JEE MAIN

EMI FORMULAE

Now that's how you REVISE

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List of Content on Eduniti YouTube Channel:

1. PYQs Video Solution Topic Wise:
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2. Rank Booster Problems for JEE Main
3. Part Test Series for JEE Main
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EMI : TOPICS COVERED

1. Faraday's Law
2. Direction of Induced Current : Lenz's law
3. Charge Flown (due to induced emf)
4. Coil rotation in uniform magnetic field
5. Motional emf
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17. Mutual Inductance
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19. LC Oscillations

1. FARADAY'S LAW, $\varepsilon = -\dfrac{d\phi}{dt}$

(a) Flux, $\phi = \vec{B}\cdot\vec{A} = BA\cos\theta$

(b) Magnitude is given by $\left|\dfrac{d\phi}{dt}\right|$

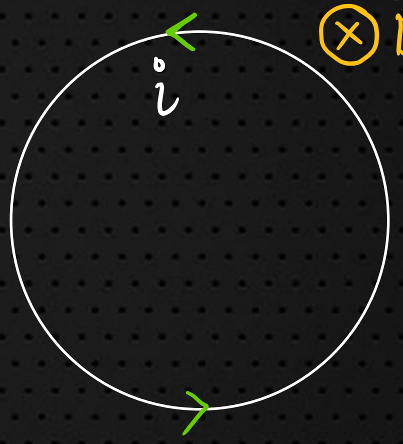
(c) Direction of induced emf (ε) or induced current ($i = \varepsilon / R$) is given by LENZ'S LAW

2. DIRECTION OF INDUCED CURRENT

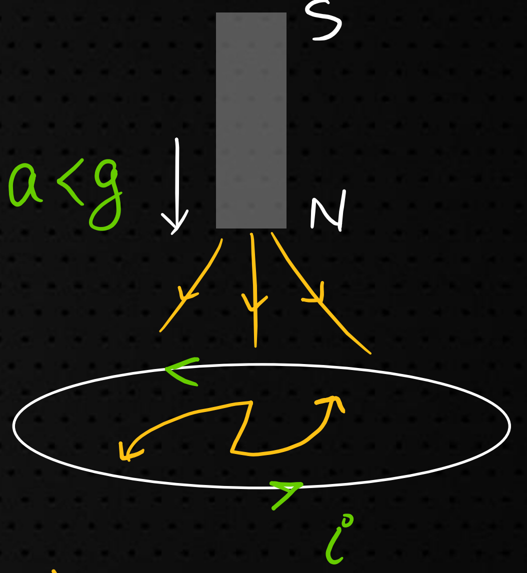
LENZ'S LAW : Direction of i will such that its effect will oppose the change in flux.

Examples:

(a) $\otimes \vec{B} \uparrow$



(b) $a<g \downarrow$

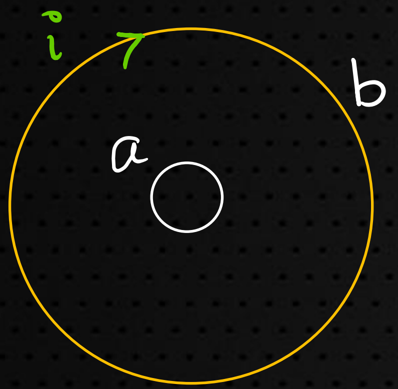


3. CHARGE FLOWN, $q_{flown} = \frac{\Delta\phi}{R}$

(a) $\Delta\phi$ is change in flux, $|\phi_f - \phi_i|$

(b) R is resistance of coil

Example:



$a << b$

If direction of i is reversed,

$$\Delta\phi = 2 \times BA$$

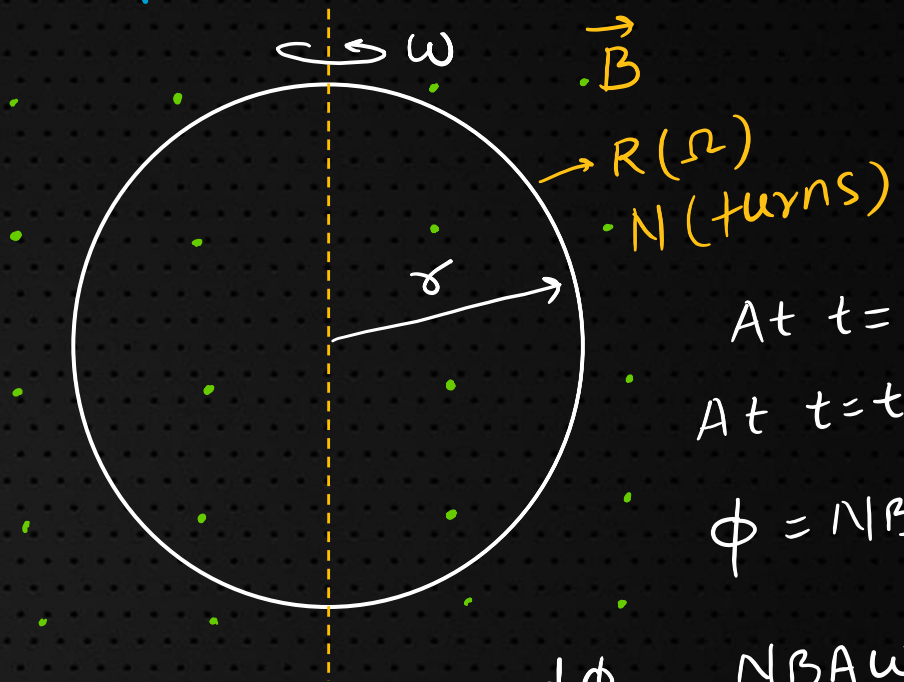
$$= 2 \times \frac{\mu_0 i}{2b} \times \pi a^2$$

