JEE MAIN | IIT JEE

ROTATION

(Part-1)

REVISION in 50 Min

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EDUNITI

PART 1 - Topics to covered

Concept + PYQs

1. Translatory vs Rotatory Motion
2. Moment of Inertia
3. Parallel & Perpendicular Axis Theorem
4. MOI of Standard Bodies (*uniform mass distribution*)
5. Addition or Subtraction of MOI
6. Radius of Gyration
7. Questions on MOI
8. Torque
9. Questions on $\Sigma \vec{\tau} = I\vec{\alpha}$
10. Question on Static Equilibrium

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Eduniti for Physics

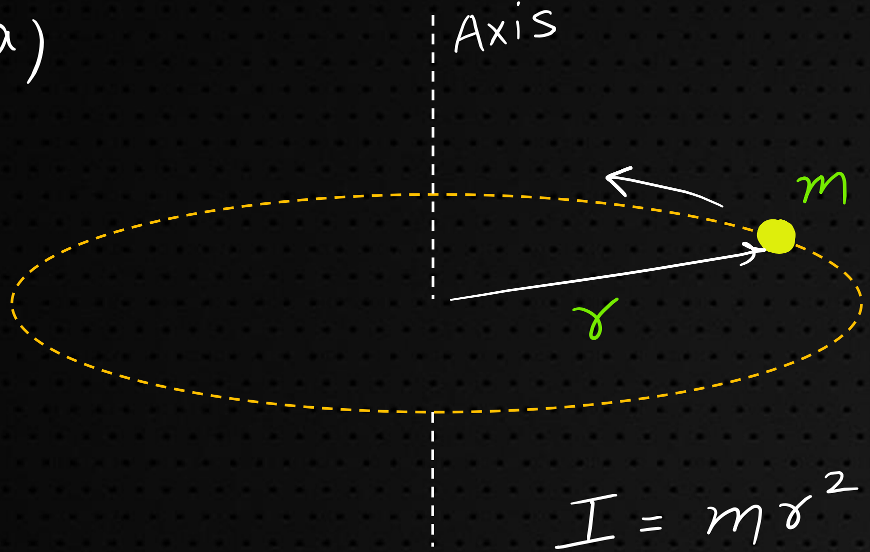
Chapter	Formulae_Concept VIDEO LINK
Unit & Dimensions	https://youtu.be/wdd-wlZF4Hk
Errors and Vectors	
Vernier Calliper	https://youtu.be/pVoN045dV8I
Screw Gauge	https://youtu.be/gYd2PtmZ0mw
Kinematics_Motion in 1d	https://youtu.be/U4NNxFaFliE
Kinematics_Motion in 2d	https://youtu.be/4_Zo5WhMf7w
Laws of Motion	https://youtu.be/7JlR8gNRQIs
Friction	https://youtu.be/Rn1bLst7eGk
Work Energy Power	https://youtu.be/kjrXoE-kDI8
Circular Motion	https://youtu.be/KnFymKHlkT0
Centre of Mass	https://youtu.be/ads35RKD618
Cons of Momentum & Collision	https://youtu.be/3f0u4L-lyyw
Rotational Motion – Part 1	
Gravitation	https://youtu.be/rAj2huLVaEk
Properties of Solids	https://youtu.be/gSXxjk89l_c
Fluids Statics (Part 1)	https://youtu.be/RFKx9B9yo3M
Fluid Dynamics (Part 2)	https://youtu.be/Y717vQpUEJQ
Fluid Properties (Part 3)	https://youtu.be/V8xUWWK2oT0
Simple Harmonic Motion	https://youtu.be/RIb7ofNG09I
Thermal Properties	https://youtu.be/OYjjyPIzddE
Heat Transfer	https://youtu.be/PyNboHgtYzM
KTG	https://youtu.be/XO1tvFhIa0I
Thermodynamics	https://youtu.be/iz_kf1jRDRw
Wave Motion -Organ Pipes and Resonance Tube	https://youtu.be/fB7pfJ77za8
Wave Motion - Doppler's Effect	https://youtu.be/9-BxOaamnwg
Electrostatics	https://youtu.be/3stXbGRMcrk
Capacitors	https://youtu.be/EXEiickNUkY
Current Electricity	https://youtu.be/gm8FUfjrX18
Moving Charges and Magnetic Effect of Current	https://youtu.be/ULD2Ok1CGJk
Earth's Magnetism	https://youtu.be/a4CT5uVwAK4
Magnetic Properties	https://youtu.be/63cwdYXNlYE
EMI	https://youtu.be/puVavm_GFRM
Alternating Current	https://youtu.be/74dTY-pzM_o
Ray Optics	https://youtu.be/BhnyTWzIlBA
Wave Optics Part 1_Interference	https://youtu.be/LG5nlE8XTeI
Wave Optics Part 2_Diffraction_Polarization	https://youtu.be/ymMyyJGGqnY
Optical Instruments	https://youtu.be/OQssbDH0A4I
Electromagnetic Waves	https://youtu.be/bcVXgEkyQZY
Semiconductors_Basics + Zener Diode	https://youtu.be/_A2JomQ7-50
Semiconductors_Transistors	https://youtu.be/psDwl84Nzb0
Semiconductors_Logic Gates	https://youtu.be/pZdQAzLbFTo
Communication Systems	https://youtu.be/8NgMqK9X79Y
Modern Physics_Part 1_Atomic Physics	https://youtu.be/9VKUnE3mpHk
Modern Physics_Part 2_Photoelectric Effect	https://youtu.be/24oTQp84jrk
Modern Physics_Part 3_Dual Nature of Light	https://youtu.be/0zoR_saMAQY
Modern Physics_Part 4_Radioactivity	https://youtu.be/AdX3YBhQyog
Modern Physics_Part 5_Nuclear Physics	https://youtu.be/VDWqVahGixc
Modern Physics_Part 6_X Rays	https://youtu.be/dSHXdzX7NX0

1. Translatory vs Rotatory Motion

Refer Animation in our Video

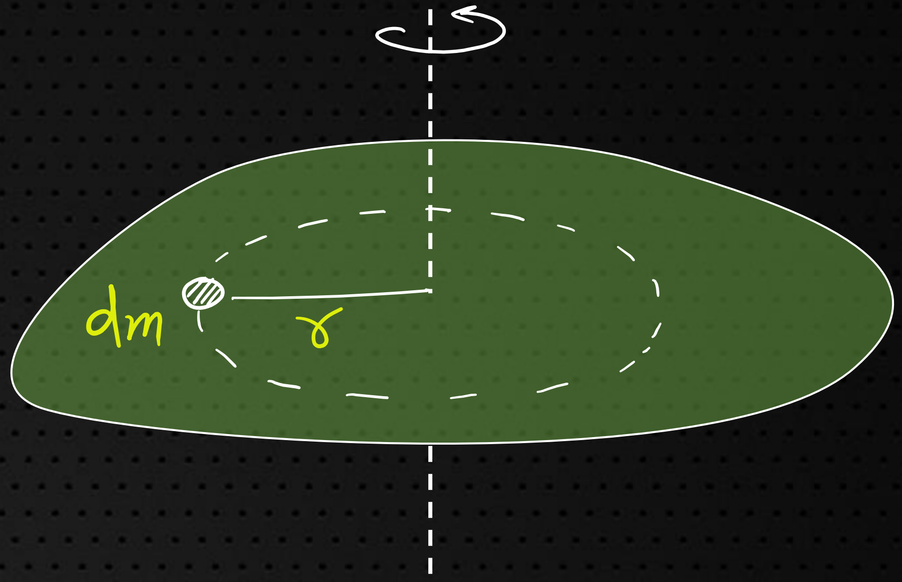
2. Moment of Inertia (I)

(a)



$$I = mr^2$$

(b)

