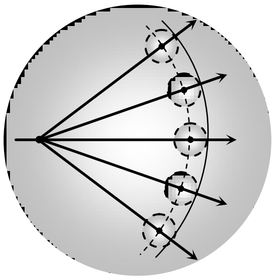


Wave Optics

Light Propagation

Light is a form of energy which generally gives the sensation of sight.

(1) **Different theories**

(2) **Optical phenomena explained (√) or not explained (×) by the different theories of light**

(3) **Wave front**

(i) Suggested by Huygens

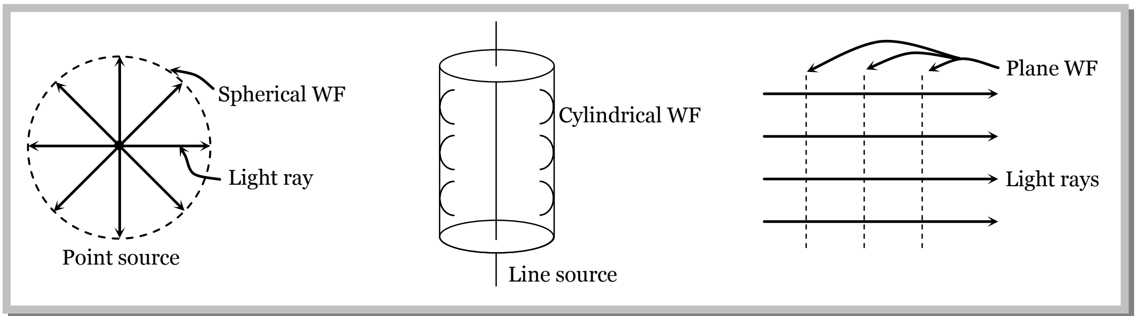
(ii) The locus of all particles in a medium, vibrating in the same phase is called Wave Front (WF)

(iii) The direction of propagation of light (ray of light) is perpendicular to the WF.

Newtons corpuscular theory	Huygen's wave theory	Maxwell's EM wave theory	Einstein's quantum theory	de-Broglie's dual theory of light
(i) Based on Rectilinear propagation of light	(i) Light travels in a hypothetical medium ether (high elasticity very low density) as waves	(i) Light travels in the form of EM waves with speed in free space $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	(i) Light is produced, absorbed and propagated as packets of energy called photons	(i) Light propagates both as particles as well as waves
(ii) Light propagates in the form of tiny particles called Corpuscles. Colour of light is due to different size of corpuscles	(ii) He proposed that light waves are of longitudinal nature. Later on it was found that they are transverse	(ii) EM waves consists of electric and magnetic field oscillation and they do not require material medium to travel	(ii) Energy associated with each photon $E = h\nu = \frac{hc}{\lambda}$ h = planks constant $= 6.6 \times 10^{-34}\ J\text{-sec}$ ν = frequency λ = wavelength	(ii) Wave nature of light dominates when light interacts with light. The particle nature of light dominates when the light interacts with matter (micro-scopic particles)

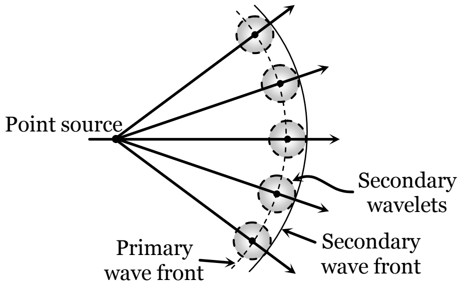
S. No.	Phenomena	Theory				
		Corpuscular	Wave	E.M. wave	Quantum	Dual
(i)	Rectilinear Propagation	√	√	√	√	√
(ii)	Reflection	√	√	√	√	√
(iii)	Refraction	√	√	√	√	√
(iv)	Dispersion	×	√	√	×	√
(v)	Interference	×	√	√	×	√
(vi)	Diffraction	×	√	√	×	√
(vii)	Polarisation	×	√	√	×	√
(viii)	Double refraction	×	√	√	×	√
(ix)	Doppler's effect	×	√	√	×	√
(x)	Photoelectric effect	×	×	×	√	√

(iv) **Types of wave front**



(v) Every point on the given wave front acts as a source of new disturbance called secondary wavelets. Which travel in all directions with the velocity of light in the medium.

A surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new wave front at that instant. This is called secondary wave front



Note : ❑ Wave front always travels in the forward direction of the medium.

❑ Light rays is always normal to the wave front.

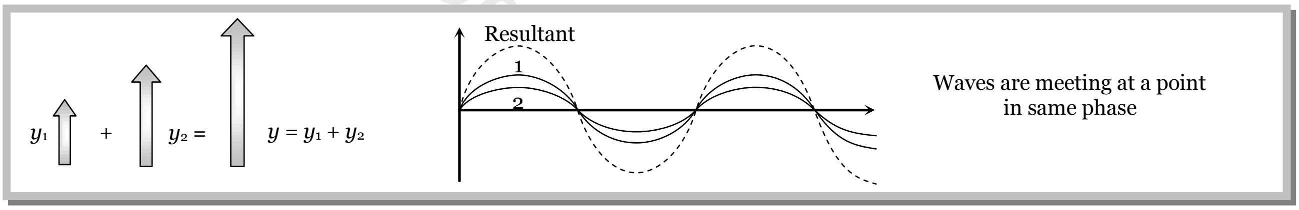
❑ The phase difference between various particles on the wave front is zero.

Principle of Super Position

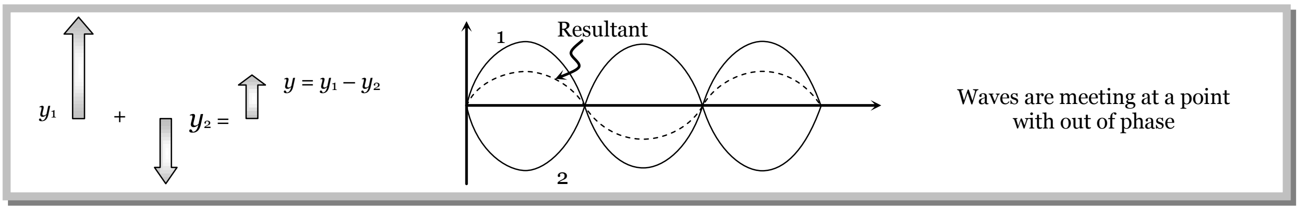
When two or more than two waves superimpose over each other at a common particle of the medium then the resultant displacement (y) of the particle is equal to the vector sum of the displacements (y_1 and y_2) produced by individual waves. *i.e.* $\vec{y} = \vec{y}_1 + \vec{y}_2$

(1) **Graphical view :**

(i)



(ii)

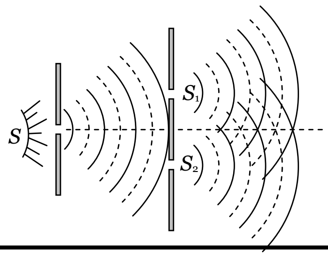
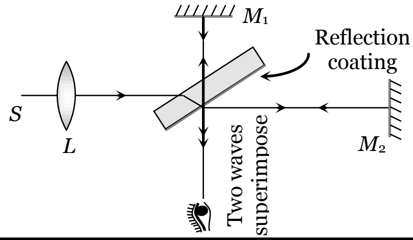


(2) **Phase / Phase difference / Path difference / Time difference**

(i) Phase : The argument of sine or cosine in the expression for displacement of a wave is defined as the phase. For displacement $y = a \sin \omega t$; term ωt = phase or instantaneous phase

(ii) Phase difference (ϕ) : The difference between the phases of two waves at a point is called phase difference *i.e.* if $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$ so phase difference = ϕ

(iii) Path difference (Δ) : The difference in path length's of two waves meeting at a point is called path difference between the waves at that point. Also $\Delta = \frac{\lambda}{2\pi} \times \phi$

Division of wave front	Division of amplitude
The light source is narrow	Light sources is extended. Light wave partly reflected (50%) and partly transmitted (50%)
The wave front emitted by a narrow source is divided in two parts by reflection of refraction.	The amplitude of wave emitted by an extend source of light is divided in two parts by partial reflection and partial refraction.
The coherent sources obtained are imaginary *e.g.* Fresnel's biprism, Llyod's mirror Youngs' double slit *etc.*	The coherent sources obtained are real *e.g.* Newtons rings, Michelson's interferrometer colours in thin films
	

(iv) Time difference (*T.D.*) : Time difference between the waves meeting at a point is $\boldsymbol{T.D.} = \frac{\boldsymbol{T}}{\boldsymbol{2\pi}} \times \boldsymbol{\phi}$

(3) **Resultant amplitude and intensity**

If suppose we have two waves $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \phi)$; where $a_1, a_2 =$ Individual amplitudes, ϕ = Phase difference between the waves at an instant when they are meeting a point. I_1, I_2 = Intensities of individual waves

Resultant amplitude : After superimposition of the given waves resultant amplitude (or the amplitude of resultant wave) is given by $\boldsymbol{A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi}}$

For the interfering waves $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \cos \omega t$, Phase difference between them is 90°. So resultant amplitude $A = \sqrt{a_1^2 + a_2^2}$

Resultant intensity : As we know intensity $\propto$ (Amplitude)² $\Rightarrow I_1 = ka_1^2, I_2 = ka_2^2$ and $I = kA^2$ (k is a proportionality constant). Hence from the formula of resultant amplitude, we get the following formula of resultant intensity $\boldsymbol{I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi}$

Note : ❑ The term $2\sqrt{I_1 I_2} \cos \phi$ is called interference term. For incoherent interference this term is zero so resultant intensity $I = I_1 + I_2$

(4) **Coherent sources**

The sources of light which emits continuous light waves of the same wavelength, same frequency and in same phase or having a constant phase difference are called coherent sources.

Two coherent sources are produced from a single source of light by adopting any one of the following two methods

Note : ❑ Laser light is highly coherent and monochromatic.

❑ Two sources of light, whose frequencies are not same and phase difference between the waves emitted by them does not remain constant *w.r.t.* time are called non-coherent.

❑ The light emitted by two independent sources (candles, bulbs *etc.*) is non-coherent and interference phenomenon cannot be produced by such two sources.

❑ The average time interval in which a photon or a wave packet is emitted from an atom is defined as the **time of coherence**. It is $\tau_c = \frac{L}{c} = \frac{\text{Distance of coherence}}{\text{Velocity of light}}$, it's value is of the order of 10^{-10} *sec*.

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Interference of Light

When two waves of exactly same frequency (coming from two coherent sources) travels in a medium, in the same direction simultaneously then due to their superposition, at some points intensity of light is maximum while at some other points intensity is minimum. This phenomenon is called Interference of light.

(1) **Types :** It is of following two types

Constructive interference	Destructive interference
(i) When the waves meets a point with same phase, constructive interference is obtained at that point (*i.e.* maximum light)	(i) When the wave meets a point with opposite phase, destructive interference is obtained at that point (*i.e.* minimum light)
(ii) Phase difference between the waves at the point of observation $\phi = 0^o$ or $2n\pi$	(ii) $\phi = 180^o$ or $(2n-1)\pi$; $n = 1, 2, ...$ or $(2n+1)\pi$; $n = 0,1,2.....$
(iii) Path difference between the waves at the point of observation $\Delta = n\lambda$ (*i.e.* even multiple of $\lambda/2$)	(iii) $\Delta = (2n-1)\frac{\lambda}{2}$ (*i.e.* odd multiple of $\lambda/2$)
(iv) Resultant amplitude at the point of observation will be maximum $a_1 = a_2 \Rightarrow A_{min} = 0$ If $a_1 = a_2 = a_0 \Rightarrow A_{max} = 2a_0$	(iv) Resultant amplitude at the point of observation will be minimum $A_{min} = a_1 - a_2$ If $a_1 = a_2 \Rightarrow A_{min} = 0$
(v) Resultant intensity at the point of observation will be maximum $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{max} = 2I_0$	(v) Resultant intensity at the point of observation will be minimum $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$ $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{min} = 0$

(2) **Resultant intensity due to two identical waves :**

For two coherent sources the resultant intensity is given by $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

For identical source $I_1 = I_2 = I_0 \Rightarrow I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi = 4I_0\cos^2\frac{\phi}{2}$ $\left[1 + \cos\theta = 2\cos^2\frac{\theta}{2}\right]$

Note : ❑ In interference redistribution of energy takes place in the form of maxima and minima.

❑ Average intensity : $I_{av} = \frac{I_{max} + I_{min}}{2} = I_1 + I_2 = a_1^2 + a_2^2$

❑ Ratio of maximum and minimum intensities :

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$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2}+1}{\sqrt{I_1/I_2}-1}\right)^2 = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2 = \left(\frac{a_1/a_2+1}{a_1/a_2-1}\right)^2 \text{ also}$$

$$\sqrt{\frac{I_1}{I_2}} = \frac{a_1}{a_2} = \left(\frac{\sqrt{\frac{I_{max}}{I_{min}}}+1}{\sqrt{\frac{I_{max}}{I_{min}}}-1}\right)$$

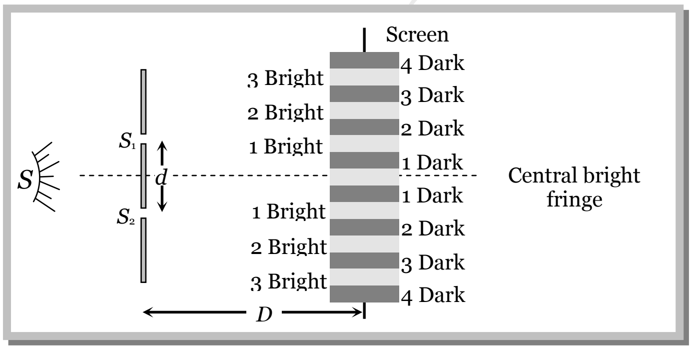
- If two waves having equal intensity ($I_1 = I_2 = I_0$) meets at two locations P and Q with path difference Δ_1 and Δ_2 respectively then the ratio of resultant intensity at point P and Q will be

$$\frac{I_P}{I_Q} = \frac{\cos^2\frac{\phi_1}{2}}{\cos^2\frac{\phi_2}{2}} = \frac{\cos^2\left(\frac{\pi\Delta_1}{\lambda}\right)}{\cos^2\left(\frac{\pi\Delta_2}{\lambda}\right)}$$

Young's Double Slit Experiment (YDSE)

Monochromatic light (single wavelength) falls on two narrow slits S_1 and S_2 which are very close together acts as two coherent sources, when waves coming from two coherent sources (S_1, S_2) superimposes on each other, an interference pattern is obtained on the screen. In YDSE alternate bright and dark bands obtained on the screen. These bands are called Fringes.

d = Distance between slits
D = Distance between slits and screen
λ = Wavelength of monochromatic light emitted from source



(1) Central fringe is always bright, because at central position $\phi = 0^o$ or $\Delta = 0$

(2) The fringe pattern obtained due to a slit is more bright than that due to a point.

(3) If the slit widths are unequal, the minima will not be complete dark. For very large width uniform illumination occurs.

(4) If one slit is illuminated with red light and the other slit is illuminated with blue light, no interference pattern is observed on the screen.

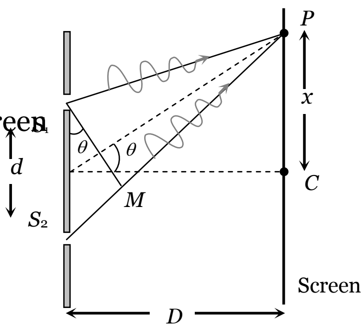
(5) If the two coherent sources consist of object and it's reflected image, the central fringe is dark instead of bright one.

(6) **Path difference**

Path difference between the interfering waves meeting at a point P on the screen is given by $\Delta = \frac{xd}{D} = d\sin\theta$

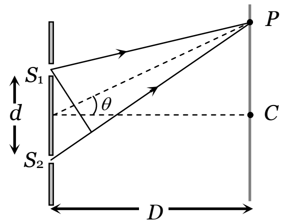
where x is the position of point P from central maxima.

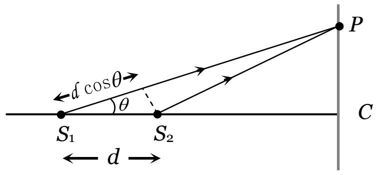
For maxima at P: $\Delta = n\lambda$; where $n = 0, \pm 1, \pm 2, ...$



and For minima at P: $\Delta = \frac{(2n-1)\lambda}{2}$; where $n = \pm 1, \pm 2, ...$

Note : ❑ If the slits are vertical, the path difference (Δ) is $d\sin\theta$, so as θ increases, Δ also increases. But if slits are horizontal path difference is $d\cos\theta$, so as θ increases, Δ decreases.





(7) **More about fringe**

(i) All fringes are of

equal width. Width of each fringe is $\boldsymbol{\beta = \frac{\lambda D}{d}}$ and angular fringe width $\boldsymbol{\theta = \frac{\lambda}{d} = \frac{\beta}{D}}$

(ii) If the whole YDSE set up is taken in another medium then λ changes so β changes

e.g. in water $\lambda_w = \frac{\lambda_a}{\mu_w} \Rightarrow \beta_w = \frac{\beta_a}{\mu_w} = \frac{3}{4}\beta_a$

(iii) Fringe width $\beta \propto \frac{1}{d}$ *i.e.* with increase in separation between the sources, β decreases.

(iv) Position of n^{th} bright fringe from central maxima $\boldsymbol{x_n = \frac{n\lambda D}{d} = n\beta}$; $n = 0, 1, 2 ....$

(v) Position of n^{th} dark fringe from central maxima $\boldsymbol{x_n = \frac{(2n-1)\lambda D}{2d} = \frac{(2n-1)\beta}{2}}$; $n = 1, 2, 3 ....$

(vi) In YDSE, if n_1 fringes are visible in a field of view with light of wavelength λ_1, while n_2 with light of wavelength λ_2 in the same field, then $\boldsymbol{n_1\lambda_1 = n_2\lambda_2}$.

(vii) Separation (Δx) between fringes

Between n^{th} bright and m^{th} bright fringes $(n > m)$	Between n^{th} bright and m^{th} dark fringe
$\Delta x = (n-m)\beta$	(a) If $n > m$ then $\Delta x = \left(n - m + \frac{1}{2}\right)\beta$ (b) If $n < m$ then $\Delta x = \left(m - n - \frac{1}{2}\right)\beta$

(8) **Identification of central bright fringe**

To identify central bright fringe, monochromatic light is replaced by white light. Due to overlapping central maxima will be white with red edges. On the other side of it we shall get a few coloured band and then uniform illumination.

(9) **Condition for observing sustained interference**

(i) The initial phase difference between the interfering waves must remain constant : Otherwise the interference will not be sustained.

(ii) The frequency and wavelengths of two waves should be equal : If not the phase difference will not remain constant and so the interference will not be sustained.

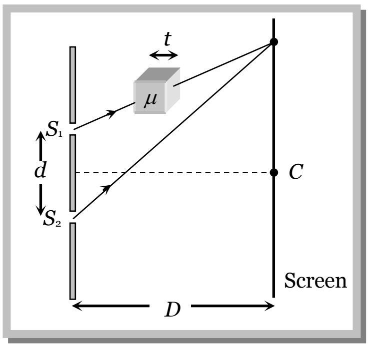
(iii) The light must be monochromatic : This eliminates overlapping of patterns as each wavelength corresponds to one interference pattern.

(iv) The amplitudes of the waves must be equal : This improves contrast with $I_{max} = 4I_0$ and $I_{min} = 0$.

(v) The sources must be close to each other : Otherwise due to small fringe width $\left(\beta \propto \frac{1}{d}\right)$ the eye can not resolve fringes resulting in uniform illumination.

(10) **Shifting of fringe pattern in YDSE**

If a transparent thin film of mica or glass is put in the path of one of the waves, then the whole fringe pattern gets shifted.



If film is put in the path of upper wave, fringe pattern shifts upward and if film is placed in the path of lower wave, pattern shift downward.

$$\text{Fringe shift} = \frac{D}{d}(\mu - 1)t = \frac{\beta}{\lambda}(\mu - 1)t$$

⇒ Additional path difference $= (\mu - 1)t$

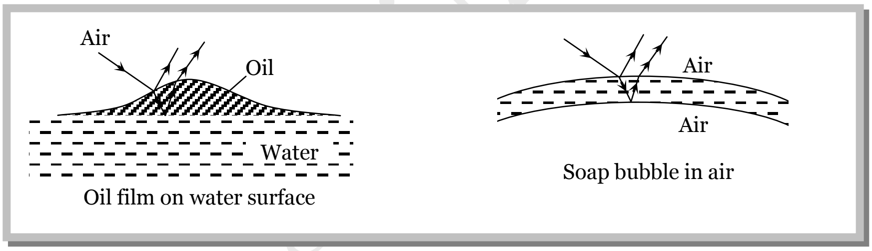
⇒ If shift is equivalent to n fringes then $n = \frac{(\mu - 1)t}{\lambda}$ or $t = \frac{n\lambda}{(\mu - 1)}$

⇒ Shift is independent of the order of fringe (*i.e.* shift of zero order maxima = shift of n^{th} order maxima.

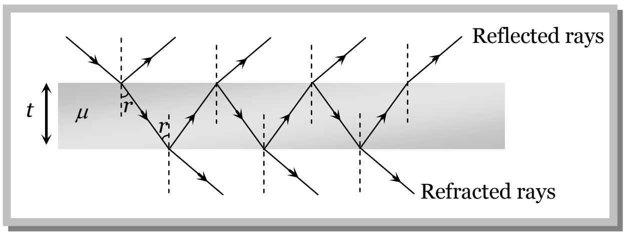
⇒ Shift is independent of wavelength.

Illustrations of Interference

Interference effects are commonly observed in thin films when their thickness is comparable to wavelength of incident light (If it is too thin as compared to wavelength of light it appears dark and if it is too thick, this will result in uniform illumination of film). Thin layer of oil on water surface and soap bubbles shows various colours in white light due to interference of waves reflected from the two surfaces of the film.



(1) **Thin films :** In thin films interference takes place between the waves reflected from it's two surfaces and waves refracted through it.



Interference in reflected light	Interference in refracted light
Condition of constructive interference (maximum intensity) $\Delta = 2\mu\, t\cos r = (2n \pm 1)\frac{\lambda}{2}$ For normal incidence r = 0 so $2\mu t = (2n \pm 1)\frac{\lambda}{2}$	Condition of constructive interference (maximum intensity) $\Delta = 2\mu t\cos r = (2n)\frac{\lambda}{2}$ For normal incidence $2\mu t = n\lambda$

Condition of destructive interference (minimum intensity) $\Delta = 2\mu t \cos r = (2n)\frac{\lambda}{2}$ For normal incidence $2\mu t = n\lambda$	Condition of destructive interference (minimum intensity) $\Delta = 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$ For normal incidence $2\mu t = (2n \pm 1)\frac{\lambda}{2}$

Doppler's Effect in Light

The phenomenon of apparent change in frequency (or wavelength) of the light due to relative motion between the source of light and the observer is called Doppler's effect.

If $\nu =$ actual frequency, $\nu' =$ Apparent frequency, v = speed of source *w.r.t* stationary observer, c = speed of light

Source of light moves towards the stationary observer ($v << c$)	**Source of light moves away from the stationary observer ($v << c$)**
(i) Apparent frequency $\nu' = \nu\left(1 + \frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1 - \frac{v}{c}\right)$	(i) Apparent frequency $\nu' = \nu\left(1 - \frac{v}{c}\right)$ and Apparent wavelength $\lambda' = \lambda\left(1 + \frac{v}{c}\right)$
(ii) Doppler's shift : Apparent wavelength < actual wavelength, So spectrum of the radiation from the source of light shifts towards the red end of spectrum. This is called Red shift Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$	(ii) Doppler's shift : Apparent wavelength > actual wavelength, So spectrum of the radiation from the source of light shifts towards the violet end of spectrum. This is called Violet shift Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$

Note : ❑ Doppler's shift $(\Delta\lambda)$ and time period of rotation (T) of a star relates as $\Delta\lambda = \frac{\lambda}{c} \times \frac{2\pi r}{T}$; r = radius of star.

Applications of Doppler effect

(i) Determination of speed of moving bodies (aeroplane, submarine etc) in RADAR and SONAR.

(ii) Determination of the velocities of stars and galaxies by spectral shift.

(iii) Determination of rotational motion of sun.

(iv) Explanation of width of spectral lines.

(v) Tracking of satellites. (vi) In medical sciences in echo cardiogram, sonography *etc.*

Concepts

☞ *The angular thickness of fringe width is defined as* $\delta = \frac{\beta}{D} = \frac{\lambda}{d}$, *which is independent of the screen distance D.*

☞ *Central maxima means the maxima formed with zero optical path difference. It may be formed anywhere on the screen.*

☞ *All the wavelengths produce their central maxima at the same position.*

☞ *The wave with smaller wavelength from its maxima before the wave with longer wavelength.*

☞ *The first maxima of violet colour is closest and that for the red colour is farthest.*

☞ *Fringes with blue light are thicker than those for red light.*

☞ *In an interference pattern, whatever energy disappears at the minimum, appears at the maximum.*

☞ *In YDSE, the nth maxima always comes before the nth minima.*

☞ *In YDSE, the ratio* $\frac{I_{\max}}{I_{\min}}$ *is maximum when both the sources have same intensity.*

☞ *For two interfering waves if initial phase difference between them is* ϕ_0 *and phase difference due to path difference between them is* ϕ'*. Then total phase difference will be* $\phi = \phi_0 + \phi' = \phi_0 + \frac{2\pi}{\lambda}\Delta$.

☞ *Sometimes maximm number of maximas or minimas are asked in the question which can be obtained on the screen. For this we use the fact that value of sin* θ *(or cos* θ*) can't be greater than 1. For example in the first case when the slits are vertical*

$$\sin\theta = \frac{n\lambda}{d} \qquad \textit{(for maximum intensity)}$$

$$\because \quad \sin\theta \not> 1 \qquad \therefore \quad \frac{n\lambda}{d} \not> 1 \quad or \quad n \not> \frac{d}{\lambda}$$

Suppose in some question d/λ *comes out say 4.6, then total number of maximuas on the screen will be 9. Corresponding to* $n = 0, \pm 1, \pm 2, \pm 3$ *and* ± 4.

☞ ***Shape of wave front***

If rays are parallel, wave front is plane. If rays are converging wave front is spherical of decreasing radius. If rays are diverging wave front is spherical of increasing radius.

Example

***Example*: 1** If two light waves having same frequency have intensity ratio 4 : 1 and they interfere, the ratio of maximum to minimum intensity in the pattern will be

(a) 9 : 1 (b) 3 : 1 (c) 25 : 9 (d) 16 : 25

Solution: (a) By using $\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right)^2 = \left(\frac{\sqrt{\frac{4}{1}}+1}{\sqrt{\frac{4}{1}}-1}\right)^2 = \frac{9}{1}$.

***Example*: 2** In Young's double slit experiment using sodium light (λ = 5898Å), 92 fringes are seen. If given colour (λ = 5461Å) is used, how many fringes will be seen

(a) 62 (b) 67 (c) 85 (d) 99

Solution: (d) By using $n_1\lambda_1 = n_2\lambda_2 \Rightarrow 92 \times 5898 = n_2 \times 5461 \Rightarrow n_2 = 99$

***Example*: 3** Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point A and π at point B. Then the difference between the resultant intensities at A and B is

(a) $2I$ (b) $4I$ (c) $5I$ (d) $7I$

Solution: (b) By using $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

At point A : Resultant intensity $I_A = I + 4I + 2\sqrt{I \times 4I}\cos\frac{\pi}{2} = 5I$

At point B : Resultant intensity $I_B = I + 4I + 2\sqrt{I \times 4I}\cos\pi = I$. Hence the difference $= I_A - I_B = 4I$

Only for students of genius academy Jamanipali, Maxwell classes Korba, and Gupta classes Kusmunda.

***Example*: 4** If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point, the amplitude of the resulting wave will be about **[MP PMT 2000]**

(a) 7 (b) 6 (c) 5 (d) 3.

Solution: (b) By using $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} \Rightarrow A = \sqrt{(4)^2 + (3)^2 + 2\times4\times3\cos\frac{\pi}{3}} = \sqrt{37} \approx 6$.

***Example*: 5** Two waves being produced by two sources S_1 and S_2. Both sources have zero phase difference and have wavelength λ. The destructive interference of both the waves will occur of point P if $(S_1P - S_2P)$ has the value

[MP PET 1987]

(a) 5λ (b) $\frac{3}{4}\lambda$ (c) 2λ (d) $\frac{11}{2}\lambda$

Solution: (d) For destructive interference, path difference the waves meeting at P (*i.e.* $S_1P - S_2P$) must be odd multiple of $\lambda/2$. Hence option (d) is correct.

***Example*: 6** Two interfering wave (having intensities are $9I$ and $4I$) path difference between them is 11 λ. The resultant intensity at this point will be

(a) I (b) $9I$ (c) $4I$ (d) $25I$

Solution: (d) Path difference $\Delta = \frac{\lambda}{2\pi}\times\phi \Rightarrow \frac{2\pi}{\lambda}\times 11\lambda = 22\pi$ *i.e.* constructive interference obtained at the same point

So, resultant intensity $I_R = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{9I} + \sqrt{4I})^2 = 25I$.

***Example*: 7** In interference if $\frac{I_{\max}}{I_{\min}} = \frac{144}{81}$ then what will be the ratio of amplitudes of the interfering wave

(a) $\frac{144}{81}$ (b) $\frac{7}{1}$ (c) $\frac{1}{7}$ (d) $\frac{12}{9}$

Solution: (b) By using $\frac{a_1}{a_2} = \left(\frac{\sqrt{\frac{I_{\max}}{I_{\min}}}+1}{\sqrt{\frac{I_{\max}}{I_{\min}}}-1}\right) = \left(\frac{\sqrt{\frac{144}{81}}+1}{\sqrt{\frac{144}{81}}-1}\right) = \left(\frac{\frac{12}{9}+1}{\frac{12}{5}-1}\right) = \frac{7}{1}$

***Example*: 8** Two interfering waves having intensities x and y meets a point with time difference $3T/2$. What will be the resultant intensity at that point

(a) $(\sqrt{x} + \sqrt{y})$ (b) $(\sqrt{x} + \sqrt{y} + \sqrt{xy})$ (c) $x + y + 2\sqrt{xy}$ (d) $\frac{x+y}{2xy}$

Solution: (c) Time difference T.D. $= \frac{T}{2\pi}\times\phi \Rightarrow \frac{3T}{2} = \frac{T}{2\pi}\times\phi \Rightarrow \phi = 3\pi$; This is the condition of constructive interference.

So resultant intensity $I_R = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}$.

***Example*: 9** In Young's double-slit experiment, an interference pattern is obtained on a screen by a light of wavelength 6000 Å, coming from the coherent sources S_1 and S_2. At certain point P on the screen third dark fringe is formed. Then the path difference $S_1P - S_2P$ in microns is **[EAMCET 2003]**

(a) 0.75 (b) 1.5 (c) 3.0 (d) 4.5

Solution: (b) For dark fringe path difference $\Delta = (2n-1)\frac{\lambda}{2}$; here $n = 3$ and $\lambda = 6000\times10^{-10}\ m$

So $\Delta = (2\times 3 - 1)\times \frac{6\times 10^{-7}}{2} = 15\times 10^{-7}\,m = 1.5\,microns.$

***Example*: 10** In a Young's double slit experiment, the slit separation is 1 *mm* and the screen is 1 *m* from the slit. For a monochromatic light of wavelength 500 *nm*, the distance of 3rd minima from the central maxima is

(a) 0.50 *mm* (b) 1.25 *mm* (c) 1.50 *mm* (d) 1.75 *mm*

Solution: (b) Distance of n^{th} minima from central maxima is given as $x = \frac{(2n-1)\lambda D}{2d}$

So here $x = \frac{(2\times 3-1)\times 500\times 10^{-9}\times 1}{2\times 10^{-3}} = 1.25\,mm$

***Example*: 11** The two slits at a distance of 1 *mm* are illuminated by the light of wavelength $6.5\times 10^{-7}\,m$. The interference fringes are observed on a screen placed at a distance of 1 *m*. The distance between third dark fringe and fifth bright fringe will be

[NCERT 1982; MP PET 1995; BVP 2003]

(a) 0.65 *mm* (b) 1.63 *mm* (c) 3.25 *mm* (d) 4.88 *mm*

Solution: (b) Distance between n^{th} bright and m^{th} dark fringe ($n > m$) is given as $x = \left(n - m + \frac{1}{2}\right)\beta = \left(n - m + \frac{1}{2}\right)\frac{\lambda D}{d}$

$\Rightarrow x = \left(5 - 3 + \frac{1}{2}\right)\times \frac{6.5\times 10^{-7}\times 1}{1\times 10^{-3}} = 1.63\,mm$.

***Example*: 12** The slits in a Young's double slit experiment have equal widths and the source is placed symmetrically relative to the slits. The intensity at the central fringes is I_0. If one of the slits is closed, the intensity at this point will be**[MP PMT 1999**

(a) I_0 (b) $I_0/4$ (c) $I_0/2$ (d) $4I_0$

Solution: (b) By using $I_R = 4I\cos^2\frac{\phi}{2}$ {where *I* = Intensity of each wave}

At central position $\phi = 0^o$, hence initially $I_0 = 4I$.

If one slit is closed, no interference takes place so intensity at the same location will be *I* only *i.e.* intensity become s $\frac{1}{4}th$ or $\frac{I_0}{4}$.

***Example*: 13** In double slit experiment, the angular width of the fringes is 0.20° for the sodium light (λ = 5890 $\mathring{A}$). In order to increase the angular width of the fringes by 10%, the necessary change in the wavelength is

(a) Increase of 589 $\mathring{A}$ (b) Decrease of 589 $\mathring{A}$ (c) Increase of 6479 $\mathring{A}$ (d) Zero

Solution: (a) By using $\theta = \frac{\lambda}{d} \Rightarrow \frac{\theta_1}{\theta_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{0.20^o}{(0.20^o + 10\%\text{ of } 0.20)} = \frac{5890}{\lambda_2} \Rightarrow \frac{0.20}{0.22} = \frac{5890}{\lambda_2} \Rightarrow \lambda_2 = 6479$

So increase in wavelength = 6479 – 5890 = 589 Å.

***Example*: 14** In Young's experiment, light of wavelength 4000 $\mathring{A}$ is used, and fringes are formed at 2 *metre* distance and has a fringe width of 0.6 *mm*. If whole of the experiment is performed in a liquid of refractive index 1.5, then width of fringe will be

[MP PMT 1994, 97]

(a) 0.2 *mm* (b) 0.3 *mm* (c) 0.4 *mm* (d) 1.2 *mm*

Solution: (c) $\beta_{medium} = \frac{\beta_{air}}{\mu} \Rightarrow \beta_{medium} = \frac{0.6}{1.5} = 0.4\,mm$.

***Example*: 15** Two identical sources emitted waves which produces intensity of *k* unit at a point on screen where path difference is λ. What will be intensity at a point on screen at which path difference is $\lambda/4$ **[RPET 1996]**

(a) $\frac{k}{4}$ (b) $\frac{k}{2}$ (c) k (d) Zero

Only for students of genius academy Jamanipali, Maxwell classes Korba, and Gupta classes Kusmunda.

Solution: (b) By using phase difference $\phi = \frac{2\pi}{\lambda}(\Delta)$

For path difference λ, phase difference $\phi_1 = 2\pi$ and for path difference $\lambda/4$, phase difference $\phi_2 = \pi/2$.

Also by using $I = 4I_0 \cos^2\frac{\phi}{2} \Rightarrow \frac{I_1}{I_2} = \frac{\cos^2(\phi_1/2)}{\cos^2(\phi_2/2)} \Rightarrow \frac{k}{I_2} = \frac{\cos^2(2\pi/2)}{\cos^2\left(\frac{\pi/2}{2}\right)} = \frac{1}{1/2} \Rightarrow I_2 = \frac{k}{2}$.

***Example*: 16** A thin mica sheet of thickness $2\times10^{-6}\,m$ and refractive index ($\mu = 1.5$) is introduced in the path of the first wave. The wavelength of the wave used is $5000 Å$. The central bright maximum will shift **[CPMT 1999]**

(a) 2 fringes upward (b) 2 fringes downward (c) 10 fringes upward (d) None of these

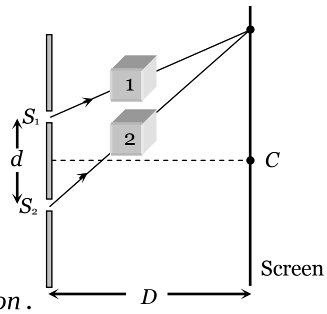
Solution: (a) By using shift $\Delta x = \frac{\beta}{\lambda}(\mu - 1)t \Rightarrow \Delta x = \frac{\beta}{5000\times10^{-10}}(1.5-1)\times2\times10^{-6} = 2\beta$

Since the sheet is placed in the path of the first wave, so shift will be 2 fringes upward.

***Example*: 17** In a *YDSE* fringes are observed by using light of wavelength 4800 Å, if a glass plate (μ = 1.5) is introduced in the path of one of the wave and another plates is introduced in the path of the (μ = 1.8) other wave. The central fringe takes the position of fifth bright fringe. The thickness of plate will be

(a) 8 *micron* (b) 80 *micron* (c) 0.8 *micron* (d) None of these

Solution: (a) Shift due to the first plate $x_1 = \frac{\beta}{\lambda}(\mu_1 - 1)t$ (Upward)



and shift due to the second $x_2 = \frac{\beta}{\lambda}(\mu_2 - 1)t$ (Downward)

Hence net shift = $x_2 - x_1 = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t$

$\Rightarrow 5\beta = \frac{\beta}{\lambda}(1.8-1.5)t \Rightarrow t = \frac{5\lambda}{0.3} = \frac{5\times4800\times10^{-10}}{0.3} = 8\times10^{-6}\,m = 8\,micron$.

***Example*: 18** In young double slit experiment $\frac{d}{D} = 10^{-4}$ (d = distance between slits, D = distance of screen from the slits). At a point P on the screen resulting intensity is equal to the intensity due to individual slit I_0. Then the distance of point P from the central maxima is (λ = 6000 Å)

(a) 2 *mm* (b) 1 *mm* (c) 0.5 *mm* (d) 4 *mm*

Solution: (a) By using shift $I = 4I_0\cos^2(\phi/2) \Rightarrow I_0 = 4I_0\cos^2(\phi/2) \Rightarrow \cos(\phi/2) = \frac{1}{2}$ or $\frac{\phi}{2} = \frac{\pi}{3} \Rightarrow \phi = \frac{2\pi}{3}$

Also path difference $\Delta = \frac{xd}{D} = \frac{\lambda}{2\pi}\times\phi \Rightarrow x\times\left(\frac{d}{D}\right) = \frac{6000\times10^{-10}}{2\pi}\times\frac{2\pi}{3} \Rightarrow x = 2\times10^{-3}\,m = 2mm.$

***Example*: 19** Two identical radiators have a separation of $d = \lambda/4$, where λ is the wavelength of the waves emitted by either source. The initial phase difference between the sources is $\pi/4$. Then the intensity on the screen at a distance point situated at an angle $\theta = 30^o$ from the radiators is (here I_0 is the intensity at that point due to one radiator)

(a) I_0 (b) $2I_0$ (c) $3I_0$ (d) $4I_0$

Solution: (a) Initial phase difference $\phi_0 = \frac{\pi}{4}$; Phase difference due to path difference $\phi' = \frac{2\pi}{\lambda}(\Delta)$

where $\Delta = d\sin\theta \Rightarrow \phi' = \frac{2\pi}{\lambda}(d\sin\theta) = \frac{2\pi}{\lambda}\times\frac{\lambda}{4}(\sin 30^o) = \frac{\pi}{4}$

Hence total phase difference $\phi = \phi_0 + \phi' = \frac{\phi}{4}$. By using $I = 4I_0 \cos^2(\phi/2) = 4I_0 \cos^2\left(\frac{\pi/2}{2}\right) = 2I_0$.

***Example*: 20** In *YDSE* a source of wavelength 6000 Å is used. The screen is placed 1 *m* from the slits. Fringes formed on the screen, are observed by a student sitting close to the slits. The student's eye can distinguish two neighbouring fringes. If they subtend an angle more than 1 minute of arc. What will be the maximum distance between the slits so that the fringes are clearly visible

(a) 2.06 *mm* (b) 2.06 *cm* (c) 2.06×10^{-3} *mm* (d) None of these

Solution: (a) According to given problem angular fringe width $\theta = \frac{\lambda}{d} \geq \frac{\pi}{180 \times 60}$ $\left[\text{As } 1' = \frac{\pi}{180 \times 60} rad\right]$

i.e. $d < \frac{6 \times 10^{-7} \times 180 \times 60}{\pi}$ *i.e.* $d < 2.06 \times 10^{-3} m \Rightarrow d_{max} = 2.06\, mm$

***Example*: 21** the maximum intensity in case of interference of *n* identical waves, each of intensity I_0, if the interference is (i) coherent and (ii) incoherent respectively are

(a) $n^2 I_0, nI_0$ (b) $nI_0, n^2 I_0$ (c) nI_0, I_0 (d) $n^2 I_0, (n-1)I_0$

Solution: (a) In case of interference of two wave $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$

(i) In case of coherent interference ϕ does not vary with time and so I will be maximum when $\cos\phi = \max = 1$

i.e. $(I_{\max})_{co} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

So for *n* identical waves each of intensity I_0 $(I_{\max})_{co} = (\sqrt{I_0} + \sqrt{I_0} + ...)^2 = (n\sqrt{I_0})^2 = n^2 I_0$

(ii)In case of incoherent interference at a given point, ϕ varies randomly with time, so $(\cos\phi)_{av} = 0$ and hence $(I_R)_{Inco} = I_1 + I_2$

So in case of *n* identical waves $(I_R)_{Inco} = I_0 + I_0 + ... = nI_0$

***Example*: 22** The width of one of the two slits in a Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit width. The ratio of the maximum to the minimum intensity in interference pattern will be

(a) $\frac{1}{a}$ (b) $\frac{9}{1}$ (c) $\frac{2}{1}$ (d) $\frac{1}{2}$

Solution: (b) $A_{\max} = 2A + A = 3A$ and $A_{\min} = 2A - A = A$. Also $\frac{I_{\max}}{I_{\min}} = \left(\frac{A_{\max}}{A_{\min}}\right)^2 = \left(\frac{3A}{A}\right)^2 = \frac{9}{1}$

***Example*: 23** A star is moving towards the earth with a speed of $4.5 \times 10^6 m/s$. If the true wavelength of a certain line in the spectrum received from the star is 5890 Å, its apparent wavelength will be about $[c = 3 \times 10^8 m/s]$

[MP PMT 1999]

(a) 5890 $\mathring{A}$ (b) 5978 $\mathring{A}$ (c) 5802 $\mathring{A}$ (d) 5896 $\mathring{A}$

Solution: (c) By using $\lambda' = \lambda\left(1 - \frac{v}{c}\right) \Rightarrow \lambda' = 5890\left(1 - \frac{4.5 \times 10^6}{3 \times 10^8}\right) = 5802\, \mathring{A}$.

***Example*: 24** Light coming from a star is observed to have a wavelength of 3737 Å, while its real wavelength is 3700 Å. The speed of the star relative to the earth is [Speed of light $= 3 \times 10^8 m/s$] **[MP PET 1997]**

(a) $3 \times 10^5 m/s$ (b) $3 \times 10^6 m/s$ (c) $3.7 \times 10^7 m/s$ (d) $3.7 \times 10^6 m/s$

Solution: (b) By using $\Delta\lambda = \lambda\frac{v}{c} \Rightarrow$ (3737-3700)= $3700 \times \frac{v}{3 \times 10^8} \Rightarrow v = 3 \times 10^6\, m/s$.

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***Example*: 25** Light from the constellation Virgo is observed to increase in wavelength by 0.4%. With respect to Earth the constellation is **[MP PMT 1994, 97; MP PET 2003]**

(a) Moving away with velocity $1.2 \times 10^6 m/s$ (b) Coming closer with velocity $1.2 \times 10^6 m/s$

(c) Moving away with velocity $4 \times 10^6 m/s$ (d) Coming closer with velocity $4 \times 10^6 m/s$

Solution: (a) By using $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$; where $\frac{\Delta\lambda}{\lambda} = \frac{0.4}{100}$ and $c = 3 \times 10^8 m/s \Rightarrow \frac{0.4}{100} = \frac{v}{3 \times 10^8} \Rightarrow v = 1.2 \times 10^6 m/s$

Since wavelength is increasing *i.e.* it is moving away.

Tricky example: 1

In *YDSE*, distance between the slits is 2×10^{-3} *m*, slits are illuminated by a light of wavelength 2000Å –9000 Å. In the field of view at a distance of 10^{-3} *m* from the central position which wavelength will be observe. Given distance between slits and screen is 2.5 *m*

(a) 40000 Å (b) 4500 Å (c) 5000 Å (d) 5500 Å

Solution : (b) $x = \frac{n\lambda D}{d} \Rightarrow \lambda = \frac{xd}{nD} = \frac{10^{-3} \times 2 \times 10^{-3}}{n \times 2.5} \Rightarrow \frac{8 \times 10^{-7}}{n} m = \frac{8000}{n} Å$

For n = 1, 2, 3……. λ = 8000 Å, 4000 Å, $\frac{8000}{3} Å$………

Hence only option (a) is correct.

Tricky example: 2

I is the intensity due to a source of light at any point *P* on the screen. If light reaches the point *P* via two different paths (a) direct (b) after reflection from a plane mirror then path difference between two paths is $3\lambda/2$, the intensity at *P* is

(a) I (b) Zero (c) $2I$ (d) $4I$

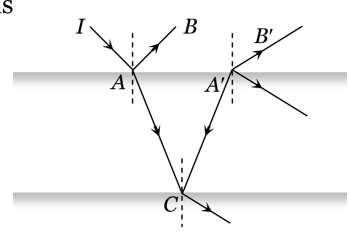
Solution : (d) Reflection of light from plane mirror gives additional path difference of $\lambda/2$ between two waves

$\therefore$ Total path difference $= \frac{3\lambda}{2} + \frac{\lambda}{2} = 2\lambda$

Which satisfies the condition of maxima. Resultant intensity $= (\sqrt{I} + \sqrt{I})^2 = 4I$.

Tricky example: 3

A ray of light of intensity *I* is incident on a parallel glass-slab at a point *A* as shown in figure. It undergoes partial reflection and refraction. At each reflection 25% of incident energy is reflected. The rays *AB* and *A′B′* undergo interference. The ratio I_{max}/I_{min} is



(a) 4 : 1

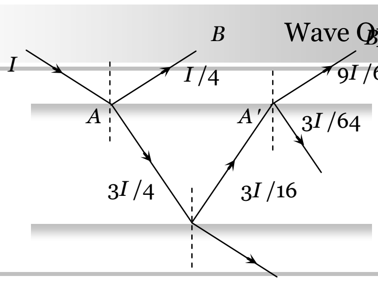
(b) 8 : 1

(c) 7 : 1

(d) 49 : 1

Solution : (d) From figure $I_1 = \frac{I}{4}$ and $I_2 = \frac{9I}{64} \Rightarrow \frac{I_2}{I_1} = \frac{9}{16}$

By using $\dfrac{I_{max}}{I_{min}} = \dfrac{\left(\sqrt{\dfrac{I_2}{I_1}}+1\right)}{\left(\sqrt{\dfrac{I_2}{I_1}}-1\right)} = \dfrac{\left(\sqrt{\dfrac{9}{16}}+1\right)}{\left(\sqrt{\dfrac{9}{16}}-1\right)} = \dfrac{49}{1}$



Fresnel's Biprism

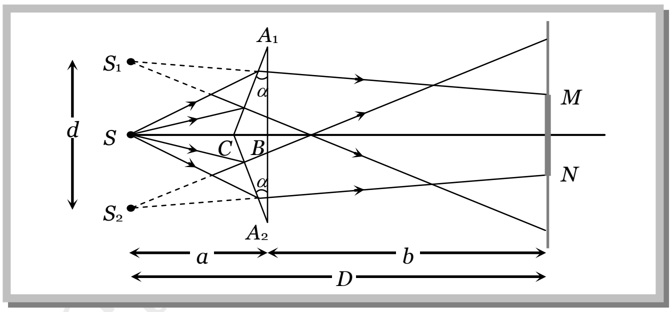
(1) It is an optical device of producing interference of light Fresnel's biprism is made by joining base to base two thin prism (A_1BC and A_2BC as shown in the figure) of very small angle or by grinding a thick glass plate.

(2) Acute angle of prism is about $1/2^o$ and obtuse angle of prism is about 179^o.

(3) When a monochromatic light source is kept in front of biprism two coherent virtual source S_1 and S_2 are produced.

(4) Interference fringes are found on the screen (in the MN region) placed behind the biprism interference fringes are formed in the limited region which can be observed with the help eye piece.

(5) Fringe width is measured by a micrometer attached to the eye piece. Fringes are of equal width and its value is $\beta = \dfrac{\lambda D}{d} \Rightarrow \lambda = \dfrac{\beta d}{D}$

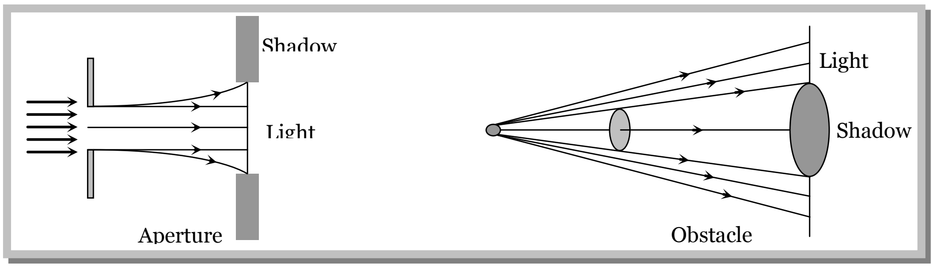


Let the separation between S_1 and S_2 be d and the distance of slits and the screen from the biprism be a and b respectively *i.e.* $D = (a + b)$. If angle of prism is α and refractive index is μ then $d = 2a(\mu - 1)\alpha$

$\therefore \quad \lambda = \dfrac{\beta[2a(\mu-1)\alpha]}{(a+b)} \Rightarrow \beta = \dfrac{(a+b)\lambda}{2a(\mu-1)\alpha}$

Diffraction of Light

It is the phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wavelength of light.



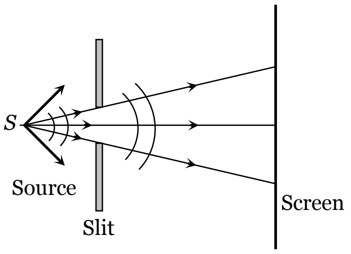
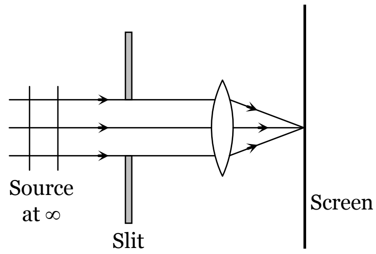
Note : ❑ Diffraction is the characteristic of all types of waves.

❑ Greater the wavelength of wave, higher will be it's degree of diffraction.

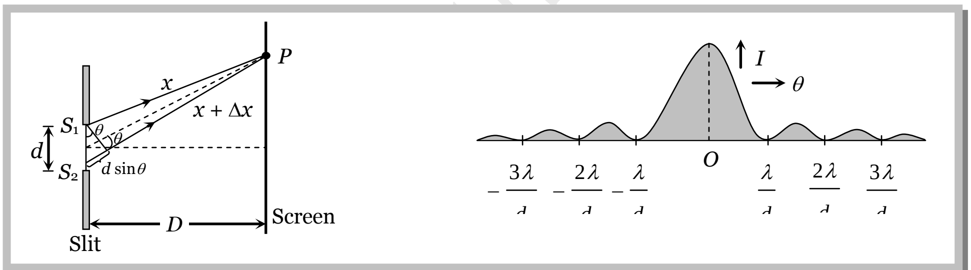
❑ Experimental study of diffraction was extended by Newton as well as Young. Most systematic study carried out by Huygens on the basis of wave theory.

- The minimum distance at which the observer should be from the obstacle to observe the diffraction of light of wavelength λ around the obstacle of size d is given by $x = \dfrac{d^2}{4\lambda}$.

(1) **Types of diffraction :** The diffraction phenomenon is divided into two types

Fresnel diffraction	Fraunhofer diffraction
(i) If either source or screen or both are at finite distance from the diffracting device (obstacle or aperture), the diffraction is called Fresnel type.	(i) In this case both source and screen are effectively at infinite distance from the diffracting device.
(ii) Common examples : Diffraction at a straight edge, narrow wire or small opaque disc etc.	(ii) Common examples : Diffraction at single slit, double slit and diffraction grating.
	

(2) **Diffraction of light at a single slit :** In case of diffraction at a single slit, we get a central bright band with alternate bright (maxima) and dark (minima) bands of decreasing intensity as shown



(i) Width of central maxima $\beta_0 = \dfrac{[illegible]}{d}$, and angular width $= \dfrac{[illegible]}{d}$

(ii) Minima occurs at a point on either side of the central maxima, such that the path difference between the waves from the two ends of the aperture is given by $\Delta = n\lambda$; where $n = 1, 2, 3 ....$

i.e. $d\sin\theta = n\lambda \Rightarrow \sin\theta = \dfrac{n\lambda}{d}$

(iii) The secondary maxima occurs, where the path difference between the waves from the two ends of the aperture is given by $\Delta = (2n+1)\dfrac{\lambda}{2}$; where $n = 1, 2, 3 ....$

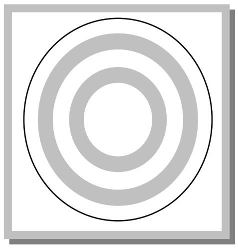
i.e. $d\sin\theta = (2n+1)\dfrac{\lambda}{2} \Rightarrow \sin\theta = \dfrac{(2n+1)\lambda}{2d}$

(3) **Comparison between interference and diffraction**

Interference	Diffraction
Results due to the superposition of waves from two coherent sources.	Results due to the superposition of wavelets from different parts of same wave front. (single coherent source)

All fringes are of same width $\beta = \frac{\lambda D}{d}$	All secondary fringes are of same width but the central maximum is of double the width $\beta_0 = 2\beta = 2\frac{\lambda D}{d}$
All fringes are of same intensity	Intensity decreases as the order of maximum increases.
Intensity of all minimum may be zero	Intensity of minima is not zero.
Positions of nth maxima and minima $x_{n(\text{Bright})} = \frac{n\lambda D}{d}$, $x_{n(\text{Dark})} = (2n-1)\frac{\lambda D}{d}$	Positions of nth secondary maxima and minima $x_{n(\text{Bright})} = (2n+1)\frac{\lambda D}{d}$, $x_{n(\text{Dark})} = \frac{n\lambda D}{d}$
Path difference for nth maxima $\Delta = n\lambda$	for nth secondary maxima $\Delta = (2n+1)\frac{\lambda}{2}$
Path difference for nth minima $\Delta = (2n-1)\lambda$	Path difference for nth minima $\Delta = n\lambda$

(4) **Diffraction and optical instruments :** The objective lens of optical instrument like telescope or microscope etc. acts like a circular aperture. Due to diffraction of light at a circular aperture, a converging lens cannot form a point image of an object rather it produces a brighter disc known as Airy disc surrounded by alternate dark and bright concentric rings.

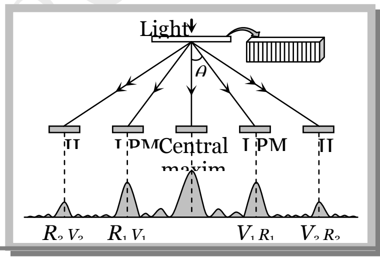


The angular half width of Airy disc $= \theta = \frac{1.22\lambda}{D}$ (where D = aperture of lens)

The lateral width of the image $= f\theta$ (where f = focal length of the lens)

Note : ❑ Diffraction of light limits the ability of optical instruments to form clear images of objects when they are close to each other.

(5) **Diffraction grating :** Consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the diffraction of principle maxima (PM) is given by $d\sin\theta = n\lambda$; where d = distance between two consecutive slits and is called grating element.

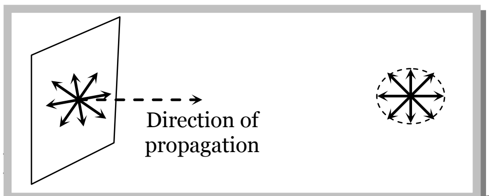


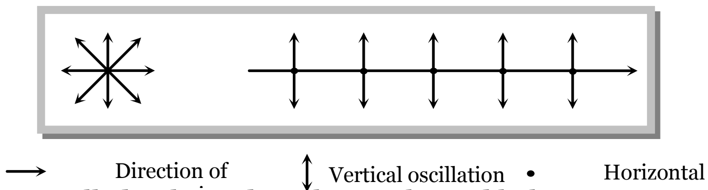
Polarisation of Light

Light propagates as transverse EM waves. The magnitude of electric field is much larger as compared to magnitude of magnetic field. We generally prefer to describe light as electric field oscillations.

(1) **Unpolarised light**

The light having electric field oscillations in all directions in the plane perpendicular to the direction of propagation is called Unpolarised light. The oscillation may be resolved into horizontal and vertical component.





(2)

The light having oscillations only in one plane is called Polarised or plane polarised light.

(i) The plane in which oscillation occurs in the polarised light is called plane of oscillation.

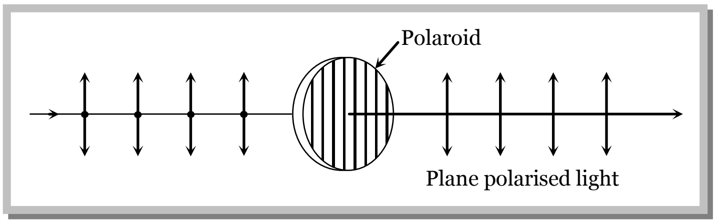
(ii) The plane perpendicular to the plane of oscillation is called plane of polarisation.

(iii) Light can be polarised by transmitting through certain crystals such as tourmaline or polaroids.

(3) **Polaroids**

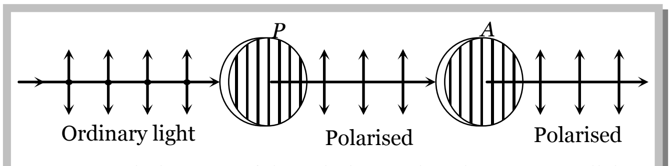
It is a device used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal. or

It is a thin film of ultramicroscopic crystals of quinine idosulphate with their optic axis parallel to each other.

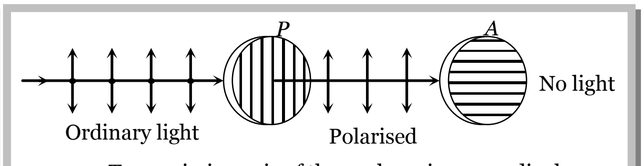


(i) Polaroids allow the light oscillations parallel to the transmission axis pass through them.

(ii) The crystal or polaroid on which unpolarised light is incident is called polariser. Crystal or polaroid on which polarised light is incident is called analyser.



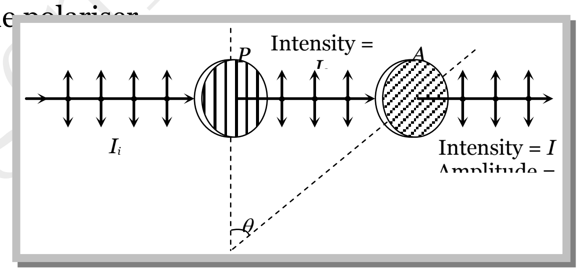
Transmission axes of the polariser and analyser are parallel to each other, so whole of the polarised light passes through



Transmission axis of the analyser is perpendicular to the polariser, hence no light passes through the

Note : ❑ When unpolarised light is incident on the polariser, the intensity of the transmitted polarised light is half the intensity of unpolarised light.

(4) **Malus law** This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.



(i) $I = I_0 \cos^2\theta$ and $A^2 = A_0^2 \cos^2\theta \Rightarrow A = A_0 \cos\theta$

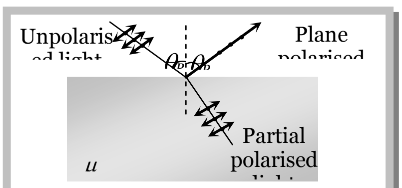
If $\theta = 0^o$, $I = I_0$, $A = A_0$, If $\theta = 45^o$, $I = \frac{I_0}{2}$, $A = \frac{A_0}{\sqrt{2}}$, If $\theta = 90^o$, $I = 0$, $A = 0$

(ii) If $I_i =$ Intensity of unpolarised light.

So $I_0 = \frac{I_i}{2}$ *i.e.* if an unpolarised light is converted into plane polarised light (say by passing it through a plaroid or a Nicol-prism), its intensity becomes half. and $I = \frac{I_i}{2}\cos^2\theta$

Note : ❑ Percentage of polarisation $= \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} \times 100$

(5) **Brewster's law :** Brewster discovered that when a beam of unpolarised light is reflected from a transparent medium (refractive index $=\mu$), the reflected light is completely plane polarised at a certain angle of incidence (called the angle of polarisation θ_p).



Also $\mu = \tan\theta_p$ Brewster's law

(i) For $i < \theta_P$ or $i > \theta_P$

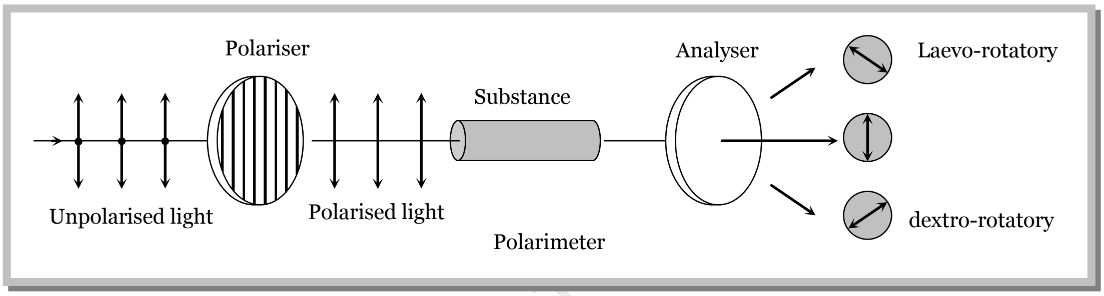
Both reflected and refracted rays becomes partially polarised

(ii) For glass $\theta_P \approx 57^o$, for water $\theta_P \approx 53^o$

(6) **Optical activity and specific rotation**

When plane polarised light passes through certain substances, the plane of polarisation of the light is rotated about the direction of propagation of light through a certain angle. This phenomenon is called optical activity or optical rotation and the substances optically active.

If the optically active substance rotates the plane of polarisation clockwise (looking against the direction of light), it is said to be *dextro-rotatory* or *right-handed*. However, if the substance rotates the plane of polarisation anti-clockwise, it is called *laevo-rotatory* or *left-handed*.



The optical activity of a substance is related to the asymmetry of the molecule or crystal as a whole, *e.g.*, a solution of cane-sugar is dextro-rotatory due to asymmetrical molecular structure while crystals of quartz are dextro or laevo-rotatory due to structural asymmetry which vanishes when quartz is fused.

Optical activity of a substance is measured with help of polarimeter in terms of 'specific rotation' which is defined as the rotation produced by a solution of length 10 *cm* (1 *dm*) and of unit concentration (*i.e.* 1 *g*/cc) for a given wavelength of light at a given temperature. *i.e.* $[\alpha]_{t^oC}^{\lambda} = \dfrac{\theta}{L \times C}$ where θ is the rotation in length L at concentration C.

(7) **Applications and uses of polarisation**

(i) By determining the polarising angle and using Brewster's law, *i.e.* $\mu = \tan\theta_P$, refractive index of dark transparent substance can be determined.

(ii) It is used to reduce glare.

(iii) In calculators and watches, numbers and letters are formed by liquid crystals through polarisation of light called liquid crystal display **(LCD)**.

(iv) In CD player polarised laser beam acts as needle for producing sound from compact disc which is an encoded digital format.

(v) It has also been used in recording and reproducing three-dimensional pictures.

(vi) Polarisation of scattered sunlight is used for navigation in solar-compass in polar regions.

(vii) Polarised light is used in optical stress analysis known as 'photoelasticity'.

(viii) Polarisation is also used to study asymmetries in molecules and crystals through the phenomenon of 'optical activity'.

Assignment

Nature of light and interference of light

1. The dual nature of light is exhibited by **[KCET 1999; AIIMS 2001; BHU 2001; Bihar CEE 2004]**

(a) Diffraction and photoelectric effect (b) Diffraction and reflection

(c) Refraction and interference (d) Photoelectric effect

2. Huygen wave theory allows us to know **[AFMC 2004]**

(a) The wavelength of the wave (b) The velocity of the wave

(c) The amplitude of the wave (d) The propagation of wave fronts

3. When a beam of light is used to determine the position of an object, the maximum accuracy is achieved if the light is **[AIIMS 2003]**

(a) Polarised (b) Of longer wavelength (c) Of shorter wavelength (d) Of high intensity

4. Which of the following phenomenon does not show the wave nature of light **[RPET 2003; MP PMT 2003]**

(a) Diffraction (b) Interference (c) Refraction (d) Photoelectric effect

5. As a result of interference of two coherent sources of light, energy is **[MP PMT 2002; KCET 2003]**

(a) Increased

(b) Redistributed and the distribution does not vary with time

(c) Decreased

(d) Redistributed and the distribution changes with time

6. To demonstrate the phenomenon of interference, we require two sources which emit radiation **[AIEEE 2003]**

(a) Of the same frequency and having a definite phase relationship

(b) Of nearly the same frequency

(c) Of the same frequency

(d) Of different wavelengths

7. Consider the following statements

Assertion (*A*): Thin films such as soap bubble or a thin layer of oil on water show beautiful colours, when illuminated by white light.

Reason (*R*) : It happens due to the interference of light reflected from the upper surface of the thin film.

Of these statements **[AIIMS 2002]**

(a) Both *A* and *R* are true but *R* is a correct explanation of *A* (b) Both *A* and *R* are true but *R* is not a correct explanation of *A*

(c) *A* is true but *R* is false (d) A is false but R is true

(e) Both *A* and *R* are false

8. When light passes from one medium into another medium, then the physical property which does not change is

[CPMT 1990; MNR 1995; AMU 1995; UPSEAT 1999, 2000; MP PET 2002; RPET 1996, 2003; AFMC 1993, 98, 2003]

(a) Velocity (b) Wavelength (c) Frequency (d) Refractive index

9. The frequency of light ray having the wavelength 3000$\mathring{A}$ is **[DPMT 2002]**

(a) 9×10^{13} *cycles/sec* (b) 10^{15} *cycles/sec* (c) 90 *cycles/sec* (d) 3000 *cycles/sec*

10. Two coherent sources of different intensities send waves which interfere. The ratio of maximum intensity to the minimum intensity is 25. The intensities of the sources are in the ratio **[RPMT 1989; UPSEAT 2002]**

(a) 25 : 1 (b) 5 : 1 (c) 9 : 4 (d) 25 : 16

11. What is the path difference of destructive interference **[AIIMS 2002]**

(a) $n\lambda$ (b) $n(\lambda+1)$ (c) $\frac{(n+1)\lambda}{2}$ (d) $\frac{(2n+1)\lambda}{2}$

12. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are

[IIT-JEE 1988; AIIMS 1997; MP PMT 1997; MP PET 1999; KCET (Engg./Med.) 2000; MP PET 2002]

genius

(a) $5I$ and I (b) $5I$ and $3I$ (c) $9I$ and I (d) $9I$ and $3I$

13. Laser beams are used to measure long distance because **[DCE 2001]**
(a) They are monochromatic (b) They are highly polarised
(c) They are coherent (d) They have high degree of parallelism

14. Wave nature of light is verified by **[RPET 2001]**
(a) Interference (b) Photoelectric effect (c) Reflection (d) Refraction

15. If the wavelength of light in vacuum be λ, the wavelength in a medium of refractive index n will be **[UPSEAT 2001; MP PET 2001]**
(a) $n\lambda$ (b) $\frac{\lambda}{n}$ (c) $\frac{\lambda}{n^2}$ (d) $n^2\lambda$

16. Newton postulated his corpuscular theory on the basis of **[UPSEAT 2001; KCET 2001]**
(a) Newton's rings (b) Colours of thin films
(c) Rectilinear propagation of light (d) Dispersion of white light

17. Two coherent sources of intensities. I_1 and I_2 produce an interference pattern. The maximum intensity in the interference pattern will be **[UPSEAT 2001; MP PET 2001]**
(a) $I_1 + I_2$ (b) $I_1^2 + I_2^2$ (c) $(I_1 + I_2)^2$ (d) $(\sqrt{I_1} + \sqrt{I_2})^2$

18. Which one among the following shows particle nature of light **[CBSE PM/PD 2001]**
(a) Photo electric effect (b) Interference (c) Refraction (d) Polarization

19. For constructive interference to take place between two monochromatic light waves of wavelength λ, the path difference should be
[MNR 1992; UPSEAT 2001]
(a) $(2n-1)\frac{\lambda}{4}$ (b) $(2n-1)\frac{\lambda}{2}$ (c) $n\lambda$ (d) $(2n+1)\frac{\lambda}{2}$

20. In a wave, the path difference corresponding to a phase difference of ϕ is **[MP PET 2000]**
(a) $\frac{\pi}{2\lambda}\phi$ (b) $\frac{\pi}{\lambda}\phi$ (c) $\frac{\lambda}{2\pi}\phi$ (d) $\frac{\lambda}{\pi}\phi$

21. A beam of monochromatic blue light of wavelength $4200Å$ in air travels in water, its wavelength in water will be **[UPSEAT 2000]**
(a) $2800Å$ (b) $5600Å$ (c) $3150Å$ (d) $4000Å$

22. Wave front originating from a point source is **[RPET 2000]**
(a) Cylindrical (b) Spherical (c) Plane (d) Cubical

23. Waves that can not be polarised are **[KCET 2000]**
(a) Transverse waves (b) Longitudinal waves (c) Light waves (d) Electromagnetic waves

24. According to Huygen's wave theory, point on any wave front may be regarded as **[J & K CET 2000]**
(a) A photon (b) An electron (c) A new source of wave (d) Neutron

25. The light produced by a laser is all the following except **[JIPMER 2000]**
(a) Incoherent (b) Monochromatic
(c) In the form of a narrow beam (d) Electromagnetic

26. The phenomena of interference is shown by **[MNR 1994; MP PMT 1997; AIIMS 1999, 2000; JIPMER 2000; UPSEAT 1994, 2000]**
(a) Longitudinal mechanical waves only (b) Transverse mechanical waves only
(c) Electromagnetic waves only (d) All the above types of waves

27. If the ratio of amplitude of two waves is 4 : 3, then the ratio of maximum and minimum intensity is
[MP PMT 1996; AFMC 1997; RPET 2000]
(a) 16 : 18 (b) 18 : 16 (c) 49 : 1 (d) 94 : 1

28. If the distance between a point source and screen is doubled, then intensity of light on the screen will become
[RPET 1997; RPMT 1999]
(a) Four times (b) Double (c) Half (d) One-fourth

29. Soap bubble appears coloured due to the phenomenon of
[CPMT 1972, 83, 86; AFMC 1995, 97; RPET 1997; CBSE PMT 1997; AFMC 1997]
(a) Interference (b) Diffraction (c) Dispersion (d) Reflection

30. Two waves are known to be coherent if they have **[RPMT 1994, 95, 97; MP PMT 1996; MNR 1995]**
(a) Same amplitude (b) Same wavelength
(c) Same amplitude and wavelength (d) Constant phase difference and same wavelength

31. An oil flowing on water seems coloured due to interference. For observing this effect, the approximate thickness of the oil film should be **[DPMT 1987; JIPMER 1997]**

(a) 100 $\mathring{A}$ (b) 10000 $\mathring{A}$ (c) 1 mm (d) 1 cm

32. If L is the coherence length and c the velocity of light, the coherent time is **[MP PMT 1996]**

(a) cL (b) $\frac{L}{c}$ (c) $\frac{c}{L}$ (d) $\frac{1}{Lc}$

33. By a monochromatic wave, we mean **[AFMC 1995]**

(a) A single ray (b) A single ray of a single colour

(c) Wave having a single wavelength (d) Many rays of a single colour

34. Two coherent sources of light produce destructive interference when phase difference between them is **[MP PMT 1994; CPMT 1995]**

(a) 2π (b) π (c) $\pi/2$ (d) 0

35. Which one of the following statements is correct **[KCET 1994]**

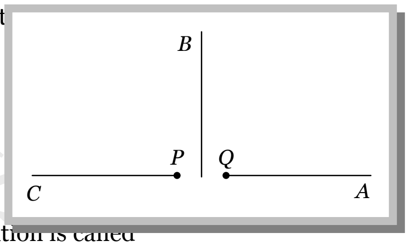
(a) In vacuum, the speed of light depends upon frequency

(b) In vacuum, the speed of light does not depend upon frequency

(c) In vacuum, the speed of light is independent of frequency and wavelength

(d) In vacuum, the speed of light depends upon wavelength

36. Figure here shows P and Q as two equally intense coherent sources emitting radiations of wavelength 20 m. The separation PQ is 5.0 m and phase of P is ahead of the phase of Q by 90°. A, B and C are three distant points of observation equidistant from the mid-point of PQ. The intensity of radiations at A, B, C will bear t **[NSEP 1994]**



(a) 0 : 1 : 4

(b) 4 : 1 : 0

(c) 0 : 1 : 2

(d) 2 : 1 : 0

37. In Huygen's wave theory, the locus of all points in the same state of vibration is called **[CBSE PMT 1993]**

(a) A half period zone (b) Vibrator (c) A wavefront (d) A ray

38. The idea of the quantum nature of light has emerged in an attempt to explain **[CPMT 1990]**

(a) Interference (b) Diffraction

(c) Radiation spectrum of a black body (d) Polarisation

39. The necessary condition for an interference by two source of light is that the **[RPMT 1988; CPMT 1989]**

(a) Two monochromatic sources should be of same amplitude but with a constant phase

(b) Two sources should be of same amplitude

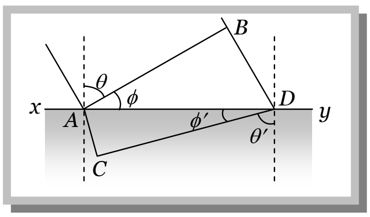
(c) Two point sources should have phase difference varying with time

(d) Two sources should be of same wavelength

40. If the intensity of the waves observed by two coherent sources is I. Then the intensity of resultant waves in constructive interference will be **[RPET 1988]**

(a) $2I$ (b) $4I$ (c) I (d) None of these

41. In figure, a wavefront AB moving in air is incident on a plane glass surface xy. Its position CD after refraction through a glass slab is shown also along with normals drawn at A and D. the refractive index of glass with respect to air will be equal to **[CPMT 19**



(a) $\frac{\sin\theta}{\sin\theta'}$

(b) $\frac{\sin\theta}{\sin\phi'}$

(c) (BD/AC)

(d) (AB/CD)

42. Four independent waves are expressed as

(i) $y_1 = a_1 \sin \omega t$ (ii) $y_2 = a_2 \sin 2\omega t$ (iii) $y_3 = a_3 \cos \omega t$ (iv) $y_4 = a_4 \sin(\omega t + \pi/3)$

The interference is possible between **[CPMT 1986]**

(a) (i) and (ii) (b) (i) and (iv) (c) (iii) and (iv) (d) Not possible at all

43. Colour of light is known by its **[MP PMT 1984]**

(a) Velocity (b) Amplitude (c) Frequency (d) Polarisation

44. Laser light is considered to be coherent because it consists of **[CPMT 1972]**

(a) Many wavelengths (b) Uncoordinated wavelengths
(c) Coordinated waves of exactly the same wavelength (d) Divergent beams

45. A laser beam may be used to measure very large distances because **[CPMT 1972]**
(a) It is unidirectional (b) It is coherent (c) It is monochromatic (d) It is not absorbed

46. Interference patterns are not observed in thick films, because
(a) Most of the incident light intensity is observed within the film
(b) A thick film has a high coefficient of reflection
(c) The maxima of interference patterns are far from the minima
(d) There is too much overlapping of colours washing out the interference pattern

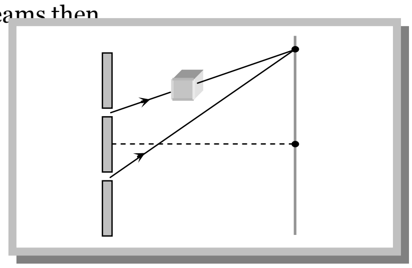
47. Phenomenon of interference is not observed by two sodium lamps of same power. It is because both waves have
(a) Not constant phase difference (b) Zero phase difference
(c) Different intensity (d) Different frequencies

Young's double slit experiment

Basic Level

48. In a Young's double slit experiment, the separation between the two slits is 0.9 *mm* and the fringes are observed one *metre* away. If it produces the second dark fringe at a distance of 1 *mm* from the central fringe, the wavelength of monochromatic source of light used is **[KCET 2004]**
(a) 500 *nm* (b) 600 *nm* (c) 450 *nm* (d) 400 *nm*

49. A monochromatic beams of light is used for the formation of fringes on the screen by illuminating the two slits in the Young's double slit mica is interposed in the path of one of the interfering beams then **[AIIMS 2004]**



(a) The fringe width increases
(b) The fringe width decreases
(c) The fringe width remains the same but the pattern shifts
(d) The fringe pattern disappears

50. In a Young's double-slit experiment the fringe width is 0.2 *mm*. If the wavelength of light used is increased by 10% and the separation between the slits is also increased by 10%, the fringe width will be **[MP PMT 2004]**
(a) 0.20 *mm* (b) 0.401 *mm* (c) 0.242 *mm* (d) 0.165 *mm*

51. In Young's experiment, the distance between the slits is reduced to half and the distance between the slit and screen is doubled, then the fringe width **[IIT 1981; MP PMT 1994; RPMT 1997; KCET (Engg./Med.) 2000; UPSEAT 2000; AMU (Engg.) 2000; C**
(a) Will not change (b) Will become half (c) Will be doubled (d) Will become four times

52. In an interference experiment, third bright fringe is obtained at a point on the screen with a light of 700 *nm*. What should be the wavelength of the light source in order obtain 5th bright fringe at the same point **[KCET 2003]**
(a) 500 *nm* (b) 630 *nm* (c) 750 *nm* (d) 420 *nm*

53. In Young's double-slit experiment the fringe width is β. If entire arrangement is placed in a liquid of refractive index n, the fringe width becomes **[KCET 2003]**
(a) $\frac{\beta}{n+1}$ (b) $n\beta$ (c) β / n (d) $\beta / n-1$

54. If the separation between slits in Young's double slit experiment is reduced to $\frac{1}{3}rd$, the fringe width becomes n times. The value of n is **[MP PET 2003]**
(a) 3 (b) $\frac{1}{3}$ (c) 9 (d) $\frac{1}{9}$

55. When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the two interfering waves of light, then the path difference changes by **[MP PMT 2002]**
(a) $(\mu+1)t$ (b) $(\mu-1)t$ (c) $\frac{(\mu+1)}{t}$ (d) $\frac{(\mu-1)}{t}$

56. In a Young's double slit experiment, the source illuminating the slits is changed from blue to violet. The width of the fringes **[Kerala CET (Med.) 2002]**

(a) Increases (b) Decreases (c) Becomes unequal (d) Remains constant

57. In Young's double slit experiment, the intensity of light coming from the first slit is double the intensity from the second slit. The ratio of the maximum intensity to the minimum intensity on the interference fringe pattern observed is **[KCET (Med.) 2002]**

(a) 34 (b) 40 (c) 25 (d) 38

58. In Young's double slit experiment the wavelength of light was changed from 7000$\AA$ to 3500$\AA$. While doubling the separation between the slits which of the following is not true for this experiment **[Orissa JEE 2002]**

(a) The width of the fringes changes

(b) The colour of bright fringes changes

(c) The separation between successive bright fringes changes

(d) The separation between successive dark fringes remains unchanged

59. In Young's double slit experiment, the central bright fringe can be identified **[KCET (Engg.) 2002]**

(a) By using white light instead of monochromatic light (b) As it is narrower than other bright fringes

(c) As it is wider than other bright fringes (d) As it has a greater intensity than the other bright fringes

60. Interference was observed in interference chamber when air was present, now the chamber is evacuated and if the same light is used, a careful observer will see **[CBSE PMT 1993; DPMT 2000; BHU 2002]**

(a) No interference

(b) Interference with bright bands

(c) Interference with dark bands

(d) Interference in which width of the fringe will be slightly increased

61. A slit of width a is illuminated by white light. For red light $(\lambda = 6500\ \AA)$. The first minima is obtained at $\theta = 30^o$. Then the value of a will be **[MP PMT 1987; CPMT 2002]**

(a) 3250 $\AA$ (b) $6.5 \times 10^{-4}\ mm$ (c) 1.24 *microns* (d) $2.6 \times 10^{-4}\ cm$

62. In the Young's double slit experiment with sodium light. The slits are 0.589 m apart. The angular separation of the third maximum from the central maximum will be (given λ = 589 mm) **[Pb. PMT 2002]**

(a) $\sin^{-1}(0.33 \times 10^{8})$ (b) $\sin^{-1}(0.33 \times 10^{-6})$ (c) $\sin^{-1}(3 \times 10^{-8})$ (d) $\sin^{-1}(3 \times 10^{-6})$

63. In the Young's double slit experiment for which colour the fringe width is least **[MP PMT 1994; UPSEAT 2001; MP PET 2001]**

(a) Red (b) Green (c) Blue (d) Yellow

64. In a Young's double slit experiment, the separation of the two slits is doubled. To keep the same spacing of fringes, the distance D of the screen from the slits should be made **[AMU (Engg.) 2001]**

(a) $\frac{D}{2}$ (b) $\frac{D}{\sqrt{2}}$ (c) $2D$ (d) $4D$

65. Consider the following statements

Assertion (A): In Young's experiment, the fringe width for dark fringes is different from that for bright fringes.

Reason (R) : In Young's double slit experiment performed with a source of white light, only black and bright fringes are observed

Of these statements **[AIIMS 2001]**

(a) Both A and R are true and R is a correct explanation of A (b) Both A and R are true but R is not a correct explanation of A

(c) Both A and R are false (d) A is false but R is true

(e) A is true but R is false

66. In a Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by **[IIT-JEE (Screening) 2001]**

(a) 12 (b) 18 (c) 24 (d) 30

67. In Young's double slit experiment, a mica slit of thickness t and refractive index μ is introduced in the ray from the first source S_1. By how much distance the fringes pattern will be displaced **[RPMT 1996, 97; JIPMER 2000]**

(a) $\frac{d}{D}(\mu - 1)t$ (b) $\frac{D}{d}(\mu - 1)t$ (c) $\frac{d}{(\mu - 1)D}$ (d) $\frac{D}{d}(\mu - 1)$

68. Young's double slit experiment is performed with light of wavelength 550 nm. The separation between the slits is 1.10 mm and screen is placed at distance of 1 m. What is the distance between the consecutive bright or dark fringes **[Pb. PMT 2000]**

(a) 1.5 *mm* (b) 1.0 *m* (c) 0.5 *mm* (d) None of these

69. In interference obtained by two coherent sources, the fringe width (β) has the following relation with wavelength (λ) **[CPMT 1997; MP PMT 2000]**

(a) $\beta \propto \lambda^2$ (b) $\beta \propto \lambda$ (c) $\beta \propto 1/\lambda$ (d) $\beta \propto \lambda^{-2}$

70. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then in the interference pattern **[IIT-JEE (Screening) 2000]**

(a) The intensities of both the maxima and the minima increase
(b) The intensity of maxima increases and the minima has zero intensity
(c) The intensity of maxima decreases and that of the minima increases
(d) The intensity of maxima decreases and the minima has zero intensity

71. In Young's double slit experiment with a source of light of wavelength 6320$\mathring{A}$, the first maxima will occur when **[Roorkee 1999]**

(a) Path difference is 9480 $\mathring{A}$ (b) Phase difference is 2π radian
(c) Path difference is 6320 $\mathring{A}$ (d) Phase difference is π radian

72. If a transparent medium of refractive index μ = 1.5 and thickness $t = 2.5 \times 10^{-5}\, m$ is inserted in front of one of the slits of Young's double slit experiment, how much will be the shift in the interference pattern? The distance between the slits is 0.5 *mm* and that between slits and screen is 100 *cm* **[AIIMS 1999]**

(a) 5 *cm* (b) 2.5 *cm* (c) 0.25 *cm* (d) 0.1 *cm*

73. If a torch is used in place of monochromatic light in Young's experiment what will happens **[MH CET (Med.) 1999; KCET (Med.) 1999]**

(a) Fringe will appear for a moment then it will disappear (b) Fringes will occur as from monochromatic light
(c) Only bright fringes will appear (d) No fringes will appear

74. When a thin metal plate is placed in the path of one of the interfering beams of light **[KCET (Engg./Med.) 1999]**

(a) Fringe width increases (b) Fringes disappear (c) Fringes become brighter (d) Fringes become blurred

75. What happens by the use of white light in Young's double slit experiment **[Similar to (AIIMS 2001; Kerala 2000); IIT-JEE 1987; RPMT 1993; MP PMT 1996; RPET 1998; UPSEAT 1999]**

(a) Bright fringes are obtained
(b) Only bright and dark fringes are obtained
(c) Central fringe is bright and two or three coloured and dark fringes are observed
(d) None of these

76. Young's experiment is performed in air and then performed in water, the fringe width **[CPMT 1990; MP PMT 1994; RPMT 1997]**

(a) Will remain same (b) Will decrease (c) Will increase (d) Will be infinite

77. In Young's experiment, one slit is covered with a blue filter and the other (slit) with a yellow filter. Then the interference pattern **[MP PET 1997]**

(a) Will be blue (b) Will be yellow (c) Will be green (d) Will not be formed

78. Two sources give interference pattern which is observed on a screen. *D* distance apart from the sources. The fringe width is $2w$. If the distance *D* is now doubled, the fringe width will **[MP PET 1997]**

(a) Become $w/2$ (b) Remain the same (c) Become w (d) Become $4w$

79. In Young's double slit experiment, angular width of fringes is 0.20° for sodium light of wavelength 5890 $\mathring{A}$. If complete system is dipped in water, then angular width of fringes becomes **[RPET 1997]**

(a) 0.11° (b) 0.15° (c) 0.22° (d) 0.30°

80. In two separate set-ups of the Young's double slit experiment, fringes of equal width are observed when lights of wavelengths in the ratio 1 : 2 are used. If the ratio of the slit separation in the two cases is 2 : 1, the ratio of the distances between the plane of the slits and the screen in the two set-ups is **[Kurukshetra CEE 1996]**

(a) 4 : 1 (b) 1 : 1 (c) 1 : 4 (d) 2 : 1

81. In a Young's double slit experiment, the central point on the screen is **[MP PMT 1996]**

(a) Bright (b) Dark (c) First bright and then dark (d) First dark and then bright

82. In Young's double slit experiment, the distance between sources is 1 *mm* and distance between the screen and source is 1*m*. If the fringe width on the screen is 0.06 *cm*, then λ = **[CPMT 1996]**

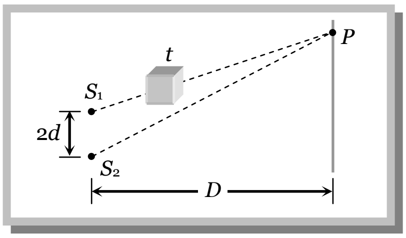
(a) 6000 $\mathring{A}$ (b) 4000 $\mathring{A}$ (c) 1200 $\mathring{A}$ (d) 2400 $\mathring{A}$

83. In a Young's double slit experiment, the distance between two coherent sources is 0.1 *mm* and the distance between the slits and the screen is 20 *cm*. If the wavelength of light is 5460 $\mathring{A}$ then the distance between two consecutive maxima is **[RPMT 1995]**

(a) 0.5 *mm* (b) 1.1 *mm* (c) 1.5 *mm* (d) 2.2 *mm*

genius

84. If a thin mica sheet of thickness t and refractive index $\mu = (5/3)$ is placed in the path of one of the interfering beams as shown in figure, then the displacement of the fringe system is **[CPMT 1995]**



(a) $\frac{Dt}{3d}$

(b) $\frac{Dt}{5d}$

(c) $\frac{Dt}{4d}$

(d) $\frac{2Dt}{5d}$

85. In a double slit experiment, the first minimum on either side of the central maximum occurs where the path difference between the two paths is **[CPMT 1995]**

(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$ (c) λ (d) 2λ

86. In Young's double slit experiment, the phase difference between the light waves reaching third bright fringe from the central fringe will be (λ = 6000 Å) **[MP PMT 1994]**

(a) Zero (b) 2π (c) 4π (d) 6π

87. Sodium light $(\lambda = 6 \times 10^{-7} m)$ is used to produce interference pattern. The observed fringe width is 0.12 *mm*. The angle between the two interfering wave trains is **[CPMT 1993]**

(a) 5×10^{-1} *rad* (b) 5×10^{-3} *rad* (c) 1×10^{-2} *rad* (d) 1×10^{-3} *rad*

88. The contrast in the fringes in any interference pattern depends on **[Roorkee 1992]**

(a) Fringe width (b) Intensity ratio of the sources

(c) Distance between the slits (d) Wavelength

89. In Young's double slit experiment, carried out with light of wavelength λ = 5000 Å, the distance between the slits is 0.2 *mm* and the screen is at 200 *cm* from the slits. The central maximum is at x = 0. The third maximum (taking the central maximum as zeroth maximum) will be at x equal to **[CBSE PMT 1992]**

(a) 1.67 *cm* (b) 1.5 *cm* (c) 0.5 *cm* (d) 5.0 *cm*

90. In a Young's experiment, two coherent sources are placed 0.90 *mm* apart and the fringes are observed one *metre* away. If it produces the second dark fringe at a distance of 1 *mm* from the central fringe, the wavelength of monochromatic light used would be **[CBSE PMT 1992]**

(a) 60×10^{-4} *cm* (b) 10×10^{-4} *cm* (c) 10×10^{-5} *cm* (d) 60×10^{-5} *cm*

91. In Fresnel's biprism, coherent sources are obtained by **[RPET 1991]**

(a) Division of wavefront (b) Division of amplitude (c) Division of wavelength (d) None of these

92. In Young's experiment, the ratio of maximum and minimum intensities in the fringe system is 9 : 1. The ratio of amplitudes of coherent sources is **[NCERT 1990]**

(a) 9 : 1 (b) 3 : 1 (c) 2 : 1 (d) 1 : 1

93. In a certain double slit experimental arrangement interference fringes of width 1.0 *mm* each are observed when light of wavelength 5000 Å is used. Keeping the set up unaltered, if the source is replaced by another source of wavelength 6000 Å, the fringe width will be **[CPMT 1988]**

(a) 0.5 *mm* (b) 1.0 *mm* (c) 1.2 *mm* (d) 1.5 *mm*

94. In Young's double slit experiment, if the slit widths are in the ratio 1 : 9, then the ratio of the intensity at minima to that at maxima will be **[MP PET 1987]**

(a) 1 (b) 1/9 (c) 1/4 (d) 1/3

95. The Young's experiment is performed with the lights of blue (λ = 4360 Å) and green colour (λ = 5460 Å). If the distance of the 4th fringe from the centre is x, then **[CPMT 1987]**

(a) $x(\text{Blue}) = x(\text{Green})$ (b) $x(\text{Blue}) > x(\text{Green})$ (c) $x(\text{Blue}) < x(\text{Green})$ (d) $\frac{x(\text{Blue})}{x(\text{Green})} = \frac{5460}{4360}$

96. In Young's experiment, keeping the distance of the slit from screen constant if the slit width is reduced to half, then **[CPMT 1986]**

(a) The fringe width will be doubled (b) The fringe width will reduce to half

(c) The fringe width will not change (d) The fringe width will become $\sqrt{2}$ times

97. In Young's experiment, if the distance between screen and the slit aperture is increased the fringe width will **[RPET 1986]**

(a) Decrease (b) Increases but intensity will decrease

(c) Increase but intensity remains unchanged (d) Remains unchanged but intensity decreases

98. In Fresnel's biprism experiment, the two coherent sources are **[RPET 1985]**

(a) Real (b) Imaginary

(c) One is real and the other is imaginary (d) None of these

99. In Fresnel's experiment, the width of the fringe depends upon the distance **[RPET 1985]**

(a) Between the prism and the slit aperture

(b) Of the prism from the screen

(c) Of screen from the imaginary light sources

(d) Of the screen from the prism and the distance from the imaginary sources

100. In the Young's double slit experiment, the ratio of intensities of bright and dark fringes is 9. This means that **[IIT-JEE 1982]**

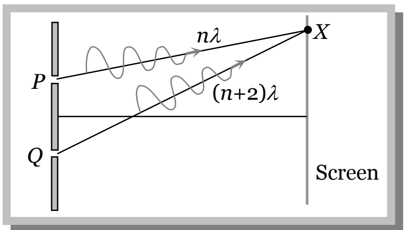
(a) The intensities of individual sources are 5 and 4 units respectively

(b) The intensities of individual sources are 4 and 1 units respectively

(c) The ratio of their amplitudes is 3

(d) The ratio of their amplitudes is 2

101. The figure below shows a double slit experiment. P and Q are the slits. The path lengths PX and QX are $n\lambda$ and $(n + 2)\lambda$ respectively where n is a whole number and λ is the wavelength. Taking the central bright fringe as zero, what is formed at X



(a) First bright

(b) First dark

(c) Second bright

(d) Second dark

102. A plate of thickness t made of a material of refractive index μ is placed in front of one of the slits in a double slit experiment. What should be the minimum thickness t which will make the intensity at the centre of the fringe pattern zero

(a) $(\mu-1)\dfrac{\lambda}{2}$ (b) $(\mu-1)\lambda$ (c) $\dfrac{\lambda}{2(\mu-1)}$ (d) $\dfrac{\lambda}{(\mu-1)}$

103. The thickness of a plate (refractive index μ for light of wavelength λ) which will introduce a path difference of $\dfrac{3\lambda}{4}$ is

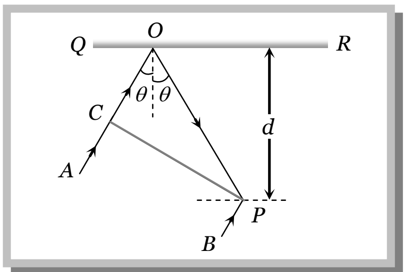
(a) $\dfrac{3\lambda}{4(\mu-1)}$ (b) $\dfrac{3\lambda}{2(\mu-1)}$ (c) $\dfrac{\lambda}{2(\mu-1)}$ (d) $\dfrac{3\lambda}{4\mu}$

Advance Level

104. In the Young's double slit experiment, if the phase difference between the two waves interfering at a point is ϕ, the intensity at that point can be expressed by the expression (where $A + B$ depends upon the amplitude of the two waves) **[MP PMT/PET 1998; MP PMT 2003]**

(a) $I=\sqrt{A^2+B^2\cos^2\phi}$ (b) $I=\dfrac{A}{B}\cos\phi$ (c) $I=A+B\cos\phi/2$ (d) $I=A+B\cos\phi$

105. In the adjacent diagram CP represents wavefronts and AO and BP the corresponding two rays. Find the condition on θ for constructive interference at P between the ray BP and reflected ray OP **[IIT-JEE (Screening) 2003]**



(a) $\cos\theta = 3\lambda/2d$

(b) $\cos\theta = \lambda/4d$

(c) $\sec\theta - \cos\theta = \lambda/d$

(d) $\sec\theta - \cos\theta = 4\lambda / d$

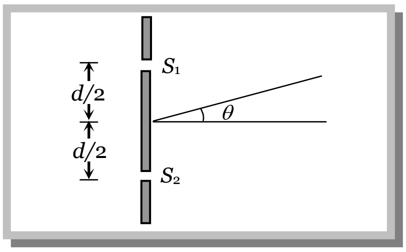
106. When one of the slits of Young's experiment is covered with a transparent sheet of thickness 4.8 *mm*, the central fringe shifts to a position originally occupied by the 30th bright fringe. What should be the thickness of the sheet if the central fringe has to shift to the position occupied by 20th bright fringe **[KCET (Engg.) 2002]**

(a) 3.8 *mm* (b) 1.6 *mm* (c) 7.6 *mm* (d) 3.2 *mm*

107. In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is **[IIT-JEE (Screening) 2002)]**

(a) 2λ (b) $\frac{2\lambda}{3}$ (c) $\frac{\lambda}{3}$ (d) λ

108. In an interference arrangement similar to Young's double slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources each of frequency 10^6 *Hz*. The sources are synchronized to have zero phase difference. The slits are separated by distance d = 150 *m*. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is maximum intensity, then $I(\theta)$ for $0 \le \theta \le 90°$ is given by **[IIT-JEE 1995]**



(a) $I(\theta) = I_0$ for $\theta = 90°$

(b) $I(\theta) = I_0 / 2$ for $\theta = 30°$

(c) $I(\theta) = I_0 / 4$ for $\theta = 90°$

(d) $I(\theta)$ is constant for all values of θ

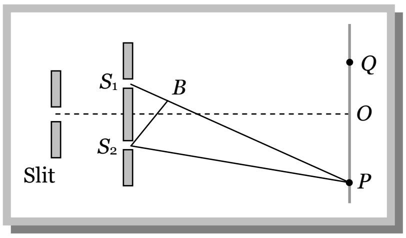
109. In Young's double slit experiment, white light is used. The separation between the slits is b. the screen is at a distance $d(d >> b)$ from the slits. Some wavelengths are missing exactly in front of one slit. These wavelengths are **[IIT-JEE 1984; AIIMS 1995]**

(a) $\lambda = \frac{b^2}{d}$ (b) $\lambda = \frac{2b^2}{d}$ (c) $\lambda = \frac{b^2}{3d}$ (d) $\lambda = \frac{2b^2}{3d}$

110. In a two slit experiment with monochromatic light fringes are obtained on a screen placed at some distance from the sits. If the screen is moved by $5 \times 10^{-2} m$ towards the slits, the change in fringe width is $3 \times 10^{-5} m$. If separation between the slits is $10^{-3} m$, the wavelength of light used is **[Roorkee 1992]**

(a) 6000 $Å$ (b) 5000 $Å$ (c) 3000 $Å$ (d) 4500 $Å$

111. In the figure is shown Young's double slit experiment. Q is the position of the first bright fringe on the right side of O. P is the 11th fringe on the other side, as measured from Q. If the wavelength of the light used is $6000 \times 10^{-10} m$, then S_1B will be equal to **[CPMT 1986, 92]**



(a) $6 \times 10^{-6} m$

(b) $6.6 \times 10^{-6} m$

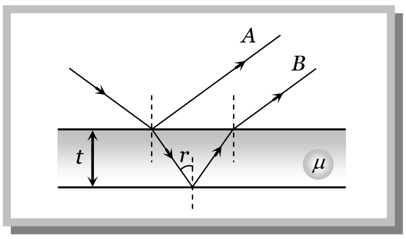
(c) $3.138 \times 10^{-7} m$

(d) $3.144 \times 10^{-7} m$

112. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same set up the two slits are of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is **[IIT-JJE 1986]**

(a) 1 : 2 (b) 2 : 1 (c) 4 : 1 (d) 1 : 1

113. When light of wavelength λ falls on a thin film of thickness t and refractive index n, the essential condition for the production of constructive interference fringes by the rays A and B are (m = 1, 2, 3, ……)



(a) $2nt\cos r = \left(m - \frac{1}{2}\right)\lambda$

(b) $2nt\cos r = m\lambda$

(c) $nt\cos r = m\lambda$

(d) $nt\cos r = (m-1)\lambda$

114. Four light waves are represented by

(i) $y = a_1 \sin\omega t$ (ii) $y = a_2 \sin(\omega t + \phi)$ (iii) $y = a_1 \sin 2\omega t$ (iv) $y = a_2 \sin 2(\omega t + \phi)$

Interference fringes may be observed due to superposition of

(a) (i) and (ii) (b) (i) and (iii) (c) (ii) and (iv) (d) (iii) and (iv)

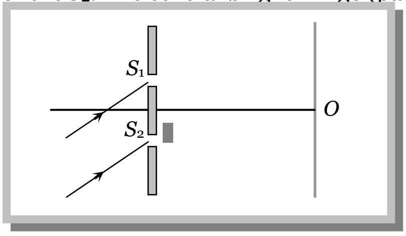
115. In Young's double slit experiment the y-coordinates of central maxima and 10th maxima are 2 cm and 5 cm respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5 the corresponding y-coordinates will be

(a) 2 cm, 7.5 cm (b) 3 cm, 6 cm (c) 2 cm, 4cm (d) 4/3 cm, 10/3 cm

116. The maximum intensity in Young's double slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 10\,d$

(a) $\frac{I_0}{2}$ (b) $\frac{3}{4}I_0$ (c) I_0 (d) $\frac{I_0}{4}$

117. A monochromatic beam of light falls on YDSE apparatus at some angle (say θ) as shown in figure. A thin sheet of glass is inserted in front of the lower slit S_2. The central bright fringe (path difference = 0) will be obtained



(a) At O

(b) Above O

(c) Below O

(d) Anywhere depending on angle θ, thickness of plate t and refractive index of glass μ

118. In Young's double slit experiment how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringe if $\lambda = 2000\ Å$ and $d = 7000\ Å$

(a) 12 (b) 7 (c) 18 (d) 4

119. Young's double slit experiment is made in a liquid. The 10th bright fringe in liquid lies where 6th dark fringe lies in vacuum. The refractive index of the liquid is approximately

(a) 1.8 (b) 1.54 (c) 1.67 (d) 1.2

120. Light of wavelength λ_0 in air enters a medium of refractive index n. If two points A and B in this medium lie along the path of this light at a distance x, then phase difference ϕ_0 between these two points is

(a) $\phi_0 = \frac{1}{n}\left(\frac{2\pi}{\lambda_0}\right)x$ (b) $\phi_0 = n\left(\frac{2\pi}{\lambda_0}\right)x$ (c) $\phi_0 = (n-1)\left(\frac{2\pi}{\lambda_0}\right)x$ (d) $\phi_0 = \frac{1}{(n-1)}\left(\frac{2\pi}{\lambda_0}\right)x$

121. In a Young's double slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelength $\lambda_0 = 750\,nm$ and $\lambda = 900\,nm$. The minimum distance from the common central bright fringe on a screen 2m from the slits where a bright fringe from one interference pattern coincides with a bright fringe from the other is

(a) 1.5 mm (b) 3 mm (c) 4.5 mm (d) 6 mm

122. In the ideal double slit experiment, when a glass plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is

(a) 2λ (b) $\frac{2\lambda}{3}$ (c) $\frac{\lambda}{3}$ (d) λ

123. Two wavelengths of light λ_1 and λ_2 are sent through a Young's double slit apparatus simultaneously. If the third order λ_1 bright fringe coincides with the fourth order λ_2 bright fringe then

(a) $\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$ (b) $\frac{\lambda_1}{\lambda_2} = \frac{3}{4}$ (c) $\frac{\lambda_1}{\lambda_2} = \frac{5}{4}$ (d) $\frac{\lambda_1}{\lambda_2} = \frac{4}{5}$

124. A flake of glass (refractive index 1.5) is placed over one of the openings of a double slit apparatus. The interference pattern displaces itself through seven successive maxima towards the side where the flake is placed. if wavelength of the diffracted light is $\lambda = 600\,nm$, then the thickness of the flake is

(a) 2100 nm (b) 4200 nm (c) 8400 nm (d) None of these

125. In a double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then in the interference pattern

(a) The intensitites of both the maxima and the minima increase

(b) The intensity of the maxima increases and minima has zero intensity

(c) The intensity of the maxima decreases and that of minima increases

(d) The intensity of the maxima decreases and the minima has zero intensity

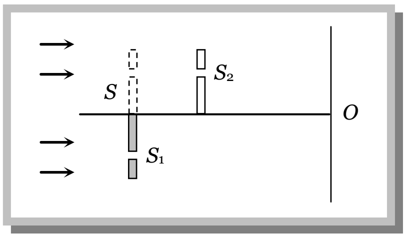
126. In Young's experiment the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of n for which the $(n+1)th$ blue bright band coincides with the n^{th} red band is

(a) 4 (b) 3 (c) 2 (d) 1

127. In a double slit experiment if 5^{th} dark fringe is formed opposite to one of the slits, the wavelength of light is

(a) $\frac{d^2}{6D}$ (b) $\frac{d^2}{5D}$ (c) $\frac{d^2}{15D}$ (d) $\frac{d^2}{9D}$

128. In a Young's double slit experiment one of the slits is advanced towards the screen by a distance $d/2$ and $d = n\lambda$ where n is an odd integer and d is the initial distance between the slits. If I_0 is the intensity of each wave from the slits, the intensity at O is



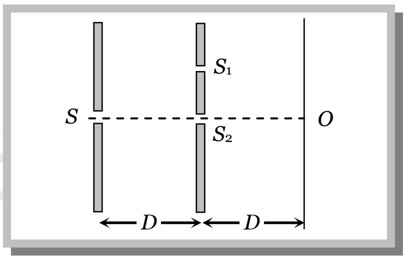
(a) I_0

(b) $\frac{I_0}{4}$

(c) 0

(d) $2I_0$

129. Two ideal slits S_1 and S_2 are at a distance d apart, and illuminated by light of wavelength λ passing through an ideal source slit S placed on the line through S_2 as shown. The distance between the planes of slits and the source slit is D. A screen is held at a distance D from the plane of the slits. The minimum value of d for which there is darkness at O is



(a) $\sqrt{\frac{3\lambda D}{2}}$

(b) $\sqrt{\lambda D}$

(c) $\sqrt{\frac{\lambda D}{2}}$

(d) $\sqrt{3\lambda D}$

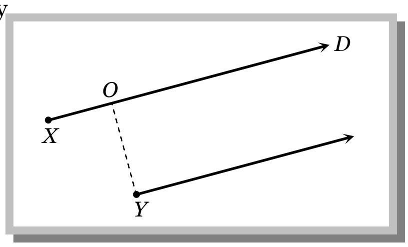
130. In a double slit experiment interference is obtained from electron waves produced in an electron gun supplied with voltage V. if λ is the wavelength of the beam, D is the distance of screen, d is the spacing between coherent source, h is Planck's constant, e is charge on electron and m is mass of electron then fringe width is given as

(a) $\frac{hD}{\sqrt{2meV}\,d}$ (b) $\frac{2hD}{\sqrt{meV}\,d}$ (c) $\frac{hd}{\sqrt{2meV}\,D}$ (d) $\frac{2hd}{\sqrt{meV}\,D}$

131. In a double slit arrangement fringes are produced using light of wavelength 4800 $\AA$. One slit is covered by a thin plate of glass of refractive index 1.4 and the other with another glass plate of same thickness but of refractive index 1.7. By doing so the central bright shifts to original fifth bright fringe from centre. Thickness of glass plate is

(a) 8 μm (b) 6 μm (c) 4 μm (d) 10 μm

132. Two point sources X and Y emit waves of same frequency and speed but Y lags in phase behind X by $2\pi l$ radian. If there is a maximum in direction D the distance XO using n as an integer is given by



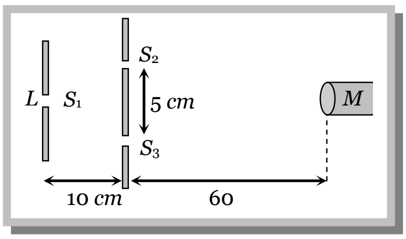
(a) $\frac{\lambda}{2}(n-l)$

(b) $\lambda(n+l)$

(c) $\frac{\lambda}{2}(n+l)$

(d) $\lambda(n-l)$

133. A student is asked to measure the wavelength of monochromatic light. He sets up the apparatus sketched below. S_1, S_2, S_3 are narrow parallel slits, L is a sodium lamp and M is a micrometer eye-piece. The student fails to observe interference fringes. You would advise him to



(a) Increase the width of S_1

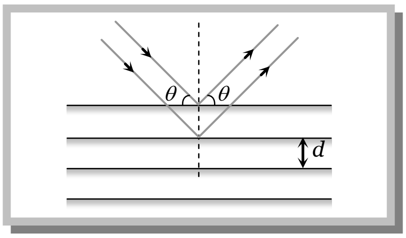
(b) Decrease the distance between S_2 and S_3

(c) Replace L with a white light source

(d) Replace M with a telescope

genius

134. A beam with wavelength λ falls on a stack of partially reflecting planes with separation d. The angle θ that the beam should make with the planes so that the beams reflected from successive planes may interfere constructively is (where n =1, 2, ……)



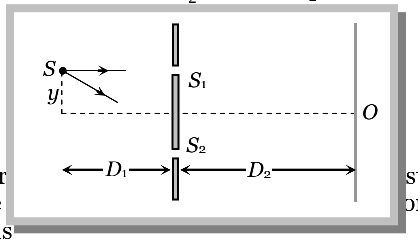
(a) $\sin^{-1}\left(\frac{n\lambda}{d}\right)$

(b) $\tan^{-1}\left(\frac{n\lambda}{d}\right)$

(c) $\sin^{-1}\left(\frac{n\lambda}{2d}\right)$

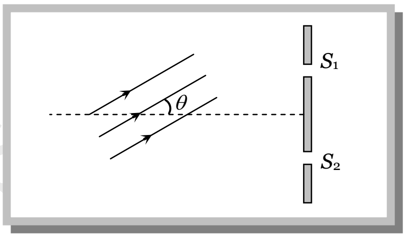
(d) $\cos^{-1}\left(\frac{n\lambda}{2d}\right)$

135. In a double slit experiment the source slit S is at a distance D_1 and the screen at a distance D_2 from the plane of ideal slit cuts S_1 and S_2 as shown. If the source slit is shifted to by parallel to S_1S_2, the central bright fringe will be shifted by



(a) y (b) $-y$ (c) $\frac{D_2}{D_1}y$ (d) $-\frac{D_2}{D_1}y$

136. A parallel beam of monochromatic light is used in a Young's double slit exper stance d and the screen is placed parallel to the plane of the slits. The angle which the ormal to the plane of the slits to produce darkness at the position of central brightness is



(a) $\cos^{-1}\frac{\lambda}{d}$

(b) $\cos^{-1}\frac{2\lambda}{d}$

(c) $\sin^{-1}\frac{\lambda}{d}$

(d) $\sin^{-1}\frac{\lambda}{2d}$

137. In a Young's double slit experiment, let β be the fringe width, and let I_0 be the intensity at the central bright fringe. At a distance x from the central bright fringe, the intensity will be

(a) $I_0\cos\left(\frac{x}{\beta}\right)$ (b) $I_0\cos^2\left(\frac{x}{\beta}\right)$ (c) $I_0\cos^2\left(\frac{\pi x}{\beta}\right)$ (d) $\left(\frac{I_0}{4}\right)\cos^2\left(\frac{\pi x}{\beta}\right)$

138. In Young's double slit experiment the distance d between the slits S_1 and S_2 is 1 mm. What should be the width of each slit be so as to obtain 10 maxima of the two slit interference pattern with in the central maximum of the single slit diffraction pattern

(a) 0.1 mm (b) 0.2 mm (c) 0.3 mm (d) 0.4 mm

Diffraction of light

139. When light is incident on a diffraction grating the zero order principal maximum will be **[KCET 2004]**

(a) One of the component colours (b) Absent

(c) Spectrum of the colours (d) White

140. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is

[IIT-JEE 1994; KCET 2004]

(a) 1.2 mm (b) 1.2 cm (c) 2.4 cm (d) 2.4 mm

141. Consider the following statements

Assertion (A): When a tiny circular obstacle is placed in the path of light from some distance, a bright spot is seen at the centre of the shadow of the obstacle.

Reason (R**)** : Destructive interference occurs at the centre of the shadow.

Of these statements **[AIIMS 2002]**

(a) Both A and R are true and R is a correct explanation of A (b) Both A and R are true but R is not a correct explanation of A

(c) A is true but R is false (d) A is false but R is true

(e) Both A and R are false

genius

142. The light of wavelength 6328 $\AA$ is incident on a slit of width 0.2 *mm* perpendicularly situated at a distance of 9 *m* and the central maxima between two minima, the angular is approximately **[MP PMT 1987; Pb. PMT 2002]**

(a) 0.36° (b) 0.18° (c) 0.72° (d) 0.08°

143. A diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light **[KCET (Eng./Med.) 2000; BHU 2001]**

(a) No change
(b) diffraction bands become narrower and crowded together
(c) Bands become broader and farther apart
(d) Bands disappear

144. Angular width (β) of central maximum of a diffraction pattern on a single slit does not depend upon **[DCE 2000, 2001]**

(a) Distance between slit and source
(b) Wavelength of light used
(c) Width of the slit
(d) Frequency of light used

145. In order to see diffraction the thickness of the film is **[J&K CEE 2001]**

(a) 100 Å (b) 10,000 Å (c) 1 *mm* (d) 1 *cm*

146. What will be the angle of diffracting for the first minimum due to Fraunhoffer diffraction with sources of light of wave length 550 *nm* and slit of width 0.55 *mm* **[Pb. PMT 2001]**

(a) 0.001 *rad* (b) 0.01 *rad* (c) 1 *rad* (d) 0.1 *rad*

147. The bending of beam of light around corners of obstacles is called **[NCERT 1990; AFMC 1995; RPET 1997; CPMT 1999; JIPMER 2000]**

(a) Reflection (b) Diffraction (c) Refraction (d) Interference

148. Diffraction effects are easier to notice in the case of sound waves than in the case of light waves because **[RPET 1978; KCET 2000]**

(a) Sound waves are longitudinal
(b) Sound is perceived by the ear
(c) Sound waves are mechanical waves
(d) Sound waves are of longer wavelength

149. Direction of the first secondary maximum in the Fraunhofer diffraction pattern at a single slit is given by (*a* is the width of the slit) **[KCET 1999]**

(a) $a\sin\theta = \frac{\lambda}{2}$ (b) $a\cos\theta = \frac{3\lambda}{2}$ (c) $a\sin\theta = \lambda$ (d) $a\sin\theta = \frac{3\lambda}{2}$

150. A slit of size 0.15 *cm* is placed at 2.1 *m* from a screen. On illuminated it by a light of wavelength $5 \times 10^{-5}\,cm$. The width of diffraction pattern will be **[RPET 1999]**

(a) 70 *mm* (b) 0.14 *mm* (c) 1.4 *cm* (d) 0.14 *cm*

151. Yellow light is used in a single slit diffraction experiment with a slit of 0.6 *mm*. If yellow light is replaced by *x*-rays, than the observed pattern will reveal **[IIT-JEE 1999]**

(a) That the central maxima is narrower
(b) More number of fringes
(c) Less number of fringes
(d) No diffraction pattern

152. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of incident beam. At the first maximum of the diffraction pattern the phase difference between the rays coming from the edges of the slit is **[IIT–JEE 1995, 98]**

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

153. Diffraction and interference of light suggest **[CPMT 1995; RPMT 1998]**

(a) Nature of light is electro-magnetic
(b) Wave nature
(c) Nature is quantum
(d) Nature of light is transverse

154. A light wave is incident normally over a slit of width $24 \times 10^{-5}\,cm$. The angular position of second dark fringe from the central maxima is 30°. What is the wavelength of light **[RPET 1995]**

(a) 6000 Å (b) 5000 Å (c) 3000 Å (d) 1500 Å

155. A beam of light of wavelength 600 *nm* from a distant source falls on a single slit 1.00 *nm* wide and the resulting diffraction pattern is observed on a screen 2 *m* away. The distance between the first dark fringes on either side of the central bright fringe is **[IIT-JEE 1994]**

(a) 1.2 *cm* (b) 1.2 *mm* (c) 2.4 *cm* (d) 2.4 *mm*

156. A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 *mm*. The light is focused by a convex lens on a screen placed on the focal plane. The first minimum will be formed for the angle of diffraction equal to **[CBSE PMT 1993]**

(a) 0° (b) 15° (c) 30° (d) 60°

157. Light appears to travel in straight lines since **[RPMT 1997; AIIMS 1998; CPMT 1987, 89, 90, 2001; KCET (Engg.) 2002; BHU 2002]**

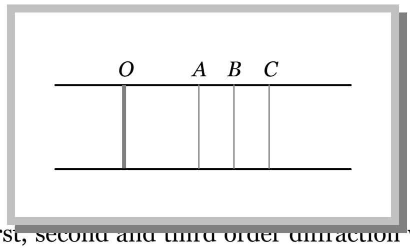
(a) It is not absorbed by the atmosphere
(b) It is reflected by the atmosphere

(c) It's wavelength is very small (d) It's velocity is very large

158. The condition for observing Fraunhofer diffraction from a single slit is that the light wavefront incident on the slit should be **[MP PMT 1987]**

(a) Spherical (b) Cylindrical (c) Plane (d) Elliptical

159. The position of the direct image obtained at O, when a monochromatic beam of light is passed through a plane transmission grating at normal incidence is shown in fig.



The diffracted images A, B and C correspond to the first, second and third order diffraction when the source is replaced by an another source of shorter wavelength **[CPMT 1986]**

(a) All the four shift in the direction C to O (b) All the four will shift in the direction O to C

(c) The images C, B and A will shift toward O (d) The images C, B and A will shift away from O

160. To observe diffraction the size of an obstacle **[CPMT 1982]**

(a) Should be of the same order as wavelength (b) Should be much larger than the wavelength

(c) Have no relation to wavelength (d) Should be exactly $\frac{\lambda}{2}$

161. The first diffraction minima due to a single slit diffraction is at $\theta = 30^o$ for a light of wavelength 5000 Å. The width of the slit is **[CPMT 1985]**

(a) $5\times10^{-5}\,cm$ (b) $1.0\times10^{-4}\,cm$ (c) $2.5\times10^{-5}\,cm$ (d) $1.25\times10^{-5}\,cm$

162. Radio waves diffract pronoucedly around buildings while light waves which are also electromagnetic waves do not because **[PPE 1978]**

(a) Wavelength of the radio waves is not comparable with the size of the obstacle

(b) Wavelength of radio waves is of the order of 200-500 *m* hence they bend more than the light waves whose wavelength is very small

(c) Light waves are transverse whereas radio waves are longitudinal

(d) None of the above

163. One cannot obtain diffraction from a wide slit illuminated by a monochromatic light because **[PPE 1978]**

(a) The half period elements contained in a wide slit are very large so the resultant effect is general illumination

(b) The half period elements contained in a wide slit are small so the resultant effect is general illumination

(c) Diffraction patterns are superimposed by interference pattern and hence the result is general illumination

(d) None of these

164. In the far field diffraction pattern of a single slit under polychromatic illumination, the first minimum with the wavelength λ_1 is found to be coincident with the third maximum at λ_2. So

(a) $3\lambda_1 = 0.3\lambda_2$ (b) $3\lambda_1 = \lambda_2$ (c) $\lambda_1 = 3.5\lambda_2$ (d) $0.3\lambda_1 = 3\lambda_2$

165. In case of Fresnel diffraction

(a) Both source and screen are at finite distance from diffracting device

(b) Source is at finite distance while screen at infinity from diffraction device

(c) Screen is at finite distance while source at infinity from diffracting device

(d) Both source and screen are effectively at infinity from diffracting device

166. Light of wavelength λ = 5000 $\mathring{A}$ falls normally on a narrow slit. A screen placed at a distance of 1 *m* from the slit and perpendicular to the direction of light. The first minima of the diffraction pattern is situated at 5 *mm* from the centre of central maximum. The width of the slit is

(a) 0.1 *mm* (b) 1.0 *mm* (c) 0.5 *mm* (d) 0.2 *mm*

167. Light falls normally on a slit of width 0.3 *mm*. A lens of focal length 40 *cm* collects the rays at its focal plane. The distance of the first dark band from the direct one is 0.8 *mm*. The wavelength of light is

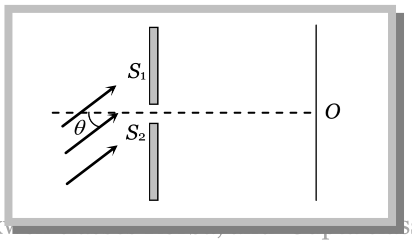
(a) 4800 Å (b) 5000 Å (c) 6000 Å (d) 5896 Å

168. A parallel monochromatic beam of light is incident at an angle θ to the normal of a slit of width e. The central point O of the screen will be dark if

(a) $e\sin\theta = n\lambda$ where n = 1, 3, 5 ...

(b) $e\sin\theta = n\lambda$ where n = 1, 2, 3 ...

(c) $e\sin\theta = (2n-1)\lambda/2$ where n = 1, 2, 3 ......



(d) $e\cos\theta = n\lambda$ where n = 1, 2, 3, 4 ....

Polarization of Light

169. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refraction index n) is **[AIEEE 2004]**

(a) $\sin^{-1}(n)$ (b) $\sin^{-1}\left(\frac{1}{n}\right)$ (c) $\tan^{-1}\left(\frac{1}{n}\right)$ (d) $\tan^{-1}(n)$

170. Through which character we can distinguish the light waves from sound waves **[CBSE PMT 1990; RPET 2002]**

(a) Interference (b) Refraction (c) Polarisation (d) Reflection

171. Which of following can not be polarised **[Kerala PMT 2001]**

(a) Radio waves (b) Ultraviolet rays (c) Infrared rays (d) Ultrasonic waves

172. A polaroid is placed at 45° to an incoming light of intensity I_0. Now the intensity of light passing through polaroid after polarisation would be **[CPMT 1995]**

(a) I_0 (b) $I_0/2$ (c) $I_0/4$ (d) Zero

173. Plane polarised light is passed through a polaroid. On viewing through the polaroid we find that when the polariod is given one complete rotation about the direction of the light, one of the following is observed **[MNR 1993]**

(a) The intensity of light gradually decreases to zero and remains at zero

(b) The intensity of light gradually increases to a maximum and remains at maximum

(c) There is no change in intensity

(d) The intensity of light is twice maximum and twice zero

174. Out of the following statements which is not correct **[CPMT 1991]**

(a) When unpolarised light passes through a Nicol's prism, the emergent light is elliptically polarised

(b) Nicol's prism works on the principle of double refraction and total internal reflection

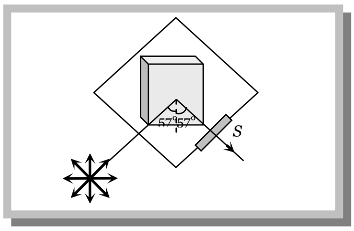
(c) Nicol's prism can be used to produce and analyse polarised light

(d) Calcite and Quartz are both doubly refracting crystals

175. A ray of light is incident on the surface of a glass plate at an angle of incidence equal to Brewster's angle ϕ. If μ represents the refractive index of glass with respect to air, then the angle between reflected and refracted rays is **[CPMT 1989]**

(a) $90+\phi$ (b) $\sin^{-1}(\mu\cos\phi)$ (c) 90° (d) $90^o - \sin^{-1}(\sin\phi/\mu)$

176. Figure represents a glass plate placed vertically on a horizontal table with a beam of unpolarised light falling on its surface at the polarising angle of 57° with the normal. The electric vector in the reflected light on screen S will vibrate with respect to the plane of incidence in a **[CPMT 1988]**



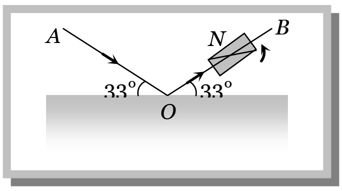
(a) Vertical plane

(b) Horizontal plane

(c) Plane making an angle of 45° with the vertical

(d) Plane making an angle of 57° with the horizontal

177. A beam of light AO is incident on a glass slab ($\mu = 1.54$) in a direction as shown in figure. The reflected ray OB is passed through a Nicol prism on viewing through a Nicole prism, we find on rotating the prism that **[CPMT 1986]**



(a) The intensity is reduced down to zero and remains zero

(b) The intensity reduces down some what and rises again

(c) There is no change in intensity

(d) The intensity gradually reduces to zero and then again increases

178. Polarised glass is used in sun glasses because **[CPMT 1981]**

(a) It reduces the light intensity to half an account of polarisation (b) It is fashionable

(c) It has good colour (d) It is cheaper

179. In the propagation of electromagnetic waves the angle between the direction of propagation and plane of polarisation is **[CPMT 1978]**

(a) 0° (b) 45° (c) 90° (d) 180°

180. The transverse nature of light is shown by

[CPMT 1972, 74, 78; RPMT 1999; MP PMT 2000; AFMC 2001; AIEEE 2002; MP PET 2004]

(a) Interference of light (b) Refraction of light (c) Polarisation of light (d) Dispersion of light

181. A calcite crystal is placed over a dot on a piece of paper and rotated, on seeing through the calcite one will be see **[CPMT 1971]**

(a) One dot (b) Two stationary dots
(c) Two rotating dots (d) One dot rotating about the other

182. In a doubly refracting crystal, optic axis is a direction along which

(a) A plane polarised beam does not suffer deviation
(b) Any beam of light does not suffer any deviation
(c) Double refraction does not take place
(d) Ordinary and extraordinary rays undergo maximum deviation

183. Which is incorrect with reference to polarisation by reflection

(a) The degree of polarisation varies with the angle of incidence
(b) Percentage of the polarising light in the reflected beam is the greatest at the angle of polarisation
(c) Reflected light is plane polarised in the plane of incidence
(d) Reflected light is plane polarised in the plane perpendicular to plane of incidence

184. Two polarising plates have polarising directions parallel so as to transmit maximum intensity of light. Through what angle must either plate be turned if the intensities of the transmitted beam is to drop by one-third

(a) $55^o18'$ (b) $144^o22'$ (c) Both of these (d) None of these

185. The polaroid is

(a) Celluloid film (b) Big crystal
(c) Cluster of small crystals arranged in a regular way (d) Cluster of small crystals arranged in a haphazard way

186. Light from the cloudless sky is

(a) Fully polarised (b) Partially polarised (c) Unpolarised (d) Can not be said

Doppler's Effect of Light

187. The observed wavelength of light coming from a distant galaxy is found to be increased by 0.5% as compared with that comparing from a terrestrial source. The galaxy is **[MP PMT 1993, 2003]**

(a) Stationary with respect to the earth
(b) Approaching the earth with velocity of light
(c) Receding from the earth with the velocity of light
(d) Receding from the earth with a velocity equal to $1.5\times10^6 m/s$

188. In hydrogen spectrum the wavelength of H_α line is 656 *nm* whereas in the spectrum of a distant galaxy. H_α line wavelength is 706*nm*. Estimated speed of the galaxy with respect to earth is **[IIT-JEE 1999; UPSEAT 2003]**

(a) $2\times10^8 m/s$ (b) $2\times10^7 m/s$ (c) $2\times10^6 m/s$ (d) $2\times10^5 m/s$

189. A star emits light of 5500 *Å* wavelength. Its appears blue to an observer on the earth, it means **[DPMT 2002]**

(a) Star is going away from the earth (b) Star is stationary
(c) Star is coming towards earth (d) None of the above

190. The 6563 Å line emitted by hydrogen atom in a star is found to be red shifted by 5 Å. The speed with which the star is receding from the earth is **[Pb. PMT 2002]**

(a) $17.29\times10^9 m/s$ (b) $4.29\times10^7 m/s$ (c) $3.39\times10^5 m/s$ (d) $2.29\times10^5 m/s$

191. Three observers *A*, *B* and *C* measure the speed of light coming from a source to be v_A, v_B and v_C. The observer *A* moves towards the source, the observer *C* moves away from the source with the same speed. The observer *B* stays stationary. The surrounding space is vacuum very where. Then **[Kerala CET (Med.) 2002]**

(a) $v_A > v_B > v_C$ (b) $v_A < v_B < v_C$ (c) $v_A = v_B = v_C$ (d) $v_A = v_B > v_C$

192. A star emitting light of wavelength 5896 Å is moving away from the earth with a speed of 3600 *km/sec*. The wavelength of light observed on earth will ($c = 3\times10^8 m/\sec$ is the speed of light) **[MP PET 1995, 2002]**

(a) Decrease by 5825.25 Å (b) Increase by 5966.75 Å (c) Decrease by 70.75 Å (d) Increase by 70.75 Å

193. The periodic time of rotation of a certain star is 22 days and its radius is $7\times10^8 m$. If the wavelength of light emitted by its surface be 4320 Å, the Doppler shift will be (1 day = 86400 *sec*) **[MP PET 2001]**

(a) $0.033\,\mathring{A}$ (b) $0.33\,\mathring{A}$ (c) $3.3\,\mathring{A}$ (d) $33\,\mathring{A}$

194. A heavenly body is receding from earth such that the fractional change in λ is 1, then its velocity is **[DCE 2000]**

(a) C (b) $\frac{3C}{5}$ (c) $\frac{C}{5}$ (d) $\frac{2C}{5}$

195. A star is going away from the earth. An observer on the earth will see the wavelength of light coming from the star **[MP PMT 1999]**

(a) Decreased

(b) Increased

(c) Neither decreased nor increased

(d) Decreased or increased depending upon the velocity of the star

196. If the shift of wavelength of light emitted by a star is towards violet, then this shows that star is **[RPET 1996; RPMT 1999]**

(a) Stationary (b) Moving towards earth (c) Moving away from earth (d) Information is incomplete

197. When the wavelength of light coming from a distant star is measured it is found shifted towards red. Then the conclusion is **[JIPMER 1999]**

(a) The star is approaching the observer (b) The star recedes away from earth

(c) There is gravitational effect on the light (d) The star remains stationary

198. In the spectrum of light of a luminous heavenly body the wavelength of a spectral line is measured to be 4747 Å while actual wavelength of the line is 4700 Å. The relative velocity of the heavenly body with respect to earth will be (velocity of light is $3\times10^8\,m/s$) **[MP PET 1997; MP PMT/PET 1998]**

(a) $3\times10^5\,m/s$ moving towards the earth (b) $3\times10^5\,m/s$ moving away from the earth

(c) $3\times10^6\,m/s$ moving towards the earth (d) $3\times10^6\,m/s$ moving away from the earth

199. The wavelength of light observed on the earth, from a moving star is found to decrease by 0.05%. Relative to the earth the star is **[MP PMT/PET 1998]**

(a) Moving away with a velocity of $1.5\times10^5\,m/s$ (b) Coming closer with a velocity of $1.5\times10^5\,m/s$

(c) Moving away with a velocity of $1.5\times10^4\,m/s$ (d) Coming closer with a velocity of $1.5\times10^4\,m/s$

200. Due to Doppler's effect, the shift in wavelength observed is 0.1 Å for a star producing wavelength 6000 Å. Velocity of recession of the star will be **[KCET 1998]**

(a) 2.5 *km/s* (b) 10 *km/s* (c) 5 *km/s* (d) 20 *km/s*

201. A rocket is going away from the earth at a speed of $10^6\,m/s$. If the wavelength of the light wave emitted by it be 5700 Å, what will be its Doppler's shift **[MP PMT 1990, 94; RPMT 1996]**

(a) 200 Å (b) 19 Å (c) 20 Å (d) 0.2 Å

202. A rocket is going away from the earth at a speed 0.2 *c*, where *c* = speed of light, it emits a signal of frequency $4\times10^7\,Hz$. What will be the frequency observed by an observer on the earth **[RPMT 1996]**

(a) $4\times10^6\,Hz$ (b) $3.3\times10^7\,Hz$ (c) $3\times10^6\,Hz$ (d) $5\times10^7\,Hz$

203. A star moves away from earth at speed 0.8 *c* while emitting light of frequency $6\times10^{14}\,Hz$. What frequency will be observed on the earth (in units of $10^{14}\,Hz$) (*c* = speed of light) **[MP PMT 1995]**

(a) 0.24 (b) 1.2 (c) 30 (d) 3.3

204. The sun is rotating about its own axis. The spectral lines emitted from the two ends of its equator, for an observer on the earth, will show **[MP PMT 1994]**

(a) Shift towards red end

(b) Shift towards violet end

(c) Shift towards red end by one line and towards violet end by other

(d) No shift

205. The time period of rotation of the sun is 25 days and its radius is $7\times10^8\,m$. The Doppler shift for the light of wavelength 6000 Å emitted from the surface of the sun will be **[MP PMT 1994]**

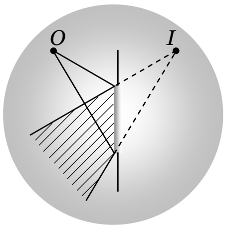
(a) $0.04\,\mathring{A}$ (b) $0.40\,\mathring{A}$ (c) $4.00\,\mathring{A}$ (d) $40.0\,\mathring{A}$

206. The apparent wavelength of the light from a star moving away from the earth is 0.01 % more than its real wavelength. Then the velocity of star is **[CPMT 1979]**

(a) 60 *km/sec* (b) 15 *km/sec* (c) 150 *km/sec* (d) 30 *km/sec*

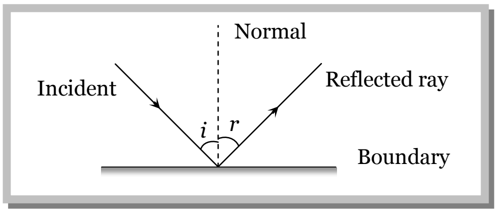
genius

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
a	**d**	**c**	**d**	**b**	**a**	**c**	**c**	**b**	**c**	**d**	**c**	**d**	**a**	**b**	**c**	**d**	**a**	**c**	**c**
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
c	**b**	**b**	**c**	**a**	**d**	**c**	**d**	**a**	**d**	**b**	**b**	**c**	**b**	**c**	**d**	**c**	**c**	**a**	**b**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
c	**d**	**c**	**c**	**a**	**d**	**a**	**b**	**c**	**a**	**d**	**d**	**c**	**a**	**b**	**b**	**a**	**d**	**a**	**d**
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
c	**d**	**c**	**c**	**c**	**b**	**b**	**c**	**b**	**a**	**b, c**	**b**	**d**	**b**	**c**	**b**	**d**	**d**	**b**	**a**
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
a	**a**	**b**	**a**	**b**	**d**	**b**	**b**	**b**	**d**	**a**	**c**	**c**	**c**	**c**	**a**	**b**	**b**	**d**	**b,d**
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
c	**c**	**a**	**d**	**b**	**d**	**a**	**a,b**	**a,c**	**a**	**a**	**b**	**a**	**a,d**	**c**	**a**	**d**	**b**	**a**	**b**
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
c	**a**	**a**	**c**	**a**	**c**	**d**	**c**	**c**	**a**	**a**	**b**	**b**	**c**	**d**	**d**	**c**	**b**	**d**	**d**
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
c	**a**	**b**	**a**	**b**	**a**	**b**	**d**	**d**	**b**	**a**	**c**	**b**	**a**	**d**	**c**	**c**	**c**	**c**	**a**
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
b	**b**	**a**	**c**	**a**	**a**	**c**	**b**	**d**	**c**	**d**	**b**	**d**	**a**	**c**	**a**	**d**	**a**	**a**	**c**
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
d	**c**	**c**	**c**	**c**	**d**	**d**	**b**	**c**	**d**	**c**	**d**	**a**	**a**	**b**	**b**	**b**	**d**	**b**	**c**
201	**202**	**203**	**204**	**205**	**206**														
b	**b**	**b**	**c**	**a**	**d**														



Reflection of Light

When a ray of light after incidenting on a boundary separating two media comes back into the same media, then this phenomenon, is called reflection of light.



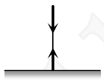
⇒ $\angle i = \angle r$

⇒ After reflection, velocity, wave length and frequency of light remains same but intensity decreases

⇒ There is a phase change of π if reflection takes place from denser medium

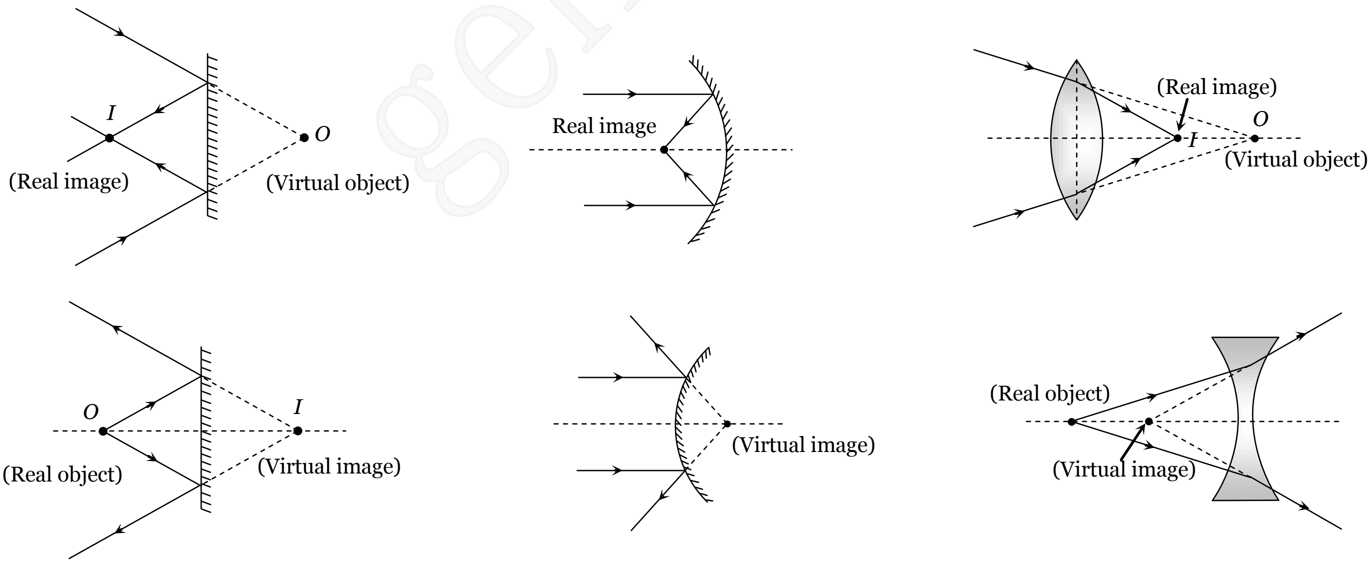
Note : ❑ After reflection velocity, wavelength and frequency of light remains same but intensity decreases.

❑ If light ray incident normally on a surface, after reflection it retraces the path.



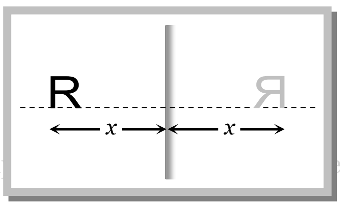
Real and virtual images

If light rays, after reflection or refraction, actually meets at a point then real image is formed and if they appears to meet virtual image is formed.

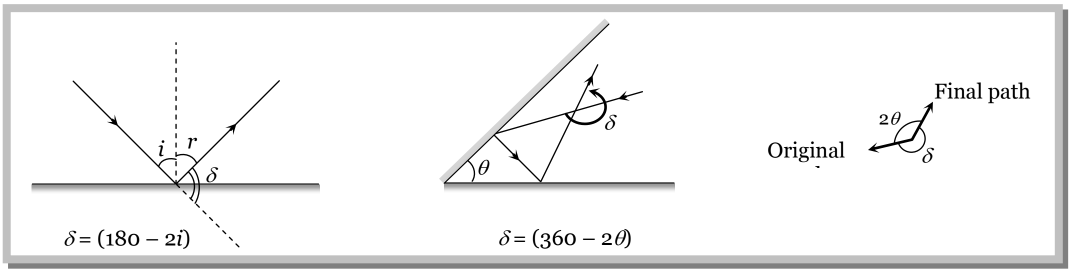


Plane Mirror

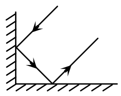
The image formed by a plane mirror is virtual, erect, laterally inverted, equal in size that of the object and at a distance equal to the distance of the object in front of the mirror.



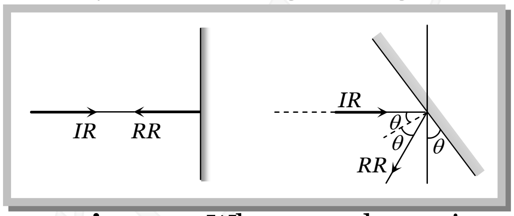
(1) **Deviation :** Deviation produced by a plane mirror and by two inclined plane mirrors.



Note **:** ❑ If two plane mirrors are inclined to each other at 90°, the emergent ray is anti-parallel to incident ray, if it suffers one reflection from each. Whatever be the angle to incidence.



(2) **Rotation :** If a plane mirror is rotated in the plane of incidence through angle θ, by keeping the incident ray fixed, the reflected ray turned through an angle 2θ.



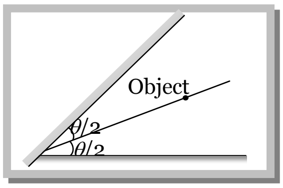
(3) **Images by two inclined plane mirrors :** When two plane mirrors are inclined to each other at an angle θ, then number of images (n) formed of an object which is kept between them.

