# Waveform Coding Techniques

3

**Syllabus :**

PCM, Uniform quantization, Baseband modulation, Noise considerations in PCM systems, DPCM, DM, ADM, LPC.

## 3.1 Introduction :

- Using the waveform coding technique we convert the analog PAM signal into a digital signal.
- This digital signal is in the form of a train or stream of binary digits 0 and 1.
- Thus with waveform coding techniques we enter into the world of digital communication.
- After sampling an analog signal, the next step in its digital transmission is the generation of the "coded version" (digital representation) of the signal.
- Pulse Code Modulation (PCM) provides one method to meet such a requirement.
- In PCM, the message signal is sampled and amplitude of each sample is approximated (rounded off) to the nearest one of a finite set of discrete levels.
- This will enable us to represent both time and amplitude in discrete form.
- Hence it is possible to transmit the message signal by means of a digital (coded) waveform.
- Conceptually PCM is simple to understand. It was the first method which was developed for the digital coding of the waveforms.
- PCM is the most applied of all the digital coding systems in use today.
- PCM is therefore widely accepted as the standard against which the other digital coder systems are calibrated.

### 3.1.1 Advantages of Digital Representation of a Signal :

The digital representation of a signal has following advantages :

- Immunity to transmission noise and interference.
- Regeneration of the coded signal along the transmission path is possible.
- Communication can be kept "private" and "secured" through the use of encryption.
- The possibility of uniform format for different kinds of baseband signals.
- It is possible to store the signal and process it further.

### 3.1.2 Disadvantages :

The advantages listed above are attained at the cost of following factors :

- Increased transmission bandwidth.
- Increased system complexity.

- PCM belongs to a class of signal coders known as **waveform coders**.
- This name is used because in PCM, an analog signal is approximated by mimicking the amplitude - versus - time waveform.

**Difference between waveform coding and source coding :**

- The waveform coders are in principle designed to be signal independent.
- The waveform coders are different from the source coders (e.g. linear predictive coders). The source coders depend on parameterization of the analog signal in accordance with an appropriate model for the generation of the signal.

## 3.2 Pulse Code Modulation (PCM) :

- PCM is a type of pulse modulation like PAM, PWM or PPM but there is an important difference between them. PAM, PWM or PPM are "analog" pulse modulation systems whereas PCM is a "digital" pulse modulation system.
- That means the PCM output is in the coded digital form. It is in the form of digital pulses of constant amplitude, width and position.
- The information is transmitted in the form of "code words". A PCM system consists of a PCM encoder (transmitter) and a PCM decoder (receiver).
- The essential operations in the PCM transmitter are sampling, quantizing and encoding.
- All these operations are usually performed in the same circuit called as analog-to-digital converter.
- It should be understood that the PCM is not modulation in the conventional sense.
- Because in modulation, one of the characteristics of the carrier is varied in proportion with the amplitude of the modulating signal. Nothing of that sort happen in PCM.

### 3.2.1 PCM Transmitter (Encoder) :

Block diagram of the PCM transmitter is as shown in Fig. 3.2.1.

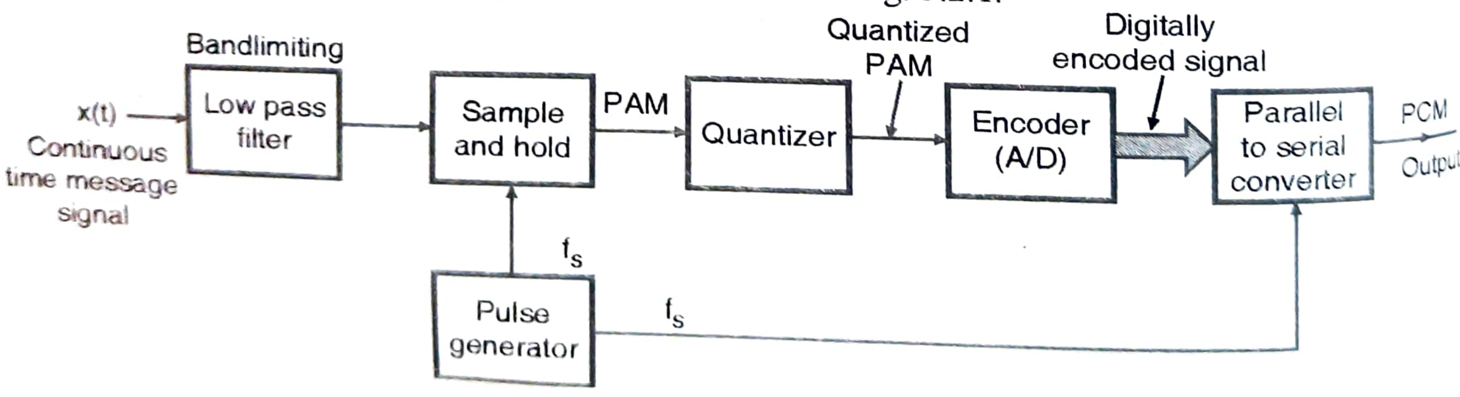



**Fig. 3.2.1 : PCM transmitter (Encoder)**

**Operation of PCM transmitter :**

Operation of the PCM transmitter is as follows :

- The analog signal x (t) is passed through a bandlimiting low pass filter, which has a cut-off frequency $f_c$ = W Hz. This will ensure that x (t) will not have any frequency component higher than "W". This will eliminate the possibility of aliasing.

- The band limited analog signal is then applied to a sample and hold circuit where it is sampled at adequately high sampling rate. Output of sample and hold block is a flat topped PAM signal.
- These samples are then subjected to the operation called "Quantization" in the "Quantizer". Quantization process is the process of approximation as will be explained later on. The quantization is used to reduce the effect of noise. The combined effect of sampling and quantization produces the quantized PAM at the quantizer output.
- The quantized PAM pulses are applied to an encoder which is basically an A to D converter. Each quantized level is converted into an N bit digital word by the A to D converter. The value of N can be 8, 16, 32, 64 etc.
- The encoder output is converted into a stream of pulses by the parallel to serial converter block. Thus at the PCM transmitter output we get a train of digital pulses.
- A pulse generator produces a train of rectangular pulses with each pulse of duration "$\tau$" seconds. The frequency of this signal is "$f_s$" Hz. This signal acts as a sampling signal for the sample and hold block. The same signal acts as "clock" signal for the parallel to serial converter. The frequency "$f_s$" is adjusted to satisfy the Nyquist criteria.

**Waveforms :**

The waveforms at various points in the PCM transmitter are as shown in Fig. 3.2.2.

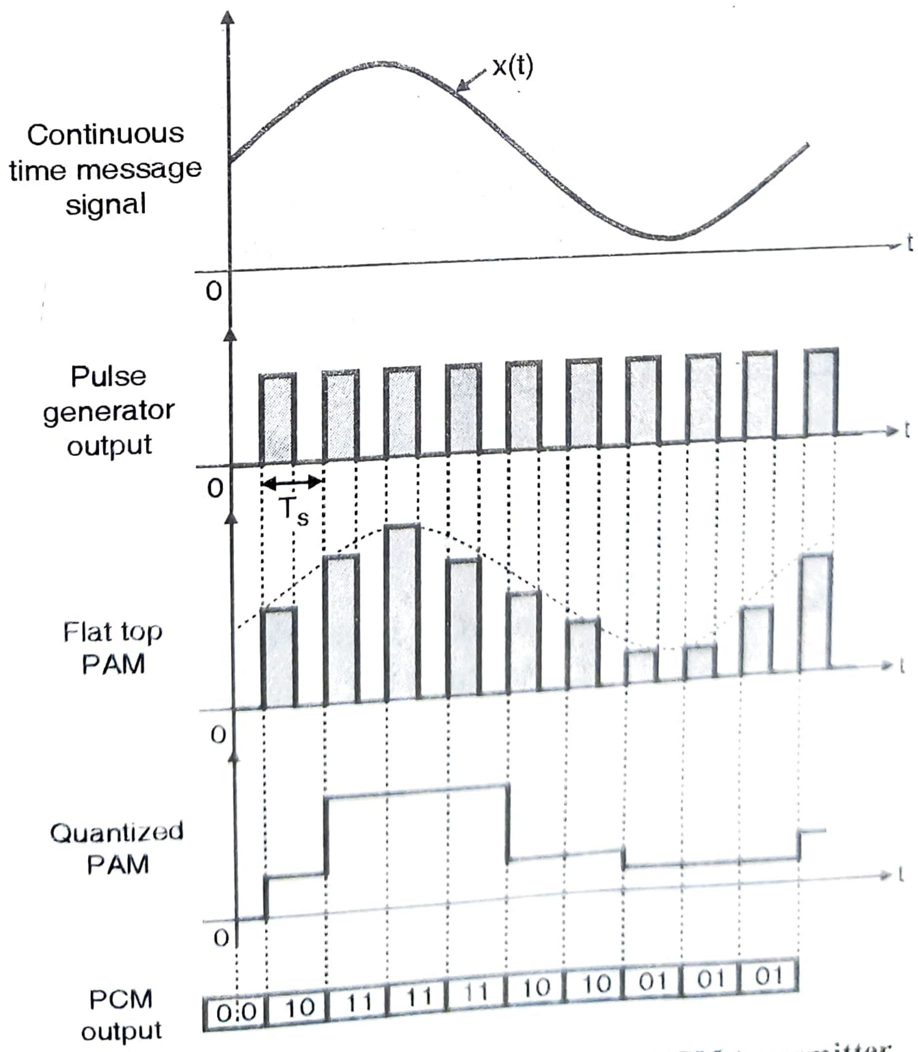



**Fig. 3.2.2 : Waveforms at different points in PCM transmitter**

### 3.2.2 Shape of the PCM Signal :

Fig. 3.2.3 shows input to and output of a PCM system. It is important to understand that the output is in the form of binary codes. Each transmitted binary code represents a particular amplitude of the input signal. Hence the "information" is contained in the "code" which is being transmitted.

- The range of input signal magnitudes is divided into 8-equal levels. Each level is denoted by a three bit digital word between 000 and 111.
- Input signal x (t) is sampled. If the sample is in the 5th - window of amplitude then a digital word 101 is transmitted. If the sample is in the 2nd - window then the transmitted word is 010 and so on.
- In this example we have converted the amplitudes into 3 bit codes, but in practice the number of bits per word can be as high as 8, 9 or 10.

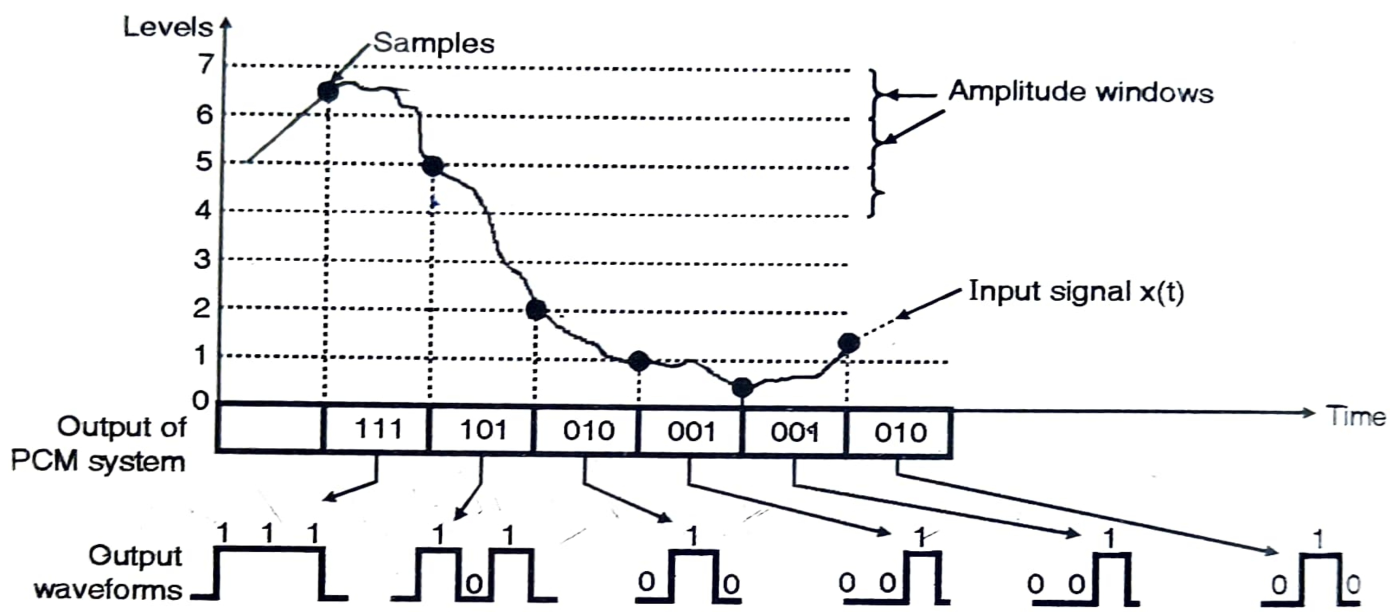



**Fig. 3.2.3 : Input and output waveforms of a PCM system**

### 3.2.3 PCM Transmission Path :

- The path between the PCM transmitter and PCM receiver overwhich the PCM signal travels is called as PCM transmission path and it is as shown in Fig. 3.2.4.

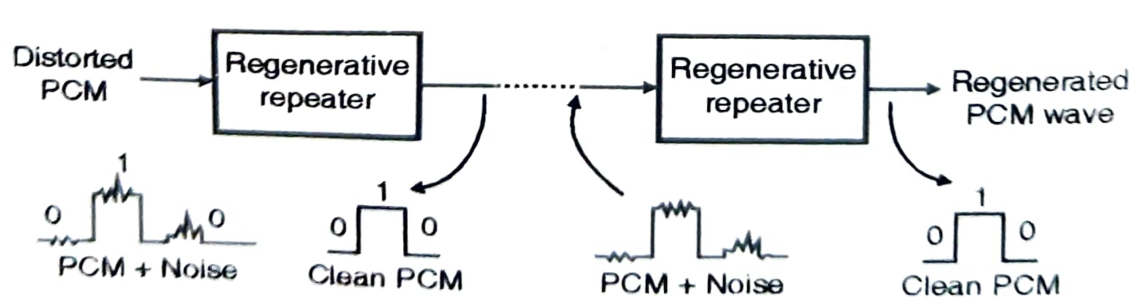



**Fig. 3.2.4 : PCM transmission path**

- The most important feature of PCM system lies in its ability to control the effects of distortion and noise when the PCM wave travels on the channel.
- PCM accomplishes this capacity by means of using a chain of regenerative repeaters as shown in Fig. 3.2.4.
- Such repeaters are spaced close enough to each other on the transmission path.
- The regenerative repeater performs three basic operations namely equalization, timing and decision making.
- So each repeater actually reproduces the clean noise free PCM signal from the PCM signal distorted by the channel noise. This improves the performance of PCM in presence of noise.

**Block diagram of a repeater :**

- The block diagram of a regenerative repeater is shown in Fig. 3.2.5.
- The amplitude equalizer shapes the distorted PCM wave so as to compensate for the effects of amplitude and phase distortions.
- The timing circuit produces a periodic pulse train that is derived from the input PCM pulses. This pulse train is then applied to the decision making device.

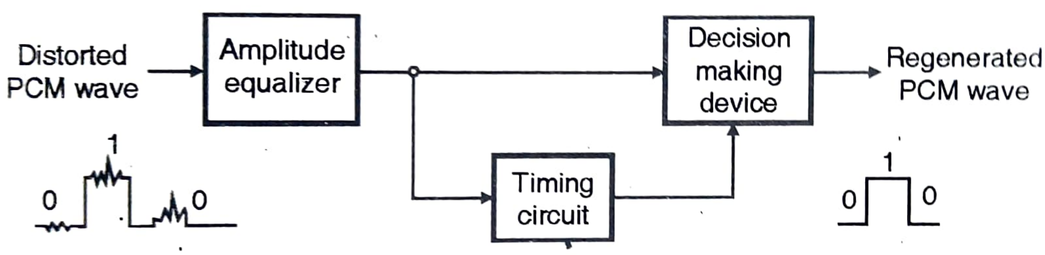



**Fig. 3.2.5 : Block diagram of a regenerative repeater**

- The decision making device uses this pulse train for sampling the equalized PCM pulses. The sampling is carried out at the instants where the signal to noise ratio is maximum.
- The decision device makes a decision about whether the equalized PCM wave at its input has a 0 value or 1 value at the instant of sampling.
- Such a decision is made by comparing equalized PCM with a reference level called decision threshold as shown in Fig. 3.2.5.
- At the output of the decision device we get a clean PCM signal without any trace of noise.

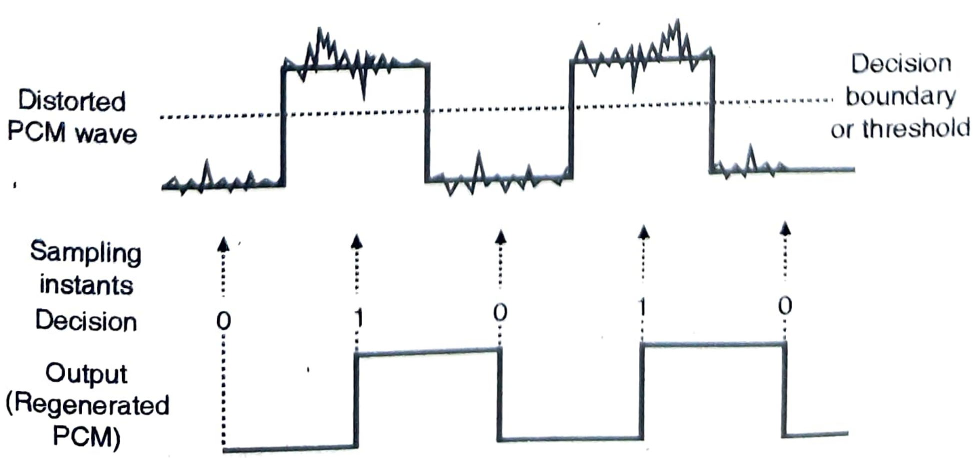



**Fig. 3.2.6 : Waveforms of regenerative repeater**

### 3.2.4 PCM Receiver (Decoder) :

Fig. 3.2.7 shows the block diagram of a PCM receiver.

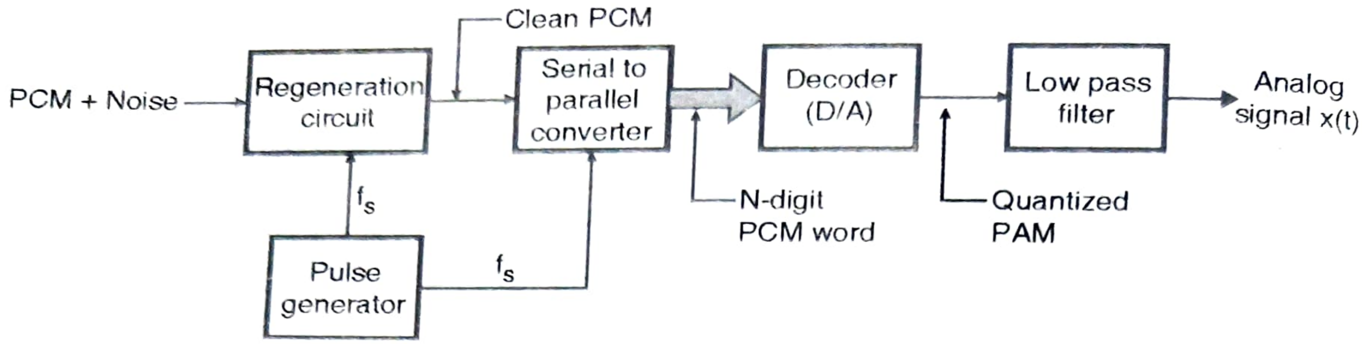



**Fig. 3.2.7 : PCM receiver (Decoder)**

**Operation of PCM receiver :**

- A PCM signal contaminated with noise is available at the receiver input.
- The regeneration circuit at the receiver will separate the PCM pulses from noise and will reconstruct the original PCM signal.
- The pulse generator has to operate in synchronization with that at the transmitter. Thus at the regeneration circuit output we get a "clean" PCM signal.
- The reconstruction of PCM signal is possible due to the digital nature of PCM signal. The reconstructed PCM signal is then passed through a serial to parallel converter.
- Output of this block is then applied to a decoder.
- The decoder is a D to A converter which performs exactly the opposite operation of the encoder.
- The decoder output is the sequence of a quantized multilevel pulses. The quantized PAM signal is thus obtained, at the output of the decoder.
- This quantized PAM signal is passed through a low pass filter to recover the analog signal, x (t).
- The low pass filter is called as the reconstruction filter and its cut off frequency is equal to the message bandwidth W.

### 3.2.5 Quantization Process :

- Quantization is a process of approximation or rounding off. The sampled signal in PCM transmitted is applied to the quantizer block.
- Quantizer converts the sampled signal into an approximate quantized signal which consists of only a finite number of predecided voltage levels.
- Each sampled value at the input of the quantizer is approximated or rounded off to the nearest standard predecided voltage level.
- These standard levels are known as the "quantization levels". Refer to Fig. 3.2.8 to understand the process of quantization.

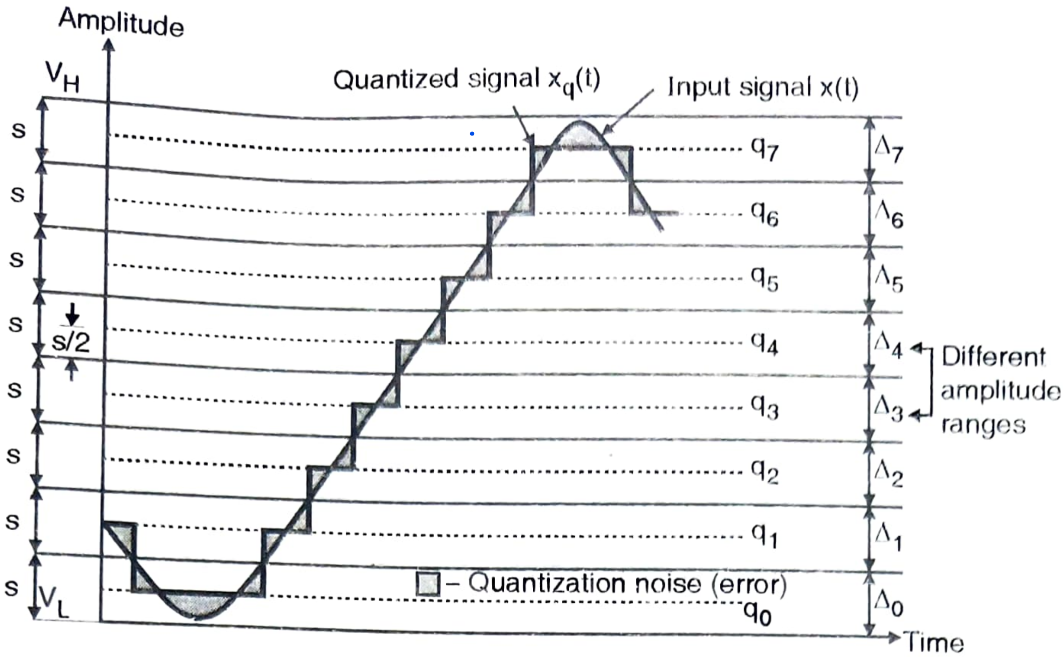



**Fig. 3.2.8 : Process of quantization**

The quantization process takes place as follows :

- The input signal x (t) is assumed to have a peak to peak swing of $V_L$ to $V_H$ volts. This entire voltage range has been divided into "Q" equal intervals each of size "s".
- "s" is called as the step size and its value is given as,

$$s = \frac{V_H - V_L}{Q} \quad ...(3.2.1)$$

In Fig. 3.2.8, the value of Q = 8

- At the center of these ranges, the quantization levels $q_0$, $q_1$,... $q_7$ are placed. Thus the number of quantization levels is Q = 8. These are also called as decision thresholds.
- $x_q$ (t) represents the quantized version of x (t). We obtain $x_q$ (t) at the output of the quantizer.
- When x (t) is in the range $\Delta_0$, then corresponding to any value of x (t), the quantizer output will be equal to "$q_0$".
- Similarly for all the values of x (t) in the range $\Delta_1$, the quantizer output is constant equal to "$q_1$".
- Thus in each range from $\Delta_0$ to $\Delta_7$, the signal x (t) is rounded off to the nearest quantization level and the quantized signal is produced.
- The quantized signal $x_q$ (t) is thus an approximation of x (t). The difference between them is called as **quantization error or quantization noise.**
- This error should be as small as possible.
- To minimize the quantization error we need to reduce the step size "s" by increasing the number of quantization levels Q.

### Why is quantization required ?

- If we do not use the quantizer block in the PCM transmitter, then we will have to convert each and every sampled value into a unique digital word.
- This will need a large number of bits per word (N). This will increase the bit rate and hence the bandwidth requirement of the channel.
- To avoid this, if we use a quantizer with only 256 quantization levels then all the sampled values will be finally approximated into only 256 distinct voltage levels.
- So we need only 8 bits per word to represent each quantized sampled value.
- Thus the number of bits per word can be reduced. This will eventually reduce the bit rate and bandwidth requirement.

