JEE MAIN | IIT JEE

ROTATION

(Part-3)

REVISION in 45 Min

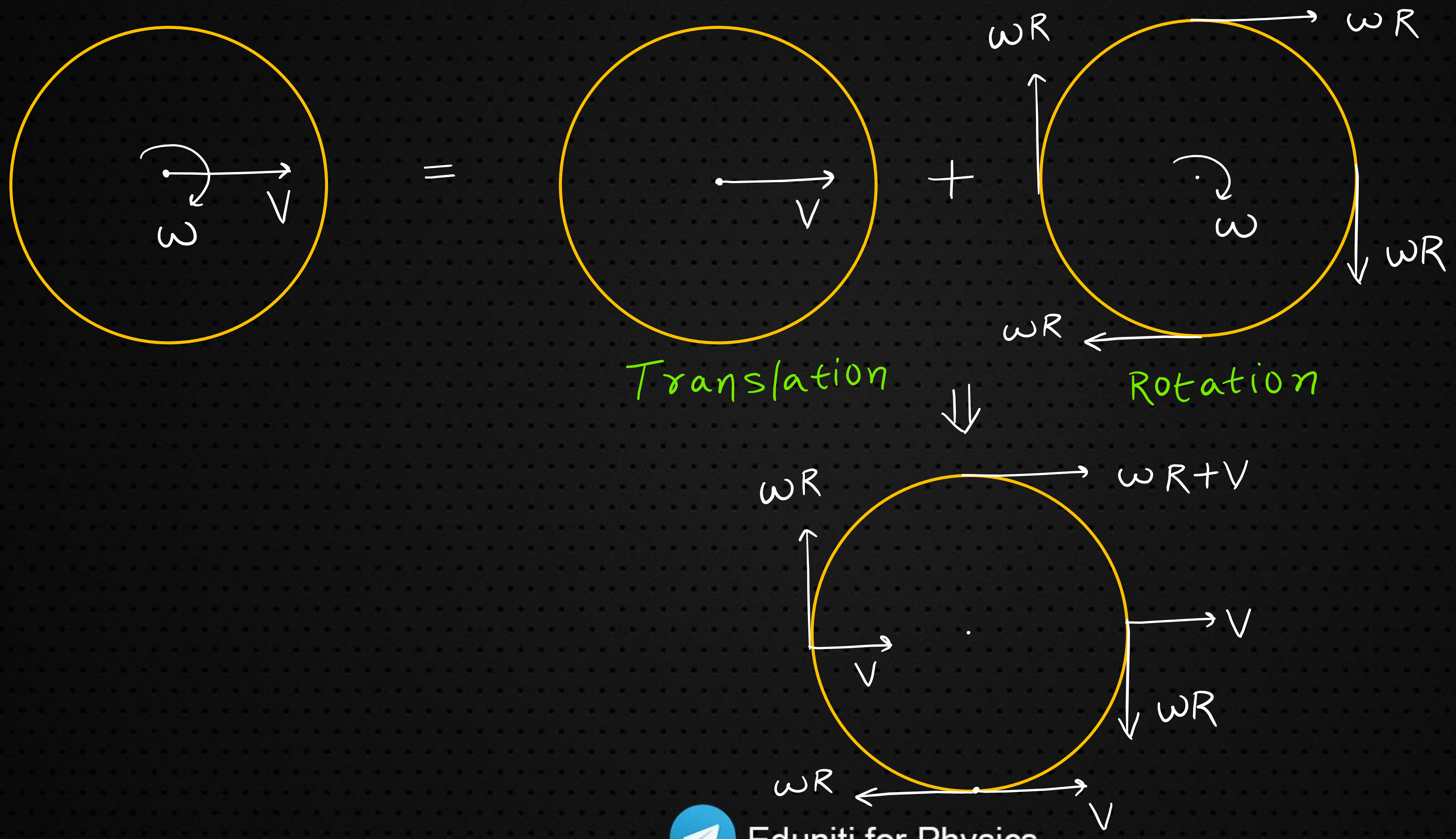
Mohit Sir, IIT Kharagpur

PART 3 - Topics to covered

1. Velocity of a Point on Circumference (Trans + Rotational)
2. Acceleration of a Point of Circumference
3. Rolling Motion (*pure rolling & rolling with slipping*)
4. Question on Vel & acc of a circumference point
5. Total Kinetic Energy in Pure Rolling
6. External Force in Rolling (*numerical included*)
7. Question on Energy Conservation (*pure rolling on Inclined*)
8. Acceleration & Friction (*Rolling on Inclined Plane*)
9. Question on Acc & Friction
10. Toppling (*Question + toppling on Incline*)

Chapter	Formulae_Concept VIDEO LINK
Unit & Dimensions	https://youtu.be/wdd-wlZF4Hk
Errors and Vectors	
Vernier Calliper	https://youtu.be/pVoN045dV8I
Screw Gauge	https://youtu.be/gYd2PtmZ0mw
Kinematics_Motion in 1d	https://youtu.be/U4NNxFaFliE
Kinematics_Motion in 2d	https://youtu.be/4_Zo5WhMf7w
Laws of Motion	https://youtu.be/7JlR8gNRQIs
Friction	https://youtu.be/Rn1bLst7eGk
Work Energy Power	https://youtu.be/kjrXoE-kDI8
Circular Motion	https://youtu.be/KnFymKHlkT0
Centre of Mass	https://youtu.be/ads35RKD618
Cons of Momentum & Collision	https://youtu.be/3f0u4L-lyyw
Rotational Motion – Part 1	https://youtu.be/O6j1mLp06XI
Rotational Motion – Part 2	https://youtu.be/OHni1DRdfAQ
Gravitation	https://youtu.be/rAj2huLVaEk
Properties of Solids	https://youtu.be/gSXxjk89l_c
Fluids Statics (Part 1)	https://youtu.be/RFKx9B9yo3M
Fluid Dynamics (Part 2)	https://youtu.be/Y717vQpUEJQ
Fluid Properties (Part 3)	https://youtu.be/V8xUWWK2oT0
Simple Harmonic Motion	https://youtu.be/RIb7ofNG09I
Thermal Properties	https://youtu.be/OYjjyPIzddE
Heat Transfer	https://youtu.be/PyNboHgtYzM
KTG	https://youtu.be/XO1tvFhIa0I
Thermodynamics	https://youtu.be/iz_kf1jRDRw
Wave Motion -Organ Pipes and Resonance Tube	https://youtu.be/fB7pfJ77za8
Wave Motion - Doppler's Effect	https://youtu.be/9-BxOaamnwg
Electrostatics	https://youtu.be/3stXbGRMcrk
Capacitors	https://youtu.be/EXEiickNUkY
Current Electricity	https://youtu.be/gm8FUfjrX18
Moving Charges and Magnetic Effect of Current	https://youtu.be/ULD2Ok1CGJk
Earth's Magnetism	https://youtu.be/a4CT5uVwAK4
Magnetic Properties	https://youtu.be/63cwdYXNlYE
EMI	https://youtu.be/puVavm_GFRM
Alternating Current	https://youtu.be/74dTY-pzM_o
Ray Optics	https://youtu.be/BhnyTWzIlBA
Wave Optics Part 1_Interference	https://youtu.be/LG5nlE8XTeI
Wave Optics Part 2_Diffraction_Polarization	https://youtu.be/ymMyyJGGqnY
Optical Instruments	https://youtu.be/OQssbDH0A4I
Electromagnetic Waves	https://youtu.be/bcVXgEkyQZY
Semiconductors_Basics + Zener Diode	https://youtu.be/_A2JomQ7-50
Semiconductors_Transistors	https://youtu.be/psDwl84Nzb0
Semiconductors_Logic Gates	https://youtu.be/pZdQAzLbFTo
Communication Systems	https://youtu.be/8NgMqK9X79Y
Modern Physics_Part 1_Atomic Physics	https://youtu.be/9VKUnE3mpHk
Modern Physics_Part 2_Photoelectric Effect	https://youtu.be/24oTQp84jrk
Modern Physics_Part 3_Dual Nature of Light	https://youtu.be/0zoR_saMAQY
Modern Physics_Part 4_Radioactivity	https://youtu.be/AdX3YBhQyog
Modern Physics_Part 5_Nuclear Physics	https://youtu.be/VDWqVahGixc
Modern Physics_Part 6_X Rays	https://youtu.be/dSHXdzX7NX0

