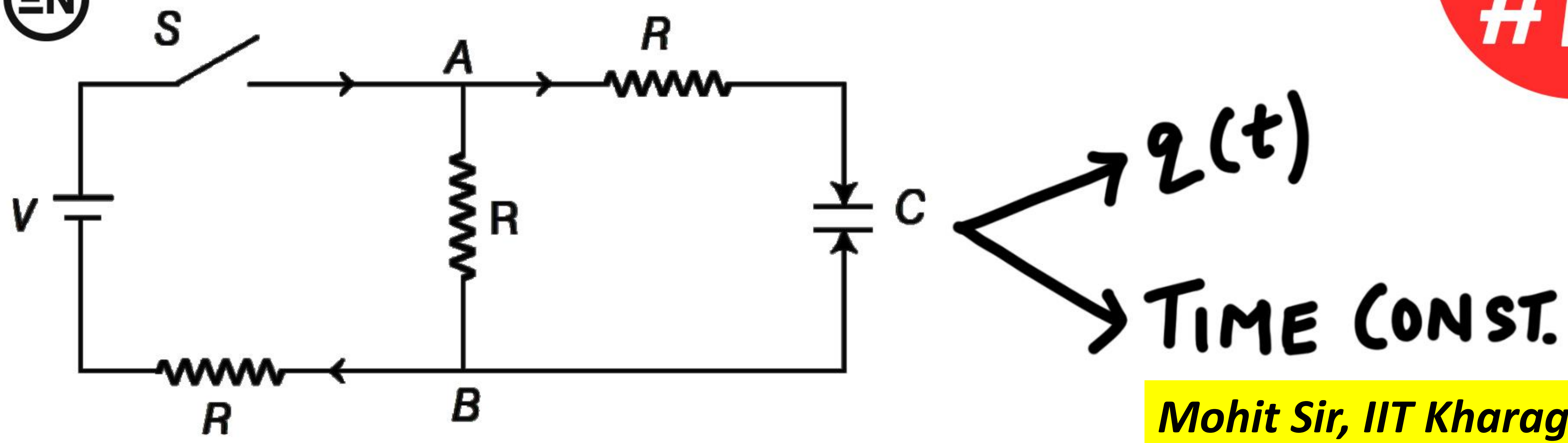


#12

Mohit Sir, IIT Kharagpur

PHD ON RC CIRCUIT

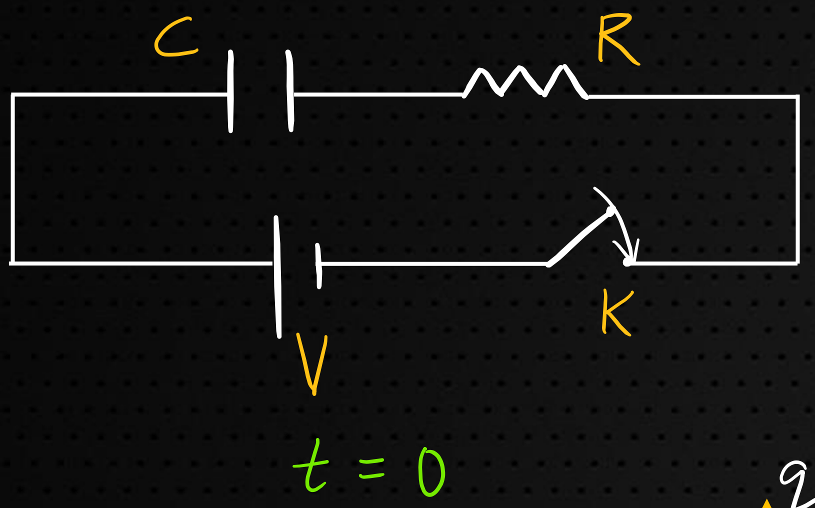
CHARGING & DISCHARGING

EDUNITI

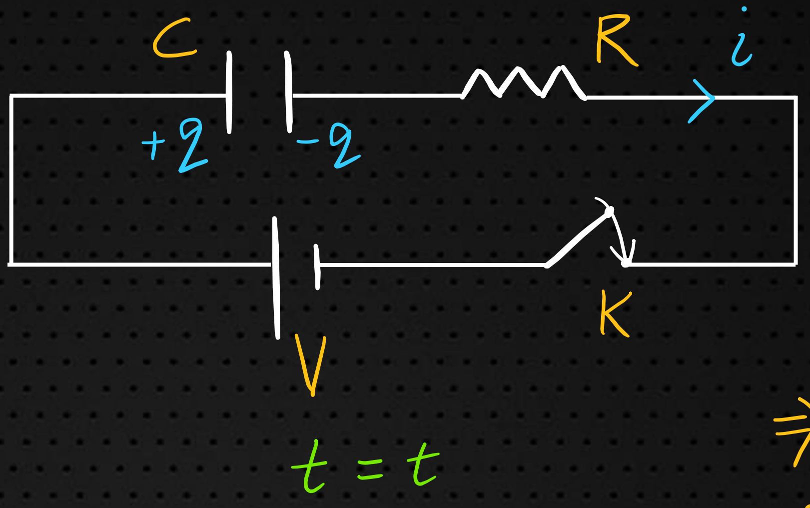
TOPICS TO BE DISCUSSED

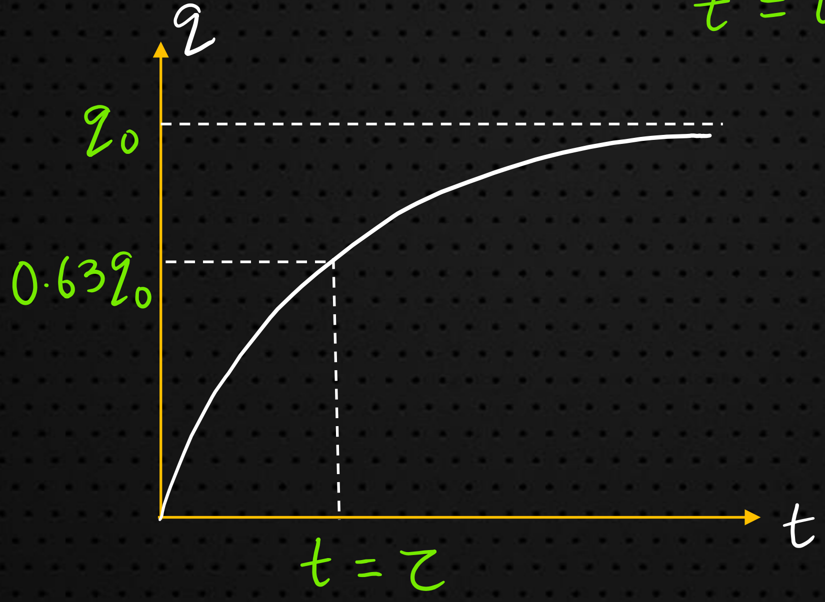
1. Charging of Capacitor (*Initially capacitor is uncharged*)
2. Charging of Capacitor (*Initially capacitor is charged*)
3. Questions on Charging
4. Discharging of Capacitor
5. Trick for finding Time Constant & Charging Equation
6. Question on finding Time Constant
7. Question on Steady State Circuit