### Quantization error or quantization noise $\in$ :

- The difference between the instantaneous values of the quantized signal and input signal is called as quantization error or quantization noise.

$$\in \; = \; x_q(t) - x(t) \qquad \text{...(3.2.2)}$$

- The quantization error is shown by shaded portions of the waveform in Fig. 3.2.8
- The maximum value of quantization error is ± s/2 where s is step size.
- Therefore to reduce the quantization error we have to reduce the step size by increasing the number of quantization levels i.e. Q. (Refer to Figs. 3.3.2 and 3.3.3 for maximum quantization error).
- The mean square value of the quantization is given by,

  $$\text{Mean square value of quantization error} = \boxed{\frac{s^2}{12}} \qquad \text{...(3.2.3)}$$

  quantization noise power

  The derivation for this expression is given later in this chapter.
- The relation between the number of quantization levels Q and the number of bits per word (N) in the transmitted signal can be found as follows :
- Because each quantized level is to be converted into a unique N bit digital word, assuming a binary coded output signal,

  The number of quantization levels Q = Number of combinations of bits/word.

  $$\therefore \quad Q = 2^N \qquad \text{...(3.2.4)}$$
- Thus if N = 4 i.e. 4 bits per word then the number of quantization levels will be $2^4$ i.e. 16.

### Signal to quantization noise ratio ($SNR_q$) :

- This ratio is the figure of merit for the PCM systems. The signal to quantization noise ratio with a sinusoidal input signal to the PCM system is expressed as,

$$\frac{S_i}{N_q} = [1.8 + 6N]\ \text{dB} \quad \textbf{For a sinusoidal signal} \qquad \text{...(3.2.5)}$$

- This equation shows that the signal to quantization noise ratio is solely dependent on the number of bits per word i.e. N.

- This ratio should be as high as possible, which can be achieved by increasing N. But this increases the bit rate and hence bandwidth of the PCM system.
- Therefore the number of bits per word is a compromise between high $SNR_q$ and bandwidth requirements.
- The derivation of Equation (3.2.5) has been given later on in this chapter.

## 3.3 Types of Quantization :

- Fig. 3.3.1 shows the classification of quantization process

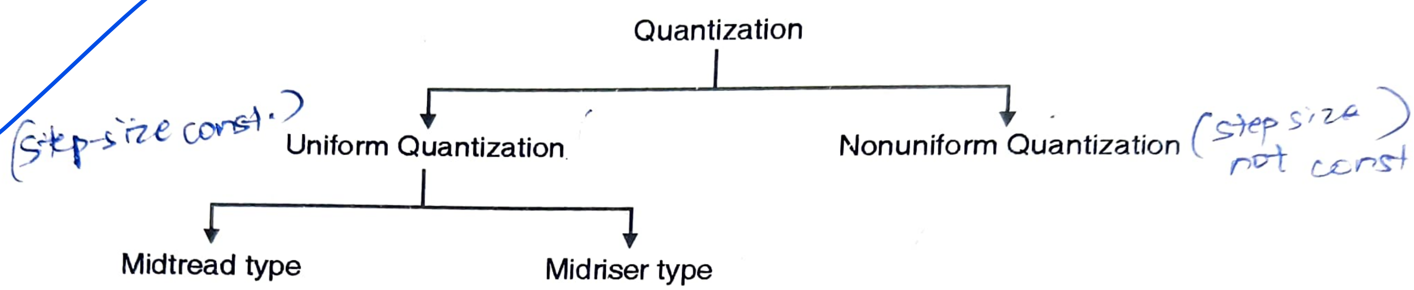



**Fig. 3.3.1 : Classification of quantization**

- The quantization process can be classified into two types as :
  1. Uniform quantization
  2. Non-uniform quantization.

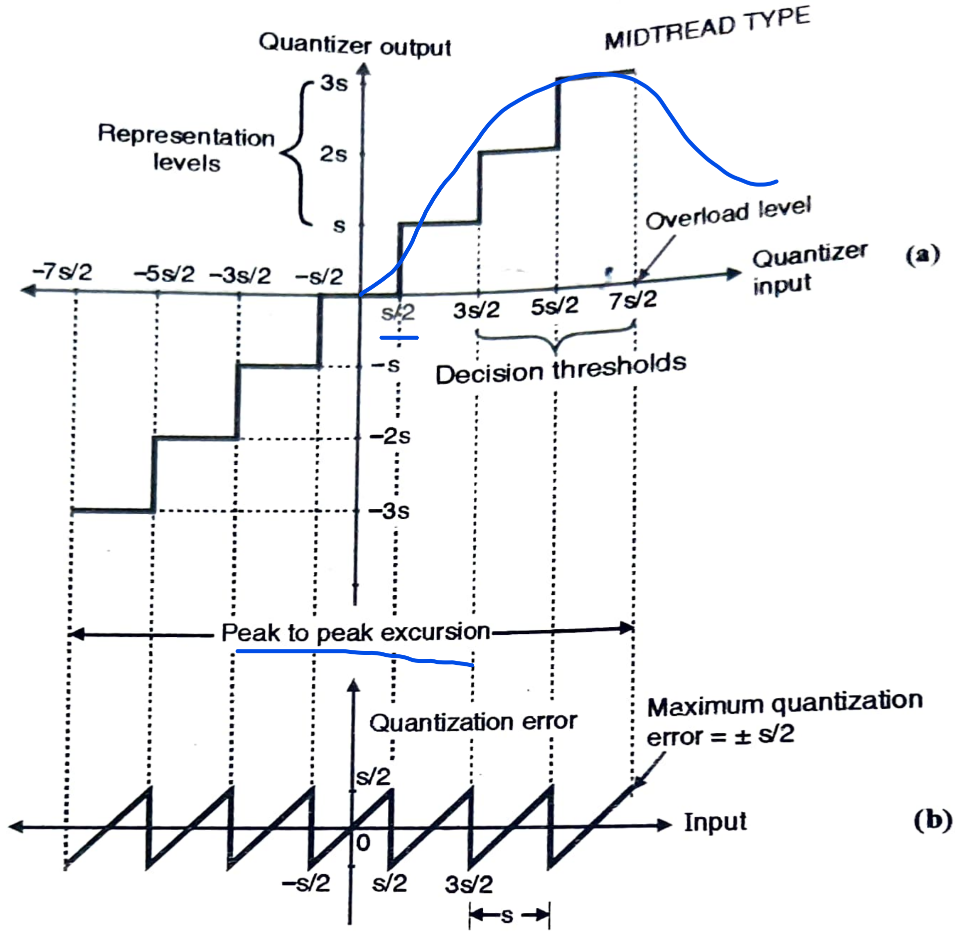
  This classification is based on the step size s defined earlier.

### 3.3.1 Uniform Quantizer :

- A quantizer is called as a uniform quantizer if the step size remains constant throughout the input range.
- However if the step size varies depending on the input then the quantizer is known as the non-uniform quantizer.

**Types of uniform quantizer :**

- There are two types of uniform quantizers :
  1. Symmetric quantizer of the midtread type
  2. Symmetric quantizer of the midriser type.




**(a) Transfer characteristic of quantizer of midtread type**

**(b) Variation of quantization error with input**

**Fig. 3.3.2**

- The transfer characteristics of the "midtread" type uniform quantizer is shown in Fig. 3.3.2(a) and the corresponding variation in the quantization error with input has been shown in Fig. 3.3.2(b).

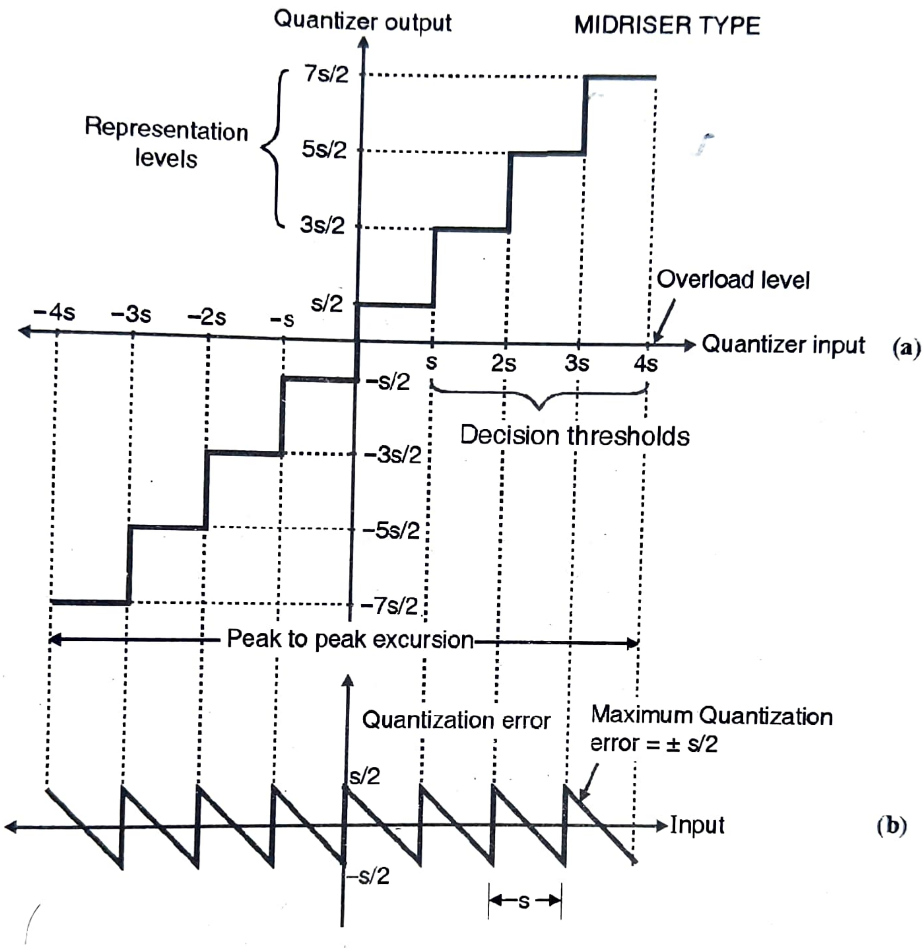
### 3.3.2 Symmetric Quantizer of Midtread Type :

- Graphically the quantizing process means that a straight line representing the relation between the input and output of a linear analog system is replaced by a transfer characteristics of staircase type.
- The quantization process has a two fold effect as follows :
  1. The peak to peak range of the input is divided into a finite set of decision levels or decision thresholds. These levels have been aligned with the "risers" of the staircase of Fig. 3.3.3(a).
  2. The output is assigned a discrete value selected from a finite set of representation levels or reconstruction value. These levels are aligned with the "treads" of the staircase in Fig. 3.3.3(a).
- The separation between the decision thresholds and the separation between the representation levels have the same value equal to step size "s".

- In Fig. 3.3.3(a), the decision thresholds are located at ± s/2, ± 3s/2, ± 5s/2 .... and the representation levels are located at 0, ± s, ± 2s ... where "s" is the step size.

### 3.3.3 Symmetric Quantizer of Midriser Type :

- The transfer characteristic of the "midriser" type uniform quantizer is as shown in Fig. 3.3.3(a) and the corresponding variation in the quantization error with input has been shown in Fig. 3.3.3(b).




**(a) Transfer characteristics of a quantizer of midriser type**

**(b) Variation of the quantization error with input**

**Fig. 3.3.3**

- This is another staircase type transfer characteristics. The decision thresholds are located at 0, ± s, ± 2s etc. and the representation levels are located at ± s/2, ± 3s/2 ...
- This is called as the midriser type characteristics because in this case the origin lies in the middle of a riser of the staircase.

**The important observations from Figs. 3.3.2(a) and 3.3.2(b) are as follows :**

1. The quantizer of Fig. 3.3.2(a) is called as midtread type because the origin lies in the middle of a tread of the staircase. Similarly the quantizer of Fig. 3.3.3(a) is called as midriser type because the origin lies in the middle of the riser of the staircase.

2. Both these quantizers are "memoryless quantizers". This is because, the output is determined only by the value of a corresponding input sample independent of the previous or next analog sample applied at the input.
3. Overload level shown in Figs. 3.3.2(a) and Fig. 3.3.3(a), has an absolute value which is one half of the peak to peak range of the input sample values.
4. Figs. 3.3.2(b) and Fig. 3.3.3(b) shows the variation of quantization error with input. For both the types we observe that the maximum instantaneous value of this error is limited to half of one step size and the total range of variation is from – s/2 to + s/2. This instantaneous variation in quantization error is plotted by subtracting the input from the output of the quantizer.

### 3.3.4 Encoding in PCM :

- We have seen that encoding is the process that follows the sampling and quantizing.
- Encoding process converts the quantized samples into a code words.
- In a binary code each symbol may have either a 0 value or a 1 value.
- There are various formats (waveforms) for representing the binary sequence. They are called as line codes.
- Fig. 3.3.4 shows two of such formats called unipolar NRZ and polar NRZ, where NRZ is stands for non-return to zero.

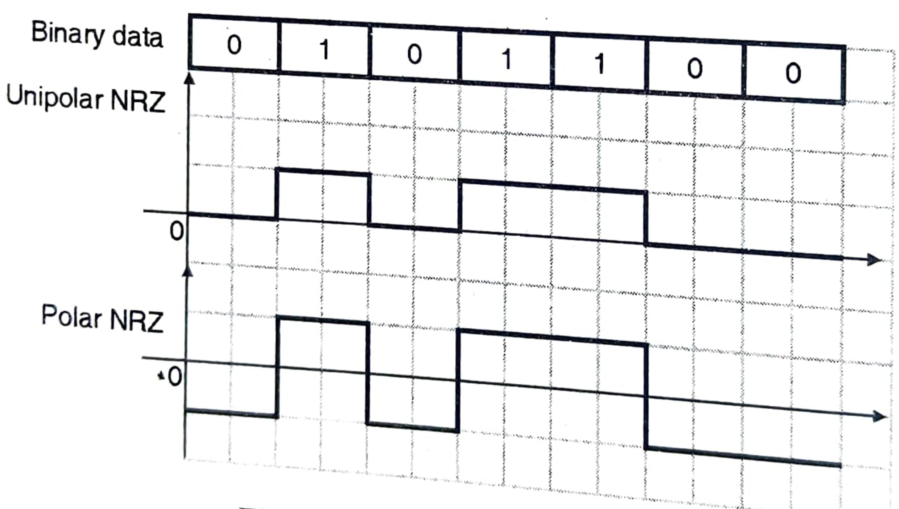



**Fig. 3.3.4 : Two binary formats**

### 3.3.5 Multiplexing in PCM Systems :

- It is possible to multiplex the PCM signals using the time division multiplexing principle.
- With increase in number of independent message sources the time interval allotted to each source has to be reduced to accommodate all the sources.
- This reduces the duration of each binary bit in a PCM code word. This increases the bandwidth requirement of the system.
- If the pulses become too short then the impairments in the transmission medium begin to interfere with the proper operation of the system.
- Therefore in practice it is necessary to restrict the number of message sources.

### 3.3.6 Synchronization in PCM :

- For a PCM system with TDM, it is necessary to synchronize the transmitter and receiver for proper operation of the system.
- For synchronization, it is necessary to synchronize the clocks at the transmitter and receivers.
- One way of synchronizing the transmitter and receiver clocks is to send a code element (one bit) or pulse at the end of each frame.
- The receiver will have a circuit which searches for this code element and thereby establish the synchronization.

### 3.3.7 Quantizer Saturation :

- The quantizer allocates Q levels of approximations. The range of input for which the difference between input and output is small is known as **operating range**.
- If the input exceeds this range, the difference between the quantized output and input becomes large and it is said that **quantizer saturation** has taken place.
- Objectionable errors are introduced due to quantizer saturation. We can avoid quantizer saturation by using the automatic gain control (AGC). This effectively extends the operating range of the converter.

## 3.4 Derivation of Expression for the Quantization Error : [redacted]

- The input signal x(t) varies between the voltage levels $V_H$ and $V_L$. Therefore the total variation in amplitude is given by,

$$\text{Total variation in amplitude} = V_H - V_L \qquad \text{...(3.4.1)}$$

- If we assume that $V_H = V$ and $V_L = -V$ then

$$\text{Total change in signal amplitude} = 2V \text{ Volts} \qquad \text{...(3.4.2)}$$

- If this range is divided into "Q" levels of quantization then the step size is given by,

$$\boxed{s = \frac{V_H - V_L}{Q} = \frac{2V}{Q}} \qquad \text{...(3.4.3)}$$

If we assume that $V_H = +1\text{Volt}$ and $V_L = -1\text{Volt}$ then

$$s = 2/Q \qquad \text{...(3.4.4)}$$

- If the step size is assumed to be sufficiently small then the quantization error can be assumed to have distributed uniformly and we can say that the quantization error is a random variable with "uniform distribution".
- As already seen the maximum quantization error is "$\pm s/2$". Therefore we can say that over the range $+s/2$ to $-s/2$, quantization error is a uniformly distributed random variable.
- The uniform distribution of quantization error is as shown in Fig. 3.4.1. The probability density function (PDF) for the quantization error "$\epsilon$" is defined as,

$$\begin{aligned} f_\epsilon(\epsilon) &= 0 && \text{for } \epsilon \le s/2 \\ &= 1/s && \text{for } -s/2 \le \epsilon \le s/2 \\ &= 0 && \text{for } \epsilon > s/2 \end{aligned} \qquad \text{... (3.4.5)}$$

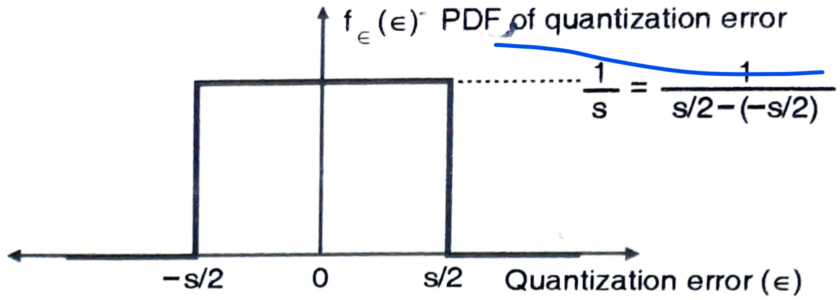



**Fig. 3.4.1 : Uniform distribution for quantization error**

- The mean value or average value of the quantization error is zero. The noise power is given by,

$$\text{Noise power} = \frac{V_n^2}{R} \qquad ...(3.4.6)$$

where $V_n^2$ = Mean square noise voltage.

- We have defined the quantization noise as a random variable, with a probability density function (PDF) equal to $f_\epsilon(\epsilon)$, we can find the mean square value of noise voltage as,

$$\text{Mean square value} = E[\epsilon^2] = \epsilon^2 \qquad ...(3.4.7)$$

$$= \int_{-\infty}^{\infty} \epsilon^2 f_\epsilon(\epsilon)\, d\epsilon \qquad ...(3.4.8)$$

- But from Fig. 3.4.1 it is clear that the PDF $f_\epsilon(\epsilon)$ exists only over the range $-s/2$ to $+s/2$. Also substitute $f_\epsilon(\epsilon) = 1/s$ and change the limits of integration to get,

$$\epsilon^2 = \int_{-s/2}^{s/2} \epsilon^2 \times \frac{1}{s}\, d\epsilon \qquad ...(3.4.9)$$

$$= \frac{1}{s}[\epsilon^3/3]_{-s/2}^{s/2} = \frac{1}{s}\left[\frac{s^3}{24} + \frac{s^3}{24}\right]$$

$$= \frac{s^2}{12} \qquad ...(3.4.10)$$

Thus, $V_n^2$ = Mean square value of noise voltage = $\frac{s^2}{12}$

- If we substitute R = 1 ohm in Equation (3.4.6), then the noise power is called "normalized" noise power.

- $\therefore$ Normalized noise power $= N_q = \frac{V_n^2}{1} = \frac{s^2}{12}$

**Normalized quantization noise power :**

$$N_q = \frac{s^2}{12} \text{ : For linear quantization} \qquad ...(3.4.11)$$

This is the required expression.

## 3.5 Expression for the Maximum Signal to Quantization Noise Ratio $[S/N_q]$ :

- The signal to quantization noise ratio can be defined as follows :

$$\left[\frac{P}{N_q}\right] = \frac{\text{Normalized signal power}}{\text{Normalized noise power}} \qquad \ldots(3.5.1)$$

- We have already obtained the value of normalized noise power ($N_q$).

$$N_q = \frac{s^2}{12} \qquad \ldots(3.5.2)$$

- Now we must obtain the expression for the normalized signal power.

Let us assume that the input signal x (t) is a sinusoidal signal with peak to peak voltage of "2V" volts as shown in Fig. 3.5.1.

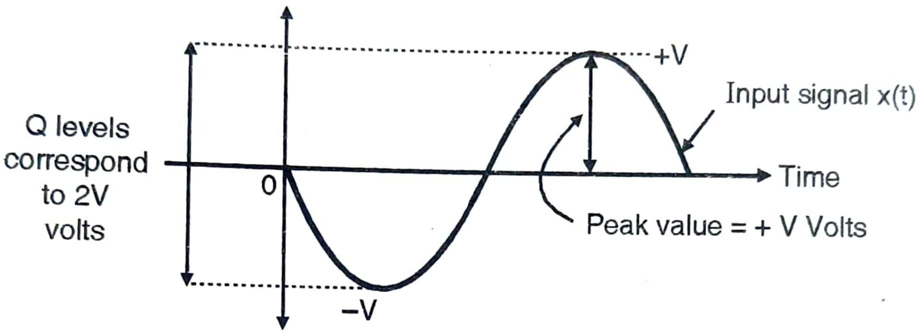



**Fig. 3.5.1 : Signal x(t)**

- The power dissipated in a resistance R by the signal voltage x (t) is given by,

$$P = \frac{(\text{Rms voltage})^2}{R} \qquad \ldots(3.5.3)$$

- The rms voltage of x (t) shown in Fig. 3.5.1 will be,

$$\text{rms voltage of x (t)} = \frac{V}{\sqrt{2}} \qquad \ldots(3.5.4)$$

- Substitute this value in Equation (3.5.3) to get,

$$\text{Signal power P} = \frac{(V/\sqrt{2})^2}{R}$$

$$\therefore \quad P = \frac{V^2}{2\,R} \qquad \ldots(3.5.5)$$

- Normalized signal power can be obtained by substituting R = 1 in Equation (3.5.5).

$$\therefore \quad \textbf{Normalized signal power, } P = \frac{V^2}{2} \qquad \ldots(3.5.6)$$

- Now substitute this value in Equation (3.4.11) along with the expression for normalized noise power "$N_q$" to get,

$$P/N_q = \frac{V^2/2}{s^2/12}$$

$$\therefore \quad [P/N_q] = \frac{6V^2}{s^2} \qquad ...(3.5.7)$$

But $\quad s = \frac{2V}{Q}$ as already proved (Equation (3.4.3)).

$$\therefore \quad \left[\frac{P}{N_q}\right] = \frac{6V^2}{4V^2/Q^2} = \frac{3}{2}Q^2 \qquad ...(3.5.8)$$

However, quantization levels $Q = 2^N$ where N is number of bits per word in PCM.

$$\therefore \quad \frac{P}{N_q} = \frac{3}{2}[2^N]^2 = \frac{3}{2}[2^{2N}] \qquad ...(3.5.9)$$

Thus maximum signal to quantization noise ratio : $\frac{P}{N_q} = \frac{3}{2} \cdot 2^{2N}$ ...(3.5.10)

$\therefore$ Signal to quantization noise ratio in dB is given as,

$$SNR_q \, dB = \left[\frac{P}{N_q}\right]_{dB} = 10 \log_{10}\left[\frac{3}{2} \times 2^{2N}\right]$$

$$= 10 \log_{10}(3/2) + 10 \log_{10} 2^{2N} = 1.76 + 6N$$

$\therefore$ $SNR_q \, dB_{(max)} \approx (1.8 + 6N)$ dB : For sinusoidal input signal ...(3.5.11)

This is the expression for the maximum signal to quantization noise ratio in dB when the signal is sinusoidal having a peak to peak voltage "2V" volts.

**Conclusion**

Equation (3.5.11) indicates that the signal to quantization noise ratio $SNR_q$ increases by 6 dB for every 1 bit increase in the number of bits in a PCM codeword.

## 3.6 Signaling Rate ( Data Transfer Rate ) and Transmission Bandwidth of PCM :

- We know that, $\quad Q = 2^N$ ...(3.6.1)

  where, Q = Number of quantization levels

  N = Number of bits per word

- The input signal x (t) is sampled at the sampling rate $f_s$, i.e. there are $f_s$ number of samples per second. Each of these samples is then converted into an N bit digital word.

  $\therefore$ Number of bits/sec. = Number of samples/sec $\times$ Number of bits/sample

  $= f_s \times N$ ...(3.6.2)

- But signaling rate is nothing but the number of bits per second.

  $\therefore$ Signaling rate of PCM $= N f_s$ ...(3.6.3)

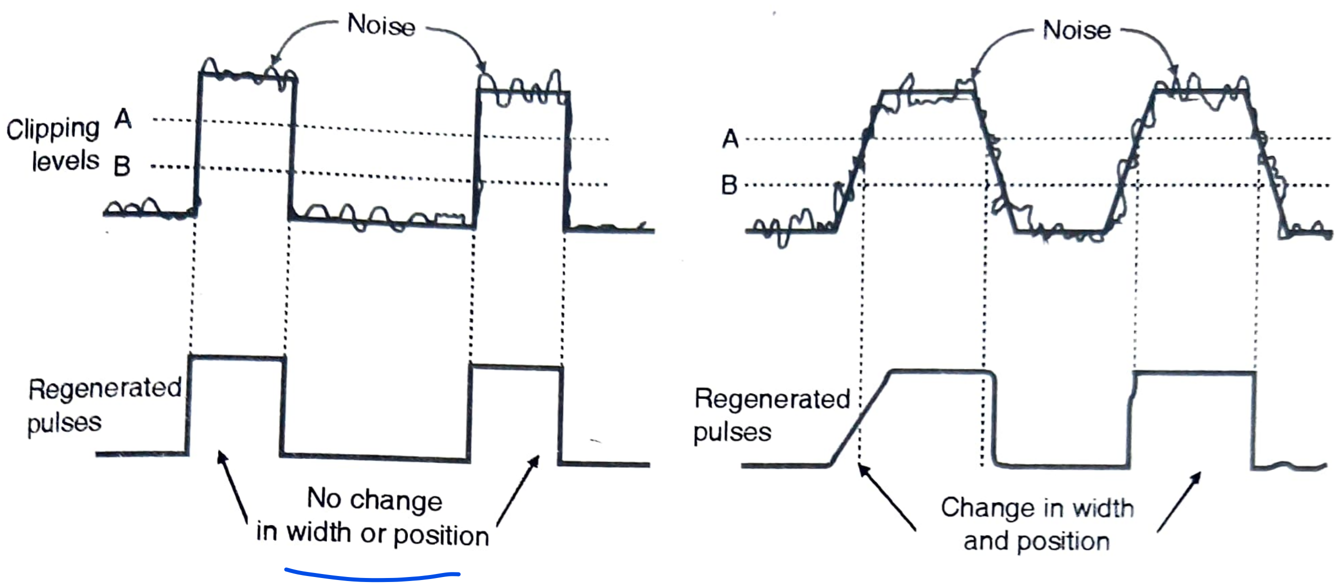
- The transmission bandwidth of PCM is equal to half the signaling rate.

