b) Current in opposite direction



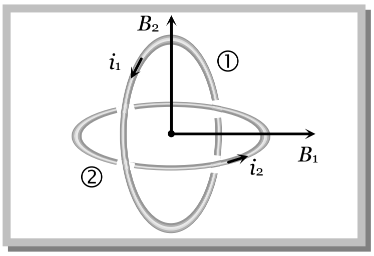
$B_2 = \frac{\mu_0}{4\pi} 2\pi i\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

Note : $\frac{B_1}{B_2} = \left(\frac{r_2 + r_1}{r_2 - r_1}\right)$

(ii) Non-coplanar and concentric : Plane of both coils are perpendicular to each other

Magnetic field at common centre

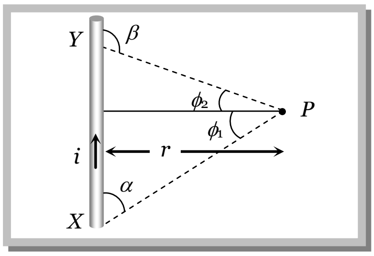
$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$



(5) **Magnetic field due to a straight current carrying wire**

Magnetic field due to a current carrying wire at a point P which lies at a perpendicular distance r from the wire as shown is given as

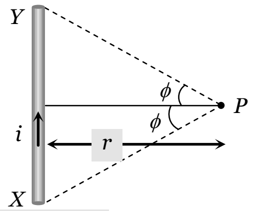
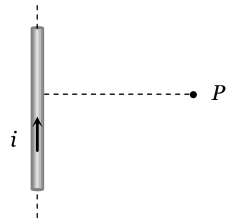
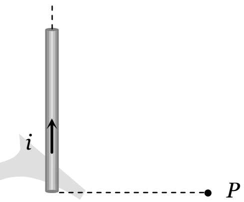
$B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$



From figure $\alpha = (90^o - \phi_1)$ and $\beta = (90^o + \phi_2)$

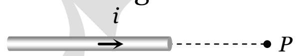
Hence $B = \frac{\mu_o}{4\pi}.\frac{i}{r}(\cos\alpha - \cos\beta)$

Different cases

***Case* 1 :** When the linear conductor XY is of finite length and the point P lies on it's perpendicular bisector as shown  $\phi_1 = \phi_2 = \phi$ So $B = \frac{\mu_0}{4\pi}.\frac{i}{r}(2\sin\phi)$	***Case* 2 :** When the linear conductor XY is of infinite length and the point P lies near the centre of the conductor  $\phi_1 = \phi_2 = 90^o$. So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 90^o] = \frac{\mu_0}{4\pi}\frac{2i}{r}$	***Case* 3 :** When the linear conductor is of infinite length and the point P lies near the end Y or X  $\phi_1 = 90^o$ and $\phi_2 = 0^o$. So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 0^o] = \frac{\mu_0}{4\pi}\frac{i}{r}$

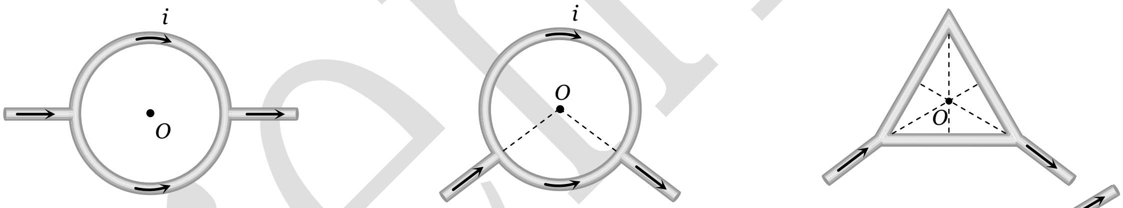
Note : ❒ When point P lies on axial position of current carrying conductor then magnetic field at P

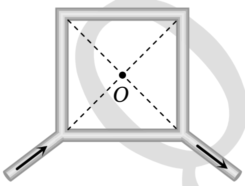
$B = 0$

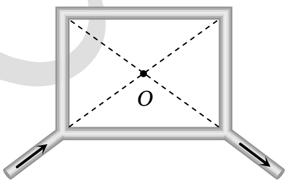


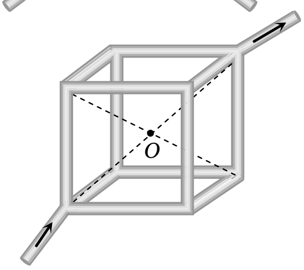
❒ The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction is zero.

(6) **Zero magnetic field :** If in a symmetrical geometry, current enters from one end and exists from the other, then magnetic field at the centre is zero.





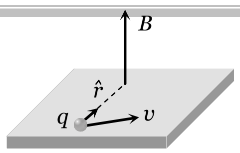




In all cases at centre $B = 0$

Concepts

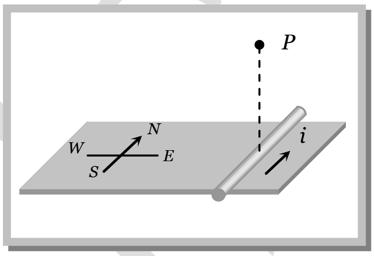
- *If a current carrying circular loop (n = 1) is turned into a coil having n identical turns then magnetic field at the centre of the coil becomes n² times the previous field i.e.* $B_{(n\ turn)} = n^2 B_{(single\ turn)}$
- *When a current carrying coil is suspended freely in earth's magnetic field, it's plane stays in* ***East-West*** *direction.*
- *Magnetic field (* $\vec{B}$ *) produced by a moving charge q is given by* $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$ *; where v = velocity of charge and v << c (speed of light).*



☞ *If an electron is revolving in a circular path of radius r with speed v then magnetic field produced at the centre of circular path* $B = \frac{\mu_0}{4\pi}.\frac{ev}{r^2}$.

Example

Example: 1 Current flows due north in a horizontal transmission line. Magnetic field at a point P vertically above it directed



(a) North wards

(b) South wards

(c) Toward east

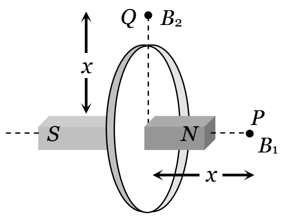
(d) Towards west

Solution : (c) By using right hand thumb rule or any other rule which helps to determine the direction of magnetic field.

Example: 2 Magnetic field due to a current carrying loop or a coil at a distant axial point P is B_1 and at an equal distance in it's plane is B_2 then $\frac{B_1}{B_2}$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

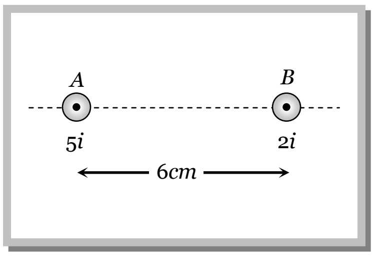
Solution : (a) Current carrying coil behaves as a bar magnet as shown in figure.



We also knows for a bar magnet, if axial and equatorial distance are same then $B_a = 2B_e$

Hence, in this equation $\frac{B_1}{B_2} = \frac{2}{1}$

Example: 3 Find the position of point from wire 'B' where net magnetic field is zero due to following current distribution



(a) 4 *cm*

(b) $\frac{30}{7}$ *cm*

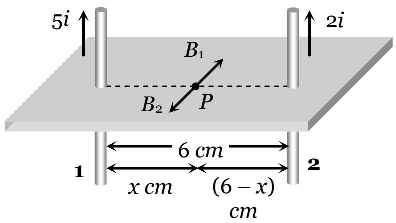
(c) $\frac{12}{7}$ *cm*

(d) 2 *cm*

Solution : (c) Suppose P is the point between the conductors where net magnetic field is zero.

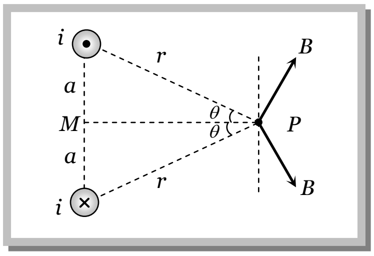
So at P |Magnetic field due to conductor 1| = |Magnetic field due to conductor 2|

i.e. $\frac{\mu_0}{4\pi}.\frac{2(5i)}{i} = \frac{\mu_0}{4\pi}.\frac{2(2i)}{(6-x)} \Rightarrow \frac{5}{x} = \frac{9}{6-x} \Rightarrow x = \frac{30}{7}\,cm$



Hence position from $B = 6 - \frac{30}{7} = \frac{12}{7}\,cm$

Example: 4 Find out the magnitude of the magnetic field at point P due to following current distribution



(a) $\frac{\mu_0 ia}{\pi r^2}$

(b) $\frac{\mu_0 ia^2}{\pi r}$

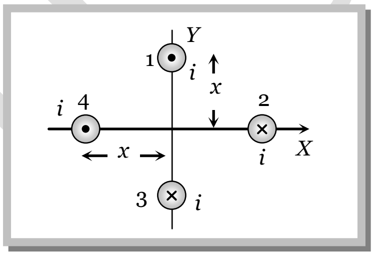
(c) $\frac{\mu_0 ia}{2\pi r^2}$

(d) $\frac{2\mu_0 ia}{\pi r^2}$

Solution : (a) Net magnetic field at P, $B_{net} = 2B\sin\theta$; where B = magnetic field due to one wire at $P = \frac{\mu_0}{4\pi}.\frac{2i}{r}$

and $\sin\theta = \frac{a}{r}$ $\therefore$ $B_{net} = 2\times\frac{\mu_0}{4\pi}.\frac{2i}{r}\times\frac{a}{r} = \frac{\mu_0 ia}{\pi r^2}$.

Example: 5 What will be the resultant magnetic field at origin due to four infinite length wires. If each wire produces magnetic field 'B' at origin



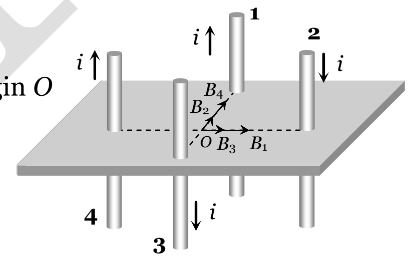
(a) $4\,B$

(b) $\sqrt{2}\,B$

(c) $2\sqrt{2}\,B$

(d) Zero

Solution : (c) Direction of magnetic field (B_1, B_2, B_3 and B_4) at origin due to wires 1, 2, 3 and 4 are shown in the following figure.



$B_1 = B_2 = B_3 = B_4 = \frac{\mu_0}{4\pi}.\frac{2i}{x} = B$. So net magnetic field at origin O

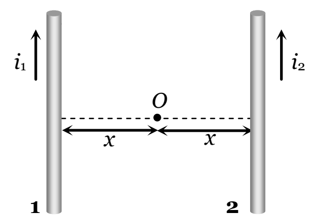
$$B_{net} = \sqrt{(B_1+B_2)^2 + (B_2+B_4)^2}$$

$$= \sqrt{(2B)^2 + (2B)^2} = 2\sqrt{2}B$$

Example: 6 Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 10 μT. If the direction of i_2 is reversed, the field becomes 30 μT. The ratio i_1/i_2 is

(a) 4 (b) 3 (c) 2 (d) 1

Solution : (c) Initially when wires carry currents in the same direction as shown.



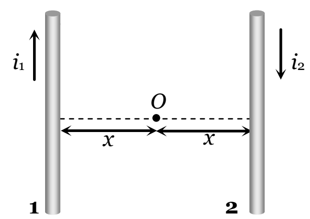
Magnetic field at mid point O due to wires 1 and 2 are respectively

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1}{x} \otimes \text{ and } B_2 = \frac{\mu_0}{4\pi}.\frac{2i_2}{x} \odot$$

Hence net magnetic field at O $B_{net} = \frac{\mu_0}{4\pi}\times\frac{2}{x}(i_1 - i_2)$

$$\Rightarrow\ 10\times10^{-6} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 - i_2) \quad .....(i)$$

If the direction of i_2 is reversed then



$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1}{x} \otimes \text{ and } B_2 = \frac{\mu_0}{4\pi}.\frac{2i_2}{x} \otimes$$

So $B_{net} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 + i_2) \Rightarrow 30 \times 10^{-6} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 + i_2)$ ......(ii)

Dividing equation (ii) by (i) $\frac{i_1 + i_2}{i_1 - i_2} = \frac{3}{1} \Rightarrow \frac{i_1}{i_2} = \frac{2}{1}$

Example: 7 A wire of fixed length is turned to form a coil of one turn. It is again turned to form a coil of three turns. If in both cases same amount of current is passed, then the ratio of the intensities of magnetic field produced at the centre of a coil will be

(a) 9 times of first case (b) $\frac{1}{9}$ times of first case (c) 3 times of first case (d) $\frac{1}{3}$ times of first case

Solution : (a) Magnetic field at the centre of n turn coil carrying current i $B = \frac{\mu_0}{4\pi}.\frac{2\pi n i}{r}$ ......(i)

For single turn $n = 1$ $B = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r}$ .....(ii)

If the same wire is turn again to form a coil of three turns *i.e.* $n = 3$ and radius of each turn $r' = \frac{r}{3}$

So new magnetic field at centre $B' = \frac{\mu_0}{4\pi}.\frac{2\pi(3)}{r'} \Rightarrow B' = 9 \times \frac{\mu_0}{4\pi}.\frac{2\pi i}{r}$ ......(iii)

Comparing equation (ii) and (iii) gives $B' = 9B$.

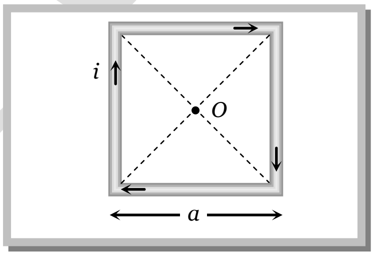
Example: 8 A wire in the form of a square of side a carries a current i. Then the magnetic induction at the centre of the square wire is (Magnetic permeability of free space = μ_0)

(a) $\frac{\mu_0 i}{2\pi a}$

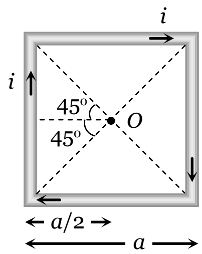
(b) $\frac{\mu_0 i\sqrt{2}}{\pi a}$

(c) $\frac{2\sqrt{2}\mu_0 i}{\pi a}$

(d) $\frac{\mu_0 i}{\sqrt{2}\pi a}$



Solution : (c) Magnetic field due to one side of the square at centre O



$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i \sin 45^o}{a/2}$$

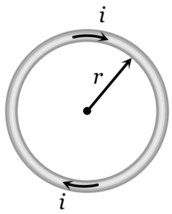
$$\Rightarrow B_1 = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\, i}{a}$$

Hence magnetic field at centre due to all side $B_{net} = 4B_1 = \frac{\mu_0(2\sqrt{2}\, i)}{\pi a}$

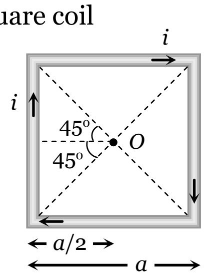
Example: 9 The ratio of the magnetic field at the centre of a current carrying circular wire and the magnetic field at the centre of a square coil made from the same length of wire will be

(a) $\frac{\pi^2}{4\sqrt{2}}$ (b) $\frac{\pi^2}{8\sqrt{2}}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{4\sqrt{2}}$

Solution : (b) Circular coil



Square coil



Length $L = 2\pi r$

Magnetic field $B = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} = \frac{\mu_0}{4\pi}.\frac{4\pi^2 i}{r}$

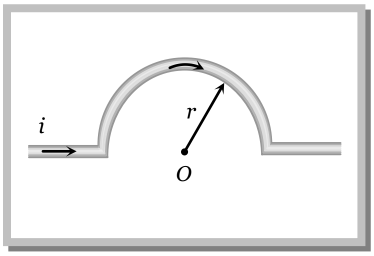
Hence $\frac{B_{circular}}{B_{square}} = \frac{\pi^2}{8\sqrt{2}}$

Length $L = 4a$

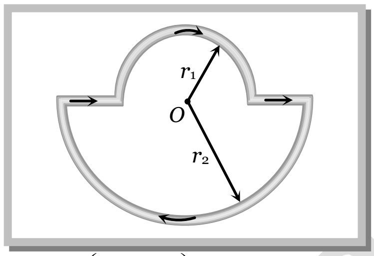
$B = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\,i}{a}$ $B = \frac{\mu_0}{4\pi}.\frac{8\sqrt{2}\,i}{a}$

***Example*: 10** Find magnetic field at centre O in each of the following figure

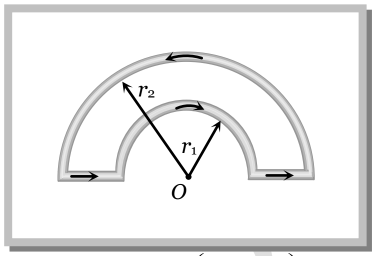
(i)



(ii)



(iii)



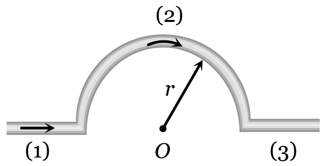
(i)
(a) $\frac{\mu_0 i}{r}\otimes$
(b) $\frac{\mu_0 i}{2r}\odot$
(c) $\frac{\mu_0 i}{4r}\otimes$
(d) $\frac{\mu_0 i}{4r}\odot$

(ii)
(a) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\otimes$
(b) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\otimes$
(c) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\odot$
(d) Zero

(iii)
(a) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\otimes$
(b) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\otimes$
(c) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\odot$
(d) Zero

Solution : (i) (c) Magnetic field at O due to parts 1 and 3, $B_1 = B_3 = 0$

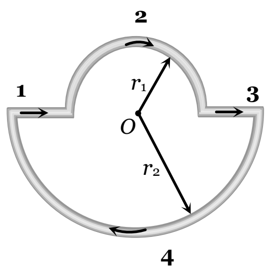
While due to part (2) $B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes$



$\therefore$ Net magnetic field at centre O,

$$B_{net} = B_1 + B_2 + B_3 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes \Rightarrow B_{net} = \frac{\mu_0 i}{4r}\otimes$$

(ii) (b) $B_1 = B_3 = 0$

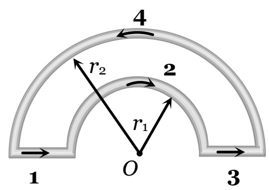


$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_1}\otimes$$

$$B_4 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_2}\otimes$$

So $B_{net} = B_2 + B_4 = \frac{\mu_0}{4\pi}.\pi i\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\otimes$

(iii) (a) $B_1 = B_3 = 0$



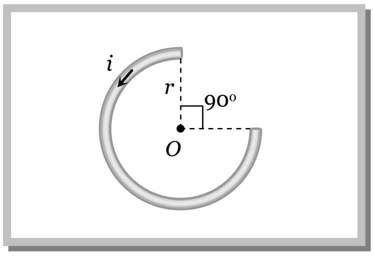
$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_1}\otimes$$

$$B_4 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_2}\otimes$$ As $|B_2| > |B_4|$

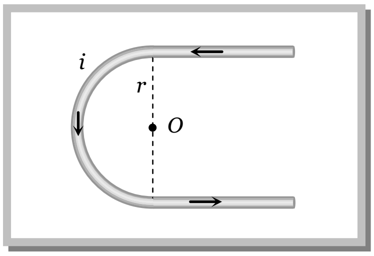
$$\text{So } B_{net} = B_2 - B_4 \Rightarrow B_{net} = \frac{\mu_0 i}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \otimes$$

Example: 11 Find magnetic field at centre O in each of each of the following figure

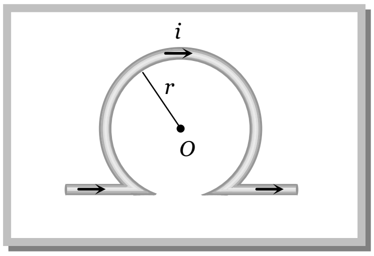
(i)



(ii)



(iii)



(i)
(a) $\frac{\mu_0 i}{2r} \odot$
(b) $\frac{\mu_0 i}{2r} \otimes$
(c) $\frac{3\mu_0 i}{8r} \otimes$
(d) $\frac{3\mu_0 i}{8r} \odot$

(ii)
(a) $\frac{\mu_0}{2\pi}\frac{i}{r}(\pi - 2) \otimes$
(b) $\frac{\mu_0 i}{4\pi}.\frac{i}{r}(\pi + 2) \odot$
(c) $\frac{\mu_0 i}{4r} \otimes$
(d) $\frac{\mu_0 i}{4r} \odot$

(iii)
(a) $\frac{\mu_0}{2r}\frac{2i}{r}(\pi + 1) \otimes$
(b) $\frac{\mu_0 i}{4r}.\frac{2i}{r}(\pi - 1) \otimes$
(c) Zero
(d) Infinite

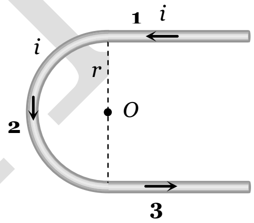
Solution : (i) (d) By using $B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \theta)i}{r} \Rightarrow B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \pi/2)i}{r} = \frac{3\mu_0 i}{8r} \odot$

(ii) (b) Magnetic field at centre O due to section 1, 2 and 3 are respectively

$$B_1 = \frac{\mu_0}{4\pi}.\frac{i}{r} \odot$$

$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r} \odot$$

$$B_3 = \frac{\mu_0}{4\pi}.\frac{i}{r} \odot$$

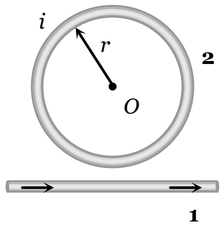


$$\Rightarrow B_{net} = B_1 + B_2 + B_3 = \frac{\mu_0}{4\pi}\frac{i}{r}(\pi + 2) \odot$$

(iii) (b) The given figure is equivalent to following figure, magnetic field at O due to long wire (part 1)

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i}{r} \odot$$

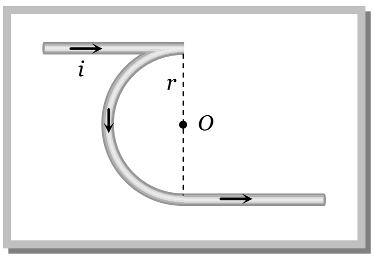
Due to circular coil $B_2 = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} \otimes$

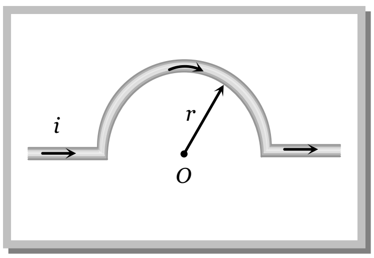


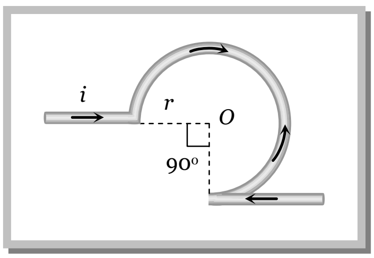
Hence net magnetic field at O

$$B_{net} = B_2 - B_1 = \frac{\mu_0}{4\pi}.\frac{2i}{r}(\pi - 1) \otimes$$

Example: 12 The field B at the centre of a circular coil of radius r is π times that due to a long straight wire at a distance r from it, for equal currents here shows three cases; in all cases the circular part has radius r and straight ones are infinitely long. For same current the field B is the centre P in cases 1, 2, 3 has the ratio**[CPMT 198**





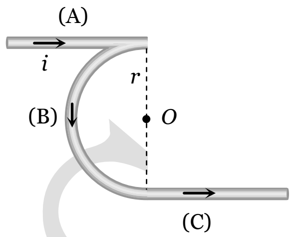


(1) **(2)** **(3)**

(a) $\left(-\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}-\frac{1}{2}\right)$ (b) $\left(-\frac{\pi}{2}+1\right):\left(\frac{\pi}{2}+1\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

(c) $-\frac{\pi}{2}:\frac{\pi}{2}:\frac{3\pi}{4}$ (d) $\left(-\frac{\pi}{2}-1\right):\left(\frac{\pi}{2}-\frac{1}{2}\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

Solution : (a) ***Case* 1 :** $B_A = \frac{\mu_0}{4\pi}.\frac{i}{r}\otimes$



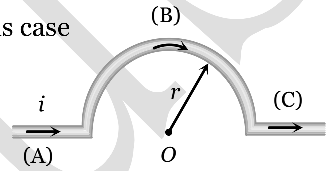
$$B_B = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$$

$$B_C = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$$

So net magnetic field at the centre of case 1

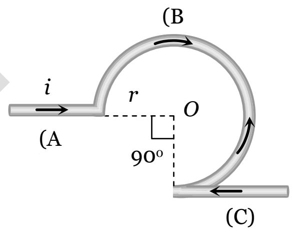
$$B_1 = B_B - (B_A + B_C) \quad \Rightarrow \quad B_1 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\odot \quad \text{..... (i)}$$

***Case* 2 :** As we discussed before magnetic field at the centre O in this case



$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes \quad \text{.....(ii)}$$

***Case* 3 :** $B_A = 0$



$$B_B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \pi/2)}{r}\otimes = \frac{\mu_0}{4\pi}.\frac{3\pi i}{2r}\otimes$$

$$B_C = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$$

So net magnetic field at the centre of case 3

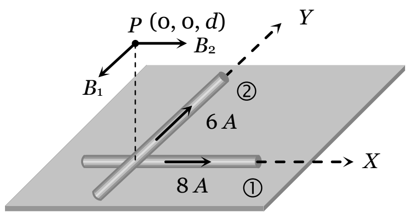
$$B_3 = \frac{\mu_0}{4\pi}.\frac{i}{r}\left(\frac{3\pi}{2}-1\right)\otimes \quad \text{....(iii)}$$

From equation (i), (ii) and (iii) $B_1 : B_2 : B_3 = \pi\odot : \pi\odot : \left(\frac{3\pi}{2}-1\right)\otimes = -\frac{\pi}{2}:\frac{\pi}{2}:\left(\frac{3\pi}{4}-\frac{1}{2}\right)$

***Example*: 13** Two infinite length wires carries currents 8A and 6A respectively and placed along X and Y-axis. Magnetic field at a point $P(0,0,d)m$ will be

(a) $\frac{7\mu_0}{\pi d}$ (b) $\frac{10\mu_0}{\pi d}$ (c) $\frac{14\mu_0}{\pi d}$ (d) $\frac{5\mu_0}{\pi d}$

Solution : (d) Magnetic field at P

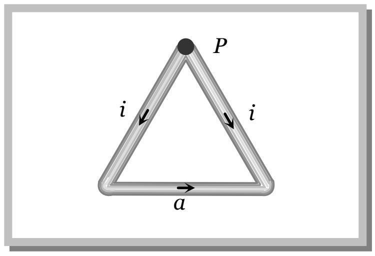


Due to wire 1, $B_1 = \frac{\mu_0}{4\pi}.\frac{2(8)}{d}$

and due to wire 2, $B_2 = \frac{\mu_0}{4\pi}.\frac{2(16)}{d}$

$$\therefore \; B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0}{4\pi}.\frac{16}{d}\right)^2 + \left(\frac{\mu_0}{4\pi}.\frac{12}{d}\right)^2} = \frac{\mu_0}{4\pi}\times\frac{2}{d}\times 10 = \frac{5\mu_0}{\pi d}$$

***Example*: 14** An equilateral triangle of side 'a' carries a current i then find out the magnetic field at point P which is vertex of triangle



(a) $\frac{\mu_0 i}{2\sqrt{3}\pi a} \otimes$

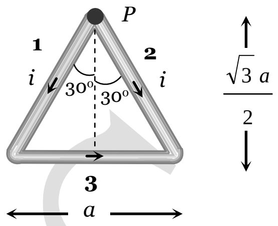
(b) $\frac{\mu_0 i}{2\sqrt{3}\pi a} \odot$

(c) $\frac{2\sqrt{3}\mu_0 i}{\pi a} \odot$

(d) Zero

Solution : (b) As shown in the following figure magnetic field at P due to side 1 and side 2 is zero.

Magnetic field at P is only due to side 3,



which is $B_1 = \frac{\mu_0}{4\pi}.\frac{2i\sin 30^o}{\frac{\sqrt{3}a}{2}} \odot$

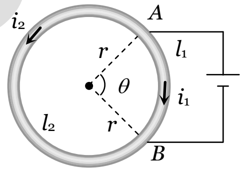
$= \frac{\mu_0}{4\pi}.\frac{2i}{\sqrt{3}a} \odot = \frac{\mu_0 i}{2\sqrt{3}\pi a} \odot$

Example: 15 A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of, the magnetic induction at the centre due to the current in the ring is **[IIT-JEE 1995]**

(a) Proportional to $2(180^o - \theta)$ (b) Inversely proportional to r

(c) Zero, only if $\theta = 180^o$ (d) Zero for all values of θ

Solution : (d) Directions of currents in two parts are different, so directions of magnetic fields due to these currents are different.



Also applying Ohm's law across AB $i_1 R_1 = i_2 R_2 \Rightarrow i_1 l_1 = i_2 l_2$ .....(i)

Also $B_1 = \frac{\mu_0}{4\pi} \times \frac{i_1 l_1}{r^2}$ and $B_2 = \frac{\mu_0}{4\pi} \times \frac{i_2 l_2}{r^2}$; $\therefore \frac{B_2}{B_1} = \frac{i_1 l_1}{i_2 l_2} = 1$ [Using (i)]

Hence, two field are equal but of opposite direction. So, resultant magnetic induction at the centre is zero and is independent of θ.

Example: 16 The earth's magnetic induction at a certain point is $7 \times 10^{-5}\ Wb/m^2$. This is to be annulled by the magnetic induction at the centre of a circular conducting loop of radius 5 *cm*. The required current in the loop is **[MP PET 1999; AIIMS 2000]**

(a) 0.56 *A* (b) 5.6 *A* (c) 0.28 *A* (d) 2.8 *A*

Solution : (b) According to the question, at centre of coil $B = B_H \Rightarrow \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} = B_H$

$\Rightarrow 10^{-7} \times \frac{2\pi i}{(5 \times 1^{-2})} = 7 \times 10^{-5} \Rightarrow i = 5.6\ amp.$

Example: 17 A particle carrying a charge equal to 100 times the charge on an electron is rotating per second in a circular path of radius 0.8 *metre*. The value of the magnetic field produced at the centre will be (μ_0 – permeability for vacuum) **[CPMT 1986]**

(a) $\frac{10^{-7}}{\mu_0}$ (b) $10^{-17}\mu_0$ (c) $10^{-6}\mu_0$ (d) $10^{-7}\mu_0$

Solution : (b) Magnetic field at the centre of orbit due to revolution of charge.

$B = \frac{\mu_0}{4\pi}.\frac{2\pi(q\nu)}{r}$; where ν = frequency of revolution of charge

So, $B = \frac{\mu_0}{4\pi} \times \frac{2\pi \times (100e \times 1)}{0.8} \Rightarrow B = 10^{-17}\mu_0$.

Example: 18 Ratio of magnetic field at the centre of a current carrying coil of radius R and at a distance of $3R$ on its axis is

(a) $10\sqrt{10}$ (b) $20\sqrt{10}$ (c) $2\sqrt{10}$ (d) $\sqrt{10}$

Solution : (a) By using $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$; where $x = 3R$ and $r = R$ $\Rightarrow \frac{B_{centre}}{B_{axis}} = (10)^{3/2} = 10\sqrt{10}$.

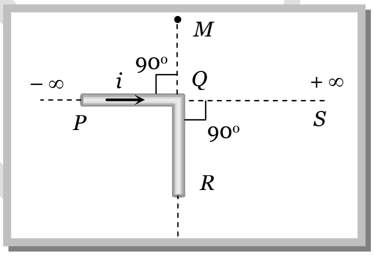
Example: 19 A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis where the magnetic induction will be $\frac{1}{8}th$ to its value at the centre of the coil, is **[MP PMT 1997]**

(a) $\frac{R}{\sqrt{3}}$ (b) $R\sqrt{3}$ (c) $2\sqrt{3}R$ (d) $\frac{2}{\sqrt{3}}R$

Solution : (b) By using $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$, given $r = R$ and $B_{axis} = \frac{1}{8}B_{centre}$

$$\Rightarrow 8 = \left(1 + \frac{x^2}{R^2}\right)^{3/2} \Rightarrow (2)^2 = \left\{\left(1 + \frac{x^2}{R^2}\right)^{1/2}\right\}^3 \Rightarrow 2 = \left(1 + \frac{x^2}{R^2}\right)^{1/2} \Rightarrow 4 = 1 + \frac{x^2}{R^2} \Rightarrow x = \sqrt{3}R$$

Example: 20 An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current is $\frac{1}{2}$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1 / H_2 is given by



(a) $\frac{1}{2}$

(b) 1

(c) $\frac{2}{3}$

(d) 2

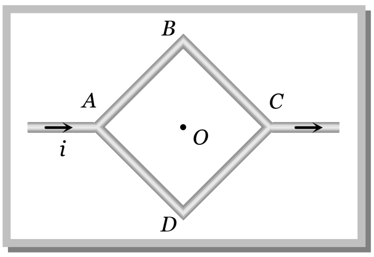
Solution : (c) Magnetic field at any point lying on the current carrying conductor is zero.

Here H_1 = magnetic field at M due to current in PQ

H_2 = magnetic field at M due to R + due to QS + due to PQ $= 0 + \frac{H_1}{2} + H_1 = \frac{3}{2}H_1$

$\therefore \frac{H_1}{H_2} = \frac{2}{3}$

Example: 21 Figure shows a square loop $ABCD$ with edge length a. The resistance of the wire ABC is r and that of ADC is $2r$. The value of magnetic field at the centre of the loop assuming uniform wire is



(a) $\frac{\sqrt{2}\,\mu_0 i}{3\pi a} \odot$

(b) $\frac{\sqrt{2}\,\mu_0 i}{3\pi a} \otimes$

(c) $\frac{\sqrt{2}\,\mu_0 i}{\pi a} \odot$

(d) $\frac{\sqrt{2}\,\mu_0 i}{\pi a} \otimes$

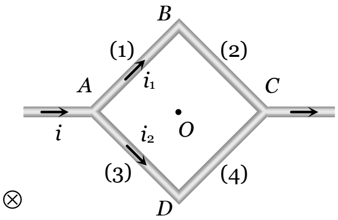
Solution : (b) According to question resistance of wire ADC is twice that of wire ABC. Hence current flows through ADC is half that of ABC *i.e.* $\frac{i_2}{i_1} = \frac{1}{2}$. Also $i_1 + i_2 = i \Rightarrow i_1 = \frac{2i}{3}$ and $i_2 = \frac{i}{3}$

Magnetic field at centre O due to wire AB and BC (part 1 and 2) $B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1 \sin 45^o}{a/2} \otimes = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\,i_1}{a} \otimes$

and magnetic field at centre O due to wires AD and DC (*i.e.* part 3 and 4) $B_3 = B_4 = \frac{\mu_0}{4\pi}\frac{2\sqrt{2}\,i_2}{a}\odot$

Also $i_1 = 2i_2$. So $(B_1 = B_2) > (B_3 = B_4)$

Hence net magnetic field at centre O

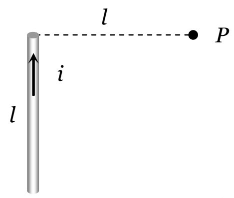


$B_{net} = (B_1 + B_2) - (B_3 + B_4)$

$$= 2 \times \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\times\left(\frac{2}{3}i\right)}{a} - \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\left(\frac{i}{3}\right)\times 2}{a} = \frac{\mu_0}{4\pi}.\frac{4\sqrt{2}\,i}{3a}(2-1)\otimes = \frac{\sqrt{2}\,\mu_0 i}{3\pi a}\otimes$$

Tricky example: 1

Figure shows a straight wire of length l current i. The magnitude of magnetic field produced by the current at point P is

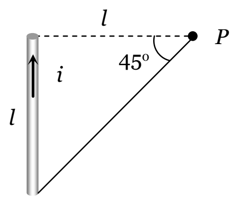


(a) $\frac{\sqrt{2}\mu_0 i}{\pi l}$ (b) $\frac{\mu_0 i}{4\pi l}$ (c) $\frac{\sqrt{2}\mu_0 i}{8\pi l}$ (d) $\frac{\mu_0 i}{2\sqrt{2}\pi l}$

Solution: (c) The given situation can be redrawn as follow.

As we know the general formula for finding the magnetic field due to a finite length wire

$$B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$$

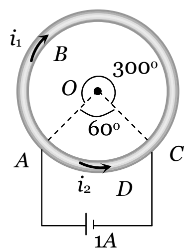


Here $\phi_1 = 0°$, $\phi = 45°$

$$\therefore\ B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin 0^o + \sin 45^o) = \frac{\mu_0}{4\pi}.\frac{i}{\sqrt{2}l} \Rightarrow B = \frac{\sqrt{2}\mu_0 i}{8\pi l}$$

Tricky example: 2

A cell is connected between the points A and C of a circular conductor $ABCD$ of centre 'O' with angle $AOC = 60°$, If B_1 and B_2 are the magnitudes of the magnetic fields at O due to the currents in ABC and ADC respectively, the ratio $\frac{B_1}{B_2}$ is **[KCET (Engg./ Med.) 1999]**



(a) 0.2

(b) 6

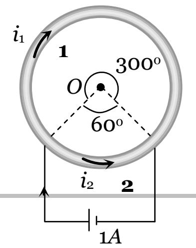
(c) 1

(d) 5

Solution: (c) $B = \frac{\mu_0}{4\pi}.\frac{\theta i}{r}$

$\Rightarrow B \propto \theta i$

$\Rightarrow \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2} \times \frac{i_1}{i_2}$

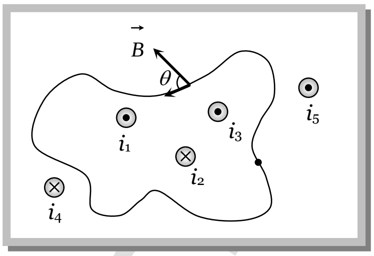


Also $\dfrac{i_1}{i_2} = \dfrac{l_2}{l_1} = \dfrac{\theta_2}{\theta_1}$ Hence $\dfrac{B_1}{B_2} = \dfrac{1}{1}$

Amperes Law

Amperes law gives another method to calculate the magnetic field due to a given current distribution.

Line integral of the magnetic field $\vec{B}$ around any closed curve is equal to μ_0 times the net current i threading through the area enclosed by the curve



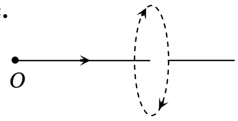
i.e. $\oint \vec{B} d\vec{l} = \mu_0 \sum i = \mu_0 (i_1 + i_3 - i_2)$

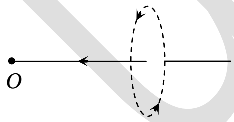
Also using $\vec{B} = \mu_0 \vec{H}$ (where $\vec{H}$ = magnetising field)

$\oint \mu_0 \vec{H}.\vec{dl} = \mu_0 \Sigma i \Rightarrow \oint \vec{H}.\vec{dl} = \Sigma i$

Note : ❒ Total current crossing the above area is $(i_1 + i_3 - i_2)$. Any current outside the area is not included in net current. (Outward $\odot \rightarrow +ve$, Inward $\otimes \rightarrow -ve$)

❒ When the direction of current is away from the observer then the direction of closed path is clockwise and when the direction of current is towards the observer then the direction of closed path is anticlockwise.

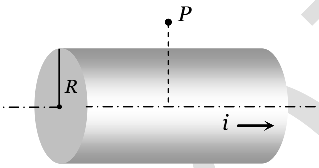




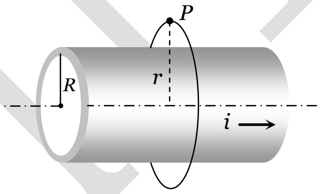
Application of Amperes law

(1) **Magnetic field due to a cylindrical wire**

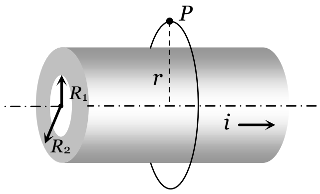
(i) **Outside the cylinder**



Solid cylinder



Thin hollow cylinder

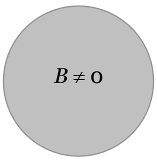


Thick hollow

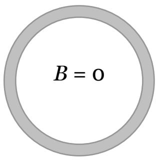
In all above cases magnetic field outside the wire at P $\oint \vec{B}.\vec{dl} = \mu_0 i \Rightarrow B\int dl = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 i \Rightarrow$ $B_{out} = \dfrac{\mu_0 i}{2\pi r}$

In all the above cases $B_{surface} = \dfrac{\mu_0 i}{2\pi R}$

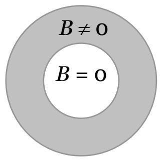
(ii) **Inside the cylinder :** Magnetic field inside the hollow cylinder is zero.



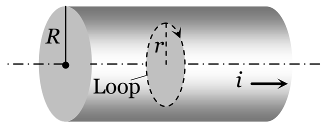
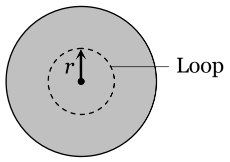
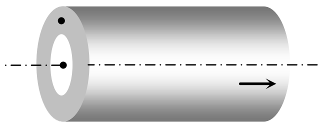
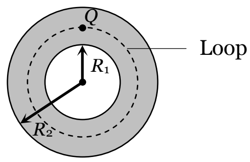
Cross sectional view Solid cylinder



Thin hollow cylinder



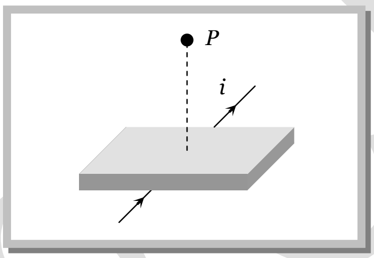
Thick hollow

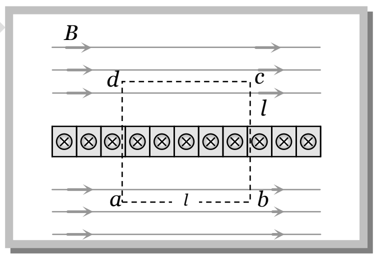
Solid cylinder	Inside the thick portion of hollow cylinder
 	 
Current enclosed by loop (i') is lesser then the total current (i) Current density is uniform *i.e.* $J = J' \Rightarrow \frac{i}{A} = \frac{i'}{A'}$ $\Rightarrow i' = i \times \frac{A'}{A} = i\left(\frac{r^2}{R^2}\right)$ Hence at point Q $\oint \vec{B}.d\vec{l} = \mu_0 i' \Rightarrow B \times 2\pi r = \frac{\mu_0 i r^2}{R^2}$ $\Rightarrow B = \frac{\mu_0}{2\pi} . \frac{ir}{R^2}$	Current enclosed by loop (i') is lesser then the total current (i) Also $i' = i \times \frac{A'}{A} = i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$ Hence at point Q $\oint \vec{B}.d\vec{l} = \mu_0 i' \Rightarrow B \times 2\pi r = \mu_0 i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$ $\Rightarrow B = \frac{\mu_0 i}{2\pi r} . \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$. If $r = R_1$ (inner surface) $B = 0$ If $r = R_2$ (outer surface) $B = \frac{\mu_0 i}{2\pi R_2}$ (max.)

Note : ❐ For all cylindrical current distributions

$B_{axis} = 0$ (min.), $B_{surface}$ = max (distance r always from axis of cylinder), $B_{out} \propto 1/r$.

(2) **Magnetic field due to an infinite sheet carrying current :** The figure shows an infinite sheet of current with linear current density j (A/m). Due to symmetry the field line pattern above and below the sheet is uniform. Consider a square loop of side l as shown in the figure.





According to Ampere's law, $\int_a^b B.dl + \int_b^c B.dl + \int_c^d B.dl + \int_d^a B.dl = \mu_0 i$.

Since $B \perp dl$ along the path $b \to c$ and $d \to a$, therefore, $\int_b^c B.dl = 0$; $\int_d^a B.dl = 0$

Also, $B \,||\, dl$ along the path $a \to b$ and $c \to d$, thus $\int_a^b B.dl + \int_d^a B.dl = 2Bl$

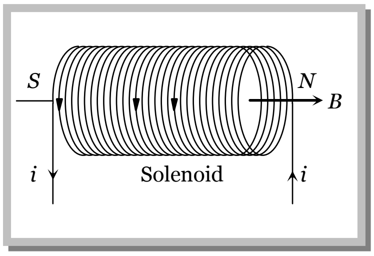
The current enclosed by the loop is $i = jl$

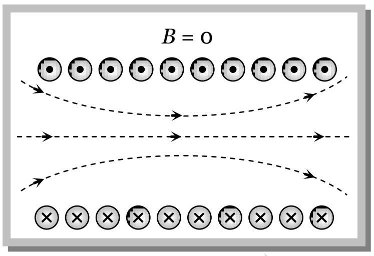
Therefore, according to Ampere's law $2Bl = \mu_0 (jl)$ or $B = \frac{\mu_0 j}{2}$

(3) **Solenoid**

A cylinderical coil of many tightly wound turns of insulated wire with generally diameter of the coil smaller than its length is called a solenoid.

One end of the solenoid behaves like the north pole and opposite end behaves like the south pole. As the length of the solenoid increases, the interior field becomes more uniform and the external field becomes weaker.



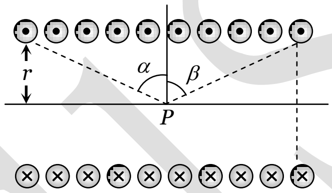


A magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of solenoid.

(i) **Finite length solenoid :** If N = total number of turns,

l = length of the solenoid

n = number of turns per unit length $= \frac{N}{l}$



Magnetic field inside the solenoid at point P is given by $B = \frac{\mu_0}{4\pi}(2\pi ni)[\sin\alpha + \sin\beta]$

(ii) **Infinite length solenoid :** If the solenoid is of infinite length and the point is well inside the solenoid *i.e.* $\alpha = \beta = (\pi/2)$.

So $\qquad \boldsymbol{B_{in} = \mu_0 ni}$

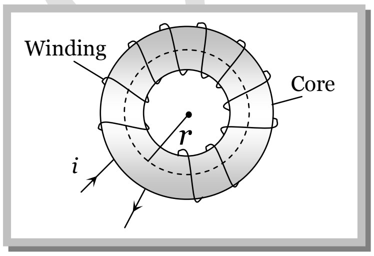
(ii) If the solenoid is of infinite length and the point is near one end *i.e.* $\alpha = 0$ and $\beta = (\pi/2)$

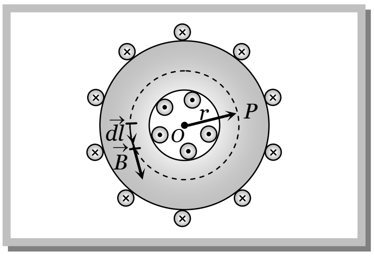
So $\qquad \boldsymbol{B_{end} = \frac{1}{2}(\mu_0 ni)}$

Note : ❒ Magnetic field outside the solenoid is zero.

❒ $B_{end} = \frac{1}{2}B_{in}$

(4) **Toroid :** A toroid can be considered as a ring shaped closed solenoid. Hence it is like an endless cylindrical solenoid.





Consider a toroid having n turns per unit length

Let i be the current flowing through the toroid (figure). The magnetic lines of force mainly remain in the core of toroid and are in the form of concentric circles. Consider such a circle of mean radius r. The circular closed path surrounds N loops of wire, each of which carries a current i therefore from $\oint \vec{B}.\vec{dl} = \mu_0 i_{net}$

$\Rightarrow\ B\times(2\pi r)=\mu_0 Ni \quad \Rightarrow B=\frac{\mu_0 Ni}{2\pi r}=\mu_o ni$ where $n=\frac{N}{2\pi r}$

For any point inside the empty space surrounded by toroid and outside the toroid, magnetic field B is zero because the net current enclosed in these spaces is zero.

Concepts

- *The line integral of magnetising field $(\vec{H})$ for any closed path called magnetomotive force (MMF). It's S.I. unit is amp.*
- *Ratio of dimension of e.m.f. to MMF is equal to the dimension of resistance.*
- *Biot-Savart law is valid for asymmetrical current distributions while Ampere's law is valid for symmetrical current distributions.*
- *Biot-Savart law is based only on the principle of magnetism while Ampere's laws is based on the principle of electromagnetism.*

Example

Example: 22 A long solenoid has 200 *turns per cm* and carries a current of 2.5 *A*. The magnetic field at its centre is [$\mu_0 = 4\pi\times 10^{-7}$ *Wb/m*²] **[MP PET 2000]**

(a) 3.14×10^{-2} *Wb/m*² (b) 6.28×10^{-2} *Wb/m*² (c) 9.42×10^{-2} *Wb/m*² (d) 12.56×10^{-2} *Wb/m*²

Solution : (b) $B=\mu_0 ni = 4\pi\times10^{-7}\times\frac{200}{10^{-2}}\times 2.5 = 6.28\times10^{-2}\,Wb/m^2$.

Example: 23 A long solenoid is formed by winding 20 *turns/cm*. The current necessary to produce a magnetic field of 20 *mili tesla* inside the solenoid will be approximately $\left(\frac{\mu_0}{4\pi}=10^{-7}\ Tesla\text{ - }metre/ampere\right)$ **[MP PMT 1994]**

(a) 8.0 *A* (b) 4.0 *A* (c) 2.0 *A* (d) 1.0 *A*

Solution : (a) $B=\mu_0 ni$; where $n=\frac{20\ turn}{10\ cm}=2000\frac{turn}{m}$. So, $20\times10^{-5}=4\pi\times2000\times i \Rightarrow i=8A$.

Example: 24 Two solenoids having lengths L and $2L$ and the number of loops N and $4N$, both have the same current, then the ratio of the magnetic field will be **[CPMT 1994]**

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

Solution : (a) $B=\mu_0\frac{N}{L}i \Rightarrow B\propto\frac{N}{L} \Rightarrow \frac{B_1}{B_2}=\frac{N_1}{N_2}\times\frac{L_2}{L_1}=\frac{N}{4N}\times\frac{2l}{L}=\frac{1}{2}$.

Example: 25 The average radius of a toroid made on a ring of non-magnetic material is 0.1 m and it has 500 turns. If it carries 0.5 ampere current, then the magnetic field produced along its circular axis inside the toroid will be

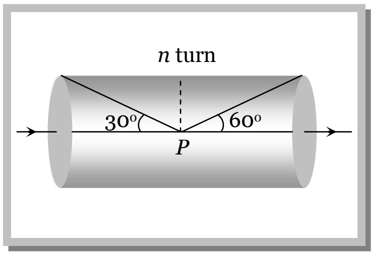
(a) 25×10^{-2} *Tesla* (b) 5×10^{-2} *Tesla* (c) 25×10^{-4} *Tesla* (d) 5×10^{-4} *Tesla*

Solution : (d) $B=\mu_0 ni$; where $n=\frac{N}{2\pi R}$ $\therefore$ $B=4\pi\times10^{-7}\times\frac{500}{2\pi\times0.1}\times0.5 = 5\times10^{-4}\,T$.

Example: 26 For the solenoid shown in figure. The magnetic field at point P is

(a) $\frac{\mu_0 ni}{4}(\sqrt{3}+1)$

(b) $\frac{\sqrt{3}\mu_0 ni}{4}$



(c) $\frac{\mu_0 ni}{2}(\sqrt{3}+1)$

(d) $\frac{\mu_0 ni}{4}(\sqrt{3}-1)$

Solution : (a) $B = \frac{\mu_0}{4\pi}.2\pi ni(\sin\alpha + \sin\beta)$. From figure $\alpha = (90^o - 30^o) = 60^o$ and $\beta = (90^o - 60^o) = 30^o$

$\therefore\ B = \frac{\mu_0 ni}{2}(\sin 60^o + \sin 30^o) = \frac{\mu_0 ni}{4}(\sqrt{3}+1)$.

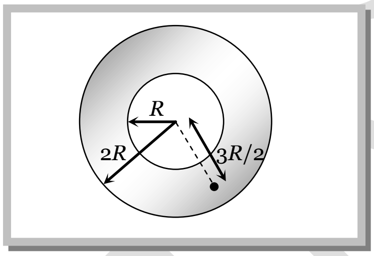
***Example*: 27** Figure shows the cress sectional view of the hollow cylindrical conductor with inner radius 'R' and outer radius '$2R$', cylinder carrying uniformly distributed current along it's axis. The magnetic induction at point 'P' at a distance $\frac{3R}{2}$ from the axis of the cylinder will be

(a) Zero

(b) $\frac{5\mu_0 i}{72\pi R}$

(c) $\frac{7\mu_0 i}{18\pi R}$

(d) $\frac{5\mu_0 i}{36\pi R}$



Solution : (d) By using $B = \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$ here $r = \frac{3R}{2}$, $a = R$, $ab = 2R \Rightarrow$ $B = \frac{\mu_0 i}{2\pi\left(\frac{3R}{2}\right)} \times \left\{\frac{\left(\frac{3R}{2}\right)^2 - R^2}{(R^2) - R^2}\right\} = \frac{5.\mu_o i}{36\pi r}$.

Tricky example: 3

A winding wire which is used to frame a solenoid can bear a maximum 10 A current. If length of solenoid is 80cm and it's cross sectional radius is 3 cm then required length of winding wire is $(B = 0.2\,T)$

(a) $1.2\times 10^2\ m$ (b) $4.8\times 10^2\ m$ (c) $2.4\times 10^3\ m$ (d) $6\times 10^3\ m$

Solution : (c) $B = \frac{\mu_0 Ni}{l}$ where N =Total number of turns, l =length of the solenoid

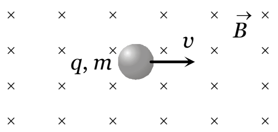
$\Rightarrow\ 0.2 = \frac{4\pi\times 10^{-7}\times N \times 10}{0.8}\ \Rightarrow\ N = \frac{4\times 10^4}{\pi}$

Since N turns are made from the winding wire so length of the wire $(L) = 2\pi r \times N$ $[2\pi r = \text{length of each turns}]$

$\Rightarrow\ L = 2\pi \times 3\times 10^{-2}\times \frac{4\times 10^4}{\pi} = 2.4\times 10^3\ m.$

Motion of Charged Particle in a Magnetic Field

If a particle carrying a positive charge q and moving with velocity v enters a magnetic field B then it experiences a force F which is given by the expression



$$F = q(\vec{v} \times \vec{B}) \Rightarrow F = qvB\sin\theta$$

Here $\vec{v} =$ velocity of the particle, $\vec{B} =$ magnetic field

(1) **Zero force**

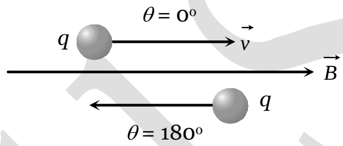
Force on charged particle will be zero (*i.e.* $F = 0$) if

(i) No field *i.e.* $B = 0 \Rightarrow F = 0$

(ii) Neutral particle *i.e.* $q = 0 \Rightarrow F = 0$

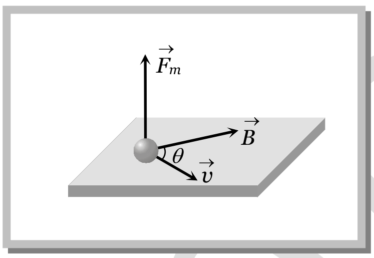
(iii) Rest charge *i.e.* $v = 0 \Rightarrow F = 0$

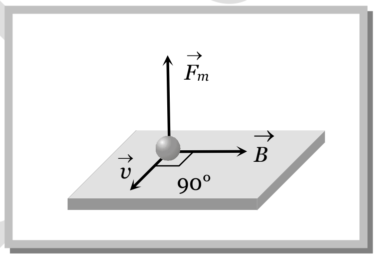
(iv) Moving charge *i.e.* $\theta = 0^o$ or $\theta = 180^o \Rightarrow F = 0$



(2) **Direction of force**

The force $\vec{F}$ is always perpendicular to both the velocity $\vec{v}$ and the field $\vec{B}$ in accordance with Right Hand Screw Rule, through $\vec{v}$ and $\vec{B}$ themselves may or may not be perpendicular to each other.



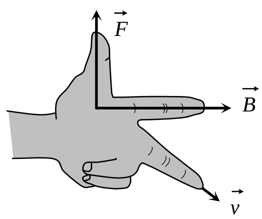


Direction of force on charged particle in magnetic field can also be find by Flemings Left Hand Rule (FLHR).

Here, *First finger* (indicates) $\rightarrow$ Direction of magnetic field

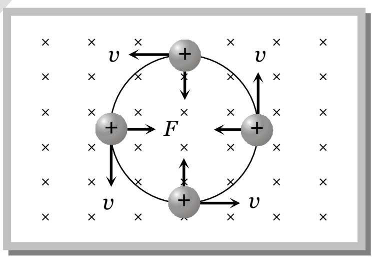
Middle finger $\rightarrow$ Direction of motion of positive charge or direction, opposite to the motion of negative charge.

Thumb $\rightarrow$ Direction of force



(3) **Circular motion of charge in magnetic field**

Consider a charged particle of charge q and mass m enters in a uniform magnetic field B with an initial velocity v perpendicular to the field.



$\theta = 90^o$, hence from $F = qvB \sin\theta$ particle will experience a maximum magnetic force $\boldsymbol{F_{max} = qvB}$ which act's in a direction perpendicular to the motion of charged particle. (By Flemings left hand rule).

(i) **Radius of the path :** In this case path of charged particle is circular and magnetic force provides the necessary centripetal force *i.e.* $qvB = \frac{mv^2}{r} \Rightarrow$ radius of path $r = \frac{mv}{qB}$

If p = momentum of charged particle and K = kinetic energy of charged particle (gained by charged particle after accelerating through potential difference V) then $p = mv = \sqrt{2mK} = \sqrt{2mqV}$

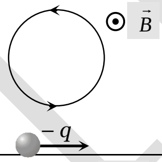
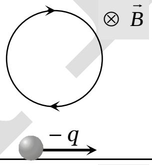
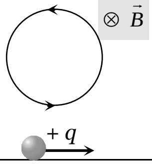
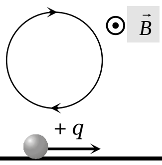
So $$\boldsymbol{r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}}$$

$r \propto v \propto p \propto \sqrt{K}$ *i.e.* with increase in speed or kinetic energy, the radius of the orbit increases.

Note : ❒ Less radius (r) means more curvature (c) *i.e.* $c \propto \frac{1}{r}$

Less : r More : c | More : r Less : c | $r = \infty$ $c = 0$

(ii) **Direction of path :** If a charge particle enters perpendicularly in a magnetic field, then direction of path described by it will be

Type of charge	Direction of magnetic field	Direction of it's circular motion
Negative	Outwards ⊙	 Anticlockwise
Negative	Inward ⊗	 Clockwise
Positive	Inward ⊗	 Anticlockwise
Positive	Outward ⊙	 Clockwise

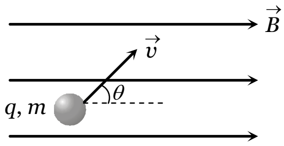
(iii) **Time period** : As in uniform circular motion $v = r\omega$, so the angular frequency of circular motion, called cyclotron or gyro-frequency, will be given by $\omega = \frac{v}{r} = \frac{qB}{m}$ and hence the time period, $T = \frac{2\pi}{\omega} = 2\pi\frac{m}{qB}$

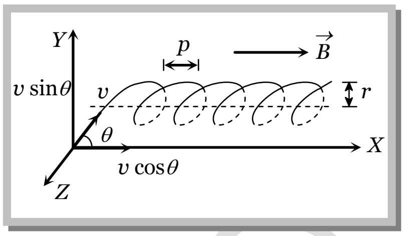
i.e., time period (or frequency) is independent of speed of particle and radius of the orbit and depends only on the field B and the nature, *i.e.*, specific charge $\left(\frac{q}{m}\right)$, of the particle.

(4) **Motion of charge on helical path**

When the charged particle is moving at an angle to the field (other than 0°, 90°, or 180°).

In this situation resolving the velocity of the particle along and perpendicular to the field, we find that the particle moves with constant velocity $v\cos\theta$ along the field (as no force acts on a charged particle when it moves parallel to the field) and at the same time it is also moving with velocity $v\sin\theta$ perpendicular to the field due to which it will describe a circle (in a plane perpendicular to the field) of radius. $r = \dfrac{m(v\sin\theta)}{qB}$





Time period and frequency do not depend on velocity and so they are given by $T = \dfrac{2\pi m}{qB}$ and $\nu = \dfrac{qB}{2\pi m}$

So the resultant path will be a *helix* with its axis parallel to the field $\vec{B}$ as shown in figure in this situation.

The *pitch* of the *helix*, (*i.e.*, linear distance travelled in one rotation) will be given by $p = T(v\cos\theta) = 2\pi\dfrac{m}{qB}(v\cos\theta)$

Note : ❒ 1 rotation $\equiv 2\pi \equiv T$ and 1 pitch $\equiv 1\,T$

❒ Number of pitches ≡ Number of rotations ≡ Number of repetition = Number of helical turns

❒ If pitch value is p, then number of pitches obtained in length l given as

Number of pitches $= \dfrac{l}{p}$ and time reqd. $t = \dfrac{l}{v\cos\theta}$

Some standard results

& Ratio of radii of path described by proton and α-particle in a magnetic field (particle enters perpendicular to the field)

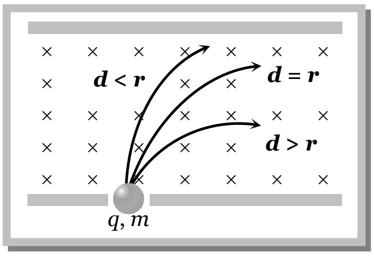
Constant quantity	Formula	Ratio of radii	Ratio of curvature (c)
v - same	$r = \dfrac{mv}{qB} \Rightarrow r \propto \dfrac{m}{q}$	$r_p : r_\alpha = 1:2$	$c_p : c_R = 2:1$
p - same	$r = \dfrac{p}{qB} \Rightarrow r \propto \dfrac{1}{q}$	$r_p : r_\alpha = 2:1$	$c_p : c_R = 1:2$
k - same	$r = \dfrac{\sqrt{2mk}}{qB} \Rightarrow r \propto \dfrac{\sqrt{m}}{q}$	$r_p : r_\alpha = 1:1$	$c_p : c_R = 1:1$
V - same	$r \propto \sqrt{\dfrac{m}{q}}$	$r_p : r_\alpha = 1:\sqrt{2}$	$c_p : c_R = \sqrt{2}:1$

& **Particle motion between two parallel plates** $(\vec{v} \perp \vec{B})$

(i) To strike the opposite plate it is essential that $d < r$.

(ii) Does not strike the opposite plate $d > r$.

(iii) To touch the opposite plate $d = r$.



(iv) To just not strike the opposite plate $d \geq r$.

(v) To just strike the opposite plate $d \leq r$.

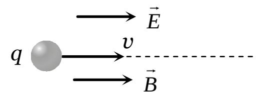
(5) **Lorentz force**

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F_e} = q\vec{E}$ and magnetic force $\vec{F_m} = q(\vec{v} \times \vec{B})$; so the net force on it will be $\boldsymbol{\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]}$. Which is the famous 'Lorentz-force equation'.

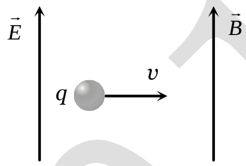
Depending on the directions of $\vec{v}, \vec{E}$ and $\vec{B}$ following situations are possible

(i) **When $\vec{v}, \vec{E}$ and $\vec{B}$ all the three are collinear :** In this situation as the particle is moving parallel or antiparallel to the field, the magnetic force on it will be zero and only electric force will act and so $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

The particle will pass through the field following a straight line path (parallel field) with change in its speed. So in this situation speed, velocity, momentum kinetic energy all will change without change in direction of motion as shown



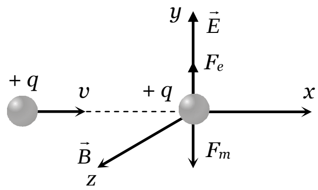
(ii) **When $\vec{E}$ is parallel to $\vec{B}$ and both these fields are perpendicular to $\vec{v}$ then :** $\vec{F_e}$ is perpendicular to $\vec{F_m}$ and they cannot cancel each other. The path of charged particle is curved in both these fields.



(iii) **$\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular :** In this situation if $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F_e} + \vec{F_m} = 0 \;\; i.e., \;\; \vec{a} = (\vec{F}/m) = 0$$

as shown in figure, the particle will pass through the field with same velocity.



And in this situation, as $F_e = F_m$ $i.e.,$ $qE = qvB$ $v = E/B$

This principle is used in 'velocity-selector' to get a charged beam having a specific velocity.

Note : ❒ From the above discussion, conclusion is as follows

❒ If E = 0, B = 0, so F = 0.

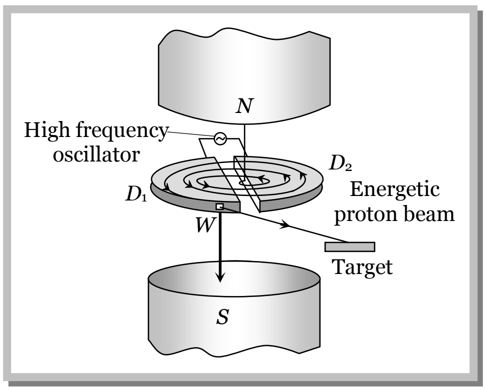
❒ If E = 0, $B \neq$ 0, so F may be zero (if $\theta = 0^o$ or 180^o).

❒ If $E \neq$ 0, $B \neq$ 0, so F = 0 (if $|\vec{F_e}| = |\vec{F_m}|$ and their directions are opposite)

❒ If $E \neq$ 0, B = 0, so $F \neq$ 0 (because $\vec{v} \neq$ constant).

Cyclotron

Cyclotron is a device used to accelerated positively charged particles (like, α-particles, deutrons *etc.*) to acquire enough energy to carry out nuclear disintegration *etc.* t is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency. Thus a small potential difference would impart if



enormously large velocities if the particle is made to traverse the potential difference a number of times.

It consists of two hollow *D*-shaped metallic chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} *mm* mercury. The whole apparatus is placed between the two poles of a strong electromagnet *NS* as shown in fig. The magnetic field acts perpendicular to the plane of the dees.

Note : ❐ The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas is used.

(1) **Cyclotron frequency :** Time taken by ion to describe *q* semicircular path is given by $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$

If T = time period of oscillating electric field then $T = 2t = \frac{2\pi m}{qB}$ the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

(2) **Maximum energy of position :** Maximum energy gained by the charged particle $E_{\max} = \left(\frac{q^2B^2}{2m}\right)r^2$

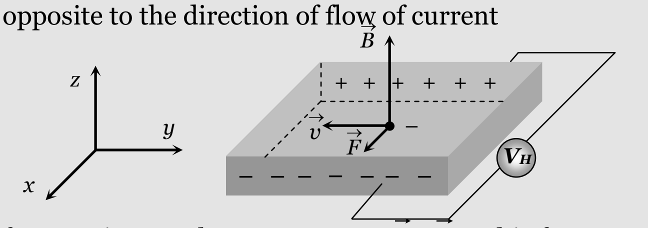
where r_0 = maximum radius of the circular path followed by the positive ion.

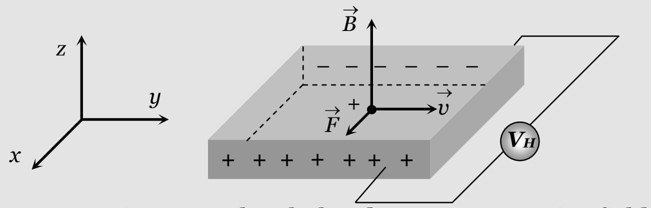
Note : ❐ Cyclotron frequency is also known as magnetic resonance frequency.

❐ Cyclotron can not accelerate electrons because they have very small mass.

Hall effect : The Phenomenon of producing a transverse emf in a current carrying conductor on applying a magnetic field perpendicular to the direction of the current is called Hall effect.

Hall effect helps us to know the nature and number of charge carriers in a conductor.

<table>
<tr><th>Negatively charged particles</th><th>Positively charged particles</th></tr>
<tr><td>Consider a conductor having electrons as current carriers. The electrons move with drift velocity $\vec{v}$ opposite to the direction of flow of current


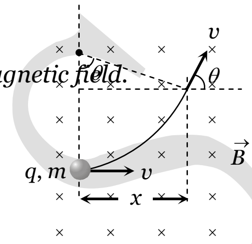
force acting on electron $F_m = -e(\vec{v} \times \vec{B})$. This force acts along *x*-axis and hence electrons will move towards face (2) and it becomes negatively charged.</td><td>Let the current carriers be positively charged holes. The hole move in the direction of current


Force acting on the hole due to magnetic field $\vec{F}_m = +e(\vec{v} \times \vec{B})$ force acts along *x*-axis and hence holes move towards face (2) and it becomes positively charged.</td></tr>
</table>

Concepts

- *The energy of a charged particle moving in a uniform magnetic field does not change because it experiences a force in a direction, perpendicular to it's direction of motion. Due to which the speed of charged particle remains unchanged and hence it's K.E. remains same.*
- *Magnetic force does no work when the charged particle is displaced while electric force does work in displacing the charged particle.*
- *Magnetic force is velocity dependent, while electric force is independent of the state of rest or motion of the charged particle.*
- *If a particle enters a magnetic field normally to the magnetic field, then it starts moving in a circular orbit. The point at which it enters the magnetic field lies on the circumference. (Most of us confuse it with the centre of the orbit)*
- ***Deviation of charged particle in magnetic field :*** *If a charged particle (q, m) enters a uniform magnetic field $\vec{B}$ (extends upto a length x) at right angles with speed v as shown in figure.*

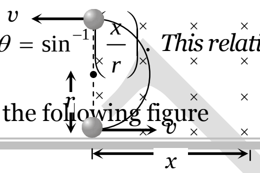
  The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field.



Deviation in terms of time t; $\theta = \omega t = \left(\frac{Bq}{m}\right)t$

Deviation in terms of length of the magnetic field ; $\theta = \sin^{-1}\left(\frac{x}{r}\right)$. *This relation can be used only when* $x \le r$.

For x > r, the deviation will be 180° as shown in the following figure



Example

Example: 28 Electrons move at right angles to a magnetic field of 1.5×10^{-2} *Tesla* with a speed of 6×10^{27} m/s. If the specific charge of the electron is 1.7×10^{11} *Coul/kg*. The radius of the circular path will be **[BHU 2003]**

(a) 2.9 *cm* (b) 3.9 *cm* (c) 2.35 *cm* (d) 3 *cm*

Solution : (c) $r = \frac{mv}{qB} \Rightarrow \frac{v}{(q/m).B} = \frac{6 \times 10^{27}}{17 \times 10^{11} \times 1.5 \times 10^{-2}} = 2.35 \times 10^{-2}\ m = 2.35\ cm.$

Example: 29 An electron (mass $= 9 \times 10^{-31}$ *kg*. charge $= 1.6 \times 10^{-19}$ *coul.*) whose kinetic energy is 7.2×10^{-18} *joule* is moving in a circular orbit in a magnetic field of 9×10^{-5} *weber* $/m^2$. The radius of the orbit is **[MP PMT 2002]**

(a) 1.25 *cm* (b) 2.5 *cm* (c) 12.5 *cm* (d) 25.0 *cm*

Solution : (d) $r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2 \times q \times 10^{-31} \times 7.2 \times 10^{-8}}}{1.6 \times 10^{-19} \times q \times 10^{-5}} = 0.25\ cm = 25\ cm$.

Example: 30 An electron and a proton enter a magnetic field perpendicularly. Both have same kinetic energy. Which of the following is true **[MP PET 1999]**

(a) Trajectory of electron is less curved (b) Trajectory of proton is less curved

(c) Both trajectories are equally curved (d) Both move on straight line path

Solution : (b) By using $r = \frac{\sqrt{2mk}}{qB}$; For both particles $q \rightarrow$ same, $B \rightarrow$ same, $k \rightarrow$ same

Hence $r \propto \sqrt{m} \Rightarrow \frac{r_e}{r_p} = \sqrt{\frac{m_e}{m_p}}$ $\because$ $m_p > m_e$ so $r_p > r_e$

Since radius of the path of proton is more, hence it's trajectory is less curved.

***Example*: 31** A proton and an α – particles enters in a uniform magnetic field with same velocity, then ratio of the radii of path describe by them

(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) None of these

Solution : (b) By using $r = \frac{mv}{qB}$; $v \to$ same, $B \to$ same $\Rightarrow r \propto \frac{m}{2} \Rightarrow \frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p} = \frac{m_p}{4m_p} \times \frac{2q_p}{q_p} = \frac{1}{2}$

***Example*: 32** A proton of mass m and charge $+e$ is moving in a circular orbit of a magnetic field with energy 1*MeV*. What should be the energy of α-particle (mass = 4 m and charge = +2e), so that it can revolve in the path of same radius **[BHU 1997]**

(a) 1 *MeV* (b) 4 *MeV* (c) 2 *MeV* (d) 0.5 *MeV*

Solution : (a) By using $r = \frac{\sqrt{2mK}}{qB}$; $r \to$ same, $B \to$ same $\Rightarrow K \propto \frac{q^2}{m}$

Hence $\frac{K_\alpha}{K_p} = \left(\frac{q_\alpha}{q_p}\right)^2 \times \frac{m_p}{m_\alpha} = \left(\frac{2q_p}{q_p}\right)^2 \times \frac{m_p}{4m_p} 1 \Rightarrow K_\alpha = K_p = 1\, meV.$

***Example*: 33** A proton and an α – particle enter a uniform magnetic field perpendicularly with the same speed. If proton takes $25\,\mu$ sec to make 5 revolutions, then the periodic time for the α – particle would be **[MP PET 1993]**

(a) $50\,\mu$ sec (b) $25\,\mu$ sec (c) $10\,\mu$ sec (d) $5\,\mu$ sec

Solution : (c) Time period of proton $T_p = \frac{25}{5} = 5\,\mu$ sec

By using $T = \frac{2\pi m}{qB} \Rightarrow \frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p} \times \frac{q_p}{q_\alpha} = \frac{4m_p}{m_p} \times \frac{q_p}{2q_p} \Rightarrow T_\alpha = 2T_p = 10\,\mu$ sec .

***Example*: 34** A particle with 10^{-11} *coulomb* of charge and 10^{-7} *kg* mass is moving with a velocity of 10^8 *m/s* along the *y*-axis. A uniform static magnetic field *B* = 0.5 *Tesla* is acting along the *x*-direction. The force on the particle is

[MP PMT 1997]

(a) 5×10^{-11} *N* along $\hat{i}$ (b) 5×10^3 *N* along $\hat{k}$ (c) 5×10^{-11} *N* along $-\hat{j}$ (d) 5×10^{-4} *N* along $-\hat{k}$

Solution : (d) By using $\vec{F} = q(\vec{v} \times \vec{B})$; where $\vec{v} = 10\hat{j}$ and $\vec{B} = 0.5\hat{i}$

$\Rightarrow \vec{F} = 10^{-11}(10^8\hat{j} \times 0.5\hat{i}) = 5 \times 10^{-4}(\hat{j} \times \hat{i}) = 5 \times 10^{-4}(-\hat{k})$ *i.e.*, $5 \times 10^{-4}\,N$ along $-\hat{k}$.

***Example*: 35** An electron is moving along positive *x*-axis. To get it moving on an anticlockwise circular path in *x-y* plane, a magnetic filed is applied

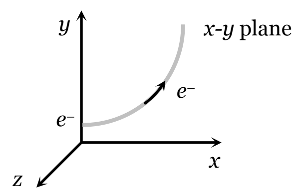
(a) Along positive *y*-axis (b) Along positive *z*-axis

(c) Along negative *y*-axis (d) Along negative *z*-axis

Solution : (a) The given situation can be drawn as follows

According to figure, for deflecting electron in *x-y* plane, force must be acting an it towards *y*-axis.

Hence according to Flemings left hand rule, magnetic field directed along positive y – axis.



***Example*: 36** A particle of charge -16×10^{-18} coulomb moving with velocity 10 *m/s* along the *x*-axis enters a region where a magnetic field of induction *B* is along the *y*-axis, and an electric field of magnitude 10^4 *V/m* is along the negative *z*-axis. If the charged particle continuous moving along the *x*-axis, the magnitude of *B* is **[AIEEE 2003]**

(a) $10^{-3}\ Wb/m^2$ (b) $10^{3}\ Wb/m^2$ (c) $10^{5}\ Wb/m^2$ (d) $10^{16}\ Wb/m^2$

Solution : (b) Particles is moving undeflected in the presence of both electric field as well as magnetic field so it's speed

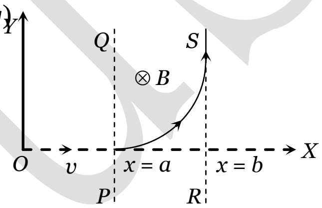
$$v=\frac{E}{B}\Rightarrow B=\frac{E}{v}=\frac{10^4}{10}=10^3\,Wb/m^2.$$

***Example*: 37** A particle of mass *m* and charge *q* moves with a constant velocity *v* along the positive *x* direction. It enters a region containing a uniform magnetic field *B* directed along the negative *z* direction extending from *x* = *a* to *x* = *b*. The minimum value of *v* required so that the particle can just enter the region *x* > *b* is

[IIT-JEE (Screening) 2002]

(a) *qbB/m* (b) *q*(*b* – *a*)*B/m* (c) *qaB/m* (d) *q*(*b*+*a*)*B/2m*

Solution : (b) As shown in the following figure, the $z-$axis points out of the paper and the magnetic fields is directed into the paper, existing in the region between *PQ* and *RS*. The particle moves in a circular path of radius *r* in the magnetic field. It can just enter the region $x>b$ for $r\ge(b-a)$



Now $r=\dfrac{mv}{qb}\ge(b-a)$

$$\Rightarrow v\ge\frac{q(b-a)B}{m}\Rightarrow v_{\min}=\frac{q(b-a)B}{m}.$$

***Example*: 38** At a certain place magnetic field vertically downwards. An electron approaches horizontally towards you and enters in this magnetic fields. It's trajectory, when seen from above will be a circle which is

(a) Vertical clockwise (b) Vertical anticlockwise

(c) Horizontal clockwise (d) Horizontal anticlockwise

Solution : (c) By using Flemings left hand rule.

***Example*: 39** When a charged particle circulates in a normal magnetic field, then the area of it's circulation is proportional to

(a) It's kinetic energy (b) It's momentum

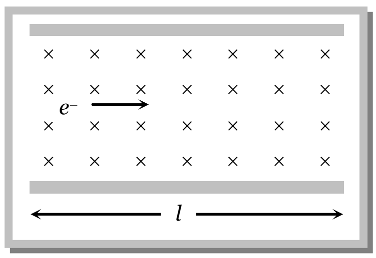
(c) It's charge (d) Magnetic fields intensity

Solution : (a) $r=\dfrac{\sqrt{2mK}}{qB}$ and $A=Aq^2\Rightarrow A=\dfrac{\pi(2mK)}{q^2b^2}\Rightarrow A\propto K.$

***Example*: 40** An electron moves straight inside a charged parallel plate capacitor at uniform charge density σ. The space between the plates is filled with constant magnetic field of induction $\vec{B}$. Time of straight line motion of the electron in the capacitor is

(a) $\dfrac{e\sigma}{\varepsilon_0 lB}$

(b) $\dfrac{\varepsilon_0 lB}{\sigma}$



(c) $\dfrac{e\sigma}{\varepsilon_0 B}$

(d) $\dfrac{\varepsilon_0 B}{e\sigma}$

Solution : (b) The net force acting on the electron is zero because it moves with constant velocity, due to it's motion on straight line.

$$\Rightarrow \vec{F}_{net} = \vec{F}_e + \vec{F}_m = 0 \Rightarrow |\vec{F}_e| = |\vec{F}_m| \Rightarrow eE = evB \Rightarrow ve = \frac{E}{B} = \frac{\sigma}{\varepsilon_0 B} \qquad \left[E = \frac{\sigma}{\varepsilon_o}\right]$$

$\therefore$ The time of motion inside the capacitor $t = \dfrac{l}{v} = \dfrac{\varepsilon_0 l B}{\sigma}$.

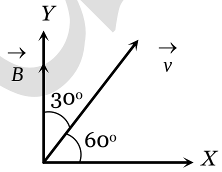
Example: 41 A proton of mass $1.67 \times 10^{-27}\,kg$ and charge $1.6 \times 10^{-19}\,C$ is projected with a speed of $2 \times 10^{6}\,m/s$ at an angle of 60° to the *X*-axis. If a uniform magnetic field of 0.104 *Tesla* is applied along *Y*-axis, the path of proton is

(a) A circle of radius = 0.2 *m* and time period $\pi \times 10^{-7}\,s$

(b) A circle of radius = 0.1 *m* and time period $2\pi \times 10^{-7}\,s$

(c) A helix of radius = 0.1 *m* and time period $2\pi \times 10^{-7}\,s$

(d) A helix of radius = 0.2 *m* and time period $4\pi \times 10^{-7}\,s$



Solution : (b) By using $r = \dfrac{mv\sin\theta}{qB} \Rightarrow r = \dfrac{1.67 \times 15^{27} \times 2 \times 10^{6} \times \sin 30°}{1.6 \times 10^{-19} \times 0.104} = 0.1m$

and it's time period $T = \dfrac{2\pi m}{qB} = \dfrac{2 \times \pi \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 0.104} = 2\pi \times 10^{-7}$ sec.

Example: 42 A charge particle, having charge *q* accelerated through a potential difference *V* enter a perpendicular magnetic field in which it experiences a force *F*. If *V* is increased to 5*V*, the particle will experience a force

(a) *F* (b) 5*F* (c) $\dfrac{F}{5}$ (d) $\sqrt{5}F$

Solution : (d) $\dfrac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\dfrac{2qV}{m}}$. Also $F = qvB$

$\Rightarrow F = qB\sqrt{\dfrac{2qV}{m}}$ hence $F \propto \sqrt{V}$ which gives $F' = \sqrt{5}F$.

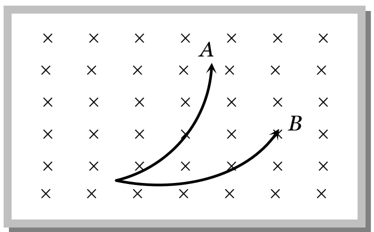
Example: 43 The magnetic field is downward perpendicular to the plane of the paper and a few charged particles are projected in it. Which of the following is true **[CPMT 1997]**

(a) A represents proton and *B* and electron

(b) Both *A* and *B* represent protons but velocity of *A* is more than that of *B*

(c) Both *A* and *B* represents protons but velocity of *B* is more than that of *A*

(d) Both *A* and *B* represent electrons, but velocity of *B* is more than that of *A*



Solution : (c) Both particles are deflecting in same direction so they must be of same sign.(*i.e.*, both *A* and *B* represents protons)

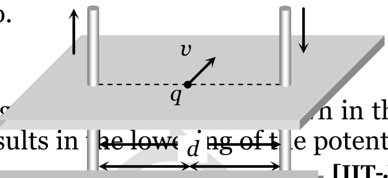
By using $r = \dfrac{mv}{qB} \Rightarrow r \propto v$

From given figure radius of the path described by particle B is more than that of A. Hence $v_B > v_A$.

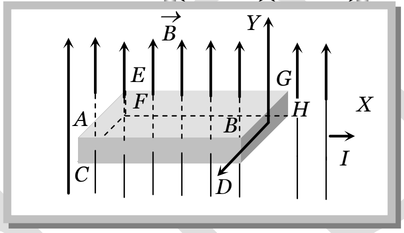
***Example*: 44** Two very long straight, particle wires carry steady currents i and $-i$ respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires. It's instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(a) $\frac{\mu_0 iqv}{2\pi d}$ (b) $\frac{\mu_0 iqv}{\pi d}$ (c) $\frac{2\mu_0 iqv}{\pi d}$ (d) Zero

Solution : (d) According to gives information following figure can be drawn, which shows that direction of magnetic field is along the direction of motion of charge so net on it is zero.



***Example*: 45** A metallic block carrying current i is subjected to a uniform magnetic induction $\vec{B}$ as shown in the figure. The moving charges experience a force F given by .... which results in the lowering of the potential of the face .... Assume the speed of the carriers to be v **[IIT-JEE 1996]**



(a) $eVB\hat{k}$, $ABCD$

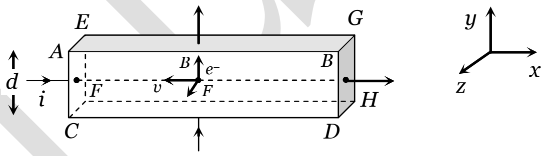
(b) $eVB\hat{k}$, $ABCD$

(c) $-eVB\hat{k}$, $ABCD$

(d) $-eVB\hat{k}$, $EFGH$

Solution : (c) As the block is of metal, the charge carriers are electrons; so for current along positive x-axis, the electrons are moving along negative x-axis, *i.e.* $\vec{v} = -vi$

and as the magnetic field is along the y-axis, *i.e.* $\vec{B} = B\hat{j}$



so $\vec{F} = q(\vec{v} \times \vec{B})$ for this case yield $\vec{F} = (-e)[-v\hat{i} \times B\hat{j}]$

i.e., $\vec{F} = evB\hat{k}$ [As $\hat{i} \times \hat{j} = \hat{k}$]

As force on electrons is towards the face $ABCD$, the electrons will accumulate on it an hence it will acquire lower potential.

Tricky example: 4

An ionised gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+ve$ x-axis and a magnetic field along the $+z$ direction then **[IIT-JEE (Screening**

(a) Positive ions deflect towards $+y$ direction and negative ions towards $-y$ direction

(b) All ions deflect towards $+y$ direction

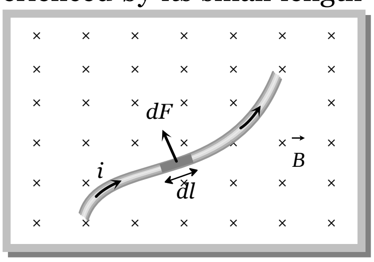
(c) All ions deflect towards $-y$ direction

(d) Positive ions deflect towards $-y$ direction and negative ions towards $+y$ direction.

Solution : (c) As the electric field is switched on, positive ion will start to move along positive x-direction and negative ion along negative x-direction. Current associated with motion of both types of ions is along positive x-direction. According to Flemings left hand rule force on both types of ions will be along negative y-direction.

Force on a Current Carrying Conductor in Magnetic Field

In case of current carrying conductor in a magnetic field force experienced by its small length element is $d\vec{F} = id\vec{l} \times \vec{B}$; $id\vec{l}$ = current element $d\vec{F} = I(d\vec{l} \times \vec{B})$



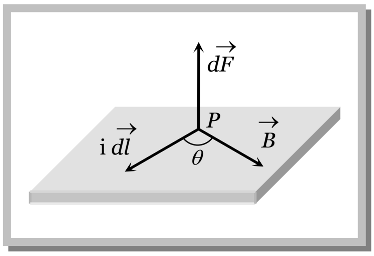
Total magnetic force $\vec{F} = \int d\vec{F} = \int i(d\vec{l} \times \vec{B})$

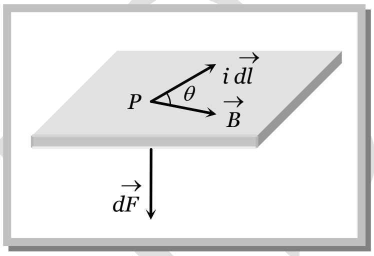
If magnetic field is uniform *i.e.*, $\vec{B}$ = constant

$$\vec{F} = i\left[\int d\vec{l}\right] \times \vec{B} = i(\vec{L}' \times \vec{B})$$

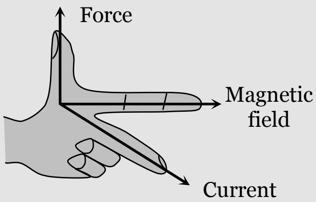
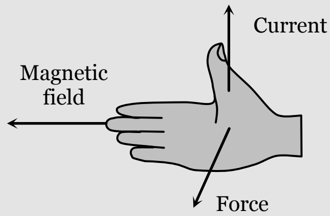
$\int d\vec{l} = \vec{L} =$ vector sum of all the length elements from initial to final point. Which is in accordance with the law of vector addition is equal to length vector $\vec{L}'$ joining initial to final point.

(1) **Direction of force :** The direction of force is always perpendicular to the plane containing $id\vec{l}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{A} \times \vec{B})$ with $\vec{A} = i\,d\vec{l}$.





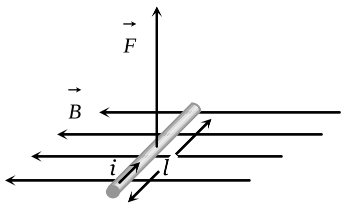
The direction of force when current element $i\,d\vec{l}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules

Fleming's left-hand rule	Right-hand palm rule
Stretch the fore-finger, central finger and thumb left hand mutually perpendicular. Then if the fore-finger points in the direction of field $\vec{B}$ and the central in the direction of current *i*, the thumb will point in the direction of force 	Stretch the fingers and thumb of right hand at right angles to each other. Then if the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current *i*, then normal to the palm will point in the direction of force 

(2) **Force on a straight wire :** If a current carrying straight conductor (length *l*) is placed in an uniform magnetic field (*B*) such that it makes an angle θ with the direction of field then force experienced by it is $F = Bil\sin\theta$

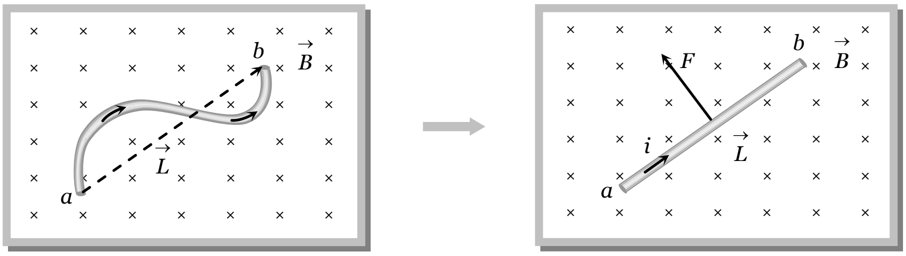
If $\theta = 0^o$, $F = 0$

If $\theta = 90^o$, $F_{\max} = Bil$



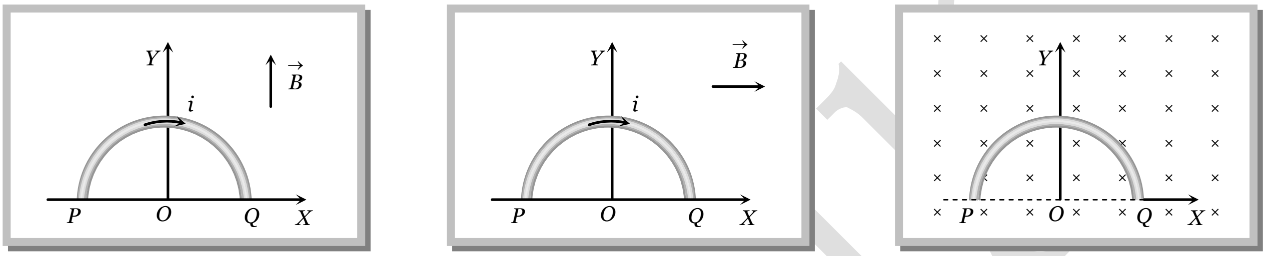
(3) **Force on a curved wire**

The force acting on a curved wire joining points *a* and *b* as shown in the figure is the same as that on a straight wire joining these points. It is given by the expression $\vec{F} = i\vec{L} \times \vec{B}$



Specific Example

The force experienced by a semicircular wire of radius R when it is carrying a current i and is placed in a uniform magnetic field of induction B as shown.



$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B\hat{i}$

So by using $\vec{F} = i(\vec{L}' \times \vec{B})$ force on the wire

$\vec{F} = i(2R)(B)(\hat{i} \times \hat{i}) \Rightarrow \vec{F} = 0$

$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B\hat{j}$

$\vec{F} = i \times 2BR(\hat{i} \times \hat{j})$

$\vec{F} = 2BiR\hat{k}$ i.e. $F = 2BiR$

(perpendicular to paper outward)

$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B(-\hat{k})$

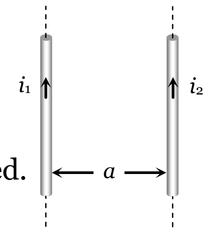
$\therefore\ \vec{F} = i \times 2BR(+\hat{j})$

$F = 2BiR$ (along Y-axis)

Force Between Two Parallel Current Carrying Conductors

When two long straight conductors carrying currents i_1 and i_2 placed parallel to each other at a distance 'a' from each other. A mutual force act between them when is given as

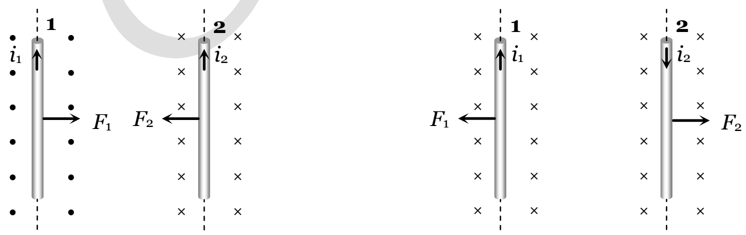
$$F_1 = F_2 = F = \frac{\mu_0}{4\pi} \cdot \frac{2 i_1 i_2}{a} \times l$$



where l is the length of that portion of the conductor on which force is to be calculated.

Hence force per unit length $\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2 i_1 i_2}{a} \frac{N}{m}$ or $\frac{F}{l} = \frac{2 i_1 i_2}{a} \frac{dyne}{cm}$

Direction of force : If conductors carries current in same direction, then force between them will be attractive. If conductor carries current in opposite direction, then force between them will be repulsive.

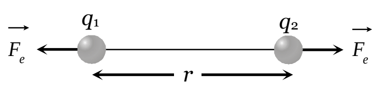


Note : ❐ If a = 1m and in free space $\frac{F}{l} = 2\times10^{-7}\ N/m$ then $i_1 = i_2 = 1 Amp$ in each identical wire.

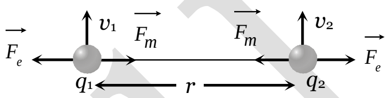
By this concept S.I. unit of Ampere is defined. This is known as **Ampere's law**.

Force Between Two Moving Charges

If two charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively and at any instant the distance between them is r, then



Stationary charges



Moving charges

Magnetic force between them is $F_m = \frac{\mu_0}{4\pi}.\frac{q_1 q_2 v_1 v_2}{r^2}$ ..... (i)

and Electric force between them is $F_e = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r^2}$ ..... (ii)

From equation (i) and (ii) $\frac{F_m}{F_e} = \mu_0\varepsilon_0 v^2$ but $\mu_0\varepsilon_0 = \frac{1}{c^2}$; where c is the velocity light in vacuum. So

$$\frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2$$

If $v << c$ then $F_m << F_e$

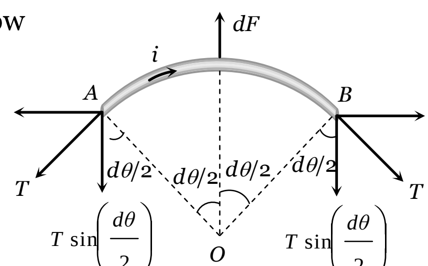
Standard Cases for Force on Current Carrying Conductors

***Case* 1 :** When an arbitrary current carrying loop placed in a magnetic field (⊥ to the plane of loop), each element of loop experiences a magnetic force due to which loop stretches and open into circular loop and tension developed in it's each part.

Specific example

In the above circular loop tension in part A and B.

In balanced condition of small part AB of the loop is shown below

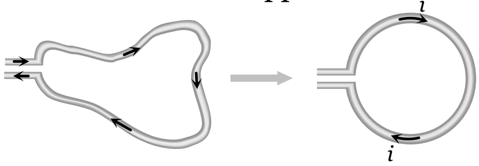


$$2T\sin\frac{d\theta}{2} = dF = Bidl \Rightarrow 2T\sin\frac{d\theta}{2} = BiRd\theta$$

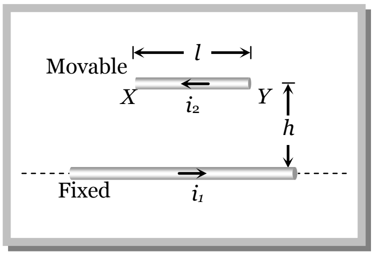
If $d\theta$ is small so, $\sin\frac{d\theta}{2} \approx \frac{d\theta}{2} \Rightarrow 2T.\frac{d\theta}{2} = BiRd\theta$

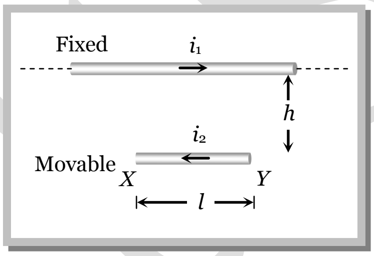
$$T = BiR, \text{ if } 2\pi R = L \text{ so } T = \frac{BiL}{2\pi}$$

Note : ❒ If no magnetic field is present, the loop will still open into a circle as in it's adjacent parts current will be in opposite direction and opposite currents repel each other.



***Case* 2 : Equilibrium of a current carrying conductor :** When a finite length current carrying wire is kept parallel to another infinite length current carrying wire, it can suspend freely in air as shown below



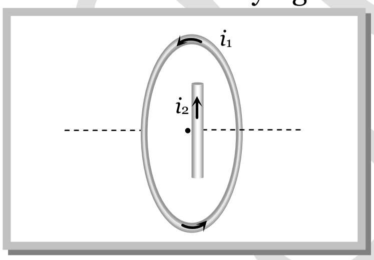


In both the situations for equilibrium of XY it's downward weight = upward magnetic force *i.e.* $\boldsymbol{mg = \frac{\mu_0}{4\pi}.\frac{2i_1i_2}{h}.l}$

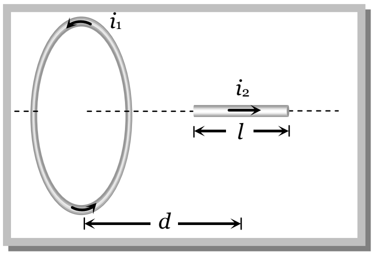
Note : ❒ In the first case if wire XY is slightly displaced from its equilibrium position, it executes SHM and it's time period is given by $T = 2\pi\sqrt{\frac{h}{g}}$.

❒ If direction of current in movable wire is reversed then it's instantaneous acceleration produced is $2g\downarrow$.

***Case* 3 : Current carrying wire and circular loop :** If a current carrying straight wire is placed in the magnetic field of current carrying circular loop.

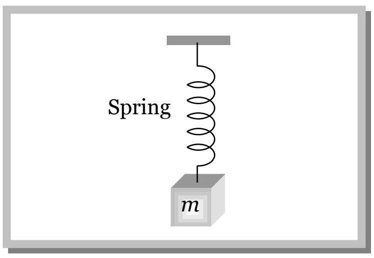


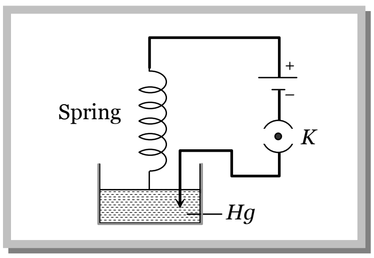
Wire is placed in the perpendicular magnetic field due to coil at it's centre, so it will experience a maximum force $F = Bil = \frac{\mu_0 i_1}{2r} \times i_2 l$



wire is placed along the axis of coil so magnetic field produced by the coil is parallel to the wire. Hence it will not experience any force.

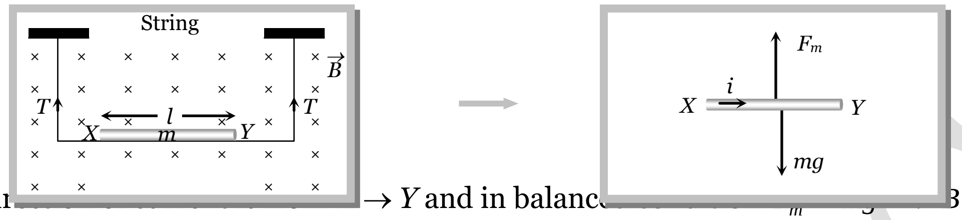
***Case* 4 : Current carrying spring :** If current is passed through a spring, then it will contract because current will flow through all the turns in the same direction.





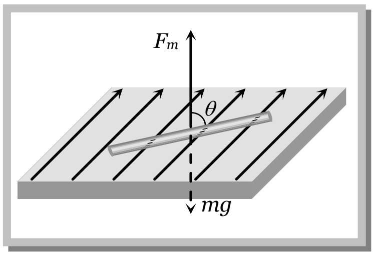
***Case* 5 : Tension less strings :** In the following figure the value and direction of current through the conductor *XY* so that strings becomes tensionless?

Strings becomes tensionless if weight of conductor *XY* balanced by magnetic force (F_m).



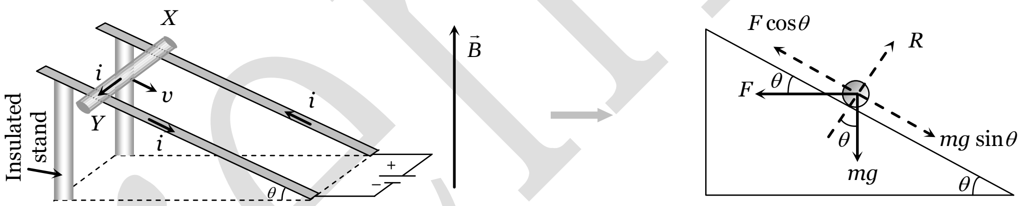
Hence di… $\rightarrow Y$ and in balanc… $Bil = mg \Rightarrow i = \frac{mg}{Bl}$

***Case* 6 :** A current carrying conductor floating in air such that it is making an angle θ with the direction of magnetic field, while magnetic field and conductor both lies in a horizontal plane.



In equilibrium $mg = Bil\sin\theta \Rightarrow i = \frac{mg}{Bl\sin\theta}$

***Case* 7 : Sliding of conducting rod on inclined rails :** When a conducting rod slides on conducting rails.

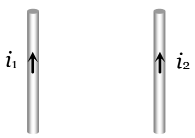


In the following situation conducting rod (*X*, *Y*) slides at constant velocity if

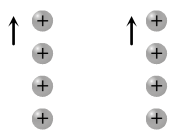
$$F\cos\theta = mg\sin\theta \Rightarrow Bil\cos\theta = mg\sin\theta \Rightarrow B = \frac{mg}{il}\tan\theta$$

Concepts

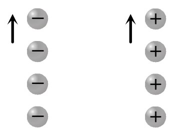
- *Electric force is an absolute concept while magnetic force is a relative concept for an observer.*
- *The nature of force between two parallel charge beams decided by electric force, as it is dominator. The nature of force between two parallel current carrying wires decided by magnetic force.*



$F_{net} = F_m$ only



$F_e \rightarrow$ repulsion
$F_m \rightarrow$ attraction
$F_{net} \rightarrow$ repulsion (Due to this force these beams diverges)



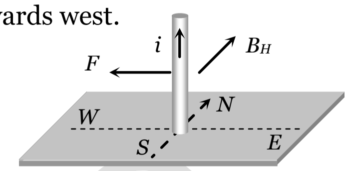
$F_e \rightarrow$ attraction
$F_m \rightarrow$ repulsion
$F_{net} \rightarrow$ attraction (Due to this force these beams converges)

Example

Example: 46 A vertical wire carrying a current in the upward direction is placed in a horizontal magnetic field directed towards north. The wire will experience a force directed towards

(a) North (b) South (c) East (d) West

Solution : (d) By applying Flemings left hand rule, direction of force is found towards west.



Example: 47 3 *A* of current is flowing in a linear conductor having a length of 40 *cm*. The conductor is placed in a magnetic field of strength 500 *gauss* and makes an angle of 30° with the direction of the field. It experiences a force of magnitude

(a) $3 \times 10^4\ N$ (b) $3 \times 10^2\ N$ (c) $3 \times 10^{-2}\ N$ (d) $3 \times 10^{-4}\ N$

Solution : (c) By using $F = Bil\sin\theta \Rightarrow F = (500 \times 10^{-4}) \times 0.4 \times \sin 30^\circ \Rightarrow 3 \times 10^{-2}\ N$.

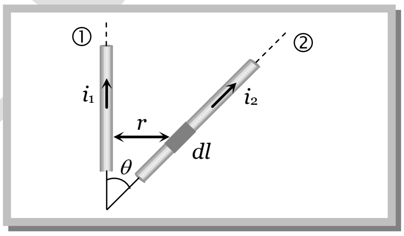
Example: 48 Wires 1 and 2 carrying currents t_1 and t_2 respectively are inclined at an angle θ to each other. What is the force on a small element *dl* of wire 2 at a distance of *r* from 1 (as shown in figure) due to the magnetic field of wire 1 **[AIEEE 2002]**

(a) $\dfrac{\mu_0}{2\pi r} i_1, i_2 dl \tan\theta$

(b) $\dfrac{\mu_0}{2\pi r} i_1, i_2\, dl \sin\theta$

(c) $\dfrac{\mu_0}{2\pi r} i_1, i_2\, dl \cos\theta$

(d) $\dfrac{\mu_0}{4\pi r} i_1, i_2\, dl \sin\theta$



Solution : (c) Length of the component *dl* which is parallel to wire (1) is *dl* cos *θ*, so force on it

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{r}(dl\cos\theta) = \frac{\mu_0 i_1 i_2 dl\cos\theta}{2\pi r}.$$

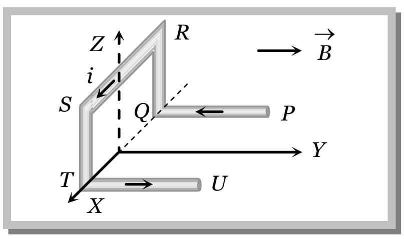
Example: 49 A conductor *PQRSTU*, each side of length *L*, bent as shown in the figure, carries a current *i* and is placed in a uniform magnetic induction *B* directed parallel to the positive *Y*-axis. The force experience by the wire and its direction are

(a) 2*iBL* directed along the negative Z-axis

(b) 5*iBL* directed along the positive *Z*-axis

(c) *iBL* direction along the positive Z-axis

(d) 2*iBL* directed along the positive Z-axis

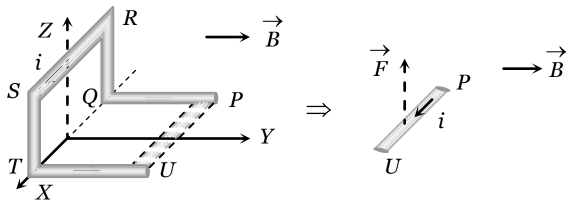


Solution : (c) As *PQ* and *UT* are parallel to *Q*, therefore $F_{PQ} = F_{UT} = 0$

The current in *TS* and *RQ* are in mutually opposite direction. Hence, $F_{TS} - F_{RQ} = 0$

Therefore the force will act only on the segment *SR* whose value is *Bil* and it's direction is +*z*.

Alternate method :



The given shape of the wire can be replaced by a straight wire of length l between P and U as shown below

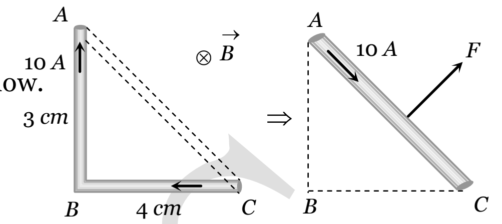
Hence force on replaced wire PU will be $F = Bil$

and according to *FLHR* it is directed towards $+z$-axis

Example: 50 A conductor in the form of a right angle ABC with $AB = 3cm$ and $BC = 4\ cm$ carries a current of $10\ A$. There is a uniform magnetic field of $5T$ perpendicular to the plane of the conductor. The force on the conductor will be

(a) $1.5\ N$ (b) $2.0\ N$ (c) $2.5\ N$ (d) $3.5\ N$

Solution : (c) According to the question figure can be drawn as shown below.



Force on the conductor ABC = Force on the conductor AC

$= 5 \times 10 \times (5 \times 10^{-2})$

$= 2.5\ N$

Example: 51 A wire of length l carries a current i along the X-axis. A magnetic field exists which is given as $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})\ T$. Find the magnitude of the magnetic force acting on the wire

(a) $B_0 il$ (b) $B_0 i l \times \sqrt{2}$ (c) $2B_0 i l$ (d) $\frac{1}{\sqrt{2}} \times B_0 i l$

Solution : (b) By using $\vec{F} = i(\vec{l} \times \vec{B}) \Rightarrow \vec{F} = i[l\hat{i} \times B_0(\hat{i} + \hat{j} + \hat{k})] = B_0 il[\hat{i} \times (\hat{i} + \hat{j} + \hat{k})]$

$\Rightarrow \quad \vec{F} = B_0 il[\hat{i} \times \hat{i} + \hat{i} \times \hat{j} + \hat{i} \times \hat{k}] = B_0 il[\hat{k} - \hat{j}] \quad \{\hat{i} \times \hat{i} = 0, \hat{i} \times \hat{j} = \hat{k}, \hat{i} \times \hat{k} = -\hat{j}\}$

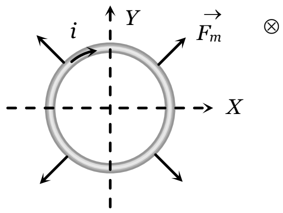
It's magnitude $F = \sqrt{2} B_0 il$

Example: 52 A conducting loop carrying a current i is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to **[IIT-JEE (Screening) 2003]]**

(a) Contract (b) Expand

(c) Move towards $+ ve\ x$-axis (d) Move towards $- ve\ x$-axis

Solution : (b) Net force on a current carrying loop in uniform magnetic field is zero. Hence the loop can't translate. So, options (c) and (d) are wrong. From Flemings left hand rule we can see that if magnetic field is perpendicular to paper inwards and current in the loop is clockwise (as shown) the magnetic force $\vec{F_m}$ on each element of the loop is radially outwards, or the loops will have a tendency to expand.



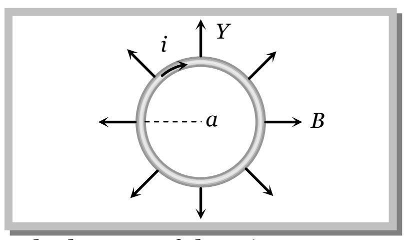
Example: 53 A circular loop of radius a, carrying a current i, is placed in a two-dimensional magnetic field. The centre of the loop coincides with the centre of the field. The strength of the magnetic field at the periphery of the loop is B. Find the magnetic force on the wire

(a) $\pi\, i\, a\, B$

(b) $4\pi\, i\, a\, B$

(c) Zero

(d) $2\pi\, i\, a\, B$

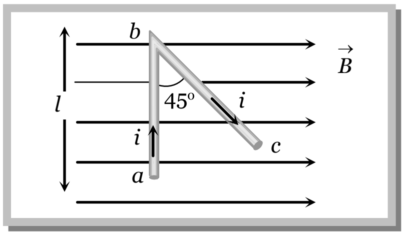


Solution : (d) The direction of the magnetic force will be vertically downwards at each element of the wire.

Thus $F = Bil = Bi\,(2\pi a) = 2\pi iaB$.

Example: 54 A wire abc is carrying current i. It is bent as shown in fig and is placed in a uniform magnetic field of magnetic induction B. Length $ab = l$ and $\angle abc = 45^o$. The ratio of force on ab and on bc is

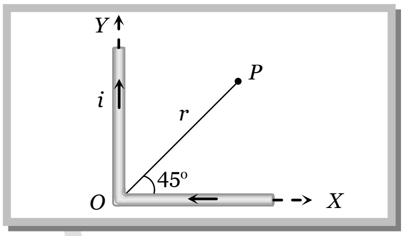
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 1 (d) $\frac{2}{3}$



Solution : (c) Force on portion *ab* of wire $F_1 = Bil \sin 90^o = Bil$

Force on portion *bc* of wire $F_2 = Bi\left(\frac{l}{\sqrt{2}}\right)\sin 45^o = Bil$. So $\frac{F_1}{F_2} = 1$.

***Example*: 55** Current *i* flows through a long conducting wire bent at right angle as shown in figure. The magnetic field at a point *P* on the right bisector of the angle *XOY* at a distance *r* from *O* is



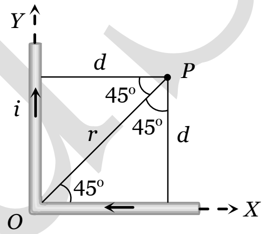
(a) $\frac{\mu_0 i}{\pi r}$

(b) $\frac{2\mu_0 i}{\pi r}$

(c) $\frac{\mu_0 i}{4\pi r}(\sqrt{2}+1)$

(d) $\frac{\mu_0}{4\pi}.\frac{2i}{r}(\sqrt{2}+1)$

Solution : (d) By using $B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$, from figure $d = r\sin 45^o = \frac{r}{\sqrt{2}}$



Magnetic field due to each wire at *P* $B = \frac{\mu_0}{4\pi}.\frac{i}{(r/\sqrt{2})}(\sin 45^o + \sin 90^o)$

$= \frac{\mu_0}{4\pi}.\frac{i}{r}(\sqrt{2}+1)$

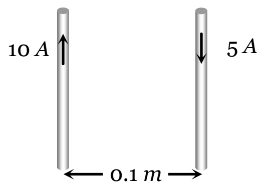
Hence net magnetic field at *P* $B_{net} = 2\times\frac{\mu_0}{4\pi}.\frac{i}{r}(\sqrt{2}+1) = \frac{\mu_0}{2\pi}.\frac{i}{r}(\sqrt{2}+1)$

***Example*: 56** A long wire A carries a current of 10 *amp*. Another long wire *B*, which is parallel to *A* and separated by 0.1 *m* from *A*, carries a current of 5 *amp*. in the opposite direction to that in *A*. What is the magnitude and nature of the force experienced per unit length of *B* $[\mu_0 = 4\pi\times 10^{-7}$ *weber/amp – m*]

(a) Repulsive force of $10^{-4}\ N/m$ (b) Attractive force of $10^{-4}\ N/m$

(c) Repulsive force of $2\pi\times 10^{-5}\ N/m$ (d) Attractive force of $2\pi\times 10^{-5}\ N/m$

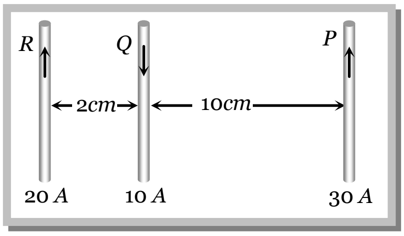
Solution : (a) By using $\frac{F}{l} = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{a}$



$\Rightarrow \frac{F}{l} = 10^{-7}\times\frac{2\times 10\times 5}{0.1} = 10^{-4}\ N$

Wires are carrying current in opposite direction so the force will be repulsive.

***Example*: 57** Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 *cm* length of wire *Q* is **[MP PET 1997]**



(a) 1.4×10^{-4} *N* towards the right

(b) 1.4×10^{-4} *N* towards the left

(c) 2.6×10^{-4} *N* to the right

(d) 2.6×10^{-4} *N* to the left

Solution : (a) Force on wire *Q* due to *R* ; $F_R = 10^{-7}\times\frac{2\times 20\times 10}{(2\times 10^{-2})}\times(10\times 10^{-2}) = 2\times 10^{-4}\ m$ (Repulsive)

Force on wire *Q* due to *P* ; $F_P = 10^{-7}\times 2\times\frac{10\times 30}{(10\times 10^{-2})}\times(10\times 10^{-2}) = 0.6\times 10^{-4}\ N$ (Repulsive)

Hence net force $F_{net} = F_R - F_P = 2 \times 10^{-4} - 0.6 \times 10^{-4} = 1.4 \times 10^{-4}\ N$ (towards right *i.e.* in the direction of $\overrightarrow{F_R}$.

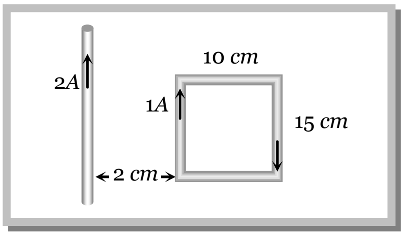
Example: 58 What is the net force on the coil **[DCE 2000]**

(a) $25 \times 10^{-7}\ N$ moving towards wire

(b) $25 \times 10^{-7}\ N$ moving away from wire

(c) $35 \times 10^{-7}\ N$ moving towards wire

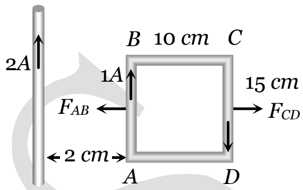
(d) $35 \times 10^{-7}\ N$ moving away from wire



Solution : (a) Force on sides *BC* and *CD* cancel each other.

Force on side *AB* $F_{AB} = 10^{-7} \times \frac{2 \times 2 \times 1}{2 \times 10^{-2}} \times 15 \times 10^{-2} = 3 \times 10^{-6}\ N$

Force on side *CD* $F_{AB} = 10^{-7} \times \frac{2 \times 2 \times 1}{12 \times 10^{-2}} \times 15 \times 10^{-2} = 0.5 \times 10^{-6}\ N$



Hence net force on loop = $F_{AB} - F_{CD} = 25 \times 10^{-7}\ N$ (towards the wire).

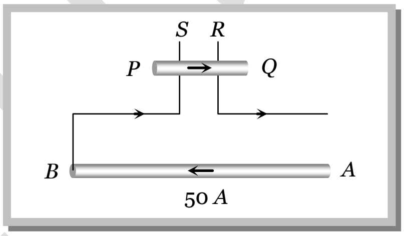
Example: 59 A long wire *AB* is placed on a table. Another wire *PQ* of mass 1.0 *g* and length 50 *cm* is set to slide on two rails *PS* and *QR*. A current of 50*A* is passed through the wires. At what distance above *AB*, will the wire *PQ* be in equilibrium

(a) 25 *mm*

(b) 50 *mm*

(c) 75 *mm*

(d) 100 *mm*



Solution : (a) Suppose in equilibrium wire *PQ* lies at a distance *r* above the wire *AB*

Hence in equilibrium $mg = Bil \Rightarrow mg = \frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right) \times il \Rightarrow\ 10^{-3} \times 10 = 10^{-7} \times \frac{2 \times (50)^2}{r} = 0.5\ \Rightarrow\ r = 25\,mm$

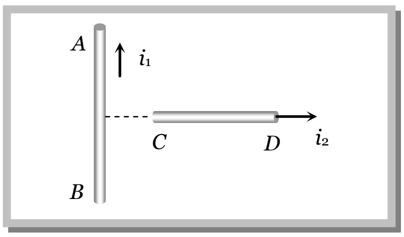
Example: 60 An infinitely long, straight conductor *AB* is fixed and a current is passed through it. Another movable straight wire *CD* of finite length and carrying current is held perpendicular to it and released. Neglect weight of the wire

(a) The rod *CD* will move upwards parallel to itself

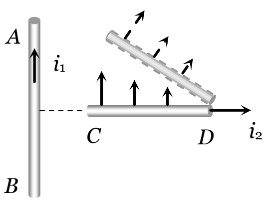
(b) The rod *CD* will move downward parallel to itself

(c) The rod *CD* will move upward and turn clockwise at the same time

(d) The rod *CD* will move upward and turn anti –clockwise at the same time



Solution : (c) Since the force on the rod *CD* is non-uniform it will experience force and torque. From the left hand side it can be seen that the force will be upward and torque is clockwise.



Tricky example: 5

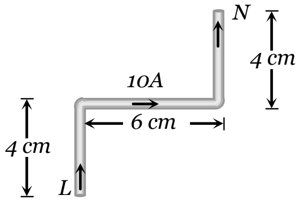
A current carrying wire LN is bent in the from shown below. If wire carries a current of 10 A and it is placed in a magnetic field a $5T$ which acts perpendicular to the paper outwards then it will experience a force

(a) Zero

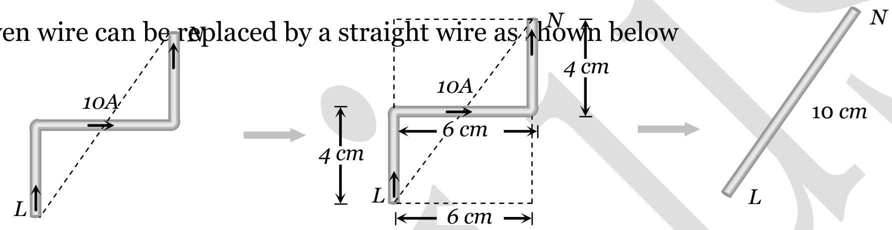
(b) 5 N

(c) 30 N

(d) 20 N



Solution : (b) The given wire can be replaced by a straight wire as shown below



Hence force experienced by the wire $F = Bil = 5 \times 10 \times 0.1 = 5\,N$

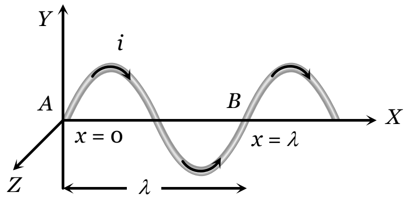
Tricky example: 6

A wire, carrying a current i, is kept in $X - Y$ plane along the curve $y = A \sin\left(\frac{2\pi}{\lambda}x\right)$. A magnetic field B exists in the Z-direction find the magnitude of the magnetic force on the portion of the wire between $x = 0$ and $x = \lambda$

(a) $i\lambda B$ (b) Zero (c) $\frac{i\lambda B}{2}$ (d) $3/2i\lambda B$

Solution : (a) The given curve is a sine curve as shown below.

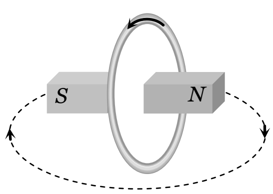
The given portion of the curved wire may be treated as a straight wire AB of length λ which experiences a magnetic force $F_m = Bi\lambda$

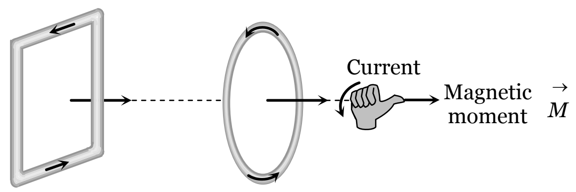


Current Loop As a Magnetic Dipole

A current carrying circular coil behaves as a bar magnet whose magnetic moment is $M = NiA$; *Where* N = Number of turns in the coil, i = Current through the coil and A = Area of the coil

Magnetic moment of a current carrying coil is a vector and it's direction is given by right hand thumb rule

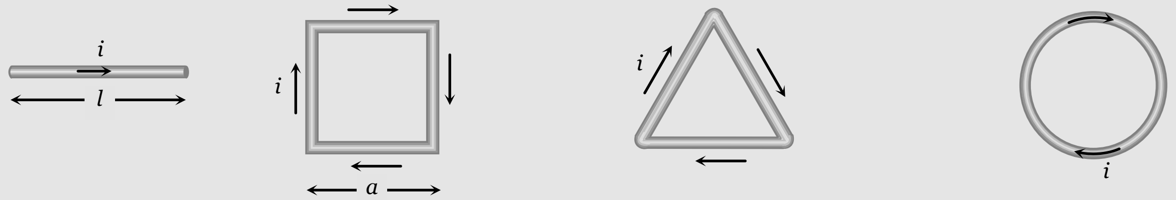




Specific examples

A given length constant current carrying straight wire moulded into different shaped loops. as shown

Linear | Square | Equilateral | Circle



$l = 4a$ | $l = 3a$ | $l = 2\pi r$

$A = a^2$ | $A = \frac{\sqrt{3}}{4}a^2$ | $A = \pi r^2$

$M = ia^2 = \frac{il^2}{16}$ | $M = i\left(\frac{\sqrt{3}}{4}a^2\right) = \frac{\sqrt{3}\,il^2}{36}$ | $M = i(\pi r^2) = \frac{il^2}{4\pi}$ ← max.

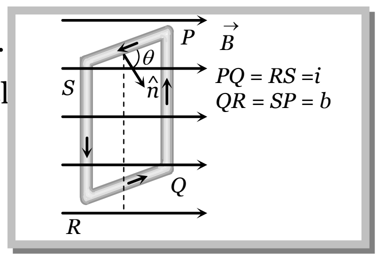
Note : ❒ For a given perimeter circular shape have maximum area. Hence maximum magnetic moment.

❒ For a any loop or coil $\vec{B}$ and $\vec{M}$ are always parallel.

Behaviour of Current loop In a Magnetic Field

(1) **Torque**

Consider a rectangular current carrying coil *PQRS* having *N* turns and area *A*, placed in a uniform field *B*, in such a way that the normal $(\hat{n})$ to the coil makes an angle θ with the direction of *B*. the coil experiences a torque given by $\tau = NBiA\sin\theta$. Vectorially $\vec{\tau} = \vec{M} \times \vec{B}$



(i) τ is zero when $\theta = 0$, *i.e.*, when the plane of the coil is perpendicular to the field.

(ii) τ is maximum when $\theta = 90^\circ$, *i.e.*, the plane of the coil is parallel to the fiel

$\Rightarrow \tau_{max} = NBiA$

The above expression is valid for coils of all shapes.

(2) **Workdone**

If coil is rotated through an angle θ from it's equilibrium position then required work. $W = MB(1 - \cos\theta)$. It is maximum when $\theta = 180^\circ \Rightarrow W_{max} = 2\,MB$

(3) **Potential energy**

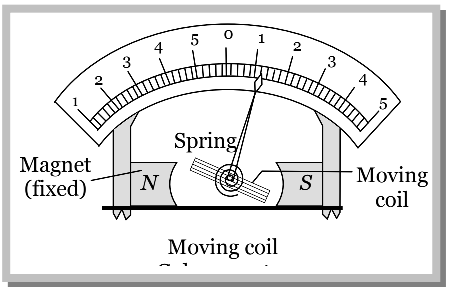
Is given by $U = -MB\cos\theta \Rightarrow U = \vec{M}.\vec{B}$

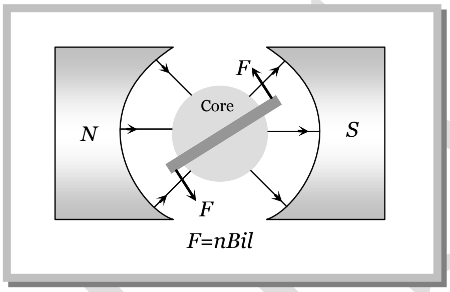
Note : ❒ Direction of $\vec{M}$ is found by using Right hand thumb rule according to which curl the fingers of right hand in the direction of circulation of conventional current, then the thumb gives the direction of $\vec{M}$.

❒ Instruments such as electric motor, moving coil galvanometer and tangent galvanometers *etc.* are based on the fact that a current-carrying coil in a uniform magnetic field experiences a torque (or couple).

Moving coil galvanometer

In a moving coil galvanometer the coil is suspended between the pole pieces of a strong horse-shoe magnet. The pole pieces are made cylinderical and a soft iron cylinderical core is placed within the coil without touching it. This makes the field radial. In such a field the plane of the coil always remains parallel to the field. Therefore $\theta = 90^{o}$ and the deflecting torque always has the maximum value.





$\tau_{\text{def}} = NBiA$ ......(i)

coil deflects, a restoring torque is set up in the suspension fibre. If α is the angle of twist, the restoring torque is

$\tau_{\text{rest}} = C\alpha$ .....(ii) where C is the torsional constant of the fibre.

When the coil is in equilibrium.

$$NBiA = C\alpha \Rightarrow i = \frac{C}{NBA}\alpha \Rightarrow i = K\alpha,$$

Where $K = \frac{C}{NBA}$ is the galvanometer constant. This linear relationship between i and α makes the moving coil galvanometer useful for current measurement and detection.

Current sensitivity : The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$S_i = \frac{\alpha}{i} = \frac{NBA}{C}$$

Thus in order to increase the sensitivity of a moving coil galvanometer, N, B and A should be increased and C should be decreased.

Quartz fibres can also be used for suspension of the coil because they have large tensile strength and very low value of k.

Voltage sensitivity (S_V) : Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit applied to it.

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{iR} = \frac{S_i}{R} = \frac{NBA}{RC}$$

Concepts

☞ *The field in a moving coil galvanometer radial in nature in order to have a linear relation between the current and the deflection.*

☞ *A rectangular current loop is in an arbitrary orientation in an external magnetic field. No work required to rotate the loop* about an axis perpendicular to it's plane.

☞ *Moving coil galvanometer can be made ballistic by using a non-conducting frame (made of ivory or bamboo) instead of a metallic frame.*

Example

***Example*: 61** A circular coil of radius 4 *cm* and 20 turns carries a current of 3 *ampere*. It is placed in a magnetic field of 0.5 *T*. The magnetic dipole moment of the coil is **[MP PMT 2001]**

(a) 0.60 $A\text{-}m^2$ (b) 0.45 $A\text{-}m^2$ (c) 0.3 $A\text{-}m^2$ (d) 0.15 $A\text{-}m^2$

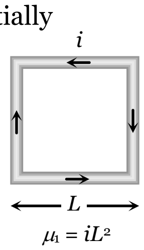
Solution : (c) $M = niA \Rightarrow M = 20 \times 3 \times \pi(4 \times 10^{-2})^2 = 0.3\ A\text{-}m^2$.

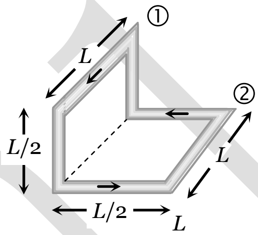
***Example*: 62** A steady current i flows in a small square loop of wire of side L in a horizontal plane. The loop is now folded about its middle such that half of it lies in a vertical plane. Let $\vec{\mu_1}$ and $\vec{\mu_2}$ respectively denote the magnetic moments due to the current loop before and after folding. Then **[IIT-JEE 1993]**

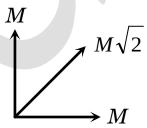
(a) $\vec{\mu_2} = 0$ (b) $\vec{\mu_1}$ and $\vec{\mu_2}$ are in the same direction

(c) $\frac{|\vec{\mu_1}|}{|\vec{\mu_2}|} = \sqrt{2}$ (d) $\frac{|\vec{\mu_1}|}{|\vec{\mu_2}|} = \left(\frac{1}{\sqrt{2}}\right)$

Solution : (c) Initially Finally







$$M = \text{magnetic moment due to each part} = i\left(\frac{L}{2}\right) \times L = \frac{iL^2}{2} = \frac{\mu_1}{2}$$

$$\therefore\ \mu_2 = M\sqrt{2} = \frac{\mu_1}{2} \times \sqrt{2} = \frac{\mu_1}{\sqrt{2}}$$

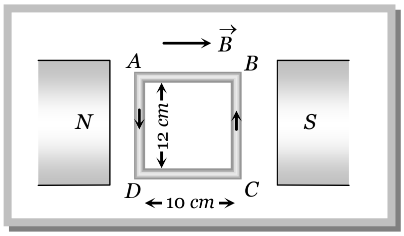
***Example*: 63** A coil of 50 turns is situated in a magnetic field b = 0.25$weber/m^2$ as shown in figure. A current of 2A is flowing in the coil. Torque acting on the coil will be

(a) 0.15 N

(b) 0.3 N

(c) 0.45 N

(d) 0.6 N



Solution : (b) Since plane of the coil is parallel to magnetic field. So $\theta = 90^o$

Hence $\tau = NBiA \sin 90^o = NBiA = 50 \times 0.25 \times 2 \times (12 \times 10^{-2} \times 10 \times 10^{-2}) = 0.3\ N$.

***Example*: 64** A circular loop of area 1 cm^2, carrying a current of 10 A, is placed in a magnetic field of 0.1 T perpendicular to the plane of the loop. The torque on the loop due to the magnetic field is

(a) Zero (b) 10^{-4} $N\text{-}m$ (c) 10^{-2} $N\text{-}m$ (d) 1 $N\text{-}m$

Solution : (a) $\tau = NBiA \sin\theta$; given $\theta = 0$ so $\tau = 0$.

***Example*: 65** A circular coil of radius 4 *cm* has 50 turns. In this coil a current of 2 A is flowing. It is placed in a magnetic field of 0.1 $weber/m^2$. The amount of work done in rotating it through 180° from its equilibrium position will be

[CPMT 1977]

(a) 0.1 J (b) 0.2 J (c) 0.4 (d) 0.8 J

Solution : (a) Work done in rotating a coil through an angle θ from it's equilibrium position is $W = MB(1 - \cos\theta)$ where $\theta = 180^o$ and $M = 50 \times 2 \times \pi(4 \times 10^{-2}) = 50.24 \times 10^{-2}$ *A-m²*. Hence $W = 0.1\, J$

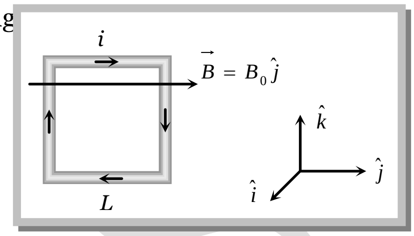
***Example*: 66** A wire of length L is bent in the form of a circular coil and current i is passed through it. If this coil is placed in a magnetic field then the torque acting on the coil will be maximum when the number of turns is

(a) As large as possible (b) Any number (c) 2 (d) 1

Solution : (d) $\tau_{\max} = MB$ or $\tau_{\max} = ni\pi a^2 B$. Let number of turns in length l is n so $l = n(2\pi a)$ or $a = \dfrac{l}{2\pi n}$

$$\Rightarrow \tau_{\max} = \frac{ni\pi Bl^2}{4\pi^2 n^2} = \frac{l^2 iB}{4\pi n_{\min}} \Rightarrow \tau_{\max} \propto \frac{1}{n_{\min}} \Rightarrow n_{\min} = 1$$

***Example*: 67** A square coil of N turns (with length of each side equal L) carrying current i is placed in a uniform magnetic field $\vec{B} = B_0\hat{j}$ as shown in figure. What is the torque acting



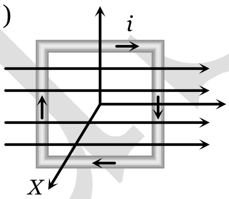
(a) $+B_0 NiL^2\hat{k}$

(b) $-B_0 NiL^2\hat{k}$

(c) $+B_0 NiL^2\hat{j}$

(d) $-B_0 NiL^2\hat{j}$

Solution : (b) The magnetic field is $\vec{B} = B_0\hat{j}$ and the magnetic moment $\vec{m} = i\vec{A} = -i(NL^2\hat{i})$



The torque is given by $\vec{\tau} = \vec{m} \times \vec{B}$

$= -iNL^2\hat{i} \times B_0\hat{j} = -iNB_0 L^2\hat{i} \times \hat{j}$

$= -iNB_0 L^2\hat{k}$

***Example*: 68** The coil of a galvanometer consists of 100 *turns* and effective area of 1 *square cm*. The restoring couple is 10^{-8} *N-m rad*. The magnetic field between the pole pieces is 5 *T*. The current sensitivity of this galvanometer will be **[MP PMT 1997]**

(a) 5×10^4 *rad/μ amp* (b) 5×10^{-6} *per amp* (c) 2×10^{-7} *per amp* (d) 5 *rad./μ amp*

Solution : (d) Current sensitivity $(S_i) = \dfrac{\theta}{i} = \dfrac{NBA}{C} \Rightarrow \dfrac{\theta}{i} = \dfrac{100 \times 5 \times 10^{-4}}{10^{-8}} = 5\, rad/\mu\, amp$.

***Example*: 69** The sensitivity of a moving coil galvanometer can be increased by **[SCRA 2000]]**

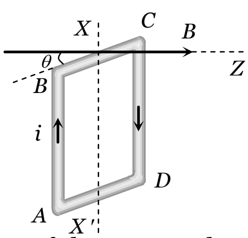
(a) Increasing the number of turns in the coil (b) Decreasing the area of the coil

(c) Increasing the current in the coil (d) Introducing a soft iron core inside the coil

Solution : (a) Sensitivity $(S_i) = \dfrac{NBA}{C} \Rightarrow S_i \propto N$.

Tricky example: 7

The square loop *ABCD*, carrying a current *i*, is placed in uniform magnetic field *B*, as shown. The loop can rotate about the axis *XX'*. The plane of the loop makes and angle $\theta\,(\theta < 90^o)$ with the direction of *B*. Through what angle will the loop rotate by itself before the torque on it becomes zero



(a) θ

(b) $90^o - \theta$

(c) $90^o + \theta$

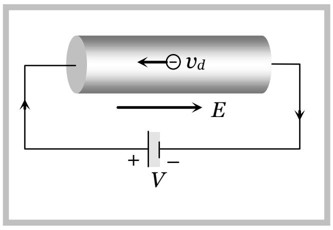
(d) $180^o - \theta$

Solution : (c) In the position shown, *AB* is outside and *CD* is inside the plane of the paper. The Ampere force on *AB* acts into the paper. The torque on the loop will be clockwise, as seen from above. The loop must rotate through an angle $(90^o + \theta)$ before the plane of the loop becomes normal to the direction of *B* and the torque becomes zero.

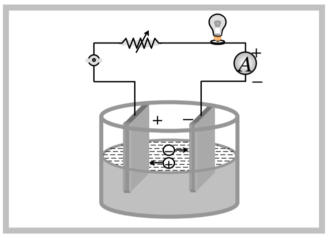
genius PHYSICS

Current Electricity

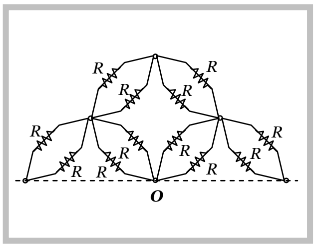
Notes by Pradeep Kshetrapal



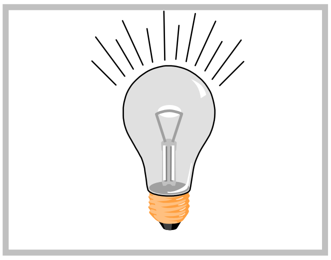
Electric current and Resistance



Cell, Kirchoff's law and Measuring instruments



Determination of resistance

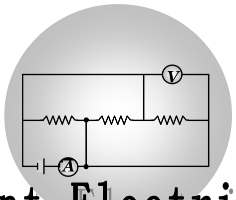


Heating effect of current

Formulas in current electricity (Direct Current)

1	Electric Current	i=q/t	"q" is charge passing in normal direction through a cross section of conductor in time "t"
2	Drift velocity V_d with Electric field	$V_d = \frac{-eE\tau}{m}$	e is charge and m is mass on electron, E is electric field, τ is relaxation time.
3	Current I with Drift velocity V_d	$I = n\,e\,A\,V_d$	n is number density with of free electrons, A is area of cross section.
4	Mobility of charge "μ"	$\mu = V_d / E = \frac{q\tau}{m}$	
5	Mobility and drift velocity	$V_d = \mu_e E$	
6	Current and Mobility	$I = Ane \times \mu_e E$	
7	Resistance, P.D., and Current	$R = V / I$	V Potential Difference, I Current .
8	Resistance R with specific Res.	$R = \rho \frac{l}{A}$	l is length of conductor and A is area of cross section
9	Specific Resistance, ρ	$\rho = R \frac{A}{l}$	
10	Resistivity with electrons	$\rho = \frac{m}{ne^2\tau}$	
11	Current density J	$\vec{J} = I / \vec{A}$	I is current, J current density, A is area of cross section
12	Current density magnitude	$J\,A\cos\theta = I$	θ is angle between $\vec{J}$ and $\vec{A}$
13	Conductance G	$G = 1/R$	
14	Conductivity σ	$\sigma = 1/\rho$	ρ is specific resistance
15	Microscopic form of Ohms Law	$J = \sigma E$	E is electric field
16	Temperature coefficient of Resistance α	$\alpha = \frac{R_t - R_0}{R_0 \times t}$	R_o is resistance at 0_o C. R_t is resistance at t^o and "t" is temperature difference.
17	Resistances in series	$R = R_1 + R_2 + R_3$	Same current through all resistances (circuit Current
	Resistances in parallel	$1/R_e = 1/R_1 + 1/R_2 + 1/R_3$	Same P.D. across each resistance (V of cell)
18	In a cell, emf and internal resistance	$I = \frac{E}{R+r}$	I is current, E is emf, R is external resistance, r is internal resistance.
19	In a circuit with a cell	$V = E - Ir$	V is terminal potential difference
20	n Cells of emf E in series	$Emf = nE$	
21	Resistance of n cells in series	$nr + R$	r is internal resistance of one cell, R external Resistance
22	Current in circuit with n cells in series	$I = \frac{nE}{R+nr}$	r is internal resistance of one cell, R external Resistance
23	n cells in parallel, then emf	$emf = E$	
24	n cells in parallel, resistance	$R + r/n$	R external resistance, r internal resistance
25	Cells in mixed group, condition for maximum current	$R = \frac{nr}{m}$	n is number of cells in one row, m is number of rows. r is internal resistance, R external resis.
26	Internal resistance of a cell	$r = \left(\frac{E-V}{V}\right) \times R$	E is emf, V is terminal Potential difference, R is external resistance.
27	Power of a circuit	$P = I.V = I^2R = V^2/R$	

28	Energy consumed	$E = I.V.\Delta T$	ΔT is time duration
29	Kirchoff Law (junction rule)	$\Sigma i = 0$	Sum of currents at junction is zero.
30	Kirchoff Law (Loop rule)	$\Sigma V = 0$	In a loop sum of all p.d.*s* is Zero

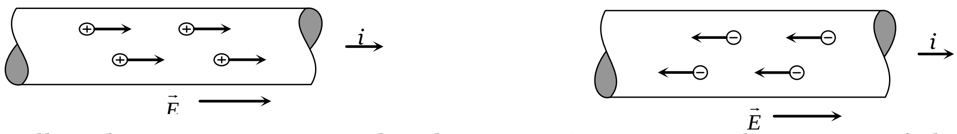


Current Electricity

Electric Current

(1) **Definition :** The time rate of flow of charge through any cross-section is called current. So if through a cross-section, ΔQ charge passes in time Δt then $i_{av} = \frac{\Delta Q}{\Delta t}$ and instantaneous current $i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$. If flow is uniform then $i = \frac{Q}{t}$. Current is a scalar quantity. It's S.I. unit is *ampere* (A) and C.G.S. unit is *emu* and is called *biot* (Bi), or *ab ampere*. 1A = (1/10) Bi (*ab amp.*)

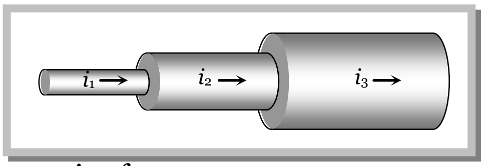
(2) **The direction of current :** The conventional direction of current is taken to be the direction of flow of positive charge, *i.e.* field and is opposite to the direction of flow of negative charge as shown below.



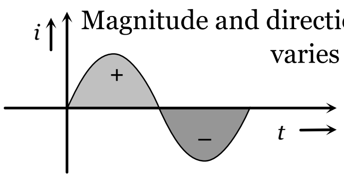
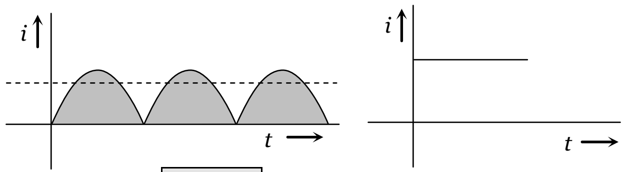
Though conventionally a direction is associated with current (Opposite to the motion of electron), it is not a vector. It is because the current can be added algebraically. Only scalar quantities can be added algebraically not the vector quantities.

(3) **Charge on a current carrying conductor :** In conductor the current is caused by electron (free electron). The no. of electron (negative charge) and proton (positive charge) in a conductor is same. Hence the net charge in a current carrying conductor is zero.

(4) **Current through a conductor of non-uniform cross-section :** For a given conductor current does not change with change in cross-sectional area. In the following figure $i_1 = i_2 = i_3$



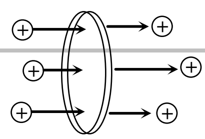
(5) **Types of current :** Electric current is of two type :

Alternating current (ac)	Direct current (dc)
(i) Magnitude and direction both varies with time  ac → Rectifier → dc	(i) (Pulsating dc) (Constant dc)  dc → Inverter → ac
(ii) Shows heating effect only	(ii) Shows heating effect, chemical effect and magnetic effect of current
(iii) It's symbol is —(~)—	(iii) It's symbol is $\overset{+}{\vert}\!\vdash\!\overset{-}{}$

Note : ❑ In our houses ac is supplied at 220V, 50Hz.

(6) **Current in difference situation :**

(i) **Due to translatory motion of charge**



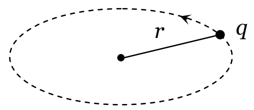
In n particle each having a charge q, pass through a given area in time t then $\boldsymbol{i} = \frac{nq}{t}$

If n particles each having a charge q pass per second per unit area, the current associated with cross-sectional area A is $\boldsymbol{i} = nqA$

If there are n particle per unit volume each having a charge q and moving with velocity v, the current thorough, cross section A is $\boldsymbol{i} = nqvA$, for electrons i= **neav$_d$**

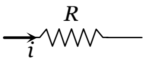
(ii) **Due to rotatory motion of charge**

If a point charge q is moving in a circle of radius r with speed v (frequency ν, angular speed ω and time period T) then corresponding currents $\boldsymbol{i} = \boldsymbol{q}\,\nu = \frac{\boldsymbol{q}}{\boldsymbol{T}} = \frac{\boldsymbol{qv}}{2\pi \boldsymbol{r}} = \frac{\boldsymbol{q}\,\omega}{2\pi}$



(iii) **When a voltage V applied across a resistance R** : Current flows through the conductor $i = \frac{V}{R}$

also by definition of power $i = \frac{P}{V}$



(7) **Current carriers :** The charged particles whose flow in a definite direction constitutes the electric current are called current carriers. In different situation current carriers are different.

(i) Solids : In solid conductors like metals current carriers are free electrons.

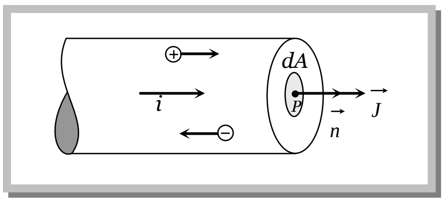
(ii) Liquids : In liquids current carriers are positive and negative ions.

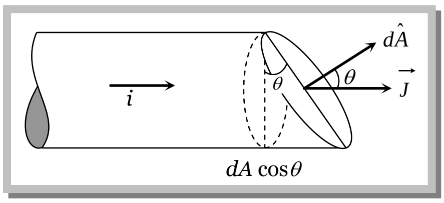
(iii) Gases : In gases current carriers are positive ions and free electrons.

(iv) Semi conductor : In semi conductors current carriers are holes and free electrons.

Current density (J)

In case of flow of charge through a cross-section, current density is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point. Current density at point P is given by $\vec{J} = \frac{di}{dA}\vec{n}$





If the cross-sectional area is not normal to the current, the cross-sectional area normal to current in accordance with following figure will be $dA\cos\theta$ and so in this situation:

$$J = \frac{di}{dA\cos\theta} \quad i.e.\ di = JdA\cos\theta \ \text{ or } di = \vec{J}.\vec{dA} \Rightarrow i = \int \vec{J}\cdot\vec{dA}$$

i.e., in terms of current density, current is the flux of current density.

Note : ❑ If current density $\vec{J}$ is uniform for a normal cross-section $\vec{A}$ then : $i = \int \vec{J}\cdot\vec{ds} = \vec{J}\cdot\int\vec{ds}$ [as $\vec{J}$ = constant]

or $i = \vec{J}\cdot\vec{A} = JA\cos 0 = JA \Rightarrow \boldsymbol{J} = \frac{\boldsymbol{i}}{\boldsymbol{A}}$ [as $\int\vec{dA} = \vec{A}$ and $\theta = 0°$]

(1) **Unit and dimension :** Current density $\vec{J}$ is a vector quantity having S.I. unit *Amp/m*^{2} and dimension.$[L^{-2}A]$

(2) **Current density in terms of velocity of charge** : In case of uniform flow of charge through a cross-section normal to it as $i = nqvA$ so, $\vec{J} = \frac{i}{A}\vec{n} = (nqv)\vec{n}$ or $\vec{J} = nq\vec{v} = \vec{v}(\rho)$ [With $\rho = \frac{\text{charge}}{\text{volume}} = nq$]

i.e., current density at a point is equal to the product of volume charge density with velocity of charge distribution at that point.

(3) **Current density in terms of electric field :** Current density relates with electric field as $\boldsymbol{J} = \sigma \boldsymbol{E} = \frac{\boldsymbol{E}}{\rho}$; where σ = conductivity and ρ = resistivity or specific resistance of substance.

(i) Direction of current density $\vec{J}$ is same as that of electric field $\vec{E}$.

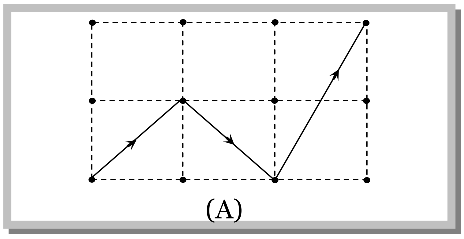
(ii) If electric field is uniform (*i.e.* $\vec{E}$ = constant) current density will be constant [as σ = constant]

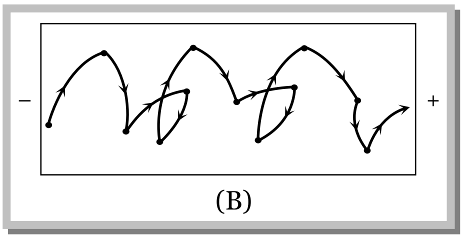
(iii) If electric field is zero (as in electrostatics inside a conductor), current density and hence current will be zero.

Conduction of Current in Metals

According to modern views, a metal consists of a '*lattice*' of fixed positively charged ions in which billions and billions of free electrons are moving randomly at speed which at room temperature (*i.e.* 300 *K*) in accordance with kinetic theory of gases is given by $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times (1.38 \times 10^{-23}) \times 300}{9.1 \times 10^{-31}}} \simeq 10^5\, m/s$

The randomly moving free electrons inside the metal collide with the lattice and follow a zig-zag path as shown in figure (A).


(A)



(B)

However, in absence of any electric field due to this random motion, the number of electrons crossing from left to right is equal to the number of electrons crossing from right to left (otherwise metal will not remain equipotential) so the net current through a cross-section is zero.
A motion of charge is possible by motion of electron or a current carrier.

<u>Velocities of charged particle (electron) in a conductor</u>
<u>thermal velocity</u> : All electrons in the atom are not capable of motion. Only a few which have little higher level of energy leave their orbit and are capable of moving around. These electrons are called "free electrons". These free electrons are in very large quantity ≈10^{29} m^{-3} in free metals. Due to temperature and thermal energy They have a *<u>thermal velocity</u>* ≈10^5 ms^{-1} . This velocity is in all directions and of magnitudes varying from zero to maximum. Due to large number of electrons we can assume that vector sum of thermal velocities at any instant is zero.

i.e. $\vec{u_1} + \vec{u_2} + \vec{u_3} + \ldots\ldots + u_n = 0$

<u>Mean Free path</u> : The fast moving electrons keep striking other atoms/ions in the conductor. They are reflected and move in other direction. They keep moving till they strike another ion/atom.
The path between two consecutive collisions is called free path. The average length of these free paths is called "Mean Free Path".

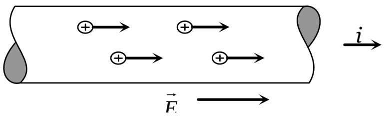
Relaxation Time :The time to travel mean free path is called Relaxation Period or Relaxation Time, denoted by Greek letter Tau "τ". If $t_1, t_2, \ldots t_n$ are the time periods for n collisions then Relaxation Time $\tau = \frac{1}{n}(t_1+t_2+\ldots t_n)$

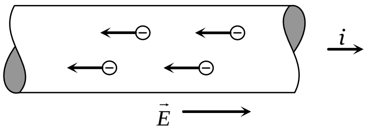
Drift Velocity :When Electric Field is applied across a conductor, the free electrons experience a force in the direction opposite to field. Dur to this force they start drifting in the direction of force. The Velocity of this drift is called drift velocity "V_d". During the drift they maintain their thermal velocity.

The drift velocity can be calculated as averaged velocity of all the electrons drifting.

Relation between drift-velocity (V_d) and electric field applied.

When electric field is applied across a conductor each electron experience a Force $\vec{F} = q\vec{E}$ in the direction of $\vec{E}$. It acquires an acceleration $\alpha = \frac{eE}{m}$ where e is charge on electron and m is its mass.





If n electrons are having initial speeds $u_1, u_2, \ldots u_n$ and their time to travel free path is $t_1, t_2, \ldots t_n$ then final velocities are

$v_1 = u_1 + \alpha t_1$,

$V_2 = u_2 + \alpha t_2$,

$V_n = u_n + \alpha t_n$ and so on.

Drift velocity is average of these velocities of charged particles. Therefore

$$
\begin{aligned}
V_d &= \frac{1}{n}(v_1 + V_2 + \ldots\ldots V_n) \\
&= \frac{1}{n}(u_1 + \alpha t_1 + u_2 + \alpha t_2 + \ldots\ldots u_n + \alpha t_n) \\
&= \frac{1}{n}(u_1 + u_2 + \ldots u_n + \alpha t_1 + \alpha t_2 + \ldots \alpha t_n) \\
&= (u_1 + u_2 + \ldots u_n) + \frac{1}{n}(\alpha t_1 + \alpha t_2 + \ldots \alpha t_n) \\
&= 0 + \alpha \frac{1}{n}(t_1 + t_2 + \ldots t_n) \\
&= \alpha\tau
\end{aligned}
$$

or $V_d = \frac{eE\tau}{m}$ $\left(\alpha = \frac{eE}{m}\right)$

$$\boxed{V_d = \frac{eE\tau}{m}}$$

<u>Relation of Current and Drift velocity :</u> When an electric field is applied, inside the conductor due to electric force the path of electron in general becomes curved (parabolic) instead of straight lines and electrons drift opposite to the field figure (B). Due to this drift the random motion of electrons get modified and there is a net transfer of electrons across a cross-section resulting in current.

Drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for current through it. Drift velocity is very small it is of the order of 10^{-4} *m/s* as compared to thermal speed $(\simeq 10^5\ m/s)$ of electrons at room temperature.

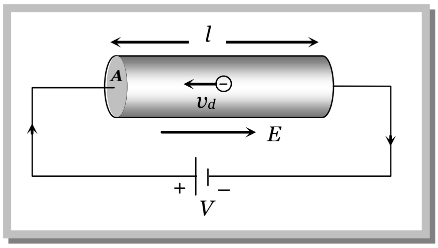
If suppose for a conductor

n = Number of electron per unit volume of the conductor

A = Area of cross-section

V = potential difference across the conductor

E = electric field inside the conductor



i = current, J = current density, ρ = specific resistance, σ = conductivity $\left(\sigma = \frac{1}{\rho}\right)$ then current relates with drift velocity as $\boldsymbol{i = neAv_d}$ we can also write $\boldsymbol{v_d = \frac{i}{neA} = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{E}{\rho ne} = \frac{V}{\rho l n e}}$.

Note : ❑ The direction of drift velocity for electron in a metal is opposite to that of applied electric field (*i.e.* current density $\vec{J}$).

❑ $v_d \propto E$ *i.e.*, greater the electric field, larger will be the drift velocity.

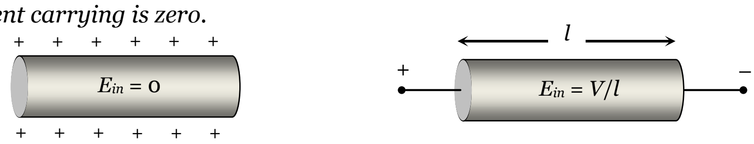
❑ When a steady current flows through a conductor of non-uniform cross-section drift velocity varies inversely with area of cross-section $\left(v_d \propto \frac{1}{A}\right)$

❑ If diameter of a conductor is doubled, then drift velocity of electrons inside it will not change.

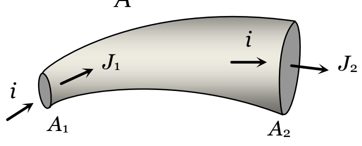
(2) **Relaxation time (τ) :** The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time $\tau = \frac{\text{mean free path}}{\text{r.m.s. velocity of electrons}} = \frac{\lambda}{v_{rms}}$ with rise in temperature v_{rms} increases consequently τ decreases.

(3) **Mobility :** Drift velocity per unit electric field is called mobility of electron *i.e.* $\mu = \frac{v_d}{E}$. It's unit is $\frac{m^2}{volt - \sec}$.

Concepts

- *Human body, though has a large resistance of the order of kΩ (say 10 kΩ), is very sensitive to minute currents even as low as a few mA. Electrocution, excites and disorders the nervous system of the body and hence one fails to control the activity of the body.*
- *1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross-section of the conductors.*
- *dc flows uniformly throughout the cross-section of conductor while ac mainly flows through the outer surface area of the conductor. This is known as skin effect.*
- *It is worth noting that electric field inside a charged conductor is zero, but it is non zero inside a current carrying conductor and is given by $E = \frac{V}{l}$ where V = potential difference across the conductor and l = length of the conductor. Electric field out side the current carrying is zero.*
- 

- *For a given conductor JA = i = constant so that $J \propto \frac{1}{A}$ i.e., $J_1A_1 = J_2A_2$; this is called equation of continuity*



- *If cross-section is constant, $I \propto J$ i.e. for a given cross-sectional area, greater the current density, larger will be current.*
- *The drift velocity of electrons is small because of the frequent collisions suffered by electrons.*
- *The small value of drift velocity produces a large amount of electric current, due to the presence of extremely large number of free electrons in a conductor. The propagation of current is almost at the speed of light and involves electromagnetic process. It is due to this reason that the electric bulb glows immediately when switch is on.*
- *In the absence of electric field, the paths of electrons between successive collisions are straight line while in presence of electric field the paths are generally curved.*
- *Free electron density in a metal is given by $n = \frac{N_A x d}{A}$ where N_A = Avogrado number, x = number of free electrons per atom, d = density of metal and A = Atomic weight of metal.*

Example

Example: 1 The potential difference applied to an *X*-ray tube is 5 *KV* and the current through it is 3.2 *mA*. Then the number of electrons striking the target per second is

(a) 2×10^{16} (b) 5×10^{6} (c) 1×10^{17} (d) 4×10^{15}

Solution : (a) $i = \frac{q}{t} = \frac{ne}{t}$ $\Rightarrow n = \frac{it}{e} = \frac{3.2 \times 10^{-3} \times 1}{1.6 \times 10^{-19}} = 2 \times 10^{16}$

Example: 2 A beam of electrons moving at a speed of 10^6 *m/s* along a line produces a current of 1.6×10^{-6} *A*. The number of electrons in the 1 *metre* of the beam is **[CPMT 2000]**

(a) 10^6 (b) 10^7 (c) 10^{13} (d) 10^{19}

Solution : (b) $i = \frac{q}{t} = \frac{q}{(x/v)} = \frac{qv}{x} = \frac{nev}{x} \Rightarrow n = \frac{ix}{ev} = \frac{1.6 \times 10^{-6} \times 1}{1.6 \times 10^{-19} \times 10^{6}} = 10^7$

Example: 3 In the Bohr's model of hydrogen atom, the electrons moves around the nucleus in a circular orbit of a radius 5×10^{-11} *metre*. It's time period is 1.5×10^{-16} *sec*. The current associated is

(a) Zero (b) 1.6×10^{-19} *A* (c) 0.17 *A* (d) 1.07×10^{-3} *A*

Solution : (d) $i = \frac{q}{T} = \frac{1.6 \times 10^{-19}}{1.5 \times 10^{-16}} = 1.07 \times 10^{-3}\,A$

Example: 4 An electron is moving in a circular path of radius 5.1×10^{-11} *m* at a frequency of 6.8×10^{15} revolution/sec. The equivalent current is approximately

(a) 5.1×10^{-3} *A* (b) 6.8×10^{-3} *A* (c) 1.1×10^{-3} *A* (d) 2.2×10^{-3} *A*

Solution : (c) $\nu = 6.8 \times 10^{15} \Rightarrow T = \frac{1}{6.8 \times 10^{15}}$ sec $\Rightarrow i = \frac{Q}{T} = 1.6 \times 10^{-19} \times 6.8 \times 10^{15} = 1.1 \times 10^{-3}\,A$

Example: 5 A copper wire of length 1*m* and radius 1*mm* is joined in series with an iron wire of length 2*m* and radius 3*mm* and a current is passed through the wire. The ratio of current densities in the copper and iron wire is

[MP PMT 1994]

(a) 18 : 1 (b) 9 : 1 (c) 6 : 1 (d) 2 : 3

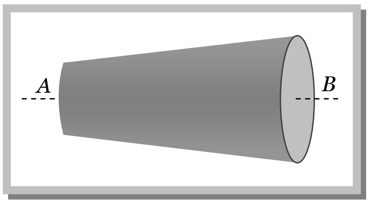
Solution : (b) We know $J = \frac{i}{A}$ when *i* = constant $J \propto \frac{1}{A} \Rightarrow \frac{J_c}{J_i} = \frac{A_i}{A_c} = \left(\frac{r_i}{r_c}\right)^2 = \left(\frac{3}{1}\right)^2 = \frac{9}{1}$

Example: 6 A conducting wire of cross-sectional area 1 cm^2 has 3×10^{23} m^{-3} charge carriers. If wire carries a current of 24 *mA*, the drift speed of the carrier is **[UPSEAT 2001]**

(a) 5×10^{-6} *m/s* (b) 5×10^{-3} *m/s* (c) 0.5 *m/s* (d) 5×10^{-2} *m/s*

Solution : (b) $v_d = \frac{i}{neA} = \frac{24 \times 10^{-3}}{3 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{-4}} = 5 \times 10^{-3}\,m/s$

Example: 7 A wire has a non-uniform cross-sectional area as shown in figure. A steady current *i* flows through it. Which one of the following statement is correct



(a) The drift speed of electron is constant (b) The drift speed increases on moving from *A* to *B*

(c) The drift speed decreases on moving from A to B (d) The drift speed varies randomly

Solution : (c) For a conductor of non-uniform cross-section $v_d \propto \dfrac{1}{\text{Area of cross - section}}$

***Example*: 8** In a wire of circular cross-section with radius r, free electrons travel with a drift velocity v, when a current i flows through the wire. What is the current in another wire of half the radius and of the some material when the drift velocity is $2v$

(a) $2i$ (b) i (c) $i/2$ (d) $i/4$

Solution : (c) $i = neAv_d = ne\pi r^2 v$ and $i' = ne\pi\left(\dfrac{r}{2}\right)^2 .2v = \dfrac{ne\pi r^2 v}{2} = \dfrac{i}{2}$

***Example*: 9** A potential difference of V is applied at the ends of a copper wire of length l and diameter d. On doubling only d, drift velocity

(a) Becomes two times (b) Becomes half (c) Does not change (d) Becomes one fourth

Solution : (c) Drift velocity doesn't depends upon diameter.

***Example*: 10** A current flows in a wire of circular cross-section with the free electrons travelling with a mean drift velocity v. If an equal current flows in a wire of twice the radius new mean drift velocity is

(a) v (b) $\dfrac{v}{2}$ (c) $\dfrac{v}{4}$ (d) None of these

Solution : (c) By using $v_d = \dfrac{i}{neA} \Rightarrow v_d \propto \dfrac{1}{A} \Rightarrow v' = \dfrac{v}{4}$

***Example*: 11** Two wires A and B of the same material, having radii in the ratio 1 : 2 and carry currents in the ratio 4 : 1. The ratio of drift speeds of electrons in A and B is

(a) 16 : 1 (b) 1 : 16 (c) 1 : 4 (d) 4 : 1

Solution : (a) As $i = neA\,v_d \Rightarrow \dfrac{i_1}{i_2} = \dfrac{A_1}{A_2} \times \dfrac{v_{d_1}}{v_{d_2}} = \dfrac{r_1^2}{r_2^2} \cdot \dfrac{v_{d_1}}{v_{d_2}} \Rightarrow \dfrac{v_{d_1}}{v_{d_2}} = \dfrac{16}{1}$

Tricky example: 1

In a neon discharge tube 2.9×10^{18} Ne^+ ions move to the right each second while 1.2×10^{18} electrons move to the left per second. Electron charge is 1.6×10^{-19} C. The current in the discharge tube **[MP PET 1999]**

(a) 1 A towards right (b) 0.66 A towards right (c) 0.66 A towards left (d) Zero

Solution: (b) Use following trick to solve such type of problem.

Trick : In a discharge tube positive ions carry q units of charge in t seconds from anode to cathode and negative carriers (electrons) carry the same amount of charge from cathode to anode in t′ second. The current in the tube is $\boldsymbol{i} = \dfrac{\boldsymbol{q}}{\boldsymbol{t}} + \dfrac{\boldsymbol{q'}}{\boldsymbol{t'}}$.

Hence in this question current $i = \dfrac{2.9 \times 10^{18} \times e}{1} + \dfrac{1.2 \times 10^{18} \times e}{1} = 0.66A$ towards right.

Tricky example: 2

If the current flowing through copper wire of 1 mm diameter is 1.1 amp. The drift velocity of electron is (Given density of Cu is 9 gm/cm^3, atomic weight of Cu is 63 $grams$ and one free electron is contributed by each atom)

(a) 0.1 mm/sec (b) 0.2 mm/sec (c) 0.3 mm/sec (d) 0.5 mm/sec

Solution: (a) 6.023×10^{23} atoms has mass = 63×10^{-3} kg

So no. of atoms per m^3 = $n = \dfrac{6.023\times 10^{23}}{63\times 10^{-3}}\times 9\times 10^{3} = 8.5\times 10^{28}$

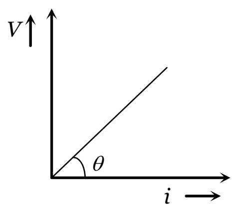
$$v_d = \frac{i}{neA} = \frac{1.1}{8.5\times 10^{28}\times 1.6\times 10^{-19}\times \pi\times(0.5\times 10^{-3})^2} = 0.1\times 10^{-3}\ m/\sec = 0.1\,mm/\sec$$

Ohm's Law

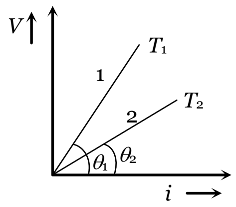
If the physical circumstances of the conductor (length, temperature, mechanical strain etc.) remains constant, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends *i.e.* $\boldsymbol{i \propto V}$

$\Rightarrow \boldsymbol{V = iR}$ or $\dfrac{V}{i} = R$; where R is a proportionality constant, known as electric resistance.

(1) Ohm's law is not a universal law, the substance which obeys ohm's law are known as ohmic substance for such ohmic substances graph between V and i is a straight line as shown. At different temperatures V-i curves are different.

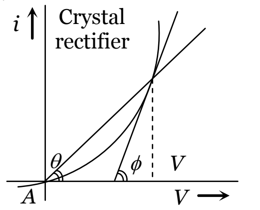


Slope of the line = $\tan\theta = \dfrac{V}{i} = R$



Here $\tan\theta_1 > \tan\theta_2$
So $R_1 > R_2$ *i.e.* $T_1 > T_2$

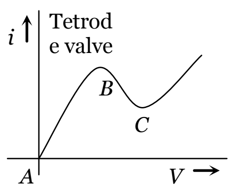
(2) The device or substances which doesn't obey ohm's law *e.g.* gases, crystal rectifiers, thermoionic valve, transistors etc. are known as non-ohmic or non-linear conductors. For these V-i curve is not linear. In these situation the ratio between voltage and current at a particular voltage is known as static resistance. While the rate of change of voltage to change in current is known as dynamic resistance.



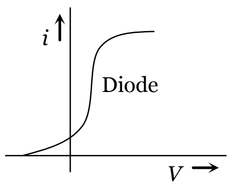
$$R_{st} = \frac{V}{i} = \frac{1}{\tan\theta}$$

while $R_{dyn} = \dfrac{\Delta V}{\Delta I} = \dfrac{1}{\tan\phi}$

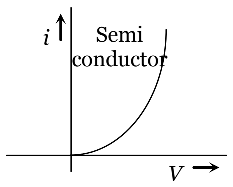
(3) Some other non-ohmic graphs are as follows :



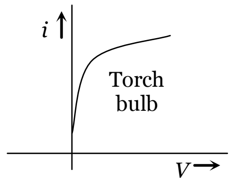
(A)



(B)



(C)



(D)

Resistance

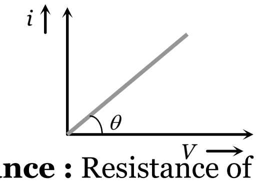
(1) **Definition :** The property of substance by virtue of which it opposes the flow of current through it, is known as the resistance.

(2) **Cause of resistance of a conductor :** It is due to the collisions of free electrons with the ions or atoms of the conductor while drifting towards the positive end of the conductor.

(3) **Formula of resistance :** For a conductor if l = length of a conductor A = Area of cross-section of conductor, n = No. of free electrons per unit volume in conductor, τ = relaxation time then resistance of conductor $\boldsymbol{R = \rho\frac{l}{A} = \frac{m}{ne^2\tau}\cdot\frac{l}{A}}$; where ρ = resistivity of the material of conductor

(4) **Unit and dimension :** It's S.I. unit is *Volt/Amp.* or *Ohm* (Ω). Also 1 *ohm* $= \frac{1 volt}{1 Amp} = \frac{10^8 \, emu \text{ of potential}}{10^{-1} \, emu \text{ of current}}$ = 10^9 *emu* of resistance. It's dimension is $[ML^2T^{-3}A^{-2}]$.

(5) **Conductance (*C*) :** Reciprocal of resistance is known as conductance. $C = \frac{1}{R}$ It's unit is $\frac{1}{\Omega}$ or Ω^{-1} or "Siemen".

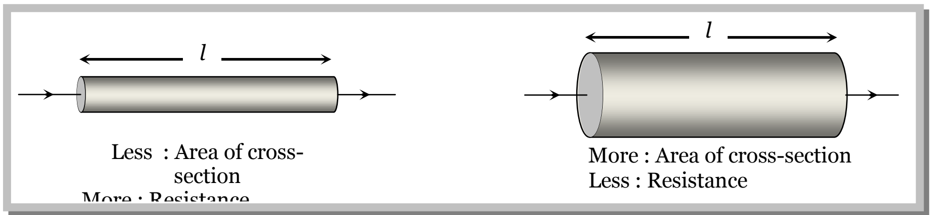


Slope = $\tan\theta = \frac{i}{V} = \frac{1}{R} = C$

(6) **Dependence of resistance :** Resistance of a conductor depends on the following factors.

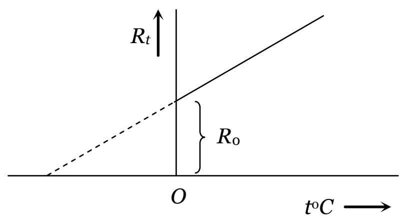
(i) Length of the conductor : Resistance of a conductor is directly proportional to it's length *i.e.* $\boldsymbol{R \propto l}$ *e.g.* a conducting wire having resistance R is cut in n equal parts. So resistance of each part will be $\frac{R}{n}$.

(ii) Area of cross-section of the conductor : Resistance of a conductor is inversely proportional to it's area of cross-section *i.e.* $R \propto \frac{1}{A}$



(iii) Material of the conductor : Resistance of conductor also depends upon the nature of material *i.e.* $R \propto \frac{1}{n}$, for different conductors n is different. Hence R is also different.

(iv) Temperature : We know that $R = \frac{m}{ne^2\tau}\cdot\frac{l}{A} \Rightarrow R \propto \frac{l}{\tau}$ when a metallic conductor is heated, the atom in the metal vibrate with greater amplitude and frequency about their mean positions. Consequently the number of collisions between free electrons and atoms increases. This reduces the relaxation time τ and increases the value of resistance R *i.e.* for a conductor **Resistance $\propto$ temperature**.



If R_0 = resistance of conductor at 0°*C*

R_t = resistance of conductor at t°*C*

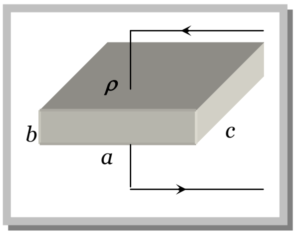
and α, β = temperature co-efficient of resistance (unit → per°*C*)

then $R_t = R_0(1 + \alpha t + \beta t^2)$ for $t > 300°C$ and $\boldsymbol{R_t = R_0(1 + \alpha t)}$ for $t \le 300°C$ or $\alpha = \frac{R_t - R_0}{R_0 \times t}$

Note : ❑ If R_1 and R_2 are the resistances at t_1°*C* and t_2°*C* respectively then $\frac{R_1}{R_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$.

- The value of α is different at different temperature. Temperature coefficient of resistance averaged over the temperature range $t_1°C$ to $t_2°C$ is given by $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$ which gives $R_2 = R_1 [1 + \alpha(t_2 - t_1)]$. This formula gives an approximate value.

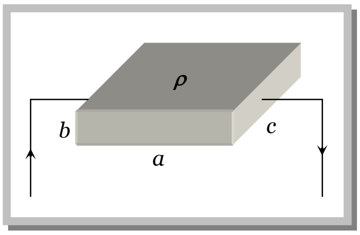
(v) **Resistance according to potential difference :** Resistance of a conducting body is not unique but depends on it's length and area of cross-section *i.e.* how the potential difference is applied. See the following figures



Length = b

Area of cross-section = $a \times c$

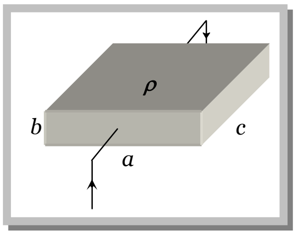
Resistance $R = \rho\left(\frac{b}{a \times c}\right)$



Length = a

Area of cross-section = $b \times c$

Resistance $R = \rho\left(\frac{a}{b \times c}\right)$



Length = c

Area of cross-section = $a \times b$

Resistance $R = \rho\left(\frac{c}{a \times b}\right)$

(7) **Variation of resistance of some electrical material with temperature :**

(i) Metals : For metals their temperature coefficient of resistance $\alpha > 0$. So resistance increases with temperature.

Physical explanation : Collision frequency of free electrons with the immobile positive ions increases

(ii) Solid non-metals : For these $\alpha = 0$. So resistance is independence of temperature.

Physical explanation : Complete absence of free electron.

(iii) Semi-conductors : For semi-conductor $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : Covalent bonds breaks, liberating more free electron and conduction increases.

(iv) Electrolyte : For electrolyte $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : The degree of ionisation increases and solution becomes less viscous.

(v) Ionised gases : For ionised gases $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : Degree of ionisation increases.

(vi) Alloys : For alloys α has a small positive values. So with rise in temperature resistance of alloys is almost constant. Further alloy resistances are slightly higher than the pure metals resistance.

Alloys are used to made standard resistances, wires of resistance box, potentiometer wire, meter bridge wire *etc.*

Commonly used alloys are : Constantan, mangnin, Nichrome etc.

(vii) Super conductors : At low temperature, the resistance of certain substances becomes exactly zero. (*e.g.* Hg below 4.2 K or Pb below 7.2 K).

These substances are called super conductors and phenomenon super conductivity. The temperature at which resistance becomes zero is called critical temperature and depends upon the nature of substance.

Resistivity or Specific Resistance (ρ)

(1) **Definition :** From $R = \rho \frac{l}{A}$; If l = 1*m*, A = 1 m^2 then $\boldsymbol{R = \rho}$ *i.e.* resistivity is numerically equal to the resistance of a substance having unit area of cross-section and unit length.

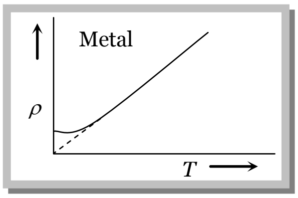
(2) **Unit and dimension :** It's S.I. unit is $ohm \times m$ and dimension is $[ML^3T^{-3}A^{-2}]$

(3) **It's formula** : $\rho = \frac{m}{ne^2\tau}$

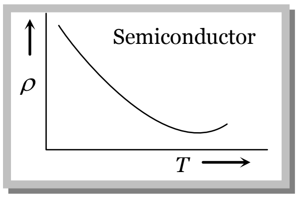
(4) **It's dependence :** Resistivity is the intrinsic property of the substance. It is independent of shape and size of the body (*i.e.* l and A). It depends on the followings :

(i) Nature of the body : For different substances their resistivity also different *e.g.* ρ_{silver} = minimum = 1.6×10^{-8} Ω-*m* and $\rho_{fused\ quartz}$ = maximum $\approx 10^{16}$ Ω-*m*

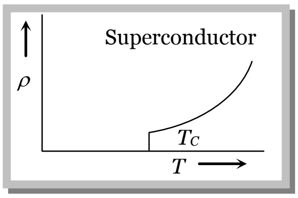
(ii) Temperature : Resistivity depends on the temperature. For metals $\rho_t = \rho_0 (1 + \alpha\Delta t)$ *i.e.* resitivity increases with temperature.



ρ increases with temperature



ρ decreases with temperature



ρ decreases with temperature and becomes zero at a certain temperature

(iii) Impurity and mechanical stress : Resistivity increases with impurity and mechanical stress.

(iv) Effect of magnetic field : Magnetic field increases the resistivity of all metals except iron, cobalt and nickel.

(v) Effect of light : Resistivity of certain substances like selenium, cadmium, sulphides is inversely proportional to intensity of light falling upon them.

(5) **Resistivity of some electrical material :** $\rho_{insulator} > \rho_{alloy} > \rho_{semi\text{-}conductor} > \rho_{conductor}$

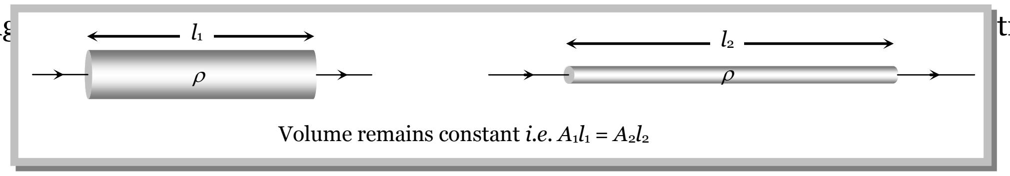
Reciprocal of resistivity is called conductivity (σ) *i.e.* $\sigma = \frac{1}{\rho}$ with unit *mho/m* and dimensions $[M^{-1}L^{-3}T^3A^2]$.

Stretching of Wire

If a conducting wire stretches, it's length increases, area of cross-section decreases so resistance increases but volume remain constant.

Suppose for a conducting wire before stretching it's length = l_1, area of cross-section = A_1, radius = r_1, diameter = d_1, and resistance $R_1 = \rho \frac{l_1}{A_1}$

Before stretching



Volume remains constant *i.e.* $A_1l_1 = A_2l_2$

tretching

After stretching length = l_2, area of cross-section = A_2, radius = r_2, diameter = d_2 and resistance = $R_2 = \rho \frac{l_2}{A_2}$

Ratio of resistances $$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^4 = \left(\frac{d_2}{d_1}\right)^4$$

(1) If length is given then $R \propto l^2 \Rightarrow \frac{R_1}{R_2} = \left(\frac{l_1}{l_2}\right)^2$

(2) If radius is given then $R \propto \frac{1}{r^4} \Rightarrow \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4$

Note : ❑ After stretching if length increases by *n* times then resistance will increase by n^2 times *i.e.* $R_2 = n^2 R_1$. Similarly if radius be reduced to $\frac{1}{n}$ times then area of cross-section decreases $\frac{1}{n^2}$ times so the resistance becomes n^4 times *i.e.* $R_2 = n^4 R_1$.

❑ After stretching if length of a conductor increases by $x\%$ then resistance will increases by $2x$ % (valid only if $x < 10\%$)

Various Electrical Conducting Material For Specific Use

(1) **Filament of electric bulb :** Is made up of tungsten which has high resistivity, high melting point.

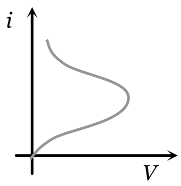
(2) **Element of heating devices (such as heater, geyser or press) :** Is made up of nichrome which has high resistivity and high melting point.

(3) **Resistances of resistance boxes (standard resistances) :** Are made up of manganin, or constantan as these materials have moderate resistivity which is practically independent of temperature so that the specified value of resistance does not alter with minor changes in temperature.

(4) **Fuse-wire :** Is made up of tin-lead alloy (63% tin + 37% lead). It should have low melting point and high resistivity. It is used in series as a safety device in an electric circuit and is designed so as to melt and thereby open the circuit if the current exceeds a predetermined value due to some fault. The function of a fuse is independent of its length.

Safe current of fuse wire relates with it's radius as $i \propto r^{3/2}$.

(5) **Thermistors** : A thermistor is a heat sensitive resistor usually prepared from oxides of various metals such as nickel, copper, cobalt, iron etc. These compounds are also semi-conductor. For thermistors α is very high which may be positive or negative. The resistance of thermistors changes very rapidly with change of temperature.



Thermistors are used to detect small temperature change and to measure very low temperature.

Concepts

☞ *In the absence of radiation loss, the time in which a fuse will melt does not depends on it's length but varies with radius as* $t \propto r^4$.

☞ *If length (l) and mass (m) of a conducting wire is given then* $R \propto \frac{l^2}{m}$.

☞ *Macroscopic form of Ohm's law is* $R = \frac{V}{i}$, *while it's microscopic form is* $J = \sigma E$.

Example

***Example*: 12** Two wires of resistance R_1 and R_2 have temperature co-efficient of resistance α_1 and α_2 respectively. These are joined in series. The effective temperature co-efficient of resistance is

(a) $\frac{\alpha_1 + \alpha_2}{2}$ (b) $\sqrt{\alpha_1 \alpha_2}$ (c) $\frac{\alpha_1 R_1 + \alpha_2 R_2}{R_1 + R_2}$ (d) $\frac{\sqrt{R_1 R_2 \alpha_1 \alpha_2}}{\sqrt{R_1^2 + R_2^2}}$

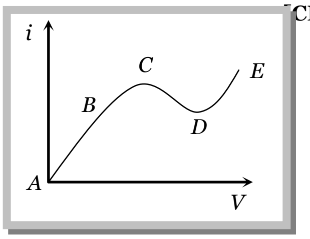
Solution : (c) Suppose at t^oC resistances of the two wires becomes R_{1t} and R_{2t} respectively and equivalent resistance becomes R_t. In series grouping $R_t = R_{1t} + R_{2t}$, also $R_{1t} = R_1(1 + \alpha_1 t)$ and $R_{2t} = R_2(1 + \alpha_2 t)$

$$R_t = R_1(1 + \alpha_1 t) + R_2(1 + \alpha_2 t) = (R_1 + R_2) + (R_1\alpha_1 + R_2\alpha_2)t = (R_1 + R_2)\left[1 + \frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2}t\right].$$

Hence effective temperature co-efficient is $\frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2}$.

***Example*: 13** From the graph between current i & voltage V shown, identity the portion corresponding to negative resistance

[CBSE PMT 1997]



(a) DE
(b) CD
(c) BC
(d) AB

Solution : (b) $R = \frac{\Delta V}{\Delta I}$, in the graph CD has only negative slope. So in this portion R is negative.

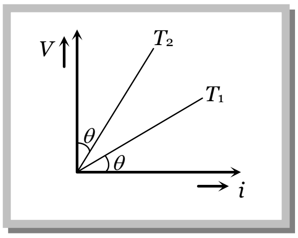
***Example*: 14** A wire of length L and resistance R is streched to get the radius of cross-section halfed. What is new resistance

[NCERT 1974; CPMT 1994; AIIMS 1997; KCET 1999; Haryana PMT 2000; UPSEAT 2001]

(a) $5R$ (b) $8R$ (c) $4R$ (d) $16R$

Solution : (d) By using $\frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4 \Rightarrow \frac{R}{R'} = \left(\frac{r/2}{r}\right)^4 \Rightarrow R' = 16R$

***Example*: 15** The V-i graph for a conductor at temperature T_1 and T_2 are as shown in the figure. $(T_2 - T_1)$ is proportional to



(a) $\cos 2\theta$
(b) $\sin\theta$
(c) $\cot 2\theta$
(d) $\tan\theta$

Solution : (c) As we know, for conductors resistance $\propto$ Temperature.

From figure $R_1 \propto T_1 \Rightarrow \tan\theta \propto T_1 \Rightarrow \tan\theta = kT_1$ .... (i) (k = constant)

and $R_2 \propto T_2 \Rightarrow \tan(90^o - \theta) \propto T_2 \Rightarrow \cot\theta = kT_2$ ....(ii)

From equation (i) and (ii) $k(T_2 - T_1) = (\cot\theta - \tan\theta)$

$$(T_2 - T_1) = \left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right) = \frac{(\cos^2\theta - \sin^2\theta)}{\sin\theta\cos\theta} = \frac{\cos 2\theta}{\sin\theta\cos\theta} = 2\cot 2\theta \Rightarrow (T_2 - T_1) \propto \cot 2\theta$$

***Example*: 16** The resistance of a wire at 20^oC is 20 Ω and at 500^oC is 60Ω. At which temperature resistance will be 25Ω

[UPSEAT 1999]

(a) 50^oC (b) 60^oC (c) 70^oC (d) 80^oC

Solution : (d) By using $\frac{R_1}{R_2} = \frac{(1+\alpha t_1)}{(1+\alpha t_2)} \Rightarrow \frac{20}{60} = \frac{1+20\alpha}{1+500\alpha} \Rightarrow \alpha = \frac{1}{220}$

Again by using the same formula for 20Ω and 25Ω ⇒ $\frac{20}{25} = \frac{\left(1+\frac{1}{220}\times 20\right)}{\left(1+\frac{1}{220}\times t\right)} \Rightarrow t = 80^oC$

***Example*: 17** The specific resistance of manganin is 50×10^{-8} Ω*m*. The resistance of a manganin cube having length $50\ cm$ is

(a) 10^{-6} Ω (b) 2.5×10^{-5} Ω (c) 10^{-8} Ω (d) 5×10^{-4} Ω

Solution : (a) $R = \rho\frac{l}{A} = \frac{50\times 10^{-8}\times 50\times 10^{-2}}{(50\times 10^{-2})^2} = 10^{-6}\,\Omega$

***Example*: 18** A rod of certain metal is 1 *m* long and 0.6 *cm* in diameter. It's resistance is 3×10^{-3}Ω. A disc of the same metal is 1 *mm* thick and 2 *cm* in diameter, what is the resistance between it's circular faces.

(a) 1.35×10^{-6}Ω (b) 2.7×10^{-7} Ω (c) 4.05×10^{-6}Ω (d) 8.1×10^{-6} Ω

Solution : (b) By using $R = \rho.\frac{l}{A}; \frac{R_{\text{disc}}}{R_{\text{rod}}} = \frac{l_{\text{disc}}}{l_{\text{rod}}} \times \frac{A_{\text{rod}}}{A_{\text{disc}}} \Rightarrow \frac{R_{\text{disc}}}{3\times 10^{-3}} = \frac{10^{-3}}{1} \times \frac{\pi(0.3\times 10^{-2})^2}{\pi(10^{-2})^2} \Rightarrow R_{\text{disc}} = 2.7 \times 10^{-7}\Omega.$

***Example*: 19** An aluminium rod of length 3.14 *m* is of square cross-section 3.14 × 3.14 mm^2. What should be the radius of 1 *m* long another rod of same material to have equal resistance

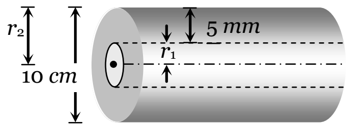
(a) 2 *mm* (b) 4 *mm* (c) 1 *mm* (d) 6 *mm*

Solution : (c) By using $R = \rho.\frac{l}{A} \Rightarrow l \propto A \Rightarrow \frac{3.14}{1} = \frac{3.14\times 3.14\times 10^{-6}}{\pi\times r^2} \Rightarrow r = 10^{-3}\,m = 1\,mm$

***Example*: 20** Length of a hollow tube is 5*m*, it's outer diameter is 10 *cm* and thickness of it's wall is 5 *mm*. If resistivity of the material of the tube is 1.7×10^{-8} Ω×*m* then resistance of tube will be

(a) 5.6×10^{-5} Ω (b) 2×10^{-5} Ω (c) 4×10^{-5} Ω (d) None of these

Solution : (a) By using $R = \rho.\frac{l}{A}$; here $A = \pi(r_2^2 - r_1^2)$



Outer radius r_2 = 5*cm*

Inner radius r_1 = 5 – 0.5 = 4.5 *cm*

So $R = 1.7\times 10^{-8} \times \frac{5}{\pi\{(5\times 10^{-2})^2 - (4.5\times 10^{-2})^2\}} = 5.6\times 10^{-5}\,\Omega$

***Example*: 21** If a copper wire is stretched to make it 0.1% longer, the percentage increase in resistance will be

[MP PMT 1996, 2000; UPSEAT 1998; MNR 1990]

(a) 0.2 (b) 2 (c) 1 (d) 0.1

Solution : (a) In case of streching $R \propto l^2$ So $\frac{\Delta R}{R} = 2\frac{\Delta l}{l} = 2\times 0.1 = 0.2$

***Example*: 22** The temperature co-efficient of resistance of a wire is $0.00125/^oC$. At 300 *K*. It's resistance is 1Ω. The resistance of the wire will be 2Ω at **[MP PMT 2001; IIT 1980]**

(a) 1154 *K* (b) 1127 *K* (c) 600 *K* (d) 1400 *K*

Solution: (b) By using $R_t = R_o(1 + \alpha\Delta t) \Rightarrow \frac{R_1}{R_2} = \frac{1+\alpha t_1}{1+\alpha t_2}$ So $\frac{1}{2} = \frac{1+(300-273)\alpha}{1+\alpha t_2} \Rightarrow t_2 = 854^oC = 1127\,K$

Example: 23 Equal potentials are applied on an iron and copper wire of same length. In order to have same current flow in the wire, the ratio $\left(\frac{r_{iron}}{r_{copper}}\right)$ of their radii must be [Given that specific resistance of iron = 1.0 × 10^{-7} Ω*m* and that of copper = 1.7 × 10^{-8} Ω*m*] **[MP PMT 2000]**

(a) About 1.2 (b) About 2.4 (c) About 3.6 (d) About 4.8

Solution: (b) V = constant., i = constant. So R = constant

$$\Rightarrow \frac{P_i l_i}{A_i} = \frac{\rho_{Cu} l_{Cu}}{A_{Cu}} \Rightarrow \frac{\rho_i l_i}{r_i^2} = \frac{\rho_{Cu} l_{Cu}}{r_{Cu}^2}$$

$$\Rightarrow \frac{r_i}{r_{Cu}} = \sqrt{\frac{\rho_i}{\rho_{Cu}}} = \sqrt{\frac{1.0 \times 10^{-7}}{1.7 \times 10^{-8}}} = \sqrt{\frac{100}{17}} \approx 2.4$$

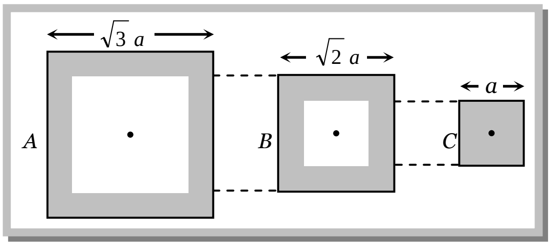
Example: 24 Masses of three wires are in the ratio 1 : 3 : 5 and their lengths are in the ratio 5 : 3 : 1. The ratio of their electrical resistance is

(a) 1 : 3 : 5 (b) 5 : 3 : 1 (c) 1 : 15 : 125 (d) 125 : 15 : 1

Solution: (d) $R = \rho\frac{l}{A} = \rho\frac{l^2}{V} = \rho\frac{l^2}{m}\sigma \quad \left(\because \sigma = \frac{m}{V}\right)$

$$R_1 : R_2 : R_3 = \frac{l_1^2}{m_1} : \frac{l_2^2}{m_2} : \frac{l_3^2}{m_3} = 25 : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$$

Example: 25 Following figure shows cross-sections through three long conductors of the same length and material, with square cross-section of edge lengths as shown. Conductor B will fit snugly within conductor A, and conductor C will fit snugly within conductor B. Relationship between their end to end resistance is



(a) $R_A = R_B = R_C$

(b) $R_A > R_B > R_C$

(c) $R_A < R_B < R$

(d) Information is not sufficient

Solution : (a) All the conductors have equal lengths. Area of cross-section of A is $\left\{(\sqrt{3}\,a)^2 - (\sqrt{2}\,a)^2\right\} = a^2$

Similarly area of cross-section of B = Area of cross-section of C = a^2

Hence according to formula $R = \rho\frac{l}{A}$; resistances of all the conductors are equal *i.e.* $R_A = R_B = R_C$

Example: 26 Dimensions of a block are 1 *cm* × 1 *cm* × 100 *cm*. If specific resistance of its material is 3 × 10^{-7} *ohm-m*, then the resistance between it's opposite rectangular faces is

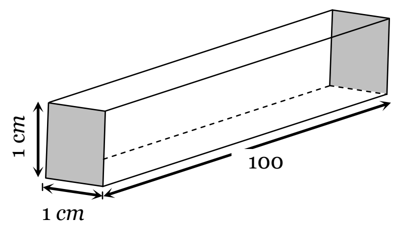
(a) 3 × 10^{-9} *ohm* (b) 3 × 10^{-7} *ohm* (c) 3 × 10^{-5} *ohm* (d) 3 × 10^{-3} *ohm*

Solution: (b) Length $l = 1\,cm = 10^{-2}\,m$

Area of cross-section A = 1 *cm* × 100 *cm*

= 100 cm^2 = 10^{-2} m^2

Resistance $R = 3 \times 10^{-7} \times \frac{10^{-2}}{10^{-2}} = 3 \times 10^{-7}\,\Omega$



Note : ❑ In the above question for calculating equivalent resistance between two opposite square faces.

$$l = 100\ cm = 1\ m, A = 1\ cm^2 = 10^{-4}\ m^2, \text{ so resistance } R = 3 \times 10^{-7} \times \frac{1}{10^{-4}} = 3 \times 10^{-3}\ \Omega$$

3. Two rods A and B of same material and length have their electric resistances are in ratio 1 : 2. When both the rods are dipped in water, the correct statement will be **[RPMT 1997]**

(a) A has more loss of weight (b) B has more loss of weight

(c) Both have same loss of weight (d) Loss of weight will be in the ratio 1 : 2

Solution: (a) $R = \rho \frac{L}{A} \Rightarrow \frac{R_1}{R_2} = \frac{A_2}{A_1}$ (ρ, L constant) $\Rightarrow \frac{A_1}{A_2} = \frac{R_2}{R_1} = 2$

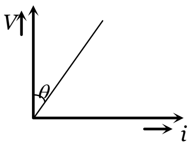
Now when a body dipped in water, loss of weight = $V\sigma_L g = AL\sigma_L g$

So $\frac{(\text{Loss of weight})_1}{(\text{Loss of weight})_2} = \frac{A_1}{A_2} = 2$; So A has more loss of weight

The V-i graph for a conductor makes an angle θ with V-axis. Here V denotes the voltage and i denotes current. The resistance of conductor is given by

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\cot\theta$

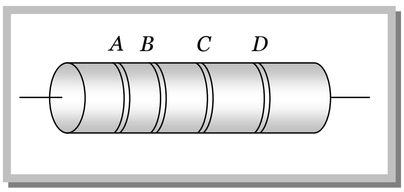
Solution: (d) At an instant approach the student will choose $\tan\theta$ will be the right answer. But it is to be seen here the curve makes the angle θ with the V-axis. So it makes an angle $(90 - \theta)$ with the i-axis. So resistance = slope = $\tan(90 - \theta) = \cot\theta$.



Colour Coding of Resistance

The resistance, having high values are used in different electrical and electronic circuits. They are generally made up of carbon, like 1 $k\Omega$, 2 $k\Omega$, 5 $k\Omega$ *etc*. To know the value of resistance colour code is used. These code are printed in form of set of rings or strips. By reading the values of colour bands, we can estimate the value of resistance.

The carbon resistance has normally four coloured rings or strips say A, B, C and D as shown in following figure.



Colour band A and B indicate the first two significant figures of resistance in *ohm*, while the C band gives the decimal multiplier *i.e.* the number of zeros that follows the two significant figures A and B.

Last band (D band) indicates the tolerance in percent about the indicated value or in other ward it represents the percentage accuracy of the indicated value.

The tolerance in the case of gold is ± 5% and in silver is ± 10%. If only three bands are marked on carbon resistance, then it indicate a tolerance of 20%.

The following table gives the colour code for carbon resistance.

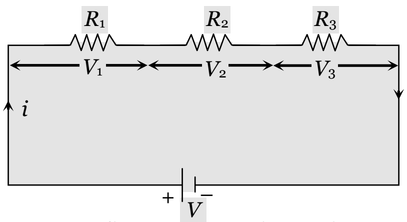
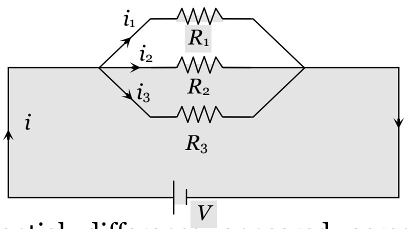
Letters as an aid to memory	Colour	Figure (*A*, *B*)	Multiplier (C)
B	Black	0	10^0
B	Brown	1	10^1
R	Red	2	10^2
O	Orange	3	10^3
Y	Yellow	4	10^4
G	Green	5	10^5
B	Blue	6	10^6
V	Violet	7	10^7
G	Grey	8	10^8
W	White	9	10^9

Colour	Tolerance (D)
Gold	5%
Silver	10%
No-colour	20%

Note : ❑ To remember the sequence of colour code following sentence should kept in memory.

B B R O Y G*reat* ***B****ritain* ***V****ery* ***G****ood* ***W****ife.*

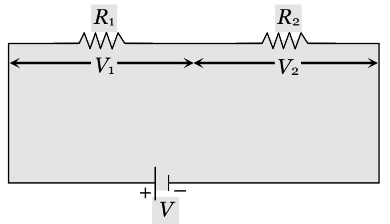
Grouping of Resistance

Series	Parallel
(1) 	(1) 
(2) Same current flows through each resistance but potential difference distributes in the ratio of resistance *i.e.* $V \propto R$ Power consumed are in the ratio of their resistance *i.e.* $P \propto R \Rightarrow P_1 : P_2 : P_3 = R_1 : R_2 : R_3$	(2) Same potential difference appeared across each resistance but current distributes in the reverse ratio of their resistance *i.e.* $i \propto \frac{1}{R}$ Power consumed are in the reverse ratio of resistance *i.e.* $P \propto \frac{1}{R} \Rightarrow P_1 : P_2 : P_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$
(3) $R_{eq} = R_1 + R_2 + R_3$ equivalent resistance is greater than the maximum value of resistance in the combination.	(3) $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ or $R_{eq} = (R_1^{-1} + R_2^{-1} + R_3^{-1})^{-1}$ or $R_{eq} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_2 R_1}$ equivalent resistance is smaller than the minimum value of resistance in the combination.
(4) For two resistance in series $R_{eq} = R_1 + R_2$	(4) For two resistance in parallel $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{\text{Multiplication}}{\text{Addition}}$

(5) Potential difference across any resistance $V' = \left(\frac{R'}{R_{eq}}\right) \cdot V$

Where R' = Resistance across which potential difference is to be calculated, R_{eq} = equivalent resistance of that line in which R' is connected, V = p.d. across that line in which R' is connected

e.g.



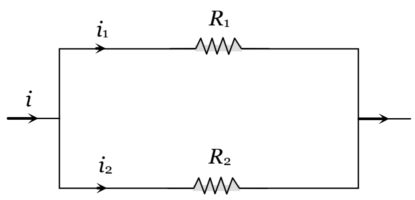
$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) \cdot V$ and $V_2 = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V$

(5) Current through any resistance

$i' = i \times \left[\frac{\text{Resistance of opposite branch}}{\text{Total resistance}}\right]$

Where i' = required current (branch current)

i = main current



$i_1 = i\left(\frac{R_2}{R_1 + R_2}\right)$ and $i_2 = i\left(\frac{R_1}{R_1 + R_2}\right)$

(6) If n identical resistance are connected in series

$R_{eq} = nR$ and p.d. across each resistance $V' = \frac{V}{n}$

(6) In n identical resistance are connected in parallel

$R_{eq} = \frac{R}{n}$ and current through each resistance $i' = \frac{i}{n}$

Note : ❑ In case of resistances in series, if one resistance gets open, the current in the whole circuit become zero and the circuit stops working. Which don't happen in case of parallel gouging.

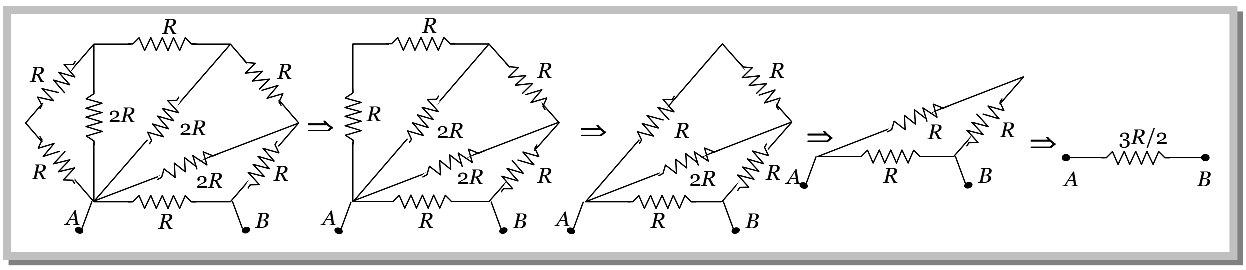
❑ Decoration of lightning in festivals is an example of series grouping whereas all household appliances connected in parallel grouping.

❑ Using n conductors of equal resistance, the number of possible combinations is 2^{n-1}.

❑ If the resistance of n conductors are totally different, then the number of possible combinations will be 2^n.

Methods of Determining Equivalent Resistance For Some Difficult Networks

(1) **Method of successive reduction :** It is the most common technique to determine the equivalent resistance. So far, we have been using this method to find out the equivalent resistances. This method is applicable only when we are able to identify resistances in series or in parallel. The method is based on the simplification of the circuit by successive reduction of the series and parallel combinations. For example to calculate the equivalent resistance between the point A and B, the network shown below successively reduced.



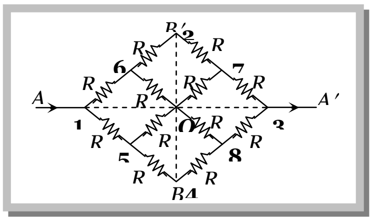
(2) **Method of equipotential points :** This method is based on identifying the points of same potential and joining them. The basic rule to identify the points of same potential is the symmetry of the network.

(i) In a given network there may be two axes of symmetry.

(a) Parallel axis of symmetry, that is, along the direction of current flow.

(b) Perpendicular axis of symmetry, that is perpendicular to the direction of flow of current.

For example in the network shown below the axis AA' is the parallel axis of symmetry, and the axis BB' is the perpendicular axis of symmetry.

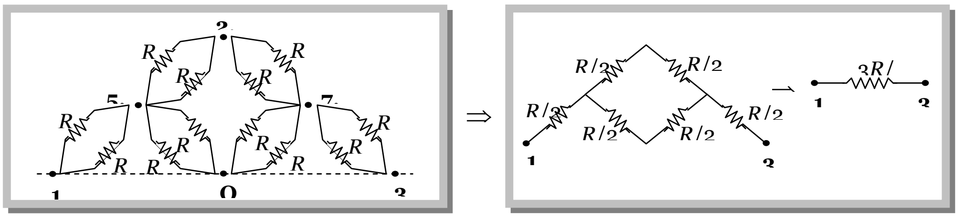


(ii) Points lying on the perpendicular axis of symmetry may have same potential. In the given network, point 2, 0 and 4 are at the same potential.

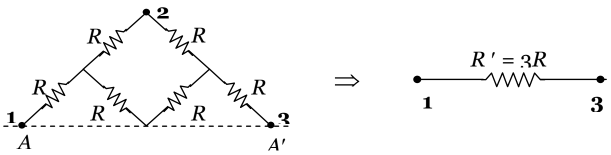
(iii) Points lying on the parallel axis of symmetry can never have same potential.

(iv) The network can be folded about the parallel axis of symmetry, and the overlapping nodes have same potential. Thus as shown in figure, the following points have same potential

(a) 5 and 6 (b) 2, 0 and 4 (c) 7 and 8



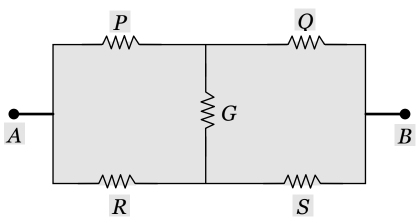
Note : ❑ Above network may be split up into two equal parts about the parallel axis of symmetry as shown in figure each part has a resistance R', then the equivalent resistance of the network will be $R = \frac{R'}{2}$.



Some Standard Results for Equivalent Resistance

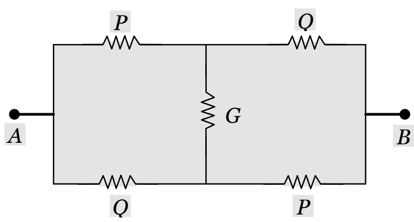
(1) **Equivalent resistance between points A and B in an unbalanced Wheatstone's bridge as shown in the diagram.**

(i)



$$R_{AB} = \frac{PQ(R+S)+(P+Q)RS+G(P+Q)(R+S)}{G(P+Q+R+S)+(P+R)(Q+S)}$$

(ii)

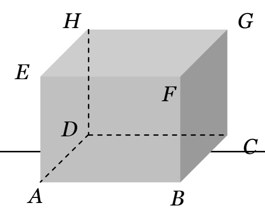


$$R_{AB} = \frac{2PQ+G(P+Q)}{2G+P+Q}$$

(2) **A cube each side have resistance R then equivalent resistance in different situations**

(i) Between E and C *i.e.* across the diagonal of the cube $R_{EC} = \frac{5}{6}R$

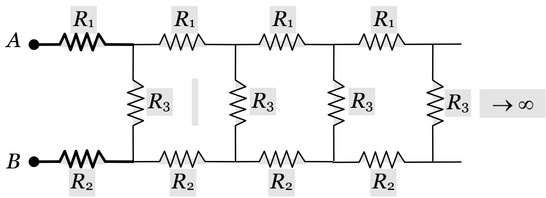
(ii) Between A and B *i.e.* across one side of the cube $R_{AB} = \frac{7}{12}R$



(iii) Between A and C *i.e.* across the diagonal of one face of the cube $R_{AC} = \frac{3}{4}R$

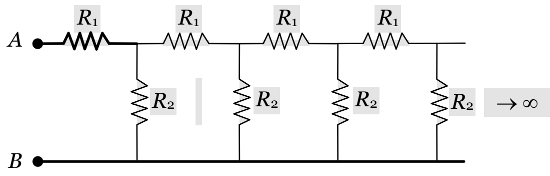
(3) **The equivalent resistance of infinite network of resistances**

(i)



$$R_{AB} = \frac{1}{2}(R_1 + R_2) + \frac{1}{2}\left[(R_1 + R_2)^2 + 4R_3(R_1 + R_2)\right]^{1/2}$$

(ii)



$$R_{AB} = \frac{1}{2}R_1\left[1 + \sqrt{1 + 4\left(\frac{R_2}{R_1}\right)}\right]$$

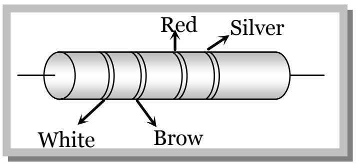
Concepts

- *If n identical resistances are first connected in series and then in parallel, the ratio of the equivalent resistance is given by* $\frac{R_p}{R_s} = \frac{n^2}{1}$.
- *If equivalent resistance of R_1 and R_2 in series and parallel be R_s and R_p respectively then* $R_1 = \frac{1}{2}\left[R_s + \sqrt{R_s^2 - 4R_sR_p}\right]$ *and* $R_2 = \frac{1}{2}\left[R_s - \sqrt{R_s^2 - 4R_sR_p}\right]$.
- *If a wire of resistance R, cut in n equal parts and then these parts are collected to form a bundle then equivalent resistance of combination will be* $\frac{R}{n^2}$.

Example

***Example*: 27** In the figure a carbon resistor has band of different colours on its body. The resistance of the following body is

(a) 2.2 $k\Omega$
(b) 3.3 $k\Omega$
(c) 5.6 $k\Omega$
(d) 9.1 $k\Omega$l



,

Solution : (d) $R = 91 \times 10^2 \pm 10\% \approx 9.1\ k\Omega$

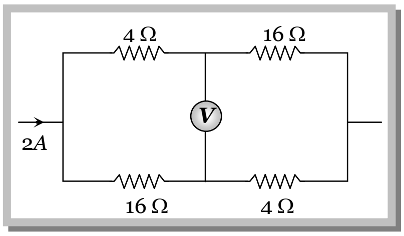
***Example*: 28** What is the resistance of a carbon resistance which has bands of colours brown, black and brown **[DCE 1999]**

(a) 100 Ω (b) 1000 Ω (c) 10 Ω (d) 1 Ω

Solution : (a) $R = 10 \times 10^1 \pm 20\% \approx 100\ \Omega$

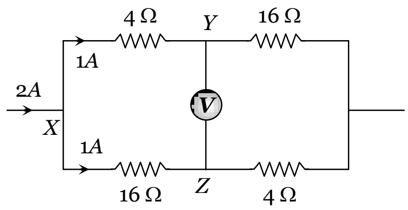
***Example*: 29** In the following circuit reading of voltmeter V is **[MP PET 2003]**

(a) 12 V
(b) 8 V
(c) 20 V
(d) 16 V



Solution : (a) P.d. between X and Y is $V_{XY} = V_X - V_Y = 1 \times 4 = 4\ V$ .... (i)

and p.d. between X and Z is $V_{XZ} = V_X - V_Z = 1 \times 16 = 16\ V$ .... (ii)



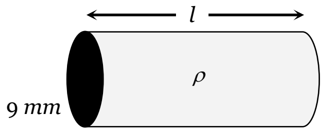
On solving equations (i) and (ii) we get potential difference between Y and Z *i.e.*, reading of voltmeter is $V_Y - V_Z = 12V$

Example: 30 An electric cable contains a single copper wire of radius 9 *mm*. It's resistance is 5 Ω. This cable is replaced by six insulated copper wires, each of radius 3 *mm*. The resultant resistance of cable will be **[CPMT 198**

(a) 7.5 Ω (b) 45 Ω (c) 90 Ω (d) 270 Ω

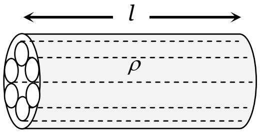
Solution : (a) Initially : Resistance of given cable

$$R = \rho \frac{l}{\pi \times (9 \times 10^{-3})^2} \quad ..... \text{(i)}$$



Finally : Resistance of each insulated copper wire is

$$R' = \rho \frac{l}{\pi \times (3 \times 10^{-3})^2}$$



Hence equivalent resistance of cable

$$R_{eq} = \frac{R'}{6} = \frac{1}{6} \times \left(\rho \frac{l}{\pi \times (3 \times 10^{-3})^2} \right) \quad ..... \text{(ii)}$$

On solving equation (i) and (ii) we get $R_{eq} = 7.5\ \Omega$

Example: 31 Two resistance R_1 and R_2 provides series to parallel equivalents as $\frac{n}{1}$ then the correct relationship is

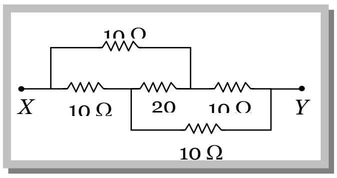
(a) $\left(\frac{R_1}{R_2}\right)^2 + \left(\frac{R_2}{R_1}\right)^2 = n^2$ (b) $\left(\frac{R_1}{R_2}\right)^{3/2} + \left(\frac{R_2}{R_1}\right)^{3/2} = n^{3/2}$

(c) $\left(\frac{R_1}{R_2}\right) + \left(\frac{R_2}{R_1}\right) = n$ (d) $\left(\frac{R_1}{R_2}\right)^{1/2} + \left(\frac{R_2}{R_1}\right)^{1/2} = n^{1/2}$

Solution : (d) Series resistance $R_S = R_1 + R_2$ and parallel resistance $R_P = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow \frac{R_S}{R_P} = \frac{(R_1 + R_2)^2}{R_1 R_2} = n$

$$\Rightarrow \frac{R_1 + R_2}{\sqrt{R_1 R_2}} = \sqrt{n} \qquad \Rightarrow \frac{\sqrt{R_1^2}}{\sqrt{R_1 R_2}} + \frac{\sqrt{R_2^2}}{\sqrt{R_1 R_2}} = \sqrt{n} \Rightarrow \sqrt{\frac{R_1}{R_2}} + \sqrt{\frac{R_2}{R_1}} = \sqrt{n}$$

Example: 32 Five resistances are combined according to the figure. The equivalent resistance between the point X and Y will be



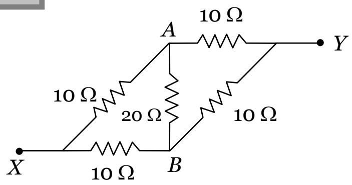
(a) 10 Ω

(b) 22 Ω

(c) 20 Ω

(d) 50 Ω

Solution : (a) The equivalent circuit of above can be drawn as



Which is a balanced wheatstone bridge.