1. Charging of Capacitor (at $t=0$, capacitor is uncharged)



$\Rightarrow$





$$V = \frac{q}{C} + iR$$

$$\Rightarrow V = \frac{q}{C} + R\frac{dq}{dt}$$

$$\Rightarrow \frac{CV-q}{C} = R\frac{dq}{dt}$$

$$\Rightarrow \int_0^q \frac{dq}{CV-q} = \int_0^t \frac{dt}{RC}$$

$$\therefore \boxed{q = CV\left(1-e^{-t/RC}\right)}$$

↳ $CV : q_0$

↳ Time Constant $\tau = RC$

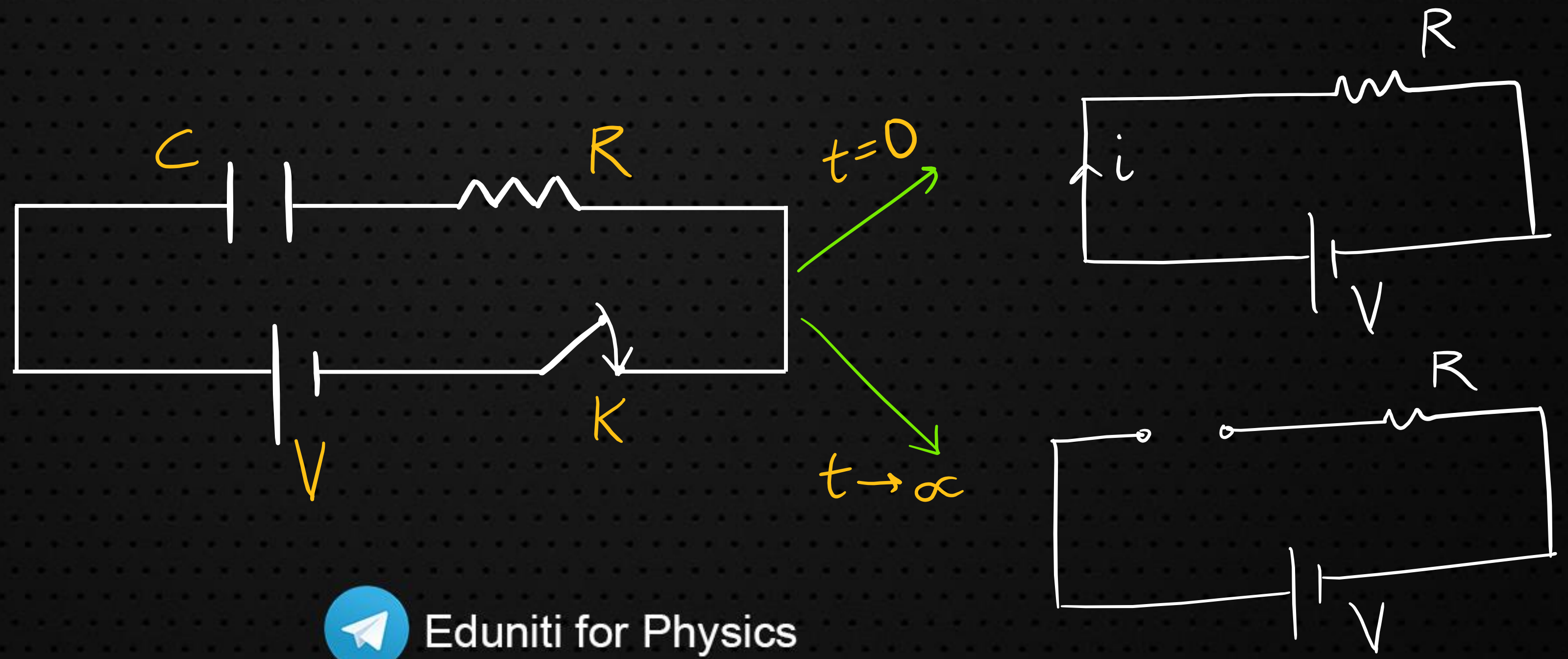
... Continued

(i) $q = CV(1 - e^{-t/RC})$ → Transient state eq^n

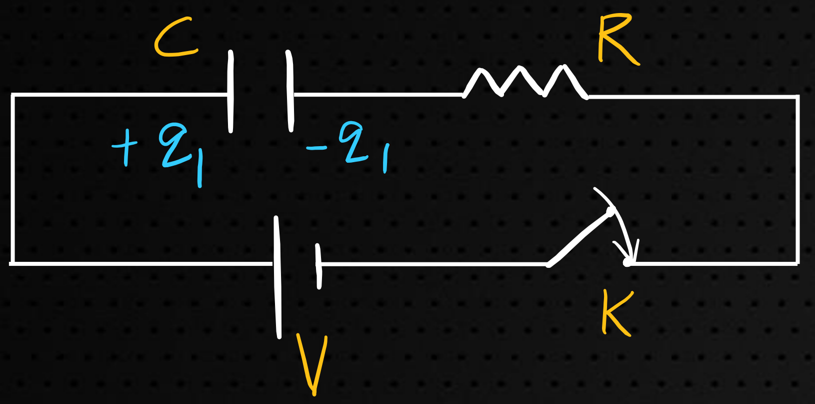
→ for $t \to \infty$, $q = CV$ (steady state when q_{max} on capacitor)

