JEE MAIN | IIT JEE

ROTATION

(Part-2)

REVISION in 50 Min

Mohit Sir, IIT Kharagpur

EDUNITI

PART 2 - Topics to covered

1. K.E. in Rotational Motion
2. Questions on Rotation about Fixed Axis (*cons of Energy*)
3. Angular Momentum (*with 2 questions)*
4. Conservation of Angular Momentum (*Impulse Equation*)
5. Standard Examples of L = Const.
6. Fixed & Free Collision (*Concept + Numerical examples*)
7. Hinge Reaction

JOIN

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Chapter	Formulae_Concept VIDEO LINK
Unit & Dimensions	https://youtu.be/wdd-wlZF4Hk
Errors and Vectors	
Vernier Calliper	https://youtu.be/pVoN045dV8I
Screw Gauge	https://youtu.be/gYd2PtmZ0mw
Kinematics_Motion in 1d	https://youtu.be/U4NNxFaFliE
Kinematics_Motion in 2d	https://youtu.be/4_Zo5WhMf7w
Laws of Motion	https://youtu.be/7JlR8gNRQIs
Friction	https://youtu.be/Rn1bLst7eGk
Work Energy Power	https://youtu.be/kjrXoE-kDI8
Circular Motion	https://youtu.be/KnFymKHlkT0
Centre of Mass	https://youtu.be/ads35RKD618
Cons of Momentum & Collision	https://youtu.be/3f0u4L-lyyw
Rotational Motion – Part 1	https://youtu.be/O6j1mLp06XI
Gravitation	https://youtu.be/rAj2huLVaEk
Properties of Solids	https://youtu.be/gSXxjk89l_c
Fluids Statics (Part 1)	https://youtu.be/RFKx9B9yo3M
Fluid Dynamics (Part 2)	https://youtu.be/Y717vQpUEJQ
Fluid Properties (Part 3)	https://youtu.be/V8xUWWK2oT0
Simple Harmonic Motion	https://youtu.be/RIb7ofNG09I
Thermal Properties	https://youtu.be/OYjjyPIzddE
Heat Transfer	https://youtu.be/PyNboHgtYzM
KTG	https://youtu.be/XO1tvFhIa0I
Thermodynamics	https://youtu.be/iz_kf1jRDRw
Wave Motion -Organ Pipes and Resonance Tube	https://youtu.be/fB7pfJ77za8
Wave Motion - Doppler's Effect	https://youtu.be/9-BxOaamnwg
Electrostatics	https://youtu.be/3stXbGRMcrk
Capacitors	https://youtu.be/EXEiickNUkY
Current Electricity	https://youtu.be/gm8FUfjrX18
Moving Charges and Magnetic Effect of Current	https://youtu.be/ULD2Ok1CGJk
Earth's Magnetism	https://youtu.be/a4CT5uVwAK4
Magnetic Properties	https://youtu.be/63cwdYXNlYE
EMI	https://youtu.be/puVavm_GFRM
Alternating Current	https://youtu.be/74dTY-pzM_o
Ray Optics	https://youtu.be/BhnyTWzIlBA
Wave Optics Part 1_Interference	https://youtu.be/LG5nlE8XTeI
Wave Optics Part 2_Diffraction_Polarization	https://youtu.be/ymMyyJGGqnY
Optical Instruments	https://youtu.be/OQssbDH0A4I
Electromagnetic Waves	https://youtu.be/bcVXgEkyQZY
Semiconductors_Basics + Zener Diode	https://youtu.be/_A2JomQ7-50
Semiconductors_Transistors	https://youtu.be/psDwl84Nzb0
Semiconductors_Logic Gates	https://youtu.be/pZdQAzLbFTo
Communication Systems	https://youtu.be/8NgMqK9X79Y
Modern Physics_Part 1_Atomic Physics	https://youtu.be/9VKUnE3mpHk
Modern Physics_Part 2_Photoelectric Effect	https://youtu.be/24oTQp84jrk
Modern Physics_Part 3_Dual Nature of Light	https://youtu.be/0zoR_saMAQY
Modern Physics_Part 4_Radioactivity	https://youtu.be/AdX3YBhQyog
Modern Physics_Part 5_Nuclear Physics	https://youtu.be/VDWqVahGixc
Modern Physics_Part 6_X Rays	https://youtu.be/dSHXdzX7NX0

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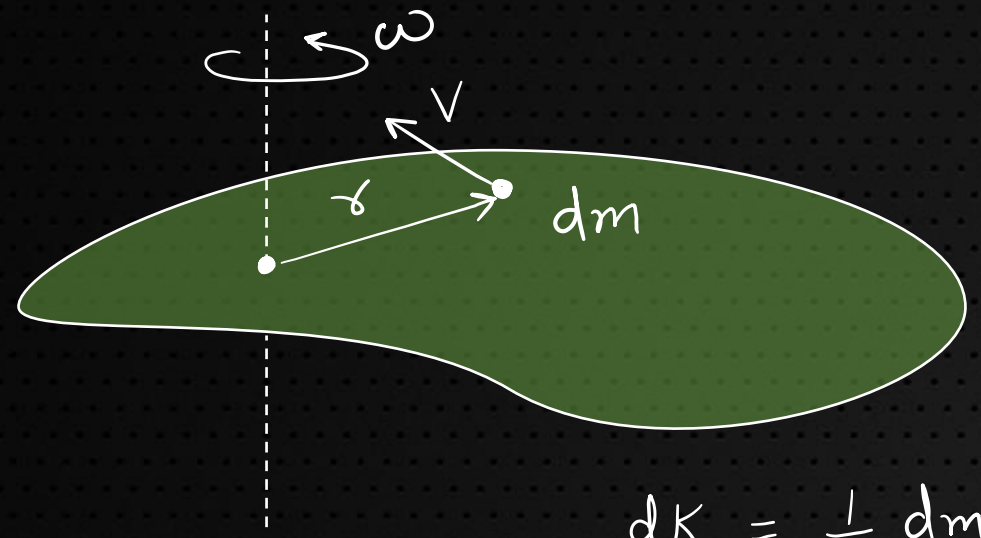
1. KE in Rotational Motion