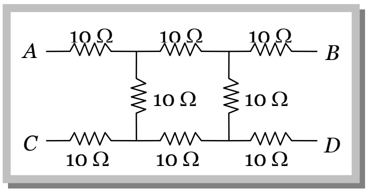
So current through AB is zero.

So $\frac{1}{R} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10} \Rightarrow R = 10\ \Omega$

Example: 33 What will be the equivalent resistance of circuit shown in figure between points A and D **[CBSE PMT 1996]**

(a) 10 Ω

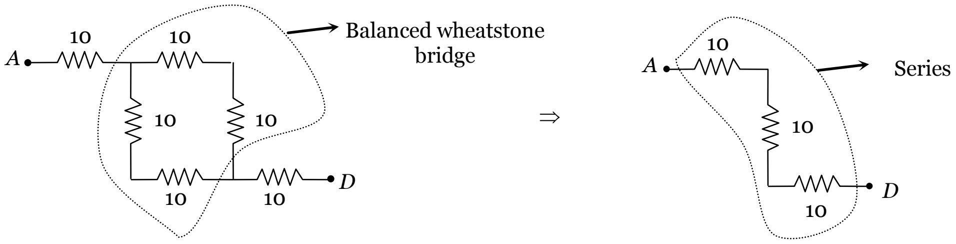
(b) 20 Ω



(c) 30 Ω

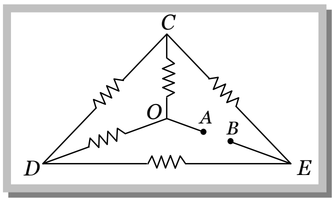
(d) 40 Ω

Solution : (c) The equivalent circuit of above fig between *A* and *D* can be drawn as



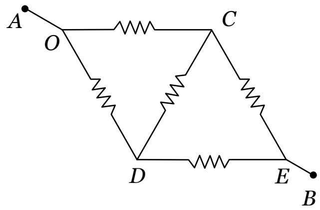
So $R_{eq} = 10 + 10 + 10 = 30\Omega$

***Example*: 34** In the network shown in the figure each of resistance is equal to 2Ω. The resistance between *A* and *B* is **[CBSE PMT 1995]**



(a) 1 Ω
(b) 2 Ω
(c) 3 Ω
(d) 4 Ω

Solution : (b) Taking the portion *COD* is figure to outside the triangle (left), the above circuit will be now as resistance of each is 2 Ω the circuit will behaves as a balanced wheatstone bridge and no current flows through *CD*. Hence $R_{AB} = 2\Omega$



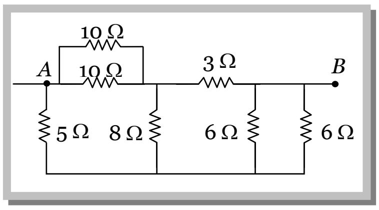
***Example*: 35** Seven resistances are connected as shown in figure. The equivalent resistance between *A* and *B* is **[MP PET 2000]**

(a) 3 Ω

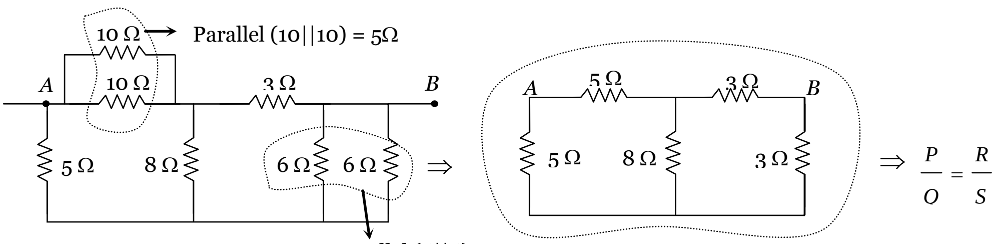
(b) 4 Ω

(c) 4.5 Ω

(d) 5 Ω



Solution : (b)

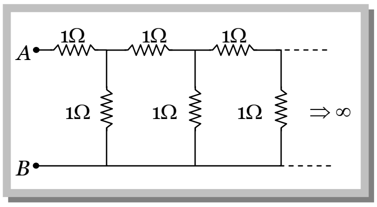


So the circuit is a balanced wheatstone bridge.
So current through 8Ω is zero $R_{eq} = (5+3)||\ (5+3) = 8||\ 8 = 4\Omega$

***Example*: 36** The equivalent resistance between points *A* and *B* of an infinite network of resistance, each of 1 Ω, connected as shown is **[CEE Haryana 1996]**

(a) Infinite

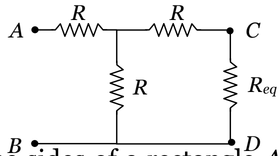
(b) 2 Ω



(c) $\frac{1+\sqrt{5}}{2}\Omega$

(d) Zero

Solution : (c) Suppose the effective resistance between A and B is R_{eq}. Since the network consists of infinite cell. If we exclude one cell from the chain, remaining network have infinite cells *i.e.* effective resistance between C and D will also R_{eq}

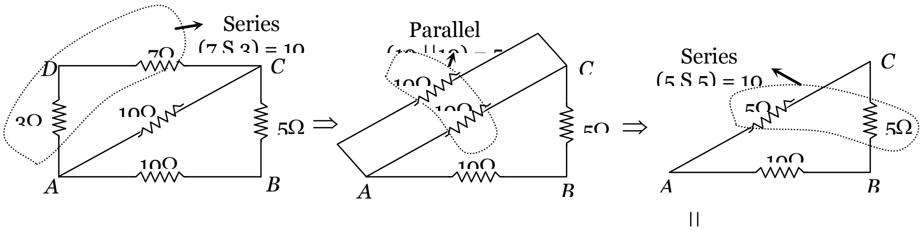


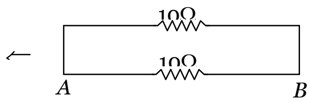
So now $R_{eq} = R_o + (R||\ R_{eq}) = R + \frac{R\,R_{eq}}{R + R_{eq}} \Rightarrow R_{eq} = \frac{1}{2}[1+\sqrt{5}]$

***Example*: 37** Four resistances 10 Ω, 5 Ω, 7 Ω and 3 Ω are connected so that they form the sides of a rectangle AB, BC, CD and DA respectively. Another resistance of 10 Ω is connected across the diagonal AC. The equivalent resistance between A & B is

(a) 2 Ω (b) 5 Ω (c) 7 Ω (d) 10 Ω

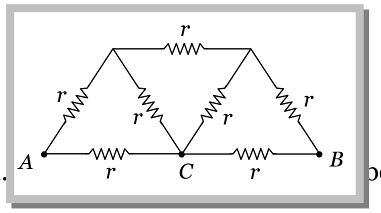
Solution : (b)





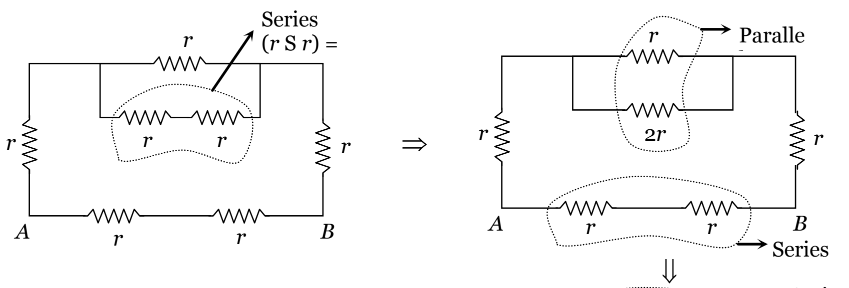
So $R_{eq} = \frac{10\times 10}{10+10} = 5\Omega$

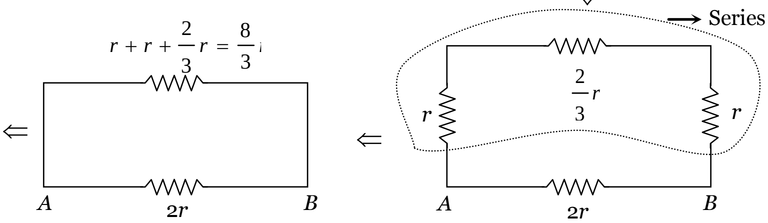
***Example*: 38** The equivalent resistance between A and B in the circuit will be



(a) $\frac{5}{4}r$ (b) $\frac{6}{5}r$ (c) $\frac{7}{6}r$ (d) $\frac{8}{7}r$

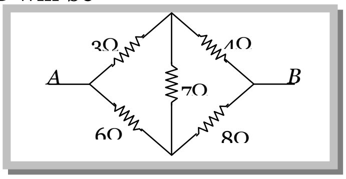
Solution : (d) In the circuit, by means of symmetry the point C is at zero potential. [illegible] be drawn as





$R_{eq} = \left(\frac{8r}{3}||\ 2r\right) = \frac{8}{7}r$

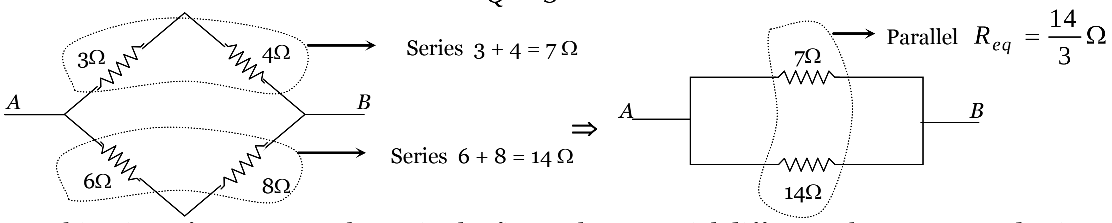
***Example*: 39** In the given figure, equivalent resistance between A and B will be **[CBSE PMT 2000]**



(a) $\frac{14}{3}\Omega$ (b) $\frac{3}{14}\Omega$

(c) $\frac{9}{14}\Omega$ (d) $\frac{14}{9}\Omega$

Solution : (a) Given Wheatstone bridge is balanced because $\frac{P}{Q}=\frac{R}{S}$. Hence the circuit can be redrawn as follows



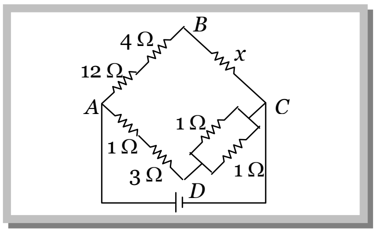
***Example*: 40** In the combination of resistances shown in the figure the potential difference between B and D is zero, when unknown resistance (x) is

(a) 4 Ω

(b) 2 Ω

(c) 3 Ω

(d) The emf of the cell is required



Solution : (b) The potential difference across B, D will be zero, when the circuit will act as a balanced wheatstone bridge and $\frac{P}{Q}=\frac{R}{S}\Rightarrow\frac{12+4}{x}=\frac{1+3}{1/2}\Rightarrow x=2\Omega$

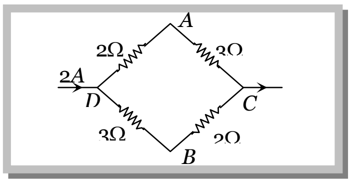
***Example*: 41** A current of 2 A flows in a system of conductors as shown. The potential difference ($V_A - V_B$) will be **[CPMT 1975, 76]**

(a) + 2V

(b) + 1V

(c) – 1 V

(d) – 2 V



Solution : (b) In the given circuit 2A current divides equally at junction D along the paths DAC and DBC (each path carry 1A current).

Potential difference between D and A, $V_D - V_A = 1\times 2 = 2$ *volt* .... (i)

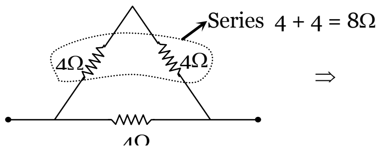
Potential difference between D and B, $V_D - V_B = 1\times 3 = 3$ *volt* ..... (ii)

On solving (i) and (ii) $V_A - V_B = +1$ *volt*

***Example*: 42** Three resistances each of 4 Ω are connected in the form of an equilateral triangle. The effective resistance between two corners is

(a) 8 Ω (b) 12 Ω (c) $\frac{3}{8}\Omega$ (d) $\frac{8}{3}\Omega$

Solution : (d)

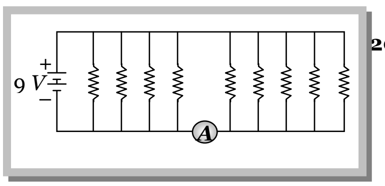


⇒ On Solving further we get equivalent resistance is $\frac{8}{3}\Omega$

***Example*: 43** If each resistance in the figure is of 9 Ω then reading of ammeter is 2000]

(a) 5 A (b) 8 A

(c) 2 A (d) 9 A

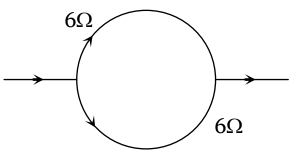


Solution : (a) Main current through the battery $i=\frac{9}{1}=9A$. Current through each resistance will be 1A and only 5 resistances on the right side of ammeter contributes for passing current through the ammeter. So reading of ammeter will be 5A.

***Example*: 44** A wire has resistance 12 Ω. It is bent in the form of a circle. The effective resistance between the two points on any diameter is equal to

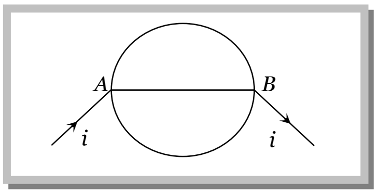
(a) 12 Ω (b) 6 Ω (c) 3 Ω (d) 24 Ω

Solution : (c) Equivalent resistance of the following circuit will be



$$R_{eq} = \frac{6}{2} = 3\Omega$$

Example: 45 A wire of resistance 0.5 Ωm^{-1} is bent into a circle of radius 1 m. The same wire is connected across a diameter AB as shown in fig. The equivalent resistance is



(a) π *ohm*

(b) $\pi(\pi + 2)$ *ohm*

(c) $\pi / (\pi + 4)$ *ohm*

(d) $(\pi + 1)$ *ohm*

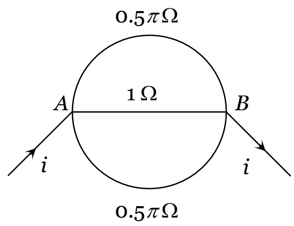
Solution : (c) Resistance of upper semicircle = Resistance of lower semicircle

$= 0.5 \times (\pi R) = 0.5\ \pi\Omega$

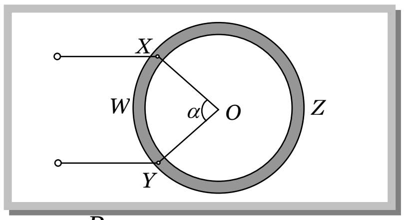
Resistance of wire AB $= 0.5 \times 2 = 1\ \Omega$

Hence equivalent resistance between A and B

$$\frac{1}{R_{AB}} = \frac{1}{0.5\pi} + \frac{1}{1} + \frac{1}{0.5\pi} \Rightarrow R_{AB} = \frac{\pi}{(\pi+4)}\Omega$$



Example: 46 A wire of resistor R is bent into a circular ring of radius r. Equivalent resistance between two points X and Y on its circumference, when angle XOY is α, can be given by



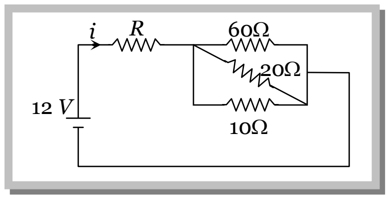
(a) $\frac{R\alpha}{4\pi^2}(2\pi - \alpha)$ (b) $\frac{R}{2\pi}(2\pi - \alpha)$

(c) $R(2\pi - \alpha)$ (d) $\frac{4\pi}{R\alpha}(2\pi - \alpha)$

Solution : (a) Here $R_{XWY} = \frac{R}{2\pi r} \times (r\alpha) = \frac{R\alpha}{2\pi}$ $\left(\because \alpha = \frac{l}{r}\right)$ and $R_{XZY} = \frac{R}{2\pi r} \times r(2\pi - \alpha) = \frac{R}{2\pi}(2\pi - \alpha)$

$$R_{eq} = \frac{R_{XWY} R_{XZY}}{R_{XWY} + R_{XZY}} = \frac{\frac{R\alpha}{2\pi} \times \frac{R}{2\pi}(2\pi - \alpha)}{\frac{R\alpha}{2\pi} + \frac{R(2\pi - \alpha)}{2\pi}} = \frac{R\alpha}{4\pi^2}(2\pi - \alpha)$$

Example: 47 If in the given figure $i = 0.25$ *amp*, then the value R will be **[RPET 2000]**



(a) 48 Ω

(b) 12 Ω

(c) 120 Ω

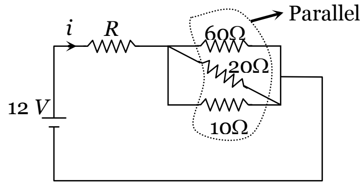
(d) 42 Ω

Solution : (d) $i = 0.25$ *amp* $V = 12\ V$ $R_{eq} = \frac{V}{i} = \frac{12}{0.25} = 48\ \Omega$

Now from the circuit $R_{eq} = R + (60 || 20 || 10)$

$= R + 6$

$\Rightarrow R = R_{eq} - 6 = 48 - 6 = 42\ \Omega$



Example: 48 Two uniform wires A and B are of the same metal and have equal masses. The radius of wire A is twice that of wire B. The total resistance of A and B when connected in parallel is **[MNR 1994]**

(a) 4 Ω when the resistance of wire A is 4.25 Ω (b) 5 Ω when the resistance of wire A is 4 Ω

(c) 4 Ω when the resistance of wire B is 4.25 Ω (d) 5 Ω when the resistance of wire B is 4 Ω

Solution : (a) Density and masses of wire are same so their volumes are same *i.e.* $A_1 l_1 = A_2 l_2$

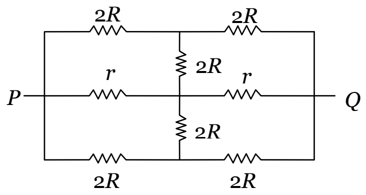
Ratio of resistances of wires A and B $\frac{R_A}{R_B} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^4$

Since $r_1 = 2r_2$ so $\frac{R_A}{R_B} = \frac{1}{16} \Rightarrow R_B = 16\, R_A$

Resistance R_A and R_B are connected in parallel so equivalent resistance $R = \frac{R_A R_B}{R_A + R_B} = \frac{16 R_A}{17}$, By checking correctness of equivalent resistance from options, only option (a) is correct.

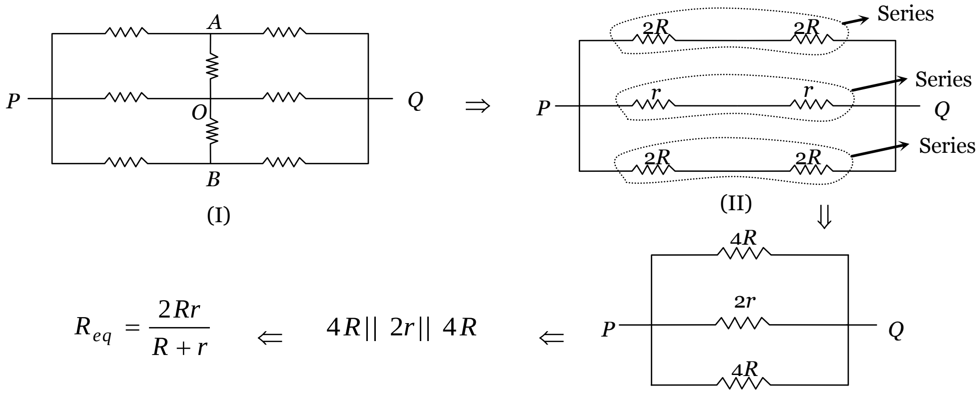
Tricky Example: 5

The effective resistance between point P and Q of the electrical circuit shown in the figure is **[IIT-JEE 1991]**



(a) $\frac{2Rr}{R+r}$ (b) $\frac{8R(R+r)}{3R+r}$ (c) $2r + 4R$ (d) $\frac{5R}{2} + 2r$

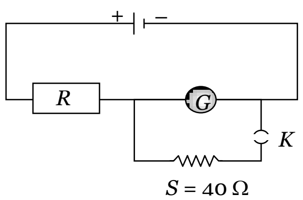
Solution : (a) The points A, O, B are at same potential. So the figure can be redrawn as follows



$R_{eq} = \frac{2Rr}{R+r} \Leftarrow 4R \,||\, 2r \,||\, 4R \Leftarrow$

Tricky Example: 6

In the following circuit if key K is pressed then the galvanometer reading becomes half. The resistance of galvanometer is

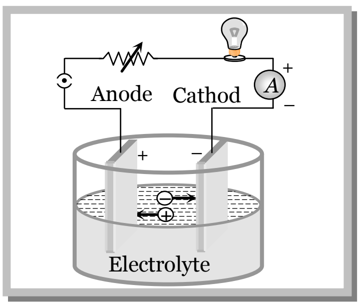


(a) 20 Ω (b) 30 Ω (c) 40 Ω (d) 50 Ω

Solution : (c) Galvanometer reading becomes half means current distributes equally between galvanometer and resistance of 40 Ω. Hence galvanometer resistance must be 40 Ω.

Cell

The device which converts chemical energy into electrical energy is known as electric cell.



(1) A cell neither creates nor destroys charge but maintains the flow of charge present at various parts of the circuit by supplying energy needed for their organised motion.

(2) Cell is a source of constant emf but not constant current.

(3) Mainly cells are of two types :

(i) Primary cell : Cannot be recharged

(ii) Secondary cell : Can be recharged

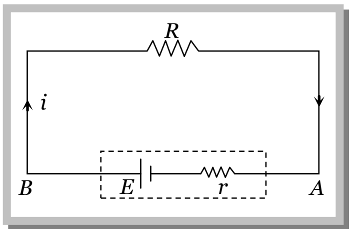
(4) The direction of flow of current inside the cell is from negative to positive electrode while outside the cell is form positive to negative electrode.

(5) A cell is said to be ideal, if it has zero internal resistance.

(6) **Emf of cell (*E*) :** The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called it's electromotive force (emf) *i.e.* emf of cell $E = \frac{W}{q}$, It's unit is *volt* or

The potential difference across the terminals of a cell when it is not given any current is called it's emf.

(7) **Potential difference (*V*) :** The energy given by the cell in the flow of unit charge in a specific part of electrical circuit (external part) is called potential difference. It's unit is also *volt* or

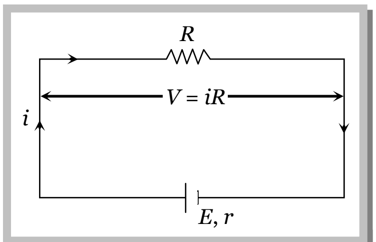


The voltage across the terminals of a cell when it is supplying current to external resistance is called potential difference or terminal voltage. Potential difference is equal to the product of current and resistance of that given part *i.e.* $V = iR$.

(8) **Internal resistance (*r*) :** In case of a cell the opposition of electrolyte to the flow of current through it is called internal resistance of the cell. The internal resistance of a cell depends on the distance between electrodes ($r \propto d$), area of electrodes [$r \propto (1/A)$] and nature, concentration ($r \propto C$) and temperature of electrolyte [$r \propto (1/\text{temp.})$]. Internal resistance is different for different types of cells and even for a given type of cell it varies from to cell.

Cell in Various Position

(1) **Closed circuit (when the cell is discharging)**



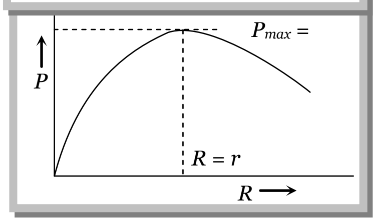
(i) Current given by the cell $i = \frac{E}{R+r}$

(ii) Potential difference across the resistance $V = iR$

(iii) Potential drop inside the cell = ir

(iv) Equation of cell $E = V + ir$ $(E > V)$

(v) Internal resistance of the cell $r = \left(\frac{E}{V} - 1\right) \cdot R$



(vi) Power dissipated in external resistance (load) $P = Vi = i^2R = \frac{V^2}{R} = \left(\frac{E}{R+r}\right)^2 .R$

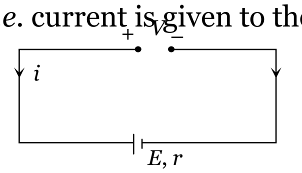
Power delivered will be maximum when $R = r$ so $P_{\max} = \frac{E^2}{4r}$.

This statement in generalised from is called "*maximum power transfer theorem*".

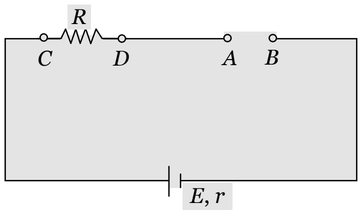
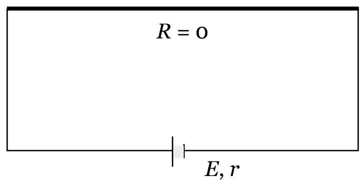
(vii) **Short trick to calculate *E* and *r* :** In the closed circuit of a cell having emf E and internal resistance r. If external resistance changes from R_1 to R_2 then current changes from i_1 to i_2 and potential difference changes from V_1 to V_2. By using following relations we can find the value of E and r.

$$E = \frac{i_1 i_2}{i_2 - i_1}(R_1 - R_2) \quad r = \left(\frac{i_2 R_2 - i_1 R_1}{i_1 - i_2}\right) = \frac{V_2 - V_1}{i_1 - i_2}$$

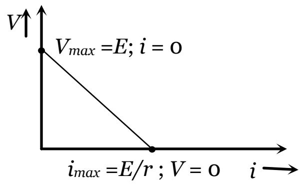
Note : ❑ When the cell is charging *i.e.* current is given to the cell then $E = V - ir$ and $E < V$.



(2) **Open circuit and short circuit**

Open circuit	**Short circuit**
	
(i) Current through the circuit $i = 0$	(i) Maximum current (called short circuit current) flows momentarily $i_{sc} = \frac{E}{r}$
(ii) Potential difference between A and B, $V_{AB} = E$	(ii) Potential difference $V = 0$
(iii) Potential difference between C and D, $V_{CD} = 0$	

Note : ❑ Above information's can be summarized by the following graph



Concepts

☞ *It is a common misconception that "current in the circuit will be maximum when power consumed by the load is maximum."*

Actually current $i = E/(R+r)$ *is maximum (= E/r) when R = min = 0 with* $P_L = (E/r)^2 \times 0 = 0$ min. *while power consumed by the load* $E^2R/(R+r)^2$ *is maximum* $(= E^2/4r)$ *when* $R = r$ *and* $i = (E/2r) \neq \max(= E/r)$.

☞ *Emf is independent of the resistance of the circuit and depends upon the nature of electrolyte of the cell while potential difference depends upon the resistance between the two points of the circuit and current flowing through the circuit.*

☞ *Emf is a cause and potential difference is an effect.*

☞ *Whenever a cell or battery is present in a branch there must be some resistance (internal or external or both) present in that branch. In practical situation it always happen because we can never have an ideal cell or battery with zero resistance.*

Example

Example: 49 A new flashlight cell of emf 1.5 *volts* gives a current of 15 *amps*, when connected directly to an ammeter of resistance 0.04 Ω. The internal resistance of cell is

(a) 0.04 Ω (b) 0.06 Ω (c) 0.10 Ω (d) 10 Ω

Solution : (b) By using $i = \frac{E}{R+r} \Rightarrow 15 = \frac{1.5}{0.04+r} \Rightarrow r = 0.06\,\Omega$

Example: 50 For a cell, the terminal potential difference is 2.2 *V* when the circuit is open and reduces to 1.8 *V*, when the cell is connected across a resistance, *R* = 5Ω. The internal resistance of the cell is

(a) $\frac{10}{9}\Omega$ (b) $\frac{9}{10}\Omega$ (c) $\frac{11}{9}\Omega$ (d) $\frac{5}{9}\Omega$

Solution : (a) In open circuit, $E = V = 2.2\,V$, In close circuit, $V = 1.8\,V$, $R = 5\Omega$

So internal resistance, $r = \left(\frac{E}{V}-1\right)R = \left(\frac{2.2}{1.8}-1\right)\times 5 \Rightarrow r = \frac{10}{9}\Omega$

Example: 51 The internal resistance of a cell of emf 2*V* is 0.1 Ω. It's connected to a resistance of 3.9 Ω. The voltage across the cell will be **[CBSE PMT 1999; AFMC 1999; MP PET 1993; CPMT 1990]**

(a) 0.5 *volt* (b) 1.9 *volt* (c) 1.95 *volt* (d) 2 *volt*

Solution : (c) By using $r = \left(\frac{E}{V}-1\right)R \Rightarrow 0.1 = \left(\frac{2}{V}-1\right)\times 3.9 \Rightarrow V = 1.95\,volt$

Example: 52 When the resistance of 2 Ω is connected across the terminal of the cell, the current is 0.5 *amp*. When the resistance is increased to 5 Ω, the current is 0.25 *amp*. The emf of the cell is

(a) 1.0 *volt* (b) 1.5 *volt* (c) 2.0 *volt* (d) 2.5 *volt*

Solution : (b) By using $E = \frac{i_1 i_2}{(i_2 - i_1)}(R_1 - R_2) = \frac{0.5\times 0.25}{(0.25-0.5)}(2-5) = 1.5\,volt$

Example: 53 A primary cell has an emf of 1.5 *volts*, when short-circuited it gives a current of 3 *amperes*. The internal resistance of the cell is

(a) 4.5 *ohm* (b) 2 *ohm* (c) 0.5 *ohm* (d) 1/4.5 *ohm*

Solution : (c) $i_{sc} = \frac{E}{r} \Rightarrow 3 = \frac{1.5}{r} \Rightarrow r = 0.5\,\Omega$

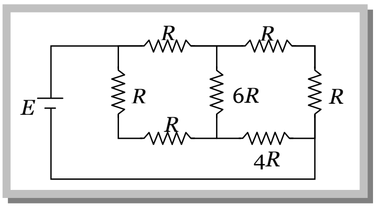
Example: 54 A battery of internal resistance 4 Ω is connected to the network of resistances as shown. In order to give the maximum power to the network, the value of *R* (in Ω) should be **[IIT-JEE 1995]**

(a) 4/9

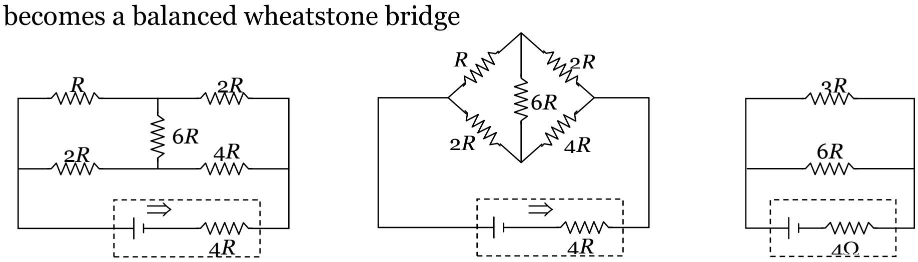
(b) 8/9

(c) 2

(d) 18



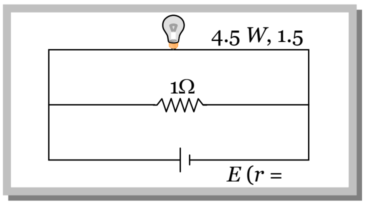
Solution : (c) The equivalent circuit becomes a balanced wheatstone bridge



For maximum power transfer, external resistance should be equal to internal resistance of source

$$\Rightarrow \frac{(R+2R)(2R+4R)}{(R+2R)+(2R+4R)} = 4 \text{ i.e. } \frac{3R \times 6R}{3R+6R} = 4 \text{ or } R = 2\Omega$$

***Example*: 55** A torch bulb rated as 4.5 *W*, 1.5 *V* is connected as shown in the figure. The emf of the cell needed to make the bulb glow at full intensity is



(a) 4.5 *V*

(b) 1.5 *V*

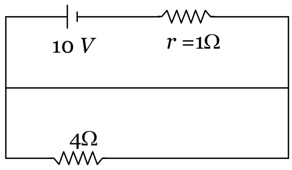
(c) 2.67 *V*

(d) 13.5 *V*

Solution : (d) When bulb glows with full intensity, potential difference across it is 1.5 *V*. So current through the bulb and resistance of 1Ω are 3 *A* and 1.5 *A* respectively. So main current from the cell $i = 3 + 1.5 = 4.5\,A$. By using $E = V + iR \Rightarrow E = 1.5 + 4.5 \times 2.67 = 13.5\,V$.

Tricky Example: 7

Potential difference across the terminals of the battery shown in figure is (*r* = internal resistance of battery)



(a) 8 *V* (b) 10 *V*

(c) 6 *V* (d) Zero

Solution : (d) Battery is short circuited so potential difference is zero.

Grouping of cell

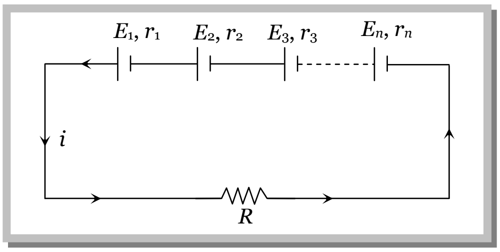
Group of cell is called a battery.

(1) **Series grouping :** In series grouping anode of one cell is connected to cathode of other cell and so on.

(i) ***n* identical cells are connected in series**

(a) Equivalent emf of the combination $E_{eq} = nE$

(b) Equivalent internal resistance $r_{eq} = nr$



(c) Main current = Current from each cell $= i = \dfrac{nE}{R+nr}$

(d) Potential difference across external resistance $V = iR$

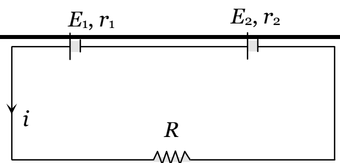
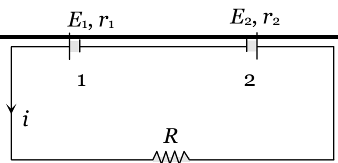
(e) Potential difference across each cell $V' = \dfrac{V}{n}$

(f) Power dissipated in the circuit $P = \left(\dfrac{nE}{R+nr}\right)^2 . R$

(g) Condition for maximum power $R = nr$ and $P_{\max} = n\left(\dfrac{E^2}{4r}\right)$

(h) This type of combination is used when $nr << R$.

(ii) **If non-identical cell are connected in series**

Cells are connected in right order	Cells are wrongly connected
	$(E_1 > E_2)$ 

(a) Equivalent emf $E_{eq} = E_1 + E_2$

(b) Current $i = \dfrac{E_{eq}}{R + r_{eq}}$

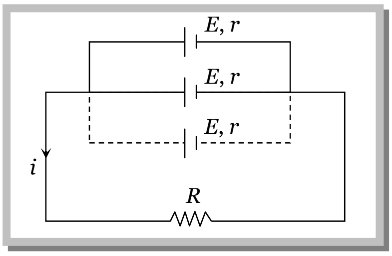
(c) Potential difference across each cell $V_1 = E_1 - ir_1$ and $V_2 = E_2 - ir_2$

(a) Equivalent emf $E_{eq} = E_1 - E_2$

(b) Current $i = \dfrac{E_1 - E_2}{R + r_{eq}}$

(c) in the above circuit cell 1 is discharging so it's equation is $E_1 = V_1 + ir_1 \Rightarrow \boldsymbol{V_1 = E_1 - ir_1}$ and cell 2 is charging so it's equation $E_2 = V_2 - ir_2 \Rightarrow \boldsymbol{V_2 = E_2 + ir_2}$

(2) **Parallel grouping :** In parallel grouping all anodes are connected at one point and all cathode are connected together at other point.



(i) **If *n* identical cells are connected in parallel**

(a) Equivalent emf $E_{eq} = E$

(b) Equivalent internal resistance $R_{eq} = r/n$

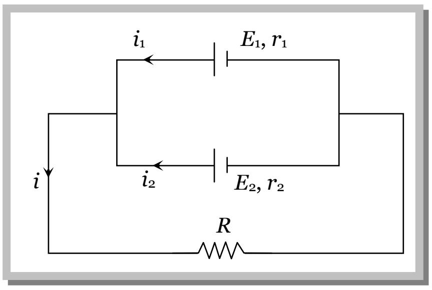
(c) Main current $i = \dfrac{E}{R + r/n}$

(d) P.d. across external resistance = p.d. across each cell = $V = iR$

(e) Current from each cell $i' = \dfrac{i}{n}$ (f) Power dissipated in the circuit $P = \left(\dfrac{E}{R + r/n}\right)^2 . R$

(g) Condition for max power $R = r/n$ and $P_{max} = n\left(\dfrac{E^2}{4r}\right)$ (h) This type of combination is used when $nr >> R$

(ii) **If non-identical cells are connected in parallel :** If cells are connected with right polarity as shown below then

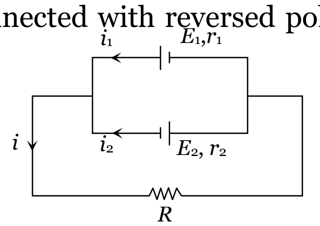


(a) Equivalent emf $E_{eq} = \dfrac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

(b) Main current $i = \dfrac{E_{eq}}{r + R_{eq}}$

(c) Current from each cell $i_1 = \dfrac{E_1 - iR}{r_1}$ and $i_2 = \dfrac{E_2 - iR}{r_2}$

Note : ❑ In this combination if cell's are connected with reversed polarity as shown in figure then :

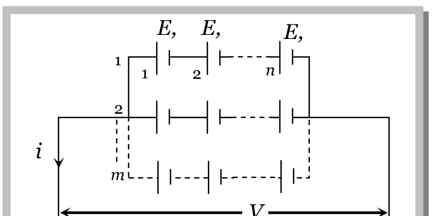


Equivalent emf $\boldsymbol{E_{eq} = \dfrac{E_1 r_2 - E_2 r_1}{r_1 + r_2}}$

(3) **Mixed Grouping :** If n identical cell's are connected in a row and such m row's are connected in parallel as shown.

(i) Equivalent emf of the combination $E_{eq} = nE$

(ii) Equivalent internal resistance of the combination $r_{eq} = \dfrac{nr}{m}$



(iii) Main current flowing through the load $i = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{mnE}{mR + nr}$

(iv) Potential difference across load $V = iR$

(v) Potential difference across each cell $V' = \dfrac{V}{n}$

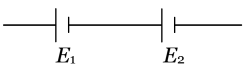
(vi) Current from each cell $i' = \dfrac{i}{n}$

(vii) Condition for maximum power $R = \dfrac{nr}{m}$ and $P_{max} = (mn)\dfrac{E^2}{4r}$

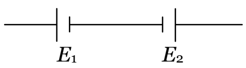
(viii) Total number of cell = mn

Concepts

☞ *In series grouping of cell's their emf's are additive or subtractive while their internal resistances are always additive. If dissimilar plates of cells are connected together their emf's are added to each other while if their similar plates are connected together their emf's are subtractive.*



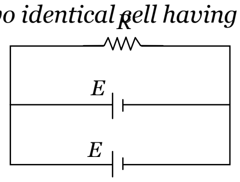
$E_{eq} = E_1 + E_2$ & $r_{eq} = r_1 + r_2$



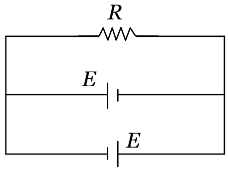
$E_{eq} = E_1 - E_2\ (E_1 > E_2)$ & $r_{eq} = r_1 + r_2$

☞ *In series grouping of identical cells. If one cell is wrongly connected then it will cancel out the effect of two cells e.g. If in the combination of n identical cells (each having emf E and internal resistance r) if x cell are wrongly connected then equivalent emf* $E_{eq} = (n - 2x)E$ *and equivalent internal resistance* $r_{eq} = nr$.

☞ *In parallel grouping of two identical cell having no internal resistance*

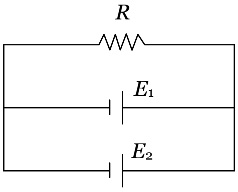


$E_{eq} = E$



$E_{eq} = 0$

☞ *When two cell's of different emf and no internal resistance are connected in parallel then equivalent emf is indeterminate, note that connecting a wire with a cell but with no resistance is equivalent to short circuiting. Therefore the total current that will be flowing will be infinity.*



Example

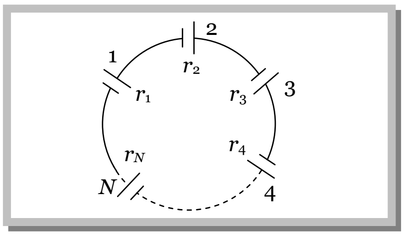
Example: 56 A group of N cells whose emf varies directly with the internal resistance as per the equation $E_N = 1.5\, r_N$ are connected as shown in the following figure. The current i in the circuit is **[KCET 2003]**

(a) 0.51 *amp*

(b) 5.1 *amp*

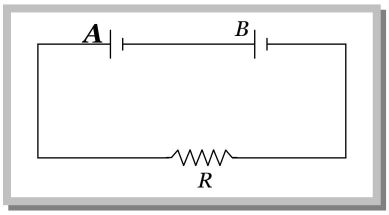
(c) 0.15 *amp*

(d) 1.5 *amp*



Solution : (d) $i = \dfrac{E_{eq}}{r_{eq}} = \dfrac{1.5r_1 + 1.5r_2 + 1.5r_3 + ...}{r_1 + r_2 + r_3 + ...} = 1.5\ amp$

Example: 57 Two batteries A and B each of emf 2 *volt* are connected in series to external resistance R = 1 Ω. Internal resistance of A is 1.9 Ω and that of B is 0.9 Ω, what is the potential difference between the terminals of battery A **[MP PET 2001]**



(a) 2 V

(b) 3.8 V

(c) 0

(d) None of these

Solution : (c) $i = \dfrac{E_1 + E_2}{R + r_1 + r_2} = \dfrac{2+2}{1+1.9+0.9} = \dfrac{4}{3.8}$. Hence $V_A = E_A - ir_A = 2 - \dfrac{4}{3.8}1.9 = 0$

Example: 58 In a mixed grouping of identical cells 5 rows are connected in parallel by each row contains 10 cell. This combination send a current i through an external resistance of 20 Ω. If the emf and internal resistance of each cell is 1.5 *volt* and 1 Ω respectively then the value of i is

(a) 0.14 (b) 0.25 (c) 0.75 (d) 0.68

Solution : (d) No. of cells in a row n = 10; No. of such rows m = 5

$$i = \frac{nE}{\left(R + \dfrac{nr}{m}\right)} = \frac{10 \times 1.5}{\left(20 + \dfrac{10 \times 1}{5}\right)} = \frac{15}{22} = 0.68\ amp$$

Example: 59 To get maximum current in a resistance of 3 Ω one can use n rows of m cells connected in parallel. If the total no. of cells is 24 and the internal resistance of a cell is 0.5 then

(a) m = 12, n = 2 (b) m = 8, n = 4 (c) m = 2, n = 12 (d) m = 6, n = 4

Solution : (a) In this question R = 3Ω, mn = 24, r = 0.5Ω and $R = \dfrac{mr}{n}$. On putting the values we get n = 2 and m = 12.

Example: 60 100 cells each of emf 5V and internal resistance 1 Ω are to be arranged so as to produce maximum current in a 25 Ω resistance. Each row contains equal number of cells. The number of rows should be**[MP PMT 1**

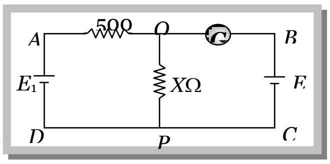
(a) 2 (b) 4 (c) 5 (d) 100

Solution : (a) Total no. of cells, = mn = 100 .... (i)

Current will be maximum when $R = \dfrac{nr}{m}$; $25 = \dfrac{n \times 1}{m} \Rightarrow n = 25\,m$ .... (ii)

From equation (i) and (ii) n = 50 and m = 2

Example: 61 In the adjoining circuit, the battery E_1 has as emf of 12 *volt* and zero internal resistance, while the battery E has an emf of 2 *volt*. If the galvanometer reads zero, then the value of resistance X *ohm* is **[NCERT 199**



(a) 10 (b) 100

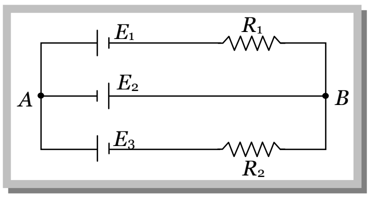
(c) 500 (d) 200

Solution : (b) For zero deflection in galvanometer the potential different across

In this condition $\dfrac{12X}{500 + X} = 2$

$\therefore$ X = 100 Ω

Example: 62 In the circuit shown here $E_1 = E_2 = E_3 = 2V$ and $R_1 = R_2 = 4$ Ω. The current flowing between point A and B through battery E_2 is



(a) Zero

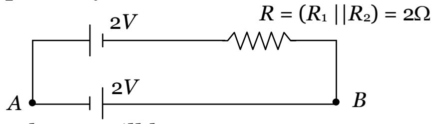
(b) 2 A from A to B

(c) 2 A from B to A

(d) None of these

Solution : (b) The equivalent circuit can be drawn as since E_1 & E_3 are parallely connected

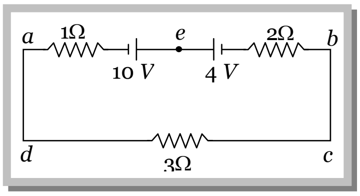
So current $i = \dfrac{2+2}{2} = 2\,Amp$ from A to B.



***Example*: 63** The magnitude and direction of the current in the circuit shown will be **[CPMT 1986, 88]**

(a) $\dfrac{7}{3}A$ from a to b through e (b) $\dfrac{7}{3}A$ from b and a through e

(c) 1.0 A from b to a through e (d) 1.0 A from a to b through e

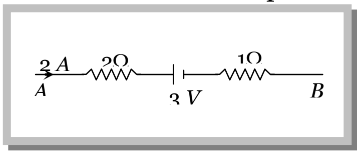


Solution : (d) Current $i = \dfrac{10-4}{3+2+1} = 1A$ from a to b via e

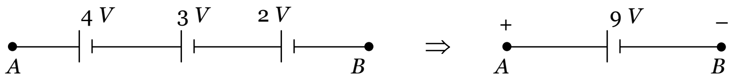
***Example*: 64** Figure represents a part of the closed circuit. The potential difference between points A and B ($V_A - V_B$) is

(a) + 9 V (b) – 9 V

(c) + 3 V (d) + 6 V



Solution : (a) The given part of a closed circuit can be redrawn as follows. It should be remember that product of current and resistance can be treated as an imaginary cell having emf = iR.



$\Rightarrow$ 7 Hence $V_A - V_B = +9\ V$

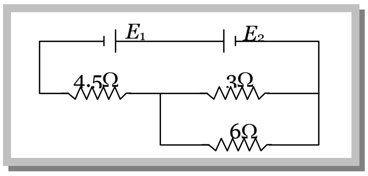
***Example*: 65** In the circuit shown below the cells E_1 and E_2 have emf's 4 V and 8 V and internal resistance 0.5 ohm and 1 ohm respectively. Then the potential difference across cell E_1 and E_2 will be

(a) 3.75 V, 7.5 V

(b) 4.25 V, 7.5 V

(c) 3.75 V, 3.5 V

(d) 4.25 V, 4.25 V



Solution : (b) In the given circuit diagram external resistance $R = \dfrac{3\times 6}{3+6} + 4.5 = 6.5\Omega$. Hence main current through the circuit $i = \dfrac{E_2 - E_1}{R + r_{eq}} = \dfrac{8-4}{6.5+0.5+0.5} = \dfrac{1}{2}\,amp.$

Cell 1 is charging so from it's emf equation $E_1 = V_1 - ir_1 \Rightarrow 4 = V_1 - \dfrac{1}{2}\times 0.5 \Rightarrow V_1 = 4.25\ volt$

Cell 2 is discharging so from it's emf equation $E_2 = V_2 + ir_2 \Rightarrow 8 = V_2 + \dfrac{1}{2}\times 1 \Rightarrow V_2 = 7.5\ volt$

***Example*: 66** A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to this current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length 2L. The temperature of wire is raised by same amount ΔT in the same time t. The value of N is

(a) 4 (b) 6 (c) 8 (d) 9

Solution : (b) Heat = $mS\Delta T = i^2Rt$

Case I : Length (L) $\Rightarrow$ Resistance = R and mass = m

Case II : Length (2L) $\Rightarrow$ Resistance = 2R and mass = 2m

So $\dfrac{m_1 S_1 \Delta T_1}{m_2 S_2 \Delta T_2} = \dfrac{i_1^2 R_1 t_1}{i_2^2 r_2 t_2} \Rightarrow \dfrac{mS\Delta T}{2mS\Delta T} = \dfrac{i_1^2 Rt}{i_2^2\, 2Rt} \Rightarrow i_1 = i_2 \Rightarrow \dfrac{(3E)^2}{12} = \dfrac{(NE)^2}{2R} \Rightarrow N = 6$

Tricky Example: 8

n identical cells, each of emf E and internal resistance r, are joined in series to form a closed circuit. The potential difference across any one cell is

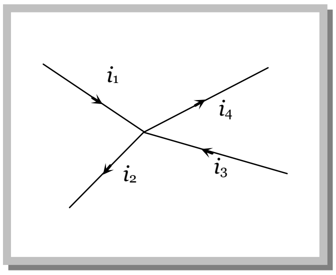
(a) Zero (b) E (c) $\frac{E}{n}$ (d) $\left(\frac{n-1}{n}\right)E$

Solution: (a) Current in the circuit $i = \frac{nE}{nr} = \frac{E}{r}$

The equivalent circuit of one cell is shown in the figure. Potential difference across the cell $= V_A - V_B = -E + ir = -E + \frac{E}{r}.r = 0$

Kirchoff's Laws

(1) **Kirchoff's first law :** This law is also known as junction rule or current law (*KCL*). According to it the algebraic sum of currents meeting at a junction is zero *i.e.* $\Sigma i = 0$.



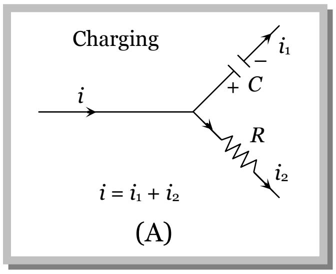
In a circuit, at any junction the sum of the currents entering the junction must equal the sum of the currents leaving the junction. $i_1 + i_3 = i_2 + i_4$

Here it is worthy to note that :

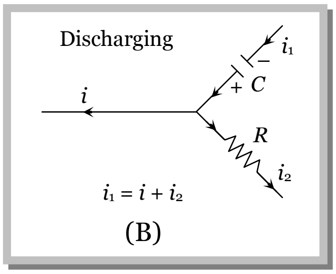
(i) If a current comes out to be negative, actual direction of current at the junction is opposite to that assumed, $i + i_1 + i_2 = 0$ can be satisfied only if at least one current is negative, *i.e.* leaving the junction.

(ii) This law is simply a statement of "*conservation of charge*" as if current reaching a junction is not equal to the current leaving the junction, charge will not be conserved.

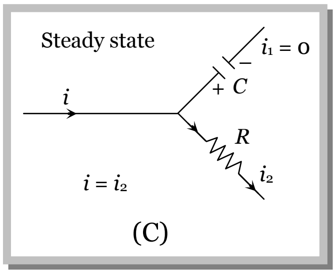
Note : ❑ This law is also applicable to a capacitor through the concept of displacement current treating the resistance of capacitor to be zero during charging or discharging and infinite in steady state as shown in figure.



(A)



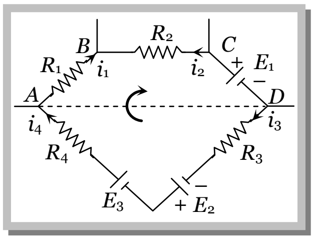
(B)



(C)

(2) **Kirchoff's second law :** This law is also known as loop rule or voltage law (KVL) and according to it "the algebraic sum of the changes in potential in complete traversal of a mesh (closed loop) is zero", *i.e.* $\Sigma V = 0$

e.g. In the following closed loop.



$$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$$

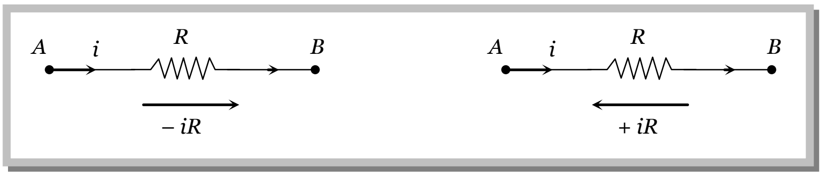
Here it is worthy to note that :

(i) This law represents "*conservation of energy*" as if the sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.

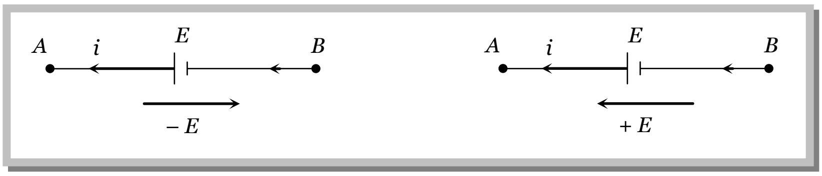
(ii) If there are n meshes in a circuit, the number of independent equations in accordance with loop rule will be $(n - 1)$.

(3) **Sign convention for the application of Kirchoff's law :** For the application of Kirchoff's laws following sign convention are to be considered

(i) The change in potential in traversing a resistance in the direction of current is – iR while in the opposite direction $+iR$

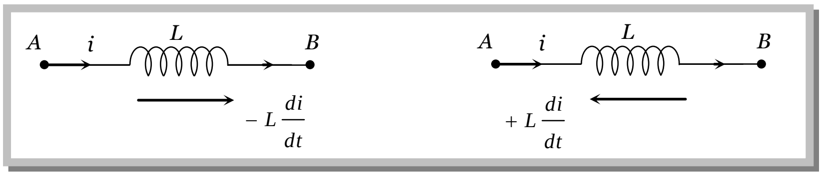


(ii) The change in potential in traversing an emf source from negative to positive terminal is $+E$ while in the opposite direction $-E$ irrespective of the direction of current in the circuit.



(iii) The change in potential in traversing a capacitor from the negative terminal to the positive terminal is $+\frac{q}{C}$ while in opposite direction $-\frac{q}{C}$.

(iv) The change in voltage in traversing an inductor in the direction of current is $-L\frac{di}{dt}$ while in opposite direction it is $+L\frac{di}{dt}$.



(4) **Guidelines to apply Kirchoff's law**

(i) Starting from the positive terminal of the battery having highest emf, distribute current at various junctions in the circuit in accordance with '*junction rule*'. It is not always easy to correctly guess the direction of current, no problem if one assumes the wrong direction.

(ii) After assuming current in each branch, we pick a point and begin to walk (mentally) around a closed loop. As we traverse each resistor, capacitor, inductor or battery we must write down, the voltage change for that element according to the above sign convention.

(iii) By applying KVL we get one equation but in order to solve the circuit we require as many equations as there are unknowns. So we select the required number of loops and apply Kirchhoff's voltage law across each such loop.

(iv) After solving the set of simultaneous equations, we obtain the numerical values of the assumed currents. If any of these values come out to be negative, it indicates that particular current is in the opposite direction from the assumed one.

Note : ❑ The number of loops must be selected so that every element of the circuit must be included in at least one of the loops.

❑ While traversing through a capacitor or battery we do not consider the direction of current.

❑ While considering the voltage drop or gain across as inductor we always assume current to be in increasing function.

(5) **Determination of equivalent resistance by Kirchoff's method :** This method is useful when we are not able to identify any two resistances in series or in parallel. It is based on the two Kirchhoff's laws. The method may be described in the following guideline.

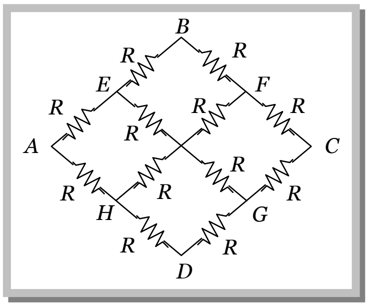
(i) Assume an imaginary battery of emf E connected between the two terminals across which we have to calculate the equivalent resistance.

(ii) Assume some value of current, say i, coming out of the battery and distribute it among each branch by applying Kirchhoff's current law.

(iii) Apply Kirchhoff's voltage law to formulate as many equations as there are unknowns. It should be noted that at least one of the equations must include the assumed battery.

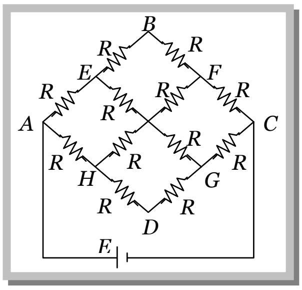
(iv) Solve the equations to determine $\frac{E}{i}$ ratio which is the equivalent resistance of the network.

e.g. Suppose in the following network of 12 identical resistances, equivalent resistance between point A and C is to be calculated.



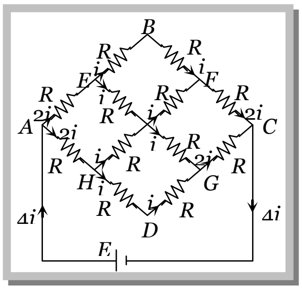
According to the above guidelines we can solve this problem as follows

Step (1)



An imaginary battery of emf E is assumed across the terminals A and C

Step (2)



The current in each branch is distributed by assuming $4i$ current coming out of the battery.

Step (3) Applying KVL along the loop including the nodes A, B, C and the battery E. Voltage equation is $-2iR - iR - iR - 2iR + E = 0$

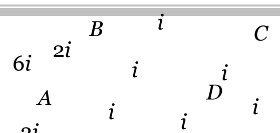
Step (4) After solving the above equation, we get $6iR = E \Rightarrow$ equivalent resistance between A and C is

$$R = \frac{E}{4i} = \frac{6iR}{4i} = \frac{3}{2}R$$

Concepts

☞ *Using Kirchoff's law while dividing the current having a junction through different arms of a network, it will be same through different arms of same resistance if the end points of these arms are equilocated w.r.t. exit point for current in network and will be different through different arms if the end point of these arms are not equilocated w.r.t. exit point for current of the network.*

e.g. In the following figure the current going in arms AB, AD and AL will be same because the location of end points B, D

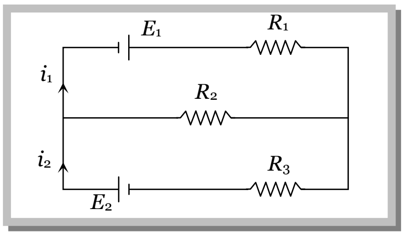


and L of these arms are symmetrically located w.r.t. exit point N of the network.

Example

Example: 67 In the following circuit $E_1 = 4V$, $R_1 = 2\Omega$ **[MP PET 2003]**

$E_2 = 6V$, $R_2 = 2\Omega$ and $R_3 = 4\Omega$. The current i_1 is



(a) 1.6 A

(b) 1.8 A

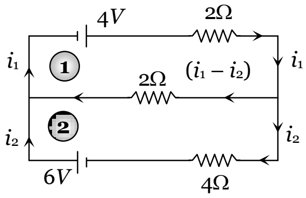
(c) 2.25 A

(d) 1 A

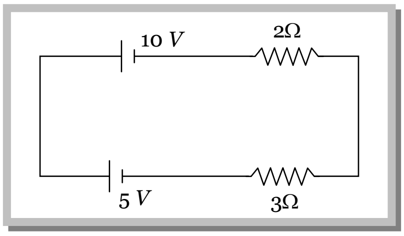
Solution : (b) For loop (1) $-2i_1 - 2(i_1 - i_2) + 4 = 0 \Rightarrow 2i_1 - i_2 = 2$ ...... (i)

For loop (2) $-4i_2 + 2(i_1 - i_2) + 6 = 0 \Rightarrow 3i_2 - i_1 = 3$ ...... (ii)

After solving equation (i) and (ii) we get $i_1 = 1.8A$ and $i_2 = 1.6A$



Example: 68 Determine the current in the following circuit



(a) 1 A

(b) 2.5 A

(c) 0.4 A

(d) 3 A

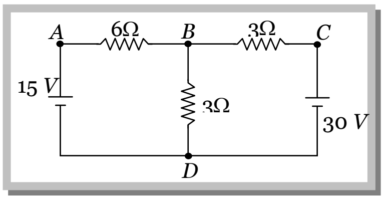
Solution : (a) Applying KVL in the given circuit we get $-2i + 10 - 5 - 3i = 0 \Rightarrow i = 1A$

Second method : Similar plates of the two batteries are connected together, so the net emf = 10 – 5 = 5V

Total resistance in the circuit = 2 + 3 = 5Ω

$$\therefore \quad i = \frac{\Sigma V}{\Sigma R} = \frac{5}{5} = 1A$$

Example: 69 In the circuit shown in figure, find the current through the branch BD



(a) 5 A (b) 0 A

(c) 3 A (d) 4 A

Solution : (a) The current in the circuit are assumed as shown in the fig.

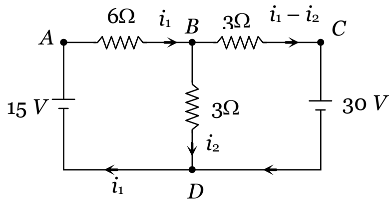
Applying KVL along the loop $ABDA$, we get

$-6i_1 - 3i_2 + 15 = 0$ or $2i_1 + i_2 = 5$ ...... (i)

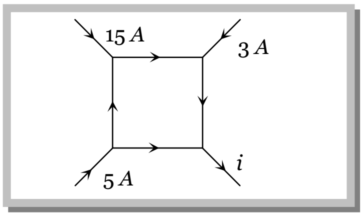
Applying KVL along the loop $BCDB$, we get

$-3(i_1 - i_2) - 30 + 3i_2 = 0$ or $-i_1 + 2i_2 = 10$ ...... (ii)

Solving equation (i) and (ii) for i_2, we get $i_2 = 5A$



Example: 70 The figure shows a network of currents. The magnitude of current is shown here. The current i will be **[MP PMT 1995]**



(a) 3 A

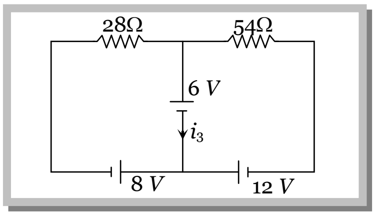
(b) 13 A

(c) 23 A

(d) – 3 A

Solution : (c) $i = 15 + 3 + 5 = 23A$

***Example*: 71** Consider the circuit shown in the figure. The current i_3 is equal to **[AMU 1995]**



(a) 5 *amp*

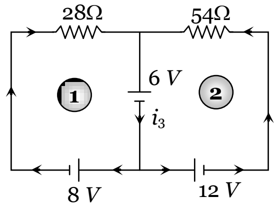
(b) 3 *amp*

(c) – 3 *amp*

(d) – 5/6 *amp*

Solution : (d) Suppose current through different paths of the circuit is as follows.

After applying *KVL* for loop (1) and loop (2)

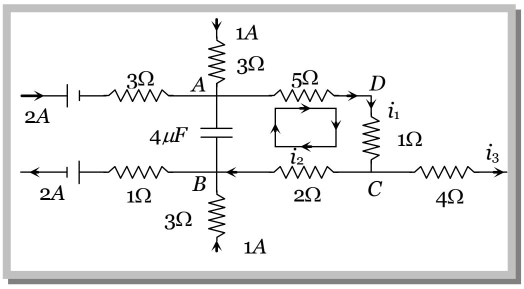


We get $28i_1 = -6 - 8 \Rightarrow i_1 = -\frac{1}{2}A$

and $54i_2 = -6 - 12 \Rightarrow i_2 = -\frac{1}{3}A$

Hence $i_3 = i_1 + i_2 = -\frac{5}{6}A$

***Example*: 72** A part of a circuit in steady state along with the current flowing in the branches, with value of each resistance is shown in figure. What will be the energy stored in the capacitor C_0



(a) $6 \times 10^{-4} J$

(b) $8 \times 10^{-4} J$

(c) $16 \times 10^{-4} J$

(d) Zero

Solution : (b) Applying Kirchhoff's first law at junctions *A* and *B* respectively we have 2 + 1 – i_1 = 0 *i.e.*, $i_1 = 3A$

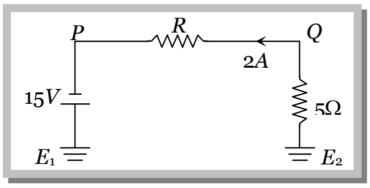
and $i_2 + 1 - 2 - 0 = 0$ *i.e.*, $i_2 = 1A$

Now applying Kirchhoff's second law to the mesh *ADCBA* treating capacitor as a seat of emf *V* in open circuit

$-3 \times 5 - 3 \times 1 - 1 \times 2 + V = 0$ *i.e.* $V(= V_A - V_B) = 20\ V$

So, energy stored in the capacitor $U = \frac{1}{2}CV^2 = \frac{1}{2}(4 \times 10^{-6}) \times (20)^2 = 8 \times 10^{-4}\ J$

***Example*: 73** In the following circuit the potential difference between *P* and *Q* is



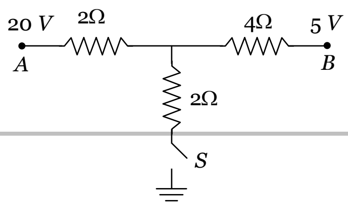
(a) 15 *V* (b) 10 *V*

(c) 5 *V* (d) 2.5 *V*

Solution : (c) By using KVL $-5 \times 2 - V_{PQ} + 15 = 0 \Rightarrow V_{PQ} = 5V$

Tricky Example: 9

As the switch *S* is closed in the circuit shown in figure, current passed through it is



(a) 4.5 A (b) 6.0 A (c) 3.0 A (d) Zero

Solution : (a) Let V be the potential of the junction as shown in figure. Applying junction law, we have

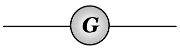
or $\frac{20-V}{2}+\frac{5-V}{4}=\frac{V-0}{2}$ or $40-2V+5-V=2V$

or $5V=45 \Rightarrow V=9V$

$\therefore \quad i_3=\frac{V}{2}=4.5A$

Different Measuring Instruments

(1) **Galvanometer :** It is an instrument used to detect small current passing through it by showing deflection. Galvanometers are of different types *e.g.* moving coil galvanometer, moving magnet galvanometer, hot wire galvanometer. In dc circuit usually moving coil galvanometer are used.

(i) **It's symbol :** 

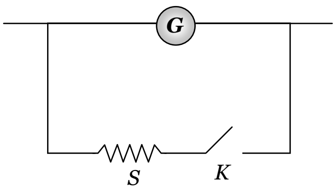
; where G is the total internal resistance of the galvanometer.

(ii) **Principle :** In case of moving coil galvanometer deflection is directly proportional to the current that passes through it *i.e.* $\boldsymbol{i \propto \theta}$.

(iii) **Full scale deflection current :** The current required for full scale deflection in a galvanometer is called full scale deflection current and is represented by i_g.

(iv) **Shunt :** The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer is known as shunt.

Merits of shunt	Demerits of shunt
(a) To protect the galvanometer coil from burning (b) It can be used to convert any galvanometer into ammeter of desired range.	Shunt resistance decreases the sensitivity of galvanometer.

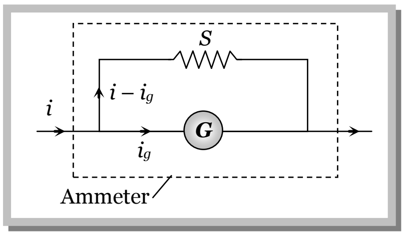


(2) **Ammeter :** It is a device used to measure current and is always connected in series with the 'element' through which current is to be measured.

(i) The reading of an ammeter is always lesser than actual current in the circuit.

(ii) Smaller the resistance of an ammeter more accurate will be its reading. An ammeter is said to be ideal if its resistance r is zero.

(iii) **Conversion of galvanometer into ammeter :** A galvanometer may be converted into an ammeter by connecting a low resistance (called shunt S) in parallel to the galvanometer G as shown in figure.



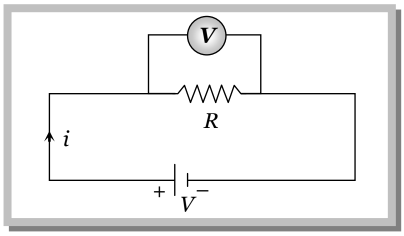
(a) Equivalent resistance of the combination $=\frac{GS}{G+S}$

(b) G and S are parallel to each other hence both will have equal potential difference *i.e.* $i_gG=(i-i_g)S$; which gives

Required shunt $\boldsymbol{S=\frac{i_g}{(i-i_g)}G}$

(c) To pass nth part of main current (*i.e.* $i_g = \frac{i}{n}$) through the galvanometer, required shunt $S = \frac{G}{(n-1)}$.

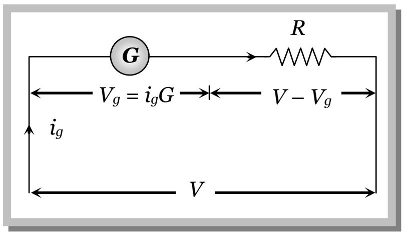
(3) **Voltmeter :** It is a device used to measure potential difference and is always put in parallel with the 'circuit element' across which potential difference is to be measured.



(i) The reading of a voltmeter is always lesser than true value.

(ii) Greater the resistance of voltmeter, more accurate will be its reading. A voltmeter is said to be ideal if its resistance is infinite, *i.e.,* it draws no current from the circuit element for its operation.

(iii) **Conversion of galvanometer into voltmeter :** A galvanometer may be converted into a voltmeter by connecting a large resistance R in series with the galvanometer as shown in the figure.



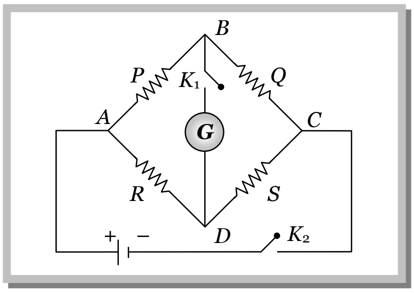
(a) Equivalent resistance of the combination = $G + R$

(b) According to ohm's law $V = i_g(G + R)$; which gives

Required series resistance $R = \frac{V}{i_g} - G = \left(\frac{V}{V_g} - 1\right)G$

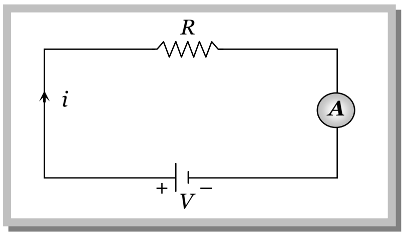
(c) If n^{th} part of applied voltage appeared across galvanometer (*i.e.* $V_g = \frac{V}{n}$) then required series resistance $R = (n-1)\,G$.

(4) **Wheatstone bridge :** Wheatstone bridge is an arrangement of four resistance which can be used to measure one of them in terms of rest. Here arms AB and BC are called ratio arm and arms AC and BD are called conjugate arms



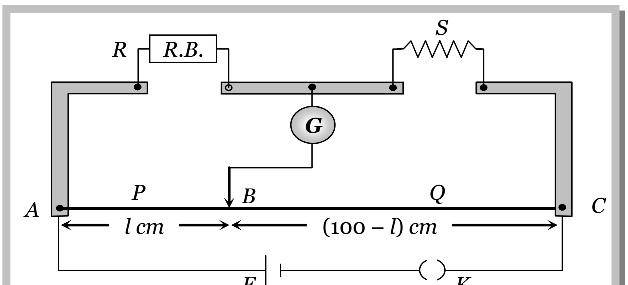
(i) **Balanced bridge :** The bridge is said to be balanced when deflection in galvanometer is zero *i.e.* no current flows through the galvanometer or in other words $V_B = V_D$. In the balanced condition $\frac{P}{Q} = \frac{R}{S}$, on mutually changing the position of cell and galvanometer this condition will not change.

(ii) **Unbalanced bridge :** If the bridge is not balanced current will flow from D to B if $V_D > V_B$ *i.e.* $(V_A - V_D) < (V_A - V_B)$ which gives $PS > RQ$.



(iii) **Applications of wheatstone bridge :** Meter bridge, post office box and Carey Foster bridge are instruments based on the principle of wheatstone bridge and are used to measure unknown resistance.

(5) **Meter bridge :** In case of meter bridge, the resistance wire AC is 100 cm long. Varying the position of tapping point B, bridge is balanced. If in balanced position of bridge $AB = l$, BC (100 – l) so that $\frac{Q}{P} = \frac{(100-l)}{l}$. Also $\frac{P}{Q} = \frac{R}{S} \Rightarrow S = \frac{(100-l)}{l}R$



Concepts

- *Wheatstone bridge is most sensitive if all the arms of bridge have equal resistances i.e. P = Q = R = S*
- *If the temperature of the conductor placed in the right gap of metre bridge is increased, then the balancing length decreases and the jockey moves towards left.*
- *In Wheatstone bridge to avoid inductive effects the battery key should be pressed first and the galvanometer key afterwards.*
- *The measurement of resistance by Wheatstone bridge is not affected by the internal resistance of the cell.*

Example

***Example:* 74** The scale of a galvanometer of resistance 100 Ω contains 25 divisions. It gives a deflection of one division on passing a current of 4×10^{-4} *A*. The resistance in *ohms* to be added to it, so that it may become a voltmeter of range 2.5 *volt* is

(a) 100 (b) 150 (c) 250 (d) 300

Solution : (b) Current sensitivity of galvanometer = 4×10^{-4} *Amp/div*.

So full scale deflection current (i_g) = Current sensitivity × Total number of division = $4 \times 10^{-4} \times 25 = 10^{-2}$ *A*

To convert galvanometer in to voltmeter, resistance to be put in series is $R = \frac{V}{i_g} - G = \frac{2.5}{10^{-2}} - 100 = 150\,\Omega$

***Example:* 75** A galvanometer, having a resistance of 50 Ω gives a full scale deflection for a current of 0.05 *A*. the length in meter of a resistance wire of area of cross-section 2.97×10^{-2} cm^2 that can be used to convert the galvanometer into an ammeter which can read a maximum of 5*A* current is : (Specific resistance of the wire = 5×10^{-7} Ω*m*) **[EAMCET 2003]**

(a) 9 (b) 6 (c) 3 (d) 1.5

Solution : (c) Given $G = 50\,\Omega$, $i_g = 0.05$ *Amp.*, $i = 5A$, $A = 2.97 \times 10^{-2}\,cm^2$ and $\rho = 5 \times 10^{-7}\,\Omega\text{-}m$

By using $\frac{i}{i_g} = 1 + \frac{G}{S} \Rightarrow S = \frac{G.i_g}{(i - i_g)} \Rightarrow \frac{\rho\, l}{A} = \frac{Gi_g}{(i - i_g)} \Rightarrow l = \frac{Gi_g A}{(i - i_g)\rho}$ on putting values $l = 3\,m$.

***Example:* 76** 100 *mA* current gives a full scale deflection in a galvanometer of resistance 2 Ω. The resistance connected with the galvanometer to convert it into a voltmeter of 5 *V* range is

[KCET 2002; UPSEAT 1998; MNR 1994 Similar to MP PMT 1999]

(a) 98 Ω (b) 52 Ω (c) 80 Ω (d) 48 Ω

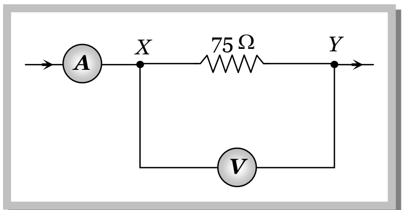
Solution : (d) $R = \frac{V}{I_g} - G = \frac{5}{100 \times 10^{-3}} - 2 = 50 - 2 = 48\,\Omega$.

***Example:* 77** A milliammeter of range 10 *mA* has a coil of resistance 1 Ω. To use it as voltmeter of range 10 *volt*, the resistance that must be connected in series with it will be

(a) 999 Ω (b) 99 Ω (c) 1000 Ω (d) None of these

Solution : (a) By using $R = \frac{V}{ig} - G \Rightarrow R = \frac{10}{10 \times 10^{-3}} - 1 = 999\Omega$

***Example:* 78** In the following figure ammeter and voltmeter reads 2 *amp* and 120 *volt* respectively. Resistance of voltmeter is



(a) 100 Ω

(b) 200 Ω

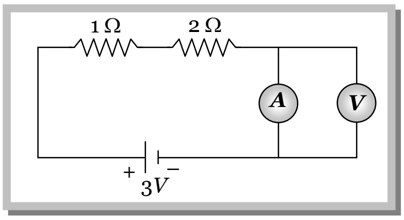
(c) 300 Ω

(d) 400 Ω

Solution : (c) Let resistance of voltmeter be R_V. Equivalent resistance between *X* and *Y* is $R_{XY} = \frac{75R_V}{75+R_V}$

Reading of voltmeter = potential difference across *X* and *Y* = 120 = $i \times R_{XY} = 2 \times \frac{75R_V}{75+R_V} \Rightarrow R_V =$ 300Ω

***Example*: 79** In the circuit shown in figure, the voltmeter reading would be



(a) Zero

(b) 0.5 *volt*

(c) 1 *volt*

(d) 2 *volt*

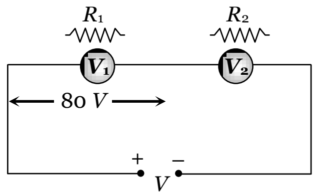
Solution : (a) Ammeter has no resistance so there will be no potential difference across it, hence reading of voltmeter is zero.

***Example*: 80** Voltmeters V_1 and V_2 are connected in series across a d.c. line. V_1 reads 80 *V* and has a per *volt* resistance of 200 Ω, V_2 has a total resistance of 32 *k*Ω. The line voltage is

(a) 120 *V* (b) 160 *V* (c) 220 *V* (d) 240 *V*

Solution : (d) Resistance of voltmeter V_1 is R_1 = 200 × 80 = 16000 Ω and resistance of voltmeter V_2 is R_2 = 32000 Ω

By using relation $V' = \left(\frac{R'}{R_{eq}}\right)V$; where *V'* = potential difference across any resistance *R'* in a series grouping.



So for voltmeter V_1 potential difference across it is

$$80 = \left(\frac{R_1}{R_1+R_2}\right).V \Rightarrow V = 240\ V$$

***Example*: 81** The resistance of 1 *A* ammeter is 0.018 Ω. To convert it into 10 *A* ammeter, the shunt resistance required will be

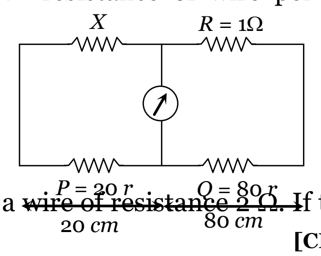
(a) 0.18 Ω (b) 0.0018 Ω (c) 0.002 Ω (d) 0.12 Ω

Solution : (c) By using $\frac{i}{i_g} = 1 + \frac{4}{S} \Rightarrow \frac{10}{1} = 1 + \frac{0.018}{S} \Rightarrow S = 0.002\ \Omega$

***Example*: 82** In meter bridge the balancing length from left and when standard resistance of 1 Ω is in right gas is found to be 20 *cm*. The value of unknown resistance is **[CBSE PMT 1999]**

(a) 0.25 Ω (b) 0.4 Ω (c) 0.5 Ω (d) 4 Ω

Solution: (a) The condition of wheatstone bridge gives $\frac{X}{R} = \frac{20\,r}{80\,r}$, *r*- resistance of wire per *cm*, *X*- unknown resistance



$$\therefore \quad X = \frac{20}{80} \times R = \frac{1}{4} \times 1 = 0.25\ \Omega$$

***Example*: 83** A galvanometer having a resistance of 8 Ω is shunted by a wire of resistance 2 Ω. If the total current is 1 *amp*, the part of it passing through the shunt will be **[CBSE PMT 1998]**

(a) 0.25 *amp* (b) 0.8 *amp* (c) 0.2 *amp* (d) 0.5 *amp*

Solution: (b) Fraction of current passing through the galvanometer

$$\frac{i_g}{i} = \frac{S}{S+G} \text{ or } \frac{i_g}{i} = \frac{2}{2+8} = 0.2$$

So fraction of current passing through the shunt

$$\frac{i_s}{i} = 1 - \frac{i_g}{i} = 1 - 0.2 = 0.8\,amp$$

***Example*: 84** A moving coil galvanometer is converted into an ammeter reading upto 0.03 *A* by connecting a shunt of resistance 4*r* across it and into an ammeter reading upto 0.06 *A* when a shunt of resistance *r* connected across it. What is the maximum current which can be through this galvanometer if no shunt is used

[MP PMT 1996]

(a) 0.01 *A* (b) 0.02 *A* (c) 0.03 *A* (d) 0.04 *A*

Solution: (b) For ammeter, $S = \frac{i_g}{(i - i_g)}G \quad \Rightarrow i_g G = (i - i_g)S$

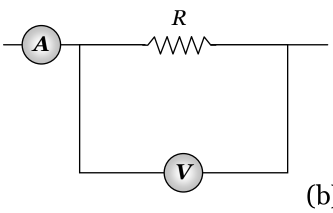
So $i_g G = (0.03 - i_g)4r$ ...... (i) and $i_g G = (0.06 - i_g)r$ ...... (ii)

Dividing equation (i) by (ii) $1 = \frac{(0.03 - i_g)4}{0.06 - i_g} \Rightarrow 0.06 - i_g = 0.12 - 4i_g$

$\Rightarrow 3i_g = 0.06 \Rightarrow i_g = 0.02\,A$

Tricky Example: 10

The ammeter *A* reads 2 *A* and the voltmeter *V* reads 20 *V*. The value of resistance *R* is



(a) Exactly 10 *ohm* (b) Less than 10 *ohm*

(c) More than 10 *ohm* (d) We cannot definitely say

Solution: (c) If current goes through the resistance *R* is *i* then $iR = 20\ volt \Rightarrow R = \frac{20}{i}$. Since $i < 2A$ so $R >$ 10Ω.

Potentiometer

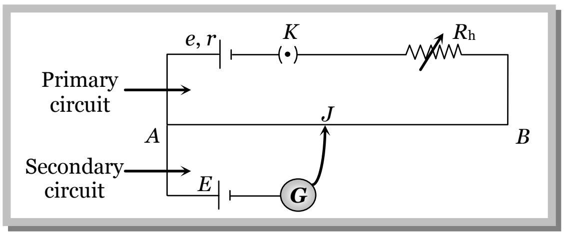
Potentiometer is a device mainly used to measure emf of a given cell and to compare emf's of cells. It is also used to measure internal resistance of a given cell.

(1) **Superiority of potentiometer over voltmeter** : An ordinary voltmeter cannot measure the emf accurately because it does draw some current to show the deflection. As per definition of emf, it is the potential difference when a cell is in open circuit or no current through the cell. Therefore voltmeter can only measure terminal voltage of a give *n* cell.

Potentiometer is based on no deflection method. When the potentiometer gives zero deflection, it does not draw any current from the cell or the circuit *i.e.* potentiometer is effectively an ideal instrument of infinite resistance for measuring the potential difference.

(2) **Circuit diagram :** Potentiometer consists of a long resistive wire *AB* of length *L* (about 6*m* to 10 *m* long) made up of mangnine or constantan. A battery of known voltage *e* and internal resistance *r* called supplier battery or driver cell. Connection of these two forms primary circuit.

One terminal of another cell (whose emf E is to be measured) is connected at one end of the main circuit and the other terminal at any point on the resistive wire through a galvanometer G. This forms the secondary circuit. Other details are as follows



J = Jockey

K = Key

R = Resistance of potentiometer wire,

ρ = Specific resistance of potentiometer wire.

R_h = Variable resistance which controls the current through the wire AB

(3) **Points to be remember**

(i) The specific resistance (ρ) of potentiometer wire must be high but its temperature coefficient of resistance (α) must be low.

(ii) All higher potential points (terminals) of primary and secondary circuits must be connected together at point A and all lower potential points must be connected to point B or jockey.

(iii) The value of known potential difference must be greater than the value of unknown potential difference to be measured.

(iv) The potential gradient must remain constant. For this the current in the primary circuit must remain constant and the jockey must not be slided in contact with the wire.

(v) The diameter of potentiometer wire must be uniform everywhere.

(4) **Potential gradient (x) :** Potential difference (or fall in potential) per unit length of wire is called potential gradient *i.e.* $x = \frac{V}{L}\frac{volt}{m}$ where $V = iR = \left(\frac{e}{R + R_h + r}\right).R$. So $\boldsymbol{x = \frac{V}{L} = \frac{iR}{L} = \frac{i\rho}{A} = \frac{e}{(R + R_h + r)}.\frac{R}{L}}$

(i) Potential gradient directly depends upon

(a) The resistance per unit length (R/L) of potentiometer wire.

(b) The radius of potentiometer wire (*i.e.* Area of cross-section)

(c) The specific resistance of the material of potentiometer wire (*i.e.* ρ)

(d) The current flowing through potentiometer wire (i)

(ii) x indirectly depends upon

(a) The emf of battery in the primary circuit (*i.e.* e)

(b) The resistance of rheostat in the primary circuit (*i.e.* R_h)

Note : ❑ When potential difference V is constant then $\frac{x_1}{x_2} = \frac{L_2}{L_1}$

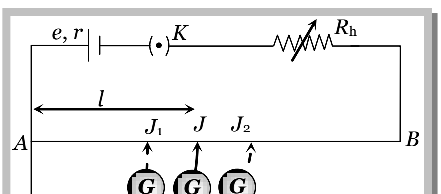
❑ Two different wire are connected in series to form a potentiometer wire then $\frac{x_1}{x_2} = \frac{R_1}{R_2}.\frac{L_2}{L_1}$

❑ If the length of a potentiometer wire and potential difference across it's ends are kept constant and if it's diameter is changed from $d_1 \to d_2$ then potential gradient remains unchanged.

❑ The value of x does not change with any change effected in the secondary circuit.

(5) **Working :** Suppose jocky is made to touch a point J on wire then potential difference between A and J will be $V = xl$

At this length (l) two potential difference are obtained



(i) V due to battery e and

(ii) E due to unknown cell

If $V > E$ then current will flow in galvanometer circuit in one directi 

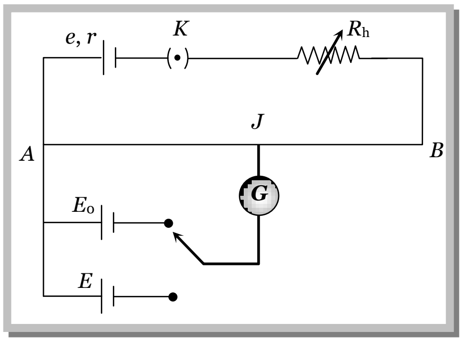
If $V < E$ then current will flow in galvanometer circuit in opposite directi 

If $V = E$ then no current will flow in galvanometer circuit this condition to known as null deflection position, length l is known as balancing length.

In balanced condition $E = xl$ or $E = xl = \frac{V}{L}l = \frac{iR}{L}l = \left(\frac{e}{R + R_h + r}\right) \cdot \frac{R}{L} \times l$

Note : ❑ If V is constant then $L \propto l \quad \Rightarrow \frac{L_1}{L_2} = \frac{l_1}{l_2}$

(6) **Standardization of potentiometer :** The process of determining potential gradient experimentally is known as standardization of potentiometer.



Let the balancing length for the standard emf E_0 is l_0 then by the principle of potentiometer $E_0 = xl_0 \Rightarrow x = \frac{E_0}{l_0}$

(7) **Sensitivity of potentiometer :** A potentiometer is said to be more sensitive, if it measures a small potential difference more accurately.

(i) The sensitivity of potentiometer is assessed by its potential gradient. The sensitivity is inversely proportional to the potential gradient.

(ii) In order to increase the sensitivity of potentiometer

(a) The resistance in primary circuit will have to be decreased.

(b) The length of potentiometer wire will have to be increased so that the length may be measured more accuracy.

(8) **Difference between voltmeter and potentiometer**

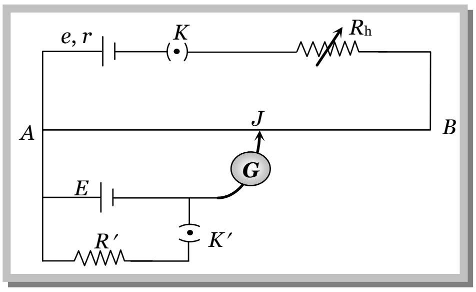
	Voltmeter	**Potentiometer**
(i)	It's resistance is high but finite	Its resistance is high but infinite
(ii)	It draws some current from source of emf	It does not draw any current from the source of known emf
(iii)	The potential difference measured by it is lesser than the actual potential difference	The potential difference measured by it is equal to actual potential difference
(iv)	Its sensitivity is low	Its sensitivity is high
(v)	It is a versatile instrument	It measures only emf or potential difference
(vi)	It is based on deflection method	It is based on zero deflection method

Application of Potentiometer

(1) **To determine the internal resistance of a primary cell**

(i) Initially in secondary circuit key K' remains open and balancing length (l_1) is obtained. Since cell E is in open circuit so it's emf balances on length l_1 i.e. $E = xl_1$ .... (i)

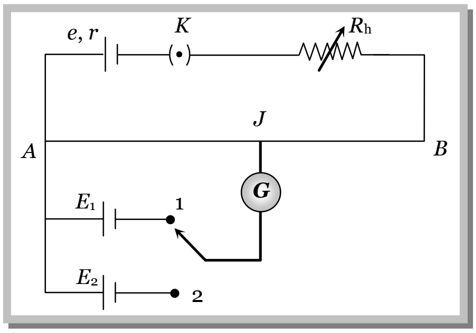
(ii) Now key K' is closed so cell E comes in closed circuit. If the process is repeated again then potential difference V balances on length l_2 *i.e.* $V = xl_2$ .... (ii)



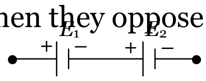
(iii) By using formula internal resistance $r = \left(\frac{E}{V} - 1\right).R'$

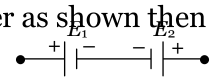
$$r = \left(\frac{l_1 - l_2}{l_2}\right).R'$$

(2) **Comparison of emf's of two cell :** Let l_1 and l_2 be the balancing lengths with the cells E_1 and E_2 respectively then $E_1 = xl_1$ and $E_2 = xl_2 \Rightarrow \frac{E_1}{E_2} = \frac{l_1}{l_2}$



Note : ❑ Let $E_1 > E_2$ and both are connected in series. If balancing length is l_1 when cell assist each other and it is l_2 when they oppose each other as shown then :



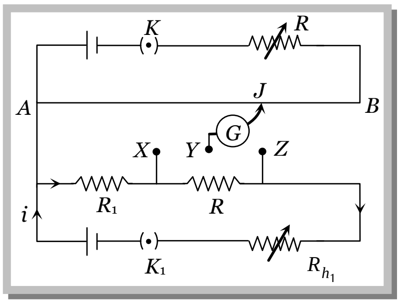


$$(E_1 + E_2) = xl_1 \qquad (E_1 - E_2) = xl_2$$

$$\Rightarrow \quad \frac{E_1 + E_2}{E_1 - E_2} = \frac{l_1}{l_2} \quad \text{or} \quad \frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

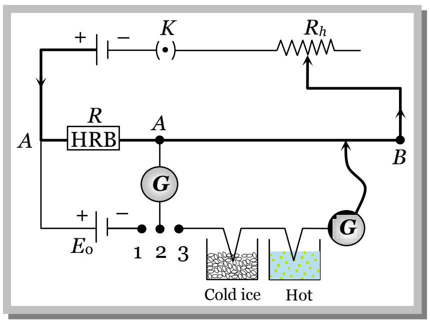
(3) **Comparison of resistances :** Let the balancing length for resistance R_1 (when XY is connected) is l_1 and let balancing length for resistance $R_1 + R_2$ (when YZ is connected) is l_2. Then $iR_1 = xl_1$ and $i(R_1 + R_2) = xl_2$

$$\Rightarrow \quad \frac{R_2}{R_1} = \frac{l_2 - l_1}{l_1}$$



(4) **To determine thermo emf**

(i) The value of thermo-emf in a thermocouple for ordinary temperature difference is very low (10^{-6} *volt*). For this the potential gradient x must be also very low (10^{-4} V/m). Hence a high resistance (R) is connected in series with the potentiometer wire in order to reduce current.



(ii) The potential difference across R must be equal to the emf of standard cell *i.e.* $iR = E_0 \therefore i = \frac{E_0}{R}$

(iii) The small thermo emf produced in the thermocouple $e = xl$

(iv) $x = i\rho = \frac{iR}{L} \quad \therefore e = \frac{iRl}{L}$ where L = length of potentiometer wire, ρ = resistance per unit length, l = balancing length for e

(5) **To calibrate ammeter and voltmeter**

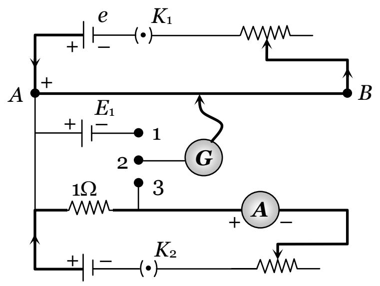
Calibration of ammeter

(i) If p.d. across 1Ω resistance is measured by potentiometer, then current through this (indirectly measured) is thus known or if R is known then $i = V/R$ can be found.

(ii) **Circuit and method**

(a) Standardisation is required and per formed as already described earlier. ($x = E_0/l_0$)

(b) The current through R or 1Ω coil is measured by the connected ammeter and same is calculated by potentiometer by finding a balancing length as described blow.



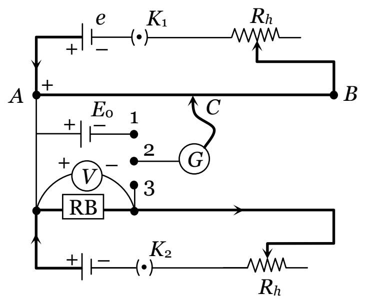
Let i' current flows through 1Ω resistance giving p.d. as $V' = i'(1) = xl_1$ where l_1 is the balancing length. So error can be found as [i (measured by ammeter) $\Delta i' = i - i'$ $\left[= xl_1 = \left(\frac{E_0}{l_0}\right)l_1\right]$

Calibration of voltmeter

(i) Practical voltmeters are not ideal, because these do not have infinite resistance. The error of such practical voltmeter can be found by comparing the voltmeter reading with calculated value of p.d. by potentiometer.

(ii) **Circuit and procedure**

(a) **Standardisation :** If l_0 is balancing length for E_0 the emf of standard cell by connecting 1 and 2 of bi-directional key, then $x = E_0/l_0$.



(b) The balancing length l_1 for unknown potential difference V' is given by (by closing 2 and 3) $V' = xl_1 = (E_0/l_0)l_1$.

If the voltmeter reading is V then the error will be $(V - V')$ which may be $+ve$, $-ve$ or zero.

Concepts

☞ *In case of zero deflection in the galvanometer current flows in the primary circuit of the potentiometer, not in the galvanometer circuit.*

☞ *A potentiometer can act as an ideal voltmeter.*

Example

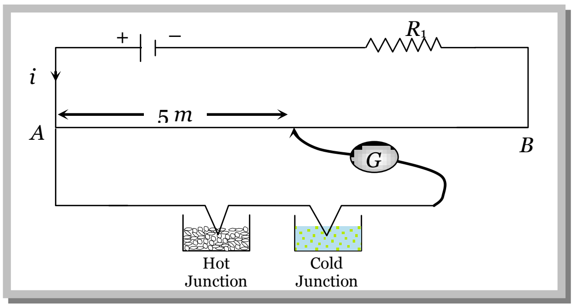
***Example*: 85** A battery with negligible internal resistance is connected with 10*m* long wire. A standard cell gets balanced on 600 *cm* length of this wire. On increasing the length of potentiometer wire by 2*m* then the null point will be displaced by

(a) 200 *cm* (b) 120 *cm* (c) 720 *cm* (d) 600 *cm*

Solution : (b) By using $\frac{L_1}{L_2} = \frac{l_1}{l_2} \Rightarrow \frac{10}{12} = \frac{600}{l_2} \Rightarrow l_2 = 720\ cm$.

Hence displacement = 720 – 600 = 120 *cm*

***Example*: 86** In the following circuit a 10 *m* long potentiometer wire with resistance 1.2 *ohm/m*, a resistance R_1 and an accumulator of emf 2 *V* are connected in series. When the emf of thermocouple is 2.4 *mV* then the deflection in galvanometer is zero. The current supplied by the accumulator will be



(a) $4 \times 10^{-4}\,A$ (b) $8 \times 10^{-4}\,A$ (c) $4 \times 10^{-3}\,A$ (d) $8 \times 10^{-3}\,A$

Solution : (a) $E = x\,l = i\rho\,l$ $\therefore\ i = \frac{E}{\rho l} = \frac{E}{\rho l} = \frac{2.4 \times 10^{-3}}{1.2 \times 5} = 4 \times 10^{-4}\,A$.

***Example*: 87** The resistivity of a potentiometer wire is $40 \times 10^{-8}\ \Omega m$ and its area of cross section is $8 \times 10^{-6}\ m^2$. If 0.2 *amp*. Current is flowing through the wire, the potential gradient will be

(a) 10^{-2} *volt*/*m* (b) 10^{-1} *volt*/*m* (c) 3.2×10^{-2} *volt*/*m* (d) 1 *volt*/*m*

Solution : (a) Potential gradient $= \frac{V}{L} = \frac{iR}{L} = \frac{i\rho L}{AL} = \frac{i\rho}{A} = \frac{0.2 \times 40 \times 10^{-8}}{8 \times 10^{-6}} = 10^{-2}\ V/m$

***Example*: 88** A deniel cell is balanced on 125 *cm* length of a potentiometer wire. When the cell is short circuited with a 2 Ω resistance the balancing length obtained is 100 *cm*. Internal resistance of the cell will be **[RPMT 1998]**

(a) 1.5 Ω (b) 0.5 Ω (c) 1.25 Ω (d) 4/5 Ω

Solution: (b) By using $r = \frac{l_1 - l_2}{l_2} \times R'$ $\Rightarrow r = \frac{125 - 100}{100} \times 2 = \frac{1}{2} = 0.5\,\Omega$

***Example*: 89** A potentiometer wire of length 10 *m* and a resistance 30 Ω is connected in series with a battery of emf 2.5 *V* and internal resistance 5 Ω and an external resistance *R*. If the fall of potential along the potentiometer wire is 50 *μV*/*mm*, the value of *R* is (in Ω)

(a) 115 (b) 80 (c) 50 (d) 100

Solution : (a) By using $x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$

$$\Rightarrow \frac{50 \times 10^{-6}}{10^{-3}} = \frac{2.5}{(30 + R + 5)} \times \frac{30}{10} \Rightarrow R = 115$$

***Example*: 90** A 2 *volt* battery, a 15 Ω resistor and a potentiometer of 100 *cm* length, all are connected in series. If the resistance of potentiometer wire is 5 Ω, then the potential gradient of the potentiometer wire is **[AIIMS 1982**

(a) 0.005 *V*/*cm* (b) 0.05 *V*/*cm* (c) 0.02 *V*/*cm* (d) 0.2 *V*/*cm*

Solution : (a) By using $x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$ $\Rightarrow x = \frac{2}{(5 + 15 + 0)} \times \frac{5}{1} = 0.5\ V/m = 0.005\ V/cm$

***Example*: 91** In an experiment to measure the internal resistance of a cell by potentiometer, it is found that the balance point is at a length of 2 *m* when the cell is shunted by a 5 Ω resistance; and is at a length of 3 *m* when the cell is shunted by a 10 Ω resistance. The internal resistance of the cell is, then

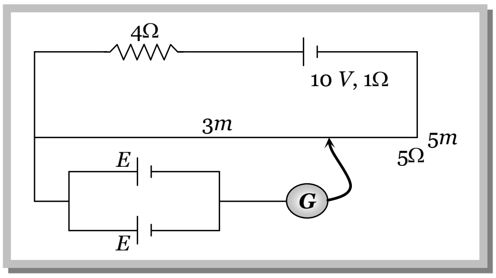
(a) 1.5 Ω (b) 10 Ω (c) 15 Ω (d) 1 Ω

Solution : (b) By using $r = \left(\frac{l_1 - l_2}{l_2}\right)R' \Rightarrow r = \left(\frac{l_1 - 2}{2}\right)\times 5$ ...... (i)

and $r = \left(\frac{l_1 - 3}{3}\right)\times 10$ .... (ii)

On solving (i) and (ii) $r = 10\ \Omega$

***Example*: 92** A resistance of 4 Ω and a wire of length 5 *metres* and resistance 5 Ω are joined in series and connected to a cell of emf 10 *V* and internal resistance 1 Ω. A parallel combination of two identical cells is balanced across 300 *cm* of the wire. The emf *E* of each cell is



(a) 1.5 *V* (b) 3.0 *V* (c) 0.67 *V* (d) 1.33 *V*

Solution : (b) By using $E_{eq} = \frac{e}{(R + R_h + r)}\cdot\frac{R}{L}\times l \Rightarrow E = \frac{10}{(5+4+1)}\times\frac{5}{5}\times 3 \Rightarrow E = 3\ volt$

***Example*: 93** A potentiometer has uniform potential gradient across it. Two cells connected in series (i) to support each other and (ii) to oppose each other are balanced over 6 *m* and 2 *m* respectively on the potentiometer wire. The emf's of the cells are in the ratio of **[MP PMT 2002; RPMT 2000]**

(a) 1 : 2 (b) 1 : 1 (c) 3 : 1 (d) 2 : 1

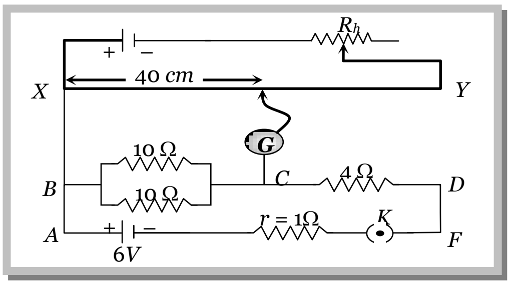
Solution : (d) If suppose emf's of the cells are E_1 and E_2 respectively then

$E_1 + E_2 = x \times 6$ .... (i) [x = potential gradient)]

and $E_1 - E_2 = x \times 2$ ....(ii)

$\Rightarrow \frac{E_1 + E_2}{E_1 - E_2} = \frac{3}{1} \Rightarrow \frac{E_1}{E_2} = \frac{2}{1}$

***Example*: 94** In the following circuit the potential difference between the points *B* and *C* is balanced against 40 *cm* length of potentiometer wire. In order to balance the potential difference between the points *C* and *D*, where should jockey be pressed



(a) 32 *cm* (b) 16 *cm* (c) 8 *cm* (d) 4 *cm*

Solution : (a) $\frac{1}{R} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$ or $R_1 = 5\ \Omega$

$R_2 = 4\Omega$, $l_1 = 40\ cm$, $l_2 = ?$ $l_2 = l_1\frac{R_2}{R_1}$ or $l_2 = \frac{40\times 4}{5} = 32\ cm$

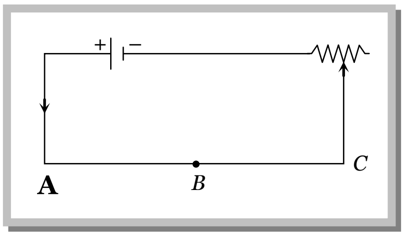
Example: 95 In the following circuit diagram fig. the lengths of the wires *AB* and *BC* are same but the radius of *AB* is three times that of *BC*. The ratio of potential gradients at *AB* and *BC* will be

(a) 1 : 9

(b) 9 : 1

(c) 3 : 1

(d) 1 : 3



Solution : (a) $x \propto R_p \propto \frac{1}{r^2} \Rightarrow \frac{x_1}{x_2} = \frac{r_2^2}{r_1^2} = \left(\frac{r}{3r}\right)^2 = \frac{1}{9}$

Example: 96 With a certain cell the balance point is obtained at 0.60 *m* from one end of the potentiometer. With another cell whose emf differs from that of the first by 0.1 *V*, the balance point is obtained at 0.55 *m*. Then, the two emf's are

(a) 1.2 *V*, 1.1 *V* (b) 1.2 *V*, 1.3 *V* (c) – 1.1 *V*, – 1.0 *V* (d) None of the above

Solution : (a) $E_1 = x\,(0.6)$ and $E_2 = E_1 - 0.1 = x\,(0.55) \Rightarrow \frac{E_1}{E_1 - 0.1} = \frac{0.6}{0.55}$

or $55\,E_1 = 60\,E_1 - 6 \Rightarrow E_1 = \frac{6}{5} = 1.2\,V$ thus $E_2 = 1.1\,V$

Tricky Example: 11

A cell of internal resistance 1.5Ω and of emf 1.5 *volt* balances 500 *cm* on a potentiometer wire. If a wire of 15Ω is connected between the balance point and the cell, then the balance point will shift

[MP PMT 1985]

(a) To zero (b) By 500 *cm*

(c) By 750 *cm* (d) None of the above

Solution : (d) In balance condition no current flows in the galvanometer circuit. Hence there will be no shift in balance point after connecting a resistance between balance point and cell.

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Formulas in current electricity (Direct Current)

1	Electric Current	i=q/t	"q" is charge passing in normal direction through a cross section of conductor in time "t"
2	Drift velocity V_d with Electric field	$V_d = \frac{-e\vec{E}\tau}{m}$	e is charge and m is mass on electron, E is electric field, τ is relaxation time.
3	Current I with Drift velocity V_d	$I = n\ e\ A\ V_d$	n is number density with of free electrons, A is area of cross section.
4	Mobility of charge "μ"	$\mu = V_d / E = \frac{q\tau}{m}$	
5	Mobility and drift velocity	$V_d = \mu_e E$	
6	Current and Mobility	I= Ane x $\mu_e E$	
7	Resistance, P.D., and Current	R = V / I	V Potential Difference, I Current .
8	Resistance R with specific Res.	$R = \rho \frac{l}{A}$	*l* is length of conductor and A is area of cross section
9	Specific Resistance, ρ	$\rho = R \frac{A}{l}$	
10	Resistivity with electrons	$\rho = \frac{m}{ne^2\tau}$	
11	Current density J	$\vec{J} = I / \vec{A}$	I is current, J current density, A is area of cross section
12	Current density magnitude	J A cosθ = I	θ is angle between $\vec{J}$ and $\vec{A}$
13	Conductance G	G = 1/R	
14	Conductivity σ	$\sigma = 1/\rho$	ρ is specific resistance
15	Microscopic form of Ohms Law	$J = \sigma E$	E is electric field
16	Temperature coefficient of Resistance α	$\alpha = \frac{Rt - R_0}{R_0\ X\ t}$	R_o is resistance at 0_o C. R_t is resistance at t^o and "t" is temperature difference.
17	Resistances in series	$R = R_1+R_2+R_3$	Same current through all resistances (circuit Current
	Resistances in parallel	$1/R_e = 1/R_1 + 1/R_2 + 1/R_3$	Same P.D. across each resistance (V of cell)
18	In a cell, emf and internal resistance	$I = \frac{E}{R+r}$	I is current, E is emf, R is external resistance, r is internal resistance.
19	In a circuit with a cell	V = E - Ir	V is terminal potential difference
20	n Cells of emf E in series	Emf = nE	
21	Resistance of n cells in series	nr + R	r is internal resistance of one cell, R external Resistance
22	Current in circuit with n cells in series	$I = \frac{nE}{R+nr}$	r is internal resistance of one cell, R external Resistance
23	n cells in parallel, then emf	emf = E	
24	n cells in parallel, resistance	R + r/n	R external resistance, r internal resistance
25	Cells in mixed group, condition for maximum current	$R = \frac{nr}{m}$	n is number of cells in one row, m is number of rows. r is internal resistance, R external resis.
26	Internal resistance of a cell	$r = (\frac{E-V}{V})\ x\ R$	E is emf, V is terminal Potential difference, R is external resistance.
27	Power of a circuit	$P = I.V = I^2R = V^2/R$	
28	Energy consumed	E = I.V.ΔT	ΔT is time duration
29	Kirchoff Law (junction rule)	$\Sigma i = 0$	Sum of currents at junction is zero.
30	Kirchoff Law (Loop rule)	$\Sigma V = 0$	In a loop sum of all p.d.s is Zero

ELECTROSTATICS : Study of Electricity in which electric charges are static i.e. not moving, is called electrostatics

- STATIC CLING
- An electrical phenomenon that accompanies dry weather, causes these pieces of papers to stick to one another and to the plastic comb.
- Due to this reason our clothes stick to our body.

- ELECTRIC CHARGE : Electric charge is characteristic developed in particle of material due to which it exert force on other such particles. It automatically accompanies the particle wherever it goes.

- Charge cannot exist without material carrying it

- It is possible to develop the charge by **rubbing two solids having friction.**

- Carrying the charges is called **electrification**.

- Electrification due to friction is called **frictional electricity**.

Since these charges are not flowing it is also called static electricity.

There are two types of charges. +ve and -ve.

- Similar charges repel each other,
- Opposite charges attract each other.

- Benjamin Franklin made this nomenclature of charges being +ve and -ve for mathematical calculations because adding them together cancel each other.

- Any particle has vast amount of charges.

- The number of positive and negative charges are **equal**, hence **matter is basically neutral.**

- Inequality of charges give the material a **net** charge which is equal to the difference of the two type of charges.

Electrostatic series :If two substances are rubbed together the former in series acquires the positive charge and later, the –ve.

(i) Glass (ii) Flannel (iii) Wool (iv) Silk (v) Hard Metal (vi) Hard rubber (vii) Sealing wax (viii) Resin (ix) Sulphur

Electron theory of Electrification

- Nucleus of atom is positively charged.
- The electron revolving around it is negatively charged.
- They are equal in numbers, hence atom is electrically neutral.
- With friction there is transfer of electrons, hence net charge is developed in the particles.

- It also explains that the charges are compulsorily developed in pairs equally . +vein one body and –ve in second.
- It establish **conservation of charges in the universe.**
- The **loss** of **electrons** develops +ve charge. While **excess** of **electrons** develop -ve charge
- A **proton** is 1837 times heavier than electron hence it cannot be transferred. Transferring lighter electron is easier.
- Therefore for electrification of matter, only **electrons** are active and responsible.

Charge and Mass relation

- Charge cannot exist without matter.
- One carrier of charge is electron which has **mass** as well.
- Hence if there is charge transfer, mass is also transferred.
- Logically, negatively charged body is heavier then positively charged body.

Conductors, Insulators and Semiconductors

- Conductors : Material in which electrons can move easily and freely.

Ex. Metals, Tap water, human body.

Brass rod in our hand, if charged by rubbing the charge will move easily to earth. Hence Brass is a conductor.

The flow of this excess charge is called **discharging**

- **Insulator** : Material in which charge cannot move freely. Ex . Glass, pure water, plastic etc.

- Electrons can be forced to move across an insulator by applying strong force (called electric field.) Then this acts like a conductor.

- **dielectric strength.**

The maximum electric field an insulator can withstand without becoming a conductor is called its dielectric strength.

- **Semiconductor** : is a material which under little stimulation (heat or Elect. Field) converts from insulator to a conductor.

Ex. Silicon, germanium.

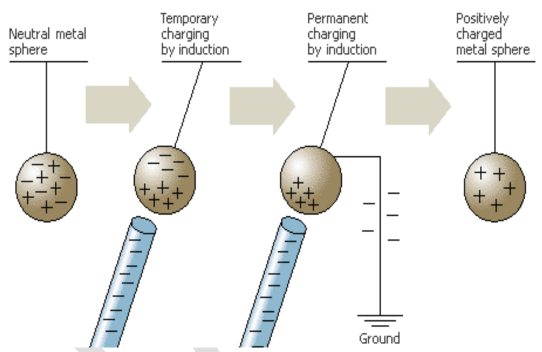
- **Superconductor** : is that material which presents no resistance to the movement of the charge through it.

The resistance is precisely zero.

Electrostatic Induction

- Phenomenon of polarization of charges in a body, when a charged body is present near it, is called electrostatic induction.
- In this process bodies are charged without touching them.

- **Charging by Induction**



A charged object will induce a charge on a nearby conductor. In this example, a negatively charged rod pushes some of the negatively charged electrons to the far side of a nearby copper sphere because like charges repel each other. The positive charges that remain on the near side of the sphere are attracted to the rod.

- If the sphere is grounded so that the electrons can escape altogether, the charge on the sphere will remain if the rod is removed.

Basic properties of Electric charge

- Additivity of Electric charges
- Quantization of Electric charge
- Conservation of Electric Charge

Additivity of Charges...

- Charges can be added by simple rules of algebra. Addition of positive and negative charge makes Zero charge

Quantization of Electric charge

- Principle: **Electric charge is not a continuous quantity, but is an integral multiple of minimum charge (e).**
- Reason of quantization:
- Minimum charge *e* exist on an electron.
- The material which is transferred during electrification is an electron, in integral numbers.
- Hence **charge transferred has to be integral multiple of *e*.**
- Charge on an electron (-e) and charge on a proton (+e) are equal and opposite, and are the **minimum.**

This minimum charge is 1.6×10^{-19} coulomb.

one electron has charge -1.6×10^{-19} C

One proton has charge $+1.6 \times 10^{-19}$ C

- Charge on a body Q is given by

$Q = \pm ne$

Where n is a whole number 1,2,3.....

and $e = 1.6 \times 10^{-19}$

- since e is smallest value of charge, it is called Elementary Charge or Fundamental charge

---

- **(Quarks** : In new theories of proton and neutrons, a required constituent particles called Quarks which carry charges $\pm(1/3)e$ or $\pm(2/3)e$.

- But because free quarks do not exist and their sum is always an integral number, it does not violet the quantization rules.)

- **Conservation of Charges**
- Like conservation of energy, and Momentum, the electric charges also follow the rules of conservation.

1. Isolated (Individual) Electric charge can neither be created nor destroyed, it can only be transferred.
2. Charges in pair can be created or destroyed.

Example for 1.

At Nuclear level : Decay of U-238

$^{238}U \quad ^{234}Th + {}^{4}He$ (Radio active decay)

Atomic number Z of radioactive material U-238 is 92. Hence it has 92 protons hence charge is 92*e*. Thorium has Z= 90, hence charge is 90e, alpha particles have charge 2e. Therefore charges before decay are 92 and after decay are 90+2=92

Example for 2. (a) Annihilation (destruction in pair)
In a nuclear process an electron -e and its antiparticle positron +e undergo annihilation process in which they transform into two gamma rays (high energy light)

$e^- + e^+ \quad y + y$

Example for 2 (b):Pair production:
is converse of annihila tion, charge is also conserved when a gamma ray transforms into an electron and a positron

$y \quad e^- + e^+$ (pair production)

Electric Force - Coulumb's Law

- Coulumb's law in Electrostatics :

Force of Interaction between two stationery point charges is

directly proportional to the product of the charges,
inversely proportional to the square of the distance between them and
acts along the straight line joining the two charges.

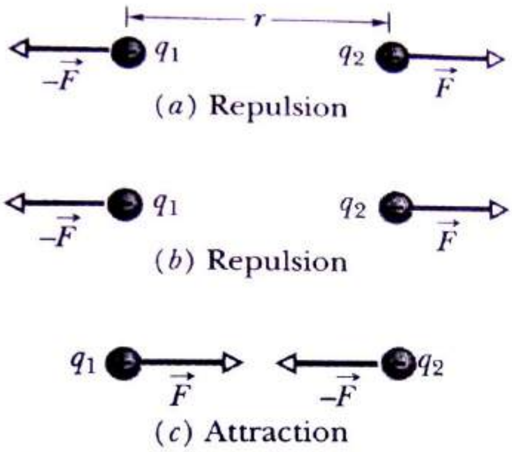


Fig. 22-6 Two charged particles, separated by distance *r*, repel each other if their charges are (*a*) both positive and (*b*) both negative. (*c*) They attract each other if their charges are of opposite signs. In each of the three situations, the force acting on one particle is equal in magnitude to the force acting on the other particle but has the opposite direction.

If two charges q_1 and q_2 are placed at distance r then,

$$F = c\,\frac{q_1 q_2}{r^2}$$

where c is a constant .

c is called Coulomb's constant and its value is

$$c = \frac{1}{4\pi\varepsilon_0} \quad \therefore \quad F = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r^2}$$

The value of c depends upon system of units and on the medium between two charges
It is seen experimentally that if two charges of 1 Coulomb each are placed at a distance of 1 meter in air or vacuum, then they attract each other with a force (F) of 9×10^9 Newton.
Accordingly value of c is 9×10^9 Newton x m^2/coul^2

e_0 is permittivity of free space or vacuum and its value is $e_0 = 8.85 \times 10^{-12}$ coul^2 / N x m^2

If point charges are immersed in a dielectric medium, then e_0 is replaced by e a quantity-characteristic of the matter involved In such case. For vacuum e = e_0

$$F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2}$$

Permittivity, Relative Permittivity and Dielectric Constant

Permittivity is a measure of the property of the medium surrounding electric charge which determine the forces between the charges.

Its value is known as Absolute permittivity of that Medium e

More is Permittivity of medium, Less is coulombs Force.

For water, permittivity is 80 times then that of vacuum, hence force between two charges in water will be 1/80 time force in vacuum (or air.)

Relative Permittivity(e_r) : It is a dimension-less characteristic constant, which express absolute permittivity of a medium w.r.t. permittivity of vacuum or air. It is also called

Dielectric constant (K) **K= e_r = e/e_0**

- This result leads to the calculation that

$$\varepsilon_r = K = \frac{F \text{ (Force in Vacuum)}}{F' \text{ (Force in Medium)}}$$

$$F = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\,\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\,\varepsilon_0 K} \frac{q_1 q_2}{r^2}$$

- Unit of charge:- In S.I. System of units, the unit of charge is **Coulomb.**
- **One coulomb** is defined as that charge, which, when placed at a distance of 1 m in air or vacuum from an equal and similar charge, repel it with a force of 9×10^9 Newton
- Charge on one electron is 1.6019×10^{-19} coul. Hence
- One coulomb is equivalent to a charge of 6.243 x 10^{18} electrons

Is electric charge a fundamental quantity?

- No, In S.I. System, the fundamental quantity is **Electric current** and its unit is Ampere. Therefore coulomb is defined in it's terms as under:
- Coulomb is that quantity of charge which passes across any section of a conductor per second when current of one ampere flows through it, i.e.
- 1 coulomb=1 Ampere x 1 sec

In **cgs electrostatic** system, the unit of charge is called as STATECOULUMB or esu of charge.

- In this system electrostatic constant c=1 for vacuum or air, $F = \frac{q_1 q_2}{r^2}$

One stat coulomb is defined that amount of charge which when placed at a distance of 1 cm in air from an equal and similar charge repel it with a force of one dyne.

In **cgs electromagnetic** system, the unit of charge is called ABCOULOMB or emu of charge

1 Coulomb = 3 x 10^9 statcoulomb

= 1/10 abcoulomb

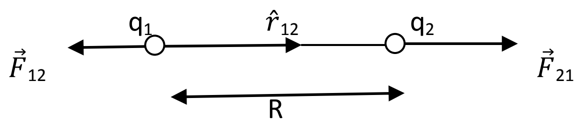
\ 1 emu = 3×10^{10} esu of charge

<u>Vector form of Coulumbs' Law</u>

Equation of Coulumbs force showing magnitude as well as direction is called Vector form of coulumbs' law.

If $\hat{r}_{12}$ is unit vector pointing from q_1 to q_2, then as per diagram $\hat{r}_{12}$ and $\vec{F}_{21}$ will be in the same direction, then

$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$ (vector equation)........ 1.



Similarly $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$ 2

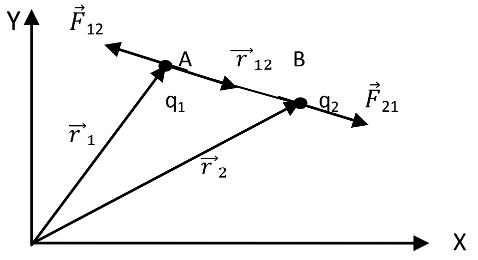
Since $\hat{r}_{21}$ = - $\hat{r}_{12}$ $\therefore$ $\vec{F}_{21}$ = - $\vec{F}_{12}$

Electrostatic Force between two point charges in terms of their position vectors.

(i).Let there be two point charges q1 and q2 at points A & B in vacume. With reference to an origin O let their position vectors be $\vec{r}_1$ (OA) and $\vec{r}_2$ (OB). Then AB=$\vec{r}_{12}$. According to triangle law of vectors :

$\vec{r}_1 + \vec{r}_{12} = \vec{r}_2 \quad \therefore \ \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ and

$\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$



(ii) According to Coulumb's law, the Force $\vec{F}_{12}$ exerted on q_1 by q_2 is given by : $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$ where $\hat{r}_{21}$ is a unit vector pointing from q_2 to q_1 . We know that $\hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|} = \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$

Hence, general Vector forms of Coulumb's equation is

$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}_1 - \vec{r}_2|^2}\overrightarrow{(r_1 - r_2)}$ and

$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{|\vec{r}_2 - \vec{r}_1|^2}\overrightarrow{(r_2 - r_1)}$

Comparison of Electrostatic and Gravitational Force

1. Identical Properties :
   - Both the forces are central forces, i.e., they act along the line joining the centers of two charged bodies.
   - Both the forces obey inverse square law, F∝ $\frac{1}{r^2}$
   - Both are conservative forces, i.e. the work done by them is independent of the path followed.
   - Both the forces are effective even in free space.
2. Non identical properties :
   a. Gravitational forces are always attractive in nature while electrostatic forces may be attractive or repulsive.
   b. Gravitational constant of proportionality does not depend upon medium, the electrical constant of proportionality depends upon medium.
   c. Electrostatic forces are extremely large as compared to gravitational forces

Qn. Compare electrostatic and gravitational force between one electron and one proton system.

Ans : $F_e = \frac{1}{4\pi\varepsilon_0}\frac{e.e}{r^2} = 9\text{x}10^9\frac{(1.6x10^{-19})^2}{r^2}$ Newton

$F_g = G\frac{me\ x\ mp}{xr^2} = 6.67\text{x}10^{-11}\frac{(9.1\ x\ 10^{-31})\ x\ (1.67\ x\ 10^{-27})}{r^2}$ Newton

$F_e / F_g = 2.26 \times 10^{39}$

Principle of Superposition of Charges :

If a number of Forces $F_{11}, F_{12}, F_{13}, \ldots\ldots F_{1n}$ are acting on a single charge q_1 then charge will experience force F_1 equal to vector sum of all these forces .

$F_1 = F_{11} + F_{12} + F_{13} + \ldots\ldots + F_{1n}$

The vector sum is obtained as usual by parallelogram law of vectors.

All electrostatics is basically about Coulomb's Law and Principle of superposition.

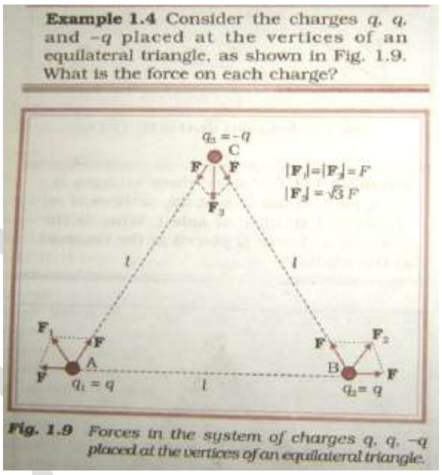

Example 1.4 Consider the charges q, q, and $-q$ placed at the vertices of an equilateral triangle, as shown in Fig. 1.9. What is the force on each charge?

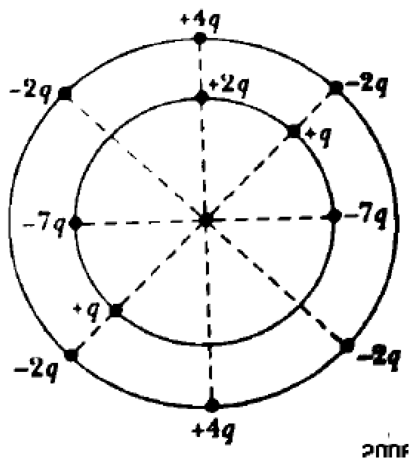
Fig. 1.9 Forces in the system of charges q, q, $-q$ placed at the vertices of an equilateral triangle.

NUMERICALS FOR PRACTICE

1.How many electrons must be removed from the sphere to give it a charge of +2 μC . Is there any change in the mass when it is given this positive charge. How much is this change?

2. Two identical charged copper spheres A and B have their centers separated by a distance of 50 cm. A third sphere of same size but uncharged is brought in contact with the first, then brought in contact with the second and finally removed from both. What is the new force of repulsion between A and B?

3. A central particle of charge –q is surrounded by two circular rings of charged particles, of radii r and R, such that R > r. What are the magnitude and direction of the net electrostatic force on the central particle due to other particles.



4.-Three equal charges each of 2.0 x 10-6 are fixed at three corners of an equilateral triangle of side 5 cm. Find the coulomb force experienced by one of the charges due to other two.

5.

✶ -------o-----------------------o---------

-3q -q

Above two charged particles are free to move. At one point, however a third charged particle can be placed such that all three particles are in equilibrium.

(a) is that point to the left of the first two particles, to their right, or between them?

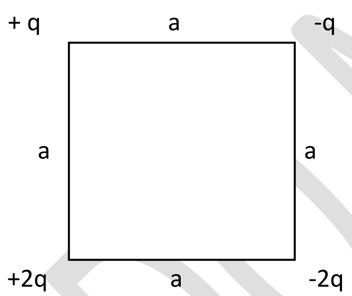
(b) Should the third particle be positively or negatively charged?

© Is the equilibrium stable or unstable?

6. A charge q is placed at the center of the line joining two equal charges Q. Show that the system of three charges will be in equilibrium if q = Q/4.

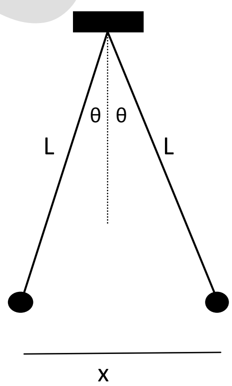
7. Two particles having charges 8q and –2q are fixed at a distance L. where, in the line joining the two charges, a proton be placed so that it is in equilibrium (the net force is zero). Is that equilibrium stable or unstable?

8. What are the horizontal and vertical components of the net electrostatic force on the charged particle in the lower left corner of the square if q = 1.0 x 10^{7}C and a = 5.0 cm?

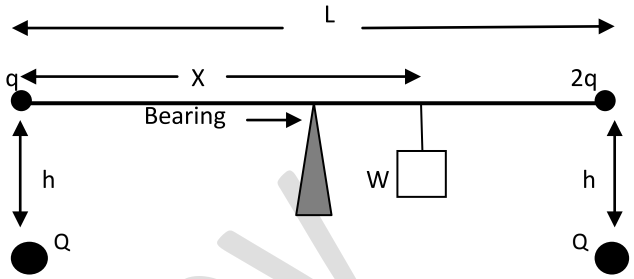


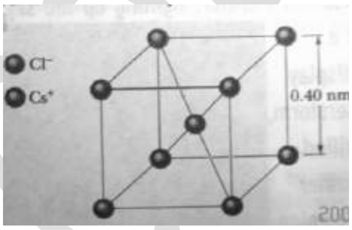
9. Two tiny conducting balls of identical mass m and identical charge q hang from non conducting threads of length L. Assume that θ is so small that tan θ can be replaced by sin θ; show that, for equilibrium,

$$X = \left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{1/3}$$



.8.A long non-conducting massless rod of length L, pivoted at its centre and balanced with a block of weight W at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges q and $2q$, respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q. a. Find the distance x when the rod is horizontal and balanced. (b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?



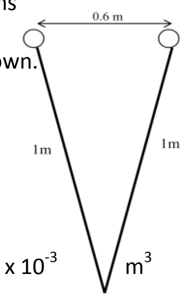


9. In the basic CsCl (Cesium chloride) crystal, Cs+ ions form the corners of a cube and a Cl- ion is at the centre of cube. Edge length is 0.40 nm.

(a) What is the magnitude of the net electrostatic force exerted on Cl- ion by the eight Cs+ ions.?

(b) If one of the Cs+ ion is missing the crystal is said to have defect. How much will be the force on chlorine ion in that case?

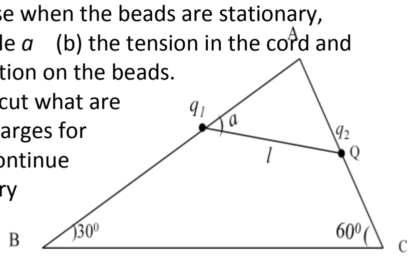
10. Two similar helium-filled spherical balloons tied to a 5 g weight with strings and each carrying a charge q float in equilibrium as shown. Find (a) the magnitude of q, assuming that the charge on each balloon is at its centre and (b) the volume of each balloon. Assume that the density of air =1.29 kg m^{-3} and the density of helium in the balloon is= 0.2 kg $^{m-3}$. Neglect the weight of the unfilled balloons. Ans: q = 5.5 x 10^{-7} V = 2.3 x 10^{-3} m^3



11. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30^0 with each other. When suspended in a liquid of density of 800 kg m^{-3}, the angle remain the same. What is the dielectric constant of the liquid? The density of the material of the sphere is 1600 kg m^{-3} Ans : K = 2

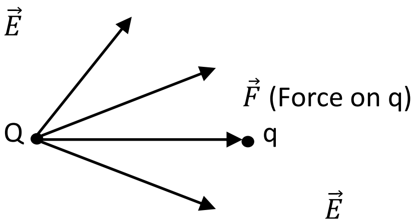
12. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane. Two beads of equal masses *m* each and carrying charges q_1 q_2 are connected by a cord of length *l* and can slide without friction on the wires. Considering the case when the beads are stationary, determine (a) angle *a* (b) the tension in the cord and (c) the normal reaction on the beads.

If the cord is now cut what are the value of the charges for which the beads continue to remain stationary



ELECTRIC FIELD

ELECTRIC FIELD-is the environment created by an electric charge (source charge) in the space around it, such that if any other electric charges(test charges)is present in this space, it will come to know of its presence and exert a force on it.



INTENSITY (OR STRENGTH) OF ELECTRIC FIELD AT A LOCATION Is the force exerted on a unit charge placed at that location

: if intensity of electric field at a location is E and a charge 'q' is placed ,then force experienced by this charges F is

$$\vec{F} = q.\vec{E} \quad \text{———} 1$$

or

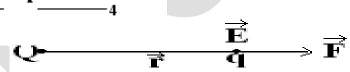
$$\boxed{\vec{E} = \frac{\vec{F}}{q}} \quad \text{———} 2$$

Direction of force F is in direction of electric field E

$$F = \frac{1}{4\pi E}\frac{Qq}{r^2}\hat{r} \quad \text{———} 3$$

By equ.1and 3 : Intensity of electric field due to Source charge Q is

$$\vec{E} = \frac{1}{4\pi E}\frac{Q}{r^2}\hat{r} \quad \text{———} 4$$



By coloumb's law we know that in similar situation if q=1 then

$$\vec{E} = \frac{1}{4\pi E}\frac{Q}{r^2}$$

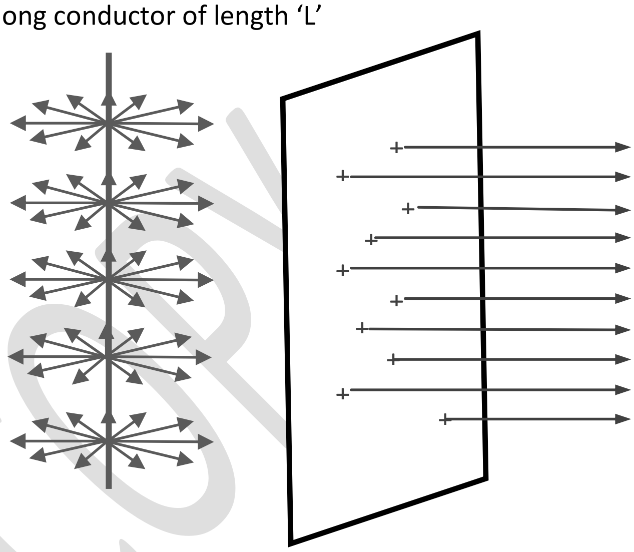
Relation in F, E and Test charge q is $E = \frac{F}{q}$

DISTRIBUTION OF CHARGE

Electric charge on a body may be concentrated at a point, then it is called a 'point charge'. If it is distributed all over, then it is called distribution of charge. Depending on shape of it is given different names

1.Linear distribution: when charge is evenly distributed over a length. In such case we use a quantity Linear charge density λ. Which has relation

$\lambda = \frac{Q}{L}$, Where 'Q' is charge distributed over a long conductor of length 'L'



2- Areal distribution: charge is evenly distributed over a surface area,S.

The surface charge density is 'σ' given by $\sigma = \frac{Q}{S}$

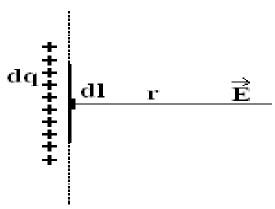
Where Q is charge given to a surface of area 'S'.

3-volumetric distribution: charge is evenly distributed throughout the body having volume 'V'Volumetric charge density is 'ρ' $\rho = \frac{Q}{V}$

GENERAL DISTRIBUTION OF ELECTRIC FIELD DUE TO DIFFERENT DISTRIBUTION OF CHARGES

1-Due to point change Q $\quad \vec{E} = \frac{1}{4\pi E}\frac{Q}{r^2}\hat{r}$

2-E due to linear distribution of electric charge

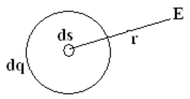


$$dq = \lambda.dl$$

$$dE = \frac{1}{4\pi E_0}\frac{\lambda.dl}{r^2}$$

$$\boxed{E = \frac{1}{4\pi E_0}\int_L \frac{\lambda.dl}{r^2}}$$

3 - E due to areal distribution of charge:

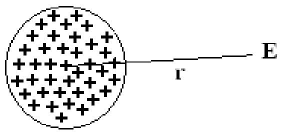


$dq = \sigma . ds$

$dE = \frac{1}{4\pi E_0} \frac{\sigma . ds}{r^2}$

$$\boxed{E = \frac{1}{4\pi E_0} \int_s \frac{\sigma . ds}{r^2}}$$

4- E due to volumetric distribution of charge



SOLID SPHERE VOLUME V

$$\boxed{E = \frac{1}{4\pi E_0} \int_V \frac{\rho . ds}{r^2}}$$

DIPOLE

1-Dipole is a system of two equal and opposite charges at finite & fixed distance.

example: molecule of electrolytic compounds.

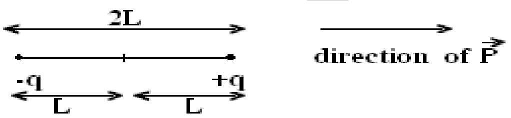
Example - HCl, H_2O.

2-CO_2 & CH_4 are non-polar because centers of –ve & +ve charges co-incide and there is no distance between them.

3-if non polar atom is placed in an elect.field a distance is created between +ve & -ve charge: it become polar.

Dipole moment:-the effectiveness or strength of a dipole is measured by the physical quantity .Dipole moment $\vec{P}$. it is calculated as $\vec{P}$= q x 2$\vec{L}$

P=q x 2L(magnitude) or $\vec{P}$= q x 2$\vec{L}$ (vector)



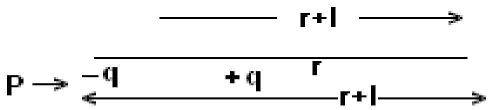
Where 'q' is each charge and '2L' is distance between them.(each charge is at a distance L from 'center' of dipole)

Dipole moment $\vec{P}$= q x 2$\vec{L}$ is a vector quantity it has magnitude p=2qL

And its direction is along line from –q to +q.

ELECTRIC FIELD DUE TO DIPOLE

ON THE AXIAL LINE



E DUE TO +q $\quad \vec{E_1} = \frac{q}{4\pi\epsilon_0 (r+l)^2} \hat{r}$

ALONG $\vec{P}$

E DUE TO –q $\quad \vec{E_1} = \frac{-q}{4\pi\epsilon_0 (r+l)^2} \hat{r}$

OPPOSITE TO $\vec{P}$

NET ELECTRIC FIELD

$$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(r-l)} \frac{1}{(r+l)^2}\right)$$

$$= \frac{q}{4\pi\epsilon_0} \frac{4rl}{(r^2-l^2)} \qquad \{2ql = P\}$$

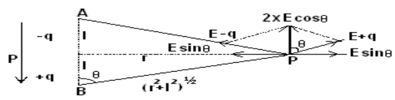
$$\boxed{E = \frac{2P.r}{4\pi\epsilon_0 (r^2-l^2)^2}}$$

SINCE $\vec{E_1} > \vec{E_2}$

: $\vec{E}$ IS IN THE DIRECTION OF $\vec{P}$

IF R>>L THE, $\quad E = \frac{2P}{4\pi\varepsilon_0 r^3}$

2 $\vec{E}$ ON EQUATORIAL LINE (TRANSVERAL LINE)



E due to +q , $\quad E_{+q} = \frac{q}{4\pi\epsilon_0 (r^2+l^2)} \hat{BP}$

E due to -q $\quad E_{-q} = \frac{q}{4\pi\epsilon_0 (r^2+l^2)} \hat{PA}$

|E+q| = |E-q| = Eq

each Eq is resolved in two direction. One along equatorial line and other in axial directions which are the Esinθ and normal direction E cosθ .

Esinθ in opposite direction cancel each other while E cosθ add up to two.

: net electric field E = 2E cosθ

E(net) = 2Ecosθ

$$= 2 \cdot \frac{q}{4\pi\epsilon_0 (r^2 + l^2)} \frac{l}{(r^2+l^2)^{1/2}}$$

$$E = \frac{P}{4\pi\varepsilon_0\,(r^2+l^2)^{3/2}}$$

IF R>>L Then, $E = \dfrac{P}{4\pi\varepsilon_0 r^3}$

The direction is opposite to that of $\vec{P}$

Electric Field at equatorial line is half of the field on axial line in strength and opposite in direction.

z

Electric Field Intensity due to a Short Electric Dipole at some General Point

(i) Let AB be a short electric dipole of dipole moment $\vec{p}$ (directed from B to A). We are interested to find the electric field at some general point P whose polar coordinates are (r, θ). The distance of observation point P w.r.t. mid point O of the dipole is r and the angle made by the line OP w.r.t. axis of dipole is θ.

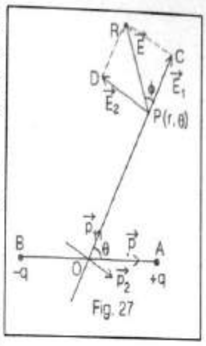


Fig. 27

(ii) We know that dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components $\vec{p}_1$ and $\vec{p}_2$ as shown in Fig. 27, so that $\vec{p} = \vec{p}_1 + \vec{p}_2$.

The magnitudes of $\vec{p}_1$ and $\vec{p}_2$ are $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$.

(iii) It is clear from figure that point P lies on the axial line of dipole with moment $\vec{p}_1$. Hence magnitude of the electric field intensity $\vec{E}_1$ at P due to $\vec{p}_1$ is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p\cos\theta}{r^3} \quad (\text{along } \vec{p}_1). \qquad \text{........(1)}$$

Similarly, P lies on the equatorial line of dipole with moment $\vec{p}_2$. Hence, magnitude of electric field intensity $\vec{E}_2$ at P due to $\vec{p}_2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\sin\theta}{r^3} \quad (\text{opposite to } \vec{p}_2) \qquad \text{........(2)}$$

Hence resultant intensity at P is : $\vec{E} = \vec{E}_1 + \vec{E}_2$

Magnitude of $\vec{E}$ is : $E = \sqrt{(E_1^2 + E_2^2)}$ (as $\vec{E}_1$ and $\vec{E}_2$ are mutually perpendicular).

or $$E = \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

or $$\boxed{E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1+3\cos^2\theta}} \qquad \text{....(3)}$$

(iv) If the resultant field intensity vector $\vec{E}$ makes an angle φ with the direction of $\vec{E}_1$, then

$$\boxed{\tan\phi = \frac{E_2}{E_1} = \frac{(p\sin\theta/4\pi\varepsilon_0 r^3)}{(2p\cos\theta/4\pi\varepsilon_0 r^3)} = \frac{1}{2}\tan\theta}$$

Electric Line of Force :

The idea of Lines of Force was given by Michel Faraday. These are imaginary lines which give visual idea of Electric field, its magnitude, and direction.

A line of force is continuous curve the tangent to which at a point gives the direction of Electric field, and its concentration gives the strength of Field.

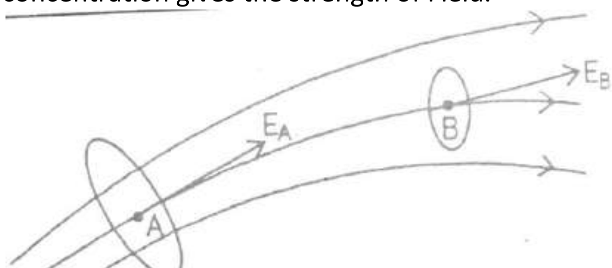


Fig. 17

Electric Field at A is stronger than field at B.

Properties of Electric Lines of Force :

Electric Lines of Force :

1.start from positive charge and end at negative.

2.Electric Lines of forces are **imaginary** but Electric field they represent is **real.**

3.The tangent drawn at any point on the line of force gives the direction of force acting on a positive charge at that point.

4.**In SI system, the number of electric lines originating or terminating on charge q is q/ε_0 . That means lines associated with unit charge are $1/\varepsilon_0$**

5.Two lines of force never cross each other, because if they do so then at the point of intersection, intensity will have two directions which is absurd.

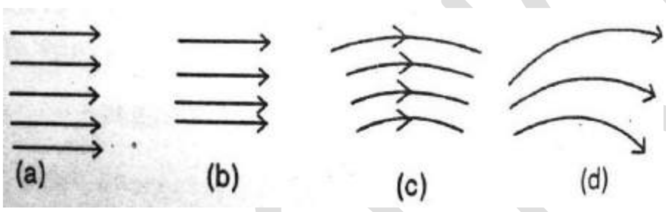
6. Electric Lines of force can never be a closed loop since they do not start and end at the same point. The lines are discontinuous, start from + and terminate at –

7. The electric line of force do not pass through a conductor as electric field inside a conductor is zero.

8. Lines of force have tendency to contract longitudinally like a stretched string, producing attraction between opposite charges and edge effect.

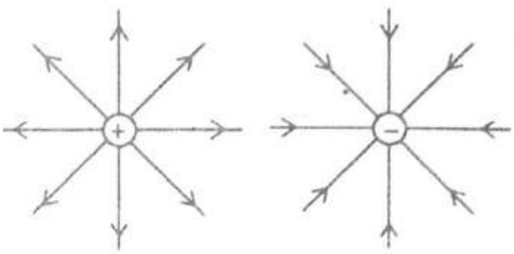
9.Electric Lines of force start and end **Normal to the surface** of conductor.

10. Crowded lines represent strong field while distant lines represent weak field. Equidistant parallel lines represent uniform field. Non-straight or non- parallel represent non-uniform field. In the diagram a is uniform while b, c, and d are non-uniform fields.

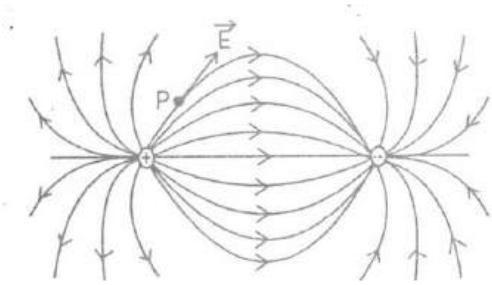


Field Lines due to some charge configurations.

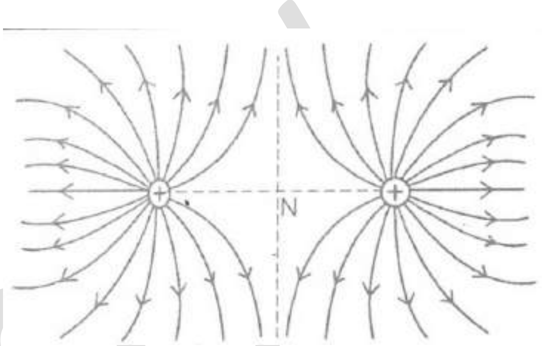
1.Single positive or negative charge



Two equal and opposite charges :



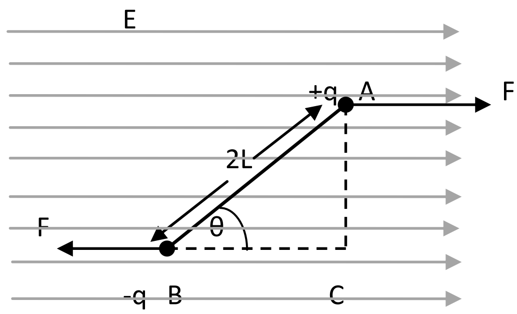
Lines of force due to Two positive charges



Elect field lines due to straight line distribution :
And Electric field lines due to very large sheet of charge are shown in the previous page.

Electric dipole in electric field

When a dipole is placed in an electric field each charge experience a force (F=qe) . Positive, in the direction of field and negative, opposite to direction of field.



Net Force on dipole : F + (-F) = 0 zero

Hence dipole will not make any linear motion.

Torque on dipole: A couple of force is acting on the body of dipole system at different points, the forces are equal and opposite in uniform field. Hence they form a couple of forces which create a **torque.** Therefore dipole is capable of rotation in a uniform electric field. The moment of forces or Torque is

$\tau = F \times AC = qE \times 2L\sin\theta = 2qL\,E\,\sin\theta = \mathbf{PE\sin\theta}$

$\vec{\ }\ \vec{\ }$

or $\vec{\tau} = P \times E$

NOTE :

1.Direction of torque is normal to the plane containing dipole moment P and electric field E and is governed by right hand screw rule.

2. If Dipole is parallel to E the torque is **Zero**.

3. Torque is **maximum** when Dipole is perpendicular to E and that torque is PE

4. This equation gives the definition of dipole moment. If E is 1 N/C then P=T.

Therefore; **Dipole Moment of a dipole is equal to the Torque experience by that dipole when placed in an electric field of strength 1 N/C at right angle to it.**

5. If a dipole experiencing a torque in electric field is allowed to rotate, then it will rotate to align itself to the Electric field. But when it reach along the direction of E the torque become zero. But due to inertia it overshoots this equilibrium condition and then starts oscillating about this mean position.

6.*Dipole in Non-Uniform Electric field* :

In case Electric field is non-uniform, magnitude of force on +q and –q will be different, hence a net force will be acting on centre of mass of dipole and it will make a linear motion. At the same time due to couple of forces acting, a torque will also be acting on it.

Work done in rotating a dipole in a uniform Electric field:

1.If a dipole is placed in a uniform electric field experience a torque. If it is rotated from its equilibrium position, work has to be done on it. If an Electric dipole with moment P is placed in electric field E making an angle α, then torque acting on it at that instant is

τ = PESinα

2. If it is rotated further by a small angle dα then work done dw = (PEsinα).dα

Then work done for rotating it through an angle θ from equilibrium position of angle 0 is :-

$$W = \int_0^{\theta} (\text{PEsin}\alpha).d\alpha = \text{PE}[-Cos\ \alpha]_0^{\theta}$$

Or, **W** = PE $\left[-cos\theta + \cos 0\right]$ = **pE** $[\mathbf{1 - Cos\ \theta}]$

3.If a dipole is **rotated through 90⁰** from the direction of the field, then work done will be

$$\boxed{W = pE\,[1 - \text{Cos}\ 90] = pE}$$

4. If the dipole is **rotated through 180⁰** from the direction of the field, then work done will be :

$$\boxed{W = pE\,[1 - \text{Cos}\ 180] = 2\,pE}$$

Potential Energy of a dipole kept in Electric field :

1. dipole in Equilibrium (P along E):-

A dipole is kept in Electric field in equilibrium condition, dipole moment P is along E

To calculate Potential Energy of dipole we calculate work done in bringing +q from zero potential i.e.∞ to location B, and add to the work done in bringing –q from ∞ to position A.

1.The work done on –q from ∞ up to A

= -(*Work done up to B* + Work done from B to A)

2.Work done on +q = +(*Work done up to B*)

Adding the two

Total work done = Work done on –q from B to A

= Force x displacement

= -qE x 2L = - 2qLE

=- P.E

This work done convert into Potential Energy of dipole

$$\mathbf{U = -\vec{P}\,.\,\vec{E}}$$

If P and E are inclined at angle θ to each other then magnitude of this Potential Energy is

U = - P E Cos θ

Electric – Potential

(1) Electric Potential is characteristic of a location in the electric field. If a unit charge is placed at that location it has potential energy (due to work done on its placement at that location). This potential energy or work done on unit charge in bringing it from infinity is called potential at that point.

(2) Potential – Difference (i) is the work done on unit charge for carrying it from one location to other location A.

V_A ⟵ q V_∞

Potential at A --------------------------V_A

Energy with q at A is q V_A

Energy with Q at B is q V_B

Difference of Energy $U_A - U_B = q(V_A - V_B)$

Using work energy theorem . $W = q((V_A - V_B)$

Or, $V_A - V_B$ = W / q & $U_A - U_B$ = W.

If $V_B = 0$ { At ∞ Potential V = 0 , Inside Earth V_E = 0}

Then V_A = W / q

This equation gives definition of potential V at point A as under :-

"Potential of a point in electric field is the work done in bringing a unit charge from infinity (Zero potential) to that point, without any acceleration."

__Expression of potential at a point due to source charge Q :-__

Let there be a charge Q which creates electric field around it. Point P is at distance 'r" from it. Let's calculate potential at this point.

A test charge 'q' is moved for a small displacement dr towards Q.

Electric field due to Q at P, E = $\frac{Q}{4\pi\epsilon_0 r^2}$

To move it against this electrical force we have to apply force in opposite direction

Hence applied force F = - $\frac{Qq}{4\pi\epsilon_0 r^2}$

Work done in moving distance dr is dw = - $\frac{Qq}{4\pi\epsilon_0 r^2}dr$

Total work done in bringing the charge from distance ∞ to distance r is

$$W = -\int_\infty^r \frac{Qq}{4\pi\epsilon_0 r^2}dr$$

$$= -\frac{Qq}{4\pi\epsilon_0}\int_\infty^r \frac{dr}{r^2}$$

$$= -\frac{Qq}{4\pi\epsilon_0}\left[\frac{r^{-1}}{-1}\right]_\infty^r = \frac{Qq}{4\pi\epsilon_0 r}$$

$$W/q = \frac{Q}{4\pi\epsilon_0 r} \quad \text{OR} \quad \mathbf{V = \frac{Q}{4\pi\epsilon_0 r}}$$

Where Q is source charge, r is distance & V_r is potential at that point.

Basically V_r is also a "potential difference" between potential of this point P and Potential at ∞ (i.e., 0).

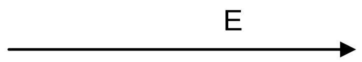
★★★★★★★★★★★

Relation between E & V

A test charge q is moved against E for a small distance dr. then work done dw by applied force -qE is dw = - qE dr

Or, dw / q = - E dr
Or, **dv = - E.dr**
Or, **E = - dv / dr**

Electric field is derivative of potential difference. –ve sign show that direction of E is opposite to direction of dv. I.e., dv decrease along the direction of E



V_A V_B

$V_A > v_B$

This also show that an electric charge experience force from high potential towards low potential if allowed to move, it will do so in this direction only.

If E and are not collinear and make angle θ between them, then according to relation of work & force

$$dv = - E\, dr \cos\theta$$

Or, $-dv/dr = E \cos\theta$

Or, $dv = -\vec{E}\cdot\vec{dr}$

Or $V = \vec{E}\cdot\vec{dr}$

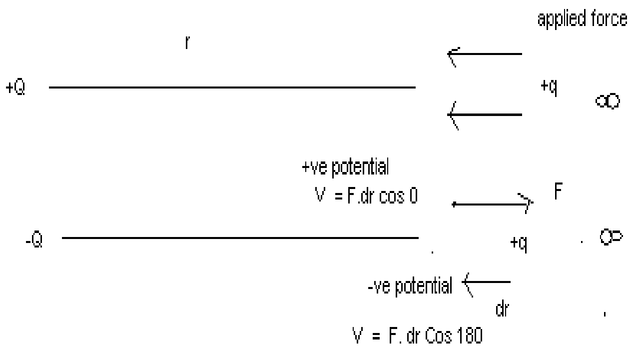
Or { Potential difference is a scalar quantity (work) given by dot product of two vector $\vec{E}$ & $\vec{dr}$.

Principle of super position:-

1) Potential at a point due to different charges is Algebric sum of potentials due to all individual charges.

$$V = V_1 + V_2 + V_3$$

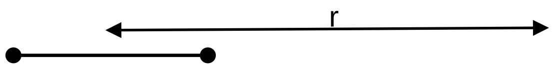
2) Potential due to +ve charge is +ve

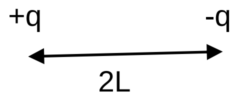


Potential due to –ve charge is –ve

<u>Potential due to a dipole</u>

1) At a point on axial line:-





At P – $V_{+q} = \dfrac{Q}{4\Pi \in_0 (r-l)}$

$V_{-q} = \dfrac{Q}{4\Pi \in_0 (r+l)}$

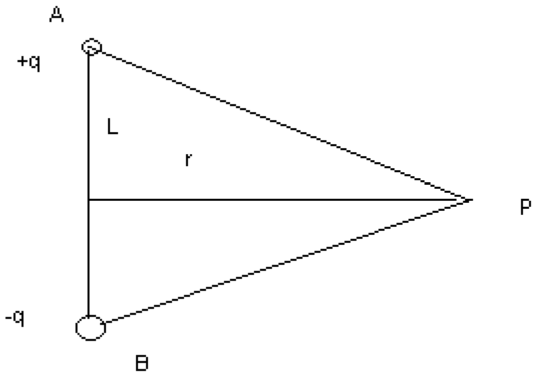
Total V =– $V_{+q} + V_{-q} = \dfrac{Q}{4\pi\epsilon_0}\left(\dfrac{1}{r-l} - \dfrac{1}{r+l}\right)$

$$= \frac{2Ql}{4\Pi \in_0 (r^2 - l^2)} = \frac{P}{4\Pi \in_0 (r^2 - l^2)}$$

If $r >> L$ Then $V = \dfrac{P}{4\Pi \in_0 r^2}$

2) <u>At a point on equatorial line</u>

- q & + q are placed at A & B. Point P is on equatorial

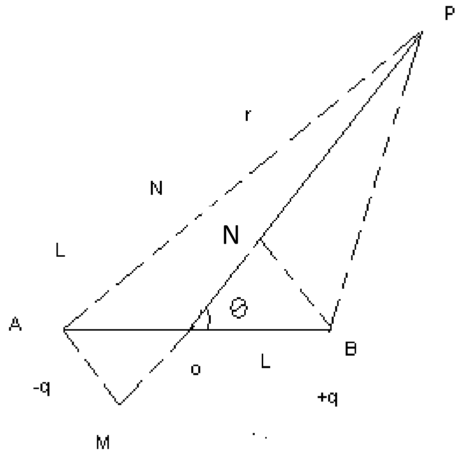


line

Every point on equatorial line is equidistant from +q & -q. Therefore +ve & -ve potential are equal **Hence net potential is zero.**

"Potential at every point on equatorial line of dipole is zero."

iii) **<u>Potential due dipole at any general point.</u>**



Draw normal from A & B on PO

PB ≈ PN = PO – ON = r – L Cos θ ------------ (i)

PA ≈ PM = PO + OM = r + L Cos θ ----------- (ii)

$$V_{+q} = \frac{Q}{4\Pi \in_0 PB} = \frac{Q}{4\Pi \in_0 (r - L\cos\theta)}$$

$$V_{-q} = \frac{-Q}{4\Pi \in_0 PA} = \frac{-Q}{4\Pi \in_0 (r + L\cos\theta)}$$

Total V = V_{+q} + V_{-q} =

$$\frac{Q}{4\Pi \in_0}\left(\frac{1}{r - LCos\theta} - \frac{1}{r - LCos\theta}\right)$$

$$= \frac{Q}{4\Pi \in_0}\left(\frac{r + L\cos\theta - r + LCos\theta}{r^2 - L^2Cos^2\theta}\right)$$

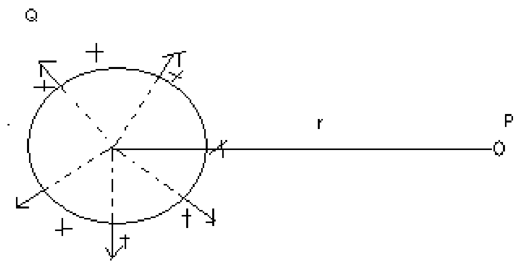
$$= \frac{QX\,2LCos\theta}{4\Pi \in_0 (r^2 - L^2Cos\theta)}$$

Or $$V = \frac{PCos\theta}{4\Pi \in_0 (r^2 - L^2Cos\theta)}$$

If r > > L

Then, Or, $$V = \frac{PCos\theta}{4\Pi \in_0 r^2}$$

Potential due to spherical shell



A spherical shell is given change Q. The electric field is directed normal to surface i.e., Radially outward. "Hence charge on the surface of a shell behaves as if all the charge is concentrated at centre.

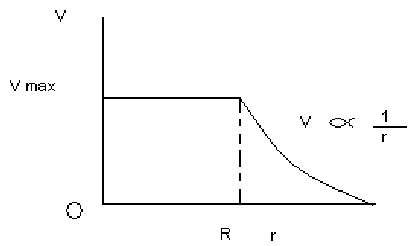
Hence potential at distance r is $V = \frac{Q}{4\Pi \in_0 r}$

Potential on the surface of shell $V = \frac{Q}{4\Pi \in_0 R}$

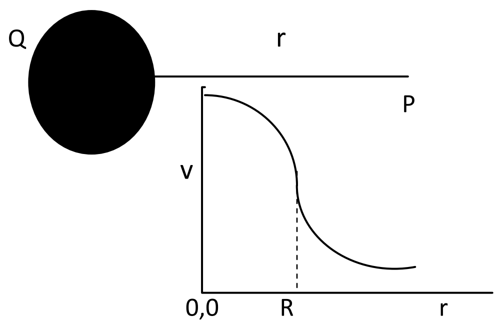
Inside shell Electric field is Zero.
Therefore change in potential dv = Zero X dr = 0 i.e., No change in potential. Hence potential inside a spherical shell is same as on the surface and it is same at every point.

It is $V = \frac{Q}{4\Pi \in_0 R}$ Where R is radius of shell.

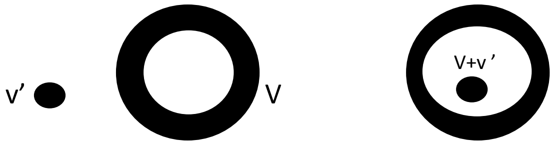
<u>Relation of V & r for spherical shell</u>



In case of non-conducting sphere of charge. potential keeps on increasing up to centre as per diagram.



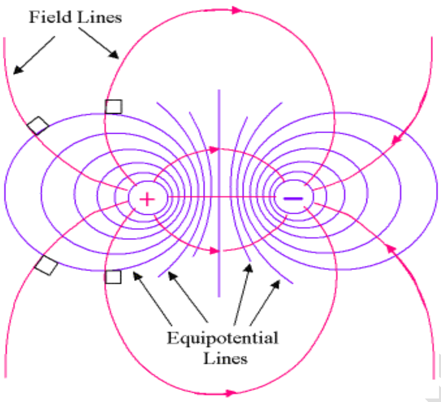
A body of potential v' is placed inside cavity of shell with potential V then potential of the body become V+v'



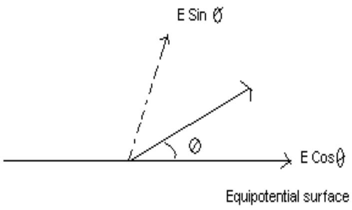
.Equipotential Surface

A real or imaginary surface in an electric field which has same potential at very point is an equipotential surface or simply, an equipotential.

Ex:- A shell having electric charge at its centre, makes an equipotential surface as it has same potential $\dfrac{Q}{4\Pi \in_0 R}$ at every point of the surface.



Electric lines of force and equipotential surface are at right angle to each other.



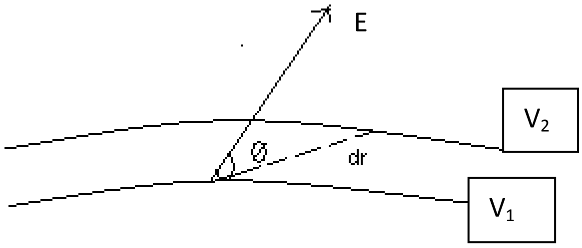
Proof:- Suppose E is not at right angle to equipotential surface, and makes angle θ with it. Then it has two components, E Cos θ along surface and E Sin θ normal to surface due to component E Cos θ, force q E Cos θ should be created on surface and it should move the charge. But we find that charges are in equilibrium. i.e.

E Cos θ = 0 ;

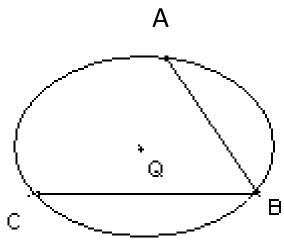
since E = ~~0~~, therefore Cos θ = 0 or $\angle\ \theta = 90^0$

Hence E is always at right angle to equip. surface.

ii) $V_2 - V_1 = dv = -E \ \cos\theta \ . \ dr$



iii) No work is done in carrying an electric charge from one point of E.P. Surface to other point (Whatever is the path)



Net work done in carrying change from A to B is Zero, B to C is Zero, because W = qV and V is same on this equipotential Surface

iv) Surface of a conductor in electrostatic field is always an equipotential surface.

Distribution of charge on uneven surface: - charge density is more on the surface which is pointed, or has smaller radius. Therefore if a conductor is brought near pointed charged surface, due to high density of charge induction will be more. Electric field set up will be very strong. This leads to construction of use of lightning arrester used on the buildings.

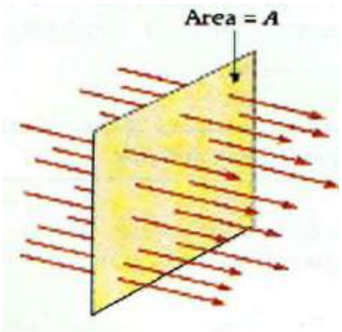
Gauss's Law

Electric Flux

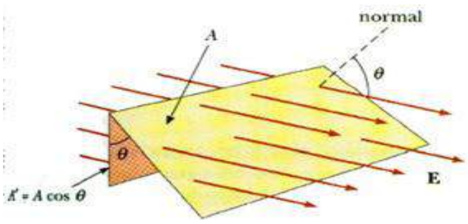
Think of air blowing in through a window. How much air comes through the window depends upon the **speed** of the air, the **direction** of the air, and the **area** of the window. We might call this air that comes through the window the "**air flux**".

We will define the **electric flux** Φ for an electric field that is perpendicular to an area as

Φ= E A



If the electric field **E** is **not** perpendicular to the area, we will have to modify this to account for that.



Think about the "air flux" of air passing through a window **at an angle** θ. The "effective area" is A cos θ or the component of the velocity perpendicular to the window is v cos θ. With this in mind, we will make a general definition of the electric flux as

- Φ = E A cos θ
- Φ You can also think of the electric flux as the **number** of electric field lines that cross the surface.

Remembering the "dot product" or the "scalar product", we can also write this as

$$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}$$

where **E** is the electric field and **A** is a vector equal to the area A and in a direction **perpendicular** to that area. Sometimes this same information is given as

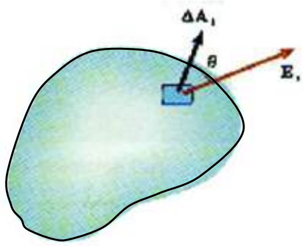
A = A **n**

where **n** is a **unit vector** pointing **perpendicular** to the area. In that case, we could also write the electric flux across an area as

$$\Phi = \mathbf{E} \cdot \mathbf{n}\, A$$

Both forms say the same thing. For this to make any sense, we must be talking about an area where the **direction** of **A** or **n** is constant.

For a curved surface, that will not be the case. For that case, we can apply this definition of the electric flux over a small area ΔA or Δ**A** or ΔA**n**.



Then the electric flux through that small area is ΔΦand

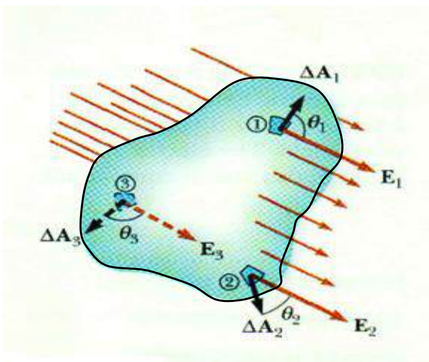
ΔΦ= E ΔA cos θ or

$$\Delta\Phi = \vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{A}}$$

To find the flux through all of a closed surface, we need to sum up all these contributions of ΔΦ over the entire surface,

$$\Phi_c = \oint \mathbf{E} \cdot \vec{dA} = \oint E_n\, dA \cos\theta$$

We will consider flux as **positive** if the electric field **E** goes from the inside to the outside of the surface and we will consider flux as **negative** if the electric field **E** goes from the outside to the inside of the surface. This is important for we will soon be interested in the **net** flux passing through a surface.



Gauss's Law : Total electric flux though a closed surface is $1/\varepsilon_0$ times the charge enclosed in the surface.

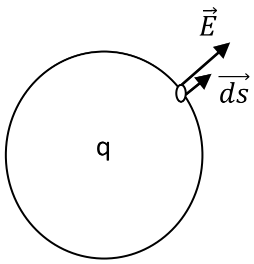
$$\Phi_E = q / \varepsilon_0$$

But we know that Electrical flux through a closed surface is $\oint \vec{E}.\vec{ds}$

$\therefore \oint \vec{E}.\vec{ds} = q / \varepsilon_0$

This is Gauss's theorem.

PROOF : Let's consider an hypothetical spherical surface having charge q placed at its centre. At every point of sphere the electrical field is radial, hence making angle 0 degree with area vector.



At the small area flux dϕ = $\oint \vec{E}.\vec{ds}$

= $\oint E.ds.Cos0°$

=$\oint \frac{q}{4\pi\epsilon_0 r^2}$ ds (E=$\frac{q}{4\pi\epsilon_0 r^2}$, Cos0=1)

= $\frac{q}{4\pi\epsilon_0 r^2}\oint ds$

For a sphere $\oint ds$ is 4πr^2.

∴ Φ=$\frac{q}{4\pi\epsilon_0 r^2}$ X 4πr^2.

Or, $\Phi = q / \varepsilon_0$

This is Guass Theorem. (Hence proved)

Application of Gauss's Law

To calculate Electric Field due to different charge distributions.

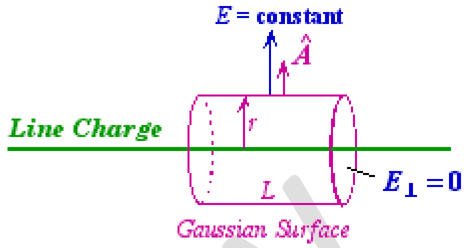
For this purpose we consider construction of a ***Guassian surface***.

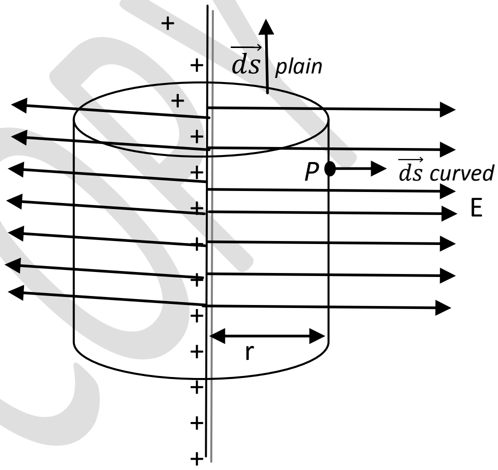
Guassian Surface : It is an imaginary surface in the electric field which is
1.closed from all sides
2. Surface is Symmetrical about the charges in it
3. Electric field $\vec{E}$ on the surface is symmetrical

Electric field due to line charge :

Electric charge is distributed on an infinite long straight conductor with linear charge density λ. We have to find Electric field on a point *P* at normal distance r.

Consider a Gaussian Surface in the shape of a cylinder having axis along conductor. It has radius r so that point P lies on the surface. Let its length be *l.*
The electric field is normal to conductor, hence it is symmetrical to the surfaces of these cylinder.





Now $\oint \vec{E}.\vec{ds}$ = $\int \vec{E}.\vec{ds}$ *for curved surface* + $\int \vec{E}.\vec{ds}$ for 2 plane surfaces.
= $\int E.ds\ cos0 + \int E.ds\ cos90$
=E $\int ds$ *for curved surface* (E is uniform)
= E2πr*l* ($\int ds$ = 2πrl, for cylindrical curved surface)

The charge enclosed within Guassian surface =λ*l*

According to Gauss theorem : $\oint \vec{E}.\vec{ds} = q / \varepsilon_0$

Putting values : E2πr*l* = λ*l* / ε_0

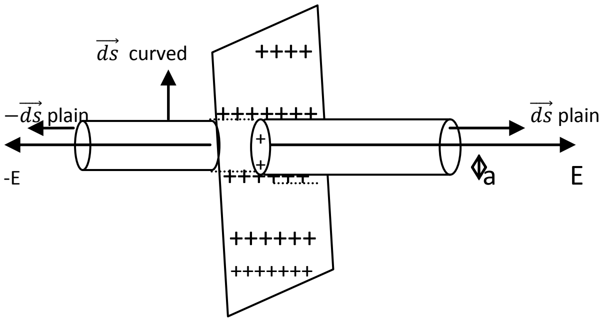
Or, $\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$

Electric field due to a plain surface : -

There is a very large plain surface having sueface density σ . There is a point P at normal distance r .

Let's consider a Gaussian surface, in shape of a cylinder which has axis normal to the sheet of charge and containing point P at its plain surface (radius a).

Electric field E is normal to the surface containing charge hence it is normal to the plain surface of cylinder and parallel to curved surface.



Now $\oint \vec{E}.\vec{ds} = \int \vec{E}.\vec{ds}$ *for curved surface* + $\int \vec{E}.\vec{ds}$ for 2 plane surfaces.

= $\int E.ds\ cos90 + \int E.ds\ cos0 + \int -E.(-ds\ cos0)$

= *for plain surfaces* 2E $\int ds$ (E is uniform)

= $2E\pi a^2$

The charge enclosed inside Gaussian surface q = σ.A

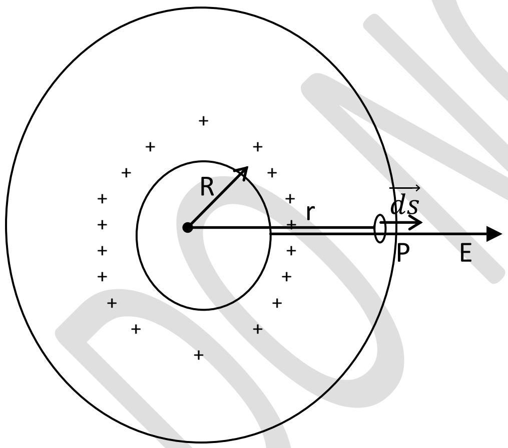
Or, $q = \sigma\pi a^2$

Applying Gauss's Law : $\oint \vec{E}.\vec{ds} = q / \varepsilon_0$

Putting values $2E\pi a^2 = \dfrac{\sigma\pi a^2}{\varepsilon_0}$

Or $\boxed{E = \dfrac{\sigma}{2\varepsilon_0}}$

Electric Field due to charge distributed over a spherical shell :-



The spherical shell or spherical conductor has total charge q, surface charge density σ , radius R. We have to find Electric Field E at a point P at distance 'r '.

Case 1. If P is outside shell.

Let's assume a Gaussian surface, which is a concentric sphere of radius r and P lies on its surface.

Electric field is normal to surface carrying charge. Hence it is radially outward. Therefore for a small area on the Gaussian surface ds E is normal to surface i.e. angle between $\vec{ds}$ and $\vec{E}$ is 0.

Now $\oint \vec{E}.\vec{ds} = \int \vec{E}.\vec{ds}$ for complete area of Gaussian surface

= $\int E.ds.Cos0$ = E$\int ds$ (E is uniform)

= E X $4\pi r^2$. (for spherical shell $\int ds = 4\pi r^2$)

Charge within Gaussian surface = q

Applying Gauss's Law : $\oint \vec{E}.\vec{ds} = q / \varepsilon_0$

Putting values E X $4\pi r^2 = q / \varepsilon_0$

Or $\boxed{E = \dfrac{q}{4\pi\epsilon_0 r^2}}$

This expression is same as electric field due to a point charge q placed at distance r from P. i.e. In this case <u>if complete charge q is placed at the centre of shell the electric field is same.</u>

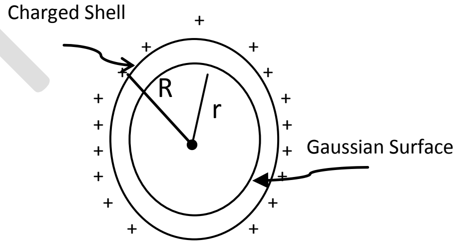
Case 2. If P is on the surface.

In above formula when r decrease to R the electric field increase.

On the surface *(replace r with R)* $\boxed{E = \dfrac{q}{4\pi\epsilon_0 R^2}}$

Hence this is electric field on the surface of a shell and its value is maximum compared to any other point.

Case 3. If P is within the surface. Or 'r'< R



Let's consider a Gaussian surface, a concentric spherical shell of radius r passing through P.

Then charge contained inside Gaussian surface is Zero.

According to Gauss's Theorem $\oint \vec{E}.\vec{ds} = q / \varepsilon_0$

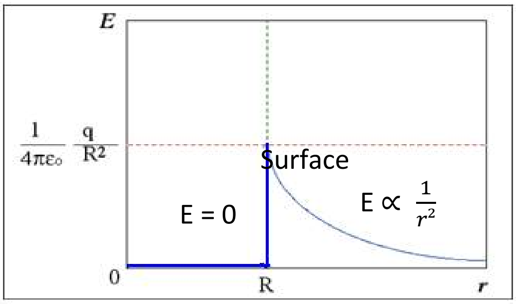
If q is zero then $\oint \vec{E}.\vec{ds} = 0$.

As *ds* is not zero then $\boxed{E = 0}$

It is very important conclusion reached by Gauss's Law that <u>Electric field inside a charged shell is zero</u>.

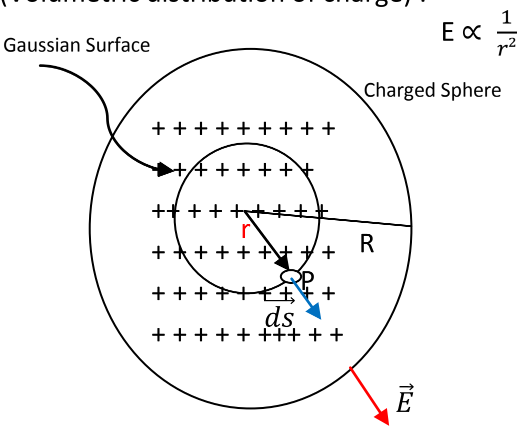
The electric field inside conductor is Zero. This phenomenon is called ***electrostatic shielding***.

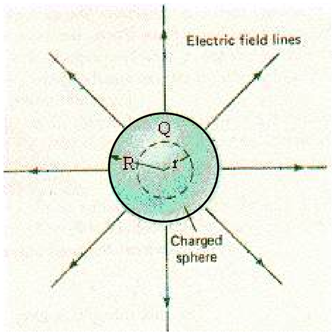
Variation of E with r (distance from centre)



Electric Field due to (filled-up) sphere of charge

(Volumetric distribution of charge) :





Case I. When P is Out side sphere. Same as in the case of charged shell E = $\frac{q}{4\pi\epsilon_0 r^2}$

Case 2. When point P is on the surface of shell: Same as in case of shell . E = $\frac{q}{4\pi\epsilon_0 R^2}$

Case 3 If point P is inside the charged sphere.

Consider Gaussian surface, a concentric spherical shell of radius r, such that point P lies on the surface. Electric field is normal to the surface.

Now $\oint \vec{E}.\vec{ds} = \int \vec{E}.\vec{ds}$ for complete area of Gaussian surface

$= \int E.ds.Cos0 = E\int ds$ (E is uniform)

= E X $4\pi r^2$. (for spherical shell $\int ds = 4\pi r^2$)

Charge within Gaussian surface = charge density x volume.

$= \rho\frac{4}{3}\pi r^3$ (where ρ is the charge per unit volume.)

Applying Gauss's Law $\oint \vec{E}.\vec{ds} = q / \varepsilon_0$

Putting values E X $4\pi r^2 = \rho\frac{4}{3}\pi r / \varepsilon_0$

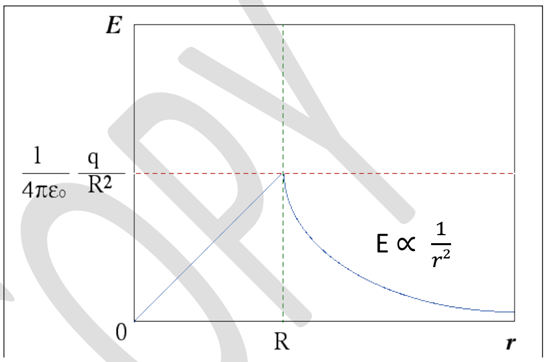
∴ $\boxed{E = \frac{\rho r}{3\varepsilon_0}}$

It shows that inside a sphere of charge, the electric field is directly proportional to distance from centre.

At centre r=0 ∴ E = 0

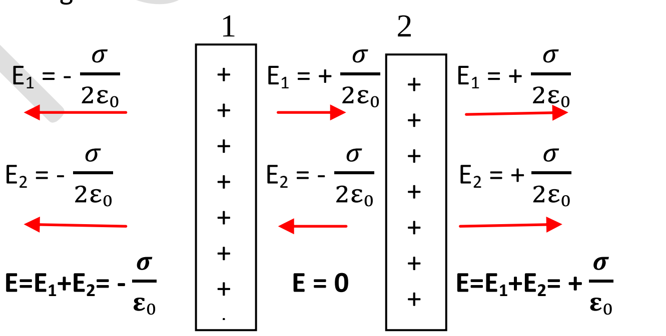
On the surface $E = \frac{\rho R}{3\varepsilon_0} = \frac{q}{4\pi\epsilon_0 R^2}$ $(\rho = q / \frac{4}{3}\pi r^3)$

Variation of E with r (distance from centre)

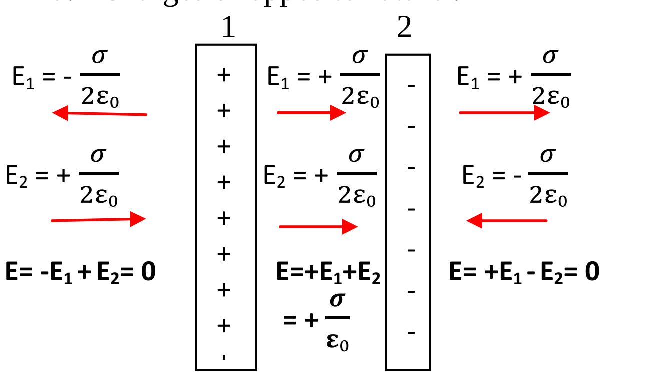


Electric field due to two charged parallel surface

Charges of similar nature



a. Charges of opposite nature :-



Equipotential Surface :

Energyof a charged particle in terms of potential:-

Work required to bring a charge q at a point of potential V is W = qV. This work done on the charged particle converts to its potential energy.

Potential energy of charge q at potential V is **U = qV**

Electron-Volt : By relation Work/energy = qV, *the smallest unit of work/energy is Electron Volt.*

One electron volt is the work done by/on one electron for moving between two points having potential difference of one Volt.

1 eV = 1.6 x 10^{-19} Joules

Potential Energy of system of charges

(i) System of Two charges :

A B

q_1 -------- r ----------------q_2

Potential due to q1 at B is potential at distance r :

V = $\frac{q_1}{4\pi\varepsilon_0 r}$ ∴ Potential Energy of system U = $\frac{q_1 q_2}{4\pi\varepsilon_0 r}$

(ii) System of three charges

We make different pairs and calculate energy as under

U = $\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}}$

(iii) System of Four charges

Four charges make six pairs : Potential Energy U=

$\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_1 q_4}{4\pi\varepsilon_0 r_{14}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}} + \frac{q_2 q_4}{4\pi\varepsilon^0 r_{24}} + \frac{q_3 q_4}{4\pi\varepsilon_0 r_{34}}$

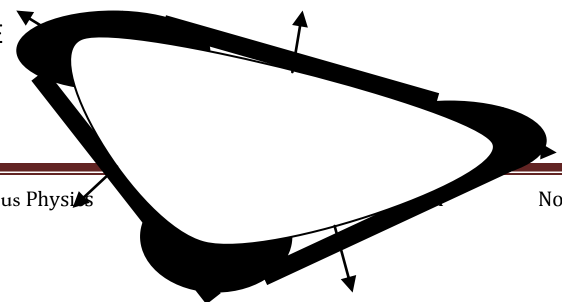
The energy is contained in the system and not by any one member. But it can be used by one or more members.

Distribution of charge on irregular shaped conductors :

Potential at each point is equal.

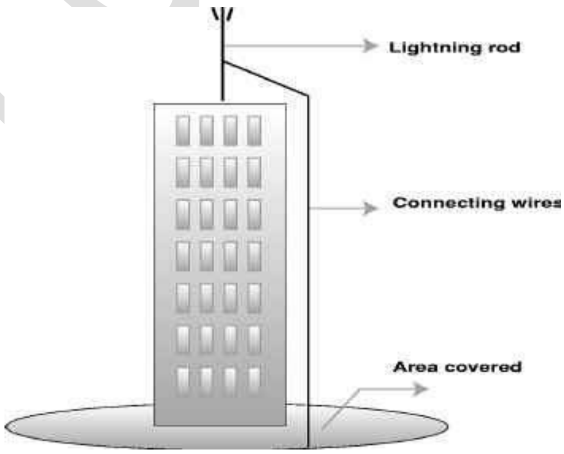
Electric field is always normal to surface.

Charge is distributed **unevenly**. Charge per unit area is more at the surface which has smaller radius. Therefore charge density is always more on the corners.



Corona discharge : when an uncharged body is brought near a charged body having sharp corners there is large number of charges at the corners. Due to induction, they induce large number of opposite charges. This creates a very strong Electric field between them. Finally the dielectric strength breaks-down and there is fast flow of charges. This Spray of charges by spiked object is called Corona discharge.

The lightening arrester work on the principles of Corona discharge where the charge pass through conductor of arrester, and the buildings are saved



Van-de-Graff generator

Introduction : It's a device used to create very high potential which is used for experiments of nuclear physics in which a charged particle with very high energy is required to hit the nucleus as target.

Principles : The following principles are involved in the device.

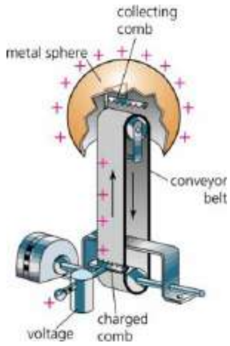
1.Charge on a conductor always move to and stay on the outer surface.

2.Pointed Corners conduct charges very effectively. (corona discharge)

3.If charge q is given to a body, its potential increases by relation V=$\frac{q}{4\pi\varepsilon_0 r}$

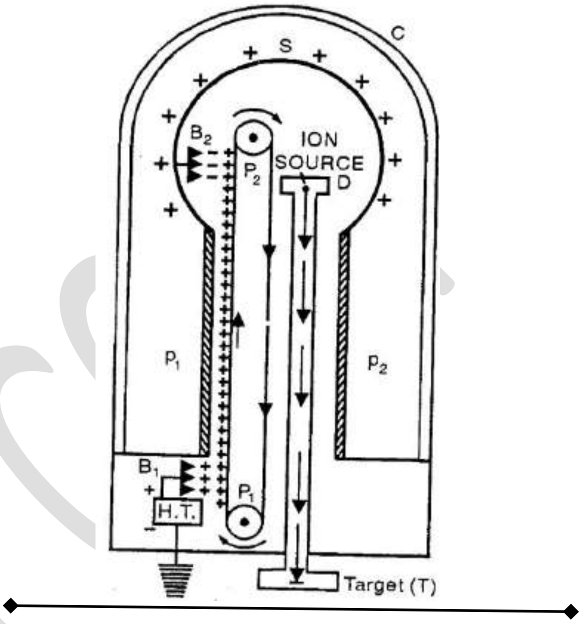
4. If a body of small potential v' is placed inside a shell having potential V, then the body acquires potential V+v'

Description : There is a large spherical conducting shell of diameter of few meters placed on a non-conducting concrete structure few meters above the ground.



A long belt of insulating material like silk rubber or rayon moves around two pulleys, driven by a motor.

Two combs with pointed heads near belt are fitted. Lower one is spray comb and the upper Collecting Comb. The spray comb is connected with a high tension source.



There is a discharge tube. One end having source of ion to be accelerated is inside the shell. Target is placed at the other end connected to earth.

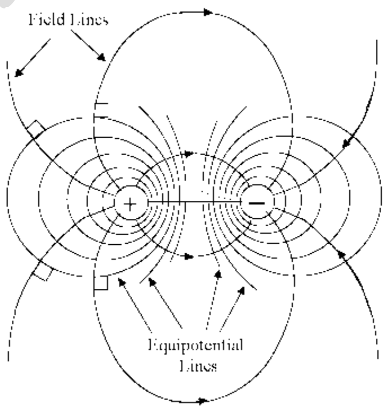
The whole system is enclosed in a steel chamber filled with nitrogen or methane at high pressure.

Working : The spray comb is given a positive potential ($\approx 10^4$Volt) w.r.t. earth by the source of high Tension. Due to sharp points there is spray of charge on belt. The belt moves up with power of motor. When the charges reach near upper comb, due to induction and corona discharge the charge on belt is transferred to comb. From comb it moves to inner layer of shell. Since charge always stay at the outer surface, it moves to outer surface and the inner surface again become without any charge, ready to receive fresh charge again. As shell receive charge it Potential increase according to relation V=$\frac{q}{4\pi\varepsilon_0 r}$. This potential is distributed all over and inside the shell.

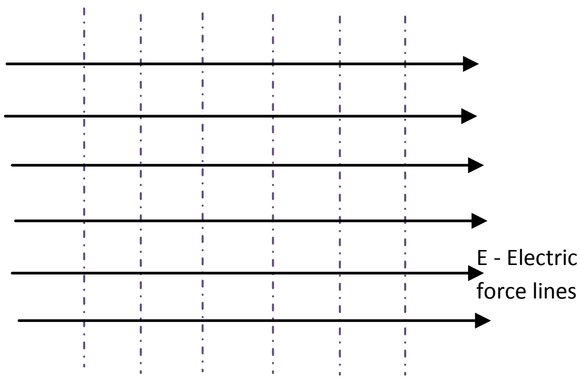
The new charged particles which are coming having small potential v' from lower comb, acquire potential V+v' due to their position inside the shell. There new potential is slightly higher than shell, therefore charges move from belt to comb to shell. This increases V further. This process keeps on repeating and V increase to a very high value, that is break-down voltage of compressed nitrogen $\approx 10^7$volt.

The ion inside discharged plate also acquires this potential due to its location inside the shell. Its energy increases by relation U = qV. The target is connected to earth at zero potential. Hence this ion gets accelerated and hits the target with very high energy.

<u>Relation between Equipotential surfaces and E-Lines</u>



Equi potential lines



CAPACITOR

It is a device to store charge and in turn store the electrical energy.

Any conductor can store charge to some extent. But we cannot give infinite charge to a conductor. When charge is given to a conductor its potential increases. But charge cannot escape the conductor because air, or medium around conductor is di-electric.

When due to increasing charge the potential increase to such extent that air touching the conductor starts getting ionized and hence charge gets leaked. No more charge can be stored and no more potential increase. This is limit of charging a conductor.

The electric field which can ionize air is 3 x 10^9 vm^{-1}.

CAPACITANCE OF A CONDUCTOR

Term capacitance of a conductor is the ratio of charge to it by rise in its Potential

$$C = \frac{q}{V}$$

In this relation if V=1 then C= q. Therefore ,

<u>*Capacitance of a conductor is equal to the charge which can change its potential by one volt.*</u>

Unit of capacitance : Unit of capacitance is farad, (symbol F).

<u>*One farad is capacitance of such a conductor whose potential increase by one volt when charge of one coulomb is given to it.*</u>

One coulomb is a very large unit. The practical smaller units are
i. Micro farad (μF) = 10^{-6}F.(used in electrical circuits)
Ii Pieco farad (pF) = 10^{-12} used in electronics circuits

Expression for capacitance of a spherical conductor :
If charge q is given to a spherical conductor of radius r, its potential rise by $V = \frac{q}{4\pi\epsilon_0 r}$
Therefore capacitance $C = \frac{q}{V} = q/\frac{q}{4\pi\epsilon_0 r} = 4\pi\epsilon_0 r$

Or for a sphere $\boxed{C = 4\pi\epsilon_0 r}$

The capacitor depends only on the radius or size of the conductor.

The capacitance of earth (radius 6400 km) is calculated to be 711 x 10^{-6} coulomb.

PARALLEL PLATE CAPACITOR : -

Since single conductor capacitor do not have large capacitance , parallel plate capacitors are constructed.

Principle : Principle of a parallel plate capacitor is that an uncharged plate brought bear a charged plate decrease the potential of charged plate and hence its capacitance ($C = \frac{q}{V}$) increase. Now it can take more charge. Now if uncharged conductor is earthed, the potential of charged plate further decreases and capacitance further increases. This arrangement of two parallel plates is called parallel plate capacitor.

Expression for capacitance :

Charge q is given to a plate

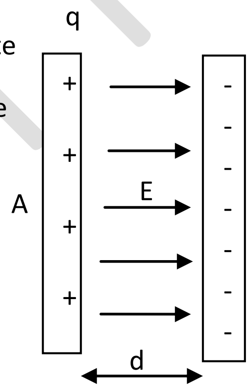
Of area 'A'. Another plate

is kept at a distance 'd'.

After induction an

Electric field E is set-up

Between the plates. Here



q = σA and $E = \frac{\sigma}{\varepsilon_0}$

The Potential difference between plates is given by

$V = Ed = \frac{\sigma}{\varepsilon_0} d$

Now $C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} = \frac{\varepsilon_0 A}{d}$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

If a dielectric of dielectric constant K is inserted between the plates, then capacitance increase by factor K and become

$$\boxed{C = \frac{\varepsilon_0 K A}{d}}$$

Note : The capacitance depends only on its configuration i.e. plate area and distance, and on the medium between them.
The other examples of parallel plate capacitors is

Cylindrical capacitor $C = \frac{4\pi\varepsilon_0 KL}{\log\ r^2/r_1}$

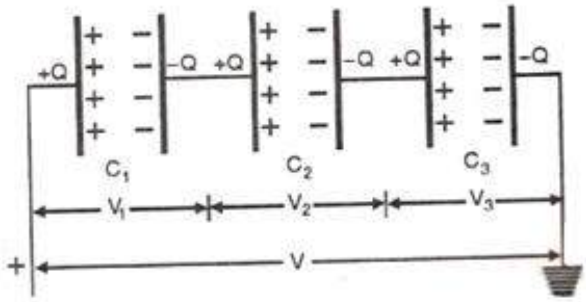
and Spherical capacitor. $C = \frac{4\pi\varepsilon_0 K r_2 r_1}{\log\ r_2 - r_1}$

Combination of capacitors

Capacitors can be combined in two ways. 1. Series and 2. Parallel.

Series Combination :

If capacitors are connected in such a way that we can proceed from one point to other by only one path passing through all capacitors then all these capacitors are said to be in series.



Here three capacitors are connected in series and are connected across a battery of P.D. 'V'.

The charge q given by battery deposits at first plate of first capacitor. Due to induction it attract –q on the opposite plate. The pairing +ve q charges are repelled to first plate of Second capacitor which in turn induce -q on the opposite plate. Same action is repeated to all the capacitors and in this way all capacitors get q charge. As a result ; the charge given by battery q, every capacitor gets charge q.

The Potential Difference V of battery is sum of potentials across all capacitors. Therefore

$V = v_1 + v_2 + v_3$

$v_1 = \frac{q_1}{c_1}$, $v_2 = \frac{q_2}{c_2}$, $v_3 = \frac{q_3}{c_3}$

Equivalent Capacitance : The equivalent capacitance across the combination can be calculated as $C_e = q/V$

Or $1/C_e = V/q$

$= (v_1 + v_2 + v_3) / q$

$= v_1/q + v_2/q + v_3/q$

Or $\mathbf{1/C_e = 1/C_1 + 1/C_2 + 1/C_3}$

The equivalent capacitance in series decrease and become smaller then smallest member.

In series q is same. Therefore by q=cv, we have

$c_1V_1 = c_2V_2 = c_3V_3$

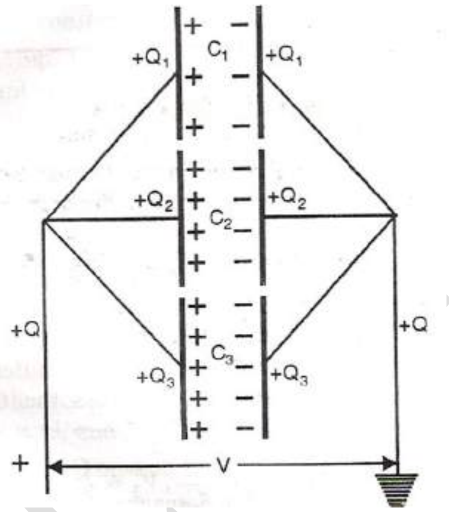
or $v \propto \frac{1}{c}$ i.e. larger c has smaller v, and smaller c has larger v across it.

For 2 capacitor system $C = \frac{c_1c_2}{c_1+c_2}$, and $v_1 = \frac{c_2}{c_1+c_2}.v$

If n capacitor of capacitance c are joint in series then equivalent capacitance $C_e = \frac{c}{n}$

Parallel combination :

If capacitors are connected in such a way that there are many paths to go from one point to other. All these paths are parallel and capacitance of each path is said to be connected in parallel.



Here three capacitors are connected in parallel and are connected across a battery of P.D. 'V'.

The potential difference across each capacitor is equal and it is same as P.D. across Battery.

The charge given by source is divided and each capacitor gets some charge. The total charge

$q = q_1 + q_2 + q_3$

Each capacitor has charge

$\underline{q_1=c_1V_1,\ q_2=c_2V_2,\ q_3=c_3V_3}$

Equivalent Capacitance : We know that

$q = q_1 + q_2 + q_3$

divide by v $\frac{q}{v} = \frac{q_1}{v} + \frac{q_2}{v} + \frac{q_3}{v}$

or, $\mathbf{C = c_1+c_2+c_3}$

The equivalent capacitance in parallel increases, and it is more than largest in parallel.

In parallel combination V is same therefore

$(v =) \quad \frac{q_1}{c_1} = \frac{q_2}{c_2} = \frac{q_3}{c_3}$

In parallel combination $q \propto c$. Larger capacitance larger is charge.

Charge distribution : $q_1=c_1v$, $q_2=c_2v$, $q_3=c_3v$.

In 2 capacitor system charge on one capacitor

$q_1 = \frac{c_1}{c_1+c_2+..}.q$

n capacitors in parallel give $C = nc$

__Energy stored in a capacitor__: When charge is added to a capacitor then charge already present on the plate repel any new incoming charge. Hence a new charge has to be sent by applying force and doing work on it. All this work done on charges become energy stored in the capacitor.

At any instant work done dw = V.dq, or $dw = \frac{q}{c}.dq$

Therefore work done in charging

the capacitor from charge 0 to q $\qquad W = \int_0^q \frac{q}{c}.dq$

$$= \frac{1}{c}\int_0^q q.dq = \frac{1}{c}\frac{q^2}{2} = \frac{q^2}{2c}$$

This work done convert into electrical Potential

Energy stored in the capacitor $\boxed{U = \frac{1}{2}\frac{q^2}{c} = \frac{1}{2}qv = \frac{1}{2}cv^2}$

This energy is stored in the form of Electric field between the plates.

Energy per unit volume $u = \frac{1}{2}cv^2/V = \frac{1}{2}\frac{\varepsilon_0 AE^2d^2}{dAd}$

Or, energy density $\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$

__Connecting two charged capacitors__ :- When two conductors are connected the charges flow from higher potential plate to lower potential plate till they reach a common potential.

__Common Potential__ : A capacitor of capacitance c_1 and potential v_1 is connected to another capacitor of capacitance c_2 and potential v_2. The charge flow from higher potential to lower potential and it reach an in between value V such that

$V = \frac{total\ charge}{Total\ capacitance}$ or $V = \frac{c_1v_1+c_2v_2}{c_1+c_2}$

__Loss of Energy on connecting two conductors__ :

A capacitor of capacitance c_1 and potential v_1 is connected to another capacitor of capacitance c_2 and potential v_2. The charge flow from higher potential to lower potential and in this process it looses some energy as charge has to do some work while passing through connecting wire. The energy is lost in form of heat of connecting wire.

Expression for energy lost : In the above two capacitors the energy contained in the two before connection, $E_1 = \frac{1}{2}c_1v_1^2 + \frac{1}{2}c_2v_2^2$ . . . . (i)

Common Potential after connection, $V = \frac{c_1v_1+c_2v_2}{c_1+c_2}$

Combined capacitance $c_1 + c_2$

Energy in combination : $\frac{1}{2}(c_1+c_2)\left(\frac{c_1v_1+c_2v_2}{c_1+c_2}\right)^2$

Hence Loss in energy : $E_1 - E_2$

$= \{\frac{1}{2}c_1v_1^2 + \frac{1}{2}c_2v_2^2\} - \{\frac{1}{2}(c_1+c_2)\left(\frac{c_1v_1+c_2v_2}{c_1+c_2}\right)^2\}$

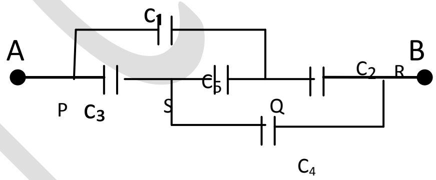
$= \frac{1}{2}\left(\frac{c_1c_2}{c_1+c_2}\right)(v_1 - v_2)^2$

It is a positive number which confirm that there is loss of energy in transfer of charges. Hence

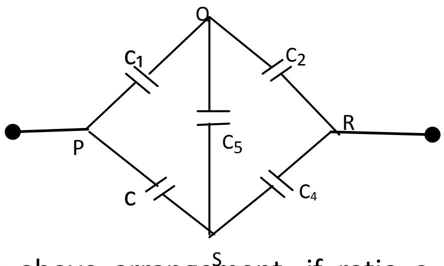
$$\boxed{\text{loss of energy} = \frac{1}{2}\left(\frac{c_1c_2}{c_1+c_2}\right)(v_1 - v_2)^2}$$

__Wheatstone bridge__ in combination of capacitors :

Five capacitors joined in following manner is called wheatstone bridge connection.



Or, it is redrawn as under :



In the above arrangement, if ratio $c_1/c_2 = c_3/c_4$ then the bridge is said to be *balanced*. In such case the potential at point Q and S are equal.

The potential across c_5 is zero hence it does not carry any charge. In this way it is not participating in storage of charges. Then it can be omitted for further calculations. Calculations are done for c_1 c_2, c_3 and c_4 only.

__Dielectrics:__ are non conducting materials. They do not have free charged particles like conductors have. They are two types.

i. Polar : The centre of +ve and –ve charges do not coincide. Example HCl, H_2O, They have their own dipole moment.

ii. Non-Polar : The centers of +ve and –ve charges coincide. Example CO_2 , C_6H_6 . They do not have their own dipole moment.

In both cases, when a dielectric slab is exposed to an electric field, the two charges experience force in opposite directions. The molecules get elongated and develops i. <u>surface charge density</u> $\boldsymbol{\sigma_p}$ and not the volumetric charge density. This leads to development of an induced electric field $\mathbf{E_p}$, which is in opposition direction of external electric field E_o . Then net electric field E is given by $E = E_o - \mathbf{E_p}$. This indicates that net electric field is decreased when dielectric is introduced.

The ratio $\frac{Eo}{E}$ = K is called **<u>dielectric constant</u>** of the dielectric.

Clearly electric field inside a dielectric is $E = \frac{Eo}{K}$.

<u>Dielectric polarization :</u> when external electric field E_0 is applied , molecules get polarized and this induced dipole moment of an atom or molecule is proportionate to applied electric field. i.e. $p \propto E_o$

$$\text{or } p = \alpha\varepsilon_0 E_0$$

here α is a constant called atomic / molecular polarizability. It has dimensions of volume (L^3) it has the order of 10^{-29} to 10^{-30} m^3.

This **<u>polarization</u>** is a vector quantity and is related to resultant electric field E as under :

$$\vec{p} = \chi_e \varepsilon E$$

Where χ_e is a constant called electric susceptibility of the dielectric.

The induced charge $\boldsymbol{\sigma_p}$ is due to this polarization, hence

$$\boldsymbol{\sigma_p} = \vec{p}.\hat{n}$$

When this dielectric is introduced between the two plates having charge density $\boldsymbol{\sigma}$ then resultant electric field can be related as

$$E.\hat{n} = E - E_p = \frac{\sigma - \sigma p}{\varepsilon_0} = \frac{\sigma - \vec{p}.\hat{n}}{\varepsilon_0}$$

$$\text{or } (\varepsilon^0 E + p).\hat{n} = \sigma$$

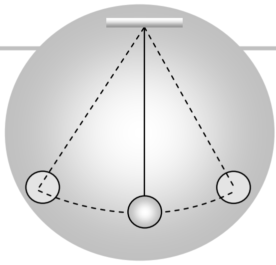
$$\text{or } \vec{D}.\hat{n} = \sigma$$

The quantity $\vec{D}$ is called **electric displacement** in dielectric.

We can prove that $K = 1 + \chi_e$

SL. NO.	quantities	FORMULA (RELATIONS)	Electrostatics
1	Quantisation of Elect. Charges (Q) on a body	Q = n.e	n is Integral Number, e is charge on electron 1.6×10^{-19} C
2	Electrostatic force constant	$1/(4\pi\epsilon_0)$	value : 9×10^9 Nm^2C^{-2}
3	Permittivity	ϵ_0	8.85×10^{-12} $C^2N^{-1}m^{-2}$
4	Coulumb's Law	$F = q_1q_2/4\pi\varepsilon_0r^2$	q_1 and q_2 are two charges placed at distance r.
5	Forces on two charges	$F_{12} = - F_{21}$	Direction of F is along r.
6	Dielectric Constant	$K = \varepsilon/\varepsilon_0 = \varepsilon_r$	ε is absolute permittivity of medium, ε_0 is permittivity of free space, ε_r is relative permittivity.
7	Electric Field at a point	E = F/q	F is force experienced by the test charge q at a point. E is called **field intensity at that point**
	Force with respect to field	F = q.E	
8	Electric field due to source charge Q at distance r	$E = Q/(4\pi\varepsilon_0r^2)$	Direction of E is along r.
9	Electric Field due to dipole on a point on axial line	$E = 2P/(4\pi\varepsilon_0r^3)$	P is dipole moment, r is distance from centre of dipole on axial line.
10	Electric Field due to dipole on a point on equitorial line	$E = P/(4\pi\varepsilon_0r^3)$	P is dipole moment, r is distance from centre of dipole on equitorial line.
11	Electric Field due to dipole at any general point, at distance r making angle θ with $\vec{P}$	$E = P\sqrt{(3\cos^2\theta+1)}/4\pi\varepsilon_0r^3$	r is distance of point from midpoint of dipole, θ is angle between direction of r and dipole moment P
	E makes angle α with r then	$\tan\alpha = \frac{1}{2}\tan\theta$	α is angle between resultant field and direction of r, θ is angle between r and P
12	E at any point on the axis of a uniformly charged ring at distance r	$qr/4\pi\varepsilon_0(r^2+a^2)^{3/2}$	
13	Torque on a dipole kept in Electric Field	$\tau = PE\sin\theta$ or $\boldsymbol{\tau} = \boldsymbol{P} \times \boldsymbol{E}$	P is dipole moment, E is electric field, Direction of Torque is normal to plain containing P and E
14	Work done for rotating dipole by angle θ	$W = PE(1- \cos\theta)$	P is dipole moment. E is electric field
15	Potential Energy of dipole in equilibrium condition when P is along E.	U = - PE	P is dipole moment. E is electric field
16	Potential energy of dipole at 90 degree to E	Zero	
17	Potential energy of dipole at 180^0	U = + PE	P is dipole moment. E is electric field
18	Electric Flux ϕ_E	$\phi_E = E.S = \int E.ds$	
19	gauss theorem	$\phi_E = \oint [\![E.ds]\!] = q/\varepsilon_0$	Flux linked to a closed surface is q/ε_0 times the charge enclosed in it.
20	Field due to infinite long straight charged conductor	$\lambda/2\pi\epsilon_0 r$	λ is linear charge density in the conductor, r is the perpendicular distance.
21	Electric field due to infinite plane sheet of charge	$\sigma/2\epsilon_0$	σ is areal charge density. Independent of distance
22	Within two parallal sheets of opposite charges	σ/ϵ_0	Outside, field is zero
23	Within two parallal sheets of similar charges	zero	Outside, field is σ/ϵ_0
24	Electric field due to spherical shell, out side shell	$E = q/(4\pi\varepsilon_0r^2)$	q is charge on shell, r distance from centre.

25	Electric field on the surface of spherical shell.	$E = q/(4\pi\varepsilon_0 R^2)$	R is radius of shell
26	Electric field inside spherical shell.	Zero	
27	Electric field inside the sphere of charge distributed uniformly all over the volume .	$E = r\rho/3\epsilon$	r is radius of sphere, ρ is volumetric charge density, is permittivity of medium
28	Potential due to charge Q at distance r	$V = Q/(4\pi\varepsilon_0 r)$	Potential is characteristic of that location
29	Potential Energy with charge q kept at a point with potential V	$U = qV = Qq/(4\pi\varepsilon_0 r)$	Potential Energy is that of the system containing Q and q.
30	Work done for in moving a charge q through a potential difference of V	$W = q(V_2-V_1)$	$V = (v_2 - v_1)$
	Energy of system of two charges	$U = q_1q_2/(4\pi\varepsilon_0 r)$	
31	Relation of E and V	$\mathbf{E = - dv/dr}$	dv is potential difference between two points at distance r where r and E are in the same direction.
32	Relation of E and V and θ	$\mathbf{E \cos\theta = - dv/dr}$	where θ is angle between dr and E
33	Potential at infinity / in earth	Zero	
34	Electric Potential due to dipole on a point on axial line	$V = P/(4\pi\varepsilon_0 r^2)$	P is dipole momentum, r is distance from centre of dipole
35	Electric Potential due to dipole on a point on equitorial line	Zero	
36	Electric Potential due to dipole at any general point,	$V = P\cos\theta / 4\pi\varepsilon_0 (r^2 - a^2 \cos^2\theta)$	P is dipole momentum, r is distance from centre of dipole, a is half length of dipole, is angle between r and P
37	Work done in moving a charge between two points of an equipotential surface	Zero	
38	Capacitance of a spherical conductor	$4\pi\varepsilon_0 R$	R is radius of the sphere
39	Capacitance of a parallal plate capacitor	$\varepsilon_0 kA/d$	A is area of each plate, d is distance between them, k is dilectric constant of the medium between plates.
40	Dielectric Constant	$k = C / C_0$	Cis capacitance with medium within plates, and C_0 is capacitance in free space.
41	Capacitance of a spherical capacitor.	$C = 4\pi\varepsilon_0 r_a r_b/(r_a-r_b)$	r_a and r_b are radius of internal and external spherical shells
42	Equivalent capacitance for Capacitors in parallal	$C = c_1 + c_2 + c_3 \ldots.$	Cis equivalent capacitance, c_1, c_2 are capacitnce of the capacitors joint together.
43	Equivalent capacitance for Capacitors in series	$1/C = 1/c_1 + 1/c_2 + 1/c_3 \ldots$	
44	Charge, capacitance, Potential Difference	$C = q/V$	q ischarge on the plate of capacitor and V is Potential Difference between the plates.
45	Energy stored in capacitor	$½cv^2$, $½qv$, $½q^2/c$	q is charge, c is capacitance, v is Pot. Difference
46	Common Potential	$V=C_1V_1+C_2V_2)/C_1+C_2$	
47	Energy loss in connecting	$\frac{1}{2}\frac{C_1C_2}{C^1+C_2}(V1-V2)^2$	c_1 at v_1 is connected to c_2 at v_2
48	C with dielectric slab inserted	$\varepsilon_0 kA/d-t(1-1/k)$	t is thickness of dielectric slab of constant k,
49	C with metal plate inserted	$\varepsilon_0 kA/(d-t)$	t is thickness of metal plate inserted,
50	Force of attraction between plates	$½q^2/\varepsilon_0 A$, $½\varepsilon_0 E^2 A$	q is charge on plate, A is area, E Elect. Field.



Simple Harmonic Motion

15.1 Periodic Motion

A motion, which repeat itself over and over again after a regular interval of time is called a periodic motion and the fixed interval of time after which the motion is repeated is called period of the motion.

Examples :

(i) Revolution of earth around the sun (period one year)

(ii) Rotation of earth about its polar axis (period one day)

(iii) Motion of hour's hand of a clock (period 12-hour)

(iv) Motion of minute's hand of a clock (period 1-hour)

(v) Motion of second's hand of a clock (period 1-minute)

(vi) Motion of moon around the earth (period 27.3 days)

15.2 Oscillatory or Vibratory Motion

Oscillatory or vibratory motion is that motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time. In such a motion, the body is confined with in well-defined limits on either side of mean position.

Oscillatory motion is also called as harmonic motion.

Example :

(i) The motion of the pendulum of a wall clock.

(ii) The motion of a load attached to a spring, when it is pulled and then released.

(iii) The motion of liquid contained in U- tube when it is compressed once in one limb and left to itself.

(iv) A loaded piece of wood floating over the surface of a liquid when pressed down and then released executes oscillatory motion.

15.3 Harmonic and Non-harmonic Oscillation

Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (*i.e.* sine or cosine function). *Example* : $y = a\sin\omega t$ or $y = a\cos\omega t$

Non-harmonic oscillation is that oscillation which can not be expressed in terms of single harmonic function. It is a combination of two or more than two harmonic oscillations. *Example* : $y = a\sin\omega t + b\sin 2\omega t$

15.4 Some Important Definitions

(1) **Time period :** It is the least interval of time after which the periodic motion of a body repeats itself.

S.I. units of time period is second.

(2) **Frequency :** It is defined as the number of periodic motions executed by body per second. S.I unit of frequency is hertz (*Hz*).

(3) **Angular Frequency :** Angular frequency of a body executing periodic motion is equal to product of frequency of the body with factor 2π. Angular frequency $\omega = 2\pi n$

S.I. units of ω is *Hz* [S.I.] ω also represents angular velocity. In that case unit will be *rad*/*sec*.

(4) **Displacement :** In general, the name displacement is given to a physical quantity which undergoes a change with time in a periodic motion.

Examples :

(i) In an oscillation of a loaded spring, displacement variable is its deviation from the mean position.

(ii) During the propagation of sound wave in air, the displacement variable is the local change in pressure

(iii) During the propagation of electromagnetic waves, the displacement variables are electric and magnetic fields, which vary periodically.

(5) **Phase :** phase of a vibrating particle at any instant is a physical quantity, which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

In oscillatory motion the phase of a vibrating particle is the argument of *sine* or *cosine* function involved to represent the generalised equation of motion of the vibrating particle.

$$y = a\sin\theta = a\sin(\omega t + \phi_0) \quad \text{here, } \theta = \omega t + \phi_0 = \text{phase of vibrating particle.}$$

(i) Initial phase or epoch : It is the phase of a vibrating particle at $t = 0$.

In $\theta = \omega t + \phi_0$, when $t = 0$; $\theta = \phi_0$ here, ϕ_0 is the angle of epoch.

