
#7
Variable Mass
" Center of Mass "
PhD Series
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The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 ms^{-2}. The gases come out at a relative speed of 500 ms^{-1} with respect to the rocket :[Use $g = 10\ m/s^2$]

(1) 6.0×10^2 kg s^{-1} (2) 500 kg s^{-1}

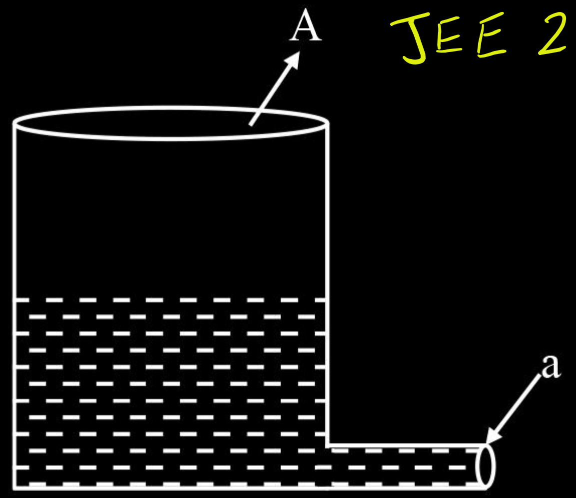
(3) 10 kg s^{-1} (4) 60 kg s^{-1}

https://youtu.be/LIKVg9txoas JEE 2021, Aug

Q1.

A light cylindrical vessel is kept on a horizontal surface. Area of base is A. A hole of cross-sectional area 'a' is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is (a < < A) :

JEE 2021, July



(1) $\frac{A}{2a}$ (2) None of these

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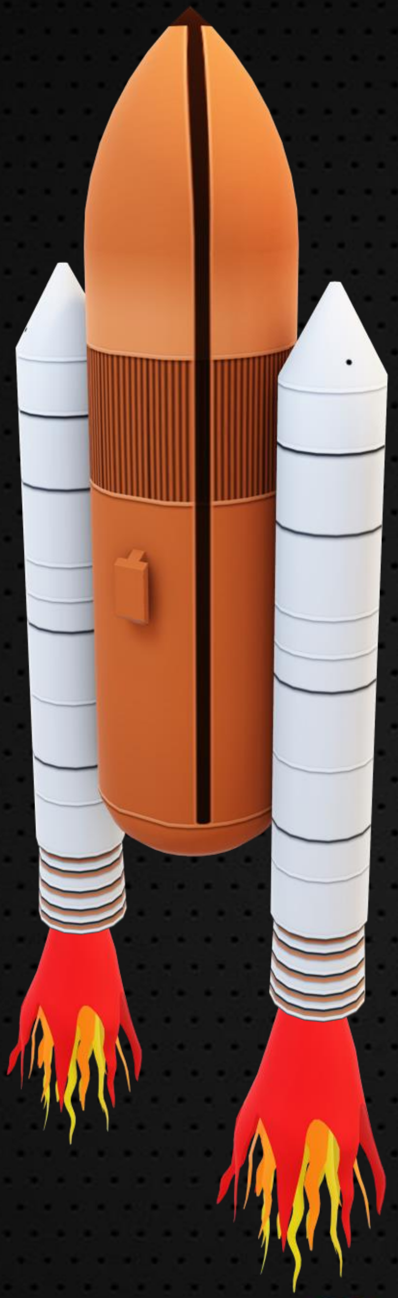
Q7.

https://youtu.be/lkwiZzOCTgE



PhD SERIES

VARIABLE MASS

1. Expression of Thrust Force
2. Rocket Propulsion Equation
3. Chain Falling on Floor
4. Water coming out of Pipe / Vessel

