JEE MAIN | IIT JEE

WAVE MOTION

String Waves (Part-1)

REVISION in 35 Min

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STRING WAVES – PART 1

1. Waves (*Transverse & Longitudinal*)
2. Travelling vs Standing Waves
3. Wave Equation
4. Path & Phase Difference
5. Direction of Wave through equation
6. Particle velocity & acceleration
7. Direction of particle velocity
8. Questions on Wave Equation
9. Wave speed on strings (*Concept + Questions*)
10. Average Power & Intensity Transmitted

EDUNITI

Chapter	Formulae_Concept VIDEO LINK
Unit & Dimensions	https://youtu.be/wdd-wlZF4Hk
Errors and Vectors	
Vernier Calliper	https://youtu.be/pVoN045dV8I
Screw Gauge	https://youtu.be/gYd2PtmZ0mw
Kinematics_Motion in 1d	https://youtu.be/U4NNxFaFliE
Kinematics_Motion in 2d	https://youtu.be/4_Zo5WhMf7w
Laws of Motion	https://youtu.be/7JlR8gNRQls
Friction	https://youtu.be/Rn1bLst7eGk
Work Energy Power	https://youtu.be/kjrXoE-kDI8
Circular Motion	https://youtu.be/KnFymKHlkT0
Centre of Mass	https://youtu.be/ads35RKD618
Cons of Momentum & Collision	https://youtu.be/3f0u4L-lyyw
Rotational Motion – Part 1	https://youtu.be/O6j1mLp06XI
Rotational Motion – Part 2	https://youtu.be/OHni1DRdfAQ
Rotational Motion_Part 3	https://youtu.be/quglqfYRCrk
Gravitation	https://youtu.be/rAj2huLVaEk
Properties of Solids	https://youtu.be/gSXxjk89l_c
Fluids Statics (Part 1)	https://youtu.be/RFKx9B9yo3M
Fluid Dynamics (Part 2)	https://youtu.be/Y717vQpUEJQ
Fluid Properties (Part 3)	https://youtu.be/V8xUWWK2oT0
Simple Harmonic Motion	https://youtu.be/RIb7ofNG09I
Thermal Properties	https://youtu.be/OYjjyPlzddE
Heat Transfer	https://youtu.be/PyNboHgtYzM
KTG	https://youtu.be/XO1tvFhIa0I
Thermodynamics	https://youtu.be/iz_kf1jRDRw
Wave Motion -Organ Pipes and Resonance Tube	https://youtu.be/fB7pfJ77za8
Wave Motion - Doppler's Effect	https://youtu.be/9-BxOaamnwg
Electrostatics	https://youtu.be/3stXbGRMcrk
Capacitors	https://youtu.be/EXEiickNUkY
Current Electricity	https://youtu.be/gm8FUfjrX18
Moving Charges and Magnetic Effect of Current	https://youtu.be/ULD2Ok1CGJk
Earth's Magnetism	https://youtu.be/a4CT5uVwAK4
Magnetic Properties	https://youtu.be/63cwdYXNlYE
EMI	https://youtu.be/puVavm_GFRM
Alternating Current	https://youtu.be/74dTY-pzM_o
Ray Optics	https://youtu.be/BhnyTWzIlBA
Wave Optics Part 1_Interference	https://youtu.be/LG5nlE8XTeI
Wave Optics Part 2_Diffraction_Polarization	https://youtu.be/ymMyyJGGqnY
Optical Instruments	https://youtu.be/OQssbDH0A4I
Electromagnetic Waves	https://youtu.be/bcVXgEkyQZY
Semiconductors_Basics + Zener Diode	https://youtu.be/_A2JomQ7-50
Semiconductors_Transistors	https://youtu.be/psDwl84Nzb0
Semiconductors_Logic Gates	https://youtu.be/pZdQAzLbFTo
Communication Systems	https://youtu.be/8NgMqK9X79Y
Modern Physics_Part 1_Atomic Physics	https://youtu.be/9VKUnE3mpHk
Modern Physics_Part 2_Photoelectric Effect	https://youtu.be/24oTQp84jrk
Modern Physics_Part 3_Dual Nature of Light	https://youtu.be/0zoR_saMAQY
Modern Physics_Part 4_Radioactivity	https://youtu.be/AdX3YBhQyog
Modern Physics_Part 5_Nuclear Physics	https://youtu.be/VDWqVahGixc
Modern Physics_Part 6_X Rays	https://youtu.be/dSHXdzX7NX0