(a)



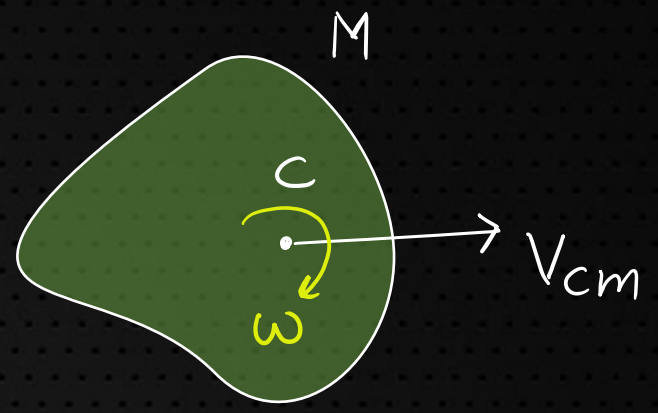
$$dK = \frac{1}{2} dm \cdot V^2$$

$$= \frac{1}{2} dm \cdot (\omega r)^2$$

$$\Rightarrow K = \frac{1}{2}\omega^2 \int dm \cdot r^2$$

$$\therefore \boxed{K = \frac{1}{2} I \omega^2}$$

(b)



here Axis of rotation passes COM

$$K = K_R + K_T$$

$$K = \frac{1}{2} I \omega^2 + \frac{1}{2} M V_{cm}^2$$

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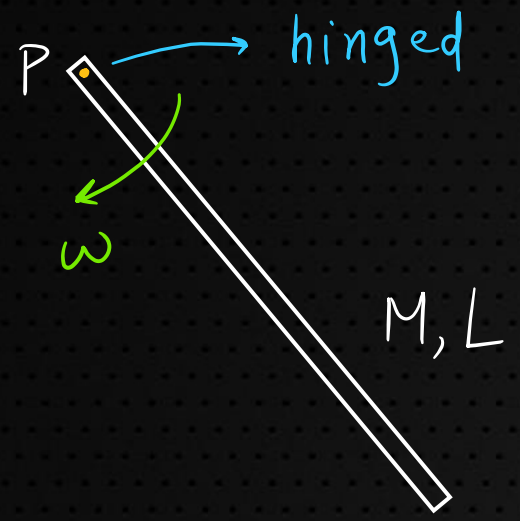
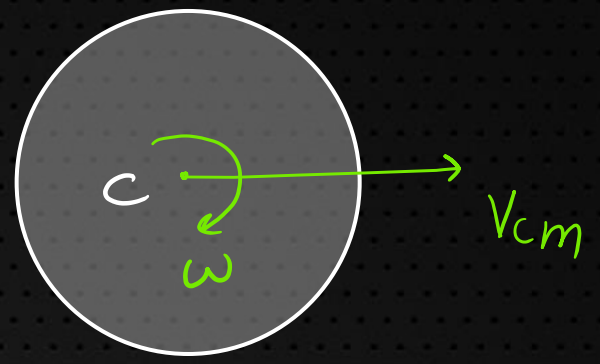
(i)



$$KE = \frac{1}{2} I_P \omega^2$$

$$= \frac{1}{2} \cdot \frac{ML^2}{3} \cdot \omega^2$$

(ii)



$$I_c = k m r^2$$

k → 1 if Ring / Hollow cyl.

$\frac{1}{2}$ if Disc / solid cyl.

$\frac{2}{3}$ if Hollow sphere

$\frac{2}{5}$ if solid sphere

$$KE = \frac{1}{2} I_c \omega^2 + \frac{1}{2} m V_{cm}^2$$

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2. Questions on Rotation about fixed axis (Energy conservation)

Ex 1. A metre stick is held vertically with one end on a rough horizontal floor. It is gently allowed to fall on the floor. Assuming that the end at the floor does not slip, find the angular speed of the rod when it hits the floor.

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2. Questions on Rotation about fixed axis
(Energy conservation)

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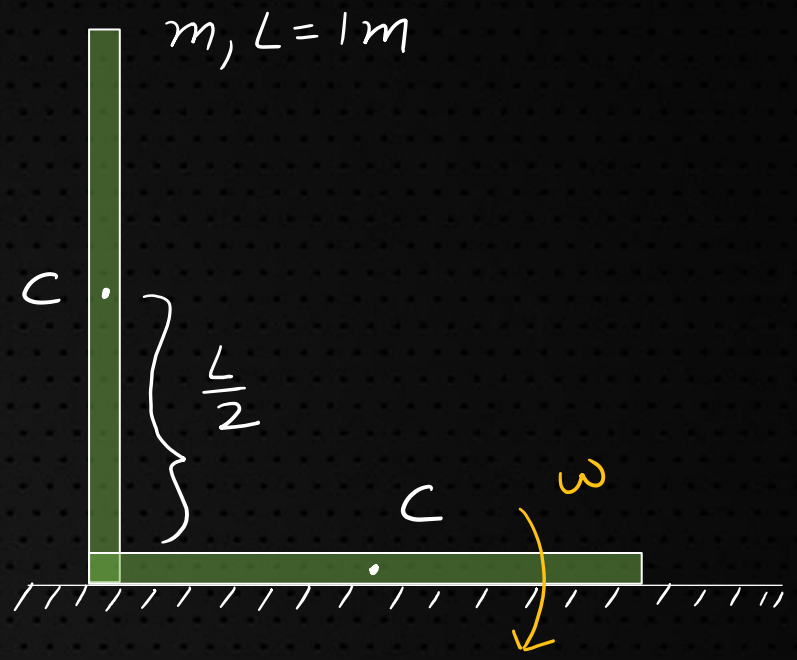
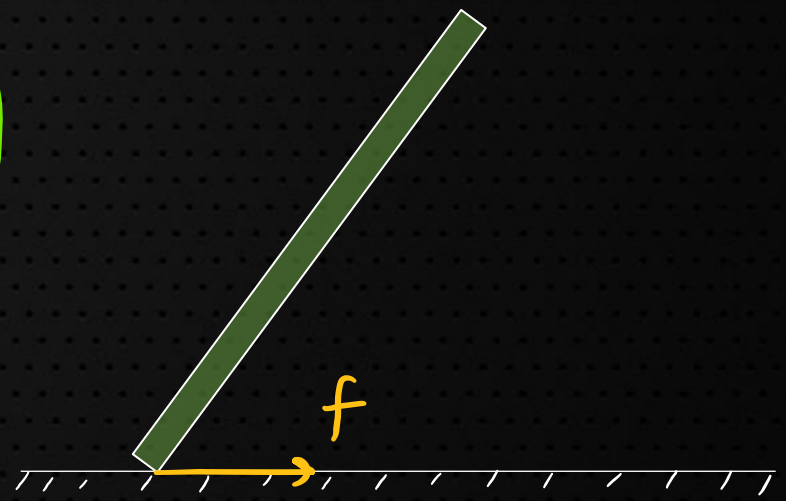