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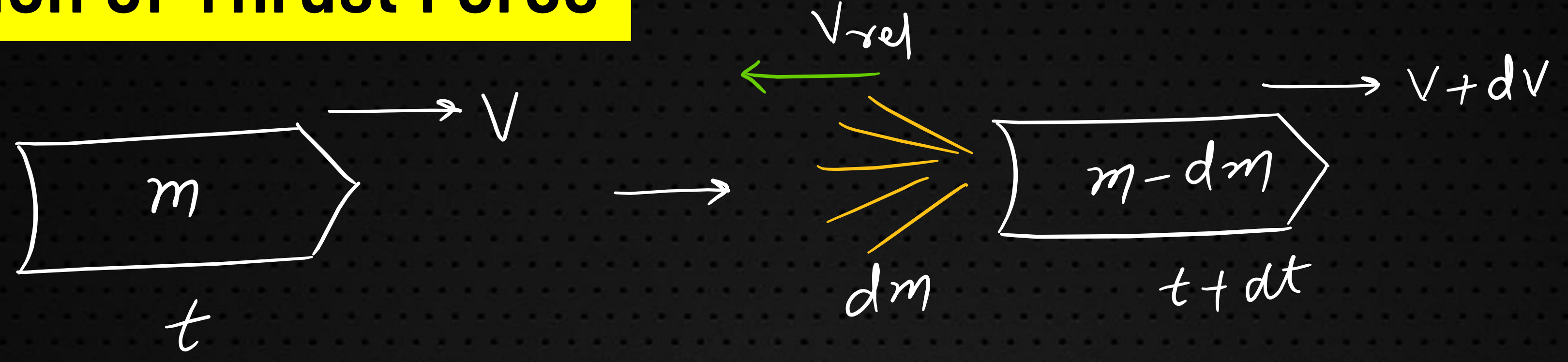
Eduniti for Physics

1. Expression of Thrust Force



Applying $P = \text{const}$ as no F_{ext}:

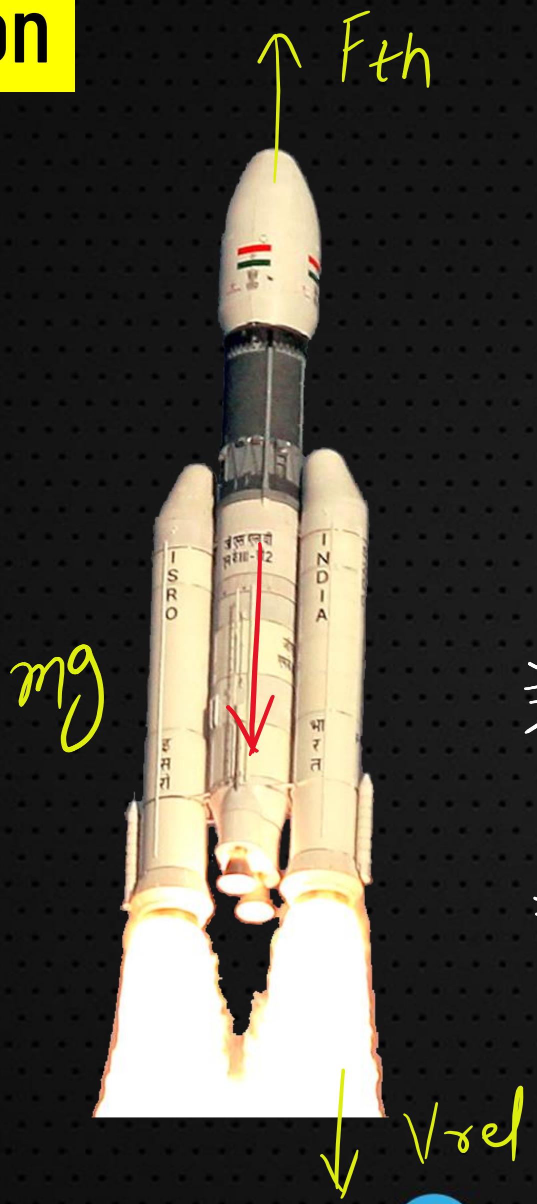
$$mV = (m-dm)(V+dV) - dm\left[V_{rel} - (V+dV)\right]$$

$$\Rightarrow \cancel{mV} = \cancel{mV} + m\,dV - \cancel{V\,dm} - \cancel{dm.dV} - V_{rel}\,dm + \cancel{V\,dm} + \cancel{dm.dV}$$

$$\Rightarrow m\,dV = V_{rel}.dm \quad \Rightarrow m\frac{dV}{dt} = V_{rel}.\frac{dm}{dt}$$

$$\Rightarrow \boxed{F_{thrust} = V_{rel}.\frac{dm}{dt}}$$

2. Rocket Propulsion



$$F_{net} = F_{th} - mg$$

$$\Rightarrow m\frac{dv}{dt} = -V_{rel}\frac{dm}{dt} - mg$$

$$\Rightarrow \int_u^V dv = -V_{rel}\int_{m_i}^{m_f}\frac{dm}{m} - g\int_0^t dt$$

$$\Rightarrow V - u = V_{rel}\ln\frac{m_i}{m_f} - gt$$

$$\Rightarrow \boxed{V = u - gt + V_{rel}\ln\frac{m_i}{m_f}}$$

Q1. A rocket with an initial mass of 1000 kg is launched vertically upwards from rest under gravity. The rocket burns the fuel at the rate of 10 kg s^{-1}. The burnt matter is ejected vertically downwards with a speed of 2000 ms^{-1} relative to the rocket. If burning is complete after 1 min, find the maximum velocity of rocket. (Given, $g = 10\ ms^{-2}$ and $\ln 2.5 = 0.916$)