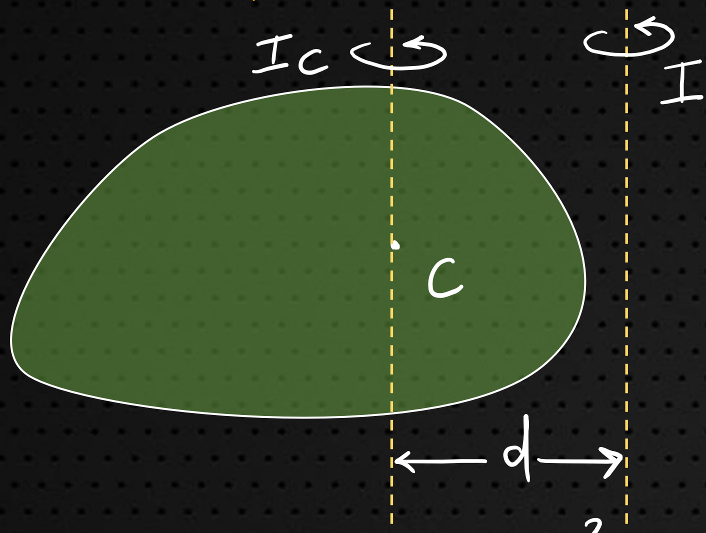


$$dI = dm \cdot r^2$$

$$\Rightarrow I = \int dm \cdot r^2$$

3. PARALLEL AND PERPENDICULAR AXIS THEOREM

Parallel Axis Theorem

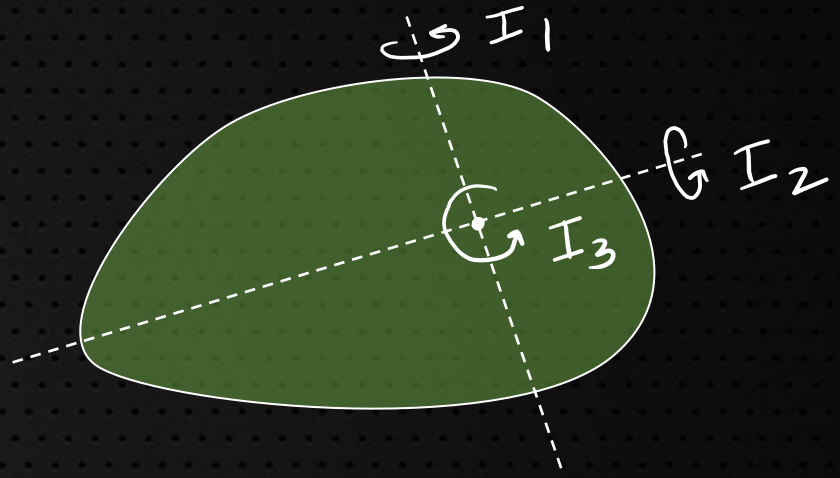


(a) $I = I_c + md^2$

(b) Applicable always

(c) I_c is MOI about an axis passing C.O.M.

Perpendicular Axis Theorem



(a) $I_3 = I_1 + I_2$

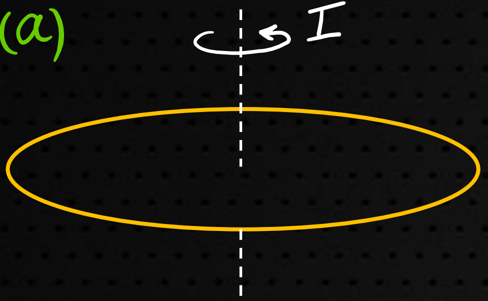
(b) Applicable for Plane-Lamina

(c) I_1 and I_2 are about axis which are in the Plane of body.

4. MOI of Standard bodies

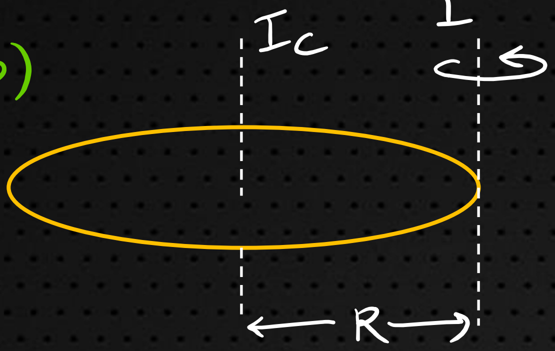
(i) MOI OF RING

(a)



$I = MR^2$

(b)



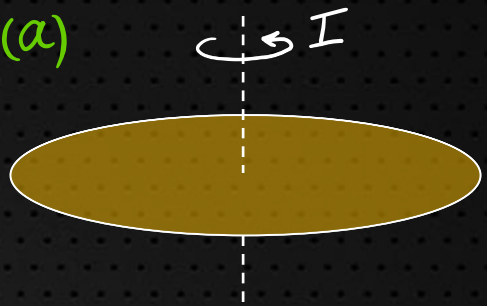
$I = 2MR^2$

(c)

$I = MR^2/2$

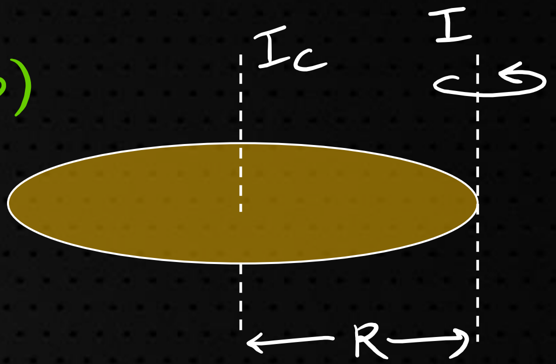
(ii) MOI OF DISC

(a)



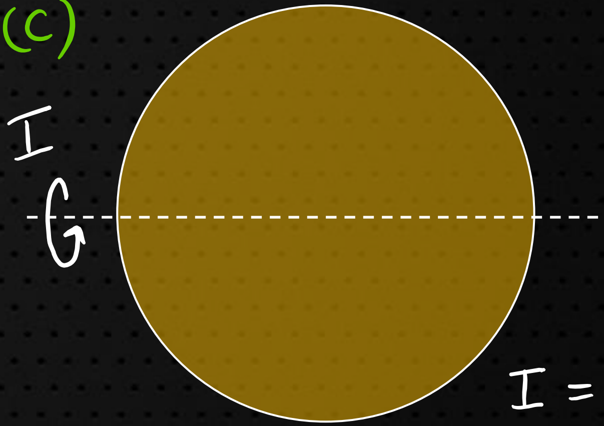
$I = MR^2/2$

(b)



$I = \frac{3MR^2}{2}$

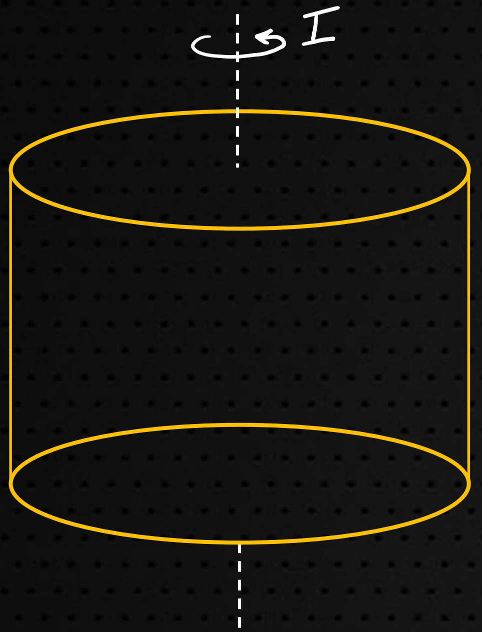
(c)