1. Velocity of Point on Circumference (Trans + Rotational)



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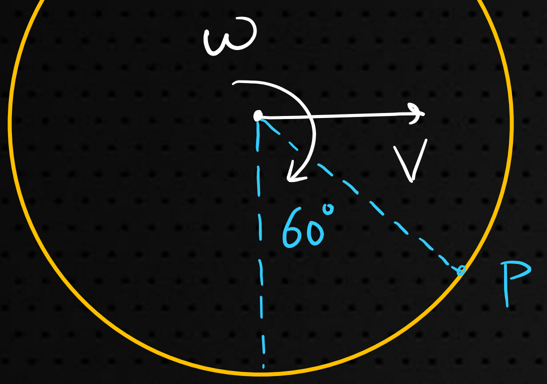
$V = 5\,m/s$, $\omega = 2.5\,rad/s$, $R = 2m$. Find V_P ?

Ex 1.



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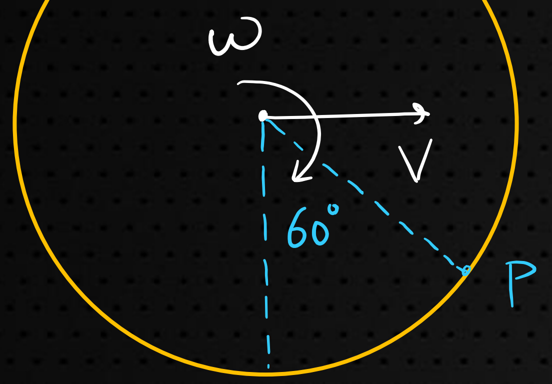
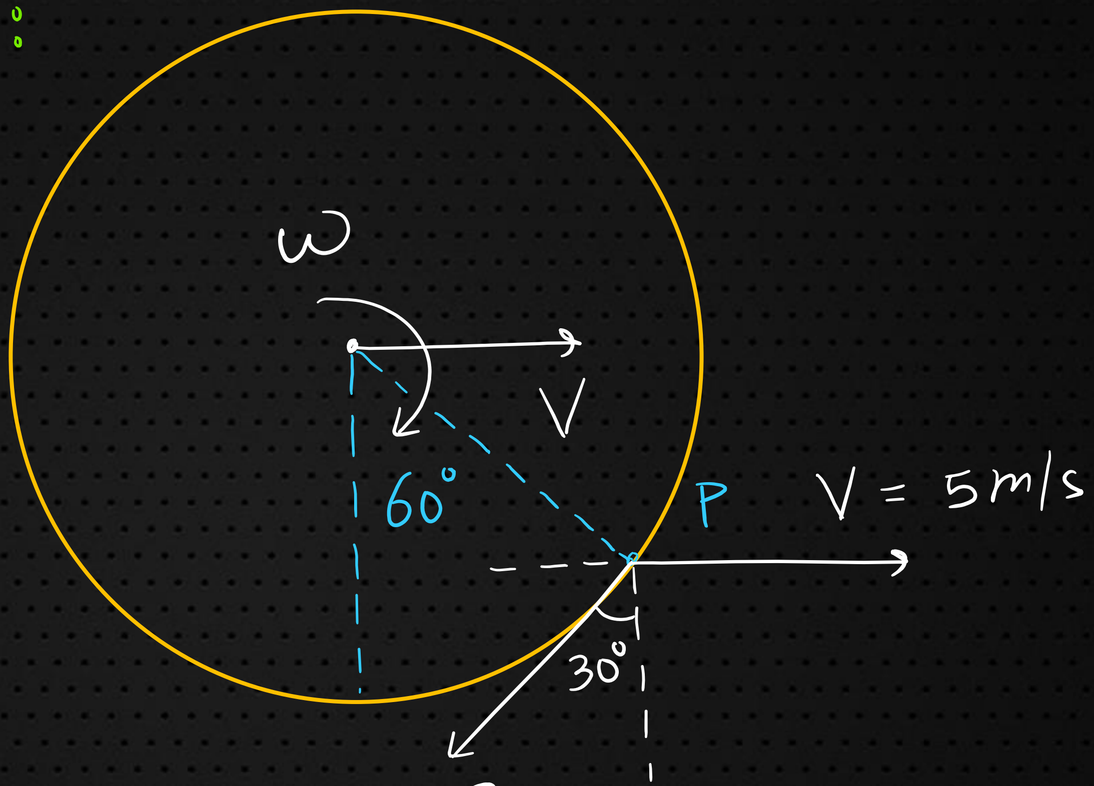


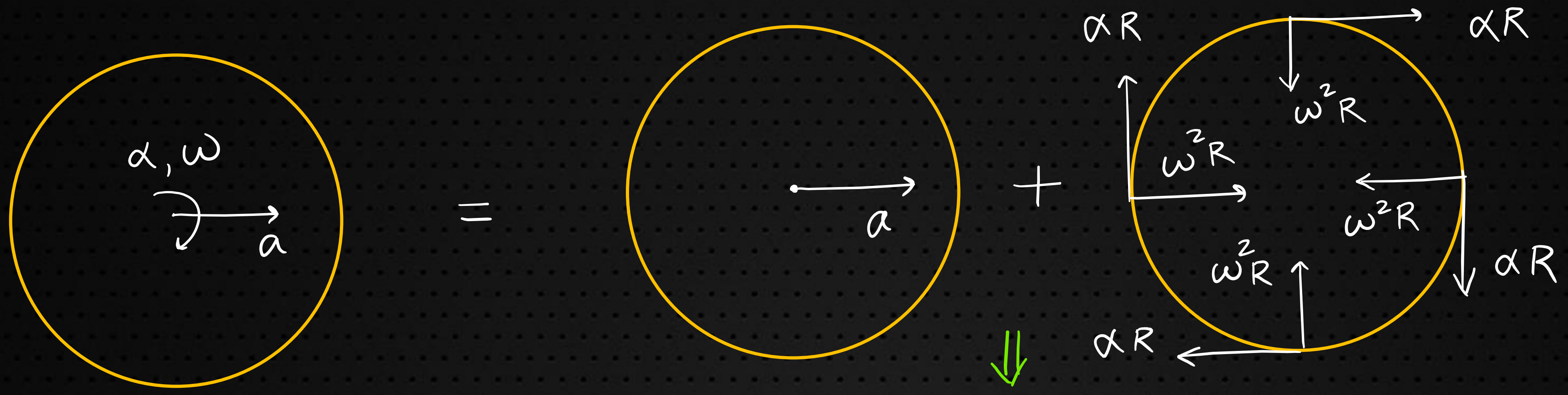
Sol^n:

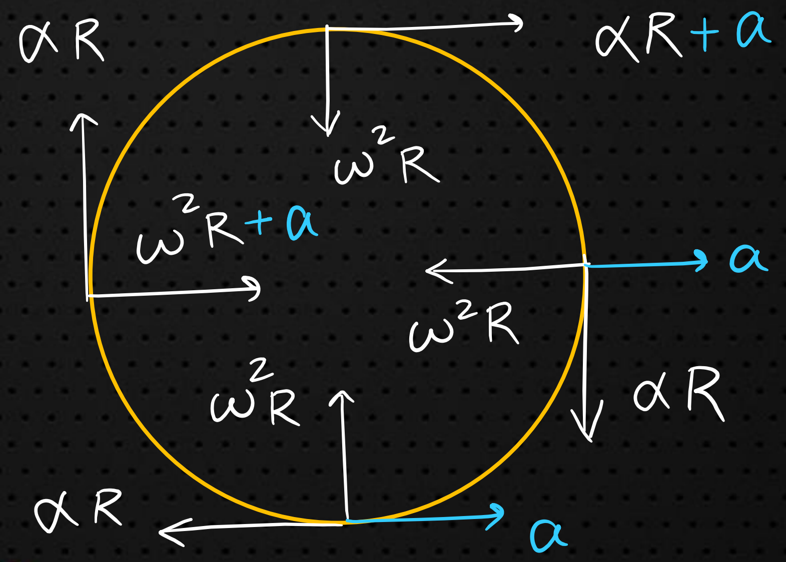


$$5\,m/s = \omega R$$

$$V_P = \sqrt{5^2 + 5^2 + 2 \cdot 5^2 \underbrace{\cos 120^\circ}_{-\frac{1}{2}}} = \boxed{5\,m/s}$$

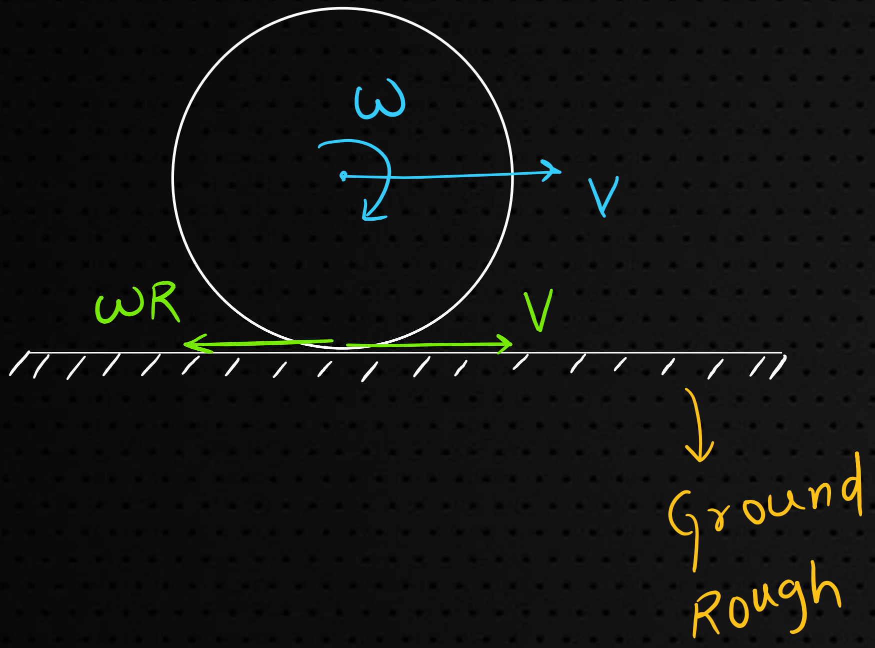
2. Acceleration of point on circumference





3. Rolling Motion

(a) Pure rolling : No relative motion of Pt. of Contact or Pt. of Contacts have same Velocity.



$\because V_{ground} = 0 \Rightarrow V - \omega R = 0 \quad \therefore \boxed{V = \omega R}$

(i) $V = \omega R$ $(\therefore a = \alpha R)$

(ii) No friction force acting

(b) Rolling with slipping : $V \neq \omega R$

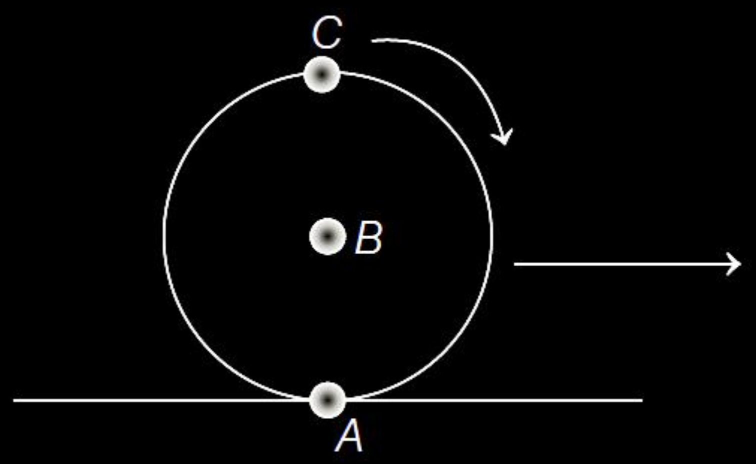
↳ If $V > \omega R \Rightarrow f_K$ acts backward

↳ If $\omega R > V \Rightarrow f_K$ acts forward

NOTE : f_K until Pure rolling starts

4. Question on Velocity & Acceleration in Pure Rolling

Ex 2. A sphere is rolling without slipping on a fixed horizontal plane surface.



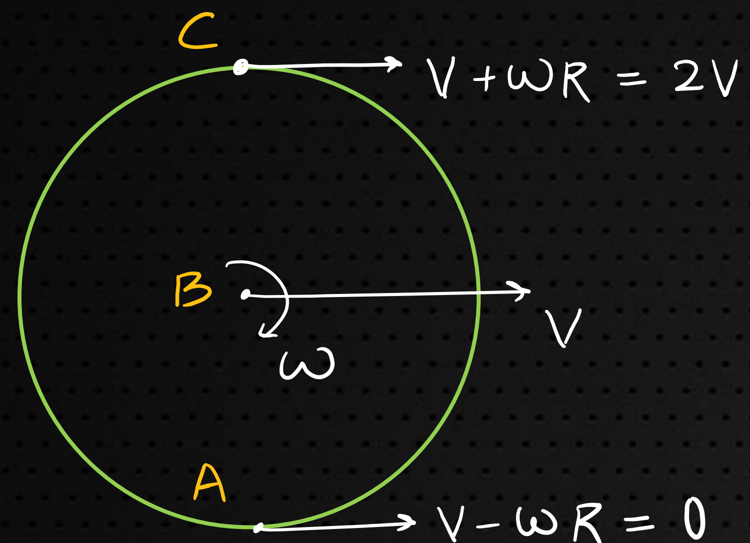
In the figure, A is the point of contact. B is the centre of the sphere and C is its topmost point. Then, **(2009)**

(a) $\mathbf{v}_C - \mathbf{v}_A = 2(\mathbf{v}_B - \mathbf{v}_C)$ (b) $\mathbf{v}_C - \mathbf{v}_B = \mathbf{v}_B - \mathbf{v}_A$

(c) $|\mathbf{v}_C - \mathbf{v}_A| = 2|\mathbf{v}_B - \mathbf{v}_C|$ (d) $|\mathbf{v}_C - \mathbf{v}_A| = 4|\mathbf{v}_B|$

4. Question on Velocity & Acceleration in Pure Rolling

Sol^n:



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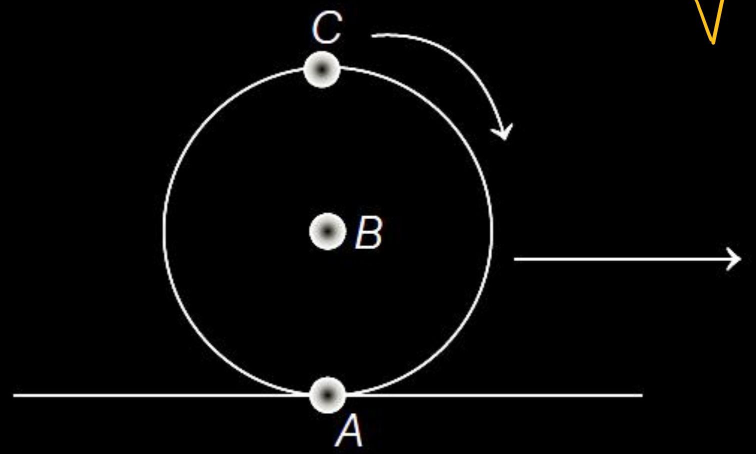

$V = \omega R$

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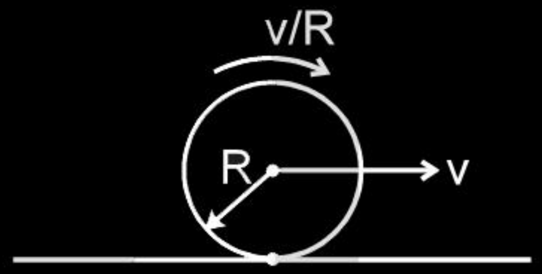
... continued

Ex 3. A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lower most point of the disc



(a) Velocity is v, acceleration is zero
(b) Velocity is zero, acceleration is zero
(c) velocity is v, acceleration is $\frac{v^2}{R}$.
(d) velocity is zero, acceleration is $\frac{v^2}{R}$

... continued

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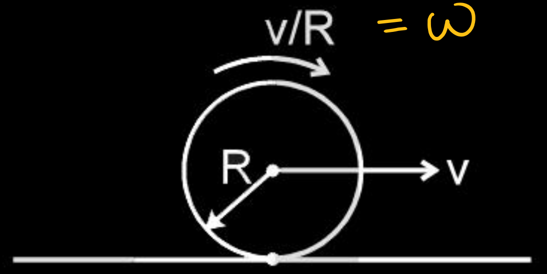

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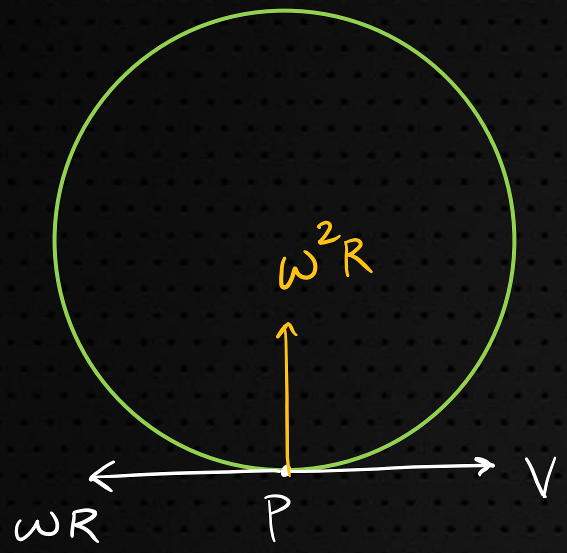
Sol^n:

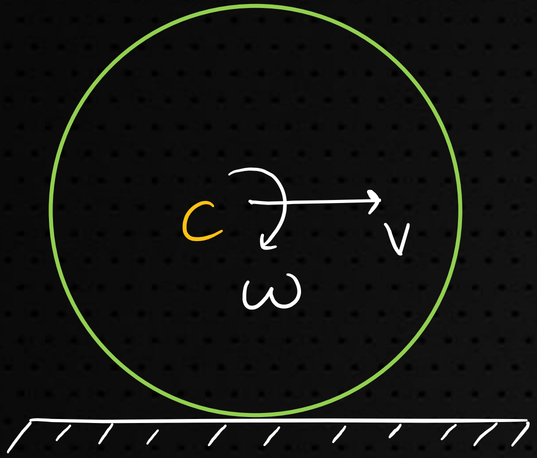


$V_P = 0$

$a_P = \omega^2 R$

$= \frac{V^2}{R}$

5. KE in Pure Rolling ($V = \omega R$)



$I_C = k m R^2$

$\longrightarrow$ 1 if Ring or Hollow cylinder

$\frac{1}{2}$ if Disc or solid cylinder

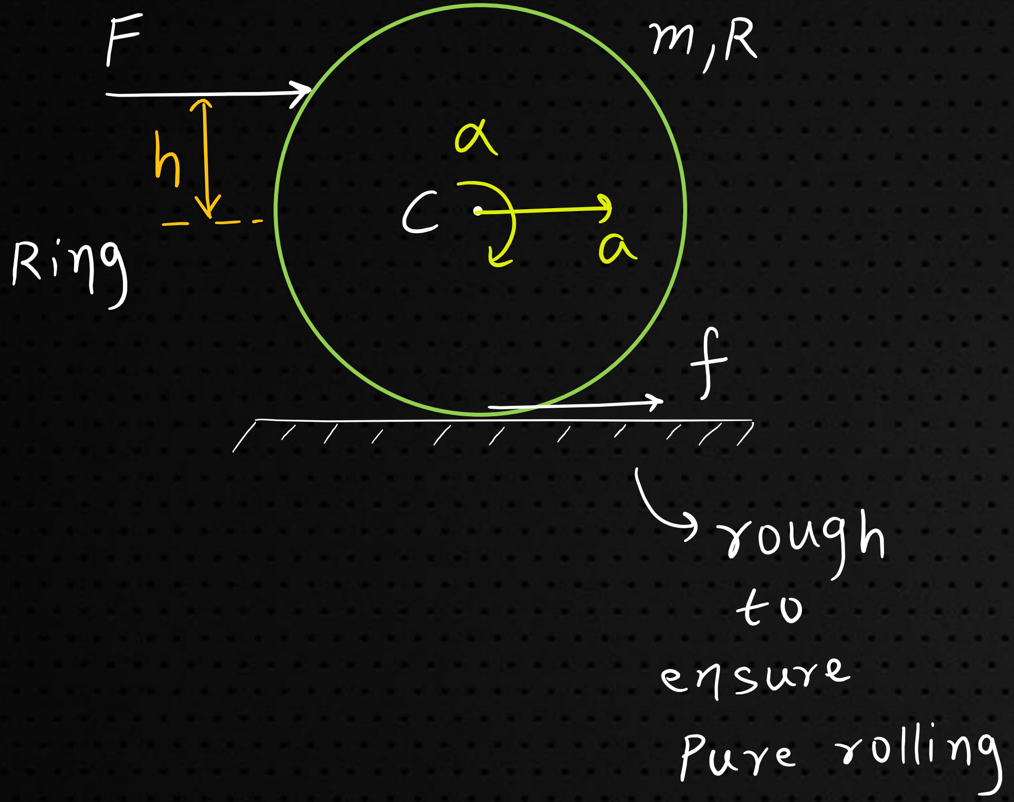
$\frac{2}{3}$ if Hollow sphere

$\frac{2}{5}$ if solid sphere

$$KE = K_T + K_R = \frac{1}{2} m V^2 + \frac{1}{2} \cdot k m R^2 \cdot \left(\frac{V}{R}\right)^2$$

$$\therefore \boxed{K.E. = \frac{1}{2} m V^2 (1 + k)}$$

6. External Force in Rolling



NOTE: (i) $\because$ Pure rolling $\Rightarrow$ f is static

(ii) $\because$ Pt. of contact is at instantaneous rest $\Rightarrow W_f = 0$

$$F + f = ma \quad \text{—} \; ①$$

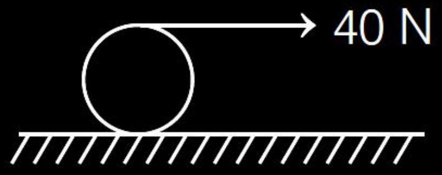
$$Fh - fR = mR^2 \cdot \alpha \qquad \left\{ \alpha = \frac{a}{R} \right\}$$

$$\Rightarrow Fh - fR = mR^2 \cdot \frac{a}{R} \quad \text{—} \; ②$$

solve ① & ② to get a and f.

