JEE MAIN

# CAPACITORS
# FORMULAE

*Now that's how you REVISE*

-Mohit Goenka, IIT Kharagpur

# List of Content on Eduniti YouTube Channel:

1. PYQs Video Solution Topic Wise:
   (a) JEE Main 2018/2020/2021 Feb & March
2. Rank Booster Problems for JEE Main
3. Part Test Series for JEE Main
4. JEE Advanced Problem Solving Series
5. Short Concept Videos
6. Tips and Tricks Videos
7. JEE Advanced PYQs

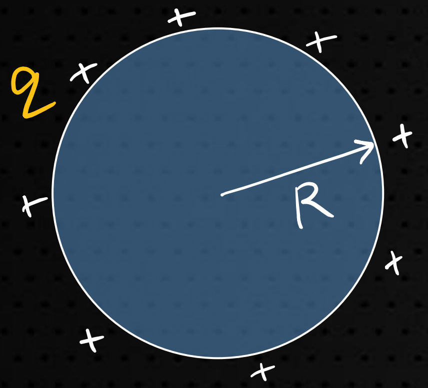
........and many more to come

# CAPACITORS

1. Spherical and Cylindrical Capacitors
2. Parallel Plate capacitors
3. Charge, Energy Stored, Work done by Battery, Heat
4. Force Between Plates
5. Series / Parallel
6. Alternative figure for plate arrangement
7. Wheatstone bridge (Balanced & Unbalanced)
8. Charge Sharing & Heat generated
9. Dielectric in Electric Field (Induced Charge)
10. Dielectric Slab in Capacitor
11. Effect of slab insertion in a capacitor (at Const. V and Q)
12. Capacitance for Multiple Dielectric Medium
13. Capacitance for Variable K
14. Capacitance for variable dimensions
15. RC – Charging and Discharging

# 1. CAPACITANCE (unit: Farad)

## ISOLATED SPHERE

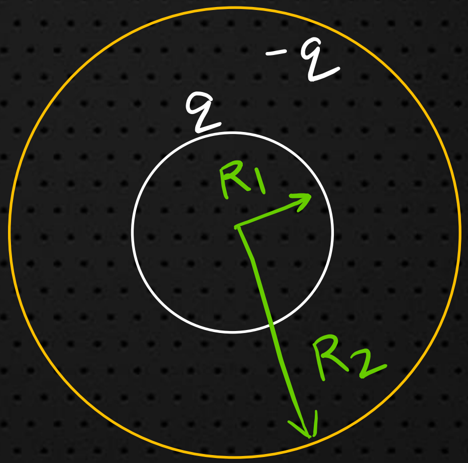



$$C = \frac{q}{V} = \frac{q}{Kq/R}$$

$$C = 4\pi\varepsilon_0 R$$

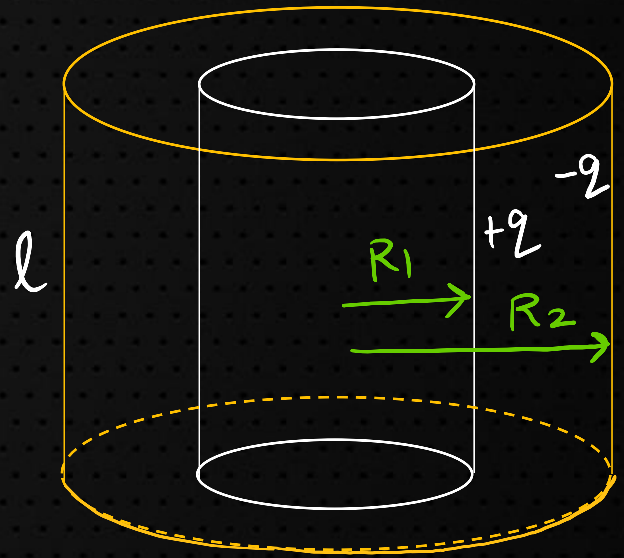
## TWO CONDUCTORS

### Spherical



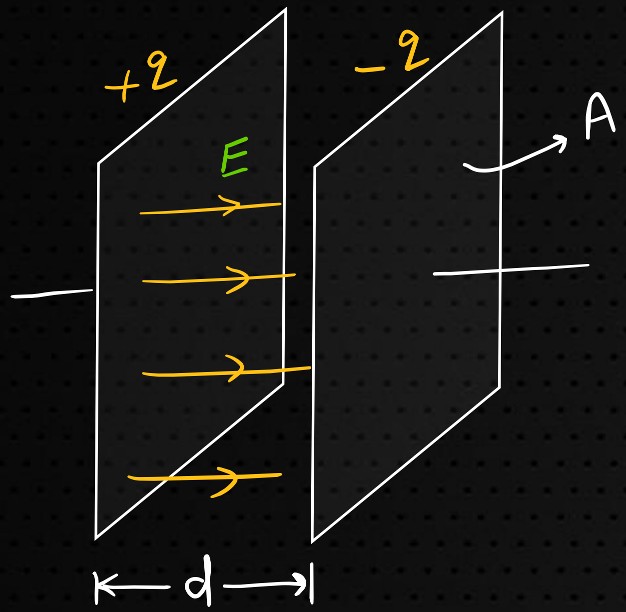

$$C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

### Cylindrical




$$C = \frac{2\pi\varepsilon_0 l}{\ln(R_2/R_1)}$$

## 2. PARALLEL PLATE CAPACITOR

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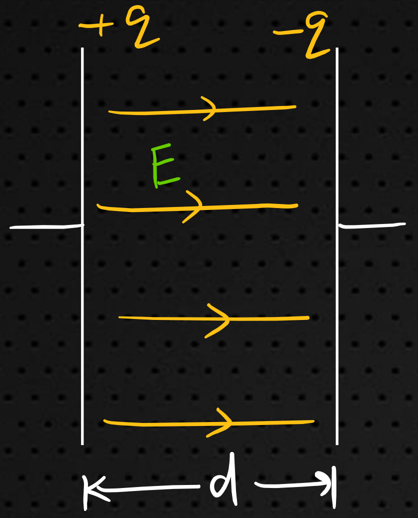
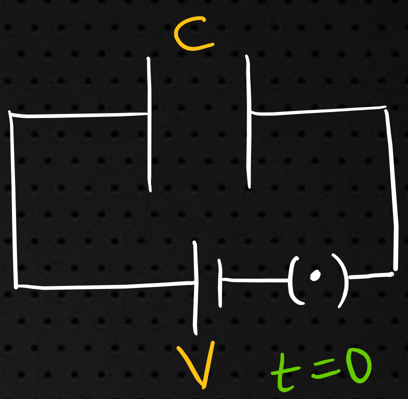