  $\therefore$ Transmission bandwidth of PCM $= \frac{1}{2} N f_s$ ...(3.6.4)

- This expression indicates that with increase in the value of N (number of bits per codeword in PCM) the required bandwidth also increases even though the signal to quantization noise ratio is decreasing.

## 3.7 Effect of Noise on the PCM System :

- Look at the two Figs. 3.7.1(a) and 3.7.1(b) which illustrate the effect of noise on the transmitted pulses.




(a) (b)

**Fig. 3.7.1 : Effect of noise on PCM**

- Consider Fig. 3.7.1(a) first. Due to the noise superimposed on the pulses, only the PAM system will be affected.
- However the PWM, PPM and PCM systems will remain unaffected. The regeneration of the ~~pulses is achieved by~~ using a clipper circuit with reference levels A and B.
- Now consider Fig. 3.7.1(b). Here the sides of the transmitted pulse are not perfectly vertical. In practice the transmitted pulses usually have slightly sloping sides (edges).
- As the noise is superimposed on them, the width and the position of the regenerated pulses is changed.
- Now this is going to distort the information contents in the PWM and PPM signals.
- But PCM is still unaffected as it does not contain any information in the width or the position of the pulses.
- Thus PCM has much better noise immunity as compared to PAM, PWM and PPM systems.

### 3.7.1 Idle Channel Noise :

- The idle channel noise is the coding noise measured at the receiver output with zero transmission input.
- The zero input condition corresponds to situations such as silences in speech.
- The average power corresponding to idle channel noise is dependent on the type of quantizer used.
- In a quantizer of midriser type, the zero input is quantized to $\pm$ s/2. If these two levels are assumed to be equiprobable, then the idle channel noise will have a zero mean and an average power of $s^2/4$.
- But if the quantizer is of midtread type, then the output is zero for zero input and hence the idle channel noise will be zero ideally.

- However for a practical midtread type quantizer, the output is never exactly equal to zero. Accordingly it is found that the average power of idle channel noise in a midtread type quantizer is also approximately equal to but slightly less than $s^2 / 4$.

## 3.8 Robust Quantization :

- In the previous section for uniform quantizer with a step size "s" it was shown that the variance of quantization noise or the normalized quantization noise power $N_q = \epsilon^2 = s^2/12$
- Thus the quantization noise is independent of the size of input signal. It is constant.
- As a result of this the signal to quantization noise ratio $SNR_q$ decreases with decrease in the input signal power level. This is highly objectionable and unacceptable.
- In certain applications where PCM is used for the transmission of speech or music signals, this problem is very serious.
- Because the same quantizer has to accommodate the input signals of varying power levels. This happens because the range of voltages covered by a speech signals from maximum to minimum has a ratio of the order of 1000 : 1.
- Hence the weak speech signals will have a small value of $SNR_q$ and hence the PCM performance will degrade.
- Hence it is desirable that $SNR_q$ should remain essentially constant over a wide range of input power level.
- A quantizer that satisfies all these requirements is called as a **Robust Quantizer**.
- Such a robust performance can be obtained by using a **nonuniform quantization**.

### 3.8.1 Nonuniform Quantization :

[redacted]

- If the quantizer characteristics is nonlinear and the step size is not constant instead if it is variable, dependent on the amplitude of input signal then the quantization is called as non uniform quantization.
- In non-uniform quantization, the step size is reduced with reduction in signal level. For weak signals (P << 1), the step size is small, therefore the quantization noise reduces, to improve the signal to quantization noise ratio for weak signals.
- The step size is thus varied according to the signal level to keep the signal to noise ratio adequately high. This is non-uniform quantization.
- The non-uniform quantization is practically achieved through a process called "companding". We will discuss companding in the next section.

**Need of non-uniform quantization for speech signals :**

- Non-uniform quantization is generally used for the speech and music signals.
- To understand the need of non-uniform quantization for the speech and music signals it is necessary to define an important parameter called "crest factor".
- Crest factor is defined as the ratio of peak amplitude to the rms amplitude of a signal.

$$\therefore \text{Crest factor} = \frac{\text{Peak value}}{\text{rms value}} \qquad \ldots(3.8.1)$$

- The value of crest factor is very high for the speech and music signals. Now let us see the effect of this high crest factor on the normalized power P.

- The destination signal power P is defined as,

$$P = \frac{\text{Mean square value of the signal}}{R}$$

$$\therefore \quad P = \frac{\overline{x^2(t)}}{R} \qquad \text{...(3.8.2)}$$

where, $\overline{x^2(t)}$ = Mean square value of the signal.

- The normalized signal power is obtained by substituting R = 1 in Equation (3.8.2).

$$\therefore \quad \text{Normalized signal power} \quad P = \overline{x^2(t)} \qquad \text{...(3.8.3)}$$

- $$\text{The crest factor} = \frac{\text{Peak value}}{\text{rms value}}$$

$$= \frac{x_{max}}{\left[\overline{x^2(t)}\right]^{1/2}} \qquad \text{...(3.8.4)}$$

- But $\overline{x^2(t)} = P$

$$\therefore \quad CF = \frac{x_{max}}{\sqrt{P}} \qquad \text{...(3.8.5)}$$

- Now if we normalize the signal i.e. if $x_{max} = 1$, then

$$CF = \frac{1}{\sqrt{P}}$$

$$\text{or } P = \frac{1}{CF^2} \qquad \text{...(3.8.6)}$$

- The maximum possible value of the normalized power P is 1. Equation (3.8.6) shows that the normalized power P for the speech and music signal will be much less than 1 (which is its maximum possible value). This happens due to the high value of the crest factor.
- Equation (3.9.2) states that the signal to quantization noise ratio, for nonsinusoidal signals is given by,

$$\frac{S}{N_q} = 3 \times 2^{2N} \times P$$

- Hence if P <<< 1 then the signal to quantization noise ratio will reduce drastically. Thus for the speech and music signal having high crest factor, the signal to quantization noise ratio is poor which leads to degradation in the quality of sound.
- This problem can be overcome by use of non-uniform quantization. This is because in non-uniform quantization, the step size reduced with reduction in signal level.
- For weak signals (P << 1), the step size is small, therefore the quantization noise reduces, to improve the signal to quantization noise ratio for weak signals.
- The step size is thus varied according to the signal level to keep the signal to noise ratio adequately high. This is non-uniform quantization.
- The non-uniform quantization is practically achieved through a process called "companding". We will discuss companding in the next section.

## 3.9 Companding (Companded PCM) :

- Companding is non-uniform quantization. It is required to be implemented to improve the signal to quantization noise ratio of weak signals.
- The quantization noise is given by,

$$N_q = s^2/12$$

- This shows that in the uniform quantization once the step size is fixed, the quantization noise power remains constant.
- But the signal power is not constant. It is proportional to the square of signal amplitude. Hence signal power will be small for weak signals, but quantization noise power is constant.
- Therefore the signal to quantization noise ratio for the weak signals is very poor. This will affect the quality of signal. The remedy is to use **companding**.
- Companding is a term derived from two words, compression and expansion.
- Companding = Compressing + Expanding
- Practically it is difficult to implement the non-uniform quantization because it is not known in advance about the changes in the signal level.
- Therefore a trick is used. The weak signals are amplified and strong signals are attenuated before applying them to a uniform quantizer.
- This process is called as "compression" and the block that provides it is called as a "compressor".
- At the receiver exactly opposite process is followed which is called expansion. The circuit used for providing expansion is called as an "expander".

**The compression of signal at the transmitter and expansion at the receiver is combined to be called as "companding".**

- The process of companding is shown in the block diagram form in Fig. 3.9.1.


**Fig. 3.9.1 : Model of companding**

### 3.9.1 Compressor Characteristics :

Fig. 3.9.2 shows the compressor characteristics.

- As shown in Fig. 3.9.2, the compressor provides a higher gain to the weak signals and smaller gain to the strong input signals.
- Thus weak signals are artificially boosted to improve the signal to quantization noise ratio.

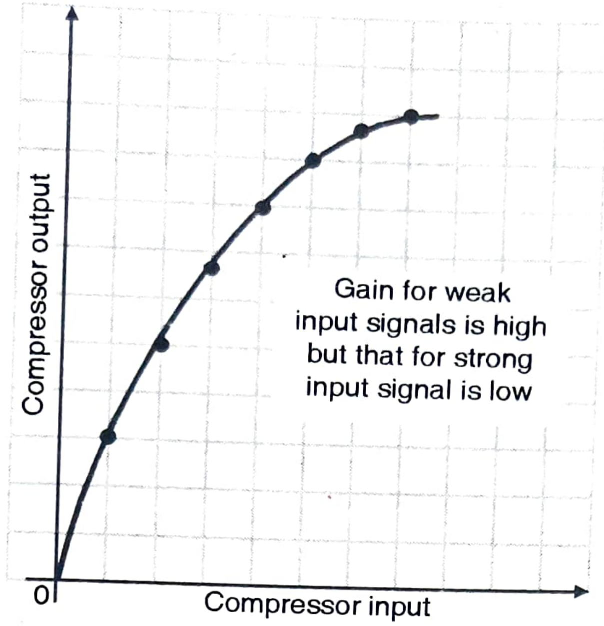
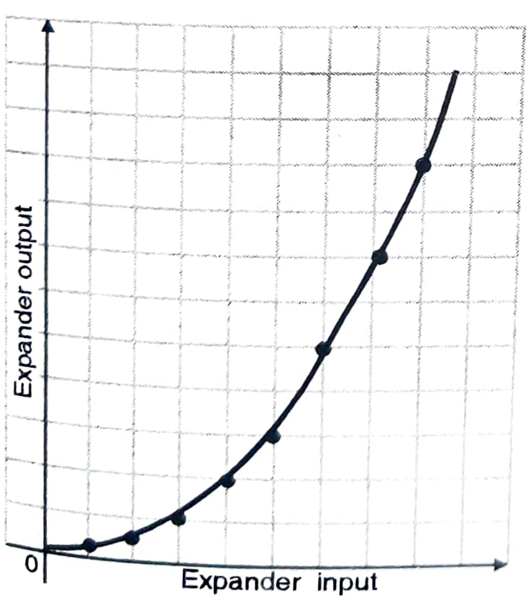



**Fig. 3.9.2 : Compressor characteristics**

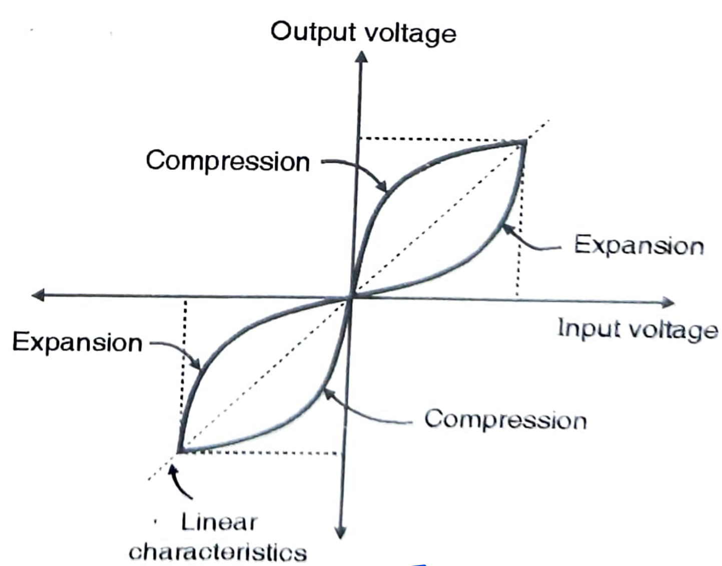
- Note that this compressor characteristics has been shown only for the positive input signal but we can draw it even for the negative input signals using the same principle.
- The compressor is included at the PCM transmitter.

## 3.9.2 Expander Characteristics :

- The expander characteristics is shown in Fig. 3.9.3.
- This characteristics is exactly the inverse of the compressor characteristics.
- This ensures that all the artificially boosted signals by the compressor are brought back to their original amplitudes at the receiver.




**(a) Expander characteristics**




**(b) Companding curves for PCM**

**Fig. 3.9.3**

### 3.9.3 Compander Characteristics :

- Fig. 3.9.3(b) shows the compander characteristics which is the combination of the compressor and expander characteristics.
- Due to the inverse nature of compressor and expander, the overall characteristics of the compander is a straight line (dotted line in Fig. 3.9.3(b)).
- This indicates that all the boosted signals are brought back to their original amplitudes.

### 3.9.4 Examples on Pulse Code Modulation :

**Ex. 3.9.1 :** A voice signal bandlimited to 3.4 kHz is to be transmitted using PCM system. The signaling rate of the PCM is not to exceed 36000 bits/sec. Find :

(a) Approximate value of $f_s$ (b) The number of quantization levels Q

(c) Number of digits (bits) per word N.

**Soln. :**

1. It is given that signaling rate $r \le 36000$

$$\therefore \; N f_s \le 36000 \quad \text{...(1)}$$

Minimum sampling frequency $f_{s\,(min)} = 2 \times f_M = 2 \times 3.4$ kHz

$$\therefore \; f_{s\,(min)} = 6.8 \text{ kHz} \quad \text{...(2)}$$

2. Substitute this value of $f_s$ in Equation (1) to get,

$$N \le \frac{36000}{6.8 \times 10^3} \qquad \therefore \; N \le 10.29$$

So let us select $N = 5$ ...Ans.

The number of quantization levels $= Q = 2^N = 2^5 = 32$ ...Ans.

3. Now with N = 5 let us calculate the maximum allowable value of sampling frequency $f_{s\,(max)}$.

$$f_{s\,(max)} = \frac{36000}{N} = \frac{36000}{5} = 7.2 \text{ kHz}$$

**Ans. :**

(a) $f_s$ should be between 6.8 kHz and 7.2 kHz

(b) Number of quantization levels = 32

(c) N = 5.

**Ex. 3.9.2 :** In a binary PCM system, the output signal to quantization noise ratio is to be held to a minimum of 40 dB. First calculate the number of binary digits per word, necessary to meet this requirement and then find the actual value of the output signal to quantization noise ratio.

**Soln. :** Assuming the signal to be sinusoidal, we can use Equation (3.5.11) to calculate the number of digits per word.

$$SNR_q \text{ dB} = 1.8 + 6N$$

The minimum value of signal to quantization noise ratio is 40 dB

$$\therefore \quad SNR_q \geq 40 \text{ dB}$$

$$\therefore \quad 1.8 + 6N \geq 40 \qquad \therefore \quad N \geq 6.36$$

Hence let us have $\quad N = 7$ ...**Ans.**

$$\therefore \quad \text{Actual value of} \left[\frac{S}{N_q}\right]_{dB} = 1.8 + (6 \times 7) = 43.8 \text{ dB}$$ ...**Ans.**

---

**Ex. 3.9.3 :** An audio signal has spectral components present in the range of 300 Hz to 3300 Hz. A PCM signal is generated by sampling this audio signal at $f_s$ = 8 kHz. The minimum value of signal to quantization noise ratio is 30 dB. Calculate :

(a) The minimum number of quantization levels Q and number of binary digits per word N.

(b) Signaling rate r.

(c) Minimum transmission bandwidth. [redacted]

**Soln. :**

$$\text{Given that} \quad f_s = 8 \text{ kHz and} \left[\frac{S}{N_q}\right] \geq 30 \text{ dB}$$

**(a) To calculate Q and N :**

Assuming that the input signal is sinusoidal we write,

$$SNR_q = 1.8 + 6N$$

$$\text{As } SNR_q \geq 30 \text{ dB} \qquad 1.8 + 6N \geq 30 \text{ dB}$$

$$\therefore \quad N \geq 4.7$$

$\therefore$ Number of digits per word, $N = 5$ ...**Ans.**

Number of quantization levels $Q = 2^N = 2^5 = 32$ ...**Ans.**

**(b) Signaling rate (r) :**

$$(r) = N f_s = 5 \times 8 \text{ kHz} = 40 \text{ Kbits/sec.}$$ ...**Ans.**

**(c) Transmission bandwidth :**

$$B_T = \frac{1}{2} r = 20 \text{ kHz}$$ ...**Ans.**

---

**Ex. 3.9.4 :** A PCM system uses a uniform quantizer followed by a 7 bit encoder. The system bit rate is 50 Mbits/sec. Calculate the maximum bandwidth of the message signal for which this system operates satisfactorily.

**Soln. :**

It has been given that : Bit rate r = 50 Mbits/sec and N = 7

We know that bit rate $r = N f_s$

$$\therefore \quad f_s = \frac{r}{N} = \frac{50 \times 10^6}{7} = 7.14 \text{ MHz}$$

$\therefore$ Maximum signal bandwidth BW $= f_s/2 = \dfrac{7.14 \text{ MHz}}{2}$

$$\therefore \quad BW = 3.57 \text{ MHz} \qquad \text{...Ans.}$$

**Ex. 3.9.5 :** The bandwidth of a video signal is 4.5 MHz. This signal is to be transmitted using PCM with the number of quantization levels Q = 1024. The sampling rate should be 20% higher than the Nyquist rate. Calculate the system bit rate.

[redacted]

**Soln. :**

$$\text{Bandwidth } W = 4.5 \text{ MHz}$$

$$\therefore \quad \text{As per Nyquist rate } f_s = 2W = 9 \text{ MHz}$$

But $f_s$ should be 20% higher than Nyquist rate

$$\therefore \quad f_s = 1.2 \times 9 \text{ MHz} = 10.8 \text{ MHz} \qquad \text{...(1)}$$

We know that, $Q = 2^N$, $\quad \therefore \quad 1024 = 2^N$

$$\therefore \quad N = 10$$

$$\therefore \quad \text{System bit rate } r = N f_s = 10 \times 10.8 \text{ MHz} \qquad \text{...(2)}$$

**Ex. 3.9.6 :** Derive the expression for the signal to quantization noise ratio of PCM system employing uniform quantization technique. Assume that input signal is of nonsinusoidal nature.

**Soln. :**

**Expression for $N_q$ :** The signal to quantization noise ratio is defined as,

$$\frac{S}{N_q} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}} \qquad \text{...(1)}$$

In section 3.5 we have already derived the expression for the normalized quantization noise power. From Equation (3.4.11),

$$\text{Normalized noise power} = N_q = s^2/12 \qquad \text{...(2)}$$

$$\text{But} \quad s = \frac{\text{Peak to peak signal amplitude}}{Q}$$

Let, $2x_{max}$ = Peak to peak signal amplitude

$$\therefore \quad s = \frac{2x_{max}}{Q}$$

But $Q = 2^N$

$$\therefore \quad s = \frac{2x_{max}}{2^N} \qquad \text{...(3)}$$

Substitute Equation (3) into Equation (2) to get,

$$\therefore \quad N_q = \left[\frac{2x_{max}}{2^N}\right]^2 / 12 = \frac{4x_{max}^2}{2^{2N} \times 12}$$

$$\therefore \quad N_q = \frac{x_{max}^2}{3 \times 2^{2N}} \qquad \ldots(4)$$

**Expression for $S_i/N_q$ :**

Let the signal power at the destination be represented by "P".

$\therefore$ Equation (1) can be written as,

$$\frac{S}{N_q} = \frac{S}{\left[x_{max}^2/3 \times 2^{2N}\right]} = \frac{3S \times 2^{2N}}{x_{max}^2}$$

$\therefore$ **Signal to quantization noise ratio for nonsinusoidal signal** $= \dfrac{\mathbf{3S}}{\mathbf{x_{max}^2}} \cdot \mathbf{2^{2N}}$ $\qquad \ldots(3.9.1)$

Now if the input signal x (t) is normalized i.e. $x_{max} = 1$ then Equation (3.9.1) gets modified to,

$$\frac{S}{N_q} = 3 \times 2^{2N} \times P \qquad \ldots(3.9.2)$$

And if the signal power "P" at the destination is also normalized,

$$\text{i.e. if} \quad P \leq 1 \qquad \ldots(3.9.3)$$

Then Equation (3.9.2) gets modified to,

$$SNR_q \leq 3 \times 2^{2N} \qquad \ldots(3.9.4)$$

Convert this equation in decibels as follows :

$$SNR_q \text{ (dB)} = 10 \log_{10} \left[\frac{S}{N_q}\right] \text{ as } \frac{S}{N_q} \text{ is a power ratio}$$

$$\therefore \quad SNR_q \text{ (dB)} \leq 10 \log_{10} \left[3 \times 2^{2N}\right]$$

$$SNR_q \text{ (dB)} \leq \left[10 \log_{10} 3 + 20\, N \log_{10} 2\right]$$

$$\therefore \quad SNR_q \text{ (dB)} \leq [4.8 + 6N] \qquad \ldots(3.9.5)$$

Hence **Maximum signal to quantization noise ratio for normalized power P and input amplitude x (t)** : $\mathbf{SNR_q = (4.8 + 6\ N)\ dB}$ $\qquad \ldots(3.9.6)$

### 3.9.5 Different Types of Compressor Characteristics :

Ideally we need a linear compressor characteristic for small amplitudes of the input signal and a logarithmic characteristic elsewhere. Practically this is achieved by using two methods :

1. μ - law companding
2. A - law companding.

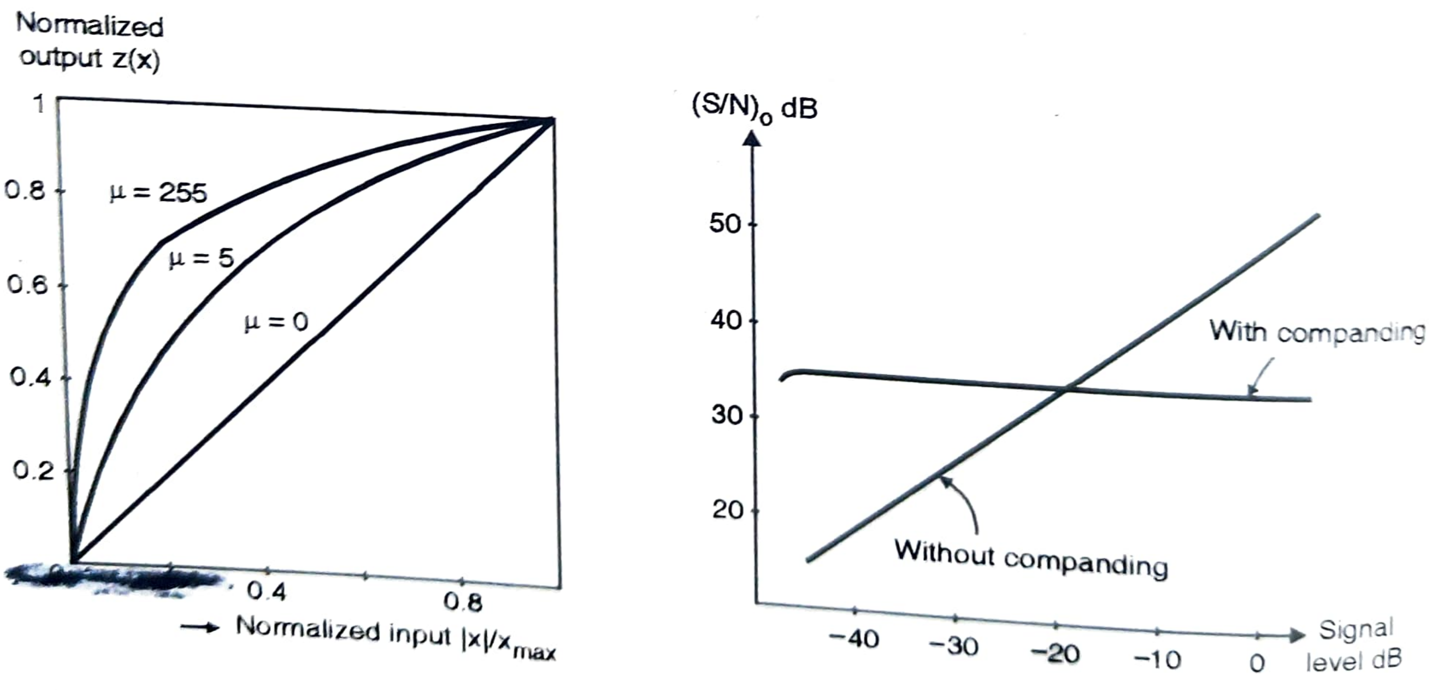
## 1. μ - Law Companding :

- In the μ-law companding, the compressor characteristic is continuous. It is approximately linear for smaller values of input levels and logarithmic for high input levels.
- The μ -law compressor characteristic is mathematically expressed as,

$$z(x) = (\text{sgn } x) \frac{\ln(1 + \mu |x| / x_{max})}{\ln(1 + \mu)} \qquad ...(3.9.7)$$

where $0 \leq |x| / x_{max} \leq 1$.