(i) $n = \left(\frac{360}{\theta} - 1\right)$; If $\frac{360}{\theta}$ = even integer

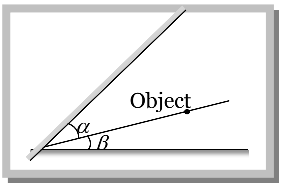
(ii) If $\frac{360}{\theta}$ = odd integer then there are two possibilities

(a) Object is placed symmetrically (b) Object is placed asymmetrically

$n = \left(\frac{360}{\theta} - 1\right)$



$n = \frac{360}{\theta}$



Note **:** ❑ If θ = 0° *i.e.* mirrors are parallel to each other so $n = \infty$ *i.e.* infinite images will be formed.

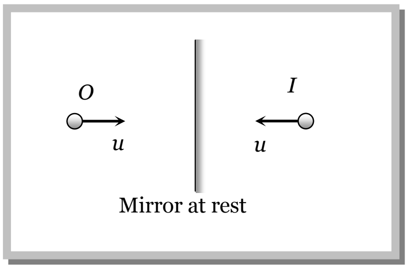
❑ If θ = 90°, $n = \frac{360}{90} - 1 = 3$

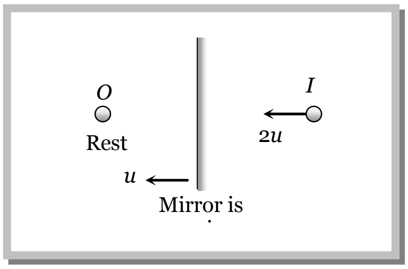
❑ If θ = 72°, $n = \frac{360}{72} - 1 = 4$ (If nothing is said object is supposed to be symmetrically placed).

(4) **Other important informations**

(i) When the object moves with speed u towards (or away) from the plane mirror then image also moves toward (or away) with speed u. But relative speed of image $w.r.t.$ object is $2u$.

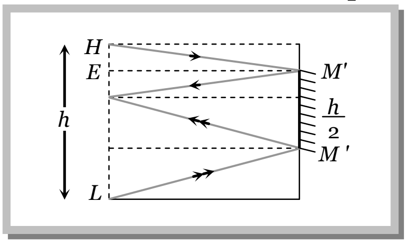
(ii) When mirror moves towards the stationary object with speed u, the image will move with speed $2u$.

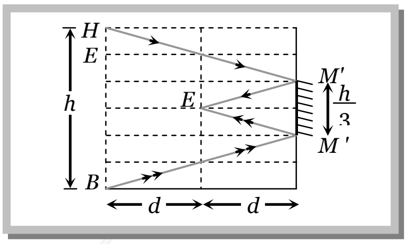




(iii) A man of height h requires a mirror of length at least equal to $h/2$, to see his own complete image.

(iv) To see complete wall behind himself a person requires a plane mirror of at least one third the height of wall. It should be noted that person is standing in the middle of the room.



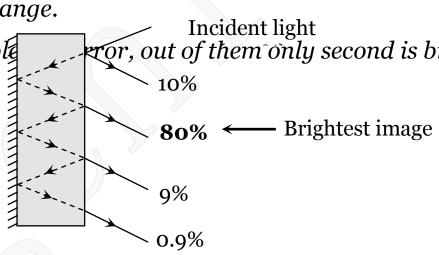


Example

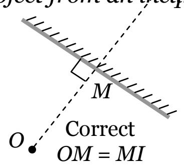
Concepts

☞ *The reflection from a denser medium causes an additional phase change of π or path change of $\lambda/2$ while reflection from rarer medium doesn't cause any phase change.*

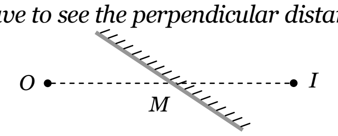
☞ *We observe number of images in a thick plane mirror, out of them only second is brightest.*



☞ *To find the location of an object from an inclined plane mirror, you have to see the perpendicular distance of the object from the mirror.*



Correct
$OM = MI$

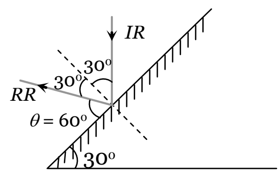


Wrong
$OM = MI$

Example: 1 A plane mirror makes an angle of 30° with horizontal. If a vertical ray strikes the mirror, find the angle between mirror and reflected ray

(a) 30° (b) 45° (c) 60° (d) 90°

Solution : (c) Since angle between mirror and normal is 90° and reflected ray (RR) makes an angle of 30° with the normal so required angle will be $\theta = 60°$.



***Example*: 2** Two vertical plane mirrors are inclined at an angle of 60° with each other. A ray of light travelling horizontally is reflected first from one mirror and then from the other. The resultant deviation is

(a) 60° (b) 120° (c) 180° (d) 240°

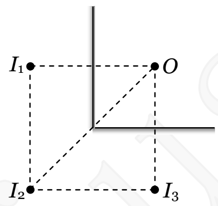
Solution : (d) By using $\delta = (360 - 2\theta)$ $\Rightarrow$ $\delta = 360 - 2 \times 60 = 240^o$

***Example*: 3** A person is in a room whose ceiling and two adjacent walls are mirrors. How many images are formed

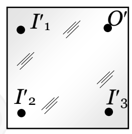
[AFMC 2002]

(a) 5 (b) 6 (c) 7 (d) 8

Solution : (c) The walls will act as two mirrors inclined to each other at 90° and so sill form $\frac{360}{90} - 1 = 3$ images of the person. Now these images with object (Person) will act as objects for the ceiling mirror and so ceiling will form 4 images as shown. Therefore total number of images formed = 3 + 4 = 7



Three images by walls



Four images by

Note : ❑ The person will see only six images of himself $(I_1, I_2, I_3, I_1', I_2', I_3')$

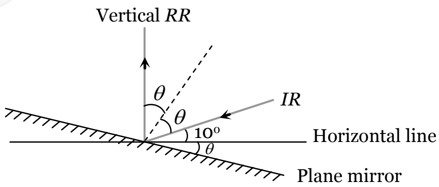
***Example*: 4** A ray of light makes an angle of 10° with the horizontal above it and strikes a plane mirror which is inclined at an angle θ to the horizontal. The angle θ for which the reflected ray becomes vertical is

(a) 40° (b) 50° (c) 80° (d) 100°

Solution : (a) From figure

$\theta + \theta + 10 = 90$

$\Rightarrow \theta = 40^o$



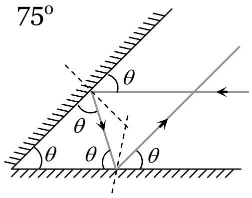
***Example*: 5** A ray of light incident on the first mirror parallel to the second and is reflected from the second mirror parallel to first mirror. The angle between two mirrors is

(a) 30° (b) 60° (c) 75° (d) 90°

Solution : (b) From geometry of figure

$\theta + \theta + \theta = 180^o$

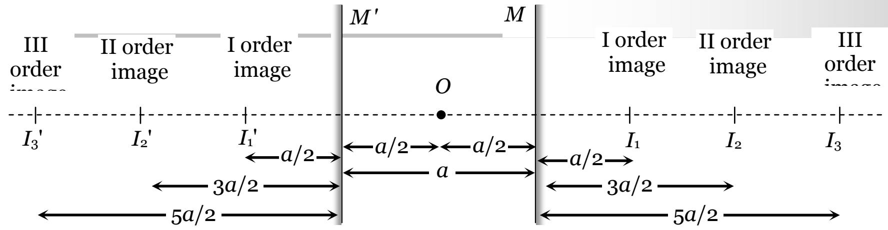
$\Rightarrow \theta = 60^o$



***Example*: 6** A point object is placed mid-way between two plane mirrors distance '*a*' apart. The plane mirror forms an infinite number of images due to multiple reflection. The distance between the *n*th order image formed in the two mirrors is

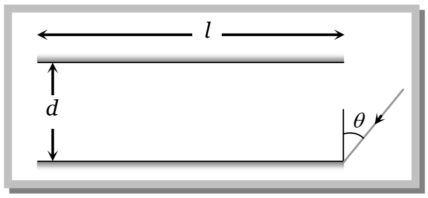
(a) *na* (b) 2*na* (c) *na*/2 (d) $n^2 a$

Solution : (b)



From above figure it can be proved that seperation between nth order image formed in the two mirrors = $2na$

***Example*: 7** Two plane mirrors P and Q are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of θ at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is



(a) $\frac{l}{d\tan\theta}$

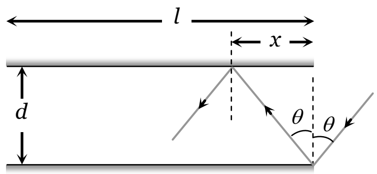
(b) $\frac{d}{l\tan\theta}$

(c) $ld\tan\theta$

(d) None of these

Solution : (a) Suppose n = Total number of reflection light ray undergoes before exist out.

x = Horizontal distance travelled by light ray in one reflection.



So $nx = l$ also $\tan\theta = \frac{x}{d}$

$\Rightarrow \quad n = \frac{l}{d\tan\theta}$

***Example*: 8** A plane mirror and a person are moving towards each other with same velocity v. Then the velocity of the image is

(a) v (b) $2v$ (c) $3v$ (d) $4v$

Solution : (c) If mirror would be at rest, then velocity of image should be $2v$. but due to the motion of mirror, velocity of image will be $2v + v = 3v$.

***Example*: 9** A ray reflected successively from two plane mirrors inclined at a certain angle undergoes a deviation of 300°. The number of images observable are

(a) 10 (b) 11 (c) 12 (d) 13

Solution : (b) By using $\delta = (360 - 2\theta) \Rightarrow 300 = 360 - 2\theta$

$\Rightarrow \theta = 30^o$. Hence number of images $= \frac{360}{30} - 1 = 11$

Tricky example: 1

A small plane mirror placed at the centre of a spherical screen of radius R. A beam of light is falling on the mirror. If the mirror makes n revolution. per second, the speed of light on the screen after reflection from the mirror will be

(a) $4\pi nR$ (b) $2\pi nR$ (c) $\frac{nR}{2\pi}$ (d) $\frac{nR}{4\pi}$

Solution : (a) When plane mirror rotates through an angle θ, the reflected ray rotates through an angle 2θ. So spot on the screen will make $2n$ revolution per second

$\therefore$ Speed of light on screen $v = \omega R = 2\pi(2n)R = 4\pi nR$

Tricky example: 2

A watch shows time as 3 : 25 when seen through a mirror, time appeared will be

[RPMT 1997; JIPMER 2001, 2002]

(a) 8 : 35 (b) 9 : 35 (c) 7 : 35 (d) 8 : 25

Solution : (a) For solving this type of problems remember

Actual time = 11 : 60 – given time

So here Actual time = 11 : 60 – 3 : 25 = 8 : 35

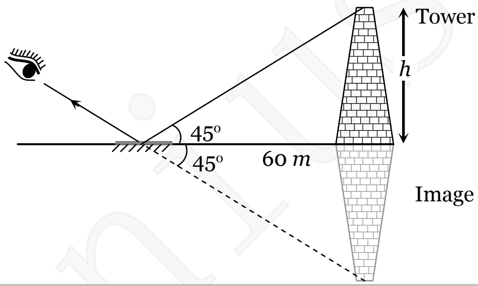
Tricky example: 3

When a plane mirror is placed horizontally on a level ground at a distance of 60 *m* from the foot of a tower, the top of the tower and its image in the mirror subtend an angle of 90° at the eye. The height of the tower will be **[CPMT 1984]**

(a) 30 *m* (b) 60 *m* (c) 90 *m* (d) 120 *m*

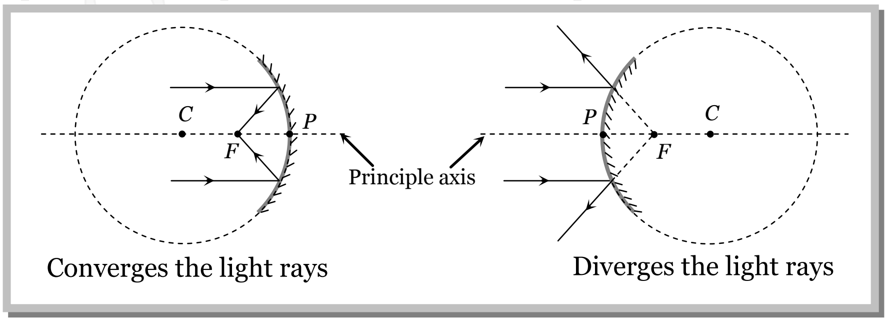
Solution : (b) Form the figure it is clear that $\frac{h}{60} = \tan 45^o$

$\Rightarrow h = 60\ m$



Curved Mirror

It is a part of a transparent hollow sphere whose one surface is polished.



(1) **Some definitions :**

(i) **Pole (P)**	:	Mid point of the mirror
(ii) Centre of curvature (C)	:	Centre of the sphere of which the mirror is a part.
(iii) Radius of curvature (R)	:	Distance between pole and centre of curvature.
		$(R_{\text{concave}} = -ve\ ,\ \ R_{\text{convex}} = +ve\ ,\ R_{\text{plane}} = \infty)$
(iv) Principle axis	:	A line passing through *P* and *C*.
(v) Focus (F)	:	An image point on principle axis for which object is at ∞
(vi) Focal length (f)	:	Distance between *P* and *F*.

(vii) Relation between f and R : $f = \frac{R}{2}$ ($f_{\text{concare}} = -ve$, $f_{\text{convex}} = +ve$, $f_{\text{plane}} = \infty$)

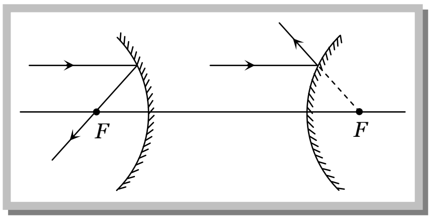
(viii) Power : The converging or diverging ability of mirror

(ix) Aperture : Effective diameter of light reflecting area. Intensity of image $\propto$ Area $\propto$ (Aperture)^2

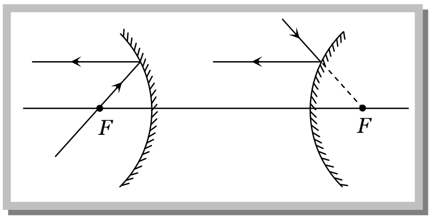
(x) Focal plane : A plane passing from focus and perpendicular to principle axis.

(2) **Rules of image formation and sign convention** :

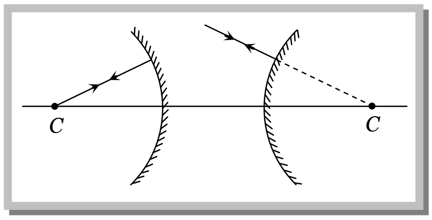
Rule (i)



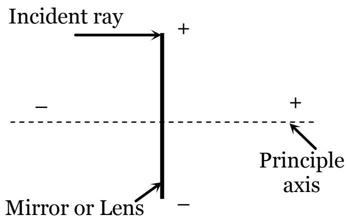
Rule (ii)



Rule (iii)



(3) **Sign conventions :**



(i) All distances are measured from the pole.

(ii) Distances measured in the direction of incident rays are taken as positive while in the direction opposite of incident rays are taken negative.

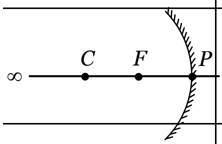
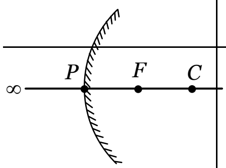
(iii) Distances above the principle axis are taken positive and below the principle axis are taken negative.

Note : ❑ Same sign convention are also valid for lenses.

Use following sign while solving the problem :

Concave mirror			Convex mirror
	Real image (u ≥ f)	**Virtual image (u< f)**	
Distance of object	$u \rightarrow -$	$u \rightarrow -$	$u \rightarrow -$
Distance of image	$v \rightarrow -$	$v \rightarrow +$	$v \rightarrow +$
Focal length	$f \rightarrow -$	$f \rightarrow -$	$f \rightarrow +$
Height of object	$O \rightarrow +$	$O \rightarrow +$	$O \rightarrow +$
Height of image	$I \rightarrow -$	$I \rightarrow +$	$I \rightarrow +$
Radius of curvature	$R \rightarrow -$	$R \rightarrow -$	$R \rightarrow +$
Magnification	$m \rightarrow -$	$m \rightarrow +$	$m \rightarrow +$

(4) **Position, size and nature of image formed by the spherical mirror**

Mirror	Location of the object	Location of the image	Magnification, Size of the image	Nature	
				Real virtual	Erect inverted
(a) Concave	At infinity *i.e.* $u = \infty$	At focus *i.e.* $v = f$	$m << 1$, diminished	Real	inverted
	Away from centre of curvature ($u > 2f$)	Between f and $2f$ *i.e.* $f < v < 2f$	$m < 1$, diminished	Real	inverted
	At centre of curvature $u = 2f$	At centre of curvature *i.e.* $v = 2f$	$m = 1$, same size as that of the object	Real	inverted
	Between centre of curvature and focus : $F < u < 2f$	Away from the centre of curvature $v > 2f$	$m > 1$, magnified	Real	inverted
	At focus *i.e.* $u = f$	At infinity *i.e.* $v = \infty$	$m = \infty$, magnified	Real	inverted
	Between pole and focus $u < f$	$v > u$	$m > 1$ magnified	Virtual	erect
(b) Convex	At infinity *i.e.* $u = \infty$	At focus *i.e.*, $v = f$	$m < 1$, diminished	Virtual	erect
	Anywhere between infinity and pole	Between pole and focus	$m < 1$, diminished	Virtual	erect

Note :
- ❑ In case of convex mirrors, as the object moves away from the mirror, the image becomes smaller and moves closer to the focus.
- ❑ Images formed by mirrors do not show chromatic aberration.
- ❑ For convex mirror maximum image distance is it's focal length.
- ❑ In concave mirror, minimum distance between a real object and it's real image is zero.
(*i.e.* when $u = v = 2f$)

Mirror formula and magnification

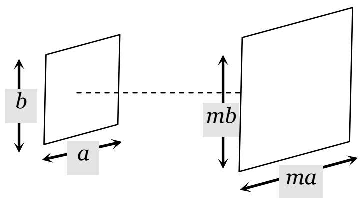
For a spherical mirror if u = Distance of object from pole, v = distance of image from pole, f = Focal length, R = Radius of curvature, O = Size of object, I = size of image, m = magnification (or linear magnification), m_s = Areal magnification, A_o = Area of object, A_i = Area of image

(1) **Mirror formula :** $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$; (use sign convention while solving the problems).

Note : ❑ **Newton's formula :** If object distance (x_1) and image distance (x_2) are measured from focus instead of pole then $f^2 = x_1 x_2$

A collection by Pradeep Kshetrapal for Physics students at genius, Maxwell and Gupta classes Only

(2) **Magnification :** m = $\dfrac{\text{Size of object}}{\text{Size of image}}$

Linear magnification		Areal magnification
Transverse	**Longitudinal**	
When a object is placed perpendicular to the principle axis, then linear magnification is called lateral or transverse magnification. It is given by $m = \dfrac{I}{O} = -\dfrac{v}{u} = \dfrac{f}{f-u} = \dfrac{f-v}{f}$ (* Always use sign convention while solving the problems)	When object lies along the principle axis then its longitudinal magnification $m = \dfrac{I}{O} = \dfrac{-(v_2 - v_1)}{(u_2 - u_1)}$ If object is small; $m = -\dfrac{dv}{du} = \left(\dfrac{v}{u}\right)^2$ Also Length of image = $\left(\dfrac{v}{u}\right)^2 \times \text{Length of object } (L_0)$ $(L_i) = \left(\dfrac{f}{u-f}\right)^2 .L_o$	 If a 2*D*-object is placed with it's plane perpendicular to principle axis It's Areal magnification $M_s = \dfrac{\text{Area of image } (A_i)}{\text{Area of object } (A_o)}$ $= \dfrac{ma \times mb}{ab} = m^2$ $\Rightarrow m_s = m^2 = \dfrac{A_i}{A_o}$

Note : ❑ Don't put the sign of quantity which is to be determined.

❑ If a spherical mirror produces an image '*m*' times the size of the object (*m* = magnification) then *u*, *v* and *f* are given by the followings

$$u = \left(\frac{m-1}{m}\right)f, \quad v = -(m-1)f \quad \text{and} \quad f = \left(\frac{m}{m-1}\right)u \quad \text{(use sign convention)}$$

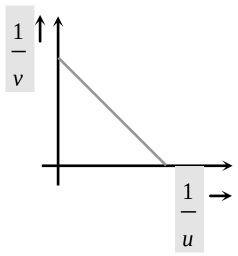
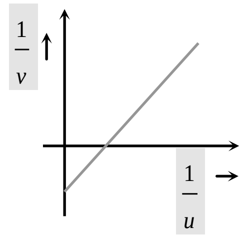
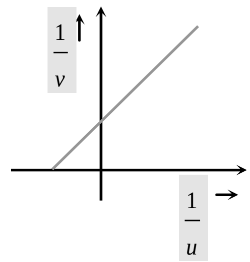
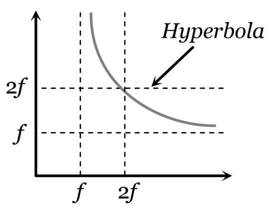
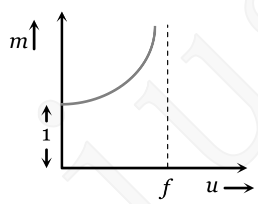
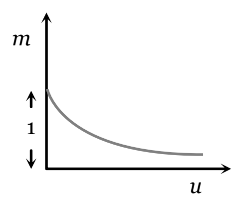
(3) **Uses of mirrors**

(i) **Concave mirror :** Used as a shaving mirror, In search light, in cinema projector, in telescope, by E.N.T. specialists etc.

(ii) **Convex mirror :** In road lamps, side mirror in vehicles *etc.*

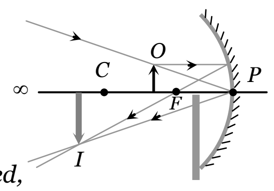
Note : ❑ Field of view of convex mirror is more than that of concave mirror.

Different graphs

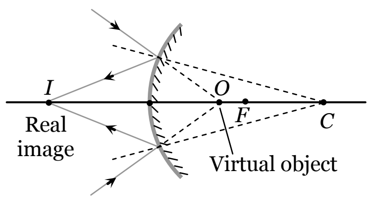
Graph between $\frac{1}{v}$ and $\frac{1}{u}$		
(a) Real image formed by concave mirror	(b) Virtual image formed by concave mirror	(c) Virtual image formed by convex mirror
		
Graph between u and v for real image of concave mirror	**Graph between u and m for virtual image by concave mirror**	**Graph between u and m for virtual image by convex mirror.**
		

Concepts

- *Focal length of a mirror is independent of material of mirror, medium in which it is placed, wavelength of incident light*
- *Divergence or Convergence power of a mirror does not change with the change in medium.*
- *If an object is moving at a speed v_o towards a spherical mirror along it's axis then speed of image away from mirror is* $v_i = -\left(\frac{f}{u-f}\right)^2 . v_o$ *(use sign convention)*



- *When object is moved from focus to infinity at constant speed, the image will move faster in the beginning and slower later on, towards the mirror.*
- *As every part of mirror forms a complete image, if a part of the mirror is obstructed, full image will be formed but intensity will be reduced.*
- *Can a convex mirror form real images?*
  yes if (distance of virtual object) $u < f$ (focal length)



Example

Example: 10 A convex mirror of focal length f forms an image which is $1/n$ times the object. The distance of the object from the mirror is

(a) $(n-1)f$ (b) $\left(\frac{n-1}{n}\right)f$ (c) $\left(\frac{n+1}{n}\right)f$ (d) $(n+1)f$

Solution : (a) By using $m = \frac{f}{f-u}$

Here $m = +\frac{1}{n}$, $f \to +f$ So, $+\frac{1}{n} = \frac{+f}{+f-u} \Rightarrow u = -(n-1)f$

Example: 11 An object 5 *cm* tall is placed 1 *m* from a concave spherical mirror which has a radius of curvature of 20 *cm*. The size of the image is

(a) 0.11 *cm* (b) 0.50 *cm* (c) 0.55 *cm* (d) 0.60 *cm*

Solution : (c) By using $\frac{I}{O} = \frac{f}{f-u}$

Here $O + 5\,cm$, $f = -\frac{R}{2} = -10\,cm$, $u = -1\,m = -100\,cm$

So, $\frac{I}{+5} = \frac{-10}{-10-(-100)}$ $\Rightarrow$ $I = -\,0.55\,cm$.

Example: 12 An object of length 2.5 *cm* is placed at a distance of 1.5 f from a concave mirror where f is the magnitude of the focal length of the mirror. The length of the object is perpendicular to the principle axis. The length of the image is

(a) 5 *cm,* erect (b) 10 *cm,* erect (c) 15 *cm,* erect (d) 5 *cm,* inverted

Solution : (d) By using $\frac{I}{O} = \frac{f}{f-u}$; where $I = ?$, $O = +2.5\,cm$. $f \to -f$, $u = -1.5f$

$\therefore$ $\frac{I}{+2.5} = \frac{-f}{-f-(-1.5f)}$ $\Rightarrow$ $I = -5\,cm$. (Negative sign indicates that image is inverted.)

Example: 13 A convex mirror has a focal length f. A real object is placed at a distance f in front of it from the pole produces an image at

(a) Infinity (b) f (c) $f/2$ (d) $2f$

Solution : (c) By using $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow$ $\frac{1}{+f} = \frac{1}{v} + \frac{1}{(-f)} \Rightarrow v = \frac{f}{2}$

Example: 14 Two objects A and B when placed one after another infront of a concave mirror of focal length 10 *cm* from images of same size. Size of object A is four times that of B. If object A is placed at a distance of 50 *cm* from the mirror, what should be the distance of B from the mirror

(a) 10 *cm* (b) 20 *cm* (c) 30 *cm* (d) 40 *cm*

Solution : (b) By using $\frac{I}{O} = \frac{f}{f-u} \Rightarrow \frac{I_A}{I_B} \times \frac{O_B}{O_A} = \frac{f-u_B}{f-u_A} \Rightarrow \frac{1}{1} \times \frac{1}{4} = \frac{-10-u_B}{-10-(-50)} \Rightarrow u_B = -20cm$.

Example: 15 A square of side 3 *cm* is placed at a distance of 25 *cm* from a concave mirror of focal length 10 *cm*. The centre of the square is at the axis of the mirror and the plane is normal to the axis. The area enclosed by the image of the wire is

(a) 4 cm^2 (b) 6 cm^2 (c) 16 cm^2 (d) 36 cm^2

Solution : (a) By using $m^2 = \frac{A_i}{A_o}$; where $m = \frac{f}{f-u}$

Hence from given values $m = \frac{-10}{-10-(-25)} = \frac{-2}{3}$ and $A_o = 9\,cm^2$ ∴ $A_i = \left(\frac{-2}{3}\right)^2 \times 9 = 4 cm^2$

Example: 16 A convex mirror of focal length 10 *cm* is placed in water. The refractive index of water is 4/3. What will be the focal length of the mirror in water

(a) 10 *cm* (b) 40/3 *cm* (c) 30/4 *cm* (d) None of these

Solution : (a) No change in focal length, because *f* depends only upon radius of curvature *R*.

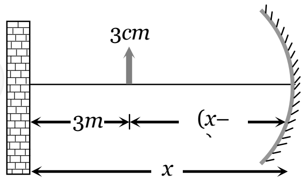
Example: 17 A candle flame 3 *cm* is placed at distance of 3 *m* from a wall. How far from wall must a concave mirror be placed in order that it may form an image of flame 9 *cm* high on the wall

(a) 225 *cm* (b) 300 *cm* (c) 450 *cm* (d) 650 *cm*

Solution : (c) Let the mirror be placed at a distance *x* from wall

By using

$\frac{I}{O} = \frac{-v}{u} \Rightarrow \frac{-9}{+3} = \frac{-(-x)}{-(x-3)} \Rightarrow x = -4.5m = -450cm.$



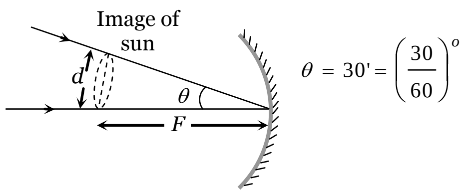
Example: 18 A concave mirror of focal length 100 *cm* is used to obtain the image of the sun which subtends an angle of 30'. The diameter of the image of the sun will be

(a) 1.74 *cm* (b) 0.87 *cm* (c) 0.435 *cm* (d) 100 *cm*

Solution : (b) Diameter of image of sun $d = f\theta$

$\Rightarrow d = 100 \times \left(\frac{30}{60}\right) \times \frac{\pi}{180}$

$\Rightarrow \quad d = 0.87\, cm.$



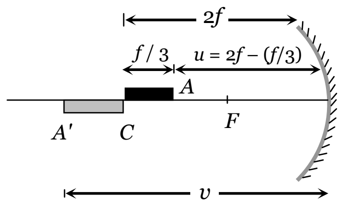
Example: 19 A thin rod of length $f/3$ lies along the axis of a concave mirror of focal length *f*. One end of its magnified image touches an end of the rod. The length of the image is **[MP PET 1995]**

(a) f (b) $\frac{1}{2}f$ (c) $2f$ (d) $\frac{1}{4}f$

Solution : (b) If end *A* of rod acts an object for mirror then it's image will be *A'* and if $u = 2f - \frac{f}{3} = \frac{5f}{3}$

So by using $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-f} = \frac{1}{v} + \frac{1}{\frac{-5f}{3}} \Rightarrow v = -\frac{5}{2}f$

∴ Length of image $= \frac{5}{2}f - 2f = \frac{f}{2}$

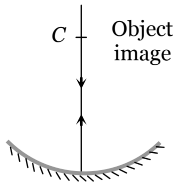


Example: 20 A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let *O* be the pole of the mirror and *C* its centre of curvature. A point object is placed at *C*. It has a real image, also located at *C*. If the mirror is now filled with water, the image will be

(a) Real, and will remain at C (b) Real, and located at a point between C and ∞

(c) Virtual and located at a point between C and O (d) Real, and located at a point between C and O

Solution : (d)



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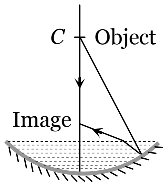


Finally

Tricky example: 4

An object is placed infront of a convex mirror at a distance of 50 *cm*. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and plane mirror is 30 *cm,* it is found that there is no parallel between the images formed by two mirrors. Radius of curvature of mirror will be

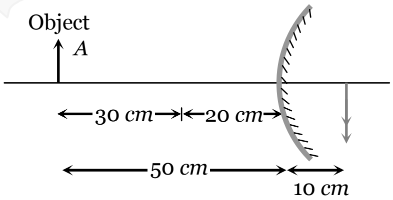
(a) 12.5 *cm* (b) 25 *cm* (c) $\frac{50}{3} cm$ (d) 18 *cm*

Solution : (b) Since there is no parallel, it means that both images (By plane mirror and convex mirror) coinciding each other.

According to property of plane mirror it will form image at a distance of 30 *cm* behind it. Hence for convex mirror $u = -50\ cm,\quad v = +10\ cm$

By using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\Rightarrow \frac{1}{f} = \frac{1}{+10} + \frac{1}{-50} = \frac{4}{50}$

$\Rightarrow \quad f = \frac{25}{2} cm$ $\Rightarrow R = 2f = 25 cm.$

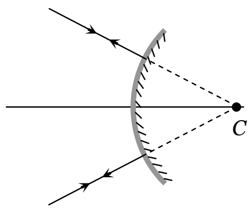


Tricky example: 5

A convergent beam of light is incident on a convex mirror so as to converge to a distance 12 *cm* from the pole of the mirror. An inverted image of the same size is formed coincident with the virtual object. What is the focal length of the mirror

(a) 24 *cm* (b) 12 *cm* (c) 6 *cm* (d) 3 *cm*

Solution : (c) Here object and image are at the same position so this position must be centre of curvature



$\therefore\ R = 12\ cm \quad \Rightarrow f = \frac{R}{2}$

Practice Questions Basic Level

1. A light bulb is placed between two mirrors (plane) inclined at an angle of 60°. Number of images formed are

[NCERT 1980; CPMT 1996, 97; SCRA 1994; AIIMS 1997; RPMT 1999; AIEEE 2002; Orissa JEE 2003; MP PET 2004]

(a) 2 (b) 4 (c) 5 (d) 6

2. Two plane mirrors are inclined at an angle of 72^o. The number of images of a point object placed between them will be **[KCET (Engg. & Med.)1999; BCECE 2003]**

(a) 2 (b) 3 (c) 4 (d) 5

3. To get three images of a single object, one should have two plane mirrors at an angle of **[AIEEE 2003]**

(a) 30^o (b) 60^o (c) 90^o (d) 120^o

4. A man of length h requires a mirror of length at least equal to, to see his own complete image **[MP PET 2003]**

(a) $\frac{h}{4}$ (b) $\frac{h}{3}$ (c) $\frac{h}{2}$ (d) h

5. Two plane mirrors are at 45^o to each other. If an object is placed between them then the number of images will be **[MP PMT 200**

(a) 5 (b) 9 (c) 7 (d) 8

6. An object is at a distance of 0.5 m in front of a plane mirror. Distance between the object and image is **[CPMT 2002]**

(a) 0.5 m (b) 1 m (c) 0.25 m (d) 1.5 m

7. A man runs towards a mirror at a speed 15 m/s. The speed of the image relative to the man is **[RPMT 1999; Kerala PET 2002]**

(a) 15 ms^{-1} (b) 30 ms^{-1} (c) 35 ms^{-1} (d) 20 ms^{-1}

8. The light reflected by a plane mirror may form a real image **[KCET (Engg. & Med.) 2002]**

(a) If the rays incident on the mirror are diverging (b) If the rays incident on the mirror are converging

(c) If the object is placed very close to the mirror (d) Under no circumstances

9. A man is 180 cm tall and his eyes are 10 cm below the top of his head. In order to see his entire height right from toe to head, he uses a plane mirror kept at a distance of 1 m from him. The minimum length of the plane mirror required is **[MP PMT 1993; DPMT 2001]**

(a) 180 cm (b) 90 cm (c) 85 cm (d) 170 cm

10. A small object is placed 10 cm infront of a plane mirror. If you stand behind the object 30 cm from the object and look at its image, the distance focused for your eye will be

(a) 60 cm (b) 20 cm (c) 40 cm (d) 80 cm

11. Two plane mirrors are at right angles to each other. A man stands between them and combs his hair with his right hand. In how many of the images will he be seen using his right hand

(a) None (b) 1 (c) 2 (d) 3

12. A man runs towards mirror at a speed of 15 m/s. What is the speed of his image **[CBSE PMT 2000]**

(a) 7.5 m/s (b) 15 m/s (c) 30 m/s (d) 45 m/s

13. A ray of light is incidenting normally on a plane mirror. The angle of reflection will be **[MP PET 2000]**

(a) 0^o (b) 90^o (c) Will not be reflected (d) None of these

14. A plane mirror produces a magnification of **[MP PMT/PET 1997]**

(a) – 1 (b) + 1 (c) Zero (d) Between 0 and + ∞

15. When a plane mirror is rotated through an angle θ, then the reflected ray turns through the angle 2θ, then the size of the image **[MP PAT 1996]**

(a) Is doubled (b) Is halved (c) Remains the same (d) Becomes infinite

16. What should be the angle between two plane mirrors so that whatever be the angle of incidence, the incident ray and the reflected ray from the two mirrors be parallel to each other

(a) 60^o (b) 90^o (c) 120^o (d) 175^o

17. Ray optics is valid, when characteristic dimensions are **[CBSE PMT 1994]**

(a) Of the same order as the wavelength of light (b) Much smaller than the wavelength of light

(c) Of the order of one millimeter (d) Much larger than the wavelength of light

18. It is desired to photograph the image of an object placed at a distance of 3 m from the plane mirror. The camera which is at a distance of 4.5 m from the mirror should be focussed for a distance of

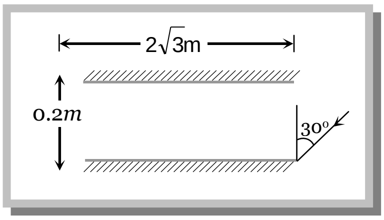
(a) 3 m (b) 4.5 m (c) 6 m (d) 7.5 m

19. Two plane mirrors are parallel to each other an spaced 20 *cm* apart. An object is kept in between them at 15 *cm* from *A*. Out of the following at which point an image is not formed in mirror *A* (distance measured from mirror *A*)

(a) 15 *cm* (b) 25 *cm* (c) 45 *cm* (d) 55 *cm*

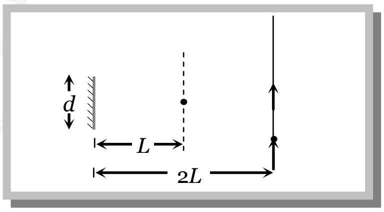
Advance Level

20. Two plane mirrors *A* and *B* are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of 30° at a point just inside one end of *A*. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is



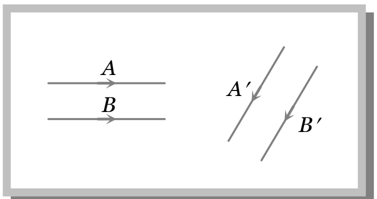
(a) 28
(b) 30
(c) 32
(d) 34

21. A point source of light *B* is placed at a distance *L* in front of the centre of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2*L* from it as shown. The greatest distance over which he can see the image of the light source in the mirror is



(a) *d*/2
(b) *d*
(c) 2*d*
(d) 3*d*

22. The figure shows two rays *A* and *B* being reflected by a mirror and going as *A*′ and *B*′. The mirror is



(a) Plane (b) Concave
(c) Convex (d) May be any spherical mirror

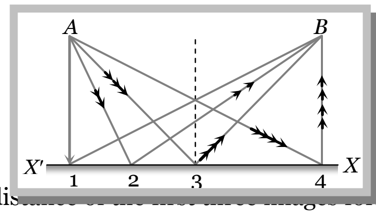
23. An object is initially at a distance of 100 *cm* from a plane mirror. If the mirror approaches the object at a speed of 5 *cm/s,* then after 6 *s* the distance between the object and its image will be

(a) 60 *cm* (b) 140 *cm* (c) 170 *cm* (d) 150 *cm*

24. An object placed in front of a plane mirror is displaced by 0.4 *m* along a straight line at an angle of 30° to mirror plane. The change in the distance between the object and its image is

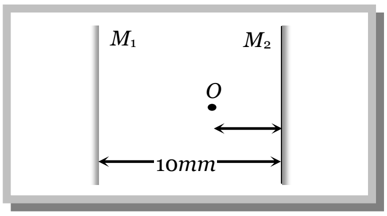
(a) 0.20 *m* (b) 0.40 *m* (c) 0.25 *m* (d) 0.80 *m*

25. A ray of light travels from *A* to *B* with uniform speed. On its way it is reflected by the surface *XX*′. The path followed by the ray to take least time is



(a) 1 (b) 2
(c) 3 (d) 4

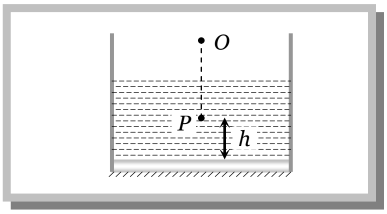
26. A point object *O* is placed between two plan mirrors as shown is fig. The distance of the first three images formed by mirror M_2 from it are



(a) 2 *mm,* 8 *mm,* 18 *mm*
(b) 2 *mm,* 18 *mm,* 28 *mm*
(c) 2 *mm,* 18 *mm,* 22 *mm*

(d) 2 *mm*, 18 *mm*, 58 *mm*

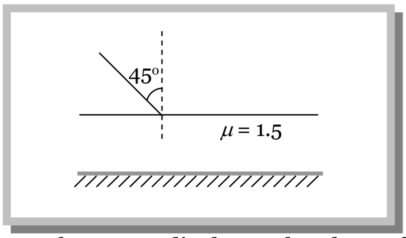
27. A plane mirror is placed at the bottom of the tank containing *a* liquid of refractive index μ . *P* is a small object at a height *h* above the mirror. An observer *O*-vertically above *P* outside the liquid see *P* and its image in the mirror. The apparent distance between these two will be



(a) $2\mu h$ (b) $\frac{2h}{\mu}$

(c) $\frac{2h}{\mu-1}$ (d) $h\left(1+\frac{1}{\mu}\right)$

28. One side of a glass slab is silvered as shown. A ray of light is incident on the other side at angle of incidence $i = 45^o$. Refractive index of glass is given as 1.5. The deviation of the ray of light from its initial path when it comes out of the slab is



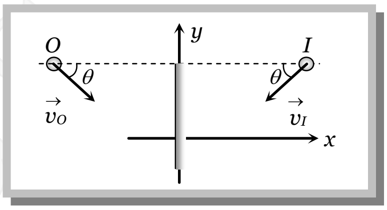
(a) 90°

(b) 180°

(c) 120°

(d) 45°

29. If an object moves towards a plane mirror with a speed *v* at an angle θ to the perpendicular to the plane of the mirror, find the relative velocity between the object and the image



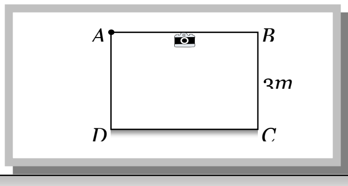
(a) *v*

(b) 2*v*

(c) $2v\cos\theta$

(d) $2v\sin\theta$

30. Figure shows a cubical room *ABCD* will the wall *CD* as a plane mirror. Each side of the room is 3*m*. We place a camera at the midpoint of the wall *AB*. At what distance should the camera be focussed to photograph an object placed at *A*



(a) 1.5 *m* (b) 3 *m*

(c) 6 *m* (d) More than 6 *m*

Reflection of light at spherical surface

Basic Level

31. A man having height 6 *m*, want to see full height in mirror. They observe image of 2*m* height erect, then used mirror is **[J & K CET 2004]**

(a) Concave (b) Convex (c) Plane (d) None of these

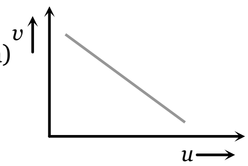
32. An object of length 6*cm* is placed on the principal axis of a concave mirror of focal length *f* at a distance of 4 *f*. The length of the image will be **[MP PET 2003]**

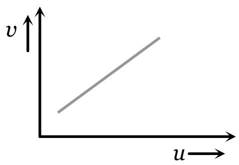
(a) 2 *cm* (b) 12 *cm* (c) 4 *cm* (d) 1.2 *cm*

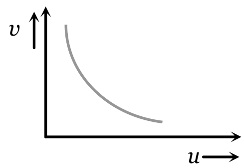
33. Convergence of concave mirror can be decreased by dipping in **[AFMC 2003]**

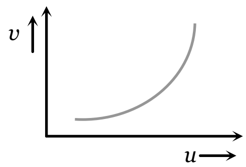
(a) Water (b) Oil (c) Both (d) None of these

34. In an experiment of find the focal length of a concave mirror a graph is drawn between the magnitudes of *u* and *v*. The graph looks like

(a) 

(b) 

(c) 

(d) 

35. An object 2.5 *cm* high is placed at a distance of 10 *cm* from a concave mirror of radius of curvature 30 *cm* The size of the image is **[BVP 2003]**

(a) 9.2 *cm* (b) 10.5 *cm* (c) 5.6 *cm* (d) 7.5 *cm*

36. A diminished virtual image can be formed only in **[MP PMT 2002]**

(a) Plane mirror (b) A concave mirror (c) A convex mirror (d) Concave-parabolic mirror

37. A point object is placed at a distance of 30 *cm* from a convex mirror of focal length 30*cm*. The image will form at **[JIPMER 2002]**

(a) Infinity (b) Focus (c) Pole (d) 15 *cm* behind the mirror

38. The focal length of a convex mirror is 20 *cm* its radius of curvature will be **[MP PMT 2001]**

(a) 10 *cm* (b) 20 *cm* (c) 30 *cm* (d) 40 *cm*

39. A concave mirror of focal length 15 *cm* forms an image having twice the linear dimensions of the object. The position of the object when the image is virtual will be

(a) 22.5 *cm* (b) 7.5 *cm* (c) 30 *cm* (d) 45 *cm*

40. Under which of the following conditions will a convex mirror of focal length f produce an image that is erect, diminished and virtual **[AMU (Engg.) 2001]**

(a) Only when $2f > u > f$ (b) Only when $u = f$ (c) Only when $u < f$ (d) Always

41. A concave mirror gives an image three times as large as the object placed at a distance of 20 *cm* from it. For the image to be real, the focal length should be **[SCRA 1998; JIPMER 2**

(a) 10 *cm* (b) 15 *cm* (c) 20 *cm* (d) 30 *cm*

42. A point object is placed at a distance of 10 *cm* and its real image is formed at a distance of 20*cm* from a concave mirror. If the object is moved by 0.1*cm* towards the mirror, the image will shift by about

(a) 0.4 *cm* away from the mirror (b) 0.4 *cm* towards the mirror

(c) 0.8 *cm* away from the mirror (d) 0.8 *cm* towards the mirror

43. The minimum distance between the object and its real image for concave mirror is **[RPMT 1999]**

(a) f (b) $2f$ (c) $4f$ (d) Zero

44. An object is placed at 20 *cm* from a convex mirror of focal length 10 *cm*. The image formed by the mirror is **[JIPMER 1999]**

(a) Real and at 20 *cm* from the mirror (b) Virtual and at 20 *cm* from the mirror

(c) Virtual and at 20/3 *cm* from the mirror (d) Real and at 20/3 *cm* from the mirror

45. An object is placed 40 *cm* from a concave mirror of focal length 20 *cm*. The image formed is **[MP PET 1986; MP PMT/PET 19**

(a) Real, inverted and same in size (b) Real, inverted and smaller

(c) Virtual, erect and larger (d) Virtual, erect and smaller

46. Match List I with List II and select the correct answer using the codes given below the lists **[SCRA 1998]**

List I (Position of the object)	List II (Magnification)
(I) An object is placed at focus before a convex mirror	(A) Magnification is $-\infty$
(II) An object is placed at centre of curvature before a concave mirror	(B) Magnification is 0.5
(III) An object is placed at focus before a concave mirror	(C) Magnification is + 1
(IV) An object is placed at centre of curvature before a convex mirror	(D) Magnification is – 1
	(E) Magnification is 0.33

Codes :

(a) I-B, II-D, III-A, IV-E (b) I-A, II-D, III-C, IV-B (c) I-C, II-B, III-A, IV-E (d) I-B, II-E, III-D, IV-C

47. In a concave mirror experiment, an object is placed at a distance x_1 from the focus and the image is formed at a distance x_2 from the focus. The focal length of the mirror would be

(a) x_1x_2 (b) $\sqrt{x_1x_2}$ (c) $\frac{x_1+x_2}{2}$ (d) $\sqrt{\frac{x_1}{x_2}}$

48. Which of the following forms a virtual and erect image for all positions of the object **[IIT-JEE 1996]**

(a) Convex lens (b) Concave lens (c) Convex mirror (d) Concave mirror

49. A convex mirror has a focal length f. A real object is placed at a distance f in front of it from the pole produces an image at **[MP PAT 1996]**

(a) Infinity (b) f (c) $f/2$ (d) $2f$

50. Radius of curvature of concave mirror is 40 *cm* and the size of image is twice as that of object, then the object distance is **[AFMC 1995]**

(a) 60 *cm* (b) 20 *cm* (c) 40 *cm* (d) 30 *cm*

51. All of the following statements are correct except **[Manipal MEE 1995]**

(a) The magnification produced by a convex mirror is always less than one

(b) A virtual, erect, same-sized image can be obtained using a plane mirror

(c) A virtual, erect, magnified image can be formed using a concave mirror

(d) A real, inverted, same-sized image can be formed using a convex mirror

52. Radius of curvature of convex mirror is 40 *cm* and the size of object is twice as that of image, then the image distance is **[AFMC 1995]**

(a) 10 *cm* (b) 20 *cm* (c) 40 *cm* (d) 30 *cm*

53. If an object is placed 10 *cm* in front of a concave mirror of focal length 20 *cm*, the image will be **[MP PMT 1995]**

(a) Diminished, upright, virtual (b) Enlarged, upright, virtual (c) Diminished, inverted, real (d) Enlarged, upright, real

54. An object 1 *cm* tall is placed 4 *cm* in front of a mirror. In order to produce an upright image of 3 *cm* height one needs a **[SCRA 1994]**

(a) Convex mirror of radius of curvature 12 *cm* (b) Concave mirror of radius of curvature 12 *cm*

(c) Concave mirror of radius of curvature 4 *cm* (d) Plane mirror of height 12 *cm*

55. The image formed by a convex mirror of a real object is larger than the object **[CPMT 1994]**

(a) When $u < 2f$ (b) When $u > 2f$ (c) For all values of u (d) For no value of u

56. An object 5 *cm* tall is placed 1 *m* from a concave spherical mirror which has a radius of curvature of 20 *cm*. The size of the image is **[MP PET 1993]**

(a) 0.11 *cm* (b) 0.50 *cm* (c) 0.55 *cm* (d) 0.60 *cm*

57. A virtual image three times the size of the object is obtained with a concave mirror of radius of curvature 36 *cm*. The distance of the object from the mirror is

(a) 5 *cm* (b) 12 *cm* (c) 10 *cm* (d) 20 *cm*

58. Given a point source of light, which of the following can produce a parallel beam of light **[CPMT 1974]**

(a) Convex mirror (b) Concave mirror

(c) Concave lens (d) Two plane mirrors inclined at an angle of 90^o

59. A convex mirror is used to form the image of an object. Then which of the following statements is wrong

(a) The images lies between the pole and the focus (b) The image is diminished in size

(c) The images is erect (d) The image is real

60. A boy stands straight infront of a mirror at a distance of 30 *cm* away from it. He sees his erect image whose height is $\frac{1}{5}^{th}$ of his real height. The mirror he is using is

(a) Plane mirror (b) Convex mirror (c) Concave mirror (d) Plano-convex mirror

61. For the largest distance of the image from a concave mirror of focal length 10*cm*, the object should be kept at

(a) 10 *cm* (b) Infinite (c) 40 *cm* (d) 60 *cm*

62. A dentist uses a small mirror that gives a magnification of 4 when it is held 0.60 *cm* from a tooth. The radius of curvature of the mirror is

(a) 1.60 *cm* (convex) (b) 0.8 *cm* (concave) (c) 1.60 *cm* (concave) (d) 0.8 *cm* (convex)

63. A dice is placed with its one edge parallel to the principal axis between the principal focus and the centre of the curvature of a concave mirror. Then the image has the shape of

(a) Cube (b) Cuboid (c) Barrel shaped (d) Spherical

Advance Level

64. A short linear object of length l lies along the axis of a concave mirror of focal length f at a distance u form the pole of the mirror. The size of the image is approximately equal to **[IIT 1988; BHU 2003]**

(a) $l\left(\frac{u-f}{f}\right)^{1/2}$ (b) $l\left(\frac{u-f}{f}\right)^{2}$ (c) $l\left(\frac{f}{u-f}\right)^{1/2}$ (d) $l\left(\frac{f}{u-f}\right)^{2}$

65. A point object is moving on the principal axis of a concave mirror of focal length 24 *cm* towards the mirror. When it is at a distance of 60 *cm* from the mirror, its velocity is 9 *cm/sec*. What is the velocity of the image at that instant

(a) 5 *cm/sec* towards the mirror (b) 4 *cm/sec* towards the mirror

(c) 4 *cm/sec* away from the mirror (d) 9 *cm/sec* away from the mirror

66. A convex mirror of focal length 10 *cm* forms an image which is half of the size of the object. The distance of the object from the mirror is

(a) 10 *cm* (b) 20 *cm* (c) 5 *cm* (d) 15 *cm*

67. A concave mirror is used to focus the image of a flower on a nearby well 120 *cm* from the flower. If a lateral magnification of 16 is desired, the distance of the flower from the mirror should be

(a) 8 *cm* (b) 12 *cm* (c) 80 *cm* (d) 120 *cm*

68. A thin rod of 5 *cm* length is kept along the axis of a concave mirror of 10 *cm* focal length such that its image is real and magnified and one end touches the rod. Its magnification will be

(a) 1 (b) 2 (c) 3 (d) 4

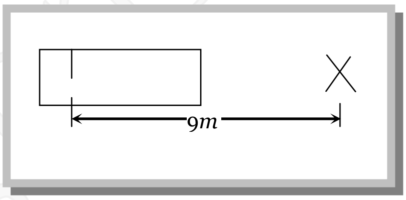
69. A luminous object is placed 20 *cm* from surface of a convex mirror and a plane mirror is set so that virtual images formed in two mirrors coincide. If plane mirror is at a distance of 12 *cm* from object, then focal length of convex mirror, is

(a) 5 *cm* (b) 10 *cm* (c) 20 *cm* (d) 40 *cm*

70. A rear mirror of a vehicle is cylindrical having radius of curvature 10 *cm*. The length of arc of curved surface is also 10 *cm*. If the eye of driver is assumed to be at large distance, from the mirror, then the field of view in radian is

(a) 0.5 (b) 1 (c) 2 (d) 4

71. A vehicle has a driving mirror of focal length 30 *cm*. Another vehicle of dimension $2 \times 4 \times 1.75\, m^3$ is 9 *m* away from the mirror of first vehicle. Position of the second vehicle as seen in the mirror of first vehicle is



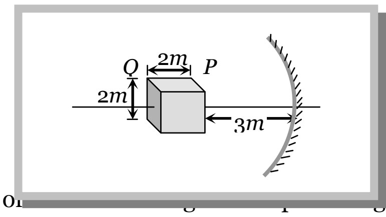
(a) 30 *cm*

(b) 60 *cm*

(c) 90 *cm*

(d) 9 *cm*

72. A cube of side 2 *m* is placed in front of a concave mirror focal length 1*m* with its face *P* at a distance of 3 *m* and face *Q* at a distance of 5 *m* from the mirror. The distance between the images of face *P* and *Q* and height of images of *P* and *Q* are



(a) 1 *m*, 0.5 *m*, 0.25 *m*

(b) 0.5 *m*, 1 *m*, 0.25 *m*

(c) 0.5 *m*, 0.25 *m*, 1*m*

(d) 0.25 *m*, 1*m*, 0.5 *m*

73. A concave mirror of radius of curvature 60 *cm* is placed at the bottom o... ht of 20 *cm*. The mirror faces upwards with its axis vertical. Solar light falls normally on the surface of water and the image of the sun is formed. If ${}_a\mu_w = \frac{4}{3}$ then with the observer in air, the distance of the image from the surface of water is

(a) 30 *cm* (b) 10 *cm* (c) 7.5 *cm* above (d) 7.5 *cm* below

74. A concave mirror forms an image of the sun at a distance of 12 *cm* from it

(a) The radius of curvature of this mirror is 6 *cm*

(b) To use it as a shaving mirror, it must be held at a distance of 8-10 *cm* from the face

(c) If an object is kept at a distance of 12 *cm* from it, the image formed will be of the same size as the object

(d) All the above a alternatives are correct

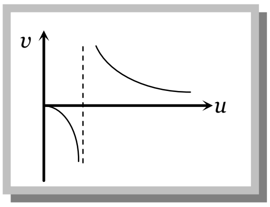
75. A small piece of wire bent into an L shape with upright and horizontal portions of equal lengths, is placed with the horizontal portion along the axis of the concave mirror whose radius of curvature is 10 *cm*. If the bend is 20 *cm* from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portions of the wire is

(a) 1 : 2 (b) 3 : 1 (c) 1 : 3 (d) 2 : 1

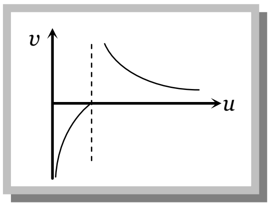
76. As the position of an object (u) reflected from a concave mirror is varied, the position of the image (v) also varies. By letting the u changes from 0 to $+\infty$ the graph between v versus u will be

77.

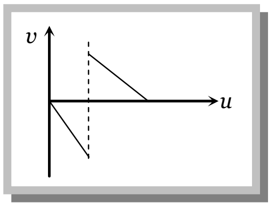
(a)



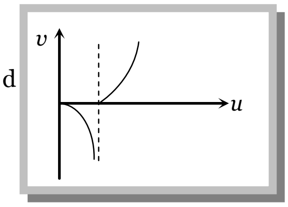
b



c



d



78. A concave mirror has a focal length 20 *cm*. The distance between the two positions of the object for which the image size is double of the object size is

(a) 20 *cm* (b) 40 *cm* (c) 30 *cm* (d) 60 *cm*

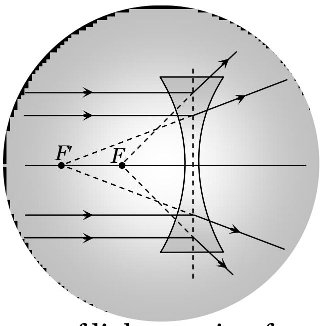
79. A concave mirror of focal length 10 *cm* and a convex mirror of focal length 15 *cm* are placed facing each other 40 *cm* apart. A point object is placed between the mirrors, on their common axis and 15 *cm* from the concave mirror. Find the position and nature of the image produced by the successive reflections, first at concave mirror and then at convex mirror

(a) 2 *cm* (b) 4 *cm* (c) 6 *cm* (d) 8 *cm*

Answer Sheet

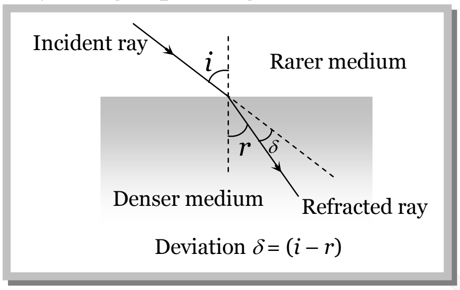
Assignments

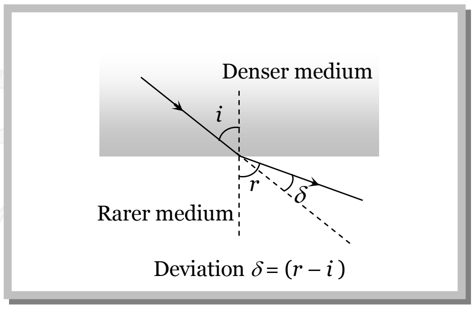
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
c	c	c	c	c	b	b	b	b	c	b	b	a	b	c	b	d	d	c	b
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
d	a	b	b	c	c	b	a	c	d	b	a	d	c	d	c	d	d	b	d
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
b	a	d	c	a	a	b	b, c	c	d	d	a	b	b	d	c	b	b	d	b
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**		
a	c	b	d	c	a	a	a	a	b	a	d	c	b	b	a	a	c		



Refraction of Light

The bending of the ray of light passing from one medium to the other medium is called refraction.





Snell's law

The ratio of sine of the angle of incidence to the angle of refraction (r) is a constant called refractive index

i.e. $\frac{\sin i}{\sin r} = \mu$ (a constant). For two media, Snell's law can be written as ${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$

$\Rightarrow \mu_1 \times \sin i = \mu_2 \times \sin r$ *i.e.* $\mu \sin\theta =$ constant

Also in vector form : $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{n}} = \boldsymbol{\mu}(\hat{\boldsymbol{r}} \times \hat{\boldsymbol{n}})$

Refractive Index

Refractive index of a medium is that characteristic which decides speed of light in it. It is a scalar, unit less and dimensionless quantity

.(1) **Types :** It is of following two types

Absolute refractive index	Relative refractive index
(i) When light travels from air to any transparent medium then R.I. of medium *w.r.t.* air is called it's absolute R.I. *i.e.* ${}_{\text{air}}\mu_{\text{medium}} = \frac{c}{v}$	(i) When light travels from medium (1) to medium (2) then R.I. of medium (2) *w.r.t.* medium (1) is called it's relative R.I. *i.e.* ${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$ (where v_1 and v_2 are the speed of light in medium 1 and 2 respectively).
(ii) Some absolute R.I.	(ii) Some relative R.I. (a) When light enters from water to glass :

$_a\mu_{\text{glass}} = \frac{3}{2} = 1.5$, $_a\mu_{water} = \frac{4}{3} = 1.33$

$_a\mu_{\text{diamond}} = 2.4$, $_a\mu_{Cs_2} = 1.62$

$_a\mu_{\text{crown}} = 1.52$, $\mu_{\text{vacuum}} = 1$, $\mu_{\text{air}} = 1.0003 \approx 1$

$_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$

(b) When light enters from glass to diamond :

$_g\mu_D = \frac{\mu_D}{\mu_g} = \frac{2.4}{1.5} = \frac{8}{5}$

Note : ❑ Cauchy's equation : $\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ...$ $(\lambda_{\text{Red}} > \lambda_{\text{violet}} \text{ so } \mu_{\text{Red}} < \mu_{\text{violet}})$

❑ If a light ray travels from medium (1) to medium (2), then $_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$ $\mu \propto \frac{1}{\lambda}$, $\mu \propto \frac{1}{v}$, $v \propto \lambda$

(2) **Dependence of Refractive index**

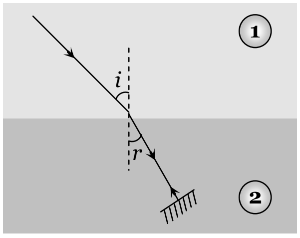
(i) Nature of the media of incidence and refraction.

(ii) Colour of light or wavelength of light.

(iii) Temperature of the media : Refractive index decreases with the increase in temperature.

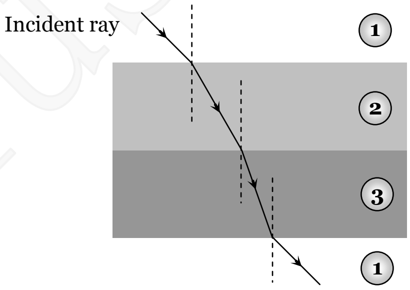
(3) **Principle of reversibility of light and refraction through several media :**

Principle of reversibility



$_1\mu_2 = \frac{1}{_2\mu_1}$

Refraction through several media

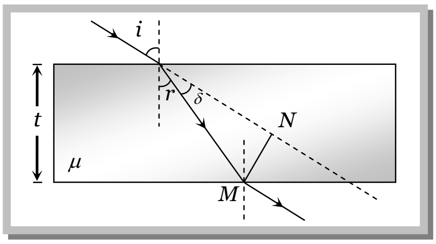


$_1\mu_2 \times {_2\mu_3} \times {_3\mu_1} = 1$

Refraction Through a Glass Slab and Optical Path

(1) **Lateral shift**

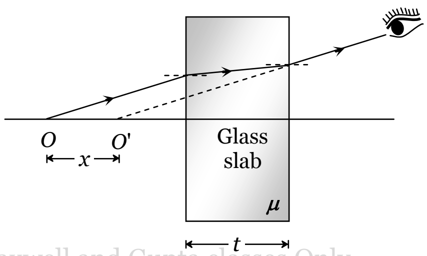
The refracting surfaces of a glass slab are parallel to each other. When a light ray passes through a glass slab it is refracted twice at the two parallel faces and finally emerges out parallel to it's incident direction *i.e.* the ray undergoes no deviation δ = 0. The angle of emergence (e) is equal to the angle of incidence (i)



The Lateral shift of the ray is the perpendicular distance between the incident and the emergent ray, and it is given by ***MN* = *t* sec *r* sin (*i* – *r*)**

Normal shift

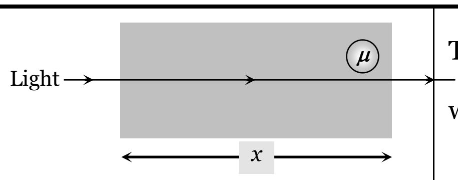
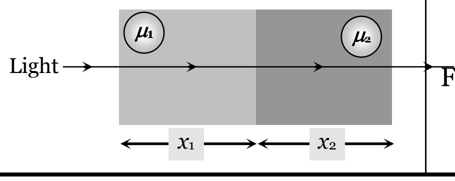
Normal shift $OO' = x = \left(1 - \frac{1}{\mu}\right)t$



Or the object appears to be shifted towards the slab by the distance x

(2) **Optical path :**

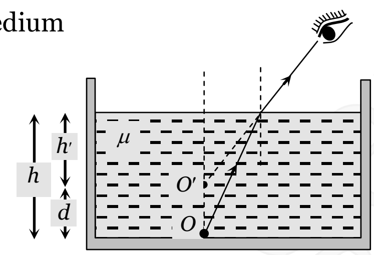
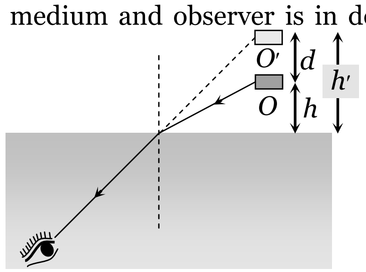
It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium.

	Time taken by light ray to pass through the medium $= \frac{\mu x}{c}$; where x = geometrical path and μx = optical path
	For two medium in contact optical path = $\mu_1 x_1 + \mu_2 x_2$

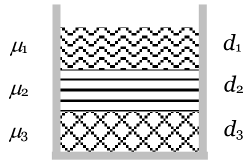
Note : ❑ Since for all media $\mu > 1$, so optical path length (μx) is always greater than the geometrical path length (x).

Real and Apparent Depth

If object and observer are situated in different medium then due to refraction, object appears to be displaced from it's real position. There are two possible conditions.

(1) When object is in denser medium and observer is in rarer medium 	(1) Object is in rarer medium and observer is in denser medium. 
(2) $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{h}{h'}$ Real depth >Apparent depth that's why a coin at the bottom of bucket (full of water) appears to be raised)	(2) $\mu = \frac{h'}{h}$ Real depth < Apparent depth that's why high flying aeroplane appears to be higher than it's actual height.
(3) Shift $d = h - h' = \left(1 - \frac{1}{\mu}\right)h$	(3) $d = (\mu - 1)h$
(4) For water $\mu = \frac{4}{3} \Rightarrow d = \frac{h}{4}$ For glass $\mu = \frac{3}{2} \Rightarrow d = \frac{h}{3}$	(4) Shift for water $d_w = \frac{h}{3}$ Shift for glass $d_g = \frac{h}{2}$

Note : ❑ If a beaker contains various immisible liquids as shown then



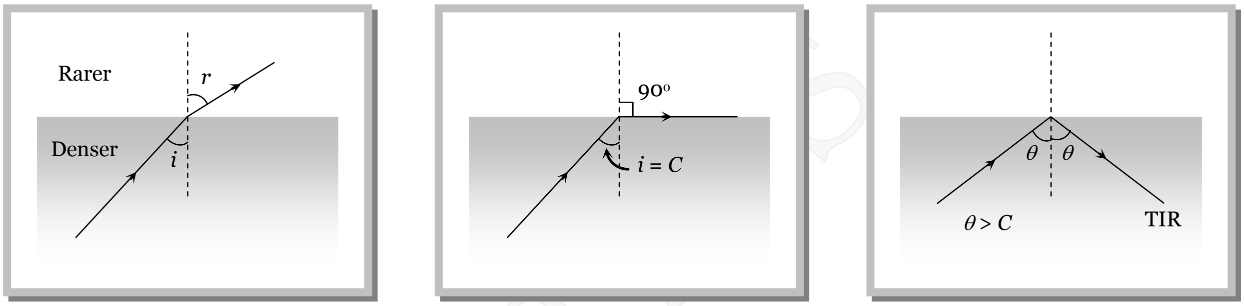
Apparent depth of bottom $= \dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2} + \dfrac{d_3}{\mu_3} + ...$

$\mu_{\text{combination}} = \dfrac{d_{AC}}{d_{App.}} = \dfrac{d_1 + d_2 + ...}{\dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2} + ...}$ (In case of two liquids if $d_1 = d_2$ than $\mu = \dfrac{2\mu_1\mu_2}{\mu_1 + \mu_2}$)

Total Internal Reflection

When a ray of light goes from denser to rarer medium it bends away from the normal and as the angle of incidence in denser medium increases, the angle of refraction in rarer medium also increases and at a certain angle, angle of refraction becomes 90°, this angle of incidence is called critical angle (*C*).

When Angle of incidence exceeds the critical angle than light ray comes back in to the same medium after reflection from interface. This phenomenon is called Total internal reflection (TIR).

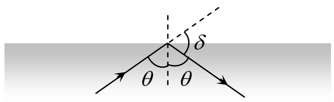


Important formula $\mu = \dfrac{1}{\sin C} = \operatorname{cosec} C$; where $\mu \rightarrow {}_{\text{Rerer}}\mu_{\text{Denser}}$

Note : ❑ When a light ray travels from denser to rarer medium, then deviation of the ray is

$\delta = \pi - 2\theta \Rightarrow \delta \rightarrow \text{max. when } \theta \rightarrow \text{min.} = C$

i.e. $\delta_{\max} = (\pi - 2C)$; $C \rightarrow$ critical angle



(1) **Dependence of critical angle**

(i) Colour of light (or wavelength of light) : Critical angle depends upon wavelength as $\lambda \propto \dfrac{1}{\mu} \propto \sin C$

(a) $\lambda_R > \lambda_V \Rightarrow C_R > C_V$

(b) Sin $C = \dfrac{1}{{}_R\mu_D} = \dfrac{\mu_R}{\mu_D} = \dfrac{\lambda_D}{\lambda_R} = \dfrac{v_D}{v_R}$ (for two media) (c) For TIR from boundary of two media $i > \sin^{-1}\dfrac{\mu_R}{\mu_D}$

(ii) Nature of the pair of media : Greater the refractive index lesser will be the critical angle.

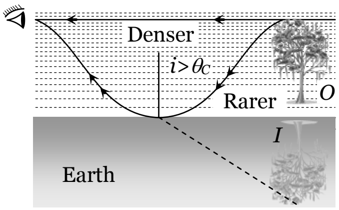
(a) For (glass- air) pair $\rightarrow C_{\text{glass}} = 42^o$ (b) For (water-air) pair $\rightarrow C_{\text{water}} = 49^o$

(c) For (diamond-air) pair $\rightarrow C_{\text{diamond}} = 24^o$

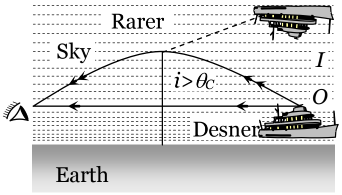
(iii) Temperature : With temperature rise refractive index of the material decreases therefore critical angle increases.

(2) **Examples of total internal reflection (TIR)**

(i)



Mirage : An optical illusion in deserts



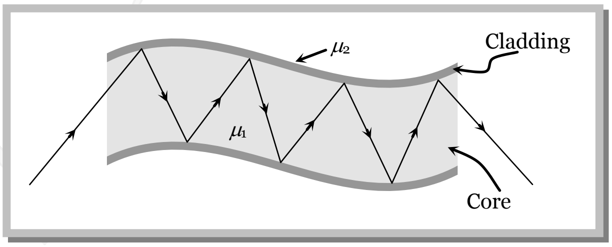
Looming : An optical illusion in cold countries

(ii) **Brilliance of diamond :** Due to repeated internal reflections diamond sparkles.

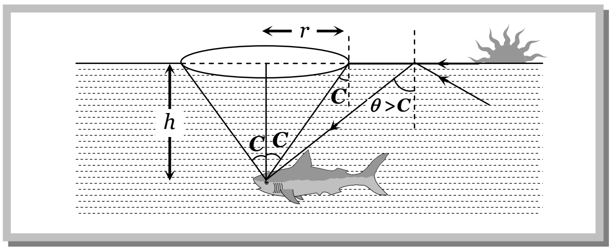
(iii) **Optical fibre :** Optical fibres consist of many long high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core (μ_1) is higher than that of the cladding (μ_2).

When the light is incident on one end of the fibre at a small angle, the light passes inside, undergoes repeated total internal reflections along the fibre and finally comes out. The angle of incidence is always larger than the critical angle of the core material with respect to its cladding. Even if the fibre is bent, the light can easily travel through along the fibre

A bundle of optical fibres can be used as a 'light pipe' in medical and optical examination. It can also be used for optical signal transmission. Optical fibres have also been used for transmitting and receiving electrical signals which are converted to light by suitable transducers.



(iv) **Field of vision of fish (or swimmer) :** A fish (diver) inside the water can see the whole world through a cone with.



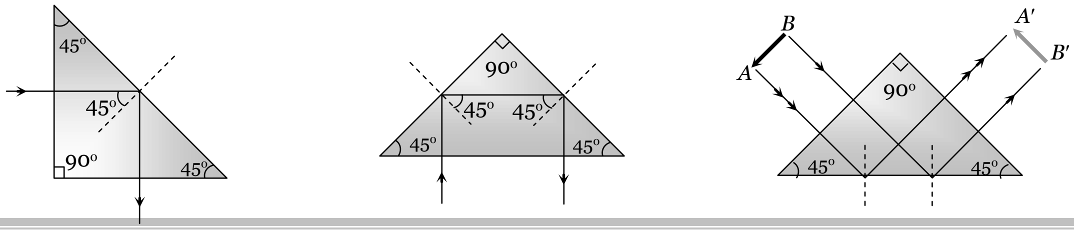
(a) Apex angle $= 2C = 98^o$

(b) Radius of base $r = h\tan C = \dfrac{h}{\sqrt{\mu^2 - 1}}$

(c) Area of base $A = \dfrac{\pi h^2}{(\mu^2 - 1)}$

Note : ❑ For water $\mu = \dfrac{4}{3}$ so $r = \dfrac{3h}{\sqrt{7}}$ and $A = \dfrac{9\pi h^2}{7}$.

(v) **Porro prism :** A right angled isosceles prism, which is used in periscopes or binoculars. It is used to deviate light rays through 90^o and 180^o and also to erect the image.



Example

***Example*: 1** A beam of monochromatic blue light of wavelength 4200 $\mathring{A}$ in air travels in water $(\mu = 4/3)$. Its wavelength in water will be

(a) 2800 $\mathring{A}$ (b) 5600 $\mathring{A}$ (c) 3150 $\mathring{A}$ (d) 4000 $\mathring{A}$

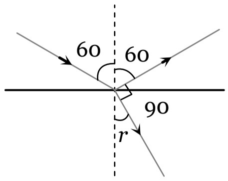
Solution: (c) $\mu \propto \frac{1}{\lambda} \Rightarrow \frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{1}{4/3} = \frac{\lambda_2}{4200} \Rightarrow \lambda_2 = 3150\ \mathring{A}$

***Example*: 2** On a glass plate a light wave is incident at an angle of 60°. If the reflected and the refracted waves are mutually perpendicular, the refractive index of material is **[MP PMT 1994; Haryana CEE 1996]**

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) $\frac{3}{2}$ (d) $\frac{1}{\sqrt{3}}$

Solution: (b) From figure $r = 30^o$

$\therefore \quad \mu = \frac{\sin i}{\sin r} = \frac{\sin 60^o}{\sin 30^o} = \sqrt{3}$



***Example*: 3** Velocity of light in glass whose refractive index with respect to air is 1.5 is $2 \times 10^8\ m/s$ and in certain liquid the velocity of light found to be $2.50 \times 10^8\ m/s$. The refractive index of the liquid with respect to air is **[CPMT 1978; MP PET/PMT 1988]**

(a) 0.64 (b) 0.80 (c) 1.20 (d) 1.44

Solution: (c) $\mu \propto \frac{1}{v} \Rightarrow \frac{\mu_{li}}{\mu_g} = \frac{v_g}{v_l} \Rightarrow \frac{\mu_l}{1.5} = \frac{2 \times 10^8}{2.5 \times 10^8} \Rightarrow \mu_l = 1.2$

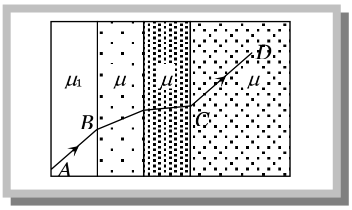
***Example*: 4** A ray of light passes through four transparent media with refractive indices $\mu_1 . \mu_2, \mu_3$, and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray *CD* is parallel to the incident ray *AB*, we must have

(a) $\mu_1 = \mu_2$

(b) $\mu_2 = \mu_3$

(c) $\mu_3 = \mu_4$

(d) $\mu_4 = \mu_1$



Solution: (d) For successive refraction through difference media $\mu \sin\theta =$ constant.

Here as θ is same in the two extreme media. Hence $\mu_1 = \mu_4$

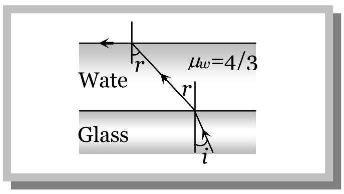
***Example*: 5** A ray of light is incident at the glass–water interface at an angle *i*, it emerges finally parallel to the surface of water, then the value of μ_g would be

(a) (4/3) sin *i*

(b) 1/ sin *i*

(c) 4/ 3

(d) 1



Solution: (b) For glass water interface ${}_g\mu_\omega = \frac{\sin i}{\sin r}$ ......(i) and For water-air interface ${}_\omega\mu_a = \frac{\sin r}{\sin 90}$ .....(ii)

$\therefore \quad {}_g\mu_\omega \times {}_\omega\mu_a = \sin i \quad \Rightarrow \quad \mu_g = \dfrac{1}{\sin i}$

Example: 6 The ratio of thickness of plates of two transparent mediums *A* and *B* is 6 : 4. If light takes equal time in passing through them, then refractive index of *B* with respect to *A* will be

(a) 1.4 (b) 1.5 (c) 1.75 (d) 1.33

Solution: (b) By using $t = \dfrac{\mu x}{c}$

$\Rightarrow \dfrac{\mu_B}{\mu_A} = \dfrac{x_A}{x_B} = \dfrac{6}{4} \Rightarrow {}_A\mu_B = \dfrac{3}{2} = 1.5$

Example: 7 A ray of light passes from vacuum into a medium of refractive index μ, the angle of incidence is found to be twice the angle of refraction. Then the angle of incidence is

(a) $\cos^{-1}(\mu/2)$ (b) $2\cos^{-1}(\mu/2)$ (c) $2\sin^{-1}(\mu)$ (d) $2\sin^{-1}(\mu/2)$

Solution: (b) By using $\mu = \dfrac{\sin i}{\sin r} \Rightarrow \mu = \dfrac{\sin 2r}{\sin r} = \dfrac{2\sin r\cos r}{\sin r}$ $(\sin 2\theta = 2\sin\theta\cos\theta)$

$\Rightarrow r = \cos^{-1}\left(\dfrac{\mu}{2}\right)$. So, $i = 2r = 2\cos^{-1}\left(\dfrac{\mu}{2}\right)$.

Example: 8 A ray of light falls on the surface of a spherical glass paper weight making an angle α with the normal and is refracted in the medium at an angle β. The angle of deviation of the emergent ray from the direction of the incident ray is

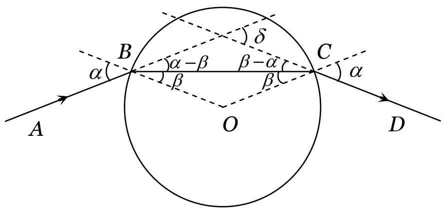
(a) $(\alpha-\beta)$ (b) $2(\alpha-\beta)$ (c) $(\alpha-\beta)/2$ (d) $(\alpha+\beta)$

Solution: (b) From figure it is clear that ΔOBC is an isosceles triangle,

Hence $\angle OCB = \beta$ and emergent angle is α

Also sum of two in terior angles = exterior angle

$\therefore \quad \delta = (\alpha-\beta)+(\alpha-\beta) = 2(\alpha-\beta)$

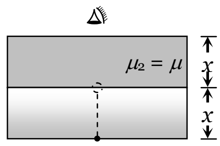


Example: 9 A rectangular slab of refractive index μ is placed over another slab of refractive index 3, both slabs being identical in dimensions. If a coin is placed below the lower slab, for what value of μ will the coin appear to be placed at the interface between the slabs when viewed from the top

(a) 1.8 (b) 2 (c) 1.5 (d) 2.5

Solution: (c) Apparent depth of coin as seen from top $= \dfrac{x}{\mu_1} + \dfrac{x}{\mu_2} = x$

$\Rightarrow \dfrac{1}{\mu_1} + \dfrac{1}{\mu_2} = 1 \quad \Rightarrow \dfrac{1}{3} + \dfrac{1}{\mu} = 1 \quad \Rightarrow \mu = 1.5$



Example: 10 A coin is kept at bottom of an empty beaker. A travelling microscope is focussed on the coin from top, now water is poured in beaker up to a height of 10 *cm*. By what distance and in which direction should the microscope be moved to bring the coin again in focus

(a) 10 *cm* up ward (b) 10 *cm* down ward (c) 2.5 *cm* up wards (d) 2.5 *cm* down wards

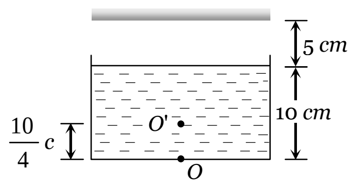
Solution: (c) When water is poured in the beaker. Coin appears to shift by a distance $d = \dfrac{h}{4} = \dfrac{10}{4} = 2.5 cm$

Hence to bring the coil again in focus, the microscope should be moved by 2.5 *cm* in upward direction.

***Example*: 11** Consider the situation shown in figure. Water $\left(\mu_w = \frac{4}{3}\right)$ is filled in a breaker upto a height of 10 *cm*. A plane mirror fixed at a height of 5 *cm* from the surface of water. Distance of image from the mirror after reflection from it of an object *O* at the bottom of the beaker is

(a) 15 *cm* (b) 12.5 *cm* (c) 7.5 *cm* (d) 10 *cm*

Solution: (b) From figure it is clear that object appears to be raised by $\frac{10}{4} cm\,(2.5\,cm)$



Hence distance between mirror and $O' = 5 + 7.5 = 12.5\,cm$

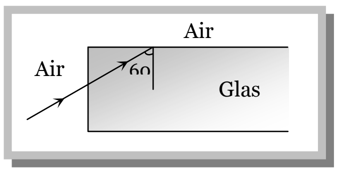
So final image will be formed at 12.5 *cm* behind the plane mirror.

***Example*: 12** The wavelength of light in two liquids '*x*' and '*y*' is 3500 Å and 7000 Å, then the critical angle of *x* relative to *y* will be

(a) 60° (b) 45° (c) 30° (d) 15°

Solution: (c) $\sin C = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{3500}{7000} = \frac{1}{2} \Rightarrow C = 30^o$

***Example*: 13** A light ray from air is incident (as shown in figure) at one end of a glass fiber (refractive index $\mu = 1.5$) making an incidence angle of 60° on the lateral surface, so that it undergoes a total internal reflection. How much time would it take to traverse the straight fiber of length 1 *km* **[Orissa JEE 2002]**



(a) 3.33 μ*sec*

(b) 6.67 μ*sec*

(c) 5.77 μ*sec*

(d) 3.85 μ*sec*

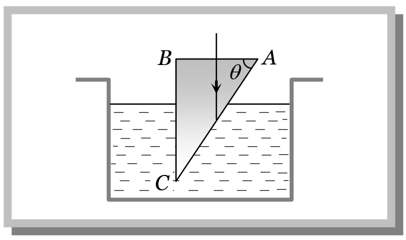
Solution: (d) When total internal reflection just takes place from lateral surface then $i = C$ *i.e.* $C = 60°$

From $\mu = \frac{1}{\sin C} \Rightarrow \mu = \frac{1}{\sin 60} = \frac{2}{\sqrt{3}}$

Hence time taken by light traverse some distance in medium $t = \frac{\mu x}{C}$

$\Rightarrow t = \frac{\frac{2}{\sqrt{3}} \times (1 \times 10^3)}{3 \times 10^8} = 3.85\,\mu\,sec.$

***Example*: 14** A glass prism of refractive index 1.5 is immersed in water $(\mu = 4/3)$. A light beam incident normally on the face *AB* is totally reflected to reach the face *BC* if



(a) $\sin\theta > 8/9$

(b) $2/3 < \sin\theta < 8/9$

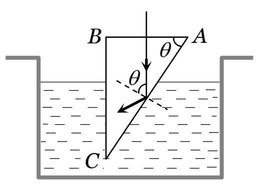
(c) $\sin\theta \le 2/3$

(d) $\cos\theta \ge 8/9$

Solution: (a) From figure it is clear that

Total internal reflection takes place at *AC*, only if $\theta > C$

$\Rightarrow \sin\theta > \sin C$ $\Rightarrow \sin\theta > \frac{1}{{}_{\omega}\mu_g}$



$\Rightarrow \sin\theta > \dfrac{1}{9/8}$ $\qquad \Rightarrow \sin\theta > \dfrac{8}{9}$

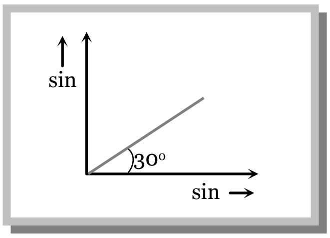
Example: 15 When light is incident on a medium at angle i and refracted into a second medium at an angle r, the graph of sin i *vs* sin r is as shown in the graph. From this, one can conclude that

(a) Velocity of light in the second medium is 1.73 times the velocity of light in the I medium

(b) Velocity of light in the I medium is 1.73 times the velocity in the II medium

(c) The critical angle for the two media is given by $\sin\, i_c = \dfrac{1}{\sqrt{3}}$

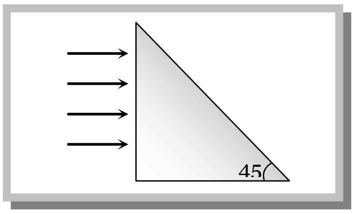
(d) $\sin\, i_c = \dfrac{1}{2}$



Solution: (b, c) From graph $\tan 30^o = \dfrac{\sin r}{\sin i} = \dfrac{1}{{}_1\mu_2} \Rightarrow {}_1\mu_2 = \sqrt{3} \Rightarrow \dfrac{\mu_2}{\mu_1} = \dfrac{v_1}{v_2} = 1.73 \quad \Rightarrow v_1 = 1.75\, v_2$

Also from $\mu = \dfrac{1}{\sin C} \Rightarrow \sin C = \dfrac{1}{{}_{Rarer}\mu_{Denser}} \Rightarrow \sin C = \dfrac{1}{{}_1\mu_2} = \dfrac{1}{\sqrt{3}}$.

Example: 16 A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive indices of the material of the prism for the above red, green and blue wavelength are 1.39, 1.44 and 1.47 respectively. The prism will



(a) Separate part of red colour from the green and the blue colours

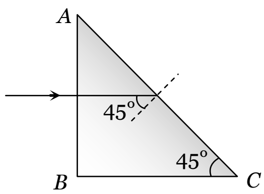
(b) Separate part of the blue colour from the red and green colours

(c) Separate all the colours from one another

(d) Not separate even partially any colour from the other two colours

Solution: (a) At face AB, i = 0 so r = 0, *i.e.*, no refraction will take place. So light will be incident on face AC at an angle of incidence of 45°. The face AC will not transmit the light for which $i > \theta_C$, *i.e.*, $\sin i > \sin\theta_C$

or $\sin 45^o > (1/\mu)$ *i.e.*, $\mu > \sqrt{2}$ $(= 1.41)$

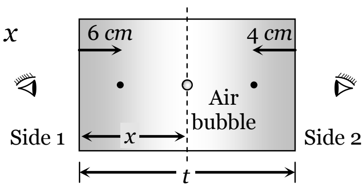


Now as $\mu_R < \mu$ while μ_G and $\mu_B > \mu$, so red will be transmitted through the face AC while green and blue will be reflected. So the prism will separate red colour from green and blue.

Example: 17 An air bubble in a glass slab $(\mu = 1.5)$ is 6 *cm* deep when viewed from one face and 4 *cm* deep when viewed from the opposite face. The thickness of the glass plate is

(a) 10 *cm* (b) 6.67 *cm* (c) 15 *cm* (d) None of these

Solution: (c) Let thickness of slab be t and distance of air bubble from one side is x



When viewed from side (1) : $1.5 = \dfrac{x}{6} \Rightarrow x = 9cm$

When viewed from side (2) : $1.5 = \dfrac{(t-x)}{4} \Rightarrow 1.5 = \dfrac{(t-9)}{4} \Rightarrow t = 15cm$

Tricky example: 1

One face of a rectangular glass plate 6 *cm* thick is silvered. An object held 8 *cm* in front of the first face, forms an image 12 *cm* behind the silvered face. The refractive index of the glass is

(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.6

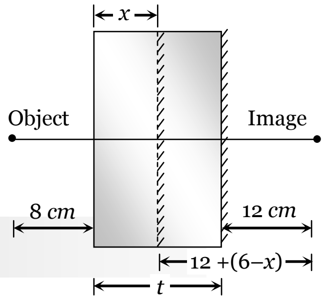
Solution : (c) From figure thickness of glass plate $t = 6\ cm$.

Let x be the apparent position of the silvered surface.

According to property of plane mirror

$x + 8 = 12 + 6 - x \quad \Rightarrow \quad x = 5\ cm$

Al $\quad t \quad 6 \quad 1.2$



Tricky example: 2

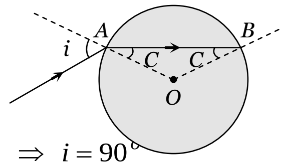
A ray of light is incident on a glass sphere of refractive index 3/2. What should be the angle of incidence so that the ray which enters the sphere doesn't come out of the sphere

(a) $\tan^{-1}\left(\frac{2}{3}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$ (c) 90° (d) $\cos^{-1}\left(\frac{1}{3}\right)$

Solution : (c) Ray doesn't come out from the sphere means TIR takes place.

Hence from figure $\angle ABO = \angle OAB = C$

$\therefore \quad \mu = \frac{1}{\sin C} \Rightarrow \sin C = \frac{1}{\mu} = \frac{2}{3}$



Applying Snell's Law at A $\quad \frac{\sin i}{\sin C} = \frac{3}{2} \Rightarrow \sin i = \frac{3}{2}\sin C = \frac{3}{2} \times \frac{2}{3} = 1 \quad \Rightarrow \quad i = 90^\circ$

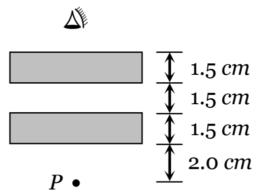
Tricky example: 3

The image of point P when viewed from top of the slabs will be

(a) 2.0 *cm* above P (b) 1.5 *cm* above P (c) 2.0 *cm* below P (d) 1 *cm* above P

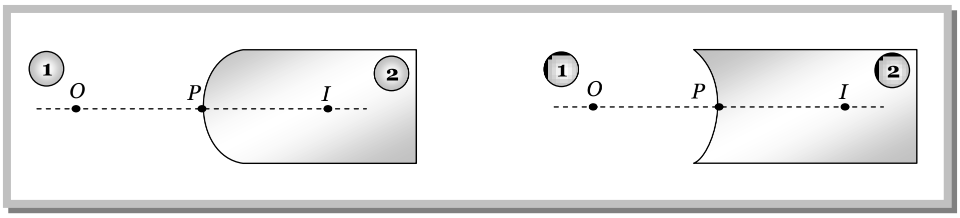
Solution: (d) The two slabs will shift the image a distance

$$d = 2\left(1 - \frac{1}{\mu}\right)t = 2\left(1 - \frac{1}{1.5}\right)(1.5) = 1\,cm$$



Therefore, final image will be 1 *cm* above point P.

Refraction From Curved Surface



μ_1 = Refractive index of the medium from which light rays are coming (from object).

μ_2 = Refractive index of the medium in which light rays are entering.

u = Distance of object, v = Distance of image, R = Radius of curvature

Refraction formula : $\frac{\mu_2 - \mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$ (use sign convention while solving the problem)

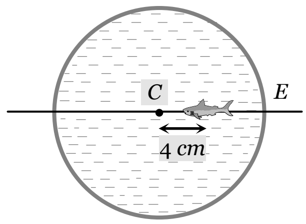
Note : ❑ Real image forms on the side of a refracting surface that is opposite to the object, and virtual image forms on the same side as the object.

❑ Lateral (Transverse) magnification $m = \frac{I}{O} = \frac{\mu_1 v}{\mu_2 u}$.

Specific Example

In a thin spherical fish bowl of radius 10 *cm* filled with water of refractive index 4/3 there is a small fish at a distance of 4 *cm* from the centre C as shown in figure. Where will the image of fish appears, if seen from E

(a) 5.2 *cm* (b) 7.2 *cm* (c) 4.2 *cm* (d) 3.2 *cm*



Solution : (a) By using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

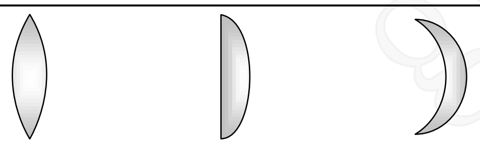
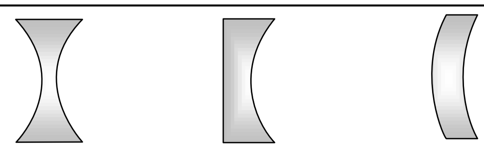
where $\mu_1 = \frac{4}{3}$, $\mu_2 = 1$, $u = -6\,cm$, $v = ?$

On putting values $v = -5.2\,cm$

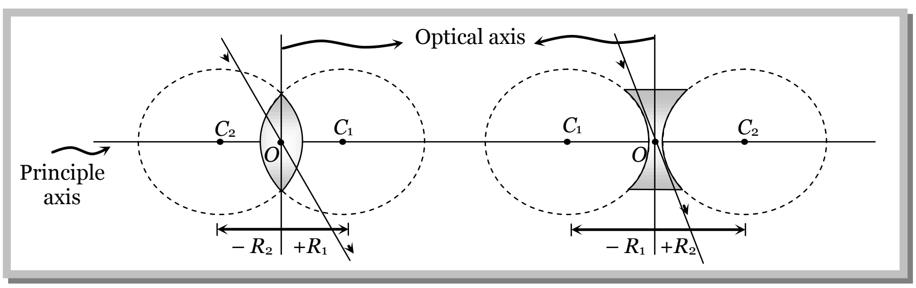
Lens

Lens is a transparent medium bounded by two refracting surfaces, such that at least one surface is spherical.

(1) **Type of lenses**

Convex lens (Converges the light rays)	Concave lens (Diverges the light rays)
 Double convex, Plano convex, Concavo convex	 Double concave, Plane concave, Convexo concave
Thick at middle	Thin at middle
It forms real and virtual images both	It forms only virtual images

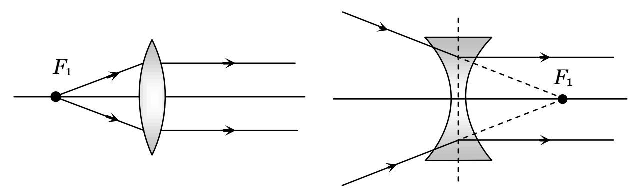
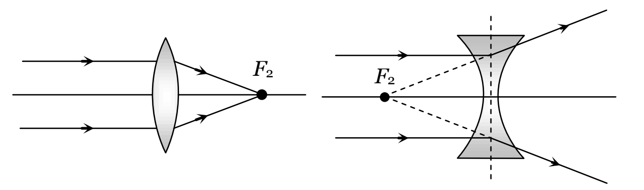
(2) **Some definitions**



C_1, C_2 – Centre of curvature,
R_1, R_2 – Radii of curvature

(i) **Optical centre (O) :** A point for a given lens through which light ray passes undeviated (Light ray passes undeviated through optical centre).

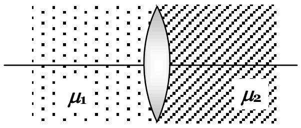
(ii) **Principle focus**

First principle focus	**Second principle focus**
	

Note : ❑ Second principle focus is the principle focus of the lens.

❑ When medium on two sides of lens is same then $|F_1| = |F_2|$.

❑ If medium on two sides of lens are not same then the ratio of two focal lengths $\frac{f_1}{f_2} = \frac{\mu_1}{\mu_2}$



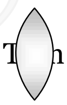
(iii) **Focal length (*f*) :** Distance of second principle focus from optical centre is called focal length

$f_{\text{convex}} \rightarrow$ positive, $f_{\text{concave}} \rightarrow$ negative, $f_{\text{plane}} \rightarrow \infty$

(iv) **Aperture :** Effective diameter of light transmitting area is called aperture. Intensity of image $\propto (\text{Aperture})^2$

(v) **Power of lens (*P*) :** Means the ability of a lens to converge the light rays. Unit of power is Diopter (*D*).

$$P = \frac{1}{f(m)} = \frac{100}{f(cm)};\ P_{\text{convex}} \rightarrow \text{positive},\ P_{\text{concave}} \rightarrow \text{negative},\ P_{\text{plane}} \rightarrow \text{zero}.$$

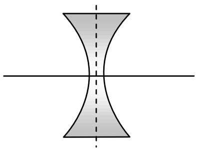
Note : ❑ Thick lens  Thin lens 

$P\uparrow f\downarrow R\downarrow$ $\quad$ $P\downarrow f\uparrow R\uparrow$

(2) **Image formation by lens**

Lens	**Location of the object**	**Location of the image**	**Nature of image**		
			Magnification	**Real / virtual**	**Erect / inverted**
Convex	At infinity *i.e.* $u = \infty$	At focus *i.e.* $v = f$	$m < 1$ diminished	Real	Inverted
	Away from 2*f* *i.e.* $(u > 2f)$	Between *f* and 2*f* *i.e.* $f < v < 2f$	$m < 1$ diminished	Real	Inverted

2*f* *f* *f* 2*f*

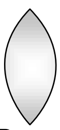
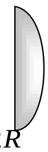
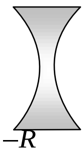
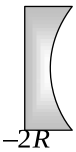
	At $2f$ or $(u = 2f)$	At $2f$ *i.e.* $(v = 2f)$	$m = 1$ same size	Real	Inverted
	Between f and $2f$ *i.e.* $f < u < 2f$	Away from $2f$ *i.e.* $(v > 2f)$	$m > 1$ magnified	Real	Inverted
	At focus *i.e.* $u = f$	At infinity *i.e.* $v = \infty$	$m = \infty$ magnified	Real	Inverted
	Between optical centre and focus, $u < f$	At a distance greater than that of object $v > u$	$m > 1$ magnified	Virtual	Erect
Concave 	At infinity *i.e.* $u = \infty$	At focus *i.e.* $v = f$	$m < 1$ diminished	Virtual	Erect
	Anywhere between infinity and optical centre	Between optical centre and focus	$m < 1$ diminished	Virtual	Erect

Note : ❑ Minimum distance between an object and it's real image formed by a convex lens is $4f$.

❑ Maximum image distance for concave lens is it's focal length.

(4) **Lens maker's formula**

The relation between f, μ, R_1 and R_2 is known as lens maker's formula and it is $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Equiconvex lens	Plano convex lens	Equi concave lens	Plano concave lens
$R_1 = R$ and $R_2 = -R$ $f = \frac{R}{2(\mu - 1)}$  for $\mu = 1.5$, $f = R$	$R_1 = \infty$, $R_2 = -R$ $f = \frac{R}{(\mu - 1)}$  for $\mu = 1.5$, $f = 2R$	$R_1 = -R$, $R_2 = +R$ $f = -\frac{R}{2(\mu - 1)}$  for $\mu = 1.5$ $f = -R$	$R_1 = \infty$, $R_2 = R$ $f = \frac{R}{2(\mu - 1)}$  for $\mu = 1.5$, $f = -2R$

(5) **Lens in a liquid**

Focal length of a lens in a liquid (f_l) can be determined by the following formula

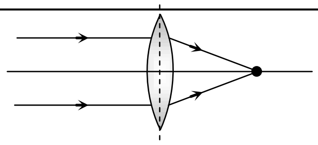
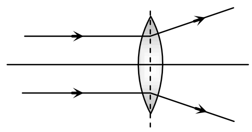
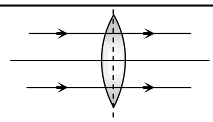
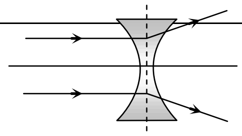
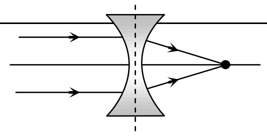
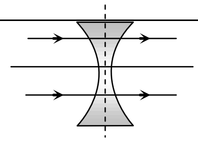
$$\frac{f_l}{f_a} = \frac{({}_a\mu_g - 1)}{({}_l\mu_g - 1)}$$ (Lens is supposed to be made of glass).

Note : ❑ Focal length of a glass lens $(\mu = 1.5)$ is f in air then inside the water it's focal length is $4f$.

❑ In liquids focal length of lens increases (↑) and it's power decreases (↓).

(6) **Opposite behaviour of a lens**

In general refractive index of lens $(\mu_L) >$ refractive index of medium surrounding it (μ_M).

$\mu_L > \mu_M$	$\mu_L < \mu_M$	$\mu_L = \mu_M$
		
		

(7) **Lens formula and magnification of lens**

(i) Lens formula : $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$; (use sign convention)

(ii) Magnification : The ratio of the size of the image to the size of object is called magnification.

(a) Transverse magnification : $m = \frac{I}{O} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$ (use sign convention while solving the problem)

(b) Longitudinal magnification : $m = \frac{I}{O} = \frac{v_2 - v_1}{u_2 - u_1}$. For very small object $m = \frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f+u}\right)^2 = \left(\frac{f-v}{f}\right)^2$

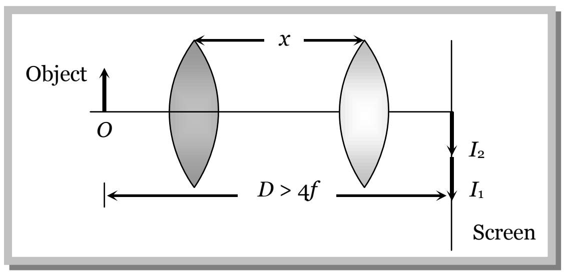
(c) Areal magnification : $m_s = \frac{A_i}{A_o} = m^2 = \left(\frac{f}{f+u}\right)^2$, ($A_i$ = Area of image, A_o = Area of object)

(8) **Relation between object and image speed**

If an object move with constant speed (V_o) towards a convex lens from infinity to focus, the image will move slower in the beginning and then faster. Also $V_i = \left(\frac{f}{f+u}\right)^2 . V_o$

(9) **Focal length of convex lens by displacement method**

(i) For two different positions of lens two images (I_1 and I_2) of an object is formed at the same location.



(ii) Focal length of the lens $f = \frac{D^2 - x^2}{4D} = \frac{x}{m_1 - m_2}$

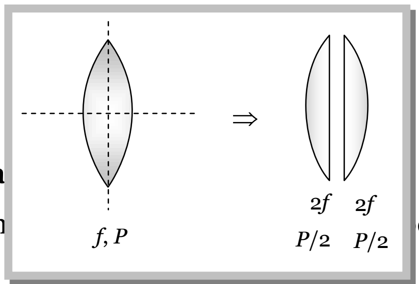
where $m_1 = \frac{I_1}{O}$ and $m_2 = \frac{I_2}{O}$

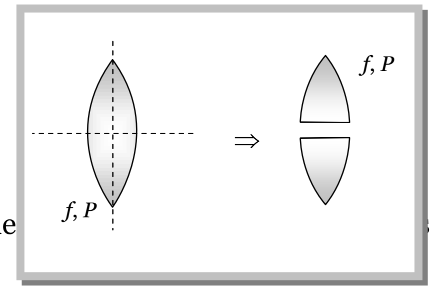
(iii) Size of object $O = \sqrt{I_1 . I_2}$

(10) **Cutting of lens**

(i) A symmetric lens is cut along optical axis in two equal parts. Intensity of image formed by each part will be same as that of complete lens.

(ii) A symmetric lens is cut along principle axis in two equal parts. Intensity of image formed by each part will be less compared as that of complete lens.(aperture of each part is $\frac{1}{\sqrt{2}}$ times that of complete lens)





(11) **Combina**

(i) For a systen cal le follows :

$P = P_1 + P_2 + P_3 \ldots\ldots\ldots$, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots\ldots$,

$m = m_1 \times m_2 \times m_3 \times \ldots\ldots\ldots$ ..

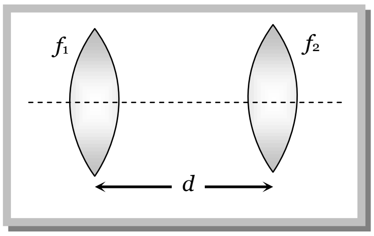
(ii) In case when two thin lens are in contact : Combination will behave as a lens, which have more power or lesser focal length.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow F = \frac{f_1 f_2}{f_1 + f_2} \quad \text{and} \quad P = P_1 + P_2$$

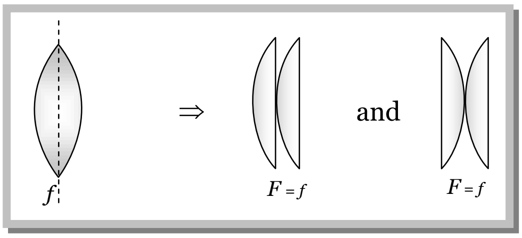
(iii) If two lens of equal focal length but of opposite nature are in contact then combination will behave as a plane glass plate and $F_{combination} = \infty$

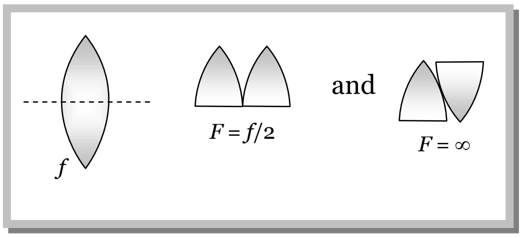
(iv) When two lenses are placed co-axially at a distance d from each other then equivalent focal length (F).

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad \text{and} \quad P = P_1 + P_2 - dP_1 P_2$$



(v) Combination of parts of a lens :



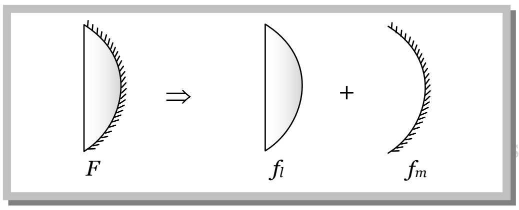


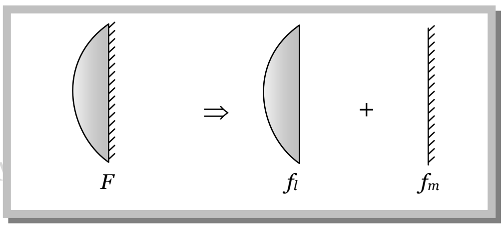
(12) **Silvering of lens**

On silvering the surface of the lens it behaves as a mirror. The focal length of the silvered lens is $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$ where f_l = focal length of lens from which refraction takes place (twice)

f_m = focal length of mirror from which reflection takes place.

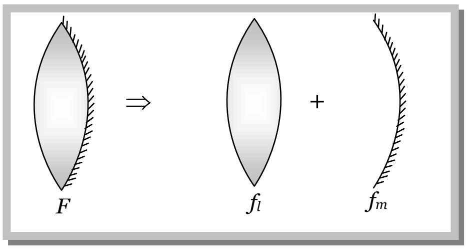
(i) Plano convex is silvered





$f_m = \frac{R}{2},\ f_l = \frac{R}{(\mu - 1)}$ so $F = \frac{R}{2\mu}$ $\qquad$ $f_m = \infty,\ f_l = \frac{R}{(\mu - 1)}$ so $F = \frac{R}{2(\mu - 1)}$

(ii) Double convex lens is silvered



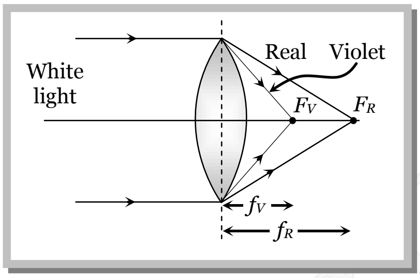
Since $f_l = \frac{R}{2(\mu - 1)},\ f_m = \frac{R}{2}$

So $F = \frac{R}{2(2\mu - 1)}$

Note : ❑ Similar results can be obtained for concave lenses.

(13) **Defects in lens**

(i) **Chromatic aberration :** Image of a white object is coloured and blurred because μ (hence f) of lens is different for different colours. This defect is called chromatic aberration.



$\mu_V > \mu_R$ so $f_R > f_V$

Mathematically chromatic aberration $= f_R - f_V = \omega f_y$

ω = Dispersion power of lens.

f_y = Focal length for mean colour $= \sqrt{f_R f_V}$

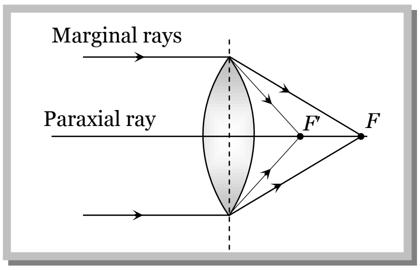
Removal : To remove this defect *i.e.* for Achromatism we use two or more lenses in contact in place of single lens.

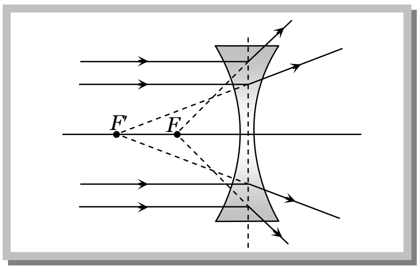
Mathematically condition of Achromatism is : $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ or $\omega_1 f_2 = -\omega_2 f_1$

Note : ❑ Component lenses of an achromatic doublet cemented by canada blasam because it is transparent and has a refractive index almost equal to the refractive of the glass.

(ii) **Spherical aberration :** Inability of a lens to form the point image of a point object on the axis is called Spherical aberration.

In this defect all the rays passing through a lens are not focussed at a single point and the image of a point object on the axis is blurred.





Removal : A simple method to reduce spherical aberration is to use a stop before and infront of the lens. (but this method reduces the intensity of the image as most of the light is cut off). Also by using plano-convex lens, using two lenses separated by distance $d = F - F'$, using crossed lens.

Note : ❑ Marginal rays : The rays farthest from the principal axis.

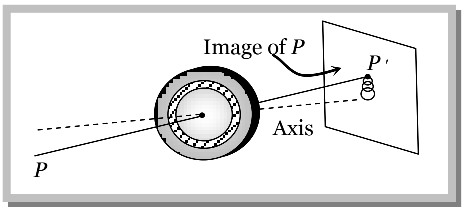
Paraxial rays : The rays close to the principal axis.

❑ Spherical aberration can be reduced by either stopping paraxial rays or marginal rays, which can be done by using a circular annular mask over the lens.

❑ Parabolic mirrors are free from spherical aberration.

(iii) **Coma :** When the point object is placed away from the principle axis and the image is received on a screen perpendicular to the axis, the shape of the image is like a comet. This defect is called Coma.

It refers to spreading of a point object in a plane ⊥ to principle axis.

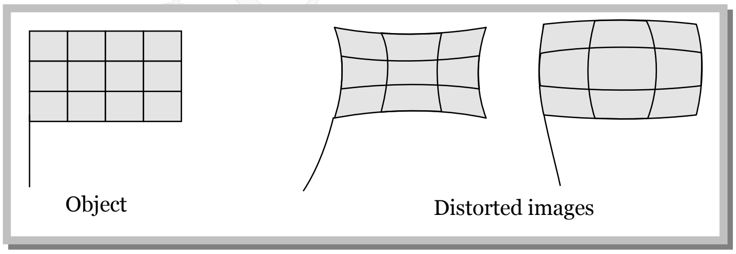


Removal : It can be reduced by properly designing radii of curvature of the lens surfaces. It can also be reduced by appropriate stops placed at appropriate distances from the lens.

(iv) **Curvature :** For a point object placed off the axis, the image is spread both along and perpendicular to the principal axis. The best image is, in general, obtained not on a plane but on a curved surface. This defect is known as Curvature.

Removal : Astigmatism or the curvature may be reduced by using proper stops placed at proper locations along the axis.

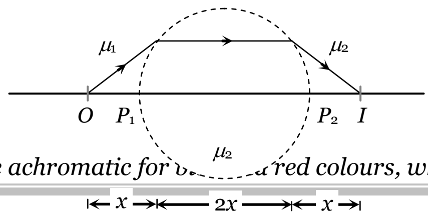
(v) **Distortion :** When extended objects are imaged, different portions of the object are in general at different distances from the axis. The magnification is not the same for all portions of the extended object. As a result a line object is not imaged into a line but into a curve.



(vi) **Astigmatism :** The spreading of image (of a point object placed away from the principal axis) along the principal axis is called Astigmatism.

Concepts

☞ *If a sphere of radius R made of material of refractive index* μ_2 *is placed in a medium of refractive index* μ_1 *, Then if the object is placed at a distance* $\left(\frac{\mu_1}{\mu_2-\mu_1}\right)R$ *from the pole, the real image formed is equidistant from the sphere.*



☞ *The lens doublets used in telescope are achromatic for ... red colours, while these used in camera are achromatic for*

μ_1

violet and green colours. The reason for this is that our eye is most sensitive between blue and red colours, while the photographic plates are most sensitive between violet and green colours.

☞ ***Position of optical centre***

Equiconvex and equiconcave	*Exactly at centre of lens*
Convexo-concave and concavo-convex	*Outside the glass position*
Plano convex and plano concave	*On the pole of curved surface*

☞ ***Composite lens :*** *If a lens is made of several materials then*

Number of images formed = Number of materials used

Here no. of images = 5

Example

Example: 18 A thin lens focal length f_1 and its aperture has diameter *d*. It forms an image of intensity *I*. Now the central part of the aperture upto diameter *d*/2 is blocked by an opaque paper. The focal length and image intensity will change to

(a) $\frac{f}{2}$ and $\frac{I}{2}$ (b) f and $\frac{I}{4}$ (c) $\frac{3f}{4}$ and $\frac{I}{2}$ (d) f and $\frac{3I}{4}$

Solution: (d) Centre part of the aperture up to diameter $\frac{d}{2}$ is blocked *i.e.* $\frac{1}{4}th$ area is blocked $\left(A=\frac{\pi d^2}{4}\right)$. Hence remaining area $A'=\frac{3}{4}A$. Also, we know that intensity ∝ Area ⇒ $\frac{I'}{I}=\frac{A'}{A}=\frac{3}{4}$ ⇒ $I'=\frac{3}{4}I$.

Focal length doesn't depend upon aperture.

Example: 19 The power of a thin convex lens $({}_a\mu_g=1.5)$ is + 5.0 *D*. When it is placed in a liquid of refractive index ${}_a\mu_l$, then it behaves as a concave lens of local length 100 *cm*. The refractive index of the liquid ${}_a\mu_l$ will be

(a) $5/3$ (b) $4/3$ (c) $\sqrt{3}$ (d) $5/4$

Solution: (a) By using $\frac{f_l}{f_a}=\frac{{}_a\mu_g-1}{{}_l\mu_g-1}$; where ${}_l\mu_g=\frac{\mu_g}{\mu_l}=\frac{1.5}{\mu_l}$ and $f_a=\frac{1}{P}=\frac{1}{5}m=20\,cm$

$$\Rightarrow \frac{-100}{20}=\frac{1.5-1}{\frac{1.5}{\mu_l}-1} \Rightarrow \mu_l=5/3$$

Example: 20 A double convex lens made of a material of refractive index 1.5 and having a focal length of 10 *cm* is immersed in liquid of refractive index 3.0. The lens will behave as **[NCERT 1973]**

(a) Diverging lens of focal length 10 *cm* (b) Diverging lens of focal length 10 / 3 *cm*

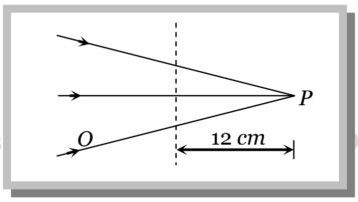
(c) Converging lens of focal length 10 / 3 *cm* (d) Converging lens of focal length 30 *cm*

Solution: (a) By using $\frac{f_l}{f_a}=\frac{{}_a\mu_g-1}{{}_l\mu_g-1} \Rightarrow \frac{f_l}{+10}=\frac{1.5-1}{\frac{1.5}{3}-1} \Rightarrow f_l=-10\,cm$ (*i.e.* diverging lens)

Example: 21 Figure given below shows a beam of light converging at point *P*. When a concave lens of focal length 16 *cm* is introduced in the path of the beam at a place *O* shown by dotted line, the beam converges at a distance *x* from the lens. The value *x* will be equal to

(a) 12 *cm*

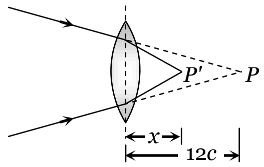
(b) 24 *cm*



(c) 36 *cm*

(d) 48 *cm*

Solution: (d) From the figure shown it is clear that



For lens : $u = 12$ *cm* and $v = x = ?$

By using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\Rightarrow \frac{1}{+16} = \frac{1}{x} - \frac{1}{+12}$ $\Rightarrow x = 48\ cm.$

***Example*: 22** A convex lens of focal length 40 *cm* is an contact with a concave lens of focal length 25 *cm*. The power of combination is

(a) $-1.5\ D$ (b) $-6.5\ D$ (c) $+6.5\ D$ (d) $+6.67\ D$

Solution: (a) By using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ $\Rightarrow \frac{1}{F} = \frac{1}{+40} + \frac{1}{-25}$

$\Rightarrow F = -\frac{200}{3} cm$, hence $P = \frac{100}{f(cm)} = \frac{100}{-200/3} = -1.5\ D$

***Example*: 23** A combination of two thin lenses with focal lengths f_1 and f_2 respectively forms an image of distant object at distance 60 *cm* when lenses are in contact. The position of this image shifts by 30 *cm* towards the combination when two lenses are separated by 10 *cm*. The corresponding values of f_1 and f_2 are **[AIIMS 1995]**

(a) 30 *cm*, – 60 *cm* (b) 20 *cm*, – 30 *cm* (c) 15 *cm*, – 20 *cm* (d) 12 *cm*, – 15 *cm*

Solution: (b) Initially $F = 60\ cm$ (Focal length of combination)

Hence by using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{60}$ $\Rightarrow \frac{f_1 f_2}{f_1 + f_2}$ ......(i)

Finally by using $\frac{1}{F'} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ where $F' = 30\,cm$ and $d = 10\ cm$ $\Rightarrow$

$\frac{1}{30} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{10}{f_1 f_2}$ ......(ii)

From equations (i) and (ii) $f_1 f_2 = -600$.

From equation (i) $f_1 + f_2 = -10$ .....(iii)

Also,difference of focal lengths can written as $f_1 - f_2 = \sqrt{(f_1 + f_2)^2 - 4 f_1 f_2}$ $\Rightarrow f_1 - f_2 = 50$.(iv)

From (iii) × (iv) $f_1 = 20$ and $f_2 = -30$

***Example*: 24** A thin double convex lens has radii of curvature each of magnitude 40 *cm* and is made of glass with refractive index 1.65. Its focal length is nearly

(a) 20 *cm* (b) 31 *cm* (c) 35 *cm* (d) 50 *cm*

Solution: (b) By using $f = \frac{R}{2(\mu - 1)}$ $\Rightarrow$ $f = \frac{40}{2(1.65 - 1)} = 30.7\,cm \approx 31\,cm.$

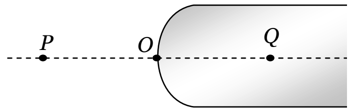
***Example*: 25** A spherical surface of radius of curvature *R* separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object *P* placed in air is found to have a real image *Q* in the glass. The line *PQ* cuts the surface at a point *O* and $PO = OQ$. The distance *PO* is equal to

[MP PMT 1994; Haryana CEE 1996]

(a) $5R$ (b) $3R$ (c) $2R$ (d) $1.5R$

Solution: (a) By using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Where $\mu_1 = 1, \quad \mu_2 = 1.5, \quad u = -OP, \quad v = OQ$



Hence $\frac{1.5}{OQ} - \frac{1}{-OP} = \frac{1.5-1}{(+R)} \Rightarrow \frac{1.5}{OP} + \frac{1}{OP} = \frac{0.5}{R}$

$\Rightarrow OP = 5R$

***Example*: 26** The distance between an object and the screen is 100 *cm*. A lens produces an image on the screen when placed at either of the positions 40 *cm* apart. The power of the lens is

(a) $3D$ (b) $5D$ (c) $7D$ (d) $9D$

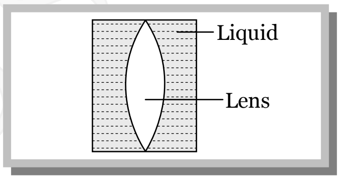
Solution: (b) By using $f = \frac{D^2 - x^2}{4D} \Rightarrow f = \frac{100^2 - 40^2}{4 \times 100} = 21\,cm$

Hence power $P = \frac{100}{F(cm)} = \frac{100}{21} \approx +5D$

***Example*: 27** Shown in figure here is a convergent lens placed inside a cell filled with a liquid. The lens has focal length +20 *cm* when in air and its material has refractive index 1.50. If the liquid has refractive index 1.60, the focal length of the system is

(a) + 80 *cm* (b) – 80 *cm*

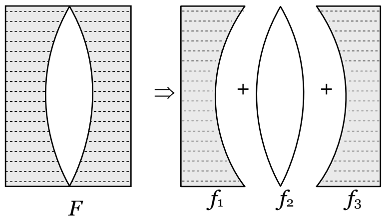
(c) – 24 *cm* (d) – 100 *cm*



Solution: (d) Here $\frac{1}{f_1} = (1.6-1)\left(\frac{1}{\infty} - \frac{1}{20}\right) = \frac{-3}{100}$ ....(i)

$\frac{1}{f_2} = (1.5-1)\left(\frac{1}{20} - \frac{1}{-20}\right) = \frac{1}{20}$ ....(ii)

$\frac{1}{f_3} = (1.6-1)\left(\frac{1}{-20} - \frac{1}{\infty}\right) = \frac{-3}{100}$ ....(iii)



By using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \Rightarrow \frac{1}{F} = \frac{-3}{100} + \frac{1}{20} - \frac{3}{100} \Rightarrow F = -100\,cm$

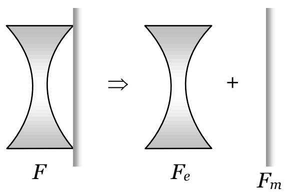
***Example*: 28** A concave lens of focal length 20 *cm* placed in contact with a plane mirror acts as a **RA 1998]**

(a) Convex mirror of focal length 10 *cm* (b) Concave mirror of focal length 40 *cm*

(c) Concave mirror of focal length 60 *cm* (d) Concave mirror of focal length 10 *cm*

Solution: (a) By using $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$

Since $f_m = \infty \Rightarrow F = \frac{f_l}{2} = \frac{20}{2} = 10\,cm$



(After silvering concave lens behave as convex mirror)

***Example*: 29** A candle placed 25 *cm* from a lens, forms an image on a screen placed 75 *cm* on the other end of the lens. The focal length and type of the lens should be

(a) + 18.75 *cm* and convex lens (b) – 18.75 *cm* and concave lens

(c) + 20.25 *cm* and convex lens (d) – 20.25 *cm* and concave lens

Solution: (a) In concave lens, image is always formed on the same side of the object. Hence the given lens is a convex lens for which $u = -25\,cm, \quad v = 75\,cm.$

By using $\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{f}=\frac{1}{(+75)}-\frac{1}{(-25)} \Rightarrow f=+18.75\ cm.$

***Example*: 30** A convex lens forms a real image of an object for its two different positions on a screen. If height of the image in both the cases be 8 *cm* and 2 *cm*, then height of the object is **[KCET (Engg./Med.) 2**

(a) 16 *cm* (b) 8 *cm* (c) 4 *cm* (d) 2 *cm*

Solution: (c) By using $O=\sqrt{I_1 I_2} \Rightarrow O=\sqrt{8\times 2}=4\ cm$

***Example*: 31** A convex lens produces a real image *m* times the size of the object. What will be the distance of the object from the lens **[JIPMER 2002]**

(a) $\left(\frac{m+1}{m}\right)f$ (b) $(m-1)f$ (c) $\left(\frac{m-1}{m}\right)f$ (d) $\frac{m+1}{f}$

Solution: (a) By using $m=\frac{f}{f+u}$ here $-m=\frac{(+f)}{(+f)+u} \Rightarrow -\frac{1}{m}=\frac{f+u}{f}=1+\frac{u}{f} \Rightarrow u=-\left(\frac{m+1}{m}\right).f$

***Example*: 32** An air bubble in a glass sphere having 4 *cm* diameter appears 1 *cm* from surface nearest to eye when looked along diameter. If ${}_a\mu_g=1.5$, the distance of bubble from refracting surface is **[CPMT 2002]**

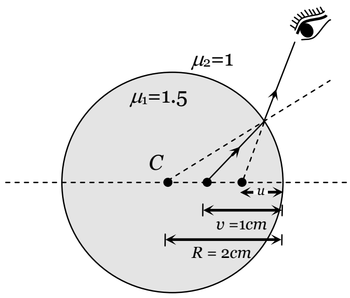
(a) 1.2 *cm* (b) 3.2 *cm* (c) 2.8 *cm* (d) 1.6 *cm*

Solution: (a) By using

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

where $u=?$, $v=-1\ cm$, $\mu_1=1.5$, $\mu_2=1$, $R=-2\ cm.$

$$\frac{1}{-1}-\frac{1.5}{u}=\frac{1-1.5}{(-2)} \Rightarrow u=-\frac{6}{5}=-1.2\ cm.$$

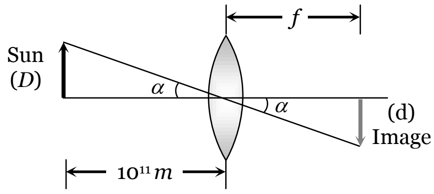


***Example*: 33** The sun's diameter is $1.4\times 10^9\ m$ and its distance from the earth is $10^{11} m$. The diameter of its image, formed by a convex lens of focal length 2*m* will be

(a) 0.7 *cm* (b) 1.4 *cm* (c) 2.8 *cm* (d) Zero (*i.e.* point image)

Solution: (c) From figure

$$\frac{D}{d}=\frac{10^{11}}{2} \Rightarrow d=\frac{2\times 1.4\times 10^9}{10^{11}}=2.8\ cm.$$



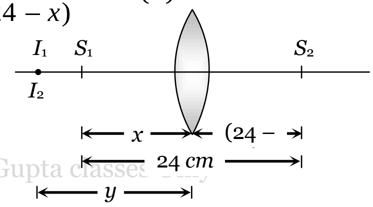
***Example*: 34** Two point light sources are 24 *cm* apart. Where should a convex lens of focal length 9 *cm* be put in between them from one source so that the images of both the sources are formed at the same place

(a) 6 *cm* (b) 9 *cm* (c) 12 *cm* (d) 15 *cm*

Solution: (a) The given condition will be satisfied only if one source (S_1) placed on one side such that $u < f$ (*i.e.* it lies under the focus). The other source (S_2) is placed on the other side of the lens such that $u > f$ (*i.e.* it lies beyond the focus).

If S_1 is the object for lens then $\frac{1}{f}=\frac{1}{-y}-\frac{1}{-x} \Rightarrow \frac{1}{y}=\frac{1}{x}-\frac{1}{f}$ ....(i)

If S_2 is the object for lens then $\frac{1}{f}=\frac{1}{+y}-\frac{1}{-(24-x)} \Rightarrow \frac{1}{y}=\frac{1}{f}-\frac{1}{(24-x)}$ ....(ii)



From equation (i) and (ii)

$$\frac{1}{x}-\frac{1}{f}=\frac{1}{f}-\frac{1}{(24-x)} \Rightarrow \frac{1}{x}+\frac{1}{(24-x)}=\frac{2}{f}=\frac{2}{9} \Rightarrow x^2-24x+108=0$$

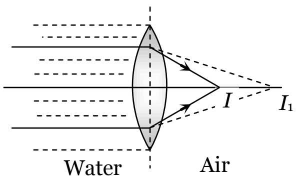
On solving the equation $x = 18\ cm$, $6\ cm$

***Example*: 35** There is an equiconvex glass lens with radius of each face as R and ${}_a\mu_g = 3/2$ and ${}_a\mu_w = 4/3$. If there is water in object space and air in image space, then the focal length is

(a) $2R$ (b) R (c) $3R/2$ (d) R^2

Solution: (c) Consider the refraction of the first surface *i.e.* refraction from rarer medium to denser medium

$$\frac{\mu_2-\mu_1}{R}=\frac{\mu_1}{-u}+\frac{\mu_2}{v_1} \Rightarrow \frac{\left(\frac{3}{2}\right)-\left(\frac{4}{3}\right)}{R}=\frac{\frac{4}{3}}{\infty}+\frac{\frac{3}{2}}{v_1} \Rightarrow v_1 = 9R$$



Now consider the refraction at the second surface of the lens *i.e.* refraction from denser medium to rarer medium

$$\frac{1-\frac{3}{2}}{-R}=-\frac{\frac{3}{2}}{9R}+\frac{1}{v_2} \Rightarrow v_2=\left(\frac{3}{2}\right)R$$

The image will be formed at a distance do $\frac{3}{2}R$. This is equal to the focal length of the lens.

Tricky example: 4

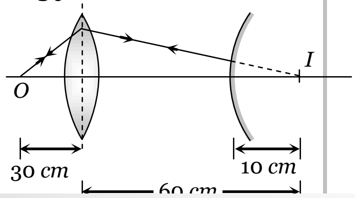
A luminous object is placed at a distance of 30 *cm* from the convex lens of focal length 20 *cm*. On the other side of the lens. At what distance from the lens a convex mirror of radius of curvature 10 *cm* be placed in order to have an upright image of the object coincident with it

[CBSE PMT 1998; JIPMER 2001, 2002]

(a) 12 *cm* (b) 30 *cm* (c) 50 *cm* (d) 60 *cm*

Solution : (c) For lens $u = 30\ cm$, $f = 20\ cm$, hence by using $\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{+20}=\frac{1}{v}-\frac{1}{-30} \Rightarrow v = 60\ cm$

The final image will coincide the object, if light ray falls normally on convex mirror as shown. From figure it is seen clear that reparation between lens and mirror is 60 – 10 = 50 *cm*.



Tricky example: 5

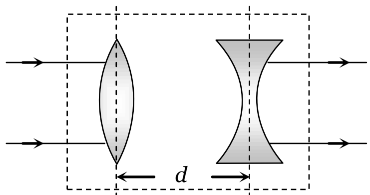
A convex lens of local length 30 *cm* and a concave lens of 10 *cm* focal length are placed so as to have the same axis. If a parallel beam of light falling on convex lens leaves concave lens as a parallel beam, then the distance between two lenses will be

(a) 40 *cm* (b) 30 *cm* (c) 20 *cm* (d) 10 *cm*

Solution : (c) According to figure the combination behaves as plane glass plate (*i.e., F*= ∞)

Hence by using $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$

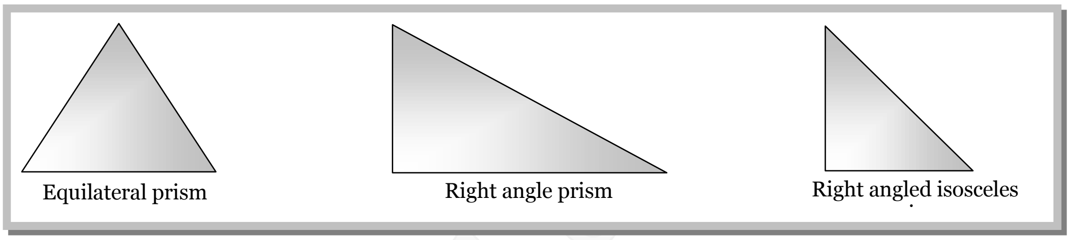
$$\Rightarrow \frac{1}{\infty}=\frac{1}{+30}+\frac{1}{-10}-\frac{d}{(30)(-10)} \Rightarrow d = 20\ cm$$



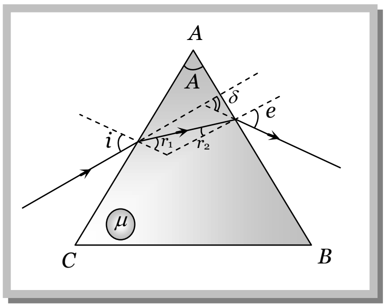
Prism

Prism is a transparent medium bounded by refracting surfaces, such that the incident surface (on which light ray is incidenting) and emergent surface (from which light rays emerges) are plane and non parallel.

Commonly used prism :



(1) **Refraction through a prism**



$A = r_1 + r_2$ and $i + e = A + \delta$

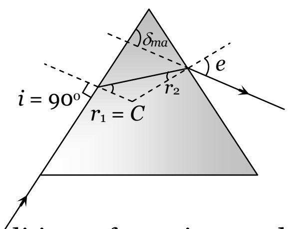
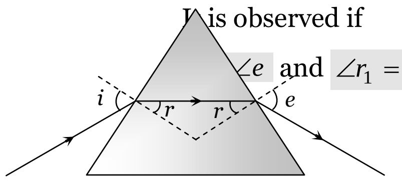
For surface AC $\mu = \frac{\sin i}{\sin r_1}$;

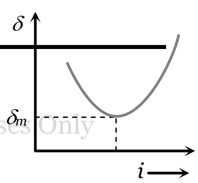
i – Angle of incidence, e – Angle of emergence, A – Angle of prism or refracting angle of prism, r_1 and r_2 – Angle of refraction, δ – Angle of

For surface AB $\mu = \frac{\sin r_2}{\sin e}$

(2) **Deviation through a prism**

For thin prism $\delta = (\mu - 1)A$. Also deviation is different for different colour light *e.g.* $\mu_R < \mu_V$ so $\delta_R < \delta_V$. And $\mu_{\text{Flint}} > \mu_{\text{Crown}}$ so $\delta_F > \delta_C$

Maximum deviation	**Minimum deviation**
 In this condition of maximum deviation $\angle i = 90^o$, $r_1 = C$, $r_2 = A - C$ and from Snell's law on emergent surface	 is observed if ∠e and $\angle r_1 = \angle r_2 = r$ (i) Refracted ray inside the prism is parallel to the base of the prism

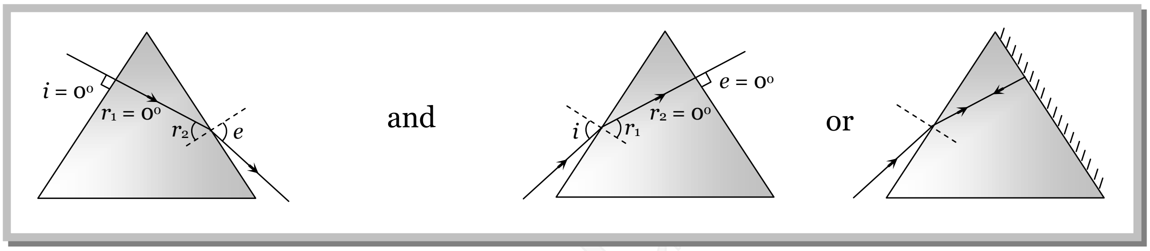


$e = \sin^{-1}\left[\dfrac{\sin(A-C)}{\sin C}\right]$	(ii) $r = \dfrac{A}{2}$ and $i = \dfrac{A+\delta_m}{2}$ (iii) $\mu = \dfrac{\sin i}{\sin A/2}$ or $\mu = \dfrac{\sin \dfrac{A+\delta_m}{2}}{\sin A/2}$

Note : ❑ If $\delta_m = A$ then $\mu = 2\cos A/2$

(3) **Normal incidence on a prism**

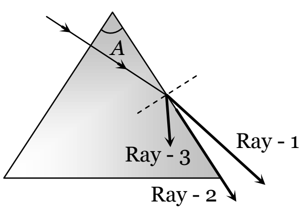
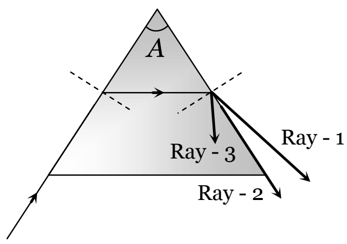
If light ray incident normally on any surface of prism as shown



In any of the above case use $\mu = \dfrac{\sin i}{\sin A}$ and $\delta = i - A$

(4) **Grazing emergence and TIR through a prism**

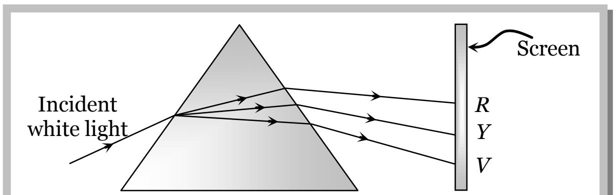
When a light ray falls on one surface of prism, it is not necessary that it will exit out from the prism. It may or may not be exit out as shown below

Normal incidence	**Grazing incidence**
 Ray –1 : General emergence $A < C$ and $\mu <$ cosec A Ray – 2: Grazing emergence $A = C$ and $\mu =$ cosec A Ray – 3: TIR $A > C$ and $\mu >$ cosec A A = angle of prism and C = Critical angle for the material of the prism	 Ray –1 : General emergence $A < 2C$ and $\mu <$ cosec $(A/2)$ Ray – 2: Grazing emergence $A = 2C$ and $\mu =$ cosec $(A/2)$ Ray – 3: TIR $A > 2C$ and $\mu >$ cosec $(A/2)$

Note : ❑ For the condition of grazing emergence. Minimum angle of incidence $i_{min} = \sin^{-1}\left[\sqrt{\mu^2 - 1}\,\sin A - \cos A\right]$.

(5) **Dispersion through a prism**

The splitting of white light into it's constituent colours is called dispersion of light.



(i) Angular dispersion (θ) : Angular separation between extreme colours *i.e.* $\boldsymbol{\theta = \delta_V - \delta_R = (\mu_V - \mu_R)A}$. It depends upon μ and A.

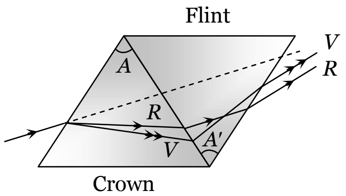
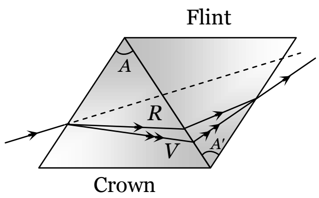
(ii) Dispersive power (ω) : $\omega = \dfrac{\theta}{\delta_y} = \dfrac{\mu_V - \mu_R}{\mu_y - 1}$ wher e $\left\{\mu_y = \dfrac{\mu_V + \mu_R}{2}\right\}$

$\Rightarrow$ It depends only upon the material of the prism *i.e.* μ and it doesn't depends upon angle of prism A

Note : ❑ Remember $\omega_{\text{Flint}} > \omega_{\text{Crown}}$.

(6) **Combination of prisms**

Two prisms (made of crown and flint material) are combined to get either dispersion only or deviation only.

Dispersion without deviation (chromatic combination)	Deviation without dispersion (Achromatic combination)
	
(i) $\dfrac{A'}{A} = -\dfrac{(\mu_y - 1)}{(\mu'_y - 1)}$	(i) $\dfrac{A'}{A} = -\dfrac{(\mu_V - \mu_R)}{(\mu'_V - \mu'_R)}$
(ii) $\theta_{\text{net}} = \theta\left(1 - \dfrac{\omega'}{\omega}\right) = (\omega\delta - \omega'\delta')$	(ii) $\delta_{\text{net}} = \delta\left(1 - \dfrac{\omega}{\omega'}\right)$

Scattering of Light

Molecules of a medium after absorbing incoming light radiations, emits them in all direction. This phenomenon is called Scattering.

(1) **According to scientist Rayleigh :** Intensity of scattered light $\propto \dfrac{1}{\lambda^4}$

(2) **Some phenomenon based on scattering :** (i) Sky looks blue due to scattering.

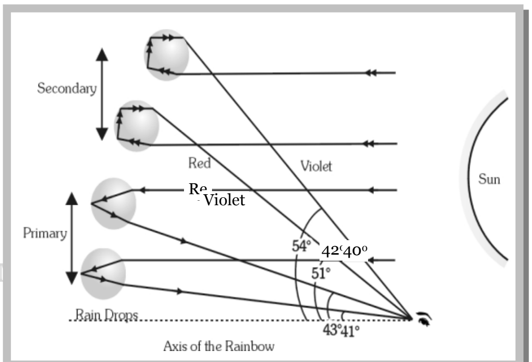
(ii) At the time of sunrise or sunset it looks reddish. (iii) Danger signals are made from red.

(3) **Elastic scattering :** When the wavelength of radiation remains unchanged, the scattering is called elastic.

(4) **Inelastic scattering (Raman's effect) :** Under specific condition, light can also suffer inelastic scattering from molecules in which it's wavelength changes.

Rainbow

Rainbow is formed due to the dispersion of light suffering refraction and TIR in the droplets present in the atmosphere.



(1) **Primary rainbow :** (i) Two refraction and one TIR. (ii) Innermost arc is violet and outermost is red. (iii) Subtends an angle of 42^{o} at the eye of the observer. (iv) More bright

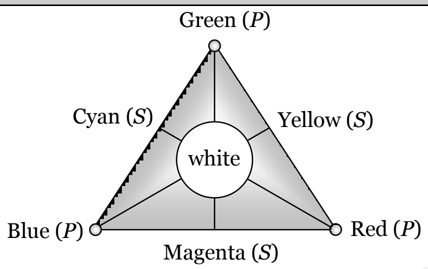
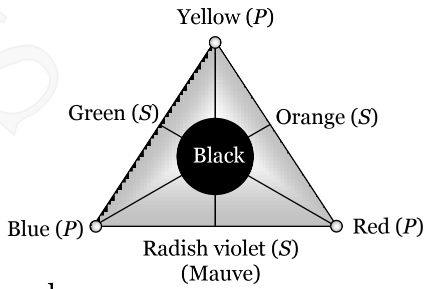
(2) **Secondary rainbow :** (i) Two refraction and two TIR. (ii) Innermost arc is red and outermost is violet.