(ii) $i = \dfrac{dq}{dt} \Rightarrow i = \dfrac{V}{R} e^{-t/RC}$ ⟶ at $t = 0$, $i = \dfrac{V}{R}$ (as if capacitor is not there)

⟶ at $t \to \infty$, $i = 0$ (C acts as open circuit)

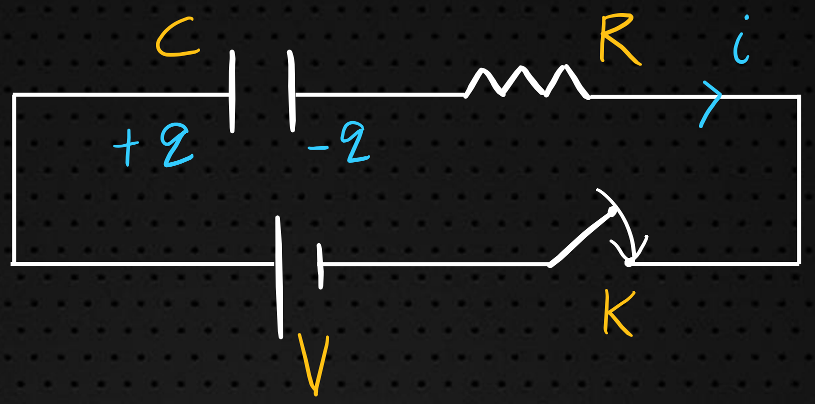


2. Charging of Capacitor (at $t=0$, capacitor has charge q_1)



$t = 0$

$\Rightarrow$



$t = t$

$$V = \frac{q}{C} + iR$$

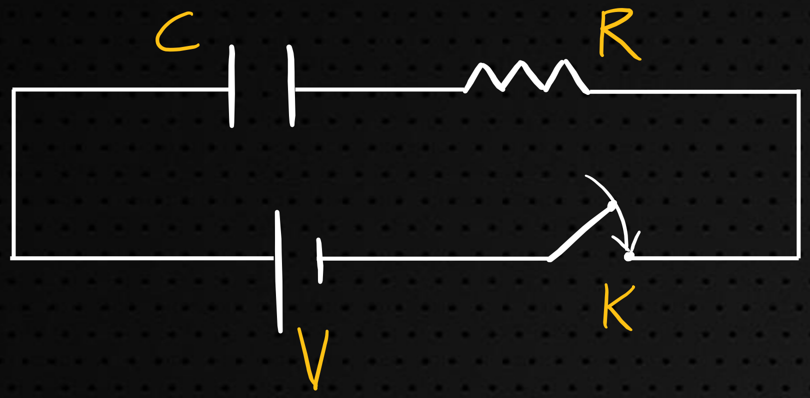
$$\Rightarrow V = \frac{q}{C} + \frac{R\,dq}{dt}$$

$$\Rightarrow \int_{q_1}^{q} \frac{dq}{CV - q} = \int_{0}^{t} \frac{dt}{RC}$$

$$\boxed{\therefore\ q = CV - (CV - q_1)\, e^{-t/RC}}$$

3. Question on Charging

Ex 1.

