JEE 2022

LR CIRCUIT

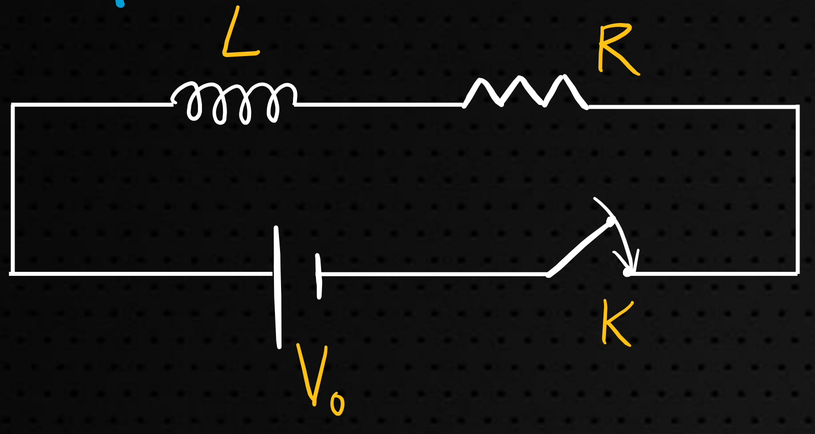
GROWTH & DECAY

MOHIT SIR-IIT KGP

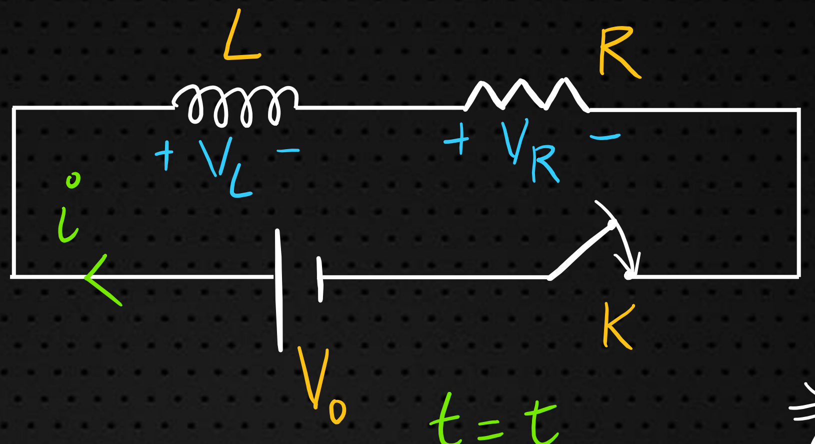
TOPICS TO BE DISCUSSED

1. Growth of Current
2. Decay of Current
3. 4 Questions (PYQs)

1. Growth of Current



$\Rightarrow$

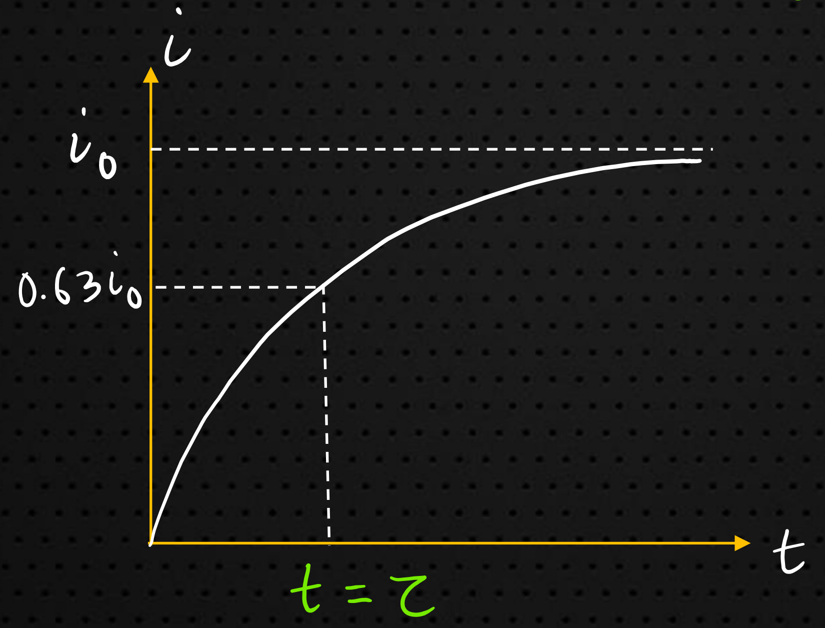


$$V_L + V_R = V_0$$

$$\Rightarrow L\frac{di}{dt} + iR = V_0$$

$$\Rightarrow \int_0^i \frac{di}{V_0 - iR} = \int_0^t \frac{dt}{L}$$

