#9

PhD Series

Work Energy Theorem

JEE Concept & PYQs

Mohit Goenka, IIT Kharagpur

Revision Series Playlist Link https://bit.ly/3eBbib9

PhD Series https://bit.ly/3cQSxPT

GoldMine Link https://bit.ly/2VhOGFF

CLICK

1.

Work-Energy Theorem

↳ Net work done = change in Kinetic energy

$$W_{net} = K_f - K_i \Rightarrow W_{net} = \Delta K$$

Work can be due to various forces :

- ↳ Conservative force (gravitational, Electrostatic, spring force)
- ↳ Non-Conservative (friction, viscous force)
- ↳ External agent (person applying forces)
- ↳ Tension, Normal

... Continued

Work-Energy Theorem

↳ Net work done = change in Kinetic energy

$$W_{net} = K_f - K_i \Rightarrow W_{net} = \Delta K$$

$$\hookrightarrow W_c + W_{NC} + W_{ext} = \Delta K$$

$$\Rightarrow -\Delta U + W_{NC} + W_{ext} = \Delta K$$

$$\Rightarrow W_{NC} + W_{ext} = \underbrace{\Delta K + \Delta U}_{\text{Total change in Mechanical Energy}}$$

... Continued

Work - Energy Theorem

$\Rightarrow \boxed{W_{NC} + W_{ext} = \Delta K + \Delta U}$

If on $W_{NC} = 0$ & $W_{ext} = 0$ (either they are absent or their work done is zero)

