FLUID MECHANICS - PART 2

FLUID DYNAMICS

REVISION in 30 Min

Now that's how you REVISE

-Mohit Goenka, IIT Kharagpur

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FLUID MECHANICS

FLUID STATICS

Fluid is at rest with respect to vessel

(Part 1)

FLUID DYNAMICS

Fluid is in motion w.r.t vessel / Pipe

(Part 2)

FLUID PROPERTIES

Surface tension and viscosity

(Part 3)

Revision Series Playlist Link https://bit.ly/3eBbib9

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Topics to cover in Fluid Dynamics

1. Types of Flow *(Streamline, Turbulent)*
2. Reynolds Number
3. Ideal Fluid *(4 assumptions)*
4. Equation of Continuity *(ex: water falling from Tap)*
5. Bernoulli's Equation
6. Pressure-speed Relation *(horizontal flow)*
7. Speed of Efflux – Torricelli Law
8. Speed of Efflux *(important other case)*
9. Vessel emptying time *(y as a function of time)*
10. Thrust on Vessel *(liquid coming out)*
11. Venturimeter

1. Streamline & Turbulent flow

(Laminar or steady flow)

↳ velocity of fluid at a point is always same

↳ random flow / mixing of flow

↳ vel of fluid at any point changes

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2. Reynolds Number (R_e)

$$R_e = \frac{\rho v d}{\eta}$$ → Dimensionless qty.

ρ → density of liquid
v → speed
d → pipe cross sec^n dia
η → coeff. of viscosity

$R_e < 1000$ → streamline flow
$R_e > 2000$ → Turbulent flow
$1000 < R_e < 2000$ → Unsteady flow
↳ As mentioned in NCERT

3. Ideal Fluid

(i) fluid is incompressible (ρ is const.)
(ii) non-viscous (no fluid friction)
(iii) streamline flow
(iv) no rotational effect

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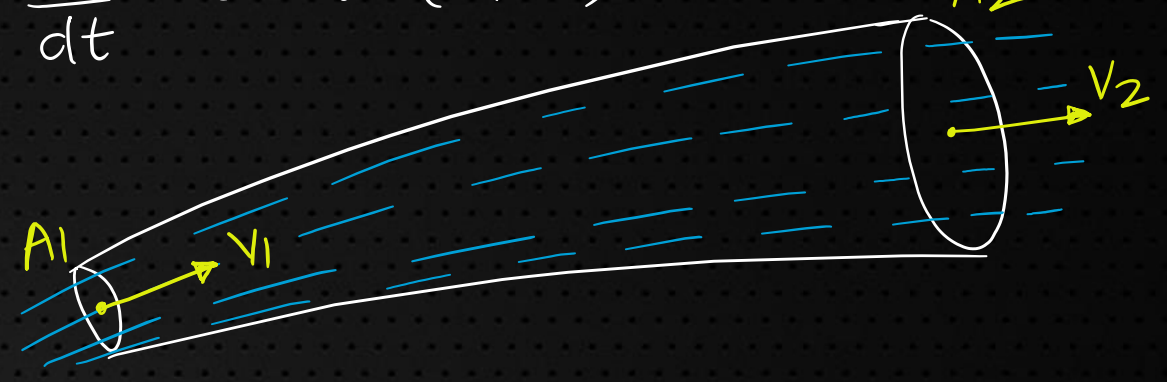
4. Eq^n of continuity (incompressible)

↳ Volume flow rate is constant, $\frac{dV}{dt} = \text{const.}$ (m^3/s)