1. Waves (Transverse & Longitudnal)

↓

Energy is transmitted from one point to another

↳ also called Travelling waves

Transverse

Disturbance travels in x-dir^n but particle moves in dir^n $\perp$ to x-axis

↳ string waves

Longitudnal

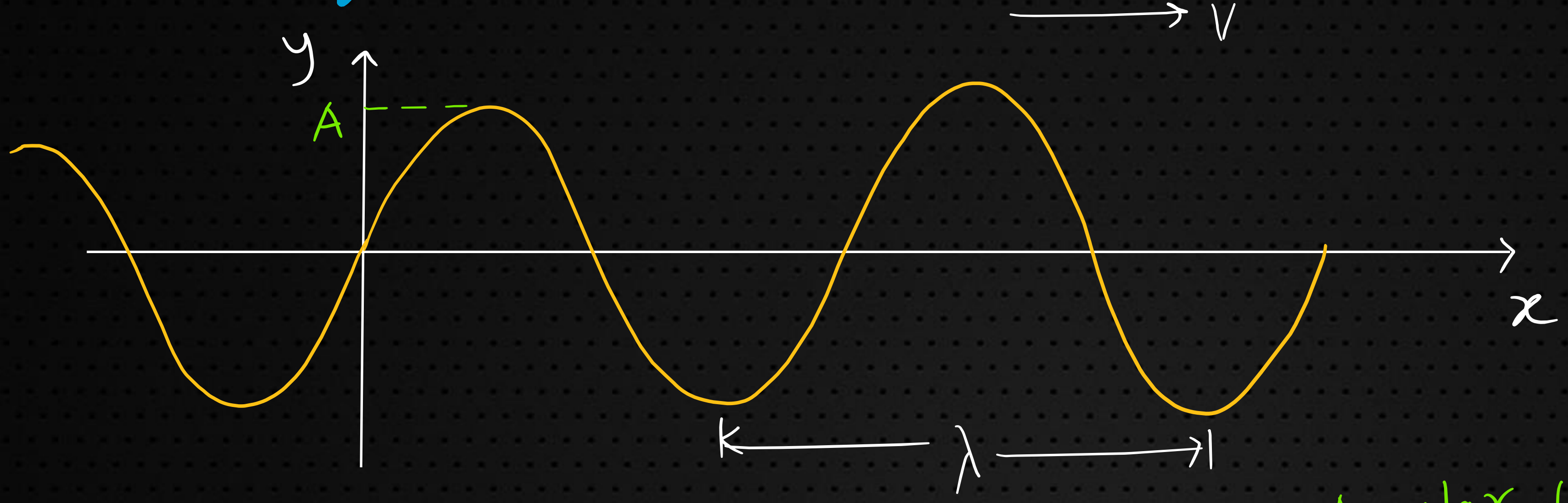
Particle moves along the same dir^n as disturbance

↳ Sound waves

2. Travelling vs Standing Waves (string waves)

See the animation in Video (YouTube)

3. Wave Equation (string waves)



Wave speed

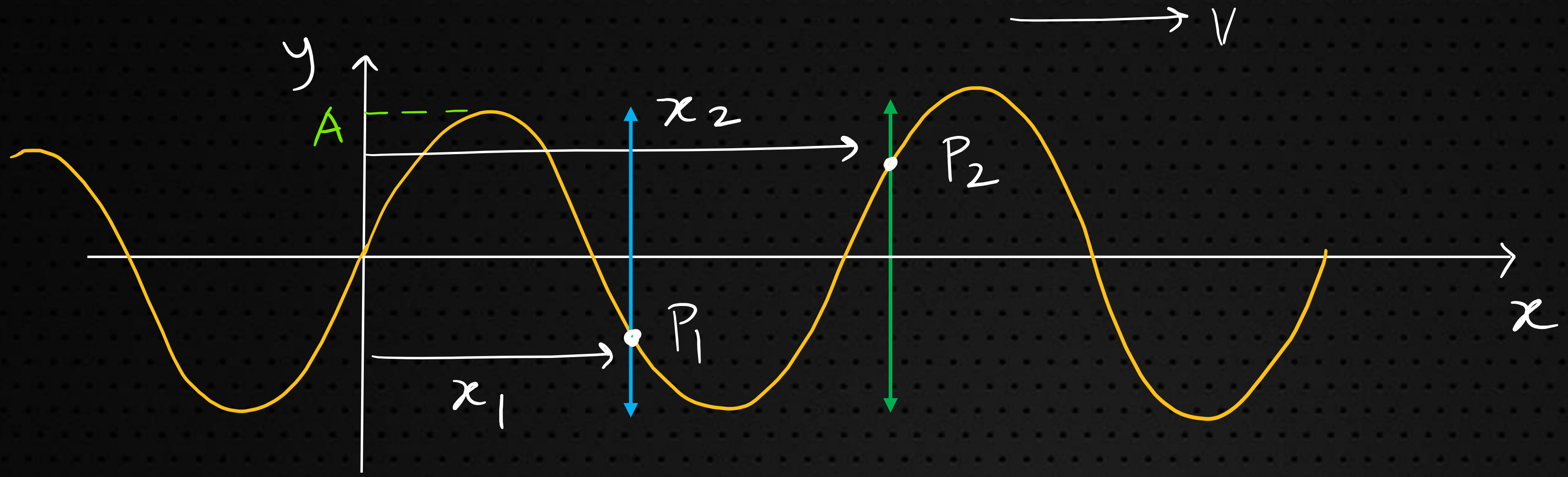
$$V = \frac{\text{Coeff of } t}{\text{Coeff of } x} = \frac{\omega}{K}$$

$$= f\lambda$$

$$y = A\sin(\omega t - Kx + \phi)$$

- A → Amplitude
- ω → Angular freq, $\omega = 2\pi f = \frac{2\pi}{T}$
- K → Angular wave no., $K = \frac{2\pi}{\lambda}$
- ϕ → Initial phase Const.

... continued



$$y = A\sin(\omega t - Kx + \phi) \longrightarrow y_1 = A\sin(\omega t - Kx_1 + \phi)$$