- Here z (x) represents the output and x is the input to the compressor. $|x| / x_{max}$ represents the normalized value of input with respect to the maximum value $x_{max}$ · (Sgn x) term represents ± 1 i.e. positive and negative values of input and output.
- The μ -law compressor characteristics for different values of μ are as shown in Fig. 3.9.4(a). The practically used value of μ is 255.
- The characteristic corresponding to μ = 0 corresponds to the uniform quantization.
- The μ-law companding is used for speech and music signals. It is used for PCM telephone systems in United States, Canada and Japan.
- Fig. 3.9.4(b) shows the variation of signal to quantization noise ratio with respect to signal level with and without companding. It is clearly seen that SNR is almost constant at all the signal levels when companding is used.




**(a) Compressor characteristic of a μ-law compressor**

**(b) PCM performance with μ-law companding**

**Fig. 3.9.4**

2. **A - Law Companding :**

- In the A-law companding, the compressor characteristic is piecewise, made up of a linear segment for low level inputs and a logarithmic segment for high level inputs.
- Fig. 3.9.5 shows the A-law compressor characteristics for different values of A. Corresponding to A = 1 we observe that the characteristic is linear which corresponds to a uniform quantization.
- The practically used value of "A" is 87.56. The A-law companding is used for PCM telephone systems in Europe.

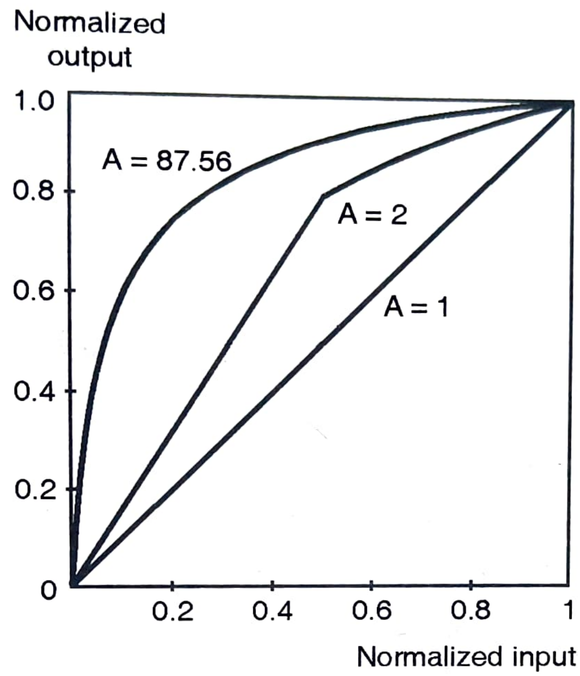



**Fig. 3.9.5 : Compressor characteristics of A-law compressor**

- The linear segment of the characteristics is for low level inputs whereas the logarithmic segment is for high level input. It is mathematically expressed as,

$$\frac{z(x)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\log_e A} & 0 \leq \dfrac{|x|}{x_{max}} \leq 1 \\[2ex] \dfrac{1+\log_e [A|x|/x_{max}]}{1+\log_e A} & \dfrac{1}{A} \leq \dfrac{|x|}{x_{max}} \leq 1 \end{cases} \qquad \text{...(3.9.8)}$$

### 3.9.6 Effect of Companding :

- Refer Fig. 3.9.6 which shows the curves of output signal to noise ratio versus input signal power for the uniform and nonuniform quantizers.
- The curve for nonuniform quantizer corresponds to the $\mu$ law companding with $\mu = 255$.

- The curves in Fig. 3.9.6 have been plotted with the following assumptions.
  1. Value of Q is 256.
  2. Parameter $\mu = 255$
  3. Let X denote the random variable representing the input. Let it has the **Laplacian distribution** which is represented mathematically as follows.

$$f_X(x) = \frac{1}{\sqrt{2}\,\sigma_x} e^{-2|x|/\sigma_x} \quad \text{...(3.9.9)}$$

     where $\sigma_x^2$ is the variance of X.
  4. The input signal ranges between $-x_{max}$ to $+x_{max}$.

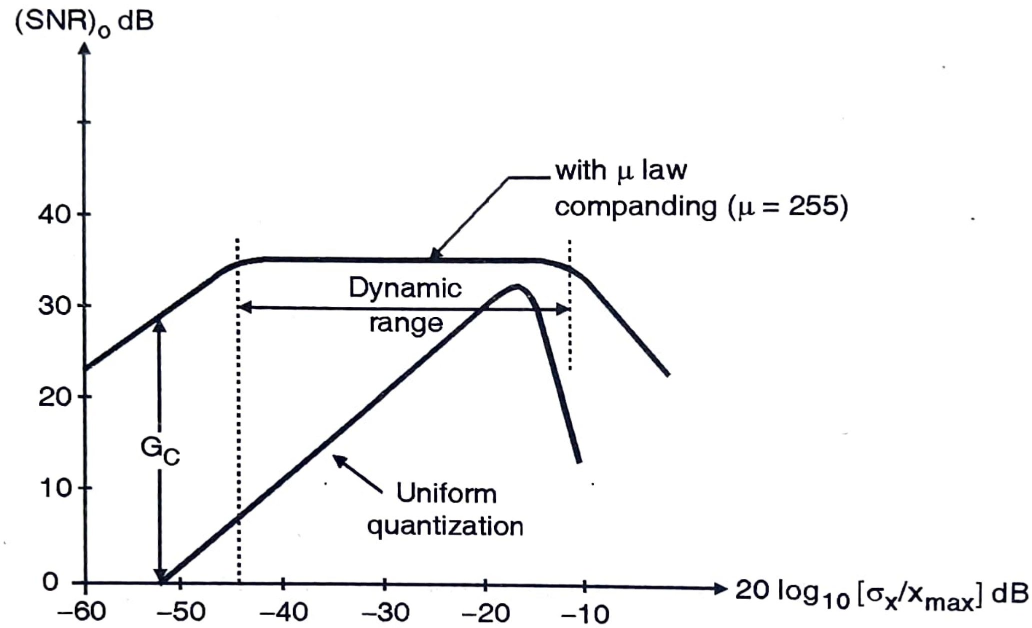



**Fig. 3.9.6 : Output SNR characteristics for uniform and nonuniform quantization**

**Dynamic range :**

- Fig. 3.9.6 indicates that the performance of uniform quantizer is highly dependent on the input whereas the $\mu$ law compander has a dynamic range of about 30 dB (from – 15 dB to – 45 dB).
- Dynamic range is defined as the range of input over which the output SNR remains within 3 dB of the maximum value of 38 dB.

### 3.9.7 Companding Gain :

- This is another important parameter used for assessing the improvement due to the companding process.
- It is denoted by $G_c$ and defined as

$$G_c = \left. \frac{dz(x)}{dx} \right|_{x \to 0} \quad \text{...(3.9.10)}$$

where z (x) represents the compressor characteristics.

- The companding gain for a μ - law compressor is given by,

$$G_c = \frac{\mu}{\log_e (1+\mu)} \quad \text{...(3.9.11)}$$

- If $\mu = 255$, then

$$G_c = \frac{255}{\log_e (256)} = 45.98 \quad \text{...(3.9.12)}$$

- Converting into dB, we get

$$G_c \text{ dB} = 20 \log_{10} G_c = 33.3 \text{ dB}$$

- The value of companding gain should be as high as possible.
- The effect of the companding gain of 33.3 dB is shown in Fig. 3.9.6. It shows that due to companding, the smallest step size with companding is smaller than the step size of a uniform quantizer by a factor equal to $G_c = 32$.

**What is the difference between μ -law and A-law companding ?**

The most important difference between the two types of compressors is that the A-law compressor has a midriser at the origin whereas the μ-law compressor has a midtread at the origin. Thus the A-law compressor has no zero value.

---

**Ex. 3.9.7 :** Plot the characteristics of a μ-law compressor. [redacted]

**Soln. :**

**To plot the characteristics of μ-law compressor :**

The expression for the normalized output of a μ-law compressor is given by,

$$Z(x) = \pm \frac{ln (1 + \mu |x|)}{ln (1+\mu)} \quad \text{..... } x \le 1$$

Let $\mu = 255$.

Note that instead of $|x| / x_{max}$ we have written only $|x|$ and restricted the values of x only upto 1. This has the same effect as that of normalizing.

$$\therefore \quad Z(x) = \pm \frac{ln (1 + 255 |x|)}{ln\ 256} \quad \text{...(1)}$$

Substitute x = 0, 0.2, 0.4, 0.6, 0.8 and 1 in Equation (1) to get the corresponding values of Z as shown in the following table.

| \|x\| | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|---|
| Z | 0 | ± 0.71 | ± 0.84 | ± 0.9 | ± 0.96 | ± 1 |

Therefore the compressor characteristics is as shown in Fig. P. 3.9.7.

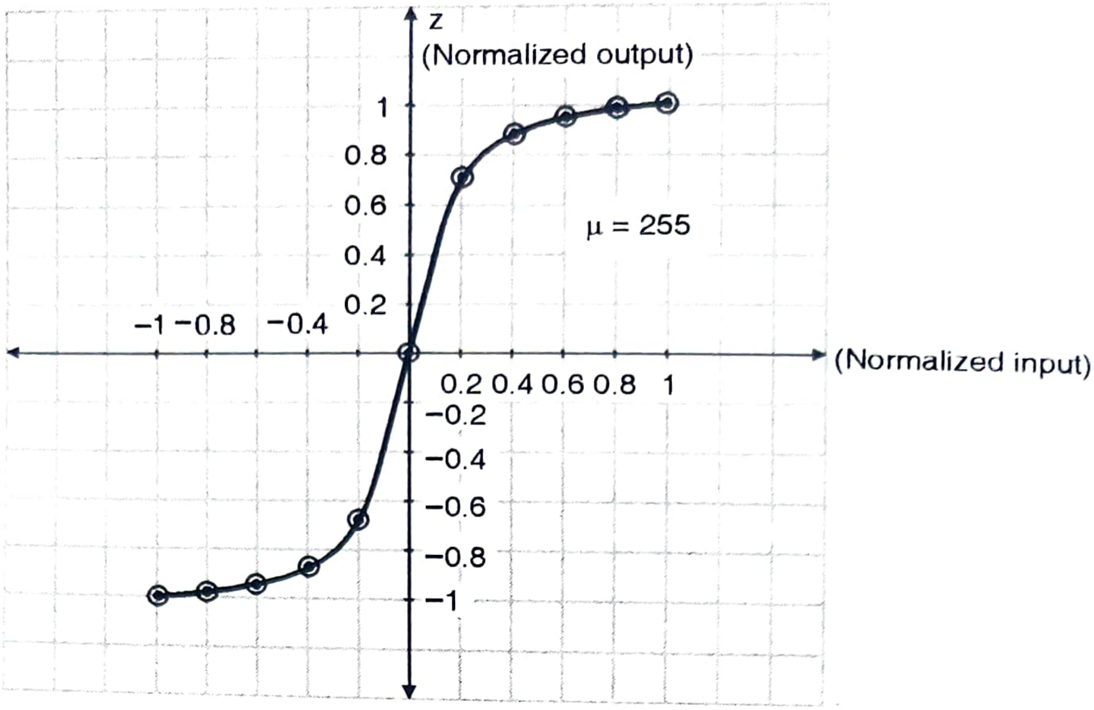



**Fig. P. 3.9.7 : μ-law compressor characteristics**

## 3.10 Noise Considerations in PCM System :

- To judge the performance of the PCM system we have to consider two major sources of noise as follows :

  1. Channel noise  2. Quantization noise.

  1. **Channel noise** may get introduced anywhere along the transmission path. It is also called as decoding noise.
  2. **Quantization noise** as we have defined earlier, is introduced at the transmitter and is carried along to the receiver output.

- Both of them are present simultaneously but we will consider them one by one to find their effect on the PCM system.

### 3.10.1 Channel Noise and its Effect :

- The major effect of channel noise is introduction of transmission errors at the receiver when the PCM signal is being reconstructed.
- Due to such errors the receiver will make mistake in making the decision about whether a 0 was received or a 1 was received.
- A 0 may be mistaken as 1 and a 1 may be mistaken as 0. Such errors should be minimized so as to improve the fidelity of PCM system.

**Error rate or probability of error ($P_e$) :**

The fidelity of a PCM system in presence of channel noise is measured in terms of error or probability of error. The probability of error or error rate is the probability that the symbol at the receiver output differs from that transmitted.

**Expression for the error probability ($P_e$) :**

- To obtain the expression for error probability ($P_e$) we will use the matched filter shown in Fig. 3.10.1 and assume that the type of noise is AWGN i.e. additive while gaussian noise.

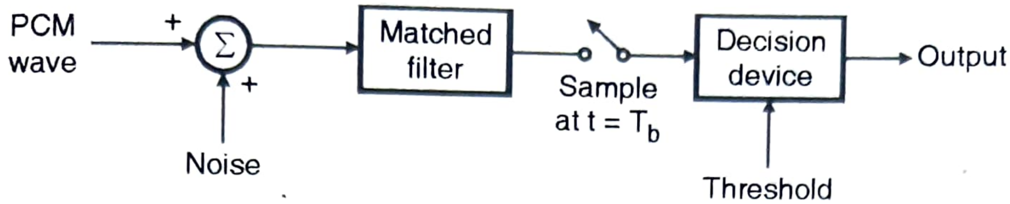



**Fig. 3.10.1 : Matched filter receiver for PCM**

- The matched filter and its performance has been discussed in chapter 5.
- The derivation for the probability of error for a PCM system ($P_e$) has been discussed earlier.
- Here we are going to only state the expression for the error probability ($P_e$) of PCM system as follows.

$$P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right] \qquad ...(3.10.1)$$

Where $E_{max}$ = Peak signal energy

$N_0$ = Noise spectral density

- We can substitute $E_{max} = P_{max}\, T_b$ where $P_{max}$ is the maximum or peak signal power and $T_b$ is the bit duration. Hence the error probability in terms of power is as follows

$$P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{1}{2}\sqrt{\frac{P_{max} \times T_b}{N_0}}\right]$$

$$P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{1}{2}\sqrt{\frac{P_{max}}{N_0 / T_b}}\right] \qquad ...(3.10.2)$$

- The ratio $N_0 / T_b$ can be seen as the average noise power contained in a transmission bandwidth equal to the bit rate ($1/T_b$).
- Hence $E_{max} / N_0$ may be viewed as the peak signal to noise power ratio.

### 3.10.2 Conclusions from Equation (3.10.2) :

1. The average probability of error in PCM receiver depends only on the ratio of peak signal energy $E_{max}$ to the noise power spectral density $N_0$ measured at the receiver input
2. The complementary error function "erfc" is a monotonically decreasing function. So $\operatorname{erfc}\sqrt{\frac{E_{max}}{N_0}}$ will decrease with increase in the ratio ($E_{max} / N_0$) as shown in Fig. 3.10.2.

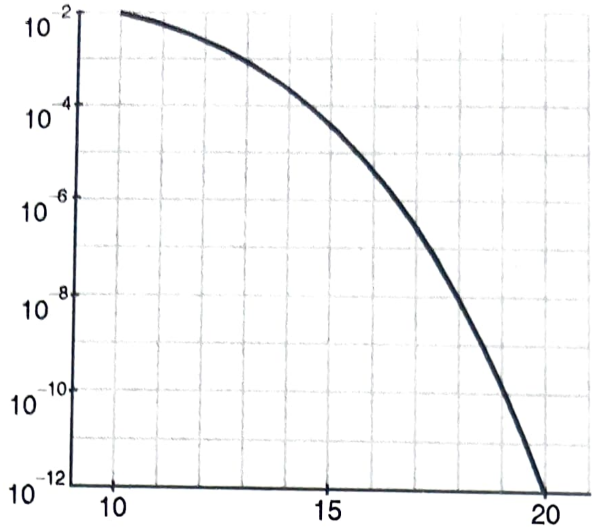


**Fig. 3.10.2 : Probability of error in a PCM system**

**Error threshold :**

- From Fig. 3.10.2, it is seen that the error probability decreases very rapidly as the value of this ratio $E_{max} / N_0$ is increased.
- A very small increase in transmitted signal energy or power will make the reception of binary pulses almost error free.
- The effect of increase in the ratio $E_{max} / N_0$ is illustrated in Table 3.10.1.

**Table 3.10.1 : Effect of $E_{max} / N_0$ on probability of error**

| $E_{max} / N_0$ | Probability of error $P_e$ | For a bit rate of $10^5$ bits per second this is about 1 error every |
|---|---|---|
| 10.3 dB | $10^{-2}$ | 1 msec |
| 14.4 dB | $10^{-4}$ | 0.1 sec |
| 16.6 dB | $10^{-6}$ | 10 sec |
| 18 dB | $10^{-8}$ | 20 min |
| 19 dB | $10^{-10}$ | 1 day |
| 20 dB | $10^{-12}$ | 3 months |

- Table 3.10.1 shows that there is an error threshold at about 17 dB.
- Above this value of $E_{max} / N_0$ the error probability is very low. Whereas below this threshold the error probability is high and hence the effect of noise is significant.
- The effect of channel noise can be reduced by using the regenerative repeaters.
- Another important characteristics of PCM system is its **ruggedness to interference**. As discussed earlier for on off signalling, there is no effect of noise unless the peak amplitude is greater than half the pulse height.
- Thus if adequate margin over the error threshold is provided, then the system can successfully withstand large amount of noise and interference.
- So we can say that PCM is a **noise resistant or rugged system**.

### 3.10.3 Average Transmitted Power for the Provision of Noise Margin :

- Consider an M-ary PCM system. It uses a codeword consisting of N code elements each one having M possible discrete amplitude values. Hence its name is M-ary PCM.
- So as to provide adequate noise margin and hence to maintain a small error rate, there has to be a certain separation between these M discrete amplitude levels.
- Let the separation between the adjacent amplitude levels be $k\sigma$ as shown in Fig. 3.10.3 where k is constant and $\sigma^2 = N_0 B$ i.e. the noise variance measured in a channel bandwidth B.

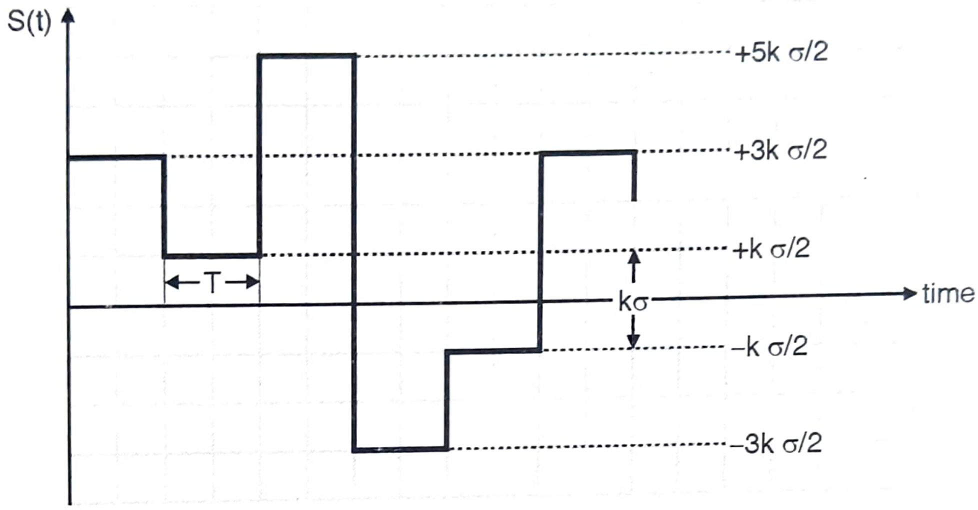



**Fig. 3.10.3**

- Let us calculate the signal power P corresponding to this signal as follows,
- All the M messages are equally likely, therefore every message i.e. every level in the Fig. 3.10.3 is likely to occur atleast once.
- Therefore the average signal power can be obtained as follows :

$$\text{Average signal power (P)} = \frac{\text{Sum of the powers due to individual messages}}{\text{Total number of messages}}$$

$$\therefore \; P = \frac{[k\sigma/2]^2 + [-k\sigma/2]^2 + [3k\sigma/2]^2 + [-3k\sigma/2]^2 + \ldots + \left[\frac{M-1}{2}k\sigma\right]^2 + \left[-\frac{M-1}{2}k\sigma\right]^2}{M} \quad \ldots(3.10.3)$$

- In the above expression, the powers of individual messages are the "normalized" powers.
- The normalized power of the first message level $(k\sigma/2)$ is the power consumed by a $1\,\Omega$ resistance hence it would be $\frac{(k\sigma/2)^2}{1}$ or $(k\sigma/2)^2$. Similarly the normalized powers can be obtained for all the messages.

$$\therefore \; P = \frac{2}{M}\left\{\left[\frac{k\sigma}{2}\right]^2 + \left[\frac{3k\sigma}{2}\right]^2 + \ldots + \left[\frac{(M-1)k\sigma}{2}\right]^2\right\} \quad \ldots(3.10.4)$$

$$\therefore \; P = \frac{M^2-1}{12}(k\sigma)^2 \quad \ldots(3.10.5)$$

**Conclusion :**

We can draw two important observations from Equation (3.10.5) as follows :

1. For a prescribed noise variance $\sigma^2$ the average transmitted power P (required to operate above the error threshold) increases rapidly with increase in the number of levels M.
2. If M = 2 corresponding to NRZ polar signaling we get

$$P = \frac{(2)^2 - 1}{12} k^2 \sigma^2$$

$$\therefore \quad P = \frac{k^2 \sigma^2}{4} \qquad ...(3.10.6)$$

Thus for the same noise margin, the use of NRZ polar signaling needs one half of the average transmitted power required for NRZ unipolar signaling.

### 3.10.4 M-ary PCM Performance :

- In this section we will see the performance of M-ary PCM system by using the Shannon's channel capacity theorem.
- To obtain the expression for channel capacity we have to first obtain the expression for M i.e. the number of levels.
- Rearrange the Equation (3.10.5) for M to get,

$$M^2 = 1 + \frac{12\,P}{(k\,\sigma)^2}$$

$$\therefore \quad M = \left[1 + \frac{12\,P}{(k\,\sigma)^2}\right]^{1/2} \qquad ...(3.10.7)$$

- The rms noise voltage is equal to $\sigma$. Hence the normalized noise power (i.e. in $1\Omega$ resistance) will be $N = \sigma^2 / 1 = \sigma^2$
- Therefore substitute $\sigma^2 = N$ i.e. noise power in the Equation (3.10.7) to get,

$$M = \left[1 + \frac{12\,P}{k^2 N}\right]^{1/2} \qquad ...(3.10.8)$$

Thus we have obtained the expression for M.

- Now substitute $N = N_0 B$ to get

$$M = \left[1 + \frac{12\,P}{k^2 N_0 B}\right]^{1/2}$$

where B is the bandwidth.

- The rate of information transmission system is given by

$$R = 2W \times N \log_2 M \text{ bits/s} \qquad ...(3.10.9)$$

- But WN = B/2

$$\therefore \quad R = 2 \times \frac{B}{2} \log_2 M$$

$$\therefore \quad R = B \log_2 M \qquad ...(3.10.10)$$

- Substituting the expression of M from Equation (3.10.7) we get,

$$\therefore \quad R = B \log_2 \left[ 1 + \frac{12}{k^2} \frac{P}{N_0 B} \right] \text{ bits/sec} \qquad ...(3.10.11)$$

- From Shannon's theorem, for acceptable probability of error,

$$C \geq R$$

- That means the lowest value of C = R. Using this identity we get,

$$\mathbf{C = R = B \log_2 \left[ 1 + \frac{12}{k^2} \frac{P}{N_0 B} \right]} \qquad ...(3.10.12)$$

This is the required result.

- The ideal system is described by Shannon's channel capacity theorem. It states the channel capacity C is given by,

$$C = B \log_2 \left[ 1 + \frac{P}{N_0 B} \right] \qquad ...(3.10.13)$$

- Now compare Equations (3.10.12) and (3.10.13). If they are to be equal then the average transmitted power of the PCM system should be increased by a factor $k^2/12$ as compare to the ideal system.

**Trade off between Bandwidth and SNR :**

By Shannon Hartley theorem we get the channel capacity as,

$$C = B \log_2 \left[ 1 + \frac{S}{N} \right]$$

Let us try to find out the maximum possible value of C. From the equation for "C" it is evident that it depends on two factors, which are the bandwidth "B" and the S/N ratio. Let us find their effect on "C" one by one.

**Effect of S/N on C :**

If the communication channel is noiseless then N = 0. Therefore $(S / N) \rightarrow \infty$ and so "C" also will tend to $\infty$. Thus the noiseless channel will have an infinite capacity.

**Effect of bandwidth B on C :**

- Now consider that some white Gaussian noise is present hence (S/N) is not infinite. Now as the bandwidth approaches infinity the channel capacity does not become infinite since $N = N_0 B$ will also increase with the bandwidth B.
- This will reduce the value of (S/N) with increase in B, assuming the signal power S to be constant.
- Thus the ideal system operates under two constraints namely, limited bandwidth B and limited power P.
- But the M-ary PCM operates under an additional constraint of a finite number of amplitude levels M.
- Fig. 3.10.4 is plotted as input signal to noise ratio $P/N_0 B$ dB versus bandwidth efficiency R/B bits per second per hertz.
- In Fig. 3.10.4 we have substituted R = C and for an M-ary PCM system for different values of M, k has been substituted equal to 7.