(iii) It subtends an angle of 52.5^{o} at the eye. (iv) Comparatively less bright.

Colours

Colour is defined as the sensation received by the eye (cone cells of the eye) due to light coming from object.

(1) **Types of colours**

Spectral colours	**Colours of pigment and dyes**
 	
(i) Complementary colours : Green and magenta Blue and yellow Red and cyan (ii) Combination : Green + red + blue = White Blue + yellow = White Red + cyan = White Green + magenta = White	(i) Complementary colours : yellow and mauve Red and green Blue and orange (ii) Combination : Yellow + red + blue = Black Blue + orange = Black Red + green = Black Yellow + mauve = Black

(2) **Colours of object :** The perception of a colour by eye depends on the nature of object and the light incident on it.

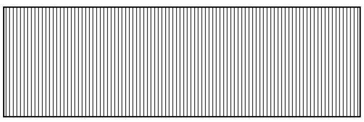
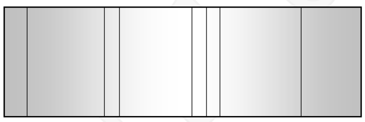
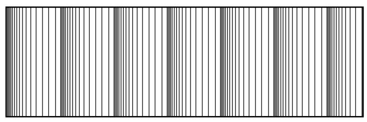
Colours of opaque object	**Colours of transparent object**
(i) Due to selective reflection.	(i) Due to selective transmission.
(ii) A rose appears red in white light because it reflects red colour and absorbs all remaining colours.	(ii) A red glass appears red because it absorbs all colours, except red which it transmits.
(iii) When yellow light falls on a bunch of flowers, then yellow and white flowers looks yellow. Other flowers looks black.	(iii) When we look on objects through a green glass or green filter then green and white objects will appear green while other black.

Note : A hot object will emit light of that colour only which it has observed when it was heated.

Spectrum

The ordered arrangements of radiations according to wavelengths or frequencies is called Spectrum. Spectrum can be divided in two parts (I) Emission spectrum and (II) Absorption spectrum.

(1) **Emission spectrum :** When light emitted by a self luminous object is dispersed by a prism to get the spectrum, the spectrum is called emission spectra.

Continuous emission spectrum	Line emission spectrum	Band emission spectrum
(i) It consists of continuously varying wavelengths in a definite wavelength range.	(i) It consist of distinct bright lines.	(iii) It consist of district bright bands.
(ii) It is produced by solids, liquids and highly compressed gases heated to high temperature.	(ii) It is produced by an excited source in atomic state.	(ii) It is produced by an excited source in molecular state.
(iii) *e.g.* Light from the sun, filament of incandescent bulb, candle flame *etc.*	(iii) *e.g.* Spectrum of excited helium, mercury vapours, sodium vapours or atomic hydrogen.	(iii) *e.g.* Spectra of molecular H_2, CO, NH_3 etc.
		

(2) **Absorption spectrum :** When white light passes through a semi-transparent solid, or liquid or gas, it's spectrum contains certain dark lines or bands, such spectrum is called absorption spectrum (of the substance through which light is passed).

(i) Substances in atomic state produces line absorption spectra. Polyatomic substances such as H_2, CO_2 and $KMnO_4$ produces band absorption spectrum.

(ii) Absorption spectra of sodium vapour have two (yellow lines) wavelengths $D_1(5890\ Å)$ and $D_2(5896\ Å)$

Note : ❑ If a substance emits spectral lines at high temperature then it absorbs the same lines at low temperature. This is Kirchoff's law.

(3) **Fraunhoffer's lines :** The central part (photosphere) of the sun is very hot and emits all possible wavelengths of the visible light. However, the outer part (chromosphere) consists of vapours of different elements. When the light emitted from the photosphere passes through the chromosphere, certain wavelengths are absorbed. Hence, in the spectrum of sunlight a large number of dark lines are seen called Fraunhoffer lines.

(i) The prominent lines in the yellow part of the visible spectrum were labelled as *D*-lines, those in blue part as *F*-lines and in red part as *C*-line.

(ii) From the study of Fraunhoffer's lines the presence of various elements in the sun's atmosphere can be identified *e.g.* abundance of hydrogen and helium.

(4) **Spectrometer :** A spectrometer is used for obtaining pure spectrum of a source in laboratory and calculation of μ of material of prism and μ of a transparent liquid.

It consists of three parts : Collimator which provides a parallel beam of light; Prism Table for holding the prism and Telescope for observing the spectrum and making measurements on it.

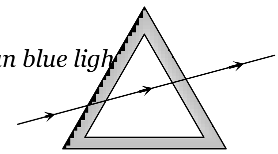
The telescope is first set for parallel rays and then collimator is set for parallel rays. When prism is set in minimum deviation position, the spectrum seen is pure spectrum. Angle of prism (A) and angle of minimum deviation (δ_m) are measured and μ of material of prism is calculated using prism formula. For μ of a transparent liquid, we take a hollow prism with thin glass sides. Fill it with the liquid and measure (δ_m) and A of liquid prism. μ of liquid is calculated using prism formula.

(5) **Direct vision spectroscope :** It is an instrument used to observe pure spectrum. It produces dispersion without deviation with the help of n crown prisms and $(n-1)$ flint prisms alternately arranged in a tabular structure.

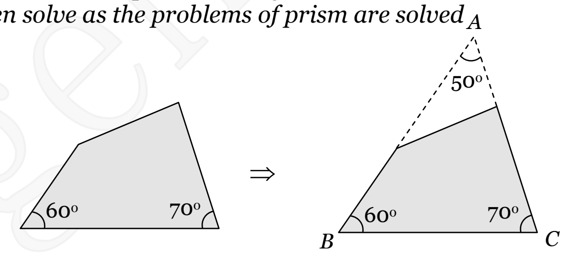
For no deviation $n(\mu-1)A = (n-1)(\mu'-1)A'$.

Concepts

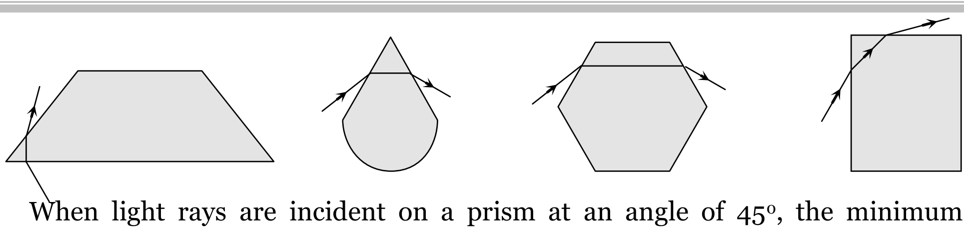
- *When a ray of white light passes through a glass prism red light is deviated less than blue light*
- *For a hollow prism $A \neq 0$ but $\delta = 0$*



- *If an opaque coloured object or crystal is crushed to fine powder it will appear white (in sun light) as it will lose it's property of selective reflection.*
- *Our eye is most sensitive to that part at the spectrum which lies between the F line (sky green) one the C-line (red) of hydrogen equal to the refractive index for the D line (yellow) of sodium. Hence for the dispersive power, the following formula is internationally accepted* $\omega = \dfrac{\mu_F - \mu_C}{\mu_D - 1}$
- *Sometimes a part of prism is given and we keep on thinking whether how should we proceed ? To solve such problems first complete the prism then solve as the problems of prism are solved*



- *Some other types of prism*



***Example*: 36** When light rays are incident on a prism at an angle of 45°, the minimum deviation is obtained. If refractive index of the material of prism is $\sqrt{2}$, then the angle of prism will be

(a) 30° (b) 40° (c) 50° (d) 60°

Solution: (d) $\mu = \dfrac{\sin i}{\sin \dfrac{A}{2}} \Rightarrow \sqrt{2} = \dfrac{\sin 45}{\sin \dfrac{A}{2}} \Rightarrow \sin \dfrac{A}{2} = \dfrac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \dfrac{1}{2} \Rightarrow \dfrac{A}{2} = 30^o \Rightarrow A = 60^o$

***Example*: 37** Angle of minimum deviation for a prism of refractive index 1.5 is equal to the angle of prism. The angle of prism is $(\cos 41^o = 0.75)$

(a) 62° (b) 41° (c) 82° (d) 31°

Solution: (c) Given $\delta_m = A$, then by using $\mu = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}} \Rightarrow \mu = \dfrac{\sin\dfrac{A+A}{2}}{\sin\dfrac{A}{2}} = \dfrac{\sin A}{\sin\dfrac{A}{2}} = 2\cos\dfrac{A}{2}$

$\left\{\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2}\right\}$

$\Rightarrow 1.5 = 2\cos\dfrac{A}{2} \Rightarrow 0.75 = \cos\dfrac{A}{2} \Rightarrow 41^o = \dfrac{A}{2} \Rightarrow A = 82^o$.

Example: 38 Angle of glass prism is 60° and refractive index of the material of the prism is 1.414,then what will be the angle of incidence, so that ray should pass symmetrically through prism

(a) 38° 61' (b) 35° 35' (c) 45° (d) 53° 8'

Solution: (c) incident ray and emergent ray are symmetrical in the cure, when prism is in minimum deviation position.

Hence in this condition $\mu = \dfrac{\sin i}{\sin\dfrac{A}{2}} \Rightarrow \sin i = \mu\sin\left(\dfrac{A}{2}\right) \Rightarrow \sin i = 1.414 \times \sin 30^o = \dfrac{1}{\sqrt{2}} \Rightarrow i = 45^o$

Example: 39 A prism $(\mu = 1.5)$ has the refracting angle of 30°. The deviation of a monochromatic ray incident normally on its one surface will be $(\sin 48^o 36' = 0.75)$

(a) 18° 36' (b) 20° 30' (c) 18° (d) 22° 1'

Solution: (a) By using $\mu = \dfrac{\sin i}{\sin A} \Rightarrow 1.5 = \dfrac{\sin i}{\sin 30} \Rightarrow \sin i = 0.75 \Rightarrow i = 48^o 36'$

Also from $\delta = i - A \Rightarrow \delta = 48^o 36' - 30^o = 18^o 36'$

Example: 40 Angle of a prism is 30° and its refractive index is $\sqrt{2}$ and one of the surface is silvered. At what angle of incidence, a ray should be incident on one surface so that after reflection from the silvered surface, it retraces its path

(a) 30° (b) 60° (c) 45° (d) $\sin^{-1}\sqrt{1.5}$

Solution: (c) This is the case when light ray is falling normally an second surface.

Hence by using $\mu = \dfrac{\sin i}{\sin A} \Rightarrow \sqrt{2} = \dfrac{\sin i}{\sin 30^o} \Rightarrow \sin i = \sqrt{2} \times \dfrac{1}{2} \Rightarrow i = 45^o$

Example: 41 The refracting angle of prism is A and refractive index of material of prism is $\cot\dfrac{A}{2}$. The angle of minimum deviation is

(a) $180^o - 3A$ (b) $180^o + 2A$ (c) $90^o - A$ (d) $180^o - 2A$

Solution: (d) By using $\mu = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}} \Rightarrow \cot\dfrac{A}{2} = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}} \Rightarrow \dfrac{\cos\dfrac{A}{2}}{\sin\dfrac{A}{2}} = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}}$

$\Rightarrow \sin\left(90 - \dfrac{A}{2}\right) = \sin\left(\dfrac{A+\delta_m}{2}\right) \Rightarrow 90 - \dfrac{A}{2} = \dfrac{A+\delta_m}{2} \Rightarrow \delta_m = 180 - 2A$

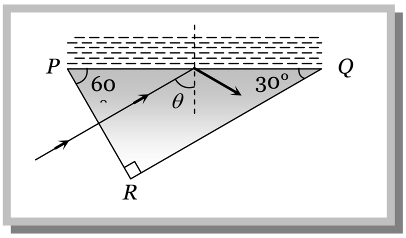
Example: 42 A ray of light passes through an equilateral glass prism in such a manner that the angle of incidence is equal to the angle of emergence and each of these angles is equal to 3/4 of the angle of the prism. The angle of deviation is

(a) 45° (b) 39° (c) 20° (d) 30°

Solution: (d) Given that $A = 60^o$ and $i = e = \frac{3}{4}A = \frac{3}{4} \times 60 = 45^o$

By using $i + e = A + \delta \Rightarrow 45 + 45 = 60 + \delta \Rightarrow \delta = 30^o$

***Example*: 43** *PQR* is a right angled prism with other angles as 60° and 30°. Refractive index of prism is 1.5. *PQ* has a thin layer of liquid. Light falls normally on the face *PR*. For total internal reflection, maximum refractive index of liquid is



(a) 1.4

(b) 1.3

(c) 1.2

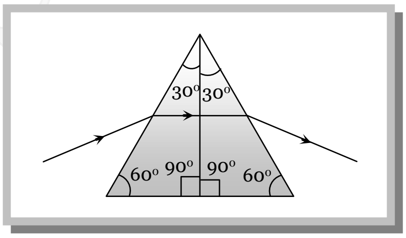
(d) 1.6

Solution: (c) For *TIR* at *PQ* $\theta < C$

From geometry of figure $\theta = 60$ *i.e.* $60 > C \Rightarrow \sin 60 > \sin C$

$\Rightarrow \frac{\sqrt{3}}{2} > \frac{\mu_{Liquid}}{\mu_{Prism}} \Rightarrow \mu_{Liquid} < \frac{\sqrt{3}}{2} \times \mu_{Prism} \Rightarrow \mu_{Liquid} < \frac{\sqrt{3}}{2} \times 1.5 \Rightarrow \mu_{Liquid} < 1.3$.

***Example*: 44** Two identical prisms 1 and 2, each will angles of 30°, 60° and 90° are placed in contact as shown in figure. A ray of light passed through the combination in the position of minimum deviation and suffers a deviation of 30°. If the prism 2 is removed, then the angle of deviation of the same ray is **[PMT (Andhra) 1995]**



(a) Equal to 15°

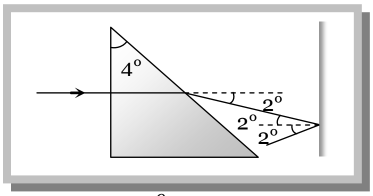
(b) Smaller than 30°

(c) More than 15°

(d) Equal to 30°

Solution: (a) $\delta = (\mu - 1)A$ as *A* is halved, so δ is also halves

***Example*: 45** A prism having an apex angle 4° and refraction index 1.5 is located in front of a vertical plane mirror as shown in figure. Through what total angle is the ray deviated after reflection from the mirror



(a) 176° (b) 4°

(c) 178° (d) 2°

Solution: (c) $\delta_{Prism} = (\mu - 1)A = (1.5 - 1)4^o = 2^o$

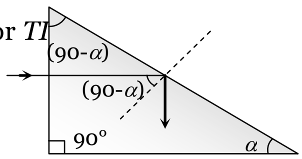
$\therefore \delta_{Total} = \delta_{Prism} + \delta_{Mirror} = (\mu - 1)A + (180 - 2i) = 2^o + (180 - 2 \times 2) = 178^o$

***Example*: 46** A ray of light is incident to the hypotenuse of a right-angled prism after travelling parallel to the base inside the prism. If μ is the refractive index of the material of the prism, the maximum value of the base angle for which light is totally reflected from the hypotenuse is **[EAMCET 2003]**

(a) $\sin^{-1}\left(\frac{1}{\mu}\right)$ (b) $\tan^{-1}\left(\frac{1}{\mu}\right)$ (c) $\sin^{-1}\left(\frac{\mu - 1}{\mu}\right)$ (d) $\cos^{-1}\left(\frac{1}{\mu}\right)$

Solution: (d) If α = maximum value of vase angle for which light is totally reflected from hypotenuse.

$(90 - \alpha) = C$ = minimum value of angle of incidence an hypotenuse for TIR



$\sin(90 - \alpha) = \sin C = \frac{1}{\mu} \Rightarrow \alpha = \cos^{-1}\left(\frac{1}{\mu}\right)$

***Example*: 47** If the refractive indices of crown glass for red, yellow and violet colours are 1.5140, 1.5170 and 1.5318 respectively and for flint glass these are 1.6434, 1.6499 and 1.6852 respectively, then the dispersive powers for crown and flint glass are respectively

A collection by Pradeep Kshetrapal for Physics students at genius, Maxwell and Gupta classes Only

(a) 0.034 and 0.064 (b) 0.064 and 0.034 (c) 1.00 and 0.064 (d) 0.034 and 1.0

Solution: (a) $\omega_{\text{Crown}} = \dfrac{\mu_v - \mu_r}{\mu_y - 1} = \dfrac{1.5318 - 1.5140}{(1.5170 - 1)} = 0.034$ and

$$\omega_{\text{Flint}} = \frac{\mu_v^{'} - \mu_r^{'}}{\mu_y^{'} - 1} = \frac{1.6852 - 1.6434}{1.6499 - 1} = 0.064$$

***Example*: 48** Flint glass prism is joined by a crown glass prism to produce dispersion without deviation. The refractive indices of these for mean rays are 1.602 and 1.500 respectively. Angle of prism of flint prism is 10^o, then the angle of prism for crown prism will be

(a) $12^o 2.4'$ (b) $12^o 4'$ (c) 1.24^o (d) 12^o

Solution: (a) For dispersion without deviation

$$\frac{A_C}{A_F} = \frac{(\mu_F - 1)}{(\mu_C - 1)} \Rightarrow \frac{A}{10} = \frac{(1.602 - 1)}{(1.500 - 1)} \Rightarrow A = 12.04^o = 12^o 2.4'$$

Tricky example: 6

An achromatic prism is made by crown glass prism $(A_C = 19^o)$ and flint glass prism $(A_F = 6^o)$. If ${}^C\mu_v = 1.5$ and ${}^F\mu_v = 1.66$, then resultant deviation for red coloured ray will be

(a) 1.04^o (b) 5^o (c) 0.96^o (d) 13.5^o

Solution : (d) For achromatic combination $w_C = -w_F \Rightarrow [(\mu_v - \mu_r)A]_C = -[(\mu_v - \mu_r)A]_F$

$\Rightarrow [\mu_r A]_C + [\mu_r A]_F = [\mu_v A]_C + [\mu_v A]_F = 1.5 \times 19 + 6 \times 1.66 = 38.5$

Resultant deviation $\delta = [(\mu_r - 1)A]_C + [(\mu_r - 1)A]_F$

$$= [\mu_r A]_C + [\mu_r A]_F - (A_C + A_F) = 38.5 - (19 + 6) = 13.5^o$$

Tricky example: 7

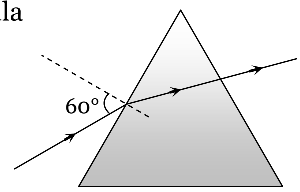
The light is incident at an angle of 60^o on a prism of which the refracting angle of prism is 30^o. The refractive index of material of prism will be

(a) $\sqrt{2}$ (b) $2\sqrt{3}$ (c) 2 (d) $\sqrt{3}$

Solution : (d) By using $i + e = A + \delta \Rightarrow 60 + e = 30 + 30 \Rightarrow e = 0$.

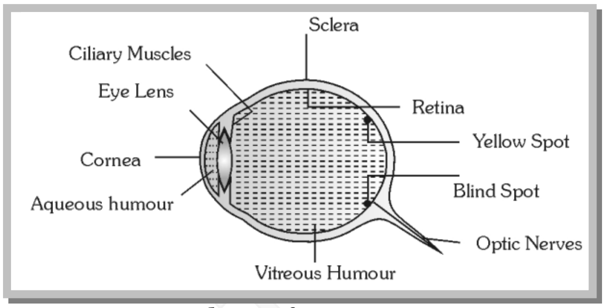
Hence ray will emerge out normally so by using the formula

$$\mu = \frac{\sin i}{\sin A} = \frac{\sin 60}{\sin 30} = \sqrt{3}$$



Optical Instruments

Human Eye



(1) **Eye lens :** Over all behaves as a convex lens of $\mu = 1.437$

(2) **Retina :** Real and inverted image of an object, obtained at retina, brain sense it erect.

(3) **Yellow spot :** It is the most sensitive part, the image formed at yellow spot is brightest.

(4) **Blind spot :** Optic nerves goes to brain through blind spot. It is not sensitive for light.

(5) **Ciliary muscles –** Eye lens is fixed between these muscles. It's both radius of curvature can be changed by applying pressure on it through ciliary muscles.

(6) **Power of accomodation :** The ability of eye to see near objects as well as far objects is called power of accomodation.

Note : ❑ When we look distant objects, the eye is relaxed and it's focal length is largest.

(7) **Range of vision :** For healthy eye it is 25 *cm* (near point) to ∞ (far point).

A normal eye can see the objects clearly, only if they are at a distance greater than 25 *cm*. This distance is called Least distance of distinct vision and is represented by *D*.

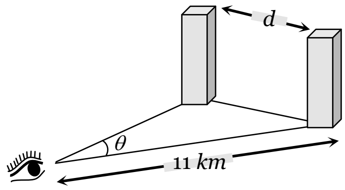
(8) **Persistence of vision :** Is 1/10 *sec*. *i.e.* if time interval between two consecutive light pulses is lesser than 0.1 *sec*., eye cannot distinguish them separately.

(9) **Binocular vision :** The seeing with two eyes is called binocular vision.

(10) **Resolving limit :** The minimum angular displacement between two objects, so that they are just resolved is called resolving limit. For eye it is $1' = \left(\frac{1}{60}\right)^o$.

Specific Example

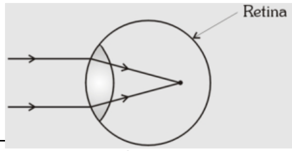
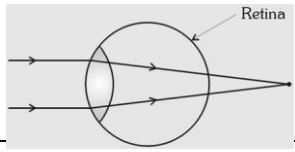
A person wishes to distinguish between two pillars located at a distances of 11 *Km*. What should be the minimum distance between the pillars.



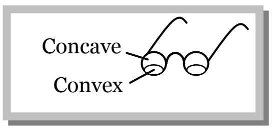
Solution : As the limit of resolution of eye is $\left(\frac{1}{60}\right)^o$

So $\theta > \left(\frac{1}{60}\right)^o$ $\Rightarrow \frac{d}{11\times 10^3} > \left(\frac{1}{60}\right) \times \frac{\pi}{180} \Rightarrow d > 3.2\,m$

(11) **Defects in eye**

Myopia (short sightness)	**Hypermetropia (long sightness)**
(i) Distant objects are not seen clearly but nearer objects are clearly visible.	(i) Distant objects are seen clearly but nearer object are not clearly visible.
(ii) Image formed before the retina. 	(ii) Image formed behind the retina. 
(iii) Far point comes closer.	(iii) Near point moves away
(iv) Reasons : (a) Focal length or radii of curvature of lens reduced or power of lens increases. (b) Distance between eye lens and retina increases.	(iv) Reasons : (a) Focal length or radii of curvature of lens increases or power of lens decreases. (b) Distance between eye lens and retina decreases.
(v) Removal : By using a concave lens of suitable focal length.	(v) Removal : By using a convex lens.
(vi) Focal length : (a) A person can see upto distance $\rightarrow x$ wants to see $\rightarrow \infty$, so focal length of used lens $f = -x = -$ (defected far point) (b) A person can see upto distance $\rightarrow x$ wants to see distance $\rightarrow y$ $(y > x)$ so $f = \frac{xy}{x-y}$	(vi) Focal length : (a) A person cannot see before distance $\rightarrow d$ wants to see the object place at distance $\rightarrow D$ so $f = \frac{dD}{d-D}$

Presbyopia : In this defect both near and far objects are not clearly visible. It is an old age disease and it is due to the loosing power of accommodation. It can be removed by using bifocal lens.



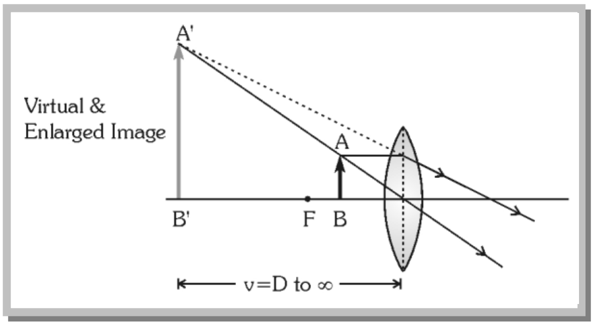
Astigmatism : In this defect eye cannot see horizontal and vertical lines clearly, simultaneously. It is due to imperfect spherical nature of eye lens. This defect can be removed by using cylindrical lens (Torric lenses).

Microscope

It is an optical instrument used to see very small objects. It's magnifying power is given by

$$m = \frac{\text{Visual angle with instrument } (\beta)}{\text{Visual angle when object is placed at least distance of distinct vision } (\alpha)}$$

(1) **Simple miscroscope**



(i) It is a single convex lens of lesser focal length.

(ii) Also called magnifying glass or reading lens.

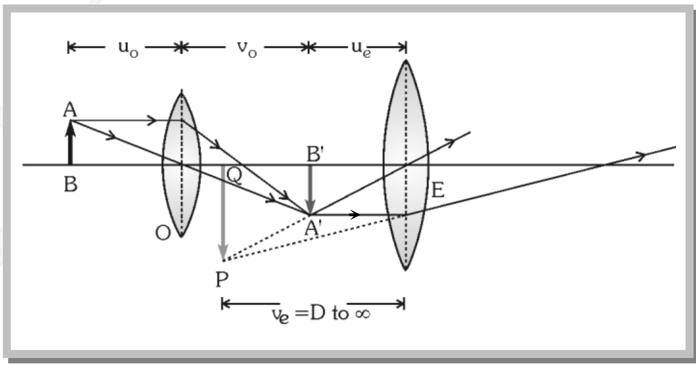
(iii) Magnification's, when final image is formed at D and ∞ (*i.e.* m_D and m_∞)

$$m_D = \left(1 + \frac{D}{f}\right)_{\max} \text{ and } m_\infty = \left(\frac{D}{f}\right)_{\min}$$

Note : ❑ $m_{\max.} - m_{\min.} = 1$

❑ If lens is kept at a distance a from the eye then $m_D = 1 + \frac{D-a}{f}$ and $m_\infty = \frac{D-a}{f}$

(2) **Compound microscope**



(i) Consist of two converging lenses called objective and eye lens.

(ii) $f_{\text{eye lens}} > f_{\text{objective}}$ and

$(\text{diameter})_{\text{eye lens}} > (\text{diameter})_{\text{objective}}$

(iii) Final image is magnified, virtual and inverted.

(iv) u_0 = Distance of object from objective (o), v_0 = Distance of image ($A'B'$) formed by objective from objective, u_e = Distance of $A'B'$ from eye lens, v_e = Distance of final image from eye lens, f_0 = Focal length of objective, f_e = Focal length of eye lens.

Magnification : $$m_D = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = -\frac{f_0}{(u_0 - f_0)}\left(1 + \frac{D}{f_e}\right) = -\frac{(v_0 - f_0)}{f_0}\left(1 + \frac{D}{f_e}\right)$$

$$m_\infty = -\frac{v_0}{u_0}.\frac{D}{F_e} = \frac{-f_0}{(u_0 - f_0)}\left(\frac{D}{f_e}\right) = -\frac{(v_0 - f_0)}{f_0}.\frac{D}{F_e}$$

Length of the tube (*i.e.* distance between two lenses)

When final image is formed at D ; $\quad L_D = v_0 + u_e = \frac{u_0 f_0}{u_0 - f_0} + \frac{f_e D}{f_e + D}$

When final images is formed at ∞ ; $\quad L_\infty = v_0 + f_e = \frac{u_0 f_0}{u_0 - f_0} + f_e$

(Do not use sign convention while solving the problems)

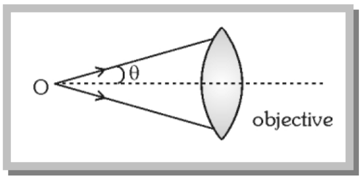
Note : ❑ $m_\infty = \frac{(L_\infty - f_0 - f_e)D}{f_0 f_e}$

❑ For maximum magnification both f_0 and f_e must be less.

❑ $m = m_{\text{objective}} \times m_{\text{eyelens}}$

❑ If objective and eye lens are interchanged, practically there is no change in magnification.

(3) **Resolving limit and resolving power :** In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (*RL*) and it's reciprocal is called Resolving power (*RP*)



$$R.L. = \frac{\lambda}{2\mu\sin\theta} \text{ and } R.P. = \frac{2\mu\sin\theta}{\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$

λ = Wavelength of light used to illuminate the object,

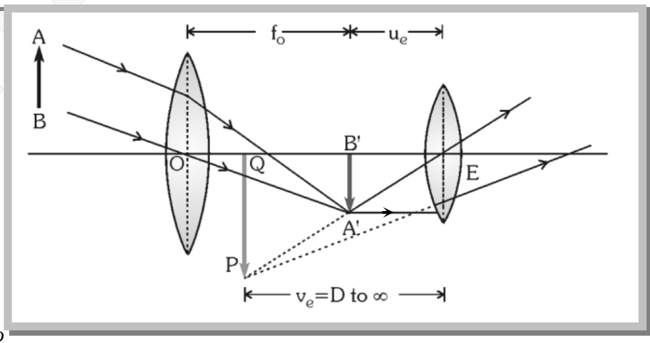
μ = Refractive index of the medium between object and objective,

θ = Half angle of the cone of light from the point object, $\mu\sin\theta$ = Numerical aperture.

Note : ❑ Electron microscope : electron beam $(\lambda \approx 1Å)$ is used in it so it's *R.P.* is approx 5000 times more than that of ordinary microscope $(\lambda \approx 5000\ Å)$

Telescope

By telescope distant objects are seen.



(1) **Astronomical telescope**

(i) Used to see heavenly bodies.

(ii) $f_{\text{objective}} > f_{\text{eyelens}}$ and $d_{\text{objective}} > d_{\text{eyelens}}$.

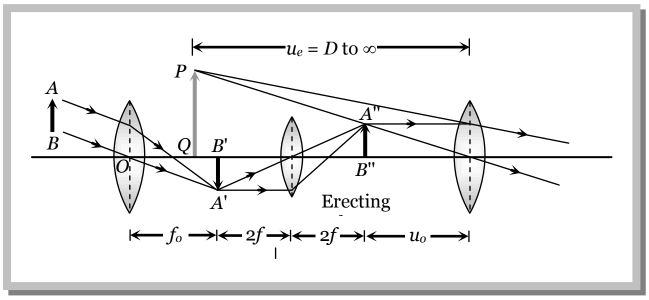
(iii) Intermediate image is real, inverted and small.

(iv) Final image is virtual, inverted and small.

(v) Magnification : $m_D = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$ and $m_\infty = -\frac{f_o}{f_e}$

(vi) Length : $L_D = f_0 + u_e = f_0 + \frac{f_e D}{f_e + D}$ and $L_\infty = f_0 + f_e$

(2) **Terrestrial telescope**



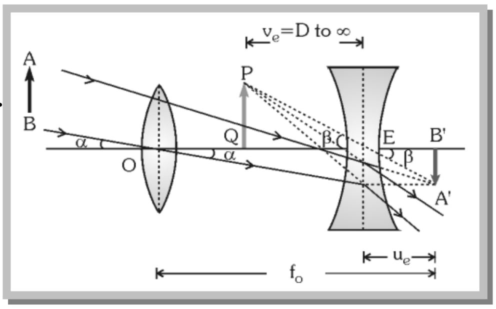
(i) Used to see far off object on the earth.

(ii) It consists of three converging lens : objective, eye lens and erecting lens.

(iii) It's final image is virtual erect and smaller.

(iv) Magnification : $m_D = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$ and $m_\infty = \frac{f_0}{f_e}$

(v) Length : $L_D = f_0 + 4f + u_e = f_0 + 4f + \dfrac{f_e D}{f_e + D}$ and $L_\infty = f_0 + 4f + f_e$



(3) **Galilean telescope**

(i) It is also a terrestrial telescope but of much smaller field of view.

(ii) Objective is a converging lens while eye lens is diverging lens.

(iii) Magnification : $m_D = \dfrac{f_0}{f_e}\left(1 - \dfrac{f_e}{D}\right)$ and $m_\infty = \dfrac{f_0}{f_e}$

(iv) Length : $L_D = f_0 - u_e$ and $L_\infty = f_0 - f_e$

(4) **Resolving limit and resolving power**

Smallest angular separations ($d\theta$) between two distant objects, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \dfrac{1.22\lambda}{a}$

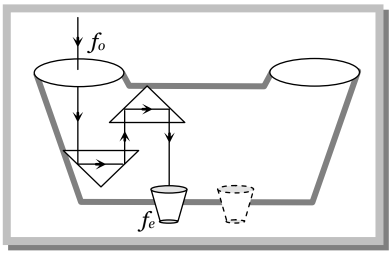
and resolving power $(RP) = \dfrac{1}{d\theta} = \dfrac{a}{1.22\lambda} \Rightarrow R.P. \propto \dfrac{1}{\lambda}$ where a = aperture of objective.

Note :Minimum separation (d) between objects, so they can just resolved by a telescope is –

$d = \dfrac{r}{R.P.}$ where r = distance of objects from telescope.

(5) **Binocular**

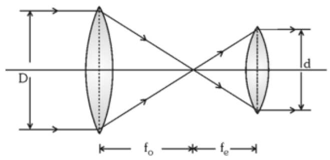
If two telescopes are mounted parallel to each other so that an object can be seen by both the eyes simultaneously, the arrangement is called 'binocular'. In a binocular, the length of each tube is reduced by using a set of totally reflecting prisms which provided intense, erect image free from lateral inversion. Through a binocular we get two images of the same object from different angles at same time. Their superposition gives the perception of depth also along with length and breadth, *i.e.*, binocular vision gives proper three-dimensional (3*D*) image.



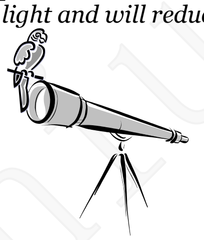
Concepts

- *As magnifying power is negative, the image seen in astronomical telescope is truly inverted, i.e., left is turned right with upside down simultaneously. However, as most of the astronomical objects are symmetrical this inversion does not affect the observations.*
- *Objective and eye lens of a telescope are interchanged, it will not behave as a microscope but object appears very small.*
- *In a telescope, if field and eye lenses are interchanged magnification will change from (f_o / f_e) to (f_e / f_o), i.e., it will change from m to (1/m), i.e., will become (1/m²) times of its initial value.*
- *As magnification for normal setting as (f_o / f_e), so to have large magnification, f_o must be as large as practically possible and f_e small. This is why in a telescope, objective is of large focal length while eye piece of small.*
- *In a telescope, aperture of the field lens is made as large as practically possible to increase its resolving power as resolving power of a telescope ∝ (D/λ)*. Large aperture of objective also helps in improving the brightness of image by gathering more light from distant object. However, it increases aberrations particularly spherical.*
- *For a telescope with increase in length of the tube, magnification decreases.*
- *In case of a telescope if object and final image are at infinity then :*

$$m = \frac{f_o}{f_e} = \frac{D}{d}$$



- *If we are given four convex lenses having focal lengths $f_1 > f_2 > f_3 > f_4$. For making a good telescope and microscope. We choose the following lenses respectively. Telescope $f_1(o), f_4(e)$ Microscope $f_4(o), f_3(e)$*
- *If a parrot is sitting on the objective of a large telescope and we look towards (or take a photograph)of distant astronomical object (say moon) through it, the parrot will not be seen but the intensity of the image will be slightly reduced as the parrot will act as obstruction to light and will reduce the aperture of the objective.*



Example

***Example*: 1** A man can see the objects upto a distance of one metre from his eyes. For correcting his eye sight so that he can see an object at infinity, he requires a lens whose power is

or

A man can see upto 100 *cm* of the distant object. The power of the lens required to see far objects will be

[MP PMT 1993, 2003]

(a) +0.5 *D* (b) +1.0 *D* (c) +2.0 *D* (d) –1.0 *D*

Solution: (d) *f* = –(defected far point) = – 100 *cm*. So power of the lens $P = \frac{100}{f} = \frac{100}{-100} = -1D$

***Example*: 2** A man can see clearly up to 3 *metres*. Prescribe a lens for his spectacles so that he can see clearly up to 12 *metres*

[DPMT 2002]

(a) – 3/4 *D* (b) 3 *D* (c) – 1/4 *D* (d) – 4 *D*

Solution: (c) By using $f = \frac{xy}{x-y} \Rightarrow f = \frac{3\times 12}{3-12} = -4m$. Hence power $P = \frac{1}{f} = -\frac{1}{4}D$

***Example*: 3** The diameter of the eye-ball of a normal eye is about 2.5 *cm*. The power of the eye lens varies from

(a) 2 *D* to 10 *D* (b) 40 *D* to 32 *D* (c) 9 *D* to 8 *D* (d) 44 *D* to 40 *D*

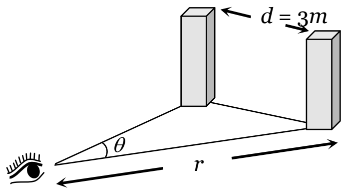
Solution: (d) An eye sees distant objects with full relaxation so $\frac{1}{2.5\times10^{-2}}-\frac{1}{-\infty}=\frac{1}{f}$ or $P=\frac{1}{f}=\frac{1}{25\times10^{-2}}=40D$

An eye sees an object at 25 *cm* with strain so $\frac{1}{2.5\times10^{-2}}-\frac{1}{-25\times10^{-2}}=\frac{1}{f}$ or $P=\frac{1}{f}=40+4=44D$

Example: 4 The resolution limit of eye is 1 *minute*. At a distance of *r* from the eye, two persons stand with a lateral separation of 3 *metre*. For the two persons to be just resolved by the naked eye, *r* should be

(a) 10 *km* (b) 15 *km* (c) 20 *km* (d) 30 *km*

Solution: (a) From figure $\theta=\frac{d}{r}$; where $\theta=1'=\left(\frac{1}{60}\right)^{o}=\left(\frac{1}{60}\right)\times\frac{\pi}{180}rad$



$\Rightarrow 1\times\frac{1}{60}\times\frac{\pi}{180}=\frac{3}{r}\Rightarrow r=10\ km$

Example: 5 Two points separated by a distance of 0.1 *mm* can just be resolved in a microscope when a light of wavelength 6000 *Å* is used. If the light of wavelength 4800 *Å* is used this limit of resolution becomes

[UPSEAT 2002]

(a) 0.08 *mm* (b) 0.10 *mm* (c) 0.12 *mm* (d) 0.06 *mm*

Solution: (a) By using resolving limit (R.L.) $\propto\lambda$ $\Rightarrow\frac{(R.L.)_1}{(R.L.)_2}=\frac{\lambda_1}{\lambda_2}\Rightarrow\frac{0.1}{(R.L.)_2}=\frac{6000}{4800}\Rightarrow(R.L.)_2=0.08\,mm$.

Example: 6 In a compound microscope, the focal lengths of two lenses are 1.5 *cm* and 6.25 *cm* an object is placed at 2 *cm* form objective and the final image is formed at 25 *cm* from eye lens. The distance between the two lenses is

[EAMCET (Med.) 2000]

(a) 6.00 *cm* (b) 7.75 *cm* (c) 9.25 *cm* (d) 11.00 *cm*

Solution: (d) It is given that $f_o=1.5\ cm$, $f_e=6.25\ cm$, $u_o=2\ cm$

When final image is formed at least distance of distinct vision, length of the tube $L_D=\frac{u_of_o}{u_o-f_o}+\frac{f_eD}{f_e+D}$

$\Rightarrow L_D=\frac{2\times1.5}{(2-1.5)}+\frac{6.25\times25}{(6.25+25)}=11\,cm$.

Example: 7 The focal lengths of the objective and the eye-piece of a compound microscope are 2.0 *cm* and 3.0 *cm* respectively. The distance between the objective and the eye-piece is 15.0 *cm*. The final image formed by the eye-piece is at infinity. The two lenses are thin. The distances in *cm* of the object and the image produced by the objective measured from the objective lens are respectively **[IIT-JEE 1995]**

(a) 2.4 and 12.0 (b) 2.4 and 15.0 (c) 2.3 and 12.0 (d) 2.3 and 3.0

Solution: (a) Given that $f_o=2\,cm$, $f_e=3\,cm$, $L_\infty=15\,cm$

By using $L_\infty=v_o+f_e\Rightarrow 15=v_o+3\Rightarrow v_o=12\,cm$. Also $\frac{v_o}{u_o}=\frac{v_o-f_o}{f_o}\Rightarrow\frac{12}{u_o}=\frac{12-2}{2}\Rightarrow u_o=2.4\,cm$.

Example: 8 The focal lengths of the objective and eye-lens of a microscope are 1 *cm* and 5 *cm* respectively. If the magnifying power for the relaxed eye is 45, then the length of the tube is

(a) 30 *cm* (b) 25 *cm* (c) 15 *cm* (d) 12 *cm*

Solution: (c) Given that $f_o = 1\,cm$, $f_e = 5\,cm$, $m_\infty = 45$

By using $m_\infty = \frac{(L_\infty - f_o - f_e)}{f_o f_e} \Rightarrow 45 = \frac{(L_\infty - 1 - 5)\times 25}{1\times 5} \Rightarrow L_\infty = 15\,cm$

***Example*: 9** If the focal lengths of objective and eye lens of a microscope are 1.2 *cm* and 3 *cm* respectively and the object is put 1.25 *cm* away from the objective lens and the final image is formed at infinity, then magnifying power of the microscope is

(a) 150 (b) 200 (c) 250 (d) 400

Solution: (b) Given that $f_o = 1.2\,cm$, $f_e = 3\,cm$, $u_o = 1.25\,cm$

By using $m_\infty = -\frac{f_o}{(u_o - f_o)}\cdot\frac{D}{f_e} \Rightarrow m_\infty = -\frac{1.2}{(1.25-1.2)}\times\frac{25}{3} = -200$.

***Example*: 10** The magnifying power of an astronomical telescope is 8 and the distance between the two lenses is 54*cm*. The focal length of eye lens and objective lens will be respectively **[MP PMT 1991; CPMT 1991**

(a) 6 *cm* and 48 *cm* (b) 48 *cm* and 6 *cm* (c) 8 *cm* and 64 *cm* (d) 64 *cm* and 8 *cm*

Solution: (a) Given that $m_\infty = 8$ and $L_\infty = 54$

By using $|m_\infty| = \frac{f_o}{f_e}$ and $L_\infty = f_o + f_e$ we get $f_o = 6\,cm$ and $f_e = 48\,cm$.

***Example*: 11** If an object subtend angle of 2° at eye when seen through telescope having objective and eyepiece of focal length $f_o = 60\,cm$ and $f_e = 5\,cm$ respectively than angle subtend by image at eye piece will be **[UPSEAT 2001]**

(a) 16° (b) 50° (c) 24° (d) 10°

Solution: (c) By using $\frac{\beta}{\alpha} = \frac{f_o}{f_e} \Rightarrow \frac{\beta}{20} = \frac{60}{5} \Rightarrow \beta = 24°$

***Example*: 12** The focal lengths of the lenses of an astronomical telescope are 50 *cm* and 5 *cm*. The length of the telescope when the image is formed at the least distance of distinct vision is

(a) 45 *cm* (b) 55 *cm* (c) $\frac{275}{6}$ *cm* (d) $\frac{325}{6}$ *cm*

Solution: (d) By using $L_D = f_o + u_e = f_o + \frac{f_e D}{f_e + D} = 50 + \frac{5\times 25}{(5+25)} = \frac{325}{6}\,cm$

***Example*: 13** The diameter of moon is $3.5\times 10^3\,km$ and its distance from the earth is $3.8\times 10^5\,km$. If it is seen through a telescope whose focal length for objective and eye lens are 4 *m* and 10 *cm* respectively, then the angle subtended by the moon on the eye will be approximately

(a) 15° (b) 20° (c) 30° (d) 35°

Solution: (b) The angle subtended by the moon on the objective of telescope $\alpha = \frac{3.5\times 10^3}{3.8\times 10^5} = \frac{3.5}{3.8}\times 10^{-2}\,rad$

Also $m = \frac{f_o}{f_e} = \frac{\beta}{\alpha} \Rightarrow \frac{400}{10} = \frac{\beta}{\alpha} \Rightarrow \beta = 40\,\alpha \Rightarrow \beta = 40\times\frac{3.5\times 10^3}{3.8\times 10^5}\times\frac{180}{\pi} = 20°$

***Example*: 14** A telescope has an objective lens of 10 *cm* diameter and is situated at a distance one *kilometre* from two objects. The minimum distance between these two objects, which can be resolved by the telescope, when the mean wavelength of light is 5000 *Å*, is of the order of

(a) 0.5 *m* (b) 5 *m* (c) 5 *mm* (d) 5*cm*

Solution: (b) Suppose minimum distance between objects is *x* and their distance from telescope is *r*

So Resolving limit $d\theta = \frac{1.22\lambda}{a} = \frac{x}{r} \Rightarrow x = \frac{1.22\lambda\times r}{a} = \frac{1.22\times(5000\times 10^{-10})\times(1\times 10^3)}{(0-1)} = 6.1\times 10^{-3} m = 6.1\,mm$

Hence, It's order is ≈ 5 *mm*.

***Example*: 15** A compound microscope has a magnifying power 30. The focal length of its eye-piece is 5 *cm*. Assuming the final image to be at the least distance of distinct vision. The magnification produced by the objective will be

(a) +5 (b) – 5 (c) +6 (d) – 6

Solution (b) Magnification produced by compound microscope $m = m_o \times m_e$

where m_o = ? and $m_e = \left(1 + \frac{D}{f_e}\right) = 1 + \frac{25}{5} = 6 \Rightarrow 30 = -m_o \times 6 \Rightarrow m_o = -5$.

Tricky Example 1 : A man is looking at a small object placed at his least distance of distinct vision. Without changing his position and that of the object he puts a simple microscope of magnifying power 10 *X* and just sees the clear image again. The angular magnification obtained is

(a) 2.5 (b) 10.0 (c) 5.0 (d) 1.0

Solution : (d) Angular magnification $= \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{I/D}{O/D} = \frac{I}{O}$

Since image and object are at the same position, $\frac{I}{O} = \frac{v}{u} = 1 \Rightarrow$ Angular magnification = 1

Tricky Example 2: A compound microscope is used to enlarge an object kept at a distance 0.03*m* from it's objective which consists of several convex lenses in contact and has focal length 0.02*m*. If a lens of focal length 0.1*m* is removed from the objective, then by what distance the eye-piece of the microscope must be moved to refocus the image

(a) 2.5 *cm* (b) 6 *cm* (c) 15 *cm* (d) 9 *cm*

Solution : (d) If initially the objective (focal length F_o) forms the image at distance v_o then $v_o = \frac{u_o f_o}{u_o - f_o} = \frac{3 \times 2}{3 - 2} = 6\, cm$

Now as in case of lenses in contact $\frac{1}{F_o} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ... = \frac{1}{f_1} + \frac{1}{F'_o}$ $\left\{\text{where } \frac{1}{F'_o} = \frac{1}{f_2} + \frac{1}{f_3} + ...\right\}$

So if one of the lens is removed, the focal length of the remaining lens system

$\frac{1}{F'_o} = \frac{1}{F_0} - \frac{1}{f_1} = \frac{1}{2} - \frac{1}{10} \Rightarrow F'_o = 2.5\, cm$

This lens will form the image of same object at a distance v'_o such that $v'_o = \frac{u_o F'_o}{u_o - F'_o} = \frac{3 \times 2.5}{(3 - 2.5)} = 15\, cm$

So to refocus the image, eye-piece must be moved by the same distance through which the image formed by the objective has shifted *i.e.* 15 – 6 = 9 *cm*.

Assignment

Human eye

80. Near and far points of human eye are **[EAMCET (Med.) 1995; MP PET 2001; Bihar CECE 2004]**

(a) 25 *cm* and infinite (b) 50 *cm* and 100 *cm* (c) 25 *cm* and 50 *cm* (d) 0 *cm* and 25 *cm*

81. A defective eye cannot see close objects clearly because their image is formed **[MP PET 2003]**

(a) On the eye lens (b) Between eye lens and retina

(c) On the retina (d) Beyond retina

82. Retina of eye acts like ..... of camera **[AFMC 2003]**

(a) Shutter (b) Film (c) Lens (d) None of these

83. A person who can see things most clearly at a distance of 10 *cm*. Requires spectacles to enable him to see clearly things at a distance of 30 *cm*. What should be the focal length of the spectacles **[BHU 2003]**

(a) 15 *cm* (concave) (b) 15 *cm* (convex) (c) 10 *cm* (d) 0

84. An astronaut is looking down on earth's surface from a space shuttle at an altitude of $400\,km$. Assuming that the astronaut's pupil diameter is $5\,mm$ and the wavelength of visible light is $500\,nm$. The astronaut will be able to resolve linear object of the size of about **[AIIMS 2003]**

(a) 0.5 *m* (b) $5\,m$ (c) 50 *m* (d) 500 *m*

85. A person uses a lens of power + 3*D* to normalise vision. Near point of hypermetropic eye is **[CPMT 2002]**

(a) 1 *m* (b) 1.66 *m* (c) 2 *m* (d) 0.66 *m*

86. The separation between two microscopic particles is measured P_A and P_B by two different lights of wavelength 2000 Å and 3000 Å respectively, then **[AIEEE 2002]**

(a) $P_A > P_B$ (b) $P_A < P_B$ (c) $P_A < 3/2P_B$ (d) $P_A = P_B$

87. To remove myopia (short sightedness) a lens of power 0.66 *D* is required. The distant point of the eye is approximately **[MP PMT 2001]**

(a) 100 *cm* (b) 150 *cm* (c) 50 *cm* (d) 25 *cm*

88. A person suffering from 'presbyopia' should use **[MP PET 2001]**

(a) A concave lens (b) A convex lens

(c) A bifocal lens whose lower portion is convex (d) A bifocal lens whose upper portion is convex

89. The resolving limit of healthy eye is about **[MP PET 1999; RPMT 1999; AIIMS 2001]**

(a) $1'$ (b) $1''$ (c) 1° (d) $\frac{1}{60}''$

90. A person uses spectacles of power + 2*D*. He is suffering from **[MP PET 2000]**

(a) Short sightedness or myopia (b) Long sightedness or hypermetropia

(c) Presbyopia (d) Astigmatism

91. The hyper metropia is a **[CBSE PMT 2000]**

(a) Short-side defect (b) Long-side defect

(c) Bad vision due to old age (d) None of these

92. A man cannot see clearly the objects beyond a distance of 20 *cm* from his eyes. To see distant objects clearly he must use which kind of lenses and of what focal length **[MP PMT 2000]**

(a) 100 *cm* convex (b) 100 *cm* concave (c) 20 *cm* convex (d) 20 *cm* concave

93. An eye specialist prescribes spectacles having a combination of convex lens of focal length 40 *cm* in contact with a concave lens of focal length 25 *cm*. The power of this lens combination in diopters is **[IIT 1997 Cancelled; DPMT 2000]**

(a) + 1.5 (b) –1.5 (c) +6.67 (d) –6.67

94. Two parallel pillars are 11 *km* away from an observer. The minimum distance between the pillars so that they can be seen separately will be **[RPET 1997; RPMT 2000]**

(a) 3.2 *m* (b) 20.8 *m* (c) 91.5 *m* (d) 183 *m*

95. A person cannot see objects clearly beyond 2.0 *m*. The power of lens required to correct his vision will be **[MP PMT/PET 1998; JIPMER 2000; KCET (Engg./Med.) 2000]**

(a) + 2.0 *D* (b) – 1.0 *D* (c) + 1.0 *D* (d) – 0.5 *D*

96. When objects at different distances are seen by the eye, which of the following remains constant **[MP PMT 1999]**

(a) The focal length of the eye lens (b) The object distance from the eye lens

(c) The radii of curvature of the eye lens (d) The image distance from the eye lens

97. A person wears glasses of power –2.0 D. The defect of the eye and the far point of the person without the glasses will be **[MP PMT 1999]**

(a) Nearsighted, 50 *cm* (b) Farsighted, 50 *cm* (c) Nearsighted, 250 *cm* (d) Astigmatism, 50 *cm*

98. A person is suffering from the defect astigmatism. Its main reason is **[MP PMT 1997]**

(a) Distance of the eye lens from retina is increased (b) Distance of the eye lens from retina is decreased

(c) The cornea is not spherical (d) Power of accommodation of the eye is decreased

99. Myopia is due to **[AFMC 1996]**

(a) Elongation of eye ball (b) Irregular change in focal length

(c) Shortening of eye ball (d) Older age

100. Human eye is most sensitive to visible light of the wavelength **[CPMT 1996]**

(a) 6050 $Å$ (b) 5500 $Å$ (c) 4500 $Å$ (d) 7500 $Å$

101. Match the List I with the List II from the combinations shown **[ISM Dhanbad 1994]**

(I) Presbiopia (A) Sphero-cylindrical lens

(II) Hypermetropia (B) Convex lens of proper power may be used close to the eye

(III) Astigmatism (C) Concave lens of suitable focal length

(IV) Myopia (D) Convex spectacle lens of suitable focal length

(a) I-A; II-C; III-B; IV-D (b) I-B; II-D; III-C; IV-A (c) I-D; II-B; III-A; IV-C (d) I-D; II-A; III-C; IV-B

102. The human eye has a lens which has a **[MP PET 1994]**

(a) Soft portion at its centre (b) Hard surface

(c) Varying refractive index (d) Constant refractive index

103. A man with defective eyes cannot see distinctly object at the distance more than 60 *cm* from his eyes. The power of the lens to be used will be **[MP PMT 1994]**

(a) + 60D (b) – 60D (c) – 1.66D (d) $\frac{1}{1.66}D$

104. A person's near point is 50 *cm* and his far point is 3 *m*. Power of the lenses he requires for

(i) Reading and (ii) For seeing distant stars are **[MP PMT 1994]**

(a) – 2D and 0.33D (b) 2D and – 0.33D (c) – 2D and 3D (d) 2D and – 3D

105. The focal length of a simple convex lens used as a magnifier is 10 *cm*. For the image to be formed at a distance of distinct vision (D = 25 *cm*), the object must be placed away from the lens at a distance of **[CPMT 1991]**

(a) 5 *cm* (b) 7.14 *cm* (c) 7.20 *cm* (d) 16.16 *cm*

106. A person is suffering from myopic defect. He is able to see clear objects placed at 15 *cm*. What type and of what focal length of lens he should use to see clearly the object placed 60 *cm* away **[MP PMT 1991]**

(a) Concave lens of 20 *cm* focal length (b) Convex lens of 20 *cm* focal length

(c) Concave lens of 12 *cm* focal length (d) Convex lens of 12 *cm* focal length

107. A person can see a thing clearly when it is at a distance of 1 *metre* only. If he wishes to see a distance star, he needs a lens of focal length **[MP PET 1990]**

(a) +100 *cm* (b) – 100 *cm* (c) +50 *cm* (d) –50 *cm*

108. A man suffering from myopia can read a book placed at 10 *cm* distance. For reading the book at a distance of 60 *cm* with relaxed vision, focal length of the lens required will be **[MP PMT 1989]**

(a) 45 *cm* (b) – 20 *cm* (c) –12 *cm* (d) 30 *cm*

109. A person can see clearly objects at 100 *cm* distance. If he wants to see objects at 40 *cm* distance, then the power of the lens he shall require is **[MP PET 1989]**

(a) +1.5 D (b) – 1.5 D (c) +3.0 D (d) –3.0 D

genius

110. If the distance of the far point for a myopia patient is doubled, the focal length of the lens required to cure it will become **[MP PET 1989]**

(a) Half (b) Double

(c) The same but a convex lens (d) The same but a concave lens

111. Image is formed for the short sighted person at **[AFMC 1988]**

(a) Retina (b) Before retina (c) Behind the retina (d) Image is not formed at all

112. A man who cannot see clearly beyond 5 *m* wants to see stars clearly. He should use a lens of focal length **[MP PET/PMT 1988]**

(a) – 100 *metre* (b) + 5 *metre* (c) – 5 *metre* (d) Very large

113. Far point of myopic eye is 250 *cm*, then the focal length of the lens to be used will be **[CPMT 1986; DPMT 2002]**

(a) + 250 *cm* (b) – 250 *cm* (c) + 250/9 *cm* (d) – 250/9 *cm*

114. One can take pictures of objects which are completely invisible to the eye using camera film which are invisible to **[MNR 1985]**

(a) Ultra-violet rays (b) Sodium light (c) Visible light (d) Infra-red rays

115. In human eye the focussing is done by **[CPMT 1983]**

(a) To and fro movement of eye lens (b) To and fro movement of the retina

(c) Change in the convexity of the lens surface (d) Change in the refractive index of the eye fluids

116. The minimum light intensity that can be perceived by the eye is about $10^{-10}\ watt/metre^2$. The number of photons of wavelength $5.6\times10^{-7}\ metre$ that must enter per second the pupil of area $10^{-4}\ metre^2$ for vision, is approximately equal to $(h = 6.6\times10^{-34}\ joule-\text{sec})$ **[NCERT 1982]**

(a) 3×10^2 *photons* (b) 3×10^6 *photons* (c) 3×10^4 *photons* (d) 3×10^5 *photons*

117. A far sighted man who has lost his spectacles, reads a book by looking through a small hole (3-4 *mm*) in a sheet of paper. The reason will be **[CPMT 1977]**

(a) Because the hole produces an image of the letters at a longer distance

(b) Because in doing so, the focal length of the eye lens is effectively increased

(c) Because in doing so, the focal length of the eye lens is effectively decreased

(d) None of these

118. The maximum focal length of the eye-lens of a person is greater than its distance from the retina. The eye is

(a) Always strained in looking at an object (b) Strained for objects at large distances only

(c) Strained for objects at short distances only (d) Unstrained for all distances

119. The focal length of a normal eye-lens is about

(a) 1 *mm* (b) 2 *cm* (c) 25 *cm* (d) 1

120. The distance of the eye-lens from the retina is *x*. For normal eye, the maximum focal length of the eye-lens is

(a) $= x$ (b) $< x$ (c) $> x$ (d) $= 2x$

121. A man wearing glasses of focal length +1*m* can clearly see beyond 1*m*

(a) If he is farsighted (b) If he is nearsighted (c) If his vision is normal (d) In each of these cases

122. The near point of a person is 50 *cm* and the far point is 1.5 *m*. The spectacles required for reading purpose and for seeing distance are respectively

(a) $+2D, -\left(\frac{2}{3}\right)D$ (b) $+\left(\frac{2}{3}\right)D - 2D$ (c) $-2D, +\left(\frac{2}{3}\right)D$ (d) $-\left(\frac{2}{3}\right)D + 2D$

123. A man, wearing glasses of power $+2D$ can read clearly a book placed at a distance of 40 *cm* from the eye. The power of the lens required so that he can read at 25 *cm* from the eye is

(a) $+4.5\ D$ (b) $+4.0\ D$ (c) $+3.5\ D$ (d) $+3.0\ D$

124. A person can see clearly between 1 *m* and 2*m*. His corrective lenses should be

(a) Bifocals with power $-0.5D$ and additional $+3.5D$ (b) Bifocals with power $-1.0D$ and additional $+3.0\ D$

(c) Concave with power 1.0 *D* (d) Convex with power 0.5 *D*

125. While reading the book a man keeps the page at a distance of 2.5 *cm* from his eye. He wants to read the book by holding the page at 25 *cm*. What is the nature of spectacles one should advice him to use to completely cure his eye sight

(a) Convex lens of focal length 25 *cm* (b) Concave lens of focal length 25 *cm*

(c) Convex lens of focal length 2.5 *cm* (d) Concave lens of focal length 2.5 *cm*

126. The blades of a rotating fan can not be distinguished from each other due to

(a) Parallex (b) Power of accommodation (c) Persistence of vision (d) Binocular vision

127. Aperture of the human eye is 2 *mm*. Assuming the mean wavelength of light to be 5000 $\AA$, the angular resolution limit of the eye is nearly

(a) 2 *minutes* (b) 1 *minute* (c) 0.5 *minute* (d) 1.5 *minutes*

128. If there had been one eye of the man, then

(a) Image of the object would have been inverted (b) Visible region would have decreased

(c) Image would have not been seen three dimensional (d) (b) and (c) both

129. A man can see the object between 15*cm* and 30*cm*. He uses the lens to see the far objects. Then due to the lens used, the near point will be at

(a) $\frac{10}{3}\ cm$ (b) 30 *cm* (c) 15 *cm* (d) $\frac{100}{3}\ cm$

130. A presbyopic patient has near point as 30 *cm* and far point as 40 *cm*. The dioptric power for the corrective lens for seeing distant objects is

(a) 40 *D* (b) 4 *D* (c) 2.5 *D* (d) 0.25 *D*

131. A man swimming under clear water is unable to see clearly because

(a) The size of the aperture decreases (b) The size of the aperture increases

(c) The focal length of eye lens increases (d) The focal length of eye lens decreases

132. The distance between retina and eye-lens in a normal eye is 2.0 *cm*. The accommodated power of eye lens range from

(a) 45 *D* to 50 *D* (b) 50 *D* to 54 *D* (c) 10 *D* to 16 *D* (d) 5 *D* to 8 *D*

133. If the eye is taken as a spherical ball of radius 1 *cm*, the range of accommodated focal length of eye-lens is

(a) 1.85 *cm* to 2.0 *cm* (b) 1.0 *cm* to 2.8 *cm* (c) 1.56 *cm* to 2.5 *cm* (d) 1.6 *cm* to 2.0 *cm*

134. A person cannot read printed matter within 100 *cm* from his eye. The power of the correcting lens required to read at 20 *cm* from his eye if the distance between the eye lens and the correcting lens is 2 *cm* is

(a) 4.8 *D* (b) 1.25 *D* (c) 4.25 *D* (d) 4.55 *D*

135. A student having $-1.5\ D$ spectacles uses a lens of focal length 5 *cm* as a simple microscope to read minute scale divisions in the laboratory. The least distance of distinct vision without glasses is 20 *cm* for the student. The maximum magnifying power he gets with spectacles on is

(a) 6 (b) 9 (c) 5 (d) 4

Microscope

136. In a compound microscope the object of f_o and eyepiece of f_e are placed at distance L such that L equals **[Kerala PMT 2004]**

(a) $f_o + f_e$ (b) $f_o - f_e$

(c) Much greater than f_o or f_e (d) Need not depend either value of focal lengths

137. In a simple microscope, if the final image is located at infinity then its magnifying power is **[CPMT 1985; MP PMT 2004]**

(a) $\frac{25}{f}$ (b) $\frac{D}{25}$ (c) $\frac{f}{25}$ (d) $\frac{f}{D^{+1}}$

138. In a simple microscope, if the final image is located at 25 *cm* from the eye placed close to the lens, then the magnifying power is

[BVP 2003]

(a) $\frac{25}{f}$ (b) $1+\frac{25}{f}$ (c) $\frac{f}{25}$ (d) $\frac{f}{25}+1$

139. The maximum magnification that can be obtained with a convex lens of focal length 2.5 *cm* is (the least distance of distinct vision is 25 *cm*) **[MP PET 2003]**

(a) 10 (b) 0.1 (c) 62.5 (d) 11

140. In a compound microscope, the intermediate image is **[IIT-JEE (Screening) 2000; AIEEE 2003]**

(a) Virtual, erect and magnified (b) Real, erect and magnified

(c) Real, inverted and magnified (d) Virtual, erect and reduced

141. A compound microscope has two lenses. The magnifying power of one is 5 and the combined magnifying power is 100. The magnifying power of the other lens is **[Kerala PMT 2002]**

(a) 10 (b) 20 (c) 50 (d) 25

142. Wavelength of light used in an optical instrument are $\lambda_1 = 4000\ \mathring{A}$ and $\lambda_2 = 5000\ \mathring{A}$, then ratio of their respective resolving power (corresponding to λ_1 and λ_2) is **[AIEEE 2002]**

(a) 16 : 25 (b) 9 : 1 (c) 4 : 5 (d) 5 : 4

143. The angular magnification of a simple microscope can be increased by increasing **[Orissa JEE 2002]**

(a) Focal length of lens (b) Size of object (c) Aperture of lens (d) Power of lens

144. The magnification produced by the objective lens and the eye lens of a compound microscope are 25 and 6 respectively. The magnifying power of this microscope is **[Manipal MEE 1995; DPMT 2002]**

(a) 19 (b) 31 (c) 150 (d) $\sqrt{150}$

145. The length of the compound microscope is 14 *cm*. The magnifying power for relaxed eye is 25. If the focal length of eye lens is 5 *cm*, then the object distance for objective lens will be **[Pb. PMT 2002]**

(a) 1.8 *cm* (b) 1.5 *cm* (c) 2.1 *cm* (d) 2.4 *cm*

146. The magnifying power of a simple microscope is 6. The focal length of its lens in *metres* will be, if least distance of distinct vision is 25 *cm* **[MP PMT 2001]**

(a) 0.05 (b) 0.06 (c) 0.25 (d) 0.12

147. Relative difference of focal lengths of objective and eye lens in the microscope and telescope is given as

[MH CET (Med.) 2001]

(a) It is equal in both (b) It is more in telescope (c) It is more in microscope (d) It may be more in any one

148. Three objective focal lengths (f_o) and two eye piece focal lengths (f_e) are available for a compound microscope. By combining these two, the magnification of microscope will be maximum when **[RPMT 2001]**

(a) $f_o = f_e$ (b) $f_o >> f_e$ (c) f_o and f_e both are small (d) $f_o >> f_e$

149. If the red light is replaced by blue light illuminating the object in a microscope the resolving power of the microscope

[DCE 2001]

(a) Decreases (b) Increases (c) Gets halved (d) Remains unchanged

150. In case of a simple microscope, the object is placed at **[UPSEAT 2000]**

(a) Focus f of the convex lens (b) A position between f and $2f$ (c) Beyond $2f$ (d) Between the lens and f

151. In a compound microscope cross-wires are fixed at the point **[EAMCET (Engg.) 2000]**

(a) Where the image is formed by the objective (b) Where the image is formed by the eye-piece

(c) Where the focal point of the objective lies (d) Where the focal point of the eye-piece lies

152. The length of the tube of a microscope is 10 *cm*. The focal lengths of the objective and eye lenses are 0.5 *cm* and 1.0 *cm*. The magnifying power of the microscope is about **[MP PMT 2000]**

(a) 5 (b) 23 (c) 166 (d) 500

153. Least distance of distinct vision is 25 *cm*. Magnifying power of simple microscope of focal length 5 *cm* is **[EAMCET (Engg.) 1995; Pb. PMT 1999]**

(a) 1/5 (b) 5 (c) 1/6 (d) 6

154. The objective of a compound microscope is essentially **[SCRA 1998]**

(a) A concave lens of small focal length and small aperture (b) Convex lens of small focal length and large aperture

(c) Convex lens of large focal length and large aperture (d) Convex lens of small focal length and small aperture

155. For relaxed eye, the magnifying power of a microscope is **[CBSE PMT 1998]**

(a) $-\frac{v_o}{u_o}\times\frac{D}{f_e}$ (b) $-\frac{v_o}{u_o}\times\frac{f_e}{D}$ (c) $\frac{u_o}{v_o}\times\frac{D}{f_e}$ (d) $\frac{u_o}{v_o}\times\left(-\frac{D}{f_e}\right)$

156. A person using a lens as a simple microscope sees an **[AIIMS 1998]**

(a) Inverted virtual image (b) Inverted real magnified image

(c) Upright virtual image (d) Upright real magnified image

157. The focal length of the objective lens of a compound microscope is **[CPMT 1985; MNR 1986; MP PET 1997]**

(a) Equal to the focal length of its eye piece (b) Less than the focal length of eye piece

(c) Greater than the focal length of eye piece (d) Any of the above three

158. To produce magnified erect image of a far object, we will be required along with a convex lens, is **[MNR 1983; MP PAT 1996]**

(a) Another convex lens (b) Concave lens (c) A plane mirror (d) A concave mirror

159. An object placed 10 *cm* in front of a lens has an image 20 *cm* behind the lens. What is the power of the lens (in *dioptres*) **[MP PMT 1995]**

(a) 1.5 (b) 3.0 (c) – 15.0 (d) +15.0

160. Resolving power of a microscope depends upon **[MP PET 1995]**

(a) The focal length and aperture of the eye lens (b) The focal lengths of the objective and the eye lens

(c) The apertures of the objective and the eye lens (d) The wavelength of light illuminating the object

161. If the focal length of the objective lens is increased then **[MP PMT 1994]**

(a) Magnifying power of microscope will increase but that of telescope will decrease

(b) Magnifying power of microscope and telescope both will increase

(c) Magnifying power of microscope and telescope both will decrease

(d) Magnifying power of microscope will decrease but that of telescope will increase

162. If in compound microscope m_1 and m_2 be the linear magnification of the objective lens and eye lens respectively, then magnifying power of the compound microscope will be **[CPMT 1985; KCET 1994]**

(a) $m_1 - m_2$ (b) $\sqrt{m_1 + m_2}$ (c) $(m_1 + m_2)/2$ (d) $m_1 \times m_2$

163. The magnifying power of a microscope with an objective of 5 *mm* focal length is 400. The length of its tube is 20 *cm*. Then the focal length of the eye-piece is **[MP PMT 1991]**

(a) 200 *cm* (b) 160 *cm* (c) 2.5 *cm* (d) 0.1 *cm*

164. In a compound microscope, if the objective produces an image I_o and the eye piece produces an image I_e, then **[MP PET 1990]**

(a) I_o is virtual but I_e is real (b) I_o is real but I_e is virtual (c) I_o and I_e are both real (d) I_o and I_e are both virtual

165. In an electron microscope if the potential is increased from 20 *kV* to 80 *kV*, the resolving power of the microscope will change from *R* to **[CPMT 1988, 89]**

(a) $R/4$ (b) $4R$ (c) $2R$ (d) $R/2$

166. When the length of a microscope tube increases, its magnifying power **[MNR 1986]**

(a) Decreases (b) Increases (c) Does not change (d) May decrease or increase

167. An electron microscope is superior to an optical microscope in **[CPMT 1984]**

(a) Having better resolving power (b) Being easy to handle

(c) Low cost (d) Quickness of observation

168. In a compound microscope magnification will be large, if the focal length of the eye piece is **[CPMT 1984]**

(a) Large (b) Smaller (c) Equal to that of objective (d) Less than that of objective

169. An electron microscope gives better resolution than optical microscope because **[CPMT 1982]**

(a) Electrons are abundant (b) Electrons can be focused nicely

(c) Effective wavelength of electron is small (d) None of these

170. A man is looking at a small object placed at his near point. Without altering the position of his eye or the object, he puts a simple microscope of magnifying power 5*X* before his eyes. The angular magnification achieved is

(a) 5 (b) 2.5 (c) 1 (d) 0.2

171. The focal length of the objective of a compound microscope is f_0 and its distance from the eyepiece is *L*. The object is placed at a distance *u* from the objective. For proper working of the instrument

(a) $L < u$ (b) $L > u$ (c) $f_0 < L < 2f_0$ (d) $L > 2f_0$

172. Find the maximum magnifying power of a compound microscope having a 25 *diopter* lens as the objective, a 5 *diopter* lens as the eyepiece and the separation 30 *cm* between the two lenses. The least distance for clear vision is 25 *cm*

(a) 8.4 (b) 7.4 (c) 9.4 (d) 10.4

173. The focal length of the objective and the eye-piece of a microscope are 2 *cm* and 5 *cm* respectively and the distance between them is 30 *cm*. If the image seen by the eye is 25 *cm* from the eye-piece, the distance of the object from the objective is

(a) 0.8 *cm* (b) 2.3 *cm* (c) 0.4 *cm* (d) 1.2 *cm*

174. The focal length of objective and eye-piece of a microscope are 1 *cm* and 5 *cm* respectively. If the magnifying power for relaxed eye is 45, then length of the tube is

(a) 6 *cm* (b) 9 *cm* (c) 12 *cm* (d) 15 *cm*

175. A microscope has an objective of focal length 1.5 *cm* and an eye-piece of focal length 2.5 *cm*. If the distance between objective and eye-piece is 25 *cm*. What is the approximate value of magnification produced for relaxed eye is

(a) 75 (b) 110 (c) 140 (d) 25

176. The magnifying power of a microscope is generally marked as 10*X*, 100 *X*, etc. These markings are for a normal relaxed eye. A microscope marked 10*X* is used by an old man having his near point at 40 *cm*. The magnifying power of the microscope for the old man with his eyes completely relaxed is

(a) 10 (b) 18 (c) 12 (d) 16

177. If the focal length of objective and eye lens are 1.2 *cm* and 3 *cm* respectively and the object is put 1.25 *cm* away from the objective lens and the final image is formed at infinity. The magnifying power of the microscope is

(a) 150 (b) 200 (c) 250 (d) 400

178. A compound microscope is adjusted for viewing the distant image of an object, the distance of the object from the object glass is now slightly increased, what re-adjustment of the instrument would be necessary for obtaining a distant image again

(a) Objective should be moved away from the eye-piece (b) Eye-piece should be moved towards the objective

(c) Both should be moved towards each other (d) Both should be moved away from each other

179. When the object is self-luminous, the resolving power of a microscope is given by the expression

(a) $\frac{2\mu\sin\theta}{\lambda}$ (b) $\frac{\mu\sin\theta}{\lambda}$ (c) $\frac{2\mu\cos\theta}{\lambda}$ (d) $\frac{2\mu}{\lambda}$

180. In a compound microscope, maximum magnification is obtained when the final image

(a) Is formed at infinity (b) Is formed at the least of distinct vision

(c) Coincides with the object (d) Coincides with the objective lens

181. How should people wearing spectacles work with a microscope

(a) They should keep on wearing their spectacles

(b) They should take off their spectacles

(c) They may keep on wearing or take off their spectacles, It makes no difference

(d) They cannot use a microscope at all

Telescope

182. The focal length of the objective and eyepiece of an astronomical telescope for normal adjustments are 50 *cm* and 5 *cm*. The length of the telescope should be **[MP PMT 2004]**

(a) 50 *cm* (b) 55 *cm* (c) 60 *cm* (d) 45 *cm*

183. The resolving power of an astronomical telescope is 0.2 *seconds*. If the central half portion of the objective lens is covered, the resolving power will be **[MP PMT 2004]**

(a) 0.1 *sec* (b) 0.2 *sec* (c) 1.0 *sec* (d) 0.6 *sec*

184. If F_o and F_e are the focal length of the objective and eye-piece respectively of a telescope, then its magnifying power will be

[CPMT 1977, 82, 97, 99, 2003; SCRA 1994; KCET (Engg./Med.) 1999; Pb. PMT 2000; BHU 2001; BCECE 2003, 2004]

(a) $F_o + F_e$ (b) $F_o \times F_e$ (c) F_o / F_e (d) $\frac{1}{2}(F_o + F_e)$

185. The length of an astronomical telescope for normal vision (relaxed eye) (f_o = focal length of objective lens and f_e = focal length of eye lens) is **[EAMCET (Med.) 1995; MP PAT 1996; CPMT 1999; BVP 2003]**

(a) $f_o \times f_e$ (b) $\frac{f_o}{f_e}$ (c) $f_o + f_e$ (d) $f_o - f_e$

186. A telescope of diameter 2*m* uses light of wavelength 5000 Å for viewing stars. The minimum angular separation between two stars whose image is just resolved by this telescope is **[MP PET 2003]**

(a) 4×10^{-4} *rad* (b) 0.25×10^{-6} *rad* (c) 0.31×10^{-6} *rad* (d) 5.0×10^{-3} *rad*

187. The aperture of the objective lens of a telescope is made large so as to **[AIEEE 2003; KCET 2003]**

(a) Increase the magnifying power of the telescope (b) Increase the resolving power of the telescope

(c) Make image aberration less (d) Focus on distant objects

188. The distance of the moon from earth is 3.8×10^5 *km*. The eye is most sensitive to light of wavelength 5500 Å. The separation of two points on the moon that can be resolved by a 500 *cm* telescope will be **[AMU (Med.) 2002]**

(a) 51 *m* (b) 60 *m* (c) 70 *m* (d) All of the above

189. To increase both the resolving power and magnifying power of a telescope **[Kerala PET 2002; KCET (Engg.) 2002]**

(a) Both the focal length and aperture of the objective has to be increased

(b) The focal length of the objective has to be increased

(c) The aperture of the objective has to be increased

(d) The wavelength of light has to be decreased

190. The focal lengths of the objective and eye lenses of a telescope are respectively 200*cm* and 5*cm*. The maximum magnifying power of the telescope will be **[MP PMT/PET 1998; JIPMER 2001, 2002]**

(a) – 40 (b) – 48 (c) – 60 (d) – 100

191. A telescope has an objective of focal length 50 *cm* and an eye piece of focal length 5 *cm*. The least distance of distinct vision is 25 *cm*. The telescope is focussed for distinct vision on a scale 200 *cm* away. The separation between the objective and the eye-piece is **[Kerala PET 2002]**

(a) 75 *cm* (b) 60 *cm* (c) 71 *cm* (d) 74 *cm*

192. In a laboratory four convex lenses L_1, L_2, L_3 and L_4 of focal lengths 2, 4, 6 and 8*cm* respectively are available. Two of these lenses form a telescope of length 10*cm* and magnifying power 4. The objective and eye lenses are **[MP PMT 2001]**

(a) L_2, L_3 (b) L_1, L_4 (c) L_3, L_2 (d) L_4, L_1

193. Four lenses of focal length + 15 *cm*, + 20 *cm*, + 150 *cm* and + 250 *cm* are available for making an astronomical telescope. To produce the largest magnification, the focal length of the eye-piece should be **[CPMT 2001; AIIMS 2001]**

(a) + 15 *cm* (b) + 20 *cm* (c) + 150 *cm* (d) + 250 *cm*

194. In a terrestrial telescope, the focal length of objective is 90 *cm*, of inverting lens is 5 *cm* and of eye lens is 6 *cm*. If the final image is at 30 *cm*, then the magnification will be **[DPMT 2001]**

(a) 21 (b) 12 (c) 18 (d) 15

195. The focal lengths of the objective and the eyepiece of an astronomical telescope are 20 *cm* and 5 *cm* respectively. If the final image is formed at a distance of 30 *cm* from the eye piece, find the separation between the lenses for distinct vision **[BHU (Med.) 2000]**

(a) 32.4 *cm* (b) 42.3 *cm* (c) 24.3 *cm* (d) 30.24 *cm*

196. Resolving power of reflecting type telescope increases with **[DPMT 2000]**

(a) Decrease in wavelength of incident light (b) Increase in wavelength of incident light

(c) Increase in diameter of objective lens (d) None of these

197. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 *m* and an eye-piece of focal length 2*cm* **[IIT-JEE 1992; Roorkee 2000]**

(a) The distance between the objective and the eye-piece is 16.02 *m*

(b) The angular magnification of the planet is 800

genius

(c) The image of the planet is inverted

(d) All of the above

198. The astronomical telescope consists of objective and eye-piece. The focal length of the objective is **[AIIMS 1998; BHU 2000]**

(a) Equal to that of the eye-piece (b) Greater than that of the eye-piece

(c) Shorter than that of the eye-piece (d) Five times shorter than that of the eye-piece

199. The diameter of the objective of a telescope is a, the magnifying power is m and wavelength of light is λ. The resolving power of the telescope is **[MP PMT 2000]**

(a) $(1.22\lambda)/a$ (b) $(1.22a)/\lambda$ (c) $\lambda m/(1.22a)$ (d) $a/(1.22\lambda m)$

200. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 *cm* and final image is formed at infinity. The focal lengths of the objective and eyepiece are respectively

[IIT-JEE 1989; MP PET 1995; JIPMER 2000]

(a) 20 *cm*, 16 *cm* (b) 50 *cm*, 10 *cm* (c) 30 *cm*, 6 *cm* (d) 45 *cm*, –9 *cm*

201. A photograph of the moon was taken with telescope. Later on, it was found that a housefly was sitting on the objective lens of the telescope. In photograph **[NCERT 1970; MP PET 1999]**

(a) The image of housefly will be reduced (b) There is a reduction in the intensity of the image

(c) There is an increase in the intensity of the image (d) The image of the housefly will be enlarged

202. The magnifying power of a telescope is M. If the focal length of eye piece is doubled, then the magnifying power will become

[Haryana CEET 1998]

(a) $2M$ (b) $M/2$ (c) $\sqrt{2}M$ (d) $3M$

203. The minimum magnifying power of a telescope is M. If the focal length of its eyelens is halved, the magnifying power will become

[MP PMT/PET 1998]

(a) $M/2$ (b) $2M$ (c) $3M$ (d) $4M$

204. The final image in an astronomical telescope is **[EAMCET (Engg.) 1998]**

(a) Real and errect (b) Virtual and inverted (c) Real and inverted (d) Virtual and errect

205. The astronomical telescope has two lenses of focal powers 0.5 D and 20 D. Its magnifying power will be **[CPMT 1997]**

(a) 40 (b) 10 (c) 100 (d) 35

206. An astronomical telescope of ten-fold angular magnification has a length of 44 *cm*. The focal length of the objective is**[CBSE PMT 19**

(a) 4 *cm* (b) 40 *cm* (c) 44 *cm* (d) 440 *cm*

207. A telescope consisting of an objective of focal length 100 *cm* and a single eyes lens of focal length 10 *cm* is focussed on a distant object in such a way that parallel rays emerge from the eye lens. If the object subtends an angle of 2° at the objective, the angular width of the image is **[JIPMER 1997]**

(a) 20° (b) 1/6° (c) 10° (d) 24°

208. When diameter of the aperture of the objective of an astronomical telescope is increased, its **[MP PMT 1997]**

(a) Magnifying power is increased and resolving power is decreased

(b) Magnifying power and resolving power both are increased

(c) Magnifying power remains the same but resolving power is increased

(d) Magnifying power and resolving power both are decreased

209. The focal length of objective and eye-piece of a telescope are 100 *cm* and 5 *cm* respectively. Final image is formed at least distance of distinct vision. The magnification of telescope is **[RPET 1997]**

(a) 20 (b) 24 (c) 30 (d) 36

210. A simple telescope, consisting of an objective of focal length 60 *cm* and single eye lens of focal length 5 *cm* is focussed on a distant object in such a way that parallel rays comes out from the eye lens. If the object subtends an angle 2° at the objective, the angular width of the image **[CPMT 1979; NCERT 1980; MP PET 1992; JIPMER 1997]**

(a) 10° (b) 24° (c) 50° (d) 1/6°

211. The diameter of the objective of the telescope is 0.1 *metre* and wavelength of light is 6000 Å. Its resolving power would be approximately **[MP PET 1997]**

(a) 7.32×10^{-6} *radian* (b) 1.36×10^{6} *radian* (c) 7.32×10^{-5} *radian* (d) 1.36×10^{5} *radian*

212. A Gallilean telescope has objective and eye-piece of focal lengths 200 *cm* and 2 *cm* respectively. The magnifying power of the telescope for normal vision is **[MP PMT 1996]**

(a) 90 (b) 100 (c) 108 (d) 198

213. All of the following statements are correct except **[Manipal MEE 1995]**

(a) The total focal length of an astronomical telescope is the sum of the focal lengths of its two lenses

(b) The image formed by the astronomical telescope is always erect because the effect of the combination of the two lenses its divergent

(c) The magnification of an astronomical telescope can be increased by decreasing the focal length of the eye-piece

(d) The magnifying power of the refracting type of astronomical telescope is the ratio of the focal length of the objective to that of the eye-piece

214. The length of a telescope is 36 *cm*. The focal length of its lenses can be **[Bihar MEE 1995]**

(a) 30 *cm*, 6 *cm* (b) – 30 *cm*, – 6 *cm* (c) – 30 *cm*, – 6 *cm* (d) – 30 *cm*, 6 *cm*

215. The diameter of the objective lens of telescope is 5.0 *m* and wavelength of light is 6000 Å. The limit of resolution of this telescope will be **[MP PMT 1994]**

(a) 0.03 *sec* (b) 3.03 *sec* (c) 0.06 *sec* (d) 0.15 *sec*

216. If tube length of astronomical telescope is 105 *cm* and magnifying power is 20 for normal setting, calculate the focal length of objective **[AFMC 1994]**

(a) 100 *cm* (b) 10 *cm* (c) 20 *cm* (d) 25 *cm*

217. Radio telescope is used to see **[AFMC 1994]**

(a) Distant start and planets (b) Sun and to measure its temperature

(c) Stars and to measures diameters (d) None of these

218. Four lenses with focal lens ± 15 *cm* and ± 150 *cm* are being placed for used as a telescopic objective. The focal length of the lens which produces the largest magnification with a given eye-piece is **[CBSE PMT 1994]**

(a) –15 *cm* (b) +150 *cm* (c) –150 *cm* (d) +15 *cm*

219. The image of a star (effectively a point source) is made by convergent lens of focal length 50 *cm* and diameter of aperture 5.0 *cm*. If the lens is ideal, and the effective wavelength in image formation is taken as 5×10^{-5} *cm*, the diameter of the image formed will be nearest to **[NSEP 1994]**

(a) Zero (b) 10^{-6} *cm* (c) 10^{-5} *cm* (d) 10^{-3} *cm*

220. To increase the magnifying power of telescope (f_o = focal length of the objective and f_e = focal length of the eye lens)

[MP PET/PMT 1988; MP PMT 1992, 94]

(a) f_o should be large and f_e should be small (b) f_o should be small and f_e should be large

(c) f_o and f_e both should be large (d) f_o and f_e both should be small

221. The limit of resolution of a 100 *cm* telescope $(\lambda = 5.5 \times 10^{-7}\ m)$ is **[BHU 1993]**

(a) 0.14" (b) 0.3" (c) 1′ (d) 1"

222. In a reflecting astronomical telescope, if the objective (a spherical mirror) is replaced by a parabolic mirror of the same focal length and aperture, then **[IIT-JEE 1993]**

(a) The final image will be erect (b) The larger image will be obtained

(c) The telescope will gather more light (d) Spherical aberration will be absent

223. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 *m* and an eyepiece of focal length 2 *cm* **[IIT-JEE 1993]**

(a) The distance between the objective and the eyepiece is 16.02 *m*

(b) The angular magnification of the planet is 800

(c) The image of the planet is inverted

(d) The objective is larger than the eyepiece

224. The average distance between the earth and moon is 38.6×10^4 *km*. The minimum separation between the two points on the surface of the moon that can be resolved by a telescope whose objective lens has a diameter of 5 *m* with λ = 6000 $\mathring{A}$ is **[MP PMT 1993]**

(a) 5.65 *m* (b) 28.25 *m* (c) 11.30 *m* (d) 56.51 *m*

225. The focal length of the objective and eye piece of a telescope are respectively 60 *cm* and 10 *cm*. The magnitude of the magnifying power when the image is formed at infinity is **[MP PET 1991]**

(a) 50 (b) 6 (c) 70 (d) 5

226. The focal length of an objective of a telescope is 3 *metre* and diameter 15 *cm*. Assuming for a normal eye, the diameter of the pupil is 3 *mm* for its complete use, the focal length of eye piece must be **[MP PET 1989]**

(a) 6 *cm* (b) 6.3 *cm* (c) 20 *cm* (d) 60 *cm*

227. An opera glass (Gallilean telescope) measures 9 *cm* from the objective to the eyepiece. The focal length of the objective is 15 *cm*. Its magnifying power is **[DPMT 1988]**

(a) 2.5 (b) 2/5 (c) 5/3 (d) 0.4

228. The focal length of objective and eye lens of a astronomical telescope are respectively 2 *m* and 5 *cm*. Final image is formed at (i) least distance of distinct vision (ii) infinity. The magnifying power in both cases will be **[MP PMT/PET 1988]**

(a) – 48, – 40 (b) – 40, – 48 (c) – 40, 48 (d) – 48, 40

229. An optical device that enables an observer to see over or around opaque objects, is called **[CPMT 1986]**

(a) Microscope (b) Telescope (c) Periscope (d) Hydrometer

230. The magnifying power of a telescope can be increased by **[CPMT 1979]**

(a) Increasing focal length of the system (b) Fitting eye piece of high power

(c) Fitting eye piece of low power (d) Increasing the distance of objects

231. An achromatic telescope objective is to be made by combining the lenses of flint and crown glasses. This proper choice is **[CPMT 1977]**

(a) Convergent of crown and divergent of flint (b) Divergent of crown and convergent of flint

(c) Both divergent (d) Both convergent

232. An observer looks at a tree of height 15 *m* with a telescope of magnifying power 10. To him, the tree appears **[CPMT 1975]**

(a) 10 times taller (b) 15 times taller (c) 10 times nearer (d) 15 times nearer

233. The magnification produced by an astronomical telescope for normal adjustment is 10 and the length of the telescope is 1.1 *m*. The magnification when the image is formed at least distance of distinct vision (*D* = 25 *cm*) is

(a) 14 (b) 6 (c) 16 (d) 18

234. The objective of a telescope has a focal length of 1.2 *m*. it is used to view a 10.0 *m* tall tower 2 *km* away. What is the height of the image of the tower formed by the objective

(a) 2 *mm* (b) 4 *mm* (c) 6 *mm* (d) 8 *mm*

235. A giant telescope in an observatory has an objective of focal length 19 *m* and an eye-piece of focal length 1.0 *cm*. In normal adjustment, the telescope is used to view the moon. What is the diameter of the image of the moon formed by the objective? The diameter of the moon is 3.5×10^6 *m* and the radius of the lunar orbit round the earth is 3.8×10^8 *m*

(a) 10 *cm* (b) 12.5 *cm* (c) 15 *cm* (d) 17.5 *cm*

236. The aperture of the largest telescope in the world is ≈ 5 *metre*. If the separation between the moon and the earth is $\approx 4 \times 10^5$ *km* and the wavelength of the visible light is $\cong 5000\ \mathring{A}$, then the minimum separation between the objects on the surface of the moon which can be just resolved is

(a) 1 *metre* approximately (b) 10 *metre* approximately (c) 50 *metre* approximately (d) 200 *metre* approximately

237. In Galileo's telescope, magnifying power for normal vision is 20 and power of eye-piece is –20 *D*. Distance between the objective and eye-piece should be

(a) 90 *cm* (b) 95 *cm* (c) 100 *cm* (d) 105 *cm*

238. The least resolve angle by a telescope using objective of aperture 5 *m* and light of wavelength = 4000 *A.U.* is nearly

(a) $\frac{1}{50}^{\circ}$ (b) $\frac{1}{50}$ *sec* (c) $\frac{1}{50}$ *minute* (d) $\frac{1}{500}$ *sec*

239. The limit of resolution of a 10 *cm* telescope for visible light of wavelength 6000 Å is approximately

(a) 0.1 *s* or arc (b) 30° (c) $\left(\frac{1}{6}\right)^{o}$ (d) None of these

240. An eye-piece of a telescope with a magnification of 100 has a power of 20 diopters. The object of this telescope has a power of

(a) 2 *diopters* (b) 0.2 *diopters* (c) 2000 *diopters* (d) 20 *diopters*

241. The Yerkes Observatory telescope has a large telescope with objective of diameter of about 1 *m*. Assuming wavelength of light to be $6\times10^{-7}m$, the angular distance θ between two stars which can just be resolved is

(a) $(7.3\times10^{-7})^{\circ}$ (b) 7.3×10^{-7} *rad* (c) $\frac{1}{40}$ of a second (d) None of these

242. A Galilean telescope measures 9 *cm* from the objective to the eye-piece. The focal length of the objective is 15 *cm*. Its magnifying power is

(a) 2.5 (b) 2/5 (c) 5/3 (d) 0.4

243. For seeing a cricket match, we prefer binoculars to the terrestrial telescope, because

(a) Binoculars give three-dimensional view (b) Terrestrial telescope gives inverted image

(c) To avoid chromatic aberration (d) To have larger magnification

244. A simple two lens telescope has an objective of focal length 50 *cm* and an eye-piece of 2.5 *cm*. The telescope is pointed at an object at a very large distance which subtends at an angle of 1 *milliradian* on the naked eye. The eye piece is adjusted so that the final virtual image is formed at infinity. The size of the real image formed by the objective is

(a) 5 *mm* (b) 1 *mm* (c) 0.5 *mm* (d) 0.1 *mm*

245. The objective of a telescope, after focussing for infinity is taken out and a slit of length *L* is placed in its position. A sharp image of the slit is formed by the eye-piece at a certain distance from it on the other side. The length of this image is *l*, then magnification of telescope is

(a) $\frac{l}{2L}$ (b) $\frac{2L}{l}$ (c) $\frac{l}{L}$ (d) $\frac{L}{l}$

246. An astronomical telescope in normal adjustment receives light from a distant source *S*. The tube length is now decreased slightly

(a) A virtual image of *S* will be formed at a finite distance

(b) No image will be formed

(c) A small, real image of *S* will be formed behind the eye-piece, close to it

(d) A large, real image of *S* will be formed behind the eye-piece, far away from it

247. A telescope consisting of object glass of power + 2 *D* and eye-glass of power + 20 *D* is focussed on an object 1*m* from the object glass. The final image is seen with completely relaxed eye. The magnifying power of the telescope is

(a) 20 (b) 41 (c) 24 (d) 49.2

248. An astronomical telescope and a Galilean telescope use identical objective lenses. They have the same magnification, when both are in normal adjustment. The eye-piece of the astronomical telescope has a focal length *f*

(a) The tube lengths of the two telescopes differ by *f* (b) The tube lengths of the two telescopes differ by 2*f*

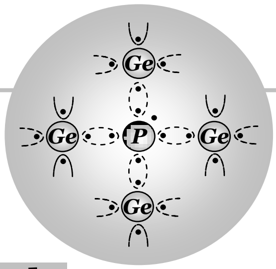
(c) The Galilean telescope has a shorter tube length (d) The Galilean telescope has a longer tube length

79	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**

A collection by Pradeep Kshetrapal for Physics students at genius, Maxwell and Gupta classes Only

a	d	b	a	c	a	b	b	c	a	b	b	d	b	a	d	d	a	c	a
99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118
b	c	c	c	b	b	c	b	c	a	b	b	c	b	d	c	c	a	a	b
119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138
a	d	a	c	a	d	c	b	d	b	c	c	b	a	d	a	c	a	b	d
139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158
c	b	d	d	c	a	a	b	c	b	d	a	d	d	d	a	c	b	b	d
159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178
d	d	d	c	b	c	a	a	b	c	c	b,d	a	b	d	c	d	b	b	a
179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198
b	b	b	c	c	c	c	b	a	a	b	c	d	a	c	c	a, c	d	b	d
199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218
c	b	b	b	b	a	b	a	c	b	b	d	b	b	a	a	a	a	b	d
219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238
a	a	d	a	d	b	a	a	a	c	b	a	c	a	c	d	c	b	b	a
239	240	241	242	243	244	245	246	247											
b	b	a	a	c	d	a	b	b, c											

genius



Semi-conductor Devices

Energy Bands

In isolated atom the valence electrons can exist only in one of the allowed orbitals each of a sharply defined energy called energy levels. But when two atoms are brought nearer to each other, there are alterations in energy levels and they spread in the form of bands.

Energy bands are of following types

(1) **Valence band**

The energy band formed by a series of energy levels containing valence electrons is known as valence band. At 0 *K*, the electrons fills the energy levels in valence band starting from lowest one.

(i) This band is always fulfill by electron.

(ii) This is the band of maximum energy.

(iii) Electrons are not capable of gaining energy from external electric field.

(iv) No flow of current due to such electrons.

(v) The highest energy level which can be occupied by an electron in valence band at 0 *K* is called fermi level.

(2) **Conduction band**

The higher energy level band is called the conduction band.

(i) It is also called empty band of minimum energy.

(ii) This band is partially filled by the electrons.

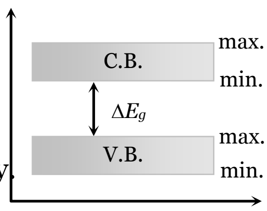
(iii) In this band the electrons can gain energy from external electric field.

(iv) The electrons in the conduction band are called the free electrons. They are able to move any where within the volume of the solid.

(v) Current flows due to such electrons.

(3) **Forbidden energy gap (ΔE_g)**

Energy gap between conduction band and valence band $\Delta E_g = (C.B.)_{\min} - (V.B.)_{\max}$



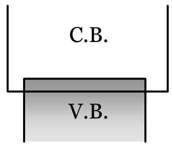
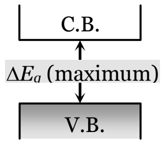
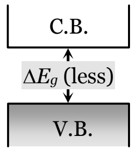
(i) No free electron present in forbidden energy gap.

(ii) Width of forbidden energy gap upon the nature of substance.

(iii) As temperature increases (↑), forbidden energy gap decreases (↓) very slightly.

Types of Solids

On the basis of band structure of crystals, solids are divided in three categories.

S.No.	Properties	Conductors	Insulators	Semiconductors
(1)	Electrical conductivity	10^2 to 10^8 $\mho/m$	10^{-8} $\mho/m$	10^{-5} to 10^0 $\mho/m$
(2)	Resistivity	10^{-2} to 10^{-8} Ω-m (negligible)	10^8 Ω-m	10^5 to 10^0 Ω-m
(3)	Band structure			
(4)	Energy gap	Zero or very small	Very large; for diamond it is 6 eV	For Ge E_g = 0.7 eV for Si E_g = 1.1 eV
(5)	Current carries	Free electrons	––	Free electrons and holes
(6)	Condition of V.B. and C.B. at ordinary temperature	V.B. and C.B. are completely filled or C.B. is some what empty	V.B. – completely filled C.B. – completely unfilled	V.B. – somewhat empty C.B. – somewhat filled
(7)	Temperature co-efficient of resistance (α)	Positive	Zero	Negative
(8)	Effect of temperature on conductivity	Decreases	—	Increases
(9)	Effect of temperature on resistance	Increases	—	Decreases
(11)	Examples	Cu, Ag, Au, Na, Pt, Hg etc.	Wood, plastic, mica, diamond, glass etc.	Ge, Si, Ga, As etc.
(12)	Electron density	$10^{29}/m^3$	—	Ge ~ 10^{19} $/m^3$ Si ~ 10^{16} $/m^3$

Holes in semiconductors

At absolute zero temperature (0 K) conduction band of semiconductor is completely empty and the semiconductor behaves as an insulator.

When temperature increases the valence electrons acquires thermal energy to jump to the conduction band (Due to the braking of covalent bond). If they jumps to C.B. they leaves behind the deficiency of electrons in the valence band. This deficiency of electron is known as ***hole*** or cotter. A hole is considered as a seat of positive charge, having magnitude of charge equal to that of an electron.

(1) Holes acts as virtual charge, although there is no physical charge on it.

(2) Effective mass of hole is more than electron.

(3) Mobility of hole is less than electron.

Types of Semiconductors

(1) **Intrinsic semiconductor**

A pure semiconductor is called intrinsic semiconductor. It has thermally generated current carriers

(i) They have four electrons in the outermost orbit of atom and atoms are held together by covalent bond

(ii) Free electrons and holes both are charge carriers and n_e (in C.B.) $= n_h$ (in V.B.)

(iii) The drift velocity of electrons (v_e) is greater than that of holes (v_h)

(iv) For them fermi energy level lies at the centre of the C.B. and V.B.

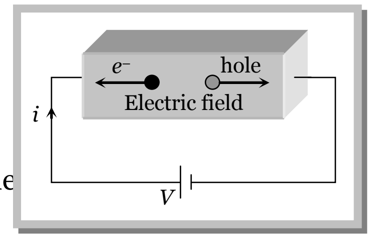
(v) In pure semiconductor, impurity must be less than 1 in 10^8 parts of semiconductor.

(vi) In intrinsic semiconductor $n_e^{(o)} = n_h^{(o)} = n_i = AT^{3/2}e^{-\Delta E_g/2KT}$; where $n_e^{(o)}$ = Electron density in conduction band, $n_h^{(o)}$ = Hole density in V.B., n_i = Density of intrinsic carriers.

(vii) Because of less number of charge carriers at room temperature, intrinsic semiconductors have low conductivity so they have no practical use.

Net current and conductivity

When some potential difference is applied across a piece of intrinsic semiconductor current flows in it due to both electron and holes *i.e.* $i = i_e + i_h \Rightarrow i = n_e eAv_e \quad i = eA[n_e v_e + n_h v_h]$



Hence conductivity of semiconductor $\sigma = e[n_e \mu_e + n_h \mu_h]$

where v_e = drift velocity of electron, v_h = drift velocity of holes,

E = Applied electric field $\mu_e = \dfrac{v_e}{E}$ = mobility of e^- and $\mu_n = \dfrac{v_h}{E}$ = mobility of hole

Note : ❑ $(ni)_{Ge} \simeq 2.4 \times 10^{19} /m^3$ and $(ni)_{Si} \simeq 1.5 \times 10^{16} /m^3$

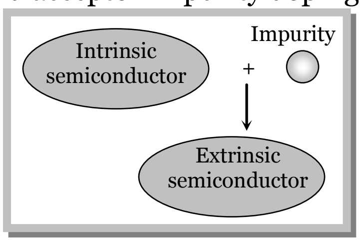
❑ At room temperature $\sigma_{Ge} > \sigma_{Si}$

❑ $\mu_e > \mu_h$

❑ Conductivity of semiconductor increases with temperature because number density of charge carriers increases.

❑ In a doped semiconductor, the number density of electrons and holes is not equal. But it can be established that $\boldsymbol{n_e n_h = n_i^2}$; where n_e, n_h are the number density of electrons and holes respectively and n_i is the number density of intrinsic curries (*i.e.* electrons or holes) in a pure semiconductor. This product is independent of donor and acceptor impurity doping.

(2) **Extrinsic semiconductor**



(i) It is also called impure semiconductor.

(ii) The process of adding impurity is called Doping.

(iii) Impurities are of two types :

Pentavalent impurity	Trivalent impurity
The elements whose atom has five valance impurities e.g. As, P, Sb etc. These are also called donor impurities. These impurities are also called donor impurities because they donates extra free electron.	*The elements whose each atom has three valance electrons are called trivalent impurities e.g. In, Ga, Al, B, etc. These impurities are also called acceptor impurities as they accept electron.*

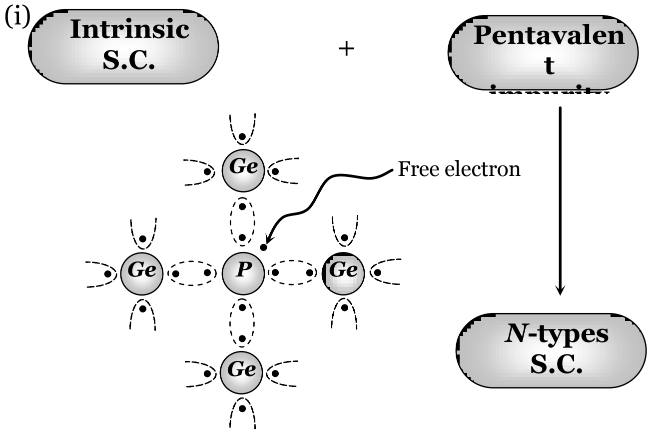
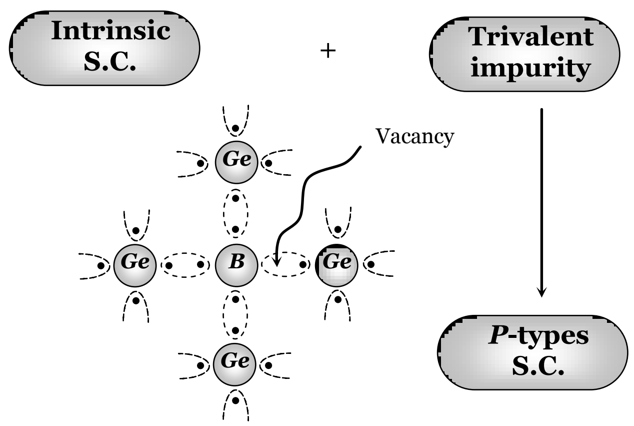
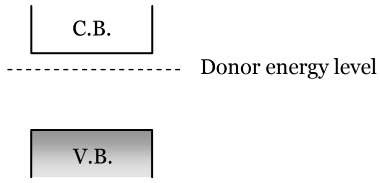
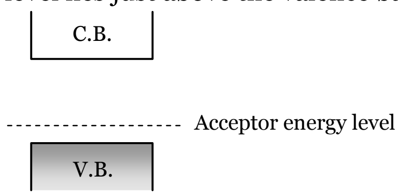
(iv) The number of atoms of impurity element is about 1 in 10^8 atoms of the semiconductor.

(v) $n_e \neq n_h$

(vi) In these fermi level shifts towards valence or conduction energy bands.

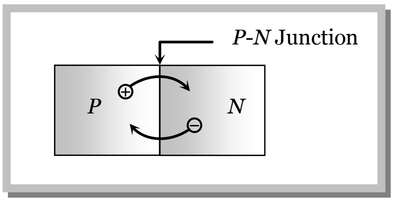
(vii) Their conductivity is high and they are practically used.

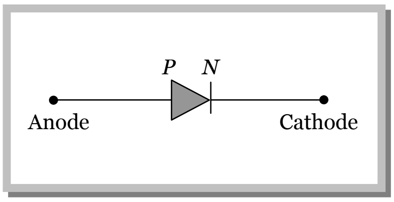
(3) **Types of extrinsic semiconductor**

***N*-type semiconductor**	***P*-type semiconductor**
(i) 	
(ii) Majority charge carriers – electrons Minority charge carriers – holes	Majority charge carriers – holes Minority charge carriers – electrons
(iii) $n_e >> n_h$; $i_e >> i_h$	$n_h >> n_e$; $i_h >> i_e$
(iv) Conductivity $\sigma \approx n_e \mu_e e$	Conductivity $\sigma \approx n_h \mu_h e$
(iv) *N*-type semiconductor is electrically neutral (not negatively charged)	*P*-type semiconductor is also electrically neutral (not positively charged)
(v) Impurity is called Donar impurity because one impurity atom generate one e^- .	Impurity is called Acceptor impurity.
(vi) Donor energy level lies just below the conduction band. 	Acceptor energy level lies just above the valence band. 

P-N Junction Diode

When a *P*-type semiconductor is suitably joined to an *N*-type semiconductor, then resulting arrangement is called *P-N* junction or *P-N* junction diode

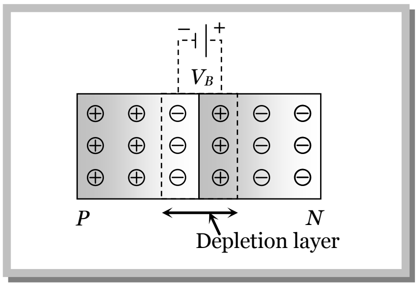




(1) **Depletion region**

On account of difference in concentration of charge carrier in the two sections of *P-N* junction, the electrons from *N*-region diffuse through the junction into *P*-region and the hole from *P* region diffuse into *N*-region.

Due to diffusion, neutrality of both N and P-type semiconductor is disturbed, a layer of negative charged ions appear near the junction in the P-crystal and a layer of positive ions appears near the junction in N-crystal. This layer is called depletion layer



(i) The thickness of depletion layer is 1 *micron* = 10^{-6} *m*.

(ii) Width of depletion layer $\propto \dfrac{1}{\text{Dopping}}$

(iii) Depletion is directly proportional to temperature.

(iv) The P-N junction diode is equivalent to capacitor in which the depletion layer acts as a dielectric.

(2) **Potential barrier**

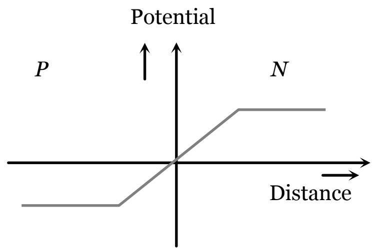
The potential difference created across the P-N junction due to the diffusion of electron and holes is called potential barrier.

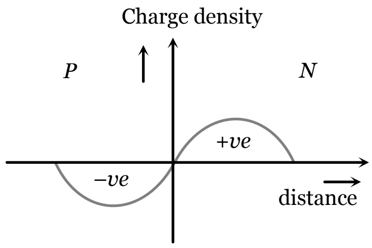
For Ge $V_B = 0.3V$ and for silicon $V_B = 0.7V$

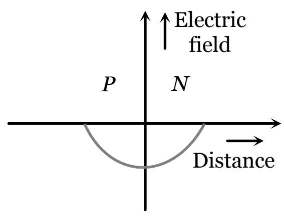
On the average the potential barrier in P-N junction is ~ 0.5 V and the width of depletion region ~ 10^{-6}.

So the barrier electric field $E = \dfrac{V}{d} = \dfrac{0.5}{10^{-6}} = 5 \times 10^{5}\ V/m$

Some important graphs







(3) **Diffusion and drift current**

Because of concentration difference holes/electron try to diffuse from their side to other side. Only these holes/electrons crosses the junction, having high kinetic energy. This diffusion results is an electric current from the P-side to the N-side known as diffusion current (i_{df})

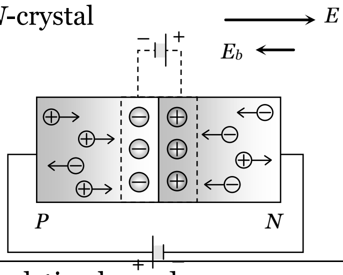
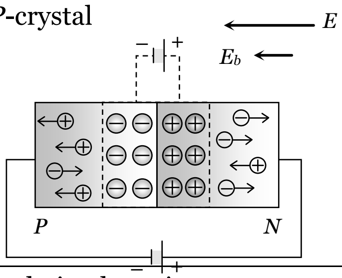
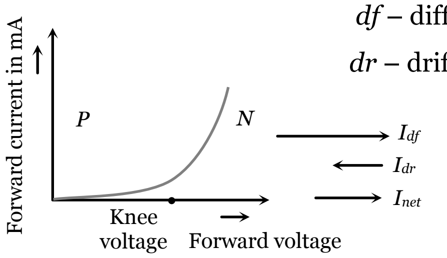
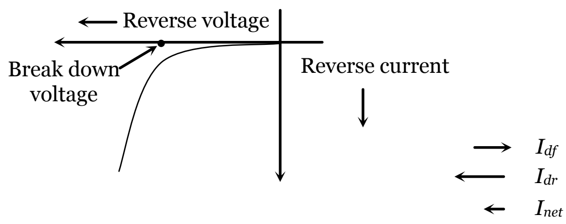
As electron hole pair (because of thermal collisions) are continuously created in the depletion region. These is a regular flow of electrons towards the N-side and of holes towards the P-side. This makes a current from the N-side to the P-side. This current is called the drift current (i_{dr}).

Note : ❑ In steady state $i_{df} = i_{dr}$ so $i_{net} = 0$

❑ When no external source is connected, diode is called unbiased.

(4) **Biasing**

Means the way of connecting *emf* source to *P-N* junction diode

<table>
<tr><th>Forward biasing</th><th>Reverse biasing</th></tr>
<tr><td>(i) Positive terminal of the battery is connected to the P-crystal and negative terminal of the battery is connected to N-crystal
</td><td>(i) Positive terminal of the battery is connected to the N-crystal and negative terminal of the battery is connected to P-crystal
</td></tr>
<tr><td>(ii) Width of depletion layer decreases</td><td>(ii) Width of depletion layer increases</td></tr>
<tr><td>(iii) $R_{\text{Forward}} \approx 10\Omega$ - 25Ω</td><td>(iii) $R_{\text{Reverse}} \approx 10^5\Omega$</td></tr>
<tr><td>(iv) Forward bias opposes the potential barrier and for $V > V_B$ a forward current is set up across the junction.</td><td>(iv) Reverse bias supports the potential barrier and no current flows across the junction due to the diffusion of the majority carriers.
(A very small reverse currents may exist in the circuit due to the drifting of minority carriers across the junction)</td></tr>
<tr><td>(v) Cut-in (Knee) voltage : The voltage at which the current starts to increase. For Ge it is 0.3 V and for Si it is 0.7 V.</td><td>(v) Break down voltage : Reverse voltage at which break down of semiconductor occurs. For Ge it is 25 V and for Si it is 35 V.</td></tr>
<tr><td>(vi) df – diffusion
dr – drift
</td><td>(vi)
</td></tr>
</table>

Reverse Breakdown and Special Purpose Diodes

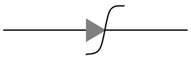
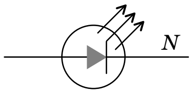
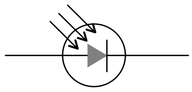
(1) **Zener breakdown**

When reverse bias is increased the electric field at the junction also increases. At some stage the electric field becomes so high that it breaks the covalent bonds creating electron, hole pairs. Thus a large number of carriers are generated. This causes a large current to flow. This mechanism is known as **Zener breakdown.**

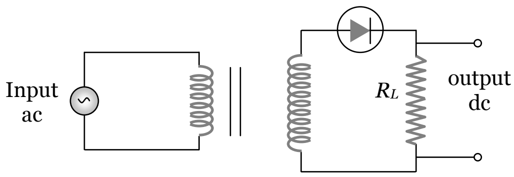
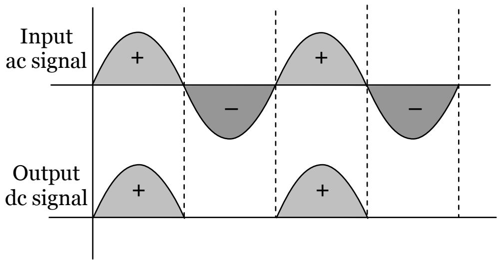
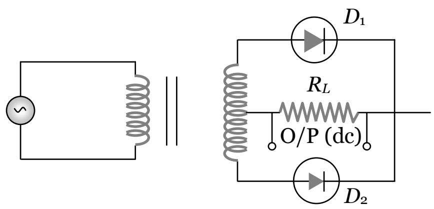
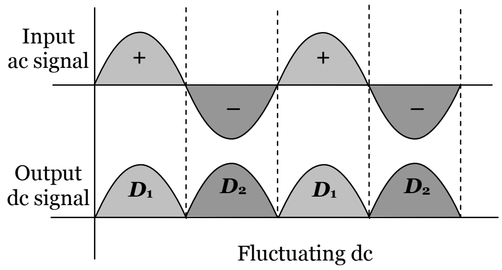
(2) **Avalanche breakdown**

At high reverse voltage, due to high electric field, the miniority charge carriers, while crossing the junction acquires very high velocities. These by collision breaks down the covalent bonds, generating more carriers. A chain reaction is established, giving rise to high current. This mechanism is called **avalanche breakdown.**

(3) **Special purpose diodes**

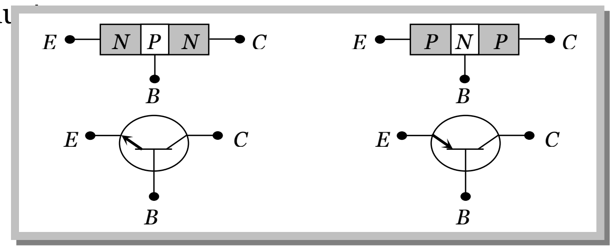
Zener diode	**Light emitting diode (LED)**	**Photo diode**	**Solar cells**
 It is a highly doped *p-n* junction which is not damaged by high reverse current. The breakdown voltage is made very sharp. In the forward bias, the zener diode acts as ordinary diode. It can be used as voltage regulator	 Specially designed diodes, which give out light radiations when forward biases. LED'S are made of *GaAsp*, *Gap* etc.	 In these diodes electron and hole pairs are created by junction photoelectric effect. That is the covalent bonds are broken by the EM radiations absorbed by the electron in the V.B. These are used for detecting light signals.	It is based on the photovoltaic effect. One of the semiconductor region is made so thin that the light incident on it reaches the *p-n* junction and gets absorbed. It converts solar energy into electrical energy.

P-N Junction Diode as a Rectifier

Half wave rectifier	**Full wave rectifier**
  During positive half cycle Diode ⟶ forward biased Output signal ⟶ obtained During negative half cycle Diode ⟶ reverse biased Output signal ⟶ not obtained	  Fluctuating dc During positive half cycle Diode : D_1 ⟶ forward biased D_2 ⟶ reverse biased Output signal ⟶ obtained due to D_1 only During negative half cycle Diode : D_1 ⟶ reverse biased D_2 ⟶ forward biased Output signal ⟶ obtained due to D_2 only **Note** : ❑ Fluctuating dc → Filter → constant dc.

Transistor

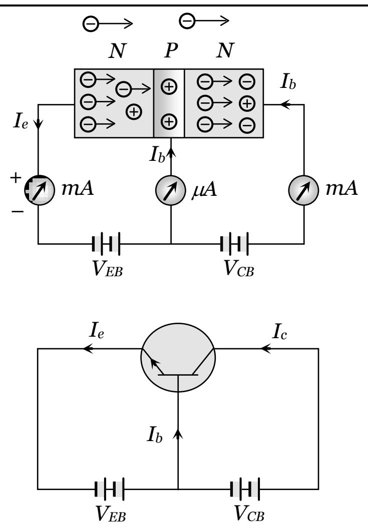
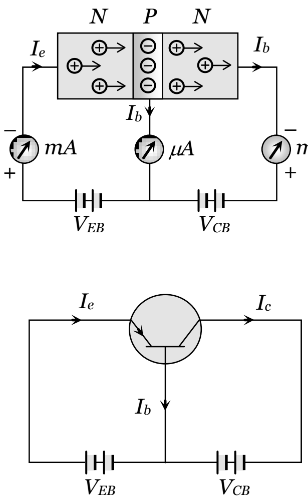
A junction transistor is formed by sandwiching a thin layer of *P*-type semiconductor between two *N*-type semiconductors or by sandwiching a thin layer of *n*-type semiconductor between two *P*-type semicondu



E – Emitter (emits majority charge carriers)
C – Collects majority charge carriers
B – Base (provide proper interaction between E and *C*)

Note : In normal operation base-emitter is forward biased and collector base junction is reverse biased.

(1) **Working of Transistor :** In both transistor emitter - base junction is forward biased and collector – base junction is reverse biased.

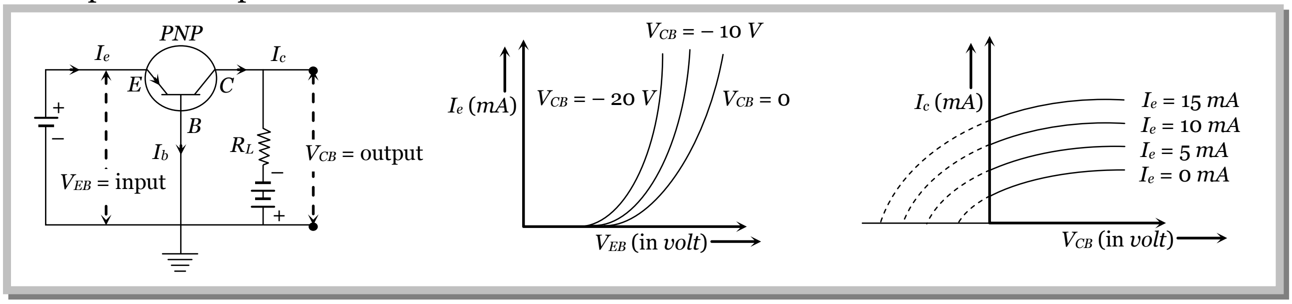
***NPN* – transistor**	***PNP* – transistor**
	
5% emitter electron combine with the holes in the base region resulting in small base current. Remaining 95% electrons enter the collector region.	5% emitter holes combine with the electrons in the base region resulting in small base current. Remaining 95% holes enter the collector region.
$I_e > I_c$, and $I_c = I_b + I_c$	$I_e > I_c$, and $I_c = I_b + I_c$

Note : ❑ In a transistor circuit the reverse bias is high as compared to the forward bias. So that it may exert a large attractive force on the charge carriers to enter the collector region.

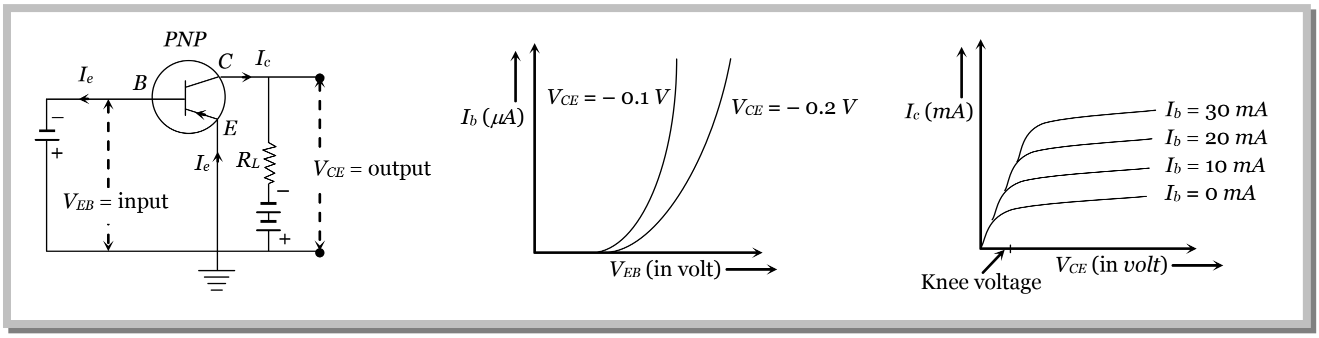
(2) **Characteristics of transistors :** A transistor can be connected in a circuit in the following three different configurations.

(i) Common base (CB) (ii) Common emitter (CE) (iii) Common collector (CC)

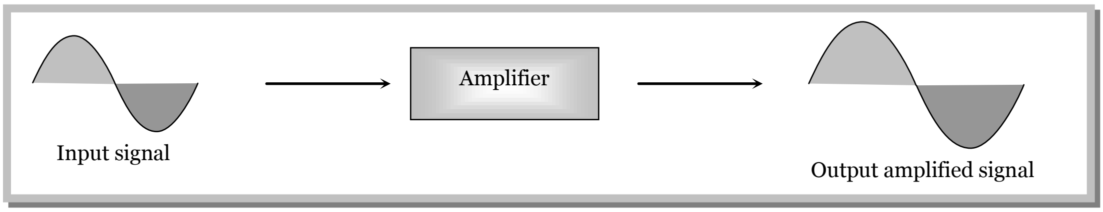
(i) CB characteristics : The graphs between voltages and currents when base of a transistor is common to input and output circuits are known as CB characteristic of a transistor.



(ii) CE characteristics : The graphs between voltages an d currents when emitter of a transistor is common to input and output circuits are known as CE characteristics of a transistor.

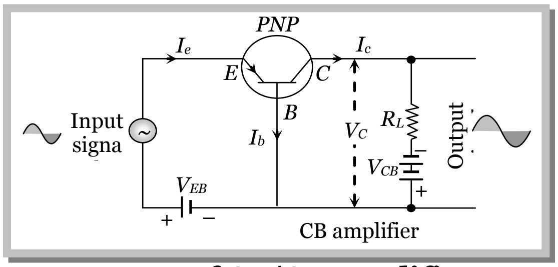


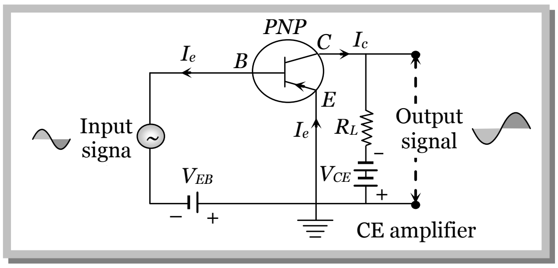
(3) **Transistor as an amplifier :** A device which increases the amplitude of the input signal is called amplifier.



The transistor can be used as an amplifier in the following three configuration

(i) CB amplifier (ii) CE amplifier (iii) CC amplifier





(4) **Parameters of CE/CB amplifiers**

Transistor as C.E. amplifier	**Transistor as C.B. amplifier**
(i) Current gain (α) (a) $\alpha_{ac} = \dfrac{\text{Small change in collector current } (\Delta i_c)}{\text{Small change in collector current } (\Delta i_e)}$; V_B (constant) (b) $\alpha_{dc}(\text{or}\,\alpha) = \dfrac{\text{Collector current}(i_c)}{\text{Emitter current } (i_e)}$ valve of α_{dc} lies between 0.95 to 0.99	(i) Current gain (β) (a) $\beta_{ac} = \left(\dfrac{\Delta i_c}{\Delta i_b}\right)$ V_{CE} = constant (b) $\beta_{dc} = \dfrac{i_c}{i_b}$ valve of β_{ac} lies between 15 and 20
(ii) Voltage gain $A_v = \dfrac{\text{Change in output voltage}(\Delta V_o)}{\text{Change in input voltage } (\Delta V_i)}$ $\Rightarrow A_v = \alpha_{ac} \times$ Resistance gain	(ii) Voltage gain $A_v = \dfrac{\Delta V_o}{\Delta V_i} = \beta_{ac} \times \text{Resistance gain}$
(iii) Power gain $= \dfrac{\text{Change in output power}(\Delta P_o)}{\text{Change in input power}(\Delta P_c)}$ $\Rightarrow$ Power gain $= \alpha_{ac}^2 \times$ Resistance gain	(iii) Power gain $= \dfrac{\Delta P_o}{\Delta P_i} = \beta_{ac}^2 \times \text{Resistance gain}$

Note : ❑ **Trans conductance (*gm*) :** The ratio of the change in collector current to the change in emitter base voltage is called trans conductance. *i.e.* $g_m = \frac{\Delta i_c}{\Delta V_{EB}}$. Also $g_m = \frac{A_V}{R_L}$; R_L = Load resistance

(5) **Relation between α and β :** $\beta = \frac{\alpha}{1-\alpha}$ or $\alpha = \frac{\beta}{1+\beta}$

(6) **Comparison between *CB*, *CE* and *CC* amplifier**

S.No.	Characteristic	Amplifier		
		CB	*CE*	*CC*
(i)	Input resistance (R_i)	≈ 50 to 200 Ω low	≈ 1 to 2 *k*Ω medium	≈ 150 – 800 *k*Ω high
(ii)	Output resistance (R_o)	≈ 1 – 2 *k*Ω high	≈ 50 *k*Ω medium	≈ *k*Ω low
(iii)	Current gain	0.8 – 0.9 low	20 – 200 high	20 – 200 high
(iv)	Voltage gain	Medium	High	Low
(v)	Power gain	Medium	High	Low
(vi)	Phase difference between input and output voltages	Zero	180°	Zero
(vii)	Used as amplifier for	current	Power	Voltage

Example

***Example*: 2** A *Ge* specimen is doped with *Al*. The concentration of acceptor atoms is ~10^{21} *atoms/m*^{3}. Given that the intrinsic concentration of electron hole pairs is ~ $10^{19}/m^3$, the concentration of electrons in the specimen is **[AIIMS 2004]**

(a) $10^{17}/m^3$ (b) $10^{15}/m^3$ (c) $10^4/m^3$ (d) $10^2/m^3$

Solution : (a) $n_i^2 = n_h n_e \Rightarrow (10^{19})^2 = 10^{21} \times n_e \Rightarrow n_e = 10^{17}\ /m^3$.

***Example*: 3** A silicon specimen is made into a *P*-type semi-conductor by doping, on an average, one Indium atom per 5×10^7 silicon atoms. If the number density of atoms in the silicon specimen is 5×10^{28} *atoms/m*^{3}, then the number of acceptor atoms in silicon will be

(a) 2.5×10^{30} *atoms/cm*^{3} (b) 1.0×10^{13} *atoms/cm*^{3} (c) 1.0×10^{15} *atoms/cm*^{3} (d) 2.5×10^{36} *atoms/cm*^{3}

Solution : (c) Number density of atoms in silicon specimen = 5×10^{28} *atom/m*^{3} = 5×10^{22} *atom/cm*^{3}

Since one atom of indium is doped in 5×10^7 *Si* atom. So number of indium atoms doped per *cm*$^{-3}$ of silicon.

$$n = \frac{5\times10^{22}}{5\times10^7} = 1\times10^{15}\ atom/cm^3.$$

***Example*: 4** A *P*-type semiconductor has acceptor levels 57 *meV* above the valence band. The maximum wavelength of light required to create a hole is (Planck's constant $h = 6.6\times10^{-34}$ *J-s*)

(a) 57 *Å* (b) 57×10^{-3} *Å* (c) 217100 *Å* (d) 11.61×10^{-33} *Å*

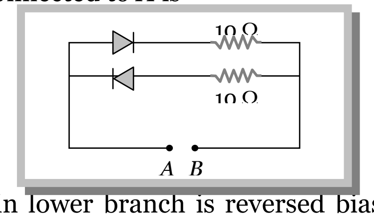
Solution : (c) $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.6\times10^{-34}\times3\times10^8}{57\times10^{-3}\times1.6\times10^{-19}} = 217100$ Å.

***Example*: 5** A potential barrier of 0.50*V* exists across a *P-N* junction. If the depletion region is $5.0\times10^{-7}\ m$ wide, the intensity of the electric field in this region is **[UPSEAT 2002]**

(a) $1.0\times10^6\ V/m$ (b) $1.0\times10^5\ V/m$ (c) $2.0\times10^5\ V/m$ (d) $2.0\times10^6\ V/m$

Solution : (a) $E = \frac{V}{d} = \frac{0.50}{5 \times 10^{-7}} = 1 \times 10^6 \ V/m.$

Example: 6 A 2V battery is connected across the points A and B as shown in the figure given below. Assuming that the resistance of each diode is zero in forward bias and infinity in reverse bias, the current supplied by the battery when its positive terminal is connected to A is

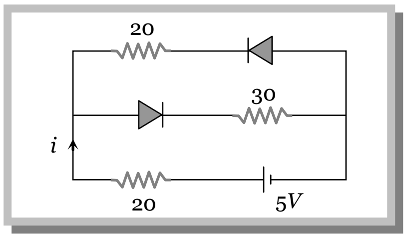


(a) 0.2 A
(b) 0.4 A
(c) Zero
(d) 0.1 A

Solution : (a) Since diode in upper branch is forward biased and in lower branch is reversed biased. So current through circuit $i = \frac{V}{R + r_d}$; here r_d = diode resistance in forward biasing = 0

So $i = \frac{V}{R} = \frac{2}{10} = 0.2 A$.

Example: 7 Current in the circuit will be

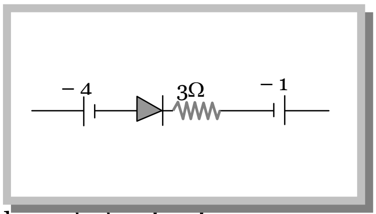


(a) $\frac{5}{40} A$
(b) $\frac{5}{50} A$
(c) $\frac{5}{10} A$
(d) $\frac{5}{20} A$

Solution : (b) The diode in lower branch is forward biased and diode in upper branch is reverse biased

$\therefore \quad i = \frac{5}{20 + 30} = \frac{5}{50} A$

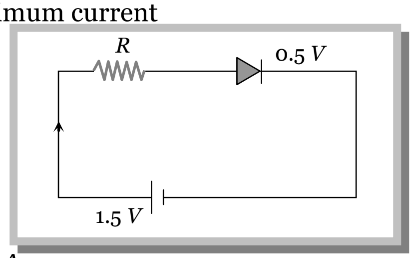
Example: 8 Find the magnitude of current in the following circuit **[RPMT 2001]**



(a) 0
(b) 1 *amp*
(c) 0.1 *amp*
(d) 0.2 *amp*

Solution : (a) Diode is reverse biased. Therefore no current will flow through the circuit.

Example: 9 The diode used in the circuit shown in the figure has a constant voltage drop of 0.5 V at all currents and a maximum power rating of 100 *milliwatts*. What should be the value of the resistor R, connected in series with the diode for obtaining maximum current



(a) 1.5 Ω
(b) 5 Ω
(c) 6.67 Ω
(d) 200 Ω

Solution : (b) The current through circuit $i = \frac{P}{V} = \frac{100 \times 10^{-3}}{0.5} = 0.2 A$

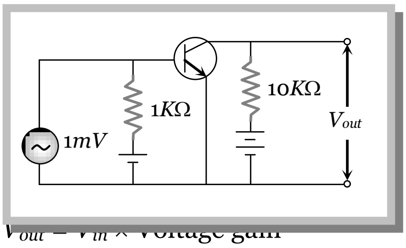
$\therefore$ voltage drop across resistance = 1.5 – 0.5 = 1 V $\Rightarrow R = \frac{1}{0.2} = 5\,\Omega$

Example: 10 For a transistor amplifier in common emitter configuration for load impedance of 1 $k\Omega$ (h_{fe} = 50 and h_{oe} = 25) the current gain is

(a) – 5.2 (b) – 15.7 (c) – 24.8 (d) – 48.78

Solution : (d) In common emitter configuration current gain $A_i = \dfrac{-h_{fe}}{1 + h_{oe} R_L} = \dfrac{-50}{1 + 25 \times 10^{-6} \times 10^3} = -48.78.$

***Example*: 11** In the following common emitter configuration an *NPN* transistor with current gain $\beta = 100$ is used. The output voltage of the amplifier will be **[AIIMS 2003]**



(a) 10 mV
(b) 0.1 V
(c) 1.0 V
(d) 10 V

Solution : (c) Voltage gain $= \dfrac{\text{Output voltage}}{\text{Input voltage}}$ $\Rightarrow V_{out} = V_{in} \times$ Voltage gain

$\Rightarrow V_{out} = V_{in} \times$ Current gain $\times$ Resistance gain $= V_{in} \times \beta \times \dfrac{R_L}{R_{BE}} = 10^{-3} \times 100 \times \dfrac{10}{1} = 1V.$

***Example*: 12** While a collector to emitter voltage is constant in a transistor, the collector current changes by 8.2 mA when the emitter current changes by 8.3 mA. The value of forward current ratio h_{fe} is

(a) 82 (b) 83 (c) 8.2 (d) 8.3

Solution : (a) $h_{fe} = \left(\dfrac{\Delta i_c}{\Delta i_b}\right)_{V_{ce}} = \dfrac{8.2}{8.3 - 8.2} = 82$

***Example*: 13** The transfer ratio of a transistor is 50. The input resistance of the transistor when used in the common-emitter configuration is 1 $K\Omega$. The peak value for an ac input voltage of 0.01 V peak is

(a) 100 μA (b) 0.01 mA (c) 0.25 mA (d) 500 μA

Solution : (d) $i_c = \beta i_b = \beta \times \dfrac{V_i}{R_i} = 50 \times \dfrac{0.01}{1000} = 500 \times 10^{-6} A = 500 \mu A$

***Example*: 14** In a common base amplifier circuit, calculate the change in base current if that in the emitter current is 2 mA and $\alpha = 0.98$ **[BHU 1995]**

(a) 0.04 mA (b) 1.96 mA (c) 0.98 mA (d) 2 mA

Solution : (a) $\Delta i_c = \alpha \Delta i_e = 0.98 \times 2 = 196\, mA$

$\therefore \Delta i_b = \Delta i_e - \Delta i_c = 2 - 1.96 = 0.04\, mA$.

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Revision Notes

1. Magnetic flux. Magnetic flus if denoted by (ϕ)

$$\phi = \vec{A} . \vec{B}$$

$$\phi = AB \cos \theta$$

Unit. Wb. or Tesla m^2. Thus 1 T = 1 W b m^2.

1. Faraday's laws of electromagnetic induction. On the basis of his experiments, Faraday gave the following laws :

(*i*) *Whenever magnetic flux linked with a circuit changes, induced e.m.f. is produced.*

(*ii*) *The induced e.m.f. lasts as long as the change in the magnetic flux continues.*

(*iii*) *The magnitude of induced e.m.f. is directly proportional to the rate of change of magnetic flux.*

Key Point

The magnetic flux linked with a loop does not change with time when

(*i*) When both loop and magnet move in same direction with same velocity.

(*ii*) magnet is rotated about its axis keeping its position from loop unchanged.

(*iii*) loop is rotated in a uniform magnet field keeping it fully within the field.

The magnitude of induced e.m.f. is given by

$$e = \frac{\phi_2 - \phi_1}{dt}.$$

where ϕ_1 and ϕ_2 are magnetic flux linked with the coil initially and after time t. The negative sign shows that induced e.m.f. opposes the change taking plane in magnetic flux/

In differential notation,

$$e = -\frac{d\phi}{dt}.$$

In CGS system, e is measured in *e.m.u.* and ϕ in maxwell while in SI system, e is measured in volt and ϕ in weber.

Note. It may be remembered that

$$1 \text{ volt} = 10^8 \text{ e.m.u. of potential,}$$

$$1 \text{ ampere} = \frac{1}{10} \textit{ e.m.u.} \text{ of current,}$$

$$1 \text{ coulomb} = \frac{1}{10} \textit{ e.m.u.} \text{ of charge,}$$

and $$1 \text{ ohm} = 10^9 \text{ e.m.u. of resistance.}$$

(*a*) **Induced current.** If the coil is a closed circuit and has a resistance R, then induced current

$$i = \frac{e}{R} = \frac{N}{R}\frac{d\phi}{dt} \text{(N = no. of turns in the coil)}$$

(*b*) **Induced charge.** Incuded charge is given by :

$$q = i \times t = \frac{N\phi}{R} \text{ where } \phi = \text{change of flux.}$$

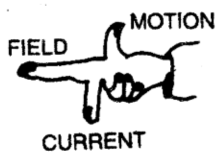
Also $$q = -\frac{N(\phi_2 - \phi_1)}{Rt} \times t$$

$$q = -\frac{N(\phi_2 - \phi_1)}{R}$$

$$q = \frac{e}{R} \times t \qquad q = \frac{N(\phi_2 - \phi_1)\,t}{R}$$

This shows that induced change is independent of **time interval.**

(*c*) **Len's law.** The direction of induced *e.m.f.* due to electromagnetic induction is such that its effect opposes the cause which has produced it.



(*d*) **Fleming's right hand rile.** *If thumb, fore–finger and the middle finger are spread perpendicular to one another (in two different⊥ planes) such that the fore–finger denotes the direction oif magnetic field and thumb, the direction of motion then the middle finger denotes the direction of induced e.m.f.*

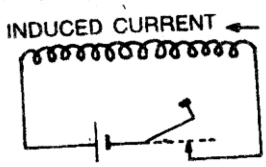
3. E.M.F. induced in a moving conductor. If a straight conductor of length l is moving perpendicular to a uniform magnetic field of flux density B with a velocity v, then

Infuced *e.m.f.* $e = Blv$

If R is the resistance of the conductor, then

Induced current $$i = \frac{e}{R} = \frac{Blv}{R}$$

4. Self inductance or coefficient of self induction. The magnetic flux linked with a coil through which current I is flowing is given by



$$\phi = LI$$

where L is called self inductance or coefficient of self induction of the coil.

The coefficient of self induction of a coil is numerically equal to the magnetic flux linked with it, when unit current flowes through it.

The instantaneous induced e.m.f. produced in the coil is given by

$$e = -L\frac{dI}{dt}$$

where $\frac{dI}{dt}$ is rate of change of current at that instant,

The coefficient of self induction of a coil is also numerically equal to the induced e.m.f set up in the coil, when the rate of change of current in the coil is unity.

The unit of self inductance is e.m.u. in CGS system and henry in SI.

Self inductance of a solenoid. Self inductance of a solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

$L = \mu_r\mu_0 n^2 lA$ where m_r is relative permeability of material of core used.

where μ_0 = permeability of free space.

N = Total no. of turns and n = no. of turns per unit length.

l = length of the coil.

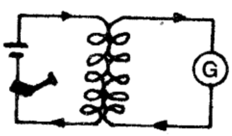
A = area of cross-section of coil.

Also $$L = \frac{\mu_0 n^2 l^2 A}{l} = \mu_0 n^2 lA.$$

5. Mutual inductance or coefficient of mutual induction. The magnetic flux linked with one coil when a current I flows through a neighbouring coil is given by

$$\phi = MI$$

where M is called *mutual inductance or coefficient of mutual induction between the two coils.*



The coefficient of mutual induction between two coils is numerically equal to the magnetic flux linked with one coil, when unit current flows through the neighbouring coil.

The instantaneous induced *e.m.f.* produced in one coil is given by

$$e = -M\frac{dI}{dt}$$

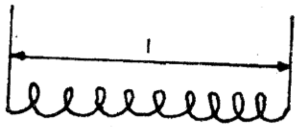
where $\frac{dI}{dt}$ is rate to change of current in the neighbouring coil at that instant.

The coefficient of mutual induction between two coils is also numerically equal to the induced e.m.f. set up in one coil, when the rate of change of current in the neighbouring coil is unity.

In CGS system, the unit of mutual inductance is *e.m.u.* and in S.I., the unit is *henry*.
1 henry = 10^9 *e.m.u.* of mutual inductance.

> Remember
>
> Self induction is called inertia of electricity.

Mutual inductance between two coils. The mutual inductance between two coils of area A, no. of turns N_1 and N_2 with length of secondary or primary as l is given by :

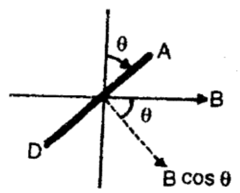


$$M = -\mu_0\frac{N_1N_2A}{l}$$

If n_1 and n_2 are the no. of turns per unit length in the two coils,
then $M = \mu_0 n_1 n_2 A l$. $\quad (\because N_1 = n_1 l$ and $N_2 = n_2 l)$

Note. In CGS system I, e, ϕ and M are measured in *e.m.u.* of current, *e.m.u.* of *e.m.f.*, maxwell and e.m.u. of mutual induction respectively, whele in SI, they are respectively measured in ampere, volt, weber and henry.

