NON-LINEAR DYNAMICS

CHUA'S PATH TO CHAOS

NON-LINEARITY IN CHUA'S CIRCUIT

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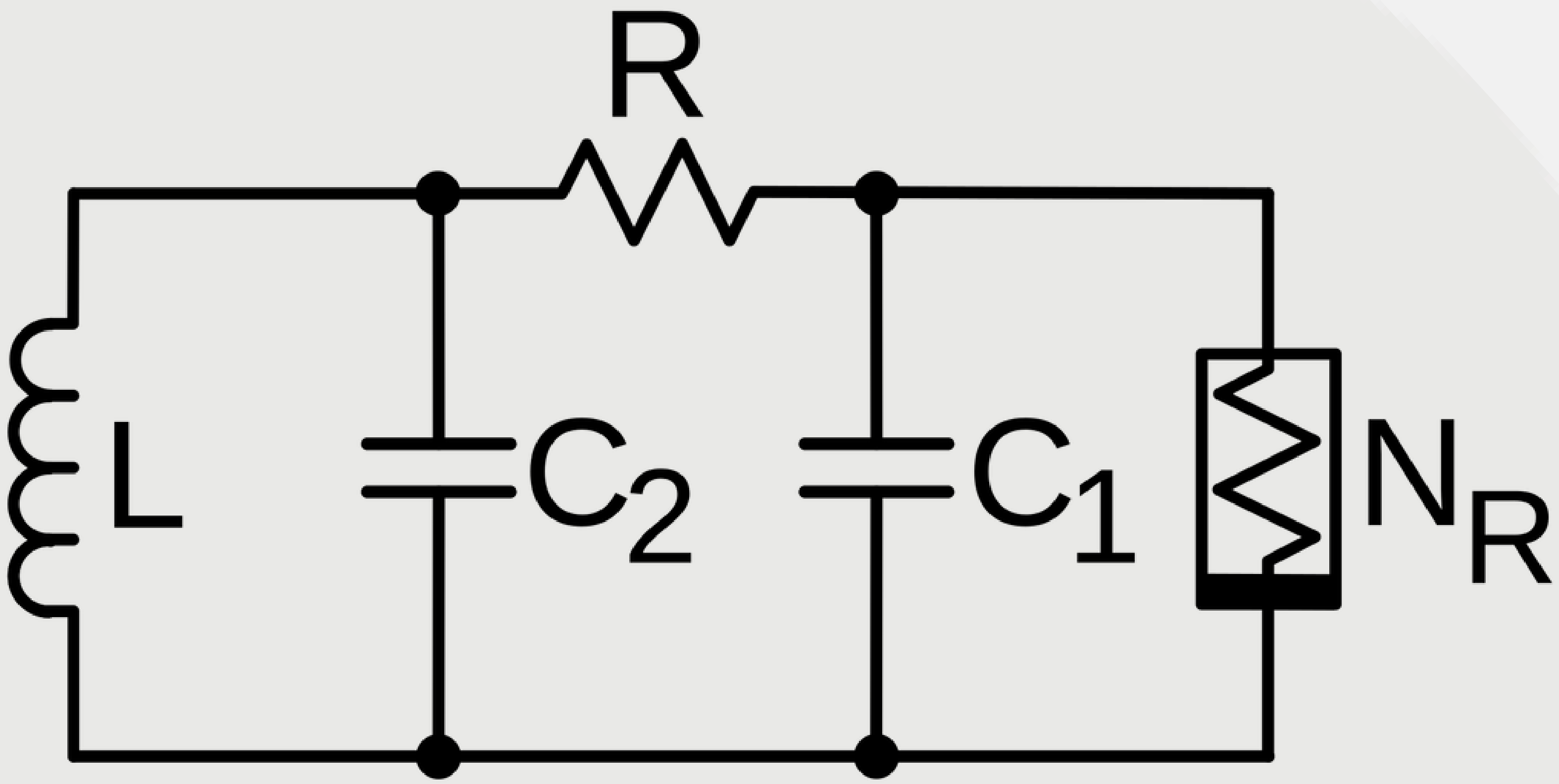
Overview

CHUA'S CIRCUIT ANALYSIS

Chua's Circuit

Chua's Circuit is a simple electronic circuit that acts as a nonperiodic oscillator exhibiting classic chaotic behavior.

