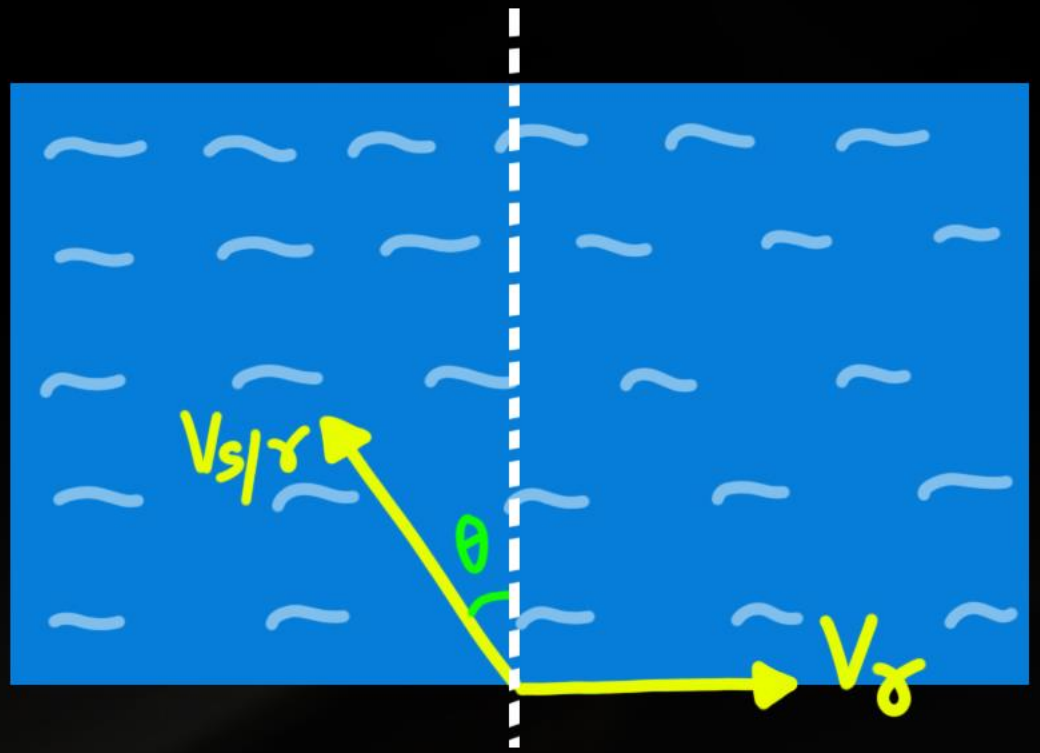
#4

PhD ON RIVER-SWIMMER

RELATIVE MOTION

CASE A,B,C



Mohit Goenka | IIT Kharagpur

EDUNITI YouTube Channel
Learn Like Never Before

List of Content on Eduniti YouTube Channel:

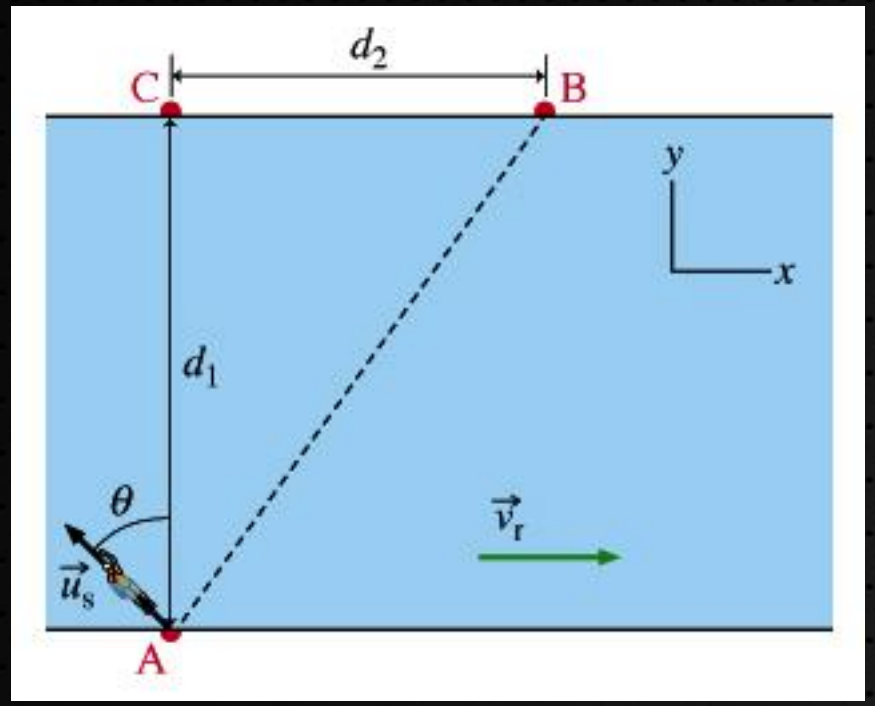
1. PYQs Video Solution Topic Wise:
   (a) JEE Main 2018/2020/2021 Feb & March
2. Rank Booster Problems for JEE Main
3. Part Test Series for JEE Main
4. JEE Advanced Problem Solving Series
5. Short Concept Videos
6. Tips and Tricks Videos
7. JEE Advanced PYQs
8. Formulae Revision Series

.....and many more to come

EDUNITI

Eduniti for Physics

EDUNITI



River-Swimmer/Boat Problems

Kinematics (Relative Velocity)

PhD SERIES

"JOIN"

Eduniti for Physics

PYQs (2020, 2021)

