UNITS AND DIMENSIONS

+4 Marks

IITJEE

CONCEPT

PYQs

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1. PYQs Video Solution Topic Wise:
   (a) JEE Main 2018/2020/2021 Feb & March
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6. Tips and Tricks Videos
7. JEE Advanced PYQs
8. Formulae Revision Series

.....and many more to come

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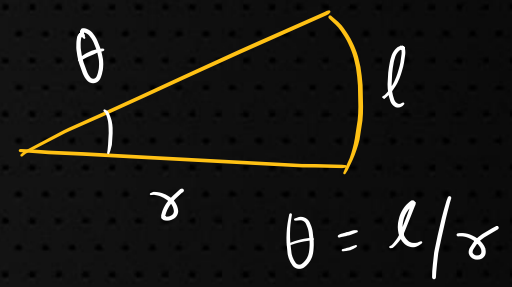
Eduniti for Physics

TOPICS COVERED

1. Fundamental Units
2. Supplementary Units
3. Must Know / Practice Dimensional Formulae
4. Principle of Homogeneity
5. Conversion of Units
6. Dimension in Terms of other Physical Quantity
7. KEY Points (Dimensionless Qty)
8. PYQs (Build your understanding)

1. FUNDAMENTAL UNITS

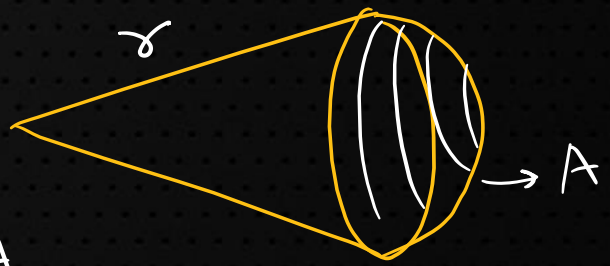
	Quantities	SI UNIT	C.G.S	SYMBOL
1.	LENGTH	m	cm	L
2.	MASS	kg	g	M
3.	TIME	s	s	T
4.	TEMP°	K		θ
5.	CURRENT	A		A
6.	INTENSITY	cd		cd
7.	AMOUNT	mol		mol



2. SUPPLEMENTARY UNITS

(i) ANGLE ⟶ Radian

(ii) SOLID ANGLE ⟶ Steradian



$$\Omega = \frac{A}{r^2}$$

3. MUST KNOW DIMENSIONAL FORMULAE

	Physical Qty	DIMENSIONAL FORMULAE
1	FORCE	MLT^{-2}
2	ENERGY	ML^2T^{-2}
3	ε_0	$M^{-1}L^{-3}T^4A^2$
4	μ_0	$MLT^{-2}A^{-2}$

Ex: Steffan boltzmann Const. (σ)

$\frac{dQ}{dt} = \sigma e A T^4 \Rightarrow \sigma = \frac{Q}{t \times A \times T^4}$

$[\sigma] = \frac{ML^2T^{-2}}{T \times L^2 \times \theta^4} = MT^{-3}\theta^{-4}$

↳ e is dimensionless.

NOTE: You may practice to find Dimension of some constants Like:

(a) Gravitational Const., $G = M^{-1}L^3T^{-2}$

(b) Gas Const., $R = ML^2T^{-2}\theta^{-1}$

(c) Boltzmann's Const., $K_B = ML^2T^{-2}\theta^{-1}$

(d) Planck Const., $h = ML^2T^{-1}$

(e) Rydberg's Const., $R_y = L^{-1}$

(f) Magnetising field, $H = \frac{B}{\mu_0} = \frac{\mu_0 i/2\pi r}{\mu_0} = \frac{i}{r} \Rightarrow AL^{-1}$

4. PRINCIPLE OF HOMOGENEITY

(i) If $a = b + c$

$\Rightarrow [a] = [b] = [c]$

Ex: $S = ut + \frac{1}{2}at^2$

$[S] = [ut] = [at^2]$

5. CONVERSION OF UNITS

$Q = nu$ → units (can be SI, CGS or Given in Question)

- Q ← amount of physical qty.
- n ↓ numeric value

Ex: Find how many POISE (C.G.S unit of viscosity) is equal to 1 POISEUILLE (in SI)

Soln: $F = 6\pi\eta r v \Rightarrow [\eta] = \frac{[F]}{[r][v]} = \frac{MLT^{-2}}{L \times LT^{-1}} = ML^{-1}T^{-1}$

$n_1 u_1 = n_2 u_2$

$\Rightarrow n_1 \times (g\,cm^{-1}s^{-1}) = 1 \times (kg\,m^{-1}s^{-1})$

$\Rightarrow n_1 = \frac{kg\,m^{-1}s^{-1}}{g\,cm^{-1}s^{-1}} = \frac{10^3\,g \times 10^{-2}\,cm^{-1} \times s^{-1}}{g\,cm^{-1}s^{-1}}$

$= \boxed{10}$

6. DIMENSION IN TERMS OF OTHER PHYSICAL QUANTITY

#Ex: Expression for time in terms of G (Gravitational Const.), h (Planck const.) and c (speed of light) is proportional to ?