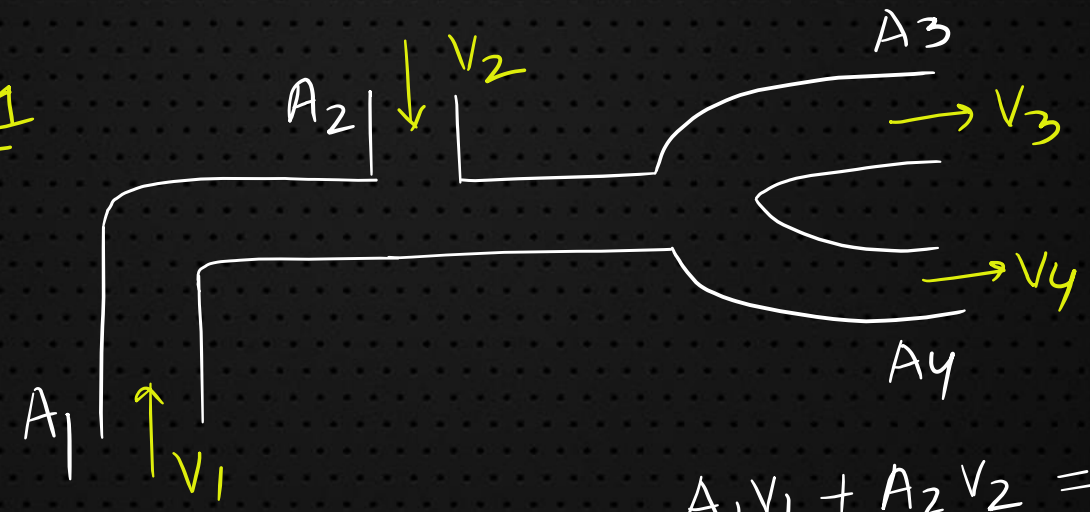
$$\Rightarrow \frac{A\,dx}{dt} = \text{const.} \Rightarrow \boxed{A v = \text{const.}}$$

$$\Downarrow$$

$$A_1 v_1 = A_2 v_2$$



Ex.1



$$A_1 V_1 + A_2 V_2 = A_3 V_3 + A_4 V_4$$

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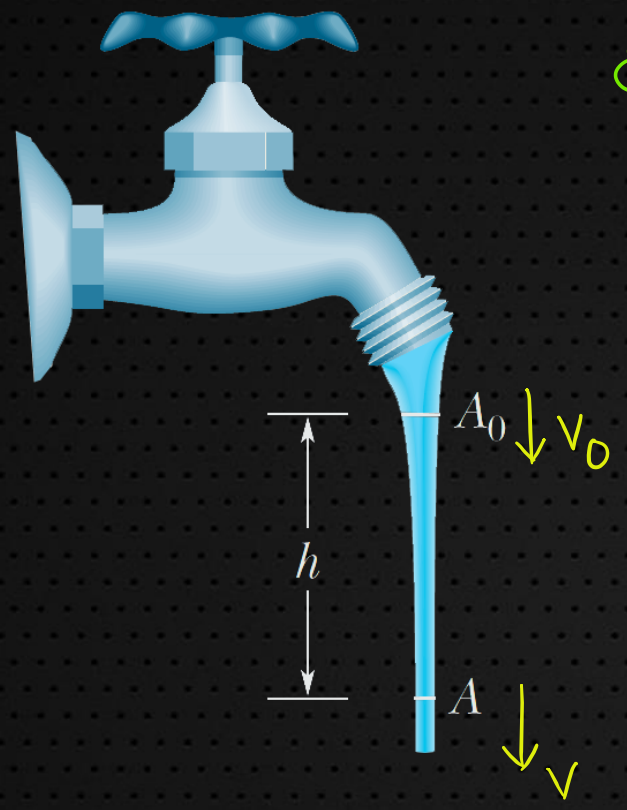
Ex.2



Eq^n of continuity: $A_0 V_0 = A V$ — (1)

$\because$ free fall : $V^2 = V_0^2 + 2gh$ — (2)