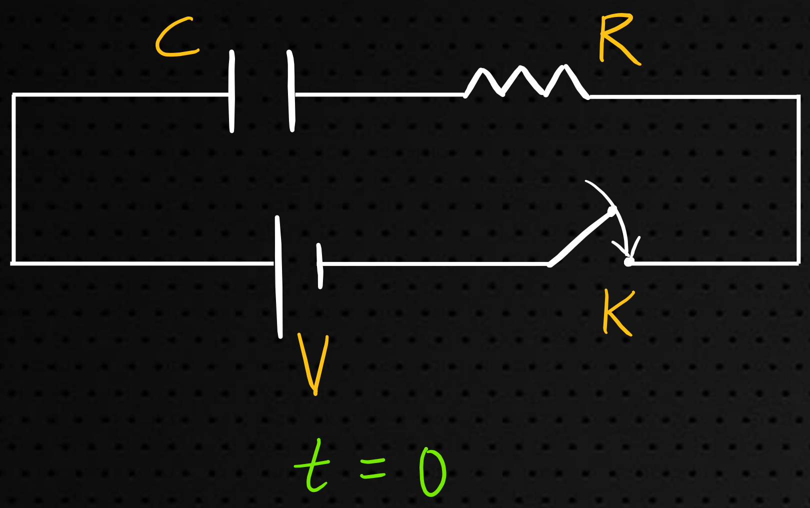


$t = 0$

At $t=0$ switch is closed. Find time when energy stored in capacitor is $\frac{1}{4}$th of max.

3. Question on Charging

Ex1.



At $t=0$ switch is closed. Find time when energy stored in capacitor is $\frac{1}{4}$th of max.

Sol^n: $q = q_0(1-e^{-t/RC})$, q_0 is q_{max} on C.

$$\Rightarrow \frac{q^2}{2C} = \frac{q_0^2}{2C}(1-e^{-t/RC})^2$$

$$\Rightarrow U = U_0(1-e^{-t/RC})^2$$

$$\Rightarrow \frac{U_0}{4} = U_0(1-e^{-t/RC})^2 \Rightarrow \frac{1}{2} = 1-e^{-t/RC}$$

$$\Rightarrow \frac{1}{2} = e^{-t/RC} \quad \therefore \boxed{t = RC\ln 2}$$

... continued

Ex 2. A 4μF capacitor and a resistance of 2.5 MΩ are in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor. [Given, ln (2) = 0.693]

(2005, 2M)

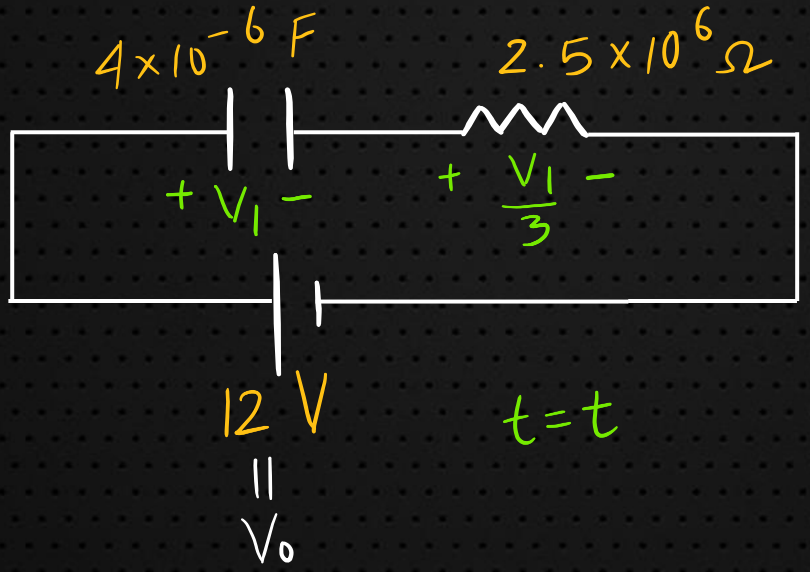
(a) 13.86 s (b) 6.93 s (c) 7 s (d) 14 s

... continued

Ex 2. A 4μF capacitor and a resistance of 2.5 MΩ are in series with 12 V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor. [Given, ln (2) = 0.693]

(2005, 2M)

