UNIT-1

Theory of Metal Cutting

Manufacturing:- It refers to the processing of raw materials (or) parts in to finished products through the use of machinery, tools, human labour etc

Machine Tool:- It is a power driven machine which is capable of supporting the job to be worked and at the same time direct & drive the cutting tool. to perform a definite metal removing operation inorder to produce desired shape and size.

functions of M/c tool: (1) To hold and support the job & cutting tool
(2) To move cutting tool in desired motion.
(3) regulate cutting speed & feed.

M/c tools

- Standar M/c
  - → (performing no. of different operations)
  - → large variety of jobs
- Special purpose M/c
  - → (performing only some desired operations)
  - → Mass production of same variety

→ Machining is a term applied to all material removal processes.

Metal cutting :-

Metal Cutting is the process in which a thin layer of excess metal (chip) is removed by wedge shaped single point (or) multipoint cutting tool, with desired geometry from a workpiece through a process of extensive plastic deformation

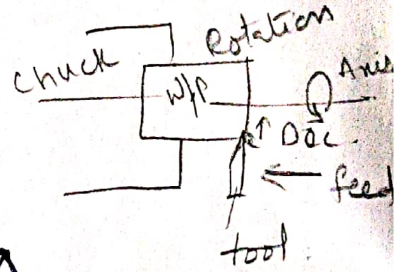
(or)

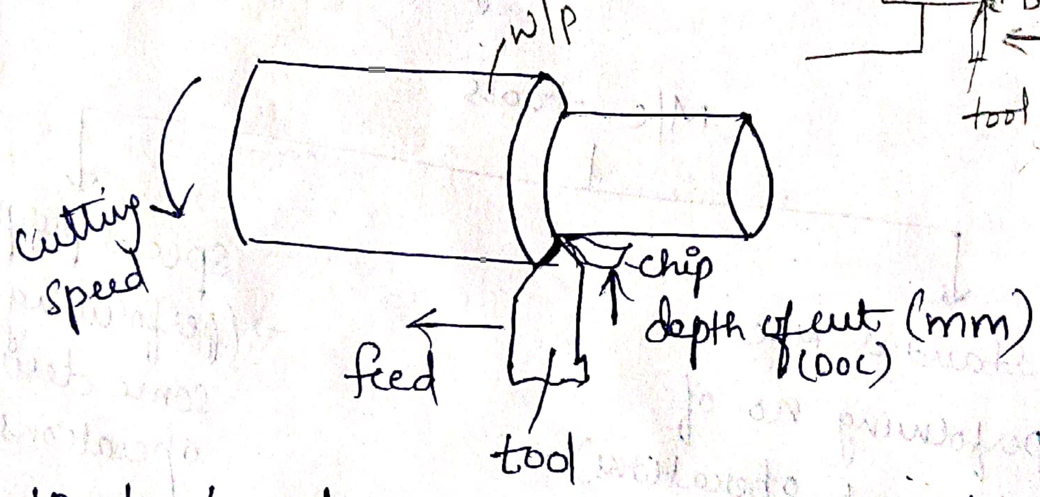
Removal of material from a part in the form of chips so as to attain a final desired shape and size.

Cutting speed :-

Cutting speed of a tool is the speed at which the metal is removed by the tool from the workpiece.

Units :- m/min.





Cutting speed depends on – Hardness of material being machined

– Material of tool bit, Toolgeometry

– feed & DOC required

– Surface quality required

Calculation

It is a peripheral speed of the work past the cutting tool

$$v = \frac{\pi D N}{1000} \text{ mm/min}$$

D = Dia. of work in mm

N = rpm of work.

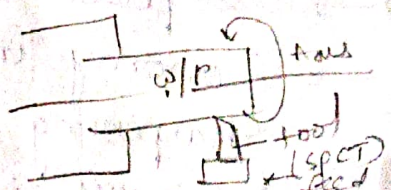
Feed rate :-

→ It is defined as the distance the tool travels during one revolution of the part.

→ Cutting speed, feed determines surface, power requirements, Material removal rate

Selection of feed depends on → material of tool & work
→ Rigidity of workpiece
→ size and condition of m/c tool

Feed rate (f) = mm/rev



Feed, $F = \frac{L}{N \times T}$ L - length, mm; N - rpm; T = Time in minute

Depth of cut :-

It is the ⊥ lar distance that tool bit moves into the work (mm)

$$DOC = \frac{D-d}{2}$$

D = Dia. of work in mm before machining

d = Dia. of work in mm after machining.

Metal cutting

Non cutting shaping processes (No chip formation)

→ metal is shaped under the action of heat, pressure.

Ex forging, drawing, spinning, rolling, extruding etc

cutting shaping

metals shaped & sized by removing excess material from parent metal in the form of chips through machining

Ex:- turning, boring, drilling, milling etc.

Objectives of efficient & economical machining practice

→ Quick metal removal

→ High class surface finish

→ Economy in tool cost

→ less power consumption

→ Economy in the cost of replacement and Sharpening of tools.

→ Minimum idle time of machine tools.

Basic elements of Machining :-

1) w/p

2) Tool

3) Chip

→ These elements represent the cutting action of a tool in 2-dimensional (or) orthogonal cutting

→ To provide cutting action, a relative motion between the tool & work piece is necessary.

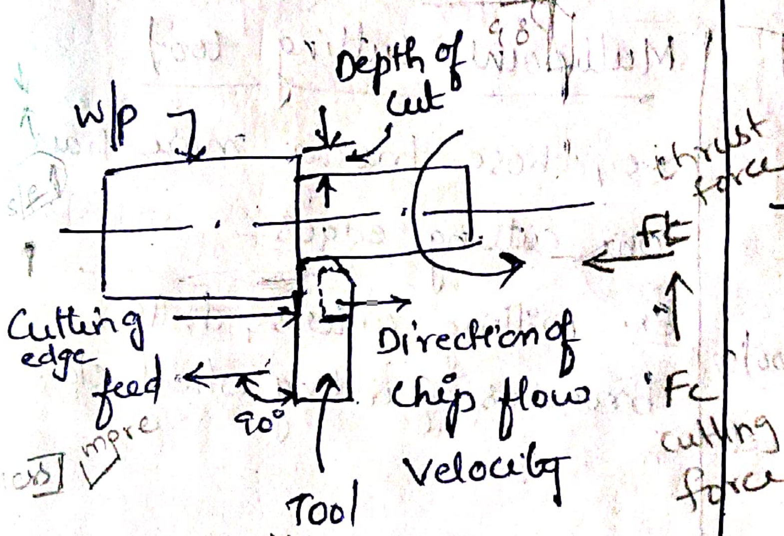
Relative motion b/w w/p & tool obtained by
- → w/p stationary tool moving
- → tool stationary, w/p moving
- → moving both relatively.