✓(a) 1232 ms^{-1}	(a) 1000 ms^{-1}
(a) 1423 ms^{-1}	(a) 1523 ms^{-1}

$$\boxed{V = u - gt + V_{rel} \ln \frac{m_i}{m_f}}$$

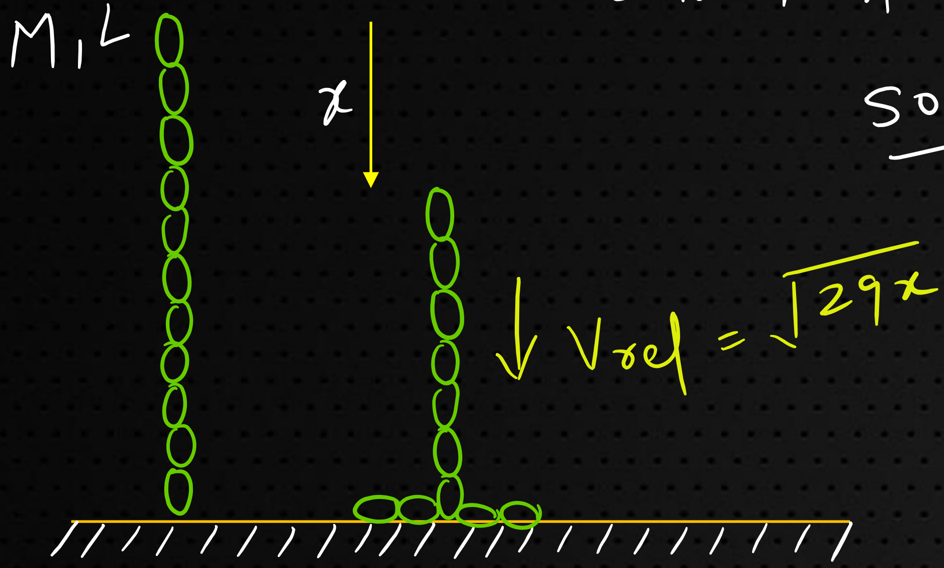
Sol^n: $u = 0$, $m_i = 1000$ kg,

$$m_f = m_i - t_0 \times \frac{dm}{dt}$$
$$= 1000 - 60 \times 10$$
$$= 400 \text{ kg}$$

$$\therefore V = 0 - 10 \times 60 + 2000 \ln \frac{1000}{400}$$
$$= -600 + 2000 \times 0.916$$
$$= 1232 \text{ m/s}$$

3. Chain Falling

Find total force acting on floor when x Length of chain has fallen.