(a) 13.86 s (b) 6.93 s (c) 7 s (d) 14 s



Sol^n: $V_1 + \frac{V_1}{3} = 12$

$\Rightarrow V_1 = 9V$

$q = CV_0(1-e^{-t/RC})$

$\therefore$ p.d across capacitor is q/C

$\Rightarrow V_C = V_0(1-e^{-t/RC})$

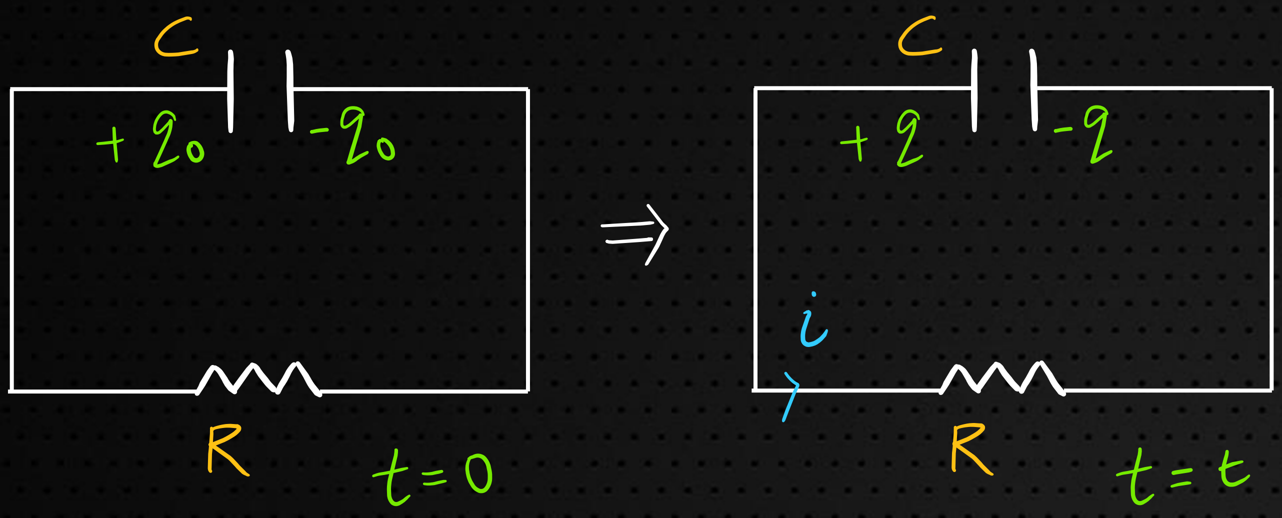
$\Rightarrow 9 = 12(1-e^{-t/10})$

$\therefore \frac{1}{4} = e^{-t/10}$

$\Rightarrow t = 20\ln 2 = 20\times0.693$

$= \boxed{13.86s}$

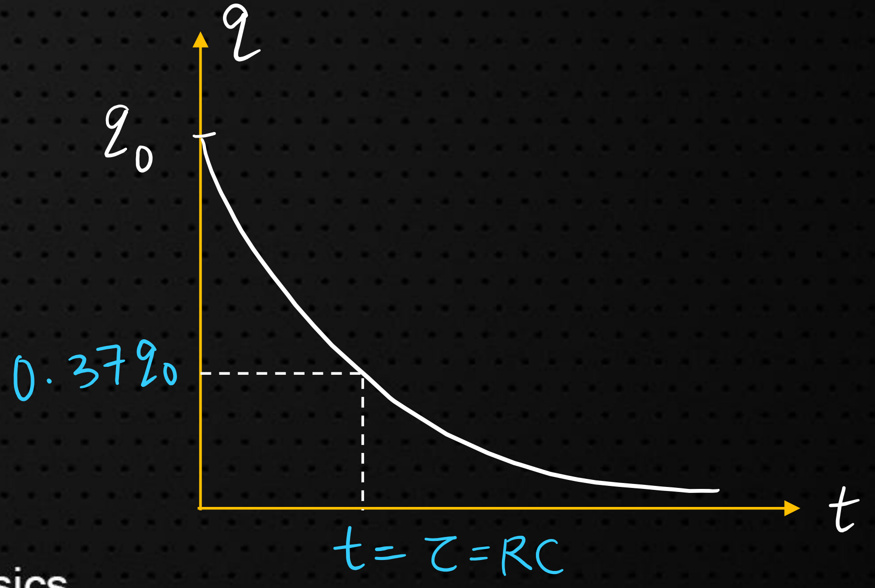
4. Discharging of Capacitor



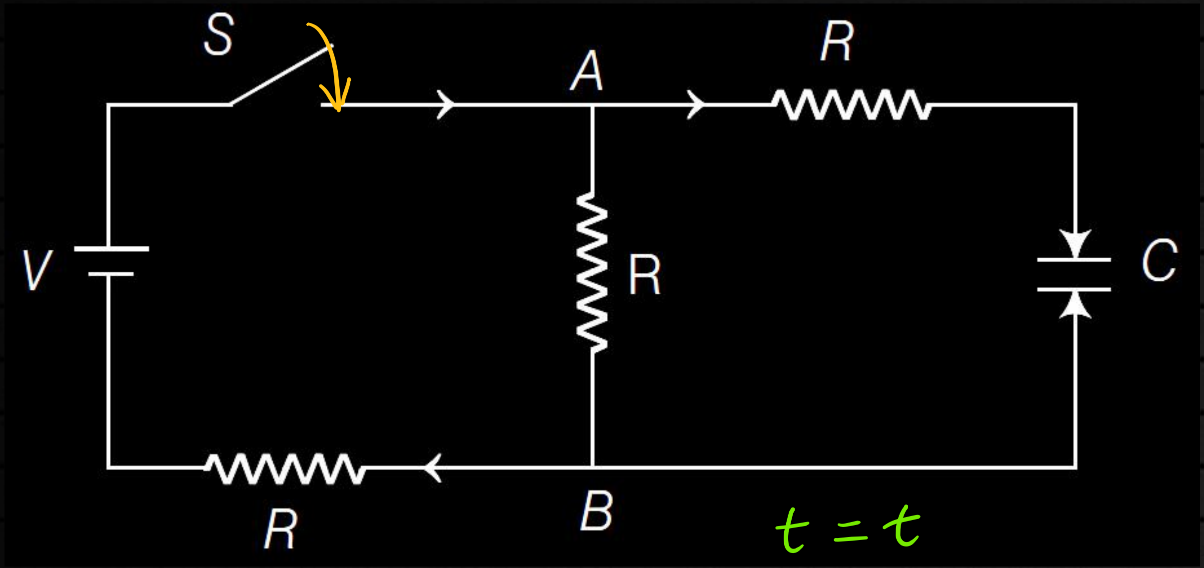