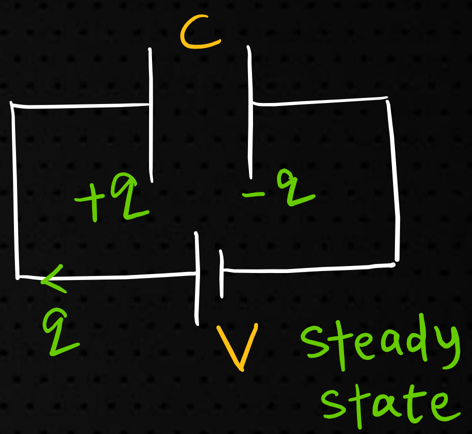

$$C = q/V = q/Ed$$

$$\left(E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}\right)$$

$$\Rightarrow \boxed{C = \frac{A\varepsilon_0}{d}}$$

## 3. CHARGE / ENERGY STORED $W_{battery}$ / HEAT DISSIPATION




(i) $q = CV$

(ii) $W_b = q_{flown} \times V = CV^2$

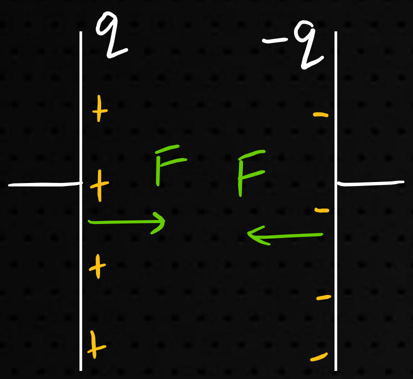
(iii) $U = \frac{1}{2}CV^2 \quad \left\{ q^2/2C \right.$

(iv) Heat Dissipated

$$= W_b - \Delta U$$

$$= CV^2 - \frac{1}{2}CV^2 = \frac{1}{2}CV^2$$

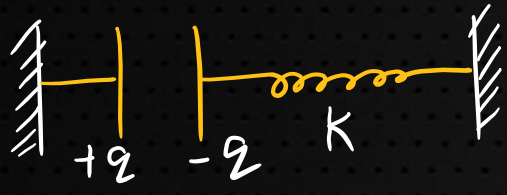
## 4. FORCE BETWEEN PLATES




$$F = q \times E_{-q} = q \times \frac{q}{2A\varepsilon_0}$$

$$\Rightarrow F = q^2/2A\varepsilon_0$$

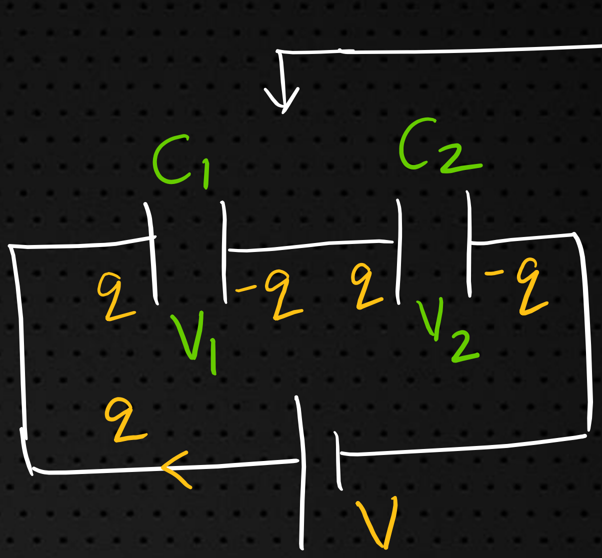
Ex:




$x$ = spring Elongation

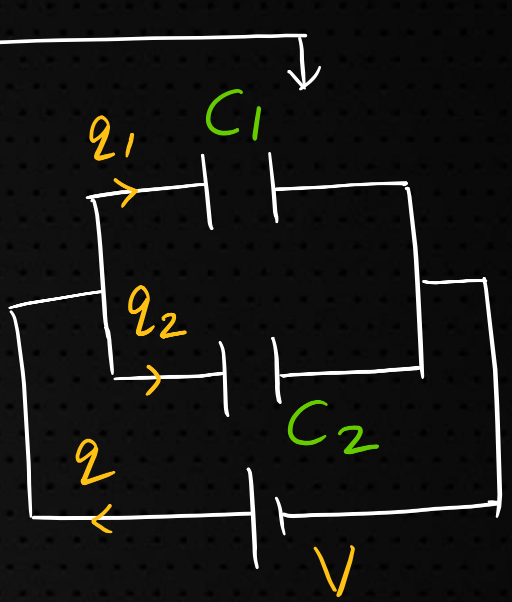
$$\therefore Kx = \frac{q^2}{2A\varepsilon_0}$$