Sol^n: F = force due to weight of x Length + F_{th}

$$= \frac{M}{L} x g + V_{rel} \frac{dm}{dt}$$

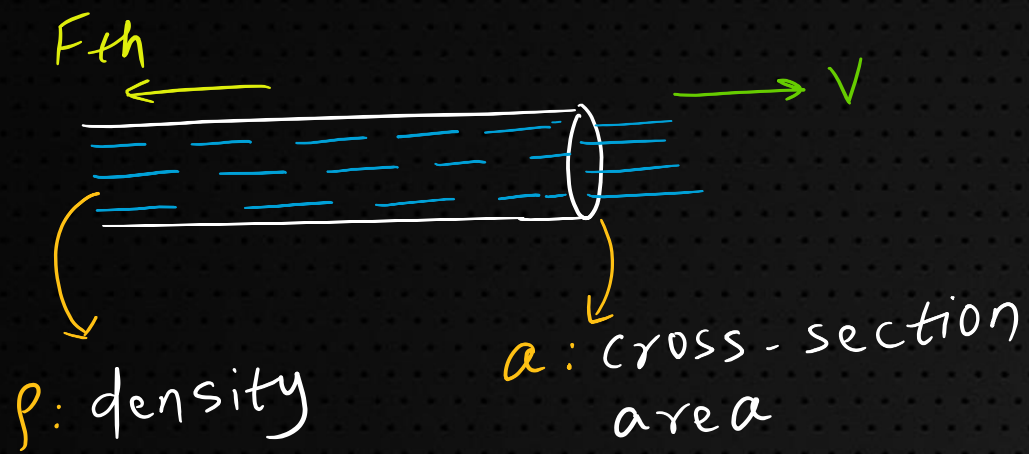
$$\lambda = \frac{M}{L}$$

$$= \frac{Mgx}{L} + V_{rel} \times \lambda \frac{dx}{dt}$$

$$= \frac{Mgx}{L} + \lambda V_{rel}^2$$

$$= \frac{Mgx}{L} + \frac{M}{L} \times 2gx = \boxed{\frac{3Mgx}{L}}$$

4. Water coming out of Pipe/Vessel



$$F_{th} = V_{rel}\frac{dm}{dt}$$

$$= v \times \rho \frac{dV}{dt}$$

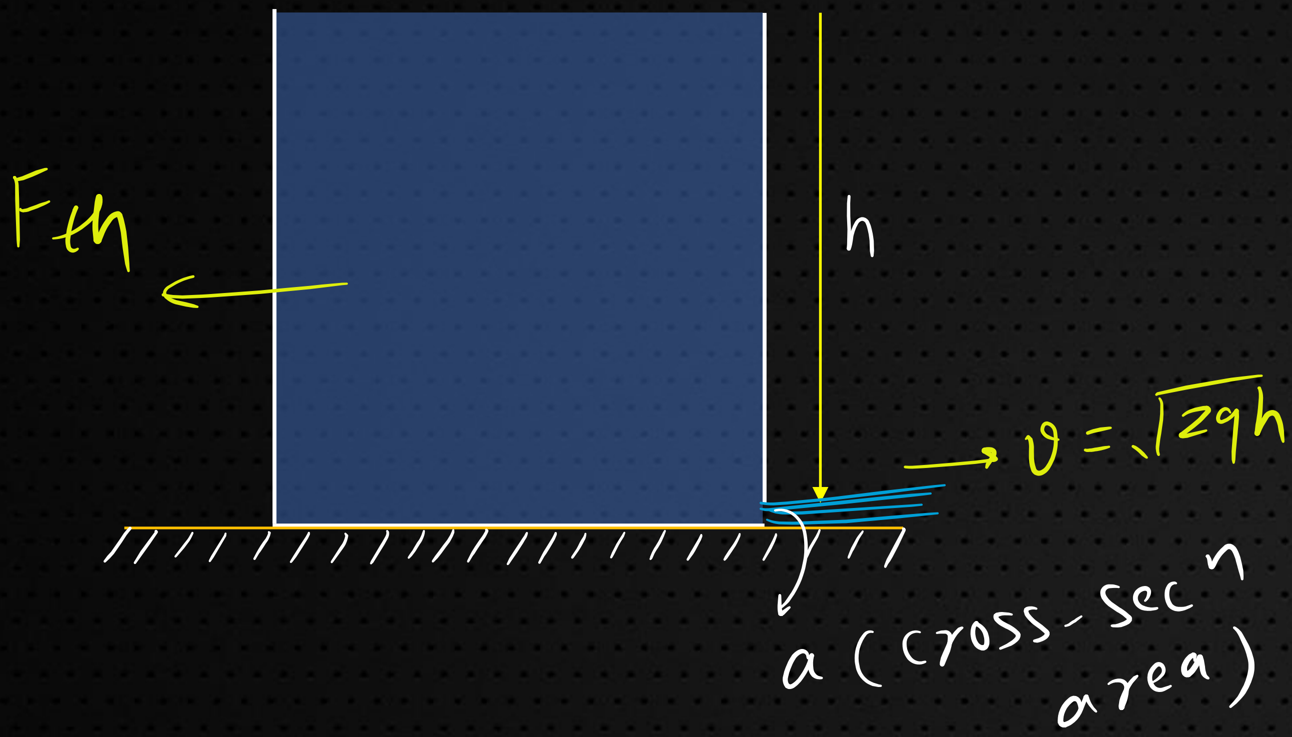
small volume

$$= v \times \rho\, a \frac{dx}{dt}$$

v

$$= \boxed{\rho a v^2}$$

4. Water coming out of Pipe/Vessel



$$F_{th} = \rho a v^2$$
$$= \rho a \times 2gh$$

Q2. A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $A/100$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass m_0. Assuming that the liquid starts flowing out horizontally through the hole at $t = 0$. Calculate **(1997 C, 5M)**