$$\frac{q}{C} = iR \Rightarrow \frac{q}{C} = R\left(-\frac{dq}{dt}\right)$$

$$\Rightarrow \int_{q_0}^{q} \frac{dq}{q} = -\int_{0}^{t} \frac{dt}{RC}$$

$$\Rightarrow \ln\left(\frac{q}{q_0}\right) = -\frac{t}{RC} \Rightarrow \boxed{q = q_0 e^{-t/RC}}$$



5. Trick to find Time Constant & Charging Eqn

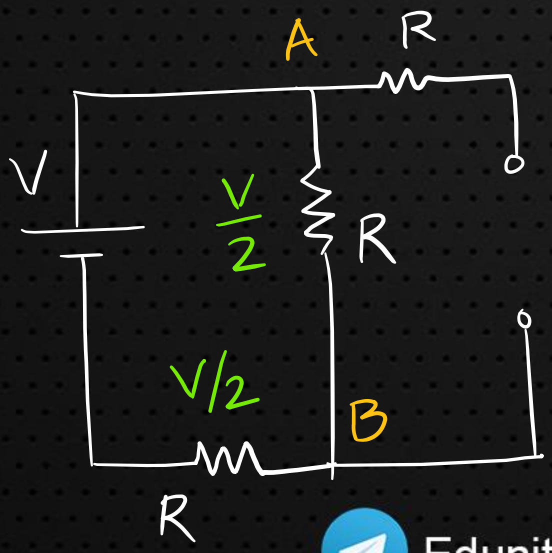


Here charging eqn is

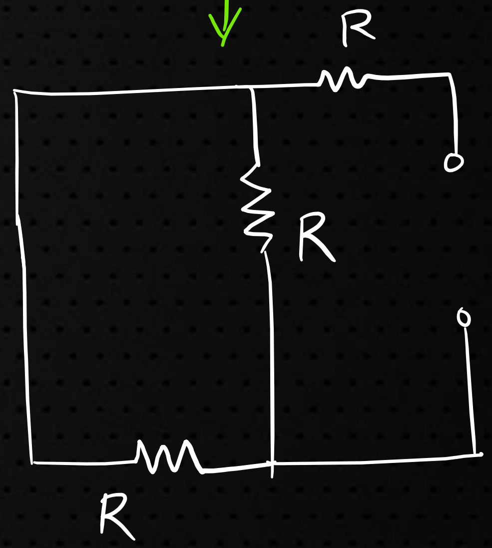
$$q = q_{max}\left(1 - e^{-t/R_{eq}C}\right)$$

charge on C at steady state

Replace cell by wire & find Req across C.