Find A : from (1) & (2)

$$A = \frac{A_0 V_0}{\sqrt{V_0^2 + 2gh}}$$

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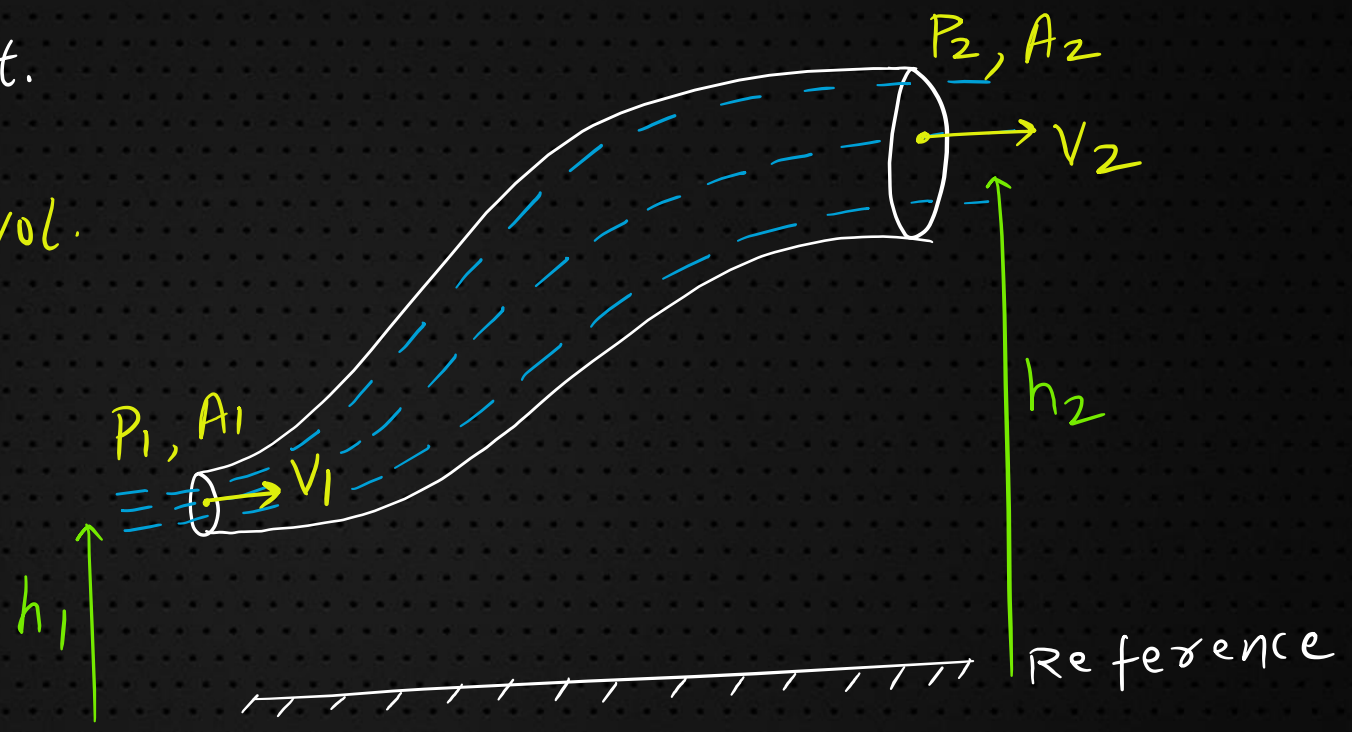
5. Bernoulli's Eqn

↳ based on conservation of mechanical energy to flow of Ideal fluid

$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const.}$

- P → Pressure Energy/vol.
- $\frac{1}{2}\rho v^2$ → K.E/vol.
- $\rho g h$ → P.E/vol.