(ii) Same phase : Two vibrating particle are said to be in same phase, if the phase difference between them is an even multiple of π or path difference is an even multiple of $(\lambda / 2)$ or time interval is an even multiple of $(T / 2)$ because 1 time period is equivalent to 2π *rad* or 1 wave length (λ)

(iii) Opposite phase : When the two vibrating particles cross their respective mean positions at the same time moving in opposite directions, then the phase difference between the two vibrating particles is 180°

Opposite phase means the phase difference between the particle is an odd multiple of π (say π, 3π, 5π, 7π.....) or the path difference is an odd multiple of λ (say $\frac{\lambda}{2}, \frac{3\lambda}{2}$,.......) or the time interval is an odd multiple of $(T / 2)$.

(iv) Phase difference : If two particles performs S.H.M and their equation are

$$y_1 = a\sin(\omega t + \phi_1) \text{ and } y_2 = a\sin(\omega t + \phi_2)$$

then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

15.5 Simple Harmonic Motion

Simple harmonic motion is a special type of periodic motion, in which a particle moves to and fro repeatedly about a mean position under a restoring force which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean position at that instant.

Restoring force $\propto$ Displacement of the particle from mean position.

$$F \propto -x$$

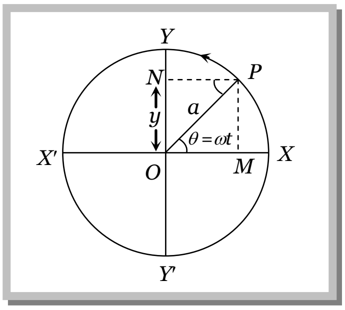
$$F = -kx$$

Where k is known as force constant. Its S.I. unit is Newton/meter and dimension is $[MT^{-2}]$.

15.6 Displacement in S.H.M.

The displacement of a particle executing S.H.M. at an instant is defined as the distance of particle from the mean position at that instant.

As we know that simple harmonic motion is defined as the projection of uniform circular motion on any diameter of circle of reference. If the projection is taken on y-axis.



then from the figure $y = a\sin\omega t$

$$y = a\sin\frac{2\pi}{T}t$$

$$y = a\sin 2\pi n t$$

$$y = a\sin(\omega t \pm \phi)$$

where a = Amplitude, ω = Angular frequency, t = Instantaneous time,

T = Time period, n = Frequency and ϕ = Initial phase of particle

If the projection of P is taken on X-axis then equations of S.H.M. can be given as

$$x = a\cos(\omega t \pm \phi)$$

$$x = a\cos\left(\frac{2\pi}{T}t \pm \phi\right)$$

$$x = a\cos(2\pi n t \pm \phi)$$

Important points

(i) $y = a\sin\omega t$ when the time is noted from the instant when the vibrating particle is at mean position.

(ii) $y = a\cos\omega t$ when the time is noted from the instant when the vibrating particle is at extreme position.

(iii) $y = a\sin(\omega t \pm \phi)$ when the vibrating particle is ϕ phase leading or lagging from the mean position.

(iv) Direction of displacement is always away from the equilibrium position, particle either is moving away from or is coming towards the equilibrium position.

(v) If t is given or phase (θ) is given, we can calculate the displacement of the particle.

If $t = \frac{T}{4}$ (or $\theta = \frac{\pi}{2}$) then from equation $y = a\sin\frac{2\pi}{T}t$, we get $y = a\sin\frac{2\pi}{T}\frac{T}{4} = a\sin\left(\frac{\pi}{2}\right) = a$

Similarly if $t = \frac{T}{2}$ (or $\theta = \pi$) then we get $y = 0$

Sample problems based on Displacement

***Problem* 1.** A simple harmonic oscillator has an amplitude A and time period T. The time required by it to travel from $x = A$ to $x = A/2$ is **[CBSE 1992; SCRA 1996]**

(a) $T/6$ (b) $T/4$ (c) $T/3$ (d) $T/2$

Solution : (a) Because the S.H.M. starts from extreme position so $y = a\cos\omega t$ form of S.H.M. should be used.

$$\frac{A}{2} = A\cos\frac{2\pi}{T}t \Rightarrow \cos\frac{\pi}{3} = \cos\frac{2\pi}{T}t \Rightarrow t = T/6$$

Problem 2. A mass m = 100 *gms* is attached at the end of a light spring which oscillates on a friction less horizontal table with an amplitude equal to 0.16 meter and the time period equal to 2 *sec*. Initially the mass is released from rest at t = 0 and displacement x = – 0.16 *meter*. The expression for the displacement of the mass at any time (t) is **[MP PMT 1995]**

(a) $x = 0.16\cos(\pi t)$ (b) $x = -0.16\cos(\pi t)$ (c) $x = 0.16\cos(\pi t + \pi)$ (d) $x = -0.16\cos(\pi t + \pi)$

Solution : (b) Standard equation for given condition

$$x = a\cos\frac{2\pi}{T}t \Rightarrow x = -0.16\cos(\pi t) \quad [\text{As } a = -0.16\text{ meter}, T = 2\text{ sec}]$$

Problem 3. The motion of a particle executing S.H.M. is given by $x = 0.01\sin 100\pi(t + .05)$. Where x is in meter and time t is in seconds. The time period is

(a) 0.01 *sec* (b) 0.02 *sec* (c) 0.1 *sec* (d) 0.2 *sec*

Solution : (b) By comparing the given equation with standard equation $y = a\sin(\omega t + \phi)$

$$\omega = 100\pi \text{ so } T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = 0.02\ sec$$

Problem 4. Two equations of two S.H.M. are $x = a\sin(\omega t - \alpha)$ and $y = b\cos(\omega t - \alpha)$. The phase difference between the two is **[MP PMT 1985]**

(a) 0° (b) α° (c) 90° (d) 180°

Solution : (c) $x = a\sin(\omega t - \alpha)$ and $y = b\cos(\omega t - \alpha) = b\sin(\omega t - \alpha + \pi/2)$

Now the phase difference = $(\omega t - \alpha + \frac{\pi}{2}) - (\omega t - \alpha) = \pi/2 = 90^o$

15.7 Velocity in S.H.M.

Velocity of the particle executing S.H.M. at any instant, is defined as the time rate of change of its displacement at that instant.

In case of S.H.M. when motion is considered from the equilibrium position

$$y = a\sin\omega t$$

so $$v = \frac{dy}{dt} = a\omega\cos\omega t$$

$\therefore$ $$v = a\omega\cos\omega t \quad \text{......(i)}$$

or $$v = a\omega\sqrt{1 - \sin^2\omega t} \quad [\text{As } \sin\omega t = y/a]$$

or $$v = \omega\sqrt{a^2 - y^2} \quad \text{.....(ii)}$$

Important points

(i) In S.H.M. velocity is maximum at equilibrium position.

From equation (i) $v_{max} = a\omega$ when $|\cos\omega t| = 1$ *i.e.* $\theta = \omega t = 0$

from equation (ii) $v_{max} = a\omega$ when $y = 0$

(ii) In S.H.M. velocity is minimum at extreme position.

From equation (i) $v_{min} = 0$ when $|\cos\omega t| = 0$ *i.e* $\theta = \omega t = \frac{\pi}{2}$

From equation (ii) $v_{min} = 0$ when $y = a$

(iii) Direction of velocity is either towards or away from mean position depending on the position of particle.

Sample problems based on Velocity

Problem 5. A body is executing simple harmonic motion with an angular frequency 2 *rad/sec*. The velocity of the body at 20 *mm* displacement. When the amplitude of motion is 60 *mm* is **[AFMC 1998]**

(a) 40 *mm/sec* (b) 60 *mm/sec* (c) 113 *mm/sec* (d) 120 *mm/sec*

Solution : (c) $v = \omega\sqrt{a^2 - y^2} = 2\sqrt{(60)^2 - (20)^2} = 113$ *mm/sec*

Problem 6. A body executing S.H.M. has equation $y = 0.30\sin(220\,t + 0.64)$ in *meter*. Then the frequency and maximum velocity of the body is

(a) $35Hz, 66m/s$ (b) $45Hz, 66m/s$ (c) $58Hz, 113m/s$ (d) $35Hz, 132m/s$

Solution : (a) By comparing with standard equation $y = a\sin(\omega t + \phi)$ we get $a = 0.30;\ \omega = 220$

$\therefore\ 2\pi n = 220 \Rightarrow n = 35Hz$ so $v_{\max} = a\omega = 0.3 \times 220 = 66m/s$

Problem 7. A particle starts S.H.M. from the mean position. Its amplitude is *A* and time period is *T*. At the time when its speed is half of the maximum speed. Its displacement *y* is

(a) $A/2$ (b) $A/\sqrt{2}$ (c) $A\sqrt{3}/2$ (d) $2A/\sqrt{3}$

Solution : (c) $v = \omega\sqrt{a^2 - y^2} \Rightarrow \frac{a\omega}{2} = \omega\sqrt{a^2 - y^2} \Rightarrow \frac{a^2}{4} = a^2 - y^2 \Rightarrow y = \frac{\sqrt{3}A}{2}$ [As $v = \frac{v_{\max}}{2} = \frac{a\omega}{2}$]

Problem 8. A particle perform simple harmonic motion. The equation of its motion is $x = 5\sin(4t - \frac{\pi}{6})$. Where *x* is its displacement. If the displacement of the particle is 3 units then its velocity is **[MP PMT 1994]**

(a) $2\pi/3$ (b) $5\pi/6$ (c) 20 (d) 16

Solution : (d) $v = \omega\sqrt{a^2 - y^2} = 4\sqrt{5^2 - 3^2} = 16$ [As $\omega = 4,\ a = 5,\ y = 3$]

Problem 9. A simple pendulum performs simple harmonic motion about *x* = 0 with an amplitude (*A*) and time period (*T*). The speed of the pendulum at $x = \frac{A}{2}$ will be **[MP PMT 1987]**

(a) $\frac{\pi A\sqrt{3}}{T}$ (b) $\frac{\pi A}{T}$ (c) $\frac{\pi A\sqrt{3}}{2T}$ (d) $\frac{3\pi^2 A}{T}$

Solution : (a) $v = \omega\sqrt{a^2 - y^2} \Rightarrow v = \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{4}} = \frac{\pi A\sqrt{3}}{T}$ [As *y* = A/2]

Problem 10. A particle is executing S.H.M. if its amplitude is 2 *m* and periodic time 2 *seconds*. Then the maximum velocity of the particle will be

(a) 6π (b) 4π (c) 2π (d) π

Solution : (c) $v_{\max} = a\omega = a\frac{2\pi}{T} = 2\frac{2\pi}{2} \Rightarrow v_{\max} = 2\pi$

Problem 11. A S.H.M. has amplitude '*a*' and time period *T*. The maximum velocity will be **[MP PMT 1985]**

(a) $\frac{4a}{T}$ (b) $\frac{2a}{T}$ (c) $2\pi\sqrt{\frac{a}{T}}$ (d) $\frac{2\pi a}{T}$

Solution : (d) $v_{\max} = a\omega = \frac{a2\pi}{T}$

***Problem* 12.** A particle executes S.H.M. with a period of 6 second and amplitude of 3 *cm* its maximum speed in *cm/sec* is **[AIIMS 1982]**

(a) $\pi / 2$ (b) π (c) 2π (d) 3π

Solution : (b) $v_{max} = a\omega = a\frac{2\pi}{T} = 3\frac{2\pi}{6} \Rightarrow v_{max} = \pi$

***Problem* 13.** A body of mass 5 *gm* is executing S.H.M. about a point with amplitude 10 *cm*. Its maximum velocity is 100 *cm/sec*. Its velocity will be 50 *cm/sec*, at a distance **[CPMT 1976]**

(a) 5 (b) $5\sqrt{2}$ (c) $5\sqrt{3}$ (d) $10\sqrt{2}$

Solution : (c) $v_{max} = a\omega = 100\ cm/\sec$ and $a = 10\ cm$ so $\omega = 10\ rad/\sec$.

$\therefore\ v = \omega\sqrt{a^2 - y^2} \Rightarrow 50 = 10\sqrt{10^2 - y^2} \Rightarrow y = 5\sqrt{3}$

15.8 Acceleration in S.H.M.

The acceleration of the particle executing S.H.M. at any instant, is defined as the rate of change of its velocity at that instant. So acceleration $A = \frac{dv}{dt} = \frac{d}{dt}(a\omega\cos\omega t)$

$$A = -\omega^2 a \sin\omega t \qquad \text{......(i)}$$

$$A = -\omega^2 y \qquad \text{......(ii)} \qquad [\text{As } y = a\sin\omega t\,]$$

Important points

(i) In S.H.M. as $|\text{Acceleration}| = \omega^2 y$ is not constant. So equations of translatory motion can not be applied.

(ii) In S.H.M. acceleration is maximum at extreme position.

From equation (i) $|A_{max}| = \omega^2 a$ when $|\sin\omega t| = \text{maximum} = 1$ *i.e.* at $t = \frac{T}{4}$ or $\omega t = \frac{\pi}{2}$

From equation (ii) $|A_{max}| = \omega^2 a$ when $y = a$

(iii) In S.H.M. acceleration is minimum at mean position

From equation (i) $A_{min} = 0$ when $\sin\omega t = 0$ *i.e.* at $t = 0$ or $t = \frac{T}{2}$ or $\omega t = \pi$

From equation (ii) $A_{min} = 0$ when $y = 0$

(iv) Acceleration is always directed towards the mean position and so is always opposite to displacement

i.e., $A \propto -y$

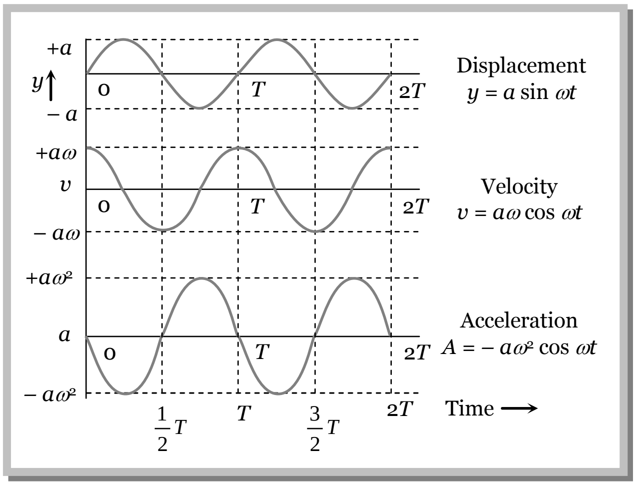
15.9 Comparative Study of Displacement, Velocity and Acceleration

Displacement $y = a\sin\omega t$

Velocity $v = a\omega\cos\omega t = a\omega\sin(\omega t + \frac{\pi}{2})$

Acceleration $A = -a\omega^2\sin\omega t = a\omega^2\sin(\omega t + \pi)$

From the above equations and graphs we can conclude that.



(i) All the three quantities displacement, velocity and acceleration show harmonic variation with time having same period.

(ii) The velocity amplitude is ω times the displacement amplitude

(iii) The acceleration amplitude is ω^2 times the displacement amplitude

(iv) In S.H.M. the velocity is ahead of displacement by a phase angle $\pi / 2$

(v) In S.H.M. the acceleration is ahead of velocity by a phase angle $\pi / 2$

(vi) The acceleration is ahead of displacement by a phase angle of π

(vii) Various physical quantities in S.H.M. at different position :

Physical quantities	Equilibrium position (y = 0)	Extreme Position ($y = \pm a$)
Displacement $y = a\sin\omega t$	Minimum (Zero)	Maximum (a)
Velocity $v = \omega\sqrt{a^2 - y^2}$	Maximum ($a\omega$)	Minimum (Zero)
Acceleration $\lvert A \rvert = \omega^2 y$	Minimum (Zero)	Maximum ($\omega^2 a$)

15.10 Energy in S.H.M.

A particle executing S.H.M. possesses two types of energy : Potential energy and Kinetic energy

(1) **Potential energy :** This is an account of the displacement of the particle from its mean position.

The restoring force $F = -ky$ against which work has to be done

So $$U = -\int dw = -\int_0^x F dx = \int_0^y ky\,dy = \frac{1}{2}ky^2$$

$\therefore$ potential Energy $$U = \frac{1}{2}m\omega^2 y^2 \qquad [\text{As } \omega^2 = k/m]$$

$$U = \frac{1}{2}m\omega^2 a^2 \sin^2\omega t \qquad [\text{As } y = a\sin\omega t]$$

Important points

(i) Potential energy maximum and equal to total energy at extreme positions

$$U_{\max} = \frac{1}{2}ka^2 = \frac{1}{2}m\omega^2 a^2 \qquad \text{when } y = \pm a;\ \omega t = \pi/2;\ t = T/4$$

(ii) Potential energy is minimum at mean position

$$U_{\min} = 0 \qquad \text{when } y = 0;\ \omega t = 0;\ t = 0$$

(2) **Kinetic energy :** This is because of the velocity of the particle

Kinetic Energy $$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}ma^2\omega^2\cos^2\omega t \qquad [\text{As } v = a\omega\cos\omega t]$$

$$K = \frac{1}{2} m\omega^2 (a^2 - y^2) \qquad [\text{As } v = \omega\sqrt{a^2 - y^2}\,]$$

(i) Kinetic energy is maximum at mean position and equal to total energy at mean position.

$$K_{max} = \frac{1}{2} m\omega^2 a^2 \qquad \text{when } y = 0\,;\ t = 0\,;\ \omega t = 0$$

(ii) Kinetic energy is minimum at extreme position.

$$K_{min} = 0 \qquad \text{when } y = a\,;\ t = T/4\,,\ \omega t = \pi/2$$

(3) **Total energy :** Total mechanical energy = Kinetic energy + Potential energy

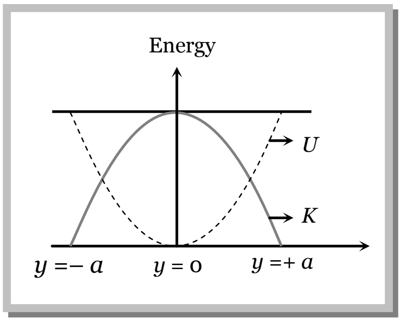
$$E = \frac{1}{2} m\omega^2 (a^2 - y^2) + \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 a^2$$

Total energy is not a position function *i.e.* it always remains constant.

(4) **Energy position graph :** Kinetic energy $(K) = \frac{1}{2} m\omega^2 (a^2 - y^2)$

Potential Energy $(U) = \frac{1}{2} m\omega^2 y^2$

Total Energy $(E) = \frac{1}{2} m\omega^2 a^2$



It is clear from the graph that

(i) Kinetic energy is maximum at mean position and minimum at extreme position

(ii) Potential energy is maximum at extreme position and minimum at mean position

(iii) Total energy always remains constant.

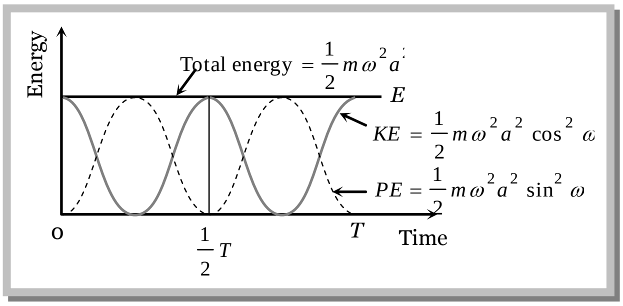
(5) Kinetic Energy $\quad K = \frac{1}{2} m\omega^2 a^2 \cos^2 \omega t = \frac{1}{4} m\omega^2 a^2 (1 + \cos 2\omega t) = \frac{1}{2} E(1 + \cos \omega' t)$

Potential Energy $\quad U = \frac{1}{2} m\omega^2 a^2 \sin^2 \omega t = \frac{1}{4} m\omega^2 a^2 (1 - \cos 2\omega t) = \frac{1}{2} E(1 - \cos \omega' t)$

where $\omega' = 2\omega$ and $E = \frac{1}{2} m\omega^2 a^2$

i.e. in S.H.M., kinetic energy and potential energy vary periodically with double the frequency of S.H.M. (*i.e.* with time period $T' = T/2$)

From the graph we note that potential energy or kinetic energy completes two vibrations in a time during which S.H.M. completes one vibration. Thus the frequency of potential energy or kinetic energy double than that of S.H.M.



Sample problems based on Energy

***Problem* 14.** A particle is executing simple harmonic motion with frequency *f*. The frequency at which its kinetic energy changes into potential energy is

(a) $f/2$ (b) f (c) $2f$ (d) $4f$

Solution : (c)

***Problem* 15.** When the potential energy of a particle executing simple harmonic motion is one-fourth of the maximum value during the oscillation, its displacement from the equilibrium position in terms of amplitude '*a*' is

[CBSE 1993; MP PMT 1994; MP PET 1995, 96; MP PMT 2000]

(a) $a/4$ (b) $a/3$ (c) $a/2$ (d) $2a/3$

Solution : (c) According to problem potential energy $= \frac{1}{4}$ maximum Energy

$$\Rightarrow \frac{1}{2}m\omega^2y^2 = \frac{1}{4}\left(\frac{1}{2}m\omega^2a^2\right) \Rightarrow y^2 = \frac{a^2}{4} \Rightarrow y = a/2$$

Problem **16.** A particle of mass 10 *grams* is executing S.H.M. with an amplitude of 0.5 *meter* and circular frequency of 10 *radian/sec*. The maximum value of the force acting on the particle during the course of oscillation is

[MP PMT 2000]

(a) 25 N (b) 5 N (c) 2.5 N (d) 0.5 N

Solution : (d) Maximum force = mass × maximum acceleration $= m\omega^2 a = 10 \times 10^{-3}(10)^2(0.5) = 0.5\,N$

Problem **17.** A body is moving in a room with a velocity of 20 *m/s* perpendicular to the two walls separated by 5 meters. There is no friction and the collision with the walls are elastic. The motion of the body is **[MP PMT 19**

(a) Not periodic (b) Periodic but not simple harmonic

(c) Periodic and simple harmonic (d) Periodic with variable time period

Solution : (b) Since there is no friction and collision is elastic therefore no loss of energy take place and the body strike again and again with two perpendicular walls. So the motion of the ball is periodic. But here, there is no restoring force. So the characteristics of S.H.M. will not satisfied.

Problem **18.** Two particles executes S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions. Each time their displacement is half of their amplitude. The phase difference between them is

(a) 30° (b) 60° (c) 90° (d) 120°

Solution : (d) Let two simple harmonic motions are $y = a\sin\omega t$ and $y = a\sin(\omega t + \phi)$

In the first case $\frac{a}{2} = a\sin\omega t \Rightarrow \sin\omega t = 1/2$ $\quad\therefore\quad \cos\omega t = \frac{\sqrt{3}}{2}$

In the second case $\frac{a}{2} = a\sin(\omega t + \phi)$

$$\Rightarrow \frac{1}{2} = [\sin\omega t.\cos\phi + \cos\omega t\sin\phi] \Rightarrow \frac{1}{2} = \left[\frac{1}{2}\cos\phi + \frac{\sqrt{3}}{2}\sin\phi\right]$$

$$\Rightarrow\ 1 - \cos\phi = \sqrt{3}\sin\phi \Rightarrow (1-\cos\phi)^2 = 3\sin^2\phi \Rightarrow (1-\cos\phi)^2 = 3(1-\cos^2\phi)$$

By solving we get $\cos\phi = +1$ or $\cos\phi = -1/2$

i.e. $\phi = 0$ or $\phi = 120^o$

Problem **19.** The acceleration of a particle performing S.H.M. is 12 *cm/sec*² at a distance of 3 *cm* from the mean position. Its time period is

(a) 0.5 *sec* (b) 1.0 *sec* (c) 2.0 *sec* (d) 3.14 *sec*

Solution : (d) $A = \omega^2 y \Rightarrow \omega = \sqrt{\frac{A}{y}} = \sqrt{\frac{12}{3}} = 2$; but $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi = 3.14$

Problem **20.** A particle of mass 10 *gm* is describing S.H.M. along a straight line with period of 2 *sec* and amplitude of 10 *cm*. Its kinetic energy when it is at 5 cm. From its equilibrium position is

(a) $37.5\pi^2 erg$ (b) $3.75\pi^2 erg$ (c) $375\pi^2 erg$ (d) $0.375\pi^2 erg$

Solution : (c) Kinetic energy $=\frac{1}{2}m\omega^2(a^2-y^2)=\frac{1}{2}10\frac{4\pi^2}{4}(10^2-5^2)=375\pi^2 ergs$.

***Problem* 21.** The total energy of the body executing S.H.M. is *E*. Then the kinetic energy when the displacement is half of the amplitude is **[RPET 1996]**

(a) $E/2$ (b) $E/4$ (c) $3E/4$ (d) $\sqrt{3}E/4$

Solution : (c) Kinetic energy $=\frac{1}{2}m\omega^2(a^2-y^2)=\frac{1}{2}m\omega^2\left(a^2-\frac{a^2}{4}\right)=\frac{3}{4}\left(\frac{1}{2}m\omega^2a^2\right)=\frac{3E}{4}$ [As $y=\frac{a}{2}$]

***Problem* 22.** A body executing simple harmonic motion has a maximum acceleration equal to 24 *m/sec*² and maximum velocity equal to 16 *meter/sec*. The amplitude of simple harmonic motion is **[MP PMT 1995]**

(a) $\frac{32}{3}meters$ (b) $\frac{3}{32}meters$ (c) $\frac{1024}{9}meters$ (d) $\frac{64}{9}meters$

Solution : (a) Maximum acceleration $\omega^2 a=24$ .....(i)

and maximum velocity $a\omega=16$ ....(ii)

Dividing (i) by (ii) $\omega=\frac{3}{2}$

Substituting this value in equation (ii) we get $a=32/3 meter$

***Problem* 23.** The displacement of an oscillating particle varies with time (in seconds) according to the equation. $y(cm)=\sin\frac{\pi}{2}\left(\frac{t}{2}+\frac{1}{3}\right)$. The maximum acceleration of the particle approximately

(a) 5.21 *cm/sec*² (b) 3.62 *cm/sec*² (c) 1.81 *cm/sec*² (d) 0.62 *cm/sec*²

Solution : (d) By comparing the given equation with standard equation, $y=a\sin(\omega t+\phi)$

We find that $a=1$ and $\omega=\pi/4$

Now maximum acceleration $=\omega^2 a=\left(\frac{\pi^2}{4}\right)=\left(\frac{3.14}{4}\right)^2=0.62 cm/\sec^2$

***Problem* 24.** The potential energy of a particle executing S.H.M. at a distance *x* from the mean position is proportional to

[Roorkee 1992]

(a) $\sqrt{x}$ (b) x (c) x^2 (d) x^3

Solution : (c)

***Problem* 25.** The kinetic energy and potential energy of a particle executing S.H.M. will be equal, when displacement is (amplitude = *a*) **[MP PMT 1987; CPMT 1990]**

(a) $a/2$ (b) $a\sqrt{2}$ (c) $a/\sqrt{2}$ (d) $\frac{a\sqrt{2}}{3}$

Solution : (c) According to problem Kinetic energy = Potential energy $\Rightarrow \frac{1}{2}m\omega^2(a^2-y^2)=\frac{1}{2}m\omega^2y^2$

$\Rightarrow a^2-y^2=y^2 \therefore y=a/\sqrt{2}$

***Problem* 26.** The phase of a particle executing S.H.M. is $\frac{\pi}{2}$ when it has

(a) Maximum velocity (b) Maximum acceleration (c) Maximum energy (d)

Solution : (b, d) Phase $\pi/2$ means extreme position. At extreme position acceleration and displacement will be maximum.

Problem 27. The displacement of a particle moving in S.H.M. at any instant is given by $y = a\sin\omega t$. The acceleration after time $t = \frac{T}{4}$ is (where T is the time period) **[MP PET 1984]**

(a) $a\omega$ (b) $-a\omega$ (c) $a\omega^2$ (d) $-a\omega^2$

Solution : (d)

Problem 28. A particle of mass m is hanging vertically by an ideal spring of force constant k, if the mass is made to oscillate vertically, its total energy is

(a) Maximum at extreme position (b) Maximum at mean position

(c) Minimum at mean position (d) Same at all position

Solution : (d)

15.11 Time Period and Frequency of S.H.M.

For S.H.M. restoring force is proportional to the displacement

$F \propto y$ or $F = -ky$ ...(i) where k is a force constant.

For S.H.M. acceleration of the body $A = -\omega^2 y$ ...(ii)

$\therefore$ Restoring force on the body $F = mA = -m\omega^2 y$ ...(iii)

From (i) and (iii) $ky = m\omega^2 y \Rightarrow \omega = \sqrt{\frac{k}{m}}$

$$\therefore \quad \text{Time period } (T) = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$\text{or} \quad \text{Frequency } (n) = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

In different types of S.H.M. the quantities m and k will go on taking different forms and names.

In general m is called inertia factor and k is called spring factor.

$$\text{Thus} \quad T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

$$\text{or} \quad n = \frac{1}{2\pi}\sqrt{\frac{\text{Spring factor}}{\text{Inertia factor}}}$$

In linear S.H.M. the spring factor stands for force per unit displacement and inertia factor for mass of the body executing S.H.M. and in Angular S.H.M. k stands for restoring torque per unit angular displacement and inertial factor for moment of inertia of the body executing S.H.M.

$$\text{For linear S.H.M.} \quad T = 2\pi\sqrt{\frac{m}{k}} = \sqrt{\frac{m}{\text{Force/Displacement}}} = 2\pi\sqrt{\frac{m \times \text{Displacement}}{m \times \text{Acceleration}}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{y}{A}}$$

$$\text{or} \quad n = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}} = \frac{1}{2\pi}\sqrt{\frac{A}{y}}$$

15.12 Differential Equation of S.H.M.

For S.H.M. (linear) Acceleration $\propto$ – (Displacement)

$A \propto -y$

or $A = -\omega^2 y$

or $\frac{d^2 y}{dt^2} = -\omega^2 y$

or $m\frac{d^2 y}{dt^2} + ky = 0$ [As $\omega = \sqrt{\frac{k}{m}}$]

For angular S.H.M. $\tau = -c\theta$ and $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$

where $\omega^2 = \frac{c}{I}$ [As c = Restoring torque constant and I = Moment of inertia]

Sample problems based on Differential equation of S.H.M.

Problem 29. A particle moves such that its acceleration a is given by $a = -bx$. Where x is the displacement from equilibrium position and b is a constant. The period of oscillation is

[NCERT 1984; CPMT 1991; MP PMT 1994; MNR 1995]

(a) $2\pi\sqrt{b}$ (b) $\frac{2\pi}{\sqrt{b}}$ (c) $\frac{2\pi}{b}$ (d) $2\sqrt{\frac{\pi}{b}}$

Solution : (b) We know that Acceleration = $-\omega^2$ (displacement) and $a = -bx$ (given in the problem)

Comparing above two equation $\omega^2 = b \Rightarrow \omega = \sqrt{b}$ $\therefore$ Time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{b}}$

Problem 30. The equation of motion of a particle is $\frac{d^2y}{dt^2} + ky = 0$ where k is a positive constant. The time period of the motion is given by

(a) $\frac{2\pi}{k}$ (b) $2\pi k$ (c) $\frac{2\pi}{\sqrt{k}}$ (d) $2\pi\sqrt{k}$

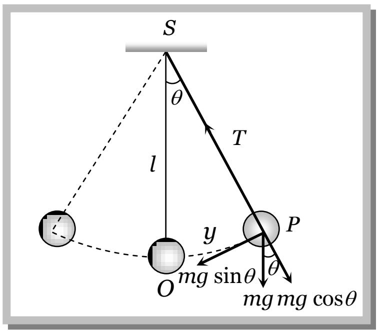
Solution : (c) Standard equation $m\frac{d^2y}{dt^2} + ky = 0$ and in a given equation m =1 and $k = k$

So, $T = 2\pi\sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{k}}$

15.13 Simple Pendulum

An ideal simple pendulum consists of a heavy point mass body suspended by a weightless, inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.

But in reality neither point mass nor weightless string exist, so we can never construct a simple pendulum strictly according to the definition.



Let mass of the bob = m

Length of simple pendulum = l

Displacement of mass from mean position (OP) = x

When the bob is displaced to position P, through a small angle θ from the vertical. Restoring force acting on the bob

$$F = -mg\sin\theta$$

or $\quad F = -mg\,\theta \quad$ (When θ is small $\sin\theta \simeq \theta = \frac{\text{Arc}}{\text{Length}} = \frac{OP}{l} = \frac{x}{l}$)

or $\quad F = -mg\,\frac{x}{l}$

$\therefore \quad \frac{F}{x} = \frac{-mg}{l} = k$ (Spring factor)

So time period $\quad T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi\sqrt{\frac{m}{mg/l}} = 2\pi\sqrt{\frac{l}{g}}$

Important points

(i) The period of simple pendulum is independent of amplitude as long as its motion is simple harmonic. But if θ is not small, $\sin\theta \neq \theta$ then motion will not remain simple harmonic but will become oscillatory. In this situation if θ_0 is the amplitude of motion. Time period

$$T = 2\pi\sqrt{\frac{l}{g}}\left[1 + \frac{1}{2^2}\sin^2\left(\frac{\theta_0}{2}\right) + ...\right] \approx T_0\left[1 + \frac{\theta_0^{\,2}}{16}\right]$$

(ii) Time period of simple pendulum is also independent of mass of the bob. This is why

(a) If the solid bob is replaced by a hollow sphere of same radius but different mass, time period remains unchanged.

(b) If a girl is swinging in a swing and another sits with her, the time period remains unchanged.

(iii) Time period $T \propto \sqrt{l}$ where l is the distance between point of suspension and center of mass of bob and is called effective length.

(a) When a sitting girl on a swinging swing stands up, her center of mass will go up and so l and hence T will decrease.

(b) If a hole is made at the bottom of a hollow sphere full of water and water comes out slowly through the hole and time period is recorded till the sphere is empty, initially and finally the center of mass will be at the center of the sphere. However, as water drains off the sphere, the center of mass of the system will first move down and then will come up. Due to this l and hence T first increase, reaches a maximum and then decreases till it becomes equal to its initial value.

(iv) If the length of the pendulum is comparable to the radius of earth then $T = 2\pi\sqrt{\frac{1}{g\left[\frac{1}{l} + \frac{1}{R}\right]}}$

(a) If $l << R$, then $\frac{1}{l} >> \frac{1}{R}$ $\quad$ so $T = 2\pi\sqrt{\frac{l}{g}}$

(b) If $l >> R (\rightarrow \infty) 1/l < 1/R$ $\quad$ so $T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4\times10^6}{10}} \cong 84.6$ *minutes*

and it is the maximum time period which an oscillating simple pendulum can have

(c) If $l = R$ $\quad$ so $\quad T = 2\pi\sqrt{\frac{R}{2g}} \cong 1\,hour$

(v) If the bob of simple pendulum is suspended by a wire then effective length of pendulum will increase with the rise of temperature due to which the time period will increase.

$l = l_0(1 + \alpha\Delta\theta)$ (If $\Delta\theta$ is the rise in temperature, l_0 = initial length of wire, l = final length of wire)

$$\frac{T}{T_0} = \sqrt{\frac{l}{l_0}} = (1 + \alpha\Delta\theta)^{1/2} \approx 1 + \frac{1}{2}\alpha\Delta\theta$$

So $\quad \frac{T}{T_0} - 1 = \frac{1}{2}\alpha\Delta\theta \quad$ *i.e.* $\quad \frac{\Delta T}{T} \approx \frac{1}{2}\alpha\Delta\theta$

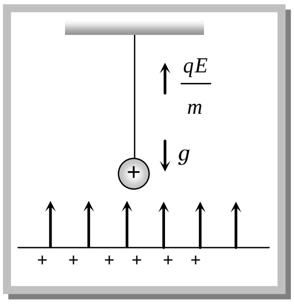
(vi) If bob a simple pendulum of density ρ is made to oscillate in some fluid of density σ (where $\sigma < \rho$) then time period of simple pendulum gets increased.

As thrust will oppose its weight therefore $\quad mg' = mg - \text{Thrust}$

or $\quad g' = g - \frac{V\sigma g}{V\rho} \quad$ *i.e.* $\quad g' = g\left[1 - \frac{\sigma}{\rho}\right] \Rightarrow \frac{g'}{g} = \frac{\rho - \sigma}{\rho}$

$\therefore \quad \frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{\rho}{\rho - \sigma}} > 1$

(vii) If a bob of mass m carries a positive charge q and pendulum is placed in a uniform electric field of strength E directed vertically upwards.



In given condition net down ward acceleration $g' = g - \frac{qE}{m}$

So $\quad T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$

If the direction of field is vertically downward then time period $T = 2\pi\sqrt{\frac{l}{g + \frac{qE}{m}}}$

(viii) Pendulum in a lift : If the pendulum is suspended from the ceiling of the lift.

(a) If the lift is at rest or moving down ward /up ward with constant velocity.

$T = 2\pi\sqrt{\frac{l}{g}} \quad$ and $\quad n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

(b) If the lift is moving up ward with constant acceleration a

$T = 2\pi\sqrt{\frac{l}{g + a}} \quad$ and $\quad n = \frac{1}{2\pi}\sqrt{\frac{g + a}{l}}$

Time period decreases and frequency increases

(c) If the lift is moving down ward with constant acceleration a

$T = 2\pi\sqrt{\frac{l}{g - a}} \quad$ and $\quad n = \frac{1}{2\pi}\sqrt{\frac{g - a}{l}}$

Time period increase and frequency decreases

(d) If the lift is moving down ward with acceleration $a = g$

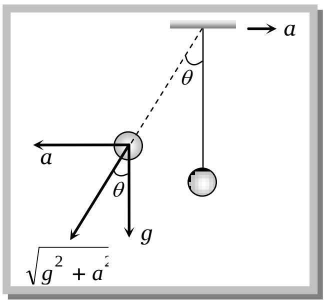
$$T = 2\pi\sqrt{\frac{l}{g-g}} = \infty \quad \text{and} \quad n = \frac{1}{2\pi}\sqrt{\frac{g-g}{l}} = 0$$

It means there will be no oscillation in a pendulum.

Similar is the case in a satellite and at the centre of earth where effective acceleration becomes zero and pendulum will stop.

(ix) The time period of simple pendulum whose point of suspension moving horizontally with acceleration a

$$T = 2\pi\sqrt{\frac{l}{(g^2+a^2)^{1/2}}} \quad \text{and} \quad \theta = \tan^{-1}(a/g)$$



(x) If simple pendulum suspended in a car that is moving with constant speed v around a circle of radius r.

$$T = 2\pi\frac{\sqrt{l}}{\sqrt{g^2+\left(\frac{v^2}{r}\right)^2}}$$

(xi) Second's Pendulum : It is that simple pendulum whose time period of vibrations is two seconds.

Putting $T = 2\ sec$ and $g = 9.8 m/\sec^2$ in $T = 2\pi\sqrt{\frac{l}{g}}$ we get

$$l = \frac{4\times 9.8}{4\pi^2} = 0.993\ m = 99.3\ cm$$

Hence length of second's pendulum is 99.3 *cm* or nearly 1 meter on earth surface.

For the moon the length of the second's pendulum will be 1/6 meter [As $g_{moon} = \frac{g_{\text{Earth}}}{6}$]

(xii) In the absence of resistive force the work done by a simple pendulum in one complete oscillation is zero.

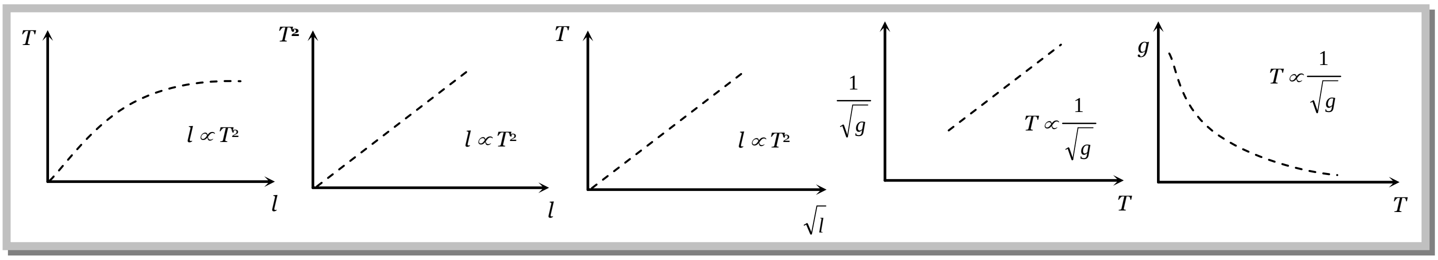
(xiii) Work done in giving an angular displacement θ to the pendulum from its mean position.

$$W = U = mgl(1-\cos\theta)$$

(xiv) Kinetic energy of the bob at mean position = work done or potential energy at extreme

$$KE_{mean} = mgl(1-\cos\theta)$$

(xv) Various graph for simple pendulum



Sample problems based on Simple pendulum

***Problem* 31.** A clock which keeps correct time at 20°*C*, is subjected to 40°*C*. If coefficient of linear expansion of the pendulum is $12 \times 10^{-6} / {}^\circ C$. How much will it gain or loose in time **[BHU 1998]**

(a) 10.3 *sec/day* (b) 20.6 *sec*/day (c) 5 *sec*/day (d) 20 *min*/day

Solution : (a) $\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta = \frac{1}{2} \times 12 \times 10^{-6} \times (40-20)$; $\Delta T = 12 \times 10^{-5} \times 86400\ sec/\text{day}$ = 10.3 *sec*/day.

***Problem* 32.** The metallic bob of simple pendulum has the relative density ρ. The time period of this pendulum is *T*. If the metallic bob is immersed in water, then the new time period is given by **[SCRA 1998]**

(a) $T\left(\frac{\rho-1}{\rho}\right)$ (b) $T\left(\frac{\rho}{\rho-1}\right)$ (c) $T\sqrt{\frac{\rho-1}{\rho}}$ (d) $T\sqrt{\frac{\rho}{\rho-1}}$

Solution : (d) Formula $\frac{T'}{T} = \sqrt{\frac{\rho}{\rho-\sigma}}$ Here $\sigma = 1$ for water so $T' = T\sqrt{\frac{\rho}{\rho-1}}$.

***Problem* 33.** The period of a simple pendulum is doubled when **[CPMT 1974; MNR 1980; AFMC 1995]**

(a) Its length is doubled

(b) The mass of the bob is doubled

(c) Its length is made four times

(d) The mass of the bob and the length of the pendulum are doubled

Solution : (c)

***Problem* 34.** A simple pendulum is executing S.H.M. with a time period *T*. if the length of the pendulum is increased by 21% the percentage increase in the time period of the pendulum is **[BHU 1994]**

(a) 10% (b) 21% (c) 30% (d) 50%

Solution : (a) As $T \propto \sqrt{l}$ $\therefore \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{1.21} \Rightarrow T_2 = 1.1\,T = T + 10\%T$.

***Problem* 35.** The length of simple pendulum is increased by 1% its time period will **[MP PET 1994]**

(a) Increase by 1% (b) Increase by 0.5% (c) Decrease by 0.5% (d) Increase by 2%

Solution : (b) $T = 2\pi\sqrt{l/g}$ hence $T \propto \sqrt{l}$

Percentage increment in $T = \frac{1}{2}$(percentage increment in *l*) = 0.5%.

***Problem* 36.** The bob of a simple pendulum of mass *m* and total energy *E* will have maximum linear momentum equal to **[MP PMT 1986]**

(a) $\sqrt{\frac{2E}{m}}$ (b) $\sqrt{2mE}$ (c) 2*mE* (d) mE^2

Solution : (b) $E = \frac{P^2}{2m}$ where *E* = Kinetic Energy, *P* = Momentum, *m* = Mass

So $P = \sqrt{2mE}$.

***Problem* 37.** The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (if it is a second's pendulum on earth) **[IIT 1973]**

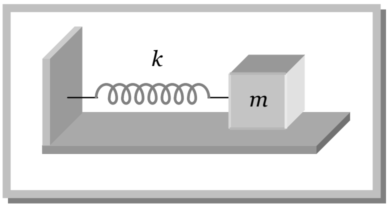
(a) $\frac{1}{\sqrt{2}}\,sec$ (b) $2\sqrt{2}\,sec$ (c) $2\,sec$ (d) $\frac{1}{2}\,sec$

Solution : (b) $g \propto \frac{M}{R^2}$; $g' = g/2$; $\frac{T'}{T} = \sqrt{\frac{g}{g'}}$ (T = 2 sec for second's pendulum)

$T' = 2\sqrt{2}$

15.14 Spring Pendulum

A point mass suspended from a mass less spring or placed on a frictionless horizontal plane attached with spring (fig.) constitutes a linear harmonic spring pendulum



Time period $T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$

$T = 2\pi\sqrt{\frac{m}{k}}$ and Frequency $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Important points

(i) Time period of a spring pendulum depends on the mass suspended

$$T \propto \sqrt{m} \quad \text{or} \quad n \propto \frac{1}{\sqrt{m}}$$

i.e. greater the mass greater will be the inertia and so lesser will be the frequency of oscillation and greater will be the time period.

(ii) The time period depends on the force constant k of the spring

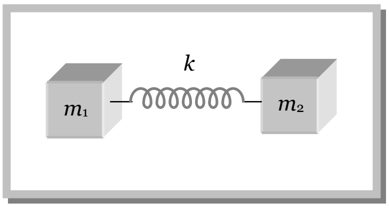
$$T \propto \frac{1}{\sqrt{k}} \quad \text{or} \quad n \propto \sqrt{k}$$

(iii) Time of a spring pendulum is independent of acceleration due to gravity. That is why a clock based on spring pendulum will keep proper time every where on a hill or moon or in a satellite and time period of a spring pendulum will not change inside a liquid if damping effects are neglected.

(iv) If the spring has a mass M and mass m is suspended from it, effective mass is given by $m_{eff} = m + \frac{M}{3}$

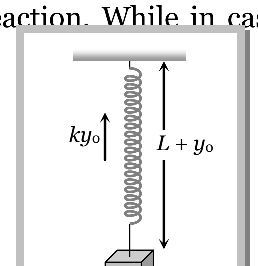
So that $T = 2\pi\sqrt{\frac{m_{eff}}{k}}$

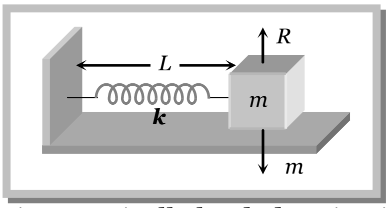
(v) If two masses of mass m_1 and m_2 are connected by a spring and made to oscillate on horizontal surface, the reduced mass m_r is given by $\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2}$



So that $T = 2\pi\sqrt{\frac{m_r}{k}}$

(vi) If a spring pendulum, oscillating in a vertical plane is made to oscillate on a horizontal surface, (or on inclined plane) time period will remain unchanged. However, equilibrium position for a spring in a horizontal plain is the position of natural length of spring as weight is balanced by reaction. While in case of vertical motion equilibrium position will be $L + y_0$ with $ky_0 = mg$



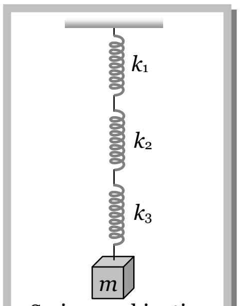


(vii) If the stretch in a vertically loaded spring is y_0 then for equilibrium of mass m, $ky_0 = mg$ *i.e.* $\frac{m}{k} = \frac{y_0}{g}$

So that $\quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$

Time period does not depends on 'g' because along with g, y_0 will also change in such a way that $\frac{y_0}{g} = \frac{m}{k}$ remains constant

(viii) Series combination : If n springs of different force constant are connected in series having force constant k_1, k_2, k_3....... respectively then



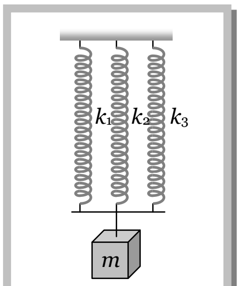
Series combination

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ...$$

If all spring have same spring constant then

$$k_{eff} = \frac{k}{n}$$

(ix) Parallel combination : If the springs are connected in parallel then



Parallel combination

$$k_{eff} = k_1 + k_2 + k_3 + ...$$

If all spring have same spring constant then

$$k_{eff} = nk$$

(x) If the spring of force constant k is divided in to n equal parts then spring constant of each part will become nk and if these n parts connected in parallel then

$$k_{eff} = n^2k$$

(xi) The spring constant k is inversely proportional to the spring length.

As $\quad k \propto \frac{1}{\text{Extension}} \propto \frac{1}{\text{Length of spring}}$

That means if the length of spring is halved then its force constant becomes double.

(xii) When a spring of length l is cut in two pieces of length l_1 and l_2 such that $l_1 = nl_2$.

If the constant of a spring is k then $\quad$ Spring constant of first part $k_1 = \frac{k(n+1)}{n}$

Spring constant of second part $k_2 = (n+1)k$

and ratio of spring constant $\frac{k_1}{k_2} = \frac{1}{n}$

Sample problems based on Spring pendulum

***Problem* 38.** A spring of force constant k is cut into two pieces such that one pieces is double the length of the other. Then the long piece will have a force constant of **[IIT-JEE 1999]**

(a) $2/3k$ (b) $3/2k$ (c) $3k$ (d) $6k$

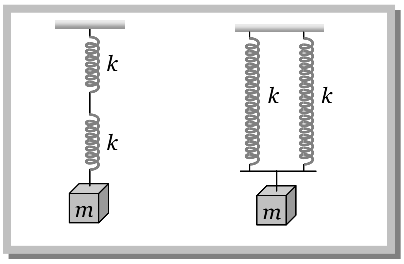
Solution : (b) If $l_1 = nl_2$ then $k_1 = \frac{(n+1)k}{n} = \frac{3}{2}k$ [As $n = 2$]

***Problem* 39.** Two bodies M and N of equal masses are suspended from two separate mass less springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of M to that of N is

(a) k_1/k_2 (b) $\sqrt{k_1/k_2}$ (c) k_2/k_1 (d) $\sqrt{k_2/k_1}$

Solution : (d) Given that maximum velocities are equal $a_1\omega_1 = a_2\omega_2 \Rightarrow a_1\sqrt{\frac{k_1}{m}} = a_2\sqrt{\frac{k_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$.

***Problem* 40.** Two identical springs of constant k are connected in series and parallel as shown in figure. A mass m is suspended from them. The ratio of their frequencies of vertical oscillation will be



(a) 2 : 1

(b) 1 : 1

(c) 1 : 2

(d) 4 : 1

Solution : (c) For series combination $n_1 \propto \sqrt{k/2}$

For parallel combination $n_2 \propto \sqrt{2k}$ so $\frac{n_1}{n_2} = \sqrt{\frac{k/2}{2k}} = \frac{1}{2}$.

***Problem* 41.** A block of mass m attached to a spring of spring constant k oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. The block has a speed v when the spring is at its natural length. Before coming to an instantaneous rest, if the block moves a distance x from the Mean position, then

(a) $x = \sqrt{m/k}$ (b) $x = \frac{1}{v}\sqrt{\frac{m}{k}}$ (c) $x = v\sqrt{m/k}$ (d) $x = \sqrt{mv/k}$

Solution : (c) Kinetic energy of block $\left(\frac{1}{2}mv^2\right)$ = Elastic potential energy of spring $\left(\frac{1}{2}kx^2\right)$

By solving we get $x = v\sqrt{\frac{m}{k}}$.

***Problem* 42.** A block is placed on a friction less horizontal table. The mass of the block is m and springs of force constant k_1, k_2 are attached on either side with if the block is displaced a little and left to oscillate, then the angular frequency of oscillation will be

(a) $\left(\frac{k_1+k_2}{m}\right)^{1/2}$ (b) $\left[\frac{k_1k_2}{m(k_1+k_2)}\right]^{1/2}$ (c) $\left[\frac{k_1k_2}{(k_1-k_2)m}\right]^{1/2}$ (d) $\left[\frac{k_1^2+k_2^2}{(k_1+k_2)m}\right]^{1/2}$

Solution : (a) Given condition match with parallel combination so $k_{eff} = k_1 + k_2$ $\therefore \omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1+k_2}{m}}$.

***Problem* 43.** A particle of mass 200 gm executes S.H.M. The restoring force is provided by a spring of force constant 80 N/m. The time period of oscillations is

(a) 0.31 sec (b) 0.15 sec (c) 0.05 sec (d) 0.02 sec

Solution : (a) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2}{80}} = \frac{2\pi}{20} = 0.31\,\text{sec}$.

***Problem* 44.** The length of a spring is l and its force constant is k when a weight w is suspended from it. Its length increases by x. if the spring is cut into two equal parts and put in parallel and the same weight W is suspended from them, then the extension will be

(a) $2x$ (b) x (c) $x/2$ (d) $x/4$

Solution : (d) As $F = kx$ so $x \propto \frac{1}{k}$ (if F = constant)

If the spring of constant k is divided in to two equal parts then each parts will have a force constant $2k$.
If these two parts are put in parallel then force constant of combination will becomes $4k$.

$$x \propto \frac{1}{k} \qquad \text{so,} \ \frac{x_2}{x_1} = \frac{k_1}{k_2} = \frac{k}{4k} \Rightarrow x_2 = \frac{x}{4}.$$

***Problem* 45.** A mass m is suspended from a string of length l and force constant k. The frequency of vibration of the mass is f_1. The spring is cut in to two equal parts and the same mass is suspended from one of the parts. The new frequency of vibration of mass is f_2. Which of the following reaction between the frequencies is correct.

[NCERT 1983; CPMT 1986; MP PMT 1991]

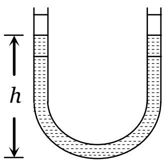
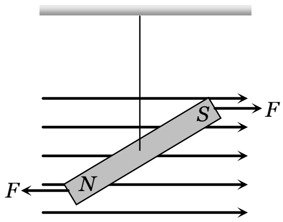
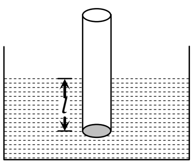
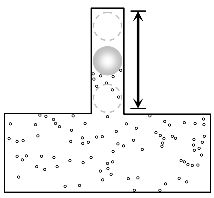
(a) $f_1 = \sqrt{2} f_2$ (b) $f_1 = f_2$ (c) $f_1 = 2f_2$ (d) $f_2 = \sqrt{2} f_1$

Solution : (d) $f \propto \sqrt{k}$

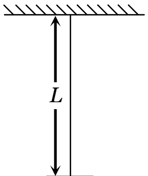
If the spring is divided in to equal parts then force constant of each part will becomes double

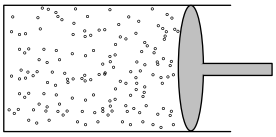
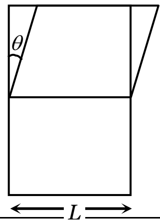
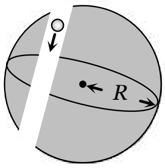
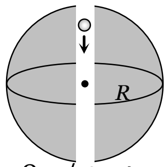
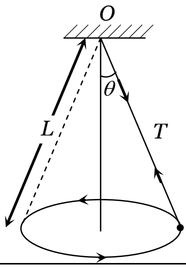
$$\frac{f_2}{f_1} = \sqrt{\frac{k_2}{k_1}} = \sqrt{2} \Rightarrow f_2 = \sqrt{2} f_1$$

15.15 Various Formulae of S.H.M.

S.H.M. of a liquid in U tube	**S.H.M. of a bar magnet in a magnetic field**
If a liquid of density ρ contained in a vertical U tube performs S.H.M. in its two limbs. Then time period $T = 2\pi\sqrt{\frac{L}{2g}} = 2\pi\sqrt{\frac{h}{g}}$ where L = Total length of liquid column, h = Height of undisturbed liquid in each limb (L=2h)  	$T = 2\pi\sqrt{\frac{I}{MB}}$ I = Moment of inertia of magnet M = Magnetic moment of magnet B = Magnetic field intensity 
S.H.M. of a floating cylinder	**S.H.M. of ball in the neck of an air chamber**
If l is the length of cylinder dipping in liquid then time period $T = 2\pi\sqrt{\frac{l}{g}}$  	$T = \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$ m = mass of the ball V = volume of air- chamber A = area of cross section of neck E = Bulk modulus for Air 
S.H.M. of a small ball rolling down in	**S.H.M. of a body suspended from a wire**

R



hemi-spherical bowl $T = 2\pi\sqrt{\frac{R-r}{g}}$ R = radius of the bowl r =radius of the ball	$T = 2\pi\sqrt{\frac{mL}{YA}}$ m = mass of the body L = length of the wire Y = young's modulus of wire A = area of cross section of wire
S.H.M. of a piston in a cylinder $T = 2\pi\sqrt{\frac{Mh}{PA}}$ M = mass of the piston A = area of cross section h = height of cylinder P = pressure in a cylinder 	**S.H.M of a cubical block** $T = 2\pi\sqrt{\frac{M}{\eta L}}$ M = mass of the block L = length of side of cube η = modulus of rigidity 
S.H.M. of a body in a tunnel dug along any chord of earth $T = 2\pi\sqrt{\frac{R}{g}} = 84.6$ minutes 	**S.H.M. of body in the tunnel dug along the diameter of earth** $T = 2\pi\sqrt{\frac{R}{g}}$ T = 84.6 minutes R = radius of the earth = 6400km g = acceleration due to gravity = 9.8m/s^2 at earth's surface 
S.H.M. of a conical pendulum $T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$ L = length of string θ = angle of string from the vertical g = acceleration due to gravity 	**S.H.M. of *L-C* circuit** $T = 2\pi\sqrt{LC}$ L = coefficient of self inductance C = capacity of condenser

15.16 Important Facts and Formulae

(1) When a body is suspended from two light springs separately. The time period of vertical oscillations are T_1 and T_2 respectively.

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}} \quad \therefore \quad k_1 = \frac{4\pi^2 m}{T_1^{\,2}} \qquad \text{and} \qquad T_2 = 2\pi\sqrt{\frac{m}{k_2}} \quad \therefore \ k_2 = \frac{4\pi^2 m}{T_2^{\,2}}$$

When these two springs are connected in series and the same mass m is attached at lower end and then for series combination $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

By substituting the values of k_1, k_2 $\quad \frac{T^2}{4\pi^2 m} = \frac{T_1^2}{4\pi^2 m} + \frac{T_2^2}{4\pi^2 m}$

Time period of the system $T = \sqrt{T_1^{\,2} + T_2^{\,2}}$

When these two springs are connected in parallel and the same mass m is attached at lower end and then for parallel combination $\quad k = k_1 + k_2$

By substituting the values of k_1, k_2 $\frac{4\pi^2 m}{T^2} = \frac{4\pi^2 m}{T_1^2} + \frac{4\pi^2 m}{T_2^2}$

Time period of the system $T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$

(2) The pendulum clock runs slow due to increase in its time period whereas it becomes fast due to decrease in time period.

(3) If infinite spring with force constant $k, 2k, 4k, 8k$.......... respectively are connected in series. The effective force constant of the spring will be $k/2$.

(4) If $y_1 = a\sin\omega t$ and $y_2 = b\cos\omega t$ are two S.H.M. then by the superimposition of these two S.H.M. we get

$$\vec{y} = \vec{y}_1 + \vec{y}_2$$

$$y = a\sin\omega t + b\cos\omega t$$

$$y = A\sin(\omega t + \phi)$$ this is also the equation of S.H.M.

where $A = \sqrt{a^2 + b^2}$ and $\phi = \tan^{-1}(b/a)$

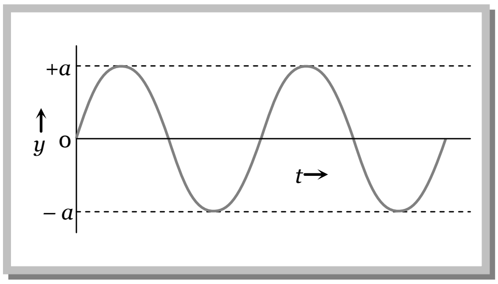
(5) If a particle performs S.H.M. whose velocity is v_1 at a x_1 distance from mean position and velocity v_2 at distance x_2

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \; ; \quad T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}} \quad a = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}} \; ; \quad v_{max} = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{x_2^2 - x_1^2}}$$

15.17 Free, Damped, Forced and Maintained Oscillation

(1) **Free oscillation**

(i) The oscillation of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations

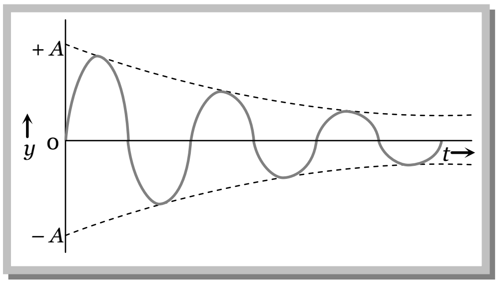


(ii) The amplitude, frequency and energy of oscillation remains constant

(iii) Frequency of free oscillation is called natural frequency because it depends upon the nature and structure of the body.

(2) **Damped oscillation**

(i) The oscillation of a body whose amplitude goes on decreasing with time are defined as damped oscillation



(ii) In these oscillation the amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, hystersis *etc.*

(iii) Due to decrease in amplitude the energy of the oscillator also goes on decreasing exponentially

(3) **Forced oscillation**

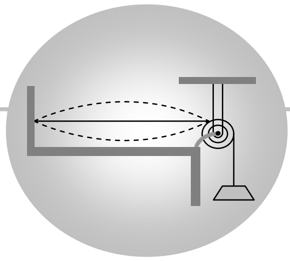
(i) The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation

(ii) The amplitude of oscillator decrease due to damping forces but on account of the energy gained from the external source it remains constant.

(iii) Resonance : When the frequency of external force is equal to the natural frequency of the oscillator. Then this state is known as the state of resonance. And this frequency is known as resonant frequency.

(4) **Maintained oscillation**

The oscillation in which the loss of oscillator is compensated by the supplying energy from an external source are known as maintained oscillation.



Wave Motion

16.1 Wave

A wave is a disturbance which propagates energy and momentum from one place to the other without the transport of matter.

(1) **Necessary properties of the medium for wave propagation :**

(i) Elasticity : So that particles can return to their mean position, after having been disturbed.

(ii) Inertia : So that particles can store energy and overshoot their mean position.

(iii) Minimum friction amongst the particles of the medium.

(iv) Uniform density of the medium.

(2) **Characteristics of wave motion :**

(i) It is a sort of disturbance which travels through a medium.

(ii) Material medium is essential for the propagation of mechanical waves.

(iii) When a wave motion passes through a medium, particles of the medium only vibrate simple harmonically about their mean position. They do leave their position and move with the disturbance.

(iv) There is a continuous phase difference amongst successive particles of the medium *i.e.*, particle 2 starts vibrating slightly later than particle 1 and so on.

(v) The velocity of the particle during their vibration is different at different position.

(vi) The velocity of wave motion through a particular medium is constant. It depends only on the nature of medium not on the frequency, wavelength or intensity.

(vii) Energy is propagated along with the wave motion without any net transport of the medium.

(3) **Mechanical waves :** The waves which require medium for their propagation are called mechanical waves.

Example : Waves on string and spring, waves on water surface, sound waves, seismic waves.

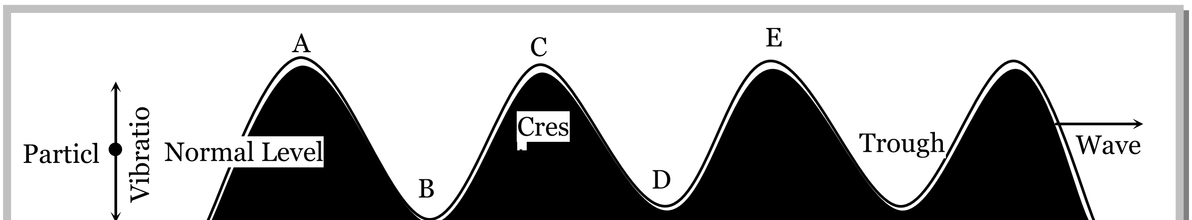
(4) **Non-mechanical waves :** The waves which do not require medium for their propagation are called non- mechanical or electromagnetic waves.

Examples : Light, heat (Infrared), radio waves, γ- rays, X-rays *etc.*

(5) **Transverse waves :** Particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.

(i) It travels in the form of crests and troughs.

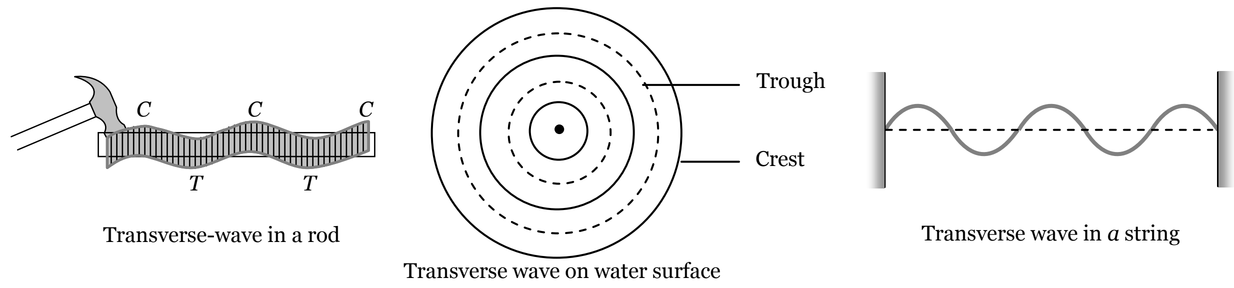
(ii) A crest is a portion of the medium which is raised temporarily above the normal position of rest of the particles of the medium when a transverse wave passes through it.



(iii) A trough is a portion of the medium which is depressed temporarily below the normal position of rest of the particles of the medium, when transverse wave passes through it.

(iv) Examples of transverse wave motion : Movement of string of a sitar or violin, movement of the membrane of a Tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.

(v) Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they can not be transmitted into liquids and gases.

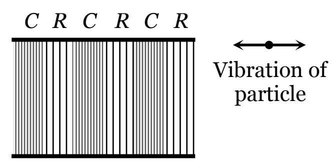


Transverse-wave in a rod

Transverse wave on water surface

Transverse wave in *a* string

(6) **Longitudinal waves :** If the particles of a medium vibrate in the direction of wave motion the wave is called longitudinal.



(i) It travels in the form of compression and rarefaction.

(ii) A compression (*C*) is a region of the medium in which particles are compressed.

(iii) A rarefaction (*R*) is a region of the medium in which particles are rarefied.

(iv) Examples sound waves travel through air in the form of longitudinal waves, Vibration of air column in organ pipes are longitudinal, Vibration of air column above the surface of water in the tube of resonance apparatus are longitudinal.

(v) These waves can be transmitted through solids, liquids and gases because for these waves propagation, volume elasticity is necessary.

(7) **One dimensional wave :** Energy is transferred in a single direction only.

Example : Wave propagating in a stretched string.

(8) **Two dimensional wave :** Energy is transferred in a plane in two mutually perpendicular directions.

Example : Wave propagating on the surface of water.

(9) **Three dimensional wave :** Energy in transferred in space in all direction.

Example : Light and sound waves propagating in space.

16.2 Important Terms Regarding Wave Motion

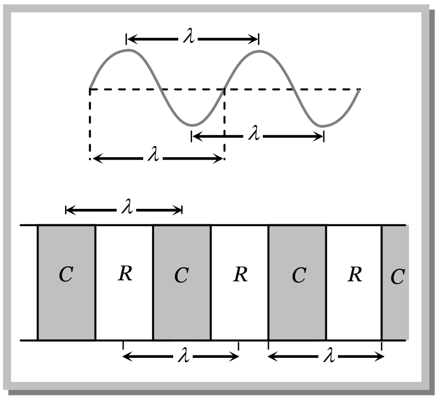
(1) **Wavelength :** (i) It is the length of one wave.

(ii) Wavelength is equal to the distance travelled by the wave during the time in which any one particle of the medium completes one vibration about its mean position.

(iii) Wavelength is the distance between any two nearest particles of the medium, vibrating in the same phase.

(iv) Distance travelled by the wave in one time period is known as wavelength.

(v) In transverse wave motion :



λ = Distance between the centres of two consecutive crests.

λ = Distance between the centres of two consecutive troughs.

λ = Distance in which one trough and one crest are contained.

(vi) In longitudinal wave motion :

λ = Distance between the centres of two consecutive compression.

λ = Distance between the centres of two consecutive rarefaction.

λ = Distance in which one compression and one rarefaction contained.

(2) **Frequency :** (i) Frequency of vibration of a particle is defined as the number of vibrations completed by particle in one second.

(ii) It is the number of complete wavelengths traversed by the wave in one second.

(iii) Units of frequency are hertz (*Hz*) and per second.

(3) **Time period :** (i) Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position.

(ii) It is the time taken by the wave to travel a distance equal to one wavelength.

(4) **Relation between frequency and time period :** Time period = 1/Frequency $\Rightarrow T = 1/n$

(5) **Relation between velocity, frequency and wavelength :** $v = n\lambda$

Velocity (v) of the wave in a given medium depends on the elastic and inertial property of the medium.

Frequency (n) is characterised by the source which produces disturbance. Different sources may produce vibration of different frequencies. Wavelength (λ) will differ to keep $n\lambda = v$ = constant

16.3 Sound Waves

The energy to which the human ears are sensitive is known as sound. In general all types of waves are produced in an elastic material medium, Irrespective of whether these are heard or not are known as sound.

According to their frequencies, waves are divided into three categories :

(1) **Audible or sound waves :** Range 20 *Hz* to 20 *KHz*. These are generated by vibrating bodies such as vocal cords, stretched strings or membrane.

(2) **Infrasonic waves :** Frequency lie below 20 *Hz*.

Example : waves produced during earth quake, ocean waves *etc.*

(3) **Ultrasonic waves :** Frequency greater than 20 *KHz*. Human ear cannot detect these waves, certain creatures such as mosquito, dog and bat show response to these. As velocity of sound in air is 332 *m*/*sec* so the wavelength of ultrasonics $\lambda < 1.66$ *cm* and for infrasonics $\lambda > 16.6$ *m*.

Note : ❑ **Supersonic speed :** An object moving with a speed greater than the speed of sound is said to move with a supersonic speed.

❑ **Mach number :** It is the ratio of velocity of source to the velocity of sound.

$$\text{Mach Number} = \frac{\text{Velocity of source}}{\text{Velocity of sound}}.$$

❑ Difference between sound and light waves :

(i) For propagation of sound wave material medium is required but no material medium is required for light waves.

(ii) Sound waves are longitudinal but light waves are transverse.

(iii) Wavelength of sound waves ranges from 1.65 *cm* to 16.5 meter and for light it ranges from 4000 Å to 2000 Å.

16.4 Velocity of Sound (Wave motion)

(1) **Speed of transverse wave motion :**

(i) On a stretched string : $v = \sqrt{\frac{T}{m}}$ T = Tension in the string; m = Linear density of string (mass per unit length).

(ii) In a solid body : $v = \sqrt{\frac{\eta}{\rho}}$ η = Modulus of rigidity; ρ = Density of the material.

(2) **Speed of longitudinal wave motion :**

(i) In a solid medium $v = \sqrt{\frac{k + \frac{4}{3}\eta}{\rho}}$ k = Bulk modulus; η = Modulus of rigidity; ρ = Density

When the solid is in the form of long bar $v = \sqrt{\frac{Y}{\rho}}$ Y = Young's modulus of material of rod

(ii) In a liquid medium $v = \sqrt{\frac{k}{\rho}}$

(iii) In gases $v = \sqrt{\frac{k}{\rho}}$

16.5 Velocity of Sound in Elastic Medium

When a sound wave travels through a medium such as air, water or steel, it will set particles of medium into vibration as it passes through it. For this to happen the medium must possess both inertia *i.e.* mass density (so that kinetic energy may be stored) and elasticity (so that PE may be stored). These two properties of matter determine the velocity of sound.

i.e. velocity of sound is the characteristic of the medium in which wave propagate.

$v = \sqrt{\frac{E}{\rho}}$ (E = Elasticity of the medium; ρ = Density of the medium)

Important points

(1) As solids are most elastic while gases least *i.e.* $E_S > E_L > E_G$. So the velocity of sound is maximum in solids and minimum in gases

$$v_{steel} > v_{water} > v_{air}$$

$$5000\ m/s > 1500\ m/s > 330\ m/s$$

As for sound $v_{water} > v_{Air}$ while for light $v_w < v_A$.

Water is rarer than air for sound and denser for light.

The concept of rarer and denser media for a wave is through the velocity of propagation (and not density). Lesser the velocity, denser is said to be the medium and vice-versa.

(2) **Newton's formula :** He assumed that when sound propagates through air temperature remains constant.(*i.e.* the process is isothermal) $v_{air} = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{P}{\rho}}$ As $K = E_\theta = P$; E_θ = Isothermal elasticity; P = Pressure.

By calculation $v_{air} = 279\ m/sec$.

However the experimental value of sound in air is 332 m/sec which is greater than that given by Newton's formula.

(3) **Laplace correction :** He modified Newton's formula assuming that propagation of sound in air as adiabatic process.

$$v = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{E_\phi}{\rho}} \qquad (\text{As } k = E_\phi = \gamma p = \text{Adiabatic elasticity})$$

$$v = \sqrt{1.41} \times 279 = 331.3\ m/s \qquad (\gamma_{\text{Air}} = 1.41)$$

(4) **Effect of density :** $v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v \propto \frac{1}{\sqrt{\rho}}$

(5) **Effect of pressure :** $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$. Velocity of sound is independent of the pressure of gas provided the temperature remains constant. ($P \propto \rho$ when T = constant)

(6) **Effect of temperature :** $v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \sqrt{T(\text{in } K)}$

When the temperature change is small then $v_t = v_0(1 + \alpha t)$

v_o = velocity of sound at 0°C, v_t = velocity of sound at t°C, α = temp-coefficient of velocity of sound.

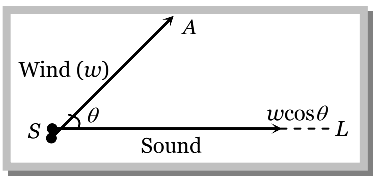
Value of $\alpha = 0.608 \frac{m/s}{^oC} = 0.61$ (Approx.)

Temperature coefficient of velocity of sound is defined as the change in the velocity of sound, when temperature changes by 1°C.

(7) **Effect of humidity :** With increase in humidity, density of air decreases. So with rise in humidity velocity of sound increases.

This is why sound travels faster in humid air (rainy season) than in dry air (summer) at the same temperature.

(8) **Effect of wind velocity :** Because wind drifts the medium (air) along its direction of motion therefore the velocity of sound in a particular direction is the algebraic sum of the velocity of sound and the component of wind velocity in that direction. Resultant velocity of sound along $SL = v + w\cos\theta$.



(9) Sound of any frequency or wavelength travels through a given medium with the same velocity.

(v = constant) For a given medium velocity remains constant. All other factors like phase, loudness pitch, quality *etc*. have practically no effect on sound velocity.

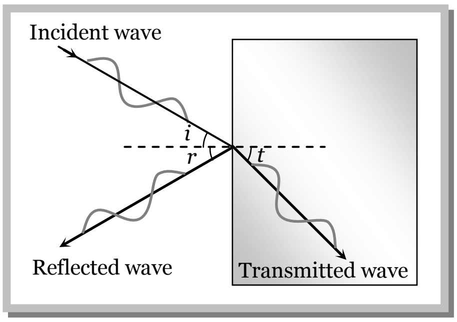
(10) Relation between velocity of sound and root mean square velocity.

$$v_{sound} = \sqrt{\frac{\gamma RT}{M}} \text{ and } v_{rms} = \sqrt{\frac{3RT}{M}} \quad \text{so} \quad \frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}} \quad \text{or} \quad v_{sound} = [\gamma/3]^{1/2} v_{rms}.$$

(11) There is no atmosphere on moon, therefore propagation of sound is not possible there. To do conversation on moon, the astronaut uses an instrument which can transmit and detect electromagnetic waves.

16.6 Reflection and Refraction of Waves

When sound waves are incident on a boundary between two media, a part of incident waves returns back into the initial medium (reflection) while the remaining is partly absorbed and partly transmitted into the second medium (refraction) In case of reflection and refraction of sound



(1) The frequency of the wave remains unchanged that means

$$\omega_i = \omega_r = \omega_t = \omega = \text{constant}$$

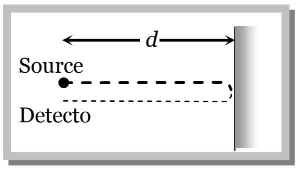
(2) The incident ray, reflected ray, normal and refracted ray all lie in the same plane.

(3) For reflection angle of incidence (i) = Angle of reflection (r)

(4) For refraction $\frac{\sin i}{\sin t} = \frac{v_i}{v_t}$

(5) In reflection from a denser medium or rigid support, phase changes by 180° and direction reverses if incident wave is $y = A_1 \sin(\omega t - kx)$ then reflected wave becomes $y = A_r \sin(\omega t + kx + \pi) = -A_r \sin(\omega t + kx)$.

(6) In reflection from a rarer medium or free end, phase does not change and direction reverses if incident wave is $y = A_I \sin(\omega t - kx)$ then reflected wave becomes $y = A_r \sin(\omega t + kx)$



(7) Echo is an example of reflection.

If there is a sound reflector at a distance d from the source then time interval between original sound and its echo at the site of source will be $t = \frac{2d}{v}$

16.7 Reflection of Mechanical Waves

Medium	Longitudinal wave	Transverse wave	Change in direction	Phase change	Time change	Path change
Reflection from rigid end/denser medium	Compression as rarefaction and vice-versa	Crest as crest and Trough as trough	Reversed	π	$\frac{T}{2}$	$\frac{\lambda}{2}$
Reflection from free end/rarer medium	Compression as compression and rarefaction as rarefaction	Crest as trough and trough as crest	No change	Zero	Zero	Zero

16.8 Progressive Wave

(1) These waves propagate in the forward direction of medium with a finite velocity.

(2) Energy and momentum are transmitted in the direction of propagation of waves without actual transmission of matter.

(3) In progressive waves, equal changes in pressure and density occurs at all points of medium.

(4) Various forms of progressive wave function.

(i) $y = A\sin(\omega t - kx)$

(ii) $y = A\sin(\omega t - \frac{2\pi}{\lambda}x)$

(iii) $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$

(iv) $y = A\sin\frac{2\pi}{\lambda}(vt - x)$

(v) $y = A\sin\omega\left(t - \frac{x}{v}\right)$

where y = displacement
A = amplitude
ω = angular frequency
n = frequency
k = propagation constant
T = time period
λ = wave length
v = wave velocity
t = instantaneous time
x = position of particle from origin

Important points

(a) If the sign between t and x terms is negative the wave is propagating along positive X-axis and if the sign is positive then the wave moves in negative X-axis direction.

(b) The coefficient of sin or cos functions *i.e.* Argument of sin or cos function *i.e.* $(\omega t - kx)$ = Phase.

(c) The coefficient of t gives angular frequency $\omega = 2\pi n = \frac{2\pi}{T} = vk$.

(d) The coefficient of x gives propagation constant or wave number $k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$.

(e) The ratio of coefficient of t to that of x gives wave or phase velocity. *i.e.* $v = \frac{\omega}{k}$.

(f) When a given wave passes from one medium to another its frequency does not change.

(g) From $v = n\lambda \Rightarrow v \propto \lambda \because n = \text{constant} \Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$.

(5) **Some terms related to progressive waves**

(i) **Wave number ($\bar{n}$) :** The number of waves present in unit length is defined as the wave number ($\bar{n}$) = $\frac{1}{\lambda}$.

Unit = *meter*$^{-1}$; Dimension = $[L^{-1}]$.

(ii) **Propagation constant (k) :** $k = \frac{\phi}{x} = \frac{\text{Phase difference between particles}}{\text{Distance between them}}$

$$k = \frac{\omega}{v} = \frac{\text{Angular velocity}}{\text{Wave velocity}} \quad \text{and} \quad k = \frac{2\pi}{\lambda} = 2\pi\bar{\lambda}$$

(iii) **Wave velocity (v) :** The velocity with which the crests and troughs or compression and rarefaction travel in a medium, is defined as wave velocity $v = \frac{\omega}{k} = n\lambda = \frac{\omega\lambda}{2\pi} = \frac{\lambda}{T}$.

(iv) **Phase and phase difference :** Phase of the wave is given by the argument of sine or cosine in the equation of wave. It is represented by $\phi(x,t)=\frac{2\pi}{\lambda}(vt-x)$.

(v) At a given position (for fixed value of x) phase changes with time (t).

$$\frac{d\phi}{dt}=\frac{2\pi v}{\lambda}=\frac{2\pi}{T} \Rightarrow d\phi=\frac{2\pi}{T}.dt \Rightarrow \text{Phase difference} = \frac{2\pi}{T}\times \text{Time difference.}$$

(vi) At a given time (for fixed value of t) phase changes with position (x).

$$\frac{d\phi}{dx}=\frac{2\pi}{\lambda} \Rightarrow d\phi=\frac{2\pi}{\lambda}\times dx \Rightarrow \text{Phase difference} = \frac{2\pi}{\lambda}\times \text{ Path difference}$$

$$\Rightarrow \text{Time difference} = \frac{T}{\lambda}\times \text{Path difference}$$

Sample problems based on Progressive wave

Problem 1. The speed of a wave in a certain medium is 960 *m/sec*. If 3600 waves pass over a certain point of the medium in 1 minute, the wavelength is **[MP PMT 2000]**

(a) 2 *meters* (b) 4 *meters* (c) 8 *meters* (d) 16 *meters*

Solution : (d) $v = 960\ m/s$; $n = \frac{3600}{60}\ Hz$. So $\lambda=\frac{v}{n}=\frac{960}{60}=$ 16 *meters*.

Problem 2. A simple harmonic progressive wave is represented by the equation $y = 8 \sin 2\pi\,(0.1x - 2t)$ where x and y are in *cm* and t is in seconds. At any instant the Phase difference between two particles separated by 2.0 *cm* in the x-direction is **[MP PMT 2000]**

(a) 18° (b) 36° (c) 54° (d) 72°

Solution : (d) $y = 8 \sin 2\pi\left(\frac{x}{10}-2t\right)$ given by comparing with standard equation $y = a \sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$

$\lambda = 10\ cm$

So Phase Difference = $\frac{2\pi}{\lambda}\times$ path difference $=\frac{2\pi}{10}\times 2=\frac{2}{5}\times 180° = 72°$

Problem 3. The frequency of sound wave is n and its velocity is v if the frequency is increased to $4n$ the velocity of the wave will be **[MP PET 2000]**

(a) v (b) $2v$ (c) $4v$ (d) $v/4$

Solution : (a) Wave velocity does not depends on the frequency. It depends upon the Elasticity and inertia of the medium.

Problem 4. The displacement of a particle is given by $x = 3\sin(5\pi t) + 4\cos(5\pi t)$ The amplitude of particle is **[MP PMT 1999]**

(a) 3 (b) 4 (c) 5 (d) 7

Solution : (c) Standard equation : $x = a\sin\omega t + b\cos\omega t$

$$x=\sqrt{a^2+b^2}\ \sin\left(\omega t+\tan^{-1}(b/a)\right)$$

Given equation $x = 3\sin(5\pi t)+4\cos(5\pi t)$

$$x=\sqrt{9+16}\ \sin\left(5\pi t+\tan^{-1}4/3\right)$$

$$x=5\sin(5\pi t+\tan^{-1}(4/3))$$

Problem 5. The equation of a transverse wave travelling on a rope is given by $y=10\sin\pi(0.01x-2.00t)$ where y and x are in *cm* and t in seconds. The maximum transverse speed of a particle in the rope is about **[MP PET 1999]**

(a) 63 *cm/sec* (b) 75 *cm/s* (c) 100 *cm/sec* (d) 121 *cm/sec*

Solution : (a) Standard eq. of travelling wave $y = A \sin (kx - \omega t)$

By comparing with the given equation $y = 10 \sin (0.01\pi x - 2\pi t)$

$A = 10\ cm,\ \omega = 2\pi$

Maximum particle velocity $= A\omega = 2\pi \times 10 = 63\ cm/sec$

Problem 6. In a wave motion $y = a \sin (kx - \omega t), y$ can represents

(a) Electric Field (b) magnetic field (c) Displacement (d) Pressure

Solution : (a,b,c,d)

Problem 7. Find the ratio of the speed of sound in nitrogen gas to that of helium gas, at 300 k is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\sqrt{\frac{3}{5}}$ (d) $\frac{4}{5}$

Solution : (c) $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{v_N}{v_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}} \frac{M_{He}}{M_{N2}}} = \sqrt{\frac{7/5}{5/3} \cdot \frac{4}{28}} = \sqrt{\frac{3}{5}}.$$

Problem 8. The displacement x (in metres) of a particle performing simple harmonic motion is related to time t (in seconds) as $x = 0.05 \cos \left(4\pi t + \frac{\pi}{4}\right)$. The frequency of the motion will be **[MP PMT / PET 1998]**

(a) 0.5 Hz (b) 1.0 Hz (c) 1.5 Hz (d) 2.0 Hz

Solution : (d) From the given equation, coefficient of $t = \omega = 4\pi$

$$\therefore \quad n = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} = 2\ Hz$$

Problem 9. A wave is represented by the equation $Y = 7 \sin \left(7\pi t - 0.04\pi x + \frac{\pi}{3}\right)$ x is in meters and t is in seconds. The speed of the wave is **[MP PET 1996]**

(a) 175 m/sec (b) 49 $\pi\ m/s$ (c) $\frac{49}{\pi} m/s$ (d) 0.28 $\pi\ m/s$

Solution : (a) Standard equation $y = A \sin (\omega t - kx + \phi_0)$

In a given equation $\omega = 7\pi, k = 0.04\pi$

$$v = \frac{\omega}{k} = \frac{7\pi}{.04\pi} = 175\ m/sec$$

Problem 10. A wave is represented by the equation y = 0.5 sin (10 t + x)m. It is a travelling wave propagating along the x direction with velocity. **[Roorkee 1995]**

(a) 10 m/s (b) 20 m/s (c) 5 m/s (d) None of these

Solution : (a) $v = \omega / k = 10/1 = 10\ m/s$

Problem 11. A transverse progressive wave on a stretched string has a velocity of 10 ms^{-1} and a frequency of 100 Hz. The phase difference between two particles of the string which are 2.5 cm apart will be

(a) $\pi/8$ (b) $\pi/4$ (c) $3\pi/8$ (d) $\pi/2$

Solution : (d) $\lambda = v/n = \frac{10}{100} = 0.1m = 10cm$

Phase difference $= \frac{2\pi}{\lambda} \times$ path difference $= \frac{2\pi}{10} \times 2.5 = \frac{\pi}{2}$

Problem 12. In a stationary wave, all particles are **[MP PMT 1994]**

(a) At rest at the same time twice in every period of oscillation

(b) At rest at the same time only once in every period of oscillation

(c) Never at rest at the same time
(d) Never at rest at all

Solution : (a)

Problem 13. The path difference between the two waves

$y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and $y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$ is **[MP PMT 1994]**

(a) $\frac{\lambda}{2\pi}\phi$ (b) $\frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$ (c) $\frac{2\pi}{\lambda}\left(\phi - \frac{\pi}{2}\right)$ (d) $\frac{2\pi}{\lambda}(\phi)$

Solution : (b) $y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$; $y_2 = a_2 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$

Phase difference $= \left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right) - \left(\omega t - \frac{2\pi x}{\lambda}\right) = \left(\phi + \frac{\pi}{2}\right)$

Path difference $= \frac{\lambda}{2\pi} \times$ Phase difference $= \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

Problem 14. A plane wave is described by the equation $y = 3\cos\left(\frac{x}{4} - 10t - \frac{\pi}{2}\right)$. The maximum velocity of the particles of the medium due to this wave is

(a) 30 (b) $3\pi/2$ (c) 3/4 (d) 40

Solution : (a) Maximum velocity $= A\omega = 3 \times 10 = 30$

Problem 15. A wave represented by the given equation $y = A \sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$ where x is in meter and t is in second. The expression represents

(a) A wave travelling in the positive x-direction with a velocity of 1.5 m/sec
(b) A wave travelling in the negative x-direction with a velocity of 1.5 m/sec
(c) A wave travelling in the negative x-direction with a wavelength of 0.2 m
(d) A wave travelling in the positive x-direction with a wavelength of 0.2 m

Solution : (b, c) By comparing with standard equation $Y = A \sin(kx + \omega t + \pi/3)$

$$K = 10\pi, \quad \omega = 15\pi$$

We know that : $v = \frac{\omega}{k} = 1.5\ m/sec$; $\lambda = \frac{2\pi}{k} = 0.2\ meter$.

Problem 16. A transverse wave is described by the equation $Y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$ The maximum particle velocity is four times the wave velocity if

(a) $\lambda = \frac{\pi y_0}{4}$ (b) $\lambda = \frac{\pi y_0}{2}$ (c) $\lambda = \pi y_0$ (d) $\lambda = 2\pi y_0$

Solution : (b) Maximum particle velocity = 4 wave velocity

$$A\omega = 4f\lambda$$

$$y_0\, 2\pi f = 4f\lambda$$

$$\lambda = \frac{\pi y_0}{2}$$

Problem 17. The equation of a wave travelling in a string can be written as $y = 3\cos\pi(100t - x)$ Its wavelength is **[MP PMT 1991, 94, 97; MNR 1985]**

(a) 100 cm (b) 2 cm (c) 5 cm (d) None of these

Solution : (b) $y = A\cos(\omega t - kx)$ – standard equation

$y = 3\cos(100\ \pi t - \pi x)$ – given equation

So $K = \pi$ and $\lambda = \frac{2\pi}{k} = 2\ cm$

***Problem* 18.** A plane wave is represented by x = 1.2 sin (314 t + 12.56 y) where x and y are distances measured along in x and y direction in meter and t is time in seconds. This wave has **[MP PET 1991]**

(a) A wave length of 0.25 m and travels m + ve x-direction

(b) A wavelength of 0.25 m and travels in + ve y-direction

(c) A wavelength of 0.5 m and travels in – ve y-direction

(d) A wavelength of 0.5 m and travels in – ve x-direction

Solution : (c) From given equation k = 12.56

$\lambda = \frac{2\pi}{k} 0.5\ m$ direction = $-y$

***Problem* 19.** A wave is reflected from a rigid support. The change in phase on reflection will be

(a) $\pi/4$ (b) $\pi/2$ (c) π (d) 2π

Solution : (c)

***Problem* 20.** The equation of displacement of two waves are given as y_1 = 10 sin $\left(3\pi t + \frac{\pi}{3}\right)$; y_2 = 5 $[\sin 3\pi t + \sqrt{3}\cos 3\pi t]$

Then what is the ratio of their amplitudes **[AIIMS 1997]**

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) None of these

Solution : (c) $y_2 = 5\ [\sin 3\pi t + \sqrt{3}\cos 3\pi t] \quad = 5\sqrt{1+3}\sin\left(3\pi t + \frac{\pi}{3}\right) = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

So, $A_1 = 10$ and $A_2 = 10$

***Problem* 21.** The equation of a wave travelling on a string is y = 4 sin $\frac{\pi}{2}\left(8t - \frac{x}{8}\right)$ if x and y are in cm, then velocity of wave is **[MP PET 1990]**

(a) 64 cm/sec in – x direction (b) 32 cm/sec in – x direction

(c) 32 cm/sec in +x direction (d) 64 cm/sec in +x direction

Solution : (d) $y = 4\sin\left(4\pi t - \frac{\pi}{16}.x\right)$

$\omega = 4\pi, k = \pi/16$

$v = \frac{\omega}{k} = \frac{4\pi}{\pi/16} = 64\ cm/\sec$ in + x direction.

***Problem* 22.** The equation of wave is $y = 2\sin\ \pi(0.5x - 200\ t)$ where x and y are expressed in cm and t in sec. The wave velocity is **[MP PMT 1986]**

(a) 100 cm/sec (b) 200 cm/sec (c) 300 cm/sec (d) 400 cm/sec

Solution : (d) $v = \frac{\omega}{k} = \frac{200\ \pi}{0.5\ \pi}$ = 400 cm/sec

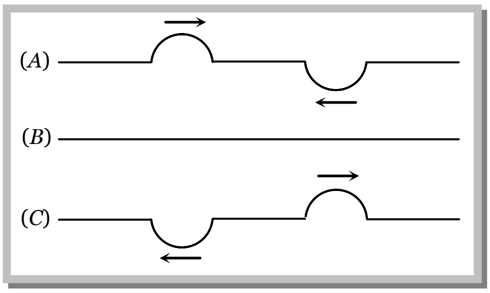
16.9 Principle of Superposition

The displacement at any time due to any number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements due each one of the waves at that point at the same time.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3$ .... are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement. $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots\ldots\ldots\ \ldots$

Examples

(i) Radio waves from different stations having different frequencies cross the antenna. But our T.V/Radio set can pick up any desired frequency.



(ii) When two pulses of equal amplitude on a string approach each other [fig. (A)], then on meeting, they superimpose to produce a resultant pulse of zero amplitude [fig (B)]. After crossing, the two pulses travel independently as shown in [fig (C)] as if nothing had happened.

Important applications of superposition principle :

(a) Interference of waves (b) Stationary waves (c) Beats.

16.10 Interference of Sound Waves

When two waves of same frequency, same wavelength, same velocity (nearly equal amplitude) moves in the same direction, Their superimposition results in the interference. Due to interference the resultant intensity of sound at that point is different from the sum of intensities due to each wave separately. This modification of intensity due to superposition of two or more waves is called interference.

Let at a given point two waves arrives with phase difference ϕ and the equation of these waves is given by

$y_1 = a_1 \sin \omega t$, $y_2 = a_2 \sin(\omega t + \phi)$ then by the principle of superposition

$\vec{y} = \vec{y}_1 + \vec{y}_2 \Rightarrow y = A\sin(\omega t + \theta)$ where $A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$ and $\tan\theta = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$

and since Intensity $\propto A^2$.

So $I = a_1^2 + a_2^2 + 2a_1, a_2 \cos\phi \Rightarrow I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Important points

(1) **Constructive interference :** Intensity will be maximum

when $\phi = 0, 2\pi, 4\pi, \ldots\ldots\ 2\pi n$; where n = 0,1,2 ....

when x = 0, λ, 2λ, ... $n\lambda$; where n = 0, 1 ....

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \propto (A_1 + A_2)^2$

It means the intensity will be maximum at those points where path difference is an integral multiple of wavelength λ. These points are called points of constructive interference or interference maxima.

(2) **Destructive interference :** Intensity will be minimum

when $\phi = \pi, 3\pi, 5\pi \ldots\ldots\ (2n-1)\pi$; where n = 1, 2, 3 ....

when x = λ/2, 3λ/2, ... $(2n-1)\lambda/2$; where n = 1, 2, 3........

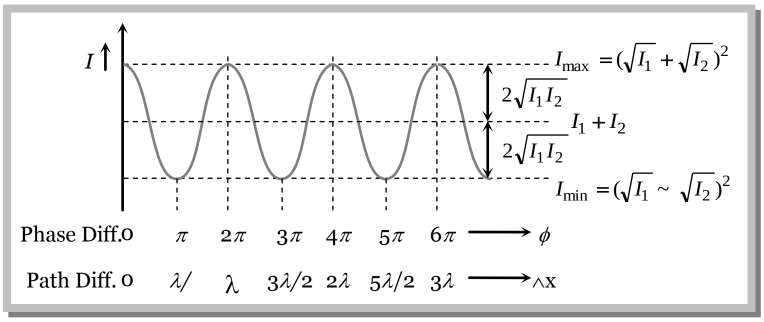
$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \Rightarrow I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \propto (A_1 \sim A_2)^2$

(3) All maxima are equally spaced and equally loud. Same is also true for minima. Also interference maxima and minima are alternate as for maximum $\Delta x = 0, \lambda, 2\lambda ....etc$. and for minimum $\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} .... etc$.

(4) $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 \sim A_2)^2}$ with $\frac{I_1}{I_2} = \frac{A_1^{\,2}}{A_2^{\,2}}$

(5) If $I_1 = I_2 = I_0$ then $I_{max} = 4I_o$ and $I_{min} = 0$

(6) In interference the intensity in maximum $\left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ exceeds the sum of individual intensities ($I_1 + I_2$) by an amount $2\sqrt{I_1 I_2}$ while in minima $\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2$ lacks $(I_1 + I_2)$ by the same amount $2\sqrt{I_1 I_2}$.



Hence in interference energy is neither created nor destroyed but is redistributed.

16.11 Standing Waves or Stationary Waves

When two sets of progressive wave trains of same type (both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

Characteristics of standing waves :

(1) The disturbance confined to a particular region between the starting point and reflecting point of the wave.

(2) There is no forward motion of the disturbance from one particle to the adjoining particle and so on, beyond this particular region.

(3) The total energy associated with a stationary wave is twice the energy of each of incident and reflected wave. But there is no flow or transference of energy along the stationary wave.

(4) There are certain points in the medium in a standing wave, which are permanently at rest. These are called nodes. The distance between two consecutive nodes is $\frac{\lambda}{2}$

(5) Points of maximum amplitude is known as antinodes. The distance between two consecutive antinodes is also $\lambda / 2$. The distance between a node and adjoining antinode is $\lambda / 4$.

(6)The medium splits up into a number of segments. Each segment is vibrating up and down as a whole.

(7) All the particles in one particular segment vibrate in the same phase. Particles in two consecutive segments differ in phase by 180°.

(8) All the particles except those at nodes, execute simple harmonic motion about their mean position with the same time period.

(9) The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.

(10) Twice during each vibration, all the particles of the medium pass simultaneously through their mean position.

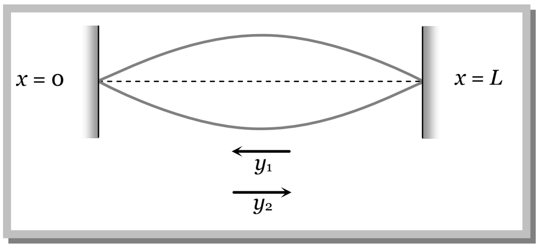
(11) The wavelength and time period of stationary waves are the same as for the component waves.

(12) Velocity of particles while crossing mean position varies from maximum at antinodes to zero at nodes.

(13) In standing waves, if amplitude of component waves are not equal. Resultant amplitude at nodes will be minimum (but not zero). Therefore, some energy will pass across nodes and waves will be partially standing.

16.12 Standing Waves on a String

When a string under tension is set into vibration, transverse harmonic waves propagate along its length. When the length of string is fixed, reflected waves will also exist. The incident and reflected waves will superimpose to produce transverse stationary waves in a string

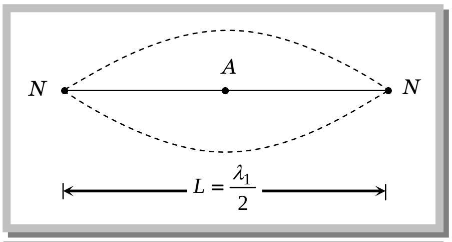


Incident wave $y_1 = a \sin \dfrac{2\pi}{\lambda}(vt + x)$

Reflected wave $y_2 = a \sin \dfrac{2\pi}{\lambda}[(vt - x) + \pi] = -a \sin \dfrac{2\pi}{\lambda}(vt - x)$

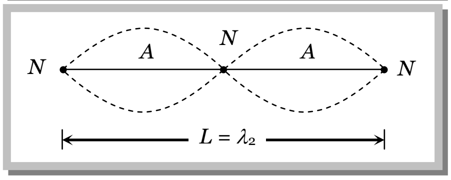
According to superposition principle : $y = y_1 + y_2 = 2\,a \cos \dfrac{2\pi vt}{\lambda} \sin \dfrac{2\pi x}{\lambda}$

General formula for wavelength $\lambda = \dfrac{2L}{n}$ where n = 1,2,3, ... correspond to 1st , 2nd, 3rd modes of vibration of the string.



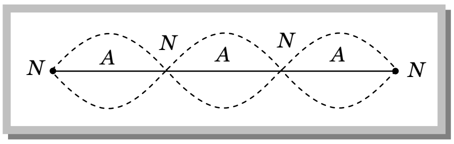
(1) First normal mode of vibration $n_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2L} \Rightarrow n_1 = \dfrac{1}{2L}\sqrt{\dfrac{T}{m}}$

This mode of vibration is called the fundamental mode and the frequency is called fundamental frequency. The sound from the note so produced is called fundamental note or first harmonic.



(2) Second normal mode of vibration : $n_2 = \dfrac{v}{\lambda_2} = \dfrac{v}{L} = \dfrac{2v}{2L} = 2(n_1)$

This is second harmonic or first over tone.



(3) Third normal mode of vibration : $n_3 = \dfrac{v}{\lambda_3} = \dfrac{3v}{2L} = 3n_1$

This is third harmonic or second overtone.

Position of nodes : $x = 0, \dfrac{L}{n}, \dfrac{2L}{n}, \dfrac{3L}{n}$.......... ..$L$

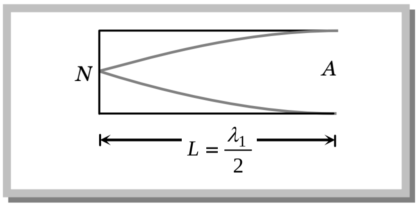
For first mode of vibration	$x = 0$, $x = L$	[Two nodes]
For second mode of vibration	$x = 0, x = \dfrac{L}{2}, x = L$	[Three nodes]
For third mode of vibration	$x = 0$, $x = \dfrac{L}{3}, x = \dfrac{2L}{3}, x = L$	[Four nodes]

Position of antinodes : $x = \dfrac{L}{2n}, \dfrac{3L}{2n}, \dfrac{5L}{2n}$.......... . $\dfrac{(2n-1)L}{2n}$

For first mode of vibration $x = L/2$	[One antinode]
For second mode of vibration x = $\dfrac{L}{4}, \dfrac{3L}{4}$	[Two antinode]

16.13 Standing Wave in a Closed Organ Pipe

Organ pipes are the musical instrument which are used for producing musical sound by blowing air into the pipe. Longitudinal stationary waves are formed on account of superimposition of incident and reflected longitudinal waves.

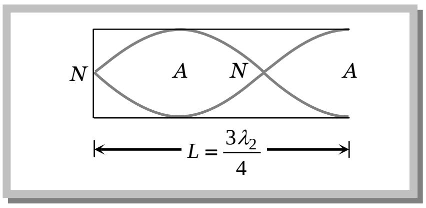


Equation of standing wave y = $2a\cos\frac{2\pi vt}{\lambda}\sin\frac{2\pi x}{\lambda}$

General formula for wavelength $\lambda = \frac{4L}{(2n-1)}$

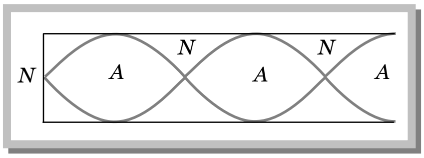
(1) First normal mode of vibration : $n_1 = \frac{v}{4L}$

This is called fundamental frequency. The note so produced is called fundamental note or first harmonic.



(2) Second normal mode of vibration : $n_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3n_1$

This is called *third harmonic* or *first overtone*.



(3) Third normal mode of vibration : $n_3 = \frac{5v}{4L} = 5n_1$

This is called *fifth harmonic* or *second overtone*.

Position of nodes : $x = 0, \frac{2L}{(2n-1)}, \frac{4L}{(2n-1)}, \frac{6L}{(2n-1)} .... \frac{2nL}{(2n-1)}$

For first mode of vibration $x = 0$ [One node]

For second mode of vibration $x = 0, x = \frac{2L}{3}$ [Two nodes]

For third mode of vibration $x = 0,\ x = \frac{2L}{5}, \frac{4L}{5}$ [Three nodes]

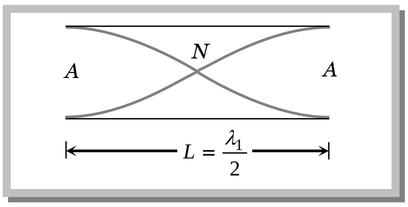
Position of antinode : $x = \frac{L}{2n-1}, \frac{3L}{2n-1}, \frac{5L}{2n-1}, ..., L$

For first mode of vibration $x = L$ [One antinode]

For second mode of vibration $x = \frac{L}{3}, x = L$ [Two antinode]

For third mode of vibration $x = \frac{L}{5}, \frac{3L}{5}, L$ [Three antinode]

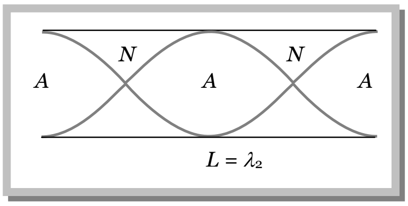
16.14 Standing Waves in Open Organ Pipes



General formula for wavelength

$\lambda = \frac{2L}{n}$ where n = 1,2,3 ....

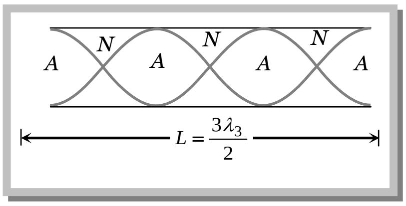
(1) First normal mode of vibration : $n_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$



This is called fundamental frequency and the note so produced is called *fundamental note* or *first harmonic*.

(2) Second normal mode of vibration $n_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2n_1 \Rightarrow n_2 = 2n_1$

This is called *second harmonic* or *first overtone*.



(3) Third normal mode of vibration $n_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}$, $n_3 = 3n_1$

This is called *third harmonic* or *second overtone*.

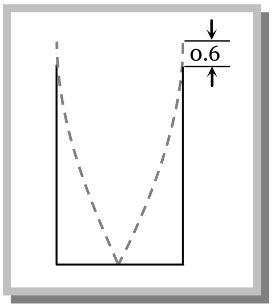
Important points

(i) Comparison of closed and open organ pipes shows that fundamental note in open organ pipe $\left(n_1 = \frac{v}{2L}\right)$ has double the frequency of the fundamental note in closed organ pipe $\left(n_1 = \frac{v}{4L}\right)$.

Further in an open organ pipe all harmonics are present whereas in a closed organ pipe, only alternate harmonics of frequencies $n_1, 3n_1, 5n_1, ...$ *etc* are present. The harmonics of frequencies $2n_1$, $4n_1$, $6n_1$ ...... are missing.

Hence musical sound produced by an open organ pipe is sweeter than that produced by a closed organ pipe.

(ii) Harmonics are the notes/sounds of frequency equal to or an integral multiple of fundamental frequency (n). Thus the first, second, third, harmonics have frequencies $n_1, 2n_1, 3n_1,$ ....

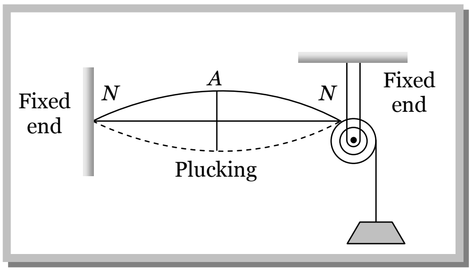


(iii) Overtones are the notes/sounds of frequency twice/thrice/ four times the fundamental frequency (n) *eg*. $2n, 3n, 4n$ .... and so on.

(iv) In organ pipe an antinode is not formed exactly at the open end rather it is formed a little distance away from the open end outside it. The distance of antinode from the open end of the pipe is known as end correction.

16.15 Vibration of a String

Fundamental frequency $n = \frac{1}{2L}\sqrt{\frac{T}{m}}$



General formula $n_p = \frac{p}{2L}\sqrt{\frac{T}{m}}$

L = Length of string, T = Tension in the string

m = Mass per unit length (linear density), p = mode of vibration

Important points

(1) As a string has many natural frequencies, so when it is excited with a tuning fork, the string will be in resonance with the given body if any of its natural frequencies concides with the body.

(2) (i) $n \propto \frac{1}{L}$ if T and m are constant (ii) $n \propto \sqrt{T}$ if L and m are constant (iii) $n \propto \frac{1}{\sqrt{m}}$ if T and L are constant

(3) If M is the mass of the string of length L, $m = \frac{M}{L}$

So $n = \frac{1}{2L}\sqrt{\frac{T}{m}} = \frac{1}{2L}\sqrt{\frac{T}{M/L}} = \frac{1}{2}\sqrt{\frac{T}{ML}} = \frac{1}{2L}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{2Lr}\sqrt{\frac{T}{\pi\rho}}$ where $m = \pi r^2\rho$ (r = Radius, ρ = Density)

16.16 Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S. No.	Parameter	Stretched string	Open organ pipe	Closed organ pipe
(1)	Fundamental frequency or 1st harmonic	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{4l}$
(2)	Frequency of 1st overtone or 2nd harmonic	$n_2 = 2n_1$	$n_2 = 2n_1$	Missing
(3)	Frequency of 2nd overtone or 3rd harmonic	$n_3 = 3n_1$	$n_3 = 3n_1$	$n_3 = 3n_1$
(4)	Frequency ratio of overtones	2 : 3 : 4...	2 : 3 : 4...	3 : 5 : 7...
(5)	Frequency ratio of harmonics	1 : 2 : 3 : 4...	1 : 2 : 3 : 4...	1 : 3 : 5 : 7...
(6)	Nature of waves	Transverse stationary	Longitudinal stationary	Longitudinal stationary

16.17 Beats

When two sound waves of slightly different frequencies, travelling in a medium along the same direction, superimpose on each other, the intensity of the resultant sound at a particular position rises and falls regularly with time. This phenomenon of regular variation in intensity of sound with time at a particular position is called beats.

Important points

(1) **One beat :** If the intensity of sound is maximum at time t = 0, one beat is said to be formed when intensity becomes maximum again after becoming minimum once in between.

(2) **Beat period :** The time interval between two successive beats (*i.e.* two successive maxima of sound) is called beat period.

(3) **Beat frequency :** The number of beats produced per second is called beat frequency.

(4) **Persistence of hearing :** The impression of sound heard by our ears persist in our mind for 1/10th of a second. If another sound is heard before 1/10 second is over, the impression of the two sound mix up and our mind cannot distinguish between the two.

So for the formation of distinct beats, frequencies of two sources of sound should be nearly equal (difference of frequencies less than 10)

(5) **Equation of beats :** If two waves of equal amplitudes 'a' and slightly different frequencies n_1 and n_2 travelling in a medium in the same direction are.

$$y_1 = a\sin\omega_1 t = a\sin 2\pi n_1 t\ ;\ y_2 = a\sin\omega_2 t = a\sin 2\pi n_2 t$$

By the principle of super position : $\vec{y} = \vec{y}_1 + \vec{y}_2$

$$y = A\sin\pi(n_1 + n_2)t \qquad \text{where } A = 2a\cos\pi(n_1 - n_2)t = \text{Amplitude of resultant wave.}$$

(6) **Beat frequency :** $n = n_1 \sim n_2$.

(7) **Beat period :** $T = \dfrac{1}{\text{Beat frequency}} = \dfrac{1}{n_1 \sim n_2}$

16.18 Determination of Unknown Frequency

Let n_2 is the unknown frequency of tuning fork B, and this tuning fork B produce x beats per second with another tuning fork of known frequency n_1.

As number of beat/sec is equal to the difference in frequencies of two sources, therefore $n_2 = n_1 \pm x$

The positive/negative sign of x can be decided in the following two ways :

By loading	By filing
If B is loaded with wax so its frequency decreases	**If B is filed, its frequency increases**
If number of beats decreases $n_2 = n_1 + x$	If number of beats decreases $n_2 = n_1 - x$
If number of beats Increases $n_2 = n_1 - x$	If number of beats Increases $n_2 = n_1 + x$
If number of beats remains unchanged $n_2 = n_1 + x$	If number of beats remains unchanged $n_2 = n_1 - x$
If number of beats becomes zero $n_2 = n_1 + x$	If number of beats becomes zero $n_2 = n_1 - x$
If A is loaded with wax its frequency decreases	**If A is filed, its frequency increases**
If number of beats decreases $n_2 = n_1 - x$	If number of beats decreases $n_2 = n_1 + x$
If number of beats increases $n_2 = n_1 + x$	If number of beats Increases $n_2 = n_1 - x$
If number of beats remains unchanged $n_2 = n_1 - x$	If number of beats remains unchanged $n_2 = n_1 + x$
If number of beats becomes zero $n_2 = n_1 - x$	If no of beats becomes zero $n_2 = n_1 + x$

Sample problems based on Superposition of waves

***Problem* 23.** The stationary wave produced on a string is represented by the equation $y = 5\cos\left(\frac{\pi x}{3}\right)\sin(40\,\pi\, t)$ where x and y are in *cm* and t is in seconds. The distance between consecutive nodes is

(a) 5 *cm* (b) π *cm* (c) 3 *cm* (d) 40 *cm*

Solution : (c) By comparing with standard equation of stationary wave

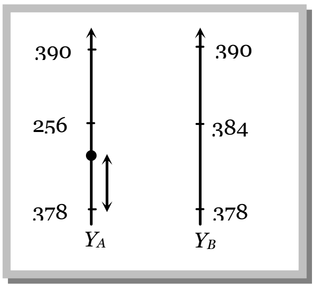
$$y = a\cos\frac{2\pi x}{\lambda}\sin\frac{2\,\pi\, vt}{\lambda}$$

We get $\frac{2\pi x}{\lambda} = \frac{\pi x}{3} \Rightarrow \lambda = 6$; Distance between two consecutive nodes $= \frac{\lambda}{2} = 3\ cm$

***Problem* 24.** On sounding tuning fork A with another tuning fork B of frequency 384 *Hz*, 6 beats are produced per second. After loading the prongs of A with wax and then sounding it again with B, 4 Beats are produced per second what is the frequency of the tuning fork A.

(a) 388 *Hz* (b) 80 *Hz* (c) 378 *Hz* (d) 390 *Hz*

Solution : (c)



Probable frequency of A is 390 *Hz* and 378 *Hz* and After loading the beats are decreasing from 6 to 4 so the original frequency of A will be $n_2 = n_1 - x = 378\ Hz$.

***Problem* 25.** Two sound waves of slightly different frequencies propagating in the same direction produces beats due to **[MP PET 2000]**

(a) Interference (b) Diffraction (c) Polarization (d) Refraction

Solution : (a)

***Problem* 26.** Beats are produced with the help of two sound waves on amplitude 3 and 5 units. The ratio of maximum to minimum intensity in the beats is **[MP PMT 1999]**

(a) 2 : 1 (b) 5 : 3 (c) 4 : 1 (d) 16 : 1

Solution : (d) $\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{5+3}{5-3}\right)^2 = 16:1$

***Problem* 27.** Two tuning forks have frequencies 380 and 384 hertz respectively. When they are sounded together, they produce 4 beats. After hearing the maximum sound, how long will it take to hear the minimum sound **[MP PMT/PET 1998]**

(a) 1/2 sec (b) 1/4 *sec* (c) 1/8 *sec* (d) 1/16 *sec*

Solution : (c) Beats period = Time interval between two minima

$$T = \frac{1}{n_1 - n_2} = \frac{1}{4}\text{ sec}$$

Time interval between maximum sound and minimum sound = $T/2 = 1/8$ *sec*

***Problem* 28.** Two tuning fork *A* and *B* give 4 beats per second when sounded together. The frequency of *A* is 320 *Hz*. When some wax is added to *B* and it is sounded with *A*, 4 beats per second are again heard. The frequency of *B* is

(a) 312 *Hz* (b) 316 *Hz* (c) 324 *Hz* (d) 328 *Hz*

Solution : (c) Since there is no change in beats. Therefore the original frequency of *B* is

$n_2 = n_1 + x = 320 + 4 = 324$

***Problem* 29.** 41 forks are so arranged that each produces 5 beat/sec when sounded with its near fork. If the frequency of last fork is double the frequency of first fork, then the frequencies of the first and last fork respectively **[MP PMT 1997]**

(a) 200, 400 (b) 205, 410 (c) 195, 390 (d) 100, 200

Solution : (a) Let the frequency of first tuning fork = n and that of last = $2n$

$n, n + 5, n + 10, n + 15 \ldots\ldots 2n$ this forms A.P.

Formula of A.P $l = a + (N - 1)\,r$ where l = Last term, a = First term, N = Number of term, r = Common difference

$$2n = n + (41 - 1)\,5$$

$$2n = n + 200$$

$$n = 200 \quad \text{and} \quad 2n = 400$$

***Problem* 30.** In stationary waves, antinodes are the points where there is **[MP PMT 1996]**

(a) Minimum displacement and minimum pressure change

(b) Minimum displacement and maximum pressure change

(c) Maximum displacement and maximum pressure change

(d) Maximum displacement and minimum pressure change

Solution : (d) At Antinodes displacement is maximum but pressure change is minimum.

Problem 31. The equation y = 0.15 sin $5x$ cos 300 t, describes a stationary wave. The wavelength of the stationary wave is

[MP PMT 1995]

(a) Zero *meter* (b) 1.256 *meter* (c) 2.512 *meter* (d) 0.628 *meter*

Solution : (b) By comparing with standard equation $\therefore \frac{2\pi x}{\lambda} = 5x \Rightarrow \lambda = \frac{2}{5} \times \pi$ = 1.256 *meter*

Problem 32. The equation of a stationary wave is y = 0.8 $\cos\left(\frac{\pi x}{20}\right) \sin 200\pi t$ where x is in *cm*. and t is in sec. The separation between consecutive nodes will be

(a) 20 *cm* (b) 10 *cm* (c) 40 *cm* (d) 30 *cm*

Solution : (a) Standard equation $y = A \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda}$

By comparing this equation with given equation. $\frac{2\pi x}{\lambda} = \frac{\pi x}{20} \Rightarrow \lambda$ = 40 *cm*

Distance Between two nodes = $\frac{\lambda}{2}$ = 20 *cm*.

Problem 33. Which of the property makes difference between progressive and stationary waves

(a) Amplitude (b) Frequency (c) Propagation of energy (d) Phase of the wave

Solution : (c) In stationary waves there is no transfer of energy.

Problem 34. If amplitude of waves at distance r from a point source is A, the amplitude at a distance $2r$ will be

[MP PMT 1985]

(a) $2A$ (b) A (c) $A/2$ (d) $A/4$

Solution : (c) $I \propto A^2$ and $I \propto \frac{1}{r^2}$ so $r \propto \frac{1}{A}$; $\frac{r_1}{r_2} = \frac{A_2}{A_1} \Rightarrow A_2 = A_1\left(\frac{r_1}{r_2}\right) = A\left(\frac{1}{2}\right) = A/2$

Problem 35. If two waves of same frequency and same amplitude respectively on superimposition produced a resultant disturbance of the same amplitude the wave differ in phase by **[MP PMT 1990]**

(a) π (b) $2\pi/3$ (c) $\pi/2$ (d) zero

Solution : (b) $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$A^2 = A^2 + A^2 + 2A^2\cos\phi$ [$A_1 = A_2 = A$ given]

$\cos\phi = -1/2 \Rightarrow \phi = 120° = \frac{2\pi}{3}$

Problem 36. The superposition takes place between two waves of frequency f and amplitude a. The total intensity is directly proportional to **[MP PMT 1986]**

(a) a (b) $2a$ (c) $2a^2$ (d) $4a^2$

Solution : (d) $I \propto (a_1 + a_2)^2$ [As $a_1 = a_2 = a$]

$I \propto 4a^2$

Problem 37. The following equation represent progressive transverse waves **[MP PET 1993]**

$z_1 = A\cos(\omega t - kx)$

$z_2 = A\cos(\omega t + kx)$

$z_3 = A\cos(\omega t + ky)$

$z_4 = A\cos(2\omega t - 2ky)$

A stationary wave will be formed by superposing

(a) z_1 and z_2 (b) z_1 and z_4 (c) z_2 and z_3 (d) z_3 and z_4

Solution : (a) The direction of wave must be opposite and frequencies will be same then by superposition, standing wave formation takes place.

Problem 38. When two sound waves with a phase difference of $\pi/2$ and each having amplitude A and frequency ω are superimposed on each other, then the maximum amplitude and frequency of resultant wave is **[MP PMT 1989]**

(a) $\frac{A}{\sqrt{2}};\omega/2$ (b) $\frac{A}{\sqrt{2}};\omega$ (c) $\sqrt{2}A;\frac{\omega}{2}$ (d) $\sqrt{2}A;\omega$

Solution : (d) Resultant Amplitude = $\sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} = \sqrt{A^2 + A^2 + 2A^2\cos\frac{\pi}{2}} = \sqrt{2}A$

and frequency remains same = ω.

Problem 39. There is a destructive interference between the two waves of wavelength λ coming from two different paths at a point. To get maximum sound or constructive interference at that point, the path of one wave is to be increased by **[MP PET 1985]**

(a) $\lambda/4$ (b) $\lambda/2$ (c) $\frac{3\lambda}{4}$ (d) λ

Solution : (b) Destructive interference means the path difference is $(2n-1)\frac{\lambda}{2}$

If it is increased by $\lambda/2$

Then new path difference $(2n-1)\frac{\lambda}{2} + \frac{\lambda}{2} = n\lambda$

which is the condition of constructive interference.

Problem 40. The tuning fork and sonometer wire were sounded together and produce 4 beats/second when the length of sonometer wire is 95 *cm* or 100 *cm*. The frequency of tuning fork is **[MP PMT 1990]**

(a) 156 *Hz* (b) 152 *Hz* (c) 148 *Hz* (d) 160 *Hz*

Solution : (a) Frequency $n \propto \frac{1}{l}$ $\therefore$ As $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

If n is the frequency of tuning fork.

$n + 4 \propto \frac{1}{95} \Rightarrow n - 4 \propto \frac{1}{100} \Rightarrow (n+4)\,95 = (n-4)\,100 \Rightarrow n = 156\ Hz.$

Problem 41. A tuning fork F_1 has a frequency of 256 *Hz* and it is observed to produce 6 beats/second with another tuning fork F_2. When F_2 is loaded with wax. It still produces 6 beats/second with F_1. The frequency of F_2 before loading was **[MP PET 1990]**

(a) 253 *Hz* (b) 262 *Hz* (c) 250 *Hz* (d) 259 *Hz*

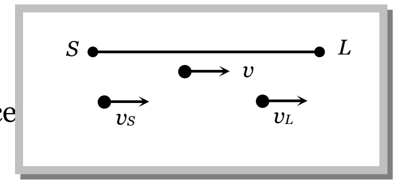
Solution : (b) No of beats does not change even after loading then $n_2 = n_1 + x = 256 + 6 = 262\ Hz$.

16.19 Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

When the distance between the source and listener is decreasing the apparent frequency increases. It means the apparent frequency is more than the actual frequency of sound. The reverse is also true.

General expression for apparent frequency $n' = \frac{[(v+v_m)-v_L]n}{[(v+v_m)-v_S]}$



Here n = Actual frequency; v_L = Velocity of listener; v_S = Velocity of source

v_m = Velocity of medium and v = Velocity of sound wave

Sign convention : All velocities

along the direction S to L are taken as positive and all velocities along the direction L to S are taken as negative. If the medium is stationary $v_m = 0$ then $n' = \left(\frac{v-v_L}{v-v_S}\right)n$

Special cases :

(1) Source is moving towards the listener, but the listener at rest $n' = \frac{v}{v-v_S}.n$

(2) Source is moving away from the listener but the listener is at rest $n' = \frac{v}{v+v_S}.n$

(3) Source is at rest and listener is moving away from the source $n' = \frac{v-v_L}{v}n$

(4) Source is at rest and listener is moving towards the source $n' = \frac{v+v_L}{v}.n$

(5) Source and listener are approaching each other $n' = \left(\frac{v+v_L}{v-v_S}\right)n$

(6) Source and listener moving away from each other $n' = \left(\frac{v-v_L}{v+v_S}\right)n$

(7) Both moves in the same direction with same velocity $n' = n$, *i.e.* there will be no Doppler effect because relative motion between source and listener is zero.

(8) Source and listener moves at right angle to the direction of wave propagation. $n' = n$

It means there is no change in frequency of sound heard if there is a small displacement of source and listener at right angle to the direction of wave propagation but for a large displacement the frequency decreases because the distance between source of sound and listener increases.

Important points

(i) If the velocity of source and listener is equal to or greater than the sound velocity then Doppler effect is not seen.

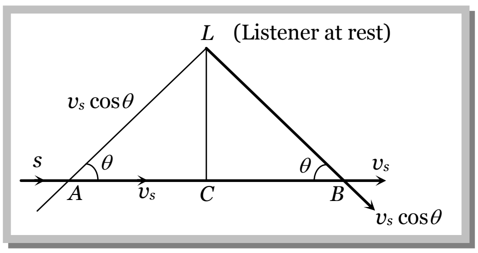
(ii) Doppler effect gives information regarding the change in frequency only. It does not says about intensity of sound.

(iii) Doppler effect in sound is asymmetric but in light it is symmetric.

16.20 Some Typical Features of Doppler's Effect in Sound

(1) **When a source is moving in a direction making an angle θ *w.r.t.* the listener :** The apparent frequency heard by listener L at rest

When source is at point A is $n' = \frac{nv}{v - v_s \cos\theta}$



As source moves along AB, value of θ increases, $\cos\theta$ decreases, n' goes on decreasing.

At point C, $\theta = 90^o$, $\cos\theta = \cos 90^o = 0$, $n' = n$.

At point B, the apparent frequency of sound becomes $n'' = \dfrac{nv}{v + v_s \cos\theta}$

(2) **When a source of sound approaches a high wall or a hill** with a constant velocity v_s, the reflected sound propagates in a direction opposite to that of direct sound. We can assume that the source and observer are approaching each other with same velocity *i.e.* $v_s = v_L$

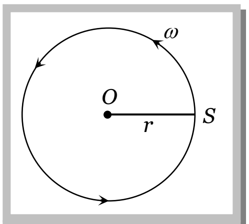
$$\therefore \quad n' = \left(\frac{v + v_L}{v - v_s}\right) n$$

(3) **When a listener moves between two distant sound sources :** Let v_L be the velocity of listener away from S_1 and towards S_2. Apparent frequency from S_1 is $n' = \dfrac{(v - v_L)n}{v}$

and apparent frequency heard from S_L is $n'' = \dfrac{(v + v_L)n}{v}$

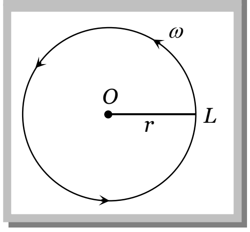
$\therefore$ Beat frequency $= n'' - n' = \dfrac{2nv_L}{v}$

(4) **When source is revolving in a circle and listener *L* is on one side**



$v_s = r\omega$ so $n_{max} = \dfrac{nv}{v - v_s}$ and $n_{min} = \dfrac{nv}{v + v_s}$

(5) **When listener *L* is moving in a circle and the source is on one side**



$v_L = r\omega$ so $n_{max} = \dfrac{(v + v_L)n}{v}$ and $n_{min} = \dfrac{(v - v_L)n}{v}$

(6) There will be no change in frequency of sound heard, if the source is situated at the centre of the circle along which listener is moving.

(7) **Conditions for no Doppler effect :** (i) When source (*S*) and listener (*L*) both are at rest.

(ii) When medium alone is moving.

(iii) When *S* and *L* move in such a way that distance between *S* and *L* remains constant.

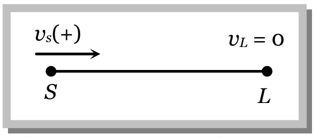
(iv) When source *S* and listener *L*, are moving in mutually perpendicular directions.

Sample problems based on Doppler effect

Problem 42. A source of sound of frequency 90 *vibration/sec* is approaching a stationary observer with a speed equal to 1/10 the speed of sound. What will be the frequency heard by the observer **[MP PMT 2000]**

(a) 80 *vibration/sec* (b) 90 *vibration/sec* (c) 100 *vibration/sec* (d) 120 *vibration/sec*

Solution : (c) $n' = \dfrac{v}{v - v_s}.n \Rightarrow n' = \dfrac{v}{v - \dfrac{v}{10}}.n \Rightarrow n' = \dfrac{10}{9}n = \dfrac{10 \times 90}{9} = 100$ vibration/sec



Problem 43. A source of sound of frequency 500 *Hz* is moving towards an observer with velocity 30 *m/s*. The speed of the sound is 330 *m/s*. The frequency heard by the observer will be **[MP PET 2000]**

(a) 550 *Hz* (b) 458.3 *Hz* (c) 530 *Hz* (d) 545.5 *Hz*

Solution : (a) $n' = \dfrac{v}{v - v_s}.n \Rightarrow n' = \dfrac{330}{330 - 30}.500 \Rightarrow n' = 550\ Hz$

Problem 44. A motor car blowing a horn of frequency 124 *vibration/sec* moves with a velocity 72 *km/hr* towards a tall wall. The frequency of the reflected sound heard by the driver will be (velocity of sound in air is 330 *m/s*) **[MP PET 1997]**

(a) 109 *vibration/sec* (b) 132 *vibration/sec* (c) 140 *vibration/sec* (d) 248 *vibration/sec*

Solution : (c) In the given condition source and listener are at the same position *i.e.* (car) for given condition

$$n' = \frac{v + v_{car}}{v - v_{car}}.n = \frac{330+20}{330-20}.n = 140 \text{ vibration/sec}$$

Problem 45. The driver of car travelling with a speed 30 *meter/sec.* towards a hill sounds a horn of frequency 600 *Hz*. If the velocity of sound in air is 330 m/s the frequency of reflected sound as heard by the driver is **[MP PMT 1996**

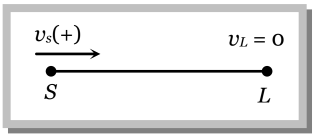
(a) 720 *Hz* (b) 555.5 *Hz* (c) 550 *Hz* (d) 500 *Hz*

Solution : (a) This question is same as that of previous one so $n' = \frac{v+v_{car}}{v-v_{car}}.n = 720\ Hz$

Problem 46. The source of sound s is moving with a velocity 50 *m/s* towards a stationary observer. The observer measures the frequency of the source as 1000 *Hz*. What will be the apparent frequency of the source when it is moving away from the observer after crossing him ? The velocity of sound in the medium is 350 *m/s* **[MP PM**

(a) 750 *Hz* (b) 857 *Hz* (c) 1143 *Hz* (d) 1333 *Hz*

Solution : (a) When source is moving towards the stationary listener.



$$n' = \frac{v}{v - v_s}n \Rightarrow 1000 = \frac{350}{350-50}.n \Rightarrow n = 857.14$$

When source is moving away from the stationary observer $n'' = \frac{v}{v+v_s} = \frac{350}{350+50} \times 857 = 750\ Hz$

Problem 47. A source and listener are both moving towards each other with speed $v/10$ where v is the speed of sound. If the frequency of the note emitted by the source is *f*, the frequency heard by the listener would be nearly **[MP PMT 1994]**

(a) 1.11 *f* (b) 1.22 *f* (c) *f* (d) 1.27 *f*

Solution : (b) $n' = \left(\frac{v+v_L}{v-v_s}\right)n \Rightarrow n' = \left(\frac{v+\frac{v}{10}}{v-\frac{v}{10}}\right)n \Rightarrow n' = \frac{11}{9} f = 1.22 f.$

Problem 48. A man is watching two trains, one leaving and the other coming in with equal speed of 4 *m/s*. If they sound their whistles, each of frequency 240 *Hz*, the number of beats heard by the man (velocity of sound in air = 320 *m/s*) will be equal to **[MP PET 1999; CPMT 1997; NCERT 1984]**

(a) 6 (b) 3 (c) 0 (d) 12

Solution : (a) App. Frequency due to train which is coming in $n_1 = \frac{v}{v-v_s}.n$

App. Frequency due to train which is leaving $n_2 = \frac{v}{v+v_s}.n$

So number of beats $n_1 - n_2 = \left(\frac{1}{316} - \frac{1}{324}\right)320 \times 240 \Rightarrow n_1 - n_2 = 6$

Problem 49. At what speed should a source of sound move so that observer finds the apparent frequency equal to half of the original frequency **[RPMT 1996]**

(a) $v/2$ (b) $2v$ (c) $v/4$ (d) v

Solution : (d) $n' = \frac{v}{v + v_s}.n \Rightarrow \frac{n}{2} = \frac{v}{v + v_s}.n \Rightarrow v_s = v$

Practice Problems

Problems based on Pressure

Basic level

1. The pressure at the bottom of a tank containing a liquid does not depend on **[Kerala (Engg.) 2002]**

(a) Acceleration due to gravity (b) Height of the liquid column

(c) Area of the bottom surface (d) Nature of the liquid

2. When a large bubble rises from the bottom of a lake to the surface. Its radius doubles. If atmospheric pressure is equal to that of column of water height H, then the depth of lake is **[AIIMS 1995; AFMC 1997]**

(a) H (b) $2H$ (c) $7H$ (d) $8H$

3. The volume of an air bubble becomes three times as it rises from the bottom of a lake to its surface. Assuming atmospheric pressure to be 75 cm of Hg and the density of water to be 1/10 of the density of mercury, the depth of the lake is

(a) 5 m (b) 10 m (c) 15 m (d) 20 m

4. The value of g at a place decreases by 2%. The barometric height of mercury

(a) Increases by 2% (b) Decreases by 2%

(c) Remains unchanged (d) Sometimes increases and sometimes decreases

5. Two stretched membranes of area 2 cm^2 and 3 cm^2 are placed in a liquid at the same depth. The ratio of pressures on them is

(a) 1 : 1 (b) 2 : 3 (c) 3 : 2 (d) $2^2 : 3^2$

6. Three identical vessels are filled to the same height with three different liquids A, B and $C(\rho_A > \rho_B > \rho_C)$. The pressure at the base will be

(a) Equal in all vessels (b) Maximum in vessel A (c) Maximum in vessel B (d) Maximum in vessel C

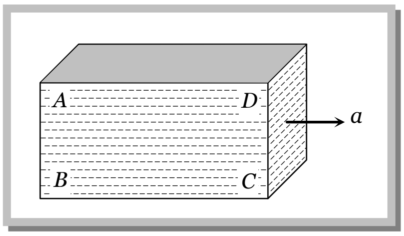
7. Three identical vessels are filled with equal masses of three different liquids A, B and C $(\rho_A > \rho_B > \rho_C)$. The pressure at the base will be

(a) Equal in all vessels (b) Maximum in vessel A (c) Maximum in vessel B (d) Maximum in vessel C

8. A barometer kept in a stationary elevator reads 76 cm. If the elevator starts accelerating up the reading will be

(a) Zero (b) Equal to 76 cm (c) More than 76 cm (d) Less than 76 cm

9. A closed rectangular tank is completely filled with water and is accelerated horizontally with an acceleration a towards right. Pressure is (i) maximum at, and (ii) minimum at



(a) (i) B (ii) D

(b) (i) C (ii) D

(c) (i) B (ii) C

(d) (i) B (ii) A

10. A beaker containing a liquid is kept inside a big closed jar. If the air inside the jar is continuously pumped out, the pressure in the liquid near the bottom of the liquid will

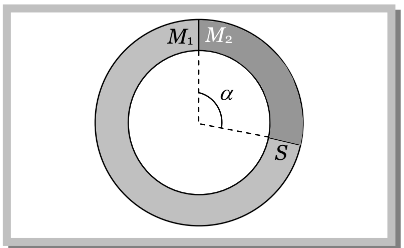
(a) Increases (b) Decreases (c) Remain constant (d) First decrease and then increase

11. A barometer tube reads 76 cm of mercury. If the tube is gradually inclined at an angle of 60° with vertical, keeping the open end immersed in the mercury reservoir, the length of the mercury column will be

(a) 152 cm (b) 76 cm (c) 38 cm (d) $38\sqrt{3}cm$

▶▶ *Advance level*

12. A ring shaped tube contains two ideal gases with equal masses and molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle α in degrees is



(a) 192

(b) 291

(c) 129

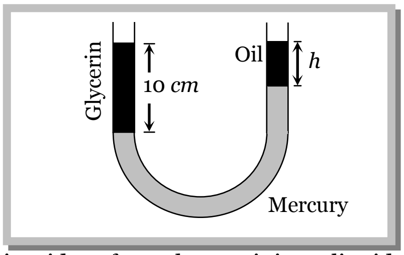
(d) 219

13. The height to which a cylindrical vessel be filled with a homogeneous liquid, to make the average force with which the liquid presses the side of the vessel equal to the force exerted by the liquid on the bottom of the vessel, is equal to

(a) Half of the radius of the vessel (b) Radius of the vessel

(c) One-fourth of the radius of the vessel (d) Three-fourth of the radius of the vessel

14. A vertical U-tube of uniform inner cross section contains mercury in both sides of its arms. A glycerin (density = 1.3 g/cm^3) column of length 10 cm is introduced into one of its arms. Oil of density 0.8 gm/cm^3 is poured into the other arm until the upper surfaces of the oil and glycerin are in the same horizontal level. Find the length of the oil column, Density of mercury = 13.6 g/cm^3



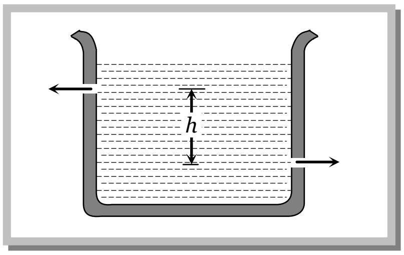
(a) 10.4 cm

(b) 8.2 cm

(c) 7.2 cm

(d) 9.6 cm

15. There are two identical small holes of area of cross-section a on the opposite sides of a tank containing a liquid of density ρ. The difference in height between the holes is h. Tank is resting on a smooth horizontal surface. Horizontal force which will has to be applied on the tank to keep it in equilibrium is



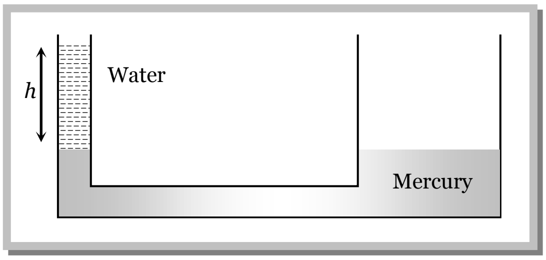
(a) $gh\rho a$

(b) $\frac{2gh}{\rho a}$

(c) $2\rho agh$

(d) $\frac{\rho gh}{a}$

16. Two communicating vessels contain mercury. The diameter of one vessel is n times larger than the diameter of the other. A column of water of height h is poured into the left vessel. The mercury level will rise in the right-hand vessel (s = relative density of mercury and ρ = density of water) by



(a) $\frac{n^2h}{(n+1)^2 s}$

(b) $\frac{h}{(n^2+1)s}$

(c) $\frac{h}{(n+1)^2 s}$

(d) $\frac{h}{n^2 s}$

17. A triangular lamina of area A and height h is immersed in a liquid of density ρ in a vertical plane with its base on the surface of the liquid. The thrust on the lamina is

(a) $\frac{1}{2}A\rho gh$ (b) $\frac{1}{3}A\rho gh$ (c) $\frac{1}{6}A\rho gh$ (d) $\frac{2}{3}A\rho gh$

Problems based on Pascal's law

18. Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel. This law was first formulated by

(a) Bernoulli (b) Archimedes (c) Boyle (d) Pascal

19. A piston of cross-section area 100 cm^2 is used in a hydraulic press to exert a force of 10^7 dynes on the water. The cross-sectional area of the other piston which supports an object having a mass 2000 kg. is

(a) 100 cm^2 (b) $10^9 cm^2$ (c) $2 \times 10^4 cm^2$ (d) $2 \times 10^{10} cm^2$

Problems based on Archimedes principle

20. A block of steel of size 5 *cm* × 5 *cm* × 5 *cm* is weighed in water. If the relative density of steel is 7, its apparent weight is **[AFMC 1997]**

(a) $6 \times 5 \times 5 \times 5\, gf$ (b) $4 \times 4 \times 4 \times 7\, gf$ (c) $5 \times 5 \times 5 \times 7\, gf$ (d) $4 \times 4 \times 4 \times 6\, gf$

21. A body is just floating on the surface of a liquid. The density of the body is same as that of the liquid. The body is slightly pushed down. What will happen to the body **[AIIMS 1980]**

(a) It will slowly come back to its earlier position (b) It will remain submerged, where it is left

(c) It will sink (d) It will come out violently

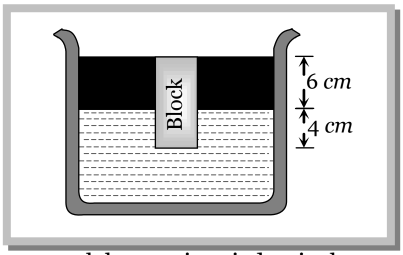
22. A uniform rod of density ρ is placed in a wide tank containing a liquid of density $\rho_0 (\rho_0 > \rho)$. The depth of liquid in the tank is half the length of the rod. The rod is in equilibrium, with its lower end resting on the bottom of the tank. In this position the rod makes an angle θ with the horizontal

(a) $\sin\theta = \frac{1}{2}\sqrt{\rho_0 / \rho}$ (b) $\sin\theta = \frac{1}{2} . \frac{\rho_0}{\rho}$ (c) $\sin\theta = \sqrt{\rho / \rho_0}$ (d) $\sin\theta = \rho_0 / \rho$

23. A cork is submerged in water by a spring attached to the bottom of a bowl. When the bowl is kept in an elevator moving with acceleration downwards, the length of spring

(a) Increases (b) Decreases (c) Remains unchanged (d) None of these

24. A cubical block of wood 10 cm on a side floats at the interface between oil and water with its lower surface horizontal and 4 cm below the interface. The density of oil is $0.6 gcm^{-1}$. The mass of block is



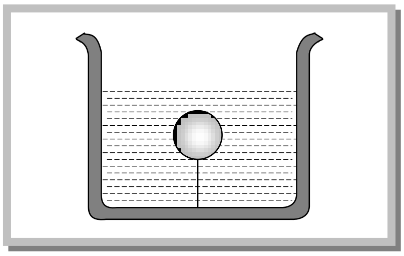
(a) 706 *g*

(b) 607 *g*

(c) 760 *g*

(d) 670 *g*

25. A solid sphere of density η (> 1) times lighter than water is suspended in a water tank by a string tied to its base as shown in fig. If the mass of the sphere is *m* then the tension in the string is given by



(a) $\left(\frac{\eta - 1}{\eta}\right) mg$

(b) ηmg

(c) $\frac{mg}{\eta - 1}$

(d) $(\eta - 1) mg$

26. A spherical ball of radius *r* and relative density 0.5 is floating in equilibrium in water with half of it immersed in water. The work done in pushing the ball down so that whole of it is just immersed in water is : (where ρ is the density of water)

(a) $\frac{5}{12}\pi r^4 \rho g$ (b) $0.5 \rho r g$ (c) $\frac{4}{3}\pi r^3 \rho g$ (d) $\frac{2}{3}\pi r^4 \rho g$

27. A hollow sphere of volume *V* is floating on water surface with *half* immersed in it. What should be the minimum volume of water poured inside the sphere so that the sphere now sinks into the water

(a) $V/2$ (b) $V/3$ (c) $V/4$ (d) V

28. A rectangular block is 5 *cm* × 5 *cm* × 10*cm* in size. The block is floating in water with 5 *cm* side vertical. If it floats with 10 *cm* side vertical, what change will occur in the level of water?

(a) No change (b) It will rise

(c) It will fall (d) It may rise or fall depending on the density of block

29. A ball whose density is $0.4 \times 10^3\ kg/m^3$ falls into water from a height of 9 *cm* . To what depth does the ball sink

(a) 9 *cm* (b) 6 *cm* (c) 4.5 *cm* (d) 2.25 *cm*

30. Two solids *A* and *B* float in water. It is observed that A floats with $\frac{1}{2}$ of its body immersed in water and *B* floats with $\frac{1}{4}$ of its volume above the water level. The ratio of the density of *A* to that of *B* is

(a) 4 : 3 (b) 2 : 3 (c) 3 : 4 (d) 1 : 2

31. A boat carrying steel balls is floating on the surface of water in a tank. If the balls are thrown into the tank one by one, how will it affect the level of water

(a) It will remain unchanged (b) It will rise (c) It will fall (d)

32. Two pieces of metal when immersed in a liquid have equal upthrust on them; then

(a) Both pieces must have equal weights (b) Both pieces must have equal densities

(c) Both pieces must have equal volumes (d) Both are floating to the same depth

33. A wooden cylinder floats vertically in water with half of its length immersed. The density of wood is

(a) Equal of that of water (b) Half the density of water

(c) Double the density of water (d) The question is incomplete

34. If *W* be the weight of a body of density ρ in vacuum then its apparent weight in air of density σ is

(a) $\frac{W\rho}{\sigma}$ (b) $W\left(\frac{\rho}{\sigma}-1\right)$ (c) $\frac{W}{\rho}\sigma$ (d) $W\left(1-\frac{\sigma}{\rho}\right)$

Problems based on Density

35. A block of ice floats on a liquid of density 1.2in a beaker then level of liquid when ice completely melt **[IIT-JEE 1994]**

(a) Remains same (b) rises (c) Lowers (d) (A), (B) or (c)

36. If two liquids of same masses but densities ρ_1 and ρ_2 respectively are mixed, then density of mixture is given by

(a) $\rho = \frac{\rho_1+\rho_2}{2}$ (b) $\rho = \frac{\rho_1+\rho_2}{2\rho_1\rho_2}$ (c) $\rho = \frac{2\rho_1\rho_2}{\rho_1+\rho_2}$ (d) $\rho = \frac{\rho_1\rho_2}{\rho_1+\rho_2}$

37. If two liquids of same volume but different densities ρ_1 and ρ_2 are mixed, then density of mixture is given by

(a) $\rho = \frac{\rho_1+\rho_2}{2}$ (b) $\rho = \frac{\rho_1+\rho_2}{2\rho_1\rho_2}$ (c) $\rho = \frac{2\rho_1\rho_2}{\rho_1+\rho_2}$ (d) $\rho = \frac{\rho_1\rho_2}{\rho_1+\rho_2}$

38. The density ρ of water of bulk modulus *B* at a depth *y* in the ocean is related to the density at surface ρ_0 by the relation

(a) $\rho = \rho_0\left[1-\frac{\rho_0 gy}{B}\right]$ (b) $\rho = \rho_0\left[1+\frac{\rho_0 gy}{B}\right]$ (c) $\rho = \rho_0\left[1+\frac{B}{\rho_0 hgy}\right]$ (d) $\rho = \rho_0\left[1-\frac{B}{\rho_0 gy}\right]$

39. With rise in temperature, density of a given body changes according to one of the following relations

(a) $\rho = \rho_0[1+\gamma\, d\theta]$ (b) $\rho = \rho_0[1-\gamma\, d\theta]$ (c) $\rho = \rho_0\gamma d\theta$ (d) $\rho = \rho_0 / \gamma d\theta$

40. Three liquids of densities $d, 2d$ and $3d$ are mixed in equal volumes. Then the density of the mixture is

(a) d (b) $2d$ (c) $3d$ (d) $5d$

41. Three liquids of densities $d, 2d$ and $3d$ are mixed in equal proportions of weights. The relative density of the mixture is

(a) $\frac{11d}{7}$ (b) $\frac{18d}{11}$ (c) $\frac{13d}{9}$ (d) $\frac{23d}{18}$

Problems based on Streamlined & turbulent flow

42. Stream-line flow is more likely for liquids with **[Pb. CET 1997]**

(a) Low density and low viscosity (b) High viscosity and high density

(c) High viscosity and low density (d) Low viscosity and high density

43. In a laminar flow the velocity of the liquid in contact with the walls of the tub is

(a) Zero (b) Maximum

(c) In between zero and maximum (d) Equal to critical velocity

44. In a turbulent flow, the velocity of the liquid molecules in contact with the walls of the tube is

(a) Zero (b) Maximum

(c) Equal to critical velocity (d) May have any value

45. Which of the following is NOT the characteristic of turbulent flow

(a) Velocity more than the critical velocity (b) Velocity less than the critical velocity

(c) Irregular flow (d) Molecules crossing from one layer to another

46. The Reynolds number of a flow is the ratio of

(a) Gravity to viscous force (b) Gravity force to pressure force

(c) Inertia forces to viscous force (d) Viscous forces to pressure forces

Problems based on Equation of Continuity

47. Water is flowing through a tube of non-uniform cross-section ratio of the radius at entry and exit end of the pipe is 3 : 2. Then the ratio of velocities at entry and exit of liquid is **[RPMT 2001]**

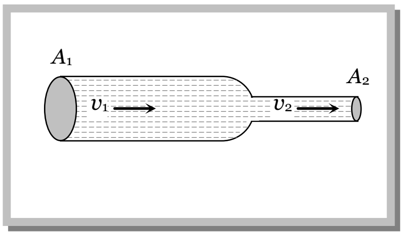
(a) 4 : 9 (b) 9 : 4 (c) 8 : 27 (d) 1 : 1

48. Water is flowing through a horizontal pipe of non-uniform cross-section. At the extreme narrow portion of the pipe, the water will have

(a) Maximum speed and least pressure (b) Maximum pressure and least speed

(c) Both pressure and speed maximum (d) Both pressure and speed least

49. A liquid flows in a tube from left to right as shown in figure. A_1 and A_2 are the cross-sections of the portions of the tube as shown. Then the ratio of speeds v_1 / v_2 will be



(a) A_1 / A_2

(b) A_2 / A_1

(c) $\sqrt{A_2} / \sqrt{A_1}$

(d) $\sqrt{A_1} / \sqrt{A_2}$

50. In a streamline flow

(a) The speed of a particle always remains same

(b) The velocity of a particle always remains same

(c) The kinetic energies of all the particles arriving at a given point are the same

(d) The moments of all the particles arriving at a given point are the same

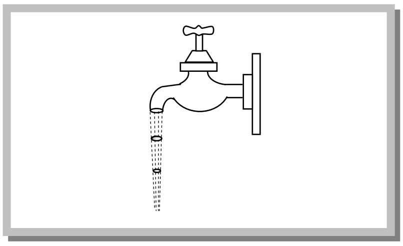
51. Water coming out of the mouth of a tap and falling vertically in streamline flow forms a tapering, column, *i.e.,* the area of cross-section of the liquid column decreases as it moves down. Which of the following is the most accurate explanation for this

(a) As the water moves down, its speed increases and hence its pressure decreases. It is then compressed by the atmosphere

(b) Falling water tries to reach a terminal velocity and hence reduces the area of cross-section to balance upward and downward forces

(c) The mass of water flowing past any cross-section must remain constant. Also, water is almost incompressible. Hence, the rate of volume flow must remain constant. As this is equal to velocity × area, the area decreases as velocity increases

(d) The surface tension causes the exposed surface area of the liquid to decrease continuously



Problems based on Equation of Bernoulli's Theorem

52. An application of Bernoulli's equation for fluid flow is found in **[IIT-JEE (Screening) 1994]**

(a) Dynamic lift of an aeroplane (b) Viscosity meter

(c) Capillary rise (d) Hydraulic press

53. The Working of an atomizer depends upon **[MP PMT 1992]**

(a) Bernoulli's theorem (b) Boyle's law (c) Archimedes principle (d) Newton's law of motion

54. The pans of a physical balance are in equilibrium. Air is blown under the right hand pan; then the right hand pan will

(a) Move up (b) Move down (c) Move erratically (d) Remain at the same level

55. According to Bernoulli's equation

$$\frac{P}{\rho g}+h+\frac{1}{2}\frac{v^2}{g}=\text{constant}$$

The terms *A*, *B* and *C* are generally called respectively:

(a) Gravitational head, pressure head and velocity head (b) Gravity, gravitational head and velocity head

(c) Pressure head, gravitational head and velocity head (d) Gravity, pressure and velocity head

56. At what speed the velocity head of a stream of water be equal to 40 *cm* of *Hg*

(a) 1032.6 *cm/sec* (b) 432.6 *cm/sec* (c) 632.6 *cm/sec* (d) 832.6 *cm/sec*

57. The weight of an aeroplane flying in air is balanced by

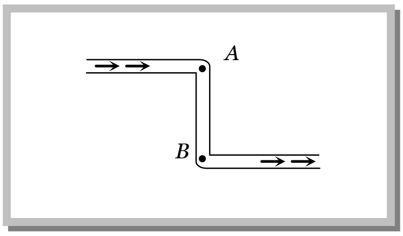
(a) Upthrust of the air which will be equal to the weight of the air having the same volume as the plane

(b) Force due to the pressure difference between the upper and lower surfaces of the wings, created by different air speeds on the surface

(c) Vertical component of the thrust created by air currents striking the lower surface of the wings

(d) Force due to the reaction of gases ejected by the revolving propeller

58. In this figure, an ideal liquid flows through the tube, which is of uniform cross-section. The liquid has velocities v_A and v_B, and pressure P_A and P_B at points *A* and *B* respectively



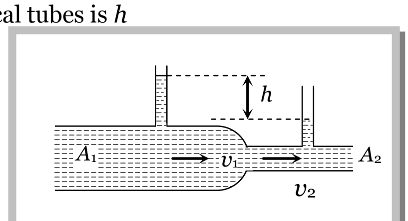
(a) $v_A=v_B$

(b) $v_B>v_A$

(c) $P_A=P_B$

(d) $P_B>P_A$

59. A liquid flows through a horizontal tube. The velocities of the liquid in the two sections, which have areas of cross-section A_1 and A_2, are v_1 and v_2 respectively. The difference in the levels of the liquid in the two vertical tubes is *h*



(a) The volume of the liquid flowing through the tube in unit time is A_1v_1

(b) $v_2 - v_1 = \sqrt{2gh}$

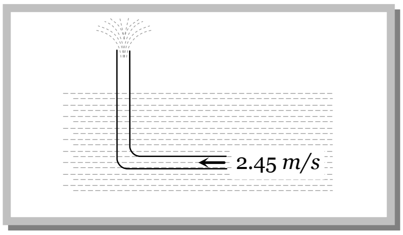
(c) $v_2^2 - v_1^2 = 2gh$

(d) The energy per unit mass of the liquid is the same in both sections of the tube

60. A sniper fires a rifle bullet into a gasoline tank making a hole 53.0 *m* below the surface of gasoline. The tank was sealed at 3.10 *atm*. The stored gasoline has a density of 660 kgm^{-3}. The velocity with which gasoline begins to shoot out of the hole is

(a) $27.8\, ms^{-1}$ (b) $41.0\, ms^{-1}$ (c) $9.6\, ms^{-1}$ (d) $19.7\, ms^{-1}$

61. An L-shaped tube with a small orifice is held in a water stream as shown in fig. The upper end of the tube is 10.6 *cm* above the surface of water. What will be the height of the jet of water coming from the orifice? Velocity of water stream is 2.45 *m/s*



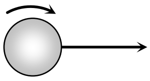
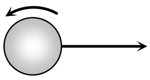
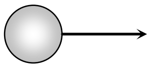
(a) Zero

(b) 20.0 *cm*

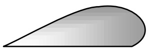
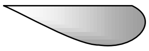
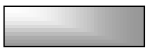
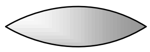
(c) 10.6 *cm*

(d) 40.0 *cm*

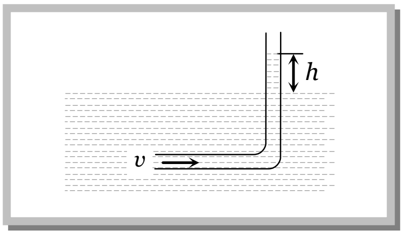
62. To get the maximum flight, a ball must be thrown as

(a)  v (b)  v (c)  v (d) Any of (a), (b) and (c)

63. Fig. represents vertical sections of four wings moving horizontally in air. In which case is the force upwards

(a)  (b)  (c)  (d) 

64. An L-shaped glass tube is just immersed in flowing water such that its opening is pointing against flowing water. If the speed of water current is v, then



(a) The water in the tube rises to height $\frac{v^2}{2g}$

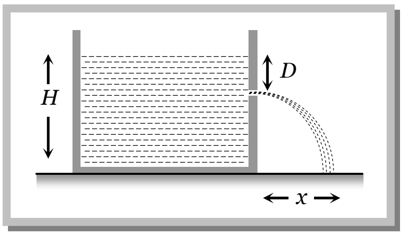
(b) The water in the tube rises to height $\frac{g}{2v^2}$

(c) The water in the tube does not rise at all

(d) None of these

Problems based on Velocity of Efflux

65. A tank is filled with water up to a height *H*. Water is allowed to come out of a hole *P* in one of the walls at a depth *D* below the surface of water. Express the horizontal distance *x* in terms of *H* and *D*



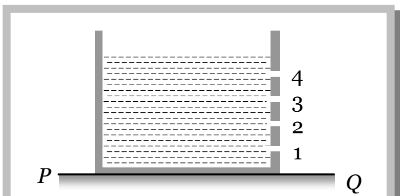
(a) $x = \sqrt{D(H-D)}$

(b) $x = \sqrt{\frac{D(H-D)}{2}}$

(c) $x = 2\sqrt{D(H-D)}$

(d) $x = 4\sqrt{D(H-D)}$

66. A cylindrical vessel of 90 *cm* height is kept filled upto the brim. It has four holes 1, 2, 3, 4 which are respectively at heights of 20 *cm*, 30 *cm*, 45 *cm* and 50 *cm* from the horizontal floor *PQ*. The water falling at the maximum horizontal distance from the vessel comes from **[CPMT 1989]**



(a) Hole number 4

(b) Hole number 3

(c) Hole number 2

(d) Hole number 1

67. A rectangular vessel when full of water takes 10 minutes to be emptied through an orifice in its bottom. How much time will it take to be emptied when half filled with water

(a) 9 *minute* (b) 7 *minute* (c) 5 *minute* (d) 3 *minute*

68. A vessel of area of cross-section *A* has liquid to a height *H*. There is a hole at the bottom of vessel having area of cross-section *a*. The time taken to decrease the level from H_1 to H_2 will be

(a) $\frac{A}{a}\sqrt{\frac{2}{g}}[\sqrt{H_1}-\sqrt{H_2}]$ (b) $\sqrt{2gh}$ (c) $\sqrt{2gh(H_1-H_2)}$ (d) $\frac{A}{a}\sqrt{\frac{g}{2}}[\sqrt{H_1}-\sqrt{H_2}]$

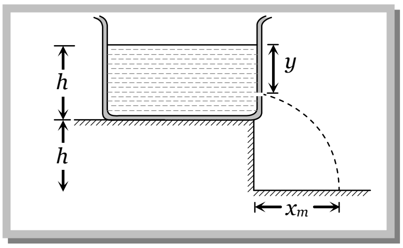
69. A streamlined body falls through air from a height h on the surface of a liquid. If *d* and (*D* > *d*) represents the densities of the material of the body and liquid respectively, then the time after which the body will be instantaneously at rest, is

(a) $\sqrt{\frac{2h}{g}}$ (b) $\sqrt{\frac{2h}{g}\cdot\frac{D}{d}}$ (c) $\sqrt{\frac{2h}{g}\cdot\frac{d}{D}}$ (d) $\sqrt{\frac{2h}{g}}\left(\frac{d}{D-d}\right)$

70. A large tank is filled with water to a height *H*. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to $\frac{H}{\eta}(\eta>1)$; and it takes T_2 time to take out the rest of water. If $T_1=T_2$, then the value of η is

(a) 2 (b) 3 (c) 4 (d) $2\sqrt{2}$

71. A tank is filled upto a height *h* with a liquid and is placed on a platform of height *h* from the ground. To get maximum range x_m a small hole is punched at a distance of *y* from the free surface of the liquid. Then



(a) $x_m=2h$

(b) $x_m=1.5h$

(c) $y=h$

(d) $y=0.75h$

Problems based on Viscosity

72. Velocity of water in a river is **[CBSE PMT 1988]**

(a) Same everywhere (b) More in the middle and less near its banks

(c) Less in the middle and more near its banks (d) Increase from one bank to other bank

73. As the temperature of water increases, its viscosity

(a) Remains unchanged (b) Decreases

(c) Increases (d) Increases or decreases depending on the external pressure

74. The coefficient of viscosity for hot air is

(a) Greater than the coefficient of viscosity for cold air (b) Smaller than the coefficient of viscosity for cold air

(c) Same as the coefficient of viscosity for cold air (d) Increases or decreases depending on the external pressure

75. A good lubricant should have

(a) High viscosity (b) Low viscosity (c) Moderate viscosity (d) High density

76. The relative velocity of two consecutive layers is 8 *cm/s*. If the perpendicular distance between the layers is 0.1 *cm*, then the velocity gradient will be

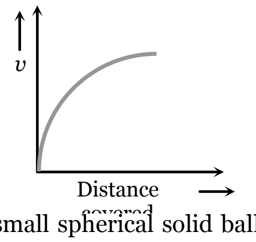
(a) 8sec^{-1} (b) 80 sec^{-1} (c) 0.8 sec^{-1} (d) 0.08 sec^{-1}

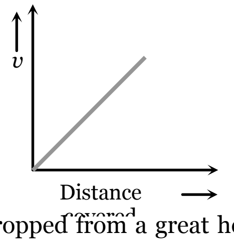
77. We have three beakers A, B and C containing glycerine, water and kerosene respectively. They are stirred vigorously and placed on a table. The liquid which comes to rest at the earliest is

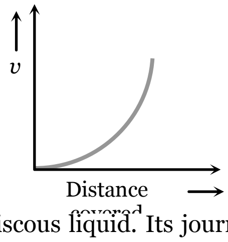
(a) Glycerine (b) Water (c) Kerosene (d) All of them at the same time

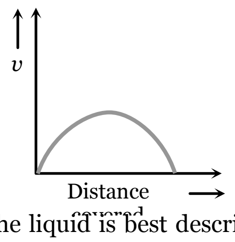
Problems based on Stoke's law and Terminal velocity

78. A lead shot of 1*mm* diameter falls through a long column of glycerine. The variation of its velocity v. with distance covered is represented by **[AIIMS 2003]**

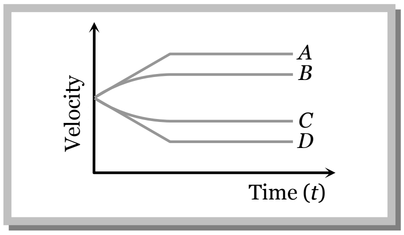
(a) 

(b) 

(c) 

(d) 

79. A small spherical solid ball is dropped from a great height in a viscous liquid. Its journey in the liquid is best described in the diagram given below by the **[CPMT 1988]**



(a) Curve A

(b) Curve B

(c) Curve C

(d) Curve D

80. A small drop of water falls from rest through a large height h in air; the final velocity is

(a) $\propto \sqrt{h}$ (b) $\propto h$ (c) $\propto (1/h)$ (d) Almost independent of h

Problems based on Poiseuille's law

81. The rate of flow of liquid in a tube of radius r, length l, whose ends are maintained at a pressure difference P is $V = \frac{\pi QP\, r^4}{\eta l}$ where η is coefficient of the viscosity and Q is **[DCE 2002]**

(a) 8 (b) $\frac{1}{8}$ (c) 16 (d) $\frac{1}{16}$

82. In Poiseuilli's method of determination of coefficient of viscosity, the physical quantity that requires greater accuracy in measurement is **[EAMCET 2001]**

(a) Pressure difference (b) Volume of the liquid collected

(c) Length of the capillary tube (d) Inner radius of the capillary tube

83. Two capillary tubes of the same length but different radii r_1 and r_2 are fitted in parallel to the bottom of a vessel. The pressure head is P. What should be the radius of a single tube that can replace the two tubes so that the rate of flow is same as before

(a) $r_1 + r_2$ (b) $r_1^2 + r_2^2$ (c) $r_1^4 + r_2^4$ (d) None of these

84. Under a constant pressure head, the rate of flow of liquid through a capillary tube is V. If the length of the capillary is doubled and the diameter of the bore is halved, the rate of flow would become

(a) $V/4$ (b) $16\,V$ (c) $V/8$ (d) $V/32$

85. Two capillaries of same length and radii in the ratio 1 : 2 are connected in series. A liquid flows through them in streamlined condition. If the pressure across the two extreme ends of the combination is 1 m of water, the pressure difference cross first capillary is

(a) 9.4 m (b) 4.9 m (c) 0.49 m (d) 0.94 m

86. Water flows in a streamlined manner through a capillary tube of radius a, the pressure difference being P and the rate of flow Q. If the radius is reduced to $a/2$ and the pressure increased to $2P$, the rate of flow becomes

(a) $4Q$ (b) Q (c) $\frac{Q}{4}$ (d) $\frac{Q}{8}$

FORMULA BANK THERMODYNAMICS

1. Relation between different scales of temperature

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80} = \frac{T_K - 273 \cdot 15}{100}$$

2. $T_C = \frac{5}{9}(T_F - 32)$

3. $T_F = \frac{9}{5}T_C + 32$

4. $-40°C$ has same value on Celcius and Fahrenheit scales.

5. Triple point of water on absolute scale of temperature is $273 \cdot 16\, k$.

6. Faulty Thermometer.

$$\frac{\text{False reading} - \text{lower point}}{\text{range}} = \frac{\text{True reading} - \text{lower point}}{\text{range}}$$

7. Co-efficient of linear expansion

$$\alpha = \frac{\Delta l}{l \Delta T}$$

$$l' = l(1 + \alpha \Delta T)$$

8. Coefficient of superficial expansion

$$\beta = \frac{\Delta S}{S \Delta T}$$

$$S' = S(1 + \beta \Delta T).$$

9. Coefficient of cubic expansion

$$\gamma = \frac{\Delta V}{V \Delta T}$$

$$V' = V(1 + \gamma \Delta T).$$

10. Relation between α, β and γ

$$6\alpha = 3\beta = 2\gamma$$

or $\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$

11. Heat supplied to a solid of mass m for increasing temperature ΔT is $Q = mC\Delta T$.

12. Heat supplied to change its state at constant temperature $Q = mL$

13. Gases possess infinite values of specific heat but we consider only two specific heats C_P and C_v.

14. Mayer's formula $C_P - C_v = R$.

15. For monoatomic gas, $f = 3$

$$C_v = \frac{3}{2}R \text{ and } C_P = \frac{5}{2}R \text{ and } \gamma = \frac{5}{3} = 1 \cdot 67$$

16. For diatomic gas $f = 5$ at room temperature

$$C_v = \frac{5}{2}R \text{ and } C_P = \frac{7}{2}R \text{ and } \gamma = \frac{7}{5} = 1 \cdot 4$$

17. For triatomic gas $f = 6$

$$Cv = 3R, C_P = 4R \text{ and } \gamma = \frac{4}{3} = 1 \cdot 33$$

18. Joules mechanical equivalent of heat

$$J = \frac{W}{Q} = 4 \cdot 186 \text{ J cal}^{-1}.$$

19. Rise in temperature of body when it falls throu height h

$$\Delta T = \frac{gh}{CJ}.$$

20. The height from which a block of ice be dropped that it melts completely on reaching ground.

$$h = \frac{JL}{g}$$

21. The velocity with whcih a ball of ice be thro against a wall so that it melts completely,

$$v = \sqrt{2JL}$$

22. Equation of isothermal process

$PV = \text{Const.}$

23. Equation of adiabatic process

(*i*) $PV^{\gamma} = \text{Const.}$

(*ii*) $TP^{\gamma - 1} = \text{Const.}$

(*iii*) $\frac{T^{\gamma}}{P^{\gamma - 1}} = \text{Const.}$

24. Work done during isothermal process

$$W = 2 \cdot 303\, RT \log_{10} \frac{V_2}{V_1}$$

$$W = 2 \cdot 303\, RT \log_{10} \frac{P_2}{P_1}$$

25. Work done during adiabatic process

$$W = \frac{R}{\gamma - 1}(T_1 - T_2)$$

$$W = \frac{R}{\gamma - 1}(P_1V_1 - P_2V_2)$$

$$W = C_v(T_1 - T_2)$$

26. Slope of adiabatic graph is γ-times more than sl of isothermal process.

27. First law of thermodynamics

$$dQ = dU + dW$$

28. Efficiency of heat engine

$$\eta = 1 - \frac{Q_2}{Q_1}$$

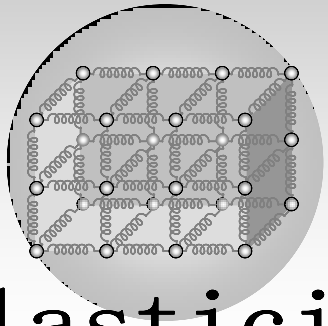
$$\eta = 1 - \frac{T_2}{T_1}$$

29. Efficiency of heat engine can never be 100%.

30. Coefficient of performance of refrigerator.

$$\beta = \frac{T_2}{T_1 - T_2} = \frac{Q_2}{Q_1 - Q_2}.$$

31. There are two dead centres per cycle for a st engine.



Elasticity

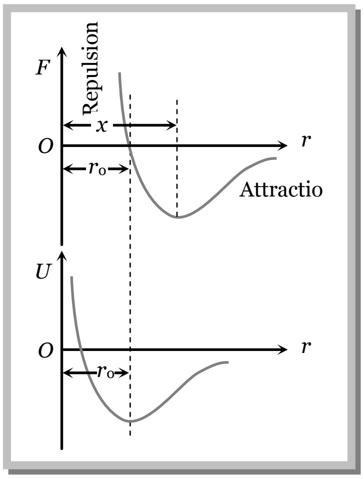
9.1 Interatomic Forces.

The forces between the atoms due to electrostatic interaction between the charges of the atoms are called interatomic forces. These forces are electrical in nature and these are active if the distance between the two atoms is of the order of atomic size *i.e.* 10^{-10} *metre*.

(1) Every atom is electrically neutral, the number of electrons (negative charge) orbiting around the nucleus is equal to the number of proton (positive charge) in the nucleus. So if two atoms are placed at a very large distance from each other then there will be a very small (negligible) interatomic force working between them.

(2) When these two atoms are brought close to each other to a distance of the order of 10^{-10} m, the distances between their positive nuclei and negative electron clouds get disturbed, and due to this, attractive interatomic force is produced between two atoms.

(3) This attractive force increases continuously with decrease in r and becomes maximum for one value of r called critical distance, represented by x (as shown in the figure). Beyond this the attractive force starts decreasing rapidly with further decrease in the value of r.



(4) When the distance between the two atoms becomes r_0, the interatomic force will be zero. This distance r_0 is called normal or equilibrium distance.

(r_0 = 0.74 Å for hydrogen).

(5) When the distance between the two atoms further decreased, the interatomic force becomes repulsive in nature and increases very rapidly with decrease in distance between two atoms.

(6) The potential energy U is related with the interatomic force F by the following relation.

$$F = \frac{-dU}{dr}$$

(i) When two atoms are at very large distance, the potential energy is negative and becomes more negative as r is decreased.

(ii) When the distance between the two atoms becomes r_0, the potential energy of the system of two atoms becomes minimum (*i.e.* attains maximum negative value). As the state of minimum potential energy is the state of equilibrium, hence the two atoms at separation r_0 will be in a state of equilibrium.

($U_0 = -7.2 \times 10^{-19}$ *Joule* for hydrogen).

(iii) When the distance between the two atoms is further decreased (*i.e.* $r < r_0$) the negative value of potential energy of the system starts decreasing. It becomes zero and then attains positive value with further decrease in r (as shown in the figure).

9.2 Intermolecular Forces.

The forces between the molecules due to electrostatic interaction between the charges of the molecules are called intermolecular forces. These forces are also called Vander Waal forces and are quite weak as compared to inter-atomic forces. These forces are also electrical in nature and these are active if the separation between two molecules is of the order of molecular size *i.e.* $\approx 10^{-9}\ m$.

(1) It is found that the force of attraction between molecules varies inversely as seventh power of the distance between them *i.e.*

$$F_{\text{att}} \propto \frac{1}{r^7} \quad \text{or} \quad F_{\text{att}} = \frac{-a}{r^7}$$

The negative sign indicates that the force is attractive in nature.

(2) When the distance between molecules becomes less than r_0, the forces becomes repulsive in nature and is found to vary inversely as ninth power of the distance between them *i.e.*

$$F_{\text{rep}} \propto \frac{1}{r^9} \quad \text{or} \quad F_{\text{rep}} = \frac{b}{r^9}.$$

Therefore force between two molecules is given by $F = F_{\text{att}} + F_{\text{rep}} = \frac{-a}{r^7} + \frac{b}{r^9}$

The value of constants a and b depend upon the structure and nature of molecules.

(3) Intermolecular forces between two molecules has the same general nature as shown in the figure for interatomic forces.

(4) Potential Energy : Potential energy can be approximately expressed by the formula $U = \frac{A}{r^n} - \frac{B}{r^m}$

where the term $\frac{A}{r^n}$ represents repulsive contribution and term $\frac{B}{r^m}$ represents the attractive contribution. Constants A, B and numbers m and n are different for different molecules.

For majority of solids $n = 12$ and $m = 6$.

So potential energy can be expressed as $U = \frac{A}{r^{12}} - \frac{B}{r^6}$

9.3 Comparison Between Inter atomic and Intermolecular Forces.

(1) **Similarities**

(i) Both the forces are electrical in origin.

(ii) Both the forces are active over short distances.

(iii) General shape of force-distance graph is similar for both the forces.

(iv) Both the forces are attractive up to certain distance between atoms/molecules and become repulsive when the distance between them become less than that value.

(2) **Dissimilarities**

(i) Interatomic force depends upon the distance between the two atoms, whereas the intermolecular force depends upon the distance between the two molecules as well as their relative orientation.

(ii) Interatomic forces are about 50 to100 times stronger than intermolecular forces.

(iii) The value of r_0 for two atoms is smaller than the corresponding value for the molecules. Therefore one molecule is not restricted to attract only one molecule, but can attract many molecule. It is not so incase of atoms, since the atoms of one molecule cannot bind the atoms of other molecules.

9.4 States of Matter

The three states of matter differ from each other due to the following two factors.

(1) The different magnitudes of the interatomic and intermolecular forces.

(2) The extent of random thermal motion of atoms and molecules of a substance (which depends upon temperature).

Comparison Chart of Solid, Liquid and Gaseous States

Property	Solid	Liquid	Gas
Shape	Definite	Not definite	Not definite
Volume	Definite	Definite	Not definite
Density	Maximum	Less than solids but more than gases.	Minimum
Compressibility	Incompressible	Less than gases but more than solids.	Compressible
Crystallinity	Crystalline	Non-crystalline	
Interatomic or intermolecular distance	Constant	Not constant	Not constant
Relation between kinetic energy K and potential energy (U)	$K < U$	$K > U$	$K >> U$
Intermolecular force	Strongest	Less than solids but more than gases.	Weakest
Freedom of motion	Molecules vibrate about their mean position but cannot move freely.	Molecules have limited free motion.	Molecules are free to move.
Effect of temperature	Matter remains in solid form below a certain temperature.	Liquids are found at temperatures more than that of solid.	These are found at temperatures greater than that of solids and liquids.

Note : ❑ The fourth state of matter in which the medium is in the form of positive and negative ions, is known as plasma. Plasma occurs in the atmosphere of stars (including the sun) and in discharge tubes.

9.5 Types of Solids

A solid is that state of matter in which its constituent atoms or molecules are held strongly at the position of minimum potential energy and it has a definite shape and volume. The solids can be classified into two categories, crystalline and glassy or amorphous solids.

Comparison chart of Crystalline and Amorphous Solids

Crystalline solids	Amorphous or glassy solids
The constituent atoms, ions or molecules are arranged in a regular repeated three dimensional pattern, within the solid.	The constituent atoms, ions or molecules are not arranged in a regular repeated three dimensional pattern, within the solid.
Definite external geometric shape.	No regularity in external shape.
All the bonds in ions, or atoms or molecules are equally strong.	All the bonds are not equally strong.
They are anisotropic.	They are isotropic.
They have sharp melting point.	They don't have no sharp melting point.
They have a long-range order of atoms or ions or molecules in them.	They don't have a long-range order.
They are considered true and stable solids.	They are not regarded as true and stable solids.

9.6 Elastic Property of Matter

(1) **Elasticity :** The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming force is called elasticity.

(2) **Plasticity :** The property of matter by virtue of which it does not regain its original shape and size after the removal of deforming force is called plasticity.

(3) **Perfectly elastic body :** If on the removal of deforming forces the body regain its original configuration completely it is said to be perfectly elastic.

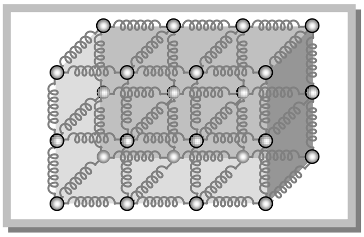
A quartz fibre and phosphor bronze (an alloy of copper containing 4% to 10% tin, 0.05% to 1% phosphorus) is the nearest approach to the perfectly elastic body.

(4) **Perfectly plastic body :** If the body does not have any tendency to recover its original configuration, on the removal of deforming force, it is said to be perfectly plastic.

Paraffin wax, wet clay are the nearest approach to the perfectly plastic body.

Practically there is no material which is either perfectly elastic or perfectly plastic and the behaviour of actual bodies lies between the two extremes.

(5) **Reason of elasticity :** In a solids, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to neighbouring molecules. These forces are known as intermolecular forces.



For simplicity, the two molecules in their equilibrium positions (at inter-molecular distance $r = r_0$) (see graph in article 9.1) are shown by connecting them with a spring.

In fact, the spring connecting the two molecules represents the inter-molecular force between them. On applying the deforming forces, the molecules either come closer or go far apart from each other and restoring forces are developed. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium position ($r = r_0$) and hence the body regains its original form.

(6) **Elastic limit :** Elastic bodies show their property of elasticity upto a certain value of deforming force. If we go on increasing the deforming force then a stage is reached when on removing the force, the body will not return to its original state. The maximum deforming force upto which a body retains its property of elasticity is called elastic limit of the material of body.

Elastic limit is the property of a body whereas elasticity is the property of material of the body.

(7) **Elastic fatigue :** The temporary loss of elastic properties because of the action of repeated alternating deforming force is called elastic fatigue.

It is due to this reason

(i) Bridges are declared unsafe after a long time of their use.

(ii) Spring balances show wrong readings after they have been used for a long time.

(iii) We are able to break the wire by repeated bending.

(8) **Elastic after effect :** The time delay in which the substance regains its original condition after the removal of deforming force is called elastic after effect. It is the time for which restoring forces are present after the removal of the deforming force it is negligible for perfectly elastic substance, like quartz, phosphor bronze and large for glass fibre.

9.7 Stress

When a force is applied on a body there will be relative displacement of the particles and due to property of elasticity an internal restoring force is developed which tends to restore the body to its original state.

The internal restoring force acting per unit area of cross section of the deformed body is called stress.

At equilibrium, restoring force is equal in magnitude to external force, stress can therefore also be defined as external force per unit area on a body that tends to cause it to deform.

If external force F is applied on the area A of a body then,

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

Unit : N/m^2 (S.I.) , $dyne\ /cm^2$ (C.G.S.)

Dimension : $[ML^{-1}T^{-2}]$

Stress developed in a body depends upon how the external forces are applied over it.

On this basis there are two types of stresses : Normal and Shear or tangential stress

(1) **Normal stress :** Here the force is applied normal to the surface.

It is again of two types : Longitudinal and Bulk or volume stress

(i) *Longitudinal stress*

(a) It occurs only in solids and comes in picture when one of the three dimensions *viz.* length, breadth, height is much greater than other two.

(b) Deforming force is applied parallel to the length and causes increase in length.

(c) Area taken for calculation of stress is area of cross section.

(d) Longitudinal stress produced due to increase in length of a body under a deforming force is called tensile stress.

(e) Longitudinal stress produced due to decrease in length of a body under a deforming force is called compressional stress.

(ii) *Bulk or Volume stress*

(a) It occurs in solids, liquids or gases.

(b) In case of fluids only bulk stress can be found.

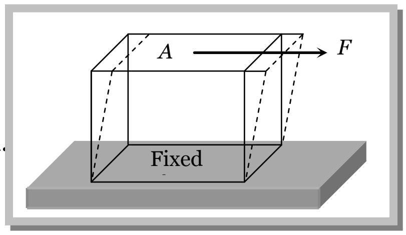
(c) It produces change in volume and density, shape remaining same.

(d) Deforming force is applied normal to surface at all points.

(e) Area for calculation of stress is the complete surface area perpendicular to the applied forces.

(f) It is equal to change in pressure because change in pressure is responsible for change in volume.

(2) **Shear or tangential stress :** It comes in picture when successive layers of solid move on each other *i.e.* when there is a relative displacement between various layers of solid.



(i) Here deforming force is applied tangential to one of the faces.

(ii) Area for calculation is the area of the face on which force is applied.

(iii) It produces change in shape, volume remaining the same.

Difference between Pressure and Stress

Pressure	Stress
Pressure is always normal to the area.	Stress can be normal or tangential.
Always compressive in nature.	May be compressive or tensile in nature.

Sample problems based on Stress

Problem 1. A and B are two wires. The radius of A is twice that of B. they are stretched by the same load. Then the stress on B is **[MP PMT 1993]**

(a) Equal to that on A (b) Four times that on A

(c) Two times that on A (d) Half that on A

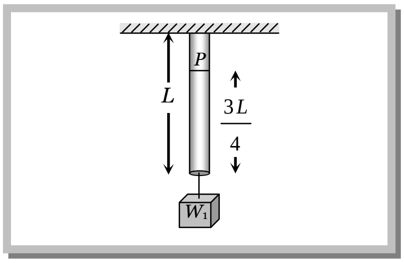
Solution : (b) Stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{\pi r^2}$

$$\therefore \text{Stress} \propto \frac{1}{r^2} \Rightarrow \frac{(\text{Stress})_B}{(\text{Stress})_A} = \left(\frac{r_A}{r_B}\right)^2 = (2)^2 \Rightarrow (\text{Stress})_B = 4 \times (\text{stress})_A \qquad [\text{As } F = \text{constant}]$$

Problem 2. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at a height $3L/4$ from its lower end is

(a) $\frac{W_1}{S}$ (b) $\frac{W_1 + (W/4)}{S}$ (c) $\frac{W_1 + (3W/4)}{S}$ (d) $\frac{W_1 + W}{S}$

Solution : (c) As the wire is uniform so the weight of wire below point P is $\frac{3W}{4}$



$\therefore$ Total force at point $P = W_1 + \frac{3W}{4}$ and area of cross-section = S

$\therefore$ Stress at point $P = \frac{\text{Force}}{\text{Area}} = \frac{W_1 + \frac{3W}{4}}{S}$

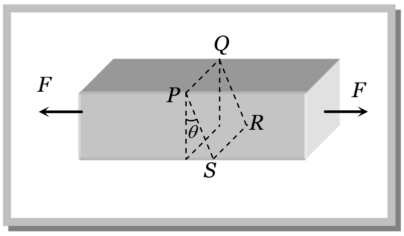
Problem 3. On suspending a weight Mg, the length l of elastic wire and area of cross-section A its length becomes double the initial length. The instantaneous stress action on the wire is

(a) Mg/A (b) $Mg/2A$ (c) $2Mg/A$ (d) $4Mg/A$

Solution : (c) When the length of wire becomes double, its area of cross section will become half because volume of wire is constant $(V = AL)$.

So the instantaneous stress = $\frac{\text{Force}}{\text{Area}} = \frac{Mg}{A/2} = \frac{2Mg}{A}$.

Problem 4. A bar is subjected to equal and opposite forces as shown in the figure. $PQRS$ is a plane making angle θ with the cross-section of the bar. If the area of cross-section be 'A', then what is the tensile stress on $PQRS$



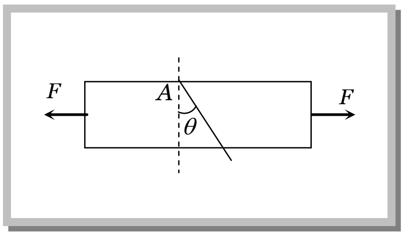
(a) F/A

(b) $F\cos\theta/A$

(c) $F\cos^2\theta/A$

(d) $F/A\cos\theta$

Solution : (c) As tensile stress = $\frac{\text{Normal force}}{\text{Area}} = \frac{F_N}{A_N}$



and here $A_N = (A/\cos\theta)$, F_N = Normal force = $F\cos\theta$

So, Tensile stress = $\frac{F\cos\theta}{A/\cos\theta} = \frac{F\cos^2\theta}{A}$

Problem 5. In the above question, what is the shearing stress on PQ

(a) $F/A\cos\theta$ (b) $F\sin 2\theta/2A$ (c) $F/2A\sin 2\theta$ (d) $F\cos\theta/A$

Solution : (b) Shear stress $= \frac{\text{Tangential force}}{\text{Area}} = \frac{F\sin\theta}{(A/\cos\theta)} = \frac{F\sin\theta\cos\theta}{A} = \frac{F\sin 2\theta}{2A}$

Problem 6. In the above question, when is the tensile stress maximum

(a) $\theta = 0^o$ (b) $\theta = 30^o$ (c) $\theta = 45^o$ (d) $\theta = 90^o$

Solution : (a) Tensile stress $= \frac{F\cos^2\theta}{A}$. It will be maximum when $\cos^2\theta = \max$. *i.e.* $\cos\theta = 1 \Rightarrow \theta = 0^o$.

Problem 7. In the above question, when is the shearing stress maximum

(a) $\theta = 0^o$ (b) $\theta = 30^o$ (c) $\theta = 45^o$ (d) $\theta = 90^o$

Solution : (c) Shearing stress $= \frac{F\sin 2\theta}{2A}$. It will be maximum when $\sin 2\theta = \max$ *i.e.* $\sin 2\theta = 1 \Rightarrow 2\theta = 90^o \Rightarrow \theta = 45^o$.

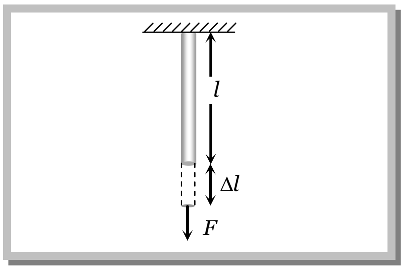
9.8 Strain

The ratio of change in configuration to the original configuration is called strain.

Being the ratio of two like quantities, it has no dimensions and units.

Strain are of three types :

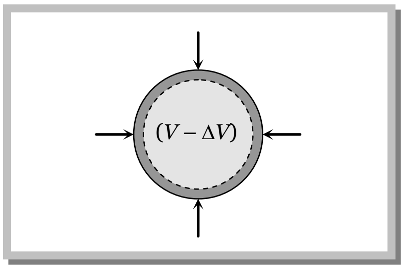
(1) **Linear strain :** If the deforming force produces a change in length alone, the strain produced in the body is called linear strain or tensile strain.



$$\text{Linear strain} = \frac{\text{Change in length}(\Delta l)}{\text{Original length}(l)}$$

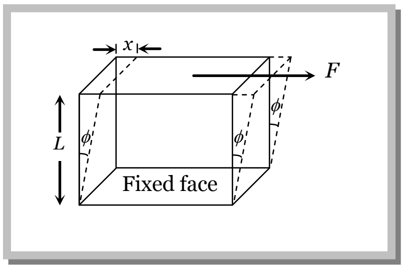
Linear strain in the direction of deforming force is called longitudinal strain and in a direction perpendicular to force is called lateral strain.

(2) **Volumetric strain :** If the deforming force produces a change in volume alone the strain produced in the body is called volumetric strain.



$$\text{Volumetric strain} = \frac{\text{Change in volume}(\Delta V)}{\text{Original volume}(V)}$$

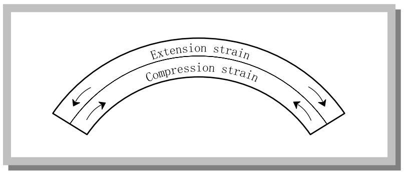
(3) **Shearing strain :** If the deforming force produces a change in the shape of the body without changing its volume, strain produced is called shearing strain.



It is defined as angle in radians through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force.

$$\phi = \frac{x}{L}$$

Note : ❑ When a beam is bent both compression strain as well as an extension strain is produced.



Sample problems based on Strain

Problem 8. A cube of aluminium of sides 0.1 *m* is subjected to a shearing force of 100 *N*. The top face of the cube is displaced through 0.02 *cm* with respect to the bottom face. The shearing strain would be **[MP PAT 1990]**

(a) 0.02 (b) 0.1 (c) 0.005 (d) 0.002

Solution : (d) Shearing strain $\phi = \frac{x}{L} = \frac{0.02cm}{0.1m} = 0.002$

Problem 9. A wire is stretched to double its length. The strain is

(a) 2 (b) 1 (c) Zero (d) 0.5

Solution : (b) Strain $= \dfrac{\text{Change in length}}{\text{Original length}} = \dfrac{2L - L}{L} = 1$

***Problem* 10.** The length of a wire increases by 1% by a load of 2 *kg-wt*. The linear strain produced in the wire will be

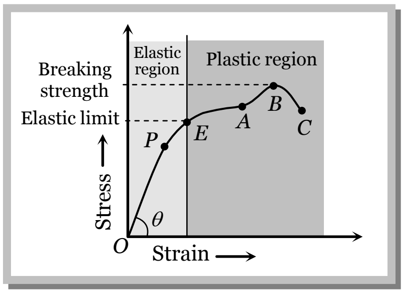
(a) 0.02 (b) 0.001 (c) 0.01 (d) 0.002

Solution : (c) Strain $= \dfrac{\text{Change in length}}{\text{Original length}} = \dfrac{1\% \text{ of } L}{\text{L}} = \dfrac{L/100}{L} = 0.01$

9.9 Stress-strain Curve

If by gradually increasing the load on a vertically suspended metal wire, a graph is plotted between stress (or load) and longitudinal strain (or elongation) we get the curve as shown in figure. From this curve it is clear that :

(1) When the strain is small (< 2%) (*i.e.*, in region *OP*) stress is proportional to strain. This is the region where the so called Hooke's law is obeyed. The point *P* is called limit of proportionality and slope of line *OP* gives the Young's modulus *Y* of the material of the wire. If θ is the angle of *OP* from strain axis then $Y = \tan\theta$.

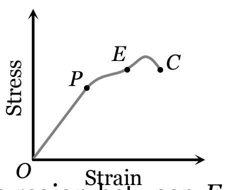
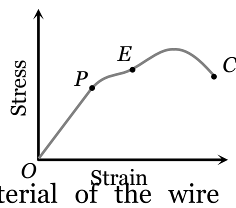
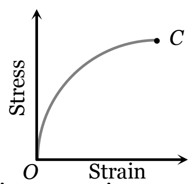


(2) If the strain is increased a little bit, *i.e.*, in the region *PE*, the stress is not proportional to strain. However, the wire still regains its original length after the removal of stretching force. This behaviour is shown up to point *E* known as elastic limit or yield-point. The region *OPE* represents the elastic behaviour of the material of wire.

(3) If the wire is stretched beyond the elastic limit *E*, *i.e.*, between *EA*, the strain increases much more rapidly and if the stretching force is removed the wire does not come back to its natural length. Some permanent increase in length takes place.

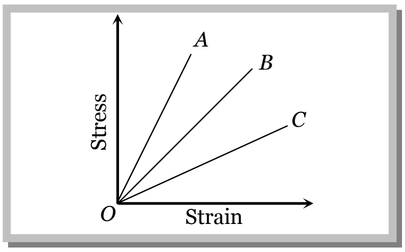
(4) If the stress is increased further, by a very small increase in it a very large increase in strain is produced (region *AB*) and after reaching point *B*, the strain increases even if the wire is unloaded and ruptures at *C*. In the region *BC* the wire literally flows. The maximum stress corresponding to *B* after which the wire begins to flow and breaks is called breaking or tensile strength. The region *EABC* represents the plastic behaviour of the material of wire.

(5) Stress-strain curve for different materials.

Brittle material	Ductile material	Elastomers
		
The plastic region between *E* and *C* is small for brittle material and it will break soon after the elastic limit is crossed.	The material of the wire have a good plastic range and such materials can be easily changed into different shapes and can be drawn into thin wires	Stress strain curve is not a straight line within the elastic limit for elastomers and strain produced is much larger than the stress applied. Such materials have no plastic range and the breaking point lies very close to elastic limit. Example rubber

Sample problems based on Stress-strain curve

***Problem* 11.** The stress-strain curves for brass, steel and rubber are shown in the figure. The lines *A*, *B* and *C* are for



(a) Rubber, brass and steel respectively

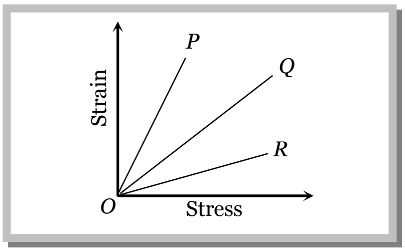
(b) Brass, steel and rubber

(c) Steel, brass and rubber respectively

(d) Steel, rubber and brass

Solution : (c) From the graph $\tan\theta_C < \tan\theta_B < \tan\theta_A \Rightarrow Y_C < Y_B < Y_A \quad \therefore Y_{\text{Rubber}} < Y_{\text{Brass}} < Y_{\text{Steel}}$

***Problem* 12.** The strain stress curves of three wires of different materials are shown in the figure. *P*, *Q* and *R* are the elastic limits of the wires. The figure shows that



(a) Elasticity of wire *P* is maximum

(b) Elasticity of wire *Q* is maximum

(c) Tensile strength of *R* is maximum

(d) None of the above is true

Solution : (d) On the graph stress is represented on *X*- axis and strain *Y*-axis

So from the graph $Y = \cot\theta = \frac{1}{\tan\theta} \propto \frac{1}{\theta}$ [where θ is the angle from stress axis]

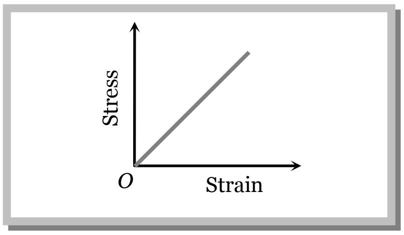
$\therefore Y_P < Y_Q < Y_R$ [As $\theta_P > \theta_Q > \theta_R$]

We can say that elasticity of wire *P* is minimum and *R* is maximum.

9.10 Hooke's law and Modulus of Elasticity

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain or $\frac{\text{stress}}{\text{strain}} = \text{constant} = E$



The constant *E* is called modulus of elasticity.

(1) It's value depends upon the nature of material of the body and the manner in which the body is deformed.

(2) It's value depends upon the temperature of the body.

(3) It's value is independent of the dimensions (length, volume etc.) of the body.

There are three modulii of elasticity namely Young's modulus (*Y*), Bulk modulus (*K*) and modulus of rigidity (η) corresponding to three types of the strain.

9.11 Young's Modulus (*Y*)

It is defined as the ratio of normal stress to longitudinal strain within limit of proportionality.

$$Y = \frac{\text{Normal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/L} = \frac{FL}{Al}$$

If force is applied on a wire of radius *r* by hanging a weight of mass *M*, then

$$Y = \frac{MgL}{\pi r^2 l}$$

Important points

(i) If the length of a wire is doubled,

Then longitudinal strain = $\frac{\text{change in length}(l)}{\text{initial length}(L)} = \frac{\text{final length} - \text{initial length}}{\text{Initial length}} = \frac{2L - L}{L} = 1$

$\therefore$ Young's modulus = $\frac{\text{stress}}{\text{strain}}$ $\Rightarrow$ Y = stress [As strain = 1]

So young's modulus is numerically equal to the stress which will double the length of a wire.

(ii) Increment in the length of wire $l = \frac{FL}{\pi r^2 Y}$ $\left[\text{As } Y = \frac{FL}{Al}\right]$

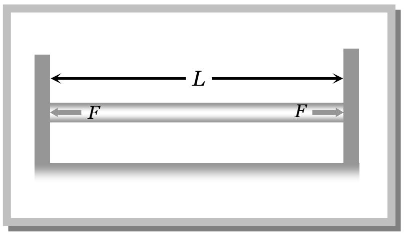
So if same stretching force is applied to different wires of same material, $l \propto \frac{L}{r^2}$ [As F and Y are constant]

i.e., greater the ratio $\frac{L}{r^2}$, greater will be the elongation in the wire.

(iii) **Elongation in a wire by its own weight** : The weight of the wire Mg act at the centre of gravity of the wire so that length of wire which is stretched will be $L/2$.

$\therefore$ Elongation $l = \frac{FL}{AY} = \frac{Mg(L/2)}{AY} = \frac{MgL}{2AY} = \frac{L^2 dg}{2Y}$ [As mass (M) = volume (AL) × density (d)]

(iv) **Thermal stress** : If a rod is fixed between two rigid supports, due to change in temperature its length will change and so it will exert a normal stress (compressive if temperature increases and tensile if temperature decreases) on the supports. This stress is called thermal stress.



As by definition, coefficient of linear expansion $\alpha = \frac{l}{L\Delta\theta}$

$\Rightarrow$ thermal strain $\frac{l}{L} = \alpha\Delta\theta$

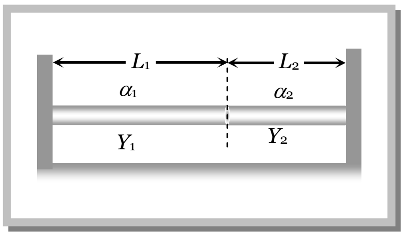
So thermal stress = $Y\alpha\,\Delta\theta$ [As Y = stress/strain]

And tensile or compressive force produced in the body = $YA\alpha\,\Delta\theta$

Note : ❑ In case of volume expansion Thermal stress = $K\gamma\Delta\theta$

Where K = Bulk modulus, γ = coefficient of cubical expansion

(v) **Force between the two rods** : Two rods of different metals, having the same area of cross section A, are placed end to end between two massive walls as shown in figure. The first rod has a length L_1, coefficient of linear expansion α_1 and young's modulus Y_1. The corresponding quantities for second rod are L_2, α_2 and Y_2. If the temperature of both the rods is now raised by T degrees.



Increase in length of the composite rod (due to heating) will be equal to

$l_1 + l_2 = [L_1\alpha_1 + L_2\alpha_2]T$ [As $l = L\,\alpha\,\Delta\theta$]

and due to compressive force F from the walls due to elasticity,

decrease in length of the composite rod will be equal to $\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right]\frac{F}{A}$ $\left[\text{As } l = \frac{FL}{AY}\right]$

as the length of the composite rod remains unchanged the increase in length due to heating must be equal to decrease in length due to compression *i.e.* $\frac{F}{A}\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right] = [L_1\alpha_1 + L_2\alpha_2]T$

or $$F = \frac{A[L_1\alpha_1 + L_2\alpha_2]T}{\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right]}$$

(vi) **Force constant of wire** : Force required to produce unit elongation in a wire is called force constant of material of wire. It is denoted by k.

$\therefore$ $$k = \frac{F}{l} \quad .....(i)$$

but from the definition of young's modulus $Y = \frac{F/A}{l/L} \Rightarrow \frac{F}{l} = \frac{YA}{L}$ .....(ii)

from (i) and (ii) $k = \frac{YA}{L}$

It is clear that the value of force constant depends upon the dimension (length and area of cross section) and material of a substance.

(vii) Actual length of the wire : If the actual length of the wire is L, then under the tension T_1, its length becomes L_1 and under the tension T_2, its length becomes L_2.

$L_1 = L + l_1 \Rightarrow L_1 = L + \frac{T_1}{k}$ ......(i) and $L_2 = L + l_2 \Rightarrow L_2 = L + \frac{T_2}{k}$ .....(ii)

From (i) and (ii) we get $L = \frac{L_1T_2 - L_2T_1}{T_2 - T_1}$

Sample problems based on Young's modulus

Problem 13. The diameter of a brass rod is 4 *mm* and Young's modulus of brass is $9\times10^{10}\ N/m^2$. The force required to stretch by 0.1% of its length is **[MP PET 1991; BVP 2003]**

(a) 360 πN (b) 36 N (c) $144\pi\times10^3\ N$ (d) $36\pi\times10^5\ N$

Solution : (a) $r = 2\times10^{-3} m,\ Y = 9\times10^{10}\ N/m^2,\ l = 0.1\%L \Rightarrow \frac{l}{L} = 0.001$

As $Y = \frac{F}{A}\frac{L}{l}$ $\therefore F = YA\frac{l}{L} = 9\times10^{10}\times\pi(2\times10^{-3})^2\times0.001 = 360\pi\ N$

Problem 14. A wire of length 2*m* is made from 10 *cm*³ of copper. A force F is applied so that its length increases by 2 *mm*. Another wire of length 8 *m* is made from the same volume of copper. If the force F is applied to it, its length will increase by **[MP PET 2003]**

(a) 0.8 *cm* (b) 1.6 *cm* (c) 2.4 *cm* (d) 3.2 *cm*

Solution : (d) $l = \frac{FL}{AY} = \frac{FL^2}{VY}$ $\therefore l \propto L^2$ [As V, Y and F are constant]

$$\frac{l_2}{l_1} = \left[\frac{L_2}{L_1}\right]^2 = \left(\frac{8}{2}\right)^2 = 16 \Rightarrow l_2 = 16\, l_1 = 16 \times 2\ mm = 32\ mm = 3.2\ cm$$

Problem 15. A wire of length L and radius r is rigidly fixed at one end. On stretching the other end of the wire with a force F, the increase in its length is l. If another wire of same material but of length $2L$ and radius $2r$ is stretched with a force of $2F$, the increase in its length will be

[AIIMS 1980; MP PAT 1990; MP PET 1989, 92; MP PET/PMT 1988; MP PMT 1996, 2002; UPSEAT 2002]

(a) l (b) $2l$ (c) $\frac{l}{2}$ (d) $\frac{l}{4}$

Solution : (a) $l = \frac{FL}{\pi r^2 Y} \Rightarrow \frac{l_2}{l_1} = \frac{F_2}{F_1}\frac{L_2}{L_1}\left(\frac{r_1}{r_2}\right)^2 = 2 \times 2 \times \left(\frac{1}{2}\right)^2 = 1 \quad \therefore l_2 = l_1$ *i.e.* the increment in length will be same.

Problem 16. Two wires A and B are of same materials. Their lengths are in the ratio 1 : 2 and diameters are in the ratio 2 : 1 when stretched by force F_A and F_B respectively they get equal increase in their lengths. Then the ratio F_A/F_B should be **[Orissa JEE 2002]**

(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 8 : 1

Solution : (d) $Y = \frac{FL}{\pi r^2 l} \qquad \therefore F = Y\pi r^2 \frac{l}{L}$

$$\frac{F_A}{F_B} = \frac{Y_A}{Y_B}\left(\frac{r_A}{r_B}\right)^2\left(\frac{l_A}{l_B}\right)\left(\frac{L_B}{L_A}\right) = 1 \times \left(\frac{2}{1}\right)^2 \times (1) \times \left(\frac{2}{1}\right) = 8$$

Problem 17. A uniform plank of Young's modulus Y is moved over a smooth horizontal surface by a constant horizontal force F. The area of cross-section of the plank is A. the compressive strain on the plank in the direction of the force is **[Kerala (Engg.) 2002]**

(a) $\frac{F}{AY}$ (b) $\frac{2F}{AY}$ (c) $\frac{1}{2}\left(\frac{F}{AY}\right)$ (d) $\frac{3F}{AY}$

Solution : (a) Compressive strain $= \frac{\text{Stress}}{\text{Young's modulus}} = \frac{F/A}{Y} = \frac{F}{AY}$

Problem 18. A wire is stretched by 0.01 m by a certain force F. Another wire of same material whose diameter and length are double to the original wire is stretched by the same force. Then its elongation will be

[EAMCET (Engg.) 1995; CPMT 2001]

(a) 0.005 m (b) 0.01 m (c) 0.02 m (d) 0.002 m

Solution : (a) $l = \frac{FL}{\pi r^2 Y} \quad \therefore\ l \propto \frac{L}{r^2}$ [As F and Y are constants]

$$\frac{l_2}{l_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{r_1}{r_2}\right)^2 = (2) \times \left(\frac{1}{2}\right)^2 = \frac{1}{2} \Rightarrow l_2 = \frac{l_1}{2} = \frac{0.01}{2} = 0.005 m.$$

Problem 19. The length of an elastic string is a *metres* when the longitudinal tension is 4 N and b *metres* when the longitudinal tension is 5 N. The length of the string in *metres* when the longitudinal tension is 9 N is

[EAMCET 2001]

(a) $a - b$ (b) $5b - 4a$ (c) $2b - \frac{1}{4}a$ (d) $4a - 3b$

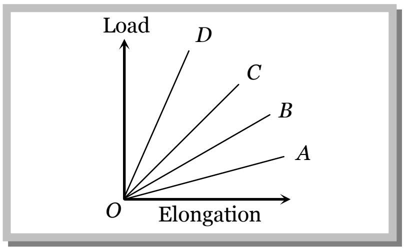
Solution : (b) Let the original length of elastic string is L and its force constant is k.

When longitudinal tension $4N$ is applied on it $\quad L + \frac{4}{k} = a \quad$ .....(i) and when longitudinal tension 5*N* is applied on it $L + \frac{5}{k} = b$ .....(ii)

By solving (i) and (ii) we get $k = \frac{1}{b-a}$ and $L = 5a - 4b$

Now when longitudinal tension 9*N* is applied on elastic string then its length = $L + \frac{9}{k} = 5a - 4b + 9(b-a) = 5b - 4a$

Problem **20.** The load versus elongation graph for four wires of the same material is shown in the figure. The thickest wire is represented by the line **[KCET (Engg./Med.) 2001]**



(a) *OD*

(b) *OC*

(c) *OB*

(d) *OA*

Solution : (a) Young's modulus $Y = \frac{FL}{Al}$ $\therefore\ l \propto \frac{1}{A}$ (As *Y*, *L* and *F* are constant)

From the graph it is clear that for same load elongation is minimum for graph *OD*.

As elongation (*l*) is minimum therefore area of cross-section (*A*) is maximum.

So thickest wire is represented by *OD*.

Problem **21.** A 5 *m* long aluminum wire $(Y = 7 \times 10^{10}\ N/m^2)$ of diameter 3 *mm* supports a 40 *kg* mass. In order to have the same elongation in a copper wire $(Y = 12 \times 10^{10}\ N/m^2)$ of the same length under the same weight, the diameter should now be, in *mm*

(a) 1.75 (b) 2.0 (c) 2.3 (d) 5.0

Solution : (c) $l = \frac{FL}{\pi r^2 Y} = \frac{4FL}{\pi d^2 Y}$ [As $r = d/2$]

If the elongation in both wires (of same length) are same under the same weight then $d^2 Y$ = constant

$$\left(\frac{d_{Cu}}{d_{Al}}\right)^2 = \frac{Y_{Al}}{Y_{Cu}} \Rightarrow d_{Cu} = d_{Al} \times \sqrt{\frac{Y_{Al}}{Y_{Cu}}} = 3 \times \sqrt{\frac{7 \times 10^{10}}{12 \times 10^{10}}} = 2.29\ mm$$

Problem **22.** On *applying* a stress of $20 \times 10^8\ N/m^2$ the length of a perfectly elastic wire is doubled. Its Young's modulus will be **[MP PET 2000]**

(a) $40 \times 10^8\ N/m^2$ (b) $20 \times 10^8\ N/m^2$ (c) $10 \times 10^8\ N/m^2$ (d) $5 \times 10^8\ N/m^2$

Solution : (b) When strain is unity then Young's modulus is equal to stress.

Problem **23.** The dimensions of four wires of the same material are given below. In which wire the increase in length will be maximum when the same tension is applied

[IIT-JEE 1981; NCERT 1976; CPMT 1983, 90; MP PMT 1992, 94, 97; MP PET/PMT 1998; MP PET 1989, 90, 99]

(a) Length 100 *cm*, diameter 1 *mm* (b) Length 200 *cm*, diameter 2 *mm*

(c) Length 300 *cm*, diameter 3 *mm* (d) Length 50 *cm*, diameter 0.5 *mm*

Solution : (d) If same force is applied on four wires of same material then elongation in each wire depends on the length and diameter of the wire and given by $l \propto \frac{L}{d^2}$ and the ratio of $\frac{L}{d^2}$ is maximum for (d) option.

***Problem* 24.** The Young's modulus of a wire of length L and radius r is $Y\ N/m^2$. If the length and radius are reduced to $L/2$ and $r/2$, then its Young's modulus will be **[MP PMT 1985; MP PET 1997; KCET (Engg./Med.) 1999]**

(a) $Y/2$ (b) Y (c) $2Y$ (d) $4Y$

Solution : (b) Young's modulus do not depend upon the dimensions of wire. It is constant for a given material of wire.

***Problem* 25.** A fixed volume of iron is drawn into a wire of length L. The extension x produced in this wire by a constant force F is proportional to

(a) $\frac{1}{L^2}$ (b) $\frac{1}{L}$ (c) L^2 (d) L

Solution : (c) $l = \frac{FL}{AY} = \frac{FL^2}{ALY} = \frac{FL^2}{VY}$ for a fixed volume $l \propto L^2$

***Problem* 26.** A rod is fixed between two points at $20^o C$. The coefficient of linear expansion of material of rod is $1.1\times10^{-5}/^oC$ and Young's modulus is $1.2\times10^{11}\ N/m^2$. Find the stress developed in the rod if temperature of rod becomes $10^o C$

(a) $1.32\times10^7\ N/m^2$ (b) $1.10\times10^{15}\ N/m^2$ (c) $1.32\times10^8\ N/m^2$ (d) $1.10\times10^6\ N/m^2$

Solution : (a) Thermal stress $\frac{F}{A} = Y\alpha\,\Delta\theta = 1.2\times10^{11}\times1.1\times10^{-5}\times(20-10) = 1.32\times10^7\ N/m^2$

***Problem* 27.** The coefficient of linear expansion of brass and steel are α_1 and α_2. If we take a brass rod of length L_1 and steel rod of length L_2 at $0^o C$, their difference in length $(L_2 - L_1)$ will remain the same at any temperature if

[EAMCET (Med.) 1995]

(a) $\alpha_1 L_2 = \alpha_2 L_1$ (b) $\alpha_1 L_2^2 = \alpha_2 L_1^2$ (c) $\alpha_1^2 L_1 = \alpha_2^2 L_2$ (d) $\alpha_1 L_1 = \alpha_2 L_2$

Solution : (d) Difference in lengths of rods will remain same if expansion is same in both the rods.