Thrust force :- The force that pushes an object forward

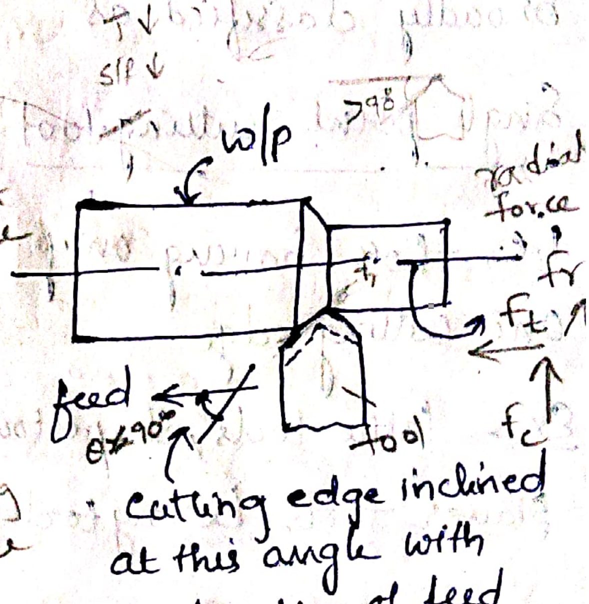
Radial force : The force that acts along the radius of a circular path

Orthogonal and Oblique Cutting

orthogonal cutting | oblique cutting



2D - Cutting



3D - cutting

Orthogonal cutting	Oblique cutting
1) cutting edge of tool is Perpendicular to the direction of tool travels	1) The cutting edge is inclined at an angle (less than 90°) to direction of tool travel.
2) Direction of chip flow is perpendicular to cutting edge	2) The chip flows on the tool face making an angle.
3) The chip coils in tight flat spiral	3) The chip flows sideways in long curl

Shear force : The force acts parallel to the surface of c/s of a material

4) Cutting forces acts on smaller area so tool life is less → Shear force per unit area is high	4) cutting force acts on large area, so tool life is ~~more~~ less
5) produces sharp corners	5) produces chamfer at the end of cut.
Ex:- broaching, slotting dia reduction.	Ex:- All machining operations i.e., taper turning, drilling, boring etc.

Classification of cutting Tools :-

All cutting tools used in metal cutting can be broadly classified as

Single point cutting tool	Multipoint cutting tool
i.e., those having only one cutting edge	i.e., those having more than one cutting edges
Ex:- lathe tools, shaper tools, planer tools, boring tools etc	Ex:- milling cutters, drills, broaches, grinding wheels etc.

Note:-

The cutting tools can also be classified according to the motion as:

a) Linear motion tools:- lathe, boring, broaching, planing, shaping tools etc.

b) Rotary motion tools:- milling cutters, grinding wheels etc

c) Linear and rotary tools - drills, honing tools, boring heads etc.

Important terms:

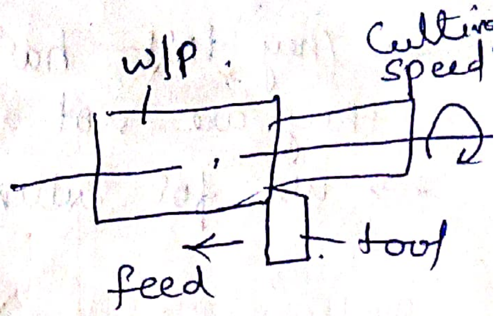
Material removal Rate (MRR):- Volume of material removed per unit time is called MRR

~~MRR~~

$$MRR = \frac{\text{volume of material removal}}{\text{Machine time.}}$$

Ex:-
In turning,

$$MRR = v.f.d$$



v = cutting speed = $\frac{mm}{min}$

f = feed rate = $\frac{mm}{rev}$

d = depth of cut = mm

$$MRR = \frac{mm}{min} \times \frac{mm}{rev} \times mm$$

Units:- $\frac{mm^3}{min}$ (or) $\frac{m^3}{sec}$

$$MRR = \frac{\text{Initial wt} - \text{final wt}}{\text{m/cing time}}$$

(or)

$$MRR = \frac{\text{Volume Removed}}{\text{cutting time}}$$

for rough surface.

$f = 0.4 - 0.125$ mm/rev
$d = 2.5 - 20$ mm

for smooth surface

$f = 0.125 - 0.4$ mm/rev
$d = 0.75 - 2.0$ mm.

(cutting power to MR)

$$\text{Power required} = \frac{MRR \times \text{specific cutting energy}}{\eta}$$

$$\frac{mm^3}{min} \times \frac{J}{mm^3}.$$

Types of cutting tool materials :-

1) **High carbon steel :-** (HCS) or plain carbon steel

→ It contain Carbon % up to 1.5, but not Suitable for production work

→ They are less costly, and easy to heat treat

→ They lost hardness at about 200°C - 250°C and they are not suitable for high speed cutting

→ use for cutting of wood & some soft materials

2) **High speed steel (HSS) :-**

→ HSS contains material like tungsten, chromium, Vanadium, cobalt & molybdenum etc upto 25% are used

→ HSS tools can operate at 2 to 3 times higher than HCS tools

→ These alloying elements ↑ its strength, toughness, wear resistance, cutting ability etc

→ These tools retain its hardness at temp's in the range of 550°C to 600°C

→ use to manufacture complex shaped tool such as reamers, drills, taps etc

3) **Cemented Carbides :-**

→ Used for mass production

→ formed by tungsten, Titanuim, with carbon and the compound is combined with cobalt and Sintered in furnance at 1400°C.

→ It possess a very high degree of hardness and wear resistance

→ diamond is the only material which is harder than this.

→ They are able to retain this hardness at elevated temp's up to 1000°C

→ They can operate at speeds 5 to 6 times more than HSS

→ expensive

4) Stellite :-

→ It is a non ferrous alloy consisting of cobalt, tungstine & chromium.

→ Tool can operate at 2 times more than HSS tool

→ The retain their hardness upto 920°C

→ It gives better tool life than HSS

5) Cemented oxides (OR) Ceramics :-

→ They are capable of withstanding high temp's without losing hardness upto 1200°C

→ more wear resistance than cemented carbide tools

→ They are more brittle and have low bending resistance

→ It is used in the form of tips.

6) Diamond :-

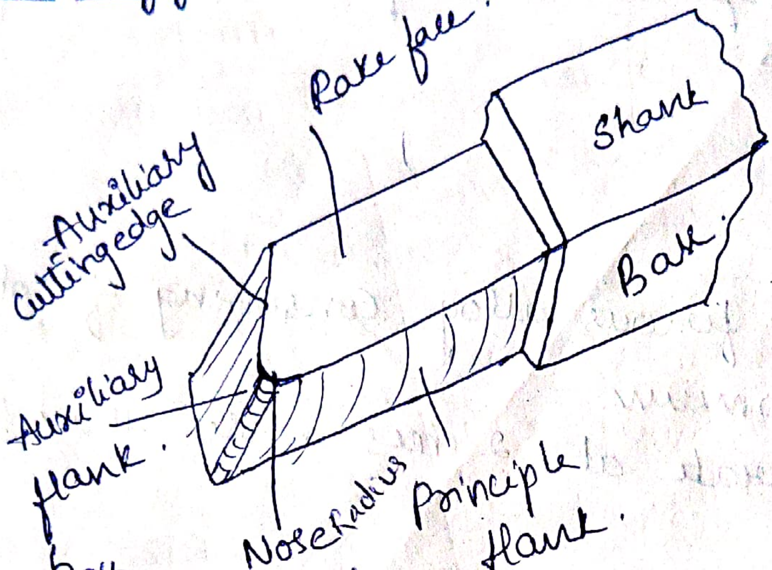
→ It is the hardest cutting tool material

→ It offers high wear resistance but low shock resistance due to brittleness

→ good s/f on non ferrous metals.