... continued

Ex 4. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)



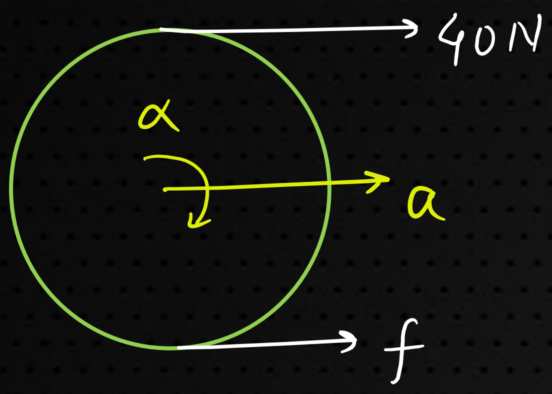
(2019 Main, 11 Jan II)

(a) $10\ \text{rad/s}^2$ (b) $16\ \text{rad/s}^2$

(c) $20\ \text{rad/s}^2$ (d) $12\ \text{rad/s}^2$

... continued

Sol^n:



$a = \alpha R$

$40 + f = ma$ — ①

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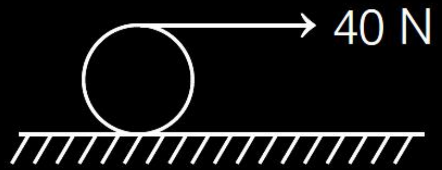

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(c) $20\ \text{rad/s}^2$ (d) $12\ \text{rad/s}^2$

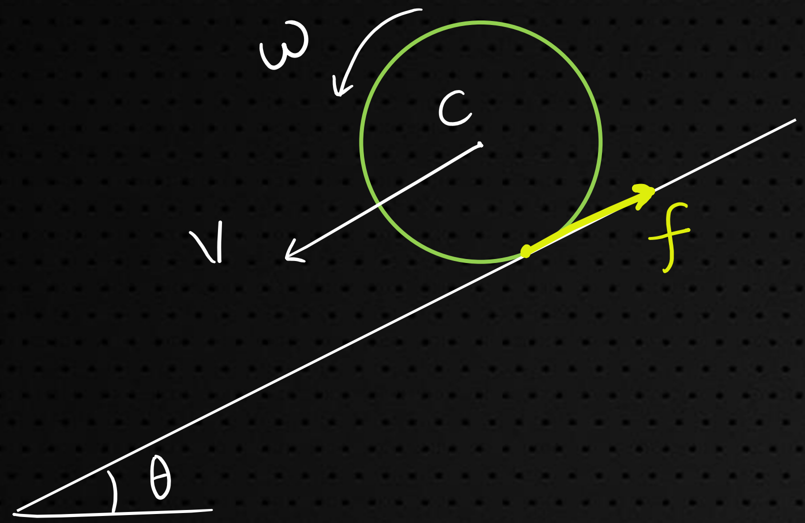
$40R - fR = mR^2 \cdot \alpha \Rightarrow 40 - f = mR\alpha$ — ②

① + ② : $80 = m(a + \alpha R) \Rightarrow \dfrac{80}{2mR} = \alpha$

$\therefore \alpha = \dfrac{80}{2 \times 5 \times 0.5} = \boxed{16\ \text{rad/s}^2}$

7. Questions on Energy Conservation (pure rolling on Inclined)

$\hookrightarrow V = \omega R$ & $a = \alpha R$



NOTE : (i) $V \uparrow$ due to gravity

(ii) Thus ω must $\uparrow$ so that $V = \omega R$

(iii) Thus f acts (creating Torque about C)

$\downarrow$

Static $\Rightarrow W_f = 0$

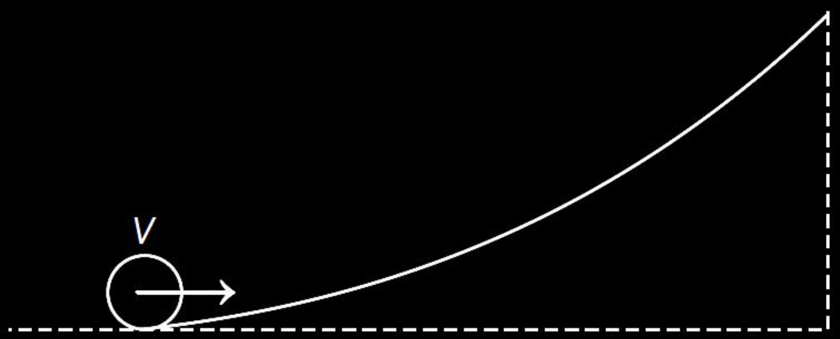
$\Downarrow$

Energy Cons. is applicable

... Continued

Ex 5. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is

(2007, 3M)



(a) ring
(b) solid sphere
(c) hollow sphere
(d) disc

... Continued

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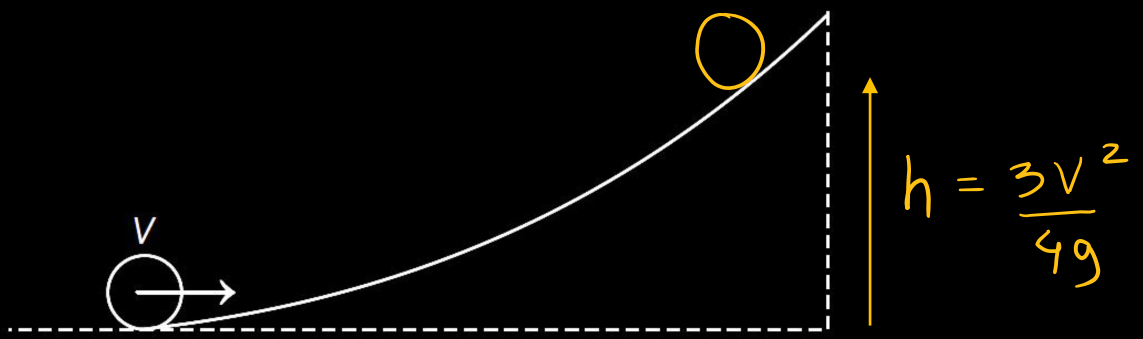


(a) ring (b) solid sphere

(c) hollow sphere (d) disc

Soln: Loss in K.E. = Gain in PE

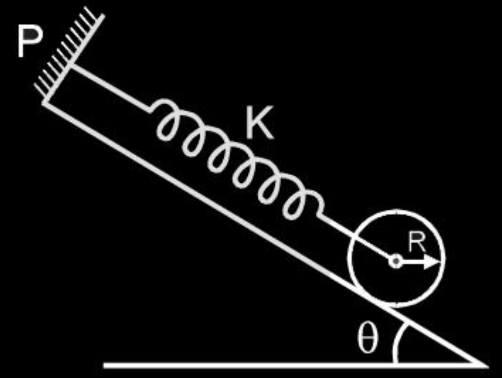
$\Rightarrow \frac{1}{2}mv^2(K+1) = mg \times \frac{3v^2}{4g}$