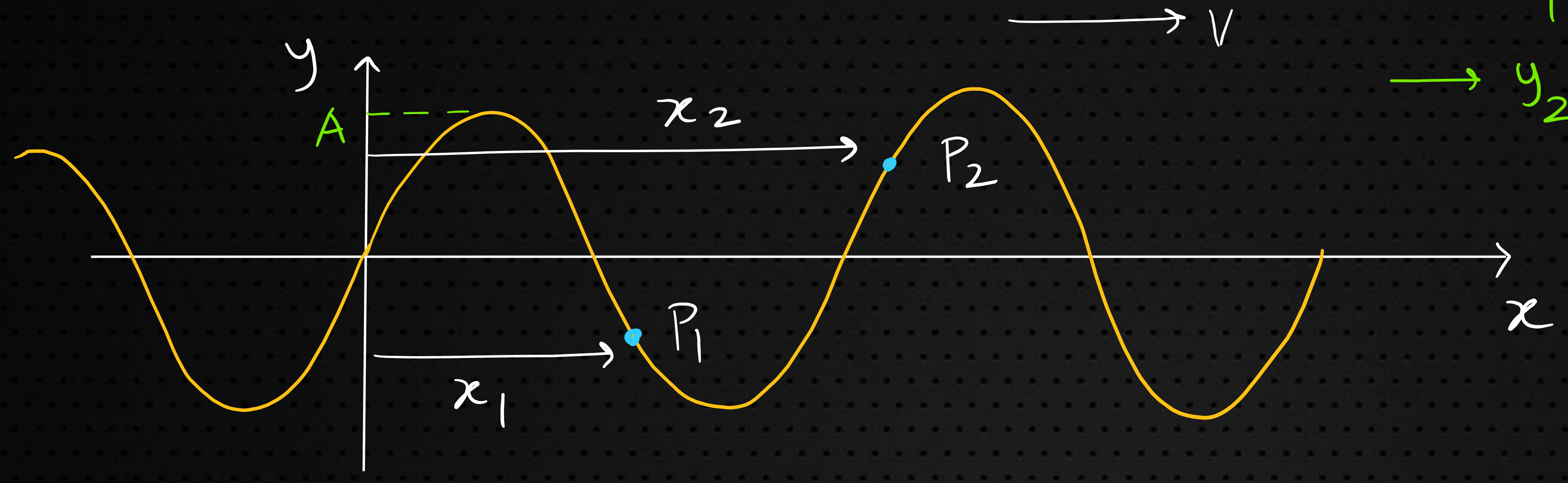
SHM eq^n of P_1

$$\longrightarrow y_2 = A\sin(\omega t - Kx_2 + \phi)$$

SHM eq^n of P_2

Amp of all particle is same.

4. Path & Phase difference



$\rightarrow y_1 = A\sin(\omega t - kx_1 + \phi)$

$\rightarrow y_2 = A\sin(\omega t - kx_2 + \phi)$

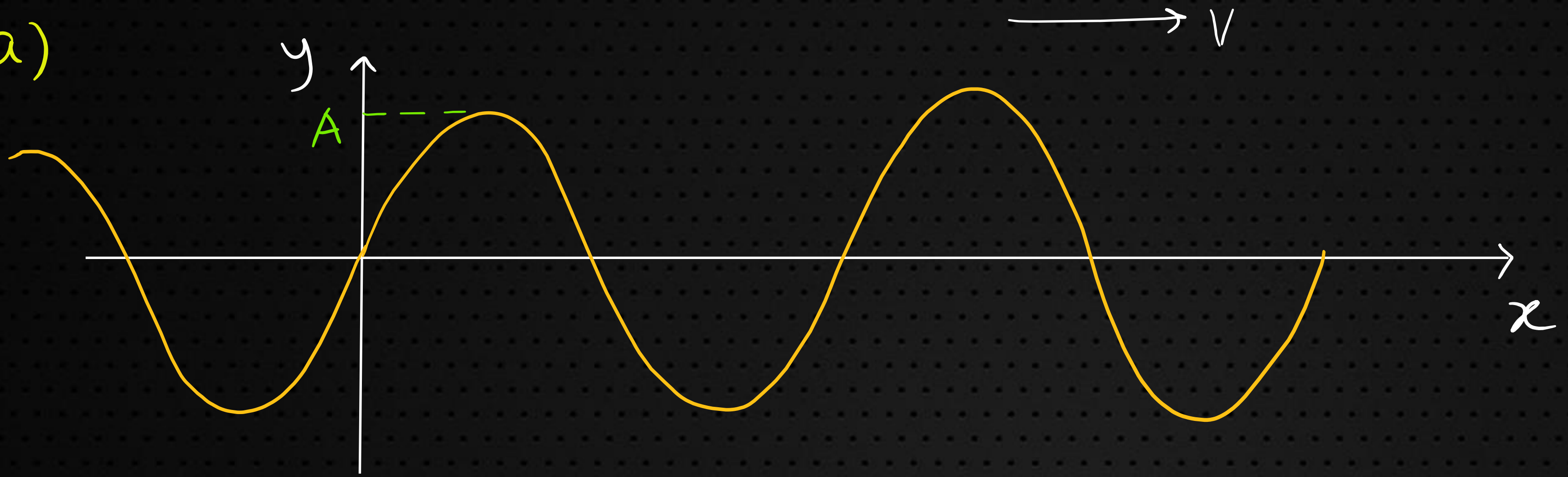
Phase difference, $\Delta\phi = \phi_1 - \phi_2 = k(x_2 - x_1)$

$$\Rightarrow \boxed{\Delta\phi = \frac{2\pi}{\lambda}\cdot\Delta x}$$

Δx is path difference

5. Direction of Wave Motion in Eqn

(a)