Single point cutting tool (SPCT)

Terminology:-



Heel :- bottom of the tool when base & flank of the tool meet

Shank :- Main body of a tool, it is a part of the tool which is gripped to the tool holder.

Face :- Top surface of the tool between the shank and the cutting point of the tool. The chips flow along this surface only.

Flank :- portion of the tool which faces the work. It is the surface adjacent to and below the cutting edge when the tool lies in a horizontal position.

Point :- It is a wedge shaped portion where the face and flank of the tool meet

Base :- Bearing surface of the tool on which it is held in a tool holder.

Nose radius :- * Cutting tip (nose) of a single point tool carries a sharp cutting point.

* Due to high stress during operation, it may loose its cutting ability.
* To prevent this effects, the nose is

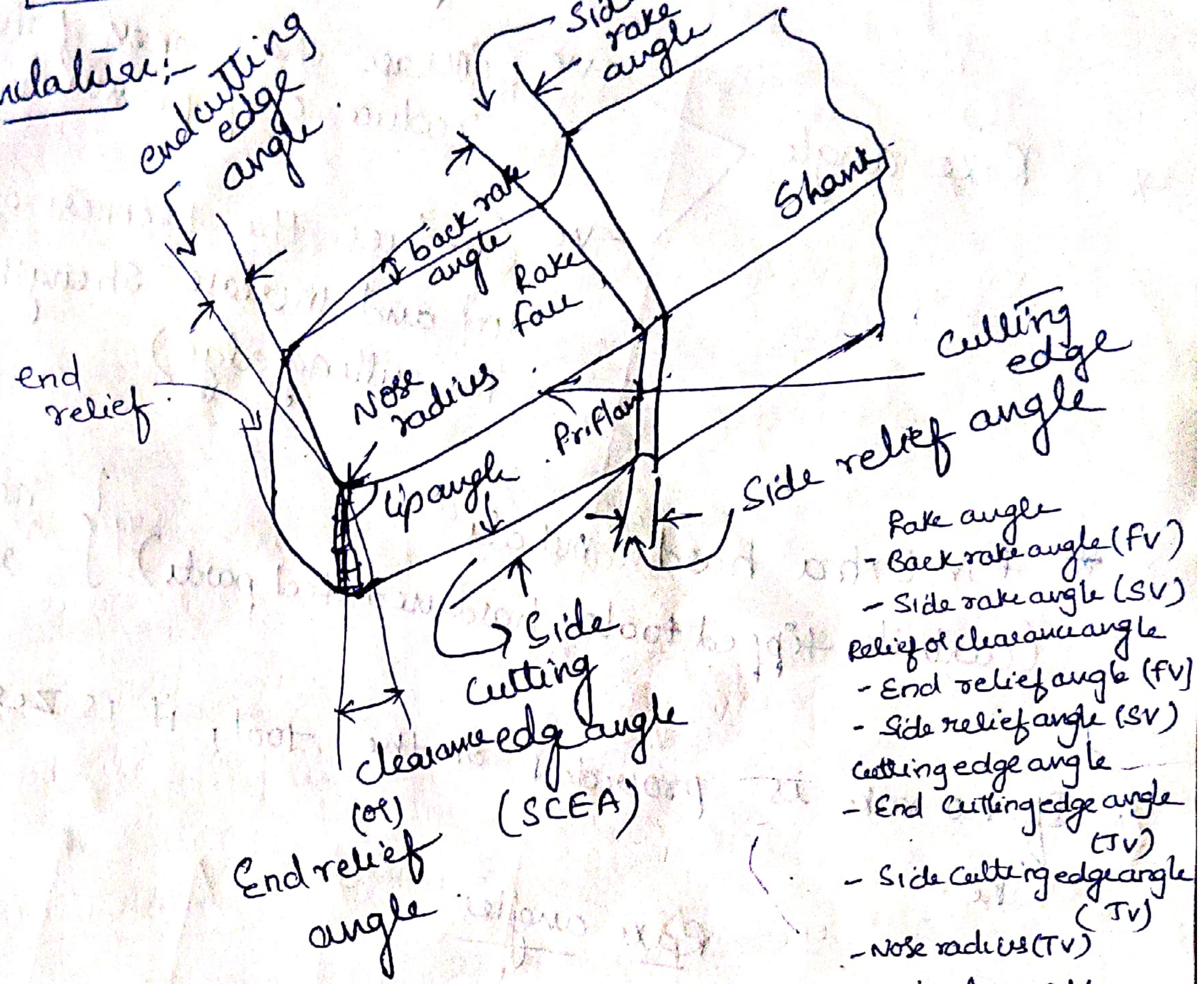
provided with a radius, called nose radius.

* It enables greater strength, increases tool life, and superior surface finish.

$\boxed{\text{Nose radius lies b/w 0.4mm to 1.6mm}}$

Too large — chatter
Too small — weakens pt

Nomenclature:-



Rake angle
- Back rake angle (FV)
- Side rake angle (SV)

Relief or clearance angle
- End relief angle (FV)
- Side relief angle (SV)

Cutting edge angle
- End cutting edge angle (TV)
- Side cutting edge angle (TV)
- Nose radius (TV)

(FV, SV)
Rake angle:— It is the angle formed between the face of the tool and a plane parallel to its base

→ Inclination is towards the shank, is known as back rake angle (or) top rake angle.

→ The inclination towards the side of the tool is called side rake angle.

* Rake angle guides the chips away from the cutting edge, thereby reducing the chip pressure on the face and increasing the keenness (quality) of the tool so that less power is required for cutting.

Note:- Increased rake angle will reduce the Strength of the cutting edge.

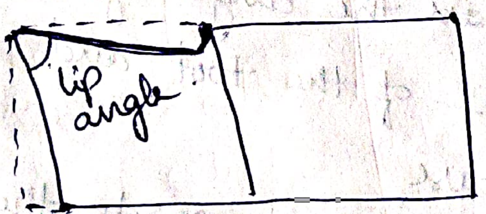
* For hard metals — Smaller rake angles } +ve rake angles.
  Softer metals — larger rake angles.

* Rake angle
  - +ve (increase the keenness of the tool, reduce chip pressure)
  - −ve (reduces the keenness of the tool and increase strength of the cutting edge)

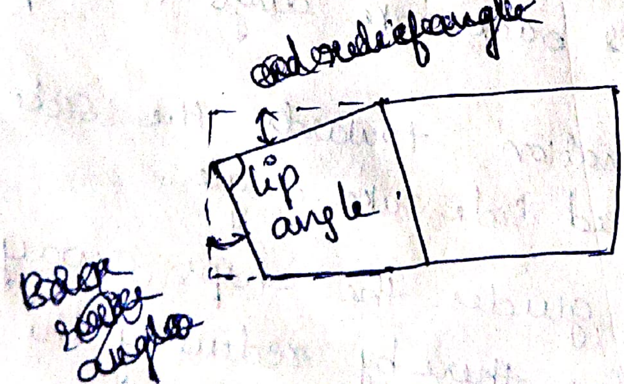
* For Extra hard metals (Carbide tipped tools, hardened steel parts) } negative rake.

* If no rake is provided on the tool, if it's Zero rake.

Rake angles.



Positive Rake
Top face slopes downward away from point



Negative Rake.
Top face slopes upward away from pt.

Zero rake.

lip angle (wedge angle or Angle of keenness)

→ The angle b/w the face and the flank of the tool is known as lip angle.