$\Rightarrow K+1 = \frac{3}{2}$ $\therefore$ $\boxed{K = \frac{1}{2}}$

Disc or solid cylinder

... Continued

Ex 6. A uniform cylinder of mass M and radius R rolls without slipping down a slope of angle θ to the horizontal. The cylinder is connected to a spring constant K while the other end of the spring is connected to a rigid support at P. The cylinder is released when the spring is unstretched. The maximum distance that the cylinder travels is



(a) $\frac{3}{4}\frac{Mg\sin\theta}{K}$ (b) $\frac{Mg\tan\theta}{K}$

(c) $\frac{2Mg\sin\theta}{K}$ (d) $\frac{4}{3}\frac{Mg\sin\theta}{K}$

... Continued

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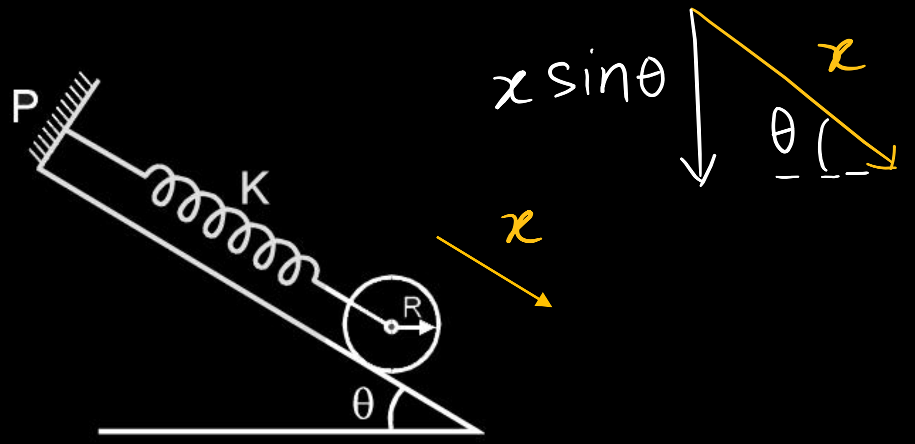


(a) $\frac{3}{4}\frac{Mg\sin\theta}{K}$ (b) $\frac{Mg\tan\theta}{K}$

(c) $\frac{2Mg\sin\theta}{K}$ (d) $\frac{4}{3}\frac{Mg\sin\theta}{K}$

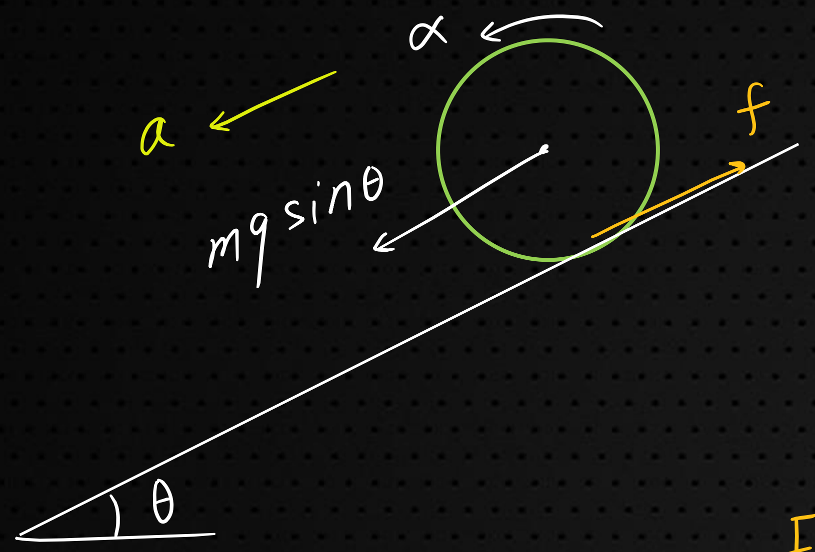
Sol^n: Initial & Finally system at rest.

Loss in Grav P.E. = Gain in Spring P.E.

$\Rightarrow mgx\sin\theta = \frac{1}{2}Kx^2$

$\therefore \boxed{x = \frac{2mg\sin\theta}{K}}$

8. Acceleration & Friction – Rolling on Inclined Plane



$$mg\sin\theta - f = ma \quad \text{— (1)}$$

$$fR = kmR^2 \cdot \alpha \qquad \left\{ \alpha = \frac{a}{R} \right\}$$

$$\Rightarrow fR = kmR^2 \cdot \frac{a}{R} \Rightarrow f = kma \quad \text{— (2)}$$

From (1) and (2) :

$$\boxed{a = \frac{g\sin\theta}{1+k}} \quad \& \quad \boxed{f = \frac{kmg\sin\theta}{1+k}}$$

EDUNITI

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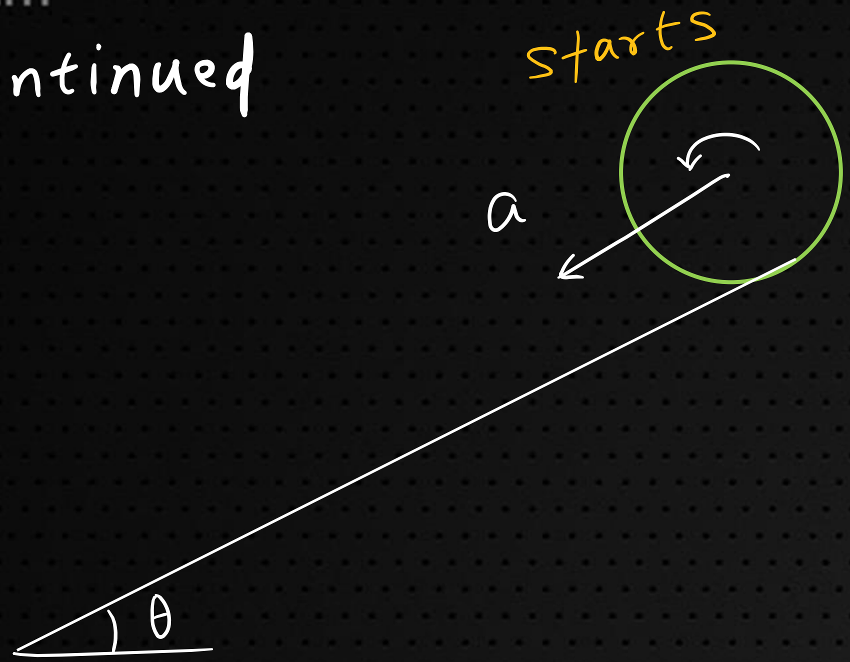

$$a = \frac{g\sin\theta}{1+k}$$ { Lower k $\Rightarrow$ higher a

Ring	Disc	H. sphere	S. sphere
$K = 1$	$K = \frac{1}{2}$	$K = \frac{2}{3}$	$K = \frac{2}{5}$

$$\Rightarrow \quad a_{S.S} > a_{disc} > a_{H.S} > a_{ring}$$

$\Downarrow$

Solid sphere reaches 1st if all starts together

9. Questions on acc^n & friction – Rolling on Incline

Ex 7. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is .... (Take, $g = 10\,\text{ms}^{-2}$) (2018 Adv)

