JEE MAIN | IIT JEE

Circular Motion

Horizontal & Vertical CM

REVISION in 55 Min

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EDUNITI

Topics to covered

1. Kinematics of Circular Motion
2. Radius of Curvature
3. Relative Angular Velocity
4. Uniform vs Non-Uniform Circular Motion
5. Non-Uniform C.M. ($\alpha = Const.$)
6. Non-Uniform C.M. ($\alpha \neq Const.$)
7. Centripetal Force
8. Conical Pendulum
9. Centrifugal Force
10. Dynamics of Vertical C.M.
11. Vertical C.M. of a Pendulum Bob (string)
12. Vertical C.M of a Light Rod Pendulum
13. Vertical C.M. Inside a Circular Track
14. Vertical C.M. Outside a Circular Track
15. Vertical C.M. in a Circular Tube
16. Banked Roads

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Chapter	Formulae_Concept VIDEO LINK
Unit & Dimensions	https://youtu.be/wdd-wlZF4Hk
Errors and Vectors	
Vernier Calliper	https://youtu.be/pVoN045dV8I
Screw Gauge	https://youtu.be/gYd2PtmZ0mw
Kinematics_Motion in 1d	https://youtu.be/U4NNxFaFliE
Kinematics_Motion in 2d	https://youtu.be/4_Zo5WhMf7w
Laws of Motion	https://youtu.be/7JlR8gNRQIs
Friction	https://youtu.be/Rn1bLst7eGk
Work Energy Power	https://youtu.be/kjrXoE-kDI8
Centre of Mass	
Centre of Mass of Standard Bodies	https://youtu.be/oCeACfryB-U
Collision	
Rotational Motion_Moment of Inertia	https://youtu.be/9ckZdOhy3z0
Gravitation	https://youtu.be/rAj2huLVaEk
Properties of Solids	https://youtu.be/gSXxjk89I_c
Fluids Statics (Part 1)	https://youtu.be/RFKx9B9yo3M
Fluid Dynamics (Part 2)	https://youtu.be/Y717vQpUEJQ
Fluid Properties (Part 3)	https://youtu.be/V8xUWWK2oT0
Simple Harmonic Motion	https://youtu.be/RIb7ofNG09I
Thermal Properties	
KTG	https://youtu.be/XO1tvFhIa0I
Thermodynamics	https://youtu.be/iz_kf1jRDRw
Wave Motion -Organ Pipes and Resonance Tube	https://youtu.be/fB7pfJ77za8
Wave Motion - Doppler's Effect	https://youtu.be/9-BxOaamnwg
Electrostatics	https://youtu.be/3stXbGRMcrk
Capacitors	https://youtu.be/EXEiickNUkY
Current Electricity	https://youtu.be/gm8FUfjrX18
Moving Charges and Magnetic Effect of Current	https://youtu.be/ULD2Ok1CGJk
Earth's Magnetism	https://youtu.be/a4CT5uVwAK4
Magnetic Properties	https://youtu.be/63cwdYXNlYE
EMI	https://youtu.be/puVavm_GFRM
Alternating Current	https://youtu.be/74dTY-pzM_o
Ray Optics	https://youtu.be/BhnyTWzIlBA
Wave Optics Part 1_Interference	https://youtu.be/LG5nlE8XTeI
Wave Optics Part 2_Diffraction_Polarization	https://youtu.be/ymMyyJGGqnY
Optical Instruments	https://youtu.be/OQssbDH0A4I
Electromagnetic Waves	https://youtu.be/bcVXgEkyQZY
Semiconductors_Basics + Zener Diode	https://youtu.be/_A2JomQ7-50
Semiconductors_Transistors	https://youtu.be/psDwl84Nzb0
Semiconductors_Logic Gates	https://youtu.be/pZdQAzLbFTo
Communication Systems	https://youtu.be/8NgMqK9X79Y
Modern Physics_Part 1_Atomic Physics	https://youtu.be/9VKUnE3mpHk
Modern Physics_Part 2_Photoelectric Effect	https://youtu.be/24oTQp84jrk
Modern Physics_Part 3_Dual Nature of Light	https://youtu.be/0zoR_saMAQY
Modern Physics_Part 4_Radioactivity	https://youtu.be/AdX3YBhQyog
Modern Physics_Part 5_Nuclear Physics	https://youtu.be/VDWqVahGixc
Modern Physics_Part 6_X Rays	https://youtu.be/dSHXdzX7NX0

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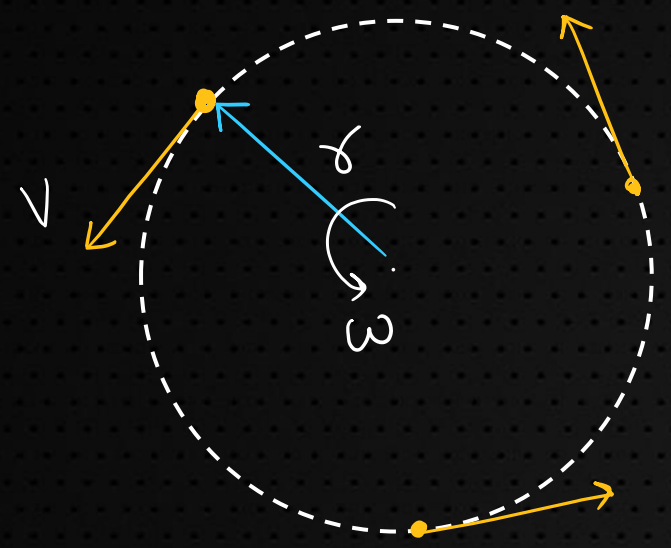
1. Kinematics of Circular Motion



(a) → Angular displacement, $\Delta\theta$ (in rad)

→ Angular velocity, ω (rad/s)

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad \omega_{inst} = \frac{d\theta}{dt}$$

→ Angular acc^n, α (rad/s^2) $= \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$

(b) Relation between V and ω : $V = \omega r$

(c) $\frac{dV}{dt} = r\frac{d\omega}{dt} \Rightarrow a_t = r\alpha$ { a_t changes speed of particle

(d) ∵ dir^n changing, $a_n = \frac{V^2}{r} = \omega^2 r$ { normal or centripetal acc^n

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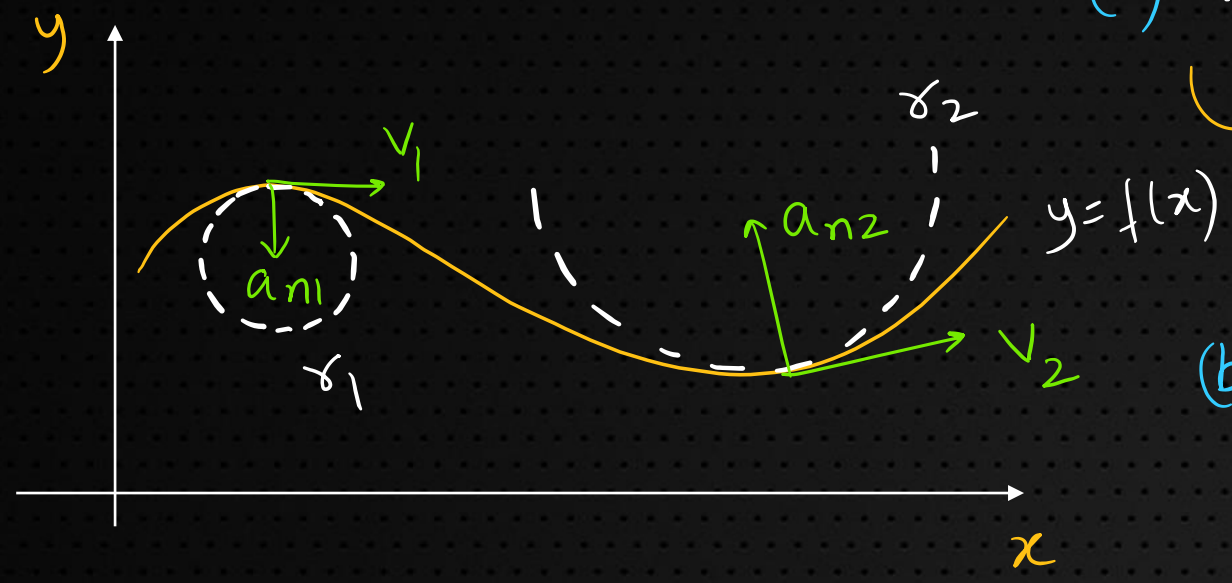
2. Radius of Curvature



(a) $r_1 = v_1^2 / a_{n1}$, $r_2 = v_2^2 / a_{n2}$

↳ here a_{n1} & a_{n2} are acc^n $\perp$ to v_1 & v_2

(b) At (x_1, y_1), radius of curvature is

$$r_c = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \Bigg/ \left|\frac{d^2y}{dx^2}\right| \Bigg|_{x_1, y_1}$$