6. Induced emf produced in a coil rotating inside a magnetic field (a.c generator). Consider coil of area A, number of induction turns N and rotating inside a magnetic field of induction B with angular velocity ω. At $t = 0$, the coil is vertical. At any time t, the plane of coil will make an angle θ equal to ωt with the vertical.



The induced *e.m.f.* produced in the coil at time t given by

$$e = NAB\omega \sin \omega t$$

where N is frequency and T is time period of rotation.

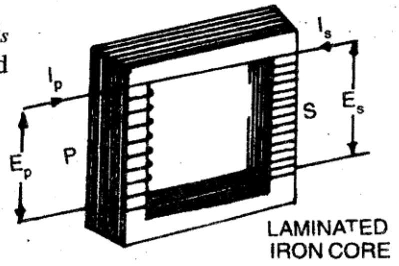
$$e_{max} = NAB\omega$$

Thus $$e = e_{max}\sin \omega t.$$

7. Transformer. *A transformer is a device of changing a low voltage alternating current into a high voltage alternating current or vice-versa.*

A transformer which increases the voltage (current will decrease), is called **step-up transformer** while another while which decreases the voltage (current will increases) is **step-down transformer.**

Suppose a transformer consists of a primary of n_p turns and the secondary coil of n_s turns [Fig.]. Let E_p and E_s be the values of *e.m.f.* across primary and secondary coil and I_p and I_s be the respective values of the current.



Then, $\frac{E_s}{E_p} = \frac{n_s}{n_p} = \frac{I_p}{I_s} = k$ where k is known as transformation ratio.

For a step-up transformer, $k > 1$ and for a step down transformer, $k > 1$
For a 100% efficient (ideal) transformer, Input power = Output power *i.e.*,

$$E_pI_p = E_sI_s.$$

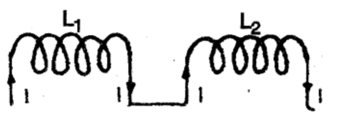
8. (*i*) An electric field produced by time varying magnetic field, which has non-vanishing closed line integral is called as non-conservative field.Here $\oint \vec{E}\, \vec{dl} \neq 0$,

(*ii*) In conservative fields $\oint E\,.\,dl = 0$.

(*iii*) The direction of induced current is given by Fleming's right hand rule.

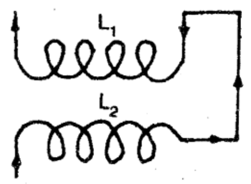
(*iv*) The circulating currents induced in metal sheets, blocks when the magnetic flux linked with them changes are called eddy currents or Focault current

(*v*) If coils are in series as shown in (a) the

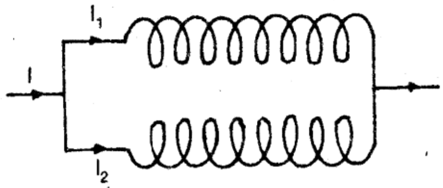


$$L = L_1 + L_2 + 2M$$

If the coils are as shown in (b), then

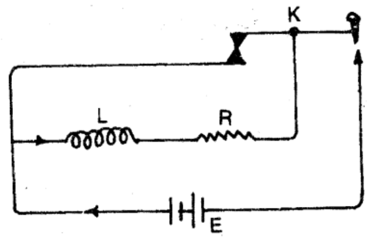


$$L = L_1 + L_2 - 2M.$$



$$\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}.$$

9. (*i*) Growth and decay of current in L–R circuit. During growth of current in L–R circuit, let R be the inductanceless resistance and L the resistanceless inductance. If I is the instantaneous value of the current at any time t, then



The maximum value of current $I_m=\frac{E}{R}$

The current I at any instant is given by

$$I=I_m\left(1-e^{-\frac{R}{L}t}\right)$$

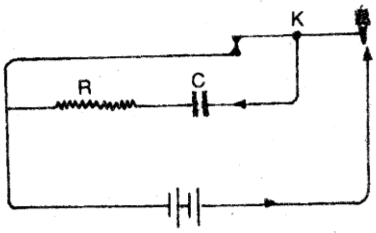
The expression L/R is called the time constant and is measured in seconds.

***(ii)* Decay of current.** On switching off the circuit without introducing and additional resistance, the current takes some time to decay from maximum to zero value. The current at any instant is given by

$$I=I_m\,e^{-\frac{R}{L}t}$$

The value of current I after L/2, 2L/R, 3L/R ...... is given by 0.3679 I_m, 0.1357 I_m, 0.0498 I_m ...... etc.

10. Charging and discharging of a condenser. (*i*) When a circuit containing capacitance and resistance in series with a battery is switched on the charge grows from zero to maximum value through the capacitor in a certain time. If q is the instantaneous value of charge and q_m the maximum value of charge.



Maximum value of charge $q_m=EC$

The instantaneous value of charge q is given by

$$q=q_m\left(1-e^{\frac{-t}{RC}}\right)$$

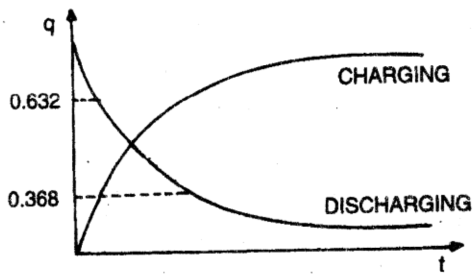
(*ii*) Similarly when the circuit is switched off without introducing any additional resistance, then the charge takes some time to decay from maximum to zero value. The value of charge at any instant is given by

$$q=q_m\cdot e^{\frac{-t}{RC}}$$

In both these cases RC is called the **time constant** as it has the dimensions of time.

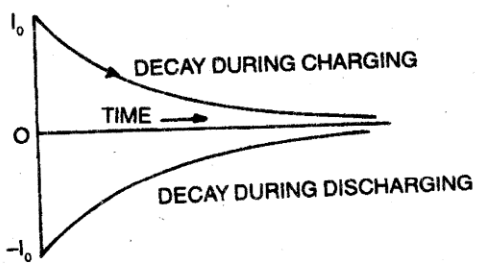
(*iii*) After t = 5RC, the capacitor gets almost fully charged.

(*iv*) Discharging of a capacitor $q=q_0e^{-t/RC}$ is graphically shows as



RC is time constant = 0.368 q_0

(*v*) Decay of current during charging and discharging is shown in Fig. below



11. Energy stored in an inductance coil. When the current grows in an inductance, work has to be done in establishing the current in it. This work is stored into the inductance as magnetic energy. When the circuit is broken, this energy is liberated.The energy stored at the make and liberated at the break is given by

$$\text{Energy} = \frac{1}{2} LI_m^2$$

$$\text{Energy at any instant} = \frac{1}{2} LI^2$$

If L is in henry and I in amperes, then energy is in Joules.

12. Circuit containing inductance and capacitance. When a condenser is allowed to discharge through an inductance, the discharge is oscillatory and is according to S.H.M. equation.

$$q = q_m \cos \omega t$$

where $$\omega = \frac{1}{\sqrt{LC}}$$

L being the inductance and C the capacitance of the circuit. The period of oscillation is given by

$$T = 2\pi \sqrt{LC}$$

and the frequency $n = \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}}$ and is called the natural frequency of L-C circuit.

13. Maximum or peak value of alternating voltage and alternating current. *The maximum value of e.m.f. in either direction is called the* **peak value** *of alternating e.m.f. It is given by*

$$E = E_0 \sin \omega t$$

where E is the instantaneous value and $\omega = 2\pi n$ in which n is the frequency of A.C.

Maximum value of the current I_0 in either direction is called the **peak value** *of the alternating current.*

$$I = I_0 \sin \omega t$$

where I is the instantaneous value of current.

14. Mean or average value of A.C. voltage and current. *The average or mean value of current is that steady current which sends the same charge through a circuit in the same time as the alternating current does in half the time period.* If I_m denotes the mean value, then

$$I_m = \frac{2I_0}{\pi} = 0.637\, I_0$$

Similarly the mean or average voltage E_m is given by

$$E_m = \frac{2E_0}{\pi} = 0.637\, E_m$$

15. Root mean square value or virtual value. *It is that steady current or voltage which produces the same heating effect in a resistance in a given time as the A.C. does in the same resistance in the same time.*

> **Key Point**
>
> D.C. ammeter and volt-meter are not capable of measuring A.C. currents of voltages. They will give zero reading when used in a.c. circuit. It is due to reason that mean value of A.C. current and voltages is zero over complete cycle.

If I_p denotes the virtual value of current, then

$$I_p = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

Similarly $$E_p = \frac{E_0}{\sqrt{2}} = 0.707\, E_0$$

16. Relation between virtual and mean values. $I_v = I_m \frac{\pi}{2\sqrt{2}}$ and $E_v = E_m \frac{\pi}{2\sqrt{2}}$

17. Impedance and reactance. The ratio of the applied voltage to the current is called the impedance (Z) of the A.C. circuit if all the three elements (R, L, C) are present in general.

When only inductance or only capacitance is present in the circuit, then the ratio of E_{rms} and i_{rms} is called reactance of inductance or of capacitance respectively (represented by X_L and X_C).

Key Point

Alternating currents/voltage a always measured by a.c. ammete and voltmeters. They alwa record their virtual values only.

18. Different types of alternating circuits : (*i*) Circuits containing only resistance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin \omega t$$

Instantaneous current and voltage are in phase always.

(*ii*) Circuit containing only inductance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin (\omega t - \pi/2)$$

The current lags behind the voltage by phase $\pi/2$. The inductive reactance $X_L = \omega L = 2\pi n L$

(*iii*) Circuit containing only capacitance :

$$E = E_0 \sin \omega t \text{ then } i = i_0 \sin (\omega t + \pi/2)$$

The current leads the voltage by a phase angle $\pi/2$ radian.

The capacitance reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi n C}$

(*iv*) Circuit containing resistance and inductance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin (\omega t - \phi)$$

where $\tan \phi = \frac{\omega L}{R}$ and $i_0 = \frac{E_0}{Z}$ and $I_v = \frac{E_v}{Z}$

The current lags behind the voltage by phase angle ϕ radian. The impedance

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{R^2 + X_L^2}$$

(*v*) Circuit containing resistance and capacitance :

$$E = E_0 \sin \omega t, \text{ then } i = i_0 \sin (\omega t - \phi)$$

where $\tan \phi = \frac{I/\omega C}{R}$

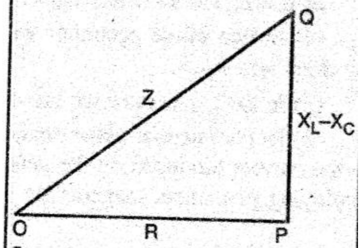
$$i_0 = \frac{E_0}{Z} \text{ and } I_v = \frac{E_v}{Z}$$

The current leads the voltage by phase angle ϕ. The impedance

$$Z = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2} = \sqrt{R^2 + X_C{}^2}$$

Key Point

The impedance triangle is given as below :



It measures the total effective resistance offered by LCR-circuit.

(*vi*) Circuit containing resistance, inductance and capacitance in series (LCR circuit) :

The impedance $$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

or $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

When $E = E_0 \sin \omega t$, then $i = i_0 \sin(\omega t - \phi)$

where $\tan \phi = \dfrac{X_L - X_C}{R} = \dfrac{\omega L - 1/\omega C}{R}$

and $i_0 = \dfrac{E_0}{Z}$ and $I_v = \dfrac{E_v}{Z}$

At resonance : $n = \dfrac{1}{2\pi\sqrt{LC}}$ and $X_L = X_C$.

$Z = R$, $\phi = 0$ and I_0 is maximum.

(*vii*) Coefficient of coupling of two coils :

$$k = \sqrt{\frac{M}{L_1 L_2}}, \ k \text{ is always less than one}$$

(*viii*) In case of L-C circuit :

$$\frac{d^2q}{dt^2} + \omega^2 q = 0$$

here $\omega^2 = \dfrac{1}{LC}$.

Key Point

(*i*) The reciprocal of reactance X_L of a coil is called susceptance.
(*ii*) The reciprocal of impedance Z of a.c. circuit is called admittance.

Power of an A.C. circuit. If $E = E_0 \sin \omega t$ is the applied *e.m.f.* and $I = I_0 \sin(\omega t + \phi)$ is the corresponding value of the current, then

Power of circuit $P = E_v \cdot I_v \cos \phi$

$\cos \phi$ is called the **power factor** of the circuit and is given by the ratio $\dfrac{R}{Z}$ where Z is the impedance of the circuit.

(*i*) For pure resistance $\phi = 0$ and $\cos \phi = 1$

$\therefore$ Power $P = E_v I_v$

(*ii*) For pure inductance and capacitance circuit $\phi = \pi/2$ and $\cos \phi = 0$. Such circuits are called **wattless circuit for which P = 0.**

(*iii*) For L–R circuit $\cos \phi = \dfrac{R}{\sqrt{R^2 + L^2\omega^2}}$.

19. (1) A genegator or a dynamo is a machine used for generating electric current by mechanical means. Here mechanical energy is converted into electrical energy.

(2) The frequency of A.C. in India is 50 Hz. In certain other countries it is 30 Hz, 50 Hz or 60 Hz.

(3) EMF is A.C. dynamo, $E = NBA\,\omega \sin \theta$ where N is the number of turns in the armature and θ is the angle between B and A (A is the area)

or $E = E_0 \sin \omega t$ where $E_0 = NBA\omega$

(4) In two phase generator we use two armatures and in three phase generator we use three armatures.

(5) In D.C. generator we use commutator. Here $E = E_0 \sin \omega t$.

(6) In commercial generators, we make use of electromagnets which are energised by the current produced by the generator itself. These are called dynamos whereas those employing permanent magnets are called magnetos.

(7) In two phase generator $E_1 = E_0 \sin \omega t$ and $E_2 = E_0 \sin\left(\omega t \pm \dfrac{\pi}{2}\right)$. In three phase generator $E_1 = E_0 \sin \omega t$, $E_2 = E_0 \sin\left(\omega t \pm \dfrac{2\pi}{3}\right)$, $E_3 = E_0 \sin\left(\omega t \pm \dfrac{4\pi}{3}\right)$.

(8) The flux at time t in the case of a generator is given by

$$\phi = NBA \cos \omega t$$

Key Point

Q factor of a resonant LCR circuit is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It is voltage multiplication factor of a a.c. circuit.

(9) The choke coil does a wonderful job in A.C. It finds extensive use in battery eliminators, ratio and T.V. sets, mercury, fluorescent lamps etc.

(10) Capacitor can also do the job done by an inductor but it is inferior to choke.

(11) Induction coil is an apparatus for obtaining high potential difference from a low potential difference supply (D.C.). It is based on the principal of mutual induction.

(12) An induction coil is used in laboratory as high voltage supply for studying discharge through gases and as a high tension supply for the spark plugs in car engines.

(13) An electric motor is a machine for converting electrical energy into mechanical energy.

(14) Efficiency of a D.C. motor

$$\eta = \frac{\text{output mechanical power}}{\text{input electrical power}} = \frac{\text{back e.m.f.}}{\text{applied e.m.f.}}$$

For efficiency to be maximum, the back e.m.f. should be half of the applied e.m.f.

(15) Since $I_0 = \frac{E_0}{\omega L}$ so for low frequency AC, choke coild with laminated soft iron cores are used and for reducing high frequency A.C., air core chokes are used.

(16) Impedance triangle is a right angled triangle whose base is ohmic resistance R, normal $(X_L - X_C)$ and hypotenuse is impedance Z.

(17) When LCR are connected in parallel

$$\frac{1}{|Z|} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

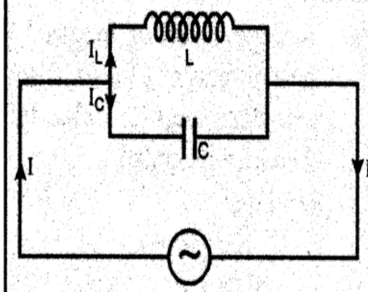
At $\omega = \omega_r = \frac{1}{\sqrt{LC}}, \frac{1}{|Z|}$ is minimum or $|Z|$ is maximum.

(18) For C-R circuit $\cos\phi = \frac{R}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}$.

(19) For L-C-R circuit $\cos\phi = \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$

Remember

Parallel resonant circuit is shown below



Formulas

2. $E = -\frac{N(\phi_2 - \phi_1)}{t} = -\frac{N\,d\phi}{dt}$
3. $E = Blv$
4. $E = E_0 \sin \omega t,\ E_0 = BAN\omega$
5. $E = -L\frac{dI}{dt},\ \phi = LI$
6. $E = -M\frac{dI}{dt},\ \phi = MI$
7. $\frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$
8. $\eta = \frac{\text{Output power}}{\text{Input power}}$
9. $L = \frac{\mu_0 N^2 A}{l}$
10. $M = \frac{\mu_0 N_1 N_2 A}{l}$
11. $f = \frac{1}{2\pi\sqrt{LC}}$
12. $I = I_0(1 - e^{-R/L\,t})$

    $I = I_0 e^{-R/L\,t},\ \tau = \frac{L}{R}$
13. $Q = Q_0(1 - e^{-t/C}R),\ Q = Q_0 e^{-t/C}R$

    $\tau = CR$
14. $I = I_0 \sin \omega t$
15. $I_{rms} = \frac{I_0}{\sqrt{2}}$
16. $X_L = \omega L$
17. $X_C = \frac{1}{\omega C}$
18. LR circuit, $I = I_0 \sin(\omega t - \theta)$

    $$= \frac{E_0}{\sqrt{R^2 + X_L^2}} \sin(\omega t - \theta)$$

    $$\theta = \frac{\tan^{-1}\omega L}{R}$$

    $$Z = \sqrt{R^2 + X_L^2}$$
19. CR-circuit, $I = I_0 \sin(\omega t + \theta)$

    $$= \frac{E_0}{\sqrt{R^2 + X_C^2}} \sin(\omega t + \theta)$$

    $$\theta = \tan^{-1}\frac{1}{\omega CR}$$

    $$Z = \sqrt{R^2 + X_C^2}$$
20. **L-C-R circuit**

    $I = I_0 \sin(\omega t - \theta)$

    $$\Rightarrow \quad I = \frac{E_0}{\sqrt{R^2 + (X_L - X_C^2)}} \sin(\omega t - \theta)$$

    $$\theta = \tan^{-1}\frac{X_L - X_C}{R}$$

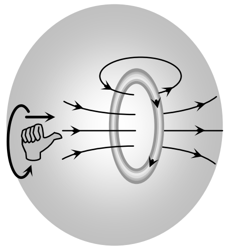
    $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
21. $P = 1/2\, E_0 I_0$

    $= E_{rms}.I_{rms}$ (Non-inductive circuit)
22. $P = 1/2\, E_0 I_0 \cos\theta$

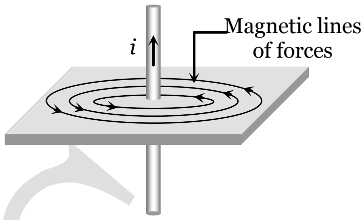
    $= E_{rms}.I_{rms} \cos\theta$ (inductive circuit)
23. $\cos\theta = \frac{\text{True power}}{\text{Apparent power}} = \frac{R}{Z}$
24. $Q = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}$

    = Quality factor

Magnetic Effect of Current



Oersted found that a magnetic field is established around a current carrying conductor.



Magnetic field exists as long as there is current in the wire.

The direction of magnetic field was found to be changed when direction of current was reversed.

Note : ❒ A moving charge produces magnetic as well as electric field, unlike a stationary charge which only produces electric field.

Biot Savart's Law

Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductors.

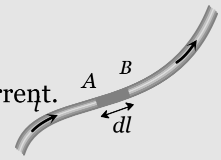
This law is although for infinitesimally small conductors yet it can be used for long conductors. In order to understand the Biot-Savart's law, we need to understand the term current-element.

Current element

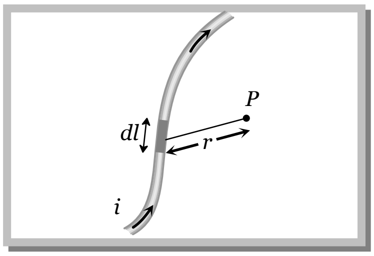
It is the product of current and length of infinitesimal segment of current carrying wire.

The current element is taken as a vector quantity. Its direction is same as the direction of current.

Current element $AB = i\vec{dl}$



In the figure shown below, there is a segment of current carrying wire and P is a point where magnetic field is to be calculated. $i\vec{dl}$ is a current element and r is the distance of the point 'P' with respect to the current element $i\vec{dl}$. According to Biot-Savart Law, magnetic field at point 'P' due to the current element $i\vec{dl}$ is given by the expression, $\boldsymbol{dB = k\dfrac{i\,dl\sin\theta}{r^2}}$ also $B = \int dB = \dfrac{\mu_0 i}{4\pi}.\int \dfrac{dl\sin\theta}{r^2}$



In C.G.S. : $k = 1 \Rightarrow dB = \dfrac{idl\sin\theta}{r^2}$ *Gauss*

In S.I. : $k = \dfrac{\mu_0}{4\pi} \Rightarrow dB = \dfrac{\mu_0}{4\pi}.\dfrac{idl\sin\theta}{r^2}$ *Tesla*

where μ_0 = Absolute permeability of air or vacuum $= 4\pi\times 10^{-7}\dfrac{Wb}{Amp-metre}$. It's other units are $\dfrac{Henry}{metre}$

or $\dfrac{N}{Amp^2}$ or $\dfrac{Tesla-metre}{Ampere}$

(1) **Different forms of Biot-Savarts law**

Vector form	**Biot-Savarts law in terms of current density**	**Biot-savarts law in terms of charge and it's velocity**
Vectorially, $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\vec{l} \times \hat{r})}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\vec{l} \times \vec{r})}{r^3}$ $\Rightarrow$ Direction of $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$. This is given by right hand screw rule.	In terms of current density $d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \vec{r}}{r^3} dV$ where $j = \frac{i}{A} = \frac{idl}{Adl} = \frac{idl}{dV}$ = current density at any point of the element, dV = volume of element	In terms of charge and it's velocity, $d\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\vec{v} \times \vec{r})}{r^3}$ $\because\ id\vec{l} = \frac{q}{dt} d\vec{l} = q\frac{d\vec{l}}{dt} = q\vec{v}$

(2) **Similarities and differences between Biot-Savart law and Coulomb's Law**

(i) The current element produces a magnetic field, whereas a point charge produces an electric field.

(ii) The magnitude of magnetic field varies as the inverse square of the distance from the current element, as does the electric field due to a point charge.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{l} \times \hat{r}}{r^2} \qquad \text{Biot-Savart Law} \qquad \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \qquad \text{Coulomb's Law}$$

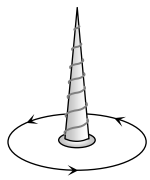
(iii) The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element $d\vec{l}$ and the unit vector $\hat{r}$.

Direction of Magnetic Field

The direction of magnetic field is determined with the help of the following simple laws :

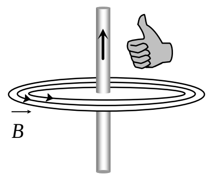
(1) **Maxwell's cork screw rule**

According to this rule, if we imagine a right handed screw placed along the current carrying linear conductor, be rotated such that the screw moves in the direction of flow of current, then the direction of rotation of the thumb gives the direction of magnetic lines of force.

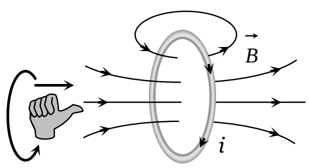


(2) **Right hand thumb rule**

According to this rule if a current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of folding fingers will represent the direction of magnetic lines of force.



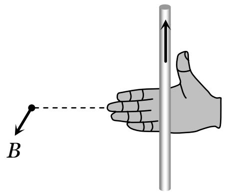
(3) **Right hand thumb rule of circular currents**



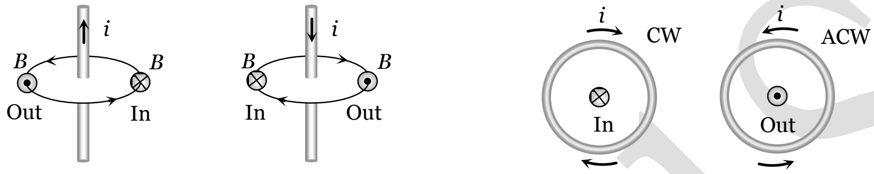
According to this rule if the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb.

(4) **Right hand palm rule**

If we stretch our right hand such that fingers point towards the point. At which magnetic field is required while thumb is in the direction of current then normal to the palm will show the direction of magnetic field.



Note : ❒ If magnetic field is directed perpendicular and into the plane of the paper it is represented by ⊗ (cross) while if magnetic field is directed perpendicular and out of the plane of the paper it is represented by ⊙ (dot)



In : Magnetic field is away from the observer or perpendicular inwards.

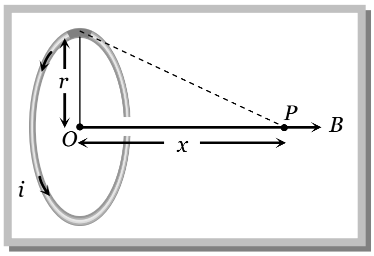
Out : Magnetic field is towards the observer or perpendicular outwards.

Application of Biot-Savarts Law

(1) **Magnetic field due to a circular current**

If a coil of radius r, carrying current i then magnetic field on it's axis at a distance x from its centre given by

$$B_{axis} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi Nir^2}{(x^2 + r^2)^{3/2}} ;$$ where N = number of turns in coil.



Different cases

***Case* 1 : Magnetic field at the centre of the coil**

(i) At centre $x = 0 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi Ni}{r} = \frac{\mu_0 Ni}{2r} = B_{\max}$

(ii) For single turn coil $N = 1 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} = \frac{\mu_0 i}{2r}$ (iii) In C.G.S. $\frac{\mu_0}{4\pi} = 1 \Rightarrow B_{centre} = \frac{2\pi i}{r}$

Note : ❒ $B_{centre} \propto N$ (i, r constant), $B_{centre} \propto i$ (N, r constant), $B_{centre} \propto \frac{1}{r}$ (N, i constant)

***Case* 2 : Ratio of B_{centre} and B_{axis}**

The ratio of magnetic field at the centre of circular coil and on it's axis is given by $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$

(i) If $x = \pm a$, $B_c = 2\sqrt{2}\, B_a$ $\quad x = \pm\frac{a}{2}$, $B_c = \frac{5\sqrt{5}}{8} B_a$ $\quad x = \pm\frac{a}{\sqrt{2}}$, $B_c = \left(\frac{3}{2}\right)^{3/2} B_a$

(ii) If $B_a = \frac{B_c}{n}$ then $x = \pm r\sqrt{(n^{2/3} - 1)}$ and if $B_a = \frac{B_c}{\sqrt{n}}$ then $x = \pm r\sqrt{(n^{1/3} - 1)}$

***Case* 3 : Magnetic field at very large/very small distance from the centre**

(i) If $x >> r$ (very large distance) $\Rightarrow B_{axis} = \frac{\mu_0}{4\pi}.\frac{2\pi Nir^2}{x^3} = \frac{\mu_0}{4\pi}.\frac{2NiA}{x^3}$ where $A = \pi r^2$ = Area of each turn of the coil.

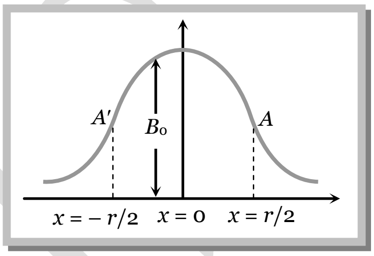
(ii) If $x << r$ (very small distance) $\Rightarrow B_{axis} \neq B_{centre}$, but by using binomial theorem and neglecting higher power of $\frac{x^2}{r^2}$; $B_{axis} = B_{centre}\left(1 - \frac{3}{2}\frac{x^2}{r^2}\right)$

***Case* 4 : *B-x* curve**

The variation of magnetic field due to a circular coil as the distance x varies as shown in the figure.

B varies non-linearly with distance x as shown in figure and is maximum when $x^2 = \min = 0$, *i.e.*, the point is at the centre of the coil and it is zero at $x = \pm\infty$.

Point of inflection (A and A') **:** Also known as points of curvature change or pints of zero curvature.



(i) At these points B varies linearly with $x \Rightarrow \frac{dB}{dx}$ = constant $\Rightarrow \frac{d^2B}{dx^2} = 0$.

(ii) They locates at $x = \pm\frac{r}{2}$ from the centre of the coil.

(iii) Separation between point of inflextion is equal to radius of coil (r)

(iv) Application of points of inflextion is "Hamholtz coils" arrangement.

Note : ❒ The magnetic field at $x = \frac{r}{2}$ is $B = \frac{4\mu_0 Ni}{5\sqrt{5}\,r}$

(2) **Helmholtz coils**

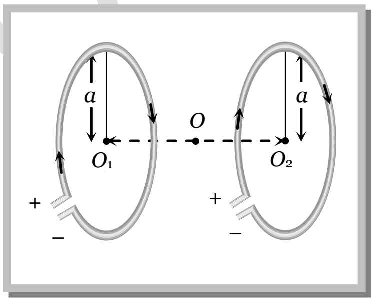
(i) This is the set-up of two coaxial coils of same radius such that distance between their centres is equal to their radius.

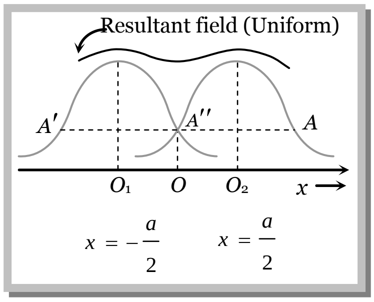
(ii) These coils are used to obtain uniform magnetic field of short range which is obtained between the coils.

(iii) At axial mid point O, magnetic field is given by $B = \frac{8\mu_0 Ni}{5\sqrt{5}R} = 0.716\frac{\mu_0 Ni}{R} = 1.432\,B$, where $B = \frac{\mu_0 Ni}{2R}$

(iv) Current direction is same in both coils otherwise this arrangement is not called Helmholtz's coil arrangement.

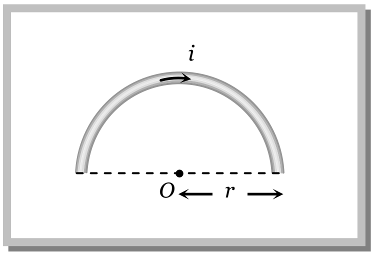
(v) Number of points of inflextion $\Rightarrow$ Three (A, A', A'')



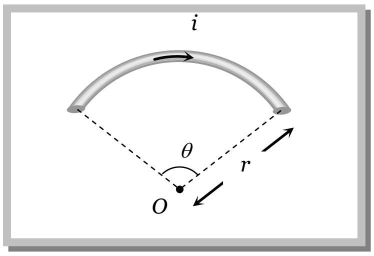


Note : ❒ The device whose working principle based on this arrangement and in which uniform magnetic field is used called as "Halmholtz galvanometer".

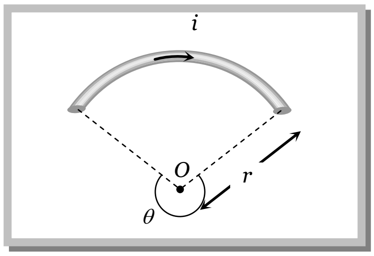
(3) **Magnetic field due to current carrying circular arc :** Magnetic field at centre O



$$B = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{r} = \frac{\mu_0 i}{4r}$$



$$B = \frac{\mu_0}{4\pi}\cdot\frac{\theta i}{r}$$



$$B = \frac{\mu_0}{4\pi}\cdot\frac{(2\pi-\theta)i}{r}$$

Special results

If magnetic field at the centre of circular coil is denoted by $B_0 \left(=\frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{r}\right)$

Magnetic field at the centre of arc which is making an angle θ at the centre is

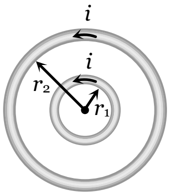
$$B_{arc} = \left(\frac{B_0}{2\pi}\right).\theta$$

Angle at centre	Magnetic field at centre in term of B_0
360° (2π)	B_0
180° (π)	B_0 / 2
120° ($2\pi/3$)	B_0 / 3
90° ($\pi/2$)	B_0 / 4
60° ($\pi/3$)	B_0 / 6
30° ($\pi/6$)	B_0 / 12

(4) **Concentric circular loops** (N = 1)

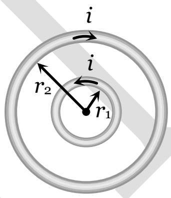
(i) Coplanar and concentric : It means both coils are in same plane with common centre

(a) Current in same direction



$$B_1 = \frac{\mu_0}{4\pi}2\pi i\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

(b) Current in opposite direction



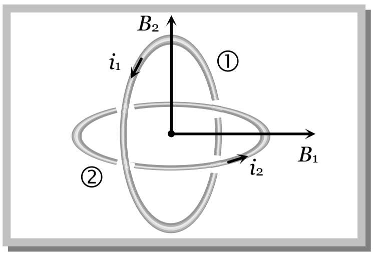
$$B_2 = \frac{\mu_0}{4\pi}2\pi i\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

Note : ❐ $\frac{B_1}{B_2} = \left(\frac{r_2+r_1}{r_2-r_1}\right)$

(ii) Non-coplanar and concentric : Plane of both coils are perpendicular to each other

Magnetic field at common centre

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$$



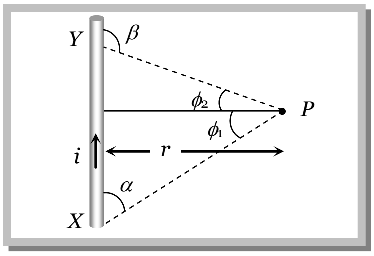
(5) **Magnetic field due to a straight current carrying wire**

Magnetic field due to a current carrying wire at a point P which lies at a perpendicular distance r from the wire as shown is given as

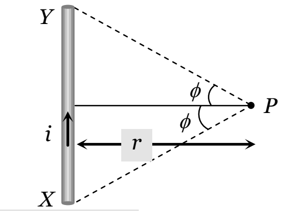
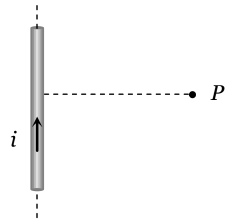
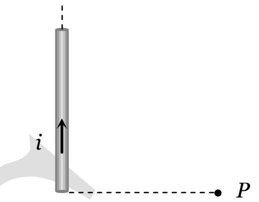
$$B = \frac{\mu_0}{4\pi}\cdot\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$$

From figure $\alpha = (90^o - \phi_1)$ and $\beta = (90^o + \phi_2)$

Hence $B = \frac{\mu_o}{4\pi}\cdot\frac{i}{r}(\cos\alpha - \cos\beta)$



Different cases

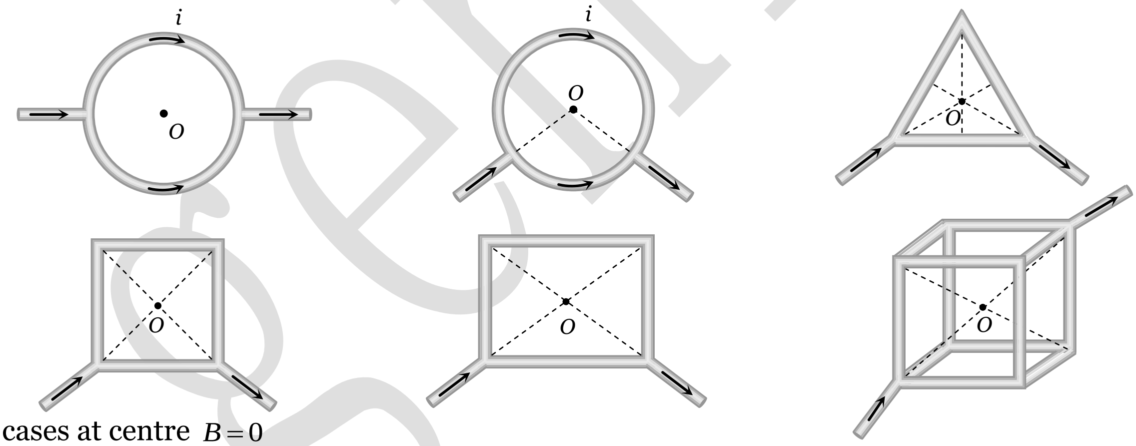
***Case* 1 :** When the linear conductor XY is of finite length and the point P lies on it's perpendicular bisector as shown  $\phi_1 = \phi_2 = \phi$ So $B = \frac{\mu_0}{4\pi}.\frac{i}{r}(2\sin\phi)$	***Case* 2 :** When the linear conductor XY is of infinite length and the point P lies near the centre of the conductor  $\phi_1 = \phi_2 = 90^o$. So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 90^o] = \frac{\mu_0}{4\pi}\frac{2i}{r}$	***Case* 3 :** When the linear conductor is of infinite length and the point P lies near the end Y or X  $\phi_1 = 90^o$ and $\phi_2 = 0^o$. So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 0^o] = \frac{\mu_0}{4\pi}\frac{i}{r}$

Note : ❒ When point P lies on axial position of current carrying conductor then magnetic field at P

$B = 0$

❒ The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction is zero.

(6) **Zero magnetic field :** If in a symmetrical geometry, current enters from one end and exists from the other, then magnetic field at the centre is zero.



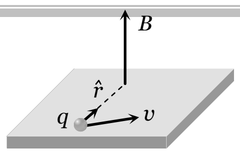
In all cases at centre $B = 0$

Concepts

☞ *If a current carrying circular loop (n = 1) is turned into a coil having n identical turns then magnetic field at the centre of the coil becomes n² times the previous field i.e.* $B_{(n\ turn)} = n^2 B_{(single\ turn)}$

☞ *When a current carrying coil is suspended freely in earth's magnetic field, it's plane stays in* ***East-West*** *direction.*

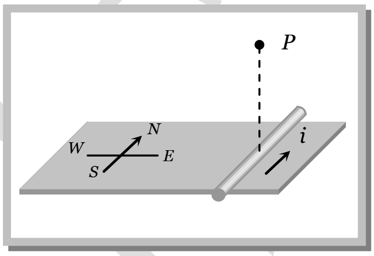
☞ *Magnetic field (* $\vec{B}$ *) produced by a moving charge q is given by* $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$ *; where v = velocity of charge and v << c (speed of light).*



☛ *If an electron is revolving in a circular path of radius r with speed v then magnetic field produced at the centre of circular path* $B = \frac{\mu_0}{4\pi} \cdot \frac{ev}{r^2}$.

Example

Example: 1 Current flows due north in a horizontal transmission line. Magnetic field at a point P vertically above it directed



(a) North wards

(b) South wards

(c) Toward east

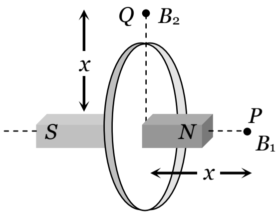
(d) Towards west

Solution : (c) By using right hand thumb rule or any other rule which helps to determine the direction of magnetic field.

Example: 2 Magnetic field due to a current carrying loop or a coil at a distant axial point P is B_1 and at an equal distance in it's plane is B_2 then $\frac{B_1}{B_2}$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

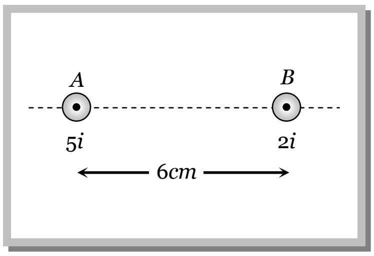
Solution : (a) Current carrying coil behaves as a bar magnet as shown in figure.



We also knows for a bar magnet, if axial and equatorial distance are same then $B_a = 2B_e$

Hence, in this equation $\frac{B_1}{B_2} = \frac{2}{1}$

Example: 3 Find the position of point from wire 'B' where net magnetic field is zero due to following current distribution



(a) 4 *cm*

(b) $\frac{30}{7}$ *cm*

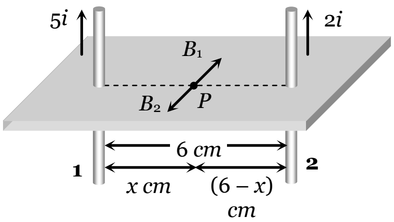
(c) $\frac{12}{7}$ *cm*

(d) 2 *cm*

Solution : (c) Suppose P is the point between the conductors where net magnetic field is zero.

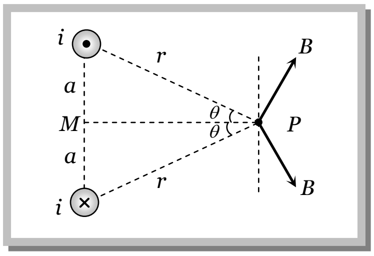
So at P |Magnetic field due to conductor 1| = |Magnetic field due to conductor 2|

i.e. $\frac{\mu_0}{4\pi} \cdot \frac{2(5i)}{i} = \frac{\mu_0}{4\pi} \cdot \frac{2(2i)}{(6-x)} \Rightarrow \frac{5}{x} = \frac{9}{6-x} \Rightarrow x = \frac{30}{7}\, cm$



Hence position from $B = 6 - \frac{30}{7} = \frac{12}{7}\, cm$

Example: 4 Find out the magnitude of the magnetic field at point P due to following current distribution



(a) $\frac{\mu_0 ia}{\pi r^2}$

(b) $\frac{\mu_0 ia^2}{\pi r}$

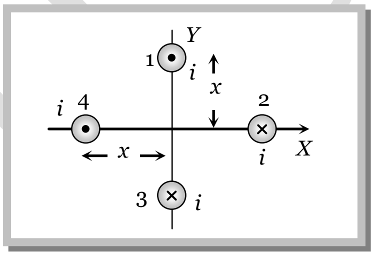
(c) $\frac{\mu_0 ia}{2\pi r^2}$

(d) $\frac{2\mu_0 ia}{\pi r^2}$

Solution : (a) Net magnetic field at P, $B_{net} = 2B\sin\theta$; where B = magnetic field due to one wire at $P = \frac{\mu_0}{4\pi}.\frac{2i}{r}$

and $\sin\theta = \frac{a}{r}$ $\therefore$ $B_{net} = 2\times\frac{\mu_0}{4\pi}.\frac{2i}{r}\times\frac{a}{r} = \frac{\mu_0 ia}{\pi r^2}$.

Example: 5 What will be the resultant magnetic field at origin due to four infinite length wires. If each wire produces magnetic field 'B' at origin



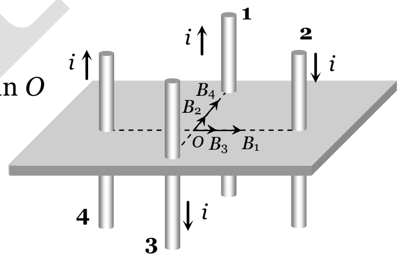
(a) $4B$

(b) $\sqrt{2}\,B$

(c) $2\sqrt{2}\,B$

(d) Zero

Solution : (c) Direction of magnetic field (B_1, B_2, B_3 and B_4) at origin due to wires 1, 2, 3 and 4 are shown in the following figure.



$B_1 = B_2 = B_3 = B_4 = \frac{\mu_0}{4\pi}.\frac{2i}{x} = B$. So net magnetic field at origin O

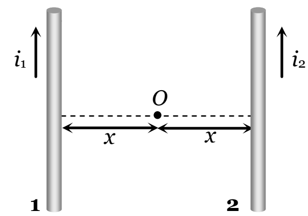
$$B_{net} = \sqrt{(B_1 + B_2)^2 + (B_2 + B_4)^2}$$

$$= \sqrt{(2B)^2 + (2B)^2} = 2\sqrt{2}B$$

Example: 6 Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 10 μT. If the direction of i_2 is reversed, the field becomes 30 μT. The ratio i_1/i_2 is

(a) 4 (b) 3 (c) 2 (d) 1

Solution : (c) Initially when wires carry currents in the same direction as shown.

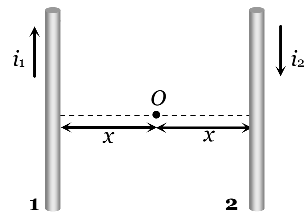


Magnetic field at mid point O due to wires 1 and 2 are respectively

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1}{x} \otimes \text{ and } B_2 = \frac{\mu_0}{4\pi}.\frac{2i_2}{x} \odot$$

Hence net magnetic field at O $B_{net} = \frac{\mu_0}{4\pi}\times\frac{2}{x}(i_1 - i_2)$

$$\Rightarrow \quad 10\times10^{-6} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 - i_2) \qquad .....(i)$$



If the direction of i_2 is reversed then

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1}{x} \otimes \text{ and } B_2 = \frac{\mu_0}{4\pi}.\frac{2i_2}{x} \otimes$$

So $B_{net} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 + i_2) \Rightarrow 30 \times 10^{-6} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 + i_2)$ ......(ii)

Dividing equation (ii) by (i) $\frac{i_1 + i_2}{i_1 - i_2} = \frac{3}{1} \Rightarrow \frac{i_1}{i_2} = \frac{2}{1}$

***Example*: 7** A wire of fixed length is turned to form a coil of one turn. It is again turned to form a coil of three turns. If in both cases same amount of current is passed, then the ratio of the intensities of magnetic field produced at the centre of a coil will be

(a) 9 times of first case (b) $\frac{1}{9}$ times of first case (c) 3 times of first case (d) $\frac{1}{3}$ times of first case

Solution : (a) Magnetic field at the centre of n turn coil carrying current i $B = \frac{\mu_0}{4\pi}.\frac{2\pi ni}{r}$ ......(i)

For single turn $n = 1$ $B = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r}$ .....(ii)

If the same wire is turn again to form a coil of three turns *i.e.* $n = 3$ and radius of each turn $r' = \frac{r}{3}$

So new magnetic field at centre $B' = \frac{\mu_0}{4\pi}.\frac{2\pi(3)}{r'}$ $\Rightarrow B' = 9 \times \frac{\mu_0}{4\pi}.\frac{2\pi i}{r}$ ......(iii)

Comparing equation (ii) and (iii) gives $B' = 9B$.

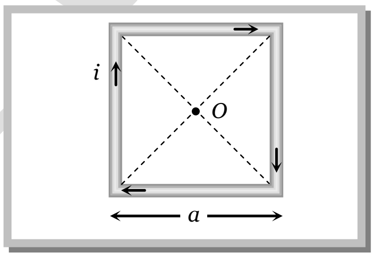
***Example*: 8** A wire in the form of a square of side a carries a current i. Then the magnetic induction at the centre of the square wire is (Magnetic permeability of free space = μ_0)

(a) $\frac{\mu_0 i}{2\pi a}$

(b) $\frac{\mu_0 i\sqrt{2}}{\pi a}$

(c) $\frac{2\sqrt{2}\mu_0 i}{\pi a}$

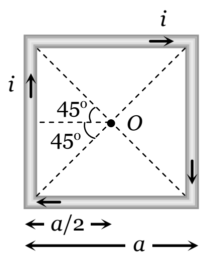
(d) $\frac{\mu_0 i}{\sqrt{2}\pi a}$



Solution : (c) Magnetic field due to one side of the square at centre O

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i\sin 45^o}{a/2}$$

$$\Rightarrow B_1 = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\,i}{a}$$

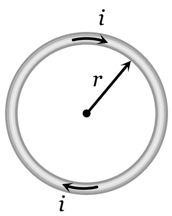


Hence magnetic field at centre due to all side $B_{net} = 4B_1 = \frac{\mu_0(2\sqrt{2}\,i)}{\pi a}$

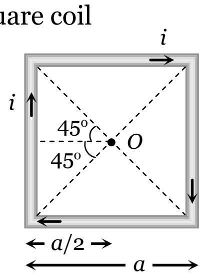
***Example*: 9** The ratio of the magnetic field at the centre of a current carrying circular wire and the magnetic field at the centre of a square coil made from the same length of wire will be

(a) $\frac{\pi^2}{4\sqrt{2}}$ (b) $\frac{\pi^2}{8\sqrt{2}}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{4\sqrt{2}}$

Solution : (b) Circular coil



Square coil



genius

Length $L = 2\pi r$

Magnetic field $B = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} = \frac{\mu_0}{4\pi}.\frac{4\pi^2 i}{r}$

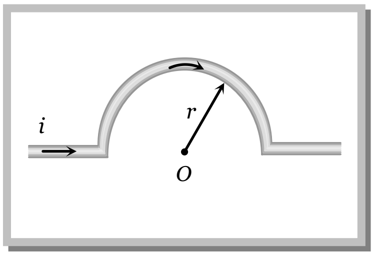
Hence $\frac{B_{circular}}{B_{square}} = \frac{\pi^2}{8\sqrt{2}}$

Length $L = 4a$

$B = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\,i}{a}$ $B = \frac{\mu_0}{4\pi}.\frac{8\sqrt{2}\,i}{a}$

***Example*: 10** Find magnetic field at centre O in each of the following figure

(i)



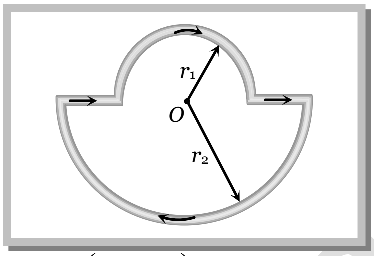
(a) $\frac{\mu_0 i}{r}\otimes$

(b) $\frac{\mu_0 i}{2r}\odot$

(c) $\frac{\mu_0 i}{4r}\otimes$

(d) $\frac{\mu_0 i}{4r}\odot$

(ii)



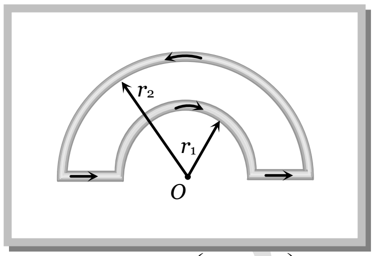
(a) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\otimes$

(b) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\otimes$

(c) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\odot$

(d) Zero

(iii)



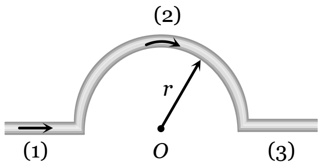
(a) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\otimes$

(b) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\otimes$

(c) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\odot$

(d) Zero

Solution : (i) (c) Magnetic field at O due to parts 1 and 3, $B_1 = B_3 = 0$

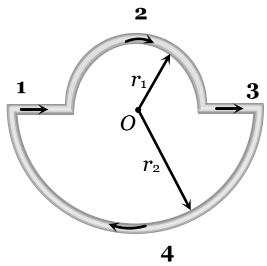


While due to part (2) $B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes$

$\therefore$ Net magnetic field at centre O,

$$B_{net} = B_1 + B_2 + B_3 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes \Rightarrow B_{net} = \frac{\mu_0 i}{4r}\otimes$$

(ii) (b) $B_1 = B_3 = 0$

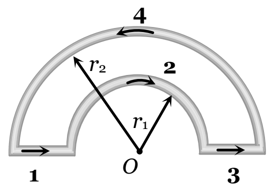


$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_1}\otimes$$

$$B_4 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_2}\otimes$$

So $B_{net} = B_2 + B_4 = \frac{\mu_0}{4\pi}.\pi i\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\otimes$

(iii) (a) $B_1 = B_3 = 0$



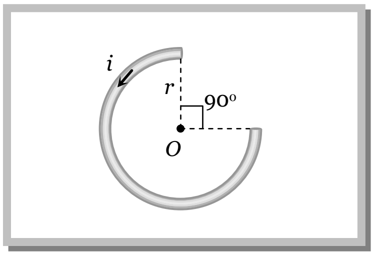
$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_1}\otimes$$

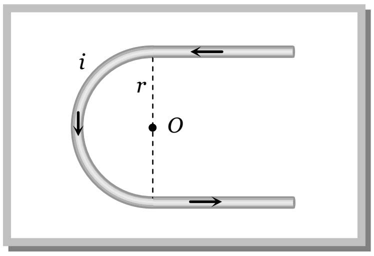
$B_4 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_2}\otimes$ As $|B_2| > |B_4|$

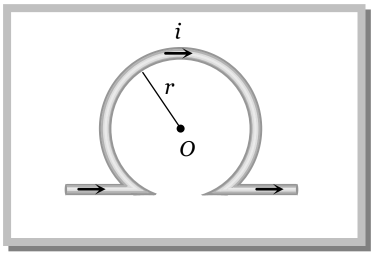
genius

So $B_{net} = B_2 - B_4 \Rightarrow B_{net} = \frac{\mu_0 i}{4}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \otimes$

***Example:* 11** Find magnetic field at centre O in each of each of the following figure

(i) 

(ii) 

(iii) 

(i)	(ii)	(iii)
(a) $\frac{\mu_0 i}{2r} \odot$	(a) $\frac{\mu_0}{2\pi}\frac{i}{r}(\pi - 2) \otimes$	(a) $\frac{\mu_0}{2r}\frac{2i}{r}(\pi + 1) \otimes$
(b) $\frac{\mu_0 i}{2r} \otimes$	(b) $\frac{\mu_0 i}{4\pi}.\frac{i}{r}(\pi + 2) \odot$	(b) $\frac{\mu_0 i}{4r}.\frac{2i}{r}(\pi - 1) \otimes$
(c) $\frac{3\mu_0 i}{8r} \otimes$	(c) $\frac{\mu_0 i}{4r} \otimes$	(c) Zero
(d) $\frac{3\mu_0 i}{8r} \odot$	(d) $\frac{\mu_0 i}{4r} \odot$	(d) Infinite

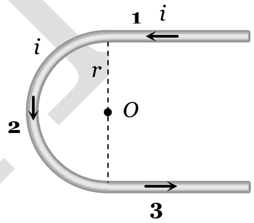
Solution : (i) (d) By using $B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \theta)i}{r} \Rightarrow B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \pi/2)i}{r} = \frac{3\mu_0 i}{8r} \odot$

(ii) (b) Magnetic field at centre O due to section 1, 2 and 3 are respectively

$B_1 = \frac{\mu_0}{4\pi}.\frac{i}{r} \odot$

$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r} \odot$

$B_3 = \frac{\mu_0}{4\pi}.\frac{i}{r} \odot$

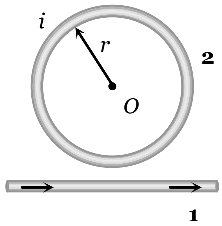


$\Rightarrow B_{net} = B_1 + B_2 + B_3 = \frac{\mu_0}{4\pi}\frac{i}{r}(\pi + 2) \odot$

(iii) (b) The given figure is equivalent to following figure, magnetic field at O due to long wire (part 1)

$B_1 = \frac{\mu_0}{4\pi}.\frac{2i}{r} \odot$

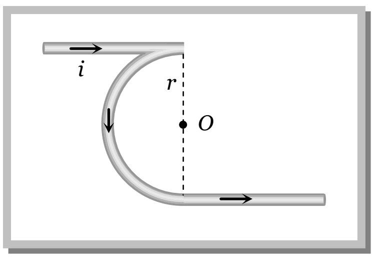
Due to circular coil $B_2 = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} \otimes$

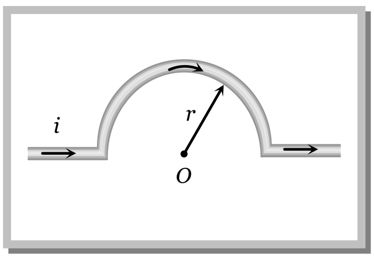


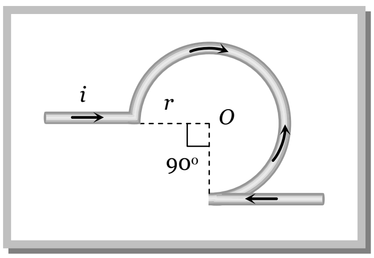
Hence net magnetic field at O

$B_{net} = B_2 - B_1 = \frac{\mu_0}{4\pi}.\frac{2i}{r}(\pi - 1) \otimes$

***Example:* 12** The field B at the centre of a circular coil of radius r is π times that due to a long straight wire at a distance r from it, for equal currents here shows three cases; in all cases the circular part has radius r and straight ones are infinitely long. For same current the field B is the centre P in cases 1, 2, 3 has the ratio**[CPMT 198**





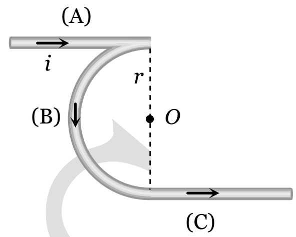


(1) **(2)** **(3)**

(a) $\left(-\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}-\frac{1}{2}\right)$ (b) $\left(-\frac{\pi}{2}+1\right):\left(\frac{\pi}{2}+1\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

(c) $-\frac{\pi}{2}:\frac{\pi}{2}:\frac{3\pi}{4}$ (d) $\left(-\frac{\pi}{2}-1\right):\left(\frac{\pi}{2}-\frac{1}{2}\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

Solution : (a) ***Case* 1 :** $B_A = \frac{\mu_0}{4\pi}.\frac{i}{r}\otimes$



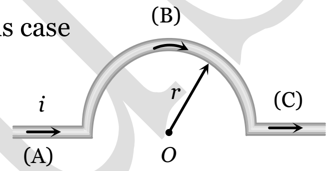
$$B_B = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$$

$$B_C = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$$

So net magnetic field at the centre of case 1

$$B_1 = B_B - (B_A + B_C) \quad \Rightarrow B_1 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\odot \quad ..... \text{(i)}$$

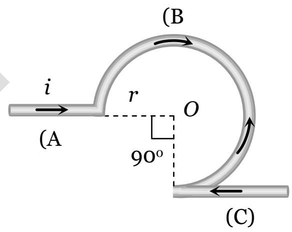
***Case* 2 :** As we discussed before magnetic field at the centre O in this case



$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes \quad .....\text{(ii)}$$

***Case* 3 :** $B_A = 0$

$$B_B = \frac{\mu_0}{4\pi}.\frac{(2\pi-\pi/2)}{r}\otimes = \frac{\mu_0}{4\pi}.\frac{3\pi i}{2r}\otimes$$



$$B_C = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$$

So net magnetic field at the centre of case 3

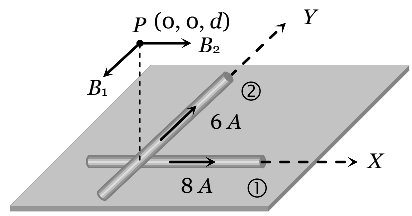
$$B_3 = \frac{\mu_0}{4\pi}.\frac{i}{r}\left(\frac{3\pi}{2}-1\right)\otimes \quad ....\text{(iii)}$$

From equation (i), (ii) and (iii) $B_1 : B_2 : B_3 = \pi\odot : \pi\odot : \left(\frac{3\pi}{2}-1\right)\otimes = -\frac{\pi}{2}:\frac{\pi}{2}:\left(\frac{3\pi}{4}-\frac{1}{2}\right)$

***Example*: 13** Two infinite length wires carries currents 8A and 6A respectively and placed along X and Y-axis. Magnetic field at a point $P(0, 0, d)m$ will be

(a) $\frac{7\mu_0}{\pi d}$ (b) $\frac{10\mu_0}{\pi d}$ (c) $\frac{14\mu_0}{\pi d}$ (d) $\frac{5\mu_0}{\pi d}$

Solution : (d) Magnetic field at P



Due to wire 1, $B_1 = \frac{\mu_0}{4\pi}.\frac{2(8)}{d}$

and due to wire 2, $B_2 = \frac{\mu_0}{4\pi}.\frac{2(16)}{d}$

$$\therefore B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0}{4\pi}.\frac{16}{d}\right)^2 + \left(\frac{\mu_0}{4\pi}.\frac{12}{d}\right)^2} = \frac{\mu_0}{4\pi}\times\frac{2}{d}\times 10 = \frac{5\mu_0}{\pi d}$$

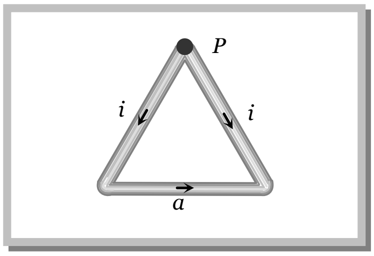
***Example*: 14** An equilateral triangle of side 'a' carries a current i then find out the magnetic field at point P which is vertex of triangle

(a) $\frac{\mu_0 i}{2\sqrt{3}\pi a} \otimes$

(b) $\frac{\mu_0 i}{2\sqrt{3}\pi a} \odot$

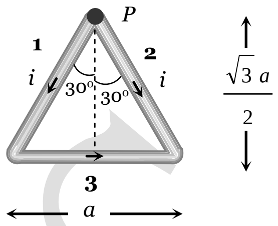
(c) $\frac{2\sqrt{3}\mu_0 i}{\pi a} \odot$

(d) Zero



Solution : (b) As shown in the following figure magnetic field at P due to side 1 and side 2 is zero.

Magnetic field at P is only due to side 3,



which is $B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2i \sin 30^o}{\frac{\sqrt{3}a}{2}} \odot$

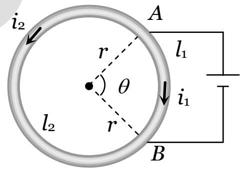
$= \frac{\mu_0}{4\pi} \cdot \frac{2i}{\sqrt{3}a} \odot = \frac{\mu_0 i}{2\sqrt{3}\pi a} \odot$

Example: 15 A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of, the magnetic induction at the centre due to the current in the ring is **[IIT-JEE 1995]**

(a) Proportional to $2(180^o - \theta)$ (b) Inversely proportional to r

(c) Zero, only if $\theta = 180^o$ (d) Zero for all values of θ

Solution : (d) Directions of currents in two parts are different, so directions of magnetic fields due to these currents are different.



Also applying Ohm's law across AB $\quad i_1 R_1 = i_2 R_2 \Rightarrow i_1 l_1 = i_2 l_2$ .....(i)

Also $B_1 = \frac{\mu_0}{4\pi} \times \frac{i_1 l_1}{r^2}$ and $B_2 = \frac{\mu_0}{4\pi} \times \frac{i_2 l_2}{r^2}$; $\therefore \frac{B_2}{B_1} = \frac{i_1 l_1}{i_2 l_2} = 1$ [Using (i)]

Hence, two field are equal but of opposite direction. So, resultant magnetic induction at the centre is zero and is independent of θ.

Example: 16 The earth's magnetic induction at a certain point is $7 \times 10^{-5}\ Wb/m^2$. This is to be annulled by the magnetic induction at the centre of a circular conducting loop of radius 5 *cm*. The required current in the loop is **[MP PET 1999; AIIMS 2000]**

(a) 0.56 *A* (b) 5.6 *A* (c) 0.28 *A* (d) 2.8 *A*

Solution : (b) According to the question, at centre of coil $B = B_H \Rightarrow \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} = B_H$

$\Rightarrow 10^{-7} \times \frac{2\pi i}{(5 \times 1^{-2})} = 7 \times 10^{-5} \Rightarrow i = 5.6\ amp.$

Example: 17 A particle carrying a charge equal to 100 times the charge on an electron is rotating per second in a circular path of radius 0.8 *metre*. The value of the magnetic field produced at the centre will be (μ_0 – permeability for vacuum) **[CPMT 1986]**

(a) $\frac{10^{-7}}{\mu_0}$ (b) $10^{-17}\,\mu_0$ (c) $10^{-6}\,\mu_0$ (d) $10^{-7}\,\mu_0$

Solution : (b) Magnetic field at the centre of orbit due to revolution of charge.

$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi(q\nu)}{r}$; where ν = frequency of revolution of charge

So, $B = \frac{\mu_0}{4\pi} \times \frac{2\pi \times (100e \times 1)}{0.8} \Rightarrow B = 10^{-17}\,\mu_0$.

Example: 18 Ratio of magnetic field at the centre of a current carrying coil of radius R and at a distance of $3R$ on its axis is

(a) $10\sqrt{10}$ (b) $20\sqrt{10}$ (c) $2\sqrt{10}$ (d) $\sqrt{10}$

Solution : (a) By using $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$; where $x = 3R$ and $r = R \Rightarrow \frac{B_{centre}}{B_{axis}} = (10)^{3/2} = 10\sqrt{10}$.

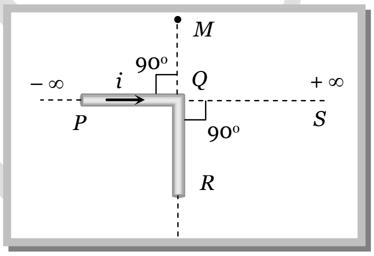
***Example*: 19** A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis where the magnetic induction will be $\frac{1}{8}th$ to its value at the centre of the coil, is **[MP PMT 1997]**

(a) $\frac{R}{\sqrt{3}}$ (b) $R\sqrt{3}$ (c) $2\sqrt{3}R$ (d) $\frac{2}{\sqrt{3}}R$

Solution : (b) By using $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$, given $r = R$ and $B_{axis} = \frac{1}{8} B_{centre}$

$$\Rightarrow 8 = \left(1 + \frac{x^2}{R^2}\right)^{3/2} \Rightarrow (2)^2 = \left\{\left(1 + \frac{x^2}{R^2}\right)^{1/2}\right\}^3 \Rightarrow 2 = \left(1 + \frac{x^2}{R^2}\right)^{1/2} \Rightarrow 4 = 1 + \frac{x^2}{R^2} \Rightarrow x = \sqrt{3}R$$

***Example*: 20** An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current is $\frac{1}{2}$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1 / H_2 is given by



(a) $\frac{1}{2}$

(b) 1

(c) $\frac{2}{3}$

(d) 2

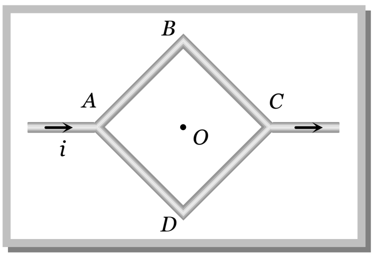
Solution : (c) Magnetic field at any point lying on the current carrying conductor is zero.

Here H_1 = magnetic field at M due to current in PQ

H_2 = magnetic field at M due to R + due to QS + due to PQ $= 0 + \frac{H_1}{2} + H_1 = \frac{3}{2}H_1$

$\therefore \frac{H_1}{H_2} = \frac{2}{3}$

***Example*: 21** Figure shows a square loop $ABCD$ with edge length a. The resistance of the wire ABC is r and that of ADC is $2r$. The value of magnetic field at the centre of the loop assuming uniform wire is



(a) $\frac{\sqrt{2}\,\mu_0 i}{3\pi a} \odot$

(b) $\frac{\sqrt{2}\,\mu_0 i}{3\pi a} \otimes$

(c) $\frac{\sqrt{2}\,\mu_0 i}{\pi a} \odot$

(d) $\frac{\sqrt{2}\,\mu_0 i}{\pi a} \otimes$

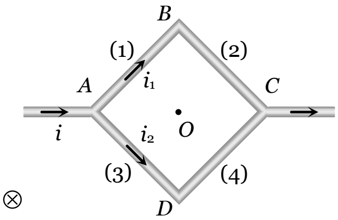
Solution : (b) According to question resistance of wire ADC is twice that of wire ABC. Hence current flows through ADC is half that of ABC *i.e.* $\frac{i_2}{i_1} = \frac{1}{2}$. Also $i_1 + i_2 = i \Rightarrow i_1 = \frac{2i}{3}$ and $i_2 = \frac{i}{3}$

Magnetic field at centre O due to wire AB and BC (part 1 and 2) $B_1 = \frac{\mu_0}{4\pi} . \frac{2i_1 \sin 45^o}{a/2} \otimes = \frac{\mu_0}{4\pi} . \frac{2\sqrt{2}\, i_1}{a} \otimes$

and magnetic field at centre O due to wires AD and DC (*i.e.* part 3 and 4) $B_3 = B_4 = \frac{\mu_0}{4\pi}\frac{2\sqrt{2}\, i_2}{a} \odot$

Also $i_1 = 2i_2$. So $(B_1 = B_2) > (B_3 = B_4)$

Hence net magnetic field at centre O

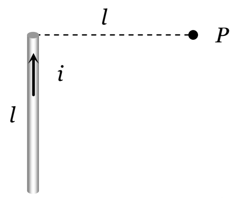


$B_{net} = (B_1 + B_2) - (B_3 + B_4)$

$$= 2 \times \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2} \times \left(\frac{2}{3} i\right)}{a} - \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}\left(\frac{i}{3}\right) \times 2}{a} = \frac{\mu_0}{4\pi} \cdot \frac{4\sqrt{2}\, i}{3a}(2-1) \otimes = \frac{\sqrt{2}\, \mu_0 i}{3\pi a} \otimes$$

Tricky example: 1

Figure shows a straight wire of length l current i. The magnitude of magnetic field produced by the current at point P is

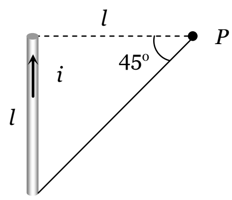


(a) $\frac{\sqrt{2}\mu_0 i}{\pi l}$ (b) $\frac{\mu_0 i}{4\pi l}$ (c) $\frac{\sqrt{2}\mu_0 i}{8\pi l}$ (d) $\frac{\mu_0 i}{2\sqrt{2}\pi l}$

Solution: (c) The given situation can be redrawn as follow.

As we know the general formula for finding the magnetic field due to a finite length wire

$B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}(\sin\phi_1 + \sin\phi_2)$

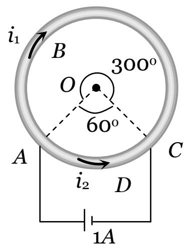


Here $\phi_1 = 0°$, $\phi = 45°$

$\therefore \; B = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}(\sin 0° + \sin 45°) = \frac{\mu_0}{4\pi} \cdot \frac{i}{\sqrt{2}l} \Rightarrow B = \frac{\sqrt{2}\mu_0 i}{8\pi l}$

Tricky example: 2

A cell is connected between the points A and C of a circular conductor $ABCD$ of centre 'O' with angle AOC = 60°, If B_1 and B_2 are the magnitudes of the magnetic fields at O due to the currents in ABC and ADC respectively, the ratio $\frac{B_1}{B_2}$ is **[KCET (Engg./ Med.) 1999]**



(a) 0.2

(b) 6

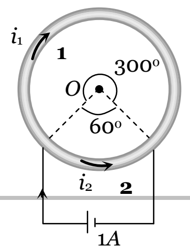
(c) 1

(d) 5

Solution: (c) $B = \frac{\mu_0}{4\pi} \cdot \frac{\theta i}{r}$

$\Rightarrow B \propto \theta i$

$\Rightarrow \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2} \times \frac{i_1}{i_2}$

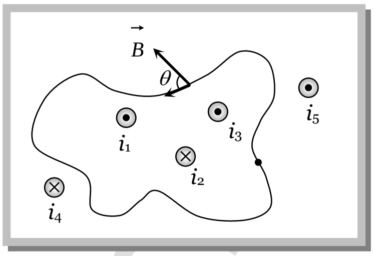


Also $\frac{i_1}{i_2} = \frac{l_2}{l_1} = \frac{\theta_2}{\theta_1}$ Hence $\frac{B_1}{B_2} = \frac{1}{1}$

Amperes Law

Amperes law gives another method to calculate the magnetic field due to a given current distribution.

Line integral of the magnetic field $\vec{B}$ around any closed curve is equal to μ_0 times the net current i threading through the area enclosed by the curve



i.e. $\oint \vec{B} d\vec{l} = \mu_0 \sum i = \mu_0 (i_1 + i_3 - i_2)$

Also using $\vec{B} = \mu_0 \vec{H}$ (where $\vec{H}$ = magnetising field)

$\oint \mu_0 \vec{H}.\vec{dl} = \mu_0 \Sigma i \Rightarrow \oint \vec{H}.\vec{dl} = \Sigma i$

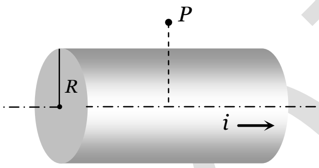
Note : ❒ Total current crossing the above area is $(i_1 + i_3 - i_2)$. Any current outside the area is not included in net current. (Outward $\odot \to +ve$, Inward $\otimes \to -ve$)

❒ When the direction of current is away from the observer then the direction of closed path is clockwise and when the direction of current is towards the observer then the direction of closed path is anticlockwise.

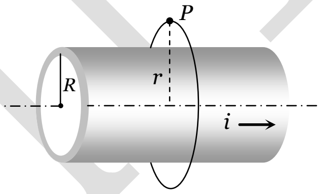
Application of Amperes law

(1) **Magnetic field due to a cylindrical wire**

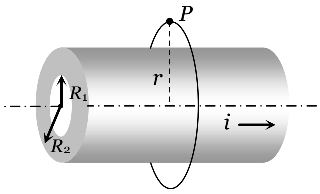
(i) **Outside the cylinder**



Solid cylinder



Thin hollow cylinder



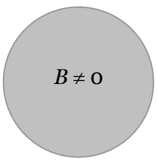
Thick hollow

In all above cases magnetic field outside the wire at P $\oint \vec{B}.\vec{dl} = \mu_0 i \Rightarrow B \oint dl = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 i \Rightarrow$

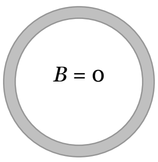
$B_{out} = \frac{\mu_0 i}{2\pi r}$

In all the above cases $B_{surface} = \frac{\mu_0 i}{2\pi R}$

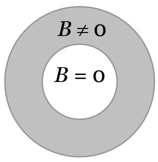
(ii) **Inside the cylinder :** Magnetic field inside the hollow cylinder is zero.



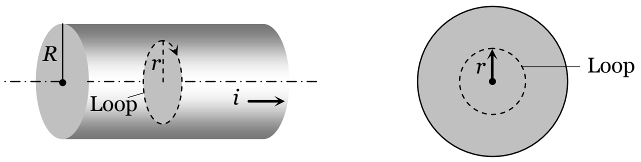
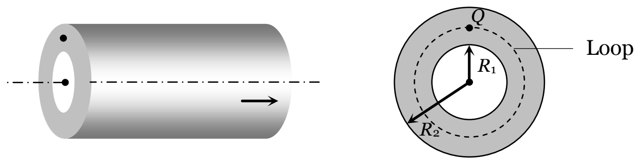
Cross sectional view Solid cylinder



Thin hollow cylinder



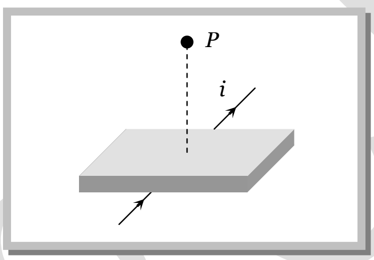
Thick hollow

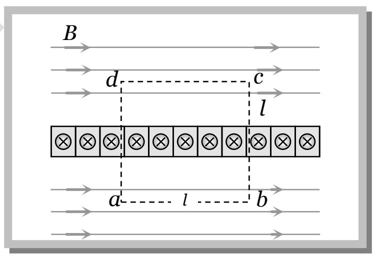
<table>
<tr><th>Solid cylinder</th><th>Inside the thick portion of hollow cylinder</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Current enclosed by loop (i') is lesser then the total current (i)
Current density is uniform i.e. $J = J' \Rightarrow \frac{i}{A} = \frac{i'}{A'}$
$\Rightarrow i' = i \times \frac{A'}{A} = i\left(\frac{r^2}{R^2}\right)$
Hence at point Q $\oint \vec{B}.\vec{dl} = \mu_0 i' \Rightarrow B \times 2\pi r = \frac{\mu_0 i r^2}{R^2}$
$\Rightarrow B = \frac{\mu_0}{2\pi}.\frac{ir}{R^2}$</td><td>Current enclosed by loop (i') is lesser then the total current (i)
Also $i' = i \times \frac{A'}{A} = i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$
Hence at point Q $\oint \vec{B}.\vec{dl} = \mu_0 i' \Rightarrow B \times 2\pi r = \mu_0 i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$
$\Rightarrow B = \frac{\mu_0 i}{2\pi r}.\frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$. If $r = R_1$ (inner surface) $B = 0$
If $r = R_2$ (outer surface) $B = \frac{\mu_0 i}{2\pi R_2}$ (max.)</td></tr>
</table>

Note : ❐ For all cylindrical current distributions

$B_{axis} = 0$ (min.), $B_{surface}$ = max (distance r always from axis of cylinder), $B_{out} \propto 1/r$.

(2) **Magnetic field due to an infinite sheet carrying current :** The figure shows an infinite sheet of current with linear current density j (A/m). Due to symmetry the field line pattern above and below the sheet is uniform. Consider a square loop of side l as shown in the figure.





According to Ampere's law, $\int_a^b B.dl + \int_b^c B.dl + \int_c^d B.dl + \int_d^a B.dl = \mu_0 i$.

Since $B \perp dl$ along the path $b \to c$ and $d \to a$, therefore, $\int_b^c B.dl = 0$; $\int_d^a B.dl = 0$

Also, $B \,||\, dl$ along the path $a \to b$ and $c \to d$, thus $\int_a^b B.dl + \int_d^a B.dl = 2Bl$

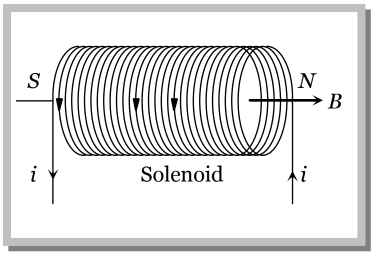
The current enclosed by the loop is $i = jl$

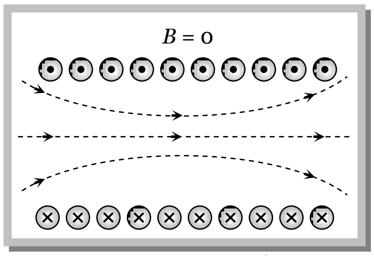
Therefore, according to Ampere's law $2Bl = \mu_0 (jl)$ or $B = \frac{\mu_0 j}{2}$

(3) **Solenoid**

A cylinderical coil of many tightly wound turns of insulated wire with generally diameter of the coil smaller than its length is called a solenoid.

One end of the solenoid behaves like the north pole and opposite end behaves like the south pole. As the length of the solenoid increases, the interior field becomes more uniform and the external field becomes weaker.



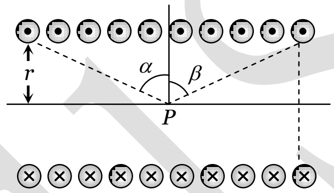


A magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of solenoid.

(i) **Finite length solenoid :** If N = total number of turns,

l = length of the solenoid

n = number of turns per unit length $= \frac{N}{l}$



Magnetic field inside the solenoid at point P is given by $B = \frac{\mu_0}{4\pi}(2\pi ni)[\sin\alpha + \sin\beta]$

(ii) **Infinite length solenoid :** If the solenoid is of infinite length and the point is well inside the solenoid *i.e.* $\alpha = \beta = (\pi/2)$.

So $\qquad \boldsymbol{B_{in} = \mu_0 ni}$

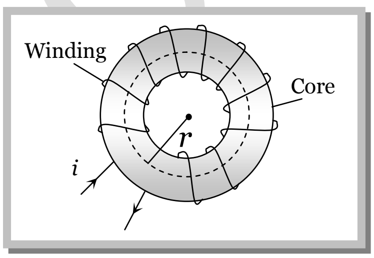
(ii) If the solenoid is of infinite length and the point is near one end *i.e.* $\alpha = 0$ and $\beta = (\pi/2)$

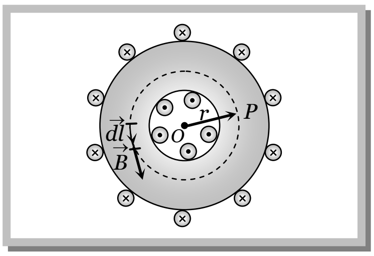
So $\qquad \boldsymbol{B_{end} = \frac{1}{2}(\mu_0 ni)}$

Note : ❒ Magnetic field outside the solenoid is zero.

❒ $B_{end} = \frac{1}{2}B_{in}$

(4) **Toroid :** A toroid can be considered as a ring shaped closed solenoid. Hence it is like an endless cylindrical solenoid.





Consider a toroid having n turns per unit length

Let i be the current flowing through the toroid (figure). The magnetic lines of force mainly remain in the core of toroid and are in the form of concentric circles. Consider such a circle of mean radius r. The circular closed path surrounds N loops of wire, each of which carries a current i therefore from $\oint \vec{B}.\vec{dl} = \mu_0 i_{net}$

$$\Rightarrow \; B \times (2\pi r) = \mu_0 Ni \qquad \Rightarrow B = \frac{\mu_0 Ni}{2\pi r} = \mu_o ni \;\text{ where } n = \frac{N}{2\pi r}$$

For any point inside the empty space surrounded by toroid and outside the toroid, magnetic field B is zero because the net current enclosed in these spaces is zero.

Concepts

- *The line integral of magnetising field $(\vec{H})$ for any closed path called magnetomotive force (MMF). It's S.I. unit is amp.*
- *Ratio of dimension of e.m.f. to MMF is equal to the dimension of resistance.*
- *Biot-Savart law is valid for asymmetrical current distributions while Ampere's law is valid for symmetrical current distributions.*
- *Biot-Savart law is based only on the principle of magnetism while Ampere's laws is based on the principle of electromagnetism.*

Example

***Example*: 22** A long solenoid has 200 *turns per cm* and carries a current of 2.5 *A*. The magnetic field at its centre is $[\mu_0 = 4\pi \times 10^{-7}\ Wb/m^2]$ **[MP PET 2000]**

(a) $3.14 \times 10^{-2}\ Wb/m^2$ (b) $6.28 \times 10^{-2}\ Wb/m^2$ (c) $9.42 \times 10^{-2}\ Wb/m^2$ (d) $12.56 \times 10^{-2}\ Wb/m^2$

Solution : (b) $B = \mu_0 ni = 4\pi \times 10^{-7} \times \frac{200}{10^{-2}} \times 2.5 = 6.28 \times 10^{-2}\ Wb/m^2$.

***Example*: 23** A long solenoid is formed by winding 20 *turns/cm*. The current necessary to produce a magnetic field of 20 *mili tesla* inside the solenoid will be approximately $\left(\frac{\mu_0}{4\pi} = 10^{-7}\ Tesla\text{-}metre/ampere\right)$ **[MP PMT 1994]**

(a) 8.0 *A* (b) 4.0 *A* (c) 2.0 *A* (d) 1.0 *A*

Solution : (a) $B = \mu_0 ni$; where $n = \frac{20}{10}\frac{turn}{cm} = 2000\frac{turn}{m}$. So, $20 \times 10^{-5} = 4\pi \times 2000 \times i \Rightarrow i = 8A$.

***Example*: 24** Two solenoids having lengths L and $2L$ and the number of loops N and $4N$, both have the same current, then the ratio of the magnetic field will be **[CPMT 1994]**

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

Solution : (a) $B = \mu_0 \frac{N}{L} i \Rightarrow B \propto \frac{N}{L} \Rightarrow \frac{B_1}{B_2} = \frac{N_1}{N_2} \times \frac{L_2}{L_1} = \frac{N}{4N} \times \frac{2l}{L} = \frac{1}{2}$.

***Example*: 25** The average radius of a toroid made on a ring of non-magnetic material is 0.1 m and it has 500 turns. If it carries 0.5 ampere current, then the magnetic field produced along its circular axis inside the toroid will be

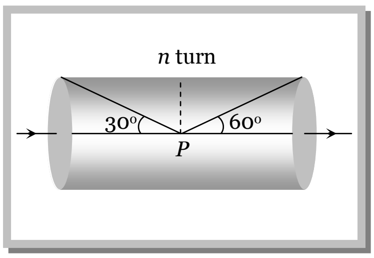
(a) 25×10^{-2} *Tesla* (b) 5×10^{-2} *Tesla* (c) 25×10^{-4} *Tesla* (d) 5×10^{-4} *Tesla*

Solution : (d) $B = \mu_0 ni$; where $n = \frac{N}{2\pi R}$ $\therefore$ $B = 4\pi \times 10^{-7} \times \frac{500}{2\pi \times 0.1} \times 0.5 = 5 \times 10^{-4}\ T$.

***Example*: 26** For the solenoid shown in figure. The magnetic field at point P is

(a) $\frac{\mu_0 ni}{4}(\sqrt{3}+1)$

(b) $\frac{\sqrt{3}\mu_0 ni}{4}$



(c) $\frac{\mu_0 ni}{2}(\sqrt{3}+1)$

(d) $\frac{\mu_0 ni}{4}(\sqrt{3}-1)$

Solution : (a) $B=\frac{\mu_0}{4\pi}.2\pi ni(\sin\alpha+\sin\beta)$. From figure $\alpha=(90^o-30^o)=60^o$ and $\beta=(90^o-60^o)=30^o$

$\therefore\ B=\frac{\mu_0 ni}{2}(\sin 60^o+\sin 30^o)=\frac{\mu_0 ni}{4}(\sqrt{3}+1)$.

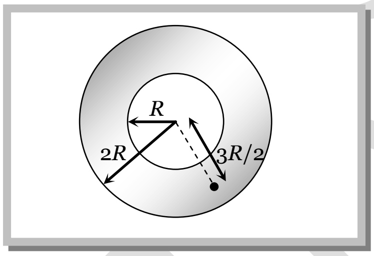
***Example*: 27** Figure shows the cress sectional view of the hollow cylindrical conductor with inner radius 'R' and outer radius '$2R$', cylinder carrying uniformly distributed current along it's axis. The magnetic induction at point 'P' at a distance $\frac{3R}{2}$ from the axis of the cylinder will be

(a) Zero

(b) $\frac{5\mu_0 i}{72\pi R}$

(c) $\frac{7\mu_0 i}{18\pi R}$

(d) $\frac{5\mu_0 i}{36\pi R}$



Solution : (d) By using $B=\frac{\mu_0 i}{2\pi r}\left(\frac{r^2-a^2}{b^2-a^2}\right)$ here $r=\frac{3R}{2}$, $a=R$, $ab=2R\Rightarrow$ $B=\frac{\mu_0 i}{2\pi\left(\frac{3R}{2}\right)}\times\left\{\frac{\left(\frac{3R}{2}\right)-R^2}{(R^2)-R^2}\right\}=\frac{5.\mu_o i}{36\pi r}$.

Tricky example: 3

A winding wire which is used to frame a solenoid can bear a maximum 10 A current. If length of solenoid is 80cm and it's cross sectional radius is 3 cm then required length of winding wire is $(B=0.2\,T)$

(a) $1.2\times10^2\ m$ (b) $4.8\times10^2\ m$ (c) $2.4\times10^3\ m$ (d) $6\times10^3\ m$

Solution : (c) $B=\frac{\mu_0 Ni}{l}$ where $N=$Total number of turns, $l=$length of the solenoid

$\Rightarrow\ 0.2=\frac{4\pi\times10^{-7}\times N\times10}{0.8}\ \Rightarrow\ N=\frac{4\times10^4}{\pi}$

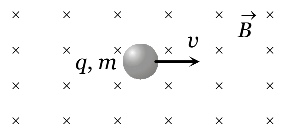
Since N turns are made from the winding wire so length of the wire $(L)=2\pi r\times N$ $[2\pi r=\text{length of each turns}]$

$\Rightarrow\ L=2\pi\times3\times10^{-2}\times\frac{4\times10^4}{\pi}=2.4\times10^3\ m.$

Motion of Charged Particle in a Magnetic Field

If a particle carrying a positive charge q and moving with velocity v enters a magnetic field B then it experiences a force F which is given by the expression

$$F = q(\vec{v} \times \vec{B}) \Rightarrow F = qvB \sin\theta$$



Here $\vec{v} =$ velocity of the particle, $\vec{B} =$ magnetic field

(1) **Zero force**

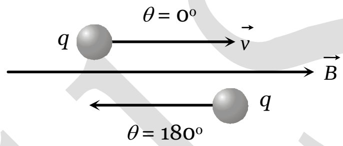
Force on charged particle will be zero (*i.e.* $F = 0$) if

(i) No field *i.e.* $B = 0 \Rightarrow F = 0$

(ii) Neutral particle *i.e.* $q = 0 \Rightarrow F = 0$

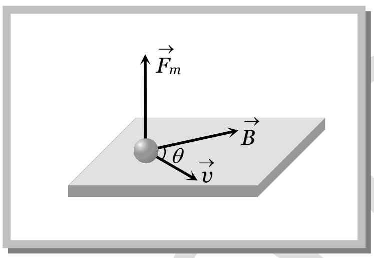
(iii) Rest charge *i.e.* $v = 0 \Rightarrow F = 0$

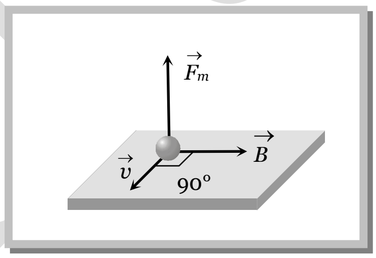
(iv) Moving charge *i.e.* $\theta = 0^o$ or $\theta = 180^o \Rightarrow F = 0$



(2) **Direction of force**

The force $\vec{F}$ is always perpendicular to both the velocity $\vec{v}$ and the field $\vec{B}$ in accordance with Right Hand Screw Rule, through $\vec{v}$ and $\vec{B}$ themselves may or may not be perpendicular to each other.



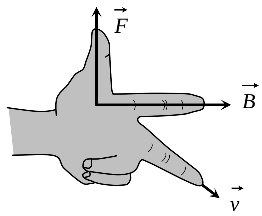


Direction of force on charged particle in magnetic field can also be find by Flemings Left Hand Rule (FLHR).

Here, *First finger* (indicates) $\rightarrow$ Direction of magnetic field

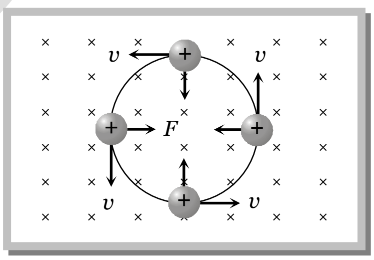
Middle finger $\rightarrow$ Direction of motion of positive charge or direction, opposite to the motion of negative charge.

Thumb $\rightarrow$ Direction of force



(3) **Circular motion of charge in magnetic field**

Consider a charged particle of charge q and mass m enters in a uniform magnetic field B with an initial velocity v perpendicular to the field.



$\theta = 90^o$, hence from $F = qvB\sin\theta$ particle will experience a maximum magnetic force $\boldsymbol{F_{max} = qvB}$ which act's in a direction perpendicular to the motion of charged particle. (By Flemings left hand rule).

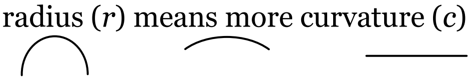
(i) **Radius of the path :** In this case path of charged particle is circular and magnetic force provides the necessary centripetal force *i.e.* $qvB = \frac{mv^2}{r}$ $\Rightarrow$ radius of path $r = \frac{mv}{qB}$

If p = momentum of charged particle and K = kinetic energy of charged particle (gained by charged particle after accelerating through potential difference V) then $p = mv = \sqrt{2mK} = \sqrt{2mqV}$

So $$\boldsymbol{r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}}$$

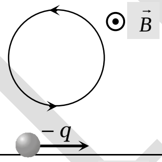
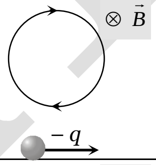
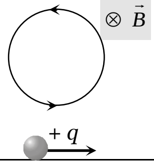
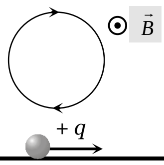
$r \propto v \propto p \propto \sqrt{K}$ *i.e.* with increase in speed or kinetic energy, the radius of the orbit increases.

Note : ❒ Less radius (r) means more curvature (c) *i.e.* $c \propto \frac{1}{r}$



Less : r More : c | More : r Less : c | $r = \infty$ $c = 0$

(ii) **Direction of path :** If a charge particle enters perpendicularly in a magnetic field, then direction of path described by it will be

Type of charge	Direction of magnetic field	Direction of it's circular motion
Negative	Outwards ⊙	 Anticlockwise
Negative	Inward ⊗	 Clockwise
Positive	Inward ⊗	 Anticlockwise
Positive	Outward ⊙	 Clockwise

(iii) **Time period** : As in uniform circular motion $v = r\omega$, so the angular frequency of circular motion, called cyclotron or gyro-frequency, will be given by $\omega = \frac{v}{r} = \frac{qB}{m}$ and hence the time period, $T = \frac{2\pi}{\omega} = 2\pi\frac{m}{qB}$

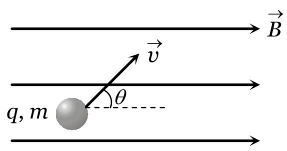
i.e., time period (or frequency) is independent of speed of particle and radius of the orbit and depends only on the field B and the nature, *i.e.*, specific charge $\left(\frac{q}{m}\right)$, of the particle.

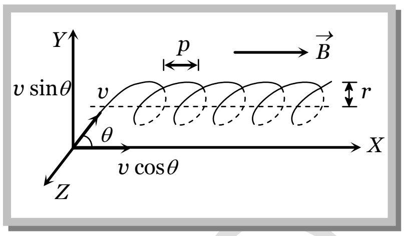
(4) **Motion of charge on helical path**

genius

When the charged particle is moving at an angle to the field (other than 0°, 90°, or 180°).

In this situation resolving the velocity of the particle along and perpendicular to the field, we find that the particle moves with constant velocity $v\cos\theta$ along the field (as no force acts on a charged particle when it moves parallel to the field) and at the same time it is also moving with velocity $v\sin\theta$ perpendicular to the field due to which it will describe a circle (in a plane perpendicular to the field) of radius. $\boldsymbol{r} = \dfrac{\boldsymbol{m(v\sin\theta)}}{\boldsymbol{qB}}$





Time period and frequency do not depend on velocity and so they are given by $T = \dfrac{2\pi m}{qB}$ and $\nu = \dfrac{qB}{2\pi m}$

So the resultant path will be a *helix* with its axis parallel to the field $\vec{B}$ as shown in figure in this situation.

The *pitch* of the *helix*, (*i.e.*, linear distance travelled in one rotation) will be given by $p = T(v\cos\theta) = 2\pi\dfrac{m}{qB}(v\cos\theta)$

Note : ❒ 1 rotation ≡ 2π ≡ T and 1 pitch ≡ 1 T

❒ Number of pitches ≡ Number of rotations ≡ Number of repetition = Number of helical turns

❒ If pitch value is p, then number of pitches obtained in length l given as

Number of pitches $= \dfrac{l}{p}$ and time reqd. $t = \dfrac{l}{v\cos\theta}$

Some standard results

& Ratio of radii of path described by proton and α-particle in a magnetic field (particle enters perpendicular to the field)

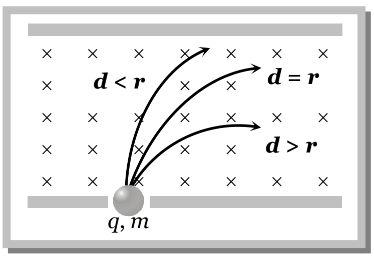
Constant quantity	Formula	Ratio of radii	Ratio of curvature (c)
v - same	$r = \dfrac{mv}{qB} \Rightarrow r \propto \dfrac{m}{q}$	$r_p : r_\alpha = 1:2$	$c_p : c_R = 2:1$
p - same	$r = \dfrac{p}{qB} \Rightarrow r \propto \dfrac{1}{q}$	$r_p : r_\alpha = 2:1$	$c_p : c_R = 1:2$
k - same	$r = \dfrac{\sqrt{2mk}}{qB} \Rightarrow r \propto \dfrac{\sqrt{m}}{q}$	$r_p : r_\alpha = 1:1$	$c_p : c_R = 1:1$
V - same	$r \propto \sqrt{\dfrac{m}{q}}$	$r_p : r_\alpha = 1:\sqrt{2}$	$c_p : c_R = \sqrt{2}:1$

& **Particle motion between two parallel plates** $(\vec{v} \perp \vec{B})$

(i) To strike the opposite plate it is essential that $d < r$.

(ii) Does not strike the opposite plate $d > r$.

(iii) To touch the opposite plate $d = r$.



(iv) To just not strike the opposite plate $d \geq r$.

(v) To just strike the opposite plate $d \leq r$.

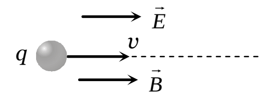
(5) **Lorentz force**

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F}_e = q\vec{E}$ and magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B})$; so the net force on it will be $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$. Which is the famous 'Lorentz-force equation'.

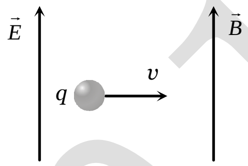
Depending on the directions of $\vec{v}, \vec{E}$ and $\vec{B}$ following situations are possible

(i) **When $\vec{v}, \vec{E}$ and $\vec{B}$ all the three are collinear :** In this situation as the particle is moving parallel or antiparallel to the field, the magnetic force on it will be zero and only electric force will act and so $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

The particle will pass through the field following a straight line path (parallel field) with change in its speed. So in this situation speed, velocity, momentum kinetic energy all will change without change in direction of motion as shown

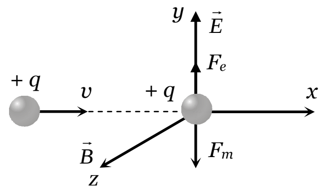


(ii) **When $\vec{E}$ is parallel to $\vec{B}$ and both these fields are perpendicular to $\vec{v}$ then :** $\vec{F}_e$ is perpendicular to $\vec{F}_m$ and they cannot cancel each other. The path of charged particle is curved in both these fields.



(iii) **$\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular :** In this situation if $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0 \ \text{i.e.,} \ \vec{a} = (\vec{F}/m) = 0$$



as shown in figure, the particle will pass through the field with same velocity.

And in this situation, as $F_e = F_m$ i.e., $qE = qvB$ $v = E/B$

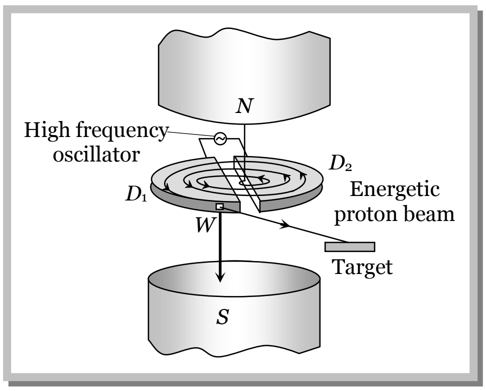
This principle is used in 'velocity-selector' to get a charged beam having a specific velocity.

Note : ❒ From the above discussion, conclusion is as follows

- ❒ If E = 0, B = 0, so F = 0.
- ❒ If E = 0, $B \neq$ 0, so F may be zero (if $\theta = 0^o$ or 180^o).
- ❒ If $E \neq$ 0, $B \neq$ 0, so F = 0 (if $|\vec{F}_e| = |\vec{F}_m|$ and their directions are opposite)
- ❒ If $E \neq$ 0, B = 0, so $F \neq$ 0 (because $\vec{v} \neq$ constant).

Cyclotron

Cyclotron is a device used to accelerated positively charged particles (like, α-particles, deutrons *etc.*) to acquire enough energy to carry out nuclear disintegration *etc.* t is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency. Thus a small potential difference would impart if



enormously large velocities if the particle is made to traverse the potential difference a number of times.

It consists of two hollow *D*-shaped metallic chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} *mm* mercury. The whole apparatus is placed between the two poles of a strong electromagnet *NS* as shown in fig. The magnetic field acts perpendicular to the plane of the dees.

Note : ❒ The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas is used.

(1) **Cyclotron frequency :** Time taken by ion to describe *q* semicircular path is given by $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$

If T = time period of oscillating electric field then $T = 2t = \frac{2\pi m}{qB}$ the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

(2) **Maximum energy of position :** Maximum energy gained by the charged particle $E_{max} = \left(\frac{q^2B^2}{2m}\right)r^2$

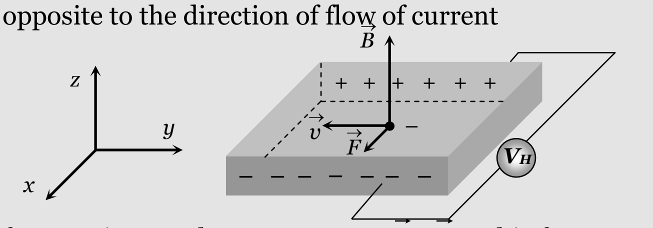
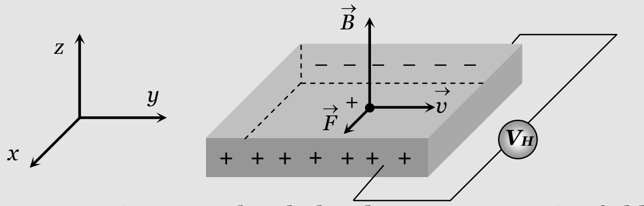
where r_0 = maximum radius of the circular path followed by the positive ion.

Note : ❒ Cyclotron frequency is also known as magnetic resonance frequency.

❒ Cyclotron can not accelerate electrons because they have very small mass.

Hall effect : The Phenomenon of producing a transverse emf in a current carrying conductor on applying a magnetic field perpendicular to the direction of the current is called Hall effect.

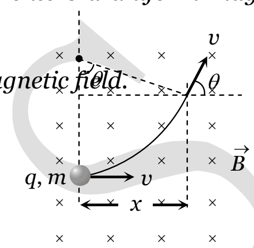
Hall effect helps us to know the nature and number of charge carriers in a conductor.

Negatively charged particles	**Positively charged particles**
Consider a conductor having electrons as current carriers. The electrons move with drift velocity $\vec{v}$ opposite to the direction of flow of current  force acting on electron $F_m = -e(\vec{v} \times \vec{B})$. This force acts along *x*-axis and hence electrons will move towards face (2) and it becomes negatively charged.	Let the current carriers be positively charged holes. The hole move in the direction of current  Force acting on the hole due to magnetic field $\vec{F}_m = +e(\vec{v} \times \vec{B})$ force acts along *x*-axis and hence holes move towards face (2) and it becomes positively charged.

Concepts

- *The energy of a charged particle moving in a uniform magnetic field does not change because it experiences a force in a direction, perpendicular to it's direction of motion. Due to which the speed of charged particle remains unchanged and hence it's K.E. remains same.*
- *Magnetic force does no work when the charged particle is displaced while electric force does work in displacing the charged particle.*
- *Magnetic force is velocity dependent, while electric force is independent of the state of rest or motion of the charged particle.*
- *If a particle enters a magnetic field normally to the magnetic field, then it starts moving in a circular orbit. The point at which it enters the magnetic field lies on the circumference. (Most of us confuse it with the centre of the orbit)*
- ***Deviation of charged particle in magnetic field :*** *If a charged particle (q, m) enters a uniform magnetic field $\vec{B}$ (extends upto a length x) at right angles with speed v as shown in figure.*

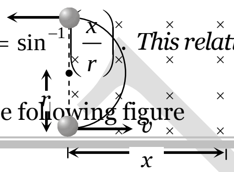
  The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field.



Deviation in terms of time t; $\theta = \omega t = \left(\frac{Bq}{m}\right)t$

Deviation in terms of length of the magnetic field ; $\theta = \sin^{-1}\left(\frac{x}{r}\right)$. *This relation can be used only when* $x \le r$.

For x > r, the deviation will be 180^o as shown in the following figure



Example

Example: 28 Electrons move at right angles to a magnetic field of 1.5×10^{-2} *Tesla* with a speed of 6×10^{27} m/s. If the specific charge of the electron is 1.7×10^{11} *Coul/kg*. The radius of the circular path will be **[BHU 2003]**

(a) 2.9 *cm* (b) 3.9 *cm* (c) 2.35 *cm* (d) 3 *cm*

Solution : (c) $r = \frac{mv}{qB} \Rightarrow \frac{v}{(q/m).B} = \frac{6\times10^{27}}{17\times10^{11}\times1.5\times10^{-2}} = 2.35\times10^{-2}\,m = 2.35\,cm.$

Example: 29 An electron (mass $=9\times10^{-31}$ *kg*. charge $=1.6\times10^{-19}$ *coul.*) whose kinetic energy is 7.2×10^{-18} *joule* is moving in a circular orbit in a magnetic field of 9×10^{-5} *weber* $/m^2$. The radius of the orbit is **[MP PMT 2002]**

(a) 1.25 *cm* (b) 2.5 *cm* (c) 12.5 *cm* (d) 25.0 *cm*

Solution : (d) $r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2\times q\times10^{-31}\times7.2\times10^{-8}}}{1.6\times10^{-19}\times q\times10^{-5}} = 0.25\,cm = 25\,cm$.

Example: 30 An electron and a proton enter a magnetic field perpendicularly. Both have same kinetic energy. Which of the following is true **[MP PET 1999]**

(a) Trajectory of electron is less curved (b) Trajectory of proton is less curved

(c) Both trajectories are equally curved (d) Both move on straight line path

Solution : (b) By using $r = \frac{\sqrt{2mk}}{qB}$; For both particles $q \to$ same, $B \to$ same, $k \to$ same

Hence $r \propto \sqrt{m} \Rightarrow \frac{r_e}{r_p} = \sqrt{\frac{m_e}{m_p}}$ $\because$ $m_p > m_e$ so $r_p > r_e$

Since radius of the path of proton is more, hence it's trajectory is less curved.

***Example*: 31** A proton and an α – particles enters in a uniform magnetic field with same velocity, then ratio of the radii of path describe by them

(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) None of these

Solution : (b) By using $r = \frac{mv}{qB}$; $v \to$ same, $B \to$ same $\Rightarrow r \propto \frac{m}{2} \Rightarrow \frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p} = \frac{m_p}{4m_p} \times \frac{2q_p}{q_p} = \frac{1}{2}$

***Example*: 32** A proton of mass m and charge $+e$ is moving in a circular orbit of a magnetic field with energy 1*MeV*. What should be the energy of α-particle (mass = 4 m and charge = +2e), so that it can revolve in the path of same radius **[BHU 1997]**

(a) 1 *MeV* (b) 4 *MeV* (c) 2 *MeV* (d) 0.5 *MeV*

Solution : (a) By using $r = \frac{\sqrt{2mK}}{qB}$; $r \to$ same, $B \to$ same $\Rightarrow K \propto \frac{q^2}{m}$

Hence $\frac{K_\alpha}{K_p} = \left(\frac{q_\alpha}{q_p}\right)^2 \times \frac{m_p}{m_\alpha} = \left(\frac{2q_p}{q_p}\right)^2 \times \frac{m_p}{4m_p} 1 \Rightarrow K_\alpha = K_p = 1\, meV.$

***Example*: 33** A proton and an α – particle enter a uniform magnetic field perpendicularly with the same speed. If proton takes $25\,\mu\,\text{sec}$ to make 5 revolutions, then the periodic time for the α – particle would be **[MP PET 1993]**

(a) $50\,\mu$ sec (b) $25\,\mu$ sec (c) $10\,\mu$ sec (d) $5\,\mu$ sec

Solution : (c) Time period of proton $T_p = \frac{25}{5} = 5\,\mu\,\text{sec}$

By using $T = \frac{2\pi m}{qB} \Rightarrow \frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p} \times \frac{q_p}{q_\alpha} = \frac{4m_p}{m_p} \times \frac{q_p}{2q_p} \Rightarrow T_\alpha = 2T_p = 10\,\mu$ sec .

***Example*: 34** A particle with 10^{-11} *coulomb* of charge and 10^{-7} *kg* mass is moving with a velocity of 10^8 *m/s* along the *y*-axis. A uniform static magnetic field *B* = 0.5 *Tesla* is acting along the *x*-direction. The force on the particle is

[MP PMT 1997]

(a) 5×10^{-11} *N* along $\hat{i}$ (b) 5×10^{3} *N* along $\hat{k}$ (c) 5×10^{-11} *N* along $-\hat{j}$ (d) 5×10^{-4} *N* along $-\hat{k}$

Solution : (d) By using $\vec{F} = q(\vec{v} \times \vec{B})$; where $\vec{v} = 10\hat{j}$ and $\vec{B} = 0.5\hat{i}$

$\Rightarrow \vec{F} = 10^{-11}(10^8\,\hat{j} \times 0.5\hat{i}) = 5 \times 10^{-4}(\hat{j} \times \hat{i}) = 5 \times 10^{-4}(-\hat{k})$ *i.e.*, $5 \times 10^{-4} N$ along $-\hat{k}$.

***Example*: 35** An electron is moving along positive *x*-axis. To get it moving on an anticlockwise circular path in *x-y* plane, a magnetic filed is applied

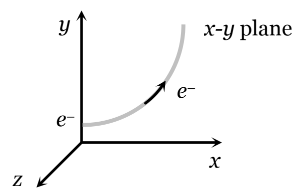
(a) Along positive *y*-axis (b) Along positive *z*-axis

(c) Along negative *y*-axis (d) Along negative *z*-axis

Solution : (a) The given situation can be drawn as follows

According to figure, for deflecting electron in *x-y* plane, force must be acting an it towards *y*-axis.

Hence according to Flemings left hand rule, magnetic field directed along positive y – axis.



Example: 36 A particle of charge -16×10^{-18} coulomb moving with velocity 10 *m/s* along the *x*-axis enters a region where a magnetic field of induction *B* is along the *y*-axis, and an electric field of magnitude 10^4 *V/m* is along the negative *z*-axis. If the charged particle continuous moving along the *x*-axis, the magnitude of *B* is **[AIEEE 2003]**

(a) $10^{-3}\ Wb/m^2$ (b) $10^3\ Wb/m^2$ (c) $10^5\ Wb/m^2$ (d) $10^{16}\ Wb/m^2$

Solution : (b) Particles is moving undeflected in the presence of both electric field as well as magnetic field so it's speed

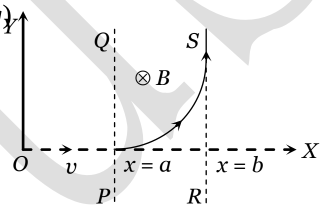
$$v = \frac{E}{B} \Rightarrow B = \frac{E}{v} = \frac{10^4}{10} = 10^3 Wb/m^2.$$

Example: 37 A particle of mass *m* and charge *q* moves with a constant velocity *v* along the positive *x* direction. It enters a region containing a uniform magnetic field *B* directed along the negative *z* direction extending from *x* = *a* to *x* = *b*. The minimum value of *v* required so that the particle can just enter the region *x* > *b* is

[IIT-JEE (Screening) 2002]

(a) *qbB/m* (b) *q*(*b* – *a*)*B/m* (c) *qaB/m* (d) *q*(*b*+*a*)*B*/2*m*

Solution : (b) As shown in the following figure, the $z-$axis points out of the paper and the magnetic fields is directed into the paper, existing in the region between *PQ* and *RS*. The particle moves in a circular path of radius *r* in the magnetic field. It can just enter the region $x > b$ for $r \geq (b-a)$



Now $r = \dfrac{mv}{qb} \geq (b-a)$

$$\Rightarrow v \geq \frac{q(b-a)B}{m} \Rightarrow v_{\min} = \frac{q(b-a)B}{m}.$$

Example: 38 At a certain place magnetic field vertically downwards. An electron approaches horizontally towards you and enters in this magnetic fields. It's trajectory, when seen from above will be a circle which is

(a) Vertical clockwise (b) Vertical anticlockwise

(c) Horizontal clockwise (d) Horizontal anticlockwise

Solution : (c) By using Flemings left hand rule.

Example: 39 When a charged particle circulates in a normal magnetic field, then the area of it's circulation is proportional to

(a) It's kinetic energy (b) It's momentum

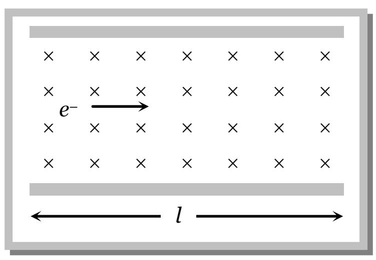
(c) It's charge (d) Magnetic fields intensity

Solution : (a) $r = \dfrac{\sqrt{2mK}}{qB}$ and $A = Aq^2 \Rightarrow A = \dfrac{\pi(2mK)}{q^2b^2} \Rightarrow A \propto K.$

Example: 40 An electron moves straight inside a charged parallel plate capacitor at uniform charge density σ. The space between the plates is filled with constant magnetic field of induction $\vec{B}$. Time of straight line motion of the electron in the capacitor is

(a) $\dfrac{e\sigma}{\varepsilon_0 lB}$

(b) $\dfrac{\varepsilon_0 lB}{\sigma}$



genius

(c) $\frac{e\sigma}{\varepsilon_0 B}$

(d) $\frac{\varepsilon_0 B}{e\sigma}$

Solution : (b) The net force acting on the electron is zero because it moves with constant velocity, due to it's motion on straight line.

$$\Rightarrow \vec{F}_{net} = \vec{F}_e + \vec{F}_m = 0 \Rightarrow |\vec{F}_e| = |\vec{F}_m| \Rightarrow eE = evB \Rightarrow ve = \frac{E}{B} = \frac{\sigma}{\varepsilon_0 B} \qquad \left[E = \frac{\sigma}{\varepsilon_o}\right]$$

$\therefore$ The time of motion inside the capacitor $t = \frac{l}{v} = \frac{\varepsilon_0 lB}{\sigma}$.

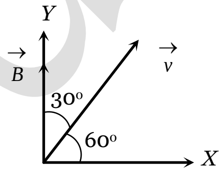
Example: 41 A proton of mass $1.67 \times 10^{-27}\ kg$ and charge $1.6 \times 10^{-19}\ C$ is projected with a speed of $2 \times 10^6\ m/s$ at an angle of 60° to the *X*-axis. If a uniform magnetic field of 0.104 *Tesla* is applied along *Y*-axis, the path of proton is

(a) A circle of radius = 0.2 *m* and time period $\pi \times 10^{-7}\ s$

(b) A circle of radius = 0.1 *m* and time period $2\pi \times 10^{-7}\ s$

(c) A helix of radius = 0.1 *m* and time period $2\pi \times 10^{-7}\ s$

(d) A helix of radius = 0.2 *m* and time period $4\pi \times 10^{-7}\ s$



Solution : (b) By using $r = \frac{mv\sin\theta}{qB} \Rightarrow r = \frac{1.67 \times 15^{27} \times 2 \times 10^6 \times \sin 30°}{1.6 \times 10^{-19} \times 0.104} = 0.1m$

and it's time period $T = \frac{2\pi m}{qB} = \frac{2 \times \pi \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 0.104} = 2\pi \times 10^{-7}$ sec.

Example: 42 A charge particle, having charge *q* accelerated through a potential difference *V* enter a perpendicular magnetic field in which it experiences a force *F*. If *V* is increased to 5*V*, the particle will experience a force

(a) *F* (b) 5*F* (c) $\frac{F}{5}$ (d) $\sqrt{5}F$

Solution : (d) $\frac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\frac{2qV}{m}}$. Also $F = qvB$

$\Rightarrow F = qB\sqrt{\frac{2qV}{m}}$ hence $F \propto \sqrt{V}$ which gives $F' = \sqrt{5}F$.

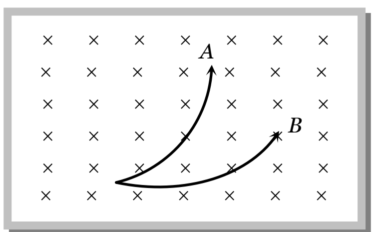
Example: 43 The magnetic field is downward perpendicular to the plane of the paper and a few charged particles are projected in it. Which of the following is true **[CPMT 1997]**

(a) *A* represents proton and *B* and electron

(b) Both *A* and *B* represent protons but velocity of *A* is more than that of *B*

(c) Both *A* and *B* represents protons but velocity of *B* is more than that of *A*

(d) Both *A* and *B* represent electrons, but velocity of *B* is more than that of *A*



Solution : (c) Both particles are deflecting in same direction so they must be of same sign.(*i.e.*, both *A* and *B* represents protons)

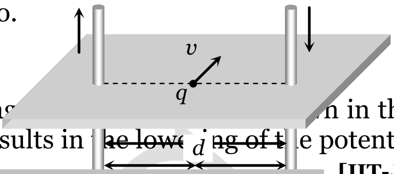
By using $r = \frac{mv}{qB} \Rightarrow r \propto v$

From given figure radius of the path described by particle B is more than that of A. Hence $v_B > v_A$.

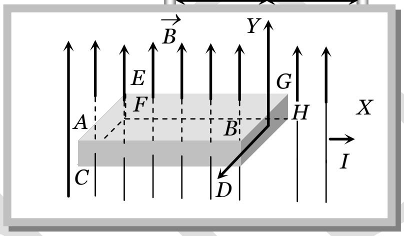
***Example*: 44** Two very long straight, particle wires carry steady currents i and $-i$ respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires, in the plane of the wires. It's instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(a) $\dfrac{\mu_0 iqv}{2\pi d}$ (b) $\dfrac{\mu_0 iqv}{\pi d}$ (c) $\dfrac{2\mu_0 iqv}{\pi d}$ (d) Zero

Solution : (d) According to gives information following figure can be drawn, which shows that direction of magnetic field is along the direction of motion of charge so net on it is zero.



***Example*: 45** A metallic block carrying current i is subjected to a uniform ma... n in the figure. The moving charges experience a force F given by .... which results in ... e potential of the face .... Assume the speed of the carriers to be v **[IIT-JEE 1996]**



(a) $eVB\hat{k}$, $ABCD$

(b) $eVB\hat{k}$, $ABCD$

(c) $-eVB\hat{k}$, $ABCD$

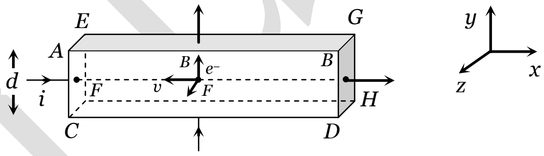
(d) $-eVB\hat{k}$, $EFGH$

Solution : (c) As the block is of metal, the charge carriers are electrons; so for current along positive x-axis, the electrons are moving along negative x-axis, *i.e.* $\vec{v} = -vi$

and as the magnetic field is along the y-axis, *i.e.* $\vec{B} = B\hat{j}$

so $\vec{F} = q(\vec{v} \times \vec{B})$ for this case yield $\vec{F} = (-e)[-v\hat{i} \times B\hat{j}]$

i.e., $\vec{F} = evB\hat{k}$ [As $\hat{i} \times \hat{j} = \hat{k}$]



As force on electrons is towards the face $ABCD$, the electrons will accumulate on it an hence it will acquire lower potential.

Tricky example: 4

An ionised gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+ve$ x-axis and a magnetic field along the $+z$ direction then **[IIT-JEE (Screening**

(a) Positive ions deflect towards $+y$ direction and negative ions towards $-y$ direction

(b) All ions deflect towards $+y$ direction

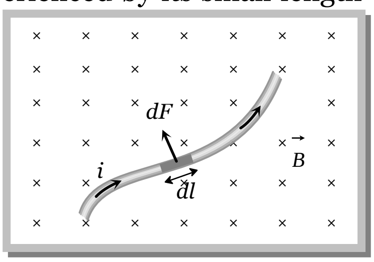
(c) All ions deflect towards $-y$ direction

(d) Positive ions deflect towards $-y$ direction and negative ions towards $+y$ direction.

Solution : (c) As the electric field is switched on, positive ion will start to move along positive x-direction and negative ion along negative x-direction. Current associated with motion of both types of ions is along positive x-direction. According to Flemings left hand rule force on both types of ions will be along negative y-direction.

Force on a Current Carrying Conductor in Magnetic Field

In case of current carrying conductor in a magnetic field force experienced by its small length element is $d\vec{F} = i d\vec{l} \times \vec{B}$; $i d\vec{l}$ = current element $d\vec{F} = l(d\vec{l} \times \vec{B})$



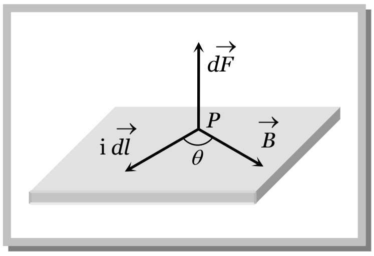
Total magnetic force $\vec{F} = \int d\vec{F} = \int i(d\vec{l} \times \vec{B})$

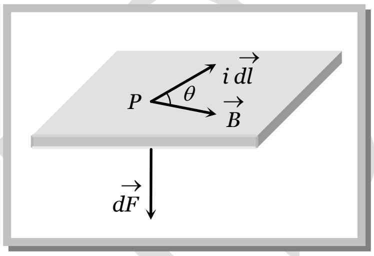
If magnetic field is uniform *i.e.*, $\vec{B}$ = constant

$$\vec{F} = i\left[\int d\vec{l}\right] \times \vec{B} = i(\vec{L}' \times \vec{B})$$

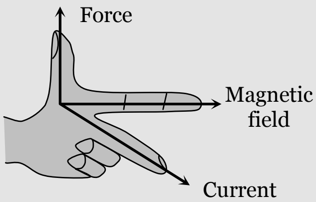
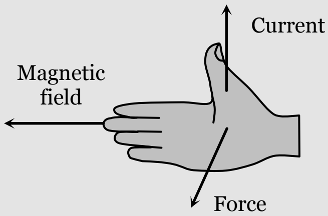
$\int d\vec{l} = \vec{L} =$ vector sum of all the length elements from initial to final point. Which is in accordance with the law of vector addition is equal to length vector $\vec{L}'$ joining initial to final point.

(1) **Direction of force :** The direction of force is always perpendicular to the plane containing $id\vec{l}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{A} \times \vec{B})$ with $\vec{A} = i\,d\vec{l}$.





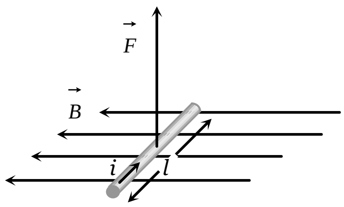
The direction of force when current element $i\,d\vec{l}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules

Fleming's left-hand rule	Right-hand palm rule
Stretch the fore-finger, central finger and thumb left hand mutually perpendicular. Then if the fore-finger points in the direction of field $\vec{B}$ and the central in the direction of current i, the thumb will point in the direction of force 	Stretch the fingers and thumb of right hand at right angles to each other. Then if the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current i, then normal to the palm will point in the direction of force 

(2) **Force on a straight wire :** If a current carrying straight conductor (length l) is placed in an uniform magnetic field (B) such that it makes an angle θ with the direction of field then force experienced by it is $F = Bil\sin\theta$

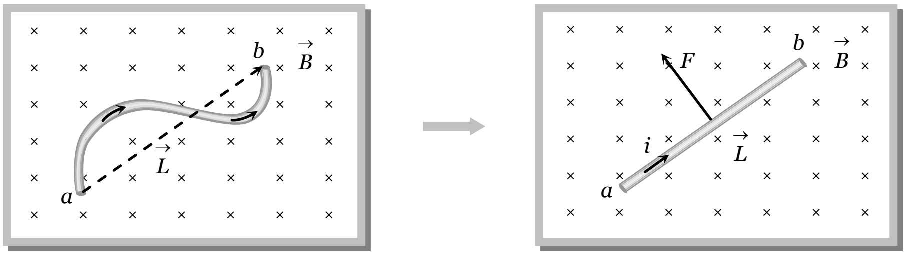
If $\theta = 0^o$, $F = 0$

If $\theta = 90^o$, $F_{max} = Bil$



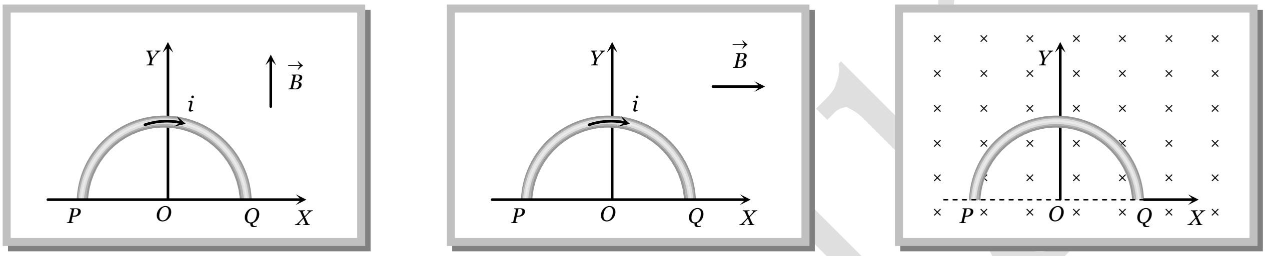
(3) **Force on a curved wire**

The force acting on a curved wire joining points a and b as shown in the figure is the same as that on a straight wire joining these points. It is given by the expression $\vec{F} = i\vec{L} \times \vec{B}$



Specific Example

The force experienced by a semicircular wire of radius R when it is carrying a current i and is placed in a uniform magnetic field of induction B as shown.



$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B\hat{i}$

So by using $\vec{F} = i(\vec{L}' \times \vec{B})$ force on the wire

$\vec{F} = i(2R)(B)(\hat{i} \times \hat{i}) \Rightarrow \vec{F} = 0$

$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B\hat{j}$

$\vec{F} = i \times 2BR(\hat{i} \times \hat{j})$

$\vec{F} = 2BiR\hat{k}$ i.e. $F = 2BiR$

(perpendicular to paper outward)

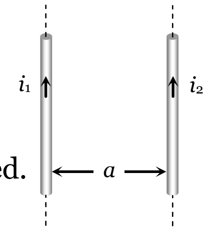
$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B(-\hat{k})$

$\therefore\ \vec{F} = i \times 2BR(+\hat{j})$

$F = 2BiR$ (along Y-axis)

Force Between Two Parallel Current Carrying Conductors

When two long straight conductors carrying currents i_1 and i_2 placed parallel to each other at a distance 'a' from each other. A mutual force act between them when is given as

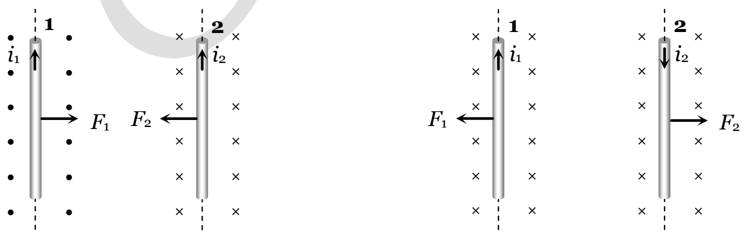


$$F_1 = F_2 = F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a} \times l$$

where l is the length of that portion of the conductor on which force is to be calculated.

Hence force per unit length $\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a} \frac{N}{m}$ or $\frac{F}{l} = \frac{2i_1 i_2}{a} \frac{dyne}{cm}$

Direction of force : If conductors carries current in same direction, then force between them will be attractive. If conductor carries current in opposite direction, then force between them will be repulsive.

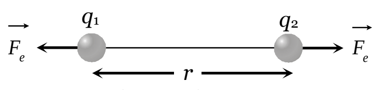


Note : ❐ If a = 1m and in free space $\frac{F}{l} = 2\times10^{-7}\, N/m$ then $i_1 = i_2 = 1 Amp$ in each identical wire.

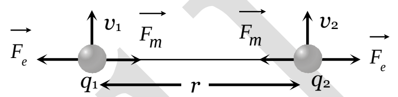
By this concept S.I. unit of Ampere is defined. This is known as **Ampere's law**.

Force Between Two Moving Charges

If two charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively and at any instant the distance between them is r, then



Stationary charges



Moving charges

Magnetic force between them is $F_m = \frac{\mu_0}{4\pi}.\frac{q_1 q_2 v_1 v_2}{r^2}$ ..... (i)

and Electric force between them is $F_e = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r^2}$ ..... (ii)

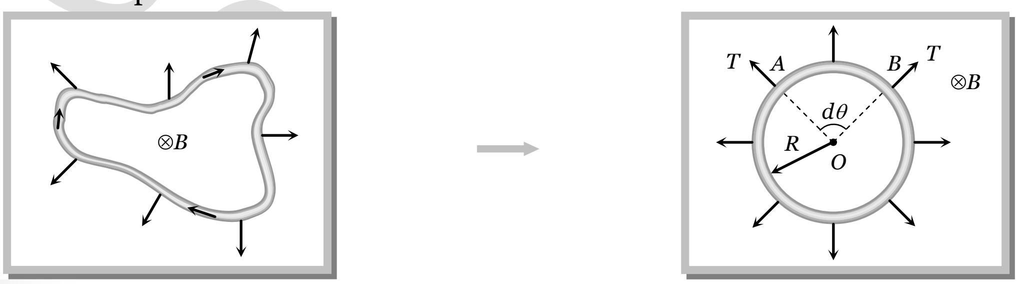
From equation (i) and (ii) $\frac{F_m}{F_e} = \mu_0 \varepsilon_0 v^2$ but $\mu_0 \varepsilon_0 = \frac{1}{c^2}$; where c is the velocity light in vacuum. So

$$\frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2$$

If $v << c$ then $F_m << F_e$

Standard Cases for Force on Current Carrying Conductors

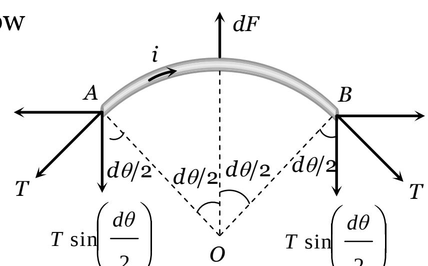
***Case* 1 :** When an arbitrary current carrying loop placed in a magnetic field (⊥ to the plane of loop), each element of loop experiences a magnetic force due to which loop stretches and open into circular loop and tension developed in it's each part.



Specific example

In the above circular loop tension in part A and B.

In balanced condition of small part AB of the loop is shown below

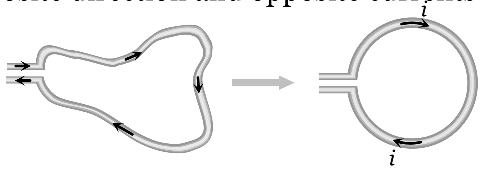


$$2T\sin\frac{d\theta}{2} = dF = Bidl \Rightarrow 2T\sin\frac{d\theta}{2} = BiRd\theta$$

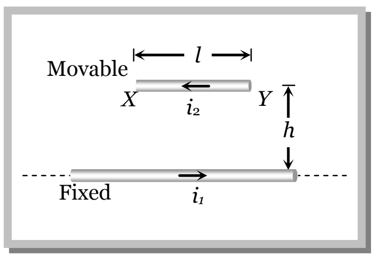
If $d\theta$ is small so, $\sin\frac{d\theta}{2} \approx \frac{d\theta}{2} \Rightarrow 2T.\frac{d\theta}{2} = BiRd\theta$

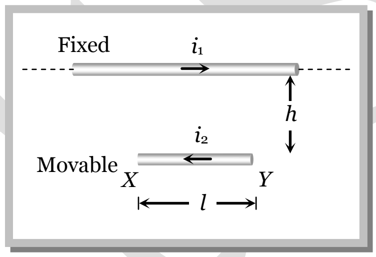
$$T = BiR,\ \text{if } 2\pi R = L \text{ so } T = \frac{BiL}{2\pi}$$

Note : ❒ If no magnetic field is present, the loop will still open into a circle as in it's adjacent parts current will be in opposite direction and opposite currents repel each other.



***Case* 2 : Equilibrium of a current carrying conductor :** When a finite length current carrying wire is kept parallel to another infinite length current carrying wire, it can suspend freely in air as shown below



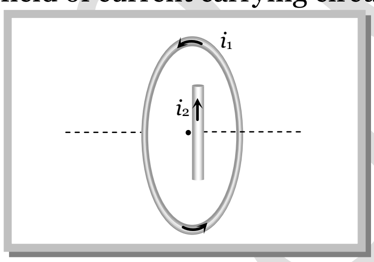


In both the situations for equilibrium of XY it's downward weight = upward magnetic force *i.e.* $mg = \frac{\mu_0}{4\pi}.\frac{2i_1i_2}{h}.l$

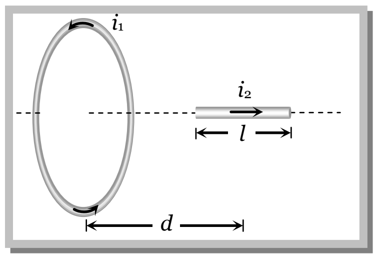
Note : ❒ In the first case if wire XY is slightly displaced from its equilibrium position, it executes SHM and it's time period is given by $T = 2\pi\sqrt{\frac{h}{g}}$.

❒ If direction of current in movable wire is reversed then it's instantaneous acceleration produced is $2g\downarrow$.

***Case* 3 : Current carrying wire and circular loop :** If a current carrying straight wire is placed in the magnetic field of current carrying circular loop.

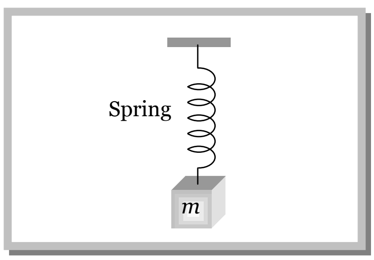


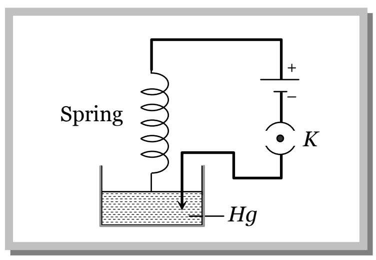
Wire is placed in the perpendicular magnetic field due to coil at it's centre, so it will experience a maximum force $F = Bil = \frac{\mu_0 i_1}{2r} \times i_2 l$



wire is placed along the axis of coil so magnetic field produced by the coil is parallel to the wire. Hence it will not experience any force.

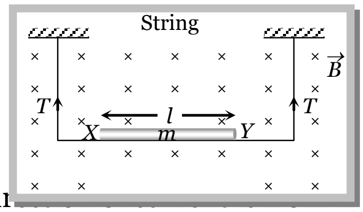
***Case* 4 : Current carrying spring :** If current is passed through a spring, then it will contract because current will flow through all the turns in the same direction.

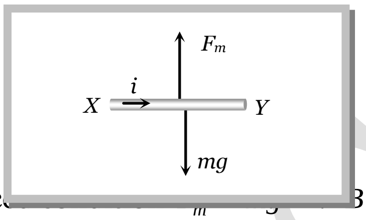




***Case* 5 : Tension less strings :** In the following figure the value and direction of current through the conductor XY so that strings becomes tensionless?

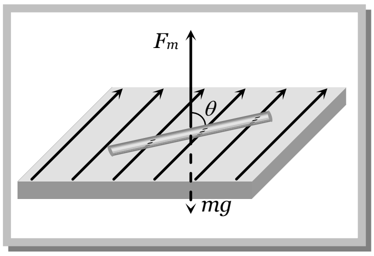
Strings becomes tensionless if weight of conductor XY balanced by magnetic force (F_m).





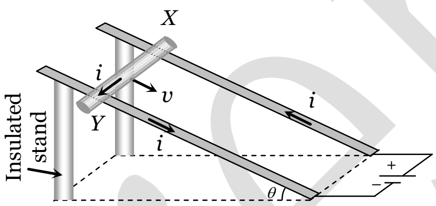
Hence di… → Y and in balanc… $Bil = mg \Rightarrow i = \frac{mg}{Bl}$

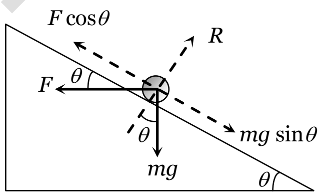
***Case* 6 :** A current carrying conductor floating in air such that it is making an angle θ with the direction of magnetic field, while magnetic field and conductor both lies in a horizontal plane.



In equilibrium $mg = Bil \sin\theta \Rightarrow i = \frac{mg}{Bl\sin\theta}$

***Case* 7 : Sliding of conducting rod on inclined rails :** When a conducting rod slides on conducting rails.



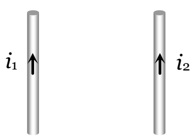


In the following situation conducting rod (X, Y) slides at constant velocity if

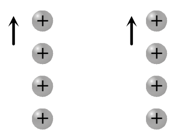
$$F\cos\theta = mg\sin\theta \Rightarrow Bil\cos\theta = mg\sin\theta \Rightarrow B = \frac{mg}{il}\tan\theta$$

Concepts

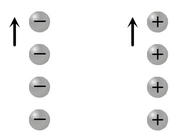
- *Electric force is an absolute concept while magnetic force is a relative concept for an observer.*
- *The nature of force between two parallel charge beams decided by electric force, as it is dominator. The nature of force between two parallel current carrying wires decided by magnetic force.*



$F_{net} = F_m$ only



$F_e \to$ repulsion
$F_m \to$ attraction
$F_{net} \to$ repulsion (Due to this force these beams diverges)



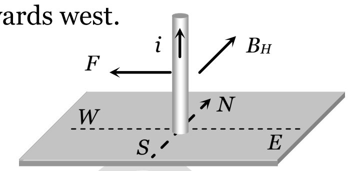
$F_e \to$ attraction
$F_m \to$ repulsion
$F_{net} \to$ attraction (Due to this force these beams converges)

Example

Example: 46 A vertical wire carrying a current in the upward direction is placed in a horizontal magnetic field directed towards north. The wire will experience a force directed towards

(a) North (b) South (c) East (d) West

Solution : (d) By applying Flemings left hand rule, direction of force is found towards west.



Example: 47 3 *A* of current is flowing in a linear conductor having a length of 40 *cm*. The conductor is placed in a magnetic field of strength 500 *gauss* and makes an angle of 30° with the direction of the field. It experiences a force of magnitude

(a) $3 \times 10^4\,N$ (b) $3 \times 10^2\,N$ (c) $3 \times 10^{-2}\,N$ (d) $3 \times 10^{-4}\,N$

Solution : (c) By using $F = Bil\sin\theta \Rightarrow F = (500 \times 10^{-4}) \times 0.4 \times \sin 30^\circ \Rightarrow 3 \times 10^{-2}\,N$.

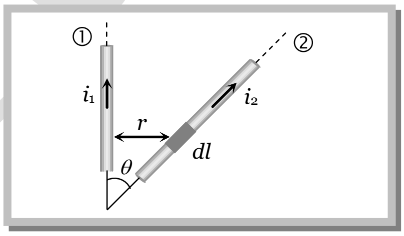
Example: 48 Wires 1 and 2 carrying currents t_1 and t_2 respectively are inclined at an angle θ to each other. What is the force on a small element *dl* of wire 2 at a distance of *r* from 1 (as shown in figure) due to the magnetic field of wire 1 **[AIEEE 2002]**

(a) $\frac{\mu_0}{2\pi r} i_1, i_2 dl \tan\theta$

(b) $\frac{\mu_0}{2\pi r} i_1, i_2\, dl \sin\theta$

(c) $\frac{\mu_0}{2\pi r} i_1, i_2\, dl \cos\theta$

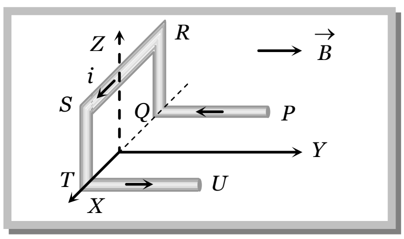
(d) $\frac{\mu_0}{4\pi r} i_1, i_2\, dl \sin\theta$



Solution : (c) Length of the component *dl* which is parallel to wire (1) is *dl* cos *θ*, so force on it

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{r}(dl\cos\theta) = \frac{\mu_0 i_1 i_2 dl\cos\theta}{2\pi r}.$$

Example: 49 A conductor *PQRSTU*, each side of length *L*, bent as shown in the figure, carries a current *i* and is placed in a uniform magnetic induction *B* directed parallel to the positive *Y*-axis. The force experience by the wire and its direction are



(a) 2*iBL* directed along the negative Z-axis

(b) 5*iBL* directed along the positive *Z*-axis

(c) *iBL* direction along the positive Z-axis

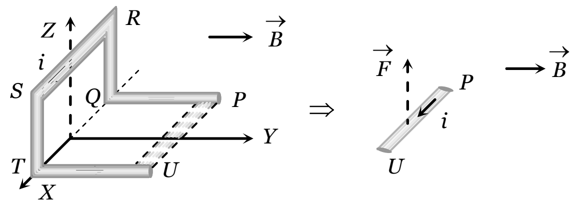
(d) 2*iBL* directed along the positive Z-axis

Solution : (c) As *PQ* and *UT* are parallel to *Q*, therefore $F_{PQ} = F_{UT} = 0$

The current in *TS* and *RQ* are in mutually opposite direction. Hence, $F_{TS} - F_{RQ} = 0$

Therefore the force will act only on the segment *SR* whose value is *Bil* and it's direction is +*z*.