Sol^n: Loss in P.E = Gain in K.E

$$\Rightarrow mg\frac{L}{2} = \frac{1}{2}\cdot\frac{mL^2}{3}\omega^2$$

$$\therefore \omega = \sqrt{\frac{3g}{L}} = \boxed{\sqrt{30}\ \text{rad/s}}$$

Ans.



but $W_f = 0$

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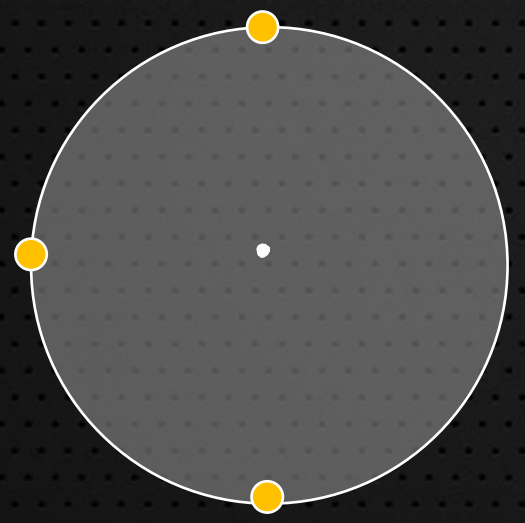
Ex2. A solid cylinder of mass M = 1kg & radius R = 0.5m is pivoted at its centre & has three particles of mass m = 0.1kg mounted at its perimeter as shown in the figure. The system is originally at rest. Find the angular speed of the cylinder, when it has swung through 90^0 in anticlockwise direction.



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Net PE Loss = Gain in KE

$$\Rightarrow mgR = \frac{1}{2}\left(\frac{MR^2}{2} + 3 \cdot mR^2\right)\omega^2$$

$$\therefore \boxed{\omega = \sqrt{5}\ \text{rad/s}}$$

Sol^n:

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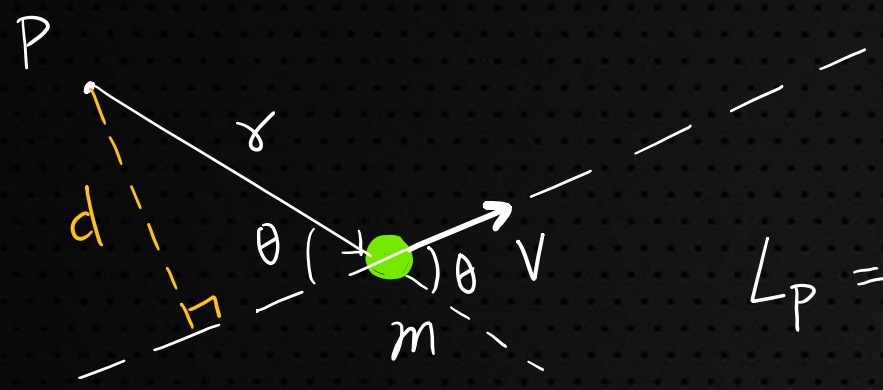
3. Angular Momentum (L)

(a) For point mass



$$L_P = mvd$$

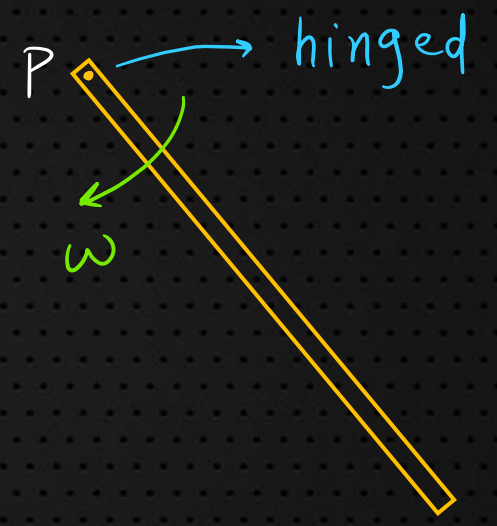
$$\# \vec{L} = \vec{r} \times \vec{P}$$
$$= \vec{r} \times m\vec{V}$$

