JEE MAIN | IIT JEE

STRING WAVES-2

Standing Wave & Sonometer

REVISION in **40** Min

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STRING WAVES – PART 2

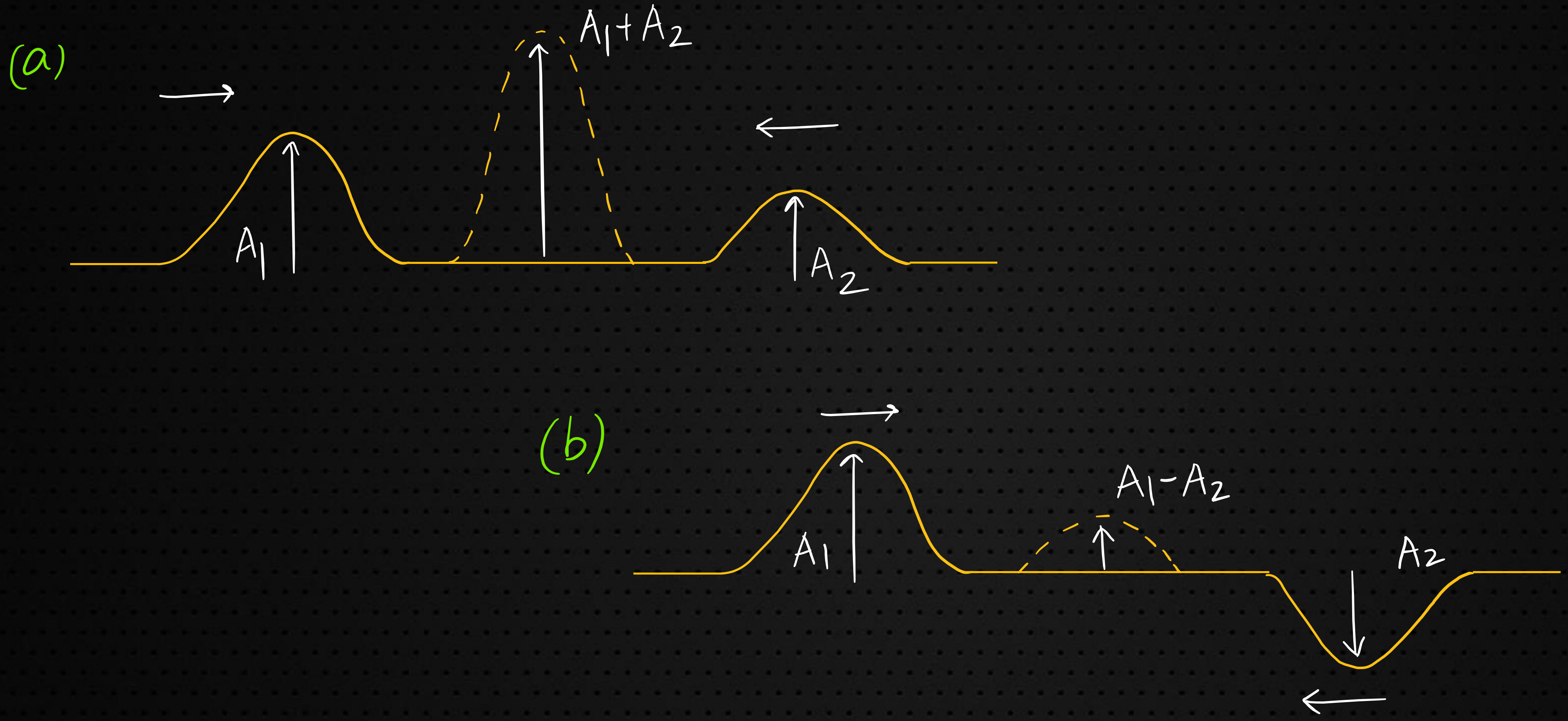
1. Superposition of Pulse
2. Reflection from fixed & free ends
3. Reflection & Transmission between two strings
4. Standing or Stationary Waves
5. Key Points in Standing Waves
6. Equation of Standing Waves
7. Question on Standing wave equation
8. Standing wave in Clamped string (*between 2 fixed ends*)
9. Standing wave in Clamped String (*1 fixed & 1 free end*)
10. Standing wave in Composite String
11. Question on Standing wave in Clamped String
12. Sonometer
13. Question on Sonometer