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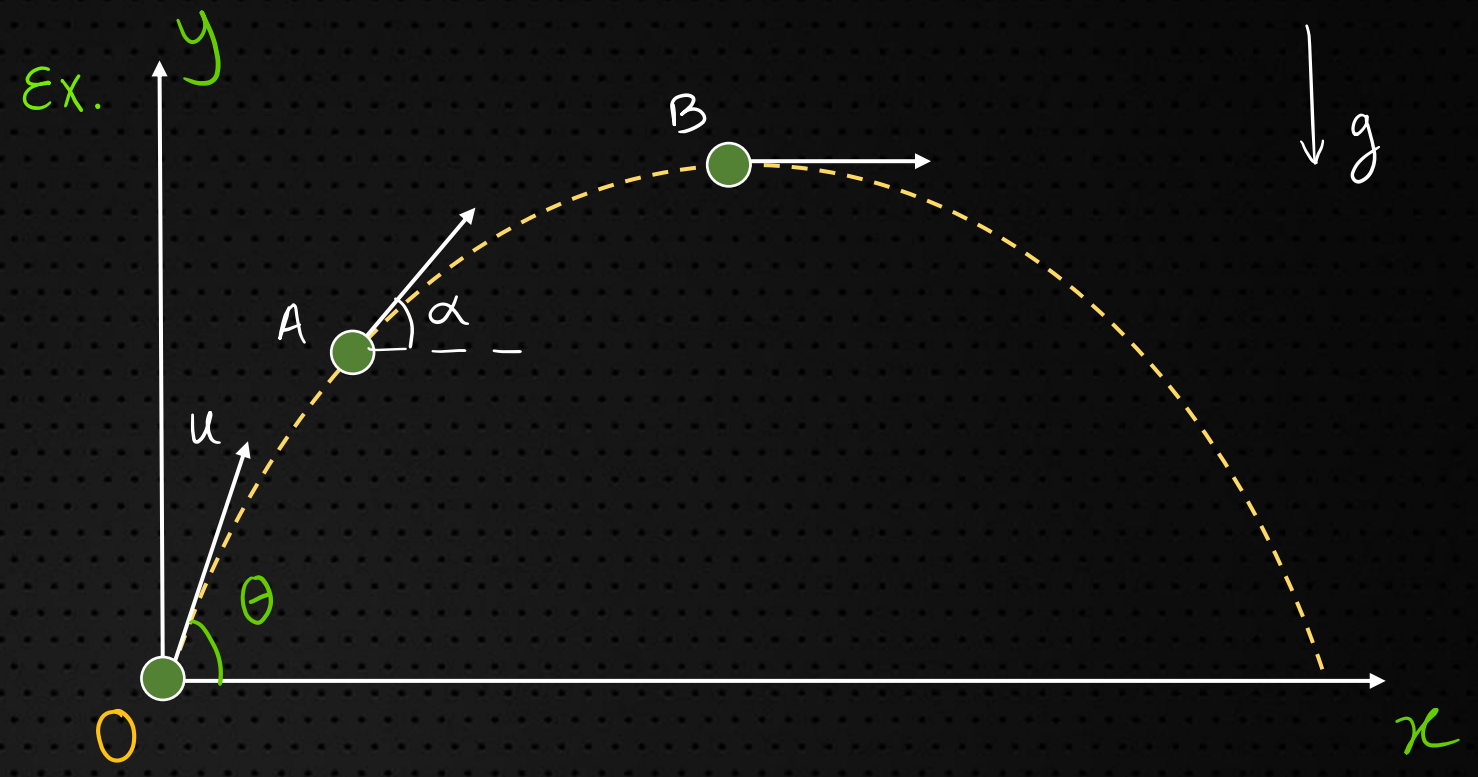
Find r_A & r_B ?



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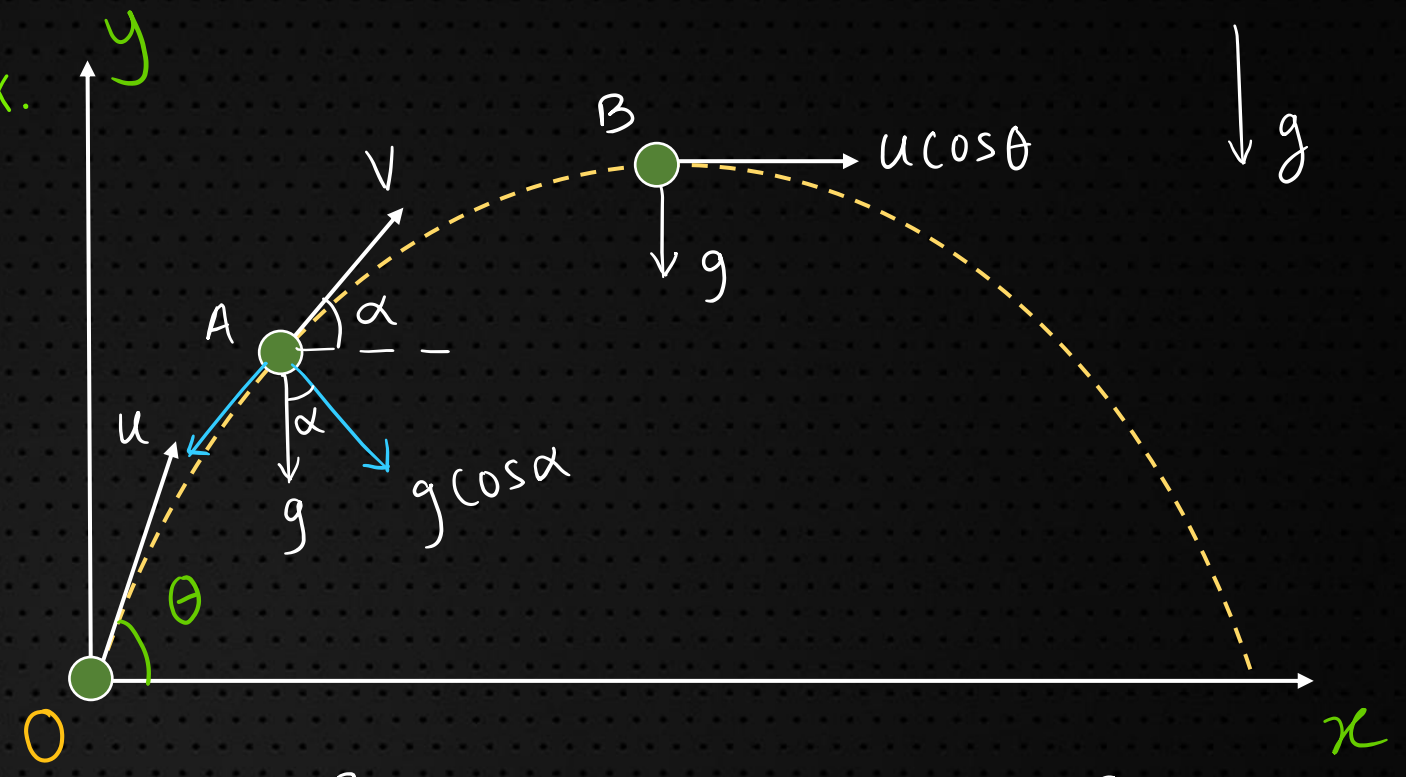
Find r_A & r_B ?

Sol^n: $r_B = \dfrac{u^2\cos^2\theta}{g}$

$V\cos\alpha = u\cos\theta \Rightarrow V = \dfrac{u\cos\theta}{\cos\alpha}$

Ex.



$$\therefore r_A = \frac{V^2}{a_n} = \frac{u^2\cos^2\theta}{\cos^2\alpha} \times \frac{1}{g\cos\alpha} = \frac{u^2\cos^2\theta}{g\cos^3\alpha}$$

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3. Relative Angular Velocity

$$\omega_{BA} = \frac{V_{rel}}{r} = \frac{V_B - V_A}{r}$$

$$\omega_{BA} = \frac{V_{rel}}{r} = \frac{V_B + V_A}{r}$$

Note:

$$\omega_{AB} = \frac{V_B \sin\theta_2 - V_A \sin\theta_1}{r}$$

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https://youtu.be/AcYJyL8VtMo

Q 7.

Ex. Two satellites revolve around a planet in coplanar circular orbits in anticlockwise direction. Their period of revolutions are 1 hour and 8 hours respectively. The radius of the orbit of nearer satellite is 2×10^3 km. The angular speed of the farther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x}$ rad h^{-1} where x is .....

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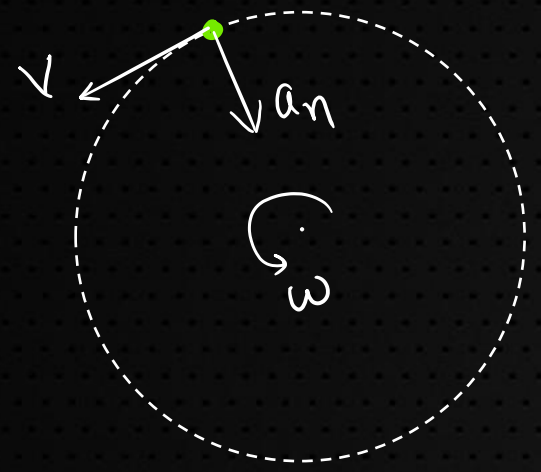
4. Uniform vs Non-uniform Circular Motion



Uniform

$\omega = \text{const.}$

v is const.