Sol^{n}: $t \propto G^a h^b c^c \Rightarrow [t] = [G]^a [h]^b [c]^c$

$\Rightarrow [M^0 L^0 T^1] = [M^{-1} L^3 T^{-2}]^a [ML^2 T^{-1}]^b [LT^{-1}]^c$

$\Rightarrow [M^0 L^0 T^1] = [M^{-a+b} \; L^{3a+2b+c} \; T^{-2a-b-c}]$

$\therefore 0 = -a+b, \; 0 = 3a+2b+c, \; 1 = -2a-b-c$

$\Rightarrow a = b = 1/2, \; c = -5/2$

$\therefore t \propto G^{1/2} h^{1/2} c^{-5/2}$ or $\boxed{t \propto \sqrt{\frac{Gh}{c^5}}}$

7. KEY POINTS (dimensionless qty)

(i) All trigonometric ratios ($\sin\theta$, $\cos\theta$ etc.)

(ii) Angle, θ ($\sin\frac{ab}{c}$, here $[ab]=[c]$)

(iii) Exponential functions, e^{x} → x must be dimensionless

($e^{-t/RC}$ → $[t]=[RC]$)

(iv) Reynolds number ($Re=\frac{\rho VD}{\eta}$), Dielectric Const. (K)

Refractive index and many more.

5 PYQs For UNDERSTANDING

1. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant 'h', speed of light 'c', Gravitational constant 'G' and ampere A is

(a) $h^{-2/3}c^{-1/3}G^{4/3}A^{-1}$ **(b)** $h^{0}c^{5}G^{-1}A^{-1}$

(c) $h^{2}G^{3/2}c^{1/3}A^{-1}$ (d) $h^{1/3}G^{2/3}c^{1/3}A^{-1}$

JEE Main 2020

Solution on Next Page

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(c) $h^2 G^{3/2} c^{1/3} A^{-1}$ (d) $h^{1/3} G^{2/3} c^{1/3} A^{-1}$

JEE Main 2020

Soln: $[V_0] = [h]^a [c]^b [G]^c [A]^d$

$[ML^2T^{-3}A^{-1}] = [ML^2T^{-1}]^a [LT^{-1}]^b \times [M^{-1}L^3T^{-2}]^c [A]^d$

$\Rightarrow ML^2T^{-3}A^{-1} = M^{a-c} L^{2a+b+3c} T^{-a-b-2c} A^d$

$\therefore a-c=1,\ 2a+b+3c=2,\ -a-b-2c=-3,\ d=-1$

solving we get:

$a=0,\ b=5,\ c=-1,\ d=-1$

$\Rightarrow h^0 c^5 G^{-1} A^{-1}$

Energy = qV

$\Rightarrow [V] = \frac{ML^2T^{-2}}{IT} = ML^2T^{-3}A^{-1}$

2. A quantity x is given by $(IF\nu^2/WL^4)$ in terms of moment of inertia I, force F, velocity v, work W and Length L. The dimensional formula for x is same as that of

(a) Planck's constant (b) Force constant

(c) Energy density (d) Coefficient of viscosity

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Solution on Next Page

2. A quantity x is given by (IFv^2/WL^4) in terms of moment of inertia I, force F, velocity v, work W and Length L. The dimensional formula for x is same as that of

(a) Planck's constant (b) Force constant

(c) Energy density (d) Coefficient of viscosity

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Soln:

$$x = IFv^2/WL^4$$

$$\Rightarrow [x] = \frac{[ML^2][MLT^{-2}][L^2T^{-2}]}{[ML^2T^{-2}][L^4]}$$

$$= ML^{-1}T^{-2}$$

(a) $[h] = [E]/[\nu] = ML^2T^{-1}$

(b) $[K] = [F]/[x] = MT^{-2}$

(c) $[u] = [E]/[\text{Volume}] = ML^{-1}T^{-2}$

(d) $[\eta] = \frac{[F]}{[\gamma][V]} = ML^{-1}T^{-1}$

3. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is

(a) $[PA^{-1}T^{-2}]$ **(b)** $[PA^{1/2}T^{-1}]$

(c) $[P^2AT^{-2}]$ (d) $[P^{1/2}AT^{-1}]$

JEE Main 2020

Solution on Next Page

3. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is

(a) $[PA^{-1}T^{-2}]$ ✓ **(b)** $[PA^{1/2}T^{-1}]$

(c) $[P^2AT^{-2}]$ (d) $[P^{1/2}AT^{-1}]$

JEE Main 2020

Soln: $[E] = [P]^a [A]^b [T]^c$

$\Rightarrow ML^2T^{-2} = [MLT^{-1}]^a [L^2]^b [T]^c$

$\Rightarrow ML^2T^{-2} = M^a L^{a+2b} T^{-a+c}$

$\therefore 1 = a, \quad 2 = a + 2b, \quad -2 = -a + c$

solving them, we get

$a = 1, \quad b = \frac{1}{2}, \quad c = -1$

$\Rightarrow [PA^{1/2}T^{-1}]$

4. The force of interaction between two atoms is given by $F = \alpha\beta \exp\left(-\frac{x^2}{\alpha k T}\right)$; where x is the distance, k is the boltzmann constant, T is temperature and α and β are two constants. The dimension of β is

(a) $M^0 L^2 T^{-4}$ (b) $M^2 L T^{-4}$

(c) $M L T^{-2}$ (d) $M^2 L^2 T^{-2}$

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4. The force of interaction between two atoms is given by $F = \alpha\beta \exp\left(-\frac{x^2}{\alpha k T}\right)$; where x is the distance, k is the boltzmann constant, T is temperature and α and β are two constants. The dimension of β is

(a) $M^0L^2T^{-4}$ ✓ (b) M^2LT^{-4}

(c) MLT^{-2} (d) $M^2L^2T^{-2}$

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Soln: since exponential terms are dimensionless,

$[x^2] = [\alpha k T]$

{ Energy $= \frac{1}{2}kT$

$\Rightarrow [L^2] = [\alpha][ML^2T^{-2}]$

$\Rightarrow [\alpha] = M^{-1}T^2$

Now, $[F] = [\alpha\beta]$

$\Rightarrow MLT^{-2} = M^{-1}T^2 \times [\beta]$

$\therefore \boxed{[\beta] = M^2LT^{-4}}$

5. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units?

(a) $[M^{-3}L^{-2}T^{8}A^{4}]$ (b) $[M^{-1}L^{-2}T^{4}A^{2}]$

(c) $[M^{-2}L^{0}T^{-4}A^{-2}]$ (d) $[M^{-2}L^{-2}T^{6}A^{3}]$

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Solution on Next Page

5. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units?

✓ **(a)** $[M^{-3}L^{-2}T^{8}A^{4}]$ (b) $[M^{-1}L^{-2}T^{4}A^{2}]$

(c) $[M^{-2}L^{0}T^{-4}A^{-2}]$ (d) $[M^{-2}L^{-2}T^{6}A^{3}]$

JEE Main 2019

Soln:

$$[Y] = \frac{[X]}{[Z^2]} = \frac{[C]}{[B^2]}$$

$$U = \frac{Q^2}{2C} \Rightarrow [C] = \frac{[Q^2]}{[U]} = \frac{A^2T^2}{ML^2T^{-2}} = M^{-1}L^{-2}T^{4}A^{2}$$

$$F = ILB \Rightarrow [B] = \frac{[F]}{[IL]} = \frac{MLT^{-2}}{AL} = MT^{-2}A^{-1}$$

$$\therefore [Y] = \frac{M^{-1}L^{-2}T^{4}A^{2}}{(MT^{-2}A^{-1})^2} = M^{-3}L^{-2}T^{8}A^{4}$$

6. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50g, the numerical value of density of the material is

(a) 40 (b) 16
(c) 640 (d) 410

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Solution on Next Page

6. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50g, the numerical value of density of the material is

(a) 40 (b) 16
(c) 640 (d) 410

JEE Main 2019

Sol$^{n^o}$ $n_1u_1 = n_2u_2$

$\Rightarrow 128 \times \left(\frac{kg}{m^3}\right) = n_2 \times \frac{50\,g}{(25\,cm)^3}$

$\Rightarrow 128 \times \frac{1000\,g}{100^3\,cm^3} = n_2 \times \frac{50}{25^3}\,\frac{g}{cm^3}$

$\Rightarrow n_2 = 128 \times \frac{1000}{50} \times \left(\frac{25}{100}\right)^3$

$= 40$