EDUNITI

Chapter	Formulae_Concept VIDEO LINK
Unit & Dimensions	https://youtu.be/wdd-wlZF4Hk
Errors and Vectors	
Vernier Calliper	https://youtu.be/pVoN045dV8I
Screw Gauge	https://youtu.be/gYd2PtmZ0mw
Kinematics_Motion in 1d	https://youtu.be/U4NNxFaFliE
Kinematics_Motion in 2d	https://youtu.be/4_Zo5WhMf7w
Laws of Motion	https://youtu.be/7JlR8gNRQIs
Friction	https://youtu.be/Rn1bLst7eGk
Work Energy Power	https://youtu.be/kjrXoE-kDI8
Circular Motion	https://youtu.be/KnFymKHlkT0
Centre of Mass	https://youtu.be/ads35RKD618
Cons of Momentum & Collision	https://youtu.be/3f0u4L-lyyw
Rotational Motion – Part 1	https://youtu.be/O6j1mLp06XI
Rotational Motion – Part 2	https://youtu.be/OHni1DRdfAQ
Rotational Motion_Part 3	https://youtu.be/quglqfYRCrk
Gravitation	https://youtu.be/rAj2huLVaEk
Properties of Solids	https://youtu.be/gSXxjk89l_c
Fluids Statics (Part 1)	https://youtu.be/RFKx9B9yo3M
Fluid Dynamics (Part 2)	https://youtu.be/Y717vQpUEJQ
Fluid Properties (Part 3)	https://youtu.be/V8xUWWK2oT0
Simple Harmonic Motion	https://youtu.be/RIb7ofNG09I
Thermal Properties	https://youtu.be/OYjjyPIzddE
Heat Transfer	https://youtu.be/PyNboHgtYzM
KTG	https://youtu.be/XO1tvFhIa0I
Thermodynamics	https://youtu.be/iz_kf1jRDRw
String Waves (Part-1)	https://youtu.be/mZWNmH19wDQ
String Waves (Part-2)	https://youtu.be/PDGq4d3xA6c
Wave Motion -Organ Pipes and Resonance Tube	https://youtu.be/fB7pfJ77za8
Wave Motion - Doppler's Effect	https://youtu.be/9-BxOaamnwg

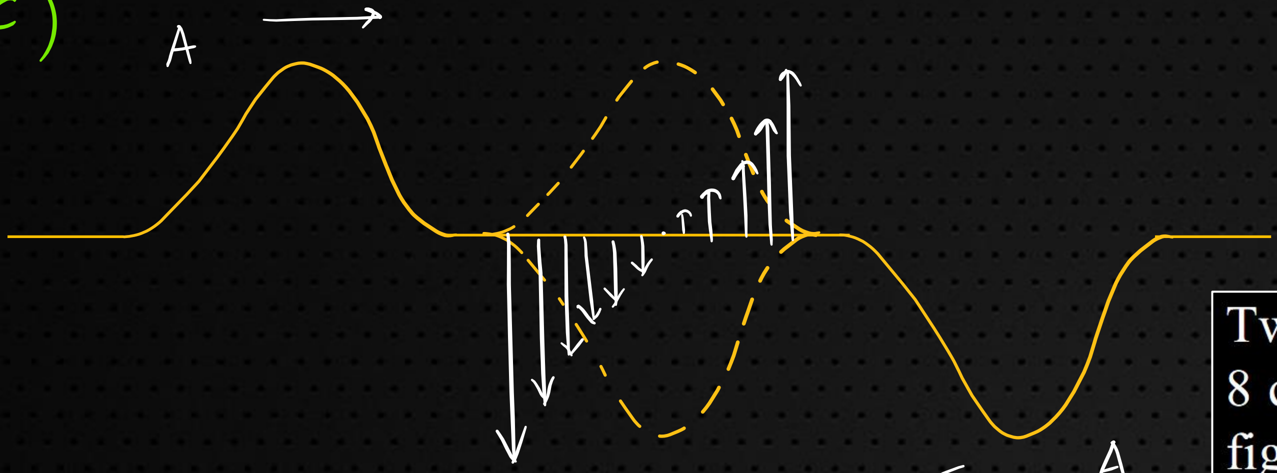
Electrostatics	https://youtu.be/3stXbGRMcrk
Capacitors	https://youtu.be/EXEiickNUkY
Current Electricity	https://youtu.be/gm8FUfjrX18
Moving Charges and Magnetic Effect of Current	https://youtu.be/ULD2Ok1CGJk
Earth's Magnetism	https://youtu.be/a4CT5uVwAK4
Magnetic Properties	https://youtu.be/63cwdYXNlYE
EMI	https://youtu.be/puVavm_GFRM
Alternating Current	https://youtu.be/74dTY-pzM_o
Ray Optics	https://youtu.be/BhnyTWzIlBA
Wave Optics Part 1_Interference	https://youtu.be/LG5nlE8XTeI
Wave Optics Part 2_Diffraction_Polarization	https://youtu.be/ymMyyJGGqnY
Optical Instruments	https://youtu.be/OQssbDH0A4I
Electromagnetic Waves	https://youtu.be/bcVXgEkyQZY
Semiconductors_Basics + Zener Diode	https://youtu.be/_A2JomQ7-50
Semiconductors_Transistors	https://youtu.be/psDwl84Nzb0
Semiconductors_Logic Gates	https://youtu.be/pZdQAzLbFTo
Communication Systems	https://youtu.be/8NgMqK9X79Y
Modern Physics_Part 1_Atomic Physics	https://youtu.be/9VKUnE3mpHk
Modern Physics_Part 2_Photoelectric Effect	https://youtu.be/24oTQp84jrk
Modern Physics_Part 3_Dual Nature of Light	https://youtu.be/0zoR_saMAQY
Modern Physics_Part 4_Radioactivity	https://youtu.be/AdX3YBhQyog
Modern Physics_Part 5_Nuclear Physics	https://youtu.be/VDWqVahGixc
Modern Physics_Part 6_X Rays	https://youtu.be/dSHXdzX7NX0