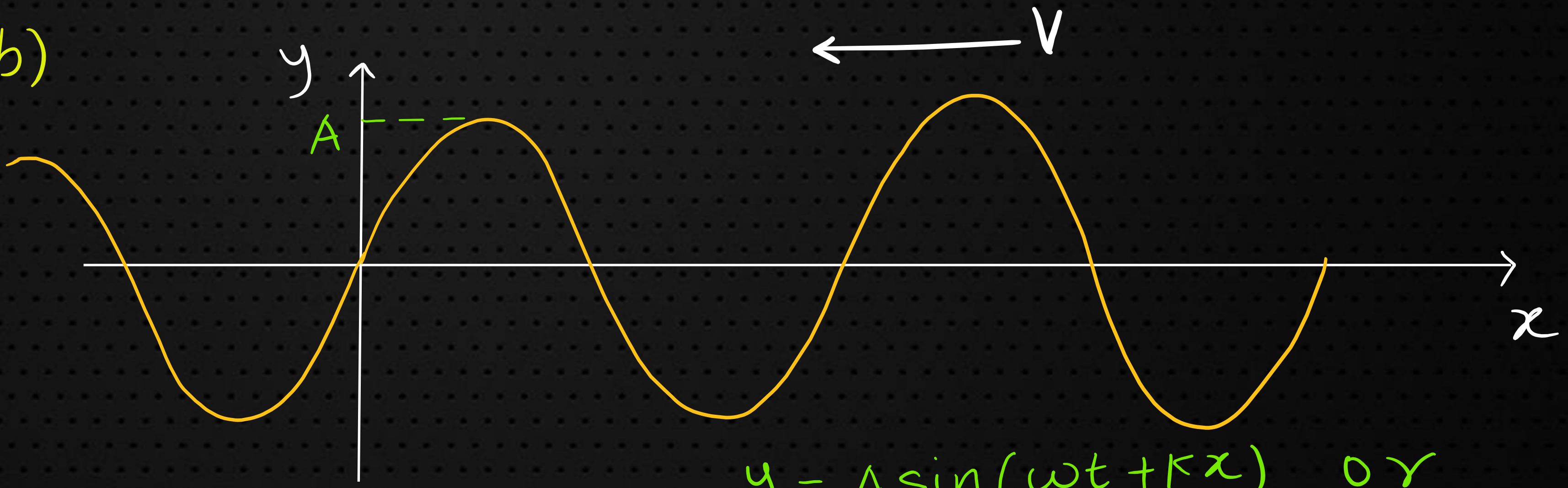


$$y = A\sin(\omega t - kx)$$

or

$$= A\sin(-\omega t + kx)$$

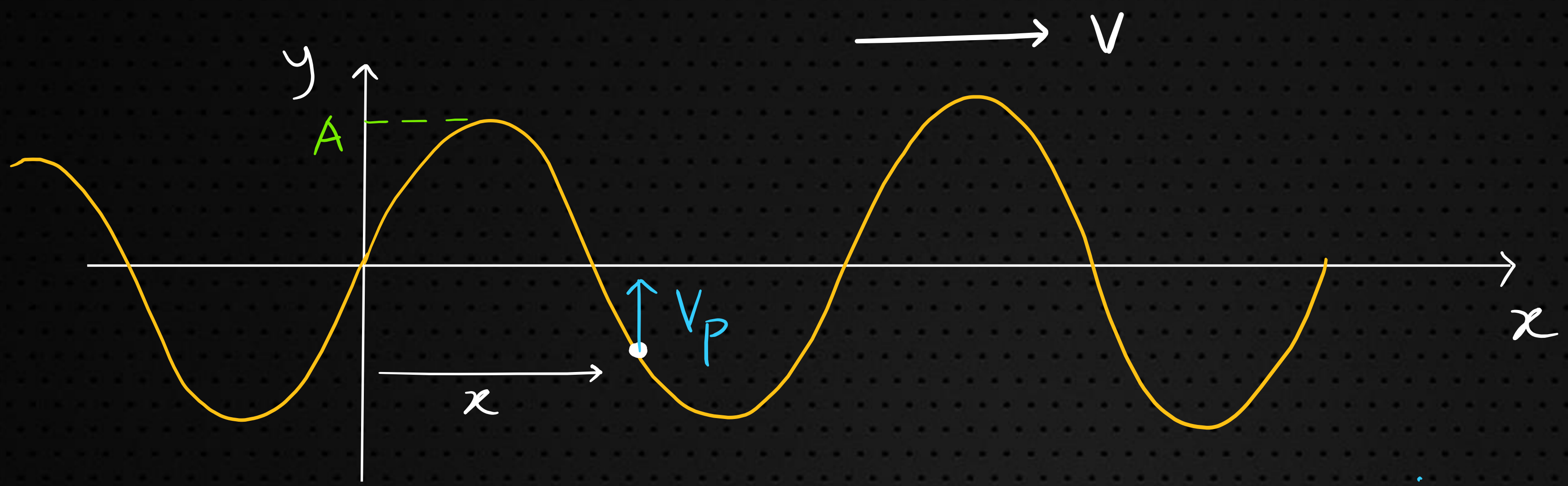
(b)