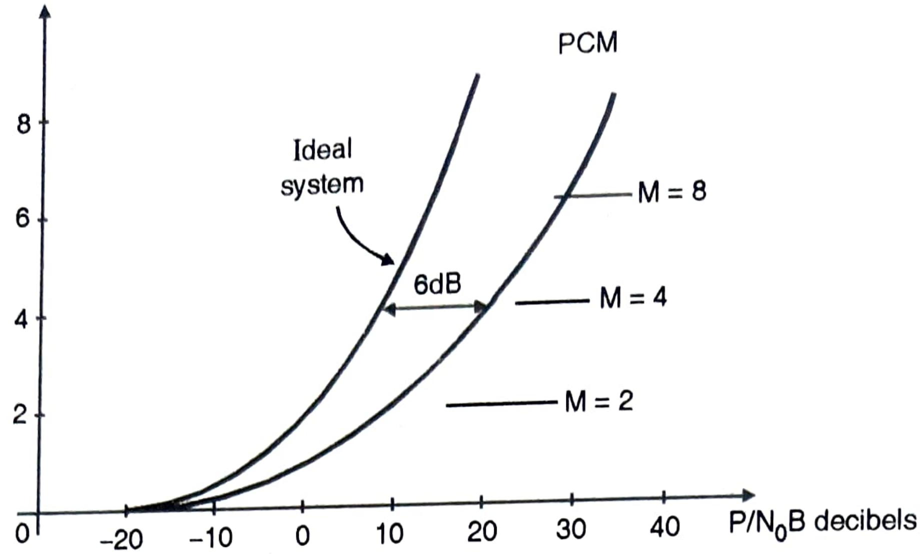



**Fig. 3.10.4 : Comparison of M-ary PCM with the ideal system**

- The constraint on M, in M-ary PCM tends to drive the M-ary PCM system into saturation when the bandwidth efficiency satisfies the following condition

$$\frac{R}{B} \leq 2 \log_2 M$$

- This expression tells us that with increase in the value of M, the saturation level goes on increasing.
- When a PCM system enters into saturations the error probability reaches its limiting value of 1.

### 3.10.5 Applications of PCM :

Some of the applications of PCM are as follows :

1. In telephony (with the advent of fibre optic cables).
2. In the space communication, space craft transmits signals to earth. Here the transmitted power is very low (10 to 15W) and the distances are huge (a few million km). Still due to the high noise immunity, only PCM systems can be used in such applications.

### 3.10.6 Advantages of PCM :

1. Very high noise immunity.
2. Due to digital nature of the signal, repeaters can be placed between the transmitter and the receivers. The repeaters actually regenerate the received PCM signal. This is not possible in analog systems. Repeaters further reduce the effect of noise.
3. It is possible to store the PCM signal due to its digital nature.
4. It is possible to use various coding techniques so that only the desired person can decode the received signal.

### 3.10.7 Disadvantages of PCM :

1. The encoding, decoding and quantizing circuitry of PCM is complex.
2. PCM requires a large bandwidth as compared to the other systems.

## 3.10.8 PCM versus Analog Modulation : [redacted]

- The threshold effect in PCM is similar to a property of analog modulation methods such as FM or PPM.
- The property is that, these systems tend to reduce the wideband noise above their threshold levels.
- The PCM also provides the wideband noise reduction if it is operated above its threshold which is given by,

$$(S/N)_D = 3\,Q^2\,S_x$$

where $Q = 2^N$ for binary PCM and $Q = M^N$ for M-ary PCM

- We assume that the sampling frequency is close to the Nyquist rate and $B_T = NW$ Hz
- Then $Q = M^N = M^b$ where $b = B_T / W$ is called as the bandwidth ratio.

$$\therefore \quad (S/N)_D = 3 \times (M^N)^2\,S_x$$

$$= 3 \times M^{2N}\,S_x$$

$$= 3\,M^{2b}\,S_x$$

Here $(S/N)_D$ = Signal to noise ratio at the destination

$S_x$ = Signal power at the destination

- Note that here the signal to noise ratio $(S/N)_D$ is proportional to $M^{2b}$ which is much higher than the $(S/N)_D$ of the wideband FM which is proportional to only b or $b^2$.
- Hence PCM performs better than FM.
- Fig. 3.10.5 shows the performance of various modulation types as a function of $\gamma$.

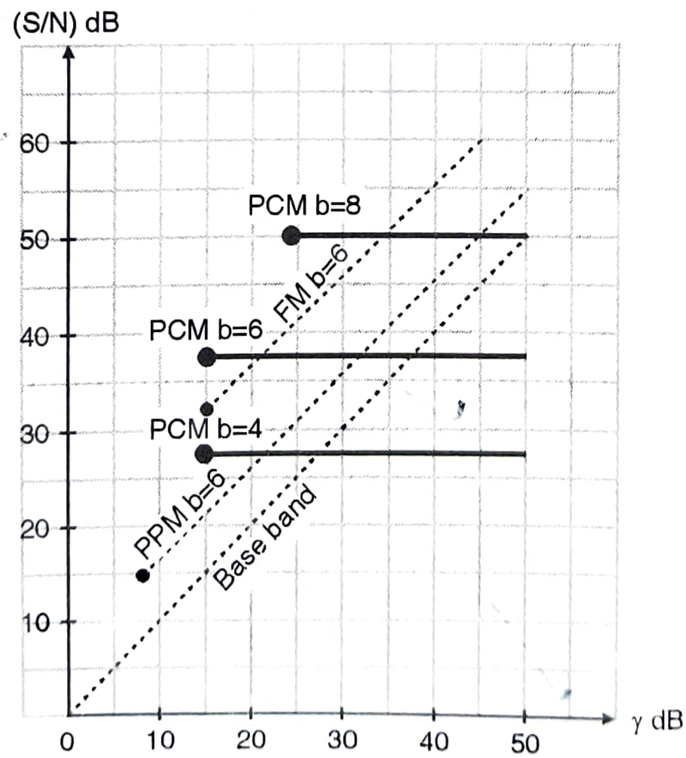



**Fig. 3.10.5 : Comparison of PCM and analog communication**

- All the curves in Fig. 3.10.5 have been plotted for $S_x = 1/2$. The dots indicate the threshold points.
- The PCM curves are drawn for $M = 2$ and $N = b$.

**Observations :**

Some of the important observations from Fig. 3.10.5 are as follows :

1. For PCM if "b" is constant, then increase in $\gamma$ beyond its the threshold value $\gamma_{th}$ (corresponding to the threshold point) does not increase $(S/N)_D$ at all. See the flat PCM curves in Fig. 3.10.5. So PCM should be operated just above the threshold.
2. Near threshold, the PCM does offer some advantage over FM and PPM, with the same values of "b" and $(S/N)_D$.
3. But this advantage is gained at the expense of more complicated and expensive circuitry.
4. The $(S/N)_D$ for FM and PPM increases linearly with increase in the value of $\gamma$ and becomes better than that of PCM for the higher values of $\gamma$.

**Benefits of PCM :**

Fig. 3.10.5 does not reveal the following benefits of using the PCM.

1. PCM allows the use of regenerative repeaters. This will improve its noise performance.
2. PCM allows the transmission of analog signals in the form of digital signals.

**Why is PCM not used for broadcasting ?**

- In radio broadcasting a relatively large signal to noise ratio (typically of the order of 60 dB) is required.
- To get this level of $(S/N)_D$ the PCM with $b > 8$ is required.
- However we can obtain the same performance with an FM system with $b = 6$ and with much simpler transmitter and receiver circuits.
- So higher bandwidth requirement and complicated circuitry are the disadvantages of PCM which does not allow its use for the radio, TV broadcasting applications.

## 3.11 Virtues Limitations and Modifications of PCM :

- The PCM is considered to be the best modulation scheme to transmit the voice and video signals.
- All the advantages of PCM are due to the fact that it uses coded pulses for the transmission of information.

### 3.11.1 Applications of PCM :

- Some of the applications of PCM are as follows :

1. In telephony (with the advent of fibre optic cables).
2. In the space communication, space craft transmits signals to earth. Here the transmitted power is very low (10 to 15W) and the distances are huge (a few million km). Still due to the high noise immunity, only PCM systems can be used in such applications.

### 3.11.2 Virtues of PCM :

1. Very high noise immunity.

2. Due to digital nature of the signal, repeaters can be placed between the transmitter and the receivers. The repeaters actually regenerate the received PCM signal. This is not possible in analog systems. Repeaters further reduce the effect of noise.
3. It is possible to store the PCM signal due to its digital nature.
4. It is possible to use various coding techniques so that only the desired person can decode the received signal. This makes the communication secure.
5. The increased channel bandwidth requirement for PCM is balanced by the improved SNR. This is the due to the fact that PCM obeys an exponential law.
6. There is a **uniform format** used for the transmission of different types of base band signals. Hence it is easy to integrate all these signals together and send them on the common network.
7. It is easy to drop or reinsert the message sources in a PCM-TDM system.

### 3.11.3 Limitations of PCM :

1. The encoding, decoding and quantizing circuitry of PCM is complex.
2. PCM requires a large bandwidth as compared to the other systems.

### 3.11.4 Modifications in PCM :

- Even though PCM is complex, it is possible to implement it using the VLSI technology.
- Due to the improvements in VLSI technology, the use of PCM for digital transmission of analog signals is going to increase.
- But if the simplicity is a more important issue, then one should use the **Delta Modulation** in place of PCM.
- The requirement of large channel bandwidth for PCM is not a real problem now, due to the availability of wideband communication channels.
- Due to the liberation from bandwidth contraint it has become possible to use the communication satellites and optical fiber communication.
- It is possible to remove the **redundancy** in PCM by means of using the data compression **techniques**. This will reduce the bit rate of transmitted data without serious degradation in the contents.
- This will increase the complexity of PCM further.

**Why is PCM not used for broadcasting ?**

- In radio broadcasting a relatively large signal to noise ratio (typically of the order of 60 dB) is required.
- To get this level of $(S/N)_D$ the PCM with $b > 8$ is required, where $b = B_T / W$ i.e. ratio of transmission bandwidth to baseband bandwidth.
- However we can obtain the same performance with an FM system with $b = 6$ and with much simpler transmitter and receiver circuits.
- So higher bandwidth requirement and complicated circuitry are the disadvantages of PCM which do not allow its use for the radio, TV broadcasting applications.

## 3.12 Baseband Transmission :

- In the preceding sections, we have learnt about transmission of analog waveforms into binary digits using PCM.

- But it is not enough to convert the analog signal into binary digits. It is necessary to have something **physical** which can **carry** the digits.
- We will represent the binary digits with electric pulses in order to transmit them through the baseband channel. Such a representation is shown in Fig. 3.12.1.
- In Fig. 3.12.1(a) the bit duration is $T_b$ and the codeword duration corresponds to four bit duration.
- Now refer Fig. 3.12.1(b), in which each binary "1" is represented by a pulse and each binary "0" is represented by the absence of pulse. This pattern of electrical pulses can now be used to transmit the information in the PCM bit stream.
- Now refer Fig. 3.12.1(c). This waveform is obtained by increasing the width of each pulse in Fig. 3.12.1(b) to its maximum possible value.
- This increases the energy content in each pulse and hence makes the "correct detection" at the receiver practically possible.
- The waveform of Fig.3.12.1(c) represents "1" by a high positive level (+ V) and "0" by a negative level (– V).

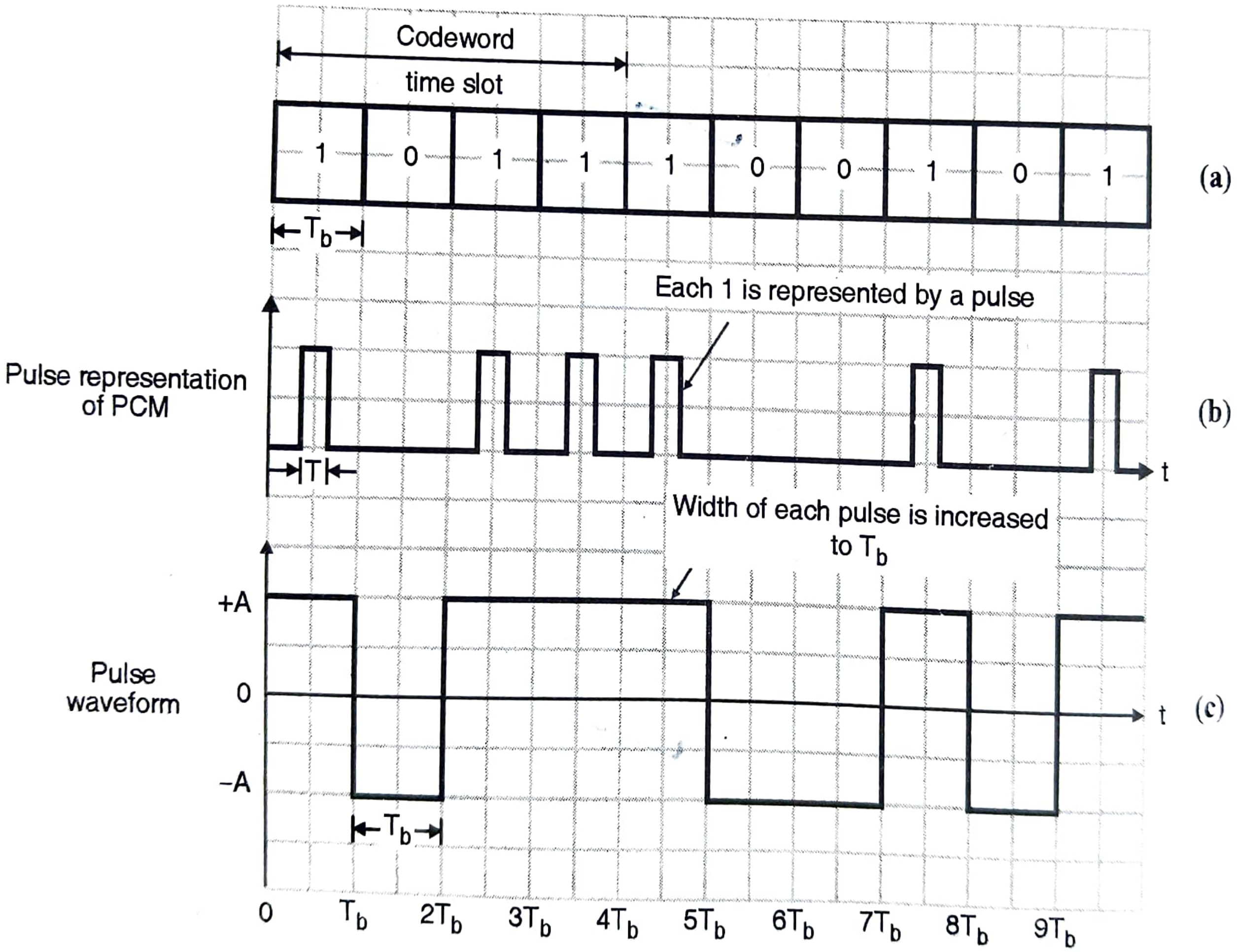



**Fig. 3.12.1 : Representation of binary digits using waveforms**

### 3.12.1 PCM Waveform Types :

- When we represent the binary PCM symbol by means of a waveform, the resulting waveform is called as a pulse code modulation (PCM) waveform.
- There are various types of PCM waveforms which are used in **telephony**. These waveforms are called as **Line codes**.

### 3.12.2 M-ary Pulse Modulation Waveforms :

- When pulse modulation is applied to a non-binary symbol the resulting waveform is called as M-ary pulse modulation waveform.
- There are several types of M-ary pulse modulation waveforms. One of the example is Pulse Amplitude Modulation (PAM). The other types are shown in Fig. 3.12.2.

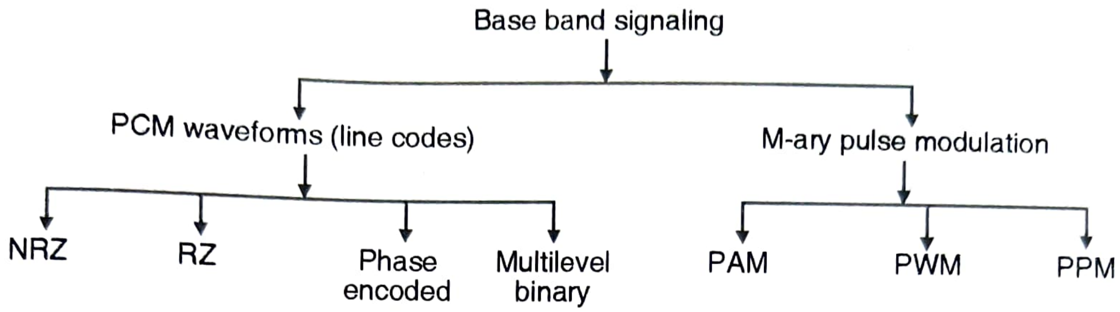



**Fig. 3.12.2 : Types of baseband digital transmission**

Thus the types of PCM waveforms (line codes) are as follows :

1. Non-return to zero (NRZ)
2. Return to zero (RZ)
3. Phase encoded
4. Multilevel binary.

### 3.12.3 Why to use Line Codes ?

- The binary data which may be coming from a digital computer or some other source cannot be put directly (as it is) on the communication channel such as a telephone line because the format of the binary signal is not suitable for its direct transmission.
- Hence the data must be converted into a suitable format or line code and then transmitted over a communication channel. Various line codes have been discussed in chapter 4.

## 3.13 Linear Delta Modulation (D.M.) :

- In PCM system, N number of binary digits are transmitted per quantized sample. Hence the signaling rate and transmission channel bandwidth of the PCM system are very large.
- These disadvantages can be overcome by using the delta modulation.

**Principle of operation :**

- Delta modulation transmits only one bit per sample instead of N bits transmitted in PCM. This reduces its signaling rate and bandwidth requirement to a great extent.
- In the basic or linear D.M., a staircase approximated version of the sampled input signal is produced as shown in Fig. 3.13.1.
- The original signal and its staircase representation are then compared to produce a difference signal.
- And this difference signal is quantized into only two levels namely $\pm\delta$ corresponding to positive and negative difference respectively.
- That means if the approximated signal $x'(t)$ lies below $x(t)$ at the sampling instant, then the approximated signal is increased by "$\delta$". (See instants 1, 2, 3, 4, 5 and 6 in Fig. 3.13.1.)
- Whereas if $x'(t)$ is greater than $x(t)$ at the sampling instant, then $x'(t)$ is decreased by "$\delta$" (see instants 7, 9 and 10 in Fig. 3.13.1.)

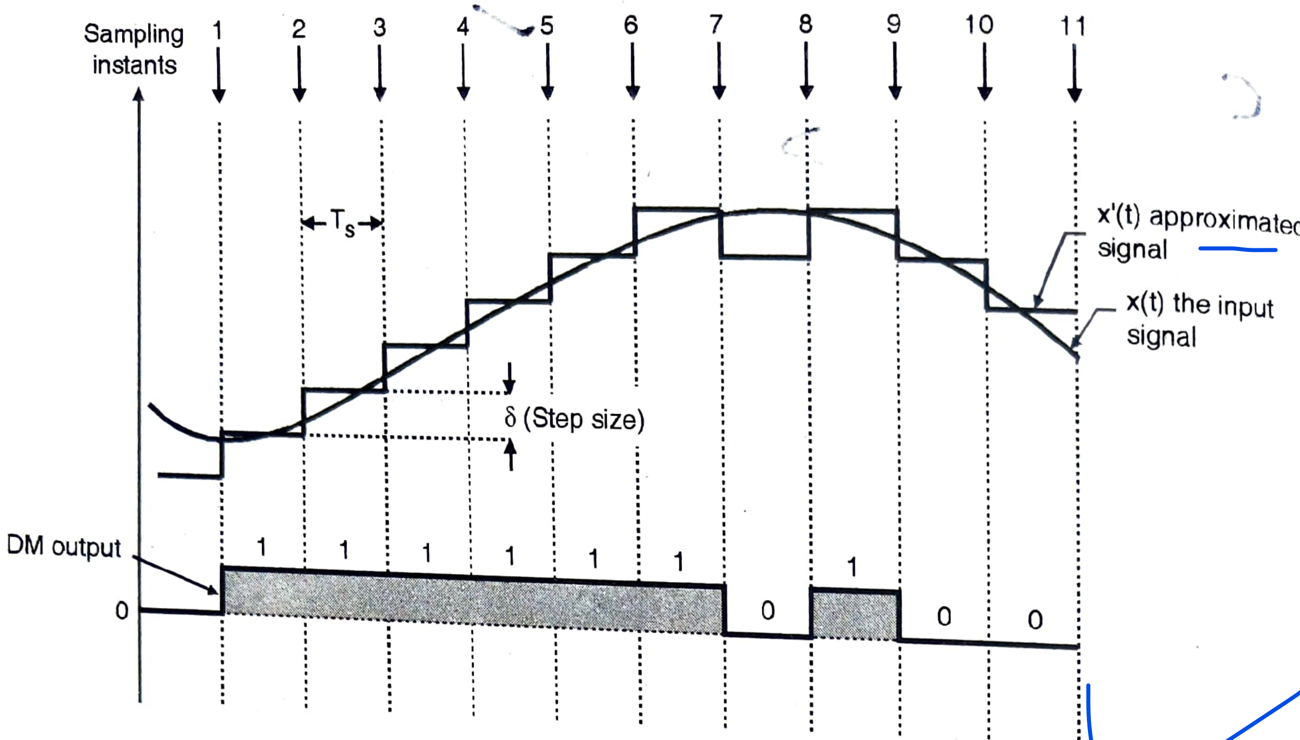



**Fig. 3.13.1 : D.M. Waveforms**

**D.M. output :**

- As shown in Fig. 3.13.1, the D.M. output is 1 if the staircase signal $x'(t)$ is increased by "$\delta$" i.e. at sampling instants 1, 2, 3, 4, 5 and 6.
- Whereas D.M. output is 0 if $x'(t)$ is decreased by "$\delta$" i.e. at sampling instants 7, 9 and 10.

**In delta modulation, the present sample value x (t) is compared with the approximate value x′ (t) and the result of this comparison is transmitted. Thus we are sending the information of whether the present sample value is higher than or lower than the approximate value. Note that the actual sampled value is not being transmitted.**

## 3.13.1 D.M. Transmitter :

- The block diagram of a D.M. transmitter is as shown in Fig. 3.13.2.

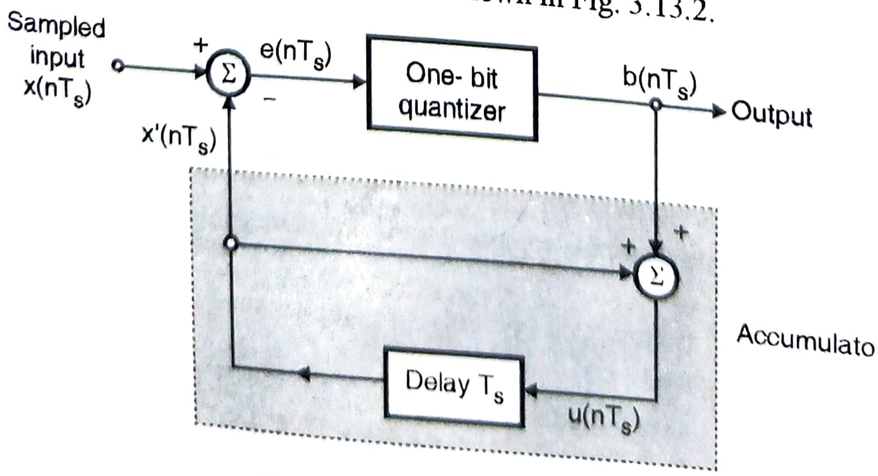



**Fig. 3.13.2 : D.M. transmitter**

- The sampled input signal $x(nT_s)$ and its approximated version $x'(nT_s)$ are compared with each other.
- The difference between them is produced as $e(nT_s)$ which is given by,

$$e(nT_s) = x(nT_s) - x'(nT_s) = x(nT_s) - u(nT_s - T_s)$$

- Because $x'(nT_s) = u(nT_s - T_s)$. The delay unit output is thus the previous value of $x(nT_s)$.
- This is applied to a one bit quantizer that quantizes this difference $e(nT_s)$ into a two bit signal $b(nT_s)$.

$$b(nT_s) = \delta \operatorname{sgn}[e(nT_s)] \quad \text{...(3.13.1)}$$

- If $e(nT_s)$ is positive then $\operatorname{sgn}[e(nT_s)]$ is +1 and if $e(nT_s)$ is negative then $\operatorname{sgn}[e(nT_s)]$ is $-1$.

$$\therefore \quad b(nT_s) = \pm\delta$$

- The input-output characteristic of the two-level quantizer is shown in Fig. 3.13.3.