$I = \frac{MR^2}{4}$

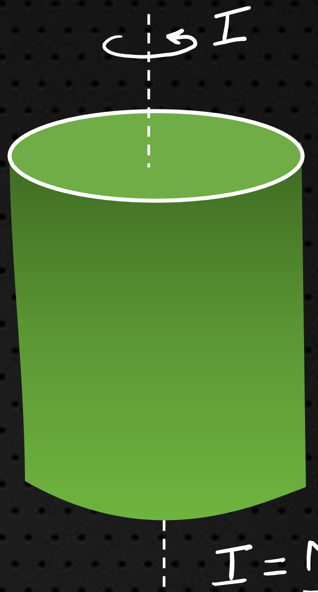
... continued

(iii) MOI OF HOLLOW CYLINDER

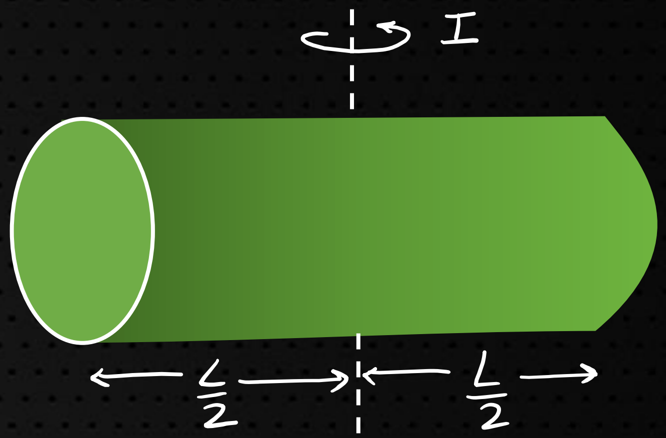


$$I = MR^2$$

(iv) MOI OF SOLID CYLINDER

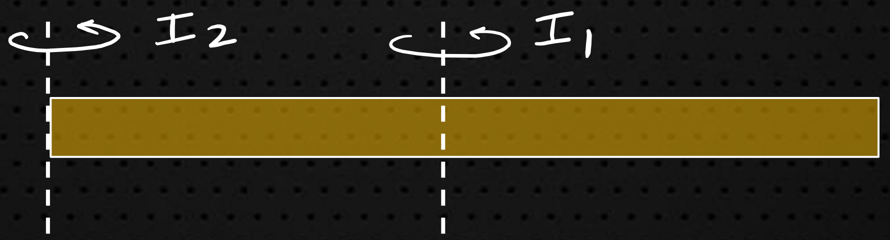


$$I = \frac{MR^2}{2}$$



$$I = \frac{MR^2}{4} + \frac{ML^2}{12}$$

(V) MOI OF THIN ROD



$$I_1 = ML^2/12$$

$$I_2 = ML^2/3$$

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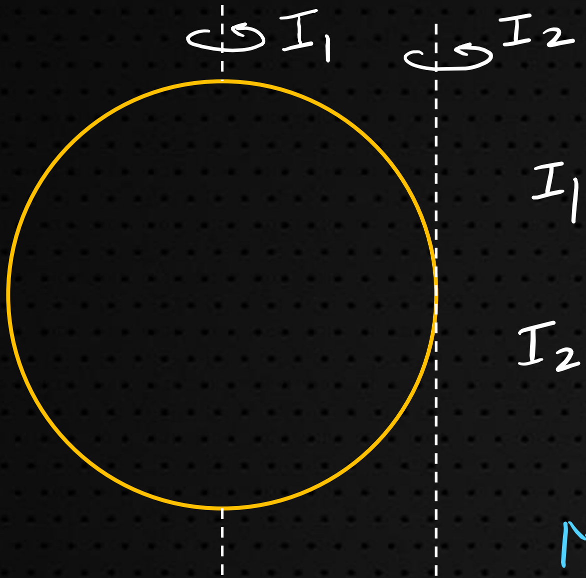
... continued

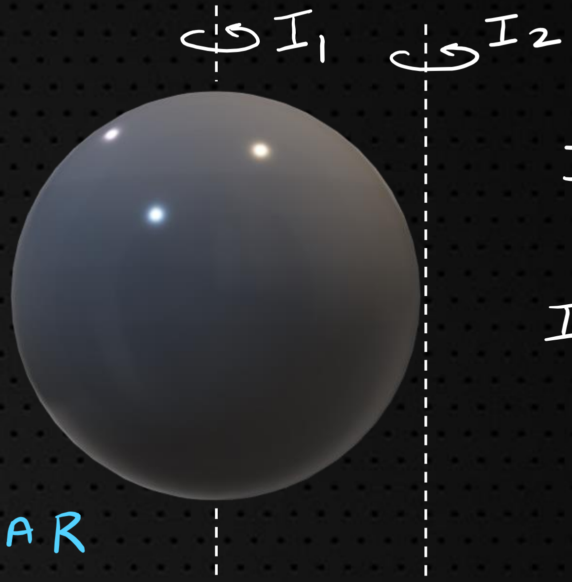
(vi) MOI OF HOLLOW SPHERE



$$I_1 = \frac{2}{3} MR^2$$

$$I_2 = \frac{5}{3} MR^2$$

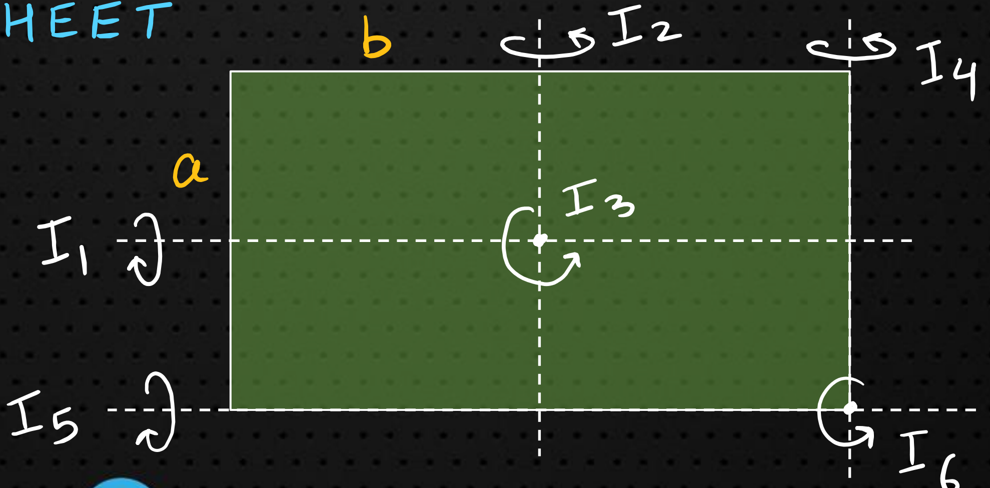
(vii) MOI OF SOLID SPHERE



$$I_1 = \frac{2}{5} MR^2$$

$$I_2 = \frac{7}{5} MR^2$$

(viii) MOI OF RECTANGULAR SHEET



$$I_1 = Ma^2/12$$

$$I_2 = Mb^2/12$$

$$I_3 = I_1 + I_2$$

$$I_4 = Mb^2/3$$

$$I_5 = Ma^2/3$$

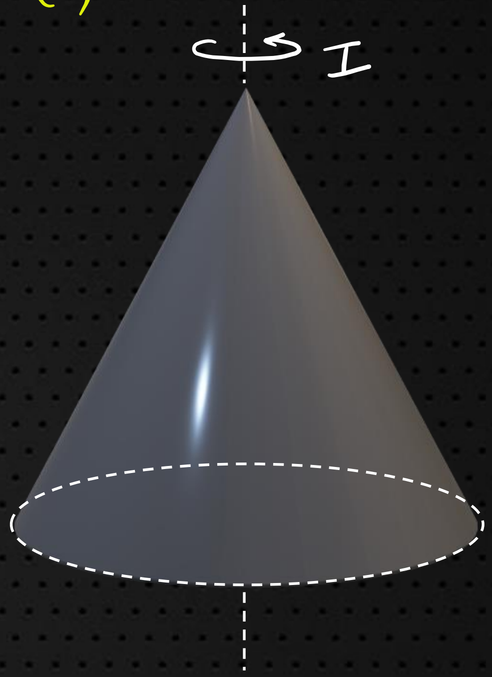
$$I_6 = I_4 + I_5$$

... continued

(ix) MOI OF HOLLOW CONE

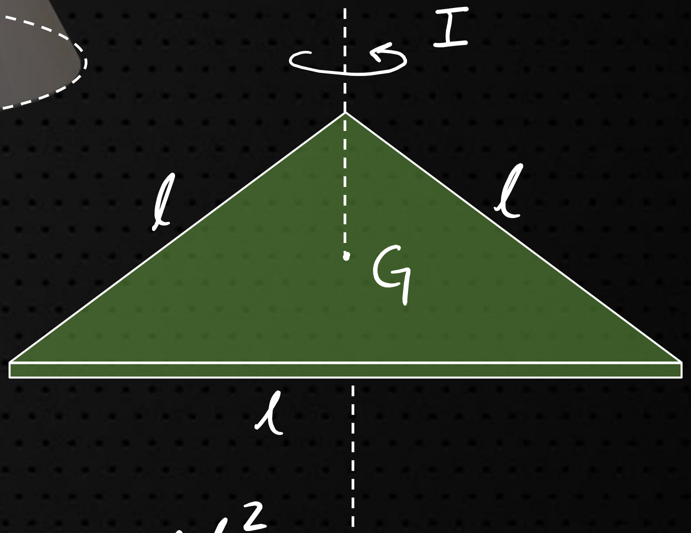
$$I = \frac{MR^2}{2}$$

(x) MOI OF SOLID CONE



$$I = \frac{3MR^2}{10}$$

(xi) MOMENT OF INERTIA OF EQUILATERAL TRIANGLE SHEET (about centroid)

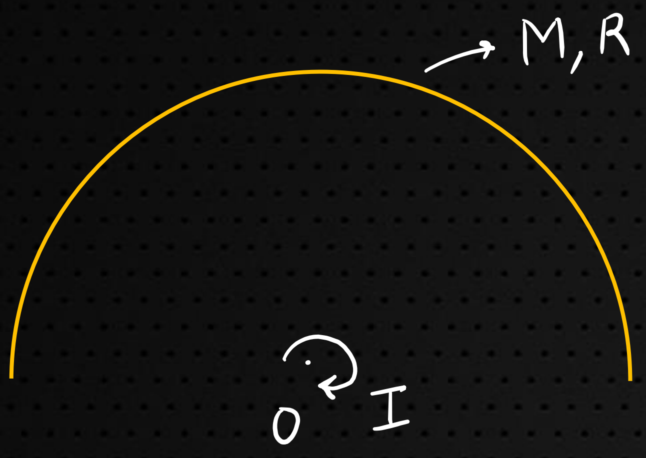


$$I = \frac{Ml^2}{12}$$

... continued

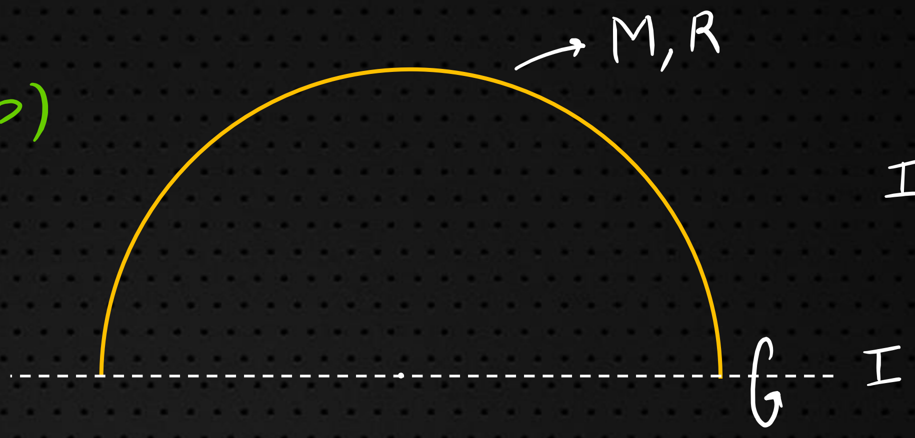
(xii) MOI OF SEMICIRCULAR RING

(a)