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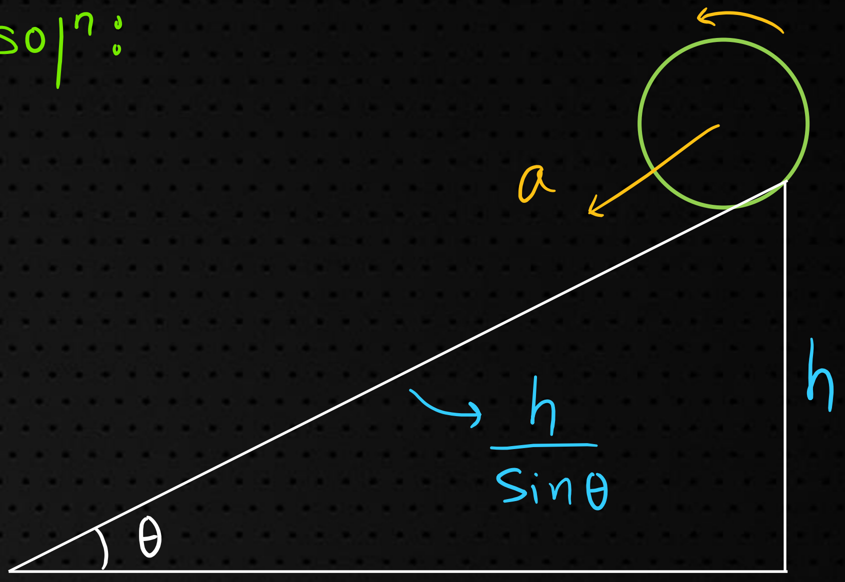
sol^n:



$$\frac{h}{\sin\theta} = \frac{1}{2} \times \frac{g\sin\theta}{1+k} \cdot t^2 \Rightarrow t = \sqrt{\frac{2h(1+k)}{g\sin^2\theta}}$$

$K = 1$, ring

$= \frac{1}{2}$, disc

$t_{disc} = \sqrt{2h/5}$

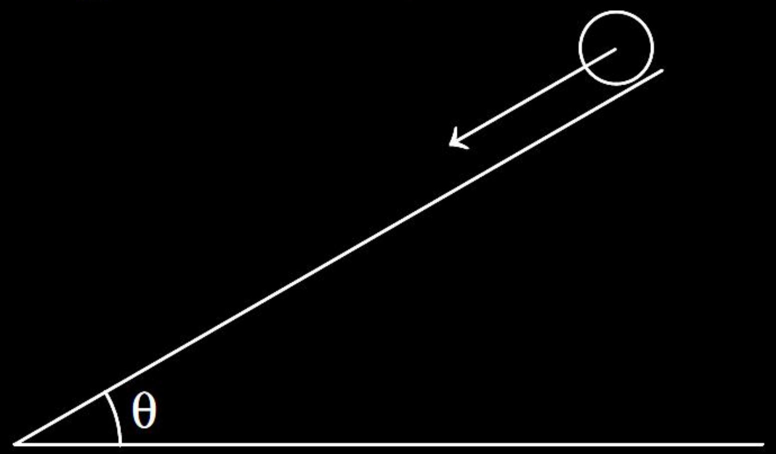
$t_{ring} = \sqrt{8h/15}$

$$\therefore \sqrt{\frac{8h}{15}} - \sqrt{\frac{2h}{5}} = \frac{2}{\sqrt{10}} - \frac{\sqrt{3}}{\sqrt{10}}$$

solving, $\boxed{h = 0.75}$

... Continued

Ex 8. A solid sphere is in pure rolling motion on an inclined surface having inclination θ **(2006, 5M)**



(a) frictional force acting on sphere is $f = \mu mg \cos\theta$

(b) f is dissipative force

(c) friction will increase its angular velocity and opposes its linear velocity

(d) If θ decreases, friction will decrease

... Continued

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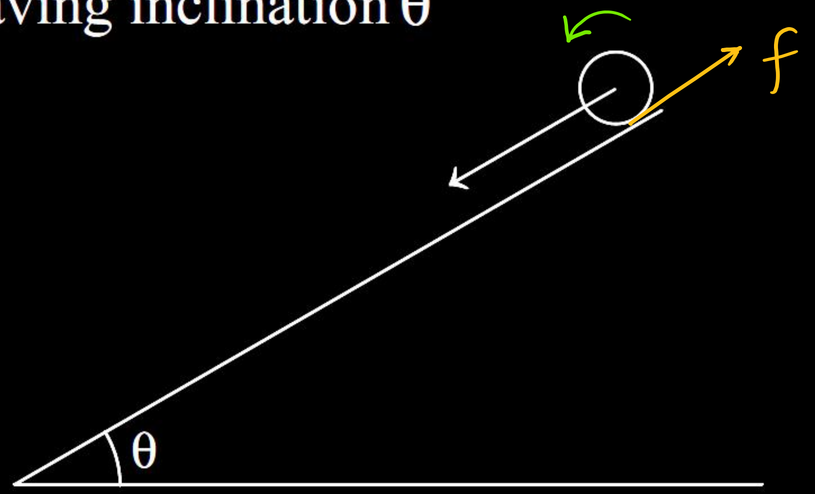


(a) frictional force acting on sphere is $f = \mu mg \cos\theta$ ×

(b) f is dissipative force ×

(c) friction will increase its angular velocity and opposes its linear velocity ✓