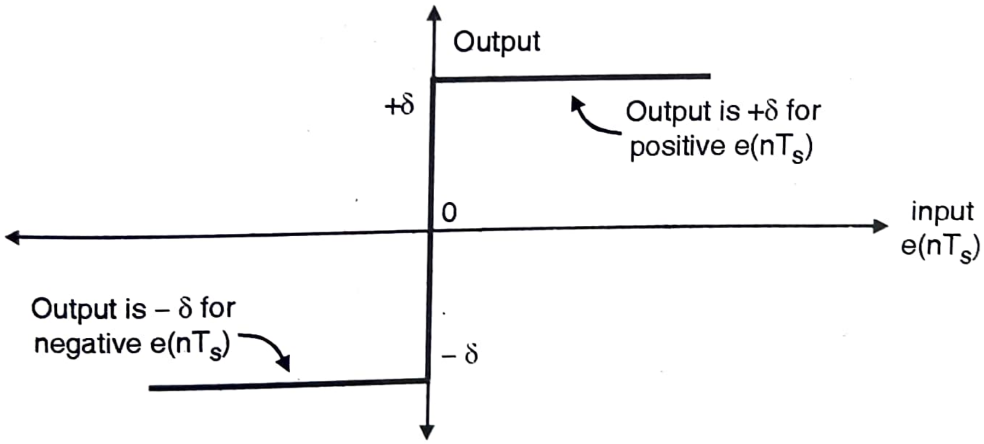



**Fig. 3.13.3 : Input-output characteristic of two level quantizer**

- The input to the delay unit in the transmitter is given by,

$$u(nT_s) = u(nT_s - T_s) + b(nT_s)$$

where $u(nT_s - T_s) = x'(nT_s)$ and $T_s$ is the sampling period.
- $e(nT_s)$ is the prediction error that represents the difference between the present sample $x(nT_s)$ and its previous approximate value $x'(nT_s)$ where $x'(nT_s) = u(nT_s - T_s)$.
- The binary quantity $b(nT_s)$ at the output of D.M. transmitter is the algebraic sign of error $e(nT_s)$ except for the scaling factor $\delta$.
- Thus $b(nT_s)$ is a single bit word transmitted by the D.M. system.
- The **accumulator** in the D.M. transmitter circuit is initially set to zero. The accumulator output is given by,

$$u(nT_s) = \delta \sum_{i=1}^{n} \text{sgn}[e(iT_s)] \qquad ...(3.13.2)$$

$$\text{But} \quad \text{sgn}[e(iT_s)] = \pm 1$$

$$\therefore \quad u(nT_s) = \delta \Sigma \pm 1 \qquad ...(3.13.3)$$

**Summary of operation of D.M. transmitter :**

1. Sampled input $x(nT_s)$ is applied at the input.
2. Accumulator produces a staircase approximated version $x'(nT_s)$ of the previous sample value.
3. The prediction error is produced by taking the difference between $x(nT_s)$ and $x'(nT_s)$.

$$e(nT_s) = x(nT_s) - x'(nT_s)$$

4. The prediction error is quantized into a single bit (0 or 1) by the one bit quantizer.
5. The quantizer output is transmitted as D.M. signal. The accumulator uses the same signal to produce the delayed approximate signal $x'(t)$.

### 3.13.2 Delta Modulator Transmitter (Another Method) : New Syll. PU - Dec. 2010

The block diagram of a delta modulator transmitter is as shown in the Fig. 3.13.4.

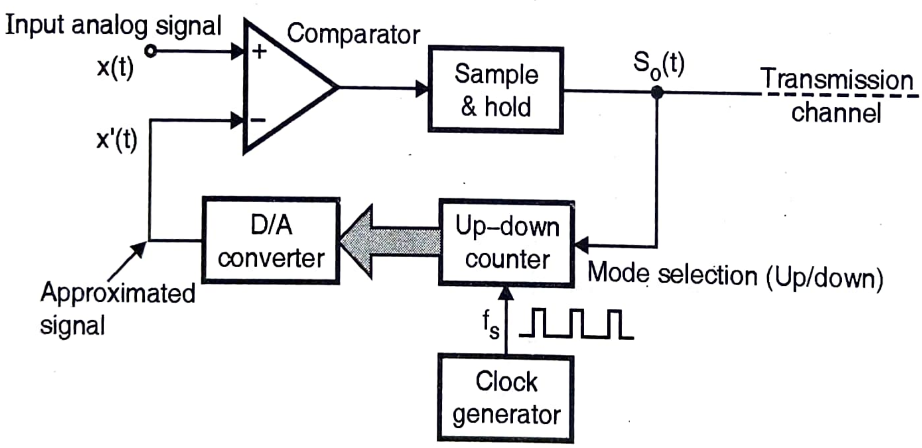



**Fig. 3.13.4 : D.M. transmitter**

**The operation of the circuit is as follows :**

- $x(t)$ is the analog input signal and $x'(t)$ is the quantized (approximated) version of $x(t)$. Both these signals are applied to a comparator.
- The comparator output goes high if $x(t) > x'(t)$ and it goes low if $x(t) < x'(t)$. Thus the comparator output is either 1 or 0. The sample and hold circuit will hold this level (0 or 1) for the entire clock cycle period.
- The output of the sample and hold circuit is transmitted as the output of the DM system. Thus in DM, the information which is transmitted is only whether $x(t) > x'(t)$ or vice versa. Also note that one bit per clock cycle is being sent. This will reduce the bit rate and hence the BW.

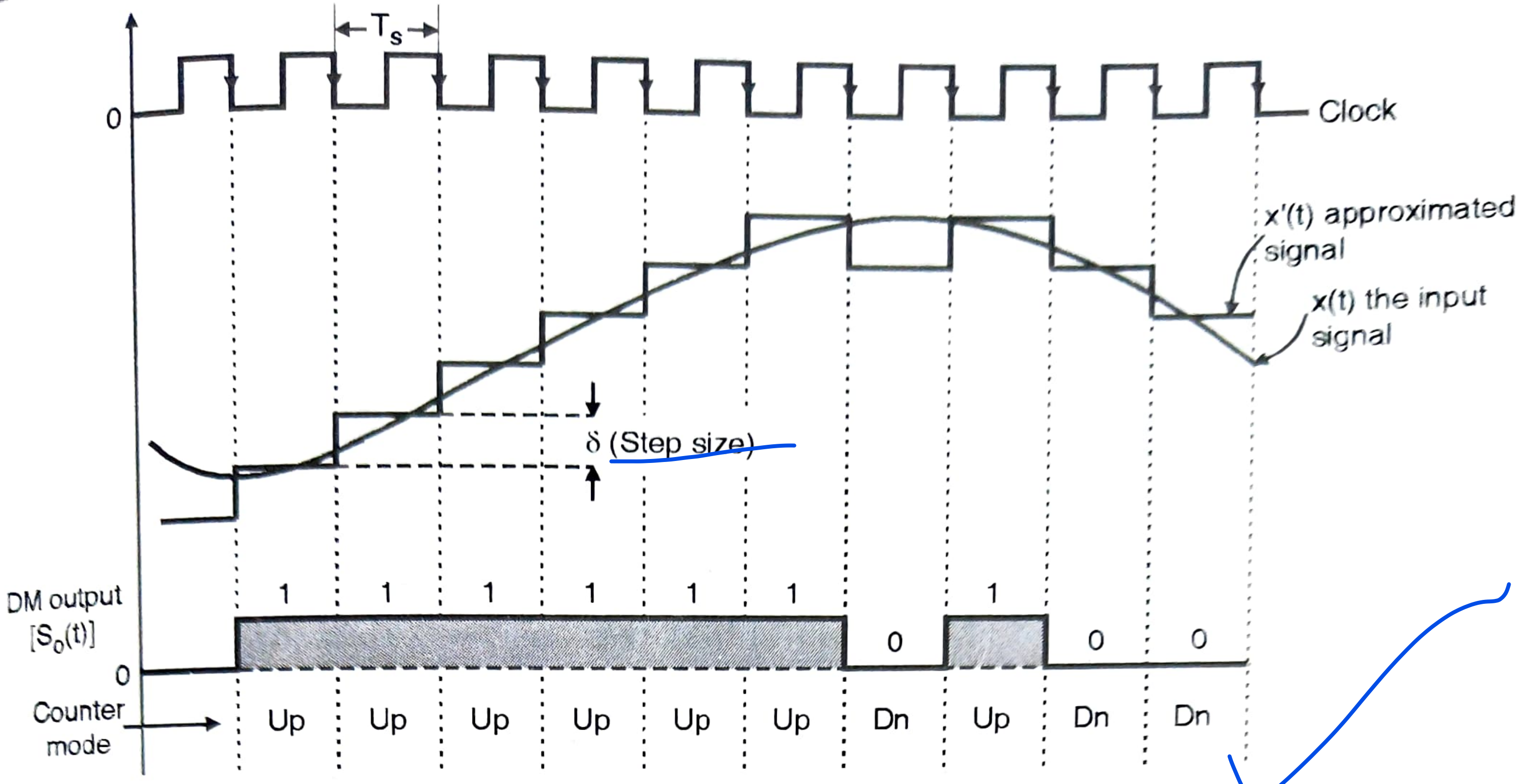



**Fig. 3.13.5 : D.M. waveforms**

- The transmitted signal is also used to decide the mode of operation of an up/down counter. The counter output increments by 1 if $S_o(t) = 1$ and it decrements by 1 if $S_o(t) = 0$, at the falling edge of each clock pulse. This is as shown in the waveform in the Fig. 3.13.5.
- The counter output is converted into analog signal by a D to A converter. Thus we get the approximated signal $x'(t)$ at the output of the D to A converter.

### 3.13.3 D.M. Receiver :

- The block schematic of D.M. receiver is shown in Fig. 3.13.6.
- The D.M. signal is passed through the accumulator to produce the staircase approximation in a manner similar to that used at the transmitter.
- The accumulator output is then applied to a low pass filter to produce the original signal.

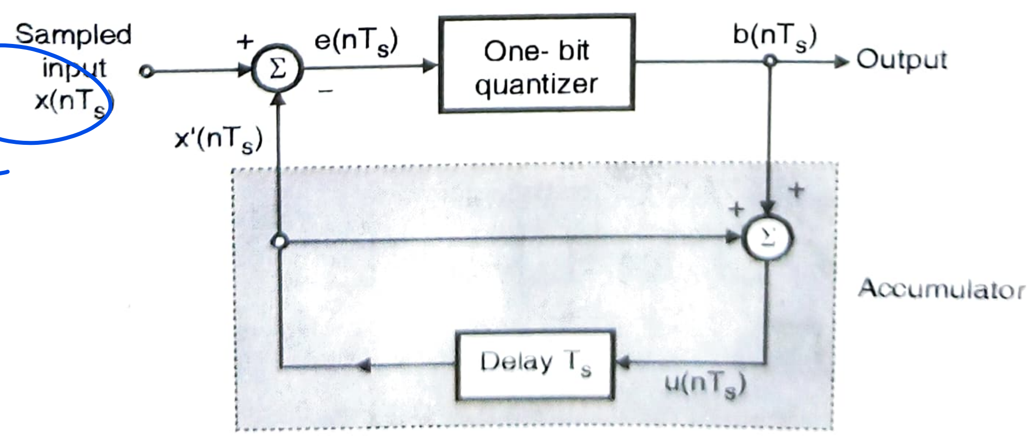



**Fig. 3.13.6 : D.M. receiver**

### 3.13.4 D.M. Receiver (Alternate Method) :

- The block diagram of the D.M. receiver is as shown in Fig. 3.13.7.

- Compare it with the transmitter block diagram, you will find that it is identical to the chain of blocks producing the signal $x'(t)$ i.e. the approximated signal.
- The original modulating signal can be recovered back by passing this signal through a low pass filter.

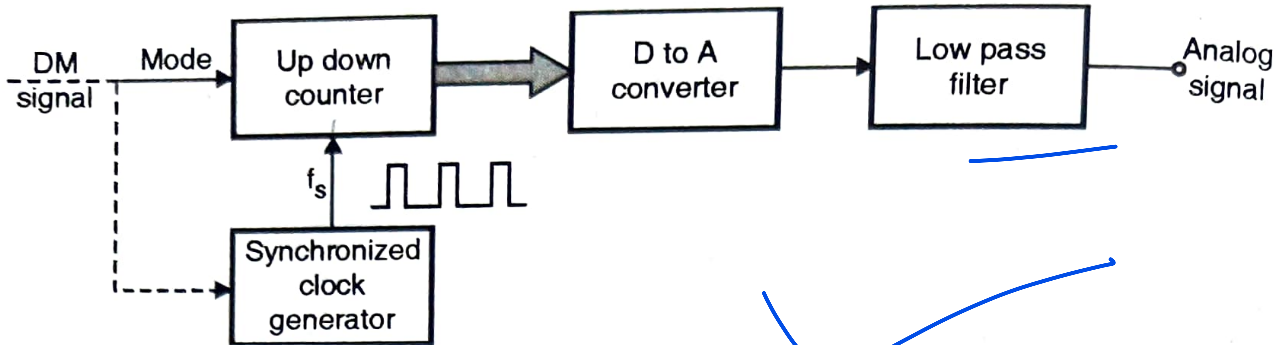



**Fig. 3.13.7 : D.M. receiver**

### 3.13.5 Comparison of D.M. and DPCM :

- The comparison of D.M. and DPCM systems revels that except for an output low pass filter, they are identical.
- So D.M. is actually a special case of DPCM.

### 3.13.6 Features of D.M. :

- A one bit codeword for output. Hence no need of framing.
- Simplicity of design for transmitter and receiver.

### 3.13.7 Applications of D.M. :

- For some types of digital communications.
- For digital voice storage.

### 3.13.8 Quantization Noise (Distortions) in the DM System :

The DM system is subjected to two types of quantization error :

1. Slope overload distortion and
2. Granular noise.

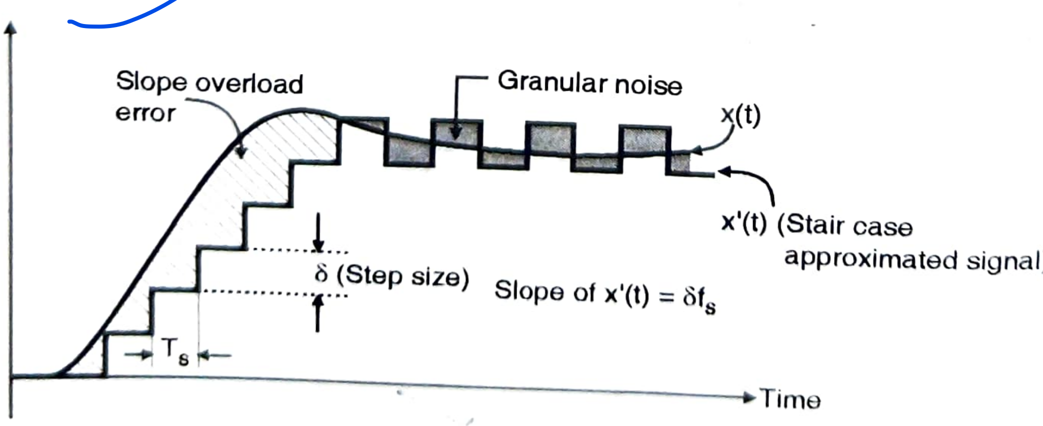



**Fig. 3.13.8 : Distortions in D.M.**

**1. Slope overload distortion :**

Look at the Fig. 3.13.8. Due to small step size ($\delta$), the slope of the approximated signal $x'(t)$ will be small.

$$\text{The slope of } x'(t) = \frac{\delta}{T_s} = \delta f_s \qquad \ldots(3.13.4)$$

- If slope of the analog signal x (t) is much higher than that of $x'(t)$ over a long duration then $x'(t)$ will not be able to follow x (t), at all.
- The difference between x (t) and $x'(t)$ is called as the slope overload distortion.
- Thus the slope overload error occurs when slope of x (t) is much larger than slope of $x'(t)$.
- The slope overload error can be reduced by increasing slope of the approximated signal $x'(t)$.
- Slope of $x'(t)$ can be increased and hence the slope overload error can be reduced by either increasing the step size "$\delta$" or by increasing the sampling frequency $f_s$.
- However with increase in $\delta$ the granular noise increases and if $f_s$ is increased, signaling rate and bandwidth requirements will go up.

**2. Granular noise :**

- When the input signal x (t) is relatively constant in amplitude, the approximated signal $x'(t)$ will hunt above and below x (t) as shown in Fig. 3.13.5.
- The granular noise is similar to the quantization noise in the PCM system.
- It increases with increase in the step size $\delta$. To reduce the granular noise, the step size should be as small as possible.
- However this will increase the slope overload distortion.
- In the linear delta modulator the step size $\delta$ is not variable. If it is made variable then the slope overload distortion and granular noise both can be controlled.
- A system with a variable step size is known as the adaptive delta modulator (ADM).

**D.M. bit rate (signaling rate) :**

- D.M. bit rate (r) = Number of bits transmitted / second

  = Number of samples/sec $\times$ Number of bits/sample $= f_s \times 1 = f_s$ ...(3.13.5)
- Thus the D.M. bit rate is (1/N) times the bit rate of a PCM system, where N is the number of bits per transmitted PCM codeword.
- Hence the channel bandwidth for the D.M. system is reduced to a great extent as compared to that for the PCM system.

### 3.13.9 Advantages of Delta Modulation : [redacted]

1. Low signaling rate and low transmission channel bandwidth, because in delta modulation, only one bit is transmitted per sample.
2. The delta modulator transmitter and receiver are less complicated to implement as compared to PCM.

### 3.13.10 Disadvantages of Delta Modulation :

1. The two distortions discussed earlier i.e. slope overload error and granular noise are present.
2. Practically the signaling rate with no slope overload error will be much higher than that of PCM.

The slope overload error can be reduced by using another type of delta modulation, called as adaptive delta modulation (ADM).

### 3.13.11 Condition for Avoiding the Slope Overload Error :

[redacted]

Refer the following example to derive the condition for avoiding the slope overload error.

---

**Ex. 3.13.1 :** Consider a sinusoidal signal $x(t) = A\cos(\omega_m t)$ applied to a delta modulator with a step size $\delta$. Show that the slope overload distortion will occur if

$$A > \frac{\delta}{\omega_m T_s} = \frac{\delta}{2\pi}\left(\frac{f_s}{f_m}\right)$$ where $T_s$ is the sampling period.

[redacted]

**Soln. :**

- Let the input signal be sinusoidal with amplitude A volts and frequency $f_m$ Hz as shown in Fig. P. 3.13.1.
- The given signal is $x(t) = A\cos\omega_m t$
- The slope of this signal will be maximum when derivative of x (t) with respect to time is maximum.

$$\therefore \quad \text{Slope of } x(t) = \frac{dx(t)}{dt} = -A\,\omega_m \sin\omega_m t$$

$$\text{The maximum value of the slope of } x(t) = -A\,\omega_m \qquad ...(1)$$

$$\text{Slope of the staircase approximated signal } x'(t) = \frac{\delta}{T_s} \qquad ...(2)$$

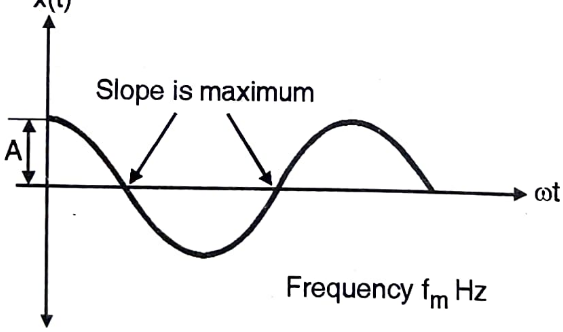



**Fig. P. 3.13.1 : Input signal x (t)**

- To avoid the slope overload distortion, it is necessary that the maximum slope of x (t) be less than the slope of x′ (t).

$$\therefore \quad \left|\frac{dx(t)}{dt}\right|_{max} \le \frac{\delta}{T_s}$$

$$\therefore \quad A\,\omega_m \le \frac{\delta}{T_s}$$

$$\therefore \quad A \le \frac{\delta}{\omega_m T_s}$$

**This is the condition for avoiding the slope overload distortion.** Therefore the slope overload distortion will occur if this condition is not satisfied i.e.

$$\textbf{If } A > \frac{\delta}{\omega_m T_s} \qquad ...(3.13.6)$$

### 3.13.12 Maximum Output Signal to Noise Ratio :

**PU : Dec. 2005, May 2007, New Syll.: May 2012**

Refer the following example to prove that the maximum signal to noise ratio of a D.M. system is given by,

$$\frac{S}{N_q} = \frac{3}{8\,\pi^2 f_m^2\, f_M\, T_s^3} \qquad \text{...(3.13.7)}$$

where $f_M$ = Cutoff frequency of the low pass filter in the D.M. receiver.

---

**Ex. 3.13.2 :** For a sinusoidal modulating signal

$m(t) = A \cos \omega_m t \qquad \omega_m = 2\pi f_m$

show that the maximum output signal to quantization noise ratio in a DM system with no slope overload distortion is given by,

$$\left(\frac{S}{N_q}\right) = \frac{3}{8\,\pi^2 f_m^2\, f_M\, T_s^3}$$

where $f_s$ = sampling frequency and $f_M$ = cutoff frequency of a low pass filter at the output of a receiver.

**Soln. :**

- From Equation (3.13.6), the condition for avoiding the slope overload error i.e.

$$A < \frac{\delta}{\omega_m T_s} = \frac{\delta}{2\pi}\left(\frac{f_s}{f_m}\right) \qquad \text{...(1)}$$

- Therefore the maximum value of the output signal power is given by,

$$P_{max} = \left[\frac{A}{\sqrt{2}}\right]^2 \quad \text{(As P is proportional to square of rms value).}$$

$$\therefore \quad P_{max} = \frac{A^2}{2} = \frac{\delta^2 f_s^2}{8\,\pi^2 f_m^2} \qquad \text{...(2)}$$

- Now we need to obtain the expression for quantization noise power. The quantizing error in D.M. is equally likely to lie anywhere in the interval $(-\delta, \delta)$.

  i.e. the maximum quantization error $\epsilon_{max} = \pm\,\delta$.

- This error can be assumed to be uniformly distributed as shown in Fig. P. 3.13.2(a).

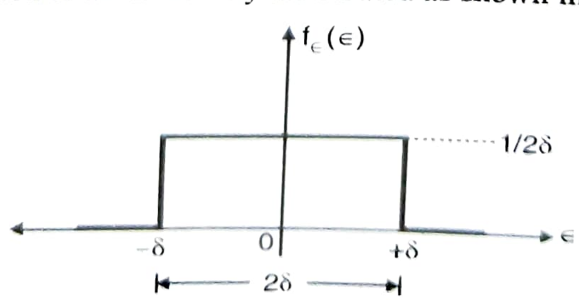



**Fig. P. 3.13.2(a) : PDF of quantization error for delta modulation**

- The PDF is thus uniform distribution defined as follows :

$$f_\epsilon(\epsilon) = \frac{1}{2\delta} \qquad \ldots \; -\delta \le f_\epsilon(\epsilon) \le +\delta$$

$$= 0 \qquad \ldots \text{ elsewhere}$$

- The mean square value or the variance of the quantization noise is given by,

$$\overline{\epsilon^2} = \int_{-\delta}^{\delta} \epsilon^2 f_\epsilon(\epsilon)\, d\epsilon = \int_{-\delta}^{\delta} \epsilon^2 \frac{1}{2\delta}\, d\epsilon = \frac{1}{2\delta}\left[\frac{\epsilon^3}{3}\right]_{-\delta}^{\delta}$$

$$= \frac{1}{2\delta}\left[\frac{\delta^3}{3} + \frac{\delta^3}{3}\right] = \frac{1}{2\delta} \times \frac{2\delta^3}{3}$$

$$\therefore \quad \overline{\epsilon^2} = \frac{\delta^2}{3} \qquad \ldots(3)$$

$\therefore$ Normalized quantization noise power $N_q = \frac{\overline{\epsilon^2}}{1} = \frac{\delta^2}{3}$ ...(4)

- The DM signal is passed through a reconstruction low pass filter at the output of a DM receiver. The bandwidth of this low pass filter is $f_M$ such that,

$$f_M \ge f_m \quad \text{and} \quad f_M << f_s$$

- The arrangement of filter is shown in Fig. P. 3.13.2(b).

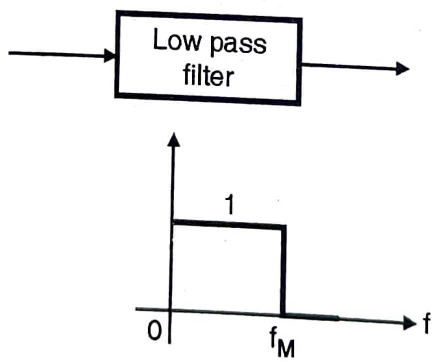



**Fig. P. 3.13.2(b) : Low pass filter at D.M. receiver**

- Now assuming that the quantization noise power $N_q$ is distributed uniformly over the frequency band upto $f_s$, the output quantization noise power within the bandwidth $f_M$ is given by,

Normalized noise power at the filter output, $N_q' = \frac{\delta^2}{3} \times \frac{f_M}{f_s}$ ...(5)

- So substituting the values from Equations (2) and (4) we get the expression for output signal to quantization noise ratio as,

$$\left[\frac{S}{N_q}\right]_o = \frac{P_{max}}{N_q'} = \frac{\delta^2 f_s^2}{8\pi^2 f_m^2} \times \frac{3 f_s}{\delta^2 f_M}$$

$$\left[\frac{S}{N_q}\right]_o = \frac{3 f_s^3}{8 \pi^2 f_m^2 f_M} \qquad ...(6)$$

- But $f_s = 1/T_s$

$$\therefore \quad \left[\frac{S}{N_q}\right]_o = \frac{3}{8 \pi^2 f_M f_m^2 T_s^3} \qquad ...(7)$$

This is the desired expression for the output signal to quantization noise ratio

### 3.13.13 Examples on D.M. :

**Ex. 3.13.3 :** A sinusoidal voice signal $x(t) = \cos(6000 \pi t)$ is to be transmitted using either PCM or DM. The sampling rate for PCM system is 8 kHz and for the transmission with DM, the step size $\delta$ is decided to be of 31.25 mV. The slope overload error is to be avoided. Assume that the number quantization levels for a PCM system is 64. Calculate the signaling rates of both these systems and comment on the result.

**Soln. :**

1. **Signaling rate of a PCM system :**

$$r = N f_s$$

$$\text{But } Q = 2^N \qquad \therefore N = \log_2 Q = \log_2 64 = 6$$

$$\therefore \text{Signaling rate of PCM} = r = 6 \times 8 \text{ kHz} = 48 \text{ kHz}$$ **...Ans.**

2. **Signaling rate of DM system :**

- The signaling rate of a DM system is equal to its sampling rate $f_s$ because in DM only one bit is transmitted per sample. We know that the condition to avoid the slope overload distortion is given by,

$$A \le \frac{\delta}{\omega_m T_s} \quad \text{or} \quad A \le \frac{\delta f_s}{2 \pi f_m}$$

- We want to calculate $f_s$

$$\therefore \quad f_s \ge \frac{2 \pi f_m A}{\delta}$$

- Substitute values to get

$$f_s \ge \frac{2 \pi \times 3 \times 10^3 \times 1}{31.25 \times 10^{-3}}$$

$$\therefore \quad f_s \ge 603.18 \text{ kHz}$$

$$\therefore \text{Signaling rate of DM} \ge 603.18 \text{ kHz}$$ **...Ans.**

**Comment :** To transmit the same voice signal, the DM needs a very large signaling rate as compared to PCM. This is the biggest disadvantage of DM, which makes it an impractical system.