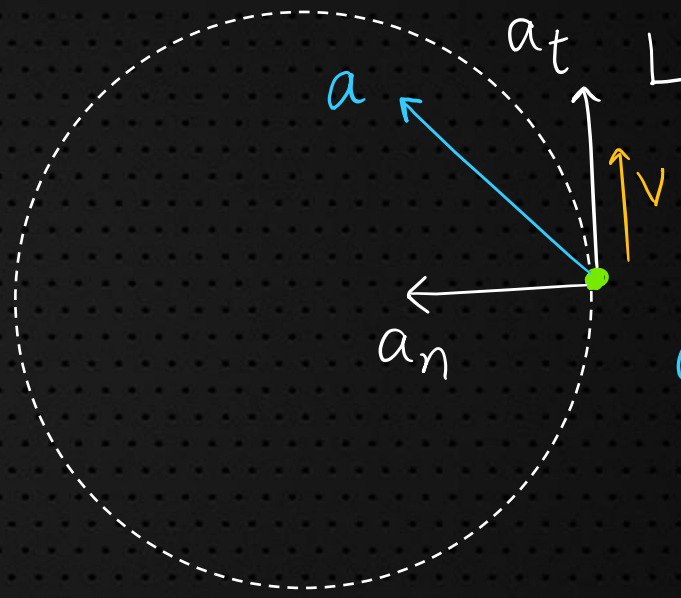
$\alpha = 0, \quad a_t = 0$

$$a_n = \frac{v^2}{r} = \omega^2 r$$

Non-Uniform

$\omega \neq \text{const.}$

↳ speed & dirn both changes



$$a = \sqrt{a_t^2 + a_n^2}$$

$a_t \rightarrow \alpha r$, $\quad a_n \rightarrow \frac{v^2}{r}$

$\alpha \neq 0$

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5. Non-Uniform CM (α = Const.)

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta_n = \omega_i + \frac{\alpha}{2}(2n-1)$$

Note:

(i) If both ω_i and α are in same sense (CW or ACW), take α +ve.

(ii) If α opp sense of ω_i, α -ve (retardation, Ex- fan switching off)

(iii) Be carefull about units

ω in rpm $\longrightarrow$ ω in rad/s

Ex. 50 rpm $\longrightarrow$ $\frac{50 \times 2\pi}{60}$ rad/s

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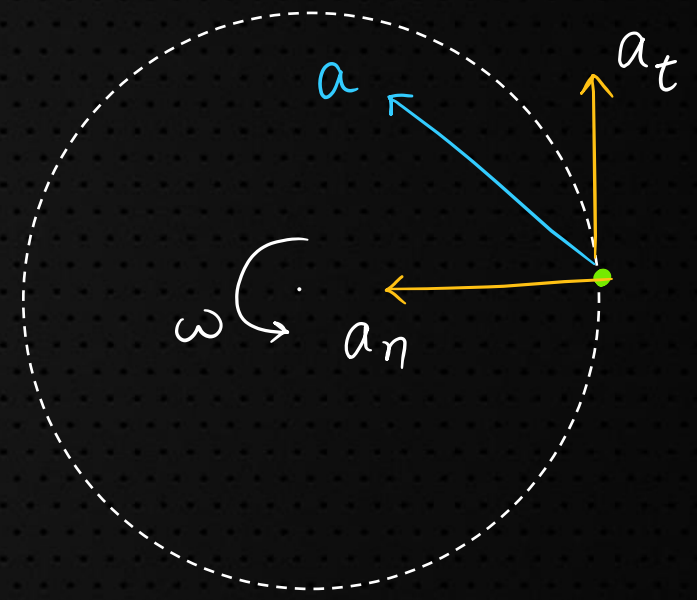
6. Non-uniform CM ($\alpha \neq$ const.)

$\hookrightarrow \omega = d\theta/dt, \quad \alpha = \dfrac{d\omega}{dt} = \omega \dfrac{d\omega}{d\theta}$

Ex. $\alpha = 2t$ and particle starts from rest. At $t = 3s$ find a ? ($r = 2m$)

Sol^n: At $t = 3s$, $\alpha = 2 \times 3 = 6\ rad/s^2$

$a_t = \alpha r = 6 \times 2 = 12\ m/s^2$



$\alpha = 2t \Rightarrow \dfrac{d\omega}{dt} = 2t$

$\Rightarrow \int_0^{\omega} d\omega = \int_0^{3} 2t\, dt \Rightarrow \omega = 9\ rad/s$

$a_n = \omega^2 r = 81 \times 2 = 162\ m/s^2$

$\therefore a = \sqrt{12^2 + 162^2} \approx 162.4\ m/s^2$

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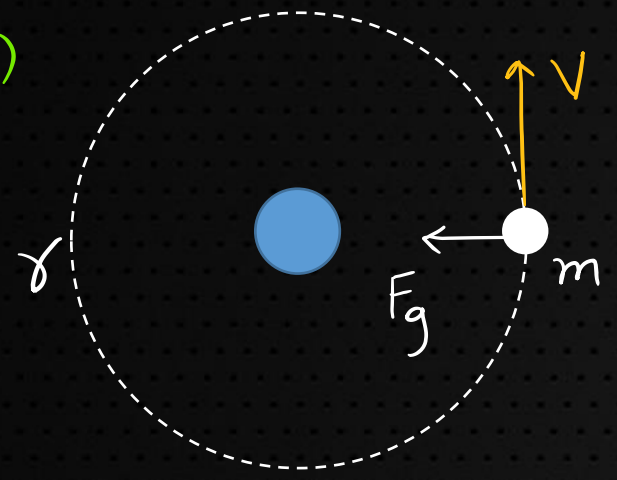
7. Centripetal Force

↳ net force towards centre of circular path, $F_{net} = m a_n = \frac{mv^2}{r}$

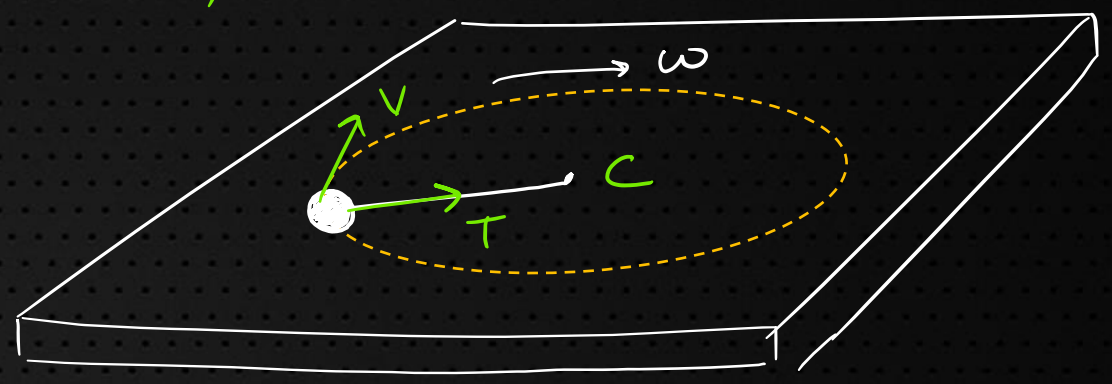
(a)



$$F_g = \frac{mv^2}{r}$$

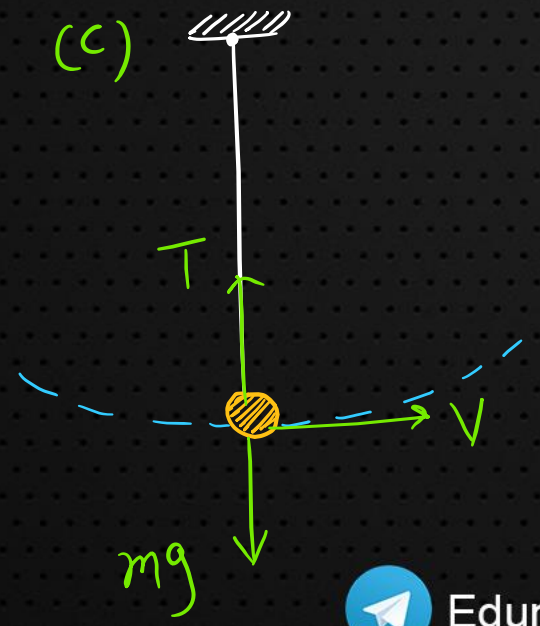
$$\Rightarrow \frac{GMm}{r^2} = \frac{mv^2}{r}$$

(b)



$$T = \frac{mv^2}{l} = m\omega^2 l$$

(c)



$$T - mg = \frac{mv^2}{l}$$

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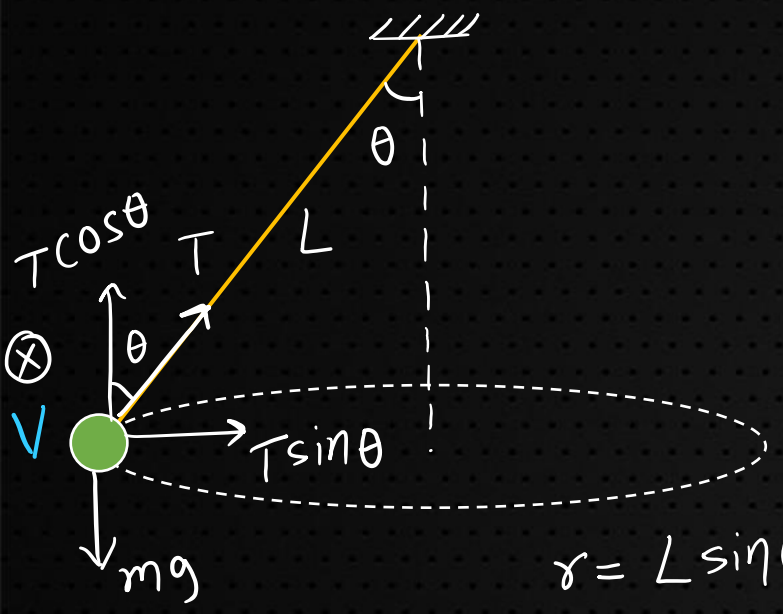
8. Conical Pendulum