## 5. COMBINATION OF CAPACITOR




(i) $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2}$

(ii) $q = C_{eq} V$




$C_{eq} = C_1 + C_2$

$q_1 = C_1 V$, $q_2 = C_2 V$

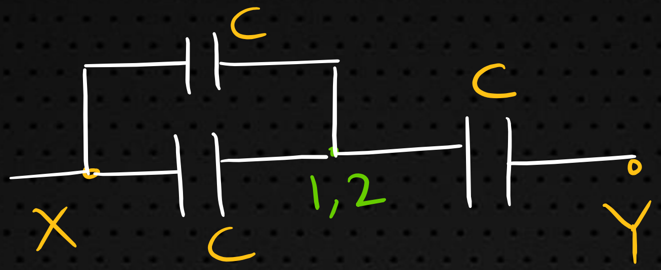
NOTE: (a) $n$ identical capacitor in series, $C_{eq} = C/n$

(b) If in parallel, $C_{eq} = nC$

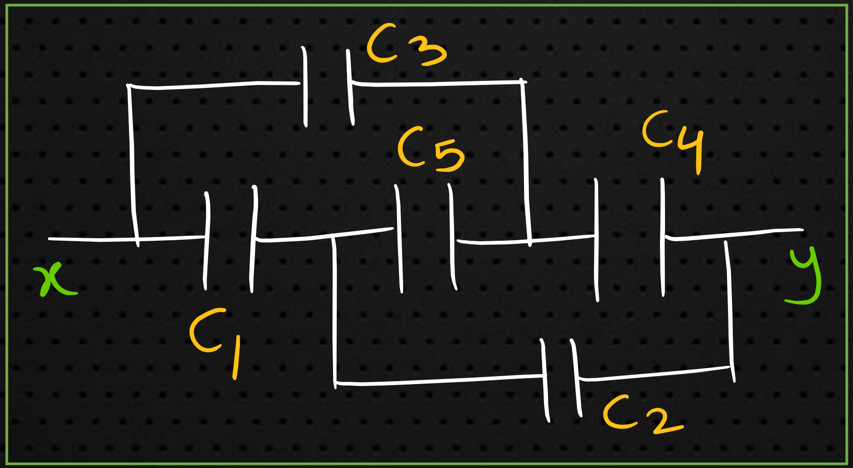
Alternatively:


Identical plates

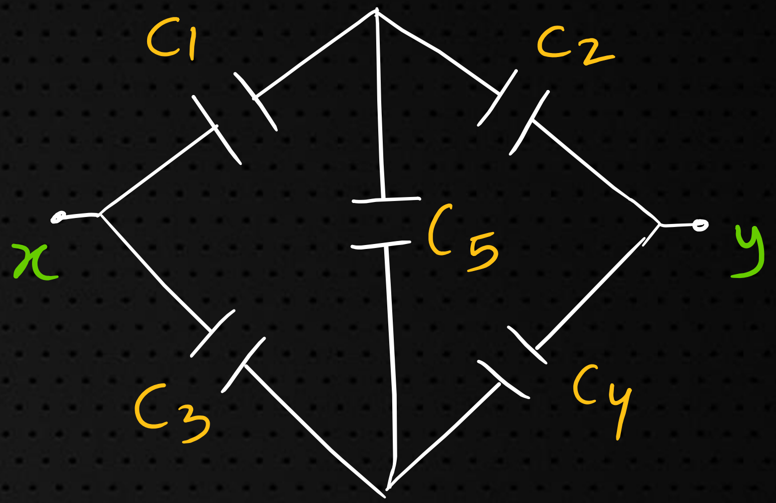
$C = A\varepsilon_0/d$



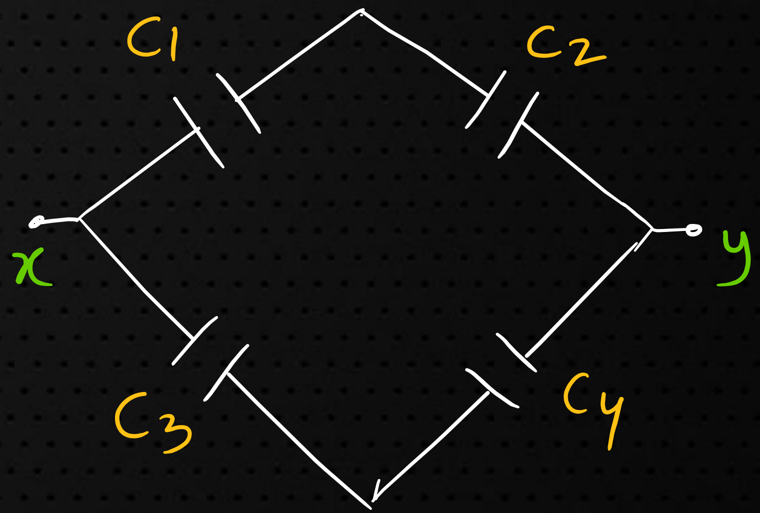

$$C_{eq} = \frac{2C}{3} = \frac{2A\varepsilon_0}{3d}$$




## 6. WHEATSTONE BRIDGE (BALANCED)




If $C_1C_4 = C_2C_3$

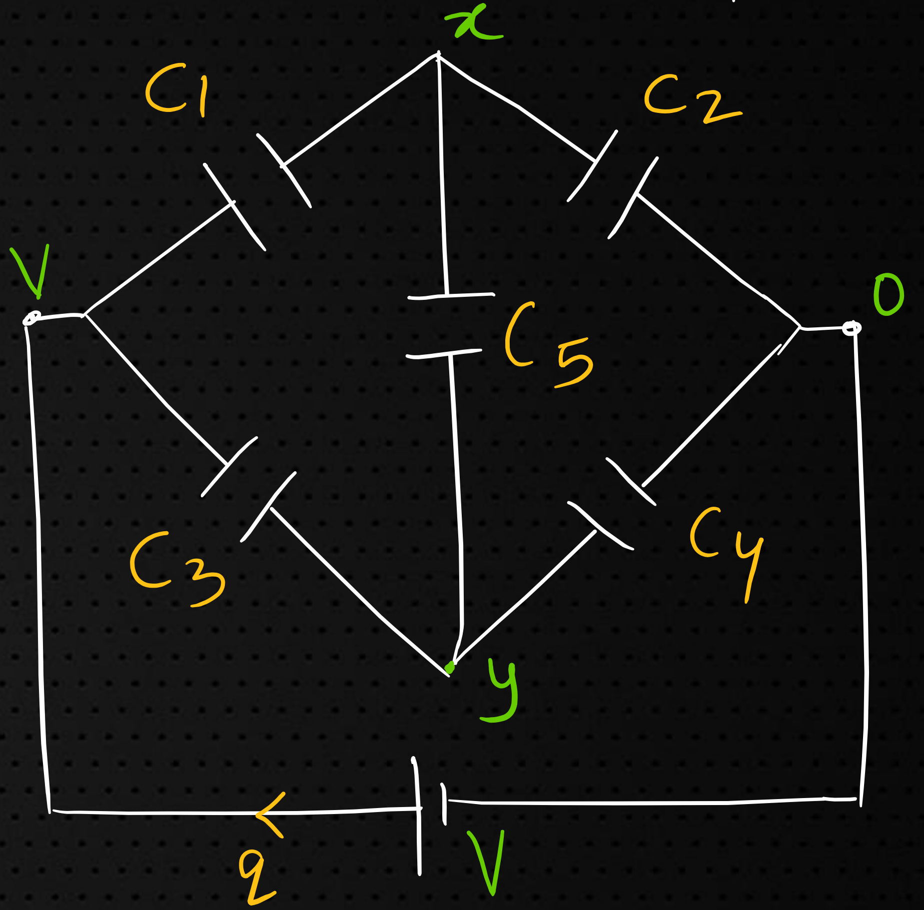


## 7. TECHNIQUE FOR UNBALANCED WHEATSTONE BRIDGE ($C_1C_4 \neq C_2C_3$)

(Point Potential + Junction Rule)

At $x$: $C_1(x-V) + C_5(x-y) + C_2(x-0) = 0$ — (1)

At $y$: $C_3(y-V) + C_5(y-x) + C_4(y-0) = 0$ — (2)




(a) solve (1) and (2) to find $x$ and $y$.

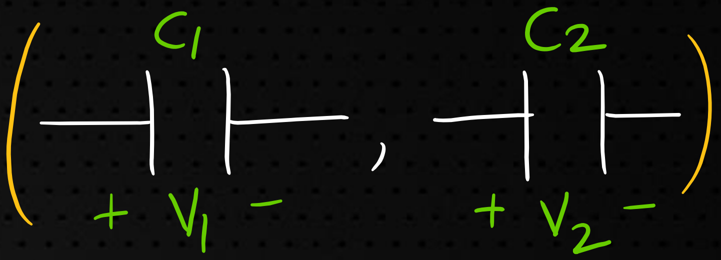
(b) Then we can find $q_1, q_2, q_3, q_4$ and $q_5$

(c) $q = q_1 + q_3$

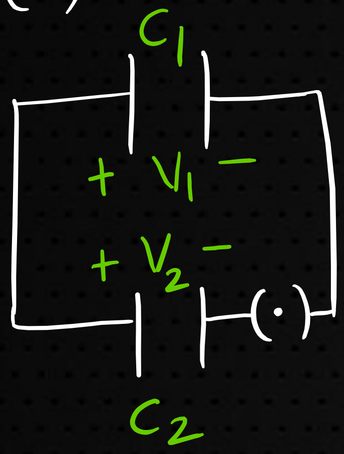
(d) $C_{eq} = \dfrac{q}{V}$

# NOTE : This method can be used to solve any kind of circuit.

# 8. CHARGE SHARING AND HEAT GENERATED

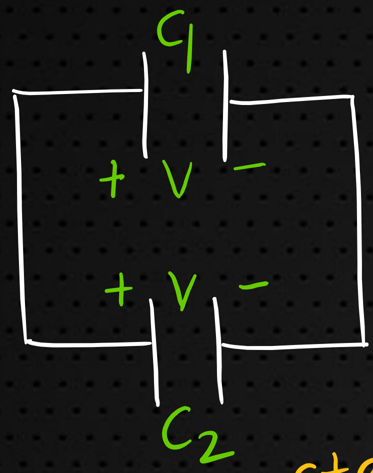



## (a) CONNECTED SAME POLARITY




$t = 0$

$\equiv$




steady state

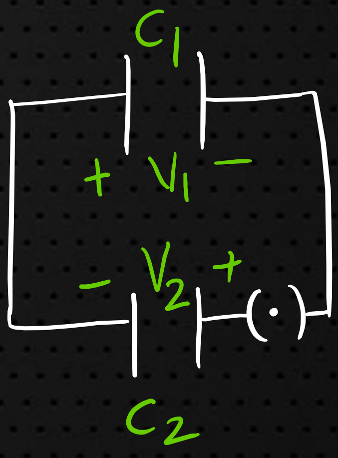
(i) $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

(ii) $H = \dfrac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$

## (b) CONNECTED OPPOSITE POLARITY

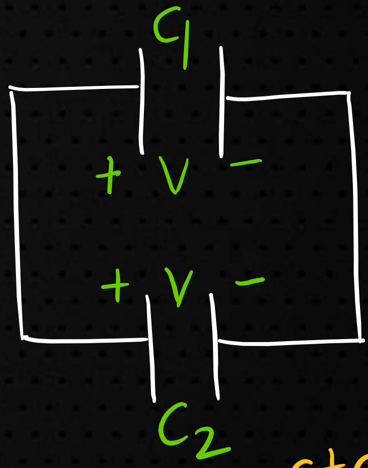
(i) $V = \dfrac{C_1V_1 - C_2V_2}{C_1 + C_2}$