1. Superposition of Pulse



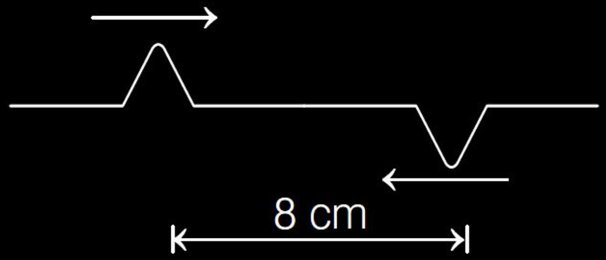
... continued

(c)



Although displacement is zero but all energy is in form of K.E.

Two pulses in a stretched string, whose centres are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 s the total energy of the pulses will be **(2001, 2M)**



(a) zero

✓(b) purely kinetic

(c) purely potential

(d) partly kinetic and partly potential

2. Reflection from fixed & free ends

(a)

$y_i = A\sin(\omega t - Kx)$

Phase change of π

$y_r = A\sin(\omega t + Kx + \pi)$

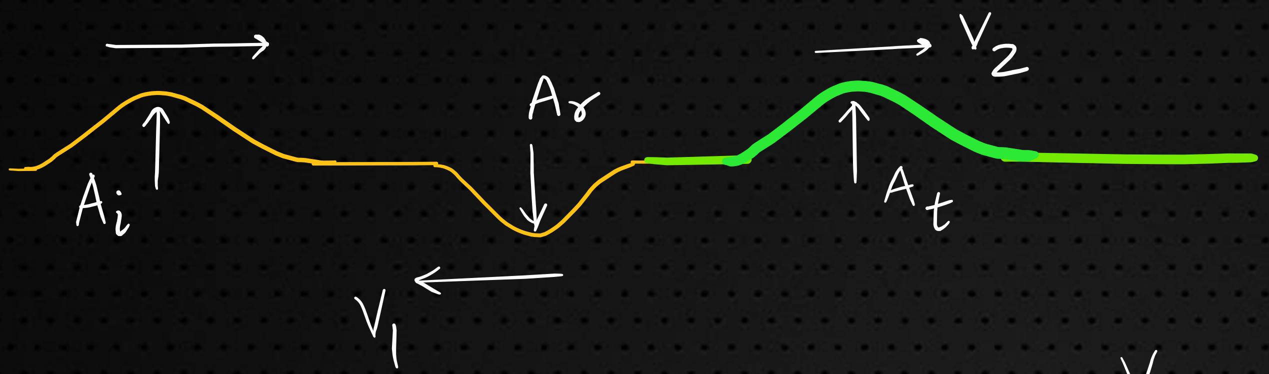
(b)

$y_i = A\sin(\omega t - Kx)$

$y_r = A\sin(\omega t + Kx)$

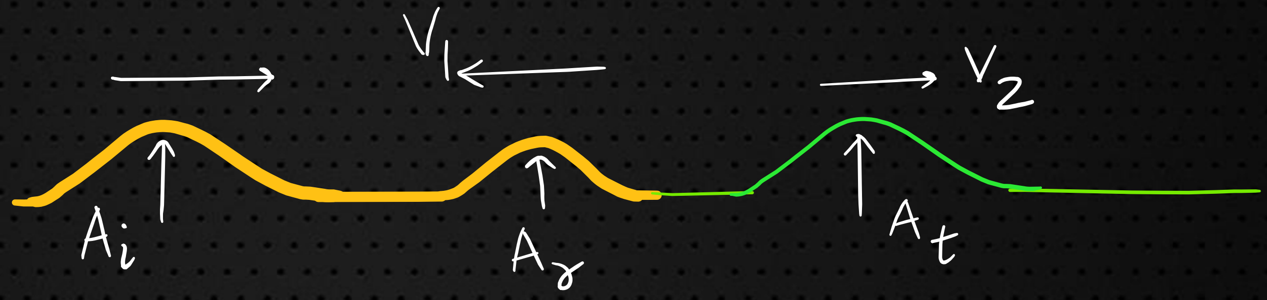
3. Reflection & transmission between two strings

(a)



$$V = \sqrt{\frac{T}{\mu}} \Rightarrow V \propto \frac{1}{\sqrt{\mu}} \Rightarrow V_1 > V_2$$

(b)