If expansion in first rod is $l_1 = L_1\alpha_1\Delta\theta$ and expansion in second rod is $l_2 = L_2\alpha_2\Delta\theta$

then $L_1\alpha_1\Delta\theta = L_2\alpha_2\Delta\theta \quad \therefore L_1\alpha_1 = L_2\alpha_2$

***Problem* 28.** The force required to stretch a steel wire of 1 cm^2 cross-section to 1.1 times its length would be $(Y = 2\times10^{11}\ Nm^{-2})$ **[MP PET 1992]**

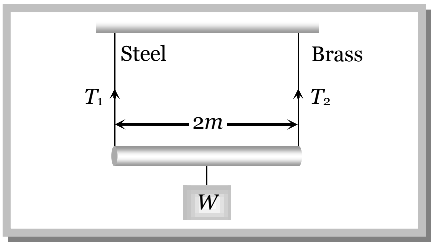
(a) $2\times10^6\ N$ (b) $2\times10^3\ N$ (c) $2\times10^{-6}\ N$ (d) $2\times10^{-7}\ N$

Solution : (a) $L_2 = 1.1\,L_1$ $\quad \therefore$ Strain $= \frac{l}{L_1} = \frac{L_2 - L_1}{L_1} = \frac{1.1L_1 - L_1}{L_1} = 0.1.$

$F = YA\frac{l}{L} = 2\times10^{11}\times1\times10^{-4}\times0.1 = 2\times10^6\ N.$

***Problem* 29.** A two *metre* long rod is suspended with the help of two wires of equal length. One wire is of steel and its cross-sectional area is 0.1 cm^2 and another wire is of brass and its cross-sectional area is 0.2 cm^2. If a load

W is suspended from the rod and stress produced in both the wires is same then the ratio of tensions in them will be



(a) Will depend on the position of W

(b) $T_1 / T_2 = 2$

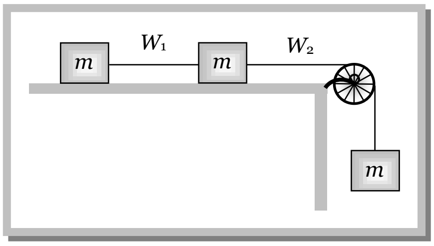
(c) $T_1 / T_2 = 1$

(d) $T_1 / T_2 = 0.5$

Solution : (d) Stress $= \dfrac{\text{Tension}}{\text{Area of cross-section}} =$ constant

$$\therefore \frac{T_1}{A_1} = \frac{T_2}{A_2} \Rightarrow \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{0.1}{0.2} = \frac{1}{2} = 0.5\,.$$

***Problem* 30.** Three blocks, each of same mass m, are connected with wires W_1 and W_2 of same cross-sectional area a and Young's modulus Y. Neglecting friction the strain developed in wire W_2 is



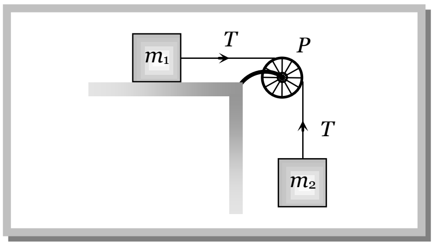
(a) $\dfrac{2}{3}\dfrac{mg}{aY}$

(b) $\dfrac{3}{2}\dfrac{mg}{aY}$

(c) $\dfrac{1}{3}\dfrac{mg}{aY}$

(d) $\dfrac{3mg}{aY}$

Solution : (a) If the system moves with acceleration a and T is the tension in the string W_2 then by comparing this condition from standard case $T = \dfrac{m_1 m_2}{m_1 + m_2} g$



In the given problem $m_1 = (m + m) = 2m$ and $m_2 = m$

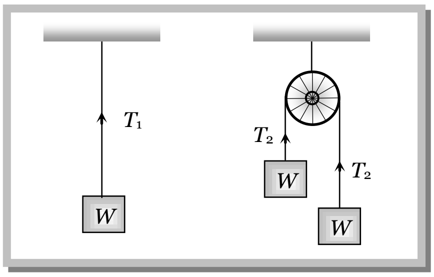
$$\therefore \quad \text{Tension} = \frac{m.2m.g}{m + 2m} = \frac{2}{3} mg$$

$$\therefore \text{ Stress} = \frac{T}{a} = \frac{2}{3a} mg \text{ and Strain} = \frac{\text{Stress}}{\text{Young's modulus}} = \frac{2}{3}\frac{mg}{aY}$$

***Problem* 31.** A wire elongates by 1.0 mm when a load W is hanged from it. If this wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be

(a) 0.5 m (b) 1.0 mm (c) 2.0 mm (d) 4.0 mm

Solution : (b) Elongation in the wire $\propto$ Tension in the wire



In first case $T_1 = W$ and in second case $T_2 = \dfrac{2W \times W}{W + W} = W$

$$\text{As } \frac{T_1}{T_2} = 1 \quad \therefore \frac{l_1}{l_2} = 1 \Rightarrow l_2 = l_1 = 1.0 mm$$

Problem 32. The Young's modulus of three materials are in the ratio 2 : 2 : 1. Three wires made of these materials have their cross-sectional areas in the ratio 1 : 2 : 3. For a given stretching force the elongation's in the three wires are in the ratio

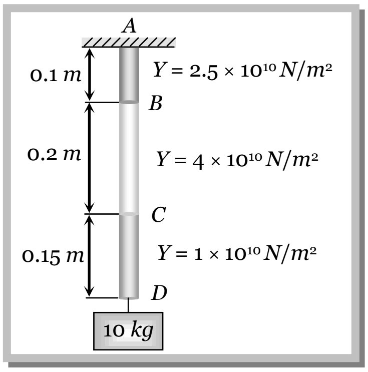
(a) 1 : 2 : 3 (b) 3 : 2 : 1 (c) 5 : 4 : 3 (d) 6 : 3 : 4

Solution : (d) $l = \frac{FL}{AY}$ and for a given stretching force $l \propto \frac{1}{AY}$

Let three wires have young's modulus $2Y$, $2Y$ and Y and their cross sectional areas are A, $2A$ and $3A$ respectively.

$$l_1 : l_2 : l_3 = \frac{1}{A_1Y_1} : \frac{1}{A_2Y_2} : \frac{1}{A_3Y_3} = \frac{1}{A \times 2Y} : \frac{1}{2A \times 2Y} : \frac{1}{3A \times Y} = \frac{1}{2} : \frac{1}{4} : \frac{1}{3} = 6:3:4\,.$$

Problem 33. A light rod with uniform cross-section of $10^{-4}\ m^2$ is shown in the adjoining figure. The rod consists of three different materials whose lengths are 0.1 m, 0.2 m and 0.15 m respectively and whose Young's modulii are $2.5 \times 10^{10}\ N/m^2$, $4 \times 10^{10}\ N/m^2$ and $1 \times 10^{10}\ N/m^2$ respectively. The displacement of point B will be



(a) $24 \times 10^{-6} m$

(b) $9 \times 10^{-6} m$

(c) $4 \times 10^{-6} m$

(d) $1 \times 10^{-6} m$

Solution : (c) Increment in the length $AB = \frac{MgL}{AY} = \frac{10 \times 10 \times 0.1}{10^{-4} \times 2.5 \times 10^{10}} = 4 \times 10^{-6} m$

$\therefore$ Displacement of point $B = 4 \times 10^{-6} m$

Problem 34. In the above problem, displacement of point C will be

(a) $24 \times 10^{-6} m$ (b) $9 \times 10^{-6} m$ (c) $4 \times 10^{-6} m$ (d) $1 \times 10^{-6} m$

Solution : (b) Increment in the length $BC = \frac{MgL}{AY} = \frac{10 \times 10 \times 0.2}{10^{-4} \times 4 \times 10^{10}} = 5 \times 10^{-6} m$

$\therefore$ Displacement of point $C = 4 \times 10^{-6} + 5 \times 10^{-6} = 9 \times 10^{-6} m$

Problem 35. In the above problem, the displacement of point D will be

(a) $24 \times 10^{-6} m$ (b) $9 \times 10^{-6} m$ (c) $4 \times 10^{-6} m$ (d) $1 \times 10^{-6} m$

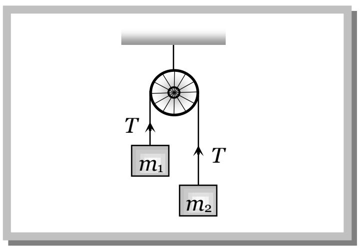
Solution : (a) Increment in the length $CD = \frac{MgL}{AY} = \frac{10 \times 10 \times 0.15}{10^{-4} \times 1 \times 10^{10}} = 15 \times 10^{-6} m$

$\therefore$ Displacement of point $D = 4 \times 10^{-6} + 5 \times 10^{-6} m + 15 \times 10^{-6} = 24 \times 10^{-6} m$.

Problem 36. Two blocks of masses m_1 and m_2 are joined by a wire of Young's modulus Y via a massless pulley. The area of cross-section of the wire is S and its length is L. When the system is released, increase in length of the wire is

(a) $\frac{m_1 m_2 gL}{YS\ (m_1 + m_2)}$

(b) $\frac{2m_1 m_2 gL}{YS\ (m_1 + m_2)}$



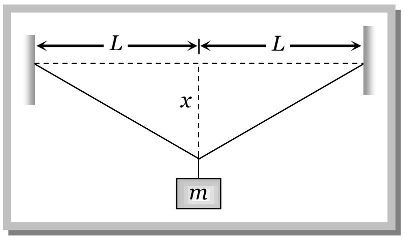
(c) $\frac{(m_1 - m_2)gL}{YS\,(m_1 + m_2)}$

(d) $\frac{4m_1 m_2 gL}{YS\,(m_1 + m_2)}$

Solution : (b) Tension in the wire $T = \frac{2m_1 m_2}{m_1 + m_2} g$ $\therefore$ stress in the wire $= \frac{T}{S} = \frac{2m_1 m_2 g}{S(m_1 + m_2)}$

$\therefore$ Strain $\frac{l}{L} = \frac{\text{Stress}}{\text{Y}} = \frac{2m_1 m_2 g}{YS\,(m_1 + m_2)} \Rightarrow l = \frac{2m_1 m_2 gL}{YS\,(m_1 + m_2)}$

Problem 37. A steel wire of diameter d, area of cross-section A and length $2L$ is clamped firmly at two points A and B which are $2L$ *metre* apart and in the same plane. A body of mass m is hung from the middle point of wire such that the middle point sags by x lower from original position. If Young's modulus is Y then m is given by



(a) $\frac{1}{2}\frac{YAx^2}{gL^2}$

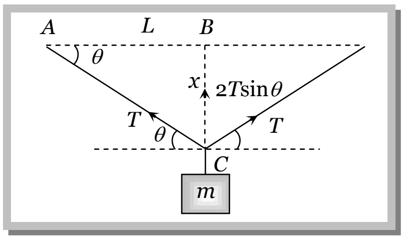
(b) $\frac{1}{2}\frac{YAL^2}{gx^2}$

(c) $\frac{YAx^3}{gL^3}$

(d) $\frac{YAL^3}{gx^2}$

Solution : (c) Let the tension in the string is T and for the equilibrium of mass m

$2T\sin\theta = mg \Rightarrow T = \frac{mg}{2\sin\theta} = \frac{mgL}{2x}$ [As θ is small then $\sin\theta = \frac{x}{L}$]



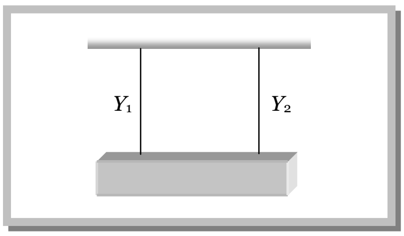
Increment in the length $l = AC - AB = \sqrt{L^2 + x^2} - L = (L^2 + x^2)^{1/2} - L$

$$= L\left[\left(1 + \frac{x^2}{L^2}\right)^{1/2} - 1\right] = L\left[1 + \frac{1}{2}\frac{x^2}{L^2} - 1\right] = \frac{x^2}{2L}$$

As Young's modulus $Y = \frac{T}{A}\frac{L}{l}$ $\therefore$ $T = \frac{YAl}{L}$

Substituting the value of T and l in the above equation we get $\frac{mgL}{2x} = \frac{YA}{L}.\frac{x^2}{2L}$ $\therefore m = \frac{YAx^3}{gL^3}$

Problem 38. Two wires of equal length and cross-section are suspended as shown. Their Young's modulii are Y_1 and Y_2 respectively. The equivalent Young's modulus will be



(a) $Y_1 + Y_2$

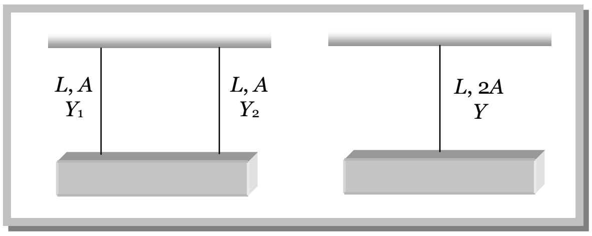
(b) $\frac{Y_1 + Y_2}{2}$

(c) $\frac{Y_1 Y_2}{Y_1 + Y_2}$

(d) $\sqrt{Y_1 Y_2}$

Solution : (b) Let the equivalent young's modulus of given combination is Y and the area of cross section is $2A$.

For parallel combination $k_1 + k_2 = k_{eq.}$



$$\frac{Y_1 A}{L} + \frac{Y_2 A}{L} = \frac{Y 2A}{L}$$

$$Y_1 + Y_2 = 2Y \text{ , } \therefore Y = \frac{Y_1 + Y_2}{2}$$

***Problem* 39.** If a load of 9*kg* is suspended on a wire, the increase in length is 4.5 *mm*. The force constant of the wire is

(a) $0.49 \times 10^4 \, N/m$ (b) $1.96 \times 10^4 \, N/m$ (c) $4.9 \times 10^4 \, N/m$ (d) $0.196 \times 10^4 \, N/m$

Solution : (b) Force constant $k = \frac{F}{l} = \frac{mg}{l} = \frac{9 \times 9.8}{4.5 \times 10^{-3}} \Rightarrow k = 1.96 \times 10^4 \, N/m$

***Problem* 40.** One end of a long metallic wire of length *L*, area of cross-section *A* and Young's modulus *Y* is tied to the ceiling. The other end is tied to a massless spring of force constant *k*. A mass *m* hangs freely from the free end of the spring. It is slightly pulled down and released. Its time period is given by

(a) $2\pi\sqrt{\frac{m}{K}}$ (b) $2\pi\sqrt{\frac{mYA}{KL}}$ (c) $2\pi\sqrt{\frac{mK}{YA}}$ (d) $2\pi\sqrt{\frac{m(KL+YA)}{KYA}}$

Solution : (d) Force constant of wire $k_1 = \frac{F}{l} = \frac{YA}{L}$ and force constant of spring $k_2 = k$ (given)

Equivalent force constant for given combination $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{L}{YA} + \frac{1}{k} \Rightarrow k_{eq} = \frac{kYA}{kL + YA}$

$\therefore$ Time period of combination $T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m(kL+YA)}{kYA}}$

***Problem* 41.** Two wires *A* and *B* have the same length and area of cross section. But Young's modulus of *A* is two times the Young's modulus of *B*. Then the ratio of force constant of *A* to that of *B* is

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\sqrt{2}$

Solution : (b) Force constant of wire $k = \frac{YA}{L} \Rightarrow \frac{k_A}{k_B} = \frac{Y_A}{Y_B} = 2$ [As *L* and *A* are same]

9.12 Work Done in Stretching a Wire

In stretching a wire work is done against internal restoring forces. This work is stored in the wire as elastic potential energy or strain energy.

If a force *F* acts along the length *L* of the wire of cross-section *A* and stretches it by *x* then

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{x/L} = \frac{FL}{Ax} \Rightarrow F = \frac{YA}{L}.x$$

So the work done for an additional small increase *dx* in length, $dw = Fdx = \frac{YA}{L}x.dx$

Hence the total work done in increasing the length by *l*, $W = \int_0^l dW = \int_0^l Fdx = \int_0^l \frac{YA}{L}.x\,dx = \frac{1}{2}\frac{YA}{L}l^2$

This work done is stored in the wire.

$\therefore$ Energy stored in wire $U = \frac{1}{2}\frac{YAl^2}{L} = \frac{1}{2}Fl$ $\quad\left[\text{As } F = \frac{YAl}{L}\right]$

Dividing both sides by volume of the wire we get energy stored in per unit volume of wire.

$U_V = \frac{1}{2}\times\frac{F}{A}\times\frac{l}{L} = \frac{1}{2}\times \text{stress}\times\text{strain} = \frac{1}{2}\times Y\times(\text{strain})^2 = \frac{1}{2Y}(\text{stress})^2$ [As AL = volume of wire]

Total energy stored in wire (U)	**Energy stored in per unit volume of wire (U_V)**
$\frac{1}{2}Fl$	$\frac{1}{2}\frac{Fl}{\text{volume}}$
$\frac{1}{2}\times\text{stress}\times\text{strain}\times\text{volume}$	$\frac{1}{2}\times\text{stress}\times\text{strain}$
$\frac{1}{2}\times Y\times(\text{strain})^2\times\text{volume}$	$\frac{1}{2}\times Y\times(\text{strain})^2$
$\frac{1}{2Y}\times(\text{stress})^2\times\text{volume}$	$\frac{1}{2Y}\times(\text{stress})^2$

Note : ❑ If the force on the wire is increased from F_1 to F_2 and the elongation in wire is l then energy stored in the wire $U = \frac{1}{2}\frac{(F_1+F_2)}{2}l$

❑ Thermal energy density = Thermal energy per unit volume = $\frac{1}{2}\times$ Thermal stress $\times$ strain

$$= \frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2}(Y\alpha\Delta\theta)(\alpha\Delta\theta) = \frac{1}{2}Y\alpha^2(\Delta\theta)^2$$

Sample problems based on Work done in Stretching a Wire

Problem 42. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm, then the elastic energy stored in the wire is

(a) 0.1 J (b) 0.2 J (c) 10 J (d) 20 J

Solution : (a) Elastic energy stored in wire = $U = \frac{1}{2}Fl = \frac{1}{2}\times 200\times 1\times 10^{-3} = 0.1\,J$

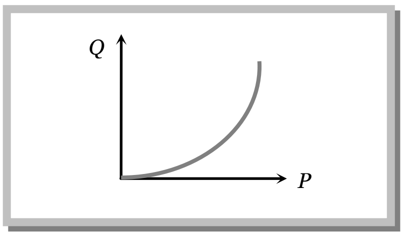
Problem 43. The graph shows the behaviour of a length of wire in the region for which the substance obeys Hooke's law. P and Q represent **[AMU 2001]**

(a) P = applied force, Q = extension

(b) P = extension, Q = applied force

(c) P = extension, Q = stored elastic energy

(d) P = stored elastic energy, Q = extension



Solution : (c) The graph between applied force and extension will be straight line because in elastic range applied force $\propto$ extension, but the graph between extension and stored elastic energy will be parabolic in nature.

As $U = \frac{1}{2}kx^2$ or $U \propto x^2$

***Problem* 44.** When a 4 *kg* mass is hung vertically on a light spring that obeys Hooke's law, the spring stretches by 2 *cms*. The work required to be done by an external agent in the stretching this spring by 5 *cms* will be (g = 9.8 m/s^2)

[MP PMT 1995]

(a) 4.900 *J* (b) 2.450 *J* (c) 0.495 *J* (d) 0.245 *J*

Solution : (b) When a 4 *kg* mass is hung vertically on a spring, it stretches by 2 *cm* $\therefore k = \frac{F}{x} = \frac{4 \times 9.8}{2 \times 10^{-2}} = 1960\, N/m$

Now work done in stretching this spring by 5 *cms* $U = \frac{1}{2}kx^2 = \frac{1}{2} \times 1960(5 \times 10^{-2})^2 = 2.45\, J.$

***Problem* 45.** A rod of iron of Young's modulus $Y = 2.0 \times 10^{11}\, N/m^2$ just fits the gap between two rigid supports 1*m* apart. If the rod is heated through $100^o C$ the strain energy of the rod is ($\alpha = 18 \times 10^{-6}\, {}^oC^{-1}$ and area of cross-section $A = 1\, cm^2$)

(a) 32.4 *J* (b) 32.4 *mJ* (c) 26.4 *J* (d) 26.4 *mJ*

Solution : (a) $U = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume} = \frac{1}{2} \times Y(\alpha\, \Delta\theta)^2 \times A \times L$ $\left(\text{Thermal strain } \frac{l}{L} = \alpha\, \Delta\theta\right)$

$= \frac{1}{2} \times (2 \times 10^{11}) \times (18 \times 10^{-6} \times 100)^2 \times 1 \times 10^{-4} \times 1 = 324 \times 10^{-1} = 32.4\, J.$

***Problem* 46.** Which of the following cases will have the greatest strain energy (F is the stretching force, A is the area of cross section and s is the strain)

(a) F = 10 N, A = 1 cm^2, s = 10^{-3} (b) F = 15 N, A = 2 cm^2, s = 10^{-3}

(c) F = 10 N, A = $\frac{1}{2}$ cm^2, s = 10^{-4} (d) F = 5 N, A = 3 cm^2, s = 10^{-3}

Solution: (b) Strain energy $= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} = \frac{1}{2} \times \frac{F}{A} \times \text{strain} \times AL = \frac{1}{2} \times F \times \text{strain} \times L$

For wire (a) $U = \frac{1}{2} \times 10 \times 10^{-3} \times L = 5 \times 10^{-3} L$; For wire (b) $U = \frac{1}{2} \times 15 \times 10^{-3} \times L = 7.5 \times 10^{-3} L$

For wire (c) $U = \frac{1}{2} \times 10 \times 10^{-4} \times L = 0.5 \times 10^{-3} L$; For wire (d) $U = \frac{1}{2} \times 5 \times 10^{-3} = 2.5 \times 10^{-3} L$

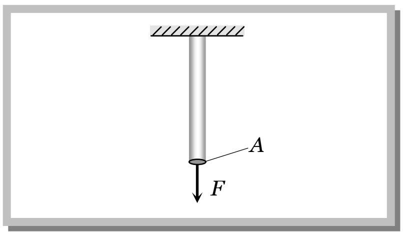
For a given length wire (b) will have greatest strain energy.

9.13 Breaking of Wire

When the wire is loaded beyond the elastic limit, then strain increases much more rapidly. The maximum stress corresponding to B (see stress-strain curve) after which the wire begin to flow and breaks, is called breaking stress or tensile strength and the force by application of which the wire breaks is called the breaking force.

(i) Breaking force depends upon the area of cross-section of the wire *i.e.*, Breaking force $\propto A$

$\therefore$ Breaking force = $P \times A$



Here P is a constant of proportionality and known as breaking stress.

(ii) Breaking stress is a constant for a given material and it does not depends upon the dimension (length or thickness) of wire.

(iii) If a wire of length L is cut into two or more parts, then again it's each part can hold the same weight. Since breaking force is independent of the length of wire.

(iv) If a wire can bear maximum force F, then wire of same material but double thickness can bear maximum force $4F$ because Breaking force $\propto \pi r^2$.

(v) The working stress is always kept lower than that of a breaking stress.

So that safety factor = $\frac{\text{breaking stress}}{\text{working stress}}$ may have large value.

(vi) Breaking of wire under its own weight.

Breaking force = Breaking stress × Area of cross section

Weight of wire = $Mg = ALdg = PA$ [As mass = volume × density = ALd]

$\Rightarrow \quad Ldg = P \quad \therefore \quad L = \frac{P}{dg}$

This is the length of wire if it breaks by its own weight.

Sample problems based on Breaking of Wire

Problem 47. A wire of diameter 1 *mm* breaks under a tension of 1000 *N*. Another wire of same material as that of the first one, but of diameter 2 *mm* breaks under a tension of

(a) 500 *N* (b) 1000 *N* (c) 10000 *N* (d) 4000 *N*

Solution : (d) Breaking force $\propto$ area of cross-section $(\pi r^2) \propto d^2$

$$\frac{F_2}{F_1} = \left(\frac{d_2}{d_1}\right)^2 \Rightarrow \frac{F_2}{1000} = \left(\frac{2mm}{1mm}\right)^2 \Rightarrow F_2 = 1000 \times 4 = 4000\,N.$$

Problem 48. In steel, the Young's modulus and the strain at the breaking point are $2 \times 10^{11}\ Nm^{-2}$ and 0.15 respectively. The stress at the breaking point for steel is therefore

(a) $1.33 \times 10^{11}\ Nm^{-2}$ (b) $1.33 \times 10^{12}\ Nm^{-2}$ (c) $7.5 \times 10^{-13}\ Nm^{-2}$ (d) $3 \times 10^{10}\ Nm^{-2}$

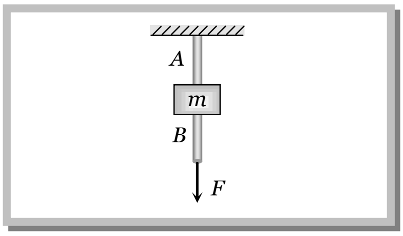
Solution : (d) $Y = \frac{\text{Stress}}{\text{Strain}} \quad \therefore$ Stress = Y × Strain $= 2 \times 10^{11} \times 0.15 = 0.3 \times 10^{11} = 3 \times 10^{10}\ N/m^2$

Problem 49. To break a wire, a force of $10^6\ N/m^2$ is required. If the density of the material is $3 \times 10^3\ kg/m^3$, then the length of the wire which will break by its own weight will be **[Roorkee 1979]**

(a) 34 *m* (b) 30 *m* (c) 300 *m* (d) 3 *m*

Solution : (a) Length of the wire which will break by its own weight $L = \frac{P}{dg} = \frac{10^6}{3 \times 10^3 \times 10} = \frac{100}{3} = 33.3\ m \simeq 34\ m$.

Problem 50. The wires A and B shown in the figure are made of the same material and have radii r_A and r_B respectively. The block between them has a mass m. When the force F is $mg/3$, one of the wires break



(a) A will break before B if $r_A = r_B$

(b) A will break before B if $r_A < 2r_B$

(c) Either A or B may break if $r_A = 2r_B$

(d) The lengths of A and B must be known to predict which wire will break

Solution :(a,b,c) When force $F = \frac{mg}{3}$ is applied at the lower end then

Stress in wire $B = \frac{F}{\pi r_B^2} = \frac{mg}{3\pi r_B^2}$ and stress in wire $A = \frac{F + mg}{\pi r_A^2} = \frac{\frac{mg}{3} + mg}{\pi r_A^2} = \frac{4}{3}\frac{mg}{\pi r_A^2}$

(i) if $r_A = r_B = r$ (Let) then stress in wire $B = \frac{mg}{3\pi r^2}$ and stress in wire $A = \frac{4}{3} \cdot \frac{mg}{\pi r^2}$

i.e. stress in wire A > stress in wire B so the A will break before B

(ii) if $r_B = r$, (let) then $r_A = 2r$

Stress in wire $B = \frac{mg}{3\pi r^2}$ and Stress in wire $A = \frac{4mg}{3\pi (2r)^2} = \frac{mg}{3\pi r^2}$

i.e. stress in wire A = stress in wire B. It means either A or B may break.

(iii) If $r_A < 2r_B$ then stress in A will be more than B. *i.e.* A will break before B.

***Problem* 51.** A body of mass 10 *kg* is attached to a wire 0.3 *m* long. Its breaking stress is $4.8 \times 10^7 \ N/m^2$. The area of cross-section of the wire is $10^{-6} m^2$. What is the maximum angular velocity with which it can be rotated in the horizontal circle

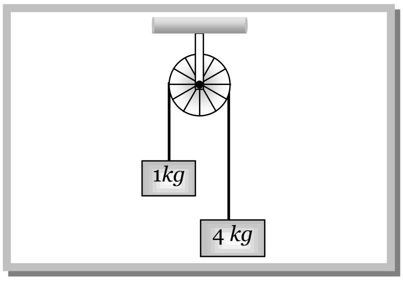
(a) 1 *rad/sec* (b) 2 *rad/sec* (c) 4 *rad/sec* (d) 8 *rad/sec*

Solution : (c) Breaking force = centrifugal force

Breaking stress × area of cross-section = $m\omega^2 l$

$4.8 \times 10^7 \times 10^{-6} = 10 \times \omega^2 \times 0.3 \Rightarrow \omega^2 = 16 \quad \Rightarrow \omega = 4 rad/\sec$

***Problem* 52.** Two block of masses 1 *kg* and 4 *kg* are connected by a metal wire going over a smooth pulley as shown in the figure. The breaking stress of the metal is $3.18 \times 10^{10} \ N/m^2$. The minimum radius of the wire so it will not break is



(a) $1 \times 10^{-5} m$

(b) $2 \times 10^{-5} m$

(c) $3 \times 10^{-5} m$

(d) $4 \times 10^{-5} m$

Solution : (d) Tension in the wire $T = \frac{2m_1 m_2}{m_1 + m_2} g \Rightarrow T = \frac{2 \times 1 \times 4}{1 + 4} \times 10 \Rightarrow T = 16N$

Breaking force = Breaking stress × Area of cross-section

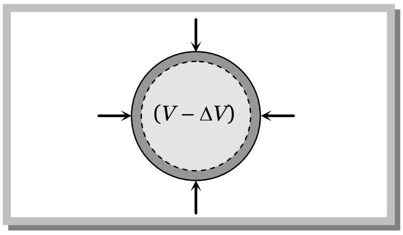
Tension in the wire = $3.18 \times 10^{10} \times \pi r^2$

$16 = 3.18 \times 10^{10} \times \pi r^2 \Rightarrow r = \sqrt{\frac{16}{3.18 \times 10^{10} \times 3.14}} = 4 \times 10^{-5} m.$

9.14 Bulk Modulus.

When a solid or fluid (liquid or gas) is subjected to a uniform pressure all over the surface, such that the shape remains the same, then there is a change in volume.

Then the ratio of normal stress to the volumetric strain within the elastic limits is called as Bulk modulus. This is denoted by K.



$$K = \frac{\text{Normal stress}}{\text{volumetric strain}}$$

$$K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$

where p = increase in pressure; V = original volume; ΔV = change in volume

The negative sign shows that with increase in pressure p, the volume decreases by ΔV *i.e.* if p is positive, ΔV is negative. The reciprocal of bulk modulus is called compressibility.

$$C = \text{compressibility} = \frac{1}{K} = \frac{\Delta V}{pV}$$

S.I. unit of compressibility is $N^{-1}m^2$ and C.G.S. unit is $dyne^{-1}\,cm^2$.

Gases have two bulk moduli, namely isothermal elasticity E_θ and adiabatic elasticity E_ϕ.

(1) **Isothermal elasticity (E_θ) :** Elasticity possess by a gas in isothermal condition is defined as isothermal elasticity.

For isothermal process, PV = constant (Boyle's law)

Differentiating both sides $PdV + VdP = 0 \Rightarrow PdV = -VdP$

$$P = \frac{dP}{(-dV/V)} = \frac{\text{stress}}{\text{strain}} = E_\theta$$

$\therefore \quad E_\theta = P$

i.e., Isothermal elasticity is equal to pressure.

(2) **Adiabatic elasticity (E_ϕ) :** Elasticity possess by a gas in adiabatic condition is defined as adiabatic elasticity.

For adiabatic process, PV^γ = constant (Poisson's law)

Differentiating both sides, $P\gamma V^{\gamma-1}dV + V^\gamma dP = 0 \Rightarrow \gamma PdV + VdP = 0$

$$\gamma P = \frac{dP}{\left(\frac{-dV}{V}\right)} = \frac{\text{stress}}{\text{strain}} = E_\phi$$

$\therefore \quad E_\phi = \gamma P$

i.e., adiabatic elasticity is equal to γ times pressure. [where $\gamma = \frac{C_p}{C_v}$]

Note: ❑ Ratio of adiabatic to isothermal elasticity $\frac{E_\phi}{E_\theta} = \frac{\gamma P}{P} = \gamma > 1 \quad \therefore E_\phi > E_\theta$

i.e., adiabatic elasticity is always more than isothermal elasticity.

9.15 Density of Compressed Liquid

If a liquid of density ρ, volume V and bulk modulus K is compressed, then its density increases.

As density $\rho = \frac{m}{V}$ so $\frac{\Delta\rho}{\rho} = \frac{-\Delta V}{V}$ .....(i)

But by definition of bulk modulus $K = \frac{-V\Delta P}{\Delta V} \Rightarrow -\frac{\Delta V}{V} = \frac{\Delta P}{K}$ .....(ii)

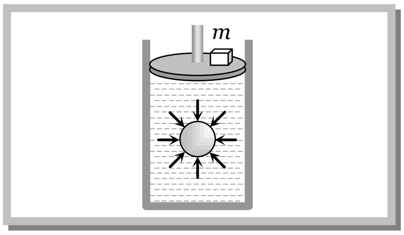
From (i) and (ii) $\frac{\Delta\rho}{\rho} = \frac{\rho' - \rho}{\rho} = \frac{\Delta P}{K}$ [As $\Delta\rho = \rho' - \rho$]

or $\rho' = \rho\left[1 + \frac{\Delta P}{K}\right] = \rho[1 + C\Delta P]$ $\left[\text{As } \frac{1}{K} = C\right]$

9.16 Fractional Change in the Radius of Sphere

A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container.

A massless piston of area A floats on the surface of the liquid.



Volume of the spherical body $V = \frac{4}{3}\pi R^3$

$$\frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

$\therefore$ $\frac{\Delta R}{R} = \frac{1}{3}\frac{\Delta V}{V}$ .....(i)

Bulk modulus $K = -V\frac{\Delta P}{\Delta V}$

$\therefore$ $\left|\frac{\Delta V}{V}\right| = \frac{\Delta P}{K} = \frac{mg}{AK}$ .....(ii) $\left[\text{As } \Delta P = \frac{mg}{A}\right]$

Substituting the value of $\frac{\Delta V}{V}$ from equation (ii) in equation (i) we get $\frac{\Delta R}{R} = \frac{1}{3}\frac{mg}{AK}$

Sample problems based on Bulk modulus

Problem 53. When a pressure of 100 atmosphere is applied on a spherical ball of rubber, then its volume reduces to 0.01%. The bulk modulus of the material of the rubber in $dyne/cm^2$ is

(a) 10×10^{12} (b) 100×10^{12} (c) 1×10^{12} (d) 20×10^{12}

Solution : (c) $1\, atm = 10^5\, N/m^2$ $\therefore 100\, atm = 10^7\, N/m^2$ and $\Delta V = 0.01\% V \therefore \frac{\Delta V}{V} = 0.0001$

$$K = \frac{P}{\Delta V / V} = \frac{10^7}{0.0001} = 1 \times 10^{11}\, N/m^2 = 1 \times 10^{12}\, \frac{Dyne}{cm^2}.$$

Problem 54. Coefficient of isothermal elasticity E_θ and coefficient of adiabatic elasticity E_ϕ are related by $(\gamma = C_p / C_v)$

[MP PET 2000]

(a) $E_\theta = \gamma E_\phi$ (b) $E_\phi = \gamma E_\theta$ (c) $E_\theta = \gamma / E_\phi$ (d) $E_\theta = \gamma^2 E_\phi$

Solution : (b) Adiabatic elasticity $= \gamma \times$ isothermal elasticity $\Rightarrow E_\phi = \gamma E_\theta$.

Problem 55. A uniform cube is subjected to volume compression. If each side is decreased by 1%, then bulk strain is **[EAMCET (Engg.) 1995; DPMT 2000]**

(a) 0.01 (b) 0.06 (c) 0.02 (d) 0.03

Solution : (d) Volume of cube $V = L^3$ $\therefore$ Percentage change in $V = 3 \times$ (percentage change in L)= $3(1\%) = 3\%$

$\therefore \Delta V = 3\%$ of $V \Rightarrow$ Volumetric strain $= \frac{\Delta V}{V} = \frac{3}{100} = 0.03$

Problem 56. A ball falling in a lake of depth 200 m shows 0.1% decrease in its volume at the bottom. What is the bulk modulus of the material of the ball **[AFMC 1997]**

(a) $19.6 \times 10^{8}\ N/m^2$ (b) $19.6 \times 10^{-10}\ N/m^2$ (c) $19.6 \times 10^{10}\ N/m^2$ (d) $19.6 \times 10^{-8}\ N/m^2$

Solution : (a) $K = \frac{P}{\Delta V / V} = \frac{hdg}{\Delta V / V} = \frac{200 \times 10^3 \times 9.8}{0.001} = 19.6 \times 10^8\ N/m^2$

Problem 57. The ratio of the adiabatic to isothermal elasticities of a triatomic gas is **[MP PET 1991]**

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 1 (d) $\frac{5}{3}$

Solution : (b) For triatomic gas $\gamma = 4/3$ $\therefore$ Ratio of adiabatic to isothermal elasticity $\gamma = \frac{4}{3}$.

Problem 58. A gas undergoes a change according to the law $P = P_0 e^{\alpha V}$. The bulk modulus of the gas is

(a) P (b) αPV (c) αP (d) $\frac{PV}{\alpha}$

Solution : (b) $P = P_o e^{\alpha V} \Rightarrow \frac{dP}{dV} = P_o e^{\alpha V} \alpha = P\alpha$ [As $P = P_o e^{\alpha V}$]

$\frac{dP}{dV} V = P\alpha V \Rightarrow \frac{dP}{(dV/V)} = P\alpha V$ $\therefore K = P\alpha V$

Problem 59. The ratio of two specific heats of gas C_p / C_v for argon is1.6 and for hydrogen is 1.4. Adiabatic elasticity of argon at pressure P is E. Adiabatic elasticity of hydrogen will also be equal to E at the pressure

(a) P (b) $\frac{8}{7}P$ (c) $\frac{7}{8}P$ (d) 1.4 P

Solution : (b) Adiabatic elasticity = γ(pressure)

For Argon $(E_\phi)_{Ar} = 1.6P$ and for Hydrogen $(E_\phi)_{H_2} = 1.4P'$

According to problem $(E_\phi)_{H_2} = (E_\phi)_{Ar} \Rightarrow 1.4P' = 1.6P \Rightarrow P' = \frac{16}{14}P = \frac{8}{7}P$.

Problem 60. The pressure applied from all directions on a cube is P. How much its temperature should be raised to maintain the original volume ? The volume elasticity of the cube is β and the coefficient of volume expansion is α

(a) $\frac{P}{\alpha\beta}$ (b) $\frac{P\alpha}{\beta}$ (c) $\frac{P\beta}{\alpha}$ (d) $\frac{\alpha\beta}{P}$

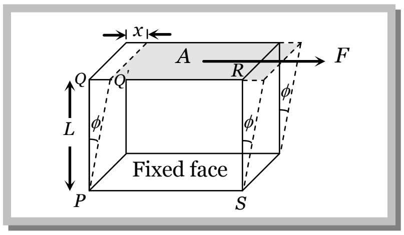
Solution : (a) Change in volume due to rise in temperature $\Delta V = V\alpha\,\Delta\theta$

$\therefore$ volumetric strain $= \dfrac{\Delta V}{V} = \alpha\,\Delta\theta$

But bulk modulus $\Rightarrow \beta = \dfrac{\text{stress}}{\text{strain}} = \dfrac{P}{\alpha\,\Delta\theta}$ $\therefore \Delta\theta = \dfrac{P}{\alpha\beta}$

9.17 Modulus of Rigidity

Within limits of proportionality, the ratio of tangential stress to the shearing strain is called modulus of rigidity of the material of the body and is denoted by η, *i.e.*



$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

In this case the shape of a body changes but its volume remains unchanged.

Consider a cube of material fixed at its lower face and acted upon by a tangential force F at its upper surface having area A. The shearing stress, then, will be

Shearing stress $= \dfrac{F_{||}}{A} = \dfrac{F}{A}$

This shearing force causes the consecutive horizontal layers of the cube to be slightly displaced or sheared relative to one another, each line such as PQ or RS in the cube is rotated through an angle ϕ by this shear. The shearing strain is defined as the angle ϕ in radians through which a line normal to a fixed surface has turned. For small values of angle,

Shearing strain $= \phi = \dfrac{QQ'}{PQ} = \dfrac{x}{L}$

So $\quad \eta = \dfrac{\text{shear stress}}{\text{shear strain}} = \dfrac{F/A}{\phi} = \dfrac{F}{A\phi}$

Only solids can exhibit a shearing as these have definite shape.

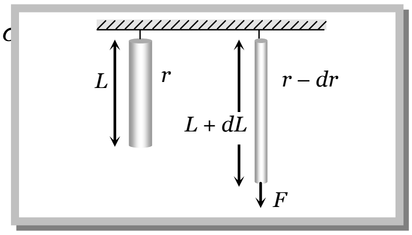
9.18 Poisson's Ratio

When a long bar is stretched by a force along its length then its length increases and the radius decreases as shown in the figure.

Lateral strain : The ratio of change in radius to the original radius is called lateral strain.

Longitudinal strain : The ratio of change in length to the original length is called longitudinal strain.

The ratio of lateral strain to longitudinal strain is called Poisson's ratio (σ)



i.e. $\quad \sigma = \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$$\sigma = \frac{-dr/r}{dL/L}$$

Negative sign indicates that the radius of the bar decreases when it is stretched.

Poisson's ratio is a dimensionless and a unitless quantity.

9.19 Relation Between Volumetric Strain, Lateral Strain and Poisson's Ratio

If a long bar have a length L and radius r then volume $V = \pi r^2 L$

Differentiating both the sides $dV = \pi r^2 dL + \pi 2rL\,dr$

Dividing both the sides by volume of bar $\frac{dV}{V} = \frac{\pi r^2 dL}{\pi r^2 L} + \frac{\pi 2rL\,dr}{\pi r^2 L} = \frac{dL}{L} + 2\frac{dr}{r}$

$\Rightarrow$ Volumetric strain = longitudinal strain + 2(lateral strain)

$\Rightarrow$ $\frac{dV}{V} = \frac{dL}{L} + 2\sigma\frac{dL}{L} = (1+2\sigma)\frac{dL}{L}$ $\left[\text{As } \sigma = \frac{dr/r}{dL/L} \Rightarrow \frac{dr}{r} = \sigma\frac{dL}{L}\right]$

or $\sigma = \frac{1}{2}\left[1 - \frac{dV}{AdL}\right]$ [where A = cross-section of bar]

Important points

(i) If a material having $\sigma = -0.5$ then $\frac{dV}{V} = [1+2\sigma]\frac{dL}{L} = 0$

$\therefore$ Volume = constant or $K = \infty$ *i.e.*, the material is incompressible.

(ii) If a material having σ = 0, then lateral strain is zero *i.e.* when a substance is stretched its length increases without any decrease in diameter *e.g.* cork. In this case change in volume is maximum.

(iii) Theoretical value of Poisson's ratio $-1 < \sigma < 0.5$.

(iv) Practical value of Poisson's ratio $0 < \sigma < 0.5$

9.20 Relation between Y, k, η and σ.

Moduli of elasticity are three, *viz.* Y, K and η while elastic constants are four, *viz*, Y, K, η and σ. Poisson's ratio σ is not modulus of elasticity as it is the ratio of two strains and not of stress to strain. Elastic constants are found to depend on each other through the relations : $Y = 3K(1-2\sigma)$ and $Y = 2\eta(1+\sigma)$

Eliminating σ or Y between these, we get $Y = \frac{9K\eta}{3K+\eta}$ and $\sigma = \frac{3K-2\eta}{6K+2\eta}$

Sample problems based on relation between Y, k, η and σ

Problem 61. Minimum and maximum values of Poisson's ratio for a metal lies between **[Orissa JEE 2003]**

(a) $-\infty$ to $+\infty$ (b) 0 to 1 (c) $-\infty$ to 1 (d) 0 to 0.5

Solution : (d)

Problem 62. For a given material, the Young's modulus is 2.4 times that of rigidity modulus. Its Poisson's ratio is **[EAMCET 1990; RPET 2001]**

(a) 2.4 (b) 1.2 (c) 0.4 (d) 0.2

Solution : (d) $Y = 2\eta(1+\sigma) \Rightarrow 2.4\eta = 2\eta(1+\sigma) \Rightarrow 1.2 = 1+\sigma \Rightarrow \sigma = 0.2$

Problem 63. There is no change in the volume of a wire due to change in its length on stretching. The Poisson's ratio of the material of the wire is

(a) + 0.50 (b) – 0.50 (c) + 0.25 (d) – 0.25

Solution : (b) $\frac{dV}{V} = \frac{dL}{L} + 2\sigma\frac{dL}{L} = (1+2\sigma)\frac{dL}{L} = 0$ [As there is no change in the volume of the wire]

$\therefore 1 + 2\sigma = 0 \Rightarrow \sigma = -\frac{1}{2}$

***Problem* 64.** The values of Young's and bulk modulus of elasticity of a material are $8\times10^{10}\ N/m^2$ and $10\times10^{10}\ N/m^2$ respectively. The value of Poisson's ratio for the material will be

(a) 0.25 (b) – 0.25 (c) 0.37 (d) – 0.37

Solution : (c) $Y = 3K(1-2\sigma) \Rightarrow 8\times10^{10} = 3\times10\times10^{10}(1-2\sigma) \Rightarrow \sigma = 0.37$

***Problem* 65.** The Poisson's ratio for a metal is 0.25. If lateral strain is 0.0125, the longitudinal strain will be

(a) 0.125 (b) 0.05 (c) 0.215 (d) 0.0125

Solution : (b) $\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$ $\therefore$ Longitudinal strain $= \frac{\text{Lateral strain}}{\sigma} = \frac{0.0125}{0.25} = 0.05$

***Problem* 66.** The 'σ' of a material is 0.20. If a longitudinal strain of 4.0×10^{-3} is caused, by what percentage will the volume change

(a) 0.48% (b) 0.32% (c) 0.24% (d) 0.50%

Solution : (c) Longitudinal strain $= 4\times10^{-3}$ or 0.4%

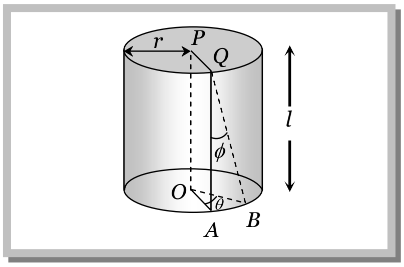
Lateral strain $= \sigma\times0.4\% = 0.2\times0.4\% = 0.08\%$

$\therefore$ Volumetric strain = longitudinal strain – 2 lateral strain $= 0.4 - 2\times(0.08)$ = 0.24%

$\therefore$ Volume will change by 0.24%.

9.21 Torsion of Cylinder.

If the upper end of a cylinder is clamped and a torque is applied at the lower end the cylinder gets twisted by angle θ. Simultaneously shearing strain ϕ is produced in the cylinder.



(i) The angle of twist θ is directly proportional to the distance from the fixed end of the cylinder.

At fixed end $\theta = 0^\circ$ and at free end θ = maximum.

(ii) The value of angle of shear ϕ is directly proportional to the radius of the cylindrical shell.

At the axis of cylinder ϕ = 0 and at the outermost shell ϕ = maximum.

(iii) Relation between angle of twist (θ) and angle of shear (ϕ)

$$AB = r\theta = \phi l \qquad \therefore \phi = \frac{r\theta}{l}$$

(iv) Twisting couple per unit twist or torsional rigidity or torque required to produce unit twist.

$$C = \frac{\pi\eta r^4}{2l} \qquad \therefore C \propto r^4 \propto A^2$$

(v) Work done in twisting the cylinder through an angle θ is $W = \frac{1}{2}C\theta^2 = \frac{\pi\eta r^4\theta^2}{4l}$

Sample problems based on Torsion

Problem 67. Mark the wrong statement **[MP PMT 2003]**

(a) Sliding of molecular layer is much easier than compression or expansion

(b) Reciprocal of bulk modulus of elasticity is called compressibility

(c) It is difficult to twist a long rod as compared to small rod

(d) Hollow shaft is much stronger than a solid rod of same length and same mass

Solution : (c)

Problem 68. A rod of length l and radius r is joined to a rod of length $l/2$ and radius $r/2$ of same material. The free end of small rod is fixed to a rigid base and the free end of larger rod is given a twist of θ, the twist angle at the joint will be **[RPET 1997]**

(a) $\theta/4$ (b) $\theta/2$ (c) $5\theta/6$ (d) $8\theta/9$

Solution : (d) If torque τ is applied at the free end of larger rod and twist θ is given to it then twist at joint is θ_1 and twist at the upper end (fixed base) θ_2

$$\tau = \frac{\pi\eta r^4(\theta - \theta_1)}{2l} = \frac{\pi\eta\left(\frac{r}{2}\right)^4(\theta_1 - \theta_2)}{2(l/2)}$$

$$\Rightarrow \quad (\theta - \theta_1) = \frac{(\theta_1 - 0)}{8} \qquad [\text{As } \theta_2 = 0]$$

$$\Rightarrow \quad 8\theta - 8\theta_1 = \theta_1 \Rightarrow 9\theta_1 = 8\theta \Rightarrow \theta_1 = \frac{8\theta}{9}.$$

Problem 69. The upper end of a wire of radius 4 *mm* and length 100 *cm* is clamped and its other end is twisted through an angle of 30^o. Then angle of shear is **[NCERT 1990; MP PMT 1996]**

(a) 12^o (b) 0.12^o (c) 1.2^o (d) 0.012^o

Solution : (b) $L\phi = r\theta \quad \therefore \phi = \frac{r\theta}{L} = \frac{4\times10^{-3}\times30^o}{1} = 0.12^o$

Problem 70. Two wires *A* and *B* of same length and of the same material have the respective radii r_1 and r_2. Their one end is fixed with a rigid support, and at the other end equal twisting couple is applied. Then the ratio of the angle of twist at the end of *A* and the angle of twist at the end of *B* will be **[AIIMS 1980]**

(a) $\frac{r_1^2}{r_2^2}$ (b) $\frac{r_2^2}{r_1^2}$ (c) $\frac{r_2^4}{r_1^4}$ (d) $\frac{r_1^4}{r_2^4}$

Solution : (c) $\tau_1 = \tau_2 \Rightarrow \frac{\pi\eta r_1^4\theta_1}{2l_1} = \frac{\pi\eta r_2^4\theta_2}{2l_2} \Rightarrow \frac{\theta_1}{\theta_2} = \left(\frac{r_2}{r_1}\right)^4$

Problem 71. The work done in twisting a steel wire of length 25 *cm* and radius 2*mm* through 45^o will be $(\eta = 8\times10^{10}\ N/m^2)$

(a) 2.48 *J* (b) 3.1 *J* (c) 15.47 *J* (d) 18.79 *J*

Solution : (a) $W = \frac{1}{2}C\theta^2 = \frac{\pi\eta r^4\theta^2}{4l} = \frac{3.14\times8\times10^{10}\times(2\times10^{-3})^4\times(\pi/4)^2}{4\times25\times10^{-2}} = 2.48\ J$

9.22 Interatomic Force Constant

Behaviour of solids with respect to external forces is such that if their atoms are connected to springs. When an external force is applied on a solid, this distance between its atoms changes and interatomic force works to restore the original dimension.

The ratio of interatomic force to that of change in interatomic distance is defined as the interatomic force constant. $K = \dfrac{F}{\Delta r}$

It is also given by $K = Y \times r_0$ [Where Y = Young's modulus, r_0 = Normal distance between the atoms of wire]

Unit of interatomic force constant is N/m and Dimension MT^{-2}

Note : ❑ The number of atoms having interatomic distance r_0 in length l of a wire, $N = l/r_0$.

❑ The number of atoms in area A of wire having interatomic separation r_0 is $N = A/r_0^2$.

Sample problems based on Interatomic Force Constant

Problem 72. The mean distance between the atoms of iron is $3 \times 10^{-10} m$ and interatomic force constant for iron is 7 N/m. The Young's modulus of elasticity for iron is **[JIPMER 2002]**

(a) $2.33 \times 10^5\ N/m^2$ (b) $23.3 \times 10^{10}\ N/m^2$ (c) $233 \times 10^{10}\ N/m^2$ (d) $2.33 \times 10^{10}\ N/m^2$

Solution : (d) $Y = \dfrac{k}{r_o} = \dfrac{7}{3 \times 10^{-10}} = 2.33 \times 10^{10}\ N/m^2$.

Problem 73. The Young's modulus for steel is $Y = 2 \times 10^{11}\ N/m^2$. If the inter-atomic distance is 3.2$Å$, the inter atomic force constant in $N/Å$ will be

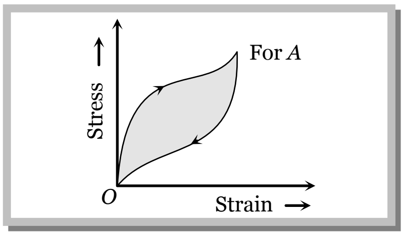
(a) 6.4×10^9 (b) 6.4×10^{-9} (c) 3.2×10^9 (d) 3.2×10^{-9}

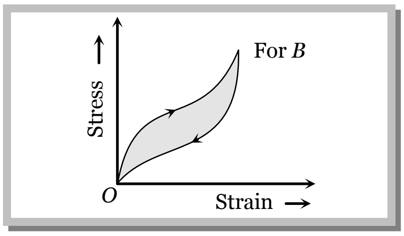
Solution : (b) $k = Y \times r_0 = 2 \times 10^{11} \times 3.2 \times 10^{-10} = 6.4 \times 10^1\ N/m = 6.4 \times 10^{-9}\ N/Å$.

9.23 Elastic Hysteresis

When a deforming force is applied on a body then the strain does not change simultaneously with stress rather it lags behind the stress. The lagging of strain behind the stress is defined as elastic hysteresis. This is the reason why the values of strain for same stress are different while increasing the load and while decreasing the load.

Hysteresis loop : The area of the stress-strain curve is called the hysteresis loop and it is numerically equal to the work done in loading the material and then unloading it.





If we have two tyres of rubber having different hysteresis loop then rubber B should be used for making the car tyres. It is because of the reason that area under the curve *i.e.* work done in case of rubber B is lesser and hence the car tyre will not get excessively heated and rubber A should be used to absorb vibration of the machinery because of the large area of the curve, a large amount of vibrational energy can be dissipated.

9.24 Factors Affecting Elasticity.

(1) Hammering and rolling : Crystal grains break up into smaller units by hammering and rolling. This result in increase in the elasticity of material.

(2) Annealing **:** The metals are annealed by heating and then cooling them slowly. Annealing results in decrease in the elasticity of material.

(3) Temperature **:** Intermolecular forces decreases with rise in temperature. Hence the elasticity decreases with rise in temperature but the elasticity of invar steel (alloy) does not change with change of temperature.

(4) Impurities **:** Due to impurities in a material elasticity can increase or decrease. The type of effect depends upon the nature of impurities present in the material.

9.25 Important Facts About Elasticity.

(1) The body which requires greater deforming force to produce a certain change in dimension is more elastic.

Example : Ivory and steel balls are more elastic than rubber.

(2) When equal deforming force is applied on different bodies then the body which shows less deformation is more elastic.

Example : (i) For same load, more elongation is produced in rubber wire than in steel wire hence steel is more elastic than rubber.

(ii) Water is more elastic than air as volume change in water is less for same applied pressure.

(iii) Four identical balls of different materials are dropped from the same height then after collision balls rises upto different heights.

The order of their height can be given by $h_{\text{ivory}} > h_{\text{steel}} > h_{\text{rubber}} > h_{\text{clay}}$ because $Y_{\text{ivory}} > Y_{\text{steel}} > Y_{\text{rubber}} > Y_{\text{clay}}$.

(3) The value of moduli of elasticity is independent of the magnitude of the stress and strain. It depends only on the nature of material of the body.

(4) For a given material there can be different moduli of elasticity depending on the type of stress applied and resulting strain.

Name of substance	Young's modulus (Y) $10^{10} N/m^2$	Bulk modulus (K) $10^{10} N/m^2$	Modulus of rigidity (η) $10^{10} N/m^2$
Aluminium	6.9	7.0	2.6
Brass	9.0	6.7	3.4
Copper	11.0	13.0	4.5
Iron	19.0	14.0	4.6
Steel	20.0	16.0	8.4
Tungsten	36.0	20.0	15.0
Diamond	83.0	55.0	34.0
Water	–	0.22	–
Glycerin	–	0.45	–
Air	–	1.01	–

(5) The moduli of elasticity has same dimensional formula and units as that of stress since strain is dimensionless. ∴ Dimensional formula $ML^{-1}T^{-2}$ while units *dyne/cm*² or *Newton/m*².

(6) Greater the value of moduli of elasticity more elastic is the material. But as $Y \propto (1/l)$, $K \propto (1/\Delta V)$ and $\eta \propto (1/\phi)$ for a constant stress, so smaller change in shape or size for a given stress corresponds to greater elasticity.

(7) The moduli of elasticity Y and η exist only for solids as liquids and gases cannot be deformed along one dimension only and also cannot sustain shear strain. However K exist for all states of matter *viz*. solid, liquid or gas.

(8) Gases being most compressible are least elastic while solids are most *i.e.* the bulk modulus of gas is very low while that for liquids and solids is very high. $K_{\text{solid}} > K_{\text{liquid}} > K_{\text{gas}}$

(9) For a rigid body l, ΔV or $\phi = 0$ so Y, K or η will be ∞, *i.e.* elasticity of a rigid body is infinite.

Diamond and carborundum are nearest approach to rigid bodies.

(10) In a suspension bridge there is a stretch in the ropes by the load of the bridge. Due to which length of rope changes. Hence Young's modulus of elasticity is involved.

(11) In an automobile tyre as the air is compressed, volume of the air in tyre changes, hence the bulk modulus of elasticity is involved.

(12) In transmitting power, an automobile shaft is sheared as it rotates, so shearing strain is set up, hence modulus of rigidity is involved.

(13) The shape of rubber heels changes under stress, so modulus of rigidity is involved.

9.26 Practical Applications of Elasticity

(i) The metallic parts of machinery are never subjected to a stress beyond elastic limit, otherwise they will get permanently deformed.

(ii) The thickness of the metallic rope used in the crane in order to lift a given load is decided from the knowledge of elastic limit of the material of the rope and the factor of safety.

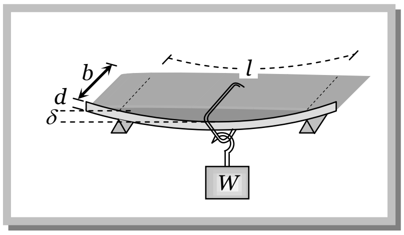
(iii) The bridges are declared unsafe after long use because during its long use, a bridge under goes quick alternating strains continuously. It results in the loss of elastic strength.

(iv) Maximum height of a mountain on earth can be estimated from the elastic behaviour of earth.

At the base of the mountain, the pressure is given by $P = h\rho g$ and it must be less than elastic limit (K) of earth's supporting material.

$$K > P > h\rho g \qquad \therefore\ h < \frac{K}{\rho g} \qquad \text{or} \qquad h_{\max} = \frac{K}{\rho g}$$

(v) In designing a beam for its use to support a load (in construction of roofs and bridges), it is advantageous to increase its depth rather than the breadth of the beam because the depression in rectangular beam.



$$\delta = \frac{Wl^3}{4Ybd^3}$$

To minimize the depression in the beam, it is designed as *I*-shaped girder.

(vi) For a beam with circular cross-section depression is given by $\delta = \frac{WL^3}{12\pi r^4 Y}$

(vii) A hollow shaft is stronger than a solid shaft made of same mass, length and material.

Torque required to produce a unit twist in a solid shaft $\tau_{\text{solid}} = \frac{\pi\eta r^4}{2l}$ ......(i)

and torque required to produce a unit twist in a hollow shaft $\tau_{\text{hollow}} = \frac{\pi\eta(r_2^4 - r_1^4)}{2l}$ .....(ii)

From (i) and (ii), $\frac{\tau_{\text{hollow}}}{\tau_{\text{solid}}} = \frac{r_2^4 - r_1^4}{r^4} = \frac{(r_2^2 + r_1^2)(r_2^2 - r_1^2)}{r^4}$ .....(iii)

Since two shafts are made from equal volume $\therefore \pi r^2 l = \pi(r_2^2 - r_1^2)l \Rightarrow r^2 = r_2^2 - r_1^2$

Substituting this value in equation (iii) we get, $\frac{\tau_{\text{hollow}}}{\tau_{\text{solid}}} = \frac{r_2^2 + r_1^2}{r^2} > 1 \quad \therefore \tau_{\text{hollow}} > \tau_{\text{solid}}$

i.e., the torque required to twist a hollow shaft is greater than the torque necessary to twist a solid shaft of the same mass, length and material through the same angle. Hence, a hollow shaft is stronger than a solid shaft.

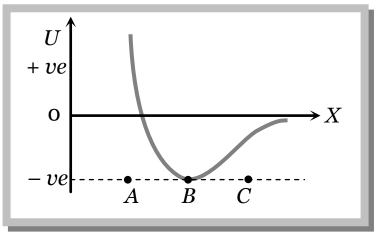
Problems based on Interatomic and Intermolecular forces

1. In solids, inter-atomic forces are **[DCE 1999]**

(a) Totally repulsive (b) Totally attractive

(c) Combination of (a) and (b) (d) None of these

2. The potential energy U between two molecules as a function of the distance X between them has been shown in the figure. The two molecules are **[CPMT 1986, 88, 91]**



(a) Attracted when x lies between A and B and are repelled when X lies between B and C

(b) Attracted when x lies between B and C and are repelled when X lies between A and B

(c) Attracted when they reach B

(d) Repelled when they reach B

3. The nature of molecular forces resembles with the nature of the

(a) Gravitational force (b) Nuclear force (c) Electromagnetic force (d) Weak force

Problems based on Stress

4. The ratio of radius of two wire of same material is 2 : 1. Stretched by same force, then the ratio of stress is **[PET 1991]**

(a) 2 : 1 (b) 1 : 2 (c) 1 : 4 (d) 4 : 1

5. If equal and opposite forces applied to a body tend to elongate it, the stress so produced is called

(a) Tensile stress (b) Compressive stress (c) Tangential stress (d) Working stress

6. A vertical hanging bar of length l and mass m per unit length carries a load of mass M at the lower end, its upper end is clamped to a rigid support. The tensile force at a distance x from support is

(a) $Mg + mg(l - x)$ (b) Mg (c) $Mg + mgl$ (d) $(M + m)g\frac{x}{l}$

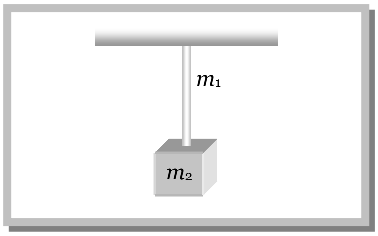
7. One end of a uniform rod of mass m_1 and cross-sectional area A is hung from a ceiling. The other end of the bar is supporting mass m_2. The stress at the midpoint is

(a) $\frac{g(m_2+2m_1)}{2A}$

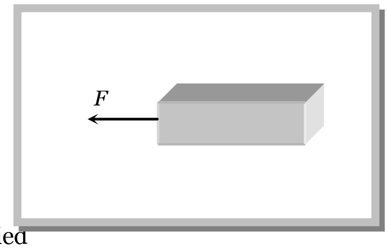
(b) $\frac{g(m_2+m_1)}{2A}$

(c) $\frac{g(2m_2+m_1)}{2A}$

(d) $\frac{g(m_2+m_1)}{A}$



8. A uniform bar of square cross-section is lying along a frictionless horizontal surface. A horizontal force is applied to pull it from one of its ends then



(a) The bar is under same stress throughout its length

(b) The bar is not under any stress because force has been applied only at one end

(c) The bar simply moves without any stress in it

(d) The stress developed reduces to zero at the end of the bar where no force is applied

Problems based on Strain

9. Which one of the following quantities does not have the unit of force per unit area **[MP PMT 1992]**

(a) Stress (b) Strain

(c) Young's modulus of elasticity (d) Pressure

10. The reason for the change in shape of a regular body is **[EAMCET 1980]**

(a) Volume stress (b) Shearing strain (c) Longitudinal strain (d) Metallic strain

11. When a spiral spring is stretched by suspending a load on it, the strain produced is called

(a) Shearing (b) Longitudinal (c) Volume (d) Transverse

12. The longitudinal strain is only possible in

(a) Gases (b) Fluids (c) Solids (d) Liquids

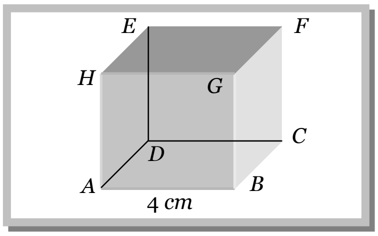
13. The face *EFGH* of the cube shown in the figure is displaced 2 *mm* parallel to itself when forces of $5\times10^5 N$ each are applied on the lower and upper faces. The lower face is fixed. The strain produced in the cube is

(a) 2

(b) 0.5

(c) 0.05

(d) 1.2×10^8



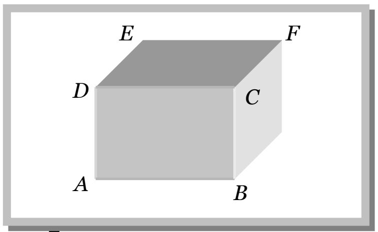
14. Forces of $10^5 N$ each are applied in opposite direction on the upper and lower faces of a cube of side 10 *cm*, shifting the upper face parallel to itself by 0.5 *cm*. If the side of the cube were 20 *cm*, the displacement would be

(a) 1 *cm*

(b) 0.5 *cm*

(c) 0.25 *cm*

(d) 0.125 *cm*



Problems based on Stress strain curve

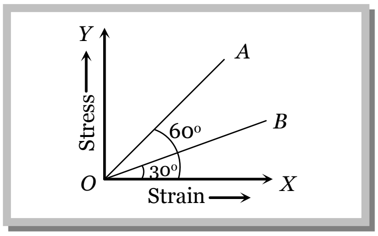
15. The stress versus strain graphs for wires of two materials A and B are as shown in the figure. If Y_A and Y_B are the Young's modulii of the materials, then **[Kerala (Engg.) 2001]**

(a) $Y_B = 2Y_A$

(b) $Y_A = Y_B$

(c) $Y_B = 3Y_A$

(d) $Y_A = 3Y_B$



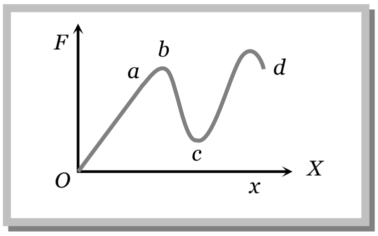
16. The graph is drawn between the applied force F and the strain (x) for a thin uniform wire. The wire behaves as a liquid in the part **[CPMT 1988]**

(a) ab

(b) bc

(c) cd

(d) oa



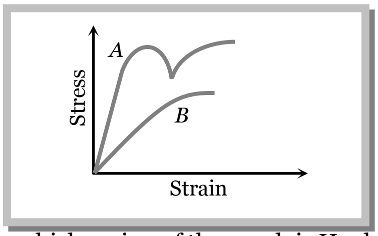
17. The diagram shows stress v/s strain curve for the materials A and B. From the curves we infer that

(a) A is brittle but B is ductile

(b) A is ductile and B is brittle

(c) Both A and B are ductile

(d) Both A and B are brittle



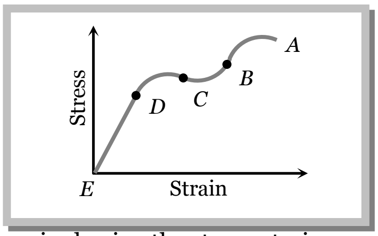
18. The figure shows the stress-strain graph of a certain substance. Over which region of the graph is Hooke's law obeyed

(a) AB

(b) BC

(c) CD

(d) ED



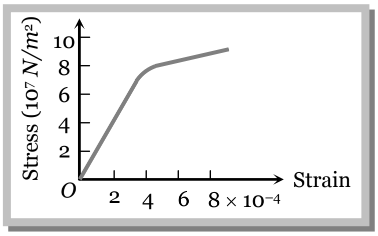
19. Which one of the following is the Young's modulus (in N/m^2) for the wire having the stress-strain curve shown in the figure

(a) 24×10^{11}

(b) 8.0×10^{11}

(c) 10×10^{11}

(d) 2.0×10^{11}



Problems based on Young's Modulus

20. The adjacent graph shows the extension (Δl) of a wire of length $1m$ suspended from the top of a roof at one end with a load W connected to the other end. If the cross sectional area of the wire is $10^{-6} m^2$, calculate the young's modulus of the material of the wire

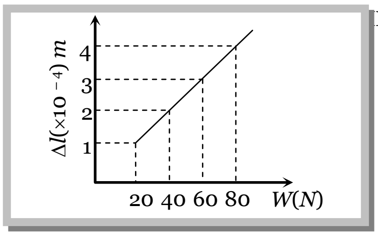
IT-JEE (Screening) 2003]

(a) $2 \times 10^{11}\ N/m^2$

(b) $2 \times 10^{-11}\ N/m^2$

(c) $3 \times 10^{-12}\ N/m^2$

(d) $2 \times 10^{-13}\ N/m^2$



21. In the Young's experiment, if length of wire and radius both are doubled then the value of Y will become **[RPET 2003]**

(a) 2 times (b) 4 times (c) Remains same (d) Half

22. A rubber cord catapult has cross-sectional area 25*mm*² and initial length of rubber cord is 10*cm*. It is stretched to 5*cm*. and then released to project a missile of mass 5*gm*. Taking $Y_{rubber} = 5\times10^8\ N/m^2$ velocity of projected missile is **[CPMT 2002]**

(a) 20 ms^{-1} (b) 100 ms^{-1} (c) 250 ms^{-1} (d) 200 ms^{-1}

23. Consider the following statements

Assertion (*A*) : Stress is the internal force per unit area of a body.

Reason (*R*) : Rubber is more elastic than steel.

Of these statements **[AIIMS 2002]**

(a) Both *A* and *R* are true and the *R* is a correct explanation of the *A*

(b) Both *A* and *R* are true but the *R* is not a correct explanation of the *A*

(c) *A* is true but the *R* is false

(d) Both *A* and *R* are false

(e) *A* is false but the *R* is true

24. The area of cross-section of a steel wire $(Y = 2.0\times10^{11}\ N/m^2)$ is 0.1 *cm*². The force required to double its length will be **[MP PET 2002]**

(a) $2\times10^{12}\,N$ (b) $2\times10^{11}\,N$ (c) $2\times10^{10}\,N$ (d) $2\times10^{6}\,N$

25. A metal bar of length *L* and area of cross-section *A* is clamped between two rigid supports. For the material of the rod, its Young's modulus is *Y* and coefficient of linear expansion is *α*. If the temperature of the rod is increased by Δt^oC, the force exerted by the rod on the supports is **[MP PMT 2001]**

(a) $YAL\,\Delta t$ (b) $YA\,\alpha\Delta t$ (c) $\frac{YL\,\alpha\Delta t}{A}$ (d) $Y\alpha AL\,\Delta t$

26. Which one of the following substances possesses the highest elasticity **[MP PMT 1992; RPMT 1999; RPET 2000; MH CET (Med.) 2001]**

(a) Rubber (b) Glass (c) Steel (d) Copper

27. There are two wires of same material and same length while the diameter of second wire is 2 times the diameter of first wire, then ratio of extension produced in the wires by applying same load will be

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 4 : 1

28. Consider the following statements

Assertion (*A*) : Rubber is more elastic than glass.

Reason (*R*) : The rubber has higher modulus of elasticity than glass.

Of these statements **[AIIMS 2000]**

(a) Both *A* and *R* are true and the *R* is a correct explanation of the *A*

(b) Both *A* and *R* are true but the *R* is not a correct explanation of the *A*

(c) *A* is true but the *R* is false

(d) Both *A* and *R* are false

(e) *A* is false but the *R* is true

29. The longitudinal extension of any elastic material is very small. In order to have an appreciable change, the material must be in the form of

(a) Thin block of any cross section (b) Thick block of any cross section

(c) Long thin wire (d) Short thin wire

30. In suspended type moving coil galvanometer, quartz suspension is used because

(a) It is good conductor of electricity (b) Elastic after effects are negligible

(c) Young's modulus is greater (d) There is no elastic limit

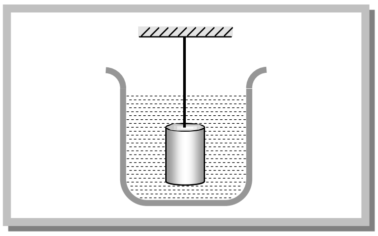
31. You are given three wires *A*, *B* and *C* of the same length and cross section. They are each stretched by applying the same force to the ends. The wire *A* is stretched least and comes back to its original length when the stretching force is removed. The wire *B* is stretched more than *A* and also comes back to its original length when the stretching force is removed. The wire *C* is stretched most and remains stretched even when stretching force is removed. The greatest Young's modulus of elasticity is possessed by the material of wire

(a) *A* (b) *B* (c) *C* (d) All have the same elasticity

32. The ratio of diameters of two wires of same material is *n* : 1. The length of wires are 4 *m* each. On applying the same load, the increase in length of thin wire will be

(a) n^2 times (b) n times (c) $2n$ times (d) None of the above

33. A wire of radius r, Young's modulus Y and length l is hung from a fixed point and supports a heavy metal cylinder of volume V at its lower end. The change in length of wire when cylinder is immersed in a liquid of density ρ is in fact



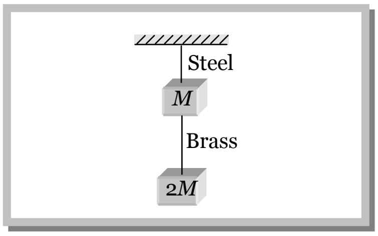
(a) Decrease by $\frac{Vl\rho g}{Y\pi r^2}$

(b) Increase by $\frac{Vr\rho g}{Y\pi l^2}$

(c) Decrease by $\frac{V\rho g}{Y\pi r}$

(d) $\frac{V\rho g}{Y\pi}$

34. If the ratio of lengths, radii and Young's modulii of steel and brass wires in the figure are a, b and c respectively. Then the corresponding ratio of increase in their lengths would be



(a) $\frac{2a^2c}{b}$

(b) $\frac{3a}{2b^2c}$

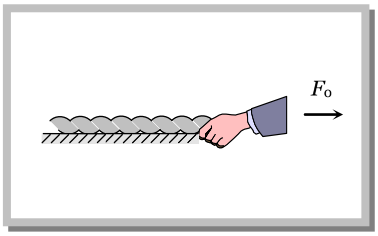
(c) $\frac{2ac}{b^2}$

(d) $\frac{3c}{2ab^2}$

35. A uniform heavy rod of weight W, cross sectional area A and length L is hung from a fixed support. Young's modulus of the material of the rod is Y. If lateral contraction is neglected, the elongation of the rod under its own weight is

(a) $\frac{2WL}{AY}$ (b) $\frac{WL}{AY}$ (c) $\frac{WL}{2AY}$ (d) Zero

36. A constant force F_0 is applied on a uniform elastic string placed over a smooth horizontal surface as shown in figure. Young's modulus of string is Y and area of cross-section is S. The strain produced in the string in the direction of force is



(a) $\frac{F_0 Y}{S}$

(b) $\frac{F_0}{SY}$

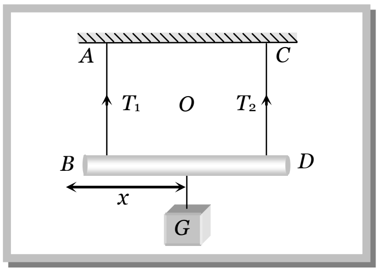
(c) $\frac{F_0}{2SY}$

(d) $\frac{F_0 Y}{2S}$

37. A uniform rod of length L has a mass per unit length λ and area of cross section A. The elongation in the rod is l due to its own weight if it is suspended from the ceiling of a room. The Young's modulus of the rod is

(a) $\frac{2\lambda gL^2}{Al}$ (b) $\frac{\lambda gL^2}{2Al}$ (c) $\frac{2\lambda gL}{Al}$ (d) $\frac{\lambda gl^2}{AL}$

38. AB is an iron wire and CD is a copper wire of same length and same cross-section. BD is a rod of length 0.8 m. A load G = 2kg-wt is suspended from the rod. At what distance x from point B should the load be suspended for the rod to remain in a horizontal position $(Y_{Cu} = 11.8 \times 10^{10} N/m^2, Y_{Fe} = 19.6 \times 10^{10} N/m^2)$



(a) 0.1 m

(b) 0.3 m

(c) 0.5 m

(d) 0.7 m

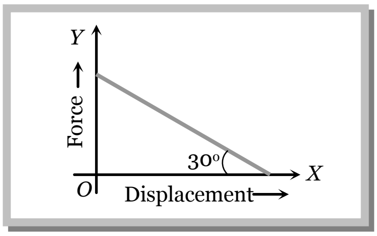
39. A slightly conical wire of length L and end radii r_1 and r_2 is stretched by two forces F, F applied parallel to length in opposite directions and normal to end faces. If Y denotes the Young's modulus, then extension produced is

(a) $\frac{FL}{\pi r_1^2 Y}$ (b) $\frac{FL}{\pi r_1 Y}$ (c) $\frac{FL}{\pi r_1 r_2 Y}$ (d) $\frac{FLY}{\pi r_1 r_2}$

40. The force constant of wire is K and its area of cross-section is A. If the force F is applied on it, then the increase in its length will be

(a) KA (b) FKA (c) $\frac{F}{K}$ (d) $\frac{FK}{AL}$

41. The value of force constant between the applied elastic force F and displacement will be



(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{2}$

(d) $\frac{\sqrt{3}}{2}$

42. The force constant of a wire does not depend on

(a) Nature of the material (b) Radius of the wire (c) Length of the wire (d) None of the above

43. A metal wire of length L, area of cross-section A and Young's modulus Y behaves as a spring. The equivalent spring constant will be

(a) $\frac{Y}{AL}$ (b) $\frac{YA}{L}$ (c) $\frac{YL}{A}$ (d) $\frac{L}{AY}$

44. A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force is applied perpendicular to one of the sides faces of A. After the force is withdrawn, block A execute small oscillations the time period of which is given by

(a) $2\pi\sqrt{M\eta L}$ (b) $2\pi\sqrt{\frac{M\eta}{L}}$ (c) $2\pi\sqrt{\frac{ML}{\eta}}$ (d) $2\pi\sqrt{\frac{M}{\eta L}}$

Problems based on Stretching a wire

45. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. It is stretched by an amount x. The work done is **[MP PET 1996; BVP 2003]**

(a) $\frac{YxA}{2L}$ (b) $\frac{Yx^2A}{L}$ (c) $\frac{Yx^2A}{2L}$ (d) $\frac{2Yx^2A}{L}$

46. Two wires of same diameter of the same material having the length l and $2l$. If the force F is applied on each, the ratio of the work done in the two wires will be **[MP PET 1989]**

(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 1 : 1

47. If the potential energy of a spring is V on stretching it by 2 cm, then its potential energy when it is stretched by 10 cm will be **[CPMT 1976]**

(a) $V/25$ (b) $5V$ (c) $V/5$ (d) $25V$

48. The strain energy stored in a body of volume V due to shear S and shear modulus η is

(a) $\frac{S^2V}{2\eta}$ (b) $\frac{SV^2}{2\eta}$ (c) $\frac{S^2V}{\eta}$ (d) $\frac{1}{2}\eta S^2V$

49. K is the force constant of a spring. The work done in increasing its extension from l_1 to l_2 will be **[MP PET 1995; MP PMT 1996]**

(a) $K(l_2-l_1)$ (b) $\frac{K}{2}(l_2+l_1)$ (c) $K(l_2^2-l_1^2)$ (d) $\frac{K}{2}(l_2^2-l_1^2)$

Problems based on Breaking of wire

50. The breaking stress of a wire depends upon **[AIIMS 2002]**

(a) Length of the wire (b) Radius of the wire (c) Material of the wire (d) Shape of the cross section

51. An aluminium rod has a breaking strain of 0.2%. The minimum cross sectional area of the rod, in m^2, in order to support a load of 10^4 *N* is $(Y = 7\times10^9\ N/m^2)$

(a) 1.4×10^{-4} (b) 7.1×10^{-4} (c) 1.4×10^{-3} (d) 7.1×10^{-5}

52. A cable is replaced by another one of the same length and material but of twice the diameter. The maximum load that the new wire can support without exceeding the elastic limit, as compared to the load that the original wire could support, is

(a) Half (b) Double (c) Four times (d) One-fourth

53. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break

(a) When the mass is at the highest point (b) When the mass is at the lowest point

(c) When the wire is horizontal (d) At an angle of $\cos^{-1}(1/3)$ from the upward vertical

54. A heavy uniform rod is hanging vertically from a fixed support. It is stretched by its own weight. The diameter of the rod is

(a) Smallest at the top and gradually increases down the rod

(b) Largest at the top and gradually decreases down the rod

(c) Uniform everywhere

(d) Maximum in the middle

Problems based on Bulk modulus

55. The isothermal bulk modulus of a gas at atmospheric pressure is **[AIIMS 2000; KCET (Engg./Med.) 1999]**

(a) 1 *mm* of *Hg* (b) 13.6 *mm* of *Hg* (c) $1.013\times10^5\ N/m^2$ (d) $2.026\times10^5\ N/m^2$

56. The specific heat at constant pressure and at constant volume for an ideal gas are C_p and C_v and its adiabatic and isothermal elasticities are E_ϕ and E_θ respectively. The ratio of E_ϕ to E_θ is **[MP PMT 1989; MP PET 1992]**

(a) C_v/C_p (b) C_p/C_v (c) C_pC_v (d) $1/C_pC_v$

57. If a rubber ball is taken at the depth of 200 *m* in a pool. Its volume decreases by 0.1%. If the density of the water is $1\times10^3\ kg/m^3$ and g = 10 m/s^2, then the volume elasticity in N/m^2 will be **[MP PMT 1991]**

(a) 10^8 (b) 2×10^8 (c) 10^9 (d) 2×10^9

58. The compressibility of water is 4×10^{-5} per unit atmospheric pressure. The decrease in volume of 100 *cubic centimetre* of water under a pressure of 100 atmosphere will be **[MP PMT 1990]**

(a) 0.4 *cc* (b) 4×10^{-5} *cc* (c) 0.025 *cc* (d) 0.004 *cc*

59. An ideal gas of mass *m*, volume *V*, pressure *p* and temperature *T* undergoes a small change in state at constant temperature. Its adiabatic exponent *i.e.*, $\frac{C_p}{C_v}$ is γ. The bulk modulus of the gas at the constant temperature process called isothermal process is

(a) p (b) γp (c) $\frac{m\gamma p}{T}$ (d) $\frac{\gamma pV}{T}$

60. An ideal gas of mass *m*, volume *V*, pressure *p* and temperature *T* undergoes a small change under a condition that heat can neither enter into it from outside nor can it leave the system. Such a process is called adiabatic process. The bulk modulus of the gas $\left(\gamma = \frac{C_p}{C_v}\right)$ is

(a) p (b) γp (c) $\frac{m\gamma p}{T}$ (d) $\frac{\gamma pV}{T}$

61. An ideal gas whose adiabatic exponent is γ is expanded according to the law $p = \alpha V$ where α is a constant. For this process the bulk modulus of the gas is

(a) p (b) $\frac{p}{\alpha}$ (c) αp (d) $(l-\alpha)p$

62. 1 *c.c.* of water is taken from the top to the bottom of a 200 *m* deep lake. What will be the change in its volume if *K* of water is $2.2\times10^9\ N/m^2$

(a) 8.8×10^{-6} *c.c.* (b) 8.8×10^{-2} *c.c.* (c) 8.8×10^{-4} *c.c.* (d) 8.8×10^{-1} *c.c*

Problems based on Modulus of rigidity

63. Modulus of rigidity of a liquid

(a) Non zero constant (b) Infinite (c) Zero (d) Cannot be predicted

64. The Young's modulus of the material of a wire is $6 \times 10^{12}\ N/m^2$ and there is no transverse strain in it, then its modulus of rigidity will be

(a) $3 \times 10^{12}\ N/m^2$ (b) $2 \times 10^{12}\ N/m^2$ (c) $10^{12}\ N/m^2$ (d) None of the above

Problems based on relation between Y, η, K and σ

65. The value of Poisson's ratio lies between **[AIIMS 1985; MP PET 1986; DPMT 2002]**

(a) -1 to $\frac{1}{2}$ (b) $-\frac{3}{4}$ to $-\frac{1}{2}$ (c) $-\frac{1}{2}$ to 1 (d) 1 to 2

66. Which of the following will be σ if $Y = 2.4\eta$ **[RPET 2001]**

(a) –1 (b) 0.2 (c) 0.1 (d) – 0.25

67. Which is correct relation **[RPET 2001]**

(a) $Y < \sigma$ (b) $Y > \sigma$ (c) $Y = \sigma$ (d) $\sigma = +1$

68. The relationship between Young's modulus Y, bulk modulus K and modulus of rigidity η is **[MP PET 1991; MP PMT 1997]**

(a) $Y = \frac{9\eta K}{\eta + 3K}$ (b) $\eta = \frac{9yK}{Y + 3K}$ (c) $Y = \frac{9\eta K}{3\eta + K}$ (d) $Y = \frac{3\eta K}{9\eta + K}$

69. The Poisson's ratio cannot have the value **[EAMCET 1989]**

(a) 0.7 (b) 0.2 (c) 0.1 (d) 0.5

70. Which of the following relations is true **[CPMT 1984]**

(a) $3Y = K(1 - \sigma)$ (b) $K = \frac{9\eta Y}{Y + \eta}$ (c) $\sigma = (6K + \eta)\ Y$ (d) $\sigma = \frac{0.5Y - \eta}{\eta}$

71. The wrong relation for modulus of rigidity (η) is

(a) $\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$ (b) Unit of η is N/m^2 (c) $\eta = \frac{Y}{2(1-\sigma)}$ (d) $\eta = \frac{Y}{2(1+\sigma)}$

Problems based on Torsion

72. A rod of 2*m* length and radius 1 *cm* is twisted at one end by 0.8 *rad* with respect to other end being clamped. The shear strain developed in its rod will be **[RPET 1997]**

(a) 0.002 (b) 0.004 (c) 0.008 (d) 0.016

73. The upper end of a wire 1 *metre* long and 2 *mm* in radius is clamped. The lower end is twisted through an angle of 45^o. The angle of shear is **[MP PMT 1990]**

(a) 0.09^o (b) 0.9^o (c) 9^o (d) 90^o

74. The end of a wire of length 0.5*m* and radius $10^{-3}m$ is twisted through 0.80 *radian*. The shearing strain at the surface of wire will be

(a) 1.6×10^{-3} (b) 1.6×10^{3} (c) 16×10^{3} (d) 16×10^{6}

75. Two cylinders A and B of the same material have same length, their radii being in the ratio of 1 : 2 respectively. The two are joined in series. The upper end of A is rigidly fixed. The lower end of B is twisted through an angle θ, the angle of twist of the cylinder A is **fig.**

(a) $\frac{15}{16}\theta$ (b) $\frac{16}{15}\theta$ (c) $\frac{16}{17}\theta$ (d) $\frac{17}{16}\theta$

Problems based on Interatomic force constant

76. If the interatomic spacing in a steel wire is 3.0$Å$ and $Y_{steel} = 20 \times 10^{10}\ N/m^2$, then force constant is

(a) $6 \times 10^{-2}\ N/Å$ (b) $6 \times 10^{-9}\ N/Å$ (c) $4 \times 10^{-5}\ N/Å$ (d) $6 \times 10^{-5}\ N/Å$

77. The Young's modulus of a metal is $1.2 \times 10^{11}\ N/m^2$ and the inter-atomic force constant is $3.6 \times 10^{-9}\ N/Å$. The mean distance between the atoms of the metal is

(a) 2$Å$ (b) 3$Å$ (c) 4.5$Å$ (d) 5$Å$

78. The interatomic distance for a metal is $3\times10^{-10}m$. If the interatomic force constant is $3.6\times10^{-9}\ N/Å$, then the Young's modulus in N/m^2 will be

(a) 1.2×10^{11} (b) 4.2×10^{11} (c) 10.8×10^{-19} (d) 2.4×10^{10}

Miscellaneous problems

79. A particle of mass *m* is under the influence of a force *F* which varies with the displacement *x* according to the relation $F = -kx + F_0$ in which *k* and F_0 are constants. The particle when disturbed will oscillate

(a) About $x = 0$, with $\omega \neq \sqrt{k/m}$ (b) About $x = 0$, with $\omega = \sqrt{k/m}$

(c) About $x = F_0/k$ with $\omega = \sqrt{k/m}$ (d) About $x = F_0/k$ with $\omega \neq \sqrt{k/m}$

80. The extension in a string obeying Hooke's law is *x*. The speed of sound in the stretched string is *v*. If the extension in the string is increased to 1.5*x*, the speed of sound will be **[IIT 1996]**

(a) 1.22 *v* (b) 0.61 *v* (c) 1.50 *v* (d) 0.75 *v*

81. Railway lines and girders for buildings, are *I* shaped, because

(a) The bending of a girder is inversely proportional to depth, hence high girder bends less

(b) The coefficient of rigidity increases by this shape

(c) Less volume strain is caused

(d) This keeps the surface smooth

82. If Young's modulus for a material is zero, then the state of material should be

(a) Solid (b) Solid but powder (c) Gas (d) None of the above

83. The elasticity of *invar*

(a) Increases with temperature rise (b) Decreases with temperature rise

(c) Does not depend on temperature (d) None of the above

84. For the same cross-sectional area and for a given load, the ratio of depressions for the beam of square cross-section and circular cross-section is

(a) $\pi : 3$ (b) $\pi : 1$ (c) $3 : \pi$ (d) $1 : \pi$

85. A uniform rod of mass *m*, length *L*, area of cross-section *A* is rotated about an axis passing through one of its ends and perpendicular to its length with constant angular velocity ω in a horizontal plane. If *Y* is the Young's modulus of the material of rod, the increase in its length due to rotation of rod is

(a) $\frac{m\omega^2L^2}{AY}$ (b) $\frac{m\omega^2L^2}{2AY}$ (c) $\frac{m\omega^2L^2}{3AY}$ (d) $\frac{2m\omega^2L^2}{AY}$

86. A steel wire is suspended vertically from a rigid support. When loaded with a weight in air, it extends by l_a and when the weight is immersed completely in water, the extension is reduced to l_w. Then the relative density of the material of the weight is

(a) $\frac{l_a}{l_w}$ (b) $\frac{l_a}{l_a - l_w}$ (c) $\frac{l_a}{l_a - l_w}$ (d) $\frac{l_w}{l_a}$

87. The twisting couple per unit twist for a solid cylinder of radius 4.9 *cm* is 0.1 *N-m*. The twisting couple per unit twist for a hollow cylinder of same material with outer and inner radii of 5 *cm* and 4 *cm* respectively, will be

(a) 0.64 *N-m* (b) 0.64×10^{-1} *N-m* (c) 0.64×10^{-2} *N-m* (d) 0.64×10^{-3} *N-m*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	b	c	c	a	a	c	b	b	b
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
a	c	c	c	d	b	b	d	d	a
21.	22.	23.	24.	25.	26.	27.	28.	29.	30.

c	c	c	d	b	c	d	d	c	b
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
a	a	a	b	c	c	b	b	c	c
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
b	d	b	d	c	a	d	d	d	c
51.	**52.**	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**
b	c	b	a	c	b	d	a	a	b
61.	**62.**	**63.**	**64.**	**65.**	**66.**	**67.**	**68.**	**69.**	**70.**
a	c	c	a	a	b	b	a	a	d
71.	**72.**	**73.**	**74.**	**75.**	**76.**	**77.**	**78.**	**79.**	**80.**
c	b	a	a	c	b	b	a	c	a
81.	**82.**	**83.**	**84.**	**85.**	**86.**	**87.**			
a	b	c	c	c	b	b			

Gravitation

8.1 Introduction

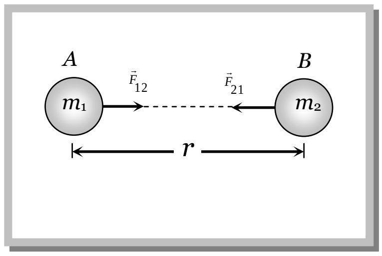
Newton at the age of twenty-three is said to have seen an apple falling down from tree in his orchid. This was the year 1665. He started thinking about the role of earth's attraction in the motion of moon and other heavenly bodies.

By comparing the acceleration due to gravity due to earth with the acceleration required to keep the moon in its orbit around the earth, he was able to arrive the Basic Law of Gravitation.

8.2 Newton's law of Gravitation

Newton's law of gravitation states that every body in this universe attracts every other body with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The direction of the force is along the line joining the particles.

Thus the magnitude of the gravitational force F that two particles of masses m_1 and m_2 separated by a distance r exert on each other is given by $F \propto \frac{m_1 m_2}{r^2}$



or $$F = G\frac{m_1 m_2}{r^2}$$

Vector form : According to Newton's law of gravitation

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r^2}\hat{r}_{21} = \frac{-Gm_1m_2}{r^3}\vec{r}_{21} = \frac{-Gm_1m_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

$\hat{r}_{12}$ = unit vector from A to B

$\hat{r}_{21}$ = unit vector from B to A,

$\vec{F}_{12}$ = gravitational force exerted on body A by body B

$\vec{F}_{21}$ = gravitational force exerted on body B by

Here negative sign indicates that the direction of $\vec{F}_{12}$ is opposite to that of $\hat{r}_{21}$.

Similarly $$\vec{F}_{21} = \frac{-Gm_1m_2}{r^2}\hat{r}_{12} = \frac{-Gm_1m_2}{r^3}\vec{r}_{12} = \frac{-Gm_1m_2}{|\vec{r}_{12}|^3}\vec{r}_{12}$$

$$= \frac{Gm_1m_2}{r^2}\hat{r}_{21} \quad [\because \hat{r}_{12} = -\hat{r}_{21}]$$

$\therefore$ It is clear that $\vec{F}_{12} = -\vec{F}_{21}$. Which is Newton's third law of motion.

Here G is constant of proportionality which is called 'Universal gravitational constant'.

If $m_1 = m_2$ and $r = 1$ then $G = F$

i.e. universal gravitational constant is equal to the force of attraction between two bodies each of unit mass whose centres are placed unit distance apart.

Important points

(i) The value of G in the laboratory was first determined by Cavendish using the torsional balance.

(ii) The value of G is 6.67×10^{-11} $N\text{–}m^2\,kg^{-2}$ in S.I. and 6.67×10^{-8} $dyne\text{-}cm^2\text{-}g^{-2}$ in C.G.S. system.

(iii) Dimensional formula $[M^{-1}L^{3}T^{-2}]$.

(iv) The value of G does not depend upon the nature and size of the bodies.

(v) It also does not depend upon the nature of the medium between the two bodies.

(vi) As G is very small hence gravitational forces are very small, unless one (or both) of the masses is huge.

8.3 Properties of Gravitational Force.

(1) It is always attractive in nature while electric and magnetic force can be attractive or repulsive.

(2) It is independent of the medium between the particles while electric and magnetic force depend on the nature of the medium between the particles.

(3) It holds good over a wide range of distances. It is found true for interplanetary to inter atomic distances.

(4) It is a central force *i.e.* acts along the line joining the centres of two interacting bodies.

(5) It is a two-body interaction *i.e.* gravitational force between two particles is independent of the presence or absence of other particles; so the principle of superposition is valid *i.e.* force on a particle due to number of particles is the resultant of forces due to individual particles *i.e.* $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$

While nuclear force is many body interaction

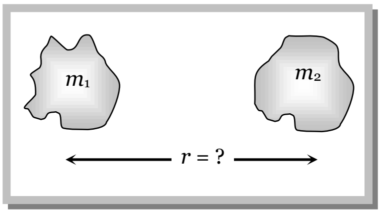
(6) It is the weakest force in nature : As $F_{nuclear} > F_{electromagnetic} > F_{gravitational}$.

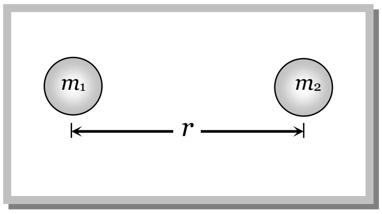
(7) The ratio of gravitational force to electrostatic force between two electrons is of the order of 10^{-43}.

(8) It is a conservative force *i.e.* work done by it is path independent or work done in moving a particle round a closed path under the action of gravitational force is zero.

(9) It is an action reaction pair *i.e.* the force with which one body (say earth) attracts the second body (say moon) is equal to the force with which moon attracts the earth. This is in accordance with Newton's third law of motion.

Note : ❑ The law of gravitation is stated for two point masses, therefore for any two arbitrary finite size bodies, as shown in the figure, It can not be applied as there is not unique value for the separation.





But if the two bodies are uniform spheres then the separation r may be taken as the distance between their centres because a sphere of uniform mass behave as a point mass for any point lying outside it.

Sample problems based on Newton's law of gravitation

Problem 1. The gravitational force between two objects does not depend on **[RPET 2003]**

(a) Sum of the masses (b) Product of the masses

(c) Gravitational constant (d) Distance between the masses

Solution : (a) $F = \dfrac{\text{Gravitational constant} \times \text{product of the masses}}{(\text{Distance between the masses})^2}$.

Problem 2. Mass M is divided into two parts xM and $(1 - x)M$. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is **[EAMCET 2001]**

(a) $\dfrac{1}{2}$ (b) $\dfrac{3}{5}$ (c) 1 (d) 2

Solution : (a) Gravitational force $F = \dfrac{Gm_1m_2}{r^2} = \dfrac{GxM(1-x)M}{r^2} = \dfrac{GM^2}{r^2}x(1-x)$

For maximum value of force $\dfrac{dF}{dx} = 0$ $\therefore$ $\dfrac{d}{dx}\left[\dfrac{GM^2x}{r^2}(1-x)\right] = 0$

$\Rightarrow \dfrac{d}{dx}(x - x^2) = 0 \Rightarrow 1 - 2x = 0 \Rightarrow x = 1/2$

Problem 3. The mass of the moon is about 1.2% of the mass of the earth. Compared to the gravitational force the earth exerts on the moon, the gravitational force the moon exerts on earth **[SCRA 1998]**

(a) Is the same (b) Is smaller (c) Is greater (d) Varies with its phase

Solution : (a) Earth and moon both exerts same force on each other.

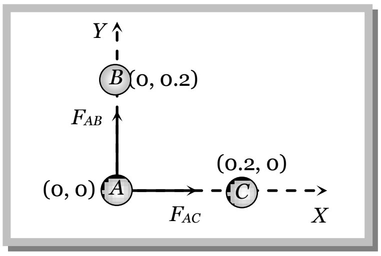
Problem 4. Three identical point masses, each of mass 1*kg* lie in the *x*-*y* plane at points (0, 0), (0, 0.2*m*) and (0.2*m*, 0). The net gravitational force on the mass at the origin is

(a) $1.67 \times 10^{-9}(\hat{j} + \hat{j})N$ (b) $3.34 \times 10^{-10}(\hat{i} + \hat{j})N$

(c) $1.67 \times 10^{-9}(\hat{i} - \hat{j})N$ (d) $3.34 \times 10^{-10}(\hat{i} + \hat{j})N$

Solution : (a) Let particle A lies at origin, particle B and C on y and x-axis respectively

$\overrightarrow{F_{AC}} = \dfrac{Gm_Am_B}{r_{AB}^2}\hat{i} = \dfrac{6.67 \times 10^{-11} \times 1 \times 1}{(0.2)^2}\hat{i} = 1.67 \times 10^{-9}\,\hat{i}\,N$



Similarly $\overrightarrow{F_{AB}} = 1.67 \times 10^{-9}\,\hat{j}\,N$

$\therefore$ Net force on particle A $\vec{F} = \vec{F}_{AC} + \vec{F}_{AB} = 1.67 \times 10^{-9}(\hat{i} + \hat{j})\,N$

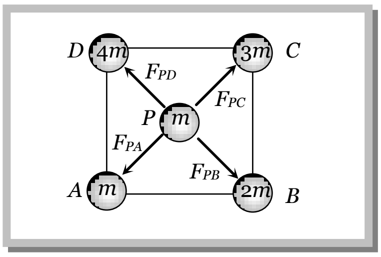
Problem 5. Four particles of masses m, $2m$, $3m$ and $4m$ are kept in sequence at the corners of a square of side a. The magnitude of gravitational force acting on a particle of mass m placed at the centre of the square will be

(a) $\dfrac{24m^2G}{a^2}$ (b) $\dfrac{6m^2G}{a^2}$ (c) $\dfrac{4\sqrt{2}Gm^2}{a^2}$ (d) Zero

Solution : (c) If two particles of mass m are placed x distance apart then force of attraction $\dfrac{Gmm}{x^2} = F$ (Let)

Now according to problem particle of mass m is placed at the centre (P) of square. Then it will experience four forces

F_{PA} = force at point P due to particle $A = \dfrac{Gmm}{x^2} = F$



Similarly $F_{PB} = \dfrac{G2mm}{x^2} = 2F$, $F_{PC} = \dfrac{G3mm}{x^2} = 3F$ and $F_{PD} = \dfrac{G4mm}{x^2} = 4F$

Hence the net force on P $\vec{F}_{net} = \vec{F}_{PA} + \vec{F}_{PB} + \vec{F}_{PC} + \vec{F}_{PD} = 2\sqrt{2}\,F$

$$\therefore \ \vec{F}_{net} = 2\sqrt{2}\frac{Gmm}{x^2} = 2\sqrt{2}\frac{Gm^2}{(a/\sqrt{2})^2} \qquad [x = \frac{a}{\sqrt{2}} = \text{half of the diagonal of the square}]$$

$$= \frac{4\sqrt{2}\,Gm^2}{a^2}.$$

8.4 Acceleration Due to Gravity

The force of attraction exerted by the earth on a body is called gravitational pull or gravity.

We know that when force acts on a body, it produces acceleration. Therefore, a body under the effect of gravitational pull must accelerate.

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity, it is denoted by g.

Consider a body of mass m is lying on the surface of earth then gravitational force on the body is given by

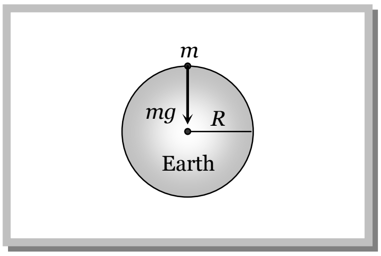
$$F = \frac{GMm}{R^2} \qquad .....(i)$$

Where M = mass of the earth and R = radius of the earth.

If g is the acceleration due to gravity, then the force on the body due to earth is given by

Force = mass × acceleration

or $\quad F = mg \qquad .....(ii)$



From (i) and (ii) we have $mg = \frac{GMm}{R^2}$

$$\therefore \quad g = \frac{GM}{R^2} \qquad .....(iii)$$

$$\Rightarrow \quad g = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right) \qquad [\text{As mass } (M) = \text{volume } (\frac{4}{3}\pi R^3) \times \text{density } (\rho)]$$

$$\therefore \quad g = \frac{4}{3}\pi\rho GR \qquad .....(iv)$$

Important points

(i) From the expression $g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR$ it is clear that its value depends upon the mass radius and density of planet and it is independent of mass, shape and density of the body placed on the surface of the planet. *i.e.* a given planet (reference body) produces same acceleration in a light as well as heavy body.

(ii) The greater the value of (M/R^2) or ρR, greater will be value of g for that planet.

(iii) Acceleration due to gravity is a vector quantity and its direction is always towards the centre of the planet.

(iv) Dimension $[g] = [LT^{-2}]$

(v) it's average value is taken to be 9.8 m/s^2 or 981 cm/sec^2 or 32 $feet/sec^2$, on the surface of the earth at mean sea level.

(vi) The value of acceleration due to gravity vary due to the following factors : (a) Shape of the earth, (b) Height above the earth surface, (c) Depth below the earth surface and (d) Axial rotation of the earth.

Sample problems based on acceleration due to gravity

***Problem* 6.** Acceleration due to gravity on moon is 1/6 of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_m) and moon (ρ_e) is $\left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3}$ then radius of moon R_m in terms of R_e will be **[MP PMT 20**

(a) $\frac{5}{18}R_e$ (b) $\frac{1}{6}R_e$ (c) $\frac{3}{18}R_e$ (d) $\frac{1}{2\sqrt{3}}R_e$

Solution : (a) Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR$ $\therefore$ $g \propto \rho R$ or $\frac{g_m}{g_e} = \frac{\rho_m}{\rho_e}.\frac{R_m}{R_e}$ [As $\frac{g_m}{g_e} = \frac{1}{6}$ and $\frac{\rho_e}{\rho_m} = \frac{5}{3}$ (given)]

$\therefore$ $\frac{R_m}{R_e} = \left(\frac{g_m}{g_e}\right)\left(\frac{\rho_e}{\rho_m}\right) = \frac{1}{6}\times\frac{5}{3}$ $\therefore$ $R_m = \frac{5}{18}R_e$

***Problem* 7.** A spherical planet far out in space has a mass M_0 and diameter D_0. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity which is equal to **[MP PMT 1987; DPMT 2002]**

(a) GM_0/D_0^2 (b) $4mGM_0/D_0^2$ (c) $4GM_0/D_0^2$ (d) GmM_0/D_0^2

Solution : (c) We know $g = \frac{GM}{R^2} = \frac{GM}{(D/2)^2} = \frac{4GM}{D^2}$

If mass of the planet $= M_0$ and diameter of the planet $= D_0$. Then $g = \frac{4GM_0}{D_0^2}$.

***Problem* 8.** The moon's radius is 1/4 that of the earth and its mass is 1/80 times that of the earth. If g represents the acceleration due to gravity on the surface of the earth, that on the surface of the moon is **[MP PMT 1997; RPET 2000; MP PET 2000, 2001]**

(a) $\frac{g}{4}$ (b) $\frac{g}{5}$ (c) $\frac{g}{6}$ (d) $\frac{g}{8}$

Solution : (b) Acceleration due to gravity $g = \frac{GM}{R^2}$ $\therefore$ $\frac{g_{moon}}{g_{earth}} = \frac{M_{moon}}{M_{earth}}.\frac{R_{earth}^2}{R_{moon}^2} = \left(\frac{1}{80}\right)\left(\frac{4}{1}\right)^2$

$g_{moon} = g_{earth}\times\frac{16}{80} = \frac{g}{5}$.

***Problem* 9.** If the radius of the earth were to shrink by 1% its mass remaining the same, the acceleration due to gravity on the earth's surface would **[IIT-JEE 1981; CPMT 1981; MP PMT 1996, 97; Roorkee 1992; MP PET 1999]**

(a) Decrease by 2% (b) Remain unchanged (c) Increase by 2% (d) Increase by 1%

Solution : (c) We know $g \propto \frac{1}{R^2}$ [As R decreases, g increases]

So % change in $g = 2$ (% change in R) $= 2\times 1\% = 2\%$

$\therefore$ acceleration due to gravity increases by 2%.

***Problem* 10.** Mass of moon is $7.34\times10^{22}kg$. If the acceleration due to gravity on the moon is $1.4m/s^2$, the radius of the moon is ($G = 6.667\times10^{-11}\,Nm^2/kg^2$) **[AFMC 1998]**

(a) 0.56×10^4m (b) 1.87×10^6m (c) 1.92×10^6m (d) 1.01×10^8m

Solution : (b) We know $g = \frac{GM}{R^2}$ $\therefore$ $R = \sqrt{\frac{GM}{g}} = \sqrt{\frac{6.67\times10^{-11}\times7.34\times10^{22}}{1.4}} = 1.87\times10^6m$.

***Problem* 11.** A planet has mass 1/10 of that of earth, while radius is 1/3 that of earth. If a person can throw a stone on earth surface to a height of 90m, then he will be able to throw the stone on that planet to a height **[RPMT 199**

(a) 90m (b) 40m (c) 100m (d) 45m

Solution : (c) Acceleration due to gravity $g = \frac{GM}{R^2}$ $\therefore \frac{g_{planet}}{g_{earth}} = \frac{M_{planet}}{M_{earth}}\left(\frac{R_{earth}}{R_{planet}}\right)^2 = \frac{1}{10}\times\left(\frac{3}{1}\right)^2 = \frac{9}{10}$

If a stone is thrown with velocity u from the surface of the planet then maximum height $H = \frac{u^2}{2g}$

$\frac{H_{planet}}{H_{earth}} = \frac{g_{earth}}{g_{planet}}$ $\Rightarrow H_{planet} = \frac{10}{9}\times H_{earth} = \frac{10}{9}\times 90 = 100$ *metre.*

***Problem* 12.** The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity at their surfaces is

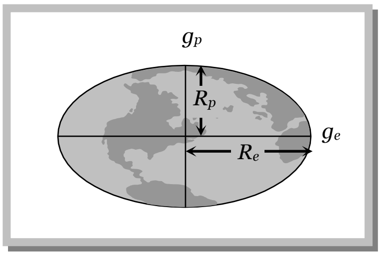
(a) $g_1 : g_2 = \frac{\rho_1}{R_1^2} : \frac{\rho_2}{R_2^2}$ (b) $g_1 : g_2 = R_1R_2 : \rho_1\rho_2$

(c) $g_1 : g_2 = R_1\rho_2 : R_2\rho_1$ (d) $g_1 : g_2 = R_1\rho_1 : R_2\rho_2$

Solution : (d) Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR$ $\therefore g_1 : g_2 = R_1\rho_1 : R_2\rho_2$.

8.5 Variation in *g* Due to Shape of Earth.

Earth is elliptical in shape. It is flattened at the poles and bulged out at the equator. The equatorial radius is about 21 *km* longer than polar radius, from $g = \frac{GM}{R^2}$



At equator $g_e = \frac{GM}{R_e^2}$ ......(i)

At poles $g_p = \frac{GM}{R_p^2}$ .....(ii)

From (i) and (ii) $\frac{g_e}{g_p} = \frac{R_p^2}{R_e^2}$

Since $R_{equator} > R_{pole}$ $\therefore g_{pole} > g_{equator}$ and $g_p = g_e + 0.018\ ms^{-2}$

Therefore the weight of body increases as it is taken from equator to the pole.

Sample problems based on variation in g due to shape of the earth

***Problem* 13.** Where will it be profitable to purchase 1 *kg* sugar (by spring balance) **[RPET 1996]**

(a) At poles (b) At equator (c) At 45° latitude (d) At 40°latitude

Solution : (b) At equator the value of *g* is minimum so it is profitable to purchase sugar at this position.

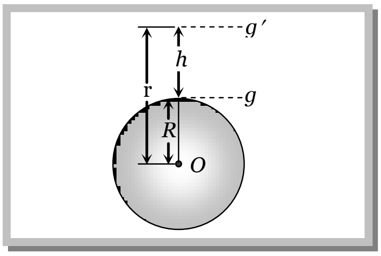
***Problem* 14.** Force of gravity is least at **[CPMT 1992]**

(a) The equator (b) The poles

(c) A point in between equator and any pole (d) None of these

Solution : (a)

8.6 Variation in *g* With Height.



Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} \quad .....(i)$$

Acceleration due to gravity at height *h* from the surface of the earth

$$g' = \frac{GM}{(R+h)^2} \quad .....(ii)$$

From (i) and (ii) $g' = g\left(\frac{R}{R+h}\right)^2$ .....(iii)

$= g\frac{R^2}{r^2}$ .....(iv) [As $r = R + h$]

Important points

(i) As we go above the surface of the earth, the value of g decreases because $g' \propto \frac{1}{r^2}$.

(ii) If $r = \infty$ then $g' = 0$, *i.e.*, at infinite distance from the earth, the value of g becomes zero.

(iii) If $h << R$ *i.e.*, height is negligible in comparison to the radius then from equation (iii) we get

$$g' = g\left(\frac{R}{R+h}\right)^2 = g\left(1+\frac{h}{R}\right)^{-2} = g\left[1-\frac{2h}{R}\right] \qquad \text{[As } h << R\text{]}$$

(iv) If $h << R$ then decrease in the value of g with height :

Absolute decrease $\Delta g = g - g' = \frac{2hg}{R}$

Fractional decrease $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{2h}{R}$

Percentage decrease $\frac{\Delta g}{g} \times 100\,\% = \frac{2h}{R} \times 100\,\%$

Sample problems based on variation in g with height

***Problem* 15.** The acceleration of a body due to the attraction of the earth (radius R) at a distance $2R$ from the surface of the earth is (g = acceleration due to gravity at the surface of the earth)

(a) $\frac{g}{9}$ (b) $\frac{g}{3}$ (c) $\frac{g}{4}$ (d) g

Solution : (a) $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{R}{R+2R}\right)^2 = \frac{1}{9}$ $\therefore\ g' = \frac{g}{9}$.

***Problem* 16.** The height of the point vertically above the earth's surface, at which acceleration due to gravity becomes 1% of its value at the surface is (Radius of the earth = R)

(a) $8R$ (b) $9R$ (c) $10\,R$ (d) $20R$

Solution : (b) Acceleration due to gravity at height h is given by $g' = g\left(\frac{R}{R+h}\right)^2$

$$\Rightarrow \frac{g}{100} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{10} \Rightarrow h = 9R.$$

***Problem* 17.** At surface of earth weight of a person is 72 N then his weight at height $R/2$ from surface of earth is (R = radius of earth) **[CBSE PMT 2000; AIIMS 2000]**

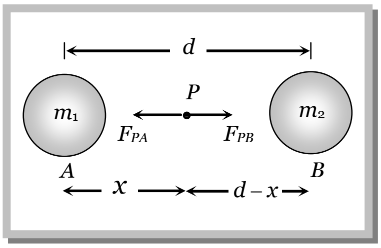
(a) $28N$ (b) $16N$ (c) $32N$ (d) $72N$

Solution : (c) Weight of the body at height R, $W' = W\left(\frac{R}{R+h}\right)^2 = W\left(\frac{R}{R+\frac{R}{2}}\right)^2 = W\left(\frac{2}{3}\right)^2 = \frac{4}{9}W = \frac{4}{9}\times 72 = 32\,N$.

Problem 18. If the distance between centres of earth and moon is D and the mass of earth is 81 times the mass of moon, then at what distance from centre of earth the gravitational force will be zero

(a) $D/2$ (b) $2D/3$ (c) $4D/3$ (d) $9D/10$

Solution : (d) If P is the point where net gravitational force is zero then $F_{PA} = F_{PB}$



$$\frac{Gm_1m}{x^2} = \frac{Gm_2m}{(d-x)^2}$$

By solving $x = \frac{\sqrt{m_1}\,d}{\sqrt{m_1}+\sqrt{m_2}}$

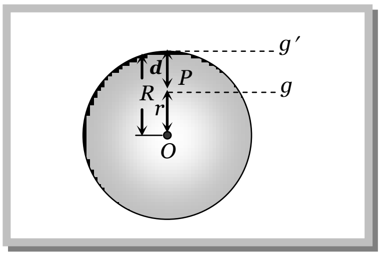
For the given problem $d = D$, $m_1 =$ earth, $m_2 =$ moon and $m_1 = 81m_2$ $\therefore m_2 = \frac{m_1}{81}$

So $x = \frac{\sqrt{m_1}\,D}{\sqrt{m_1}+\sqrt{m_2}} = \frac{\sqrt{m_1}\,D}{\sqrt{m_1}+\sqrt{\frac{m_1}{81}}} = \frac{D}{1+\frac{1}{9}} = \frac{9D}{10}$

8.7 Variation in g With Depth.

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR \qquad .....(i)$$



Acceleration due to gravity at depth d from the surface of the earth

$$g' = \frac{4}{3}\pi\rho G(R-d) \qquad .....(ii)$$

From (i) and (ii) $g' = g\left[1-\frac{d}{R}\right]$

Important points

(i) The value of g decreases on going below the surface of the earth. From equation (ii) we get $g' \propto (R-d)$.

So it is clear that if d increase, the value of g decreases.

(ii) At the centre of earth $d = R$ $\therefore$ $g' = 0$, *i.e.*, the acceleration due to gravity at the centre of earth becomes zero.

(iii) Decrease in the value of g with depth

Absolute decrease $\Delta g = g - g' = \frac{dg}{R}$

Fractional decrease $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{d}{R}$

Percentage decrease $\frac{\Delta g}{g} \times 100\,\% = \frac{d}{R} \times 100\,\%$

(iv) The rate of decrease of gravity outside the earth (if $h << R$) is double to that of inside the earth.

Sample problems based on variation in g with depth

Problem 19. Weight of a body of mass m decreases by 1% when it is raised to height h above the earth's surface. If the body is taken to a depth h in a mine, change in its weight is **[KCET 2003; MP PMT 2003]**

(a) 2% decrease (b) 0.5% decrease (c) 1% increase (d) 0.5% increase

Solution : (b) Percentage change in g when the body is raised to height h, $\frac{\Delta g}{g} \times 100\% = \frac{2h \times 100}{R} = 1\%$

Percentage change in g when the body is taken into depth d, $\frac{\Delta g}{g} \times 100\% = \frac{d}{R} \times 100\% = \frac{h}{R} \times 100\%$ [As $d = h$]

$\therefore$ Percentage decrease in weight $= \frac{1}{2}\left(\frac{2h}{R} \times 100\right) = \frac{1}{2}(1\%) = 0.5\%$.

Problem 20. The depth at which the effective value of acceleration due to gravity is $\frac{g}{4}$ is (R = radius of the earth) **[MP PET 2003]**

(a) R (b) $\frac{3R}{4}$ (c) $\frac{R}{2}$ (d) $\frac{R}{4}$

Solution : (b) $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{4} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \frac{3R}{4}$

Problem 21. Assuming earth to be a sphere of a uniform density, what is the value of gravitational acceleration in a mine 100 km below the earth's surface (Given R = 6400km)

(a) $9.66 m/s^2$ (b) $7.64 m/s^2$ (c) $5.06 m/s^2$ (d) $3.10 m/s^2$

Solution : (a) Acceleration due to gravity at depth d, $g' = g\left[1 - \frac{d}{R}\right] = g\left[1 - \frac{100}{6400}\right] = 9.8\left[1 - \frac{1}{64}\right]$

$= 9.8 \times \frac{63}{64} = 9.66 m/s^2$.

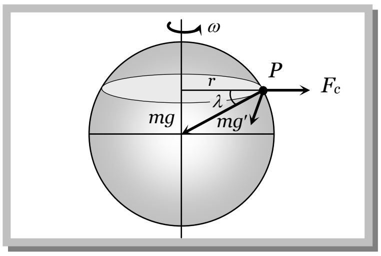
Problem 22. The depth d at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface, is [R = radius of the earth]

(a) $\frac{R}{n}$ (b) $R\left(\frac{n-1}{n}\right)$ (c) $\frac{R}{n^2}$ (d) $R\left(\frac{n}{n+1}\right)$

Solution : (b) $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{d}{R} = 1 - \frac{1}{n} \Rightarrow d = \left(\frac{n-1}{n}\right)R$

8.8 Variation in g Due to Rotation of Earth

As the earth rotates, a body placed on its surface moves along the circular path and hence experiences centrifugal force, due to it, the apparent weight of the body decreases.



Since the magnitude of centrifugal force varies with the latitude of the place, therefore the apparent weight of the body varies with latitude due to variation in the magnitude of centrifugal force on the body.

If the body of mass m lying at point P, whose latitude is λ, then due to rotation of earth its apparent weight can be given by $\overrightarrow{mg'} = \overrightarrow{mg} + \overrightarrow{F_c}$

or $$mg' = \sqrt{(mg)^2 + (F_c)^2 + 2mg\,F_c\cos(180 - \lambda)}$$

$\Rightarrow$ $$mg' = \sqrt{(mg)^2 + (m\omega^2 R\cos\lambda)^2 + 2mg\,m\omega^2 R\cos\lambda\,(-\cos\lambda)}$$ [As $F_c = m\omega^2 r = m\omega^2 R\cos\lambda$]

By solving we get $g' = g - \omega^2 R\cos^2\lambda$

Note: ❑ The latitude at a point on the surface of the earth is defined as the angle, which the line joining that point to the centre of earth makes with equatorial plane. It is denoted by λ.

❑ For the poles $\lambda = 90^o$ and for equator $\lambda = 0^o$

Important points

(i) Substituting $\lambda = 90^o$ in the above expression we get $g_{pole} = g - \omega^2 R\cos^2 90^o$

$\therefore$ $g_{pole} = g$ .....(i)

i.e., there is no effect of rotational motion of the earth on the value of g at the poles.

(ii) Substituting $\lambda = 0^o$ in the above expression we get $g_{eqator} = g - \omega^2 R\cos^2 0^o$

$\therefore$ $g_{equator} = g - \omega^2 R$ .....(ii)

i.e., the effect of rotation of earth on the value of g at the equator is maximum.

From equation (i) and (ii) $g_{pole} - g_{equator} = R\omega^2 = 0.034\,m/s^2$

(iii) When a body of mass m is moved from the equator to the poles, its weight increases by an amount

$$m(g_p - g_e) = m\omega^2 R$$

(iv) Weightlessness due to rotation of earth : As we know that apparent weight of the body decreases due to rotation of earth. If ω is the angular velocity of rotation of earth for which a body at the equator will become weightless

$$g' = g - \omega^2 R\cos^2\lambda$$

$\Rightarrow$ $0 = g - \omega^2 R\cos^2 0^o$ [As $\lambda = 0^o$ for equator]

$\Rightarrow$ $g - \omega^2 R$ $\therefore$ $\omega = \sqrt{\dfrac{g}{R}}$

or time period of rotation of earth $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$

Substituting the value of $R = 6400 \times 10^3 m$ and $g = 10 m/s^2$ we get

$$\omega = \frac{1}{800} = 1.25 \times 10^{-3} \frac{rad}{sec} \quad \text{and} \quad T = 5026.5\ sec = 1.40\ hr.$$

Note : ❑ This time is about $\frac{1}{17}$ times the present time period of earth. Therefore if earth starts rotating 17 times faster then all objects on equator will become weightless.

❑ If earth stops rotation about its own axis then at the equator the value of g increases by $\omega^2 R$ and consequently the weight of body lying there increases by $m\omega^2 R$.

❑ After considering the effect of rotation and elliptical shape of the earth, acceleration due to gravity at the poles and equator are related as

$$g_p = g_e + 0.034 + 0.018\ m/s^2 \quad \therefore g_p = g_e + 0.052\ m/s^2$$

Sample problems based on variation in g due to rotation of the earth

***Problem* 23.** The angular velocity of the earth with which it has to rotate so that acceleration due to gravity on 60° latitude becomes zero is (Radius of earth = 6400 *km*. At the poles $g = 10\ ms^{-2}$)

(a) $2.5\times10^{-3} rad/sec$ (b) $5.0\times10^{-1} rad/sec$ (c) $10 \times 10^1 rad/sec$ (d) $7.8 \times 10^{-2} rad/sec$

Solution : (a) Effective acceleration due to gravity due to rotation of earth $g' = g - \omega^2 R\cos^2\lambda$

$$\Rightarrow 0 = g - \omega^2 R\cos^2 60^o \Rightarrow \frac{\omega^2 R}{4} = g \Rightarrow \omega = \sqrt{\frac{4g}{R}} = 2\sqrt{\frac{g}{R}} = \frac{2}{800}\frac{rad}{sec} \quad [\text{As } g' = 0 \text{ and } \lambda = 60^o]$$

$$\Rightarrow \omega = \frac{1}{400} = 2.5 \times 10^{-3} \frac{rad}{\text{sec}}.$$

***Problem* 24.** If earth stands still what will be its effect on man's weight

(a) Increases (b) Decreases (c) Remains same (d) None of these

Solution : (a) When earth stops suddenly, centrifugal force on the man becomes zero so its effective weight increases.

***Problem* 25.** If the angular speed of earth is increased so much that the objects start flying from the equator, then the length of the day will be nearly

(a) 1.5 hours (b) 8 hours (c) 18 hours (d) 24 hours

Solution : (a) Time period for the given condition $T = 2\pi\sqrt{\frac{R}{g}} = 1.40\ hr \approx 1.5\ hr$ nearly.

8.9 Mass and Density of Earth

Newton's law of gravitation can be used to estimate the mass and density of the earth.

As we know $g = \frac{GM}{R^2}$, so we have $M = \frac{gR^2}{G}$

$$\therefore \quad M = \frac{9.8 \times (6.4 \times 10^6)^2}{6.67 \times 10^{-11}} = 5.98 \times 10^{24} kg \approx 10^{25} kg$$

and as we know $g = \frac{4}{3}\pi\rho GR$, so we have $\rho = \frac{3g}{4\pi GR}$

$$\therefore \qquad \rho = \frac{3 \times 9.8}{4 \times 3.14 \times 6.67 \times 10^{-11} \times 6.4 \times 10^{6}} = 5478.4\ kg/m^3$$

8.10 Inertial and Gravitational Masses.

(1) **Inertial mass** : It is the mass of the material body, which measures its inertia.

If an external force F acts on a body of mass m_i, then according to Newton's second law of motion

$$F = m_i a \text{ or } m_i = \frac{F}{a}$$

Hence inertial mass of a body may be measured as the ratio of the magnitude of the external force applied on it to the magnitude of acceleration produced in its motion.

Important points

(i) It is the measure of ability of the body to oppose the production of acceleration in its motion by an external force.

(ii) Gravity has no effect on inertial mass of the body.

(iii) It is proportional to the quantity of matter contained in the body.

(iv) It is independent of size, shape and state of body.

(v) It does not depend on the temperature of body.

(vi) It is conserved when two bodies combine physically or chemically.

(vii) When a body moves with velocity v, its inertial mass is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$, where m_0 = rest mass of body, c = velocity of light in vacuum,

(2) **Gravitational Mass :** It is the mass of the material body, which determines the gravitational pull acting upon it.

If M is the mass of the earth and R is the radius, then gravitational pull on a body of mass m_g is given by

$$F = \frac{GMm_g}{R^2} \text{ or } m_g = \frac{F}{(GM/R^2)} = \frac{F}{E}$$

Here m_g is the gravitational mass of the body, if $E = 1$ then $m_g = F$

Thus the gravitational mass of a body is defined as the gravitational pull experienced by the body in a gravitational field of unit intensity,

(3) **Comparison between inertial and gravitational mass**

(i) Both are measured in the same units.

(ii) Both are scalars

(iii) Both do not depends on the shape and state of the body

(iv) Inertial mass is measured by applying Newton's second law of motion where as gravitational mass is measured by applying Newton's law of gravitation.

(v) Spring balance measure gravitational mass and inertial balance measure inertial mass.

(4) **Comparison between mass and weight of the body**

Mass *(m)*	**Weight *(W)***
It is a quantity of matter contained in a body.	It is the attractive force exerted by earth on any body.
Its value does not change with *g*	Its value changes with *g*.
Its value can never be zero for any material particle.	At infinity and at the centre of earth its value is zero.
Its unit is kilogram and its dimension is [*M*].	Its unit is Newton or *kg-wt* and dimension are $[MLT^{-2}]$
It is determined by a physical balance.	It is determined by a spring balance.
It is a scalar quantity.	It is a vector quantity.

Sample problems based on inertial and gravitational mass

Problem 26. Gravitational mass is proportional to gravitational **[AIIMS 1998]**

(a) Field (b) Force (c) Intensity (d) All of these

Solution : (d)

Problem 27. The ratio of the inertial mass to gravitational mass is equal to **[CPMT 1978]**

(a) 1/2 (b) 1 (c) 2 (d) No fixed number

Solution : (b)

8.11 Gravitational Field.

The space surrounding a material body in which gravitational force of attraction can be experienced is called its gravitational field.

Gravitational field intensity : The intensity of the gravitational field of a material body at any point in its field is defined as the force experienced by a unit mass (test mass) placed at that point, provided the unit mass (test mass) itself does not produce any change in the field of the body.

So if a test mass m at a point in a gravitational field experiences a force $\vec{F}$ then

$$\vec{I} = \frac{\vec{F}}{m}$$

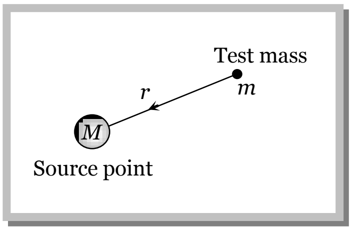
Important points

(i) It is a vector quantity and is always directed towards the centre of gravity of body whose gravitational field is considered.

(ii) Units : *Newton/kg* or m/s^2

(iii) Dimension : $[M^0LT^{-2}]$

(iv) If the field is produced by a point mass M and the test mass m is at a distance r from it then by Newton's law of gravitation $F = \frac{GMm}{r^2}$

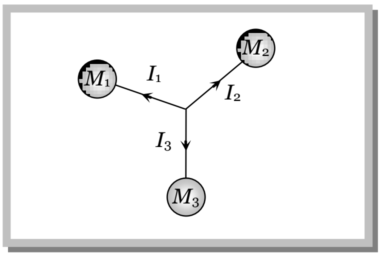


then intensity of gravitational field $I = \frac{F}{m} = \frac{GMm/r^2}{m}$

$$\therefore \qquad I = \frac{GM}{r^2}$$

(v) As the distance (r) of test mass from the point mass (M), increases, intensity of gravitational field decreases

$$I = \frac{GM}{r^2}; \quad \therefore \quad I \propto \frac{1}{r^2}$$



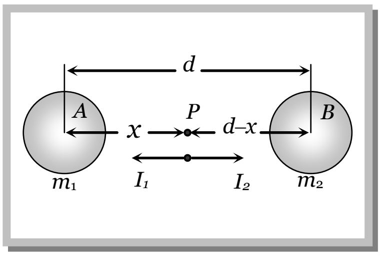
(vi) Intensity of gravitational field $I = 0$, when $r = \infty$.

(vii) Intensity at a given point (*P*) due to the combined effect of different point masses can be calculated by vector sum of different intensities

$$\overrightarrow{I_{net}} = \vec{I_1} + \vec{I_2} + \vec{I_3} + \ldots\ldots$$

(viii) Point of zero intensity : If two bodies *A* and *B* of different masses m_1 and m_2 are d distance apart.

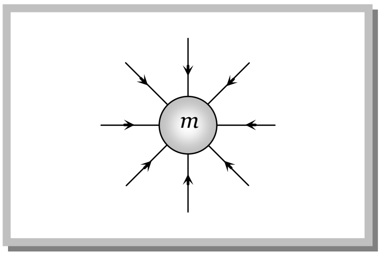
Let P be the point of zero intensity *i.e.*, the intensity at this point is equal and apposite due to two bodies A and B and if any test mass placed at this point it will not experience any force.



For point P $\quad \vec{I_1} + \vec{I_2} = 0 \Rightarrow \frac{-Gm_1}{x^2} + \frac{Gm_2}{(d-x)^2} = 0$

By solving $\quad x = \frac{\sqrt{m_1}\, d}{\sqrt{m_1} + \sqrt{m_2}}$ and $(d - x) = \frac{\sqrt{m_2}\, d}{\sqrt{m_1} + \sqrt{m_2}}$

(ix) Gravitational field line is a line, straight or curved such that a unit mass placed in the field of another mass would always move along this line. Field lines for an isolated mass m are radially inwards.



(x) As $I = \frac{GM}{r^2}$ and also $g = \frac{GM}{R^2}$ $\therefore \quad I = g$

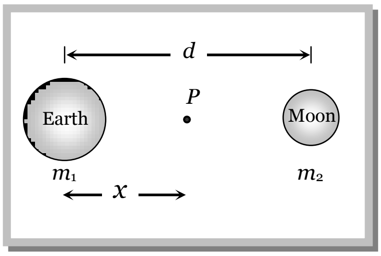
Thus the intensity of gravitational field at a point in the field is equal to acceleration of test mass placed at that point.

Sample problems based on gravitational field

***Problem* 28.** Knowing that mass of Moon is $\frac{M}{81}$ where *M* is the mass of Earth, find the distance of the point where gravitational field due to Earth and Moon cancel each other, from the Moon. Given that distance between Earth and Moon is 60 *R*. Where *R* is the radius of Earth

(a) 2 *R* (b) 4 *R* (c) 6 *R* (d) 8 *R*

Solution : (c) Point of zero intensity $x = \frac{\sqrt{m_1}\, d}{\sqrt{m_1} + \sqrt{m_2}}$



mass of the earth $m_1 = M$, Mass of the moon $m_2 = \frac{M}{81}$

and distance between earth & moon $d = 60R$

Point of zero intensity from the Earth $x = \frac{\sqrt{M} \times 60R}{\sqrt{M} + \sqrt{\frac{M}{81}}} = \frac{9}{10} \times 60R = 54R$

So distance from the moon $= 60R - 54R = 6R$.

Problem 29. The gravitational potential in a region is given by $V = (3x + 4y + 12z)\,J/kg$. The modulus of the gravitational field at $(x = 1, y = 0, z = 3)$ is

(a) $20\,N\,kg^{-1}$ (b) $13\,N\,kg^{-1}$ (c) $12\,N\,kg^{-1}$ (d) $5\,N\,kg^{-1}$

Solution : (b) $I = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = -(3\hat{i} + 4\hat{j} + 12\hat{k})$ [As $V = (3x + 4y + 12z)$ (given)]

It is uniform field Hence its value is same every where $|I| = \sqrt{3^2 + 4^2 + 12^2} = 13\,Nkg^{-1}$.

Problem 30. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$ (b) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$ (d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

Solution : (a, b) We know that gravitational force $\propto$ Intensity $\propto \frac{1}{r^2}$ when $r > R$ [As $I = \frac{GM}{r^2}$]

$\therefore \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

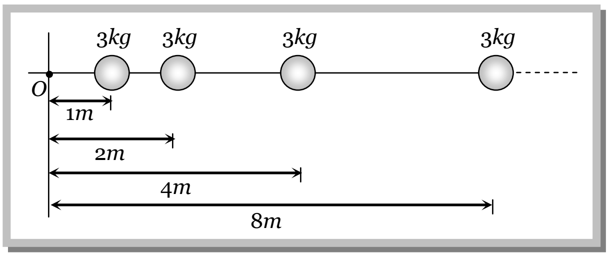
and gravitational force $\propto$ Intensity $\propto r$ when $r < R$ [As $I = \frac{4}{3}\pi\rho G r$]

$\therefore \frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$.

Problem 31. Infinite bodies, each of mass 3*kg* are situated at distances 1*m*, 2*m*, 4*m*, 8*m*....... respectively on *x*-axis. The resultant intensity of gravitational field at the origin will be

(a) G (b) $2G$ (c) $3G$ (d) $4G$

Solution : (d) Intensity at the origin $I = I_1 + I_2 + I_3 + I_4 + ...$



$$= \frac{GM}{r_1^2} + \frac{GM}{r_2^2} + \frac{GM}{r_3^2} + \frac{GM}{r_4^2} + ...$$

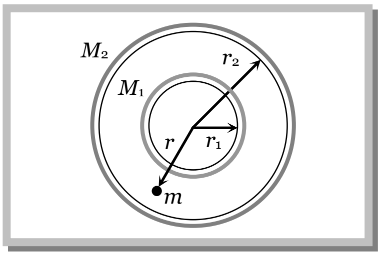
$$= GM\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + ...\right]$$

$$= GM\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + ...\right]$$

$$= GM\left(\frac{1}{1 - \frac{1}{4}}\right)$$ [As sum of G.P. $= \frac{a}{1-r}$]

$$= GM \times \frac{4}{3} = G \times 3 \times \frac{4}{3} = 4G$$ [As $M = 3kg$ given]

Problem 32. Two concentric shells of mass M_1 and M_2 are having radii r_1 and r_2. Which of the following is the correct expression for the gravitational field on a mass m.



(a) $I = \frac{G(M_1 + M_2)}{r^2}$ for $r < r_1$

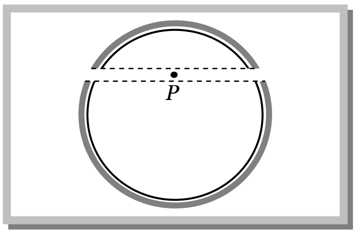
(b) $I = \frac{G(M_1 + M_2)}{r^2}$ for $r < r_2$

(c) $I = G\frac{M_2}{r^2}$ for $r_1 < r < r_2$

(d) $I = \frac{GM_1}{r^2}$ for $r_1 < r < r_2$

Solution : (d) Gravitational field on a mass m due to outer shell (radius r_2) will be zero because the mass is placed inside this shell. But the inner shell (radius r_1) behaves like point mass placed at the centre so $I = \frac{GM_1}{r^2}$ for $r_1 < r < r_2$

***Problem* 33.** A spherical shell is cut into two pieces along a chord as shown in the figure. P is a point on the plane of the chord. The gravitational field at P due to the upper part is I_1 and that due to the lower part is I_2. What is the relation between them



(a) $I_1 > I_2$

(b) $I_1 < I_2$

(c) $I_1 = I_2$

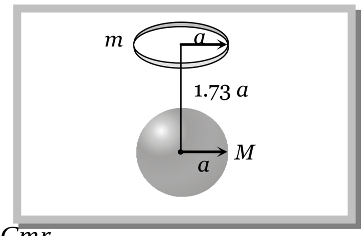
(d) No definite relation

Solution : (c) Intensity at P due to upper part $= I_1$ and Intensity at P due to lower part $= I_2$

Net Intensity at p due to spherical shell $\vec{I_1} + \vec{I_2} = 0$

$\therefore \vec{I_1} = -\vec{I_2}$

***Problem* 34.** A uniform ring of mass m is lying at a distance 1.73 a from the centre of a sphere of mass M just over the sphere where a is the small radius of the ring as well as that of the sphere. Then gravitational force exerted is



(a) $\frac{GMm}{8a^2}$ (b) $\frac{GMm}{(1.73\,a)^2}$

(c) $\sqrt{3}\frac{GMm}{a^2}$ (d) $1.73\frac{GMm}{8a^2}$

Solution : (d) Intensity due to uniform circular ring at a point on its axis $I = \frac{Gmr}{(a^2 + r^2)^{3/2}}$

$\therefore$ Force on sphere $F = \frac{GMmr}{(a^2 + r^2)^{3/2}} = \frac{GMm\sqrt{3}a}{(a^2 + (\sqrt{3}a)^2)^{3/2}} = \frac{GMm\sqrt{3}a}{(4a^2)^{3/2}} = \frac{\sqrt{3}GMm}{8a^2}$ [As $r = \sqrt{3}a$]

8.13 Gravitational Potential.

At a point in a gravitational field potential V is defined as negative of work done per unit mass in shifting a test mass from some reference point (usually at infinity) to the given point *i.e.*,

$$V = -\frac{W}{m} = -\int \frac{\vec{F}.\vec{dr}}{m} = -\int \vec{I}.\vec{dr} \qquad \left[\text{As } \frac{F}{m} = I\right]$$

$$\therefore \qquad I = -\frac{dV}{dr}$$

i.e., negative gradient of potential gives intensity of field or potential is a scalar function of position whose space derivative gives intensity. Negative sign indicates that the direction of intensity is in the direction where the potential decreases.

Important points

(i) It is a scalar quantity because it is defined as work done per unit mass.

(ii) Unit : *Joule/kg* or m^2/sec^2

(iii) Dimension : $[M^0L^2T^{-2}]$

(iv) If the field is produced by a point mass then $V = -\int I\,dr = -\int \left(-\frac{GM}{r^2}\right) dr$ $\quad \left[\text{As } I = -\frac{GM}{r^2}\right]$

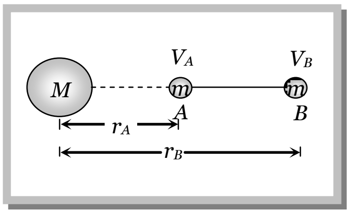
$$\therefore \qquad V = -\frac{GM}{r} + c \qquad [\text{Here } c = \text{constant of integration}]$$

Assuming reference point at ∞ and potential to be zero there we get

$$0 = -\frac{GM}{\infty} + c \Rightarrow c = 0$$

$$\therefore \qquad \text{Gravitational potential } V = -\frac{GM}{r}$$

(v) Gravitational potential difference : It is defined as the work done to move a unit mass from one point to the other in the gravitational field. The gravitational potential difference in bringing unit test mass m from point A to point B under the gravitational influence of source mass M is



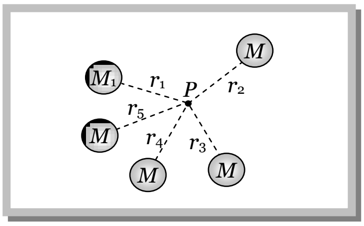
$$\Delta V = V_B - V_A = \frac{W_{A\to B}}{m} = -GM\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

(vi) Potential due to large numbers of particle is given by scalar addition of all the potentials.

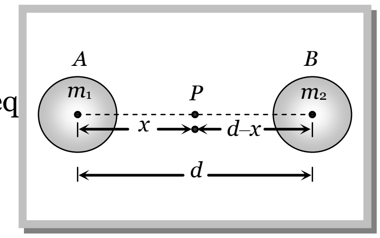
$$V = V_1 + V_2 + V_3 + \dots\dots\dots$$

$$= -\frac{GM}{r_1} - \frac{GM}{r_2} - \frac{GM}{r_3} \dots\dots\dots$$

$$= -G\sum_{i=1}^{i=n} \frac{M_i}{r_i}$$



(vii) Point of zero potential : It is that point in the gravitational field, if the unit mass is shifted from infinity to that point then net work done will be eq



Let m_1 and m_2 are two masses placed at d distance apart and P is the point of zero potential in between the two masses.

Net potential for point $P = V_A + V_B = 0$

$\Rightarrow \quad -\frac{Gm_1}{x} - \frac{Gm_2}{d-x} = 0$ By solving $x = \frac{m_1 d}{m_1 - m_2}$

Sample problems based on gravitational potential

Problem 35. In some region, the gravitational field is zero. The gravitational potential in this region **[BVP 2003]**

(a) Must be variable (b) Must be constant (c) Cannot be zero (d) Must be zero

Solution : (b) As $I = -\frac{dV}{dx}$, if $I = 0$ then $V =$ constant.

Problem 36. The gravitational field due to a mass distribution is $E = K/x^3$ in the x - direction (K is a constant). Taking the gravitational potential to be zero at infinity, its value at a distance x is

(a) K/x (b) $K/2x$ (c) K/x^2 (d) $K/2x^2$

Solution : (d) $V = -\int E\,dx = -\int \frac{K}{x^3}dx = \frac{K}{2x^2}$.

Problem 37. The intensity of gravitational field at a point situated at a distance of 8000 *km* from the centre of the earth is $6N/kg$. The gravitational potential at that point is – (in *Joule / kg*)

(a) 8×10^6 (b) 2.4×10^3 (c) 4.8×10^7 (d) 6.4×10^{14}

Solution : (c) Gravitational intensity at point *P*, $I = \frac{GM}{r^2}$ and gravitational potential $V = -\frac{GM}{r}$

$\therefore \ V = I \times r = 6\ N/kg \times 8000\ km = 4.8 \times 10^7 \frac{Joule}{kg}$.

Problem 38. The gravitational potential due to the earth at infinite distance from it is zero. Let the gravitational potential at a point *P* be $-5J/kg$. Suppose, we arbitrarily assume the gravitational potential at infinity to be + 10 J/kg, then the gravitational potential at *P* will be

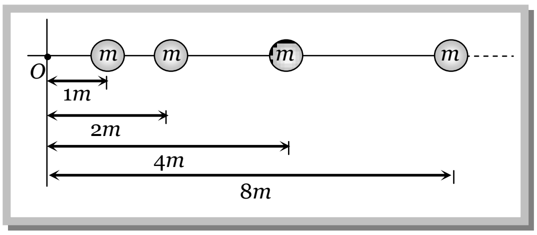
(a) $-5\ J/kg$ (b) $+5\ J/kg$ (c) $-15\ J/kg$ (d) $+15\ J/kg$

Solution : (b) Potential increases by $+10J/kg$ every where so it will be $+10-5 = +5J/kg$ at P

Problem 39. An infinite number of point masses each equal to *m* are placed at *x* =1. *x* = 2, *x* = 4, *x* = 8 .... What is the total gravitational potential at *x* = 0

(a) $-Gm$ (b) $-2Gm$ (c) $-4Gm$ (d) $-8Gm$

Solution : (b) Net potential at origin $V = -\left[\frac{Gm}{r_1} + \frac{Gm}{r_2} + \frac{Gm}{r_3} + ...\right]$



$= -Gm\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right] = -Gm\left(\frac{1}{1-\frac{1}{2}}\right) = -2Gm$

Problem 40. Two bodies of masses *m* and *M* are placed a distance *d* apart. The gravitational potential at the position where the gravitational field due to them is zero is *V*, then

(a) $V = -\frac{G}{d}(m+M)$ (b) $V = -\frac{Gm}{d}$ (c) $V = -\frac{GM}{d}$ (d) $V = -\frac{G}{d}(\sqrt{m} + \sqrt{M})^2$

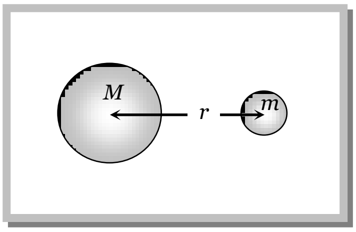
Solution : (d) If P is the point of zero intensity, then $x = \frac{\sqrt{M}}{\sqrt{M} + \sqrt{m}}.d$ and $d - x = \frac{\sqrt{m}}{\sqrt{M} + \sqrt{m}}d$

Now potential at point P, $V = V_1 + V_2 = -\frac{GM}{x} - \frac{GM}{d-x}$

Substituting the value of x and $d-x$ we get $V = -\frac{G}{d}\left(\sqrt{m} + \sqrt{M}\right)^2$.

8.15 Gravitational Potential Energy.

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.



$$W = \int_{\infty}^{r} \frac{GMm}{x^2}dx = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$W = -\frac{GMm}{r}$$

This work done is stored inside the body as its gravitational potential energy

$$\therefore U = -\frac{GMm}{r}$$

Important points

(i) Potential energy is a scalar quantity.

(ii) *Unit* : Joule

(iii) *Dimension* : $[ML^2T^{-2}]$

(iv) Gravitational potential energy is always negative in the gravitational field because the force is always attractive in nature.

(v) As the distance r increases, the gravitational potential energy becomes less negative i.e., it increases.

(vi) If $r = \infty$ then it becomes zero (maximum)

(vii) In case of discrete distribution of masses

Gravitational potential energy $U = \sum u_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + ...\right]$

(viii) If the body of mass m is moved from a point at a distance r_1 to a point at distance $r_2 (r_1 > r_2)$ then change in potential energy $\Delta U = \int_{r_1}^{r_2} \frac{GMm}{x^2}dx = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$ or $\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

As r_1 is greater than r_2, the change in potential energy of the body will be negative. It means that if a body is brought closer to earth it's potential energy decreases.

(ix) Relation between gravitational potential energy and potential $U = -\frac{GMm}{r} = m\left[\frac{-GM}{r}\right]$

$\therefore \qquad U = mV$

(x) Gravitational potential energy at the centre of earth relative to infinity.

$$U_{centre} = m\,V_{centre} = m\left(-\frac{3}{2}\frac{GM}{R}\right) = -\frac{3}{2}\frac{GMm}{R}$$

(xi) Gravitational potential energy of a body at height h from the earth surface is given by

$$U_h = -\frac{GMm}{R+h} = -\frac{gR^2m}{R+h} \equiv -\frac{mgR}{1+\frac{h}{R}}$$

8.16 Work Done Against Gravity

If the body of mess m is moved from the surface of earth to a point at distance h above the surface of earth, then change in potential energy or work done against gravity will be

$$W = \Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\Rightarrow \quad W = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right] \qquad [\text{As } r_1 = R \text{ and } r_2 = R+h]$$

$$\Rightarrow \quad W = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)} = \frac{mgh}{1+\frac{h}{R}} \qquad [\text{As } \frac{GM}{R^2} = g]$$

Important points

(i) When the distance h is not negligible and is comparable to radius of the earth, then we will use above formula.

(ii) If $h = nR$ then $W = mgR\left(\frac{n}{n+1}\right)$

(iii) If $h = R$ then $W = \frac{1}{2}mgR$

(iv) If h is very small as compared to radius of the earth then term h/R can be neglected

From $\boxed{W = \frac{mgh}{1+h/R} = mgh}$ $\left[\text{As } \frac{h}{R} \to 0\right]$

Sample problems based on potential Energy

Problem 41. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

(a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{3R^2}$ (c) $\frac{GMm}{8R}$ (d) $\frac{GMm}{6R}$

Solution : (d) Work done = Change in potential energy $= U_2 - U_1 = \left[-\frac{GMm}{r_2}\right] - \left[-\frac{GMm}{r_1}\right] = -\frac{GMm}{3R} + \frac{GMm}{2R}$

$= \frac{GMm}{6R}$.

Problem 42. A body of mass m kg. starts falling from a point $2R$ above the earth's surface. Its kinetic energy when it has fallen to a point 'R' above the earth's surface [R-Radius of earth, M-Mass of earth, G-Gravitational constant]

[MP PMT 2002]

(a) $\frac{1}{2}\frac{GMm}{R}$ (b) $\frac{1}{6}\frac{GMm}{R}$ (c) $\frac{2}{3}\frac{GMm}{R}$ (d) $\frac{1}{3}\frac{GMm}{R}$

Solution : (b) When body starts falling toward earth's surface its potential energy decreases so kinetic energy increases.

Increase in kinetic energy = Decrease in potential energy

Final kinetic energy – Initial kinetic energy = Initial potential energy – Final potential energy

Final kinetic energy – 0 $= \left(-\frac{GMm}{r_1}\right) - \left(-\frac{GMm}{r_2}\right)$

$\therefore$ Final kinetic energy $= \left(\frac{-GMm}{R+h_1}\right) - \left(-\frac{GMm}{R+h_2}\right)$

$= \left(-\frac{GMm}{R+2R}\right) - \left(-\frac{GMm}{R+R}\right) = -\frac{GMm}{3R} + \frac{GMm}{2R} = \frac{1}{6}\frac{GMm}{R}$.

Problem 43. A body of mass m is taken from earth surface to the height h equal to radius of earth, the increase in potential energy will be **[CPMT 1971, 97; IIT-JEE 1983; CBSE PMT 1991; Haryana CEE 1996; CEET Bihar 1995; MNR 1998; RPET 2000]**

(a) mgR (b) $\frac{1}{2}mgR$ (c) $2mgR$ (d) $\frac{1}{4}mgR$

Solution : (b) Work done $= \frac{mgh}{1+h/R}$, If $h = R$ then work done $= \frac{mgR}{1+R/R} = \frac{1}{2}mgR$.

Problem 44. If mass of earth is M, radius is R and gravitational constant is G, then work done to take 1 kg mass from earth surface to infinity will be

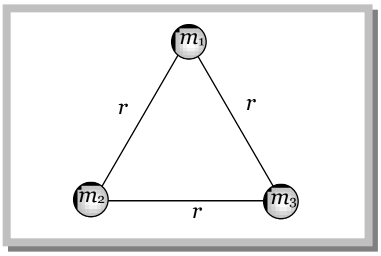
(a) $\sqrt{\frac{GM}{2R}}$ (b) $\frac{GM}{R}$ (c) $\sqrt{\frac{2GM}{R}}$ (d) $\frac{GM}{2R}$

Solution : (b) Work done $= U_{final} - U_{initial} = U_\infty - U_R = 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$ [As $m = 1kg$]

Problem 45. Three particles each of mass 100 gm are brought from a very large distance to the vertices of an equilateral triangle whose side is 20 cm in length. The work done will be

(a) 0.33×10^{-11} *Joule* (b) -0.33×10^{-11} *Joule* (c) 1.00×10^{-11} *Joule* (d) -1.00×10^{-11} *Joule*

Solution : (d) Potential energy of three particles system



$$U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_1m_3}{r_{13}}$$

Given $m_1 = m_2 = m_3 = 100\ gm$ and $r_{12} = r_{23} = r_{13} = 2cm$

$$\therefore\ U = 3\left[\frac{-6.67\times10^{-11}\times(10^{-1})\times(10^{-1})}{20\times10^{-2}}\right] = -1.00\times10^{-11}\ Joule.$$

Problem 46. A boy can jump to a height h on ground level. What should be the radius of a sphere of density d such that on jumping on it, he escapes out of the gravitational field of the sphere

(a) $\left[\frac{4\pi}{3}\frac{Gd}{gh}\right]^{1/2}$ (b) $\left[\frac{4\pi}{3}\frac{gh}{Gd}\right]^{1/2}$ (c) $\left[\frac{3}{4\pi}\frac{gh}{Gd}\right]^{1/2}$ (d) $\left[\frac{3}{4\pi}\frac{Gd}{gh}\right]^{1/2}$

Solution : (c) When a boy jumps from a ground level up to height h then its velocity of jumping $v = \sqrt{2gh}$ .....(i)

and for the given condition this will become equal to escape velocity $v_{escape} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\left(\frac{4}{3}\pi R^3.d\right)}$

.....(ii)

Equating (i) and (ii) $\sqrt{2gh} = R\sqrt{\frac{8}{3}G\pi d} \Rightarrow R = \left[\frac{3}{4\pi}\frac{gh}{Gd}\right]^{1/2}$.

8.17 Escape Velocity.

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

The work done to displace a body from the surface of earth ($r = R$) to infinity ($r = \infty$) is

$$W = \int_R^\infty \frac{GMm}{x^2}dx = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$\Rightarrow \qquad W = \frac{GMm}{R}$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth.

If v_e is the required escape velocity, then kinetic energy which should be given to the body is $\frac{1}{2}mv_e^2$

$$\therefore \quad \frac{1}{2}mv_e^2 = \frac{GMm}{R} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow v_e = \sqrt{2gR} \qquad [\text{As } GM = gR^2]$$

$$\text{or} \quad v_e = \sqrt{2 \times \frac{4}{3}\pi\rho GR \times R} \Rightarrow v_e = R\sqrt{\frac{8}{3}\pi G\rho} \qquad [\text{As } g = \frac{4}{3}\pi\rho GR]$$

Important points

(i) Escape velocity is independent of the mass and direction of projection of the body.

(ii) Escape velocity depends on the reference body. Greater the value of (M/R) or (gR) for a planet, greater will be escape velocity.

(iii) For the earth as $g = 9.8 m/s^2$ and $R = 6400\ km$

$$\therefore v_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11.2 km/\sec$$

(iv) A planet will have atmosphere if the velocity of molecule in its atmosphere $\left[v_{rms} = \sqrt{\frac{3RT}{M}}\right]$ is lesser than escape velocity. This is why earth has atmosphere (as at earth $v_{rms} < v_e$) while moon has no atmosphere (as at moon $v_{rms} < v_e$)

(v) If body projected with velocity lesser than escape velocity ($v < v_e$) it will reach a certain maximum height and then may either move in an orbit around the planet or may fall down back to the planet.

(vi) Maximum height attained by body : Let a projection velocity of body (mass m) is v, so that is attains a maximum height h. At maximum height, the velocity of particle is zero, so kinetic energy is zero.

By the law of conservation of energy

Total energy at surface = Total energy at height h.

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

$$\Rightarrow \quad \frac{v^2}{2} = GM\left[\frac{1}{R} - \frac{1}{R+h}\right] = \frac{GMh}{R(R+h)}$$

$$\Rightarrow \quad \frac{2GM}{v^2R} = \frac{R+h}{h} = 1 + \frac{R}{h}$$

$$\Rightarrow \quad h = \frac{R}{\left(\frac{2GM}{v^2R} - 1\right)} = \frac{R}{\frac{v_e^2}{v^2} - 1} = R\left[\frac{v^2}{v_e^2 - v^2}\right] \qquad \left[\text{As } v_e = \sqrt{\frac{2GM}{R}} \therefore \frac{2GM}{R} = v_e^2\right]$$

(vii) If a body is project with velocity greater than escape velocity $(v > v_e)$ then by conservation of energy.

Total energy at surface = Total energy at infinite

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m(v')^2 + 0$$

i.e., $(v')^2 = v^2 - \frac{2GM}{R} \Rightarrow v'^2 = v^2 - v_e^2$ [As $\frac{2GM}{R} = v_e^2$]

$\therefore$ $v' = \sqrt{v^2 - v_e^2}$

i.e, the body will move in interplanetary or inter stellar space with velocity $\sqrt{v^2 - v_e^2}$.

(viii) Energy to be given to a stationary object on the surface of earth so that its total energy becomes zero, is called escape energy.

Total energy at the surface of the earth $= KE + PE = 0 - \frac{GMm}{R}$

$\therefore$ Escape energy $= \frac{GMm}{R}$

(ix) If the escape velocity of a body is equal to the velocity of light then from such bodies nothing can escape, not even light. Such bodies are called black holes.

The radius of a black hole is given as

$$R = \frac{2GM}{C^2} \qquad \text{[As } C = \sqrt{\frac{2GM}{R}}\text{, where } C \text{ is the velocity of light]}$$

Sample problems based on escape velocity

Problem 47. For a satellite escape velocity is $11 km/s$. If the satellite is launched at an angle of 60^o with the vertical, then escape velocity will be

(a) $11 km/s$ (b) $11\sqrt{3}\ km/s$ (c) $\frac{11}{\sqrt{3}} km/s$ (d) $33 km/s$

Solution : (a) Escape velocity does not depend upon the angle of projection.

Problem 48. The escape velocity from the earth is about $11 km/s$. The escape velocity from a planet having twice the radius and the same mean density as the earth, is **[MP PMT 1987; UPSEAT 1999; AIIMS 2001; MP PET 2001, 2003]**

(a) 22 *km/s* (b) 11 *km/s* (c) 5.5 *km/s* (d) 15.5 *km/s*

Solution : (a) $v_e = \sqrt{\frac{2Gm}{R}} = \sqrt{\frac{8}{3}\pi\rho GR^2}$ $\therefore v_e \propto R$ if $\rho =$ constant. Since the planet having double radius in comparison to earth therefore the escape velocity becomes twice *i.e.* $22 km/s$.

Problem 49. A projectile is projected with velocity kv_e in vertically upward direction from the ground into the space. (v_e is escape velocity and $k < 1$). If air resistance is considered to be negligible then the maximum height from the centre of earth to which it can go, will be (R = radius of earth) **[Roorkee 1999; RPET 1**

(a) $\frac{R}{k^2+1}$ (b) $\frac{R}{k^2-1}$ (c) $\frac{R}{1-k^2}$ (d) $\frac{R}{k+1}$

Solution : (c) From the law of conservation of energy

Difference in potential energy between ground and maximum height = Kinetic energy at the point of projection

$$\frac{mgh}{1+h/R} = \frac{1}{2}m(kv_e)^2 = \frac{1}{2}mk^2v_e^2 = \frac{1}{2}mk^2(\sqrt{2gR})^2 \qquad \text{[As } v_e = \sqrt{2gR}\text{]}$$

By solving height from the surface of earth $h = \frac{Rk^2}{1-k^2}$

So height from the centre of earth $r = R + h = R + \frac{Rk^2}{1-k^2} = \frac{R}{1-k^2}$.

Problem 50. If the radius of earth reduces by 4% and density remains same then escape velocity will **[MP PET 1991; MP PMT 1995]**

(a) Reduce by 2% (b) Increase by 2% (c) Reduce by 4% (d) Increase by 4%

Solution : (c) Escape velocity $v_e \propto R\sqrt{\rho}$ and if density remains constant $v_e \propto R$

So if the radius reduces by 4% then escape velocity also reduces by 4%.

Problem 51. A rocket of mass M is launched vertically from the surface of the earth with an initial speed V. Assuming the radius of the earth to be R and negligible air resistance, the maximum height attained by the rocket above the surface of the earth is

(a) $\frac{R}{\left(\frac{gR}{2V^2}-1\right)}$ (b) $R\left(\frac{gR}{2V^2}-1\right)$ (c) $\frac{R}{\left(\frac{2gR}{V^2}-1\right)}$ (d) $R\left(\frac{2gR}{V^2}-1\right)$

Solution : (c) Kinetic energy given to rocket at the surface of earth = Change in potential energy of the rocket in reaching from ground to highest point

$$\Rightarrow \frac{1}{2}mv^2 = \frac{mgh}{1+h/R} \Rightarrow \frac{v^2}{2} = \frac{g}{\frac{1}{h}+\frac{1}{R}} \Rightarrow$$

$$\frac{1}{h}+\frac{1}{R} = \frac{2g}{v^2} \Rightarrow \frac{1}{h} = \frac{2g}{v^2} - \frac{1}{R} \Rightarrow \frac{1}{h} = \frac{2gR - v^2}{v^2R} \Rightarrow h = \frac{v^2R}{2gR - v^2}$$

$$\Rightarrow h = \frac{R}{\left(\frac{2gR}{v^2}-1\right)}$$

Problem 52. A body of mass m is situated at a distance $4R_e$ above the earth's surface, where R_e is the radius of earth. How much minimum energy be given to the body so that it may escape

(a) mgR_e (b) $2mgR_e$ (c) $\frac{mgR_e}{5}$ (d) $\frac{mgR_e}{16}$

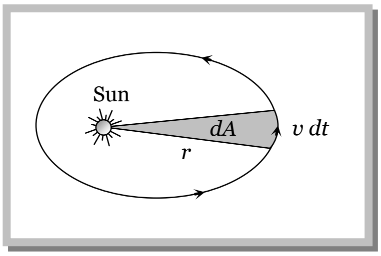
Solution : (c) Potential energy of the body at a distance $4R_e$ from the surface of earth

$$U = -\frac{mgR_e}{1+h/R_e} = -\frac{mgR_e}{1+4} = -\frac{mgR_e}{5} \qquad [\text{As } h = 4R_e \text{ (given)}]$$

So minimum energy required to escape the body will be $\frac{mgR_e}{5}$.

8.18 Kepler's Laws of Planetary Motion.

Planets are large natural bodies rotating around a star in definite orbits. The planetary system of the star sun called solar system consists of nine planets, viz., Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto. Out of these planets Mercury is the smallest, closest to the sun and so hottest. Jupiter is largest and has maximum moons (12). Venus is closest to Earth and brightest. Kepler after a life time study work out three empirical laws which govern the motion of these planets and are known as *Kepler's laws of planetary motion*. These are,



(1) **The law of Orbits** : Every planet moves around the sun in an elliptical orbit with sun at one of the foci.

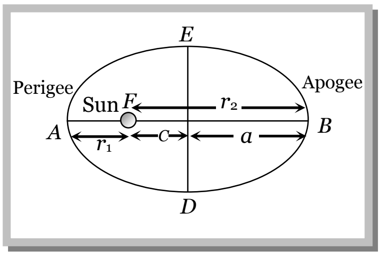
(2) **The law of Area** : The line joining the sun to the planet sweeps out equal areas in equal interval of time. *i.e.* areal velocity is constant. According to this law planet will move slowly when it is farthest from sun and more rapidly when it is nearest to sun. It is similar to law of conservation of angular momentum.

Areal velocity $= \frac{dA}{dt} = \frac{1}{2}\frac{r(vdt)}{dt} = \frac{1}{2}rv$

$\therefore \quad \frac{dA}{dt} = \frac{L}{2m}$ [As $L = mvr$; $rv = \frac{L}{m}$]

(3) **The law of periods** : The square of period of revolution (T) of any planet around sun is directly proportional to the cube of the semi-major axis of the orbit.

$$T^2 \propto a^3 \text{ or } T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3$$



Proof : From the figure $AB = AF + FB$

$2a = r_1 + r_2 \quad \therefore a = \frac{r_1 + r_2}{2}$ where a = semi-major axis

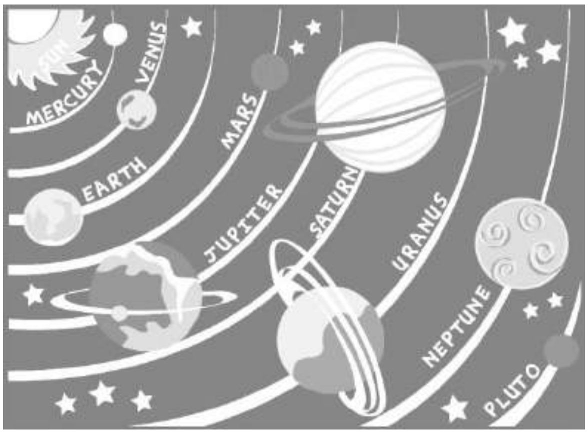
r_1 = Shortest distance of planet from sun (perigee).

r_2 = Largest distance of planet from sun (apogee).

Important data

Planet	Semi-major axis *a* (10^{10} *meter*)	Period *T(year)*	T^2/a^3 (10^{-34} $year^2/meter^3$)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Solar System



Note: ❑ Kepler's laws are valid for satellites also.

8.19 Velocity of a Planet in Terms of Eccentricity

Applying the law of conservation of angular momentum at perigee and apogee

$$mv_p r_p = mv_a r_a$$

$\Rightarrow \quad \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{a+c}{a-c} = \frac{1+e}{1-e}$ [As $r_p = a - c$, $r_a = a + c$ and eccentricity $e = \frac{c}{a}$]

Applying the conservation of mechanical energy at perigee and apogee

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a} \Rightarrow v_p^2 - v_a^2 = 2GM\left[\frac{1}{r_p} - \frac{1}{r_a}\right]$$

$\Rightarrow \quad v_a^2\left[\frac{r_a^2 - r_p^2}{r_p^2}\right] = 2GM\left[\frac{r_a - r_p}{r_a r_p}\right]$ [As $v_p = \frac{v_a r_a}{r_p}$]

$$\Rightarrow \qquad v_a^2 = \frac{2GM}{r_a + r_p}\left[\frac{r_p}{r_a}\right] \Rightarrow v_a^2 = \frac{2GM}{a}\left(\frac{a-c}{a+c}\right) = \frac{2GM}{a}\left(\frac{1-e}{1+e}\right)$$

Thus the speeds of planet at apogee and perigee are

$$v_a = \sqrt{\frac{2GM}{a}\left(\frac{1-e}{1+e}\right)}, \qquad v_p = \sqrt{\frac{2GM}{a}\left(\frac{1+e}{1-e}\right)}$$

Note: ❑ The gravitational force is a central force so torque on planet relative to sun is always zero, hence angular momentum of a planet or satellite is always constant irrespective of shape of orbit.

8.20 Some Properties of the Planet.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mean distacne from sun, 10^6 *km*	57.9	108	150	228	778	1430	2870	4500	5900
Period of revolution, *year*	0.241	0.615	1.00	1.88	11.9	29.5	84.0	165	248
Orbital speed, *km/s*	47.9	35.0	29.8	24.1	13.1	9.64	6.81	5.43	4.74
Equatiorial diameter, *km*	4880	12100	12800	6790	143000	120000	51800	49500	2300
Mass (Earth =1)	0.0558	0.815	1.000	0.107	318	95.1	14.5	17.2	0.002
Density (Water =1)	5.60	5.20	5.52	3.95	1.31	0.704	1.21	1.67	2.03
Surface value of *g*, m/s^2	3.78	8.60	9.78	3.72	22.9	9.05	7.77	11.0	0.5
Escape velocity, *km/s*	4.3	10.3	11.2	5.0	59.5	35.6	21.2	23.6	1.1
Known satellites	0	0	1	2	16+ring	18+rings	17+rings	8+rings	1

Sample problems based on Kepler's law

Problem 53. The distance of a planet from the sun is 5 times the distance between the earth and the sun. The Time period of the planet is **[UPSEAT 2003]**

(a) $5^{3/2}$ *years* (b) $5^{2/3}$ *years* (c) $5^{1/3}$ *years* (d) $5^{1/2}$ *years*

Solution : (a) According to Kepler's law $T \propto R^{3/2}$ $\therefore$ $T_{planet} = (5)^{3/2}\, T_{earth} = 5^{(3/2)} \times 1\, year = 5^{3/2}\, years$.

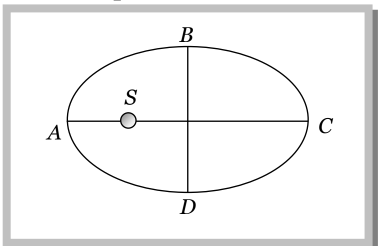
Problem 54. In planetary motion the areal velocity of position vector of a planet depends on angular velocity (ω) and the distance of the planet from sun (r). If so the correct relation for areal velocity is

(a) $\frac{dA}{dt} \propto \omega r$ (b) $\frac{dA}{dt} \propto \omega^2 r$ (c) $\frac{dA}{dt} \propto \omega r^2$ (d) $\frac{dA}{dt} \propto \sqrt{\omega r}$

Solution : (c) $\frac{dA}{dt} = \frac{L}{2m} = \frac{mvr}{2m} = \frac{1}{2}\omega r^2$ [As Angular momentum $L = mvr$ and $v = r\omega$]

$\therefore \frac{dA}{dt} \propto \omega r^2$.

Problem 55. The planet is revolving around the sun as shown in elliptical path. The correct option is **[UPSEAT 2002]**



(a) The time taken in travelling *DAB* is less than that for *BCD*

(b) The time taken in travelling *DAB* is greater than that for *BCD*

(c) The time taken in travelling *CDA* is less than that for *ABC*

(d) The time taken in travelling *CDA* is greater than that for *ABC*

Solution : (a) When the planet passes nearer to sun then it moves fast and vice-versa. Hence the time taken in travelling *DAB* is less than that for *BCD*.

Problem 56. The distance of Neptune and Saturn from sun are nearly 10^{13} and 10^{12} meters respectively. Assuming that they move in circular orbits, their periodic times will be in the ratio **[NCERT 1975; CBSE PMT 1994; MP PET 2**

(a) $\sqrt{10}$ (b) 100 (c) $10\sqrt{10}$ (d) $1/\sqrt{10}$

Solution : (c) Kepler's third law $T^2 \propto R^3$ $\therefore \frac{T_{Neptune}}{T_{Saturn}} = \left(\frac{R_{Neptune}}{R_{Saturn}}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10\sqrt{10}$.

Problem 57. The maximum and minimum distance of a comet from the sun are $8\times10^{12}\,m$ and $1.6\times10^{12}\,m$. If its velocity when nearest to the sun is 60 *m/s,* what will be its velocity in *m/s* when it is farthest

(a) 12 (b) 60 (c) 112 (d) 6

Solution : (a) According to conservation of angular momentum $mv_{\min}r_{\max} = mv_{\max}r_{\min}$ = constant

$$\therefore v_{\min} = v_{\max}\times\frac{r_{\min}}{r_{\max}} = 60\times\left(\frac{1.6\times10^{12}}{8\times10^{12}}\right) = 12m/s$$

Problem 58. A satellite *A* of mass *m* is at a distance of *r* from the centre of the earth. Another satellite *B* of mass 2*m* is at distance of 2*r* from the earth's centre. Their time periods are in the ratio of

(a) 1 : 2 (b) 1 : 16 (c) 1 : 32 (d) $1:2\sqrt{2}$

Solution : (d) Time period does not depend upon the mass of satellite, it only depends upon the orbital radius.

According to Kepler's law $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \frac{1}{2\sqrt{2}}$.

Problem 59. A planet moves around the sun. At a given point *P,* it is closed from the sun at a distance d_1 and has a speed v_1. At another point *Q*. when it is farthest from the sun at a distance d_2, its speed will be **[MP PMT 1987]**

(a) $\frac{d_1^2v_1}{d_2^2}$ (b) $\frac{d_2v_1}{d_1}$ (c) $\frac{d_1v_1}{d_2}$ (d) $\frac{d_2^2v_1}{d_1^2}$

Solution : (c) According to law of conservation of angular momentum $mv_1d_1 = mv_2d_2$ $\therefore v_2 = \frac{d_1v_1}{d_2}$.

8.21 Orbital Velocity of Satellite

Satellites are natural or artificial bodies describing orbit around a planet under its gravitational attraction. Moon is a natural satellite while INSAT-1B is an artificial satellite of earth. Condition for establishment of artificial satellite is that the centre of orbit of satellite must coincide with centre of earth or satellite must move around great circle of earth.

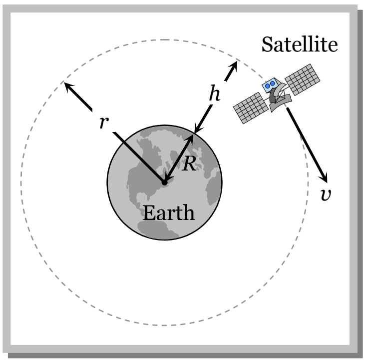
Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth.

For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}} \qquad [\text{As } GM = gR^2 \text{ and } r = R+h]$$



Important points

(i) Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit *i.e.,* satellites of deferent masses have same orbital velocity, if they are in the same orbit.

(ii) Orbital velocity depends on the mass of central body and radius of orbit.

(iii) For a given planet, greater the radius of orbit, lesser will be the orbital velocity of the satellite $\left(v \propto 1/\sqrt{r}\right)$.

(iv) Orbital velocity of the satellite when it revolves very close to the surface of the planet

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} \quad \therefore\ v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \qquad [\text{As } h = 0 \text{ and } GM = gR^2]$$

For the earth $v = \sqrt{9.8 \times 6.4 \times 10^6} = 7.9\, km/s \approx 8\, km/\sec$

(v) Close to the surface of planet $v = \sqrt{\frac{GM}{R}}$ [As $v_e = \sqrt{\frac{2GM}{R}}$]

$$\therefore \qquad v = \frac{v_e}{\sqrt{2}} \qquad i.e., \ v_{escape} = \sqrt{2}\ v_{orbital}$$

It means that if the speed of a satellite orbiting close to the earth is made $\sqrt{2}$ times (or increased by 41%) then it will escape from the gravitational field.

(vi) If the gravitational force of attraction of the sun on the planet varies as $F \propto \frac{1}{r^n}$ then the orbital velocity varies as $v \propto \frac{1}{\sqrt{r^n - 1}}$.

Sample problems based on orbital velocity

Problem 60. Two satellites A and B go round a planet P in circular orbits having radii $4R$ and R respectively. If the speed of the satellite A is $3V$, the speed of the satellite B will be

(a) 12 V (b) 6 V (c) 3/2 V (d) 3/2 V

Solution : (b) Orbital velocity of satellite $v = \sqrt{\frac{GM}{r}}$ $\therefore\ v \propto \frac{1}{\sqrt{r}} \Rightarrow \frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} \Rightarrow \frac{v_B}{3V} = \sqrt{\frac{4R}{R}} \Rightarrow v_B = 6V$.

Problem 61. A satellite is moving around the earth with speed v in a circular orbit of radius r. If the orbit radius is decreased by 1%, its speed will

(a) Increase by 1% (b) Increase by 0.5% (c) Decrease by 1% (d) Decrease by 0.5%

Solution : (b) Orbital velocity $v = \sqrt{\frac{Gm}{r}}$ $\therefore\ v \propto \frac{1}{\sqrt{r}}$ [If r decreases then v increases]

Percentage change in $v = \frac{1}{2}$ (Percentage change in r) $= \frac{1}{2}$ (1%) $= 0.5\%$ $\therefore$ orbital velocity increases by 0.5%.

Problem 62. If the gravitational force between two objects were proportional to $1/R$; where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to **[CBSE PMT 1994; JIPMER 2001, 02]**

(a) $1/R^2$ (b) R^0 (c) R^1 (d) $1/R$

Solution : (b) If $F \propto \frac{1}{R^n}$ then $v \propto \frac{1}{\sqrt{R^{n-1}}}$; here $n = 1$ $\therefore v \propto \frac{1}{\sqrt{R^{1-1}}} \propto R^0$.

Problem 63. The distance between centre of the earth and moon is 384000 km. If the mass of the earth is $6 \times 10^{24}\ kg$ and $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$. The speed of the moon is nearly **[MH CET 2002]**

(a) 1 km/sec (b) 4 km/sec (c) 8 km/sec (d) 11.2 km/sec

Solution : (a) Orbital velocity $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{38400 \times 10^3}}$ $v = 1.02\ km/sec = 1 km/sec$ (Approx.)

8.22 Time Period of Satellite

It is the time taken by satellite to go once around the earth.

$$\therefore \qquad T = \frac{\text{Circumference of the orbit}}{\text{orbital velocity}}$$

$\Rightarrow \quad T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}}$ [As $v = \sqrt{\frac{GM}{r}}$]

$\Rightarrow \quad T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{gR^2}}$ [As $GM = gR^2$]

$\Rightarrow \quad T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}\left(1+\frac{h}{R}\right)^{3/2}$ [As $r = R+h$]

Important points

(i) From $T = 2\pi\sqrt{\frac{r^3}{GM}}$ it is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit

(ii) $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\Rightarrow \quad T^2 = \frac{4\pi^2}{GM}r^3$ *i.e.*, $T^2 \propto r^3$

This is in accordance with Kepler's third law of planetary motion r becomes a (semi major axis) if the orbit is elliptic.

(iii) Time period of nearby satellite,

From $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$ [As $h = 0$ and $GM = gR^2$]

For earth $R = 6400\,km$ and $g = 9.8\,m/s^2$

$T = 84.6$ minute $\approx 1.4\ hr$

(iv) Time period of nearby satellite in terms of density of planet can be given as

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{GM}} = \frac{2\pi(R^3)^{1/2}}{\left[G.\frac{4}{3}\pi R^3\rho\right]^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$$

(v) If the gravitational force of attraction of the sun on the planet varies as $F \propto \frac{1}{r^n}$ then the time period varies as $T \propto r^{\frac{n+1}{2}}$

(vi) If there is a satellite in the equatorial plane rotating in the direction of earth's rotation from west to east, then for an observer, on the earth, angular velocity of satellite will be $(\omega_S - \omega_E)$. The time interval between the two consecutive appearances overhead will be

$$T = \frac{2\pi}{\omega_S - \omega_E} = \frac{T_S T_E}{T_E - T_S} \qquad \left[\text{As } T = \frac{2\pi}{\omega}\right]$$

If $\omega_S = \omega_E$, $T = \infty$ *i.e.* satellite will appear stationary relative to earth. Such satellites are called geostationary satellites.

Sample problems based on time period

Problem 64. A satellite is launched into a circular orbit of radius 'R' around earth while a second satellite is launched into an orbit of radius 1.02 R. The percentage difference in the time periods of the two satellites is **[EAMCET 2003]**

(a) 0.7 (b) 1.0 (c) 1.5 (d) 3

Solution : (d) Orbital radius of second satellite is 2% more than first satellite

So from $T \propto (r)^{3/2}$, Percentage increase in time period $= \frac{3}{2}$ (Percentage increase in orbital radius)

$$= \frac{3}{2}(2\%) = 3\%.$$

Problem 65. Periodic time of a satellite revolving above Earth's surface at a height equal to R, where R the radius of Earth, is [g is acceleration due to gravity at Earth's surface]

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $4\sqrt{2}\pi\sqrt{\frac{R}{g}}$ (c) $2\pi\sqrt{\frac{R}{g}}$ (d) $8\pi\sqrt{\frac{R}{g}}$

Solution : (b) $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+R)^3}{gR^2}} = 2\pi\sqrt{\frac{8R}{g}} = 4\sqrt{2}\pi\sqrt{\frac{R}{g}}$ [As $h = R$ (given)].

Problem 66. An earth satellite S has an orbit radius which is 4 times that of a communication satellite C. The period of revolution of S is **[MP PMT 1994; DCE 1999]**

(a) 4 days (b) 8 days (c) 16 days (d) 32 days

Solution : (b) Orbital radius of satellite $r_s = 4r_c$ (given)

From Kepler's law $T \propto r^{3/2}$ $\therefore$ $\frac{T_s}{T_c} = \left(\frac{r_s}{s_c}\right)^{3/2} = (4)^{3/2} \Rightarrow T_s = 8T_c = 8 \times 1$ day = 8 days.

Problem 67. One project after deviation from its path, starts moving round the earth in a circular path at radius equal to nine times the radius at earth R, its time period will be

(a) $2\pi\sqrt{\frac{R}{g}}$ (b) $27 \times 2\pi\sqrt{\frac{R}{g}}$ (c) $\pi\sqrt{\frac{R}{g}}$ (d) $8 \times 2\pi\sqrt{\frac{R}{g}}$

Solution : (b) $T = 2\pi\sqrt{\frac{r^3}{gR^2}} = 2\pi\sqrt{\frac{(9R)^3}{gR^2}} = 2\pi(9)^{3/2}\sqrt{\frac{R}{g}} = 27 \times 2\pi\sqrt{\frac{R}{g}}$ [As $r = 9R$ (given)].

Problem 68. A satellite A of mass m is revolving round the earth at a height 'r' from the centre. Another satellite B of mass $2m$ is revolving at a height $2r$. The ratio of their time periods will be **[CBSE PMT 1993]**

(a) 1 : 2 (b) 1 : 16 (c) 1 : 32 (d) $1:2\sqrt{2}$

Solution : (d) Time period depends only upon the orbital radius $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \frac{1}{2\sqrt{2}}$.

8.23 Height of Satellite.

As we know, time period of satellite $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

By squaring and rearranging both sides $\frac{gR^2T^2}{4\pi^2} = (R+h)^3$

$$\Rightarrow \quad h = \left(\frac{T^2gR^2}{4\pi^2}\right)^{1/3} - R$$

By knowing the value of time period we can calculate the height of satellite the surface of the earth.

Sample problems based on height

***Problem* 69.** Given radius of earth 'R' and length of a day 'T' the height of a geostationary satellite is

[G – Gravitational constant, M – Mass of earth] **[MP PMT 2002]**

(a) $\left(\frac{4\pi^2 GM}{T^2}\right)^{1/3}$ (b) $\left(\frac{4\pi GM}{R^2}\right)^{1/3} - R$ (c) $\left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$ (d) $\left(\frac{GMT^2}{4\pi^2}\right)^{1/3} + R$

Solution : (c) From the expression $h = \left(\frac{T^2 gR^2}{4\pi^2}\right)^{1/3} - R \quad \therefore \; h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$ [As $gR^2 = GM$]

***Problem* 70.** A satellite is revolving round the earth in circular orbit at some height above surface of earth. It takes 5.26×10^3 seconds to complete a revolution while its centripetal acceleration is $9.92\, m/s^2$. Height of satellite above surface of earth is (Radius of earth $6.37 \times 10^6\ m$) **[MP PET 1993]**

(a) 70 km (b) 120 km (c) 170 km (d) 220 km

Solution : (c) Centripetal acceleration $(a_c) = \frac{v^2}{r}$ and $T = \frac{2\pi r}{v}$

From equation (i) and (ii) $r = \frac{a_c T^2}{4\pi^2} \Rightarrow R + h = \frac{9.32 \times (5.26 \times 10^3)^2}{4 \times \pi^2}$

$h = 6.53 \times 10^6 - R = 6.53 \times 10^6 - 6.37 \times 10^6 = 160 \times 10^3 m = 160 km \approx 170 km$.

8.24 Geostationary Satellite.

The satellite which appears stationary relative to earth is called geostationary or geosynchronous satellite, communication satellite.

A geostationary satellite always stays over the same place above the earth such a satellite is never at rest. Such a satellite appears stationary due to its zero relative velocity *w.r.t.* that place on earth.

The orbit of a geostationary satellite is known as the parking orbit.

Important points

(i) It should revolve in an orbit concentric and coplanar with the equatorial plane.

(ii) It sense of rotation should be same as that of earth about its own axis *i.e.,* in anti-clockwise direction (from west to east).

(iii) Its period of revolution around the earth should be same as that of earth about its own axis.

$$\therefore \; T = 24 \ hr = 86400 \ \text{sec}$$

(iv) Height of geostationary satellite

As $\quad T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 24\, hr$

Substituting the value of G and M we get $R + h = r = 42000 \ km = 7R$

$\therefore$ height of geostationary satellite from the surface of earth $h = 6R = 36000 \ km$

(v) Orbital velocity of geo stationary satellite can be calculated by $v = \sqrt{\frac{GM}{r}}$

Substituting the value of G and M we get $v = 3.08\ km/\sec$

8.25 Angular Momentum of Satellite

Angular momentum of satellite $L = mvr$

$\Rightarrow \qquad L = m\sqrt{\frac{GM}{r}}\, r \qquad$ [As $v = \sqrt{\frac{GM}{r}}$]

$\therefore \qquad L = \sqrt{m^2 GMr}$

i.e., Angular momentum of satellite depend on both the mass of orbiting and central body as well as the radius of orbit.

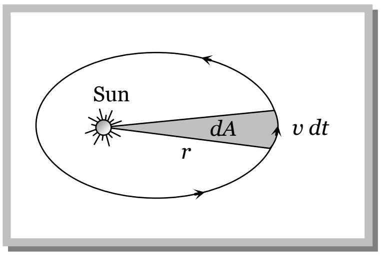
Important points

(i) In case of satellite motion, force is central so torque = 0 and hence angular momentum of satellite is conserved *i.e.*, $L =$ constant

(ii) In case of satellite motion as areal velocity

$$\frac{dA}{dt} = \frac{1}{2}\frac{(r)(vdt)}{dt} = \frac{1}{2}rv$$

$\Rightarrow \qquad \frac{dA}{dt} = \frac{L}{2m} \qquad$ [As $L = mvr$]



But as $L =$ constant, $\therefore$ areal velocity (dA/dt) = constant which is Kepler's II law

i.e., Kepler's II law or constancy of areal velocity is a consequence of conservation of angular momentum.

Sample problems based on angular momentum

***Problem* 71.** The orbital angular momentum of a satellite revolving at a distance r from the centre is L. If the distance is increased to $16r$, then new angular momentum will be

(a) $16\,L$ (b) $64\,L$ (c) $\frac{L}{4}$ (d) $4\,L$

Solution : (d) Angular momentum $L = \sqrt{m^2 GMr}$ $\therefore\ L \propto \sqrt{r}$

$$\frac{L_2}{L_1} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{16r}{r}} = 4$$

$L_2 = 4L_1 = 4L$

***Problem* 72.** Angular momentum of a planet of mass m orbiting around sun is J, areal velocity of its radius vector will be

(a) $\frac{1}{2}mJ$ (b) $\frac{J}{2m}$ (c) $\frac{m}{2J}$ (d) $\frac{1}{2mJ}$

Solution : (b)

8.26 Energy of Satellite

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of earth) and kinetic energy (due to orbital motion).

(1) Potential energy : $U = mV = \frac{-GMm}{r} = \frac{-L^2}{mr^2}$ $\left[\text{As } V = \frac{-GM}{r}, L^2 = m^2GMr\right]$

(2) Kinetic energy : $K = \frac{1}{2}mv^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$ $\left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$

(3) Total energy : $E = U + K = \frac{-GMm}{r} + \frac{GMm}{2r} = \frac{-GMm}{2r} = \frac{-L^2}{2mr^2}$

Important points

(i) Kinetic energy, potential energy or total energy of a satellite depends on the mass of the satellite and the central body and also on the radius of the orbit.

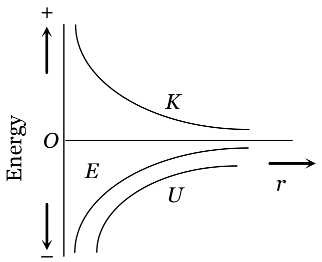
(ii) From the above expressions we can say that

Kinetic energy (K) = – (Total energy)

Potential energy (U) = 2 (Total energy)

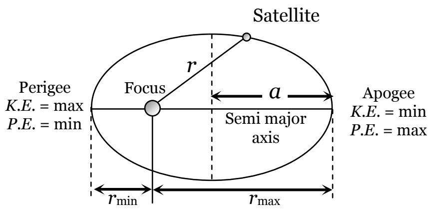
Potential energy (K) = – 2 (Kinetic energy)

(iii) Energy graph for a satellite



(A)

(iv) Energy distribution in elliptical orbit



(B)

(v) If the orbit of a satellite is elliptic then

(a) Total energy $(E) = \frac{-GMm}{2a}$ = constant ; where a is semi-major axis .

(b) Kinetic energy (K) will be maximum when the satellite is closest to the central body (at perigee) and maximum when it is farthest from the central body (at apogee)

(c) Potential energy (U) will be minimum when kinetic energy = maximum i.e., the satellite is closest to the central body (at perigee) and maximum when kinetic energy = minimum i.e., the satellite is farthest from the central body (at apogee).

(vi) Binding Energy : Total energy of a satellite in its orbit is negative. Negative energy means that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity. The energy required to remove the satellite from its orbit to infinity is called Binding Energy of the system, *i.e.*,

Binding Energy (B.E.) $= -E = \frac{GMm}{2r}$

8.Change in the orbit of Satelite

When the satellite is transferred to a higher orbit $(r_2 > r_1)$ then variation in different quantities can be shown by the following table

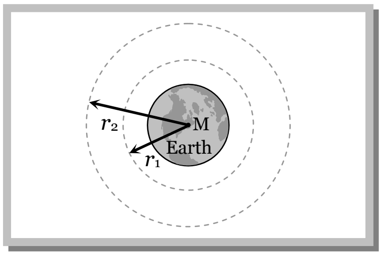
Quantities	Variation	Relation with *r*
Orbital velocity	Decreases	$v \propto \frac{1}{\sqrt{r}}$
Time period	Increases	$T \propto r^{3/2}$

Linear momentum	Decreases	$P \propto \frac{1}{\sqrt{r}}$
Angular momentum	Increases	$L \propto \sqrt{r}$
Kinetic energy	Decreases	$K \propto \frac{1}{r}$
Potential energy	Increases	$U \propto -\frac{1}{r}$
Total energy	Increases	$E \propto -\frac{1}{r}$
Binding energy	Decreases	$BE \propto \frac{1}{r}$

Note: ❑ Work done in changing the orbit $W = E_2 - E_1$

$$W = \left(-\frac{GMm}{2r_2}\right) - \left(-\frac{GMm}{2r_1}\right)$$

$$W = \frac{GMm}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$



Sample problems based on Energy

Problem 73. Potential energy of a satellite having mass 'm' and rotating at a height of $6.4 \times 10^6\ m$ from the earth centre is

[AIIMS 2000; CBSE PMT 2001; BHU 2001]

(a) $-0.5\, mgR_e$ (b) $-mgR_e$ (c) $-2\, mgR_e$ (d) $4\, mgR_e$

Solution : (a) Potential energy $= -\frac{GMm}{r} = -\frac{GMm}{R_e + h} = -\frac{GMm}{2R_e}$ [As $h = R_e$ (given)]

$\therefore$ Potential energy $= -\frac{gR_e^2 m}{2R_e} = -0.5 mgR_e$ [As $GM = gR^2$]

Problem 74. In a satellite if the time of revolution is T, then kinetic energy is proportional to

(a) $\frac{1}{T}$ (b) $\frac{1}{T^2}$ (c) $\frac{1}{T^3}$ (d) $T^{-2/3}$

Solution : (d) Time period $T \propto r^{3/2} \Rightarrow r \propto T^{2/3}$ and Kinetic energy $\propto \frac{1}{r} \propto \frac{1}{T^{2/3}} \propto T^{-2/3}$.

Problem 75. Two satellites are moving around the earth in circular orbits at height R and $3R$ respectively, R being the radius of the earth, the ratio of their kinetic energies is

(a) 2 (b) 4 (c) 8 (d) 16

Solution : (a) $r_1 = R + h_1 = R + R = 2R$ and $r_2 = R + h_2 = R + 3R = 4R$

Kinetic energy $\propto \frac{1}{r}$ $\therefore \frac{(KE)_1}{(KE)_2} = \frac{r_2}{r_1} = \frac{4R}{2R} = \frac{2}{1}$.

8.28 Weightlessness

The weight of a body is the force with which it is attracted towards the centre of earth. When a body is stationary with respect to the earth, its weight equals the gravity. This weight of the body is known as its static or true weight.

We become conscious of our weight, only when our weight (which is gravity) is opposed by some other object. Actually, the secret of measuring the weight of a body with a weighing machine lies in the fact that as we place the body on the machine, the weighing machine opposes the weight of the body. The reaction of the weighing machine to the body gives the measure of the weight of the body.

The state of weightlessness can be observed in the following situations.

(1) **When objects fall freely under gravity** : For example, a lift falling freely, or an airship showing a feat in which it falls freely for a few seconds during its flight, are in state of weightlessness.

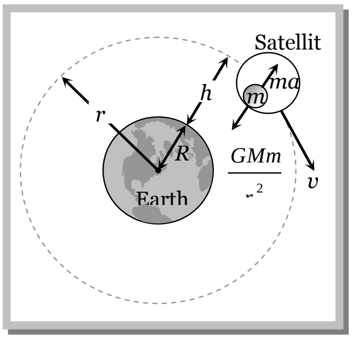
(2) **When a satellite revolves in its orbit around the earth** : Weightlessness poses many serious problems to the astronauts. It becomes quite difficult for them to control their movements. Everything in the satellite has to be kept tied down. Creation of artificial gravity is the answer to this problem.

(3) **When bodies are at null points in outer space** : On a body projected up, the pull of the earth goes on decreasing, but at the same time the gravitational pull of the moon on the body goes on increasing. At one particular position, the two gravitational pulls may be equal and opposite and the net pull on the body becomes zero. This is zero gravity region or the null point and the body in question is said to appear weightless.

8.29 Weightlessness in a Satellite

A satellite, which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. The acceleration of satellite is $\frac{GM}{r^2}$ towards the centre of earth.

If a body of mass m placed on a surface inside a satellite moving around the earth. Then force on the body are



(i) The gravitational pull of earth $= \frac{GMm}{r^2}$

(ii) The reaction by the surface $= R$

By Newton's law $\frac{GmM}{r^2} - R = m\ a$

$$\frac{GmM}{r^2} - R = m\left(\frac{GM}{r^2}\right) \qquad \therefore \qquad R = 0$$

Thus the surface does not exert any force on the body and hence its apparent weight is zero.

A body needs no support to stay at rest in the satellite and hence all position are equally comfortable. Such a state is called weightlessness.

Important points

(i) One will find it difficult to control his movement, without weight he will tend to float freely. To get from one spot to the other he will have to push himself away from the walls or some other fixed objects.

(ii) As everything is in free fall, so objects are at rest relative to each other, *i.e.*, if a table is withdrawn from below an object, the object will remain where it was without any support.

(iii) If a glass of water is tilted and glass is pulled out, the liquid in the shape of container will float and will not flow because of surface tension.

(iv) If one tries to strike a match, the head will light but the stick will not burn. This is because in this situation convection currents will not be set up which supply oxygen for combustion

(v) If one tries to perform simple pendulum experiment, the pendulum will not oscillate. It is because there will not be any restoring torque and so $T = 2\pi\sqrt{(L/g')} = \infty$. [As $g' = 0$]

(vi) Condition of weightlessness can be experienced only when the mass of satellite is negligible so that it does not produce it own gravity.

e.g. Moon is a satellite of earth but due to its own weight it applies gravitational force of attraction on the body placed on its surface and hence weight of the body will not be equal to zero at the surface of the moon.

Sample problems based on weightlessness in satellite

Problem 76. The time period of a simple pendulum on a freely moving artificial satellite is

(a) Zero (b) 2 *sec* (c) 3 *sec* (d) Infinite

Solution : (d) $T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{0}} = \infty$ [As $g' = 0$ in the satellite]

Problem 77. The weight of an astronaut, in an artificial satellite revolving around the earth, is

(a) Zero (b) Equal to that on the earth

(c) More than that on the earth (d) Less than that on the earth

Solution : (a)

Problem 78. A ball is dropped from a spacecraft revolving around the earth at a height of 120 *km*. What will happen to the ball

(a) It will continue to move with velocity *v* along the original orbit of spacecraft

(b) If will move with the same speed tangentially to the spacecraft

(c) It will fall down to the earth gradually

(d) It will go very far in the space

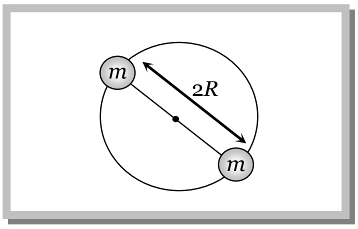
Solution : (a) Because ball possess same initial tangential speed as that of space craft.. So it also feels the condition of weightlessness.

Sample problems (Miscellaneous)

Problem 79. Two particles of equal mass go round a circle of radius *R* under the action of their mutual gravitational attraction. The speed of each particle is **[CBSE PMT 1995]**

(a) $v = \frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (b) $v = \sqrt{\frac{Gm}{2R}}$ (c) $v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$ (d) $v = \sqrt{\frac{4Gm}{R}}$

Solution : (c) Both the particles moves diametrically opposite position along the circular path of radius *R* and the gravitational force provides required centripetal force



$$\frac{mv^2}{R} = \frac{Gmm}{(2R)^2} \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

Problem 80. Two types of balances, the beam balance and the spring balance are commonly used for measuring weight in shops. If we are on the moon, we can continue to use

(a) Only the beam type balance without any change

(b) Only the spring balance without any change

(c) Both the balances without any change

(d) Neither of the two balances without making any change

Solution : (a) Because in beam type balance effect of less gravitation force works on both the Pans. So it is neutralizes but in spring balance weight of the body decreases so apparent weight varies with actual weight.

Problem 81. During a journey from earth to the moon and back, the greatest energy required from the space-ship rockets is to overcome **[CBSE PMT 1991]**

(a) The earth's gravity at take off

(b) The moon's gravity at lunar landing

(c) The moon's gravity at lunar take off

(d) The point where the pull of the earth and moon are equal but opposite

Solution : (a)

Problem 82. If the radius of earth contracts $\frac{1}{n}$ of its present value, the length of the day will be approximately

(a) $\frac{24}{n}h$ (b) $\frac{24}{n^2}h$ (c) $24\,n\,h$ (d) $24\,n^2\,h$

Solution : (b) Conservation of angular momentum $L = I\omega = = \frac{2}{5}MR^2 \times \frac{2\pi}{T}$ = constant $\therefore\ T \propto R^2$ [If M remains same]

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^2 = \left(\frac{R/n}{R}\right)^2 = \frac{1}{n^2} \Rightarrow T_2 = \frac{24}{n^2}hr \qquad [\text{As } T_1 = 24\,hr].$$

Problem 83. A body released from a height h takes time t to reach earth's surface. The time taken by the same body released from the same height to reach the moon's surface is

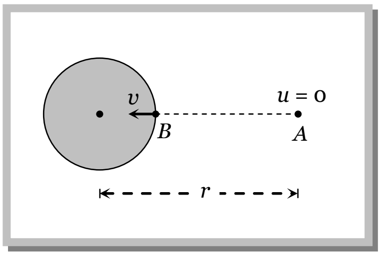
(a) t (b) $6t$ (c) $\sqrt{6}t$ (d) $\frac{t}{6}$

Solution : (c) If body falls from height h then time of descent $t = \sqrt{\frac{2h}{g}} \Rightarrow \frac{t_{moon}}{t_{earth}} = \sqrt{\frac{g_{earth}}{g_{moon}}} = \sqrt{6} \Rightarrow t_{moon} = \sqrt{6}\ t$.

Problem 84. A satellite is revolving round the earth with orbital speed v_0. If it stops suddenly, the speed with which it will strike the surface of earth would be (v_e = escape velocity of a particle on earth's surface)

(a) $\frac{v_e^2}{v_0}$ (b) v_0 (c) $\sqrt{v_e^2 - v_0^2}$ (d) $\sqrt{v_e^2 - 2v_0^2}$

Solution : (d) Applying conservation of mechanical energy between A and B point



$$-\frac{GMm}{r} = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right);\ \frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{r}$$

$$v^2 = \frac{2Gm}{R} - \frac{2Gm}{r} = v_e^2 - 2v_0^2 \Rightarrow v = \sqrt{v_e^2 - 2v_0^2}$$

[As escape velocity $v_e = \sqrt{\frac{2Gm}{R}}$, orbital velocity $v_0 = \sqrt{\frac{Gm}{r}}$]

Problem 85. The escape velocity for a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(a) v_e (b) $\frac{v_e}{\sqrt{2}}$ (c) $\frac{v_e}{2}$ (d) Zero

Solution : (b) Gravitational potential at the surface of the earth $V_s = -\frac{GM}{R}$

Gravitational potential at the centre of earth $V_c = -\frac{3GM}{2R}$

By the conservation of energy $\frac{1}{2}mv^2 = m(V_s - V_c)$

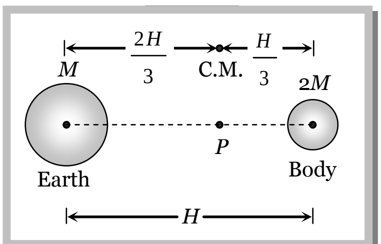
$$v^2 = 2\frac{GM}{R}\left(\frac{3}{2} - 1\right) = \frac{GM}{R} = gR = \frac{v_e^2}{2} \qquad [\text{As } v_e = \sqrt{2gR}\,]$$

$$\therefore\ v = \frac{v_e}{\sqrt{2}}$$

Problem 86. A small body of superdense material, whose mass is twice the mass of the earth but whose size is very small compared to the size of the earth, starts from rest at a height $H << R$ above the earth's surface, and reaches the earth's surface in time t. Then t is equal to

(a) $\sqrt{2H/g}$ (b) $\sqrt{H/g}$ (c) $\sqrt{2H/3g}$

Solution : (c) As the masses of the body and the earth are comparable, they will move towards their centre of mass, which remains stationary.



Hence the body of mass $2m$ move through distance $\frac{H}{3}$.

and time to reach the earth surface $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2H/3}{g}} = \sqrt{\frac{2H}{3g}}$

Practice Problems

Problems based on Newton's law of gravitation

▶ *Basic level*

1. The force of gravitation is **[AIIMS 2002]**

(a) Repulsive (b) Electrostatic (c) Conservative (d) Non - conservative

2. If the distance between two masses is doubled, the gravitational attraction between them **[CPMT 1973; AMU (Med.) 2000]**

(a) Is doubled (b) Becomes four times (c) Is reduced to half (d) Is reduced to a quarter

3. A mass M is split into two parts, m and $M - m$, which are then separated by a certain distance. What ratio of m/M maximizes the gravitational force between the two parts **[AMU 2000]**

(a) 1/3 (b) 1/2 (c) 1/4 (d) 1/5

4. Three particles each of mass m are placed at the three corners of an equilateral triangle. The centre of the triangle is at a distance x from either corner. If a mass M be placed at the centre, what will be the net gravitational force on it

(a) Zero (b) $3GMm/x^2$ (c) $2GMm/x^2$ (d) GMm/x^2

5. Two identical spheres are placed in contact with each other. The force of gravitation between the spheres will be proportional to (R = radius of each sphere)

(a) R (b) R^2 (c) R^4 (d) None of these

▶▶ *Advance level*

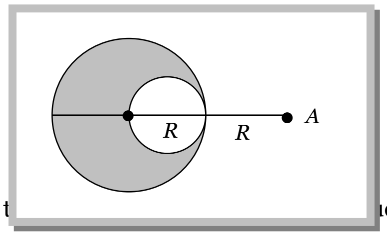
6. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at A, distance $2R$ from the centre of the sphere. A spherical cavity of radius $R/2$ is now made in the sphere as shown in the figure. The sphere with cavity now applies a gravitational force F_2 on the same particle placed at A. The ratio F_2/F_1 will be

(a) 1/2

(b) 3

(c) 7

(d) 7/9



7. Three uniform spheres of mass M and radius R each are kept in such a way that each t … de of the gravitational force on any of the spheres due to the other two is

(a) $\frac{\sqrt{3}}{4}\frac{GM^2}{R^2}$ (b) $\frac{3}{2}\frac{GM^2}{R^2}$ (c) $\frac{\sqrt{3}GM^2}{R^2}$ (d) $\frac{\sqrt{3}}{2}\frac{GM^2}{R^2}$

8. A mass of 10*kg* is balanced on a sensitive physical balance. A 1000 *kg* mass is placed below 10 *kg* mass at a distance of 1*m*. How much additional mass will be required for balancing the physical balance

(a) $66 \times 10^{-15} kg$ (b) $6.7 \times 10^{-8} kg$ (c) $66 \times 10^{-12} kg$ (d) $6.7 \times 10^{-6} kg$

Problems based on acceleration due to gravity

▶ *Basic level*

9. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is **[CPMT 1990; CBSE 1995; BHU 1998; MH CET (Med.) 1999; Kerala PMT 2002]**

(a) $4\pi G / 3gR$ (b) $3\pi R / 4gG$ (c) $3g / 4\pi RG$ (d) $\pi Rg / 12G$

10. A mass 'm' is taken to a planet whose mass is equal to half that of earth and radius is four times that of earth. The mass of the body on this planet will be **[RPMT 1989, 97]**

(a) $m/2$ (b) $m/8$ (c) $m/4$ (d) m

11. The diameters of two planets are in the ratio 4 : 1 and their mean densities in the ratio 1: 2. The acceleration due to gravity on the planets will be in ratio **[ISM Dhanbad 1994]**

(a) 1 : 2 (b) 2 : 3 (c) 2 : 1 (d) 4 : 1

12. The acceleration due to gravity on the moon is only one sixth that of earth. If the earth and moon are assumed to have the same density, the ratio of the radii of moon and earth will be

(a) $\frac{1}{6}$ (b) $\frac{1}{(6)^{1/3}}$ (c) $\frac{1}{36}$ (d) $\frac{1}{(6)^{2./3}}$

▶▶ Advance level

13. Let g be the acceleration due to gravity at earth's surface and K be the rotational kinetic energy of the earth. Suppose the earth's radius decreases by 2% keeping all other quantities same, then **[BHU 1994; JIPMER 2000]**

(a) g decreases by 2% and K decreases by 4% (b) g decreases by 4% and K increases by 2%

(c) g increases by 4% and K decreases by 4% (d) g decreases by 4% and K increase by 4%

14. Clock A based on spring oscillations and a clock B based on oscillations of simple pendulum are synchronised on earth. Both are taken to mars whose mass is 0.1 times the mass of earth and radius is half that of earth. Which of the following statement is correct

(a) Both will show same time

(b) Time measured in clock A will be greater than that in clock B

(c) Time measured in clock B will be greater than that in clock A

(d) Clock A will stop and clock B will show time as it shows on earth

Problems based on variation in g with height

▶ Basic level

15. A body weight W *Newton* at the surface of the earth. Its weight at a height equal to half the radius of the earth will be **[UPSEAT 2002]**

(a) $\frac{W}{2}$ (b) $\frac{2W}{3}$ (c) $\frac{4W}{9}$ (d) $\frac{8W}{27}$

16. The value of g on the earth's surface is 980 cm/sec^2. Its value at a height of 64 km from the earth's surface is (Radius of the earth R = 6400 *Kilometers*) **[MP PMT 1995]**

(a) $960.40\,cm/\sec^2$ (b) $984.90\,cm/\sec^2$ (c) $982.45\,cm/\sec^2$ (d) $977.55\,cm/\sec^2$

17. The decrease in the value of g at height h from earth's surface is

(a) $\frac{2h}{R}$ (b) $\frac{2h}{R}g$ (c) $\frac{h}{R}g$ (d) $\frac{R}{2hg}$

▶▶ Advance level

18. A simple pendulum has a time period T_1 when on earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of earth. The value of T_2/T_1 is **[IIT-JEE (Screening) 2001]**

(a) 1 (b) $\sqrt{2}$ (c) 4 (d) 2

19. A pendulum clock is set to give correct time at the sea level. This clock is moved to hill station at an altitude of 2500m above the sea level. In order to keep correct time of the hill station, the length of the pendulum **[SCRA 1994]**

(a) Has to be reduced (b) Has to be increased

(c) Needs no adjustment (d) Needs no adjustment but its mass has to be increased

20. Which of the following correctly indicates the approximate effective values of g on various parts of a journey to the moon (values are in $metres/sec^2$)

Before take off from earth	One minute after lift – off	In earth orbit	on the moon

(a)	9.80	9.80	0	1.6
(b)	9.80	0.98	0	1.6
(c)	9.80	0.00	0	9.8×6
(d)	9.80	7.00	0	1.6

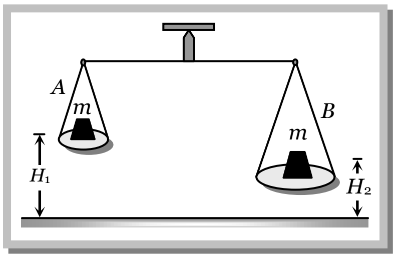
21. Two blocks of masses m each are hung from a balance. The scale pan A is at height H_1 whereas scale pan B is at height H_2. The error in weighing when $H_1 > H_2$ and R being the radius of earth is

(a) $mg\left(\frac{1-2H_1}{R}\right)$

(b) $2mg\left(\frac{H_1}{R}-\frac{H_2}{R}\right)$

(c) $2mg\left(\frac{H_2}{R}-\frac{H_1}{R}\right)$

(d) $2mg\,\frac{H_2H_1}{H_1+H_2}$



Problems based on variation in g with depth

▶ Basic level

22. If the value of 'g' acceleration due to gravity, at earth surface is $10\,m/s^2$, its value in m/s^2 at the centre of the earth, which is assumed to be a sphere of radius 'R' *metre* and uniform mass density is **[AIIMS 2002]**

(a) 5 (b) $10/R$ (c) 10/2R (d) Zero

23. The loss in weight of a body taken from earth's surface to a height h is 1%. The change in weight taken into a mine of depth h will be

(a) 1% loss (b) 1% gain (c) 0.5% gain (d) 0.5% loss

24. The weight of body at earth's surface is W. At a depth half way to the centre of the earth, it will be (assuming uniform density in earth)

(a) W (b) $W/2$ (c) $W/4$ (d) $W/8$

▶▶ Advance level

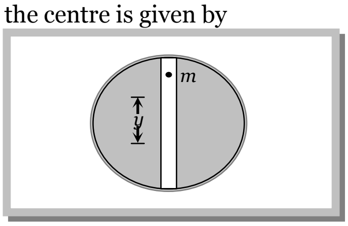
25. A particle would take a time t to move down a straight tunnel from the surface of earth (supposed to be a homogeneous sphere) to its centre. If gravity were to remain constant this time would be t'. The ratio of $\frac{t}{t'}$ will be

(a) $\frac{\pi}{2\sqrt{2}}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{\sqrt{3}}$

26. Suppose a vertical tunnel is dug along the diameter of earth assumed to be a sphere of uniform mass having density ρ. If a body of mass m is thrown in this tunnel, its acceleration at a distance y from the centre is given by

(a) $\frac{4\pi}{3}G\rho y m$ (b) $\frac{3}{4}\pi G\rho y$

(c) $\frac{4}{3}\pi\rho y$ (d) $\frac{4}{3}\pi G\rho y$



27. A tunnel is dug along the diameter of the earth. If a particle of mass m is situated in the tunnel at a distance x from the centre of earth then gravitational force acting on it, will be

(a) $\frac{GM_e m}{R_e^3}x$ (b) $\frac{GM_e m}{R_e^2}$ (c) $\frac{GM_e m}{x^2}$ (d) $\frac{GM_e m}{(R_e+x)^2}$

Problems based on variation in g due to shape of the earth

▶ Basic level

28. The acceleration due to gravity at pole and equator can be related as **[DPMT 2002]**

(a) $g_p < g_e$ (b) $g_p = g_e = g$ (c) $g_p = g_e < g$ (d) $g_p > g_e$

29. Weight of a body is maximum at **[AFMC 2001]**

(a) Moon (b) Poles of earth (c) Equator of earth (d) Centre of earth

30. The value of 'g' at a particular point is $9.8 m/s^2$. Suppose the earth suddenly shrinks uniformly to half its present size without losing any mass. The value of 'g' at the same point (assuming that the distance of the point from the centre of earth does not shrink) will now be **[NCERT 1984; DPMT 1999]**

(a) $4.9 m/\sec^2$ (b) $3.1 m/\sec^2$ (c) $9.8 m/\sec^2$ (d) $19.6 m/\sec^2$

▶▶ *Advance level*

31. The acceleration due to gravity increases by 0.5% when we go from the equator to the poles. What will be the time period of the pendulum at the equator which beats seconds at the poles

(a) 1.950 s (b) 1.995 s (c) 2.050 s (d) 2.005 s

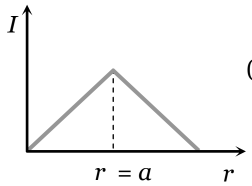
Problems based on gravitational field

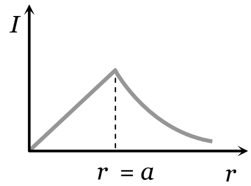
▶ *Basic level*

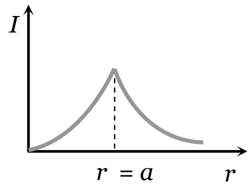
32. There are two bodies of masses 100 kg and 10000 kg separated by a distance 1m. At what distance from the smaller body, the intensity of gravitational field will be zero **[BHU 1997]**

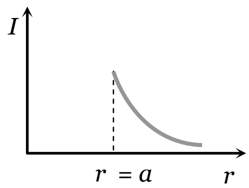
(a) $\frac{1}{9} m$ (b) $\frac{1}{10} m$ (c) $\frac{1}{11} m$ (d) $\frac{10}{11} m$

33. Which one of the following graphs represents correctly the variation of the gravitational field (F) with the distance (r) from the centre of a spherical shell of mass M and radius a

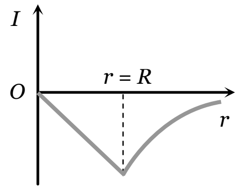
(a) 

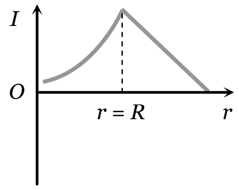
(b) 

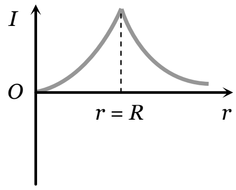
(c) 

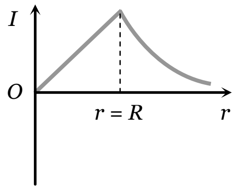
(d) 

34. The curve depicting the dependence of intensity of gravitational field on the distance r from the centre of the earth is

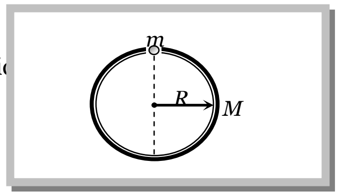
(a) 

(b) 

(c) 

(d) 

35. A thin spherical shell of mass M and radius R has a small hole. A particle of mass m is released at the mouth of the hole. Then



(a) The particle will execute simple harmonic motion inside the shell

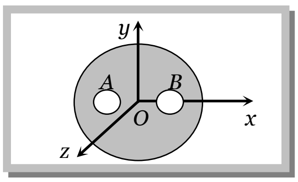
(b) The particle will oscillate inside the small, but the oscillations are not simple harmonic

(c) The particle will not oscillate, but the speed of the particle will go on increasing

(d) None of these

▶▶ *Advance level*

36. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit with their centres at $A(-2, 0, 0)$ and $B(2, 0, 0)$ respectively are taken out of the solid leaving behind spherical cavities as shown in figure **[IIT-JEE 1993]**



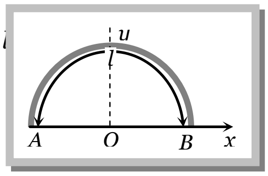
(a) The gravitational force due to this object at the origin is zero

(b) The gravitational force at the point $B(2, 0, 0)$ is zero

(c) The gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$

(d) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$

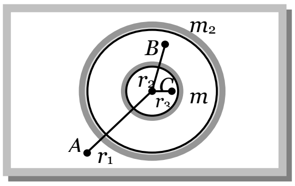
37. Gravitational field at the centre of a semicircle formed by a thin wire AB of mass m and length l



(a) $\frac{Gm}{l}$ along x axis (b) $\frac{Gm}{\pi l}$ along y axis

(c) $\frac{2\pi Gm}{l^2}$ along x axis (d) $\frac{2\pi Gm}{l^2}$ along y axis

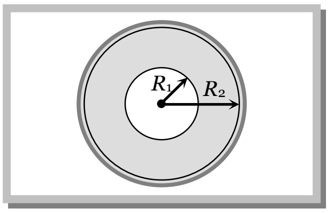
38. Two concentric shells of different masses m_1 and m_2 are having a sliding particle of mass m. The forces on the particle at position A, B and C are

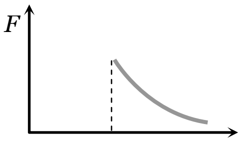


(a) $0, \frac{Gm_1}{r_2^2}, \frac{G(m_1+m_2)m}{r_1^2}$ (b) $\frac{Gm_2}{r_2^2}, 0, \frac{Gm_1}{r_1^2}$

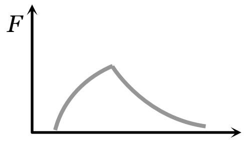
(c) $\frac{G(m_1+m_2)m}{r_1^2}, \frac{Gm_2}{r_2^2}, 0$ (d) $\frac{G(m_1+m_2)m}{r_1^2}, \frac{Gm_1}{r_2^2}, 0$

39. A sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in figure. The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as $(0 \le r \le \infty)$

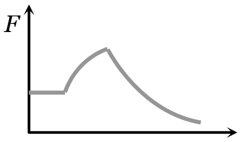




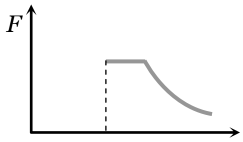
(a)



(b)

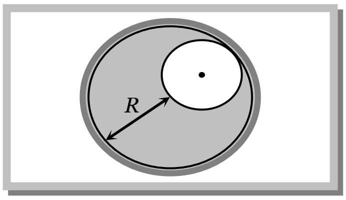


(c)



(d)

40. A spherical hole is made in a solid sphere of radius R. The mass of the sphere before hollowing was M. The gravitational field at the centre of the hole due to the remaining mass is



(a) Zero (b) $\frac{GM}{8R^2}$

(c) $\frac{GM}{2R^2}$ (d) $\frac{GM}{R^2}$

41. A point P lies on the axis of a ring of mass M and radius a, at a distance a from its centre C. A small particle starts from P and reaches C under gravitational attraction only. Its speed at C will be

(a) $\sqrt{\frac{2GM}{a}}$ (b) $\sqrt{\frac{2GM}{a}\left(1-\frac{1}{\sqrt{2}}\right)}$ (c) $\sqrt{\frac{2GM}{a}(\sqrt{2}-1)}$ (d) Zero

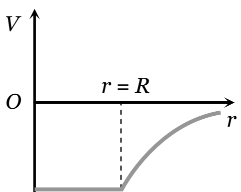
Problems based on gravitational potential

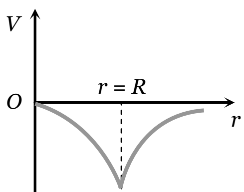
▶ *Basic level*

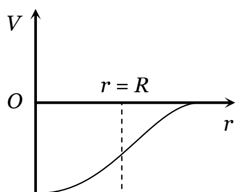
42. If V is the gravitational potential on the surface of the earth, then what is its value at the centre of the earth

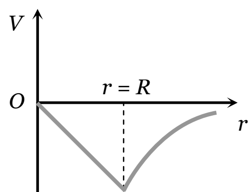
(a) $2V$ (b) $3V$ (c) $\frac{3}{2}V$ (d) $\frac{2}{3}V$

43. The diagram showing the variation of gravitational potential of earth with distance from the centre of earth is



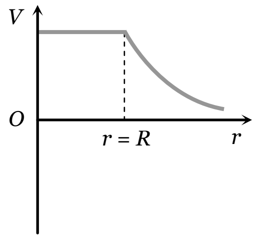


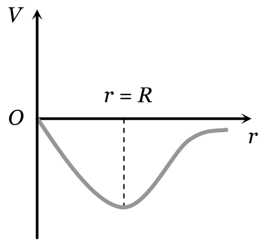


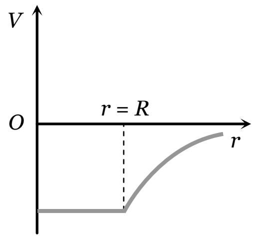


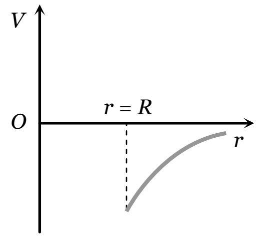
(a) (b) (c) (d)

44. By which curve will the variation of gravitational potential of a hollow sphere of radius R with distance be depicted

(a) 

(b) 

(c) 

(d) 

45. Two concentric shells have mass M and m and their radii are R and r respectively, where $R > r$. What is the gravitational potential at their common centre

(a) $-\frac{GM}{R}$ (b) $-\frac{Gm}{r}$ (c) $-G\left[\frac{M}{R}-\frac{m}{r}\right]$ (d) $-G\left[\frac{M}{R}+\frac{m}{r}\right]$

▶▶ *Advance level*

46. A person brings a mass of 1 *kg* from infinity to a point *A*. Initially the mass was at rest but it moves with a speed of 2 *m/s* as it reaches *A*. The work done by the person on the mass is – 3 *J*. The potential of *A* is

(a) – 3 *J/kg* (b) – 2 *J/kg* (c) – 5 *J/kg* (d) – 7 *J/kg*

47. A thin rod of length L is bent to form a semicircle. The mass of the rod is M. What will be the gravitational potential at the centre of the circle

(a) $-\frac{GM}{L}$ (b) $-\frac{GM}{2\pi L}$ (c) $-\frac{\pi GM}{2L}$ (d) $-\frac{\pi GM}{L}$

Problems based on escape velocity

▶ *Basic level*

48. The escape velocity of a planet having mass 6 times and radius 2 times as that of earth is **[CPMT 1999; MP PET 2003]**

(a) $\sqrt{3}V_e$ (b) $3V_e$ (c) $\sqrt{2}V_e$ (d) $2V_e$

49. The escape velocity of a particle of mass m varies as **[CPMT 1978; RPMT 1999; AIEEE 2002]**

(a) m^2 (b) m (c) m^0 (d) m^{-1}

50. How many times is escape velocity (v_e), of orbital velocity (v_0) for a satellite revolving near earth **[RPMT 2000]**

(a) $\sqrt{2}$ *times* (b) 2 *times* (c) 3 *times* (d) 4 *times*

51. The orbital velocity of a satellite at a height h above the surface of earth is v. The value of escape velocity from the same location is given by **[J&K CET 2000]**

(a) $\sqrt{2}\,v$ (b) v (c) $\frac{v}{\sqrt{2}}$ (d) $\frac{v}{2}$

52. How much energy will be necessary for making a body of 500 *kg* escape from the earth $[g = 9.8\,m/s^2$, radius of earth $= 6.4\times 10^6\,m]$

[MP PET 1999]

(a) About $9.8\times 10^6\,J$ (b) About $6.4\times 10^8\,J$ (c) About $3.1\times 10^{10}\,J$ (d) About $27.4\times 10^{12}\,J$

53. The escape velocity of a body on the surface of the earth is $11.2\,km/s$. If the earth's mass increases to twice its present value and the radius of the earth becomes half, the escape velocity would become **[CBSE PMT 1997]**

(a) $5.6\,km/s$ (b) $11.2\,km/s$ (remain unchanged)

(c) $22.4\ km/s$ (d) $44.8\ km/s$

54. A rocket is launched with velocity 10 *km/s*. If radius of earth is *R*, then maximum height attained by it will be **[RPET 1997]**

(a) $2R$ (b) $3R$ (c) $4R$ (d) $5R$

55. A missile is launched with a velocity less then the escape velocity. The sum of its kinetic and potential energy is **[MP PET 1995]**

(a) Positive

(b) Negative

(c) Zero

(d) May be positive or negative depending upon its initial velocity

56. v_e and v_p denotes the escape velocity from the earth and another planet having twice the radius and the same mean density as the earth. Then **[NCERT 1974; MP PMT 1994]**

(a) $v_e = v_p$ (b) $v_e = v_p/2$ (c) $v_e = 2v_p$ (d) $v_e = v_p/4$

57. The magnitude of the potential energy per unit mass of the object at the surface of earth is *E*. Then the escape velocity of the object is

(a) $\sqrt{2E}$ (b) $4E^2$ (c) $\sqrt{E}$ (d) $\sqrt{E/2}$

▶▶ *Advance level*

58. A ball of mass *m* is fired vertically upwards from the surface of the earth with velocity nv_e, where v_e is the escape velocity and $n < 1$. Neglecting air resistance, to what height will the ball rise? (Take radius of the earth as R)

(a) R/n^2 (b) $R/(1-n^2)$ (c) $Rn^2/(1-n^2)$ (d) Rn^2

59. The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2 respectively. Their centres are distance *d* apart. The minimum velocity with which a particle of mass *m* should be projected from a point midway between their centres so that it escape to infinity is **[MP PET 1997]**

(a) $2\sqrt{\frac{G}{d}(M_1+M_2)}$ (b) $2\sqrt{\frac{2G}{d}(M_1+M_2)}$ (c) $2\sqrt{\frac{Gm}{d}(M_1+M_2)}$ (d) $2\sqrt{\frac{Gm(M_1+M_2)}{d(R_1+R_2)}}$

60. A body is projected with a velocity $2v_e$, where v_e is the escape velocity. Its velocity when it escapes the gravitational field of the earth is

(a) $\sqrt{7}v_e$ (b) $\sqrt{5}v_e$ (c) $\sqrt{3}v_e$ (d) v_e

Problems based on energy

▶ *Basic level*

61. Escape velocity of a body of 1 *kg* mass on a planet is 100 *m/sec*. Gravitational potential energy of the body at the planet is **[MP PMT 2002]**

(a) $-5000\ J$ (b) $-1000\ J$ (c) $-2400\ J$ (d) $5000\ J$

62. A body of mass *m* rises to a height $h = \frac{R}{5}$ from the earth's surface where *R* is earth's radius. If *g* is acceleration due to gravity at the earth's surface, the increase in potential energy is **[CPMT 1989]**

(a) mgh (b) $\frac{4}{5}mgh$ (c) $\frac{5}{6}mgh$ (d) $\frac{6}{7}mgh$

63. The work done is bringing three particles each of mass 10 *g* from large distances to the vertices of an equilateral triangle of side 10 *cm*.