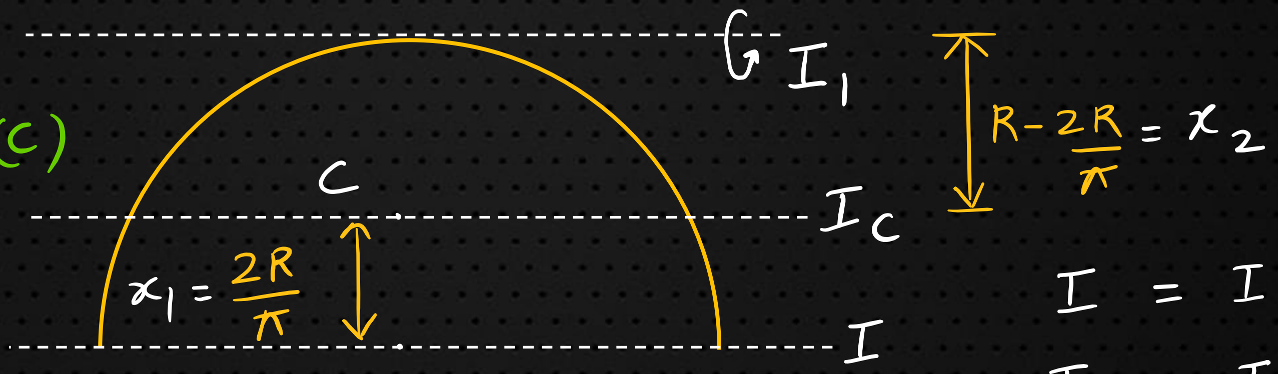
$I = MR^2$

(b)



$$I = \frac{MR^2}{2}$$

(c)



Find I_1 ?

$$I = I_c + M x_1^2 \quad \text{— (1)}$$

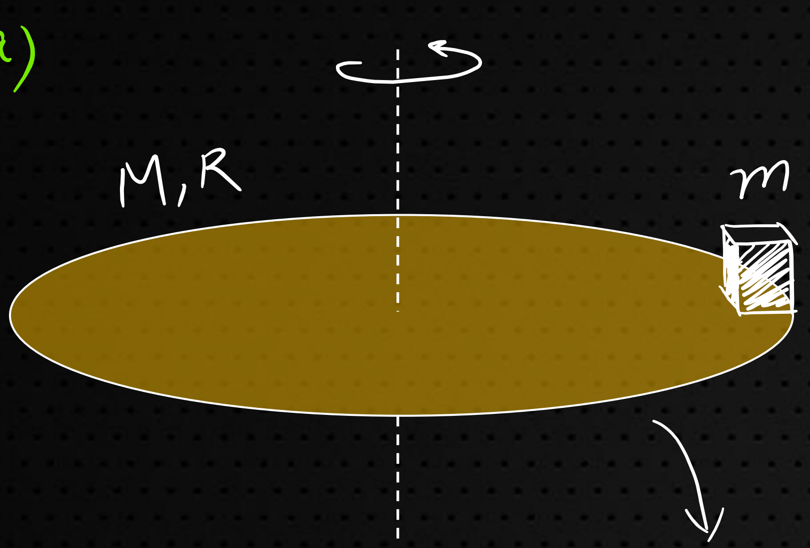
$$I_1 = I_c + M x_2^2 \quad \text{— (2)}$$

from (2) and (1) :

$$I_1 = I + M\left(x_2^2 - x_1^2\right)$$

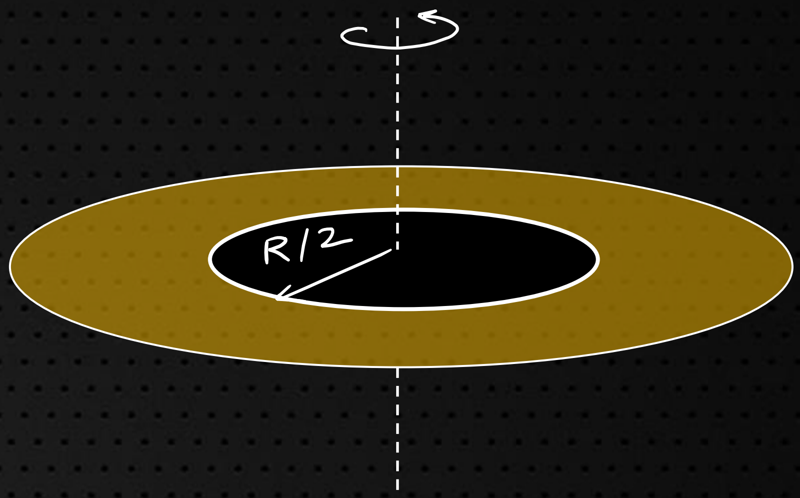
5. Addition or Subraction of MOI

(a)



$$I = \frac{MR^2}{2} + mR^2$$

(b)

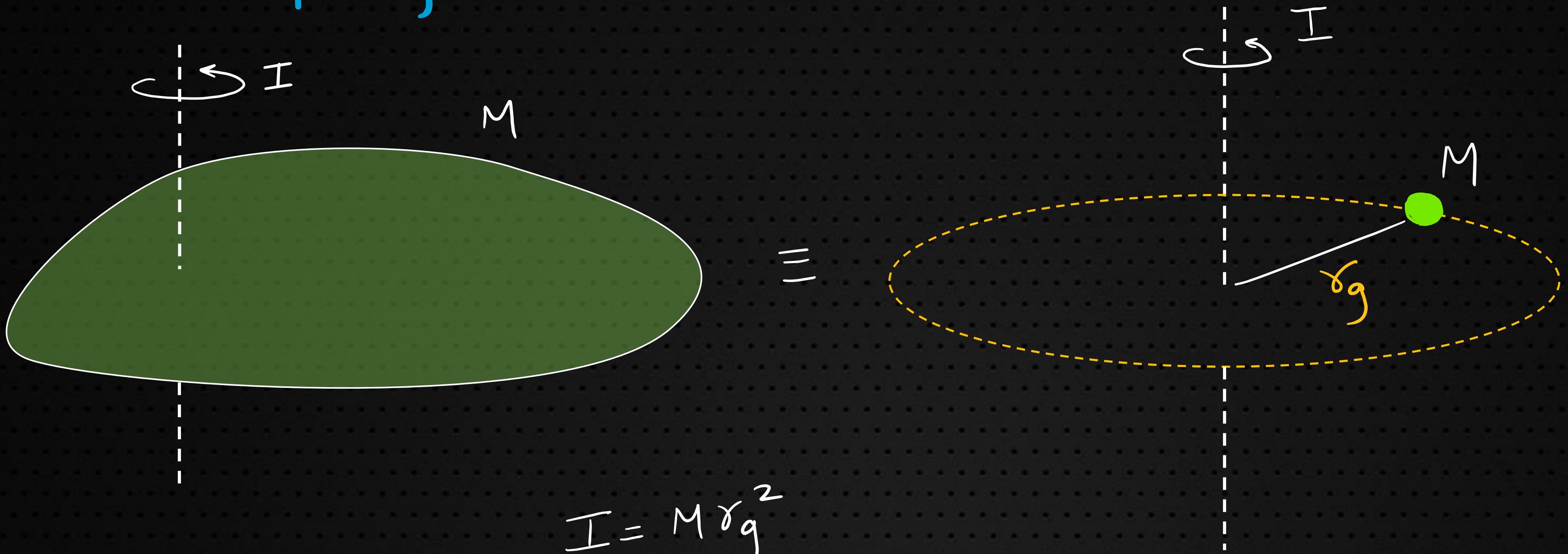


$$I = \frac{MR^2}{2} - \text{MOI of removed Disc}$$

$$= \frac{MR^2}{2} - \left(\frac{M}{\pi R^2} \times \frac{\pi R^2}{4}\right)\left(\frac{R}{2}\right)^2 \times \frac{1}{2}$$

$$= \frac{MR^2}{2} - \frac{MR^2}{32} = \frac{15}{32} MR^2$$

6. Radius of Gyration

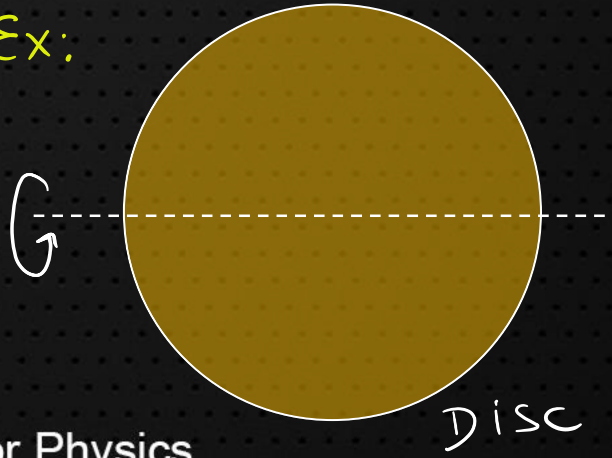


$I = M r_g^2$

$\Rightarrow r_g = \sqrt{\frac{I}{M}}$

Ex:

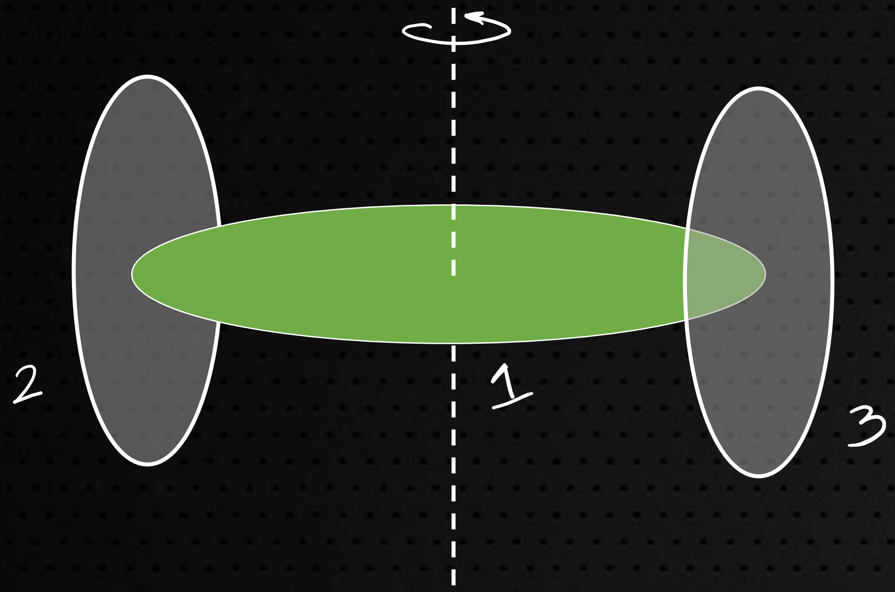
$\frac{MR^2}{4} = I$



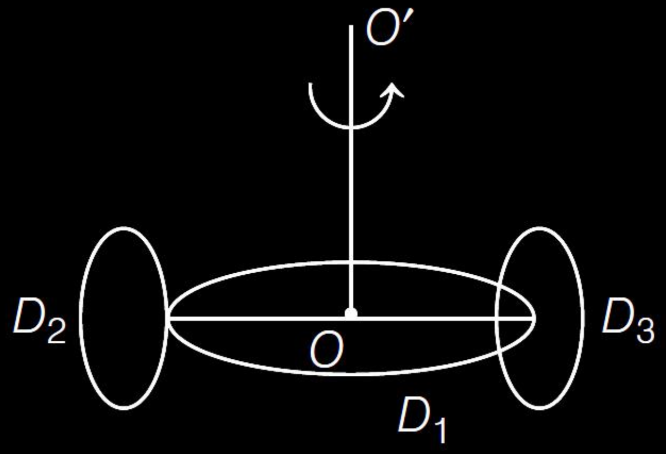
$r_g = \sqrt{\frac{MR^2/4}{M}}$

$= \frac{R}{2}$

7. Questions on MOI



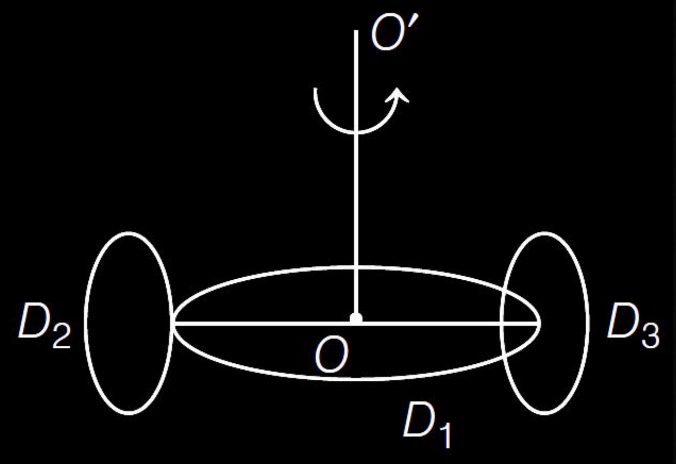
Ex 1. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO' passing through the centre of D_1, as shown in the figure will be **(2019 Main, 11 Jan II)**



(a) $\frac{2}{3}MR^2$ (b) $\frac{4}{5}MR^2$ (c) $3MR^2$ (d) MR^2

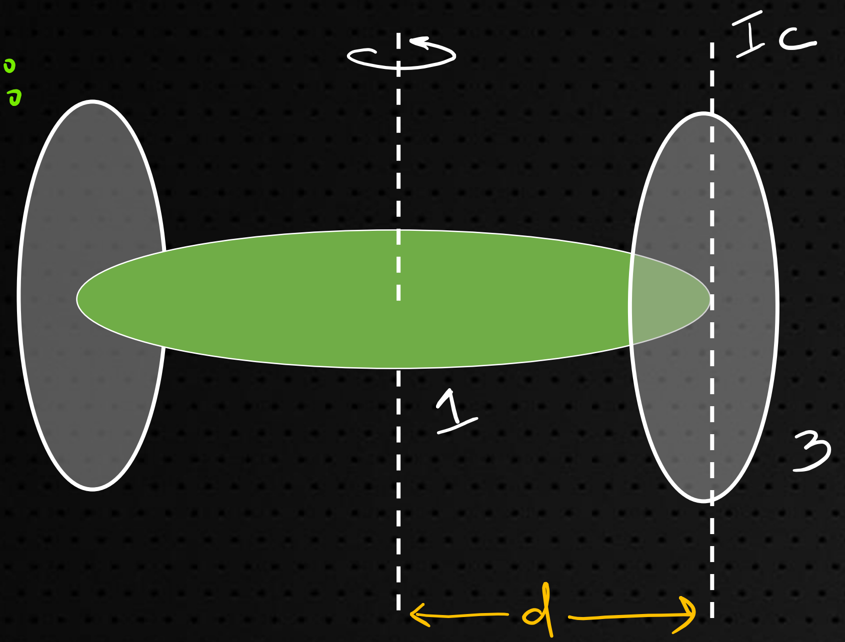
7. Questions on MOI

Ex 1. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO' passing through the centre of D_1, as shown in the figure will be **(2019 Main, 11 Jan II)**



(a) $\frac{2}{3}MR^2$ (b) $\frac{4}{5}MR^2$ (c) $3MR^2$ (d) MR^2

Sol^n:



$$I = I_1 + 2\cdot I_3 \Rightarrow I = \frac{MR^2}{2} + 2\left(I_c + Md^2\right)$$

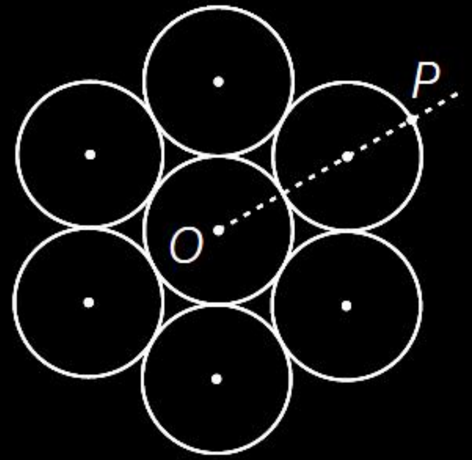
$$\Rightarrow I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$$

$$\therefore \boxed{I = 3MR^2}$$

... Continued

Ex 2. Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown in the figure. The moment of inertia of the arrangement about an axis normal to the plane and passing through the point P is