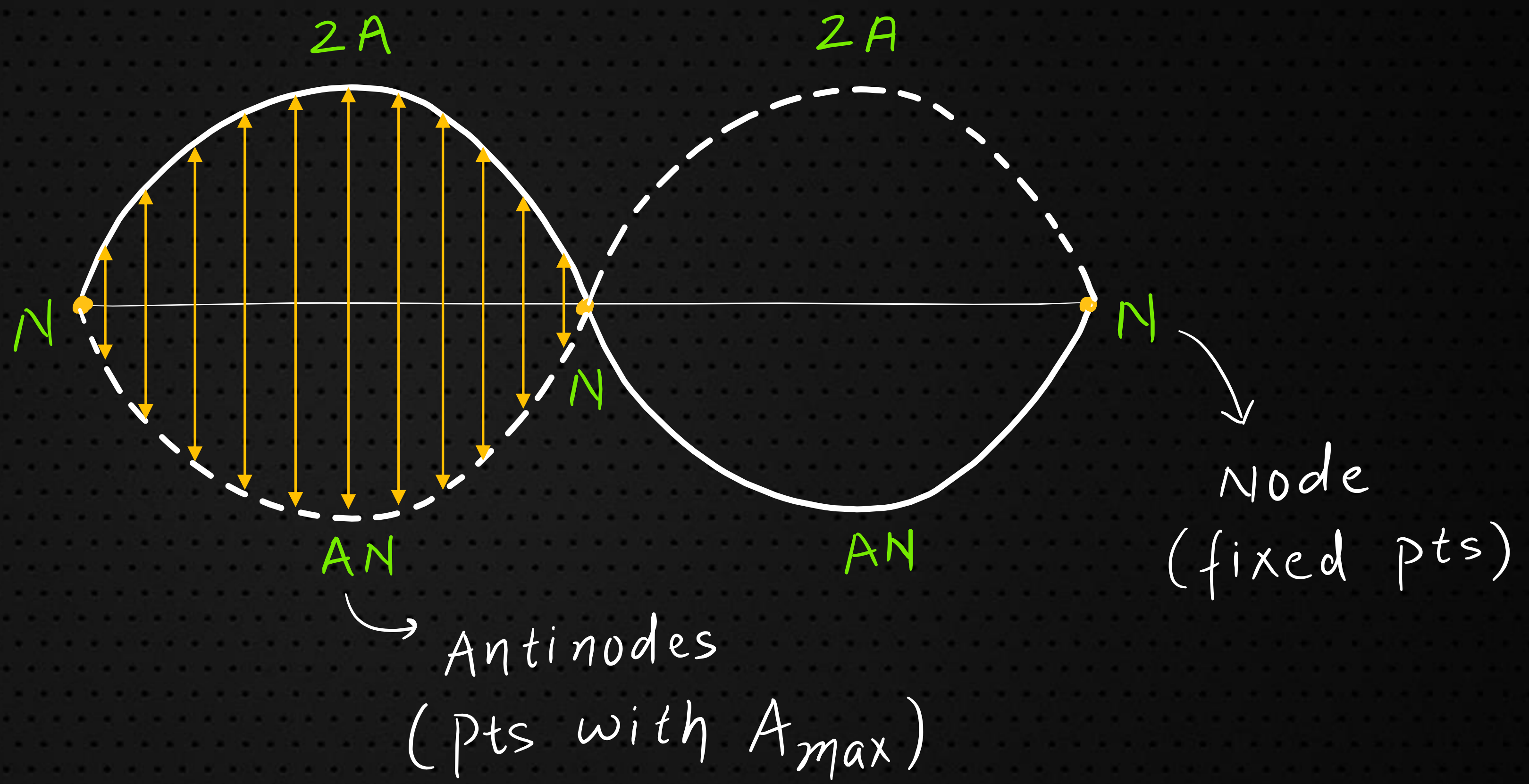
$V_1 < V_2$

$$A_t = \frac{2V_2}{V_1+V_2} A_i \quad , \quad A_r = \frac{V_2 - V_1}{V_2+V_1} A_i$$

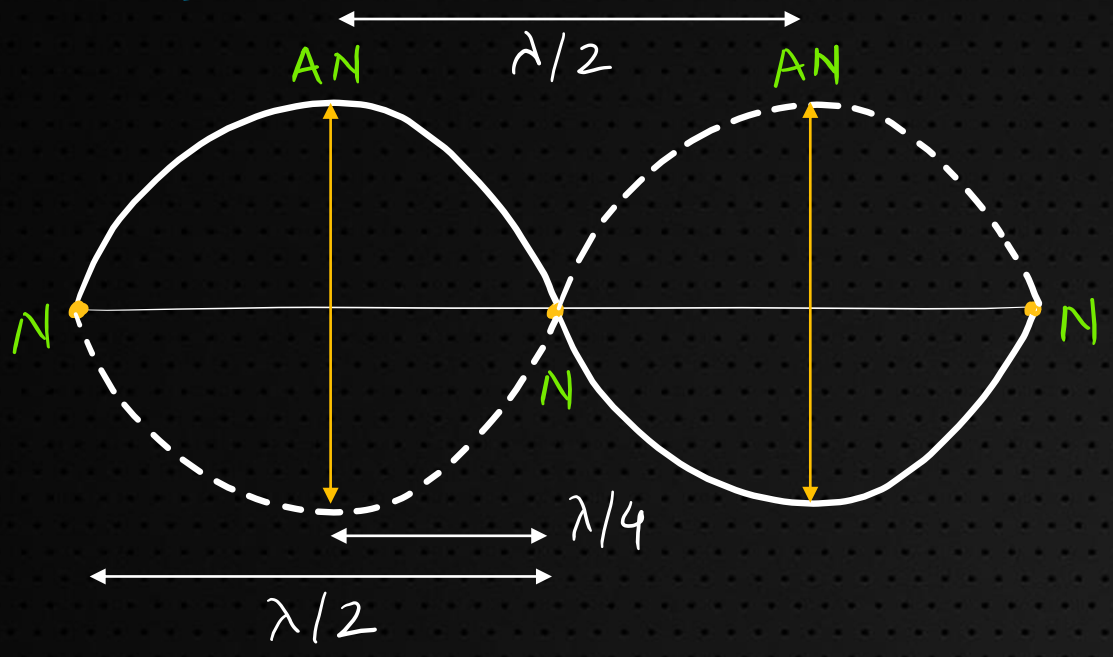
4. Standing waves or Stationary waves

↳ formed due to superposⁿ of two coherent waves travelling in opp direction

↳ both waves had 'A' Amp.