Concept Videos

Advanced Problems

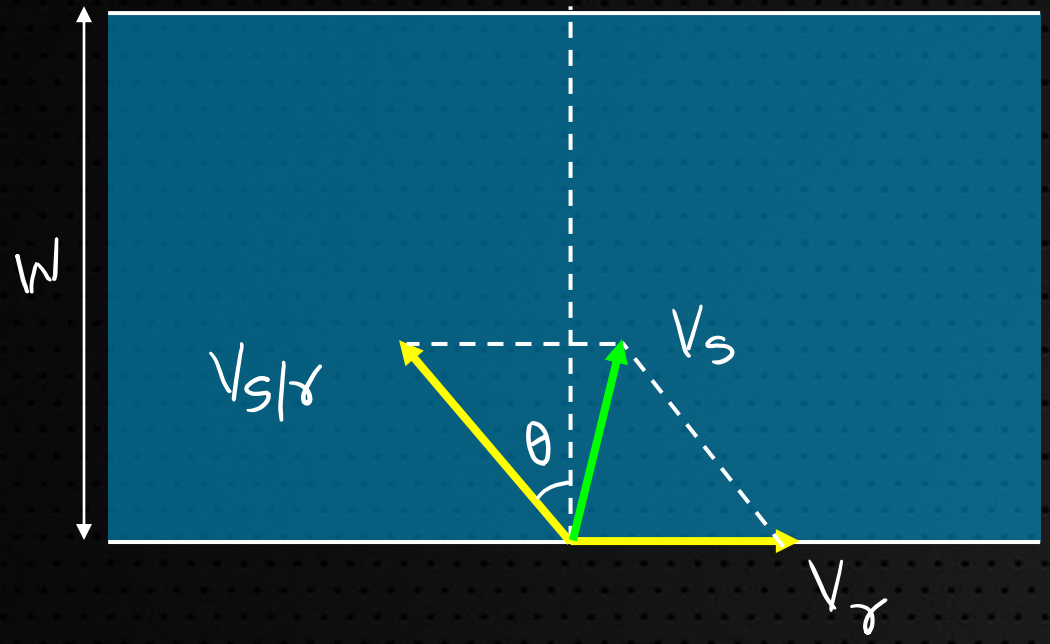
Part and Full Test

CLICK HERE

GOLD Mine Link -

https://bit.ly/2VhOGFF

River-Swimmer



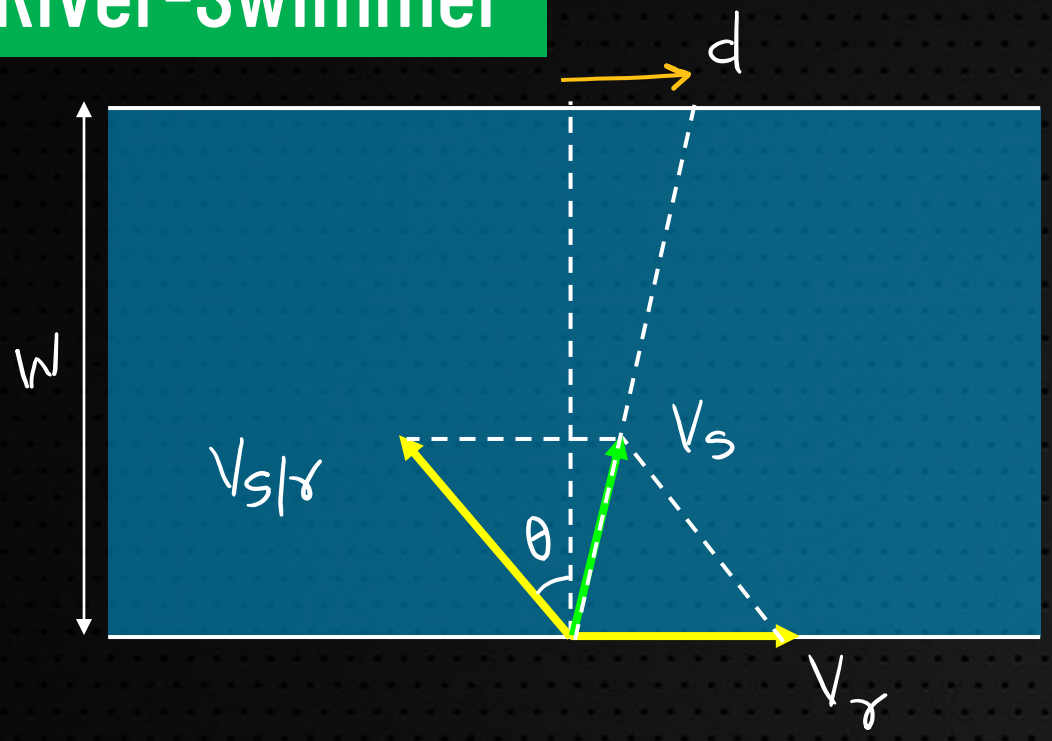
River Crossing

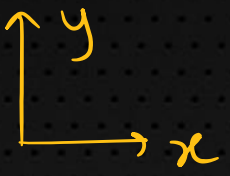
$\vec{V_r}$: Velocity of river w.r.t Ground

$\vec{V_{s/r}}$: Velocity of swimmer w.r.t river (vel of swimmer in still river)