→ Strength of cutting edge directly effected by this angle

→ Larger lipangle – Stronger cutting edge.

$$\text{lip angle} \propto \frac{1}{\text{rake angle}}$$

→ for harder metals – Stronger tool is required. rake angle is reduced – lip angle is increased.

→ So cutting speed should reduced which is a disadvantage

→ The lip angle is kept as low as possible without making the cutting edge so weak.

Relief (or) clearance angle :-

→ Ground on the end and side faces of a tool to prevent it from rubbing on the workpiece

→ To enable only the cutting edge to touch the work piece and cut freely without rubbing against the surface of the job

cutting edge angle

→ Ground on a tool so that it can be mounted in the correct position for various machining operations

Side cutting edge angle (SCEA)

Allow flank of the tool (15°) to approach the w/p first

end cutting edge angle (ECEA)

Allow the cutting tool to m/c close to the w/p during turning operation usually (20-30°)

Tool Signature :-

* It is the system of designating the principal angles of a single point cutting tool
* There are two systems widely used are
  1) ASA system (American Standard Association (or) M/C Reference system)
  2) ORS system (orthogonal rake system)

Reference planes :-

They are used to describe the geometry and locate the different parameters of a single point cutting tool.

1) The coordinate system
2) The orthogonal system.

Tool geometry in coordinate system (or) ASA system

It is adopted by ASA system.

SPCT is designated as

$\alpha_b - \alpha_s - \theta_e - \theta_s - C_e - C_s - r$

- α_b → Back rake angle
- α_s → side rake angle
- θ_e → End relief angle
- θ_s → side relief angle
- C_e → ECEA
- C_s → SCEA
- r → Nose radius

eg:- Describe a tool with 8, 10, 6, 6, 10, 2 signature in ASA System

Back rake angle - 8°

Side rake angle - 10°

End relief angle - 6°

Side relief angle - 6°

End cutting edge angle - 6°

Side cutting edge angle - 10°

Nose radius - 2mm.