5. Key points in standing waves



(i) All Points between two successive NODES are in same phase

(ii) All points across a NODE are in opp phase $(\Delta\phi = \pi)$

(iii) separation bet^n two consecutive NODES or ANTINODES is $\lambda/2$.

(iv) and bet^n consecutive NODES and ANTINODE is $\lambda/4$.

6. Equation of standing Waves

$$\hookrightarrow \; y = \underbrace{2A\sin(Kx+\phi_1)}_{\text{Amplitude of a point at distance } x} \sin(\omega t+\phi_2)$$

$\phi_1 = 0$

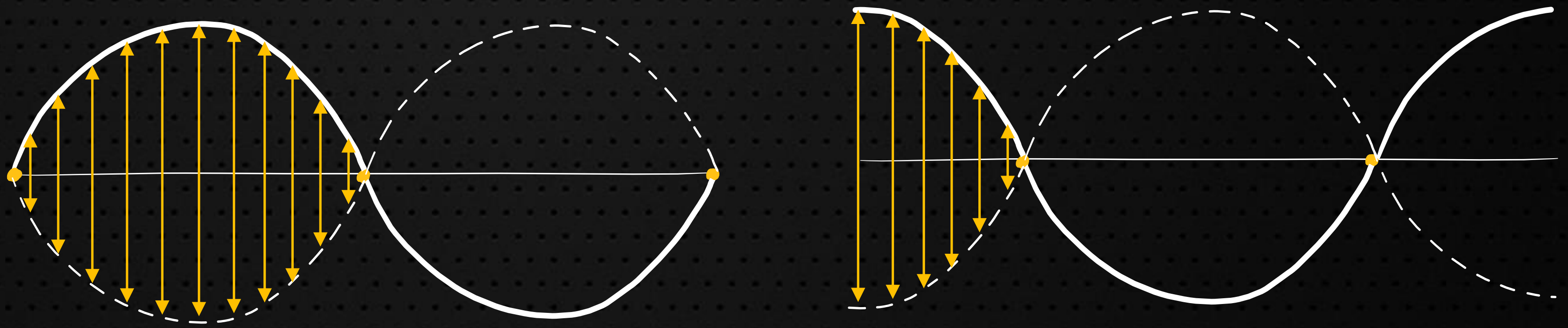
$\Rightarrow y = 2A\sin Kx \sin(\omega t+\phi_2)$

(at $x=0$, NODE)

$\phi_1 = \pi/2$

$y = 2A\cos Kx \sin(\omega t+\phi_2)$

(at $x=0$, AN)



7. Question on standing wave eqn

Ex1. Two travelling waves produces a standing wave represented by equation,

$$y = 1.0 \text{ mm} \cos(1.57 \text{ cm}^{-1})\, x \sin(78.5 \text{ s}^{-1})t.$$

The node closest to the origin in the region $x > 0$ will be at $x =$ .... cm.

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The node closest to the origin in the region $x > 0$ will be at $x =$ .... cm.

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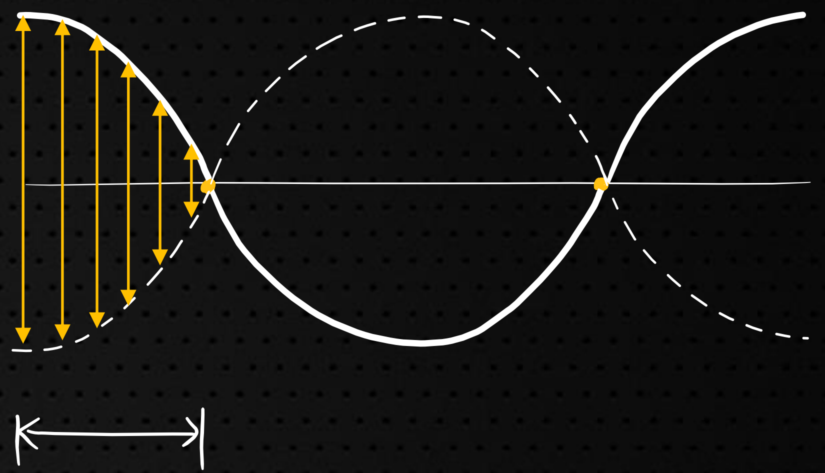
AN is at $x = 0$

Solⁿ:



$\frac{\lambda}{4}$

$K = 1.57$

$\Rightarrow \frac{2\pi}{\lambda} = 1.57 \Rightarrow \frac{\lambda}{2} = \frac{\pi}{1.57}$

$\therefore \frac{\lambda}{4} = \frac{\pi}{1.57 \times 2} = \boxed{1 \text{ cm}}$

8. Standing Wave in clamped string (fixed at both ends)

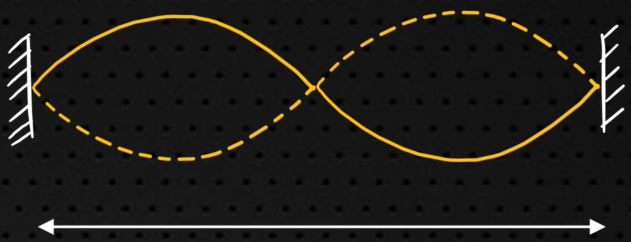
(a)

$L = \lambda/2$

$$f_1 = \frac{V}{\lambda} \Rightarrow f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

↳ fundamental freq
or
1st Harmonic

(b)



$L = \lambda$

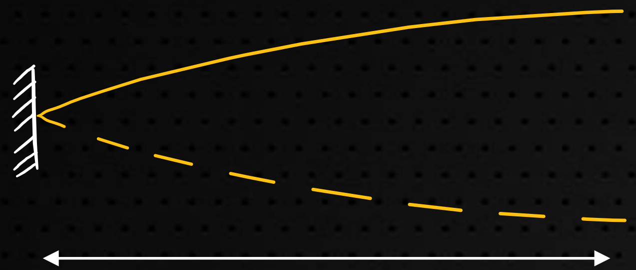
$$f_2 = \frac{1}{L}\sqrt{\frac{T}{\mu}} = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$$

→ 2nd Harmonic
→ 1st overtone

n^{th} Harmonic or $(n-1)^{th}$ Overtone $\Rightarrow f = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = nf_1$

NOTE: If $f_1 = 100\,Hz \Rightarrow$ String will be in resonance with tuning fork of 100, 200, 300 Hz...

9. Standing Wave in clamped string (fixed at one end)

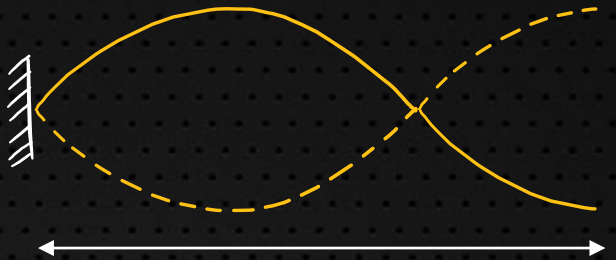


(a)

$L = \lambda/4$

$$f_1 = \frac{v}{\lambda} = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$$

→ Fundamental freq or 1st Harmonic



(b)

$L = 3\lambda/4$

$$f_3 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$$

→ 3rd Harmonic

→ 1st overtone

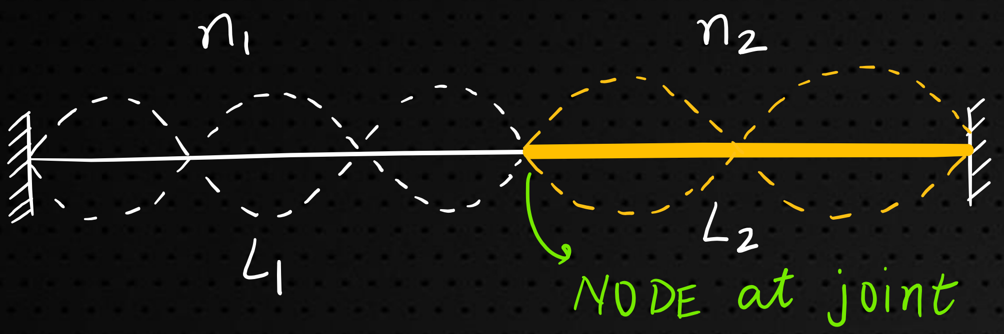
$$\boxed{f = \frac{(2n+1)}{4L}\sqrt{\frac{T}{\mu}} = (2n+1) f_1}$$

→ $(2n+1)^{th}$ Harmonic

→ n^{th} overtone

NOTE: If $f_1 = 100$ Hz ⇒ String will be in resonance with tuning fork of 100, 300, 500 Hz...