Alternate method :



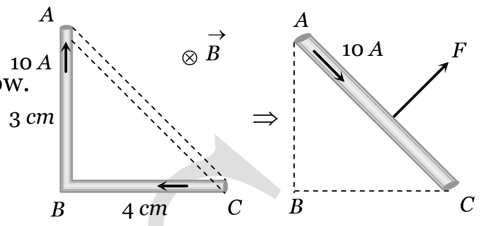
The given shape of the wire can be replaced by a straight wire of length l between P and U as shown below

Hence force on replaced wire PU will be $F = Bil$

and according to *FLHR* it is directed towards $+z$-axis

Example: 50 A conductor in the form of a right angle ABC with $AB = 3cm$ and $BC = 4\ cm$ carries a current of $10\ A$. There is a uniform magnetic field of $5T$ perpendicular to the plane of the conductor. The force on the conductor will be

(a) $1.5\ N$ (b) $2.0\ N$ (c) $2.5\ N$ (d) $3.5\ N$



Solution : (c) According to the question figure can be drawn as shown below.

Force on the conductor ABC = Force on the conductor AC

$= 5 \times 10 \times (5 \times 10^{-2})$

$= 2.5\ N$

Example: 51 A wire of length l carries a current i along the X-axis. A magnetic field exists which is given as $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})\ T$. Find the magnitude of the magnetic force acting on the wire

(a) $B_0 il$ (b) $B_0 i l \times \sqrt{2}$ (c) $2B_0 i l$ (d) $\frac{1}{\sqrt{2}} \times B_0 i l$

Solution : (b) By using $\vec{F} = i(\vec{l} \times \vec{B}) \Rightarrow \vec{F} = i[l\hat{i} \times B_0(\hat{i} + \hat{j} + \hat{k})] = B_0 il[\hat{i} \times (\hat{i} + \hat{j} + \hat{k})]$

$\Rightarrow \quad \vec{F} = B_0 il[\hat{i} \times \hat{i} + \hat{i} \times \hat{j} + \hat{i} \times \hat{k}] = B_0 il[\hat{k} - \hat{j}] \qquad \{\hat{i} \times \hat{i} = 0, \hat{i} \times \hat{j} = \hat{k}, \hat{i} \times \hat{k} = -\hat{j}\}$

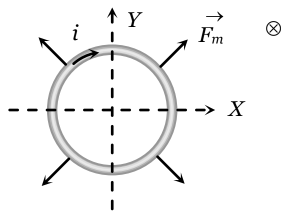
It's magnitude $F = \sqrt{2} B_0 il$

Example: 52 A conducting loop carrying a current i is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to **[IIT-JEE (Screening) 2003]]**

(a) Contract (b) Expand

(c) Move towards $+ve$ x-axis (d) Move towards $-ve$ x-axis

Solution : (b) Net force on a current carrying loop in uniform magnetic field is zero. Hence the loop can't translate. So, options (c) and (d) are wrong. From Flemings left hand rule we can see that if magnetic field is perpendicular to paper inwards and current in the loop is clockwise (as shown) the magnetic force $\vec{F_m}$ on each element of the loop is radially outwards, or the loops will have a tendency to expand.



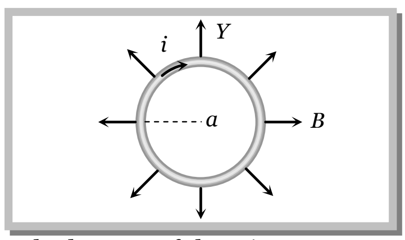
Example: 53 A circular loop of radius a, carrying a current i, is placed in a two-dimensional magnetic field. The centre of the loop coincides with the centre of the field. The strength of the magnetic field at the periphery of the loop is B. Find the magnetic force on the wire

(a) $\pi\, i\, a\, B$

(b) $4\pi\, i\, a\, B$

(c) Zero

(d) $2\pi\, i\, a\, B$

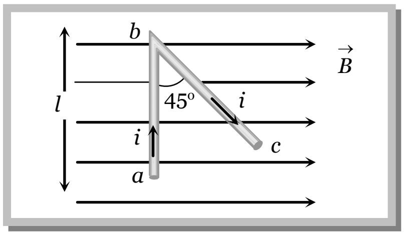


Solution : (d) The direction of the magnetic force will be vertically downwards at each element of the wire.

Thus $F = Bil = Bi\,(2\pi a) = 2\pi iaB$.

Example: 54 A wire abc is carrying current i. It is bent as shown in fig and is placed in a uniform magnetic field of magnetic induction B. Length $ab = l$ and $\angle abc = 45^{\circ}$. The ratio of force on ab and on bc is

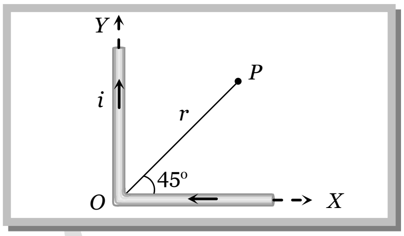
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 1 (d) $\frac{2}{3}$



Solution : (c) Force on portion *ab* of wire $F_1 = Bil \sin 90° = Bil$

Force on portion *bc* of wire $F_2 = Bi\left(\frac{l}{\sqrt{2}}\right)\sin 45^o = Bil$. So $\frac{F_1}{F_2} = 1$.

***Example*: 55** Current i flows through a long conducting wire bent at right angle as shown in figure. The magnetic field at a point P on the right bisector of the angle XOY at a distance r from O is



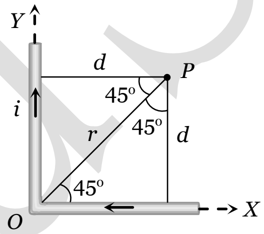
(a) $\frac{\mu_0 i}{\pi r}$

(b) $\frac{2\mu_0 i}{\pi r}$

(c) $\frac{\mu_0 i}{4\pi r}(\sqrt{2}+1)$

(d) $\frac{\mu_0}{4\pi}.\frac{2i}{r}(\sqrt{2}+1)$

Solution : (d) By using $B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$, from figure $d = r\sin 45^o = \frac{r}{\sqrt{2}}$



Magnetic field due to each wire at P $B = \frac{\mu_0}{4\pi}.\frac{i}{(r/\sqrt{2})}(\sin 45^o + \sin 90^o)$

$= \frac{\mu_0}{4\pi}.\frac{i}{r}(\sqrt{2}+1)$

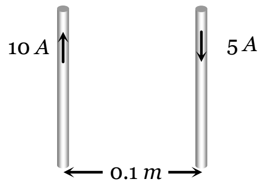
Hence net magnetic field at P $B_{net} = 2\times\frac{\mu_0}{4\pi}.\frac{i}{r}(\sqrt{2}+1) = \frac{\mu_0}{2\pi}.\frac{i}{r}(\sqrt{2}+1)$

***Example*: 56** A long wire A carries a current of 10 *amp*. Another long wire *B*, which is parallel to *A* and separated by 0.1 *m* from *A*, carries a current of 5 *amp*. in the opposite direction to that in *A*. What is the magnitude and nature of the force experienced per unit length of *B* $[\mu_0 = 4\pi\times 10^{-7}\ weber/amp - m]$

(a) Repulsive force of $10^{-4}\ N/m$ (b) Attractive force of $10^{-4}\ N/m$

(c) Repulsive force of $2\pi\times 10^{-5}\ N/m$ (d) Attractive force of $2\pi\times 10^{-5}\ N/m$

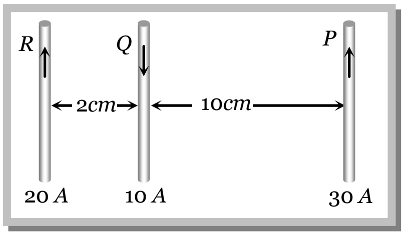
Solution : (a) By using $\frac{F}{l} = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{a}$



$\Rightarrow \frac{F}{l} = 10^{-7}\times\frac{2\times 10\times 5}{0.1} = 10^{-4}\ N$

Wires are carrying current in opposite direction so the force will be repulsive.

***Example*: 57** Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 *cm* length of wire *Q* is **[MP PET 1997]**



(a) 1.4×10^{-4} *N* towards the right

(b) 1.4×10^{-4} *N* towards the left

(c) 2.6×10^{-4} *N* to the right

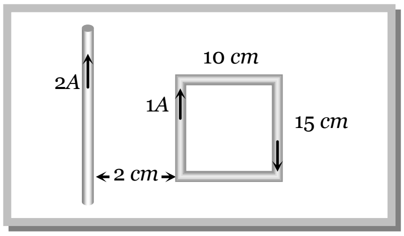
(d) 2.6×10^{-4} *N* to the left

Solution : (a) Force on wire *Q* due to *R* ; $F_R = 10^{-7}\times\frac{2\times 20\times 10}{(2\times 10^{-2})}\times(10\times 10^{-2}) = 2\times 10^{-4}\ m$ (Repulsive)

Force on wire *Q* due to *P* ; $F_P = 10^{-7}\times 2\times\frac{10\times 30}{(10\times 10^{-2})}\times(10\times 10^{-2}) = 0.6\times 10^{-4}\ N$ (Repulsive)

Hence net force $F_{net} = F_R - F_P = 2 \times 10^{-4} - 0.6 \times 10^{-4} = 1.4 \times 10^{-4}\,N$ (towards right *i.e.* in the direction of $\overrightarrow{F_R}$.

***Example*: 58** What is the net force on the coil **[DCE 2000]**



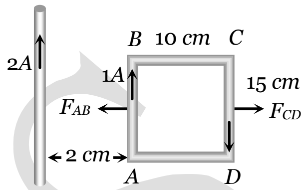
(a) $25 \times 10^{-7}\,N$ moving towards wire

(b) $25 \times 10^{-7}\,N$ moving away from wire

(c) $35 \times 10^{-7}\,N$ moving towards wire

(d) $35 \times 10^{-7}\,N$ moving away from wire

Solution : (a) Force on sides *BC* and *CD* cancel each other.

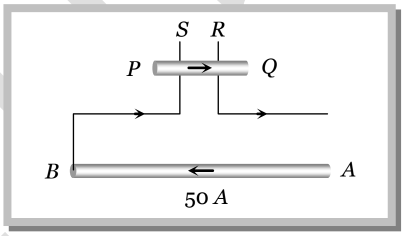


Force on side *AB* $F_{AB} = 10^{-7} \times \dfrac{2 \times 2 \times 1}{2 \times 10^{-2}} \times 15 \times 10^{-2} = 3 \times 10^{-6}\,N$

Force on side *CD* $F_{AB} = 10^{-7} \times \dfrac{2 \times 2 \times 1}{12 \times 10^{-2}} \times 15 \times 10^{-2} = 0.5 \times 10^{-6}\,N$

Hence net force on loop = $F_{AB} - F_{CD} = 25 \times 10^{-7}\,N$ (towards the wire).

***Example*: 59** A long wire *AB* is placed on a table. Another wire *PQ* of mass 1.0 *g* and length 50 *cm* is set to slide on two rails *PS* and *QR*. A current of 50*A* is passed through the wires. At what distance above *AB*, will the wire *PQ* be in equilibrium



(a) 25 *mm*

(b) 50 *mm*

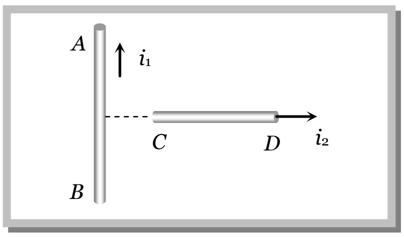
(c) 75 *mm*

(d) 100 *mm*

Solution : (a) Suppose in equilibrium wire *PQ* lies at a distance *r* above the wire *AB*

Hence in equilibrium $mg = Bil \Rightarrow mg = \dfrac{\mu_0}{4\pi}\left(\dfrac{2i}{r}\right) \times il \Rightarrow\ 10^{-3} \times 10 = 10^{-7} \times \dfrac{2 \times (50)^2}{r} = 0.5 \Rightarrow r = 25\,mm$

***Example*: 60** An infinitely long, straight conductor *AB* is fixed and a current is passed through it. Another movable straight wire *CD* of finite length and carrying current is held perpendicular to it and released. Neglect weight of the wire



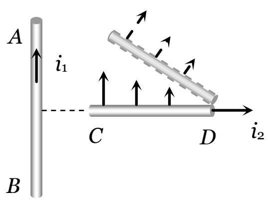
(a) The rod *CD* will move upwards parallel to itself

(b) The rod *CD* will move downward parallel to itself

(c) The rod *CD* will move upward and turn clockwise at the same time

(d) The rod *CD* will move upward and turn anti –clockwise at the same time

Solution : (c) Since the force on the rod *CD* is non-uniform it will experience force and torque. From the left hand side it can be seen that the force will be upward and torque is clockwise.



Tricky example: 5

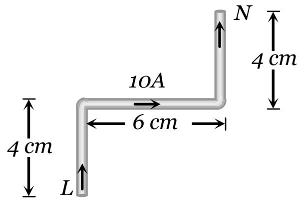
A current carrying wire LN is bent in the from shown below. If wire carries a current of 10 A and it is placed in a magnetic field a 5T which acts perpendicular to the paper outwards then it will experience a force

(a) Zero

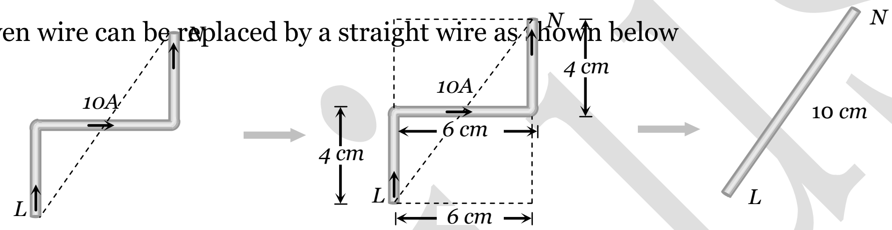
(b) 5 N

(c) 30 N

(d) 20 N



Solution : (b) The given wire can be replaced by a straight wire as shown below



Hence force experienced by the wire $F = Bil = 5 \times 10 \times 0.1 = 5\ N$

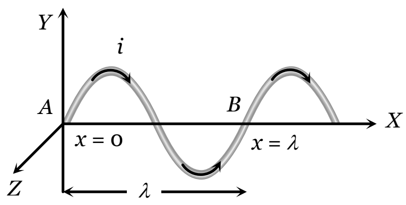
Tricky example: 6

A wire, carrying a current i, is kept in $X - Y$ plane along the curve $y = A\sin\left(\frac{2\pi}{\lambda}x\right)$. A magnetic field B exists in the Z-direction find the magnitude of the magnetic force on the portion of the wire between $x = 0$ and $x = \lambda$

(a) $i\lambda B$ (b) Zero (c) $\frac{i\lambda B}{2}$ (d) $3/2i\lambda B$

Solution : (a) The given curve is a sine curve as shown below.

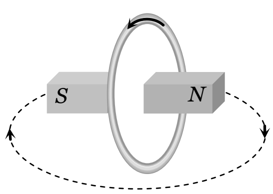
The given portion of the curved wire may be treated as a straight wire AB of length λ which experiences *a* magnetic force $F_m = Bi\lambda$

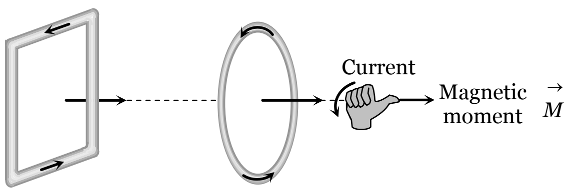


Current Loop As a Magnetic Dipole

A current carrying circular coil behaves as a bar magnet whose magnetic moment is $M = NiA$; *Where* N = Number of turns in the coil, i = Current through the coil and A = Area of the coil

Magnetic moment of a current carrying coil is a vector and it's direction is given by right hand thumb rule

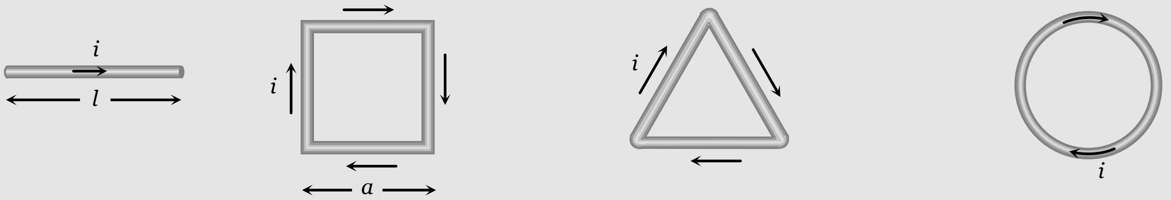




Specific examples

A given length constant current carrying straight wire moulded into different shaped loops. as shown

Linear | Square | Equilateral | Circle



Linear	Square	Equilateral	Circle
	$l = 4a$	$l = 3a$	$l = 2\pi r$
	$A = a^2$	$A = \frac{\sqrt{3}}{4}a^2$	$A = \pi r^2$
	$M = ia^2 = \frac{il^2}{16}$	$M = i\left(\frac{\sqrt{3}}{4}a^2\right) = \frac{\sqrt{3}\,il^2}{36}$	$M = i(\pi r^2) = \frac{il^2}{4\pi}$ ← max.

Note : ❒ For a given perimeter circular shape have maximum area. Hence maximum magnetic moment.

❒ For a any loop or coil $\vec{B}$ and $\vec{M}$ are always parallel.

Behaviour of Current loop In a Magnetic Field

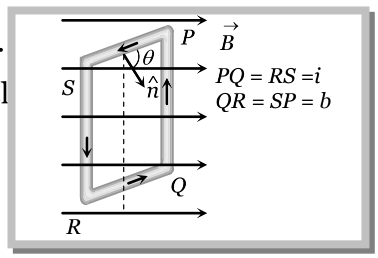
(1) **Torque**

Consider a rectangular current carrying coil *PQRS* having *N* turns and area *A*, placed in a uniform field *B*, in such a way that the normal $(\hat{n})$ to the coil makes an angle θ with the direction of *B*. the coil experiences a torque given by $\boldsymbol{\tau = NBiA \sin\theta}$. Vectorially $\vec{\tau} = \vec{M} \times \vec{B}$

(i) τ is zero when $\theta = 0$, *i.e.*, when the plane of the coil is perpendicular to the field.

(ii) τ is maximum when $\theta = 90^\circ$, *i.e.*, the plane of the coil is parallel to the fiel

$\Rightarrow \tau_{max} = NBiA$



The above expression is valid for coils of all shapes.

(2) **Workdone**

If coil is rotated through an angle θ from it's equilibrium position then required work. $W = MB(1 - \cos\theta)$. It is maximum when $\theta = 180^\circ \Rightarrow W_{max} = 2\,MB$

(3) **Potential energy**

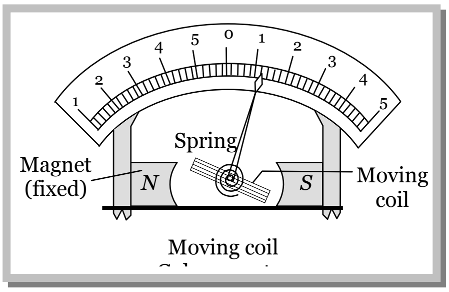
Is given by $U = -MB\cos\theta \Rightarrow U = \vec{M}.\vec{B}$

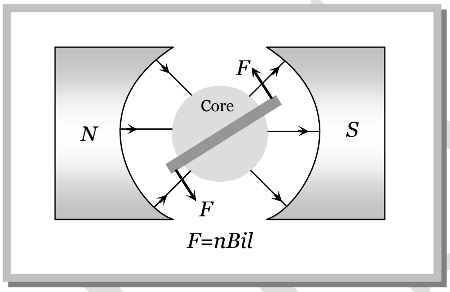
Note : ❒ Direction of $\vec{M}$ is found by using Right hand thumb rule according to which curl the fingers of right hand in the direction of circulation of conventional current, then the thumb gives the direction of $\vec{M}$.

❒ Instruments such as electric motor, moving coil galvanometer and tangent galvanometers *etc.* are based on the fact that a current-carrying coil in a uniform magnetic field experiences a torque (or couple).

Moving coil galvanometer

In a moving coil galvanometer the coil is suspended between the pole pieces of a strong horse-shoe magnet. The pole pieces are made cylinderical and a soft iron cylinderical core is placed within the coil without touching it. This makes the field radial. In such a field the plane of the coil always remains parallel to the field. Therefore $\theta = 90^o$ and the deflecting torque always has the maximum value.





$\tau_{def} = NBiA$ ......(i)

coil deflects, a restoring torque is set up in the suspension fibre. If α is the angle of twist, the restoring torque is

$\tau_{rest} = C\alpha$ .....(ii) where C is the torsional constant of the fibre.

When the coil is in equilibrium.

$$NBiA = C\alpha \Rightarrow i = \frac{C}{NBA}\alpha \Rightarrow i = K\alpha,$$

Where $K = \frac{C}{NBA}$ is the galvanometer constant. This linear relationship between i and α makes the moving coil galvanometer useful for current measurement and detection.

Current sensitivity : The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$S_i = \frac{\alpha}{i} = \frac{NBA}{C}$$

Thus in order to increase the sensitivity of a moving coil galvanometer, N, B and A should be increased and C should be decreased.

Quartz fibres can also be used for suspension of the coil because they have large tensile strength and very low value of k.

Voltage sensitivity (S_V) : Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit applied to it.

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{iR} = \frac{S_i}{R} = \frac{NBA}{RC}$$

Concepts

☞ *The field in a moving coil galvanometer radial in nature in order to have a linear relation between the current and the deflection.*

☞ *A rectangular current loop is in an arbitrary orientation in an external magnetic field. No work required to rotate the loop* *about an axis perpendicular to it's plane.*

☞ *Moving coil galvanometer can be made ballistic by using a non-conducting frame (made of ivory or bamboo) instead of a metallic frame.*

Example

Example: 61 A circular coil of radius 4 *cm* and 20 turns carries a current of 3 *ampere*. It is placed in a magnetic field of 0.5 *T*. The magnetic dipole moment of the coil is **[MP PMT 2001]**

(a) 0.60 $A\text{-}m^2$ (b) 0.45 $A\text{-}m^2$ (c) 0.3 $A\text{-}m^2$ (d) 0.15 $A\text{-}m^2$

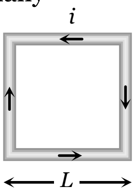
Solution : (c) $M = niA \Rightarrow M = 20 \times 3 \times \pi(4 \times 10^{-2})^2 = 0.3\ A\text{-}m^2$.

Example: 62 A steady current i flows in a small square loop of wire of side L in a horizontal plane. The loop is now folded about its middle such that half of it lies in a vertical plane. Let $\vec{\mu}_1$ and $\vec{\mu}_2$ respectively denote the magnetic moments due to the current loop before and after folding. Then **[IIT-JEE 1993]**

(a) $\vec{\mu}_2 = 0$ (b) $\vec{\mu}_1$ and $\vec{\mu}_2$ are in the same direction

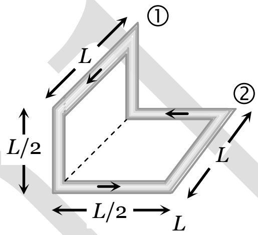
(c) $\dfrac{|\vec{\mu}_1|}{|\vec{\mu}_2|} = \sqrt{2}$ (d) $\dfrac{|\vec{\mu}_1|}{|\vec{\mu}_2|} = \left(\dfrac{1}{\sqrt{2}}\right)$

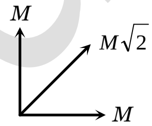
Solution : (c) Initially



$\mu_1 = iL^2$

Finally





$$M = \text{magnetic moment due to each part} = i\left(\frac{L}{2}\right) \times L = \frac{iL^2}{2} = \frac{\mu_1}{2}$$

$$\therefore\ \mu_2 = M\sqrt{2} = \frac{\mu_1}{2} \times \sqrt{2} = \frac{\mu_1}{\sqrt{2}}$$

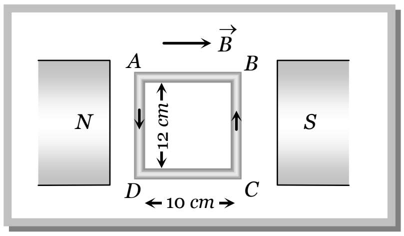
Example: 63 A coil of 50 turns is situated in a magnetic field $b = 0.25 weber/m^2$ as shown in figure. A current of $2A$ is flowing in the coil. Torque acting on the coil will be

(a) 0.15 *N*

(b) 0.3 *N*

(c) 0.45 *N*

(d) 0.6 *N*



Solution : (b) Since plane of the coil is parallel to magnetic field. So $\theta = 90^o$

Hence $\tau = NBiA \sin 90^o = NBiA = 50 \times 0.25 \times 2 \times (12 \times 10^{-2} \times 10 \times 10^{-2}) = 0.3\ N$.

Example: 64 A circular loop of area 1 cm^2, carrying a current of 10 *A*, is placed in a magnetic field of 0.1 *T* perpendicular to the plane of the loop. The torque on the loop due to the magnetic field is

(a) Zero (b) 10^{-4} *N-m* (c) 10^{-2} *N-m* (d) 1 *N-m*

Solution : (a) $\tau = NBiA \sin\theta$; given $\theta = 0$ so $\tau = 0$.

Example: 65 A circular coil of radius 4 *cm* has 50 turns. In this coil a current of 2 *A* is flowing. It is placed in a magnetic field of 0.1 $weber/m^2$. The amount of work done in rotating it through 180° from its equilibrium position will be **[CPMT 1977]**

(a) 0.1 *J* (b) 0.2 *J* (c) 0.4 (d) 0.8 *J*

Solution : (a) Work done in rotating a coil through an angle θ from it's equilibrium position is $W = MB(1 - \cos\theta)$ where $\theta = 180^o$ and $M = 50 \times 2 \times \pi(4 \times 10^{-2}) = 50.24 \times 10^{-2}$ A-m^2. Hence $W = 0.1\ J$

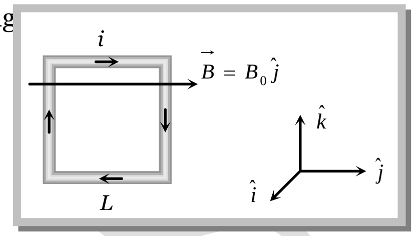
Example: 66 A wire of length L is bent in the form of a circular coil and current i is passed through it. If this coil is placed in a magnetic field then the torque acting on the coil will be maximum when the number of turns is

(a) As large as possible (b) Any number (c) 2 (d) 1

Solution : (d) $\tau_{\max} = MB$ or $\tau_{\max} = ni\pi a^2 B$. Let number of turns in length l is n so $l = n(2\pi a)$ or $a = \dfrac{l}{2\pi n}$

$$\Rightarrow \tau_{\max} = \frac{ni\pi B l^2}{4\pi^2 n^2} = \frac{l^2 iB}{4\pi n_{\min}} \Rightarrow \tau_{\max} \propto \frac{1}{n_{\min}} \Rightarrow n_{\min} = 1$$

Example: 67 A square coil of N turns (with length of each side equal L) carrying current i is placed in a uniform magnetic field $\vec{B} = B_0 \hat{j}$ as shown in figure. What is the torque acting



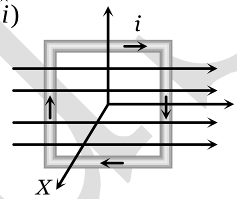
(a) $+B_0 NiL^2 \hat{k}$

(b) $-B_0 NiL^2 \hat{k}$

(c) $+B_0 NiL^2 \hat{j}$

(d) $-B_0 NiL^2 \hat{j}$

Solution : (b) The magnetic field is $\vec{B} = B_0 \hat{j}$ and the magnetic moment $\vec{m} = i\vec{A} = -i(NL^2 \hat{i})$



The torque is given by $\vec{\tau} = \vec{m} \times \vec{B}$

$= -iNL^2 \hat{i} \times B_0 \hat{j} = -iNB_0 L^2 \hat{i} \times \hat{j}$

$= -iNB_0 L^2 \hat{k}$

Example: 68 The coil of a galvanometer consists of 100 *turns* and effective area of 1 *square cm*. The restoring couple is 10^{-8} *N-m rad*. The magnetic field between the pole pieces is 5 *T*. The current sensitivity of this galvanometer will be **[MP PMT 1997]**

(a) 5×10^4 *rad/μ amp* (b) 5×10^{-6} *per amp* (c) 2×10^{-7} *per amp* (d) 5 *rad./μ amp*

Solution : (d) Current sensitivity $(S_i) = \dfrac{\theta}{i} = \dfrac{NBA}{C} \Rightarrow \dfrac{\theta}{i} = \dfrac{100 \times 5 \times 10^{-4}}{10^{-8}} = 5\ rad/\mu\ amp$.

Example: 69 The sensitivity of a moving coil galvanometer can be increased by **[SCRA 2000]]**

(a) Increasing the number of turns in the coil (b) Decreasing the area of the coil

(c) Increasing the current in the coil (d) Introducing a soft iron core inside the coil

Solution : (a) Sensitivity $(S_i) = \dfrac{NBA}{C} \Rightarrow S_i \propto N$.

Tricky example: 7

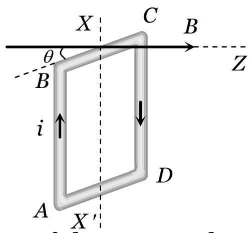
The square loop $ABCD$, carrying a current i, is placed in uniform magnetic field B, as shown. The loop can rotate about the axis XX'. The plane of the loop makes and angle θ ($\theta < 90^o$) with the direction of B. Through what angle will the loop rotate by itself before the torque on it becomes zero

(a) θ

(b) $90^o - \theta$

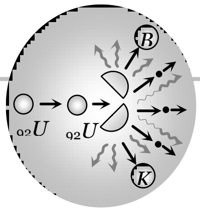
(c) $90^o + \theta$

(d) $180^o - \theta$



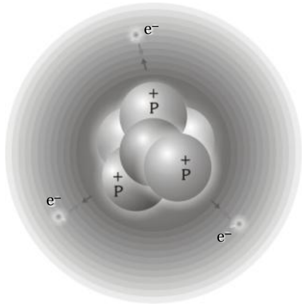
Solution : (c) In the position shown, AB is outside and CD is inside the plane of the paper. The Ampere force on AB acts into the paper. The torque on the loop will be clockwise, as seen from above. The loop must rotate through an angle $(90^o + \theta)$ before the plane of the loop becomes normal to the direction of B and the torque becomes zero.

genius



Nuclear Physics & Radioactivity

Rutherford's α-scattering experiment established that the mass of atom is concentrated with small positively charged region at the centre which is called 'nucleus'.



Nuclei are made up of proton and neutron. The number of protons in a nucleus (called the atomic number or proton number) is represented by the symbol Z. The number of neutrons (neutron number) is represented by N. The total number of neutrons and protons in a nucleus is called it's mass number A so $A = Z + N$.

Neutrons and proton, when described collectively are called ***nucleons.***

Nucleus contains two types of particles : Protons and neutrons

Nuclides are represented as ${}_Z X^A$; where X denotes the chemical symbol of the element.

Neutron

Neutron is a fundamental particle which is essential constituent of all nuclei except that of hydrogen atom. It was discovered by Chadwick.

(1) The charge of neutron : It is neutral

(2) The mass of neutron : 1.6750×10^{-27} kg

(3) It's spin angular momentum : $\frac{1}{2} \times \left(\frac{h}{2\pi}\right) J\text{-}s$

(4) It's magnetic moment : 9.57×10^{-27} $J/Tesla$

(5) It's half life : 12 minutes

(6) Penetration power : High

(7) Types : Neutrons are of two types slow neutron and fast neutron, both are fully capable of penetrating a nucleus and causing artificial disintegration.

A free neutron outside the nucleus is unstable and decays into proton and electron.

$${}_0n^1 \rightarrow \underset{\text{Proton}}{{}_1H^1} + \underset{\text{Electron}}{{}_{-1}\beta^0} + \underset{\text{Antinutrino}}{\bar{\nu}}$$

Thermal neutrons

Fast neutrons can be converted into slow neutrons by certain materials called moderator's (Paraffin wax, heavy water, graphite) when fast moving neutrons pass through a moderator, they collide with the molecules of the moderator, as a result of this, the energy of moving neutron decreases while that of the molecules of the moderator increases. After sometime they both attains same energy. The neutrons are then in thermal equilibrium with the molecules of the moderator and are called thermal neutrons.

Note : ❑ Energy of thermal neutron is about 0.025 eV and speed is about 2.2 km/s.

Nucleus

(1) **Different types of nuclei**

The nuclei have been classified on the basis of the number of protons (atomic number) or the total number of nucleons (mass number) as follows

(i) **Isotopes :** The atoms of element having same atomic number but different mass number are called isotopes. All isotopes have the same chemical properties. The isotopes of some elements are the following

$${}_1H^1,\ {}_1H^2,\ {}_1H^3 \quad {}_8O^{16},\ {}_8O^{17},\ {}_8O^{18} \quad {}_2He^3,\ {}_2He^4 \quad {}_{17}Cl^{35},\ {}_{17}Cl^{37} \quad {}_{92}U^{235},\ {}_{92}U^{238}$$

(ii) **Isobars :** The nuclei which have the same mass number (*A*) but different atomic number (*Z*) are called isobars. Isobars occupy different positions in periodic table so all isobars have different chemical properties. Some of the examples of isobars are

$$_1H^3 \text{ and } _2He^3, \quad _6C^{14} \text{ and } _7N^{14}, \; _8O^{17} \text{ and } _9F^{17}$$

(iii) **Isotones :** The nuclei having equal number of neutrons are called isotones. For them both the atomic number (*Z*) and mass number (*A*) are different, but the value of (*A* – *Z*) is same. Some examples are

$$_4Be^9 \text{ and } _5B^{10}, {}_6C^{13} \text{ and } _7N^{14}, {}_8O^{18} \text{ and } _9F^{19}, {}_3Li^7 \text{ and } _4Be^8, {}_1H^3 \text{ and } _2He^4$$

(iv) **Mirror nuclei :** Nuclei having the same mass number *A* but with the proton number (*Z*) and neutron number (*A* – *Z*) interchanged (or whose atomic number differ by 1) are called mirror nuclei for example.

$$_1H^3 \text{ and } _2He^3, \; _3Li^7 \text{ and } _4Be^7$$

(2) **Size of nucleus**

(i) Nuclear radius : Experimental results indicates that the nuclear radius is proportional to $A^{1/3}$, where *A* is the mass number of nucleus *i.e.* $R \propto A^{1/3} \Rightarrow R = R_0A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}\, m = 1.2\, fm$.

Note : ❑ Heavier nuclei are bigger in size than lighter nuclei.

(ii) Nuclear volume : The volume of nucleus is given by $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \Rightarrow V \propto A$

(iii) Nuclear density : Mass per unit volume of a nucleus is called nuclear density.

$$\text{Nuclear density}(\rho) = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi(R_0A^{1/3})^3}$$

where *m* = Average of mass of a nucleon (= mass of proton + mass of neutron = $1.66 \times 10^{-27}\, kg$)

and *mA* = Mass of nucleus

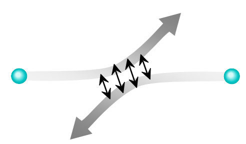
$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3} = 2.38 \times 10^{17}\, kg/m^3$$

Note : ❑ ρ is independent of *A*, it means ρ is same of all atoms.

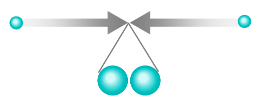
❑ Density of a nucleus is maximum at it's centre and decreases as we move outwards from the nucleus.

(3) **Nuclear force**

Forces that keep the nucleons bound in the nucleus are called nuclear forces.



At low speeds, electromagnetic repulsion prevents the collision of nuclei



At high speeds, nuclei come close enough for the strong force to bind them together.

(i) Nuclear forces are short range forces. These do not exist at large distances greater than $10^{-15}\, m$.

(ii) Nuclear forces are the strongest forces in nature.

(iii) These are attractive force and causes stability of the nucleus.

(iv) These forces are charge independent.

(v) Nuclear forces are non-central force.

Nuclear forces are exchange forces

According to scientist Yukawa the nuclear force between the two nucleons is the result of the exchange of particles called mesons between the nucleons.

π- mesons are of three types – Positive π meson (π^+), negative π meson (π^-), neutral π *meson* (π^0)

The force between neutron and proton is due to exchange of charged meson between them *i.e.*

$$p \rightarrow \pi^+ + n, \quad n \rightarrow p + \pi^-$$

The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^0) between them *i.e.* $p \rightarrow p' + \pi^0$ and $n \rightarrow n' + \pi^0$

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the nuclear forces.

Dog-Bone analogy

The above interactions can be explained with the dog bone analogy according to which we consider the two interacting nucleons to be two dogs having a common bone clenched in between their teeth very firmly. Each one of these dogs wants to take the bone and hence they cannot be separated easily. They seem to be bound to each other with a strong attractive force (which is the bone) though the dogs themselves are strong enemies. The meson plays the same role of the common bone in between two nucleons.

(4) **Atomic mass unit (*amu*)**

The unit in which atomic and nuclear masses are measured is called atomic mass unit (*amu*)

1 *amu* (or 1*u*) $= \frac{1}{12} th$ of mass of ${}_6C^{12}$ atom = 1.66×10^{-27} *kg*

Masses of electron, proton and neutrons

Mass of electron (m_e) = 9.1×10^{-31} *kg* = 0.0005486 *amu*, Mass of proton (m_p) = 1.6726×10^{-27} *kg* = 1.007276 *amu*

Mass of neutron (m_n) = 1.6750×10^{-27} *kg* = 1.00865 *amu*, Mass of hydrogen atom ($m_e + m_p$) = 1.6729×10^{-27} *kg* = 1.0078 *amu*

Mass-energy equivalence

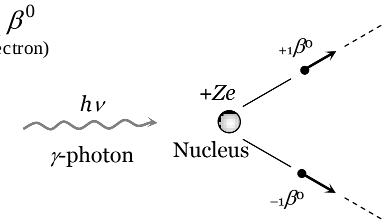
According to Einstein, mass and energy are inter convertible. The Einstein's mass energy relationship is given by $E = mc^2$

If $m = 1$ *amu*, $c = 3 \times 10^8$ *m/sec* then $E = 931$ *MeV* *i.e.* 1 *amu* is equivalent to 931 *MeV* or **1 *amu* (or 1 *u*) = 931 *MeV***

(5) **Pair production and pair-annihilation**

When an energetic γ-ray photon falls on a heavy substance. It is absorbed by some nucleus of the substance and an electron and a positron are produced. This phenomenon is called pair production and may be represented by the following equation

$$\underset{(\gamma-\text{photon})}{h\nu} = \underset{(\text{Positron})}{{}_1\beta^0} + \underset{(\text{Electron})}{{}_{-1}\beta^0}$$



The rest-mass energy of each of positron and electron is

$$E_0 = m_0c^2 = (9.1 \times 10^{-31} \text{ kg}) \times (3.0 \times 10^8 \text{ m/s})^2$$

$$= 8.2 \times 10^{-14} J = \mathbf{0.51\ MeV}$$

Hence, for pair-production it is essential that the energy of γ-photon must be at least 2 × 0.51 = 1.02 *MeV*. If the energy of γ-photon is less than this, it would cause photo-electric effect or Compton effect on striking the matter.

The converse phenomenon pair-annihilation is also possible. Whenever an electron and a positron come very close to each other, they annihilate each other by combining together and two γ-photons (energy) are produced. This phenomenon is called pair annihilation and is represented by the following equation.

$$\underset{\text{(Positron)}}{{}_{+1}\beta^0} + \underset{\text{(Electron)}}{{}_{-1}\beta^0} = \underset{(\gamma\text{-photon})}{h\nu} + \underset{(\gamma\text{-photon})}{h\nu}$$

(6) **Nuclear stability**

Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α, β-particles and γ- EM waves. (This process is called radioactivity). The stability of nucleus is determined by many factors. Few such factors are given below :

(i) Neutron-proton ratio $\left(\frac{N}{Z}\text{ Ratio}\right)$

The chemical properties of an atom are governed entirely by the number of protons (*Z*) in the nucleus, the stability of an atom appears to depend on both the number of protons and the number of neutrons.

For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal ($N \approx Z$) *i.e.* $\frac{N}{Z} = 1$

Heavy nuclei are stable only when they have more neutrons than protons. Thus heavy nuclei are neutron rich compared to lighter nuclei (for heavy nuclei, more is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore more neutrons are added to provide the strong attractive forces necessary to keep the nucleus stable.)

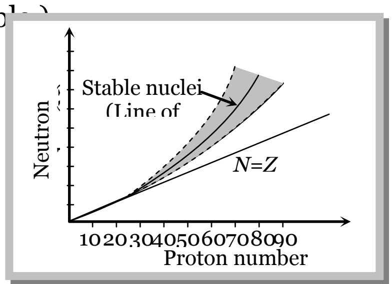
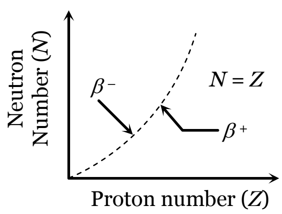


Figure shows a plot of *N* verses *Z* for the stable nuclei. For mass number upto about *A* = 40. For larger value of *Z* the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At *Bi* (*Z* = 83, *A* = 209), the neutron excess in $N - Z = 43$. There are no stable nuclides with $Z > 83$.

Note : ❑ The nuclide ${}_{83}Bi^{209}$ is the heaviest stable nucleus.

❑ A nuclide above the line of stability *i.e.* having excess neutrons, decay through β^- emission (neutron changes into proton). Thus increasing atomic number *Z* and decreasing neutron number *N*. In β^- emission, $\frac{N}{Z}$ ratio decreases.

A nuclide below the line of stability have excess number of protons. It decays by β^+ emission, results in decreasing *Z* and increasing *N*. In β^+ emission, the $\frac{N}{Z}$ ratio increases.



(ii) Even or odd numbers of Z or N : The stability of a nuclide is also determined by the consideration whether it contains an even or odd number of protons and neutrons.

It is found that an even-even nucleus (even Z and even N) is more stable (60% of stable nuclide have even Z and even N).

An even-odd nucleus (even Z and odd N) or odd-even nuclide (odd Z and even N) is found to be lesser sable while the odd-odd nucleus is found to be less stable.

Only five stable odd-odd nuclides are known : ${}_1H^2$, ${}_3Li^6$, ${}_5Be^{10}$, ${}_7N^{14}$ and ${}_{75}Ta^{180}$

(iii) Binding energy per nucleon : The stability of a nucleus is determined by value of it's binding energy per nucleon. In general higher the value of binding energy per nucleon, more stable the nucleus is

Mass Defect and Binding Energy

(1) **Mass defect (Δm)**

It is found that the mass of a nucleus is always less than the sum of masses of it's constituent nucleons in free state. This difference in masses is called mass defect. Hence mass defect

$$\Delta m = \text{Sum of masses of nucleons} - \text{Mass of nucleus}$$

$$= \{Zm_p + (A-Z)m_n\} - M = \{Zm_p + Zm_e + (A-Z)m_z\} - M'$$

where m_p = Mass of proton, m_n = Mass of each neutron, m_e = Mass of each electron

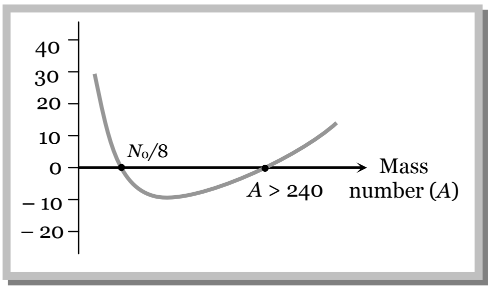
M = Mass of nucleus, Z = Atomic number, A = Mass number, M' = Mass of atom as a whole.

Note : ❑ The mass of a typical nucleus is about 1% less than the sum of masses of nucleons.

(2) **Packing fraction**

Mass defect per nucleon is called packing fraction

Packing fraction $(f) = \dfrac{\Delta m}{A} = \dfrac{M-A}{A}$ where M = Mass of nucleus, A = Mass number



Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, larger is the stability of the nucleus.

(i) Packing fraction may be of positive, negative or zero value.

(iii) At $A = 16, f \to$ Zero

(3) **Binding energy (B.E.)**

The neutrons and protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them infinitely apart (or the same energy is released during the formation of the nucleus). This energy is called the binding energy of the nucleus.

or

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect then according to Einstein's mass energy relation

Binding energy = $\Delta m \cdot c^2 = [\{m_p Z + m_n(A-Z)\} - M] \cdot c^2$

(This binding energy is expressed in *joule*, because Δm is measured in *kg*)

If Δm is measured in *amu* then binding energy = $\Delta m\ amu = [\{m_p Z + m_n(A - Z)\} - M]\ amu = \Delta m \times 931$ *MeV*

(4) **Binding energy per nucleon**

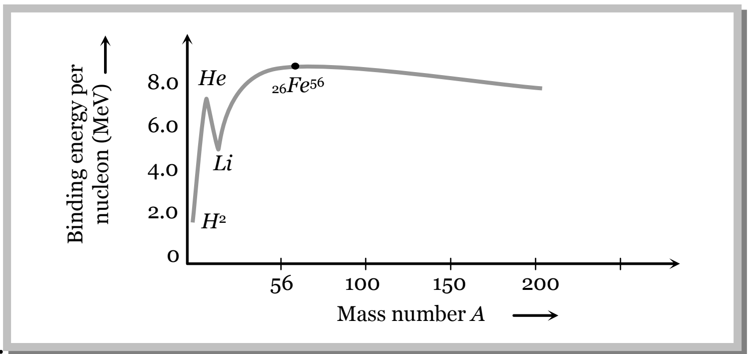
The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

$$\text{Binding energy per nucleon} = \frac{\text{Total binding energy}}{\text{Mass number } (i.e. \text{ total number of nucleons})} = \frac{\Delta m \times 931}{A}\ \frac{MeV}{Nucleon}$$

Binding energy per nucleon $\propto$ Stability of nucleus

Binding Energy Curve

It is the graph between binding energy per nucleon and total number of nucleons (*i.e.* mass number *A*)



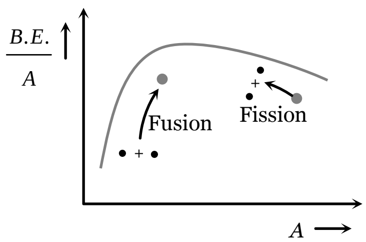
(1) Some nuclei with mass number $A < 20$ have large binding energy per nucleon than their neighbour nuclei. For example ${}_2He^4$, ${}_4Be^8$, ${}_6C^{12}$, ${}_8O^{16}$ and ${}_{10}Ne^{20}$. These nuclei are more stable than their neighbours.

(2) The binding energy per nucleon is maximum for nuclei of mass number $A = 56$ (${}_{26}Fe^{56}$). It's value is 8.8 *MeV* per nucleon.

(3) For nuclei having $A > 56$, binding energy per nucleon gradually decreases for uranium ($A = 238$), the value of binding energy per nucleon drops to 7.5 *MeV*.

Note : ❑ When a heavy nucleus splits up into lighter nuclei, then binding energy per nucleon of lighter nuclei is more than that of the original heavy nucleus. Thus a large amount of energy is liberated in this process (nuclear fission).

❑ When two very light nuclei combines to form a relatively heavy nucleus, then binding energy per nucleon increases. Thus, energy is released in this process (nuclear fusion).



Nuclear Reactions

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction. The general expression for the nuclear reaction is as follows.

$$\underset{\text{(Parent nucleus)}}{X} + \underset{\text{(Incident particle)}}{a} \longrightarrow \underset{\text{(Compound nucleus)}}{C} \longrightarrow \underset{\text{(Compound nucleus)}}{Y} + \underset{\text{(Product particles)}}{b} + \underset{\text{(Energy)}}{Q}$$

Here *X* and *a* are known as reactants and *Y* and *b* are known as products. This reaction is known as (*a*, *b*) reaction and can be represented as *X*(*a*, *b*) *Y*

(1) ***Q* value or energy of nuclear reaction**

The energy absorbed or released during nuclear reaction is known as *Q*-value of nuclear reaction.

Q-value = (Mass of reactants – mass of products)c^2 *Joules*

= (Mass of reactants – mass of products) *amu*

If $Q < 0$, The nuclear reaction is known as endothermic. (The energy is absorbed in the reaction)

If $Q > 0$, The nuclear reaction is known as exothermic (The energy is released in the reaction)

(2) **Law of conservation in nuclear reactions**

(i) Conservation of mass number and charge number : In the following nuclear reaction

$$_2He^4 + _7N^{14} \rightarrow _8O^{17} + _1H^1$$

Mass number (A) $\rightarrow$	Before the reaction	After the reaction
	4 +14 = 18	17 + 1 = 18
Charge number (Z) $\rightarrow$	2 + 7 = 9	8 + 1 = 9

(ii) Conservation of momentum : Linear momentum/angular momentum of particles before the reaction is equal to the linear/angular momentum of the particles after the reaction. That is $\Sigma p = 0$

(iii) Conservation of energy : Total energy before the reaction is equal to total energy after the reaction. Term Q is added to balance the total energy of the reaction.

(3) **Common nuclear reactions**

The nuclear reactions lead to artificial transmutation of nuclei. Rutherford was the first to carry out artificial transmutation of nitrogen to oxygen in the year 1919.

$$_2He^4 + _7N^{14} \rightarrow _9F^{18} \rightarrow _8O^{17} + _1H^1$$

It is called (α, p) reaction. Some other nuclear reactions are given as follows.

(p, n) reaction $\Rightarrow$ $_1H^1 + _5B^{11} \rightarrow _6C^{12} \rightarrow _6C^{11} + _0n^1$

(p, α) reaction $\Rightarrow$ $_1H^1 + _3Li^{11} \rightarrow _4Be^8 \rightarrow _2He^4 + _2He^4$

(p, γ) reaction $\Rightarrow$ $_1H^1 + _6C^{12} \rightarrow _7N^{13} \rightarrow _7N^{13} + \gamma$

(n, p) reaction $\Rightarrow$ $_0n^1 + _7N^{14} \rightarrow _7N^{15} \rightarrow _6C^{14} + _1H^1$

(γ, n) reaction $\Rightarrow$ $\gamma + _1H^2 \rightarrow _1H^1 + _0n^1$

Nuclear Fission and Fusion

Nuclear fission

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy is called nuclear fission.

The phenomenon of nuclear fission was discovered by scientist Ottohann and F. Strassman and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.

(1) **Fission reaction of U^{235}**

(i) Nuclear reaction :

$$_{92}U^{235} + {_0}n^1 \rightarrow \underset{\text{(unstable nucleus)}}{_{92}U^{236}} \rightarrow {_{56}}Ba^{141} + {_{36}}Kr^{92} + 3{_0}n^1 + Q$$

(ii) The energy released in U^{235} fission is about 200 *MeV* or 0.8 *MeV* per nucleon.

(iii) By fission of $_{92}U^{235}$, on an average 2.5 neutrons are liberated. These neutrons are called fast neutrons and their energy is about 2 *MeV* (for each). These fast neutrons can escape from the reaction so as to proceed the chain reaction they are need to slow down.

(iv) Fission of U^{235} occurs by slow neutrons only (of energy about 1*eV*) or even by thermal neutrons (of energy about 0.025 *eV*).

(v) 50 *kg* of U^{235} on fission will release $\approx 4 \times 10^{15}$ *J* of energy. This is equivalence to 20,000 tones of *TNT* explosion. The nuclear bomb dropped at Hiroshima had this much explosion power.

(vi) The mass of the compound nucleus must be greater than the sum of masses of fission products.

(vii) The $\frac{\text{Binding energy}}{\text{A}}$ of compound nucleus must be less than that of the fission products.

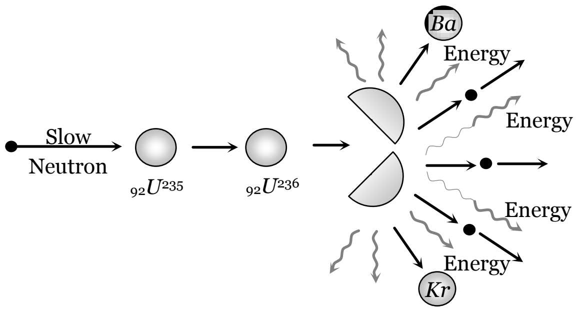
(viii) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments $_{56}Ba$ and $_{36}Kr$ are formed but they may be any stable isotopes of middle weight atoms.

Same other U^{235} fission reactions are

$$_{92}U^{235} + {_0}n^1 \rightarrow {_{54}}Xe^{140} + {_{38}}Sr^{94} + 2{_0}n^1$$
$$\rightarrow {_{57}}La^{148} + {_{35}}Br^{85} + 3\,{_0}n^1$$
$$\rightarrow \text{Many more}$$

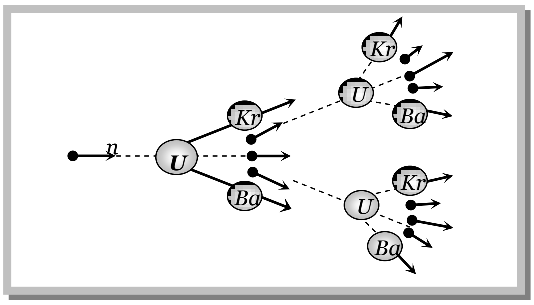
(ix) The neutrons released during the fission process are called prompt neutrons.

(x) Most of energy released appears in the form of kinetic energy of fission fragments.



(2) **Chain reaction**

In nuclear fission, three neutrons are produced along with the release of large energy. Under favourable conditions, these neutrons can cause further fission of other nuclei, producing large number of neutrons. Thus a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.



In the chain reaction, the number of nuclei undergoing fission increases very fast. So, the energy produced takes a tremendous magnitude very soon.

Difficulties in chain reaction

(i) Absorption of neutrons by U^{238}, the major part in natural uranium is the isotope U^{238} (99.3%), the isotope U^{235} is very little (0.7%). It is found that U^{238} is fissionable with fast neutrons, whereas U^{235} is fissionable with slow neutrons. Due to the large percentage of U^{238}, there is more possibility of collision of neutrons with U^{238}. It is found that the neutrons get slowed on coliding with U^{238}, as a result of it further fission of U^{238} is not possible (Because they are slow and they are absorbed by U^{238}). This stops the chain reaction.

Removal : (i) To sustain chain reaction $_{92}U^{235}$ is separated from the ordinary uranium. Uranium so obtained $\left(_{92}U^{235}\right)$ is known as enriched uranium, which is fissionable with the fast and slow neutrons and hence chain reaction can be sustained.

(ii) If neutrons are slowed down by any method to an energy of about 0.3 *eV*, then the probability of their absorption by U^{238} becomes very low, while the probability of their fissioning U^{235} becomes high. This job is done by moderators. Which reduce the speed of neutron rapidly graphite and heavy water are the example of moderators.

(iii) Critical size : The neutrons emitted during fission are very fast and they travel a large distance before being slowed down. If the size of the fissionable material is small, the neutrons emitted will escape the fissionable material before they are slowed down. Hence chain reaction cannot be sustained.

Removal : The size of the fissionable material should be large than a critical size.

The chain reaction once started will remain steady, accelerate or retard depending upon, a factor called neutron reproduction factor (*k*). It is defined as follows.

$$k = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

→ If $k = 1$, the chain reaction will be steady. The size of the fissionable material used is said to be the critical size and it's mass, the critical mass.

→ If $k > 1$, the chain reaction accelerates, resulting in an explosion. The size of the material in this case is super critical. (Atom bomb)

→ If $k < 1$, the chain reaction gradually comes to a halt. The size of the material used us said to be sub-critical.

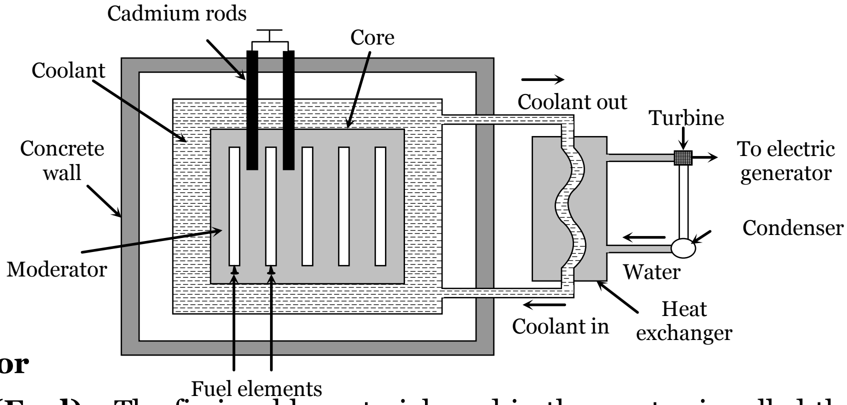
Types of chain reaction : Chain reactions are of following two types

Controlled chain reaction	Uncontrolled chain reaction
Controlled by artificial method	No control over this type of nuclear reaction
All neurons are absorbed except one	More than one neutron takes part into reaction
It's rate is slow	Fast rate
Reproduction factor $k = 1$	Reproduction factor $k > 1$
Energy liberated in this type of reaction is always less than explosive energy	A large amount of energy is liberated in this type of reaction
Chain reaction is the principle of nuclear reactors	Uncontrolled chain reaction is the principle of atom bomb.

Note : ❑ The energy released in the explosion of an atom bomb is equal to the energy released by 2000 tonn of TNT and the temperature at the place of explosion is of the order of 10^7 ^oC.

Nuclear Reactor

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction. It is also called an atomic pile. It is thus a source of controlled energy which is utilised for many useful purposes.



(1) **Parts of nuclear reactor**

(i) **Fissionable material (Fuel) :** The fissionable material used in the reactor is called the fuel of the reactor. Uranium isotope (U^{235}) Thorium isotope (Th^{232}) and Plutonium isotopes (Pu^{239}, Pu^{240} and Pu^{241}) are the most commonly used fuels in the reactor.

(ii) **Moderator :** Moderator is used to slow down the fast moving neutrons. Most commonly used moderators are graphite and heavy water (D_2O).

(iii) **Control Material :** Control material is used to control the chain reaction and to maintain a stable rate of reaction. This material controls the number of neutrons available for the fission. For example, cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rods in or out of the core of the reactor.

(iv) **Coolant :** Coolant is a cooling material which removes the heat generated due to fission in the reactor. Commonly used coolants are water, CO_2 nitrogen *etc.*

(v) **Protective shield :** A protective shield in the form a concrete thick wall surrounds the core of the reactor to save the persons working around the reactor from the hazardous radiations.

Note : ❑ It may be noted that Plutonium is the best fuel as compared to other fissionable material. It is because fission in Plutonium can be initiated by both slow and fast neutrons. Moreover it can be obtained from U^{238}.

❑ Nuclear reactor is firstly devised by fermi. ❑ Apsara was the first Indian nuclear reactor.

(2) **Uses of nuclear reactor**

(i) In electric power generation.

(ii) To produce radioactive isotopes for their use in medical science, agriculture and industry.

(iii) In manufacturing of PU^{239} which is used in atom bomb.

(iv) They are used to produce neutron beam of high intensity which is used in the treatment of cancer and nuclear research.

Note : ❑ A type of reactor that can produce more fissile fuel than it consumes is the breeder reactor.

Nuclear fusion

In nuclear fusion two or more than two lighter nuclei combine to form a single heavy nucleus. The mass of single nucleus so formed is less than the sum of the masses of parent nuclei. This difference in mass results in the release of tremendous amount of energy

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4\,MeV$$

$${}_1H^3 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1 + 17.6\,MeV$$

or $${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + 24\,MeV$$

For fusion high pressure ($\approx 10^6$ *atm*) and high temperature (of the order of 10^7 *K* to 10^8 *K*) is required and so the reaction is called thermonuclear reaction.

Fusion energy is greater then fission energy fission of one uranium atom releases about 200 *MeV* of energy. But the fusion of a deutron $({}_1H^2)$ and triton $({}_1H^3)$ releases about 17.6 *MeV* of energy. However the energy released per nucleon in fission is about 0.85 *MeV* but that in fusion is 4.4 *MeV*. So for the same mass of the fuel, the energy released in fusion is much larger than in fission.

Plasma : The temperature of the order of 10^8 *K* required for thermonuclear reactions leads to the complete ionisation of the atom of light elements. The combination of base nuclei and electron cloud is called plasma. The enormous gravitational field of the sun confines the plasma in the interior of the sun.

The main problem to carryout nuclear fusion in the laboratory is to contain the plasma at a temperature of 10^8K. No solid container can tolerate this much temperature. If this problem of containing plasma is solved, then the large quantity of deuterium present in sea water would be able to serve as in-exhaustible source of energy.

Note : ❑ To achieve fusion in laboratory a device is used to confine the plasma, called **Tokamak**.

Stellar Energy

Stellar energy is the energy obtained continuously from the sun and the stars. Sun radiates energy at the rate of about 10^{26} *joules* per *second*.

Scientist Hans Bethe suggested that the fusion of hydrogen to form helium (thermo nuclear reaction) is continuously taking place in the sun (or in the other stars) and it is the source of sun's (star's) energy.

The stellar energy is explained by two cycles

Proton-proton cycle	Carbon-nitrogen cycle
${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_1e^0 + Q_1$	${}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} + Q_1$
${}_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + Q_2$	${}_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e^0$
${}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1 + Q_3$	${}_1H^1 + {}_6C^{13} \rightarrow {}_7N^{14} + Q_2$
$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\gamma + 26.7\ MeV$	${}_1H^1 + {}_7N^{14} \rightarrow {}_8O^{15} + Q_3$
	${}_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 + Q_4$
	${}_1H^1 + {}_7N^{15} \rightarrow {}_6C^{12} + {}_2He^4$
	$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_1e^0 + 24.7\ MeV$

About 90% of the mass of the sun consists of hydrogen and helium.

Nuclear Bomb Based on uncontrolled nuclear reactions.

Atom bomb	Hydrogen bomb
Based on fission process it involves the fission of U^{235}	Based on fusion process. Mixture of deutron and tritium is used in it
In this critical size is important	There is no limit to critical size
Explosion is possible at normal temperature and pressure	High temperature and pressure are required
Less energy is released compared to hydrogen bomb	More energy is released as compared to atom bomb so it is more dangerous than atom bomb

Concepts

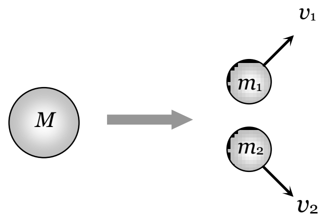
- *A test tube full of base nuclei will weight heavier than the earth.*
- *The nucleus of hydrogen contains only one proton. Therefore we may say that the proton is the nucleus of hydrogen atom.*
- *If the relative abundance of isotopes in an element has a ratio $n_1 : n_2$ whose atomic masses are m_1 and m_2 then atomic mass of the element is* $M = \dfrac{n_1m_1 + n_2m_2}{n_1 + n_2}$

Example

***Example*: 1** A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio 8 : 1. The ratio of radii of the fragments is

(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 2 : 1

Solution : (a)



By conservation of momentum $m_1v_1 = m_2v_2$

$\Rightarrow \frac{v_1}{v_2} = \frac{8}{1} = \frac{m_2}{m_1}$ ...... (i)

Also from $r \propto A^{1/3} \Rightarrow \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{1}{8}\right)^{1/3} = \frac{1}{2}$

***Example*: 2** The ratio of radii of nuclei ${}^{27}_{13}Al$ and ${}^{125}_{52}Te$ is approximately **[J & K CET 2000]**

(a) 6 : 10 (b) 13 : 52 (c) 40 : 177 (d) 14 : 7

Solution : (a) By using $r \propto A^{1/3} \Rightarrow \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{8}{5} = \frac{6}{10}$

***Example*: 3** If Avogadro's number is 6×10^{23} then the number of protons, neutrons and electrons in 14 *g* of ${}_6C^{14}$ are respectively

(a) $36 \times 10^{23}, 48 \times 10^{23}, 36 \times 10^{23}$ (b) $36 \times 10^{23}, 36 \times 10^{23}, 36 \times 10^{21}$

(c) $48 \times 10^{23}, 36 \times 10^{23}, 48 \times 10^{21}$ (d) $48 \times 10^{23}, 48 \times 10^{23}, 36 \times 10^{21}$

Solution : (a) Since the number of protons, neutrons and electrons in an atom of ${}_6C^{14}$ are 6, 8 and 6 respectively. As 14 *gm* of ${}_6C^{14}$ contains 6×10^{23} atoms, therefore the numbers of protons, neutrons and electrons in 14 *gm* of ${}_6C^{14}$ are $6 \times 6 \times 10^{23} = 36 \times 10^{23}$, $8 \times 6 \times 10^{23} = 48 \times 10^{23}$, $6 \times 6 \times 10^{23} = 36 \times 10^{23}$.

***Example*: 4** Two Cu^{64} nuclei touch each other. The electrostatics repulsive energy of the system will be

(a) 0.788 *MeV* (b) 7.88 *MeV* (c) 126.15 *MeV* (d) 788 *MeV*

Solution : (c) Radius of each nucleus $R = R_0(A)^{1/3} = 1.2\,(64)^{1/3} = 4.8\ fm$

Distance between two nuclei (r) = 2R

So potential energy $U = \frac{k \cdot q^2}{r} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19} \times 29)^2}{2 \times 4.8 \times 10^{-15} \times 1.6 \times 10^{-19}} = 126.15\ MeV$.

***Example*: 5** When ${}_{92}U^{235}$ undergoes fission. 0.1% of its original mass is changed into energy. How much energy is released if 1 *kg* of ${}_{92}U^{235}$ undergoes fission **[MP PET 1994; MP PMT/PET 1998; BHU 2001; BVP 2003]**

(a) $9 \times 10^{10}\ J$ (b) $9 \times 10^{11}\ J$ (c) $9 \times 10^{12}\ J$ (d) $9 \times 10^{13}\ J$

Solution : (d) By using $E = \Delta m \cdot c^2 \Rightarrow E = \left(\frac{0.1}{100} \times 1\right)(3 \times 10^8)^2 = 9 \times 10^{13}\ J$

***Example*: 6** 1 *g* of hydrogen is converted into 0.993 *g* of helium in a thermonuclear reaction. The energy released is **[EAMCET (Med.) 1995; CPMT 1999]**

(a) $63 \times 10^7\ J$ (b) $63 \times 10^{10}\ J$ (c) $63 \times 10^{14}\ J$ (d) $63 \times 10^{20}\ J$

Solution : (b) $\Delta m = 1 - 0.993 = 0.007\ gm$

$\therefore E = \Delta mc^2 = 0.007 \times 10^{-3} \times (3 \times 10^8)^2 = 63 \times 10^{10}\ J$

***Example*: 7** The binding energy per nucleon of deuteron $({}^2_1H)$ and helium nucleus $({}^4_2He)$ is 1.1 *MeV* and 7 *MeV* respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is **[MP PMT 1992; Roorkee 1994; IIT-JEE 1996; AIIMS 1997; Haryana PMT 2000; Pb PMT 2001; CPMT 2001; AIEEE 2004]**

(a) 13.9 *MeV* (b) 26.9 *MeV* (c) 23.6 *MeV* (d) 19.2 *MeV*

Solution : (c) ${}_1H^2 + {}_1H^2 \to {}_2He^4 + Q$

Total binding energy of helium nucleus = $4 \times 7 = 28\ MeV$

Total binding energy of each deutron = $2 \times 1.1 = 2.2\ MeV$

Hence energy released = $28 - 2 \times 2.2 = 23.6\ MeV$

***Example*: 8** The masses of neutron and proton are 1.0087 *amu* and 1.0073 *amu* respectively. If the neutrons and protons combine to form a helium nucleus (alpha particles) of mass 4.0015 *amu*. The binding energy of the helium nucleus will be [1 *amu*= 931 *MeV*] **[CPMT 1986; MP PMT 1995; CBSE 2003]**

(a) 28.4 *MeV* (b) 20.8 *MeV* (c) 27.3 *MeV* (d) 14.2 *MeV*

Solution : (a) Helium nucleus consist of two neutrons and two protons.

So binding energy $E = \Delta m$ $amu = \Delta m \times 931$ MeV

$\Rightarrow E = (2 \times m_p + 2m_n - M) \times 931\ MeV = (2 \times 1.0073 + 2 \times 1.0087 - 4.0015) \times 931 = 28.4\ MeV$

***Example*: 9** A atomic power reactor furnace can deliver 300 *MW*. The energy released due to fission of each of uranium atom U^{238} is 170 *MeV*. The number of uranium atoms fissioned per hour will be

(a) 5×10^{15} (b) 10×10^{20} (c) 40×10^{21} (d) 30×10^{25}

Solution : (c) By using $P = \frac{W}{t} = \frac{n \times E}{t}$ where n = Number of uranium atom fissioned and E = Energy released due to each fission so $300 \times 10^6 = \frac{n \times 170 \times 10^6 \times 1.6 \times 10^{-19}}{3600}$ $\Rightarrow n = 40 \times 10^{21}$

***Example*: 10** The binding energy per nucleon of O^{16} is 7.97 *MeV* and that of O^{17} is 7.75 *MeV*. The energy (in *MeV*) required to remove a neutron from O^{17} is **[IIT-JEE 1995]**

(a) 3.52 (b) 3.64 (c) 4.23 (d) 7.86

Solution : (c) $O^{17} \rightarrow O^{16} + {}_0n^1$

$\therefore$ Energy required = Binding of O^{17} – binding energy of O^{16} = $17 \times 7.75 - 16 \times 7.97 = 4.23$ *MeV*

***Example*: 11** A gamma ray photon creates an electron-positron pair. If the rest mass energy of an electron is 0.5 *MeV* and the total kinetic energy of the electron-positron pair is 0.78 *MeV*, then the energy of the gamma ray photon must be **[MP PMT 1991]**

(a) 0.78 *MeV* (b) 1.78 *MeV* (c) 1.28 *MeV* (d) 0.28 *MeV*

Solution : (b) Energy of γ-rays photon = 0.5 + 0.5 +0.78 = 1.78 *MeV*

***Example*: 12** What is the mass of one Curie of U^{234} **[MNR 1985]**

(a) 3.7×10^{10} *gm* (b) 2.348×10^{23} *gm* (c) 1.48×10^{-11} *gm* (d) 6.25×10^{-34} *gm*

Solution : (c) 1 *curie* = 3.71×10^{10} disintegration/*sec* and mass of 6.02×10^{23} atoms of U^{234} = 234 *gm*

$\therefore$ Mass of 3.71×10^{10} atoms $= \frac{234 \times 3.71 \times 10^{10}}{6.02 \times 10^{23}} = 1.48 \times 10^{-11} gm$

***Example*: 13** In the nuclear fusion reaction ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n,$ given that the repulsive potential energy between the two nuclei is $-7.7 \times 10^{-14}\,J$, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}\,J/K$] **[AIEEE 2003]**

(a) $10^9 K$ (b) $10^7 K$ (c) $10^5 K$ (d) $10^3 K$

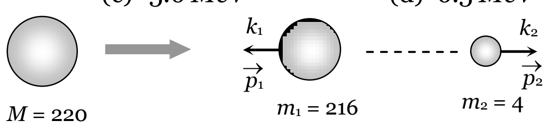
Solution : (a) Kinetic energy of molecules of a gas at a temperature *T* is 3/2 *kT*

$\therefore$ To initiate the reaction $\frac{3}{2}kT = 7.7 \times 10^{-14}\,J$ $\Rightarrow T = 3.7 \times 10^9\ K.$

***Example*: 14** A nucleus with mass number 220 initially at rest emits an α-particle. If the *Q* value of the reaction is 5.5 *MeV*. Calculate the kinetic energy of the α-particle **[IIT-JEE (Screening) 2003]**

(a) 4.4 *MeV* (b) 5.4 *MeV* (c) 5.6 *MeV* (d) 6.5 *MeV*

Solution : (b)



Q-value of the reaction is 5.5 *eV* *i.e.* $k_1 + k_2 = 5.5\ MeV$ ......(i)

By conservation of linear momentum $p_1 = p_2 \Rightarrow \sqrt{2(216)k_1} = \sqrt{2(4)k_2} \Rightarrow k_2 = 54\ k_1$ ......(ii)

On solving equation (i) and (ii) we get k_2 = 5.4 *MeV*.

***Example*: 15** Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a ${}_{10}^{20}Ne$ nucleus and M_2 the mass of a ${}_{20}^{40}Ca$ nucleus. Then **[IIT 1998; DPMT 2000]**

(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$ (c) $M_2 < 2M_1$ (d) $M_1 < 10(m_n + m_p)$

Solution : (c, d) Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less then the sum of masses of it's constituent particles ${}^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons. Therefore, mass of ${}^{20}_{10}Ne$ nucleus $M_1 < 10\,(m_p + m_n)$

Also heavier the nucleus, more is he mass defect thus $20\,(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

or $10\,(m_p + m_n) > M_2 - M_1$

$\Rightarrow$ $M_2 < M_1 + 10\,(m_p + m_n) \Rightarrow M_2 < M_1 + M_1 \Rightarrow M_2 < 2M_1$

Tricky example: 1

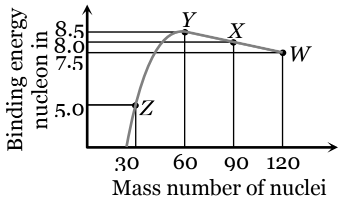
Binding energy per nucleon *vs* mass number curve for nuclei is shown in the figure. *W*, *X*, *Y* and *Z* are four nuclei indicated on the curve. The process that would release energy is **[IIT-JEE 1999]**

(a) $Y \to 2Z$

(b) $W \to X + Z$

(c) $W \to 2Y$

(d) $X \to Y + Z$



Solution : (c) Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleon) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (c) this happens.

Given $W \to 2Y$

Binding energy of reactants = 120 × 7.5 = 900 *MeV*

and binding energy of products = 2 (60 × 8.5) = 1020 *MeV* > 900 *MeV*

Radioactivity

The phenomenon of spontaneous emission of radiatons by heavy elements is called radioactivity. The elements which shows this phenomenon are called radioactive elements.

(1) Radioactivity was discovered by Henery Becquerel in uranium salt in the year 1896.

(2) After the discovery of radioactivity in uranium, Piere Curie and Madame Curie discovered a new radioactive element called radium (which is 10^6 times more radioactive than uranium)

(3) Some examples of radio active substances are : Uranium, Radium, Thorium, Polonium, Neptunium *etc.*

(4) Radioactivity of a sample cannot be controlled by any physical (pressure, temperature, electric or magnetic field) or chemical changes.

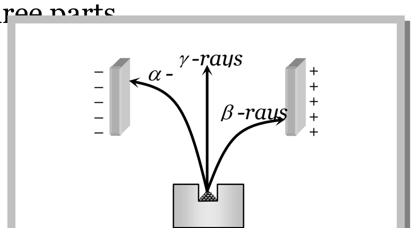
(5) All the elements with atomic number (Z) > 82 are naturally radioactive.

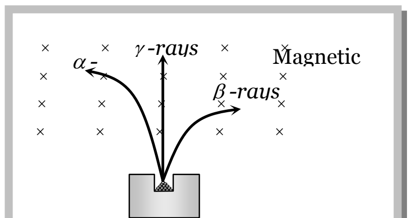
(6) The conversion of lighter elements into radioactive elements by the bombardment of fast moving particles is called artificial or induced radioactivity.

(7) Radioactivity is a nuclear event and not atomic. Hence electronic configuration of atom don't have any relationship with radioactivity.

Nuclear radiatons

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only. When the radiation enters into the external electric field, they splits into three parts





(i) Radiations which deflects towards negative plate are called α-rays (stream of positively charged particles)

(ii) Radiations which deflects towards positive plate are called β particles (stream of negatively charged particles)

(iii) Radiations which are undeflected called γ-*rays*. (E.M. waves or photons)

Note : ❑ Exactly same results were obtained when these radiations were subjected to magnetic field.

❑ No radioactive substance emits both α and β particles simultaneously. Also γ-rays are emitted after the emission of α or β-particles.

❑ β-particles are not orbital electrons they come from nucleus. The neutron in the nucleus decays into proton and an electron. This electron is emitted out of the nucleus in the form of β-rays.

Properties of α, β and γ-rays

Features	α- particles	β - particles	γ - rays
1. Identity	Helium nucleus or doubly ionised helium atom ($_2He^4$)	Fast moving electron $(-\beta^0$ or $\beta^-)$	Photons (E.M. waves)
2. Charge	+ 2e	– e	Zero
3. Mass 4 m_p (m_p = mass of proton = 1.87×10^{-27}	4 m_p	m_e	Massless
4. Speed	$\approx 10^7\ m/s$	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 *MeV* to 9 *MeV*	All possible values between a minimum certain value to 1.2 *MeV*	Between a minimum value to 2.23 *MeV*
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 *cm* of iron (or *Pb*) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete
10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	${}_Z X^A \xrightarrow{\alpha-decay} {}_{Z-2}Y^{A-4} + {}_2He^4$ ${}_Z X^A \xrightarrow{n_\alpha} {}_{Z'}Y^{A'}$	${}_Z X^A \to {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$ ${}_Z X^A \xrightarrow{n_\beta} {}_{Z'}X^A$ $\Rightarrow \boldsymbol{n_\beta = (2n_\alpha - Z + Z')}$	${}_z X^A \to {}_z X^a + \gamma$

	$\Rightarrow n_\alpha = \frac{A' - A}{4}$		

Radioactive Disintegration

(1) **Law of radioactive disintegration**

According to Rutherford and Soddy law for radioactive decay is as follows.

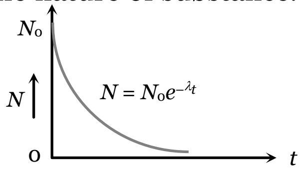
"At any instant the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant" *i.e.* $-\frac{dN}{dt} \propto N \quad \Rightarrow \frac{dN}{dt} = -\lambda N$. It can be proved that $\boldsymbol{N = N_0 e^{-\lambda t}}$

This equation can also be written in terms of mass *i.e.* $M = M_0 e^{-\lambda t}$

where N = Number of atoms remains undecayed after time t, N_0 = Number of atoms present initially (*i.e.* at t = 0), M = Mass of radioactive nuclei at time t, M_0 = Mass of radioactive nuclei at time t = 0, $N_0 - N$ = Number of disintegrated nucleus in time t

$\frac{dN}{dt}$ = rate of decay, λ = Decay constant or disintegration constant or radioactivity constant or Rutherford Soddy's constant or the probability of decay per unit time of a nucleus.

Note : ❑ λ depends only on the nature of substance. It is independent of time and any physical or chemical changes.



(2) **Activity**

It is defined as the rate of disintegration (or count rate) of the substance (or the number of atoms of any material decaying per second) *i.e.* $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$

where A_0 = Activity of t = 0, A = Activity after time t

Units of activity (Radioactivity)

It's units are *Becqueral* (*Bq*), Curie (*Ci*) and Rutherford (*Rd*)

1 *Becquerel* = 1 *disintegration/sec*, 1 *Rutherford* = 10^6 *dis/sec*, 1 *Curie* = 3.7×10^{11} *dis/sec*

Note : ❑ Activity per *gm* of a substance is known as specific activity. The specific activity of 1 *gm* of radium – 226 is 1 *Curie*.

❑ 1 *millicurie* = 37 *Rutherford*

❑ The activity of a radioactive substance decreases as the number of undecayed nuclei decreases with time.

❑ Activity $\propto \frac{1}{\text{Half life}}$

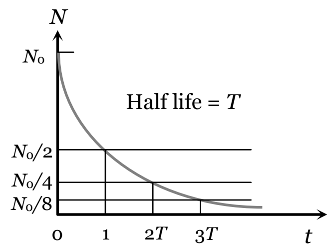
(3) **Half life ($T_{1/2}$)**

Time interval in which the mass of a radioactive substance or the number of it's atom reduces to half of it's initial value is called the half life of the substance.

i.e. if $N = \frac{N_0}{2}$ then $t = T_{1/2}$

Hence from $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda (T_{1/2})} \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$



Time (t)	Number of undecayed atoms (N) (N_0 = Number of initial atoms)	Remaining fraction of active atoms (N/N_0) probability of survival	Fraction of atoms decayed ($N_0 - N$) $/N_0$ probability of decay
t = 0	N_0	1 (100%)	0

$t = T_{1/2}$	$\frac{N_0}{2}$	$\frac{1}{2}$ (50%)	$\frac{1}{2}$ (50%)
$t = 2(T_{1/2})$	$\frac{1}{2} \times \frac{N_0}{2} = \frac{N_0}{(2)^2}$	$\frac{1}{4}$ (25%)	$\frac{3}{4}$ (75%)
$t = 3(T_{1/2})$	$\frac{1}{2} \times \frac{N_0}{(2)} = \frac{N_0}{(2)^3}$	$\frac{1}{8}$ (12.5%)	$\frac{7}{8}$ (87.5%)
$t = 10\ (T_{1/2})$	$\frac{N_0}{(2)^{10}}$	$\left(\frac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = n\ (N_{1/2})$	$\frac{N}{(2)^2}$	$\left(\frac{1}{2}\right)^{n}$	$\left\{1 - \left(\frac{1}{2}\right)^{n}\right\}$

Useful relation

After n half-lives, number of undecayed atoms $N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

(4) **Mean (or average) life (τ)**

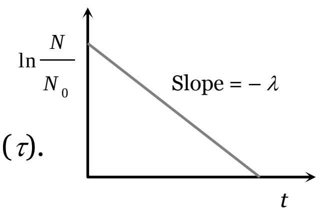
The time for which a radioactive material remains active is defined as mean (average) life of that material.

Other definitions

(i) It is defined as the sum of lives of all atoms divided by the total number of atoms

$$i.e.\ \tau = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}} = \frac{1}{\lambda}$$

(ii) From $N = N_0 e^{-\lambda t} \Rightarrow \dfrac{\ln \frac{N}{N_0}}{t} = -\lambda$ slope of the line shown in the graph



i.e. the magnitude of inverse of slope of $\ln \frac{N}{N_0}$ *vs* t curve is known as mean life (τ).

(iii) From $N = N_0 e^{-\lambda t}$

If $t = \frac{1}{\lambda} = \tau \Rightarrow N = N_0 e^{-1} = N_0\left(\frac{1}{e}\right) = 0.37\, N_0 = 37\%$ of N_0.

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\frac{1}{e}$ times or 0.37 times or 37% of original number of atoms. or

It is the time in which number of decayed atoms ($N_0 - N$) becomes $\left(1 - \frac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(iv) From $T_{1/2} = \frac{0.693}{\lambda} \Rightarrow \frac{1}{\lambda} = \tau = \frac{1}{0.693}.(t_{1/2}) = 1.44\,(T_{1/2})$

i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$

Note : ❑ Half life and mean life of a substance doesn't change with time or with pressure, temperature *etc.*

Radioactive Series

If the isotope that results from a radioactive decay is itself radioactive then it will also decay and so on.

The sequence of decays is known as radioactive decay series. Most of the radio-nuclides found in nature are members of four radioactive series. These are as follows

Mass number	Series (Nature)	Parent	Stable and product	Integer n	Number of lost particles
$4n$	Thorium (natural)	${}_{90}Th^{232}$	${}_{82}Pb^{208}$	52	$\alpha = 6, \beta = 4$
$4n + 1$	Neptunium (Artificial)	${}_{93}Np^{237}$	${}_{83}Bi^{209}$	52	$\alpha = 8, \beta = 5$

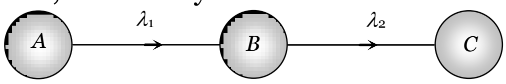
$4n + 2$	Uranium (Natural)	$_{92}U^{238}$	$_{82}Pb^{206}$	51	$\alpha = 8, \beta = 6$
$4n + 3$	Actinium (Natural)	$_{89}Ac^{227}$	$_{82}Pb^{207}$	51	$\alpha = 7, \beta = 4$

Note : ❑ The $4n + 1$ series starts from $_{94}PU^{241}$ but commonly known as neptunium series because neptunium is the longest lived member of the series.

❑ The $4n + 3$ series actually starts from $_{92}U^{235}$.

Successive Disintegration and Radioactive Equilibrium

Suppose a radioactive element A disintegrates to form another radioactive element B which intern disintegrates to still another element C; such decays are called successive disintegration.



Rate of disintegration of $A = \frac{dN_1}{dt} = -\lambda_1 N_1$ (which is also the rate of formation of B)

Rate of disintegration of $B = \frac{dN_2}{dt} = -\lambda_2 N_2$

$\therefore$ Net rate of formation of B = Rate of disintegration of A – Rate of disintegration of B

$= \lambda_1 N_1 - \lambda_2 N_2$

Equilibrium

In radioactive equilibrium, the rate of decay of any radioactive product is just equal to it's rate of production from the previous member.

i.e. $\lambda_1 N_1 = \lambda_2 N_2 \quad \Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_2} = \frac{\tau_2}{\tau_1} = \frac{(T_{1/2})}{(T_{1/2})_1}$

Note : ❑ In successive disintegration if N_0 is the initial number of nuclei of A at $t = 0$ then number of nuclei of product B at time t is given by $N_2 = \frac{\lambda_1 N_0}{(\lambda_2 - \lambda_1)}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$ where $\lambda_1 \lambda_2$ – decay constant of A and B.

Uses of radioactive isotopes

(1) **In medicine**

(i) For testing blood-chromium - 51

(ii) For testing blood circulation - Na - 24

(iii) For detecting brain tumor- Radio mercury - 203

(iv) For detecting fault in thyroid gland - Radio iodine - 131

(v) For cancer - cobalt - 60

(vi) For blood - Gold - 189

(vii) For skin diseases - Phospohorous - 31

(2) **In Archaeology**

(i) For determining age of archaeological sample (carbon dating) C^{14}

(ii) For determining age of meteorites - K^{40}

(iii) For determining age of earth-Lead isotopes

(3) **In agriculture**

(i) For protecting potato crop from earthworm- CO^{60}

(ii) For art As fertilizers - P^{32}

(4) **As tracers** - (Tracer) : Very small quantity of radioisotopes present in a mixture is known as tracer

(i) Tracer technique is used for studying biochemical reaction in tracer and animals.

(5) **In industries**

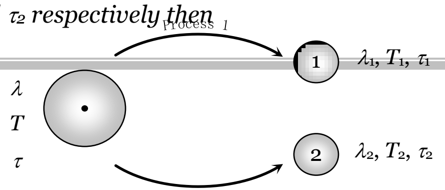
(i) For detecting leakage in oil or water pipe lines

(ii) For determining the age of planets.

Concept

☞ *If a nuclide can decay simultaneously by two different process which have decay constant λ_1 and λ_2, half life T_1 and T_2 and mean lives τ_1 and τ_2 respectively then*

$\Rightarrow \lambda = \lambda_1 + \lambda_2$



$$\Rightarrow \quad T = \frac{T_1 T_2}{T_1 + T_2}$$

$$\Rightarrow \quad \tau = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$$

***Example*: 16** When ${}_{90}Th^{228}$ transforms to ${}_{83}Bi^{212}$, then the number of the emitted α–and β–particles is, respectively **[MP PET 2002]**

(a) $8\alpha, 7\beta$ (b) $4\alpha, 7\beta$ (c) $4\alpha, 4\beta$ (d) $4\alpha, 1\beta$

Solution : (d) ${}_{Z=90}Th^{A=228} \to {}_{Z'=83}Bi^{A'=212}$

Number of α-particles emitted $n_\alpha = \frac{A - A'}{4} = \frac{228 - 212}{4} = 4$

Number of β-particles emitted $n_\beta = 2n_\alpha - Z + Z' = 2 \times 4 - 90 + 83 = 1.$

***Example*: 17** A radioactive substance decays to 1/16th of its initial activity in 40 days. The half-life of the radioactive substance expressed in days is

(a) 2.5 (b) 5 (c) 10 (d) 20

Solution : (c) By using $N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{N}{N_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^{40/T_{1/2}} \Rightarrow T_{1/2} = 10$ days.

***Example*: 18** A sample of radioactive element has a mass of 10 *gm* at an instant *t* = 0. The approximate mass of this element in the sample after two mean lives is **[CBSE PMT 2003]**

(a) 2.50 *gm* (b) 3.70 *gm* (c) 6.30 *gm* (d) 1.35 *gm*

Solution : (d) By using $M = M_0 e^{-\lambda t} \Rightarrow M = 10e^{-\lambda(2\tau)} = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 = 1.359\ gm$

***Example*: 19** The half-life of ${}^{215}At$ is 100 μs. The time taken for the radioactivity of a sample of ${}^{215}At$ to decay to 1/16th of its initial value is **[IIT-JEE (Screening) 2002]**

(a) 400 μs (b) 6.3 μs (c) 40 μs (d) 300 μs

Solution : (a) By using $N = N_0 \left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{t/100} \Rightarrow t = 400\ \mu\,sec.$

***Example*: 20** The mean lives of a radioactive substance for α and β emissions are 1620 years and 405 years respectively. After how much time will the activity be reduced to one fourth **[RPET 1999]**

(a) 405 year (b) 1620 year (c) 449 year (d) None of these

Solution : (c) $\lambda_\alpha = \frac{1}{1620}$ *per year* and $\lambda_\beta = \frac{1}{405}$ *per year* and it is given that the fraction of the remained activity $\frac{A}{A_0} = \frac{1}{4}$

Total decay constant $\lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ *per year*

We know that $A = A_0 e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda}\log_e \frac{A_0}{A} \Rightarrow t = \frac{1}{\lambda}\log_e 4 = \frac{2}{\lambda}\log_e 2 = 324 \times 2 \times 0.693 = 449$ *years.*

***Example*: 21** At any instant the ratio of the amount of radioactive substances is 2 : 1. If their half lives be respectively 12 and 16 hours, then after two days, what will be the ratio of the substances

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

Solution : (a) By using $N = N_0 \left(\frac{1}{2}\right)^n \Rightarrow \frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2} \times \frac{(1/2)^{n_1}}{(1/2)^{n_2}} = \frac{2}{1} \times \frac{\left(\frac{1}{2}\right)^{\frac{2\times 24}{12}}}{\left(\frac{1}{2}\right)^{\frac{2\times 24}{16}}} = \frac{1}{1}$

***Example*: 22** From a newly formed radioactive substance (Half-life 2 *hours*), the intensity of radiation is 64 times the permissible safe level. The minimum time after which work can be done safely from this source is **[IIT 1983; SCRA 1996]**

(a) 6 *hours* (b) 12 *hours* (c) 24 *hours* (d) 128 *hours*

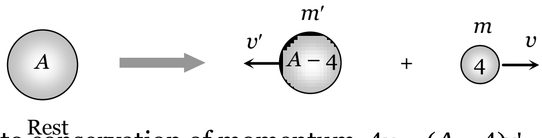
Solution : (b) By using $A = A_0 \left(\frac{1}{2}\right)^n \Rightarrow \frac{A}{A_0} = \frac{1}{64} = \left(\frac{1}{2}\right)^0 = \left(\frac{1}{2}\right)^n \Rightarrow n = 6$

$\Rightarrow \frac{t}{T_{1/2}} = 6 \Rightarrow t = 6 \times 2 = 12$ *hours.*

***Example*: 23** nucleus of mass number A, originally at rest, emits an α-particle with speed v. The daughter nucleus recoils with a speed **[DCE 2000; AIIMS 2004]**

(a) $2v/(A+4)$ (b) $4v/(A+4)$ (c) $4v/(A-4)$ (d) $2v/(A-4)$

Solution : (c)



According to conservation of momentum $4v = (A-4)v' \Rightarrow v' = \dfrac{4v}{A-4}$.

***Example*: 24** The counting rate observed from a radioactive source at t = 0 second was 1600 counts per second and at t = 8 seconds it was 100 counts per second. The counting rate observed as counts per second at t = 6 seconds will be **[MP PET 1996; UPSEAT 2000]**

(a) 400 (b) 300 (c) 200 (d) 150

Solution : (c) By using $A = A_0\left(\dfrac{1}{2}\right)^n \Rightarrow 100 = 1600\left(\dfrac{1}{2}\right)^{8/T_{1/2}} \Rightarrow \dfrac{1}{16} = \left(\dfrac{1}{2}\right)^{8/T_{1/2}} \Rightarrow T_{1/2} = 2\ sec$

Again by using the same relation the count rate at t = 6 *sec* will be $A = 1600\left(\dfrac{1}{2}\right)^{6/2} = 200$.

***Example*: 25** The kinetic energy of a neutron beam is 0.0837 *eV*. The half-life of neutrons is 693*s* and the mass of neutrons is 1.675×10^{-27} *kg*. The fraction of decay in travelling a distance of 40*m* will be

(a) 10^{-3} (b) 10^{-4} (c) 10^{-5} (d) 10^{-6}

Solution : (c) $v = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2 \times 0.0837 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}} = 4 \times 10^3\ m/sec$

$\therefore$ Time taken by neutrons to travel a distance of 40 *m* $\Delta t' = \dfrac{40}{4 \times 10^3} = 10^{-2}\ sec$

$\therefore \dfrac{dN}{dt} = \lambda N \Rightarrow \dfrac{dN}{N} = \lambda\, dt$

$\therefore$ Fraction of neutrons decayed in Δt *sec* in $\dfrac{\Delta N}{N} = \lambda \Delta t = \dfrac{0.693}{T}\Delta t = \dfrac{0.693}{693} \times 10^{-2} = 10^{-5}$

***Example*: 26** The fraction of atoms of radioactive element that decays in 6 days is 7/8. The fraction that decays in 10 days will be

(a) 77/80 (b) 71/80 (c) 31/32 (d) 15/16

Solution : (c) By using $N = N_0\left(\dfrac{1}{2}\right)^{t/T_{1/2}} \Rightarrow t = \dfrac{T_{1/2}\log_e\left(\dfrac{N_0}{N}\right)}{\log_e(2)} \Rightarrow t \propto \log_e \dfrac{N_0}{N} \Rightarrow \dfrac{t_1}{t_2} = \dfrac{\left(\log_e \dfrac{N_0}{N}\right)_1}{\left(\log_e \dfrac{N_0}{N}\right)_2}$

Hence $\dfrac{6}{10} = \dfrac{\log_e(8/1)}{\log_e(N_0/N)} \Rightarrow \log_e \dfrac{N_0}{N} = \dfrac{10}{6}\log_e(8) = \log_e 32 \Rightarrow \dfrac{N_0}{N} = 32$.

So fraction that decays $= 1 - \dfrac{1}{32} = \dfrac{31}{32}$.

Tricky example: 2

Half-life of a substance is 20 minutes. What is the time between 33% decay and 67% decay **[AIIMS 200**

(a) 40 minutes (b) 20 minutes (c) 30 minutes (d) 25 minutes

Solution : (b) Let N_0 be the number of nuclei at beginning

$\therefore$ Number of undecayed nuclei after 33% decay = 0.67 N_0

and number of undecayed nuclei after 67% of decay = 0.33 N_0

$\because$ 0.33 $N_0 \simeq \dfrac{0.67 N_0}{2}$ and in the half-life time the number of undecayed nuclei becomes half.

ELECTROSTATICS : Study of Electricity in which electric charges are static i.e. not moving, is called electrostatics

- STATIC CLING
- An electrical phenomenon that accompanies dry weather, causes these pieces of papers to stick to one another and to the plastic comb.
- Due to this reason our clothes stick to our body.

- ELECTRIC CHARGE : Electric charge is characteristic developed in particle of material due to which it exert force on other such particles. It automatically accompanies the particle wherever it goes.
- Charge cannot exist without material carrying it
- It is possible to develop the charge by **rubbing two solids having friction.**
- Carrying the charges is called **electrification**.
- Electrification due to friction is called **frictional electricity**.

Since these charges are <u>not flowing</u> it is also called static electricity.

There are two types of charges. +ve and ‑ve.

- Similar charges repel each other,
- Opposite charges attract each other.

- Benjamin Franklin made this nomenclature of charges being +ve and ‑ve for mathematical calculations because adding them together cancel each other.
- Any particle has vast amount of charges.
- The number of positive and negative charges are **equal**, hence **matter is basically neutral.**
- Inequality of charges give the material a <u>**net**</u> charge which is equal to the difference of the two type of charges.

<u>**Electrostatic series**</u> :If two substances are rubbed together the former in series acquires the positive charge and later, the –ve.

(i) Glass (ii) Flannel (iii) Wool (iv) Silk (v) Hard Metal (vi) Hard rubber (vii) Sealing wax (viii) Resin (ix) Sulphur

<u>**Electron theory of Electrification**</u>

- Nucleus of atom is positively charged.
- The electron revolving around it is negatively charged.
- They are equal in numbers, hence atom is electrically neutral.
- With friction there is transfer of electrons, hence net charge is developed in the particles.

- It also explains that the charges are compulsorily developed in pairs equally . +vein one body and –ve in second.
- It establish **conservation of charges in the universe.**
- The **loss** of **electrons** develops +ve charge. While **excess** of **electrons** develop ‑ve charge
- A **proton** is 1837 times heavier than electron hence it cannot be transferred. Transferring lighter electron is easier.
- Therefore for electrification of matter, only **electrons** are active and responsible.

<u>**Charge and Mass relation**</u>

- Charge cannot exist without matter.
- One carrier of charge is electron which has **mass** as well.
- Hence if there is charge transfer, mass is also transferred.
- Logically, negatively charged body is heavier then positively charged body.

<u>**Conductors, Insulators and Semiconductors**</u>

- Conductors : Material in which electrons can move easily and freely.

Ex. Metals, Tap water, human body.

Brass rod in our hand, if charged by rubbing the charge will move easily to earth. Hence Brass is a conductor.

The flow of this excess charge is called **discharging**

- **Insulator** : Material in which charge cannot move freely. Ex . Glass, pure water, plastic etc.

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- Electrons can be forced to move across an insulator by applying strong force (called electric field.) Then this acts like a conductor.

- **dielectric strength.**

The maximum electric field an insulator can withstand without becoming a conductor is called its dielectric strength.

- **Semiconductor** : is a material which under little stimulation (heat or Elect. Field) converts from insulator to a conductor.

Ex. Silicon, germanium.

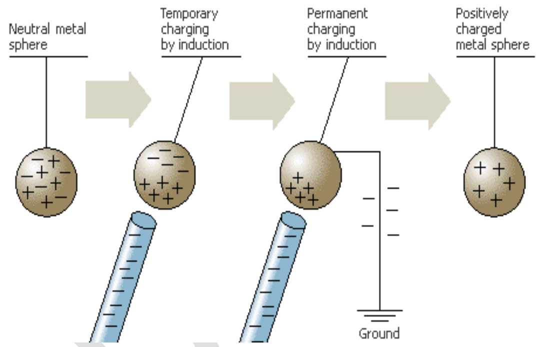
- **Superconductor** : is that material which presents no resistance to the movement of the charge through it.

The resistance is precisely zero.

Electrostatic Induction

- Phenomenon of polarization of charges in a body, when a charged body is present near it, is called electrostatic induction.
- In this process bodies are charged without touching them.

- **Charging by Induction**



A charged object will induce a charge on a nearby conductor. In this example, a negatively charged rod pushes some of the negatively charged electrons to the far side of a nearby copper sphere because like charges repel each other. The positive charges that remain on the near side of the sphere are attracted to the rod.

- If the sphere is grounded so that the electrons can escape altogether, the charge on the sphere will remain if the rod is removed.

Basic properties of Electric charge

- Additivity of Electric charges
- Quantization of Electric charge
- Conservation of Electric Charge

Additivity of Charges...

- Charges can be added by simple rules of algebra. Addition of positive and negative charge makes Zero charge

Quantization of Electric charge

- Principle: **Electric charge is not a continuous quantity, but is an integral multiple of minimum charge (e).**
- Reason of quantization:
- Minimum charge *e* exist on an electron.
- The material which is transferred during electrification is an electron, in integral numbers.
- Hence **charge transferred has to be integral multiple of *e*.**
- Charge on an electron (-e) and charge on a proton (+e) are equal and opposite, and are the **minimum.**

This minimum charge is 1.6×10^{-19} coulomb.

one electron has charge -1.6×10^{-19} C

One proton has charge $+1.6 \times 10^{-19}$ C

- Charge on a body Q is given by

$Q = \pm ne$

Where n is a whole number 1,2,3.....

and $e = 1.6 \times 10^{-19}$

- since e is smallest value of charge, it is called Elementary Charge or Fundamental charge

---

- (**Quarks** : In new theories of proton and neutrons, a required constituent particles called Quarks which carry charges $\pm(1/3)$e or $\pm(2/3)$e.

- But because free quarks do not exist and their sum is always an integral number, it does not violet the quantization rules.)

- **Conservation of Charges**
- Like conservation of energy, and Momentum, the electric charges also follow the rules of conservation.

1. Isolated (Individual) Electric charge can neither be created nor destroyed, it can only be transferred.
2. Charges in pair can be created or destroyed.

Example for 1.

At Nuclear level : Decay of U-238

^{238}U $^{234}Th + ^{4}He$ (Radio active decay)

Atomic number Z of radioactive material U-238 is 92. Hence it has 92 protons hence charge is 92*e*. Thorium has Z= 90, hence charge is 90e, alpha particles have charge 2e. Therefore charges before decay are 92 and after decay are 90+2=92

Example for 2. (a) Annihilation (destruction in pair)
In a nuclear process an electron -e and its antiparticle positron +e undergo annihilation process in which they transform into two gamma rays (high energy light)

$e^- + e^+$ $y + y$

Example for 2 (b):Pair production:
is converse of annihila tion, charge is also conserved when a gamma ray transforms into an electron and a positron

y $e^- + e^+$ (pair production)

Electric Force - Coulumb's Law

- Coulumb's law in Electrostatics :

Force of Interaction between two stationery point charges is

directly proportional to the product of the charges,
inversely proportional to the square of the distance between them and
acts along the straight line joining the two charges.

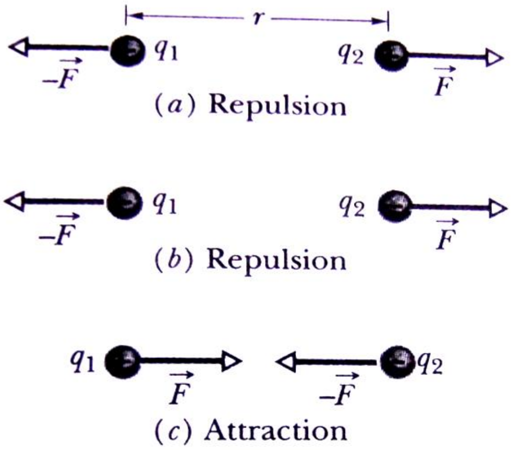


Fig. 22-6 Two charged particles, separated by distance *r*, repel each other if their charges are (*a*) both positive and (*b*) both negative. (*c*) They attract each other if their charges are of opposite signs. In each of the three situations, the force acting on one particle is equal in magnitude to the force acting on the other particle but has the opposite direction.

If two charges q_1 and q_2 are placed at distance r then,

$$F = c\,\frac{q_1 q_2}{r^2}$$

where c is a constant .

c is called Coulomb's constant and its value is

$$c = \frac{1}{4\pi\varepsilon_0} \quad \therefore \quad F = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r^2}$$

The value of c depends upon system of units and on the medium between two charges
It is seen experimentally that if two charges of 1 Coulomb each are placed at a distance of 1 meter in air or vacuum, then they attract each other with a force (F) of 9×10^9 Newton.
Accordingly value of c is 9×10^9 Newton x $m^2/coul^2$

e_0 is permittivity of free space or vacuum and its value is $e_0 = 8.85 \times 10^{-12}$ coul^2 / N x m^2

If point charges are immersed in a dielectric medium, then e_0 is replaced by e a quantity-characteristic of the matter involved In such case. For vacuum e = e_0

$$F = \frac{1}{4\pi\varepsilon}\ \frac{q_1 q_2}{r^2}$$

Permittivity, Relative Permittivity and Dielectric Constant

Permittivity is a measure of the property of the medium surrounding electric charge which determine the forces between the charges.

Its value is known as Absolute permittivity of that Medium e

More is Permittivity of medium, Less is coulombs Force.

For water, permittivity is 80 times then that of vacuum, hence force between two charges in water will be 1/80 time force in vacuum (or air.)

Relative Permittivity(e_r) : It is a dimension-less characteristic constant, which express absolute permittivity of a medium w.r.t. permittivity of vacuum or air. It is also called

Dielectric constant (K) **K= e_r = e/e_0**

- This result leads to the calculation that

$$\varepsilon_r = K = \frac{F\ (\text{Force in Vacuum})}{F'\ (\text{Force in Medium})}$$

$$F = \frac{1}{4\pi\varepsilon}\ \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\ \varepsilon_0 \varepsilon_r}\ \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\ \varepsilon_0 K}\ \frac{q_1 q_2}{r^2}$$

- Unit of charge:- In S.I. System of units, the unit of charge is **Coulomb.**
- **<u>One coulomb</u>** is defined as that charge, which, when placed at a distance of 1 m in air or vacuum from an equal and similar charge, repel it with a force of 9×10^9 Newton
- Charge on one electron is 1.6019×10^{-19} coul. Hence
- One coulomb is equivalent to a charge of 6.243 x 10^{18} electrons

Is electric charge a fundamental quantity?

- No, In S.I. System, the fundamental quantity is **Electric current** and its unit is Ampere. Therefore coulomb is defined in it's terms as under:
- Coulomb is that quantity of charge which passes across any section of a conductor per second when current of one ampere flows through it, i.e.
- 1 coulomb=1 Ampere x 1 sec

In **cgs electrostatic** system, the unit of charge is called as STATECOULUMB or esu of charge.

- In this system electrostatic constant c=1 for vacuum or air, $F = \frac{q_1 q_2}{r^2}$

One stat coulomb is defined that amount of charge which when placed at a distance of 1 cm in air from an equal and similar charge repel it with a force of one dyne.

In **cgs electromagnetic** system, the unit of charge is called ABCOULOMB or emu of charge

1 Coulomb = 3 x 10^9 statcoulomb

= 1/10 abcoulomb

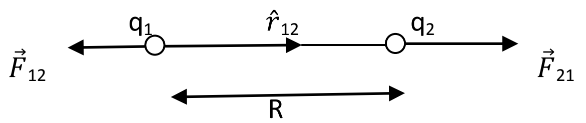
\ 1 emu = 3×10^{10} esu of charge

<u>**Vector form of Coulumbs' Law**</u>

Equation of Coulumbs force showing magnitude as well as direction is called Vector form of coulumbs' law.

If $\hat{r}_{12}$ is unit vector pointing from q_1 to q_2, then as per diagram $\hat{r}_{12}$ and $\vec{F}_{21}$ will be in the same direction, then

$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\ \frac{q_1 q_2}{r^2}\ \hat{r}_{12}$ (vector equation)........ 1.



Similarly $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\ \frac{q_1 q_2}{r^2}\ \hat{r}_{21}$ 2

Since $\hat{r}_{21} = -\ \hat{r}_{12}$ $\therefore$ $\vec{F}_{21} = -\ \vec{F}_{12}$

Electrostatic Force between two point charges in terms of their position vectors.

(i).Let there be two point charges q1 and q2 at points A & B in vacume. With reference to an origin O let their

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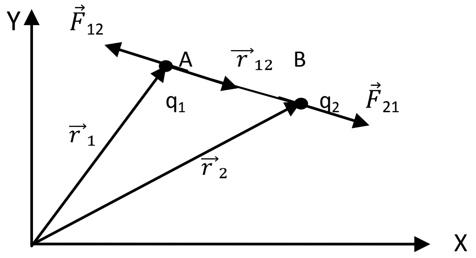
position vectors be $\vec{r}_1$ (OA) and $\vec{r}_2$ (OB). Then AB=$\vec{r}_{12}$. According to triangle law of vectors :

$\vec{r}_1 + \vec{r}_{12} = \vec{r}_2$ $\therefore$ $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ and

$\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$



(ii) According to Coulumb's law, the Force $\vec{F}_{12}$ exerted on q_1 by q_2 is given by : $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_{21}|^2} \hat{r}_{21}$ where $\hat{r}_{21}$ is a unit vector pointing from q_2 to q_1 . We know that $\hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|} = \frac{(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|}$

Hence, general Vector forms of Coulumb's equation is

$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} (\vec{r}_1 - \vec{r}_2)$ and

$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^2} (\vec{r}_2 - \vec{r}_1)$

Comparison of Electrostatic and Gravitational Force

1. Identical Properties :
   - Both the forces are central forces, i.e., they act along the line joining the centers of two charged bodies.
   - Both the forces obey inverse square law, F$\propto \frac{1}{r^2}$
   - Both are conservative forces, i.e. the work done by them is independent of the path followed.
   - Both the forces are effective even in free space.
2. Non identical properties :

   a. Gravitational forces are always attractive in nature while electrostatic forces may be attractive or repulsive.

   b. Gravitational constant of proportionality does not depend upon medium, the electrical constant of proportionality depends upon medium.

   c. Electrostatic forces are extremely large as compared to gravitational forces

Qn. Compare electrostatic and gravitational force between one electron and one proton system.

Ans : $F_e = \frac{1}{4\pi\varepsilon_0} \frac{e.e}{r^2} = 9\text{x}10^9 \frac{(1.6x10^{-19})^2}{r^2}$ Newton

$F_g = G\frac{me\ x\ mp}{xr^2} = 6.67\text{x}10^{-11} \frac{(9.1\ x\ 10^{-31})\ x\ (1.67\ x\ 10^{-27})}{r^2}$ Newton

$F_e / F_g = 2.26 \text{ x } 10^{39}$

Principle of Superposition of Charges :

If a number of Forces F_{11}, F_{12},F_{13},......F_{1n} are acting on a single charge q_1 then charge will experience force F_1 equal to vector sum of all these forces .

$F_1 = F_{11} + F_{12} + F_{13} + \ldots\ldots + F_{1n}$

The vector sum is obtained as usual by parallelogram law of vectors.

All electrostatics is basically about Coulomb's Law and Principle of superposition.

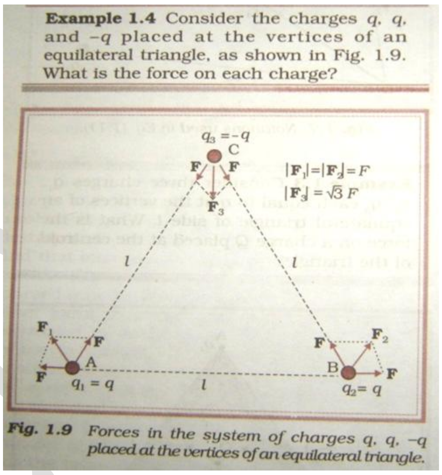

Example 1.4 Consider the charges q, q, and −q placed at the vertices of an equilateral triangle, as shown in Fig. 1.9. What is the force on each charge?

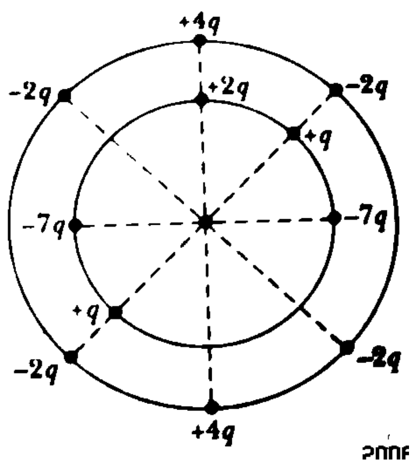
Fig. 1.9 Forces in the system of charges q, q, −q placed at the vertices of an equilateral triangle.

NUMERICALS FOR PRACTICE

1.How many electrons must be removed from the sphere to give it a charge of +2 μC . Is there any change in the mass when it is given this positive charge. How much is this change?

2. Two identical charged copper spheres A and B have their centers separated by a distance of 50 cm. A third sphere of same size but uncharged is brought in contact with the first, then brought in contact with the second and finally removed from both. What is the new force of repulsion between A and B?

3. A central particle of charge –q is surrounded by two circular rings of charged particles, of radii r and R, such that R > r. What are the magnitude and direction of the net electrostatic force on the central particle due to other particles.



4.-Three equal charges each of 2.0 x 10-6 are fixed at three corners of an equilateral triangle of side 5 cm. Find the coulomb force experienced by one of the charges due to other two.

5.

✶ --------o------------------------o---------

-3q -q

Above two charged particles are free to move. At one point, however a third charged particle can be placed such that all three particles are in equilibrium.

(a) is that point to the left of the first two particles, to their right, or between them?

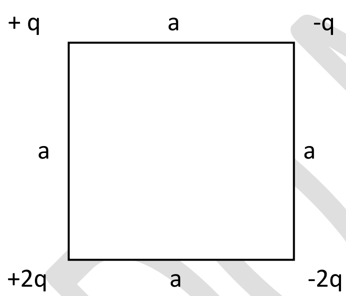
(b) Should the third particle be positively or negatively charged?

© Is the equilibrium stable or unstable?

6. A charge q is placed at the center of the line joining two equal charges Q. Show that the system of three charges will be in equilibrium if q = Q/4.

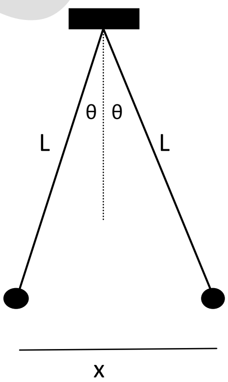
7. Two particles having charges 8q and –2q are fixed at a distance L. where, in the line joining the two charges, a proton be placed so that it is in equilibrium (the net force is zero). Is that equilibrium stable or unstable?

8. What are the horizontal and vertical components of the net electrostatic force on the charged particle in the lower left corner of the square if q = 1.0 x 10^7C and a = 5.0 cm?

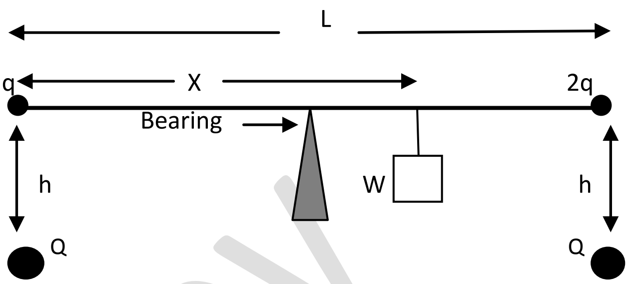


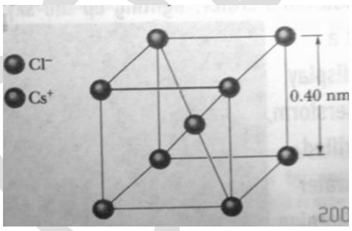
9. Two tiny conducting balls of identical mass m and identical charge q hang from non conducting threads of length L. Assume that θ is so small that tan θ can be replaced by sin θ; show that, for equilibrium,

$$X = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}$$



.8.A long non-conducting massless rod of length L, pivoted at its centre and balanced with a block of weight W at a distance x from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges q and $2q$, respectively. A distance h directly beneath each of these spheres is a fixed sphere with positive charge Q. a. Find the distance x when the rod is horizontal and balanced. (b) What value should h have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?



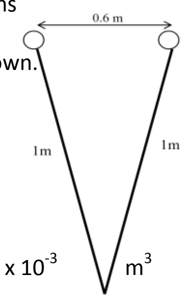


9. In the basic CsCl (Cesium chloride) crystal, Cs+ ions form the corners of a cube and a Cl- ion is at the centre of cube. Edge length is 0.40 nm.

(a) What is the magnitude of the net electrostatic force exerted on Cl- ion by the eight Cs+ ions.?

(b) If one of the Cs+ ion is missing the crystal is said to have defect. How much will be the force on chlorine ion in that case?

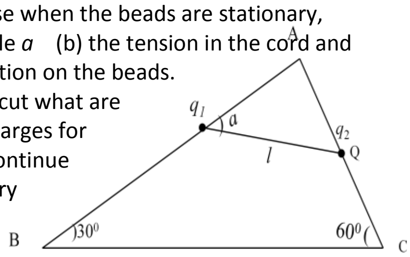
10. Two similar helium-filled spherical balloons tied to a 5 g weight with strings and each carrying a charge q float in equilibrium as shown. Find (a) the magnitude of q, assuming that the charge on each balloon is at its centre and (b) the volume of each balloon. Assume that the density of air =1.29 kg m^{-3} and the density of helium in the balloon is= 0.2 kg $^{m-3}$. Neglect the weight of the unfilled balloons. Ans: q = 5.5 x 10^{-7} V = 2.3 x 10^{-3} m^3



11. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30^0 with each other. When suspended in a liquid of density of 800 kg m^{-3}, the angle remain the same. What is the dielectric constant of the liquid? The density of the material of the sphere is 1600 kg m^{-3} Ans : K = 2

12. A rigid insulated wire frame in the form of a right angled triangle ABC, is set in a vertical plane. Two beads of equal masses *m* each and carrying charges q_1 q_2 are connected by a cord of length *l* and can slide without friction on the wires. Considering the case when the beads are stationary, determine (a) angle *a* (b) the tension in the cord and (c) the normal reaction on the beads.

If the cord is now cut what are the value of the charges for which the beads continue to remain stationary



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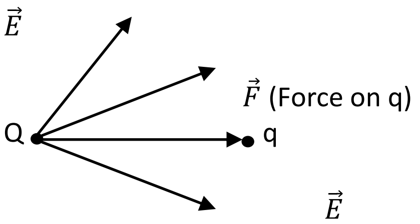
ELECTRIC FIELD

ELECTRIC FIELD-is the environment created by an electric charge (source charge) in the space around it, such that if any other electric charges(test charges)is present in this space, it will come to know of its presence and exert a force on it.



INTENSITY (OR STRENGTH) OF ELECTRIC FIELD AT A LOCATION Is the force exerted on a unit charge placed at that location

: if intensity of electric field at a location is E and a charge 'q' is placed ,then force experienced by this charges F is

$$\vec{F} = q.\vec{E} \quad \text{———} 1$$

or

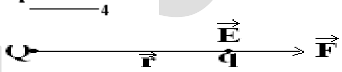
$$\boxed{\vec{E} = \frac{\vec{F}}{q}} \quad \text{———} 2$$

Direction of force F is in direction of electric field E

$$F = \frac{1}{4\pi E}\frac{Qq}{r^2}\hat{r} \quad \text{———} 3$$

By equ.1and 3 : Intensity of electric field due to Source charge Q is

$$\vec{E} = \frac{1}{4\pi E}\frac{Q}{r^2}\hat{r} \quad \text{———} 4$$



By coloumb's law we know that in similar situation if q=1 then

$$\vec{E} = \frac{1}{4\pi E}\frac{Q}{r^2}$$

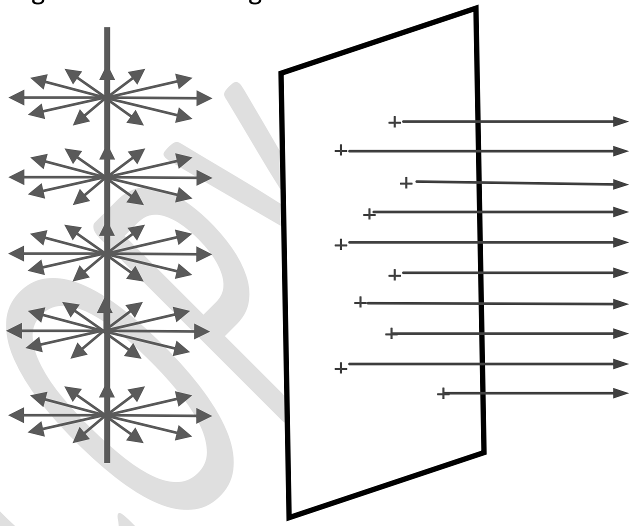
Relation in F, E and Test charge q is $E = \frac{F}{q}$

DISTRIBUTION OF CHARGE

Electric charge on a body may be concentrated at a point, then it is called a 'point charge'. If it is distributed all over, then it is called distribution of charge. Depending on shape of it is given different names

1.<u>Linear distribution</u>: when charge is evenly distributed over a length. In such case we use a quantity <u>Linear charge density λ.</u> Which has relation

$\lambda = \frac{Q}{L}$, Where 'Q' is charge distributed over a long conductor of length 'L'



2- <u>Areal distribution:</u> charge is evenly distributed over a surface area,S.

The surface charge density is 'σ' given by $\sigma = \frac{Q}{S}$

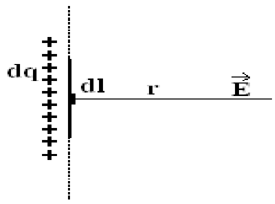
Where Q is charge given to a surface of area 'S'.

3-<u>volumetric distribution:</u> charge is evenly distributed throughout the body having volume 'V'Volumetric charge density is 'ρ' $\rho = \frac{Q}{V}$

<u>GENERAL DISTRIBUTION OF ELECTRIC FIELD DUE TO DIFFERENT DISTRIBUTION OF CHARGES</u>

1-Due to point change Q $\quad \vec{E} = \frac{1}{4\pi E}\frac{Q}{r^2}\hat{r}$

2-E due to linear distribution of electric charge



$$dq = \lambda.dl$$

$$dE = \frac{1}{4\pi E_0}\frac{\lambda.dl}{r^2}$$

$$\boxed{E = \frac{1}{4\pi E_0}\int_L \frac{\lambda.dl}{r^2}}$$

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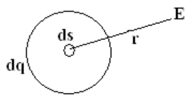
3 - E due to areal distribution of charge:

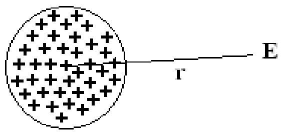


$dq = \sigma . ds$

$$dE = \frac{1}{4\pi E_0} \frac{\sigma . ds}{r^2}$$

$$\boxed{E = \frac{1}{4\pi E_0} \int_s \frac{\sigma . ds}{r^2}}$$

4- E due to volumetric distribution of charge



SOLID SPHERE VOLUME V

$$\boxed{E = \frac{1}{4\pi E_0} \int_V \frac{\rho . ds}{r^2}}$$

DIPOLE

1-Dipole is a system of two equal and opposite charges at finite & fixed distance.

example: molecule of electrolytic compounds.

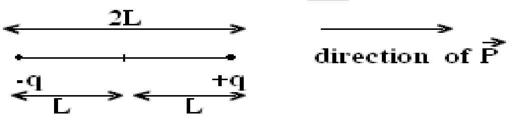
Example - HCl, H_2O.

2-CO_2 & CH_4 are non-polar because centers of –ve & +ve charges co-incide and there is no distance between them.

3-if non polar atom is placed in an elect.field a distance is created between +ve & -ve charge: it become polar.

Dipole moment:-the effectiveness or strength of a dipole is measured by the physical quantity .Dipole moment $\vec{P}$. it is calculated as $\vec{P}$= q x 2$\vec{L}$

P=q x 2L(magnitude) or $\vec{P}$= q x 2$\vec{L}$ (vector)



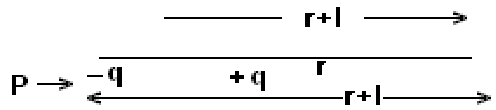
<u>Where 'q' is each charge and '2L' is distance between them.(each charge is at a distance L from 'center' of dipole)</u>

Dipole moment $\vec{P}$= q x 2$\vec{L}$ is a vector quantity it has magnitude p=2qL

And its direction is along line from –q to +q.

ELECTRIC FIELD DUE TO DIPOLE

ON THE AXIAL LINE



E DUE TO +q $\vec{E}_1 = \frac{q}{4\pi\epsilon_0 (r+l)^2} \hat{r}$

ALONG $\vec{P}$

E DUE TO –q $\vec{E}_1 = \frac{-q}{4\pi\epsilon_0 (r+l)^2} \hat{r}$

OPPOSITE TO $\vec{P}$

NET ELECTRIC FIELD

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(r-l)} \frac{1}{(r+l)^2}\right)$$

$$= \frac{q}{4\pi\epsilon_0} \frac{4rl}{(r^2 l^2)} \qquad \left\{2ql = P\right\}$$

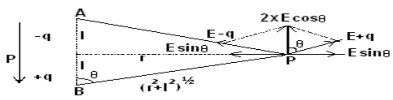
$$\boxed{E = \frac{2P.r}{4\pi\epsilon_0 (r^2 l^2)^2}}$$

SINCE $\vec{E}_1 > \vec{E}_2$

: $\vec{E}$ IS IN THE DIRECTION OF $\vec{P}$

IF R>>L THE, $E = \frac{2P}{4\pi\varepsilon_0 r^3}$

2 $\vec{E}$ ON EQUATORIAL LINE (TRANSVERAL LINE)



E due to +q , $E_{+q} = \frac{q}{4\pi\epsilon_0 (r^2 l^2)} \hat{BP}$

E due to -q $E_{-q} = \frac{q}{4\pi\epsilon_0 (r^2 l^2)} \hat{PA}$

|E+q| = |E-q| = Eq

each Eq is resolved in two direction. One along equatorial line and other in axial directions which are the Esinθ and normal direction E cosθ .

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Esinθ in opposite direction cancel each other while E cosθ add up to two.

: net electric field E = 2E cosθ

E(net) = 2Ecosθ

$$= 2 \cdot \frac{q}{4\pi\epsilon_0 (r^2+l^2)} \cdot \frac{l}{(r^2+l^2)^{1/2}}$$

$$E = \frac{P}{4\pi\varepsilon_0\,(r^2+l^2)^{3/2}}$$

IF R>>L Then, $E = \dfrac{P}{4\pi\varepsilon_0 r^3}$

The direction is opposite to that of $\vec{P}$

<u>Electric Field at equatorial line is half of the field on axial line in strength and opposite in direction</u>.

z

Electric Field Intensity due to a Short Electric Dipole at some General Point

(i) Let AB be a short electric dipole of dipole moment $\vec{p}$ (directed from B to A). We are interested to find the electric field at some general point P whose polar coordinates are (r, θ). The distance of observation point P w.r.t. mid point O of the dipole is r and the angle made by the line OP w.r.t. axis of dipole is θ.

(ii) We know that dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components $\vec{p}_1$ and $\vec{p}_2$ as shown in Fig. 27, so that $\vec{p} = \vec{p}_1 + \vec{p}_2$.

The magnitudes of $\vec{p}_1$ and $\vec{p}_2$ are $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$.

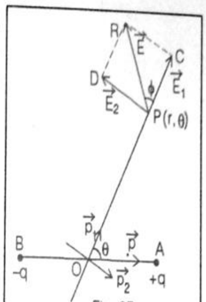

Fig. 27

(iii) It is clear from figure that point P lies on the axial line of dipole with moment $\vec{p}_1$. Hence magnitude of the electric field intensity $\vec{E}_1$ at P due to $\vec{p}_1$ is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p\cos\theta}{r^3} \quad (\text{along } \vec{p}_1). \qquad \text{........(1)}$$

Similarly, P lies on the equatorial line of dipole with moment $\vec{p}_2$. Hence, magnitude of electric field intensity $\vec{E}_2$ at P due to $\vec{p}_2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\sin\theta}{r^3} \quad (\text{opposite to } \vec{p}_2) \qquad \text{........(2)}$$

Hence resultant intensity at P is : $\vec{E} = \vec{E}_1 + \vec{E}_2$

Magnitude of $\vec{E}$ is : $E = \sqrt{(E_1^2 + E_2^2)}$ (as $\vec{E}_1$ and $\vec{E}_2$ are mutually perpendicular).

or $$E = \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

or $$\boxed{E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1+3\cos^2\theta}} \qquad \text{....(3)}$$

(iv) If the resultant field intensity vector $\vec{E}$ makes an angle ϕ with the direction of $\vec{E}_1$, then

$$\boxed{\tan\phi = \frac{E_2}{E_1} = \frac{(p\sin\theta/4\pi\varepsilon_0 r^3)}{(2p\cos\theta/4\pi\varepsilon_0 r^3)} = \frac{1}{2}\tan\theta}$$

<u>Electric Line of Force :</u>

The idea of Lines of Force was given by Michel Faraday. These are imaginary lines which give visual idea of Electric field, its magnitude, and direction.

A line of force is continuous curve the tangent to which at a point gives the direction of Electric field, and its concentration gives the strength of Field.

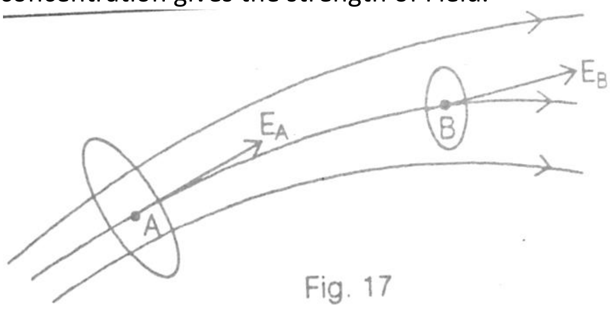

Fig. 17

Electric Field at A is stronger than field at B.

Properties of Electric Lines of Force :

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Electric Lines of Force :

1.start from positive charge and end at negative.

2.Electric Lines of forces are **imaginary** but Electric field they represent is **real.**

3.The tangent drawn at any point on the line of force gives the direction of force acting on a positive charge at that point.

4.**In SI system, the number of electric lines originating or terminating on charge q is q/ε_0 . That means lines associated with unit charge are $1/\varepsilon_0$**

5.Two lines of force never cross each other, because if they do so then at the point of intersection, intensity will have two directions which is absurd.

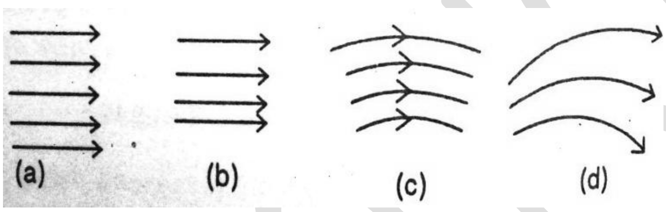
6. Electric Lines of force can never be a closed loop since they do not start and end at the same point. The lines are discontinuous, start from + and terminate at –

7. The electric line of force do not pass through a conductor as electric field inside a conductor is zero.

8. Lines of force have tendency to contract longitudinally like a stretched string, producing attraction between opposite charges and edge effect.

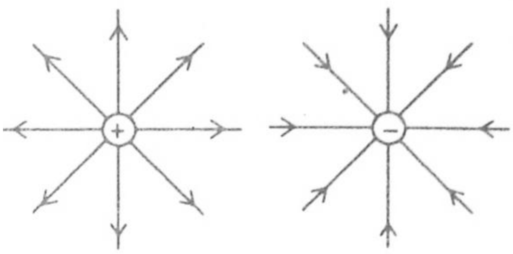
9.Electric Lines of force start and end **Normal to the surface** of conductor.

10. Crowded lines represent strong field while distant lines represent weak field. Equidistant parallel lines represent uniform field. Non-straight or non- parallel represent non-uniform field. In the diagram a is uniform while b, c, and d are non-uniform fields.

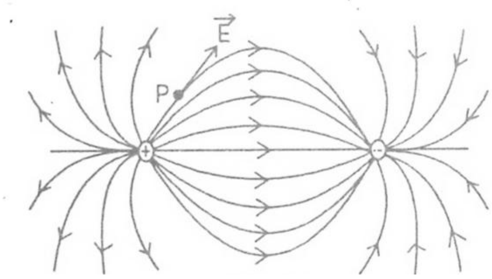


Field Lines due to some charge configurations.

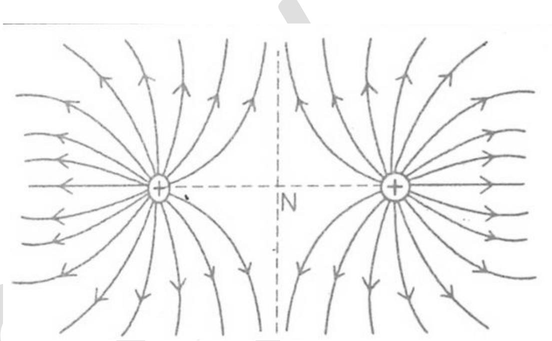
1.Single positive or negative charge



Two equal and opposite charges :



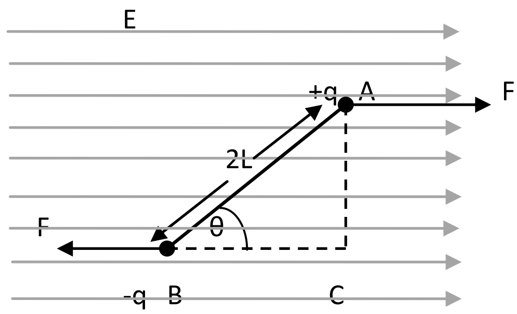
Lines of force due to Two positive charges



Elect field lines due to straight line distribution :
And Electric field lines due to very large sheet of charge are shown in the previous page.

Electric dipole in electric field

When a dipole is placed in an electric field each charge experience a force (F=qe) . Positive, in the direction of field and negative, opposite to direction of field.



Net Force on dipole : F + (-F) = 0 zero

Hence dipole will not make any linear motion.

Torque on dipole: A couple of force is acting on the body of dipole system at different points, the forces are equal and opposite in uniform field. Hence they form a couple of forces which create a **torque.** Therefore dipole is capable of rotation in a uniform electric field. The moment of forces or Torque is

$\tau = F \times AC = qE\times 2L\sin\theta = 2qL\,E\,\sin\theta = \mathbf{PE\sin\theta}$

or $\vec{\tau} = P \times E$

NOTE :

1.Direction of torque is normal to the plane containing dipole moment P and electric field E and is governed by right hand screw rule.

2. If Dipole is parallel to E the torque is **Zero**.

3. Torque is **maximum** when Dipole is perpendicular to E and that torque is PE

4. This equation gives the definition of dipole moment. If E is 1 N/C then P=T.

Therefore; **Dipole Moment of a dipole is equal to the Torque experience by that dipole when placed in an electric field of strength 1 N/C at right angle to it.**

5. If a dipole experiencing a torque in electric field is allowed to rotate, then it will rotate to align itself to the Electric field. But when it reach along the direction of E the torque become zero. But due to inertia it overshoots this equilibrium condition and then starts oscillating about this mean position.

6.*Dipole in Non-Uniform Electric field* :

In case Electric field is non-uniform, magnitude of force on +q and –q will be different, hence a net force will be acting on centre of mass of dipole and it will make a linear motion. At the same time due to couple of forces acting, a torque will also be acting on it.

Work done in rotating a dipole in a uniform Electric field:

1.If a dipole is placed in a uniform electric field experience a torque. If it is rotated from its equilibrium position, work has to be done on it. If an Electric dipole with moment P is placed in electric field E making an angle α, then torque acting on it at that instant is

τ = PESinα

2. If it is rotated further by a small angle dα then work done dw = (PEsinα).dα

Then work done for rotating it through an angle θ from equilibrium position of angle 0 is :-

$$W = \int_0^\theta (\text{PEsin}\alpha).d\alpha = \text{PE}[-Cos\ \alpha]_0^\theta$$

Or, **W** = PE $[-cos\theta + \cos 0]$ = **pE** $[\mathbf{1 - Cos\ \theta}]$

3.If a dipole is **rotated through 90^0** from the direction of the field, then work done will be

$$\boxed{W = pE\,[1 - \text{Cos } 90] = pE}$$

4. If the dipole is **rotated through 180^0** from the direction of the field, then work done will be :

$$\boxed{W = pE\,[1 - \text{Cos } 180] = 2\,pE}$$

<u>Potential Energy of a dipole kept in Electric field :</u>

1. dipole in Equilibrium (P along E):-

A dipole is kept in Electric field in equilibrium condition, dipole moment P is along E

To calculate Potential Energy of dipole we calculate work done in bringing +q from zero potential i.e.∞ to location B, and add to the work done in bringing –q from ∞ to position A.

1.The work done on –q from ∞ up to A

= -(*Work done up to B* + Work done from B to A)

2.Work done on +q = +(*Work done up to B*)

Adding the two

Total work done = Work done on –q from B to A

= Force x displacement

= -qE x 2L = - 2qLE

=- P.E

This work done convert into Potential Energy of dipole

$$\mathbf{U = -\vec{P}\cdot\vec{E}}$$

If P and E are inclined at angle θ to each other then magnitude of this Potential Energy is

U = - P E Cos θ

<u>*Electric – Potential*</u>

(1) Electric Potential is characteristic of a location in the electric field. If a unit charge is placed at that location it has potential energy (due to work done on its placement at that location). This potential energy or work done on unit charge in bringing it from infinity is called potential at that point.

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(2) Potential – Difference (i) is the work done on unit charge for carrying it from one location to other location A.

$V_A \longleftarrow q\ V_\infty$

Potential at A --------------------------V_A

Energy with q at A is $q\ V_A$

Energy with Q at B is $q\ V_B$

Difference of Energy $U_A - U_B = q\,(V_A - V_B)$

Using work energy theorem . $W = q\,((V_A - V_B)$

Or, $V_A - V_B = W/q$ & $U_A - U_B = W$.

If $V_B = 0$ { At ∞ Potential $V = 0$, Inside Earth $V_E = 0$}

Then $V_A = W/q$

This equation gives definition of potential V at point A as under :-

"Potential of a point in electric field is the work done in bringing a unit charge from infinity (Zero potential) to that point, without any acceleration."

<u>Expression of potential at a point due to source charge Q :-</u>

Let there be a charge Q which creates electric field around it. Point P is at distance 'r" from it. Let's calculate potential at this point.

A test charge 'q' is moved for a small displacement dr towards Q.

Electric field due to Q at P, $E = \frac{Q}{4\pi\epsilon_0 r^2}$

To move it against this electrical force we have to apply force in opposite direction

Hence applied force $F = -\frac{Qq}{4\pi\epsilon_0 r^2}$

Work done in moving distance dr is $dw = -\frac{Qq}{4\pi\epsilon_0 r^2}dr$

Total work done in bringing the charge from distance ∞ to distance r is

$$W = -\int_\infty^r \frac{Qq}{4\pi\epsilon_0 r^2}dr$$

$$= -\frac{Qq}{4\pi\epsilon_0}\int_\infty^r \frac{dr}{r^2}$$

$$= -\frac{Qq}{4\pi\epsilon_0}\left[\frac{r^{-1}}{-1}\right]_\infty^r = \frac{Qq}{4\pi\epsilon_0 r}$$

$$W/q = \frac{Q}{4\pi\epsilon_0 r} \quad \text{OR} \quad \mathbf{V = \frac{Q}{4\pi\epsilon_0 r}}$$

Where Q is source charge, r is distance & V_r is potential at that point.

Basically V_r is also a "potential difference" between potential of this point P and Potential at ∞ (i.e., 0).

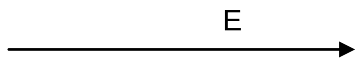
★★★★★★★★★★★

Relation between E & V

A test charge q is moved against E for a small distance dr. then work done dw by applied force -qE is $dw = -qE\ dr$

Or, $dw/q = -E\ dr$
Or, $\mathbf{dv = -E.dr}$
Or, $\mathbf{E = -dv/dr}$

Electric field is derivative of potential difference. –ve sign show that direction of E is opposite to direction of dv. I.e., dv decrease along the direction of E



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V_A $\qquad\qquad\qquad$ V_B

$V_A \quad > \quad v_B$

This also show that an electric charge experience force from high potential towards low potential if allowed to move, it will do so in this direction only.

If E and are not collinear and make angle θ between them, then according to relation of work & force

dv = - E dr Cos θ

Or, - dv / dr = E Cos θ

Or, dv = - E . dr

Or V = $\vec{E} \cdot \vec{dr}$

Or { Potential difference is a scalar quantity (work) given by dot product of two vector

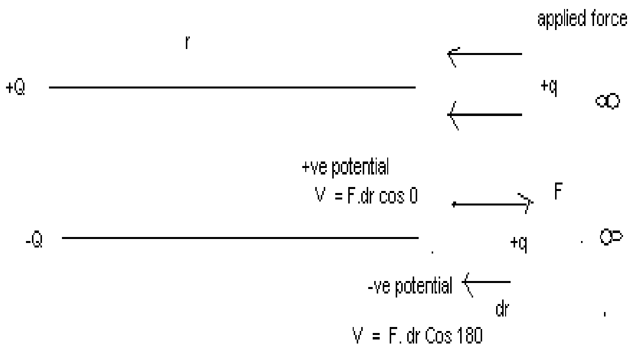
$\vec{E}$ & $\vec{dr}$.

Principle of super position:-

1) Potential at a point due to different charges is Algebric sum of potentials due to all individual charges.