$\Rightarrow \boxed{0 = \Delta K + \Delta U}$

→ Total change in Mech Energy is zero

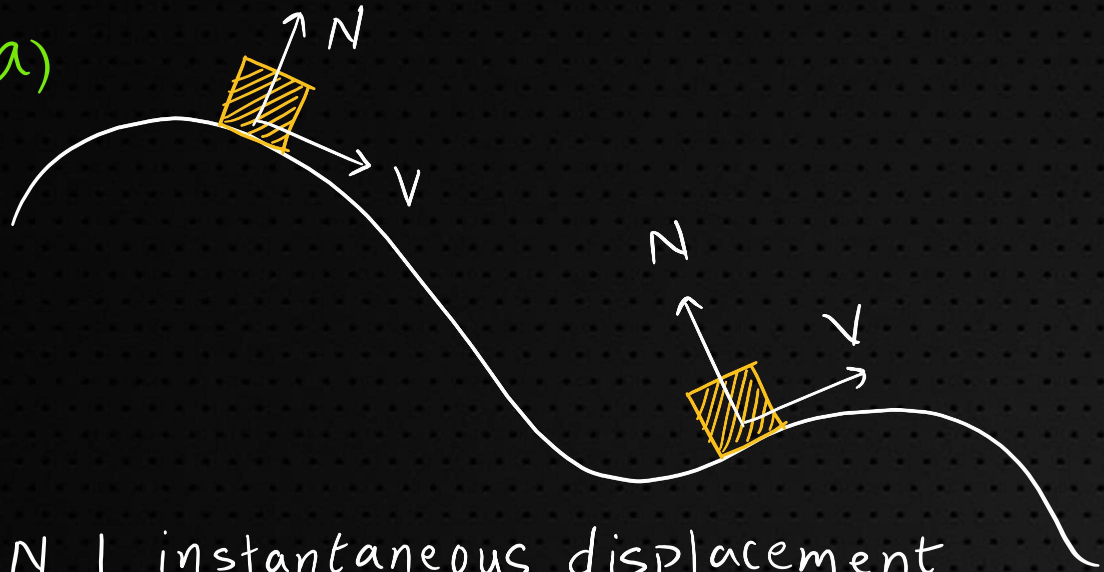
$\Rightarrow$ Energy Conservation

or $0 = K_f - K_i + U_f - U_i$

$\Rightarrow \boxed{K_i + U_i = K_f + U_f}$

2. Sitiuation where $W = 0$

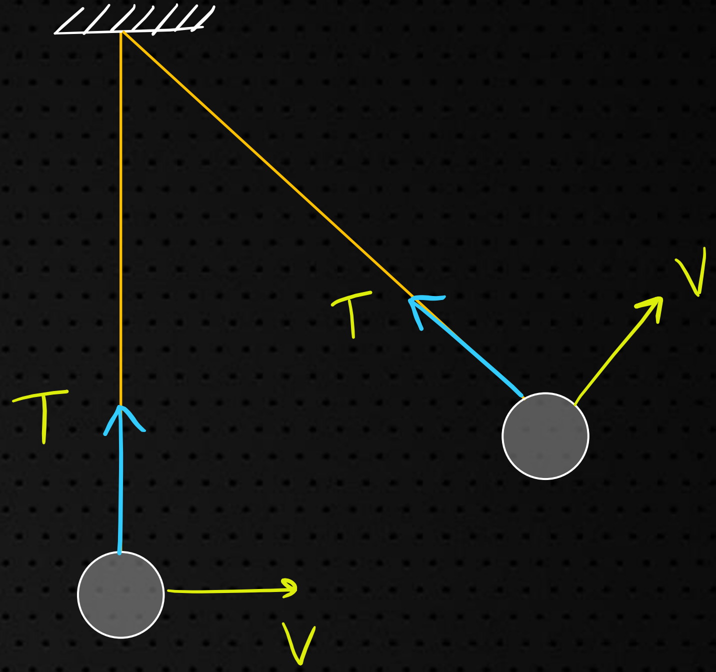
(a)



$N \perp$ instantaneous displacement

$\Rightarrow W_N = 0$

(b)

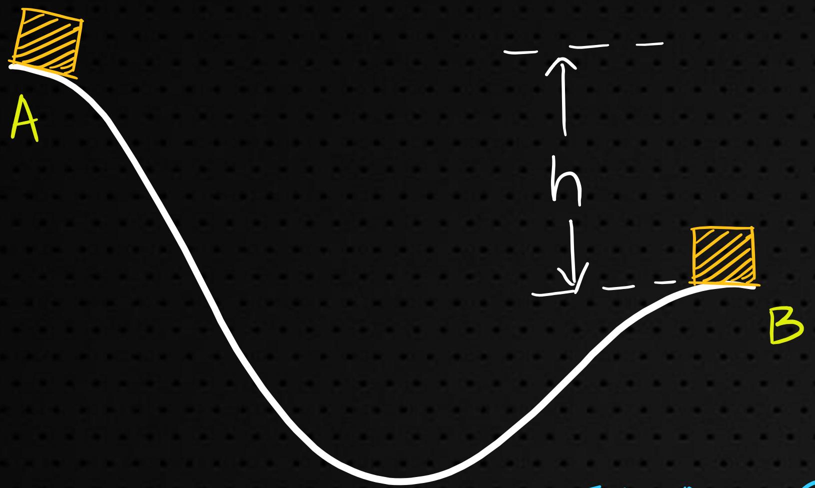


$T \perp$ to instantaneous displacement of bob

$\Rightarrow W_T = 0$

3. How to write ΔU

(a)