$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

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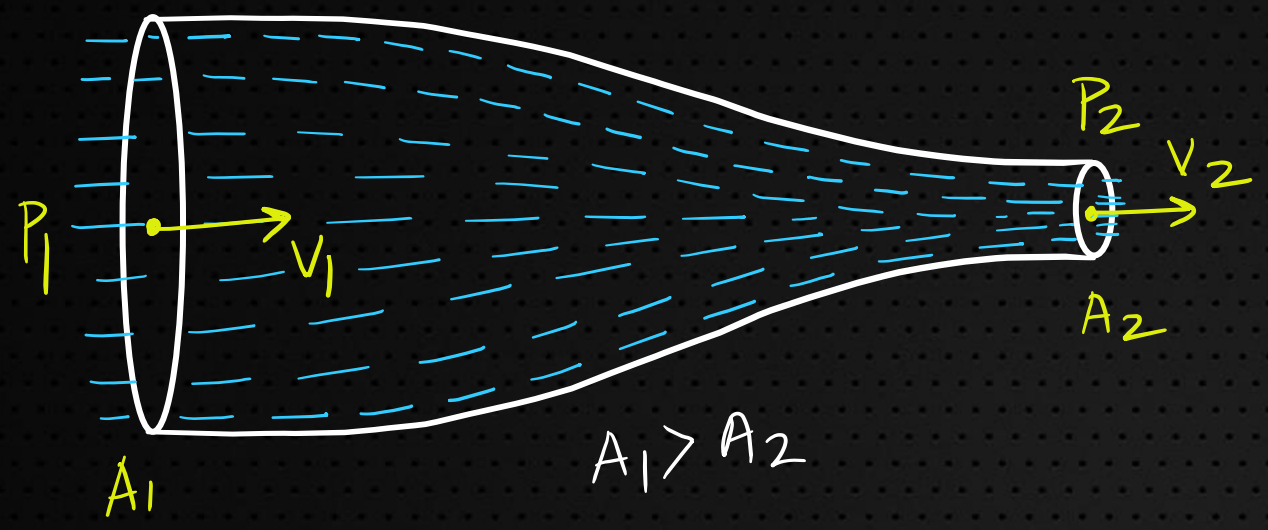
6. P-v Relation (horizontal flow)



$\because$ flow is horizontal

$\Rightarrow P + \frac{1}{2}\rho v^2 = \text{const.}$

Also $A_1 V_1 = A_2 V_2$, so $V_1 < V_2$

$\therefore \boxed{P_1 > P_2}$

* Note: Loss in pressure energy / vol. = gain in KE / vol.

(for horizontal flow)

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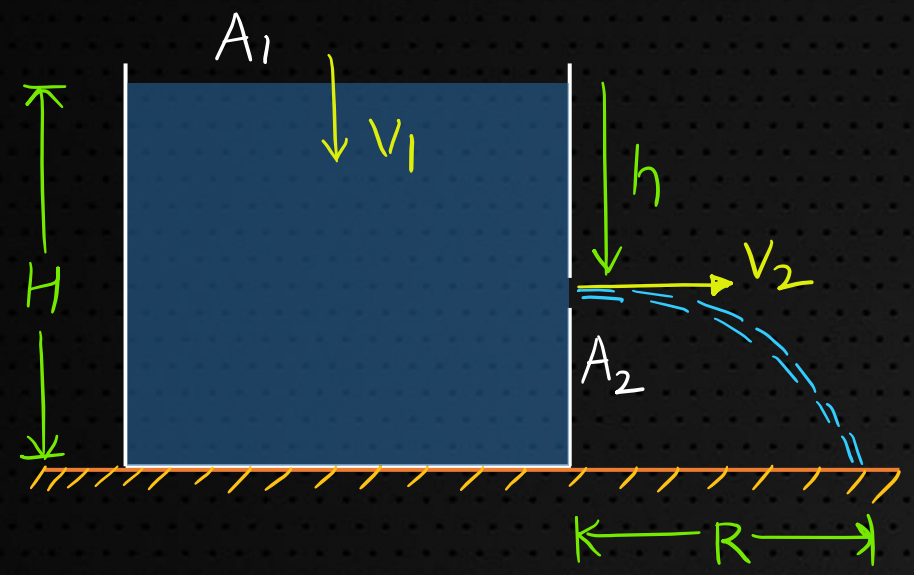
7. Speed of Efflux : Toricellis Law

↳ speed of efflux from an open tank is identical to freely falling body.