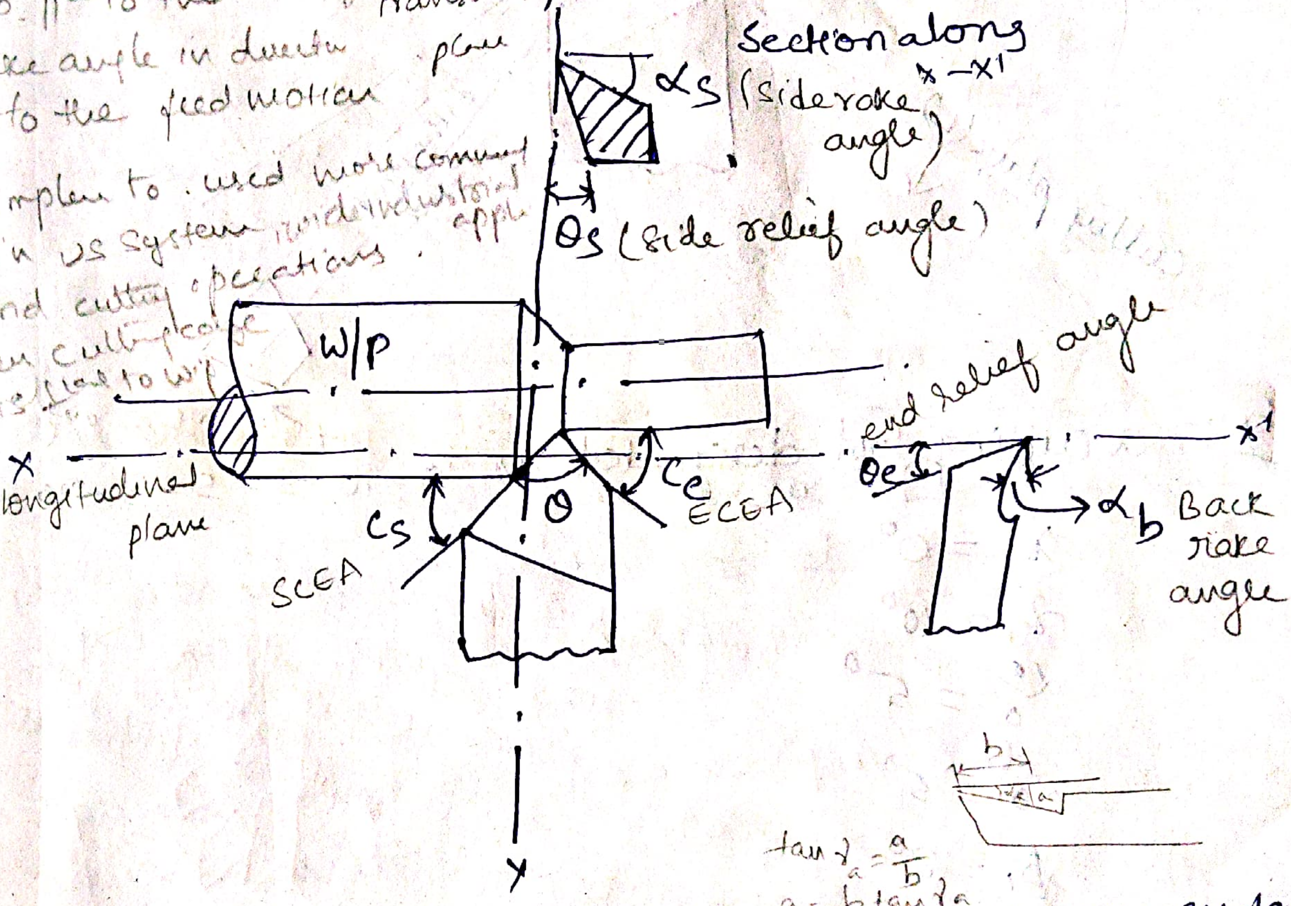
R.P. ‖le to the base of the tool

Rake angle in direction transverse plane ‖le to the feed motion

Simpler to used more common in US system understand appl

and cutting operations

when cutting edge is ‖ to w/p



$\tan \gamma_a = \frac{a}{b}$

$a = b \tan \gamma_a$

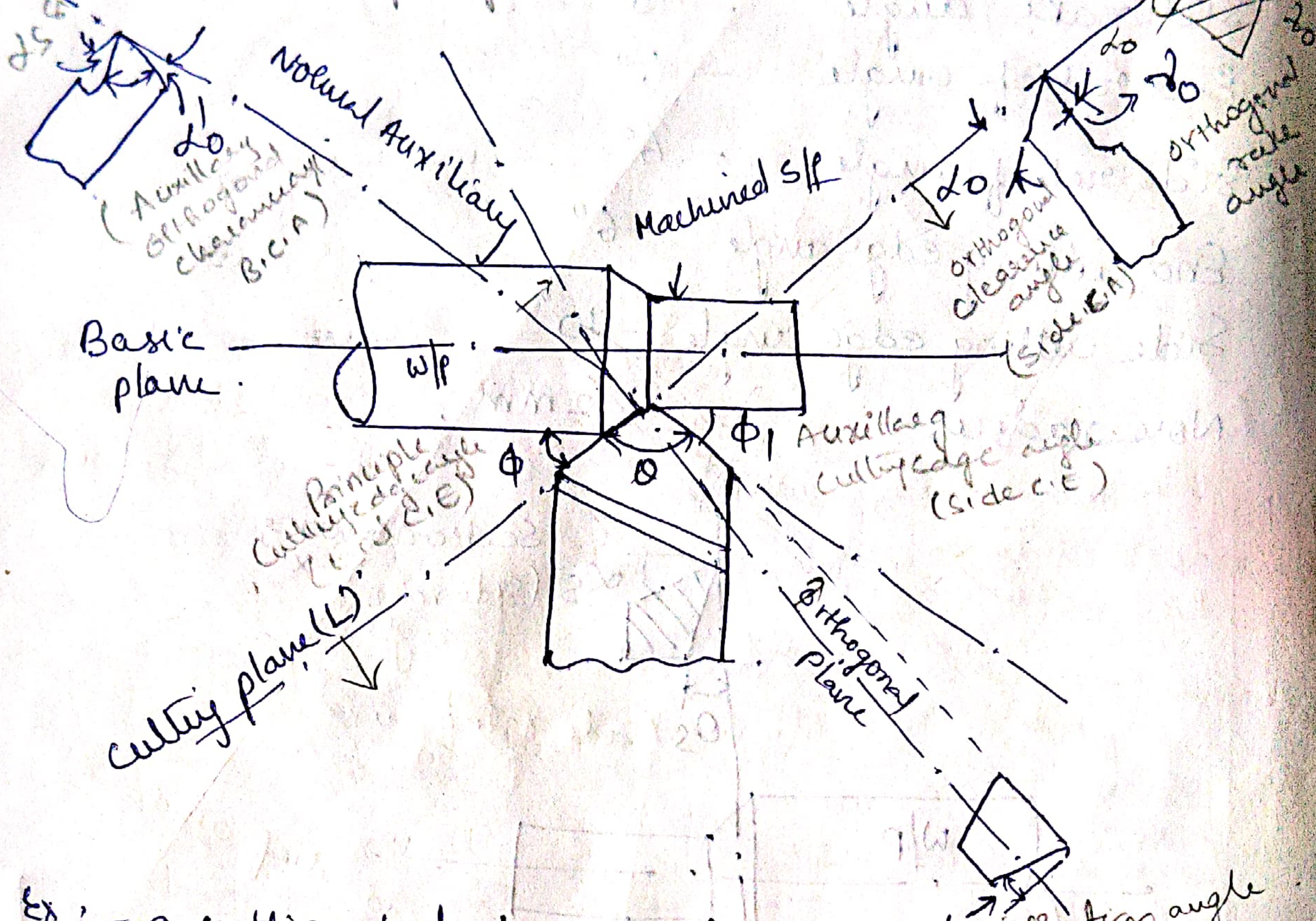
Tool geometry in orthogonal system (OS) ORS system

$\lambda \quad \gamma_0 \quad \alpha_0 \quad \alpha_0' \quad \phi_1 \quad \phi \quad r(mm)$

- $r(mm)$ → Nose radius
- ϕ → Principle cutting edge angle
- ϕ_1 → Auxillary cutting edge angle
- α_0' → end relief angle → Auxillary orthogonal Clearance angle.
- α_0 → Side relief angle → Orthogonal clearance angle
- γ_0 → Orthogonal rake angle
- λ → Inclination angle

In ORS R. plane ⊥lar to cutting edge

Rake angle in the direction of normal to the cutting edge



Ex :- a cutting tool designated as 0 – 10 – 5 – 5 – 8 – 90 – 1

$\lambda = 0^\circ$

$\gamma_o = 10^\circ$

$\alpha_o = 5^\circ$

$\alpha_o' = 5^\circ$

$\phi_1 = 8^\circ$

$\phi = 90^\circ$

$R = 1\,mm$

Adv

ORS is useful when the cutting edge is not ⊥lar to w/p making it more versatile & used for oblique cutting

– R.P ∥le to the base of tool & w/p
– Rake angle ∥le to the feed motion
– Adv: Simpler to use and more common in U.S
– used for strd cutting edge operation when cutting edge is ⊥lar to feed motion
– more practical & widely used in industrial setting in U.S

Ref. plane ⊥lar to the cutting edge
– Rake angle direction normal to the cutting edge
Adv
– used when cutting edge is not ⊥lar to the w/p, & more versatile in dealing with oblique cutting
– more theoretical & analytical

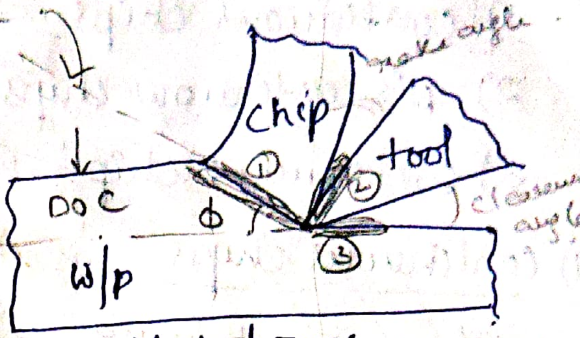
Mechanism of chip formation (or) Mechanics of metal cutting

In machining process basic elements are

w/p

Tool

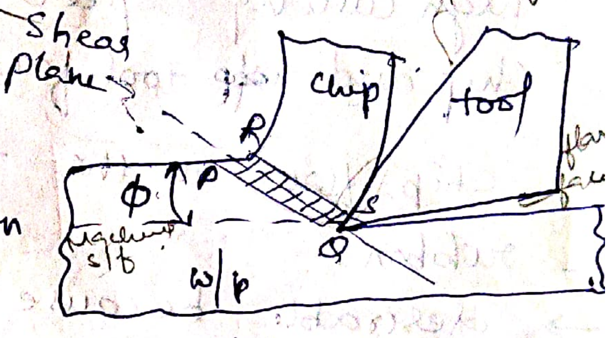
Chip.



heat zone
1 - Shear zone (60%) Primary deformation zone
2 - lateral shear zone (30%) Secondary deformation zone
3 - sharpness of tool (10%)

heat Generation zones

The relative motion b/w w/p and tool may be provided with
(w/piece constant - tool is rotating)
(or) (tool is stationary - w/p rotating)



For example

Consider a orthogonal cutting
It is a schematic representation of shaping operation.

→ workpiece - constant, and tool is rotating

→ As the tool advances in to the w/p towards the left,

→ Thus the metal gets compressed very severely, causes shear stress

→ The stress is maximum along the plane is shear plane.

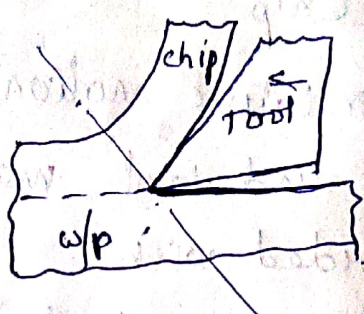
→ material w/p is ductile - material flows plastically along the plane forming the chip, which flows upward along the face of the tool.

Types of chips

The chips produced during machining can be broadly classified into three types.

1) continuous chips
2) Discontinuous chips (or) segmental chips
3) continuous chips with build up edge.