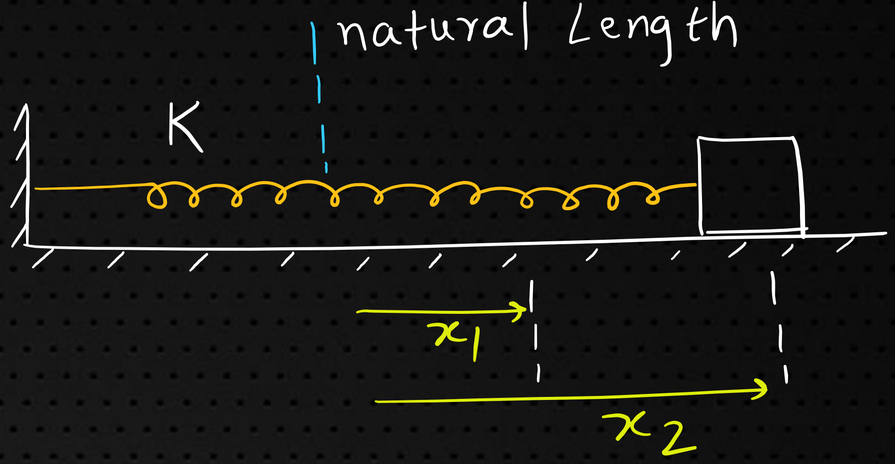
If $A \rightarrow B$

$$\Delta U = -mgh$$

If $B \rightarrow A$

$$\Delta U = mgh$$

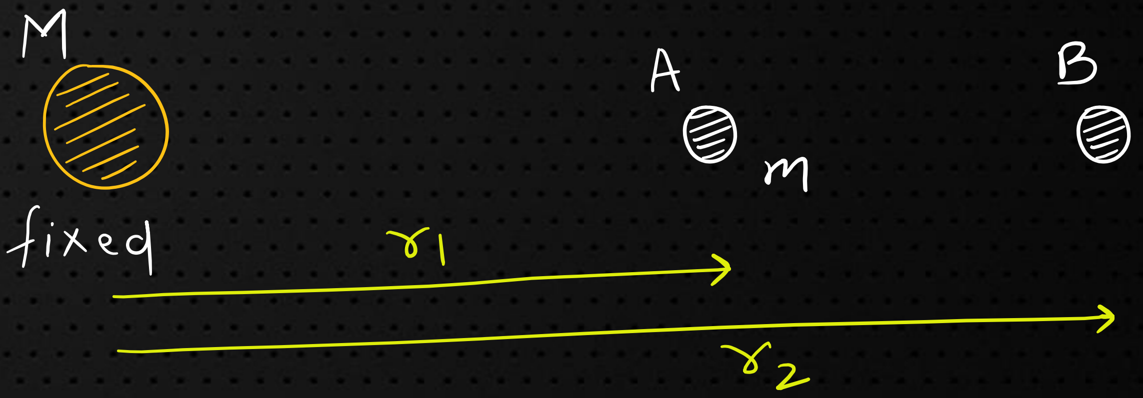
(b)



From $x_1 \rightarrow x_2$

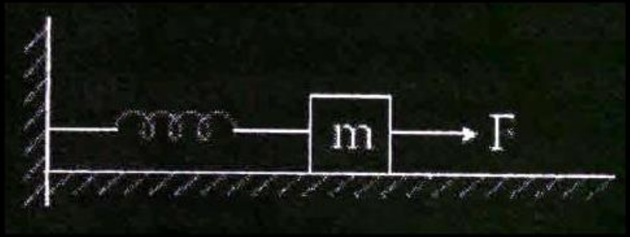
$$\Delta U = \frac{1}{2}Kx_2^2 - \frac{1}{2}Kx_1^2$$

(c)



$A \rightarrow B$, $\Delta U = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$

Ex 1. A block of mass m, lying, on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is JEE 2019

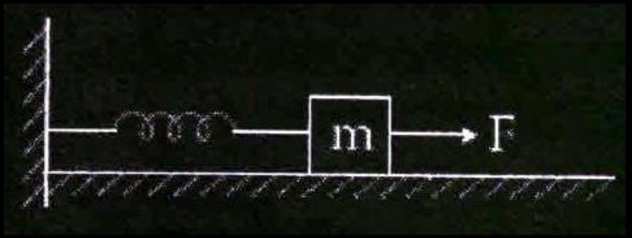


(a) $\dfrac{2F}{\sqrt{mk}}$ (b) $\dfrac{F}{\pi\sqrt{mk}}$

(c) $\dfrac{\pi F}{\sqrt{mk}}$ **(d)** $\dfrac{F}{\sqrt{mk}}$

Ex 1. A block of mass m, lying, on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is