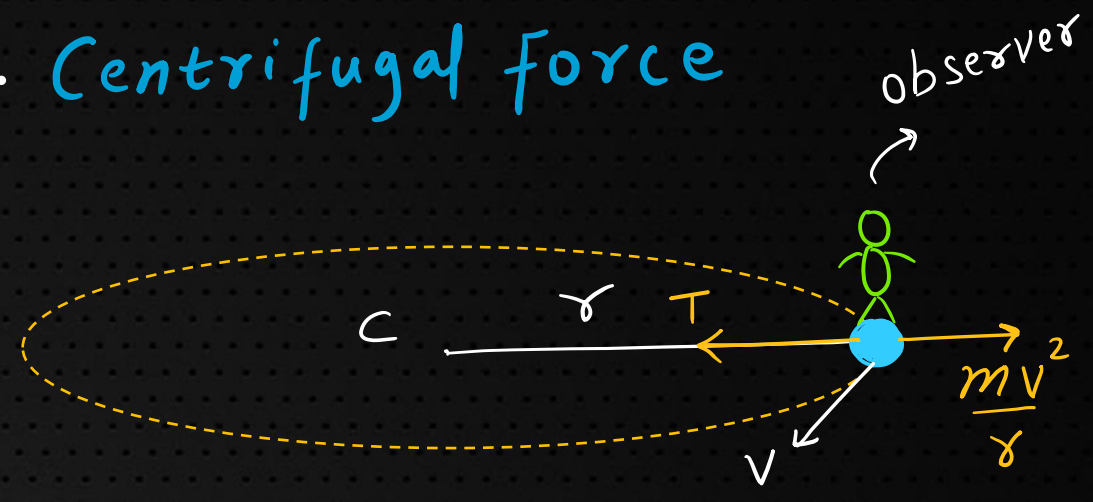
$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

$$\Rightarrow \boxed{\tan\theta = \frac{v^2}{rg}}$$

$$r = L\sin\theta$$

9. Centrifugal Force



W.r.t obs → particle at rest

$$\Rightarrow T = \frac{mv^2}{r}$$

↳ Here $\frac{mv^2}{r}$ is centrifugal force (which is pseudo force)

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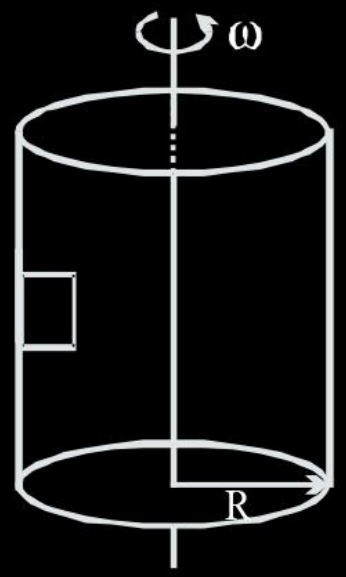
Ex. A hollow cylindrical drum of radius R rotates about the vertical axis. Find the minimum value of its angular velocity ω so that a body kept on inside the wall does not slip down. The coefficient of static friction is μ_s.



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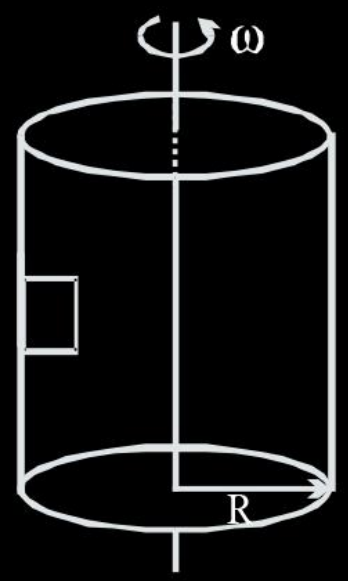
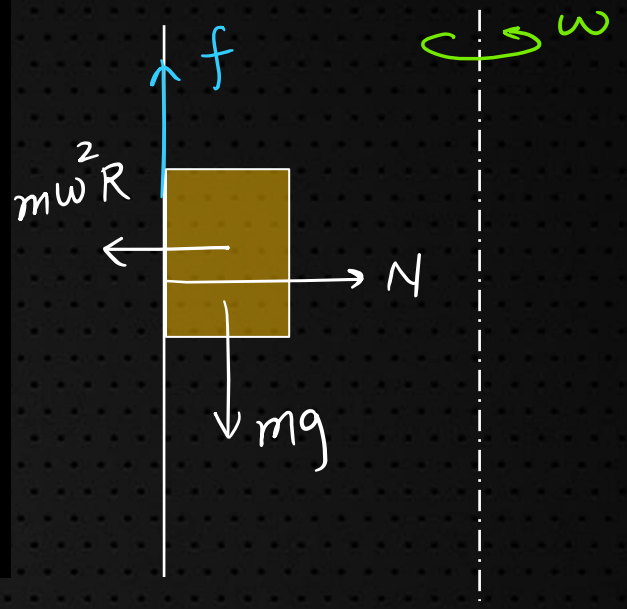


Soln: FBD in rotating frame



$$f_{max} \geq mg$$

$$\Rightarrow \mu_s N \geq mg \Rightarrow \mu_s m\omega^2 R \geq mg$$

$$\Rightarrow \omega \geq \sqrt{\frac{g}{\mu_s R}} \quad \therefore \omega_{min} = \sqrt{\frac{g}{\mu_s R}}$$

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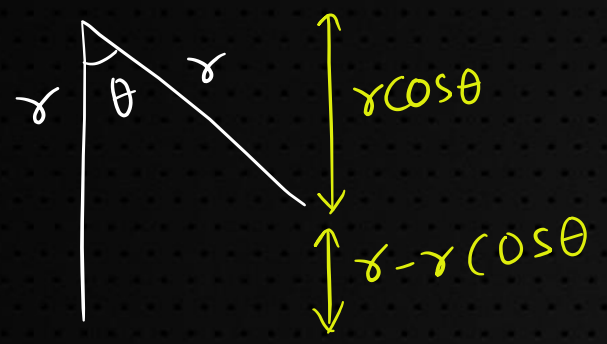
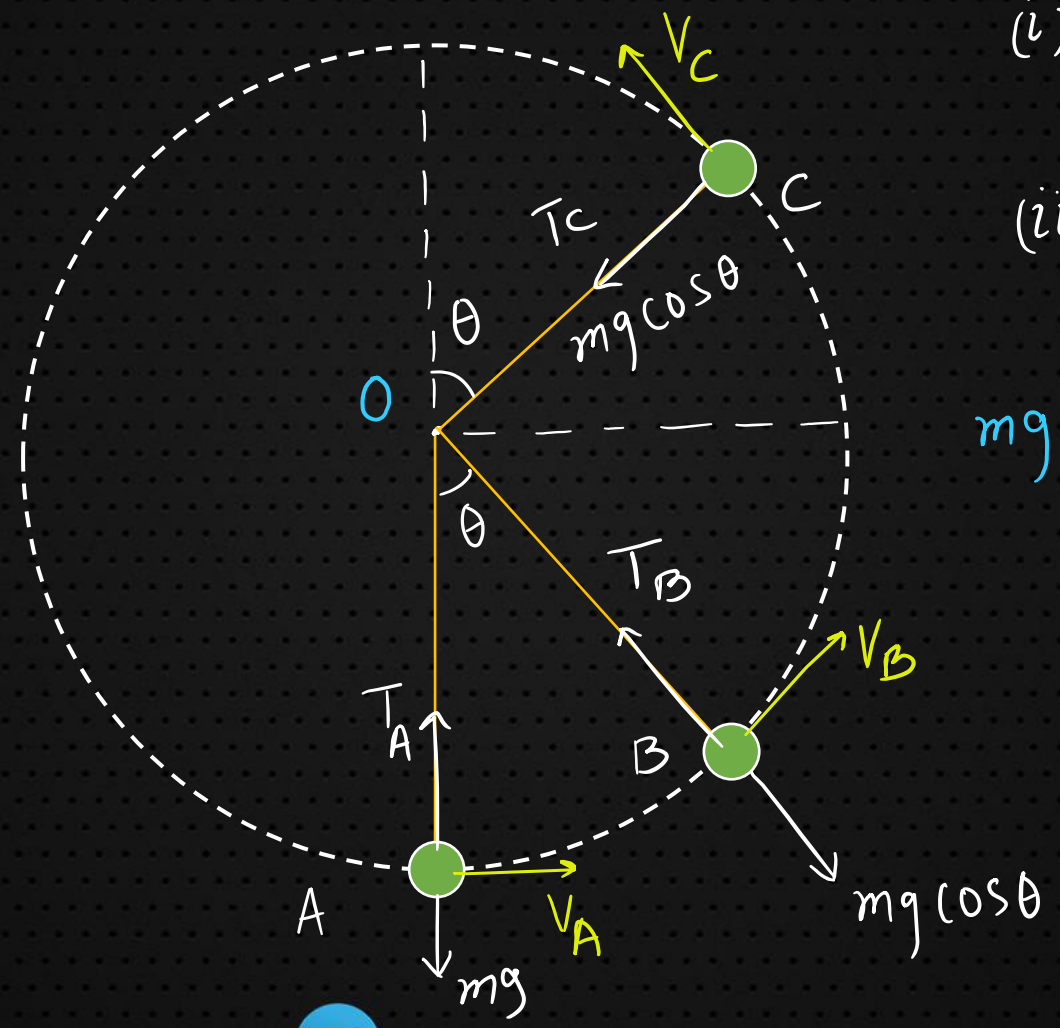
10. Dynamics of Vertical Circular Motion