$\vec{V_s}$: Velocity of swimmer w.r.t Ground

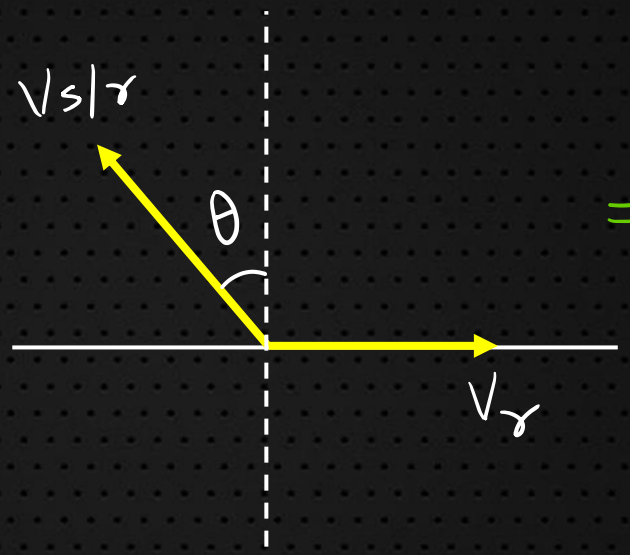
River-Swimmer

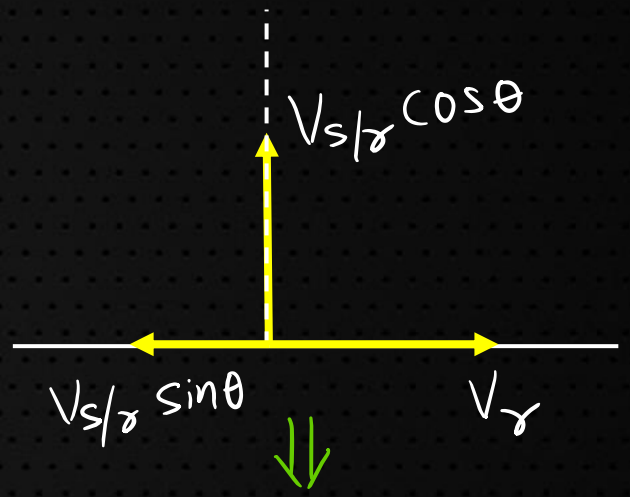


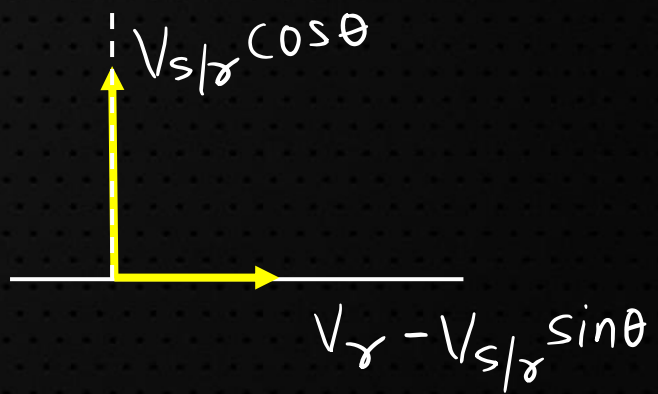


River Crossing

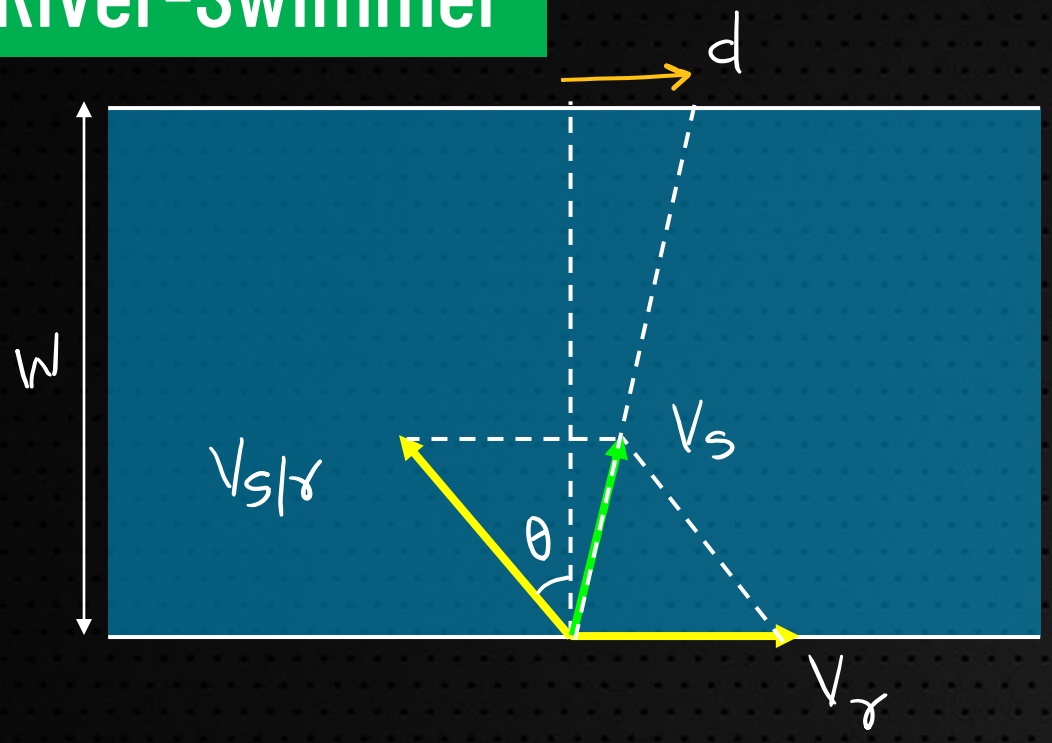
1. Drift (d) : Displacement along x-axis

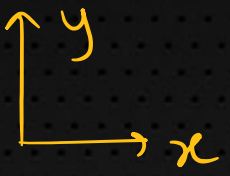






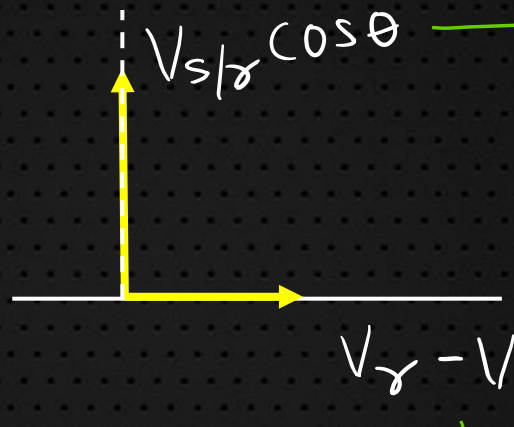
River-Swimmer





River Crossing

1. Drift (d) : Displacement along x-axis



$V_{s/r}\cos\theta$ → Responsible for crossing river $\left(t = \frac{W}{V_{s/r}\cos\theta}\right)$