$$y = A\sin(\omega t + kx)$$ or

$$= A\sin(-\omega t - kx)$$

6. Particle Velocity & Acceleration



$$y = A\sin(\omega t - kx + \phi)$$

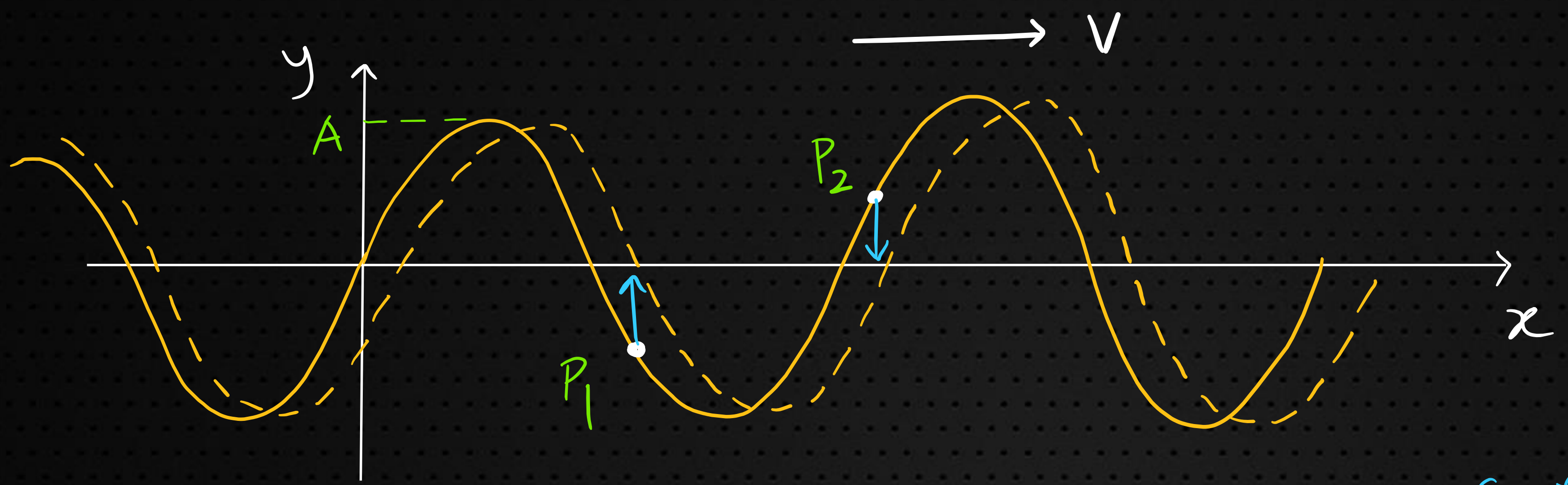
$$V_p = \partial y / \partial t$$

$$\Rightarrow V_p = A\omega\cos(\omega t - kx + \phi)$$

or, $\boxed{V_p = \omega\sqrt{A^2 - y^2}}$ $\left\{ \begin{array}{l} y \text{ is pos}^n \text{ of particle at} \\ x \text{ distance at time } t \end{array} \right.$

and, $a_p = \dfrac{\partial V_p}{\partial t} \Rightarrow \boxed{a_p = -\omega^2 y}$

7. Particle Direction at time t



Method 1 : $\boxed{v_p = -V \times \text{slope}}$

$\left\{ \begin{array}{l} \text{at } P_1 \text{ slope is } -ve \\ \therefore v_p \text{ is } +ve \Rightarrow \uparrow \end{array} \right.$

Method 2 : Draw wave after dt time

↳ same result.

8. Questions on Wave Eqⁿ

Ex 1. A wave is represented by the equation ;

$$y = A \sin(10\pi x + 15\pi t + \pi/3)$$

where, x is in metre and t is in second. The expression represents

(1990, 2M)

(a) a wave travelling in the positive x-direction with a velocity 1.5 m/s

(b) a wave travelling in the negative x-direction with a velocity 1.5 m/s

(c) a wave travelling in the negative x-direction with a wavelength 0.2 m

(d) a wave travelling in the positive x-direction with a wavelength 0.2 m

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(c) a wave travelling in the negative x-direction with a wavelength 0.2 m ✓

(d) a wave travelling in the positive x-direction with a wavelength 0.2 m

Sol^n:

(i) travelling in $-$ve x dir^n

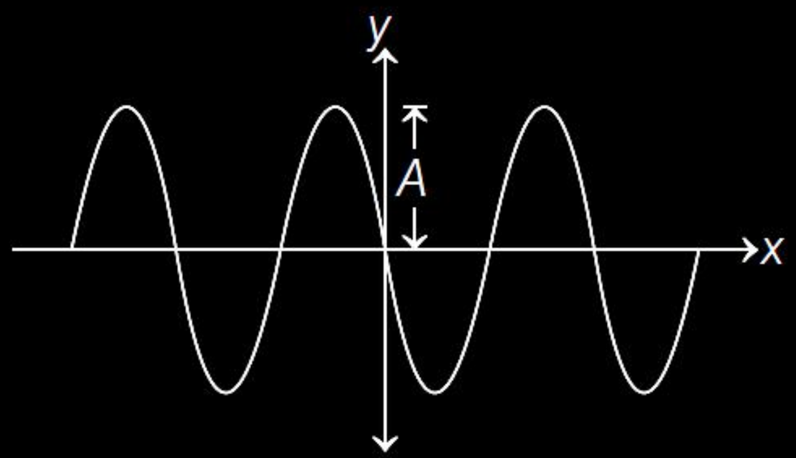
(ii) $K = \frac{2\pi}{\lambda} \Rightarrow 10\pi = \frac{2\pi}{\lambda}$

$\therefore \lambda = 0.2\,m$

(iii) $V = \frac{\omega}{K} = \frac{15\pi}{10\pi} = 1.5\,m/s$

... Continued

Ex 2. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure. **(2019 Main, 12 April I)**



For this wave, the phase ϕ is

(a) $-\frac{\pi}{2}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

... Continued

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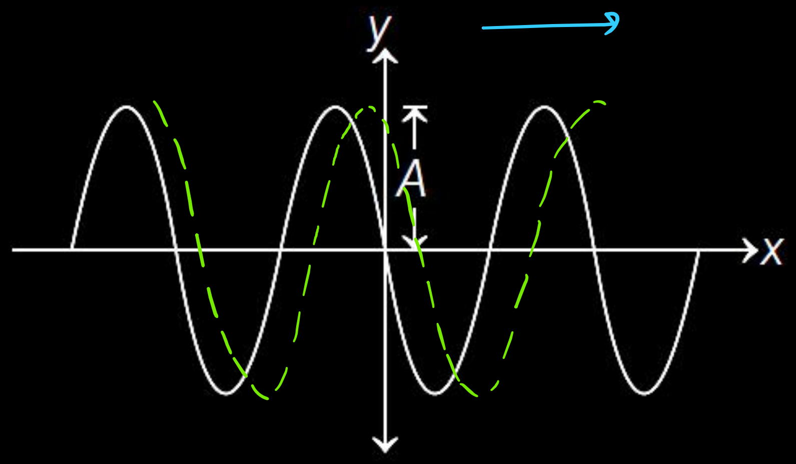