**Ex. 3.13.4 :** Determine the output signal to noise ratio of a linear delta modulation system for a 2 kHz sinusoidal input signal sampled at 64 kHz. Slope overload error is not present and the post reconstruction filter has a bandwidth of 4 kHz.

**Soln. :**

$$(SNR)_o = \frac{3 f_s^3}{8 \pi^2 f_m^2 f_M}$$

Here, $f_s = 64$ kHz, $f_m = 2$ kHz and $f_M = 4$ kHz

$$\therefore \quad (SNR)_o = \frac{3 \times [64 \times 10^3]^3}{8 \pi^2 \times (2 \times 10^3)^2 \times 4 \times 10^3}$$

$$\therefore \quad (SNR)_o = 622.51 = 27.94 \text{ dB}$$

...**Ans.**

**Ex. 3.13.5 :** For the same sinusoidal input of Ex. 3.13.4 calculate the signal to quantization noise ratio of a PCM system which has the same data rate of 64 Kbits/s. The sampling frequency is 8 kHz and the number of bits per sample is N = 8. Comment on the result.

**Soln. :**

The signal to noise ratio of a PCM system is given by,

$$(SNR)_q = (1.8 + 6 N) \text{ dB} = 1.8 + (6 \times 8)$$

$$= 49.8 \text{ dB}$$

...**Ans.**

**Comment :** The SNR of a DM system is 27.94 dB which is to poor as compared to 49.8 dB of an 8 bit PCM system. Thus for all the simplicity of DM, it cannot perform as well as an 8 bit PCM.

## 3.13.14 Delta-Sigma Modulation :

Dec. 2007, May 2009, New Syll. : Dec. 2010

**Drawback of the conventional delta modulator :**

- The quantizer input in the conventional delta modulator can be considered as an approximation to the derivative of the input message signal.
- So the noise results in an accumulated error in the demodulated signal.
- This is the drawback of the conventional delta modulator.
- This drawback can be overcome by using the delta sigma modulator.
- In the delta sigma modulator, the input sign is passed through an integrator before applying it to the delta modulator circuit.

**Advantages of the integrator :**

The use of integrator has the following advantages :

1. The low frequency components in the input signal are boosted (pre-emphasized).

2. The correlation between the adjacent samples of delta modulator is increased. This improves the overall system performance, due to reduction in the variance of the error signal at the quantizer input.

3. It simplifies the receiver design.

**Block diagram :**

- Fig. 3.13.9(a) shows the block diagram of a delta sigma modulator.
- m(t) is the message signal which is a continuous time signal. It is first passed through an integrator and a comparator.
- The comparator output is applied to the pulse modulator block. It consists of a hard limiter, and a product, modulator.

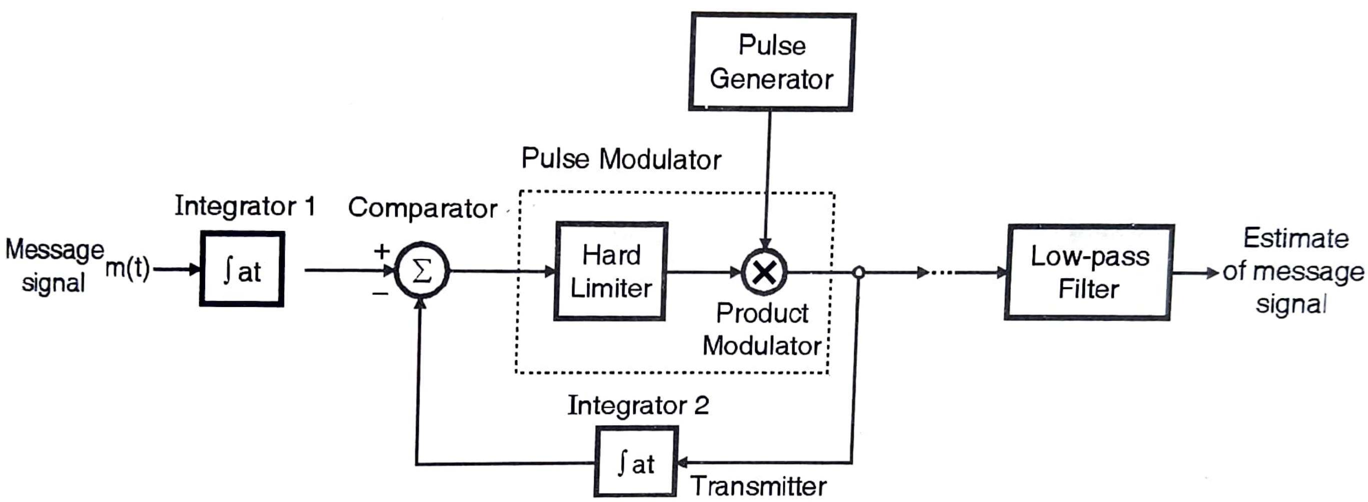



**Fig. 3.13.9(a) : Delta-sigma modulator**

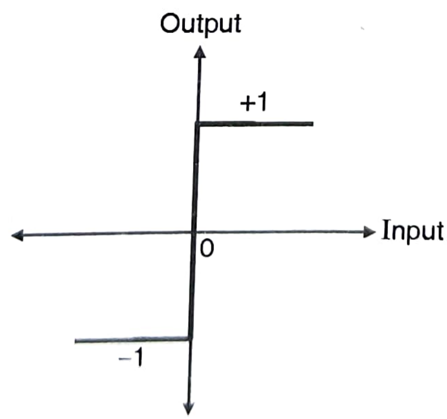



**Fig. 3.13.9(b) : Input output characteristics of hard limiter**

- The input output characteristics of the hard limiter is shown in Fig. 3.13.9(b). So if the input to this block is negative it will produce a – 1 output and for all the positive inputs it produces a + 1 output.
- The hard limiter output (± 1) is applied to a multiplier (product modulator). The other input to the multiplier is the clock pulses produced by the external pulse generator.
- The frequency of clock pulses should be higher than the Nyquist rate.
- At the output of the product modulator we get the sampled version of limiter output. It is transmitted over the communication channel. Thus we transmit a one bit encoded signal.
- The same output signal is applied to the second integrator.
- The output of integrator 2 is compared with the output of integrator 1 with the help of the comparator.

**Receiver :**

Fig. 3.13.10 shows that the receiver of sigma delta system is simply a low pass filter i.e. an integrator.

**Simplified sigma-delta system :**

- Since integration is a linear operation, we can combine the integrators 1 and 2 into one integrator which is placed after the comparator.

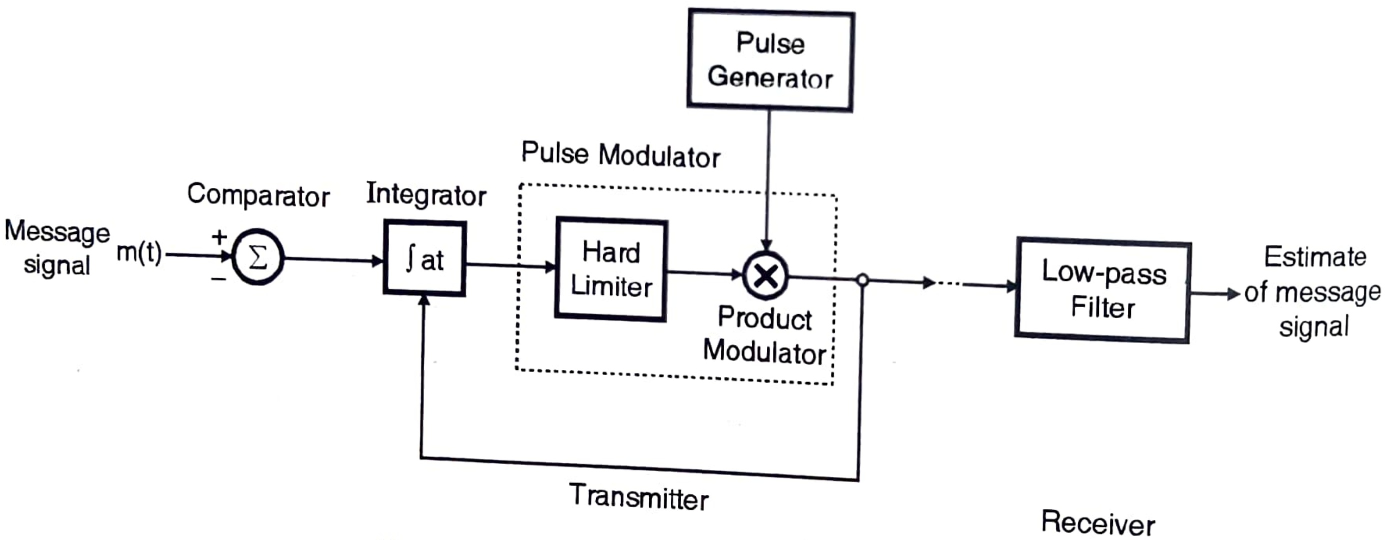



**Fig. 3.13.10 : Simplified sigma-delta system**

- This will simplify the transmitter design to some extent.
- The simplified sigma-delta system is shown in Fig. 3.13.10.

**Advantages of sigma-delta system :**

The sigma-delta system has following advantages over the conventional delta modulator.

## 3.15 Differential Pulse Code Modulation :

- In a PCM system, the signal x (t) is sampled at a rate which is slightly higher than the Nyquist rate.
- It is observed that the resulting sampled signal exhibits a high correlation between the adjacent samples.
- That means there is a correlation between the adjacent samples. It is observed that generally the signal x (t) does not change rapidly from one sample to the next.
- Hence the difference in amplitudes of adjacent samples is very small, as shown in Fig. 3.15.1.
- When these highly correlated samples are encoded in a standard PCM system, the resulting encoded signal contains redundant information.
- By removing this redundancy before encoding, we can obtain a more efficiently coded signal. The DPCM system operates on this principle. In DPCM system a special circuit called "predictor" is used.
- The "predictor" can actually predict the values of the future samples of x (t). This helps in reducing the redundancy.

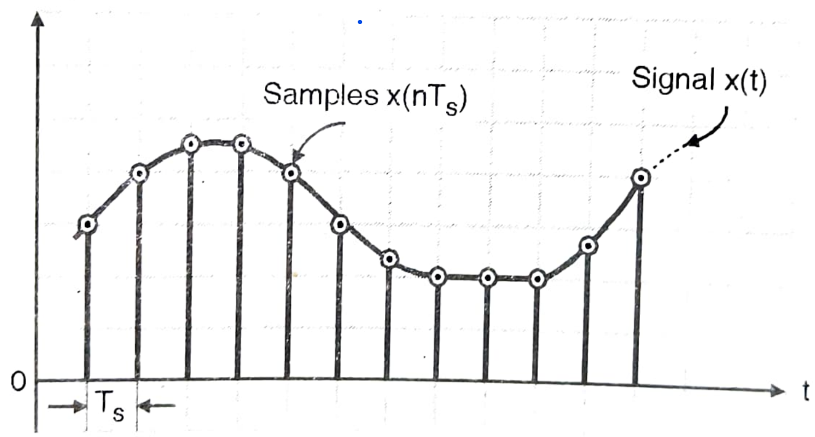



**Fig. 3.15.1 : Samples have a correlation between them**

### 3.15.1 Role of a Predictor :

- It is observed that if the sampling takes place at a rate which is higher than the Nyquist rate, then there is a correlation between successive samples of the signal $x(nT_s)$.
- Hence a knowledge of past sample values or differences helps us to predict the range of next required increment or decrement at the predictor output.
- This reduces the difference or error between $x(nT_s)$ and $x(nT_s)$. Therefore to encode this small value of error the DPCM system requires less number of bits which will ultimately reduce the bit rate. This is the role predictor in DPCM system.

## 3.15.2 DPCM Transmitter :

The DPCM transmitter is shown in Fig. 3.15.2.

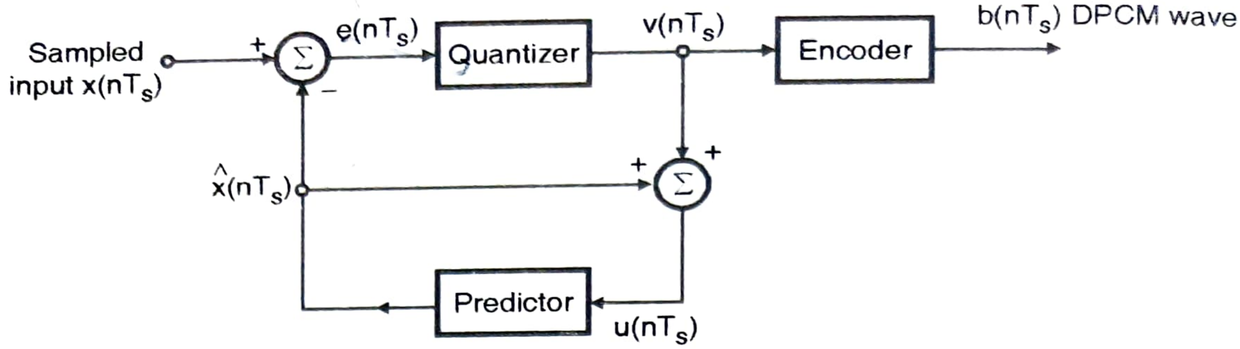



**Fig. 3.15.2 : DPCM transmitter**

**Operation :**

- Suppose that a baseband signal x (t) is sampled at a rate $f_s = 1/T_s$ to produce the sampled signal $x(nT_s)$. This signal acts as the input signal to the DPCM transmitter.
- Let the sequence of such samples be denoted by $\{x(nT_s)\}$. Where n is an integer.
- Let the predictor produce a predicted version of the sampled input and let the predictor output be denoted by $\hat{x}(nT_s)$.
- The predictor output is subtracted from the sampled input to obtain a difference signal $e(nT_s)$ as follows :

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{...(3.15.1)}$$

- The predictor value $\hat{x}(nT_s)$ is produced by the predictor whose input consists of quantized version of input signal $x(nT_s)$.
- The difference signal $e(nT_s)$ is called as **prediction error**, because it represents the difference between the sample and its predicted value.
- The quantizer output $v(nT_s)$ is encoded to obtain the digital pulses i.e. DPCM signal.
- Let the input output characteristics of the quantizer be denoted by a nonlinear function $Q(\cdot)$.
- So referring to Fig. 3.15.2 we get the quantizer output as,

$$v(nT_s) = Q[e(nT_s)]$$
$$= e(nT_s) + q(nT_s) \quad \text{...(3.15.2)}$$

where $q(nT_s)$ is the quantization error.

- Referring to Fig. 3.15.2 the predictor input is given by,

$$u(nT_s) = \hat{x}(nT_s) + v(nT_s) \quad \text{...(3.15.3)}$$

- Substituting the expression for $v(nT_s)$ we get

$$u(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \quad \text{...(3.15.4)}$$

- But $\hat{x}(nT_s) + e(nT_s) = x(nT_s)$

$$\therefore \quad u(nT_s) = x(nT_s) + q(nT_s) \quad \text{...(3.15.5)}$$

This is nothing but the quantized version of input $x(nT_s)$.

- Thus the quantized signal $u(nT_s)$ at the predictor input differs from the original input signal by $q(nT_s)$ i.e. the quantization error.

### 3.15.3 DPCM Receiver :

- The block diagram of a DPCM receiver is shown in Fig. 3.15.3.

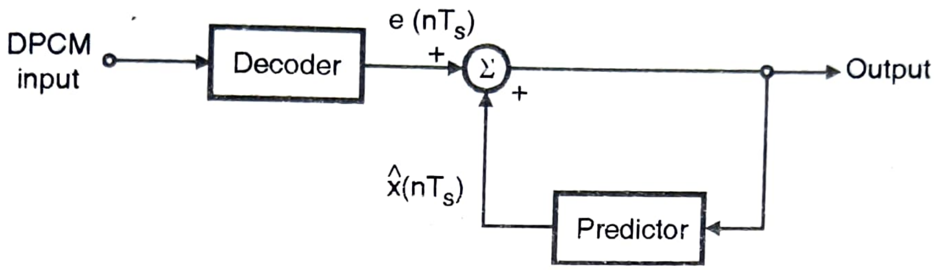



**Fig. 3.15.3 : DPCM receiver**

- The DPCM signal is applied to the decoder for reconstructing the quantized version of the input.
- The decoder output is actually the reconstructed quantized error signal.
- This signal is then added to the predictor output to produce the original signal.
- The predictor used at the receiver is same as that at the transmitter.

$$\text{Receiver output} = e(nT_s) + \hat{x}(nT_s)$$
$$= x(nT_s) \qquad ...(3.15.6)$$

### 3.15.4 Output Signal to Noise Ratio :

- The output signal to quantization noise ratio for a DPCM system can be defined in a similar way as that for a PCM system.

$$\therefore \quad (SNR) = \frac{\text{Mean square value of signal}}{\text{Mean square value of quantization noise}}$$

- But mean square value is equal to the variance.
- Hence,

$$(SNR) = \frac{\sigma_X^2}{\sigma_Q^2} \qquad ...(3.15.7)$$

Where $\sigma_X^2$ is the variance of original input signal $x(nT_s)$ and $\sigma_Q^2$ is the variance of the quantization error $q(nT_s)$.

- We can rearrange the above expression as follows

$$SNR = \frac{\sigma_X^2}{\sigma_E^2} \times \frac{\sigma_E^2}{\sigma_Q^2} \qquad ...(3.15.8)$$

Where $\sigma_E^2$ is the variance of the prediction error $e(nT_s)$

$$\therefore \quad SNR = G_p\,(SNR)_p \qquad ...(3.15.9)$$

Where $G_p = (\sigma_X^2 / \sigma_E^2)$ and called as prediction gain and $(SNR)_p = (\sigma_E^2 / \sigma_Q^2)$ is the prediction error-to-quantization noise ratio.

**Importance of prediction gain :**

- The prediction gain $G_p$ is defined as follows

$$G_p = \frac{\sigma_X^2}{\sigma_E^2} \qquad \text{...(3.15.10)}$$

- The prediction gain should be as high as possible. For a given baseband signal, the variance $\sigma_X^2$ is fixed hence to maximize $G_p$, we have to minimize the variance $\sigma_E^2$ of the prediction error $e(nT_s)$.
- The predictor should be designed accordingly.

### 3.15.5 Types of Predictors :

The predictors used for DPCM are

1. One-tap predictors
2. N-tap predictors

### 3.15.6 Advantage of DPCM :

- As the difference between $x(nT_s)$ and $\hat{x}(nT_s)$ is being encoded and transmitted by the PCM technique, a small difference voltage is to be quantized and encoded.
- This will need less number of quantization levels and hence less number of bits to represent them.
- Thus signalling rate and bandwidth of a DPCM system will be less than that of PCM.

## 3.16 Adaptive Delta Modulation (ADM) :

PU : Dec. 2007

- In the ADM system, the step size is not constant. Rather when the slope overload occurs the step size becomes progressive larger and therefore $x'(t)$ will catch up with $x(t)$ more rapidly.
- Whenever the slope of input signal is large, the step size of the staircase approximated signal $x'(t)$ is increased.
- On the other hand when the input signal is varying slowly the step size is reduced.
- Thus the step size is adapted as per the level of input signal.

### 3.16.1 Types of ADM :

- There are various types of ADM systems available depending on the type of scheme used for adjusting the step size.
- In one type a discrete set of values is provided for the step size whereas in another type a continuous range of step size variation is provided.
- We will discuss the first type here.

### 3.16.2 ADM Transmitter :

- Fig. 3.16.1 shows the ADM transmitter block diagram.
- If you compare this block diagram with that of the linear D.M., then you will find that the block named "logic for step size control" is the only addition. The remaining block schematic is same as that of linear D.M.

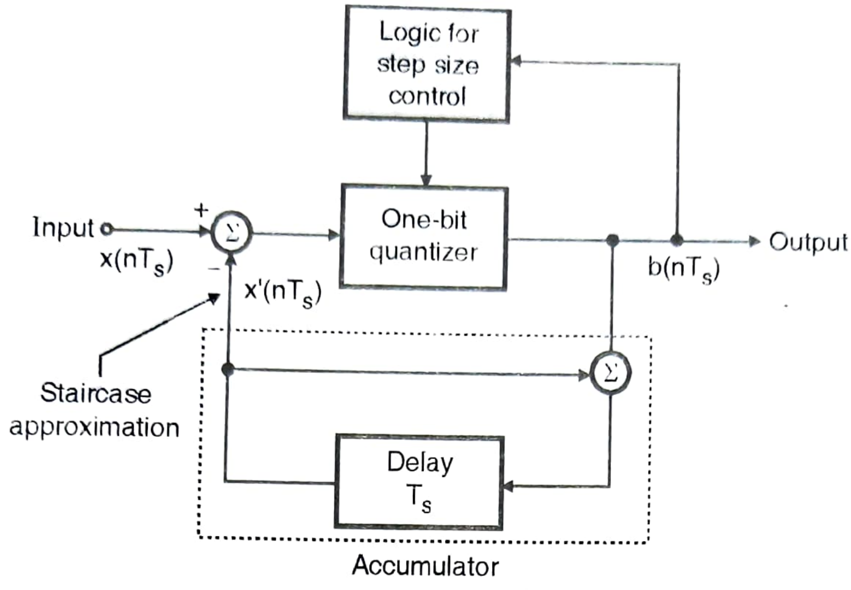



**Fig. 3.16.1 : ADM transmitter**

- Fig. 3.16.2 shows the staircase approximated waveform for an ADM system.

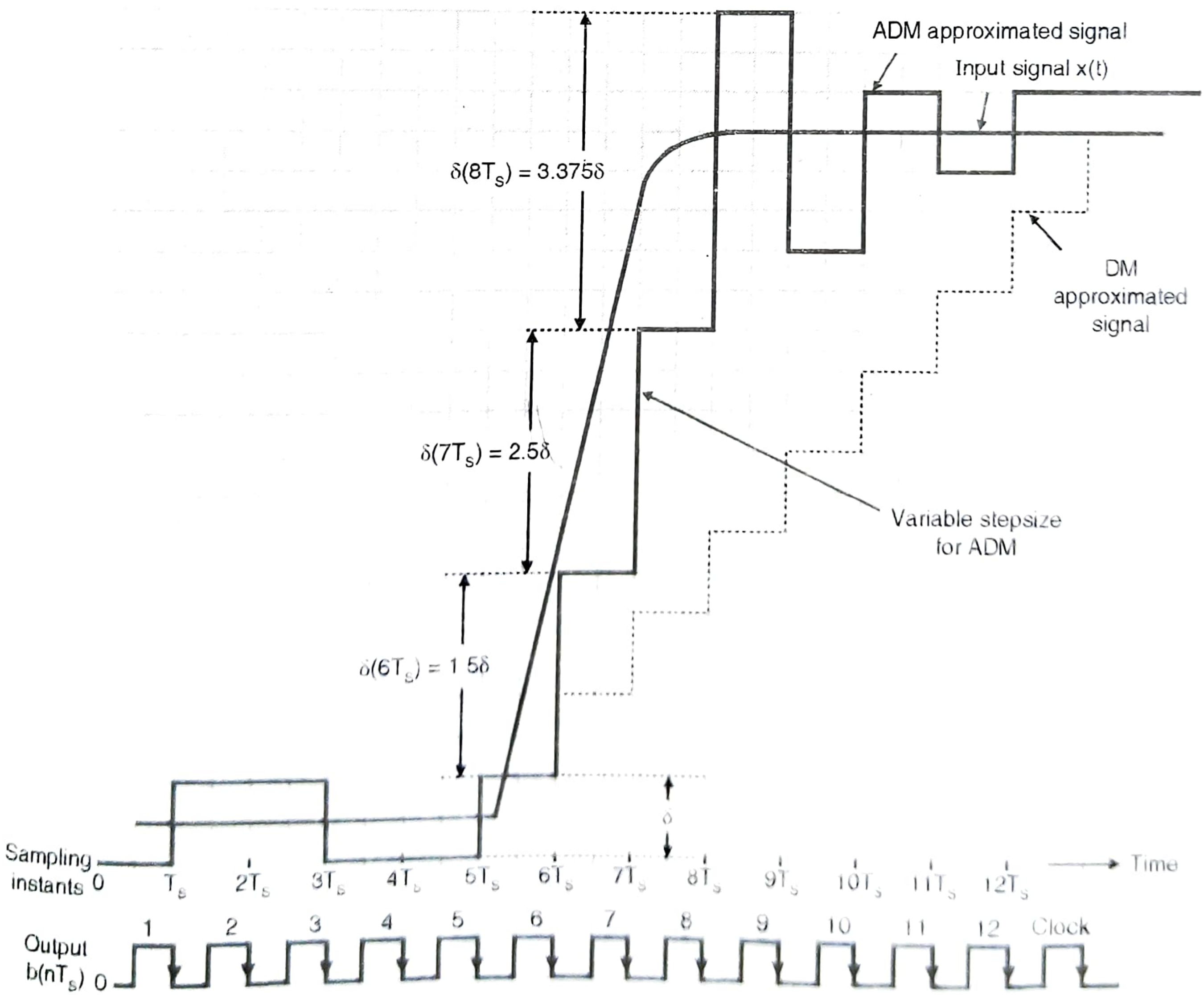



**Fig. 3.16.2 : Waveforms of ADM**

- Let the stepsize at various sampling instants be denoted by $\delta(n T_s)$ with "n" an integer. So from Fig. 3.16.2. the stepsize at n = 1 is $\delta(T_s) = \delta$ and the step size at n = 6 is $\delta(6 T_s) = 2\delta$. Similarly $\delta(7 T_s) = 3\delta$ and so on.