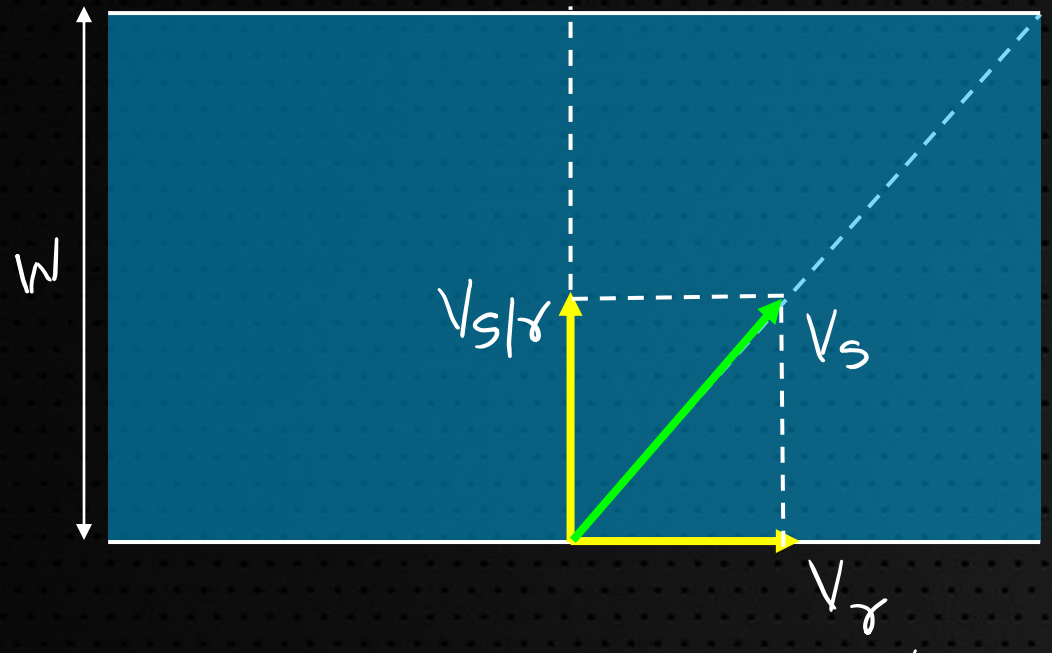
$V_r - V_{s/r}\sin\theta$ → Responsible for Drift

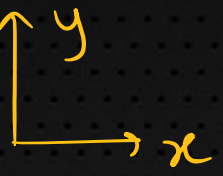
$$d = (V_r - V_{s/r}\sin\theta)\cdot t$$

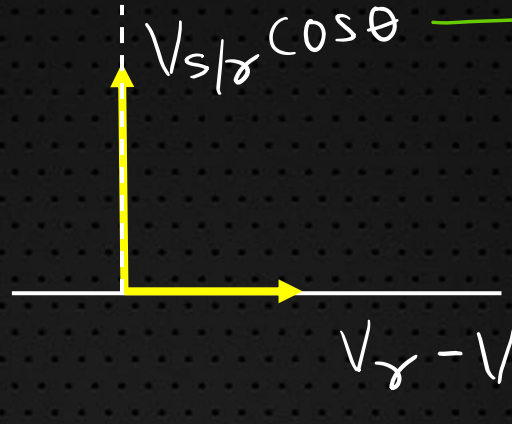
EDUNITI

River-Swimmer

River Crossing Case A





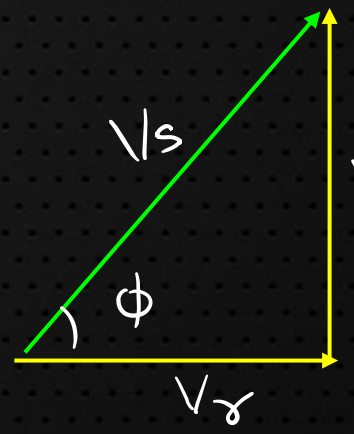


$V_{s/r}\cos\theta \rightarrow$ Responsible for crossing river $\left(t = \frac{W}{V_{s/r}\cos\theta}\right)$

Condition to cross in minimum time:

For t_{min}, $\cos\theta = 1 \Rightarrow \theta = 0^\circ \Rightarrow$ $V_{s/r}$ should be $\perp$ to V_r