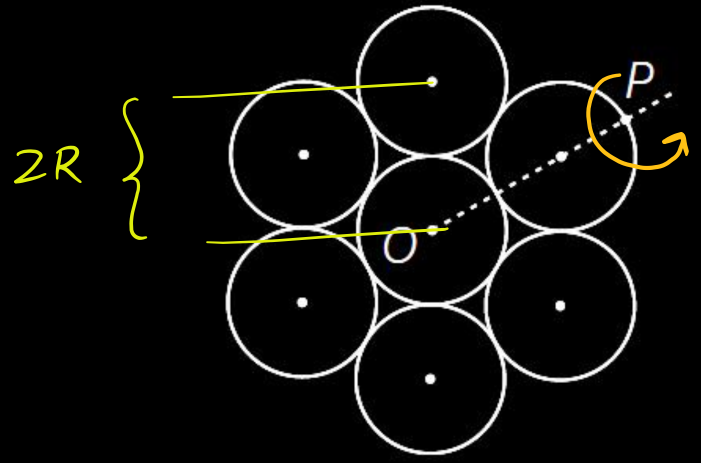
(2018 Main)



(a) $\frac{181}{2}MR^2$ (b) $\frac{19}{2}MR^2$ (c) $\frac{55}{2}MR^2$ (d) $\frac{73}{2}MR^2$

... Continued

Ex 2. Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown in the figure. The moment of inertia of the arrangement about an axis normal to the plane and passing through the point P is (2018 Main)



(a) $\frac{181}{2}MR^2$ (b) $\frac{19}{2}MR^2$ (c) $\frac{55}{2}MR^2$ (d) $\frac{73}{2}MR^2$

Sol^n: 1st Find I_0 of system.

$$I_0 = \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M(2R)^2\right)$$

$$= \frac{55}{2}MR^2$$

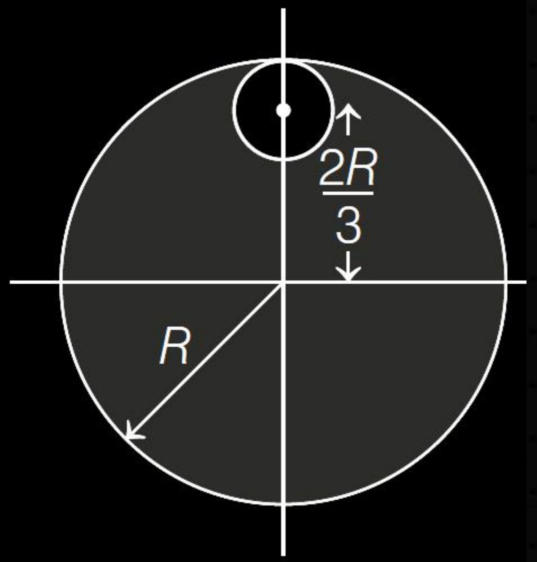
$$\therefore I_P = I_0 + 7M \cdot d^2$$

$$= \frac{55}{2}MR^2 + 7M \times (3R)^2$$

$$= \boxed{\frac{181}{2}MR^2}$$

... Continued

Ex 3. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is **(2018 Main)**

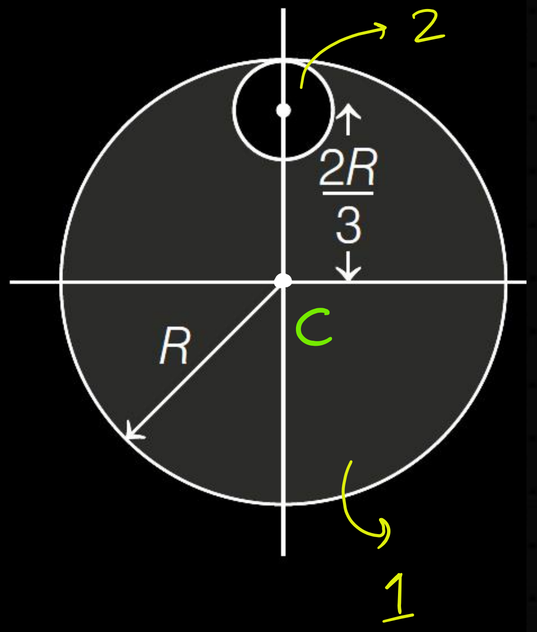


(a) $\frac{37}{9}MR^2$ (b) $4MR^2$

(c) $\frac{40}{9}MR^2$ (d) $10MR^2$

... Continued

Ex 3. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is **(2018 Main)**



(a) $\frac{37}{9}MR^2$ (b) $4MR^2$

(c) $\frac{40}{9}MR^2$ (d) $10MR^2$

Sol^n: mass of cut out disc,

$$m = \frac{9M}{\pi R^2} \times \pi \frac{R^2}{9} = M$$

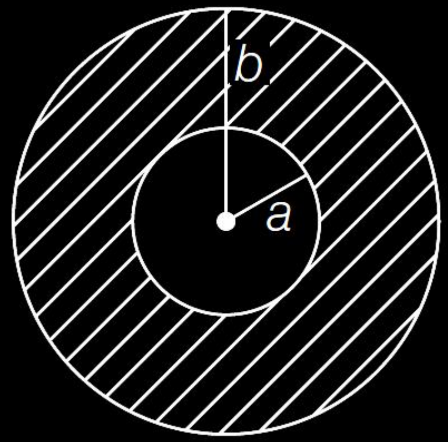
$$I_C = \text{MOI of } 9M - \text{MOI of } m$$

$$= 9M\frac{R^2}{2} - \left[\frac{M(R/3)^2}{2} + M\left(\frac{2R}{3}\right)^2\right]$$

$$\therefore \boxed{I_C = 4MR^2}$$

... Continued

Ex4. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is **(2019 Main, 12 April I)**

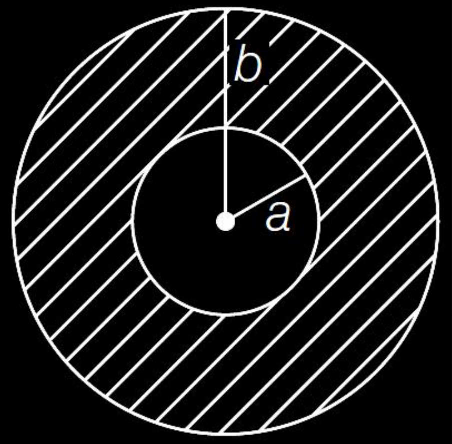


(a) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (b) $\frac{a+b}{2}$

(c) $\sqrt{\frac{a^2+b^2+ab}{3}}$ (d) $\frac{a+b}{3}$

... Continued

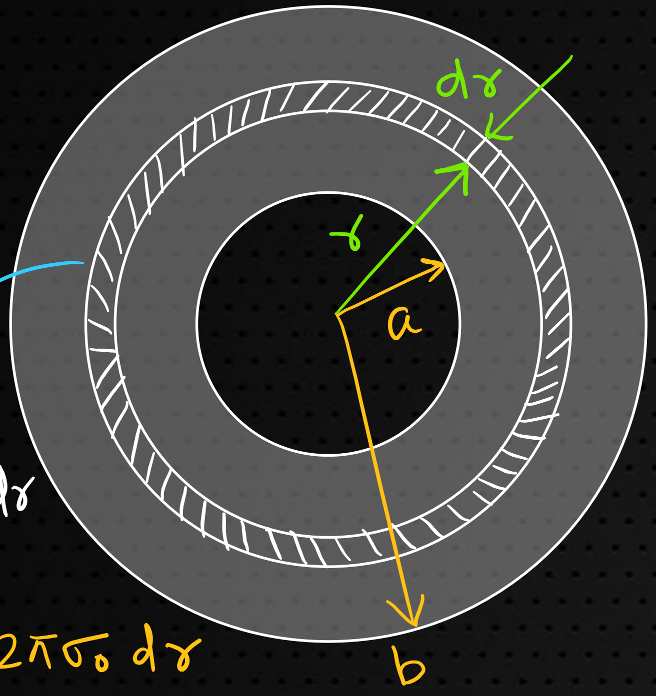
Ex4. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is **(2019 Main, 12 April I)**



(a) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (b) $\frac{a+b}{2}$

(c) $\sqrt{\frac{a^2+b^2+ab}{3}}$ (d) $\frac{a+b}{3}$

Sol^n:



$dm = \frac{\sigma_0}{r} \cdot 2\pi r\, dr$

$\therefore dm = 2\pi\sigma_0\, dr$

$r_g = \sqrt{\frac{I}{M}}$

$I = \int_a^b dm \cdot r^2 \Rightarrow I = \int_a^b 2\pi\sigma_0 r^2\, dr \Rightarrow I = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$