$$\Rightarrow i = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right)$$



(i) $\tau = L/R$

(ii) $i_0 = \frac{V_0}{R}$ at $t \to \infty$

... Continued

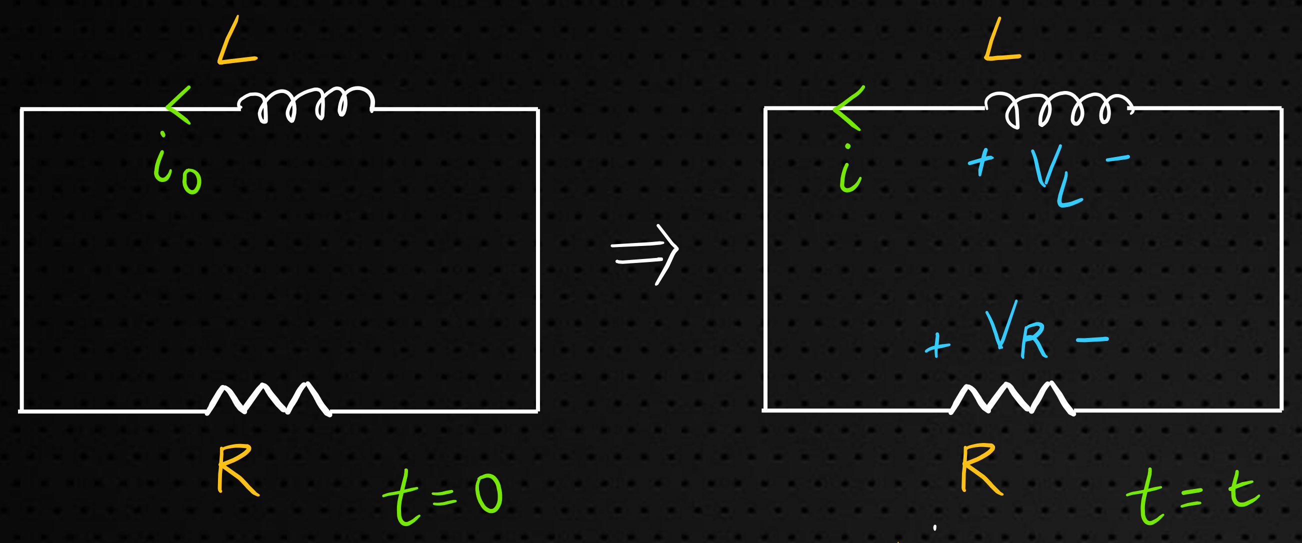
Key points

$$i = i_0 \left(1 - e^{-Rt/L}\right)$$

→ At $t = 0$, $i = 0$ (inductor behaves as open circuit)

→ At $t \to \infty$, $i = i_0 = \frac{V_0}{R}$ (steady state, inductor behaves as conducting wire)