q_{flown} in small coil

$$= \frac{\Delta\phi}{R}$$

R is resistance of small coil

4. COIL ROTATION IN UNIFORM MAGNETIC FIELD



At $t=0$, $\vec{A} \parallel \vec{B}$

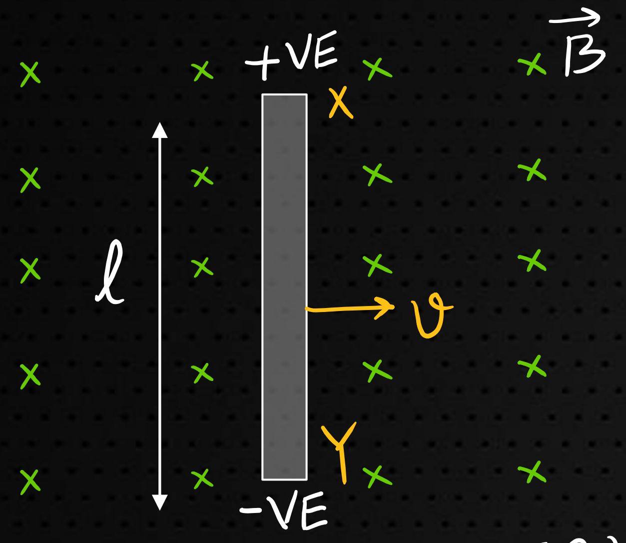
At $t=t$, $\theta = \omega t$

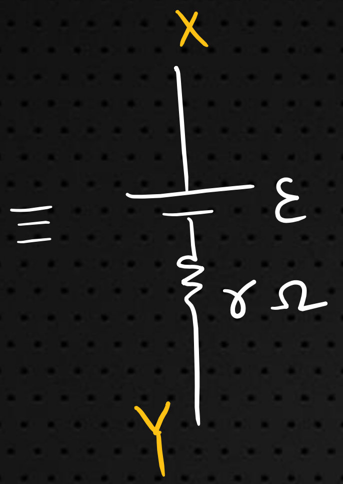
$$\phi = NBA\cos\omega t$$

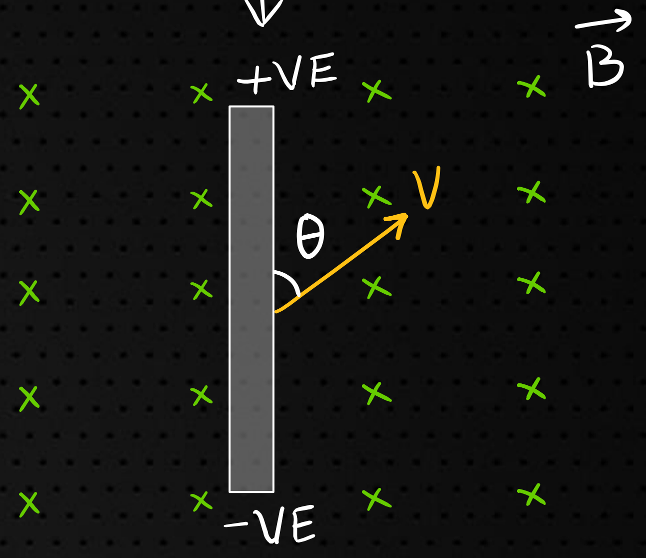
$$\varepsilon = -\frac{d\phi}{dt} = \underbrace{NBA\omega}_{\varepsilon_0} \sin\omega t$$

$$i = \underbrace{\frac{\varepsilon_0}{R}}_{i_0} \sin\omega t$$

5. MOTIONAL EMF







(a) $E = VB$

(b) $\varepsilon = B\ell V$

(c) applicable only if V, B and ℓ are mutually perpendicular

NOTE: If any two are parallel $\varepsilon = 0$

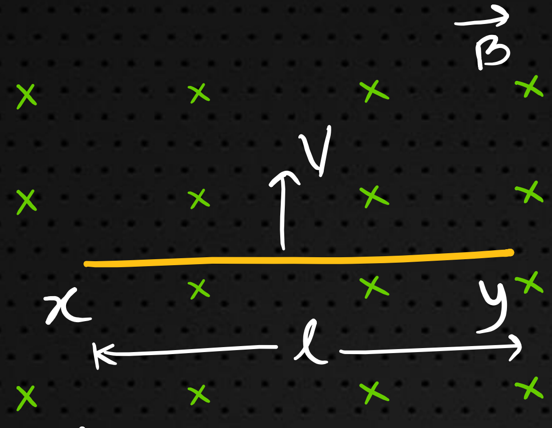
$\varepsilon = B\ell$ (component of V $\perp$ to ℓ)

$= B\ell V \sin\theta$

+VE Polarity is towards direction of $\vec{V} \times \vec{B}$.