(a) $1\times10^{-13}\ J$ (b) $2\times10^{-13}\ J$ (c) $4\times10^{-11}\ J$ (d) $1\times10^{-11}\ J$

64. The potential energy due to gravitational field of earth will be maximum at

(a) Infinite distance (b) The poles of earth (c) The centre of earth (d) The equator of earth

▶▶ *Advance level*

65. The radius and mass of earth are increased by 0.5%. Which of the following statement is false at the surface of the earth **[Roorkee 2000]**

(a) g will increase (b) g will decrease

(c) Escape velocity will remain unchanged (d) Potential energy will remain unchanged

66. Two identical thin rings each of radius R are coaxially placed at a distance R. If the rings have a uniform mass distribution and each has mass m_1 and m_2 respectively, then the work done in moving a mass m from centre of one ring to that of the other is

(a) Zero (b) $\frac{Gm(m_1 - m_2)(\sqrt{2}-1)}{\sqrt{2}R}$ (c) $\frac{Gm\sqrt{2}(m_1 - m_2)}{R}$ (d) $\frac{Gm_1m_2(\sqrt{2}+1)}{m_2R}$

Problems based on orbital velocity of satellite

▶ *Basic level*

67. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is **[Kerala (Engg.) 2001]**

(a) $\frac{3}{2}V$ (b) $\sqrt{\frac{3}{2}}V$ (c) $\sqrt{\frac{2}{3}}V$ (d) $\frac{2}{3}V$

68. The speed of a satellite is v while revolving in an elliptical orbit and is at nearest distance 'a' from earth. The speed of satellite at farthest distance 'b' will be **[RPMT 1995]**

(a) $(b/a)v$ (b) $(a/b)v$ (c) $(\sqrt{a/b})v$ (d) $(\sqrt{b/a})v$

69. For the moon to cease to remain the earth's satellite its orbital velocity has to increase by a factor of **[MP PET 1994]**

(a) 2 (b) $\sqrt{2}$ (c) $1/\sqrt{2}$ (d) $\sqrt{3}$

70. Two artificial satellites A and B are at a distances r_A and r_B above the earth's surface. If the radius of earth is R, then the ratio of their speeds will be

(a) $\left(\frac{r_B + R}{r_A + R}\right)^{1/2}$ (b) $\left(\frac{r_B + R}{r_A + R}\right)^2$ (c) $\left(\frac{r_B}{r_A}\right)^2$ (d) $\left(\frac{r_B}{r_A}\right)^{1/2}$

▶▶ *Advance level*

71. When a satellite going round earth in a circular orbit of radius r and speed v, losses some of its energy. Then r and v change as

[EAMCET (Med.) 2000]

(a) r and v both will increase (b) r and v both will decrease

(c) r will decrease and v will increase (d) r will increase and v will decrease

72. A satellite is revolving around a planet of mass M in an elliptical orbit of semi-major axis a. The orbital velocity of the satellite at a distance r from the focus will be

(a) $\left[GM\left(\frac{2}{r}-\frac{1}{a}\right)\right]^{1/2}$ (b) $\left[GM\left(\frac{1}{r}-\frac{2}{a}\right)\right]^{1/2}$ (c) $\left[GM\left(\frac{2}{r^2}-\frac{1}{a^2}\right)\right]^{1/2}$ (d) $\left[GM\left(\frac{1}{r^2}-\frac{2}{a^2}\right)\right]^{1/2}$

Problems based on time period of satellite

▶ *Basic level*

73. A geo-stationary satellite is orbiting the earth at a height of 6 R above the surface of earth, R being the radius of earth. The time period of another satellite at a height of 2.5 R from the surface of earth is **[UPSEAT 2002; AMU (Med.) 2002]**

(a) 10 hr (b) $(6/\sqrt{2})hr$ (c) 6 hr (d) $6\sqrt{2}$ hr

74. Time period of revolution of a satellite around a planet of radius R is T. Period of revolution around another planet. Whose radius is $3R$ but having same density is **[CPMT 1981]**

(a) T (b) $3T$ (c) $9T$ (d) $3\sqrt{3}\,T$

75. A satellite is orbiting around the earth with a period T. If the earth suddenly shrinks to half its radius without change in mass, the period of revolution of the satellite will be

(a) $T/\sqrt{2}$ (b) $T/2$ (c) T (d) $2T$

76. A satellite is orbiting around the earth in the equitorial plane rotating from west to east as the earth does. If ω_e be the angular speed of the earth and ω_s be that of satellite, then the satellite will repeatedly appear at the some location after a time t =

(a) $\frac{2\pi}{\omega_s - \omega_c}$ (b) $\frac{2\pi}{\omega_s + \omega_c}$ (c) $\frac{\pi}{\omega_s - \omega_c}$ (d) $\frac{\pi}{\omega_s + \omega_c}$

77. Suppose the gravitational force varies inversely as the *nth* power of distance. Then, the time period of a planet in circular orbit of radius *R* around the sun will be proportional to

(a) R^n (b) $R^{\frac{n+1}{2}}$ (c) $R^{\frac{n+1}{2}}$ (d) R^{-n}

Advance level

78. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 *km*. Then, the time period of a satellite orbiting a few hundred *kilometres* above the earth's surface $(R_{Earth} = 6400\ km)$ will approximately be **[IIT-JEE (Screening) 2002]**

(a) 1/2 *h* (b) 1 *h* (c) 2 *h* (d) 4 *h*

79. If the distance between the earth and the sun becomes half its present value, the number of days in a year would have been **[IIT-JEE 1996; RPET 1996]**

(a) 64.5 (b) 129 (c) 182.5 (d) 730

80. A satellite is launched into a circular orbit of radius *R* around the earth. A second satellite is launched into an orbit of radius (1.01) *R*. The period of the second satellite is larger than that of the first one by approximately **[IIT-JEE 1995]**

(a) 0.5% (b) 1.0% (c) 1.5% (d) 3.0%

81. A satellite moves eastwards very near the surface of the earth in the equatorial plane of the earth with speed v_0. Another satellite moves at the same height with the same speed in the equatorial plane but westwards. If *R* = radius of the earth about its own axis, then the difference in the two time period as observed on the earth will be approximately equal to

(a) $\frac{4\pi R v_0}{R^2\omega^4 - v_0^2}$ (b) $\frac{4\pi R v_0}{R^2\omega^2 - v_0^2}$ (c) $\frac{4\pi R v_0}{R^2\omega^2 + v_0^2}$ (d) $\frac{2\pi R v_0}{R^2\omega^2 + v_0^2}$

82. A "*double star*" is a composite system of two stars rotating about their centre of mass under their mutual gravitational attraction. Let us consider such a "*double star*" which has two stars of masses *m* and 2*m* at separation *l*. If *T* is the time period of rotation about their centre of mass then,

(a) $T = 2\pi\sqrt{\frac{l^3}{mG}}$ (b) $T = 2\pi\sqrt{\frac{l^3}{2mG}}$ (c) $T = 2\pi\sqrt{\frac{l^3}{3mG}}$ (d) $T = 2\pi\sqrt{\frac{l^3}{4mG}}$

83. A space probe projected from the earth moves round the moon in a circular orbit at a distance equal to its radius $R_{moon} = \frac{R}{4}$ where *R* = radius of the earth. Its rocket launcher moves in circular orbit around the earth at a distance equal to *R* from its surface. The ratio of the times taken for one revolution by the probe and the rocket launcher is $\left(M_{moon} = \frac{M}{80}, \text{where } M = \text{mass of the earth}\right)$

(a) $\sqrt{3}:2$ (b) $\sqrt{5}:2$ (c) $1:1$ (d) $2:\sqrt{3}$

Problems based on height of satellite

Basic level

84. The distance of a geo-stationary satellite from the centre of the earth (Radius *R* = 6400 *km*) is nearest to **[AFMC 2001]**

(a) 5 *R* (b) 7 *R* (c) 10 *R* (d) 18 *R*

85. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the escape speed from the earth. If *R* is the radius of the earth then the height of the satellite above the surface of the earth is

(a) $\frac{R}{2}$ (b) $\frac{2R}{3}$ (c) R (d) $2R$

Advance level

86. If the angular velocity of a planet about its own axis is halved, the distance of geostationary satellite of this planet from the centre of the planet will become

(a) $(2)^{1/3}$ times (b) $(2)^{3/2}$ times (c) $(2)^{2/3}$ times (d) 4 times

Problems based on energy of satellite

▶ Basic level

87. A satellite moves around the earth in a circular orbit with speed v. If m is the mass of the satellite, its total energy is **[CBSE PMT 1991]**

(a) $-\frac{1}{2}mv^2$ (b) $\frac{1}{2}mv^2$ (c) $\frac{3}{2}mv^2$ (d) $\frac{1}{4}mv^2$

88. The minimum energy required to launch a satellite of mass m from the surface of earth of radius R in a circular orbit at an altitude $2R$ is (mass of earth is M)

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$ (c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{3R}$

89. The masses of moon and earth are $7.36\times10^{22}\ kg$ and $5.98\times10^{24}\ kg$ respectively and their mean separation is $3.82\times10^{5}\ km$. The energy required to break the earth-moon system is

(a) $12.4\times10^{32}\ J$ (b) $3.84\times10^{28}\ J$ (c) $5.36\times10^{24}\ J$ (d) $2.96\times10^{20}\ J$

90. A body placed at a distance R_0 form the centre of earth, starts moving from rest. The velocity of the body on reaching at the earth's surface will be (R_e = radius of earth and M_e = mass of earth)

(a) $GM_e\left(\frac{1}{R_e}-\frac{1}{R_0}\right)$ (b) $2GM_e\left(\frac{1}{R_e}-\frac{1}{R_0}\right)$ (c) $GM_e\sqrt{\frac{1}{R_e}-\frac{1}{R_0}}$ (d) $\sqrt{2GM_e\left(\frac{1}{R_e}-\frac{1}{R_0}\right)}$

91. If total energy of an earth satellite is zero, it means that

(a) The satellite is bound to earth

(b) The satellite may no longer be bound to earth's field

(c) The satellite moves away from the orbit along a parabolic path

(d) The satellite escapes in a hyperbolic path

▶▶ Advance level

92. By what percent the energy of a satellite has to be increased to shift it from an orbit of radius r to $\frac{3}{2}r$

(a) 66.7% (b) 33.3% (c) 15% (d) 20.3%

93. A mass m is raised from the surface of the earth to a point distant $\beta R(\beta>1)$ from the centre of the earth and then put into a circular orbit to make it an artificial satellite. The total work done to complete this job is

(a) $mgR(2\beta-1)$ (b) $mgR(2\beta+1)$ (c) $mgR(\beta+1)$ (d) $mgR\frac{2\beta-1}{2\beta}$

Problems based on angular momentum of satellite

▶ Basic level

94. A satellite of mass m is circulating around the earth with constant angular velocity. If radius of the orbit is R_0 and mass of the earth M, the angular momentum about the centre of the earth is **[MP PMT 1996; RPMT 2000]**

(a) $m\sqrt{GMR_0}$ (b) $M\sqrt{GmR_0}$ (c) $m\sqrt{\frac{GM}{R_0}}$ (d) $M\sqrt{\frac{GM}{R_0}}$

95. A planet of mass m is moving in an elliptical path about the sun. Its maximum and minimum distances from the sun are r_1 and r_2 respectively. If M_s is the mass of sun then the angular momentum of this planet about the center of sun will be

(a) $\sqrt{\frac{2GM_s}{(r_1+r_2)}}$ (b) $2GM_s m\sqrt{\frac{r_1r_2}{(r_1+r_2)}}$ (c) $m\sqrt{\frac{2GM_s r_1r_2}{(r_1+r_2)}}$ (d) $m\sqrt{\frac{2GM_s m(r_1+r_2)}{r_1r_2}}$

Problems based on weightlessness in satellite

▶ Basic level

96. Reaction of weightlessness in a satellite is **[RPMT 2000]**

(a) Zero gravity (b) Centre of mass

(c) Zero reaction force by satellite surface (d) None of these

97. A body suspended from a spring balance is placed in a satellite. Reading in balance is W_1 when the satellite moves in an orbit of radius R. Reading in balance is W_2 when the satellite moves in an orbit of radius $2R$. Then

(a) $W_1=W_2$ (b) $W_1>W_2$ (c) $W_1<W_2$ (d) $W_1=2W_2$

98. An astronaut feels weightlessness because

(a) Gravity is zero there

(b) Atmosphere is not there

(c) Energy is zero in the chamber of a rocket

(d) The fictitious force in rotating frame of reference cancels the effect or weight

99. Inside a satellite orbiting very close to the earth's surface, water does not fall out of a glass when it is inverted. Which of the following is the best explanation for this

(a) The earth does not exert any force on the water

(b) The earth's force of a attraction on the water is exactly balanced by the force created by the satellites motion

(c) The water and the glass have the same acceleration, equal to g, towards the centre of the earth, and hence there is no relative motion between them

(d) The gravitational attraction between the glass and the water balances the earth's attraction on the water

100. To overcome the effect of weightlessness in an artificial satellite

(a) The satellite is rotated its axis with compartment of astronaut at the centre of the satellite

(b) The satellite is shaped like a wheel

(c) The satellite is rotated around and around till weightlessness disappears

(d) The compartment of astronaut is kept on the periphery of rotating wheel like satellite

Problems based on Kepler's laws

Basic level

101. Which of the following astronomer first proposed that sun is static and earth rounds sun **[AFMC 2002]**

(a) Copernicus (b) Kepler (c) Galilio (d) None

102. The period of a satellite in a circular orbit of radius R is T, the period of another satellite in a circular orbit of radius $4R$ is **[CPMT 1982; MP PET/PMT 1998; AIIMS 2000; CBSE 2002]**

(a) $4T$ (b) $T/4$ (c) $8T$ (d) $T/8$

103. Kepler's second law is based on **[AIIMS 2002]**

(a) Newton's first law (b) Newton's second law

(c) Special theory of relativity (d) Conservation of angular momentum

104. Two planets at mean distance d_1 and d_2 from the sun and their frequencies are n_1 and n_2 respectively then **[Kerala (Med.) 2002]**

(a) $n_1^2 d_1^2 = n_2 d_2^2$ (b) $n_2^2 d_2^3 = n_1^2 d_1^3$ (c) $n_1 d_1^2 = n_2 d_2^2$ (d) $n_1^2 d_1 = n_2^2 d_2$

105. Earth needs one year to complete one revolution round the sun. If the distance between sun and earth is doubled then the period of revolution of earth will become **[PM PMT 1997]**

(a) $2\sqrt{2}$ yrs (b) 8 yrs (c) $\frac{1}{2}$ yrs (d) 1 yrs

106. The eccentricity of earth's orbit is 0.0167. The ratio of its maximum speed in its orbit to its minimum speed is **[NCERT 1973]**

(a) 2.507 (b) 1.033 (c) 8.324 (d) 1.000

107. For a planet around the sun in an elliptical orbit of semi – major and semi – minor axes a and b, respectively, and period T

(A) The torque acting on the planet about the sun is non – zero

(B) The angular momentum of the planet about the sun is constant

(C) The areal velocity is $\pi ab/T$

(D) The planet moves with a constant speed around the sun

(a) A, B (b) B, C (c) C, D (d) D, A

108. A planet moves in an elliptical orbit around one of the foci. The ratio of maximum velocity $v_{\max}$ and minimum velocity $v_{\min}$ in terms of eccentricity e of the ellipse is given by

(a) $\frac{1-e}{1+e}$ (b) $\frac{e-1}{e+1}$ (c) $\frac{1+e}{1-e}$ (d) $\frac{e}{e-1}$

109. The satellites S_1 and S_2 describe circular orbits of radii r and $2r$ respectively around a planet. If the orbital angular velocity of S_1 is ω, that of S_2 is

(a) $\frac{\omega}{2\sqrt{2}}$ (b) $\omega\sqrt{2}$ (c) $\frac{\omega}{\sqrt{2}}$ (d) $\frac{\omega\sqrt{2}}{3}$

▶▶ *Advance level*

110. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between planet and star is proportional to $R^{-5/2}$, then T^2 is proportional to

[IIT-JEE 1989; RPMT 1997]

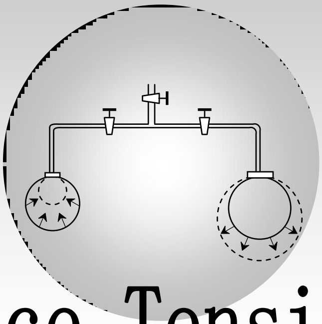
(a) R^3 (b) $R^{7/2}$ (c) $R^{5/2}$ (d) $R^{3/2}$

111. A binary star has stars of masses m and nm (where n is a numerical factor) having separation of their centres as r. If these stars revolve because of gravitational force of each other, the period of revolution is given by

(a) $\frac{2\pi r^{3/2}}{\left(\frac{Gnm^2}{(n+1)m}\right)^{\frac{1}{2}}}$ (b) $\frac{2\pi r^{1/2}}{\left(\frac{G(n+1)m}{nm}\right)^{\frac{1}{2}}}$ (c) $\frac{2\pi r^3}{\frac{2}{3}GMn}$ (d) $\frac{2\pi r^{3/2}}{\left(\frac{2}{3}GMn\right)^{2/3}}$

<u>Answers of Practice problems</u>

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
c	d	b	a	c	d	a	b	c	d
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
c	a	c	b	c	a	b	d	a	d
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
b	d	d	b	a	d	a	d	b	c
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
d	c	d	b	d	a, c, d	d	d	b	c
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
b	c	c	c	d	c	d	a	c	a
51.	**52.**	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**
b	c	c	c	b	b	a	c	a	c
61.	**62.**	**63.**	**64.**	**65.**	**66.**	**67.**	**68.**	**69.**	**70.**
a	c	b	a	a	b	c	b	b	a
71.	**72.**	**73.**	**74.**	**75.**	**76.**	**77.**	**78.**	**79.**	**80.**
c	a	d	a	c	a	b	c	b	c
81.	**82.**	**83.**	**84.**	**85.**	**86.**	**87.**	**88.**	**89.**	**90.**
b	c	b	b	c	c	a	a	b	d
91.	**92.**	**93.**	**94.**	**95.**	**96.**	**97.**	**98.**	**99.**	**100.**
c	b	d	a	c	c	a	d	c	d
101.	**102.**	**103.**	**104.**	**105.**	**106.**	**107.**	**108.**	**109.**	**110.**
a	c	d	b	a	b	b	c	a	b
111.									
a									



Surface Tension

10.1 Intermolecular Force

The force of attraction or repulsion acting between the molecules are known as intermolecular force. The nature of intermolecular force is electromagnetic.

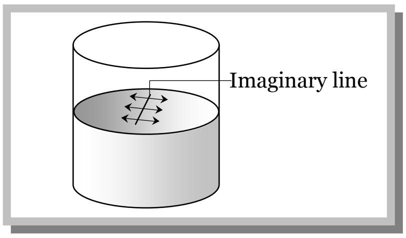
The intermolecular forces of attraction may be classified into two types.

Cohesive force	Adhesive force
The force of attraction between molecules of same substance is called the force of cohesion. This force is lesser in liquids and least in gases.	The force of attraction between the molecules of the different substances is called the force of adhesion.
Ex. (i) Two drops of a liquid coalesce into one when brought in mutual contact. (ii) It is difficult to separate two sticky plates of glass welded with water. (iii) It is difficult to break a drop of mercury into small droplets because of large cohesive force between the mercury molecules.	Ex. (i) Adhesive force enables us to write on the blackboard with a chalk. (ii) A piece of paper sticks to another due to large force of adhesion between the paper and gum molecules. (iii) Water wets the glass surface due to force of adhesion.

Note : ❑ Cohesive or adhesive forces are inversely proportional to the eighth power of distance between the molecules.

10.2 Surface Tension

The property of a liquid due to which its free surface tries to have minimum surface area and behaves as if it were under tension some what like a stretched elastic membrane is called surface tension. A small liquid drop has spherical shape, as due to surface tension the liquid surface tries to have minimum surface area and for a given volume, the sphere has minimum surface area.



Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid, the direction of this force being perpendicular to the line and tangential to the free surface of liquid. So if F is the force acting on one side of imaginary line of length L, then $T = (F/L)$

(1) It depends only on the nature of liquid and is independent of the area of surface or length of line considered.

(2) It is a scalar as it has a unique direction which is not to be specified.

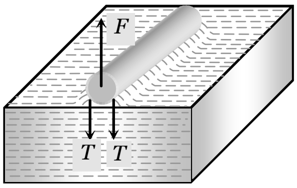
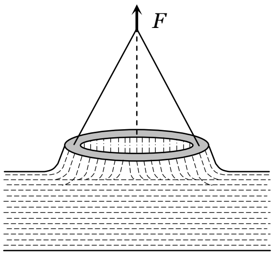
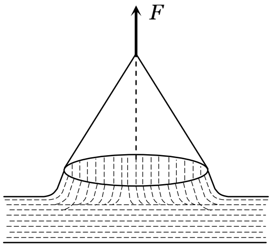
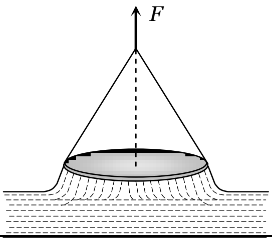
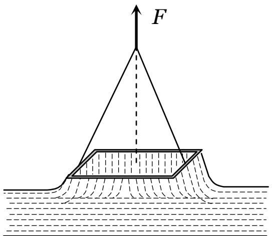
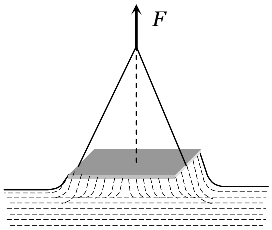
(3) Dimension : $[MT^{-2}]$. (Similar to force constant)

(4) Units : N/m (S.I.) and $Dyne/cm$ [C.G.S.]

(5) It is a molecular phenomenon and its root cause is the electromagnetic forces.

10.3 Force Due to Surface Tension

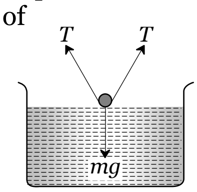
If a body of weight W is placed on the liquid surface, whose surface tension is T. If F is the minimum force required to pull it away from the water then value of F for different bodies can be calculated by the following table.

Body	Figure	Force
Needle (Length = l)		$F = 2l\,T + W$
Hollow disc (Inner radius = r_1 Outer radius = r_2)		$F = 2\pi(r_1 + r_2)T + W$
Thin ring (Radius = r)		$F = 2\pi(r + r)T + W$ $F = 4\pi rT + W$
Circular plate or disc (Radius = r)		$F = 2\pi rT + W$
Square frame (Side = l)		$F = 8l\,T + W$
Square plate		$F = 4l\,T + W$

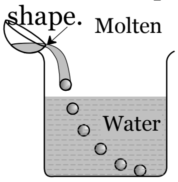
10.4 Examples of Surface Tension

(1) When mercury is split on a clean glass plate, it forms globules. Tiny globules are spherical on the account of surface tension because force of gravity is negligible. The bigger globules get flattened from the middle but have round shape near the edges, figure

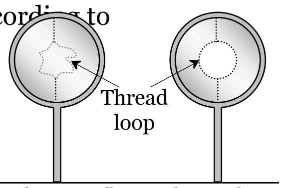
(2) When a greased iron needle is placed gently on the surface of water at rest, so that it does not prick the water surface, the needle floats on the surface of water despite it being heavier because the weight of needle is balanced by the vertical components of the forces of surface tension. If the water surface is pricked by one end of the needle, the needle sinks down.



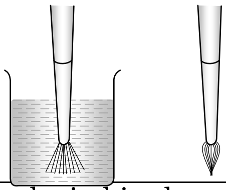
(3) When a molten metal is poured into water from a suitable height, the falling stream of metal breaks up and the detached portion of the liquid in small quantity acquire the spherical shape.



(4) Take a frame of wire and dip it in soap solution and take it out, a soap film will be formed in the frame. Place a loop of wet thread gently on the film. It will remain in the form, we place it on the film according to figure. Now, piercing the film with a pin at any point inside the loop, It immediately takes the circular form as shown in figure.



(5) Hair of shaving brush/painting brush when dipped in water spread out, but as soon as it is taken out, its hair stick together.



(6) If a small irregular piece of camphor is floated on the surface of pure water, it does not remain steady but dances about on the surface. This is because, irregular shaped camphor dissolves unequally and decreases the surface tension of the water locally. The unbalanced forces make it move haphazardly in different directions.

(7) Rain drops are spherical in shape because each drop tends to acquire minimum surface area due to surface tension, and for a given volume, the surface area of sphere is minimum.

(8) Oil drop spreads on cold water. Whereas it may remain as a drop on hot water. This is due to the fact that the surface tension of oil is less than that of cold water and is more than that of hot water.

10.5 Factors Affecting Surface Tension

(1) **Temperature :** The surface tension of liquid decreases with rise of temperature. The surface tension of liquid is zero at its boiling point and it vanishes at critical temperature. At critical temperature, intermolecular forces for liquid and gases becomes equal and liquid can expand without any restriction. For small temperature differences, the variation in surface tension with temperature is linear and is given by the relation

$$T_t = T_0(1 - \alpha t)$$

where T_t, T_0 are the surface tensions at $t^o C$ and $0^o C$ respectively and α is the temperature coefficient of surface tension.

Examples : (i) Hot soup tastes better than the cold soup.

(ii) Machinery parts get jammed in winter.

(2) **Impurities :** The presence of impurities either on the liquid surface or dissolved in it, considerably affect the force of surface tension, depending upon the degree of contamination. A highly soluble substance like sodium chloride when dissolved in water, increases the surface tension of water. But the sparingly soluble substances like phenol when dissolved in water, decreases the surface tension of water.

10.6 Applications of Surface Tension

(1) The oil and grease spots on clothes cannot be removed by pure water. On the other hand, when detergents (like soap) are added in water, the surface tension of water decreases. As a result of this, wetting power of soap solution increases. Also the force of adhesion between soap solution and oil or grease on the clothes increases. Thus, oil, grease and dirt particles get mixed with soap solution easily. Hence clothes are washed easily.

(2) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension, the antiseptics spreads properly over wound.

(3) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(4) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(5) A rough sea can be calmed by pouring oil on its surface.

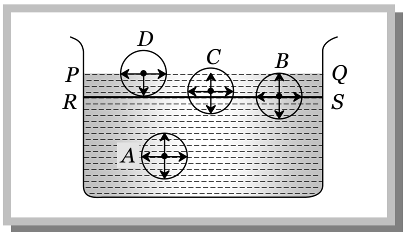
(6) In soldering, addition of 'flux' reduces the surface tension of molten tin, hence, it spreads.

10.7 Molecular Theory of Surface Tension

The maximum distance upto which the force of attraction between two molecules is appreciable is called molecular range $(\approx 10^{-9} m)$. A sphere with a molecule as centre and radius equal to molecular range is called the sphere of influence. The liquid enclosed between free surface (*PQ*) of the liquid and an imaginary plane (*RS*) at a distance *r* (equal to molecular range) from the free surface of the liquid form a liquid film.

To understand the tension acting on the free surface of a liquid, let us consider four liquid molecules like *A*, *B*, *C* and *D*. Their sphere of influence are shown in the figure.

(1) Molecule *A* is well within the liquid, so it is attracted equally in all directions. Hence the net force on this molecule is zero and it moves freely inside the liquid.



(2) Molecule *B* is little below the free surface of the liquid and it is also attracted equally in all directions. Hence the resultant force on it is also zero.

(3) Molecule *C* is just below the upper surface of the liquid film and the part of its sphere of influence is outside the free liquid surface. So the number of molecules in the upper half (attracting the molecules upward) is less than the number of molecule in the lower half (attracting the molecule downward). Thus the molecule *C* experiences a net downward force.

(4) Molecule *D* is just on the free surface of the liquid. The upper half of the sphere of influence has no liquid molecule. Hence the molecule *D* experiences a maximum downward force.

Thus all molecules lying in surface film experiences a net downward force. Therefore, free surface of the liquid behaves like a stretched membrane.

Sample problems based on Surface tension

***Problem* 1.** A wooden stick $2m$ long is floating on the surface of water. The surface tension of water $0.07\ N/m$. By putting soap solution on one side of the sticks the surface tension is reduced to $0.06\ N/m$. The net force on the stick will be **[Pb. PMT 2002]**

(a) $0.07\ N$ (b) $0.06\ N$ (c) $0.01\ N$ (d) $0.02\ N$

Solution : (d) Force on one side of the stick $F_1 = T_1 \times L \quad = 0.07 \times 2 = 0.14 N$

and force on other side of the stick $F_2 = T_2 \times L = 0.06 \times 2 = 0.12 N$

So net force on the stick $= F_1 - F_2 = 0.14 - 0.12 = 0.02 N$

***Problem* 2.** A thin metal disc of radius r floats on water surface and bends the surface downwards along the perimeter making an angle θ with vertical edge of disc. If the disc displaces a weight of water W and surface tension of water is T, then the weight of metal disc is **[AMU (Med.) 1999]**

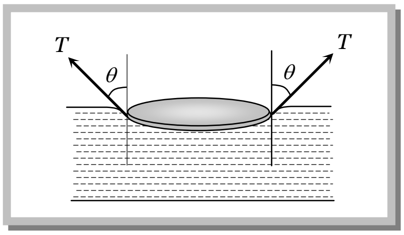
(a) $2\pi rT + W$ (b) $2\pi rT\cos\theta - W$ (c) $2\pi rT\cos\theta + W$ (d) $W - 2\pi rT\cos\theta$

Solution : (c) Weight of metal disc = total upward force

= upthrust force + force due to surface tension

= weight of displaced water + $T\cos\theta(2\pi r)$

$= W + 2\pi rT\cos\theta$



***Problem* 3.** A 10 *cm* long wire is placed horizontally on the surface of water and is gently pulled up with a force of $2\times10^{-2} N$ to keep the wire in equilibrium. The surface tension in Nm^{-1} of water is

(a) 0.1 N/m (b) 0.2 N/m (c) 0.001 N/m (d) 0.002 N/m

Solution : (a) Force on wire due to surface tension $F = T \times 2l$

$$\therefore T = \frac{F}{2l} = \frac{2\times10^{-2}}{2\times10\times10^{-2}} = 0.1 \, N/m$$

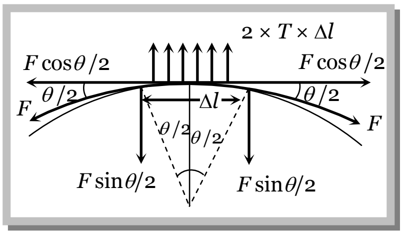
***Problem* 4.** There is a horizontal film of soap solution. On it a thread is placed in the form of a loop. The film is pierced inside the loop and the thread becomes a circular loop of radius R. If the surface tension of the loop be T, then what will be the tension in the thread

(a) $\pi R^2 / T$ (b) $\pi R^2 T$ (c) $2\pi RT$ (d) $2RT$

Solution : (d) Suppose tension in thread is F, then for small part Δl of thread

$\Delta l = R\theta$ and $2F\sin\theta/2 = 2T\Delta l = 2TR\theta$

$$\Rightarrow F = \frac{TR\theta}{\sin\theta/2} = \frac{TR\theta}{\theta/2} = 2TR \quad (\sin\theta/2 \approx \theta/2)$$



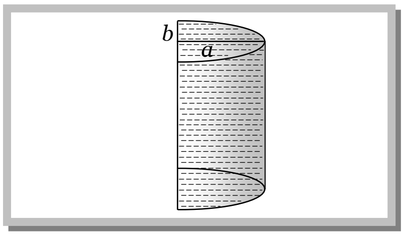
***Problem* 5.** A liquid is filled into a tube with semi-elliptical cross-section as shown in the figure. The ratio of the surface tension forces on the curved part and the plane part of the tube in vertical position will be

(a) $\frac{\pi(a+b)}{4b}$

(b) $\frac{2\pi a}{b}$

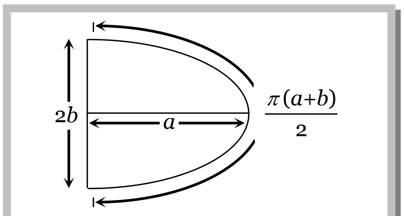
(c) $\frac{\pi a}{4b}$

(d) $\frac{\pi(a-b)}{4b}$



Solution : (a) From the figure Curved part = semi perimeter $= \frac{\pi(a+b)}{2}$

and the plane part = minor axis = $2b$

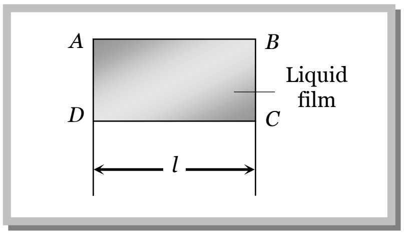


$\therefore$ Force on curved part = $T \times \frac{\pi(a+b)}{2}$

and force on plane part = $T \times 2b$

$\therefore$ Ratio $= \frac{\pi(a+b)}{4b}$

Problem 6. A liquid film is formed over a frame *ABCD* as shown in figure. Wire *CD* can slide without friction. The mass to be hung from *CD* to keep it in equilibrium is



(a) $\frac{Tl}{g}$

(b) $\frac{2Tl}{g}$

(c) $\frac{g}{2Tl}$

(d) $T \times l$

Solution : (b) Weight of the body hung from wire (mg) = upward force due to surface tension $(2Tl) \Rightarrow m = \frac{2Tl}{g}$

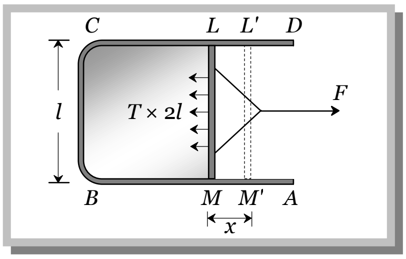
10.8 Surface Energy.

The molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to be done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface energy.

Unit : $Joule/m^2$ (S.I.) erg/cm^2 (C.G.S.)

Dimension : $[MT^{-2}]$

If a rectangular wire frame *ABCD*, equipped with a sliding wire *LM* dipped in soap solution, a film is formed over the frame. Due to the surface tension, the film will have a tendency to shrink and thereby, the sliding wire *LM* will be pulled in inward direction. However, the sliding wire can be held in this position under a force *F*, which is equal and opposite to the force acting on the sliding wire *LM* all along its length due to surface tension in the soap film.



If *T* is the force due to surface tension per unit length, then $F = T \times 2l$

Here, *l* is length of the sliding wire *LM*. The length of the sliding wire has been taken as 2*l* for the reason that the film has got two free surfaces.

Suppose that the sliding wire *LM* is moved through a small distance *x*, so as to take the position $L'M'$. In this process, area of the film increases by $2l \times x$ (on the two sides) and to do so, the work done is given by

$$W = F \times x = (T \times 2l) \times x = T \times (2lx) = T \times \Delta A$$

$\therefore \; W = T \times \Delta A$ [ΔA = Total increase in area of the film from both the sides]

If temperature of the film remains constant in this process, this work done is stored in the film as its surface energy.

From the above expression $T = \frac{W}{\Delta A}$ or $T = W$ [If $\Delta A = 1$]

i.e. surface tension may be defined as the amount of work done in increasing the area of the liquid surface by unity against the force of surface tension at constant temperature.

10.9 Work Done in Blowing a Liquid Drop or Soap Bubble.

(1) If the initial radius of liquid drop is r_1 and final radius of liquid drop is r_2 then

$W = T \times$ Increment in surface area

$W = T \times 4\pi[r_2^2 - r_1^2]$ [drop has only one free surface]

(2) In case of soap bubble

$W = T \times 8\pi[r_2^2 - r_1^2]$ [Bubble has two free surfaces]

10.10 Splitting of Bigger Drop.

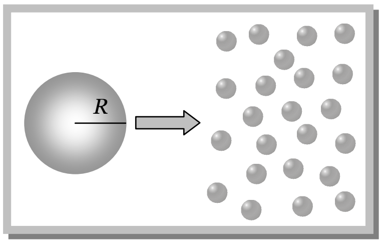
When a drop of radius R splits into n smaller drops, (each of radius r) then surface area of liquid increases. Hence the work is to be done against surface tension.

Since the volume of liquid remains constant therefore $\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3 \quad \therefore \; R^3 = nr^3$

Work done = $T \times \Delta A = T$ [Total final surface area of n drops – surface area of big drop] = $T[n4\pi r^2 - 4\pi R^2]$

Various formulae of work done

$4\pi T[nr^2 - R^2]$	$4\pi R^2 T[n^{1/3} - 1]$	$4\pi T r^2 n^{2/3}[n^{1/3} - 1]$	$4\pi TR^3\left[\frac{1}{r} - \frac{1}{R}\right]$



If the work is not done by an external source then internal energy of liquid decreases, subsequently temperature decreases. This is the reason why spraying causes cooling.

By conservation of energy, Loss in thermal energy = work done against surface tension

$$JQ = W$$

$$\Rightarrow \quad JmS\,\Delta\theta = 4\pi TR^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\Rightarrow \quad J\frac{4}{3}\pi R^3 d\,S\Delta\theta = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right] \qquad \left[\text{As } m = V \times d = \frac{4}{3}\pi R^3 \times d\right]$$

$\therefore$ Decrease in temperature $\quad \Delta\theta = \frac{3T}{JSd}\left[\frac{1}{r} - \frac{1}{R}\right]$

where J = mechanical equivalent of heat, S = specific heat of liquid, d = density of liquid.

10.11 Formation of Bigger Drop

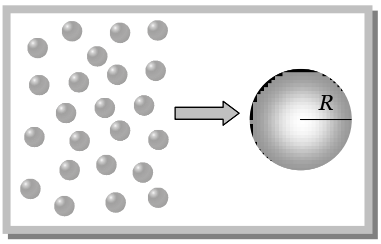
If n small drops of radius r coalesce to form a big drop of radius R then surface area of the liquid decreases.

Amount of surface energy released = Initial surface energy – final surface energy

$$E = n4\pi r^2 T - 4\pi R^2 T$$

Various formulae of released energy

$4\pi T[nr^2 - R^2]$	$4\pi R^2 T(n^{1/3} - 1)$	$4\pi T r^2 n^{2/3}(n^{1/3} - 1)$	$4\pi TR^3\left[\frac{1}{r} - \frac{1}{R}\right]$



(i) If this released energy is absorbed by a big drop, its temperature increases and rise in temperature can be given by $\Delta\theta = \frac{3T}{JSd}\left[\frac{1}{r} - \frac{1}{R}\right]$

(ii) If this released energy is converted into kinetic energy of a big drop without dissipation then by the law of conservation of energy.

$$\frac{1}{2}mv^2 = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right] \Rightarrow \frac{1}{2}\left[\frac{4}{3}\pi R^3 d\right]v^2 = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right] \Rightarrow v^2 = \frac{6T}{d}\left[\frac{1}{r} - \frac{1}{R}\right]$$

$$\therefore \quad v = \sqrt{\frac{6T}{d}\left(\frac{1}{r} - \frac{1}{R}\right)}$$

Sample problems based on Surface energy

Problem 7. Two small drops of mercury, each of radius R, coalesce to form a single large drop. The ratio of the total surface energies before and after the change is **[AIIMS 2003]**

(a) $1 : 2^{1/3}$ (b) $2^{1/3} : 1$ (c) $2 : 1$ (d) $1 : 2$

Solution : (b) As $R = n^{1/3}r = 2^{1/3}r \Rightarrow R^2 = 2^{2/3}r^2 \Rightarrow \frac{r^2}{R^2} = 2^{-2/3}$

$$\frac{\text{Initial surface energy}}{\text{Final surface energy}} = \frac{2(4\pi r^2 T)}{(4\pi R^2 T)} = 2\left(\frac{r^2}{R^2}\right) = 2 \times 2^{-2/3} = 2^{1/3}$$

Problem 8. Radius of a soap bubble is increased from R to $2R$ work done in this process in terms of surface tension is

[CPMT 1991; RPET 2001; BHU 2003]

(a) $24\pi R^2 S$ (b) $48\pi R^2 S$ (c) $12\pi R^2 S$ (d) $36\pi R^2 S$

Solution : (a) $W = 8\pi T\left(R_2^2 - R_1^2\right) = 8\pi S[(2R)^2 - (R)^2] = 24\pi R^2 S$

Problem 9. The work done in blowing a soap bubble of 10cm radius is (surface tension of the soap solution is $\frac{3}{100} N/m$)

[MP PMT 1995; MH CET 2002]

(a) $75.36 \times 10^{-4} J$ (b) $37.68 \times 10^{-4} J$ (c) $150.72 \times 10^{-4} J$ (d) $75.36 J$

Solution : (a) $W = 8\pi R^2 T = 8\pi(10 \times 10^{-2})^2 \frac{3}{100} = 75.36 \times 10^{-4} J$

Problem 10. A drop of mercury of radius 2mm is split into 8 identical droplets. Find the increase in surface energy. (Surface tension of mercury is 0.465 J/m^2)

(a) 23.4μJ (b) 18.5μJ (c) 26.8μJ (d) 16.8μJ

Solution : (a) Increase in surface energy $= 4\pi R^2 T(n^{1/3} - 1) = 4\pi(2 \times 10^{-3})^2(0.465)(8^{1/3} - 1) = 23.4 \times 10^{-6} J = 23.4\mu J$

Problem 11. The work done in increasing the size of a soap film from 10cm × 6cm to 10cm × 11cm is $3 \times 10^{-4} J$. The surface tension of the film is **[MP PET 1999; MP PMT 2000; AIIMS 2000; JIPMER 2001, 02]**

(a) $1.5 \times 10^{-2} Nm^{-1}$ (b) $3.0 \times 10^{-2} Nm^{-1}$ (c) $6.0 \times 10^{-2} Nm^{-1}$ (d) $11.0 \times 10^{-2} Nm^{-1}$

Solution : (b) $A_1 = 10 \times 6 = 60 cm^2 = 60 \times 10^{-4} m^2$, $A_2 = 10 \times 11 = 110 cm^2 = 110 \times 10^{-4} m^2$

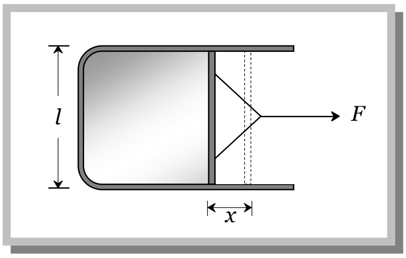
As the soap film has two free surfaces $\therefore W = T \times 2\Delta A$

$$\Rightarrow W = T \times 2 \times (A_2 - A_1) \Rightarrow T = \frac{W}{2 \times 50 \times 10^{-4}} = \frac{3 \times 10^{-4}}{2 \times 50 \times 10^{-4}} = 3 \times 10^{-2}\, N/m$$

***Problem* 12.** A film of water is formed between two straight parallel wires of length 10*cm* each separated by 0.5*cm*. If their separation is increased by 1 *mm* while still maintaining their parallelism, how much work will have to be done (Surface tension of water $= 7.2 \times 10^{-2}\, N/m$) **[Roorkee 1986; MP PET 2001]**

(a) $7.22 \times 10^{-6} J$ (b) $1.44 \times 10^{-5} J$ (c) $2.88 \times 10^{-5} J$ (d) $5.76 \times 10^{-5} J$

Solution : (b) As film have two free surfaces $W = T \times 2\Delta A$



$$W = T \times 2l \times x$$

$$= 7.2 \times 10^{-2} \times 2 \times 0.1 \times 1 \times 10^{-3}$$

$$= 1.44 \times 10^{-5} J$$

***Problem* 13.** If the work done in blowing a bubble of volume V is W, then the work done in blowing the bubble of volume $2V$ from the same soap solution will be **[MP PET 1989]**

(a) $W/2$ (b) $\sqrt{2}\, W$ (c) $\sqrt[3]{2}\, W$ (d) $\sqrt[3]{4}\, W$

Solution : (d) As volume of the bubble $V = \frac{4}{3}\pi R^3 \Rightarrow R = \left(\frac{3}{4\pi}\right)^{1/3} V^{1/3} \Rightarrow R^2 = \left(\frac{3}{4\pi}\right)^{2/3} V^{2/3} \Rightarrow R^2 \propto V^{2/3}$

Work done in blowing a soap bubble $W = 8\pi R^2 T \Rightarrow W \propto R^2 \propto V^{2/3}$

$$\therefore \frac{W_2}{W_1} = \left(\frac{V_2}{V_1}\right)^{2/3} = \left(\frac{2V}{V}\right)^{2/3} = (2)^{2/3} = (4)^{1/3} \Rightarrow W_2 = \sqrt[3]{4}\, W$$

***Problem* 14.** Several spherical drops of a liquid of radius r coalesce to form a single drop of radius R. If T is surface tension and V is volume under consideration, then the release of energy is

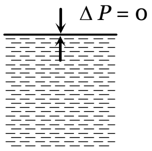
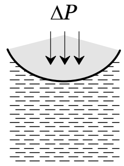
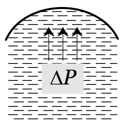
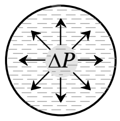
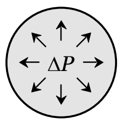
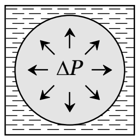
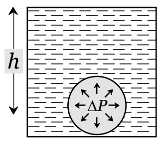
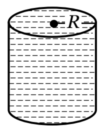
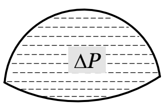
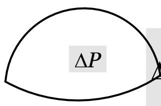
(a) $3VT\left(\frac{1}{r} + \frac{1}{R}\right)$ (b) $3VT\left(\frac{1}{r} - \frac{1}{R}\right)$ (c) $VT\left(\frac{1}{r} - \frac{1}{R}\right)$ (d) $VT\left(\frac{1}{r^2} + \frac{1}{R^2}\right)$

Solution : (b) Energy released $= 4\pi TR^3\left[\frac{1}{r} - \frac{1}{R}\right] = 3\left(\frac{4}{3}\pi R^3\right)T\left[\frac{1}{r} - \frac{1}{R}\right] = 3VT\left[\frac{1}{r} - \frac{1}{R}\right]$

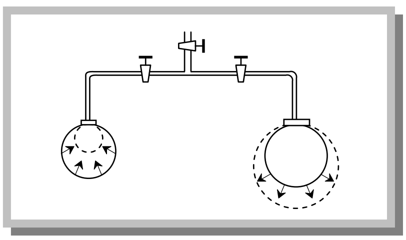
10.12 Excess Pressure

Due to the property of surface tension a drop or bubble tries to contract and so compresses the matter enclosed. This in turn increases the internal pressure which prevents further contraction and equilibrium is achieved. So in equilibrium the pressure inside a bubble or drop is greater than outside and the difference of pressure between two sides of the liquid surface is called excess pressure. In case of a drop excess pressure is provided by hydrostatic pressure of the liquid within the drop while in case of bubble the gauge pressure of the gas confined in the bubble provides it.

Excess pressure in different cases is given in the following table :

Plane surface	Concave surface
 $\Delta P = 0$	 $\Delta P = \frac{2T}{R}$
Convex surface	**Drop**
 $\Delta P = \frac{2T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble in air	**Bubble in liquid**
 $\Delta P = \frac{4T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble at depth *h* below the free surface of liquid of density *d*	**Cylindrical liquid surface**
 $\Delta P = \frac{2T}{R} + hdg$	 $\Delta P = \frac{T}{R}$
Liquid surface of unequal radii	**Liquid film of unequal radii**
 $\Delta P = T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$	 $\Delta P = 2T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

Note : ❑ Excess pressure is inversely proportional to the radius of bubble (or drop), *i.e.*, pressure inside a smaller bubble (or drop) is higher than inside a larger bubble (or drop). This is why when two bubbles of different sizes are put in communication with each other, the air will rush from smaller to larger bubble, so that the smaller will shrink while the larger will expand till the smaller bubble reduces to droplet.



Sample problems based on Excess pressure

***Problem* 15.** The pressure inside a small air bubble of radius 0.1*mm* situated just below the surface of water will be equal to (Take surface tension of water $70 \times 10^{-3}\, Nm^{-1}$ and atmospheric pressure $= 1.013 \times 10^{5}\, Nm^{-2}$) **[AMU (Med.) 2002]**

(a) $2.054 \times 10^{3}\, Pa$ (b) $1.027 \times 10^{3}\, Pa$ (c) $1.027 \times 10^{5}\, Pa$ (d) $2.054 \times 10^{5}\, Pa$

Solution : (c) Pressure inside a bubble when it is in a liquid $= P_o + \frac{2T}{R} = 1.013 \times 10^{5} + 2 \times \frac{70 \times 10^{-3}}{0.1 \times 10^{-3}} = 1.027 \times 10^{5}\, Pa$.

***Problem* 16.** If the radius of a soap bubble is four times that of another, then the ratio of their excess pressures will be **[AIIMS 2000]**

(a) 1 : 4 (b) 4 : 1 (c) 16 : 1 (d) 1 : 16

Solution : (a) Excess pressure inside a soap bubble $\Delta P = \frac{4T}{r} \Rightarrow \frac{\Delta P_1}{\Delta P_2} = \frac{r_2}{r_1} = 1:4$

***Problem* 17.** Pressure inside two soap bubbles are 1.01 and 1.02 atmospheres. Ratio between their volumes is **[MP PMT 1991]**

(a) 102 : 101 (b) $(102)^3 : (101)^3$ (c) 8 : 1 (d) 2 : 1

Solution : (c) Excess pressure $\Delta P = P_{in} - P_{\text{out}} = 1.01 atm - 1 atm = 0.01 atm$ and similarly $\Delta P_2 = 0.02 atm$

and volume of air bubble $V = \frac{4}{3}\pi r^3 \quad \therefore V \propto r^3 \propto \frac{1}{(\Delta P)^3}$ [as $\Delta P \propto \frac{1}{r}$ or $r \propto \frac{1}{\Delta P}$]

$$\therefore \frac{V_1}{V_2} = \left(\frac{\Delta P_2}{\Delta P_1}\right)^3 = \left(\frac{0.02}{0.01}\right)^3 = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$$

***Problem* 18.** The excess pressure inside an air bubble of radius *r* just below the surface of water is P_1. The excess pressure inside a drop of the same radius just outside the surface is P_2. If *T* is surface tension then

(a) $P_1 = 2P_2$ (b) $P_1 = P_2$ (c) $P_2 = 2P_1$ (d) $P_2 = 0,\ P_1 \neq 0$

Solution : (b) Excess pressure inside a bubble just below the surface of water $P_1 = \frac{2T}{r}$

and excess pressure inside a drop $P_2 = \frac{2T}{r} \qquad \therefore P_1 = P_2$

10.13 Shape of Liquid Meniscus.

We know that a liquid assumes the shape of the vessel in which it is contained *i.e.* it can not oppose permanently any force that tries to change its shape. As the effect of force is zero in a direction perpendicular to it, the free surface of liquid at rest adjusts itself at right angles to the resultant force.

When a capillary tube is dipped in a liquid, the liquid surface becomes curved near the point of contact. This curved surface is due to the resultant of two forces *i.e.* the force of cohesion and the force of adhesion. The curved surface of the liquid is called meniscus of the liquid.

If liquid molecule *A* is in contact with solid (*i.e.* wall of capillary tube) then forces acting on molecule *A* are

(i) Force of adhesion F_a (acts outwards at right angle to the wall of the tube).

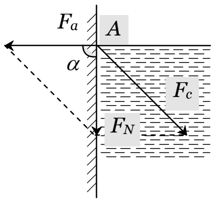
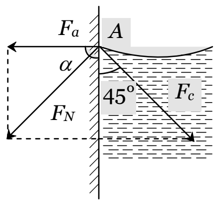
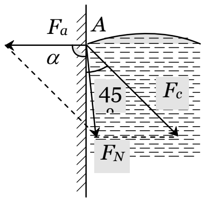
(ii) Force of cohesion F_c (acts at an angle 45^o to the vertical).

Resultant force F_N depends upon the value of F_a and F_c.

If resultant force F_N make an angle α with F_a.

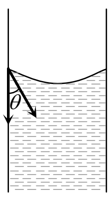
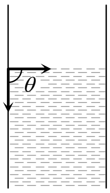
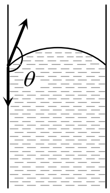
Then $\tan\alpha = \dfrac{F_c \sin 135^o}{F_a + F_c \cos 135^o} = \dfrac{F_c}{\sqrt{2}F_a - F_c}$

By knowing the direction of resultant force we can find out the shape of meniscus because the free surface of the liquid adjust itself at right angle to this resultant force.

If $F_c = \sqrt{2}Fa$	$F_c < \sqrt{2}Fa$	$F_c > \sqrt{2}Fa$
$\tan\alpha = \infty$ $\therefore$ $\alpha = 90^o$	$\tan\alpha$ = positive $\therefore$ α is acute angle	$\tan\alpha$ = negative $\therefore$ α is obtuse angle
i.e. the resultant force acts vertically downwards. Hence the liquid meniscus must be horizontal.	*i.e.* the resultant force directed outside the liquid. Hence the liquid meniscus must be concave upward.	*i.e.* the resultant force directed inside the liquid. Hence the liquid meniscus must be convex upward.
		
Example: Pure water in silver coated capillary tube.	Example: Water in glass capillary tube.	Example: Mercury in glass capillary tube.

10.14 Angle of Contact

Angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both the tangents being drawn at the point of contact of the liquid with the solid.

$\theta < 90^o$ $F_a > \dfrac{F_c}{\sqrt{2}}$ concave meniscus.  Liquid wets the solid surface	$\theta = 90^o$ $F_a = \dfrac{F_c}{\sqrt{2}}$ plane meniscus.  Liquid does not wet the solid surface.	$\theta > 90^o$ $F_a < \dfrac{F_c}{\sqrt{2}}$ convex meniscus.  Liquid does not wet the solid surface.

Important points

(i) Its value lies between 0° and 180°

$\theta = 0^\circ$ for pure water and glass, $\theta = 8^\circ$ for tap water and glass, $\theta = 90^\circ$ for water and silver

$\theta = 138^\circ$ for mercury and glass, $\theta = 160^\circ$ for water and chromium

(ii) It is particular for a given pair of liquid and solid. Thus the angle of contact changes with the pair of solid and liquid.

(iii) It does not depends upon the inclination of the solid in the liquid.

(iv) On increasing the temperature, angle of contact decreases.

(v) Soluble impurities increases the angle of contact.

(vi) Partially soluble impurities decreases the angle of contact.

10.15 Capillarity

If a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called capillarity.

The root cause of capillarity is the difference in pressures on two sides of (concave and convex) curved surface of liquid.

Examples of capillarity :

(i) Ink rises in the fine pores of blotting paper leaving the paper dry.

(ii) A towel soaks water.

(iii) Oil rises in the long narrow spaces between the threads of a wick.

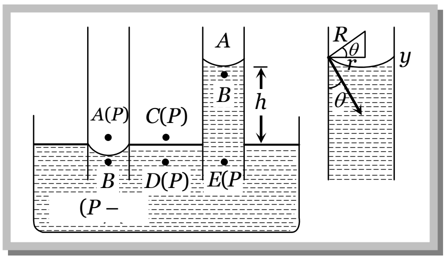
(iv) Wood swells in rainy season due to rise of moisture from air in the pores.

(v) Ploughing of fields is essential for preserving moisture in the soil.

(vi) Sand is drier soil than clay. This is because holes between the sand particles are not so fine as compared to that of clay, to draw up water by capillary action.

10.16 Ascent Formula

When one end of capillary tube of radius r is immersed into a liquid of density d which wets the sides of the capillary tube (water and capillary tube of glass), the shape of the liquid meniscus in the tube becomes concave upwards.



R = radius of curvature of liquid meniscus.

T = surface tension of liquid

P = atmospheric pressure

Pressure at point A = P, Pressure at point B = $P - \frac{2T}{R}$

Pressure at points C and D just above and below the plane surface of liquid in the vessel is also P (atmospheric pressure). The points B and D are in the same horizontal plane in the liquid but the pressure at these points is different.

In order to maintain the equilibrium the liquid level rises in the capillary tube upto height h.

Pressure due to liquid column = pressure difference due to surface tension

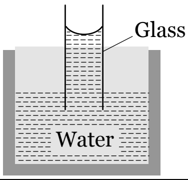
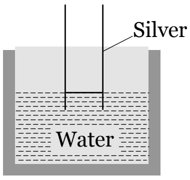
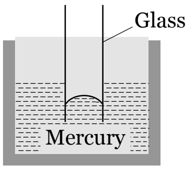
$$\Rightarrow \qquad hdg = \frac{2T}{R}$$

$$\therefore \qquad h = \frac{2T}{Rdg} = \frac{2T\cos\theta}{rdg} \qquad \left[\text{As } R = \frac{r}{\cos\theta}\right]$$

Important points

(i) The capillary rise depends on the nature of liquid and solid both *i.e.* on T, d, θ and R.

(ii) Capillary action for various liquid-solid pair.

	Meniscus	Angle of contact	Level
	Concave	$\theta < 90^o$	Rises
	Plane	$\theta = 90^o$	No rise no fall
	Convex	$\theta > 90^o$	Fall

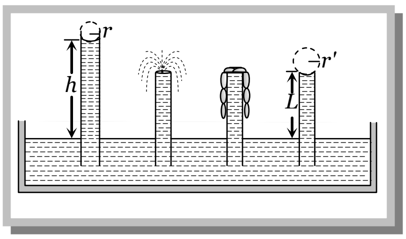
(iii) For a given liquid and solid at a given place

$$h \propto \frac{1}{r} \qquad \text{[As } T,\ \theta,\ d \text{ and } g \text{ are constant]}$$

i.e. lesser the radius of capillary greater will be the rise and vice-versa. This is called Jurin's law.

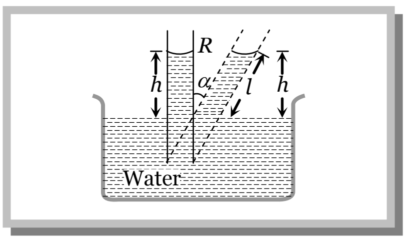
(iv) If the weight of the liquid contained in the meniscus is taken into consideration then more accurate ascent formula is given by

$$h = \frac{2T\cos\theta}{rdg} - \frac{r}{3}$$



(v) In case of capillary of insufficient length, *i.e.*, $L < h$, the liquid will neither overflow from the upper end like a fountain nor will it tickle along the vertical sides of the tube. The liquid after reaching the upper end will increase the radius of its meniscus without changing nature such that :

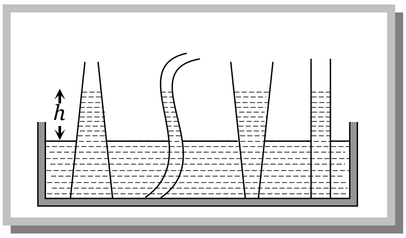
$$hr = Lr' \quad \because\ L < h \qquad \therefore\ r' > r$$



(vi) If a capillary tube is dipped into a liquid and tilted at an angle α from vertical, then the vertical height of liquid column remains same whereas the length of liquid column (l) in the capillary tube increases.

$$h = l\cos\alpha \text{ or } l = \frac{h}{\cos\alpha}$$

(vii) It is important to note that in equilibrium the height h is independent of the shape of capillary if the radius of meniscus remains the same. That is why the vertical height h of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same.



Sample problems based on Capillarity

***Problem* 19.** Water rises to a height of 10*cm* in a capillary tube and mercury falls to a depth of 3.5*cm* in the same capillary tube. If the density of mercury is 13.6 *gm/cc* and its angle of contact is 135^o and density of water is 1 *gm/cc* and its angle of contact is 0^o, then the ratio of surface tensions of the two liquids is $(\cos 135^o = 0.7)$

[MP PMT 1988; EAMCET (Med.) 2003]

(a) 1 : 14 (b) 5 : 34 (c) 1 : 5 (d) 5 : 27

Solution : (b) $h = \frac{2T\cos\theta}{rdg}$ $\therefore \frac{h_W}{h_{Hg}} = \frac{T_W}{T_{Hg}} \frac{\cos\theta_W}{\cos\theta_{Hg}} \frac{d_{Hg}}{d_W}$ [as r and g are constants]

$$\Rightarrow \frac{10}{3.5} = \frac{T_W}{T_{Hg}} . \frac{\cos 0^o}{\cos 135} \frac{13.6}{1} \Rightarrow \frac{T_W}{T_{Hg}} = \frac{10 \times 0.7}{3.5 \times 13.6} = \frac{20}{136} = \frac{5}{34}$$

***Problem* 20.** Water rises in a vertical capillary tube upto a height of 2.0 *cm*. If the tube is inclined at an angle of 60^o with the vertical, then upto what length the water will rise in the tube **[UPSEAT 2002]**

(a) 2.0 *cm* (b) 4.0 *cm* (c) $\frac{4}{\sqrt{3}}$ *cm* (d) $2\sqrt{2}$ *cm*

Solution : (b) The height upto which water will rise $l = \frac{h}{\cos\alpha} = \frac{2cm}{\cos 60} = 4cm$. [$h$ = vertical height, α = angle with vertical]

***Problem* 21.** Two capillary tubes of same diameter are kept vertically one each in two liquids whose relative densities are 0.8 and 0.6 and surface tensions are 60 and 50 *dyne/cm* respectively. Ratio of heights of liquids in the two tubes $\frac{h_1}{h_2}$ is **[MP PMT 2002]**

(a) $\frac{10}{9}$ (b) $\frac{3}{10}$ (c) $\frac{10}{3}$ (d) $\frac{9}{10}$

Solution : (d) $h = \frac{2T\cos\theta}{rdg}$ [If diameter of capillaries are same and taking value of θ same for both liquids]

$$\therefore \frac{h_1}{h_2} = \left(\frac{T_1}{T_2}\right)\left(\frac{d_2}{d_1}\right) = \left(\frac{60}{50}\right) \times \left(\frac{0.6}{0.8}\right) = \left(\frac{36}{40}\right) = \frac{9}{10}.$$

***Problem* 22.** A capillary tube of radius R is immersed in water and water rises in it to a height H. Mass of water in the capillary tube is M. If the radius of the tube is doubled, mass of water that will rise in the capillary tube will now be **[RPMT 1997; RPET 1999; CPMT 2002]**

(a) M (b) $2M$ (c) $M/2$ (d) $4M$

Solution : (b) Mass of the liquid in capillary tube $M = V\rho = (\pi r^2 h)\rho$ $\therefore M \propto r^2 h \propto r$ [As $h \propto \frac{1}{r}$]

So if radius of the tube is doubled, mass of water will becomes $2M$, which will rise in capillary tube.

***Problem* 23.** Water rises to a height h in a capillary at the surface of earth. On the surface of the moon the height of water column in the same capillary will be **[MP PMT 2001]**

(a) $6h$ (b) $\frac{1}{6}h$ (c) h (d) Zero

Solution : (a) $h = \frac{2T\cos\theta}{rdg}$ $\therefore h \propto \frac{1}{g}$ [If other quantities remains constant]

$\frac{h_{moon}}{h_{earth}} = \frac{g_{earth}}{g_{moon}} = 6 \Rightarrow h_{moon} = 6h$ [As $g_{earth} = 6g_{moon}$]

***Problem* 24.** Water rises upto a height h in a capillary on the surface of earth in stationary condition. Value of h increases if this tube is taken

(a) On sun (b) On poles
(c) In a lift going upward with acceleration (d) In a lift going downward with acceleration

Solution : (d) $h \propto \frac{1}{g}$. In a lift going downward with acceleration (a), the effective acceleration decreases. So h increases.

***Problem* 25.** If the surface tension of water is 0.06 N/m, then the capillary rise in a tube of diameter 1mm is ($\theta = 0^o$) **[AFMC 1998]**

(a) 1.22 cm (b) 2.44 cm (c) 3.12 cm (d) 3.86 cm

Solution : (b) $h = \frac{2T\cos\theta}{rdg}$, [$\theta$=0, $r = \frac{1}{2}mm = 0.5\times10^{-3}m$, $T = 0.06 N/m$, $d = 10^3 kg/m^3$, $g = 9.8\ m/s^2$]

$$h = \frac{2\times0.06\times\cos\theta}{0.5\times10^{-3}\times10^3\times9.8} = 0.0244m = 2.44cm$$

***Problem* 26.** Two capillaries made of same material but of different radii are dipped in a liquid. The rise of liquid in one capillary is 2.2cm and that in the other is 6.6cm. The ratio of their radii is **[MP PET 1990]**

(a) 9 : 1 (b) 1 : 9 (c) 3 : 1 (d) 1 : 3

Solution : (c) As $h \propto \frac{1}{r}$ $\therefore \frac{h_1}{h_2} = \frac{r_2}{r_1}$ or $\frac{r_1}{r_2} = \frac{h_2}{h_1} = \frac{6.6}{2.2} = \frac{3}{1}$

***Problem* 27.** The lower end of a capillary tube is at a depth of 12cm and the water rises 3cm in it. The mouth pressure required to blow an air bubble at the lower end will be X cm of water column where X is **[CPMT 1989]**

(a) 3 (b) 9 (c) 12 (d) 15

Solution : (d) The lower end of capillary tube is at a depth of 12 + 3 = 15 cm from the free surface of water in capillary tube.

So, the pressure required = 15 cm of water column.

***Problem* 28.** The lower end of a capillary tube of radius r is placed vertically in water. Then with the rise of water in the capillary, heat evolved is

(a) $+\frac{\pi^2r^2h^2}{J}dg$ (b) $+\frac{\pi r^2h^2dg}{2J}$ (c) $-\frac{\pi r^2h^2dg}{2J}$ (d) $-\frac{\pi r^2h^2dg}{J}$

Solution : (b) When the tube is placed vertically in water, water rises through height h given by $h = \frac{2T\cos\theta}{rdg}$

Upward force $= 2\pi r \times T\cos\theta$

Work done by this force in raising water column through height h is given by

$$\Delta W = (2\pi rT\cos\theta)h = (2\pi rh\cos\theta)T = (2\pi rh\cos\theta)\left(\frac{rhdg}{2\cos\theta}\right) = \pi r^2h^2dg$$

However, the increase in potential energy ΔE_p of the raised water column $= mg\frac{h}{2}$

where m is the mass of the raised column of water $\therefore m = \pi r^2 hd$

So, $\Delta E_P = (\pi r^2 hd)\left(\frac{hg}{2}\right) = \frac{\pi r^2 h^2 dg}{2}$

Further, $\Delta W - \Delta E_p = \frac{\pi r^2 h^2 dg}{2}$

The part $(\Delta W - \Delta E_P)$ is used in doing work against viscous forces and frictional forces between water and glass surface and appears as heat. So heat released $= \frac{\Delta W - \Delta E_p}{J} = \frac{\pi r^2 h^2 dg}{2J}$

Problem 29. Water rises in a capillary tube to a certain height such that the upward force due to surface tension is balanced by $75 \times 10^{-4} N$ force due to the weight of the liquid. If the surface tension of water is $6 \times 10^{-2}\ N/m$, the inner circumference of the capillary must be

(a) $1.25 \times 10^{-2}\ m$ (b) $0.50 \times 10^{-2}\ m$ (c) $6.5 \times 10^{-2}\ m$ (d) $12.5 \times 10^{-2}\ m$

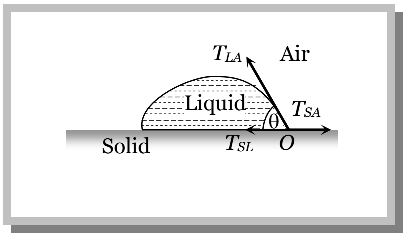
Solution : (d) Weight of liquid = upward force due to surface tension

$75 \times 10^{-4} = 2\pi r T$

Circumference $2\pi r = \frac{75 \times 10^{-4}}{T} = \frac{75 \times 10^{-4}}{6 \times 10^{-2}} = 0.125 = 12.5 \times 10^{-2} m$

10.17 Shape of Drops

Whether the liquid will be in equilibrium in the form of a drop or it will spread out; depends on the relative strength of the force due to surface tension at the three interfaces.

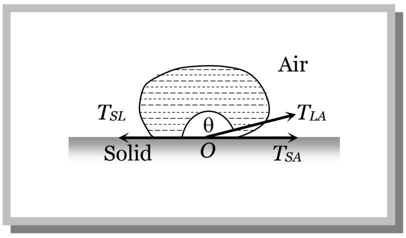


T_{LA} = surface tension at liquid-air interface, T_{SA} = surface tension at solid-air interface.

T_{SL} = surface tension at solid-liquid interface, θ = angle of contact between the liquid and solid.

For the equilibrium of molecule

$T_{SL} + T_{LA}\cos\theta = T_{SA}$ or $\cos\theta = \frac{T_{SA} - T_{SL}}{T_{LA}}$ .....(i)



Special Cases

$T_{SA} > T_{SL}$, $\cos\theta$ is positive *i.e.* $0^o < \theta < 90^o$.

This condition is fulfilled when the molecules of liquid are strongly attracted to that of solid.

Example : (i) Water on glass.

(ii) Kerosene oil on any surface.

$T_{SA} < T_{SL}$, $\cos\theta$ is negative *i.e.* $90^o < \theta < 180^o$.

This condition is fulfilled when the molecules of the liquid are strongly attracted to themselves and relatively weakly to that of solid.

Example : (i) Mercury on glass surface.

(ii) Water on lotus leaf (or a waxy or oily surface)

(T_{SL} + T_{LA} cosθ) > T_{SA}

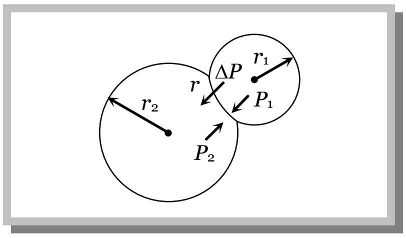
In this condition, the molecule of liquid will not be in equilibrium and experience a net force at the interface. As a result, the liquid spreads.

Example : (i) Water on a clean glass plate.

10.18 Useful Facts and Formulae

(1) Formation of double bubble : If r_1 and r_2 are the radii of smaller and larger bubble and P_0 is the atmospheric pressure, then the pressure inside them will be $P_1 = P_0 + \frac{4T}{r_1}$ and $P_2 = P_0 + \frac{4T}{r_2}$.

Now as $r_1 < r_2 \;\therefore\; P_1 > P_2$



So for interface $\Delta P = P_1 - P_2 = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ .....(i)

As excess pressure acts from concave to convex side, the interface will be concave towards the smaller bubble and convex towards larger bubble and if r is the radius of interface.

$$\Delta P = \frac{4T}{r} \quad .....(ii)$$

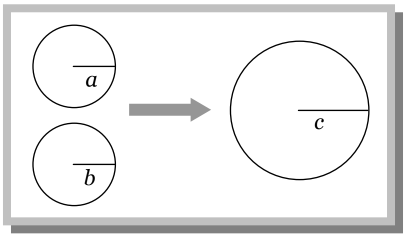
From (i) and (ii) $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$

$\therefore$ Radius of the interface $r = \frac{r_1 r_2}{r_2 - r_1}$

(2) Formation of a single bubble

(i) Under isothermal condition two soap bubble of radii 'a' and 'b' coalesce to form a single bubble of radius 'c'.

If the external pressure is P_0 then pressure inside bubbles



$$P_a = \left(P_0 + \frac{4T}{a}\right),\; P_b = \left(P_0 + \frac{4T}{b}\right) \text{ and } P_c = \left(P_0 + \frac{4T}{c}\right)$$

and volume of the bubbles

$$V_a = \frac{4}{3}\pi a^3,\; V_b = \frac{4}{3}\pi b^3,\; V_c = \frac{4}{3}\pi c^3$$

Now as mass is conserved $\mu_a + \mu_b = \mu_c \Rightarrow \frac{P_a V_a}{RT_a} + \frac{P_b V_b}{RT_b} = \frac{P_c V_c}{RT_c}$ $\left[\text{As } PV = \mu RT,\; i.e.,\; \mu = \frac{PV}{RT}\right]$

$\Rightarrow$ $P_a V_a + P_b V_b = P_c V_c$ .....(i) [As temperature is constant, *i.e.*, $T_a = T_b = T_c$]

Substituting the value of pressure and volume

$$\Rightarrow \left[P_0 + \frac{4T}{a}\right]\left[\frac{4}{3}\pi a^3\right] + \left[P_0 + \frac{4T}{b}\right]\left[\frac{4}{3}\pi b^3\right] = \left[P_0 + \frac{4T}{c}\right]\left[\frac{4}{3}\pi c^3\right]$$

$$\Rightarrow 4T(a^2 + b^2 - c^2) = P_0(c^3 - a^3 - b^3)$$

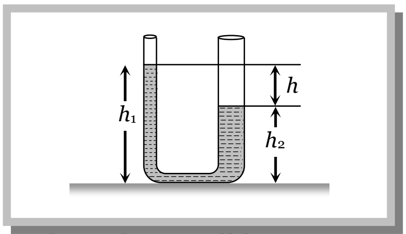
$\therefore$ Surface tension of the liquid $T = \frac{P_0(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$

(ii) If two bubble coalesce in vacuum then by substituting $P_0 = 0$ in the above expression we get

$$a^2 + b^2 - c^2 = 0 \quad \therefore \; c^2 = a^2 + b^2$$

Radius of new bubble $= c = \sqrt{a^2 + b^2}$ or can be expressed as $r = \sqrt{r_1^2 + r_2^2}$.

(3) The difference of levels of liquid column in two limbs of u-tube of unequal radii r_1 and r_2 is



$$h = h_1 - h_2 = \frac{2T\cos\theta}{dg}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

(4) A large force (F) is required to draw apart normally two glass plate enclosing a thin water film because the thin water film formed between the two glass plates will have concave surface all around. Since on the concave side of a liquid surface, pressure is more, work will have to be done in drawing the plates apart.

$F = \dfrac{2AT}{t}$ where T= surface tension of water film, t= thickness of film, A = area of film.

(5) When a soap bubble is charged, then its size increases due to outward force on the bubble.

(6) The materials, which when coated on a surface and water does not enter through that surface are known as water proofing agents. For example wax *etc*. Water proofing agent increases the angle of contact.

(7) Values of surface tension of some liquids.

Liquid	**Surface tension *Newton/metre***
Mercury	0.465
Water	0.075
Soap solution	0.030
Glycerine	0.063
Carbon tetrachloride	0.027
Ethyl alcohol	0.022

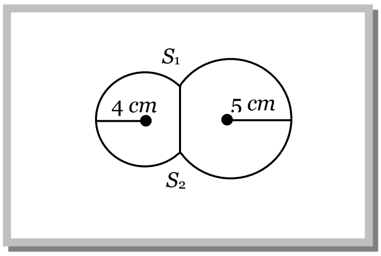
Sample problems (Miscellaneous)

***Problem* 30.** The radii of two soap bubbles are r_1 and r_2. In isothermal conditions, two meet together in vacuum. Then the radius of the resultant bubble is given by **[RPET 1999; MP PMT 2001; EAMCET 2003]**

(a) $R = (r_1 + r_2)/2$ (b) $R = r_1(r_1r_2 + r_2)$ (c) $R^2 = r_1^2 + r_2^2$ (d) $R = r_1 + r_2$

Solution : (c) Under isothermal condition surface energy remain constant $\therefore\; 8\pi r_1^2 T + 8\pi r_2^2 T = 8\pi R^2 T \;\Rightarrow\; R^2 = r_1^2 + r_2^2$

***Problem* 31.** Two soap bubbles of radii r_1 and r_2 equal to 4*cm* and 5*cm* are touching each other over a common surface S_1S_2 (shown in figure). Its radius will be **[MP PMT 2002]**



(a) 4 *cm*

(b) 20 *cm*

(c) 5 *cm*

(d) 4.5 *cm*

Solution : (b) Radius of curvature of common surface of double bubble $r = \frac{r_2 r_1}{r_2 - r_1} = \frac{5 \times 4}{5 - 4} = 20 cm$

***Problem* 32.** An air bubble in a water tank rises from the bottom to the top. Which of the following statements are true **[Roorkee 2000]**

(a) Bubble rises upwards because pressure at the bottom is less than that at the top

(b) Bubble rises upwards because pressure at the bottom is greater than that at the top

(c) As the bubble rises, its size increases

(d) As the bubble rises, its size decreases

Solution : (b, c)

***Problem* 33.** The radii of two soap bubbles are R_1 and R_2 respectively. The ratio of masses of air in them will be

(a) $\frac{R_1^3}{R_2^3}$ (b) $\frac{R_2^3}{R_1^3}$ (c) $\left(\frac{P + \frac{4T}{R_1}}{P + \frac{4T}{R_2}}\right)\frac{R_1^3}{R_2^3}$ (d) $\left(\frac{P + \frac{4T}{R_2}}{P + \frac{4T}{R_1}}\right)\frac{R_2^3}{R_1^3}$

Solution : (c) From $PV = \mu RT$.

At a given temperature, the ratio masses of air $\frac{\mu_1}{\mu_2} = \frac{P_1 V_1}{P_2 V_2} = \frac{\left(P + \frac{4T}{R_1}\right)\frac{4}{3}\pi R_1^3}{\left(P + \frac{4T}{R_2}\right)\frac{4}{3}\pi R_2^3} = \frac{\left(P + \frac{4T}{R_1}\right)}{\left(P + \frac{4T}{R_2}\right)}\frac{R_1^3}{R_2^3}$.

***Problem* 34.** On dipping one end of a capillary in liquid and inclining the capillary at an angles 30^o and 60^o with the vertical, the lengths of liquid columns in it are found to be l_1 and l_2 respectively. The ratio of l_1 and l_2 is

(a) $1 : \sqrt{3}$ (b) $1 : \sqrt{2}$ (c) $\sqrt{2} : 1$ (d) $\sqrt{3} : 1$

Solution : (a) $l_1 = \frac{h}{\cos\alpha_1}$ and $l_2 = \frac{h}{\cos\alpha_2}$ $\therefore \frac{l_1}{l_2} = \frac{\cos\alpha_2}{\cos\alpha_1} = \frac{\cos 60^o}{\cos 30^o} = \frac{1/2}{\sqrt{3}/2} = 1 : \sqrt{3}$

***Problem* 35.** A drop of water of volume V is pressed between the two glass plates so as to spread to an area A. If T is the surface tension, the normal force required to separate the glass plates is

(a) $\frac{TA^2}{V}$ (b) $\frac{2TA^2}{V}$ (c) $\frac{4TA^2}{V}$ (d) $\frac{TA^2}{2V}$

Solution : (b) Force required to separate the glass plates $F = \frac{2AT}{t} \times \frac{A}{A} = \frac{2TA^2}{(A \times t)} = \frac{2TA^2}{V}$.

Problems based on Cohesive and adhesive force

1. Mercury does not wet glass, wood or iron because **[MP PMT 1995; MP PET 1997]**

(a) Cohesive force is less than adhesive force (b) Cohesive force is greater than adhesive force

(c) Angle of contact is less than 90° (d) Cohesive force is equal to adhesive force

2. The force of cohesion is **[CPMT 1996]**

(a) Maximum in solids (b) Maximum in liquid (c) Same in different matters (d) Maximum in gases

3. What enables us to write on the black board with chalk

(a) Gravity (b) Cohesion (c) Adhesion (d) None of the above

4. Intermolecular forces decrease rapidly as the distance between the molecules increases and do so much more

(a) Slowly than demanded by the inverse square law of the distance

(b) Rapidly than anticipated through the inverse square law of the distance

(c) According to inverse square law

(d) It actually remains the same for all the distances

Problems based on Surface tension

5. The spherical shape of rain-drop is due to

[CPMT 1976, 90; CPMT 2001; NCERT 1982; AIIMS 1998; MHCET 2000; DCE 1999; AFMC 1999, 2001]

(a) Density of the liquid (b) Surface tension (c) Atmospheric pressure (d) Gravity

6. At which of the following temperatures, the value of surface tension of water is minimum **[MP PMT/PET 1998]**

(a) $4^{\circ}C$ (b) $25^{\circ}C$ (c) $50^{\circ}C$ (d) $75^{\circ}C$

7. Force necessary to pull a circular plate of 5*cm* radius from water surface for which surface tension is 75 *dynes*/*cm*, is **[MP PMT 1991]**

(a) 30 *dynes* (b) 60 *dynes* (c) 750 *dynes* (d) 750π *dynes*

8. A square frame of side L is dipped in a liquid. On taking it out, a membrane is formed. If the surface tension of the liquid is T, the force acting on the frame will be **[MP PMT 1990]**

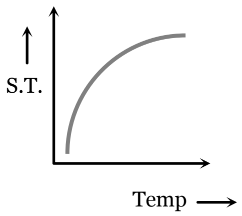
(a) $2TL$ (b) $4TL$ (c) $8TL$ (d) $10TL$

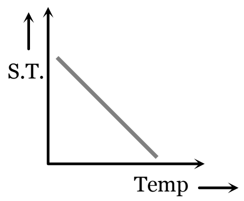
9. Ball pen and fountain pen depend respectively upon the principle of

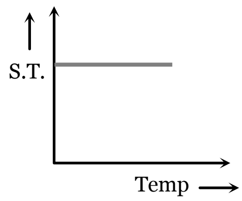
(a) Surface tension and viscosity (b) Surface tension and gravity

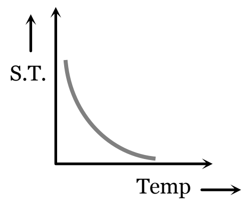
(c) Gravitation and surface tension (d) Surface tension and surface tension

10. Which graph represents the variation of surface tension with temperature over small temperature ranges for water

(a) 

(b) 

(c) 

(d) 

11. The material of a wire has a density of 1.4 g per cm^3. If it is not wetted by a liquid of surface tension 44 *dyne* per *cm*, then the maximum radius of the wire which can float on the surface of the liquid is

(a) $\frac{1}{7}\ cm$ (b) 0.7 *cm* (c) $\frac{10}{14}\ cm$ (d) $\frac{10}{28}\ cm$

12. A water drop of 0.05cm^3 is squeezed between two glass plates and spreads into area of 40cm^2. If the surface tension of water is 70 *dyne*/*cm* then the normal force required to separate the glass plates from each other will be

(a) 90 N (b) 45 N (c) 22.5 N (d) 450 N

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(a) The liquid surface has a tendency to contract but the stretched membrane does not

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16. A hollow disc of aluminum whose external and internal radii are R and r respectively, is floating on the surface of a liquid whose surface tension is T. The maximum weight of disc can be

(a) $2\pi(R + r)\,T$ (b) $2\pi(R - r)\,T$ (c) $4\pi(R + r)\,T$ (d) $4\pi(R - r)\,T$

Problems based on Surface energy

17. 8000 identical water drops are combined to form a big drop. Then the ratio of the final surface energy to the initial surface energy of all the drops together is

(a) 1 : 10 (b) 1 : 15 (c) 1 : 20 (d) 1 : 25

18. 8 mercury drops coalesce to form one mercury drop, the energy changes by a factor of **[DCE 2000]**

(a) 1 (b) 2 (c) 4 (d) 6

19. Which of the following statements are true in case when two water drops coalesce and make a bigger drop **[Roorkee 1999]**

(a) Energy is released

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20. An oil drop of radius 1*cm* is sprayed into 1000 small equal drops of same radius. If the surface tension of oil drop is 50 *dyne/cm* then the work done is **[RPET 1990]**

(a) $18\pi\,ergs$ (b) $180\pi\,ergs$ (c) $1800\pi\,ergs$ (d) $18000\pi\,ergs$

21. If work W is done in blowing a bubble of radius R from a soap solution, then the work done in blowing a bubble of radius $2R$ from the same solution is **[MP PET 1990]**

(a) $W/2$ (b) $2W$ (c) $4W$ (d) $2\frac{1}{3}W$

22. A liquid drop of radius R is broken up into N small droplets. The work done is proportional to

(a) N (b) $N^{2/3}$ (c) $N^{1/3}$ (d) N^0

23. The work done in increasing the volume of a soap bubble of radius R and surface tension T by 700% will be

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24. 1000 drops of water all of same size join together to form a single drop and the energy released raises the temperature of the drop. Given that T is the surface tension of water, r the radius of each small drop, ρ the density of liquid, J the mechanical equivalent of heat. What is the rise in the temperature

(a) T/Jr (b) $10T/Jr$ (c) $100T/Jr$ (d) None of these

Problems based on Excess pressure

25. Two bubbles A and B ($A > B$) are joined through a narrow tube. Then **[UPSEAT 2001; Kerala (Med.) 2002]**

(a) The size of A will increase (b) The size of B will increase

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26. Excess pressure of one soap bubble is four times more than the other. Then the ratio of volume of first bubble to another one is **[CPMT 1997; MH CET 2000]**

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27. The pressure of air in a soap bubble of 0.7*cm* diameter is 8*mm* of water above the pressure outside. The surface tension of the soap solution is **[MP PET 1991; MP PMT 1997]**

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28. An air bubble of radius r in water is at a depth h below the water surface at some instant. If P is atmospheric pressure, d and T are density and surface tension of water respectively, the pressure inside the bubble will be **[Roorkee 1990]**

(a) $P + h\,dg - \frac{4T}{r}$ (b) $P + h\,dg + \frac{2T}{r}$ (c) $P + h\,dg - \frac{2T}{r}$ (d) $P + h\,dg + \frac{4T}{r}$

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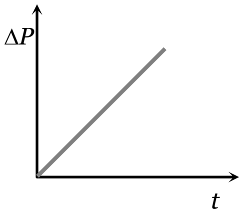
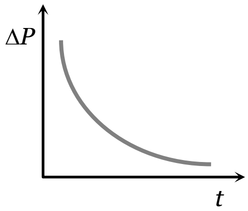
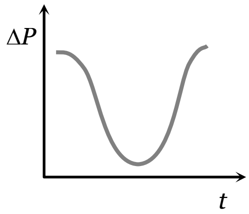
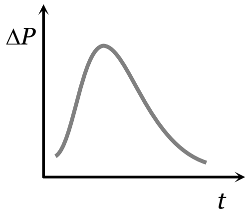
(a) 

(b) 

(c) 

(d) 

Problems based on Angle of contact

30. A liquid does not wet the sides of a solid, if the angle of contact is

[MP PAT 1990; AFMC 1988, MNR 1998, KCET 1998, Haryana CEE 1998; RPMT 1999; 2003]

(a) Zero (b) Obtuse (More than 90°) (c) Acute (Less than 90°) (d) 90^{oo}

31. The meniscus of mercury in the capillary tube is **[MP PET/PMT 1988]**

(a) Convex (b) Concave (c) Plane (d) Uncertain

32. The angle of contact between glass and mercury is **[MP PMT 1987]**

(a) 0^{o} (b) 30^{o} (c) 90^{o} (d) 135^{o}

33. When the temperature is increased the angle of contact of a liquid

(a) Increases (b) Decreases

(c) Remains the same (d) First increases and then decreases

34. For those liquids which do not wet the solid surface, the ratio of cohesive force and adhesive force will be

(a) Greater than $\frac{1}{\sqrt{2}}$ (b) Greater than $\sqrt{2}$ (c) Lesser than $\frac{1}{\sqrt{2}}$ (d) Lesser than $\sqrt{2}$

35. The water proofing agent makes an angle of contact

(a) From acute angle to obtuse angle (b) From obtuse angle to acute angle

(c) From obtuse angle to right angle (d) From acute angle to right angle

36. A glass plate is partly dipped vertically in the mercury and the angle of contact is measured. If the plate is inclined, then the angle of contact will

(a) Increase (b) Remain unchanged (c) Increase or decrease (d) Decrease

Problems based on Capillarity

37. The surface tension for pure water in a capillary tube experiment is **[MH CET 2002]**

(a) $\frac{\rho g}{2hr}$ (b) $\frac{2}{hr\rho g}$ (c) $\frac{r\rho g}{2h}$ (d) $\frac{hr\rho g}{2}$

38. If capillary experiment is performed in vacuum then for a liquid there

(a) It will rise (b) Will remain same (c) It will fall (d) Rise to the top

39. A surface tension experiment with a capillary tube in water is repeated in an artificial satellite. Which is revolving around the earth, water will rise in the capillary tube upto a height of **[Roorkee 1992]**

(a) 0.1 *m* (b) 0.2 *m*
(c) 0.98 *m* (d) Full length of the capillary tube

40. When a capillary is dipped in water, water rises to a height *h*. If the length of the capillary is made less than *h*, then **[MP PAT 1990]**

(a) The water will come out (b) The water will not come out
(c) The water will not rise (d) The water will rise but less than height of capillary

41. A long cylindrical glass vessel has a small hole of radius '*r*' at its bottom. The depth to which the vessel can be lowered vertically in the deep water bath (surface tension *T*) without any water entering inside is **[MP PMT 1990]**

(a) $4T/\rho rg$ (b) $3T/\rho rg$ (c) $2T/\rho rg$ (d) $T/\rho rg$

42. Water rises to a height of 10*cm* in capillary tube and mercury falls to a depth of 3.112*cm* in the same capillary tube. If the density of mercury is 13.6 and the angle of contact for mercury is 135^o, the ratio of surface tension of water and mercury is**[MP PET/PMT 1988]**

(a) 1 : 0.15 (b) 1 : 3 (c) 1 : 6 (d) 1.5 : 1

43. Water can rise to a height *h* in a capillary tube lowered vertically into water. If the height of tube above the surface of water be *l* and *l* < *h*, then water will rise in the capillary to a height

(a) h (b) l (c) $l-h$ (d) $l+h$

44. The height upto which water will rise in a capillary tube will be

(a) Maximum when water temperature is $4^o C$ (b) Maximum when water temperature is $0^o C$
(c) Minimum when water temperature is $4^o C$ (d) Same at all temperatures

45. The exact expression for surface tension of liquid which rises up in the capillary tube is

(a) $T = rhdg/2$ (b) $T = rhdg/2\cos\theta$ (c) $T = \frac{r(h+r/3)dg}{2}$ (d) $T = \frac{r(h+r/3)dg}{2\cos\theta}$

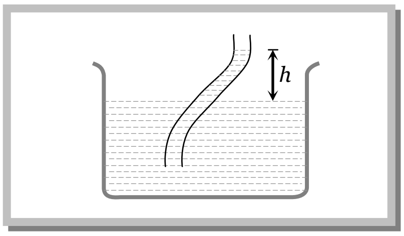
46. If a wax coated capillary tube is dipped in water, then water in it will

(a) Rise up (b) Depress
(c) Sometimes rise and sometimes fall (d) Rise up and come out as a fountain

47. Capillaries made from various materials but having the same bore are dipped in the same liquid, then

(a) Liquid will not rise in any of them
(b) Liquid will rise in all upto same height
(c) Liquid will not rise in all upto same height
(d) Liquid will rise in all and height of liquid columns will be inversely proportional to the density of material used

48. A straight capillary tube is immersed in water and the water rises to 5*cm*. If the capillary is bent as shown in figure then the height of water column will be



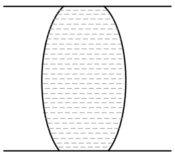
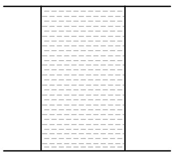
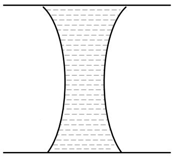
(a) 5*cm*
(b) Less than 5*cm*
(c) Greater than 5*cm*
(d) 4 cos α

49. Water rises in a capillary tube through a height *h*. If the tube is inclined to the liquid surface at 30^o, the liquid will rise in the tube upto its length equal to

(a) $\frac{h}{2}$ (b) h (c) $2h$ (d) $4h$

Problems (Miscellaneous)

50. If a water drop is kept between two glass plates, then its shape is

(a)  (b)  (c)  (d) None of these

51. When two soap bubbles of radius r_1 and r_2 $(r_2 > r_1)$ coalesce, the radius of curvature of common surface is **[MP PMT 1996]**

(a) $r_2 - r_1$ (b) $\frac{r_2 - r_1}{r_1 r_2}$ (c) $\frac{r_1 r_2}{r_2 - r_1}$ (d) $r_2 + r_1$

52. Two soap bubbles of radius 1*cm* and 2*cm* coalesce to form a single drop under isothermal conditions. The total energy possessed by them if surface tension is 30 *dyne cm*$^{-1}$, will be

(a) 400 π *ergs* (b) 600 π *ergs* (c) 1000 π *ergs* (d) 1200 π *ergs*

53. In the above question, the radius of the bigger drop will be

(a) $\sqrt{3}$ *cm* (b) $\sqrt{5}$ *cm* (c) $\sqrt{7}$ *cm* (d) $\sqrt{8}$ *cm*

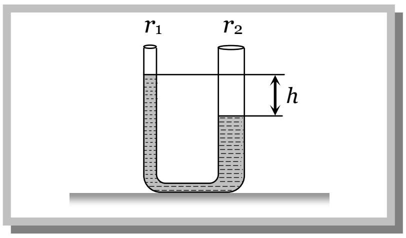
54. In a *U*-tube the radii of two columns are respectively r_1 and r_2 and if a liquid of density *d* filled in it has level difference of *h* then the surface tension of the liquid is

(a) $T = \frac{hdg}{r_2 - r_1}$

(b) $T = \frac{(r_2 - r_1)hdg}{2}$

(c) $T = \frac{(r_1 + r_2)hdg}{2}$

(d) $T = \frac{hdg}{2}\frac{(r_1 r_2)}{r_2 - r_1}$



1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
b	a	c	b	b	d	d	c	c	b
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
a	b	b	a	c	a	c	c	a, d	c
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
c	c	b	d	a	a	b	b	b	b
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
a	d	b	b	a	b	d	a	d	b
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
c	c	b	c	d	b	c	a	c	c
51.	**52.**	**53.**	**54.**						
c	d	b	d						

Practice Problems

Problems based on Cohesive and adhesive force

1. Mercury does not wet glass, wood or iron because **[MP PMT 1995; MP PET 1997]**

(a) Cohesive force is less than adhesive force (b) Cohesive force is greater than adhesive force

(c) Angle of contact is less than 90^o (d) Cohesive force is equal to adhesive force

2. The force of cohesion is **[CPMT 1996]**

(a) Maximum in solids (b) Maximum in liquid (c) Same in different matters (d) Maximum in gases

3. What enables us to write on the black board with chalk

(a) Gravity (b) Cohesion (c) Adhesion (d) None of the above

4. Intermolecular forces decrease rapidly as the distance between the molecules increases and do so much more

(a) Slowly than demanded by the inverse square law of the distance

(b) Rapidly than anticipated through the inverse square law of the distance

(c) According to inverse square law

(d) It actually remains the same for all the distances

Problems based on Surface tension

5. The spherical shape of rain-drop is due to

[CPMT 1976, 90; CPMT 2001; NCERT 1982; AIIMS 1998; MHCET 2000; DCE 1999; AFMC 1999, 2001]

(a) Density of the liquid (b) Surface tension (c) Atmospheric pressure (d) Gravity

6. At which of the following temperatures, the value of surface tension of water is minimum **[MP PMT/PET 1998]**

(a) $4^o C$ (b) $25^o C$ (c) $50^o C$ (d) $75^o C$

7. Force necessary to pull a circular plate of 5*cm* radius from water surface for which surface tension is 75 *dynes*/*cm*, is **[MP PMT 1991]**

(a) 30 *dynes* (b) 60 *dynes* (c) 750 *dynes* (d) 750π *dynes*

8. A square frame of side L is dipped in a liquid. On taking it out, a membrane is formed. If the surface tension of the liquid is T, the force acting on the frame will be **[MP PMT 1990]**

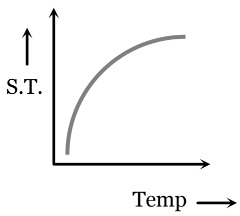
(a) $2TL$ (b) $4TL$ (c) $8TL$ (d) $10TL$

9. Ball pen and fountain pen depend respectively upon the principle of

(a) Surface tension and viscosity (b) Surface tension and gravity

(c) Gravitation and surface tension (d) Surface tension and surface tension

10. Which graph represents the variation of surface tension with temperature over small temperature ranges for water

(a)


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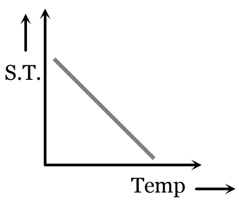

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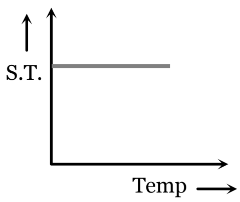

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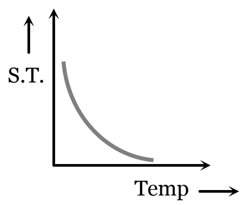

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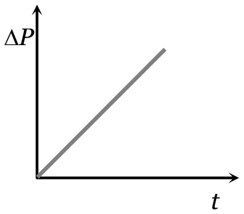

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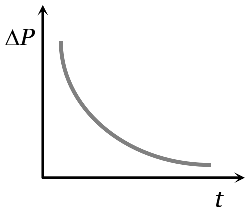

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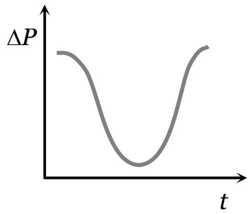

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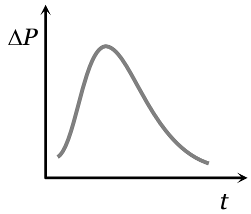

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(a) Convex (b) Concave (c) Plane (d) Uncertain

32. The angle of contact between glass and mercury is **[MP PMT 1987]**

(a) 0^o (b) 30^o (c) 90^o (d) 135^o

33. When the temperature is increased the angle of contact of a liquid

(a) Increases (b) Decreases

(c) Remains the same (d) First increases and then decreases

34. For those liquids which do not wet the solid surface, the ratio of cohesive force and adhesive force will be

(a) Greater than $\frac{1}{\sqrt{2}}$ (b) Greater than $\sqrt{2}$ (c) Lesser than $\frac{1}{\sqrt{2}}$ (d) Lesser than $\sqrt{2}$

35. The water proofing agent makes an angle of contact

(a) From acute angle to obtuse angle (b) From obtuse angle to acute angle

(c) From obtuse angle to right angle (d) From acute angle to right angle

36. A glass plate is partly dipped vertically in the mercury and the angle of contact is measured. If the plate is inclined, then the angle of contact will

(a) Increase (b) Remain unchanged (c) Increase or decrease (d) Decrease

Problems based on Capillarity

37. The surface tension for pure water in a capillary tube experiment is **[MH CET 2002]**

(a) $\frac{\rho g}{2hr}$ (b) $\frac{2}{hr\rho g}$ (c) $\frac{r\rho g}{2h}$ (d) $\frac{hr\rho g}{2}$

38. If capillary experiment is performed in vacuum then for a liquid there

(a) It will rise (b) Will remain same (c) It will fall (d) Rise to the top

39. A surface tension experiment with a capillary tube in water is repeated in an artificial satellite. Which is revolving around the earth, water will rise in the capillary tube upto a height of **[Roorkee 1992]**

(a) 0.1 m (b) 0.2 m

(c) 0.98 m (d) Full length of the capillary tube

40. When a capillary is dipped in water, water rises to a height h. If the length of the capillary is made less than h, then **[MP PAT 1990]**

(a) The water will come out (b) The water will not come out

(c) The water will not rise (d) The water will rise but less than height of capillary

41. A long cylindrical glass vessel has a small hole of radius 'r' at its bottom. The depth to which the vessel can be lowered vertically in the deep water bath (surface tension T) without any water entering inside is **[MP PMT 1990]**

(a) $4T/\rho rg$ (b) $3T/\rho rg$ (c) $2T/\rho rg$ (d) $T/\rho rg$

42. Water rises to a height of 10cm in capillary tube and mercury falls to a depth of 3.112cm in the same capillary tube. If the density of mercury is 13.6 and the angle of contact for mercury is 135^o, the ratio of surface tension of water and mercury is **[MP PET/PMT 1988]**

(a) 1 : 0.15 (b) 1 : 3 (c) 1 : 6 (d) 1.5 : 1

43. Water can rise to a height h in a capillary tube lowered vertically into water. If the height of tube above the surface of water be l and $l < h$, then water will rise in the capillary to a height

(a) h (b) l (c) $l - h$ (d) $l + h$

44. The height upto which water will rise in a capillary tube will be

(a) Maximum when water temperature is $4^o C$ (b) Maximum when water temperature is $0^o C$

(c) Minimum when water temperature is $4^o C$ (d) Same at all temperatures

45. The exact expression for surface tension of liquid which rises up in the capillary tube is

(a) $T = rhdg/2$ (b) $T = rhdg/2\cos\theta$ (c) $T = \frac{r(h+r/3)dg}{2}$ (d) $T = \frac{r(h+r/3)dg}{2\cos\theta}$

46. If a wax coated capillary tube is dipped in water, then water in it will

(a) Rise up (b) Depress

(c) Sometimes rise and sometimes fall (d) Rise up and come out as a fountain

47. Capillaries made from various materials but having the same bore are dipped in the same liquid, then

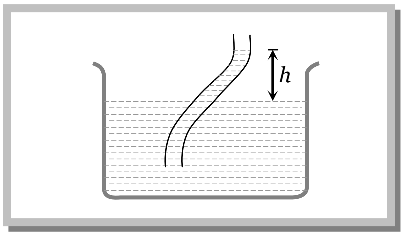
(a) Liquid will not rise in any of them

(b) Liquid will rise in all upto same height

(c) Liquid will not rise in all upto same height

(d) Liquid will rise in all and height of liquid columns will be inversely proportional to the density of material used

48. A straight capillary tube is immersed in water and the water rises to 5cm. If the capillary is bent as shown in figure then the height of water column will be



(a) 5cm

(b) Less than 5cm

(c) Greater than 5cm

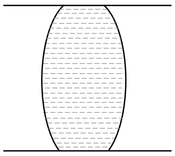
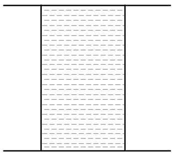
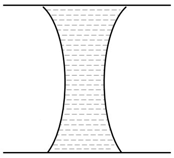
(d) 4 cos α

49. Water rises in a capillary tube through a height h. If the tube is inclined to the liquid surface at 30^o, the liquid will rise in the tube upto its length equal to

(a) $\frac{h}{2}$ (b) h (c) $2h$ (d) $4h$

Problems (Miscellaneous)

50. If a water drop is kept between two glass plates, then its shape is

(a)  (b)  (c)  (d) None of these

51. When two soap bubbles of radius r_1 and r_2 $(r_2 > r_1)$ coalesce, the radius of curvature of common surface is **[MP PMT 1996]**

(a) $r_2 - r_1$ (b) $\frac{r_2 - r_1}{r_1 r_2}$ (c) $\frac{r_1 r_2}{r_2 - r_1}$ (d) $r_2 + r_1$

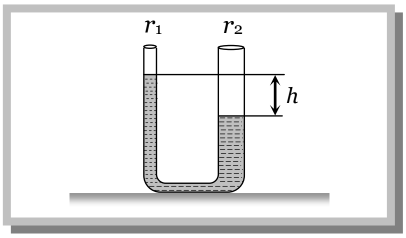
52. Two soap bubbles of radius 1*cm* and 2*cm* coalesce to form a single drop under isothermal conditions. The total energy possessed by them if surface tension is 30 *dyne cm*$^{-1}$, will be

(a) $400\ \pi\,ergs$ (b) $600\ \pi\,ergs$ (c) $1000\ \pi\,ergs$ (d) $1200\ \pi\,ergs$

53. In the above question, the radius of the bigger drop will be

(a) $\sqrt{3}\ cm$ (b) $\sqrt{5}\ cm$ (c) $\sqrt{7}\ cm$ (d) $\sqrt{8}\ cm$

54. In a *U*-tube the radii of two columns are respectively r_1 and r_2 and if a liquid of density d filled in it has level difference of h then the surface tension of the liquid is

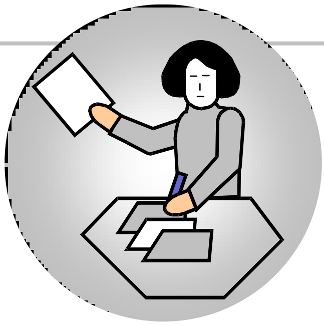


(a) $T = \frac{hdg}{r_2 - r_1}$

(b) $T = \frac{(r_2 - r_1)hdg}{2}$

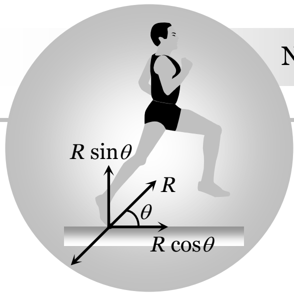
(c) $T = \frac{(r_1 + r_2)hdg}{2}$

(d) $T = \frac{hdg}{2}\frac{(r_1 r_2)}{r_2 - r_1}$



Answer Sheet (Practice problems)

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
b	a	c	b	b	d	d	c	c	b
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
a	b	b	a	c	a	c	c	a, d	c
21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
c	c	b	d	a	a	b	b	b	b
31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
a	d	b	b	a	b	d	a	d	b
41.	42.	43.	44.	45.	46.	47.	48.	49.	50.
c	c	b	c	d	b	c	a	c	c
51.	52.	53.	54.						
c	d	b	d						



Newton's Laws of Motion

4.1 Point Mass

(1) An object can be considered as a point object if during motion in a given time, it covers distance much greater than its own size.

(2) Object with zero dimension considered as a point mass.

(3) Point mass is a mathematical concept to simplify the problems.

4.2 Inertia

(1) Inherent property of all the bodies by virtue of which they cannot change their state of rest or uniform motion along a straight line by their own is called inertia.

(2) Inertia is not a physical quantity, it is only a property of the body which depends on mass of the body.

(3) Inertia has no units and no dimensions

(4) Two bodies of equal mass, one in motion and another is at rest, possess same inertia because it is a factor of mass only and does not depend upon the velocity.

4.3 Linear Momentum

(1) Linear momentum of a body is the quantity of motion contained in the body.

(2) It is measured in terms of the force required to stop the body in unit time.

(3) It is measured as the product of the mass of the body and its velocity *i.e.*, Momentum = mass × velocity.

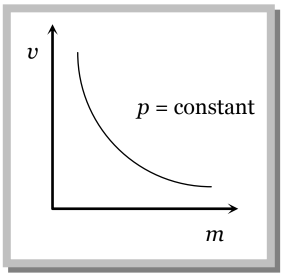
If a body of mass m is moving with velocity $\vec{v}$ then its linear momentum $\vec{p}$ is given by $\vec{p} = m\vec{v}$

(4) It is a vector quantity and it's direction is the same as the direction of velocity of the body.

(5) Units : *kg-m/sec* [S.I.], *g-cm/sec* [C.G.S.]

(6) Dimension : $[MLT^{-1}]$

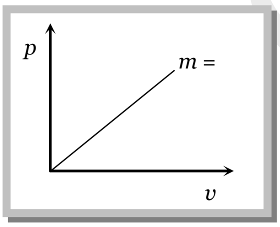
(7) If two objects of different masses have same momentum, the lighter body possesses greater velocity.



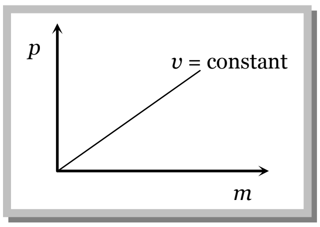
$$p = m_1 v_1 = m_2 v_2 = \text{constant}$$

$\therefore \quad \frac{v_1}{v_2} = \frac{m_2}{m_1}$ *i.e.* $v \propto \frac{1}{m}$ [As p is constant]

(8) For a given body $p \propto v$



(9) For different bodies at same velocities $p \propto m$



4.4 Newton's First Law

A body continue to be in its state of rest or of uniform motion along a straight line, unless it is acted upon by some external force to change the state.

(1) If no net force acts on a body, then the velocity of the body cannot change *i.e.* the body cannot accelerate.

(2) Newton's first law defines inertia and is rightly called the law of inertia. Inertia are of three types :

Inertia of rest, Inertia of motion, Inertia of direction

(3) **Inertia of rest :** It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

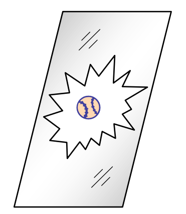
Example : (i) A person who is standing freely in bus, thrown backward, when bus starts suddenly.

When a bus suddenly starts, the force responsible for bringing bus in motion is also transmitted to lower part of body, so this part of the body comes in motion along with the bus. While the upper half of body (say above the waist) receives no force to overcome inertia of rest and so it stays in its original position. Thus there is a relative displacement between the two parts of the body and it appears as if the upper part of the body has been thrown backward.

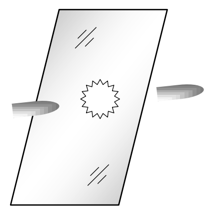
Note : ❑ If the motion of the bus is slow, the inertia of motion will be transmitted to the body of the person uniformly and so the entire body of the person will come in motion with the bus and the person will not experience any jerk.

(ii) When a horse starts suddenly, the rider tends to fall backward on account of inertia of rest of upper part of the body as explained above.

(iii) A bullet fired on a window pane makes a clean hole through it while a stone breaks the whole window because the bullet has a speed much greater than the stone. So its time of contact with glass is small. So in case of bullet the motion is transmitted only to a small portion of the glass in that small time. Hence a clear hole is created in the glass window, while in case of ball, the time and the area of contact is large. During this time the motion is transmitted to the entire window, thus creating the cracks in the entire window.



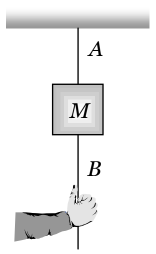
Cracks by the ball



Hole by the bullet

(iv) In the arrangement shown in the figure :

(a) If the string B is pulled with a sudden jerk then it will experience tension while due to inertia of rest of mass M this force will not be transmitted to the string A and so the string B will break.



(b) If the string B is pulled steadily the force applied to it will be transmitted from string B to A through the mass M and as tension in A will be greater than in B by Mg (weight of mass M) the string A will break.

(v) If we place a coin on smooth piece of card board covering a glass and strike the card board piece suddenly with a finger. The cardboard slips away and the coin falls into the glass due to inertia of rest.

(vi) The dust particles in a durree falls off when it is beaten with a stick. This is because the beating sets the durree in motion whereas the dust particles tend to remain at rest and hence separate.

(4) **Inertia of motion :** It is the inability of a body to change itself its state of uniform motion *i.e.,* a body in uniform motion can neither accelerate nor retard by its own.

Example : (i) When a bus or train stops suddenly, a passenger sitting inside tends to fall forward. This is because the lower part of his body comes to rest with the bus or train but the upper part tends to continue its motion due to inertia of motion.

(ii) A person jumping out of a moving train may fall forward.

(iii) An athlete runs a certain distance before taking a long jump. This is because velocity acquired by running is added to velocity of the athlete at the time of jump. Hence he can jump over a longer distance.

(5) **Inertia of direction :** It is the inability of a body to change by itself direction of motion.

Example : (i) When a stone tied to one end of a string is whirled and the string breaks suddenly, the stone flies off along the tangent to the circle. This is because the pull in the string was forcing the stone to move in a circle. As soon as the string breaks, the pull vanishes. The stone in a bid to move along the straight line flies off tangentially.

(ii) The rotating wheel of any vehicle throw out mud, if any, tangentially, due to directional inertia.

(iii) When a car goes round a curve suddenly, the person sitting inside is thrown outwards.

Sample problem based on Newton's first law

***Problem* 1.** When a bus suddenly takes a turn, the passengers are thrown outwards because of **[AFMC 1999; CPMT 2000, 2001]**

(a) Inertia of motion (b) Acceleration of motion

(c) Speed of motion (d) Both (b) and (c)

Solution : (a)

***Problem* 2.** A person sitting in an open car moving at constant velocity throws a ball vertically up into air. The ball fall **[EAMCET (Med.) 1995]**

(a) Outside the car (b) In the car ahead of the person

(c) In the car to the side of the person (d) Exactly in the hand which threw it up

Solution : (d) Because the horizontal component of velocity are same for both car and ball so they cover equal horizontal distances in given time interval.

4.5 Newton's Second Law

(1) The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.

(2) If a body of mass m, moves with velocity $\vec{v}$ then its linear momentum can be given by $\vec{p} = m\vec{v}$ and if force $\vec{F}$ is applied on a body, then

$$\vec{F} \propto \frac{d\vec{p}}{dt} \Rightarrow F = K\frac{d\vec{p}}{dt}$$

or $$\vec{F} = \frac{d\vec{p}}{dt}$$ ($K = 1$ in C.G.S. and S.I. units)

or $$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$ (As $a = \frac{d\vec{v}}{dt}$ = acceleration produced in the body)

$\therefore$ $$\vec{F} = m\vec{a}$$

Force = mass × acceleration

Sample problem based on Newton's second law

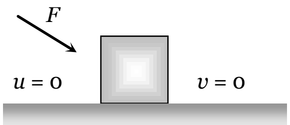
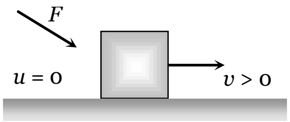
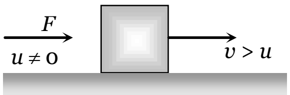
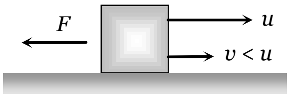
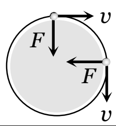
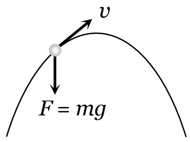
***Problem* 3.** A train is moving with velocity 20 m/sec. on this, dust is falling at the rate of 50 kg/min. The extra force required to move this train with constant velocity will be **[RPET 1999]**

(a) 16.66 N (b) 1000 N (c) 166.6 N (d) 1200 N

Solution : (a) Force $F = v\frac{dm}{dt} = 20 \times \frac{50}{60} = 16.66\,N$

***Problem* 4.** A force of 10 Newton acts on a body of mass 20 kg for 10 seconds. Change in its momentum is **[MP PET 2002]**

(a) 5 *kg m/s* (b) 100 *kg m/s* (c) 200 *kg m/s* (d) 1000 *kg m/s*

	Body remains at rest. Here force is trying to change the state of rest.
	Body starts moving. Here force changes the state of rest.
	In a small interval of time, force increases the magnitude of speed and direction of motion remains same.
	In a small interval of time, force decreases the magnitude of speed and direction of motion remains same.
	In uniform circular motion only direction of velocity changes, speed remains constant. Force is always perpendicular to velocity.
	In non-uniform circular motion, elliptical, parabolic or hyperbolic motion force acts at an angle to the direction of motion. In all these motions. Both magnitude and direction of velocity changes.

1) Force is an external effect in the form of a push or pulls which

(i) Produces or tries to produce motion in a body at rest.

(ii) Stops or tries to stop a moving body.

(iii) Changes or tries to change the direction of motion of the body.

(2) Dimension : Force = mass × acceleration

$$[F]=[M][LT^{-2}]=[MLT^{-2}]$$

(3) Units : Absolute units : (i) *Newton* (S.I.) (ii) *Dyne* (C.G.S)

Gravitational units : (i) Kilogram-force (M.K.S.) (ii) Gram-force (C.G.S)

Newton : One Newton is that force which produces an acceleration of $1 m/s^2$ in a body of mass 1 *Kilogram*. $\therefore 1\,Newton = 1 kg\,m/s^2$

Dyne : One dyne is that force which produces an acceleration of $1 cm/s^2$ in a body of mass 1 *gram*. $\therefore$ 1 $Dyne = 1 gm\ cm/\sec^2$

Relation between absolute units of force $1\,Newton = 10^5\,Dyne$

Kilogram-force : It is that force which produces an acceleration of $9.8 m/s^2$ in a body of mass 1 *kg*. $\therefore$ 1 *kg-f* = 9.81 *Newton*

Gram-force : It is that force which produces an acceleration of $980\,cm/s^2$ in a body of mass 1*gm*. $\therefore$ 1 *gm-f* = 980 *Dyne*

Relation between gravitational units of force : 1 *kg-f* = 10^7 *gm-f*

Solu tion : (b)

Pro ble m 5.

(a)

Solu tion : (a)

Force = *mc* / *d*

= 100(0 – 5 / 0.1

F = –5000

4.6 For ce

(

(4) $\vec{F} = m\vec{a}$ formula is valid only if force is changing the state of rest or motion and the mass of the body is constant and finite.

(5) If m is not constant $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$

(6) If force and acceleration have three component along x, y and z axis, then

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \text{ and } \vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

From above it is clear that $F_x = ma_x$, $F_y = ma_y, F_z = ma_z$

(7) No force is required to move a body uniformly along a straight line.

$\vec{F} = ma$ $\therefore \vec{F} = 0$ (As $a = 0$)

(8) When force is written without direction then positive force means repulsive while negative force means attractive.

Example : *Positive force* – Force between two similar charges

Negative force – Force between two opposite charges

(9) Out of so many natural forces, for distance 10^{-15} *metre*, nuclear force is strongest while gravitational force weakest. $F_{\text{nuclear}} > F_{\text{electromagnetic}} > F_{\text{gravitational}}$

(10) Ratio of electric force and gravitational force between two electron $F_e / F_g = 10^{43}$ $\therefore F_e >> F_g$

(11) Constant force : If the direction and magnitude of a force is constant. It is said to be a constant force.

(12) Variable or dependent force :

(i) *Time dependent force* : In case of impulse or motion of a charged particle in an alternating electric field force is time dependent.

(ii) *Position dependent force* : Gravitational force between two bodies $\frac{Gm_1m_2}{r^2}$

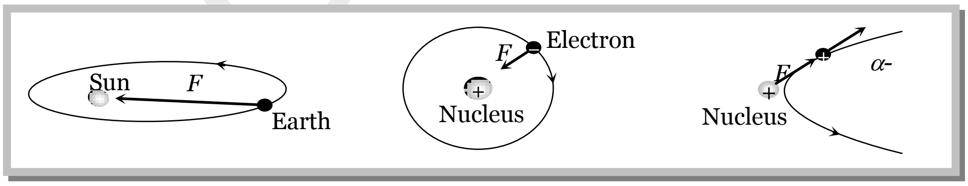
or Force between two charged particles $= \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$.

(iii) *Velocity dependent force* : Viscous force $(6\pi\eta rv)$

Force on charged particle in a magnetic field $(qvB\sin\theta)$

(13) Central force : If a position dependent force is always directed towards or away from a fixed point it is said to be central otherwise non-central.

Example : Motion of earth around the sun. Motion of electron in an atom. Scattering of α-particles from a nucleus.



(14) Conservative or non conservative force : If under the action of a force the work done in a round trip is zero or the work is path independent, the force is said to be conservative otherwise non conservative.

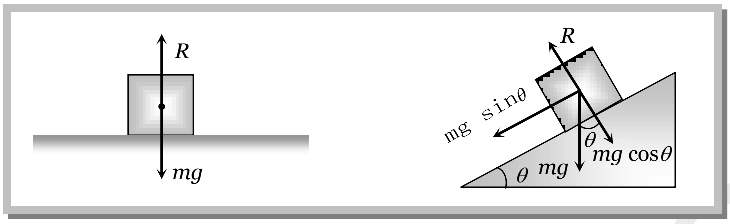
Example : Conservative force : Gravitational force, electric force, elastic force.

Non conservative force : Frictional force, viscous force.

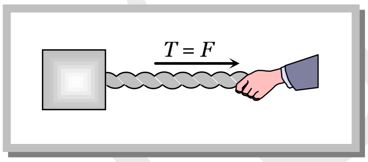
(15) Common forces in mechanics :

(i) *Weight* : Weight of an object is the force with which earth attracts it. It is also called the force of gravity or the gravitational force.

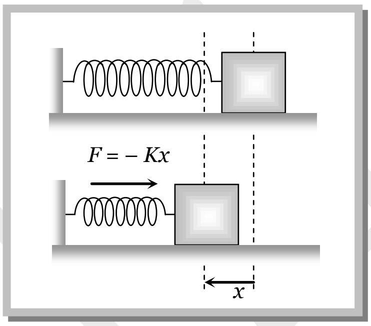
(ii) *Reaction or Normal force :* When a body is placed on a rigid surface, the body experiences a force which is perpendicular to the surfaces in contact. Then force is called 'Normal force' or 'Reaction'.



(iii) *Tension* : The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension. The direction of tension is so as to pull the body.



(iv) *Spring force :* Every spring resists any attempt to change its length. This resistive force increases with change in length. Spring force is given by $F = -Kx$; where x is the change in length and K is the spring constant (unit N/m).



4.7 Equilibrium of Concurrent Force

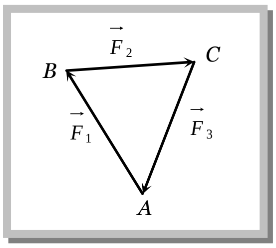
(1) If all the forces working on a body are acting on the same point, then they are said to be concurrent.

(2) A body, under the action of concurrent forces, is said to be in equilibrium, when there is no change in the state of rest or of uniform motion along a straight line.

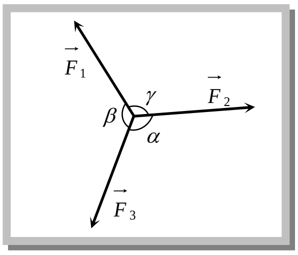
(3) The necessary condition for the equilibrium of a body under the action of concurrent forces is that the vector sum of all the forces acting on the body must be zero.

(4) Mathematically for equilibrium $\sum F_{\text{net}} = 0$ or $\sum F_x = 0$; $\sum F_y = 0$; , $\sum F_z = 0$

(5) Three concurrent forces will be in equilibrium, if they can be represented completely by three sides of a triangle taken in order.



(6) Lami's Theorem : For concurrent forces $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$



Sample problem based on force and equilibrium

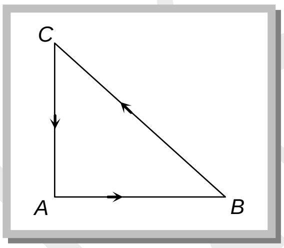
***Problem* 6.** Three forces starts acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC (as shown). The particle will now move with velocity

(a) $\vec{v}$ remaining unchanged

(b) Less than $\vec{v}$

(c) Greater than $\vec{v}$

(d) $\vec{v}$ in the direction of the largest force BC

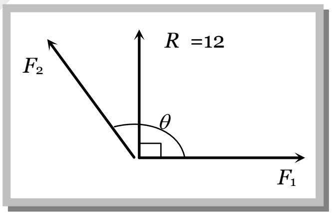


Solution : (a) Given three forces are in equilibrium *i.e.* net force will be zero. It means the particle will move with same velocity.

***Problem* 7.** Two forces are such that the sum of their magnitudes is 18 N and their resultant is perpendicular to the smaller force and magnitude of resultant is 12. Then the magnitudes of the forces are **[AIEEE 2002]**

(a) 12 N, 6 N (b) 13 N, 5N (c) 10 N, 8 N (d) 16 N, 2 N

Solution : (b) Let two forces are F_1 and $F_2 (F_1 < F_2)$.



According to problem: $F_1 + F_2 = 18$ .....(i)

Angle between F_1 and resultant (R) is 90°

$$\therefore \quad \tan 90 = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta} = \infty$$

$$\Rightarrow F_1 + F_2\cos\theta = 0$$

$$\Rightarrow \cos\theta = -\frac{F_1}{F_2} \qquad \text{.....(ii)}$$

and $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$$144 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \qquad \text{.....(iii)}$$

by solving (i), (ii) and (iii) we get $F_1 = 5N$ and $F_2 = 13N$

***Problem* 8.** The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is **[KCET (Engg./Med.) 2002]**

(a) 60^o (b) 120^o (c) 150^o (d) 90^o

Solution: (b) Let forces are F and $2F$ and angle between them is θ and resultant makes an angle α with the force F.

$$\tan\alpha = \frac{2F\sin\theta}{F + 2F\cos\theta} = \tan 90 = \infty$$

$$\Rightarrow F + 2F\cos\theta = 0 \qquad \therefore \cos\theta = -1/2 \text{ or } \theta = 120^\circ$$

Problem 9. A weightless ladder, 20 *ft* long rests against a frictionless wall at an angle of 60° with the horizontal. A 150 pound man is 4 *ft* from the top of the ladder. A horizontal force is needed to prevent it from slipping. Choose the correct magnitude from the following **[CBSE PMT 1998]**

(a) 175 *lb* (b) 100 *lb* (c) 70 *lb* (d) 150 *lb*

Solution: (c) Since the system is in equilibrium therefore $\sum F_x = 0$ and $\sum F_y = 0$ $\therefore$ $F = R_2$ and $W = R_1$

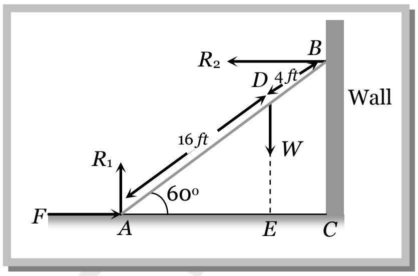
Now by taking the moment of forces about point *B*.

$F.(BC) + W.(EC) = R_1(AC)$ [from the figure EC= 4 cos 60]

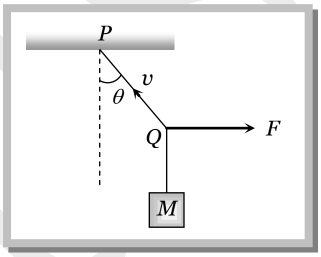
$F.(20\sin 60) + W(4\cos 60) = R_1(20\cos 60)$

$10\sqrt{3}F + 2W = 10R_1$ [As $R_1 = W$]

$\therefore\ F = \dfrac{8W}{10\sqrt{3}} = \dfrac{8\times 150}{10\sqrt{3}} = 70 lb$



Problem 10. A mass *M* is suspended by a rope from a rigid support at *P* as shown in the figure. Another rope is tied at the end *Q*, and it is pulled horizontally with a force *F*. If the rope *PQ* makes angle θ with the vertical then the tension in the string PQ is



(a) $F\sin\theta$

(b) $F/\sin\theta$

(c) $F\cos\theta$

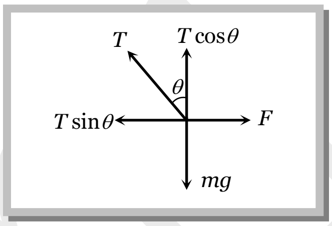
(d) $F/\cos\theta$

Solution: (b) From the figure

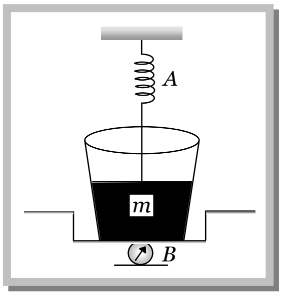
For horizontal equilibrium

$T\sin\theta = F$

$\therefore\ T = \dfrac{F}{\sin\theta}$



Problem 11. A spring balance A shows a reading of 2 *kg*, when an aluminium block is suspended from it. Another balance *B* shows a reading of 5 *kg*, when a beaker full of liquid is placed in its pan. The two balances are arranged such that the *Al* – block is completely immersed inside the liquid as shown in the figure. Then **[IIT-JEE**



(a) The reading of the balance *A* will be more than 2 *kg*

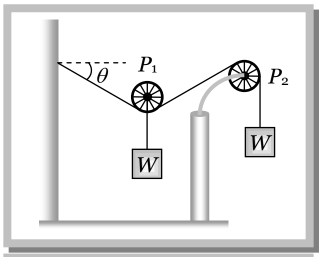
(b) The reading of the balance *B* will be less than 5 *kg*

(c) The reading of the balance *A* will be less than 2 *kg*. and that of *B* will be more than 5 *kg*

(d) The reading of balance A will be 2 *kg*. and that of *B* will be 5 *kg*.

Solution: (c) Due to buoyant force on the aluminium block the reading of spring balance *A* will be less than 2 kg but it increase the reading of balance *B*.

Problem 12. In the following diagram, pulley P_1 is movable and pulley P_2 is fixed. The value of angle θ will be



(a) 60°

(b) 30°

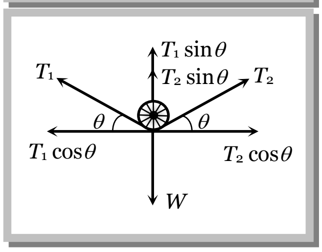
(c) 45°

(d) 15°

Solution: (b) Free body diagram of pulley P_1 is shown in the figure

For horizontal equilibrium $T_1\cos\theta = T_2\cos\theta$ $\therefore$ $T_1 = T_2$

and $T_1 = T_2 = W$

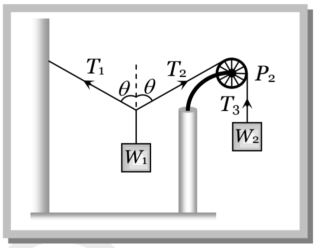


For vertical equilibrium

$T_1 \sin\theta + T_2 \sin\theta = W \Rightarrow W\sin\theta + W\sin\theta = W$

$\therefore \sin\theta = \dfrac{1}{2}$ or $\theta = 30°$

Problem 13. In the following figure, the pulley is massless and frictionless. The relation between T_1, T_2 and T_3 will be



(a) $T_1 = T_2 \neq T_3$

(b) $T_1 \neq T_2 = T_3$

(c) $T_1 \neq T_2 \neq T_3$

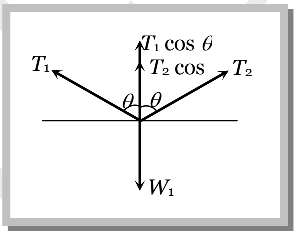
(d) $T_1 = T_2 = T_3$

Solution : (d) Since through a single string whole system is attached so $W_2 = T_3 = T_2 = T_1$

Problem 14. In the above problem (13), the relation between W_1 and W_2 will be

(a) $W_2 = \dfrac{W_1}{2\cos\theta}$ (b) $2W_1 \cos\theta$ (c) $W_2 = W_1$ (d) $W_2 = \dfrac{2\cos\theta}{W_1}$

Solution : (a) For vertical equilibrium

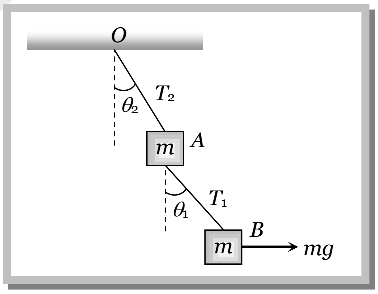


$T_1 \cos\theta + T_2 \cos\theta = W_1$ $[\text{As } T_1 = T_2 = W_2]$

$2W_2 \cos\theta = W_1$

$\therefore W_2 = \dfrac{W_1}{2\cos\theta}$.

Problem 15. In the following figure the masses of the blocks *A* and *B* are same and each equal to *m*. The tensions in the strings *OA* and *AB* are T_2 and T_1 respectively. The system is in equilibrium with a constant horizontal force mg on *B*. The T_1 is



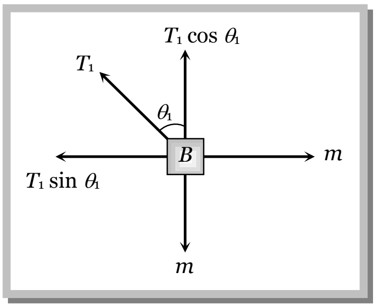
(a) mg

(b) $\sqrt{2}\, mg$

(c) $\sqrt{3}\, mg$

(d) $\sqrt{5}\, mg$

Solution : (b) From the free body diagram of block *B*



$T_1 \cos\theta_1 = mg$ ......(i)

$T_1 \sin\theta_1 = -mg$ .....(ii)

by squaring and adding $T_1^2\left(\sin^2\theta_1 + \cos^2\theta_1\right) = 2(mg)^2$

$\therefore T_1 = \sqrt{2}mg$

Problem 16. In the above problem (15), the angle θ_1 is

(a) 30° (b) 45° (c) 60° (d) $\tan^{-1}\left(\dfrac{1}{2}\right)$

Solution : (b) From the solution (15) by dividing equation(ii) by equation (i)

$$\frac{T_1 \sin\theta_1}{T_1 \cos\theta_1} = \frac{mg}{mg}$$

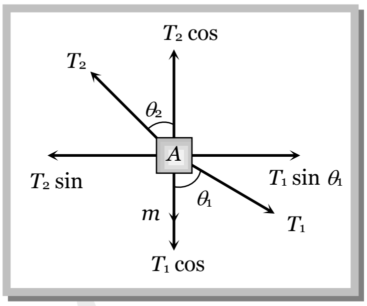
$\therefore \tan\theta_1 = 1$ or $\theta_1 = 45°$

Problem 17. In the above problem (15) the tension T_2 will be

(a) mg (b) $\sqrt{2}\,mg$ (c) $\sqrt{3}\,mg$ (d) $\sqrt{5}\,mg$

Solution : (d) From the free body diagram of block A

For vertical equilibrium $T_2\cos\theta_2 = mg + T_1\cos\theta_1$



$$T_2\cos\theta_2 = mg + \sqrt{2}mg\cos 45°$$

$$T_2\cos\theta_2 = 2mg \quad .....(i)$$

For horizontal equilibrium $T_2\sin\theta_2 = T_1\sin\theta_1 = \sqrt{2}mg\sin 45°$

$$T_2\sin\theta_2 = mg \quad .....(ii)$$

by squaring and adding (i) and (ii) equilibrium

$$T_2^2 = 5(mg)^2 \text{ or } T_2 = \sqrt{5}mg$$

Problem 18, In the above problem (15) the angle θ_2 will be

(a) 30° (b) 45° (c) 60° (d) $\tan^{-1}\left(\frac{1}{2}\right)$

Solution : (d) From the solution (17) by dividing equation(ii) by equation (i)

$$\frac{\sin\theta_2}{\cos\theta_2} = \frac{mg}{2mg} \Rightarrow \tan\theta_2 = \frac{1}{2} \qquad \therefore \theta_2 = \tan^{-1}\left[\frac{1}{2}\right]$$

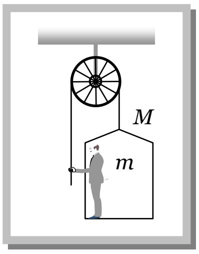
Problem 19. A man of mass m stands on a crate of mass M. He pulls on a light rope passing over a smooth light pulley. The other end of the rope is attached to the crate. For the system to be in equilibrium, the force exerted by the men on the rope will be

(a) $(M+m)g$

(b) $\frac{1}{2}(M+m)g$

(c) Mg

(d) mg

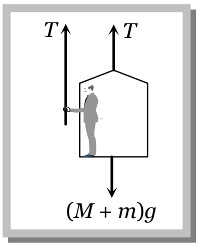


Solution : (b) From the free body diagram of man and crate system:

For vertical equilibrium

$2T = (M+m)g$

$\therefore T = \dfrac{(M+m)g}{2}$



Problem 20. Two forces, with equal magnitude F, act on a body and the magnitude of the resultant force is $\frac{F}{3}$. The angle between the two forces is

(a) $\cos^{-1}\left(-\frac{17}{18}\right)$ (b) $\cos^{-1}\left(-\frac{1}{3}\right)$ (c) $\cos^{-1}\left(\frac{2}{3}\right)$ (d) $\cos^{-1}\left(\frac{8}{9}\right)$

Solution : (a) Resultant of two vectors A and B, which are working at an angle θ, can be given by

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$ [As $A = B = F$ and $R = \frac{F}{3}$]

$$\left(\frac{F}{3}\right)^2 = F^2 + F^2 + 2F^2\cos\theta$$

$$\frac{F^2}{9} = 2F^2 + 2F^2\cos\theta \quad \Rightarrow \frac{-17}{9}F^2 = 2F^2\cos\theta \Rightarrow \cos\theta = \left(\frac{-17}{18}\right) \text{ or } \theta = \cos^{-1}\left(\frac{-17}{18}\right)$$

***Problem* 21.** A cricket ball of mass 150 *gm* is moving with a velocity of 12 *m/s* and is hit by a bat so that the ball is turned back with a velocity of 20 *m/s*. The force of blow acts for 0.01*s* on the ball. The average force exerted by the bat on the ball is

(a) 480 *N* (b) 600 *N* (c) 500 *N* (d) 400 *N*

Solution : (a) $v_1 = -12 m/s$ and $v_2 = +20 m/s$ [because direction is reversed]

$m = 150 gm = 0.15 kg$, $t = 0.01$ sec

Force exerted by the bat on the ball $F = \frac{m[v_2 - v_1]}{t} = \frac{0.15[20-(-12)]}{0.01}$ = 480 *Newton*

4.8 Newton's Third Law

To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

(1) When a body exerts a force on any other body, the second body also exerts an equal and opposite force on the first.

(2) Forces in nature always occurs in pairs. A single isolated force is not possible.

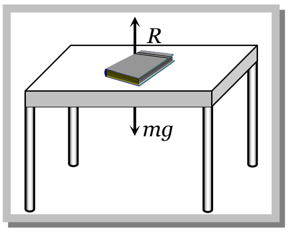
(3) Any agent, applying a force also experiences a force of equal magnitude but in opposite direction. The force applied by the agent is called '*Action*' and the counter force experienced by it is called '*Reaction*'.

(4) Action and reaction never act on the same body. If it were so the total force on a body would have always been zero *i.e.* the body will always remain in equilibrium.

(5) If $\vec{F}_{AB}$ = force exerted on body *A* by body *B* (Action) and $\vec{F}_{BA}$ = force exerted on body *B* by body *A* (Reaction)

Then according to Newton's third law of motion $\vec{F}_{AB} = -\vec{F}_{BA}$

(6) Example : (i) A book lying on a table exerts a force on the table which is equal to the weight of the book. This is the force of action.



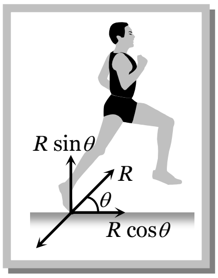
The table supports the book, by exerting an equal force on the book. This is the force of reaction.

As the system is at rest, net force on it is zero. Therefore force of action and reaction must be equal and opposite.

(ii) Swimming is possible due to third law of motion.

(iii) When a gun is fired, the bullet moves forward (action). The gun recoils backward (reaction)

(iv) Rebounding of rubber ball takes place due to third law of motion.



(v) While walking a person presses the ground in the backward direction (action) by his feet. The ground pushes the person in forward direction with an equal force (reaction). The component of reaction in horizontal direction makes the person move forward.

(vi) It is difficult to walk on sand or ice.

(vii) Driving a nail into a wooden block without holding the block is difficult.

Sample problem based on Newton's third law

***Problem* 22.** You are on a frictionless horizontal plane. How can you get off if no horizontal force is exerted by pushing against the surface

(a) By jumping (b) By splitting or sneezing

(c) By rolling your body on the surface (d) By running on the plane

Solution : (b) By doing so we can get push in backward direction in accordance with *Newton's* third law of motion.

4.9 Frame of Reference

(1) A frame in which an observer is situated and makes his observations is known as his 'Frame of reference'.

(2) The reference frame is associated with a co-ordinate system and a clock to measure the position and time of events happening in space. We can describe all the physical quantities like position, velocity, acceleration etc. of an object in this coordinate system.

(3) Frame of reference are of two types : (i) Inertial frame of reference (ii) Non-inertial frame of reference.

(i) **Inertial frame of reference :**

(a) A frame of reference which is at rest or which is moving with a uniform velocity along a straight line is called an inertial frame of reference.

(b) In inertial frame of reference Newton's laws of motion holds good.

(c) Inertial frame of reference are also called unaccelerated frame of reference or Newtonian or Galilean frame of reference.

(d) Ideally no inertial frame exist in universe. For practical purpose a frame of reference may be considered as inertial if it's acceleration is negligible with respect to the acceleration of the object to be observed.

(e) To measure the acceleration of a falling apple, earth can be considered as an inertial frame.

(f) To observe the motion of planets, earth can not be considered as an inertial frame but for this purpose the sun may be assumed to be an inertial frame.

Example : The lift at rest, lift moving (up or down) with constant velocity, car moving with constant velocity on a straight road.

(ii) **Non inertial frame of reference :**

(a) Accelerated frame of references are called non-inertial frame of reference.

(b) Newton's laws of motion are not applicable in non-inertial frame of reference.

Example : Car moving in uniform circular motion, lift which is moving upward or downward with some acceleration, plane which is taking off.

4.10 Impulse

(1) When a large force works on a body for very small time interval, it is called impulsive force.

An impulsive force does not remain constant, but changes first from zero to maximum and then from maximum to zero. In such case we measure the total effect of force.

(2) Impulse of a force is a measure of total effect of force.

(3) $\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt$.

(4) Impulse is a vector quantity and its direction is same as that of force.

(5) Dimension : $[MLT^{-1}]$

(6) Units : *Newton-second* or $Kg\text{-}m\text{-}s^{-1}$ (S.I.) and *Dyne-second* or $gm\text{-}cm\text{-}s^{-1}$ (C.G.S.)

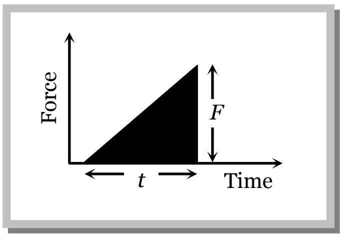
(7) Force-time graph : Impulse is equal to the area under *F-t* curve.

If we plot a graph between force and time, the area under the curve and time axis gives the value of impulse.

I = Area between curve and time axis

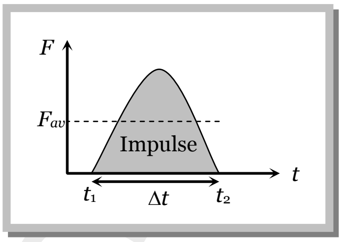
$= \frac{1}{2} \times$ Base $\times$ Height

$= \frac{1}{2} F\,t$



(8) If F_{av} is the average magnitude of the force then

$$I = \int_{t_1}^{t_2} F\,dt = F_{av} \int_{t_1}^{t_2} dt = F_{av} \Delta t$$



(9) From Newton's second law $\vec{F} = \frac{d\vec{p}}{dt}$

or $$\int_{t_1}^{t_2} \vec{F}\,dt = \int_{p_1}^{p_2} d\vec{p} \Rightarrow \vec{I} = \vec{p}_2 - \vec{p}_1 = \overrightarrow{\Delta p}$$

i.e. The impulse of a force is equal to the change in momentum.

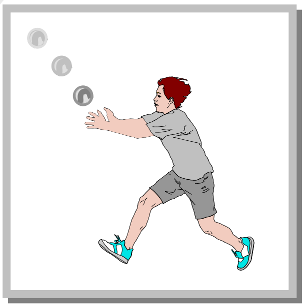
This statement is known as *Impulse momentum theorem*.

(10) *Examples* : Hitting, kicking, catching, jumping, diving, collision *etc.*

In all these cases an impulse acts. $I = \int F\,dt = F_{av}.\Delta t = \Delta p =$ constant

So if time of contact Δt is increased, average force is decreased (or diluted) and vice-versa.

(i) In hitting or kicking a ball we decrease the time of contact so that large force acts on the ball producing greater acceleration.



(ii) In catching a ball a player by drawing his hands backwards increases the time of contact and so, lesser force acts on his hands and his hands are saved from getting hurt.

(iii) In jumping on sand (or water) the time of contact is increased due to yielding of sand or water so force is decreased and we are not injured. However if we jump on cemented floor the motion stops in a very short interval of time resulting in a large force due to which we are seriously injured.

(iv) An athlete is advised to come to stop slowly after finishing a fast race. So that time of stop increases and hence force experienced by him decreases.

(v) China wares are wrapped in straw or paper before packing.

Sample problem based on Impulse

Problem 23. A ball of mass 150*g* moving with an acceleration $20 m/s^2$ is hit by a force, which acts on it for 0.1 *sec*. The impulsive force is **[AFMC 1999]**

(a) 0.5 *N-s* (b) 0.1 *N-s* (c) 0.3 *N-s* (d) 1.2 *N-s*

Solution : (c) Impulsive force = force × time $= m\,a \times t = 0.15 \times 20 \times 0.1 = 0.3$ *N-s*

Problem 24. A force of 50 *dynes* is acted on a body of mass 5 *g* which is at rest for an interval of 3 *seconds*, then impulse is

(a) $0.15 \times 10^{-3}\, N\text{-}s$ (b) $0.98 \times 10^{-3}\, N\text{-}s$ (c) $1.5 \times 10^{-3}\, N\text{-}s$ (d) $2.5 \times 10^{-3}\, N\text{-}s$

Solution : (c) Impulse = force × time $= 50 \times 10^{-5} \times 3 = 1.5 \times 10^{-3}\, N\text{-}s$

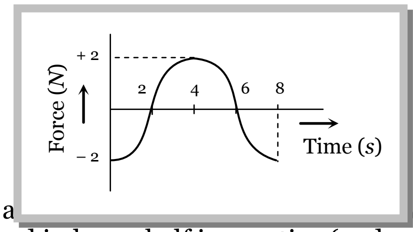
***Problem* 25.** The force-time ($F - t$) curve of a particle executing linear motion is as shown in the figure. The momentum acquired by the particle in time interval from zero to 8 *second* will be

(a) -2 *N-s*

(b) $+4$ *N-s*

(c) 6 *N-s*

(d) Zero



Solution : (d) Momentum acquired by the particle is numerically equal to the area enclosed between the curve and time Axis. For the given diagram area in a upper half is positive and in lower half is negative (and equal to the upper half). So net area is zero. Hence the momentum acquired by the particle will be zero.

4.11 Law of Conservation of Linear Momentum

If no external force acts on a system (called isolated) of constant mass, the total momentum of the system remains constant with time.

(1) According to this law for a system of particles $\vec{F} = \dfrac{d\vec{p}}{dt}$

In the absence of external force $\vec{F} = 0$ then $\vec{p} =$ constant

i.e., $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ... =$ constant.

or $m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ... =$ constant

This equation shows that in absence of external force for a closed system the linear momentum of individual particles may change but their sum remains unchanged with time.

(2) Law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.

(3) Conservation of linear momentum is equivalent to Newton's third law of motion.

For a system of two particles in absence of external force by law of conservation of linear momentum.

$\vec{p}_1 + \vec{p}_2 =$ constant. $\quad\therefore\quad m_1\vec{v}_1 + m_2\vec{v}_2 =$ constant.

Differentiating above with respect to time

$$m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = 0 \Rightarrow m_1\vec{a}_1 + m_2\vec{a}_2 = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0$$

$\therefore \quad \vec{F}_2 = -\vec{F}_1$

i.e. for every action there is equal and opposite reaction which is Newton's third law of motion.

(4) Practical applications of the law of conservation of linear momentum

(i) When a man jumps out of a boat on the shore, the boat is pushed slightly away from the shore.

(ii) A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.

(iii) **Recoiling of a gun :** For bullet and gun system, the force exerted by trigger will be internal so the momentum of the system remains unaffected.

Let m_G = mass of gun, m_B = mass of bullet,

v_G = velocity of gun, v_B = velocity of bullet

Initial momentum of system = 0

Final momentum of system $= m_G\vec{v}_G + m_B\vec{v}_B$

By the law of conservation linear momentum

$$m_G\vec{v}_G + m_B\vec{v}_B = 0$$

So recoil velocity $\vec{v}_G = -\frac{m_B}{m_G}\vec{v}_B$

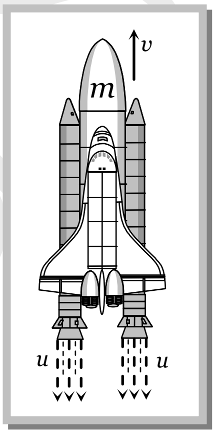
(a) Here negative sign indicates that the velocity of recoil $\vec{v}_G$ is opposite to the velocity of the bullet.

(b) $v_G \propto \frac{1}{m_G}$ *i.e.* higher the mass of gun, lesser the velocity of recoil of gun.

(c) While firing the gun must be held tightly to the shoulder, this would save hurting the shoulder because in this condition the body of the shooter and the gun behave as one body. Total mass become large and recoil velocity becomes too small.

$$v_G \propto \frac{1}{m_G + m_{\text{man}}}$$

(iv) **Rocket propulsion :** The initial momentum of the rocket on its launching pad is zero. When it is fired from the launching pad, the exhaust gases rush downward at a high speed and to conserve momentum, the rocket moves upwards.



Let m_0 = initial mass of rocket,

m = mass of rocket at any instant 't' (instantaneous mass)

m_r = residual mass of empty container of the rocket

u = velocity of exhaust gases,

v = velocity of rocket at any instant 't' (instantaneous velocity)

$\frac{dm}{dt}$ = rate of change of mass of rocket = rate of fuel consumption

= rate of ejection of the fuel.

(a) Thrust on the rocket : $F = -u\frac{dm}{dt} - mg$

Here negative sign indicates that direction of thrust is opposite to the direction of escaping gases.

$$F = -u\frac{dm}{dt} \text{ (if effect of gravity is neglected)}$$

(b) Acceleration of the rocket : $a = \frac{u}{m}\frac{dm}{dt} - g$

and if effect of gravity is neglected $a = \frac{u}{m}\frac{dm}{dt}$

(c) Instantaneous velocity of the rocket : $v = u\log_e\left(\frac{m_0}{m}\right) - gt$

and if effect of gravity is neglected $v = u\log_e\left(\frac{m_0}{m}\right) = 2.303\,u\log_{10}\left(\frac{m_0}{m}\right)$

(d) Burnt out speed of the rocket : $v_b = v_{\max} = u\log_e\left(\frac{m_0}{m_r}\right)$

The speed attained by the rocket when the complete fuel gets burnt is called burnt out speed of the rocket. It is the maximum speed acquired by the rocket.

Sample Problem based on conservation of momentum

Problem **26.** A wagon weighing 1000 *kg* is moving with a velocity 50 *km/h* on smooth horizontal rails. A mass of 250 *kg* is dropped into it. The velocity with which it moves now is

(a) 12.5 *km/hour* (b) 20 *km/hour* (c) 40 *km/hour* (d) 50 *km/hour*

Solution : (c) Initially the wagon of mass 1000 *kg* is moving with velocity of 50 *km/h*

So its momentum $= 1000 \times 50 \dfrac{kg \times km}{h}$

When a mass 250*kg* is dropped into it. New mass of the system $= 1000 + 250 = 1250kg$

Let v is the velocity of the system.

By the conservation of linear momentum : Initial momentum = Final momentum $\Rightarrow$ $1000 \times 50 = 1250 \times v$

$\therefore \qquad v = \dfrac{50{,}000}{1250} = 40 km/h.$

Problem 27. The kinetic energy of two masses m_1 and m_2 are equal. The ratio of their linear momentum will be **[RPET 1988]**

(a) m_1/m_2 (b) m_2/m_1 (c) $\sqrt{m_1/m_2}$ (d) $\sqrt{m_2/m_1}$

Solution : (c) Relation between linear momentum (P), man (m) and kinetic energy (E)

$P = \sqrt{2mE} \Rightarrow P \propto \sqrt{m}$ [as E is constant] $\therefore \dfrac{P_1}{P_2} = \sqrt{\dfrac{m_1}{m_2}}$

Problem 28. Which of the following has the maximum momentum

(a) A 100 *kg* vehicle moving at 0.02 ms^{-1} (b) A 4 *g* weight moving at 10000 cms^{-1}

(c) A 200 *g* weight moving with kinetic energy 10^{-6} *J* (d) A 20 *g* weight after falling 1 *kilometre*

Solution : (d) Momentum of body for given options are :

(a) $P = mv = 100 \times 0.02 = 2 kgm/\sec$ (b) $P = mv = 4 \times 10^{-3} \times 100 = 0.4 kgm/\sec$

(c) $P = \sqrt{2mE} = \sqrt{2 \times 0.2 \times 10^{-6}} = 6.3 \times 10^{-4} kgm/\sec$

(d) $P = m\sqrt{2gh} = 20 \times 10^{-3} \times \sqrt{2 \times 10 \times 10^3} = 2.82 kgm/\sec$

So for option (d) momentum is maximum.

Problem 29. A rocket with a lift-off mass 3.5×10^4 *kg* is blasted upwards with an initial acceleration of $10 m/s^2$. Then the initial thrust of the blast is

(a) $1.75 \times 10^5 N$ (b) $3.5 \times 10^5 N$ (c) $7.0 \times 10^5 N$ (d) $14.0 \times 10^5 N$

Solution : (c) Initial thrust on the rocket $F = m(g + a) = 3.5 \times 10^4 (10 + 10) = 7.0 \times 10^5 N$

Problem 30. In a rocket of mass 1000 *kg* fuel is consumed at a rate of 40 *kg/s*. The velocity of the gases ejected from the rocket is $5 \times 10^4 m/s$. The thrust on the rocket is

(a) $2 \times 10^3 N$ (b) $5 \times 10^4 N$ (c) $2 \times 10^6 N$ (d) $2 \times 10^9 N$

Solution : (c) Thrust on the rocket $F = \dfrac{udm}{dt} = 5 \times 10^4 (40) = 2 \times 10^6 N$

Problem 31. If the force on a rocket moving with a velocity of 300 *m/s* is 210 *N*, then the rate of combustion of the fuel is

(a) 0.7 *kg/s* (b) 1.4 *kg/s* (c) 0.07 *kg/s* (d) 10.7 *kg/s*

Solution : (a) Force on the rocket $= \dfrac{udm}{dt}$ $\therefore$ Rate of combustion of fuel $\left(\dfrac{dm}{dt}\right) = \dfrac{F}{u} = \dfrac{210}{300} = 0.7 kg/s.$

Problem 32. A rocket has a mass of 100 *kg*. 90% of this is fuel. It ejects fuel vapours at the rate of 1 *kg/sec* with a velocity of 500 *m/sec* relative to the rocket. It is supposed that the rocket is outside the gravitational field. The initial upthrust on the rocket when it just starts moving upwards is **[NCERT 1978]**

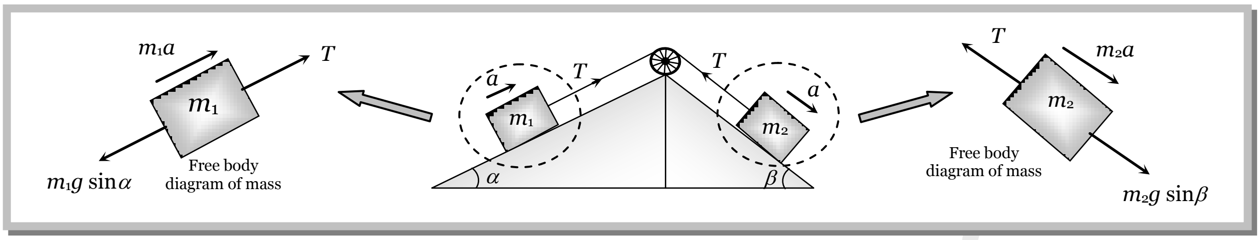
(a) Zero (b) 500 *N* (c) 1000 *N* (d) 2000 *N*

Solution : (b) Up thrust force $F = u\left(\frac{dm}{dt}\right) = 500 \times 1 = 500\ N$

4.12 Free Body Diagram

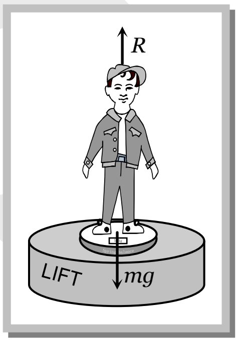
In this diagram the object of interest is isolated from its surroundings and the interactions between the object and the surroundings are represented in terms of forces.

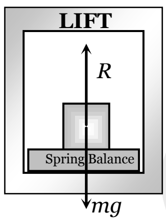
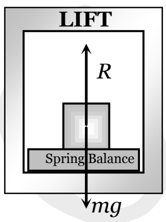
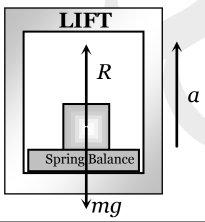
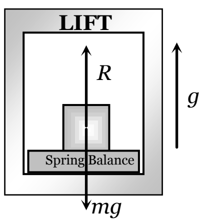
Example :

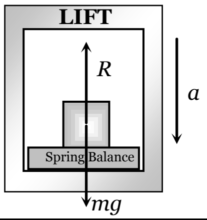
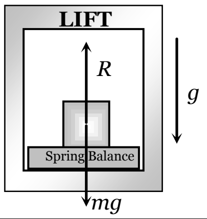
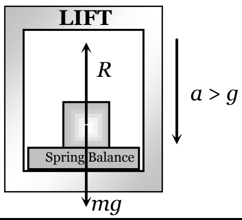


4.13 Apparent Weight of a Body in a Lift

When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg. This acts on a weighing machine which offers a reaction R given by the reading of weighing machine. This reaction exerted by the surface of contact on the body is the *apparent weight* of the body.



Condition	Figure	Velocity	Acceleration	Reaction	Conclusion
Lift is at rest		$v = 0$	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift moving upward or downward with constant velocity		v = constant	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift accelerating upward at the rate of 'a'		v = variable	$a < g$	$R - mg = ma$ $\therefore R = m(g + a)$	Apparent weight > Actual weight
Lift accelerating upward at the rate of 'g'		v = variable	$a = g$	$R - mg = mg$ $R = 2mg$	Apparent weight = 2 Actual weight

Lift accelerating downward at the rate of 'a'		v = variable	$a < g$	$mg - R = ma$ $\therefore R = m(g - a)$	Apparent weight < Actual weight
Lift accelerating downward at the rate of 'g'		v = variable	$a = g$	$mg - R = mg$ $R = 0$	Apparent weight = Zero (weightlessness)
Lift accelerating downward at the rate of a(>g)		v = variable	$a > g$	$mg - R = ma$ $R = mg - ma$ $R = -ve$	Apparent weight negative means the body will rise from the floor of the lift and stick to the ceiling of the lift.

Sample problems based on lift

Problem 33. A man weighs $80kg$. He stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of $5m/s^2$. What would be the reading on the scale. $(g = 10m/s^2)$

(a) 400 N (b) 800 N (c) 1200 N (d) Zero

Solution : (c) Reading of weighing scale $= m(g + a) = 80\,(10 + 5) = 1200\,N$

Problem 34. A body of mass 2 kg is hung on a spring balance mounted vertically in a lift. If the lift descends with an acceleration equal to the acceleration due to gravity 'g', the reading on the spring balance will be

(a) 2 kg (b) $(4 \times g)$ kg (c) $(2 \times g)$ kg (d) Zero

Solution : (d) $R = m(g - a) = (g - g) = 0$ [because the lift is moving downward with $a = g$]

Problem 35. In the above problem, if the lift moves up with a constant velocity of 2 m/sec, the reading on the balance will

Be (a) 2 kg (b) 4 kg (c) Zero (d) 1 kg

Solution : (a) $R = mg = 2g$ *Newton* or $2kg$ [because the lift is moving with the zero acceleration]

Problem 36. If the lift in problem, moves up with an acceleration equal to the acceleration due to gravity, the reading on the spring balance will be

(a) 2 kg (b) $(2 \times g)$ kg (c) $(4 \times g)$ kg (d) 4 kg

Solution : (d) $R = m(g + a) = m\,(g + g)$ [because the lift is moving upward with $a = g$]

$= 2mg$ $R = 2 \times 2g\,N = 4g\,N$ or $4\,kg$

Problem 37. A man is standing on a weighing machine placed in a lift, when stationary, his weight is recorded as 40 kg. If the lift is accelerated upwards with an acceleration of $2\,m/s^2$, then the weight recorded in the machine will be $(g = 10\,m/s^2)$ **[MP PMT 1994]**

(a) 32 kg (b) 40 kg (c) 42 kg (d) 48 kg

Solution : (d) $R = m(g+a) = 40(10+2) = 480\ N$ or $48 kg$

Problem 38. An elevator weighing 6000 *kg* is pulled upward by a cable with an acceleration of $5 ms^{-2}$. Taking *g* to be $10 ms^{-2}$, then the tension in the cable is **[Manipal MEE 1995]**

(a) 6000 *N* (b) 9000 *N* (c) 60000 *N* (d) 90000 *N*

Solution : (d) $T = m(g+a) = 6000(10+5)$ $T = 90,000\ N$

Problem 39. The ratio of the weight of a man in a stationary lift and when it is moving downward with uniform acceleration '*a*' is 3 : 2. The value of '*a*' is (*g*- Acceleration due to gravity on the earth)

(a) $\frac{3}{2}g$ (b) $\frac{g}{3}$ (c) $\frac{2}{3}g$ (d) g

Solution : (b) $\frac{\text{weight of a man in stationary lift}}{\text{weight of a man in downward moving lift}} = \frac{mg}{m(g-a)} = \frac{3}{2}$

$\therefore \frac{g}{g-a} = \frac{3}{2} \Rightarrow 2g = 3g - 3a$ or $a = \frac{g}{3}$

Problem 40. A 60 *kg* man stands on a spring scale in the lift. At some instant he finds, scale reading has changed from 60 *kg* to 50 *kg* for a while and then comes back to the original mark. What should we conclude

(a) The lift was in constant motion upwards

(b) The lift was in constant motion downwards

(c) The lift while in constant motion upwards, is stopped suddenly

(d) The lift while in constant motion downwards, is suddenly stopped

Solution : (c) For retarding motion of a lift $R = m(g+a)$ for downward motion

$R = m(g-a)$ for upward motion

Since the weight of the body decrease for a while and then comes back to original value it means the lift was moving upward and stops suddenly.

Note : ❑ Generally we use $R = m(g+a)$ for upward motion

$R = m(g-a)$ for downward motion

here *a*= acceleration, but for the given problem *a*= retardation

Problem 41. A bird is sitting in a large closed cage which is placed on a spring balance. It records a weight placed on a spring balance. It records a weight of 25 *N*. The bird (mass = 0.5*kg*) flies upward in the cage with an acceleration of $2m/s^2$. The spring balance will now record a weight of **[MP PMT 1999]**

(a) 24 *N* (b) 25 *N* (c) 26 *N* (d) 27 *N*

Solution : (b) Since the cage is closed and we can treat bird cage and air as a closed (Isolated) system. In this condition the force applied by the bird on the cage is an internal force due to this reading of spring balance will not change.

Problem 42. A bird is sitting in a wire cage hanging from the spring balance. Let the reading of the spring balance be W_1. If the bird flies about inside the cage, the reading of the spring balance is W_2. Which of the following is true

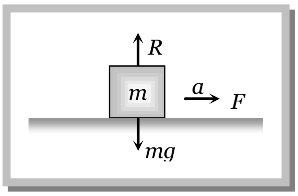
(a) $W_1 = W_2$ (b) $W_1 > W_2$

(c) $W_1 < W_2$ (d) Nothing definite can be predicted

Solution : (b) In this problem the cage is wire-cage the momentum of the system will not be conserved and due to this the weight of the system will be lesser when the bird is flying as compared to the weight of the same system when bird is resting is $W_2 < W_1$.

4.14 Acceleration of Block on Horizontal Smooth Surface

(1) **When a pull is horizontal**

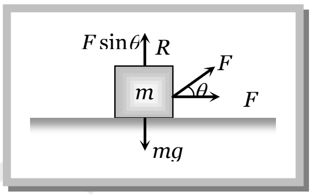


$$R = mg$$

and $F = ma$

$\therefore$ $a = F/m$

(2) **When a pull is acting at an angle (θ) to the horizontal (upward)**



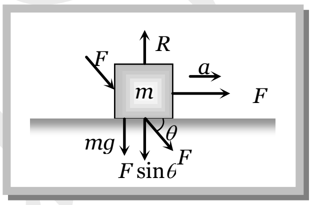
$$R + F\sin\theta = mg$$

$\Rightarrow$ $R = mg - F\sin\theta$

and $F\cos\theta = ma$

$\therefore$ $a = \dfrac{F\cos\theta}{m}$

(3) **When a push is acting at an angle (θ) to the horizontal (downward)**



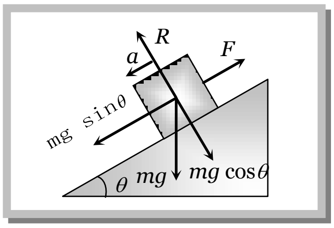
$$R = mg + F\sin\theta$$

and $F\cos\theta = ma$

$$a = \frac{F\cos\theta}{m}$$

4.15 Acceleration of Block on Smooth Inclined Plane

(1) **When inclined plane is at rest**



Normal reaction $R = mg\cos\theta$

Force along a inclined plane

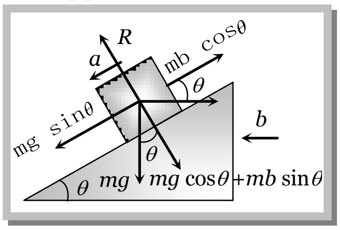
$$F = mg\sin\theta$$

$$ma = mg\sin\theta$$

$\therefore$ $a = g\sin\theta$

(2) **When a inclined plane given a horizontal acceleration '*b*'**

Since the body lies in an accelerating frame, an inertial force (mb) acts on it in the opposite direction.



Normal reaction $R = mg\cos\theta + mb\sin\theta$

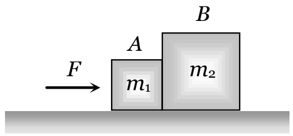
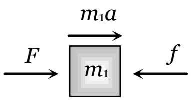
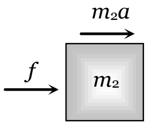
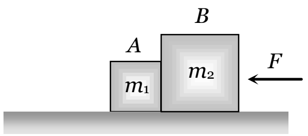
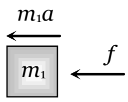
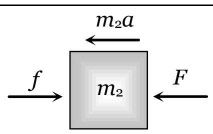
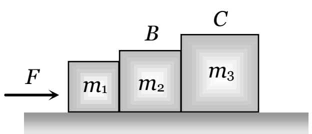
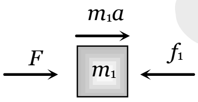
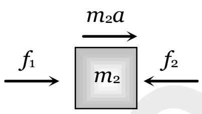
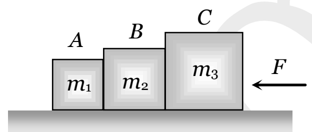
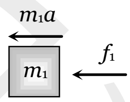
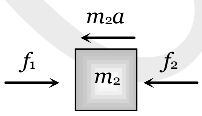
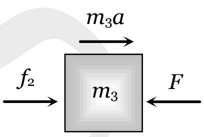
and $ma = mg\sin\theta - mb\cos\theta$

$\therefore$ $a = g\sin\theta - b\cos\theta$

Note : ❑The condition for the body to be at rest relative to the inclined plane : $a = g\sin\theta - b\cos\theta = 0$

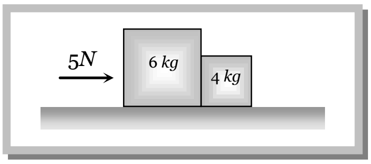
$\therefore$ $b = g\tan\theta$

4.16 Motion of Blocks in Contact

Condition	Free body diagram	Equation	Force and acceleration
		$F-f=m_1a$	$a=\dfrac{F}{m_1+m_2}$
		$f=m_2a$	$f=\dfrac{m_2F}{m_1+m_2}$
		$f=m_1a$	$a=\dfrac{F}{m_1+m_2}$
		$F-f=m_2a$	$f=\dfrac{m_1F}{m_1+m_2}$
		$F-f_1=m_1a$	$a=\dfrac{F}{m_1+m_2+m_3}$
		$f_1-f_2=m_2a$	$f_1=\dfrac{(m_2+m_3)F}{m_1+m_2+m_3}$
		$f_2=m_3a$	$f_2=\dfrac{m_3F}{m_1+m_2+m_3}$
		$f_1=m_1a$	$a=\dfrac{F}{m_1+m_2+m_3}$
		$f_2-f_1=m_2a$	$f_1=\dfrac{m_1F}{m_1+m_2+m_3}$
		$F-f_2=m_3a$	$f_2=\dfrac{(m_1+m_2)F}{m_1+m_2+m_3}$

Sample problems based on motion of blocks in contact

***Problem* 43.** Two blocks of mass 4 *kg* and 6 *kg* are placed in contact with each other on a frictionless horizontal surface. If we apply a push of 5 *N* on the heavier mass, the force on the lighter mass will be



(a) $5\,N$
(b) $4\,N$
(c) $2\,N$
(d) None of the above

Solution : (c) Let $m_1 = 6kg, m_2 = 4kg$ and $F = 5N$ (given)

Force on the lighter mass = $\dfrac{m_2 \times F}{m_1 + m_2} = \dfrac{4 \times 5}{6 + 4} = 2N$

***Problem* 44.** In the above problem, if a push of $5\,N$ is applied on the lighter mass, the force exerted by the lighter mass on the heavier mass will be

(a) $5\,N$ (b) $4\,N$ (c) $2\,N$ (d) None of the above

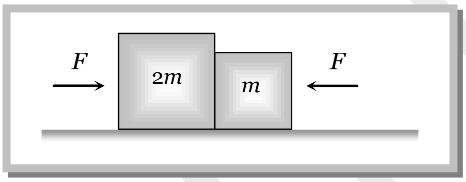
Solution : (d) Force on the heavier mass $= \dfrac{m_1 F}{m_1 + m_2} = \dfrac{6 \times 5}{6 + 4} = 3N$

***Problem* 45.** In the above problem, the acceleration of the lighter mass will be

(a) $0.5\,ms^{-2}$ (b) $\dfrac{5}{4}\,ms^{-2}$ (c) $\dfrac{5}{6}\,ms^{-2}$ (d) None of the above

Solution : (a) Acceleration $= \dfrac{\text{Net force on the system}}{\text{Total mass of the system}} = \dfrac{5}{10} = 0.5\,m/s^2$

***Problem* 46.** Two blocks are in contact on a frictionless table one has a mass m and the other $2\,m$ as shown in figure. Force F is applied on mass $2m$ then system moves towards right. Now the same force F is applied on m. The ratio of force of contact between the two blocks will be in the two cases respectively.



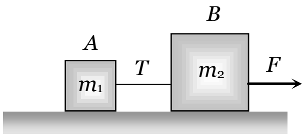
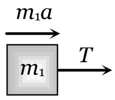
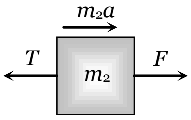
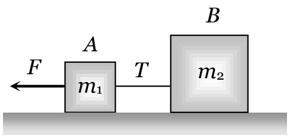
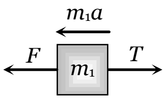
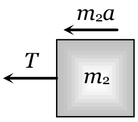
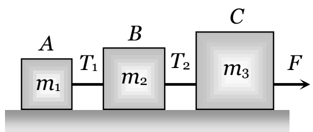
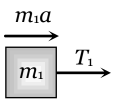
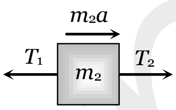
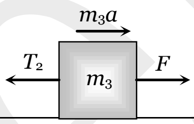
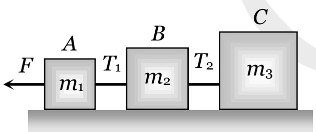
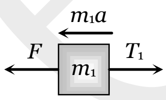
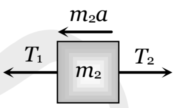
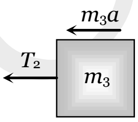
(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

Solution : (b) When the force is applied on mass $2m$ contact force $f_1 = \dfrac{m}{m + 2m} g = \dfrac{g}{3}$

When the force is applied on mass m contact force $f_2 = \dfrac{2m}{m + 2m} g = \dfrac{2}{3} g$

Ratio of contact forces $\dfrac{f_1}{f_2} = \dfrac{1}{2}$

4.17 Motion of Blocks Connected by Mass Less String

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$
		$F - T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_1 - T_2 = m_2 a$	$T_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$T_2 = m_3 a$	$T_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

Sample problems based on motion of blocks connected by mass less string

***Problem* 47.** A monkey of mass 20 *kg* is holding a vertical rope. The rope will not break when a mass of 25 *kg* is suspended from it but will break if the mass exceeds 25 *kg*. What is the maximum acceleration with which the monkey can climb up along the rope $(g = 10 m/s^2)$

(a) $10\ m/s^2$ (b) $25\ m/s^2$ (c) $2.5 m/s^2$ (d) $5m/s^2$

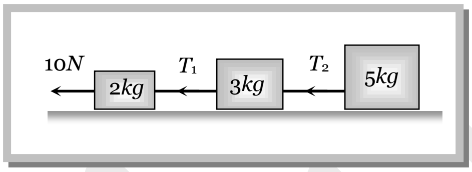
Solution : (c) Maximum tension that string can bear $(T_{max}) = 25 \times g\ N = 250\ N$

Tension in rope when the monkey climb up $T = m(g + a)$

For limiting condition $T = T_{\max} \Rightarrow m(g + a) = 250 \Rightarrow 20(10 + a) = 250 \quad \therefore a = 2.5\ m/s^2$

***Problem* 48.** Three blocks of masses 2 *kg*, 3 *kg* and 5 *kg* are connected to each other with light string and are then placed on a frictionless surface as shown in the figure. The system is pulled by a force $F = 10N$, then tension $T_1 =$

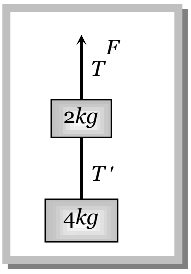
[Orissa JEE 2002]



(a) 1*N* (b) 5 *N* (c) 8 *N* (d) 10 *N*

Solution : (c) By comparing the above problem with general expression. $T_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3} = \frac{(3+5)10}{2+3+5} = 8\ Newton$

***Problem* 49.** Two blocks are connected by a string as shown in the diagram. The upper block is hung by another string. A force *F* applied on the upper string produces an acceleration of $2m/s^2$ in the upward direction in both the blocks. If *T* and T' be the tensions in the two parts of the string, then **[AMU (Engg.) 2000]**



(a) $T = 70.8N$ and $T' = 47.2N$

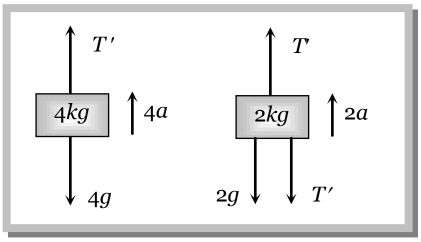
(b) $T = 58.8N$ and $T' = 47.2N$

(c) $T = 70.8N$ and $T' = 58.8N$

(d) $T = 70.8N$ and $T' = 0$

Solution : (a) From F.B.D. of mass 4 *kg* $\quad 4a = T' - 4g$ .....(i)

From F.B.D. of mass 2 *kg* $\quad 2a = T - T' - 2g$ .....(ii)



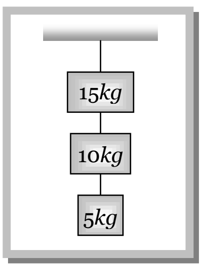
For total system upward force

$$F = T = (2+4)(g+a) = 6(18+2)N = 70.8\text{ N}$$

by substituting the value of T in equation (i) and (ii)

and solving we get $T' = 47.2N$

***Problem* 50.** Three masses of 15 *kg*. 10 *kg* and 5 *kg* are suspended vertically as shown in the fig. If the string attached to the support breaks and the system falls freely, what will be the tension in the string between 10 *kg* and 5 *kg* masses. Take $g = 10\ ms^{-2}$. It is assumed that the string remains tight during the motion



(a) 300 N (b) 250 N (c) 50 N (d) Zero

Solution : (d) In the condition of free fall, tension becomes zero.

***Problem* 51.** A sphere is accelerated upwards with the help of a cord whose breaking strength is five times its weight. The maximum acceleration with which the sphere can move up without cord breaking is

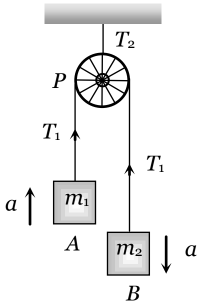
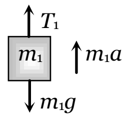
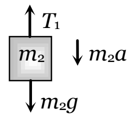
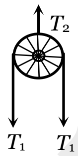
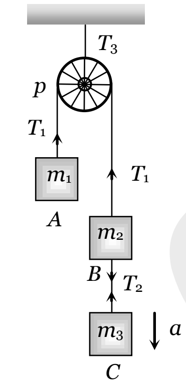
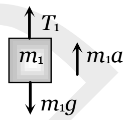
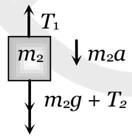
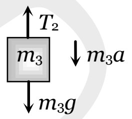
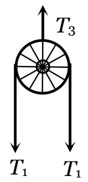
(a) $4g$ (b) $3g$ (c) $2g$ (d) g

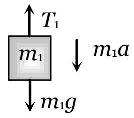
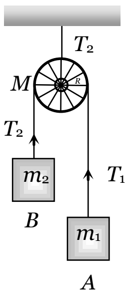
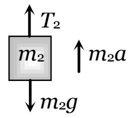
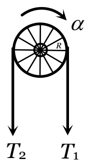
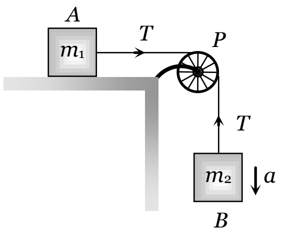
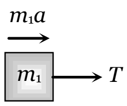
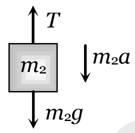
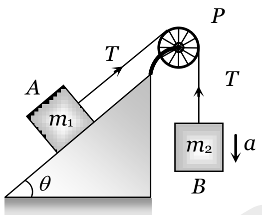
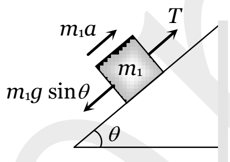
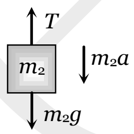
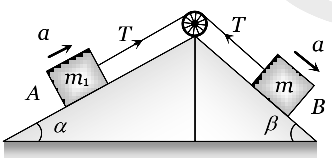
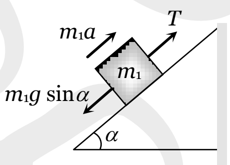
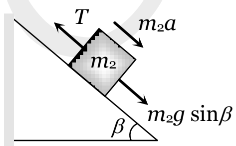
Solution : (a) Tension in the cord = $m(g+a)$ and breaking strength = 5 mg

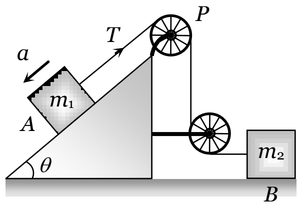
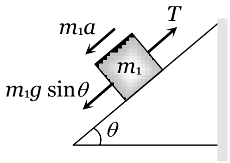
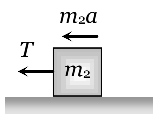
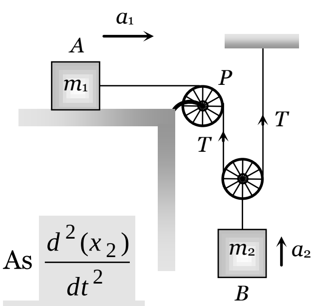
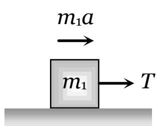
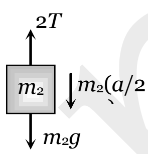
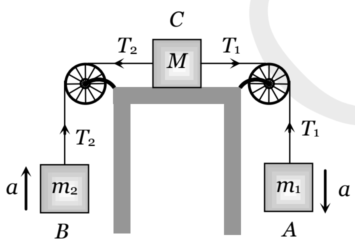
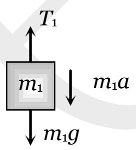
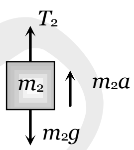
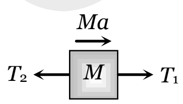
For critical condition $m(g+a) = 5mg \Rightarrow a = 4g$

This is the maximum acceleration with which the sphere can move up with cord breaking.

4.18 Motion of Connected Block Over a Pulley

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1 a = T_1 - m_1 g$	$T_1 = \frac{2m_1 m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g - T_1$	$T_2 = \frac{4m_1 m_2}{m_1 + m_2} g$
		$T_2 = 2T_1$	$a = \left[\frac{m_2 - m_1}{m_1 + m_2}\right] g$
		$m_1 a = T_1 - m_1 g$	$T_1 = \frac{2m_1[m_2 + m_3]}{m_1 + m_2 + m_3} g$
		$m_2 a = m_2 g + T_2 - T_1$	$T_2 = \frac{2m_1 m_3}{m_1 + m_2 + m_3} g$
		$m_3 a = m_3 g - T_2$	$T_3 = \frac{4m_1[m_2 + m_3]}{m_1 + m_2 + m_3} g$
		$T_3 = 2T_1$	$a = \frac{[(m_2 + m_3) - m_1]g}{m_1 + m_2 + m_3}$

Condition	Free body diagram	Equation	Tension and acceleration
When pulley have a finite mass M and radius R then tension in two segments of string are different		$m_1 a = m_1 g - T_1$	$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{M}{2}}$
		$m_2 a = T_2 - m_2 g$	$T_1 = \frac{m_1\left[2m_2 + \frac{M}{2}\right]}{m_1 + m_2 + \frac{M}{2}} g$
		Torque $= (T_1 - T_2)R = I\alpha$ $(T_1 - T_2)R = I\frac{a}{R}$ $(T_1 - T_2)R = \frac{1}{2}MR^2\frac{a}{R}$ $T_1 - T_2 = \frac{Ma}{2}$	$T_2 = \frac{m_2\left[2m_1 + \frac{M}{2}\right]}{m_1 + m_2 + \frac{M}{2}} g$
		$T = m_1 a$	$a = \frac{m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2}{m_1 + m_2} g$
		$m_1 a = T - m_1 g \sin\theta$	$a = \left[\frac{m_2 - m_1 \sin\theta}{m_1 + m_2}\right] g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2 (1 + \sin\theta)}{m_1 + m_2} g$
		$T - m_1 g \sin\alpha = m_1 a$	$a = \frac{(m_2 \sin\beta - m_1 \sin\alpha)}{m_1 + m_2} g$
		$m_2 a = m_2 g \sin\beta - T$	$T = \frac{m_1 m_2 (\sin\alpha + \sin\beta)}{m_1 + m_2} g$

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1 g \sin\theta - T = m_1 a$	$a = \frac{m_1 g \sin\theta}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{2m_1 m_2}{4m_1 + m_2} g$
 As $\frac{d^2(x_2)}{dt^2} = \frac{1}{2}\frac{d^2(x_1)}{dt^2}$ $\therefore a_2 = \frac{a_1}{2}$ $a_1 =$ acceleration of block A $a_2 =$ acceleration of block B		$T = m_1 a$	$a_1 = a = \frac{2m_2 g}{4m_1 + m_2}$
		$m_2 \frac{a}{2} = m_2 g - 2T$	$a_2 = \frac{m_2 g}{4m_1 + m_2}$ $T = \frac{2m_1 m_2 g}{4m_1 + m_2}$
		$m_1 a = m_1 g - T_1$	$a = \frac{(m_1 - m_2)}{[m_1 + m_2 + M]} g$
		$m_2 a = T_2 - m_2 g$	$T_1 = \frac{m_1(2m_2 + M)}{[m_1 + m_2 + M]} g$
		$T_1 - T_2 = Ma$	$T_2 = \frac{m_2(2m_2 + M)}{[m_1 + m_2 + M]} g$

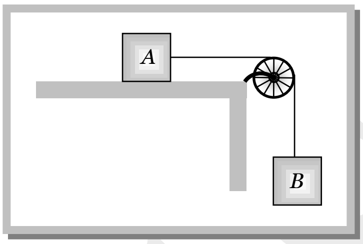
Sample problems based on motion of blocks over pulley

Problem 52. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$ then the ratio of the masses is **[AIEEE 2002]**

(a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3

Solution : (b) $a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \frac{g}{8}$; by solving $\frac{m_2}{m_1} = 9/7$

Problem 53. A block A of mass 7 kg is placed on a frictionless table. A thread tied to it passes over a frictionless pulley and carries a body B of mass 3 kg at the other end. The acceleration of the system is (given $g = 10\,ms^{-2}$) **[Kerala (Engg.) 2000]**



(a) $100\,ms^{-2}$

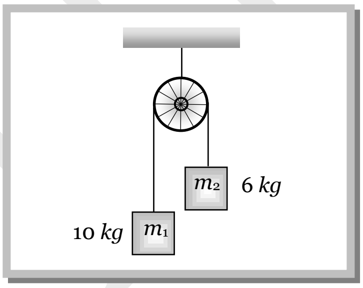
(b) $3ms^{-2}$

(c) $10ms^{-2}$

(d) $30ms^{-2}$

Solution : (b) $a = \left(\frac{m_2}{m_1 + m_2}\right)g = \left(\frac{3}{7+3}\right)10 = 3m/s^2$

Problem 54. Two masses m_1 and m_2 are attached to a string which passes over a frictionless smooth pulley. When $m_1 = 10\,kg$, $m_2 = 6\,kg$, the acceleration of masses is **[Orissa JEE 2002]**



(a) $20\,m/s^2$

(b) $5m/s^2$

(c) $2.5\,m/s^2$

(d) $10\,m/s^2$

Solution : (c) $a = \frac{m_1 - m_2}{m_1 + m_2}g = \left(\frac{10-6}{10+6}\right)10 = 2.5\,m/s^2$

Problem 55. Two weights W_1 and W_2 are suspended from the ends of a light string passing over a smooth fixed pulley. If the pulley is pulled up with an acceleration g, the tension in the string will be

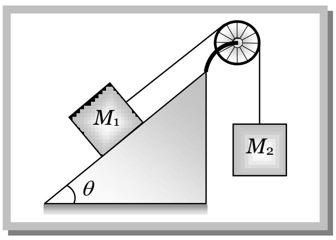
(a) $\frac{4W_1W_2}{W_1 + W_2}$ (b) $\frac{2W_1W_2}{W_1 + W_2}$ (c) $\frac{W_1W_2}{W_1 + W_2}$ (d) $\frac{W_1W_2}{2(W_1 + W_2)}$

Solution : (a) When the system is at rest tension in string $T = \frac{2m_1m_2}{(m_1 + m_2)}g$

If the system moves upward with acceleration g then $T = \frac{2m_1m_2}{m_1 + m_2}(g + g) = \frac{4m_1m_2}{m_1 + m_2}g$ or $T = \frac{4w_1w_2}{w_1 + w_2}$

Problem 56. Two masses M_1 and M_2 are attached to the ends of a string which passes over a pulley attached to the top of an inclined plane. The angle of inclination of the plane in θ. Take $g = 10\ ms^{-2}$.

If $M_1 = 10\ kg$, $M_2 = 5\ kg$, $\theta = 30^o$, what is the acceleration of mass M_2



(a) $10ms^{-2}$ (b) $5ms^{-2}$

(c) $\frac{2}{3}ms^{-2}$ (d) Zero

Solution : (d) Acceleration $= \frac{m_2 - m_1 \sin\theta}{m_1 + m_2}g = \frac{5 - 10\,.\sin 30}{5+10}g = \frac{5-5}{5+10}g = 0$

Problem 57. In the above problem, what is the tension in the string

(a) 100 N (b) 50 N (c) 25 N (d) Zero

Solution : (b) $T = \frac{m_1 m_2(1+\sin\theta)}{m_1+m_2}g = \frac{10 \times 5(1+\sin 30).10}{10+5} = 50N$

Problem 58. In the above problem, given that $M_2 = 2M_1$ and M_2 moves vertically downwards with acceleration a. If the position of the masses are reversed the acceleration of M_2 down the inclined plane will be

(a) $2a$ (b) a (c) $a/2$ (d) None of the above

Solution : (d) If $m_2 = 2m_1$, then m_2 moves vertically downward with acceleration

$$a = \frac{m_2 - m_1 \sin\theta}{m_1+m_2}g = \frac{2m_1 - m_1\sin 30}{m_1+2m_1}g = g/2$$

If the position of masses are reversed then m_2 moves downward with acceleration

$$a' = \frac{m_2\sin\theta - m_1}{m_1+m_2}g = \frac{2m_1\sin 30 - m_1}{m_1+2m_1}.g = 0 \qquad [\text{As } m_2 = 2m_1]$$

i.e. the m_2 will not move.

Problem 59. In the above problem, given that $M_2 = 2M_1$ and the tension in the string is T. If the positions of the masses are reversed, the tension in the string will be

(a) $4T$ (b) $1T$ (c) T (d) $T/2$

Solution : (c) Tension in the string $T = \frac{m_1 m_2(1+\sin\theta)}{m_1+m_2}g$

If the position of the masses are reversed then there will be no effect on tension.

Problem 60. In the above problem, given that $M_1 = M_2$ and $\theta = 30^o$. What will be the acceleration of the system

(a) $10ms^{-2}$ (b) $5ms^{-2}$ (c) $2.5ms^{-2}$ (d) Zero

Solution : (c) $a = \frac{m_2 - m_1\sin\theta}{m_1+m_2}g = \frac{1-\sin 30}{2}g = \frac{g}{4} = 2.5\, m/s^2$ [As $m_1 = m_2$]

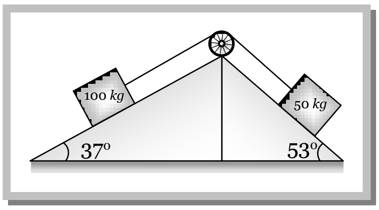
Problem 61. In the above problem, given that $M_1 = M_2 = 5\,kg$ and $\theta = 30^o$. What is tension in the string

(a) $37.5\,N$ (b) $25\,N$ (c) $12.5\,N$ (d) Zero

Solution : (a) $T = \frac{m_1 m_2(1+\sin\theta)}{m_1+m_2}g = \frac{5\times 5(1+\sin 30)}{5+5}\times 10 = 37.5\,N$

Problem 62. Two blocks are attached to the two ends of a string passing over a smooth pulley as shown in the figure. The acceleration of the block will be (in m/s^2) ($\sin 37^o = 0.60$, $\sin 53^o = 0.80$)

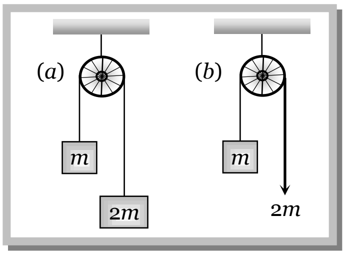
(a) 0.33
(b) 0.133
(c) 1
(d) 0.066



Solution : (b) $a = \frac{m_2\sin\beta - m_1\sin\alpha}{m_1+m_2}g = \frac{50\sin 53^\circ - 100\sin 37^\circ}{100+50}g = -0.133\, m/s^2$

Problem 63. The two pulley arrangements shown in the figure are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass $2m$ to the other end of the rope. In (b). m is lifted up by pulling the other end of the rope with a constant downward force of $2mg$. The ratio of accelerations in two cases will be

(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4



Solution : (c) For first case $a_1 = \frac{m_2 - m_1}{m_1+m_2}g = \frac{2m-m}{m+2m} = \frac{g}{3}$ .....(i)

For second case

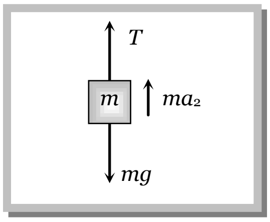
from free body diagram of m

$ma_2 = T - mg$

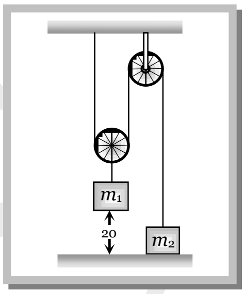
$ma_2 = 2mg - mg$ [As $T = 2mg$]

$a_2 = g$ ....(ii)

From (i) and (ii) $\frac{a_1}{a_2} = \frac{g/3}{g} = 1/3$



Problem 64. In the adjoining figure $m_1 = 4m_2$. The pulleys are smooth and light. At time $t = 0$, the system is at rest. If the system is released and if the acceleration of mass m_1 is a, then the acceleration of m_2 will be



(a) g

(b) a

(c) $\frac{a}{2}$

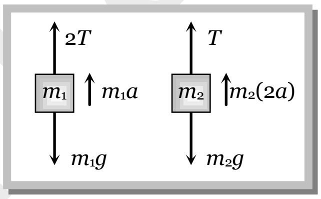
(d) $2a$

Solution : (d) Since the mass m_2 travels double distance in comparison to mass m_1 therefore its acceleration will be double *i.e.* $2a$

Problem 65. In the above problem (64), the value of a will be

(a) g (b) $\frac{g}{2}$ (c) $\frac{g}{4}$ (d) $\frac{g}{8}$

Solution : (c) By drawing the FBD of m_1 and m_2



$m_1 a = m_1 g - 2T$ .....(i)

$m_2(2a) = T - m_2 g$ .....(ii)

by solving these equation $a = g/4$

Problem 66. In the above problem, the tension T in the string will be

(a) m_2g (b) $\frac{m_2 g}{2}$ (c) $\frac{2}{3}m_2 g$ (d) $\frac{3}{2}m_2 g$

Solution : (d) From the solution (65) by solving equation

$$T = \frac{3}{2}m_2 g$$

Problem 67. In the above problem, the time taken by m_1 in coming to rest position will be

(a) 0.2 *s* (b) 0.4 *s* (c) 0.6 *s* (d) 0.8 *s*

Solution : (b) Time taken by mass m_2 to cover the distance 20 *cm*

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2 \times 0.2}{g/4}} = \sqrt{\frac{2 \times 0.2}{2.5}} = 0.4 \text{ sec}$$

Problem 68. In the above problem, the distance covered by m_2 in 0.4 *s* will be

(a) 40 *cm* (b) 20 *cm* (c) 10 *cm* (d) 80 *cm*

Solution : (a) Since the m_2 mass cover double distance therefore $S = 2 \times 20 = 40$ *cm*

Problem 69. In the above problem, the velocity acquired by m_2 in 0.4 second will be

(a) 100 *cm/s* (b) 200 *cm/s* (c) 300 *cm/s* (d) 400 *cm/s*

Solution : (b) Velocity acquired by mass m_2 in 0.4 sec

From $v = u + at$ [As $a = g/2 = \frac{10}{2} = 5\, m/s^2$]

$v = 0 + 5 \times 0.4 = 2 m/s = 200\, cm/sec$.

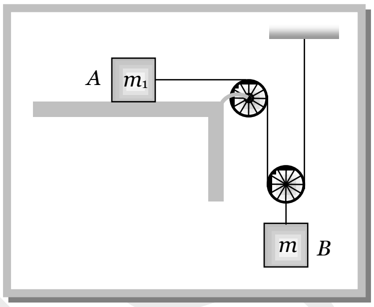
***Problem* 70.** In the above problem, the additional distance traversed by m_2 in coming to rest position will be

(a) 20 *cm* (b) 40 *cm* (c) 60 *cm* (d) 80 *cm*

Solution : (a) When m_2 mass acquired velocity 200 cm/sec it will move upward till its velocity becomes zero.

$$H = \frac{u^2}{2g} = \frac{(200)^2}{2 \times 100} = 20\ cm$$

***Problem* 71.** The acceleration of block *B* in the figure will be



(a) $\frac{m_2 g}{(4m_1 + m_2)}$

(b) $\frac{2m_2 g}{(4m_1 + m_2)}$

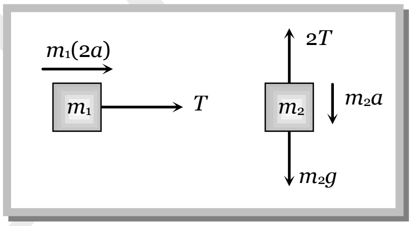
(c) $\frac{2m_1 g}{(m_1 + 4m_2)}$

(d) $\frac{2m_1 g}{(m_1 + m_2)}$

Solution : (a) When the block m_2 moves downward with acceleration a, the acceleration of mass m_1 will be $2a$ because it covers double distance in the same time in comparison to m_2.

Let T is the tension in the string.

By drawing the free body diagram of *A* and *B*



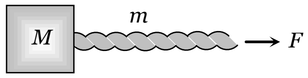
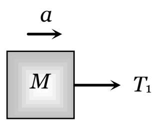
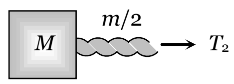
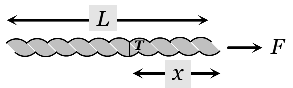
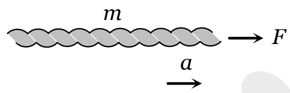
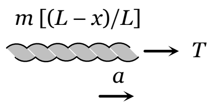
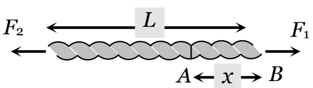
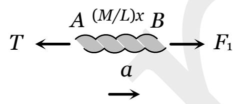
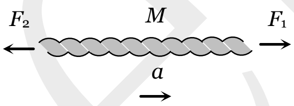
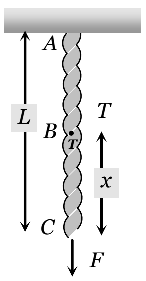
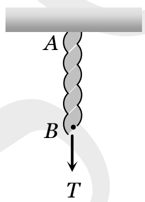
$T = m_1 2a$ ....(i)

$m_2 g - 2T = m_2 a$ ....(ii)

by solving (i) and (ii)

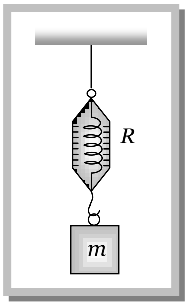
$$a = \frac{m_2 g}{(4m_1 + m_2)}$$

4.19 Motion of Massive String

Condition	Free body diagram	Equation	Tension and acceleration
	 T_1 = force applied by the string on the block	$F=(M+m)a$ $T_1=Ma$	$a=\frac{F}{M+m}$ $T_1=M\frac{F}{(M+m)}$ $T_2=\frac{(2M+m)}{2(M+m)}F$
	 T_2 = Tension at mid point of the rope	$T_2=\left(M+\frac{m}{2}\right)a$	
 m = Mass of string T = Tension in string at a distance x from the end where the force is applied		$F=ma$	$a=F/m$ $T=\left(\frac{L-x}{L}\right)F$
		$T=m\left(\frac{L-x}{L}\right)a$	
 M = Mass of uniform rod L = Length of rod		$F_1-T=\frac{Mxa}{L}$	$a=\frac{F_1-F_2}{M}$ $T=F_1\left(1-\frac{x}{L}\right)+F_2\left(\frac{x}{L}\right)$
		$F_1-F_2=Ma$	
 Mass of segment BC $=\left(\frac{M}{L}\right)x$		$T=\left(\frac{L-x}{L}\right)F$	$T=\left(\frac{L-x}{L}\right)F$

4.20 Spring Balance and Physical Balance

(1) **Spring balance :** When its upper end is fixed with rigid support and body of mass m hung from its lower end. Spring is stretched and the weight of the body can be measured by the reading of spring balance $R = W = mg$

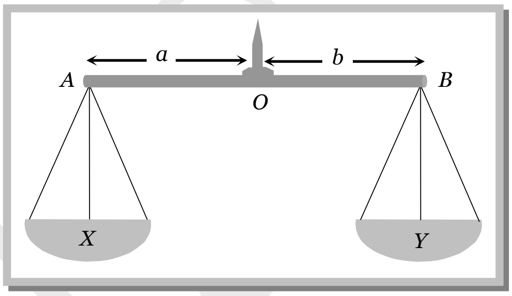


The mechanism of weighing machine is same as that of spring balance.

Effect of frame of reference : In inertial frame of reference the reading of spring balance shows the actual weight of the body but in non-inertial frame of reference reading of spring balance increases or decreases in accordance with the direction of acceleration

[for detail refer Article (4.13)]

(2) **Physical balance :** In physical balance actually we compare the mass of body in both the pans. Here we does not calculate the absolute weight of the body.



Here X and Y are the mass of the empty pan.

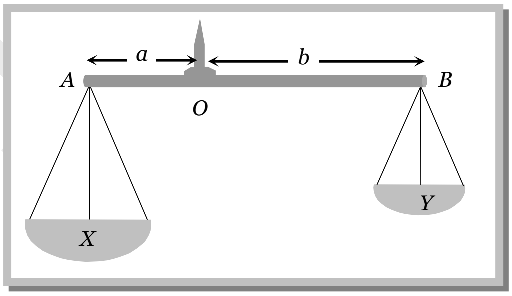
(i) Perfect physical balance :

Weight of the pan should be equal *i.e.* $X = Y$

and the needle must in middle of the beam *i.e.* $a = b$.

Effect of frame of reference : If the physical balance is perfect then there will be no effect of frame of reference (either inertial or non-inertial) on the measurement. It is always errorless.

(ii) False balance : When the masses of the pan are not equal then balance shows the error in measurement. False balance may be of two types



(a) If the beam of physical balance is horizontal (when the pans are empty) but the arms are not equal

$X > Y$ and $a < b$

For rotational equilibrium about point 'O'

$$Xa = Yb \qquad \text{......(i)}$$

In this physical balance if a body of weight W is placed in pan X then to balance it we have to put a weight W_1 in pan Y.

For rotational equilibrium about point 'O'

$$(X + W)a = (Y + W_1)b \qquad \text{.....(ii)}$$

Now if the pans are changed then to balance the body we have to put a weight W_2 in pan X.

For rotational equilibrium about point 'O'

$$(X + W_2)a = (Y + W)b \qquad \text{......(iii)}$$

From (i), (ii) and (iii)

True weight $W = \sqrt{W_1 W_2}$

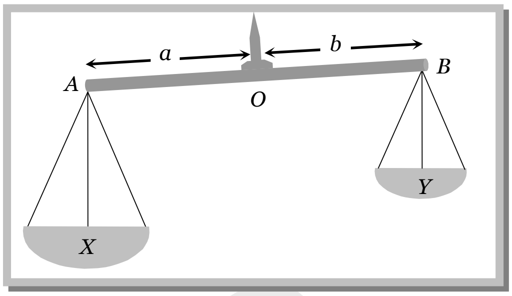
(b) If the beam of physical balance is not horizontal (when the pans are empty) and the arms are equal

i.e. $X > Y$ and $a = b$

In this physical balance if a body of weight W is placed in X Pan then to balance it.

We have to put a weight W_1 in Y Pan

For equilibrium $X + W = Y + W_1$ .....(i)



Now if pans are changed then to balance the body we have to put a weight W_2 in X Pan.

For equilibrium $X + W_2 = Y + W$ .....(ii)

From (i) and (ii)

True weight $W = \dfrac{W_1 + W_2}{2}$

Sample problems (Miscellaneous)

Problem 72. A body weighs 8 *gm*, when placed in one pan and 18*gm*, when placed in the other pan of a false balance. If the beam is horizontal (when both the pans are empty), the true weight of the body is

(a) 13 *gm* (b) 12 *gm* (c) 15.5 *gm* (d) 15 *gm*

Solution : (b) For given condition true weight = $\sqrt{W_1 W_2} = \sqrt{8 \times 18} = 12$ *gm*.

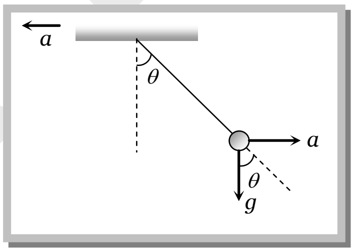
Problem 73. A plumb line is suspended from a ceiling of a car moving with horizontal acceleration of a. What will be the angle of inclination with vertical **[Orissa JEE 2003]**

(a) $\tan^{-1}(a/g)$ (b) $\tan^{-1}(g/a)$ (c) $\cos^{-1}(a/g)$ (d) $\cos^{-1}(g/a)$

Solution : (a) From the figure

$\tan\theta = \dfrac{a}{g}$

$\theta = \tan^{-1}(a/g)$



Problem 74. A block of mass $5\,kg$ is moving horizontally at a speed of 1.5 *m/s*. A perpendicular force of 5 *N* acts on it for 4 *sec*. What will be the distance of the block from the point where the force started acting **[Pb PMT 2002]**

(a) 10 *m* (b) 8 *m* (c) 6 *m* (d) 2 *m*

Solution : (a) In the given problem force is working in a direction perpendicular to initial velocity. So the body will move under the effect of constant velocity in horizontal direction and under the effect of force in vertical direction.

$S_x = u_x \times t = 1.5 \times 4 = 6m$

$S_y = u_y t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(F/m)t^2 = \frac{1}{2}(5/5)(4)^2 = 8m$

$\therefore\ S = \sqrt{S_x^2 + S_y^2} = \sqrt{36 + 64} = \sqrt{100} = 10m$

Problem 75. The velocity of a body of rest mass m_0 is $\dfrac{\sqrt{3}}{2}c$ (where c is the velocity of light in vacuum). Then mass of this body is **[Orissa JEE 2002]**

(a) $(\sqrt{3}/2)m_0$ (b) $(1/2)m_0$ (c) $2m_0$ (d) $(2/\sqrt{3})m_0$

Solution : (c) From Einstein's formula $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{m_0}{\sqrt{1-\dfrac{\left(\sqrt{\dfrac{3}{2}}C\right)^2}{C^2}}} = \dfrac{m_0}{\sqrt{1-\dfrac{3}{4}}} = 2m_0$

Problem 76. Three weights W, $2W$ and $3W$ are connected to identical springs suspended form rigid horizontal rod. The assembly of the rod and the weights fall freely. The positions of the weights from the rod are such that

[Roorkee 1999]

(a) 3 W will be farthest (b) W will be farthest
(c) All will be at the same distance (d) 2 W will be farthest

Solution : (c) For W, $2W$, $3W$ apparent weight will be zero because the system is falling freely. So there will be no extension in any spring *i.e.* the distances of the weight from the rod will be same.