$$t_{min} = \frac{W}{V_{s/r}}$$



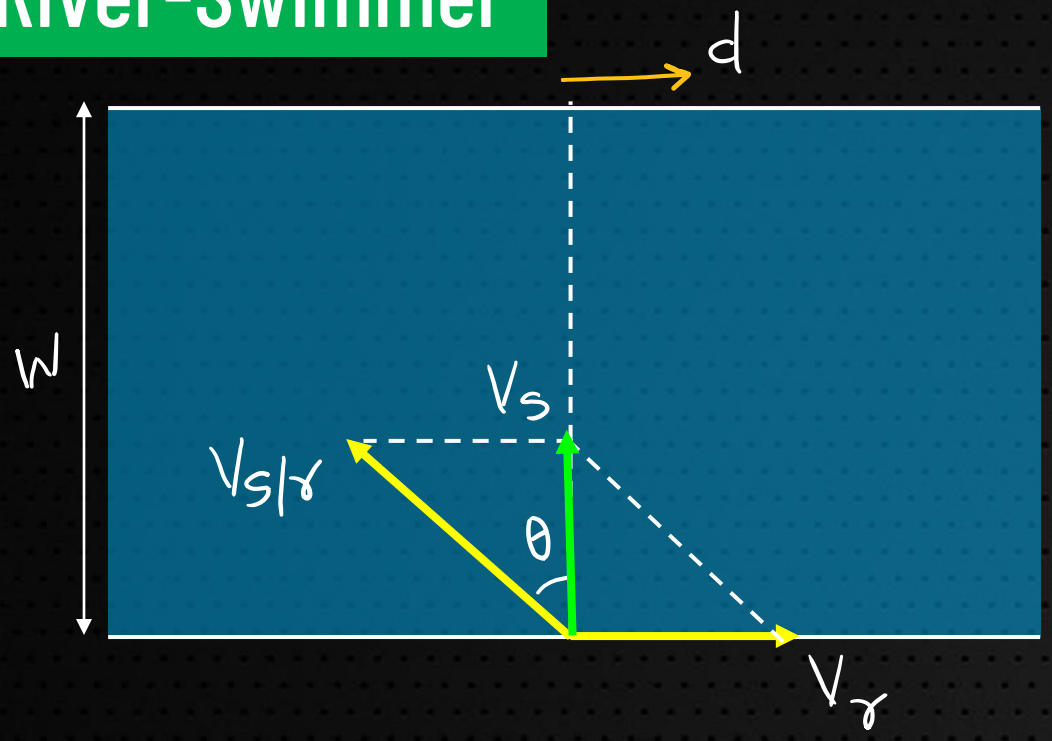
$$\vec{V_s} = \vec{V_{s/r}} + \vec{V_r}$$

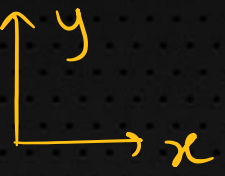
$$V_s = \sqrt{V_{s/r}^2 + V_r^2}$$

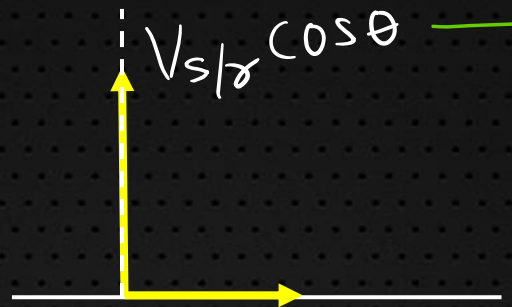
$$\tan\phi = \frac{V_{s/r}}{V_r}$$

River-Swimmer

River Crossing Case B







$V_{s/r}\cos\theta \rightarrow$ Responsible for crossing river $\left(t = \frac{W}{V_{s/r}\cos\theta}\right)$

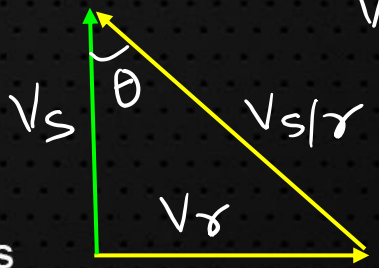
$V_r - V_{s/r}\sin\theta \rightarrow$ Causes drift

$d = (V_r - V_{s/r}\sin\theta)\cdot t$

Condition for drift to be zero (shortest path)

For $d = 0$, $V_r - V_{s/r}\sin\theta = 0 \Rightarrow V_r = V_{s/r}\sin\theta$

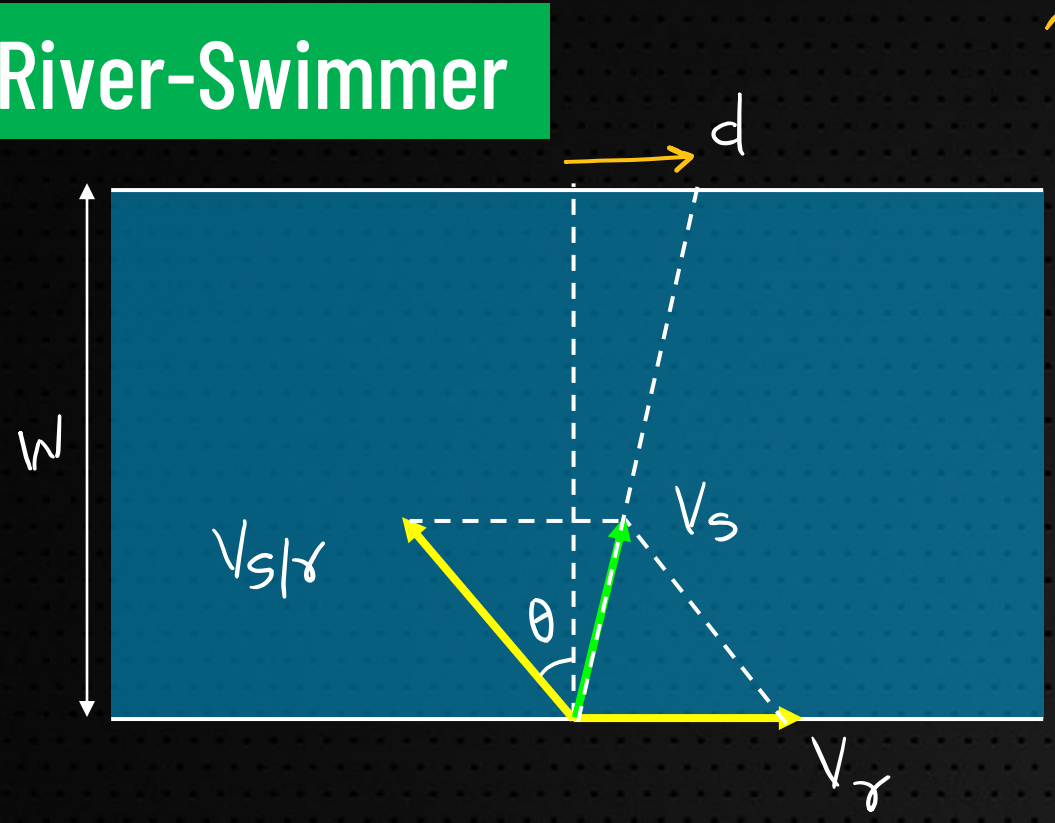
also, $\sin\theta = \frac{V_r}{V_{s/r}}$ $\because \sin\theta \leq 1 \Rightarrow \frac{V_r}{V_{s/r}} \leq 1 \Rightarrow \boxed{V_r \leq V_{s/r}}$