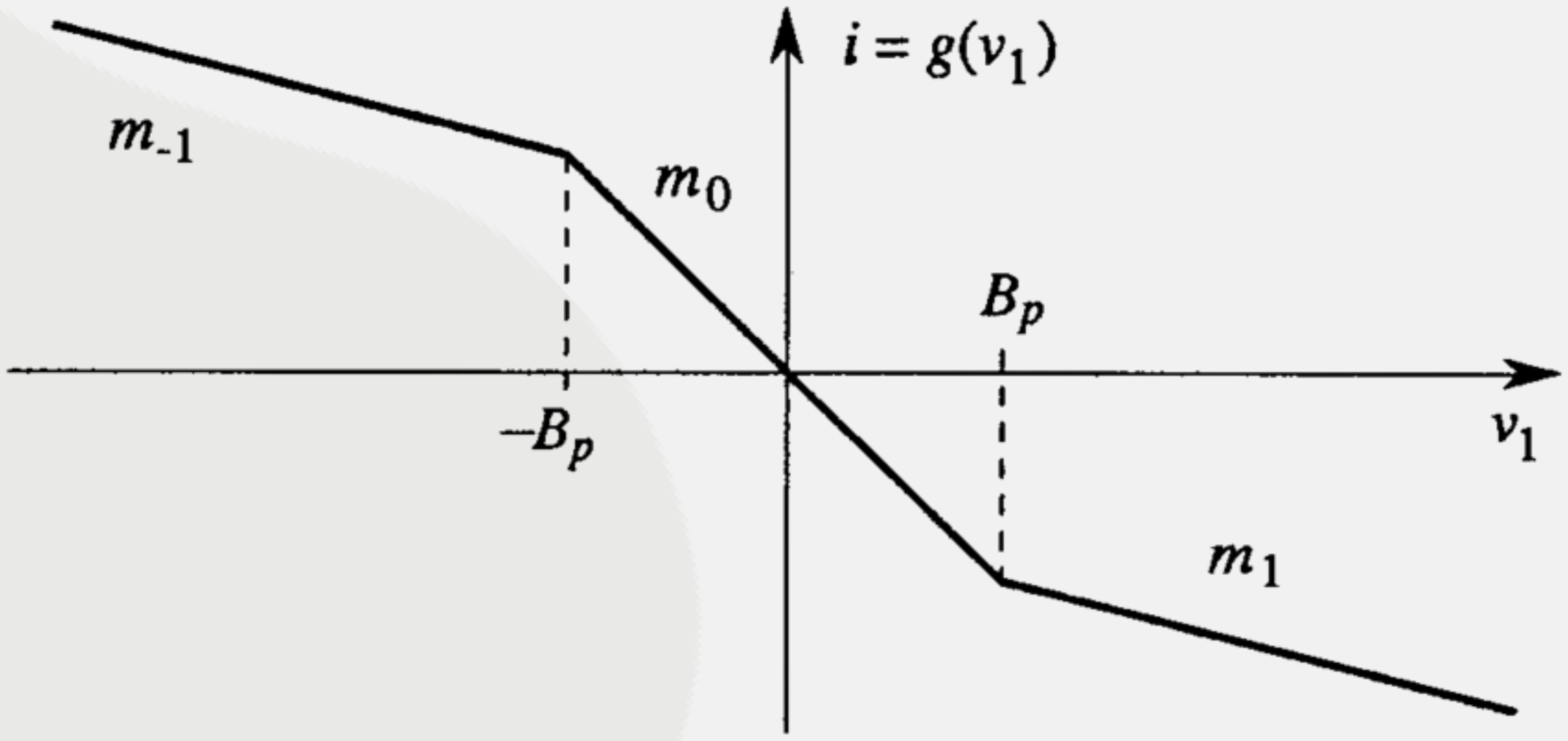


$$C_1 \frac{dv_1}{dt} = \frac{1}{R}(v_2 - v_1) - g(v_1)$$

$$C_2 \frac{dv_2}{dt} = \frac{1}{R}(v_1 - v_2) + i_L$$

$$L \frac{di_L}{dt} = -v_2$$

Chua’s Circuit



$$g(v_1) = \begin{cases} m_1 v_1 + (m_0 - m_1)E, & \text{if } v_1 \leq -E \\ m_0 v_1, & \text{if } -E < v_1 < E \\ m_1 v_1 + (m_0 - m_1)(-E), & \text{if } v_1 \geq E \end{cases}$$