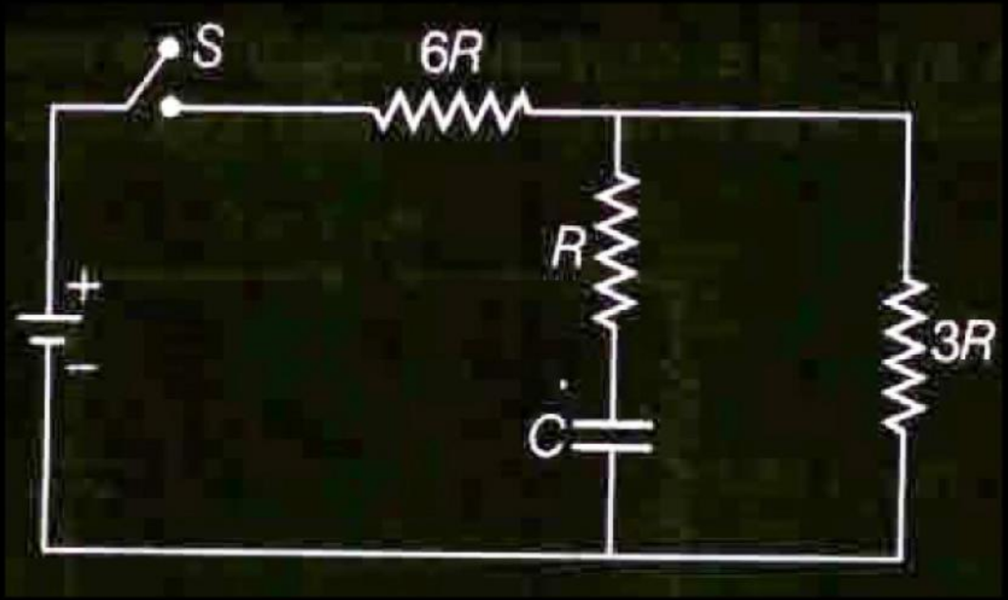
$$\therefore q_{max} = \frac{CV}{2}$$



$$R_{eq} = \frac{R}{2} + R = \frac{3R}{2}$$

6. Question to find Time Constant ($\tau = R_{eq} C$)

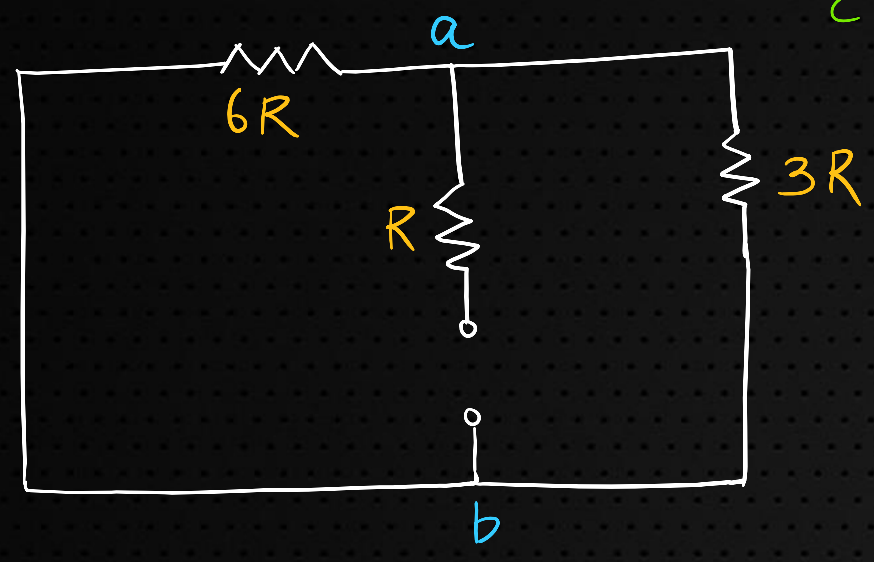
Ex 3. For the given circuit shown in figure, time constant is