↳ Particle along circular path in vertical plane





(i) $T_A - mg = \frac{mV_A^2}{r}$

(ii) $T_B - mg\cos\theta = \frac{mV_B^2}{r}$

$mgr(1-\cos\theta) = \frac{1}{2}mV_A^2 - \frac{1}{2}mV_B^2$

(iii) $T_C + mg\cos\theta = \frac{mV_C^2}{r}$

$mgr(1+\cos\theta) = \frac{1}{2}mV_A^2 - \frac{1}{2}mV_C^2$

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11. Vertical C.M. of a Pendulum Bob

(a) $u \leq \sqrt{2gl}$

Gain in P.E. = Loss in K.E

$\Rightarrow \cancel{m}gl(1-\cos\theta) = \frac{1}{2}\cancel{m}u^2$

$\therefore \cos\theta = \frac{2gl-u^2}{2gl}$ $\longrightarrow$ (i) $u = \sqrt{2gl}$

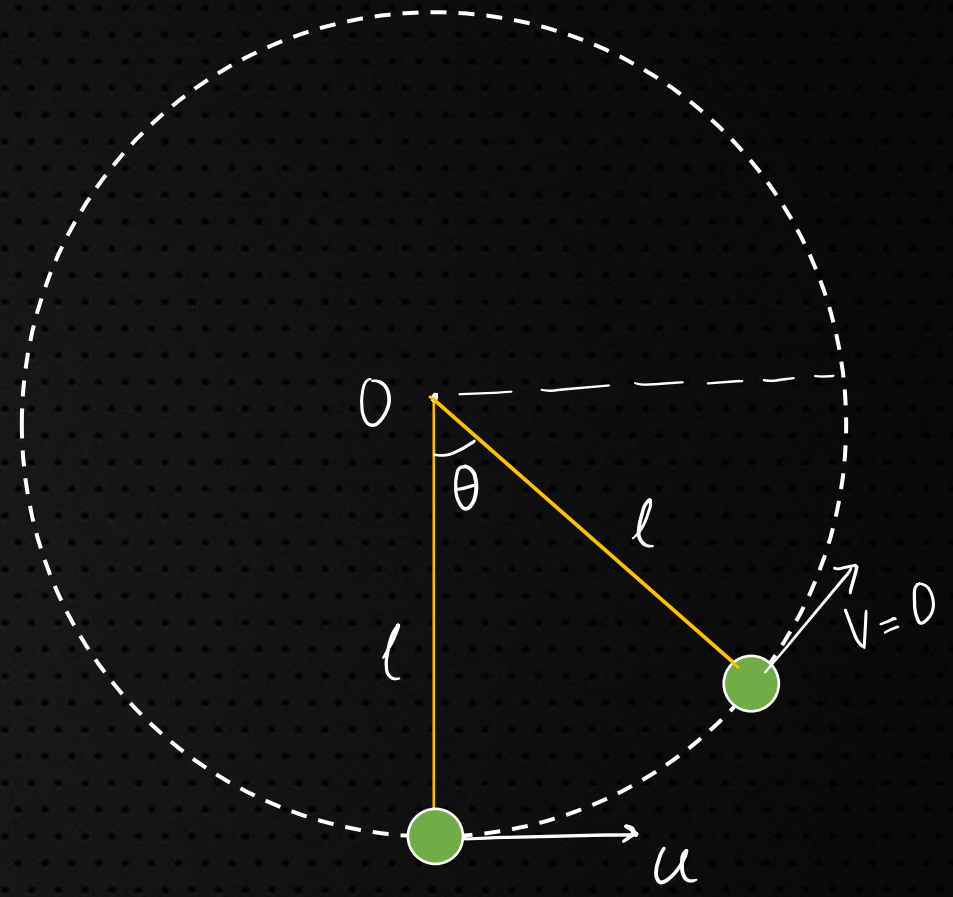
$\Rightarrow \theta = 90^\circ$

(ii) use this if

$u \leq \sqrt{2gl}$

Ex. $u = \sqrt{gl}$

$\Rightarrow \theta = 60^\circ$



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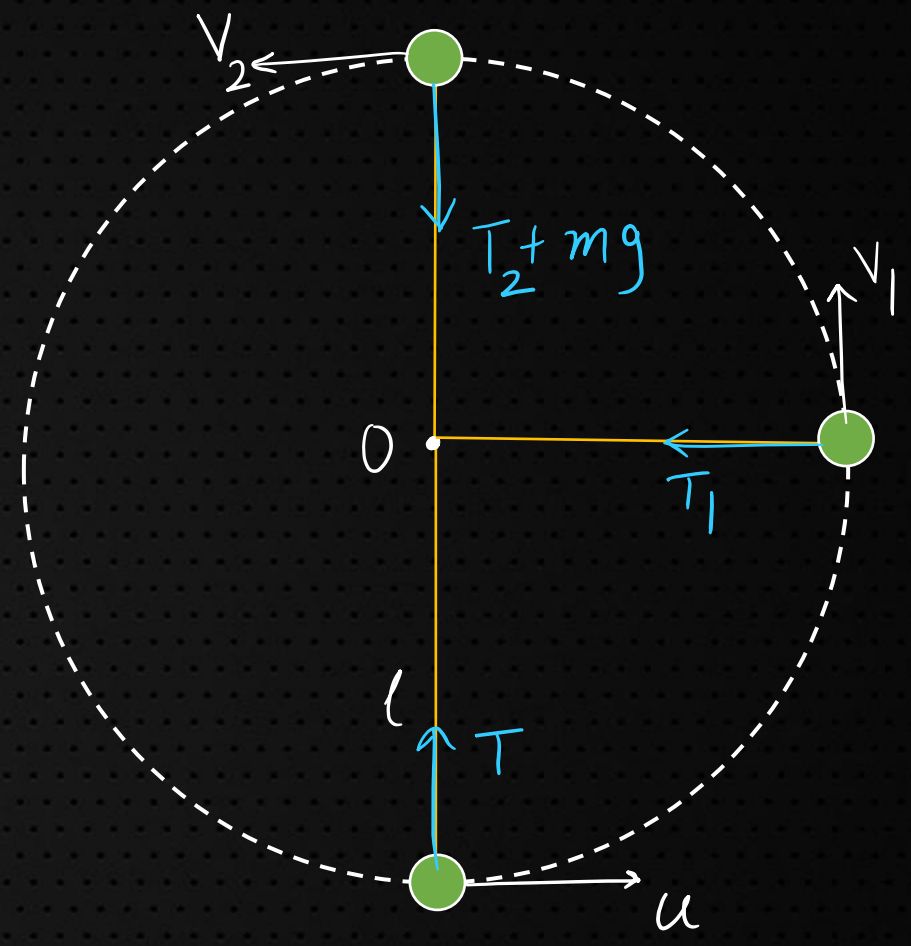
(b) $u \geq \sqrt{5gl}$

(i) bob does complete circle

(ii) If $u = \sqrt{5gl} \Rightarrow T_2 = 0$ but $v_2 \neq 0$

(iii) $T - T_1 = 3mg$ &

$T - T_2 = 6mg$



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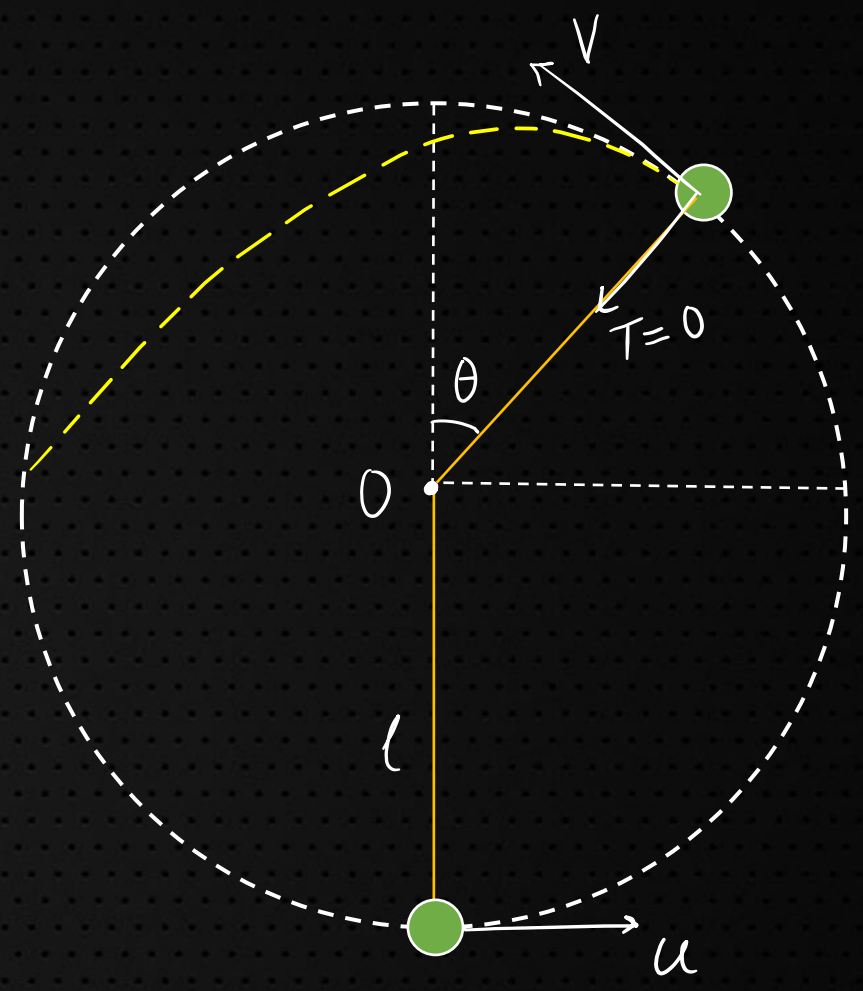
(c) $\sqrt{2gl} < u < \sqrt{5gl}$