Problem 77. A bird is sitting on stretched telephone wires. If its weight is W then the additional tension produced by it in the wires will be

(a) $T = W$ (b) $T > W$ (c) $T < W$ (d) $T = 0$

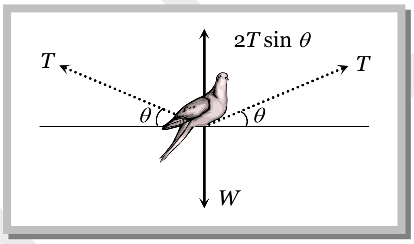
Solution : (b) For equilibrium

$2T\sin\theta = W$

$T = \dfrac{W}{2\sin\theta}$

θ lies between 0 to 90° *i.e.* $\sin\theta < 1$

$\therefore\ T > W$



Problem 78. With what minimum acceleration can a fireman slides down a rope while breaking strength of the rope is $\dfrac{2}{3}$ his weight

[CPMT 1979]

(a) $\dfrac{2}{3}g$ (b) g (c) $\dfrac{1}{3}g$ (d) Zero

Solution : (c) When fireman slides down, Tension in the rope $T = m(g-a)$

For critical condition $m(g-a) = 2/3\ mg \Rightarrow mg - ma = \dfrac{2}{3}mg \quad \therefore\ a = \dfrac{g}{3}$

So, this is the minimum acceleration by which a fireman can slides down on a rope.

Problem 79. A car moving at a speed of 30 *kilo meters* per hour's is brought to a halt in 8 *metres* by applying brakes. If the same car is moving at 60 *km*. per hour, it can be brought to a halt with same braking power in

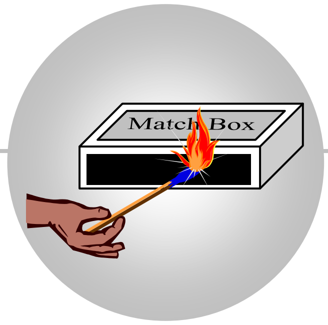
(a) 8 *metres* (b) 16 *metres* (c) 24 *metres* (d) 32 *metres*

Solution : (d) From $v^2 = u^2 - 2as$

$0 = u^2 - 2as$

$s = \dfrac{u^2}{2a} \Rightarrow s \propto u^2$ (if a = constant)

$\dfrac{s_2}{s_1} = \left(\dfrac{u_2}{u_1}\right)^2 = \left(\dfrac{60}{30}\right)^2 = 4 \Rightarrow s_2 = 4s_1 = 4\times 8 = 32$ *metres.*



Friction

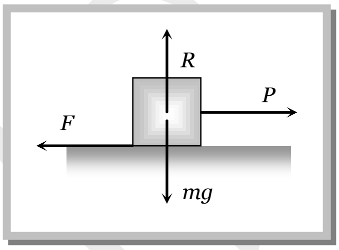
5.1 Introduction

If we slide or try to slide a body over a surface the motion is resisted by a bonding between the body and the surface. This resistance is represented by a single force and is called friction.

The force of friction is parallel to the surface and opposite to the direction of intended motion.

5.2 Types of Friction

(1) **Static friction :** The opposing force that comes into play when one body tends to move over the surface of another, but the actual motion has yet not started is called static friction.



(i) If applied force is P and the body remains at rest then static friction $F = P$.

(ii) If a body is at rest and no pulling force is acting on it, force of friction on it is zero.

(iii) Static friction is a self-adjusting force because it changes itself in accordance with the applied force.

(2) **Limiting friction :** If the applied force is increased the force of static friction also increases. If the applied force exceeds a certain (maximum) value, the body starts moving. This maximum value of static friction upto which body does not move is called limiting friction.

(i) The magnitude of limiting friction between any two bodies in contact is directly proportional to the normal reaction between them.

$$F_l \propto R \text{ or } F_l = \mu_s R$$

(ii) Direction of the force of limiting friction is always opposite to the direction in which one body is at the verge of moving over the other

(iii) Coefficient of static friction : (a) μ_s is called coefficient of static friction and defined as the ratio of force of limiting friction and normal reaction $\mu_s = \dfrac{F}{R}$

(b) Dimension : $[M^0 L^0 T^0]$

(c) Unit : It has no unit.

(d) Value of μ_s lies in between 0 and 1

(e) Value of μ depends on material and nature of surfaces in contact that means whether dry or wet ; rough or smooth polished or non-polished.

(f) Value of μ does not depend upon apparent area of contact.

(3) **Kinetic or dynamic friction :** If the applied force is increased further and sets the body in motion, the friction opposing the motion is called kinetic friction.

(i) Kinetic friction depends upon the normal reaction.

$$F_k \propto R \text{ or } F_k = \mu_k R \text{ where } \mu_k \text{ is called the coefficient of kinetic friction}$$

(ii) Value of μ_k depends upon the nature of surface in contact.

(iii) Kinetic friction is always lesser than limiting friction $F_k < F_l \quad \therefore \mu_k < \mu_s$

i.e. coefficient of kinetic friction is always less than coefficient of static friction. Thus we require more force to start a motion than to maintain it against friction. This is because once the motion starts actually ; inertia of rest has been overcome. Also when motion has actually started, irregularities of one surface have little time to get locked again into the irregularities of the other surface.

(iv) Types of kinetic friction

(a) **Sliding friction** : The opposing force that comes into play when one body is actually sliding over the surface of the other body is called sliding friction. *e.g.* A flat block is moving over a horizontal table.

(b) **Rolling friction** : When objects such as a wheel (disc or ring), sphere or a cylinder rolls over a surface, the force of friction comes into play is called rolling friction.

❑ Rolling friction is directly proportional to the normal reaction (R) and inversely proportional to the radius (r) of the rolling cylinder or wheel.

$$F_{rolling} = \mu_r \frac{R}{r}$$

μ_r is called coefficient of rolling friction. It would have the dimensions of length and would be measured in *metre.*

❑ Rolling friction is often quite small as compared to the sliding friction. That is why heavy loads are transported by placing them on carts with wheels.

❑ In rolling the surfaces at contact do not rub each other.

❑ The velocity of point of contact with respect to the surface remains zero all the times although the centre of the wheel moves forward.

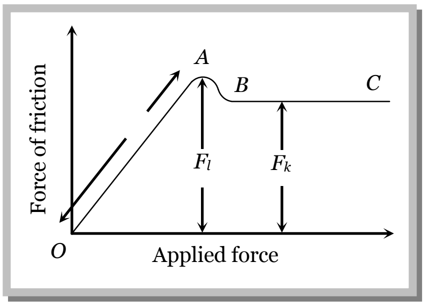
5.3 Graph Between Applied Force and Force of Friction

(1) Part OA of the curve represents static friction (F_s). Its value increases linearly with the applied force

(2) At point A the static friction is maximum. This represent limiting friction (F_l).

(3) Beyond A, the force of friction is seen to decrease slightly. The portion BC of the curve therefore represents the kinetic friction (F_k).

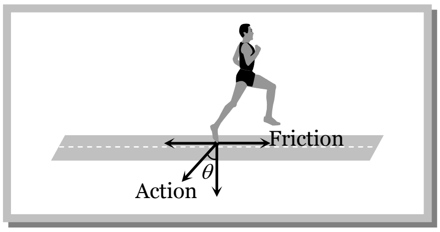
(4) As the portion BC of the curve is parallel to x-axis therefore kinetic friction does not change with the applied force, it remains constant, whatever be the applied force.



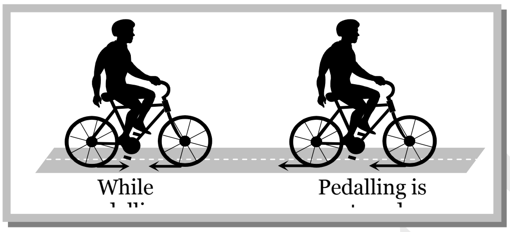
5.4 Friction is a Cause of Motion

It is a general misconception that friction always opposes the motion. No doubt friction opposes the motion of a moving body but in many cases it is also the cause of motion. For example :

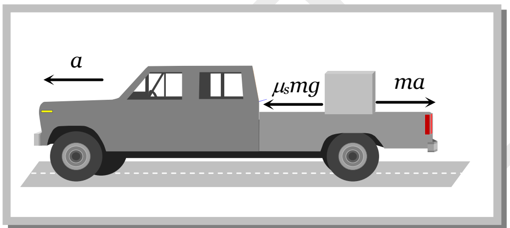
(1) In moving, a person or vehicle pushes the ground backwards (action) and the rough surface of ground reacts and exerts a forward force due to friction which causes the motion. If there had been no friction there will be slipping and no motion.



(2) In cycling, the rear wheel moves by the force communicated to it by pedalling while front wheel moves by itself. So, when pedalling a bicycle, the force exerted by rear wheel on ground makes force of friction act on it in the forward direction (like walking). Front wheel moving by itself experience force of friction in backward direction (like rolling of a ball). [However, if pedalling is stopped both wheels move by themselves and so experience force of friction in backward direction.]



(3) If a body is placed in a vehicle which is accelerating, the force of friction is the cause of motion of the body along with the vehicle (*i.e.*, the body will remain at rest in the accelerating vehicle until $ma < \mu_s mg$). If there had been no friction between body and vehicle the body will not move along with the vehicle.



From these examples it is clear that without friction motion cannot be started, stopped or transferred from one body to the other.

Sample problems based on fundamentals of friction

***Problem* 1.** If a ladder weighing 250N is placed against a smooth vertical wall having coefficient of friction between it and floor is 0.3, then what is the maximum force of friction available at the point of contact between the ladder and the floor **[AIIMS 2002]**

(a) 75 N (b) 50 N (c) 35 N (d) 25 N

Solution : (a) Maximum force of friction $F_l = \mu_s R = 0.3 \times 250 = 75\,N$

***Problem* 2.** On the horizontal surface of a truck (μ = 0.6), a block of mass 1 kg is placed. If the truck is accelerating at the rate of 5m/sec^2 then frictional force on the block will be

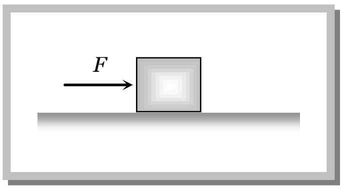
(a) 5 N (b) 6 N (c) 5.88 N (d) 8 N

Solution : (a) Limiting friction $= \mu_s R = \mu_s mg = 0.6 \times 1 \times 9.8 = 5.88\,N$

When truck accelerates in forward direction at the rate of $5m/s^2$ a pseudo force (ma) of 5N works on block in back ward direction. Here the magnitude of pseudo force is less than limiting friction So, static friction works in between the block and the surface of the truck and as we know, static friction = Applied force = 5N.

***Problem* 3.** A block of mass 2 kg is kept on the floor. The coefficient of static friction is 0.4. If a force F of 2.5 N is applied on the block as shown in the figure, the frictional force between the block and the floor will be **[MP PET 2**

(a) 2.5 N



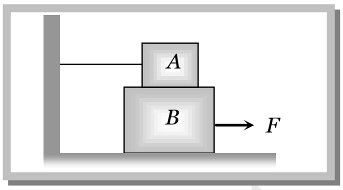
(b) 5 N
(c) 7.84 N
(d) 10 N

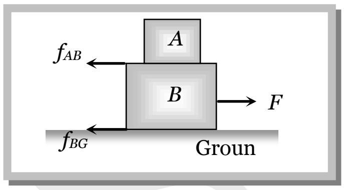
Solution : (a) Applied force = 2.5 N and limiting friction = μmg = 0.4 × 2 × 9.8 = 7.84 N

As applied force is less than limiting friction. So, for the given condition static friction will work.
Static friction on a body = Applied force = 2.5 N.

***Problem* 4.** A block A with mass 100 kg is resting on another block B of mass 200 kg. As shown in figure a horizontal rope tied to a wall holds it. The coefficient of friction between A and B is 0.2 while coefficient of friction between B and the ground is 0.3. The minimum required force F to start moving B will be

(a) 900 N
(b) 100 N
(c) 1100 N
(d) 1200 N





Solution : (c) Two frictional force will work on block B.

$F = f_{AB} + f_{BG} = \mu_{AB} m_a g + \mu_{BG}(m_A + m_B) g$

= 0.2 × 100 × 10 + 0.3 (300) × 10

= 200 + 900 = 1100N. (This is the required minimum force)

***Problem* 5.** A 20 kg block is initially at rest on a rough horizontal surface. A horizontal force of 75 N is required to set the block in motion. After it is in motion, a horizontal force of 60 N is required to keep the block moving with constant speed. The coefficient of static friction is

(a) 0.38 (b) 0.44 (c) 0.52 (d) 0.60

Solution : (a) Coefficient of static friction $\mu_S = \dfrac{F_l}{R} = \dfrac{75}{20 \times 9.8} = 0.38$.

***Problem* 6.** A block of mass M is placed on a rough floor of a lift. The coefficient of friction between the block and the floor is μ. When the lift falls freely, the block is pulled horizontally on the floor. What will be the force of friction

(a) μMg (b) $\mu Mg/2$ (c) $2\mu Mg$ (d) None of these

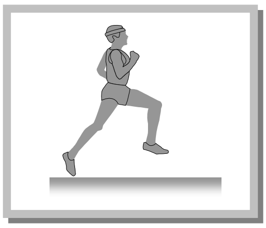
Solution : (d) When the lift moves down ward with acceleration 'a' then effective acceleration due to gravity

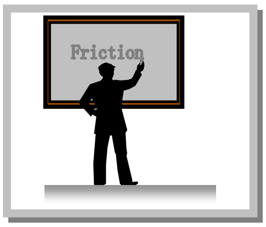
$g' = g - a$

$\therefore g' = g - g = 0$ [As the lift falls freely, so $a = g$]

So force of friction $= \mu mg' = 0$

5.5 Advantages and Disadvantages of Friction





(1) **Advantages of friction**

(i) Walking is possible due to friction.

(ii) Two body sticks together due to friction.

(iii) Brake works on the basis of friction.

(iv) Writing is not possible without friction.

(v) The transfer of motion from one part of a machine to other part through belts is possible by friction.

(2) **Disadvantages of friction**

(i) Friction always opposes the relative motion between any two bodies in contact. Therefore extra energy has to be spent in over coming friction. This reduces the efficiency of machine.

(ii) Friction causes wear and tear of the parts of machinery in contact. Thus their lifetime reduces.

(iii) Frictional force result in the production of heat, which causes damage to the machinery.

5.6 Methods of Changing Friction

We can reduce friction

(1) By polishing.

(2) By lubrication.

(3) By proper selection of material.

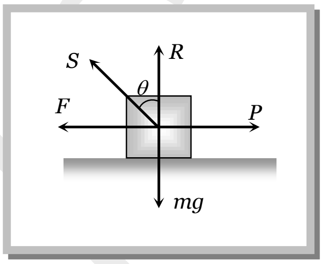
(4) By streamlining the shape of the body.

(5) By using ball bearing.

Also we can increase friction by throwing some sand on slippery ground. In the manufacturing of tyres, synthetic rubber is preferred because its coefficient of friction with the road is larger.

5.7 Angle of Friction

Angle of friction may be defined as the angle which the resultant of limiting friction and normal reaction makes with the normal reaction.



By definition angle θ is called the angle of friction

$$\tan\theta = \frac{F}{R}$$

$\therefore$ $\tan\theta = \mu$ [As we know $\frac{F}{R} = \mu$]

or $\theta = \tan^{-1}(\mu)$

Hence coefficient of limiting friction is equal to tangent of the angle of friction.

5.8 Resultant Force Exerted by Surface on Block

In the above figure resultant force $S = \sqrt{F^2 + R^2}$

$$S = \sqrt{(\mu mg)^2 + (mg)^2}$$

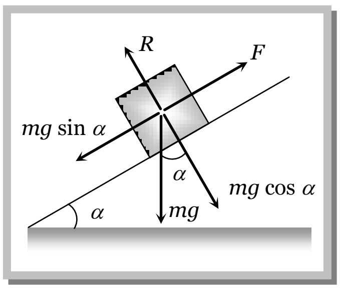
$$S = mg\sqrt{\mu^2 + 1}$$

when there is no friction $(\mu = 0)$ S will be minimum *i.e.*, $S = mg$

Hence the range of S can be given by, $mg \le S \le mg\sqrt{\mu^2 + 1}$

5.9 Angle of Repose

Angle of repose is defined as the angle of the inclined plane with horizontal such that a body placed on it is just begins to slide.



By definition α is called the angle of repose.

In limiting condition $F = mg\sin\alpha$

and $R = mg\cos\alpha$

So $\frac{F}{R} = \tan\alpha$

$\therefore$ $\frac{F}{R} = \mu = \tan\theta = \tan\alpha$ [As we know $\frac{F}{R} = \mu = \tan\theta$]

Thus the coefficient of limiting friction is equal to the tangent of angle of repose.

As well as $\alpha = \theta$ *i.e.* angle of repose = angle of friction.

Sample problems based on angle of friction and angle of repose

***Problem* 7.** A body of 5 *kg* weight kept on a rough inclined plane of angle 30° starts sliding with a constant velocity. Then the coefficient of friction is (assume g = 10 m/s^2)

(a) $1/\sqrt{3}$ (b) $2/\sqrt{3}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

Solution : (a) Here the given angle is called the angle of repose

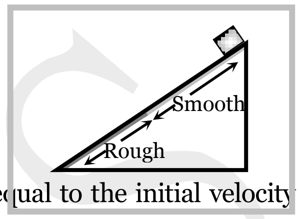
So, $\mu = \tan 30^o = \dfrac{1}{\sqrt{3}}$

***Problem* 8.** The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A body starting from the rest at top comes back to rest at the bottom if the coefficient of friction for the lower half is given **[Pb PMT 2000]**

(a) $\mu = \sin\theta$ (b) $\mu = \cot\theta$ (c) $\mu = 2\cos\theta$ (d) $\mu = 2\tan\theta$

Solution : (d) For upper half by the equation of motion $v^2 = u^2 + 2as$

$v^2 = 0^2 + 2(g\sin\theta)l/2 = gl\sin\theta$ [As $u = 0, s = l/2, a = g\sin\theta$]



For lower half

$0 = u^2 + 2g(\sin\theta - \mu\cos\theta)\ l/2$ [As $v = 0, s = l/2, a = g(\sin\theta - \mu\cos\theta)$]

$\Rightarrow\ 0 = gl\sin\theta + gl(\sin\theta - \mu\cos\theta)$ [As final velocity of upper half will be equal to the initial velocity of lower half]

$\Rightarrow 2\sin\theta = \mu\cos\theta \Rightarrow \mu = 2\tan\theta$

5.10 Calculation of Necessary Force in Different Conditions

If W = weight of the body, θ = angle of friction, $\mu = \tan\theta$ = coefficient of friction

then we can calculate necessary force for different condition in the following manner :

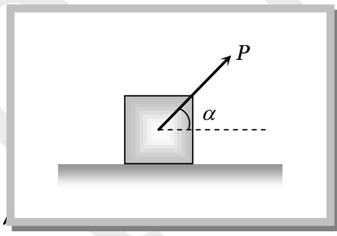
(1) **Minimum pulling force *P* at an angle α from the horizontal**

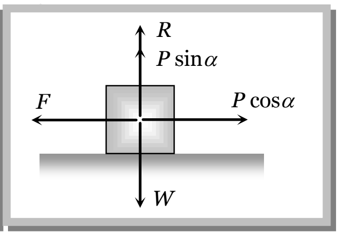
By resolving P in horizontal and vertical direction (as shown in figure)

For the condition of equilibrium

$F = P\cos\alpha$ and $R = W - P\sin\alpha$

By substituting these value in $F = \mu R$





$P\cos\alpha = \mu(W - P\sin\alpha)$

$\Rightarrow\quad P\cos\alpha = \dfrac{\sin\theta}{\cos\theta}(W - P\sin\alpha)$ [As

$\Rightarrow\quad P = \dfrac{W\sin\theta}{\cos(\alpha - \theta)}$

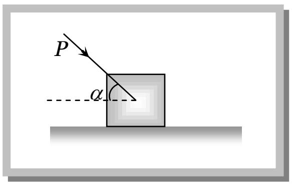
(2) **Minimum pushing force *P* at an angle α from the horizontal**

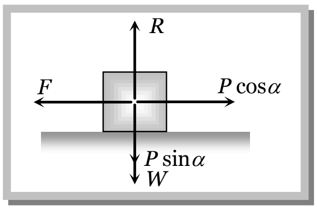
By Resolving P in horizontal and vertical direction (as shown in the figure)

For the condition of equilibrium

$F = P\cos\alpha$ and $R = W + P\sin\alpha$

By substituting these value in $F = \mu R$





$\Rightarrow\quad P\cos\alpha = \mu(W + P\sin\alpha)$

$\Rightarrow\quad P\cos\alpha = \dfrac{\sin\theta}{\cos\theta}(W + P\sin\alpha)$ [As $\mu = \tan\theta$]

$\Rightarrow\quad P = \dfrac{W\sin\theta}{\cos(\alpha + \theta)}$

(3) **Minimum pulling force *P* to move the body up an inclined plane**

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

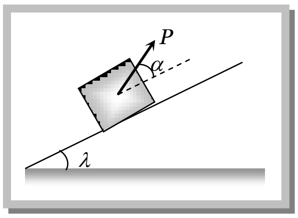
For the condition of equilibrium

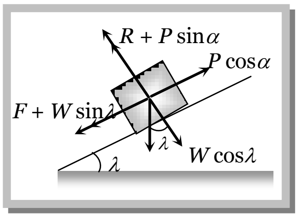
$$R + P\sin\alpha = W\cos\lambda$$

$\therefore$ $$R = W\cos\lambda - P\sin\alpha$$

and $$F + W\sin\lambda = P\cos\alpha$$

$\therefore$ $$F = P\cos\alpha - W\sin\lambda$$





By substituting these values in $F = \mu R$ and solving we get

$$P = \frac{W\sin(\theta+\lambda)}{\cos(\alpha-\theta)}$$

(4) **Minimum force on body in downward direction along the surface of inclined plane to start its motion**

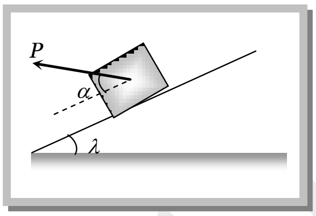
By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

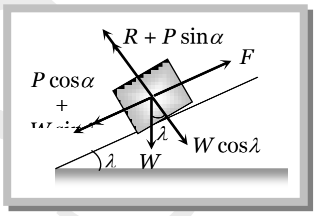
For the condition of equilibrium

$$R + P\sin\alpha = W\cos\lambda$$

$\therefore$ $$R = W\cos\lambda - P\sin\alpha$$

and $$F = P\cos\alpha + W\sin\lambda$$





By substituting these values in $F = \mu R$ and solving we get

$$P = \frac{W\sin(\theta-\lambda)}{\cos(\alpha-\theta)}$$

(5) **Minimum force to avoid sliding a body down an inclined plane**

By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure)

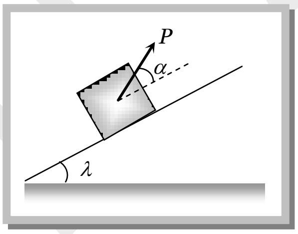
For the condition of equilibrium

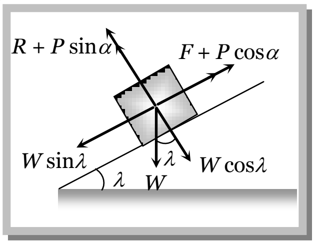
$$R + P\sin\alpha = W\cos\lambda$$

$\therefore$ $$R = W\cos\lambda - P\sin\alpha$$

and $$P\cos\alpha + F = W\sin\lambda$$

$\therefore$ $$F = W\sin\lambda - P\cos\alpha$$





By substituting these values in $F = \mu R$ and solving we get

$$P = W\left[\frac{\sin(\lambda-\theta)}{\cos(\theta+\alpha)}\right]$$

(6) **Minimum force for motion and its direction**

Let the force P be applied at an angle α with the horizontal.

By resolving P in horizontal and vertical direction (as shown in figure)

For vertical equilibrium

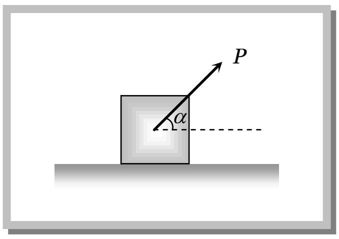
$$R + P\sin\alpha = mg$$

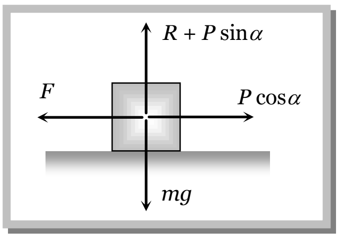
$\therefore$ $$R = mg - P\sin\alpha \quad ....(i)$$

and for horizontal motion

$$P\cos\alpha \geq F$$

i.e. $$P\cos\alpha \geq \mu R \quad ....(ii)$$





Substituting value of R from (i) in (ii)

$$P\cos\alpha \geq \mu(mg - P\sin\alpha)$$

$$P \geq \frac{\mu mg}{\cos\alpha + \mu\sin\alpha} \qquad \text{....(iii)}$$

For the force P to be minimum $(\cos\alpha + \mu\sin\alpha)$ must be maximum *i.e.*

$$\frac{d}{d\alpha}[\cos\alpha + \mu\sin\alpha] = 0 \Rightarrow -\sin\alpha + \mu\cos\alpha = 0$$

$\therefore$ $\tan\alpha = \mu$

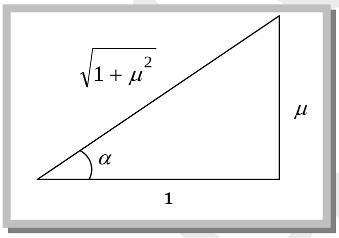
or $\alpha = \tan^{-1}(\mu) = \text{angle of friction}$

i.e. For minimum value of P its angle from the horizontal should be equal to angle of friction

As $\tan\alpha = \mu$ so from the figure $\sin\alpha = \frac{\mu}{\sqrt{1+\mu^2}}$ and $\cos\alpha = \frac{1}{\sqrt{1+\mu^2}}$

By substituting these value in equation (iii)

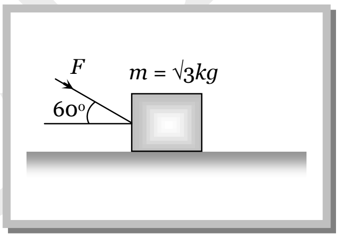
$$P \geq \frac{\mu mg}{\frac{1}{\sqrt{1+\mu^2}} + \frac{\mu^2}{\sqrt{1+\mu^2}}} \geq \frac{\mu mg}{\sqrt{1+\mu^2}}$$

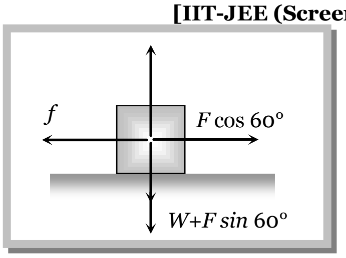


$\therefore$ $$P_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Sample problems based on force against friction

Problem 9. What is the maximum value of the force F such that the block shown in the arrangement, does not move $(\mu = 1/2\sqrt{3})$ **[IIT-JEE (Screening) 2003]**





(a) 20 N

(b) 10 N

(c) 12 N

(d) 15 N

Solution : (a) Frictional force $f = \mu R$

$$\Rightarrow F\cos 60 = \mu(W + F\sin 60)$$

$$\Rightarrow F\cos 60 = \frac{1}{2\sqrt{3}}\left(\sqrt{3}g + F\sin 60\right)$$

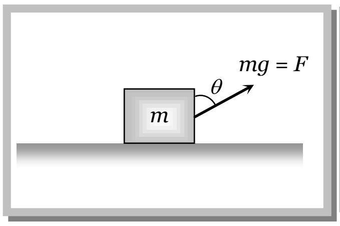
$$\Rightarrow F = 20N.$$

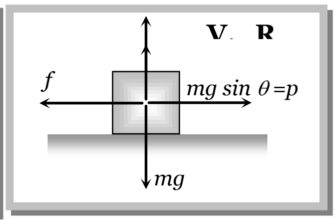
Problem 10. A block of mass m rests on a rough horizontal surface as shown in the figure. Coefficient of friction between the block and the surface is μ. A force F = mg acting at angle θ with the vertical side of the block pulls it. In which of the following cases the block can be pulled along the surface

(a) $\tan\theta \geq \mu$

(b) $\cot\theta \geq \mu$

(c) $\tan\theta/2 \geq \mu$





(d) $\cot\theta/2 \geq \mu$

Solution : (d) For pulling of block $P \geq f$

$\Rightarrow mg\sin\theta \geq \mu R \Rightarrow mg\sin\theta \geq \mu(mg - mg\cos\theta)$

$\Rightarrow \sin\theta \geq \mu(1-\cos\theta)$

$\Rightarrow 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \mu\left(2\sin^2\frac{\theta}{2}\right) \Rightarrow \cot\left(\frac{\theta}{2}\right) \geq \mu$

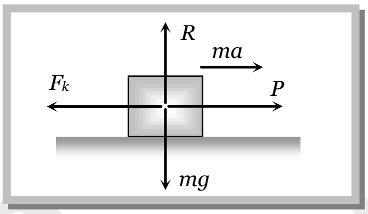
5.11 Acceleration of a Block Against Friction

(1) **Acceleration of a block on horizontal surface**

When body is moving under application of force P, then kinetic friction opposes its motion.

Let a is the net acceleration of the body

From the figure

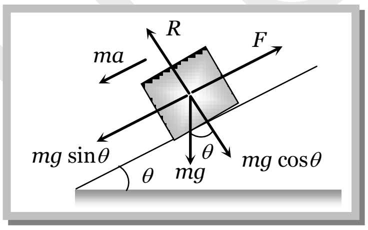


$$ma = P - F_k$$

$$\therefore \quad a = \frac{P - F_k}{m}$$

(2) **Acceleration of a block down a rough inclined plane**

When angle of inclined plane is more than angle of repose, the body placed on the inclined plane slides down with an acceleration a.



From the figure $ma = mg\sin\theta - F$

$\Rightarrow \quad ma = mg\sin\theta - \mu R$

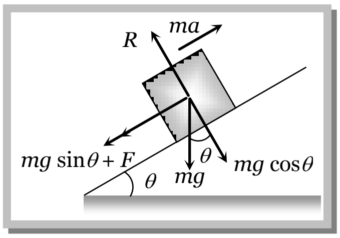
$\Rightarrow \quad ma = mg\sin\theta - \mu mg\cos\theta$

$\therefore$ Acceleration $a = g[\sin\theta - \mu\cos\theta]$

Note : ❑ For frictionless inclined plane $\mu = 0 \ \therefore a = g\sin\theta$.

(3) **Retardation of a block up a rough inclined plane**

When angle of inclined plane is less than angle of repose, then for the upward motion



$ma = mg\sin\theta + F$

$ma = mg\sin\theta + \mu mg\cos\theta$

Retardation $a = g[\sin\theta + \mu\cos\theta]$

Note : ❑ For frictionless inclined plane $\mu = 0 \ \therefore a = g\sin\theta$

Sample problems based on acceleration against friction

***Problem* 11.** A body of mass 10 *kg* is lying on a rough plane inclined at an angle of 30° to the horizontal and the coefficient of friction is 0.5. The minimum force required to pull the body up the plane is

(a) 914 *N* (b) 91.4 *N* (c) 9.14 *N* (d) 0.914 *N*

Solution : (b) $F = mg(\sin\theta + \mu\cos\theta) = 10 \times 9.8\,(\sin 30 + 0.5\cos 30) = 91.4\ N$

***Problem* 12.** A block of mass 10 *kg* is placed on a rough horizontal surface having coefficient of friction μ = 0.5. If a horizontal force of 100 *N* is acting on it, then acceleration of the block will be **[AIIMS 2002]**

(a) 0.5 m/s^2 (b) 5 m/s^2 (c) 10 m/s^2 (d) 15 m/s^2

Solution : (b) $a = \frac{\text{Applied force} - \text{kinetic friction}}{\text{mass}} = \frac{100 - 0.5 \times 10 \times 10}{10} = 5 m/s^2$.

***Problem* 13.** A body of weight 64 *N* is pushed with just enough force to start it moving across a horizontal floor and the same force continues to act afterwards. If the coefficients of static and dynamic friction are 0.6 and 0.4 respectively, the acceleration of the body will be (Acceleration due to gravity = g) **[EAMCET 2001]**

(a) $\frac{g}{6.4}$ (b) 0.64 g (c) $\frac{g}{32}$ (d) 0.2 g

Solution : (d) Limiting friction = $F_l = \mu_s R \Rightarrow 64 = 0.6\, m\, g \Rightarrow m = \frac{64}{0.6g}$.

Acceleration = $\frac{\text{Applied force} - \text{Kinetic friction}}{\text{Mass of the body}} = \frac{64 - \mu_K mg}{m} = \frac{64 - 0.4 \times \frac{64}{0.6}}{\frac{64}{0.6g}} = 0.2g$

***Problem* 14.** If a block moving up at $\theta = 30^o$ with a velocity 5 m/s, stops after 0.5 sec, then what is μ

(a) 0.5 (b) 1.25 (c) 0.6 (d) None of these

Solution : (c) From $v = u - at \Rightarrow 0 = u - at \;\; \therefore t = \frac{u}{a}$

for upward motion on an inclined plane $a = g(\sin\theta + \mu\cos\theta)$ $\therefore t = \frac{u}{g(\sin\theta + \mu\cos\theta)}$

Substituting the value of $\theta = 30^o, t = 0.5$ sec and $u = 5m/s$, we get $\mu = 0.6$

5.12 Work Done Against Friction

(1) **Work done over a rough inclined surface**

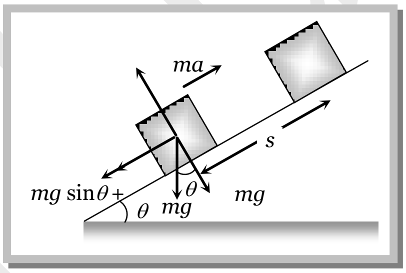
If a body of mass m is moved up on a rough inclined plane through distance s, then

Work done = force $\times$ distance

$= ma \times s$

$= mg\,[\sin\theta + \mu\cos\theta]s$

$= mg\, s[\sin\theta + \mu\cos\theta]$



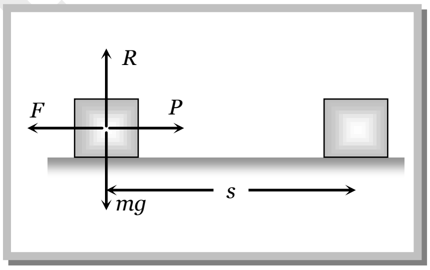
(2) **Work done over a horizontal surface**

In the above expression if we put $\theta = 0$ then

Work done = force $\times$ distance

$= F \times s$

$= \mu\, mg\, s$



It is clear that work done depends upon

(i) Weight of the body.

(ii) Material and nature of surface in contact.

(iii) Distance moved.

Sample problems based on work done against friction

***Problem* 15.** A body of mass 5kg rests on a rough horizontal surface of coefficient of friction 0.2. The body is pulled through a distance of 10m by a horizontal force of 25 N. The kinetic energy acquired by it is (g = 10 ms^2) **[EAMCET (Med.) 2000]**

(a) 330 J (b) 150 J (c) 100 J (d) 50 J

Solution : (b) Kinetic energy acquired by body = Total work done on the body – Work done against friction

$= F \times S - \mu mgS = 25 \times 10 - 0.2 \times 5 \times 10 \times 10 = 250 - 100 = 150\,J$.

***Problem* 16.** 300 *Joule* of work is done in sliding a 2 kg. block up an inclined plane to a height of 10 *meters*. Taking value of acceleration due to gravity 'g' to be 10 m/s^2, work done against friction is **[MP PMT 2002]**

(a) 100 J (b) 200 J (c) 300 J (d) Zero

Solution : (a) Work done against gravity $= mgh = 2 \times 10 \times 10 = 200J$

Work done against friction = Total work done – Work done against gravity = 300 – 200 = 100J.

Problem 17. A block of mass 1 *kg* slides down on a rough inclined plane of inclination 60° starting from its top. If the coefficient of kinetic friction is 0.5 and length of the plane is 1 *m*, then work done against friction is (Take $g = 9.8\ m/s^2$) **[AFMC 2000; KCET (Engg./Med.) 2001]**

(a) 9.82 *J* (b) 4.94 *J* (c) 2.45*J* (d) 1.96 *J*

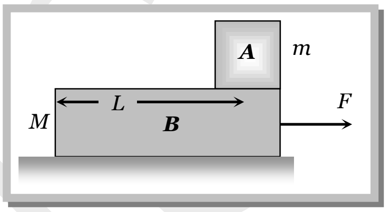
Solution : (c) $W = \mu mg \cos\theta . S = 0.5 \times 1 \times 9.8 \times \frac{1}{2} = 2.45\ J.$

Problem 18. A block of mass 50 *kg* slides over a horizontal distance of 1 *m*. If the coefficient of friction between their surfaces is 0.2, then work done against friction is **[CBSE PMT 1999, 2000; AIIMS 2000; BHU 2001]**

(a) 98 *J* (b) 72*J* (c) 56 *J* (d) 34 *J*

Solution : (a) $W = \mu mgS = 0.2 \times 50 \times 9.8 \times 1 = 98\,J$.

5.13 Motion of Two Bodies One Resting on the Other

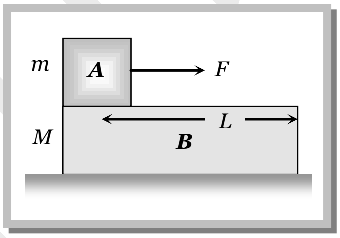


When a body *A* of mass *m* is resting on a body *B* of mass *M* then two conditions are possible

(1) A force *F* is applied to the upper body, (2) A force *F* is applied to the lower body

We will discuss above two cases one by one in the following manner :

(1) A force *F* is applied to the upper body, then following four situations are possible



(i) **When there is no friction**

(a) The body *A* will move on body *B* with acceleration (*F*/*m*).

$$a_A = F/m$$

(b) The body *B* will remain at rest

$$a_B = 0$$

(c) If *L* is the length of *B* as shown in figure *A* will fall from *B* after time *t*

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{F}} \qquad \left[\text{As } s = \frac{1}{2}a\,t^2 \text{ and } a = F/m\right]$$

(ii) **If friction is present between *A* and *B* only and applied force is less than limiting friction ($F < F_l$)**

(F = Applied force on the upper body, F_l = limiting friction between *A* and *B*, F_k = Kinetic friction between *A* and *B*)

(a) The body *A* will not slide on body *B* till $F < F_l$ *i.e.* $F < \mu_s mg$

(b) Combined system (*m* + *M*) will move together with common acceleration $a_A = a_B = \dfrac{F}{M+m}$

(iii) **If friction is present between *A* and *B* only and applied force is greater than limiting friction ($F > F_l$)**

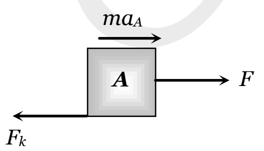
In this condition the two bodies will move in the same direction (*i.e.* of applied force) but with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of *A* while will cause the motion of *B*.

$F - F_k = m\,a_A$

i.e. $a_A = \dfrac{F - F_k}{m}$

$a_A = \dfrac{(F - \mu_k mg)}{m}$

Free body diagram of *A*

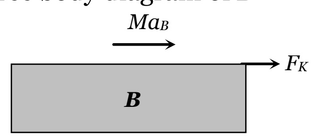


$F_k = M\,a_B$

i.e. $a_B = \dfrac{F_k}{M}$

$\therefore$ $a_B = \dfrac{\mu_k mg}{M}$

Free body diagram of *B*



Note : ❑ As both the bodies are moving in the same direction.

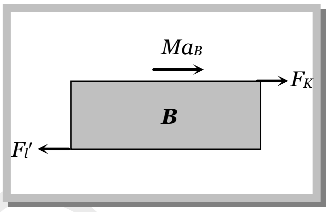
Acceleration of body A relative to B will be $a = a_A - a_B = \frac{MF - \mu_k mg\,(m+M)}{mM}$

So, A will fall from B after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2\,m\,ML}{MF - \mu_k mg\,(m+M)}}$

(iv) **If there is friction between B and floor**

(where $F_l' = \mu'(M+m)g$ = limiting friction between B and floor, F_k = kinetic friction between A and B)

B will move only if $F_k > F_l'$ and then $F_k - F_l' = M\,a_B$



However if B does not move then static friction will work (not limiting friction) between body B and the floor *i.e.* friction force = applied force (= F_k) not F_l'.

(2) **A force F is applied to the lower body, then following four situations are possible**

(i) **When there is no friction**

(a) B will move with acceleration (F/M) while A will remain at rest (relative to ground) as there is no pulling force on A.

$$a_B = \left(\frac{F}{M}\right) \text{ and } a_A = 0$$

(b) As relative to B, A will move backwards with acceleration (F/M) and so will fall from it in time t.

$$\therefore\ t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F}}$$

(ii) **If friction is present between A and B only and $F' < F_l$**

(where F' = Pseudo force on body A and F_l = limiting friction between body A and B)

(a) Both the body will move together with common acceleration $a = \frac{F}{M+m}$

(b) Pseudo force on the body A, $F' = ma = \frac{mF}{m+M}$ and $F_l = \mu_s mg$

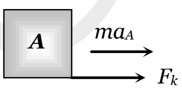
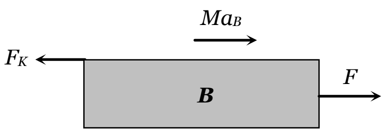
(c) $F' < F_l \Rightarrow \frac{mF}{m+M} < \mu_s mg \Rightarrow F < \mu_s (m+M)g$

So both bodies will move together with acceleration $a_A = a_B = \frac{F}{m+M}$ if $F < \mu_s [m+M]g$

(iii) **If friction is present between A and B only and $F > F_l'$**

(where $F_l' = \mu_s\,(m + M)g$ = limiting friction between body B and surface)

Both the body will move with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of B while will cause the motion of A.

$ma_A = \mu_k mg$ *i.e.* $a_A = \mu_k g$ Free body diagram of A 	$F - F_k = Ma_B$ *i.e.* $a_B = \frac{[F - \mu_k mg]}{M}$ Free body diagram of B 

Note : ❑ As both the bodies are moving in the same direction

Acceleration of body A relative to B will be

$$a = a_A - a_B = -\left[\frac{F - \mu_k g(m+M)}{M}\right]$$

Negative sign implies that relative to B, A will move backwards and will fall it after time

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F - \mu_k g(m+M)}}$$

(iv) **If there is friction between *B* and floor :** The system will move only if $F > F_l'$ then replacing F by $F - F_l'$. The entire case (iii) will be valid.

However if $F < F_l'$ the system will not move and friction between B and floor will be F while between A and B is zero.

Sample problems based on body resting on another

***Problem* 19.** A 4 *kg* block *A* is placed on the top of a 8 *kg* block *B* which rests on a smooth table. *A* just slips on *B* when a force of 12 *N* is applied on *A*. Then the maximum horizontal force on B to make both *A* and *B* move together, is

(a) 12 *N* (b) 24 *N* (c) 36 *N* (d) 48 *N*

Solution : (c) Maximum friction *i.e.* limiting friction between *A* and *B*, F_l = 12 *N*.

If *F* is the maximum value of force applied on lower body such that both body move together

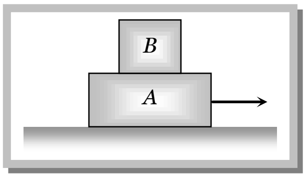
It means Pseudo force on upper body is just equal to limiting friction

$$F' = F_l \Rightarrow m\left(\frac{F}{m+M}\right) = \left(\frac{4}{4+8}\right)F = 12 \quad \therefore F = 36N.$$

***Problem* 20.** A body *A* of mass 1 *kg* rests on a smooth surface. Another body *B* of mass 0.2 *kg* is placed over *A* as shown. The coefficient of static friction between *A* and *B* is 0.15. *B* will begin to slide on *A* if *A* is pulled with a force greater than

(a) 1.764 *N* (b) 0.1764 *N*

(c) 0.3 *N* (d) It will not slide for any *F*



Solution : (a) *B* will begin to slide on *A* if Pseudo force is more than limiting friction

$$F' > F_l \Rightarrow m\left(\frac{F}{m+M}\right) > \mu_s R \Rightarrow m\left(\frac{F}{m+M}\right) > 0.15mg \quad \therefore F > 1.764N$$

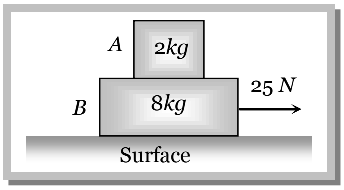
***Problem* 21.** A block *A* of mass 2 *kg* rests on another block *B* of mass 8 *kg* which rests on a horizontal floor. The coefficient of friction between *A* and *B* is 0.2, while that between *B* and floor is 0.5. When a horizontal force of 25 *N* is applied on the block *B*, the force of friction between *A* and *B* is **[IIT-JEE 1993]**

(a) Zero (b) 3.9 *N* (c) 5.0 *N* (d) 49 *N*

Solution : (a) Limiting friction between the block *B* and the surface

$$F_{BS} = \mu_{BS}.R = 0.5(m+M)g = 0.5(2+8)10 = 50N$$

but the applied force is 25 *N* so the lower block will not move *i.e.* there is no pseudo force on upper block *A*. Hence there will be no force of friction between *A* and *B*.

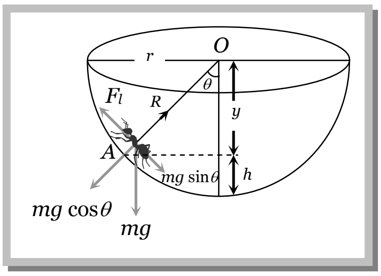


5.14 Motion of an Insect in the Rough Bowl

The insect crawl up the bowl up to a certain height h only till the component of its weight along the bowl is balanced by limiting frictional force.

Let m = mass of the insect, r = radius of the bowl, μ = coefficient of friction

for limiting condition at point A



$$R = mg\cos\theta \quad ......(i) \quad \text{and} \quad F_l = mg\sin\theta \quad ......(ii)$$

Dividing (ii) by (i)

$$\tan\theta = \frac{F_l}{R} = \mu \qquad [\text{As } F_l = \mu R]$$

$$\therefore \qquad \frac{\sqrt{r^2 - y^2}}{y} = \mu \quad \text{or} \quad y = \frac{r}{\sqrt{1+\mu^2}}$$

So $\qquad h = r - y = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$, $\therefore\ h = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$

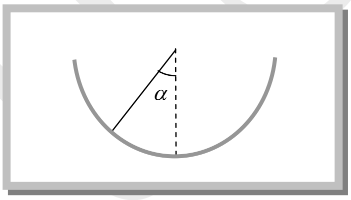
***Problem* 22.** An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by **[IIT-JEE (Screening) 2001]**

(a) $\cot\alpha = 3$

(b) $\tan\alpha = 3$

(c) $\sec\alpha = 3$

(d) $\text{cosec}\ \alpha = 3$



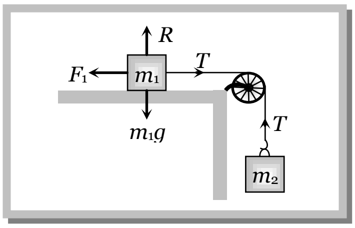
Solution : (a) From the above expression, for the equilibrium $R = mg\cos\alpha$ and $F = mg\sin\alpha$.

Substituting these value in $F = \mu R$ we get $\tan\alpha = \mu$ or $\cot\alpha = \frac{1}{\mu} = 3$.

5.15 Minimum Mass Hung From the String to Just Start the Motion

(1) **When a mass m_1 placed on a rough horizontal plane :** Another mass m_2 hung from the string connected by pulley, the tension (T) produced in string will try to start the motion of mass m_1.

At limiting condition



$$T = F_l$$

$$\Rightarrow \qquad m_2 g = \mu R$$

$$\Rightarrow \qquad m_2 g = \mu m_1 g$$

$\therefore\ m_2 = \mu m_1$ this is the minimum value of m_2 to start the motion.

Note : ❑ In the above condition Coefficient of friction $\mu = \frac{m_2}{m_1}$

(2) **When a mass m_1 placed on a rough inclined plane :** Another mass m_2 hung from the string connected by pulley, the tension (T) produced in string will try to start the motion of mass m_1.

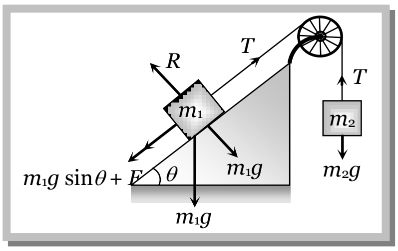
At limiting condition

For $m_2 \quad T = m_2 g \qquad ......$ (i)

For $m_1 \quad T = m_1 g\sin\theta + F \Rightarrow T = m_1 g\sin\theta + \mu R$

$\Rightarrow \quad T = m_1 g\sin\theta + \mu m_1 g\cos\theta \qquad ......$(ii)

From equation (i) and (ii) $\quad m_2 = m_1[\sin\theta + \mu\cos\theta]$



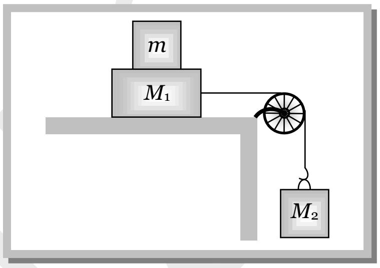
this is the minimum value of m_2 to start the motion

Note : ❑ In the above condition Coefficient of friction

$$\mu = \left[\frac{m_2}{m_1 \cos\theta} - \tan\theta\right]$$

Sample problems based on hung mass

Problem 23. Two blocks of mass M_1 and M_2 are connected with a string passing over a pulley as shown in the figure. The block M_1 lies on a horizontal surface. The coefficient of friction between the block M_1 and horizontal surface is μ. The system accelerates. What additional mass m should be placed on the block M_1 so that the system does not accelerate



(a) $\frac{M_2 - M_1}{\mu}$

(b) $\frac{M_2}{\mu} - M_1$

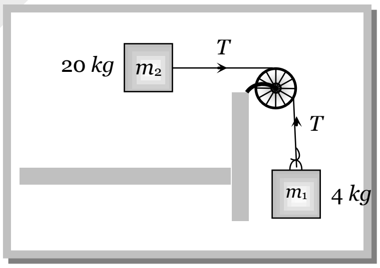
(c) $M_2 - \frac{M_1}{\mu}$

(d) $(M_2 - M_1)\mu$

Solution : (b) By comparing the given condition with general expression

$$\mu = \frac{M_2}{m + M_1} \Rightarrow m + M_1 = \frac{M_2}{\mu} \Rightarrow m = \frac{M_2}{\mu} - M_1$$

Problem 24. The coefficient of kinetic friction is 0.03 in the diagram where mass $m_2 = 20\,kg$ and $m_1 = 4\,kg$. The acceleration of the block shall be $(g = 10ms^{-2})$

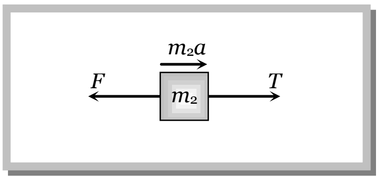


(a) $1.8\,ms^{-2}$

(b) $0.8\,ms^{-2}$

(c) $1.4\,ms^{-2}$

(d) $0.4\,ms^{-2}$

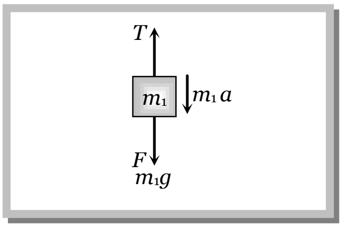


Solution : (c) Let the acceleration of the system is a

From the F.B.D. of m_2

$T - F = m_2 a \Rightarrow T - \mu m_2 g = m_2 a$

$\Rightarrow T - 0.03 \times 20 \times 10 = 20a \Rightarrow T - 6 = 20a$ .....(i)



From the FBD of m_1

$m_1 g - T = m_1 a$

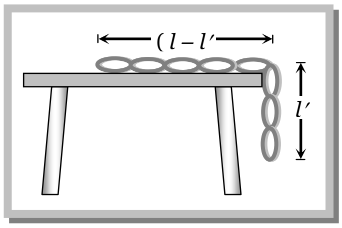
$\Rightarrow 4 \times 10 - T = 4a \Rightarrow 40 - T = 4a$ ....(ii)

Solving (i) and (ii) $a = 1.4 m/s^2$.

5.16 Maximum Length of Hung Chain

A uniform chain of length l is placed on the table in such a manner that its l' part is hanging over the edge of table with out sliding. Since the chain have uniform linear density therefore the ratio of mass or ratio of length for any part of the chain will be equal.

We know $\mu = \dfrac{m_2}{m_1} = \dfrac{\text{mass hanging from the table}}{\text{mass lying on the table}}$ [From article 5.15]



$\therefore$ For this expression we can rewrite above expression in the following manner

$\mu = \dfrac{\text{length hanging from the table}}{\text{length lying on the table}}$ [As chain have uniform linear density]

$\therefore \ \mu = \dfrac{l'}{l - l'}$

by solving $l' = \dfrac{\mu l}{(\mu + 1)}$

***Problem* 25.** A heavy uniform chain lies on a horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is **[CBSE PMT 1990]**

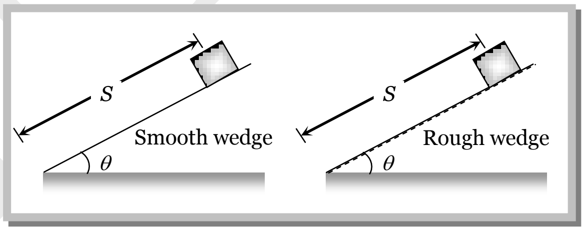
(a) 20% (b) 25% (c) 35% (d) 15%

Solution : (a) From the expression $l' = \left(\dfrac{\mu}{\mu + 1}\right) l = \left(\dfrac{0.25}{0.25 + 1}\right) l$ [As μ = 0.25]

$\Rightarrow \ l' = \dfrac{0.25}{1.25} l = \dfrac{l}{5}$ = 20% of the length of the chain.

5.17 Coefficient of Friction Between Body and Wedge

A body slides on a smooth wedge of angle θ and its time of descent is t.



If the same wedge made rough then time taken by it to come down becomes n times more (*i.e.* nt) The length of path in both the cases are same.

For smooth wedge

$$S = ut + \frac{1}{2}at^2$$

$$S = \frac{1}{2}(g\sin\theta)t^2 \quad .....(i)$$

[As $u = 0$ and $a = g\sin\theta$]

For rough wedge

$$S = ut + \frac{1}{2}at^2$$

$$S = \frac{1}{2}g(\sin\theta - \mu\cos\theta)(nt)^2 \quad .....(ii)$$

[As $u = 0$ and $a = g(\sin\theta - \mu\cos\theta)$]

From equation (i) and (ii) $\dfrac{1}{2}(g\sin\theta)t^2 = \dfrac{1}{2}g(\sin\theta - \mu\cos\theta)(nt)^2$

$$\Rightarrow \quad \sin\theta = (\sin\theta - \mu\cos\theta)n^2$$

$$\Rightarrow \quad \mu = \tan\theta\left[1 - \frac{1}{n^2}\right]$$

Problem 26. A body takes just twice the time as long to slide down a plane inclined at 30° to the horizontal as if the plane were frictionless. The coefficient of friction between the body and the plane is **[JIPMER 1999]**

(a) $\frac{\sqrt{3}}{4}$ (b) $\sqrt{3}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

Solution : (a) $\mu = \tan\theta\left(1 - \frac{1}{n^2}\right) = \tan 30\left(1 - \frac{1}{2^2}\right) = \frac{\sqrt{3}}{4}$.

5.18 Stopping of Block Due to Friction

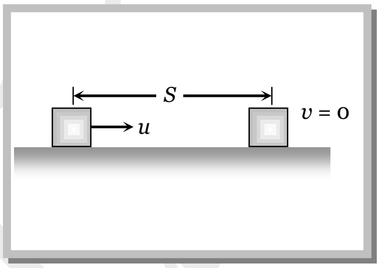
(1) **On horizontal road**

(i) **Distance travelled before coming to rest :** A block of mass m is moving initially with velocity u on a rough surface and due to friction it comes to rest after covering a distance S.

Retarding force $F = ma = \mu R$

$\Rightarrow \quad ma = \mu mg$

$\therefore \quad a = \mu g$.



From $v^2 = u^2 - 2aS \Rightarrow 0 = u^2 - 2\mu g S$ [As $v = 0$, $a = \mu g$]

$\therefore \quad S = \frac{u^2}{2\mu g}$

or $S = \frac{P^2}{2\mu m^2 g}$ [As momentum $P = mu$]

(ii) **Time taken to come to rest**

From equation $v = u - at \Rightarrow 0 = u - \mu g t$ [As $v = 0, a = \mu g$]

$\therefore t = \frac{u}{\mu g}$

(iii) **Force of friction acting on the body**

We know, $F = ma$

So, $F = m\frac{(v-u)}{t}$

$F = \frac{mu}{t}$ [As v = 0]

$F = \mu mg$ $\left[\text{As } t = \frac{u}{\mu g}\right]$

(2) **On inclined road :** When block starts with velocity u its kinetic energy will be converted into potential energy and some part of it goes against friction and after travelling distance S it comes to rest *i.e.* v = 0.

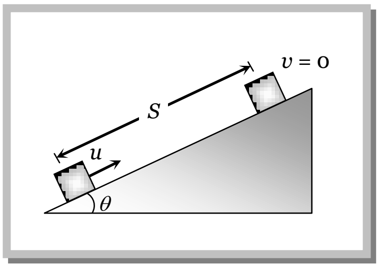
And we know that retardation $a = g[\sin\theta + \mu\cos\theta]$

By substituting the value of v and a in the following equation

$v^2 = u^2 - 2aS$

$\Rightarrow \quad 0 = u^2 - 2g[\sin\theta + \mu\cos\theta]S$

$\therefore S = \frac{u^2}{2g(\sin\theta + \mu\cos\theta)}$



Sample problems based on motion of body on rough surface

Problem 27. A marble block of mass 2 *kg* lying on ice when given a velocity of 6 *m/s* is stopped by friction in 10*s*. Then the coefficient of friction is

(a) 0.01 (b) 0.02 (c) 0.03 (d) 0.06

Solution : (d) $v = u - at = u - \mu g t = 0$ $\therefore \mu = \frac{u}{gt} = \frac{6}{10 \times 10} = 0.06$.

Problem 28. A 2 *kg* mass starts from rest on an inclined smooth surface with inclination 30° and length 2 *m*. How much will it travel before coming to rest on a surface with coefficient of friction 0.25

(a) 4 *m* (b) 6 *m* (c) 8 *m* (d) 2 *m*

Solution : (a) $v^2 = u^2 + 2aS = 0 + 2 \times g \sin 30 \times 2$ $v = \sqrt{20}$

Let it travel distance '*S*' before coming to rest

$$S = \frac{v^2}{2\mu g} = \frac{20}{2 \times 0.25 \times 10} = 4m.$$

5.19 Stopping of Two Blocks Due to Friction

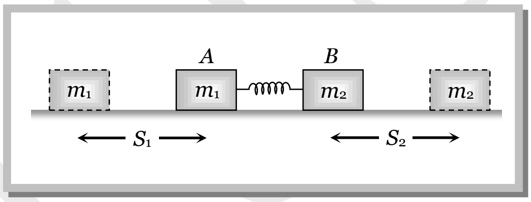
When two masses compressed towards each other and suddenly released then energy acquired by each block will be dissipated against friction and finally block comes to rest

i.e., $F \times S = E$ [Where F = Friction, S = Distance covered by block, E = Initial kinetic energy of the block]

$\Rightarrow F \times S = \frac{P^2}{2m}$ [Where P = momentum of block]

$\Rightarrow \mu mg \times S = \frac{P^2}{2m}$ [As $F = \mu mg$]

$\Rightarrow S = \frac{P^2}{2\mu m^2 g}$



In a given condition P and μ are same for both the blocks.

So $S \propto \frac{1}{m^2}$ $\therefore \frac{S_1}{S_2} = \left[\frac{m_2}{m_1}\right]^2$

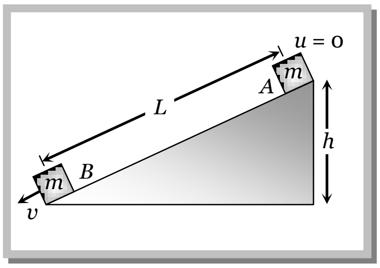
5.20 Velocity at the Bottom of Rough Wedge

A body of mass m which is placed at the top of the wedge (of height h) starts moving downward on a rough inclined plane.

Loss of energy due to friction = FL (Work against friction)

PE at point $A = mgh$

KE at point $B = \frac{1}{2}mv^2$



By the law of conservation of energy

i.e. $\frac{1}{2}mv^2 = mgh - FL$

$$v = \sqrt{\frac{2}{m}(mgh - FL)}$$

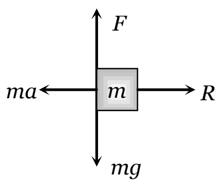
5.21 Sticking of a Block With Accelerated Cart

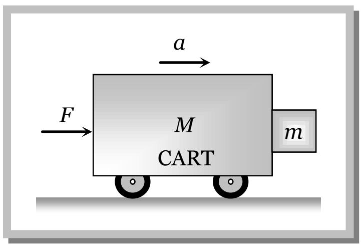
When a cart moves with some acceleration toward right then a pseudo force (ma) acts on block toward left.

This force (ma) is action force by a block on cart.

Now block will remain static *w.r.t.* block. If friction force $\mu R \geq mg$

$\Rightarrow$ $\mu ma \geq mg$ [As $R = ma$]





$\Rightarrow \quad a \geq \frac{g}{\mu}$

$\therefore \quad a_{\min} = \frac{g}{\mu}$

This is the minimum acceleration of the cart so that block does not fall.
and the minimum force to hold the block together

$F_{\min} = (M+m)a_{\min}$ OR $F_{\min} = (M+m)\frac{g}{\mu}$

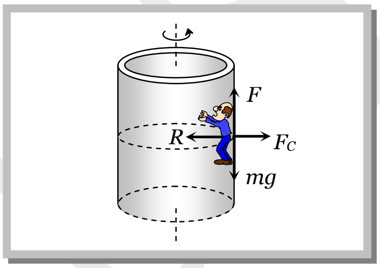
5.22 Sticking of a Person With the Wall of Rotor

A person with a mass m stands in contact against the wall of a cylindrical drum (rotor). The coefficient of friction between the wall and the clothing is μ.

If Rotor starts rotating about its axis, then person thrown away from the centre due to centrifugal force at a particular speed w, the person stuck to the wall even the floor is removed, because friction force balances its weight in this condition.

From the figure.

Friction force (F) = weight of person (mg)



$\Rightarrow \quad \mu R = mg$

$\Rightarrow \quad \mu F_c = mg$ [Here, F_c= centrifugal force]

$\Rightarrow \quad \mu m \omega_{\min}^2 r = mg$

$\therefore \quad \omega_{\min} = \sqrt{\frac{g}{\mu r}}$

Sample problems (Miscellaneous)

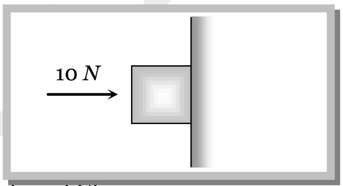
Problem 29. A motorcycle is travelling on a curved track of radius 500m if the coefficient of friction between road and tyres is 0.5. The speed avoiding skidding will be **[MH CET (Med.) 2001]**

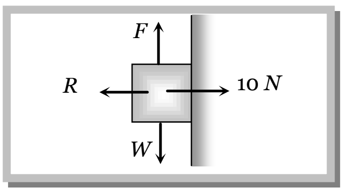
(a) 50 m/s (b) 75 m/s (c) 25 m/s (d) 35 m/s

Solution : (a) $v = \sqrt{\mu r g} = \sqrt{0.5 \times 500 \times 10} = 50\, m/s$.

Problem 30. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is **[AIEEE 2003]**

(a) 2 N
(b) 20 N
(c) 50 N
(d) 100 N





Solution : (a) For equilibrium

Weight (W) = Force of friction (F)

$W = \mu R = 0.2 \times 10 = 2N$

Problem 31. A body of mass 2 kg is kept by pressing to a vertical wall by a force of 100 N. The friction between wall and body is 0.3. Then the frictional force is equal to **[Orissa JEE 2003]**

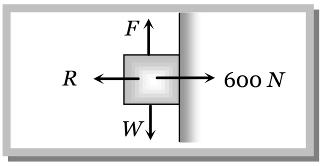
(a) 6 N (b) 20 N (c) 600 N (d) 700 N

Solution : (b) For the given condition Static friction = Applied force = Weight of body = 2 × 10 = 20 N.

Problem 32. A fireman of mass 60kg slides down a pole. He is pressing the pole with a force of 600 N. The coefficient of friction between the hands and the pole is 0.5, with what acceleration will the fireman slide down (g = 10 m/s^2) **[Pb. PMT 2002]**

(a) 1 m/s^2 (b) 2.5 m/s^2 (c) 10 m/s^2 (d) 5 m/s^2

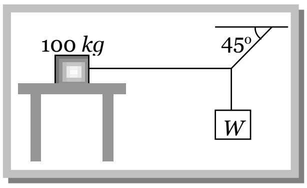
Solution : (d) Friction = $\mu R = 0.5 \times 600 = 300\ N$, Weight = $600\ N$



$ma = W - F \Rightarrow a = \frac{W - F}{m} = \frac{600 - 300}{60}$

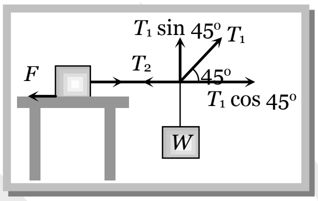
$\therefore\ a = 5\ m/s^2$

***Problem* 33.** The system shown in the figure is in equilibrium. The maximum value of W, so that the maximum value of static frictional force on 100 *kg*. body is 450 *N*, will be



(a) 100 *N*
(b) 250 *N*
(c) 450 *N*
(d) 1000 *N*

Solution : (c) For vertical equilibrium $T_1 \sin 45^o = W$ $\therefore T_1 = \frac{W}{\sin 45^o}$



For horizontal equilibrium $T_2 = T_1 \cos 45^o = \frac{W}{\sin 45^o} \cos 45^o = W$

and for the critical condition $T_2 = F$ $\therefore W = T_2 = F = 450\ N$

Practice Problems

▶ Basic level

1. When a body is moving on a surface, the force of friction is called **[MP PET 2002]**

(a) Static friction (b) Dynamic friction (c) Limiting friction (d) Rolling friction

2. Which one of the following is not used to reduce friction **[Kerala (Engg.) 2001]**

(a) Oil (b) Ball bearings (c) Sand (d) Graphite

3. A block of mass 10 *kg* is placed on an inclined plane. When the angle of inclination is 30°, the block just begins to slide down the plane. The force of static friction is

(a) 10 *kg wt* (b) 89 *kg wt* (c) 49 *kg wt* (d) 5 *kg wt*

4. A vehicle of mass m is moving on a rough horizontal road with momentum P. If the coefficient of friction between the tyres and the road be μ, then the stopping distance is **[CBSE PMT 2001]**

(a) $\frac{P}{2\mu m g}$ (b) $\frac{P^2}{2\mu m g}$ (c) $\frac{P}{2\mu m^2 g}$ (d) $\frac{P^2}{2\mu m^2 g}$

5. A box is lying on an inclined plane what is the coefficient of static friction if the box starts sliding when an angle of inclination is 60°

[KCET (Engg./Med.) 2000]

(a) 1.173 (b) 1.732 (c) 2.732 (d) 1.677

6. A brick of mass 2 *kg* begins to slide down on a plane inclined at an angle of 45° with the horizontal. The force of friction will be

[CPMT 2000]

(a) $19.6 \sin 45^{o}$ (b) $19.6 \cos 45^{o}$ (c) $9.8 \sin 45^{o}$ (d) $9.8 \cos 45^{o}$

7. To avoid slipping while walking on ice, one should take smaller steps because of the **[BHU 1999]**

(a) Friction of ice is large (b) Larger normal reaction

(c) Friction of ice is small (d) Smaller normal reaction

8. Two bodies having the same mass, 2 *kg* each have different surface areas $50\,m^2$ and $100\,m^2$ in contact with a horizontal plane. If the coefficient of friction is 0.2, the forces of friction that come into play when they are in motion will be in the ratio **[EAMCET (Med.) 1999]**

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

9. Starting from rest, a body slides down a 45^{o} inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is **[CBSE PMT 1990]**

(a) 0.33 (b) 0.25 (c) 0.75 (d) 0.80

10. Brakes of very small contact area are not used although friction is independent of area, because friction

(a) Resists motion (b) Causes wear and tear

(c) Depends upon the nature of materials (d) Operating in this case is sliding friction

11. The angle between frictional force and the instantaneous velocity of the body moving over a rough surface is

(a) Zero (b) $\pi/2$

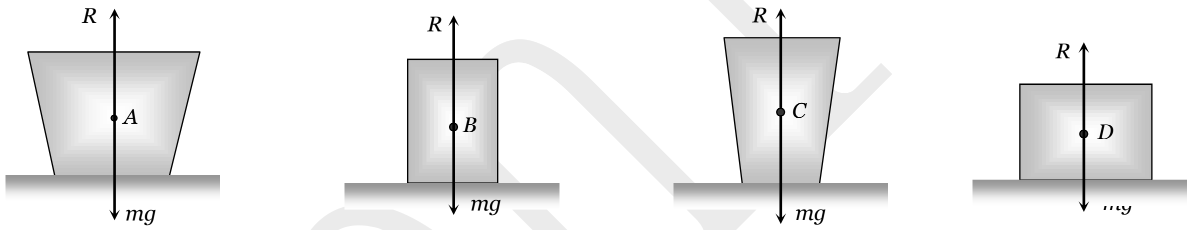
(c) π (d) Equal to the angle of friction

12. What happens to the coefficient of friction, when the normal reaction is halved

(a) Halved (b) Doubled

(c) No change (d) Depends on the nature of the surface

13. What can be inferred regarding the limiting frictional force in the following four figures



(a) $F_A = F_B = F_C = F_D$ (b) $F_A > F_B > F_C > F_D$ (c) $F_A < F_B < F_C < F_D$ (d) $F_A = F_B < F_C < F_D$

14. A force of 98 Newton is required to drag a body of mass 100 *kg* on ice. The coefficient of friction will be

(a) 0.98 (b) 0.89 (c) 0.49 (d) 0.1

15. A 60 *kg* body is pushed with just enough force to start it moving across a floor and the same force continues to act afterwards. The coefficients of static and sliding friction are 0.5 and 0.4 respectively. The acceleration of the body is

(a) $6\,m/\sec^2$ (b) $4.9\,m/\sec^2$ (c) $3.92\,m/\sec^2$ (d) $1\,m/\sec^2$

16. A particle is projected along a line of greatest slope up a rough plane inclined at an angle of 45^{o} with the horizontal. If the coefficient of friction is $\frac{1}{2}$, then the retardation is

(a) $\frac{g}{\sqrt{2}}$ (b) $\frac{g}{2\sqrt{2}}$ (c) $\frac{g}{\sqrt{2}}\left[1+\frac{1}{2}\right]$ (d) $\frac{g}{\sqrt{2}}\left[1-\frac{1}{2}\right]$

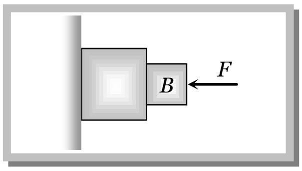
17. A block moves down a smooth inclined plane of inclination θ. Its velocity on reaching the bottom is v. If it slides down a rough inclined plane of same inclination its velocity on reaching the bottom is v/n, where n is a number greater than 0. The coefficient of friction μ is given by

18. (a) $\mu = \tan\theta\left[1-\frac{1}{n^2}\right]$ (b) $\mu = \cot\theta\left[1-\frac{1}{n^2}\right]$ (c) $\mu = \tan\theta\left[1-\frac{1}{n^2}\right]^{\frac{1}{2}}$ (d) $\mu = \cot\theta\left[1-\frac{1}{n^2}\right]^{\frac{1}{2}}$

19. Consider a car moving along a straight horizontal road with a speed of 72 *km/hr*. If the coefficient of static friction between the tyres and the road is 0.5, the shortest distance in which the car can be stopped is $(g = 10 m/s^2)$

(a) 30 *m* (b) 40 *m* (c) 72 *m* (d) 20 *m*

20. All the surfaces shown in the figure are rough. The direction of friction on *B* due to *A* is



(a) Zero

(b) To the left

(c) Upwards

(d) Downwards

21. A body of mass *M* just starts sliding down an inclined plane (rough) with inclination θ, such that $\tan\theta = 1/3$. The force acting on the body down the plane in this position is

(a) Mg (b) $\frac{Mg}{3}$ (c) $\frac{2}{3}Mg$ (d) $\frac{Mg}{\sqrt{10}}$

▶▶ Advance level

22. Consider the following statements

Assertion (*A*) : It is difficult to move a cycle along the road with its brakes on.

Reason (*R*) : Sliding friction is greater than rolling friction.

Of these statements **[AIIMS 2002]**

(a) Both *A* and *R* are true and the *R* is a correct explanation of the *A*

(b) Both *A* and *R* are true but the *R* is not a correct explanation of the *A*

(c) *A* is true but the *R* is false

(d) Both *A* and *R* are false

(e) *A* is false but the *R* is true

23. A body is sliding down an inclined plane having coefficient of friction 0.5. If the normal reaction is twice that of the resultant downward force along the incline, the angle between the inclined plane and the horizontal is **[EAMCET (Engg.) 2000]**

(a) 15° (b) 30° (c) 45° (d) 60°

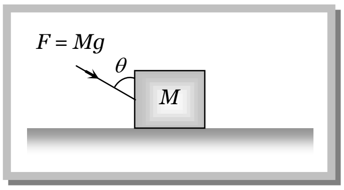
24. A block of mass 2 *kg* rests on a rough inclined plane making an angle of 30^o with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is **[IIT-JEE 1980]**

(a) 9.8 *N* (b) $0.7 \times 9.8 \times \sqrt{3}\ N$ (c) $9.8 \times \sqrt{3}\ N$ (d) $0.7 \times 9.8\ N$

25. A body of weight *W* is lying at rest on a rough horizontal surface. If the angle of friction is θ, then the minimum force required to move the body along the surface will be

(a) $W\tan\theta$ (b) $W\cos\theta$ (c) $W\sin\theta$ (d) $W\cos\theta$

26. A block of mass *M* is placed on a rough horizontal surface as shown in the figure. A force $F = Mg$ acts on the block. It is inclined to the vertical at an angle θ. The coefficient of friction is μ. The block can be pushed along the surface only when



(a) $\tan\theta \geq \mu$

(b) $\cot\theta \geq \mu$

(c) $\tan\theta/2 \geq \mu$

(d) $\cot\theta/2 \geq \mu$

27. A plane is inclined at an angle θ with the horizontal. A body of mass *m* rests on it. If the coefficient of friction is μ, then the minimum force that has to be applied parallel to the inclined plane to make the body just move up the inclined plane is

(a) $mg\sin\theta$ (b) $\mu mg\cos\theta$

(c) $\mu mg\cos\theta - mg\sin\theta$ (d) $\mu mg\cos\theta + mg\sin\theta$

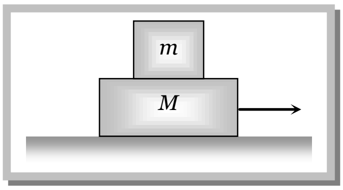
28. A block of mass m is placed on another block of mass M which itself is lying on a horizontal surface. The coefficient of friction between the two blocks is μ_1 and that between the block of mass M and horizontal surface is μ_2. What maximum horizontal force can be applied to the lower block so that the two blocks move without separation

(a) $(M+m)(\mu_2-\mu_1)g$

(b) $(M-m)(\mu_2-\mu_1)g$

(c) $(M-m)(\mu_2+\mu_1)g$

(d) $(M+m)(\mu_2+\mu_1)g$



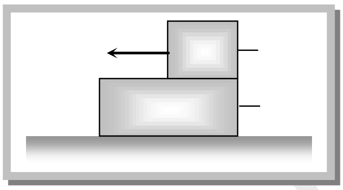
29. A block of mass M_1 is placed on a slab of mass M_2. The slab lies on a frictionless horizontal surface. The coefficient of static friction between the block and slab is μ_1 and that of dynamic friction is μ_2. A force F acts on the block M_1. Take g = 10 ms^{-2}. If M_1 = 10 kg, M_2 = 30 kg, μ_1 = 0.5, μ_2 = 0.15 and F = 40 N, what will be the acceleration with which the slab will move

(a) 5 ms^{-2}

(b) 2 ms^{-2}

(c) 1 ms^{-2}

(d) Zero



30. In the above problem if F = 100 N, what will be the acceleration with which the slab will move

(a) 5 ms^{-2} (b) 2 ms^{-2} (c) 1 ms^{-2} (d) None of these

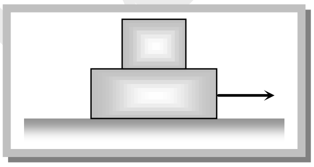
31. A block X of mass 4 kg is lying on another block Y of mass 8 kg. As shown in the figure. When the force acting on X is 12N, block X is on the verge of slipping on Y. The force F in Newton necessary to make both X and Y move simultaneously will be

(a) 36

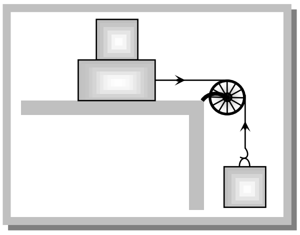
(b) 3.6

(c) 0.36

(d) 3.6



32. Two masses 10 kg and 5kg are connected by a string passing over a pulley as shown. If the coefficient of friction be 0.15, then the minimum weight that may be placed on 10 kg to stop motion is



(a) 18.7 kg

(b) 23.3 kg

(c) 32.5 kg

(d) 44.3 kg

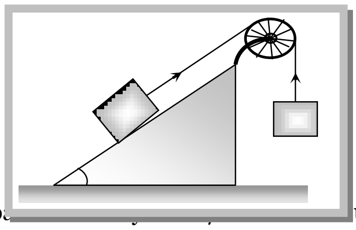
33. Two blocks of mass M_1 and M_2 are connected with a string which passes over a smooth pulley. The mass M_1 is placed on a rough inclined plane as shown in the figure. The coefficient of friction between the block and the inclined plane is μ. What should be the maximum mass M_2 so that block M_1 slides downwards

(a) $M_2 = M_1(\sin\theta + \mu\cos\theta)$

(b) $M_2 = M_1(\sin\theta - \mu\cos\theta)$

(c) $M_2 = M_1/(\sin\theta + \mu\cos\theta)$

(d) $M_2 = M_1/(\sin\theta - \mu\cos\theta)$



34. A car starts from rest to cover a distance s. the coefficient of friction between the ro... um time in which the car can cover the distance is proportional to

(a) μ (b) $\sqrt{\mu}$ (c) $\frac{1}{\mu}$ (d) $\frac{1}{\sqrt{\mu}}$

35. An engine of mass 50,000 kg pulls a coach of mass 40,000 kg. If there is a resistance of 1 N per 100 kg acting on both the engine and the coach, and if the driving force of the engine be 4,500 N, then the acceleration of the engine is

(a) 0.08 m/s^2 (b) Zero (c) 0.04 m/s^2 (d) None of these

36. In the above question, then tension in the coupling is

(a) 2,000 N (b) 1,500 N (c) 500 N (d) 1000 N

37. An aeroplane requires for take off a speed of 72 km/h. The run of the ground is 100m.The mass of the plane is $10^4\ kg$ and the coefficient of friction between the plane and the ground is 0.2. The plane accelerates uniformly during take off. What is the acceleration of the plane

(a) 1 m/s^2 (b) 2 m/s^2 (c) 3 m/s^2 (d) 4 m/s^2

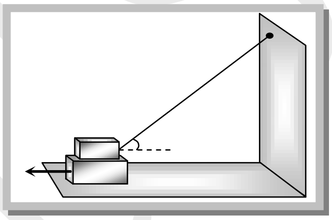
38. The force required to just move a body up an inclined plane is double the force required to just prevent it from sliding down. If ϕ is angle of friction and θ is the angle which incline makes with the horizontal then

(a) $\tan\theta = \tan\phi$ (b) $\tan\theta = 2\tan\phi$ (c) $\tan\theta = 3\tan\phi$ (d) $\tan\phi = 3\tan\theta$

39. A body is on a rough horizontal plane. A force is applied to the body direct towards the plane at an angle ϕ with the vertical. If θ is the angle of friction then for the body to move along the plane

(a) $\phi > \theta$ (b) $\phi < \theta$ (c) $\phi = \theta$ (d) ϕ can take up any value

40. In the arrangement shown $W_1 = 200\ N$, $W_2 = 100\ N$, $\mu = 0.25$ for all surfaces in contact. The block W_1 just slides under the block W_2



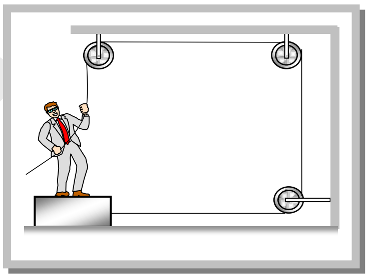
(a) A pull of $50\ N$ is to be applied on W_1

(b) A pull of $90\ N$ is to be applied on W_1

(c) Tension in the string AB is $10\sqrt{2}N$

(d) Tension in the string AB is $20\sqrt{2}N$

41. A board of mass m is placed on the floor and a man of mass M is standing on the board as shown. The coefficient of friction between the board and the floor is μ. The maximum force that the can exert on he rope so that the board does not slip on the floor is



(a) $F = \mu(M+m)g$

(b) $F = \mu mg$

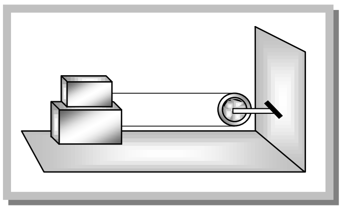
(c) $F = \dfrac{\mu Mg}{\mu+1}$

(d) $F = \dfrac{\mu(M+m)g}{\mu+1}$

42. A body slides over an inclined plane forming an angle of 45° with the horizontal. The distance x travelled by the body in time t is described by the equation $x = kt^2$, where $k = 1.732$. The coefficient of friction between the body and the plane has a value

(a) $\mu = 0.5$ (b) $\mu = 1$ (c) $\mu = 0.25$ (d) $\mu = 0.75$

43. Two blocks A and B of masses m and M respectively are placed on each other and their combination rests on a fixed horizontal surface C. A light string passing over the smooth light pulley is used to connect A and B as shown. The coefficient of sliding friction between all surfaces in contact is μ. If A is dragged with a force F then for both A and B to move with a uniform speed we have



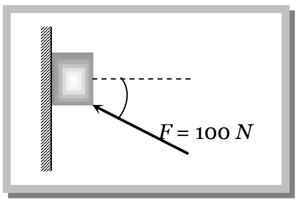
(a) $F = \mu(M+m)g$

(b) $F = \mu mg$

(c) $F = \mu(3M+m)g$

(d) $F = \mu(3m+M)g$

44. A force of 100 N is applied on a block of mass 3 kg as shown in figure. The coefficient of friction between the surface of the block is 1/4. The friction force acting on the block is



(a) 15 N downwards (b) 25 N upwards (c) 20 N downwards (d) 20 N upwards

Answer Sheet (Practice problems)

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
b	c	d	d	b	a	c	a	c	b
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
c	c	a	d	d	c	a	b	c	d
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
a	c	a	a	c	d	d	c	d	a
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
b	b	d	c	a	b	c	a	b, d	d
41.	**42.**	**43.**	**44**						
a	d	c	b						

Motion In Two Dimension

The motion of an object is called two dimensional, if two of the three co-ordinates are required to specify the position of the object in space changes *w.r.t* time.

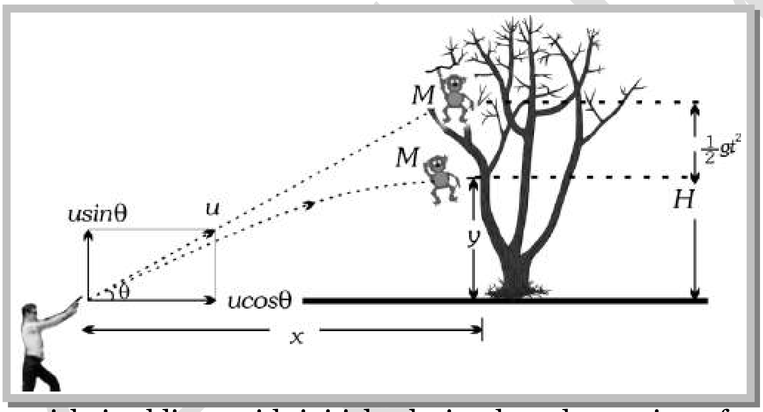
In such a motion, the object moves in a plane. For example, a billiard ball moving over the billiard table, an insect crawling over the floor of a room, earth revolving around the sun *etc.*

Two special cases of motion in two dimension are 1. Projectile motion 2. Circular motion

PROJECTILE MOTION

3.1 Introduction

A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. If the monkey remains in his position, he will be safe but at the instant the bullet leaves the barrel of gun, if the monkey drops from the tree, the bullet will hit the monkey because the bullet will not follow the linear path.



The path of motion of a as projectile motion.

If the force acting on a particle is oblique with initial velocity then the motion of particle is called projectile motion.

3.2 Projectile

A body which is in flight through the atmosphere but is not being propelled by any fuel is called projectile.

Example: (i) A bomb released from an aeroplane in level flight

(ii) A bullet fired from a gun

(iii) An arrow released from bow

(iv) A Javelin thrown by an athlete

3.3 Assumptions of Projectile Motion

(1) There is no resistance due to air.

(2) The effect due to curvature of earth is negligible.

(3) The effect due to rotation of earth is negligible.

(4) For all points of the trajectory, the acceleration due to gravity 'g' is constant in magnitude and direction.

3.4 Principles of Physical Independence of Motions

(1) The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts. Horizontal motion and vertical motion. These two motions take place independent of each other. This is called the principle of physical independence of motions.

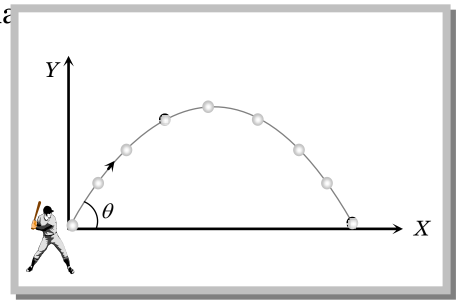
(2) The velocity of the particle can be resolved into two mutually perpendicular components. Horizontal component and vertical component.

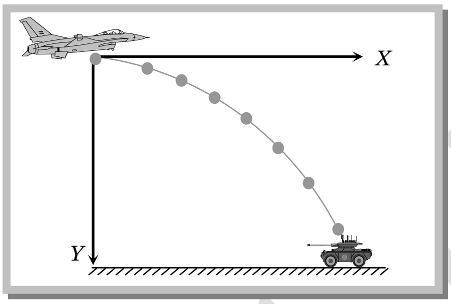
(3) The horizontal component remains unchanged throughout the flight. The force of gravity continuously affects the vertical component.

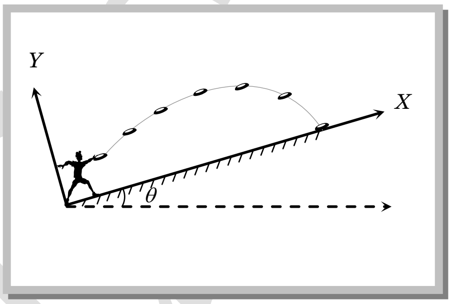
(4) The horizontal motion is a uniform motion and the vertical motion is a uniformly accelerated retarded motion.

3.5 Types of Projectile Motion

(1) Oblique projectile motion (2) Horizontal projectile motion (3) Projectile motion on an inclined plane



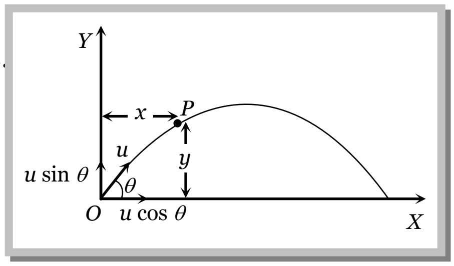




3.6 Oblique Projectile

In projectile motion, horizontal component of velocity ($u\cos\theta$), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity ($u\sin\theta$), momentum, kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.

(1) **Equation of trajectory :** A projectile thrown with velocity u at an angle θ with the horizontal. The velocity u can be resolved into two rectangular components.



$v\cos\theta$ component along X–axis and $u\sin\theta$ component along Y–axis.

For horizontal motion $x = u\cos\theta \times t \Rightarrow t = \dfrac{x}{u\cos\theta}$ .... (i)

For vertical motion $y = (u\sin\theta)t - \dfrac{1}{2}gt^2$ .... (ii)

From equation (i) and (ii) $y = u\sin\theta\left(\dfrac{x}{u\cos\theta}\right) - \dfrac{1}{2}g\left(\dfrac{x^2}{u^2\cos^2\theta}\right)$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

This equation shows that the trajectory of projectile is parabolic because it is similar to equation of parabola

$$y = ax - bx^2$$

Note : ❑ Equation of oblique projectile also can be written as

$$y = x\tan\theta\left[1 - \frac{x}{R}\right] \qquad \text{(where } R = \text{horizontal range} = \frac{u^2\sin 2\theta}{g}\text{)}$$

Sample problems based on trajectory

***Problem* 1.** The trajectory of a projectile is represented by $y=\sqrt{3}x-gx^2/2$. The angle of projection is

(a) 30° (b) 45° (c) 60° (d) None of these

Solution : (c) By comparing the coefficient of x in given equation with standard equation $y=x\tan\theta-\dfrac{gx^2}{2u^2\cos^2\theta}$

$\tan\theta=\sqrt{3}\quad\therefore\theta=60°$

***Problem* 2.** The path followed by a body projected along y-axis is given as by $y=\sqrt{3}x-(1/2)x^2$, if g = 10 m/s, then the initial velocity of projectile will be – (x and y are in m)

(a) $3\sqrt{10}\ m/s$ (b) $2\sqrt{10}\ m/s$ (c) $10\sqrt{3}\ m/s$ (d) $10\sqrt{2}\ m/s$

Solution : (b) By comparing the coefficient of x^2 in given equation with standard equation $y=x\tan\theta-\dfrac{gx^2}{2u^2\cos^2\theta}$.

$$\frac{g}{2u^2\cos^2\theta}=\frac{1}{2}$$

Substituting θ = 60° we get $u=2\sqrt{10}\ m/\sec$.

***Problem* 3.** The equation of projectile is $y=16x-\dfrac{5x^2}{4}$. The horizontal range is

(a) 16 m (b) 8 m (c) 3.2 m (d) 12.8 m

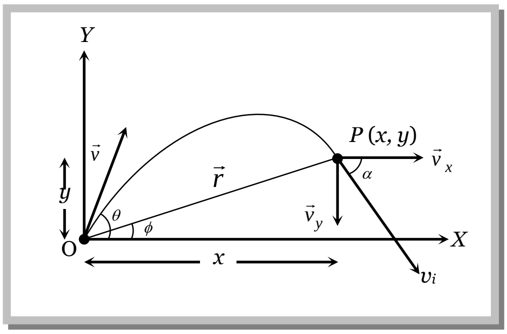
Solution : (d) Standard equation of projectile motion $y=x\tan\theta\left[1-\dfrac{x}{R}\right]$

Given equation : $y=16x-\dfrac{5x^2}{4}$ or $y=16x\left[1-\dfrac{x}{64/5}\right]$

By comparing above equations $R=\dfrac{64}{5}$ =12.8 m.

(2) **Displacement of projectile** $(\vec{r})$ **:** Let the particle acquires a position P having the coordinates (x, y) just after time t from the instant of projection. The corresponding position vector of the particle at time t is $\vec{r}$ as shown in the figure.

$$\vec{r}=x\hat{i}+y\hat{j} \quad ....(i)$$



The horizontal distance covered during time t is given as

$$x=v_x\,t\Rightarrow x=u\cos\theta\ t \quad ....(ii)$$

The vertical velocity of the particle at time t is given as

$$v_y=(v_0)_y-gt, \quad ....(iii)$$

Now the vertical displacement y is given as

$$y=u\sin\theta\,t-1/2\,gt^2 \quad ....(iv)$$

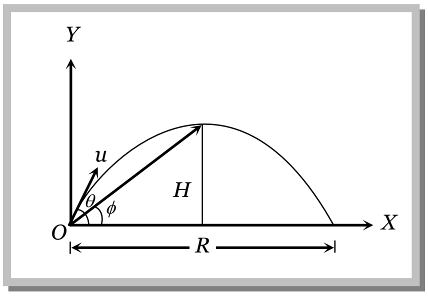
Putting the values of x and y from equation (ii) and equation (iv) in equation (i) we obtain the position vector at any time t as

$$\vec{r}=(u\cos\theta)t\,\hat{i}+\left((u\sin\theta)t-\frac{1}{2}gt^2\right)\hat{j}\ \Rightarrow\ r=\sqrt{(u\,t\cos\theta)^2+\left((u\,t\sin\theta)-\frac{1}{2}gt^2\right)^2}$$

$$r=u\,t\sqrt{1+\left(\frac{gt}{2u}\right)^2-\frac{gt\sin\theta}{u}}\ \text{and}\ \phi=\tan^{-1}(y/x)\ =\tan^{-1}\left(\frac{ut\sin\theta-1/2gt^2}{(u\,t\cos\theta)}\right)\ \text{or}\ \phi=\tan^{-1}\left(\frac{2u\sin\theta-gt}{2u\cos\theta}\right)$$

Note : ❑ The angle of elevation ϕ of the highest point of the projectile and the angle of projection θ are related to each other as

$$\tan\phi = \frac{1}{2}\tan\theta$$

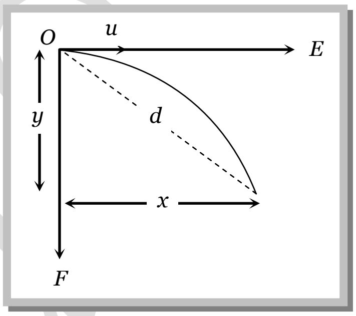


Sample problems based on displacement

Problem 4. A body of mass 2 *kg* has an initial velocity of 3 *m/s* along *OE* and it is subjected to a force of 4 Newton's in *OF* direction perpendicular to *OE*. The distance of the body from *O* after 4 seconds will be

(a) 12 *m* (b) 28 *m* (c) 20 *m* (d) 48 *m*

Solution : (c) Body moves horizontally with constant initial velocity 3 *m/s* upto 4 seconds $\therefore \; x = ut = 3\times 4 = 12\,m$



and in perpendicular direction it moves under the effect of constant force with zero initial velocity upto 4 seconds.

$$\therefore \quad y = ut + \frac{1}{2}(a)t^2 = 0 + \frac{1}{2}\left(\frac{F}{m}\right)t^2 = \frac{1}{2}\left(\frac{4}{2}\right)4^2 = 16\,m$$

So its distance from *O* is given by $d = \sqrt{x^2+y^2} = \sqrt{(12)^2+(16)^2}$

$\therefore \quad d = 20\,m$

Problem 5. A body starts from the origin with an acceleration of 6 m/s^2 along the *x*-axis and 8 m/s^2 along the *y*-axis. Its distance from the origin after 4 seconds will be **[MP PMT 1999]**

(a) 56 *m* (b) 64 *m* (c) 80 *m* (d) 128 *m*

Solution : (c) Displacement along *X*- axis : $x = u_x t + \frac{1}{2}a_x t^2 = \frac{1}{2}\times 6\times(4)^2 = 48\,m$

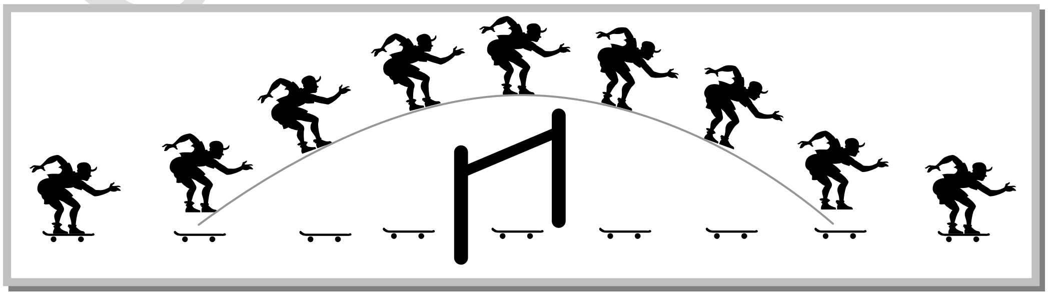
Displacement along *Y*- axis : $y = u_y t + \frac{1}{2}a_y t^2 = \frac{1}{2}\times 8\times(4)^2 = 64\,m$

Total distance from the origin $= \sqrt{x^2+y^2} = \sqrt{(48)^2+(64)^2} = 80\,m$

(3) **Instantaneous velocity v :** In projectile motion, vertical component of velocity changes but horizontal component of velocity remains always constant.

Example : When a man jumps over the hurdle leaving behind its skateboard then vertical component of his velocity is changing, but not the horizontal component, which matches with the skateboard velocity.

As a result, the skateboard stays underneath him, allowing him to land on it.



Let v_i be the instantaneous velocity of projectile at time *t* direction of this velocity is along the tangent to the trajectory at point *P*.

$$\vec{v}_i = v_x i + v_y \hat{j} \Rightarrow v_i = \sqrt{v_x^2+v_y^2} = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

$$v_i = \sqrt{u^2 + g^2t^2 - 2u\,gt\sin\theta}$$

Direction of instantaneous velocity $\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$ or $\alpha = \tan^{-1}\left[\tan\theta - \frac{gt}{u}\sec\theta\right]$

(4) **Change in velocity :** Initial velocity (at projection point) $\vec{u}_i = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Final velocity (at highest point) $\vec{u}_f = u\cos\theta\,\hat{i} + 0\,\hat{j}$

(i) Change in velocity (Between projection point and highest point) $\Delta u = \vec{u}_f - \vec{u}_i = -u\sin\theta\,\hat{j}$

When body reaches the ground after completing its motion then final velocity $\vec{u}_f = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

(ii) Change in velocity (Between complete projectile motion) $\Delta u = u_f - u_i = -2u\sin\theta\,\hat{i}$

Sample problems based on velocity

***Problem* 6.** In a projectile motion, velocity at maximum height is

(a) $\frac{u\cos\theta}{2}$ (b) $u\cos\theta$ (c) $\frac{u\sin\theta}{2}$ (d) None of these

Solution : (b) In a projectile motion at maximum height body possess only horizontal component of velocity *i.e.* $u\cos\theta$.

***Problem* 7.** A body is thrown at angle 30° to the horizontal with the velocity of 30 *m/s*. After 1 sec, its velocity will be (in *m/s*) (g = 10 *m/s*²)

(a) $10\sqrt{7}$ (b) $700\sqrt{10}$ (c) $100\sqrt{7}$ (d) $\sqrt{40}$

Solution : (a) From the formula of instantaneous velocity $v = \sqrt{u^2 + g^2t^2 - 2u\,g\,t\sin\theta}$

$$v = \sqrt{(30)^2 + (10)^2 \times 1^2 - 2\times 30\times 10\times 1\times \sin 30^o} = 10\sqrt{7}\ m/s$$

***Problem* 8.** A projectile is fired at 30° to the horizontal. The vertical component of its velocity is 80 *ms*⁻¹. Its time of flight is *T*. What will be the velocity of the projectile at $t = T/2$

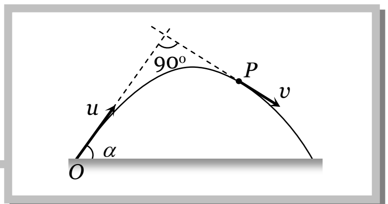
(a) 80 ms^{-1} (b) $80\sqrt{3}$ ms^{-1} (c) $(80/\sqrt{3})$ ms^{-1} (d) 40 ms^{-1}

Solution : (b) At half of the time of flight, the position of the projectile will be at the highest point of the parabola and at that position particle possess horizontal component of velocity only.

Given $u_{vertical} = u\sin\theta = 80 \Rightarrow u = \frac{80}{\sin 30^o} = 160\ m/s$

$\therefore\quad u_{horizontal} = u\cos\theta = 160\ \cos 30^o = 80\sqrt{3}\ m/s.$

***Problem* 9.** A particle is projected from point *O* with velocity *u* in a direction making an angle α with the horizontal. At any instant its position is at point *P* at right angles to the initial direction of projection. Its velocity at point *P* is

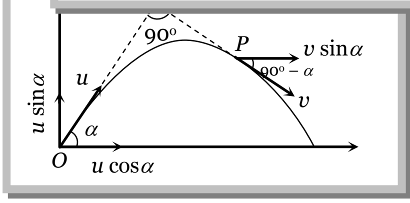


(a) $u\tan\alpha$ (b) $u\cot\alpha$ (c) $u\,\mathrm{cosec}\alpha$ (d) $u\sec\alpha$

Solution : (b) Horizontal velocity at point $'O' = u\cos\alpha$

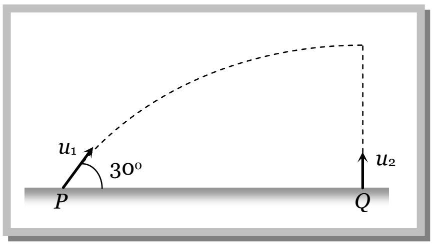
Horizontal velocity at point $'P' = v\sin\alpha$

In projectile motion horizontal component of velocity remains constant throughout the motion



$\therefore\ v\sin\alpha = u\cos\alpha \Rightarrow v = u\cot\alpha$

Problem 10. A particle P is projected with velocity u_1 at an angle of 30° with the horizontal. Another particle Q is thrown vertically upwards with velocity u_2 from a point vertically below the highest point of path of P. The necessary condition for the two particles to collide at the highest point is



(a) $u_1 = u_2$

(b) $u_1 = 2u_2$

(c) $u_1 = \frac{u_2}{2}$

(d) $u_1 = 4u_2$

Solution : (b) Both particle collide at the highest point it means the vertical distance travelled by both the particle will be equal, *i.e.* the vertical component of velocity of both particle will be equal

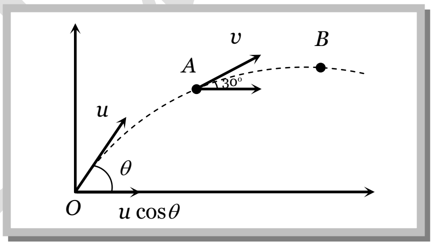
$u_1\sin 30° = u_2 \Rightarrow \frac{u_1}{2} = u_2 \quad \therefore\ u_1 = 2u_2$

Problem 11. Two seconds after projection a projectile is travelling in a direction inclined at 30° to the horizontal after one more sec, it is travelling horizontally, the magnitude and direction of its velocity are

(a) $2\sqrt{20}\ m/\text{sec}, 60^o$ (b) $20\sqrt{3}\ m/\text{sec}, 60^o$ (c) $6\sqrt{40}\ m/\text{sec}, 30^o$ (d) $40\sqrt{6}\ m/\text{sec}, 30^o$

Solution : (b) Let in 2 *sec* body reaches upto point A and after one more *sec* upto point B.

Total time of ascent for a body is given 3 *sec i.e.* $t = \frac{u\sin\theta}{g} = 3$



$\therefore u\sin\theta = 10\times 3 = 30$ .....(i)

Horizontal component of velocity remains always constant

$u\cos\theta = v\cos 30°$ .....(ii)

For vertical upward motion between point O and A

$v\sin 30^o = u\sin\theta - g\times 2$ [Using $v = u - gt$]

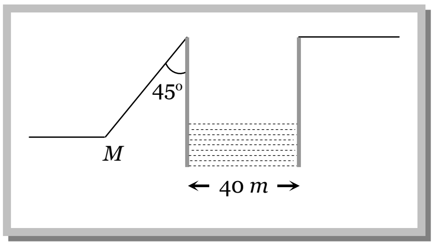
$v\sin 30^o = 30 - 20$ [As $u\sin\theta = 30$]

$\therefore v = 20\ m/s.$

Substituting this value in equation (ii) $u\cos\theta = 20\cos 30^o = 10\sqrt{3}$ .....(iii)

From equation (i) and (iii) $u = 20\sqrt{3}$ and $\theta = 60°$

Problem 12. A body is projected up a smooth inclined plane (length = $20\sqrt{2}\ m$) with velocity u from the point M as shown in the figure. The angle of inclination is 45° and the top is connected to a well of diameter 40 m. If the body just manages to cross the well, what is the value of v



(a) $40\ ms^{-1}$

(b) $40\sqrt{2}\ ms^{-1}$

(c) $20\ ms^{-1}$

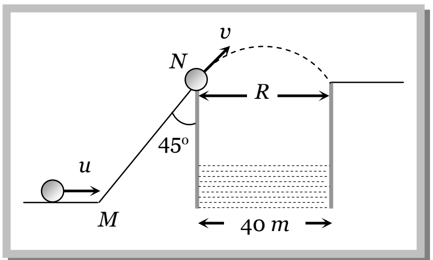
(d) $20\sqrt{2}\ ms^{-1}$

Solution : (d) At point N angle of projection of the body will be 45°. Let velocity of projection at this point is v.

If the body just manages to cross the well then Range = Diameter of well

$\frac{v^2\sin 2\theta}{g} = 40$ [As $\theta = 45°$]

$v^2 = 400 \Rightarrow v = 20\ m/s$



But we have to calculate the velocity (u) of the body at point M.

For motion along the inclined plane (from M to N)

Final velocity (v) = 20 m/s,

acceleration (a) = $-g\sin\alpha = -g\sin 45^o$, distance of inclined plane (s) = $20\sqrt{2}\ m$

$(20)^2 = u^2 - 2\frac{g}{\sqrt{2}}.20\sqrt{2}$ [Using $v^2 = u^2 + 2as$]

$u^2 = 20^2 + 400 \Rightarrow u = 20\sqrt{2}\ m/s.$

Problem 13. A projectile is fired with velocity u making angle θ with the horizontal. What is the change in velocity when it is at the highest point

(a) $u\cos\theta$ (b) u (c) $u\sin\theta$ (d) $(u\cos\theta - u)$

Solution : (c) Since horizontal component of velocity remain always constant therefore only vertical component of velocity changes.

Initially vertical component $u\sin\theta$

Finally it becomes zero. So change in velocity $= u\sin\theta$

(5) **Change in momentum :** Simply by the multiplication of mass in the above expression of velocity (Article-4).

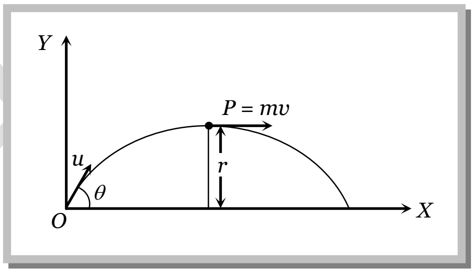
(i) Change in momentum (Between projection point and highest point) $\Delta p = \vec{p}_f - \vec{p}_i = -mu\sin\theta\,\hat{j}$

(ii) Change in momentum (For the complete projectile motion) $\Delta p = \vec{p}_f - \vec{p}_i = -2mu\sin\theta\,\hat{j}$

(6) **Angular momentum :** Angular momentum of projectile at highest point of trajectory about the point of projection is given by

$$L = mvr \qquad \left[\text{Here } r = H = \frac{u^2\sin^2\theta}{2g}\right]$$

$$\therefore \quad L = m\,u\cos\theta\frac{u^2\sin^2\theta}{2g} = \frac{m\,u^3\cos\theta\sin^2\theta}{2g}$$



Sample problems based on momentum and angular momentum

Problem 14. A body of mass 0.5 kg is projected under gravity with a speed of 98 m/s at an angle of 30^o with the horizontal. The change in momentum (in magnitude) of the body is

(a) 24.5 $N–s$ (b) 49.0 $N–s$ (c) 98.0 $N–s$ (d) 50.0 $N–s$

Solution : (b) Change in momentum between complete projectile motion = $2mu\sin\theta = 2\times 0.5\times 98\times\sin 30° = 49\ N–s$.

Problem 15. A particle of mass 100 g is fired with a velocity 20 $m\ sec^{-1}$ making an angle of 30^o with the horizontal. When it rises to the highest point of its path then the change in its momentum is

(a) $\sqrt{3}\,kg\ m\ sec^{-1}$ (b) 1/2 $kg\ m\ sec^{-1}$ (c) $\sqrt{2}\,kg\ m\ sec^{-1}$ (d) 1 $kg\ m\ sec^{-1}$

Solution : (d) Horizontal momentum remains always constant

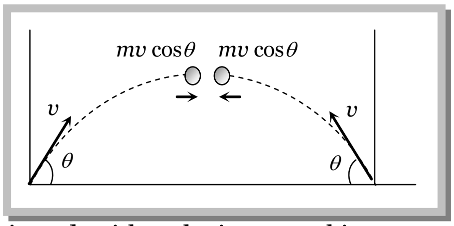
So change in vertical momentum ($\Delta\vec{p}$) = Final vertical momentum – Initial vertical momentum $= 0 - mu\sin\theta$

$|\Delta P| = 0.1\times 20\times\sin 30^o = 1\,kg\ m/sec$.

Problem 16. Two equal masses (m) are projected at the same angle (θ) from two points separated by their range with equal velocities (v). The momentum at the point of their collision is

(a) Zero (b) $2\,mv\cos\theta$ (c) $-2\,mv\cos\theta$ (d) None of these

Solution : (a) Both masses will collide at the highest point of their trajectory with equal and opposite momentum. So net momentum of the system will be zero.



Problem 17. A particle of mass m is projected with velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the particle about the point of projection when the particle is at its maximum height is (where g = acceleration due to gravity)

(a) Zero (b) $mv^3/(4\sqrt{2}g)$ (c) $mv^3/(\sqrt{2}g)$ (d) $mv^2/2g$

Solution : (b) $L = \dfrac{m\,u^3\cos\theta\sin^2\theta}{2g} = \dfrac{mv^3}{(4\sqrt{2}\,g)}$ [As $\theta = 45°$]

Problem 18. A body is projected from the ground with some angle to the horizontal. What happens to the angular momentum about the initial position in this motion

(a) Decreases (b) Increases

(c) Remains same (d) First increases and then decreases

Solution : (b)

Problem 19. In case of a projectile, where is the angular momentum minimum

(a) At the starting point

(b) At the highest point

(c) On return to the ground

(d) At some location other than those mentioned above

Solution : (a)

(7) **Time of flight :** The total time taken by the projectile to go up and come down to the same level from which it was projected is called time of flight.

For vertical upward motion $0 = u\sin\theta - gt \Rightarrow t = (u\sin\theta/g)$

Now as time taken to go up is equal to the time taken to come down so

Time of flight $T = 2t = \dfrac{2u\sin\theta}{g}$

(i) Time of flight can also be expressed as : $T = \dfrac{2.u_y}{g}$ (where u_y is the vertical component of initial velocity).

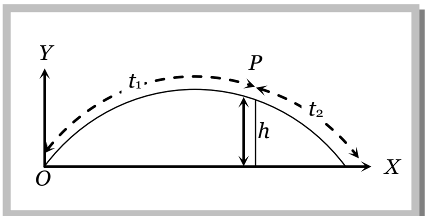
(ii) For complementary angles of projection θ and $90° - \theta$

(a) Ratio of time of flight $= \dfrac{T_1}{T_2} = \dfrac{2u\sin\theta/g}{2u\sin(90-\theta)/g} = \tan\theta \Rightarrow \dfrac{T_1}{T_2} = \tan\theta$

(b) Multiplication of time of flight $= T_1T_2 = \dfrac{2u\sin\theta}{g}\dfrac{2u\cos\theta}{g} \Rightarrow T_1T_2 = \dfrac{2R}{g}$

(iii) If t_1 is the time taken by projectile to rise upto point p and t_2 is the time taken in falling from point p to ground level then $t_1 + t_2 = \dfrac{2u\sin\theta}{g}$ = time of flight

or $u\sin\theta = \dfrac{g(t_1+t_2)}{2}$



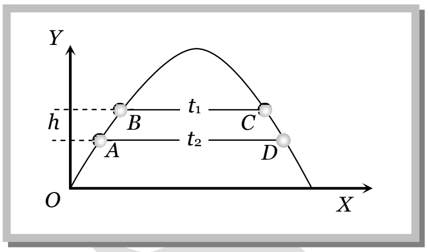
and height of the point p is given by $h = u\sin\theta\, t_1 - \frac{1}{2}gt_1^2$

$$h = g\frac{(t_1 + t_2)}{2}t_1 - \frac{1}{2}gt_1^2$$

by solving $h = \frac{g\,t_1 t_2}{2}$

(iv) If B and C are at the same level on trajectory and the time difference between these two points is t_1, similarly A and D are also at the same level and the time difference between these two positions is t_2 then

$$t_2^2 - t_1^2 = \frac{8h}{g}$$



Sample problems based on time of flight

Problem 20. For a given velocity, a projectile has the same range R for two angles of projection if t_1 and t_2 are the times of flight in the two cases then

(a) $t_1t_2 \propto R^2$ (b) $t_1t_2 \propto R$ (c) $t_1t_2 \propto \frac{1}{R}$ (d) $t_1t_2 \propto \frac{1}{R^2}$

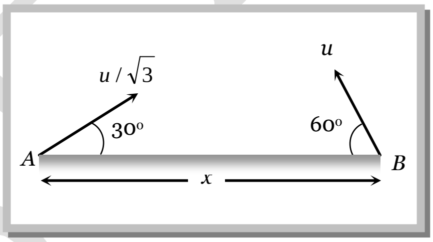
Solution : (b) As we know for complementary angles $t_1t_2 = \frac{2R}{g}$ $\therefore$ $t_1t_2 \propto R$.

Problem 21. A body is thrown with a velocity of 9.8 m/s making an angle of 30° with the horizontal. It will hit the ground after a time **[JIPMER 2001, 2002; KCET (Engg.) 2001]**

(a) 1.5 s (b) 1 s (c) 3 s (d) 2 s

Solution : (b) $T = \frac{2u\sin\theta}{g} = \frac{2\times 9.8\times \sin 30^o}{9.8} = 1sec$

Problem 22. Two particles are separated at a horizontal distance x as shown in figure. They are projected at the same time as shown in figure with different initial speed. The time after which the horizontal distance between the particles become zero is



(a) $u/2x$ (b) x/u (c) $2u/x$ (d) u/x

Solution : (b) Let x_1 and x_2 are the horizontal distances travelled by particle A and B respectively in time t.

$$x_1 = \frac{u}{\sqrt{3}}.\cos 30^\circ \times t \quad .....(i) \qquad \text{and} \qquad x_2 = u\cos 60^o \times t \quad ......(ii)$$

$$x_1 + x_2 = \frac{u}{\sqrt{3}}.\cos 30^o \times t + u\cos 60^o \times t = ut \Rightarrow x = ut \;\therefore\; t = x/u$$

Problem 23. A particle is projected from a point O with a velocity u in a direction making an angle α upward with the horizontal. After some time at point P it is moving at right angle with its initial direction of projection. The time of flight from O to P is

(a) $\frac{u \sin\alpha}{g}$ (b) $\frac{u \operatorname{cosec}\alpha}{g}$ (c) $\frac{u \tan\alpha}{g}$ (d) $\frac{u \sec\alpha}{g}$

Solution : (b) When body projected with initial velocity $\vec{u}$ by making angle α with the horizontal. Then after time *t*, (at point *P*) it's direction is perpendicular to $\vec{u}$.

Magnitude of velocity at point *P* is given by $v = u\cot\alpha$. (from sample problem no. 9)

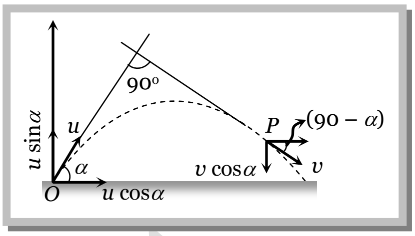
For vertical motion : Initial velocity (at point *O*) $= u\sin\alpha$

Final velocity (at point *P*) $= -v\cos\alpha = -u\cot\alpha\cos\alpha$

Time of flight (from point *O* to *P*) = *t*

Applying first equation of motion $v = u - gt$

$-u\cot\alpha\cos\alpha = u\sin\alpha - gt$



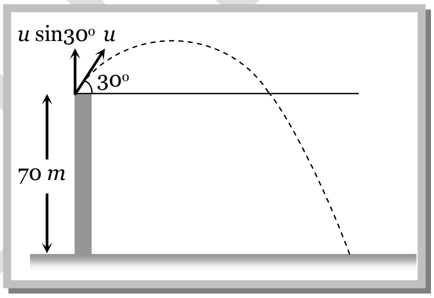
$\therefore\ t = \frac{u\sin\alpha + u\cot\alpha\cos\alpha}{g} = \frac{u}{g\sin\alpha}\left[\sin^2\alpha + \cos^2\alpha\right] = \frac{u \operatorname{cosec}\alpha}{g}$

***Problem* 24.** A ball is projected upwards from the top of tower with a velocity 50 ms^{-1} making angle 30° with the horizontal. The height of the tower is 70 *m*. After how many seconds from the instant of throwing will the ball reach the ground

(a) 2.33 *sec* (b) 5.33 *sec* (c) 6.33 *sec* (d) 9.33 *sec*

Solution : (c) Formula for calculation of time to reach the body on the ground from the tower of height '*h*' (If it is thrown vertically up with velocity *u*) is given by

$t = \frac{u}{g}\left[1 + \sqrt{1 + \frac{2gh}{u^2}}\right]$



So we can resolve the given velocity in vertical direction and can apply the above formula.

Initial vertical component of velocity $u\sin\theta = 50\sin 30 = 25\, m/s$.

$\therefore\quad t = \frac{25}{9.8}\left[1 + \sqrt{1 + \frac{2\times 9.8\times 70}{(25)^2}}\right] = 6.33$ *sec.*

***Problem* 25.** If for a given angle of projection, the horizontal range is doubled, the time of flight becomes

(a) 4 times (b) 2 times (c) $\sqrt{2}$ times (d) $1/\sqrt{2}$ times

Solution : (c) $R = \frac{u^2\sin 2\theta}{g}$ and $T = \frac{2u\sin\theta}{g}$

$\therefore\ R \propto u^2$ and $T \propto u$ (If θ and *g* are constant).

In the given condition to make range double, velocity must be increased upto $\sqrt{2}$ times that of previous value. So automatically time of flight will becomes $\sqrt{2}$ times.

***Problem* 26.** A particle is thrown with velocity *u* at an angle α from the horizontal. Another particle is thrown with the same velocity at an angle α from the vertical. The ratio of times of flight of two particles will be

(a) $\tan 2\alpha : 1$ (b) $\cot 2\alpha : 1$ (c) $\tan\alpha : 1$ (d) $\cot\alpha : 1$

Solution : (c) For first particles angle of projection from the horizontal is α. So $T_1 = \frac{2u\sin\alpha}{g}$

For second particle angle of projection from the vertical is α. it mean from the horizontal is $(90-\alpha)$.

$\therefore T_2 = \frac{2u\sin(90-\alpha)}{g} = \frac{2u\cos\alpha}{g}$. So ratio of time of flight $\frac{T_1}{T_2} = \tan\alpha$.

***Problem* 27.** The friction of the air causes vertical retardation equal to one tenth of the acceleration due to gravity (Take *g* = 10 ms^{-2}). The time of flight will be decreased by

(a) 0% (b) 1% (c) 9% (d) 11%

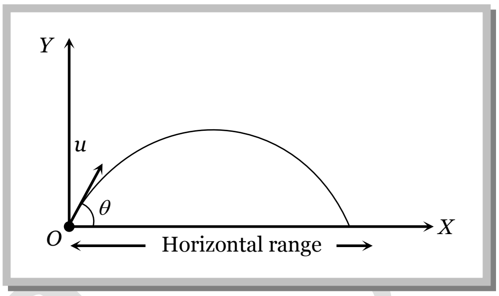
Solution : (c) $T = \frac{2u\sin\theta}{g}$ $\therefore \frac{T_1}{T_2} = \frac{g_2}{g_1} = \frac{g + \frac{g}{10}}{g} = \frac{11}{10}$

Fractional decrease in time of flight $= \frac{T_1 - T_2}{T_1} = \frac{1}{11}$

Percentage decrease = 9%

(8) **Horizontal range :** It is the horizontal distance travelled by a body during the time of flight.

So by using second equation of motion



$$R = u\cos\theta \times T = u\cos\theta \times (2u\sin\theta / g) = \frac{u^2\sin 2\theta}{g}$$

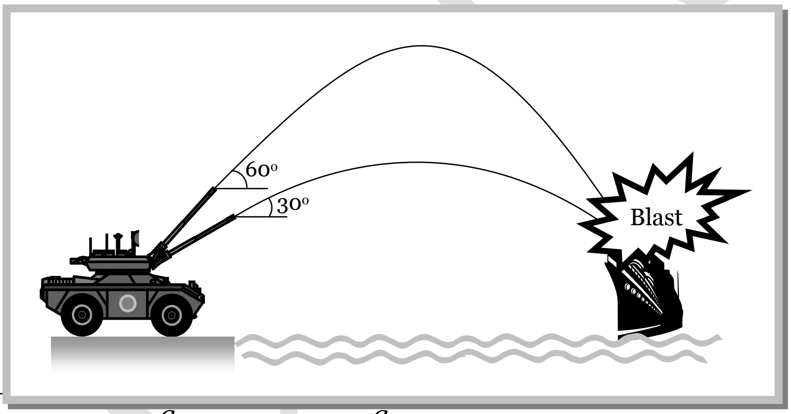
$$R = \frac{u^2\sin 2\theta}{g}$$

(i) Range of projectile can also be expressed as :

$$R = u\cos\theta \times T = u\cos\theta\frac{2u\sin\theta}{g} = \frac{2u\cos\theta\; u\sin\theta}{g} = \frac{2u_x u_y}{g}$$

$\therefore \; R = \frac{2u_x u_y}{g}$ (where u_x and u_y are the horizontal and vertical component of initial velocity)

(ii) If angle of projection is changed from θ to $\theta' = (90 - \theta)$ then range remains unchanged.



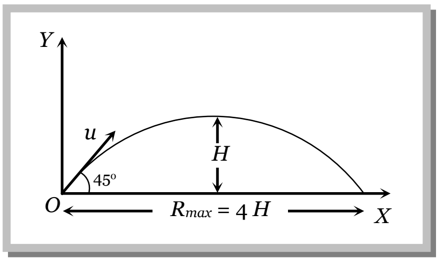
$R' = \frac{u^2\sin 2\theta}{g}$

So a projectile has same range at angles of projection θ and $(90 - \theta)$, though time of flight, maximum height and trajectories are different.

These angles θ and $90° - \theta$ are called complementary angles of projection and for complementary angles of projection ratio of range $\frac{R_1}{R_2} = \frac{u^2\sin 2\theta / g}{u^2\sin[2(90° - \theta)]/g} = 1 \Rightarrow \frac{R_1}{R_2} = 1$

(iii) For angle of projection $\theta_1 = (45 - \alpha)$ and $\theta_2 = (45 + \alpha)$, range will be same and equal to $u^2\cos 2\alpha/g$. θ_1 and θ_2 are also the complementary angles.

(iv) Maximum range : For range to be maximum



$$\frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{u^2\sin 2\theta}{g}\right] = 0$$

$\Rightarrow \cos 2\theta = 0$ *i.e.* $2\theta = 90° \Rightarrow \theta = 45°$ and $R_{max} = (u^2/g)$

i.e., *a* projectile will have maximum range when it is projected at an angle of 45° to the horizontal and the maximum range will be (u^2/g).

When the range is maximum, the height H reached by the projectile

$$H=\frac{u^2\sin^2\theta}{2g}=\frac{u^2\sin^2 45}{2g}=\frac{u^2}{4g}=\frac{R_{\max}}{4}$$

i.e., if a person can throw a projectile to a maximum distance R_{max}, The maximum height to which it will rise is $\left(\frac{R_{\max}}{4}\right)$.

(v) Relation between horizontal range and maximum height : $R=\frac{u^2\sin 2\theta}{g}$ and $H=\frac{u^2\sin^2\theta}{2g}$

$$\therefore \quad \frac{R}{H}=\frac{u^2\sin 2\theta/g}{u^2\sin^2\theta/2g}=4\cot\theta \qquad \Rightarrow R=4H\cot\theta$$

(vi) If in case of projectile motion range R is n times the maximum height H

i.e. $\quad R=nH \quad \Rightarrow \frac{u^2\sin 2\theta}{g}=n\frac{u^2\sin^2\theta}{2g} \Rightarrow \tan\theta=[4/n]$ or $\theta=\tan^{-1}[4/n]$

The angle of projection is given by $\theta=\tan^{-1}[4/n]$

Note : ❑ If $R=H$ then $\theta=\tan^{-1}(4)$ or $\theta=76^o$.

If $R=4H$ then $\theta=\tan^{-1}(1)$ or $\theta=45^o$.

Sample problem based on horizontal range

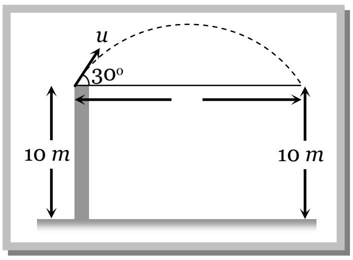
***Problem* 28.** A boy playing on the roof of a 10m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground (g = 10 m/s^2, $\sin 30^o=\frac{1}{2}$, $\cos 30^o=\frac{\sqrt{3}}{2}$) **[AIEEE 2003]**

(a) 8.66 m (b) 5.20 m (c) 4.33 m (d) 2.60 m

Solution : (a) Simply we have to calculate the range of projectile

$$R=\frac{u^2\sin 2\theta}{g}=\frac{(10)^2\sin(2\times 30^o)}{10}$$

$$R=5\sqrt{3}=8.66\,meter$$



***Problem* 29.** Which of the following sets of factors will affect the horizontal distance covered by an athlete in a long–jump event **[AMU (Engg.) 2001]**

(a) Speed before he jumps and his weight

(b) The direction in which he leaps and the initial speed

(c) The force with which he pushes the ground and his speed

(d) The direction in which he leaps and the weight

Solution : (b) Because range $=\frac{(\text{Velocity of projection})^2\times\sin 2(\text{Angle of projection})}{g}$

***Problem* 30.** For a projectile, the ratio of maximum height reached to the square of flight time is (g = 10 ms^{-2}) **[EAMCET (Med.) 2000]**

(a) 5 : 4 (b) 5 : 2 (c) 5 : 1 (d) 10 : 1

Solution : (a) $H = \frac{u^2 \sin^2\theta}{2g}$ and $T = \frac{2u\sin\theta}{g}$ $\therefore \frac{H}{T^2} = \frac{u^2 \sin^2\theta / 2g}{4u^2 \sin^2\theta / g^2} = \frac{g}{8} = \frac{10}{8} = \frac{5}{4}$

Problem 31. A cricketer can throw a ball to a maximum horizontal distance of 100 *m*. The speed with which he throws the ball is (to the nearest integer)

(a) 30 ms^{-1} (b) 42 ms^{-1} (c) 32 ms^{-1} (d) 35 ms^{-1}

Solution : (c) $R_{\max} = \frac{u^2}{g} = 100$ (when $\theta = 45°$)

$\therefore u = \sqrt{1000} = 31.62\, m/s.$

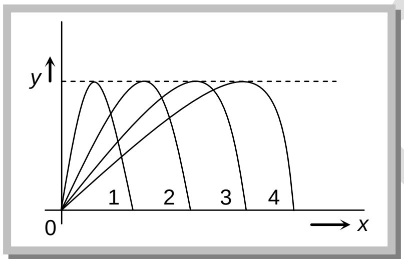
Problem 32. If two bodies are projected at 30° and 60° respectively, with the same velocity, then **[CBSE PMT 2000; JIPMER 2002]**

(a) Their ranges are same (b) Their heights are same

(c) Their times of flight are same (d) All of these

Solution : (a) Because these are complementary angles.

Problem 33. Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to initial horizontal velocity component, highest first



(a) 1, 2, 3, 4 (b) 2, 3, 4, 1 (c) 3, 4, 1, 2 (d) 4, 3, 2, 1

Solution : (d) Range $\propto$ horizontal component of velocity. Graph 4 shows maximum range, so football possess maximum horizontal velocity in this case.

Problem 34. Four bodies *P, Q, R* and *S* are projected with equal velocities having angles of projection 15°, 30°, 45° and 60° with the horizontal respectively. The body having shortest range is **[EAMCET (Engg.) 2000]**

(a) *P* (b) *Q* (c) *R* (d) *S*

Solution : (a) Range of projectile will be minimum for that angle which is farthest from 45°.

Problem 35. A particle covers 50 *m* distance when projected with an initial speed. On the same surface it will cover a distance, when projected with double the initial speed **[RPMT 2000]**

(a) 100 *m* (b) 150 *m* (c) 200 *m* (d) 250 *m*

Solution : (c) $R = \frac{u^2 \sin 2\theta}{g}$ $\therefore R \propto u^2$ so $\frac{R_2}{R_1} = \left(\frac{u_2}{u_1}\right)^2 = \left(\frac{2u}{u}\right)^2 \Rightarrow R_2 = 4R_1 = 4 \times 50 = 200\, m$

Problem 36. A bullet is fired from a canon with velocity 500 *m/s*. If the angle of projection is 15° and *g* = 10 *m/s*². Then the range is **[CPMT 1997]**

(a) $25 \times 10^3\, m$ (b) $12.5 \times 10^3\, m$ (c) $50 \times 10^2\, m$ (d) $25 \times 10^2\, m$

Solution : (b) $Range\,(R) = \frac{u^2 \sin 2\theta}{g} = \frac{(500)^2 \sin(2 \times 15)}{10} = 12500\, m = 12.5 \times 10^3\, m$

Problem 37. A projectile thrown with a speed *v* at an angle θ has a range *R* on the surface of earth. For same *v* and θ, its range on the surface of moon will be

(a) *R*/6 (b) 6 *R* (c) *R*/36 (d) 36 *R*

Solution : (b) $R = \frac{u^2 \sin 2\theta}{g}$ $\therefore R \propto 1/g$

$\frac{R_{Moon}}{R_{Earth}} = \frac{g_{Earth}}{g_{Moon}} = 6$ $\left[\because g_{Moon} = \frac{1}{6} g_{Earth}\right]$

$\therefore R_{Moon} = 6\,R_{Earth} = 6R$

Problem 38. A projectile is thrown into space so as to have maximum horizontal range R. Taking the point of projection as origin, the co-ordinates of the point where the speed of the particle is minimum are

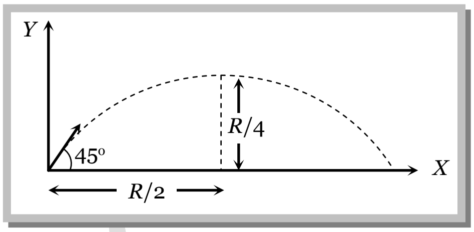
(a) (R, R) (b) $\left(R, \frac{R}{2}\right)$ (c) $\left(\frac{R}{2}, \frac{R}{4}\right)$ (d) $\left(R, \frac{R}{4}\right)$

Solution : (c) For maximum horizontal Range $\theta = 45°$

From $R = 4H\cot\theta = 4H$ [As $\theta = 45°$, for maximum range.]

Speed of the particle will be minimum at the highest point of parabola.

So the co-ordinate of the highest point will be $(R/2, R/4)$



Problem 39. The speed of a projectile at the highest point becomes $\frac{1}{\sqrt{2}}$ times its initial speed. The horizontal range of the projectile will be

(a) $\frac{u^2}{g}$ (b) $\frac{u^2}{2g}$ (c) $\frac{u^2}{3g}$ (d) $\frac{u^2}{4g}$

Solution : (a) Velocity at the highest point is given by $u\cos\theta = \frac{u}{\sqrt{2}}$ (given) $\therefore \theta = 45°$

Horizontal range $R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin(2\times 45^o)}{g} = \frac{u^2}{g}$

Problem 40. A large number of bullets are fired in all directions with same speed u. What is the maximum area on the ground on which these bullets will spread

(a) $\pi\frac{u^2}{g}$ (b) $\pi\frac{u^4}{g^2}$ (c) $\pi^2\frac{u^4}{g^2}$ (d) $\pi^2\frac{u^2}{g^2}$

Solution : (b) The maximum area will be equal to area of the circle with radius equal to the maximum range of projectile

Maximum area $\pi r^2 = \pi(R_{max})^2 = \pi\left(\frac{u^2}{g}\right)^2 = \pi\frac{u^4}{g^2}$ [As $r = R_{max} = u^2/g$ for $\theta = 45°$]

Problem 41. A projectile is projected with initial velocity $(6\hat{i} + 8\hat{j})m/sec$. If $g = 10\ ms^{-2}$, then horizontal range is

(a) 4.8 *metre* (b) 9.6 *metre* (c) 19.2 *metre* (d) 14.0 *metre*

Solution : (b) Initial velocity $= (6\hat{i} + 8\hat{j})m/s$ (given)

Magnitude of velocity of projection $u = \sqrt{u_x^2 + u_y^2} = \sqrt{6^2 + 8^2} = 10\ m/s$

Angle of projection $\tan\theta = \frac{u_y}{u_x} = \frac{8}{6} = \frac{4}{3}$ $\therefore \sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$

Now horizontal range $R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2\, 2\sin\theta\cos\theta}{g} = \frac{(10)^2 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10} = 9.6\ meter$

Problem 42. A projectile thrown with an initial speed u and angle of projection 15° to the horizontal has a range R. If the same projectile is thrown at an angle of 45° to the horizontal with speed $2u$, its range will be

(a) $12R$ (b) $3R$ (c) $8R$ (d) $4R$

Solution : (c) $R = \dfrac{u^2 \sin 2\theta}{g}$ $\therefore R \propto u^2 \sin 2\theta$

$$\frac{R_2}{R_1} = \left(\frac{u_2}{u_1}\right)^2 \left(\frac{\sin 2\theta_2}{\sin 2\theta_1}\right) \Rightarrow R_2 = R_1\left(\frac{2u}{u}\right)^2\left(\frac{\sin 90^o}{\sin 30^o}\right) = 8R_1$$

Problem 43. The velocity at the maximum height of a projectile is half of its initial velocity of projection u. Its range on the horizontal plane is **[MP PET 1993]**

(a) $\sqrt{3}u^2/2g$ (b) $u^2/3g$ (c) $3u^2/2g$ (d) $3u^2/g$

Solution : (a) If the velocity of projection is u then at the highest point body posses only $u\cos\theta$

$u\cos\theta = \dfrac{u}{2}$ (given) $\therefore \theta = 60°$

Now $R = \dfrac{u^2 \sin(2\times 60°)}{g} = \dfrac{\sqrt{3}}{2}\dfrac{u^2}{g}$

Problem 44. A projectile is thrown from a point in a horizontal place such that its horizontal and vertical velocity component are 9.8 m/s and 19.6 m/s respectively. Its horizontal range is

(a) 4.9 m (b) 9.8 m (c) 19.6 m (d) 39.2 m

Solution : (d) We know $R = \dfrac{2u_x u_y}{g} = \dfrac{2\times 9.8\times 19.6}{9.8} = 39.2\,m$

Where u_x = horizontal component of initial velocity, u_y = vertical component of initial velocity.

Problem 45. A particle is projected with a velocity v such that its range on the horizontal plane is twice the greatest height attained by it. The range of the projectile is (where g is acceleration due to gravity)

(a) $\dfrac{4v^2}{5g}$ (b) $\dfrac{4g}{5v^2}$ (c) $\dfrac{v^2}{g}$ (d) $\dfrac{4v^2}{\sqrt{5g}}$

Solution : (a) We know $R = 4H\cot\theta$

$2H = 4H\cot\theta \Rightarrow \cot\theta = \dfrac{1}{2}$; $\sin\theta = \dfrac{2}{\sqrt{5}}$; $\cos\theta = \dfrac{1}{\sqrt{5}}$ [As $R = 2H$ given]

$$\text{Range} = \frac{u^2 .2.\sin\theta.\cos\theta}{g} = \frac{2u^2 \frac{2}{\sqrt{5}}.\frac{1}{\sqrt{5}}}{g} = \frac{4u^2}{5g}$$

Problem 46. The range R of projectile is same when its maximum heights are h_1 and h_2. What is the relation between R and h_1 and h_2 **[EAMCET (Med.) 2000]**

(a) $R = \sqrt{h_1 h_2}$ (b) $R = \sqrt{2h_1 h_2}$ (c) $R = 2\sqrt{h_1 h_2}$ (d) $R = 4\sqrt{h_1 h_2}$

Solution : (d) For equal ranges body should be projected with angle θ or $(90^o - \theta)$ from the horizontal.

And for these angles : $h_1 = \dfrac{u^2 \sin^2\theta}{2g}$ and $h_2 = \dfrac{u^2\cos^2\theta}{2g}$

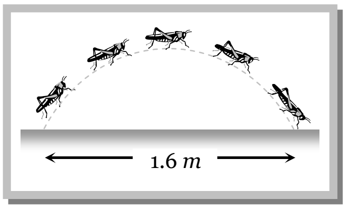
by multiplication of both height : $h_1 h_2 = \dfrac{u^2 \sin^2\theta\cos^2\theta}{4g^2} = \dfrac{1}{16}\left(\dfrac{u^2\sin 2\theta}{g}\right)^2$

$\Rightarrow 16h_1 h_2 = R^2 \Rightarrow R = 4\sqrt{h_1 h_2}$

Problem 47. A grasshopper can jump maximum distance 1.6 m. It spends negligible time on the ground. How far can it go in 10 *seconds*

(a) $5\sqrt{2}\,m$ (b) $10\sqrt{2}\,m$ (c) $20\sqrt{2}\,m$ (d) $40\sqrt{2}\,m$

Solution : (c) Horizontal distance travelled by grasshopper will be maximum for $\theta = 45°$



$R_{\max} = \frac{u^2}{g} = 1.6\,m \Rightarrow u = 4\,m/s.$

Horizontal component of velocity of grasshopper $u\cos\theta = 4\cos 45 = 2\sqrt{2}\,m/s$

Total distance covered by it in10 *sec.* $S = u\cos\theta \times t = 2\sqrt{2} \times 10 = 20\sqrt{2}\,m$

Problem 48. A projectile is thrown with an initial velocity of $v = a\hat{i} + b\hat{j}$, if the range of projectile is double the maximum height reached by it then

(a) $a = 2b$ (b) $b = a$ (c) $b = 2a$ (d) $b = 4a$

Solution : (c) Angle of projection $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{b}{a}$ $\therefore \tan\theta = \frac{b}{a}$ ...(i)

From formula $R = 4H\cot\theta = 2H \Rightarrow \cot\theta = \frac{1}{2}$ $\therefore \tan\theta = 2$ ...(ii) [As $R = 2H$ given]

From equation (i) and (ii) $b = 2a$

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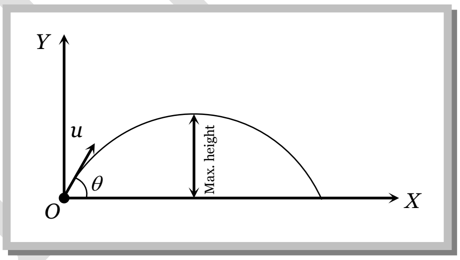
(9) **Maximum height :** It is the maximum height from the point of projection, a projectile can reach.

So, by using $v^2 = u^2 + 2as$



$$0 = (u\sin\theta)^2 - 2gH$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

(i) Maximum height can also be expressed as

$H = \frac{u_y^2}{2g}$ (where u_y is the vertical component of initial velocity).

(ii) $H_{\max} = \frac{u^2}{2g}$ (when $\sin^2\theta = \max = 1$ *i.e.*, $\theta = 90°$)

i.e., for maximum height body should be projected vertically upward. So it falls back to the point of projection after reaching the maximum height.

(iii) For complementary angles of projection θ and $90° - \theta$

Ratio of maximum height $= \frac{H_1}{H_2} = \frac{u^2\sin^2\theta/2g}{u^2\sin^2(90° - \theta)2g} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$

$\therefore \quad \frac{H_1}{H_2} = \tan^2\theta$

Sample problem based on maximum height

Problem 49. A cricketer can throw a ball to a maximum horizontal distance of 100 *m*. With the same effort, he throws the ball vertically upwards. The maximum height attained by the ball is **[UPSEAT 2002]**

(a) 100 *m* (b) 80 *m* (c) 60 *m* (d) 50 *m*

Solution : (d) $R_{\max} = \frac{u^2}{g} = 100\,m$ (when $\theta = 45°$)

$\therefore u^2 = 100 \times 10 = 1000$

$H_{max} = \dfrac{u^2}{2g} = \dfrac{1000}{2 \times 10} = 50\ metre$. (when $\theta = 90°$)

Problem 50. A ball thrown by one player reaches the other in 2 *sec*. the maximum height attained by the ball above the point of projection will be about **[Pb. PMT 2002]**

(a) 10 *m* (b) 7.5 *m* (c) 5 *m* (d) 2.5 *m*

Solution : (c) $T = \dfrac{2u\sin\theta}{g} = 2\ sec$ (given) $\therefore u\sin\theta = 10$

Now $H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{(10)^2}{2 \times 10} = 5\,m$.

Problem 51. Two stones are projected with the same magnitude of velocity, but making different angles with horizontal. The angle of projection of one is $\pi/3$ and its maximum height is *Y*, the maximum height attained by the other stone with as $\pi/6$ angle of projection is

(a) *Y* (b) 2 *Y* (c) 3 *Y* (d) $\dfrac{Y}{3}$

Solution : (d) When two stones are projected with same velocity then for complementary angles θ and $(90° - \theta)$

Ratio of maximum heights : $\dfrac{H_1}{H_2} = \tan^2\theta = \tan^2\dfrac{\pi}{3} = 3 \Rightarrow H_2 = \dfrac{H_1}{3} = \dfrac{Y}{3}$

Problem 52. If the initial velocity of a projectile be doubled. Keeping the angle of projection same, the maximum height reached by it will

(a) Remain the same (b) Be doubled (c) Be quadrupled (d) Be halved

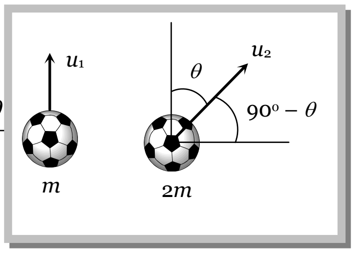
Solution : (c) $H = \dfrac{u^2\sin 2\theta}{2g}$ $\therefore H \propto u^2$ [As θ = constant]

If initial velocity of a projectile be doubled then *H* will becomes 4 times.

Problem 53. Pankaj and Sudhir are playing with two different balls of masses *m* and 2*m* respectively. If Pankaj throws his ball vertically *up* and Sudhir at an angle *θ*, both of them stay in our view for the same period. The height attained by the two balls are in the ratio

(a) 2 : 1 (b) 1 : 1 (c) 1 : cos*θ* (d) 1 : sec*θ*

Solution : (b) Time of flight for the ball thrown by Pankaj $T_1 = \dfrac{2u_1}{g}$



Time of flight for the ball thrown by Sudhir $T_2 = \dfrac{2u_2\sin(90° - \theta)}{g} = \dfrac{2u_2\cos\theta}{g}$

According to problem $T_1 = T_2 \Rightarrow \dfrac{2u_1}{g} = \dfrac{2u_2\cos\theta}{g} \Rightarrow u_1 = u_2\cos\theta$

Height of the ball thrown by Pankaj $H_1 = \dfrac{u_1^2}{2g}$

Height of the ball thrown by Sudhir $H_2 = \dfrac{u_2^2\sin^2(90° - \theta)}{2g} = \dfrac{u_2^2\cos^2\theta}{2g}$

$\therefore \dfrac{H_1}{H_2} = \dfrac{u_1^2/2g}{u_2^2\cos^2\theta/2g} = 1$ [As $u_1 = u_2\cos\theta$]

Short Trick :
Maximum height $H \propto T^2$

$\dfrac{H_1}{H_2} = \left(\dfrac{T_1}{T_2}\right)^2$

$\therefore \dfrac{H_1}{H_2} = 1$ (As $T_1 = T_2$)

Problem 54. A boy aims a gun at a bird from a point, at a horizontal distance of 100 *m*. If the gun can impart a velocity of 500 ms^{-1} to the bullet. At what height above the bird must he aim his gun in order to hit it (take g = 10 ms^{-2})

[CPMT 1996]

(a) 20 *cm* (b) 10 *cm* (c) 50 *cm* (d) 100 *cm*

Solution : (a) Time taken by bullet to travel a horizontal distance of 100 *m* is given by $t = \frac{100}{500} = \frac{1}{5}\,sec$

In this time the bullet also moves downward due to gravity its vertical displacement

$h = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times\left(\frac{1}{5}\right)^2 = 1/5\,m = 20\ cm$

So bullet should be fired aiming 20 *cm* above the bird to hit it.

Problem **55.** The maximum horizontal range of a projectile is 400 *m*. The maximum height attained by it will be

(a) 100 *m* (b) 200 *m* (c) 400 *m* (d) 800 *m*

Solution : (a) $R_{max} = 400\,m$ [when $\theta = 45°$]

So from the Relation $R = 4H\cot\theta \Rightarrow 400 = 4H\cot 45° \Rightarrow H = 100\,m.$

Problem **56.** Two bodies are projected with the same velocity. If one is projected at an angle of 30° and the other at an angle of 60° to the horizontal, the ratio of the maximum heights reached is

[EAMCET (Med.) 1995; Pb. PMT 2000; AIIMS 2001]

(a) 3 : 1 (b) 1 : 3 (c) 1 : 2 (d) 2 : 1

Solution : (b) $\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2} = \frac{\sin^2 30^o}{\sin^2 60^o} = \frac{1}{3}$

Problem **57.** If time of flight of a projectile is 10 seconds. Range is 500 *m*. The maximum height attained by it will be

[RPMT 1997; RPET 1998]

(a) 125 *m* (b) 50 *m* (c) 100 *m* (d) 150 *m*

Solution : (a) $T = \frac{2u\sin\theta}{g} = 10\text{ sec} \Rightarrow u\sin\theta = 50$ so $H = \frac{u^2\sin^2\theta}{2g} = \frac{(50)^2}{2\times 10} = 125\,m.$

Problem **58.** A man can throw a stone 80 *m*. The maximum height to which he can raise the stone is

(a) 10 *m* (b) 15 *m* (c) 30 *m* (d) 40 *m*

Solution : (d) The problem is different from problem no. (54). In that problem for a given angle of projection range was given and we had find maximum height for that angle.

But in this problem angle of projection can vary, $R_{max} = \frac{u^2}{g} = 80\,m$ [for $\theta = 45°$]

But height can be maximum when body projected vertically *up* $H_{max} = \frac{u^2\sin^2 90^o}{2g} = \frac{u^2}{2g} = \frac{1}{2}\left(\frac{u^2}{g}\right) = 40$ *m*

Problem **59.** A ball is thrown at different angles with the same speed *u* and from the same points and it has same range in both the cases. If y_1 and y_2 be the heights attained in the two cases, then $y_1 + y_2 =$

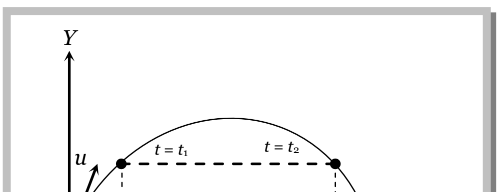
(a) $\frac{u^2}{g}$ (b) $\frac{2u^2}{g}$ (c) $\frac{u^2}{2g}$ (d) $\frac{u^2}{4g}$

Solution : (c) Same ranges can be obtained for complementary angles *i.e.* θ and 90° – θ

$y_1 = \frac{u^2\sin^2\theta}{2g}$ and $y_2 = \frac{u^2\cos^2\theta}{2g}$ $\quad\therefore y_1 + y_2 = \frac{u^2\sin^2\theta}{2g} + \frac{u^2\cos^2\theta}{2g} = \frac{u^2}{2g}$

(10) **Projectile passing through two different points on same height at time t_1 and t_2 :** If the particle passes two points situated at equal height *y* at $t = t_1$ and $t = t_2$, then

(i) **Height (*y*):** $y = (u\sin\theta)t_1 - \frac{1}{2}gt_1^2$ .....(i)



and $y = (u\sin\theta)t_2 - \frac{1}{2}gt_2^2$ .....(ii)

Comparing equation (i) with equation (ii)

$$u\sin\theta = \frac{g(t_1 + t_2)}{2}$$

Substituting this value in equation (i)

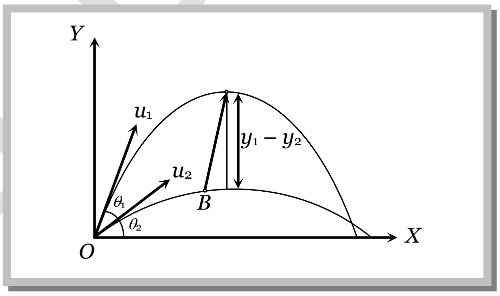
$$y = g\left(\frac{t_1 + t_2}{2}\right)t_1 - \frac{1}{2}gt_1^2 \qquad \Rightarrow y = \frac{gt_1t_2}{2}$$

(ii) **Time (t_1 and t_2):** $y = u\sin\theta\, t - \frac{1}{2}gt^2$

$$t^2 - \frac{2u\sin\theta}{g}t + \frac{2y}{g} = 0 \qquad \Rightarrow t = \frac{u\sin\theta}{g}\left[1 \pm \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

$$t_1 = \frac{u\sin\theta}{g}\left[1 + \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right] \text{ and } t_2 = \frac{u\sin\theta}{g}\left[1 - \sqrt{1 - \left(\frac{\sqrt{2gy}}{u\sin\theta}\right)^2}\right]$$

(11) **Motion of a projectile as observed from another projectile :** Suppose two balls A and B are projected simultaneously from the origin, with initial velocities u_1 and u_2 at angle θ_1 and θ_2, respectively with the horizontal.



The instantaneous positions of the two balls are given by

Ball A : $x_1 = (u_1\cos\theta_1)t$ $\qquad y_1 = (u_1\sin\theta_1)t - \frac{1}{2}gt^2$

Ball B : $x_2 = (u_2\cos\theta_2)t$ $\qquad y_2 = (u_2\sin\theta_2)t - \frac{1}{2}gt^2$

The position of the ball A with respect to ball B is given by

$$x = x_1 - x_2 = (u_1\cos\theta_1 - u_2\cos\theta_2)t$$

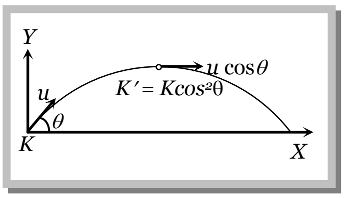
$$y = y_1 - y_2 = (u_1\sin\theta_1 - u_2\sin\theta_2)t$$

Now $\frac{y}{x} = \left(\frac{u_1\sin\theta_1 - u_2\sin\theta_2}{u_1\cos\theta_1 - u_2\cos\theta_2}\right) = \text{constant}$

Thus motion of a projectile relative to another projectile is a straight line.

(12) **Energy of projectile :** When a projectile moves upward its kinetic energy decreases, potential energy increases but the total energy always remain constant.

If a body is projected with initial kinetic energy $K(=1/2\ mu^2)$, with angle of projection θ with the horizontal then at the highest point of trajectory



(i) **Kinetic energy** $= \frac{1}{2}m(u\cos\theta)^2 = \frac{1}{2}mu^2\cos^2\theta$

$\therefore \qquad K' = K\cos^2\theta$

(ii) **Potential energy** $= mgH = mg\frac{u^2\sin^2\theta}{2g} = \frac{1}{2}mu^2\sin^2\theta \qquad \left(\text{As } H = \frac{u^2\sin^2\theta}{2g}\right)$

(iii) **Total energy** = Kinetic energy + Potential energy $=\frac{1}{2}mu^2\cos^2\theta+\frac{1}{2}mu^2\sin^2\theta$

$$=\frac{1}{2}mu^2 = \text{Energy at the point of projection.}$$

This is in accordance with the law of conservation of energy.

Sample problems based on energy

***Problem* 60.** A projectile is projected with a kinetic energy K. Its range is R. It will have the minimum kinetic energy, after covering a horizontal distance equal to **[UPSEAT 2002]**

(a) 0.25 R (b) 0.5 R (c) 0.75 R (d) R

Solution : (b) Projectile possess minimum kinetic energy at the highest point of the trajectory *i.e.* at a horizontal distance $R/2$.

***Problem* 61.** A projectile is fired at 30° with momentum p. Neglecting friction, the change in kinetic energy when it returns to the ground will be

(a) Zero (b) 30% (c) 60% (d) 100%

Solution : (a) According to law of conservation of energy, projectile acquire same kinetic energy when it comes at same level.

***Problem* 62.** A particle is projected making angle 45° with horizontal having kinetic energy K. The kinetic energy at highest point will be **[CBSE PMT 2000, 01; AIEEE 2002]**

(a) $\frac{K}{\sqrt{2}}$ (b) $\frac{K}{2}$ (c) $2K$ (d) K

Solution : (b) Kinetic energy at the highest point $K' = K\cos^2\theta = K\cos^2 45^\circ = K/2$

***Problem* 63.** Two balls of same mass are projected one vertically upwards and the other at angle 60° with the vertical. The ratio of their potential energy at the highest point is

(a) 3 : 2 (b) 2 : 1 (c) 4 : 1 (d) 4 : 3

Solution : (c) Potential energy at the highest point is given by $PE=\frac{1}{2}mu^2\sin^2\theta$

For first ball $\theta=90^\circ$ $\therefore$ $(PE)_1=\frac{1}{2}mu^2$

For second ball $\theta=(90^\circ-60^\circ)=30^\circ$ from the horizontal $\therefore$ $(PE)_2=\frac{1}{2}mu^2\sin^2 30^\circ=\frac{1}{8}mu^2$

$\therefore$ $\frac{(PE)_I}{(PE)_{II}}=4:1$

***Problem* 64.** In the above problem, the kinetic energy at the highest point for the second ball is K. What is the kinetic energy for the first ball

(a) 4 K (b) 3 K (c) 2 K (d) Zero

Solution : (d) KE at the highest point $KE=\frac{1}{2}mu^2\cos^2\theta$

For first ball $\theta = 90^\circ$ $\therefore$ $KE = 0$

***Problem* 65.** A ball is thrown at an angle θ with the horizontal. Its initial kinetic energy is 100 J and it becomes 30 J at the highest point. The angle of projection is

(a) 45° (b) 30° (c) $\cos^{-1}(3/10)$ (d) $\cos^{-1}(\sqrt{3/10})$

Solution : (d) KE at highest point $K'=K\cos^2\theta$

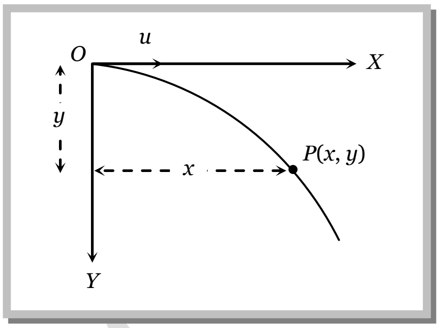
$$30=100\cos^2\theta \Rightarrow \cos^2\theta=\frac{3}{10} \Rightarrow \theta=\cos^{-1}\left(\sqrt{\frac{3}{10}}\right)$$

3.7 Horizontal Projectile

A body be projected horizontally from a certain height 'y' vertically above the ground with initial velocity u. If friction is considered to be absent, then there is no other horizontal force which can affect the horizontal motion. The horizontal velocity therefore remains constant and so the object covers equal distance in horizontal direction in equal intervals of time.

(1) **Trajectory of horizontal projectile :** The horizontal displacement x is governed by the equation

$$x = ut \Rightarrow t = \frac{x}{u} \quad \text{.... (i)}$$



The vertical displacement y is governed by $y = \frac{1}{2}gt^2$ .... (ii)

(since initial vertical velocity is zero)

By substituting the value of t in equation (ii) $y = \frac{1}{2}\frac{gx^2}{u^2}$

Sample problems based on trajectory

***Problem* 66.** An aeroplane is flying at a constant horizontal velocity of 600 km/hr at an elevation of 6 km towards a point directly above the target on the earth's surface. At an appropriate time, the pilot releases a ball so that it strikes the target at the earth. The ball will appear to be falling

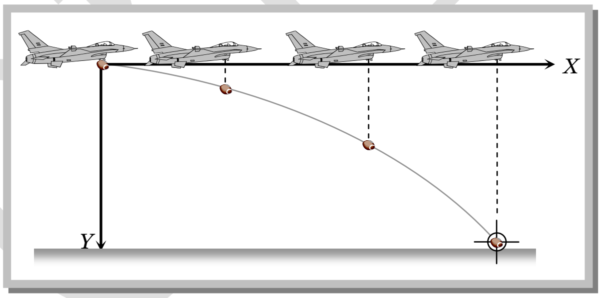
(a) On a parabolic path as seen by pilot in the plane

(b) Vertically along a straight path as seen by an observer on the ground near the target

(c) On a parabolic path as seen by an observer on the ground near the target

(d) On a zig-zag path as seen by pilot in the plane

Solution : (c)



The path of the ball appears parabolic to a observer near the target because it is at rest. But to a Pilot the path appears straight line because the horizontal velocity of aeroplane and the ball are equal, so the relative horizontal displacement is zero.

***Problem* 67.** The barrel of a gun and the target are at the same height. As soon as the gun is fired, the target is also released. In which of the following cases, the bullet will not strike the target

(a) Range of projectile is less than the initial distance between the gun and the target

(b) Range of projectile is more than the initial distance between the gun and the target

(c) Range of projectile is equal to the initial distance between the gun and target

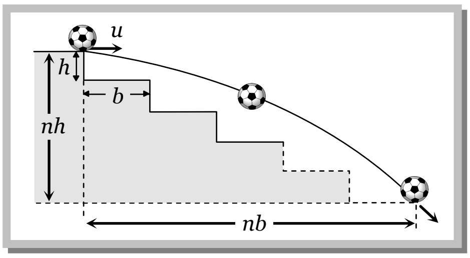
(d) Bullet will always strike the target

Solution : (a) Condition for hitting of bullet with target initial distance between the gun and target $\leq$ Range of projectile.

***Problem* 68.** A ball rolls off top of a staircase with a horizontal velocity u m/s. If the steps are h *metre* high and b *mere* wide, the ball will just hit the edge of nth step if n equals to

(a) $\frac{hu^2}{gb^2}$ (b) $\frac{u^2 8}{gb^2}$ (c) $\frac{2hu^2}{gb^2}$ (d) $\frac{2u^2 g}{hb^2}$

Solution : (c) By using equation of trajectory $y = \frac{gx^2}{2u^2}$ for given condition



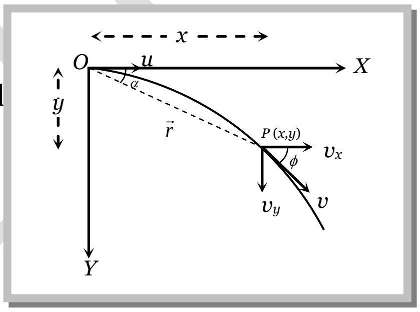
$$nh = \frac{g(nb)^2}{2u^2} \quad \therefore n = \frac{2hu^2}{gb^2}$$

(2) **Displacement of Projectile ($\vec{r}$)** : After time *t*, horizontal displacement $x = ut$ and vertical displacement $y = \frac{1}{2}gt^2$.

So, the position vector $\vec{r} = ut\hat{i} - \frac{1}{2}gt^2\hat{j}$

Therefore $\quad r = ut\sqrt{1+\left(\frac{gt}{2u}\right)^2}$ and $\alpha = \tan^{-1}\left(\frac{gt}{2u}\right)$

$$\alpha = \tan^{-1}\left(\sqrt{\frac{gy}{2}}\Big/u\right) \qquad \left(\text{as } t = \sqrt{\frac{2y}{g}}\right)$$



(3) **Instantaneous velocity :** Throughout the motion, the horizontal component of the velocity is $v_x = u$.

The vertical component of velocity increases with time and is given by

$$v_y = 0 + g\,t = g\,t \qquad (\text{From } v = u + g\,t)$$

So, $\quad \vec{v} = v_x\hat{i} - v_y\hat{j} = \vec{v} = u\hat{i} - g\,t\,\hat{j}$

i.e. $\quad v = \sqrt{u^2 + (gt)^2} = u\sqrt{1+\left(\frac{gt}{u}\right)^2}$

Again $\quad \vec{v} = u\hat{i} - \sqrt{2gy}\,\hat{j}$

i.e. $\quad v = \sqrt{u^2 + 2gy}$

Direction of instantaneous velocity : $\tan\phi = \frac{v_y}{v_x} \Rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{\sqrt{2gy}}{u}\right)$ or $\phi = \tan^{-1}\left(\frac{gt}{u}\right)$

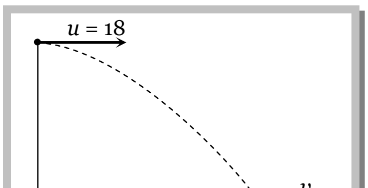
Where ϕ is the angle of instantaneous velocity from the horizontal.

Sample problems based on velocity

***Problem* 69.** A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at angle 45°. What is the vertical component of velocity when it strikes the ground

(a) 9 ms^{-1} (b) $9\sqrt{2}$ ms^{-1} (c) 18 ms^{-1} (d) $18\sqrt{2}$ ms^{-1}

Solution : (c) When the body strikes the ground



$$\tan 45^o = \frac{v_y}{v_x} = \frac{v_y}{18} = 1$$

$v_y = 18\, m/s.$

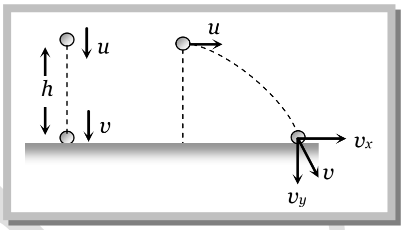
***Problem* 70.** A man standing on the roof of a house of height h throws one particle vertically downwards and another particle horizontally with the same velocity u. The ratio of their velocities when they reach the earth's surface will be

(a) $\sqrt{2gh+u^2} : u$ (b) 1 : 2 (c) 1 : 1 (d) $\sqrt{2gh+u^2} : \sqrt{2gh}$

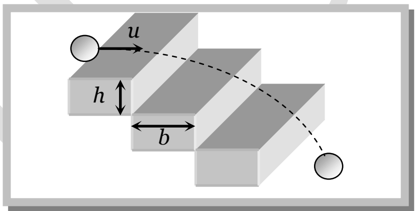
Solution : (c) For first particle : $v^2 = u^2 + 2gh \Rightarrow v = \sqrt{u^2 + 2gh}$

For second particle : $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + (\sqrt{2gh})^2} = \sqrt{u^2 + 2gh}$

So the ratio of velocities will be 1 : 1.



***Problem* 71.** A staircase contains three steps each 10 *cm* high and 20 *cm* wide. What should be the minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit directly the lowest plane



(a) 0.5 *m/s* (b) 1 *m/s* (c) 2 *m/s* (d) 4 *m/s*

Solution : (c) Formula for this condition is given by $n = \frac{2hu^2}{gb^2}$ where h = height of each step, b = width of step, u = horizontal velocity of projection, n = number of step.

$\Rightarrow 3 = \frac{2 \times 10 \times u^2}{10 \times 20^2} \Rightarrow u^2 = 200\, cm/sec = 2\, m/sec$

(4) **Time of flight :** If a body is projected horizontally from a height h with velocity u and time taken by the body to reach the ground is T, then

$$h = 0 + \frac{1}{2} gT^2 \text{ (for vertical motion)}$$

$$T = \sqrt{\frac{2h}{g}}$$

Sample problems based on time of flight

***Problem* 72.** Two bullets are fired simultaneously, horizontally and with different speeds from the same place. Which bullet will hit the ground first

(a) The faster one (b) Depends on their mass

(c) The slower one (d) Both will reach simultaneously

Solution : (d)

***Problem* 73.** An aeroplane is flying at a height of 1960 m in horizontal direction with a velocity of 360 km/hr. When it is vertically above the point. A on the ground, it drops a bomb. The bomb strikes a point B on the ground, then the time taken by the bomb to reach the ground is

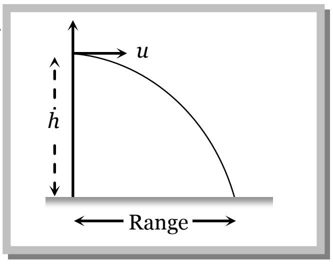
(a) $20\sqrt{2}$ *sec* (b) 20 *sec* (c) $10\sqrt{2}$ *sec* (d) 10 *sec*

Solution : (b) $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20\ sec$

(5) **Horizontal range :** Let R is the horizontal distance travelled by the body

$$R = uT + \frac{1}{2}\, 0\, T^2 \text{ (for horizontal motion)}$$

$$R = u\sqrt{\frac{2h}{g}}$$



Sample problems based on horizontal range

***Problem* 74.** A bomb is dropped on an enemy post by an aeroplane flying with a horizontal velocity of 60 *km/hr* and at a height of 490 *m*. How far the aeroplane must be from the enemy post at time of dropping the bomb, so that it may directly hit the target. (g = 9.8 m/s^2)

(a) $\frac{100}{3}m$ (b) $\frac{500}{3}m$ (c) $\frac{200}{3}m$ (d) $\frac{400}{3}m$

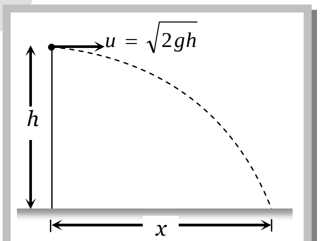
Solution : (b) $S = u \times t = u \times \sqrt{\frac{2h}{g}} = 60 \times \frac{5}{18} \times \sqrt{\frac{2 \times 490}{9.8}} = \frac{500}{3}m$

***Problem* 75.** A body is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of tower at a distance x from the tower. The value of x is

(a) h (b) $\frac{h}{2}$ (c) $2h$ (d) $\frac{2h}{3}$

Solution : (c) $x = u \times \sqrt{\frac{2h}{g}} = \sqrt{2gh} \times \sqrt{\frac{2h}{g}}$

$\therefore\ x = 2h$



***Problem* 76.** An aeroplane moving horizontally with a speed of 720 *km/h* drops a fo … t a height of 396.9 *m*. The time taken by a food packet to reach the ground and its horizontal range is (Take g = 9.8 m/sec^2)

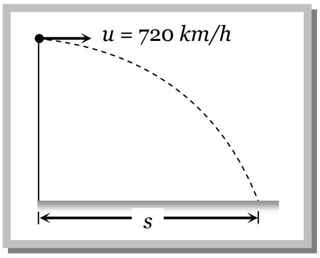
(a) 3 *sec* and 2000 *m* (b) 5 *sec* and 500 *m* (c) 8 *sec* and 1500 *m* (d) 9 *sec* and 1800 *m*

Solution : (d) Time of descent $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 396.9}{9.8}}$

$\Rightarrow t = 9\ sec$

and horizontal distance $S = u \times t$

$S = \left(\frac{720 \times 5}{18}\right) \times 9 = 1800\ m$

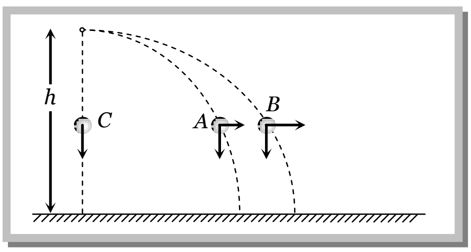


(6) If projectiles A and B are projected horizontally with different initial velocity from same height and third particle C is dropped from same point then

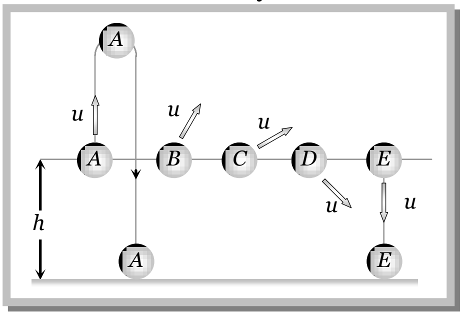
(i) All three particles will take equal time to reach the ground.

(ii) Their net velocity would be different but all three particle possess same vertical component of velocity.

(iii) The trajectory of projectiles A and B will be straight line *w.r.t.* particle C.



(7) If various particles thrown with same initial velocity but indifferent direction then



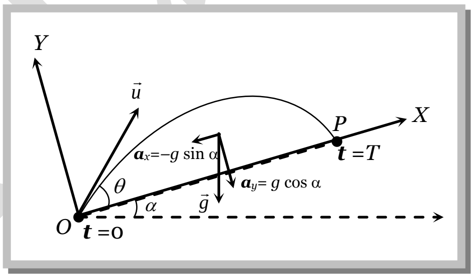
(i) They strike the ground with same speed at different times irrespective of their initial direction of velocities.

(ii) Time would be least for particle *E* which was thrown vertically downward.

(iii) Time would be maximum for particle *A* which was thrown vertically upward.

3.8 Projectile Motion on an Inclined Plane

Let a particle be projected up with a speed *u* from an inclined plane which makes an angle α with the horizontal velocity of projection makes an angle θ with the inclined plane.



We have taken reference *x*-axis in the direction of plane.

Hence the component of initial velocity parallel and perpendicular to the plane are equal to $u\cos\theta$ and $u\sin\theta$ respectively *i.e.*

$u_{||} = u\cos\theta$ and $u_{\perp} = u\sin\theta$.

The component of *g* along the plane is $g\sin\alpha$ and perpendicular to the plane is $g\cos\alpha$ as shown in the figure *i.e.* $a_{||} = -g\sin\alpha$ and $a_{\perp} = g\cos\alpha$.

Therefore the particle decelerates at a rate of $g\sin\alpha$ as it moves from *O* to *P*.

(1) **Time of flight :** We know for oblique projectile motion $T = \dfrac{2u\sin\theta}{g}$

or we can say $T = \dfrac{2u_{\perp}}{a_{\perp}}$

$\therefore$ Time of flight on an inclined plane $T = \dfrac{2u\sin\theta}{g\cos\alpha}$

(2) **Maximum height :** We know for oblique projectile motion $H = \dfrac{u^2\sin^2\theta}{2g}$

or we can say $H = \dfrac{u_{\perp}^2}{2a_{\perp}}$

$\therefore$ Maximum height on an inclined plane $H = \dfrac{u^2\sin^2\theta}{2g\cos\alpha}$

(3) **Horizontal range :** For one dimensional motion $s = ut + \dfrac{1}{2}at^2$

Horizontal range on an inclined plane $R = u_{\parallel} T + \frac{1}{2} a_{\parallel} T^2$

$$R = u\cos\theta\, T - \frac{1}{2} g\sin\alpha\; T^2$$

$$R = u\cos\theta\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2} g\sin\alpha\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

By solving $R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$

(i) Maximum range occurs when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$

(ii) The maximum range along the inclined plane when the projectile is thrown upwards is given by

$$R_{\max} = \frac{u^2}{g(1+\sin\alpha)}$$

(iii) The maximum range along the inclined plane when the projectile is thrown downwards is given by

$$R_{\max} = \frac{u^2}{g(1-\sin\alpha)}$$

Sample problem based on inclined projectile

***Problem* 77.** For a given velocity of projection from a point on the inclined plane, the maximum range down the plane is three times the maximum range up the incline. Then, the angle of inclination of the inclined plane is

(a) 30^o (b) 45^o (c) 60^o (d) 90^o

Solution : (a) Maximum range up the inclined plane $(R_{\max})_{up} = \frac{u^2}{g(1+\sin\alpha)}$

Maximum range down the inclined plane $(R_{\max})_{down} = \frac{u^2}{g(1-\sin\alpha)}$

and according to problem : $\frac{u^2}{g(1-\sin\alpha)} = 3\times\frac{u^2}{g(1+\sin\alpha)}$

By solving $\alpha = 30^o$

***Problem* 78.** A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30^o$, and the angle of the barrel to the horizontal $\beta = 60^o$. The initial velocity v of the shell is 21 *m/sec*. Then distance of point from the gun at which shell will fall

(a) 10 *m* (b) 20 *m* (c) 30 *m* (d) 40 *m*

Solution : (c) Here $u = 21\ m/sec$, $\alpha = 30^o$, $\theta = \beta - \alpha = 60^o - 30^o = 30^o$

Maximum range $R = \frac{2u^2}{g}\frac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha} = \frac{2\times(21)^2\times\sin 30^o\cos 60^o}{9.8\times\cos^2 30^o} = 30\,m$

***Problem* 79.** The maximum range of rifle bullet on the horizontal ground is 6 *km* its maximum range on an inclined of 30^o will be

(a) 1 *km* (b) 2 *km* (c) 4 *km* (d) 6 *km*

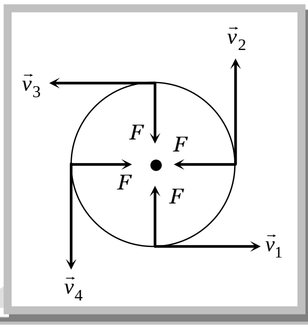
Solution : (c) Maximum range on horizontal plane $R = \frac{u^2}{g} = 6km$ (given)

Maximum range on a inclined plane $R_{\max} = \frac{u^2}{g(1+\sin\alpha)}$

Putting $\alpha = 30^o$ $R_{\max} = \frac{u^2}{g(1+\sin 30^o)} = \frac{2}{3}\left(\frac{u^2}{g}\right) = \frac{2}{3}\times 6 = 4\,km$.

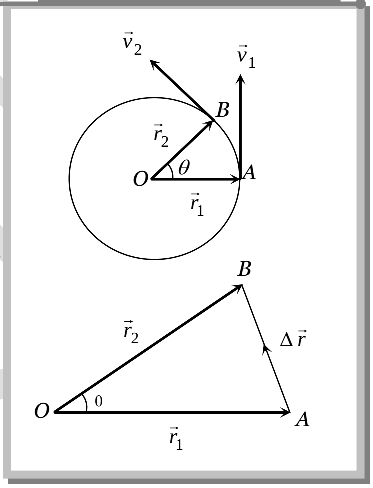
CIRCULAR MOTION

Circular motion is another example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.



Since this force is always at right angles to the displacement due to the initial velocity therefore no work is done by the force on the particle. Hence, its kinetic energy and thus speed is unaffected. But due to simultaneous action of the force and the velocity the particle follows resultant path, which in this case is a circle. Circular motion can be classified into two types – Uniform circular motion and non-uniform circular motion.

3.9 Variables of Circular Motion



(1) **Displacement and distance :** When particle moves in a circular path describing an angle θ during time t (as shown in the figure) from the position A to the position B, we see that the magnitude of the position vector $\vec{r}$ (that is equal to the radius of the circle) remains constant. *i.e.*, $|\vec{r}_1| = |\vec{r}_2| = r$ and the direction of the position vector changes from time to time.

(i) Displacement **:** The change of position vector or the displacement $\Delta\vec{r}$ of the particle from position A to the position B is given by referring the figure.

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$\Rightarrow \Delta r = |\Delta\vec{r}| = |\vec{r}_2 - \vec{r}_1| \quad \Delta r = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\theta}$

Putting $r_1 = r_2 = r$ we obtain

$$\Delta r = \sqrt{r^2 + r^2 - 2r.r\cos\theta}$$

$$\Rightarrow \Delta r = \sqrt{2r^2(1-\cos\theta)} = \sqrt{2r^2\left(2\sin^2\frac{\theta}{2}\right)}$$

$$\Delta r = 2r\sin\frac{\theta}{2}$$

(ii) Distance **:** The distanced covered by the particle during the time t is given as

d = length of the arc $AB = r\,\theta$

(iii) Ratio of distance and displacement **:** $\dfrac{d}{\Delta r} = \dfrac{r\theta}{2r\sin\theta/2} = \dfrac{\theta}{2}\operatorname{cosec}(\theta/2)$

Sample problems based on distance and displacement

Problem 80. A particle is rotating in a circle of radius r. The distance traversed by it in completing half circle would be

(a) r (b) πr (c) $2\pi r$ (d) Zero

Solution : (b) Distance travelled by particle = Semi-circumference = πr.

Problem 81. An athlete completes one round of a circular track of radius 10 *m* in 40 *sec*. The distance covered by him in 2 min 20 sec is **[Kerala PMT 2002]**

(a) 70 *m* (b) 140 *m* (c) 110 *m* (d) 220 *m*

Solution : (d) No. of revolution (n) = $\dfrac{\text{Total time of motion}}{\text{Time period}} = \dfrac{140\text{ sec}}{40\text{ sec}} = 3.5$

Distance covered by an athlete in revolution = $n(2\pi r) = 3.5(2\pi r) = 3.5 \times 2 \times \frac{22}{7} \times 10$ = 220 *m*.

Problem 82. A wheel covers a distance of 9.5 *km* in 2000 revolutions. The diameter of the wheel is **[RPMT 1999; BHU 2000]**

(a) 15 *m* (b) 7.5 *m* (c) 1.5 *m* (d) 7.5 *m*

Solution : (c) Distance $= n(2\pi r) \Rightarrow 9.5 \times 10^3 = 2000 \times (\pi D) \Rightarrow D = \dfrac{9.5 \times 10^3}{2000 \times \pi} = 1.5\, m.$

(2) **Angular displacement (θ) :** The angle turned by a body moving on a circle from some reference line is called angular displacement.

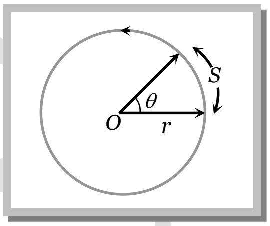
(i) Dimension = $[M^0L^0T^0]$ (as $\theta = arc / \text{radius}$) .

(ii) Units = Radian or Degree. It is some times also specified in terms of fraction or multiple of revolution.

(iii) 2π rad $= 360^o = 1$ Revolution

(iv) Angular displacement is a axial vector quantity.

Its direction depends upon the sense of rotation of the object and can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.



(v) Relation between linear displacement and angular displacement $\vec{s} = \vec{\theta} \times \vec{r}$

or $s = r\theta$

Sample problem based on angular displacement

***Problem* 83.** A flywheel rotates at constant speed of 3000 *rpm*. The angle described by the shaft in radian in one second is

(a) 2 π (b) 30 π (c) 100 π (d) 3000 π

Solution : (c) Angular speed $= 3000\, rpm = 50\, rps = 50 \times 2\pi\, rad/\sec = 100\pi\, rad/\sec$

i.e. angle described by the shaft in one second is $100\,\pi\, rad.$

***Problem* 84.** A particle completes 1.5 revolutions in a circular path of radius 2 *cm*. The angular displacement of the particle will be – (in radian)

(a) 6π (b) 3π (c) 2π (d) π

Solution : (b) 1 revolution mean the angular displacement of $2\pi\, rad$

$\therefore 1.5$ revolution means $1.5 \times 2\pi = 3\pi\, rad$.

(3) **Angular velocity (ω) :** Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

(i) Angular velocity $\omega = \dfrac{\text{angle traced}}{\text{time taken}} = \underset{\Delta t \to 0}{Lt} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

$$\therefore \qquad \omega = \frac{d\theta}{dt}$$

(ii) Dimension : $[M^0L^0T^{-1}]$

(iii) Units : Radians per second ($rad.s^{-1}$) or Degree per second.

(iv) Angular velocity is an axial vector.

(v) Relation between angular velocity and linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$

Its direction is the same as that of $\Delta\theta$. For anticlockwise rotation of the point object on the circular path, the direction of ω, according to Right hand rule is along the axis of circular path directed upwards. For clockwise rotation of the point object on the circular path, the direction of ω is along the axis of circular path directed downwards.

Note : ❑ It is important to note that nothing actually moves in the direction of the angular velocity vector $\vec{\omega}$. The direction of $\vec{\omega}$ simply represents that the rotational motion is taking place in a plane perpendicular to it.

(vi) For uniform circular motion ω remains constant where as for non-uniform motion ω varies with respect to time.

Sample problems based on angular velocity

Problem 85. A scooter is going round a circular road of radius 100 *m* at a speed of 10 *m/s*. The angular speed of the scooter will be

(a) 0.01 *rad/s* (b) 0.1 *rad/s* (c) 1 *rad/s* (d) 10 *rad/s*

Solution : (b) $\omega = \frac{v}{r} = \frac{10}{100} = 0.1\, rad/sec$

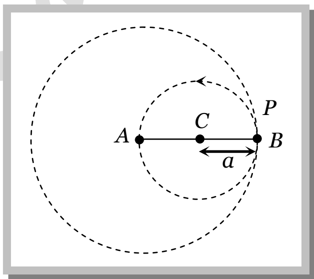
Problem 86. The ratio of angular velocity of rotation of minute hand of a clock with the angular velocity of rotation of the earth about its own axis is

(a) 12 (b) 6 (c) 24 (d) None of these

Solution : (c) $\omega_{\text{Minute hand}} = \frac{2\pi}{60}\frac{rad}{\min}$ and $\omega_{Earth} = \frac{2\pi}{24}\frac{Rad}{hr} = \frac{2\pi}{24\times 60}\frac{rad}{\min}$ $\therefore \frac{\omega_{\text{Minute hand}}}{\omega_{Earth}} = 24:1$

Problem 87. A particle *P* is moving in a circle of radius '*a*' with a uniform speed *v*. *C* is the centre of the circle and *AB* is a diameter. When passing through *B* the angular velocity of *P* about *A* and *C* are in the ratio **[NCERT 1982]**

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 4 : 1



Solution : (b) Angular velocity of *P* about *A*

$\omega_A = \frac{v}{2a}$ Angular velocity of *P* about *C* $\omega_C = \frac{v}{a}$ $\therefore \frac{\omega_A}{\omega_C} = 1:2$

Problem 88. The length of the seconds hand of a watch is 10 *mm*. What is the change in the angular speed of the watch after 15 *seconds*

(a) Zero (b) $(10\pi/2)\, mms^{-1}$ (c) $(20/\pi)\, mms^{-1}$ (d) $10\sqrt{2}\; mms^{-1}$

Solution : (a) Angular speed of seconds hand of watch is constant and equal to $\frac{2\pi}{60}\frac{rad}{sec} = \frac{\pi}{30} rad/sec$. So change in angular speed will be zero.

(4) **Change in velocity :** We want to know the magnitude and direction of the change in velocity of the particle which is performing uniform circular motion as it moves from *A* to *B* during time *t* as shown in figure. The change in velocity vector is given as

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

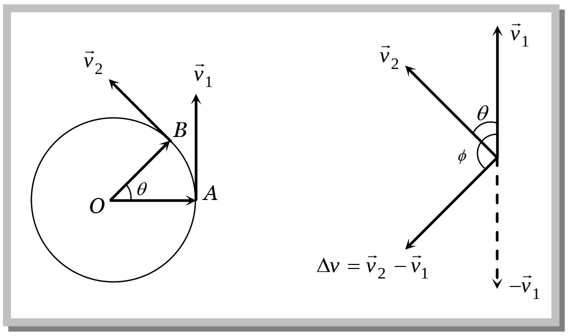
or $|\Delta\vec{v}| = |\vec{v}_2 - \vec{v}_1| \Rightarrow \Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$

For uniform circular motion $v_1 = v_2 = v$

So $\Delta v = \sqrt{2v^2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}$

The direction of $\Delta\vec{v}$ is shown in figure that can be given as

$$\phi = \frac{180^o - \theta}{2} = (90^o - \theta/2)$$



Note : ❑ Relation between linear velocity and angular velocity.

In vector form $\vec{v} = \vec{\omega}\times\vec{r}$

Sample problems based on velocity

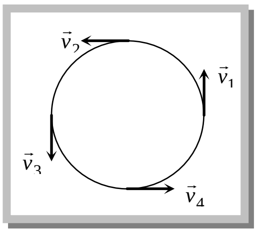
Problem **89.** If a particle moves in a circle describing equal angles in equal times, its velocity vector **[CPMT 1972, 74; JIPMER 1997]**

(a) Remains constant (b) Changes in magnitude

(c) Changes in direction (d) Changes both in magnitude and direction

Solution : (c) In uniform circular motion velocity vector changes in direction but its magnitude always remains constant.

$$|\vec{v_1}| = |\vec{v_2}| = |\vec{v_3}| = |\vec{v_4}| = \text{constant}$$



Problem **90.** A body is whirled in a horizontal circle of radius 20 *cm*. It has angular velocity of 10 *rad/s*. What is its linear velocity at any point on circular path **[CBSE PMT 1996; JIPMER 2000]**

(a) 10 m/s (b) 2 m/s (c) 20 m/s (d) $\sqrt{2}\ m/s$

Solution : (b) $v = r\omega = 0.2 \times 10 = 2\, m/s$

Problem **91.** The linear velocity of a point on the equator is nearly (radius of the earth is 6400 *km*)

(a) 800 km/hr (b) 1600 km/hr (c) 3200 km/hr (d) 6400 km/hr

Solution : (b) $v = rw = 6400\ km \times \dfrac{2\pi\ rad}{24\ \ hr} = 1675\ km/hr = 1600\ km/hr$

Problem **92.** A particle moves along a circle with a uniform speed *v*. After it has made an angle of 60° its speed will be

(a) $v\sqrt{2}$ (b) $\dfrac{v}{\sqrt{2}}$ (c) $\dfrac{v}{\sqrt{3}}$ (d) v

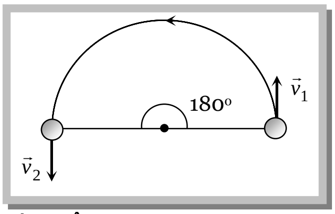
Solution : (d) Uniform speed means speed of the particle remains always constant.

Problem **93.** A particle is moving along a circular path of radius 2 *m* and with uniform speed of 5 ms^{-1}. What will be the change in velocity when the particle completes half of the revolution

(a) Zero (b) 10 ms^{-1} (c) $10\sqrt{2}\, ms^{-1}$ (d) $10/\sqrt{2}\, ms^{-1}$

Solution : (b) $\Delta v = 2v \sin\left(\dfrac{\theta}{2}\right) = 2 \times 5 \sin\left(\dfrac{180^o}{2}\right)$

$= 2 \times 5 \sin 90^o = 10\, m/s$



Problem **94.** What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$ **[Pb. PMT 2000]**

(a) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (b) $18\hat{i} + 13\hat{j} - 2\hat{k}$ (c) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (d) $6\hat{i} - 2\hat{j} + 8\hat{k}$

Solution : (b) $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$

$\vec{v} = (-24+6)i - (18-5)\hat{j} + (-18+20)\hat{k} = 18\hat{i} + 13\hat{j} - 2\hat{k}$

Problem **95.** A particle comes round circle of radius 1 *m* once. The time taken by it is 10 *sec*. The average velocity of motion is **[JIPMER 1999]**

(a) 0.2 $\pi\, m/s$ (b) 2 $\pi\, m/s$ (c) 2 m/s (d) Zero

Solution : (d) In complete revolution total displacement becomes zero. So the average velocity will be zero.

Problem 96. Two particles of mass M and m are moving in a circle of radii R and r. If their time-periods are same, what will be the ratio of their linear velocities **[CBSE PMT 2001]**

(a) $MR : mr$ (b) $M : m$ (c) $R : r$ (d) 1 : 1

Solution : (c) $\frac{v_1}{v_2} = \frac{r_1\omega_1}{r_2\omega_2}$. Time periods are equal *i.e.* $\omega_1 = \omega_2 \therefore \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{R}{r}$

(5) **Time period (*T*) :** In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.

(i) Units : second. (ii) Dimension : $[M^0L^0T]$

(iii) Time period of second's hand of watch = 60 *second*. (iv) Time period of minute's hand of watch = 60 *minute* (v) Time period of hour's hand of watch = 12 *hour*

(6) **Frequency (*n*) :** In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.

(i) Units : s^{-1} or hertz (*Hz*). (ii) Dimension : $[M^0L^0T^{-1}]$

Note : ❑ Relation between time period and frequency : If n is the frequency of revolution of an object in circular motion, then the object completes n revolutions in 1 second. Therefore, the object will complete one revolution in $1/n$ second.

$$\therefore T = 1/n$$

❑ Relation between angular velocity, frequency and time period : Consider a point object describing a uniform circular motion with frequency n and time period T. When the object completes one revolution, the angle traced at its axis of circular motion is 2π radians. It means, when time $t = T$, $\theta = 2\pi$ radians. Hence, angular velocity $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi n$ $(\because T = 1/n)$

$$\omega = \frac{2\pi}{T} = 2\pi n$$

❑ If two particles are moving on same circle or different coplanar concentric circles in same direction with different uniform angular speeds ω_A and ω_B respectively, the angular velocity of B relative to A will be

$$\omega_{rel} = \omega_B - \omega_A$$

So the time taken by one to complete one revolution around O with respect to the other (i.e., time in which B complete one revolution around O with respect to the other (*i.e.*, time in which B completes one more or less revolution around O than A)

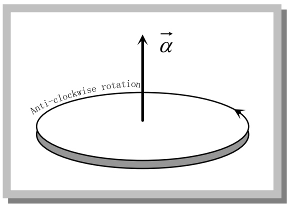
$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1T_2}{T_1 - T_2} \qquad \left[\text{as } T = \frac{2\pi}{\omega}\right]$$

Special case : If $\omega_B = \omega_A, \omega_{rel} = 0$ and so $T = \infty$., particles will maintain their position relative to each other. This is what actually happens in case of geostationary satellite ($\omega_1 = \omega_2$ = constant)

(7) **Angular acceleration (α) :** Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

(i) If $\Delta\omega$ be the change in angular velocity of the object in time interval t and $t + \Delta t$, while moving on a circular path, then angular acceleration of the object will be

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$



(ii) Units : *rad*. s^{-2} (iii) Dimension : $[M^0L^0T^{-2}]$

(iv) Relation between linear acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

(v) For uniform circular motion since ω is constant so $\alpha = \frac{d\omega}{dt} = 0$

(vi) For non-uniform circular motion $\alpha \neq 0$

Note :
- Relation between linear (tangential) acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$
- For uniform circular motion angular acceleration is zero, so tangential acceleration also is equal to zero.
- For non-uniform circular motion $a \neq 0$ (because $\alpha \neq 0$).

Sample problems based on angular acceleration

Problem 97. A body is revolving with a uniform speed v in a circle of radius r. The angular acceleration of the body is

(a) $\frac{v}{r}$ (b) Zero (c) $\frac{v^2}{r}$ along the radius and towards the centre (d) $\frac{v^2}{r}$ along the radius and away from the centre

Solution : (b) In uniform circular motion ω constant so $\alpha = \frac{d\omega}{dt} = 0$

Problem 98. The linear acceleration of a car is 10m/s^2. If the wheels of the car have a diameter of 1m, the angular acceleration of the wheels will be

(a) 10 rad/sec^2 (b) 20 rad/sec^2 (c) 1 rad/sec^2 (d) 2 rad/sec^2

Solution : (b) Angular acceleration $= \frac{\text{linear acceleration}}{\text{radius}} = \frac{10}{0.5} = 20\, rad/sec^2$

Problem 99. The angular speed of a motor increases from 600 rpm to 1200 rpm in 10 s. What is the angular acceleration of the motor

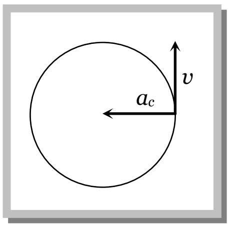
(a) $600\ rad\ sec^{-2}$ (b) $60\pi\ rad\ sec^{-2}$ (c) $60\ rad\ sec^{-2}$ (d) $2\pi\ rad\ sec^{-2}$

Solution : (d) $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi(1200 - 600)}{10 \times 60}\frac{rad}{sec^2} = 2\pi\, rad/sec^2$

3.10 Centripetal Acceleration

(1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

(2) It always acts on the object along the radius towards the centre of the circular path.



(3) Magnitude of centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r = 4\pi n^2 r = \frac{4\pi^2}{T^2} r$

(4) Direction of centripetal acceleration : It is always the same as that of $\Delta\vec{\upsilon}$. When Δt decreases, $\Delta\theta$ also decreases. Due to which $\Delta\vec{\upsilon}$ becomes more and more perpendicular to $\vec{\upsilon}$. When $\Delta t \to 0$, $\Delta\vec{\upsilon}$ becomes perpendicular to the velocity vector. As the velocity vector of the particle at an instant acts along the tangent to the circular path, therefore $\Delta\vec{\upsilon}$ and hence the centripetal acceleration vector acts along the radius of the circular path at that point and is directed towards the centre of the circular path.

Sample problems based on centripetal acceleration

Problem 100. If a cycle wheel of radius 4 m completes one revolution in two seconds. Then acceleration of the cycle will be

(a) $\pi^2 m/s^2$ (b) $2\pi^2 m/s^2$ (c) $4\pi^2 m/s^2$ (d) $8\pi\, m/s^2$ **[Pb. PMT 2001]**

Solution : (c) Given $r = 4\,m$ and $T = 2$ seconds.

$$\therefore a_c = \frac{4\pi^2}{T^2} r = \frac{4\pi^2}{(2)^2} 4 = 4\pi^2\, m/s^2$$

***Problem* 101.** A stone is tied to one end of a spring 50 *cm* long is whirled in a horizontal circle with a constant speed. If the stone makes 10 revolutions in 20 *s*, what is the magnitude of acceleration of the stone **[Pb. PMT 2000]**

(a) 493 *cm/sec*² (b) 720 *cm/sec*² (c) 860 *cm/sec*² (d) 990 *cm/sec*²

Solution : (a) Time period $= \frac{\text{Total time}}{\text{No. of revolution}} = \frac{20}{10} = 2\,sec$

$$\therefore a_c = \frac{4\pi^2}{T^2}.r = \frac{4\pi^2}{(2)^2} \times (1/2) m/s^2 = 4.93\, m/s^2 = 493\, cm/s^2$$

***Problem* 102.** A particle moves with a constant speed v along a circular path of radius r and completes the circle in time T. What is the acceleration of the particle **[Orissa JEE 2002]**

(a) mg (b) $\frac{2\pi v}{T}$ (c) $\frac{\pi r^2}{T}$ (d) $\frac{\pi v^2}{T}$

Solution : (b) $a_c = \frac{v^2}{r} = \omega^2 r = v\omega = v\left(\frac{2\pi}{T}\right) = \frac{2\pi v}{T}$

***Problem* 103.** If the speed of revolution of a particle on the circumference of a circle and the speed gained in falling through a distance equal to half the radius are equal, then the centripetal acceleration will be

(a) $\frac{g}{2}$ (b) $\frac{g}{4}$ (c) $\frac{g}{3}$ (d) g

Solution : (d) Speed gain by body falling through a distance h is equal to $v = \sqrt{2gh} = \sqrt{2g\frac{r}{2}}$ [As $h = \frac{r}{2}$ given]

$$\Rightarrow v = \sqrt{gr} \Rightarrow \frac{v^2}{r} = g$$

***Problem* 104.** Two cars going round curve with speeds one at 90 *km/h* and other at 15 *km/h*. Each car experiences same acceleration. The radii of curves are in the ratio of **[EAMCET (Med.) 1998]**

(a) 4 : 1 (b) 2 : 1 (c) 16 : 1 (d) 36 : 1

Solution : (d) Centripetal acceleration $= \frac{v_1^2}{r_1} = \frac{v_2^2}{r_2}$ (given)

$$\therefore \frac{r_1}{r_2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{90}{15}\right)^2 = \frac{36}{1}$$

***Problem* 105.** A wheel of radius $0.20m$ is accelerated from rest with an angular acceleration of $1\, rad/s^2$. After a rotation of 90^o the radial acceleration of a particle on its rim will be

(a) $\pi\ m/s^2$ (b) $0.5\ \pi\ m/s^2$ (c) $2.0\pi\ m/s^2$ (d) $0.2\ \pi\ m/s^2$

Solution : (d) From the equation of motion

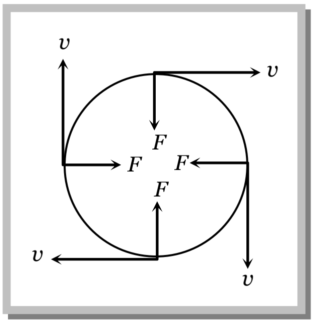
Angular speed acquired by the wheel, $\omega_2^2 = \omega_1^2 + 2\alpha\theta = 0 + 2\times 1\times\frac{\pi}{2} \Rightarrow \omega_2^2 = \pi$

Now radial acceleration $\omega^2 r = \pi\times 0.2 = 0.2\pi\, m/s^2$

3.11 Centripetal Force

According to Newton's first law of motion, whenever a body moves in a straight line with uniform velocity, no force is required to maintain this velocity. But when a body moves along a circular path with uniform speed, its direction changes continuously *i.e.* velocity keeps on changing on account of a change in direction. According to Newton's second law of motion, a change in the direction of motion of the body can take place only if some external force acts on the body.

Due to inertia, at every point of the circular path; the body tends to move along the tangent to the circular path at that point (in figure). Since every body has directional inertia, a velocity cannot change by itself and as such we have to apply a force. But this force should be such that it changes the direction of velocity and not its magnitude. This is possible only if the force acts perpendicular to the direction of velocity. Because the velocity is along the tangent, this force must be along the radius (because the radius of a circle at any point is perpendicular to the tangent at that point). Further, as this force is to move the body in a circular path, it must acts towards the centre. This centre-seeking force is called the centripetal force.



Hence, centripetal force is that force which is required to move a body in a circular path with uniform speed. The force acts on the body along the radius and towards centre.

(1) Formulae for centripetal force : $F = \frac{mv^2}{r} = m\omega^2 r = m\,4\pi^2 n^2 r = \frac{m4\pi^2 r}{T^2}$

(2) Centripetal force in different situation

Situation	Centripetal Force
A particle tied to a string and whirled in a horizontal circle	Tension in the string
Vehicle taking a turn on a level road	Frictional force exerted by the road on the tyres
A vehicle on a speed breaker	Weight of the body or a component of weight
Revolution of earth around the sun	Gravitational force exerted by the sun
Electron revolving around the nucleus in an atom	Coulomb attraction exerted by the protons in the nucleus

A charged particle describing a circular path in a magnetic field	Magnetic force exerted by the agent that sets up the magnetic field

3.12 Centrifugal Force

It is an imaginary force due to incorporated effects of inertia. When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer *A* who is not sharing the motion along the circular path, the body appears to fly off tangential at the point of release. To another observer *B*, who is sharing the motion along the circular path (*i.e.*, the observer *B* is also rotating with the body with the same velocity), the body appears to be stationary before it is released. When the body is released, it appears to *B*, as if it has been thrown off along the radius away from the centre by some force. In reality no force is actually seen to act on the body. In absence of any real force the body tends to continue its motion in a straight line due to its inertia. The observer *A* easily relates this events to be due to inertia but since the inertia of both the observer *B* and the body is same, the observer *B* can not relate the above happening to inertia. When the centripetal force ceases to act on the body, the body leaves its circular path and continues to moves in its straight-line motion but to observer *B* it appears that a real force has actually acted on the body and is responsible for throwing the body radially out-words. This imaginary force is given a name to explain the effects on inertia to the observer who is sharing the circular motion of the body. This inertial force is called centrifugal force. Thus centrifugal force is a fictitious force which has significance only in a rotating frame of reference.

~~*Sample problems based on centripetal and centrifugal force*~~

Problem **106.** A ball of mass 0.1 *kg* is whirled in a horizontal circle of radius 1 *m* by means of a string at an initial speed of 10 *r.p.m.* Keeping the radius constant, the tension in the string is reduced to one quarter of its initial value. The new speed is

(a) 5 *r.p.m.* (b) 10 *r.p.m.* (c) 20 *r.p.m.* (d) 14 *r.p.m.*

Solution : (a) Tension in the string $T = m\omega^2 r = m4\pi^2 n^2 r$

$T \propto n^2$ or $n \propto \sqrt{T}$ [As *m* and *r* are constant]

$$\therefore \frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{T/4}{T}} \Rightarrow n_2 = \frac{n_1}{2} = \frac{10}{2} = 5\,rpm$$

Problem **107.** A cylindrical vessel partially filled with water is rotated about its vertical central axis. It's surface will **[RPET 2000]**

(a) Rise equally (b) Rise from the sides (c) Rise from the middle (d) Lowered equally

Solution : (b) Due to the centrifugal force.

Problem **108.** A proton of mass 1.6×10^{-27} *kg* goes round in a circular orbit of radius 0.10 *m* under a centripetal force of 4×10^{-13} *N*. then the frequency of revolution of the proton is about **[Kerala PMT 2002]**

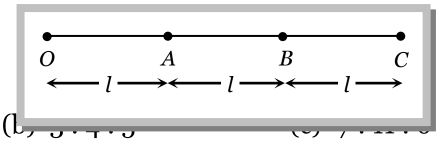
(a) 0.08×10^8 *cycles per sec* (b) 4×10^8 *cycles per* sec

(c) 8×10^8 *cycles per sec* (d) 12×10^8 *cycles per sec*

Solution : (a) $F = 4 \times 10^{-13}\ N$; $m = 1.6 \times 10^{-27}\ kg$; $r = 0.1 m$

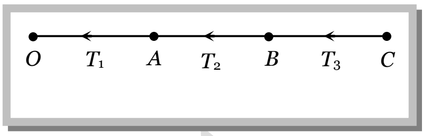
Centripetal force $F = m4\pi^2 n^2 r \therefore n = \sqrt{\frac{F}{4m\pi^2 r}} = 8 \times 10^6\ cycles/\sec = 0.08 \times 10^8\ cycle/\sec$.

***Problem* 109.** Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is



(a) 3 : 5 : 7 (b) 3 : 4 : 5 (c) 7 : 11 : 6 (d) 3 : 5 : 6

Solution : (d) Let the angular speed of the thread is ω



For particle '*C*' $\Rightarrow$ $T_3 = m\omega^2 3l$

For particle '*B*' $T_2 - T_3 = m\omega^2 2l \Rightarrow T_2 = m\omega^2 5l$

For particle '*C*' $T_1 - T_2 = m\omega^2 l \Rightarrow T_1 = m\omega^2 6l$

$\therefore\ T_3 : T_2 : T_1 = 3 : 5 : 6$

***Problem* 110.** A stone of mass 1 *kg* tied to the end of a string of length 1 *m*, is whirled in a horizontal circle with a uniform angular velocity of 2 *rad/s*. The tension of the string is (in *N*) **[KCET 1998]**

(a) 2 (b) $\frac{1}{3}$ (c) 4 (d) $\frac{1}{4}$

Solution : (c) $T = m\omega^2 r = 1 \times (2)^2 \times (1) = 4$ *Newton*

***Problem* 111.** A cord can bear a maximum force of 100 *N* without breaking. A body of mass 1 *kg* tied to one end of a cord of length 1 *m* is revolved in a horizontal plane. What is the maximum linear speed of the body so that the cord does not break

(a) 10 *m/s* (b) 20 *m/s* (c) 25 *m/s* (d) 30 *m/s*

Solution : (a) Tension in cord appears due to centrifugal force $T = \frac{mv^2}{r}$ and for critical condition this tension will be equal to breaking force (100 *N*) $\therefore \frac{mv_{max}^2}{r} = 100 \Rightarrow v_{max}^2 = \frac{100 \times 1}{1} \Rightarrow v_{max} = 10\, m/s$

***Problem* 112.** A mass is supported on a frictionless horizontal surface. It is attached to a string and rotates about a fixed centre at an angular velocity ω_0. If the length of the string and angular velocity are doubled, the tension in the string which was initially T_0 is now **[AIIMS 1985]**

(a) T_0 (b) $T_0/2$ (c) $4T_0$ (d) $8T_0$

Solution : (d) $T = m\omega^2 l \therefore \frac{T_2}{T_1} = \left(\frac{\omega_2}{\omega_1}\right)^2 \left(\frac{l_2}{l_1}\right) \Rightarrow \frac{T_2}{T_0} = \left(\frac{2\omega}{\omega}\right)^2 \left(\frac{2l}{l}\right) \Rightarrow T_2 = 8T_0$

***Problem* 113.** A stone is rotated steadily in a horizontal circle with a period *T* by a string of length *l*. If the tension in the string is kept constant and *l* increases by 1%, what is the percentage change in *T*

(a) 1% (b) 0.5% (c) 2% (d) 0.25%

Solution : (b) Tension $= \frac{m\, 4\pi^2 l}{T^2} \therefore\ l \propto T^2$ or $T \propto \sqrt{l}$ [Tension and mass are constant]

Percentage change in Time period $= \frac{1}{2}$ (percentage change in length) [If % change is very small]

$$= \frac{1}{2}(1\%) = 0.5\%$$

***Problem* 114.** If mass speed and radius of rotation of a body moving in a circular path are all increased by 50%, the necessary force required to maintain the body moving in the circular path will have to be increased by

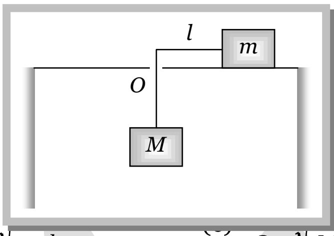
(a) 225% (b) 125% (c) 150% (d) 100%

Solution : (b) Centripetal force $F = \frac{mv^2}{r}$

If m, v and r are increased by 50% then let new force $F' = \frac{\left(m+\frac{m}{2}\right)\left(v+\frac{v}{2}\right)^2}{\left(r+\frac{r}{2}\right)} = \frac{9}{4}\frac{mv^2}{r} = \frac{9}{4}F$

Percentage increase in force $\frac{\Delta F}{F}\times 100 = \frac{F'-F}{F}\times 100\% = \frac{500}{4}\% = 125\%$

***Problem* 115.** Two masses m and M are connected by a light string that passes through a smooth hole O at the centre of a table. Mass m lies on the table and M hangs vertically. m is moved round in a horizontal circle with O as the centre. If l is the length of the string from O to m then the frequency with which m should revolve so that M remains stationary is



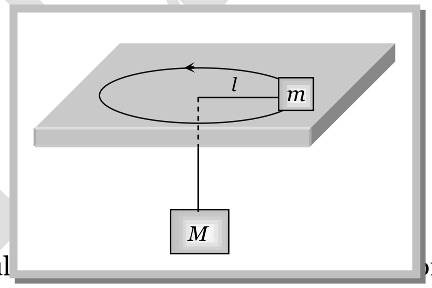
(a) $\frac{1}{2\pi}\sqrt{\frac{Mg}{m\,l}}$ (b) $\frac{1}{\pi}\sqrt{\frac{Mg}{m\,l}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{m\,l}{Mg}}$ (d) $\frac{1}{\pi}\sqrt{\frac{m\,l}{Mg}}$

Solution : (a) 'm' Mass performs uniform circular motion on the table. Let n is the frequency of revolution then centrifugal force $= m\,4\pi^2n^2l$

For equilibrium this force will be equal to weight Mg

$m\,4\pi^2n^2l = Mg$

$\therefore n = \frac{1}{2\pi}\sqrt{\frac{Mg}{m\,l}}$



***Problem* 116.** A particle of mass M moves with constant speed along a circul... n of a force F. Its speed is **[MP PMT 2002]**

(a) $\sqrt{\frac{rF}{m}}$ (b) $\sqrt{\frac{F}{r}}$ (c) $\sqrt{Fmr}$ (d) $\sqrt{\frac{F}{mr}}$

Solution : (a) Centripetal force $F = \frac{mv^2}{r}$ $\therefore v = \sqrt{\frac{rF}{m}}$

***Problem* 117.** In an atom for the electron to revolve around the nucleus, the necessary centripetal force is obtained from the following force exerted by the nucleus on the electron

(a) Nuclear force (b) Gravitational force (c) Magnetic force (d) Electrostatic force

Solution : (d)

***Problem* 118.** A motor cycle driver doubles its velocity when he is having a turn. The force exerted outwardly will be **[AFMC 2002]**

(a) Double (b) Half (c) 4 times (d) $\frac{1}{4}$ times

Solution : (c) $F = \frac{mv^2}{r}$ $\therefore F \propto v^2$ or $\frac{F_2}{F_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{2v}{v}\right)^2 = 4 \Rightarrow F_2 = 4F_1$

***Problem* 119.** A bottle of soda water is grasped by the neck and swing briskly in a vertical circle. Near which portion of the bottle do the bubbles collect

(a) Near the bottom (b) In the middle of the bottle

(c) Near the neck (d) Uniformly distributed in the bottle

Solution : (c) Due to the lightness of the gas bubble they feel less centrifugal force so they get collect near the neck of the bottle. They collect near the centre of circular motion *i.e.* near the neck of the bottle.

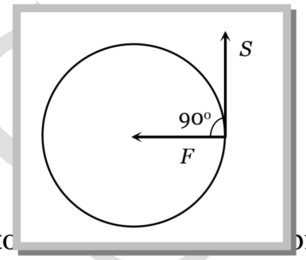
Problem 120. A body is performing circular motion. An observer O_1 is sitting at the centre of the circle and another observer O_2 is sitting on the body. The centrifugal force is experienced by the observer

(a) O_1 only (b) O_2 only (c) Both by O_1 and O_2 (d) None of these

Solution : (b) Centrifugal force is a pseudo force, which is experienced only by that observer who is attached with the body performing circular motion.

3.13 Work done by Centripetal Force

The work done by centripetal force is always zero as it is perpendicular to velocity and hence instantaneous displacement.



Work done = Increment in kinetic energy of revolving body

Work done = 0

Also $W = \vec{F}.\vec{S} = F \cdot S\cos\theta$

$= F{\cdot}S\cos 90^\circ = 0$

Example : (i) When an electron revolve around the nucleus in hydrogen atom in a particular orbit, it neither absorb nor emit any energy means its energy remains constant.

(ii) When a satellite established once in a orbit around the earth and it starts revolving with particular speed, then no fuel is required for its circular motion.

Sample problem based on work done

Problem 121. A particle does uniform circular motion in a horizontal plane. The radius of the circle is 20 *cm*. The centripetal force acting on the particle is 10 *N*. It's kinetic energy is

(a) 0.1 *Joule* (b) 0.2 *Joule* (c) 2.0 *Joule* (d) 1.0 *Joule*

Solution : (d) $\frac{mv^2}{r} = 10\,N$ (given) $\Rightarrow mv^2 = 10\times r = 10\times 0.2 = 2$

Kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}(2) = 1\,Joule$.

Problem 122. A body of mass 100 *g* is rotating in a circular path of radius *r* with constant velocity. The work done in one complete revolution is **[AFMC 1998]**

(a) 100*r Joule* (b) (*r*/100) *Joule* (c) (100/*r*) *Joule* (d) Zero

Solution : (d) Because in uniform circular motion work done by the centripetal force is always zero.

Problem 123. A particle of mass *m* is describing a circular path of radius *r* with uniform speed. If *L* is the angular momentum of the particle about the axis of the circle, the kinetic energy of the particle is given by **[CPMT 1995]**

(a) L^2/mr^2 (b) $L^2/2mr^2$ (c) $2L^2/mr^2$ (d) mr^2L

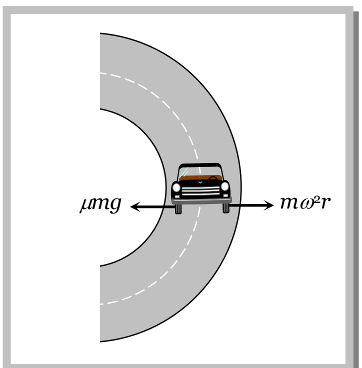
Solution : (b) Rotational kinetic energy $E = \frac{L^2}{2I} = \frac{L^2}{2mr^2}$ (As for a particle $I = mr^2$)

3.14 Skidding of Vehicle on a Level Road

When a vehicle turns on a circular path it requires centripetal force.

If friction provides this centripetal force then vehicle can move in circular path safely if

Friction force ≥ Required centripetal force



$$\mu mg \geq \frac{mv^2}{r}$$

$\therefore$ $$v_{safe} \leq \sqrt{\mu rg}$$

This is the maximum speed by which vehicle can turn in a circular path of radius *r*, where coefficient of friction between the road and tyre is μ.