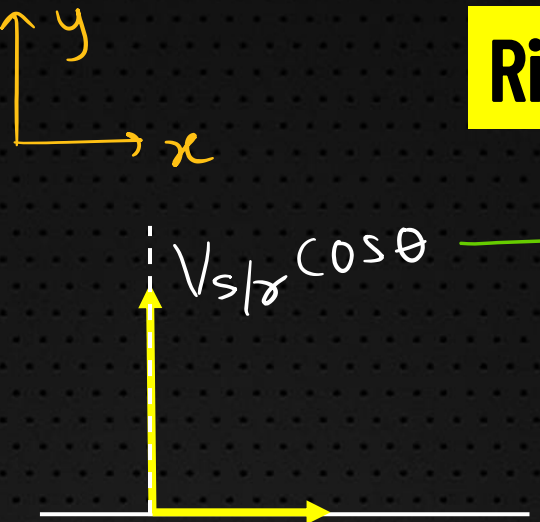


$V_s = \sqrt{V_{s/r}^2 - V_r^2}$

River-Swimmer

River Crossing Case C





$V_{s/r}\cos\theta$ → Responsible for crossing river $\left(t = \frac{W}{V_{s/r}\cos\theta}\right)$

$V_r - V_{s/r}\sin\theta$ → Causes drift

$d = (V_r - V_{s/r}\sin\theta)\cdot t$

If $V_r > V_{s/r}$ d can never be zero ∴ Cond^n for d_{min} ?

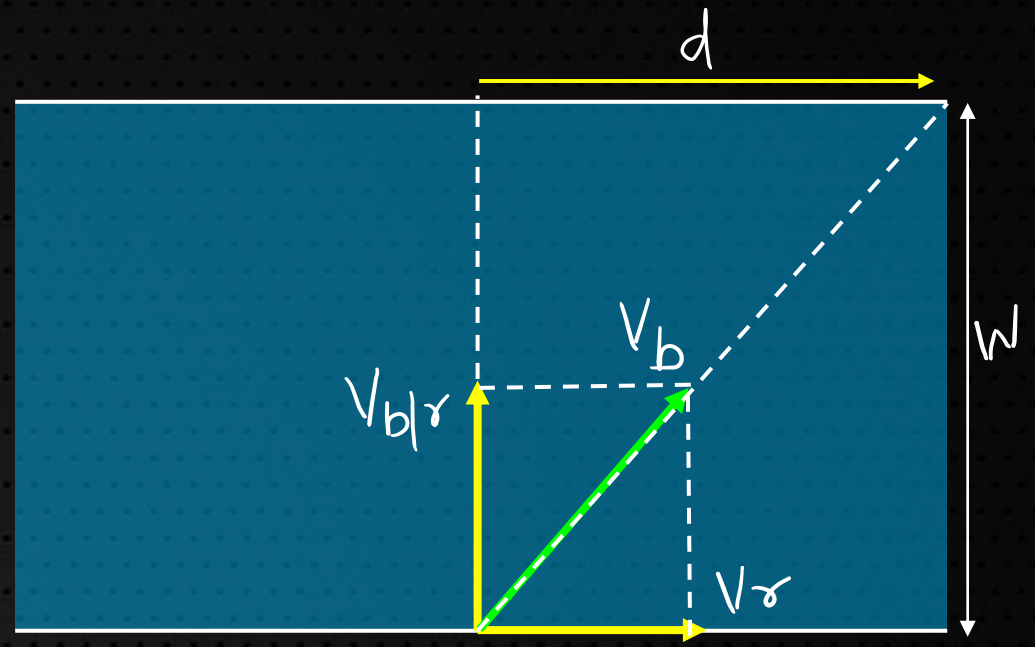
so, $d = (V_r - V_{s/r}\sin\theta)\cdot\frac{W}{V_{s/r}\cos\theta}$

Differentiate w.r.t θ and equate to zero.

We get : $\sin\theta = \frac{V_{s/r}}{V_r} \Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{V_{s/r}}{V_r}\right)}$

Q1. A river 400 m wide is flowing at a rate of 2·0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river. (a) Find the time taken by the boat to reach the opposite bank. (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank ?

Q1. A river 400 m wide is flowing at a rate of 2·0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river. (a) Find the time taken by the boat to reach the opposite bank. (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank ?



