LC OSCILLATION

ALTERNATING CURRENT

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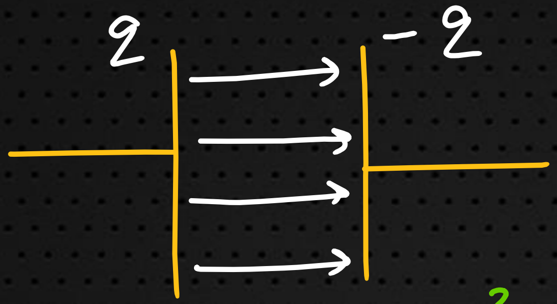
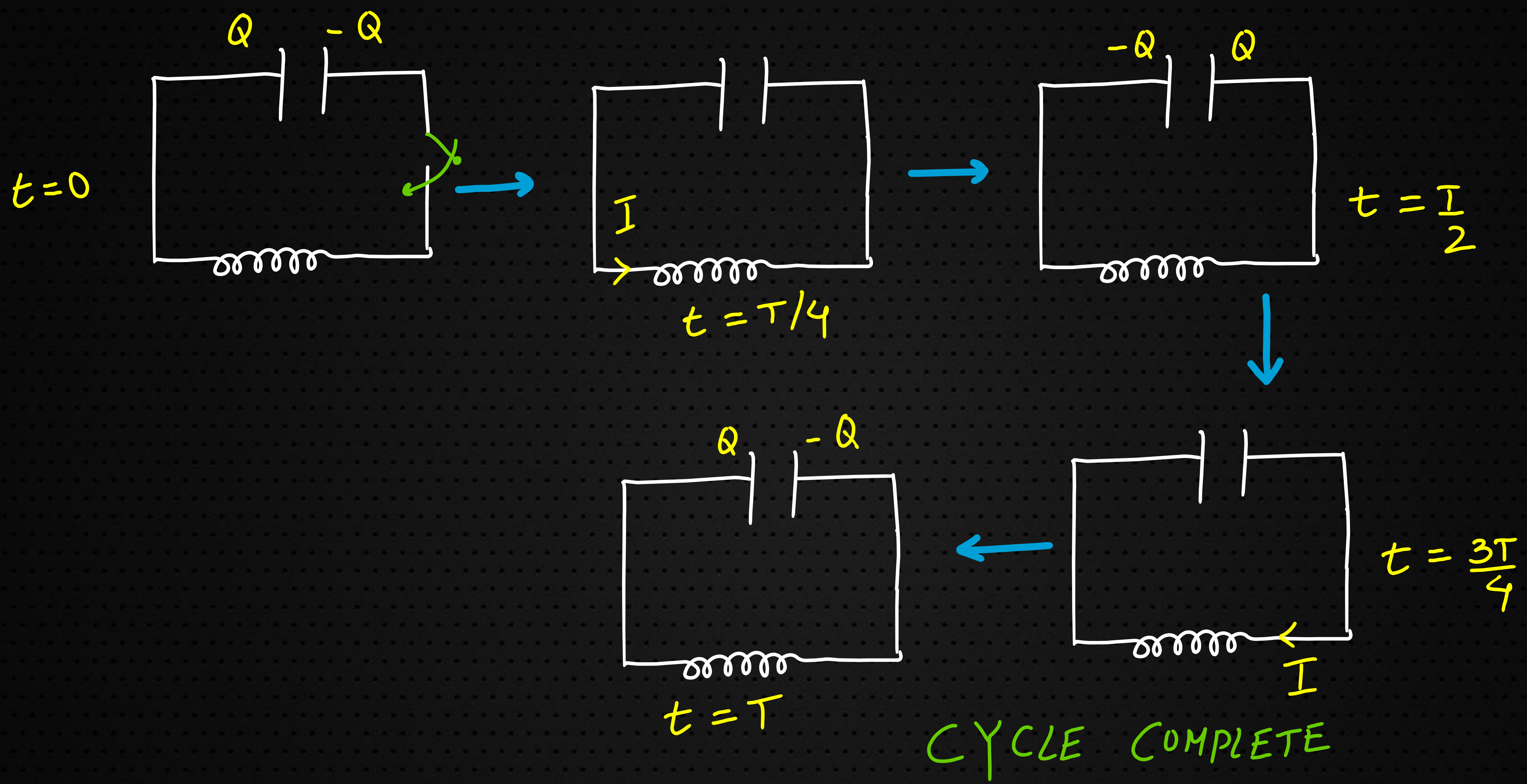
LC OSCILLATIONS

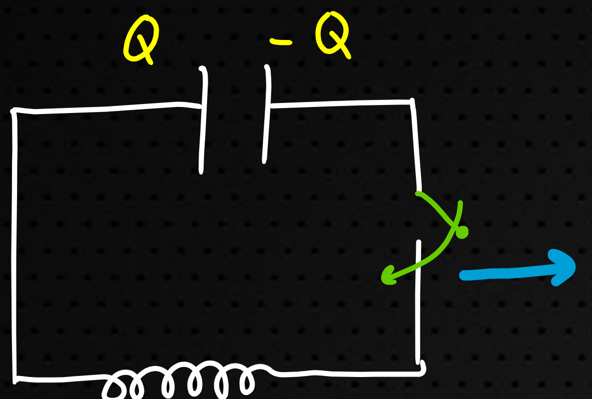
(i) Oscillation between Electrostatic energy and magnetic energy.