Analogy

- **An electrical circuit equation:**

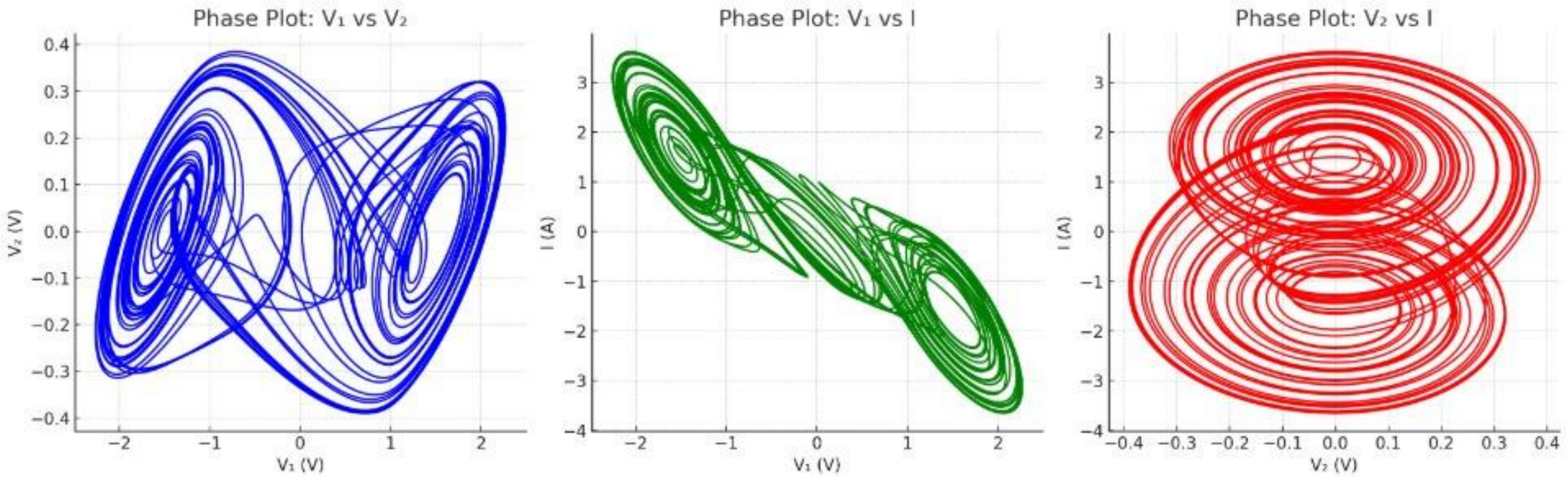
$$LC\ddot{v} + RC\dot{v} + v + g(v) = 0$$

- **A mechanical oscillator equation:**

$$m\ddot{x} + r\dot{x} + kx + f(x) = 0$$

Electrical System	Mechanical System
Voltage v	Displacement x
Current derivative $\dot{v}$	Velocity $\dot{x}$
$LC\ddot{v}$	$m\ddot{x}$
$RC\dot{v}$	$r\dot{x}$
v	kx
$g(v)$	$f(x)$

Phase Space diagrams



Fixed Points

NORMALIZED
EQUATIONS

$$\dot{x} = \alpha(y - x - g(x))$$
$$\dot{y} = x - y + z$$
$$\dot{z} = -\beta y$$

$$g(x) = \begin{cases} m_1 x + (m_0 - m_1), & x > 1 \\ m_0 x, & |x| \leq 1 \\ m_1 x - (m_0 - m_1), & x < -1 \end{cases}$$

SOLUTIO
N

$$x + g(x) = 0$$
$$y = 0$$
$$z = -x$$

Fixed Points

$$g(x) = \begin{cases} m_1 x + (m_0 - m_1), & x > 1 \\ m_0 x, & |x| \leq 1 \\ m_1 x - (m_0 - m_1), & x < -1 \end{cases}$$

Region 1: $|x| \leq 1 \Rightarrow g(x) = m_0 x$

$$x + m_0 x = 0 \Rightarrow x(1 + m_0) = 0 \Rightarrow x = 0$$

Region 2: $x > 1 \Rightarrow g(x) = m_1 x + (m_0 - m_1)$

$$x + m_1 x + (m_0 - m_1) = 0 \Rightarrow x(1 + m_1) + (m_0 - m_1) = 0 \Rightarrow x = \frac{-(m_0 - m_1)}{1 + m_1}$$

Region 3: $x < -1 \Rightarrow g(x) = m_1 x - (m_0 - m_1)$

$$x + m_1 x - (m_0 - m_1) = 0 \Rightarrow x(1 + m_1) = (m_0 - m_1) \Rightarrow x = \frac{m_0 - m_1}{1 + m_1}$$