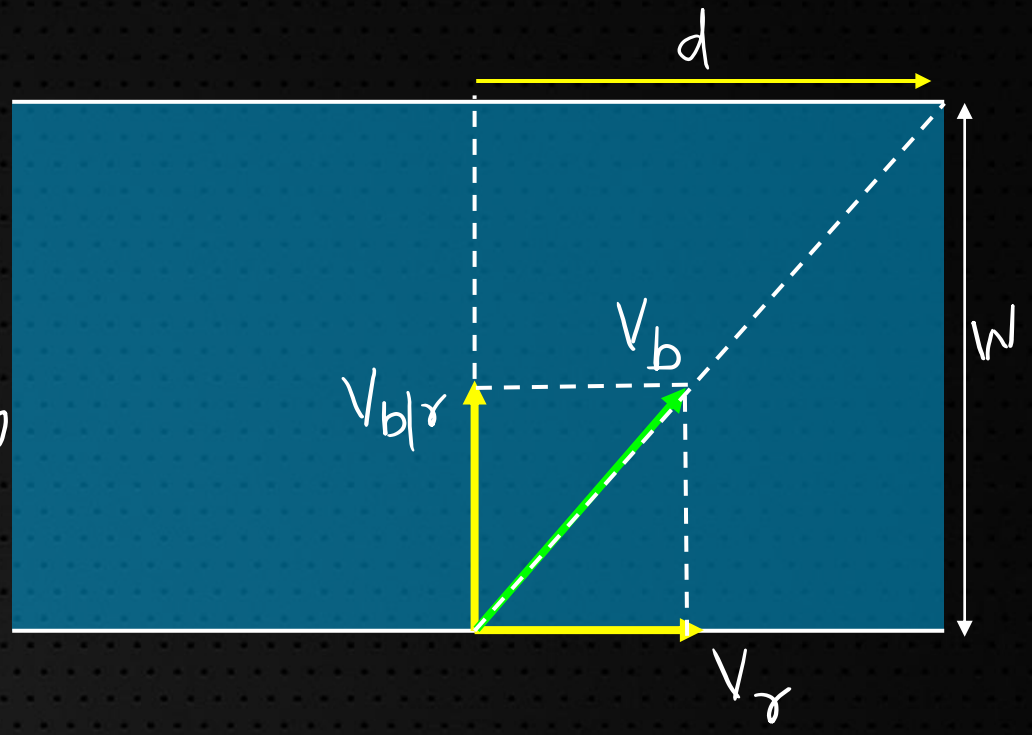
soln: $W = 400\,m$, $V_r = 2\,m/s$, $V_{b/r} = 10\,m/s$

(a) $t = \frac{W}{V_{b/r}} = \frac{400}{10} = \boxed{40\,s}$

(b) $d = V_r \cdot t = 2 \times 40 = \boxed{80\,m}$

Q2. A man wishes to cross a river in a boat. If he crosses the river in minimum time, he takes 10 min with a drift of 120 m. If he crosses the river taking shortest route, he takes 12.5 min Find the velocity of boat with respect to water. (in m/min)

Q2. A man wishes to cross a river in a boat. If he crosses the river in minimum time, he takes 10 min with a drift of 120 m. If he crosses the river taking shortest route, he takes 12.5 min Find the velocity of boat with respect to water. (in m/min)



Soln: For t_{min}: $d = V_r \cdot t \Rightarrow V_r = \frac{120}{10} = 12$ m/min

Also, $t_{min} = \frac{W}{V_{b/r}} \Rightarrow W = 10\, V_{b/r}$ — (1)

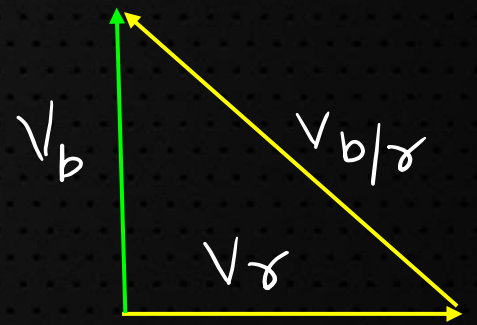
$V_b \cdot t = W$

$\Rightarrow \sqrt{V_{b/r}^2 - V_r^2} \cdot t = 10\, V_{b/r}$

$\Rightarrow (V_{b/r}^2 - 144)\frac{625}{4} = 100\, V_{b/r}$

$\therefore \boxed{V_{b/r} = 20 \text{ m/min}}$

Ans



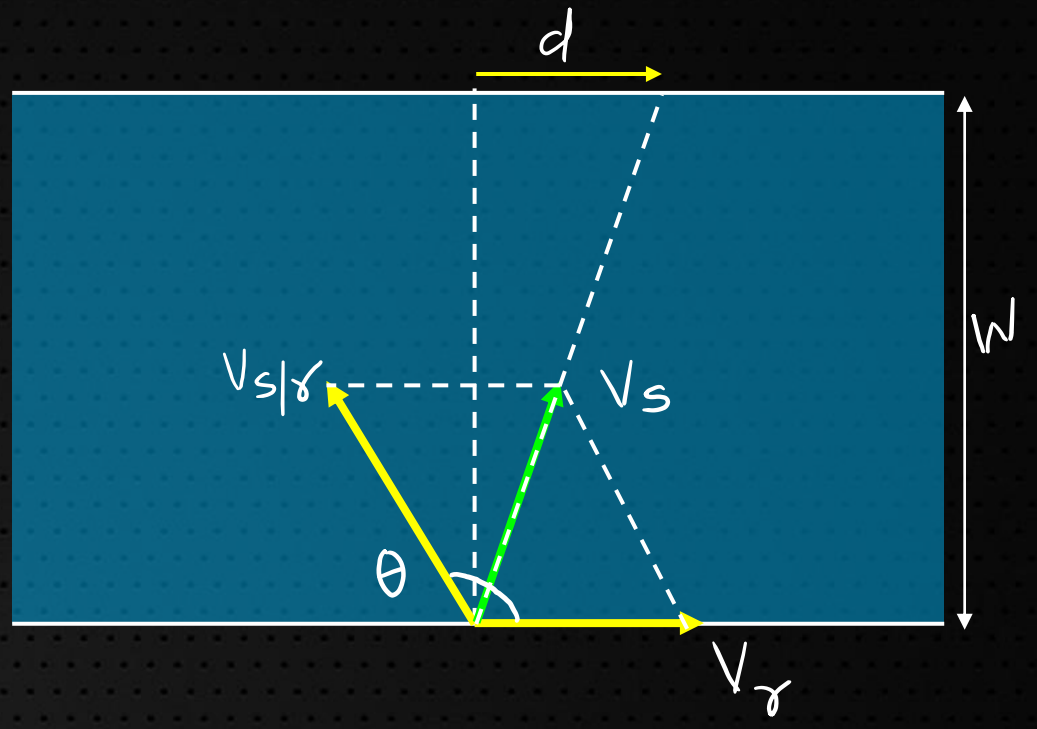
For shortest Path

Q3. A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h.
(a) If he heads in a direction making an angle θ with the flow, find the time he takes to cross the river.
(b) Find the shortest possible time to cross the river.