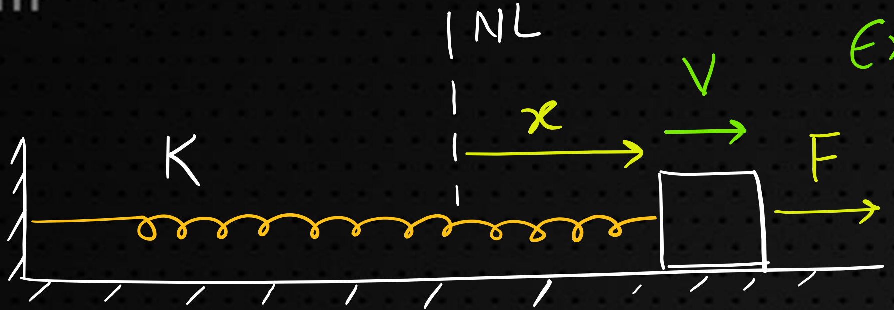
JEE 2019



(a) $\dfrac{2F}{\sqrt{mk}}$ (b) $\dfrac{F}{\pi\sqrt{mk}}$

(c) $\dfrac{\pi F}{\sqrt{mk}}$ **(d)** $\dfrac{F}{\sqrt{mk}}$

Sol^n:



V is max when $F = F_s$ (spring force)

$F = Kx \Rightarrow x = \dfrac{F}{K}$

$W_{ext} = \Delta K + \Delta U$

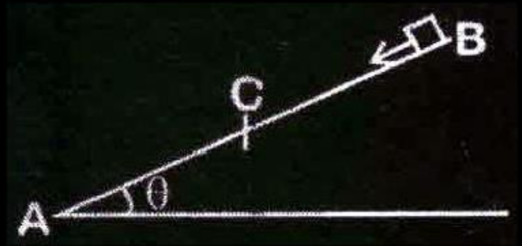
$\Rightarrow Fx = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$

$\Rightarrow \dfrac{F^2}{K} = \dfrac{1}{2}mv^2 + \dfrac{F^2}{2K}$

$\Rightarrow \boxed{V = \dfrac{F}{\sqrt{mK}}}$

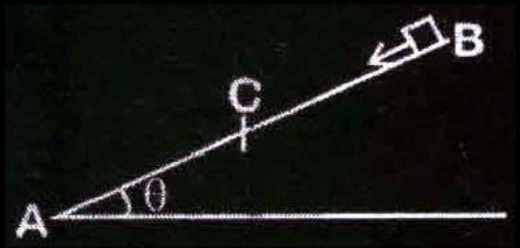
Ex 2. A small block starts slipping down from a point B on an inclined plane AB, which is making an

JEE 2020



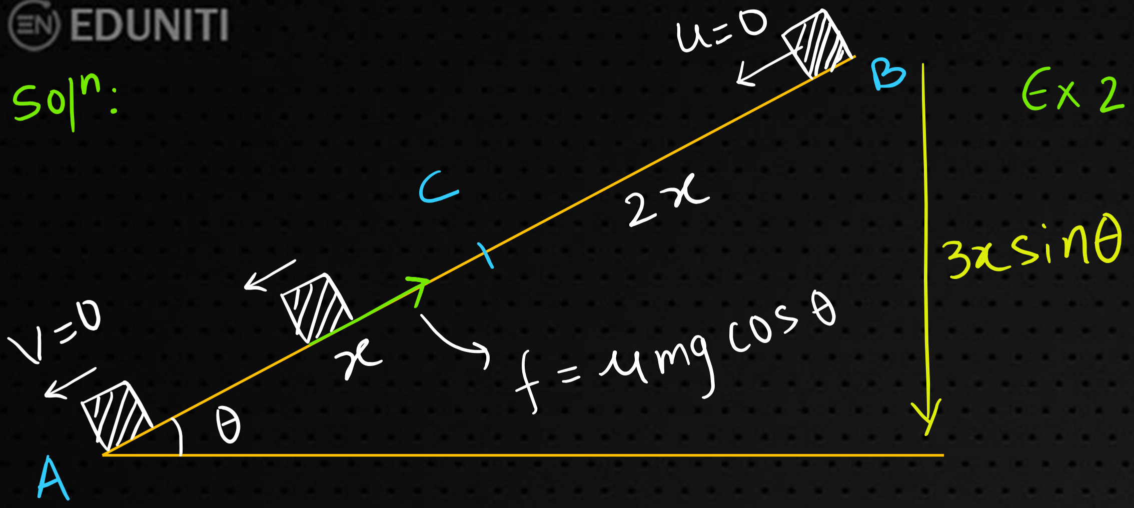
angle θ with the horizontal. Section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k \tan\theta$. The value of k is