(ii) $H = \dfrac{C_1C_2}{2(C_1+C_2)}(V_1 + V_2)^2$




$t = 0$

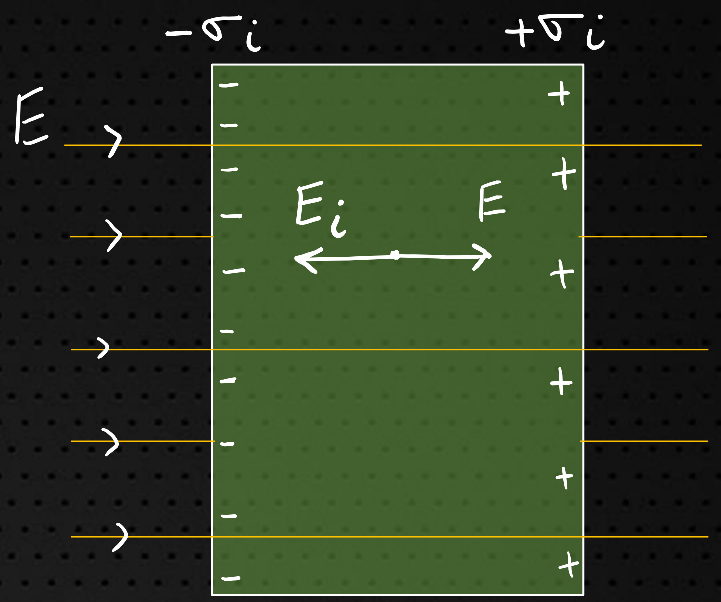
$\equiv$




steady state

## 9. DIELECTRIC IN EXTERNAL ELECTRIC FIELD

- Insulators (gets polarized in E)
- Dielectric Constant ($K$ or $\varepsilon_r$)
  - for air/vacuum $K = 1$
  - for metal $K \to \infty$



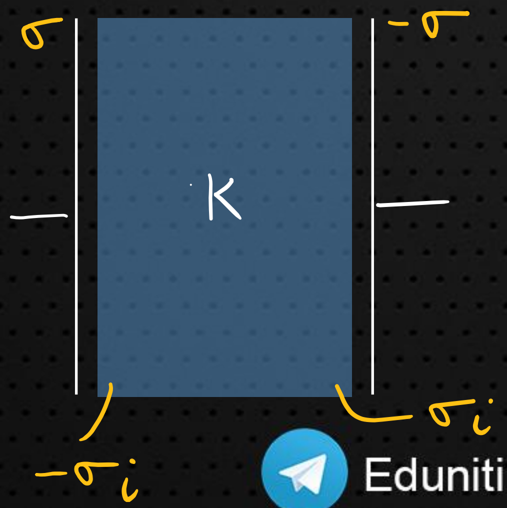

$E_{net} = E - E_i$

$\Rightarrow \dfrac{E}{K} = E - \dfrac{\sigma_i}{\varepsilon_0}$

$\Rightarrow \sigma_i = \varepsilon_0 E\left(1 - \dfrac{1}{K}\right)$

\# Induced charge density, $\sigma_i = \dfrac{Q_i}{A}$

## 10. SLAB IN CAPACITOR



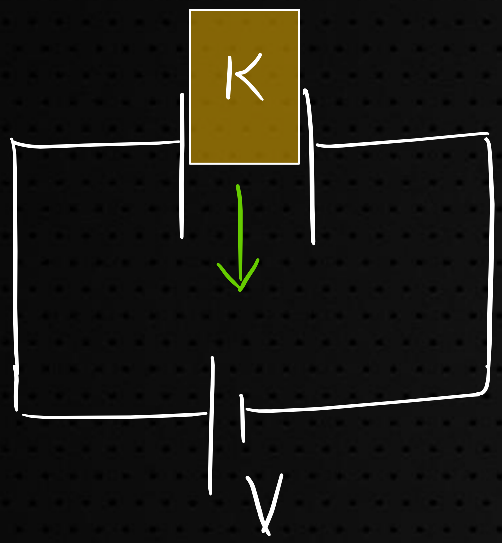

(i) $C = KA\varepsilon_0/d$

(ii) $\sigma_i = \sigma\left(1 - \dfrac{1}{K}\right)$

$Q_i = Q\left(1 - \dfrac{1}{K}\right)$

# 11. EFFECT OF INSERTING DIELECTRIC IN CAPACITOR

## (a) At constant V (Battery connected)




(i) $C \longrightarrow KC$ $(C\uparrow)$

(ii) $q \longrightarrow Kq$ $(q\uparrow)$

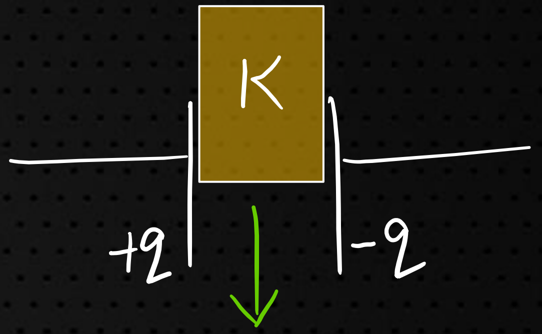
(iii) $V$ is const.

(iv) $E$ is const. $\left(E = \frac{V}{d}\right)$

(V) $U \rightarrow KU$ $(U\uparrow)$

$\left(U = \frac{1}{2}CV^2\right)$

## (b) At constant charge (Battery removed)




(i) $q$ is const.