(d) If θ decreases, friction will decrease ✓

Sol^n: $f_{static} = \dfrac{k\,mg\sin\theta}{1+k}$

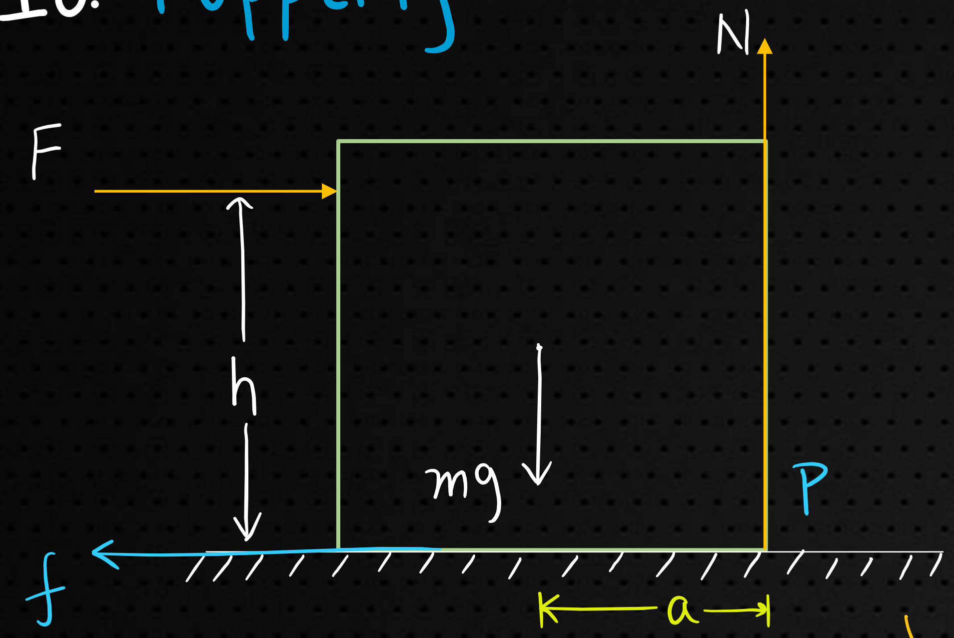
$= \dfrac{2}{7} mg\sin\theta$

$\rightarrow W_f = 0$

$\rightarrow \theta\downarrow \Rightarrow \sin\theta\downarrow$

$\Rightarrow f\downarrow$

10. Toppling



friction enough to prevent sliding

Condition for Toppling to begin

↳ at limiting case N passes pt. P

for toppling about P

$$\tau_F > \tau_{mg}$$

$$\Rightarrow Fh > mg\,a$$

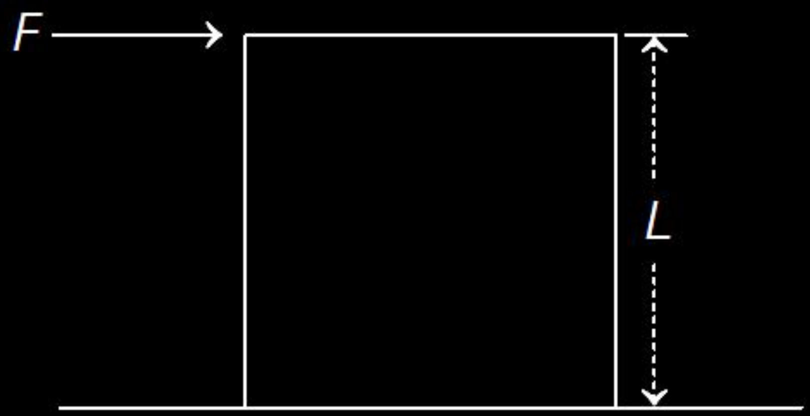
$$\Rightarrow F > mg\frac{a}{h}$$

$$\therefore \boxed{F_{min} = \frac{mga}{h}}$$

... Continued

Ex 9. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high, so that the block does not slide before toppling, the minimum force required to topple the block is

(2000)



(a) infinitesimal

(b) $mg/4$

(c) $mg/2$

(d) $mg(1-\mu)$

... Continued

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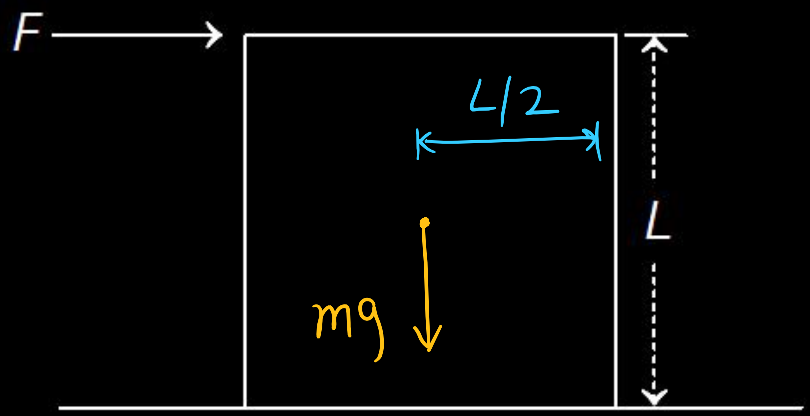


(a) infinitesimal
(b) $mg/4$
(c) $mg/2$
(d) $mg(1-\mu)$

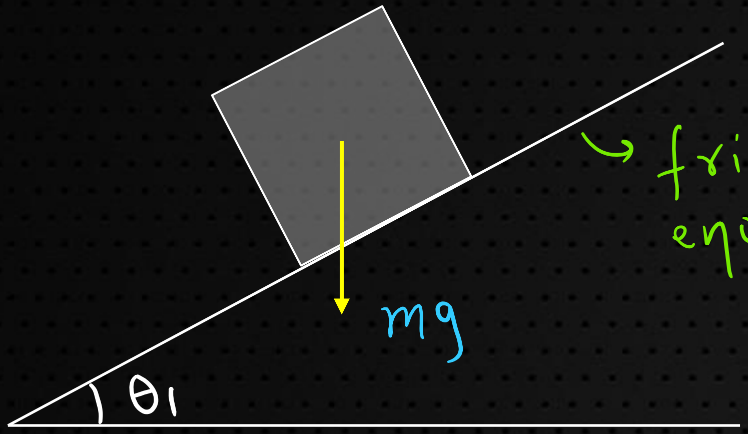
Sol^n:

$$F_{min} = \frac{mg \times L/2}{L}$$

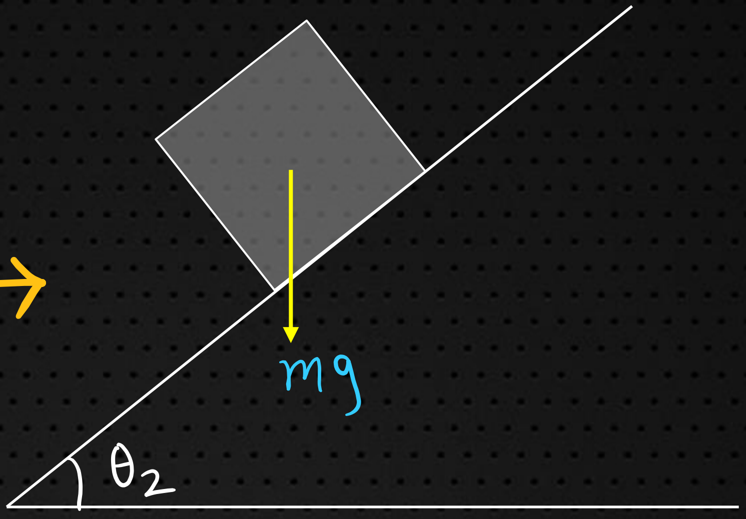
$$= \boxed{\frac{mg}{2}}$$

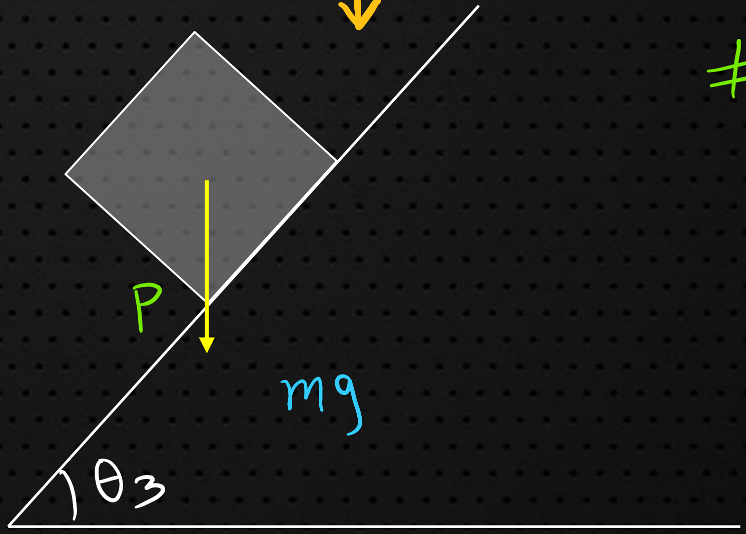
... Continued

Increase θ (Toppling on incline)



friction enough





θ_3 is limiting angle.

PYQs LINKS (JEE MAIN)

2021 Feb	2021 March	2021 July	2021 August	2020	Selected Problems
https://youtu.be/2hqJnity2_o	https://youtu.be/6CsOsazSvmA	https://youtu.be/H_jH3GtD4Bs	https://youtu.be/wmlghKW_PUQ	https://youtu.be/zWddLCElwWE	https://youtu.be/wjBRVQih400

CLICK (Practice these Questions)

HCV Rotation Solution Q1-Q70

https://bit.ly/3fM9fCC

Revision Series Playlist Link https://bit.ly/3eBbib9

JEE Main PYQs Link https://bit.ly/2S54jzh Chapter wise 2021, 2020, 2018

GoldMine Link https://bit.ly/2VhOGFF

CLICK