V = V_1 + V_2 + V_3

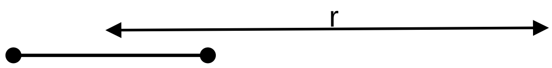
2) Potential due to +ve charge is +ve

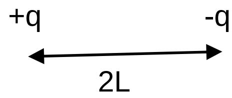


Potential due to –ve charge is –ve

Potential due to a dipole

1) At a point on axial line:-





At P – $\quad V_{+q} = \dfrac{Q}{4\Pi \in_0 (r-l)}$

$V_{-q} = \dfrac{Q}{4\Pi \in_0 (r+l)}$

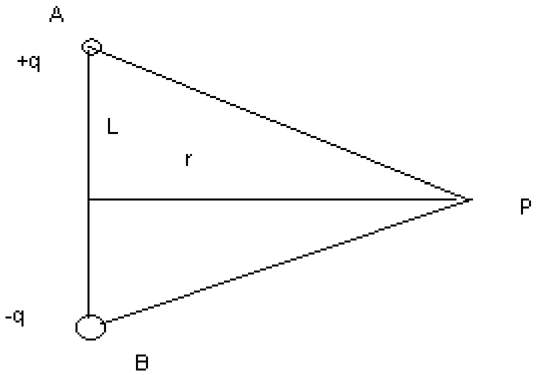
Total V =– V_{+q} + V_{-q} $= \dfrac{\boldsymbol{Q}}{\boldsymbol{4\pi\epsilon_0}}\left(\dfrac{1}{r-l} - \dfrac{1}{r+l}\right)$

$= \dfrac{2Ql}{4\Pi \in_0 (r^2 - l^2)} = \dfrac{P}{4\Pi \in_0 (r^2 - l^2\ )}$

If r > > L $\qquad$ Then V = $\dfrac{P}{4\Pi \in_0 r^2}$

2) <u>At a point on equatorial line</u>

- q & + q are placed at A & B. Point P is on equatorial



line

Every point on equatorial line is equidistant from +q & -q. Therefore +ve & -ve potential are equal **Hence net potential is zero.**

"Potential at every point on equatorial line of dipole is zero."

iii) **<u>Potential due dipole at any general point.</u>**

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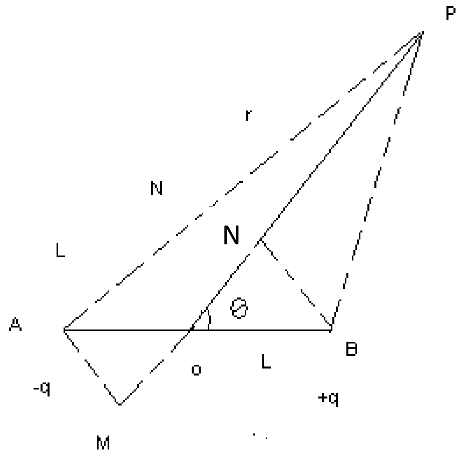


Draw normal from A & B on PO

PB ≈ PN = PO – ON = r – L Cos θ ------------ (i)

PA ≈ PM = PO + OM = r + L Cos θ ----------- (ii)

$$V_{+q} = \frac{Q}{4\Pi \in_0 PB} = \frac{Q}{4\Pi \in_0 (r - L\cos\theta)}$$

$$V_{-q} = \frac{-Q}{4\Pi \in_0 PA} = \frac{-Q}{4\Pi \in_0 (r + L\cos\theta)}$$

Total V = V_{+q} + V_{-q} =

$$\frac{Q}{4\Pi \in_0}\left(\frac{1}{r - LCos\theta} - \frac{1}{r - LCos\theta}\right)$$

$$= \frac{Q}{4\Pi \in_0}\left(\frac{r + L\cos\theta - r + LCos\theta}{r^2 - L^2Cos^2\theta}\right)$$

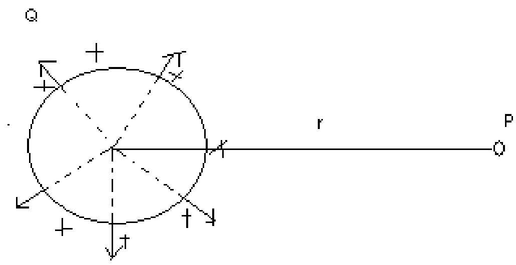
$$= \frac{QX\,2LCos\theta}{4\Pi \in_0 (r^2 - L^2Cos\theta)}$$

Or $V = \dfrac{PCos\theta}{4\Pi \in_0 (r^2 - L^2Cos\theta)}$

If r > > L

Then, Or, $V = \dfrac{PCos\theta}{4\Pi \in_0 r^2}$

Potential due to spherical shell



A spherical shell is given change Q. The electric field is directed normal to surface i.e., Radially outward. "Hence charge on the surface of a shell behaves as if all the charge is concentrated at centre.

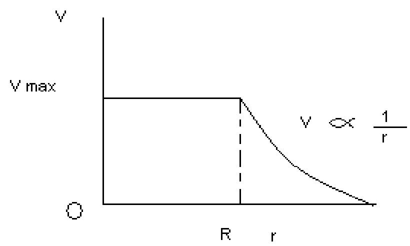
Hence potential at distance r is $V = \dfrac{Q}{4\Pi \in_0 r}$

Potential on the surface of shell $V = \dfrac{Q}{4\Pi \in_0 R}$

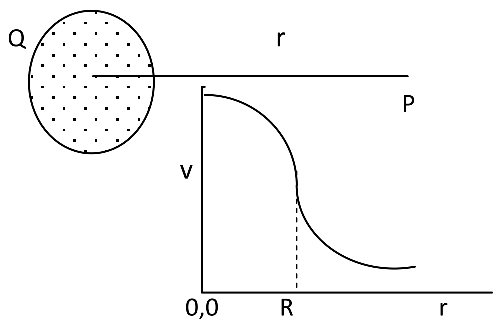
Inside shell Electric field is Zero.
Therefore change in potential dv = Zero X dr = 0 i.e., No change in potential. Hence potential inside a spherical shell is same as on the surface and it is same at every point.

It is $V = \dfrac{Q}{4\Pi \in_0 R}$ Where R is radius of shell.

Relation of V & r for spherical shell



In case of non-conducting sphere of charge. potential keeps on increasing up to centre as per diagram.



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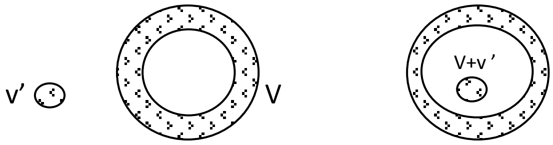
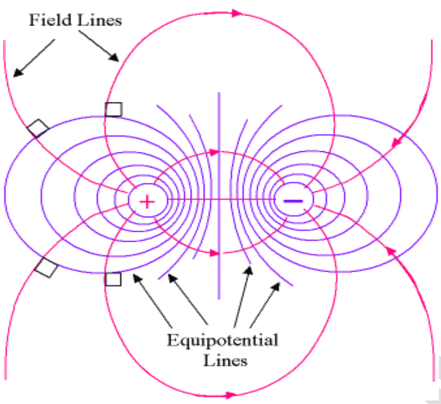
A body of potential v′ is placed inside cavity of shell with potential V then potential of the body become V+v′



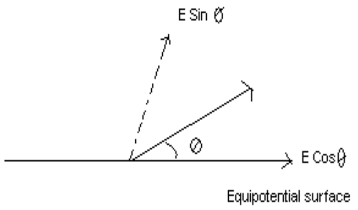
.Equipotential Surface

A real or imaginary surface in an electric field which has same potential at very point is an equipotential surface or simply, an equipotential.

Ex:- A shell having electric charge at its centre, makes an equipotential surface as it has same potential $\frac{Q}{4\Pi \in_0 R}$ at every point of the surface.



Electric lines of force and equipotential surface are at right angle to each other.



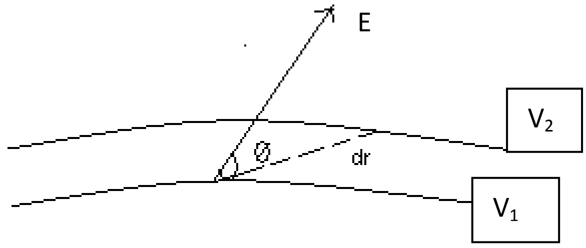
Proof:- Suppose E is not at right angle to equipotential surface, and makes angle θ with it. Then it has two components, E Cos θ along surface and E Sin θ normal to surface due to component E Cos θ , force q E Cos θ should be created on surface and it should move the charge. But we find that charges are in equilibrium. i.e.

E Cos θ = 0 ;

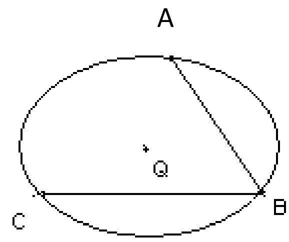
since E = $\not{0}$, therefore Cos θ = 0 or $\angle\ \theta = 90^0$

Hence E is always at right angle to equip. surface.

ii) $V_2 - V_1 = dv = -E \cos\theta \cdot dr$



iii) No work is done in carrying an electric charge from one point of E.P. Surface to other point (Whatever is the path)



Net work done in carrying change from A to B is Zero, B to C is Zero, because W = qV and V is same on this equipotential Surface

iv) Surface of a conductor in electrostatic field is always an equipotential surface.

Distribution of charge on uneven surface: - charge density is more on the surface which is pointed, or has smaller radius. Therefore if a conductor is brought near pointed charged surface, due to high density of charge induction will be more. Electric field set up will be very strong. This leads to construction of use of lightning arrester used on the buildings.

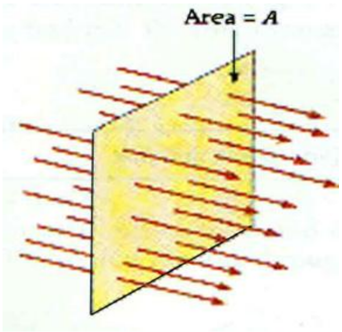
Gauss's Law

Electric Flux

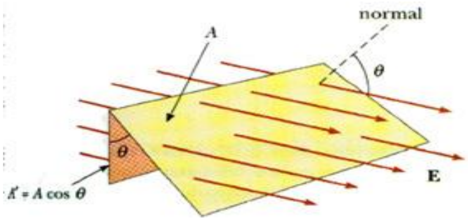
Think of air blowing in through a window. How much air comes through the window depends upon the **speed** of the air, the **direction** of the air, and the **area** of the window. We might call this air that comes through the window the "**air flux**".

We will define the **electric flux** $\Phi$for an electric field that is perpendicular to an area as

Φ= E A



If the electric field **E** is **not** perpendicular to the area, we will have to modify this to account for that.



Think about the "air flux" of air passing through a window **at an angle** θ. The "effective area" is A cos θ or the component of the velocity perpendicular to the window is v cos θ. With this in mind, we will make a general definition of the electric flux as

- Φ = E A cos θ
- Φ You can also think of the electric flux as the **number** of electric field lines that cross the surface.

Remembering the "dot product" or the "scalar product", we can also write this as

$$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}$$

where **E** is the electric field and **A** is a vector equal to the area A and in a direction **perpendicular** to that area. Sometimes this same information is given as

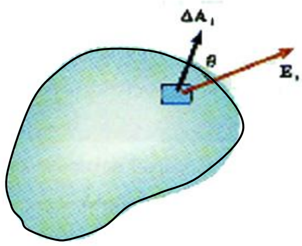
$$\mathbf{A} = A\,\mathbf{n}$$

where **n** is a **unit vector** pointing **perpendicular** to the area. In that case, we could also write the electric flux across an area as

$$\Phi = \mathbf{E} \cdot \mathbf{n}\, A$$

Both forms say the same thing. For this to make any sense, we must be talking about an area where the **direction** of **A** or **n** is constant.

For a curved surface, that will not be the case. For that case, we can apply this definition of the electric flux over a small area ΔA or **ΔA** or ΔA**n**.



Then the electric flux through that small area is ΔΦand

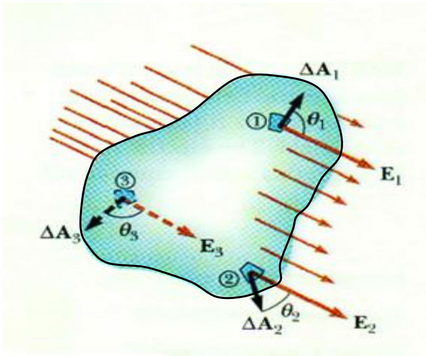
$\Delta\Phi = E\,\Delta A \cos\theta$ or

$$\Delta\Phi = \vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{A}}$$

To find the flux through all of a closed surface, we need to sum up all these contributions of ΔΦ over the entire surface,

$$\Phi_c = \oint \mathbf{E} \cdot \vec{d\mathbf{A}} = \oint E_n\, dA \text{ Cos}\theta$$

We will consider flux as **positive** if the electric field **E** goes from the inside to the outside of the surface and we will consider flux as **negative** if the electric field **E** goes from the outside to the inside of the surface. This is important for we will soon be interested in the **net** flux passing through a surface.



Gauss's Law : Total electric flux though a closed surface is $1/\varepsilon_0$ times the charge enclosed in the surface.

$$\Phi_E = q / \varepsilon_0$$

But we know that Electrical flux through a closed surface is $\oint \vec{E}.\vec{ds}$

$\therefore \oint \vec{E}.\vec{ds} = q / \varepsilon_0$

This is Gauss's theorem.

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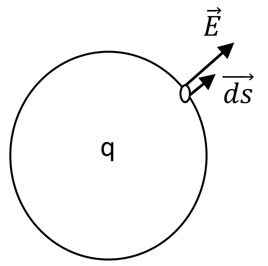
PROOF : Let's consider an hypothetical spherical surface having charge q placed at its centre. At every point of sphere the electrical field is radial, hence making angle 0 degree with area vector.



At the small area flux dϕ = $\oint \vec{E}.\vec{ds}$

= $\oint E.ds.Cos0°$

=$\oint \frac{q}{4\pi\epsilon_0 r^2}$ ds (E=$\frac{q}{4\pi\epsilon_0 r^2}$, Cos0=1)

= $\frac{q}{4\pi\epsilon_0 r^2}\oint ds$

For a sphere $\oint ds$ is 4πr^2.

∴ Φ=$\frac{q}{4\pi\epsilon_0 r^2}$ X 4πr^2.

Or, Φ = q / ε_0

This is Guass Theorem. (Hence proved)

Application of Gauss's Law

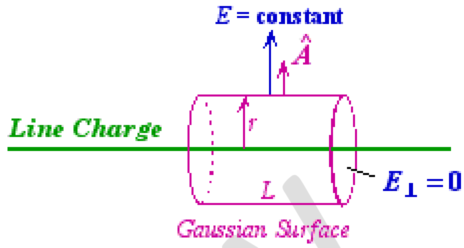
To calculate Electric Field due to different charge distributions.

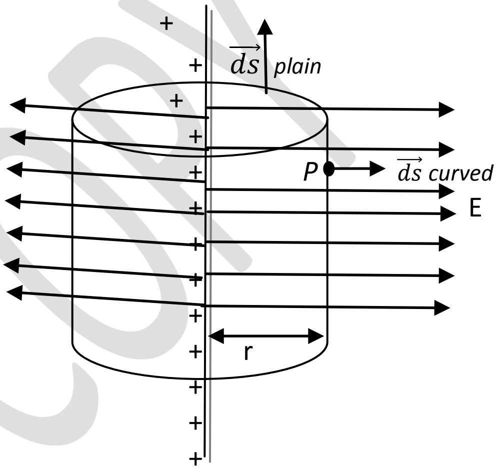
For this purpose we consider construction of a ***Guassian surface***.

Guassian Surface : It is an imaginary surface in the electric field which is
1.closed from all sides
2. Surface is Symmetrical about the charges in it
3. Electric field $\vec{E}$ on the surface is symmetrical

Electric field due to line charge :
Electric charge is distributed on an infinite long straight conductor with linear charge density λ. We have to find Electric field on a point *P* at normal distance r.

Consider a Gaussian Surface in the shape of a cylinder having axis along conductor. It has radius r so that point P lies on the surface. Let its length be *l.*
The electric field is normal to conductor, hence it is symmetrical to the surfaces of these cylinder.





Now $\oint \vec{E}.\vec{ds}$ = $\int \vec{E}.\vec{ds}$ *for curved surface* + $\int \vec{E}.\vec{ds}$ for 2 plane surfaces.
= $\int E.ds\ cos0 + \int E.ds\ cos90$
=E $\int ds$ *for curved surface* (E is uniform)
= E2πr*l*   ($\int ds$ = 2πrl, for cylindrical curved surface)

The charge enclosed within Guassian surface =λ*l*

According to Gauss theorem : $\oint \vec{E}.\vec{ds}$ = q / ε_0

Putting values : E2πr*l* = λ*l* / ε_0

Or, $\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$

Electric field due to a plain surface : -

There is a very large plain surface having sueface density σ . There is a point P at normal distance r .

Let's consider a Gaussian surface, in shape of a cylinder which has axis normal to the sheet of charge and containing point P at its plain surface (radius a).

Electric field E is normal to the surface containing charge hence it is normal to the plain surface of cylinder and parallel to curved surface.

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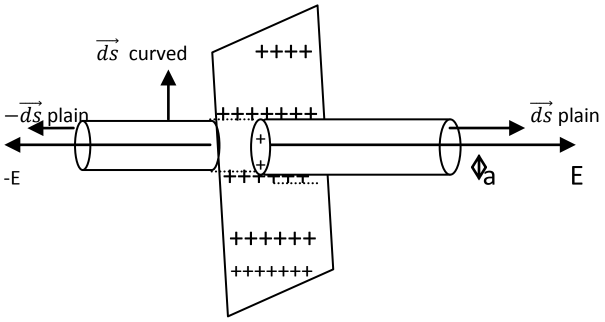


Now $\oint \vec{E}.\vec{ds}$ = $\int \vec{E}.\vec{ds}$ *for curved surface* + $\int \vec{E}.\vec{ds}$ for 2 plane surfaces.

= $\int E.ds\ cos90 + \int E.ds\ cos0 + \int -E.(-ds\ cos0)$

= *for plain surfaces* 2E $\int ds$ (E is uniform)

= 2Eπa^2

The charge enclosed inside Gaussian surface q = σ.A

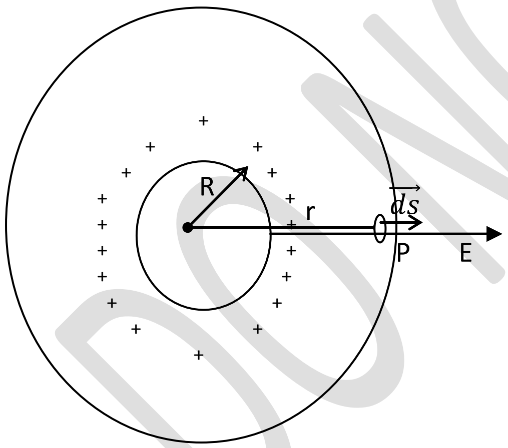
Or, q = σπa^2

Applying Gauss's Law : $\oint \vec{E}.\vec{ds}$ = q / ε_0

Putting values 2Eπa^2 = $\frac{\sigma\pi a^2}{\varepsilon_0}$

Or $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$

Electric Field due to charge distributed over a spherical shell :-



The spherical shell or spherical conductor has total charge q, surface charge density σ , radius R. We have to find Electric Field E at a point P at distance 'r '.

Case 1. If P is outside shell.

Let's assume a Gaussian surface, which is a concentric sphere of radius r and P lies on its surface.

Electric field is normal to surface carrying charge. Hence it is radially outward. Therefore for a small area on the Gaussian surface ds E is normal to surface i.e. angle between $\vec{ds}$ and $\vec{E}$ is 0.

Now $\oint \vec{E}.\vec{ds}$ = $\int \vec{E}.\vec{ds}$ for complete area of Gaussian surface

= $\int E.ds.Cos0$ = E$\int ds$ (E is uniform)

= E X 4πr^2. (for spherical shell $\int ds$ = 4πr^2)

Charge within Gaussian surface = q

Applying Gauss's Law : $\oint \vec{E}.\vec{ds}$ = q / ε_0

Putting values E X 4πr^2 = q / ε_0

Or $\boxed{E = \frac{q}{4\pi\epsilon_0 r^2}}$

This expression is same as electric field due to a point charge q placed at distance r from P. i.e. In this case if complete charge q is placed at the centre of shell the electric field is same.

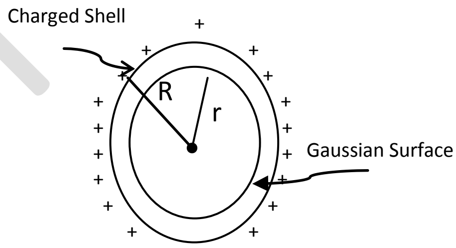
Case 2. If P is on the surface.

In above formula when r decrease to R the electric field increase.

On the surface *(replace r with R)* $\boxed{E = \frac{q}{4\pi\epsilon_0 R^2}}$

Hence this is electric field on the surface of a shell and its value is maximum compared to any other point.

Case 3. If P is within the surface. Or 'r'< R



Let's consider a Gaussian surface, a concentric spherical shell of radius r passing through P.

Then charge contained inside Gaussian surface is Zero.

According to Gauss's Theorem $\oint \vec{E}.\vec{ds}$ = q / ε_0

If q is zero then $\oint \vec{E}.\vec{ds}$ = 0.

As *ds* is not zero then $\boxed{E = 0}$

It is very important conclusion reached by Gauss's Law that Electric field inside a charged shell is zero.

The electric field inside conductor is Zero. This phenomenon is called ***electrostatic shielding***.

Variation of E with r (distance from centre)

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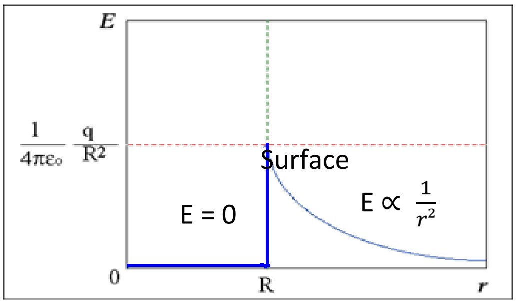
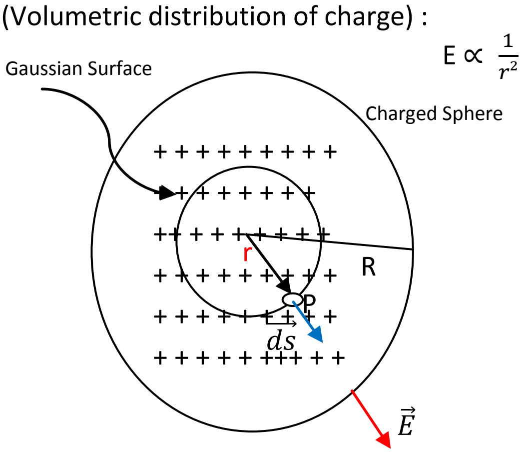
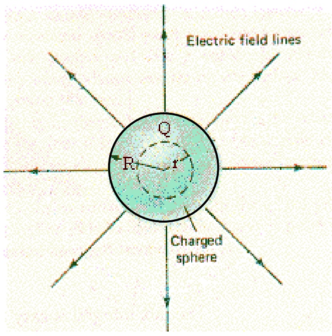


Electric Field due to (filled-up) sphere of charge

(Volumetric distribution of charge) :





Case I. When P is Out side sphere. Same as in the case of charged shell E = $\frac{q}{4\pi\epsilon_0 r^2}$

Case 2. When point P is on the surface of shell: Same as in case of shell . E = $\frac{q}{4\pi\epsilon_0 R^2}$

Case 3 If point P is inside the charged sphere.

Consider Gaussian surface, a concentric spherical shell of radius r, such that point P lies on the surface. Electric field is normal to the surface.

Now $\oint \vec{E}.\vec{ds}$ = $\int \vec{E}.\vec{ds}$ for complete area of Gaussian surface

= $\int E.ds.Cos0$ = E$\int ds$ (E is uniform)

= E X $4\pi r^2$. (for spherical shell $\int ds$ = $4\pi r^2$)

Charge within Gaussian surface = charge density x volume.

= $\rho\frac{4}{3}\pi r^3$ (where ρ is the charge per unit volume.)

Applying Gauss's Law $\oint \vec{E}.\vec{ds}$ = q / ε_0

Putting values E X $4\pi r^2$ = $\rho\frac{4}{3}\pi r$ / ε_0

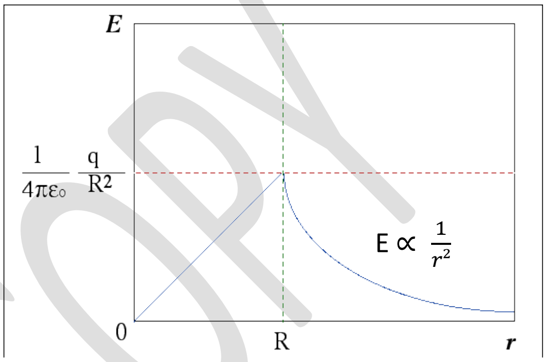
∴ $\boxed{E = \frac{\rho r}{3\varepsilon_0}}$

It shows that inside a sphere of charge, the electric field is directly proportional to distance from centre.

At centre r=0 ∴ E = 0

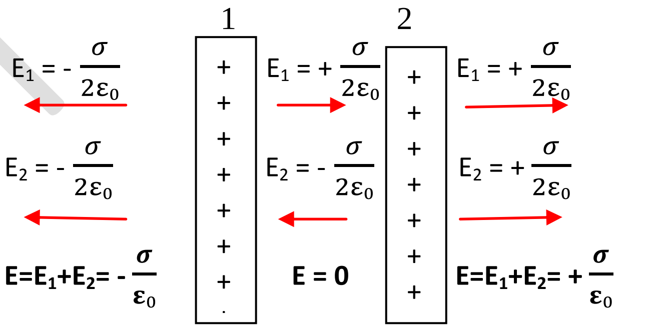
On the surface E = $\frac{\rho R}{3\varepsilon_0}$ = $\frac{q}{4\pi\epsilon_0 R^2}$ ($\rho = q / \frac{4}{3}\pi r^3$)

Variation of E with r (distance from centre)

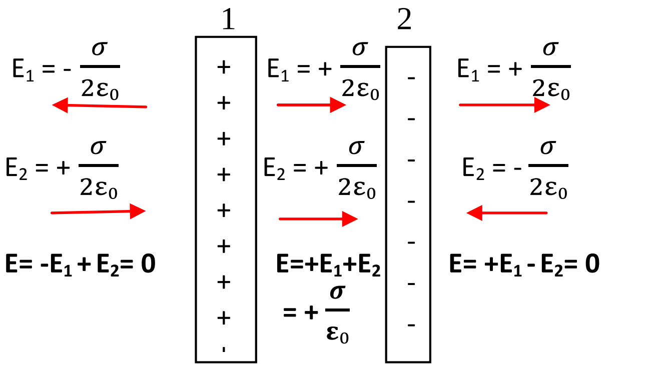


Electric field due to two charged parallel surface

Charges of similar nature



a. Charges of opposite nature :-



<u>Equipotential Surface :</u>

<u>Energyof a charged particle in terms of potential:-</u>

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Work required to bring a charge q at a point of potential V is W = qV. This work done on the charged particle converts to its potential energy.

Potential energy of charge q at potential V is **U = qV**

Electron-Volt : By relation Work/energy = qV, *the smallest unit of work/energy is Electron Volt.*

One electron volt is the work done by/on one electron for moving between two points having potential difference of one Volt.

1 eV = 1.6 x 10^{-19} Joules

Potential Energy of system of charges

(i) System of Two charges :

A B

q_1 -------- r ----------------q_2

Potential due to q1 at B is potential at distance r :

V = $\frac{q_1}{4\pi\varepsilon_0 r}$ ∴ Potential Energy of system U = $\frac{q_1 q_2}{4\pi\varepsilon_0 r}$

(ii) System of three charges

We make different pairs and calculate energy as under

U = $\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}}$

(iii) System of Four charges

Four charges make six pairs : Potential Energy U=

$\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\varepsilon_0 r_{13}} + \frac{q_1 q_4}{4\pi\varepsilon_0 r_{14}} + \frac{q_2 q_3}{4\pi\varepsilon_0 r_{23}} + \frac{q_2 q_4}{4\pi\varepsilon^0 r_{24}} + \frac{q_3 q_4}{4\pi\varepsilon_0 r_{34}}$

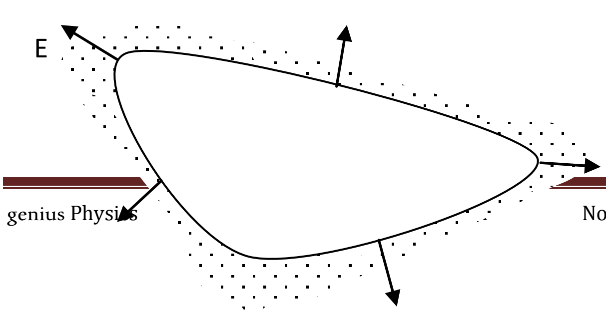
The energy is contained in the system and not by any one member. But it can be used by one or more members.

Distribution of charge on irregular shaped conductors :

Potential at each point is equal.

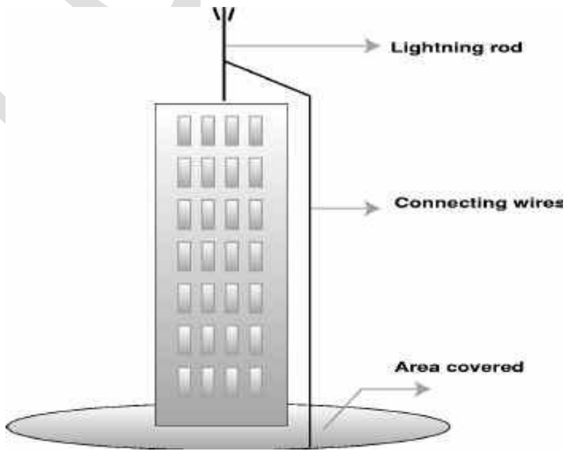
Electric field is always normal to surface.

Charge is distributed **unevenly**. Charge per unit area is more at the surface which has smaller radius. Therefore charge density is always more on the corners.



Corona discharge : when an uncharged body is brought near a charged body having sharp corners there is large number of charges at the corners. Due to induction, they induce large number of opposite charges. This creates a very strong Electric field between them. Finally the dielectric strength breaks-down and there is fast flow of charges. This Spray of charges by spiked object is called Corona discharge.

The lightening arrester work on the principles of Corona discharge where the charge pass through conductor of arrester, and the buildings are saved



Van-de-Graff generator

Introduction : It's a device used to create very high potential which is used for experiments of nuclear physics in which a charged particle with very high energy is required to hit the nucleus as target.

Principles : The following principles are involved in the device.

1.Charge on a conductor always move to and stay on the outer surface.

2.Pointed Corners conduct charges very effectively. (corona discharge)

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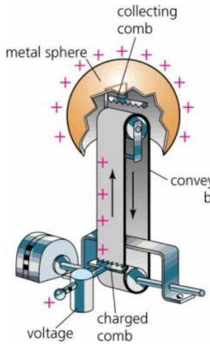
3.If charge q is given to a body, its potential increases by relation V=$\frac{q}{4\pi\varepsilon_0 r}$

4. If a body of small potential v' is placed inside a shell having potential V, then the body acquires potential V+v'



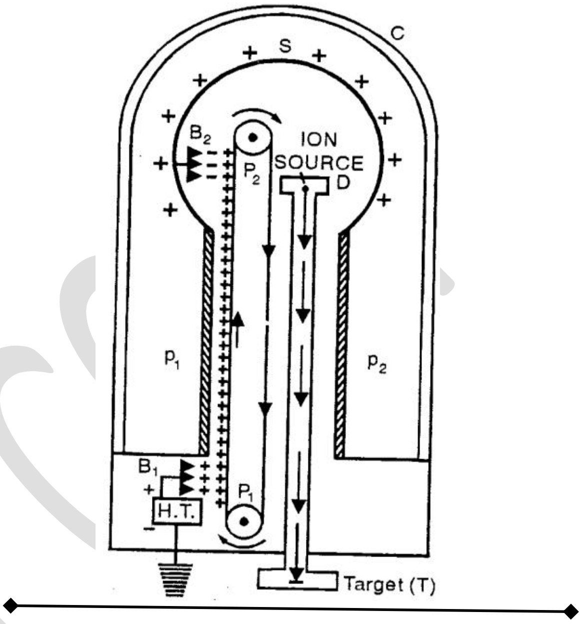
Description : There is a large spherical conducting shell of diameter of few meters placed on a non-conducting concrete structure few meters above the ground.

A long belt of insulating material like silk rubber or rayon moves around two pulleys, driven by a motor.

Two combs with pointed heads near belt are fitted. Lower one is spray comb and the upper Collecting Comb. The spray comb is connected with a high tension source.

There is a discharge tube. One end having source of ion to be accelerated is inside the shell. Target is placed at the other end connected to earth.

The whole system is enclosed in a steel chamber filled with nitrogen or methane at high pressure.

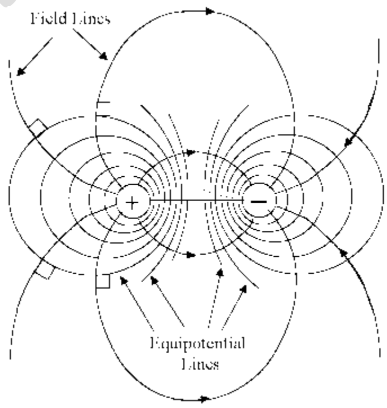


Working : The spray comb is given a positive potential ($\approx 10^4$Volt) w.r.t. earth by the source of high Tension. Due to sharp points there is spray of charge on belt. The belt moves up with power of motor. When the charges reach near upper comb, due to induction and corona discharge the charge on belt is transferred to comb. From comb it moves to inner layer of shell. Since charge always stay at the outer surface, it moves to outer surface and the inner surface again become without any charge, ready to receive fresh charge again. As shell receive charge it Potential increase according to relation V=$\frac{q}{4\pi\varepsilon_0 r}$. This potential is distributed all over and inside the shell.

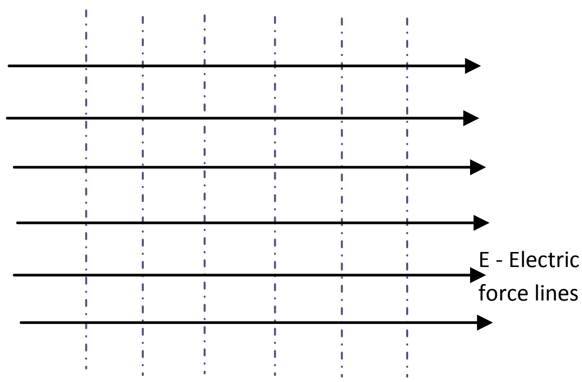
The new charged particles which are coming having small potential v' from lower comb, acquire potential V+v' due to their position inside the shell. There new potential is slightly higher than shell, therefore charges move from belt to comb to shell. This increases V further. This process keeps on repeating and V increase to a very high value, that is break-down voltage of compressed nitrogen $\approx 10^7$volt.

The ion inside discharged plate also acquires this potential due to its location inside the shell. Its energy increases by relation U = qV. The target is connected to earth at zero potential. Hence this ion gets accelerated and hits the target with very high energy.

Relation between Equipotential surfaces and E-Lines



Equi potential lines



CAPACITOR

It is a device to store charge and in turn store the electrical energy.

Any conductor can store charge to some extent. But we cannot give infinite charge to a conductor. When charge is given to a conductor its potential increases. But charge cannot escape the conductor because air, or medium around conductor is di-electric.

When due to increasing charge the potential increase to such extent that air touching the conductor starts getting ionized and hence charge gets leaked. No more charge can be stored and no more potential increase. This is limit of charging a conductor.

The electric field which can ionize air is 3 x 10^9 vm^{-1}.

CAPACITANCE OF A CONDUCTOR

Term capacitance of a conductor is the ratio of charge to it by rise in its Potential

$$C = \frac{q}{V}$$

In this relation if V=1 then C= q. Therefore ,

Capacitance of a conductor is equal to the charge which can change its potential by one volt.

Unit of capacitance : Unit of capacitance is farad, (symbol F).

One farad is capacitance of such a conductor whose potential increase by one volt when charge of one coulomb is given to it.

One coulomb is a very large unit. The practical smaller units are
i. Micro farad (µF) = 10^{-6}F.(used in electrical circuits)
Ii Pieco farad (pF) = 10^{-12} used in electronics circuits

Expression for capacitance of a spherical conductor :
If charge q is given to a spherical conductor of radius r, its potential rise by $V = \frac{q}{4\pi\epsilon_0 r}$
Therefore capacitance $C = \frac{q}{V} = q/\frac{q}{4\pi\epsilon_0 r} = 4\pi\epsilon_0 r$

Or for a sphere $\boxed{C = 4\pi\epsilon_0 r}$

The capacitor depends only on the radius or size of the conductor.

The capacitance of earth (radius 6400 km) is calculated to be 711 x 10^{-6} coulomb.

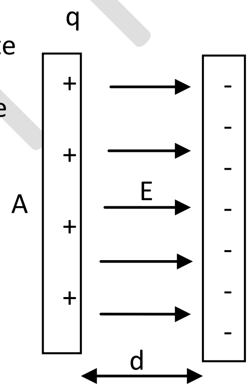
PARALLEL PLATE CAPACITOR : -

Since single conductor capacitor do not have large capacitance , parallel plate capacitors are constructed.

Principle : Principle of a parallel plate capacitor is that an uncharged plate brought bear a charged plate decrease the potential of charged plate and hence its capacitance ($C = \frac{q}{V}$) increase. Now it can take more charge. Now if uncharged conductor is earthed, the potential of charged plate further decreases and capacitance further increases. This arrangement of two parallel plates is called parallel plate capacitor.

Expression for capacitance :

Charge q is given to a plate Of area 'A'. Another plate is kept at a distance 'd'. After induction an Electric field E is set-up Between the plates. Here



$q = \sigma A$ and $E = \frac{\sigma}{\varepsilon_0}$

The Potential difference between plates is given by

$V = Ed = \frac{\sigma}{\varepsilon_0} d$

Now $C = \frac{q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} = \frac{\varepsilon_0 A}{d}$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

If a dielectric of dielectric constant K is inserted between the plates, then capacitance increase by factor K and become

$$\boxed{C = \frac{\varepsilon_0 K A}{d}}$$

Note : The capacitance depends only on its configuration i.e. plate area and distance, and on the medium between them.
The other examples of parallel plate capacitors is

Cylindrical capacitor $C = \frac{4\pi\varepsilon_0 KL}{\log r^2/r_1}$

and Spherical capacitor. $C = \frac{4\pi\varepsilon_0 K r_2 r_1}{\log r_2 - r_1}$

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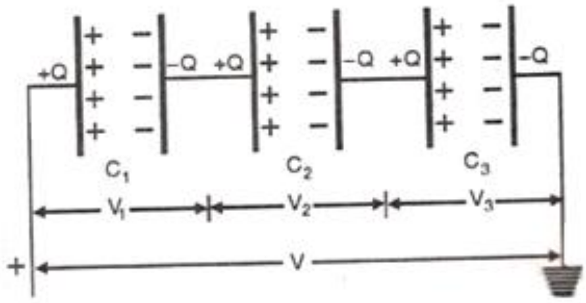
Combination of capacitors

Capacitors can be combined in two ways. 1. Series and 2. Parallel.

Series Combination :

If capacitors are connected in such a way that we can proceed from one point to other by only one path passing through all capacitors then all these capacitors are said to be in series.



Here three capacitors are connected in series and are connected across a battery of P.D. 'V'.

The charge q given by battery deposits at first plate of first capacitor. Due to induction it attract –q on the opposite plate. The pairing +ve q charges are repelled to first plate of Second capacitor which in turn induce -q on the opposite plate. Same action is repeated to all the capacitors and in this way all capacitors get q charge. As a result ; the charge given by battery q, every capacitor gets charge q.

The Potential Difference V of battery is sum of potentials across all capacitors. Therefore

$V = v_1 + v_2 + v_3$

$$v_1 = \frac{q_1}{c_1},\ v_2 = \frac{q_2}{c_2},\ v_3 = \frac{q_3}{c_3}$$

Equivalent Capacitance : The equivalent capacitance across the combination can be calculated as $C_e = q/V$

Or $1/C_e = V/q$

$= (v_1 + v_2 + v_3)/q$

$= v_1/q + v_2/q + v_3/q$

Or $\mathbf{1/C_e = 1/C_1 + 1/C_2 + 1/C_3}$

The equivalent capacitance in series decrease and become smaller then smallest member.

In series q is same. Therefore by q=cv, we have

$c_1v_1 = c_2v_2 = c_3v_3$

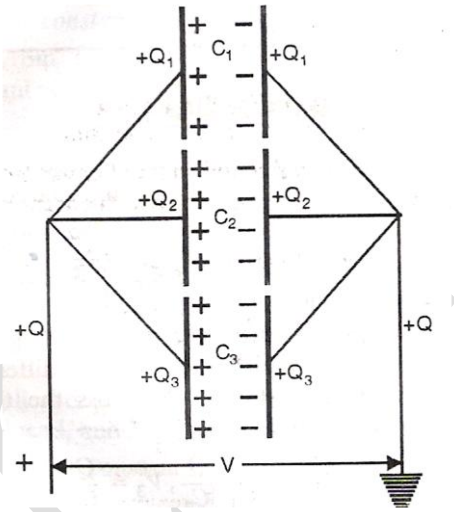
or $v \propto \frac{1}{c}$ i.e. larger c has smaller v, and smaller c has larger v across it.

For 2 capacitor system $C = \frac{c_1c_2}{c_1 + c_2}$, and $v_1 = \frac{c_2}{c_1 + c_2}.v$

If n capacitor of capacitance c are joint in series then equivalent capacitance $C_e = \frac{c}{n}$

Parallel combination :

If capacitors are connected in such a way that there are many paths to go from one point to other. All these paths are parallel and capacitance of each path is said to be connected in parallel.



Here three capacitors are connected in parallel and are connected across a battery of P.D. 'V'.

The potential difference across each capacitor is equal and it is same as P.D. across Battery.

The charge given by source is divided and each capacitor gets some charge. The total charge

$$q = q_1 + q_2 + q_3$$

Each capacitor has charge

<u>$q_1=c_1v_1$, $q_2=c_2v_2$, $q_3=c_3v_3$</u>

Equivalent Capacitance : We know that

$q = q_1 + q_2 + q_3$

divide by v $\frac{q}{v} = \frac{q_1}{v} + \frac{q_2}{v} + \frac{q_3}{v}$

or, $\mathbf{C = c_1+c_2+c_3}$

The equivalent capacitance in parallel increases, and it is more than largest in parallel.

In parallel combination V is same therefore

(v =) $\frac{q_1}{c_1} = \frac{q_2}{c_2} = \frac{q_3}{c_3}$

In parallel combination $q \propto c$. Larger capacitance larger is charge.

Charge distribution : $q_1=c_1v$, $q_2=c_2v$, $q_3=c_3v$.

In 2 capacitor system charge on one capacitor

$q_1 = \frac{c_1}{c_1+c_2+..}.q$

n capacitors in parallel give $C = nc$

Energy stored in a capacitor: When charge is added to a capacitor then charge already present on the plate repel any new incoming charge. Hence a new charge has to be sent by applying force and doing work on it. All this work done on charges become energy stored in the capacitor.

At any instant work done $dw = V.dq$, or $dw = \frac{q}{c}.dq$

Therefore work done in charging the capacitor from charge 0 to q $\quad W = \int_0^q \frac{q}{c}.dq$

$$= \frac{1}{c}\int_0^q q.dq = \frac{1}{c}\frac{q^2}{2} = \frac{q^2}{2c}$$

This work done convert into electrical Potential

Energy stored in the capacitor $\boxed{U = \frac{1}{2}\frac{q^2}{c} = \frac{1}{2}qv = \frac{1}{2}cv^2}$

This energy is stored in the form of Electric field between the plates.

Energy per unit volume $u = \frac{1}{2}cv^2/V = \frac{1}{2}\frac{\varepsilon_0 AE^2d^2}{dAd}$

Or, energy density $\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$

Connecting two charged capacitors :- When two conductors are connected the charges flow from higher potential plate to lower potential plate till they reach a common potential.

Common Potential : A capacitor of capacitance c_1 and potential v_1 is connected to another capacitor of capacitance c_2 and potential v_2. The charge flow from higher potential to lower potential and it reach an in between value V such that

$$V = \frac{total\ charge}{Total\ capacitance} \quad \text{or} \quad V = \frac{c_1v_1+c_2v_2}{c_1+c_2}$$

Loss of Energy on connecting two conductors :

A capacitor of capacitance c_1 and potential v_1 is connected to another capacitor of capacitance c_2 and potential v_2. The charge flow from higher potential to lower potential and in this process it looses some energy as charge has to do some work while passing through connecting wire. The energy is lost in form of heat of connecting wire.

Expression for energy lost : In the above two capacitors the energy contained in the two before connection, $E_1 = \frac{1}{2}c_1v_1^2 + \frac{1}{2}c_2v_2^2$ . . . . (i)

Common Potential after connection, $V = \frac{c_1v_1+c_2v_2}{c_1+c_2}$

Combined capacitance $c_1 + c_2$

Energy in combination : $\frac{1}{2}(c_1+c_2)\left(\frac{c_1v_1+c_2v_2}{c_1+c_2}\right)^2$

Hence Loss in energy : $E_1 - E_2$

$$= \left\{\frac{1}{2}c_1v_1^2 + \frac{1}{2}c_2v_2^2\right\} - \left\{\frac{1}{2}(c_1+c_2)\left(\frac{c_1v_1+c_2v_2}{c_1+c_2}\right)^2\right\}$$

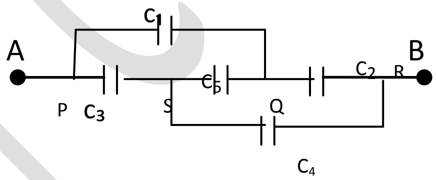
$$= \frac{1}{2}\left(\frac{c_1c_2}{c_1+c_2}\right)(v_1 - v_2)^2$$

It is a positive number which confirm that there is loss of energy in transfer of charges. Hence

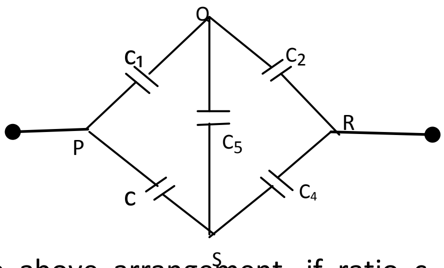
$$\boxed{\text{loss of energy} = \frac{1}{2}\left(\frac{c_1c_2}{c_1+c_2}\right)(v_1 - v_2)^2}$$

Wheatstone bridge in combination of capacitors :

Five capacitors joined in following manner is called wheatstone bridge connection.



Or, it is redrawn as under :



In the above arrangement, if ratio $c_1/c_2 = c_3/c_4$ then the bridge is said to be *balanced*. In such case the potential at point Q and S are equal.

The potential across c_5 is zero hence it does not carry any charge. In this way it is not participating in storage of charges. Then it can be omitted for further calculations. Calculations are done for c_1 c_2, c_3 and c_4 only.

Dielectrics: are non conducting materials. They do not have free charged particles like conductors have. They are two types.

i. Polar : The centre of +ve and –ve charges do not coincide. Example HCl, H_2O, They have their own dipole moment.

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ii. Non-Polar : The centers of +ve and –ve charges coincide. Example CO_2 , C_6H_6 . They do not have their own dipole moment.

In both cases, when a dielectric slab is exposed to an electric field, the two charges experience force in opposite directions. The molecules get elongated and develops i. surface charge density **σ_p** and not the volumetric charge density. This leads to development of an induced electric field **E_p** , which is in opposition direction of external electric field E_o . Then net electric field E is given by E = E_o - **E_p** . This indicates that net electric field is decreased when dielectric is introduced.

The ratio $\frac{Eo}{E}$ = K is called **dielectric constant** of the dielectric.
Clearly electric field inside a dielectric is $E=\frac{Eo}{K}$.

Dielectric polarization : when external electric field E_0 is applied , molecules get polarized and this induced dipole moment of an atom or molecule is proportionate to applied electric field. i.e. $p \propto E_o$

$$\text{or } p = \alpha\varepsilon_0 E_0$$

here α is a constant called atomic / molecular polarizability. It has dimensions of volume (L^3) it has the order of 10^{-29} to 10^{-30} m^3.

This **polarization** is a vector quantity and is related to resultant electric field E as under :

$$\vec{p} = \chi_e \varepsilon E$$

Where χ_e is a constant called electric susceptibility of the dielectric.

The induced charge **σ_p** is due to this polarization, hence

$$\boldsymbol{\sigma_p} = \vec{p}.\hat{n}$$

When this dielectric is introduced between the two plates having charge density **σ** then resultant electric field can be related as

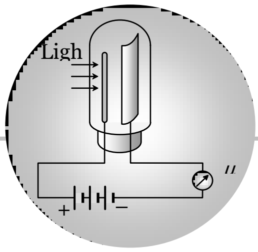
$$E.\hat{n} = E - E_p = \frac{\sigma-\sigma p}{\varepsilon_0} = \frac{\sigma-\vec{p}.\hat{n}}{\varepsilon_0}$$

$$\text{or } (\varepsilon^0 E + p).\hat{n} = \sigma$$

$$\text{or } \vec{D}.\hat{n} = \sigma$$

The quantity $\vec{D}$ is called **electric displacement** in dielectric.

We can prove that K = 1+ χ_e



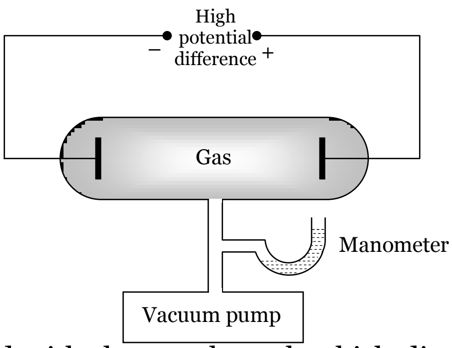
Electron, Photon, Photoelectric Effect and X-rays

Electric Discharge Through Gases

At normal atmospheric pressure, the gases are poor conductor of electricity. If we establish a potential difference (of the order of 30 *kV*) between two electrodes placed in air at a distance of few *cm* from each other, electric conduction starts in the form of sparks.

The passage of electric current through air is called electric discharge through the air.

The discharge of electricity through gases can be systematically studied with the help of discharge tube shown below



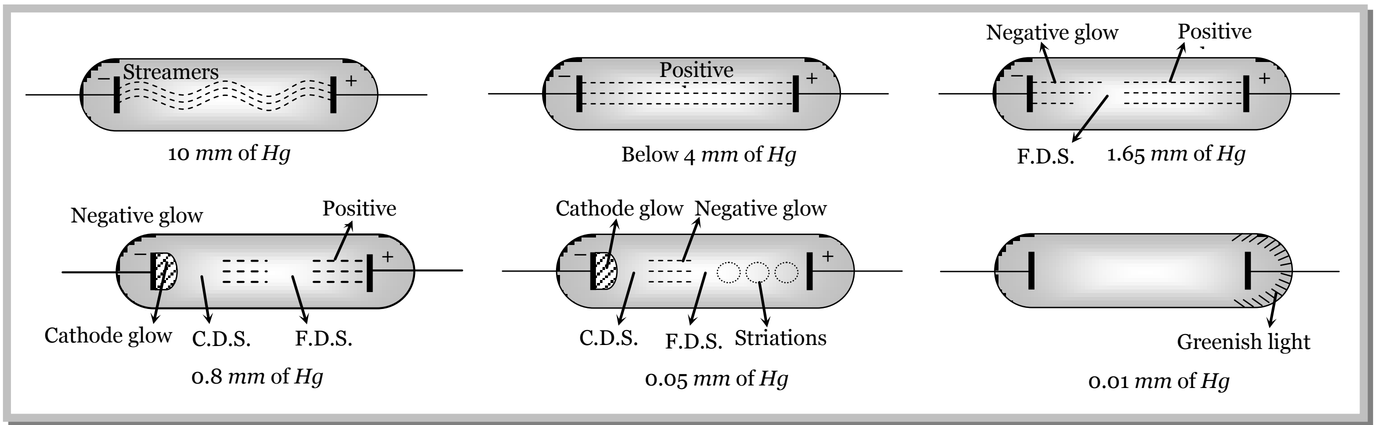
Length of discharge tube ≈ 30 to 40 *cm*

Diameter of the tube ≈ 4*cm*

The discharge tube is filled with the gas through which discharge is to be studied. The pressure of the enclosed gas can be reduced with the help of a vacuum pump and it's value is read by manometer.

Sequence of phenomenon

As the pressure inside the discharge tube is gradually reduced, the following is the sequence of phenomenon that are observed.



(1) At normal pressure no discharge takes place.

(2) At the pressure 10 *mm* of *Hg*, a zig-zag thin red spark runs from one electrode to other and cracking sound is heard.

(3) At the pressure 4 *mm*. of *Hg*, an illumination is observed at the electrodes and the rest of the tube appears dark. This type of discharge is called dark discharge.

(4) When the pressure falls below 4 *mm* of *Hg* then the whole tube is filled with bright light called positive column and colour of light depends upon the nature of gas in the tube as shown in the following table.

Gas	Colour
Air	Purple red

H_2	Blue
N_2	Red
Cl_2	Green
CO_2	Bluish white
Na	Yellow
Neon	Dark red

(5) At a pressure of 1.65 *mm* of *Hg* :

(i) Sky colour light is produced at the cathode it is called as negative glow.

(ii) Positive column shrinks towards the anode and the dark space between positive column and negative glow is called Faradays dark space (FDS)

(6) At a pressure of 0.8 *mm Hg* : At this pressure, negative glow is detached from the cathode and moves towards the anode. The dark space created between cathode and negative glow is called as Crook's dark space length of positive column further reduced. A glow appear at cathode called cathode glow.

(7) At a pressure of 0.05 *mm* of *Hg* : The positive column splits into dark and bright disc of light called striations.

(8) At the pressure of 0.01 or 10^{-2} *mm* of *Hg* some invisible particle move from cathode which on striking with the glass tube of the opposite side of cathode cause the tube to glow. These invisible rays emerging from cathode are called cathode rays.

(9) Finally when pressure drops to nearly 10^{-4} *mm* of *Hg*, there is no discharge in tube.

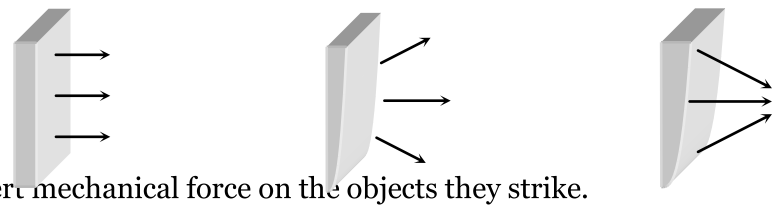
Cathode Rays

Cathode rays, discovered by sir Willium Crooke are the stream of electrons. They can be produced by using a discharge tube containing gas at a low pressure of the order of 10^{-2} *mm* of *Hg*. At this pressure the gas molecules ionise and the emitted electrons travel towards positive potential of anode. The positive ions hit the cathode to cause emission of electrons from cathode. These electrons also move towards anode. Thus the cathode rays in the discharge tube are the electrons produced due to ionisation of gas and that emitted by cathode due to collision of positive ions.

(1) **Properties of cathode rays**

(i) Cathode rays travel in straight lines (cast shadows of objects placed in their path)

(ii) Cathode rays emit normally from the cathode surface. Their direction is independent of the position of the anode.



(iii) Cathode rays exert mechanical force on the objects they strike.

(iv) Cathode rays produce heat when they strikes a material surface.

(v) Cathode rays produce fluorescence.

(vi) When cathode rays strike a solid object, specially a metal of high atomic weight and high melting point *X*-rays are emitted from the objects.

(vii) Cathode rays are deflected by an electric field and also by a magnetic field.

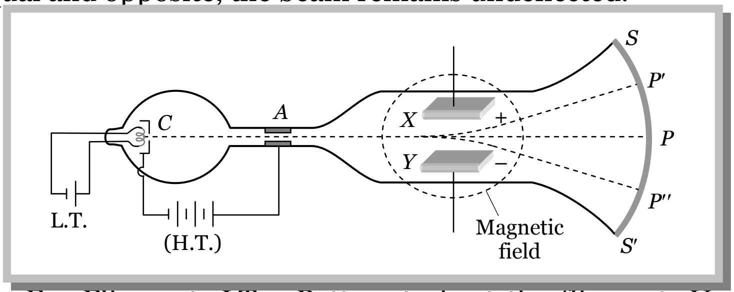
(viii) Cathode rays ionise the gases through which they are passed.

(ix) Cathode rays can penetrate through thin foils of metal.

(x) Cathode rays are found to have velocity ranging $\frac{1}{30}th$ to $\frac{1}{10}th$ of velocity of light.

(2) **J.J. Thomson's method to determine specific charge of electron**

It's working is based on the fact that if a beam of electron is subjected to the crossed electric field $\vec{E}$ and magnetic field $\vec{B}$, it experiences a force due to each field. In case the forces on the electrons in the electron beam due to these fields are equal and opposite, the beam remains undeflected.



C = Cathode, A = Anode, F = Filament, LT = Battery to heat the filament, V = potential difference to accelerate the electrons, SS' = ZnS coated screen, XY = metallic plates (Electric field produced between them)

(i) When no field is applied, the electron beam produces illuminations at point P.

(ii) In the presence of any field (electric and magnetic) electron beam deflected up or down (illumination at P' or P'')

(iii) If both the fields are applied simultaneously and adjusted such that electron beam passes undeflected and produces illumination at point P.

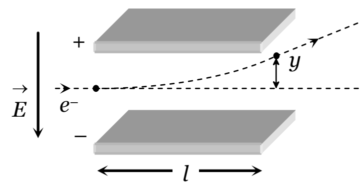
In this case; Electric force = Magnetic force $\Rightarrow eE = evB \Rightarrow v = \frac{E}{B}$; v = velocity of electron

As electron beam accelerated from cathode to anode its potential energy at the cathode appears as gain in the K.E. at the anode. If suppose V is the potential difference between cathode and anode then, potential energy $= eV$

And gain in kinetic energy at anode will be K.E. $= \frac{1}{2}mv^2$ i.e. $eV = \frac{1}{2}mv^2 \Rightarrow \frac{e}{m} = \frac{v^2}{2V} \Rightarrow \frac{e}{m} = \frac{E^2}{2VB^2}$

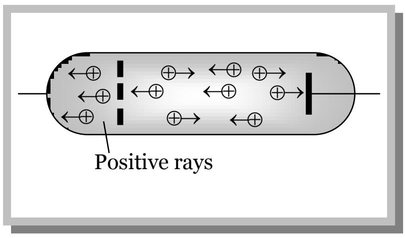
Thomson found, $\frac{e}{m} = 1.77 \times 10^{11}\, C/kg$.

Note : ❑ The deflection of an electron in a purely electric field is given by $y = \frac{1}{2}\left(\frac{eE}{m}\right).\frac{l^2}{v^2}$; where l length of each plate, y = deflection of electron in the field region, v = speed of the electron.



Positive Rays

Positive rays are sometimes known as the canal rays. These were discovered by Goldstein. If the cathode of a discharge tube has holes in it and the pressure of the gas is around 10^{-3} *mm* of *Hg* then faint luminous glow comes out from each hole on the backside of the cathode. It is said positive rays which are coming out from the holes.



(1) **Origin of positive rays**

When potential difference is applied across the electrodes, electrons are emitted from the cathode. As they move towards anode, they gain energy. These energetic electrons when collide with the atoms of the gas in the discharge tube, they ionize the atoms. The positive ions so formed at various places between cathode and anode, travel towards the cathode. Since during their motion, the positive ions when reach the cathode, some pass through the holes in the cathode. These streams are the positive rays.

(2) **Properties of positive rays**

(i) These are positive ions having same mass if the experimental gas does not have isotopes. However if the gas has isotopes then positive rays are group of positive ions having different masses.

(ii) They travels in straight lines and cast shadows of objects placed in their path. But the speed of the positive rays is much smaller than that of cathode rays.

(iii) They are deflected by electric and magnetic fields but the deflections are small as compared to that for cathode rays.

(iv) They show a spectrum of velocities. Different positive ions move with different velocities. Being heavy, their velocity is much less than that of cathode rays.

(v) q/m ratio of these rays depends on the nature of the gas in the tube (while in case of the cathode rays q/m is constant and doesn't depend on the gas in the tube). q/m for hydrogen is maximum.

(vi) They carry energy and momentum. The kinetic energy of positive rays is more than that of cathode rays.

(vii) The value of charge on positive rays is an integral multiple of electronic charge.

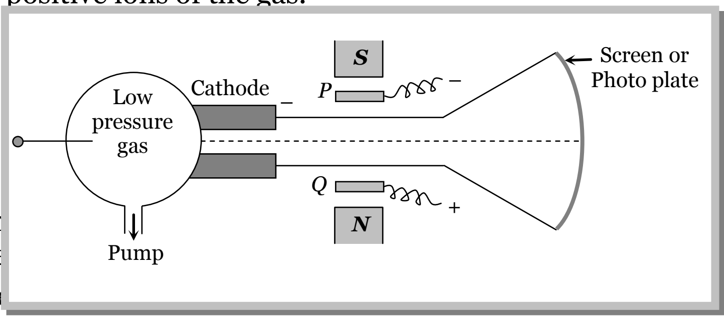
(viii) They cause ionisation (which is much more than that produced by cathode rays).

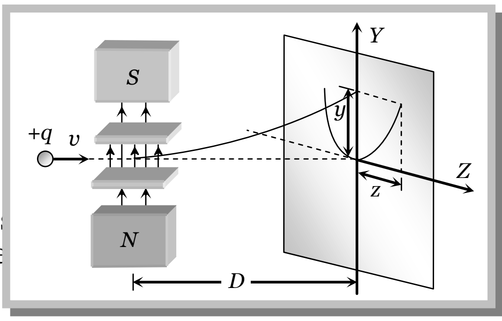
Mass Spectrograph

It is a device used to determine the mass or (q/m) of positive ions.

(1) **Thomson mass spectrograph**

It is used to measure atomic masses of various isotopes in gas. This is done by measuring q/m of singly ionised positive ions of the gas.





Th… hand … rds cathode… in the … s is subject… owed … but $\vec{E}$ or $\vec{B} \perp \vec{v}$).

If the initial motion of the ions is in $+x$ direction and electric and magnetic fields are applied along $+y$ axis then force due to electric field is in the direction of y-axis and due to magnetic field it is along z-direction.

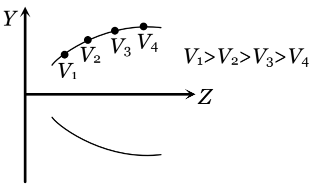
The deflection due to electric field alone $y = \dfrac{qELD}{mv^2}$ ....(i)

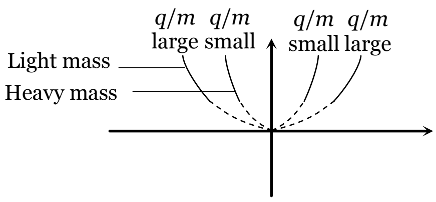
The deflection due to magnetic field alone $z = \dfrac{qBLD}{mv}$ ....(ii)

From equation (i) and (ii)

$z^2 = k\left(\dfrac{q}{m}\right)y$, where $k = \dfrac{B^2LD}{E}$; This is the equation of parabola. It means all the charged particles moving with different velocities but of same q/m value will strike the screen placed in yz plane on a parabolic track as shown in the above figure.

Note : ❑ All the positive ions of same. q/m moving with different velocity lie on the same parabola. Higher is the velocity lower is the value of y and z. The ions of different specific charge will lie on different parabola.





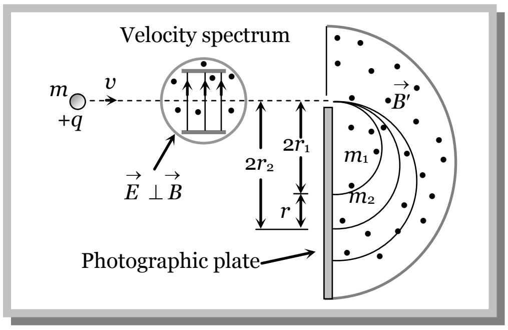
❑ The number of parabola tells the number of isotopes present in the given ionic beam.

(2) **Bainbridge mass spectrograph**

In Bainbridge mass spectrograph, field particles of same velocity are selected by using a velocity selector and then they are subjected to a uniform magnetic field perpendicular to the velocity of the particles. The particles corresponding to different isotopes follow different circular paths as shown in the figure.

(i) **Velocity selector :** The positive ions having a certain velocity v gets isolated from all other velocity particles. In this chamber the electric and magnetic fields are so balanced that the particle moves undeflected. For this the necessary condition is $v = \frac{E}{B}$.

(ii) **Analysing chamber :** In this chamber magnetic field B is applied perpendicular to the direction of motion of the particle. As a result the particles move along a circular path of radius



$$r = \frac{mE}{qBB'} \Rightarrow \frac{q}{m} = \frac{E}{BB'r} \text{ also } \frac{r_1}{r_2} = \frac{m_1}{m_2}$$

In this way the particles of different masses gets deflected on circles of different radii and reach on different points on the photo plate.

Note : ❑ Separation between two traces $= d = 2r_2 - 2r_1 \Rightarrow d = \frac{2v(m_2 - m_1)}{qB'}$.

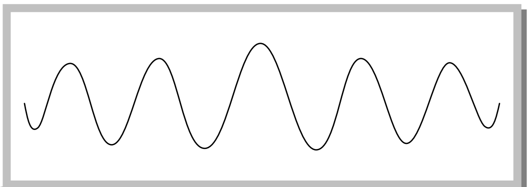
Matter waves (de-Broglie Waves)

According to de-Broglie a moving material particle sometimes acts as a wave and sometimes as a particle.

or

A wave is associated with moving material particle which control the particle in every respect.

The wave associated with moving particle is called matter wave or de-Broglie wave and it propagates in the form of wave packets with group velocity.



(1) **de-Broglie wavelength**

According to de-Broglie theory, the wavelength of de-Broglie wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \qquad \Rightarrow \lambda \propto \frac{1}{p} \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}}$$

Where h = Plank's constant, m = Mass of the particle, v = Speed of the particle, E = Energy of the particle.

The smallest wavelength whose measurement is possible is that of γ-rays.

The wavelength of matter waves associated with the microscopic particles like electron, proton, neutron, α-particle *etc.* is of the order of $10^{-10}\ m$.

(i) **de-Broglie wavelength associated with the charged particles.**

The energy of a charged particle accelerated through potential difference V is $E = \frac{1}{2}mv^2 = qV$

Hence de-Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

$$\lambda_{electron} = \frac{12.27}{\sqrt{V}}\mathring{A}, \quad \lambda_{proton} = \frac{0.286}{\sqrt{V}}\mathring{A}, \quad \lambda_{deutron} = \frac{0.202 \times 10^{-10}}{\sqrt{V}}\mathring{A}, \quad \lambda_{\alpha-particle} = \frac{0.101}{\sqrt{V}}\mathring{A}$$

(ii) **de-Broglie wavelength associated with uncharged particles.**

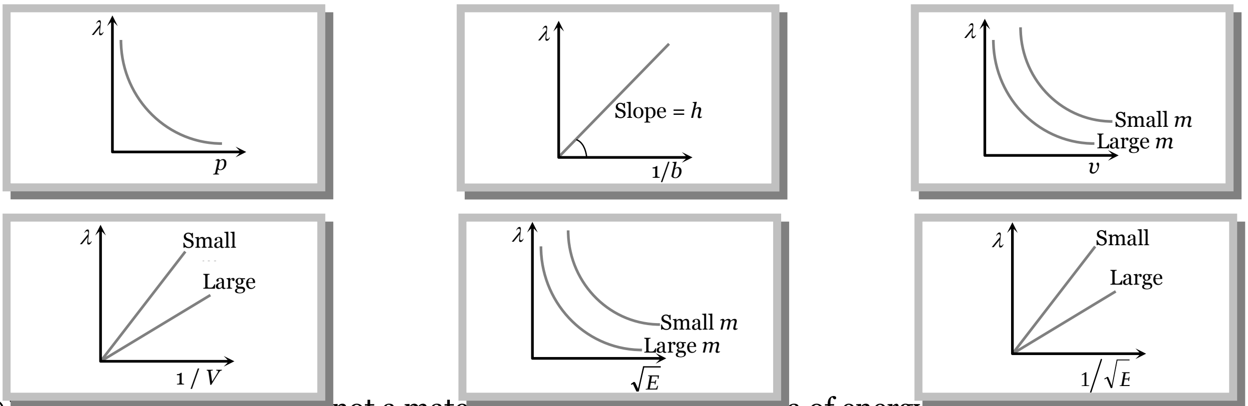
For *Neutron* de-Broglie wavelength is given as $\lambda_{Neutron} = \frac{0.286 \times 10^{-10}}{\sqrt{E(\text{in } eV)}} m = \frac{0.286}{\sqrt{E(\text{in } eV)}}\mathring{A}$

Energy of thermal neutrons at ordinary temperature

$\because \; E = kT \Rightarrow \lambda = \dfrac{h}{\sqrt{2mkT}}$; where k = Boltzman's constant = 1.38×10^{-23} *Joules/kelvin*, T = Absolute temp.

So $\lambda_{\text{Thermal Neutron}} = \dfrac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.07 \times 10^{-17} \times 1.38 \times 10^{-23} T}} = \dfrac{30.83}{\sqrt{T}} \text{Å}$

(2) **Some graphs**



Note : ❑ A photon is not a material particle. It is a quanta of energy.

❑ When a particle exhibits wave nature, it is associated with a wave packet, rather then a wave.

(3) **Characteristics of matter waves**

(i) Matter wave represents the probability of finding a particle in space.

(ii) Matter waves are not electromagnetic in nature.

(iii) de-Brogile or matter wave is independent of the charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).

(iv) Practical observation of matter waves is possible only when the de-Broglie wavelength is of the order of the size of the particles is nature.

(v) Electron microscope works on the basis of de-Broglie waves.

(vi) The electric charge has no effect on the matter waves or their wavelength.

(vii) The phase velocity of the matter waves can be greater than the speed of the light.

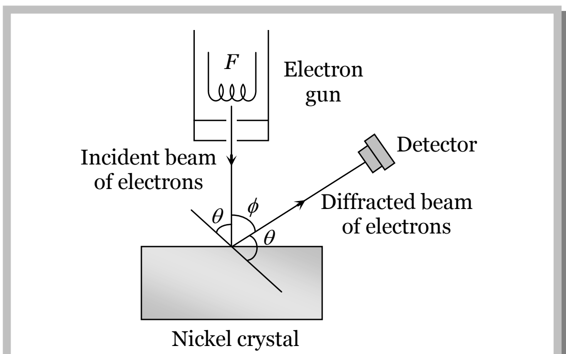
(viii) Matter waves can propagate in vacuum, hence they are not mechanical waves.

(ix) The number of de-Broglie waves associated with n^{th} orbital electron is n.

(x) Only those circular orbits around the nucleus are stable whose circumference is integral multiple of de-Broglie wavelength associated with the orbital electron.

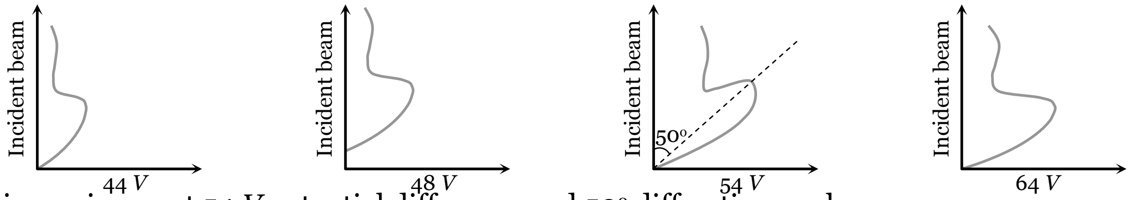
(4) **Davision and Germer experiment**

It is used to study the scattering of electron from a solid or to verify the wave nature of electron. A beam of electrons emitted by electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle. *Ni* crystal behaves like a three dimensional diffraction grating and it diffracts the electron beam obtained from electron gun.



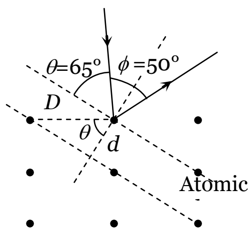
The diffracted beam of electrons [illegible] positioned at any angle by rotating it about the point of incidence. The energy of the incident beam of electrons can also be varied by changing the applied voltage to the electron gun.

According to classical physics, the intensity of scattered beam of electrons at all scattering angle will be same but Davisson and Germer, found that the intensity of scattered beam of electrons was not the same but different at different angles of scattering.



Intensity is maximum at 54 V potential difference and 50° diffraction angle.

If the de-Broglie waves exist for electrons then these should be diffracted as X-rays. Using the Bragg's formula $2d\sin\theta = n\lambda$, we can determine the wavelength of these waves.



Where d = distance between diffracting planes, $\theta = \frac{(180-\phi)}{2}$ = glancing angle for incident beam = Bragg's angle.

The distance between diffraction planes in Ni-crystal for this experiment is d = 0.91$Å$ and the Bragg's angle = 65°. This gives for n = 1, $\lambda = 2\times 0.91\times 10^{-10}\sin 65^o = 1.65\ Å$

Now the de-Broglie wavelength can also be determined by using the formula $\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} = 1.67\ Å$.

Thus the de-Broglie hypothesis is verified.

Heisenberg Uncertainty Principle

According to Heisenberg's uncertainty principle, it is impossible to measure simultaneously both the position and the momentum of the particle.

Let Δx and Δp be the uncertainty in the simultaneous measurement of the position and momentum of the particle, then $\Delta x\Delta p = \hbar$; where $\hbar = \frac{h}{2\pi}$ and h = 6.63 × 10^{-34} J-s is the Planck's constant.

If Δx = 0 then $\Delta p = \infty$

and if Δp = 0 then $\Delta x = \infty$ *i.e.,* if we are able to measure the exact position of the particle (say an electron) then the uncertainty in the measurement of the linear momentum of the particle is infinite. Similarly, if we are able to measure the exact linear momentum of the particle *i.e.*, Δp = 0, then we can not measure the exact position of the particle at that time.

Photon

According to Eienstein's quantum theory light propagates in the bundles (packets or quanta) of energy, each bundle being called a photon and possessing energy.

(1) **Energy of photon**

Energy of each photon is given by $E = h\nu = \frac{hc}{\lambda}$; where c = Speed of light, h = Plank's constant = 6.6 × 10^{-34} J-sec, ν = Frequency in Hz, λ = Wavelength of light

Energy of photon in electron volt $E(eV) = \frac{hc}{e\lambda} = \frac{12375}{\lambda(Å)} \approx \frac{12400}{\lambda(Å)}$

(2) **Mass of photon**

Actually rest mass of the photon is zero. But it's effective mass is given as

$E = mc^2 = h\nu \Rightarrow m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{c\lambda}$. This mass is also known as kinetic mass of the photon

(3) **Momentum of the photon**

Momentum $p = m\times c = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

(4) **Number of emitted photons**

The number of photons emitted per second from a source of monochromatic radiation of wavelength λ and power P is given as $(n) = \frac{P}{E} = \frac{P}{h\nu} = \frac{P\lambda}{hc}$; where E = energy of each photon

(5) **Intensity of light (*I*)**

Energy crossing per unit area normally per second is called intensity or energy flux

i.e. $\quad I = \frac{E}{At} = \frac{P}{A} \quad \left(\frac{E}{t} = P = \text{radiation power}\right)$

At a distance r from a point source of power P intensity is given by $I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}$

Concepts

- *Discovery of positive rays helps in discovering of isotopes.*
- *The de-Broglie wavelength of electrons in first Bohr orbit of an atom is equal to circumference of orbit.*
- *A particle having zero rest mass and non zero energy and momentum must travels with a speed equal to speed of light.*
- ***de-Broglie wave length associates with gas molecules** is given as $\lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3\,mkT}}$ (Energy of gas molecules at temperature T is $E = \frac{3}{2}kT$)*

***Example*: 1** The ratio of specific charge of an α -particle to that of a proton is **[BCECE 2003]**

(a) 2 : 1 (b) 1 : 1 (c) 1 : 2 (d) 1 : 3

Solution : (c) Specific charge $= \frac{q}{m}$; Ratio $= \frac{(q/m)_\alpha}{(q/m)_p} = \frac{q_\alpha}{q_p} \times \frac{m_p}{m_\alpha} = \frac{1}{2}$.

***Example*: 2** The speed of an electron having a wavelength of $10^{-10} m$ is **[AIIMS 2002]**

(a) $7.25 \times 10^6 m/s$ (b) $6.26 \times 10^6 m/s$ (c) $5.25 \times 10^6 m/s$ (d) $4.24 \times 10^6 m/s$

Solution : (a) By using $\lambda_{electron} = \frac{h}{m_e v} \Rightarrow v = \frac{h}{m_e \lambda_e} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}} = 7.25 \times 10^6 m/s$.

***Example*: 3** In Thomson experiment of finding *e/m* for electrons, beam of electron is replaced by that of muons (particle with same charge as of electrons but mass 208 times that of electrons). No deflection condition in this case satisfied if **[Orissa (Engg.) 2002]**

(a) *B* is increased 208 times (b) *E* is increased 208 times

(c) *B* is increased 14.4 times (d) None of these

Solution : (c) In the condition of no deflection $\frac{e}{m} = \frac{E^2}{2VB^2}$. If *m* is increased to 208 times then *B* should be increased by $\sqrt{208} = 14.4$ times.

***Example*: 4** In a Thomson set-up for the determination of *e/m,* electrons accelerated by 2.5 *kV* enter the region of crossed electric and magnetic fields of strengths $3.6 \times 10^4 Vm^{-1}$ and $1.2 \times 10^{-3} T$ respectively and go through undeflected. The measured value of *e/m* of the electron is equal to **[AMU 2002]**

(a) 1.0×10^{11} $C\text{-}kg^{-1}$ (b) 1.76×10^{11} $C\text{-}kg^{-1}$ (c) 1.80×10^{11} $C\text{-}kg^{-1}$ (d) 1.85×10^{11} $C\text{-}kg^{-1}$

Solution : (c) By using $\frac{e}{m} = \frac{E^2}{2VB^2} \Rightarrow \frac{e}{m} = \frac{(3.6 \times 10^4)^2}{2 \times 2.5 \times 10^3 \times (1.2 \times 10^{-3})^2} = 1.8 \times 10^{11} C/kg$.

***Example*: 5** In Bainbridge mass spectrograph a potential difference of 1000 *V* is applied between two plates distant 1 *cm* apart and magnetic field in *B* = 1*T*. The velocity of undeflected positive ions in *m/s* from the velocity selector is

(a) $10^7 m/s$ (b) $10^4 m/s$ (c) $10^5 m/s$ (d) $10^2 m/s$

Solution : (c) By using $v = \frac{E}{B}$; where $E = \frac{V}{d} = \frac{1000}{1 \times 10^{-2}} = 10^5 V/m \Rightarrow v = \frac{10^5}{1} = 10^5 m/s$.

***Example*: 6** An electron and a photon have same wavelength. It *p* is the momentum of electron and *E* the energy of photon. The magnitude of *p*/ *E* in S.I. unit is

(a) 3.0×10^8 (b) 3.33×10^{-9} (c) 9.1×10^{-31} (d) 6.64×10^{-34}

Solution : (b) $\lambda = \frac{h}{p}$ (for electron) or $p = \frac{h}{\lambda}$ and $E = \frac{hc}{\lambda}$ (for photon)

$\therefore \frac{p}{E} = \frac{1}{c} = \frac{1}{3 \times 10^8 \, m/s} = 3.33 \times 10^{-9} \, s/m$

***Example*: 7** The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is E. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio λ_1/λ_2 is proportional to **[UPSEAT 2003; IIT-JEE (Screening) 2004]**

(a) E^0 (b) $E^{1/2}$ (c) E^{-1} (d) E^{-2}

Solution : (b) For photon $\lambda_2 = \frac{hc}{E}$ .... (i) and For proton $\lambda_1 = \frac{h}{\sqrt{2mE}}$ ....(ii)

Therefore $\frac{\lambda_1}{\lambda_2} = \frac{E^{1/2}}{\sqrt{2m}\, c} \Rightarrow \frac{\lambda_1}{\lambda_2} \propto E^{1/2}$.

***Example*: 8** The de-Broglie wavelength of an electron having 80*eV* of energy is nearly ($1eV = 1.6 \times 10^{-19} J$, Mass of electron $9 \times 10^{-31} kg$ and Plank's constant 6.6×10^{-34} *J-sec*)

(a) 140 $\mathring{A}$ (b) 0.14 $\mathring{A}$ (c) 14 $\mathring{A}$ (d) 1.4 $\mathring{A}$

Solution : (d) By using $\lambda = \frac{h}{\sqrt{2mE}} = \frac{12.27}{\sqrt{V}}$. If energy is 80 *eV* then accelerating potential difference will be 80 *V*. So $\lambda = \frac{12.27}{\sqrt{80}} = 1.37 \approx 1.4 \, \mathring{A}$.

***Example*: 9** The kinetic energy of electron and proton is $10^{-32} J$. Then the relation between their de-Broglie wavelengths is

(a) $\lambda_p < \lambda_e$ (b) $\lambda_p > \lambda_e$ (c) $\lambda_p = \lambda_e$ (d) $\lambda_p = 2\lambda_e$

Solution : (a) By using $\lambda = \frac{h}{\sqrt{2mE}}$ $E = 10^{-32} J$ = Constant for both particles. Hence $\lambda \propto \frac{1}{\sqrt{m}}$

Since $m_p > m_e$ so $\lambda_p < \lambda_e$.

***Example*: 10** The energy of a proton and an α particle is the same. Then the ratio of the de-Broglie wavelengths of the proton and the α is **[RPET 1991]**

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

Solution : (b) By using $\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$ (E – same) $\Rightarrow \frac{\lambda_{proton}}{\lambda_{\alpha-particle}} = \sqrt{\frac{m_\alpha}{m_p}} = \frac{2}{1}$.

***Example*: 11** The de-Broglie wavelength of a particle accelerated with 150 *volt* potential is 10^{-10} *m*. If it is accelerated by 600 *volts* p.d., its wavelength will be **[RPET 1988]**

(a) 0.25 $\mathring{A}$ (b) 0.5 $\mathring{A}$ (c) 1.5 $\mathring{A}$ (d) 2 $\mathring{A}$

Solution : (b) By using $\lambda \propto \frac{1}{\sqrt{V}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}} \Rightarrow \frac{10^{-10}}{\lambda_2} = \sqrt{\frac{600}{150}} = 2 \Rightarrow \lambda_2 = 0.5 \, \mathring{A}$.

***Example*: 12** The de-Broglie wavelength of an electron in an orbit of circumference $2\pi r$ is **[MP PET 1987]**

(a) $2\pi r$ (b) πr (c) $1/2\pi r$ (d) $1/4\pi r$

Solution : (a) According to Bohr's theory $mvr = n\frac{h}{2\pi} \Rightarrow 2\pi r = n\left(\frac{h}{mv}\right) = n\lambda$

For $n = 1$ $\lambda = 2\pi r$

***Example*: 13** The number of photons of wavelength 540 *nm* emitted per second by an electric bulb of power 100*W* is (taking $h = 6 \times 10^{-34}$ *J-sec*) **[Kerala (Engg.) 2002]**

(a) 100 (b) 1000 (c) 3×10^{20} (d) 3×10^{18}

Solution : (c) By using $n = \frac{P\lambda}{hc} = \frac{100 \times 540 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 3 \times 10^{20}$

***Example*: 14** A steel ball of mass 1*kg* is moving with a velocity 1 *m/s*. Then its de-Broglie waves length is equal to

(a) h (b) $h/2$ (c) Zero (d) $1/h$

Solution : (a) By using $\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{\lambda}{1 \times 1} = h$.

***Example*: 15** The de-Broglie wavelength associated with a hydrogen atom moving with a thermal velocity of 3 km/s will be

(a) $1\,\mathring{A}$ (b) $0.66\,\mathring{A}$ (c) $6.6\,\mathring{A}$ (d) $66\,\mathring{A}$

Solution : (b) By using $\lambda = \dfrac{h}{mv_{rms}} \Rightarrow \lambda = \dfrac{6.6\times10^{-34}}{2\times1.67\times10^{-27}\times3\times10^{3}} = 0.66\,\mathring{A}$

***Example*: 16** When the momentum of a proton is changed by an amount P_0, the corresponding change in the de-Broglie wavelength is found to be 0.25%. Then, the original momentum of the proton was **[CPMT 2002]**

(a) p_0 (b) $100\,p_0$ (c) $400\,p_0$ (d) $4\,p_0$

Solution : (c) $\lambda \propto \dfrac{1}{p} \Rightarrow \dfrac{\Delta p}{p} = -\dfrac{\Delta\lambda}{\lambda} \Rightarrow \left|\dfrac{\Delta p}{p}\right| = \left|\dfrac{\Delta\lambda}{\lambda}\right| \Rightarrow \dfrac{p_0}{p} = \dfrac{0.25}{100} = \dfrac{1}{400} \Rightarrow p = 400\,p_0.$

***Example*: 17** If the electron has same momentum as that of a photon of wavelength 5200Å, then the velocity of electron in m/sec is given by

(a) 10^3 (b) 1.4×10^3 (c) 7×10^{-5} (d) 7.2×10^6

Solution : (b) $\lambda = \dfrac{h}{mv} \Rightarrow v = \dfrac{h}{m\lambda} = \dfrac{6.6\times10^{-34}}{9.1\times10^{-31}\times5200\times10^{-10}} \Rightarrow v = 1.4\times10^3\ m/s.$

***Example*: 18** The de-Broglie wavelength of a neutron at 27°C is λ. What will be its wavelength at 927°C

(a) $\lambda/2$ (b) $\lambda/3$ (c) $\lambda/4$ (d) $\lambda/9$

Solution : (a) $\lambda_{neutron} \propto \dfrac{1}{\sqrt{T}} \Rightarrow \dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{T_2}{T_1}} \Rightarrow \dfrac{\lambda}{\lambda_2} = \sqrt{\dfrac{(273+927)}{(273+27)}} = \sqrt{\dfrac{1200}{300}} = 2 \Rightarrow \lambda_2 = \dfrac{\lambda}{2}.$

***Example*: 19** The de-Broglie wavelength of a vehicle is λ. Its load is changed such that its velocity and energy both are doubled. Its new wavelength will be

(a) λ (b) $\dfrac{\lambda}{2}$ (c) $\dfrac{\lambda}{4}$ (d) 2λ

Solution : (a) $\lambda = \dfrac{h}{mv}$ and $E = \dfrac{1}{2}mv^2 \Rightarrow \lambda = \dfrac{hv}{2E}$ when v and E both are doubled, λ remains unchanged *i.e.* $\lambda' = \lambda$.

***Example*: 20** In Thomson mass spectrograph when only electric field of strength 20 kV/m is applied, then the displacement of the beam on the screen is 2 cm. If length of plates = 5 cm, distance from centre of plate to the screen = 20 cm and velocity of ions = $10^6\ m/s$, then q/m of the ions is

(a) $10^6\ C/kg$ (b) $10^7\ C/Kg$ (c) $10^8\ C/kg$ (d) $10^{11}\ C/kg$

Solution : (c) By using $y = \dfrac{qELD}{mv^2}$; where y = deflection on screen due to electric field only

$$\Rightarrow \frac{q}{m} = \frac{yv^2}{ELD} = \frac{2\times10^{-2}\times(10^6)^2}{20\times10^3\times5\times10^{-2}\times0.2} = 10^8\ C/kg.$$

***Example*: 21** The minimum intensity of light to be detected by human eye is $10^{-10}W/m^2$. The number of photons of wavelength $5.6\times10^{-7}m$ entering the eye, with pupil area $10^{-6}m^2$, per second for vision will be nearly

(a) 100 (b) 200 (c) 300 (d) 400

Solution : (c) By using $I = \dfrac{P}{A}$; where P = radiation power

$$\Rightarrow P = I\times A \Rightarrow \frac{nhc}{t\lambda} = IA \Rightarrow \frac{n}{t} = \frac{IA\lambda}{hc}$$

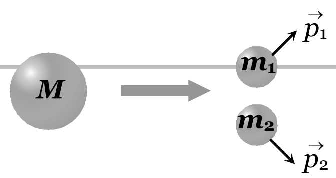
Hence number of photons entering per sec the eye $\left(\dfrac{n}{t}\right) = \dfrac{10^{-10}\times10^{-6}\times5.6\times10^{-7}}{6.6\times10^{-34}\times3\times10^{8}} = 300.$

<u>Example 1</u>. A particle of mass M at rest decays into two particles of masses m_1 and m_2, having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles, λ_1/λ_2 is **[IIT-JEE 1999]**

(a) m_1/m_2 (b) m_2/m_1 (c) 1.0 (d) $\sqrt{m_1}/\sqrt{m_1}$

Solution : (c) According to conservation of momentum *i.e.* $p_1 = p_2$

Hence from $\lambda = \dfrac{h}{p} \Rightarrow \dfrac{\lambda_1}{\lambda_2} = \dfrac{p_1}{p_2} = \dfrac{1}{1}$



The curve drawn between velocity and frequency of photon in vacuum will be a **[MP PET 2000]**

(a) Straight line parallel to frequency axis
(b) Straight line parallel to velocity axis
(c) Straight line passing through origin and making an angle of 45^o with frequency axis
(d) Hyperbola

Solution : (a) Velocity of photon (*i.e.* light) doesn't depend upon frequency. Hence the graph between velocity of photon and frequency will be as follows.

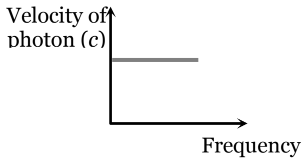


Photo-electric Effect

It is the phenomenon of emission of electrons from the surface of metals, when light radiations (Electromagnetic radiations) of suitable frequency fall on them. The emitted electrons are called photoelectrons and the current so produced is called photoelectric current.

This effect is based on the principle of conservation of energy.

(1) **Terms related to photoelectric effect**

(i) **Work function (or threshold energy) (W_0) :** The minimum energy of incident radiation, required to eject the electrons from metallic surface is defined as work function of that surface.

$$W_0 = h\nu_0 = \frac{hc}{\lambda_0} \textit{ Joules}\ ; \quad \nu_0 = \text{Threshold frequency};\ \lambda_0 = \text{Threshold wavelength}$$

Work function in electron volt $W_0(eV) = \dfrac{hc}{e\lambda_0} = \dfrac{12375}{\lambda_0(\text{Å})}$

Note : ❑ By coating the metal surface with a layer of barium oxide or strontium oxide it's work function is lowered.

(ii) **Threshold frequency (ν_0) :** The minimum frequency of incident radiations required to eject the electron from metal surface is defined as threshold frequency.

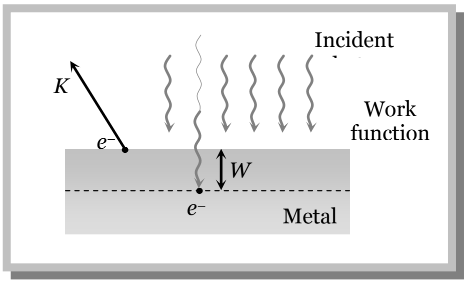
If incident frequency $\nu < \nu_0 \Rightarrow$ No photoelectron emission

(iii) **Threshold wavelength (λ_0) :** The maximum wavelength of incident radiations required to eject the electrons from a metallic surface is defined as threshold wavelength.

If incident wavelength $\lambda > \lambda_0 \Rightarrow$ No photoelectron emission

(2) **Einstein's photoelectric equation**

According to Einstein, photoelectric effect is the result of one to one inelastic collision between photon and electron in which photon is completely absorbed. So if an electron in a metal absorbs a photon of energy E ($= h\nu$), it uses the energy in three following ways.



(i) Some energy (say W) is used in shifting the electron from interior to the surface of the metal.

(ii) Some energy (say W_0) is used in making the surface electron free from the metal.

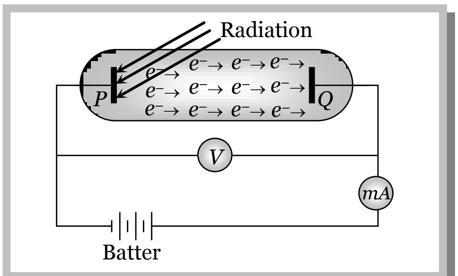
(iii) Rest energy will appear as kinetic energy (K) of the emitted photoelectrons.

Hence $E = W + W_0 + K$

For the electrons emitting from surface $W = 0$ so kinetic energy of emitted electron will be max.

Hence $\boldsymbol{E = W_0 + K_{max}}$; This is the Einstein's photoelectric equation

(3) **Experimental arrangement to observe photoelectric effect**



When light radiations of suitable frequency (or suitable wavelength and suitable energy) falls on plate P, photoelectrons are emitted from P.

(i) If plate Q is at zero potential *w.r.t.* P, very small current flows in the circuit because of some electrons of high kinetic energy are reaching to plate Q, but this current has no practical utility.

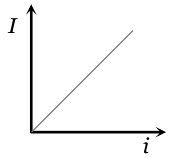
(ii) If plate Q is kept at positive potential *w.r.t.* P current starts flowing through the circuit because more electrons are able to reach upto plate Q.

(iii) As the positive potential of plate Q increases, current through the circuit increases but after some time constant current flows through the circuit even positive potential of plate Q is still increasing, because at this condition all the electrons emitted from plate P are already reached up to plate Q. This constant current is called **saturation current.**

(iv) To increase the photoelectric current further we will have to increase the intensity of incident light.

Photoelectric current (i) depends upon

(a) Potential difference between electrodes (till saturation)
(b) Intensity of incident light (I)
(c) Nature of surface of metal



(v) To decrease the photoelectric current plate Q is maintained at negative potential *w.r.t.* P, as the anode Q is made more and more negative, fewer and fewer electrons will reach the cathode and the photoelectric current decreases.

(vi) At a particular negative potential of plate Q no electron will reach the plate Q and the current will become zero, this negative potential is called **stopping potential** denoted by V_0.

(vii) If we increase further the energy of incident light, kinetic energy of photoelectrons increases and more negative potential should be applied to stop the electrons to reach upto plate Q. Hence $\boldsymbol{eV_0 = K_{max}}$.

Note : ❑ Stopping potential depends only upon frequency or wavelength or energy of incident radiation. It doesn't depend upon intensity of light.
We must remember that intensity of incident light radiation is inversely proportional to the square of distance between source of light and photosensitive plate P *i.e.*, $I \propto \frac{1}{d^2}$ so $I \propto i \propto \frac{1}{d^2}$)

Important formulae

$\Rightarrow h\nu = h\nu_0 + K_{\max}$

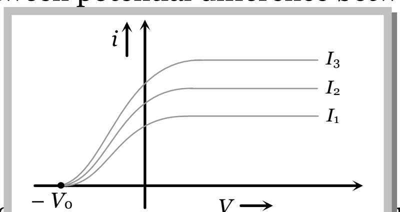
$$\Rightarrow K_{\max} = eV_0 = h(\nu - \nu_0) \;\Rightarrow \frac{1}{2}mv^2_{\max} = h(\nu - \nu_0) \;\Rightarrow v_{\max} = \sqrt{\frac{2h(\nu - \nu_0)}{m}}$$

$$\Rightarrow K_{\max} = \frac{1}{2}mv^2_{\max} = eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = hc\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right) \Rightarrow v_{\max} = \sqrt{\frac{2hc}{m}\frac{(\lambda - \lambda_0)}{\lambda\lambda_0}}$$

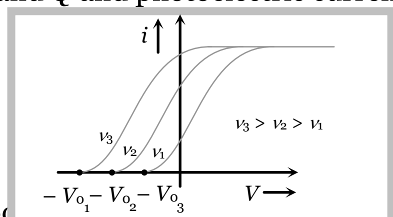
$$\Rightarrow V_0 = \frac{h}{e}(\nu - \nu_0) = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = 12375\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

(4) **Different graphs**

(i) Graph between potential difference between the plates P and Q and photoelectric current

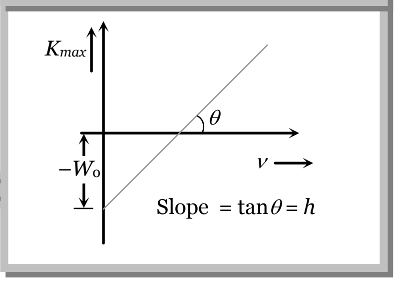


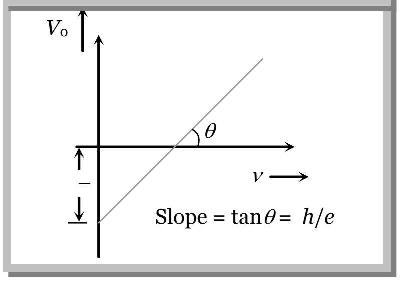
For different intensities of incident light



For different Frequencies of incident light

(ii) Graph be … rgy / stopping p… d frequency of incident light

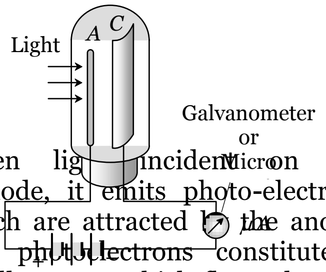
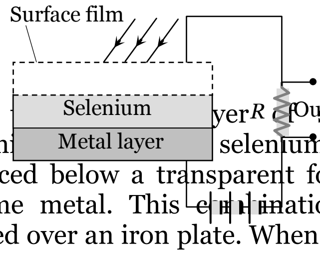
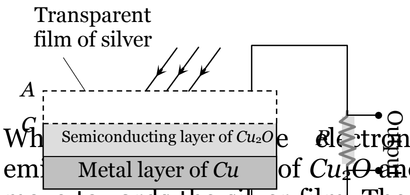




Photoelectric C

A device which converts light energy into electrical energy is called photoelectric cell. It is also known as photocell or electric eye.

Photoelectric cell are mainly of three types

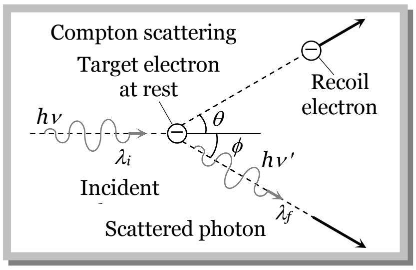
Photo-emissive cell	Photo-conductive cell	Photo-voltaic cell
It consists of an evacuated glass or quartz bulb containing anode *A* and cathode *C*. The cathode is semi-cylindrical metal on which a layer of photo-sensitive material is coated.  When light is incident on the cathode, it emits photo-electrons which are attracted by the anode. The photoelectrons constitute a small current which flows through the external circuit.	It is based on the principle that conductivity of a semiconductor increases with increase in the intensity of incident light.  In this cell a layer of some semiconductor (like selenium) is placed below a transparent foil of some metal. This combination is fixed over an iron plate. When light is incident on the transparent foil, the electrical resistance of the semiconductor layer is reduced. Hence a current starts flowing in the battery circuit connected.	It consists of a *Cu* plate coated with a thin layer of cuprous oxide (Cu_2O). On this plate is laid a semi transparent thin film of silver.  When light is incident, the electrons emitted from the layer of Cu_2O and move towards the silver film. Then the silver film becomes negatively charged and copper plate becomes positively charged. A potential difference is set up between these two and current is set up in the external resistance.

Note : ❑ The photoelectric current can be increased by filling some inert gas like Argon into the bulb. The photoelectrons emitted by cathode ionise the gas by collision and hence the current is increased.

Compton effect

The scattering of a photon by an electron is called Compton effect. The energy and momentum is conserved. Scattered photon will have less energy (more wavelength) as compare to incident photon (less wavelength). The energy lost by the photon is taken by electron as kinetic energy.

The change in wavelength due to Compton effect is called Compton shift. Compton shift $\lambda_f - \lambda_i = \dfrac{h}{m_0 c}(1 - \cos\theta)$



Note : ❑ Compton effect shows

X-rays

X-rays was discovered by scientist Rontgen that's why they are also called Rontgen rays.

Rontgen discovered that when pressure inside a discharge tube kept 10^{-3} *mm* of *Hg* and potential difference is 25 *kV* then some unknown radiations (X-rays) are emitted by anode.

(1) **Production of X-rays**

There are three essential requirements for the production of X-rays

(i) A source of electron

(ii) An arrangement to accelerate the electrons

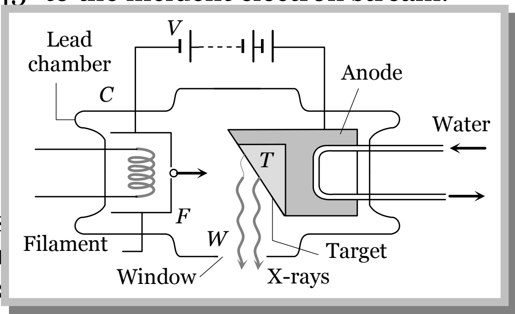
(iii) A target of suitable material of high atomic weight and high melting point on which these high speed electrons strike.

(2) **Coolidge X-ray tube**

It consists of a highly evacuated glass tube containing cathode and target. The cathode consist of a tungsten filament. The filament is coated with oxides of barium or strontium to have an emission of electrons even at low temperature. The filament is surrounded by a molybdenum cylinder kept at negative potential *w.r.t.* the target.

The target (it's material of high atomic weight, high melting point and high thermal conductivity) made of tungsten or molybdenum is embedded in a copper block.

The face of the target is set at 45° to the incident electron stream.



The filament is heated by pass ntial difference (≈ 10 kV to 80 kV) is applied between the target and cath e emitted by filament. The stream of highly energetic electrons are focus

Most of the energy of the electrons is converted into heat (above 98%) and only a fraction of the energy of the electrons (about 2%) is used to produce X-rays.

During the operation of the tube, a huge quantity of heat is produced in this target, this heat is conducted through the copper anode to the cooling fins from where it is dissipated by radiation and convection.

(i) **Control of intensity of X-rays** : Intensity implies the number of X-ray photons produced from the target. The intensity of X-rays emitted is directly proportional to the electrons emitted per second from the filament and this can be increased by increasing the filament current. So *intensity of X-rays* ∝ *Filament current*

(ii) **Control of quality or penetration power of *X*-rays :** Quality of X-rays implies the penetrating power of X-rays, which can be controlled by varying the potential difference between the cathode and the target.

For large potential difference, energy of bombarding electrons will be large and hence larger is the penetration power of X-rays.

Depending upon the penetration power, X-rays are of two types

Hard X-rays	Soft X-rays
More penetration power	Less penetration power
More frequency of the order of ≈ 10^{19} *Hz*	Less frequency of the order of ≈ 10^{16} *Hz*
Lesser wavelength range (0.1Å – 4Å)	More wavelength range (4Å – 100Å)

Note : ❑ Production of X-ray is the reverse phenomenon of photoelectric effect.

(3) **Properties of X-rays**

(i) X-rays are electromagnetic waves with wavelength range 0.1Å – 100Å.

(ii) The wavelength of X-rays is very small in comparison to the wavelength of light. Hence they carry much more energy (This is the only difference between X-rays and light)

(iii) X-rays are invisible.

(iv) They travel in a straight line with speed of light.

(v) X-rays are measured in Rontgen (measure of ionization power).

(vi) X-rays carry no charge so they are not deflected in magnetic field and electric field.

(vii) $\lambda_{Gama\ rays} < \lambda_{X\text{-}rays} < \lambda_{UV\ rays}$

(viii) They used in the study of crystal structure.

(ix) They ionise the gases

(x) X-rays do not pass through heavy metals and bones.

(xi) They affect photographic plates.

(xii) Long exposure to X-rays is injurious for human body.

(xiii) Lead is the best absorber of X-rays.

(xiv) For X-ray photography of human body parts, $BaSO_4$ is the best absorber.

(xv) They produce photoelectric effect and Compton effect

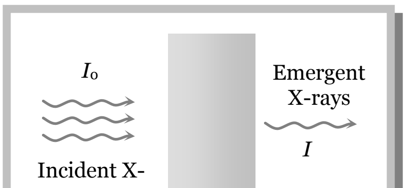
(xvi) X-rays are not emitted by hydrogen atom.

(xvii) These cannot be used in Radar because they are not reflected by the target.

(xviii) They show all the important properties of light rays like; reflection, refraction, interference, diffraction and polarization *etc*.

(4) **Absorption of X-rays**

X-rays are absorbed when they incident on substance.



Intensity of emergent X-rays $I = I_0 e^{-\mu x}$

So intensity of absorbed X-rays $I' = I_0 - I = I_0(1 - e^{-\mu x})$

where x = thickness of absorbing medium, μ = absorption coefficient

Note : ❑ The thickness of medium at which intensity of emergent X-rays becomes half *i.e.* $I' = \frac{I_0}{2}$ is called half value thickness ($x_{1/2}$) and it is given as $x_{1/2} = \frac{0.693}{\mu}$.

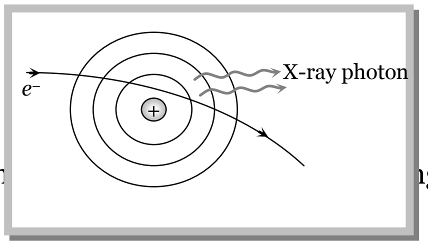
Classification of X-rays

In X-ray tube, when high speed electrons strikes the target, they penetrate the target. They loses their kinetic energy and comes to rest inside the metal. The electron before finally being stopped makes several collisions with the atoms in the target. At each collision one of the following two types of X-rays may get form.

(1) **Continuous X-rays**

As an electron passes close to the positive nucleus of atom, the electron is deflected from it's path as shown in figure. This results in deceleration of the electron. The loss in energy of the electron during deceleration is emitted in the form of X-rays.

The X-ray photons emitted so form the continuous X-ray spectrum.



Note : ❑ Continuos X-rays are produced due to the phen ng". It means slowing down or braking radiation.

Minimum wavelength

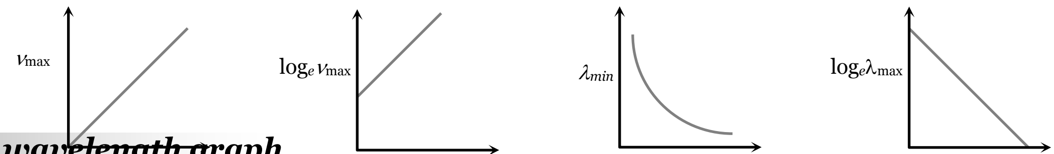
When the electron looses whole of it's energy in a single collision with the atom, an X-ray photon of maximum energy $h\nu_{max}$ is emitted *i.e.* $\frac{1}{2}mv^2 = eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$

where v = velocity of electron before collision with target atom, V = potential difference through which electron is accelerated, c = speed of light = $3 \times 10^8\ m/s$

Maximum frequency of radiations (X-rays) $\nu_{max} = \frac{eV}{h}$

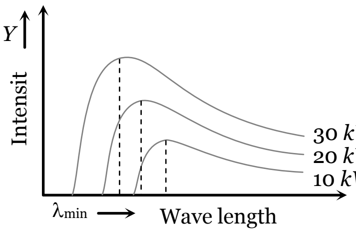
Minimum wave length = cut off wavelength of X-ray $\lambda_{min} = \frac{hc}{eV} = \frac{12375}{V}\ Å$

Note : ❑ Wavelength of continuous X-ray photon ranges from certain minimum (λ_{min}) to infinity.



Intensity wavelength graph

The continuous X-ray spectra consist of all the wavelengths over a given range. These wavelength are of different intensities. Following figure shows the intensity variation of different wavelengths for various accelerating voltages applied to X-ray tube.



For each voltage, the intensity curve starts at a particular minimum wavelength (λ_{min}). Rises rapidly to a maximum and then drops gradually.

The wavelength at which the intensity is maximum depends on the accelerating voltage, being shorter for higher voltage and vice-versa.

(2) **Characteristic X-rays**

Few of the fast moving electrons having high velocity penetrate the surface atoms of the target material and knock out the tightly bound electrons even from the inner most shells of the atom. Now when the electron is knocked out, a vacancy is created at that place. To fill this vacancy electrons from higher shells jump to fill the created vacancies, we know that when an electron jumps from a higher energy orbit E_1 to lower energy orbit E_2, it radiates energy ($E_1 - E_2$). Thus this energy difference is radiated in the form of X-rays of very small but definite wavelength which depends upon the target material. The X-ray spectrum consist of sharp lines and is called characteristic X-ray spectrum.

e^-

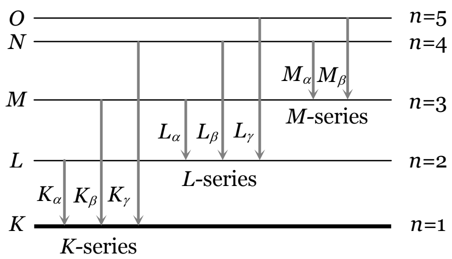
X-ray photon

e^-

e^-

K, L, M, ... series

If the electron striking the target eject an electron from the *K*-shell of the atom, a vacancy is crated in the *K*-shell. Immediately an electron from one of the outer shell, say *L*-shell jumps to the *K*-shell, emitting an X-ray photon of energy equal to the energy difference between the two shells. Similarly, if an electron from the *M*-shell jumps to the *K*-shell, X-ray photon of higher energy is emitted. The X-ray photons emitted due to the jump of electron from the *L*, *M*, *N* shells to the K-shells gives K_α, K_β, K_γ lines of the *K*-series of the spectrum.



If the electron striking the target ejects an electron from the *L*-shell of the target atom, an electron from the *M*, *N* ..... shells jumps to the *L*-shell so that X-rays photons of lesser energy are emitted. These photons form the lesser energy emission. These photons form the *L*-series of the spectrum. In a similar way the formation of *M* series, *N* series *etc.* may be explained.

Energy and wavelength of different lines

Series	Transition	Energy	Wavelength
K_α	$\underset{(2)}{L} \rightarrow \underset{(1)}{K}$	$E_L - E_K = h\nu_{K\alpha}$	$\lambda_{K\alpha} = \dfrac{hc}{E_L - E_K} = \dfrac{12375}{(E_L - E_K)eV}$ Å
K_β	$\underset{(3)}{M} \rightarrow \underset{(1)}{K}$	$E_M - E_K = h\nu_{K\beta}$	$\lambda_{K\beta} = \dfrac{hc}{E_M - E_K} = \dfrac{12375}{(E_M - E_K)eV}$ Å
L_α	$\underset{(3)}{M} \rightarrow \underset{(2)}{L}$	$E_M - E_L = h\nu_{L\alpha}$	$\lambda_{L\alpha} = \dfrac{hc}{E_M - E_L} = \dfrac{12375}{(E_M - E_L)eV}$ Å

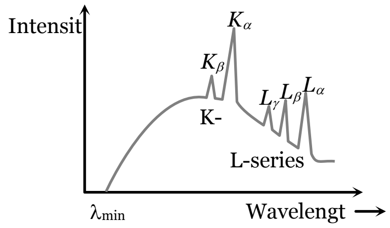
Note : ❑ The wavelength of characteristic X-ray doesn't depend on accelerating voltage. It depends on the atomic number (*Z*) of the target material.

❑ $\lambda_{K\alpha} < \lambda_{L\alpha} < \lambda_{M\alpha}$ and $\nu_{K\alpha} > \nu_{L\alpha} > \nu_{M\alpha}$

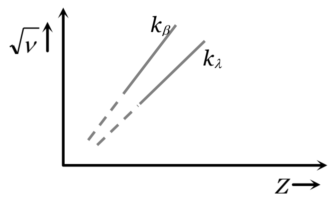
❑ $\lambda_{K\alpha} > \lambda_{L\beta} < \lambda_{K\gamma}$

Intensity-wavelength graph

At certain sharply defined wavelengths, the intensity of X-rays is very large as marked K_α, K_β .... As shown in figure. These X-rays are known as characteristic X-rays. At other wavelengths the intensity varies gradually and these X-rays are called continuous X-rays.



Mosley's law



Mosley studied the characteristic X-ray spectrum of a number of a heavy elements and concluded that the spectra of different elements are very similar and with increasing atomic number, the spectral lines merely shift towards higher frequencies.

He also gave the following relation $\sqrt{\nu} = a(Z - b)$

where ν = Frequency of emitted line, Z = Atomic number of target, a = Proportionality constant, b = Screening constant.

Note : ❑ a and b doesn't depend on the nature of target. Different values of b are as follows

$b = 1$ for K-series
$b = 7.4$ for L-series
$b = 19.2$ for M-series

- $(Z - b)$ is called effective atomic number.

More about Mosley's law

(i) It supported Bohr's theory

(ii) It experimentally determined the atomic number (Z) of elements.

(iii) This law established the importance of ordering of elements in periodic table by atomic number and not by atomic weight.

(iv) Gaps in Moseley's data for A = 43, 61, 72, 75 suggested existence of new elements which were later discovered.

(v) The atomic numbers of Cu, Ag and Pt were established to be 29, 47 and 78 respectively.

(vi) When a vacancy occurs in the K-shell, there is still one electron remaining in the K-shell. An electron in the L-shell will feel an effective charge of $(Z - 1)e$ due to $+ Ze$ from the nucleus and $- e$ from the remaining K-shell electron, because L-shell orbit is well outside the K-shell orbit.

(vii) Wave length of characteristic spectrum $\frac{1}{\lambda} = R(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ and energy of X-ray radiations.

$$\Delta E = h\nu = \frac{hc}{\lambda} = Rhc(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(viii) If transition takes place from $n_2 = 2$ to $n_1 = 1$ (K_α - line)

(a) $a = \sqrt{\frac{3RC}{4}} = 2.47 \times 10^{15}\ Hz$

(b) $\nu_{K\alpha} = RC(Z-1)^2\left(1 - \frac{1}{2^2}\right) = \frac{3RC}{4}(Z-1)^2 = 2.47 \times 10^{15}(Z-1)^2\ Hz$

(c) In general the wavelength of all the K-lines are given by $\frac{1}{\lambda_K} = R(Z-1)^2\left(1 - \frac{1}{n^2}\right)$ where n = 2, 3, 4, ...

While for K_α line $\lambda_{K\alpha} = \frac{1216}{(Z-1)}\ Å$

(d) $E_{K\alpha} = 10.2(Z-1)^2\ eV$

Uses of X-rays

(i) In study of crystal structure : Structure of DNA was also determined using X-ray diffraction.

(ii) In medical science. (iii) In radiograph

(iv) In radio therapy (v) In engineering

(vi) In laboratories (vii) In detective department

(viii) In art the change occurring in old oil paintings can be examined by X-rays.

Concepts

- *Nearly all metals emits photoelectrons when exposed to UV light. But alkali metals like lithium, sodium, potassium, rubidium and cesium emit photoelectrons even when exposed to visible light.*
- *Oxide coated filament in vacuum tubes is used to emit electrons at relatively lower temperature.*
- *Conduction of electricity in gases at low pressure takes because colliding electrons acquire higher kinetic energy due to increase in mean free path.*
- *Kinetic energy of cathode rays depends on both voltage and work function of cathode.*
- *Photoelectric effect is due to the particle nature of light.*
- *Hydrogen atom does not emit X-rays because it's energy levels are too close to each other.*
- *The essential difference between X-rays and of γ-rays is that, γ-rays emits from nucleus while X-rays from outer part of atom.*
- *There is no time delay between emission of electron and incidence of photon i.e. the electrons are emitted out as soon as the light falls on metal surface.*
- *If light were wave (not photons) it will take about an year take about an year to eject a photoelectron out of the metal surface.*
- *Doze of X-ray are measured in terms of produced ions or free energy via ionisaiton.*
- *Safe doze for human body per week is one Rontgen (One Rontgon is the amount of X-rays which emits 2.5×10^4 J free ...rough ionization of 1 gm air at NTP*

Example

Example: 22 The work function of a substance is 4.0 *eV*. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **[AIEEE 2004]**

(a) 540 *nm* (b) 400 *nm* (c) 310 *nm* (d) 220 *nm*

Solution : (c) By using $\lambda_0 = \frac{12375}{W_0(eV)} \Rightarrow \lambda_0 = \frac{12375}{4} = 3093.7 Å \simeq 310\, nm$

Example: 23 Photo-energy 6 *eV* are incident on a surface of work function 2.1 *eV*. What are the stopping potential **[MP PMT 2004]**

(a) $-5V$ (b) $-1.9\, V$ (c) $-3.9\, V$ (d) $-8.1\, V$

Solution : (c) By using Einstein's equation $E = W_0 + K_{max} \Rightarrow 6 = 2.1 + K_{\max} \Rightarrow K_{\max} = 3.9\, eV$

Also $V_0 = -\frac{K_{\max}}{\rho} = -3.9\, V.$

Example: 24 When radiation of wavelength λ is incident on a metallic surface the stopping potential is 4.8 *volts*. If the same surface is illuminated with radiation of double the wavelength, then the stopping potential becomes 1.6 *volts*. Then the threshold wavelength for the surface is

(a) 2λ (b) 4λ (c) 6λ (d) 8λ

Solution : (b) By using $V_0 = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$

$$4.8 = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \quad \text{...... (i)} \quad \text{and} \quad 1.6 = \frac{hc}{e}\left[\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right] \quad \text{...... (ii)}$$

From equation (i) and (ii) $\lambda_0 = 4\lambda$.

Example: 25 When radiation is incident on a photoelectron emitter, the stopping potential is found to be 9 volts. If *e/m* for the electron is $1.8 \times 10^{11}\, Ckg^{-1}$ the maximum velocity of the ejected electrons is

(a) $6 \times 10^5 ms^{-1}$ (b) $8 \times 10^5 ms^{-1}$ (c) $1.8 \times 10^6 ms^{-1}$ (d) $1.8 \times 10^5 ms^{-1}$

Solution : (c) $\frac{1}{2} m v_{\max}^2 = eV_0 \Rightarrow v_{\max} = \sqrt{2\left(\frac{e}{m}\right).V_0} = \sqrt{2 \times 1.8 \times 10^{11} \times 9} = 1.8 \times 10^6\, m/s$.

Example: 26 The lowest frequency of light that will cause the emission of photoelectrons from the surface of a metal (for which work function is 1.65 *eV*) will be **[JIPMER 2002]**

(a) $4 \times 10^{10}\, Hz$ (b) $4 \times 10^{11}\, Hz$ (c) $4 \times 10^{14}\, Hz$ (d) $4 \times 10^{-10}\, Hz$

Solution : (c) Threshold wavelength $\lambda_0 = \frac{12375}{W_0(eV)} = \frac{12375}{1.65} = 7500\, Å$.

$\therefore$ so minimum frequency $\nu_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{7500 \times 10^{-10}} = 4 \times 10^{14}\, Hz$.

***Example*: 27** Light of two different frequencies whose photons have energies 1 *eV* and 2.5 *eV* respectively, successively illuminates a metal of work function 0.5 *eV*. The ratio of maximum kinetic energy of the emitted electron will be

(a) 1 : 5 (b) 1 : 4 (c) 1 : 2 (d) 1 : 1

Solution : (b) By using $K_{\max} = E - W_0 \Rightarrow \frac{(K_{\max})_1}{(K_{\max})_2} = \frac{1-0.5}{2.5-0.5} = \frac{0.5}{2} = \frac{1}{4}$.

***Example*: 28** Photoelectric emission is observed from a metallic surface for frequencies ν_1 and ν_2 of the incident light rays $(\nu_1 > \nu_2)$. If the maximum values of kinetic energy of the photoelectrons emitted in the two cases are in the ratio of 1 : *k*, then the threshold frequency of the metallic surface is **[EAMCET (Engg.) 2001]**

(a) $\frac{\nu_1 - \nu_2}{k-1}$ (b) $\frac{k\nu_1 - \nu_2}{k-1}$ (c) $\frac{k\nu_2 - \nu_1}{k-1}$ (d) $\frac{\nu_2 - \nu_1}{k-1}$

Solution : (b) By using $h\nu - h\nu_0 = k_{\max} \Rightarrow h(\nu_1 - \nu_0) = k_1$ and $h(\nu_1 - \nu_0) = k_2$

Hence $\frac{\nu_1 - \nu_0}{\nu_2 - \nu_0} = \frac{k_1}{k_2} = \frac{1}{k} \Rightarrow \nu_0 = \frac{k\nu_1 - \nu_2}{k-1}$

***Example*: 29** Light of frequency $8 \times 10^{15}\, Hz$ is incident on a substance of photoelectric work function 6.125 *eV*. The maximum kinetic energy of the emitted photoelectrons is **[AFMC 2001]**

(a) 17 *eV* (b) 22 *eV* (c) 27 *eV* (d) 37 *eV*

Solution : (c) Energy of incident photon $E = h\nu = 6.6 \times 10^{-34} \times 8 \times 10^{15} = 5.28 \times 10^{-18}\, J = 33\, eV$.

From $E = W_0 + K_{\max} \Rightarrow K_{\max} = E - W_0 = 33 - 6.125 = 26.87\, eV \approx 27\, eV$.

***Example*: 30** A photo cell is receiving light from a source placed at a distance of 1 *m*. If the same source is to be placed at a distance of 2 *m*, then the ejected electron **[MNR 1986; UPSEAT 2000, 2001]**

(a) Moves with one-fourth energy as that of the initial energy
(b) Moves with one fourth of momentum as that of the initial momentum
(c) Will be half in number
(d) Will be one-fourth in number

Solution : (d) Number of photons $\propto$ Intensity $\propto \frac{1}{(\text{distance})^2}$

$\Rightarrow \frac{N_1}{N_2} = \left(\frac{d_2}{d_1}\right)^2 \Rightarrow \frac{N_1}{N_2} - \left(\frac{2}{1}\right)^2 \Rightarrow N_2 = \frac{N_1}{4}$.

***Example*: 31** When yellow light incident on a surface no electrons are emitted while green light can emit. If red light is incident on the surface then **[MNR 1998; MH CET 2000; MP PET 2000]**

(a) No electrons are emitted (b) Photons are emitted
(c) Electrons of higher energy are emitted (d) Electrons of lower energy are emitted

Solution : (a) $\lambda_{\text{Green}} < \lambda_{\text{Yellow}} < \lambda_{\text{Red}}$

According to the question λ_{Green} is the maximum wavelength for which photoelectric emission takes place. Hence no emission takes place with red light.

***Example*: 32** When a metal surface is illuminated by light of wavelengths 400 *nm* and 250 *nm* the maximum velocities of the photoelectrons ejected are *v* and 2*v* respectively. The work function of the metal is (*h* = Planck's constant, *c* = velocity of light in air) **[EMCET (Engg.) 2000]**

(a) $2hc \times 10^6\, J$ (b) $1.5hc \times 10^6\, J$ (c) $hc \times 10^6\, J$ (d) $0.5hc \times 10^6\, J$

Solution : (a) By using $E = W_0 + K_{\max} \Rightarrow \frac{hc}{\lambda} = W_0 + \frac{1}{2}mv^2$

$\frac{hc}{400 \times 10^{-9}} = W_0 + \frac{1}{2}mv^2$ ......(i) and $\frac{hc}{250 \times 10^{-9}} = W_0 + \frac{1}{2}m(2v)^2$ ......(ii)

From equation (i) and (ii) $W_0 = 2hc \times 10^6\, J$.

***Example*: 33** The work functions of metals *A* and *B* are in the ratio 1 : 2. If light of frequencies *f* and 2*f* are incident on the surfaces of *A* and *B* respectively, the ratio of the maximum kinetic energies of photoelectrons emitted is (*f* is greater than threshold frequency of *A*, 2*f* is greater than threshold frequency of *B*) **[EAMCET (Med.) 2000]**

(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

Solution : (b) By using $E = W_0 + K_{\max} \Rightarrow E_A = hf = W_A + K_A$ and $E_B = h(2f) = W_B + K_B$

So, $\frac{1}{2} = \frac{W_A + K_A}{W_B + K_B}$ ......(i) also it is given that $\frac{W_A}{W_B} = \frac{1}{2}$ ....(ii)

From equation (i) and (ii) we get $\frac{K_A}{K_B} = \frac{1}{2}$.

***Example*: 34** When a point source of monochromatic light is at a distance of 0.2*m* from a photoelectric cell, the cut-off voltage and the saturation current are 0.6 *volt* and 18 *mA* respectively. If the same source is placed 0.6 *m* away from the photoelectric cell, then **[IIT-JEE 1992; MP PMT 1999]**

(a) The stopping potential will be 0.2 *V* (b) The stopping potential will be 0.6 *V*
(c) The saturation current will be 6 *mA* (d) The saturation current will be 18 *mA*

Solution : (b) Photoelectric current (i) $\propto$ Intensity $\propto \dfrac{1}{(\text{distance})^2}$. If distance becomes 0.6 *m* (*i.e.* three times) so current becomes $\dfrac{1}{9}$ times *i.e.* 2*mA*.

Also stopping potential is independent of intensity *i.e.* it remains 0.6 *V*.

***Example*: 35** In a photoemissive cell with exciting wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be **[CBSE 1998]**

(a) $v(3/4)^{1/2}$ (b) $v(4/3)^{1/2}$ (c) Less then $v(4/3)^{1/2}$ (d) Greater then $v(4/3)^{1/2}$

Solution : (d) From $E = W_0 + \frac{1}{2}mv_{\max}^2 \Rightarrow v_{\max} = \sqrt{\dfrac{2E}{m} - \dfrac{2W_0}{m}}$ (where $E = \dfrac{hc}{\lambda}$)

If wavelength of incident light charges from λ to $\dfrac{3\lambda}{4}$ (decreases)

Let energy of incident light charges from E to E' and speed of fastest electron changes from v to v' then

$$v = \sqrt{\frac{2E}{m} - \frac{2W_0}{m}} \quad .....(i) \quad \text{and} \quad v' = \sqrt{\frac{2E'}{m} - \frac{2W_0}{m}} \quad ....(ii)$$

As $E \propto \dfrac{1}{\lambda} \Rightarrow E' = \dfrac{4}{3}E$ hence $v' = \sqrt{\dfrac{2\left(\frac{4}{3}E\right)}{m} - \dfrac{2W_0}{m}} \Rightarrow v' = \left(\dfrac{4}{3}\right)^{1/2}\sqrt{\dfrac{2E}{m} - \dfrac{2W_0}{m\left(\frac{4}{3}\right)^{1/2}}}$

$$\Rightarrow v' = \left(\frac{4}{3}\right)^{1/2} \quad X = \sqrt{\frac{2E}{m} - \frac{2W_0}{m\left(\frac{4}{3}\right)^{1/2}}} > v \text{ so } v' > \left(\frac{4}{3}\right)^{1/2} v.$$

***Example*: 36** The minimum wavelength of *X*-rays produced in a coolidge tube operated at potential difference of 40 *kV* is **[BCECE 2003]**

(a) 0.31Å (b) 3.1Å (c) 31Å (d) 311Å

Solution : (a) $\lambda_{\min} = \dfrac{12375}{40 \times 10^3} = 0.309\,Å \approx 0.31\ Å$

***Example*: 37** The *X*-ray wavelength of L_α line of platinum (Z = 78) is $1.30Å$. The X–ray wavelength of L_a line of Molybdenum (Z = 42) is **[EAMCET (Engg.) 2000]**

(a) 5.41Å (b) 4.20Å (c) 2.70Å (d) 1.35 Å

Solution : (a) The wave length of L_α line is given by $\dfrac{1}{\lambda} = R(z-7.4)^2\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) \Rightarrow \lambda \propto \dfrac{1}{(z-7.4)^2}$

Tricky example: 3

***Example*: 38** The cut off wavelength of continuous X-ray from two coolidge tubes operating at 30 *kV* but using different target materials (molybdenum *Z*= 42 and tungsten *Z* = 74) are

(a) 1Å, 3Å (b) 0.3 Å, 0.2 Å (c) 0.414 Å, 0.8 Å (d) 0.414 Å, 0.414 Å

Solution : (d) Cut off wavelength of continuous X-rays depends solely on the voltage applied and does not depend on the material of the target. Hence the two tubes will have the same cut off wavelength.

$$Ve = h\nu = \frac{hc}{\lambda} \quad \text{or} \quad \lambda = \frac{hc}{Ve} = \frac{6.627 \times 10^{-34} \times 3 \times 10^8}{30 \times 10^3 \times 1.6 \times 10^{-19}} m = 414 \times 10^{-10} m = 0.414\ Å.$$

Tricky example: 4

Two photons, each of energy $2.5 eV$ are simultaneously incident on the metal surface. If the work function of the metal is $4.5\ eV$, then from the surface of metal

(a) Two electrons will be emitted
(b) Not even a single electron will be emitted
(c) One electron will be emitted
(d) More than two electrons will be emitted

Solution : (b) Photoelectric effect is the phenomenon of one to one elastic collision between incident photon and an electron. Here in this question one electron absorbs one photon and gets energy $2.5\ eV$ which is lesser than $4.5\ eV$. Hence no photoelectron emission takes place.

Tricky example: 5

In X-ray tube when the accelerating voltage V is halved, the difference between the wavelength of K_α line and minimum wavelength of continuous X-ray spectrum

(a) Remains constant
(b) Becomes more than two times
(c) Becomes half
(d) Becomes less than two times

Solution : (c) $\Delta\lambda = \lambda_{K_\alpha} - \lambda_{\min}$ when V is halved $\lambda_{\min}$ becomes two times but λ_{K_α} remains the same.

$\therefore \quad \Delta\lambda' = \lambda_{K_\alpha} - 2\lambda_{\min} = 2(\Delta\lambda) - \lambda_{K_a}$

Tricky example: 6

Molybdenum emits K_α-photons of energy $18.5\ keV$ and iron emits K_α photons of energy $34.7\ keV$. The times taken by a molybdenum K_α photon and an iron K_α photon to travel $300\ m$ are

(a) ($3\ \mu s$, $15\ \mu s$)
(b) ($15\ \mu s$, $3 \mu s$)
(c) ($1\ \mu s$, $1\ \mu s$)
(d) ($1\ \mu s$, $5 \mu s$)

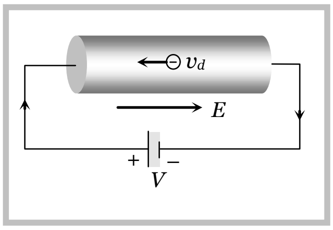
Solution : (c) Photon have the same speed whatever be their energy, frequency, wavelength, and origin.

$\therefore$ time of travel of either photon $= \dfrac{300}{3 \times 10^8} = 10^{-6}\, s = 1 \mu\, s$

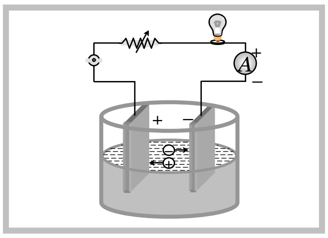
genius PHYSICS

Current Electricity

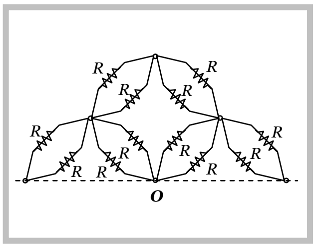
Notes by Pradeep Kshetrapal



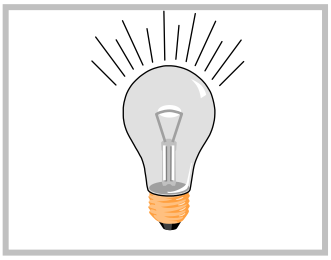
Electric current and Resistance



Cell, Kirchoff's law and Measuring instruments



Determination of resistance



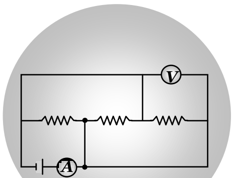
Heating effect of current

Not for circulation. Only for the students of genius PHYSICS
at Rao classes,Jamanipali, genius Physics Ghantaghar Korba, and Gupta classes Kusmunda

Formulas in current electricity (Direct Current)

1	Electric Current	i=q/t	"q" is charge passing in normal direction through a cross section of conductor in time "t"
2	Drift velocity V_d with Electric field	$V_d = \frac{-eE\tau}{m}$	e is charge and m is mass on electron, E is electric field, τ is relaxation time.
3	Current I with Drift velocity V_d	$I = n\,e\,A\,V_d$	n is number density with of free electrons, A is area of cross section.
4	Mobility of charge "μ"	$\mu = V_d / E = \frac{q\tau}{m}$	
5	Mobility and drift velocity	$V_d = \mu_e E$	
6	Current and Mobility	$I = Ane \times \mu_e E$	
7	Resistance, P.D., and Current	$R = V / I$	V Potential Difference, I Current .
8	Resistance R with specific Res.	$R = \rho \frac{l}{A}$	l is length of conductor and A is area of cross section
9	Specific Resistance, ρ	$\rho = R \frac{A}{l}$	
10	Resistivity with electrons	$\rho = \frac{m}{ne^2\tau}$	
11	Current density J	$\vec{J} = I / \vec{A}$	I is current, J current density, A is area of cross section
12	Current density magnitude	$J\,A\cos\theta = I$	θ is angle between $\vec{J}$ and $\vec{A}$
13	Conductance G	$G = 1/R$	
14	Conductivity σ	$\sigma = 1/\rho$	ρ is specific resistance
15	Microscopic form of Ohms Law	$J = \sigma E$	E is electric field
16	Temperature coefficient of Resistance α	$\alpha = \frac{Rt-R_o}{R_o \times t}$	R_o is resistance at 0_o C. R_t is resistance at t^o and "t" is temperature difference.
17	Resistances in series	$R = R_1+R_2+R_3$	Same current through all resistances (circuit Current
	Resistances in parallel	$1/R_e = 1/R_1 + 1/R_2 + 1/R_3$	Same P.D. across each resistance (V of cell)
18	In a cell, emf and internal resistance	$I = \frac{E}{R+r}$	I is current, E is emf, R is external resistance, r is internal resistance.
19	In a circuit with a cell	$V = E - Ir$	V is terminal potential difference
20	n Cells of emf E in series	$Emf = nE$	
21	Resistance of n cells in series	$nr + R$	r is internal resistance of one cell, R external Resistance
22	Current in circuit with n cells in series	$I = \frac{nE}{R+nr}$	r is internal resistance of one cell, R external Resistance
23	n cells in parallel, then emf	$emf = E$	
24	n cells in parallel, resistance	$R + r/n$	R external resistance, r internal resistance
25	Cells in mixed group, condition for maximum current	$R = \frac{nr}{m}$	n is number of cells in one row, m is number of rows. r is internal resistance, R external resis.
26	Internal resistance of a cell	$r = (\frac{E-V}{V}) \times R$	E is emf, V is terminal Potential difference, R is external resistance.
27	Power of a circuit	$P = I.V = I^2R = V^2/R$	

28	Energy consumed	E = I.V.ΔT	ΔT is time duration
29	Kirchoff Law (junction rule)	$\Sigma i = 0$	Sum of currents at junction is zero.
30	Kirchoff Law (Loop rule)	$\Sigma V = 0$	In a loop sum of all p.d.*s* is Zero

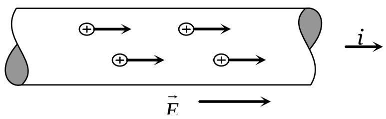


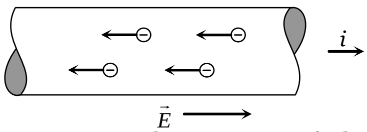
Current Electricity

Electric Current

(1) **Definition :** The time rate of flow of charge through any cross-section is called current. So if through a cross-section, ΔQ charge passes in time Δt then $i_{av} = \frac{\Delta Q}{\Delta t}$ and instantaneous current $i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$. If flow is uniform then $i = \frac{Q}{t}$. Current is a scalar quantity. It's S.I. unit is *ampere* (A) and C.G.S. unit is *emu* and is called *biot* (Bi), or *ab ampere*. $1A = (1/10)\ Bi$ (*ab amp.*)

(2) **The direction of current :** The conventional direction of current is taken to be the direction of flow of positive charge, *i.e.* field and is opposite to the direction of flow of negative charge as shown below.

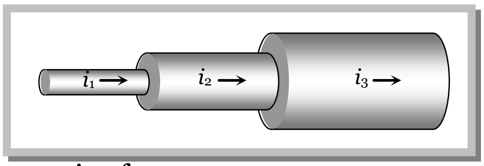




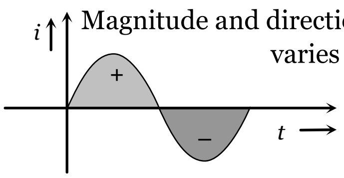
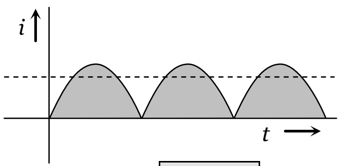
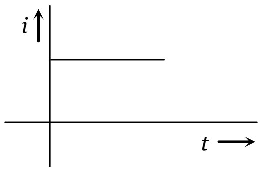
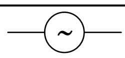
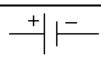
Though conventionally a direction is associated with current (Opposite to the motion of electron), it is not a vector. It is because the current can be added algebraically. Only scalar quantities can be added algebraically not the vector quantities.

(3) **Charge on a current carrying conductor :** In conductor the current is caused by electron (free electron). The no. of electron (negative charge) and proton (positive charge) in a conductor is same. Hence the net charge in a current carrying conductor is zero.

(4) **Current through a conductor of non-uniform cross-section :** For a given conductor current does not change with change in cross-sectional area. In the following figure $i_1 = i_2 = i_3$



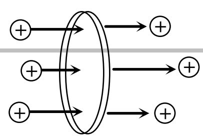
(5) **Types of current :** Electric current is of two type :

Alternating current (ac)	Direct current (dc)
(i)  ac → $\boxed{\text{Rectifier}}$ → dc	(i) (Pulsating dc)  (Constant dc)  dc → $\boxed{\text{Inverter}}$ → ac
(ii) Shows heating effect only	(ii) Shows heating effect, chemical effect and magnetic effect of current
(iii) It's symbol is 	(iii) It's symbol is 

Note : ❑ In our houses ac is supplied at $220V$, $50Hz$.

(6) **Current in difference situation :**

(i) **Due to translatory motion of charge**



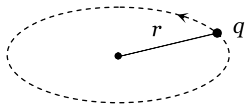
In n particle each having a charge q, pass through a given area in time t then $\boldsymbol{i} = \frac{\boldsymbol{nq}}{\boldsymbol{t}}$

If n particles each having a charge q pass per second per unit area, the current associated with cross-sectional area A is $\boldsymbol{i = nqA}$

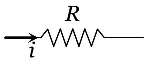
If there are n particle per unit volume each having a charge q and moving with velocity v, the current thorough, cross section A is $\boldsymbol{i = nqvA}$, for electrons i= $\mathbf{neav_d}$

(ii) **Due to rotatory motion of charge**

If a point charge q is moving in a circle of radius r with speed v (frequency ν, angular speed ω and time period T) then corresponding currents $\boldsymbol{i = q\,\nu = \frac{q}{T} = \frac{qv}{2\pi r} = \frac{q\,\omega}{2\pi}}$



(iii) **When a voltage V applied across a resistance R** : Current flows through the conductor $i = \frac{V}{R}$ also by definition of power $i = \frac{P}{V}$



(7) **Current carriers :** The charged particles whose flow in a definite direction constitutes the electric current are called current carriers. In different situation current carriers are different.

(i) Solids : In solid conductors like metals current carriers are free electrons.

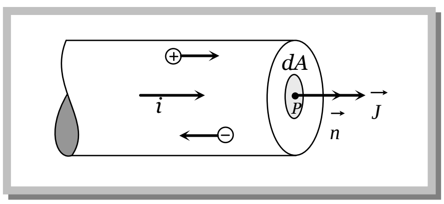
(ii) Liquids : In liquids current carriers are positive and negative ions.

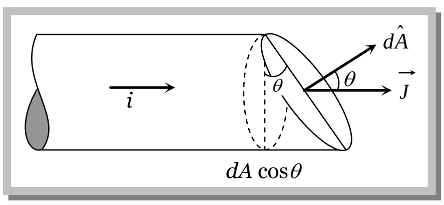
(iii) Gases : In gases current carriers are positive ions and free electrons.

(iv) Semi conductor : In semi conductors current carriers are holes and free electrons.

Current density (*J*)

In case of flow of charge through a cross-section, current density is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point. Current density at point P is given by $\vec{J} = \frac{di}{dA}\vec{n}$





If the cross-sectional area is not normal to the current, the cross-sectional area normal to current in accordance with following figure will be $dA\cos\theta$ and so in this situation:

$$J = \frac{di}{dA\cos\theta} \quad \text{i.e. } di = JdA\cos\theta \text{ or } di = \vec{J}.\vec{dA} \Rightarrow i = \int \vec{J}\cdot\vec{dA}$$

i.e., in terms of current density, current is the flux of current density.

Note : ❑ If current density $\vec{J}$ is uniform for a normal cross-section $\vec{A}$ then : $i = \int \vec{J}\cdot\vec{ds} = \vec{J}\cdot\int\vec{ds}$ [as $\vec{J}$ = constant]

or $i = \vec{J}\cdot\vec{A} = JA\cos 0 = JA \Rightarrow \boldsymbol{J = \frac{i}{A}}$ [as $\int\vec{dA} = \vec{A}$ and $\theta = 0^{\circ}$]

(1) **Unit and dimension :** Current density $\vec{J}$ is a vector quantity having S.I. unit *Amp/m*² and dimension.[$L^{-2}A$]

(2) **Current density in terms of velocity of charge** : In case of uniform flow of charge through a cross-section normal to it as $i = nqvA$ so, $\vec{J} = \frac{i}{A}\vec{n} = (nqv)\vec{n}$ or $\vec{J} = nq\vec{v} = \vec{v}\,(\rho)$ [With $\rho = \frac{\text{charge}}{\text{volume}} = nq$]

i.e., current density at a point is equal to the product of volume charge density with velocity of charge distribution at that point.

(3) **Current density in terms of electric field :** Current density relates with electric field as $\boldsymbol{J} = \sigma\boldsymbol{E} = \frac{\boldsymbol{E}}{\rho}$; where σ = conductivity and ρ = resistivity or specific resistance of substance.

(i) Direction of current density $\vec{J}$ is same as that of electric field $\vec{E}$.