$$\therefore L = mv\underbrace{r\sin\theta}_{d}$$

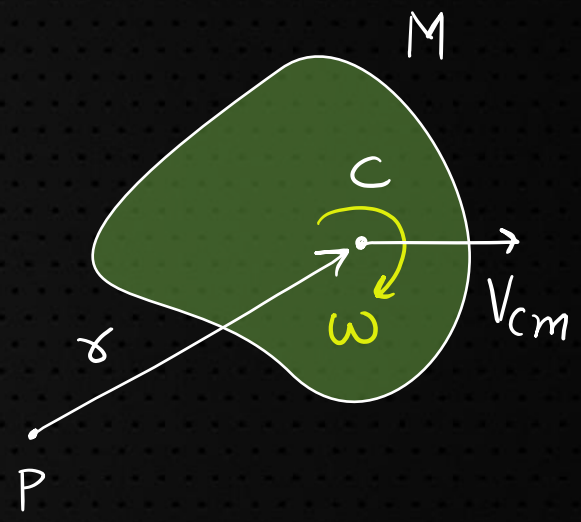
↳ For above fig: dir^n ⊙

(b) Rigid body

Pure rotation | rotation + translation



$$L_P = I_P \omega$$



$$\vec{L}_P = I_{cm}\vec{\omega} + \vec{r} \times \vec{P}_{cm}$$

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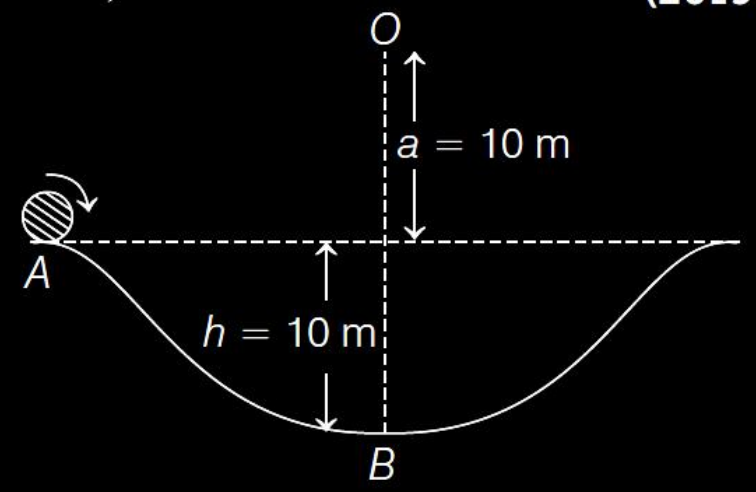
Ex 3. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be (Take, $g = 10\ \mathrm{m/s^2}$)

(2019 Main, 12 Jan II)



(a) $8\ \mathrm{kg\text{-}m^2/s}$

(b) $3\ \mathrm{kg\text{-}m^2/s}$

(c) $2\ \mathrm{kg\text{-}m^2/s}$

(d) $6\ \mathrm{kg\text{-}m^2/s}$

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...continued

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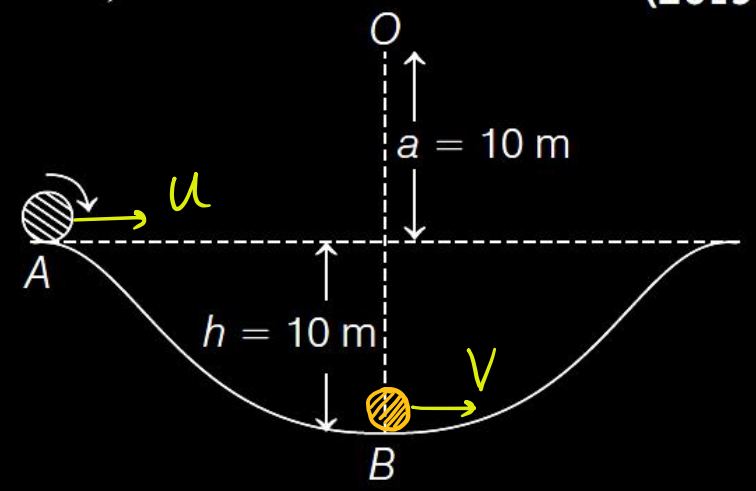

(a) 8 kg-m²/s
(b) 3 kg-m²/s
(c) 2 kg-m²/s
(d) 6 kg-m²/s

Soln: $A \rightarrow B$ Loss in PE = Gain in K.E

$$\Rightarrow mgh = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\therefore V = \sqrt{u^2 + 2gh}$$

For L_0, $d = a + h$

$$\Rightarrow L_0 = mvd = m\sqrt{u^2+2gh}\,(a+h)$$

$$= 0.02\sqrt{25+200} \times 20$$

$$= \boxed{6\ kg\,m^2/s}$$

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..continued

Ex4. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is

(a) zero

(b) $mv^3/(4\sqrt{2}\,g)$ **(1990, 2M)**

(c) $mv^3/(\sqrt{2}\,g)$

(d) $m\sqrt{2\,gh^3}$

..continued

Ex4. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is

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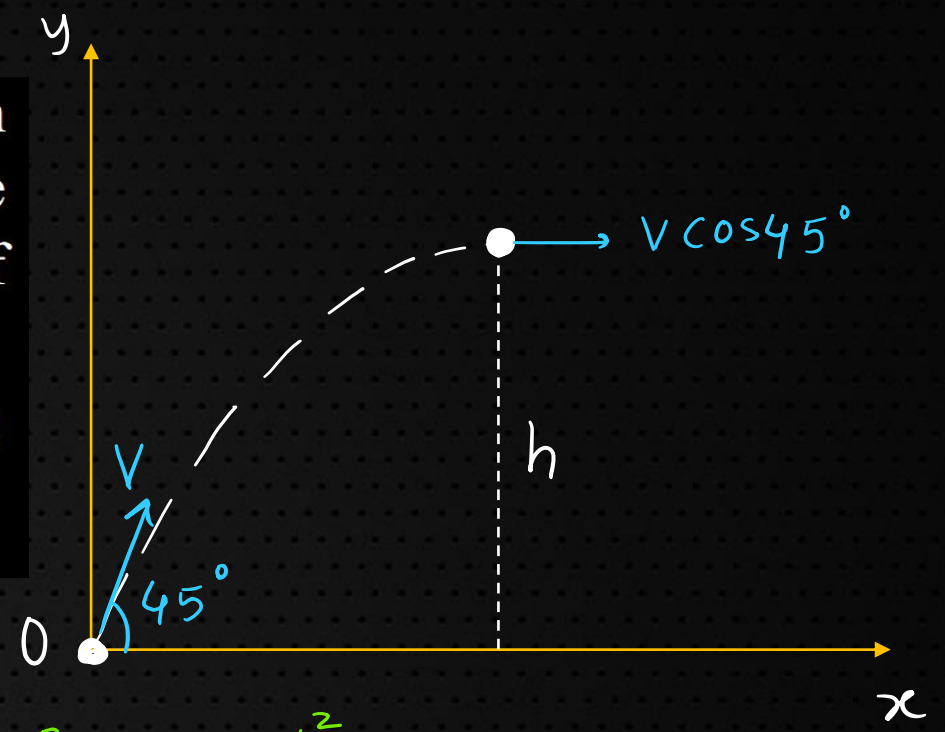
(c) $mv^3/(\sqrt{2}\,g)$ (d) $m\sqrt{2gh^3}$