$A_1 >> A_2$

$\Rightarrow V_2 >> V_1$

↳ speed of efflux

→ Just to left and right of hole :

Loss in Press. Energy / vol = Gain of KE / vol.

$$\Rightarrow (P_0 + \rho g h) - (P_0) = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho \underbrace{V_1^2}_{\approx 0}$$

$$\Rightarrow \boxed{V_2 = \sqrt{2gh}}$$

(i) Range, $R = V_2 \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$

(ii) For R to be max:

$$\frac{dR}{dh} = 0 \Rightarrow h = H/2$$

and $R_{max} = H$

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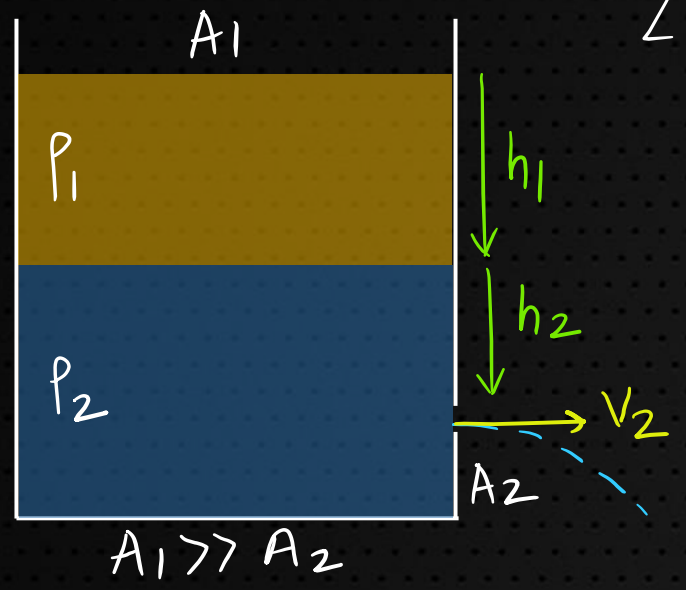
8. Speed of Efflux (other cases)

(a)

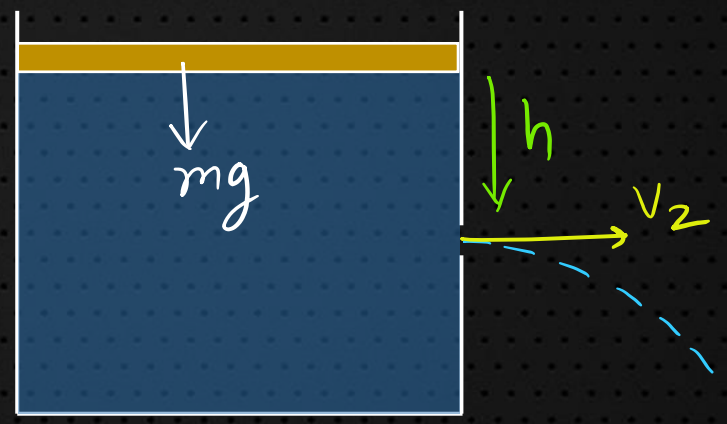


Loss in Press. Energy/vol = Gain of KE/vol.

$$(P_0 + \rho_1 g h_1 + \rho_2 g h_2) - P_0 = \frac{1}{2}\rho_2 V_2^2 \quad \{V_1 \text{ neglected}$$

$$\Rightarrow \rho_1 g h_1 + \rho_2 g h_2 = \frac{1}{2}\rho_2 V_2^2$$

(b)