2. Decay of current

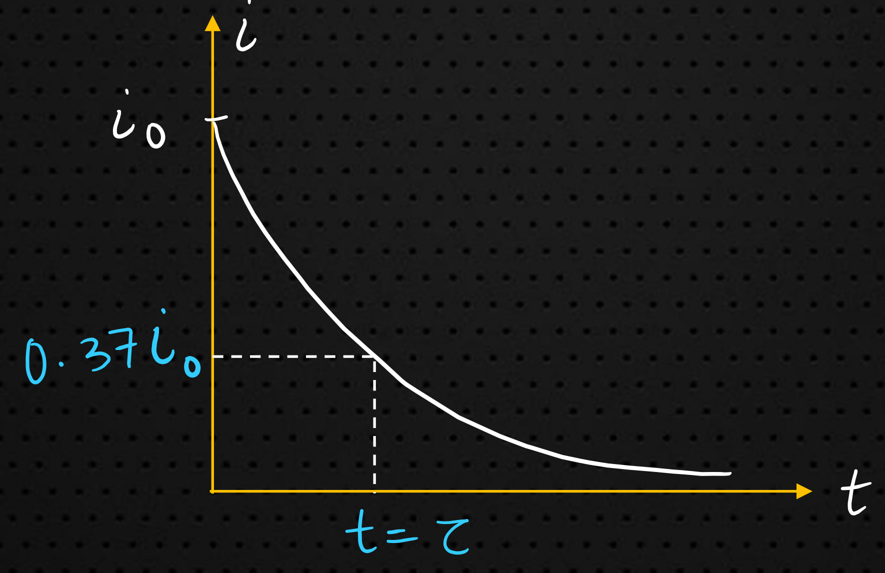


$$V_L = V_R \Rightarrow -L\frac{di}{dt} = iR$$

$$\Rightarrow \int_{i_0}^{i} \frac{di}{i} = \int_{0}^{t} -\frac{R\,dt}{L}$$

$$\Rightarrow i = i_0 e^{-Rt/L}$$

$$\hookrightarrow \tau = L/R$$



Q1. A coil of self inductance 10 mH and resistance $0.1\,\Omega$ is connected through a switch to a battery of internal resistance $0.9\,\Omega$. After the switch is closed, the time taken for the current to attain 80% of the saturation value is [Take, $\ln 5 = 1.6$]

(Main 2019, 10 April II)

(a) 0.002 s (b) 0.324 s

(c) 0.103 s (d) 0.016 s

Q1. A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is [Take, ln 5 = 1.6]

(Main 2019, 10 April II)

(a) 0.002 s (b) 0.324 s

(c) 0.103 s (d) 0.016 s

Sol^n: $i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$

$R = 0.1 + 0.9 = 1\Omega$

$L = 10\,mH = 10^{-2}\,H$

$i = 0.8\, i_0$

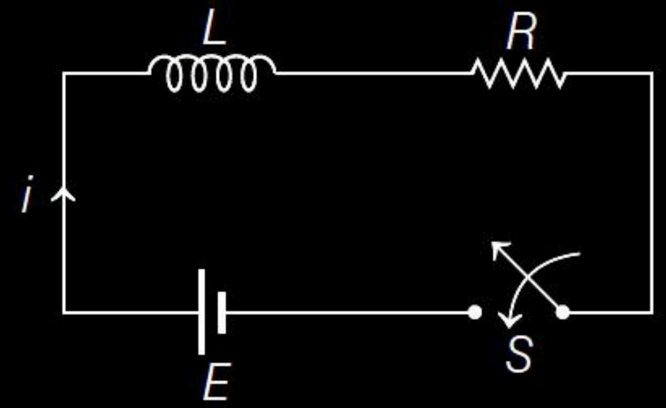
$\Rightarrow 0.8\, i_0 = i_0\left(1 - e^{-\frac{t}{10^{-2}}}\right)$

$\Rightarrow 5^{-1} = e^{-100t}$ $\Rightarrow \ln 5 = 100t$

$\therefore t = \frac{1.6}{100} = \boxed{0.016 s}$

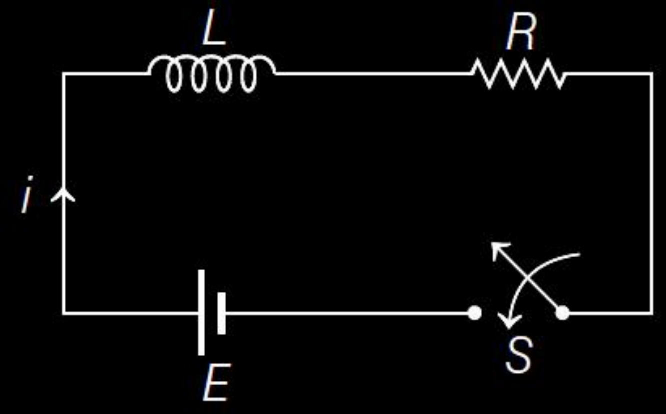
EDUNITI

Q2. Consider the L-R circuit shown in the figure. If the switch S is closed at $t = 0$, then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is **(Main 2019, 12 April II)**