Sol^n:

$$L_0 = mv\cos 45^\circ \cdot h$$

$$= \frac{mvh}{\sqrt{2}}$$

$$\left\{ h = \frac{u^2\sin^2\theta}{2g} = \frac{v^2}{4g} \right.$$

$$\therefore L_0 = \frac{mv}{\sqrt{2}} \times \frac{v^2}{4g} = \boxed{\frac{mv^3}{4\sqrt{2}g}}$$

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4. Conservation of Angular Momentum

(a) $\tau = \frac{dL}{dt}$

If $\tau_{ext} = 0 \Rightarrow L$ is const. or conserved

{similar to $F = \frac{dP}{dt}$}

(b) If $\tau_{ext} \neq 0$

$\Rightarrow \tau\, dt = dL$

or $\boxed{\int \tau\, dt = \Delta L}$

Angular impulse $= \Delta L$

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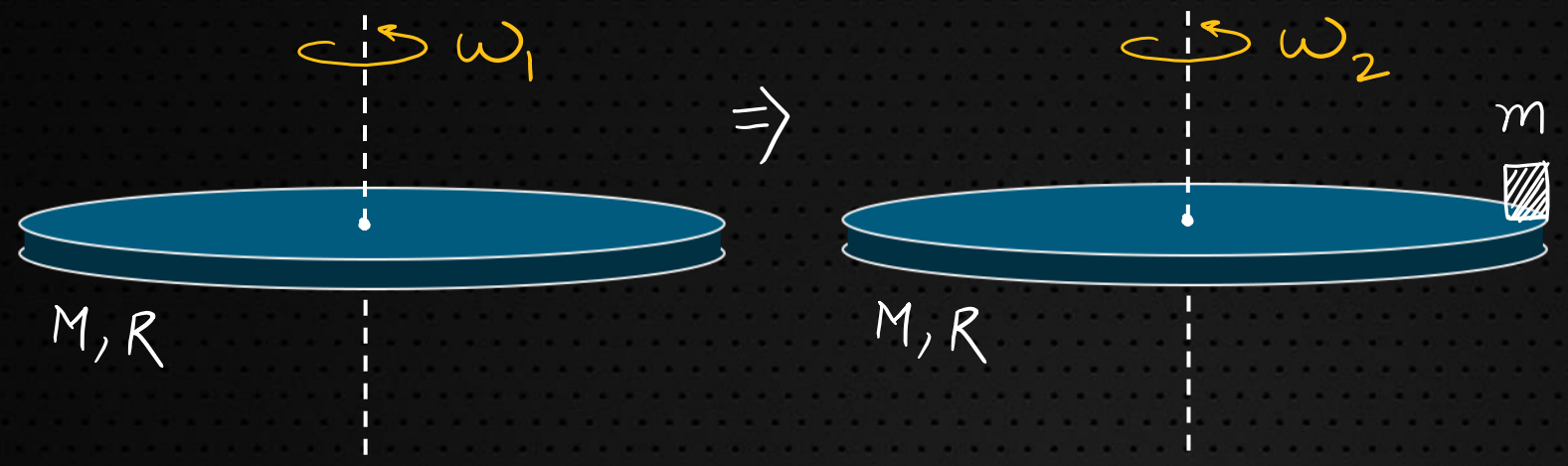
5. Standard Examples of L = const.

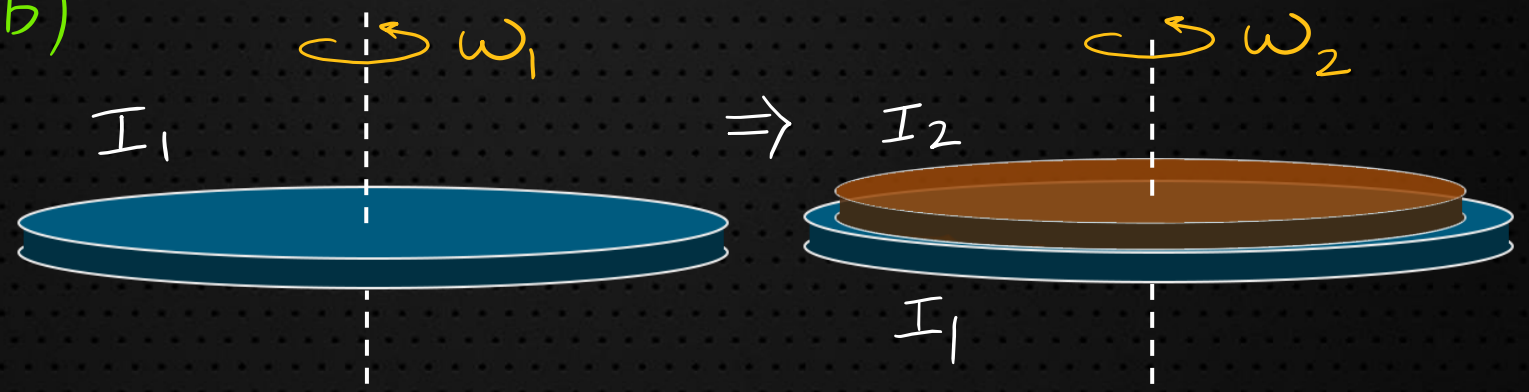
(a)



$$L_i = L_f$$

$$\Rightarrow \frac{MR^2}{2}\omega_1 = \left(\frac{MR^2}{2} + mR^2\right)\omega_2$$