10. Standing Wave in Composite String



$$freq_{,1} = freq_{,2}$$

$$\Rightarrow \frac{n_1}{2L_1}\sqrt{\frac{T}{\mu_1}} = \frac{n_2}{2L_2}\sqrt{\frac{T}{\mu_2}}$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{L_1}{L_2} \times \sqrt{\frac{\mu_1}{\mu_2}}$$

will Learn its application in Example ahead

11. Questions on Standing Waves in Clamped string

Ex2. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to

(2019 Main, 10 Jan I)

(a) 16.6 cm (b) 33.3 cm (c) 10.0 cm (d) 20.0 cm

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(a) 16.6 cm (b) 33.3 cm (c) 10.0 cm (d) 20.0 cm

Soln: $\lambda = \frac{V}{f} = \frac{1}{f}\sqrt{\frac{T}{\mu}} = \frac{1}{100}\sqrt{\frac{8}{5\times10^{-3}/1}} = 0.4\,m$

$\therefore$ separation betn successsive nodes $= \frac{\lambda}{2} = 0.2\,m$

$= \boxed{20\,cm}$

↳ 5th Harmonic

... Continued

Ex3. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is (All quantities are in SI units) **(Main 2019, 9 April I)**

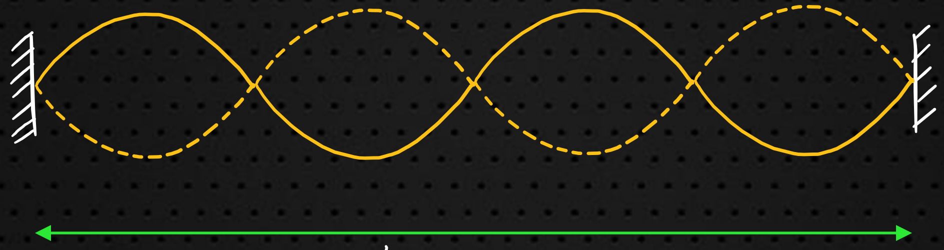
(a) 60 m (b) 40 m (c) 80 m (d) 20 m

... Continued

Ex3. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is (All quantities are in SI units) **(Main 2019, 9 April I)**

(a) 60 m (b) 40 m (c) 80 m (d) 20 m

Sol^n:



$L = 2\lambda$

$K = 0.157$

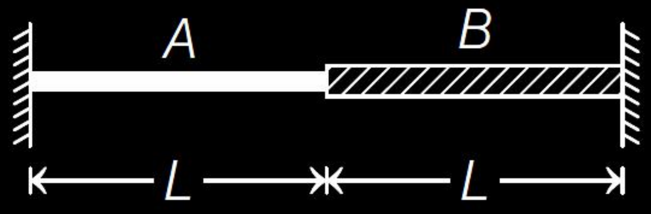
$\Rightarrow \frac{2\pi}{\lambda} = 0.157$

$\Rightarrow \lambda = \frac{2\pi}{0.157} = 40\,m$

$\therefore L = 2\lambda = \boxed{80\,m}$

... Continued

Ex 4. A wire of length $2L$, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q, then the ratio $p:q$ is **(2019 Main, 8 April I)**



(a) 3 : 5 (b) 4 : 9 (c) 1 : 2 (d) 1 : 4

... Continued

$f = \frac{n}{2L}\sqrt{\frac{T}{\rho\pi r^2}}$ $\mu = \frac{\rho\pi r^2 L}{L}$

f, T, ρ, L is same

$\Rightarrow n \propto \sqrt{r^2} \Rightarrow n \propto r$

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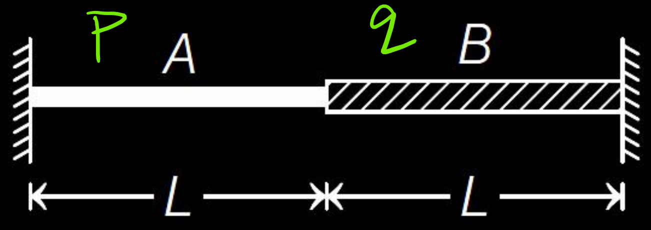
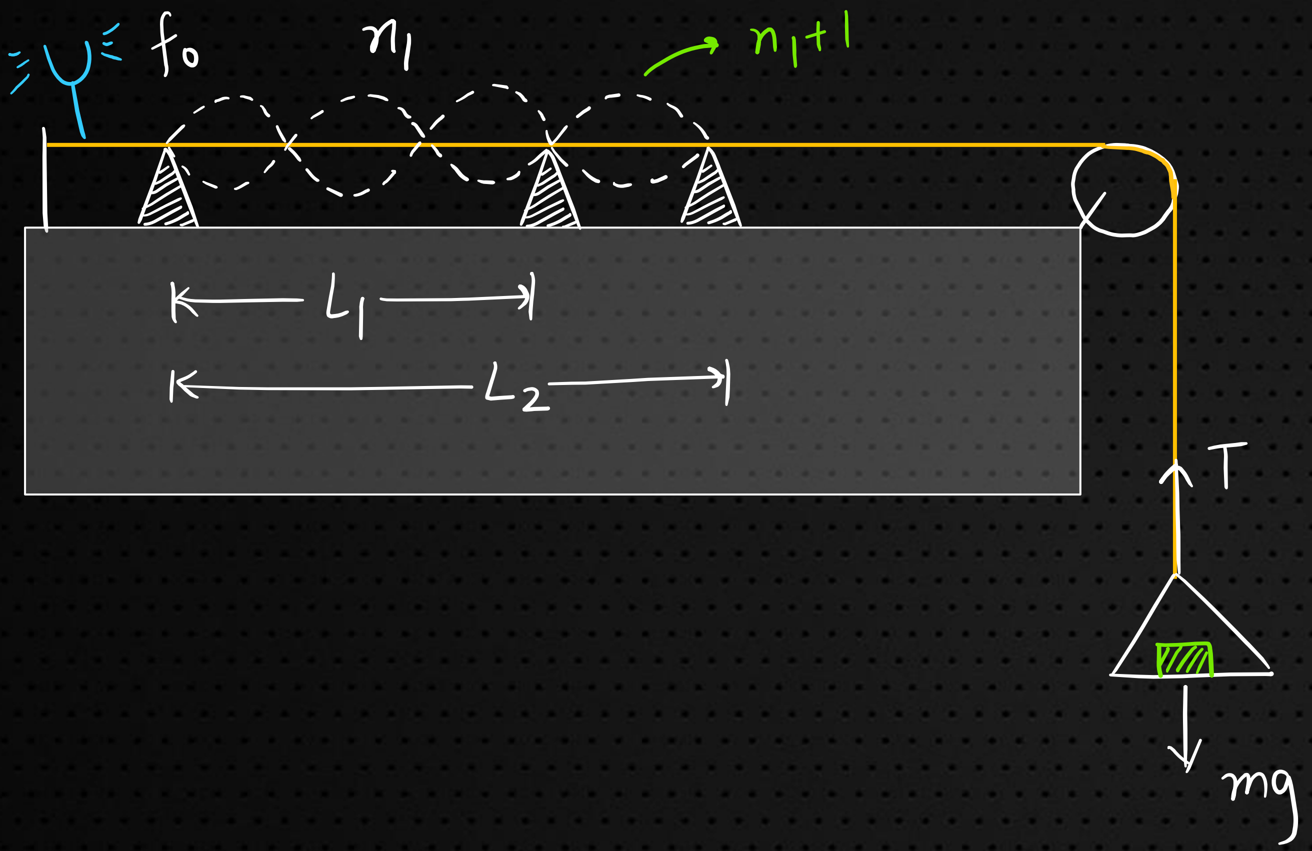