(ii) $C \longrightarrow KC$ $(C\uparrow)$

(iii) $V \longrightarrow \frac{V}{K}$ $(V\downarrow)$

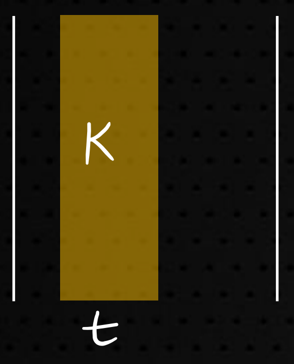
$(V = q/C)$

(iv) $E \rightarrow E/K$ $(E\downarrow)$

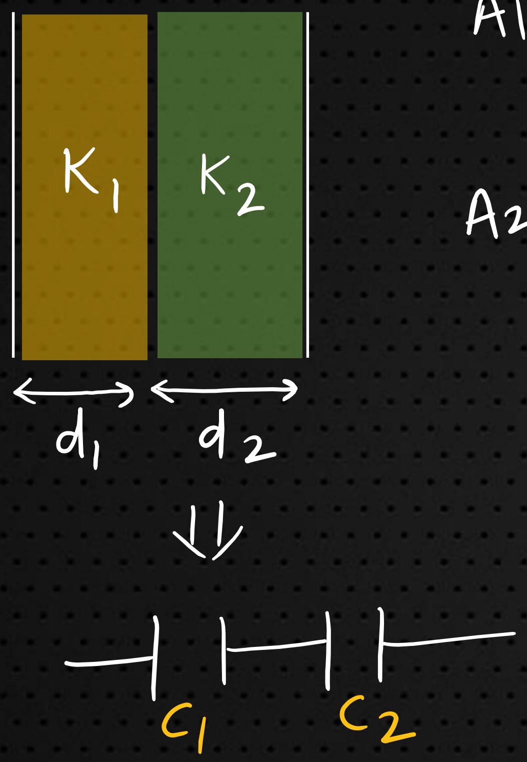
(V) $U \rightarrow U/K$ $(U\downarrow)$

$U = \frac{q^2}{2C}$

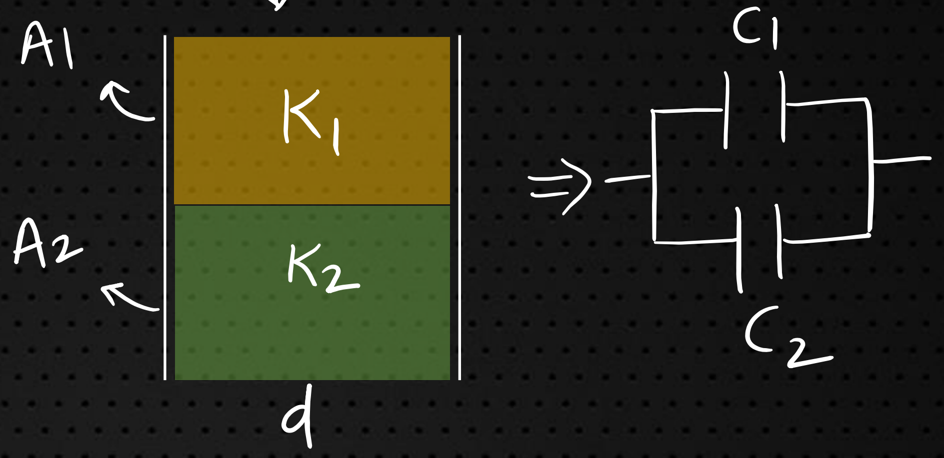
# 12. CAPACITANCE FOR MULTIPLE DIELECTRIC MEDIUM




$$C = \frac{A\varepsilon_0}{d - t + t/K}$$

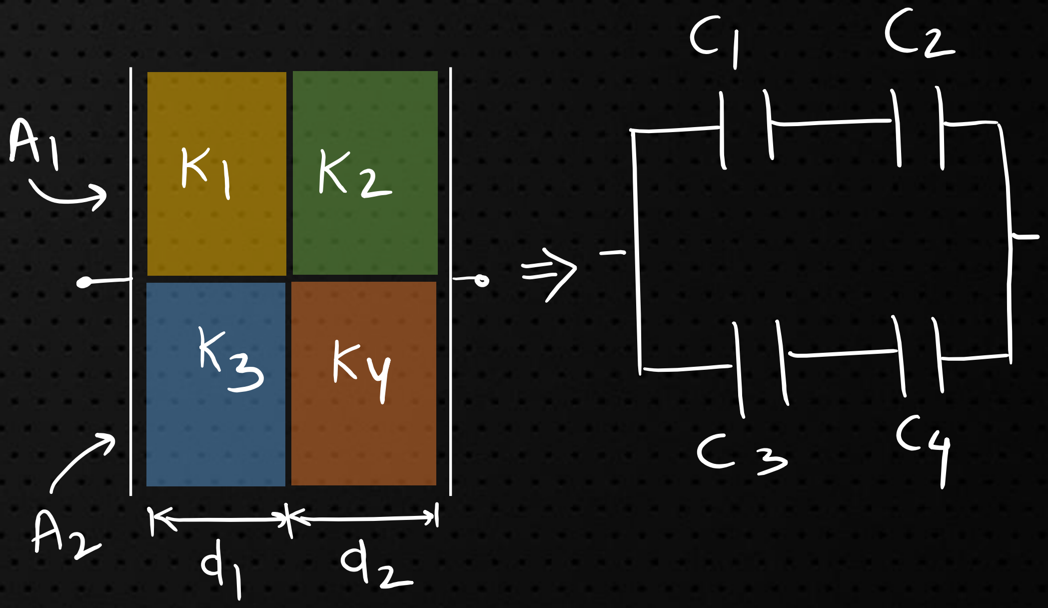



$$C_1 = \frac{K_1 A \varepsilon_0}{d_1}, \quad C_2 = \frac{K_2 A \varepsilon_0}{d_2}$$