(a) $\frac{2.7EL}{R^2}$ (b) $\frac{EL}{2.7R^2}$ (c) $\frac{7.3EL}{R^2}$ (d) $\frac{EL}{7.3R^2}$

Q2. Consider the L-R circuit shown in the figure. If the switch S is closed at $t = 0$, then the amount of charge that passes through the battery between $t = 0$ and $t = \dfrac{L}{R}$ is **(Main 2019, 12 April II)**



(a) $\dfrac{2.7EL}{R^2}$ (b) $\dfrac{EL}{2.7R^2}$ (c) $\dfrac{7.3EL}{R^2}$ (d) $\dfrac{EL}{7.3R^2}$

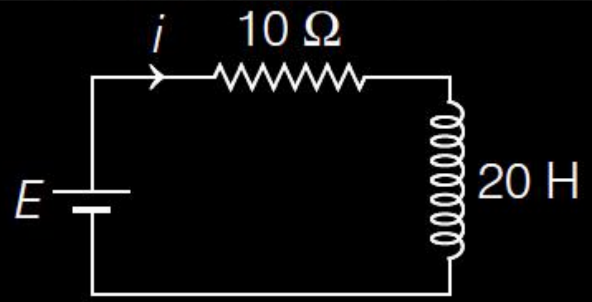
Sol^n: $i = i_0(1 - e^{-Rt/L})$

$$\Rightarrow \frac{dq}{dt} = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$$

$$\Rightarrow \int_0^q dq = \frac{E}{R}\int_0^{L/R}\left(1 - e^{-\frac{Rt}{L}}\right)dt$$

$$\Rightarrow q = \frac{E}{R}\left(t + \frac{L}{R}e^{-\frac{Rt}{L}}\right)_0^{L/R} \Rightarrow q = \frac{EL}{eR^2} = \boxed{\frac{EL}{2.7R^2}}$$

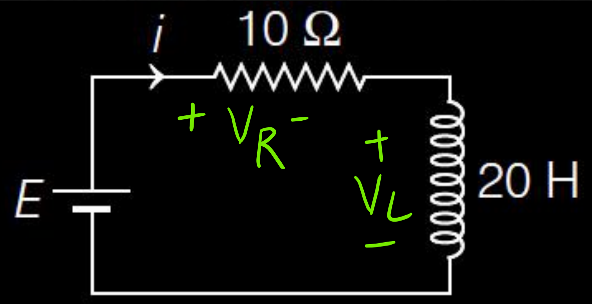
Q3. A 20 H inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is



(Main 2019, 10 Jan I)

(a) $\frac{2}{\ln 2}$ (b) $\frac{1}{2}\ln 2$ (c) $2 \ln 2$ (d) $\ln 2$

Q3. A 20 H inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is



(Main 2019, 10 Jan I)

(a) $\dfrac{2}{\ln 2}$ (b) $\dfrac{1}{2}\ln 2$ (c) $2\ln 2$ (d) $\ln 2$

sol^n:

$$P_R = P_L$$

$$\Rightarrow iV_R = iV_L \Rightarrow V_R = V_L$$

$$\therefore V_R = E/2$$

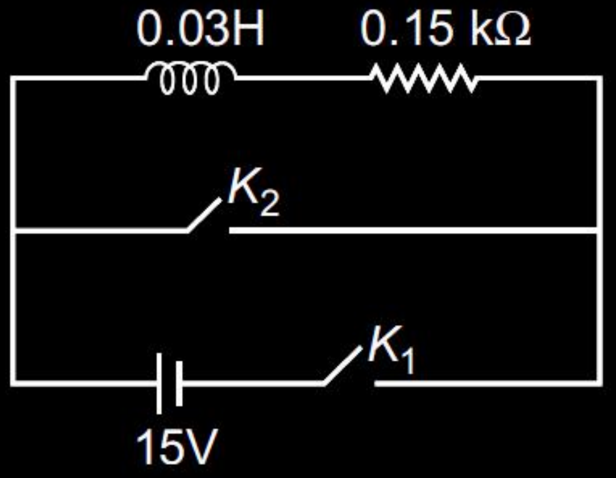
$$\Rightarrow iR = E/2 \Rightarrow i_0\left(1-e^{-\frac{Rt}{L}}\right)R = \frac{E}{2}$$

$$\Rightarrow \frac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)R = \frac{E}{2}$$

$$\therefore e^{-\frac{Rt}{L}} = \frac{1}{2} \Rightarrow t = \frac{L}{R}\ln 2$$

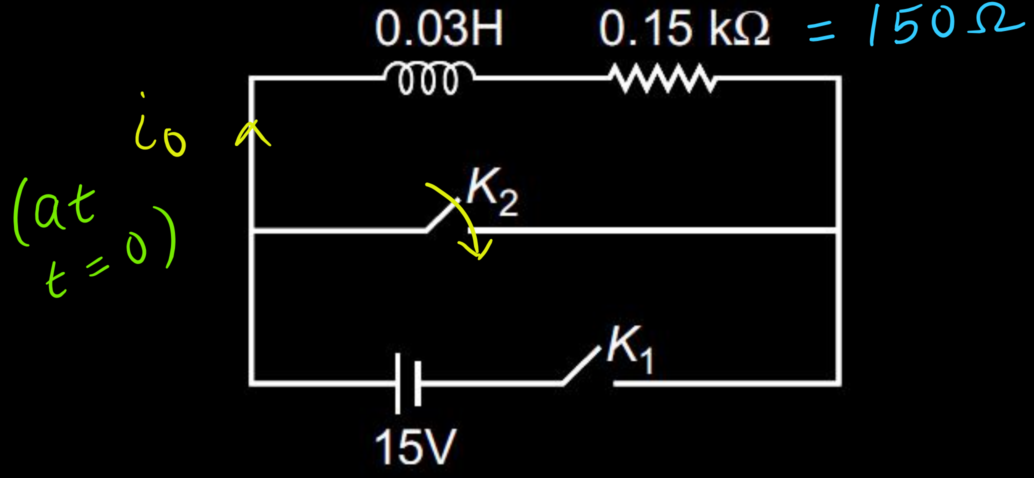
$$= \boxed{2\ln 2}$$

Q4. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15\,\text{k}\Omega$) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ms, the current in the circuit will be ($e^5 \cong 150$) **(2015 Main)**



(a) 100 mA (b) 67 mA (c) 0.67mA (d) 6.7 mA

Q4. An inductor ($L = 0.03$ H) and a resistor ($R = 0.15$ kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ms, the current in the circuit will be ($e^5 \cong 150$) **(2015 Main)**



(a) 100 mA (b) 67 mA (c) 0.67mA (d) 6.7 mA

Sol^n: K_1 closed

$\hookrightarrow i_0 = \frac{E}{R} = \frac{15}{150} = \frac{1}{10}$ A

K_1 open, K_2 closed:

$i_0 = \frac{1}{10}$ A $\qquad i = i_0 e^{-Rt/L}$

$\Rightarrow i = \frac{1}{10} e^{-\frac{150 \times 10^{-3}}{3 \times 10^{-2}}}$

$\therefore i = 0.1 \times e^{-5}$

$= 0.1 \times \frac{1}{e^5}$

$= 0.1 \times \frac{1}{150}$ A $\approx \boxed{0.67 \text{mA}}$

Revision Series Playlist Link https://bit.ly/3eBbib9

PhD Series https://bit.ly/3cQSxPT

GoldMine Link https://bit.ly/2VhOGFF

CLICK