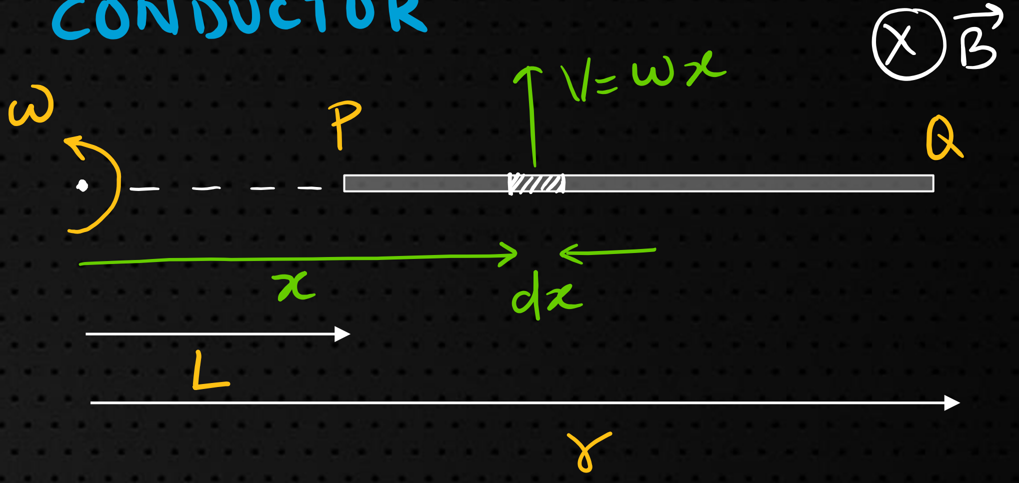
6. MOTIONAL EMF IN RANDOM SHAPED WIRE

$\equiv$



$\varepsilon = B\ell V$

7. MOTIONAL EMF OF ROTATING CONDUCTOR



$d\varepsilon = B\,dx \times \omega x$ $d\varepsilon$

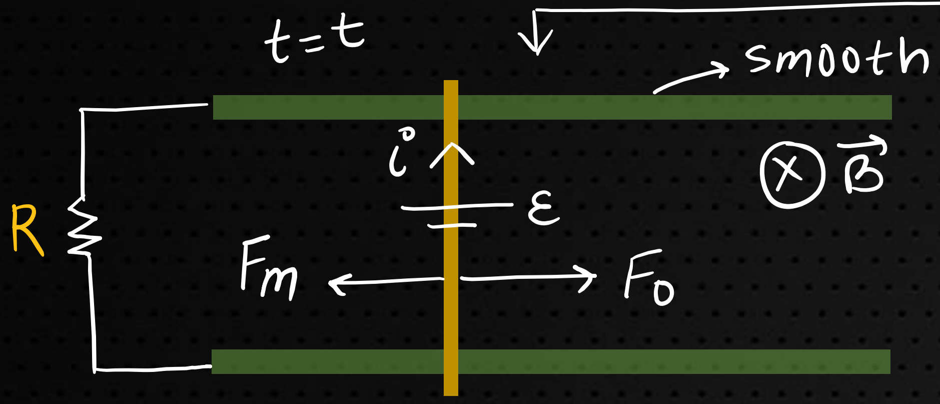
$\Rightarrow \varepsilon = B\omega \int_L^r x\,dx$

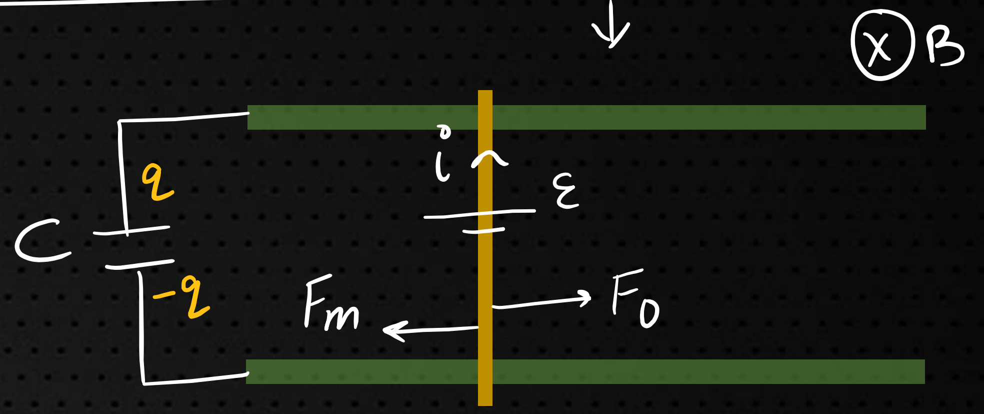
$\Rightarrow \varepsilon = \frac{1}{2} B\omega (r^2 - L^2)$

NOTE: (a) If $L = 0$, $\varepsilon = \frac{1}{2} B\omega r^2$

(b) For above fig. P at higher potential.