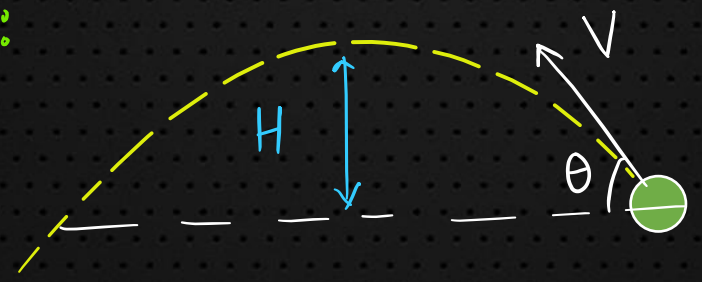
↳ rises above horizontal and somewhere $T = 0$ (string slacks) and bob does Projectile motion

(i) $\cos\theta = \dfrac{u^2 - 2gl}{3gl}$ & from Conserv. Energy

$$V = \sqrt{u^2 - 2gl(1+\cos\theta)}$$



NOTE :



$$H = \frac{V^2 \sin^2\theta}{2g}$$

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12. Vertical C.M. of a Light Rod Pendulum

↳ Here there is no issue of $T=0$ as rod can't slack (massless rod)

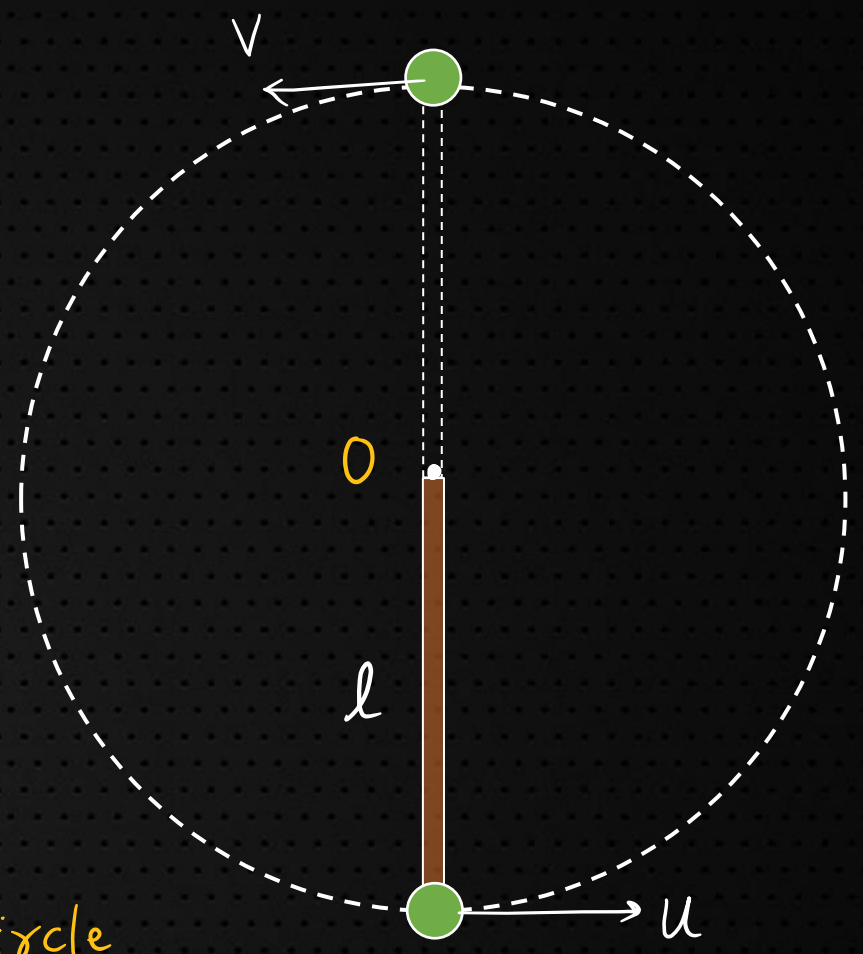
↳ For $u \geq \sqrt{4gl}$ ⟶ Complete Circle

NOTE:

(i) For $u = \sqrt{4gl}$

$V = 0$

↳ Due to inertia it completes circle



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13. Vertical C.M. Inside a Circular Track

↳ All cases are same as pendulum Bob with string.
(instead of T its N here)

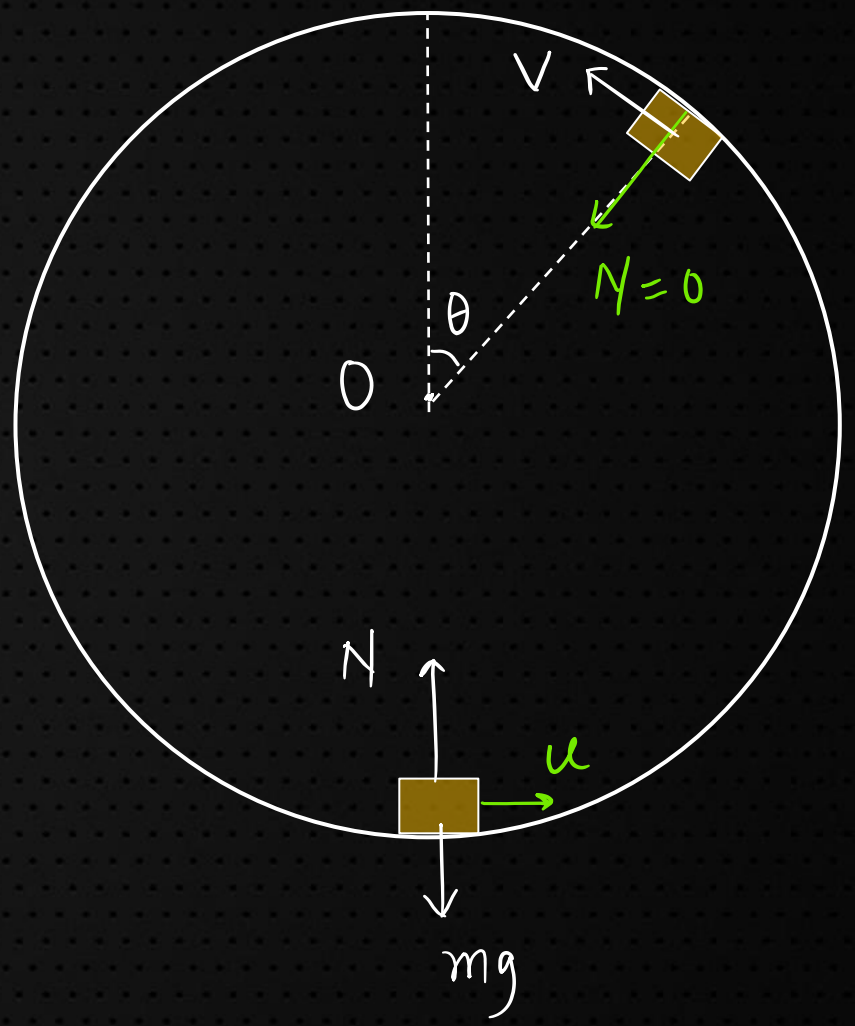
(i) $u \geq \sqrt{5gr}$ ⟶ complete circle

(ii) $\sqrt{2gr} < u < \sqrt{5gr}$ ⟶ cross horizontal and at some θ with vertical $N = 0$

$$\cos\theta = \frac{u^2 - 2gr}{3gr} \quad \text{and}$$

$$V = \sqrt{u^2 - 2gr(1+\cos\theta)}$$

↳ use this θ & V for projectile motion



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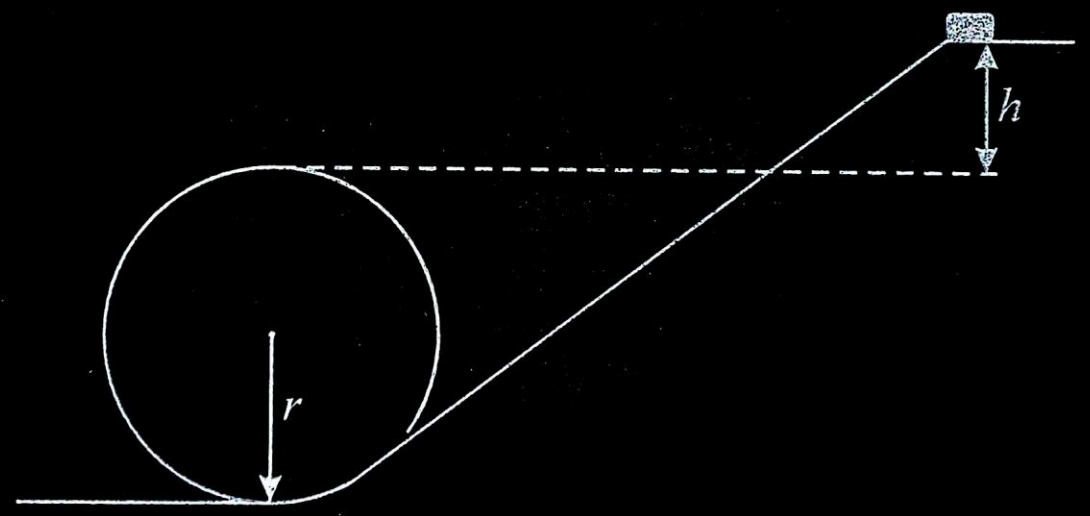
Ex. Figure shows a loop track of radius r. A box starts sliding from a platform at a distance h above the top of the loop and goes around the loop without falling off the track. Find the minimum value of h for a successful looping. Friction is negligible at all surfaces.