Ex 2. A small block starts slipping down from a point B on an inclined plane AB, which is making an **JEE 2020**



angle θ with the horizontal. Section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k \tan\theta$. The value of k is

Sol^n:



$$W_f = \Delta K + \Delta U$$

$$\Rightarrow -\mu mg\cos\theta \cdot x = 0 - mg \cdot 3x\sin\theta$$

$$\Rightarrow \mu = 3\tan\theta$$

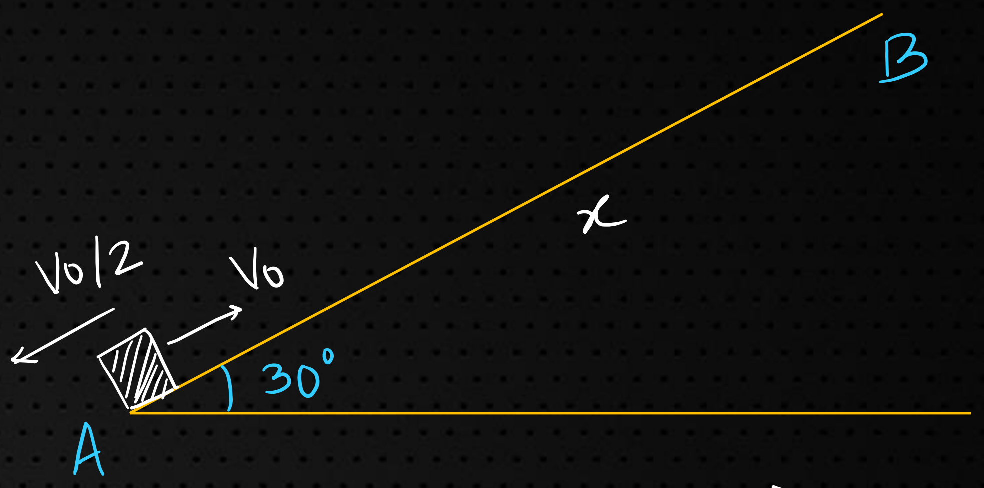
$$\therefore \boxed{k = 3}$$

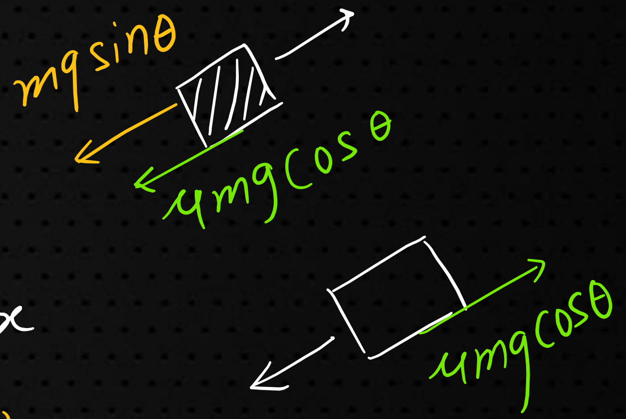
Ex 3. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $v_0/2$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $I/1000$. The nearest integer to I is________.

JEE 2020

Ex 3. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $v_0/2$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $I/1000$. The nearest integer to I is________.