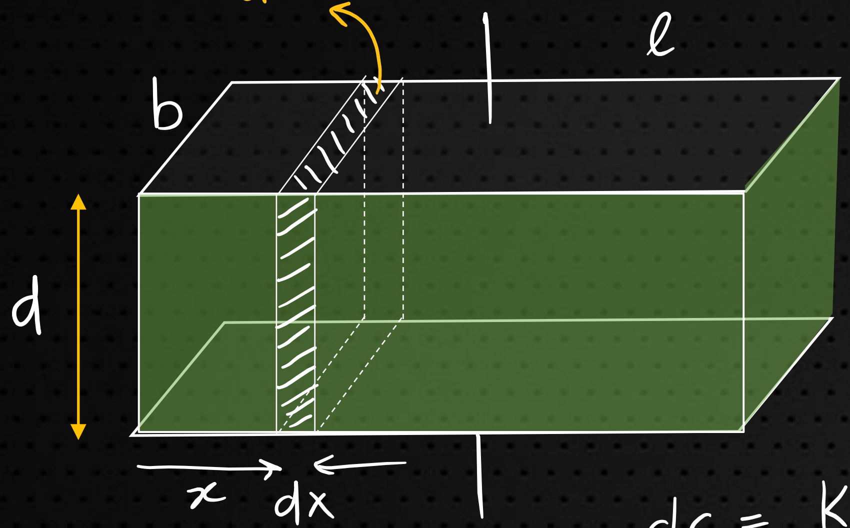

$$C_1 = \frac{K_1 A_1 \varepsilon_0}{d}$$

$$C_2 = \frac{K_2 A_2 \varepsilon_0}{d}$$



# 13. CAPACITANCE WITH VARIABLE K

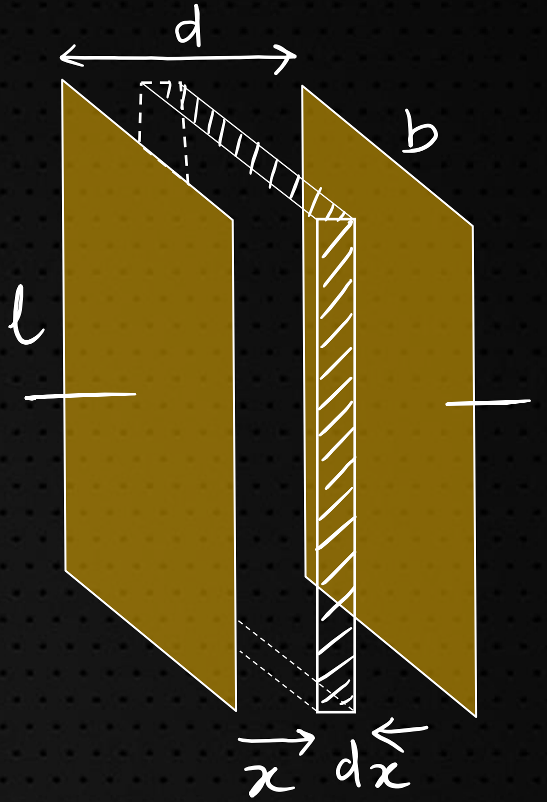
(a)




$$dC = \frac{K(x)\, b\, dx\, \varepsilon_0}{d}$$

$$\Rightarrow C = \frac{b\,\varepsilon_0}{d}\int_0^{\ell} K(x)\,dx$$

(b)

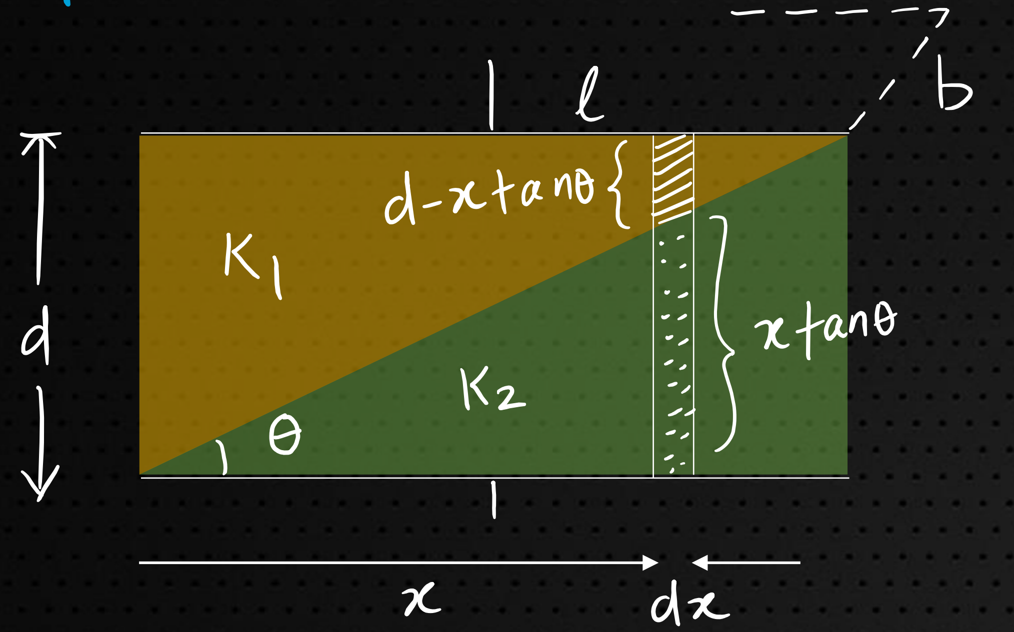



$$dC = \frac{K(x)\, A\, \varepsilon_0}{dx}$$

$$\Rightarrow \frac{1}{C} = \int \frac{1}{dC} = \frac{1}{A\varepsilon_0}\int_0^{d} \frac{dx}{K(x)}$$

# 14. CAPACITANCE WITH VARIABLE DIMENSIONS

(a)