8. PARALLEL RAIL TRACK PROBLEMS





$$F_m = ilB = \frac{Blv}{R} \times lB$$

(i) $$\frac{dV}{dt} = \frac{F_0 - F_m}{m}$$

$$V = \frac{F_0 R}{B^2 l^2}\left(1 - e^{-\frac{B^2 l^2 t}{mR}}\right)$$

At $t \to \infty$ (steady state)

Terminal Vel, $V_t = \dfrac{F_0 R}{B^2 l^2}$

$P = F_0 V_t$ = Power across R

$$q = C\varepsilon \Rightarrow q = CBlV$$

$$\Rightarrow i = \frac{dq}{dt} = CBla$$

$$F_0 - F_m = ma$$

$$\Rightarrow F_0 - (CBla)lB = ma$$

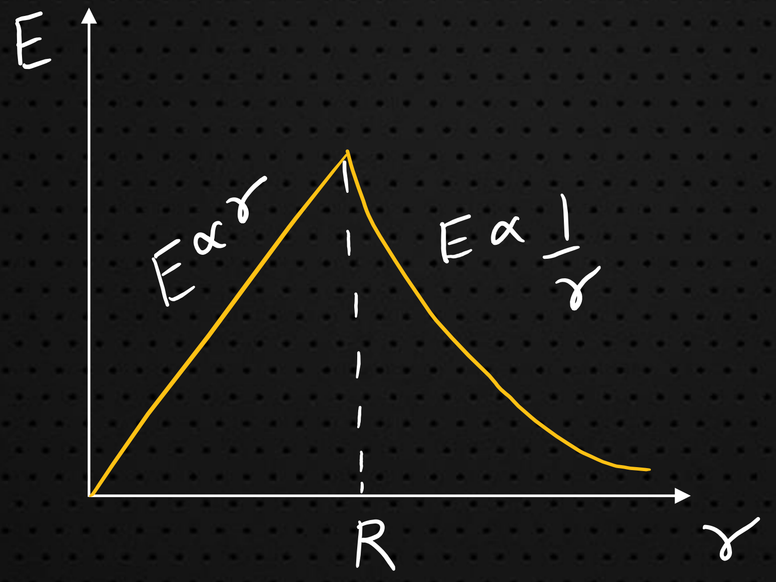
$$\therefore a = \frac{F_0}{m + B^2 l^2 C}$$

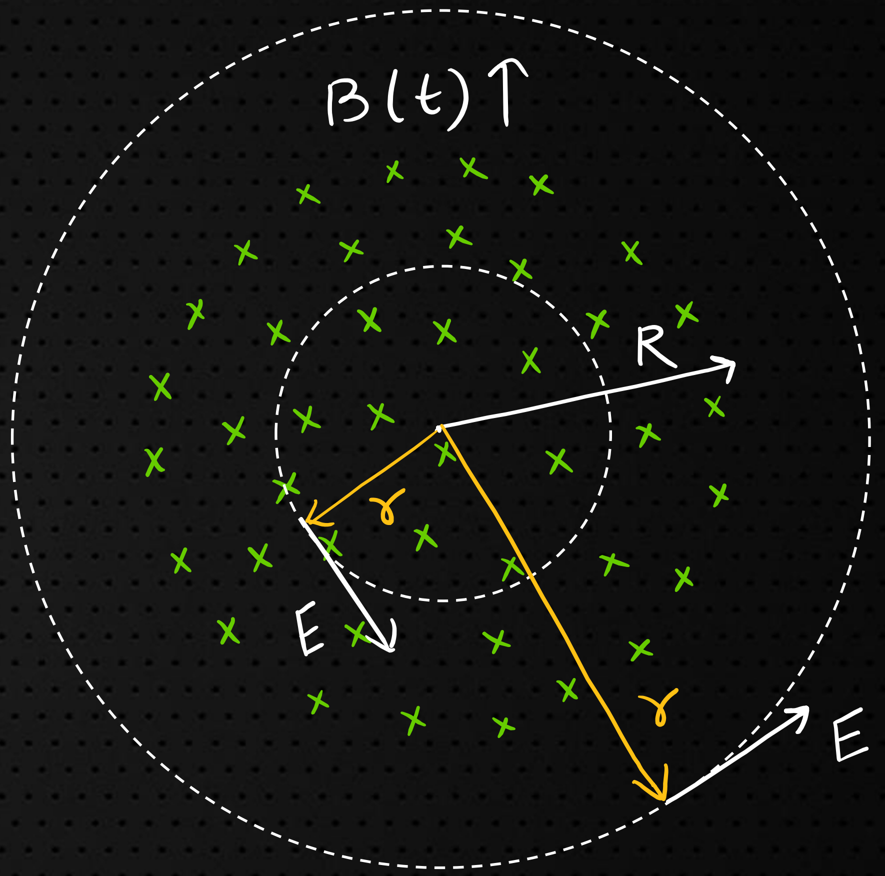
↳ a is const.