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... continued

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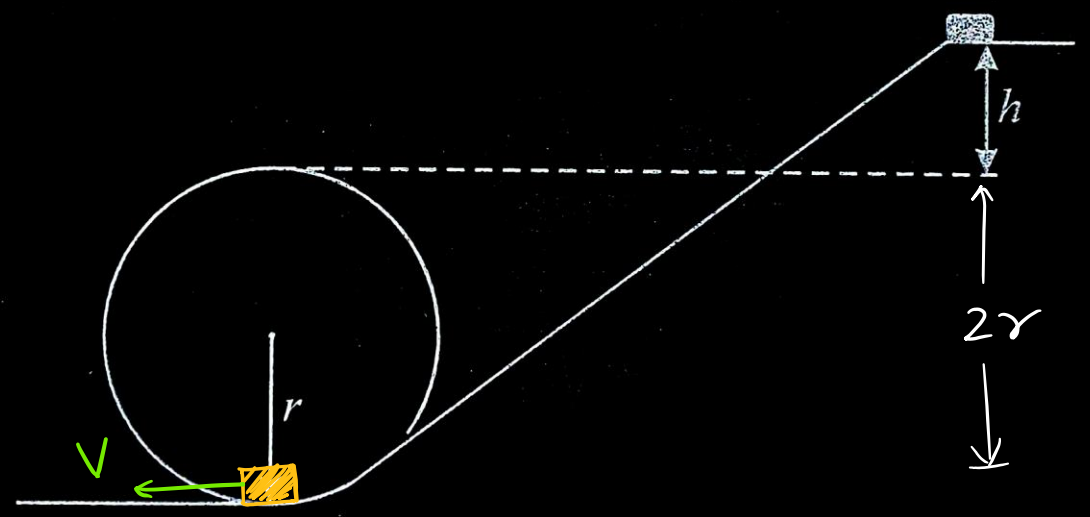

Sol^n: For looping box must have $\sqrt{5gr}$ speed at loops bottom. $\rightarrow V$

$\therefore$ Loss in PE = Gain in KE

$\Rightarrow mg(h+2r) = \frac{1}{2} m \times 5gr$

$\boxed{h = r/2}$

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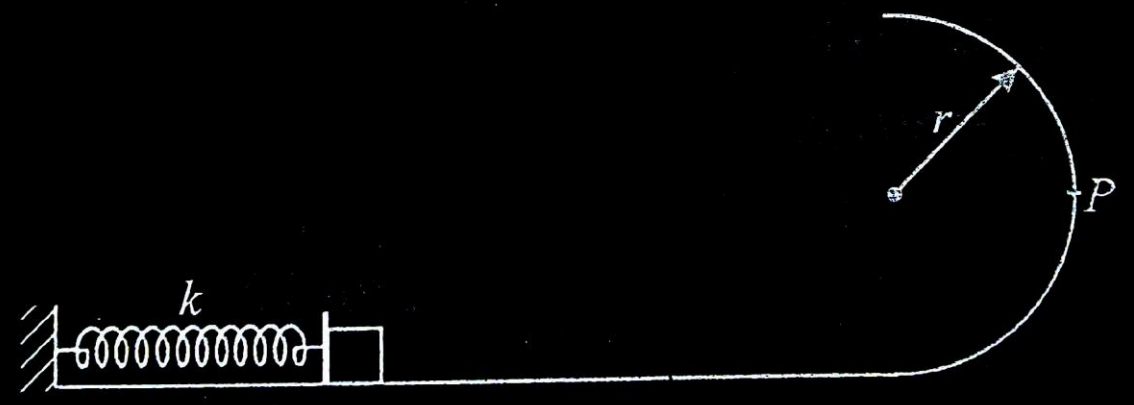
... continued

Ex. Figure shows a smooth track, a part of which is a circle of radius r. A block of mass m is pushed against a spring of spring constant k fixed at the left end and is then released. Find the initial compression of the spring so that the block presses the track with a force mg when it reaches the point P, where the radius of the track is horizontal.



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... continued

Ex. Figure shows a smooth track, a part of which is a circle of radius r. A block of mass m is pushed against a spring of spring constant k fixed at the left end and is then released. Find the initial compression of the spring so that the block presses the track with a force mg when it reaches the point P, where the radius of the track is horizontal.

Sol^n:

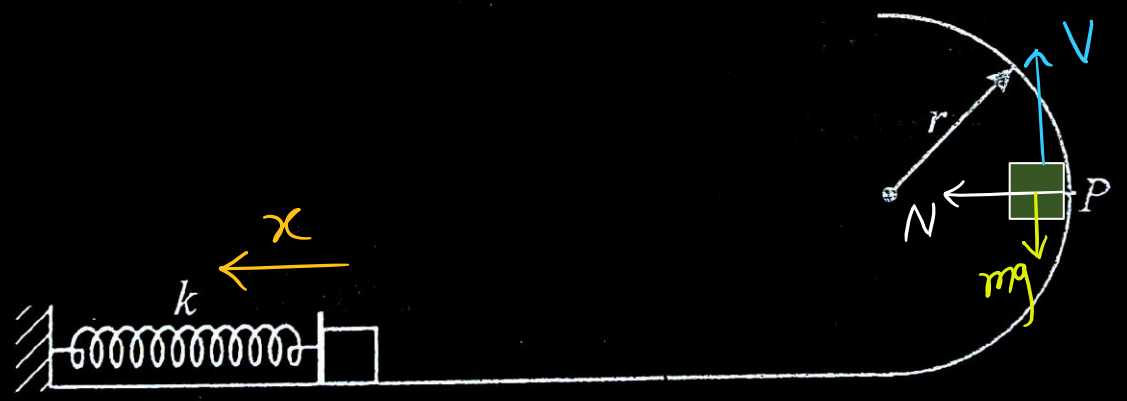
Loss in Spring PE = Gain in GPE + Gain in KE

$$\Rightarrow \frac{1}{2}Kx^2 = mgr + \frac{1}{2}mv^2 \quad \text{—(i)}$$

At P: $N = \frac{mv^2}{r} \Rightarrow mg = \frac{mv^2}{r}$

$$\Rightarrow mgr = mv^2 \quad \text{—(ii)}$$

Put (ii) in (i)



$$\Rightarrow \frac{1}{2}Kx^2 = mgr + \frac{mgr}{2} \qquad \therefore \boxed{x = \sqrt{\frac{3mgr}{K}}}$$

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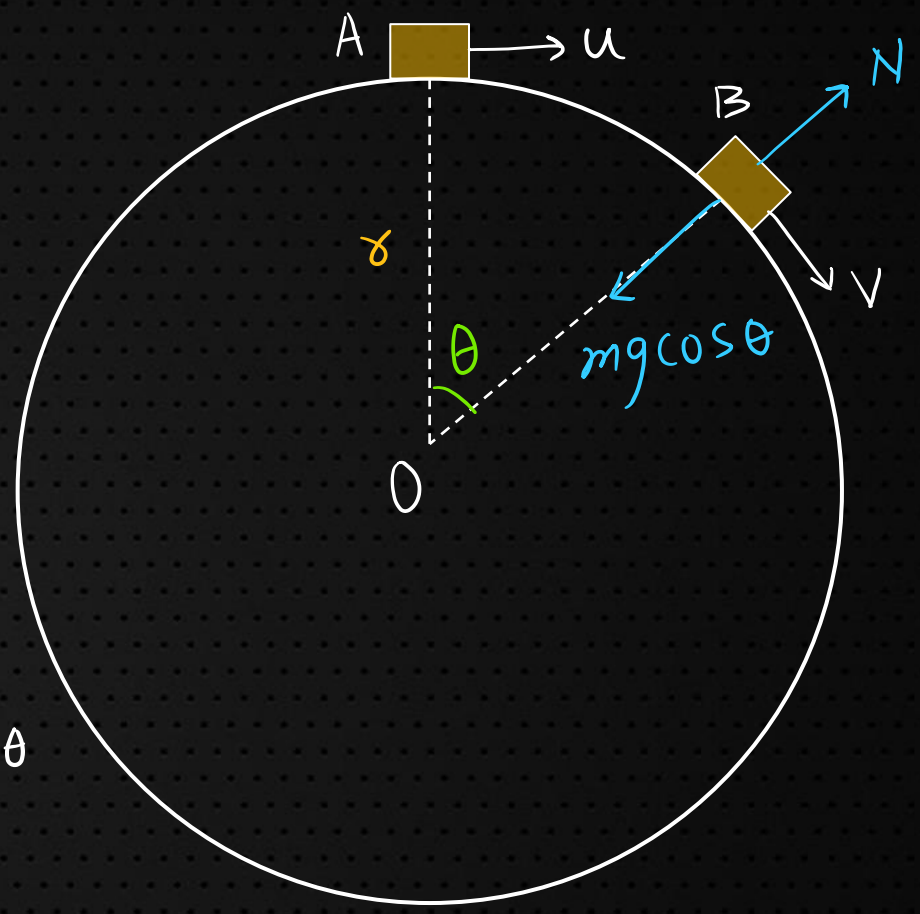
14. Vertical C.M. Outside a Circular Track