(b)



(i) $I_1\omega_1 = (I_1 + I_2)\omega_2$

(ii) Loss in KE $= \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}(I_1 + I_2)\omega_2^2$

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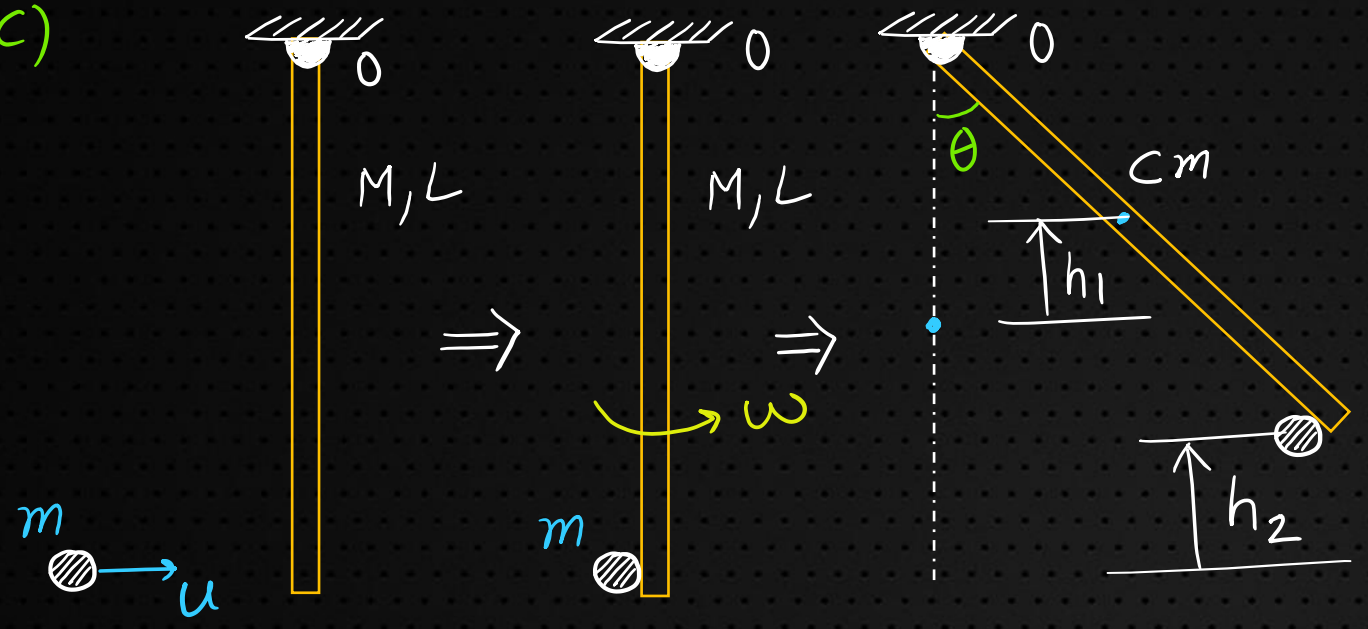
(C)



$$muL = \left(\frac{ML^2}{3} + mL^2\right)\omega \quad \text{—— (i)}$$

Gain in PE = Loss in KE

$$\Rightarrow Mgh_1 + mgh_2 = \frac{1}{2}\left(\frac{ML^2}{3} + mL^2\right)\omega^2$$

$h_1 \rightarrow \frac{L}{2}(1-\cos\theta)$

$h_2 \rightarrow L(1-\cos\theta)$

↳ ∵ τ_{net} about O is zero during impact

⇒ L is same just before & just after

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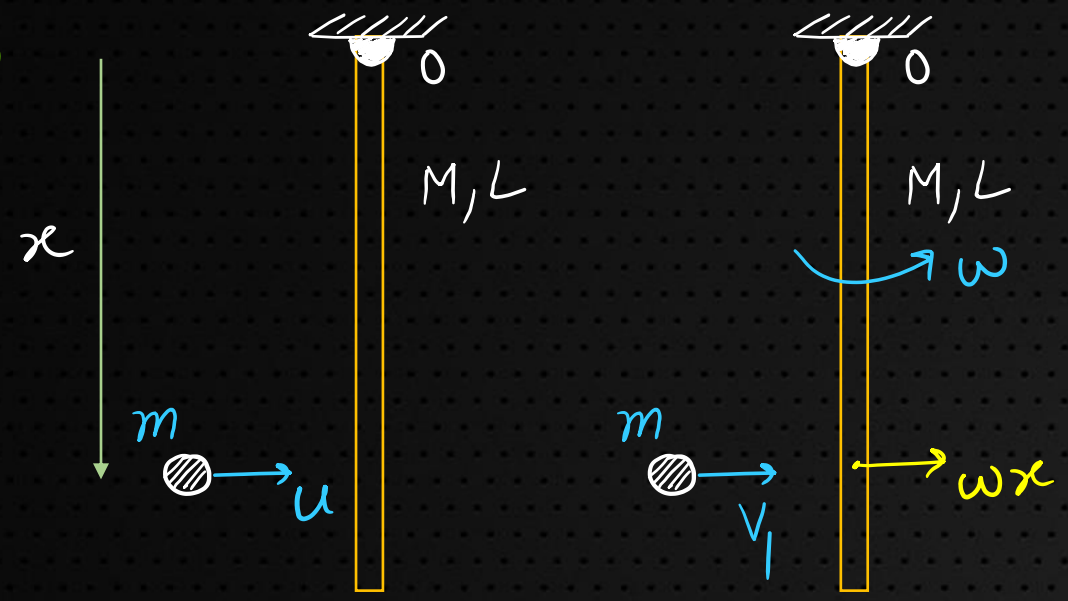
6. Fixed & Free Collision (e = Coefficient of restitution)