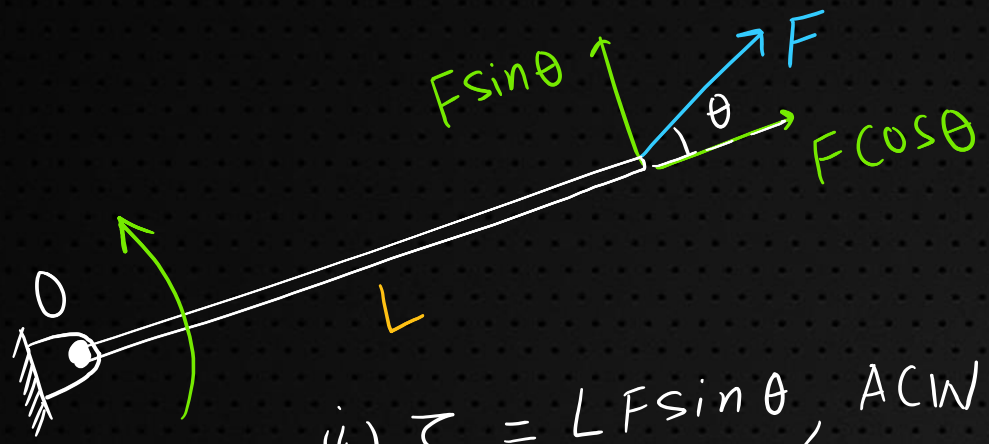
$M = \int_a^b 2\pi\sigma_0\, dr \Rightarrow M = 2\pi\sigma_0(b-a)$

$\therefore r_g = \sqrt{\frac{b^3 - a^3}{3(b-a)}}$

$b^3 - a^3 = (b-a)(a^2+b^2+ab)$

8. Torque ($\vec{\tau} = \vec{r} \times \vec{F}$), N-m

(a)



(i) $\tau = LF\sin\theta$, ACW

(ii) $F\cos\theta$ passes through O, can't have rotational effect

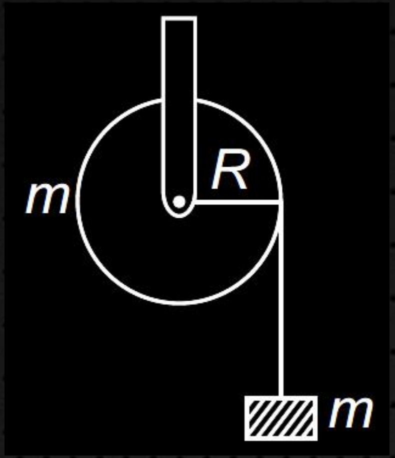
$\sum \vec{\tau} = I\vec{\alpha}$ → angular accn

static Equillibrium problems

$\sum \vec{\tau} = 0$ and $\sum \vec{F} = 0$

both ω and $V_{cm} = 0$

9. Questions on $\vec{\tau} = I\vec{\alpha}$



Ex 5. A mass m supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **(2014 Main)**

(a) $2g/3$ (b) $g/2$

(c) $5g/6$ (d) g

9. Questions on $\vec{\tau} = I\vec{\alpha}$

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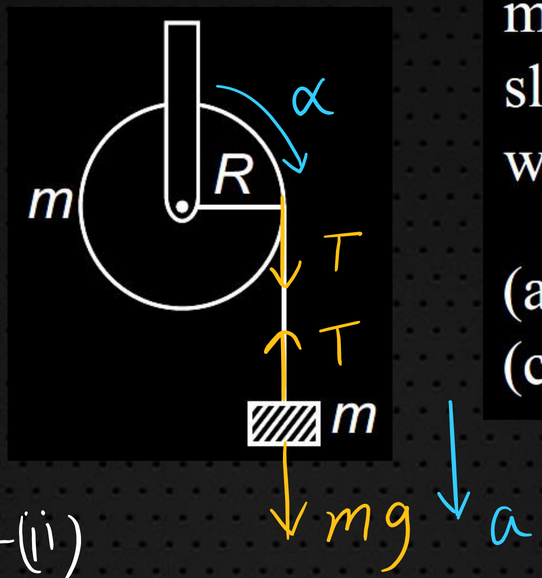


Solⁿ:

$$mg - T = ma \quad \text{—(i)}$$

$$TR = I\alpha$$

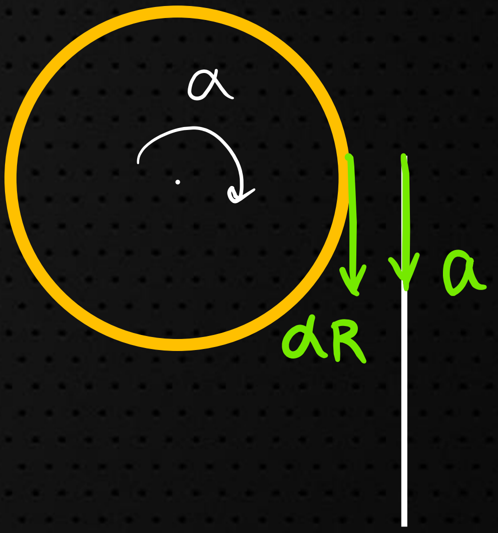
$$\Rightarrow TR = mR^2 \cdot \alpha$$

$$\Rightarrow TR = mR^2 \cdot \frac{a}{R}$$

$$\text{or} \quad T = ma \quad \text{—(ii)}$$

No slipping

$$a = \alpha R$$

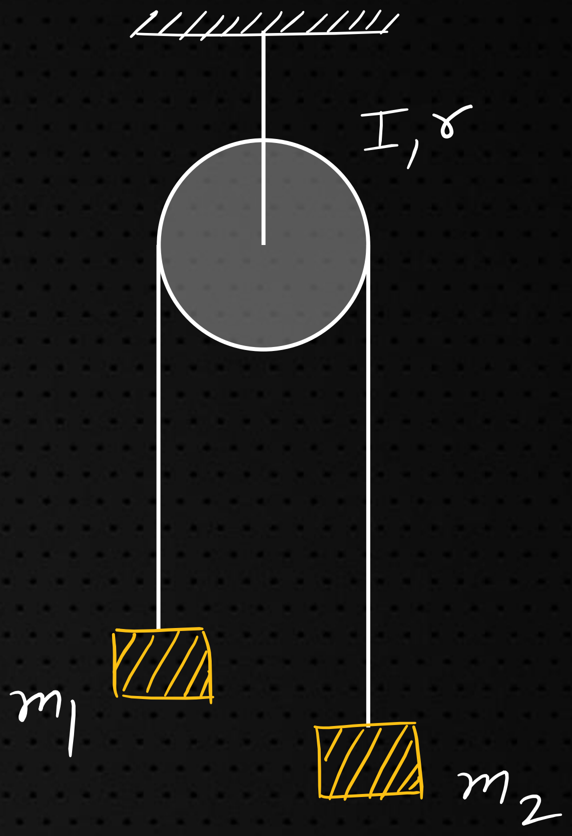


solving (i) & (ii)

$$\boxed{a = \frac{g}{2}}$$

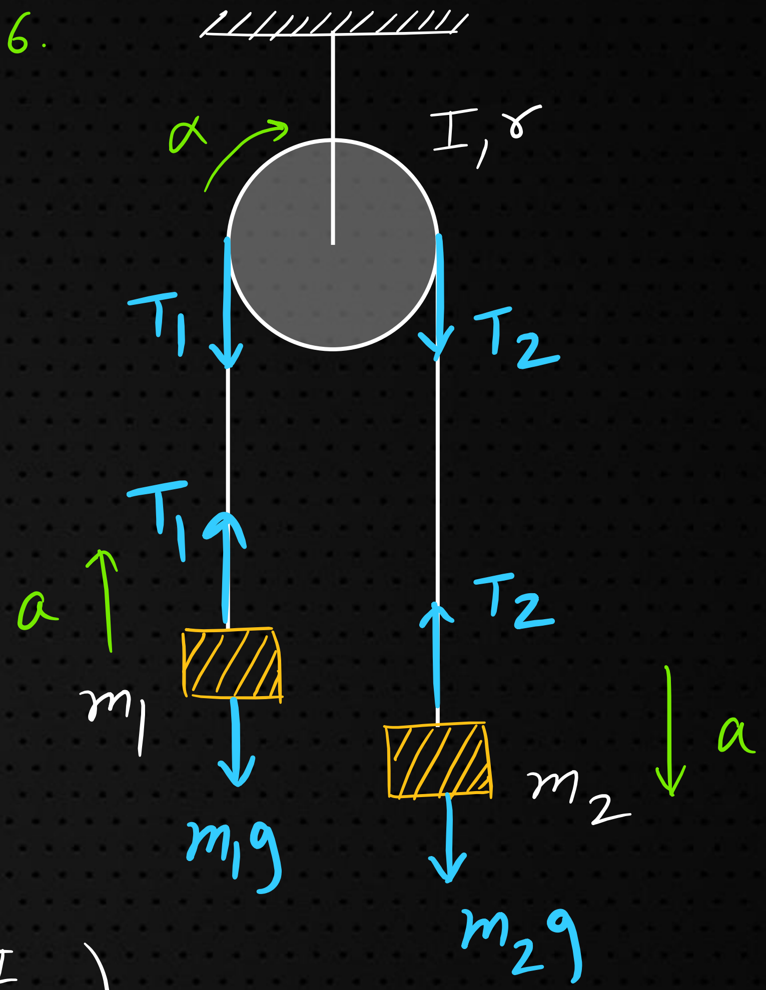
... Continued

Ex 6.