(a) 3 : 5 (b) 4 : 9 (c) 1 : 2 (d) 1 : 4

Soln:

$\Rightarrow \frac{n_1}{n_2} = \frac{r_1}{r_2} \Rightarrow \frac{p}{q} = \frac{r}{2r} \Rightarrow \boxed{\frac{p}{q} = \frac{1}{2}}$

12. Sonometer (to find wave speed in strings)



$$\therefore\ L_2 - L_1 = \frac{\lambda}{2}$$

$$\Rightarrow \lambda = 2(L_2 - L_1)$$

$$\therefore\ V = f_0\lambda = 2f_0(L_2 - L_1)$$

↳ Simply saying, it is standing waves betn 2 fixed ends.

13. Questions on Sonometer

Ex5. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M. The wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is **(2002, 2M)**

(a) 25 kg (b) 5 kg (c) 12.5 kg (d) 1/25 kg

13. Questions on Sonometer

Ex5. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M. The wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is **(2002, 2M)**

(a) 25 kg (b) 5 kg (c) 12.5 kg (d) 1/25 kg

Sol^n: $f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

Here f, L, μ are same

$\Rightarrow n\sqrt{T} = \text{const.}$

$\Rightarrow T \propto \frac{1}{n^2}$ $(T = Mg)$

$\Rightarrow M \propto \frac{1}{n^2}$

$\therefore \frac{M_2}{M_1} = \left(\frac{n_1}{n_2}\right)^2$

$\Rightarrow \frac{M}{9} = \left(\frac{5}{3}\right)^2$ $\therefore \boxed{M = 25\ \text{Kg}}$

... Continued

Ex 6. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively?

(a) 188.5 Hz (b) 178.2 Hz **(2013 Main)**

(c) 200.5 Hz (d) 770 Hz

... Continued

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(a) 188.5 Hz (b) 178.2 Hz **(2013 Main)**
(c) 200.5 Hz (d) 770 Hz

Sol^n:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$L = 1.5 + 0.01 \times 1.5 \approx 1.5m$$

$$\mu = \frac{m}{L} = \frac{\rho A L}{L} = \rho A$$

$$\frac{T}{A} = Y.\text{strain} \Rightarrow T = AY.\text{strain}$$

$$\therefore f = \frac{1}{2L}\sqrt{\frac{Y.\text{strain}}{\rho}} = \frac{1}{2 \times 1.5}\sqrt{\frac{2.2 \times 10^{11} \times 0.01}{7.7 \times 10^3}}$$

$$= \frac{1000}{3}\sqrt{\frac{2}{7}} \approx \boxed{178.2 \text{ Hz}}$$

PYQs LINKS (JEE MAIN)

2021 Feb	2021 March	2021 July	2021 August	2020
https://youtu.be/xcTzAJr-nOI	https://youtu.be/2BDGi9GUS3s	https://youtu.be/igmJzh_O_xc?t=14935	https://youtu.be/TQCDCnDhZ_I	https://youtu.be/Sr7YoXYAsjI

CLICK

(Practice these Questions)

Revision Series Playlist Link https://bit.ly/3eBbib9

JEE Main PYQs Link https://bit.ly/2S54jzh

Chapter wise 2021, 2020, 2018

GoldMine Link https://bit.ly/2VhOGFF

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