(a)



P is not Conserved

$L_i = L_f$

$$\Rightarrow mux = mV_1 x + \frac{ML^2}{3}\omega \quad \text{—(i)}$$

$$e = \frac{\text{Vel of seperation}}{\text{Vel of approach}}$$

$$e = \frac{\omega x - V_1}{u} \quad \text{—(ii)}$$

Hence solve (i) & (ii) to get V_1 and ω

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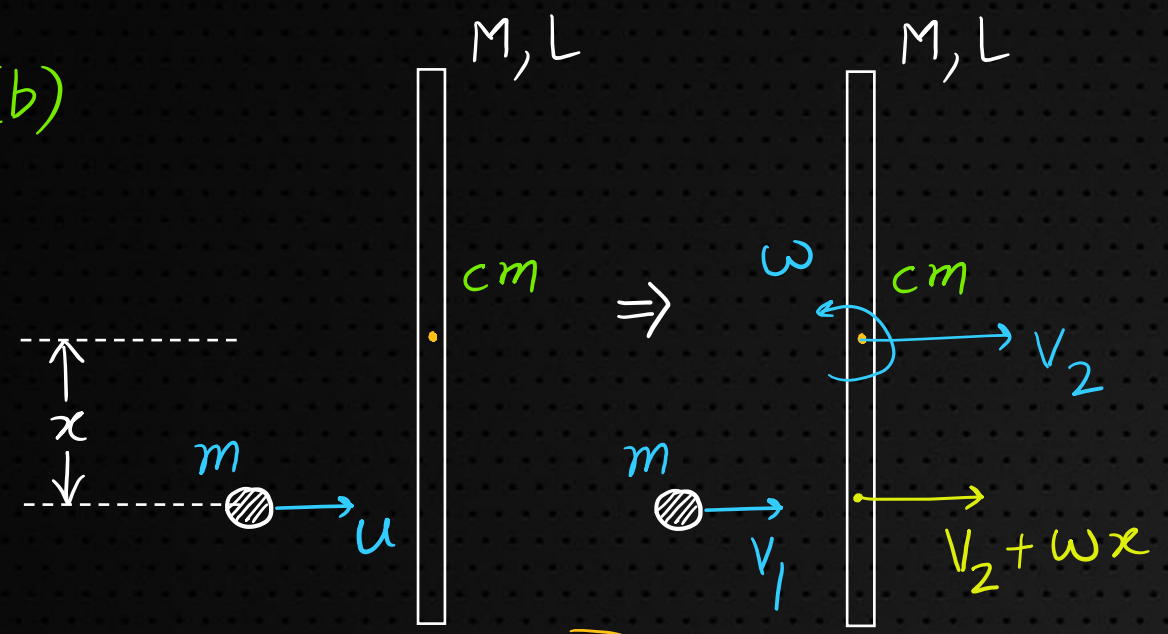
(b)



system on smooth Horizontal surface

(i) L is cons. about any point

About cm : $L_i = L_f$

$$\Rightarrow mux = mV_1 x + \frac{ML^2}{12}\omega \quad \text{— (1)}$$

(ii) P is cons. (As $F_{net} = 0$)

$$P_i = P_f \Rightarrow mu = mV_1 + MV_2 \quad \text{— (2)}$$

(iii) $$e = \frac{V_2 + \omega x - V_1}{u} \quad \text{— (3)}$$

solving (1), (2) & (3) we get V_1, V_2 and ω

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Ex 5.

A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of 'x' will be _____.

JEE 2021 - July Attempt

https://youtu.be/H_jH3GtD4Bs

Click for solution (Q2 in Video)

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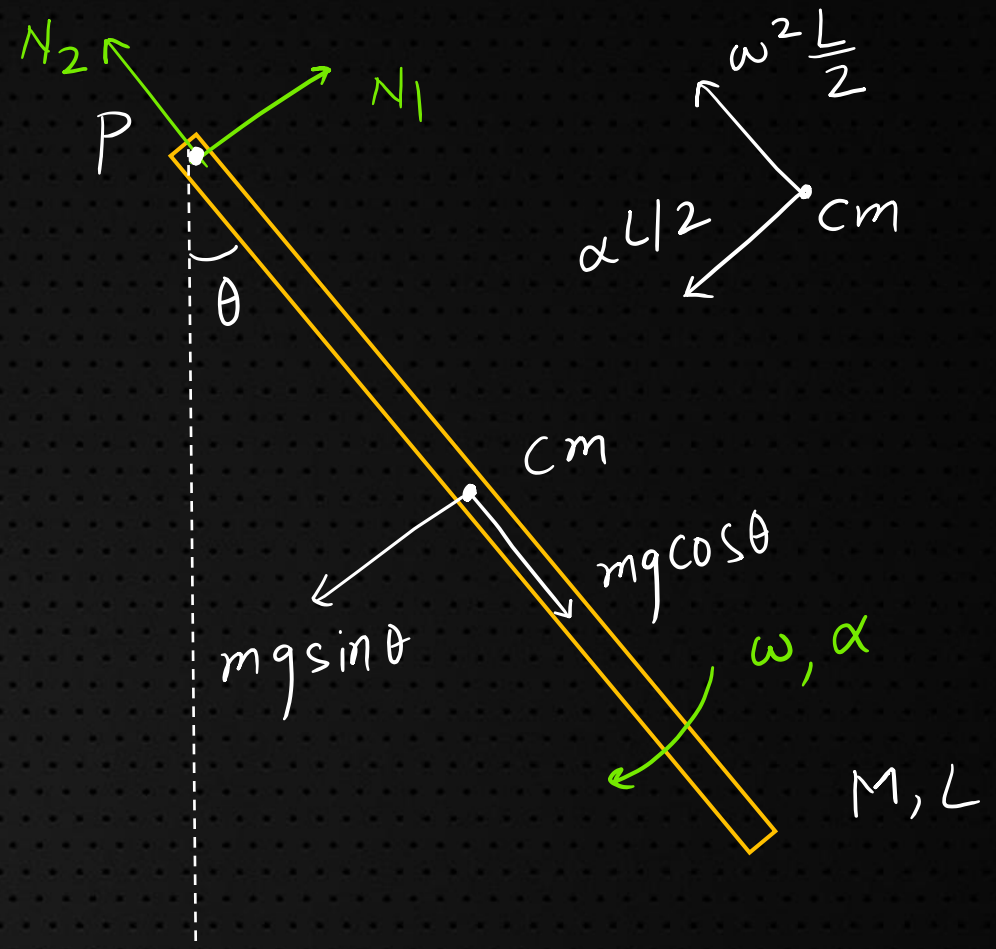
7. Hinge reaction