Bifurcation

$$\frac{-(m_0 - m_1)}{1 + m_1} > 1$$

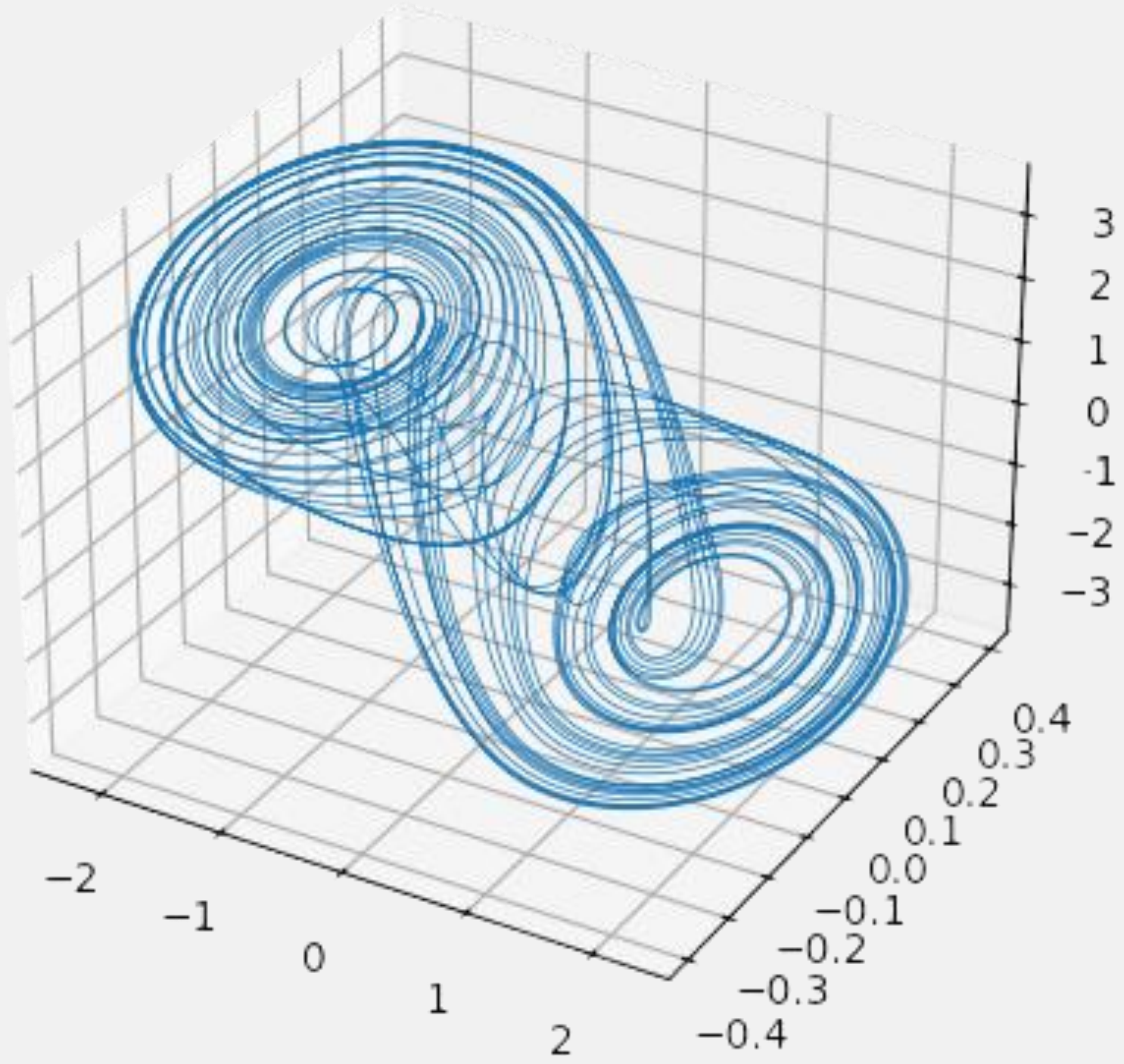
$$-(m_0 - m_1) > 1 + m_1 \Rightarrow -m_0 + m_1 > 1 + m_1 \Rightarrow -m_0 > 1 \Rightarrow m_0 < -1$$