1) Continuous chips



→ when machining ductile materials (low carbon steel, mild steel, copper, alluminum etc) Continuous chips are formed,

– under favourable conditions, such as high cutting speed and mini friction b/w chip and ~~w/p~~ tool face (large rake angle, sharp cutting edge)

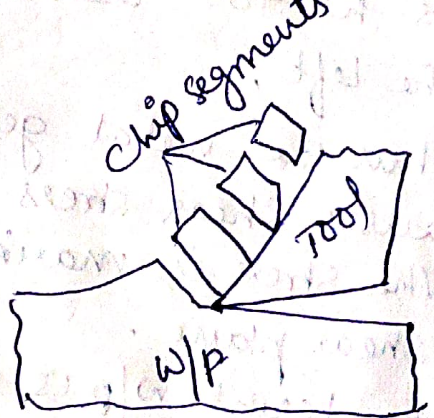
→ chip flow on the tool face in the form of ribbon.

→ desirable because a smooth s/f is obtained

→ high tool life

→ lower power consumption.

2) Discontinuous chips :-



→ This type of chip produced during machining of brittle materials like castiron & bronze (low rake angle)

→ chips are in the form of small segments.

→ The following factors favours the formation of discontinuous chips

1) Low to medium cutting speed

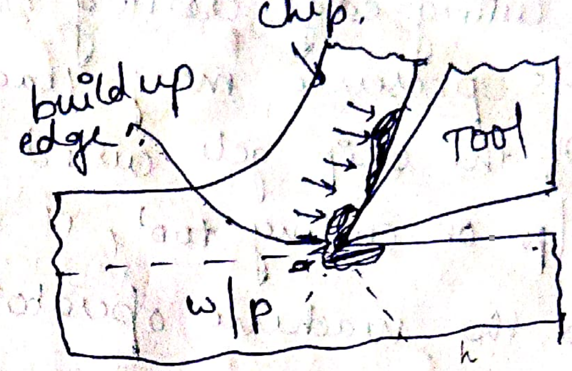
2) large feeds & depth of cut

3) Absence of cutting fluid.

→ Most of the heat is carried by chip, tool is heated to a lower temp. Thus tool life is longer.

3) <u>continuous chips with buildupedge</u>:-

→ This chips are formed when machining ductile materials with a cutting of smaller rake angle at lower cutting speed.



→ The other conditions are

1) Higher values of depth of cut & feed

2) High friction

3) poor lubrication

4) High cutting pressure and temperatures in shear zone.

→ These chips remain attached to the machined surface, causes poor surface finish of w/surface.

→ increases power consumption.

<u>Chip Breakers</u>:-

→ The chips produced during higher speeds in machining of high tensile strength materials, need to be effectively controlled.

→ when carbide tipped tools are used for machining because of higher cutting speeds, due to high temperatures, the resulting chip will be continuous, blue in colour & take the shape of a coil

→ Such a chip, if not broken into parts & removed from the surroundings of metal cutting area, is likely to adverse affect the machining results.

1. It may adversely effect the tool life by spoiling the cutting edge creating crater & raising temp.
2. Its presence may lead to poor s/f on the w/p
3. If the chip gets curled around the rotating w/p or cutting tool, it may be hazardous to the machine operator
4. Very large coils offer a lot of difficulty in their removal.

→ To prevent the adverse effects, chip breakers are used.

Common methods used for chip breaking are

1) By control of tool geometry

i.e, grinding proper back rake and side rake according to the feeds & speeds to be used

2) By obstruction method:

i.e., by interposing a metallic obstruction in the path of the coil.

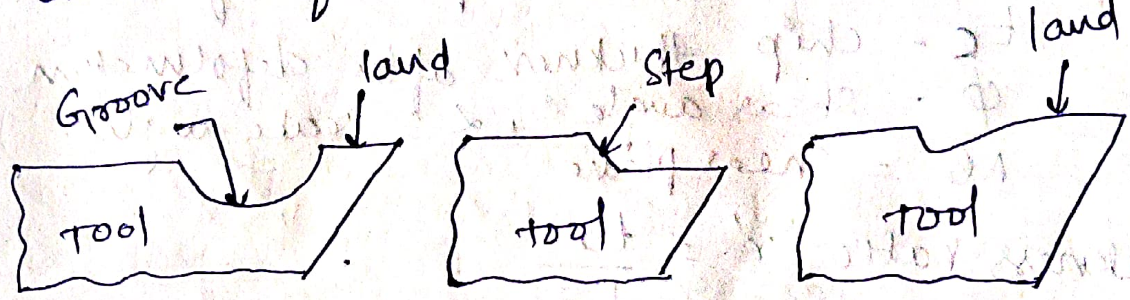
Some strict chip control is desired, then the following type of chip breakers are used

1) Groove type :- Grinding ~~proper~~ a groove on the face of the tool, leaving small land near the tip.

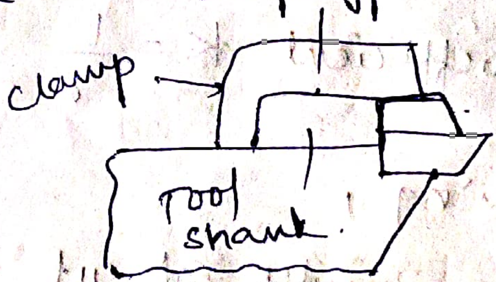
2) Step type :- Grinding a step on the face of the tool, adjacent to the cutting edge

3) Secondary rake type :- providing a secondary rake on the tool through grinding ; together with a small step.

4) Clamp type :- very common with carbide tipped tools. chip breaker is a thin and small plate which is either brazed to or held mechanically on the tool face.



① Groove type. ② Step type Secondary rake type



③ Clamp type

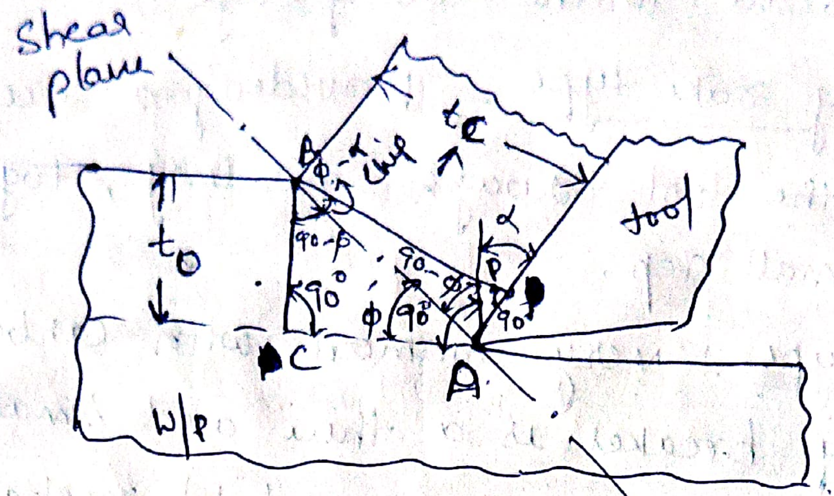
Chip thickness Ratio :-

→ During the cutting action of a metal it is observed that the thickness of the deformed chip is more than the actual depth of cut.

→ It is becoz the chip flows upwards at a slower rate than the velocity of the cut.

→ The velocity of chip flow is directly effected by the shear plane angle.

$t_c > t_o$



t_o = chip thickness prior to deformation

t_c – chip thickness after deformation.

ϕ = Shear angle, α = rake angle.

AB = Shear plane

thickness ratio 'r' $= \dfrac{t_o}{t_c}$.