9. INDUCED ELECTRIC FIELD IN TIME VARYING MAGNETIC FIELD (Cylindrical region)

For $r < R$, $E = \dfrac{r}{2}\dfrac{dB}{dt}$

For $r \geq R$, $E = \dfrac{R^2}{2r}\dfrac{dB}{dt}$

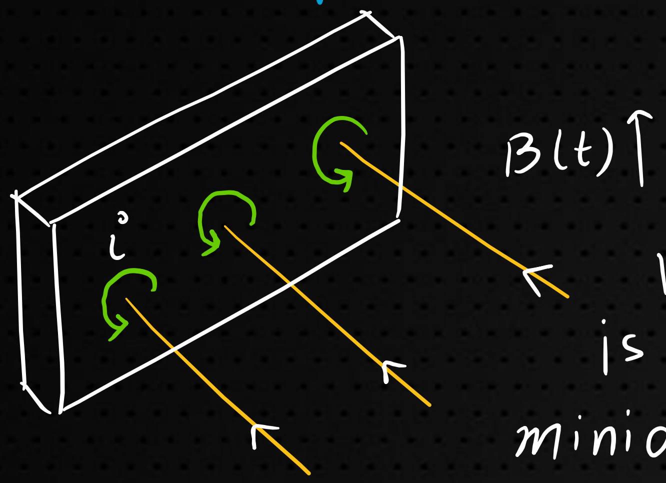




(i) E is in closed Loop

(ii) Here it is Non-Conservative in nature.

10. EDDY CURRENTS



When a metal body is placed in TVMF, miniature currents are induced.

Due to this current heat is dissipated.

11. SELF INDUCTION

↳ A property of coil by which it opposes the change in itself.

(a) For any coil carrying current I,

$\phi \propto I$ (self flux linkage due to own I)

$\Rightarrow \phi = LI$

↳ Inductance

UNIT: Henry, H

(b) If I varies, emf induced

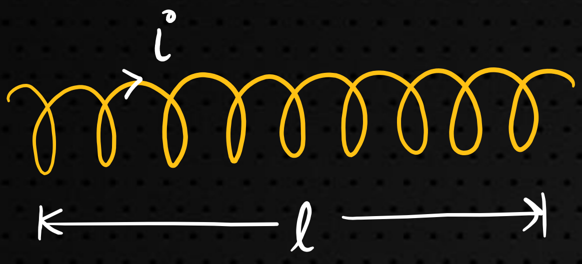
$$\varepsilon = -L\frac{dI}{dt} \quad \text{or} \quad \varepsilon = \left|L\frac{dI}{dt}\right|$$

↳ Polarity of ε can be found by LENZ's LAW

12. HOW TO FIND L ?

Example:

INDUCTOR or SOLENOID



N Turns

A : Area

$$\phi = NBA \Rightarrow \phi = N \times \frac{\mu_0 N}{l} i A$$

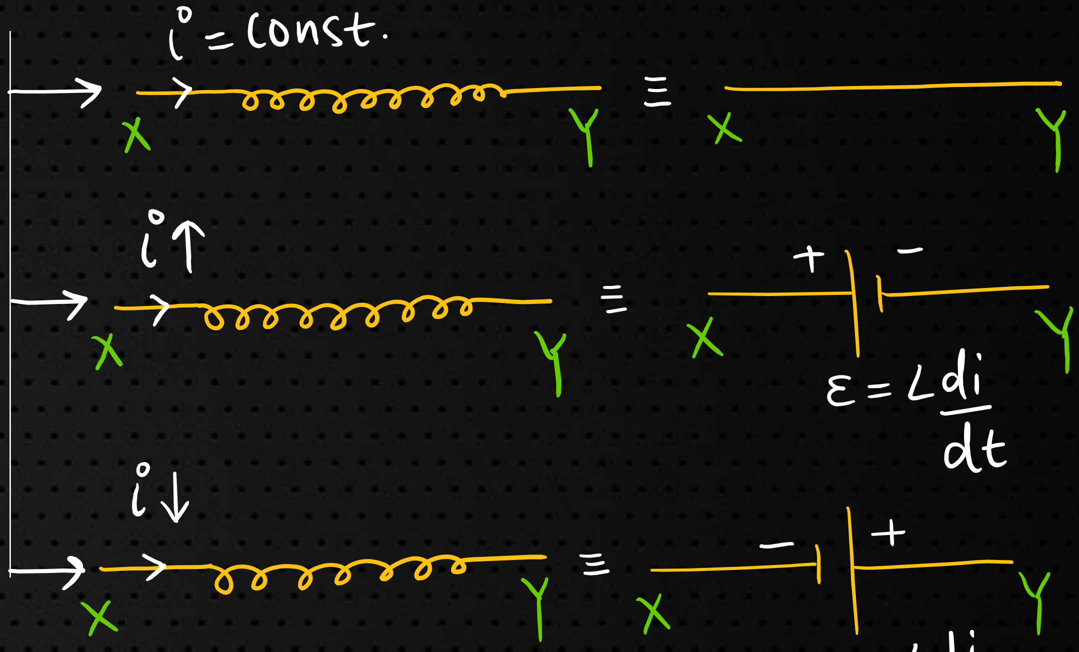
$$\Rightarrow \phi = \left(\frac{\mu_0 N^2 A}{l}\right) i$$