For this wave, the phase ϕ is

(a) $-\frac{\pi}{2}$ ✓(b) π (c) 0 (d) $\frac{\pi}{2}$

Sol^n: $V_p = \frac{\partial y}{\partial t} = -A\omega \cos(kx - \omega t + \phi)$

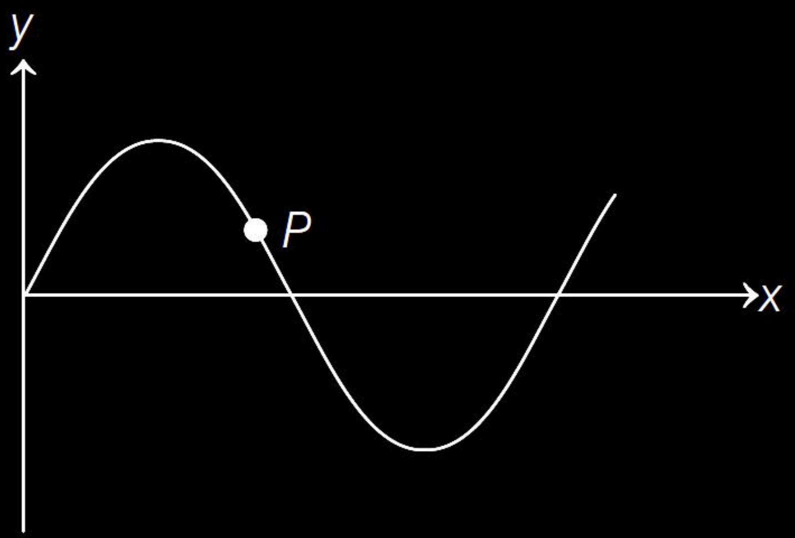
$\Rightarrow$ At $t = 0$ particle at origin was moving up.

$V_p = -A\omega \cos(\phi)$ $\{x \,\&\, t = 0$

$\because$ V_p is +VE $\Rightarrow$ $\boxed{\phi = \pi}$

... Continued

Ex 3. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is **(2008)**



(a) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (b) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (c) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s (d) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

... Continued

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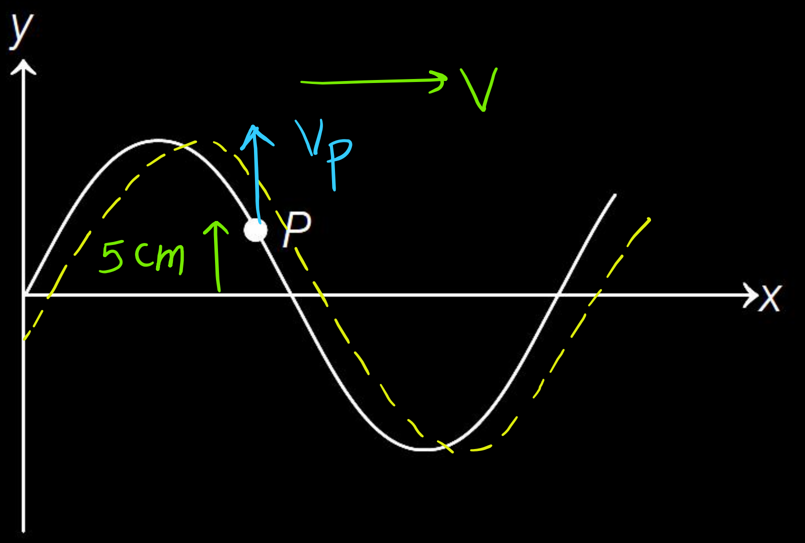


(a) $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (b) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s (c) $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s (d) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

Sol^n: $V_P = \omega\sqrt{A^2 - y_P^2}$ $\left\{ V = \frac{\omega}{K} \right.$

$\Rightarrow V_P = V \cdot \frac{2\pi}{\lambda}\sqrt{A^2 - y_P^2}$

$= 0.1 \times \frac{2\pi}{0.5}\sqrt{(0.1)^2 - (0.05)^2}$

$= \sqrt{3}\pi/50$ m/s $(\hat{j})$

... Continued

Ex4. The amplitude of a wave disturbance travelling in the positive x-direction is given by $y = \frac{1}{1+x^2}$ at time $t = 0$ and by $y = \frac{1}{[1+(x-1)^2]}$ at $t = 2$ s, where x and y are in metre.

The shape of the wave disturbance does not change during the propagation. The velocity of the wave is ...... m/s.