JEE 2020





Sol^n:

$$W_f = \Delta K + \Delta U$$

$$\Rightarrow 2\times(-\mu mg\cos\theta\cdot x) = \frac{1}{2}m\left(\frac{V_0}{2}\right)^2 - \frac{1}{2}mV_0^2 + 0$$

$$\Rightarrow -2\mu mg\cos\theta\cdot x = -\frac{3}{8}mV_0^2 \quad \text{— (i)}$$

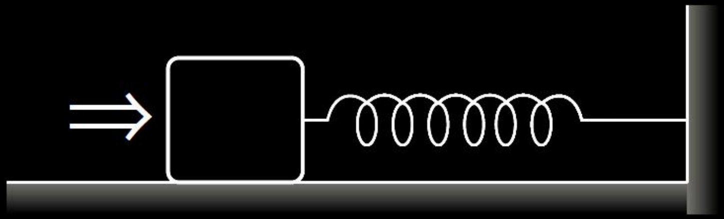
$$A\rightarrow B: \quad 0 = V_0^2 - 2(g\sin\theta + \mu g\cos\theta)\,x$$

$$\Rightarrow V_0^2 = 2g(\sin\theta + \mu\cos\theta)\cdot x \quad \text{— (ii)}$$

from (i) & (ii) $\therefore \mu = \frac{3}{5}\tan\theta \Rightarrow \mu = \frac{3}{5}\times\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{5}$

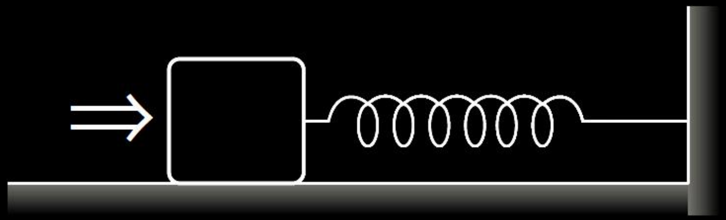
$$\Rightarrow \frac{\sqrt{3}}{5} = \frac{I}{1000} \Rightarrow \boxed{I \approx 346}$$

Ex 4. A block of mass 0.18 kg is attached to a spring of force constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $v = \frac{N}{10}$. Then N is



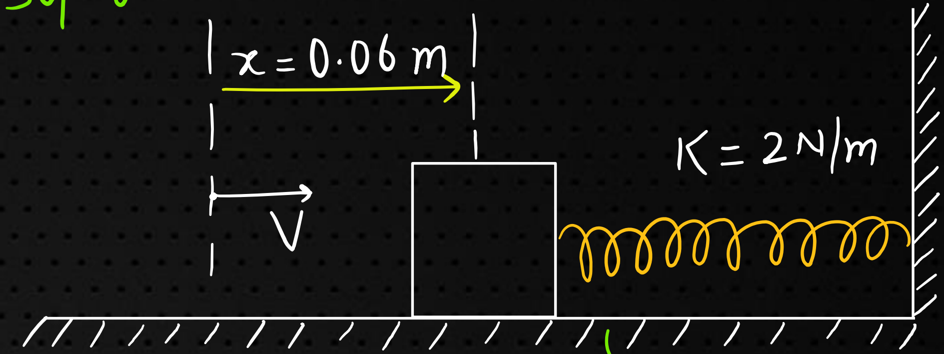
(2011)

Ex 4. A block of mass 0.18 kg is attached to a spring of force constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $v = \frac{N}{10}$. Then N is



(2011)

Sol^n:



$$W_f = \Delta U + \Delta K$$

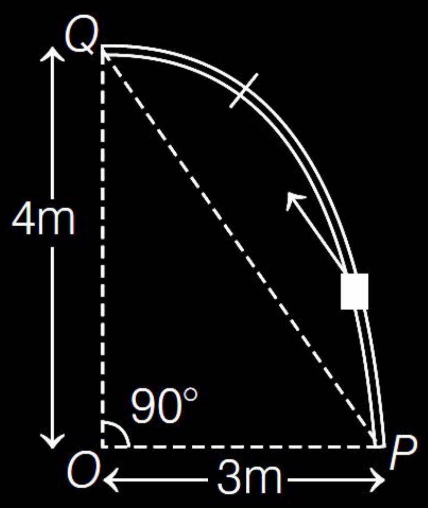
$$\Rightarrow -\mu m g x = \frac{1}{2}Kx^2 + \left(0 - \frac{1}{2}mV^2\right)$$