$$\therefore L = \frac{\mu_0 N^2 A}{l}$$

NOTE: If any medium inside

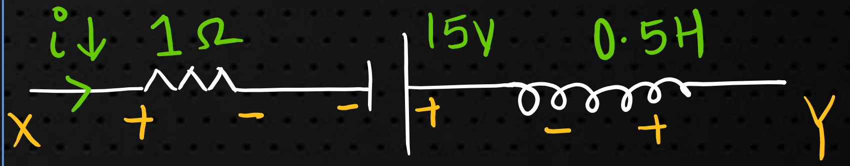
$$L = \frac{\mu_0 \mu_r N^2 A}{l}$$

13. BATTERY POLARITY INDUCED IN INDUCTOR



$\varepsilon = L\frac{di}{dt}$

$\varepsilon = L\frac{di}{dt}$



$i = 5\text{ A}$

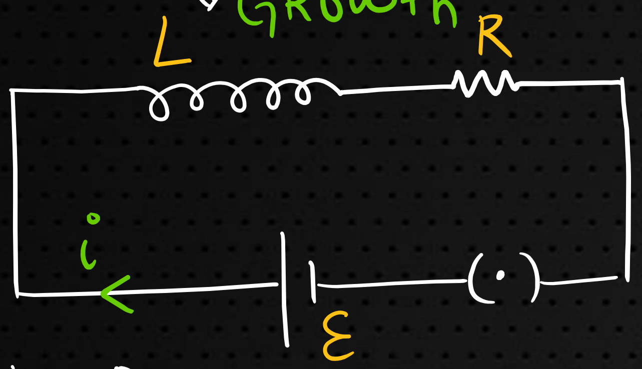
$\frac{di}{dt} = 10\text{ A/s}$

$$V_x - V_y = 5 - 15 - 5 = -15\text{ V}$$

14. GROWTH AND DECAY OF CURRENT (LR)

Growth

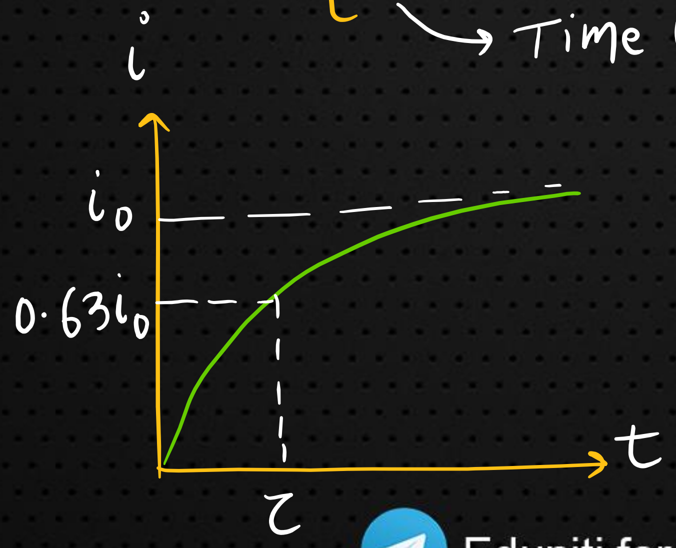
at time t



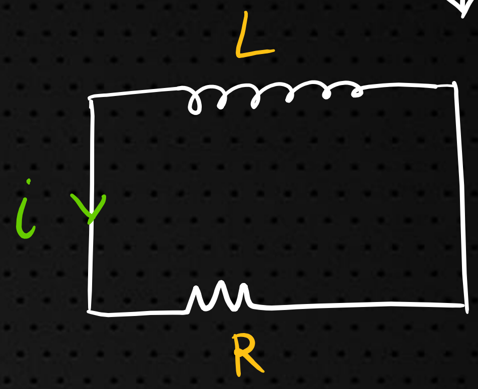
at $t=0$, $i=0$

at $t=t$, $i = i_0\left(1-e^{-Rt/L}\right)$

$\left\{\begin{array}{l} i_0 = \varepsilon/R \\ \tau = L/R \end{array}\right.$ → Time const.