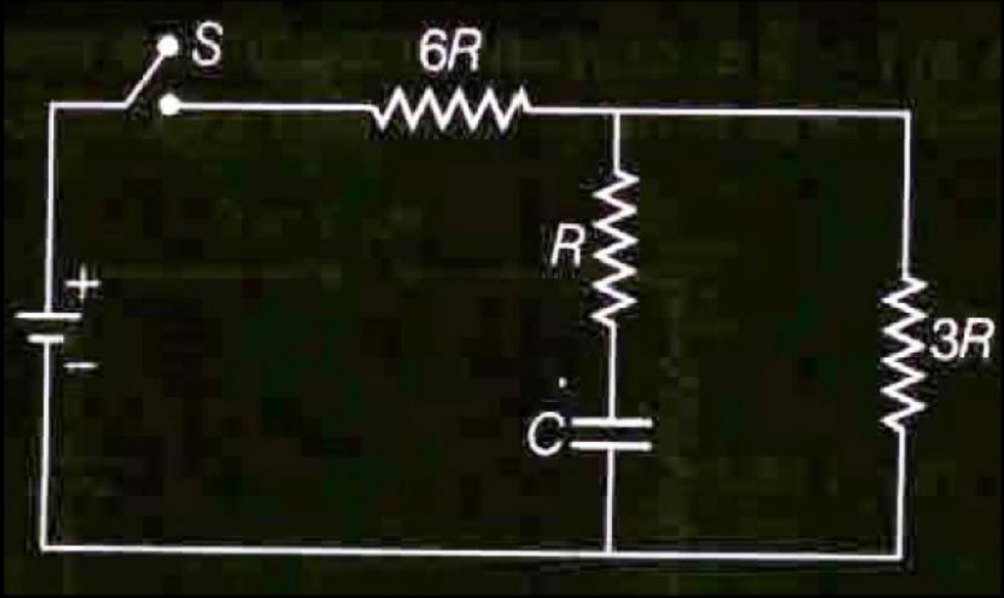
(a) $\tau = RC$ (b) $\tau = 2.1RC$

(c) $\tau = 27/4RC$ **(d)** $\tau = 3RC$

6. Question to find Time Constant ($\tau = R_{eq} C$)



Ex 3. For the given circuit shown in figure, time constant is



(a) $\tau = RC$ (b) $\tau = 2.1RC$

(c) $\tau = 27/4RC$ ✓(d) $\tau = 3RC$

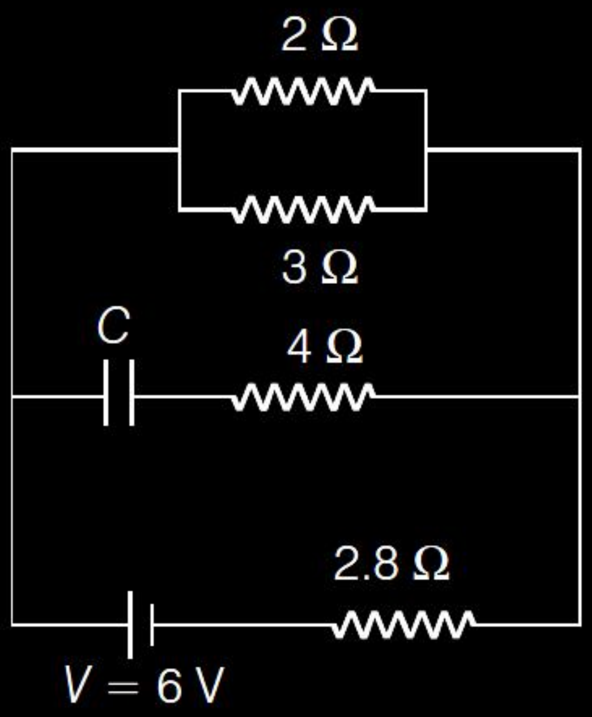
Sol^n $R_{eq} = \frac{3R \times 6R}{3R + 6R} + R = 3R$

$\therefore \boxed{\tau = 3RC}$

7. Question on steady state

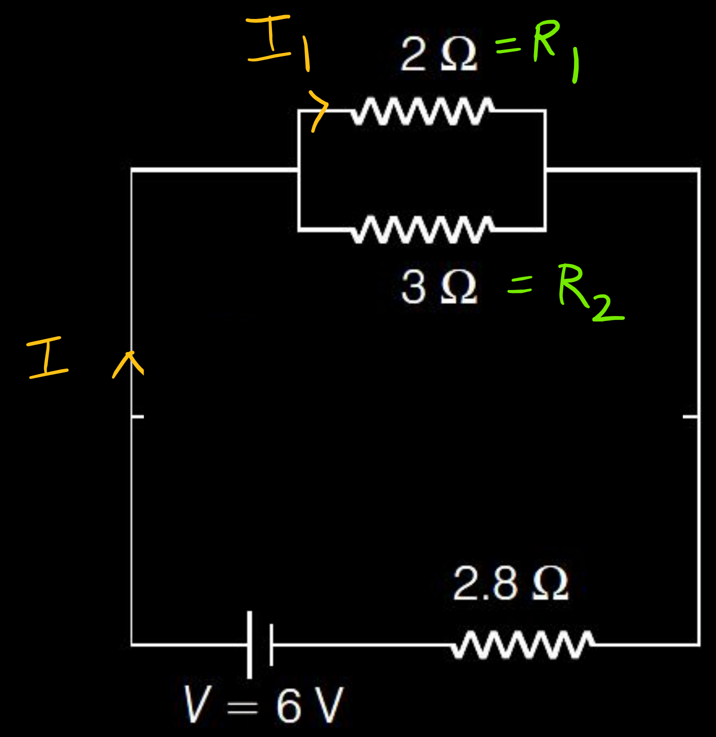
Ex 4. Calculate the steady state current in the 2 Ω resistor shown in the circuit (see figure). The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2 μF.

(1982, 5M)



7. Question on steady state

Ex 4. Calculate the steady state current in the 2 Ω resistor shown in the circuit (see figure). The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2 μF. **(1982, 5M)**



Soln:

$$R_{eq} = \frac{2 \times 3}{2+3} + 2.8 = 4\,\Omega$$

$$\therefore I = V/R_{eq} = 6/4 = 1.5\text{ A}$$

$$\Rightarrow I_1 = \frac{IR_2}{R_1+R_2} = \frac{1.5 \times 3}{5} = \boxed{0.9\text{ A}}$$ Ans

Revision Series Playlist Link https://bit.ly/3eBbib9

PhD Series https://bit.ly/3cQSxPT

GoldMine Link https://bit.ly/2VhOGFF

CLICK