(i) Always take 2 ⊥ components of Hinge rxn

↳ $mg\sin\theta - N_1 = m\alpha\frac{L}{2}$

↳ $N_2 - mg\cos\theta = m\omega^2\frac{L}{2}$

$\therefore$ rxn $= \sqrt{N_1^2 + N_2^2}$



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Ex 6.

Rod is released from rest. Find hinge rxn when rod becomes vertical.

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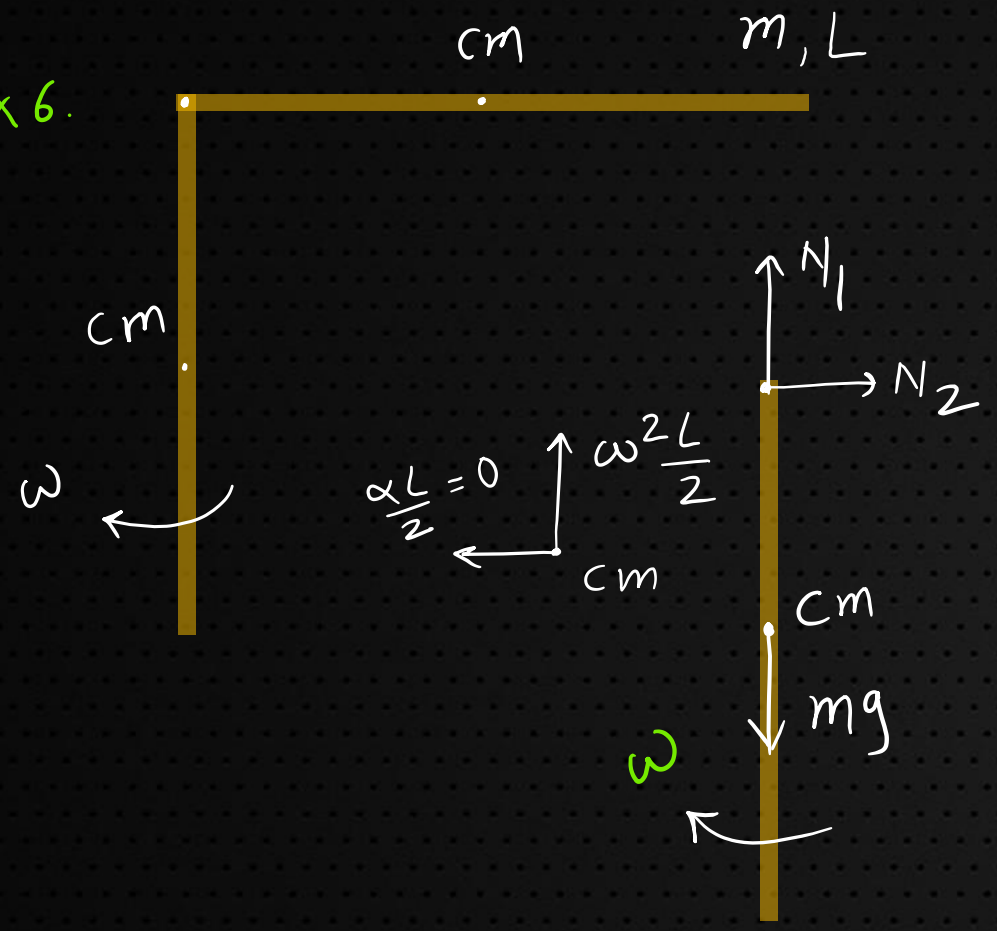
Ex 6.



Rod is released from rest. Find hinge rkⁿ when rod becomes vertical.

Solⁿ: $mg\frac{L}{2} = \frac{1}{2}\frac{mL^2}{3}\omega^2 \Rightarrow \omega^2 = 3g/L$ — ①

∵ τ due to N_1, N_2 & mg about hinge is zero ⇒ $\boxed{\alpha = 0}$

$\Rightarrow N_1 - mg = m\omega^2\frac{L}{2}$

$\Rightarrow N_1 = m\frac{3g}{L}\cdot\frac{L}{2} + mg = \boxed{\frac{5}{2}mg}$

and, $\boxed{N_2 = 0}$

∴ rkⁿ = N_1

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PYQs LINKS (JEE MAIN)

2021 Feb	2021 March	2021 July	2021 August	2020	Selected Problems
https://youtu.be/2hqJnity2_o	https://youtu.be/6CsOsazSvmA	https://youtu.be/H_jH3GtD4Bs	https://youtu.be/wmlghKWPUQ	https://youtu.be/zWddLCElwWE	https://youtu.be/wjBRVQih400

CLICK

(Practice these Questions)

HCV Rotation Solution Q1-Q70

https://bit.ly/3fM9fCC

Revision Series Playlist Link https://bit.ly/3eBbib9

JEE Main PYQs Link https://bit.ly/2S54jzh Chapter wise 2021, 2020, 2018

GoldMine Link https://bit.ly/2VhOGFF

CLICK