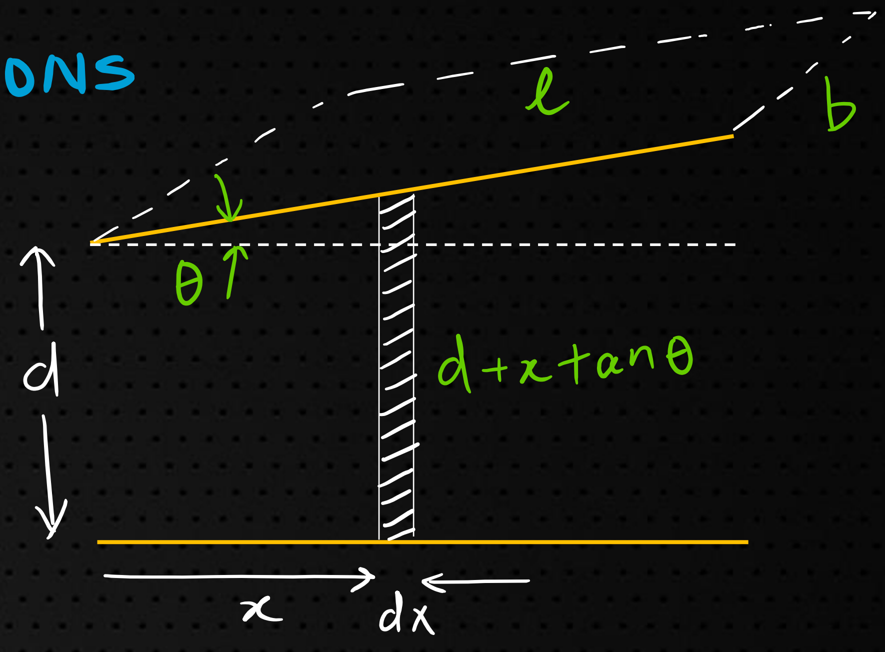



$$dC_1 = \frac{K_1 b\,dx\,\varepsilon_0}{d - x\tan\theta}, \quad dC_2 = \frac{K_2 b\,dx}{x\tan\theta}$$

$$dC_{eq} = \frac{dC_1 \times dC_2}{dC_1 + dC_2}$$

$$C_{eq} = \int_0^l dC_{eq}$$

(b)

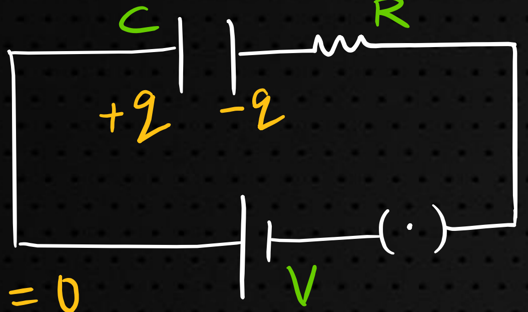



$$dc = \frac{b\,dx\,\varepsilon_0}{d + x\tan\theta}$$

$$\therefore C = b\varepsilon_0 \int_0^l \frac{dx}{d + x\tan\theta}$$

# 15. RC, CHARGING AND DISCHARGING

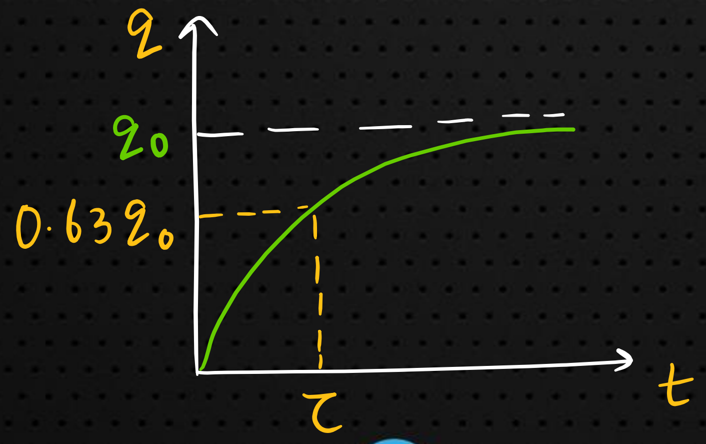
## CHARGING



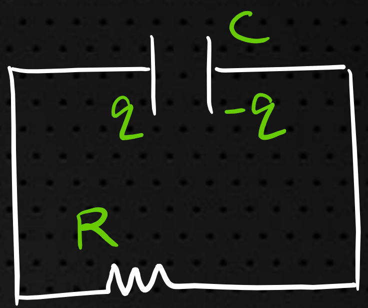

At $t=0$, $q=0$

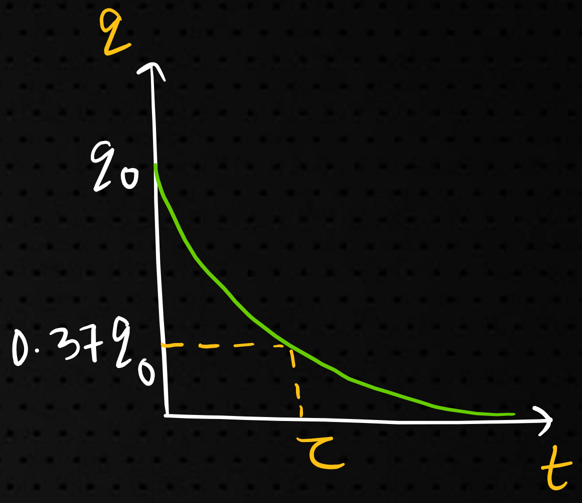
At $t=t$, $q = q_0\left(1-e^{-t/RC}\right)$ $\quad q_0 = CV$

$RC = \tau$, time const.




## DISCHARGING




at $t=0$, $q=q_0$

at $t=t$, $q = q_0 e^{-t/RC}$

**NOTE:**

(1.) At $t=0$, capacitor behaves as conducting wire

(2) At steady state it acts as open circuit.