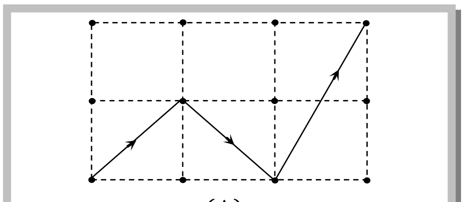
(ii) If electric field is uniform (*i.e.* $\vec{E} = \text{constant}$) current density will be constant [as σ = constant]

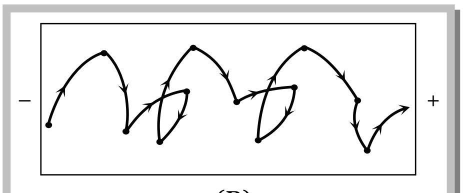
(iii) If electric field is zero (as in electrostatics inside a conductor), current density and hence current will be zero.

Conduction of Current in Metals

According to modern views, a metal consists of a '*lattice*' of fixed positively charged ions in which billions and billions of free electrons are moving randomly at speed which at room temperature (*i.e.* 300 *K*) in accordance with kinetic theory of gases is given by $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\times(1.38\times10^{-23})\times300}{9.1\times10^{-31}}} \simeq 10^5\ m/s$

The randomly moving free electrons inside the metal collide with the lattice and follow a zig-zag path as shown in figure (A).


(A)



(B)

However, in absence of any electric field due to this random motion, the number of electrons crossing from left to right is equal to the number of electrons crossing from right to left (otherwise metal will not remain equipotential) so the net current through a cross-section is zero.

A motion of charge is possible by motion of electron or a current carrier.

<u>Velocities of charged particle (electron) in a conductor</u>

<u>thermal velocity</u> : All electrons in the atom are not capable of motion. Only a few which have little higher level of energy leave their orbit and are capable of moving around. These electrons are called "free electrons". These free electrons are in very large quantity $\approx 10^{29}\ m^{-3}$ in free metals. Due to temperature and thermal energy They have a *<u>thermal velocity</u>* $\approx 10^5\ ms^{-1}$. This velocity is in all directions and of magnitudes varying from zero to maximum. Due to large number of electrons we can assume that vector sum of thermal velocities at any instant is zero.

i.e. $\vec{u_1} + \vec{u_2} + \vec{u_3} + \ldots\ldots + u_n = 0$

<u>Mean Free path</u> : The fast moving electrons keep striking other atoms/ions in the conductor. They are reflected and move in other direction. They keep moving till they strike another ion/atom.

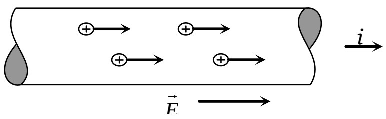
The path between two consecutive collisions is called free path. The average length of these free paths is called "Mean Free Path".

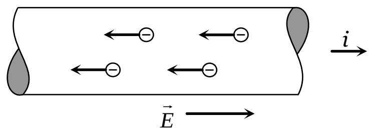
<u>*Relaxation Time*</u> :The time to travel mean free path is called Relaxation Period or Relaxation Time, denoted by Greek letter Tau "τ". If $t_1, t_2, ...t_n$ are the time periods for n collisions then Relaxation Time $\tau = \frac{1}{n}(t_1+t_2+...t_n)$

<u>***Drift Velocity***</u> :When Electric Field is applied across a conductor, the free electrons experience a force in the direction opposite to field. Dur to this force they start drifting in the direction of force. The Velocity of this drift is called drift velocity "V_d". During the drift they maintain their thermal velocity.
The drift velocity can be calculated as averaged velocity of all the electrons drifting.

<u>***Relation between drift-velocity (V_d) and electric field applied.***</u>

When electric field is applied across a conductor each electron experience a Force $\vec{F} = q\vec{E}$ in the direction of $\vec{E}$. It acquires an acceleration $\alpha = \frac{e\vec{E}}{m}$ where e is charge on electron and m is its mass.





If n electrons are having initial speeds $u_1, u_2,...u_n$ and their time to travel free path is $t_1, t_2, ...t_n$ then final velocities are

$v_1 = u_1 + \alpha t_1$,

$V_2 = u_2 + \alpha t_2$,

$V_n = u_n + \alpha t_n$ and so on.

Drift velocity is average of these velocities of charged particles. Therefore

$$
\begin{aligned}
V_d &= \frac{1}{n}(v_1 + V_2 + ... V_n) \\
&= \frac{1}{n}(u_1 + \alpha t_1 + u_2 + \alpha t_2 + ... u_n + \alpha t_n) \\
&= \frac{1}{n}(u_1 + u_2 + ... u_n + \alpha t_1 + \alpha t_2 + ... \alpha t_n) \\
&= (u_1 + u_2 + ... u_n) + \frac{1}{n}(\alpha t_1 + \alpha t_2 + ... \alpha t_n) \\
&= 0 + \alpha \frac{1}{n}(t_1 + t_2 + ... t_n) \\
&= \alpha \tau
\end{aligned}
$$

or $V_d = \frac{e\vec{E}\tau}{m}$ $\left(\alpha = \frac{e\vec{E}}{m}\right)$

$$\boxed{V_d = \frac{e\vec{E}\tau}{m}}$$

<u>**Relation of Current and Drift velocity :**</u> When an electric field is applied, inside the conductor due to electric force the path of electron in general becomes curved (parabolic) instead of straight lines and electrons drift opposite to the field figure (B). Due to this drift the random motion of electrons get modified and there is a net transfer of electrons across a cross-section resulting in current.

Drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for current through it. Drift velocity is very small it is of the order of 10^{-4} *m/s* as compared to thermal speed $(\simeq 10^5\ m/s)$ of electrons at room temperature.

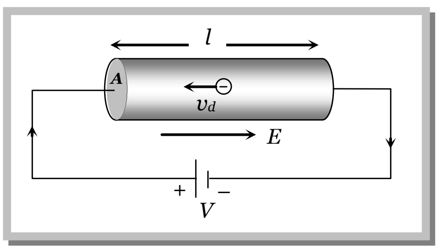
If suppose for a conductor

n = Number of electron per unit volume of the conductor

A = Area of cross-section

V = potential difference across the conductor

E = electric field inside the conductor



i = current, *J* = current density, ρ = specific resistance, σ = conductivity $\left(\sigma = \frac{1}{\rho}\right)$ then current relates with drift velocity as $\boldsymbol{i = neAv_d}$ we can also write $\boldsymbol{v_d = \frac{i}{neA} = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{E}{\rho ne} = \frac{V}{\rho l n e}}$.

Note : ❑ The direction of drift velocity for electron in a metal is opposite to that of applied electric field (*i.e.* current density $\vec{J}$).

❑ $v_d \propto E$ *i.e.*, greater the electric field, larger will be the drift velocity.

❑ When a steady current flows through a conductor of non-uniform cross-section drift velocity varies inversely with area of cross-section $\left(v_d \propto \frac{1}{A}\right)$

❑ If diameter of a conductor is doubled, then drift velocity of electrons inside it will not change.

(2) **Relaxation time (τ) :** The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time $\tau = \frac{\text{mean free path}}{\text{r.m.s. velocity of electrons}} = \frac{\lambda}{v_{rms}}$ with rise in temperature v_{rms} increases consequently τ decreases.

(3) **Mobility :** Drift velocity per unit electric field is called mobility of electron *i.e.* $\mu = \frac{v_d}{E}$. It's unit is $\frac{m^2}{volt - \sec}$.

Concepts

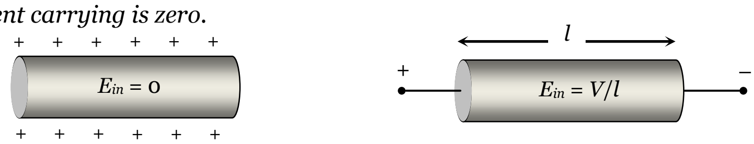
☞ *Human body, though has a large resistance of the order of kΩ (say 10 kΩ), is very sensitive to minute currents even as low as a few mA. Electrocution, excites and disorders the nervous system of the body and hence one fails to control the activity of the body.*

☞ *1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross-section of the conductors.*

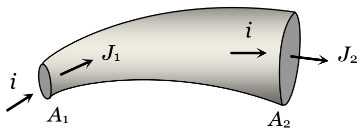
☞ *dc flows uniformly throughout the cross-section of conductor while ac mainly flows through the outer surface area of the conductor. This is known as skin effect.*

☞ *It is worth noting that electric field inside a charged conductor is zero, but it is non zero inside a current carrying conductor and is given by $E = \frac{V}{l}$ where V = potential difference across the conductor and l = length of the conductor. Electric field out side the current carrying is zero.*

☞



☞ *For a given conductor JA = i = constant so that $J \propto \frac{1}{A}$ i.e., $J_1A_1 = J_2A_2$; this is called equation of continuity*



☞ *If cross-section is constant, $I \propto J$ i.e. for a given cross-sectional area, greater the current density, larger will be current.*

☞ *The drift velocity of electrons is small because of the frequent collisions suffered by electrons.*

☞ *The small value of drift velocity produces a large amount of electric current, due to the presence of extremely large number of free electrons in a conductor. The propagation of current is almost at the speed of light and involves electromagnetic process. It is due to this reason that the electric bulb glows immediately when switch is on.*

☞ *In the absence of electric field, the paths of electrons between successive collisions are straight line while in presence of electric field the paths are generally curved.*

☞ *Free electron density in a metal is given by $n = \frac{N_A x\, d}{A}$ where N_A = Avogrado number, x = number of free electrons per atom, d = density of metal and A = Atomic weight of metal.*

Example

***Example*: 1** The potential difference applied to an X-ray tube is 5 KV and the current through it is 3.2 mA. Then the number of electrons striking the target per second is

(a) 2×10^{16} (b) 5×10^{6} (c) 1×10^{17} (d) 4×10^{15}

Solution : (a) $i = \dfrac{q}{t} = \dfrac{ne}{t}$ $\Rightarrow n = \dfrac{it}{e} = \dfrac{3.2 \times 10^{-3} \times 1}{1.6 \times 10^{-19}} = 2 \times 10^{16}$

***Example*: 2** A beam of electrons moving at a speed of 10^6 m/s along a line produces a current of 1.6×10^{-6} A. The number of electrons in the 1 *metre* of the beam is **[CPMT 2000]**

(a) 10^6 (b) 10^7 (c) 10^{13} (d) 10^{19}

Solution : (b) $i = \dfrac{q}{t} = \dfrac{q}{(x/v)} = \dfrac{qv}{x} = \dfrac{nev}{x} \Rightarrow n = \dfrac{ix}{ev} = \dfrac{1.6 \times 10^{-6} \times 1}{1.6 \times 10^{-19} \times 10^{6}} = 10^7$

***Example*: 3** In the Bohr's model of hydrogen atom, the electrons moves around the nucleus in a circular orbit of a radius 5×10^{-11} *metre*. It's time period is 1.5×10^{-16} *sec*. The current associated is

(a) Zero (b) 1.6×10^{-19} A (c) 0.17 A (d) 1.07×10^{-3} A

Solution : (d) $i = \dfrac{q}{T} = \dfrac{1.6 \times 10^{-19}}{1.5 \times 10^{-16}} = 1.07 \times 10^{-3}\, A$

***Example*: 4** An electron is moving in a circular path of radius 5.1×10^{-11} m at a frequency of 6.8×10^{15} revolution/sec. The equivalent current is approximately

(a) 5.1×10^{-3} A (b) 6.8×10^{-3} A (c) 1.1×10^{-3} A (d) 2.2×10^{-3} A

Solution : (c) $\nu = 6.8 \times 10^{15} \Rightarrow T = \dfrac{1}{6.8 \times 10^{15}}$ sec $\Rightarrow i = \dfrac{Q}{T} = 1.6 \times 10^{-19} \times 6.8 \times 10^{15} = 1.1 \times 10^{-3}\, A$

***Example*: 5** A copper wire of length 1m and radius 1mm is joined in series with an iron wire of length 2m and radius 3mm and a current is passed through the wire. The ratio of current densities in the copper and iron wire is

[MP PMT 1994]

(a) 18 : 1 (b) 9 : 1 (c) 6 : 1 (d) 2 : 3

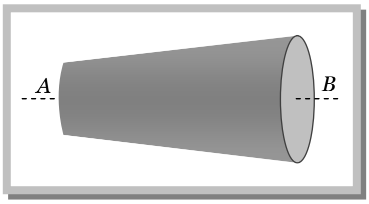
Solution : (b) We know $J = \dfrac{i}{A}$ when i = constant $J \propto \dfrac{1}{A} \Rightarrow \dfrac{J_c}{J_i} = \dfrac{A_i}{A_c} = \left(\dfrac{r_i}{r_c}\right)^2 = \left(\dfrac{3}{1}\right)^2 = \dfrac{9}{1}$

***Example*: 6** A conducting wire of cross-sectional area 1 cm^2 has 3×10^{23} m^{-3} charge carriers. If wire carries a current of 24 mA, the drift speed of the carrier is **[UPSEAT 2001]**

(a) 5×10^{-6} m/s (b) 5×10^{-3} m/s (c) 0.5 m/s (d) 5×10^{-2} m/s

Solution : (b) $v_d = \dfrac{i}{neA} = \dfrac{24 \times 10^{-3}}{3 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{-4}} = 5 \times 10^{-3}\, m/s$

***Example*: 7** A wire has a non-uniform cross-sectional area as shown in figure. A steady current i flows through it. Which one of the following statement is correct



(a) The drift speed of electron is constant (b) The drift speed increases on moving from A to B

(c) The drift speed decreases on moving from A to B (d) The drift speed varies randomly

Solution : (c) For a conductor of non-uniform cross-section $v_d \propto \frac{1}{\text{Area of cross - section}}$

***Example*: 8** In a wire of circular cross-section with radius r, free electrons travel with a drift velocity v, when a current i flows through the wire. What is the current in another wire of half the radius and of the some material when the drift velocity is $2v$

(a) $2i$ (b) i (c) $i/2$ (d) $i/4$

Solution : (c) $i = neAv_d = ne\pi r^2 v$ and $i' = ne\pi\left(\frac{r}{2}\right)^2 .2v = \frac{ne\pi r^2 v}{2} = \frac{i}{2}$

***Example*: 9** A potential difference of V is applied at the ends of a copper wire of length l and diameter d. On doubling only d, drift velocity

(a) Becomes two times (b) Becomes half (c) Does not change (d) Becomes one fourth

Solution : (c) Drift velocity doesn't depends upon diameter.

***Example*: 10** A current flows in a wire of circular cross-section with the free electrons travelling with a mean drift velocity v. If an equal current flows in a wire of twice the radius new mean drift velocity is

(a) v (b) $\frac{v}{2}$ (c) $\frac{v}{4}$ (d) None of these

Solution : (c) By using $v_d = \frac{i}{neA} \Rightarrow v_d \propto \frac{1}{A} \Rightarrow v' = \frac{v}{4}$

***Example*: 11** Two wires A and B of the same material, having radii in the ratio 1 : 2 and carry currents in the ratio 4 : 1. The ratio of drift speeds of electrons in A and B is

(a) 16 : 1 (b) 1 : 16 (c) 1 : 4 (d) 4 : 1

Solution : (a) As $i = neA\,v_d \Rightarrow \frac{i_1}{i_2} = \frac{A_1}{A_2} \times \frac{v_{d_1}}{v_{d_2}} = \frac{r_1^2}{r_2^2} . \frac{v_{d_1}}{v_{d_2}} \Rightarrow \frac{v_{d_1}}{v_{d_2}} = \frac{16}{1}$

Tricky example: 1

In a neon discharge tube 2.9×10^{18} Ne^+ ions move to the right each second while 1.2×10^{18} electrons move to the left per second. Electron charge is 1.6×10^{-19} C. The current in the discharge tube **[MP PET 1999]**

(a) 1 A towards right (b) 0.66 A towards right (c) 0.66 A towards left (d) Zero

Solution: (b) Use following trick to solve such type of problem.

Trick : In a discharge tube positive ions carry q units of charge in t seconds from anode to cathode and negative carriers (electrons) carry the same amount of charge from cathode to anode in t′ second. The current in the tube is $\boldsymbol{i = \frac{q}{t} + \frac{q'}{t'}}$.

Hence in this question current $i = \frac{2.9 \times 10^{18} \times e}{1} + \frac{1.2 \times 10^{18} \times e}{1} = 0.66A$ towards right.

Tricky example: 2

If the current flowing through copper wire of 1 *mm* diameter is 1.1 *amp*. The drift velocity of electron is (Given density of *Cu* is 9 gm/cm^3, atomic weight of *Cu* is 63 *grams* and one free electron is contributed by each atom)

(a) 0.1 *mm/sec* (b) 0.2 *mm/sec* (c) 0.3 *mm/sec* (d) 0.5 *mm/sec*

Solution: (a) 6.023×10^{23} atoms has mass $= 63 \times 10^{-3}$ *kg*

So no. of atoms per m^3 = $n = \dfrac{6.023\times10^{23}}{63\times10^{-3}}\times 9\times10^{3} = 8.5\times10^{28}$

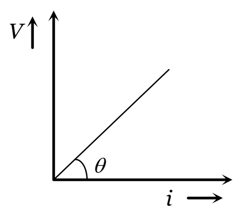
$$v_d = \frac{i}{neA} = \frac{1.1}{8.5\times10^{28}\times1.6\times10^{-19}\times\pi\times(0.5\times10^{-3})^2} = 0.1\times10^{-3}\ m/\sec = 0.1\,mm/\sec$$

Ohm's Law

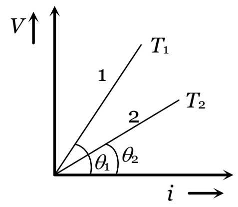
If the physical circumstances of the conductor (length, temperature, mechanical strain etc.) remains constant, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends *i.e.* $\boldsymbol{i \propto V}$

$\Rightarrow \boldsymbol{V = iR}$ or $\dfrac{V}{i} = R$; where R is a proportionality constant, known as electric resistance.

(1) Ohm's law is not a universal law, the substance which obeys ohm's law are known as ohmic substance for such ohmic substances graph between V and i is a straight line as shown. At different temperatures V-i curves are different.

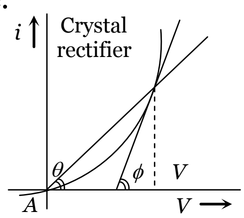


Slope of the line = $\tan\theta = \dfrac{V}{i} = R$



Here $\tan\theta_1 > \tan\theta_2$
So $R_1 > R_2$ *i.e.* $T_1 > T_2$

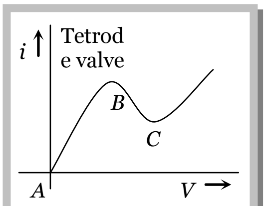
(2) The device or substances which doesn't obey ohm's law *e.g.* gases, crystal rectifiers, thermoionic valve, transistors etc. are known as non-ohmic or non-linear conductors. For these V-i curve is not linear. In these situation the ratio between voltage and current at a particular voltage is known as static resistance. While the rate of change of voltage to change in current is known as dynamic resistance.



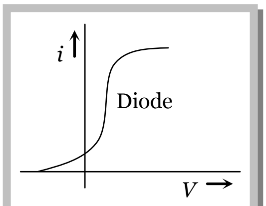
$$R_{st} = \frac{V}{i} = \frac{1}{\tan\theta}$$

while $R_{dyn} = \dfrac{\Delta V}{\Delta I} = \dfrac{1}{\tan\phi}$

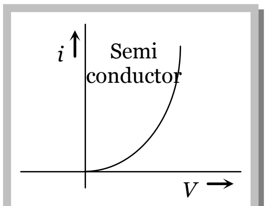
(3) Some other non-ohmic graphs are as follows :



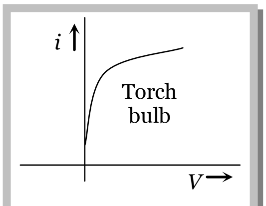
(A)



(B)



(C)



(D)

Resistance

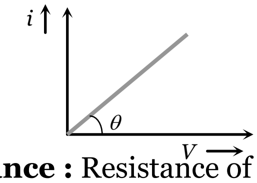
(1) **Definition :** The property of substance by virtue of which it opposes the flow of current through it, is known as the resistance.

(2) **Cause of resistance of a conductor :** It is due to the collisions of free electrons with the ions or atoms of the conductor while drifting towards the positive end of the conductor.

(3) **Formula of resistance :** For a conductor if l = length of a conductor A = Area of cross-section of conductor, n = No. of free electrons per unit volume in conductor, τ = relaxation time then resistance of conductor $\boldsymbol{R} = \rho\dfrac{\boldsymbol{l}}{\boldsymbol{A}} = \dfrac{\boldsymbol{m}}{\boldsymbol{ne^2\tau}}.\dfrac{\boldsymbol{l}}{\boldsymbol{A}}$; where ρ = resistivity of the material of conductor

(4) **Unit and dimension :** It's S.I. unit is *Volt/Amp.* or *Ohm* (Ω). Also 1 *ohm* $= \dfrac{1 volt}{1 Amp} = \dfrac{10^8 \, emu \text{ of potential}}{10^{-1} \, emu \text{ of current}}$ = 10^9 *emu* of resistance. It's dimension is $[ML^2T^{-3}A^{-2}]$.

(5) **Conductance (*C*) :** Reciprocal of resistance is known as conductance. $C = \dfrac{1}{R}$ It's unit is $\dfrac{1}{\Omega}$ or Ω^{-1} or "Siemen".

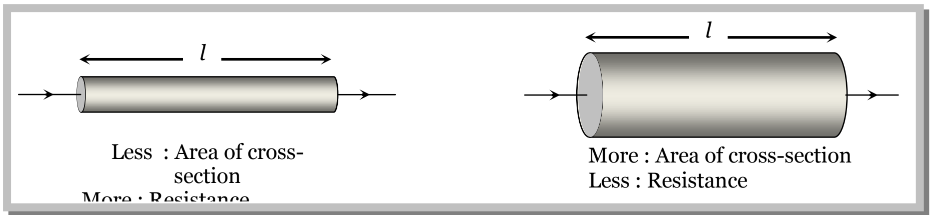


Slope = $\tan\theta = \dfrac{i}{V} = \dfrac{1}{R} = C$

(6) **Dependence of resistance :** Resistance of a conductor depends on the following factors.

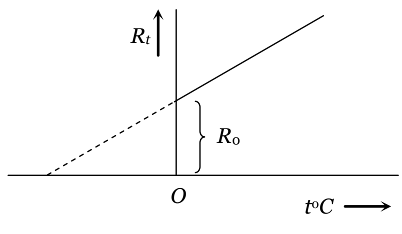
(i) Length of the conductor : Resistance of a conductor is directly proportional to it's length *i.e.* $\boldsymbol{R \propto l}$ *e.g.* a conducting wire having resistance R is cut in n equal parts. So resistance of each part will be $\dfrac{R}{n}$.

(ii) Area of cross-section of the conductor : Resistance of a conductor is inversely proportional to it's area of cross-section *i.e.* $R \propto \dfrac{1}{A}$



(iii) Material of the conductor : Resistance of conductor also depends upon the nature of material *i.e.* $R \propto \dfrac{1}{n}$, for different conductors n is different. Hence R is also different.

(iv) Temperature : We know that $R = \dfrac{m}{ne^2\tau}.\dfrac{l}{A} \Rightarrow R \propto \dfrac{l}{\tau}$ when a metallic conductor is heated, the atom in the metal vibrate with greater amplitude and frequency about their mean positions. Consequently the number of collisions between free electrons and atoms increases. This reduces the relaxation time τ and increases the value of resistance R *i.e.* for a conductor **Resistance $\propto$ temperature.**



If R_0 = resistance of conductor at 0°*C*

R_t = resistance of conductor at t°*C*

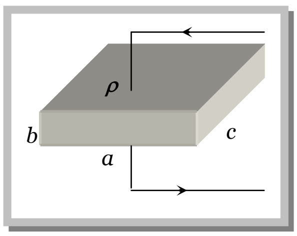
and α, β = temperature co-efficient of resistance (unit → per°*C*)

then $R_t = R_0(1 + \alpha t + \beta t^2)$ for $t > 300°C$ and $\boldsymbol{R_t = R_0(1 + \alpha t)}$ for $t \le 300°C$ or $\alpha = \dfrac{R_t - R_0}{R_0 \times t}$

Note : ❑ If R_1 and R_2 are the resistances at t_1°*C* and t_2°*C* respectively then $\dfrac{R_1}{R_2} = \dfrac{1 + \alpha t_1}{1 + \alpha t_2}$.

- The value of α is different at different temperature. Temperature coefficient of resistance averaged over the temperature range $t_1°C$ to $t_2°C$ is given by $\alpha = \dfrac{R_2 - R_1}{R_1(t_2 - t_1)}$ which gives $R_2 = R_1 [1 + \alpha(t_2 - t_1)]$. This formula gives an approximate value.

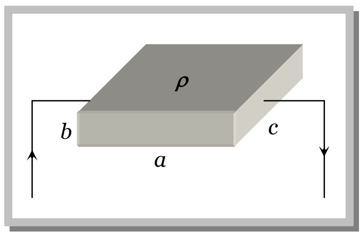
(v) **Resistance according to potential difference :** Resistance of a conducting body is not unique but depends on it's length and area of cross-section *i.e.* how the potential difference is applied. See the following figures



Length = b

Area of cross-section = $a \times c$

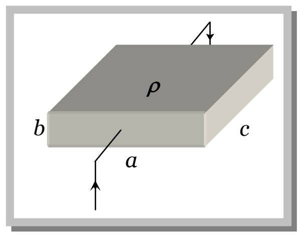
Resistance $R = \rho\left(\dfrac{b}{a \times c}\right)$



Length = a

Area of cross-section = $b \times c$

Resistance $R = \rho\left(\dfrac{a}{b \times c}\right)$



Length = c

Area of cross-section = $a \times b$

Resistance $R = \rho\left(\dfrac{c}{a \times b}\right)$

(7) **Variation of resistance of some electrical material with temperature :**

(i) Metals : For metals their temperature coefficient of resistance $\alpha > 0$. So resistance increases with temperature.

Physical explanation : Collision frequency of free electrons with the immobile positive ions increases

(ii) Solid non-metals : For these $\alpha = 0$. So resistance is independence of temperature.

Physical explanation : Complete absence of free electron.

(iii) Semi-conductors : For semi-conductor $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : Covalent bonds breaks, liberating more free electron and conduction increases.

(iv) Electrolyte : For electrolyte $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : The degree of ionisation increases and solution becomes less viscous.

(v) Ionised gases : For ionised gases $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : Degree of ionisation increases.

(vi) Alloys : For alloys α has a small positive values. So with rise in temperature resistance of alloys is almost constant. Further alloy resistances are slightly higher than the pure metals resistance.

Alloys are used to made standard resistances, wires of resistance box, potentiometer wire, meter bridge wire *etc.*

Commonly used alloys are : Constantan, mangnin, Nichrome etc.

(vii) Super conductors : At low temperature, the resistance of certain substances becomes exactly zero. (*e.g.* *Hg* below 4.2 *K* or *Pb* below 7.2 *K*).

These substances are called super conductors and phenomenon super conductivity. The temperature at which resistance becomes zero is called critical temperature and depends upon the nature of substance.

Resistivity or Specific Resistance (ρ)

(1) **Definition :** From $R = \rho \frac{l}{A}$; If $l = 1m$, $A = 1\ m^2$ then $\boldsymbol{R = \rho}$ *i.e.* resistivity is numerically equal to the resistance of a substance having unit area of cross-section and unit length.

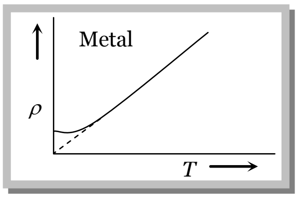
(2) **Unit and dimension :** It's S.I. unit is $ohm \times m$ and dimension is $[ML^3T^{-3}A^{-2}]$

(3) **It's formula** : $\rho = \frac{m}{ne^2\tau}$

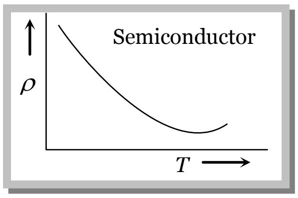
(4) **It's dependence :** Resistivity is the intrinsic property of the substance. It is independent of shape and size of the body (*i.e.* l and A). It depends on the followings :

(i) Nature of the body : For different substances their resistivity also different *e.g.* ρ_{silver} = minimum = $1.6 \times 10^{-8}\ \Omega\text{-}m$ and $\rho_{\text{fused quartz}}$ = maximum $\approx 10^{16}\ \Omega\text{-}m$

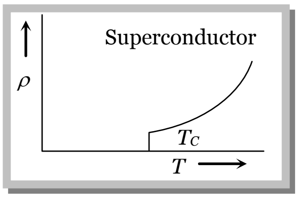
(ii) Temperature : Resistivity depends on the temperature. For metals $\rho_t = \rho_0\,(1 + \alpha\Delta t)$ *i.e.* resitivity increases with temperature.



ρ increases with temperature



ρ decreases with temperature



ρ decreases with temperature and becomes zero at a certain temperature

(iii) Impurity and mechanical stress : Resistivity increases with impurity and mechanical stress.

(iv) Effect of magnetic field : Magnetic field increases the resistivity of all metals except iron, cobalt and nickel.

(v) Effect of light : Resistivity of certain substances like selenium, cadmium, sulphides is inversely proportional to intensity of light falling upon them.

(5) **Resistivity of some electrical material :** $\rho_{\text{insulator}} > \rho_{\text{alloy}} > \rho_{\text{semi-conductor}} > \rho_{\text{conductor}}$

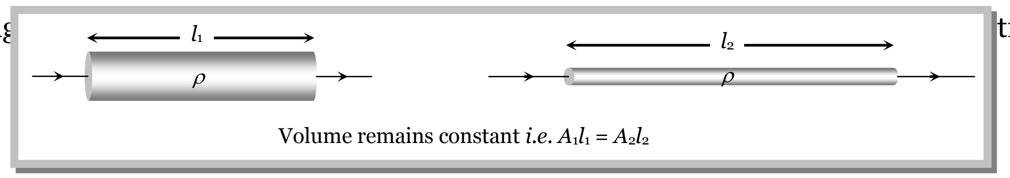
Reciprocal of resistivity is called conductivity (σ) *i.e.* $\sigma = \frac{1}{\rho}$ with unit mho/m and dimensions $[M^{-1}L^{-3}T^3A^2]$.

Stretching of Wire

If a conducting wire stretches, it's length increases, area of cross-section decreases so resistance increases but volume remain constant.

Suppose for a conducting wire before stretching it's length = l_1, area of cross-section = A_1, radius = r_1, diameter = d_1, and resistance $R_1 = \rho \frac{l_1}{A_1}$

Before stretchin



Volume remains constant *i.e.* $A_1l_1 = A_2l_2$

tretching

After stretching length = l_2, area of cross-section = A_2, radius = r_2, diameter = d_2 and resistance = $R_2 = \rho \frac{l_2}{A_2}$

Ratio of resistances $$\frac{\boldsymbol{R_1}}{\boldsymbol{R_2}} = \frac{\boldsymbol{l_1}}{\boldsymbol{l_2}} \times \frac{\boldsymbol{A_2}}{\boldsymbol{A_1}} = \left(\frac{\boldsymbol{l_1}}{\boldsymbol{l_2}}\right)^2 = \left(\frac{\boldsymbol{A_2}}{\boldsymbol{A_1}}\right)^2 = \left(\frac{\boldsymbol{r_2}}{\boldsymbol{r_1}}\right)^4 = \left(\frac{\boldsymbol{d_2}}{\boldsymbol{d_1}}\right)^4$$

(1) If length is given then $R \propto l^2 \Rightarrow \frac{R_1}{R_2} = \left(\frac{l_1}{l_2}\right)^2$

(2) If radius is given then $R \propto \frac{1}{r^4} \Rightarrow \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4$

Note : ❑ After stretching if length increases by n times then resistance will increase by n^2 times *i.e.* $R_2 = n^2 R_1$. Similarly if radius be reduced to $\frac{1}{n}$ times then area of cross-section decreases $\frac{1}{n^2}$ times so the resistance becomes n^4 times *i.e.* $R_2 = n^4 R_1$.

❑ After stretching if length of a conductor increases by x% then resistance will increases by $2x$ % (valid only if $x < 10\%$)

Various Electrical Conducting Material For Specific Use

(1) **Filament of electric bulb :** Is made up of tungsten which has high resistivity, high melting point.

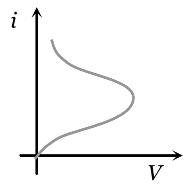
(2) **Element of heating devices (such as heater, geyser or press) :** Is made up of nichrome which has high resistivity and high melting point.

(3) **Resistances of resistance boxes (standard resistances) :** Are made up of manganin, or constantan as these materials have moderate resistivity which is practically independent of temperature so that the specified value of resistance does not alter with minor changes in temperature.

(4) **Fuse-wire :** Is made up of tin-lead alloy (63% tin + 37% lead). It should have low melting point and high resistivity. It is used in series as a safety device in an electric circuit and is designed so as to melt and thereby open the circuit if the current exceeds a predetermined value due to some fault. The function of a fuse is independent of its length.

Safe current of fuse wire relates with it's radius as $i \propto r^{3/2}$.

(5) **Thermistors** : A thermistor is a heat sensitive resistor usually prepared from oxides of various metals such as nickel, copper, cobalt, iron etc. These compounds are also semi-conductor. For thermistors α is very high which may be positive or negative. The resistance of thermistors changes very rapidly with change of temperature.



Thermistors are used to detect small temperature change and to measure very low temperature.

Concepts

☞ *In the absence of radiation loss, the time in which a fuse will melt does not depends on it's length but varies with radius as* $t \propto r^4$.

☞ *If length (l) and mass (m) of a conducting wire is given then* $R \propto \frac{l^2}{m}$.

☞ *Macroscopic form of Ohm's law is* $R = \frac{V}{i}$, *while it's microscopic form is* $J = \sigma E$.

Example

***Example*: 12** Two wires of resistance R_1 and R_2 have temperature co-efficient of resistance α_1 and α_2 respectively. These are joined in series. The effective temperature co-efficient of resistance is

(a) $\frac{\alpha_1+\alpha_2}{2}$ (b) $\sqrt{\alpha_1\alpha_2}$ (c) $\frac{\alpha_1R_1+\alpha_2R_2}{R_1+R_2}$ (d) $\frac{\sqrt{R_1R_2\alpha_1\alpha_2}}{\sqrt{R_1^2+R_2^2}}$

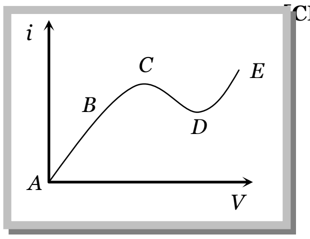
Solution : (c) Suppose at t^oC resistances of the two wires becomes R_{1t} and R_{2t} respectively and equivalent resistance becomes R_t. In series grouping $R_t = R_{1t} + R_{2t}$, also $R_{1t} = R_1(1+\alpha_1 t)$ and $R_{2t} = R_2(1+\alpha_2 t)$

$$R_t = R_1(1+\alpha_1 t) + R_2(1+\alpha_2 t) = (R_1+R_2) + (R_1\alpha_1 + R_2\alpha_2)t = (R_1+R_2)\left[1+\frac{R_1\alpha_1+R_2\alpha_2}{R_1+R_2}t\right].$$

Hence effective temperature co-efficient is $\frac{R_1\alpha_1+R_2\alpha_2}{R_1+R_2}$.

***Example*: 13** From the graph between current i & voltage V shown, identity the portion corresponding to negative resistance

[CBSE PMT 1997]



(a) DE

(b) CD

(c) BC

(d) AB

Solution : (b) $R = \frac{\Delta V}{\Delta I}$, in the graph CD has only negative slope. So in this portion R is negative.

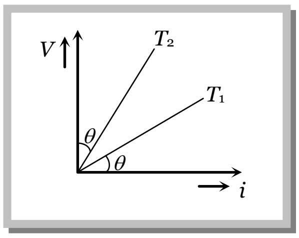
***Example*: 14** A wire of length L and resistance R is streched to get the radius of cross-section halfed. What is new resistance

[NCERT 1974; CPMT 1994; AIIMS 1997; KCET 1999; Haryana PMT 2000; UPSEAT 2001]

(a) $5R$ (b) $8R$ (c) $4R$ (d) $16R$

Solution : (d) By using $\frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^4 \Rightarrow \frac{R}{R'} = \left(\frac{r/2}{r}\right)^4 \Rightarrow R' = 16R$

***Example*: 15** The V-i graph for a conductor at temperature T_1 and T_2 are as shown in the figure. $(T_2 - T_1)$ is proportional to



(a) $\cos 2\theta$

(b) $\sin\theta$

(c) $\cot 2\theta$

(d) $\tan\theta$

Solution : (c) As we know, for conductors resistance $\propto$ Temperature.

From figure $R_1 \propto T_1 \Rightarrow \tan\theta \propto T_1 \Rightarrow \tan\theta = kT_1$ .... (i) (k = constant)

and $R_2 \propto T_2 \Rightarrow \tan(90^o - \theta) \propto T_2 \Rightarrow \cot\theta = kT_2$ ....(ii)

From equation (i) and (ii) $k(T_2 - T_1) = (\cot\theta - \tan\theta)$

$$(T_2 - T_1) = \left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right) = \frac{(\cos^2\theta - \sin^2\theta)}{\sin\theta\cos\theta} = \frac{\cos 2\theta}{\sin\theta\cos\theta} = 2\cot 2\theta \Rightarrow (T_2 - T_1) \propto \cot 2\theta$$

***Example*: 16** The resistance of a wire at 20^oC is 20 Ω and at 500^oC is 60Ω. At which temperature resistance will be 25Ω

[UPSEAT 1999]

(a) 50^oC (b) 60^oC (c) 70^oC (d) 80^oC

Solution : (d) By using $\frac{R_1}{R_2} = \frac{(1+\alpha t_1)}{(1+\alpha t_2)} \Rightarrow \frac{20}{60} = \frac{1+20\alpha}{1+500\alpha} \Rightarrow \alpha = \frac{1}{220}$

Again by using the same formula for 20Ω and 25Ω ⇒ $\frac{20}{25} = \frac{\left(1+\frac{1}{220}\times 20\right)}{\left(1+\frac{1}{220}\times t\right)} \Rightarrow t = 80^oC$

***Example*: 17** The specific resistance of manganin is $50 \times 10^{-8}\ \Omega m$. The resistance of a manganin cube having length $50\ cm$ is

(a) $10^{-6}\ \Omega$ (b) $2.5 \times 10^{-5}\ \Omega$ (c) $10^{-8}\ \Omega$ (d) $5 \times 10^{-4}\ \Omega$

Solution : (a) $R = \rho\frac{l}{A} = \frac{50\times 10^{-8}\times 50\times 10^{-2}}{(50\times 10^{-2})^2} = 10^{-6}\ \Omega$

***Example*: 18** A rod of certain metal is 1 *m* long and 0.6 *cm* in diameter. It's resistance is $3 \times 10^{-3}\Omega$. A disc of the same metal is 1 *mm* thick and 2 *cm* in diameter, what is the resistance between it's circular faces.

(a) $1.35 \times 10^{-6}\Omega$ (b) $2.7 \times 10^{-7}\ \Omega$ (c) $4.05 \times 10^{-6}\Omega$ (d) $8.1 \times 10^{-6}\ \Omega$

Solution : (b) By using $R = \rho.\frac{l}{A};\frac{R_{\text{disc}}}{R_{\text{rod}}} = \frac{l_{\text{disc}}}{l_{\text{rod}}}\times\frac{A_{\text{rod}}}{A_{\text{disc}}} \Rightarrow \frac{R_{\text{disc}}}{3\times 10^{-3}} = \frac{10^{-3}}{1}\times\frac{\pi(0.3\times 10^{-2})^2}{\pi(10^{-2})^2} \Rightarrow R_{\text{disc}} = 2.7 \times 10^{-7}\Omega$.

***Example*: 19** An aluminium rod of length 3.14 *m* is of square cross-section $3.14 \times 3.14\ mm^2$. What should be the radius of 1 *m* long another rod of same material to have equal resistance

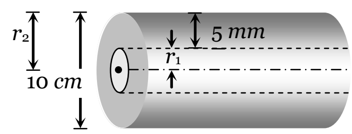
(a) 2 *mm* (b) 4 *mm* (c) 1 *mm* (d) 6 *mm*

Solution : (c) By using $R = \rho.\frac{l}{A} \Rightarrow l \propto A \Rightarrow \frac{3.14}{1} = \frac{3.14\times 3.14\times 10^{-6}}{\pi\times r^2} \Rightarrow r = 10^{-3}\ m = 1\ mm$

***Example*: 20** Length of a hollow tube is 5*m*, it's outer diameter is 10 *cm* and thickness of it's wall is 5 *mm*. If resistivity of the material of the tube is $1.7 \times 10^{-8}\ \Omega\times m$ then resistance of tube will be

(a) $5.6 \times 10^{-5}\ \Omega$ (b) $2 \times 10^{-5}\ \Omega$ (c) $4 \times 10^{-5}\ \Omega$ (d) None of these

Solution : (a) By using $R = \rho.\frac{l}{A}$; here $A = \pi(r_2^2 - r_1^2)$



Outer radius $r_2 = 5cm$

Inner radius $r_1 = 5 - 0.5 = 4.5\ cm$

So $R = 1.7\times 10^{-8}\times\frac{5}{\pi\{(5\times 10^{-2})^2 - (4.5\times 10^{-2})^2\}} = 5.6\times 10^{-5}\ \Omega$

***Example*: 21** If a copper wire is stretched to make it 0.1% longer, the percentage increase in resistance will be

[MP PMT 1996, 2000; UPSEAT 1998; MNR 1990]

(a) 0.2 (b) 2 (c) 1 (d) 0.1

Solution : (a) In case of streching $R \propto l^2$ So $\frac{\Delta R}{R} = 2\frac{\Delta l}{l} = 2\times 0.1 = 0.2$

***Example*: 22** The temperature co-efficient of resistance of a wire is $0.00125/^oC$. At 300 *K*. It's resistance is 1Ω. The resistance of the wire will be 2Ω at **[MP PMT 2001; IIT 1980]**

(a) 1154 *K* (b) 1127 *K* (c) 600 *K* (d) 1400 *K*

Solution: (b) By using $R_t = R_o(1+\alpha\Delta t) \Rightarrow \frac{R_1}{R_2} = \frac{1+\alpha t_1}{1+\alpha t_2}$ So $\frac{1}{2} = \frac{1+(300-273)\alpha}{1+\alpha t_2} \Rightarrow t_2 = 854^oC = 1127\ K$

***Example*: 23** Equal potentials are applied on an iron and copper wire of same length. In order to have same current flow in the wire, the ratio $\left(\frac{r_{iron}}{r_{copper}}\right)$ of their radii must be [Given that specific resistance of iron = 1.0 × 10^{-7} Ωm and that of copper = 1.7 × 10^{-8} Ωm] **[MP PMT 2000]**

(a) About 1.2 (b) About 2.4 (c) About 3.6 (d) About 4.8

Solution: (b) V = constant., i = constant. So R = constant

$$\Rightarrow \frac{P_i l_i}{A_i} = \frac{\rho_{Cu} l_{Cu}}{A_{Cu}} \Rightarrow \frac{\rho_i l_i}{r_i^2} = \frac{\rho_{Cu} l_{Cu}}{r_{Cu}^2}$$

$$\Rightarrow \frac{r_i}{r_{Cu}} = \sqrt{\frac{\rho_i}{\rho_{Cu}}} = \sqrt{\frac{1.0 \times 10^{-7}}{1.7 \times 10^{-8}}} = \sqrt{\frac{100}{17}} \approx 2.4$$

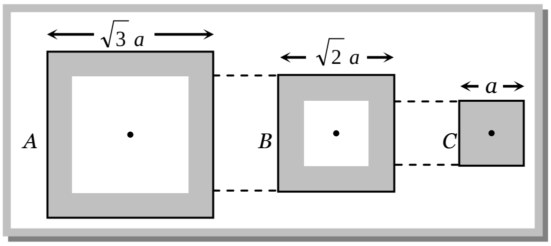
***Example*: 24** Masses of three wires are in the ratio 1 : 3 : 5 and their lengths are in the ratio 5 : 3 : 1. The ratio of their electrical resistance is

(a) 1 : 3 : 5 (b) 5 : 3 : 1 (c) 1 : 15 : 125 (d) 125 : 15 : 1

Solution: (d) $R = \rho\frac{l}{A} = \rho\frac{l^2}{V} = \rho\frac{l^2}{m}\sigma \qquad \left(\because \sigma = \frac{m}{V}\right)$

$$R_1 : R_2 : R_3 = \frac{l_1^2}{m_1} : \frac{l_2^2}{m_2} : \frac{l_3^2}{m_3} = 25 : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$$

***Example*: 25** Following figure shows cross-sections through three long conductors of the same length and material, with square cross-section of edge lengths as shown. Conductor B will fit snugly within conductor A, and conductor C will fit snugly within conductor B. Relationship between their end to end resistance is



(a) $R_A = R_B = R_C$

(b) $R_A > R_B > R_C$

(c) $R_A < R_B < R$

(d) Information is not sufficient

Solution : (a) All the conductors have equal lengths. Area of cross-section of A is $\left\{(\sqrt{3}\,a)^2 - (\sqrt{2}\,a)^2\right\} = a^2$

Similarly area of cross-section of B = Area of cross-section of C = a^2

Hence according to formula $R = \rho\frac{l}{A}$; resistances of all the conductors are equal *i.e.* $R_A = R_B = R_C$

***Example*: 26** Dimensions of a block are 1 *cm* × 1 *cm* × 100 *cm*. If specific resistance of its material is 3 × 10^{-7} *ohm-m*, then the resistance between it's opposite rectangular faces is

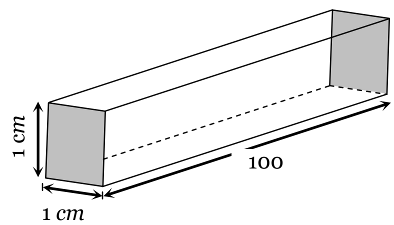
(a) 3 × 10^{-9} *ohm* (b) 3 × 10^{-7} *ohm* (c) 3 × 10^{-5} *ohm* (d) 3 × 10^{-3} *ohm*

Solution: (b) Length l = 1 *cm* $= 10^{-2} m$

Area of cross-section A = 1 *cm* × 100 *cm*

= 100 cm^2 = 10^{-2} m^2

Resistance $R = 3 \times 10^{-7} \times \frac{10^{-2}}{10^{-2}} = 3 \times 10^{-7}\,\Omega$



Note : ❑ In the above question for calculating equivalent resistance between two opposite square faces.

$$l = 100\ cm = 1\ m,\ A = 1\ cm^2 = 10^{-4}\ m^2, \text{ so resistance } R = 3 \times 10^{-7} \times \frac{1}{10^{-4}} = 3 \times 10^{-3}\ \Omega$$

3. Two rods A and B of same material and length have their electric resistances are in ratio 1 : 2. When both the rods are dipped in water, the correct statement will be **[RPMT 1997]**

(a) A has more loss of weight (b) B has more loss of weight

(c) Both have same loss of weight (d) Loss of weight will be in the ratio 1 : 2

Solution: (a) $R = \rho\frac{L}{A} \Rightarrow \frac{R_1}{R_2} = \frac{A_2}{A_1}$ (ρ, L constant) $\Rightarrow \frac{A_1}{A_2} = \frac{R_2}{R_1} = 2$

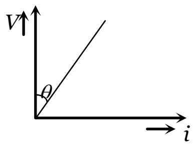
Now when a body dipped in water, loss of weight = $V\sigma_L g = AL\sigma_L g$

So $\frac{(\text{Loss of weight})_1}{(\text{Loss of weight})_2} = \frac{A_1}{A_2} = 2$; So A has more loss of weight

The V-i graph for a conductor makes an angle θ with V-axis. Here V denotes the voltage and i denotes current. The resistance of conductor is given by

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\cot\theta$

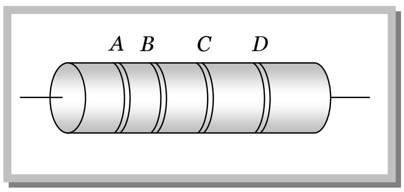
Solution: (d) At an instant approach the student will choose $\tan\theta$ will be the right answer. But it is to be seen here the curve makes the angle θ with the V-axis. So it makes an angle $(90 - \theta)$ with the i-axis. So resistance = slope = $\tan(90 - \theta) = \cot\theta$.



Colour Coding of Resistance

The resistance, having high values are used in different electrical and electronic circuits. They are generally made up of carbon, like 1 $k\Omega$, 2 $k\Omega$, 5 $k\Omega$ *etc*. To know the value of resistance colour code is used. These code are printed in form of set of rings or strips. By reading the values of colour bands, we can estimate the value of resistance.

The carbon resistance has normally four coloured rings or strips say A, B, C and D as shown in following figure.



Colour band A and B indicate the first two significant figures of resistance in *ohm*, while the C band gives the decimal multiplier *i.e.* the number of zeros that follows the two significant figures A and B.

Last band (D band) indicates the tolerance in percent about the indicated value or in other ward it represents the percentage accuracy of the indicated value.

The tolerance in the case of gold is ± 5% and in silver is ± 10%. If only three bands are marked on carbon resistance, then it indicate a tolerance of 20%.

The following table gives the colour code for carbon resistance.

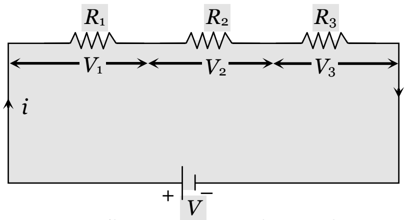
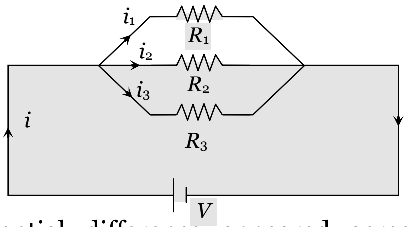
Letters as an aid to memory	Colour	Figure (A, B)	Multiplier (C)
B	Black	0	10^0
B	Brown	1	10^1
R	Red	2	10^2
O	Orange	3	10^3
Y	Yellow	4	10^4
G	Green	5	10^5
B	Blue	6	10^6
V	Violet	7	10^7
G	Grey	8	10^8
W	White	9	10^9

Colour	Tolerance (D)
Gold	5%
Silver	10%
No-colour	20%

Note : ❑ To remember the sequence of colour code following sentence should kept in memory.

***B B R O Y G**reat **B**ritain **V**ery **G**ood **W**ife.*

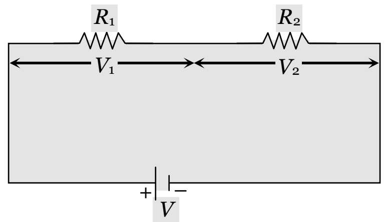
Grouping of Resistance

Series	Parallel
(1) 	(1) 
(2) Same current flows through each resistance but potential difference distributes in the ratio of resistance *i.e.* $V \propto R$ Power consumed are in the ratio of their resistance *i.e.* $P \propto R \Rightarrow P_1 : P_2 : P_3 = R_1 : R_2 : R_3$	(2) Same potential difference appeared across each resistance but current distributes in the reverse ratio of their resistance *i.e.* $i \propto \frac{1}{R}$ Power consumed are in the reverse ratio of resistance *i.e.* $P \propto \frac{1}{R} \Rightarrow P_1 : P_2 : P_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$
(3) $R_{eq} = R_1 + R_2 + R_3$ equivalent resistance is greater than the maximum value of resistance in the combination.	(3) $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ or $R_{eq} = (R_1^{-1} + R_2^{-1} + R_3^{-1})^{-1}$ or $R_{eq} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_2 R_1}$ equivalent resistance is smaller than the minimum value of resistance in the combination.
(4) For two resistance in series $R_{eq} = R_1 + R_2$	(4) For two resistance in parallel $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{\text{Multiplication}}{\text{Addition}}$

(5) Potential difference across any resistance $V' = \left(\frac{R'}{R_{eq}}\right) \cdot V$

Where R' = Resistance across which potential difference is to be calculated, R_{eq} = equivalent resistance of that line in which R' is connected, V = p.d. across that line in which R' is connected

e.g.



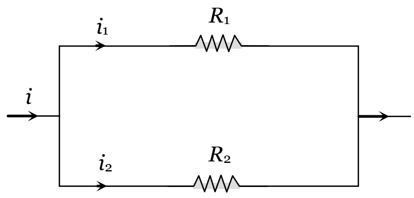
$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) \cdot V$ and $V_2 = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V$

(5) Current through any resistance

$$i' = i \times \left[\frac{\text{Resistance of opposite branch}}{\text{Total resistance}}\right]$$

Where i' = required current (branch current)

i = main current



$i_1 = i\left(\frac{R_2}{R_1 + R_2}\right)$ and $i_2 = i\left(\frac{R_1}{R_1 + R_2}\right)$

(6) If n identical resistance are connected in series

$R_{eq} = nR$ and p.d. across each resistance $V' = \frac{V}{n}$

(6) In n identical resistance are connected in parallel

$R_{eq} = \frac{R}{n}$ and current through each resistance $i' = \frac{i}{n}$

Note : ❑ In case of resistances in series, if one resistance gets open, the current in the whole circuit become zero and the circuit stops working. Which don't happen in case of parallel gouging.

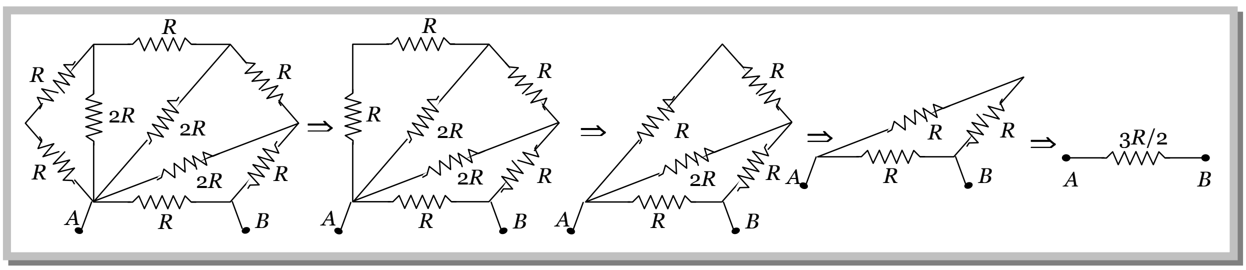
❑ Decoration of lightning in festivals is an example of series grouping whereas all household appliances connected in parallel grouping.

❑ Using n conductors of equal resistance, the number of possible combinations is 2^{n-1}.

❑ If the resistance of n conductors are totally different, then the number of possible combinations will be 2^n.

Methods of Determining Equivalent Resistance For Some Difficult Networks

(1) **Method of successive reduction :** It is the most common technique to determine the equivalent resistance. So far, we have been using this method to find out the equivalent resistances. This method is applicable only when we are able to identify resistances in series or in parallel. The method is based on the simplification of the circuit by successive reduction of the series and parallel combinations. For example to calculate the equivalent resistance between the point A and B, the network shown below successively reduced.



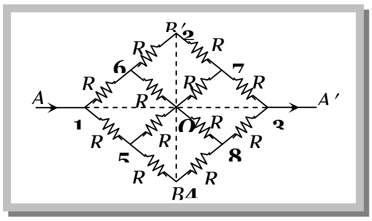
(2) **Method of equipotential points :** This method is based on identifying the points of same potential and joining them. The basic rule to identify the points of same potential is the symmetry of the network.

(i) In a given network there may be two axes of symmetry.

(a) Parallel axis of symmetry, that is, along the direction of current flow.

(b) Perpendicular axis of symmetry, that is perpendicular to the direction of flow of current.

For example in the network shown below the axis AA' is the parallel axis of symmetry, and the axis BB' is the perpendicular axis of symmetry.

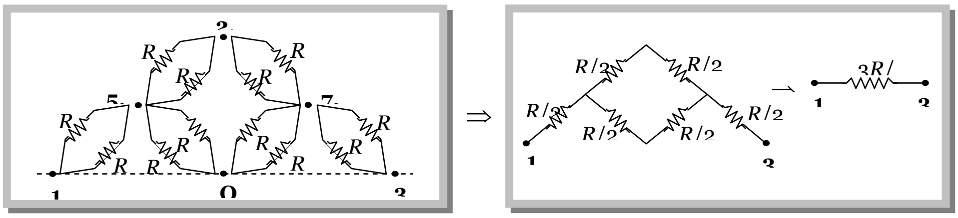


(ii) Points lying on the perpendicular axis of symmetry may have same potential. In the given network, point 2, 0 and 4 are at the same potential.

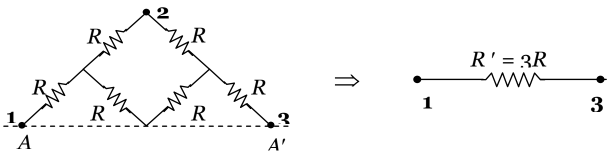
(iii) Points lying on the parallel axis of symmetry can never have same potential.

(iv) The network can be folded about the parallel axis of symmetry, and the overlapping nodes have same potential. Thus as shown in figure, the following points have same potential

(a) 5 and 6 (b) 2, 0 and 4 (c) 7 and 8



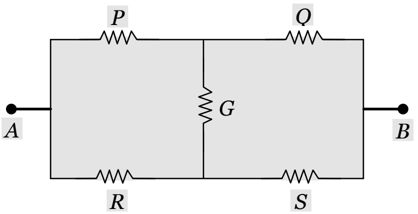
Note : ❑ Above network may be split up into two equal parts about the parallel axis of symmetry as shown in figure each part has a resistance R', then the equivalent resistance of the network will be $R = \frac{R'}{2}$.



Some Standard Results for Equivalent Resistance

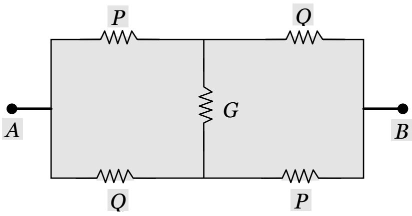
(1) **Equivalent resistance between points A and B in an unbalanced Wheatstone's bridge as shown in the diagram.**

(i)



$$R_{AB} = \frac{PQ(R+S)+(P+Q)RS + G(P+Q)(R+S)}{G(P+Q+R+S)+(P+R)(Q+S)}$$

(ii)

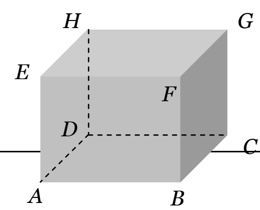


$$R_{AB} = \frac{2PQ + G(P+Q)}{2G+P+Q}$$

(2) **A cube each side have resistance R then equivalent resistance in different situations**

(i) Between E and C *i.e.* across the diagonal of the cube $R_{EC} = \frac{5}{6}R$

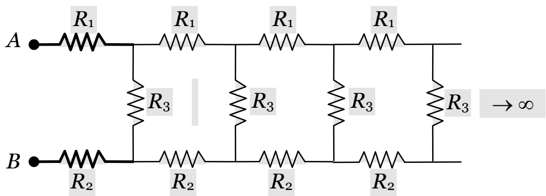
(ii) Between A and B *i.e.* across one side of the cube $R_{AB} = \frac{7}{12}R$



(iii) Between A and C *i.e.* across the diagonal of one face of the cube $R_{AC} = \frac{3}{4}R$

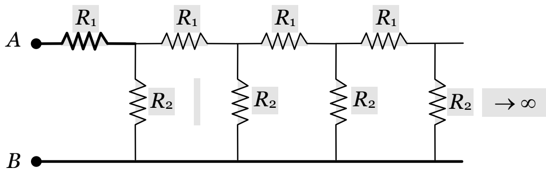
(3) **The equivalent resistance of infinite network of resistances**

(i)



$$R_{AB} = \frac{1}{2}(R_1 + R_2) + \frac{1}{2}\left[(R_1 + R_2)^2 + 4R_3(R_1 + R_2)\right]^{1/2}$$

(ii)



$$R_{AB} = \frac{1}{2}R_1\left[1 + \sqrt{1 + 4\left(\frac{R_2}{R_1}\right)}\right]$$

Concepts

☞ *If n identical resistances are first connected in series and then in parallel, the ratio of the equivalent resistance is given by* $\frac{R_p}{R_s} = \frac{n^2}{1}$.

☞ *If equivalent resistance of R_1 and R_2 in series and parallel be R_s and R_p respectively then* $R_1 = \frac{1}{2}\left[R_s + \sqrt{R_s^2 - 4R_sR_p}\right]$ *and* $R_2 = \frac{1}{2}\left[R_s - \sqrt{R_s^2 - 4R_sR_p}\right]$.

☞ *If a wire of resistance R, cut in n equal parts and then these parts are collected to form a bundle then equivalent resistance of combination will be* $\frac{R}{n^2}$.

Example

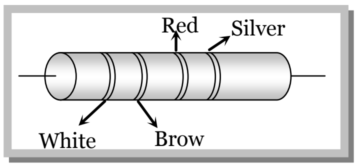
***Example*: 27** In the figure a carbon resistor has band of different colours on its body. The resistance of the following body is

(a) 2.2 $k\Omega$

(b) 3.3 $k\Omega$

(c) 5.6 $k\Omega$

(d) 9.1 $k\Omega$l



,

Solution : (d) $R = 91 \times 10^2 \pm 10\% \approx 9.1\ k\Omega$

***Example*: 28** What is the resistance of a carbon resistance which has bands of colours brown, black and brown **[DCE 1999]**

(a) 100 Ω (b) 1000 Ω (c) 10 Ω (d) 1 Ω

Solution : (a) $R = 10 \times 10^1 \pm 20\% \approx 100\ \Omega$

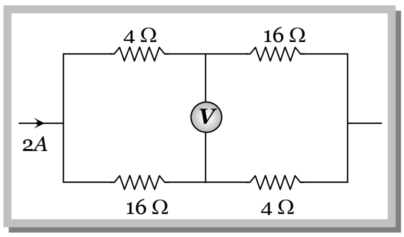
***Example*: 29** In the following circuit reading of voltmeter V is **[MP PET 2003]**

(a) 12 V

(b) 8 V

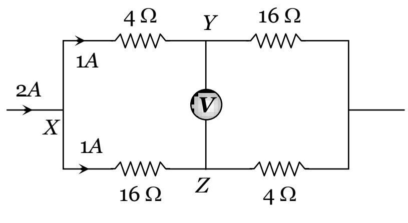
(c) 20 V

(d) 16 V



Solution : (a) P.d. between X and Y is $V_{XY} = V_X - V_Y = 1 \times 4 = 4\ V$ .... (i)

and p.d. between X and Z is $V_{XZ} = V_X - V_Z = 1 \times 16 = 16\ V$ .... (ii)

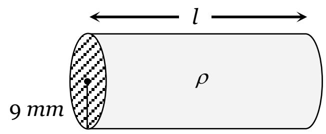


On solving equations (i) and (ii) we get potential difference between Y and Z *i.e.*, reading of voltmeter is $V_Y - V_Z = 12V$

***Example*: 30** An electric cable contains a single copper wire of radius 9 *mm*. It's resistance is 5 Ω. This cable is replaced by six insulated copper wires, each of radius 3 *mm*. The resultant resistance of cable will be **[CPMT 198**

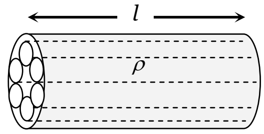
(a) 7.5 Ω (b) 45 Ω (c) 90 Ω (d) 270 Ω

Solution : (a) Initially : Resistance of given cable



$$R = \rho \frac{l}{\pi \times (9 \times 10^{-3})^2} \quad \text{..... (i)}$$

Finally : Resistance of each insulated copper wire is



$$R' = \rho \frac{l}{\pi \times (3 \times 10^{-3})^2}$$

Hence equivalent resistance of cable

$$R_{eq} = \frac{R'}{6} = \frac{1}{6} \times \left(\rho \frac{l}{\pi \times (3 \times 10^{-3})^2} \right) \quad \text{..... (ii)}$$

On solving equation (i) and (ii) we get $R_{eq} = 7.5\ \Omega$

***Example*: 31** Two resistance R_1 and R_2 provides series to parallel equivalents as $\frac{n}{1}$ then the correct relationship is

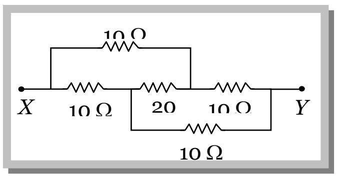
(a) $\left(\frac{R_1}{R_2}\right)^2 + \left(\frac{R_2}{R_1}\right)^2 = n^2$ (b) $\left(\frac{R_1}{R_2}\right)^{3/2} + \left(\frac{R_2}{R_1}\right)^{3/2} = n^{3/2}$

(c) $\left(\frac{R_1}{R_2}\right) + \left(\frac{R_2}{R_1}\right) = n$ (d) $\left(\frac{R_1}{R_2}\right)^{1/2} + \left(\frac{R_2}{R_1}\right)^{1/2} = n^{1/2}$

Solution : (d) Series resistance $R_S = R_1 + R_2$ and parallel resistance $R_P = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow \frac{R_S}{R_P} = \frac{(R_1 + R_2)^2}{R_1 R_2} = n$

$$\Rightarrow \frac{R_1 + R_2}{\sqrt{R_1 R_2}} = \sqrt{n} \qquad \Rightarrow \frac{\sqrt{R_1^2}}{\sqrt{R_1 R_2}} + \frac{\sqrt{R_2^2}}{\sqrt{R_1 R_2}} = \sqrt{n} \Rightarrow \sqrt{\frac{R_1}{R_2}} + \sqrt{\frac{R_2}{R_1}} = \sqrt{n}$$

***Example*: 32** Five resistances are combined according to the figure. The equivalent resistance between the point X and Y will be



(a) 10 Ω

(b) 22 Ω

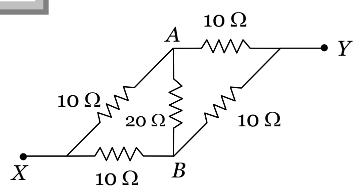
(c) 20 Ω

(d) 50 Ω

Solution : (a) The equivalent circuit of above can be drawn as

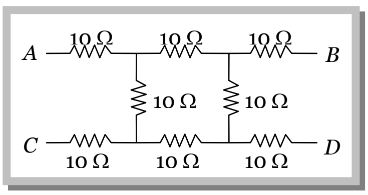
Which is a balanced wheatstone bridge.

So current through AB is zero.



So $\frac{1}{R} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10} \Rightarrow R = 10\ \Omega$

***Example*: 33** What will be the equivalent resistance of circuit shown in figure between points A and D **[CBSE PMT 1996]**



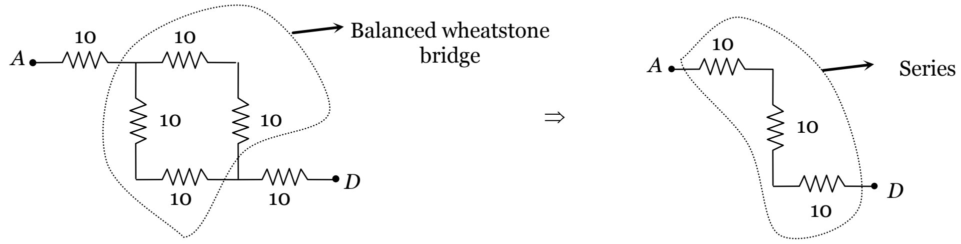
(a) 10 Ω

(b) 20 Ω

(c) 30 Ω

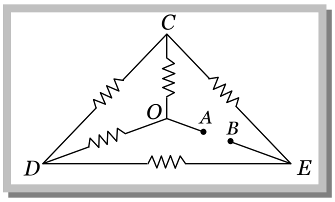
(d) 40 Ω

Solution : (c) The equivalent circuit of above fig between *A* and *D* can be drawn as



So $R_{eq} = 10 + 10 + 10 = 30\Omega$

***Example*: 34** In the network shown in the figure each of resistance is equal to 2Ω. The resistance between *A* and *B* is **[CBSE PMT 1995]**



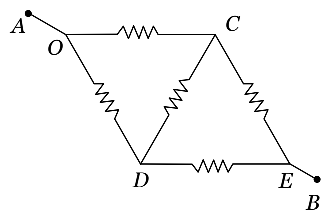
(a) 1 Ω

(b) 2 Ω

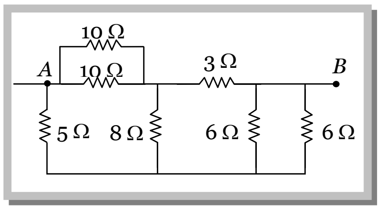
(c) 3 Ω

(d) 4 Ω

Solution : (b) Taking the portion *COD* is figure to outside the triangle (left), the above circuit will be now as resistance of each is 2 Ω the circuit will behaves as a balanced wheatstone bridge and no current flows through *CD*. Hence R_{AB} = 2Ω



***Example*: 35** Seven resistances are connected as shown in figure. The equivalent resistance between *A* and *B* is **[MP PET 2000]**



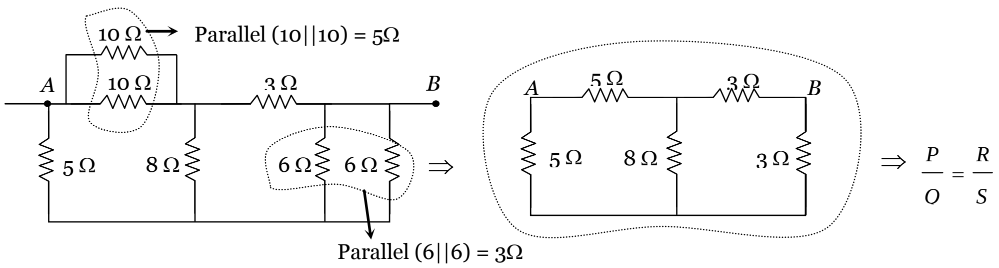
(a) 3 Ω

(b) 4 Ω

(c) 4.5 Ω

(d) 5 Ω

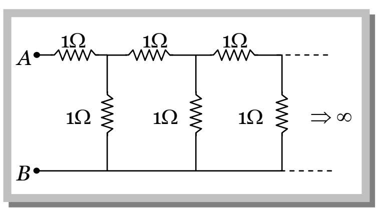
Solution : (b)



So the circuit is a balanced wheatstone bridge.

So current through 8Ω is zero $R_{eq} = (5+3)||\ (5+3) = 8||\ 8 = 4\Omega$

***Example*: 36** The equivalent resistance between points *A* and *B* of an infinite network of resistance, each of 1 Ω, connected as shown is **[CEE Haryana 1996]**



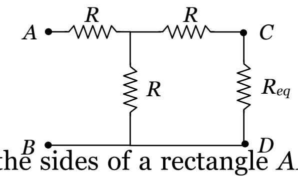
(a) Infinite

(b) 2 Ω

(c) $\frac{1+\sqrt{5}}{2}\,\Omega$

(d) Zero

Solution : (c) Suppose the effective resistance between A and B is R_{eq}. Since the network consists of infinite cell. If we exclude one cell from the chain, remaining network have infinite cells *i.e.* effective resistance between C and D will also R_{eq}

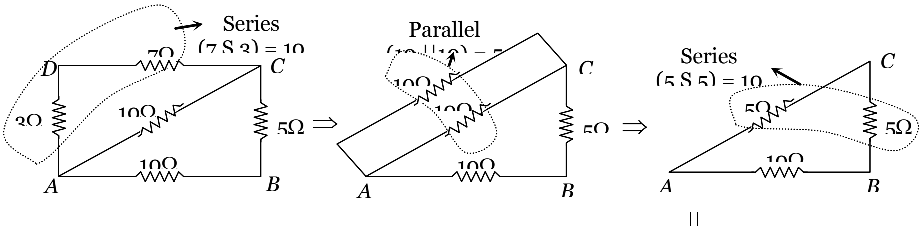


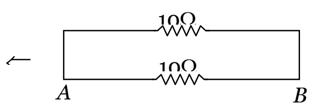
So now $R_{eq} = R_o + (R||\ R_{eq}) = R + \frac{R\,R_{eq}}{R + R_{eq}} \Rightarrow R_{eq} = \frac{1}{2}[1+\sqrt{5}]$

***Example*: 37** Four resistances 10 Ω, 5 Ω, 7 Ω and 3 Ω are connected so that they form the sides of a rectangle AB, BC, CD and DA respectively. Another resistance of 10 Ω is connected across the diagonal AC. The equivalent resistance between A & B is

(a) 2 Ω (b) 5 Ω (c) 7 Ω (d) 10 Ω

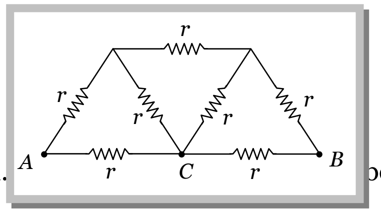
Solution : (b)





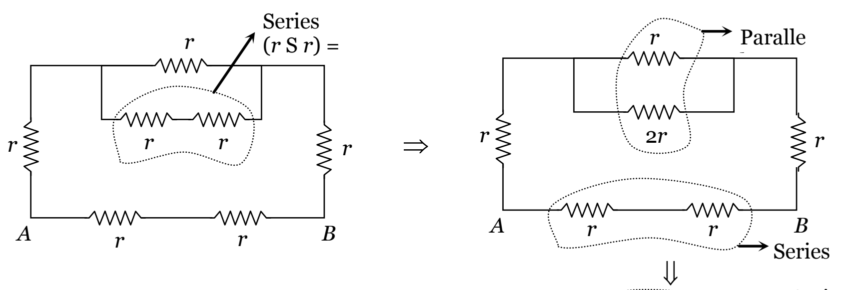
So $R_{eq} = \frac{10\times 10}{10+10} = 5\Omega$

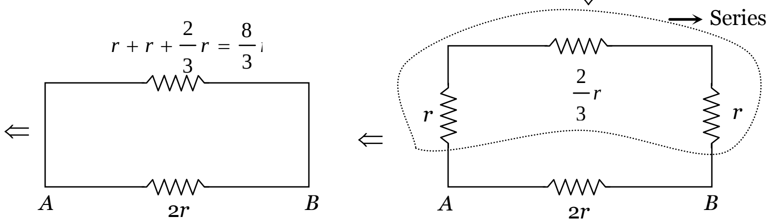
***Example*: 38** The equivalent resistance between A and B in the circuit will be



(a) $\frac{5}{4}r$ (b) $\frac{6}{5}r$ (c) $\frac{7}{6}r$ (d) $\frac{8}{7}r$

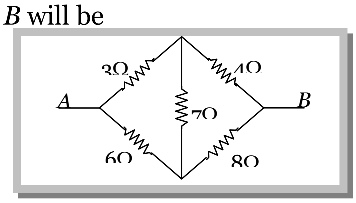
Solution : (d) In the circuit, by means of symmetry the point C is at zero potential. be drawn as





$R_{eq} = \left(\frac{8r}{3} ||\ 2r\right) = \frac{8}{7}r$

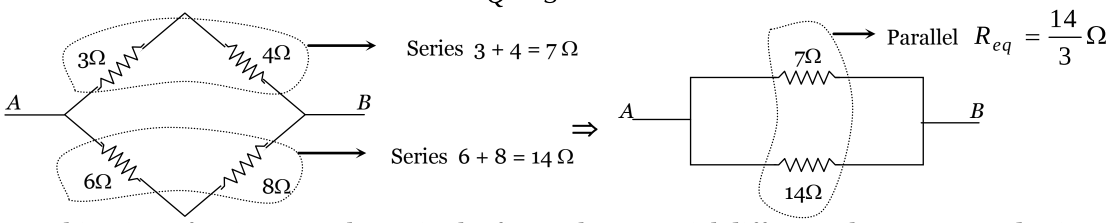
***Example*: 39** In the given figure, equivalent resistance between A and B will be **[CBSE PMT 2000]**



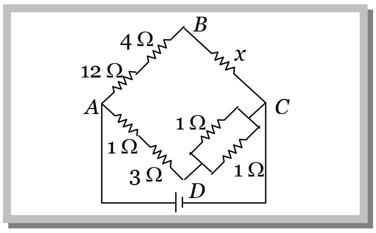
(a) $\frac{14}{3}\Omega$ (b) $\frac{3}{14}\Omega$

(c) $\frac{9}{14}\Omega$ (d) $\frac{14}{9}\Omega$

Solution : (a) Given Wheatstone bridge is balanced because $\frac{P}{Q} = \frac{R}{S}$. Hence the circuit can be redrawn as follows



***Example*: 40** In the combination of resistances shown in the figure the potential difference between *B* and *D* is zero, when unknown resistance (*x*) is



(a) 4 Ω

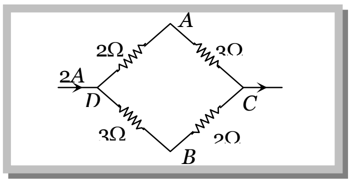
(b) 2 Ω

(c) 3 Ω

(d) The emf of the cell is required

Solution : (b) The potential difference across *B*, *D* will be zero, when the circuit will act as a balanced wheatstone bridge and $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{12+4}{x} = \frac{1+3}{1/2} \Rightarrow x = 2\Omega$

***Example*: 41** A current of 2 *A* flows in a system of conductors as shown. The potential difference $(V_A - V_B)$ will be **[CPMT 1975, 76]**



(a) + 2*V*

(b) + 1*V*

(c) − 1 *V*

(d) − 2 *V*

Solution : (b) In the given circuit 2*A* current divides equally at junction *D* along the paths *DAC* and *DBC* (each path carry 1*A* current).

Potential difference between *D* and *A*, $V_D - V_A = 1 \times 2 = 2\ volt$ .... (i)

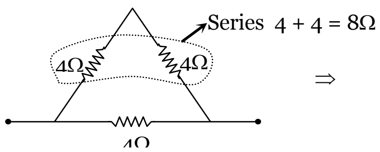
Potential difference between *D* and *B*, $V_D - V_B = 1 \times 3 = 3\ volt$ ..... (ii)

On solving (i) and (ii) $V_A - V_B = +1\ volt$

***Example*: 42** Three resistances each of 4 Ω are connected in the form of an equilateral triangle. The effective resistance between two corners is

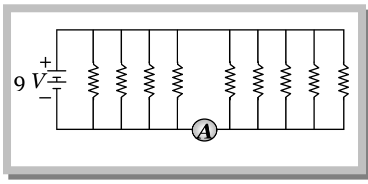
(a) 8 Ω (b) 12 Ω (c) $\frac{3}{8}\Omega$ (d) $\frac{8}{3}\Omega$

Solution : (d)



⇒ On Solving further we get equivalent resistance is $\frac{8}{3}\Omega$

***Example*: 43** If each resistance in the figure is of 9 Ω then reading of ammeter is 2000]



(a) 5 *A* (b) 8 *A*

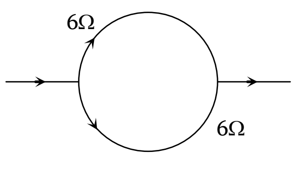
(c) 2 *A* (d) 9 *A*

Solution : (a) Main current through the battery $i = \frac{9}{1} = 9A$. Current through each resistance will be 1*A* and only 5 resistances on the right side of ammeter contributes for passing current through the ammeter. So reading of ammeter will be 5*A*.

***Example*: 44** A wire has resistance 12 Ω. It is bent in the form of a circle. The effective resistance between the two points on any diameter is equal to

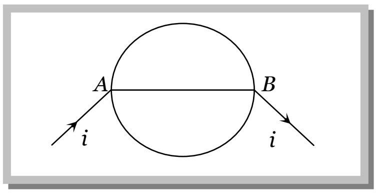
(a) 12 Ω (b) 6 Ω (c) 3 Ω (d) 24 Ω

Solution : (c) Equivalent resistance of the following circuit will be



$$R_{eq} = \frac{6}{2} = 3\Omega$$

***Example*: 45** A wire of resistance 0.5 Ωm^{-1} is bent into a circle of radius 1 *m*. The same wire is connected across a diameter *AB* as shown in fig. The equivalent resistance is



(a) π *ohm*

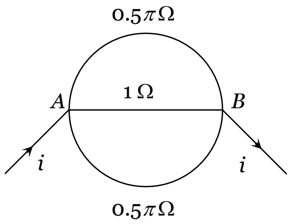
(b) $\pi(\pi + 2)$ *ohm*

(c) $\pi / (\pi + 4)$ *ohm*

(d) $(\pi + 1)$ *ohm*

Solution : (c) Resistance of upper semicircle = Resistance of lower semicircle

$= 0.5 \times (\pi R) = 0.5\ \pi\Omega$

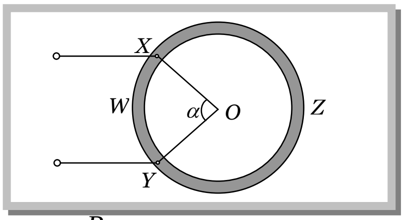


Resistance of wire *AB* $= 0.5 \times 2 = 1\ \Omega$

Hence equivalent resistance between *A* and *B*

$$\frac{1}{R_{AB}} = \frac{1}{0.5\pi} + \frac{1}{1} + \frac{1}{0.5\pi} \Rightarrow R_{AB} = \frac{\pi}{(\pi + 4)}\Omega$$

***Example*: 46** A wire of resistor *R* is bent into a circular ring of radius *r*. Equivalent resistance between two points *X* and *Y* on its circumference, when angle *XOY* is α, can be given by



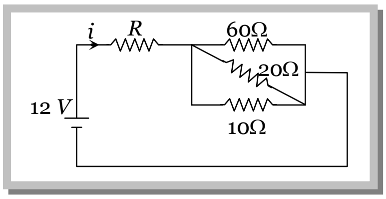
(a) $\frac{R\alpha}{4\pi^2}(2\pi - \alpha)$ (b) $\frac{R}{2\pi}(2\pi - \alpha)$

(c) $R(2\pi - \alpha)$ (d) $\frac{4\pi}{R\alpha}(2\pi - \alpha)$

Solution : (a) Here $R_{XWY} = \frac{R}{2\pi r} \times (r\alpha) = \frac{R\alpha}{2\pi}$ $\left(\because \alpha = \frac{l}{r}\right)$ and $R_{XZY} = \frac{R}{2\pi r} \times r(2\pi - \alpha) = \frac{R}{2\pi}(2\pi - \alpha)$

$$R_{eq} = \frac{R_{XWY} R_{XZY}}{R_{XWY} + R_{XZY}} = \frac{\frac{R\alpha}{2\pi} \times \frac{R}{2\pi}(2\pi - \alpha)}{\frac{R\alpha}{2\pi} + \frac{R(2\pi - \alpha)}{2\pi}} = \frac{R\alpha}{4\pi^2}(2\pi - \alpha)$$

***Example*: 47** If in the given figure $i = 0.25$ *amp*, then the value *R* will be **[RPET 2000]**



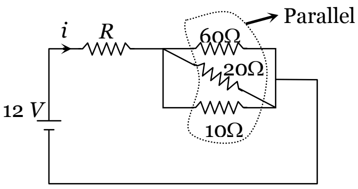
(a) 48 Ω

(b) 12 Ω

(c) 120 Ω

(d) 42 Ω

Solution : (d) $i = 0.25$ *amp* $V = 12\ V$ $R_{eq} = \frac{V}{i} = \frac{12}{0.25} = 48\ \Omega$



Now from the circuit $R_{eq} = R + (60 || 20 || 10)$

$= R + 6$

$\Rightarrow R = R_{eq} - 6 = 48 - 6 = 42\ \Omega$

***Example*: 48** Two uniform wires *A* and *B* are of the same metal and have equal masses. The radius of wire *A* is twice that of wire *B*. The total resistance of *A* and *B* when connected in parallel is **[MNR 1994]**

(a) 4 Ω when the resistance of wire *A* is 4.25 Ω (b) 5 Ω when the resistance of wire *A* is 4 Ω

(c) 4 Ω when the resistance of wire *B* is 4.25 Ω (d) 5 Ω when the resistance of wire *B* is 4 Ω

Solution : (a) Density and masses of wire are same so their volumes are same *i.e.* $A_1 l_1 = A_2 l_2$

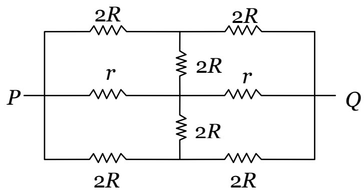
Ratio of resistances of wires A and B $\frac{R_A}{R_B} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^4$

Since $r_1 = 2r_2$ so $\frac{R_A}{R_B} = \frac{1}{16} \Rightarrow R_B = 16\,R_A$

Resistance R_A and R_B are connected in parallel so equivalent resistance $R = \frac{R_A R_B}{R_A + R_B} = \frac{16 R_A}{17}$, By checking correctness of equivalent resistance from options, only option (a) is correct.

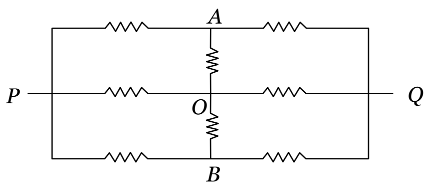
Tricky Example: 5

The effective resistance between point P and Q of the electrical circuit shown in the figure is **[IIT-JEE 1991]**



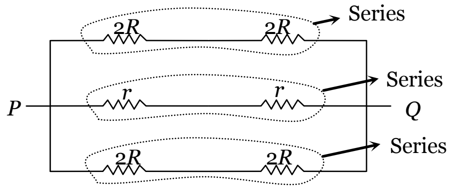
(a) $\frac{2Rr}{R+r}$ (b) $\frac{8R(R+r)}{3R+r}$ (c) $2r + 4R$ (d) $\frac{5R}{2} + 2r$

Solution : (a) The points A, O, B are at same potential. So the figure can be redrawn as follows



(I)

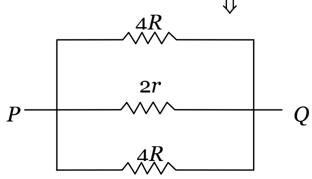
$\Rightarrow$



(II)

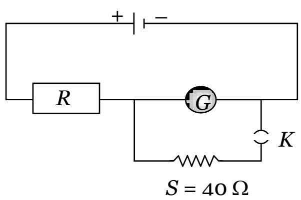
$\Downarrow$

$R_{eq} = \frac{2Rr}{R+r} \Leftarrow 4R \,||\, 2r \,||\, 4R \Leftarrow$



Tricky Example: 6

In the following circuit if key K is pressed then the galvanometer reading becomes half. The resistance of galvanometer is

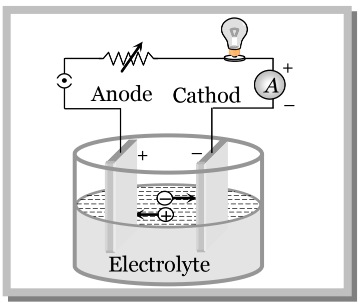


(a) 20 Ω (b) 30 Ω (c) 40 Ω (d) 50 Ω

Solution : (c) Galvanometer reading becomes half means current distributes equally between galvanometer and resistance of 40 Ω. Hence galvanometer resistance must be 40 Ω.

Cell

The device which converts chemical energy into electrical energy is known as electric cell.



(1) A cell neither creates nor destroys charge but maintains the flow of charge present at various parts of the circuit by supplying energy needed for their organised motion.

(2) Cell is a source of constant emf but not constant current.

(3) Mainly cells are of two types :

(i) Primary cell : Cannot be recharged

(ii) Secondary cell : Can be recharged

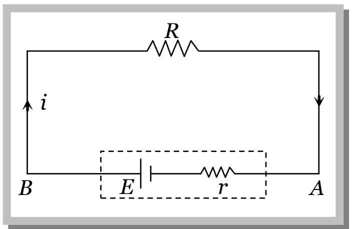
(4) The direction of flow of current inside the cell is from negative to positive electrode while outside the cell is form positive to negative electrode.

(5) A cell is said to be ideal, if it has zero internal resistance.

(6) **Emf of cell (*E*) :** The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called it's electromotive force (emf) *i.e.* emf of cell $E = \frac{W}{q}$, It's unit is *volt* or

The potential difference across the terminals of a cell when it is not given any current is called it's emf.

(7) **Potential difference (*V*) :** The energy given by the cell in the flow of unit charge in a specific part of electrical circuit (external part) is called potential difference. It's unit is also *volt* or

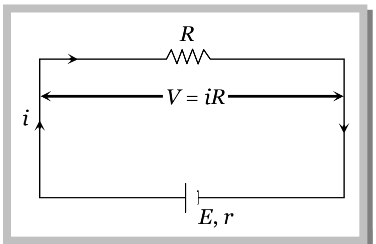


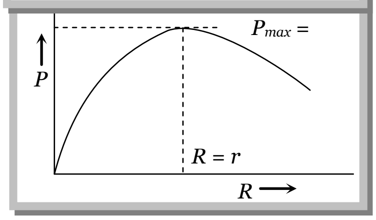
The voltage across the terminals of a cell when it is supplying current to external resistance is called potential difference or terminal voltage. Potential difference is equal to the product of current and resistance of that given part *i.e.* $V = iR$.

(8) **Internal resistance (*r*) :** In case of a cell the opposition of electrolyte to the flow of current through it is called internal resistance of the cell. The internal resistance of a cell depends on the distance between electrodes ($r \propto d$), area of electrodes [$r \propto (1/A)$] and nature, concentration ($r \propto C$) and temperature of electrolyte [$r \propto$ (1/temp.)]. Internal resistance is different for different types of cells and even for a given type of cell it varies from to cell.

Cell in Various Position

(1) **Closed circuit (when the cell is discharging)**





(i) Current given by the cell $i = \frac{E}{R + r}$

(ii) Potential difference across the resistance $V = iR$

(iii) Potential drop inside the cell = *ir*

(iv) Equation of cell $E = V + ir$ $(E > V)$

(v) Internal resistance of the cell $r = \left(\frac{E}{V} - 1\right) \cdot R$

(vi) Power dissipated in external resistance (load) $P = Vi = i^2 R = \frac{V^2}{R} = \left(\frac{E}{R + r}\right)^2 .R$

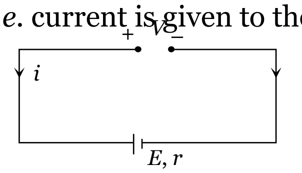
Power delivered will be maximum when $R = r$ so $P_{\max} = \frac{E^2}{4r}$.

This statement in generalised from is called "*maximum power transfer theorem*".

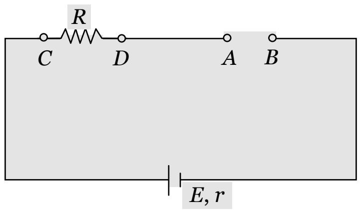
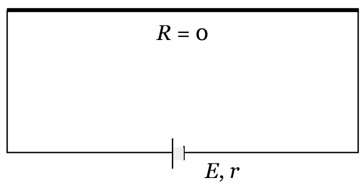
(vii) **Short trick to calculate E and r :** In the closed circuit of a cell having emf E and internal resistance r. If external resistance changes from R_1 to R_2 then current changes from i_1 to i_2 and potential difference changes from V_1 to V_2. By using following relations we can find the value of E and r.

$$E = \frac{i_1 i_2}{i_2 - i_1}(R_1 - R_2) \quad r = \left(\frac{i_2 R_2 - i_1 R_1}{i_1 - i_2}\right) = \frac{V_2 - V_1}{i_1 - i_2}$$

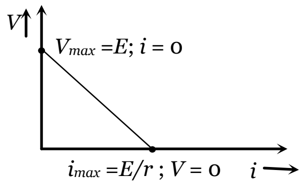
Note : ❑ When the cell is charging *i.e.* current is given to the cell then $E = V - ir$ and $E < V$.



(2) **Open circuit and short circuit**

Open circuit	**Short circuit**
	
(i) Current through the circuit $i = 0$	(i) Maximum current (called short circuit current) flows momentarily $i_{sc} = \frac{E}{r}$
(ii) Potential difference between A and B, $V_{AB} = E$	(ii) Potential difference $V = 0$
(iii) Potential difference between C and D, $V_{CD} = 0$	

Note : ❑ Above information's can be summarized by the following graph



Concepts

☞ *It is a common misconception that "current in the circuit will be maximum when power consumed by the load is maximum."*

Actually current $i = E/(R+r)$ is maximum (= E/r) when R = min = 0 with $P_L = (E/r)^2 \times 0 = 0$ min. while power consumed by the load $E^2R/(R+r)^2$ is maximum (= $E^2/4r$) when $R = r$ and $i = (E/2r) \neq \max(= E/r)$.

☞ *Emf is independent of the resistance of the circuit and depends upon the nature of electrolyte of the cell while potential difference depends upon the resistance between the two points of the circuit and current flowing through the circuit.*

☞ *Emf is a cause and potential difference is an effect.*

☞ *Whenever a cell or battery is present in a branch there must be some resistance (internal or external or both) present in that branch. In practical situation it always happen because we can never have an ideal cell or battery with zero resistance.*

Example

Example: 49 A new flashlight cell of emf 1.5 *volts* gives a current of 15 *amps*, when connected directly to an ammeter of resistance 0.04 Ω. The internal resistance of cell is

(a) 0.04 Ω (b) 0.06 Ω (c) 0.10 Ω (d) 10 Ω

Solution : (b) By using $i = \frac{E}{R+r} \Rightarrow 15 = \frac{1.5}{0.04+r} \Rightarrow r = 0.06\,\Omega$

Example: 50 For a cell, the terminal potential difference is 2.2 *V* when the circuit is open and reduces to 1.8 *V*, when the cell is connected across a resistance, $R = 5\Omega$. The internal resistance of the cell is

(a) $\frac{10}{9}\Omega$ (b) $\frac{9}{10}\Omega$ (c) $\frac{11}{9}\Omega$ (d) $\frac{5}{9}\Omega$

Solution : (a) In open circuit, $E = V = 2.2\,V$, In close circuit, $V = 1.8\,V$, $R = 5\Omega$

So internal resistance, $r = \left(\frac{E}{V} - 1\right)R = \left(\frac{2.2}{1.8} - 1\right) \times 5 \Rightarrow r = \frac{10}{9}\Omega$

Example: 51 The internal resistance of a cell of emf 2*V* is 0.1 Ω. It's connected to a resistance of 3.9 Ω. The voltage across the cell will be **[CBSE PMT 1999; AFMC 1999; MP PET 1993; CPMT 1990]**

(a) 0.5 *volt* (b) 1.9 *volt* (c) 1.95 *volt* (d) 2 *volt*

Solution : (c) By using $r = \left(\frac{E}{V} - 1\right)R \Rightarrow 0.1 = \left(\frac{2}{V} - 1\right) \times 3.9 \Rightarrow V = 1.95\,volt$

Example: 52 When the resistance of 2 Ω is connected across the terminal of the cell, the current is 0.5 *amp*. When the resistance is increased to 5 Ω, the current is 0.25 *amp*. The emf of the cell is

(a) 1.0 *volt* (b) 1.5 *volt* (c) 2.0 *volt* (d) 2.5 *volt*

Solution : (b) By using $E = \frac{i_1 i_2}{(i_2 - i_1)}(R_1 - R_2) = \frac{0.5 \times 0.25}{(0.25 - 0.5)}(2 - 5) = 1.5\,volt$

Example: 53 A primary cell has an emf of 1.5 *volts*, when short-circuited it gives a current of 3 *amperes*. The internal resistance of the cell is

(a) 4.5 *ohm* (b) 2 *ohm* (c) 0.5 *ohm* (d) 1/4.5 *ohm*

Solution : (c) $i_{sc} = \frac{E}{r} \Rightarrow 3 = \frac{1.5}{r} \Rightarrow r = 0.5\,\Omega$

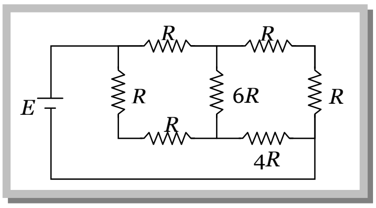
Example: 54 A battery of internal resistance 4 Ω is connected to the network of resistances as shown. In order to give the maximum power to the network, the value of *R* (in Ω) should be **[IIT-JEE 1995]**

(a) 4/9

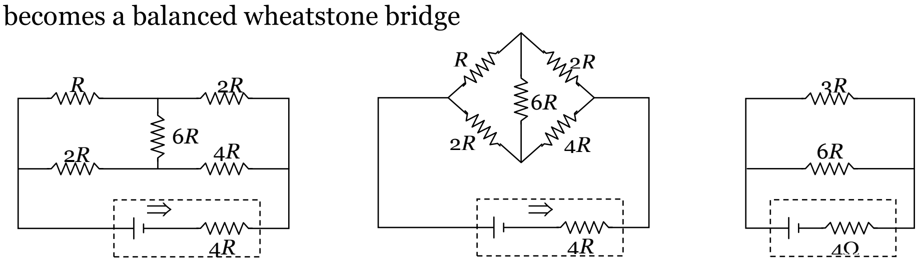
(b) 8/9

(c) 2

(d) 18



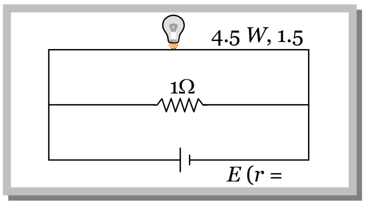
Solution : (c) The equivalent circuit becomes a balanced wheatstone bridge



For maximum power transfer, external resistance should be equal to internal resistance of source

$$\Rightarrow \frac{(R+2R)(2R+4R)}{(R+2R)+(2R+4R)} = 4 \;\; i.e. \;\; \frac{3R \times 6R}{3R+6R} = 4 \;\; \text{or } R = 2\Omega$$

***Example*: 55** A torch bulb rated as 4.5 *W*, 1.5 *V* is connected as shown in the figure. The emf of the cell needed to make the bulb glow at full intensity is



(a) 4.5 *V*

(b) 1.5 *V*

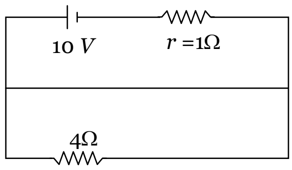
(c) 2.67 *V*

(d) 13.5 *V*

Solution : (d) When bulb glows with full intensity, potential difference across it is 1.5 *V*. So current through the bulb and resistance of 1Ω are 3 *A* and 1.5 *A* respectively. So main current from the cell i = 3 + 1.5 = 4.5 *A*. By using $E = V + iR \Rightarrow E$ = 1.5 + 4.5 × 2.67 = 13.5 *V*.

Tricky Example: 7

Potential difference across the terminals of the battery shown in figure is (r = internal resistance of battery)



(a) 8 *V* (b) 10 *V*

(c) 6 *V* (d) Zero

Solution : (d) Battery is short circuited so potential difference is zero.

Grouping of cell

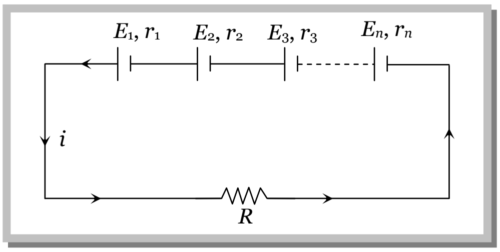
Group of cell is called a battery.

(1) **Series grouping :** In series grouping anode of one cell is connected to cathode of other cell and so on.

(i) *n* identical cells are connected in series

(a) Equivalent emf of the combination $E_{eq} = nE$

(b) Equivalent internal resistance $r_{eq} = nr$



(c) Main current = Current from each cell $= i = \dfrac{nE}{R+nr}$

(d) Potential difference across external resistance $V = iR$

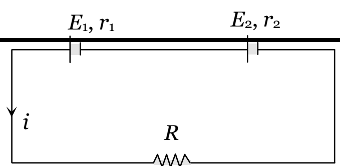
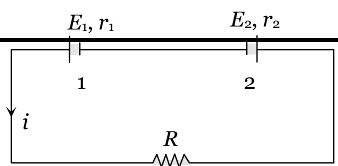
(e) Potential difference across each cell $V' = \dfrac{V}{n}$

(f) Power dissipated in the circuit $P = \left(\dfrac{nE}{R+nr}\right)^2 . R$

(g) Condition for maximum power $R = nr$ and $P_{\max} = n\left(\dfrac{E^2}{4r}\right)$

(h) This type of combination is used when $nr << R$.

(ii) If non-identical cell are connected in series

Cells are connected in right order	Cells are wrongly connected
	$(E_1 > E_2)$
	

(a) Equivalent emf $E_{eq} = E_1 + E_2$

(b) Current $i = \dfrac{E_{eq}}{R + r_{eq}}$

(c) Potential difference across each cell $V_1 = E_1 - ir_1$ and $V_2 = E_2 - ir_2$

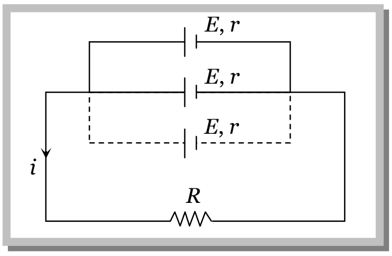
(a) Equivalent emf $E_{eq} = E_1 - E_2$

(b) Current $i = \dfrac{E_1 - E_2}{R + r_{eq}}$

(c) in the above circuit cell 1 is discharging so it's equation is $E_1 = V_1 + ir_1 \Rightarrow \boldsymbol{V_1 = E_1 - ir_1}$ and cell 2 is charging so it's equation

$$E_2 = V_2 - ir_2 \Rightarrow \boldsymbol{V_2 = E_2 + ir_2}$$

(2) **Parallel grouping :** In parallel grouping all anodes are connected at one point and all cathode are connected together at other point.



(i) **If *n* identical cells are connected in parallel**

(a) Equivalent emf $E_{eq} = E$

(b) Equivalent internal resistance $R_{eq} = r/n$

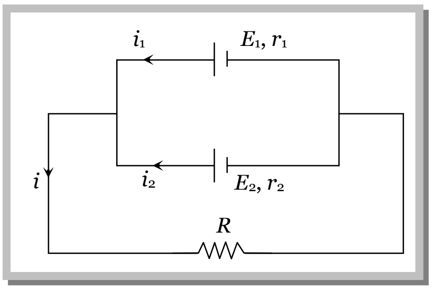
(c) Main current $i = \dfrac{E}{R + r/n}$

(d) P.d. across external resistance = p.d. across each cell = $V = iR$

(e) Current from each cell $i' = \dfrac{i}{n}$ (f) Power dissipated in the circuit $P = \left(\dfrac{E}{R + r/n}\right)^2 . R$

(g) Condition for max power $R = r/n$ and $P_{\max} = n\left(\dfrac{E^2}{4r}\right)$ (h) This type of combination is used when $nr >> R$

(ii) **If non-identical cells are connected in parallel :** If cells are connected with right polarity as shown below then

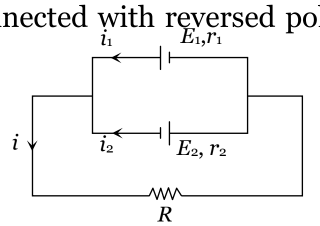


(a) Equivalent emf $E_{eq} = \dfrac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

(b) Main current $i = \dfrac{E_{eq}}{r + R_{eq}}$

(c) Current from each cell $i_1 = \dfrac{E_1 - iR}{r_1}$ and $i_2 = \dfrac{E_2 - iR}{r_2}$

Note : ❑ In this combination if cell's are connected with reversed polarity as shown in figure then :

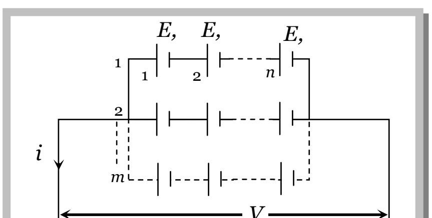


Equivalent emf $\boldsymbol{E_{eq} = \dfrac{E_1 r_2 - E_2 r_1}{r_1 + r_2}}$

(3) **Mixed Grouping :** If n identical cell's are connected in a row and such m row's are connected in parallel as shown.

(i) Equivalent emf of the combination $E_{eq} = nE$

(ii) Equivalent internal resistance of the combination $r_{eq} = \dfrac{nr}{m}$



(iii) Main current flowing through the load $i = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{mnE}{mR + nr}$

(iv) Potential difference across load $V = iR$

(v) Potential difference across each cell $V' = \dfrac{V}{n}$

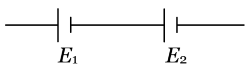
(vi) Current from each cell $i' = \dfrac{i}{n}$

(vii) Condition for maximum power $R = \dfrac{nr}{m}$ and $P_{\max} = (mn)\dfrac{E^2}{4r}$

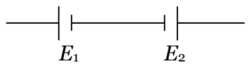
(viii) Total number of cell = mn

Concepts

☞ *In series grouping of cell's their emf's are additive or subtractive while their internal resistances are always additive. If dissimilar plates of cells are connected together their emf's are added to each other while if their similar plates are connected together their emf's are subtractive.*



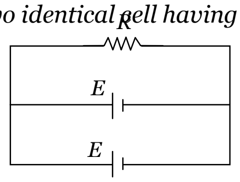
$E_{eq} = E_1 + E_2$ & $r_{eq} = r_1 + r_2$



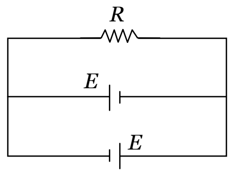
$E_{eq} = E_1 - E_2\ (E_1 > E_2)$ & $r_{eq} = r_1 + r_2$

☞ *In series grouping of identical cells. If one cell is wrongly connected then it will cancel out the effect of two cells e.g. If in the combination of n identical cells (each having emf E and internal resistance r) if x cell are wrongly connected then equivalent emf* $E_{eq} = (n - 2x)E$ *and equivalent internal resistance* $r_{eq} = nr$.

☞ *In parallel grouping of two identical cell having no internal resistance*

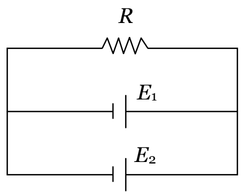


$E_{eq} = E$



$E_{eq} = 0$

☞ *When two cell's of different emf and no internal resistance are connected in parallel then equivalent emf is indeterminate, note that connecting a wire with a cell but with no resistance is equivalent to short circuiting. Therefore the total current that will be flowing will be infinity.*



Example

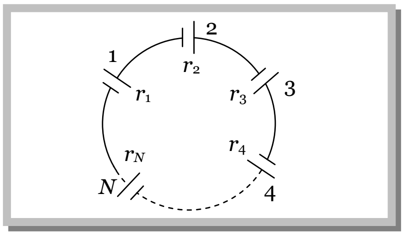
***Example*: 56** A group of N cells whose emf varies directly with the internal resistance as per the equation $E_N = 1.5\, r_N$ are connected as shown in the following figure. The current i in the circuit is **[KCET 2003]**

(a) 0.51 *amp*

(b) 5.1 *amp*

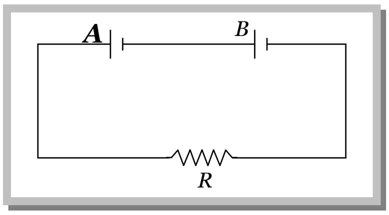
(c) 0.15 *amp*

(d) 1.5 *amp*



Solution : (d) $i = \dfrac{E_{eq}}{r_{eq}} = \dfrac{1.5r_1 + 1.5r_2 + 1.5r_3 + ...}{r_1 + r_2 + r_3 + ...} = 1.5\ amp$

Example: 57 Two batteries A and B each of emf 2 *volt* are connected in series to external resistance R = 1 Ω. Internal resistance of A is 1.9 Ω and that of B is 0.9 Ω, what is the potential difference between the terminals of battery A **[MP PET 2001]**



(a) 2 V

(b) 3.8 V

(c) 0

(d) None of these

Solution : (c) $i = \frac{E_1 + E_2}{R + r_1 + r_2} = \frac{2+2}{1+1.9+0.9} = \frac{4}{3.8}$. Hence $V_A = E_A - ir_A = 2 - \frac{4}{3.8}1.9 = 0$

Example: 58 In a mixed grouping of identical cells 5 rows are connected in parallel by each row contains 10 cell. This combination send a current i through an external resistance of 20 Ω. If the emf and internal resistance of each cell is 1.5 *volt* and 1 Ω respectively then the value of i is

(a) 0.14 (b) 0.25 (c) 0.75 (d) 0.68

Solution : (d) No. of cells in a row n = 10; No. of such rows m = 5

$$i = \frac{nE}{\left(R + \frac{nr}{m}\right)} = \frac{10 \times 1.5}{\left(20 + \frac{10 \times 1}{5}\right)} = \frac{15}{22} = 0.68\ amp$$

Example: 59 To get maximum current in a resistance of 3 Ω one can use n rows of m cells connected in parallel. If the total no. of cells is 24 and the internal resistance of a cell is 0.5 then

(a) $m = 12, n = 2$ (b) $m = 8, n = 4$ (c) $m = 2, n = 12$ (d) $m = 6, n = 4$

Solution : (a) In this question $R = 3\Omega$, $mn = 24$, $r = 0.5\Omega$ and $R = \frac{mr}{n}$. On putting the values we get n = 2 and m = 12.

Example: 60 100 cells each of emf $5V$ and internal resistance 1 Ω are to be arranged so as to produce maximum current in a 25 Ω resistance. Each row contains equal number of cells. The number of rows should be **[MP PMT 1**

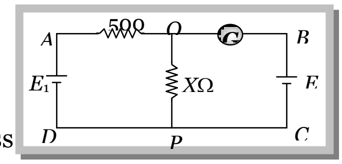
(a) 2 (b) 4 (c) 5 (d) 100

Solution : (a) Total no. of cells, = mn = 100 .... (i)

Current will be maximum when $R = \frac{nr}{m}$; $25 = \frac{n \times 1}{m} \Rightarrow n = 25\,m$ .... (ii)

From equation (i) and (ii) n = 50 and m = 2

Example: 61 In the adjoining circuit, the battery E_1 has as emf of 12 *volt* and zero internal resistance, while the battery E has an emf of 2 *volt*. If the galvanometer reads zero, then the value of resistance X *ohm* is **[NCERT 199**



(a) 10 (b) 100

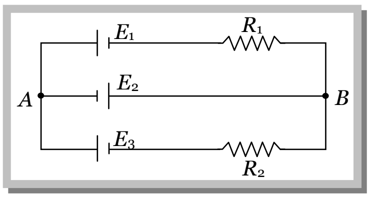
(c) 500 (d) 200

Solution : (b) For zero deflection in galvanometer the potential different across

In this condition $\frac{12X}{500 + X} = 2$

$\therefore$ $X = 100\ \Omega$

Example: 62 In the circuit shown here $E_1 = E_2 = E_3 = 2V$ and $R_1 = R_2 = 4\ \Omega$. The current flowing between point A and B through battery E_2 is



(a) Zero

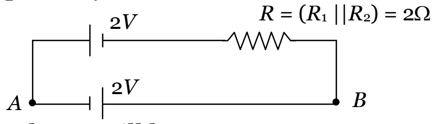
(b) 2 A from A to B

(c) 2 A from B to A

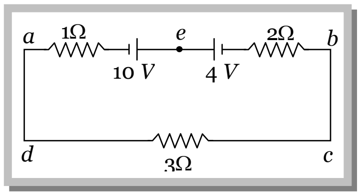
(d) None of these

Solution : (b) The equivalent circuit can be drawn as since E_1 & E_3 are parallely connected

So current $i = \dfrac{2+2}{2} = 2Amp$ from A to B.



***Example*: 63** The magnitude and direction of the current in the circuit shown will be **[CPMT 1986, 88]**

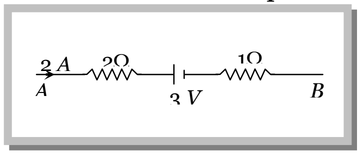


(a) $\dfrac{7}{3}A$ from a to b through e (b) $\dfrac{7}{3}A$ from b and a through e

(c) 1.0 A from b to a through e (d) 1.0 A from a to b through e

Solution : (d) Current $i = \dfrac{10-4}{3+2+1} = 1A$ from a to b via e

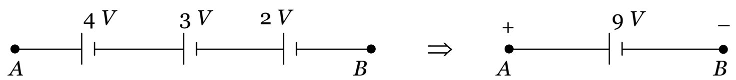
***Example*: 64** Figure represents a part of the closed circuit. The potential difference between points A and B ($V_A - V_B$) is



(a) + 9 V (b) − 9 V

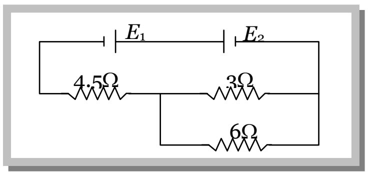
(c) + 3 V (d) + 6 V

Solution : (a) The given part of a closed circuit can be redrawn as follows. It should be remember that product of current and resistance can be treated as an imaginary cell having emf = iR.



$\Rightarrow$ 7 Hence $V_A - V_B = +9\ V$

***Example*: 65** In the circuit shown below the cells E_1 and E_2 have emf's 4 V and 8 V and internal resistance 0.5 *ohm* and 1 *ohm* respectively. Then the potential difference across cell E_1 and E_2 will be



(a) 3.75 V, 7.5 V

(b) 4.25 V, 7.5 V

(c) 3.75 V, 3.5 V

(d) 4.25 V, 4.25 V

Solution : (b) In the given circuit diagram external resistance $R = \dfrac{3\times 6}{3+6} + 4.5 = 6.5\Omega$. Hence main current through the circuit $i = \dfrac{E_2 - E_1}{R + r_{eq}} = \dfrac{8-4}{6.5+0.5+0.5} = \dfrac{1}{2}\,amp.$

Cell 1 is charging so from it's emf equation $E_1 = V_1 - ir_1 \Rightarrow 4 = V_1 - \dfrac{1}{2}\times 0.5 \Rightarrow V_1 = 4.25\ volt$

Cell 2 is discharging so from it's emf equation $E_2 = V_2 + ir_2 \Rightarrow 8 = V_2 + \dfrac{1}{2}\times 1 \Rightarrow V_2 = 7.5\ volt$

***Example*: 66** A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to this current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length 2L. The temperature of wire is raised by same amount ΔT in the same time t. The value of N is

(a) 4 (b) 6 (c) 8 (d) 9

Solution : (b) Heat = $mS\Delta T = i^2Rt$

Case I : Length (L) $\Rightarrow$ Resistance = R and mass = m

Case II : Length (2L) $\Rightarrow$ Resistance = 2R and mass = 2m

So $\dfrac{m_1 S_1 \Delta T_1}{m_2 S_2 \Delta T_2} = \dfrac{i_1^2 R_1 t_1}{i_2^2 r_2 t_2} \Rightarrow \dfrac{mS\Delta T}{2mS\Delta T} = \dfrac{i_1^2 Rt}{i_2^2\, 2Rt} \Rightarrow i_1 = i_2 \Rightarrow \dfrac{(3E)^2}{12} = \dfrac{(NE)^2}{2R} \Rightarrow N = 6$

Tricky Example: 8

n identical cells, each of emf E and internal resistance r, are joined in series to form a closed circuit. The potential difference across any one cell is

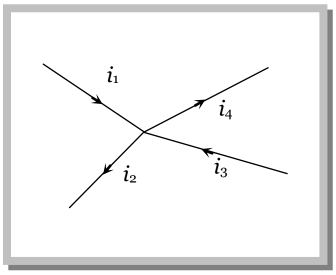
(a) Zero (b) E (c) $\frac{E}{n}$ (d) $\left(\frac{n-1}{n}\right)E$

Solution: (a) Current in the circuit $i = \frac{nE}{nr} = \frac{E}{r}$

The equivalent circuit of one cell is shown in the figure. Potential difference across the cell $= V_A - V_B = -E + ir = -E + \frac{E}{r}.r = 0$

Kirchoff's Laws

(1) **Kirchoff's first law :** This law is also known as junction rule or current law (*KCL*). According to it the algebraic sum of currents meeting at a junction is zero *i.e.* $\boldsymbol{\Sigma i = 0}$.



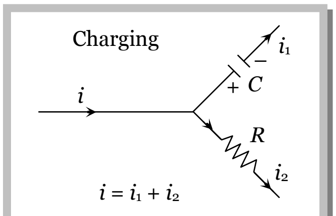
In a circuit, at any junction the sum of the currents entering the junction must equal the sum of the currents leaving the junction. $i_1 + i_3 = i_2 + i_4$

Here it is worthy to note that :

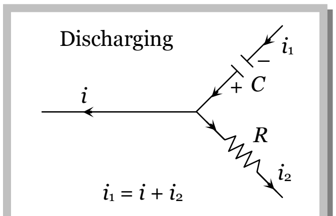
(i) If a current comes out to be negative, actual direction of current at the junction is opposite to that assumed, $i + i_1 + i_2 = 0$ can be satisfied only if at least one current is negative, *i.e.* leaving the junction.

(ii) This law is simply a statement of "*conservation of charge*" as if current reaching a junction is not equal to the current leaving the junction, charge will not be conserved.

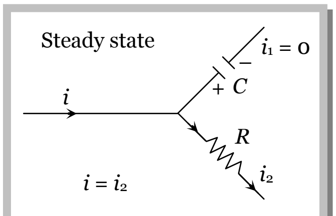
Note : ❑ This law is also applicable to a capacitor through the concept of displacement current treating the resistance of capacitor to be zero during charging or discharging and infinite in steady state as shown in figure.



(A)



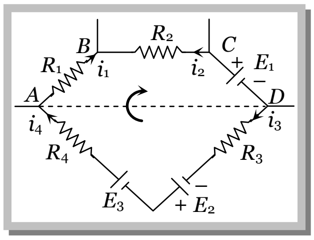
(B)



(C)

(2) **Kirchoff's second law :** This law is also known as loop rule or voltage law (KVL) and according to it "the algebraic sum of the changes in potential in complete traversal of a mesh (closed loop) is zero", *i.e.* $\boldsymbol{\Sigma V = 0}$

e.g. In the following closed loop.



$$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$$

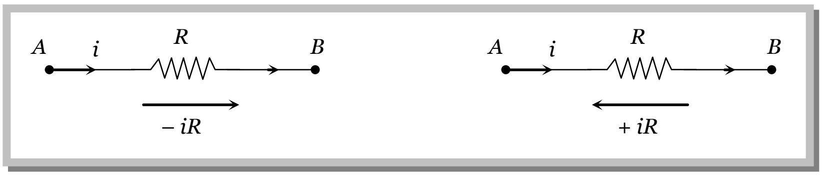
Here it is worthy to note that :

(i) This law represents "*conservation of energy*" as if the sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.

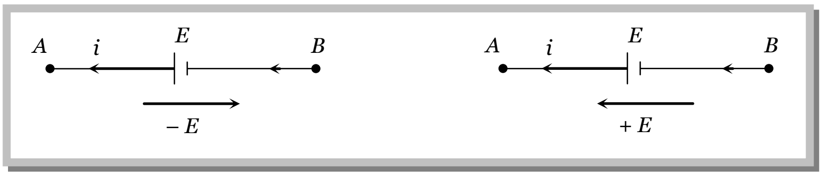
(ii) If there are n meshes in a circuit, the number of independent equations in accordance with loop rule will be $(n - 1)$.

(3) **Sign convention for the application of Kirchoff's law :** For the application of Kirchoff's laws following sign convention are to be considered

(i) The change in potential in traversing a resistance in the direction of current is $-iR$ while in the opposite direction $+iR$

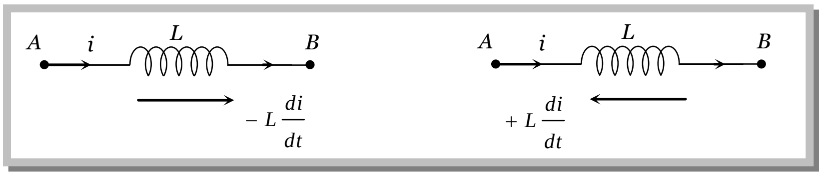


(ii) The change in potential in traversing an emf source from negative to positive terminal is $+E$ while in the opposite direction $-E$ irrespective of the direction of current in the circuit.



(iii) The change in potential in traversing a capacitor from the negative terminal to the positive terminal is $+\frac{q}{C}$ while in opposite direction $-\frac{q}{C}$.

(iv) The change in voltage in traversing an inductor in the direction of current is $-L\frac{di}{dt}$ while in opposite direction it is $+L\frac{di}{dt}$.



(4) **Guidelines to apply Kirchoff's law**

(i) Starting from the positive terminal of the battery having highest emf, distribute current at various junctions in the circuit in accordance with '*junction rule*'. It is not always easy to correctly guess the direction of current, no problem if one assumes the wrong direction.

(ii) After assuming current in each branch, we pick a point and begin to walk (mentally) around a closed loop. As we traverse each resistor, capacitor, inductor or battery we must write down, the voltage change for that element according to the above sign convention.

(iii) By applying KVL we get one equation but in order to solve the circuit we require as many equations as there are unknowns. So we select the required number of loops and apply Kirchhoff's voltage law across each such loop.

(iv) After solving the set of simultaneous equations, we obtain the numerical values of the assumed currents. If any of these values come out to be negative, it indicates that particular current is in the opposite direction from the assumed one.

Note : ❑ The number of loops must be selected so that every element of the circuit must be included in at least one of the loops.

❑ While traversing through a capacitor or battery we do not consider the direction of current.

- While considering the voltage drop or gain across as inductor we always assume current to be in increasing function.

(5) **Determination of equivalent resistance by Kirchoff's method :** This method is useful when we are not able to identify any two resistances in series or in parallel. It is based on the two Kirchhoff's laws. The method may be described in the following guideline.

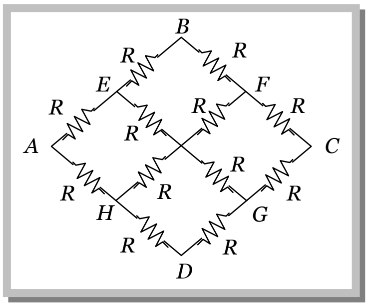
(i) Assume an imaginary battery of emf E connected between the two terminals across which we have to calculate the equivalent resistance.

(ii) Assume some value of current, say i, coming out of the battery and distribute it among each branch by applying Kirchhoff's current law.

(iii) Apply Kirchhoff's voltage law to formulate as many equations as there are unknowns. It should be noted that at least one of the equations must include the assumed battery.

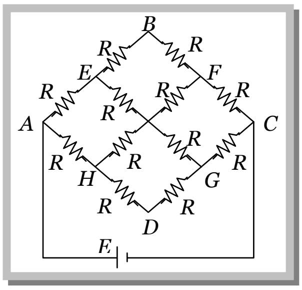
(iv) Solve the equations to determine $\frac{E}{i}$ ratio which is the equivalent resistance of the network.

e.g. Suppose in the following network of 12 identical resistances, equivalent resistance between point A and C is to be calculated.



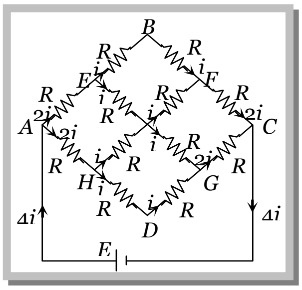
According to the above guidelines we can solve this problem as follows

Step (1)



An imaginary battery of emf E is assumed across the terminals A and C

Step (2)



The current in each branch is distributed by assuming $4i$ current coming out of the battery.

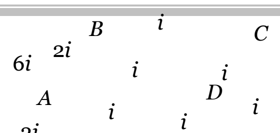
Step (3) Applying KVL along the loop including the nodes A, B, C and the battery E. Voltage equation is $-2iR - iR - iR - 2iR + E = 0$

Step (4) After solving the above equation, we get $6iR = E \Rightarrow$ equivalent resistance between A and C is $R = \frac{E}{4i} = \frac{6iR}{4i} = \frac{3}{2}R$

Concepts

☞ *Using Kirchoff's law while dividing the current having a junction through different arms of a network, it will be same through different arms of same resistance if the end points of these arms are equilocated w.r.t. exit point for current in network and will be different through different arms if the end point of these arms are not equilocated w.r.t. exit point for current of the network.*

e.g. In the following figure the current going in arms AB, AD and AL will be same because the location of end points B, D

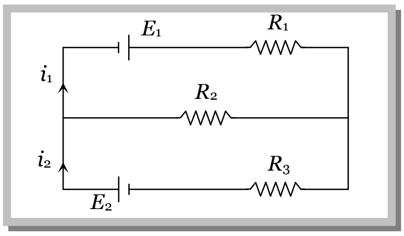


and L of these arms are symmetrically located w.r.t. exit point N of the network.

Example

Example: 67 In the following circuit $E_1 = 4V$, $R_1 = 2\Omega$ **[MP PET 2003]**

$E_2 = 6V$, $R_2 = 2\Omega$ and $R_3 = 4\Omega$. The current i_1 is



(a) 1.6 A

(b) 1.8 A

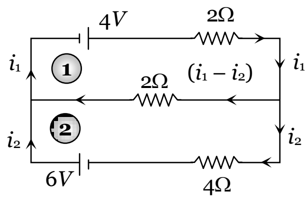
(c) 2.25 A

(d) 1 A

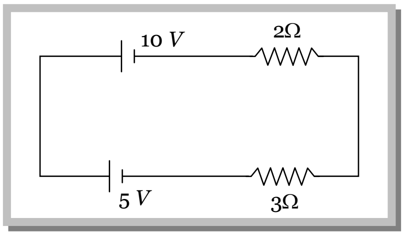
Solution : (b) For loop (1) $-2i_1 - 2(i_1 - i_2) + 4 = 0 \Rightarrow 2i_1 - i_2 = 2$ ...... (i)

For loop (2) $-4i_2 + 2(i_1 - i_2) + 6 = 0 \Rightarrow 3i_2 - i_1 = 3$ ...... (ii)

After solving equation (i) and (ii) we get $i_1 = 1.8A$ and $i_2 = 1.6A$



Example: 68 Determine the current in the following circuit



(a) 1 A

(b) 2.5 A

(c) 0.4 A

(d) 3 A

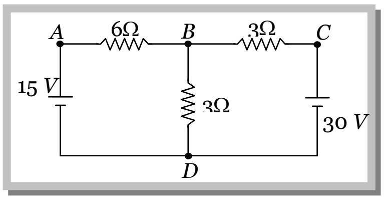
Solution : (a) Applying KVL in the given circuit we get $-2i + 10 - 5 - 3i = 0 \Rightarrow i = 1A$

Second method : Similar plates of the two batteries are connected together, so the net emf = 10 – 5 = 5V

Total resistance in the circuit = 2 + 3 = 5Ω

$\therefore \quad i = \dfrac{\Sigma V}{\Sigma R} = \dfrac{5}{5} = 1A$

Example: 69 In the circuit shown in figure, find the current through the branch BD



(a) 5 A (b) 0 A

(c) 3 A (d) 4 A

Solution : (a) The current in the circuit are assumed as shown in the fig.

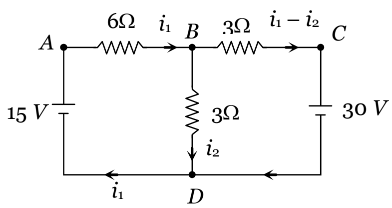
Applying KVL along the loop $ABDA$, we get

$-6i_1 - 3i_2 + 15 = 0$ or $2i_1 + i_2 = 5$ ...... (i)

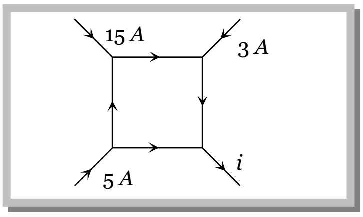
Applying KVL along the loop $BCDB$, we get

$-3(i_1 - i_2) - 30 + 3i_2 = 0$ or $-i_1 + 2i_2 = 10$ ...... (ii)

Solving equation (i) and (ii) for i_2, we get $i_2 = 5A$



Example: 70 The figure shows a network of currents. The magnitude of current is shown here. The current i will be **[MP PMT 1995]**



(a) 3 A

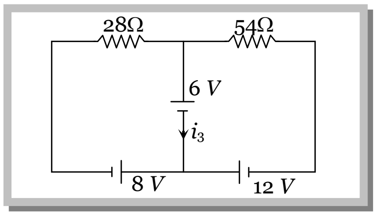
(b) 13 A

(c) 23 A

(d) – 3 A

Solution : (c) $i = 15 + 3 + 5 = 23A$

***Example*: 71** Consider the circuit shown in the figure. The current i_3 is equal to **[AMU 1995]**



(a) 5 *amp*

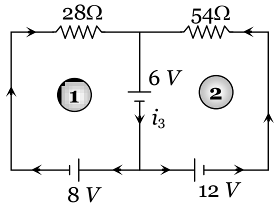
(b) 3 *amp*

(c) – 3 *amp*

(d) – 5/6 *amp*

Solution : (d) Suppose current through different paths of the circuit is as follows.

After applying *KVL* for loop (1) and loop (2)

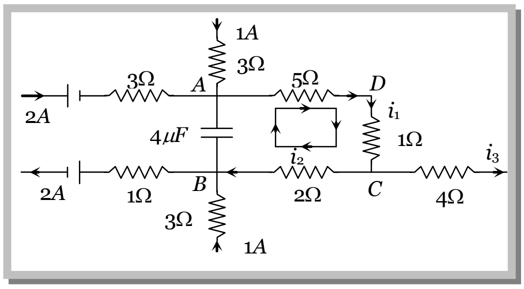


We get $28i_1 = -6 - 8 \Rightarrow i_1 = -\frac{1}{2}A$

and $54i_2 = -6 - 12 \Rightarrow i_2 = -\frac{1}{3}A$

Hence $i_3 = i_1 + i_2 = -\frac{5}{6}A$

***Example*: 72** A part of a circuit in steady state along with the current flowing in the branches, with value of each resistance is shown in figure. What will be the energy stored in the capacitor C_0



(a) $6 \times 10^{-4} J$

(b) $8 \times 10^{-4} J$

(c) $16 \times 10^{-4} J$

(d) Zero

Solution : (b) Applying Kirchhoff's first law at junctions *A* and *B* respectively we have $2 + 1 - i_1 = 0$ *i.e.*, $i_1 = 3A$

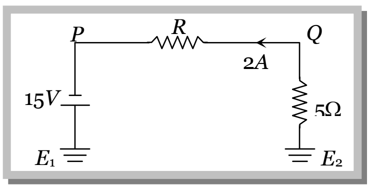
and $i_2 + 1 - 2 - 0 = 0$ *i.e.*, $i_2 = 1A$

Now applying Kirchhoff's second law to the mesh *ADCBA* treating capacitor as a seat of emf *V* in open circuit

$-3 \times 5 - 3 \times 1 - 1 \times 2 + V = 0$ *i.e.* $V(= V_A - V_B) = 20\ V$

So, energy stored in the capacitor $U = \frac{1}{2}CV^2 = \frac{1}{2}(4 \times 10^{-6}) \times (20)^2 = 8 \times 10^{-4}\ J$

***Example*: 73** In the following circuit the potential difference between *P* and *Q* is



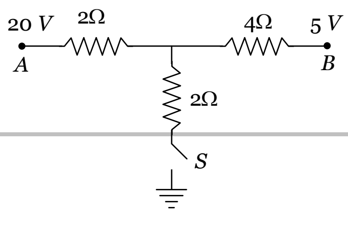
(a) 15 *V* (b) 10 *V*

(c) 5 *V* (d) 2.5 *V*

Solution : (c) By using KVL $-5 \times 2 - V_{PQ} + 15 = 0 \Rightarrow V_{PQ} = 5V$

Tricky Example: 9

As the switch *S* is closed in the circuit shown in figure, current passed through it is



(a) $4.5\,A$ (b) $6.0\,A$ (c) $3.0\,A$ (d) Zero

Solution : (a) Let V be the potential of the junction as shown in figure. Applying junction law, we have

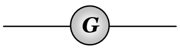
or $\frac{20-V}{2}+\frac{5-V}{4}=\frac{V-0}{2}$ or $40-2V+5-V=2V$

or $5V=45 \Rightarrow V=9V$

$\therefore\ i_3=\frac{V}{2}=4.5A$

Different Measuring Instruments

(1) **Galvanometer :** It is an instrument used to detect small current passing through it by showing deflection. Galvanometers are of different types *e.g.* moving coil galvanometer, moving magnet galvanometer, hot wire galvanometer. In dc circuit usually moving coil galvanometer are used.

(i) **It's symbol :** 

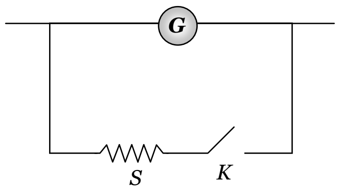
; where G is the total internal resistance of the galvanometer.

(ii) **Principle :** In case of moving coil galvanometer deflection is directly proportional to the current that passes through it *i.e.* $\boldsymbol{i \propto \theta}$.

(iii) **Full scale deflection current :** The current required for full scale deflection in a galvanometer is called full scale deflection current and is represented by i_g.

(iv) **Shunt :** The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer is known as shunt.

Merits of shunt	Demerits of shunt
(a) To protect the galvanometer coil from burning (b) It can be used to convert any galvanometer into ammeter of desired range.	Shunt resistance decreases the sensitivity of galvanometer.

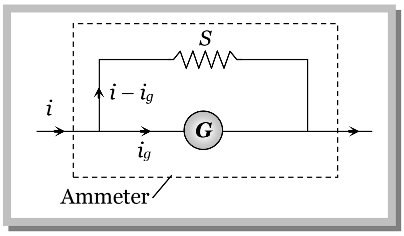


(2) **Ammeter :** It is a device used to measure current and is always connected in series with the 'element' through which current is to be measured.

(i) The reading of an ammeter is always lesser than actual current in the circuit.

(ii) Smaller the resistance of an ammeter more accurate will be its reading. An ammeter is said to be ideal if its resistance r is zero.

(iii) **Conversion of galvanometer into ammeter :** A galvanometer may be converted into an ammeter by connecting a low resistance (called shunt S) in parallel to the galvanometer G as shown in figure.



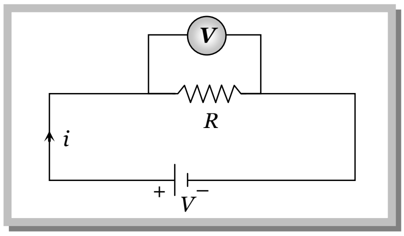
(a) Equivalent resistance of the combination $=\frac{GS}{G+S}$

(b) G and S are parallel to each other hence both will have equal potential difference *i.e.* $i_g G=(i-i_g)S$; which gives

Required shunt $\boldsymbol{S=\frac{i_g}{(i-i_g)}G}$

(c) To pass *n*th part of main current (*i.e.* $i_g = \frac{i}{n}$) through the galvanometer, required shunt $\boldsymbol{S = \frac{G}{(n-1)}}$.

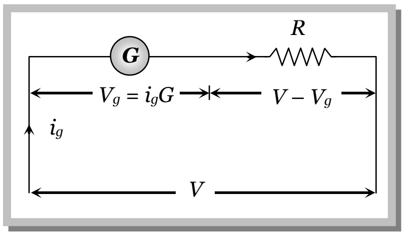
(3) **Voltmeter :** It is a device used to measure potential difference and is always put in parallel with the 'circuit element' across which potential difference is to be measured.



(i) The reading of a voltmeter is always lesser than true value.

(ii) Greater the resistance of voltmeter, more accurate will be its reading. A voltmeter is said to be ideal if its resistance is infinite, *i.e.,* it draws no current from the circuit element for its operation.

(iii) **Conversion of galvanometer into voltmeter :** A galvanometer may be converted into a voltmeter by connecting a large resistance *R* in series with the galvanometer as shown in the figure.



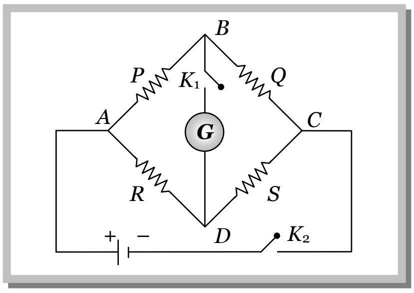
(a) Equivalent resistance of the combination = $G + R$

(b) According to ohm's law $V = i_g\,(G + R)$; which gives

Required series resistance $\boldsymbol{R = \frac{V}{i_g} - G = \left(\frac{V}{V_g} - 1\right) G}$

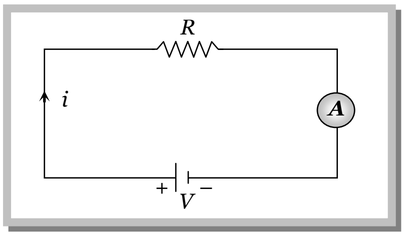
(c) If n^{th} part of applied voltage appeared across galvanometer (*i.e.* $V_g = \frac{V}{n}$) then required series resistance $\boldsymbol{R = (n-1)\,G}$.

(4) **Wheatstone bridge :** Wheatstone bridge is an arrangement of four resistance which can be used to measure one of them in terms of rest. Here arms *AB* and *BC* are called ratio arm and arms *AC* and *BD* are called conjugate arms



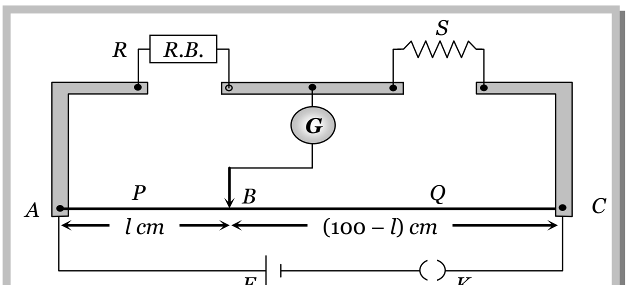
(i) **Balanced bridge :** The bridge is said to be balanced when deflection in galvanometer is zero *i.e.* no current flows through the galvanometer or in other words $V_B = V_D$. In the balanced condition $\boldsymbol{\frac{P}{Q} = \frac{R}{S}}$, on mutually changing the position of cell and galvanometer this condition will not change.

(ii) **Unbalanced bridge :** If the bridge is not balanced current will flow from *D* to *B* if $V_D > V_B$ *i.e.* $(V_A - V_D) < (V_A - V_B)$ which gives $PS > RQ$.



(iii) **Applications of wheatstone bridge :** Meter bridge, post office box and Carey Foster bridge are instruments based on the principle of wheatstone bridge and are used to measure unknown resistance.

(5) **Meter bridge :** In case of meter bridge, the resistance wire *AC* is 100 *cm* long. Varying the position of tapping point *B*, bridge is balanced. If in balanced position of bridge *AB* = *l*, *BC* (100 – *l*) so that $\frac{Q}{P} = \frac{(100 - l)}{l}$. Also $\frac{P}{Q} = \frac{R}{S} \Rightarrow S = \frac{(100 - l)}{l} R$



Concepts

- *Wheatstone bridge is most sensitive if all the arms of bridge have equal resistances i.e. $P = Q = R = S$*
- *If the temperature of the conductor placed in the right gap of metre bridge is increased, then the balancing length decreases and the jockey moves towards left.*
- *In Wheatstone bridge to avoid inductive effects the battery key should be pressed first and the galvanometer key afterwards.*
- *The measurement of resistance by Wheatstone bridge is not affected by the internal resistance of the cell.*

Example

***Example*: 74** The scale of a galvanometer of resistance 100 Ω contains 25 divisions. It gives a deflection of one division on passing a current of 4×10^{-4} *A*. The resistance in *ohms* to be added to it, so that it may become a voltmeter of range 2.5 *volt* is

(a) 100 (b) 150 (c) 250 (d) 300

Solution : (b) Current sensitivity of galvanometer = 4×10^{-4} *Amp/div.*

So full scale deflection current (i_g) = Current sensitivity × Total number of division = $4 \times 10^{-4} \times 25 = 10^{-2}$ *A*

To convert galvanometer in to voltmeter, resistance to be put in series is $R = \frac{V}{i_g} - G = \frac{2.5}{10^{-2}} - 100 = 150\,\Omega$

***Example*: 75** A galvanometer, having a resistance of 50 Ω gives a full scale deflection for a current of 0.05 *A*. the length in meter of a resistance wire of area of cross-section 2.97×10^{-2} cm^2 that can be used to convert the galvanometer into an ammeter which can read a maximum of 5*A* current is : (Specific resistance of the wire = 5×10^{-7} Ω*m*) **[EAMCET 2003]**

(a) 9 (b) 6 (c) 3 (d) 1.5

Solution : (c) Given $G = 50\,\Omega$, $i_g = 0.05$ *Amp.*, $i = 5A$, $A = 2.97 \times 10^{-2}\ cm^2$ and $\rho = 5 \times 10^{-7}\Omega\text{-}m$

By using $\frac{i}{i_g} = 1 + \frac{G}{S} \Rightarrow S = \frac{G.i_g}{(i - i_g)} \Rightarrow \frac{\rho\, l}{A} = \frac{Gi_g}{(i - i_g)} \Rightarrow l = \frac{Gi_gA}{(i - i_g)\rho}$ on putting values $l = 3\ m$.