DECAY

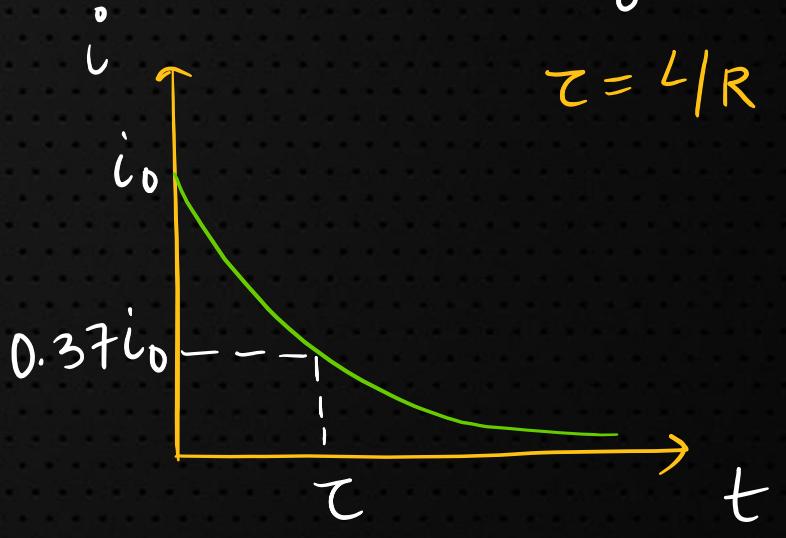


→ at time t

at $t=0$, $i=i_0$ | at $t=t$

$i = i_0 e^{-Rt/L}$

$\tau = L/R$

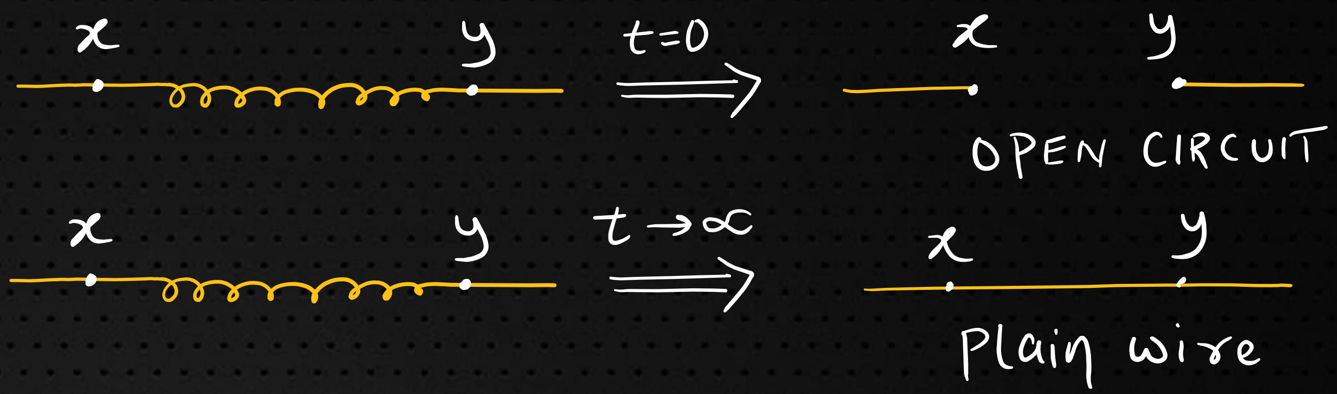


15. ENERGY STORED

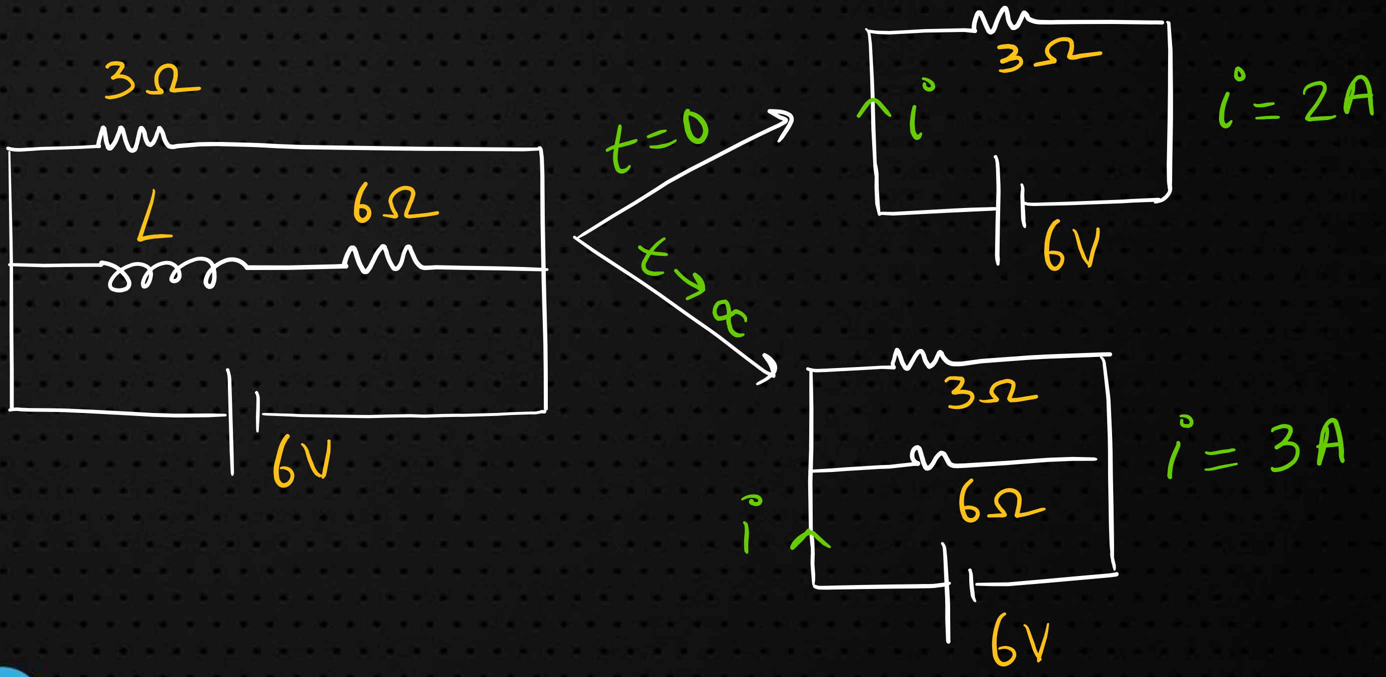
Magnetic energy stored is,

$$U = \frac{1}{2} L i^2$$

16. BEHAVIOR OF L AT $t=0$ AND $t \to \infty$



Example:



17. MUTUAL INDUCTION

Property of pair of coils due to which a change in current in 1 coil is opposed by emf induced in other coil because of ϕ LINKAGE.

$\phi_2 = M i_1$

↳ M is Mutual Inductance

↳ $\mathcal{E}_2 = -M \dfrac{di_1}{dt}$

NOTE

$M = K\sqrt{L_1 L_2}$ K: coupling factor

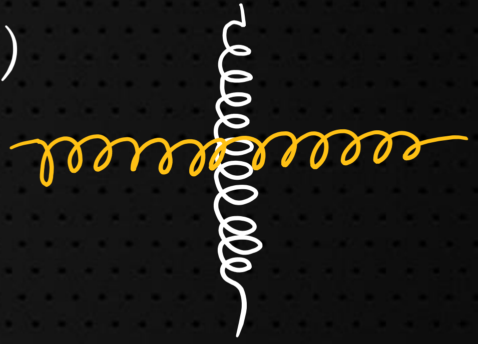
$0 \leq K \leq 1$

L_1 and L_2 are self inductance of coil 1 and coil 2

→ K gives idea of % flux linkage.

Example:

(i)



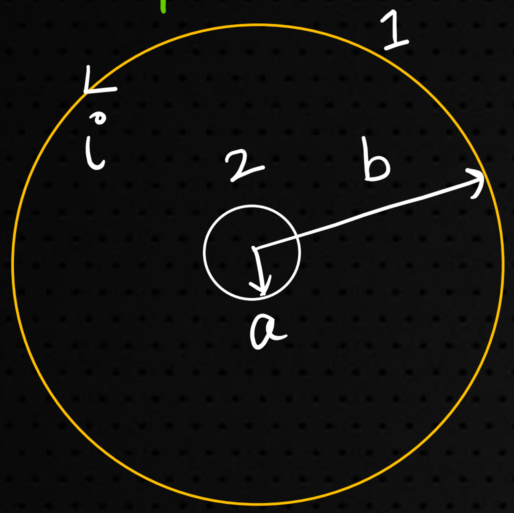
$K = 0$

(ii)

$K = 1$

18. HOW TO FIND M ?

Example:



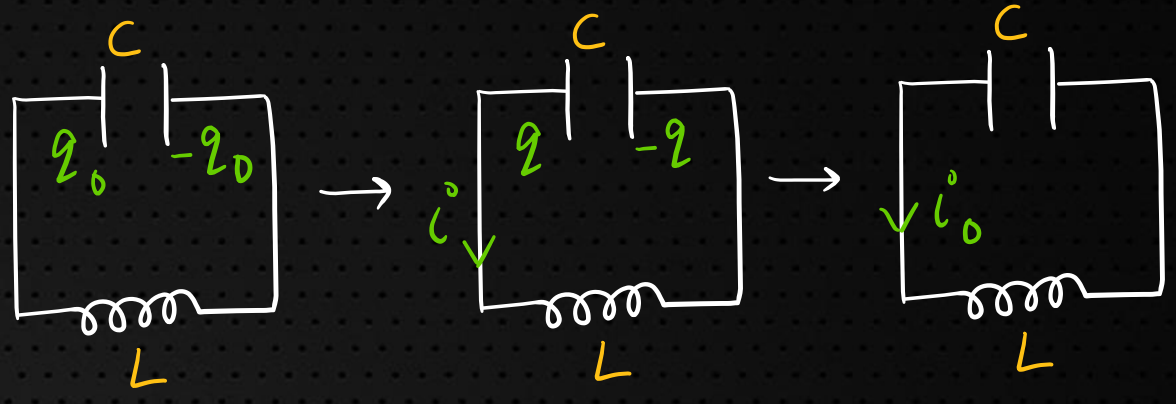
$a << b$

$$\phi_2 = \frac{\mu_0 i}{2b} \times \pi a^2$$

$$\Rightarrow \phi_2 = \left(\frac{\mu_0 \pi a^2}{2b}\right) i$$

$$\therefore M = \frac{\mu_0 \pi a^2}{2b}$$

19. LC OSCILLATION



(a) Total energy is const.

$$\frac{q_0^2}{2C} = \frac{q^2}{2C} + \frac{1}{2} L i^2 = \frac{1}{2} L i_0^2$$

(b) $\omega = \frac{1}{\sqrt{LC}}$, $T = 2\pi\sqrt{LC}$

(c) General Equation

$q = q_0 \sin(\omega t + \phi)$, $i = i_0 \cos(\omega t + \phi)$