$$\left(P_0 + \rho g h + \frac{mg}{A}\right) - P_0 = \frac{1}{2}\rho V_2^2$$

$$\Rightarrow \rho g h + \frac{mg}{A} = \frac{1}{2}\rho V_2^2$$

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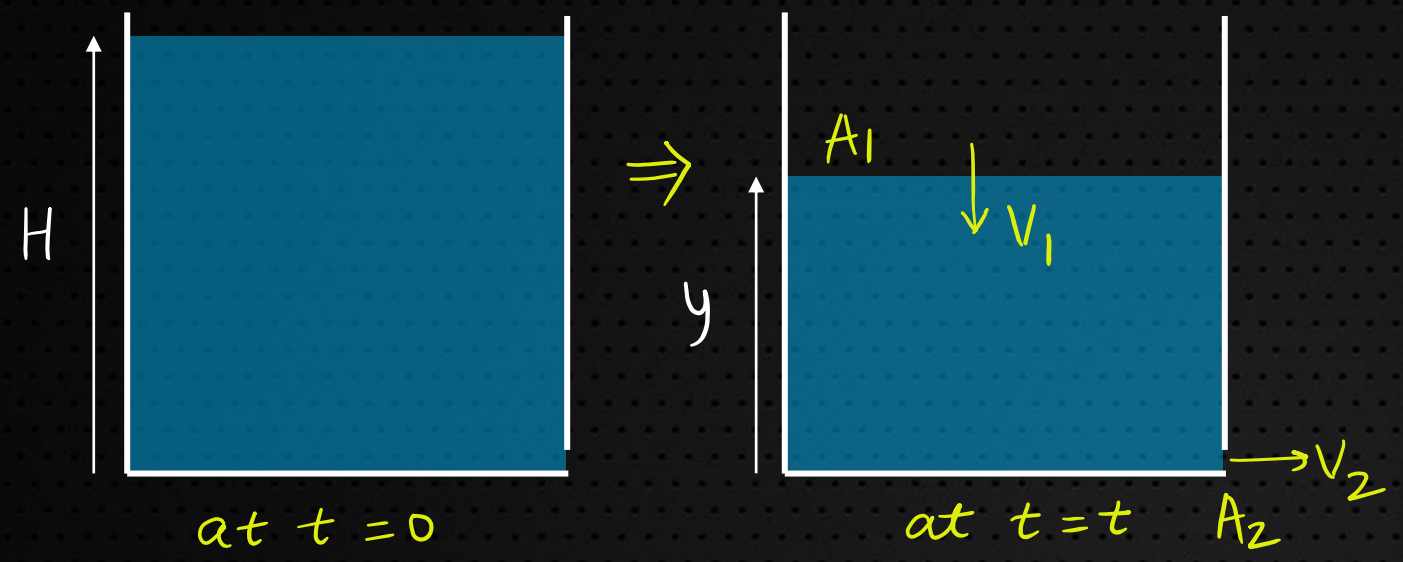
9. Vessel Emptying time



$$A_1 V_1 = A_2 V_2 \Rightarrow A_1\left(-\frac{dy}{dt}\right) = A_2\sqrt{2gy}$$

$$\Rightarrow \int_H^y \frac{dy}{\sqrt{y}} = -\frac{A_2}{A_1}\sqrt{2g}\int_0^t dt$$

$$\Rightarrow 2(\sqrt{y}-\sqrt{H}) = -\frac{A_2}{A_1}\sqrt{2g}\cdot t$$

$$\therefore \boxed{t = \frac{A_1}{A_2}\sqrt{\frac{2}{g}}\left(\sqrt{H}-\sqrt{y}\right)}$$

To empty $y=0$ at $t=t_0$

$$\Rightarrow \boxed{t_0 = \frac{A_1}{A_2}\sqrt{\frac{2H}{g}}}$$

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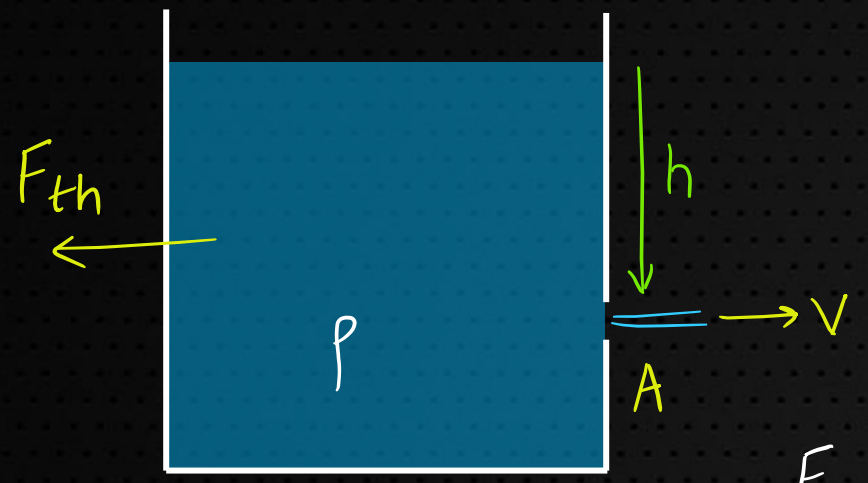
10. Thrust on Vessel (Liquid coming out)



$$F_{th} = \rho A v^2$$

$$\Rightarrow F_{th} = \rho A \times 2gh$$

Q7.