Chip reduction coefficient $K = \dfrac{1}{r}$.

In orthogonal cutting,

width of chip = width of cut.

$$\boxed{t_c = t_o}.$$

and product of the chip thickness and its length will be equal,

If L_1 and L_2 are the lengths of the metal cut and chip resp.

$$t_o \cdot L_1 = t_c \cdot L_2$$

$$\frac{t_o}{t_c} = \frac{L_2}{L_1}$$

But, $\frac{t_o}{t_c} = r$

$$r = \frac{t_o}{t_c} = \frac{L_2}{L_1}$$

$$\boxed{K = \frac{1}{r} = \frac{t_c}{t_o} = \frac{L_1}{L_2}}$$

We have two right angled Δles APB and ACB.

Consider $\lfloor ACB$

$$\frac{BC}{AB} = \sin CAB = \sin\phi.$$

$$\sin\phi = \frac{t_o}{AB} \qquad (\because BC = t_o)$$

$$\boxed{AB = \frac{t_o}{\sin\phi}} \quad \text{—— (1)}$$

Consider $\lfloor APB$.

$$\sin(90 - \phi + \alpha) = \frac{BP}{AB}$$

$$\cos(\phi - \alpha) = \frac{BP}{AB} \qquad (BP = t_c)$$

$$\cos(\phi - \alpha) = \frac{t_c}{AB}$$

$$\boxed{AB = \frac{t_c}{\cos(\phi - \alpha)}} \quad \text{—— (2)}$$

From eqn (1) & (2), we get

$$\frac{t_o}{\sin\phi} = \frac{t_c}{\cos(\phi-\alpha)}$$

$$\frac{t_o}{t_c} = \frac{\sin\phi}{\cos(\phi-\alpha)}$$

$$r = \frac{\sin\phi}{\cos(\phi-\alpha)}$$

$$r = \frac{\sin\phi}{\cos\phi\cos\alpha + \sin\phi\sin\alpha}.$$

$$r(\cos\phi\cos\alpha) + r(\sin\phi\sin\alpha) = \sin\phi$$

$$\frac{r(\cos\phi\cdot\cos\alpha)}{\sin\phi} + \frac{r(\sin\phi\sin\alpha)}{\sin\phi} = 1$$

$$\frac{r\cdot\cos\alpha}{\tan\phi} + r\sin\alpha = 1$$

$$\frac{r\cos\alpha}{\tan\phi} = 1 - r\sin\alpha$$

$$\boxed{\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}}$$

Sub, r in terms of to & tc.

$$\boxed{\tan\phi = \frac{\frac{t_o}{t_c}\cos\alpha}{1 - \frac{t_o}{t_c}\sin\alpha}}$$

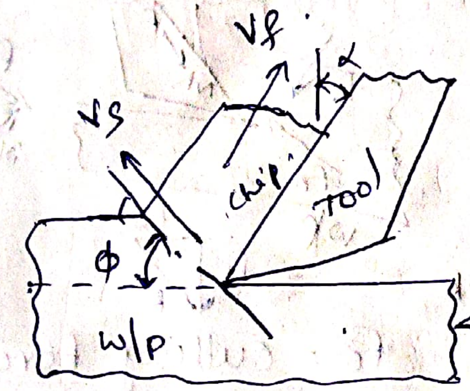
Velocity Relations :-

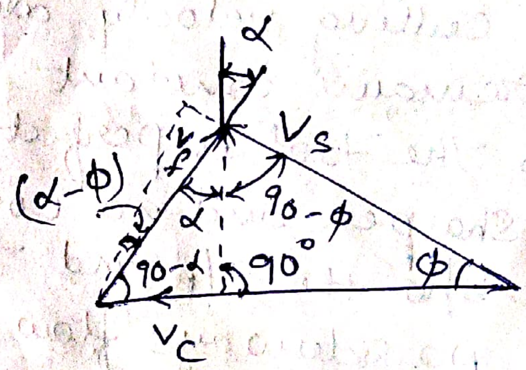
For orthogonal cutting,

let the V_c = Cutting velocity (or)
vel. of tool relative to work.

V_f = chip flow velocity (or)
vel. of chip flow relative to tool

V_s = velocity of shear (or)
vel. of displacement of the chip along the shear plane relative to work.





According to sine rule

$$\frac{V_c}{\sin(90-\phi+\alpha)} = \frac{V_f}{\sin\phi} = \frac{V_s}{\sin(90-\alpha)}$$

$$\frac{V_c}{\sin(90-(\phi-\alpha))} = \frac{V_f}{\sin\phi} = \frac{V_s}{\cos\alpha}$$

$$\frac{V_c}{\cos(\phi-\alpha)} = \frac{V_f}{\sin\phi} = \frac{V_s}{\cos\alpha}$$

V_s and V_f can be determined in terms of known velocity V_c.

$$V_s = V_c \cdot \frac{\cos\alpha}{\cos(\phi-\alpha)}$$

$$V_f = V_c \cdot \frac{\sin\phi}{\cos(\phi-\alpha)}$$

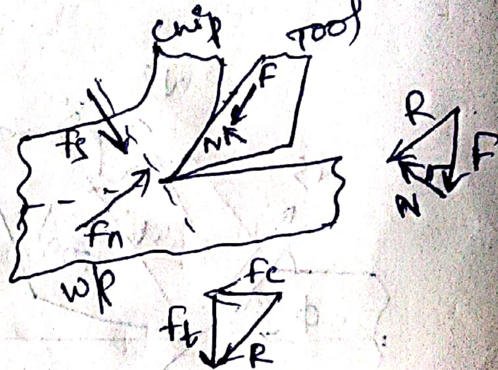
We know that,

$$r = \frac{\sin\phi}{\cos(\phi-\alpha)}$$

$$\boxed{V_f = V_c \times r}$$

Cutting forces Relations in orthogonal cutting

(FBD).



Assumptions

→ Cutting velocity always remains constant

→ The tool is perfectly sharpen and no contact b/w the w/p and tool flank.

- No sideways flow of chip
- Only continuous chip is produced
- There is no built up edge

F_c = cutting force

F_t - Thrust force

F - friction force

N - Normal force

F_s - Shear force

F_n - normal to shear force

To study further relationship, the two Δ^{les} of forces of above FBD have been combined together, called Merchant's ○le diagram.

Benefits:
- study machinability characteristics of the work material.
- Estimation of cutting power consumption.
- Selection of power source.
- Evalulate role of the machining parameter on force (Feed, velocity, doP what is change on force)

Merchant's circle diagram:- study force acting on a tool during metal cutting process

F_c = Cutting force

F_t = Thrust force.

F_c, F_t, F_R — Measured from force dynamometers.

α — rake angle.