Consider the situation of the previous problem. The man has to reach the other shore at the point directly opposite to his starting point. If he reaches the other shore somewhere else, he has to walk down to this point. Find the minimum distance that he has to walk.

Find time in (min) and distances in (Km).

Q3. A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h.
(a) If he heads in a direction making an angle θ with the flow, find the time he takes to cross the river.
(b) Find the shortest possible time to cross the river.
Consider the situation of the previous problem. The man has to reach the other shore at the point directly opposite to his starting point. If he reaches the other shore somewhere else, he has to walk down to this point. Find the minimum distance that he has to walk.



Soln: $W = 500\,m$, $V_r = 5\,km/h$, $V_{s/r} = 3\,km/h$

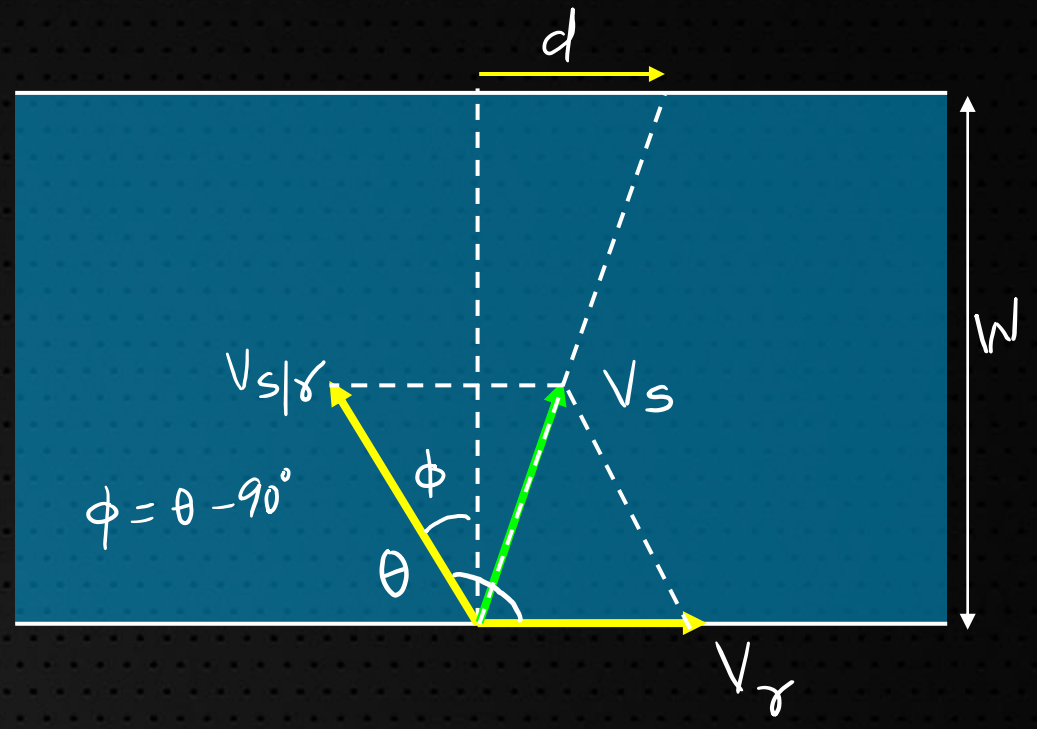
(a) $t = \dfrac{W}{V_{s/r}\cos(\theta - 90^\circ)} = \dfrac{500}{3\times\frac{1000}{60}\sin\theta} = \dfrac{10}{\sin\theta}$ min

(b) For t_{min}, $\theta = 90^\circ \Rightarrow t_{min} = 10$ min

(c) $\because V_r > V_{s/r} \Rightarrow$ d can never be zero.

d_{min} occurs at $\sin(\theta - 90^\circ) = \dfrac{V_{s/r}}{V_r} \Rightarrow \cos\theta = \dfrac{3}{5}$

Q3. A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h.
(a) If he heads in a direction making an angle θ with the flow, find the time he takes to cross the river.
(b) Find the shortest possible time to cross the river.
Consider the situation of the previous problem. The man has to reach the other shore at the point directly opposite to his starting point. If he reaches the other shore somewhere else, he has to walk down to this point. Find the minimum distance that he has to walk.



Soln: $W = 500\,m$, $V_r = 5\,km/h$, $V_{s/r} = 3\,km/h$

(a) $t = \dfrac{W}{V_{s/r}\cos(\theta - 90°)} = \dfrac{500}{3 \times \frac{1000}{60} \sin\theta} = \dfrac{10}{\sin\theta}$ min

(c) $\because V_r > V_{s/r} \Rightarrow$ d can never be zero.

d_{min} occurs at $\cos\theta = -3/5$

$d_{min} = (V_r + V_{s/r}\cos\theta) \times \dfrac{10}{\sin\theta \times 60} = \left(5 - 3 \times \dfrac{3}{5}\right) \times \dfrac{5}{4 \times 6}$

$= \boxed{2/3\ Km}$