Sample problem based on skidding of vehicle on a level road

Problem 124. Find the maximum velocity for overturn for a car moved on a circular track of radius $100m$. The coefficient of friction between the road and tyre is 0.2

(a) $0.14\ m/s$ (b) $140\ m/s$ (c) $1.4\ km/s$ (d) $14\ m/s$

Solution : (d) $v_{\max} = \sqrt{\mu r g} = \sqrt{0.2 \times 100 \times 10} = 10\sqrt{2} = 14\ m/s$

Problem 125. When the road is dry and the coefficient of friction is μ, the maximum speed of a car in a circular path is $10\ m/s$. If the road becomes wet and $\mu' = \frac{\mu}{2}$, what is the maximum speed permitted

(a) $5\ m/s$ (b) $10\ m/s$ (c) $10\sqrt{2}\ m/s$ (d) $5\sqrt{2}\ m/s$

Solution : (d) $v \propto \sqrt{\mu} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{\mu_2}{\mu_1}} = \sqrt{\frac{\mu/2}{\mu}} = \frac{1}{\sqrt{2}} \Rightarrow v_2 = \frac{1}{\sqrt{2}} v_1 \Rightarrow v_2 = \frac{10}{\sqrt{2}} = 5\sqrt{2}\ m/s$

Problem 126. The coefficient of friction between the tyres and the road is 0.25. The maximum speed with which a car can be driven round a curve of radius 40 *m* with skidding is (assume $g = 10\ ms^{-2}$)

(a) $40\ ms^{-1}$ (b) $20\ ms^{-1}$ (c) $15\ ms^{-1}$ (d) $10\ ms^{-1}$

Solution : (d) $v_{\max} = \sqrt{\mu r g} = \sqrt{0.25 \times 40 \times 10} = 10\ m/s$

3.15 Skidding of Object on a Rotating Platform

On a rotating platform, to avoid the skidding of an object (mass *m*) placed at a distance *r* from axis of rotation, the centripetal force should be provided by force of friction.

Centripetal force = Force of friction

$$m\omega^2 r = \mu mg$$

$\therefore$ $$\omega_{\max} = \sqrt{(\mu g/r)},$$

Hence maximum angular velocity of rotation of the platform is $\sqrt{(\mu g/r)}$, so that object will not skid on it.

3.16 Bending of a Cyclist

A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track, while going round a curve. Consider a cyclist of weight *mg* taking a turn of radius *r* with velocity *v*. In order to provide the necessary centripetal force, the cyclist leans through angle θ inwards as shown in figure.

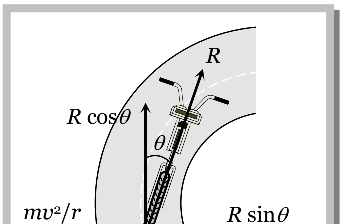
The cyclist is under the action of the following forces :

The weight *mg* acting vertically downward at the centre of gravity of cycle and the cyclist.

The reaction *R* of the ground on cyclist. It will act along a line-making angle θ with the vertical.

The vertical component $R\cos\theta$ of the normal reaction *R* will balance the weight of the cyclist, while the horizontal component $R\sin\theta$ will provide the necessary centripetal force to the cyclist.

$$R\sin\theta = \frac{mv^2}{r} \quad .....(i)$$



and $R\cos\theta = mg$ .....(ii)

Dividing equation (i) by (ii), we have

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2/r}{mg}$$

or $\tan\theta = \frac{v^2}{rg}$ ....(iii)

Therefore, the cyclist should bend through an angle $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

It follows that the angle through which cyclist should bend will be greater, if

(i) The radius of the curve is small *i.e.* the curve is sharper

(ii) The velocity of the cyclist is large.

Note : ❑ For the same reasons, an ice skater or an aeroplane has to bend inwards, while taking a turn.

Sample problem based on bending of cyclist

Problem **127.** A boy on a cycle pedals around a circle of 20 metres radius at a speed of 20 *metres/sec*. The combined mass of the boy and the cycle is $90 kg$. The angle that the cycle makes with the vertical so that it may not fall is $(g = 9.8\ m/\sec^2)$ **[MP PMT 1995]**

(a) 60.25^o (b) 63.90^o (c) 26.12^o (d) 30.00^o

Solution : (b) $r = 20\,m,\ \ v = 20\,m/s,\ \ m = 90\,kg,\ \ g = 9.8\,m/s^2$ (given)

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{20\times 20}{20\times 10}\right) = \tan^{-1}(2) = 63.90°$$

Problem **128.** If a cyclist moving with a speed of $4.9\ m/s$ on a level road can take a sharp circular turn of radius $4m$, then coefficient of friction between the cycle tyres and road is

(a) 0.41 (b) 0.51 (c) 0.71 (d) 0.61

Solution : (d) $v = 4.9\,m/s,\ r = 4\,m$ and $g = 9.8\,m/s^2$ (given)

$$\mu = \frac{v^2}{rg} = \frac{4.9\times 4.9}{4\times 9.8} = 0.61$$

Problem **129.** A cyclist taking turn bends inwards while a car passenger taking same turn is thrown outwards. The reason is **[NCERT 1972]**

(a) Car is heavier than cycle

(b) Car has four wheels while cycle has only two

(c) Difference in the speed of the two

(d) Cyclist has to counteract the centrifugal force while in the case of car only the passenger is thrown by this force

Solution : (d)

3.17 Banking of a Road

For getting a centripetal force cyclist bend towards the centre of circular path but it is not possible in case of four wheelers.

Therefore, outer bed of the road is raised so that a vehicle moving on it gets automatically inclined towards the centre.

In the figure (A) shown reaction R is resolved into two components, the component $R\cos\theta$ balances weight of vehicle

$\therefore \quad R\cos\theta = mg \quad$ ......(i)

and the horizontal component $R\sin\theta$ provides necessary centripetal force as it is directed towards centre of desired circle

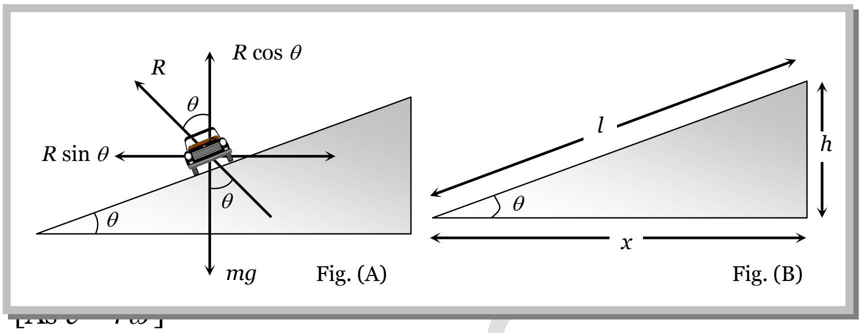


Fig. (A) Fig. (B)

Thus $\quad R\sin\theta = \dfrac{mv^2}{r} \quad$ ......(ii)

Dividing (ii) by (i), we have

$$\tan\theta = \frac{v^2}{rg} \quad \text{...... (iii)}$$

or $\tan\theta = \dfrac{\omega^2 r}{g} = \dfrac{v\omega}{rg} \quad$ ...... (iv) $\quad$ [As $v = r\omega$]

If l = width of the road, h = height of the outer edge from the ground level then from the figure (B)

$$\tan\theta = \frac{h}{x} = \frac{h}{l} \quad \text{.......(v)} \qquad \text{[since } \theta \text{ is very small]}$$

From equation (iii), (iv) and (v) $\qquad \tan\theta = \dfrac{v^2}{rg} = \dfrac{\omega^2 r}{g} = \dfrac{v\omega}{rg} = \dfrac{h}{l}$

Note : ❑ If friction is also present between the tyres and road then $\dfrac{v^2}{rg} = \dfrac{\mu + \tan\theta}{1 - \mu\tan\theta}$

❑ Maximum safe speed on a banked frictional road $v = \sqrt{\dfrac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}$

Sample problems based on banking of a road

***Problem* 130.** For traffic moving at $60\ km/hr$ along a circular track of radius $0.1\ km$, the correct angle of banking is **[MNR 1993]**

(a) $\dfrac{(60)^2}{0.1}$ (b) $\tan^{-1}\left[\dfrac{(50/3)^2}{100 \times 9.8}\right]$ (c) $\tan^{-1}\left[\dfrac{100 \times 9.8}{(50/3)^2}\right]$ (d) $\tan^{-1}\sqrt{60 \times 0.1 \times 9.8}$

Solution : (b) $\quad v = 60\ km/hr = \dfrac{50}{3} m/s$, $r = 0.1 km = 100\ m$, $g = 9.8\ m/s^2$ (given)

Angle of banking $\tan\theta = \dfrac{v^2}{rg}$ or $\theta = \tan^{-1}\left(\dfrac{v^2}{rg}\right) = \tan^{-1}\left[\dfrac{(50/3)^2}{100 \times 9.8}\right]$

***Problem* 131.** A vehicle is moving with a velocity v on a curved road of width b and radius of curvature R. For counteracting the centrifugal force on the vehicle, the difference in elevation required in between the outer and inner edges of the road is

(a) $\dfrac{v^2 b}{Rg}$ (b) $\dfrac{rb}{Rg}$ (c) $\dfrac{vb^2}{Rg}$ (d) $\dfrac{vb}{R^2 g}$

Solution : (a) $\quad$ For Banking of road $\tan\theta = \dfrac{v^2}{rg}$ and $\tan\theta = \dfrac{h}{l}$

$$\therefore \frac{v^2}{rg} = \frac{h}{l} \Rightarrow h = \frac{v^2 l}{rg} = \frac{v^2 b}{Rg} \qquad \text{[As } l = b \text{ and } r = R \text{ given]}$$

Problem 132. The radius of curvature of a road at a certain turn is $50m$. The width of the road is $10m$ and its outer edge is $1.5m$ higher than the inner edge. The safe speed for such an inclination will be

(a) $6.5\ m/s$ (b) $8.6\ m/s$ (c) $8\ m/s$ (d) $10\ m/s$

Solution : (b) $h = 1.5\,m,\ r = 50\,m,\ l = 10\,m,\ g = 10\,m/s^2$ (given)

$$\frac{v^2}{rg} = \frac{h}{l} \Rightarrow v = \sqrt{\frac{hrg}{l}} = \sqrt{\frac{1.5 \times 50 \times 10}{10}} = 8.6\,m/s$$

Problem 133. Keeping the banking angle same to increase the maximum speed with which a car can travel on a curved road by 10%, the radius of curvature of road has to be changed from $20m$ to **[EAMCET 1991]**

(a) $16m$ (b) $18m$ (c) $24.25m$ (d) $30.5\ m$

Solution : (c) $\tan\theta = \frac{v^2}{rg} \Rightarrow r \propto v^2$ (if θ is constant)

$$\frac{r_2}{r_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{1.1v}{v}\right)^2 = 1.21 \Rightarrow r_2 = 1.21 \times r_1 = 1.21 \times 20 = 24.2\ m$$

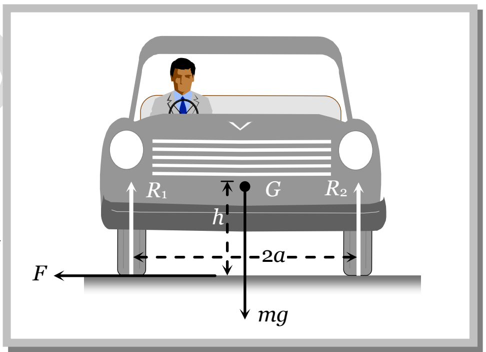
Problem 134. The slope of the smooth banked horizontal road is p. If the radius of the curve be r, the maximum velocity with which a car can negotiate the curve is given by

(a) prg (b) $\sqrt{prg}$ (c) p/rg (d) $\sqrt{p/rg}$

Solution : (b) $\tan\theta = \frac{v^2}{rg} \Rightarrow p = \frac{v^2}{rg}$ $\therefore\ v = \sqrt{prg}$

3.18 Overturning of Vehicle

When a car moves in a circular path with speed more than maximum speed then it overturns and it's inner wheel leaves the ground first



Weight of the car = mg

Speed of the car = v

Radius of the circular path = r

Distance between the centre of wheels of the car = $2a$

Height of the centre of gravity (G) of the car from the road level = h

Reaction on the inner wheel of the car by the ground = R_1

Reaction on the outer wheel of the car by the ground = R_2

When a car move in a circular path, horizontal force F provides the required centripetal force

i.e., $F = \frac{mv^2}{R}$ ....(i)

For rotational equilibrium, by taking the moment of forces R_1, R_2 and F about G

$Fh + R_1 a = R_2 a$ ....(ii)

As there is no vertical motion so $R_1 + R_2 = mg$ ....(iii)

By solving (i), (ii) and (iii)

$$R_1 = \frac{1}{2}M\left[g - \frac{v^2 h}{ra}\right] \quad ....(iv)$$

and $$R_2 = \frac{1}{2}M\left[g + \frac{v^2 h}{ra}\right] \quad ....(v)$$

It is clear from equation (iv) that if v increases value of R_1 decreases and for $R_1 = 0$

$$\frac{v^2 h}{ra} = g \quad \text{or} \quad v = \sqrt{\frac{gra}{h}}$$

i.e. the maximum speed of a car without overturning on a flat road is given by $v = \sqrt{\frac{gra}{h}}$

Sample problems based on overturning of vehicle

***Problem* 135.** The distance between two rails is $1.5m$. The centre of gravity of the train at a height of $2m$ from the ground. The maximum speed of the train on a circular path of radius $120m$ can be

(a) $10.5\ m/s$ (b) $42\ m/s$ **(c)** $21\ m/s$ (d) $84\ m/s$

Solution : (c) Height of centre of gravity from the ground $h = 2m$, Acceleration due to gravity $g = 10\,m/s^2$,
Distance between two rails $2a = 1.5m$, Radius of circular path $r = 120\ m$ (given)

$$v_{\max} = \sqrt{\frac{gra}{h}} \Rightarrow v_{\max} = \sqrt{\frac{10\times120\times0.75}{2}} = 21.2\,m/s$$

***Problem* 136.** A car sometimes overturns while taking a turn. When it overturns, it is **[AFMC 1988]**

(a) The inner wheel which leaves the ground first
(b) The outer wheel which leaves the ground first
(c) Both the wheels leave the ground simultaneously
(d) Either wheel leaves the ground first

Solution : (a)

***Problem* 137.** A car is moving on a circular path and takes a turn. If R_1 and R_2 be the reactions on the inner and outer wheels respectively, then

(a) $R_1 = R_2$ (b) $R_1 < R_2$ (c) $R_1 > R_2$ (d) $R_1 \geq R_2$

Solution : (b) Reaction on inner wheel $R_1 = \frac{M}{2}\left[g - \frac{v^2 h}{ra}\right]$ and Reaction on outer wheel $R_2 = \frac{M}{2}\left[g + \frac{v^2 h}{ra}\right]$

$\therefore\ R_1 < R_2$.

***Problem* 138.** A train A runs from east to west and another train B of the same mass runs from west to east at the same speed along the equator. A presses the track with a force F_1 and B presses the track with a force F_2

(a) $F_1 > F_2$
(b) $F_1 < F_2$
(c) $F_1 = F_2$
(d) The information is insufficient to find the relation between F_1 and F_2

Solution : (a) We know that earth revolves about its own axis from west to east. Let its angular speed is ω_e and the angular speed of the train is ω_t

For train A : Net angular speed = $(\omega_e - \omega_t)$ because the sense of rotation of train is opposite to that of earth

So reaction of track $R_1 = F_1 = mg - m(\omega_e - \omega_t)^2 R$

For train B : Net angular speed = $(\omega_e + \omega_t)$ because the sense of rotation of train is same as that of earth

So reaction of track $R_2 = F_2 = mg - m(\omega_e + \omega_t)^2 R$

So it is clear that $F_1 > F_2$

3.19 Motion of Charged Particle in Magnetic Field

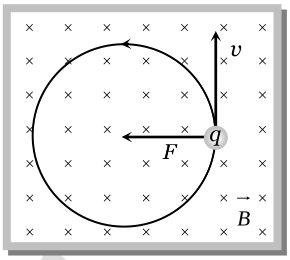
When a charged particle having mass m, charge q enters perpendicularly in a magnetic field B, with velocity v then it describes a circular path of radius r.

Because magnetic force (qvB) works in the perpendicular direction of v and it provides required centripetal force

Magnetic force = Centripetal force

$$qvB = \frac{mv^2}{r}$$

$\therefore$ radius of the circular path $r = \frac{mv}{qB}$

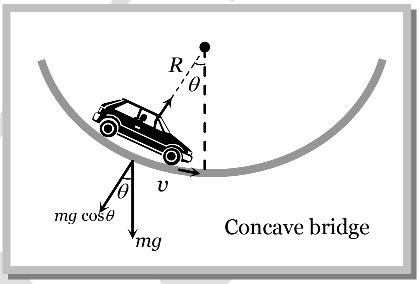


3.20 Reaction of Road on Car

(1) When car moves on a concave bridge then

Centripetal force = $R - mg\cos\theta = \frac{mv^2}{r}$

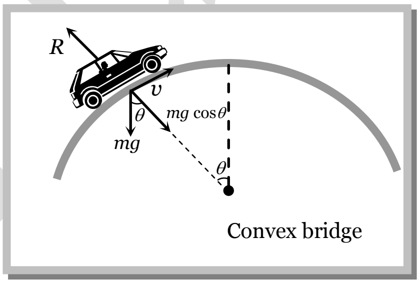
and reaction $R = mg\cos\theta + \frac{mv^2}{r}$



(2) When car moves on a convex bridge

Centripetal force = $mg\cos\theta - R = \frac{mv^2}{r}$

and reaction $R = mg\cos\theta - \frac{mv^2}{r}$



Sample problem based on reaction of road

***Problem* 139.** The road way bridge over a canal is in the form of an arc of a circle of radius $20m$. What is the minimum speed with which a car can cross the bridge without leaving contact with the ground at the highest point $(g = 9.8\ m/s^2)$

(a) $7\ m/s$ (b) $14\ m/s$ (c) $289\ m/s$ (d) $5\ m/s$

Solution : (b) At the highest point of the bridge for critical condition $mg - \frac{mv^2}{r} = 0 \Rightarrow \frac{mv^2}{r} = mg$

$\therefore v_{max} = \sqrt{gr} = \sqrt{9.8 \times 20} = \sqrt{196} = 14\ m/s$

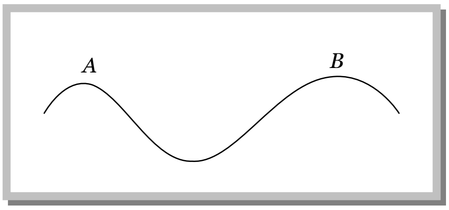
***Problem* 140.** A car moves at a constant speed on a road as shown in the figure. The normal force exerted by the road on the car is N_A and N_B when it is at the points A and B respectively

(a) $N_A = N_B$

(b) $N_A > N_B$

(c) $N_A < N_B$

(d) All possibilities are there



Solution : (c) From the formula $N = mg - \frac{mv^2}{r}$ $\therefore N \propto r$

As $r_A < r_B$ $\therefore$ $N_A < N_B$

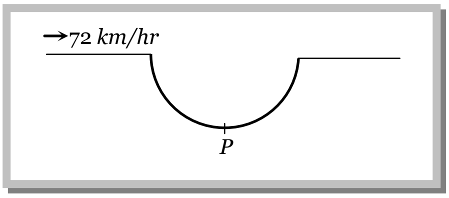
Problem 141. A car while travelling at a speed of $72\ km/hr$. Passes through a curved portion of road in the form of an arc of a radius $10m$. If the mass of the car is $500\ kg$ the reaction of the car at the lowest point P is

(a) $25\ kN$

(b) $50\ kN$

(c) $75\ kN$

(d) None of these



Solution : (a) $v = 72\frac{km}{h} = 20\ m/s,\quad r = 10\ m,\quad m = 500\ kg$ (given)

Reaction at lowest point $R = mg + \frac{mv^2}{r}$

$$= 500 \times 10 + \frac{500 \times (20)^2}{10} = 25000\ N = 25\ KN$$

3.21 Non-Uniform Circular Motion

If the speed of the particle in a horizontal circular motion changes with respect to time, then its motion is said to be non-uniform circular motion.

Consider a particle describing a circular path of radius r with centre at O. Let at an instant the particle be at P and $\vec{\upsilon}$ be its linear velocity and $\vec{\omega}$ be its angular velocity.

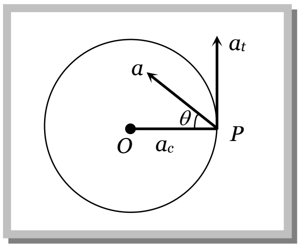
Then, $\vec{\upsilon} = \vec{\omega} \times \vec{r}$ .....(i)

Differentiating both sides of *w.r.t.* time t we have

$\frac{d\vec{\upsilon}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$ .....(ii) Here, $\frac{d\vec{v}}{dt} = \vec{a}$, (Resultant acceleration)

$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\upsilon}$ $\qquad \frac{d\vec{\omega}}{dt} = \vec{\alpha}$ (Angular acceleration)

$\vec{a} = \vec{a}_t + \vec{a}_c$ .....(iii) $\qquad \frac{d\vec{r}}{dt} = \vec{\upsilon}$ (Linear velocity)



Thus the resultant acceleration of the particle at P has two component accelerations

(1) **Tangential acceleration :** $\vec{a}_t = \vec{\alpha} \times \vec{r}$

It acts along the tangent to the circular path at P in the plane of circular path.

According to right hand rule since $\vec{\alpha}$ and $\vec{r}$ are perpendicular to each other, therefore, the magnitude of tangential acceleration is given by

$$|\vec{a}_t| = |\vec{\alpha} \times \vec{r}| = \alpha r \sin 90^o = \alpha r.$$

(2) **Centripetal (Radial) acceleration :** $\overrightarrow{a_c} = \vec{\omega} \times \vec{v}$

It is also called centripetal acceleration of the particle at P.

It acts along the radius of the particle at P.

According to right hand rule since $\vec{\omega}$ and $\vec{\upsilon}$ are perpendicular to each other, therefore, the magnitude of centripetal acceleration is given by

$$|\vec{a}_c| = |\vec{\omega} \times \vec{\upsilon}| = \omega \upsilon \sin 90^o = \omega \upsilon = \omega(\omega r) = \omega^2 r = \upsilon^2 / r$$

(3) **Tangential and centripetal acceleration in different motions**

Centripetal acceleration	Tangential acceleration	Net acceleration	Type of motion
$a_c = 0$	$a_t = 0$	$a = 0$	Uniform translatory motion
$a_c = 0$	$a_t \neq 0$	$a = a_t$	Accelerated translatory motion
$a_c \neq 0$	$a_t = 0$	$a = a_c$	Uniform circular motion
$a_c \neq 0$	$a_t \neq 0$	$a = \sqrt{a_c^2 + a_t^2}$	Non-uniform circular motion

Note : ❑ Here a_t governs the magnitude of $\vec{v}$ while $\vec{a}_c$ its direction of motion.

(4) **Force :** In non-uniform circular motion the particle simultaneously possesses two forces

Centripetal force : $F_c = ma_c = \dfrac{mv^2}{r} = mr\omega^2$

Tangential force : $F_t = ma_t$

Net force : $F_{\text{net}} = ma = m\sqrt{a_c^2 + a_t^2}$

Note : ❑ In non-uniform circular motion work done by centripetal force will be zero since $\vec{F}_c \perp \vec{v}$

❑ In non uniform circular motion work done by tangential of force will not be zero since $F_t \neq 0$

❑ Rate of work done by net force in non-uniform circular = rate of work done by tangential force

i.e. $P = \dfrac{dW}{dt} = \vec{F}_t . \vec{v}$

Sample problems based on non-uniform circular motion

***Problem* 142.** The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered. It is given as $K.E. = as^2$ where a is a constant. The force acting on the particle is **[MNR 1992; JIPMER 2001, 2002]**

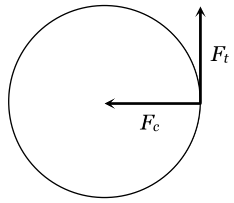
(a) $2a\dfrac{s^2}{R}$ (b) $2as\left(1 + \dfrac{s^2}{R^2}\right)^{1/2}$ (c) $2as$ (d) $2a\dfrac{R^2}{s}$

Solution : (b) In non-uniform circular motion two forces will work on a particle F_c and F_t

So the net force $F_{Net} = \sqrt{F_c^2 + F_t^2}$ ....(i)

Centripetal force $F_c = \dfrac{mv^2}{R} = \dfrac{2as^2}{R}$ ....(ii) [As kinetic energy $\dfrac{1}{2}mv^2 = as^2$ given]

Again from : $\frac{1}{2}mv^2 = as^2 \Rightarrow v^2 = \frac{2as^2}{m} \Rightarrow v = s\sqrt{\frac{2a}{m}}$



Tangential acceleration $a_t = \frac{dv}{dt} = \frac{dv}{ds}.\frac{ds}{dt} \Rightarrow a_t = \frac{d}{ds}\left[s\sqrt{\frac{2a}{m}}\right].v$

$a_t = v\sqrt{\frac{2a}{m}} = s\sqrt{\frac{2a}{m}}\sqrt{\frac{2a}{m}} = \frac{2as}{m}$

and $F_t = ma_t = 2as$ ....(iii)

Now substituting value of F_c and F_t in equation (i) $\therefore F_{Net} = \sqrt{\left(\frac{2as^2}{R}\right)^2 + (2as)^2} = 2as\left[1 + \frac{s^2}{R^2}\right]^{1/2}$

Problem 143. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is

(a) $2\pi mk^2r^2t$ (b) mk^2r^2t (c) $\frac{mk^4r^2t^5}{3}$ (d) Zero

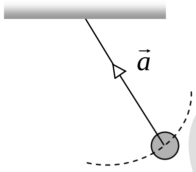
Solution : (b) $a_c = k^2rt^2 \Rightarrow \frac{v^2}{r} = k^2rt^2 \Rightarrow v^2 = k^2r^2t^2 \Rightarrow v = krt$

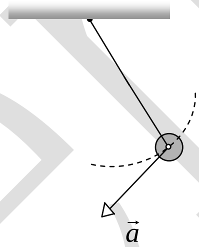
Tangential acceleration $a_t = \frac{dv}{dt} = kr$

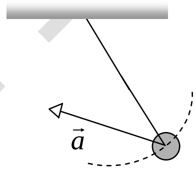
As centripetal force does not work in circular motion.

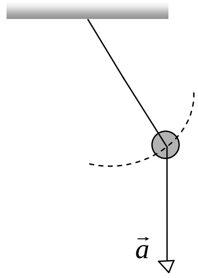
So power delivered by tangential force $P = F_tv = ma_tv = m(kr)\,krt = mk^2r^2t$

Problem 144. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in

(a) 

(b) 

(c) 

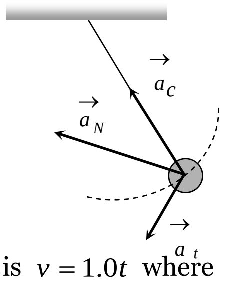
(d) 

Solution : (c) a_c = centripetal acceleration,

a_t = tangential acceleration,

a_N = net acceleration = Resultant of a_c and a_t

$a_N = \sqrt{a_c^2 + a_t^2}$



Problem 145. The speed of a particle moving in a circle of radius $0.1m$ is $v = 1.0t$ where t is time in second. The resultant acceleration of the particle at $t = 5s$ will be

(a) $10\ m/s^2$ (b) $100\ m/s^2$ (c) $250\ m/s^2$ (d) $500\ m/s^2$

Solution : (c) $v = 1.0t \Rightarrow a_t = \frac{dv}{dt} = 1m/s^2$

and $a_c = \frac{v^2}{r} = \frac{(5)^2}{0.1} = 250\ m/s^2$ [At $t = 5$ sec, $v = 5\ m/s$]

$\therefore a_N = \sqrt{a_c^2 + a_t^2} = \sqrt{(250)^2 + 1^2} \Rightarrow a_N = 250\ m/s^2$ (approx.)

Problem 146. A particle moving along the circular path with a speed v and its speed increases by 'g' in one second. If the radius of the circular path be r, then the net acceleration of the particle is

(a) $\frac{v^2}{r}+g$ (b) $\frac{v^2}{r^2}+g^2$ (c) $\left[\frac{v^4}{r^2}+g^2\right]^{\frac{1}{2}}$ (d) $\left[\frac{v^2}{r}+g\right]^{\frac{1}{2}}$

Solution : (c) $a_t = g$ (given) and $a_c = \frac{v^2}{r}$ and $a_N = \sqrt{a_t^2+a_c^2} = \sqrt{\left(\frac{v^2}{r}\right)^2+g^2} = \sqrt{\frac{v^4}{r^2}+g^2}$

Problem 147. A car is moving with speed 30 m/sec on a circular path of radius 500 m. Its speed is increasing at the rate of 2m/sec^2. What is the acceleration of the car **[Roorkee 1982; RPET 1996; MH CET 2002; MP PMT 2003]**

(a) 2 m/s^2 (b) 2.7 m/s^2 (c) 1.8 m/s^2 (d) 9.8 m/s^2

Solution : (b) $a_t = 2m/s^2$ and $a_c = \frac{v^2}{r} = \frac{30\times 30}{500} = 1.8\, m/s^2$ $\therefore$ $a = \sqrt{a_t^2+a_c^2} = \sqrt{2^2+(1.8)^2} = 2.7 m/s^2$.

Problem 148. For a particle in circular motion the centripetal acceleration is **[CPMT 1998]**

(a) Less than its tangential acceleration (b) Equal to its tangential acceleration

(c) More than its tangential acceleration (d) May be more or less than its tangential acceleration

Solution : (d)

Problem 149. A particle is moving along a circular path of radius 3 *meter* in such a way that the distance travelled measured along the circumference is given by $S = \frac{t^2}{2}+\frac{t^3}{3}$. The acceleration of particle when $t = 2$ sec is

(a) 1.3 m/s^2 (b) 13 m/s^2 (c) 3 m/s^2 (d) 10 m/s^2

Solution : (b) $s = \frac{t^2}{2}+\frac{t^3}{3} \Rightarrow v = \frac{ds}{dt} = t+t^2$ and $a_t = \frac{dv}{dt} = \frac{d}{dt}(t+t^2) = 1+2t$

At t = 2 sec, v = 6 m/s and $a_t = 5\, m/s^2$, $a_c = \frac{v^2}{r} = \frac{36}{3} = 12\, m/s^2$

$a_N = \sqrt{a_c^2+a_t^2} = \sqrt{(12)^2+(5)^2} = 13\, m/s^2$.

3.22 Equations of Circular Motion

For accelerated motion	For retarded motion
$\omega_2 = \omega_1 + \alpha t$	$\omega_2 = \omega_1 - \alpha t$
$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$	$\theta = \omega_1 t - \frac{1}{2}\alpha t^2$
$\omega_2^2 = \omega_1^2 + 2\alpha\theta$	$\omega_2^2 = \omega_1^2 - 2\alpha\theta$
$\theta_n = \omega_1 + \frac{\alpha}{2}(2n-1)$	$\theta_n = \omega_1 - \frac{\alpha}{2}(2n-1)$

Where

ω_1 = Initial angular velocity of particle

ω_2 = Final angular velocity of particle

α = Angular acceleration of particle

θ = Angle covered by the particle in time t

θ_n = Angle covered by the particle in n^{th} second

~~Sample problems based on equation of circular motion~~

Problem 150. The angular velocity of a particle is given by $\omega = 1.5\, t - 3t^2 + 2$, the time when its angular acceleration ceases to be zero will be

(a) 25 sec (b) 0.25 sec (c) 12 sec (d) 1.2 sec

Solution : (b) $\omega = 1.5t - 3t^2 + 2$ and $\alpha = \frac{d\omega}{dt} = 1.5 - 6t \Rightarrow 0 = 1.5 - 6t \therefore t = \frac{1.5}{6} = 0.25\,\text{sec}$

***Problem* 151.** A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first $2\,\text{sec}$, it rotates through an angle θ_1. In the next $2\,\text{sec}$, it rotates through an additional angle θ_2. The ratio of θ_1/θ_2 is **[AIIMS 1982]**

(a) 1 (b) 2 (c) 3 (d) 5

Solution : (c) From equation of motion $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$\theta_1 = 0 + \frac{1}{2}\alpha(2)^2 = 2\alpha$ .....(i) [As $\omega_1 = 0,\ t = 2\,\text{sec},\ \theta = \theta_1$]

For second condition

$\theta_1 + \theta_2 = 0 + \frac{1}{2}\alpha(4)^2$ [As $\omega_1 = 0,\ t = 2 + 2 = 4\,\text{sec},\ \theta = \theta_1 + \theta_2$]

$\theta_1 + \theta_2 = 8\alpha$ ....(ii)

From (i) and (ii) $\theta_1 = 2\alpha,\ \theta_2 = 6\alpha \therefore \frac{\theta_2}{\theta_1} = 3$

***Problem* 152.** If the equation for the displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after $2\,\text{sec}$ from its start is **[AIIMS 1998]**

(a) $8\ rad/\text{sec}$ (b) $12\ rad/\text{sec}$ (c) $24\ rad/\text{sec}$ (d) $36\ rad/\text{sec}$

Solution : (c) $\theta = 2t^3 + 0.5$ and $\omega = \frac{d\theta}{dt} = 6t^2$

at $t = 2\ sec,\ \omega = 6(2)^2 = 24\ rad/sec$

***Problem* 153.** A grinding wheel attained a velocity of 20 *rad/sec* in 5 sec starting from rest. Find the number of revolutions made by the wheel

(a) $\frac{\pi}{25}$ *rev/ sec* (b) $\frac{1}{\pi}$ *rev/sec* (c) $\frac{25}{\pi}$ *rev/sec* (d) None of these

Solution : (c) $\omega_1 = 0,\ \omega_2 = 20\ rad/sec,\ t = 5\ sec$

$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{20 - 0}{5} = 4\ rad/sec^2$

From the equation $\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(4).(5)^2 = 50\ rad$

$2\pi\ rad$ means 1 revolution. $\therefore$ 50 *Radian* means $\frac{50}{2\pi}$ or $\frac{25}{\pi}$ *rev.*

***Problem* 154.** A grind stone starts from rest and has a constant angular acceleration of 4.0 *rad/sec²*. The angular displacement and angular velocity, after 4 *sec.* will respectively be

(a) 32 *rad*, 16 *rad/sec* (b) 16 *rad*, 32 *rad/sec* (c) 64 *rad*, 32 *rad/sec* (d) 32 *rad*, 64 *rad/sec*

Solution : (a) $\omega_1 = 0,\ \alpha = 4\ rad/\text{sec}^2,\ t = 4\ sec$

Angular displacement $\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}4(4)^2 = 32\ rad.$

$\therefore$ Final angular $\omega_2 = \omega_1 + \alpha t = 0 + 4 \times 4 = 16\ rad/sec$

***Problem* 155.** An electric fan is rotating at a speed of 600 *rev/minute*. When the power supply is stopped, it stops after 60 revolutions. The time taken to stop is

(a) 12 *s* (b) 30 *s* (c) 45 *s* (d) 60 *s*

Solution : (a) $\omega_1 = 600 \ rev/\min = 10 \, rev/sec, \ \omega_2 = 0$ and $\theta = 60 \ rev$

From the equation $\omega_2^2 = \omega_1^2 - 2\alpha\theta \Rightarrow 0 = (10)^2 - 2\alpha 60 \therefore \alpha = \frac{100}{120} = \frac{5}{6}$

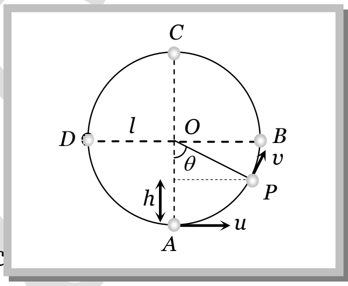
Again $\omega_2 = \omega_1 - \alpha t \Rightarrow 0 = \omega_1 - \alpha t$

$t = \frac{\omega_1}{\alpha} = \frac{10 \times 6}{5} = 12 \, sec$.

3.23 Motion in Vertical Circle

This is an example of non-uniform circular motion. In this motion body is under the influence of gravity of earth. When body moves from lowest point to highest point. Its speed decrease and becomes minimum at highest point. Total mechanical energy of the body remains conserved and *KE* converts into *PE* and vice versa.

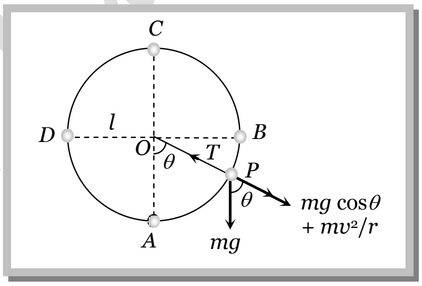
(1) **Velocity at any point on vertical loop :** If *u* is the initial velocity imparted to body at lowest point then. Velocity of body at height *h* is given by



$$v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2gl(1 - \cos\theta)} \qquad [\text{As } h = l - l\cos\theta = l(1 - \cos\theta)]$$

where *l* in the length of the string

(2) **Tension at any point on vertical loop** : Tension at general point *P*, Ac d law of motion.



Net force towards centre = centripetal force

$$T - mg \cos\theta = \frac{mv^2}{l} \quad \text{or} \quad T = mg\cos\theta + \frac{mv^2}{l}$$

$$T = \frac{m}{l}[u^2 - gl(2 - 3\cos\theta)] \qquad [\text{As } v = \sqrt{u^2 - 2gl(1 - \cos\theta)}\,]$$

(3) **Velocity and tension in a vertical loop at different positions**

Position	Angle	Velocity	Tension
A	0°	u	$\frac{mu^2}{l} + mg$
B	90°	$\sqrt{u^2 - 2gl}$	$\frac{mu^2}{l} - 2mg$
C	180°	$\sqrt{u^2 - 4gl}$	$\frac{mu^2}{l} - 5mg$
D	270°	$\sqrt{u^2 - 2gl}$	$\frac{mu^2}{l} - 2mg$

It is clear from the table that : $T_A > T_B > T_C$ and $T_B = T_D$

$T_A - T_B = 3mg$,

$T_A - T_C = 6mg$

and $T_B - T_C = 3mg$

(4) **Various conditions for vertical motion :**

Velocity at lowest point	Condition
$u_A > \sqrt{5gl}$	Tension in the string will not be zero at any of the point and body will continue the circular motion.
$u_A = \sqrt{5gl}$,	Tension at highest point C will be zero and body will just complete the circle.
$\sqrt{2gl} < u_A < \sqrt{5gl}$,	Particle will not follow circular motion. Tension in string become zero somewhere between points *B* and *C* whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory.
$u_A = \sqrt{2gl}$	Both velocity and tension in the string becomes zero between *A* and *B* and particle will oscillate along semi-circular path.
$u_A < \sqrt{2gl}$	velocity of particle becomes zero between *A* and *B* but tension will not be zero and the particle will oscillate about the point *A*.

Note : ❑ *K.E.* of a body moving in horizontal circle is same throughout the path but the *K.E.* of the body moving in vertical circle is different at different places.

❑ If body of mass *m* is tied to a string of length *l* and is projected with a horizontal velocity *u* then :

Height at which the velocity vanishes is $h = \frac{u^2}{2g}$

Height at which the tension vanishes is $h = \frac{u^2 + gl}{3g}$

(5) **Critical condition for vertical looping :** If the tension at *C* is zero, then body will just complete revolution in the vertical circle. This state of body is known as critical state. The speed of body in critical state is called as critical speed.

From the above table $T_C = \frac{mu^2}{l} - 5mg = 0 \Rightarrow u = \sqrt{5gl}$

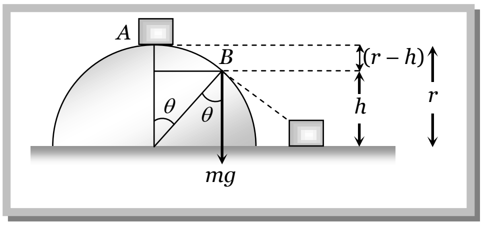
It means to complete the vertical circle the body must be projected with minimum velocity of $\sqrt{5gl}$ at the lowest point.

(6) **Various quantities for a critical condition in a vertical loop at different positions :**

Quantity	Point *A*	Point *B*	Point *C*	Point *D*	Point *P*
Linear velocity (v)	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$	$\sqrt{3gl}$	$\sqrt{gl(3+2\cos\theta)}$
Angular velocity (ω)	$\sqrt{\frac{5g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}}$	$\sqrt{\frac{3g}{l}}$	$\sqrt{\frac{g}{l}(3+2\cos\theta)}$
Tension in String (T)	6 mg	3 mg	0	3 mg	$3mg(1+\cos\theta)$
Kinetic Energy (KE)	$\frac{5}{2}mgl$	$\frac{3}{2}mgl$	$\frac{1}{2}mgl$	$\frac{3}{2}mgl$	$\frac{mgl}{2}(3+2\cos\theta)$
Potential Energy (PE)	0	mgl	2 mgl	mgl	$mgl(1-\cos\theta)$
Total Energy (TE)	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$	$\frac{5}{2}mgl$

(7) **Motion of a block on frictionless hemisphere** : A small block of mass *m* slides down from the top of a frictionless hemisphere of radius *r*. The component of the force of gravity ($mg\cos\theta$) provides required centripetal force but at point *B* it's circular motion ceases and the block lose contact with the surface of the sphere.

For point B, by equating the forces, $mg\cos\theta = \frac{mv^2}{r}$ .....(i)



For point A and B, by law of conservation of energy

Total energy at point A = Total energy at point B

K.E.$_{(A)}$ + P.E.$_{(A)}$ = K.E.$_{(B)}$ + P.E.$_{(B)}$

$$0 + mgr = \frac{1}{2}mv^2 + mgh \Rightarrow v = \sqrt{2g(r-h)} \text{ .....(ii)}$$

and from the given figure $h = r\cos\theta$ .....(iii)

By substituting the value of v and h from eq^n (ii) and (iii) in eq^n (i)

$$mg\left(\frac{h}{r}\right) = \frac{m}{r}\left(\sqrt{2g(r-h)}\right)^2$$

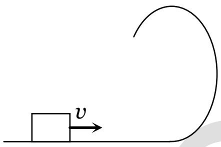
$$\Rightarrow \quad h = 2(r-h) \Rightarrow h = \frac{2}{3}r$$

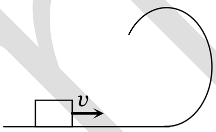
i.e. the block lose contact at the height of $\frac{2}{3}r$ from the ground.

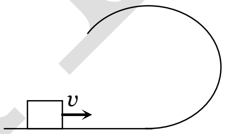
and angle from the vertical can be given by $\cos\theta = \frac{h}{r} = \frac{2}{3}$ $\quad \therefore \; \theta = \cos^{-1}\frac{2}{3}$.

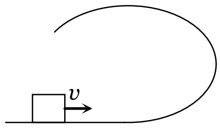
Sample problems based on vertical looping

***Problem* 156.** A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in

(a) 

(b) 

(c) 

(d) 

Solution : (a) Normal reaction at the highest point of the path $R = \frac{mv^2}{r} - mg$

For maximum R, value of the radius of curvature (r) should be minimum and it is minimum in first condition.

***Problem* 157.** A stone tied to string is rotated in a vertical circle. The minimum speed with which the string has to be rotated

[EAMCET (Engg.) 1998; CBSE PMT 1999]

(a) Decreases with increasing mass of the stone (b) Is independent of the mass of the stone

(c) Decreases with increasing in length of the string (d) Is independent of the length of the string

Solution : (b) $v = \sqrt{5gr}$ for lowest point of vertical loop.

$v \propto m^0$ *i.e.* it does not depends on the mass of the body.

***Problem* 158.** A mass m is revolving in a vertical circle at the end of a string of length 20 *cms*. By how much does the tension of the string at the lowest point exceed the tension at the topmost point

(a) 2 mg (b) 4 mg (c) 6 mg (d) 8 mg

Solution : (c) $T_{\text{Lowest point}} - T_{\text{Highest point}} = 6\,mg$ (Always)

Problem 159. In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. The string breaks when it makes an angle θ with the vertical

(a) $\theta = \cos^{-1}(1/3)$ (b) $\theta = 60^o$ (c) $\theta = \cos^{-1}(2/3)$ (d) $\theta = 0^o$

Solution : (c) Let the string breaks at point *B*.

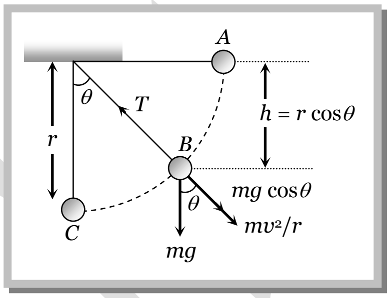
Tension $= mg\cos\theta + \dfrac{m v_B^2}{r}$ = Breaking strength

$$= mg\cos\theta + \frac{m v_B^2}{r} = 2mg \quad ....(i)$$

If the bob is released from rest (from point *A*) then velocity acquired by it at point *B*

$$v_B = \sqrt{2gh}$$

$$v_B = \sqrt{2gr\cos\theta} \quad ....(ii) \quad [\text{As } h = r\cos\theta]$$

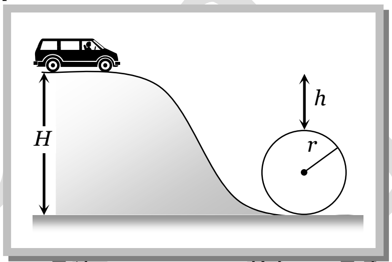


By substituting this value in equation (i)

$$mg\cos\theta + \frac{m}{r}(2gr\cos\theta) = 2mg$$

or $3mg\cos\theta = 2mg \Rightarrow \cos\theta = \dfrac{2}{3}$ $\therefore \theta = \cos^{-1}\left(\dfrac{2}{3}\right)$

Problem 160. A toy car rolls down the inclined plane as shown in the fig. It goes around the loop at the bottom. What is the relation between H and h



(a) $\dfrac{H}{h} = 2$ (b) h (c) h (d) $\dfrac{H}{h} = 5$

Solution : (d) When car rolls down the inclined plane from height *H*, then velocity acquired by it at the lowest point

$$v = \sqrt{2gH} \quad ....(i)$$

and for looping of loop, velocity at the lowest point should be $v = \sqrt{5gr}$ ....(ii)

From eq^n (i) and (ii) $v = \sqrt{2gH} = \sqrt{5gr}$ $\therefore H = \dfrac{5r}{2}$ ....(iii)

From the figure $H = h + 2r \Rightarrow r = \dfrac{H-h}{2}$

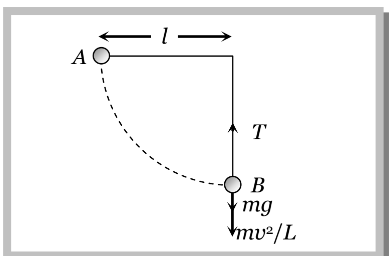
Substituting the value of *r* in equation (iii) we get $H = \dfrac{5}{2}\left[\dfrac{H-h}{2}\right] \Rightarrow \dfrac{H}{h} = 5$

Problem 161. The mass of the bob of a simple pendulum of length L is m. If the bob is left from its horizontal position then the speed of the bob and the tension in the thread in the lowest position of the bob will be respectively.

(a) $\sqrt{2gL}$ and $3mg$ (b) $3mg$ and $\sqrt{2gL}$ (c) $2mg$ and $\sqrt{2gl}$ (d) $2gl$ and $3mg$

Solution : (a) By the conservation of energy

Potential energy at point *A* = Kinetic energy at point *B*



$mg\,l = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gl}$

and tension $= mg + \frac{mv^2}{l} \Rightarrow T = mg + \frac{m}{l}(2gl) \Rightarrow T = 3mg$

Problem 162. A stone of mass m is tied to a string and is moved in a vertical circle of radius r making n revolutions per minute. The total tension in the string when the stone is at its lowest point is **[Kerala (Engg.) 2001]**

(a) $m\{g + (\pi^2 n^2 r)/900\}$ (b) $m(g + \pi nr^2)$ (c) $m(g + \pi nr)$ (d) $m(g + n^2 r^2)$

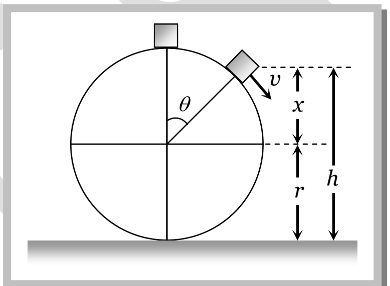
Solution : (a) Tension at lowest point $T = mg + m\omega^2 r = mg + m4\pi^2 n^2 r$

If n is revolution per minute then $T = mg + m4\pi^2 \frac{n^2}{3600} r = mg + \frac{m\pi^2 n^2 r}{900} = m\left[g + \frac{\pi^2 n^2 r}{900}\right]$

Problem 163. A particle is kept at rest at the top of a sphere of diameter 42m. When disturbed slightly, it slides down. At what height h from the bottom, the particle will leave the sphere **[IMS-BHU 2003]**

(a) 14 m (b) 28 m (c) 35 m (d) 7 m

Solution : (c) Let the particle leave the sphere at height 'h' from the bottom



We know for given condition $x = \frac{2}{3}r$

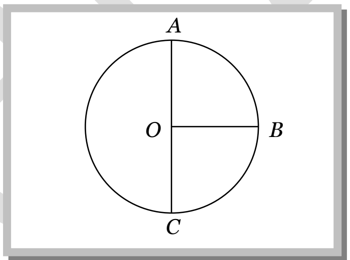
and $h = r + x = r + \frac{2}{3}r = \frac{5}{3}r = \frac{5}{3} \times 21 = 35\,m$ [As r= 21 m]

Problem 164. A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill, when the bucket is at the highest position (Take g = 10 m/sec^2)

(a) $4\,m/\text{sec}$ (b) $6.25\,m/sec$ (c) $16m/sec$ (d) None of these

Solution : (a) $v = \sqrt{gr} = \sqrt{10 \times 1.6} = \sqrt{16} = 4\,m/s$

Problem 165. The ratio of velocities at points A, B and C in vertical circular motion is



(a) $1:9:25$ (b) $1:2:3$ (c) $1:3:5$ (d) $1:\sqrt{3}:\sqrt{5}$

Solution : (d) $v_A : v_B : v_c = \sqrt{gr} : \sqrt{3gr} : \sqrt{5gr} = 1 : \sqrt{3} : \sqrt{5}$

Problem 166. The minimum speed for a particle at the lowest point of a vertical circle of radius R, to describe the circle is 'v'. If the radius of the circle is reduced to one-fourth its value, the corresponding minimum speed will be

[EAMCET (Engg.) 1999]

(a) $\frac{v}{4}$ (b) $\frac{v}{2}$ (c) $2v$ (d) $4v$

Solution : (b) $v = \sqrt{5gr}$ $\therefore v \propto \sqrt{r}$ So $\frac{v_2}{v_1} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{r/4}{r}} = \frac{1}{2} \Rightarrow v_2 = v/2$

Problem 167. A body slides down a frictionless track which ends in a circular loop of diameter D, then the minimum height h of the body in term of D so that it may just complete the loop, is **[AIIMS 2000]**

(a) $h = \frac{5D}{2}$ (b) $h = \frac{5D}{4}$ (c) $h = \frac{3D}{4}$ (d) $h = \frac{D}{4}$

Solution : (b) We know $h = \frac{5}{2}r = \frac{5}{2}\left(\frac{D}{2}\right) = \frac{5D}{4}$ [For critical condition of vertical looping]

Problem **168.** A can filled with water is revolved in a vertical circle of radius 4*m* and the water just does not fall down. The time period of revolution will be **[CPMT 1985; RPET 1999]**

(a) 1 *sec* (b) 10 *sec* (c) 8 *sec* (d) 4 *sec*

Solution : (d) At highest point $mg = m\omega^2 r \Rightarrow g = \frac{4\pi^2}{T^2}r \Rightarrow 10 = \frac{4\pi^2 4}{T^2} \Rightarrow T^2 = 16 \quad \therefore T = 4\ sec$

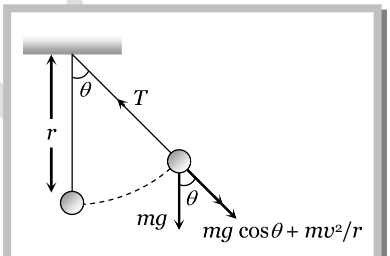
Problem **169.** A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from vertical (lowest position) are T_1 and T_2 respectively, Then

(a) $T_1 = T_2$ (b) $T_1 > T_2$ (c) $T_1 < T_2$ (d) $T_1 \geq T_2$

Solution : (b) $T = mg\cos\theta + \frac{mv^2}{r}$

As θ increases T decreases

So $T_1 > T_2$



Problem **170.** A mass of $2kg$ is tied to the end of a string of length 1*m*. It is, th… with a constant speed of $5\ ms^{-1}$. Given that $g = 10\ ms^{-2}$. At which of the following locations of tension in the string will be $70\ N$

(a) At the top (b) At the bottom

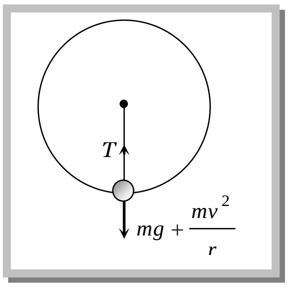
(c) When the string is horizontal (d) At none of the above locations

Solution : (b) Centrifugal force $F = \frac{mv^2}{r} = \frac{2 \times (5)^2}{1} = 50\ Newton$

Weight $= mg = 2 \times 10 = 20\ Newton$

Tension = 70 *N* (sum of above two forces)

i.e. the mass is at the bottom of the vertical circular path



Problem **171.** With what angular velocity should a 20 *m* long cord be rotated such that tension in it, while reaching the highest point, is zero

(a) 0.5 *rad/sec* (b) 0.2 *rad/sec* (c) 7.5 *rad/sec* (d) 0.7 *rad/sec*

Solution : (d) $\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{10}{20}} = \sqrt{0.5} = 0.7\ rad/sec$

Problem **172.** A body of mass of $100g$ is attached to a $1m$ long string and it is revolving in a vertical circle. When the string makes an angle of 60^o with the vertical then its speed is $2\ m/s$. The tension in the string at $\theta = 60^o$ will be

(a) $89\ N$ (b) $0.89\ N$ (c) $8.9\ N$ (d) $0.089\ N$

Solution : (b) $T = mg\cos\theta + \frac{mv^2}{r} = 0.1 \times 9.8 \times \cos 60 + \frac{0.1 \times (2)^2}{1} = 0.49 + 0.4 = 0.89\ Newton$

Problem **173.** A body of mass $2kg$ is moving in a vertical circle of radius $2m$. The work done when it moves from the lowest point to the highest point is

(a) 80 J (b) 40 J (c) 20 J (d) 0

Solution : (a) work done = change in potential energy $= 2mgr = 2 \times 2 \times 10 \times 2 = 80\ J$

Problem **174.** A body of mass m is tied to one end of a string of length l and revolves vertically in a circular path. At the lowest point of circle, what must be the $K.E.$ of the body so as to complete the circle **[RPMT 1996]**

(a) 5 mgl (b) 4 mgl (c) 2.5 mgl (d) 2 mgl

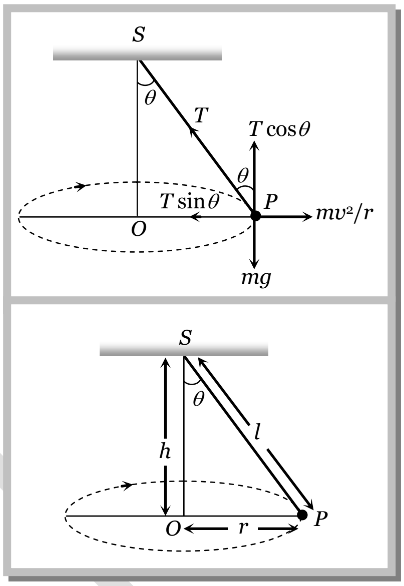
Solution : (c) Minimum velocity at lowest point to complete vertical loop $= \sqrt{5gl}$

So minimum kinetic energy $= \frac{1}{2}m(v^2) = \frac{1}{2}m(\sqrt{5gl})^2 = \frac{5}{2}mgl = 2.5\,mgl$

3.24 Conical Pendulum

This is the example of uniform circular motion in horizontal plane.

A bob of mass m attached to a light and in-extensible string rotates in a horizontal circle of radius r with constant angular speed ω about the vertical. The string makes angle θ with vertical and appears tracing the surface of a cone. So this arrangement is called conical pendulum.



The force acting on the bob are tension and weight of the bob.

From the figure $T\sin\theta = \dfrac{mv^2}{r}$ ....(i)

and $T\cos\theta = mg$ ....(ii)

(1) Tension in the string : $T = mg\sqrt{1+\left(\dfrac{v^2}{rg}\right)^2}$

$$T = \frac{mg}{\cos\theta} = \frac{mgl}{\sqrt{l^2-r^2}} \qquad \left[\text{As } \cos\theta = \frac{h}{l} = \frac{\sqrt{l^2-r^2}}{l}\right]$$

(2) Angle of string from the vertical : $\tan\theta = \dfrac{v^2}{rg}$

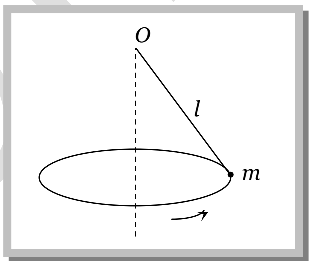
(3) Linear velocity of the bob : $v = \sqrt{gr\tan\theta}$

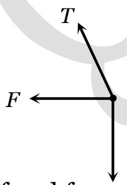
(4) Angular velocity of the bob : $\omega = \sqrt{\dfrac{g}{r}\tan\theta} = \sqrt{\dfrac{g}{h}} = \sqrt{\dfrac{g}{l\cos\theta}}$

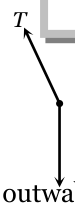
(5) Time period of revolution : $T_P = 2\pi\sqrt{\dfrac{l\cos\theta}{g}} = 2\pi\sqrt{\dfrac{h}{g}} = 2\pi\sqrt{\dfrac{l^2-r^2}{g}} = 2\pi\sqrt{\dfrac{r}{g\tan\theta}}$

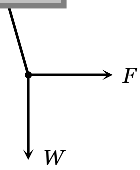
Sample problems based on conical pendulum

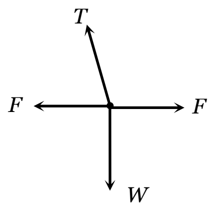
Problem **175.** A point mass m is suspended from a light thread of length l, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are



(a) 

(b) 

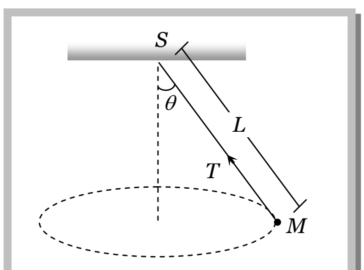


(d) 

Solution : (c) Centrifugal force (F) works radially outward,

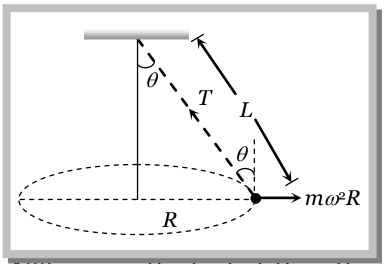
Weight (w) works downward Tension (T) work along the string and towards the point of suspension

Problem **176.** A string of length L is fixed at one end and carries a mass M at the other end. The string makes $2/\pi$ revolutions per *second* around the vertical axis through the fixed end as shown in the figure, then tension in the string is **[BHU 2002]**



(a) ML (b) $2\ ML$ (c) $4\ ML$ (d) $16\ ML$

Solution : (d) $T\sin\theta = M\omega^2 R$ ..... (i)



$T\sin\theta = M\omega^2 L\sin\theta$ ..... (ii)

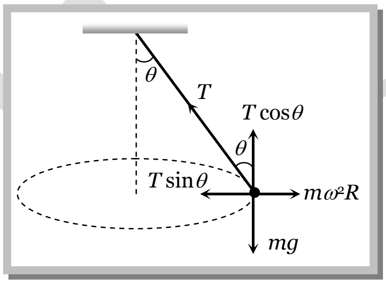
From (i) and (ii)

$T = M\omega^2 L = M4\pi^2 n^2 L = M4\pi^2\left(\frac{2}{\pi}\right)^2 L = 16ML$

Problem 177. A string of length $1m$ is fixed at one end and a mass of $100gm$ is attached at the other end. The string makes $2/\pi$ rev/sec around a vertical axis through the fixed point. The angle of inclination of the string with the vertical is ($g = 10\ m/\sec^2$)

(a) $\tan^{-1}\frac{5}{8}$ (b) $\tan^{-1}\frac{8}{5}$ (c) $\cos^{-1}\frac{8}{5}$ (d) $\cos^{-1}\frac{5}{8}$

Solution : (d) For the critical condition, in equilibrium



$T\sin\theta = m\omega^2 r$ and $T\cos\theta = mg$

$\therefore \tan\theta = \frac{\omega^2 r}{g}$

$\Rightarrow \frac{4\pi^2 n^2 r}{g} = \frac{4\pi^2 (2/\pi)^2 .1}{10} = \frac{8}{5}$

Sample problems (Miscellaneous)

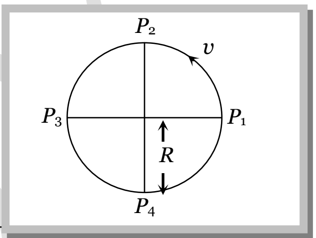
Problem 178. If the frequency of the rotating platform is f and the distance of a boy from the centre is r, which is the area swept out per second by line connecting the boy to the centre

(a) $\pi r f$ (b) $2\pi r f$ (c) $\pi r^2 f$ (d) $2\pi r^2 f$

Solution : (c) Area swept by line in complete revolution $= \pi r^2$

If frequency of rotating platform is f per second, then Area swept will be $\pi r^2 f$ per second.

Problem 179. Figure below shows a body of mass M moving with uniform speed v along a circle of radius R. What is the change in speed in going from P_1 to P_2



(a) Zero (b) $\sqrt{2}v$ (c) $v/\sqrt{2}$ (d) $2v$

Solution : (a) In uniform circular motion speed remain constant. $\therefore$ change in speed is zero.

Problem 180. In the above problem, what is change in velocity in going from P_1 to P_2

(a) Zero (b) $\sqrt{2}v$ (c) $v/\sqrt{2}$ (d) $2\ v$

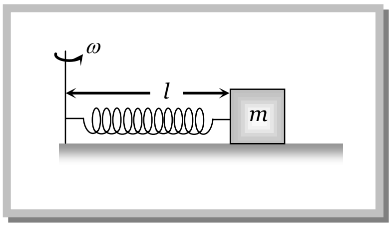
Solution : (b) Change in velocity $= 2v\sin(\theta/2) = 2v\sin\left(\frac{90}{2}\right) = 2v\sin 45 = \frac{2v}{\sqrt{2}} = \sqrt{2}\ v$

Problem 181. In the above problem, what is the change in angular velocity in going from P_1 to P_2

(a) Zero (b) $\sqrt{2}\ v/R$ (c) $v/\sqrt{2}\ R$ (d) $2v/R$

Solution : (a) Angular velocity remains constant, so change in angular velocity = Zero.

Problem 182. A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length l. The system is rotated about the other end of the spring with an angular velocity ω, in gravity free space. The increase in length of the spring will be



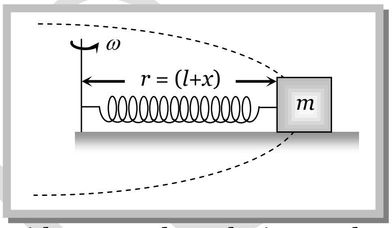
(a) $\dfrac{m\omega^2 l}{k}$

(b) $\dfrac{m\omega^2 l}{k - m\omega^2}$

(c) $\dfrac{m\omega^2 l}{k + m\omega^2}$

(d) None of these

Solution : (b) In the given condition elastic force will provides the required centripetal force



$$k\,x = m\,\omega^2 r$$

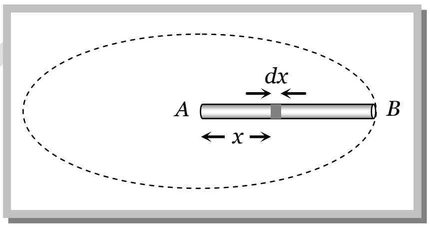
$$k\,x = m\,\omega^2 (l + x) \Rightarrow k\,x = m\,\omega^2 l + m\,\omega^2 x \;\Rightarrow\; x(k - m\,\omega^2) = m\,\omega^2 l$$

$$\therefore\; x = \frac{m\,\omega^2 l}{k - m\,\omega^2}$$

Problem 183. A uniform rod of mass m and length l rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is

(a) $\dfrac{1}{2} m\omega^2 x$ (b) $\dfrac{1}{2} m\omega^2 \dfrac{x^2}{l}$ (c) $\dfrac{1}{2} m\omega^2 l\left(1 - \dfrac{x}{l}\right)$ (d) $\dfrac{1}{2}\dfrac{m\omega^2}{l}[l^2 - x^2]$

Solution : (d) Let rod *AB* performs uniform circular motion about point *A*. We have to calculate the tension in the rod at a distance *x* from the axis of rotation. Let mass of the small segment at a distance *x* is *dm*



So $dT = dm\,\omega^2 x = \left(\dfrac{m}{l}\right) dx\,.\,\omega^2 x = \dfrac{m\,\omega^2}{l}\,[x\,d\,x]$

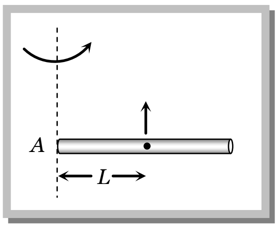
Integrating both sides $\displaystyle\int_x^l dT = \frac{m\,\omega^2}{l}\int_x^l x\,dx \Rightarrow T = \frac{m\,\omega^2}{l}\left[\frac{x^2}{2}\right]_x^l$

$$\therefore\; T = \frac{m\,\omega^2}{2l}\left[l^2 - x^2\right]$$

Problem 184. A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

(a) $\sqrt{\dfrac{\mu}{\alpha}}$ (b) $\dfrac{\mu}{\sqrt{\alpha}}$ (c) $\dfrac{1}{\sqrt{\mu\alpha}}$ (d) Infinitesimal

Solution : (a) Let the bead starts slipping after time *t*



For critical condition

Frictional force provides the centripetal force $m\,\omega^2 L = \mu\,R = \mu\,m \times a_t = \mu m L\alpha$

$m\,(\alpha t)^2 L = \mu m L\alpha \Rightarrow t = \sqrt{\dfrac{\mu}{\alpha}}$ (As $\omega = \alpha t$)

Problem 185. A smooth table is placed horizontally and an ideal spring of spring constant $k = 1000\ N/m$ and unextended length of $0.5m$ has one end fixed to its centre. The other end is attached to a mass of $5kg$ which is moving in a circle with constant speed $20m/s$. Then the tension in the spring and the extension of this spring beyond its normal length are

(a) 500 N, 0.5 m (b) 600 N, 0.6 m (c) 700 N, 0.7 m (d) 800 N, 0.8 m

Solution : (a) $k = 1000,\ m = 5\,kg,\ l = 0.5\,m,\ v = 20\,m/s$ (given)

Restoring force = $kx = \frac{mv^2}{r} = \frac{mv^2}{l+x} \Rightarrow 1000\,x = \frac{5\,(20)^2}{0.5+x} \Rightarrow x = 0.5\,m$

and Tension in the spring = $kx = 1000 \times \frac{1}{2} = 500\ N$

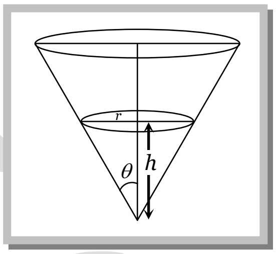
***Problem* 186.** A particle describes a horizontal circle at the mouth of a funnel type vessel as shown in figure. The surface of the funnel is frictionless. The velocity v of the particle in terms of r and θ will be

(a) $v = \sqrt{rg/\tan\theta}$

(b) $v = \sqrt{rg\,\tan\theta}$

(c) $v = \sqrt{rg\,\cot\theta}$

(d) $v = \sqrt{rg/\cot\theta}$

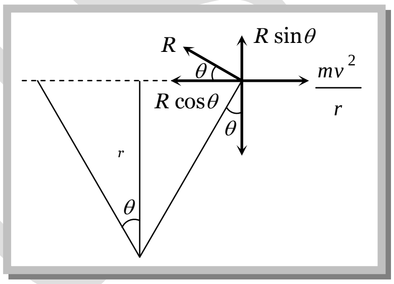


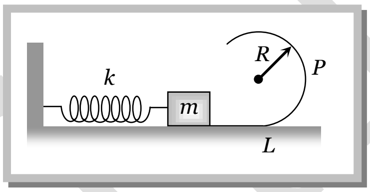
Solution : (c) For uniform circular motion of a particle $\frac{mv^2}{r} = R\cos\theta$ ....(i)

and $mg = R\sin\theta$ ....(ii)

Dividing (i) by (ii)

$\frac{v^2}{rg} = \cot\theta \Rightarrow v = \sqrt{rg\cot\theta}$





***Problem* 187.** Figure shows a smooth track, a part of which is a circle of radius R. A block of mass m is pushed against a spring constant k fixed at the left end and is then released. Find the initial compression of the spring so that the block presses the track with a force mg when it reaches the point P [see. Fig], where the radius of the track is horizontal

(a) $\sqrt{\frac{mgR}{3k}}$ (b) $\sqrt{\frac{3gR}{mk}}$ (c) $\sqrt{\frac{3mgR}{k}}$ (d) $\sqrt{\frac{3mg}{kR}}$ *Solution* :

(c) For the given condition, centrifugal force at P should be equal to mg *i.e.* $\frac{mv_P^2}{R} = mg \therefore v_P = \sqrt{Rg}$

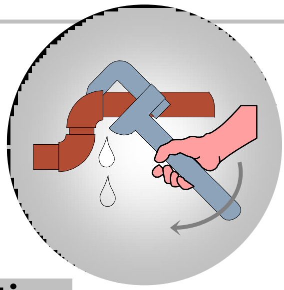
From this we can easily calculate the required velocity at the lowest point of circular track.

$v_p^2 = v_L^2 - 2gR$ (by using formula : $v^2 = u^2 - 2gh$)

$v_L = \sqrt{v_P^2 + 2gR} = \sqrt{Rg + 2gR} = \sqrt{3gR}$

It means the block should possess kinetic energy $= \frac{1}{2}mv_L^2 = \frac{1}{2}m \times 3gR$

And by the law of conservation of energy $\frac{1}{2}kx^2 = \frac{1}{2}3m \times g\,R \Rightarrow x = \sqrt{\frac{3m\,g\,R}{k}}$.



Rotational Motion

7.1 Introduction

Translation is motion along a straight line but rotation is the motion of wheels, gears, motors, planets, the hands of a clock, the rotor of jet engines and the blades of helicopters. First figure shows a skater gliding across the ice in a straight line with constant speed. Her motion is called translation but second figure shows her spinning at a constant rate about a vertical axis. Here motion is called rotation.

Up to now we have studied translatory motion of a point mass. In this chapter we will study the rotatory motion of rigid body about a fixed axis.

(1) Rigid body : A rigid body is a body that can rotate with all the parts locked together and without any change in its shape.

(2) System : A collection of any number of particles interacting with one another and are under consideration during analysis of a situation are said to form a system.

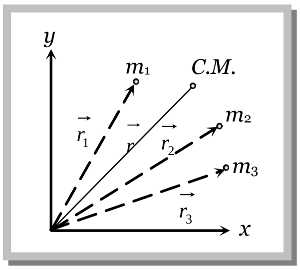
(3) Internal forces : All the forces exerted by various particles of the system on one another are called internal forces. These forces are alone enable the particles to form a well defined system. Internal forces between two particles are mutual (equal and opposite).

(4) External forces : To move or stop an object of finite size, we have to apply a force on the object from outside. This force exerted on a given system is called an external force.

7.2 Centre of Mass

Centre of mass of a system (body) is a point that moves as though all the mass were concentrated there and all external forces were applied there.

(1) **Position vector of centre of mass for *n* particle system :** If a system consists of *n* particles of masses $m_1, m_2, m_3 ...... m_n$, whose positions vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3 .... \vec{r}_n$ respectively then position vector of centre of mass



$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ... m_n\vec{r}_n}{m_1 + m_2 + m_3 + ... m_n}$$

Hence the centre of mass of *n* particles is a weighted average of the position vectors of *n* particles making up the system.

(2) **Position vector of centre of mass for two particle system :** $\vec{r} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$

and the centre of mass lies between the particles on the line joining them.

If two masses are equal *i.e.* $m_1 = m_2$, then position vector of centre of mass $\vec{r} = \dfrac{\vec{r}_1 + \vec{r}_2}{2}$

(3) **Important points about centre of mass**

(i) The position of centre of mass is independent of the co-ordinate system chosen.

(ii) The position of centre of mass depends upon the shape of the body and distribution of mass.

Example : The centre of mass of a circular disc is within the material of the body while that of a circular ring is outside the material of the body.

(iii) In symmetrical bodies in which the distribution of mass is homogenous, the centre of mass coincides with the geometrical centre or centre of symmetry of the body.

(iv) Position of centre of mass for different bodies

S. No.	**Body**	**Position of centre of mass**
(a)	Uniform hollow sphere	Centre of sphere
(b)	Uniform solid sphere	Centre of sphere
(c)	Uniform circular ring	Centre of ring
(d)	Uniform circular disc	Centre of disc
(e)	Uniform rod	Centre of rod
(f)	A plane lamina (Square, Rectangle, Parallelogram)	Point of inter section of diagonals
(g)	Triangular plane lamina	Point of inter section of medians
(h)	Rectangular or cubical block	Points of inter section of diagonals
(i)	Hollow cylinder	Middle point of the axis of cylinder
(j)	Solid cylinder	Middle point of the axis of cylinder
(k)	Cone or pyramid	On the axis of the cone at point distance $\frac{3h}{4}$ from the vertex where h is the height of cone

(v) The centre of mass changes its position only under the translatory motion. There is no effect of rotatory motion on centre of mass of the body.

(vi) If the origin is at the centre of mass, then the sum of the moments of the masses of the system about the centre of mass is zero *i.e.* $\Sigma m_i \vec{r_i} = 0$.

(vii) If a system of particles of masses $m_1, m_2, m_3, ...$ move with velocities $v_1, v_2, v_3, ...$

then the velocity of centre of mass $v_{cm} = \frac{\Sigma m_i v_i}{\Sigma m_i}$.

(viii) If a system of particles of masses $m_1, m_2, m_3, ...$ move with accelerations $a_1, a_2, a_3, ...$

then the acceleration of centre of mass $A_{cm} = \frac{\Sigma m_i a_i}{\Sigma m_i}$

(ix) If $\vec{r}$ is a position vector of centre of mass of a system

then velocity of centre of mass $\vec{v}_{cm} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(\frac{m_1 \vec{r_1} + m_2 \vec{r_2} + m_3 \vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...}\right)$

(x) Acceleration of centre of mass $\vec{A}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2}{dt^2}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...}{m_1 + m_2 + m_3 + ...}\right)$

(xi) Force on a rigid body $\vec{F} = M\vec{A}_{cm} = M\frac{d^2\vec{r}}{dt^2}$

(xii) For an isolated system external force on the body is zero

$$\vec{F} = M\frac{d}{dt}\left(\vec{v}_{cm}\right) = 0 \Rightarrow \vec{v}_{cm} = \text{constant.}$$

i.e., centre of mass of an isolated system moves with uniform velocity along a straight-line path.

Sample problems based on centre of mass

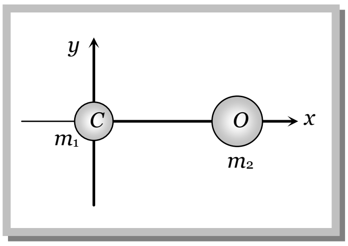
***Problem* 1.** The distance between the carbon atom and the oxygen atom in a carbon monoxide molecule is 1.1 Å. Given, mass of carbon atom is 12 *a.m.u.* and mass of oxygen atom is 16 *a.m.u.*, calculate the position of the center of mass of the carbon monoxide molecule

(a) 6.3 Å from the carbon atom (b) 1 Å from the oxygen atom

(c) 0.63 Å from the carbon atom (d) 0.12 Å from the oxygen atom

Solution : (c) Let carbon atom is at the origin and the oxygen atom is placed at *x*-axis

$m_1 = 12$, $m_2 = 16$, $\vec{r}_1 = 0\hat{i} + 0\hat{j}$ and $\vec{r}_2 = 1.1\hat{i} + 0\hat{j}$



$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{16 \times 1.1}{28}\hat{i}$$

$\vec{r} = 0.63\hat{i}$ *i.e.* 0.63 Å from carbon atom.

***Problem* 2.** The velocities of three particles of masses 20*g*, 30*g* and 50 *g* are $10\vec{i}, 10\vec{j}$, and $10\vec{k}$ respectively. The velocity of the centre of mass of the three particles is **[EAMCET 2001]**

(a) $2\vec{i} + 3\vec{j} + 5\vec{k}$ (b) $10(\vec{i} + \vec{j} + \vec{k})$ (c) $20\vec{i} + 30\vec{j} + 5\vec{k}$ (d) $2\vec{i} + 30\vec{j} + 50\vec{k}$

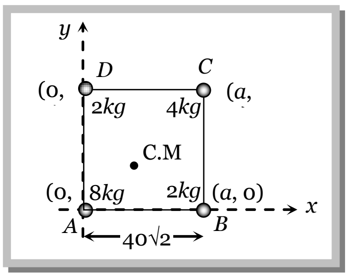
Solution : (a) Velocity of centre of mass

$$v_{cm} = \frac{m_1v_1 + m_2v_2 + m_3v_3}{m_1 + m_2 + m_3} = \frac{20 \times 10\hat{i} + 30 \times 10\hat{j} + 50 \times 10\hat{k}}{100} = 2\hat{i} + 3\hat{j} + 5\hat{k}.$$

***Problem* 3.** Masses 8, 2, 4, 2 *kg* are placed at the corners *A, B, C, D* respectively of a square *ABCD* of diagonal 80 *cm*. The distance of centre of mass from *A* will be

(a) 20 *cm* (b) 30 *cm* (c) 40 *cm* (d) 60 *cm*

Solution : (b) Let corner A of square *ABCD* is at the origin and the mass 8 *kg* is placed at this corner (given in problem) Diagonal of square $d = a\sqrt{2} = 80\,cm \Rightarrow a = 40\sqrt{2}cm$



$m_1 = 8kg$, $m_2 = 2kg$, $m_3 = 4kg$, $m_4 = 2kg$

Let $\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4$ are the position vectors of respective masses

$$\vec{r}_1 = 0\hat{i} + 0\hat{j}, \ \vec{r}_2 = a\hat{i} + 0\hat{j}, \ \vec{r}_3 = a\hat{i} + a\hat{j}, \ \vec{r}_4 = 0\hat{i} + a\hat{j}$$

From the formula of centre of mass

$$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4}{m_1 + m_2 + m_3 + m_4} = 15\sqrt{2}\hat{i} + 15\sqrt{2}\hat{j}$$

$\therefore$ co-ordinates of centre of mass $= (15\sqrt{2}, 15\sqrt{2})$ and co-ordination of the corner $= (0, 0)$

From the formula of distance between two points (x_1, y_1) and (x_2, y_2)

distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(15\sqrt{2} - 0)^2 + (15\sqrt{2} - 0)^2} = \sqrt{900} = 30cm$

***Problem* 4.** The coordinates of the positions of particles of mass $7, 4$ and $10\ gm$ are $(1, 5, -3)$, $(2, 5, 7)$ and $(3, 3, -1)\,cm$ respectively. The position of the centre of mass of the system would be

(a) $\left(-\frac{15}{7}, \frac{85}{17}, \frac{1}{7}\right) cm$ (b) $\left(\frac{15}{7}, -\frac{85}{17}, \frac{1}{7}\right) cm$ (c) $\left(\frac{15}{7}, \frac{85}{21}, -\frac{1}{7}\right) cm$ (d) $\left(\frac{15}{7}, \frac{85}{21}, \frac{7}{3}\right) cm$

Solution: (c) $m_1 = 7gm$, $m_2 = 4gm$, $m_3 = 10gm$ and $\vec{r_1} = (\hat{i} + 5\hat{j} - 3\hat{k})$, $r_2 = (2i + 5j + 7k)$, $r_3 = (3\hat{i} + 3\hat{j} - \hat{k})$

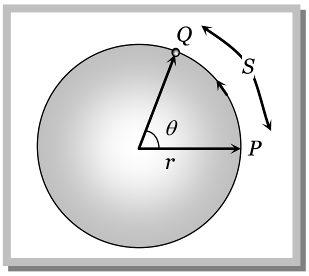
Position vector of center mass

$$\vec{r} = \frac{7(\hat{i} + 5\hat{j} - 3\hat{k}) + 4(2\hat{i} + 5\hat{j} + 7\hat{k}) + 10(3\hat{i} + 3\hat{j} - \hat{k})}{7 + 4 + 10} = \frac{(45\hat{i} + 85\hat{j} - 3\hat{k})}{21}$$

$\Rightarrow \vec{r} = \frac{15}{7}\hat{i} + \frac{85}{21}\hat{j} - \frac{1}{7}\hat{k}$. So coordinates of centre of mass $\left[\frac{15}{7}, \frac{85}{21}, \frac{-1}{7}\right]$.

7.3 Angular Displacement

It is the angle described by the position vector $\vec{r}$ about the axis of rotation.



Angular displacement $(\theta) = \frac{\text{Linear displacement } (s)}{\text{Radius } (r)}$

(1) Unit : radian

(2) Dimension : $[M^0 L^0 T^0]$

(3) Vector form $\vec{S} = \vec{\theta} \times \vec{r}$

i.e., angular displacement is a vector quantity whose direction is given by right hand rule. It is also known as axial vector. For anti-clockwise sense of rotation direction of θ is perpendicular to the plane, outward and along the axis of rotation and vice-versa.

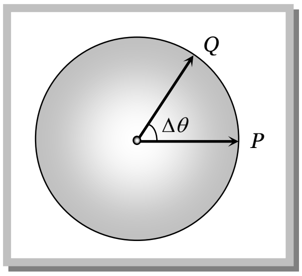
(4) 2π radian $= 360° = 1$ revolution.

(5) If a body rotates about a fixed axis then all the particles will have same angular displacement (although linear displacement will differ from particle to particle in accordance with the distance of particles from the axis of rotation).

7.4 Angular Velocity

The angular displacement per unit time is defined as angular velocity.

If a particle moves from P to Q in time Δt, $\omega = \frac{\Delta\theta}{\Delta t}$ where $\Delta\theta$ is the angular displacement.



(1) Instantaneous angular velocity $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

(2) Average angular velocity $\omega_{av} = \frac{\text{total angular displacement}}{\text{total time}} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$

(3) Unit : *Radian/sec*

(4) Dimension : $[M^0 L^0 T^{-1}]$ which is same as that of frequency.

(5) Vector form $\vec{v} = \vec{\omega} \times \vec{r}$ [where $\vec{v}$ = linear velocity, $\vec{r}$ = radius vector]

$\vec{\omega}$ is a axial vector, whose direction is normal to the rotational plane and its direction is given by right hand screw rule.

(6) $\omega = \frac{2\pi}{T} = 2\pi n$ [where T = time period, n = frequency]

(7) The magnitude of an angular velocity is called the angular speed which is also represented by ω.

7.5 Angular Acceleration

The rate of change of angular velocity is defined as angular acceleration.

If particle has angular velocity ω_1 at time t_1 and angular velocity ω_2 at time t_2 then,

Angular acceleration $\vec{\alpha} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1}$

(1) Instantaneous angular acceleration $\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$.

(2) Unit : rad/sec^2

(3) Dimension : $[M^0 L^0 T^{-2}]$.

(4) If $\alpha = 0$, circular or rotational motion is said to be uniform.

(5) Average angular acceleration $\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$.

(6) Relation between angular acceleration and linear acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$.

(7) It is an axial vector whose direction is along the change in direction of angular velocity *i.e.* normal to the rotational plane, outward or inward along the axis of rotation (depends upon the sense of rotation).

7.6 Equations of Linear Motion and Rotational Motion

	Linear Motion	Rotational Motion
(1)	If linear acceleration is 0, u = constant and $s = u\,t$.	If angular acceleration is 0, ω = constant and $\theta = \omega t$
(2)	If linear acceleration a = constant,	If angular acceleration α = constant then
	(i) $s = \frac{(u+v)}{2}t$	(i) $\theta = \frac{(\omega_1 + \omega_2)}{2}t$
	(ii) $a = \frac{v-u}{t}$	(ii) $\alpha = \frac{\omega_2 - \omega_1}{t}$
	(iii) $v = u + at$	(iii) $\omega_2 = \omega_1 + \alpha t$
	(iv) $s = ut + \frac{1}{2}at^2$	(iv) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
	(v) $v^2 = u^2 + 2as$	(v) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$
	(vi) $s_{nth} = u + \frac{1}{2}a(2n-1)$	(vi) $\theta_{nth} = \omega_1 + (2n-1)\frac{\alpha}{2}$
(3)	If acceleration is not constant, the above equation will not be applicable. In this case	If acceleration is not constant, the above equation will not be applicable. In this case

(i) $v = \frac{dx}{dt}$	(i) $\omega = \frac{d\theta}{dt}$
(ii) $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	(ii) $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
(iii) $v dv = a\, ds$	(iii) $\omega d\omega = \alpha\, d\theta$

Sample problems based on angular displacement, velocity and acceleration

Problem 5. The angular velocity of *seconds* hand of a watch will be

(a) $\frac{\pi}{60} rad/sec$ (b) $\frac{\pi}{30} rad/sec$ (c) $60\pi\ rad/sec$ (d) $30\pi\ rad/sec$

Solution : (b) We know that second's hand completes its revolution (2π) in 60 *sec* $\therefore\ \omega = \frac{\theta}{t} = \frac{2\pi}{60} = \frac{\pi}{30} rad/sec$

Problem 6. The wheel of a car is rotating at the rate of 1200 revolutions per *minute*. On pressing the accelerator for 10 *sec* it starts rotating at 4500 revolutions per *minute*. The angular acceleration of the wheel is **[MP PET 2001]**

(a) 30 *radians/sec²* (b) 1880 *degrees/sec²* (c) 40 *radians/sec²* (d) 1980 *degrees/sec²*

Solution: (d) Angular acceleration (α) = rate of change of angular speed

$$= \frac{2\pi(n_2 - n_1)}{t} = \frac{2\pi\left(\frac{4500 - 1200}{60}\right)}{10} = \frac{2\pi\frac{3300}{60}}{10} \times \frac{360}{2\pi}\frac{degree}{sec^2} = 1980\ degree/sec^2.$$

Problem 7. Angular displacement (θ) of a flywheel varies with time as $\theta = at + bt^2 + ct^3$ then angular acceleration is given by

(a) $a + 2bt - 3ct^2$ (b) $2b - 6t$ (c) $a + 2b - 6t$ (d) $2b + 6ct$

Solution: (d) Angular acceleration $\alpha = \frac{d^2\theta}{dt^2} = \frac{d^2}{dt^2}(at + bt^2 + ct^3) = 2b + 6ct$

Problem 8. A wheel completes 2000 rotations in covering a distance of $9.5\ km$. The diameter of the wheel is **[RPMT 199**

(a) $1.5\ m$ (b) $1.5\ cm$ (c) $7.5\ m$ (d) $7.5\ cm$

Solution: (a) Distance covered by wheel in 1 rotation = $2\pi r = \pi D$ (Where $D = 2r$ = diameter of wheel)

$\therefore$ Distance covered in 2000 rotation = 2000 $\pi D = 9.5 \times 10^3 m$ (given)

$\therefore D = 1.5\ meter$

Problem 9. A wheel is at rest. Its angular velocity increases uniformly and becomes 60 *rad/sec* after 5 *sec*. The total angular displacement is

(a) $600\ rad$ (b) $75\ rad$ (c) $300\ rad$ (d) $150\ rad$

Solution: (d) Angular acceleration $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{60 - 0}{5} = 12 rad/sec^2$

Now from $\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(12)(5)^2 = 150\ rad.$

Problem 10. A wheel initially at rest, is rotated with a uniform angular acceleration. The wheel rotates through an angle θ_1 in first one second and through an additional angle θ_2 in the next one second. The ratio $\frac{\theta_2}{\theta_1}$ is

(a) 4 (b) 2 (c) 3 (d) 1

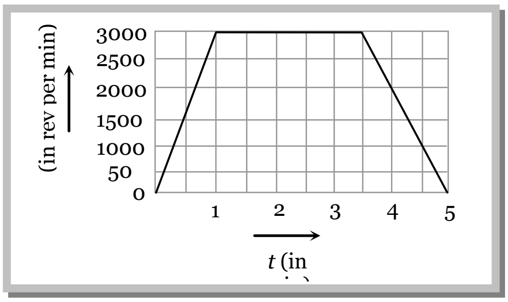
Solution: (c) Angular displacement in first one second $\theta_1 = \frac{1}{2}\alpha(1)^2 = \frac{\alpha}{2}$ ......(i) [From $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$]

Now again we will consider motion from the rest and angular displacement in total two seconds

$$\theta_1 + \theta_2 = \frac{1}{2}\alpha(2)^2 = 2\alpha \quad \text{......(ii)}$$

Solving (i) and (ii) we get $\theta_1 = \frac{\alpha}{2}$ and $\theta_2 = \frac{3\alpha}{2}$ $\therefore \frac{\theta_2}{\theta_1} = 3$.

***Problem* 11.** As a part of a maintenance inspection the compressor of a jet engine is made to spin according to the graph as shown. The number of revolutions made by the compressor during the test is

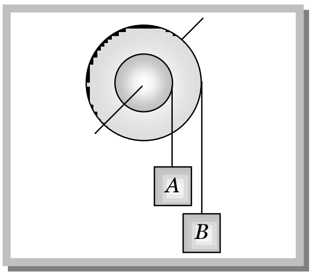


(a) 9000 (b) 16570 (c) 12750 (d) 11250

Solution: (d) Number of revolution = Area between the graph and time axis = Area of trapezium

$$= \frac{1}{2}\times(2.5+5)\times 3000 = 11250 \text{ revolution.}$$

***Problem* 12.** Figure shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates about the common axis. The strings supporting *A* and *B* do not slip on the wheels. If *x* and *y* be the distances travelled by *A* and *B* in the same time interval, then



(a) $x = 2y$

(b) $x = y$

(c) $y = 2x$

(d) None of these

Solution: (c) Linear displacement (S) = Radius (r) × Angular displacement (θ)

$\therefore S \propto r$ (if $\theta =$ constant)

$$\frac{\text{Distance travelled by mass } A(x)}{\text{Distance travelled by mass } B(y)} = \frac{\text{Radius of pulley concerned with mass } A(r)}{\text{Radius of pulley concerned with mass } B(2r)} = \frac{1}{2} \Rightarrow y = 2x.$$

***Problem* 13.** If the position vector of a particle is $\vec{r} = (3\hat{i} + 4\hat{j})$ meter and its angular velocity is $\vec{\omega} = (\hat{j} + 2\hat{k})$ *rad/sec* then its linear velocity is (in *m/s*)

(a) $(8\hat{i} - 6\hat{j} + 3\hat{k})$ (b) $(3\hat{i} + 6\hat{j} + 8\hat{k})$ (c) $-(3\hat{i} + 6\hat{j} + 6\hat{k})$ (d) $(6\hat{i} + 8\hat{j} + 3\hat{k})$

Solution: (a) $\vec{v} = \vec{\omega}\times\vec{r} = (3\hat{i} + 4\hat{j} + 0\hat{k})\times(0\hat{i} + \hat{j} + 2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 0 & 1 & 2 \end{vmatrix} = 8\hat{i} - 6\hat{j} + 3\hat{k}$

7.7 Moment of Inertia

Moment of inertia plays the same role in rotational motion as mass plays in linear motion. It is the property of a body due to which it opposes any change in its state of rest or of uniform rotation.

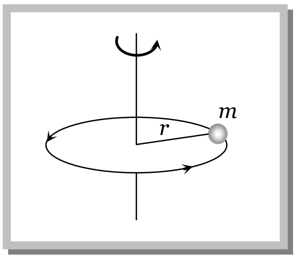
(1) Moment of inertia of a particle $I = mr^2$; where r is the perpendicular distance of particle from rotational axis.

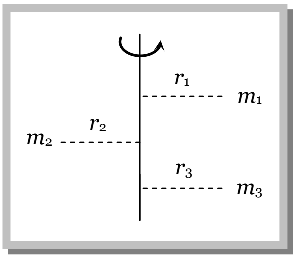
(2) Moment of inertia of a body made up of number of particles (discrete distribution)

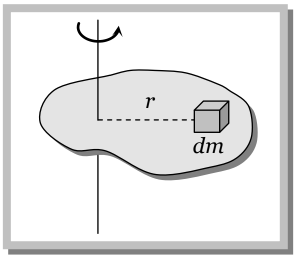
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ...$$

(3) Moment of inertia of a continuous distribution of mass, treating the element of mass dm at position r as particle

$$dI = dm\, r^2 \quad i.e., \; I = \int r^2 dm$$







(4) Dimension : $[ML^2T^0]$

(5) S.I. unit : kgm^2.

(6) Moment of inertia depends on mass, distribution of mass and on the position of axis of rotation.

(7) Moment of inertia does not depend on angular velocity, angular acceleration, torque, angular momentum and rotational kinetic energy.

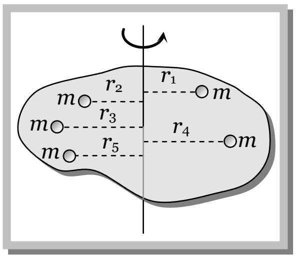
(8) It is not a vector as direction (clockwise or anti-clockwise) is not to be specified and also not a scalar as it has different values in different directions. Actually it is a tensor quantity.

(9) In case of a hollow and solid body of same mass, radius and shape for a given axis, moment of inertia of hollow body is greater than that for the solid body because it depends upon the mass distribution.

7.8 Radius of Gyration

Radius of gyration of a body about a given axis is the perpendicular distance of a point from the axis, where if whole mass of the body were concentrated, the body shall have the same moment of inertia as it has with the actual distribution of mass.

When square of radius of gyration is multiplied with the mass of the body gives the moment of inertia of the body about the given axis.



$$I = Mk^2 \text{ or } k = \sqrt{\frac{I}{M}}.$$

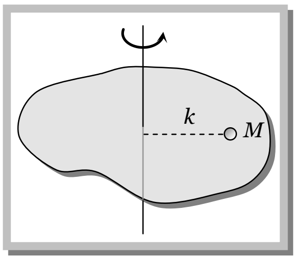
Here k is called radius of gyration.

From the formula of discrete distribution

$$I = mr_1^2 + mr_2^2 + mr_3^2 + ... + mr_n^2$$

If $m_1 = m_2 = m_3 = ... = m$ then

$$I = m(r_1^2 + r_2^2 + r_3^2 + ... r_n^2) \qquad ....(i)$$



From the definition of Radius of gyration,

$$I = Mk^2 \qquad ....(ii)$$

By equating (i) and (ii)

$$Mk^2 = m(r_1^2 + r_2^2 + r_3^2 + ...\ .. + r_n^2)$$

$$nmk^2 = m(r_1^2 + r_2^2 + r_3^2 + ...\ + r_n^2) \qquad [\text{As } M = nm]$$

$$\therefore \qquad k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + ...\ . + r_n^2}{n}}$$

Hence radius of gyration of a body about a given axis is equal to root mean square distance of the constituent particles of the body from the given axis.

(1) Radius of gyration (k) depends on shape and size of the body, position and configuration of the axis of rotation, distribution of mass of the body *w.r.t.* the axis of rotation.

(2) Radius of gyration (k) does not depends on the mass of body.

(3) Dimension $[M^0L^1T^0]$.

(4) S.I. unit : *Meter*.

(5) Significance of radius of gyration : Through this concept a real body (particularly irregular) is replaced by a point mass for dealing its rotational motion.

Example : In case of a disc rotating about an axis through its centre of mass and perpendicular to its plane

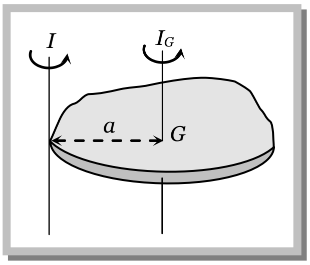
$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{(1/2)MR^2}{M}} = \frac{R}{\sqrt{2}}$$

So instead of disc we can assume a point mass M at a distance $(R/\sqrt{2})$ from the axis of rotation for dealing the rotational motion of the disc.

Note : ❑ For a given body inertia is constant whereas moment of inertia is variable.

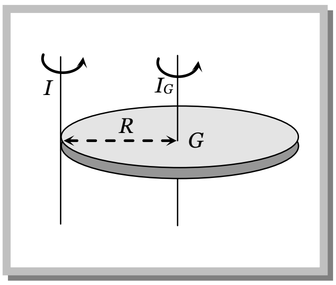
7.9 Theorem of Parallel Axes

Moment of inertia of a body about a given axis I is equal to the sum of moment of inertia of the body about an axis parallel to given axis and passing through centre of mass of the body I_g and Ma^2 where M is the mass of the body and a is the perpendicular distance between the two axes.



$$I = I_g + Ma^2$$

Example: Moment of inertia of a disc about an axis through its centre and perpendicular to the plane is $\frac{1}{2}MR^2$, so moment of inertia about an axis through its tangent and perpendicular to the plane will be



$$I = I_g + Ma^2$$

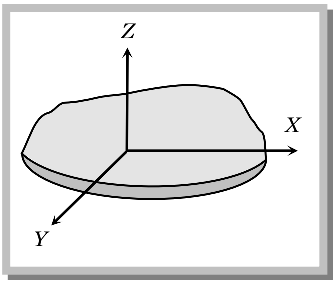
$$I = \frac{1}{2}MR^2 + MR^2$$

$$\therefore \qquad I = \frac{3}{2}MR^2$$

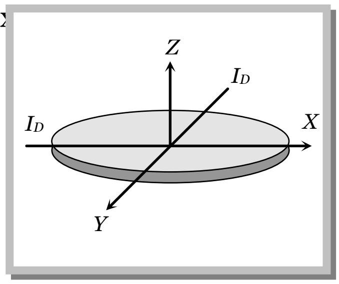
7.10 Theorem of Perpendicular Axes

According to this theorem the sum of moment of inertia of a plane lamina about two mutually perpendicular axes lying in its plane is equal to its moment of inertia about an axis perpendicular to the plane of lamina and passing through the point of intersection of first two axes.

$$I_z = I_x + I_y$$



Example : Moment of inertia of a disc about an axis through its centre of mass and perpendicular to its plane is $\frac{1}{2}MR^2$, so if the disc is in x–y plane then by theorem of perpendicular ax



i.e. $I_z = I_x + I_y$

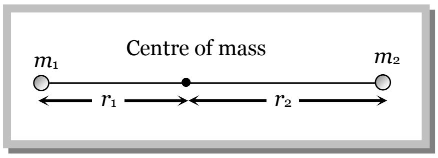
$\Rightarrow$ $\frac{1}{2}MR^2 = 2I_D$ [As ring is symmetrical body so $I_x = I_y = I_D$]

$\Rightarrow$ $I_D = \frac{1}{4}MR^2$

Note : ❑ In case of symmetrical two-dimensional bodies as moment of inertia for all axes passing through the centre of mass and in the plane of body will be same so the two axes in the plane of body need not be perpendicular to each other.

7.11 Moment of Inertia of Two Point Masses About Their Centre of Mass

Let m_1 and m_2 be two masses distant r from each-other and r_1 and r_2 be the distances of their centre of mass from m_1 and m_2 respectively, then



(1) $r_1 + r_2 = r$

(2) $m_1 r_1 = m_2 r_2$

(3) $r_1 = \frac{m_2}{m_1 + m_2} r$ and $r_2 = \frac{m_1}{m_1 + m_2} r$

(4) $I = m_1 r_1^2 + m_2 r_2^2$

(5) $I = \left[\frac{m_1 m_2}{m_1 + m_2}\right] r^2 = \mu r^2$ [where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is known as reduced mass $\mu < m_1$ and $\mu < m_2$.]

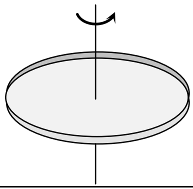
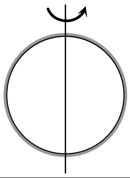
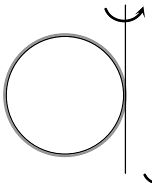
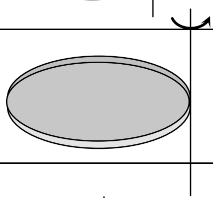
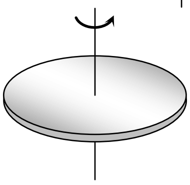
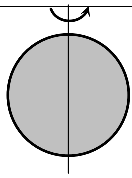
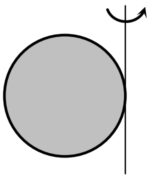
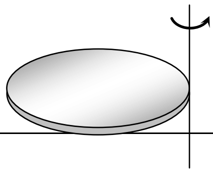
(6) In diatomic molecules like H_2, HCl *etc.* moment of inertia about their centre of mass is derived from above formula.

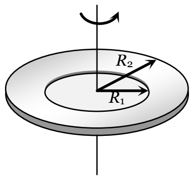
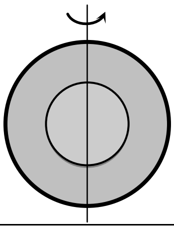
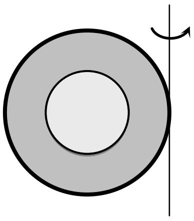
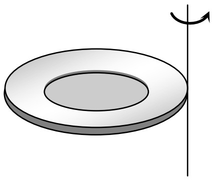
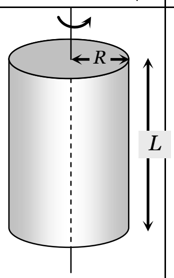
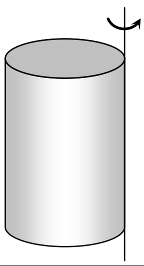
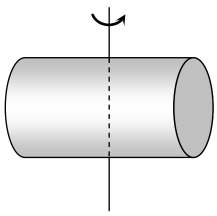
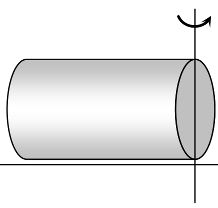
7.12 Analogy Between Tranlatory Motion and Rotational Motion

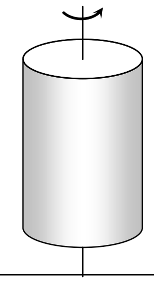
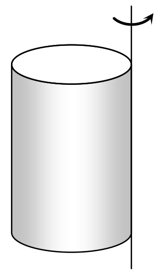
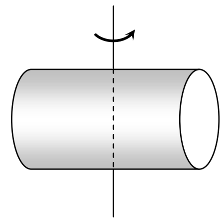
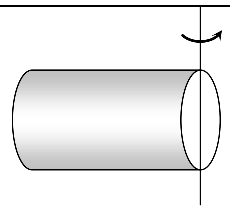
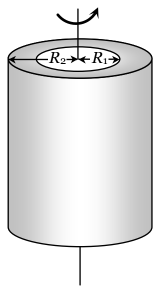
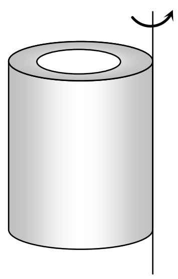
Translatory motion		**Rotatory motion**	
Mass	(m)	Moment of Inertia	(I)
Linear momentum	$P = mv$ $P = \sqrt{2mE}$	Angular Momentum	$L = I\omega$ $L = \sqrt{2IE}$
Force	$F = ma$	Torque	$\tau = I\alpha$

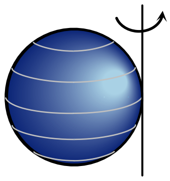
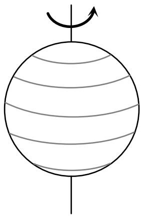
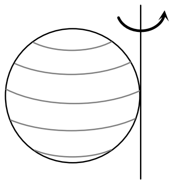
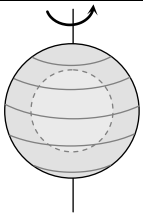
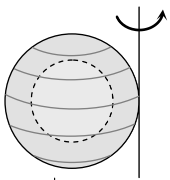
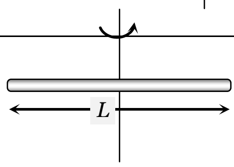
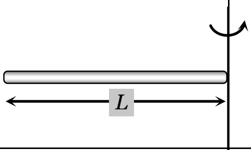
Kinetic energy	$E = \frac{1}{2}mv^2$ $E = \frac{P^2}{2m}$	$E = \frac{1}{2}I\omega^2$ $E = \frac{L^2}{2I}$

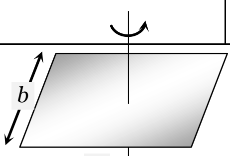
7.13 Moment of Inertia of Some Standard Bodies About Different Axes

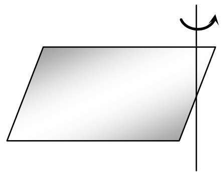
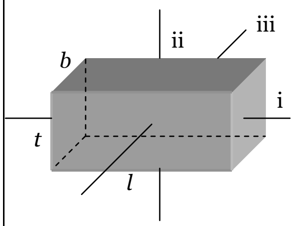
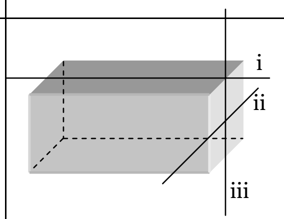
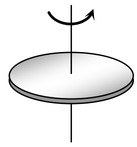
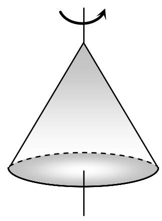
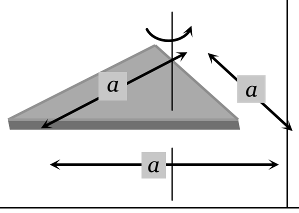
Body	Axis of Rotation	Figure	Moment of inertia	*k*	*k*²/*R*²
Ring	About an axis passing through C.G. and perpendicular to its plane		MR^2	R	1
Ring	About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Ring	About a tangential axis in its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Ring	About a tangential axis perpendicular to its own plane		$2MR^2$	$\sqrt{2}R$	2
Disc	About an axis passing through C.G. and perpendicular to its plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Disc	About its Diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$
Disc	About a tangential axis in its own plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$	$\frac{5}{4}$
Disc	About a tangential axis perpendicular to		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$

	its own plane				
Annular disc inner radius = R_1 and outer radius = R_2	Passing through the centre and perpendicular to the plane		$\frac{M}{2}[R_1^2 + R_2^2]$	–	–
Annular disc	Diameter		$\frac{M}{4}[R_1^2 + R_2^2]$	–	–
Annular disc	Tangential and Parallel to the diameter		$\frac{M}{4}[5R_1^2 + R_2^2]$	–	–
Annular disc	Tangential and perpendicular to the plane		$\frac{M}{2}[3R_1^2 + R_2^2]$	–	–
Solid cylinder	About its own axis		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Solid cylinder	Tangential (Generator)		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Solid cylinder	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{12} + \frac{R^2}{4}}$	
Solid cylinder	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3} + \frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{3} + \frac{R^2}{4}}$	

Cylindrical shell	About its own axis		MR^2	R	1
Cylindrical shell	Tangential (Generator)		$2MR^2$	$\sqrt{2}R$	2
Cylindrical shell	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12}+\frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{12}+\frac{R^2}{2}}$	
Cylindrical shell	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3}+\frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{3}+\frac{R^2}{2}}$	
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Axis of cylinder		$\frac{M}{2}(R_1^2+R_2^2)$		
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Tangential		$\frac{M}{2}(R_1^2+3R_2^2)$		

Solid Sphere	About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$
Solid sphere	About a tangential axis		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$	$\frac{7}{5}$
Spherical shell	About its diametric axis		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$	$\frac{2}{3}$
Spherical shell	About a tangential axis		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$	$\frac{5}{3}$
Hollow sphere of inner radius R_1 and outer radius R_2	About its diametric axis		$\frac{2}{5}M\left[\frac{R_2^5-R_1^5}{R_2^3-R_1^3}\right]$		
Hollow sphere	Tangential		$\frac{2M[R_2^5-R_1^5]}{5(R_2^3-R_1^3)}+MR_2^2$		
Long thin rod	About on axis passing through its centre of mass and perpendicular to the rod.		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$	
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$	
Rectangular lamina of length l and	Passing through the centre of mass		$\frac{M}{12}[l^2+b^2]$		



breadth b	and perpendicular to the plane				
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of breadth		$\frac{M}{12}[4l^2+b^2]$		
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of length		$\frac{M}{12}[l^2+4b^2]$		
Rectangular parallelopiped length l, breadth b, thickness t	Passing through centre of mass and parallel to (i) Length (x) (ii) breadth (z) (iii) thickness (y)		(i) $\frac{M[b^2+t^2]}{12}$ (ii) $\frac{M[l^2+t^2]}{12}$ (iii) $\frac{M[b^2+l^2]}{12}$		
Rectangular parallelepiped length l, breath b, thickness t	Tangential and parallel to (i) length (x) (ii) breadth (y) (iii) thickness(z)		(i) $\frac{M}{12}[3l^2+b^2+t^2]$ (ii) $\frac{M}{12}[l^2+3b^2+t^2]$ (iii) $\frac{M}{12}[l^2+b^2+3t^2]$		
Elliptical disc of semimajor axis = a and semiminor axis = b	Passing through CM and perpendicular to the plane		$\frac{M}{4}[a^2+b^2]$		
Solid cone of radius R and height h	Axis joining the vertex and centre of the base		$\frac{3}{10}MR^2$		
Equilateral triangular lamina with side a	Passing through CM and perpendicular to the plane		$\frac{Ma^2}{6}$		

Right angled triangular lamina of sides a, b, c	Along the edges		(1) $\frac{Mb^2}{6}$ (2) $\frac{Ma^2}{6}$ (3) $\frac{M}{6}\left[\frac{a^2b^2}{a^2+b^2}\right]$		

Sample problem based on moment of inertia

Problem 14. Five particles of mass = 2 *kg* are attached to the rim of a circular disc of radius 0.1 *m* and negligible mass. Moment of inertia of the system about the axis passing through the centre of the disc and perpendicular to its plane is

(a) 1 $kg\ m^2$ (b) 0.1 $kg\ m^2$ (c) 2 $kg\ m^2$ (d) 0.2 $kg\ m^2$

Solution: (b) We will not consider the moment of inertia of disc because it doesn't have any mass so moment of inertia of five particle system $I = 5\,mr^2 = 5\times 2\times(0.1)^2 = 0.1\,kg\text{-}m^2$.

Problem 15. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia I_X and I_Y is **[AIEEE 2003]**

(a) $I_Y = 64I_X$ (b) $I_Y = 32I_X$ (c) $I_Y = 16I_X$ (d) $I_Y = I_X$

Solution: (a) Moment of Inertia of disc $I = \frac{1}{2}MR^2 = \frac{1}{2}(\pi R^2 t\rho)R^2 = \frac{1}{2}\pi t\rho R^4$

[As $M = V\times\rho = \pi R^2 t\rho$ where $t =$ thickness, $\rho =$ density]

$\therefore \quad \frac{I_y}{I_x} = \frac{t_y}{t_x}\left(\frac{R_y}{R_x}\right)^4$ [If ρ = constant]

$\Rightarrow \quad \frac{I_y}{I_x} = \frac{1}{4}(4)^4 = 64$ [Given $R_y = 4R_x$, $t_y = \frac{t_x}{4}$]

$\Rightarrow \quad I_y = 64I_x$

Problem 16. Moment of inertia of a uniform circular disc about a diameter is I. Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be **[UPSEAT 2002]**

(a) $5\,I$ (b) $6\,I$ (c) $3\,I$ (d) $4\,I$

Solution: (b) Moment of inertia of disc about a diameter = $\frac{1}{4}MR^2 = I$ (given) $\therefore\ MR^2 = 4I$

Now moment of inertia of disc about an axis perpendicular to its plane and passing through a point on its rim

$$= \frac{3}{2}MR^2 = \frac{3}{2}(4I) = 6I.$$

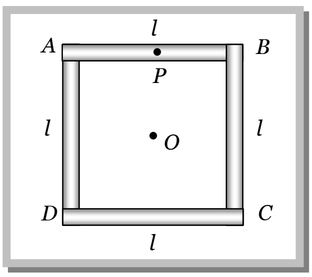
Problem 17. Four thin rods of same mass M and same length l, form a square as shown in figure. Moment of inertia of this system about an axis through centre O and perpendicular to its plane is

(a) $\frac{4}{3}Ml^2$

(b) $\frac{Ml^2}{3}$

(c) $\frac{Ml^2}{6}$

(d) $\frac{2}{3}Ml^2$



Solution: (a) Moment of inertia of rod AB about point $P = \frac{1}{12}Ml^2$

M.I. of rod AB about point $O = \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{1}{3}Ml^2$ [by the theorem of parallel axis]

and the system consists of 4 rods of similar type so by the symmetry $I_{System} = \frac{4}{3}Ml^2$.

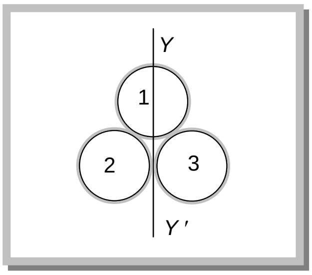
***Problem* 18.** Three rings each of mass M and radius R are arranged as shown in the figure. The moment of inertia of the system about YY' will be

(a) $3\,MR^2$

(b) $\frac{3}{2}MR^2$

(c) $5\,MR^2$

(d) $\frac{7}{2}MR^2$



Solution: (d) M.I of system about YY' $I = I_1 + I_2 + I_3$

where I_1 = moment of inertia of ring about diameter, $I_2 = I_3$ = M.I. of inertia of ring about a tangent in a plane

$$\therefore \quad I = \frac{1}{2}mR^2 + \frac{3}{2}mR^2 + \frac{3}{2}mR^2 = \frac{7}{2}mR^2$$

***Problem* 19.** Let l be the moment of inertia of an uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

[IIT-JEE 1998]

(a) l (b) $l\sin^2\theta$ (c) $l\cos^2\theta$ (d) $l\cos^2\frac{\theta}{2}$

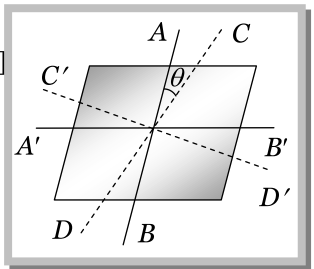
Solution: (a) Let I_Z is the moment of inertia of square plate about the axis which is passing through the centre and perpendicular to the plane.

$I_Z = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$ [By the theorem of perpendicular axis]

$I_Z = 2I_{AB} = 2I_{A'B'} = 2I_{CD} = 2I_{C'D'}$

[As AB, $A'B'$ and CD, $C'D'$ are symmetric axis]

Hence $I_{CD} = I_{AB} = l$



***Problem* 20.** Three rods each of length L and mass M are placed along X, Y and Z-axes in such a way that one end of each of the rod is at the origin. The moment of inertia of this system about Z axis is

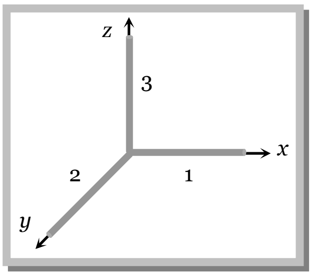
(a) $\frac{2ML^2}{3}$ (b) $\frac{4ML^2}{3}$ (c) $\frac{5ML^2}{3}$ (d) $\frac{ML^2}{3}$

Solution: (a) Moment of inertia of the system about z-axis can be find out by calculating the moment of inertia of individual rod about z-axis

$I_1 = I_2 = \frac{ML^2}{3}$ because z-axis is the edge of rod 1 and 2

and $I_3 = 0$ because rod in lying on z-axis

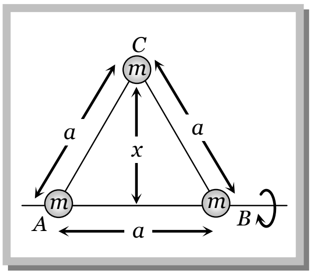
$$\therefore \quad I_{system} = I_1 + I_2 + I_3 = \frac{ML^2}{3} + \frac{ML^2}{3} + 0 = \frac{2ML^2}{3}.$$



Problem 21. Three point masses each of mass m are placed at the corners of an equilateral triangle of side a. Then the moment of inertia of this system about an axis passing along one side of the triangle is **[AIIMS 1995]**

(a) ma^2 (b) $3ma^2$ (c) $\frac{3}{4}ma^2$ (d) $\frac{2}{3}ma^2$

Solution: (c) The moment of inertia of system about AB side of triangle

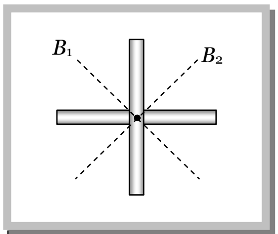


$I = I_A + I_B + I_C$

$= 0 + 0 + mx^2$

$= m\left(\frac{a\sqrt{3}}{2}\right)^2 = \frac{3}{4}ma^2$

Problem 22. Two identical rods each of mass M. and length l are joined in crossed position as shown in figure. The moment of inertia of this system about a bisector would be



(a) $\frac{Ml^2}{6}$ (b) $\frac{Ml^2}{12}$

(c) $\frac{Ml^2}{3}$ (d) $\frac{Ml^2}{4}$

Solution: (b) Moment of inertia of system about an axes which is perpendicular to plane of rods and passing through the common centre of rods $I_z = \frac{Ml^2}{12} + \frac{Ml^2}{12} = \frac{Ml^2}{6}$

Again from perpendicular axes theorem $I_z = I_{B_1} + I_{B_2} = 2I_{B_1} = 2I_{B_2} = \frac{Ml^2}{6}$ [As $I_{B_1} = I_{B_2}$]

$\therefore \quad I_{B_1} = I_{B_2} = \frac{Ml^2}{12}$.

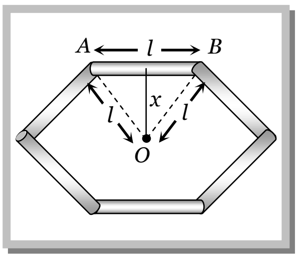
Problem 23. The moment of inertia of a rod of length l about an axis passing through its centre of mass and perpendicular to rod is I. The moment of inertia of hexagonal shape formed by six such rods, about an axis passing through its centre of mass and perpendicular to its plane will be

(a) 16I (b) 40 I (c) 60 I (d) 80 I

Solution: (c) Moment of inertia of rod AB about its centre and perpendicular to the length $= \frac{ml^2}{12} = I \quad \therefore$

$ml^2 = 12I$

Now moment of inertia of the rod about the axis which is passing through O and perpendicular to the plane of hexagon $I_{rod} = \frac{ml^2}{12} + mx^2$ [From the theorem of parallel axes]



$= \frac{ml^2}{12} + m\left(\frac{\sqrt{3}}{2}l\right)^2 = \frac{5ml^2}{6}$

Now the moment of inertia of system $I_{system} = 6 \times I_{rod} = 6 \times \frac{5ml^2}{6} = 5ml^2$

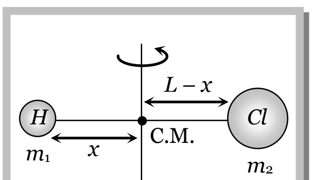
$I_{system} = 5\,(12\,I) = 60\,I$ [As $ml^2 = 12I$]

Problem 24. The moment of inertia of HCl molecule about an axis passing through its centre of mass and perpendicular to the line joining the H^+ and Cl^- ions will be, if the interatomic distance is 1 Å

(a) $0.61 \times 10^{-47}\ kg.m^2$ (b) $1.61 \times 10^{-47}\ kg.m^2$ (c) $0.061 \times 10^{-47}\ kg.m^2$ (d) 0

Solution: (b) If r_1 and r_2 are the respective distances of particles m_1 and m_2 from the centre of mass then

$m_1 r_1 = m_2 r_2 \Rightarrow 1 \times x = 35.5 \times (L - x) \Rightarrow x = 35.5(1 - x)$



$\Rightarrow x = 0.973\,\text{Å}$ and $L - x = 0.027\,\text{Å}$

Moment of inertia of the system about centre of mass $I = m_1x^2 + m_2(L-x)^2$

$I = 1amu \times (0.973\,\text{Å})^2 + 35.5\,amu \times (0.027\,\text{Å})^2$

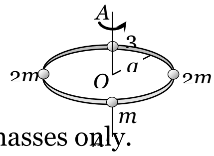
Substituting 1 $a.m.u.$ = 1.67×10^{-27} kg and 1 Å = 10^{-10} m

$I = 1.62 \times 10^{-47} kg\ m^2$

Problem 25. Four masses are joined to a light circular frame as shown in the figure. The radius of gyration of this system about an axis passing through the centre of the circular frame and perpendicular to its plane would be

(a) $a/\sqrt{2}$ (b) $a/2$

(c) a (d) $2a$



Solution: (c) Since the circular frame is massless so we will consider moment of inertia of four masses only.

$I = ma^2 + 2ma^2 + 3ma^2 + 2ma^2 = 8ma^2$ .....(i)

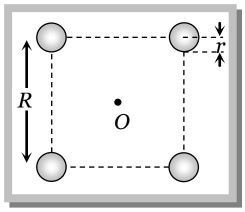
Now from the definition of radius of gyration $I = 8mk^2$ .....(ii)

comparing (i) and (ii) radius of gyration $k = a$.

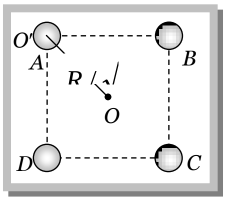
Problem 26. Four spheres, each of mass M and radius r are situated at the four corners of square of side R. The moment of inertia of the system about an axis perpendicular to the plane of square and passing through its centre will be

(a) $\frac{5}{2}M(4r^2 + 5R^2)$ (b) $\frac{2}{5}M(4r^2 + 5R^2)$

(c) $\frac{2}{5}M(4r^2 + 5r^2)$ (d) $\frac{5}{2}M(4r^2 + 5r^2)$



Solution: (b) $M.I.$ of sphere A about its diameter $I_{O'} = \frac{2}{5}Mr^2$



Now $M.I.$ of sphere A about an axis perpendicular to the plane of square and passing through its centre will be

$I_O = I_{O'} + M\left(\frac{R}{\sqrt{2}}\right)^2 = \frac{2}{5}Mr^2 + \frac{MR^2}{2}$ [by the theorem of parallel axis]

Moment of inertia of system (i.e. four sphere)= $4I_O = 4\left[\frac{2}{5}Mr^2 + \frac{MR^2}{2}\right] = \frac{2}{5}M\left[4r^2 + 5R^2\right]$

Problem 27. The moment of inertia of a solid sphere of density ρ and radius R about its diameter is

(a) $\frac{105}{176}R^5\rho$ (b) $\frac{105}{176}R^2\rho$ (c) $\frac{176}{105}R^5\rho$ (d) $\frac{176}{105}R^2\rho$

Solution: (c) Moment of inertia of sphere about it diameter $I = \frac{2}{5}MR^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3\rho\right)R^2$ [As $M = V\rho = \frac{4}{3}\pi R^3\rho$]

$I = \frac{8\pi}{15}R^5\rho = \frac{8 \times 22}{15 \times 7}R^5\rho = \frac{176}{105}R^5\rho$

Problem 28. Two circular discs A and B are of equal masses and thickness but made of metals with densities d_A and d_B $(d_A > d_B)$. If their moments of inertia about an axis passing through centres and normal to the circular faces be I_A and I_B, then

(a) $I_A = I_B$ (b) $I_A > I_B$ (c) $I_A < I_B$ (d) $I_A >=< I_B$

Solution : (c) Moment of inertia of circular disc about an axis passing through centre and normal to the circular face

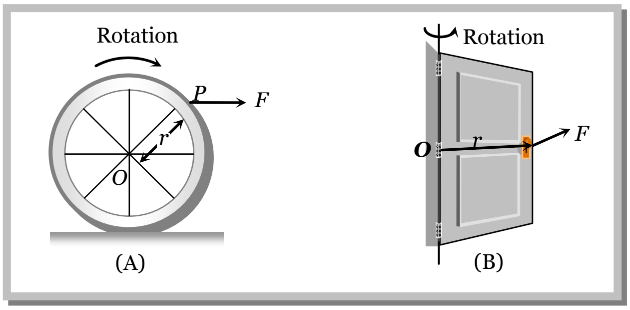
$$I = \frac{1}{2}MR^2 = \frac{1}{2}M\left(\frac{M}{\pi t\rho}\right) \qquad \left[\text{As } M = V\rho = \pi R^2 t\rho \ \therefore\ R^2 = \frac{M}{\pi t \rho}\right]$$

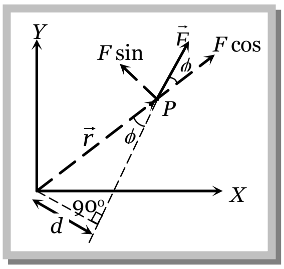
$$\Rightarrow \quad I = \frac{M^2}{2\pi t\rho} \quad \text{or} \quad I \propto \frac{1}{\rho} \text{ If mass and thickness are constant.}$$

So, in the problem $\frac{I_A}{I_B} = \frac{d_B}{d_A}$ $\quad \therefore I_A < I_B$ $\quad$ [As $d_A > d_B$]

7.14 Torque

If a pivoted, hinged or suspended body tends to rotate under the action of a force, it is said to be acted upon by a torque. or The turning effect of a force about the axis of rotation is called moment of force or torque due to the force.





If the particle rotating in xy plane about the origin under the effect of force $\vec{F}$ and at any instant the position vector of the particle is $\vec{r}$ then,

$$\text{Torque } \vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF\sin\phi$$

[where ϕ is the angle between the direction of $\vec{r}$ and $\vec{F}$]

(1) Torque is an axial vector. *i.e.,* its direction is always perpendicular to the plane containing vector $\vec{r}$ and $\vec{F}$ in accordance with right hand screw rule. For a given figure the sense of rotation is anti-clockwise so the direction of torque is perpendicular to the plane, outward through the axis of rotation.

(2) Rectangular components of force

$$\vec{F}_r = F\cos\phi = \text{radial component of force}, \quad \vec{F}_\phi = F\sin\phi = \text{transverse component of force}$$

As $\quad \tau = rF\sin\phi$

or $\quad \tau = rF_\phi$ = (position vector) × (transverse component of force)

Thus the magnitude of torque is given by the product of transverse component of force and its perpendicular distance from the axis of rotation *i.e.*, Torque is due to transverse component of force only.

(3) As $\quad \tau = rF\sin\phi$

or $\quad \tau = F(r\sin\phi) = Fd \quad$ [As $d = r\sin\phi$ from the figure]

i.e. Torque = Force × Perpendicular distance of line of action of force from the axis of rotation.

Torque is also called as moment of force and d is called moment or lever arm.

(4) Maximum and minimum torque : As $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = rF\sin\phi$

$\tau_{maximum} = rF$	When $\lvert \sin\phi \rvert = \max = 1$ *i.e.*, $\phi = 90°$	$\vec{F}$ is perpendicular to $\vec{r}$
$\tau_{minimum} = 0$	When $\lvert \sin\phi \rvert = \min = 0$ *i.e.* $\phi = 0°$ or $180°$	$\vec{F}$ is collinear to $\vec{r}$

(5) For a given force and angle, magnitude of torque depends on r. The more is the value of r, the more will be the torque and easier to rotate the body.

Example : (i) Handles are provided near the free edge of the Planck of the door.

(ii) The handle of screw driver is taken thick.

(iii) In villages handle of flourmill is placed near the circumference.

(iv) The handle of hand-pump is kept long.

(v) The arm of wrench used for opening the tap, is kept long.

(6) Unit : *Newton-metre* (M.K.S.) and *Dyne-cm* (C.G.S.)

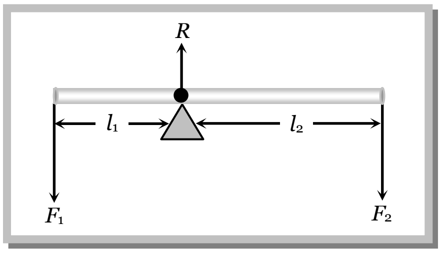
(7) Dimension : $[ML^2T^{-2}]$.

(8) If a body is acted upon by more than one force, the total torque is the vector sum of each torque.

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + ...$$

(9) A body is said to be in rotational equilibrium if resultant torque acting on it is zero *i.e.* $\Sigma \vec{\tau} = 0$.

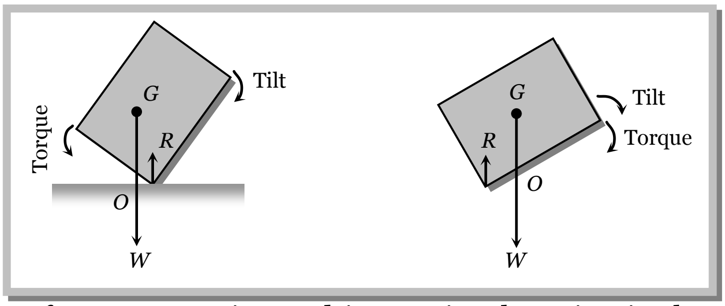
(10) In case of beam balance or see-saw the system will be in rotational equilibrium if,



$\vec{\tau}_1 + \vec{\tau}_2 = 0$ or $F_1l_1 - F_2l_2 = 0$ $\therefore$ $F_1l_1 = F_2l_2$

However if, $\vec{\tau}_1 > \vec{\tau}_2$, L.H.S. will move downwards and if $\vec{\tau}_1 < \vec{\tau}_2$. R.H.S. will move downward. and the system will not be in rotational equilibrium.

(11) On tilting, a body will restore its initial position due to torque of weight about the point O till the line of action of weight passes through its base on tilting, a body will topple due to torque of weight about O, if the line of action of weight does not pass through the base.



(12) Torque is the cause of rotatory motion and in rotational motion it plays same role as force plays in translatory motion *i.e.*, torque is rotational analogue of force. This all is evident from the following correspondences between rotatory and translatory motion.

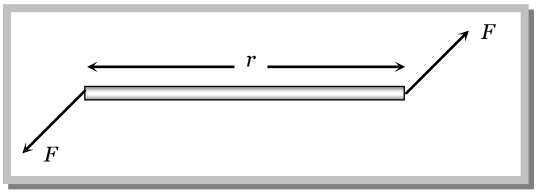
Rotatory Motion	**Translatory Motion**
$\vec{\tau} = I\vec{\alpha}$	$\vec{F} = m\vec{a}$
$W = \int \vec{\tau} \cdot d\vec{\theta}$	$W = \int \vec{F} \cdot d\vec{s}$

$P = \vec{\tau} \cdot \vec{\omega}$	$P = \vec{F} \cdot \vec{v}$
$\vec{\tau} = \dfrac{d\vec{L}}{dt}$	$\vec{F} = \dfrac{d\vec{P}}{dt}$

7.15 Couple

A special combination of forces even when the entire body is free to move can rotate it. This combination of forces is called a couple.

(1) A couple is defined as combination of two equal but oppositely directed force not acting along the same line. The effect of couple is known by its moment of couple or torque by a couple $\vec{\tau} = \vec{r} \times \vec{F}$.

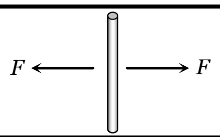
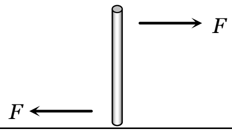
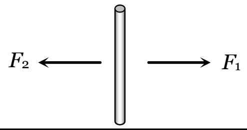
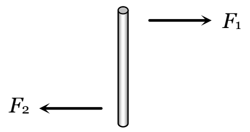


(2) Generally both couple and torque carry equal meaning. The basic difference between torque and couple is the fact that in case of couple both the forces are externally applied while in case of torque one force is externally applied and the other is reactionary.

(3) Work done by torque in twisting the wire $W = \frac{1}{2} C\theta^2$.

Where $\tau = C\theta$; C is known as twisting coefficient or couple per unit twist.

7.16 Translatory and Rotatory Equilibrium

Forces are equal and act along the same line.		$\sum F = 0$ and $\sum \tau = 0$	Body will remain stationary if initially it was at rest.
Forces are equal and does not act along the same line.		$\sum F = 0$ and $\sum \tau \neq 0$	Rotation *i.e.* spinning.
Forces are unequal and act along the same line.		$\sum F \neq 0$ and $\sum \tau = 0$	Translation *i.e.* slipping or skidding.
Forces are unequal and does not act along the same line.		$\sum F \neq 0$ and $\sum \tau \neq 0$	Rotation and translation both *i.e.* rolling.

Sample problems based on torque and couple

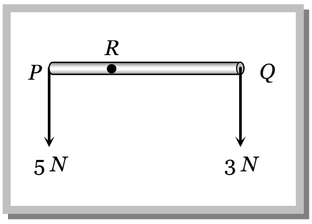
***Problem* 29.** A force of $(2\hat{i} - 4\hat{j} + 2\hat{k})N$ acts at a point $(3\hat{i} + 2\hat{j} - 4\hat{k})$ *metre* from the origin. The magnitude of torque is

(a) Zero (b) 24.4 *N-m* (c) 0.244 *N-m* (d) 2.444 *N-m*

Solution: (b) $\vec{F} = (2\hat{i} - 4\hat{j} + 2\hat{k})\,N$ and $\vec{r} = (3i + 2 - 4\hat{k})$ *meter*

Torque $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -4 \\ 2 & -4 & 2 \end{vmatrix} \Rightarrow \vec{\tau} = -12\hat{i} - 14\hat{j} - 16\hat{k}$ and $|\vec{\tau}| = \sqrt{(-12)^2 + (-14)^2 + (-16)^2} =$ 24.4 N-m

***Problem* 30.** The resultant of the system in the figure is a force of $8N$ parallel to the given force through R. The value of PR equals to



(a) $1/4\ RQ$

(b) $3/8\ RQ$

(c) $3/5\ RQ$

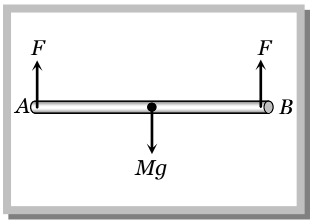
(d) $2/5\ RQ$

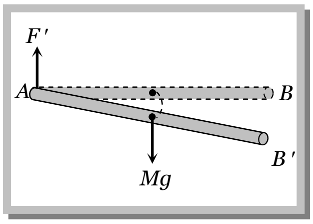
Solution: (c) By taking moment of forces about point R, $5 \times PR - 3 \times RQ = 0 \Rightarrow PR = \frac{3}{5} RQ$.

***Problem* 31.** A horizontal heavy uniform bar of weight W is supported at its ends by two men. At the instant, one of the men lets go off his end of the rod, the other feels the force on his hand changed to

(a) W (b) $\frac{W}{2}$ (c) $\frac{3W}{4}$ (d) $\frac{W}{4}$

Solution: (d) Let the mass of the rod is M $\therefore$ Weight (W) = Mg





Initially for the equilibrium $F + F = Mg \Rightarrow F = Mg/2$

When one man withdraws, the torque on the rod

$$\tau = I\alpha = Mg\frac{l}{2}$$

$$\Rightarrow \frac{Ml^2}{3}\alpha = Mg\frac{l}{2} \qquad [\text{As } I = Ml^2/3]$$

$\Rightarrow$ Angular acceleration $\alpha = \frac{3}{2}\frac{g}{l}$

and linear acceleration $a = \frac{l}{2}\alpha = \frac{3g}{4}$

Now if the new normal force at A is F' then $Mg - F' = Ma$

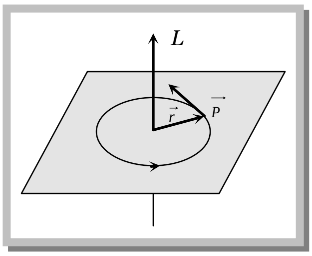
$$\Rightarrow F' = Mg - Ma = Mg - \frac{3Mg}{4} = \frac{Mg}{4} = \frac{W}{4}.$$

7.17 Angular Momentum

The turning momentum of particle about the axis of rotation is called the angular momentum of the particle.

or

The moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If $\vec{P}$ is the linear momentum of particle and $\vec{r}$ its position vector from the point of rotation then angular momentum.



$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = r P \sin\phi\, \hat{n}$$

Angular momentum is an axial vector *i.e.* always directed perpendicular to the plane of rotation and along the axis of rotation.

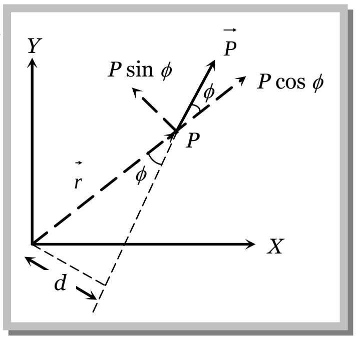
(1) S.I. Unit : *kg-m²-s⁻¹* or *J-sec.*

(2) Dimension : $[ML^2T^{-1}]$ and it is similar to Planck's constant (h).

(3) In cartesian co-ordinates if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

Then $\vec{L} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix} = (yP_z - zP_y)\hat{i} - (xP_z - zP_x)\hat{j} + (xP_y - yP_x)\hat{k}$

(4) As it is clear from the figure radial component of momentum $\vec{P}_r = P\cos$



Transverse component of momentum $\vec{P}_\phi = P\sin\phi$

So magnitude of angular momentum $L = r\,P\sin\phi$

$$L = r\,P_\phi$$

$\therefore$ Angular momentum = Position vector × Transverse component of angular momentum

i.e., The radial component of linear momentum has no role to play in angular momentum.

(5) Magnitude of angular momentum $L = P \ (r\sin\phi) = L = Pd$ [As $d = r\sin\phi$ from the figure.]

$\therefore$ Angular momentum = (Linear momentum) × (Perpendicular distance of line of action of force from the axis of rotation)

(6) Maximum and minimum angular momentum : We know $\vec{L} = \vec{r} \times \vec{P}$

$\therefore$ $\vec{L} = m[\vec{r} \times \vec{v}] = m\,v\,r\sin\phi = P\,r\sin\phi$ [As $\vec{P} = m\,\vec{v}$]

$L_{\text{maximum}} = mvr$	When $\lvert\sin\phi\rvert = \max = 1$ *i.e.*, $\phi = 90°$	$\vec{v}$ is perpendicular to $\vec{r}$
$L_{\text{minimum}} = 0$	When $\lvert\sin\phi\rvert = \min = 0$ *i.e.* $\phi = 0°$ or $180°$	$\vec{v}$ is parallel or anti-parallel to $\vec{r}$

(7) A particle in translatory motion always have an angular momentum unless it is a point on the line of motion because $L = mvr\sin\phi$ and $L > 1$ if $\phi \neq 0^o$ or 180^o

(8) In case of circular motion, $\vec{L} = \vec{r} \times \vec{P} = m(\vec{r} \times \vec{v}) = mvr\sin\phi$

$\therefore$ $L = mvr = mr^2\omega$ [As $\vec{r} \perp \vec{v}$ and $v = r\omega$]

or $L = I\omega$ [As $mr^2 = I$]

In vector form $\vec{L} = I\vec{\omega}$

(9) From $\vec{L} = I\vec{\omega}$ $\therefore$ $\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha} = \vec{\tau}$ [As $\frac{d\vec{\omega}}{dt} = \vec{\alpha}$ and $\vec{\tau} = I\vec{\alpha}$]

i.e. the rate of change of angular momentum is equal to the net torque acting on the particle. [Rotational analogue of Newton's second law]

(10) If a large torque acts on a particle for a small time then 'angular impulse' of torque is given by

$$\vec{J} = \int \vec{\tau}\, dt = \vec{\tau}_{av} \int_{t_1}^{t_2} dt$$

or Angular impulse $\vec{J} = \vec{\tau}_{av}\, \Delta t = \Delta \vec{L}$

$\therefore$ Angular impulse = Change in angular momentum

(11) The angular momentum of a system of particles is equal to the vector sum of angular momentum of each particle *i.e.*, $\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + ... + \vec{L}_n$.

(12) According to Bohr theory angular momentum of an electron in n^{th} orbit of atom can be taken as,

$$L = n\frac{h}{2\pi}$$ [where n is an integer used for number of orbit]

7.18 Law of Conservation of Angular Momentum

Newton's second law for rotational motion $\vec{\tau} = \frac{d\vec{L}}{dt}$

So if the net external torque on a particle (or system) is zero then $\frac{d\vec{L}}{dt} = 0$

i.e. $\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + ...$ = constant.

Angular momentum of a system (may be particle or body) remains constant if resultant torque acting on it zero.

As $L = I\omega$ so if $\vec{\tau} = 0$ then $I\omega = \text{constant}$ $\therefore$ $I \propto \frac{1}{\omega}$

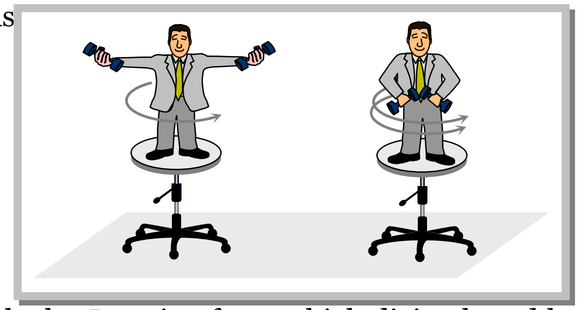
Since angular momentum $I\omega$ remains constant so when I decreases, angular velocity ω increases and vice-versa.

Examples of law of conservation of angular momentum :

(1) The angular velocity of revolution of a planet around the sun in an elliptical orbit increases when the planet come closer to the sun and vice-versa because when planet comes closer to the sun, it's moment of inertia I decreases there fore ω increases.

(2) A circus acrobat performs feats involving spin by bringing his arms and legs closer to his body or vice-versa. On bringing the arms and legs closer to body, his moment of inertia I decreases. Hence ω increases.

(3) A person-carrying heavy weight in his hands and standing on a rotating platform can change the speed of platform. When the person suddenly folds his arms. Its moment of inertia decreases and in accordance the angular speed increas



(4) A diver performs somersaults by Jumping from a high diving board keeping his legs and arms out stretched first and then curling his body.

(5) Effect of change in radius of earth on its time period

Angular momentum of the earth $L = I\omega = \text{constant}$

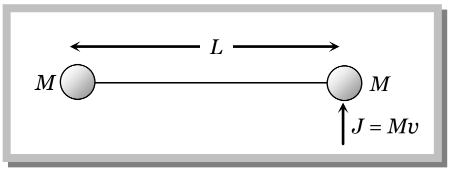
$$L = \frac{2}{5}MR^2 \times \frac{2\pi}{T} = \text{constant}$$

$\therefore$ $T \propto R^2$ [if M remains constant]

If R becomes half then time period will become one-fourth *i.e.* $\frac{24}{4} = 6 hrs.$

Sample problems based on angular momentum

***Problem* 32.** Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its ends, what would be its angular velocity **[IIT-JEE (Screening) 2003]**



(a) v/L (b) $2v/L$

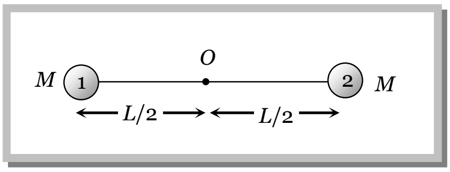
(c) $v/3L$ (d) $v/4L$

Solution: (a) Initial angular momentum of the system about point O

= Linear momentum × Perpendicular distance of linear momentum from the axis of rotation

$= Mv\left(\frac{L}{2}\right)$ ....(i)

Final angular momentum of the system about point O



$= I_1\omega + I_2\omega = (I_1 + I_2)\omega = \left[M\left(\frac{L}{2}\right)^2 + M\left(\frac{L}{2}\right)^2\right]\omega$ ....(ii)

Applying the law of conservation of angular momentum

$\Rightarrow$ $Mv\left(\frac{L}{2}\right) = 2M\left(\frac{L}{2}\right)^2\omega$ $\Rightarrow$ $\omega = \frac{v}{L}$

***Problem* 33.** A thin circular ring of mass M and radius R is rotating about its axis with a constant angular velocity ω. Four objects each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(a) $\frac{M\omega}{M + 4m}$ (b) $\frac{(M + 4m)\omega}{M}$ (c) $\frac{(M - 4m)\omega}{M + 4m}$ (d) $\frac{M\omega}{4m}$

Solution: (a) Initial angular momentum of ring $= I\omega = MR^2\omega$

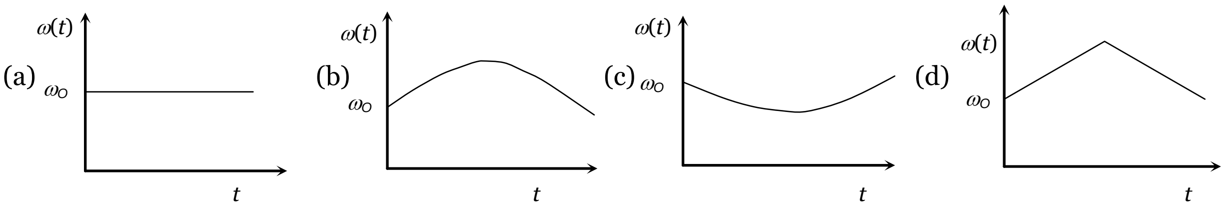
If four object each of mass m, and kept gently to the opposite ends of two perpendicular diameters of the ring then final angular momentum = $(MR^2 + 4mR^2)\omega'$

By the conservation of angular momentum

Initial angular momentum = Final angular momentum

$$MR^2\omega = (MR^2 + 4mR^2)\omega' \Rightarrow \omega' = \left(\frac{M}{M+4m}\right)\omega .$$

***Problem* 34.** A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its center. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as



Solution: (b) The angular momentum (L) of the system is conserved *i.e.* $L = I\omega$ = constant

When the tortoise walks along a chord, it first moves closer to the centre and then away from the centre. Hence, M.I. first decreases and then increases. As a result, ω will first increase and then decrease. Also the change in ω will be non-linear function of time.

***Problem* 35.** The position of a particle is given by : $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k})$ and momentum $\vec{P} = (3\hat{i} + 4\hat{j} - 2\hat{k})$. The angular momentum is perpendicular to

(a) X-axis (b) Y-axis

(c) Z-axis (d) Line at equal angles to all the three axes

Solution: (a) $\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = 0\hat{i} - \hat{j} - 2\hat{k} = -\hat{j} - 2\hat{k}$ and the X- axis is given by $i + 0\hat{j} + 0\hat{k}$

Dot product of these two vectors is zero *i.e.* angular momentum is perpendicular to X-axis.

***Problem* 36.** Two discs of moment of inertia I_1 and I_2 and angular speeds ω_1 and ω_2 are rotating along collinear axes passing through their centre of mass and perpendicular to their plane. If the two are made to rotate together along the same axis the rotational KE of system will be

(a) $\dfrac{I_1\omega_1 + I_2\omega_2}{2(I_1 + I_2)}$ (b) $\dfrac{(I_1 + I_2)(\omega_1 + \omega_2)^2}{2}$ (c) $\dfrac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$ (d) None of these

Solution: (c) By the law of conservation of angular momentum $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

Angular velocity of system $\omega = \dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

Rotational kinetic energy $= \dfrac{1}{2}(I_1 + I_2)\omega^2 = \dfrac{1}{2}(I_1 + I_2)\left(\dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2 = \dfrac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$.

***Problem* 37.** A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with angular velocity ω_0 about an axis perpendicular to the rod and passing through the mid point of the rod (see figure). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is

(a) ω_0 (b) $\frac{M\omega_0}{M+12m}$

(c) $\frac{M\omega_0}{M+2m}$ (d) $\frac{M\omega_0}{M+6m}$

Solution: (d) Since there are no external forces therefore the angular momentum of the system remains constant.

Initially when the beads are at the centre of the rod angular momentum $L_1 = \left(\frac{ML^2}{12}\right)\omega_0$ .....(i)

When beads reach the ends of the rod then angular

momentum $= \left(m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 + \frac{ML^2}{12}\right)\omega'$ ..(ii)

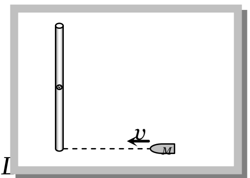
Equating (i) and (ii) $\frac{ML^2}{12}\omega_0 = \left(\frac{mL^2}{2} + \frac{ML^2}{12}\right)\omega' \Rightarrow \omega' = \frac{M\omega_o}{M+6m}$.

***Problem* 38.** Moment of inertia of uniform rod of mass M and length L about an axis through its centre and perpendicular to its length is given by $\frac{ML^2}{12}$. Now consider one such rod pivoted at its centre, free to rotate in a vertical plane. The rod is at rest in the vertical position. A bullet of mass M moving horizontally at a speed v strikes and embedded in one end of the rod. The angular velocity of the rod just after the collision will be

(a) v/L (b) $2v/L$ (c) $3v/2L$ (d) $6v/L$

Solution: (c)Initial angular momentum of the system = Angular momentum of bullet before collision $= Mv\left(\frac{L}{2}\right)$

.....(i)

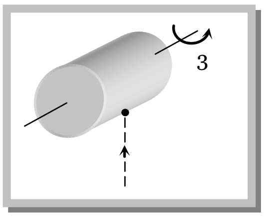


let the rod rotates with angular velocity ω.

Final angular momentum of the system $= \left(\frac{ML^2}{12}\right)\omega + M\left(\frac{L}{2}\right)^2\omega$ ......(ii)

By equation (i) and (ii) $Mv\frac{L}{2} = \left(\frac{ML^2}{12} + \frac{ML^2}{4}\right)\omega$ or $\omega = 3v/2L$

***Problem* 39.** A solid cylinder of mass 2 *kg* and radius $0.2\,m$ is rotating about its own axis without friction with angular velocity $3\,rad/s$. A particle of mass 0.5 *kg* and moving with a velocity 5 *m/s* strikes the cylinder and sticks to it as shown in figure. The angular momentum of the cylinder before collision will be



(a) 0.12 *J-s* (b) 12 *J-s*

(c) 1.2 *J-s* (d) 1.12 *J-s*

Solution: (a) Angular momentum of the cylinder before collision $L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}(2)(0.2)^2 \times 3$ = 0.12 *J-s.*

Problem **40.** In the above problem the angular velocity of the system after the particle sticks to it will be

(a) 0.3 *rad/s* (b) 5.3 *rad/s* (c) 10.3 *rad/s* (d) 89.3 *rad/s*

Solution: (c) Initial angular momentum of bullet + initial angular momentum of cylinder

= Final angular momentum of (bullet + cylinder) system

$$\Rightarrow mvr + I_1\omega = (I_1 + I_2)\omega'$$

$$\Rightarrow mvr + I_1\omega = \left(\frac{1}{2}Mr^2 + mr^2\right)\omega'$$

$$\Rightarrow 0.5 \times 5 \times 0.2 + 0.12 = \left(\frac{1}{2}2(0.2)^2 + (0.5)(0.2)^2\right)\omega'$$

$$\therefore \quad \omega' = 10.3 \ rad/sec.$$

7.19 Work, Energy and Power for Rotating Body

(1) **Work :** If the body is initially at rest and angular displacement is $d\theta$ due to torque then work done on the body.

$$W = \int \tau \, d\theta$$ [Analogue to work in translatory motion $W = \int F\,dx$]

(2) **Kinetic energy :** The energy, which a body has by virtue of its rotational motion is called rotational kinetic energy. A body rotating about a fixed axis possesses kinetic energy because its constituent particles are in motion, even though the body as a whole remains in place.

Rotational kinetic energy	Analogue to translatory kinetic energy
$K_R = \frac{1}{2}I\omega^2$	$K_T = \frac{1}{2}mv^2$
$K_R = \frac{1}{2}L\omega$	$K_T = \frac{1}{2}Pv$
$K_R = \frac{L^2}{2I}$	$K_T = \frac{P^2}{2m}$

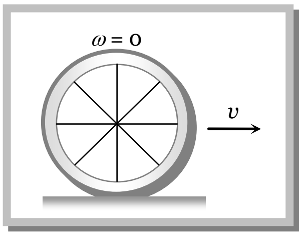
(3) **Power :** Rate of change of kinetic energy is defined as power

$$P = \frac{d}{dt}(K_R) = \frac{d}{dt}\left[\frac{1}{2}I\omega^2\right] = I\omega\frac{d\omega}{dt} = I\omega\alpha = I\alpha\omega = \tau\omega$$

In vector form Power = $\vec{\tau} \cdot \vec{\omega}$ [Analogue to power in translatory motion $P = \vec{F} \cdot \vec{v}$]

7.20 Slipping, Spinning and Rolling

(1) **Slipping :** When the body slides on a surface without rotation then its motion is called slipping motion.

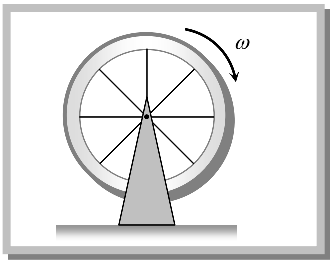


In this condition friction between the body and surface $F = 0$.

Body possess only translatory kinetic energy $K_T = \frac{1}{2}mv^2$.

Example : Motion of a ball on a frictionless surface.

(2) **Spinning :** When the body rotates in such a manner that its axis of rotation does not move then its motion is called spinning motion.



In this condition axis of rotation of a body is fixed.

Example : Motion of blades of a fan.

In spinning, body possess only rotatory kinetic energy $K_R = \frac{1}{2} I\omega^2$.

or $$K_R = \frac{1}{2} mK^2 \frac{v^2}{R^2} = \frac{1}{2} mv^2 \left(\frac{K^2}{R^2}\right)$$

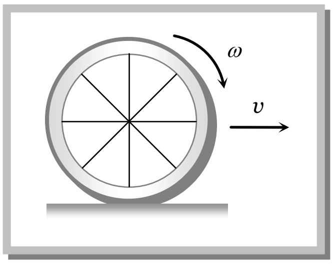
i.e., Rotatory kinetic energy = $\left(\frac{K^2}{R^2}\right)$ times translatory kinetic energy.

Here $\frac{K^2}{R^2}$ is a constant for different bodies. Value of $\frac{K^2}{R^2} = 1$ (ring), $\frac{K^2}{R^2} = \frac{1}{2}$ (disc) and $\frac{K^2}{R^2} = \frac{1}{2}$ (solid sphere)

(3) **Rolling :** If in case of rotational motion of a body about a fixed axis, the axis of rotation also moves, the motion is called combined translatory and rotatory.

Example : (i) Motion of a wheel of cycle on a road.

(ii) Motion of football rolling on a surface.



In this condition friction between the body and surface $F \neq 0$.

Body possesses both translational and rotational kinetic energy.

Net kinetic energy = (Translatory + Rotatory) kinetic energy.

$$K_N = K_T + K_R = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} mv^2 + \frac{1}{2} mv^2 \frac{K^2}{R^2}$$

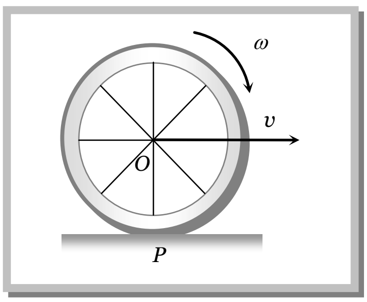
$$\therefore \quad K_N = \frac{1}{2} mv^2 \left(1 + \frac{K^2}{R^2}\right)$$

7.21 Rolling Without Slipping

In case of combined translatory and rotatory motion if the object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping.

Friction is responsible for this type of motion but work done or dissipation of energy against friction is zero as there is no relative motion between body and surface at the point of contact.

Rolling motion of a body may be treated as a pure rotation about an axis through point of contact with same angular velocity ω.



By the law of conservation of energy

$$K_N = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \qquad [\therefore \text{ As } v = R\omega]$$

$$= \frac{1}{2} mR^2\omega^2 + \frac{1}{2} I\omega^2$$

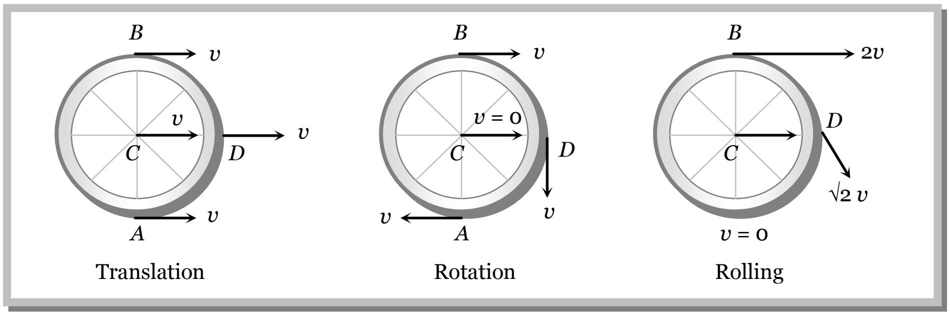
$$= \frac{1}{2}\omega^2 [mR^2 + I]$$

$$= \frac{1}{2}\omega^2[I + mR^2] = \frac{1}{2}I_P\omega^2 \quad [\text{As } I_P = I + mR^2]$$

By theorem of parallel axis, where I = moment of inertia of rolling body about its centre 'O' and I_P = moment of inertia of rolling body about point of contact 'P'.

(1) **Linear velocity of different points in rolling :** In case of rolling, all points of a rigid body have same angular speed but different linear speed.

Let A, B, C and D are four points then their velocities are shown in the following figure.



(2) **Energy distribution table for different rolling bodies :**

Body	$\frac{K^2}{R^2}$	Translatory (K_T) $\frac{1}{2}mv^2$	Rotatory (K_R) $\frac{1}{2}mv^2\frac{K^2}{R^2}$	Total (K_N) $\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$	$\frac{K_T}{K_N}$ (%)	$\frac{K_R}{K_N}$ (%)
Ring Cylindrical shell	1	$\frac{1}{2}mv^2$	$\frac{1}{2}mv^2$	mv^2	$\frac{1}{2}$ (50%)	$\frac{1}{2}$ (50%)
Disc solid cylinder	$\frac{1}{2}$	$\frac{1}{2}mv^2$	$\frac{1}{4}mv^2$	$\frac{3}{4}mv^2$	$\frac{2}{3}$ (66.6%)	$\frac{1}{3}$ (33.3%)
Solid sphere	$\frac{2}{5}$	$\frac{1}{2}mv^2$	$\frac{1}{5}mv^2$	$\frac{7}{10}mv^2$	$\frac{5}{7}$ (71.5%)	$\frac{2}{7}$ (28.5%)
Hollow sphere	$\frac{2}{3}$	$\frac{1}{2}mv^2$	$\frac{1}{3}mv^2$	$\frac{5}{6}mv^2$	$\frac{3}{5}$ (60%)	$\frac{2}{5}$ (40%)

Sample problems based on kinetic energy, work and power

***Problem* 41.** A ring of radius 0.5 *m* and mass 10 *kg* is rotating about its diameter with an angular velocity of 20 *rad/s*. Its kinetic energy is

(a) 10 *J* (b) 100 *J* (c) 500 *J* (d) 250 *J*

Solution: (d) Rotational kinetic energy $\frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}\left(\frac{1}{2}\times 10\times(0.5)^2\right)(20)^2 = 250\ J$

***Problem* 42.** An automobile engine develops 100 *kW* when rotating at a speed of 1800 rev/*min*. What torque does it deliver **[CBSE PMT 2000]**

(a) 350 *N-m* (b) 440 *N-m* (c) 531 *N-m* (d) 628 *N-m*

Solution: (c) $P = \tau\omega \Rightarrow \tau = \dfrac{100\times 10^3}{2\pi\dfrac{1800}{60}} = 531\ N\text{-}m$

***Problem* 43.** A body of moment of inertia of 3 *kg-m²* rotating with an angular velocity of 2 *rad/sec* has the same kinetic energy as a mass of 12 *kg* moving with a velocity of

(a) 8 *m/s* (b) 0.5 *m/s* (c) 2 *m/s* (d) 1 *m/s*

Solution: (d) Rotational kinetic energy of the body = $\frac{1}{2}I\omega^2$ and translatory kinetic energy = $\frac{1}{2}mv^2$

According to problem $= \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 \Rightarrow \frac{1}{2}\times 3\times(2)^2 = \frac{1}{2}\times 12\times v^2 \Rightarrow v = 1\,m/s$.

***Problem* 44.** A disc and a ring of same mass are rolling and if their kinetic energies are equal, then the ratio of their velocities will be

(a) $\sqrt{4}:\sqrt{3}$ (b) $\sqrt{3}:\sqrt{4}$ (c) $\sqrt{3}:\sqrt{2}$ (d) $\sqrt{2}:\sqrt{3}$

Solution: (a) $K_{disc} = \frac{1}{2}mv_d^2\left(1+\frac{k^2}{R^2}\right) = \frac{3}{4}mv_d^2$ $\left[\text{As } \frac{k^2}{R^2} = \frac{1}{2} \text{ for disc}\right]$

$K_{ring} = \frac{1}{2}mv_r\left(1+\frac{k^2}{R^2}\right) = mv_r^2$ $\left[\text{As } \frac{k^2}{R^2} = 1 \text{ for ring}\right]$

According to problem $K_{disc} = K_{ring} \Rightarrow \frac{3}{4}mv_d^2 = mv_r^2 \Rightarrow \frac{v_d}{v_r} = \sqrt{\frac{4}{3}}$.

***Problem* 45.** A wheel is rotating with an angular speed of $20\,rad/sec$. It is stopped to rest by applying a constant torque in $4s$. If the moment of inertia of the wheel about its axis is 0.20 *kg-m*², then the work done by the torque in two seconds will be

(a) 10 *J* (b) 20 *J* (c) 30 *J* (d) 40 *J*

Solution: (c) $\omega_1 = 20\ rad/sec$, $\omega_2 = 0, t = 4\,sec$. So angular retardation $\alpha = \frac{\omega_1-\omega_2}{t} = \frac{20}{4} = 5\,rad/sec^2$

Now angular speed after 2 *sec* $\omega_2 = \omega_1 - \alpha t = 20 - 5\times 2 = 10\ rad/sec$

Work done by torque in 2 *sec* = loss in kinetic energy $= \frac{1}{2}I\left(\omega_1^2-\omega_2^2\right) = \frac{1}{2}(0.20)((20)^2-(10)^2)$

$= \frac{1}{2}\times 0.2\times 300 = 30\,J$.

***Problem* 46.** If the angular momentum of a rotating body is increased by 200%, then its kinetic energy of rotation will be increased by

(a) 400% (b) 800% (c) 200% (d) 100%

Solution: (b) As $E = \frac{L^2}{2I} \Rightarrow \frac{E_2}{E_1} = \left(\frac{L_2}{L_1}\right)^2 = \left(\frac{3L_1}{L_1}\right)^2$ [As $L_2 = L_1 + 200\%.L_1 = 3L_1$]

$\Rightarrow E_2 = 9E_1 = E_1 + 800\%$ of E_1

***Problem* 47.** A ring, a solid sphere and a thin disc of different masses rotate with the same kinetic energy. Equal torques are applied to stop them. Which will make the least number of rotations before coming to rest

(a) Disc (b) Ring

(c) Solid sphere (d) All will make same number of rotations

Solution: (d) As $W = \tau\theta = \text{Energy} \Rightarrow \theta = \frac{\text{Energy}}{\tau} = 2n\pi$

So, if energy and torque are same then all the bodies will make same number of rotation.

***Problem* 48.** The angular velocity of a body is $\vec{\omega} = 2\hat{i}+3\hat{j}+4\hat{k}$ and a torque $\vec{\tau} = \hat{i}+2\hat{j}+3\hat{k}$ acts on it. The rotational power will be

(a) 20 *W* (b) 15 *W* (c) $\sqrt{17}$ *W* (d) $\sqrt{14}$ *W*

Solution: (a) Power $(P) = \vec{\tau}.\vec{\omega} = (i+2\hat{j}+3\hat{k}).(2\hat{i}+3\hat{j}+4\hat{k}) = 2+6+12 = 20\,W$

***Problem* 49.** A flywheel of moment of inertia 0.32 *kg-m*² is rotated steadily at $120\,rad/\text{sec}$ by a $50\,W$ electric motor. The kinetic energy of the flywheel is

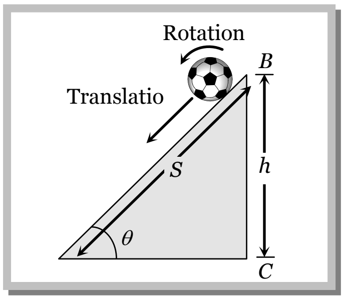
(a) 4608 *J* (b) 1152 *J* (c) 2304 *J* (d) 6912 *J*

Solution: (c) Kinetic energy $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.32)(120)^2 = 2304\,J$.

7.22 Rolling on an Inclined Plane

When a body of mass m and radius R rolls down on inclined plane of height 'h' and angle of inclination θ, it loses potential energy. However it acquires both linear and angular speeds and hence, gain kinetic energy of translation and that of rotation.

By conservation of mechanical energy $mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$



(1) **Velocity at the lowest point :** $v = \sqrt{\dfrac{2gh}{1+\dfrac{k^2}{R^2}}}$

(2) **Acceleration in motion :** From equation $v^2 = u^2 + 2aS$

By substituting $u = 0,\ S = \dfrac{h}{\sin\theta}$ and $v = \sqrt{\dfrac{2gh}{1+\dfrac{k^2}{R^2}}}$ we get

$$a = \frac{g\sin\theta}{1+\dfrac{k^2}{R^2}}$$

(3) **Time of descent :** From equation $v = u + at$

By substituting u = 0 and value of v and a from above expressions

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left[1+\frac{k^2}{R^2}\right]}$$

From the above expressions it is clear that, $v \propto \dfrac{1}{\sqrt{1+\dfrac{k^2}{R^2}}}$; $a \propto \dfrac{1}{1+\dfrac{k^2}{R^2}}$; $t \propto \sqrt{1+\dfrac{k^2}{R^2}}$

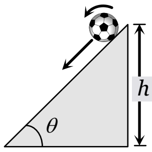
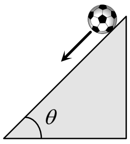
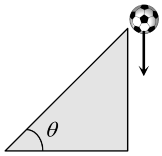
Note :
- Here factor $\left(\dfrac{k^2}{R^2}\right)$ is a measure of moment of inertia of a body and its value is constant for given shape of the body and it does not depend on the mass and radius of a body.
- Velocity, acceleration and time of descent (for a given inclined plane) all depends on $\dfrac{k^2}{R^2}$.

  Lesser the moment of inertia of the rolling body lesser will be the value of $\dfrac{k^2}{R^2}$. So greater will be its velocity and acceleration and lesser will be the time of descent.
- If a solid and hollow body of same shape are allowed to roll down on inclined plane then as $\left(\dfrac{k^2}{R^2}\right)_S < \left(\dfrac{k^2}{R^2}\right)_H$, solid body will reach the bottom first with greater velocity.
- If a ring, cylinder, disc and sphere runs a race by rolling on an inclined plane then as $\left(\dfrac{k^2}{R^2}\right)_{\text{sphere}}$ = minimum while $\left(\dfrac{k^2}{R^2}\right)_{\text{Ring}}$ = maximum, the sphere will reach the bottom first with greatest velocity while ring at last with least velocity.
- Angle of inclination has no effect on velocity, but time of descent and acceleration depends on it.

  velocity $\propto \theta^{\circ}$, time of decent $\propto \theta^{-1}$ and acceleration $\propto \theta$.

7.23 Rolling Sliding and Falling of a Body

	Figure	Velocity	Acceleration	Time

Rolling	$\frac{k^2}{R^2} \neq 0$		$\sqrt{\frac{2gh}{1+k^2/R^2}}$	$\frac{g\sin\theta}{1+K^2/R^2}$	$\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)}$
Sliding	$\frac{k^2}{R^2} = 0$		$\sqrt{2gh}$	$g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$
Falling	$\frac{k^2}{R^2} = 0$ $\theta = 90^o$		$\sqrt{2gh}$	g	$\sqrt{\frac{2h}{g}}$

7.24 Velocity, Acceleration and Time for Different Bodies

Body	$\frac{k^2}{R^2}$	Velocity $v = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$	Acceleration $a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$	Time of descent $t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)}$
Ring or Hollow cylinder	1	$\sqrt{gh}$	$\frac{1}{2}g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{4h}{g}}$
Disc or solid cylinder	$\frac{1}{2}$ or 0.5	$\sqrt{\frac{4gh}{3}}$	$\frac{2}{3}g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{3h}{g}}$
Solid sphere	$\frac{2}{5}$ or 0.4	$\sqrt{\frac{10}{7}gh}$	$\frac{5}{7}g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{14}{5}\frac{h}{g}}$
Hollow sphere	$\frac{2}{3}$ or 0.66	$\sqrt{\frac{6}{5}gh}$	$\frac{3}{5}g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{10}{3}\frac{h}{g}}$

Sample problems based on rolling on an inclined plane

***Problem* 50.** A solid cylinder of mass M and radius R rolls without slipping down an inclined plane of length L and height h. What is the speed of its centre of mass when the cylinder reaches its bottom

(a) $\sqrt{\frac{3}{4}gh}$ (b) $\sqrt{\frac{4}{3}gh}$ (c) $\sqrt{4\,gh}$ (d) $\sqrt{2\,gh}$

Solution: (b) Velocity at the bottom $(v) = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}} = \sqrt{\frac{2gh}{1+\frac{1}{2}}} = \sqrt{\frac{4}{3}gh}$.

***Problem* 51.** A sphere rolls down on an inclined plane of inclination θ. What is the acceleration as the sphere reaches bottom **[Orissa JEE 2003]**

(a) $\frac{5}{7}g\sin\theta$ (b) $\frac{3}{5}g\sin\theta$ (c) $\frac{2}{7}g\sin\theta$ (d) $\frac{2}{5}g\sin\theta$

Solution: (a) Acceleration $(a) = \frac{g\sin\theta}{1+\frac{K^2}{R^2}} = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7}g\sin\theta$.

***Problem* 52.** A ring solid sphere and a disc are rolling down from the top of the same height, then the sequence to reach on surface is **[RPMT 1999]**

(a) Ring, disc, sphere (b) Sphere, disc, ring (c) Disc, ring, sphere (d) Sphere, ring, disc

Solution: (b) Time of descent $\propto$ moment of inertia $\propto \frac{k^2}{R^2}$

$$\left(\frac{k^2}{R^2}\right)_{sphere} = 0.4\,,\ \left(\frac{k^2}{R^2}\right)_{disc} = 0.5\,,\ \left(\frac{k^2}{R^2}\right)_{ring} = 1 \quad \therefore\ t_{sphere} < t_{disc} < t_{ring}\,.$$

***Problem* 53.** A thin uniform circular ring is rolling down an inclined plane of inclination 30° without slipping. Its linear acceleration along the inclined plane will be

(a) $g/2$ (b) $g/3$ (c) $g/4$ (d) $2g/3$

Solution: (c) $a = \dfrac{g\sin\theta}{1+\dfrac{k^2}{R^2}} = \dfrac{g\sin 30^o}{1+1} = \dfrac{g}{4}$ [As $\dfrac{k^2}{R^2} = 1$ and $\theta = 30^o$]

***Problem* 54.** A solid sphere and a disc of same mass and radius starts rolling down a rough inclined plane, from the same height the ratio of the time taken in the two cases is

(a) 15 : 14 (b) $\sqrt{15} : \sqrt{14}$ (c) 14 : 15 (d) $\sqrt{14} : \sqrt{15}$

Solution: (d) Time of descent $t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left(1+\dfrac{k^2}{R^2}\right)}$ $\therefore \dfrac{t_{\text{shpere}}}{t_{\text{disc}}} = \sqrt{\dfrac{\left(1+\dfrac{k^2}{R^2}\right)_{\text{sphere}}}{\left(1+\dfrac{k^2}{R^2}\right)_{\text{disc}}}} = \sqrt{\dfrac{1+\dfrac{2}{5}}{1+\dfrac{1}{2}}} = \sqrt{\dfrac{7}{5}\times\dfrac{2}{3}} = \sqrt{\dfrac{14}{15}}$

***Problem* 55.** A solid sphere of mass $0.1\,kg$ and radius $2\,cm$ rolls down an inclined plane $1.4\,m$ in length (slope 1 in 10). Starting from rest its final velocity will be

(a) $1.4\ m/\text{sec}$ (b) $0.14\ m/\text{sec}$ (c) $14\ m/\text{sec}$ (d) $0.7\ m/\text{sec}$

Solution: (a) $v = \sqrt{\dfrac{2gh}{1+\dfrac{k^2}{R^2}}} = \sqrt{\dfrac{2\times 9.8\times l\sin\theta}{1+\dfrac{2}{5}}}$ [As $\dfrac{k^2}{R^2} = \dfrac{2}{5}$, $l = \dfrac{h}{\sin\theta}$ and $\sin\theta = \dfrac{1}{10}$ given]

$$\Rightarrow v = \sqrt{\frac{2\times 9.8\times 1.4\times\frac{1}{10}}{7/5}} = 1.4\,m/s.$$

***Problem* 56.** A solid sphere rolls down an inclined plane and its velocity at the bottom is v_1. Then same sphere slides down the plane (without friction) and let its velocity at the bottom be v_2. Which of the following relation is correct

(a) $v_1 = v_2$ (b) $v_1 = \frac{5}{7}v_2$ (c) $v_1 = \frac{7}{5}v_2$ (d) None of these

Solution: (d) When solid sphere rolls down an inclined plane the velocity at bottom $v_1 = \sqrt{\frac{10}{7}gh}$

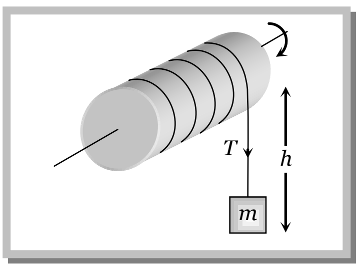
but, if there is no friction then it slides on inclined plane and the velocity at bottom $v_2 = \sqrt{2gh}$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{5}{7}}\,.$$

7.25 Motion of Connected Mass

A point mass is tied to one end of a string which is wound round the solid body [cylinder, pulley, disc]. When the mass is released, it falls vertically downwards and the solid body rotates unwinding the string

m = mass of point-mass, M = mass of a rigid body
R = radius of a rigid body, I = moment of inertia of rotating body



(1) **Downwards acceleration of point mass** $a = \dfrac{g}{1+\dfrac{I}{mR^2}}$

(2) **Tension in string** $T = mg\left[\dfrac{I}{I+mR^2}\right]$

(3) **Velocity of point mass** $v = \sqrt{\dfrac{2gh}{1 + \dfrac{I}{mR^2}}}$ (4) **Angular velocity of rigid body** $\omega = \sqrt{\dfrac{2mgh}{I + mR^2}}$

Sample problems based on motion of connected mass

Problem 57. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and moment of inertia about it is I. A weight mg is attached to the end of the cord and falls from rest. After falling through a distance h, the angular velocity of the wheel will be

(a) $\sqrt{\dfrac{2gh}{I + mr}}$ (b) $\sqrt{\dfrac{2mgh}{I + mr^2}}$ (c) $\sqrt{\dfrac{2mgh}{I + 2mr^2}}$ (d) $\sqrt{2gh}$

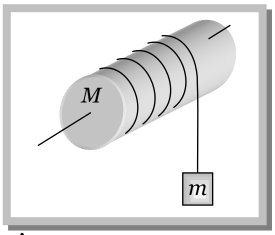
Solution : (b) According to law of conservation of energy $mgh = \frac{1}{2}(I + mr^2)\omega^2 \Rightarrow \omega = \sqrt{\dfrac{2mgh}{I + mr^2}}$.

Problem 58. In the following figure, a body of mass m is tied at one end of a light string and this string is wrapped around the solid cylinder of mass M and radius R. At the moment $t = 0$ the system starts moving. If the friction is negligible, angular velocity at time t would be

(a) $\dfrac{mgRt}{(M+m)}$ (b) $\dfrac{2Mgt}{(M+2m)}$

(c) $\dfrac{2mgt}{R(M-2m)}$ (d) $\dfrac{2mgt}{R(M+2m)}$

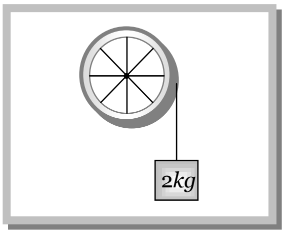
Solution : (d) We know the tangential acceleration $a = \dfrac{g}{1 + \dfrac{I}{mR^2}} = \dfrac{g}{1 + \dfrac{1/2MR^2}{mR^2}} = \dfrac{2mg}{2m + M}$ [As $I = \frac{1}{2}MR^2$ for cylinder]



After time t, linear velocity of mass m, $v = u + at = 0 + \dfrac{2mgt}{2m + M}$

So angular velocity of the cylinder $\omega = \dfrac{v}{R} = \dfrac{2mgt}{R(M + 2m)}$.

Problem 59. A block of mass $2\,kg$ hangs from the rim of a wheel of radius $0.5\,m$. On releasing from rest the block falls through $5\,m$ height in $2\,s$. The moment of inertia of the wheel will be



(a) 1 $kg\text{-}m^2$ (b) 3.2 $kg\text{-}m^2$
(c) 2.5 $kg\text{-}m^2$ (d) 1.5 $kg\text{-}m^2$

Solution : (d) On releasing from rest the block falls through 5m height in 2 *sec*.

$5 = 0 + \frac{1}{2}a(2)^2$ [As $S = ut + \frac{1}{2}at^2$] $\therefore a = 2.5\,m/s^2$

Substituting the value of a in the formula $a = \dfrac{g}{1 + \dfrac{I}{mR^2}}$ and by solving we get

$\Rightarrow 2.5 = \dfrac{10}{1 + \dfrac{I}{2 \times (0.5)^2}} \Rightarrow I = 1.5 kg - m^2$

Analogy between Linear motion and Rotational motion

Linear Motion	Rotational Motion
1. Distance (x)	1. Angle (θ)
2. Linear velocity $\left(v = \frac{dx}{dt}\right)$	2. Ang. velocity $\left(\omega = \frac{d\theta}{dt}\right)$
$v = \omega r$	
3. Linear acceleration $\left(a = \frac{dv}{dt}\right)$	3. Angular acceleration $\left(\alpha = \frac{d\omega}{dt}\right)$
$a = \alpha r$	
4. $v = u + at$	4. $\omega = \omega_0 + \alpha t$
5. $S = ut + \frac{1}{2}at^2$	5. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
6. $v^2 - u^2 = 2aS$	6. $\omega^2 - \omega_0^2 = 2\alpha\theta$
7. $x_{nth} = u + \frac{1}{2}a(2n-1)$	7. $\theta_{nth} = \omega_0 + \frac{1}{2}\alpha(2n-1)$
8. Mass/inertia (M)	8. Moment of Inertia (I)
9. Linear momentum $P = mV$	9. Angular momentum $L = I\omega$
10. Force (F)	10. Moment of force/ Torque (τ)
11. $\vec{F} = \frac{d\vec{P}}{dt}$	11. $\vec{\tau} = \frac{d\vec{L}}{dt}$
12. $F = ma$	12. $\tau = I\alpha$
13. Work = Force × distance	13. Work = Torque × angle
14. Power $P = Fv$	14. Power $P = \tau\omega$
15. Linear K.E. $= \frac{1}{2}mv^2$	15. $= \frac{1}{2}I\omega^2$
16. Law of conservation linear momentum – when F = 0 then P is constant.	16. Law of conservation of angular momentum, when = 0, then L is constant.
17. Impulse, $I = Ft = p_2 - p_1$	17. Angular Impulse $= \tau.\ t = L_2 - L_1$

1. Position vector of centre of mass is

$$\vec{r} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3} + \ldots + m_n\vec{r_n}}{m_1 + m_2 + m_3 \ldots + m_n}.$$

2. Sum of moments of all particles about centre of mass is zero.
3. When no external force acts then velocity centre of mass remains constant.
4. Torque τ = Force × ⊥ distance of line of action of force from axis of rotation.
5. $\vec{\tau} = \vec{r} \times \vec{F}$

   The direction of $\vec{\tau}$ is ⊥ to the plane containing $\vec{r}$ and $\vec{F}$.
6. Torque due to radial force is always zero.
7. Angular momentum L = linear momentum × ⊥ distance of the line of action linear momentum from axis of rotation.
8. $\vec{L} = \vec{r} \times \vec{p}$
9. L = 2 mass × areal velocity.

(*iii*) About tangent in its plane $= \frac{5}{4}MR^2$.

(*iv*) About tangent ⊥ to its plane $= \frac{3}{2}MR^2$.

17. Moment of Inertia of rod.
    (*i*) About an axis passing throug centre and ⊥ to it length $I = \frac{1}{12}ML^2$.
    (*ii*) About an axis passing through one edge of ro and ⊥ to its length $= \frac{1}{3}ML^2$.
18. Moment of Inertia of solid sphere.
    (*i*) About diameter $I = \frac{2}{5}MR^2$.
    (*ii*) About tagent $I = \frac{7}{5}MR^2$.
19. Moment of Inertia of a spherical shell.
    (*i*) About diameter $= \frac{2}{3}MR^2$.
    (*ii*) About tangent $= \frac{5}{3}MR^2$.
20. Conservation of Angular Momentum.

= mass × areal velocity.

10. $\vec{\tau} = \frac{d\vec{L}}{dt}$

11. Moment of Inertia $I = \sum_{i=1}^{n} m_i r_i^2$.

12. $I = MK^2$ where K is radius of gyration.

13. Therom perpendicular axis $I_z = I_x + I_y$.

14. Therom of parallel axis $I = I_c + MR^2$.

15. Moment of Ineria of a ring.

(*i*) About an axis passing through centre and ⊥ to its plane $I = MR^2$.

(*ii*) About diameter $I_d = \frac{1}{2} MR^2$.

(*iii*) About tangent in its plane $I_t = \frac{3}{2} MR^2$.

(*iv*) About tangent ⊥ to its plane = $2\ MR^2$.

16. Moment of Inertia of disc.

(*i*) About an axis passing throug centre and ⊥ to its plane $I = \frac{1}{2} MR^2$.

(*ii*) About diameter $I_d = \frac{1}{4} MR^2$.

20. Conservation of Angular Momentum.

$$I_1\omega_1 = I_2\omega_2.$$

21. K.E. of rotation $= \frac{1}{2} I\omega^2$.

22. Motion of point mass attached to a string wound on a cylinder.

(*i*) $a = \frac{g}{1 + \frac{I}{mr^2}}$ clearly $a < g$.

(*ii*) $T = \frac{mg}{1 + \frac{mr^2}{I}}$ $T < mg$.

23. Acceleration of body rolling without slipping down the iclined

$$a = \frac{g \sin\theta}{1 + \frac{I}{mr^2}}$$

(*i*) For cylinder $I = \frac{1}{2} mr^2$

$\therefore \quad a = \frac{2}{3} g \sin\theta = 0.6\ g \sin\theta.$

(*ii*) For ring $I = mr^2$

$\therefore \quad a = \frac{1}{2} g \sin\theta = 0.5\ g \sin\theta.$

(*iii*) For solid sphere $I = \frac{2}{5} mr^2$

$\therefore \quad a = \frac{5}{7} g \sin\theta = 0.7\ g \sin\theta.$

24. Condition for cylinder to roll without slopping

$$\tan\theta \le \frac{1}{3} \mu_s.$$

25. Total kinetic energy of solid sphere rolling without slipping $= \frac{7}{10} mv^2$.

26. Total kinetic energy of disc or solid cylinder rolling without slipping $= \frac{3}{4} mv^2$.

27. Total kinetic energy of a ring rolling without slipping $= mv^2$.

28. Maximum velocity with which a vehicle can take circular turn on a level road $v_{max} = \sqrt{\mu_s rg}$.

29. Velocity with which a vehicle can take circular turn on a banked road.

$$v = \sqrt{\frac{rg\,(\mu + \tan\theta)}{1 - \mu \tan\theta}}$$

To avoid wear and tear of tyres.

$$v = \sqrt{rg \tan\theta} \text{ or } \tan\theta = \frac{v^2}{rg}.$$

30. Bending of cyclist $\tan\theta = \frac{v^2}{rg}$.

31. For motion of a body in a vertical circle the difference of tension at lowest pt and at highest point $= 6\ mg$.

32. Minimum velocity for looping the loop at bottom $= \sqrt{5gr}$.

33. Minimum velocity for looping the loop at highest point $= \sqrt{gr}$.

Work, Energy, Power and Collision

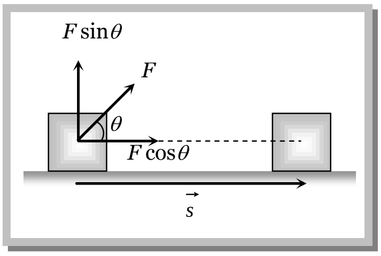
6.1 Introduction

The terms 'work', 'energy' and 'power' are frequently used in everyday language. A farmer clearing weeds in his field is said to be working hard. A woman carrying water from a well to her house is said to be working. In a drought affected region she may be required to carry it over large distances. If she can do so, she is said to have a large stamina or energy. Energy is thus the capacity to do work. The term power is usually associated with speed. In karate, a powerful punch is one delivered at great speed. In physics we shall define these terms very precisely. We shall find that there is at best a loose correlation between the physical definitions and the physiological pictures these terms generate in our minds.

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

6.2 Work Done by a Constant Force

Let a constant force $\vec{F}$ be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance s



By resolving force $\vec{F}$ into two components :

(i) $F\cos\theta$ in the direction of displacement of the body.

(ii) $F\sin\theta$ in the perpendicular direction of displacement of the body.

Since body is being displaced in the direction of $F\cos\theta$, therefore work done by the force in displacing the body through a distance s is given by

$$W = (F\cos\theta)s = Fs\cos\theta$$

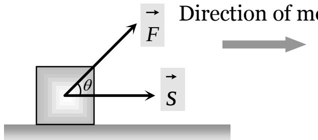
or $$W = \vec{F}.\vec{s}$$

Thus work done by a force is equal to the scalar or dot product of the force and the displacement of the body.

If a number of force $\vec{F}_1, \vec{F}_2, \vec{F}_3 ...... \vec{F}_n$ are acting on a body and it shifts from position vector $\vec{r}_1$ to position vector $\vec{r}_2$ then $W = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... \vec{F}_n).(\vec{r}_2 - \vec{r}_1)$

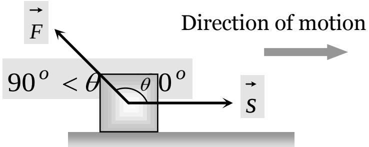
6.3 Nature of Work Done

Positive work	Negative work
Positive work means that force (or its component) is parallel to displacement	Negative work means that force (or its component) is opposite to displacement *i.e.*



$0^o \le \theta < 90^o$

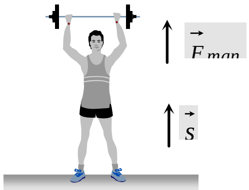
The positive work signifies that the external force favours the motion of the body.



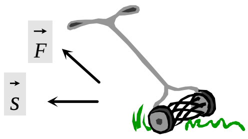
$90^o < \theta$ ... 0^o

The negative work signifies that the external force opposes the motion of the body.

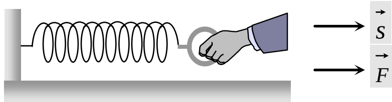
Example: (i) When a person lifts a body from the ground, the work done by the (upward) lifting force is positive



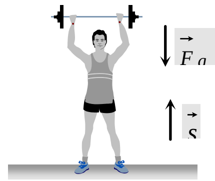
(ii) When a lawn roller is pulled by applying a force along the handle at an acute angle, work done by the applied force is positive.



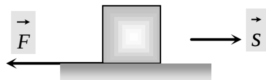
(iii) When a spring is stretched, work done by the external (stretching) force is positive.



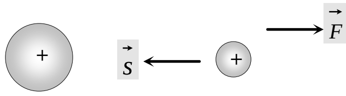
Example: (i) When a person lifts a body from the ground, the work done by the (downward) force of gravity is negative.



(ii) When a body is made to slide over a rough surface, the work done by the frictional force is negative.



(iii) When a positive charge is moved towards another positive charge. The work done by electrostatic force between them is negative.



Maximum work : $W_{max} = F\,s$

When $\cos\theta = \text{maximum} = 1$ *i.e.* $\theta = 0^o$

It means force does maximum work when angle between force and displacement is zero.

Minimum work : $W_{min} = -F\,s$

When $\cos\theta = \text{minimum} = -1$ *i.e* $\theta = 180^o$

It means force does minimum [maximum negative] work when angle between force and displacement is 180°.

Zero work

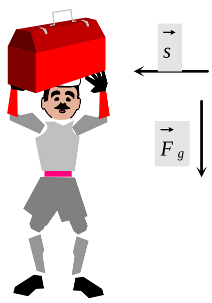
Under three condition, work done becomes zero $W = Fs\cos\theta = 0$

(1) **If the force is perpendicular to the displacement** $[\vec{F} \perp \vec{s}\,]$

Example: (i) When a coolie travels on a horizontal platform with a load on his head, work done against gravity by the coolie is zero.(Work done against friction is +ve)

(ii) When a body moves in a circle the work done by the centripetal force is always zero.

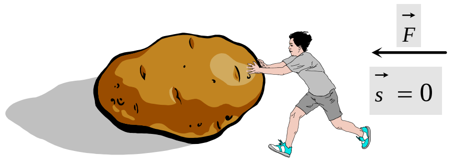
(iii) In case of motion of a charged particle in a magnetic field as force $[\vec{F} = q(\vec{v} \times \vec{B})]$ is always perpendicular to motion, work done by this force is always zero.



(2) **If there is no displacement [s = 0]**

Example: (i) When a person tries to displace a wall or heavy stone by applying a force then it does not move, the work done is zero.

(ii) A weight lifter does work in lifting the weight off the ground but does not work in holding it up.



(3) **If there is no force acting on the body [F = 0]**

Example: Motion of an isolated body in free space.

Sample Problems based on work done by constant force

Problem 1. A body of mass 5 kg is placed at the origin, and can move only on the x-axis. A force of 10 N is acting on it in a direction making an angle of 60^o with the x-axis and displaces it along the x-axis by 4 *metres*. The work done by the force is

(a) 2.5 J (b) 7.25 J (c) 40 J (d) 20 J

Solution : (d) Work done $= \vec{F}.\vec{s} = F\,s\cos\theta = 10 \times 4 \times \cos 60^o = 20\,J$

Problem 2. A force $F = (5\hat{i} + 3\hat{j})\,N$ is applied over a particle which displaces it from its origin to the point $r = (2\hat{i} - 1\hat{j})$ *metres*. The work done on the particle is

(a) $-7\,J$ (b) $+13\,J$ (c) $+7\,J$ (d) $+11\,J$

Solution : (c) Work done $= \vec{F}.\vec{r} = (5i + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = +7\,J$

Problem 3. A horizontal force of 5 N is required to maintain a velocity of 2 m/s for a block of 10 kg mass sliding over a rough surface. The work done by this force in one minute is

(a) 600 J (b) 60 J (c) 6 J (d) 6000 J

Solution : (a) Work done = Force × displacement = $F \times s = F \times v \times t = 5 \times 2 \times 60 = 600\,J$.

Problem 4. A box of mass 1 kg is pulled on a horizontal plane of length 1 m by a force of 8 N then it is raised vertically to a height of 2m, the net work done is

(a) 28 J (b) 8 J (c) 18 J (d) None of above

Solution : (a) Work done to displace it horizontally = $F \times s = 8 \times 1 = 8\,J$

Work done to raise it vertically $F \times s = mgh = 1 \times 10 \times 2 = 20\,J$

$\therefore$ Net work done = 8 +20 = 28 J

Problem 5. A 10 kg satellite completes one revolution around the earth at a height of 100 km in 108 *minutes*. The work done by the gravitational force of earth will be

(a) $108 \times 100 \times 10\,J$ (b) $\frac{108 \times 10}{100}\,J$ (c) $\frac{100 \times 10}{108}\,J$ (d) Zero

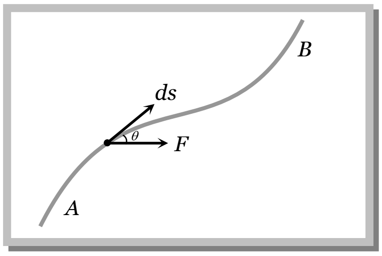
Solution : (d) Work done by centripetal force in circular motion is always equal to zero.

6.4 Work Done by a Variable Force

When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement is given by $dW = \vec{F}.d\vec{s}$

The total work done in going from A to B as shown in the figure is

$$W = \int_A^B \vec{F}.\vec{ds} = \int_A^B (F\cos\theta)ds$$



In terms of rectangular component $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\therefore\ W = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

or $W = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$

Sample Problems based on work done by variable force

Problem 6. A position dependent force $\vec{F} = (7 - 2x + 3x^2)N$ acts on a small abject of mass 2 kg to displace it from $x = 0$ to $x = 5m$. The work done in joule is **[CBSE PMT 1994]**

(a) 70 J (b) 270 J (c) 35 J (d) 135 J

Solution : (d) Work done $= \int_{x_1}^{x_2} F\,dx = \int_0^5 (7 - 2x + 3x^2)dx = [7x - x^2 + x^3]_0^5 = 35 - 25 + 125 = 135\ J$

Problem 7. A particle moves under the effect of a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done in the process is **[CPMT 1982]**

(a) Cx_1^2 (b) $\frac{1}{2}Cx_1^2$ (c) Cx_1 (d) Zero

Solution : (b) Work done $= \int_{x_1}^{x_2} F\,dx = \int_0^{x_1} Cx\,dx = C\left[\frac{x^2}{2}\right]_0^{x_1} = \frac{1}{2}C x_1^2$

Problem 8. The vessels A and B of equal volume and weight are immersed in water to a depth h. The vessel A has an opening at the bottom through which water can enter. If the work done in immersing A and B are W_A and W_B respectively, then

(a) $W_A = W_B$ (b) $W_A < W_B$ (c) $W_A > W_B$ (d) $W_A >=< W_B$

Solution : (b) When the vessels are immersed in water, work has to be done against up-thrust force but due to opening at the bottom in vessel A, up-thrust force goes on decreasing. So work done will be less in this case.

Problem 9. Work done in time t on a body of mass m which is accelerated from rest to a speed v in time t_1 as a function of time t is given by

(a) $\frac{1}{2}m\frac{v}{t_1}t^2$ (b) $m\frac{v}{t_1}t^2$ (c) $\frac{1}{2}\left(\frac{mv}{t_1}\right)^2 t^2$ (d) $\frac{1}{2}m\frac{v^2}{t_1^2}t^2$

Solution : (d) Work done $= F.s = ma.\left(\frac{1}{2}at^2\right) = \frac{1}{2}m a^2 t^2 = \frac{1}{2}m\left(\frac{v}{t_1}\right)^2 t^2$ $\left[\text{As acceleration } (a) = \frac{v}{t_1} \text{ given}\right]$

6.5 Dimension and Units of Work

Dimension : As work = Force × displacement

$$\therefore\quad [W] = [\text{Force}] \times [\text{Displacement}]$$

$$= [MLT^{-2}] \times [L] = [ML^2T^{-2}]$$

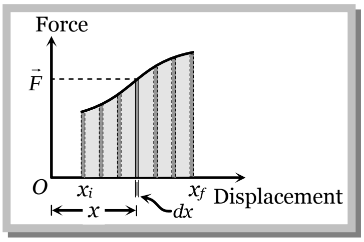
Units : The units of work are of two types

Absolute units	Gravitational units
Joule [S.I.]: Work done is said to be one *Joule*, when 1 *Newton* force displaces the body through 1 *meter* in its own direction. From $W = F.s$ 1 *Joule* = 1 *Newton* × 1 *metre*	*kg-m* [S.I.]: 1 *Kg-m* of work is done when a force of 1*kg-wt*. displaces the body through 1*m* in its own direction. From $W = F s$ 1 *kg-m* = 1 *kg-wt* × 1 *metre* = 9.81 *N* × 1 *metre* = 9.81 *Joule*
Erg [C.G.S.] : Work done is said to be one *erg* when 1 *dyne* force displaces the body through 1 *cm* in its own direction. From $W = F s$ 1 *Erg* = 1*Dyne* × 1*cm* *Relation between Joule and erg* 1 *Joule* = 1 *N* × 1 *m* = 10^5 *dyne* × 10^2 *cm* = 10^7 *dyne* × *cm* = 10^7 *Erg*	*gm-cm* [C.G.S.] : 1 *gm-cm* of work is done when a force of 1*gm-wt* displaces the body through 1*cm* in its own direction. From $W = F s$ 1 *gm-cm* = 1*gm-wt* × 1*cm*. = 981 *dyne* × 1*cm* = 981 *erg*

6.6 Work Done Calculation by Force Displacement Graph

Let a body, whose initial position is x_i, is acted upon by a variable force (whose magnitude is changing continuously) and consequently the body acquires its final position x_f.

Let $\vec{F}$ be the average value of variable force within the interval *dx* from position *x* to (*x* + *dx*) *i.e.* for small displacement *dx*. The work done will be the area of the shaded strip of width *dx*. The work done on the body in displacing it from position x_i to x_f will be equal to the sum of areas of all the such strips



$$dW = \vec{F}\,dx$$

$$\therefore W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} \vec{F}\,dx$$

$$\therefore W = \int_{x_i}^{x_f} (\text{Area of strip of width } dx)$$

$$\therefore W = \text{Area under curve Between } x_i \text{ and } x_f$$

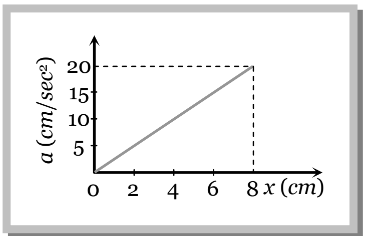
i.e. Area under force displacement curve with proper algebraic sign represents work done by the force.

Sample problems based on force displacement graph

Problem 10. A 10 *kg* mass moves along *x*-axis. Its acceleration as a function of its position is shown in the figure. What is the total work done on the mass by the force as the mass moves from $x = 0$ to $x = 8\,cm$ **[AMU (Med.) 2000]**

(a) $8 \times 10^{-2}\ J$ (b) $16 \times 10^{-2}\ J$

(c) $4 \times 10^{-4}\ J$ (d) $1.6 \times 10^{-3}\ J$



Solution : (a) Work done on the mass = mass × covered area between the graph and displacement axis on *a-t* graph.

$$= 10 \times \frac{1}{2}(8 \times 10^{-2}) \times 20 \times 10^{-2} = 8 \times 10^{-2}\ J.$$

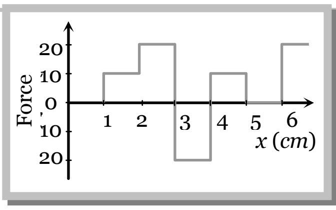
Problem 11. The relationship between force and position is shown in the figure given (in one dimensional case). The work done by the force in displacing a body from $x = 1\ cm$ to $x = 5\ cm$ is

(a) 20 *ergs*

(b) 60 *ergs*

(c) 70 *ergs*

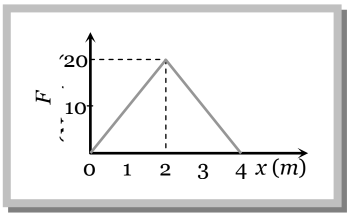
(d) 700 *ergs*



Solution : (a) Work done = Covered area on force-displacement graph = 1 × 10 + 1 × 20 – 1 × 20 + 1 × 10 = 20 *erg*.

***Problem* 12.** The graph between the resistive force *F* acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 *kg* and initial velocity is 2 *m/s*. When the distance covered by the body is $5m$, its kinetic energy would be

(a) 50 *J*
(b) 40 *J*
(c) 20 *J*
(d) 10 *J*

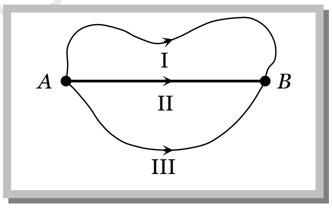


Solution : (d) Initial kinetic energy of the body $=\frac{1}{2}mu^2 = \frac{1}{2}\times 25\times(2)^2 = 50\ J$

Final kinetic energy = Initial energy – work done against resistive force (Area between graph and displacement axis) $= 50 - \frac{1}{2}\times 4\times 20 = 50 - 40 = 10\ J.$

6.7 Work Done in Conservative and Non-Conservative Field

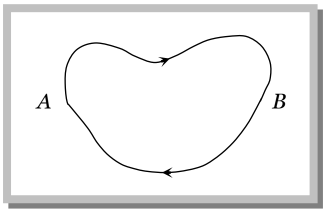
(1) In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.d\vec{l}$) is independent of the path followed between any two points.



$$\underset{\text{Path I}}{W_{A\to B}} = \underset{\text{Path II}}{W_{A\to B}} = \underset{\text{Path III}}{W_{A\to B}}$$

or $$\underset{\text{Path I}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path II}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path III}}{\int \vec{F}.d\vec{l}}$$

(2) In conservative field work done by the force (line integral of the force *i.e.* $\int \vec{F}.d\vec{l}$) over a closed path/loop is zero.



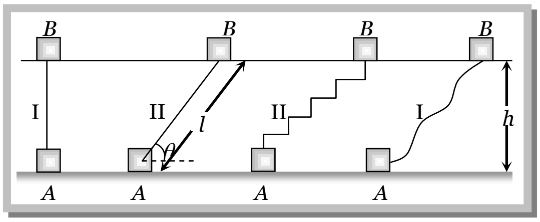
$$W_{A\to B} + W_{B\to A} = 0$$

or $$\oint \vec{F}.d\vec{l} = 0$$

Conservative force : The forces of these type of fields are known as conservative forces.

Example : Electrostatic forces, gravitational forces, elastic forces, magnetic forces *etc* and all the central forces are conservative in nature.

If a body of man *m* lifted to height *h* from the ground level by different path as shown in the figure



Work done through different paths

$$W_I = F.s = mg\times h = mgh$$

$$W_{II} = F.s = mg\sin\theta\times l = mg\sin\theta\times\frac{h}{\sin\theta} = mgh$$

$$W_{III} = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4 = mg(h_1+h_2+h_3+h_4) = mgh$$

$$W_{IV} = \int \vec{F}.d\vec{s} = mgh$$

It is clear that $W_I = W_{II} = W_{III} = W_{IV} = mgh$.

Further if the body is brought back to its initial position *A*, similar amount of work (energy) is released from the system it means $W_{AB} = mgh$

and $W_{BA} = -mgh$.

Hence the net work done against gravity over a round strip is zero.

$$W_{Net} = W_{AB} + W_{BA}$$
$$= mgh + (-mgh) = 0$$

i.e. the gravitational force is conservative in nature.

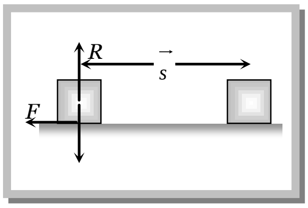
Non-conservative forces : A force is said to be non-conservative if work done by or against the force in moving a body from one position to another, depends on the path followed between these two positions and for complete cycle this work done can never be a zero.

Example: Frictional force, Viscous force, Airdrag *etc*.

If a body is moved from position *A* to another position *B* on a rough table, work done against frictional force shall depends on the length of the path between *A* and *B* and not only on the position *A* and *B*.

$$W_{AB} = \mu mgs$$

Further if the body is brought back to its initial position *A*, work has to be done against the frictional force, which always opposes the motion. Hence the net work done against the friction over a round trip is not zero.



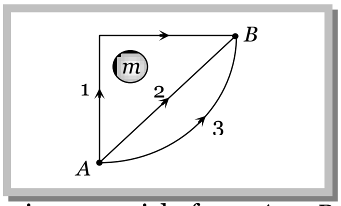
$$W_{BA} = \mu mgs.$$

$\therefore W_{Net} = W_{AB} + W_{BA} = \mu mgs + \mu mgs = 2\mu mgs \neq 0.$

i.e. the friction is a non-conservative force.

Sample problems based on work done in conservative and non-conservative field

Problem 13. If W_1, W_2 and W_3 represent the work done in moving a particle from *A* to *B* along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass *m*, find the correct relation



(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$

Solution : (b) As gravitational field is conservative in nature. So work done in moving a particle from *A* to *B* does not depends upon the path followed by the body. It always remains same.

Problem 14. A particle of mass 0.01 *kg* travels along a curve with velocity given by $4\hat{i} + 16\hat{k}$ ms^{-1}. After some time, its velocity becomes $8\hat{i} + 20\hat{j}\ ms^{-1}$ due to the action of a conservative force. The work done on particle during this interval of time is

(a) 0.32 *J* (b) 6.9 *J* (c) 9.6 *J* (d) 0.96 *J*

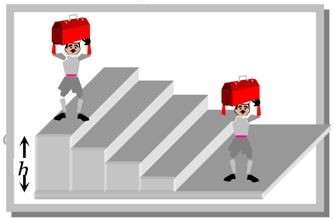
Solution : (d) $v_1 = \sqrt{4^2 + 16^2} = \sqrt{272}$ and $v_2 = \sqrt{8^2 + 20^2} = \sqrt{464}$

Work done = Increase in kinetic energy $= \frac{1}{2}m[v_2^2 - v_1^2] = \frac{1}{2} \times 0.01[464 - 272] = 0.96\,J$.

6.8 Work Depends on Frame of Reference

With change of frame of reference (inertial) force does not change while displacement may change. So the work done by a force will be different in different frames.

Examples : (1) If a porter with a suitcase on his head moves up a staircase, work done by the upward lifting force relative to him will be zero



(as displacement relative to him is zero) while relative to a person on the ground will be mgh.

(2) If a person is pushing a box inside a moving train, the work done in the frame of train will $\vec{F}.\vec{s}$ while in the frame of earth will be $\vec{F}.(\vec{s}+\vec{s}_0)$ where $\vec{s}_0$ is the displacement of the train relative to the ground.

6.9 Energy

The energy of a body is defined as its capacity for doing work.

(1) Since energy of a body is the total quantity of work done therefore it is a scalar quantity.

(2) Dimension: $[ML^2T^{-2}]$ it is same as that of work or torque.

(3) Units : *Joule* [S.I.], *erg* [C.G.S.]

Practical units : *electron volt* (*eV*), Kilowatt hour (*KWh*), Calories (*Cal*)

Relation between different units: $1\ Joule = 10^7\ erg$

$1\ eV = 1.6\times 10^{-19}\ Joule$

$1\ KWh = 3.6\times 10^6\ Joule$

$1\ Calorie = 4.18\ Joule$

(4) Mass energy equivalence : Einstein's special theory of relativity shows that material particle itself is a form of energy.

The relation between the mass of a particle m and its equivalent energy is given as

$E = mc^2$ where c = velocity of light in vacuum.

If $m = 1\,amu = 1.67\times 10^{-27}\ kg$ then $E = 931\ MeV = 1.5\times 10^{-10}\ Joule$.

If $m = 1 kg$ then $E = 9\times 10^{16}\ Joule$

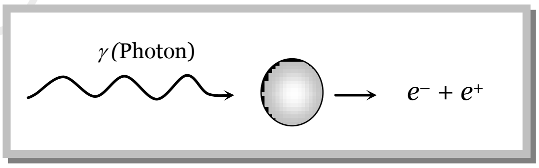
Examples : (i) Annihilation of matter when an electron (e^-) and a positron (e^+) combine with each other, they annihilate or destroy each other. The masses of electron and positron are converted into energy. This energy is released in the form of γ-rays.

$$e^- + e^+ \rightarrow \gamma + \gamma$$

Each γ photon has energy = 0.51 *MeV*.

Here two γ photons are emitted instead of one γ photon to conserve the linear momentum.

(ii) Pair production : This process is the reverse of annihilation of matter. In this case, a photon (γ) having energy equal to 1.02 *MeV* interacts with a nucleus and give rise to electron (e^-) and positron (e^+). This energy is converted into matter.



(iii) Nuclear bomb : When the nucleus is split up due to mass defect (The difference in the mass of nucleons and the nucleus) energy is released in the form of γ-radiations and heat.

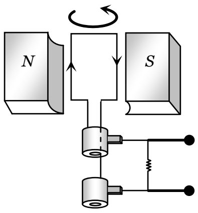
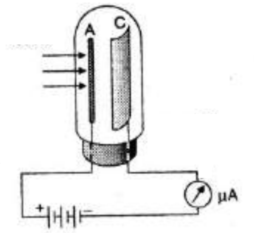
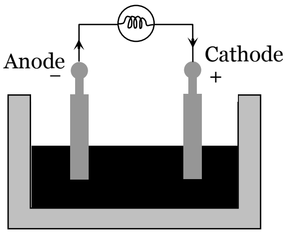
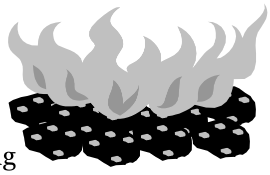
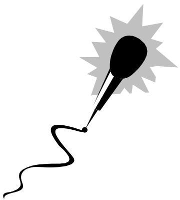
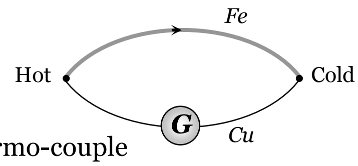
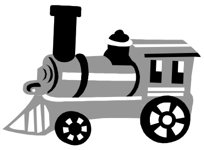
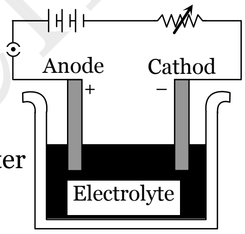
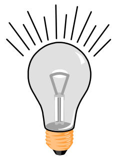
(5) Various forms of energy

(i) Mechanical energy (Kinetic and Potential) (ii) Chemical energy (iii) Electrical energy

(iv) Magnetic energy (v) Nuclear energy (vi) Sound energy

(vii) Light energy (viii) Heat energy

(6) Transformation of energy : Conversion of energy from one form to another is possible through various devices and processes.

Mechanical → electrical	Light → Electrical	Chemical → electrical
Dynamo 	Photoelectric cell 	Primary cell 
Chemical → heat	**Sounds → Electrical**	**Heat → electrical**
Coal Burning 	Microphone 	Thermo-couple 
Heat → Mechanical	**Electrical → Mechanical**	**Electrical → Heat**
Engine 	Motor 	Heater 
Electrical → Sound	**Electrical → Chemical**	**Electrical → Light**
Speaker 	Voltameter 	Bulb 

Sample problems based on energy

***Problem* 15.** A particle of mass 'm' and charge 'q' is accelerated through a potential difference of 'V' *volt*. Its energy is

[UPSEAT 2001]

(a) qV (b) mqV (c) $\left(\frac{q}{m}\right)V$ (d) $\frac{q}{mV}$

Solution : (a) Energy of charged particle = charge × potential difference = qV

***Problem* 16.** An ice cream has a marked value of 700 *kcal*. How many kilowatt hour of energy will it deliver to the body as it is digested

(a) 0.81 *kWh* (b) 0.90 *kWh* (c) 1.11 *kWh* (d) 0.71 *kWh*

Solution : (a) $700\ k\,cal = 700 \times 10^3 \times 4.2\,J = \frac{700 \times 10^3 \times 4.2}{3.6 \times 10^6} = 0.81\,kWh$ [As $3.6 \times 10^6\ J = 1\,kWh$]

***Problem* 17.** A metallic wire of length L *metres* extends by l *metres* when stretched by suspending a weight Mg to it. The mechanical energy stored in the wire is

(a) $2Mgl$ (b) Mgl (c) $\frac{Mgl}{2}$ (d) $\frac{Mgl}{4}$

Solution : (c) Elastic potential energy stored in wire $U = \frac{1}{2}Fx = \frac{Mgl}{2}$.

6.10 Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy.

Examples : (i) Flowing water possesses kinetic energy which is used to run the water mills.

(ii) Moving vehicle possesses kinetic energy.

(iii) Moving air (*i.e.* wind) possesses kinetic energy which is used to run wind mills.

(iv) The hammer possesses kinetic energy which is used to drive the nails in wood.

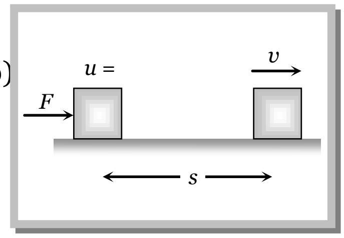
(v) A bullet fired from the gun has kinetic energy and due to this energy the bullet penetrates into a target.

(1) **Expression for kinetic energy :** Let

m = mass of the body, u = Initial velocity of the body (= 0)

F = Force acting on the body, a = Acceleration of the body

s = Distance travelled by the body, v = Final velocity of the body



From $v^2 = u^2 + 2as$

$$\Rightarrow \quad v^2 = 0 + 2as \quad \therefore s = \frac{v^2}{2a}$$

Since the displacement of the body is in the direction of the applied force, then work done by the force is

$$W = F \times s = ma \times \frac{v^2}{2a}$$

$$\Rightarrow \quad W = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE = W = \frac{1}{2}mv^2$

(2) **Calculus method :** Let a body is initially at rest and force $\vec{F}$ is applied on the body to displace it through $d\bar{s}$ along its own direction then small work done

$$dW = \vec{F}.d\vec{s} = F\,ds$$

$$\Rightarrow \quad dW = m\,a\,ds \qquad [\text{As } F = ma]$$

$$\Rightarrow \quad dW = m\frac{dv}{dt}ds \qquad \left[\text{As } a = \frac{dv}{dt}\right]$$

$$\Rightarrow \quad dW = mdv.\frac{ds}{dt}$$

$$\Rightarrow \quad dW = m\,v\,dv \qquad \text{.......(i)} \quad \left[\text{As } \frac{ds}{dt} = v\right]$$

Therefore work done on the body in order to increase its velocity from zero to v is given by

$$W = \int_0^v mv\,dv = m\int_0^v v\,dv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as the kinetic energy of the body $KE = \frac{1}{2}mv^2$.