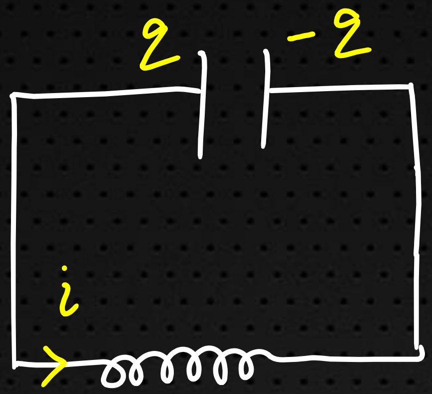
$$U_E = \frac{q^2}{2C}$$

$$U_B = \frac{1}{2} L i^2$$


Q
- Q
t = 0
I
t = T/4
-Q
Q
t = T/2
Q
- Q
t = T
t = 3T/4
I
CYCLE COMPLETE



$t = 0$



$t = t$

$$\frac{q^2}{2C} + \frac{1}{2}Li^2 = \text{Const}$$

$$\Rightarrow \frac{q}{C}\cancel{\frac{dq}{dt}} + L\cancel{i}\frac{di}{dt} = 0$$

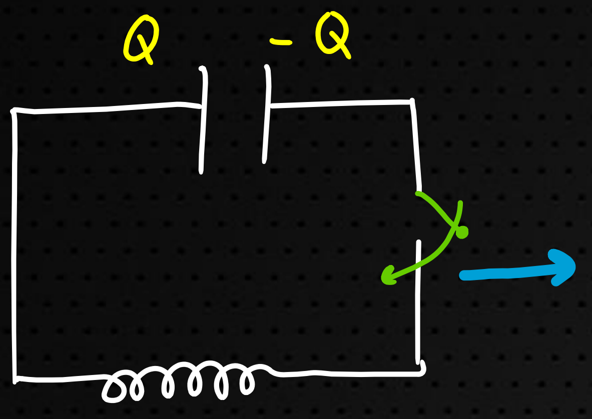
$$\Rightarrow \frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{q}{LC} = 0$$

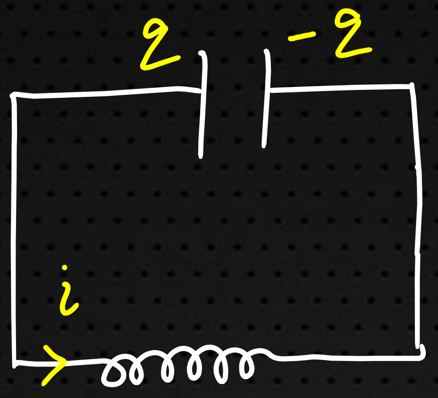
$$\left\{\frac{d^2x}{dt^2} + \omega^2 x = 0\right\}$$

$$\therefore \omega = \frac{1}{\sqrt{LC}} \Rightarrow T = 2\pi\sqrt{LC}$$

and, $\boxed{q = Q\sin(\omega t + \phi)}$



$t = 0$



$t = t$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$q = Q \sin(\omega t + \phi)$$