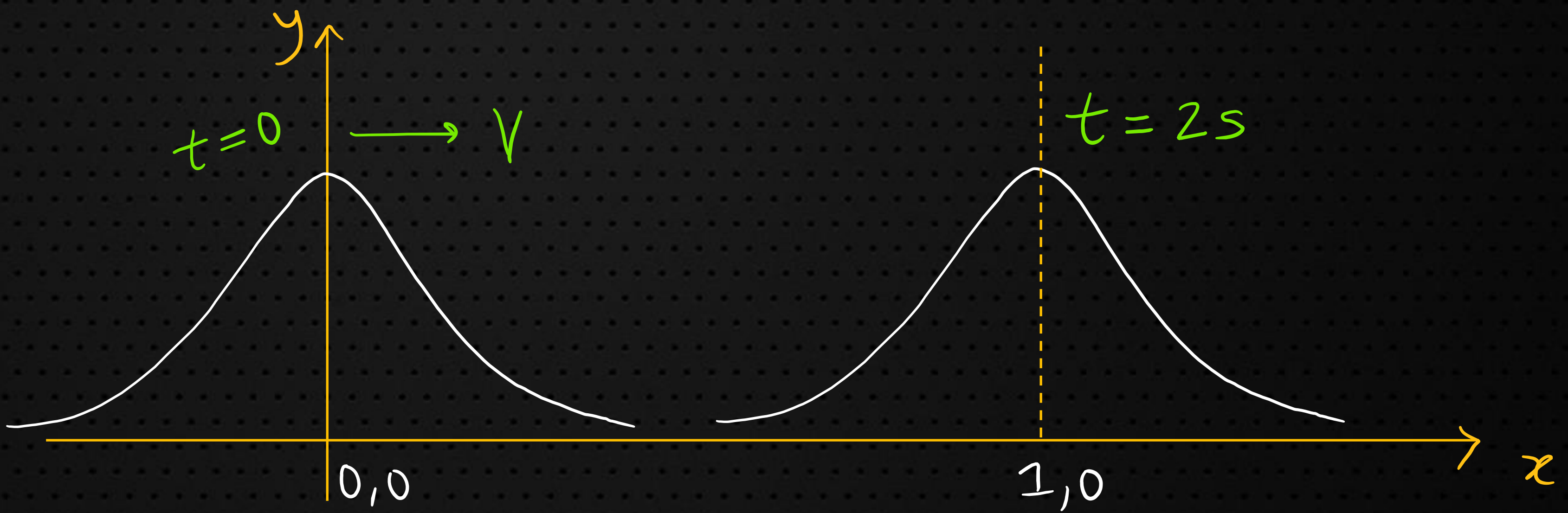
(1990, 2M)

... Continued

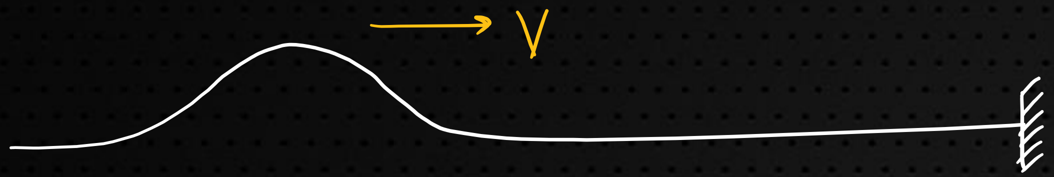
Ex4. The amplitude of a wave disturbance travelling in the positive x-direction is given by $y = \frac{1}{1+x^2}$ at time $t = 0$ and by $y = \frac{1}{[1+(x-1)^2]}$ at $t = 2$ s, where x and y are in metre. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is …… m/s. **(1990, 2M)**

Sol^n:
↳ concept used is graph shifting.

$$V = \frac{1}{2} = \boxed{0.5\, m/s}$$



9. Wave speed on strings

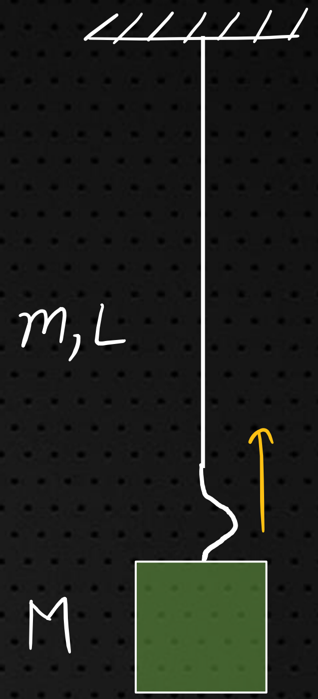


$$v = \sqrt{\frac{T}{\mu}}$$

T → Tension in string

μ → mass/Length

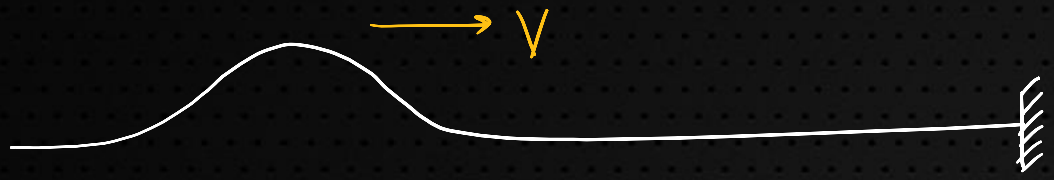
Ex 5.



$M >> m$

Find time to reach top.

9. Wave speed on strings



$$V = \sqrt{\frac{T}{\mu}}$$

$T \rightarrow$ Tension in string

$\mu \rightarrow$ mass/Length

Ex5.

$M >> m$

Find time to reach top.