A → B : Loss in PE = Gain in KE

$$\Rightarrow mgr(1-\cos\theta) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\# \quad v = \sqrt{u^2 + 2gr(1-\cos\theta)}$$

and, $mg\cos\theta - N = \frac{mv^2}{r}$

→ Contact Loose : $(N=0)$

$$v^2 = rg\cos\theta$$

$$\Rightarrow u^2 + 2gr(1-\cos\theta) = rg\cos\theta$$

$$\# \quad \cos\theta = \frac{u^2 + 2gr}{3gr}$$

Ex: If $u=0$, $\cos\theta = \frac{2}{3}$ or $\theta = \cos^{-1}\frac{2}{3}$

→ Important

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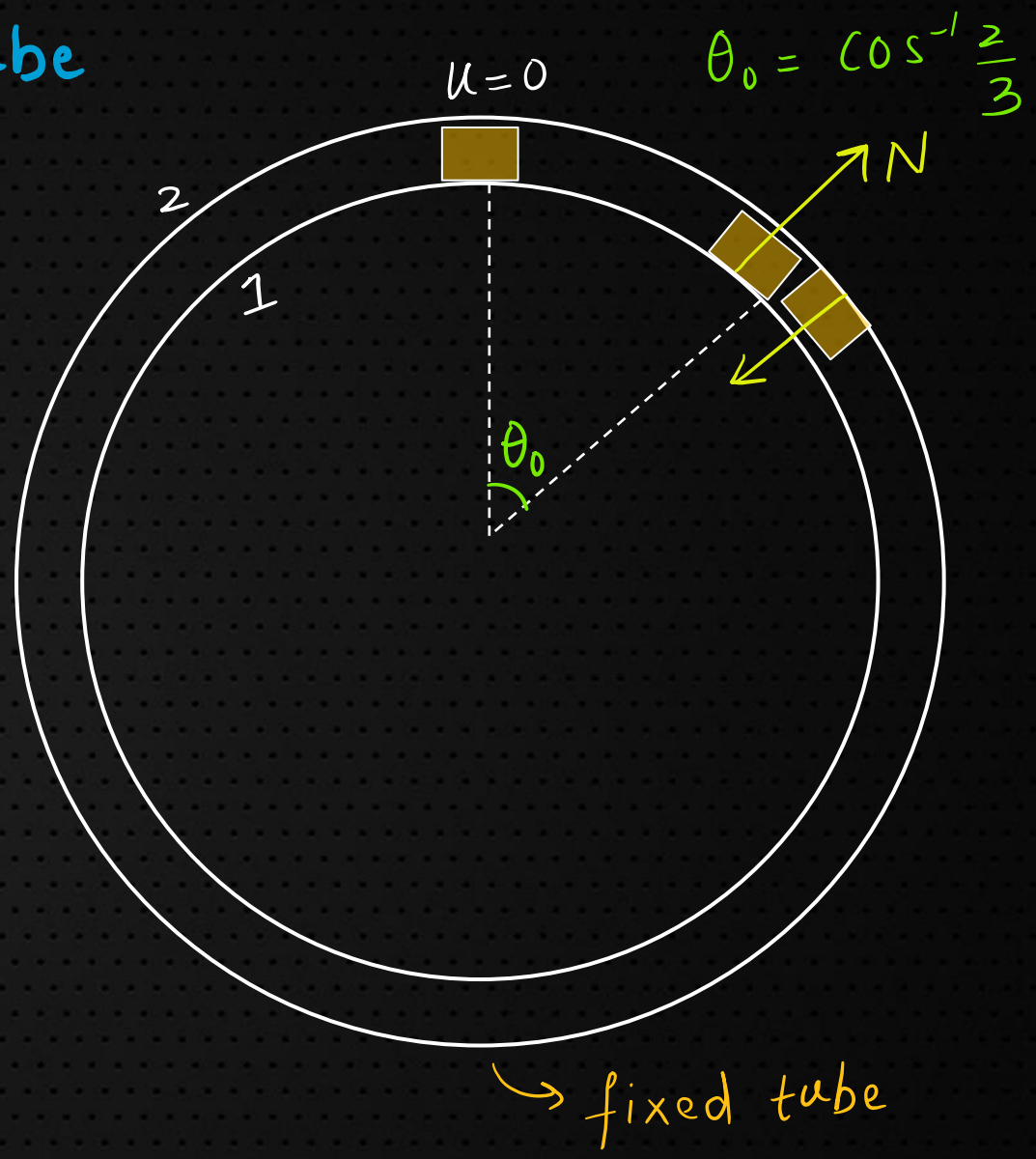
15. Vertical C.M. Inside a Circular Tube

↳ till $\theta < \cos^{-1}\frac{2}{3}$, block on 1.
(N on block radially outwards)

↳ for $\theta > \cos^{-1}\frac{2}{3}$, block on 2
(N on block radially inwards)



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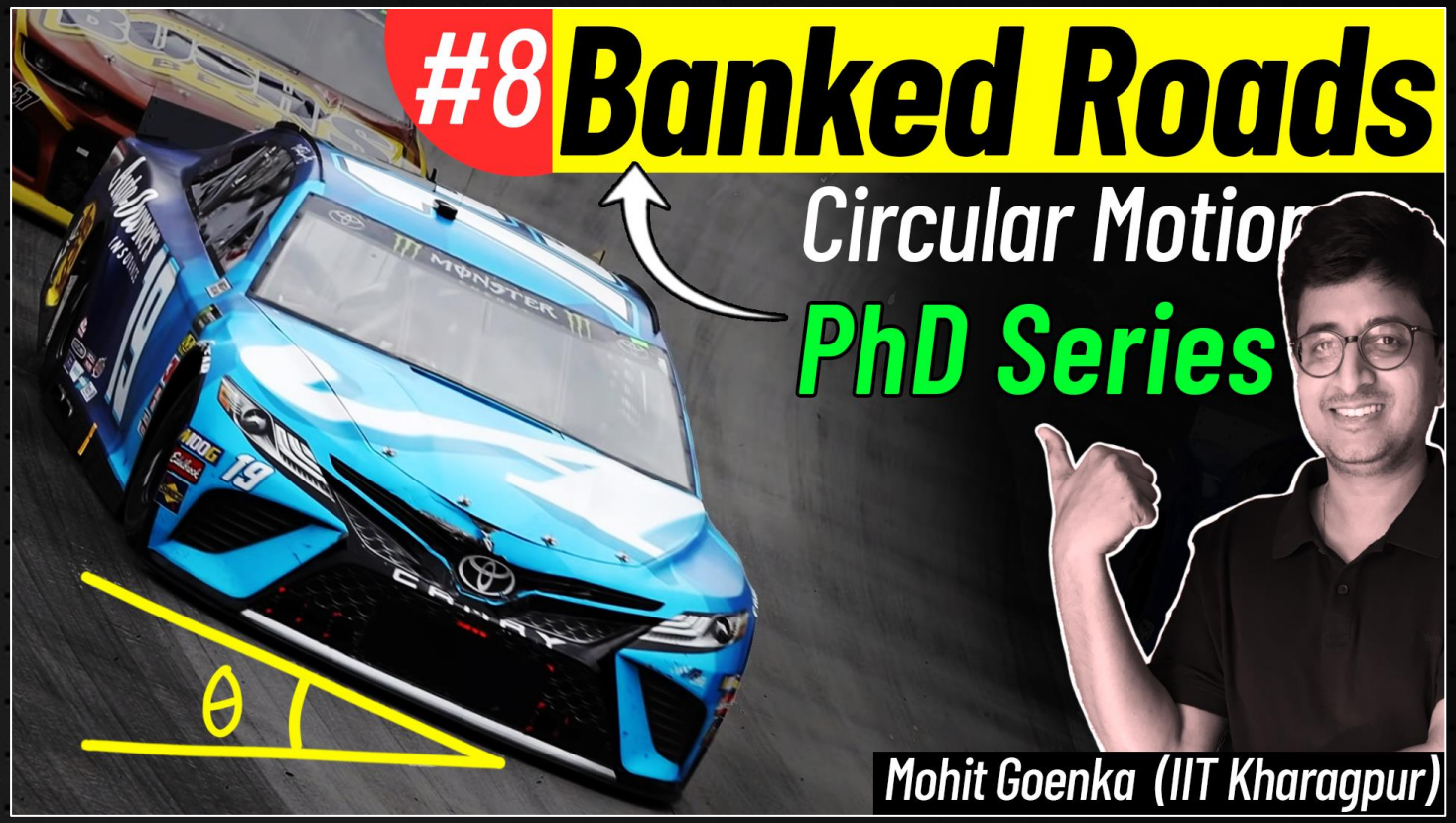
16. Banked Roads



https://youtu.be/WbdcQYw63xc

Click

PYQs LINKS (JEE MAIN)

	2021 Feb	2021 March	2021 July	2021 August	2020	Top PYQs / Mix Years
NLM	https://youtu.be/2xuXY66RQWM	https://youtu.be/zUu5ccnsB_4	https://youtu.be/hCShMjLXnFo	https://youtu.be/VqOrOiq0N_I	https://youtu.be/AxW4Ibc0KkY	https://youtu.be/ZUQ1hfF7Ov4
WEP	https://youtu.be/JBndilWvPwA	https://youtu.be/ocJJa_lXm64	https://youtu.be/pcP3So46Gug	https://youtu.be/YzSKbc2_ODY		

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(Practice these Questions)

Revision Series Playlist Link https://bit.ly/3eBbib9

JEE Main PYQs Link https://bit.ly/2S54jzh Chapter wise 2021, 2020, 2018

GoldMine Link https://bit.ly/2VhOGFF

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