... Continued

Ex 6.



Sol^n:

$$m_2 g - T_2 = m_2 a \quad \text{---(i)}$$

$$T_1 - m_1 g = m_1 a \quad \text{---(ii)}$$

$$T_2 r - T_1 r = I\alpha$$

$$\Rightarrow (T_2 - T_1) r = I \cdot \frac{a}{r}$$

$$T_2 - T_1 = \frac{Ia}{r^2} \quad \text{---(iii)}$$

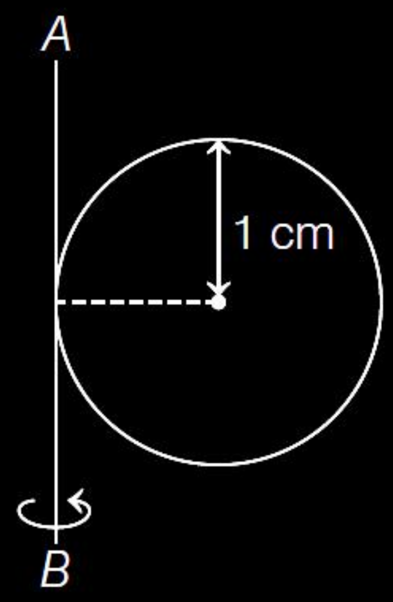
(i) + (ii) + (iii)

$$(m_2 - m_1) g = a\left(m_1 + m_2 + \frac{I}{r^2}\right)$$

$$\therefore a = \frac{(m_2 - m_1) g}{m_1 + m_2 + \frac{I}{r^2}}$$

... Continued

Ex 7. A metal coin of mass 5g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to **(2019 Main, 10 April II)**

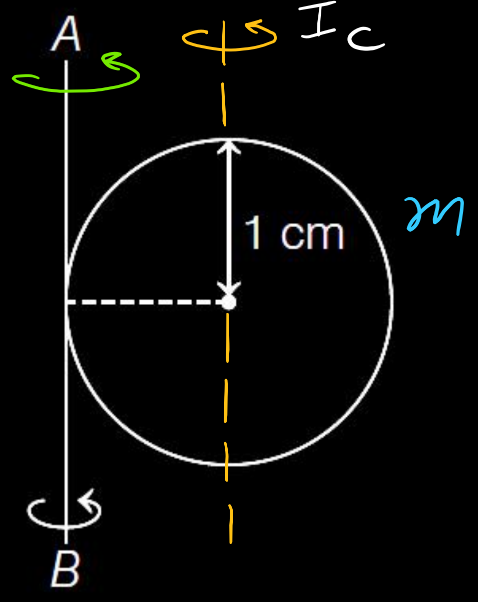


(a) 4.0×10^{-6} N-m (b) 2.0×10^{-5} N-m

(c) 1.6×10^{-5} N-m (d) 7.9×10^{-6} N-m

... Continued

Ex 7. A metal coin of mass 5g and radius 1 cm is fixed to a thin stick *AB* of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about *AB* at 25 rotations per second in 5s, is close to **(2019 Main, 10 April II)**



(a) 4.0×10^{-6} N-m (b) 2.0×10^{-5} N-m

(c) 1.6×10^{-5} N-m (d) 7.9×10^{-6} N-m

Sol^n: $\omega_i = 0$, $\omega_f = 25$ rps
$= 25 \times 2\pi$ rad/s
$= 50\pi$ rad/s

$\omega_f = \omega_i + \alpha t \Rightarrow \alpha = \frac{50\pi}{5} = 10\pi \text{ rad/s}^2$

$\tau = I\alpha \Rightarrow \tau = (I_c + md^2)\alpha$

$\Rightarrow \tau = \left(\frac{mR^2}{4} + mR^2\right)\alpha$

$= \frac{5}{4} \times 5 \times 10^{-3} \times (10^{-2})^2 \times 10\pi$

$\approx \boxed{2 \times 10^{-5} \text{ Nm}}$

10. Question on Static Equilibrium

Ex 8. Two persons of equal height are carrying a long uniform wooden beam of length ℓ. They are at distance $\ell/4$ and $\ell/6$ from nearest ends of the rod. The ratio of normal reactions at their heads is:

(a) 2 : 3
(b) 1 : 3
(c) 4 : 3
(d) 1 : 2

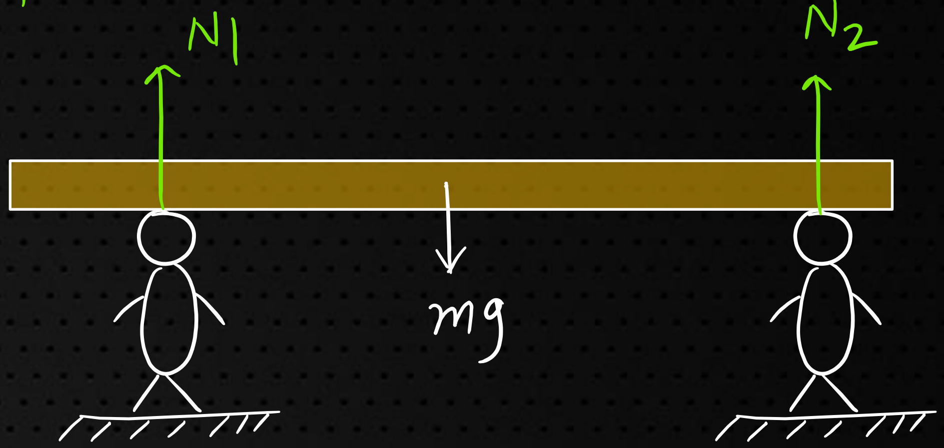
10. Question on Static Equilibrium

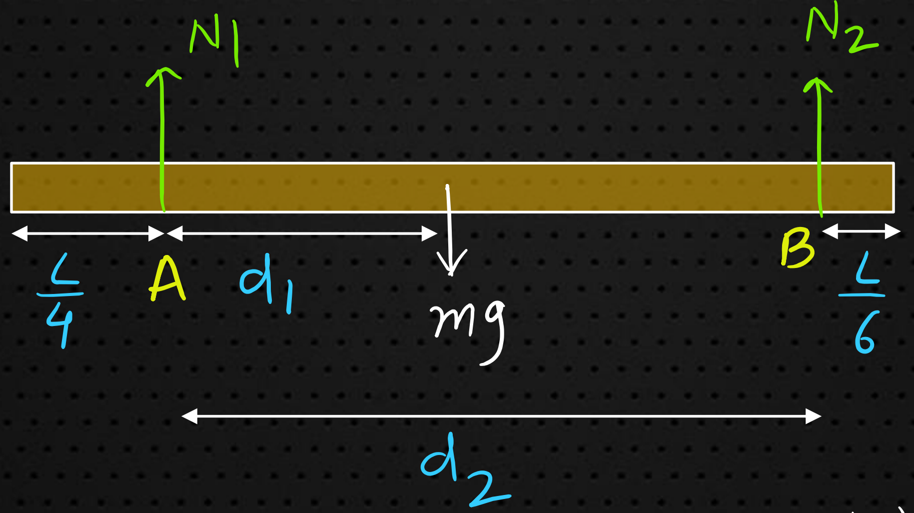
Ex8. Two persons of equal height are carrying a long uniform wooden beam of length ℓ. They are at distance $\ell/4$ and $\ell/6$ from nearest ends of the rod. The ratio of normal reactions at their heads is:

(a) 2 : 3 (b) 1 : 3

(c) 4 : 3 (d) 1 : 2

Sol^n:





$d_1 = \frac{L}{2} - \frac{L}{4} = \frac{L}{4}$, $d_2 = L - \left(\frac{L}{4} + \frac{L}{6}\right) = \frac{7L}{12}$

$\Sigma \tau_A = 0$

$\Rightarrow N_2 \times \frac{7L}{12} = mg \times \frac{L}{4}$

$\Rightarrow N_2 = 3mg/7$

$N_1 + N_2 = mg$

$\therefore N_1 = 4mg/7$

$$\boxed{\Rightarrow N_1/N_2 = 4/3}$$

EDUNITI

PYQs LINKS (JEE MAIN)

2021 Feb	2021 March	2021 July	2021 August	2020	Selected Problems
https://youtu.be/2hqJnity2_o	https://youtu.be/6CsOsazSvmA	https://youtu.be/H_jH3GtD4Bs	https://youtu.be/wmlghKW_PUQ	https://youtu.be/zWddLCElwWE	https://youtu.be/wjBRVQih400

CLICK

(Practice these Questions)

HCV Rotation Solution Q1-Q70

https://bit.ly/3fM9fCC

Eduniti for Physics

Revision Series Playlist Link https://bit.ly/3eBbib9

JEE Main PYQs Link https://bit.ly/2S54jzh Chapter wise 2021, 2020, 2018

GoldMine Link https://bit.ly/2VhOGFF

CLICK