***Example*: 76** 100 *mA* current gives a full scale deflection in a galvanometer of resistance 2 Ω. The resistance connected with the galvanometer to convert it into a voltmeter of 5 *V* range is

[KCET 2002; UPSEAT 1998; MNR 1994 Similar to MP PMT 1999]

(a) 98 Ω (b) 52 Ω (c) 80 Ω (d) 48 Ω

Solution : (d) $R = \frac{V}{I_g} - G = \frac{5}{100 \times 10^{-3}} - 2 = 50 - 2 = 48\,\Omega$.

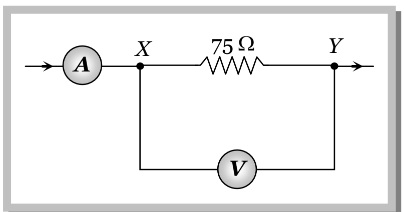
***Example*: 77** A milliammeter of range 10 *mA* has a coil of resistance 1 Ω. To use it as voltmeter of range 10 *volt*, the resistance that must be connected in series with it will be

(a) 999 Ω (b) 99 Ω (c) 1000 Ω (d) None of these

Solution : (a) By using $R = \frac{V}{ig} - G \Rightarrow R = \frac{10}{10 \times 10^{-3}} - 1 = 999\Omega$

***Example*: 78** In the following figure ammeter and voltmeter reads 2 *amp* and 120 *volt* respectively. Resistance of voltmeter is

(a) 100 Ω



(b) 200 Ω

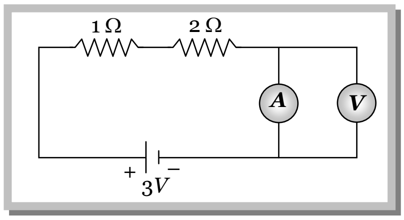
(c) 300 Ω

(d) 400 Ω

Solution : (c) Let resistance of voltmeter be R_V. Equivalent resistance between X and Y is $R_{XY} = \frac{75R_V}{75 + R_V}$

Reading of voltmeter = potential difference across X and Y = 120 = $i \times R_{XY}$ = $2 \times \frac{75R_V}{75 + R_V} \Rightarrow R_V =$ 300Ω

***Example*: 79** In the circuit shown in figure, the voltmeter reading would be



(a) Zero

(b) 0.5 *volt*

(c) 1 *volt*

(d) 2 *volt*

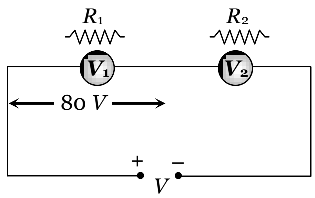
Solution : (a) Ammeter has no resistance so there will be no potential difference across it, hence reading of voltmeter is zero.

***Example*: 80** Voltmeters V_1 and V_2 are connected in series across a d.c. line. V_1reads 80 V and has a per *volt* resistance of 200 Ω, V_2 has a total resistance of 32 kΩ. The line voltage is

(a) 120 V (b) 160 V (c) 220 V (d) 240 V

Solution : (d) Resistance of voltmeter V_1 is R_1 = 200 × 80 = 16000 Ω and resistance of voltmeter V_2 is R_2 = 32000 Ω

By using relation $V' = \left(\frac{R'}{R_{eq}}\right)V$; where V' = potential difference across any resistance R' in a series grouping.



So for voltmeter V_1 potential difference across it is

$$80 = \left(\frac{R_1}{R_1 + R_2}\right).V \Rightarrow V = 240\ V$$

***Example*: 81** The resistance of 1 A ammeter is 0.018 Ω. To convert it into 10 A ammeter, the shunt resistance required will be

(a) 0.18 Ω (b) 0.0018 Ω (c) 0.002 Ω (d) 0.12 Ω

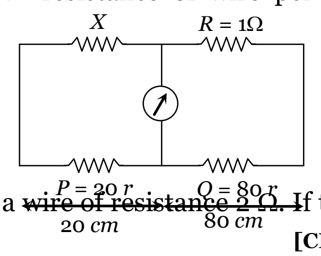
Solution : (c) By using $\frac{i}{i_g} = 1 + \frac{4}{S} \Rightarrow \frac{10}{1} = 1 + \frac{0.018}{S} \Rightarrow S = 0.002\ \Omega$

***Example*: 82** In meter bridge the balancing length from left and when standard resistance of 1 Ω is in right gas is found to be 20 cm. The value of unknown resistance is **[CBSE PMT 1999]**

(a) 0.25 Ω (b) 0.4 Ω (c) 0.5 Ω (d) 4 Ω

Solution: (a) The condition of wheatstone bridge gives $\frac{X}{R} = \frac{20\,r}{80\,r}$, r- resistance of wire per cm, X- unknown resistance

$$\therefore \quad X = \frac{20}{80} \times R = \frac{1}{4} \times 1 = 0.25\ \Omega$$



***Example*: 83** A galvanometer having a resistance of 8 Ω is shunted by a wire of resistance 2 Ω. If the total current is 1 *amp*, the part of it passing through the shunt will be **[CBSE PMT 1998]**

(a) 0.25 *amp* (b) 0.8 *amp* (c) 0.2 *amp* (d) 0.5 *amp*

Solution: (b) Fraction of current passing through the galvanometer

$$\frac{i_g}{i} = \frac{S}{S+G} \text{ or } \frac{i_g}{i} = \frac{2}{2+8} = 0.2$$

So fraction of current passing through the shunt

$$\frac{i_s}{i} = 1 - \frac{i_g}{i} = 1 - 0.2 = 0.8\,amp$$

***Example*: 84** A moving coil galvanometer is converted into an ammeter reading upto 0.03 A by connecting a shunt of resistance $4r$ across it and into an ammeter reading upto 0.06 A when a shunt of resistance r connected across it. What is the maximum current which can be through this galvanometer if no shunt is used

[MP PMT 1996]

(a) 0.01 A (b) 0.02 A (c) 0.03 A (d) 0.04 A

Solution: (b) For ammeter, $S = \frac{i_g}{(i-i_g)}G \Rightarrow i_gG = (i-i_g)S$

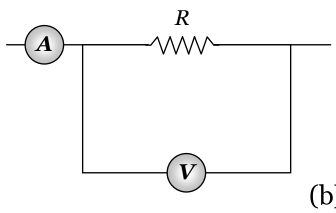
So $i_gG = (0.03 - i_g)4r$ ...... (i) and $i_gG = (0.06 - i_g)r$ ...... (ii)

Dividing equation (i) by (ii) $1 = \frac{(0.03-i_g)4}{0.06-i_g} \Rightarrow 0.06 - i_g = 0.12 - 4i_g$

$\Rightarrow 3i_g = 0.06 \Rightarrow i_g = 0.02\,A$

Tricky Example: 10

The ammeter A reads 2 A and the voltmeter V reads 20 V. The value of resistance R is



(a) Exactly 10 *ohm* (b) Less than 10 *ohm*

(c) More than 10 *ohm* (d) We cannot definitely say

Solution: (c) If current goes through the resistance R is i then $iR = 20\,volt \Rightarrow R = \frac{20}{i}$. Since $i < 2A$ so $R >$ 10Ω.

Potentiometer

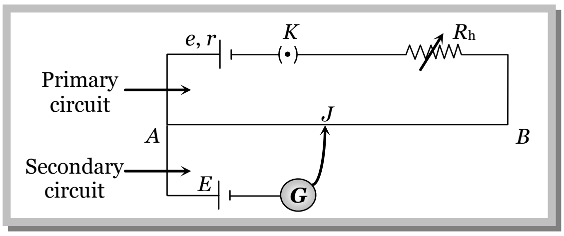
Potentiometer is a device mainly used to measure emf of a given cell and to compare emf's of cells. It is also used to measure internal resistance of a given cell.

(1) **Superiority of potentiometer over voltmeter** : An ordinary voltmeter cannot measure the emf accurately because it does draw some current to show the deflection. As per definition of emf, it is the potential difference when a cell is in open circuit or no current through the cell. Therefore voltmeter can only measure terminal voltage of a give *n* cell.

Potentiometer is based on no deflection method. When the potentiometer gives zero deflection, it does not draw any current from the cell or the circuit *i.e.* potentiometer is effectively an ideal instrument of infinite resistance for measuring the potential difference.

(2) **Circuit diagram :** Potentiometer consists of a long resistive wire AB of length L (about 6m to 10 m long) made up of mangnine or constantan. A battery of known voltage e and internal resistance r called supplier battery or driver cell. Connection of these two forms primary circuit.

One terminal of another cell (whose emf E is to be measured) is connected at one end of the main circuit and the other terminal at any point on the resistive wire through a galvanometer G. This forms the secondary circuit. Other details are as follows



J = Jockey

K = Key

R = Resistance of potentiometer wire,

ρ = Specific resistance of potentiometer wire.

R_h = Variable resistance which controls the current through the wire AB

(3) **Points to be remember**

(i) The specific resistance (ρ) of potentiometer wire must be high but its temperature coefficient of resistance (α) must be low.

(ii) All higher potential points (terminals) of primary and secondary circuits must be connected together at point A and all lower potential points must be connected to point B or jockey.

(iii) The value of known potential difference must be greater than the value of unknown potential difference to be measured.

(iv) The potential gradient must remain constant. For this the current in the primary circuit must remain constant and the jockey must not be slided in contact with the wire.

(v) The diameter of potentiometer wire must be uniform everywhere.

(4) **Potential gradient (x) :** Potential difference (or fall in potential) per unit length of wire is called potential gradient *i.e.* $x = \frac{V}{L}\frac{volt}{m}$ where $V = iR = \left(\frac{e}{R + R_h + r}\right).R$. So $\boldsymbol{x = \frac{V}{L} = \frac{iR}{L} = \frac{i\rho}{A} = \frac{e}{(R + R_h + r)}.\frac{R}{L}}$

(i) Potential gradient directly depends upon

(a) The resistance per unit length (R/L) of potentiometer wire.

(b) The radius of potentiometer wire (*i.e.* Area of cross-section)

(c) The specific resistance of the material of potentiometer wire (*i.e.* ρ)

(d) The current flowing through potentiometer wire (i)

(ii) x indirectly depends upon

(a) The emf of battery in the primary circuit (*i.e.* e)

(b) The resistance of rheostat in the primary circuit (*i.e.* R_h)

Note : ❑ When potential difference V is constant then $\frac{x_1}{x_2} = \frac{L_2}{L_1}$

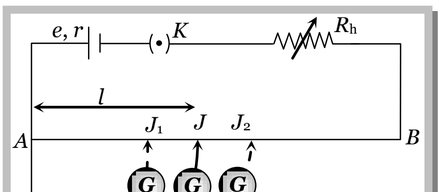
❑ Two different wire are connected in series to form a potentiometer wire then $\frac{x_1}{x_2} = \frac{R_1}{R_2}.\frac{L_2}{L_1}$

❑ If the length of a potentiometer wire and potential difference across it's ends are kept constant and if it's diameter is changed from $d_1 \to d_2$ then potential gradient remains unchanged.

❑ The value of x does not change with any change effected in the secondary circuit.

(5) **Working :** Suppose jocky is made to touch a point J on wire then potential difference between A and J will be $V = xl$

At this length (l) two potential difference are obtained



(i) V due to battery e and

(ii) E due to unknown cell

If $V > E$ then current will flow in galvanometer circuit in one directi

If $V < E$ then current will flow in galvanometer circuit in opposite directi

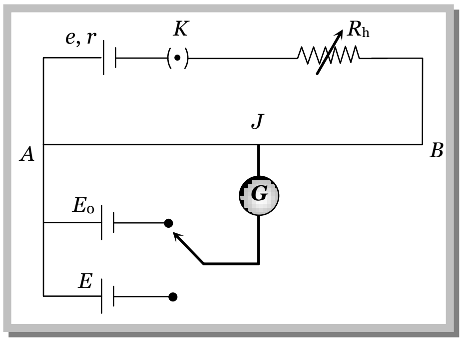
If $V = E$ then no current will flow in galvanometer circuit this condition to known as null deflection position, length l is known as balancing length.

In balanced condition $\boldsymbol{E = xl}$ or $\boldsymbol{E = xl = \frac{V}{L}l = \frac{iR}{L}l = \left(\frac{e}{R + R_h + r}\right).\frac{R}{L} \times l}$

Note : ❑ If V is constant then $L \propto l \quad \Rightarrow \frac{L_1}{L_2} = \frac{l_1}{l_2}$

(6) **Standardization of potentiometer :** The process of determining potential gradient experimentally is known as standardization of potentiometer.

Let the balancing length for the standard emf E_0 is l_0 then by the principle of potentiometer $E_0 = xl_0 \Rightarrow x = \frac{E_0}{l_0}$



(7) **Sensitivity of potentiometer :** A potentiometer is said to be more sensitive, if it measures a small potential difference more accurately.

(i) The sensitivity of potentiometer is assessed by its potential gradient. The sensitivity is inversely proportional to the potential gradient.

(ii) In order to increase the sensitivity of potentiometer

(a) The resistance in primary circuit will have to be decreased.

(b) The length of potentiometer wire will have to be increased so that the length may be measured more accuracy.

(8) **Difference between voltmeter and potentiometer**

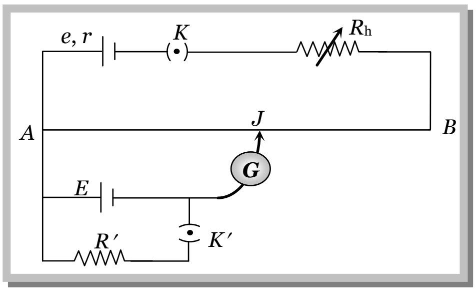
	Voltmeter	**Potentiometer**
(i)	It's resistance is high but finite	Its resistance is high but infinite
(ii)	It draws some current from source of emf	It does not draw any current from the source of known emf
(iii)	The potential difference measured by it is lesser than the actual potential difference	The potential difference measured by it is equal to actual potential difference
(iv)	Its sensitivity is low	Its sensitivity is high
(v)	It is a versatile instrument	It measures only emf or potential difference
(vi)	It is based on deflection method	It is based on zero deflection method

Application of Potentiometer

(1) **To determine the internal resistance of a primary cell**

(i) Initially in secondary circuit key K' remains open and balancing length (l_1) is obtained. Since cell E is in open circuit so it's emf balances on length l_1 *i.e.* $E = xl_1$ .... (i)

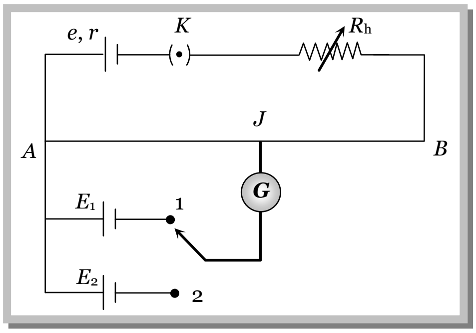
(ii) Now key K' is closed so cell E comes in closed circuit. If the process is repeated again then potential difference V balances on length l_2 *i.e.* $V = xl_2$ .... (ii)



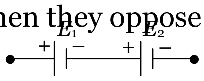
(iii) By using formula internal resistance $r = \left(\frac{E}{V} - 1\right).R'$

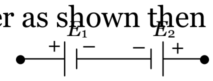
$$r = \left(\frac{l_1 - l_2}{l_2}\right).R'$$

(2) **Comparison of emf's of two cell :** Let l_1 and l_2 be the balancing lengths with the cells E_1 and E_2 respectively then $E_1 = xl_1$ and $E_2 = xl_2 \Rightarrow \frac{E_1}{E_2} = \frac{l_1}{l_2}$



Note : ❑ Let $E_1 > E_2$ and both are connected in series. If balancing length is l_1 when cell assist each other and it is l_2 when they oppose each other as shown then :

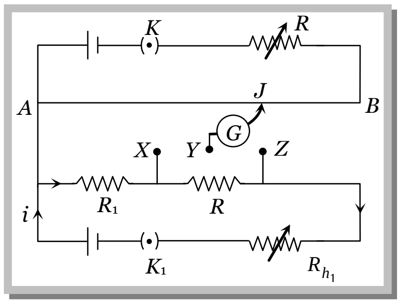




$$(E_1 + E_2) = xl_1 \qquad (E_1 - E_2) = xl_2$$

$$\Rightarrow \frac{E_1 + E_2}{E_1 - E_2} = \frac{l_1}{l_2} \quad \text{or} \quad \frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

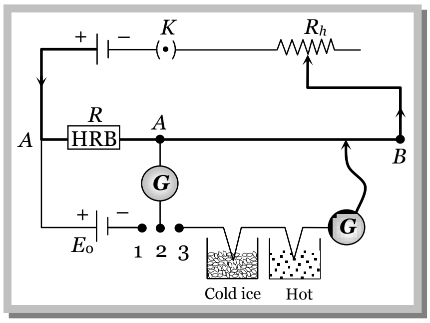
(3) **Comparison of resistances :** Let the balancing length for resistance R_1 (when XY is connected) is l_1 and let balancing length for resistance $R_1 + R_2$ (when YZ is connected) is l_2. Then $iR_1 = xl_1$ and $i(R_1 + R_2) = xl_2$



$$\Rightarrow \frac{R_2}{R_1} = \frac{l_2 - l_1}{l_1}$$

(4) **To determine thermo emf**

(i) The value of thermo-emf in a thermocouple for ordinary temperature difference is very low (10^{-6} *volt*). For this the potential gradient x must be also very low (10^{-4} V/m). Hence a high resistance (R) is connected in series with the potentiometer wire in order to reduce current.



(ii) The potential difference across R must be equal to the emf of standard cell *i.e.* $iR = E_0 \therefore i = \frac{E_0}{R}$

(iii) The small thermo emf produced in the thermocouple $e = xl$

(iv) $x = i\rho = \frac{iR}{L} \quad \therefore e = \frac{iRl}{L}$ where L = length of potentiometer wire, ρ = resistance per unit length, l = balancing length for e

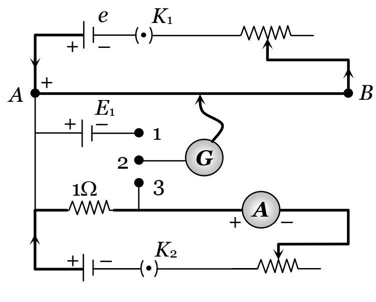
(5) To calibrate ammeter and voltmeter

Calibration of ammeter

(i) If p.d. across 1Ω resistance is measured by potentiometer, then current through this (indirectly measured) is thus known or if R is known then $i = V/R$ can be found.

(ii) **Circuit and method**

(a) Standardisation is required and per formed as already described earlier. ($x = E_0/l_0$)



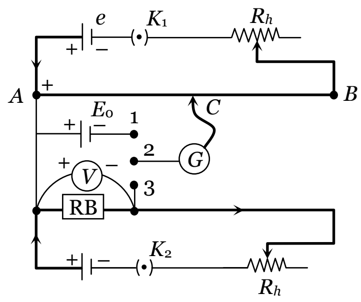
(b) The current through R or 1Ω coil is measured by the connected ammeter and same is calculated by potentiometer by finding a balancing length as described blow.

Let i' current flows through 1Ω resistance giving p.d. as $V' = i'(1) = xl_1$ where l_1 is the balancing length. So error can be found as [i (measured by ammeter) $\Delta i' = i - i'$ $\left[= xl_1 = \left(\frac{E_0}{l_0}\right)l_1\right]$

Calibration of voltmeter

(i) Practical voltmeters are not ideal, because these do not have infinite resistance. The error of such practical voltmeter can be found by comparing the voltmeter reading with calculated value of p.d. by potentiometer.

(ii) **Circuit and procedure**



(a) **Standardisation :** If l_0 is balancing length for E_0 the emf of standard cell by connecting 1 and 2 of bi-directional key, then $x = E_0/l_0$.

(b) The balancing length l_1 for unknown potential difference V' is given by (by closing 2 and 3) $V' = xl_1 = (E_0/l_0)l_1$.

If the voltmeter reading is V then the error will be ($V - V'$) which may be $+ve$, $-ve$ or zero.

Concepts

☞ *In case of zero deflection in the galvanometer current flows in the primary circuit of the potentiometer, not in the galvanometer circuit.*

☞ *A potentiometer can act as an ideal voltmeter.*

Example

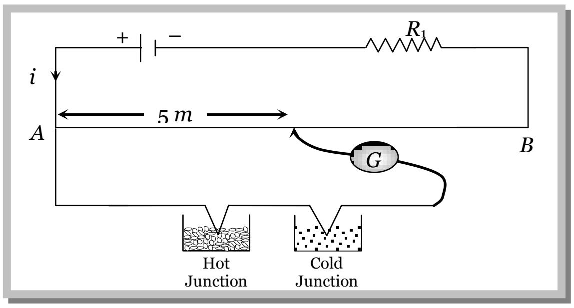
***Example*: 85** A battery with negligible internal resistance is connected with 10m long wire. A standard cell gets balanced on 600 cm length of this wire. On increasing the length of potentiometer wire by 2m then the null point will be displaced by

(a) 200 cm (b) 120 cm (c) 720 cm (d) 600 cm

Solution : (b) By using $\frac{L_1}{L_2} = \frac{l_1}{l_2} \Rightarrow \frac{10}{12} = \frac{600}{l_2} \Rightarrow l_2 = 720\ cm$.

Hence displacement = 720 – 600 = 120 cm

***Example*: 86** In the following circuit a 10 m long potentiometer wire with resistance 1.2 ohm/m, a resistance R_1 and an accumulator of emf 2 V are connected in series. When the emf of thermocouple is 2.4 mV then the deflection in galvanometer is zero. The current supplied by the accumulator will be



(a) $4 \times 10^{-4} A$ (b) $8 \times 10^{-4} A$ (c) $4 \times 10^{-3} A$ (d) $8 \times 10^{-3} A$

Solution : (a) $E = x\,l = i\rho\,l$ $\quad\therefore\quad i = \frac{E}{\rho l} = \frac{E}{\rho l} = \frac{2.4 \times 10^{-3}}{1.2 \times 5} = 4 \times 10^{-4}\,A$.

***Example*: 87** The resistivity of a potentiometer wire is $40 \times 10^{-8}\ \Omega m$ and its area of cross section is $8 \times 10^{-6}\ m^2$. If 0.2 *amp*. Current is flowing through the wire, the potential gradient will be

(a) 10^{-2} *volt/m* (b) 10^{-1} *volt/m* (c) 3.2×10^{-2} *volt/m* (d) 1 *volt/m*

Solution : (a) Potential gradient $= \frac{V}{L} = \frac{iR}{L} = \frac{i\rho L}{AL} = \frac{i\rho}{A} = \frac{0.2 \times 40 \times 10^{-8}}{8 \times 10^{-6}} = 10^{-2}\ V/m$

***Example*: 88** A deniel cell is balanced on 125 *cm* length of a potentiometer wire. When the cell is short circuited with a 2 Ω resistance the balancing length obtained is 100 *cm*. Internal resistance of the cell will be **[RPMT 1998]**

(a) 1.5 Ω (b) 0.5 Ω (c) 1.25 Ω (d) 4/5 Ω

Solution: (b) By using $r = \frac{l_1 - l_2}{l_2} \times R'$ $\Rightarrow r = \frac{125 - 100}{100} \times 2 = \frac{1}{2} = 0.5\,\Omega$

***Example*: 89** A potentiometer wire of length 10 *m* and a resistance 30 Ω is connected in series with a battery of emf 2.5 *V* and internal resistance 5 Ω and an external resistance *R*. If the fall of potential along the potentiometer wire is 50 $\mu V/mm$, the value of *R* is (in Ω)

(a) 115 (b) 80 (c) 50 (d) 100

Solution : (a) By using $x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$

$$\Rightarrow \frac{50 \times 10^{-6}}{10^{-3}} = \frac{2.5}{(30 + R + 5)} \times \frac{30}{10} \Rightarrow R = 115$$

***Example*: 90** A 2 *volt* battery, a 15 Ω resistor and a potentiometer of 100 *cm* length, all are connected in series. If the resistance of potentiometer wire is 5 Ω, then the potential gradient of the potentiometer wire is **[AIIMS 1982**

(a) 0.005 *V/cm* (b) 0.05 *V/cm* (c) 0.02 *V/cm* (d) 0.2 *V/cm*

Solution : (a) By using $x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$ $\Rightarrow x = \frac{2}{(5 + 15 + 0)} \times \frac{5}{1} = 0.5\ V/m = 0.005\ V/cm$

***Example*: 91** In an experiment to measure the internal resistance of a cell by potentiometer, it is found that the balance point is at a length of 2 *m* when the cell is shunted by a 5 Ω resistance; and is at a length of 3 *m* when the cell is shunted by a 10 Ω resistance. The internal resistance of the cell is, then

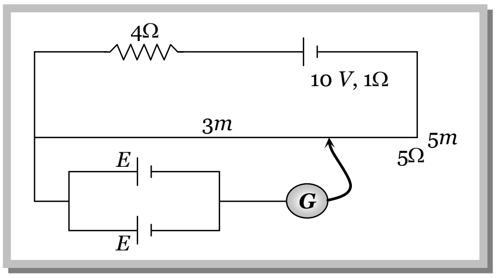
(a) 1.5 Ω (b) 10 Ω (c) 15 Ω (d) 1 Ω

Solution : (b) By using $r = \left(\frac{l_1 - l_2}{l_2}\right)R' \Rightarrow r = \left(\frac{l_1 - 2}{2}\right) \times 5$ ...... (i)

and $r = \left(\frac{l_1 - 3}{3}\right) \times 10$ .... (ii)

On solving (i) and (ii) $r = 10\ \Omega$

***Example*: 92** A resistance of 4 Ω and a wire of length 5 *metres* and resistance 5 Ω are joined in series and connected to a cell of emf 10 *V* and internal resistance 1 Ω. A parallel combination of two identical cells is balanced across 300 *cm* of the wire. The emf *E* of each cell is



(a) 1.5 *V* (b) 3.0 *V* (c) 0.67 *V* (d) 1.33 *V*

Solution : (b) By using $E_{eq} = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L} \times l \Rightarrow E = \frac{10}{(5+4+1)} \times \frac{5}{5} \times 3 \Rightarrow E = 3\ volt$

***Example*: 93** A potentiometer has uniform potential gradient across it. Two cells connected in series (i) to support each other and (ii) to oppose each other are balanced over 6 *m* and 2 *m* respectively on the potentiometer wire. The emf's of the cells are in the ratio of **[MP PMT 2002; RPMT 2000]**

(a) 1 : 2 (b) 1 : 1 (c) 3 : 1 (d) 2 : 1

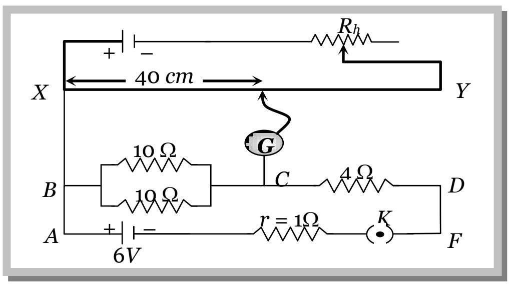
Solution : (d) If suppose emf's of the cells are E_1 and E_2 respectively then

$E_1 + E_2 = x \times 6$ .... (i) [x = potential gradient)]

and $E_1 - E_2 = x \times 2$ ....(ii)

$\Rightarrow \frac{E_1 + E_2}{E_1 - E_2} = \frac{3}{1} \Rightarrow \frac{E_1}{E_2} = \frac{2}{1}$

***Example*: 94** In the following circuit the potential difference between the points *B* and *C* is balanced against 40 *cm* length of potentiometer wire. In order to balance the potential difference between the points *C* and *D*, where should jockey be pressed

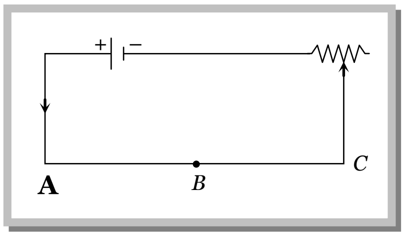


(a) 32 *cm* (b) 16 *cm* (c) 8 *cm* (d) 4 *cm*

Solution : (a) $\frac{1}{R} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$ or $R_1 = 5\ \Omega$

$R_2 = 4\Omega$, $l_1 = 40\ cm$, $l_2 = ?$ $l_2 = l_1 \frac{R_2}{R_1}$ or $l_2 = \frac{40 \times 4}{5} = 32\ cm$

***Example*: 95** In the following circuit diagram fig. the lengths of the wires *AB* and *BC* are same but the radius of *AB* is three times that of *BC*. The ratio of potential gradients at *AB* and *BC* will be



(a) 1 : 9

(b) 9 : 1

(c) 3 : 1

(d) 1 : 3

Solution : (a) $x \propto R_p \propto \frac{1}{r^2} \Rightarrow \frac{x_1}{x_2} = \frac{r_2^2}{r_1^2} = \left(\frac{r}{3r}\right)^2 = \frac{1}{9}$

***Example*: 96** With a certain cell the balance point is obtained at 0.60 *m* from one end of the potentiometer. With another cell whose emf differs from that of the first by 0.1 *V*, the balance point is obtained at 0.55 *m*. Then, the two emf's are

(a) 1.2 *V*, 1.1 *V* (b) 1.2 *V*, 1.3 *V* (c) – 1.1 *V*, – 1.0 *V* (d) None of the above

Solution : (a) $E_1 = x\,(0.6)$ and $E_2 = E_1 - 0.1 = x\,(0.55) \Rightarrow \frac{E_1}{E_1 - 0.1} = \frac{0.6}{0.55}$

or $55\,E_1 = 60\,E_1 - 6 \Rightarrow E_1 = \frac{6}{5} = 1.2\,V$ thus $E_2 = 1.1\,V$

Tricky Example: 11

A cell of internal resistance 1.5Ω and of emf 1.5 *volt* balances 500 *cm* on a potentiometer wire. If a wire of 15Ω is connected between the balance point and the cell, then the balance point will shift

[MP PMT 1985]

(a) To zero (b) By 500 *cm*

(c) By 750 *cm* (d) None of the above

Solution : (d) In balance condition no current flows in the galvanometer circuit. Hence there will be no shift in balance point after connecting a resistance between balance point and cell.

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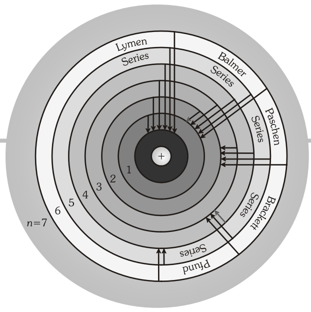


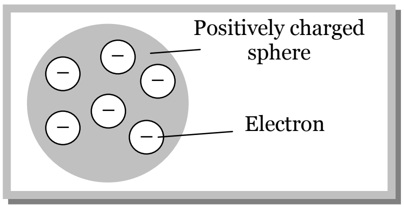
Atomic Structure

Important Atomic Models

(1) **Thomson's model**

J.J. Thomson gave the first idea regarding structure of atom. According to this model.

(i) An atom is a solid sphere in which entire and positive charge and it's mass is uniformly distributed and in which negative charge (*i.e.* electron) are embedded like seeds in watermelon.



Success and failure

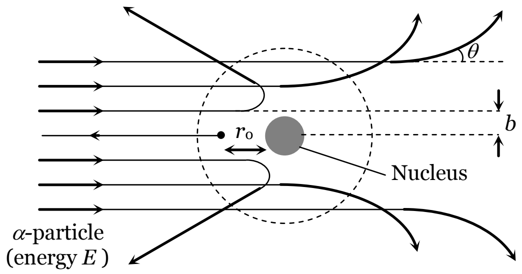
Explained successfully the phenomenon of thermionic emission, photoelectric emission and ionization.

The model fail to explain the scattering of α- particles and it cannot explain the origin of spectral lines observed in the spectrum of hydrogen and other atoms.

(2) **Rutherford's model**

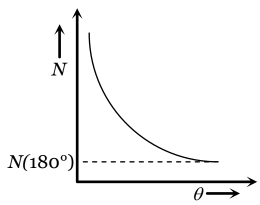
Rutherford's α-particle scattering experiment

Rutherford performed experiments on the scattering of alpha particles by extremely thin gold foils and made the following observations



Number of scattered particles :

$$N \propto \frac{1}{\sin^4(\theta/2)}$$



(i) Most of the α-particles pass through the foil straight away undeflected.

(ii) Some of them are deflected through small angles.

(iii) A few α-particles (1 in 1000) are deflected through the angle more than 90^o.

(iv) A few α -particles (very few) returned back *i.e.* deflected by 180^o.

(v) Distance of closest approach (Nuclear dimension)

The minimum distance from the nucleus up to which the α-particle approach, is called the distance of closest approach (r_o). From figure $r_0 = \frac{1}{4\pi\varepsilon_0}.\frac{Ze^2}{E}$; $E = \frac{1}{2}mv^2 =$ K.E. of α-particle

(vi) Impact parameter (b) : The perpendicular distance of the velocity vector ($\vec{v}$) of the α-particle from the centre of the nucleus when it is far away from the nucleus is known as impact parameter. It is given as

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)} \Rightarrow b \propto \cot(\theta/2)$$

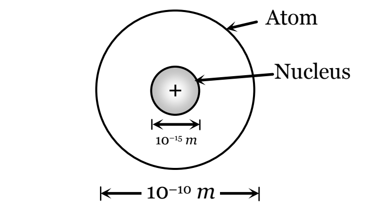
Note : ❑ If t is the thickness of the foil and N is the number of α-particles scattered in a particular direction (θ = constant), it was observed that $\frac{N}{t}$ = constant $\Rightarrow \frac{N_1}{N_2} = \frac{t_1}{t_2}$.

After Rutherford's scattering of α-particles experiment, following conclusions were made as regard as atomic structure :

(a) Most of the mass and all of the charge of an atom concentrated in a very small region is called atomic nucleus.

(b) Nucleus is positively charged and it's size is of the order of 10^{-15} $m \approx 1$ *Fermi*.

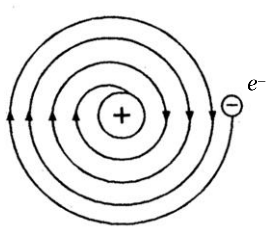
(c) In an atom there is maximum empty space and the electrons revolve around the nucleus in the same way as the planets revolve around the sun.



Size of the nucleus = 1 Fermi = 10^{-15} m
Size of the atom 1 Å = 10^{-10} m

Draw backs

(i) Stability of atom : It could not explain stability of atom because according to classical electrodynamic theory an accelerated charged particle should continuously radiate energy. Thus an electron moving in an circular path around the nucleus should also radiate energy and thus move into smaller and smaller orbits of gradually decreasing radius and it should ultimately fall into nucleus.



Instability of atom

(ii) According to this model the spectrum of atom must be continuous where as practically it is a line spectrum.

(iii) It did not explain the distribution of electrons outside the nucleus.

(3) **Bohr's model**

Bohr proposed a model for hydrogen atom which is also applicable for some lighter atoms in which a single electron revolves around a stationary nucleus of positive charge *Ze* (called hydrogen like atom)

Bohr's model is based on the following postulates.

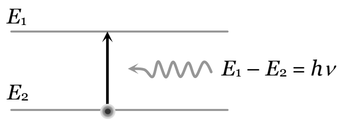
(i) The electron can revolve only in certain discrete non-radiating orbits, called stationary orbits, for which total angular momentum of the revolving electrons is an integral multiple of $\frac{h}{2\pi}$ $(= \hbar)$

i.e. $L = n\left(\frac{h}{2\pi}\right) = mvr$; where n = 1, 2, 3, ...= Principal quantum number

(ii) The radiation of energy occurs only when an electron jumps from one permitted orbit to another.

When electron jumps from higher energy orbit (E_1) to lower energy orbit (E_2) then difference of energies of these orbits *i.e.* $E_1 - E_2$ emits in the form of photon. But if electron goes from E_2 to E_1 it absorbs the same amount of energy.

Emission



Absorption

Note : ❑ According to Bohr theory the momentum of an e^- revolving in second orbit of H_2 atom will be $\frac{h}{\pi}$

❑ For an electron in the n^{th} orbit of hydrogen atom in Bohr model, circumference of orbit $= n\lambda$; where λ = de-Broglie wavelength.

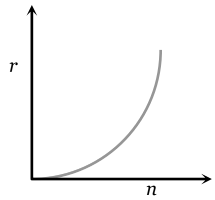
Bohr's Orbits (For Hydrogen and H_2-Like Atoms)

(1) **Radius of orbit**

For an electron around a stationary nucleus the electrostatics force of attraction provides the necessary centripetal force

i.e. $\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r}$ .... (i) also $mvr = \frac{nh}{2\pi}$ ....(ii)

From equation (i) and (ii) radius of n^{th} orbit



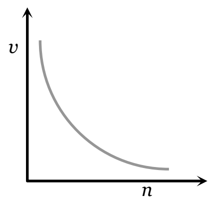
$$r_n = \frac{n^2h^2}{4\pi^2 kZme^2} = \frac{n^2h^2\varepsilon_0}{\pi mZe^2} = 0.53\frac{n^2}{Z}\text{Å} \qquad \left[\text{where } k = \frac{1}{4\pi\varepsilon_0}\right]$$

$$\Rightarrow \boldsymbol{r_n \propto \frac{n^2}{Z}}$$

Note : ❑ The radius of the innermost orbit (n = 1) hydrogen atom (z = 1) is called Bohr's radius a_0 *i.e.* $a_0 = 0.53\,\text{Å}$.

(2) **Speed of electron**

From the above relations, speed of electron in n^{th} orbit can be calculated as



$$v_n = \frac{2\pi kZe^2}{nh} = \frac{Ze^2}{2\varepsilon_0 nh} = \left(\frac{c}{137}\right).\frac{Z}{n} = 2.2\times 10^6\frac{Z}{n}\,m/sec$$

where (c = speed of light 3×10^8 m/s)

Note : ❑ The ratio of speed of an electron in ground state in Bohr's first orbit of hydrogen atom to velocity of light in air is equal to $\frac{e^2}{2\varepsilon_0 ch} = \frac{1}{137}$ (where c = speed of light in air)

(3) **Some other quantities**

For the revolution of electron in n^{th} orbit, some other quantities are given in the following table

Quantity	Formula	Dependency on n and Z
(1) Angular speed	$\omega_n = \frac{v_n}{r_n} = \frac{\pi mz^2e^4}{2\varepsilon_0^2 n^3h^3}$	$\omega_n \propto \frac{Z^2}{n^3}$
(2) Frequency	$\nu_n = \frac{\omega_n}{2\pi} = \frac{mz^2e^4}{4\varepsilon_0^2n^3h^3}$	$\nu_n \propto \frac{Z^2}{n^3}$
(3) Time period	$T_n = \frac{1}{\nu_n} = \frac{4\varepsilon_0^2n^3h^3}{mz^2e^4}$	$T_n \propto \frac{n^3}{Z^2}$
(4) Angular momentum	$L_n = mv_nr_n = n\left(\frac{h}{2\pi}\right)$	$L_n \propto n$
(5) Corresponding current	$i_n = e\nu_n = \frac{mz^2e^5}{4\varepsilon_0^2n^3h^3}$	$i_n \propto \frac{Z^2}{n^3}$
(6) Magnetic moment	$M_n = i_nA = i_n(\pi r_n^2)$	$M_n \propto n$

	(where $\mu_0 = \frac{eh}{4\pi m}$ = Bohr magneton)	
(7) Magnetic field	$B = \frac{\mu_0 i_n}{2r_n} = \frac{\pi m^2 z^3 e^7 \mu_0}{8\varepsilon_0^3 n^5 h^5}$	$B \propto \frac{Z^3}{n^5}$

(4) **Energy**

(i) **Potential energy :** An electron possesses some potential energy because it is found in the field of nucleus potential energy of electron in n^{th} orbit of radius r_n is given by $U = k.\frac{(Ze)(-e)}{r_n} = -\frac{kZe^2}{r_n}$

(ii) **Kinetic energy :** Electron posses kinetic energy because of it's motion. Closer orbits have greater kinetic energy than outer ones.

As we know $\frac{mv^2}{r_n} = \frac{k.(Ze)(e)}{r_n^2}$ $\Rightarrow$ Kinetic energy $K = \frac{kZe^2}{2r_n} = \frac{|U|}{2}$

(iii) **Total energy :** Total energy (E) is the sum of potential energy and kinetic energy *i.e.* $E = K + U$

$\Rightarrow E = -\frac{kZe^2}{2r_n}$ also $r_n = \frac{n^2h^2\varepsilon_0}{\pi mze^2}$. Hence $E = -\left(\frac{me^4}{8\varepsilon_0^2h^2}\right).\frac{z^2}{n^2} = -\left(\frac{me^4}{8\varepsilon_0^2 ch^3}\right)ch\frac{z^2}{n^2} = -Rch\frac{Z^2}{n^2} = -13.6\frac{Z^2}{n^2}eV$

where $R = \frac{me^4}{8\varepsilon_0^2 ch^3}$ = Rydberg's constant = 1.09×10^7 *per metre*

Note : ❑ Each Bohr orbit has a definite energy

❑ For hydrogen atom ($Z = 1$) $\Rightarrow E_n = -\frac{13.6}{n^2}eV$

❑ The state with $n = 1$ has the lowest (most negative) energy. For hydrogen atom it is $E_1 = -13.6$ *eV*.

❑ Rch = Rydberg's energy $\simeq 2.17 \times 10^{-18} J \simeq 31.6\,eV$.

❑ $E = -K = \frac{U}{2}$.

(iv) **Ionisation energy and potential :** The energy required to ionise an atom is called ionisation energy. It is the energy required to make the electron jump from the present orbit to the infinite orbit.

Hence $E_{ionisation} = E_\infty - E_n = 0 - \left(-13.6\frac{Z^2}{n^2}\right) = +\frac{13.6Z^2}{n^2}eV$

For H_2-atom in the ground state $E_{ionisation} = \frac{+13.6(1)^2}{n^2} = 13.6\,eV$

The potential through which an electron need to be accelerated so that it acquires energy equal to the ionisation energy is called ionisation potential. $V_{ionisation} = \frac{E_{ionisation}}{e}$

(v) **Excitation energy and potential :** When the electron is given energy from external source, it jumps to higher energy level. This phenomenon is called excitation.

The minimum energy required to excite an atom is called excitation energy of the particular excited state and corresponding potential is called exciting potential.

$$E_{Excitation} = E_{Final} - E_{Initial} \text{ and } V_{Excitation} = \frac{E_{excitation}}{e}$$

(vi) **Binding energy (B.E.) :** Binding energy of a system is defined as the energy released when it's constituents are brought from infinity to form the system. It may also be defined as the energy needed to separate it's constituents to large distances. If an electron and a proton are initially at rest and brought from large distances to form a hydrogen atom, 13.6 *eV* energy will be released. The binding energy of a hydrogen atom is therefore 13.6 *eV*.

Note : ❑ For hydrogen atom principle quantum number $n = \sqrt{\frac{13.6}{(\text{B.E.})}}$.

(5) **Energy level diagram**

The diagrammatic description of the energy of the electron in different orbits around the nucleus is called energy level diagram.

Energy level diagram of hydrogen/hydrogen like atom

Principle quantum number	Orbit	Excited state	Energy for H_2 – atom	Energy for H_2 – like atom	Ionisation energy from this level (for H_2 – atom)
$n = \infty$	Infinite	Infinite	$E_\infty = 0\ eV$	$0\ eV$	$0\ eV$
$n = 4$	Fourth	Third	$E_4 = -0.85\ eV$	$-0.85\ Z^2$	$+0.85\ eV$
$n = 3$	Third	Second	$E_3 = -1.51\ eV$	$-1.51\ Z^2$	$+1.51\ eV$
$n = 2$	Second	First	$E_2 = -3.4\ eV$	$-3.4\ Z^2$	$+3.4\ eV$
$n = 1$	First	Ground	$E_1 = -13.6\ eV$	$-13.6\ Z^2$	$+13.6\ eV$

Note : ❑ In hydrogen atom excitation energy to excite electron from ground state to first excited state will be $-3.4-(-13.6) = 10.2\ eV$.

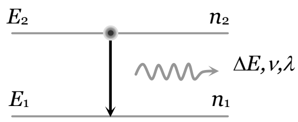
and from ground state to second excited state it is [$-1.51-(-13.6) = 12.09\ eV$].

❑ In an H_2 atom when e^- makes a transition from an excited state to the ground state it's kinetic energy increases while potential and total energy decreases.

(6) **Transition of electron**

When an electron makes transition from higher energy level having energy $E_2(n_2)$ to a lower energy level having energy $E_1\ (n_1)$ then a photon of frequency ν is emitted

(i) **Energy of emitted radiation**



Emission

$$\Delta E = E_2 - E_1 = \frac{-Rc\,h\,Z^2}{n_2^2} - \left(-\frac{Rch\,Z^2}{n_1^2}\right) = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(ii) **Frequency of emitted radiation**

$$\Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h} = Rc\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(iii) **Wave number/wavelength**

Wave number is the number of waves in unit length $\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c}$

$$\Rightarrow \frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{13.6Z^2}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(iv) **Number of spectral lines** : If an electron jumps from higher energy orbit to lower energy orbit it emits raidations with various spectral lines.

If electron falls from orbit n_2 to n_1 then the number of spectral lines emitted is given by

$$N_E = \frac{(n_2 - n_1 + 1)(n_2 - n_1)}{2}$$

If electron falls from n^{th} orbit to ground state (*i.e.* $n_2 = n$ and $n_1 = 1$) then number of spectral lines emitted $\boldsymbol{N_E = \frac{n(n-1)}{2}}$

Note : ❑ Absorption spectrum is obtained only for the transition from lowest energy level to higher energy levels. Hence the number of absorption spectral lines will be $(n - 1)$.

(v) **Recoiling of an atom** : Due to the transition of electron, photon is emitted and the atom is recoiled

Recoil momentum of atom = momentum of photon $= \frac{h}{\lambda} = hRZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Also recoil energy of atom $= \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$ (where m = mass of recoil atom)

(7) **Drawbacks of Bohr's atomic model**

(i) It is valid only for one electron atoms, *e.g.* : H, He^+, Li^{+2}, Na^{+1} *etc.*

(ii) Orbits were taken as circular but according to Sommerfield these are elliptical.

(iii) Intensity of spectral lines could not be explained.

(iv) Nucleus was taken as stationary but it also rotates on its own axis.

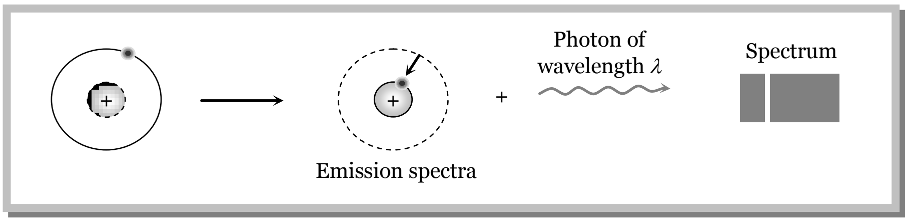
(v) It could not be explained the minute structure in spectrum line.

(vi) This does not explain the Zeeman effect (splitting up of spectral lines in magnetic field) and Stark effect (splitting up in electric field)

(vii) This does not explain the doublets in the spectrum of some of the atoms like sodium (5890$\mathring{A}$ & 5896$\mathring{A}$)

Hydrogen Spectrum and Spectral Series

When hydrogen atom is excited, it returns to its normal unexcited (or ground state) state by emitting the energy it had absorbed earlier. This energy is given out by the atom in the form of radiations of different wavelengths as the electron jumps down from a higher to a lower orbit. Transition from different orbits cause different wavelengths, these constitute spectral series which are characteristic of the atom emitting them. When observed through a spectroscope, these radiations are imaged as sharp and straight vertical lines of a single colour.



Spectral series

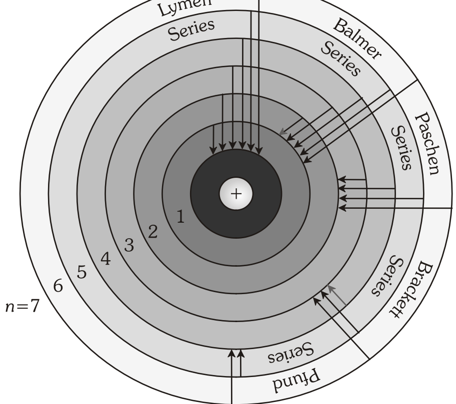
The spectral lines arising from the transition of electron forms a spectra series.

(i) Mainly there are five series and each series is named after it's discover as Lymen series, Balmer series, Paschen series, Bracket series and Pfund series.

(ii) According to the Bohr's theory the wavelength of the radiations emitted from hydrogen atom is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where n_2 = outer orbit (electron jumps from this orbit), n_1 = inner orbit (electron falls in this orbit)



(iii) First line of the series is called first member, for this line wavelength is maximum (λ_{max})

(iv) Last line of the series ($n_2 = \infty$) is called series limit, for this line wavelength is minimum (λ_{min})

Spectral series	**Transition**	**Wavelength (λ)** $= \frac{n_1^2 n_2^2}{(n_2^2 - n_1^2)R} = \frac{n_1^2}{\left(1 - \frac{n_1^2}{n_2^2}\right)R}$		$\frac{\lambda_{max}}{\lambda_{min}} = \frac{(n+1)^2}{(2n+1)}$	**Region**
		Maximum wavelength $(n_1 = n$ **and** $n_2 = n+1)$ $\lambda_{max} = \frac{n^2(n+1)^2}{(2n+1)R}$	**Minimum wavelength** $(n_2 = \infty, n_1 = n)$ $\lambda_{min} = \frac{n^2}{R}$		
1. Lymen series	$n_2 = 2, 3, 4 \ldots\infty$ $n_1 = 1$	$\lambda_{max} = \frac{(1)^2(1+1)^2}{(2\times1+1)R} = \frac{4}{3R}$	$n_1 = n = 1$ $\lambda_{min} = \frac{1}{R}$	$\frac{4}{3}$	Ultraviolet region
2.Balmer series	$n_2 = 3, 4, 5 \ldots\infty$ $n_1 = 2$	$n_1 = n = 2$, $n_2 = 2 + 1 = 3$ $\lambda_{max} = \frac{36}{5R}$	$\lambda_{min} = \frac{4}{R}$	$\frac{9}{5}$	Visible region
3. Paschen series	$n_2 = 4, 5, 6 \ldots\infty$ $n_1 = 3$	$n_1 = n = 3$, $n_2 = 3 + 1 = 4$ $\lambda_{max} = \frac{144}{7R}$	$n_1 = n = 3$ $\lambda_{min} = \frac{9}{R}$	$\frac{16}{7}$	Infrared region
4. Bracket series	$n_2 = 5, 6, 7 \ldots \infty$ $n_1 = 4$	$n_1 = n = 4$, $n_2 = 4 + 1 = 5$ $\lambda_{max} = \frac{400}{9R}$	$n_1 = n = 4$ $\lambda_{min} = \frac{16}{R}$	$\frac{25}{9}$	Infrared region
5. Pfund series	$n_2 = 6, 7, 8 \ldots \infty$ $n_1 = 5$	$n_1 = \lambda = 5$, $n_2 = 5 + 1 = 6$ $\lambda_{max} = \frac{900}{11R}$	$\lambda_{min} = \frac{25}{R}$	$\frac{36}{11}$	Infrared region

Quantum Numbers

An atom contains large number of shells and subshells. These are distinguished from one another on the basis of their size, shape and orientation (direction) in space. The parameters are expressed in terms of different numbers called quantum number.

Quantum numbers may be defined as a set of four number with the help of which we can get complete information about all the electrons in an atom. It tells us the address of the electron *i.e.* location, energy, the type of orbital occupied and orientation of that orbital.

(1) **Principal Quantum number (n) :** This quantum number determines the main energy level or shell in which the electron is present. The average distance of the electron from the nucleus and the energy of the electron depends on it. $E_n \propto \frac{1}{n^2}$ and $r_n \propto n^2$ (in H-atom)

The principal quantum number takes whole number values, $n = 1, 2, 3, 4, \ldots\ \infty$

(2) **Orbital quantum number (l) or azimuthal quantum number (l)**

This represents the number of subshells present in the main shell. These subsidiary orbits within a shell will be denoted as 1, 2, 3, 4 … or s, p, d, f … This tells the shape of the subshells.

The orbital angular momentum of the electron is given as $L = \sqrt{l(l+1)}\frac{h}{2\pi}$ (for a particular value of n).

For a given value of n the possible values of l are $l = 0, 1, 2, \ldots$ upto $(n - 1)$

(3) **Magnetic quantum number (m_l) :** An electron due to it's angular motion around the nucleus generates an electric field. This electric field is expected to produce a magnetic field. Under the influence of external magnetic field, the electrons of a subshell can orient themselves in certain preferred regions of space around the nucleus called orbitals.

The magnetic quantum number determines the number of preferred orientations of the electron present in a subshell.

The angular momentum quantum number *m* can assume all integral value between – *l* to +*l* including zero. Thus m_l can be – 1, 0, + 1 for *l* = 1. Total values of m_l associated with a particular value of *l* is given by (2*l* + 1).

(4) **Spin (magnetic) quantum number (m_s) :** An electron in atom not only revolves around the nucleus but also spins about its own axis. Since an electron can spin either in clockwise direction or in anticlockwise direction. Therefore for any particular value of magnetic quantum number, spin quantum number can have two values, *i.e.* $m_s = \frac{1}{2}$ (Spin up) or $m_s = -\frac{1}{2}$ (Spin down)

This quantum number helps to explain the magnetic properties of the substance.

Electronic Configurations of Atoms

The distribution of electrons in different orbitals of an atom is called the electronic configuration of the atom. The filling of electrons in orbitals is governed by the following rules.

(1) **Pauli's exclusion principle**

"It states that no two electrons in an atom can have all the four quantum number (*n*, *l*, m_l and m_s) the same."

It means each quantum state of an electron must have a different set of quantum numbers *n*, *l*, m_l and m_s. This principle sets an upper limit on the number of electrons that can occupy a shell.

$N_{\max}$ in one shell = $2n^2$; Thus $N_{\max}$ in *K*, *L*, *M*, *N* .... shells are 2, 8, 18, 32,

Note : ❑ The maximum number of electrons in a subshell with orbital quantum number *l* is 2(2*l* + 1).

(2) **Aufbau principle**

Electrons enter the orbitals of lowest energy first.

As a general rule, a new electron enters an empty orbital for which (*n* + *l*) is minimum. In case the value $(n + l)$ is equal for two orbitals, the one with lower value of *n* is filled first.

Thus the electrons are filled in subshells in the following order (memorize)

1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, 3*d*, 4*p*, 5*s*, 4*d*, 5*p*, 6*s*, 4*f*, 5*d*, 6*p*, 7*s*, 5*f*, 6*d*, 7*p*, ...

(3) **Hund's Rule**

When electrons are added to a subshell where more than one orbital of the same energy is available, their spins remain parallel. They occupy different orbitals until each one of them has at least one electron. Pairing starts only when all orbitals are filled up.

Pairing takes place only after filling 3, 5 and 7 electrons in *p*, *d* and *f* orbitals, respectively.

Concepts

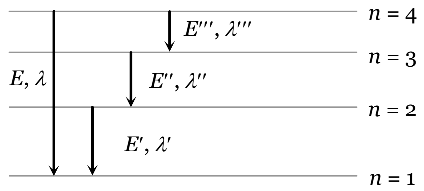
☞ *With the increase in principal quantum number the energy difference between the two successive energy level decreases, while wavelength of spectral line increases.*

$E' > E'' > E'''$

$\lambda' < \lambda'' < \lambda'''$

$E = E' + E'' + E'''$

$$\frac{1}{\lambda} = \frac{1}{\lambda'} + \frac{1}{\lambda''} + \frac{1}{\lambda'''}$$



☞ ***Rydberg constant is different for different elements***

$R(=1.09 \times 10^{7}\ m^{-1})$ is the value of Rydberg constant when the nucleus is considered to be infinitely massive as compared to the revolving electron. In other words, the nucleus is considered to be stationary.

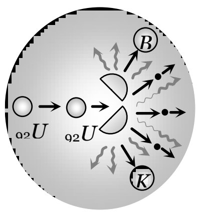
In case, the nucleus is not infinitely massive or stationary, then the value of Rydberg constant is given as $\boldsymbol{R' = \dfrac{R}{1 + \dfrac{m}{M}}}$

where m is the mass of electron and M is the mass of nucleus.

☞ ***Atomic spectrum is a line spectrum***

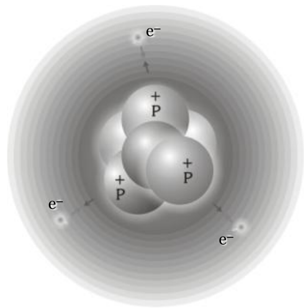
Each atom has it's own characteristic allowed orbits depending upon the electronic configuration. Therefore photons emitted during transition of electrons from one allowed orbit to inner allowed orbit are of some definite energy only. They do not have a continuous graduation of energy. Therefore the spectrum of the emitted light has only some definite lines and therefore atomic spectrum is line spectrum.

☞ *Just as dots of light of only three colours combine to form almost every conceivable colour on T.V. screen, only about 100 distinct kinds of atoms combine to form all the materials in the universe.*



Nuclear Physics & Radioactivity

Rutherford's α-scattering experiment established that the mass of atom is concentrated with small positively charged region at the centre which is called 'nucleus'.



Nuclei are made up of proton and neutron. The number of protons in a nucleus (called the atomic number or proton number) is represented by the symbol Z. The number of neutrons (neutron number) is represented by N. The total number of neutrons and protons in a nucleus is called it's mass number A so $A = Z + N$.

Neutrons and proton, when described collectively are called ***nucleons.***

Nucleus contains two types of particles : Protons and neutrons

Nuclides are represented as ${}_ZX^A$; where X denotes the chemical symbol of the element.

Neutron

Neutron is a fundamental particle which is essential constituent of all nuclei except that of hydrogen atom. It was discovered by Chadwick.

(1) The charge of neutron : It is neutral

(2) The mass of neutron : $1.6750 \times 10^{-27}\ kg$

(3) It's spin angular momentum : $\frac{1}{2} \times \left(\frac{h}{2\pi}\right) J\text{-}s$

(4) It's magnetic moment : $9.57 \times 10^{-27}\ J/Tesla$

(5) It's half life : 12 minutes

(6) Penetration power : High

(7) Types : Neutrons are of two types slow neutron and fast neutron, both are fully capable of penetrating a nucleus and causing artificial disintegration.

A free neutron outside the nucleus is unstable and decays into proton and electron.

$${}_0n^1 \rightarrow \underset{\text{Proton}}{{}_1H^1} + \underset{\text{Electron}}{{}_{-1}\beta^0} + \underset{\text{Antinutrino}}{\bar{\nu}}$$

Thermal neutrons

Fast neutrons can be converted into slow neutrons by certain materials called moderator's (Paraffin wax, heavy water, graphite) when fast moving neutrons pass through a moderator, they collide with the molecules of the moderator, as a result of this, the energy of moving neutron decreases while that of the molecules of the moderator increases. After sometime they both attains same energy. The neutrons are then in thermal equilibrium with the molecules of the moderator and are called thermal neutrons.

Note : ❑ Energy of thermal neutron is about $0.025\ eV$ and speed is about $2.2\ km/s$.

Nucleus

(1) **Different types of nuclei**

The nuclei have been classified on the basis of the number of protons (atomic number) or the total number of nucleons (mass number) as follows

(i) **Isotopes :** The atoms of element having same atomic number but different mass number are called isotopes. All isotopes have the same chemical properties. The isotopes of some elements are the following

$${}_1H^1,\ {}_1H^2,\ {}_1H^3 \quad {}_8O^{16},\ {}_8O^{17},\ {}_8O^{18} \quad {}_2He^3,\ {}_2He^4 \quad {}_{17}Cl^{35},\ {}_{17}Cl^{37} \quad {}_{92}U^{235},\ {}_{92}U^{238}$$

(ii) **Isobars :** The nuclei which have the same mass number (A) but different atomic number (Z) are called isobars. Isobars occupy different positions in periodic table so all isobars have different chemical properties. Some of the examples of isobars are

$${}_1H^3 \text{ and } {}_2He^3,\quad {}_6C^{14} \text{ and } {}_7N^{14},\ {}_8O^{17} \text{ and } {}_9F^{17}$$

(iii) **Isotones :** The nuclei having equal number of neutrons are called isotones. For them both the atomic number (Z) and mass number (A) are different, but the value of ($A - Z$) is same. Some examples are

$${}_4Be^9 \text{ and } {}_5B^{10},\ {}_6C^{13} \text{ and } {}_7N^{14},\ {}_8O^{18} \text{ and } {}_9F^{19},\ {}_3Li^7 \text{ and } {}_4Be^8,\ {}_1H^3 \text{ and } {}_2He^4$$

(iv) **Mirror nuclei :** Nuclei having the same mass number A but with the proton number (Z) and neutron number ($A - Z$) interchanged (or whose atomic number differ by 1) are called mirror nuclei for example.

$$_1H^3 \text{ and } _2He^3,\ _3Li^7 \text{ and } _4Be^7$$

(2) **Size of nucleus**

(i) Nuclear radius : Experimental results indicates that the nuclear radius is proportional to $A^{1/3}$, where A is the mass number of nucleus *i.e.* $R \propto A^{1/3} \Rightarrow R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}\ m = 1.2\ fm$.

Note : ❑ Heavier nuclei are bigger in size than lighter nuclei.

(ii) Nuclear volume : The volume of nucleus is given by $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \Rightarrow V \propto A$

(iii) Nuclear density : Mass per unit volume of a nucleus is called nuclear density.

$$\text{Nuclear density}(\rho) = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi (R_0 A^{1/3})^3}$$

where m = Average of mass of a nucleon (= mass of proton + mass of neutron = $1.66 \times 10^{-27}\ kg$)

and mA = Mass of nucleus

$$\Rightarrow \rho = \frac{3m}{4\pi R_0^3} = 2.38 \times 10^{17}\ kg/m^3$$

Note : ❑ ρ is independent of A, it means ρ is same of all atoms.

❑ Density of a nucleus is maximum at it's centre and decreases as we move outwards from the nucleus.

(3) **Nuclear force**

Forces that keep the nucleons bound in the nucleus are called nuclear forces.

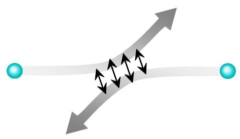
(i) Nuclear forces are short range forces. These do not exist at large distances greater than $10^{-15}\ m$.

(ii) Nuclear forces are the strongest forces in nature.

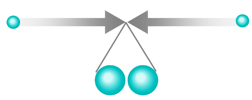
(iii) These are attractive force and causes stability of the nucleus.

(iv) These forces are charge independent.

(v) Nuclear forces are non-central force.



At low speeds, electromagnetic repulsion prevents the collision of nuclei



At high speeds, nuclei come close enough for the strong force to bind them together.

Nuclear forces are exchange forces

According to scientist Yukawa the nuclear force between the two nucleons is the result of the exchange of particles called mesons between the nucleons.

π - mesons are of three types – Positive π meson (π^+), negative π meson (π^-), neutral π *meson* (π^0)

The force between neutron and proton is due to exchange of charged meson between them *i.e.*

$$p \to \pi^+ + n, \quad n \to p + \pi^-$$

The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^0) between them *i.e.* $p \to p' + \pi^0$ and $n \to n' + \pi^0$

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the nuclear forces.

Dog-Bone analogy

The above interactions can be explained with the dog bone analogy according to which we consider the two interacting nucleons to be two dogs having a common bone clenched in between their teeth very firmly. Each one of these dogs wants to take the bone and hence they cannot be separated easily. They seem to be bound to each other with a strong attractive force (which is the bone) though the dogs themselves are strong enemies. The meson plays the same role of the common bone in between two nucleons.

(4) **Atomic mass unit (*amu*)**

The unit in which atomic and nuclear masses are measured is called atomic mass unit (*amu*)

1 *amu* (or 1*u*) = $\frac{1}{12}th$ of mass of ${}_6C^{12}$ atom = 1.66×10^{-27} *kg*

Masses of electron, proton and neutrons

Mass of electron (m_e) = 9.1×10^{-31} *kg* = 0.0005486 *amu*, Mass of proton (m_p) = 1.6726×10^{-27} *kg* = 1.007276 *amu*

Mass of neutron (m_n) = 1.6750×10^{-27} *kg* = 1.00865 *amu*, Mass of hydrogen atom ($m_e + m_p$) = 1.6729×10^{-27} *kg* = 1.0078 *amu*

Mass-energy equivalence

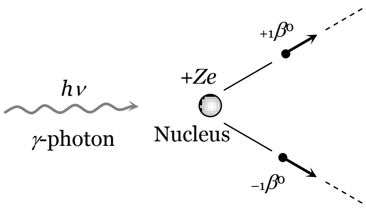
According to Einstein, mass and energy are inter convertible. The Einstein's mass energy relationship is given by $E = mc^2$

If $m = 1$ *amu*, $c = 3 \times 10^8$ *m/sec* then $E = 931$ *MeV* *i.e.* 1 *amu* is equivalent to 931 *MeV* or **1 *amu* (or 1 *u*) = 931 *MeV***

(5) **Pair production and pair-annihilation**

When an energetic γ-ray photon falls on a heavy substance. It is absorbed by some nucleus of the substance and an electron and a positron are produced. This phenomenon is called pair production and may be represented by the following equation

$$\underset{(\gamma-\text{photon})}{h\nu} = \underset{(\text{Positron})}{{}_{1}\beta^0} + \underset{(\text{Electron})}{{}_{-1}\beta^0}$$



The rest-mass energy of each of positron and electron is

$$E_0 = m_0c^2 = (9.1 \times 10^{-31}\ kg) \times (3.0 \times 10^8\ m/s)^2$$

$$= 8.2 \times 10^{-14}\ J = \mathbf{0.51\ MeV}$$

Hence, for pair-production it is essential that the energy of γ-photon must be at least $2 \times 0.51 = 1.02$ *MeV*. If the energy of γ-photon is less than this, it would cause photo-electric effect or Compton effect on striking the matter.

The converse phenomenon pair-annihilation is also possible. Whenever an electron and a positron come very close to each other, they annihilate each other by combining together and two γ-photons (energy) are produced. This phenomenon is called pair annihilation and is represented by the following equation.

$$\underset{(\text{Positron})}{{}_{+1}\beta^0} + \underset{(\text{Electron})}{{}_{-1}\beta^0} = \underset{(\gamma\text{-photon})}{h\nu} + \underset{(\gamma\text{-photon})}{h\nu}$$

(6) **Nuclear stability**

Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α, β-particles and γ - EM waves. (This process is called radioactivity). The stability of nucleus is determined by many factors. Few such factors are given below :

(i) Neutron-proton ratio $\left(\frac{N}{Z}\text{ Ratio}\right)$

The chemical properties of an atom are governed entirely by the number of protons (Z) in the nucleus, the stability of an atom appears to depend on both the number of protons and the number of neutrons.

For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal ($N \approx Z$) *i.e.* $\frac{N}{Z} = 1$

Heavy nuclei are stable only when they have more neutrons than protons. Thus heavy nuclei are neutron rich compared to lighter nuclei (for heavy nuclei, more is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore more neutrons are added to provide the strong attractive forces necessary to keep the nucleus stable.)

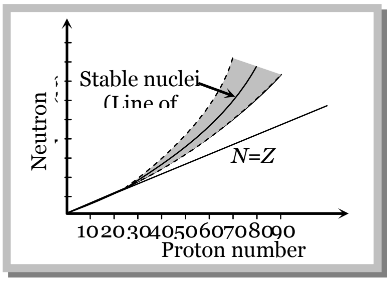
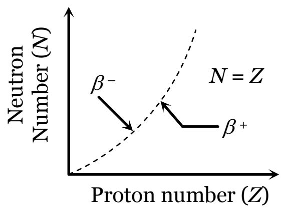


Figure shows a plot of *N* verses *Z* for the stable nuclei. For mass number upto about *A* = 40. For larger value of *Z* the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons. At *Bi* (*Z* = 83, *A* = 209), the neutron excess in *N* – *Z* = 43. There are no stable nuclides with *Z* > 83.

Note : ❑ The nuclide $_{83}Bi^{209}$ is the heaviest stable nucleus.

❑ A nuclide above the line of stability *i.e.* having excess neutrons, decay through β^- emission (neutron changes into proton). Thus increasing atomic number *Z* and decreasing neutron number *N*. In β^- emission, $\frac{N}{Z}$ ratio decreases.

A nuclide below the line of stability have excess number of protons. It decays by β^+ emission, results in decreasing *Z* and increasing *N*. In β^+ emission, the $\frac{N}{Z}$ ratio increases.



(ii) Even or odd numbers of *Z* or *N* : The stability of a nuclide is also determined by the consideration whether it contains an even or odd number of protons and neutrons.

It is found that an even-even nucleus (even *Z* and even *N*) is more stable (60% of stable nuclide have even *Z* and even *N*).

An even-odd nucleus (even *Z* and odd *N*) or odd-even nuclide (odd *Z* and even *N*) is found to be lesser sable while the odd-odd nucleus is found to be less stable.

Only five stable odd-odd nuclides are known : $_1H^2$, $_3Li^6$, $_5Be^{10}$, $_7N^{14}$ and $_{75}Ta^{180}$

(iii) Binding energy per nucleon : The stability of a nucleus is determined by value of it's binding energy per nucleon. In general higher the value of binding energy per nucleon, more stable the nucleus is

Mass Defect and Binding Energy.

(1) **Mass defect (Δm)**

It is found that the mass of a nucleus is always less than the sum of masses of it's constituent nucleons in free state. This difference in masses is called mass defect. Hence mass defect

Δm = Sum of masses of nucleons – Mass of nucleus

$$= \{Zm_p + (A-Z)m_n\} - M = \{Zm_p + Zm_e + (A-Z)m_z\} - M'$$

where m_p = Mass of proton, m_n = Mass of each neutron, m_e = Mass of each electron

M = Mass of nucleus, Z = Atomic number, A = Mass number, M' = Mass of atom as a whole.

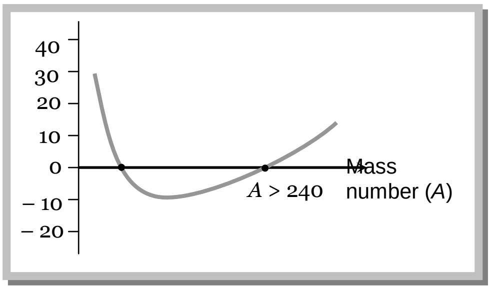
Note : ❑ The mass of a typical nucleus is about 1% less than the sum of masses of nucleons.

(2) **Packing fraction**

Mass defect per nucleon is called packing fraction

Packing fraction $(f) = \dfrac{\Delta m}{A} = \dfrac{M-A}{A}$ where M = Mass of nucleus, A = Mass number

Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, larger is the stability of the nucleus.



(i) Packing fraction may be of positive, negative or zero value.

(iii) At $A = 16, f \to$ Zero

(3) **Binding energy (B.E.)**

The neutrons and protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them infinitely apart (or the same energy is released during the formation of the nucleus). This energy is called the binding energy of the nucleus.

or

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect then according to Einstein's mass energy relation

Binding energy = $\Delta m \cdot c^2$ = $[\{m_p Z + m_n(A-Z)\} - M]\cdot c^2$

(This binding energy is expressed in *joule*, because Δm is measured in *kg*)

If Δm is measured in *amu* then binding energy = Δm *amu* = $[\{m_p Z + m_n(A-Z)\} - M]$ *amu* = $\Delta m \times 931$ *MeV*

(4) **Binding energy per nucleon**

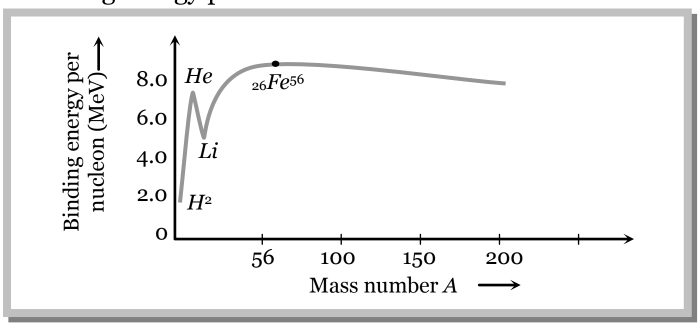
The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

$$\text{Binding energy per nucleon} = \frac{\text{Total binding energy}}{\text{Mass number } (i.e. \text{ total number of nucleons})} = \frac{\Delta m \times 931}{A}\frac{MeV}{Nucleon}$$

Binding energy per nucleon $\propto$ Stability of nucleus

Binding Energy Curve

It is the graph between binding energy per nucleon and total number of nucleons (*i.e.* mass number *A*)



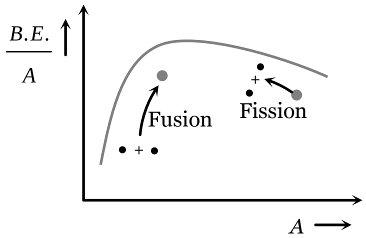
(1) Some nuclei with mass number $A < 20$ have large binding energy per nucleon than their neighbour nuclei. For example ${}_2He^4$, ${}_4Be^8$, ${}_6C^{12}$, ${}_8O^{16}$ and ${}_{10}Ne^{20}$. These nuclei are more stable than their neighbours.

(2) The binding energy per nucleon is maximum for nuclei of mass number A = 56 $({}_{26}Fe^{56})$. It's value is 8.8 MeV per nucleon.

(3) For nuclei having A > 56, binding energy per nucleon gradually decreases for uranium (A = 238), the value of binding energy per nucleon drops to 7.5 MeV.

Note : ❑ When a heavy nucleus splits up into lighter nuclei, then binding energy per nucleon of lighter nuclei is more than that of the original heavy nucleus. Thus a large amount of energy is liberated in this process (nuclear fission).

❑ When two very light nuclei combines to form a relatively heavy nucleus, then binding energy per nucleon increases. Thus, energy is released in this process (nuclear fusion).



Nuclear Reactions.

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction. The general expression for the nuclear reaction is as follows.

$$\underset{\text{(Parent nucleus)}}{X} + \underset{\text{(Incident particle)}}{a} \longrightarrow \underset{\text{(Compound nucleus)}}{C} \longrightarrow \underset{\text{(Compound nucleus)}}{Y} + \underset{\text{(Product particles)}}{b} + \underset{\text{(Energy)}}{Q}$$

Here X and a are known as reactants and Y and b are known as products. This reaction is known as (a, b) reaction and can be represented as $X(a, b)\,Y$

(1) ***Q* value or energy of nuclear reaction**

The energy absorbed or released during nuclear reaction is known as Q-value of nuclear reaction.

Q-value = (Mass of reactants – mass of products)c^2 *Joules*

= (Mass of reactants – mass of products) *amu*

If Q < 0, The nuclear reaction is known as endothermic. (The energy is absorbed in the reaction)

If Q > 0, The nuclear reaction is known as exothermic (The energy is released in the reaction)

(2) **Law of conservation in nuclear reactions**

(i) Conservation of mass number and charge number : In the following nuclear reaction

$${}_2He^4 + {}_7N^{14} \rightarrow {}_8O^{17} + {}_1H^1$$

Mass number (A) →	Before the reaction	After the reaction
	4 +14 = 18	17 + 1 = 18
Charge number (Z) →	2 + 7 = 9	8 + 1 = 9

(ii) Conservation of momentum : Linear momentum/angular momentum of particles before the reaction is equal to the linear/angular momentum of the particles after the reaction. That is $\Sigma p = 0$

(iii) Conservation of energy : Total energy before the reaction is equal to total energy after the reaction. Term Q is added to balance the total energy of the reaction.

(3) **Common nuclear reactions**

The nuclear reactions lead to artificial transmutation of nuclei. Rutherford was the first to carry out artificial transmutation of nitrogen to oxygen in the year 1919.

$${}_2He^4 + {}_7N^{14} \rightarrow {}_9F^{18} \rightarrow {}_8O^{17} + {}_1H^1$$

It is called (α, p) reaction. Some other nuclear reactions are given as follows.

(p, n) reaction $\Rightarrow$ ${}_1H^1 + {}_5B^{11} \to {}_6C^{12} \to {}_6C^{11} + {}_0n^1$

(p, α) reaction $\Rightarrow$ ${}_1H^1 + {}_3Li^{11} \to {}_4Be^8 \to {}_2He^4 + {}_2He^4$

(p, γ) reaction $\Rightarrow$ ${}_1H^1 + {}_6C^{12} \to {}_7N^{13} \to {}_7N^{13} + \gamma$

(n, p) reaction $\Rightarrow$ ${}_0n^1 + {}_7N^{14} \to {}_7N^{15} \to {}_6C^{14} + {}_1H^1$

(γ, n) reaction $\Rightarrow$ $\gamma + {}_1H^2 \to {}_1H^1 + {}_0n^1$

Nuclear Fission and Fusion

Nuclear fission

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy is called nuclear fission.

The phenomenon of nuclear fission was discovered by scientist Ottohann and F. Strassman and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.

(1) **Fission reaction of U^{235}**

(i) Nuclear reaction :

$$ {}_{92}U^{235} + {}_0n^1 \to \underset{\text{(unstable nucleus)}}{{}_{92}U^{236}} \to {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + Q $$

(ii) The energy released in U^{235} fission is about 200 MeV or 0.8 MeV per nucleon.

(iii) By fission of ${}_{92}U^{235}$, on an average 2.5 neutrons are liberated. These neutrons are called fast neutrons and their energy is about 2 MeV (for each). These fast neutrons can escape from the reaction so as to proceed the chain reaction they are need to slow down.

(iv) Fission of U^{235} occurs by slow neutrons only (of energy about 1eV) or even by thermal neutrons (of energy about 0.025 eV).

(v) 50 kg of U^{235} on fission will release $\approx 4 \times 10^{15}$ J of energy. This is equivalence to 20,000 tones of TNT explosion. The nuclear bomb dropped at Hiroshima had this much explosion power.

(vi) The mass of the compound nucleus must be greater than the sum of masses of fission products.

(vii) The $\dfrac{\text{Binding energy}}{\text{A}}$ of compound nucleus must be less than that of the fission products.

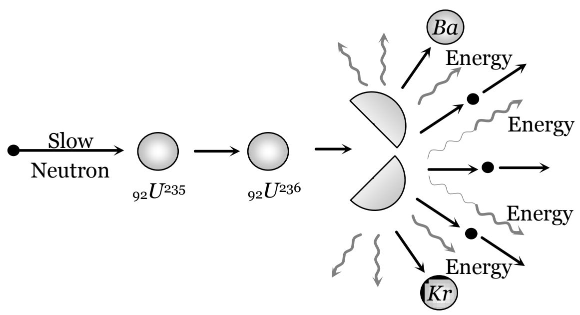
(viii) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments ${}_{56}Ba$ and ${}_{36}Kr$ are formed but they may be any stable isotopes of middle weight atoms.

Same other U^{235} fission reactions are

$$ {}_{92}U^{235} + {}_0n^1 \to {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1 $$
$$ \to {}_{57}La^{148} + {}_{35}Br^{85} + 3\,{}_0n^1 $$
$$ \to \text{Many more} $$

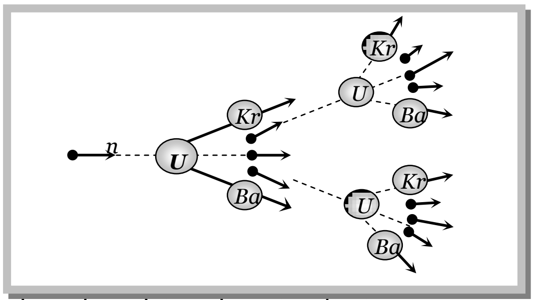
(ix) The neutrons released during the fission process are called prompt neutrons.

(x) Most of energy released appears in the form of kinetic energy of fission fragments.



(2) **Chain reaction**

In nuclear fission, three neutrons are produced along with the release of large energy. Under favourable conditions, these neutrons can cause further fission of other nuclei, producing large number of neutrons. Thus a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.



In the chain reaction, the number of nuclei undergoing fission increases very fast. So, the energy produced takes a tremendous magnitude very soon.

Difficulties in chain reaction

(i) Absorption of neutrons by U^{238}, the major part in natural uranium is the isotope U^{238} (99.3%), the isotope U^{235} is very little (0.7%). It is found that U^{238} is fissionable with fast neutrons, whereas U^{235} is fissionable with slow neutrons. Due to the large percentage of U^{238}, there is more possibility of collision of neutrons with U^{238}. It is found that the neutrons get slowed on coliding with U^{238}, as a result of it further fission of U^{238} is not possible (Because they are slow and they are absorbed by U^{238}). This stops the chain reaction.

Removal : (i) To sustain chain reaction ${}_{92}U^{235}$ is separated from the ordinary uranium. Uranium so obtained $\left({}_{92}U^{235}\right)$ is known as enriched uranium, which is fissionable with the fast and slow neutrons and hence chain reaction can be sustained.

(ii) If neutrons are slowed down by any method to an energy of about 0.3 eV, then the probability of their absorption by U^{238} becomes very low, while the probability of their fissioning U^{235} becomes high. This job is done by moderators. Which reduce the speed of neutron rapidly graphite and heavy water are the example of moderators.

(iii) Critical size : The neutrons emitted during fission are very fast and they travel a large distance before being slowed down. If the size of the fissionable material is small, the neutrons emitted will escape the fissionable material before they are slowed down. Hence chain reaction cannot be sustained.

Removal : The size of the fissionable material should be large than a critical size.

The chain reaction once started will remain steady, accelerate or retard depending upon, a factor called neutron reproduction factor (k). It is defined as follows.

$$k = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

→ If $k = 1$, the chain reaction will be steady. The size of the fissionable material used is said to be the critical size and it's mass, the critical mass.

→ If $k > 1$, the chain reaction accelerates, resulting in an explosion. The size of the material in this case is super critical. (Atom bomb)

→ If $k < 1$, the chain reaction gradually comes to a halt. The size of the material used us said to be sub-critical.

Types of chain reaction : Chain reactions are of following two types

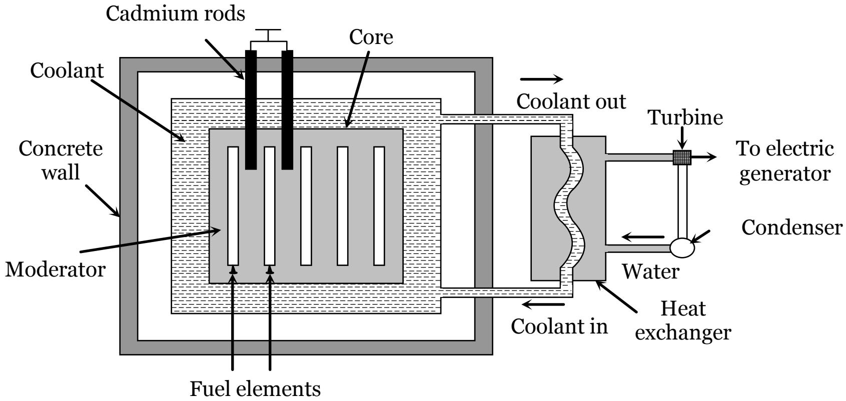
Controlled chain reaction	Uncontrolled chain reaction
Controlled by artificial method	No control over this type of nuclear reaction
All neurons are absorbed except one	More than one neutron takes part into reaction
It's rate is slow	Fast rate

Reproduction factor $k = 1$	Reproduction factor $k > 1$
Energy liberated in this type of reaction is always less than explosive energy	A large amount of energy is liberated in this type of reaction
Chain reaction is the principle of nuclear reactors	Uncontrolled chain reaction is the principle of atom bomb.

Note : ❑ The energy released in the explosion of an atom bomb is equal to the energy released by 2000 tonn of TNT and the temperature at the place of explosion is of the order of $10^7\ ^oC$.

Nuclear Reactor

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction. It is also called an atomic pile. It is thus a source of controlled energy which is utilised for many useful purposes.



(1) **Parts of nuclear reactor**

(i) **Fissionable material (Fuel) :** The fissionable material used in the reactor is called the fuel of the reactor. Uranium isotope (U^{235}) Thorium isotope (Th^{232}) and Plutonium isotopes (Pu^{239}, Pu^{240} and Pu^{241}) are the most commonly used fuels in the reactor.

(ii) **Moderator :** Moderator is used to slow down the fast moving neutrons. Most commonly used moderators are graphite and heavy water (D_2O).

(iii) **Control Material :** Control material is used to control the chain reaction and to maintain a stable rate of reaction. This material controls the number of neutrons available for the fission. For example, cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rods in or out of the core of the reactor.

(iv) **Coolant :** Coolant is a cooling material which removes the heat generated due to fission in the reactor. Commonly used coolants are water, CO_2 nitrogen *etc.*

(v) **Protective shield :** A protective shield in the form a concrete thick wall surrounds the core of the reactor to save the persons working around the reactor from the hazardous radiations.

Note : ❑ It may be noted that Plutonium is the best fuel as compared to other fissionable material. It is because fission in Plutonium can be initiated by both slow and fast neutrons. Moreover it can be obtained from U^{238}.

❑ Nuclear reactor is firstly devised by fermi. ❑ Apsara was the first Indian nuclear reactor.

(2) **Uses of nuclear reactor**

(i) In electric power generation.

(ii) To produce radioactive isotopes for their use in medical science, agriculture and industry.

(iii) In manufacturing of PU^{239} which is used in atom bomb.

(iv) They are used to produce neutron beam of high intensity which is used in the treatment of cancer and nuclear research.

Note : ❑ A type of reactor that can produce more fissile fuel than it consumes is the breeder reactor.

Nuclear fusion

In nuclear fusion two or more than two lighter nuclei combine to form a single heavy nucleus. The mass of single nucleus so formed is less than the sum of the masses of parent nuclei. This difference in mass results in the release of tremendous amount of energy

$$_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4\,MeV$$

$$_1H^3 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1 + 17.6\,MeV$$

or $$_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + 24\,MeV$$

For fusion high pressure ($\approx 10^6$ *atm*) and high temperature (of the order of 10^7 *K* to 10^8 *K*) is required and so the reaction is called thermonuclear reaction.

Fusion energy is greater then fission energy fission of one uranium atom releases about 200 *MeV* of energy. But the fusion of a deutron $({}_1H^2)$ and triton $({}_1H^3)$ releases about 17.6 *MeV* of energy. However the energy released per nucleon in fission is about 0.85 *MeV* but that in fusion is 4.4 *MeV*. So for the same mass of the fuel, the energy released in fusion is much larger than in fission.

Plasma : The temperature of the order of 10^8 *K* required for thermonuclear reactions leads to the complete ionisation of the atom of light elements. The combination of base nuclei and electron cloud is called plasma. The enormous gravitational field of the sun confines the plasma in the interior of the sun.

The main problem to carryout nuclear fusion in the laboratory is to contain the plasma at a temperature of 10^8*K*. No solid container can tolerate this much temperature. If this problem of containing plasma is solved, then the large quantity of deuterium present in sea water would be able to serve as in-exhaustible source of energy.

Note : ❑ To achieve fusion in laboratory a device is used to confine the plasma, called **Tokamak**.

Stellar Energy

Stellar energy is the energy obtained continuously from the sun and the stars. Sun radiates energy at the rate of about 10^{26} *joules* per *second*.

Scientist Hans Bethe suggested that the fusion of hydrogen to form helium (thermo nuclear reaction) is continuously taking place in the sun (or in the other stars) and it is the source of sun's (star's) energy.

The stellar energy is explained by two cycles

Proton-proton cycle	Carbon-nitrogen cycle
$_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + {}_1e^0 + Q_1$	$_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} + Q_1$
$_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + Q_2$	$_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e^0$
$_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1 + Q_3$	$_1H^1 + {}_6C^{13} \rightarrow {}_7N^{14} + Q_2$
$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_{+1}e^0 + 2\gamma + 26.7\,MeV$	$_1H^1 + {}_7N^{14} \rightarrow {}_8O^{15} + Q_3$
	$_8O^{15} \rightarrow {}_7N^{15} + {}_1e^0 + Q_4$
	$_1H^1 + {}_7N^{15} \rightarrow {}_6C^{12} + {}_2He^4$
	$4\,{}_1H^1 \rightarrow {}_2He^4 + 2\,{}_1e^0 + 24.7\,MeV$

About 90% of the mass of the sun consists of hydrogen and helium.

Nuclear Bomb Based on uncontrolled nuclear reactions.

Atom bomb	Hydrogen bomb
Based on fission process it involves the fission of U^{235}	Based on fusion process. Mixture of deutron and tritium is used in it
In this critical size is important	There is no limit to critical size
Explosion is possible at normal temperature and pressure	High temperature and pressure are required
Less energy is released compared to hydrogen bomb	More energy is released as compared to atom bomb so it is more dangerous than atom bomb

Concepts

☞ *A test tube full of base nuclei will weight heavier than the earth.*

☞ *The nucleus of hydrogen contains only one proton. Therefore we may say that the proton is the nucleus of hydrogen atom.*

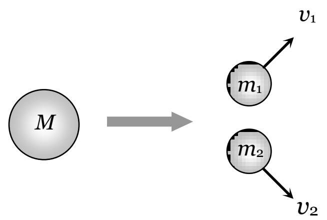
☞ *If the relative abundance of isotopes in an element has a ratio $n_1 : n_2$ whose atomic masses are m_1 and m_2 then atomic mass of the element is* $M = \dfrac{n_1m_1 + n_2m_2}{n_1 + n_2}$

Examples

***Example*: 1** A heavy nucleus at rest breaks into two fragments which fly off with velocities in the ratio 8 : 1. The ratio of radii of the fragments is

(a) 1 : 2 (b) 1 : 4 (c) 4 : 1 (d) 2 : 1

Solution : (a)



By conservation of momentum $m_1v_1 = m_2v_2$

$\Rightarrow \dfrac{v_1}{v_2} = \dfrac{8}{1} = \dfrac{m_2}{m_1}$ ...... (i)

Also from $r \propto A^{1/3} \Rightarrow \dfrac{r_1}{r_2} = \left(\dfrac{A_1}{A_2}\right)^{1/3} = \left(\dfrac{1}{8}\right)^{1/3} = \dfrac{1}{2}$

***Example*: 2** The ratio of radii of nuclei ${}^{27}_{13}Al$ and ${}^{125}_{52}Te$ is approximately **[J & K CET 2000]**

(a) 6 : 10 (b) 13 : 52 (c) 40 : 177 (d) 14 : 7

Solution : (a) By using $r \propto A^{1/3} \Rightarrow \dfrac{r_1}{r_2} = \left(\dfrac{A_1}{A_2}\right)^{1/3} = \left(\dfrac{27}{125}\right)^{1/3} = \dfrac{8}{5} = \dfrac{6}{10}$

***Example*: 3** If Avogadro's number is 6×10^{23} then the number of protons, neutrons and electrons in 14 g of ${}_6C^{14}$ are respectively

(a) $36 \times 10^{23}, 48 \times 10^{23}, 36 \times 10^{23}$ (b) $36 \times 10^{23}, 36 \times 10^{23}, 36 \times 10^{21}$

(c) $48 \times 10^{23}, 36 \times 10^{23}, 48 \times 10^{21}$ (d) $48 \times 10^{23}, 48 \times 10^{23}, 36 \times 10^{21}$

Solution : (a) Since the number of protons, neutrons and electrons in an atom of ${}_6C^{14}$ are 6, 8 and 6 respectively. As 14 gm of ${}_6C^{14}$ contains 6×10^{23} atoms, therefore the numbers of protons, neutrons and electrons in 14 gm of ${}_6C^{14}$ are $6 \times 6 \times 10^{23} = 36 \times 10^{23}$, $8 \times 6 \times 10^{23} = 48 \times 10^{23}$, $6 \times 6 \times 10^{23} = 36 \times 10^{23}$.

***Example*: 4** Two Cu^{64} nuclei touch each other. The electrostatics repulsive energy of the system will be

(a) 0.788 MeV (b) 7.88 MeV (c) 126.15 MeV (d) 788 MeV

Solution : (c) Radius of each nucleus $R = R_0(A)^{1/3} = 1.2\,(64)^{1/3} = 4.8\ fm$

Distance between two nuclei (r) = 2R

So potential energy $U = \dfrac{k \cdot q^2}{r} = \dfrac{9 \times 10^9 \times (1.6 \times 10^{-19} \times 29)^2}{2 \times 4.8 \times 10^{-15} \times 1.6 \times 10^{-19}} = 126.15\ MeV$.

***Example*: 5** When ${}_{92}U^{235}$ undergoes fission. 0.1% of its original mass is changed into energy. How much energy is released if 1 kg of ${}_{92}U^{235}$ undergoes fission **[MP PET 1994; MP PMT/PET 1998; BHU 2001; BVP 2003]**

(a) $9 \times 10^{10}\ J$ (b) $9 \times 10^{11}\ J$ (c) $9 \times 10^{12}\ J$ (d) $9 \times 10^{13}\ J$

Solution : (d) By using $E = \Delta m \cdot c^2 \Rightarrow E = \left(\dfrac{0.1}{100} \times 1\right)(3 \times 10^8)^2 = 9 \times 10^{13}\ J$

***Example*: 6** 1 g of hydrogen is converted into 0.993 g of helium in a thermonuclear reaction. The energy released is **[EAMCET (Med.) 1995; CPMT 1999]**

(a) $63 \times 10^7\ J$ (b) $63 \times 10^{10}\ J$ (c) $63 \times 10^{14}\ J$ (d) $63 \times 10^{20}\ J$

Solution : (b) $\Delta m = 1 - 0.993 = 0.007\ gm$

$\therefore E = \Delta mc^2 = 0.007 \times 10^{-3} \times (3 \times 10^8)^2 = 63 \times 10^{10}\ J$

***Example*: 7** The binding energy per nucleon of deuteron $({}_1^2H)$ and helium nucleus $({}_2^4He)$ is 1.1 *MeV* and 7 *MeV* respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is

[MP PMT 1992; Roorkee 1994; IIT-JEE 1996; AIIMS 1997; Haryana PMT 2000; Pb PMT 2001; CPMT 2001; AIEEE 2004]

(a) 13.9 *MeV* (b) 26.9 *MeV* (c) 23.6 *MeV* (d) 19.2 *MeV*

Solution : (c) ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + Q$

Total binding energy of helium nucleus = 4 × 7 = 28 *MeV*

Total binding energy of each deutron = 2 × 1.1 = 2.2 *MeV*

Hence energy released = 28 – 2 × 2.2 = 23.6 *MeV*

***Example*: 8** The masses of neutron and proton are 1.0087 *amu* and 1.0073 *amu* respectively. If the neutrons and protons combine to form a helium nucleus (alpha particles) of mass 4.0015 *amu*. The binding energy of the helium nucleus will be [1 *amu*= 931 *MeV*] **[CPMT 1986; MP PMT 1995; CBSE 2003]**

(a) 28.4 *MeV* (b) 20.8 *MeV* (c) 27.3 *MeV* (d) 14.2 *MeV*

Solution : (a) Helium nucleus consist of two neutrons and two protons.

So binding energy $E = \Delta m$ *amu* $= \Delta m \times 931$ *MeV*

$\Rightarrow E = (2 \times m_p + 2m_n - M) \times 931\ MeV = (2 \times 1.0073 + 2 \times 1.0087 - 4.0015) \times 931 = 28.4\ MeV$

***Example*: 9** A atomic power reactor furnace can deliver 300 *MW*. The energy released due to fission of each of uranium atom U^{238} is 170 *MeV*. The number of uranium atoms fissioned per hour will be

(a) 5×10^{15} (b) 10×10^{20} (c) 40×10^{21} (d) 30×10^{25}

Solution : (c) By using $P = \frac{W}{t} = \frac{n \times E}{t}$ where n = Number of uranium atom fissioned and E = Energy released due to each fission so $300 \times 10^6 = \frac{n \times 170 \times 10^6 \times 1.6 \times 10^{-19}}{3600}$ $\Rightarrow n = 40 \times 10^{21}$

***Example*: 10** The binding energy per nucleon of O^{16} is 7.97 *MeV* and that of O^{17} is 7.75 *MeV*. The energy (in *MeV*) required to remove a neutron from O^{17} is **[IIT-JEE 1995]**

(a) 3.52 (b) 3.64 (c) 4.23 (d) 7.86

Solution : (c) $O^{17} \rightarrow O^{16} + {}_0n^1$

$\therefore$ Energy required = Binding of O^{17} – binding energy of O^{16} = 17 × 7.75 – 16 × 7.97 = 4.23 *MeV*

***Example*: 11** A gamma ray photon creates an electron-positron pair. If the rest mass energy of an electron is 0.5 *MeV* and the total kinetic energy of the electron-positron pair is 0.78 *MeV*, then the energy of the gamma ray photon must be **[MP PMT 1991]**

(a) 0.78 *MeV* (b) 1.78 *MeV* (c) 1.28 *MeV* (d) 0.28 *MeV*

Solution : (b) Energy of γ-rays photon = 0.5 + 0.5 +0.78 = 1.78 *MeV*

***Example*: 12** What is the mass of one Curie of U^{234} **[MNR 1985]**

(a) 3.7×10^{10} *gm* (b) 2.348×10^{23} *gm* (c) 1.48×10^{-11} *gm* (d) 6.25×10^{-34} *gm*

Solution : (c) 1 *curie* = 3.71×10^{10} disintegration/*sec* and mass of 6.02×10^{23} atoms of U^{234} = 234 *gm*

$\therefore$ Mass of 3.71×10^{10} atoms $= \frac{234 \times 3.71 \times 10^{10}}{6.02 \times 10^{23}} = 1.48 \times 10^{-11}\ gm$

***Example*: 13** In the nuclear fusion reaction ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + n$, given that the repulsive potential energy between the two nuclei is $-7.7 \times 10^{-14}\ J$, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}\ J/K$] **[AIEEE 2003]**

(a) $10^9\ K$ (b) $10^7\ K$ (c) $10^5\ K$ (d) $10^3\ K$

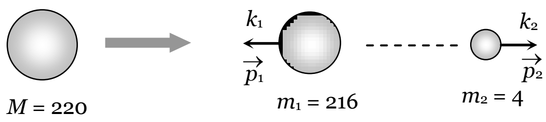
Solution : (a) Kinetic energy of molecules of a gas at a temperature T is 3/2 kT

$\therefore$ To initiate the reaction $\frac{3}{2}kT = 7.7 \times 10^{-14}\ J \Rightarrow T = 3.7 \times 10^9\ K.$

***Example*: 14** A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 *MeV*. Calculate the kinetic energy of the α-particle **[IIT-JEE (Screening) 2003]**

(a) 4.4 MeV (b) 5.4 MeV (c) 5.6 MeV (d) 6.5 MeV

Solution : (b)



Q-value of the reaction is 5.5 eV *i.e.* $k_1 + k_2 = 5.5\ MeV$ ......(i)

By conservation of linear momentum $p_1 = p_2 \Rightarrow \sqrt{2(216)k_1} = \sqrt{2(4)k_2} \Rightarrow k_2 = 54\ k_1$ ......(ii)

On solving equation (i) and (ii) we get $k_2 = 5.4\ MeV$.

***Example*: 15** Let m_p be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a $^{20}_{10}Ne$ nucleus and M_2 the mass of a $^{40}_{20}Ca$ nucleus. Then **[IIT 1998; DPMT 2000]**

(a) $M_2 = 2M_1$ (b) $M_2 > 2M_1$ (c) $M_2 < 2M_1$ (d) $M_1 < 10(m_n + m_p)$

Solution : (c, d) Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less then the sum of masses of it's constituent particles $^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons. Therefore, mass of $^{20}_{10}Ne$ nucleus $M_1 < 10\,(m_p + m_n)$

Also heavier the nucleus, more is he mass defect thus $20\,(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

or $10\,(m_p + m_n) > M_2 - M_1$

$\Rightarrow$ $M_2 < M_1 + 10\,(m_p + m_n) \Rightarrow M_2 < M_1 + M_1 \Rightarrow M_2 < 2M_1$

Tricky example: 1

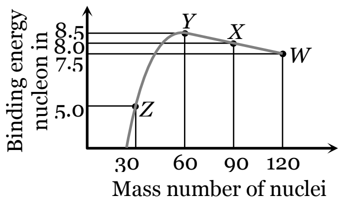
Binding energy per nucleon *vs* mass number curve for nuclei is shown in the figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is **[IIT-JEE 1999]**

(a) $Y \to 2Z$

(b) $W \to X + Z$

(c) $W \to 2Y$

(d) $X \to Y + Z$



Solution : (c) Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon × number of nucleon) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (c) this happens.

Given $W \to 2Y$

Binding energy of reactants = 120 × 7.5 = 900 MeV

and binding energy of products = 2 (60 × 8.5) = 1020 MeV > 900 MeV

Radioactivity

The phenomenon of spontaneous emission of radiatons by heavy elements is called radioactivity. The elements which shows this phenomenon are called radioactive elements.

(1) Radioactivity was discovered by Henery Becquerel in uranium salt in the year 1896.

(2) After the discovery of radioactivity in uranium, Piere Curie and Madame Curie discovered a new radioactive element called radium (which is 10^6 times more radioactive than uranium)

(3) Some examples of radio active substances are : Uranium, Radium, Thorium, Polonium, Neptunium *etc.*

(4) Radioactivity of a sample cannot be controlled by any physical (pressure, temperature, electric or magnetic field) or chemical changes.

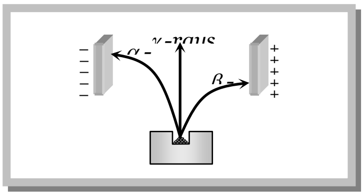
(5) All the elements with atomic number (Z) > 82 are naturally radioactive.

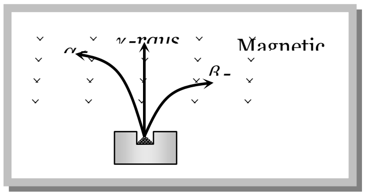
(6) The conversion of lighter elements into radioactive elements by the bombardment of fast moving particles is called artificial or induced radioactivity.

(7) Radioactivity is a nuclear event and not atomic. Hence electronic configuration of atom don't have any relationship with radioactivity.

Nuclear radiatons

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only. When the radiation enters into the external electric field, they splits into three parts





(i) Radiations which deflects towards negative plate are called α-rays (stream of positively charged particles)

(ii) Radiations which deflects towards positive plate are called β particles (stream of negatively charged particles)

(iii) Radiations which are undeflected called γ-*rays*. (E.M. waves or photons)

Note : ❑ Exactly same results were obtained when these radiations were subjected to magnetic field.

❑ No radioactive substance emits both α and β particles simultaneously. Also γ-rays are emitted after the emission of α or β-particles.

❑ β-particles are not orbital electrons they come from nucleus. The neutron in the nucleus decays into proton and an electron. This electron is emitted out of the nucleus in the form of β-rays.

Properties of α, β and γ-rays

Features	α- particles	β - particles	γ - rays
1. Identity	Helium nucleus or doubly ionised helium atom (${}_2He^4$)	Fast moving electron $(-\beta^0$ or $\beta^-)$	Photons (E.M. waves)
2. Charge	+ 2*e*	− *e*	Zero
3. Mass 4 m_p (m_p = mass of proton = 1.87×10^{-27}	4 m_p	m_e	Massless
4. Speed	$\approx 10^7\ m/s$	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 *MeV* to 9 *MeV*	All possible values between a minimum certain value to 1.2 *MeV*	Between a minimum value to 2.23 *MeV*
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 *cm* of iron (or *Pb*) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete
10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	${}_ZX^A \xrightarrow{\alpha-decay} {}_{Z-2}Y^{A-4} + {}_2He^4$	${}_ZX^A \to {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$ ${}_ZX^A \xrightarrow{n_\beta} {}_{Z'}X^A$	${}_ZX^A \to {}_ZX^a + \gamma$

	${}_{Z}X^{A} \xrightarrow{n_\alpha} {}_{Z'}Y^{A'}$ $\Rightarrow \boldsymbol{n_\alpha = \frac{A' - A}{4}}$	$\Rightarrow \boldsymbol{n_\beta = (2n_\alpha - Z + Z')}$	

Radioactive Disintegration

(1) **Law of radioactive disintegration**

According to Rutherford and Soddy law for radioactive decay is as follows.

"At any instant the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant"

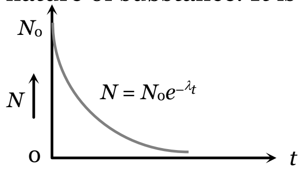
i.e. $-\frac{dN}{dt} \propto N \quad \Rightarrow \frac{dN}{dt} = -\lambda N$. It can be proved that $\boldsymbol{N = N_0 e^{-\lambda t}}$

This equation can also be written in terms of mass *i.e.* $M = M_0 e^{-\lambda t}$

where N = Number of atoms remains undecayed after time t, N_0 = Number of atoms present initially (*i.e.* at t = 0), M = Mass of radioactive nuclei at time t, M_0 = Mass of radioactive nuclei at time t = 0, $N_0 - N$ = Number of disintegrated nucleus in time t

$\frac{dN}{dt}$ = rate of decay, λ = Decay constant or disintegration constant or radioactivity constant or Rutherford Soddy's constant or the probability of decay per unit time of a nucleus.

Note : ❑ λ depends only on the nature of substance. It is independent of time and any physical or chemical changes.



(2) **Activity**

It is defined as the rate of disintegration (or count rate) of the substance (or the number of atoms of any material decaying per second) *i.e.* $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$

where A_0 = Activity of t = 0, A = Activity after time t

Units of activity (Radioactivity)

It's units are *Becqueral* (*Bq*), Curie (*Ci*) and Rutherford (*Rd*)

1 *Becquerel* = 1 *disintegration/sec*, 1 *Rutherford* = 10^6 *dis/sec*, 1 *Curie* = 3.7×10^{11} *dis/sec*

Note : ❑ Activity per *gm* of a substance is known as specific activity. The specific activity of 1 *gm* of radium – 226 is 1 *Curie.*

❑ 1 *millicurie* = 37 *Rutherford*

❑ The activity of a radioactive substance decreases as the number of undecayed nuclei decreases with time.

❑ Activity $\propto \frac{1}{\text{Half life}}$

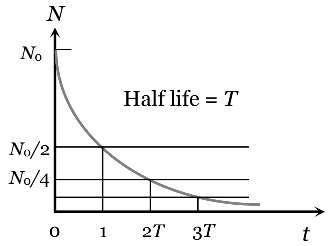
(3) **Half life ($T_{1/2}$)**

Time interval in which the mass of a radioactive substance or the number of it's atom reduces to half of it's initial value is called the half life of the substance.

i.e. if $N = \frac{N_0}{2}$ then $t = T_{1/2}$

Hence from $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda (T_{1/2})} \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$



Time (t)	Number of undecayed atoms (N) (N_0 = Number of initial atoms)	Remaining fraction of active atoms (N/N_0) probability of survival	Fraction of atoms decayed ($N_0 - N$) $/N_0$ probability of decay
t = 0	N_0	1 (100%)	0
$t = T_{1/2}$	$\frac{N_0}{2}$	$\frac{1}{2}$ (50%)	$\frac{1}{2}$ (50%)

$t = 2(T_{1/2})$	$\frac{1}{2} \times \frac{N_0}{2} = \frac{N_0}{(2)^2}$	$\frac{1}{4}$ (25%)	$\frac{3}{4}$ (75%)
$t = 3(T_{1/2})$	$\frac{1}{2} \times \frac{N_0}{(2)} = \frac{N_0}{(2)^3}$	$\frac{1}{8}$ (12.5%)	$\frac{7}{8}$ (87.5%)
$t = 10\ (T_{1/2})$	$\frac{N_0}{(2)^{10}}$	$\left(\frac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = n\ (N_{1/2})$	$\frac{N}{(2)^2}$	$\left(\frac{1}{2}\right)^{n}$	$\left\{1 - \left(\frac{1}{2}\right)^{n}\right\}$

Useful relation

After n half-lives, number of undecayed atoms $N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$

(4) **Mean (or average) life (τ)**

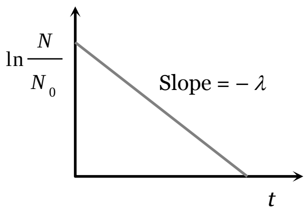
The time for which a radioactive material remains active is defined as mean (average) life of that material.

Other definitions

(i) It is defined as the sum of lives of all atoms divided by the total number of atoms

$$i.e.\ \tau = \frac{\text{Sum of the lives of all the atoms}}{\text{Total number of atoms}} = \frac{1}{\lambda}$$

(ii) From $N = N_0 e^{-\lambda t} \Rightarrow \frac{\ln \frac{N}{N_0}}{t} = -\lambda$ slope of the line shown in the graph



i.e. the magnitude of inverse of slope of $\ln \frac{N}{N_0}$ *vs* t curve is known as mean life (τ).

(iii) From $N = N_0 e^{-\lambda t}$

If $t = \frac{1}{\lambda} = \tau \Rightarrow N = N_0 e^{-1} = N_0\left(\frac{1}{e}\right) = 0.37\, N_0 = 37\%$ of N_0.

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\frac{1}{e}$ times or 0.37 times or 37% of original number of atoms. or

It is the time in which number of decayed atoms ($N_0 - N$) becomes $\left(1 - \frac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(iv) From $T_{1/2} = \frac{0.693}{\lambda} \Rightarrow \frac{1}{\lambda} = \tau = \frac{1}{0.693} \cdot (t_{1/2}) = 1.44\,(T_{1/2})$

i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$

Note : ❑ Half life and mean life of a substance doesn't change with time or with pressure, temperature *etc.*

Radioactive Series

If the isotope that results from a radioactive decay is itself radioactive then it will also decay and so on.

The sequence of decays is known as radioactive decay series. Most of the radio-nuclides found in nature are members of four radioactive series. These are as follows

Mass number	Series (Nature)	Parent	Stable and product	Integer n	Number of lost particles
$4n$	Thorium (natural)	${}_{90}Th^{232}$	${}_{82}Pb^{208}$	52	$\alpha = 6, \beta = 4$
$4n + 1$	Neptunium (Artificial)	${}_{93}Np^{237}$	${}_{83}Bi^{209}$	52	$\alpha = 8, \beta = 5$

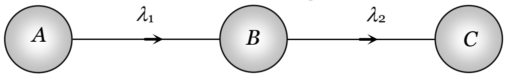
$4n + 2$	Uranium (Natural)	$_{92}U^{238}$	$_{82}Pb^{206}$	51	$\alpha = 8, \beta = 6$
$4n + 3$	Actinium (Natural)	$_{89}Ac^{227}$	$_{82}Pb^{207}$	51	$\alpha = 7, \beta = 4$

Note : ❑ The $4n + 1$ series starts from $_{94}PU^{241}$ but commonly known as neptunium series because neptunium is the longest lived member of the series.

❑ The $4n + 3$ series actually starts from $_{92}U^{235}$.

Successive Disintegration and Radioactive Equilibrium

Suppose a radioactive element *A* disintegrates to form another radioactive element *B* which intern disintegrates to still another element *C*; such decays are called successive disintegration.



Rate of disintegration of $A = \frac{dN_1}{dt} = -\lambda_1 N_1$ (which is also the rate of formation of *B*)

Rate of disintegration of $B = \frac{dN_2}{dt} = -\lambda_2 N_2$

$\therefore$ Net rate of formation of *B* = Rate of disintegration of *A* – Rate of disintegration of *B*

$= \lambda_1 N_1 - \lambda_2 N_2$

Equilibrium

In radioactive equilibrium, the rate of decay of any radioactive product is just equal to it's rate of production from the previous member.

i.e. $\lambda_1 N_1 = \lambda_2 N_2 \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_2} = \frac{\tau_2}{\tau_1} = \frac{(T_{1/2})}{(T_{1/2})_1}$

Note : ❑ In successive disintegration if N_0 is the initial number of nuclei of *A* at $t = 0$ then number of nuclei of product *B* at time *t* is given by $N_2 = \frac{\lambda_1 N_0}{(\lambda_2 - \lambda_1)}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$ where $\lambda_1 \lambda_2$ – decay constant of *A* and *B*.

Uses of radioactive isotopes

(1) **In medicine**

(i) For testing blood-chromium - 51

(ii) For testing blood circu

(iii) For detecting brain tumor- Radio mercury - 203

(iv) For detecting fault in thyroid gland - Radio iodine - 131

(v) For cancer - cobalt – 60

(vi) For blood - Gold - 189

(vii) For skin diseases - Phospohorous - 31

(2) **In Archaeology**

(i) For determining age of archaeological sample (carbon dating) C^{14}

(ii) For determining age of meteorites - K^{40}

(iii) For determining age of earth-Lead isotopes

(3) **In agriculture**

(i) For protecting potato crop from earthworm- CO^{60}

(ii) For artificial rains - *AgI*

(iii) As fertilizers - P^{32}

(4) **As tracers** - (Tracer) : Very small quantity of radioisotopes present in a mixture is known as tracer

(i) Tracer technique is used for studying biochemical reaction in tracer and animals.

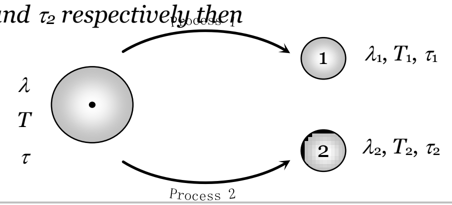
(5) **In industries**

(i) For detecting leakage in oil or water pipe lines

(ii) For determining the age of planets.

Concept

☞ *If a nuclide can decay simultaneously by two different process which have decay constant λ_1 and λ_2, half life T_1 and T_2 and mean lives τ_1 and τ_2 respectively then*



$\Rightarrow \lambda = \lambda_1 + \lambda_2$

$\Rightarrow T = \frac{T_1 T_2}{T_1 + T_2}$

$\Rightarrow \tau = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$

***Example*: 16** When $_{90}Th^{228}$ transforms to $_{83}Bi^{212}$, then the number of the emitted α–and β–particles is, respectively **[MP PET 2002]**

(a) $8\alpha, 7\beta$ (b) $4\alpha, 7\beta$ (c) $4\alpha, 4\beta$ (d) $4\alpha, 1\beta$

Solution : (d) $_{Z=90}Th^{A=228} \rightarrow {}_{Z'=83}Bi^{A'=212}$

Number of α-particles emitted $n_\alpha = \frac{A - A'}{4} = \frac{228 - 212}{4} = 4$

Number of β-particles emitted $n_\beta = 2n_\alpha - Z + Z' = 2 \times 4 - 90 + 83 = 1.$

***Example*: 17** A radioactive substance decays to 1/16th of its initial activity in 40 days. The half-life of the radioactive substance expressed in days is

(a) 2.5 (b) 5 (c) 10 (d) 20

Solution : (c) By using $N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{N}{N_0} = \frac{1}{16} = \left(\frac{1}{2}\right)^{40/T_{1/2}} \Rightarrow T_{1/2} = 10$ days.

***Example*: 18** A sample of radioactive element has a mass of 10 *gm* at an instant *t* = 0. The approximate mass of this element in the sample after two mean lives is **[CBSE PMT 2003]**

(a) 2.50 *gm* (b) 3.70 *gm* (c) 6.30 *gm* (d) 1.35 *gm*

Solution : (d) By using $M = M_0 e^{-\lambda t} \Rightarrow M = 10e^{-\lambda(2\tau)} = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 = 1.359\ gm$

***Example*: 19** The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is **[IIT-JEE (Screening) 2002]**

(a) 400 μs (b) 6.3 μs (c) 40 μs (d) 300 μs

Solution : (a) By using $N = N_0 \left(\frac{1}{2}\right)^n \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{t/100} \Rightarrow t = 400\ \mu sec.$

***Example*: 20** The mean lives of a radioactive substance for α and β emissions are 1620 years and 405 years respectively. After how much time will the activity be reduced to one fourth **[RPET 1999]**

(a) 405 year (b) 1620 year (c) 449 year (d) None of these

Solution : (c) $\lambda_\alpha = \frac{1}{1620}$ *per year* and $\lambda_\beta = \frac{1}{405}$ *per year* and it is given that the fraction of the remained activity $\frac{A}{A_0} = \frac{1}{4}$

Total decay constant $\lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ *per year*

We know that $A = A_0 e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda}\log_e \frac{A_0}{A} \Rightarrow t = \frac{1}{\lambda}\log_e 4 = \frac{2}{\lambda}\log_e 2 = 324 \times 2 \times 0.693 = 449$ *years.*

***Example*: 21** At any instant the ratio of the amount of radioactive substances is 2 : 1. If their half lives be respectively 12 and 16 hours, then after two days, what will be the ratio of the substances

(a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

Solution : (a) By using $N = N_0 \left(\frac{1}{2}\right)^n \Rightarrow \frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2} \times \frac{(1/2)^{n_1}}{(1/2)^{n_2}} = \frac{2}{1} \times \frac{\left(\frac{1}{2}\right)^{\frac{2\times 24}{12}}}{\left(\frac{1}{2}\right)^{\frac{2\times 24}{16}}} = \frac{1}{1}$

***Example*: 22** From a newly formed radioactive substance (Half-life 2 *hours*), the intensity of radiation is 64 times the permissible safe level. The minimum time after which work can be done safely from this source is **[IIT 1983; SCRA 1996]**

(a) 6 *hours* (b) 12 *hours* (c) 24 *hours* (d) 128 *hours*

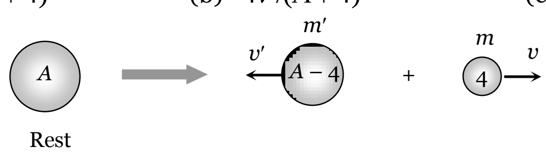
Solution : (b) By using $A = A_0 \left(\frac{1}{2}\right)^n \Rightarrow \frac{A}{A_0} = \frac{1}{64} = \left(\frac{1}{2}\right)^0 = \left(\frac{1}{2}\right)^n \Rightarrow n = 6$

$\Rightarrow \frac{t}{T_{1/2}} = 6 \Rightarrow t = 6 \times 2 = 12$ *hours.*

***Example*: 23** nucleus of mass number *A*, originally at rest, emits an α-particle with speed *v*. The daughter nucleus recoils with a speed **[DCE 2000; AIIMS 2004]**

(a) $2v/(A+4)$ (b) $4v/(A+4)$ (c) $4v/(A-4)$ (d) $2v/(A-4)$

Solution : (c)



According to conservation of momentum $4v = (A-4)v' \Rightarrow v' = \frac{4v}{A-4}$.

***Example*: 24** The counting rate observed from a radioactive source at t = 0 second was 1600 counts per second and at t = 8 seconds it was 100 counts per second. The counting rate observed as counts per second at t = 6 seconds will be **[MP PET 1996; UPSEAT 2000]**

(a) 400 (b) 300 (c) 200 (d) 150

Solution : (c) By using $A = A_0\left(\frac{1}{2}\right)^n \Rightarrow 100 = 1600\left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{8/T_{1/2}} \Rightarrow T_{1/2} = 2\,sec$

Again by using the same relation the count rate at t = 6 *sec* will be $A = 1600\left(\frac{1}{2}\right)^{6/2} = 200$.

***Example*: 25** The kinetic energy of a neutron beam is 0.0837 *eV*. The half-life of neutrons is 693*s* and the mass of neutrons is 1.675×10^{-27} *kg*. The fraction of decay in travelling a distance of 40*m* will be

(a) 10^{-3} (b) 10^{-4} (c) 10^{-5} (d) 10^{-6}

Solution : (c) $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times0.0837\times1.6\times10^{-19}}{1.675\times10^{-27}}} = 4\times10^3\ m/sec$

$\therefore$ Time taken by neutrons to travel a distance of 40 *m* $\Delta t' = \frac{40}{4\times10^3} = 10^{-2}\,sec$

$\therefore \frac{dN}{dt} = \lambda N \Rightarrow \frac{dN}{N} = \lambda\, dt$

$\therefore$ Fraction of neutrons decayed in Δt *sec* in $\frac{\Delta N}{N} = \lambda\Delta t = \frac{0.693}{T}\Delta t = \frac{0.693}{693}\times10^{-2} = 10^{-5}$

***Example*: 26** The fraction of atoms of radioactive element that decays in 6 days is 7/8. The fraction that decays in 10 days will be

(a) 77/80 (b) 71/80 (c) 31/32 (d) 15/16

Solution : (c) By using $N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow t = \frac{T_{1/2}\log_e\left(\frac{N_0}{N}\right)}{\log_e(2)} \Rightarrow t \propto \log_e\frac{N_0}{N} \Rightarrow \frac{t_1}{t_2} = \frac{\left(\log_e\frac{N_0}{N}\right)_1}{\left(\log_e\frac{N_0}{N}\right)_2}$

Hence $\frac{6}{10} = \frac{\log_e(8/1)}{\log_e(N_0/N)} \Rightarrow \log_e\frac{N_0}{N} = \frac{10}{6}\log_e(8) = \log_e 32 \Rightarrow \frac{N_0}{N} = 32$.

So fraction that decays $= 1 - \frac{1}{32} = \frac{31}{32}$.

Tricky example: 2

Half-life of a substance is 20 minutes. What is the time between 33% decay and 67% decay **[AIIMS 200**

(a) 40 minutes (b) 20 minutes (c) 30 minutes (d) 25 minutes

Solution : (b) Let N_0 be the number of nuclei at beginning

$\therefore$ Number of undecayed nuclei after 33% decay = 0.67 N_0

and number of undecayed nuclei after 67% of decay = 0.33 N_0

$\because$ 0.33 $N_0 \simeq \frac{0.67N_0}{2}$ and in the half-life time the number of undecayed nuclei becomes half.

Example

***Example*: 1** The ratio of areas within the electron orbits for the first excited state to the ground state for hydrogen atom is

(a) 16 : 1 (b) 18 : 1 (c) 4 : 1 (d) 2 : 1

Solution : (a) For a hydrogen atom

Radius $r \propto n^2 \Rightarrow \frac{r_1^2}{r_2^2} = \frac{n_1^4}{n_2^4} \Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{n_1^4}{n_2^4} \Rightarrow \frac{A_1}{A_2} = \frac{n_1^4}{n_2^4} = \frac{2^4}{1^4} = 16 \Rightarrow \frac{A_1}{A_2} = \frac{16}{1}$

***Example*: 2** The electric potential between a proton and an electron is given by $V = V_0 \ln \frac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with *n*, *n* being the principal quantum number

[IIT-JEE (Screening) 2003]

(a) $r_n \propto n$ (b) $r_n \propto 1/n$ (c) $r_n \propto n^2$ (d) $r_n \propto 1/n^2$

Solution : (a) Potential energy $U = eV = eV_0 \ln \frac{r}{r_0}$

$\therefore$ Force $F = -\left|\frac{dU}{dr}\right| = \frac{eV_0}{r}$. The force will provide the necessary centripetal force. Hence

$\frac{mv^2}{r} = \frac{eV_0}{r} \Rightarrow v = \sqrt{\frac{eV_0}{m}}$ .....(i) and $mvr = \frac{nh}{2\pi}$ .....(ii)

Dividing equation (ii) by (i) we have $mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}}$ or $r \propto n$

***Example*: 3** The innermost orbit of the hydrogen atom has a diameter 1.06 $Å$. The diameter of tenth orbit is

[UPSEAT 2002]

(a) 5.3 $Å$ (b) 10.6 $Å$ (c) 53 $Å$ (d) 106 $Å$

Solution : (d) Using $r \propto n^2 \Rightarrow \frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2$ or $\frac{d_2}{d_1} = \left(\frac{n_2}{n_1}\right)^2 \Rightarrow \frac{d_2}{1.06} = \left(\frac{10}{1}\right)^2 \Rightarrow d = 106\,Å$

***Example*: 4** Energy of the electron in n^{th} orbit of hydrogen atom is given by $E_n = -\frac{13.6}{n^2} eV$. The amount of energy needed to transfer electron from first orbit to third orbit is

(a) 13.6 *eV* (b) 3.4 *eV* (c) 12.09 *eV* (d) 1.51 *eV*

Solution : (c) Using $E = -\frac{13.6}{n^2} eV$

For $n = 1$, $E_1 = \frac{-13.6}{1^2} = -13.6\, eV$ and for $n = 3$ $E_3 = -\frac{13.6}{3^2} = -1.51\, eV$

So required energy $= E_3 - E_1 = -1.51 - (-13.6) = 12.09\, eV$

***Example*: 5** If the binding energy of the electron in a hydrogen atom is 13.6 *eV*, the energy required to remove the electron from the first excited state of Li^{++} is **[AIEEE 2003]**

(a) 122.4 *eV* (b) 30.6 *eV* (c) 13.6 *eV* (d) 3.4 *eV*

Solution : (b) Using $E_n = -\frac{13.6 \times Z^2}{n^2} eV$

For first excited state *n* = 2 and for $Li^{++}, Z = 3$

$\therefore E = -\frac{13.6}{2^2} \times 3^2 = -\frac{13.6 \times 9}{4} = -30.6\, eV$. Hence, remove the electron from the first excited state of Li^{++} be 30.6 *eV*

***Example*: 6** The ratio of the wavelengths for 2 → 1 transition in Li^{++}, He^{+} and *H* is **[UPSEAT 2003]**

(a) 1 : 2 : 3 (b) 1 : 4 : 9 (c) 4 : 9 : 36 (d) 3 : 2 : 1

Solution : (c) Using $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \lambda \propto \frac{1}{Z^2} \Rightarrow \lambda_{Li} : \lambda_{He^+} : \lambda_H = \frac{1}{9} : \frac{1}{4} : \frac{1}{1} = 4 : 9 : 36$

***Example*: 7** Energy *E* of a hydrogen atom with principal quantum number *n* is given by $E = \frac{-13.6}{n^2} eV$. The energy of a photon ejected when the electron jumps *n* = 3 state to *n* = 2 state of hydrogen is approximately

[CBSE PMT/PDT Screening 2004]

(a) 1.9 eV (b) 1.5 eV (c) 0.85 eV (d) 3.4 eV

Solution : (a) $\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{36} = 1.9\ eV$

***Example*: 8** In the Bohr model of the hydrogen atom, let R, v and E represent the radius of the orbit, the speed of electron and the total energy of the electron respectively. Which of the following quantity is proportional to the quantum number n **[KCET 2002]**

(a) R/E (b) E/v (c) RE (d) vR

Solution : (d) Rydberg constant $R = \frac{\varepsilon_0 n^2 h^2}{\pi m Ze^2}$

Velocity $v = \frac{Ze^2}{2\varepsilon_0 nh}$ and energy $E = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2}$

Now, it is clear from above expressions $R.v \propto n$

***Example*: 9** The energy of hydrogen atom in nth orbit is E_n, then the energy in nth orbit of singly ionised helium atom will be

(a) $4E_n$ (b) $E_n/4$ (c) $2E_n$ (d) $E_n/2$

Solution : (a) By using $E = -\frac{13.6\,Z^2}{n^2} \Rightarrow \frac{E_H}{E_{He}} = \left(\frac{Z_H}{Z_{He}}\right)^2 = \left(\frac{1}{2}\right)^2 \Rightarrow E_{He} = 4E_n$.

***Example*: 10** The wavelength of radiation emitted is λ_0 when an electron jumps from the third to the second orbit of hydrogen atom. For the electron jump from the fourth to the second orbit of the hydrogen atom, the wavelength of radiation emitted will be **[SCRA 1998; MP PET 2001]**

(a) $\frac{16}{25}\lambda_0$ (b) $\frac{20}{27}\lambda_0$ (c) $\frac{27}{20}\lambda_0$ (d) $\frac{25}{16}\lambda_0$

Solution : (b) Wavelength of radiation in hydrogen atom is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \frac{1}{\lambda_0} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5}{36}R \quad .....(i)$$

and $$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = R\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3R}{16} \quad .....(ii)$$

From equation (i) and (ii) $\frac{\lambda'}{\lambda} = \frac{5R}{36} \times \frac{16}{3R} = \frac{20}{27} \Rightarrow \lambda' = \frac{20}{27}\lambda_0$

***Example*: 11** If scattering particles are 56 for 90^o angle then this will be at 60^o angle **[RPMT 2000]**

(a) 224 (b) 256 (c) 98 (d) 108

Solution : (a) Using Scattering formula

$$N \propto \frac{1}{\sin^4(\theta/2)} \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_2}{2}\right)}\right]^4 \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin\left(\frac{90^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}\right]^4 = \left[\frac{\sin 45^\circ}{\sin 30^\circ}\right]^4 = 4 \Rightarrow N_2 = 4N_1 = 4 \times 56 = 224$$

***Example*: 12** When an electron in hydrogen atom is excited, from its 4th to 5th stationary orbit, the change in angular momentum of electron is (Planck's constant: $h = 6.6 \times 10^{-34}$ J–s) **[AFMC 2000]**

(a) 4.16×10^{-34} J- s (b) 3.32×10^{-34} J-s (c) 1.05×10^{-34} J-s (d) 2.08×10^{-34} J-s

Solution : (c) Change in angular momentum

$$\Delta L = L_2 - L_1 = \frac{n_2 h}{2\pi} - \frac{n_1 h}{2\pi} \Rightarrow \Delta L = \frac{h}{2\pi}(n_2 - n_1) = \frac{6.6 \times 10^{-34}}{2 \times 3.14}(5-4) = 1.05 \times 10^{-34}\ J\text{-}s$$

***Example*: 13** In hydrogen atom, if the difference in the energy of the electron in n = 2 and n = 3 orbits is E, the ionization energy of hydrogen atom is

(a) 13.2 E (b) 7.2 E (c) 5.6 E (d) 3.2 E

Solution : (b) Energy difference between $n = 2$ and $n = 3$; $E = K\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = K\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}K$ .....(i)

Ionization energy of hydrogen atom $n_1 = 1$ and $n_2 = \infty$; $E' = K\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = K$ .....(ii)

From equation (i) and (ii) $E' = \frac{36}{5}E = 7.2\,E$

***Example*: 14** In Bohr model of hydrogen atom, the ratio of periods of revolution of an electron in $n = 2$ and $n = 1$ orbits is **[EAMCET (Engg.) 2000]**

(a) 2 : 1 (b) 4 : 1 (c) 8 : 1 (d) 16 : 1

Solution : (c) According to Bohr model time period of electron $T \propto n^3 \Rightarrow \frac{T_2}{T_1} = \frac{n_2^3}{n_1^3} = \frac{2^3}{1^3} = \frac{8}{1} \Rightarrow T_2 = 8T_1$.

***Example*: 15** A double charged lithium atom is equivalent to hydrogen whose atomic number is 3. The wavelength of required radiation for emitting electron from first to third Bohr orbit in Li^{++} will be (Ionisation energy of hydrogen atom is 13.6 eV)

(a) 182.51 $Å$ (b) 177.17 $Å$ (c) 142.25 $Å$ (d) 113.74 $Å$

Solution : (d) Energy of a electron in nth orbit of a hydrogen like atom is given by

$E_n = -13.6\frac{Z^2}{n^2}eV$, and $Z = 3$ for Li

Required energy for said transition

$$\Delta E = E_3 - E_1 = 13.6Z^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 13.6 \times 3^2\left[\frac{8}{9}\right] = 108.8\,eV = 108.8 \times 1.6 \times 10^{-19}\,J$$

Now using $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} \Rightarrow \lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{108.8 \times 1.6 \times 10^{-19}} = 0.11374 \times 10^{-7}\,m \Rightarrow \lambda = 113.74\,Å$

***Example*: 16** The absorption transition between two energy states of hydrogen atom are 3. The emission transitions between these states will be

(a) 3 (b) 4 (c) 5 (d) 6

Solution : (d) Number of absorption lines = $(n - 1) \Rightarrow 3 = (n - 1) \Rightarrow n = 4$

Hence number of emitted lines $= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

***Example*: 17** The energy levels of a certain atom for 1st, 2nd and 3rd levels are E, $4E/3$ and $2E$ respectively. A photon of wavelength λ is emitted for a transition $3 \to 1$. What will be the wavelength of emissions for transition $2 \to 1$

[CPMT 1996]

(a) $\lambda/3$ (b) $4\lambda/3$ (c) $3\lambda/4$ (d) 3λ

Solution : (d) For transition $3 \to 1$ $\Delta E = 2E - E = \frac{hc}{\lambda} \Rightarrow E = \frac{hc}{\lambda}$ .....(i)

For transition $2 \to 1$ $\frac{4E}{3} - E = \frac{hc}{\lambda'} \Rightarrow E = \frac{3hc}{\lambda'}$ .....(ii)

From equation (i) and (ii) $\lambda' = 3\lambda$

***Example*: 18** Hydrogen atom emits blue light when it changes from $n = 4$ energy level to $n = 2$ level. Which colour of light would the atom emit when it changes from $n = 5$ level to $n = 2$ level **[KCET 1993]**

(a) Red (b) Yellow (c) Green (d) Violet

Solution : (d) In the transition from orbits $5 \to 2$ more energy will be liberated as compared to transition from $4 \to 2$. So emitted photon would be of violet light.

***Example*: 19** A single electron orbits a stationary nucleus of charge $+Ze$, where Z is a constant. It requires 47.2 eV to excited electron from second Bohr orbit to third Bohr orbit. Find the value of Z **[IIT-JEE 1981]**

(a) 2 (b) 5 (c) 3 (d) 4

Solution : (b) Excitation energy of hydrogen like atom for $n_2 \to n_1$

$$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)eV \Rightarrow 47.2 = 13.6Z^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6 \times \frac{5}{36}Z^2 \Rightarrow Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 24.98 \simeq 25$$

$\Rightarrow Z = 5$

***Example*: 20** The first member of the Paschen series in hydrogen spectrum is of wavelength 18,800 Å. The short wavelength limit of Paschen series is **[EAMCET (Med.) 2000]**

(a) 1215 Å (b) 6560 Å (c) 8225 Å (d) 12850 Å

Solution : (c) First member of Paschen series mean it's $\lambda_{\max} = \frac{144}{7R}$

Short wavelength of Paschen series means $\lambda_{\min} = \frac{9}{R}$

Hence $\frac{\lambda_{\max}}{\lambda_{\min}} = \frac{16}{7} \Rightarrow \lambda_{\min} = \frac{7}{16} \times \lambda_{\max} = \frac{7}{16} \times 18{,}800 = 8225\,Å$.

***Example*: 21** Ratio of the wavelengths of first line of Lyman series and first line of Balmer series is

[EAMCET (Engg.) 1995; MP PMT 1997]

(a) 1 : 3 (b) 27 : 5 (c) 5 : 27 (d) 4 : 9

Solution : (c) For Lyman series $\frac{1}{\lambda_{L_1}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$ .....(i)

For Balmer series $\frac{1}{\lambda_{B_1}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$ .....(ii)

From equation (i) and (ii) $\frac{\lambda_{L_1}}{\lambda_{B_1}} = \frac{5}{27}$.

***Example*: 22** The third line of Balmer series of an ion equivalent to hydrogen atom has wavelength of 108.5 nm. The ground state energy of an electron of this ion will be **[RPET 1997]**

(a) 3.4 eV (b) 13.6 eV (c) 54.4 eV (d) 122.4 eV

Solution : (c) Using $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{108.5 \times 10^{-9}} = 1.1 \times 10^7 \times Z^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$

$$\Rightarrow \frac{1}{108.5 \times 10^{-9}} = 1.1 \times 10^7 \times Z^2 \times \frac{21}{100} \Rightarrow Z^2 = \frac{100}{108.5 \times 10^{-9} \times 1.1 \times 10^{-7} \times 21} = 4 \Rightarrow Z = 2$$

Now Energy in ground state $E = -13.6Z^2\ eV = -13.6 \times 2^2\ eV = -54.4\ eV$

***Example*: 23** Hydrogen (H), deuterium (D), singly ionized helium (He^+) and doubly ionized lithium (Li^{++}) all have one electron around the nucleus. Consider $n = 2$ to $n = 1$ transition. The wavelengths of emitted radiations are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively. Then approximately **[KCET 1994]**

(a) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (b) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (c) $\lambda_1 = 2\lambda_2 = 2\sqrt{2}\lambda_3 = 3\sqrt{2}\lambda_4$ (d) $\lambda_1 = \lambda_2 = 2\lambda_3 = 3\lambda_4$

Solution : (a) Using $\Delta E \propto Z^2$ ($\because$ n_1 and n_2 are same)

$$\Rightarrow \frac{hc}{\lambda} \propto Z^2 \Rightarrow \lambda Z^2 = \text{constant} \Rightarrow \lambda_1 Z_1^2 = \lambda_2 Z_2^2 = \lambda_3 Z_3^2 = \lambda_4 Z^4 \Rightarrow \lambda_1 \times 1 = \lambda_2 \times 1^2 = \lambda_3 \times 2^2 = \lambda_4 \times 3^3$$

$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$.

***Example*: 24** Hydrogen atom in its ground state is excited by radiation of wavelength 975 $Å$. How many lines will be there in the emission spectrum

(a) 2 (b) 4 (c) 6 (d) 8

Solution : (c) Using $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \frac{1}{975 \times 10^{-10}} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow n = 4$

Now number of spectral lines $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$.

***Example*: 25** A photon of energy 12.4 *eV* is completely absorbed by a hydrogen atom initially in the ground state so that it is excited. The quantum number of the excited state is

(a) $n = 1$ (b) $n = 3$ (c) $n = 4$ (d) $n = \infty$

Solution : (c) Let electron absorbing the photon energy reaches to the excited state n. Then using energy conservation

$\Rightarrow -\frac{13.6}{n^2} = -13.6 + 12.4 \Rightarrow -\frac{13.6}{n^2} = -1.2 \Rightarrow n^2 = \frac{13.6}{1.2} = 12 \Rightarrow n = 3.46 \simeq 4$

***Example*: 26** The wave number of the energy emitted when electron comes from fourth orbit to second orbit in hydrogen is 20,397 cm^{-1}. The wave number of the energy for the same transition in He^+ is

(a) 5,099 cm^{-1} (b) 20,497 cm^{-1} (c) 40,994 cm^{-1} (d) 81,998 cm^{-1}

Solution : (d) Using $\frac{1}{\lambda} = \bar{\nu} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \bar{\nu} \propto Z^2 \Rightarrow \frac{\bar{\nu}_2}{\bar{\nu}_1} = \left(\frac{Z_2}{Z_1}\right)^2 = \left(\frac{Z}{1}\right)^2 = 4 \Rightarrow \bar{\nu}_2 = \bar{\nu} \times 4 = 81588\ cm^{-1}$.

***Example*: 27** In an atom, the two electrons move round the nucleus in circular orbits of radii R and $4R$. the ratio of the time taken by them to complete one revolution is

(a) 1/4 (b) 4/1 (c) 8/1 (d) 1/8

Solution : (d) Time period $T \propto \frac{n^3}{Z^2}$

For a given atom (Z = constant) So $T \propto n^3$ .....(i) and radius $R \propto n^2$ .....(ii)

$\therefore$ From equation (i) and (ii) $T \propto R^{3/2} \Rightarrow \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2} = \frac{1}{8}$.

***Example*: 28** Ionisation energy for hydrogen atom in the ground state is E. What is the ionisation energy of Li^{++} atom in the 2nd excited state

(a) E (b) $3E$ (c) $6E$ (d) $9E$

Solution : (a) Ionisation energy of atom in nth state $E_n = \frac{Z^2}{n^2}$

For hydrogen atom in ground state (n = 1) and Z = 1 $\Rightarrow E = E_0$ .....(i)

For Li^{++} atom in 2nd excited state n = 3 and Z = 3, hence $E' = \frac{E_0}{3^2} \times 3^2 = E_0$ .....(ii)

From equation (i) and (ii) $E' = E$.

***Example*: 29** An electron jumps from n = 4 to n = 1 state in H-atom. The recoil momentum of H-atom (in eV/C) is

(a) 12.75 (b) 6.75 (c) 14.45 (d) 0.85

Solution : (a) The H-atom before the transition was at rest. Therefore from conservation of momentum

Photon momentum = Recoil momentum of H-atom or $P_{recoil} = \frac{h\nu}{c} = \frac{E_4 - E_1}{c} = \frac{-0.85eV - (-13.6\ eV)}{c} = 12.75\frac{eV}{c}$

***Example*: 30** If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be

[IIT-JEE 1983; CBSE PMT 1991, 93; MP PET 1999; RPET 1993, 2001; RPMT 1999, 2003; J & K CET 2004]

(a) 60 (b) 32 (c) 4 (d) 64

Solution : (a) Maximum value of n = 4

So possible (maximum) no. of elements

$$N = 2\times1^2 + 2\times2^2 + 2\times3^2 + 2\times4^2 = 2+8+18+32 = 60\,.$$

Tricky example: 1

If the atom ${}_{100}Fm^{257}$ follows the Bohr model and the radius of ${}_{100}Fm^{257}$ is n times the Bohr radius, then find n

[IIT-JEE (Screening) 2003]

(a) 100 (b) 200 (c) 4 (d) 1/4

Solution : (d) $(r_m) = \left(\frac{m^2}{Z}\right)(0.53\,Å) = (n \times 0.53\,Å) \Rightarrow \frac{m^2}{Z} = n$

m = 5 for ${}_{100}Fm^{257}$ (the outermost shell) and z = 100

$\therefore \quad n = \frac{(5)^2}{100} = \frac{1}{4}$

Tricky example: 2

An energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy (in eV) required to remove both the electrons from a neutral helium atom is

(a) 79.0 (b) 51.8 (c) 49.2 (d) 38.2

Solution : (a) After the removal of first electron remaining atom will be hydrogen like atom.

So energy required to remove second electron from the atom $E = 13.6 \times \frac{2^2}{1} = 54.4\,eV$

$\therefore$ Total energy required = 24.6 + 54.4 = 79 eV