This happens only when:

$$\boxed{m_1 < -1}$$

This leads to Pitchfork Bifurcation

Chua Attractor



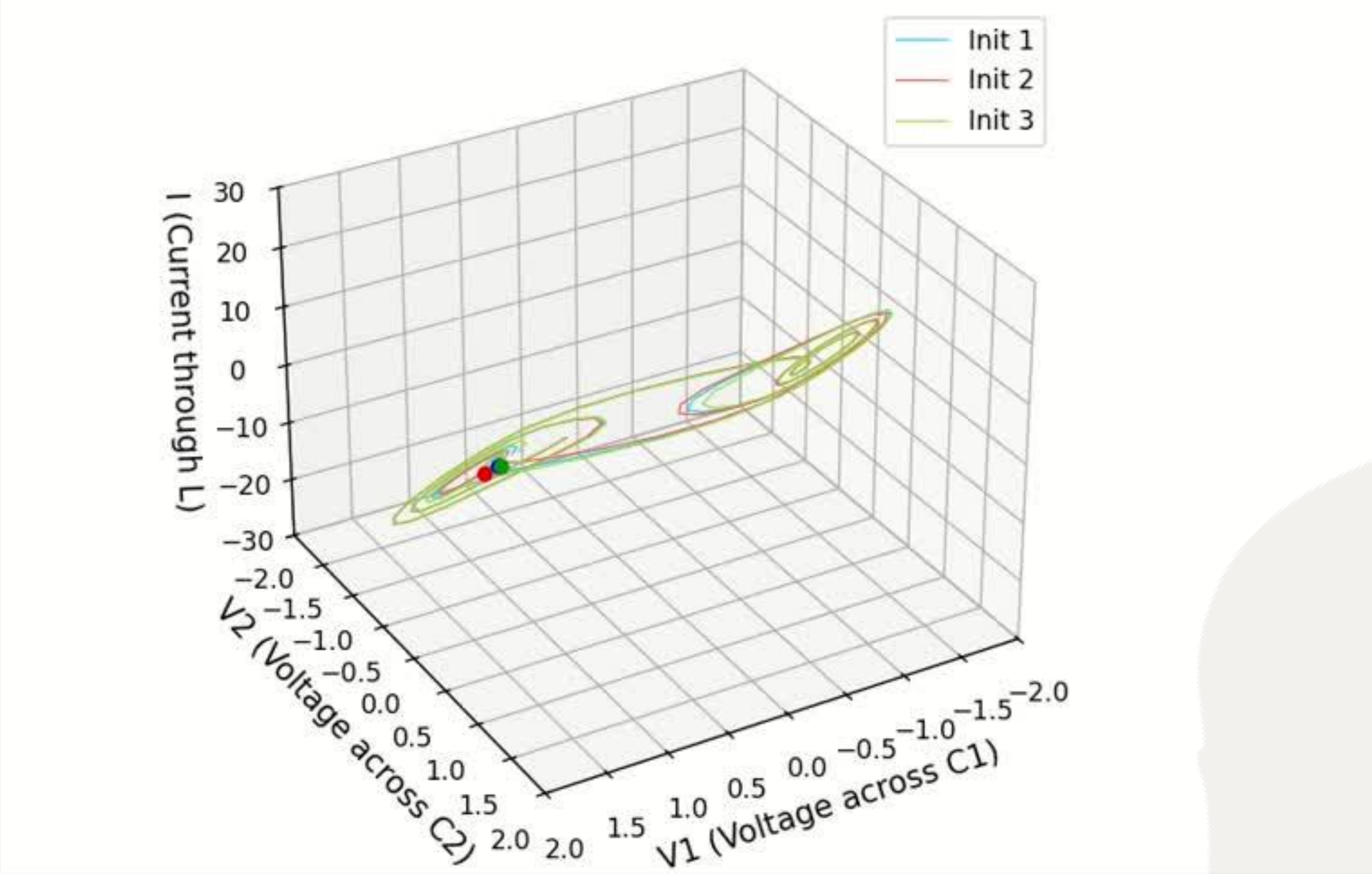
- The Chua attractor is a strange attractor representing the chaotic, bounded trajectory of Chua's circuit in phase space

INITIAL CONDITIONS:

- [v1, v2, i]=[0.7, 0, 0]
- No.of steps=10000

DETERMINISTIC BUT UNPREDICTABLE

- Behavior is governed by equations but highly sensitive to initial conditions


Init 1
Init 2
Init 3
I (Current through L)
30
20
10
0
−10
−20
−30
V2 (Voltage across C2)
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
V1 (Voltage across C1)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0

THANK YOU