Sol^n: $\because M >> m \Rightarrow T$ in string $\simeq Mg$

$$\therefore V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{m/L}} = \sqrt{\frac{MgL}{m}}$$

$$\Rightarrow t = \frac{L}{V} = \sqrt{\frac{mL}{Mg}}$$

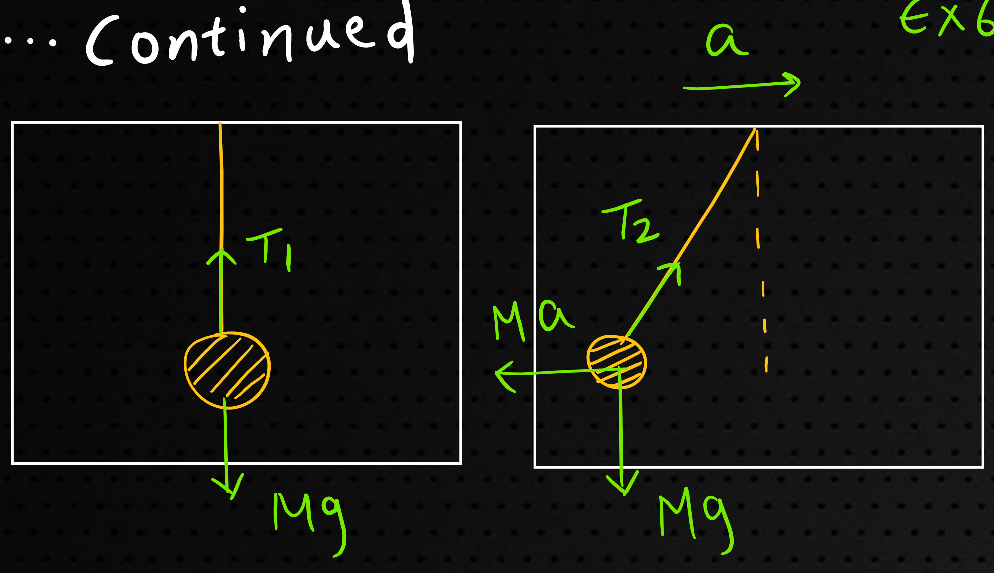
... Continued

Ex6. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $m(m << M)$. When the car is at rest, the speed of transverse waves in the string is $60\ ms^{-1}$. When the car has acceleration a, the wave speed increases to $60.5\ ms^{-1}$. The value of a, in terms of gravitational acceleration g is closest to **(2019 Main, 9 Jan I)**

(a) $\frac{g}{20}$ (b) $\frac{g}{5}$ (c) $\frac{g}{30}$ (d) $\frac{g}{10}$

Sol^n:

... Continued



$T_1 = Mg$ $\qquad$ $T_2 = M\sqrt{g^2+a^2}$

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(a) $\frac{g}{20}$ ✓(b) $\frac{g}{5}$ (c) $\frac{g}{30}$ (d) $\frac{g}{10}$

Sol^n: $V \propto \sqrt{T} \Rightarrow \frac{V_2}{V_1} = \left(\frac{T_2}{T_1}\right)^{1/2} = \left(\frac{M\sqrt{g^2+a^2}}{Mg}\right)^{1/2}$

$\Rightarrow \frac{V_2}{V_1} = \left(1+\frac{a^2}{g^2}\right)^{1/4} \Rightarrow \frac{60.5}{60} = \left(1+\frac{a^2}{g^2}\right)^{1/4}$

$\Rightarrow \left(1+\frac{0.5}{60}\right)^4 = 1+\frac{a^2}{g^2} \Rightarrow 1+\frac{2}{60} = 1+\frac{a^2}{g^2} \quad \therefore a = \frac{g}{\sqrt{30}}$

$(1+x)^n \sim 1+nx$

... Continued

Ex 7.

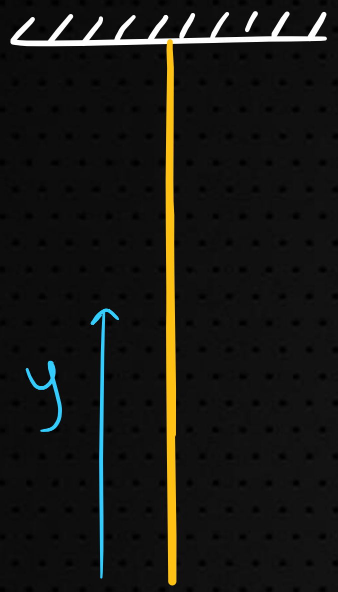
Find time taken to reach top.

... Continued

Ex 7.

Find time taken to reach top.

Soln:



$$T = \frac{M}{L} y \cdot g$$

$$\Rightarrow v(y) = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{Myg/L}{M/L}} = \sqrt{gy}$$

$\because$ at $y = 0,\ T = 0 \Rightarrow u = 0$

$$\Rightarrow s = ut + \frac{1}{2} a t^2$$

$$\Rightarrow L = \frac{1}{2} \cdot \frac{g}{2} \cdot t^2$$

$$\therefore a = v \frac{dv}{dy} = \sqrt{gy} \cdot \sqrt{g} \cdot \frac{1}{2\sqrt{y}}$$

$$\boxed{a = g/2}$$

$$\therefore \boxed{t = 2\sqrt{\frac{L}{g}}}$$ Ans.

10. Avg. Power & Intensity Transmitted

$\hookrightarrow P_{av} = \frac{1}{2} \rho s v \omega^2 A^2$

or

$P_{av} = 2\pi^2 f^2 A^2 \rho v s$

ρ : density of string

s : Cross-sec^n area of string

v : wave speed, $\sqrt{T/\mu}$

ω : $2\pi f$

A : Amplitude

$\hookrightarrow I = P_{av}/s = 2\pi^2 f^2 A^2 \rho v$

PYQs LINKS (JEE MAIN)

2021 Feb	2021 March	2021 July	2021 August	2020
https://youtu.be/xcTzAJr-nOI	https://youtu.be/2BDGi9GUS3s	https://youtu.be/igmJzh_O_xc?t=14935	https://youtu.be/TQCDCnDhZ_I	https://youtu.be/Sr7YoXYAsjI

CLICK

(Practice these Questions)

Revision Series Playlist Link https://bit.ly/3eBbib9

JEE Main PYQs Link https://bit.ly/2S54jzh Chapter wise 2021, 2020, 2018

GoldMine Link https://bit.ly/2VhOGFF

CLICK