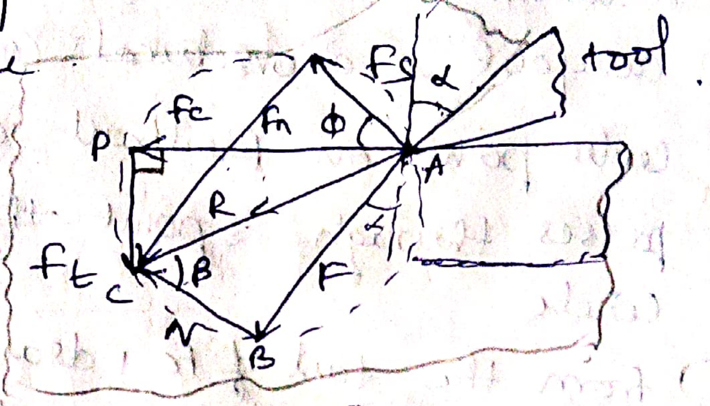
ϕ — Shear angle

F, N — Forces by Chip & tool

F — Friction force

N — Normal to friction force

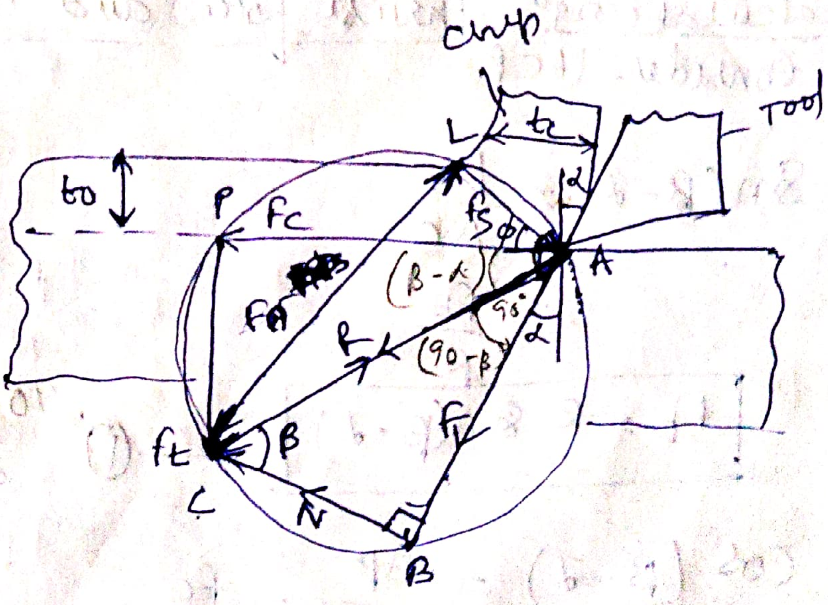
β = friction angle.



Assumption — Orthogonal cutting - 2D cutting

t_o = Uncut chip thickness

t_c = Chip thickness after deformation



Procedure:-

→ It is a graphical representation of the no. of forces acting on a w/p when it is subjected to orthogonal cutting.

① Draw a vertical line $\parallel^{le}$ to yanis and draw draw tool face making an angle with vertical line.

② Draw a cutting force f_c (which is measured from dynamometer with proper scale)

③ Draw thrust force $\perp^{lar}$ to cutting force & draw resultant

④ carryout a geometrical construction i.e, locate centre point on Resultant and draw the circle passes through 3 point this is called merchant circle

⑤ from the tail of f_c draw the shear force (f_s) along the shear plane at an angle ϕ

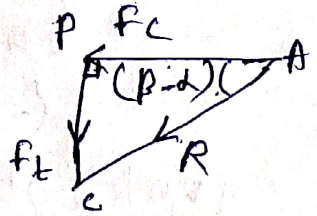
⑥ Draw frictional force f $\perp^{lar}$ to the shear force & acting along the tool - chip interface

⑦ And draw friction normal force N $\perp^{lar}$ to the friction force ~~at the tail of A~~.

Calculating thrust force and cutting force by Analytical method

Consider $\angle ACP$.

$$\sin(\beta-\alpha) = \frac{PC}{AC} = \frac{f_t}{R}$$



$$\boxed{f_t = R\sin(\beta-\alpha)} \longrightarrow ①$$

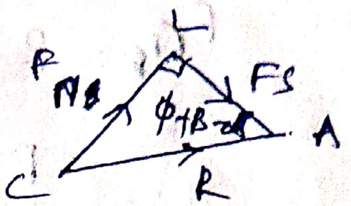
$$90-[(90-\beta)+\alpha]$$
$$90-90+\beta-\alpha=(\beta-\alpha)$$

$$\cos(\beta-\alpha) = \frac{AP}{AC} = \frac{f_c}{R}$$

$$\boxed{f_c = R\cos(\beta-\alpha)} \longrightarrow ②$$

Calculating shear force and Normal to shear force.

Consider $\angle ACL$



$$\cos(\phi+\beta-\alpha) = \frac{AL}{AC} = \frac{Fs}{R}$$

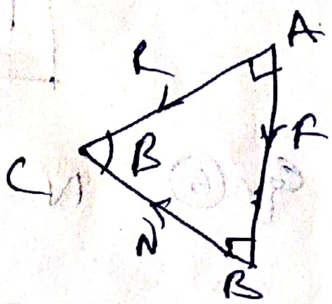
$$\boxed{Fs = R\cos(\phi+\beta-\alpha)} \quad \text{— (3)}$$

$$\sin(\phi+\beta-\alpha) = \frac{CL}{AC} = \frac{F_{N8}}{R}$$

$$\boxed{F_{N8} = R\sin(\phi+\beta-\alpha)} \quad \text{— (4)}$$

Calculating frictional force & Normal to frictional force

Consider $\angle ABC$



$$\sin\beta = \frac{AB}{AC} = \frac{F}{R}$$

$$\boxed{F = R\sin\beta} \quad \text{— (5)}$$

$$\cos\beta = \frac{BC}{AC} = \frac{N}{R}$$

$$\boxed{N = R\cos\beta} \quad \text{— (6)}$$

from all the above forces,

from eq (3) $\quad Fs = R\cos(\phi+\beta-\alpha)$

$= R\cos\phi\cos(\beta-\alpha) - \sin\phi\sin(\beta-\alpha)$

$= R\cos(\beta-\alpha)\cos\phi - R\sin(\beta-\alpha)\sin\phi$

[we know $\cos(A+B)$
$\cos A\cos B - \sin A\sin B$
$A = \phi$
$B = (\beta-\alpha)$]

$$\boxed{Fs = F_c\cos\phi - F_t\sin\phi}$$

from eqn (4), we

$$F_N = R\sin(\phi+\beta-\alpha)$$

$$= R\sin\phi\cos(\beta-\alpha) + R\cos\phi\sin(\beta-\alpha)$$

$$= R\cos(\beta-\alpha)\sin\phi + R\sin(\beta-\alpha)\cos\phi$$

$$\boxed{F_N = F_c\sin\phi + F_t\cos\phi}$$

from Eqn (5) $F = R\sin\beta = R\sin(\alpha+\beta-\alpha)$

$$= R\sin\alpha\cos(\beta-\alpha) + R\cos\alpha\sin(\beta-\alpha)$$

$$\boxed{F = F_c\sin\alpha + F_t\cos\alpha}$$

Eqn (6) $N = R\cos\beta = R\cos(\alpha+\beta-\alpha)$

$$= R\cos\alpha\cos(\beta-\alpha) - R\sin\alpha\sin(\beta-\alpha)$$

$$\boxed{N = F_c\cos\alpha - F_t\sin\alpha}$$

we, know that

Coefficient of friction, $\mu = \frac{F}{N} = \tan\beta$

$$= \frac{F_c\sin\alpha + F_t\cos\alpha}{F_c\cos\alpha - F_t\sin\alpha}$$

divide by $\cos\alpha$, numerator & denominator.

$$\mu = \frac{F_c\tan\alpha + F_t}{F_c - F_t\tan\alpha}$$