the acceleration of the container

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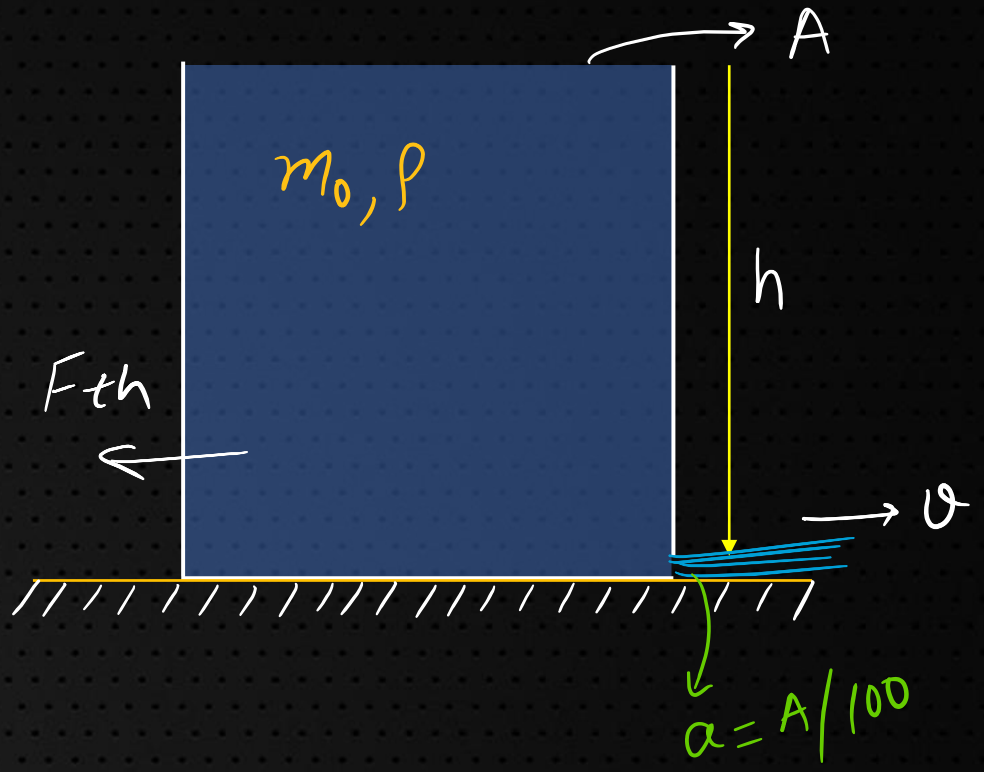

Sol^n:

$$F_{net} = F_{th}$$

$$\Rightarrow m_0 \times acc^n = \rho \times \frac{A}{100} \times v^2$$

$$\Rightarrow \cancel{\rho} \times \cancel{A}\cancel{h} \times acc^n = \cancel{\rho} \frac{\cancel{A}}{100} \times 2g\cancel{h}$$

$$\Rightarrow \boxed{acc^n = \frac{g}{50}}$$

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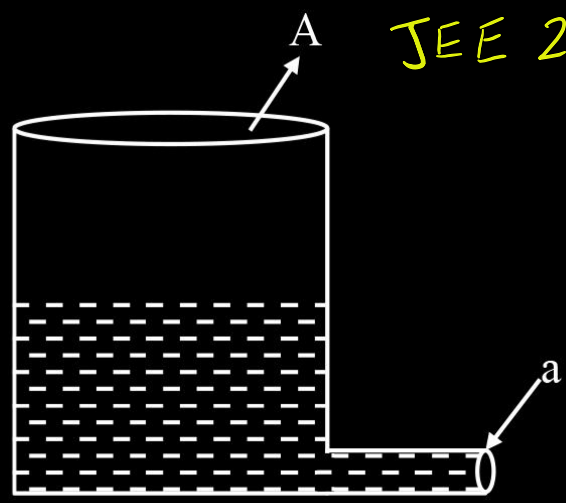

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PYQs (2020, 2021)

Concept Videos

Advanced problems

Part and Full Test

PhD Series

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