In this Example at $t = 0$, $q = Q$

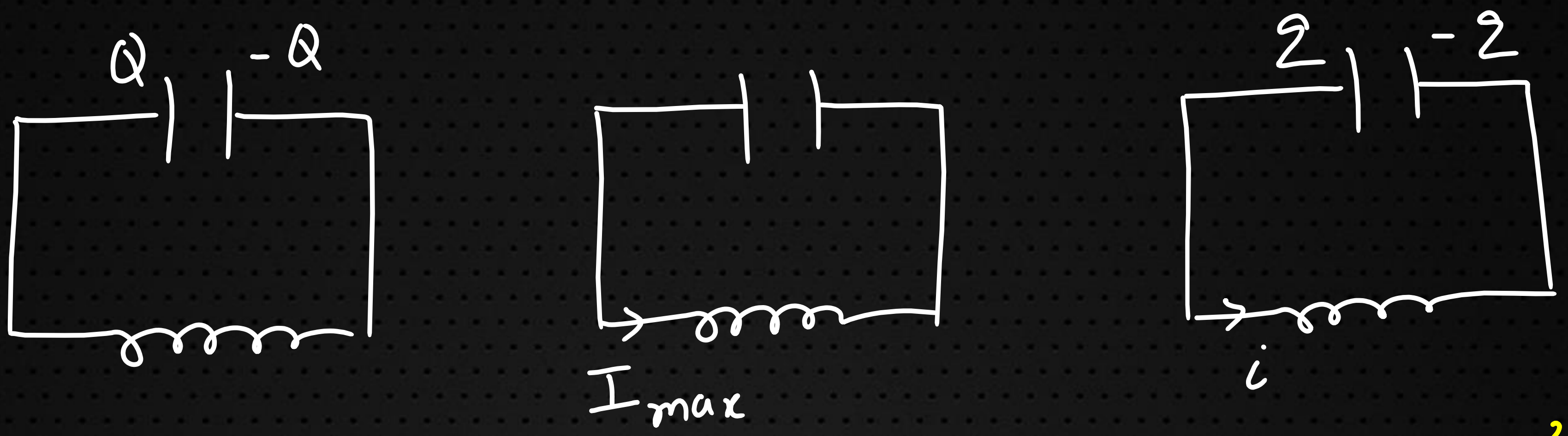
$$\therefore Q = Q \sin(0 + \phi)$$

$$\Rightarrow 1 = \sin\phi \quad \Rightarrow \phi = \frac{\pi}{2}$$

So, $\boxed{q = Q \cos\omega t}$

and $i = \dfrac{dq}{dt}$

Summary:



(1.) $\dfrac{Q^2}{2C} = \dfrac{1}{2} L I_{max}^2 = \dfrac{q^2}{2C} + \dfrac{1}{2} L i^2$

(2) $q = Q \sin(\omega t + \phi)$, $\omega = \dfrac{1}{\sqrt{LC}}$

(3) Complete U_E to U_B and Vice-Versa in $T/4$ time.

(4) U_E to U_E in $T/2$.

Q1. In a certain oscillating LC circuit, the total energy is converted from electrical energy in the capacitor to magnetic energy in the inductor in $1.50\ \mu s$.

(a) What is the time period of oscillation?

(b) How long after the magnetic energy is maximum will it be maximum again?

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Soln:

(a) $U_E \longrightarrow U_B$ in $1.5\,\mu s = T/4$

$\Rightarrow T = 6\,\mu s$

(b) $U_B \longrightarrow U_B$ in $\frac{T}{2} \Rightarrow 3\,\mu s$

Q2. In an oscillating LC circuit, in terms of the maximum charge Q_0 on the capacitor, what is the charge on it when the energy in the electric field is one third of that in the magnetic field?

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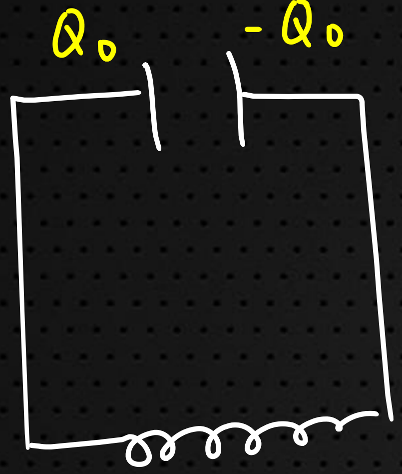
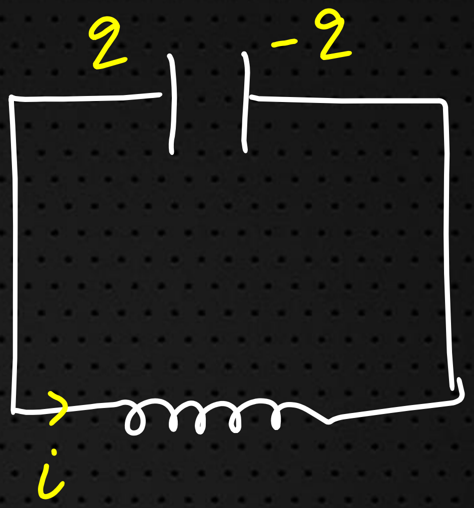
Sol^n:





$$\frac{Q_0^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}Li^2$$

$$\left\{ \frac{q^2}{2C} = \frac{1}{3} \times \frac{1}{2}Li^2 \right.$$

$$\Rightarrow \frac{3q^2}{2C} = \frac{1}{2}Li^2$$

$$\Rightarrow \frac{Q_0^2}{2C} = \frac{q^2}{2C} + \frac{3q^2}{2C}$$

$$\Rightarrow \boxed{q = \frac{Q_0}{2}}$$