In vector form $KE = \frac{1}{2}m(\vec{v}.\vec{v})$

As m and $\vec{v}.\vec{v}$ are always positive, kinetic energy is always positive scalar *i.e.* kinetic energy can never be negative.

(3) **Kinetic energy depends on frame of reference :** The kinetic energy of a person of mass *m*, sitting in a train moving with speed *v*, is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.

(4) **Kinetic energy according to relativity :** As we know $E = \frac{1}{2}mv^2$.

But this formula is valid only for ($v << c$) If v is comparable to c (speed of light in free space = $3 \times 10^8\ m/s$) then according to Einstein theory of relativity

$$E = \frac{mc^2}{\sqrt{1-(v^2/c^2)}} - mc^2$$

(5) **Work-energy theorem:** From equation (i) $dW = mv\, dv$.

Work done on the body in order to increase its velocity from u to v is given by

$$W = \int_u^v mv\, dv = m\int_u^v v\, dv = m\left[\frac{v^2}{2}\right]_u^v$$

$$\Rightarrow \qquad W = \frac{1}{2}m[v^2 - u^2]$$

Work done = change in kinetic energy

$W = \Delta E$

This is work energy theorem, it states that work done by a force acting on a body is equal to the change produced in the kinetic energy of the body.

This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).

If kinetic energy of the body increases, work is positive *i.e.* body moves in the direction of the force (or field) and if kinetic energy decreases work will be negative and object will move opposite to the force (or field).

Examples : (i) In case of vertical motion of body under gravity when the body is projected up, force of gravity is opposite to motion and so kinetic energy of the body decreases and when it falls down, force of gravity is in the direction of motion so kinetic energy increases.

(ii) When a body moves on a rough horizontal surface, as force of friction acts opposite to motion, kinetic energy will decrease and the decrease in kinetic energy is equal to the work done against friction.

(6) **Relation of kinetic energy with linear momentum:** As we know

$$E = \frac{1}{2}mv^2 = \frac{1}{2}\left[\frac{P}{v}\right]v^2 \qquad [\text{As } P = mv]$$

$$\therefore \qquad E = \frac{1}{2}Pv$$

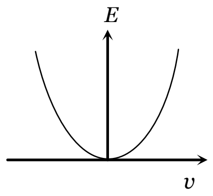
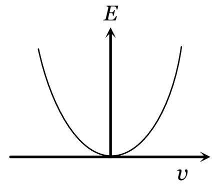
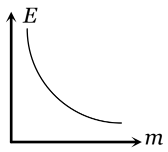
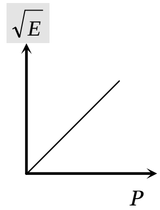
$$\text{or} \qquad E = \frac{P^2}{2m} \qquad \left[\text{As } v = \frac{P}{m}\right]$$

So we can say that kinetic energy $E = \frac{1}{2}mv^2 = \frac{1}{2}Pv = \frac{p^2}{2m}$

and Momentum $P = \frac{2E}{v} = \sqrt{2mE}$.

From above relation it is clear that a body can not have kinetic energy without having momentum and vice-versa.

(7) Various graphs of kinetic energy

	$E \propto v^2$ m = constant		$E \propto P^2$ m = constant
	$E \propto \frac{1}{m}$ P = constant		$P \propto \sqrt{E}$ m = constant

Sample problem based on kinetic energy

***Problem* 18.** Consider the following two statements

1. Linear momentum of a system of particles is zero

2. Kinetic energy of a system of particles is zero

Then **[AIEEE 2003]**

(a) 1 implies 2 and 2 implies 1 (b) 1 does not imply 2 and 2 does not imply 1

(c) 1 implies 2 but 2 does not imply 1 (d) 1 does not imply 2 but 2 implies 1

Solution : (d) Momentum is a vector quantity whereas kinetic energy is a scalar quantity. If the kinetic energy of a system is zero then linear momentum definitely will be zero but if the momentum of a system is zero then kinetic energy may or may not be zero.

***Problem* 19.** A running man has half the kinetic energy of that of a boy of half of his mass. The man speeds up by 1 m/s so as to have same $K.E.$ as that of boy. The original speed of the man will be

(a) $\sqrt{2}\,m/s$ (b) $(\sqrt{2}-1)m/s$ (c) $\frac{1}{(\sqrt{2}-1)}m/s$ (d) $\frac{1}{\sqrt{2}}m/s$

Solution : (c) Let m = mass of the boy, M = mass of the man, v = velocity of the boy and V = velocity of the man

Initial kinetic energy of man $= \frac{1}{2}MV^2 = \frac{1}{2}\left[\frac{1}{2}mv^2\right] = \frac{1}{2}\left[\frac{1}{2}\left(\frac{M}{2}\right)v^2\right]$ $\left[\text{As } m = \frac{M}{2} \text{ given}\right]$

$$\Rightarrow V^2 = \frac{v^2}{4} \Rightarrow V = \frac{v}{2} \qquad .....(i)$$

When the man speeds up by 1 m/s, $\frac{1}{2}M(V+1)^2 = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{M}{2}\right)v^2 \Rightarrow (V+1)^2 = \frac{v^2}{2}$

$$\Rightarrow V+1 = \frac{v}{\sqrt{2}} \qquad .....(ii)$$

From (i) and (ii) we get speed of the man $V = \frac{1}{\sqrt{2}-1}m/s$.

***Problem* 20.** A body of mass 10 kg at rest is acted upon simultaneously by two forces 4N and 3N at right angles to each other. The kinetic energy of the body at the end of 10 sec is **[Kerala (Engg.) 2001]**

(a) 100 J (b) 300 J (c) 50 J (d) 125 J

Solution : (d) As the forces are working at right angle to each other therefore net force on the body $F = \sqrt{4^2 + 3^2} = 5N$

Kinetic energy of the body = work done = $F \times s$

$= F \times \frac{1}{2} a t^2 = F \times \frac{1}{2}\left(\frac{F}{m}\right) t^2 = 5 \times \frac{1}{2}\left(\frac{5}{10}\right)(10)^2 = 125\,J.$

Problem 21. If the momentum of a body increases by 0.01%, its kinetic energy will increase by

(a) 0.01% (b) 0.02 % (c) 0.04 % (d) 0.08 %

Solution : (b) Kinetic energy $E = \frac{P^2}{2m}$ $\therefore$ $E \propto P^2$

Percentage increase in kinetic energy = 2(% increase in momentum) [If change is very small]

= 2(0.01%) = 0.02%.

Problem 22. If the momentum of a body is increased by 100 %, then the percentage increase in the kinetic energy is **[NCERT 1990; BHU 1999; Pb. PMT 1999; CPMT 1999, 2000; CBSE PMT 2001]**

(a) 150 % (b) 200 % (c) 225 % (d) 300 %

Solution : (d) $E = \frac{P^2}{2m} \Rightarrow \frac{E_2}{E_1} = \left(\frac{P_2}{P_1}\right)^2 = \left(\frac{2P}{P}\right)^2 = 4$

$E_2 = 4\,E_1 = E_1 + 3E_1 = E_1 + 300\,\%$ of E_1.

Problem 23. A body of mass 5 kg is moving with a momentum of 10 kg-m/s. A force of 0.2 N acts on it in the direction of motion of the body for 10 seconds. The increase in its kinetic energy is

(a) 2.8 J (b) 3.2 J (c) 3.8 J (d) 4.4 J

Solution : (d) Change in momentum $= P_2 - P_1 = F \times t \Rightarrow P_2 = P_1 + F \times t = 10 + 0.2 \times 10 = 12 kg\text{-}m/s$

Increase in kinetic energy $E = \frac{1}{2m}[P_2^2 - P_1^2]$

$= \frac{1}{2m}[(12)^2 - (10)^2] = \frac{1}{2 \times 5}[144 - 100] = \frac{44}{10} = 4.4\,J.$

Problem 24. Two masses of 1g and 9g are moving with equal kinetic energies. The ratio of the magnitudes of their respective linear momenta is

(a) 1 : 9 (b) 9 : 1 (c) 1 : 3 (d) 3 : 1

Solution : (c) $P = \sqrt{2mE}$ $\therefore$ $P \propto \sqrt{m}$ if E = constant . So $\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$.

Problem 25. A body of mass 2 kg is thrown upward with an energy 490 J. The height at which its kinetic energy would become half of its initial kinetic energy will be **[** $g = 9.8\,m/s^2$ **]**

(a) 35 m (b) 25 m (c) 12.5 m (d) 10 m

Solution : (c) If the kinetic energy would become half, then Potential energy = $\frac{1}{2}$(Initial kinetic energy)

$$\Rightarrow mgh = \frac{1}{2}[490] \Rightarrow 2 \times 9.8 \times h = \frac{1}{2}[490] \Rightarrow h = 12.5\,m$$

***Problem* 26.** A 300 *g* mass has a velocity of $(3\hat{i} + 4\hat{j})\ m/sec$ at a certain instant. What is its kinetic energy

(a) 1.35 *J* (b) 2.4 *J* (c) 3.75 *J* (d) 7.35 *J*

Solution : (c) $\vec{v} = (3\hat{i} + 4\hat{j})$ $\therefore\ v = \sqrt{3^2 + 4^2} = 5\,m/s$. So kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.3 \times (5)^2 = 3.75\ J$

6.11 Stopping of Vehicle by Retarding Force

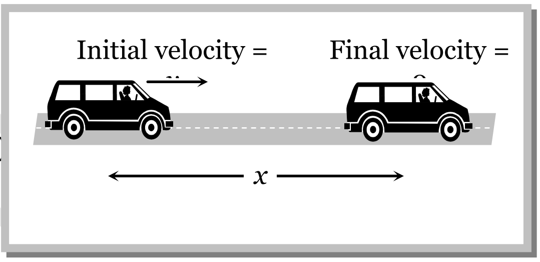
If a vehicle moves with some initial velocity and due to some retarding force it stops after covering some distance after some time.

(1) **Stopping distance :** Let m = Mass of vehicle, v = Velocity, P = Momentum, E = Kinetic energy

F = Stopping force, x = Stopping distance, t = Stopping time

Then, in this process stopping force does work on the vehicle and destroy the motion.

By the work- energy theorem



$$W = \Delta K = \frac{1}{2}mv^2$$

$\Rightarrow$ Stopping force (F) $\times$ Distance (x) = Kinetic energy (E)

$\Rightarrow$ Stopping distance (x) $= \dfrac{\text{Kinetic energy } (E)}{\text{Stopping force } (F)}$

$$\Rightarrow \quad x = \frac{mv^2}{2F} \quad .....(i)$$

(2) **Stopping time :** By the impulse-momentum theorem

$$F \times t = \Delta P \Rightarrow F \times t = P$$

$$\therefore \quad t = \frac{P}{F}$$

$$\text{or} \quad t = \frac{mv}{F} \quad .....(ii)$$

(3) **Comparison of stopping distance and time for two vehicles :** Two vehicles of masses m_1 and m_2 are moving with velocities v_1 and v_2 respectively. When they are stopped by the same retarding force (F).

The ratio of their stopping distances $\dfrac{x_1}{x_2} = \dfrac{E_1}{E_2} = \dfrac{m_1 v_1^2}{m_2 v_2^2}$

and the ratio of their stopping time $\dfrac{t_1}{t_2} = \dfrac{P_1}{P_2} = \dfrac{m_1 v_1}{m_2 v_2}$

If vehicles possess same velocities		
$v_1 = v_2$	$\dfrac{x_1}{x_2} = \dfrac{m_1}{m_2}$	$\dfrac{t_1}{t_2} = \dfrac{m_1}{m_2}$
If vehicle possess same kinetic momentum		

$P_1 = P_2$	$\frac{x_1}{x_2} = \frac{E_1}{E_2} = \left(\frac{P_1^2}{2m_1}\right)\left(\frac{2m_2}{P_2^2}\right) = \frac{m_2}{m_1}$	$\frac{t_1}{t_2} = \frac{P_1}{P_2} = 1$
If vehicle possess same kinetic energy		
$E_1 = E_2$	$\frac{x_1}{x_2} = \frac{E_1}{E_2} = 1$	$\frac{t_1}{t_2} = \frac{P_1}{P_2} = \frac{\sqrt{2m_1E_1}}{\sqrt{2m_2E_2}} = \sqrt{\frac{m_1}{m_2}}$

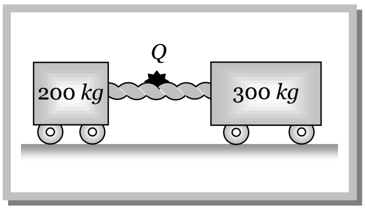
Note : ❑ If vehicle is stopped by friction then

Stopping distance $x = \dfrac{\frac{1}{2}mv^2}{F} = \dfrac{\frac{1}{2}mv^2}{ma} = \dfrac{v^2}{2\mu g}$ [As $a = \mu g$]

Stopping time $t = \dfrac{mv}{F} = \dfrac{mv}{m\mu g} = \dfrac{v}{\mu g}$

Sample problems based on stopping of vehicle

Problem 27. Two carts on horizontal straight rails are pushed apart by an explosion of a powder charge Q placed between the carts. Suppose the coefficients of friction between the carts and rails are identical. If the 200 *kg* cart travels a distance of 36 *metres* and stops, the distance covered by the cart weighing 300 *kg* is **[CPMT 1989]**



(a) 32 *metres* (b) 24 *metres*
(c) 16 *metres* (d) 12 *metres*

Solution : (c) Kinetic energy of cart will goes against friction. $\therefore\ E = \frac{P^2}{2m} = \mu mg \times s \Rightarrow s = \frac{P^2}{2\mu gm^2}$

As the two carts pushed apart by an explosion therefore they possess same linear momentum and coefficient of friction is same for both carts (given). Therefore the distance covered by the cart before coming to rest is given by

$$s \propto \frac{1}{m^2} \quad \therefore \frac{s_2}{s_1} = \left(\frac{m_1}{m_2}\right)^2 = \left(\frac{200}{300}\right)^2 = \frac{4}{9} \Rightarrow S_2 = \frac{4}{9} \times 36 = 16\ metres\ .$$

Problem 28. An unloaded bus and a loaded bus are both moving with the same kinetic energy. The mass of the latter is twice that of the former. Brakes are applied to both, so as to exert equal retarding force. If x_1 and x_2 be the distance covered by the two buses respectively before coming to a stop, then

(a) $x_1 = x_2$ (b) $2x_1 = x_2$ (c) $4x_1 = x_2$ (d) $8x_1 = x_2$

Solution : (a) If the vehicle stops by retarding force then the ratio of stopping distance $\frac{x_1}{x_2} = \frac{E_1}{E_2}$.

But in the given problem kinetic energy of bus and car are given same *i.e.* $E_1 = E_2$. $\therefore x_1 = x_2$.

Problem 29. A bus can be stopped by applying a retarding force F when it is moving with a speed v on a level road. The distance covered by it before coming to rest is s. If the load of the bus increases by 50 % because of passengers, for the same speed and same retarding force, the distance covered by the bus to come to rest shall be

(a) 1.5 *s* (b) 2 *s* (c) 1 *s* (d) 2.5 *s*

Solution : (a) Retarding force (F) × distance covered (x) = Kinetic energy $\left(\frac{1}{2}mv^2\right)$

If v and F are constants then $x \propto m$ $\therefore \frac{x_2}{x_1} = \frac{m_2}{m_1} = \frac{1.5\,m}{m} = 1.5 \Rightarrow x_2 = 1.5\,s$.

***Problem* 30.** A vehicle is moving on a rough horizontal road with velocity v. The stopping distance will be directly proportional to

(a) $\sqrt{v}$ (b) v (c) v^2 (d) v^3

Solution : (c) As $s = \frac{v^2}{2a}$ $\therefore$ $s \propto v^2$.

6.12 Potential Energy

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types : Elastic potential energy, Electric potential energy and Gravitational potential energy *etc.*

(1) **Change in potential energy :** Change in potential energy between any two points is defined in the terms of the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}.d\vec{r} = -W \qquad ......(i)$$

We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point. Whenever and wherever possible, we take the reference point at infinite and assume potential energy to be zero there, *i.e.* if take $r_1 = \infty$ and $r_2 = r$ then from equation (i)

$$U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W$$

In case of conservative force (field) potential energy is equal to negative of work done in shifting the body from reference position to given position.

This is why in shifting a particle in a conservative field (say gravitational or electric), if the particle moves opposite to the field, work done by the field will be negative and so change in potential energy will be positive *i.e.* potential energy will increase. When the particle moves in the direction of field, work will be positive and change in potential energy will be negative *i.e.* potential energy will decrease.

(2) **Three dimensional formula for potential energy:** For only conservative fields $\vec{F}$ equals the negative gradient $(-\vec{\nabla})$ of the potential energy.

So $\vec{F} = -\vec{\nabla} U$ ($\vec{\nabla}$ read as Del operator or Nabla operator and $\vec{\nabla} = \frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}$)

$$\Rightarrow \quad \vec{F} = -\left[\frac{dU}{dx}\hat{i} + \frac{dU}{dy}\hat{j} + \frac{dU}{dz}\hat{k}\right]$$

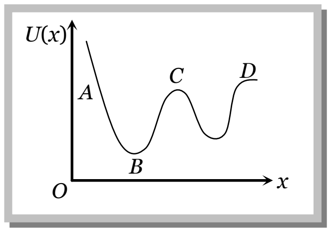
where $\frac{dU}{dx}$ = Partial derivative of U *w.r.t.* x (keeping y and z constant)

$\frac{dU}{dy}$ = Partial derivative of U *w.r.t.* y (keeping x and z constant)

$\frac{dU}{dz}$ = Partial derivative of U *w.r.t.* z (keeping x and y constant)

(3) **Potential energy curve :** A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve.

Figure shows a graph of potential energy function $U(x)$ for one dimensional motion.



As we know that negative gradient of the potential energy gives force.

$\therefore \; -\dfrac{dU}{dx} = F$

(4) **Nature of force :**

(i) Attractive force : On increasing x, if U increases $\dfrac{dU}{dx} =$ positive

then F is negative in direction *i.e.* force is attractive in nature. In graph this is represented in region BC.

(ii) Repulsive force : On increasing x, if U decreases $\dfrac{dU}{dx} =$ negative

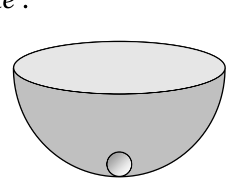
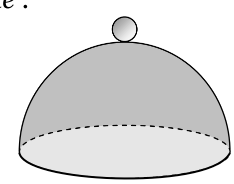
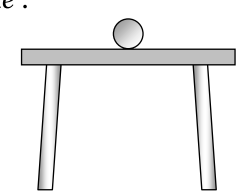
then F is positive in direction *i.e.* force is repulsive in nature. In graph this is represented in region AB.

(iii) Zero force : On increasing x, if U does not changes $\dfrac{dU}{dx} = 0$

then F is zero *i.e.* no force works on the particle. Point B, C and D represents the point of zero force or these points can be termed as position of equilibrium.

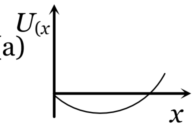
(5) **Types of equilibrium :** If net force acting on a particle is zero, it is said to be in equilibrium.

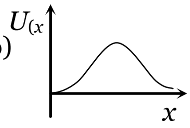
For equilibrium $\dfrac{dU}{dx} = 0$, but the equilibrium of particle can be of three types :

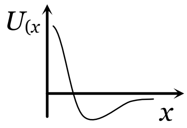
Stable	**Unstable**	**Neutral**
When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.	When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.	When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.
Potential energy is minimum.	Potential energy is maximum.	Potential energy is constant.
$F = -\dfrac{dU}{dx} = 0$	$F = -\dfrac{dU}{dx} = 0$	$F = -\dfrac{dU}{dx} = 0$
$\dfrac{d^2U}{dx^2} =$ positive *i.e.* rate of change of $\dfrac{dU}{dx}$ is positive.	$\dfrac{d^2U}{dx^2} =$ negative *i.e.* rate of change of $\dfrac{dU}{dx}$ is negative.	$\dfrac{d^2U}{dx^2} = 0$ *i.e.* rate of change of $\dfrac{dU}{dx}$ is zero.
Example :  A marble placed at the bottom of a hemispherical bowl.	*Example* :  A marble balanced on top of a hemispherical bowl.	*Example* :  A marble placed on horizontal table.

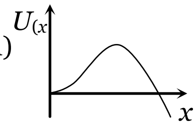
Sample problems based on potential energy

Problem 31. A particle which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional from of the potential energy $U_{(x)}$ of the particle is **[IIT-JEE (Screening) 2002]**

(a) 

(b) 

(c) 

(d) 

Solution : (d) $F = -\frac{dU}{dx} \Rightarrow dU = -F.dx \Rightarrow U = -\int_0^x (-kx + ax^3)dx \Rightarrow U = \frac{kx^2}{2} - \frac{ax^4}{4}$

$\therefore$ We get $U = 0$ at $x = 0$ and $x = \sqrt{\frac{2k}{a}}$ Also we get $U =$ negative for $x > \sqrt{\frac{2k}{a}}$

From the given function we can see that $F = 0$ at $x = 0$ *i.e.* slope of U-x graph is zero at $x = 0$.

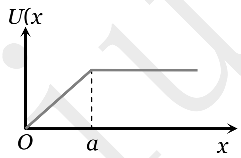
Problem 32. The potential energy of a body is given by $A - Bx^2$ (where x is the displacement). The magnitude of force acting on the particle is

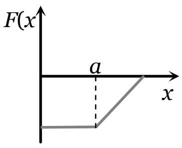
(a) Constant (b) Proportional to x

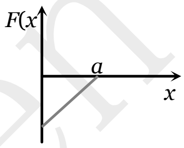
(c) Proportional to x^2 (d) Inversely proportional to x

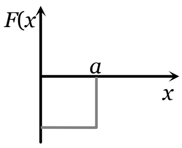
Solution : (b) $F = \frac{-dU}{dx} = -\frac{d}{dx}(A - Bx^2) = 2Bx \quad \therefore F \propto x$.

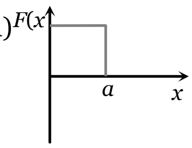
Problem 33. The potential energy of a system is represented in the first figure. The force acting on the system will be represented by



(a) 

(b) 

(c) 

(d) 

Solution : (c) As slope of problem graph is positive and constant upto distance a then it becomes zero. Therefore from $F = -\frac{dU}{dx}$ we can say that upto distance a force will be constant (negative) and suddenly it becomes zero.

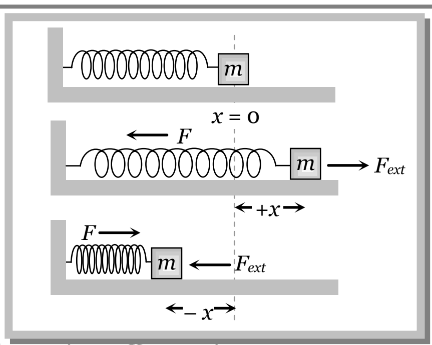
Problem 34. A particle moves in a potential region given by $U = 8x^2 - 4x + 400$ J. Its state of equilibrium will be

(a) $x = 25\,m$ (b) $x = 0.25\,m$ (c) $x = 0.025\,m$ (d) $x = 2.5\,m$

Solution : (b) $F = -\frac{dU}{dx} = -\frac{d}{dx}(8x^2 - 4x + 400)$

For the equilibrium condition $F = -\frac{dU}{dx} = 0 \Rightarrow 16x - 4 = 0 \Rightarrow x = 4/16 \therefore x = 0.25\,m$.

6.13 Elastic Potential Energy



(1) **Restoring force and spring constant :** When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance x, then a restoring force is produced in the spring to bring it to the normal position.According to Hooke's law this restoring force is proportional to the displacement x and its direction is always opposite to the displacement.

i.e. $\vec{F} \propto -\vec{x}$

or $\vec{F} = -k\,\vec{x}$ .....(i)

where k is called spring constant.

If $x = 1$, $F = k$ (Numerically)

or $k = F$

Hence spring constant is numerically equal to force required to produce unit displacement (compression or extension) in the spring. If required force is more, then spring is said to be more stiff and vice-versa.

Actually k is a measure of the stiffness/softness of the spring.

Dimension : As $k = \frac{F}{x}$ $\therefore [k] = \frac{[F]}{[x]} = \frac{[MLT^{-2}]}{L} = [MT^{-2}]$

Units : S.I. unit *Newton/metre*, C.G.S unit *Dyne/cm*.

Note : ❑ Dimension of force constant is similar to surface tension.

(2) **Expression for elastic potential energy :** When a spring is stretched or compressed from its normal position ($x = 0$), work has to be done by external force against restoring force. $\vec{F}_{ext} = \vec{F}_{restoring} = k\vec{x}$

Let the spring is further stretched through the distance dx, then work done

$$dW = \vec{F}_{ext}.d\vec{x} = F_{ext}.dx \cos 0^o = kx\, dx \qquad [\text{As } \cos 0^o = 1]$$

Therefore total work done to stretch the spring through a distance x from its mean position is given by

$$W = \int_0^x dW = \int_0^x kx\, dx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$$

This work done is stored as the potential energy of the stretched spring.

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2$

$$U = \frac{1}{2}Fx \qquad \left[\text{As } k = \frac{F}{x}\right]$$

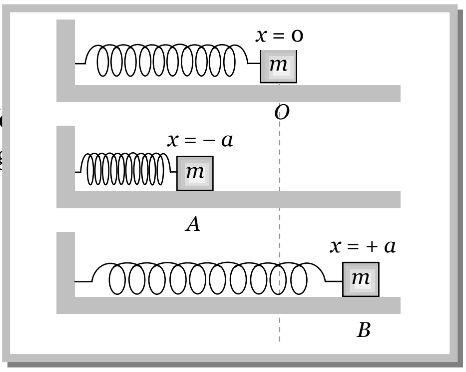
$$U = \frac{F^2}{2k} \qquad \left[\text{As } x = \frac{F}{k}\right]$$

$\therefore$ Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

Note : ❑If spring is stretched from initial position x_1 to final position x_2 then work done

= Increment in elastic potential energy $= \frac{1}{2}k(x_2^2 - x_1^2)$

(3) **Energy graph for a spring :** If the mass attached with spring performs simple harmonic motion about its mean position then its potential energy at any position (x) can be given by



$$U = \frac{1}{2}kx^2 \qquad ....(i)$$

So for the extreme position

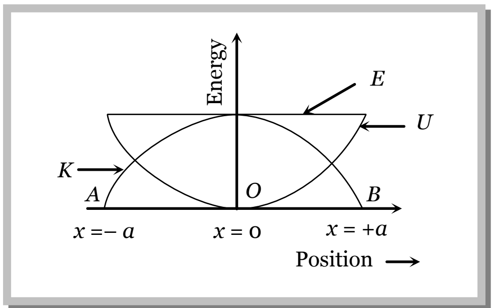
$$U = \frac{1}{2}ka^2 \qquad [\text{As } x = \pm a \text{ for extreme}]$$

This is maximum potential energy or the total energy of mass.

$\therefore$ Total energy $E = \frac{1}{2}ka^2$ ....(ii)

[Because velocity of mass = 0 at extreme $\therefore K = \frac{1}{2}mv^2 = 0$]

Now kinetic energy at any position $K = E - U = \frac{1}{2}ka^2 - \frac{1}{2}kx^2$



$$K = \frac{1}{2}k(a^2 - x^2) \qquad \text{....(iii)}$$

From the above formula we can check that

$U_{max} = \frac{1}{2}ka^2$ [At extreme $x = \pm a$] and $U_{min} = 0$ [At mean $x = 0$]

$K_{max} = \frac{1}{2}ka^2$ [At mean $x = 0$]and $K_{min} = 0$ [At extreme $x = \pm a$]

$E = \frac{1}{2}ka^2 =$ constant (at all positions)

It mean kinetic energy changes parabolically *w.r.t.* position but total energy remain always constant irrespective to position of the mass

Sample problems based on elastic potential energy

***Problem* 35.** A long spring is stretched by 2 *cm*, its potential energy is *U*. If the spring is stretched by 10 *cm*, the potential energy stored in it will be

(a) $U/25$ (b) $U/5$ (c) $5U$ (d) $25U$

Solution : (d) Elastic potential energy of a spring $U = \frac{1}{2}kx^2$ $\therefore U \propto x^2$

So $\frac{U_2}{U_1} = \left(\frac{x_2}{x_1}\right)^2 \Rightarrow \frac{U_2}{U} = \left(\frac{10\,cm}{2\,cm}\right)^2 \Rightarrow U_2 = 25U$

***Problem* 36.** A spring of spring constant $5 \times 10^3 N/m$ is stretched initially by 5 *cm* from the unstretched position. Then the work required to stretch it further by another 5 *cm* is

(a) 6.25 *N-m* (b) 12.50 *N-m* (c) 18.75 *N-m* (d) 25.00 *N-m*

Solution : (c) Work done to stretch the spring from x_1 to x_2

$$W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}5 \times 10^3[(10 \times 10^{-2})^2 - (5 \times 10^{-2})^2] = \frac{1}{2} \times 5 \times 10^3 \times 75 \times 10^{-4} = 18.75\ N.m\ .$$

***Problem* 37.** Two springs of spring constants $1500\ N/m$ and $3000\ N/m$ respectively are stretched with the same force. They will have potential energy in the ratio

(a) 4 : 1 (b) 1 : 4 (c) 2 : 1 (d) 1 : 2

Solution : (c) Potential energy of spring $U = \frac{F^2}{2k} \Rightarrow \frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{3000}{1500} = 2:1$ [If *F* = constant]

***Problem* 38.** A body is attached to the lower end of a vertical spiral spring and it is gradually lowered to its equilibrium position. This stretches the spring by a length *x*. If the same body attached to the same spring is allowed to fall suddenly, what would be the maximum stretching in this case

(a) *x* (b) 2*x* (c) 3*x* (d) *x*/2

Solution : (b) When spring is gradually lowered to it's equilibrium position

$$kx = mg \ \therefore\ x = \frac{mg}{k}.$$

When spring is allowed to fall suddenly it oscillates about it's mean position

Let *y* is the amplitude of vibration then at lower extreme, by the conservation of energy

$$\Rightarrow \frac{1}{2}ky^2 = mgy \Rightarrow y = \frac{2mg}{k} = 2x.$$

***Problem* 39.** Two equal masses are attached to the two ends of a spring of spring constant *k*. The masses are pulled out symmetrically to stretch the spring by a length *x* over its natural length. The work done by the spring on each mass is

(a) $\frac{1}{2}kx^2$ (b) $-\frac{1}{2}kx^2$ (c) $\frac{1}{4}kx^2$ (d) $-\frac{1}{4}kx^2$

Solution : (d) If the spring is stretched by length x, then work done by two equal masses = $\frac{1}{2}kx^2$

So work done by each mass on the spring = $\frac{1}{4}kx^2$ $\therefore$ Work done by spring on each mass = $-\frac{1}{4}kx^2$.

6.14 Electrical Potential Energy

It is the energy associated with state of separation between charged particles that interact via electric force. For two point charge q_1 and q_2, separated by distance r.

$$U = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r}$$

While for a point charge q at a point in an electric field where the potential is V

$$U = qV$$

As charge can be positive or negative, electric potential energy can be positive or negative.

Sample problems based on electrical potential energy

Problem 40. A proton has a positive charge. If two protons are brought near to one another, the potential energy of the system will

(a) Increase (b) Decrease

(c) Remain the same (d) Equal to the kinetic energy

Solution : (a) As the force is repulsive in nature between two protons. Therefore potential energy of the system increases.

Problem 41. Two protons are situated at a distance of 100 *fermi* from each other. The potential energy of this system will be in eV

(a) 1.44 (b) 1.44×10^3 (c) 1.44×10^2 (d) 1.44×10^4

Solution : (d) $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{100\times10^{-15}} = 2.304\times10^{-15}\ J = \frac{2.304\times10^{-15}}{1.6\times10^{-19}}eV = 1.44\times10^4\ eV$

Problem 42. ${}_{80}Hg^{208}$ nucleus is bombarded by α-particles with velocity $10^7\ m/s$. If the α-particle is approaching the Hg nucleus head-on then the distance of closest approach will be

(a) $1.115\times10^{-13}\ m$ (b) $11.15\times10^{-13}\ m$ (c) $111.5\times10^{-13}\ m$ (d) Zero

Solution : (a) When α particle moves towards the mercury nucleus its kinetic energy gets converted in potential energy of the system. At the distance of closest approach $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$\Rightarrow \frac{1}{2}\times(1.6\times10^{-27})(10^7)^2 = 9\times10^9\frac{(2.e)(80\,e)}{r} \Rightarrow r = 1.115\times10^{-13}\ m.$

Problem 43. A charged particle A moves directly towards another charged particle B. For the $(A+B)$ system, the total momentum is P and the total energy is E

(a) P and E are conserved if both A and B are free to move

(b) (a) is true only if A and B have similar charges

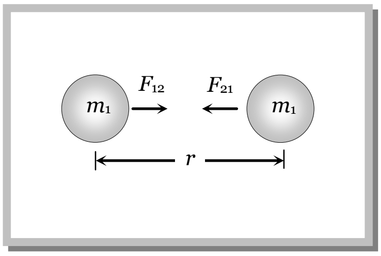
(c) If B is fixed, E is conserved but not P

(d) If B is fixed, neither E nor P is conserved

Solution : (a, c) If A and B are free to move, no external forces are acting and hence P and E both are conserved but when B is fixed (with the help of an external force) then E is conserved but P is not conserved.

6.15 Gravitational Potential Energy

It is the usual form of potential energy and is the energy associated with the state of separation between two bodies that interact via gravitational force.



For two particles of masses m_1 and m_2 separated by a distance r

Gravitational potential energy $U = -\dfrac{Gm_1m_2}{r}$

(1) If a body of mass m at height h relative to surface of earth then

Gravitational potential energy $U = \dfrac{mgh}{1+\dfrac{h}{R}}$

Where R = radius of earth, g = acceleration due to gravity at the surface of the earth.

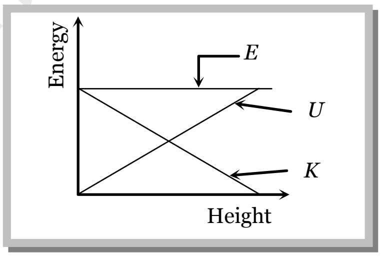
(2) If $h << R$ then above formula reduces to $U = mgh$.

(3) If V is the gravitational potential at a point, the potential energy of a particle of mass m at that point will be

$$U = mV$$

(4) Energy height graph : When a body projected vertically upward from the ground level with some initial velocity then it possess kinetic energy but its potential energy is zero.

As the body moves upward its potential energy increases due to increase in height but kinetic energy decreases (due to decrease in velocity). At maximum height its kinetic energy becomes zero and potential energy maximum but through out the complete motion total energy remains constant as shown in the figure.



Sample problems based on gravitational potential energy

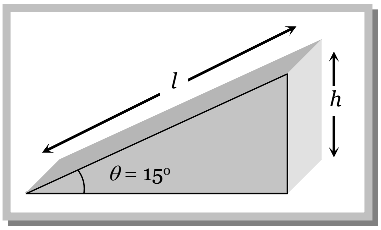
Problem 44. The work done in pulling up a block of wood weighing 2kN for a length of 10 m on a smooth plane inclined at an angle of 15^o with the horizontal is (sin 15° = 0.259)

(a) 4.36 kJ (b) 5.17 kJ (c) 8.91 kJ (d) 9.82 kJ

Solution : (b) Work done = $mg \times h$

$= 2\times 10^3 \times l \sin\theta$

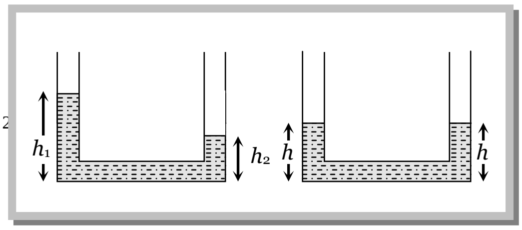
$= 2\times 10^3 \times 10 \times \sin 15^o = 5176\ J = 5.17\ kJ$



Problem 45. Two identical cylindrical vessels with their bases at same level each contains a liquid of density d. The height of the liquid in one vessel is h_1 and that in the other vessel is h_2. The area of either vases is A. The work done by gravity in equalizing the levels when the two vessels are connected, is **[SCRA 1996]**

(a) $(h_1-h_2)gd$ (b) $(h_1-h_2)gAd$ (c) $\frac{1}{2}(h_1-h_2)^2\,gAd$ (d) $\frac{1}{4}(h_1-h_2)^2\,gAd$

Solution : (d) Potential energy of liquid column is given by $mg\frac{h}{2} = Vdg\frac{h}{2} = Ahdg\frac{h}{2} = \frac{1}{2}Adgh^2$



Initial potential energy $= \frac{1}{2}Adgh_1^2 + \frac{1}{2}Adgh_2^2$

Final potential energy $= \frac{1}{2}Adgh^2 + \frac{1}{2}Adh^2g = Adgh^2$

Work done by gravity = change in potential energy

$$W = \left[\frac{1}{2}Adgh_1^2 + \frac{1}{2}Adgh_2^2\right] - Adgh^2$$

$$= Adg\left[\frac{h_1^2}{2} + \frac{h_2^2}{2}\right] - Adg\left(\frac{h_1 + h_2}{2}\right)^2 \qquad [\text{As } h = \frac{h_1 + h_2}{2}]$$

$$= Adg\left[\frac{h_1^2}{2} + \frac{h_2^2}{2} - \left(\frac{h_1^2 + h_2^2 + 2h_1h_2}{4}\right)\right] = \frac{Adg}{4}(h_1 - h_2)^2$$

***Problem* 46.** If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an abject of mass m raised from the surface of earth to a height equal to the radius of the earth R, is **[IIT-JEE1983]**

(a) $\frac{1}{2}mgR$ (b) $2mgR$ (c) mgR (d) $\frac{1}{4}mgR$

Solution : (a) Work done = gain in potential energy $= \frac{mgh}{1 + h/R} = \frac{mgR}{1 + R/R} = \frac{1}{2}mgR$ [As $h = R$ (given)]

***Problem* 47.** The work done in raising a mass of 15 *gm* from the ground to a table of 1*m* height is

(a) 15 *J* (b) 152 *J* (c) 1500 *J* (d) 0.15 *J*

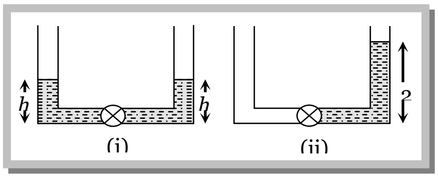
Solution : (d) $W = mgh = 15 \times 10^{-3} \times 10 \times 1 = 0.15\ J.$

***Problem* 48.** A body is falling under gravity. When it loses a gravitational potential energy by U, its speed is v. The mass of the body shall be

(a) $\frac{2U}{v}$ (b) $\frac{U}{2v}$ (c) $\frac{2U}{v^2}$ (d) $\frac{U}{2v^2}$

Solution : (c) Loss in potential energy = gain in kinetic energy $\Rightarrow U = \frac{1}{2}mv^2 \therefore m = \frac{2U}{v^2}$.

***Problem* 49.** A liquid of density d is pumped by a pump P from situation (i) to situation (ii) as shown in the diagram. If the cross-section of each of the vessels is a, then the work done in pumping (neglecting friction effects) is



(a) 2*dgh* (b) *dgha*
(c) 2*dgh*²*a* (d) *dgh*²*a*

Solution : (d) Potential energy of liquid column in first situation $= Vdg\frac{h}{2} + Vdg\frac{h}{2} = Vdgh = ahdgh = dgh^2a$

[As centre of mass of liquid column lies at height $\frac{h}{2}$]

Potential energy of the liquid column in second situation $= Vdg\left(\frac{2h}{2}\right) = (A \times 2h)dgh = 2dgh^2a$

Work done pumping = Change in potential energy = $2dgh^2a - dgh^2a = dgh^2a$.

Problem 50. The mass of a bucket containing water is 10 *kg*. What is the work done in pulling up the bucket from a well of depth 10 *m* if water is pouring out at a uniform rate from a hole in it and there is loss of 2*kg* of water from it while it reaches the top $(g = 10\,m/\sec^2)$

(a) 1000 *J* (b) 800 *J* (c) 900 *J* (d) 500 *J*

Solution : (c) Gravitational force on bucket at starting position = *mg* = 10 × 10 = 100 *N*

Gravitational force on bucket at final position = 8 × 10 = 80 *N*

So the average force through out the vertical motion = $\frac{100+80}{2} = 90\ N$

∴ Work done = Force × displacement = 90 × 10 = 900 *J*.

Problem 51. A rod of mass *m* and length *l* is lying on a horizontal table. The work done in making it stand on one end will be

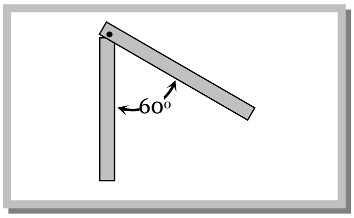
(a) mgl (b) $\frac{mgl}{2}$ (c) $\frac{mgl}{4}$ (d) $2mgl$

Solution : (b) When the rod is lying on a horizontal table, its potential energy = 0

But when we make its stand vertical its centre of mass rises upto high $\frac{l}{2}$. So it's potential energy $= \frac{mgl}{2}$

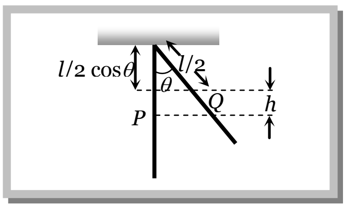
∴ Work done = charge in potential energy $= mg\frac{l}{2} - 0 = \frac{mgl}{2}$.

Problem 52. A *metre* stick, of mass 400 *g*, is pivoted at one end displaced through an angle 60^o. The increase in its potential energy is



(a) 1 *J* (b) 10 *J*
(c) 100 *J* (d) 1000 *J*

Solution : (a) Centre of mass of a stick lies at the mid point and when the stick is displaced through an angle 60° it rises upto height '*h*' from the initial position.



From the figure $h = \frac{l}{2} - \frac{l}{2}\cos\theta = \frac{l}{2}(1-\cos\theta)$

Hence the increment in potential energy of the stick = *mgh*

$$= mg\frac{l}{2}(1-\cos\theta) = 0.4\times 10\times\frac{1}{2}(1-\cos 60^o) = 1\ J$$

Problem 53. Once a choice is made regarding zero potential energy reference state, the changes in potential energy

(a) Are same (b) Are different

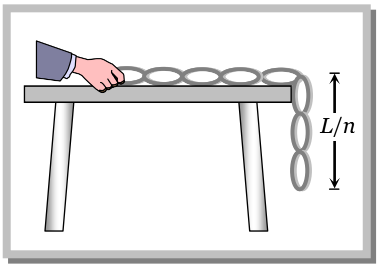
(c) Depend strictly on the choice of the zero of potential energy

(d) Become indeterminate

Solution : (a) Potential energy is a relative term but the difference in potential energy is absolute term. If reference level is fixed once then change in potential energy are same always.

6.16 Work Done in Pulling the Chain Against Gravity

A chain of length L and mass M is held on a frictionless table with $(1/n)^{th}$ of its length hanging over the edge.



Let $m = \frac{M}{L}$ = mass per unit length of the chain and y is the length of the chain hanging over the edge. So the mass of the chain of length y will be ym and the force acting on it due to gravity will be mgy.

The work done in pulling the dy length of the chain on the table.

$dW = F(-dy)$ [As y is decreasing]

i.e. $dW = mgy\,(-dy)$

So the work done in pulling the hanging portion on the table.

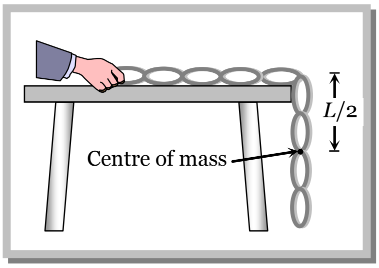
$$W = -\int_{L/n}^{0} mgy\, dy = mg\left[\frac{y^2}{2}\right]_{L/n}^{0} = \frac{mg\,L^2}{2n^2}$$

$\therefore$ $W = \frac{MgL}{2n^2}$ [As $m = M/L$]

Alternative method :

If point mass m is pulled through a height h then work done $W = mgh$

Similarly for a chain we can consider its centre of mass at the middle point of the hanging part *i.e.* at a height of $L/(2n)$ from the lower end and mass of the hanging part of chain $= \frac{M}{n}$

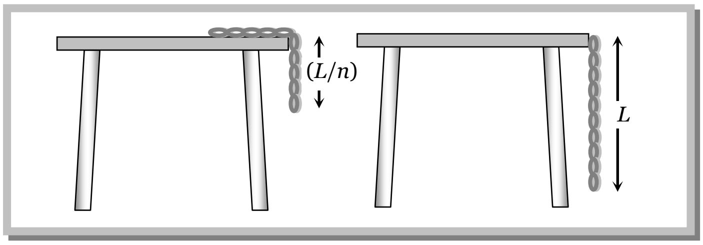


So work done to raise the centre of mass of the chain on the table is given by

$W = \frac{M}{n} \times g \times \frac{L}{2n}$ [As $W = mgh$]

or $W = \frac{MgL}{2n^2}$

6.17 Velocity of Chain While Leaving the Table



Taking surface of table as a reference level (zero potential energy)

Potential energy of chain when $1/n^{th}$ length hanging from the edge $= \frac{-MgL}{2n^2}$

Potential energy of chain when it leaves the table $= -\frac{MgL}{2}$

Kinetic energy of chain = loss in potential energy

$\Rightarrow$ $\frac{1}{2}Mv^2 = \frac{MgL}{2} - \frac{MgL}{2n^2}$

$$\Rightarrow \quad \frac{1}{2}Mv^2 = \frac{MgL}{2}\left[1 - \frac{1}{n^2}\right]$$

$\therefore$ Velocity of chain $v = \sqrt{gL\left(1 - \frac{1}{n^2}\right)}$

Sample problem based on chain

***Problem* 54.** A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is

(a) MgL (b) $\frac{MgL}{3}$ (c) $\frac{MgL}{9}$ (d) $\frac{MgL}{18}$

Solution : (d) As 1/3 part of the chain is hanging from the edge of the table. So by substituting $n = 3$ in standard expression

$$W = \frac{MgL}{2n^2} = \frac{MgL}{2(3)^2} = \frac{MgL}{18}.$$

***Problem* 55.** A chain is placed on a frictionless table with one fourth of it hanging over the edge. If the length of the chain is 2m and its mass is 4kg, the energy need to be spent to pull it back to the table is

(a) 32 J (b) 16 J (c) 10 J (d) 2.5 J

Solution : (d) $W = \frac{MgL}{2n^2} = \frac{4 \times 10 \times 2}{2 \times (4)^2} = 2.5\ J.$

***Problem* 56.** A uniform chain of length 2m is held on a smooth horizontal table so that half of it hangs over the edge. If it is released from rest, the velocity with which it leaves the table will be nearest to

(a) 2 m/s (b) 4 m/s (c) 6 m/s (d) 8 m/s

Solution : (b) $v = \sqrt{gL\left(1 - \frac{1}{n^2}\right)} = \sqrt{10 \times 2\left(1 - \frac{1}{(2)^2}\right)} = \sqrt{15} = 3.87 \simeq 4\ m/s$ (approx.)

6.18 Law of Conservation of Energy

(1) **Law of conservation of energy**

For a body or an isolated system by work-energy theorem we have $K_2 - K_1 = \int \vec{F}.d\vec{r}$ .....(i)

But according to definition of potential energy in a conservative field $U_2 - U_1 = -\int \vec{F}.d\vec{r}$ .....(ii)

So from equation (i) and (ii) we have

$$K_2 - K_1 = -(U_2 - U_1)$$

or $K_2 + U_2 = K_1 + U_1$

i.e. $K + U =$ constant.

For an isolated system or body in presence of conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depends upon time. This is known as the law of conservation of mechanical energy.

$\Delta(K + U) = \Delta E = 0$ [As E is constant in a conservative field]

$\therefore$ $\Delta K + \Delta U = 0$

i.e. if the kinetic energy of the body increases its potential energy will decrease by an equal amount and vice-versa.

(2) **Law of conservation of total energy :** If some non-conservative force like friction is also acting on the particle, the mechanical energy is no more constant. It changes by the amount of work done by the frictional force.

$\Delta(K + U) = \Delta E = W_f$ [where W_f is the work done against friction]

The lost energy is transformed into heat and the heat energy developed is exactly equal to loss in mechanical energy.

We can, therefore, write $\Delta E + Q = 0$ [where Q is the heat produced]

This shows that if the forces are conservative and non-conservative both, it is not the mechanical energy alone which is conserved, but it is the total energy, may be heat, light, sound or mechanical *etc.*, which is conserved.

In other words : "*Energy may be transformed from one kind to another but it cannot be created or destroyed. The total energy in an isolated system is constant*". This is the law of conservation of energy.

Sample problems based on conservation of energy

Problem 57. Two stones each of mass 5*kg* fall on a wheel from a height of 10*m*. The wheel stirs 2*kg* water. The rise in temperature of water would be

(a) $2.6° C$ (b) $1.2° C$ (c) $0.32° C$ (d) $0.12° C$

Solution : (d) For the given condition potential energy of the two masses will convert into heat and temperature of water will increase $W = JQ \Rightarrow 2m \times g \times h = J(m_w\ S\ \Delta t) \Rightarrow$

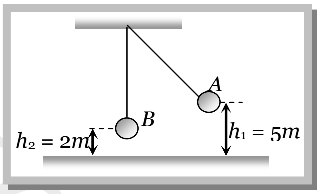
$$2 \times 5 \times 10 \times 10 = 4.2(2 \times 10^3 \times \Delta t)$$

$$\therefore\ \Delta t = \frac{1000}{8.4 \times 10^3} = 0.119^o C = 0.12^o C.$$

Problem 58. A boy is sitting on a swing at a maximum height of 5*m* above the ground. When the swing passes through the mean position which is 2*m* above the ground its velocity is approximately

(a) 7.6 *m/s* (b) 9.8 *m/s* (c) 6.26 *m/s* (d) None of these

Solution : (a) By the conservation of energy Total energy at point *A* = Total energy at point *B*

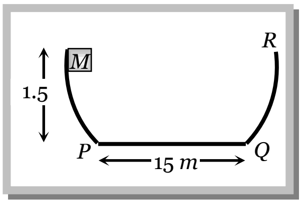


$$\Rightarrow\ mgh_1 = mgh_2 + \frac{1}{2}mv^2$$

$$\Rightarrow\ 9.8 \times 5 = 9.8 \times 2 + \frac{1}{2}v^2$$

$$\Rightarrow\ v^2 = 58.8\ \therefore\ v = 7.6\, m/s$$

Problem 59. A block of mass *M* slides along the sides of a bowl as shown in the figure. The walls of the bowl are frictionless and the base has coefficient of friction 0.2. If the block is released from the top of the side, which is 1.5 *m* high, where will the block come to rest ?
Given that the length of the base is 15 *m*



(a) 1 *m* from *P*
(b) Mid point
(c) 2 *m* from *P*
(d) At *Q*

Solution : (b) Potential energy of block at starting point = Kinetic energy at point *P* = Work done against friction in traveling a distance *s* from point *P*.

$$\therefore\quad mgh = \mu mgs\ \Rightarrow s = \frac{h}{\mu} = \frac{1.5}{0.2} = 7.5\, m$$

i.e. block come to rest at the mid point between *P* and *Q*.

Problem 60. If we throw a body upwards with velocity of $4\, ms^{-1}$ at what height its kinetic energy reduces to half of the initial value ? Take $g = 10\, m/s^2$

(a) 4*m* (b) 2 *m* (c) 1 *m* (d) None of these

Solution : (d) We know kinetic energy $K = \frac{1}{2}mv^2\ \therefore\ v \propto \sqrt{K}$

When kinetic energy of the body reduces to half its velocity becomes $v = \frac{u}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ m/s$

From the equation $v^2 = u^2 - 2gh \Rightarrow (2\sqrt{2})^2 = (4)^2 - 2 \times 10\, h\quad \therefore\ h = \frac{16-8}{20} = 0.4\, m$.

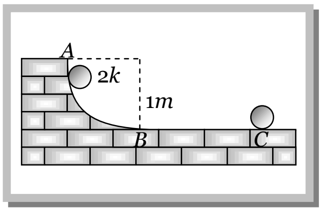
Problem 61. A 2*kg* block is dropped from a height of 0.4 *m* on a spring of force constant $K = 1960\ Nm^{-1}$. The maximum compression of the spring is

(a) 0.1 *m* (b) 0.2 *m* (c) 0.3 *m* (d) 0.4 *m*

Solution : (a) When a block is dropped from a height, its potential energy gets converted into kinetic energy and finally spring get compressed due to this energy.
∴ Gravitational potential energy of block = Elastic potential energy of spring

$$\Rightarrow\ mgh = \frac{1}{2}Kx^2\ \Rightarrow x = \sqrt{\frac{2mgh}{K}} = \sqrt{\frac{2 \times 2 \times 10 \times 0.4}{1960}} = 0.09\, m \simeq 0.1\, m.$$

Problem 62. A block of mass 2*kg* is released from *A* on the track that is one quadrant of a circle of radius 1*m*. It slides down the track and reaches *B* with a speed of $4\,ms^{-1}$ and finally stops at *C* at a distance of 3*m* from *B*. The work done against the force of friction is



(a) 10 *J*
(b) 20 *J*
(c) 2 *J*
(d) 6 *J*

Solution : (b) Block possess potential energy at point $A = mgh = 2 \times 10 \times 1 = 20\ J$

Finally block stops at point *C*. So its total energy goes against friction *i.e.* work done against friction is 20 *J*.

Problem 63. A stone projected vertically upwards from the ground reaches a maximum height *h*. When it is at a height $\frac{3h}{4}$, the ratio of its kinetic and potential energies is

(a) 3 : 4 (b) 1 : 3 (c) 4 : 3 (d) 3 : 1

Solution : (b) At the maximum height, Total energy = Potential energy = mgh

At the height $\frac{3h}{4}$, Potential energy $= mg\frac{3h}{4} = \frac{3}{4}mgh$

and Kinetic energy = Total energy – Potential energy $= mgh - 3\frac{mgh}{4} = \frac{1}{4}mgh$

$\therefore \quad \frac{\text{Kinetic energy}}{\text{Potential energy}} = \frac{1}{3}$.

6.19 Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av.}) = \frac{\Delta W}{\Delta t} = \frac{W}{t}$

Instantaneous power $(P_{inst.}) = \frac{dW}{dt} = \frac{\vec{F}.d\vec{s}}{dt}$ [As $dW = \vec{F}.d\vec{s}$]

$P_{inst} = \vec{F}.\vec{v}$ [As $\vec{v} = \frac{d\vec{s}}{dt}$]

i.e. power is equal to the scalar product of force with velocity.

Important points

(1) Dimension : $[P] = [F][v] = [MLT^{-2}][LT^{-1}]$

$\therefore \quad [P] = [ML^2T^{-3}]$

(2) Units : *Watt* or *Joule/sec* [S.I.]

Erg/sec [C.G.S.]

Practical units : *Kilowatt* (*kW*), *Mega watt* (*MW*) and Horse power (*hp*)

Relations between different units : $1\,watt = 1\,Joule/\sec = 10^7\,erg/\sec$

$1hp = 746\ Watt$

$1\ MW = 10^6\ Watt$

$1\ kW = 10^3\ Watt$

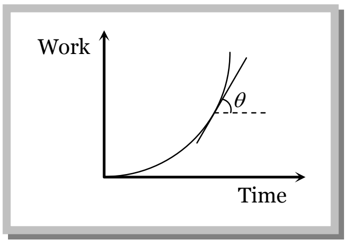
(3) If work done by the two bodies is same then power $\propto \frac{1}{\text{time}}$

i.e. the body which perform the given work in lesser time possess more power and vice-versa.

(4) As power = work/time, any unit of power multiplied by a unit of time gives unit of work (or energy) and not power, *i.e.* Kilowatt-hour or watt-day are units of work or energy.

$$1\,KWh = 10^3 \frac{J}{sec} \times (60 \times 60\, sec) = 3.6 \times 10^6\ Joule$$

(5) The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan\theta$



(6) Area under power time curve gives the work done as $P = \frac{dW}{dt}$

$$\therefore\ W = \int P\, dt$$

$\therefore$ W = Area under P-t curve

6.20 Position and Velocity of an Automobile *w.r.t* Time

An automobile of mass m accelerates, starting from rest, while the engine supplies constant power P, its position and velocity changes *w.r.t* time.

(1) **Velocity :** As $Fv = P$ = constant

i.e. $$m\frac{dv}{dt}v = P \qquad \left[\text{As } F = \frac{mdv}{dt}\right]$$

or $$\int v\, dv = \int \frac{P}{m} dt$$

By integrating both sides we get $\frac{v^2}{2} = \frac{P}{m}t + C_1$

As initially the body is at rest *i.e.* $v = 0$ at $t = 0$, so $C_1 = 0$

$$\therefore \quad v = \left(\frac{2Pt}{m}\right)^{1/2}$$

(2) **Position :** From the above expression $v = \left(\frac{2Pt}{m}\right)^{1/2}$

or $$\frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2} \qquad \left[\text{As } v = \frac{ds}{dt}\right]$$

i.e. $$\int ds = \int \left(\frac{2Pt}{m}\right)^{1/2} dt$$

By integrating both sides we get $$s = \left(\frac{2P}{m}\right)^{1/2} . \frac{2}{3} t^{3/2} + C_2$$

Now as at $t = 0, s = 0$, so $C_2 = 0$

$\therefore$ $$s = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$$

Sample problems based on power

Problem 64. A car of mass 'm' is driven with acceleration 'a' along a straight level road against a constant external resistive force 'R'. When the velocity of the car is 'v', the rate at which the engine of the car is doing work will be **[MP PMT/PET 1998; JIMPER 2000]**

(a) Rv (b) mav (c) $(R+ma)v$ (d) $(ma-R)v$

Solution : (c) The engine has to do work against resistive force R as well as car is moving with acceleration a.

Power = Force × velocity = $(R+ma)v$.

Problem 65. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

(a) v (b) v^2 (c) v^3 (d) v^4

Solution : (c) Force $= v\frac{dm}{dt} = v\frac{d}{dt}(V \times \rho) = v\rho\frac{d}{dt}[A \times l] = v\rho A\frac{dl}{dt} = \rho A v^2$

Power = $F \times v = \rho A v^2 \times v = \rho A v^3$ $\therefore P \propto v^3$.

Problem 66. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain twice as much water from the same pipe in the same time, power of the motor has to be increased to

(a) 16 times (b) 4 times (c) 8 times (d) 2 times

Solution : (d) $P = \frac{\text{work done}}{\text{time}} = \frac{mgh}{t}$ $\therefore P \propto m$

i.e. To obtain twice water from the same pipe in the same time, the power of motor has to be increased to 2 times.

Problem 67. A force applied by an engine of a train of mass $2.05 \times 10^6\, kg$ changes its velocity from 5 m/s to 25 m/s in 5 minutes. The power of the engine is **[EAMCET 2001]**

(a) 1.025 MW (b) 2.05 MW (c) 5MW (d) 5 MW

Solution : (b) Power $= \frac{\text{Work done}}{\text{time}} = \frac{\text{Increase in kinetic energy}}{\text{time}} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{t} = \frac{\frac{1}{2} \times 2.05 \times 10^6 \times [25^2 - 5^2]}{5 \times 60}$

$= 2.05 \times 10^6\, watt = 2.05\, MW$

Problem 68. From a water fall, water is falling at the rate of 100 kg/s on the blades of turbine. If the height of the fall is 100m then the power delivered to the turbine is approximately equal to **[BHU 1997]**

(a) 100kW (b) 10 kW (c) 1kW (d) 1000 kW

Solution : (a) Power $= \frac{\text{Work done}}{t} = \frac{mgh}{t} = 100 \times 10 \times 100 = 10^5\, watt = 100\, kW$ $\left[\text{As } \frac{m}{t} = 100\frac{kg}{\text{sec}} \text{(given)}\right]$

Problem 69. A particle moves with a velocity $\vec{v} = 5\hat{i} - 3\hat{j} + 6\hat{k}\, ms^{-1}$ under the influence of a constant force $\vec{F} = 10\hat{i} + 10\hat{j} + 20\hat{k}\, N$. The instantaneous power applied to the particle is

(a) 200 $J\text{-}s^{-1}$ (b) 40 $J\text{-}s^{-1}$ (c) 140 $J\text{-}s^{-1}$ (d) 170 $J\text{-}s^{-1}$

Solution : (c) $P = \vec{F}.\vec{v} = (10\hat{i} + 10\hat{j} + 20\hat{k}).(5\hat{i} - 3\hat{j} + 6\hat{k}) = 50 - 30 + 120 = 140\, J\text{-}s^{-1}$

***Problem* 70.** A car of mass 1250 *kg* experience a resistance of 750 *N* when it moves at $30ms^{-1}$. If the engine can develop 30*kW* at this speed, the maximum acceleration that the engine can produce is

(a) $0.8ms^{-2}$ (b) $0.2ms^{-2}$ (c) $0.4ms^{-1}$ (d) $0.5ms^{-2}$

Solution : (b) Power = Force × velocity = (Resistive force + Accelerating force) × velocity

$\Rightarrow \; 30\times10^3 = (750+ma)\times 30 \Rightarrow ma = 1000-750 \Rightarrow a = \frac{250}{1250} = 0.2\,ms^{-2}$.

***Problem* 71.** A bus weighing 100 quintals moves on a rough road with a constant speed of 72*km/h*. The friction of the road is 9% of its weight and that of air is 1% of its weight. What is the power of the engine. Take $g = 10m/s^2$

(a) 50 *kW* (b) 100 *kW* (c) 150 *kW* (d) 200 *kW*

Solution : (d) Weight of a bus = mass × g $= 100\times100\,kg\times10\,m/s^2 = 10^5\,N$

Total friction force = 10% of weight = 10^4 *N*

∴ Power = Force × velocity $= 10^4\,N\times72\,km/h = 10^4\times20\,watt = 2\times10^5\,watt = 200\,kW$.

***Problem* 72.** Two men with weights in the ratio 5 : 3 run up a staircase in times in the ratio 11 : 9. The ratio of power of first to that of second is

(a) $\frac{15}{11}$ (b) $\frac{11}{15}$ (c) $\frac{11}{9}$ (d) $\frac{9}{11}$

Solution : (a) Power $(P) = \frac{mgh}{t}$ or $P\propto\frac{m}{t} \Rightarrow \frac{P_1}{P_2} = \frac{m_1}{m_2}\frac{t_2}{t_1} = \left(\frac{5}{3}\right)\left(\frac{9}{11}\right) = \frac{45}{33} = \frac{15}{11}$ (g and h are constants)

***Problem* 73.** A dam is situated at a height of 550 *metre* above sea level and supplies water to a power house which is at a height of 50 *metre* above sea level. 2000 *kg* of water passes through the turbines per second. The maximum electrical power output of the power house if the whole system were 80% efficient is

(a) 8 *MW* (b) 10 *MW* (c) 12.5 *MW* (d) 16 *MW*

Solution : (a) Power $= \frac{\text{work done}}{\text{time}} = \frac{mg\Delta h}{t} = \frac{2000\times10\times(550-50)}{1} = 10\,MW$

But the system is 80% efficient ∴ Power output = 10 × 80% = 8 *MW*.

***Problem* 74.** A constant force *F* is applied on a body. The power (*P*) generated is related to the time elapsed (*t*) as

(a) $P\propto t^2$ (b) $P\propto t$ (c) $P\propto\sqrt{t}$ (d) $P\propto t^{3/2}$

Solution : (b) $F = \frac{mdv}{dt}$ ∴ $F\,dt = mdv \Rightarrow v = \frac{F}{m}t$

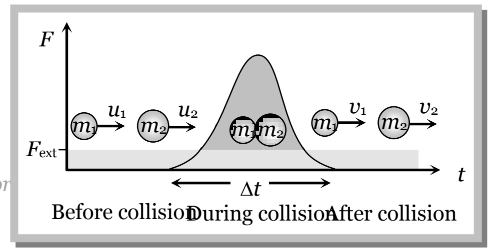
Now $P = F\times v = F\times\frac{F}{m}t = \frac{F^2t}{m}$. If force and mass are constants then $P\propto t$.

6.21 Collision

Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

In collision particles may or may not come in real touch e.g. in collision between two billiard balls or a ball and bat there is physical contact while in collision of alpha particle by a nucleus (*i.e.* Rutherford scattering experiment) there is no physical contact.

(1) **Stages of collision :** There are three distinct identifiable stages in collision, namely, before, during and after. In the before and after stage the interaction forces are zero. Between these two stages, the interaction forces are very large and often the dominating forces governing the motion of bodies. The magnitude of the interacting force is often unknown, therefore,



Newton's second law cannot be used, the law of conservation of momentum is useful in relating the initial and final velocities.

(2) **Momentum and energy conservation in collision :**

(i) Momentum conservation : In a collision the effect of external forces such as gravity or friction are not taken into account as due to small duration of collision (Δt) average impulsive force responsible for collision is much larger than external force acting on the system and since this impulsive force is 'Internal' therefore the total momentum of system always remains conserved.

(ii) Energy conservation : In a collision 'total energy' is also always conserved. Here total energy includes all forms of energy such as mechanical energy, internal energy, excitation energy, radiant energy or even mass energy.

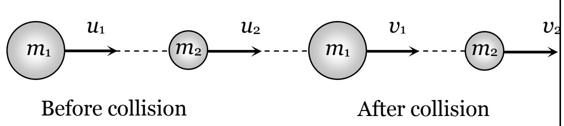
These laws are the fundamental laws of physics and applicable for any type of collision but this is not true for conservation of kinetic energy.

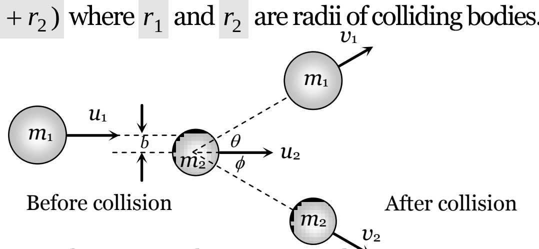
(3) **Types of collision :** (i) On the basis of conservation of kinetic energy.

Perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic.	If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic.	If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.
Coefficient of restitution $e = 1$	Coefficient of restitution $0 < e < 1$	Coefficient of restitution $e = 0$
$(KE)_{final} = (KE)_{initial}$	Here kinetic energy appears in other forms. In some cases $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released	The term 'perfectly inelastic' does not necessarily mean that all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be. (Consistent with momentum conservation).
Examples : (1) Collision between atomic particles (2) Bouncing of ball with same velocity after the collision with earth.	*Examples* : (1) Collision between two billiard balls. (2) Collision between two automobile on a road. In fact all majority of collision belong to this category.	*Example* : Collision between a bullet and a block of wood into which it is fired. When the bullet remains embeded in the block.

(ii) On the basis of the direction of colliding bodies

Head on or one dimensional collision	Oblique collision
In a collision if the motion of colliding particles before and after the collision is along the same line the collision is said to be head on or one dimensional.	If two particle collision is 'glancing' *i.e.* such that their directions of motion after collision are not along the initial line of motion, the collision is called oblique. If in oblique collision the particles before and after collision are in same plane, the collision is called 2-dimensional otherwise 3-dimensional.
Impact parameter b is zero for this type of collision.	Impact parameter b lies between 0 and $(r_1 + r_2)$ *i.e.* $0 < b < (r_1 + r_2)$ where r_1 and r_2 are radii of colliding bodies.

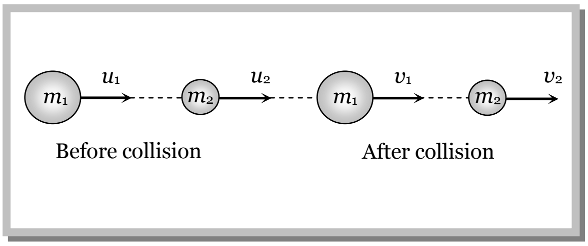




Example : collision of two gliders on an air track.	*Example* : Collision of billiard balls.

6.22 Perfectly Elastic Head on Collision

Let two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are v_1 and v_2 respectively.



According to law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad ......(i)$$

$$\Rightarrow \quad m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad ......(ii)$$

According to law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad .....(iii)$$

$$\Rightarrow \quad m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \qquad .....(iv)$$

Dividing equation (iv) by equation (ii)

$$v_1 + u_1 = v_2 + u_2 \qquad .....(v)$$

$$\Rightarrow \quad u_1 - u_2 = v_2 - v_1 \qquad .....(vi)$$

Relative velocity of approach = Relative velocity of separation

Note : ❑ The ratio of relative velocity of separation and relative velocity of approach is defined as coefficient of restitution. $e = \frac{v_2 - v_1}{u_1 - u_2}$ or $v_2 - v_1 = e(u_1 - u_2)$

❑ For perfectly elastic collision $e = 1$ $\therefore v_2 - v_1 = u_1 - u_2$ (As shown in eq. (vi)

❑ For perfectly inelastic collision $e = 0$ $\therefore v_2 - v_1 = 0$ or $v_2 = v_1$

It means that two body stick together and move with same velocity.

❑ For inelastic collision $0 < e < 1$ $\therefore v_2 - v_1 = e(u_1 - u_2)$

In short we can say that e is the degree of elasticity of collision and it is dimension less quantity.

Further from equation (v) we get $v_2 = v_1 + u_1 - u_2$

Substituting this value of v_2 in equation (i) and rearranging we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \qquad ......(vii)$$

Similarly we get $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$ ......(viii)

(1) **Special cases of head on elastic collision**

(i) **If projectile and target are of same mass *i.e.* $m_1 = m_2$**

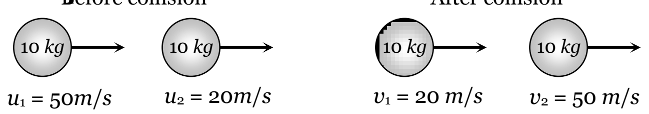
Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2}{m_1 + m_2}u_2$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

Substituting $m_1 = m_2$ we get

$v_1 = u_2$ and $v_2 = u_1$

It means when two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

Example : Collision of two billiard balls



Sub case : $u_2 = 0$ *i.e.* target is at rest
$v_1 = 0$ and $v_2 = u_1$

(ii) **If massive projectile collides with a light target *i.e.* $m_1 >> m_2$**

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

Substituting $m_2 = 0$, we get

$v_1 = u_1$ and $v_2 = 2u_1 - u_2$

Example : Collision of a truck with a cyclist

Sub case : $u_2 = 0$ i.e. target is at rest

Before collision After collision

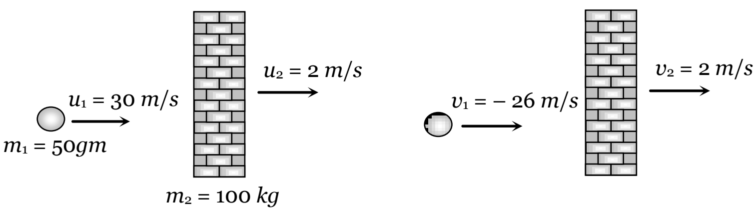
(iii) **If light projectile collides with a very heavy target *i.e.* $m_1 << m_2$**

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2 u_2}{m_1 + m_2}$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1 u_1}{m_1 + m_2}$

Substituting $m_1 = 0$, we get

$v_1 = -u_1 + 2u_2$ and $v_2 = u_2$

Example : Collision of a ball with a massive wall.



Before collision After collision

Sub case : $u_2 = 0$ i.e. target is at rest

$v_1 = -u_1$ and $v_2 = 0$

i.e. the ball rebounds with same speed in opposite direction when it collide with stationary and very massive wall.

(2) **Kinetic energy transfer during head on elastic collision**

Kinetic energy of projectile before collision $K_i = \frac{1}{2}m_1 u_1^2$

Kinetic energy of projectile after collision $K_f = \frac{1}{2}m_1v_1^2$

Kinetic energy transferred from projectile to target ΔK = decrease in kinetic energy in projectile

$$\Delta K = \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

Fractional decrease in kinetic energy $\frac{\Delta K}{K} = \frac{\frac{1}{2}m_1(u_1^2 - v_1^2)}{\frac{1}{2}m_1u_1^2} = 1 - \left(\frac{v_1}{u_1}\right)^2$ .....(i)

We can substitute the value of v_1 from the equation $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$

If the target is at rest *i.e.* $u_2 = 0$ then $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$

From equation (i) $\frac{\Delta K}{K} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$ .....(ii)

or $\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 + m_2)^2}$ .....(iii)

or $\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 - m_2)^2 + 4m_1m_2}$ .....(iv)

Note :
- Greater the difference in masses less will be transfer of kinetic energy and vice versa
- Transfer of kinetic energy will be maximum when the difference in masses is minimum

  i.e. $m_1 - m_2 = 0$ or $m_1 = m_2$ then $\frac{\Delta K}{K} = 1 = 100\,\%$

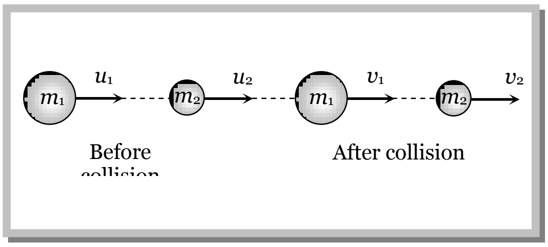
  So the transfer of kinetic energy in head on elastic collision (when target is at rest) is maximum when the masses of particles are equal i.e. mass ratio is 1 and the transfer of kinetic energy is 100%.
- If $m_2 = nm_1$ then from equation (iii) we get $\frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$
- Kinetic energy retained by the projectile $\left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1 -$ kinetic energy transferred by projectile

  $\Rightarrow$

$$\left(\frac{\Delta K}{K}\right)_{\text{Retained}} = 1 - \left[1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2\right] = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

(3) **Velocity, momentum and kinetic energy of stationary target after head on elastic collision**

(i) Velocity of target : We know $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$



$\Rightarrow v_2 = \frac{2m_1u_1}{m_1 + m_2} = \frac{2u_1}{1 + m_2/m_1}$ [As $u_2 = 0$ and Let $\frac{m_2}{m_1} = n$]

$\therefore \quad v_2 = \frac{2u_1}{1+n}$

(ii) Momentum of target : $P_2 = m_2v_2 = \frac{2nm_1u_1}{1+n}$ $\left[\text{As } m_2 = m_1 n \text{ and } v_2 = \frac{2u_1}{1+n}\right]$

$\therefore \quad P_2 = \frac{2m_1u_1}{1+(1/n)}$

(iii) Kinetic energy of target : $K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}nm_1\left(\frac{2u_1}{1+n}\right)^2 = \frac{2m_1u_1^2 n}{(1+n)^2}$

$= \frac{4(K_1)n}{(1-n)^2 + 4n}$ $\left[\text{As } K_1 = \frac{1}{2}m_1u_1^2\right]$

(iv) Relation between masses for maximum velocity, momentum and kinetic energy

Velocity	$v_2 = \frac{2u_1}{1+n}$	For v_2 to be maximum n must be minimum i.e. $n = \frac{m_2}{m_1} \to 0 \therefore m_2 << m_1$	Target should be very light.
Momentum	$P_2 = \frac{2m_1u_1}{(1+1/n)}$	For P_2 to be maximum, $(1/n)$ must be minimum or n must be maximum. i.e. $n = \frac{m_2}{m_1} \to \infty \therefore m_2 >> m_1$	Target should be massive.
Kinetic energy	$K_2 = \frac{4K_1 n}{(1-n)^2 + 4n}$	For K_2 to be maximum $(1-n)^2$ must be minimum. i.e. $1 - n = 0 \Rightarrow n = 1 = \frac{m_2}{m_1} \therefore m_2 = m_1$	Target and projectile should be of equal mass.

Sample problem based on head on elastic collision

Problem 75. n small balls each of mass m impinge elastically each second on a surface with velocity u. The force experienced by the surface will be **[MP PMT/PET 1998; RPET 2001; BHU 2001; MP PMT 2003]**

(a) mnu (b) $2\,mnu$ (c) $4\,mnu$ (d) $\frac{1}{2}mnu$

Solution : (b) As the ball rebounds with same velocity therefore change in velocity = $2u$ and the mass colliding with the surface per second = nm

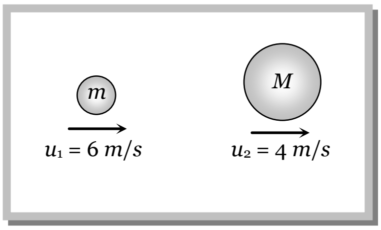
Force experienced by the surface $F = m\frac{dv}{dt}$ $\therefore F = 2\,mnu$.

Problem 76. A particle of mass m moving with horizontal speed 6 m/sec. If $m << M$ then for one dimensional elastic collision, the speed of lighter particle after collision will be **[MP PMT 2003]**

(a) 2 *m/sec* in original direction (b) 2 *m/sec* opposite to the original direction

(c) 4 *m/sec* opposite to the original direction (d) 4 *m/sec* in original direction

Solution : (a) $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$



Substituting $m_1 = 0$, $v_1 = -u_1 + 2u_2$

$\Rightarrow v_1 = -6 + 2(4) = 2m/s$

i.e. the lighter particle will move in original direction with the speed of 2 m/s.

Problem 77. A body of mass m moving with velocity v makes a head-on collision with another body of mass $2m$ which is initially at rest. The loss of kinetic energy of the colliding body (mass m) is **[MP PMT 1996; RPET 1999; AI**

(a) $\frac{1}{2}$ of its initial kinetic energy (b) $\frac{1}{9}$ of its initial kinetic energy

(c) $\frac{8}{9}$ of its initial kinetic energy (d) $\frac{1}{4}$ of its initial kinetic energy

Solution : (c) Loss of kinetic energy of the colliding body $\frac{\Delta K}{K} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = 1 - \left(\frac{m - 2m}{m + 2m}\right)^2 = 1 - \left(\frac{1}{3}\right)^2$

$\Delta K = \left(1 - \frac{1}{9}\right)K = \frac{8}{9}K$ $\therefore$ Loss of kinetic energy is $\frac{8}{9}$ of its initial kinetic energy.

Problem 78. A ball of mass m moving with velocity V, makes a head on elastic collision with a ball of the same mass moving with velocity $2V$ towards it. Taking direction of V as positive velocities of the two balls after collision are

(a) $-V$ and $2V$ (b) $2V$ and $-V$ (c) V and $-2V$ (d) $-2V$ and V

Solution : (d) Initial velocities of balls are $+V$ and $-2V$ respectively and we know that for given condition velocities get interchanged after collision. So the velocities of two balls after collision are $-2V$ and V respectively.

Problem 79. Consider the following statements

Assertion (A) : In an elastic collision of two billiard balls, the total kinetic energy is conserved during the short time of collision of the balls (*i.e.*, when they are in contact)

Reason (R) : Energy spent against friction does not follow the law of conservation of energy of these statements **[AIIMS 2002]**

(a) Both A and R are true and the R is a correct explanation of A

(b) Both A and R are true but the R is not a correct explanation of the A

(c) A is true but the R is false

(d) Both A and R are false

Solution : (d) (i) When they are in contact some part of kinetic energy may convert in potential energy so it is not conserved during the short time of collision. (ii) Law of conservation of energy is always true.

Problem 80. A big ball of mass M, moving with velocity u strikes a small ball of mass m, which is at rest. Finally small ball attains velocity u and big ball v. Then what is the value of v **[RPET 2001]**

(a) $\frac{M - m}{M + m}u$ (b) $\frac{m}{M + m}u$ (c) $\frac{2m}{M + m}u$ (d) $\frac{M}{M + m}u$

Solution : (a) From the standard equation $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 = \left(\frac{M - m}{M + m}\right)u$.

Problem 81. A car of mass 400*kg* and travelling at 72 *kmph* crashes into a truck of mass 4000*kg* and travelling at 9 *kmph*, in the same direction. The car bounces back at a speed of 18 *kmph*. The speed of the truck after the impact is

(a) 9 *kmph* (b) 18 *kmph* (c) 27 *kmph* (d) 36 *kmph*

Solution : (b) By the law of conservation of linear momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow$ $400 \times 72 + 4000 \times 9 = 400 \times (-18) + 4000 \times v_2$ $\Rightarrow v_2 = 18\, km/h$.

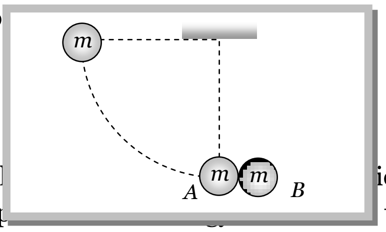
Problem 82. A smooth sphere of mass *M* moving with velocity *u* directly collides elastically with another sphere of mass *m* at rest. After collision their final velocities are *V* and *v* respectively. The value of *v* is **[MP PET 1995]**

(a) $\frac{2uM}{m}$ (b) $\frac{2um}{M}$ (c) $\frac{2u}{1 + \frac{m}{M}}$ (d) $\frac{2u}{1 + \frac{M}{m}}$

Solution : (c) Final velocity of the target $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$

As initially target is at rest so by substituting $u_2 = 0$ we get $v_2 = \frac{2Mu}{M + m} = \frac{2u}{1 + \frac{m}{M}}$.

Problem 83. A sphere of mass 0.1 *kg* is attached to a cord of 1*m* length. Starting from the height of its point of suspension this sphere hits a block of same mass at rest on a frictionless table, If the impact is elastic, then the kinetic energy of the block after the collisio



(a) 1 *J* (b) 10 *J*

(c) 0.1 *J* (d) 0.5 *J*

Solution : (a) As two blocks are of same mass and the collision is perfectl ies gets interchanged *i.e.* the block *A* comes into rest and comp to block *B*.

Now kinetic energy of block *B* after collision = Kinetic energy of block *A* before collision

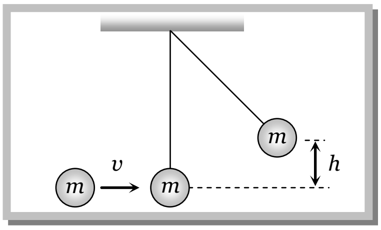
= Potential energy of block *A* at the original height

= $mgh = 0.1 \times 10 \times 1 = 1\, J$.

Problem 84. A ball moving horizontally with speed *v* strikes the bob of a simple pendulum at rest. The mass of the bob is equal to that of the ball. If the collision is elastic the bob will rise to a height

(a) $\frac{v^2}{g}$ (b) $\frac{v^2}{2g}$ (c) $\frac{v^2}{4g}$ (d) $\frac{v^2}{8g}$

Solution : (b) Total kinetic energy of the ball will transfer to the bob of simple pendulum. Let it rises to height '*h*' by the law of conservation of energy.



$\frac{1}{2}mv^2 = mgh$

$\therefore\ h = \frac{v^2}{2g}$

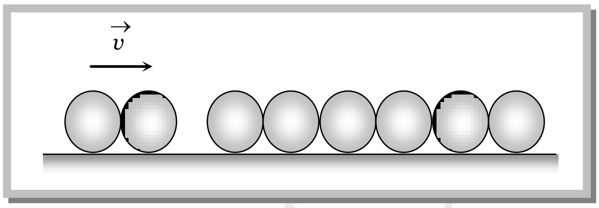
Problem 85. A moving body with a mass m_1 strikes a stationary body of mass m_2. The masses m_1 and m_2 should be in the ratio $\frac{m_1}{m_2}$ so as to decrease the velocity of the first body 1.5 times assuming a perfectly elastic impact. Then the ratio $\frac{m_1}{m_2}$ is

(a) 1/ 25 (b) 1/5 (c) 5 (d) 25

Solution : (c) $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$ [As $u_2 = 0$ and $\left(v_1 = \frac{u_1}{1.5}\right)$ given]

$\Rightarrow \frac{u_1}{1.5} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 \Rightarrow m_1 + m_2 = 1.5(m_1 - m_2) \Rightarrow \frac{m_1}{m_2} = 5$.

Problem 86. Six identical balls are lined in a straight groove made on a horizontal frictionless surface as shown. Two similar balls each moving with a velocity v collide with the row of 6 balls from left. What will happen



(a) One ball from the right rolls out with a speed $2v$ and the remaining balls will remain at rest

(b) Two balls from the right roll out with speed v each and the remaining balls will remain stationary

(c) All the six balls in the row will roll out with speed $v/6$ each and the two colliding balls will come to rest

(d) The colliding balls will come to rest and no ball rolls out from right

Solution : (b) Only this condition satisfies the law of conservation of linear momentum.

Problem 87. A moving mass of 8 *kg* collides elastically with a stationary mass of 2 *kg*. If E be the initial kinetic energy of the mass, the kinetic energy left with it after collision will be

(a) 0.80 E (b) 0.64 E (c) 0.36 E (d) 0.08 E

Solution : (c) Kinetic energy retained by projectile $\frac{\Delta K}{K} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \Rightarrow \Delta K = \left(\frac{8-2}{8+2}\right)^2 E == \frac{9}{25}E = 0.36E$.

Problem 88. A neutron travelling with a velocity v and K.E. E collides perfectly elastically head on with the nucleus of an atom of mass number A at rest. The fraction of total energy retained by neutron is

(a) $\left(\frac{A-1}{A+1}\right)^2$ (b) $\left(\frac{A+1}{A-1}\right)^2$ (c) $\left(\frac{A-1}{A}\right)^2$ (d) $\left(\frac{A+1}{A}\right)^2$

Solution : (a) Fraction of kinetic energy retained by projectile $\frac{\Delta K}{K} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$

Mass of neutron (m_1) = 1 and Mass of atom (m_2) = A $\therefore \frac{\Delta K}{K} = \left(\frac{1-A}{1+A}\right)^2$ or $\left(\frac{A-1}{A+1}\right)^2$.

Problem 89. A neutron with 0.6MeV kinetic energy directly collides with a stationary carbon nucleus (mass number 12). The kinetic energy of carbon nucleus after the collision is

(a) 1.7 *MeV* (b) 0.17 *MeV* (c) 17 *MeV* (d) Zero

Solution : (b) Kinetic energy transferred to stationary target (carbon nucleus) $\frac{\Delta K}{K}=\left[1-\left(\frac{m_1-m_2}{m_1+m_2}\right)^2\right]$

$$\frac{\Delta K}{K}=\left[1-\left(\frac{1-12}{1+12}\right)^2\right]=\left[1-\frac{121}{169}\right]=\frac{48}{169} \qquad \therefore\ \Delta K=\frac{48}{169}\times(0.6\,MeV)=0.17\,MeV.$$

***Problem* 90.** A body of mass m moving along a straight line collides with a body of mass nm which is also moving with a velocity kv in the same direction. If the first body comes to rest after the collision, then the velocity of second body after the collision would be

(a) $\frac{nv}{(1+nk)}$ (b) $\frac{nv}{(1-nk)}$ (c) $\frac{(1-nk)v}{n}$ (d) $\frac{(1+nk)v}{n}$

Solution : (d) Initial momentum $= mv+nm(kv)$ and final momentum $=0+nm\,V$

By the conservation of momentum, $mv+nm(kv)=0+nm\,V$

$$\Rightarrow v+nkv=nV \Rightarrow nV=(1+nk)v \Rightarrow V=\frac{(1+nk)v}{n}$$

***Problem* 91.** Which one of the following statement does not hold good when two balls of masses m_1 and m_2 undergo elastic collision

(a) When $m_1<m_2$ and m_2 at rest, there will be maximum transfer of momentum

(b) When $m_1>m_2$ and m_2 at rest, after collision the ball of mass m_2 moves with four times the velocity of m_1

(c) When $m_1=m_2$ and m_2 at rest, there will be maximum transfer of kinetic energy

(d) When collision is oblique and m_2 at rest with $m_1=m_2$, after collision the balls move in opposite directions

Solution : (b, d) We know that transfer of momentum will be maximum when target is massive and transfer of kinetic energy will be maximum when target and projectile are having same mass. It means statement (a) and (c) are correct, but statement (b) and (d) are incorrect because when target is very light, then after collision it will move with double the velocity of projectile and when collision is oblique and m_2 at rest with $m_1=m_2$, after collision the ball move perpendicular to each other.

6.23 Perfectly Elastic Oblique Collision

Let two bodies moving as shown in figure.

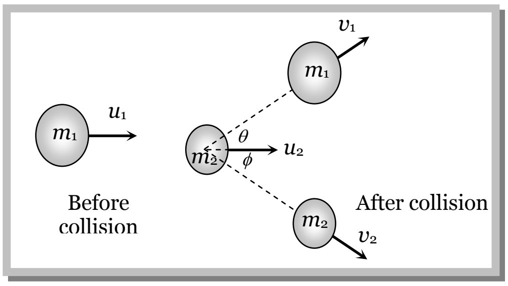
By law of conservation of momentum

Along x-axis, $m_1u_1+m_2u_2=m_1v_1\cos\theta+m_2v_2\cos\phi$ .....(i)

Along y-axis, $0=m_1v_1\sin\theta-m_2v_2\sin\phi$ .....(ii)

By law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2 \qquad .....(iii)$$



In case of oblique collision it becomes difficult to solve problem when some experimental data are provided as in these situations more unknown variables are involved than equations formed.

Special condition : If $m_1=m_2$ and $u_2=0$ substituting these values in equation (i), (ii) and (iii) we get

$$u_1=v_1\cos\theta+v_2\cos\phi \qquad .....(iv)$$

$$0=v_1\sin\theta-v_2\sin\phi \qquad .....(v)$$

and $$u_1^2=v_1^2+v_2^2 \qquad .....(vi)$$

Squaring (iv) and (v) and adding we get

$$u_1^2 = v_1^2 + v_2^2 + 2v_1u_2\cos(\theta + \phi) \qquad .....(vii)$$

Using (vi) and (vii) we get $\cos(\theta + \phi) = 0$

$\therefore \qquad \theta + \phi = \pi / 2$

i.e. after perfectly elastic oblique collision of two bodies of equal masses (if the second body is at rest), the scattering angle $\theta + \phi$ would be 90^o.

Sample problems based on oblique elastic collision

***Problem* 92.** A ball moving with velocity of $9m/s$ collides with another similar stationary ball. After the collision both the balls move in directions making an angle of 30^o with the initial direction. After the collision their speed will be

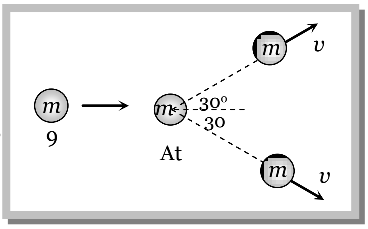
(a) $2.6m/s$ (b) $5.2m/s$ (c) $0.52m/s$ (d) $52m/s$

Solution : (b) Initial horizontal momentum of the system = $m \times 9$

Final horizontal momentum of the system = $2mv\cos 30^o$

According to law of conservation of momentum, $m \times 9 = 2mv\cos 30^o$

$\Rightarrow \quad v = 5.2\ m/s$



***Problem* 93.** A ball of mass $1kg$, moving with a velocity of $0.4m/s$ collides with another stationary ball. After the collision, the first ball moves with a velocity of $0.3m/s$ in a direction making an angle of 90^o with its initial direction. The momentum of second ball after collision will be (in *kg-m/s*)

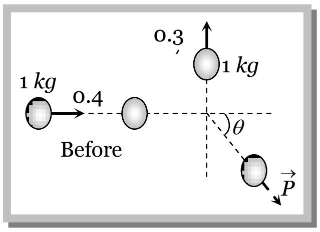
(a) 0.1 (b) 0.3 (c) 0.5 (d) 0.7

Solution : (c) Let second ball moves with momentum P making an angle θ from the horizontal (as shown in the figure).

By the conservation of horizontal momentum $1 \times 0.4 = P\cos\theta$ ......(i)

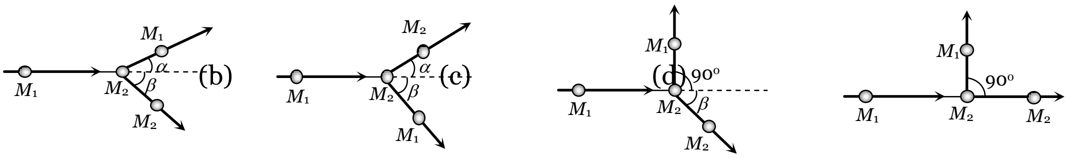
By the conservation of vertical momentum $0.3 = P\sin\theta$ ......(ii)

From (i) and (ii) we get $P = 0.5\ kg\text{-}m/s$



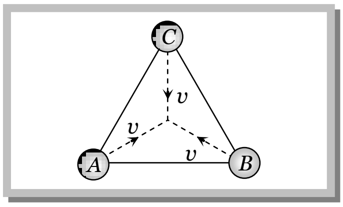
***Problem* 94.** Keeping the principle of conservation of momentum in mind which of the following collision diagram is not correct

(a) (b) (c) (d)



Solution : (d) In this condition the final resultant momentum makes some angle with *x*-axis. Which is not possible because initial momentum is along the *x*-axis and according to law of conservation of momentum initial and final momentum should be equal in magnitude and direction both.

***Problem* 95.** Three particles *A*, *B* and *C* of equal mass are moving with the same velocity *v* along the medians of an equilateral triangle. These particle collide at the centre *G* of triangle. After collision *A* becomes stationary, *B* retraces its path with velocity *v* then the magnitude and direction of velocity of *C* will be



(a) v and opposite to B

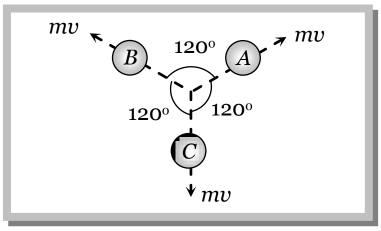
(b) v and in the direction of A

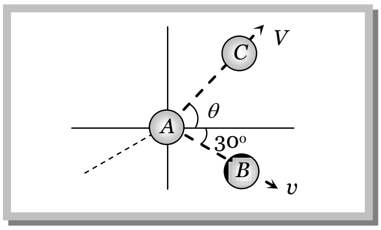
(c) v and in the direction of C

(d) v and in the direction of B

Solution : (d) From the figure (I) it is clear that before collision initial momentum of the system = 0

After the collision, A becomes stationary, B retraces its path with velocity v. Let C moves with velocity V making an angle θ from the horizontal. As the initial momentum of the system is zero, therefore horizontal and vertical momentum after the collision should also be equal to zero.





From figure (II) Horizontal momentum $v\cos\theta + v\cos 30^o = 0$ .....(i)

Vertical momentum $v\sin\theta - v\sin 30^o = 0$ .....(ii)

By solving (i) and (ii) we get $\theta = -30^o$ and $V = v$ *i.e.* the C will move with velocity v in the direction of B.

***Problem* 96.** A ball B_1 of mass M moving northwards with velocity v collides elastically with another ball B_2 of same mass but moving eastwards with the same velocity v. Which of the following statements will be true

(a) B_1 comes to rest but B_2 moves with velocity $\sqrt{2}v$

(b) B_1 moves with velocity $\sqrt{2}v$ but B_2 comes to rest

(c) Both move with velocity $v/\sqrt{2}$ in north east direction

(d) B_1 moves eastwards and B_2 moves north wards

Solution : (d) Horizontal momentum and vertical momentum both should remain conserve before and after collision. This is possible only for the (d) option.

6.24 Head on Inelastic Collision

(1) **Velocity after collision :** Let two bodies A and B collide inelastically and coefficient of restitution is e.

Where $$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$\Rightarrow$ $$v_2 - v_1 = e(u_1 - u_2)$$

$\therefore$ $$v_2 = v_1 + e(u_1 - u_2) \quad ......(i)$$

From the law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad ......(ii)$$

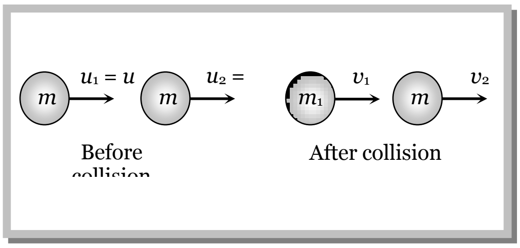
By solving (i) and (ii) we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2$$

Similarly $$v_2 = \left[\frac{(1+e)m_1}{m_1 + m_2}\right]u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2$$

By substituting $e = 1$, we get the value of v_1 and u_2 for perfectly elastic head on collision.

(2) **Ratio of velocities after inelastic collision :** A sphere of mass m moving with velocity u hits inelastically with another stationary sphere of same mass.



$\therefore \quad e = \dfrac{v_2 - v_1}{u_1 - u_2} = \dfrac{v_2 - v_1}{u - 0}$

$\Rightarrow \quad v_2 - v_1 = eu$ ......(i)

By conservation of momentum :

Momentum before collision = Momentum after collision

$mu = mv_1 + mv_2$

$\Rightarrow \quad v_1 + v_2 = u$ ......(ii)

Solving equation (i) and (ii) we get $v_1 = \dfrac{u}{2}(1-e)$ and $v_2 = \dfrac{u}{2}(1+e)$

$\therefore \quad \dfrac{v_1}{v_2} = \dfrac{1-e}{1+e}$

(3) **Loss in kinetic energy**

Loss (ΔK) = Total initial kinetic energy – Total final kinetic energy

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

Substituting the value of v_1 and v_2 from the above expression

$$\text{Loss } (\Delta K) = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(1-e^2)(u_1-u_2)^2$$

By substituting $e = 1$ we get $\Delta K = 0$ *i.e.* for perfectly elastic collision loss of kinetic energy will be zero or kinetic energy remains constant before and after the collision.

Sample problems based on inelastic collision

***Problem* 97.** A body of mass $40 kg$ having velocity $4 m/s$ collides with another body of mass $60 kg$ having velocity $2 m/s$. If the collision is inelastic, then loss in kinetic energy will be **[CPMT 1996; UP PMT 1996; Pb. P**

(a) $440\,J$ (b) $392\,J$ (c) $48\,J$ (d) $144\,J$

Solution : (c) Loss of K.E. in inelastic collision

$$\Delta K = \frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(u_1-u_2)^2 = \frac{1}{2}\frac{40\times 60}{(40+60)}(4-2)^2 = \frac{1}{2}\frac{2400}{100}4 = 48\,J.$$

***Problem* 98.** One sphere collides with another sphere of same mass at rest inelastically. If the value of coefficient of restitution is $\frac{1}{2}$, the ratio of their speeds after collision shall be **[RPMT 1998]**

(a) $1:2$ (b) $2:1$ (c) $1:3$ (d) $3:1$

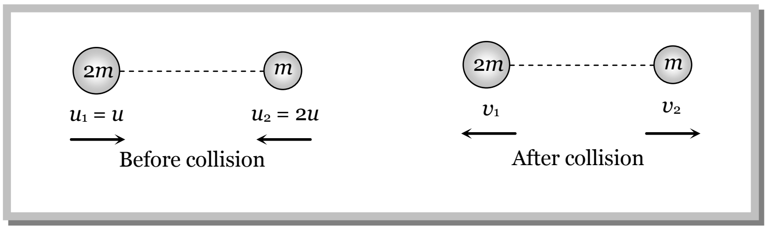
Solution : (c) $\dfrac{v_1}{v_2} = \dfrac{1-e}{1+e} = \dfrac{1-1/2}{1+1/2} = \dfrac{1/2}{3/2} = \dfrac{1}{3}$.

***Problem* 99.** The ratio of masses of two balls is 2 : 1 and before collision the ratio of their velocities is 1 : 2 in mutually opposite direction. After collision each ball moves in an opposite direction to its initial direction. If $e = (5/6)$, the ratio of speed of each ball before and after collision would be

(a) (5/6) times (b) Equal

(c) Not related (d) Double for the first ball and half for the second ball

Solution : (a) Let masses of the two ball are $2m$ and m, and their speeds are u and $2u$ respectively.



By conservation of momentum $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2 \Rightarrow 2mu - 2mu = mv_2 - 2mv_1 \Rightarrow v_2 = 2v_1$

Coefficient of restitution $= -\frac{(\vec{v}_2 - \vec{v}_1)}{(\vec{u}_2 - \vec{u}_1)} = -\frac{(2v_1 + v_1)}{(-2u - u)} = \frac{-3v_1}{-3u} = \frac{v_1}{u} = \frac{5}{6}$ [As $e = \frac{5}{6}$ given]

$\Rightarrow \frac{v_1}{u_1} = \frac{5}{6}$ = ratio of the speed of first ball before and after collision.

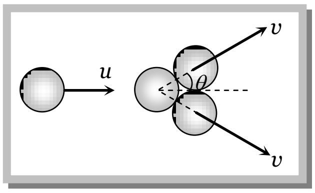
Similarly we can calculate the ratio of second ball before and after collision, $\frac{v_2}{u_2} = \frac{2v_1}{2u} = \frac{v_1}{u} = \frac{5}{6}$.

***Problem* 100.** Two identical billiard balls are in contact on a table. A third identical ball strikes them symmetrically and come to rest after impact. The coefficient of restitution is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{\sqrt{3}}{2}$

Solution : (a) $\sin\theta = \frac{r}{2r} = \frac{1}{2} \Rightarrow \theta = 30^o$

From conservation of linear momentum $mu = 2mv\cos 30^o$ or $v = \frac{u}{\sqrt{3}}$



Now $e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$ in common normal direction.

Hence, $e = \frac{v}{u\cos 30^o} = \frac{u/\sqrt{3}}{u\sqrt{3}/2} = \frac{2}{3}$

***Problem* 101.** A body of mass $3kg$, moving with a speed of $4ms^{-1}$, collides head on with a stationary body of mass $2kg$. Their relative velocity of separation after the collision is $2ms^{-1}$. Then

(a) The coefficient of restitution is 0.5 (b) The impulse of the collision is 7.2 *N-s*

(c) The loss of kinetic energy due to collision is 3.6 *J* (d) The loss of kinetic energy due to collision is 7.2 *J*

Solution: (a,b,c) $m_1 = 3kg$, $m_2 = 2kg$, $u_1 = 4m/s$, $u_2 = 0$

Relative velocity of approach $u_1 - u_2 = 4m/s$

Relative velocity of separation $v_2 - v_1 = 2m/s$ (given)

Coefficient of restitution $e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}} = \frac{2}{4} = \frac{1}{2} = 0.5$

Loss in kinetic energy $= \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(1-e)^2(u_1-u_2)^2 = \frac{1}{2}\frac{3\times 2}{3+2}\left[1-\left(\frac{1}{2}\right)^2\right](4)^2 = 7.2J$

Final velocity of m_1 mass, $v_1 = \left(\frac{m_1 - em_2}{m_1+m_2}\right)u_1 + \left[\frac{(1+e)m_2}{m_1+m_2}\right]u_2 = \frac{(3-0.5\times 2)}{3+2}\times 4 + 0 = \frac{8}{5}\, m/s$

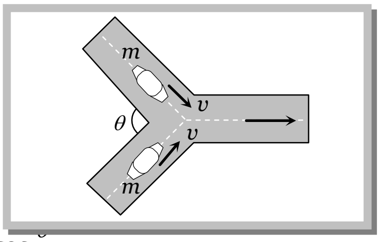
Impulse of collision = change in momentum of mass m_1 (or m_2) = $m_1v_1 - m_1u_1$

$$= 3\times\frac{8}{5} - 3\times 4 = \frac{24}{5} - 12 = 4.8 - 12 = -7.2\ N\text{-}s\ .$$

***Problem* 102.** Two cars of same mass are moving with same speed v on two different roads inclined at an angle θ with each other, as shown in the figure. At the junction of these roads the two cars collide inelastically and move simultaneously with the same speed. The speed of these cars would be

(a) $v\cos\frac{\theta}{2}$ (b) $\frac{v}{2}\cos\theta$

(c) $\frac{v}{2}\cos\frac{\theta}{2}$ (d) $2v\cos\theta$



Solution : (a) Initial horizontal momentum of the system $= mv\cos\frac{\theta}{2} + mv\cos\frac{\theta}{2}$.

If after the collision cars move with common velocity V then final horizontal momentum of the system = $2mV$.

By the law of conservation of momentum, $2mV = mv\cos\frac{\theta}{2} + mv\cos\frac{\theta}{2} \Rightarrow V = v\cos\frac{\theta}{2}$.

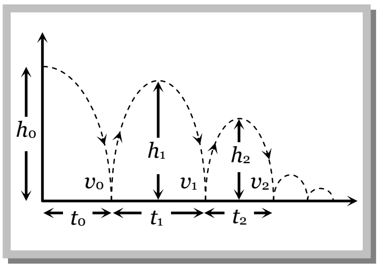
6.25 Rebounding of Ball After Collision With Ground

If a ball is dropped from a height h on a horizontal floor, then it strikes with the floor with a speed.

$$v_0 = \sqrt{2gh_0} \qquad [\text{From } v^2 = u^2 + 2gh]$$

and it rebounds from the floor with a speed

$$v_1 = e\,v_0 = e\sqrt{2gh_0} \qquad \left[\text{As } e = \frac{\text{velocity after collision}}{\text{velocity before collision}}\right]$$



(1) **First height of rebound :** $h_1 = \frac{v_1^2}{2g} = e^2 h_0$

$\therefore \qquad h_1 = e^2 h_0$

(2) **Height of the ball after n^{th} rebound :** Obviously, the velocity of ball after n^{th} rebound will be

$$v_n = e^n v_0$$

Therefore the height after n^{th} rebound will be $h_n = \frac{v_n^2}{2g} = e^{2n}h_0$

$\therefore \qquad h_n = e^{2n}h_0$

(3) **Total distance travelled by the ball before it stops bouncing**

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 + ... = h_0 + 2e^2h_0 + 2e^4h_0 + 2e^6h_0 + ...$$

$$H = h_0[1 + 2e^2(1 + e^2 + e^4 + e^6 ....)] = h_0\left[1 + 2e^2\left(\frac{1}{1-e^2}\right)\right]$$

$\left[\text{As } 1 + e^2 + e^4 + ... = \frac{1}{1-e^2}\right]$

$\therefore \qquad H = h_0\left[\dfrac{1+e^2}{1-e^2}\right]$

(4) **Total time taken by the ball to stop bouncing**

$$T = t_0 + 2t_1 + 2t_2 + 2t_3 + ... = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + ...$$

$$= \sqrt{\frac{2h_0}{g}}\,[1 + 2e + 2e^2 + ...] \qquad [\text{As } h_1 = e^2 h_0\,;\ h_2 = e^4 h_0\,]$$

$$= \sqrt{\frac{2h_0}{g}}\,[1 + 2e(1 + e + e^2 + e^3 + ...)] = \sqrt{\frac{2h_0}{g}}\left[1 + 2e\left(\frac{1}{1-e}\right)\right] = \sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)$$

$$\therefore \qquad T = \left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h_0}{g}}$$

Sample problems based on rebound of ball after collision with ground

Problem **103.** The change of momentum in each ball of mass $60\,gm$, moving in opposite directions with speeds $4\,m/s$ collide and rebound with the same speed, is

(a) $0.98\,kg\text{-}m/s$ (b) $0.73\,kg\text{-}m/s$ (c) $0.48\,kg\text{-}m/s$ (d) $0.22\,kg\text{-}m/s$

Solution : (c) Momentum before collision = mv, Momentum after collision = $-\,mv$

$\therefore$ Change in momentum $= 2mv = 2 \times 60 \times 10^{-3} \times 4 = 480 \times 10^{-3}\,kg\text{-}m/s = 0.48\,kg\text{-}m/s$

Problem **104.** A body falling from a height of 20m rebounds from hard floor. If it loses 20% energy in the impact, then coefficient of restitution is

(a) 0.89 (b) 0.56 (c) 0.23 (d) 0.18

Solution : (a) It loses 20% energy in impact and only 80% energy remains with the ball

So ball will rise upto height $h_2 = 80\%$ of $h_1 = \dfrac{80}{100} \times 20 = 16\,m$

Now coefficient of restitution $e = \sqrt{\dfrac{h_2}{h_1}} = \sqrt{\dfrac{16}{20}} = \sqrt{0.8} = 0.89.$

Problem **105.** A rubber ball is dropped from a height of $5m$ on a planet where the acceleration due to gravity is not known. On bouncing, it rises to $1.8m$. The ball loses its velocity on bouncing by a factor of **[CBSE PMT 1998]**

(a) 16/25 (b) 2/5 (c) 3/5 (d) 9/25

Solution : (c) If ball falls from height h_1, then it collides with ground with speed $v_1 = \sqrt{2gh_1}$ .....(i)

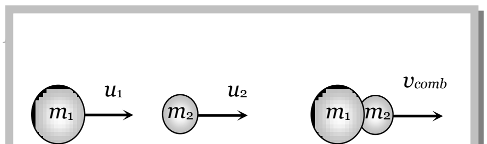
and if it rebound with velocity v_2, then it goes upto height h_2 from ground, $v_2 = \sqrt{2gh_2}$ .....(ii)

From (i) and (ii) $\dfrac{v_2}{v_1} = \sqrt{\dfrac{2gh_2}{2gh_1}} = \sqrt{\dfrac{h_2}{h_1}} = \sqrt{\dfrac{1.8}{5}} = \sqrt{\dfrac{9}{25}} = \dfrac{3}{5}$.

6.26 Perfectly Inelastic Collision

In such types of collisions the bodies move independently before collision but after collision as a one single body.

(1) **When the colliding bodies are moving in the same direction**



By the law of conservation of momentum

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v_{\text{comb}}$$

$$\Rightarrow \quad v_{\text{comb}} = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

Loss in kinetic energy $\Delta K = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)v_{comb}^2$

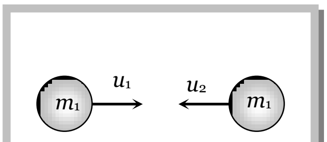
$$\Delta K = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2 \qquad \text{[By substituting the value of } v_{comb}]$$

(2) **When the colliding bodies are moving in the opposite direction**

By the law of conservation of momentum

$m_1u_1 + m_2(-u_2) = (m_1 + m_2)v_{\text{comb}}$ (Taking left to right as positive)

$\therefore \; v_{\text{comb}} = \dfrac{m_1u_1 - m_2u_2}{m_1 + m_2}$



Before collision

when $m_1u_1 > m_2u_2$ then $v_{\text{comb}} > 0$ (positive)

i.e. the combined body will move along the direction of motion of mass m_1.

when $m_1u_1 < m_2u_2$ then $v_{\text{comb}} < 0$ (negative)

i.e. the combined body will move in a direction opposite to the motion of mass m_1.

(3) **Loss in kinetic energy**

ΔK = Initial kinetic energy – Final kinetic energy

$$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}(m_1 + m_2)v_{\text{comb}}^2\right)$$

$$= \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 + u_2)^2$$

Sample problems based on perfectly inelastic collision

***Problem* 106.** Which of the following is not a perfectly inelastic collision

(a) Striking of two glass balls (b) A bullet striking a bag of sand

(c) An electron captured by a proton (d) A man jumping onto a moving cart

Solution : (a) For perfectly elastic collision relative velocity of separation should be zero *i.e.* the colliding body should move together with common velocity.

***Problem* 107.** A metal ball of mass $2kg$ moving with a velocity of $36 km/h$ has an head-on collision with a stationary ball of mass $3kg$. If after the collision, the two balls move together, the loss in kinetic energy due to collision is

[CBSE 1997; AIIMS 2001]

(a) $40 J$ (b) $60 J$ (c) $100 J$ (d) $140 J$

Solution : (b) Loss in kinetic energy $\Delta K = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 - u_2)^2 = \frac{1}{2}\frac{2\times 3}{2+3}(10-0)^2 = 60 J$.

Problem 108. A mass of $20 kg$ moving with a speed of $10 m/s$ collides with another stationary mass of 5 kg. As a result of the collision, the two masses stick together. The kinetic energy of the composite mass will be **[MP PMT 2000]**

(a) $600 J$ (b) $800 J$ (c) $1000 J$ (d) $1200 J$

Solution : (b) By conservation of momentum $m_1u_1 + m_2u_2 = (m_1 + m_2)V$

Velocity of composite mass $V = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{20\times10+5\times0}{20+5} = 8\, m/s$

$\therefore$ Kinetic energy of composite mass $= \frac{1}{2}(m_1 + m_2)V^2 = \frac{1}{2}(20+5)\times 8^2 = 800\, J.$

Problem 109. A neutron having mass of $1.67\times10^{-27} kg$ and moving at $10^8 m/s$ collides with a deutron at rest and sticks to it. If the mass of the deutron is $3.34\times10^{-27} kg$; the speed of the combination is **[CBSE PMT 2000]**

(a) $2.56\times10^3 m/s$ (b) $2.98\times10^5 m/s$ (c) $3.33\times10^7 m/s$ (d) $5.01\times10^9 m/s$

Solution : (c) $m_1 = 1.67\times10^{-27}\, kg$, $u_1 = 10^8\, m/s$, $m_2 = 3.34\times10^{-27}\, kg$ and $u_2 = 0$

Speed of the combination $V = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{1.67\times10^{-27}\times10^8 + 0}{1.67\times10^{-27} + 3.34\times10^{-27}} = 3.33\times10^7\, m/s.$

Problem 110. A particle of mass m moving eastward with a speed v collides with another particle of the same mass moving northward with the same speed v. The two particles coalesce on collision. The new particle of mass $2m$ will move in the north-easterly direction with a velocity

(a) $v/2$ (b) $2v$ (c) $v/\sqrt{2}$ (d) v

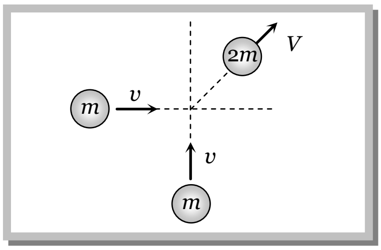
Solution : (c) Initially both the particles are moving perpendicular to each other with momentum mv.

So the net initial momentum $= \sqrt{(mv)^2 + (mv)^2} = \sqrt{2}\, mv$.

After the inelastic collision both the particles (system) moves with velocity V, so linear momentum = $2mV$

By the law of conservation of momentum $\sqrt{2}\, mv = 2mV$

$\therefore\ V = v/\sqrt{2}$.



Problem 111. A particle of mass 'm' moving with velocity 'v' collides inelastically with a stationary particle of mass '$2m$'. The speed of the system after collision will be

(a) $\frac{v}{2}$ (b) $2v$ (c) $\frac{v}{3}$ (d) $3v$

Solution : (c) By the conservation of momentum $mv + 2m\times0 = 3mV$ $\therefore$ $V = \frac{v}{3}$.

Problem 112. A ball moving with speed v hits another identical ball at rest. The two balls stick together after collision. If specific heat of the material of the balls is S, the temperature rise resulting from the collision is **[Roorkee 1999]**

(a) $\frac{v^2}{8S}$ (b) $\frac{v^2}{4S}$ (c) $\frac{v^2}{2S}$ (d) $\frac{v^2}{S}$

Solution : (b) Kinetic energy of ball will raise the temperature of the system $\frac{1}{2}mv^2 = (2m)S\,\Delta t \Rightarrow \Delta t = \frac{v^2}{4S}$.

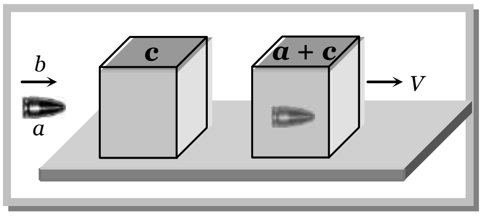
Problem 113. A bullet of mass a is fired with velocity b in a large block of mass c. The final velocity of the system will be

(a) $\frac{c}{a+c}$ (b) $\frac{ab}{a+c}$ (c) $\frac{(a+b)}{c}$ (d) $\frac{(a+c)}{a}b$

Solution : (b) Initially bullet moves with velocity b and after collision bullet get embedded in block and both move together with common velocity.

By the conservation of momentum $a \times b + 0 = (a + c)\, V$

$\therefore\ V = \frac{ab}{a+c}$



Problem 114. A particle of mass 1*g* having velocity $3\hat{i} - 2\hat{j}$ has a glued impact with another particle of mass 2*g* and velocity as $4\hat{j} - 6\hat{k}$. Velocity of the formed particle is

(a) $5.6 ms^{-1}$ (b) 0 (c) $6.4 ms^{-1}$ (d) $4.6 ms^{-1}$

Solution : (d) By conservation of momentum $m\vec{u}_1 + m_2\vec{u}_2 = (m_1 + m_2)\vec{V}$

$$\therefore\ \vec{V} = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m_2} = \frac{1(3i - 2\hat{j}) + 2(4j - 6\hat{k})}{m_1 + m_2} = \frac{3i + 6j - 12\hat{k}}{(1+2)} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$|\vec{V}| = \sqrt{(1)^2 + (2)^2 + (-4)^2} = \sqrt{1 + 4 + 16} = 4.6 ms^{-1}$.

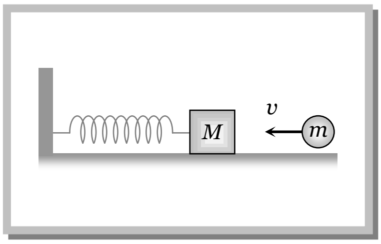
Problem 115. A body of mass 2*kg* is placed on a horizontal frictionless surface. It is connected to one end of a spring whose force constant is $250\, N/m$. The other end of the spring is joined with the wall. A particle of mass 0.15*kg* moving horizontally with speed v sticks to the body after collision. If it compresses the spring by $10 cm$, the velocity of the particle is

(a) $3m/s$ (b) $5m/s$ (c) $10m/s$ (d) $15m/s$

Solution : (d) By the conservation of momentum

Initial momentum of particle = Final momentum of system ⇒ $m \times v = (m + M)\, V$

$\therefore$ velocity of system $V = \frac{mv}{(m+M)}$



Now the spring compresses due to kinetic energy of the system so by the conservation of energy

$$\frac{1}{2}kx^2 = \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$$

$$\Rightarrow kx^2 = \frac{m^2v^2}{m+M} \Rightarrow v = \sqrt{\frac{kx^2(m+M)}{m^2}} = \frac{x}{m}\sqrt{k(m+M)}$$

Putting $m = 0.15\ kg$, $M = 2\ kg$, $k = 250\ N/m$, $x = 0.1\ m$ we get $v = 15\ m/s$.

6.27 Collision Between Bullet and Vertically Suspended Block

A bullet of mass m is fired horizontally with velocity u in block of mass M suspended by vertical thread.

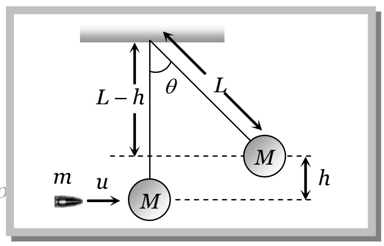
After the collision bullet gets embedded in block. Let the combined system raised upto height h and the string makes an angle θ with the vertical.

(1) **Velocity of system**

Let v be the velocity of the system (block + bullet) just after the collision.

Momentum$_{\text{bullet}}$ + Momentum$_{\text{block}}$ = Momentum$_{\text{bullet and block system}}$

$$mu + 0 = (m + M)v$$



$\therefore \quad v = \dfrac{mu}{(m+M)}$ ......(i)

(2) **Velocity of bullet :** Due to energy which remains in the bullet block system, just after the collision, the system (bullet + block) rises upto height h.

By the conservation of mechanical energy $\frac{1}{2}(m+M)v^2 = (m+M)gh \Rightarrow v = \sqrt{2gh}$

Now substituting this value in the equation (i) we get $\sqrt{2gh} = \dfrac{mu}{m+M}$

$$\therefore \quad u = \left[\frac{(m+M)\sqrt{2gh}}{m}\right]$$

(3) **Loss in kinetic energy :** We know the formula for loss of kinetic energy in perfectly inelastic collision

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 + u_2)^2$$

$$\therefore \quad \Delta K = \frac{1}{2}\frac{mM}{m+M}u^2 \qquad [\text{As } u_1 = u,\ u_2 = 0,\ m_1 = m \text{ and } m_2 = M]$$

(4) **Angle of string from the vertical**

From the expression of velocity of bullet $u = \left[\dfrac{(m+M)\sqrt{2gh}}{m}\right]$ we can get $h = \dfrac{u^2}{2g}\left(\dfrac{m}{m+M}\right)^2$

From the figure $\cos\theta = \dfrac{L-h}{L} = 1 - \dfrac{h}{L} = 1 - \dfrac{u^2}{2gL}\left(\dfrac{m}{m+M}\right)^2$

or $\quad \theta = \cos^{-1}\left[1 - \dfrac{1}{2gL}\left(\dfrac{mu}{m+M}\right)^2\right]$

Problems based on collision between bullet and block

***Problem* 116.** A bullet of mass m moving with velocity v strikes a block of mass M at rest and gets embeded into it. The kinetic energy of the composite block will be

(a) $\frac{1}{2}mv^2 \times \frac{m}{(m+M)}$ (b) $\frac{1}{2}mv^2 \times \frac{M}{(m+M)}$ (c) $\frac{1}{2}mv^2 \times \frac{(M+m)}{M}$ (d) $\frac{1}{2}Mv^2 \times \frac{m}{(m+M)}$

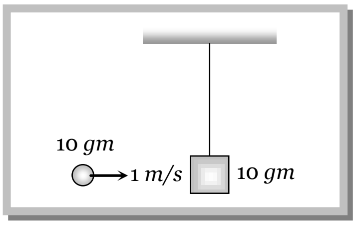
Solution : (a) By conservation of momentum,

Momentum of the bullet (mv) = momentum of the composite block $(m+M)V$

$\Rightarrow$ Velocity of composite block $V = \dfrac{mv}{m+M}$

$\therefore$ Kinetic energy $= \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2}\frac{m^2v^2}{m+M} = \frac{1}{2}mv^2\left(\frac{m}{m+M}\right)$.

***Problem* 117.** A mass of $10\,gm$, moving horizontally with a velocity of $100\,cm/\sec$, strikes the bob of a pendulum and strikes to it. The mass of the bob is also $10\,gm$ (see fig.) The maximum height to which the system can be raised is ($g = 10\,m/\sec^2$)



(a) Zero (b) $5\,cm$

(c) $2.5cm$ (d) $1.25cm$

Solution : (d) By the conservation of momentum,

Momentum of the bullet = Momentum of system $\Rightarrow 10\times 1=(10+10)\times v \Rightarrow v=\frac{1}{2}m/s$

Now maximum height reached by system $H_{max}=\frac{v^2}{2g}=\frac{(1/2)^2}{2\times 10}m=1.25cm$.

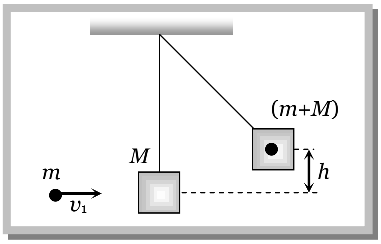
***Problem* 118.** A bullet of mass m moving with a velocity v strikes a suspended wooden block of mass M as shown in the figure and sticks to it. If the block rises to a height h the initial velocity of the bullet is **[MP PMT 1997]**

(a) $\frac{m+M}{m}\sqrt{2gh}$

(b) $\sqrt{2gh}$

(c) $\frac{M+m}{M}\sqrt{2gh}$

(d) $\frac{m}{M+m}\sqrt{2gh}$



Solution : (a) By the conservation of momentum $mv=(m+M)V$

and if the system goes upto height h then $V=\sqrt{2gh}$

$\therefore\ mv=(m+M)\sqrt{2gh} \Rightarrow v=\frac{m+M}{m}\sqrt{2gh}$.

***Problem* 119.** A bag P (mass M) hangs by a long thread and a bullet (mass m) comes horizontally with velocity v and gets caught in the bag. Then for the combined (beg + bullet) system the

(a) Momentum is $\frac{mvM}{M+m}$ (b) Kinetic energy $\frac{mV^2}{2}$

(c) Momentum is $\frac{mv(M+m)}{M}$ (d) Kinetic energy is $\frac{m^2V^2}{2(M+m)}$

Solution : (d) Velocity of combined system $V=\frac{mv}{m+M}$

Momentum for combined system $=(m+M)V=(m+M)\frac{mv}{m+M}$

Kinetic energy for combined system $=\frac{1}{2}(m+M)V^2=\frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2=\frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2}=\frac{m^2v^2}{2(m+M)}$.

***Problem* 120.** A wooden block of mass M is suspended by a cord and is at rest. A bullet of mass m, moving with a velocity v pierces through the block and comes out with a velocity $v/2$ in the same direction. If there is no loss in kinetic energy, then upto what height the block will rise

(a) $m^2v^2/2M^2g$ (b) $m^2v^2/8M^2g$ (c) $m^2v^2/4Mg$ (d) $m^2v^2/2Mg$

Solution : (b) By the conservation of momentum

Initial momentum = Final momentum

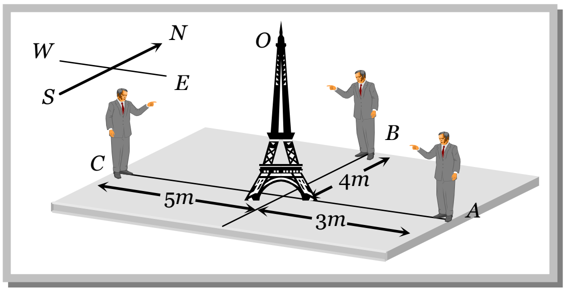
$mv+M\times 0=m\frac{v}{2}+M\times V \Rightarrow V=\frac{m}{2M}v$

If block rises upto height h then $h = \frac{V^2}{2g} = \frac{(mv/2M)^2}{2g} = \frac{m^2v^2}{8M^2g}$.

Motion In One Dimension

2.1 Position

Any object is situated at point O and three observers from three different places are looking for same object, then all three observers will have different observations about the position of point O and no one will be wrong. Because they are observing the object from their different positions.



Observer 'A' says : Point O is 3 m away in west direction.

Observer 'B' says : Point O is 4 m away in south direction.

Observer 'C' says : Point O is 5 m away in east direction.

Therefore position of any point is completely expressed by two factors: Its distance from the observer and its direction with respect to observer.

That is why position is characterised by a vector known as position vector.

Let point P is in a xy plane and its coordinates are (x, y). Then position vector $(\vec{r})$ of point will be $x\hat{i} + y\hat{j}$ and if the point P is in a space and its coordinates are (x, y, z) then position vector can be expressed as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

2.2 Rest and Motion

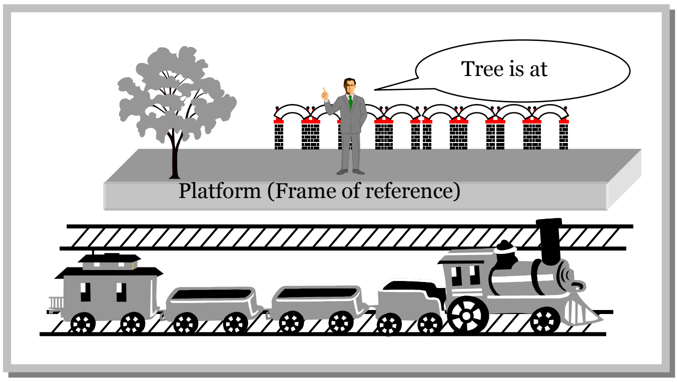
If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

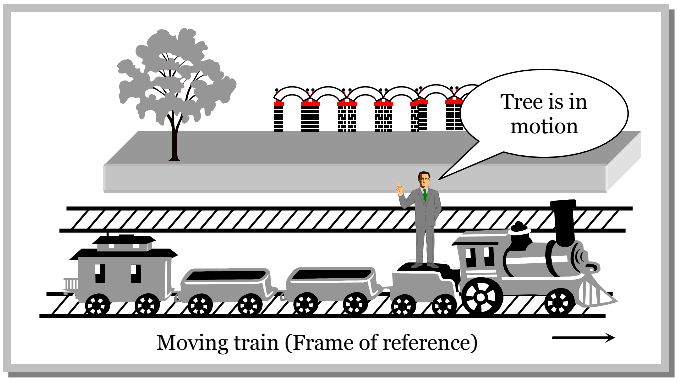
And if a body changes its position as time passes with respect to frame of reference, it is said to be in motion.

Frame of Reference : It is a system to which a set of coordinates are attached and with reference to which observer describes any event.

A passenger standing on platform observes that tree on a platform is at rest. But when the same passenger is passing away in a train through station, observes that tree is in motion. In both conditions observer is right. But observations are different because in first situation observer stands on a platform, which is reference frame at rest and in second situation observer moving in train, which is reference frame in motion.

So rest and motion are relative terms. It depends upon the frame of references.





2.3 Types of Motion

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
e.g.. Motion of car on a straight road. Motion of freely falling body.	*e.g.* Motion of car on a circular turn. Motion of billiards ball.	*e.g.*. Motion of flying kite. Motion of flying insect.

2.4 Particle or Point Mass

The smallest part of matter with zero dimension which can be described by its mass and position is defined as a particle.

If the size of a body is negligible in comparison to its range of motion then that body is known as a particle.

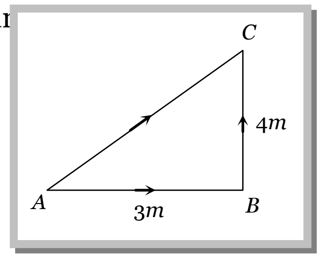
A body (Group of particles) to be known as a particle depends upon types of motion. For example in a planetary motion around the sun the different planets can be presumed to be the particles.

In above consideration when we treat body as particle, all parts of the body undergo same displacement and have same velocity and acceleration.

2.5 Distance and Displacement

(1) **Distance :** It is the actual path length covered by a moving particle in a given interval of time.

(i) If a particle starts from A and reach to C through point B as shown in the figur



Then distance travelled by particle $= AB + BC = 7\ m$

(ii) Distance is a scalar quantity.

(iii) Dimension : $[M^0L^1T^0]$

(iv) Unit : *metre* (S.I.)

(2) **Displacement :** Displacement is the change in position vector *i.e.*, A vector joining initial to final position.

(i) Displacement is a vector quantity

(ii) Dimension : $[M^0L^1T^0]$

(iii) Unit : *metre* (S.I.)

(iv) In the above figure the displacement of the particle $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$$\Rightarrow |AC| = \sqrt{(AB)^2 + (BC)^2 + 2(AB)(BC)\cos 90^o} = 5m$$

(v) If $\vec{S}_1, \vec{S}_2, \vec{S}_3 .... \vec{S}_n$ are the displacements of a body then the total (net) displacement is the vector sum of the individuals. $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + ... + \vec{S}_n$

(3) **Comparison between distance and displacement :**

(i) The magnitude of displacement is equal to minimum possible distance between two positions.

So distance ≥ |Displacement|.

(ii) For a moving particle distance can never be negative or zero while displacement can be.

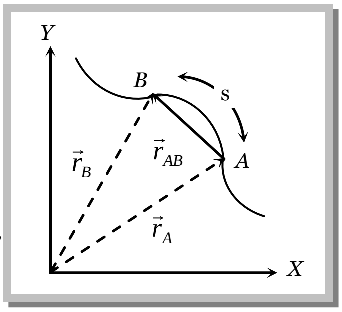
(zero displacement means that body after motion has came back to initial position)

i.e., Distance > 0 but Displacement > = or < 0

(iii) For motion between two points displacement is single valued while distance depends on actual path and so can have many values.

(iv) For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.

(v) In general magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.



(vi) If $\vec{r}_A$ and $\vec{r}_B$ are the position vectors of particle initially and finally.

Then displacement of the particle

$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

and s is the distance travelled if the particle has gone through the path *APB*

Sample problems based on distance and displacement

Problem 1. A man goes 10*m* towards North, then 20*m* towards east then displacement is

[KCET (Med.) 1999; JIPMER 1999; AFMC 2003]

(a) 22.5*m* (b) 25*m* (c) 25.5*m* (d) 30*m*

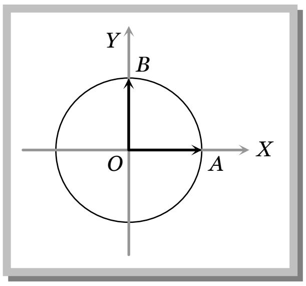
Solution : (a) If we take east as x – axis and north as y – axis, then displacement $= 20\hat{i} + 10\hat{j}$

So, magnitude of displacement $= \sqrt{20^2 + 10^2} = 10\sqrt{5} = 22.5\ m.$

Problem 2. A body moves over one fourth of a circular arc in a circle of radius *r*. The magnitude of distance travelled and displacement will be respectively

(a) $\frac{\pi r}{2}, r\sqrt{2}$ (b) $\frac{\pi r}{4}, r$ (c) $\pi r, \frac{r}{\sqrt{2}}$ (d) $\pi r, r$

Solution : (a) Let particle start from *A*, its position vector $\vec{r}_{OA} = r\hat{i}$



After one quarter position vector $\vec{r}_{OB} = r\hat{j}$.

So displacement $= r\hat{j} - r\hat{i}$

Magnitude of displacement $= r\sqrt{2}$.

and distance = one fourth of circumference $= \frac{2\pi r}{4} = \frac{\pi r}{2}$

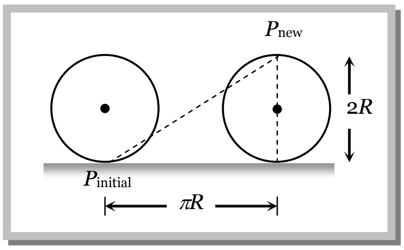
Problem 3. The displacement of the point of the wheel initially in contact with the ground, when the wheel roles forward half a revolution will be (radius of the wheel is *R*)

(a) $\frac{R}{\sqrt{\pi^2 + 4}}$ (b) $R\sqrt{\pi^2 + 4}$ (c) $2\pi R$ (d) πR

Solution : (b) Horizontal distance covered by the wheel in half revolution = πR

So the displacement of the point which was initially in contact with a ground = $\sqrt{(\pi R)^2 + (2R)^2}$

$= R\sqrt{\pi^2 + 4}$.



2.6 Speed and Velocity

(1) **Speed :** Rate of distance covered with time is called speed.

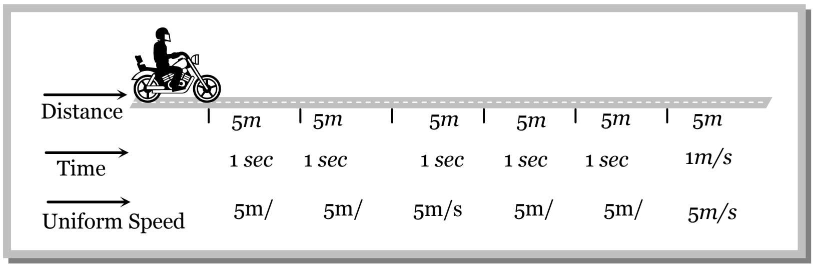
(i) It is a scalar quantity having symbol υ.

(ii) Dimension : $[M^0L^1T^{-1}]$

(iii) Unit : *metre/second* (S.I.), *cm/second* (C.G.S.)

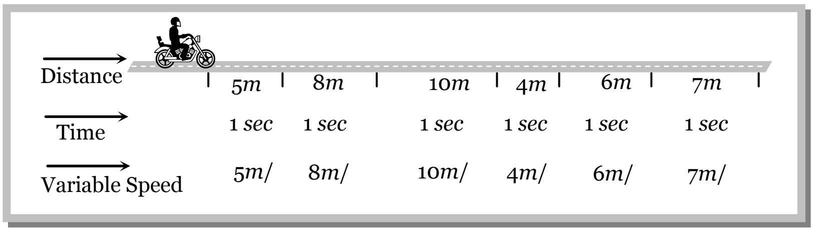
(iv) Types of speed :

(a) **Uniform speed :** When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In given illustration motorcyclist travels equal distance (= 5*m*) in each second. So we can say that particle is moving with uniform speed of 5 *m/s*.



(b) **Non-uniform (variable) speed :** In non-uniform speed particle covers unequal distances in equal intervals of time. In the given illustration motorcyclist travels 5*m* in 1st second, 8*m* in 2nd second, 10*m* in 3rd second, 4*m* in 4th second *etc.*

Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.



(c) **Average speed :** The average speed of a particle for a given 'Interval of time' is defined as the ratio of distance travelled to the time taken.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}} \; ; \; v_{av} = \frac{\Delta s}{\Delta t}$$

❑ *Time average speed* : When particle moves with different uniform speed υ_1, υ_2, υ_3 ... *etc* in different time intervals t_1, t_2, t_3, ... *etc* respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{\upsilon_1 t_1 + \upsilon_2 t_2 + \upsilon_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

Special case : When particle moves with speed υ_1 upto half time of its total motion and in rest time it is moving with speed υ_2 then $v_{av} = \frac{v_1 + v_2}{2}$

❑ *Distance averaged speed* : When a particle describes different distances $d_1, d_2, d_3, ...$ with different time intervals $t_1, t_2, t_3, ...$ with speeds $v_1, v_2, v_3 ......$ respectively then the speed of particle averaged over the total distance can be given as

$$\upsilon_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{d_1 + d_2 + d_3 + ...}{\frac{d_1}{\upsilon_1} + \frac{d_2}{\upsilon_2} + \frac{d_3}{\upsilon_3} + ...}$$

❑ When particle moves the first half of a distance at a speed of υ_1 and second half of the distance at speed υ_2 then

$$v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

❑ When particle covers one-third distance at speed υ_1, next one third at speed υ_2 and last one third at speed υ_3, then

$$v_{av} = \frac{3v_1 v_2 v_3}{v_1 v_2 + v_2 v_3 + v_3 v_1}$$

(d) **Instantaneous speed :** It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (*i.e.*, $\Delta t \to 0$). Thus

Instantaneous speed $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

(2) **Velocity :** Rate of change of position *i.e.* rate of displacement with time is called velocity.

(i) It is a scalar quantity having symbol v .

(ii) Dimension : $[M^0 L^1 T^{-1}]$

(iii) Unit : *metre/second* (S.I.), *cm/second* (C.G.S.)

(iv) Types

(a) **Uniform velocity :** A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

(b) **Non-uniform velocity :** A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

(c) **Average velocity** : It is defined as the ratio of displacement to time taken by the body

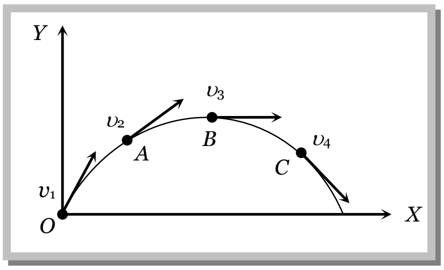
$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}} \; ; \quad \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

(d) **Instantaneous velocity :** Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

Instantaneous velocity $\vec{v} = \lim_{t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

(v) **Comparison between instantaneous speed and instantaneous velocity**

(a) instantaneous velocity is always tangential to the path followed by the particle.



When a stone is thrown from point O then at point of projection the instantaneous velocity of stone is v_1, at point A the instantaneous velocity of stone is v_2, similarly at point B and C are v_3 and v_4 respectively.

Direction of these velocities can be found out by drawing a tangent on the trajectory at a given point.

(b) A particle may have constant instantaneous speed but variable instantaneous velocity.

Example : When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

(c) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(d) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(e) If displacement is given as a function of time, then time derivative of displacement will give velocity.

Let displacement $\vec{x} = A_0 - A_1 t + A_2 t^2$

Instantaneous velocity $\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}(A_0 - A_1 t + A_2 t^2)$

$$\vec{v} = -A_1 + 2A_2 t$$

For the given value of t, we can find out the instantaneous velocity.

e.g. for $t = 0$,Instantaneous velocity $\vec{v} = -A_1$ and Instantaneous speed $|\vec{v}| = A_1$

(vi) **Comparison between average speed and average velocity**

(a) Average speed is scalar while average velocity is a vector both having same units (m/s) and dimensions $[LT^{-1}]$.

(b) Average speed or velocity depends on time interval over which it is defined.

(c) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

(d) If after motion body comes back to its initial position then $\vec{v}_{av} = \vec{0}$ (as $\Delta\vec{r} = 0$) but $v_{av} > \vec{0}$ and finite as $(\Delta s > 0)$.

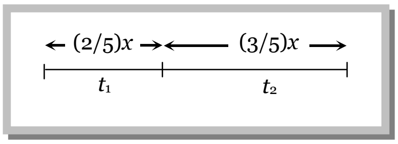
(e) For a moving body average speed can never be negative or zero (unless $t \to \infty$) while average velocity can be *i.e.* $v_{av} > 0$ while $\vec{v}_{av}$ = or < 0.

Sample problems based on speed and velocity

Problem 4. If a car covers $2/5^{th}$ of the total distance with v_1 speed and $3/5^{th}$ distance with v_2 then average speed is **[MP PMT 2003]**

(a) $\frac{1}{2}\sqrt{v_1 v_2}$ (b) $\frac{v_1 + v_2}{2}$ (c) $\frac{2v_1 v_2}{v_1 + v_2}$ (d) $\frac{5v_1 v_2}{3v_1 + 2v_2}$

Solution : (d) Average speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{x}{t_1 + t_2}$



$$= \frac{x}{\frac{(2/5)x}{v_1} + \frac{(3/5)x}{v_2}} = \frac{5v_1 v_2}{2v_2 + 3v_1}$$

Problem 5. A car accelerated from initial position and then returned at initial point, then **[AIEEE 2002]**

(a) Velocity is zero but speed increases (b) Speed is zero but velocity increases

(c) Both speed and velocity increase (d) Both speed and velocity decrease

Solution : (a) As the net displacement = 0

Hence velocity = 0 ; but speed increases.

Note : ❑ $\frac{|\text{Average velocity}|}{|\text{Average speed}|} \le 1 \Rightarrow |\text{Av.speed}| \ge |\text{Av.velocity}|$

Problem 6. A man walks on a straight road from his home to a market 2.5 *km* away with a speed of 5 *km/h*. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 *km/h*. The average speed of the man over the interval of time 0 to 40 *min*. is equal to

(a) 5 *km/h* (b) $\frac{25}{4}$ *km/h* (c) $\frac{30}{4}$ *km/h* (d) $\frac{45}{8}$ *km/h*

Solution : (d) Time taken in going to market $= \frac{2.5}{5} = \frac{1}{2} hr = 30$ min.

As we are told to find average speed for the interval 40 min., so remaining time for consideration of motion is 10 min.

So distance travelled in remaining 10 min $= 7.5 \times \frac{10}{60} = 1.25 km$.

Hence, average speed $= \frac{\text{Total distance}}{\text{Total time}} = \frac{(2.5 + 1.25) km}{(40/60) hr.} = \frac{45}{8} km/hr$.

Problem 7. The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction where x is in *metres* and *t* in sec. The displacement, when velocity is zero, is

(a) 24 *metres* (b) 12 *metres* (c) 5 *metres* (d) Zero

Solution : (d) $3t = \sqrt{3x} + 6 \Rightarrow \sqrt{3x} = (3t - 6) \Rightarrow 3x = (3t - 6)^2 \Rightarrow x = 3t^2 - 12t + 12$

$$\therefore v = \frac{dx}{dt} = \frac{d}{dt}(3t^2 - 12t + 12) = 6t - 12$$

If velocity = 0 then, $6t - 12 = 0 \Rightarrow t = 2 sec$

Hence at $t = 2$, $x = 3(2)^2 - 12(2) + 12 = 0$ *metres*.

Problem **8.** The motion of a particle is described by the equation $x = a + bt^2$ where $a = 15\,cm$ and $b = 3\,cm$. Its instantaneous velocity at time 3 *sec* will be **[AMU (Med.) 2000]**

(a) 36 *cm/sec* (b) 18 *cm/sec* (c) 16 *cm/sec* (d) 32 *cm/sec*

Solution : (b) $x = a + bt^2$ $\therefore\ v = \frac{dx}{dt} = 0 + 2bt$

At $t = 3sec$, $v = 2\times3\times3 = 18cm/sec$ (As $b = 3cm$)

Problem **9.** A train has a speed of 60 *km/h* for the first one hour and 40 *km/h* for the next half hour. Its average speed in *km/h* is **[JIPMER 1999]**

(a) 50 (b) 53.33 (c) 48 (d) 70

Solution : (b) Total distance travelled $= 60\times1 + 40\times\frac{1}{2} = 80\,km$ and Total time taken $= 1\,hr + \frac{1}{2}hr = \frac{3}{2}hr$

$\therefore$ Average speed $= \frac{80}{3/2} = 53.33\ km/h$

Problem **10.** A person completes half of its his journey with speed υ_1 and rest half with speed υ_2. The average speed of the person is **[RPET 1993; MP PMT 2001]**

(a) $\upsilon = \frac{\upsilon_1+\upsilon_2}{2}$ (b) $\upsilon = \frac{2\upsilon_1\upsilon_2}{\upsilon_1+\upsilon_2}$ (c) $\upsilon = \frac{\upsilon_1\upsilon_2}{\upsilon_1+\upsilon_2}$ (d) $\upsilon = \sqrt{\upsilon_1\upsilon_2}$

Solution : (b) In this problem total distance is divided into two equal parts. So

$$\upsilon_{av} = \frac{d_1+d_2}{\frac{d_1}{\upsilon_1}+\frac{d_2}{\upsilon_2}} = \frac{\frac{d}{2}+\frac{d}{2}}{\frac{d/2}{\upsilon_1}+\frac{d/2}{\upsilon_2}} \Rightarrow \upsilon_{av} = \frac{2}{\frac{1}{\upsilon_1}+\frac{1}{\upsilon_2}} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1+\upsilon_2}$$

Problem **11.** A car moving on a straight road covers one third of the distance with 20 *km/hr* and the rest with 60 *km/hr*. The average speed is

(a) 40 *km/hr* (b) 80 *km/hr* (c) $46\frac{2}{3}\,km/hr$ (d) 36 *km/hr*

Solution : (d) Let total distance travelled = x and total time taken $t_1 + t_2 = \frac{x/3}{20} + \frac{2x/3}{60}$

$\therefore$ Average speed $= \frac{x}{\frac{(1/3)x}{20}+\frac{(2/3)x}{60}} = \frac{1}{\frac{1}{60}+\frac{2}{180}} = 36km/hr$

2.7 Acceleration

The time rate of change of velocity of an object is called acceleration of the object.

(1) It is a vector quantity. It's direction is same as that of change in velocity (Not of the velocity)

(2) There are three possible ways by which change in velocity may occur

When only direction of velocity changes	**When only magnitude of velocity changes**	**When both magnitude and direction of velocity changes**
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
e.g. Uniform circular motion	*e.g.* Motion under gravity	*e.g.* Projectile motion

(3) Dimension : $[M^0L^1T^{-2}]$

(4) Unit : *metre/second*^{2} (S.I.); *cm/second*^{2} (C.G.S.)

(5) Types of acceleration :

(i) **Uniform acceleration :** A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note : ❑ If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. *e.g.* Projectile motion.

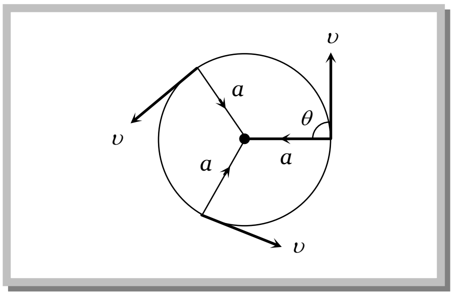
(ii) **Non-uniform acceleration :** A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

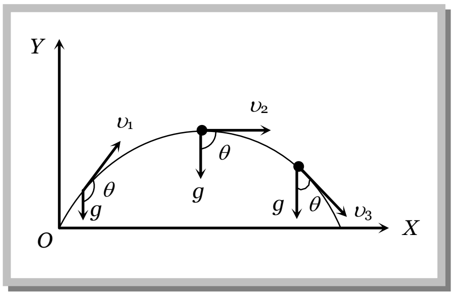
(iii) **Average acceleration :** $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta\vec{t}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

(iv) **Instantaneous acceleration** = $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

(v) For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.





e.g. (a) In uniform circular motion $\theta = 90^o$ always

(b) In a projectile motion θ is variable for every point of trajectory.

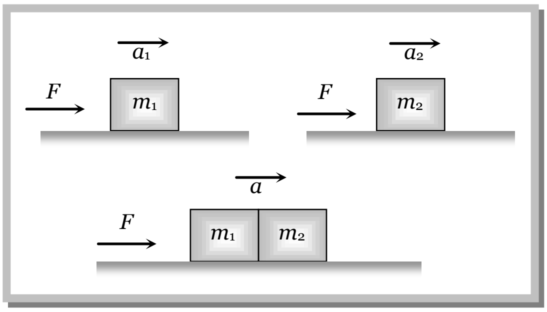
(vi) If a force $\vec{F}$ acts on a particle of mass *m*, by Newton's 2nd law, acceleration $\vec{a} = \frac{\vec{F}}{m}$

(vii) By definition $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$ $\left[\text{As } \vec{v} = \frac{d\vec{x}}{dt}\right]$

i.e., if *x* is given as a function of time, second time derivative of displacement gives acceleration

(viii) If velocity is given as a function of position, then by chain rule $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v.\frac{d\upsilon}{dx}$ $\left[\text{as } v = \frac{dx}{dt}\right]$

(ix) If a particle is accelerated for a time t_1 by acceleration a_1 and for time t_2 by acceleration a_2 then average acceleration is $a_{av} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$



(x) If same force is applied on two bodies of different masses m_1 and m_2 separately then it produces accelerations a_1 and a_2 respectively. Now these bodies are attached together and form a combined system and same force is applied on that system so that a be the acceleration of the combined system, then

$$F = (m_1 + m_2)a \Rightarrow \frac{F}{a} = \frac{F}{a_1} + \frac{F}{a_2}$$

So, $\frac{1}{a} = \frac{1}{a_1} + \frac{1}{a_2} \Rightarrow a = \frac{a_1 a_2}{a_1 + a_2}$

(xi) Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.

(xii) For motion of a body under gravity, acceleration will be equal to "g", where g is the acceleration due to gravity. Its normal value is $9.8\ m/s^2$ or $980\ cm/s^2$ or $32\ feet/s^2$.

Sample problems based on acceleration

***Problem* 12.** The displacement of a particle, moving in a straight line, is given by $s = 2t^2 + 2t + 4$ where s is in *metres* and t in seconds. The acceleration of the particle is **[CPMT 2001]**

(a) 2 m/s^2 (b) 4 m/s^2 (c) 6 m/s^2 (d) 8 m/s^2

Solution : (b) Given $s = 2t^2 + 2t + 4$ $\therefore$ velocity $(v) = \frac{ds}{dt} = 4t + 2$ and acceleration $(a) = \frac{dv}{dt} = 4(1) + 0 = 4 m/s^2$

***Problem* 13.** The position x of a particle varies with time t as $x = at^2 - bt^3$. The acceleration of the particle will be zero at time t equal to **[CBSE PMT 1997; BHU 1999; DPMT 2000; KCET (Med.) 2000]**

(a) $\frac{a}{b}$ (b) $\frac{2a}{3b}$ (c) $\frac{a}{3b}$ (d) Zero

Solution : (c) Given $x = at^2 - bt^3$ $\therefore$ velocity $(v) = \frac{dx}{dt} = 2at - 3bt^2$ and acceleration $(a) = \frac{dv}{dt} = 2a - 6bt$.

When acceleration = 0 $\Rightarrow 2a - 6bt = 0 \Rightarrow t = \frac{2a}{6b} = \frac{a}{3b}$.

***Problem* 14.** The displacement of the particle is given by $y = a + bt + ct^2 - dt^4$. The initial velocity and acceleration are respectively **[CPMT 1999, 2003]**

(a) $b, -4d$ (b) $-b, 2c$ (c) $b, 2c$ (d) $2c, -4d$

Solution : (c) Given $y = a + bt + ct^2 - dt^4$ $\therefore v = \frac{dy}{dt} = 0 + b + 2ct - 4dt^3$

Putting $t = 0$, $v_{initial} = b$

So initial velocity = b

Now, acceleration $(a) = \frac{dv}{dt} = 0 + 2c - 12\,d\,t^2$

Putting $t = 0$, $a_{initial} = 2c$

***Problem* 15.** The relation between time t and distance x is $t = \alpha x^2 + \beta x$, where α and β are constants. The retardation is (v is the velocity) **[NCERT 1982]**

(a) $2\alpha v^3$ (b) $2\beta v^3$ (c) $2\alpha\beta v^3$ (d) $2\beta^2 v^3$

Solution : (a) differentiating time with respect to distance $\frac{dt}{dx} = 2\alpha x + \beta \Rightarrow v = \frac{dx}{dt} = \frac{1}{2\alpha x + \beta}$

So, acceleration $(a) = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx} = \frac{-v.2\alpha}{(2\alpha x + \beta)^2} = -2\alpha.v.v^2 = -2\alpha v^3$

Problem 16. If displacement of a particle is directly proportional to the square of time. Then particle is moving with **[RPET 1999]**

(a) Uniform acceleration (b) Variable acceleration

(c) Uniform velocity (d) Variable acceleration but uniform velocity

Solution : (a) Given that $x \propto t^2$ or $x = Kt^2$ (where K= constant)

Velocity $(\upsilon) = \frac{dx}{dt} = 2Kt$ and Acceleration $(a) = \frac{d\upsilon}{dt} = 2K$

It is clear that velocity is time dependent and acceleration does not depend on time.

So we can say that particle is moving with uniform acceleration but variable velocity.

Problem 17. A particle is moving eastwards with velocity of 5 *m/s*. In 10 *sec* the velocity changes to 5 *m/s* northwards. The average acceleration in this time is **[IIT-JEE 1982]**

(a) Zero (b) $\frac{1}{\sqrt{2}}$ m/s^2 toward north-west

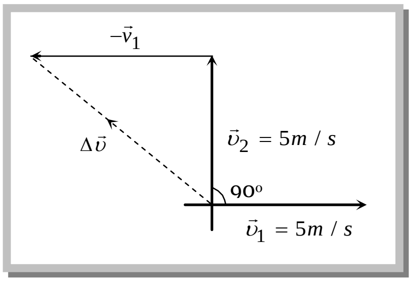
(c) $\frac{1}{\sqrt{2}}$ m/s^2 toward north-east (d) $\frac{1}{2}$ m/s^2 toward north-west

Solution : (b) $\Delta\vec{\upsilon} = \vec{\upsilon}_2 - \vec{\upsilon}_1$

$\Delta\upsilon = \sqrt{\upsilon_1^2 + \upsilon_2^2 - 2\upsilon_1\upsilon_2 \cos 90^o} = \sqrt{5^2 + 5^2} = 5\sqrt{2}$

$\Delta\upsilon = 5\sqrt{2}$

Average acceleration $= \frac{\Delta\upsilon}{\Delta t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s^2 toward north-west (As clear from the figure).



Problem 18. A body starts from the origin and moves along the *x*-axis such that velocity at any instant is given by $(4t^3 - 2t)$, where *t* is in second and velocity is in *m/s*. What is the acceleration of the particle, when it is 2*m* from the origin?

(a) 28 m/s^2 (b) 22 m/s^2 (c) 12 m/s^2 (d) 10 m/s^2

Solution : (b) Given that $\upsilon = 4t^3 - 2t$

$x = \int \upsilon\, dt$, $x = t^4 - t^2 + C$, at $t = 0, x = 0 \Rightarrow C = 0$

When particle is 2*m* away from the origin

$2 = t^4 - t^2 \Rightarrow t^4 - t^2 - 2 = 0 \Rightarrow (t^2 - 2)(t^2 + 1) = 0 \Rightarrow t = \sqrt{2}\ sec$

$a = \frac{d\upsilon}{dt} = \frac{d}{dt}\left(4t^3 - 2t\right) = 12t^2 - 2 \Rightarrow a = 12t^2 - 2$

for $t = \sqrt{2}\ sec \Rightarrow a = 12 \times \left(\sqrt{2}\right)^2 - 2 \Rightarrow a = 22\ m/s^2$

***Problem* 19.** A body of mass 10 *kg* is moving with a constant velocity of 10 *m/s*. When a constant force acts for 4 *sec* on it, it moves with a velocity 2 *m/sec* in the opposite direction. The acceleration produced in it is **[MP PET 1997]**

(a) $3\ m/s^2$ (b) $-3\ m/s^2$ (c) $0.3\ m/s^2$ (d) $-0.3\ m/s^2$

Solution : (b) Let particle moves towards east and by the application of constant force it moves towards west

$\vec{v}_1 = +10\ m/s$ and $\vec{v}_2 = -2\ m/s$. Acceleration $= \dfrac{\text{Change in velocity}}{\text{Time}} = \dfrac{\vec{v}_2 - \vec{v}_1}{t}$

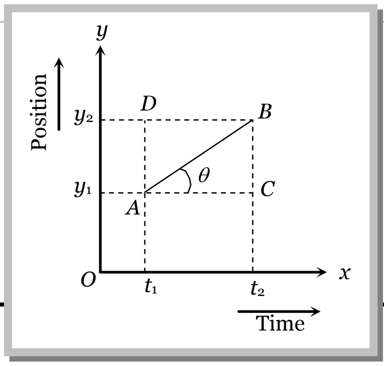
$$\Rightarrow a = \frac{(-2)-(10)}{4} = \frac{-12}{4} = -3\ m/s^2$$

2.8 Position Time Graph

During motion of the particle its parameters of kinematical analysis (u, v, a, r) changes with time. This can be represented on the graph.

Position time graph is plotted by taking time t along x-axis and position of the particle on y-axis.

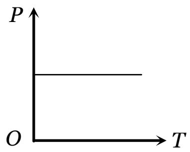
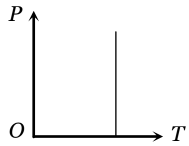
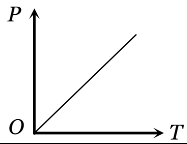
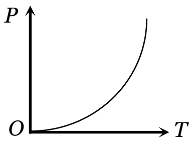
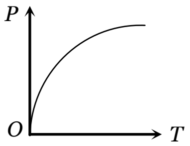
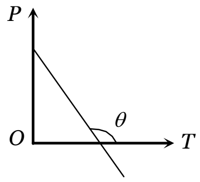
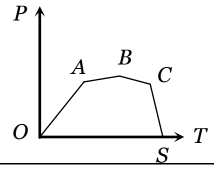
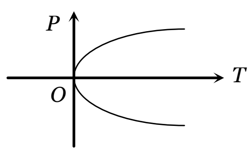
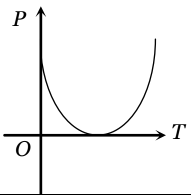
Let AB is a position-time graph for any moving particle



As Velocity = $\dfrac{\text{Change in position}}{\text{Time taken}} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ...(i)

From triangle ABC $\tan\theta = \dfrac{BC}{AC} = \dfrac{AD}{AC} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ....(ii)

By comparing (i) and (ii) Velocity = $\tan\theta$

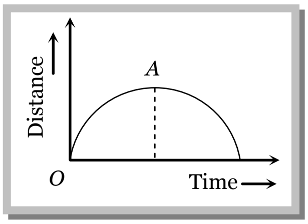
	$\theta = 0^\circ$ so $v = 0$ *i.e.*, line parallel to time axis represents that the particle is at rest.
	$\theta = 90^\circ$ so $v = \infty$ *i.e.*, line perpendicular to time axis represents that particle is changing its position but time does not changes it means the particle possesses infinite velocity. Practically this is not possible.
	θ = constant so v = constant, $a = 0$ *i.e.*, line with constant slope represents uniform velocity of the particle.
	θ is increasing so v is increasing, a is positive. *i.e.*, line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.
	θ is decreasing so v is decreasing, a is negative *i.e.*, line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.
	θ constant but > 90° so v will be constant but negative *i.e.*, line with negative slope represent that particle returns towards the point of reference. (negative displacement).
	Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.
	This graph shows that at one instant the particle has two positions. Which is not possible.
	The graph shows that particle coming towards origin initially and after that it is moving away from origin.

$$v = \tan\theta$$

It is clear that slope of position-time graph represents the velocity of the particle.

Various position – time graphs and their interpretation

Note : ❑ If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decrease with time. Hence such type of distance time graph is valid up to point A only, after point A it is not valid as shown in the figure.



❑ For two particles having displacement time graph with slopes θ_1 and θ_2 possesses velocities v_1 and v_2 respectively then $\frac{v_1}{v_2} = \frac{\tan\theta_1}{\tan\theta_2}$

Sample problems based on position-time graph

***Problem* 20.** The position of a particle moving along the x-axis at certain times is given below :

$t\,(s)$	0	1	2	3
$x\,(m)$	– 2	0	6	16

Which of the following describes the motion correctly

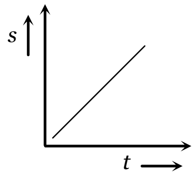
(a) Uniform, accelerated (b) Uniform, decelerated

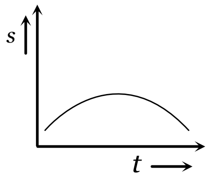
(c) Non-uniform, accelerated (d) There is not enough data for generalisation

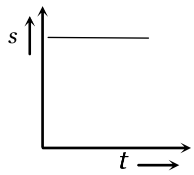
Solution : (a) Instantaneous velocity $v = \frac{\Delta x}{\Delta t}$, By using the data from the table

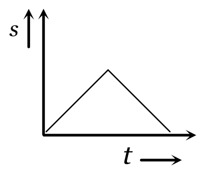
$v_1 = \frac{0-(-2)}{1} = 2m/s$, $v_2 = \frac{6-0}{1} = 6m/s$ and $v_3 = \frac{16-6}{1} = 10\,m/s$ *i.e.* the speed is increasing at a constant rate so motion is uniformly accelerated.

***Problem* 21.** Which of the following graph represents uniform motion

(a) 

(b) 

(c) 

(d) 

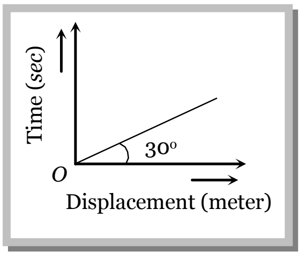
Solution : (a) When distance time graph is a straight line with constant slope than motion is uniform.

***Problem* 22.** The displacement-time graph for two particles A and B are straight lines inclined at angles of 30° and 60° with the time axis. The ratio of velocities of $v_A : v_B$ is **[CPMT 1990; MP PET 1999; MP PET 2001]**

(a) 1 : 2 (b) $1 : \sqrt{3}$ (c) $\sqrt{3} : 1$ (d) 1 : 3

Solution : (d) $v = \tan\theta$ from displacement graph. So $\frac{v_A}{v_B} = \frac{\tan 30^o}{\tan 60^o} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{\sqrt{3}\times\sqrt{3}} = \frac{1}{3}$

***Problem* 23.** From the following displacement time graph find out the velocity of a moving body



(a) $\frac{1}{\sqrt{3}}\ m/s$

(b) $3\ m/s$

(c) $\sqrt{3}\ m/s$

(d) $\frac{1}{3}$

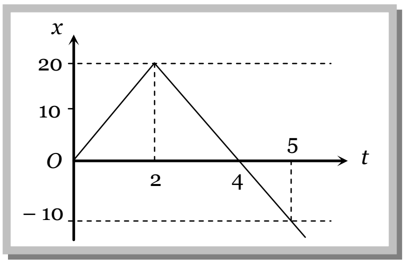
Solution : (c) In first instant you will apply $\upsilon = \tan\theta$ and say, $\upsilon = \tan 30^o = \frac{1}{\sqrt{3}}\ m/s$.

But it is wrong because formula $\upsilon = \tan\theta$ is valid when angle is measured with time axis.

Here angle is taken from displacement axis. So angle from time axis $= 90^o - 30^o = 60^o$.

Now $\upsilon = \tan 60^o = \sqrt{3}$

***Problem* 24.** The diagram shows the displacement-time graph for a particle moving in a straight line. The average velocity for the interval $t = 0, t = 5$ is



(a) 0

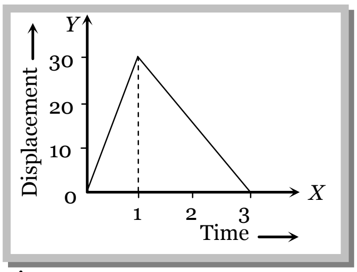
(b) 6 ms^{-1}

(c) – 2 ms^{-1}

(d) 2 ms^{-1}

Solution : (c) Average velocity = $\frac{\text{Total displacement}}{\text{Total time}} = \frac{(20)+(-20)+(-10)}{5} = -2\ m/s$

***Problem* 25.** Figure shows the displacement time graph of a body. What is the ratio of the speed in the first second and that in the next two seconds



(a) 1 : 2

(b) 1 : 3

(c) 3 : 1

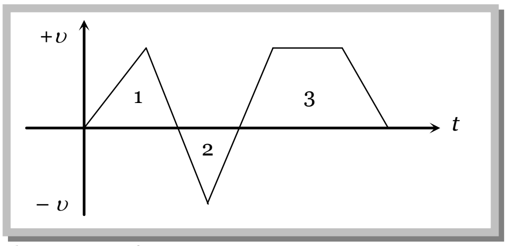
(d) 2 : 1

Solution: (d) Speed in first second = 30 and Speed in next two seconds = 15. So that ratio 2 : 1

2.9 Velocity Time Graph

The graph is plotted by taking time t along x-axis and velocity of the particle on y-axis.

Distance and displacement : The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.



Then Total distance $= |A_1| + |A_2| + |A_3|$

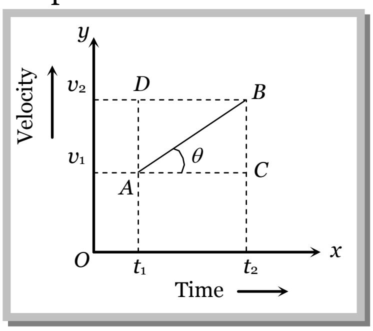
= Addition of modulus of different area. *i.e.* $s = \int |\upsilon|\, dt$

Total displacement $= A_1 + A_2 + A_3$

= Addition of different area considering their sign. *i.e.* $r = \int \upsilon\, dt$

here A_1 and A_2 are area of triangle 1 and 2 respectively and A_3 is the area of trapezium .

Acceleration : Let AB is a velocity-time graph for any moving particle



As Acceleration = $\frac{\text{Change in velocity}}{\text{Time taken}} = \frac{v_2 - v_1}{t_2 - t_1}$ ...(i)

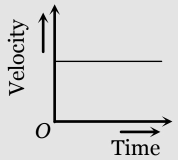
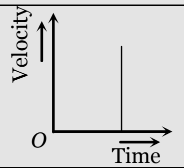
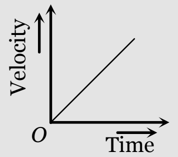
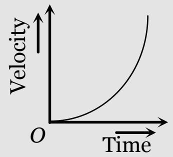
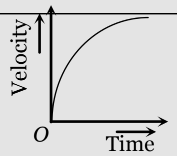
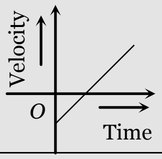
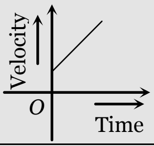
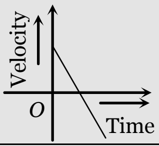
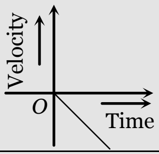
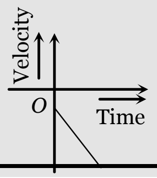
From triangle ABC, $\tan\theta = \frac{BC}{AC} = \frac{AD}{AC} = \frac{v_2 - v_1}{t_2 - t_1}$ ....(ii)

By comparing (i) and (ii)

Acceleration $(a) = \tan\theta$

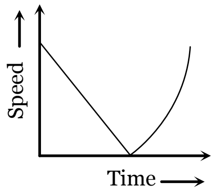
It is clear that slope of velocity-time graph represents the acceleration of the particle.

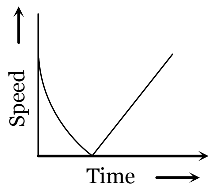
Various velocity – time graphs and their interpretation

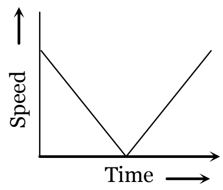
	$\theta = 0$, $a = 0$, v = constant *i.e.*, line parallel to time axis represents that the particle is moving with constant velocity.
	$\theta = 90^o$, $a = \infty$, v = increasing *i.e.*, line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible.
	θ=constant, so a = constant and v is increasing uniformly with time *i.e.*, line with constant slope represents uniform acceleration of the particle.
	θ increasing so acceleration increasing *i.e.*, line bending towards velocity axis represent the increasing acceleration in the body.
	θ decreasing so acceleration decreasing *i.e.* line bending towards time axis represents the decreasing acceleration in the body
	Positive constant acceleration because θ is constant and < 90^o but initial velocity of the particle is negative.
	Positive constant acceleration because θ is constant and < 90^o but initial velocity of particle is positive.
	Negative constant acceleration because θ is constant and > 90^o but initial velocity of the particle is positive.
	Negative constant acceleration because θ is constant and > 90^o but initial velocity of the particle is zero.
	Negative constant acceleration because θ is constant and > 90^o but initial velocity of the particle is negative.

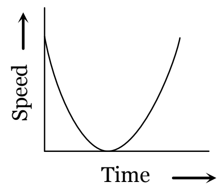
Sample problems based on velocity-time graph

Problem 26. A ball is thrown vertically upwards. Which of the following plots represents the speed-time graph of the ball during its flight if the air resistance is not ignored

(a) 

(b) 

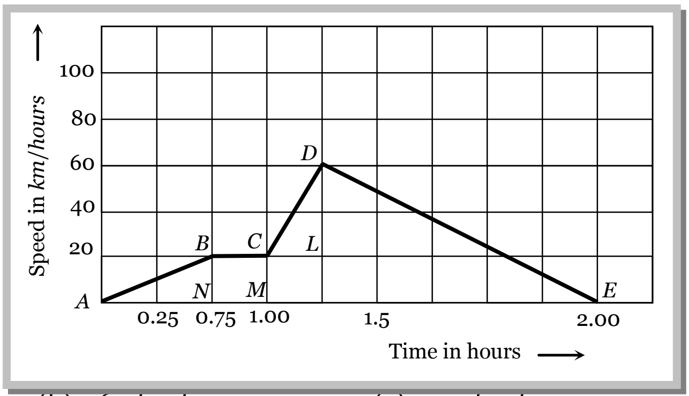
(c) 

(d) 

Solution : (c) In first half of motion the acceleration is uniform & velocity gradually decreases, so slope will be negative but for next half acceleration is positive. So slope will be positive. Thus graph '*C*' is correct.

Not ignoring air resistance means upward motion will have acceleration ($a + g$) and the downward motion will have ($g - a$).

Problem 27. A train moves from one station to another in 2 hours time. Its speed-time graph during this motion is shown in the figure. The maximum acceleration during the journey is **[Kerala (Engg.) 2002]**

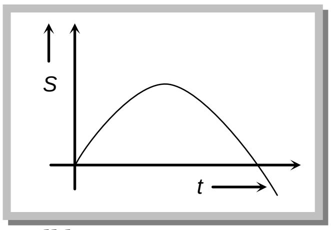


(a) 140 $km\ h^{-2}$ (b) 160 $km\ h^{-2}$ (c) 100 $km\ h^{-2}$ (d) 120 $km\ h^{-2}$

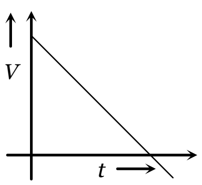
Solution : (b) Maximum acceleration means maximum slope in speed – time graph.

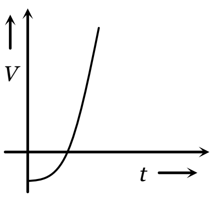
that slope is for line *CD*. So, a_{max} = slope of $CD = \frac{60-20}{1.25-1.00} = \frac{40}{0.25} = 160 km\ h^{-2}$.

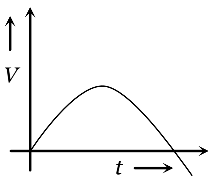
Problem 28. The graph of displacement *v/s* time is

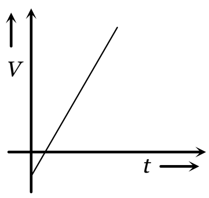


Its corresponding velocity-time graph will be **[DCE 2001]**

(a) 

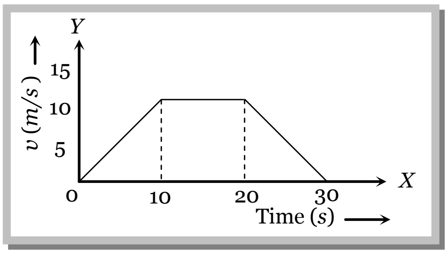
(b) 

(c) 

(d) 

Solution : (a) We know that the velocity of body is given by the slope of displacement – time graph. So it is clear that initially slope of the graph is positive and after some time it becomes zero (corresponding to the peak of the graph) and then it will be negative.

Problem 29. In the following graph, distance travelled by the body in *metres* is **[EAMCET 1994]**



(a) 200

(b) 250

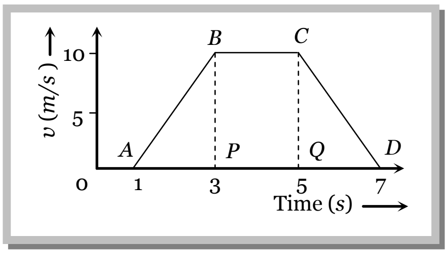
(c) 300

(d) 400

Solution : (a) Distance = The area under $v-t$ graph

$$S=\frac{1}{2}(30+10)\times 10 = 200 \text{ metre}$$

***Problem* 30.** For the velocity-time graph shown in figure below the distance covered by the body in last two seconds of its motion is what fraction of the total distance covered by it in all the seven seconds **[MP PMT/PET 1998; RPET 2001]**



(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

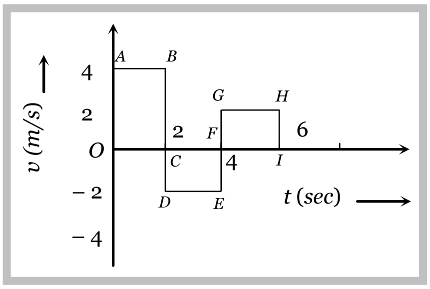
(d) $\frac{2}{3}$

Solution : (b) Distance covered in total 7 seconds = Area of trapezium $ABCD = \frac{1}{2}(2+6)\times 10 = 40\ m$

Distance covered in last 2 second = area of triangle $CDQ = \frac{1}{2}\times 2\times 10 = 10\ m$

So required fraction $= \frac{10}{40} = \frac{1}{4}$

***Problem* 31.** The velocity time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 6 *sec* are respectively



(a) $8\,m, 16\,m$

(b) $16\,m, 8\,m$

(c) $16\,m, 16\,m$

(d) $8\,m, 8\,m$

Solution : (a) Area of rectangle $ABCO = 4\times 2 = 8\ m$

Area of rectangle $CDEF = 2\times(-2) = -4\ m$

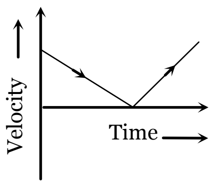
Area of rectangle $FGHI = 2\times 2 = 4\ m$

Displacement = sum of area with their sign = 8 + (– 4) + 4 = 8 *m*

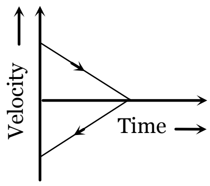
Distance = sum of area with out sign = 8 + 4 + 4 = 16 *m*

***Problem* 32.** A ball is thrown vertically upward which of the following graph represents velocity time graph of the ball during its flight (air resistance is neglected) **[CPMT 1993; AMU (Engg.) 2000]**

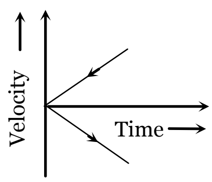
(a)



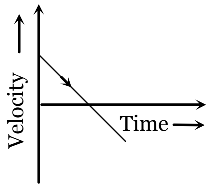
(b)



(c)



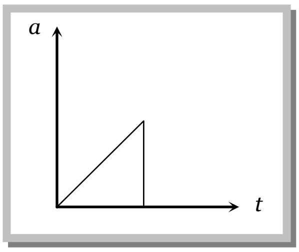
(d)



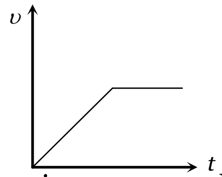
Solution : (d) In the positive region the velocity decreases linearly (during rise) and in negative region velocity increase linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region.

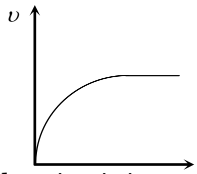
c

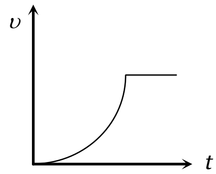
***Problem* 34.** The acceleration-time graph of a body is shown below –

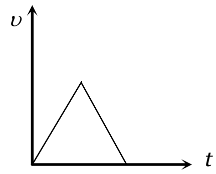


The most probable velocity-time graph of the body is

(a) 

(b) 

(c) 

(d) 

Solution : (c) From given $a-t$ graph acceleration is increasing at constant rate

$$\therefore \frac{da}{dt}=k \text{ (constant)} \Rightarrow a=kt \text{ (by integration)}$$

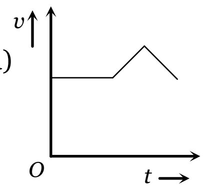
$$\Rightarrow \frac{dv}{dt}=kt \Rightarrow dv=ktdt \Rightarrow \int dv=k\int tdt \Rightarrow v=\frac{kt^2}{2}$$

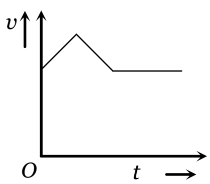
i.e., v is dependent on time parabolically and parabola is symmetric about v-axis.

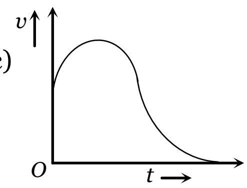
and suddenly acceleration becomes zero. i.e. velocity becomes constant.

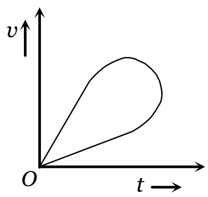
Hence (c) is most probable graph.

***Problem* 35.** Which of the following velocity time graphs is not possible

(a) 

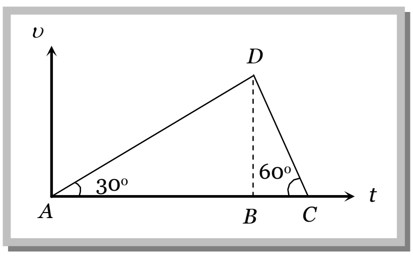
(b) 

(c) 

(d) 

Solution : (d) Particle can not possess two velocities at a single instant so graph (d) is not possible.

***Problem* 36.** For a certain body, the velocity-time graph is shown in the figure. The ratio of applied forces for intervals *AB* and *BC* is



(a) $+\frac{1}{2}$

(b) $-\frac{1}{2}$

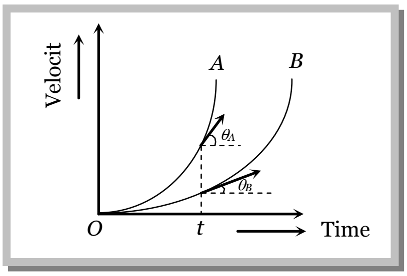
(c) $+\frac{1}{3}$

(d) $-\frac{1}{3}$

Solution : (d) Ratio of applied force = Ratio of acceleration

$$= \frac{a_{AB}}{a_{BC}} = \frac{\tan 30}{\tan(120)} = \frac{1/\sqrt{3}}{-\sqrt{3}} = -1/3$$

Problem **37.** Velocity-time graphs of two cars which start from rest at the same time, are shown in the figure. Graph shows, that



(a) Initial velocity of A is greater than the initial velocity of B

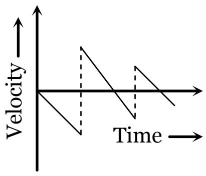
(b) Acceleration in A is increasing at lesser rate than in B

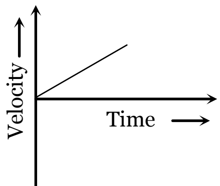
(c) Acceleration in A is greater than in B

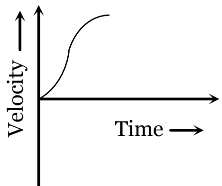
(d) Acceleration in B is greater than in A

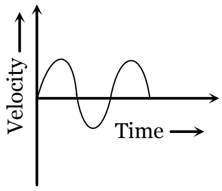
Solution : (c) At a certain instant t slope of A is greater than B ($\theta_A > \theta_B$), so acceleration in A is greater than B

Problem **38.** Which one of the following graphs represent the velocity of a steel ball which fall from a height on to a marble floor? (Here υ represents the velocity of the particle and t the time)

(a) 

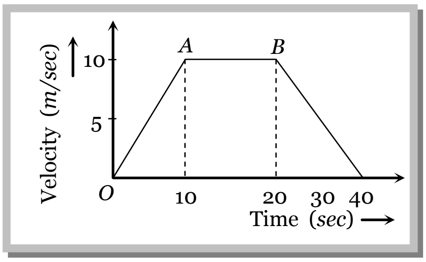
(b) 

(c) 

(d) 

Solution : (a) Initially when ball falls from a height its velocity is zero and goes on increasing when it comes down. Just after rebound from the earth its velocity decreases in magnitude and its direction gets reversed. This process is repeated untill ball comes to at rest. This interpretation is well explained in graph (a).

Problem **39.** The adjoining curve represents the velocity-time graph of a particle, its acceleration values along OA, AB and BC in $metre/sec^2$ are respectively



(a) 1, 0, – 0.5

(b) 1, 0, 0.5

(c) 1, 1, 0.5

(d) 1, 0.5, 0

Solution : (a) Acceleration along $OA = \frac{v_2 - v_1}{t} = \frac{10-0}{10} = 1 m/s^2$

Acceleration along $OB = \frac{0}{10} = 0$

Acceleration along $BC = \frac{0-10}{20} = -0.5\ m/s^2$

2.10 Equations of Kinematics

These are the various relations between u, v, a, t and s for the moving particle where the notations are used as :

u = Initial velocity of the particle at time t = 0 *sec*

v = Final velocity at time t *sec*

a = Acceleration of the particle

s = Distance travelled in time t *sec*

s_n = Distance travelled by the body in n^{th} *sec*

(1) **When particle moves with zero acceleration**

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) $v = u$, $s = u\,t$ [As a = 0]

(2) **When particle moves with constant acceleration**

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion in scalar from

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \left(\frac{u+v}{2}\right)t$$

$$s_n = u + \frac{a}{2}(2n-1)$$

Equation of motion in vector from

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$$

$$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$$

$$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

(3) **Important points for uniformly accelerated motion**

(i) If a body starts from rest and moves with uniform acceleration then distance covered by the body in t *sec* is proportional to t^2 (*i.e.* $s \propto t^2$).

So we can say that the ratio of distance covered in 1 *sec*, 2 *sec* and 3 *sec* is $1^2 : 2^2 : 3^2$ or 1 : 4 : 9.

(ii) If a body starts from rest and moves with uniform acceleration then distance covered by the body in nth sec is proportional to $(2n-1)$ (*i.e.* $s_n \propto (2n-1)$

So we can say that the ratio of distance covered in I *sec*, II *sec* and III *sec* is 1 : 3 : 5.

(iii) A body moving with a velocity u is stopped by application of brakes after covering a distance s. If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of n^2s.

As $v^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s = \dfrac{u^2}{2a}$, $s \propto u^2$ [since a is constant]

So we can say that if u becomes n times then s becomes n^2 times that of previous value.

(iv) A particle moving with uniform acceleration from A to B along a straight line has velocities v_1 and v_2 at A and B respectively. If C is the mid-point between A and B then velocity of the particle at C is equal to

$$\upsilon = \sqrt{\frac{\upsilon_1^2 + \upsilon_2^2}{2}}$$

Sample problems based on uniform acceleration

***Problem* 40.** A body A moves with a uniform acceleration a and zero initial velocity. Another body B, starts from the same point moves in the same direction with a constant velocity υ. The two bodies meet after a time t. The value of t is **[MP PET 2003]**

(a) $\frac{2\upsilon}{a}$ (b) $\frac{\upsilon}{a}$ (c) $\frac{\upsilon}{2a}$ (d) $\sqrt{\frac{\upsilon}{2a}}$

Solution : (a) Let they meet after time 't'. Distance covered by body A = $\frac{1}{2}at^2$; Distance covered by body B = vt

and $\frac{1}{2}at^2 = vt \;\therefore\; t = \frac{2v}{a}$.

***Problem* 41.** A student is standing at a distance of 50metres from the bus. As soon as the bus starts its motion with an acceleration of $1 ms^{-2}$, the student starts running towards the bus with a uniform velocity u. Assuming the motion to be along a straight road, the minimum value of u, so that the students is able to catch the bus is **[KCET 2003]**

(a) 5 ms^{-1} (b) 8 ms^{-1} (c) 10 ms^{-1} (d) 12 ms^{-1}

Solution : (c) Let student will catch the bus after t *sec*. So it will cover distance ut.

Similarly distance travelled by the bus will be $\frac{1}{2}at^2$ for the given condition

$$ut = 50 + \frac{1}{2}at^2 = 50 + \frac{t^2}{2} \Rightarrow u = \frac{50}{t} + \frac{t}{2} \quad (\text{As } a = 1\, m/s^2)$$

To find the minimum value of u, $\frac{du}{dt} = 0$, so we get t = 10 *sec*

then u = 10 m/s.

***Problem* 42.** A car, moving with a speed of 50 km/hr, can be stopped by brakes after at least 6m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is

(a) 6m (b) 12m (c) 18m (d) 24m

Solution : (d) $v^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a} \Rightarrow s \propto u^2$ (As a = constant)

$$\frac{s_2}{s_1} = \left(\frac{u_2}{u_1}\right)^2 = \left(\frac{100}{50}\right)^2 \Rightarrow s_2 = 4s_1 = 4 \times 12 = 24\, m.$$

***Problem* 43.** The velocity of a bullet is reduced from 200m/s to 100m/s while travelling through a wooden block of thickness 10cm. The retardation, assuming it to be uniform, will be **[AIIMS 2001]**

(a) 10×10^4 m/s^2 (b) 12×10^4 m/s^2 (c) 13.5×10^4 m/s^2 (d) 15×10^4 m/s^2

Solution : (d) $u = 200\, m/s$, $v = 100\, m/s$, $s = 0.1\, m$

$$a = \frac{u^2 - v^2}{2s} = \frac{(200)^2 - (100)^2}{2 \times 0.1} = 15 \times 10^4\, m/s^2$$

***Problem* 44.** A body A starts from rest with an acceleration a_1. After 2 seconds, another body *B* starts from rest with an acceleration a_2. If they travel equal distances in the 5th second, after the start of *A*, then the ratio $a_1 : a_2$ is equal to **[AIIMS 2001]**

(a) $5:9$ (b) $5:7$ (c) $9:5$ (d) $9:7$

Solution : (a) By using $S_n = u + \frac{a}{2}(2n-1)$, Distance travelled by body *A* in 5th second = $0 + \frac{a_1}{2}(2\times 5 - 1)$

Distance travelled by body B in 3rd second is = $0 + \frac{a_2}{2}(2\times 3 - 1)$

According to problem : $0 + \frac{a_1}{2}(2\times 5 - 1) = 0 + \frac{a_2}{2}(2\times 3 - 1) \Rightarrow 9a_1 = 5a_2 \Rightarrow \frac{a_1}{a_2} = \frac{5}{9}$

***Problem* 45.** The average velocity of a body moving with uniform acceleration travelling a distance of 3.06 *m* is 0.34 ms^{-1}. If the change in velocity of the body is 0.18ms^{-1} during this time, its uniform acceleration is **[EAMCET (Med.) 2000]**

(a) 0.01 ms^{-2} (b) 0.02 ms^{-2} (c) 0.03 ms^{-2} (d) 0.04 ms^{-2}

Solution : (b) $\text{Time} = \frac{\text{Distance}}{\text{Average velocity}} = \frac{3.06}{0.34} = 9\ sec$

and $\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}} = \frac{0.18}{9} = 0.02\ m/s^2$.

***Problem* 46.** A particle travels 10*m* in first 5 *sec* and 10*m* in next 3 *sec*. Assuming constant acceleration what is the distance travelled in next 2 *sec*

(a) 8.3 *m* (b) 9.3 *m* (c) 10.3 *m* (d) None of above

Solution : (a) Let initial $(t = 0)$ velocity of particle = u

for first 5 sec of motion $s_5 = 10\ metre$, so by using $s = ut + \frac{1}{2}at^2$

$10 = 5u + \frac{1}{2}a(5)^2 \Rightarrow 2u + 5a = 4$ .... (i)

for first 8 sec of motion $s_8 = 20\ metre$

$20 = 8u + \frac{1}{2}a(8)^2 \Rightarrow 2u + 8a = 5$ .... (ii)

By solving (i) and (ii) $u = \frac{7}{6}m/s \quad a = \frac{1}{3}m/s^2$

Now distance travelled by particle in total 10 *sec*. $s_{10} = u\times 10 + \frac{1}{2}a(10)^2$

by substituting the value of u and a we will get $s_{10} = 28.3\ m$

So the distance in last 2 *sec* = $s_{10} - s_8 = 28.3 - 20 = 8.3\ m$

***Problem* 47.** A body travels for 15 *sec* starting from rest with constant acceleration. If it travels distances S_1, S_2 and S_3 in the first five seconds, second five seconds and next five seconds respectively the relation between S_1, S_2 and S_3 is **[AMU (Engg.) 2000]**

(a) $S_1 = S_2 = S_3$ (b) $5S_1 = 3S_2 = S_3$ (c) $S_1 = \frac{1}{3}S_2 = \frac{1}{5}S_3$ (d) $S_1 = \frac{1}{5}S_2 = \frac{1}{3}S_3$

Solution : (c) Since the body starts from rest. Therefore $u = 0$.

$S_1 = \frac{1}{2}a(5)^2 = \frac{25a}{2}$

$S_1 + S_2 = \frac{1}{2}a(10)^2 = \frac{100a}{2} \Rightarrow S_2 = \frac{100a}{2} - S_1 = 75\frac{a}{2}$

$S_1 + S_2 + S_3 = \frac{1}{2}a(15)^2 = \frac{225a}{2} \Rightarrow S_3 = \frac{225a}{2} - S_2 - S_1 = \frac{125a}{2}$

Thus Clearly $S_1 = \frac{1}{3}S_2 = \frac{1}{5}S_3$

***Problem* 48.** If a body having initial velocity zero is moving with uniform acceleration $8\,m/\sec^2$, the distance travelled by it in fifth second will be

(a) 36 *metres* (b) 40 *metres* (c) 100 *metres* (d) Zero

Solution : (a) $S_n = u + \frac{1}{2}a(2n-1) = 0 + \frac{1}{2}(8)\,[2\times 5 - 1] = 36\,metres$

***Problem* 49.** The engine of a car produces acceleration 4m/sec² in the car, if this car pulls another car of same mass, what will be the acceleration produced **[RPET 1996]**

(a) 8 m/s^2 (b) 2 m/s^2 (c) 4 m/s^2 (d) $\frac{1}{2}m/s^2$

Solution : (b) $F = ma \quad a \propto \frac{1}{m}$ if F = constant. Since the force is same and the effective mass of system becomes double

$\frac{a_2}{a_1} = \frac{m_1}{m_2} = \frac{m}{2m}$, $a_2 = \frac{a_1}{2} = 2\,\text{m/s}^2$

***Problem* 50.** A body starts from rest. What is the ratio of the distance travelled by the body during the 4th and 3rd second. **[CBSE PMT 1993]**

(a) 7/5 (b) 5/7 (c) 7/3 (d) 3/7

Solution : (a) As $S_n \propto (2n-1)$, $\frac{S_4}{S_3} = \frac{7}{5}$

2.11 Motion of Body Under Gravity (Free Fall)

The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude ($h << R$) is called free fall.

An ideal one-dimensional motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

(1) **If a body dropped from some height (initial velocity zero)**

(i) Equation of motion : Taking initial position as origin and direction of motion (*i.e.*, downward direction) as a positive, here we have

$u = 0$ [As body starts from rest]

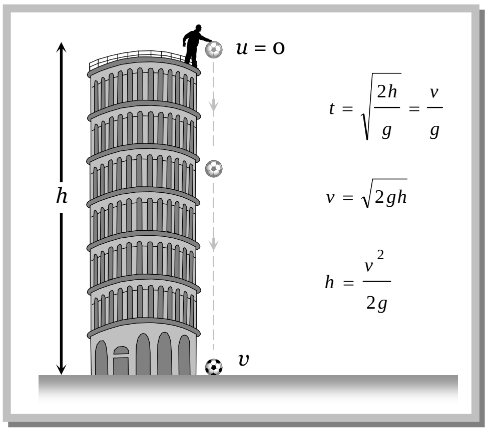
$a = +g$ [As acceleration is in the direction of motion]

$v = g\,t$ ...(i)

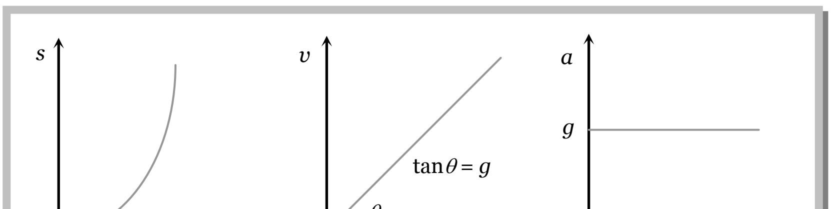
$h = \frac{1}{2}gt^2$ ...(ii)

$v^2 = 2gh$ ...(iii)

$h_n = \frac{g}{2}(2n-1)$ ...(iv)



(ii) Graph of distance velocity and acceleration with respect to time :



(iii) As $h = (1/2)gt^2$, *i.e.*, $h \propto t^2$, distance covered in time t, $2t$, $3t$, *etc.*, will be in the ratio of $1^2 : 2^2 : 3^2$, *i.e.*, square of integers.

(iv) The distance covered in the *nth sec*, $h_n = \frac{1}{2}g(2n-1)$

So distance covered in I, II, III *sec*, *etc.*, will be in the ratio of 1 : 3 : 5, *i.e.*, odd integers only.

(2) **If a body is projected vertically downward with some initial velocity**

Equation of motion : $v = u + gt$

$$h = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh$$

$$h_n = u + \frac{g}{2}(2n-1)$$

(3) **If a body is projected vertically upward**

(i) Equation of motion : Taking initial position as origin and direction of motion (*i.e.*, vertically up) as positive

$a = -g$ [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$$v = u - gt; \quad h = ut - \frac{1}{2}gt^2; \quad v^2 = u^2 - 2gh; \quad h_n = u - \frac{g}{2}(2n-1)$$

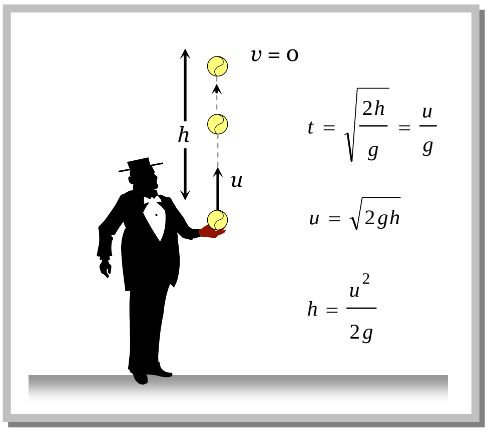
(ii) For maximum height $v = 0$

So from above equation

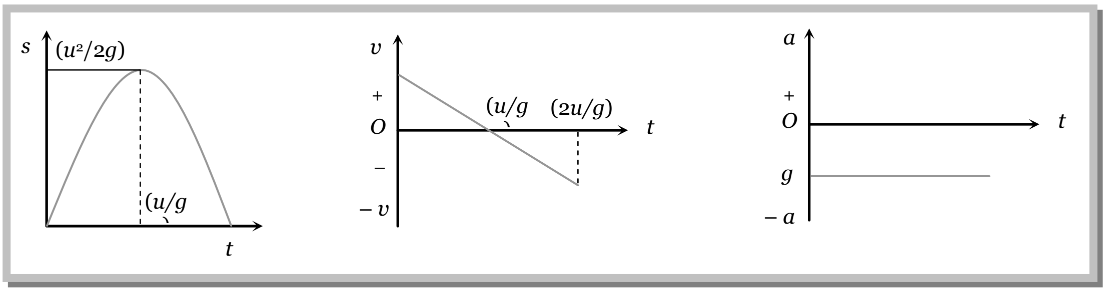
$u = gt,$

$$h = \frac{1}{2}gt^2$$

and $u^2 = 2gh$



(iii) Graph of distance, velocity and acceleration with respect to time (for maximum height) :



It is clear that both quantities do not depend upon the mass of the body or we can say that in absence of air resistance, all bodies fall on the surface of the earth with the same rate.

(4) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remain constant while speed, velocity, momentum, kinetic energy and potential energy change.

(5) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity *i.e.*, $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.

(6) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_1) = time of ascent (t_2) = u/g

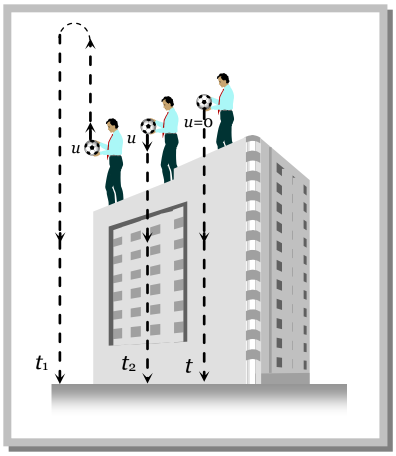
$\therefore$ Total time of flight $T = t_1 + t_2 = \dfrac{2u}{g}$

(7) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

(8) A ball is dropped from a building of height h and it reaches after t seconds on earth. From the same building if two ball are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively then

$$t = \sqrt{t_1 t_2}$$



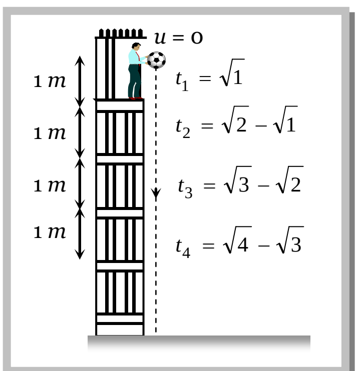
(9) A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$

Let u is the initial velocity of body then time of ascent $t_1 = \dfrac{u}{g+a}$ and $h = \dfrac{u^2}{2(g+a)}$

where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will work vertically downward.

For downward motion a and g will work in opposite direction because a always work in direction opposite to motion and g always work vertically downward.

So $$h = \frac{1}{2}(g-a)t_2^2 \Rightarrow \frac{u^2}{2(g+a)} = \frac{1}{2}(g-a)t_2^2 \Rightarrow t_2 = \frac{u}{\sqrt{(g+a)(g-a)}}$$



Comparing t_1 and t_2 we can say that $t_2 > t_1$ since $(g + a) > (g - a)$

(10) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1*m* each will then be in the ratio of the difference in the square roots of the integers *i.e.*

$$\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}).......(\sqrt{4}-\sqrt{3}),.........$$

Sample problems based on motion under gravity

Problem 51. If a body is thrown up with the velocity of 15 m/s then maximum height attained by the body is (g = 10 m/s^2) **[MP PMT 2003]**

(a) 11.25 m (b) 16.2 m (c) 24.5 m (d) 7.62 m

Solution : (a) $H_{max} = \dfrac{u^2}{2g} = \dfrac{(15)^2}{2\times 10} = 11.25m$

Problem 52. A body falls from rest in the gravitational field of the earth. The distance travelled in the fifth second of its motion is $(g = 10m/s^2)$ **[MP PET 2003]**

(a) 25m (b) 45m (c) 90m (d) 125m

Solution : (b) $h_n = \dfrac{g}{2}(2n-1) \Rightarrow h_{5th} = \dfrac{10}{2}(2\times 5-1) = 45\ m.$

Problem 53. If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is **[CBSE 2003]**

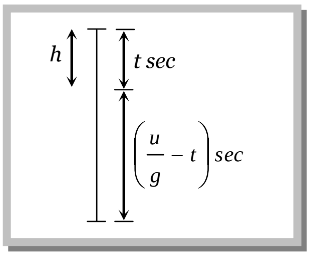
(a) $\dfrac{1}{2}gt^2$ (b) $ut - \dfrac{1}{2}gt^2$ (c) $(u-gt)t$ (d) ut

Solution : (a) If ball is thrown with velocity u, then time of flight $= \dfrac{u}{g}$

velocity after $\left(\dfrac{u}{g} - t\right)$ sec : $v = u - g\left(\dfrac{u}{g} - t\right) = gt.$

So, distance in last 't' *sec* : $0^2 = (gt)^2 - 2(g)h.$

$\Rightarrow h = \dfrac{1}{2}gt^2.$



Problem 54. A man throws balls with the same speed vertically upwards one after the other at an interval of 2 seconds. What should be the speed of the throw so that more than two balls are in the sky at any time (Given $g = 9.8m/s^2$)

(a) At least 0.8 m/s (b) Any speed less than 19.6 m/s

(c) Only with speed 19.6 m/s (d) More than 19.6 m/s

Solution : (d) Interval of ball throw = 2 *sec*.

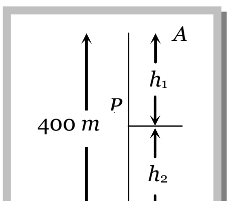
If we want that minimum three (more than two) ball remain in air then time of flight of first ball must be greater than 4 *sec*. *i.e.* $T > 4$ *sec* or $\dfrac{2U}{g} > 4\ sec \Rightarrow u > 19.6\ m/s$

It is clear that for $u = 19.6$ First ball will just strike the ground (in sky), second ball will be at highest point (in sky), and third ball will be at point of projection or on ground (not in sky).

Problem 55. A man drops a ball downside from the roof of a tower of height 400 *meters*. At the same time another ball is thrown upside with a velocity 50 *meter/sec*. from the surface of the tower, then they will meet at which height from the surface of the tower **[CPMT 2003]**

(a) 100 *meters* (b) 320 *meters* (c) 80 *meters* (d) 240 *meters*

Solution : (c) Let both balls meet at point P after time t.



The distance travelled by ball A $(h_1) = \frac{1}{2}gt^2$ .....(i)

The distance travelled by ball B $(h_2) = ut - \frac{1}{2}gt^2$ .....(ii)

By adding (i) and (ii) $h_1 + h_2 = ut$ = 400 (Given $h = h_1 + h_2 = 400.$)

$\therefore$ $t = 400/50 = 8\,sec$ and $h_1 = 320\,m$, $h_2 = 80\,m$

Problem 56. A very large number of balls are thrown vertically upwards in quick succession in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 5*m*, the number of ball thrown per minute is (take $g = 10\,ms^{-2}$) **[KCET (Med.) 2002]**

(a) 120 (b) 80 (c) 60 (d) 40

Solution : (c) Maximum height of ball = 5*m*, So velocity of projection $\Rightarrow u = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10\,m/s$

time interval between two balls (time of ascent) = $\frac{u}{g} = 1\,sec = \frac{1}{60}\,min.$

So no. of ball thrown per min = 60

Problem 57. A particle is thrown vertically upwards. If its velocity at half of the maximum height is 10 *m/s*, then maximum height attained by it is (Take $g = 10\ m/s^2$)

(a) 8 *m* (b) 10 *m* (c) 12 *m* (d) 16 *m*

Solution : (b) Let particle thrown with velocity *u* and its maximum height is H then $H = \frac{u^2}{2g}$

When particle is at a height $H/2$, then its speed is $10\,m/s$

From equation $v^2 = u^2 - 2gh$, $(10)^2 = u^2 - 2g\left(\frac{H}{2}\right) = u^2 - 2g\frac{u^2}{4g} \Rightarrow u^2 = 200$

$\therefore$ Maximum height $H = \frac{u^2}{2g} = \frac{200}{2 \times 10}$ = 10*m*

Problem 58. A stone is shot straight upward with a speed of 20 *m/sec* from a tower 200 *m* high. The speed with which it strikes the ground is approximately **[AMU (Engg.) 1999]**

(a) 60 *m/sec* (b) 65 *m/sec* (c) 70 *m/sec* (d) 75 *m/sec*

Solution : (b) Speed of stone in a vertically upward direction is 20 *m/s*. So for vertical downward motion we will consider $u = -20\,m/s$

$v^2 = u^2 + 2gh = (-20)^2 + 2 \times 10 \times 200 \Rightarrow v = 65\ m/s$

Problem 59. A body freely falling from the rest has a velocity '*v*' after it falls through a height '*h*'. The distance it has to fall down for its velocity to become double, is **[BHU 1999]**

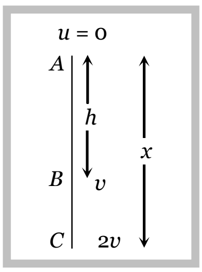
(a) $2h$ (b) $4h$ (c) $6h$ (d) $8h$

Solution : (b) Let at point A initial velocity of body is equal to zero

For path AB : $v^2 = 0 + 2gh$ ... (i)

For path AC: $(2v)^2 = 0 + 2gx \Rightarrow 4v^2 = 2gx$ ... (ii)

Solving (i) and (ii) $x = 4h$



Problem 60. A body sliding on a smooth inclined plane requires 4 *seconds* to reach the bottom starting from rest at the top. How much time does it take to cover one-fourth distance starting from rest at the top

(a) 1 *s* (b) 2 *s* (c) 4 *s* (d) 16 *s*

Solution : (b) $S = \frac{1}{2}at^2 \Rightarrow t \propto \sqrt{s}$ (As a = constant)

$$\frac{t_2}{t_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{s/4}{s}} = \frac{1}{2} \Rightarrow t_2 = \frac{t_1}{2} = \frac{4}{2} = 2\,s$$

Problem 61. A stone dropped from a building of height h and it reaches after t seconds on earth. From the same building if two stones are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively, then **[CPMT 1997; UPSEAT 2002; KCET (Engg./Med.) 20**

(a) $t = t_1 - t_2$ (b) $t = \frac{t_1 + t_2}{2}$ (c) $t = \sqrt{t_1 t_2}$ (d) $t = t_1^2\, t_2^2$

Solution : (c) For first case of dropping $h = \frac{1}{2}gt^2$.

For second case of downward throwing $h = -ut_1 + \frac{1}{2}gt_1^2 = \frac{1}{2}gt^2$

$\Rightarrow -ut_1 = \frac{1}{2}g(t^2 - t_1^2)$ ......(i)

For third case of upward throwing $h = ut_2 + \frac{1}{2}gt_2^2 = \frac{1}{2}gt^2$

$\Rightarrow ut_2 = \frac{1}{2}g(t^2 - t_2^2)$ ....(ii)

on solving these two equations : $-\frac{t_1}{t_2} = \frac{t^2 - t_1^2}{t^2 - t_2^2} \Rightarrow t = \sqrt{t_1 t_2}$.

Problem 62. By which velocity a ball be projected vertically downward so that the distance covered by it in 5th second is twice the distance it covers in its 6th second ($g = 10\,m/s^2$)

(a) $58.8\,m/s$ (b) $49\,m/s$ (c) $65\,m/s$ (d) $19.6\,m/s$

Solution : (c) By formula $h_n = u + \frac{1}{2}g(2n-1) \Rightarrow u - \frac{10}{2}[2\times 5 - 1] = 2\{u - \frac{10}{2}[2\times 6 - 1]\}$

$\Rightarrow u - 45 = 2\times(u - 55) \Rightarrow u = 65\,m/s$.

Problem 63. Water drops fall at regular intervals from a tap which is 5 *m* above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant **[CBSE PMT 1995]**

(a) 2.50 *m* (b) 3.75 *m* (c) 4.00 *m* (d) 1.25 *m*

Solution : (b) Let the interval be t then from question

For first drop $\frac{1}{2}g(2t)^2 = 5$ .....(i) For second drop $x = \frac{1}{2}gt^2$ .....(ii)

By solving (i) and (ii) $x = \frac{5}{4}$ and hence required height $h = 5 - \frac{5}{4} = 3.75m$.

Problem 64. A balloon is at a height of 81 *m* and is ascending upwards with a velocity of $12\,m/s$. A body of 2 *kg* weight is dropped from it. If $g = 10\,m/s^2$, the body will reach the surface of the earth in **[MP PMT 1994]**

(a) 1.5 *s* (b) 4.025 *s* (c) 5.4 *s* (d) 6.75 *s*

Solution : (c) As the balloon is going up we will take initial velocity of falling body $= -12m/s$, $h = 81m$, $g = +10m/s^2$

By applying $h = ut + \frac{1}{2}gt^2$; $81 = -12t + \frac{1}{2}(10)t^2 \Rightarrow 5t^2 - 12t - 81 = 0$

$$\Rightarrow t = \frac{12 \pm \sqrt{144 + 1620}}{10} = \frac{12 \pm \sqrt{1764}}{10} \approx 5.4 \text{ sec}.$$

Problem 65. A particle is dropped under gravity from rest from a height $h (g = 9.8\,m/s^2)$ and it travels a distance $9h/25$ in the last second, the height h is **[MNR 1987]**

(a) 100 *m* (b) 122.5 *m* (c) 145 *m* (d) 167.5 *m*

Solution : (b) Distance travelled in n sec $= \frac{1}{2}gn^2 = h$ .....(i)

Distance travelled in n^{th} sec $= \frac{g}{2}(2n-1) = \frac{9h}{25}$ .....(ii)

Solving (i) and (ii) we get. $h = 122.5\,m$.

Problem 66. A stone thrown upward with a speed u from the top of the tower reaches the ground with a velocity $3u$. The height of the tower is

(a) $3u^2/g$ (b) $4u^2/g$ (c) $6u^2/g$ (d) $9u^2/g$

Solution : (b) For vertical downward motion we will consider initial velocity = – u.

By applying $v^2 = u^2 + 2gh$, $(3u)^2 = (-u)^2 + 2gh \Rightarrow h = \frac{4u^2}{g}$.

Problem 67. A stone dropped from the top of the tower touches the ground in 4 *sec*. The height of the tower is about

[MP PET 1986; AFMC 1994; CPMT 1997; BHU 1998; DPMT 1999; RPET 1999]

(a) 80 *m* (b) 40 *m* (c) 20 *m* (d) 160 *m*

Solution : (a) $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 4^2 = 80m$.

Problem 68. A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

(a) 4.9 *m* (b) 9.8 *m* (c) 19.6 *m* (d) 24.5 *m*

Solution : (d) The separation between two bodies, two second after the release of second body is given by :

$s = \frac{1}{2}g(t_1^2 - t_2^2) = \frac{1}{2} \times 9.8 \times (3^2 - 2^2) = 24.5\ m$.

2.12 Motion With Variable Acceleration

(i) If acceleration is a function of time

$a = f(t)$ then $v = u + \int_0^t f(t)dt$ and $s = ut + \int\left(\int f(t)dt\right)dt$

(ii) If acceleration is a function of distance

$a = f(x)$ then $v^2 = u^2 + 2\int_{x_0}^{x} f(x)dx$

(iii) If acceleration is a function of velocity

$a = f(v)$ then $t = \int_u^v \frac{dv}{f(v)}$ and $x = x_0 + \int_u^v \frac{vdv}{f(v)}$

Sample problems based on variable acceleration

Problem 69. An electron starting from rest has a velocity that increases linearly with the time that is $v = kt$, where $k = 2\,m/\sec^2$. The distance travelled in the first 3 *seconds* will be **[NCERT 1982]**

(a) 9 *m* (b) 16 *m* (c) 27 *m* (d) 36 *m*

Solution : (a) $x = \int_{t_1}^{t_2} v\,dt = \int_0^3 2t\,dt = 2\left[\frac{t^2}{2}\right]_0^3 = 9\ m.$

***Problem* 70.** The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time t will be **[CBSE PMT 1995]**

(a) $v_0 t + \frac{1}{3}bt^2$ (b) $v_0 t + \frac{1}{3}bt^3$ (c) $v_0 t + \frac{1}{6}bt^3$ (d) $v_0 t + \frac{1}{2}bt^2$

Solution : (c) $\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt = \int_{t_1}^{t_2} (bt)dt$

$$\Rightarrow v_2 - v_1 = \left(\frac{bt^2}{2}\right)_{t_1}^{t_2}$$

$$\Rightarrow v_2 = v_1 + \left(\frac{bt^2}{2}\right)_0^t = v_0 + \frac{bt^2}{2}$$

$$\Rightarrow S = \int v_0 dt + \int \frac{bt^2}{2}dt = v_0 t + \frac{1}{6}bt^3$$

***Problem* 71.** The motion of a particle is described by the equation $u = at$. The distance travelled by the particle in the first 4 seconds **[DCE 2000]**

(a) $4a$ (b) $12a$ (c) $6a$ (d) $8a$

Solution : (d) $u = at \Rightarrow \frac{ds}{dt} = at$

$$\Rightarrow s = \int_0^4 at\,dt = a\left[\frac{t^2}{2}\right]_0^4 = 8a.$$