$$\Rightarrow V = \sqrt{\frac{Kx^2}{m} + 2\mu g x}$$

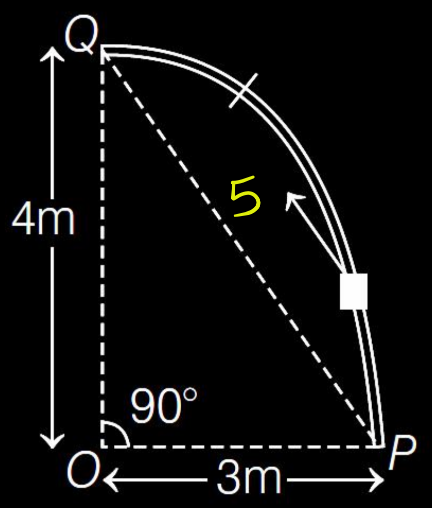
put $K = 2\,N/m$, $x = 0.06\,m$, $m = 0.18\,Kg$, $\mu = 0.1$, $g = 10\,m/s^2$

$$\Rightarrow V = \frac{4}{10} \quad \therefore \boxed{N = 4}$$

Ex5. Consider an elliptically shaped rail PQ in the vertical plane with $OP = 3$ m and $OQ = 4$ m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see figure). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ J. The value of n is (take acceleration due to gravity $= 10 \text{ ms}^{-2}$) **(2014 Adv.)**



Ex5. Consider an elliptically shaped rail PQ in the vertical plane with $OP = 3$ m and $OQ = 4$ m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see figure). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ J. The value of n is (take acceleration due to gravity $= 10\ \mathrm{ms^{-2}}$) **(2014 Adv.)**

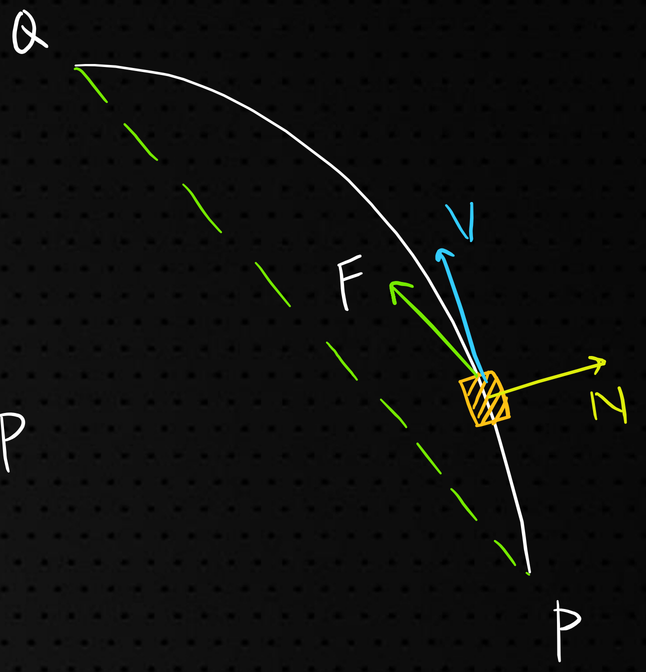


Sol^n:

(i) $N \perp V$

$\Rightarrow W_N = 0$

(ii) $F \parallel QP$



$$W_F = \Delta K + \Delta U \Rightarrow \vec{F}\cdot\vec{s} = K_f + mgh$$

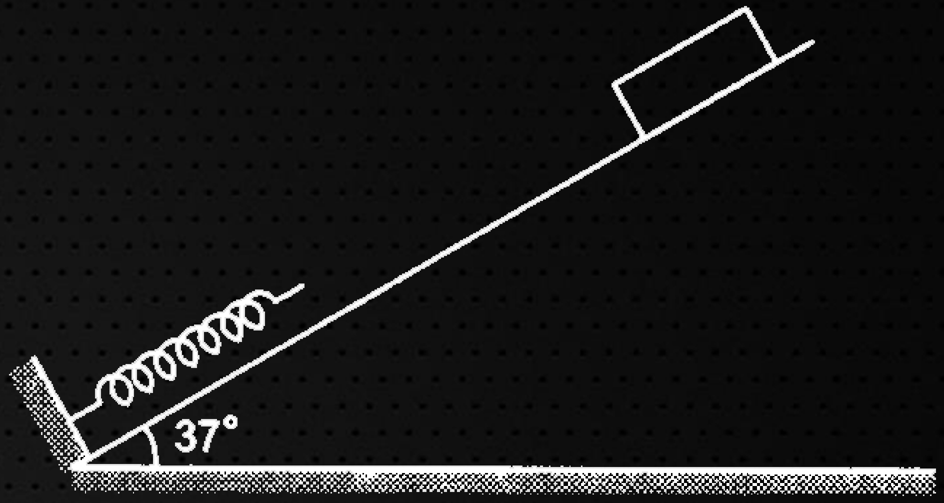
$$\Rightarrow 18 \times 5 = K_f + 1 \times 10 \times 4$$

$$\therefore K_f = 50\ \mathrm{J} \Rightarrow \boxed{n = 5}$$

Ex 6. Figure (8-E7) shows a spring fixed at the bottom end of an incline of inclination 37°. A small block of mass 2 kg starts slipping down the incline from a point 4·8 m away from the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance of 1 m up the incline. Find (a) the friction coefficient between the plane and the block

Take $g = 10\ \text{m/s}^2$.

HCV - WEP - Q43

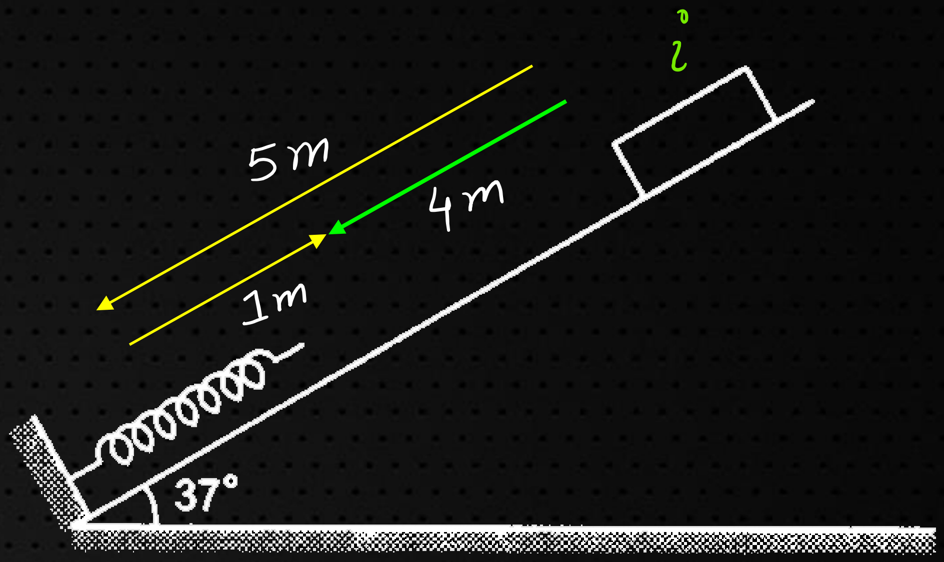


Ex 6. Figure (8-E7) shows a spring fixed at the bottom end of an incline of inclination 37°. A small block of mass 2 kg starts slipping down the incline from a point 4·8 m away from the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance of 1 m up the incline. Find (a) the friction coefficient between the plane and the block

Take $g = 10\ m/s^2$.

0.2m

HCV - WEP - Q43



Soln:

$$W_f = \Delta U + \Delta K \Rightarrow -\mu mg\cos\theta \times (5+1) = -mg \times 4\sin\theta + 0$$

$$\Rightarrow \mu = \frac{2}{3}\tan\theta = \frac{2}{3} \times \frac{3}{4} = \boxed{0.5}$$

Revision Series Playlist Link https://bit.ly/3eBbib9

PhD Series https://bit.ly/3cQSxPT

GoldMine Link https://bit.ly/2VhOGFF

CLICK