https://youtu.be/lkwiZzOCTgE

A light cylindrical vessel is kept on a horizontal surface. Area of base is A. A hole of cross-sectional area 'a' is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is (a << A) :

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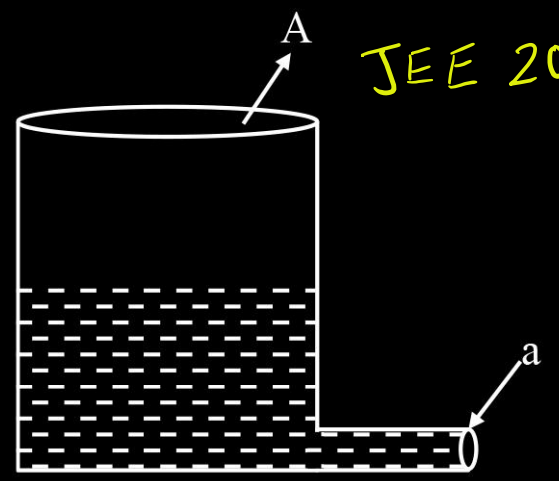

(1) $\frac{A}{2a}$

(2) None of these

(3) $\frac{2a}{A}$

(4) $\frac{a}{A}$

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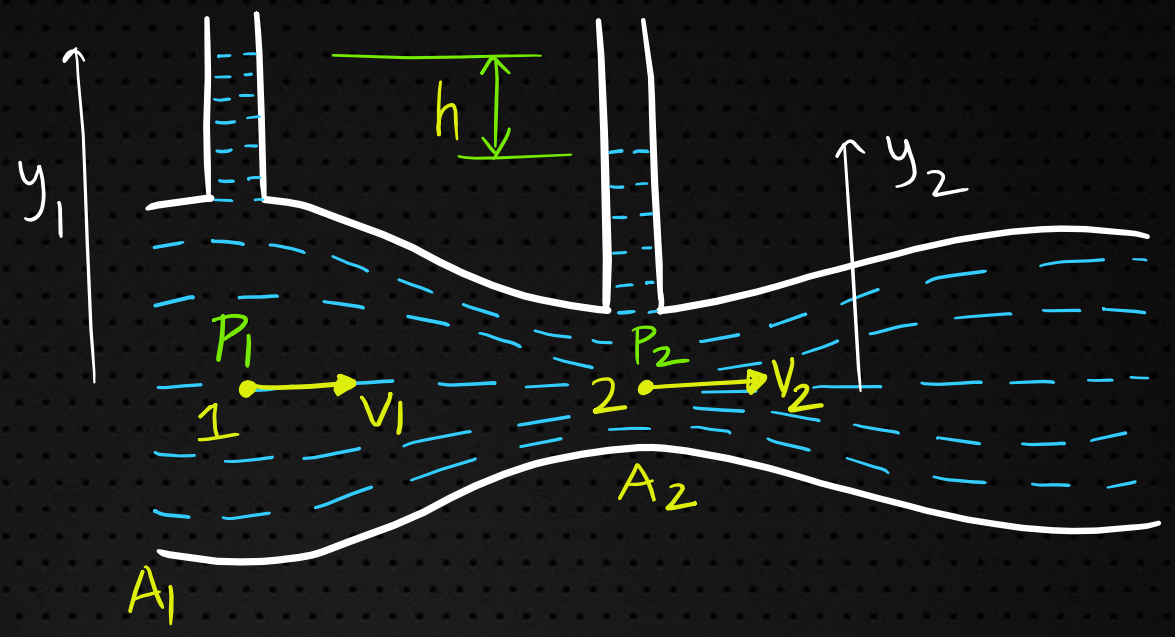
11. Venturimeter

↳ A device/setup to measure flow speed

(a) Setup 1:



$A_1 > A_2$

$\Rightarrow V_1 < V_2$

$\Rightarrow P_1 > P_2$

$V_2 = \frac{A_1 V_1}{A_2}$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 \Rightarrow \rho g y_1 - \rho g y_2 = \frac{1}{2}\rho\left(\frac{A_1 V_1}{A_2}\right)^2 - \frac{1}{2}\rho V_1^2$$

$$\Rightarrow \rho g h = \frac{1}{2}\rho V_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$\therefore \boxed{V_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}}$$

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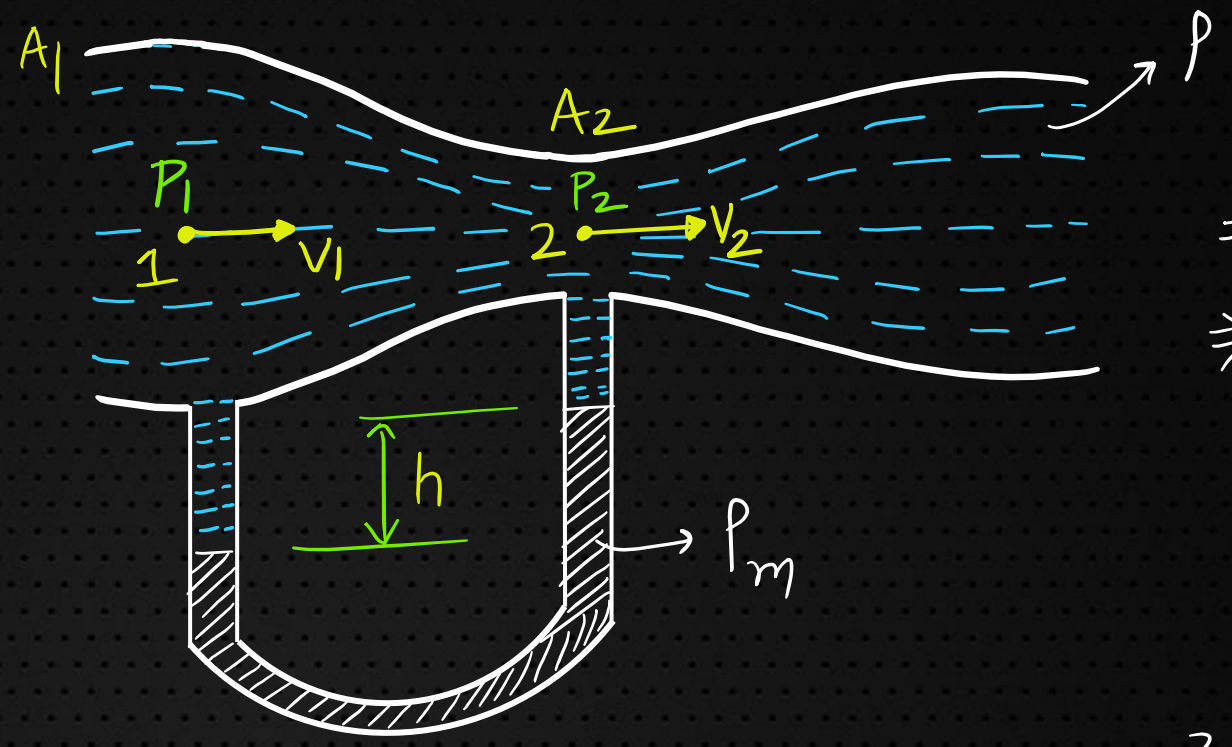
↳ A device/setup to measure flow speed

#(b) Setup 2:



$A_1 > A_2$

$\Rightarrow V_1 < V_2$

$\Rightarrow P_1 > P_2$

$V_2 = \frac{A_1 V_1}{A_2}$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 \Rightarrow P_m g h = \frac{1}{2}\rho\left(\frac{A_1 V_1}{A_2}\right)^2 - \frac{1}{2}\rho V_1^2$$

$$\therefore \boxed{V_1 = \sqrt{\frac{2 P_m g h}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}}$$

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JEE Main PYQs Link https://bit.ly/2S54jzh Chapter wise 2021, 2020, 2018

GoldMine Link https://bit.ly/2VhOGFF

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