

Probability

1. Total = 15 marbles

$$\Rightarrow {}^{15}C_2 = \frac{15!}{2!(15-2)!} \Rightarrow \frac{15 \times 14 \times 13 \times 12 \times 11}{2 \times 1} = 105$$

at least one green = 1 - prob of no green

$$\text{non-green} = 3 + 4 + 2 = 9$$

$$\Rightarrow {}^9C_2 = \frac{9 \times 8 \times 7}{2 \times 1} = 36 \Rightarrow \frac{36}{105} = \frac{12}{35}$$

$$\Rightarrow 1 - \frac{12}{35} = \frac{35-12}{35} = \frac{23}{35}$$

2. Total marbles = 15

$${}^{15}C_3 = 455$$

blue or yellow = 3 + 2 = 5

$$\Rightarrow {}^5C_2 = \frac{5 \times 4 \times 3}{2 \times 1} = 10 \Rightarrow \frac{10}{455} = \frac{2}{91}$$

3. Total number of marbles = 15

4 marbles = ${}^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365$

No. of non-blue = 4 + 6 + 2 = 12

$$\Rightarrow {}^{12}C_4 = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = 495$$

$$\Rightarrow \frac{495}{1365} = \frac{99}{273} = \frac{33}{91}$$

4. A leap year has 366 days

366 days = 52 weeks and 2 extra
 2 extra days can be some pair
 (S,M) (M,T) (T,W) (W,F) (F,S)

there are 7 possible combination for 2 events (Sunday, Monday) out of 7 possible outcome
 $= \frac{1}{7}$

$$\text{total books} = 10$$

no. of ways to arrange 9 units $\Rightarrow 9!$
 the two books within the pair can be arranged in $2!$ ways $\Rightarrow 9! \times 2!$

$$\begin{aligned} \text{total ways to arrange 10 books} &= 10! \\ \Rightarrow \frac{9! \times 2!}{10!} &= \frac{9! \times 2!}{10 \times 9!} \Rightarrow \frac{2}{10} = \frac{1}{5} \end{aligned}$$

6. two consecutive integers is always even $n, n+1$

$$\text{sum is } n + (n+1) = 2n+1$$

$2n$ is even, $2n+1$ is odd

∴ sum of two integers is always odd
 the probability is $\frac{1}{2}$

$$7. \text{total marbles} = 15 \quad {}^5C_3 = \frac{15 \times 14 \times 13}{3 \times 2} = 455$$

$$2 \text{ red marbles} = 4$$

$$1 \text{ yellow marble from 2} = {}^2C_1 = 2$$

$$2 \text{ red marbles from 4} = {}^4C_2 = \frac{4 \times 3}{2} = 6$$

$$(1 \text{ yellow} \times 2 \text{ red}) = {}^2C_1 \times {}^4C_2 = 2 \times 6 = 12$$

$$\Rightarrow \frac{{}^2C_1 \times {}^4C_2}{15C_3} = \frac{12}{455}$$

$$8. \text{total persons} = 10$$

$$\text{no. of graduates} = 4$$

$$\text{non-graduates} = 10 - 4 = 6$$

$${}^{10}C_3 = \frac{10 \times 9 \times 8}{3 \times 2} = 120$$

$$\Rightarrow 6C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \Rightarrow 20 \Rightarrow 20 = \frac{120}{120} = \frac{1}{6}$$

probability of least one = $1 - \frac{1}{6} \Rightarrow \frac{5}{6}$

9. Total no. of couples = 5

No. of people chosen = 5

$$\Rightarrow 10C_5 \Rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} \Rightarrow 252$$

$\Rightarrow 12$ couples, $8C_2 = 10$ ways

10. $p = 0.2$

Ticket not being a prize $1 - p = 1 - 0.2 = 0.8$

3. Probability of no prize in ticket

$$= 0.8 \times 0.8 \times 0.8 \times 0.8 = (0.8)^4$$

Probability of winning at least one ticket $= 1 - 0.4096 \Rightarrow 0.5904$

11. Box 1: 39 red balls

Box 2: 26 green balls

$N_1 = 42$

$N_2 = 42$

Total probability of red ball = $P(\text{Box 1}) \times P(\text{Red}) + P(\text{Box 2}) \times P(\text{Red})$

$$= \frac{1}{2} \times \frac{R_1}{N_1} + \frac{1}{2} \times \frac{R_2}{N_2}$$

R_1 and R_2

We have 39 red balls & 26 green balls

placing one red ball in box 1

$$80, R_1 = 1, N_1 = 1$$

$$(39 - 1) = 38$$

so, Box 2 = $R_2 = 38$, $n_2 = 38+26 = 64$
 $P(\text{red}) = \frac{42}{64} + \frac{1}{3} \times \frac{38}{64} \Rightarrow \frac{51}{64}$

total no. of balls = $6+8+7 = 21$ balls

prob of red ball = $P = \frac{6}{21} = \frac{2}{7}$

no. of red ball = $2 \times 9 = 18 - 21/7 = 51/7$

No. of trials = $n = 521 + n \times 2N$

binomial prob = $P(X=k) = C(n, k) p^k q^{n-k}$

if $P(X=3) = {}^5C_3 = 10 \Rightarrow 10 \times \left(\frac{2}{7}\right)^3 \times \left(\frac{5}{7}\right)^2 = \frac{2000}{16807}$

if $P(X=4) = {}^5C_4 = 5 \Rightarrow \frac{400}{16807}$

if $P(X=5) = {}^5C_5 = 1 = \frac{32}{16807} = \frac{2432}{16807}$

$\rightarrow P(N=1) = 2432$

$\rightarrow P(N=2) = 280$

$\rightarrow P(N=3) = 28$

$\rightarrow P(N=4) = 2$

$\rightarrow P(N=5) = 0.2$

$\rightarrow P(N=6) = 0.02$

$\rightarrow P(N=7) = 0.002$

$\rightarrow P(N=8) = 0.0002$

$\rightarrow P(N=9) = 0.00002$

$\rightarrow P(N=10) = 0.000002$

$\rightarrow P(N=11) = 0.0000002$

$\rightarrow P(N=12) = 0.00000002$

$\rightarrow P(N=13) = 0.000000002$

$\rightarrow P(N=14) = 0.0000000002$

$\rightarrow P(N=15) = 0.00000000002$

$\rightarrow P(N=16) = 0.000000000002$

$\rightarrow P(N=17) = 0.0000000000002$

$\rightarrow P(N=18) = 0.00000000000002$

$\rightarrow P(N=19) = 0.000000000000002$

$\rightarrow P(N=20) = 0.0000000000000002$