**Stepsize adjustment :**

The step size at any given sampling instant "$n T_s$" is obtained by using the following adaptation rule.

$$\delta(n T_s) = g(n T_s)\, \delta(n T_s - T_s) \qquad ...(3.16.1)$$

where $g(n T_s)$ is the time varying multiplier and its value depends on the present output $b(n T_s)$ and the previous output $b(n T_s - T_s)$ as follows :

$$g(n T_s) = K \qquad ... \text{if } b(n T_s) = b(n T_s - T_s) \qquad ...(3.16.2)$$

$$\text{and} \quad g(n T_s) = K^{-1} \qquad ... \text{if } b(n T_s) \neq b(n T_s - T_s) \qquad ...(3.16.3)$$

This adaptation algorithm is known as a constant factor ADM with one bit memory because we are utilizing only one previous bit $b(n T_s - T_s)$ to obtain the value of $g(n T_s)$.

**Illustration :**

- Let us illustrate now the use of Equation (3.16.1) in calculating the step size.
- Refer the waveforms of Fig. 3.16.2. Take n = 6 i.e. the 6[th] sampling instant, $t = 6 T_s$. The corresponding step size is given by,

$$\delta(6 T_s) = g(6 T_s)\, \delta(6 T_s - T_s)$$

$$= g(6 T_s)\, \delta(5 T_s) \qquad ...(3.16.4)$$

- But $g(6 T_s) = K$ ...since $b(6 T_s) = b(5 T_s)$

$$\therefore \quad \delta(6 T_s) = K\, \delta(5 T_s) \qquad ...(3.16.5)$$

- But $\delta(5 T_s) = \delta$ from Fig. 3.16.2

$$\therefore \quad \delta(6 T_s) = k\, \delta \qquad ...(3.16.6)$$

- The value of k is adjusted between 1 and 2. Practically it is observed that k = 1.5 yields the most desired results.

$$\therefore \quad \delta(6 T_s) = 1.5\, \delta \qquad ...(3.16.7)$$

- Similarly we can prove that

$$\delta(7 T_s) = 1.5\, \delta(6 T_s)$$

$$= 1.5 \times 1.5\, \delta = 2.25\, \delta$$

$$\text{and} \quad \delta(8 T_s) = k\, \delta(7 T_s) = 1.5 \times 2.25\, \delta = 3.375\, \delta$$

- As shown in Fig. 3.16.2, due to variable step size, the slope overload error is reduced. But quantization error is increased.

- Due to the adjustable step size, the slope overload problem is solved. Hence ADM system has a low bit rate than the PCM system.
- Therefore the BW required is also less than a comparable PCM system.

### 3.16.3 Adaptive Delta Modulation (Alternate Method) :

- In the ADM system, the step size is not constant. Rather when the slope overload occurs the step size becomes progressive larger and therefore $x'(t)$ will catch up with $x(t)$ more rapidly.
- The ADM transmitter is as shown in Fig. 3.16.3.

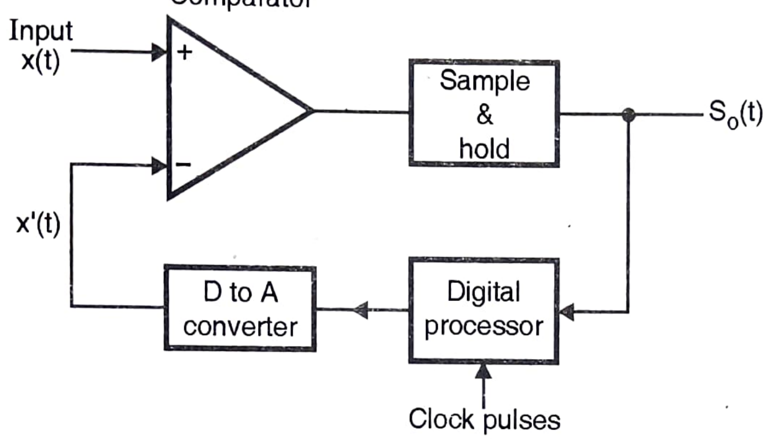



**Fig. 3.16.3 : ADM transmitter**

- If you compare this block diagram with that of the linear delta modulator, then you will find that except for the counter being replaced by the digital processor, the remaining blocks are identical.
- Let us understand the operation of the digital processor. For that carefully see the waveforms of Fig. 3.16.3.

**Operation :**

- In response to the $k^{th}$ clock pulse trailing edge, the processor generates a step which is equal in magnitude to the step generated in response to the previous i.e. $(k-1)^{th}$ clock edge.
- If the direction of both the steps is same, then the processor will increase the magnitude of the present step by "$\delta$". If the directions are opposite then the processor will decrease the magnitude of the present step by "$\delta$".
- $S_o(t)$ in the Fig. 3.16.3, i.e. the output of the ADM system is given as,

$$S_o(t) = +1 \text{ if } x(t) > x'(t) \text{ just before } k^{th} \text{ clock edge.}$$

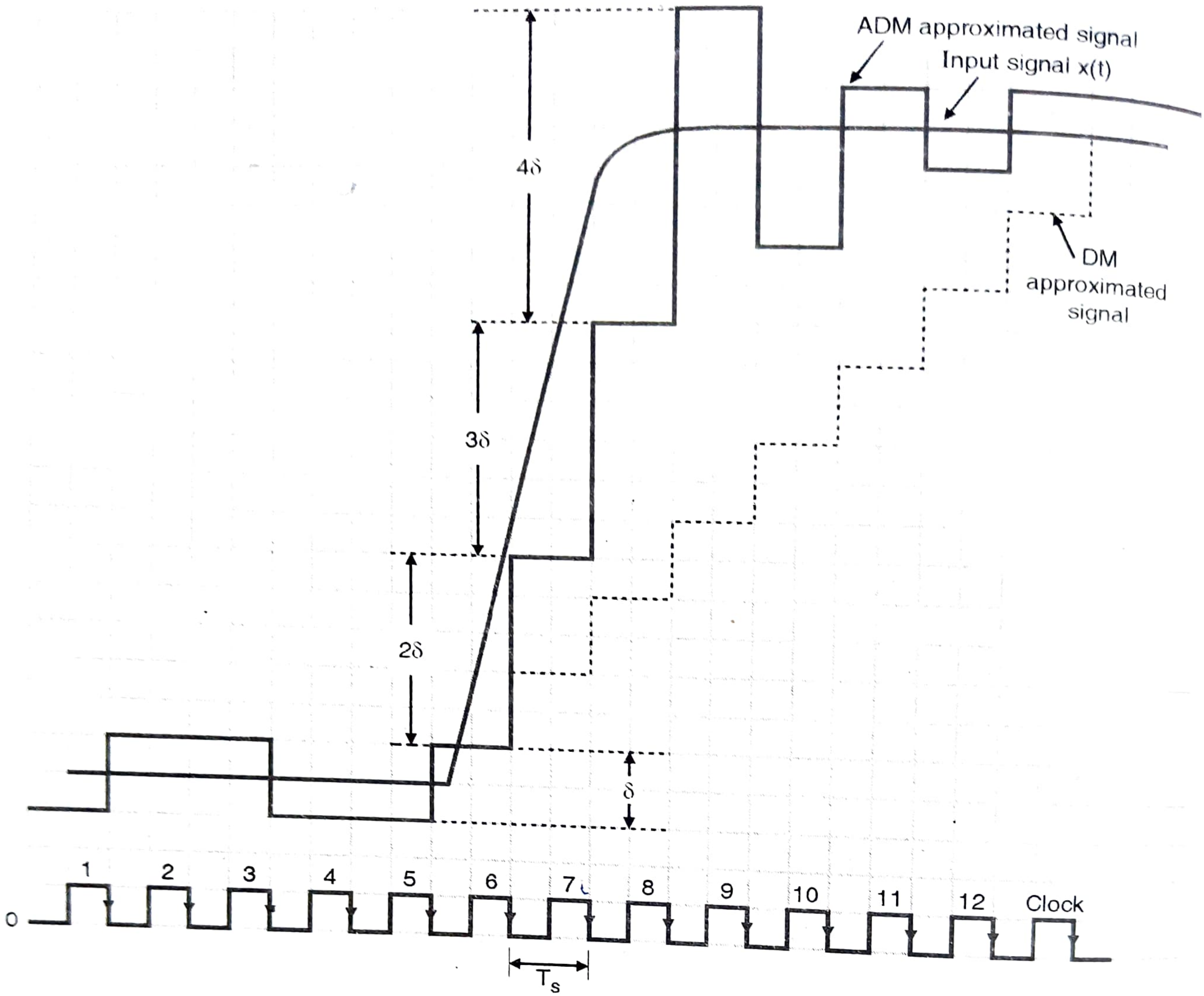



**Fig. 3.16.4 : Waveforms of ADM**

and $S_o(t) = -1$ if $x(t) < x'(t)$ just before the $k^{th}$ clock edge.

- Then the step size at the sampling instant k is given by,

$$\delta(k) = [\delta(k-1)]\; S_o(k) + \delta\; S_o(k-1) \qquad ...(3.16.8)$$

| $\delta(k)$ | $[\delta(k-1)]$ | $S_o(k)$ | $\delta$ | $S_o(k-1)$ |
|---|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ | ↓ |
| Step size at $k^{th}$ clock edge | Step size at $(k-1)^{th}$ clock edge | Output at $k^{th}$ edge | Basic step size | Output at $(k-1)^{th}$ clock edge |

- Let us take an example :

  Refer to the waveforms of Fig. 3.16.4. Let us assume $k = 6$, i.e. consider the $6^{th}$ clock edge.

  $\therefore \quad k - 1 = 5.$

$$\therefore \quad \delta(k-1) = \delta(5) = \delta$$

$$S_o(k) = S_o(6) = +1$$

$$S_o(k-1) = S_o(5) = +1$$

Substitute in Equation (3.16.8) to get,

$$\delta(6) = \delta + \delta = 2\delta \qquad \text{...(3.16.9)}$$

Look at the Fig. 3.16.4, the step size at the $6^{th}$ clock edge is $2\delta$.

- As shown in Fig. 3.16.4, due to variable step size, the slope overload error is reduced. But quantization error is increased. Due to the adjustable step size, the slope overload problem is solved. Hence ADM system has a low bit rate than the PCM system. Therefore the BW required is also less than a comparable PCM system.

### 3.16.4 Advantages of Adaptive Delta Modulation :

The advantages of ADM over DM are as follows :

1. Reduction in slope overload distortion and granular noise.
2. Improvement in signal to noise ratio.
3. Wide dynamic range due to variable step size.
4. Better utilization of bandwidth as compared to delta modulation.
5. Low signaling rate
6. Simplicity of implementation

### 3.16.5 ADM Receiver :

- The block diagram of ADM receiver is shown in Fig. 3.16.5.

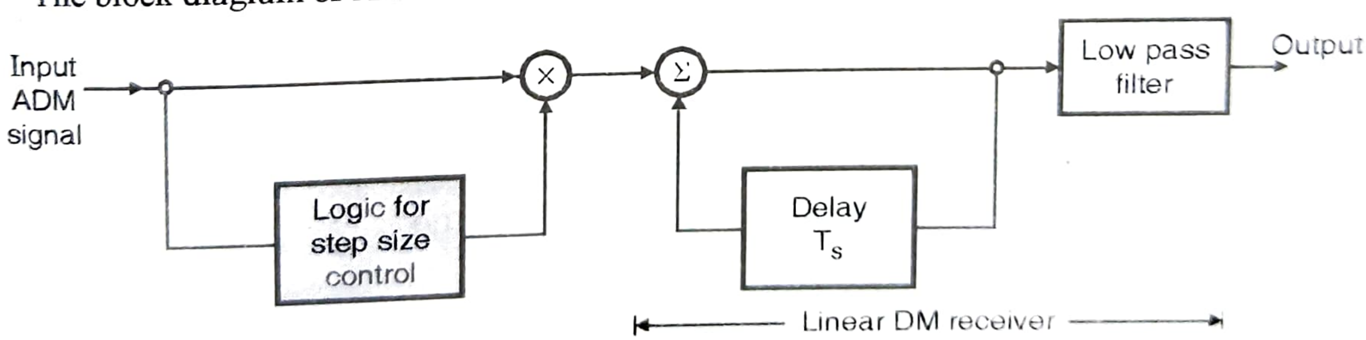



**Fig. 3.16.5 : ADM receiver**

- The ADM signal is first converted into a D.M. signal with the help of the step size control logic and then applied to a D.M. receiver.
- At the output of low pass filter we get the original signal back.

## 3.19 Comparison of Digital Pulse Modulation Systems :

The PCM, DM, ADM and DPCM all are digital pulse modulation systems. Table 3.19.1 shows the comparison of these systems.

**Table 3.19.1 : Comparison of PCM, DM, ADM and DPCM**

| Sr. No. | Parameter | PCM | DM | ADM | DPCM |
|---|---|---|---|---|---|
| 1. | Number of bits per sample | N can be 4, 8, 16, 32, 64 etc. | N = 1 | N = 1 | N is more than 1 but less than that for PCM |
| 2. | Step size | Depends on the number of Q levels | Step size is fixed | Step size is variable | Step size is fixed |

| Sr. No. | Parameter | PCM | DM | ADM | DPCM |
|---|---|---|---|---|---|
| 3. | Distortions / errors | Quantization error | Slope overload and granular noise | Granular noise | Slope overload and granular noise |
| 4. | Signaling rate and bandwidth | Highest | Low, if the input is slow varying | Lowest | Lower than PCM |
| 5. | System complexity | Complex | Simple | Simple | Simple |
| 6. | Feedback from output | No feedback | Feedback is present | Feedback is present | Feedback is present |
| 7. | Noise immunity | Very good | Very good | Very good | Very good |
| 8. | Use of repeaters | Possible | Possible | Possible | Possible |

## 3.20 Solved Examples :

**Ex. 3.20.1 :** A low pass signal of 3 kHz bandwidth and amplitude over – 5 Volts to + 5 Volts range is sampled at Nyquist rate and converted to 8-bit PCM using uniform quantization. The mean squared value of message signal is 2 Volt-squared.

Calculate :

1. The normalised power for quantization noise.
2. The bit transmission rate.
3. The signal to quantization noise ratio in dB.

Derive the expressions used in 1 and 3. **Dec. 1998, Dec. 2004, 12 Marks**

**Soln. :**

**Given :** $W = 3$ kHz, $V_L = -5$ V, $V_H = +5$ V, $N = 8$

Uniform quantization used.

$$\overline{x^2(t)} = \text{Mean square value of message signal is 2 Volt}^2$$

**1. Normalized power for quantization noise ($N_q$) :**

$$N_q = \frac{S^2}{12} \quad \text{where } S = \text{Step size} \qquad ...(1)$$

But
$$S = \frac{V_H - V_L}{Q} = \frac{V_H - V_L}{2^N}$$

$$\therefore \quad S = \frac{5-(-5)}{2^8} = \frac{10}{256} = 39.06 \text{ mV} \qquad ...(2)$$

Substitute in Equation (1) to get,

$$N_q = \frac{(39.06 \times 10^{-3})^2}{12} = 127.15 \times 10^{-6} \text{ W} \qquad \text{...Ans.}$$

2. **The bit transmission rate (r) :**

The bit transmission rate or signaling rate is the number of bits transmitted by the PCM system per second.

$$\therefore \quad r = N f_s$$

As the signal is sampled at Nyquist rate, $f_s = 2W$.

$$\therefore \quad r = 8 \times 2W$$

$$\therefore \quad r = 16 \times 3 \text{ kHz} = 48 \text{ kbits/sec}$$

...Ans.

3. **The signal to quantization noise ratio in dB :**

$$\text{The normalized signal power P} = \frac{\text{Mean square value of signal}}{1\Omega}$$

$$\therefore \quad P = \frac{2 \text{ Volt}^2}{1\,\Omega} = 2\text{Watt}$$

$$\therefore \quad (SNR)_q = \frac{P}{N_q} = \frac{2}{127.15 \times 10^{-6}} = 15728.64 \qquad ...(3)$$

$$\text{And } (SNR)_q \text{ in dB} = 10 \log_{10} (15728.64)$$

$$(SNR)_q = 41.96 \text{ dB}$$

...Ans.

---

**Ex. 3.20.2 :** A delta modulator systems is designed to operate at 5 times the Nyquist rate for a signal with 3 kHz bandwidth. Determine the maximum amplitude of 1.2 kHz input sinusoid for which the delta modulator does not have slope overload. Quantising step size is 250 mV. Derive the expression used.

[redacted]

**Soln. :**

It has been given that,

$W = 3$ kHz, $\quad f_m = 1.2$ kHz, $\quad f_s = 5 \times 2W = 30$ kHz, $\quad \delta = 250$ mV.

Let the maximum amplitude of 1.2 kHz input sinusoid for no slope overload error be "A". We have already derived the condition to avoid slope overload as,

$$A \le \frac{\delta}{\omega_m T_s}$$

$$\therefore \quad \text{Maximum value of A} = \frac{\delta}{\omega_m T_s} = \frac{\delta f_s}{2 \pi f_m} \qquad ...(1)$$

Substituting the values we get,

$$A = \frac{250 \times 10^{-3} \times 30 \times 10^3}{2 \pi \times 1.2 \times 10^3}$$

$$\therefore \quad A = 0.994 \text{ Volt}$$

For the derivation of the expression stated in Equation (1), refer to Ex. 3.13.1.

...Ans.

**Ex. 3.20.3 :** Consider audio signal comprised of sinusoid term s (t) = 3 cos 500 $\pi$ t.

1. Find signal to quantization noise ratio when this is quantised using 10 bit PCM.
2. How many bits of quantization are needed to achieve signal to quantization noise ratio of at least 40 dB ? [illegible]

**Soln. :**

**Given :** $s(t) = 3 \cos 500\pi t = 3 \cos (2\pi \times 250\, t)$

$N = 10,\ (SNR)_q = 40$ dB.

**1. Signal to quantization noise ratio :**

From the expression of s (t) it is clear that the input signal is purely sinusoidal. Hence the expression for signal to quantization noise ratio is given by :

$$(SNR)_q = (1.8 + 6\,N)\text{ dB}$$

$$= (1.8 + 6 \times 10) = 61.8\text{ dB} \qquad \text{...Ans.}$$

**2. Value of N for $(SNR)_q \geq 40$ dB :**

$$SNR_q \geq 40\text{ dB}$$

$$\therefore \quad (1.8 + 6N) \geq 40\text{ dB}$$

$$\therefore \quad 6N \geq 38.2\text{ dB}$$

$$\therefore \quad N \geq 6.36 \qquad \therefore N = 7 \qquad \text{...Ans.}$$

Hence the number of bits of quantization to achieve signal to quantization noise ratio of at least 40 dB is N = 7.

---

**Ex. 3.20.4 :** For a full scale sinusoidal modulating signal with peak value A, show that, output signal to quantization noise ratio in binary PCM system is given by, S/N = 1.76 + 20 log M dB where M = Number of quantization levels. A compact disc recording system samples each of the two stereo signals with a 16 bit A/D converter at 44.1 kb/sec.

(a) Determine output S/N ratio for a full scale sinusoid.

(b) The bit stream of digitized data is augmented by addition of error correcting bits, clock extraction bits etc. and these additional bits represent 100% overhead. Determine output bit rate of CD system.

(c) The CD can record an hour's worth of music. Determine number of bits recorded on CD.

**Soln. :**

**Given :** There are two stereo channels.

$N = 16,\ f_s = 44.1$ kbits/sec.

**(a) Output signal to noise ratio for full scale sinusoid :**

$$\left[\frac{S}{N_q}\right] = 1.76 + 6\,N = 1.76 + (6 \times 16)$$

$$= 97.76\text{ dB} \qquad \text{...Ans.}$$

**(b) Output bit rate of the CD system :**

The bit rate for each of two stereo channels $= N f_s$

$\therefore$ The bit rate of two channels $= 2 N f_s$

$= 2 \times 16 \times 44.1 \times 10^3$

$= 1.4112$ Mbits/sec ...(1)

Including the additional 100% overhead, the output bit rate is,

$2 \times 1.4112 \times 10^6$ b/s $= 2.822$ Mbits/sec ...**Ans.**

**(c) Number of bits recorded on CD :**

The CD can record an hour's worth of music.

$\therefore$ Number of bits recorded on CD = Bit rate $\times$ Number of seconds/hour

$= 2.822 \times 10^6 \times 3600$

$= 10.16 \times 10^9$ bits or 10.16 gigabits ...**Ans.**

---

**Ex. 3.20.5 :** Show that output $S/N_q$ ratio in a DM under system the assumptions of no slope overload is given by $S/N_q = 3 f_s^3 / 8 \pi^2 f^2 \cdot B$.

Where $f_s = 1/t_s$ = sampling rate, B = cut-off frequency of low pass filter at the output of receiver and f = frequency of I/P sinusoid. If the DM system is designed to operate 3 kHz BW and quantizing step of 250 mV, calculate maximum amplitude of 1 kHz input sinusoid for which delta modulator does not show slope overload.

**Soln. :**

**Maximum amplitude for no slope overload :**

We have already derived a condition to avoid the slope overload which states that the maximum amplitude of the input signal "A" should be such that :

$$A \leq \frac{\delta}{\omega_m T_s}$$

$$\therefore \text{Maximum value of A} = \frac{\delta}{2 \pi f_m T_s} = \frac{\delta f_s}{2 \pi f_m} \quad ...(1)$$

Here it is given that $\delta = 250$ mV, $f_m = 1$ kHz.

$B = 3$ kHz

$\therefore f_s = 2B = 6$ kHz.

$$\therefore \text{Maximum value of A} = \frac{250 \times 10^{-3} \times 6 \times 10^3}{2 \pi \times 1 \times 10^3}$$

$= 0.2387$ Volt ...**Ans.**

---

**Ex. 3.20.6 :** Determine the output SNR in a DM system for 1 kHz sinusoid, sampled at 32 kHz without slope overload and followed by a 4 kHz post construction filter. Derive the formula used.

**Soln. :**

**Given :** $f_m = 1$ kHz, $f_s = 32$ kHz, $B = 4$ kHz.

No slope overload.

$\therefore$ The output signal to noise ratio in a DM system is given by,

$$(SNR)_o = \frac{3 f_s^3}{8 \pi^2 f_m^2 W}$$

$$\therefore \quad (SNR)_o = \frac{3 \times (32 \times 10^3)^3}{8 \pi^2 \times (1 \times 10^3)^2 \times 4 \times 10^3}$$

$$\therefore \quad (SNR)_o = 311.25 \text{ or } 24.93 \text{ dB}$$

...**Ans.**

---

**Ex. 3.20.7 :** If a voice frequency signal is sampled at the rate of 32,000 samples/sec and characterized by peak value of 2 Volts, determine the value of step size to avoid slope overload. What is quantization noise power $N_q$ and corresponding SNR ? Assume bandwidth of signal as 4 kHz. [redacted]

**Soln. :**

**Given :** $f_s = 32{,}000$ samples/sec.

Peak value of the signal $A = 2$V.

Bandwidth $B = 4$ kHz.

1. **Step size $\delta$ to avoid slope overload :**

To avoid slope overload the following condition should be satisfied.

$$A \leq \frac{\delta}{2 \pi f_m T_s}$$

$$= \frac{\delta f_s}{2 \pi f_m}$$

Substituting the values we get,

$$2 \leq \frac{\delta \times 32000}{2 \pi \times 4 \times 10^3}$$

$$\therefore \quad \delta \geq \frac{2 \times 2 \pi \times 4 \times 10^3}{32000}$$

$$\therefore \quad \delta \geq 1.57 \text{ Volt}$$

...**Ans.**

2. **Quantization noise power ($N_q$) :**

The quantization noise power for a delta modulator is given by,

$$N_q = \frac{\delta^2}{3}$$

$$= \frac{(1.57)^2}{3}$$

$$\therefore \quad N_q = 0.822 \text{ W}$$ ...Ans.

3. **Signal to noise ratio :**

$$\text{SNR} = \frac{3 f_s^3}{8 \pi^2 f_m^2 B}$$

$$= \frac{3 \times (32 \times 10^3)^3}{8 \pi^2 \times (4 \times 10^3)^2 \times 4 \times 10^3}$$

$$= 19.45$$

---

**Ex. 3.20.8 :** A compact disc (CD) records audio signals digitally by PCM. Assume audio signal's bandwidth to be 15 kHz. If signals are sampled at a rate 20% above Nyquist rate for practical reasons and the samples are quantised into 65,536 levels, determine bits/sec required to encode the signal and minimum bandwidth required to transmit encoded signal. [redacted]

**Soln. :**

[redacted] = 15 kHz,

$$f_s = 1.2 \times 2 \text{ W} = 2.4 \times 15 \text{ kHz} = 36 \text{ kHz},$$

$$Q = 65{,}536.$$

1. **Signaling rate (r) :**

$$\text{We know that } Q = 2^N$$

$$\therefore \quad N = \log_2 Q$$

$$\therefore \quad N = \frac{\log_{10}(65{,}536)}{\log_{10} 2} = 16 \quad \text{...(1)}$$

$$\text{Signaling rate } r = N f_s$$

$$= 16 \times 36 \text{ kHz}$$

$$= 576 \text{ kbits/sec.}$$ ...Ans.

Thus the signaling rate r is 576 kbits/sec.

2. **Minimum bandwidth :**

$$B_T = \frac{1}{2}$$

$$\text{Signaling rate} = \frac{576}{2} \text{ kbits/sec}$$

$$\therefore \quad \text{Minimum bandwidth } B_T = 288 \text{ kHz}$$ ...Ans.

**Ex. 3.20.9 :** In a single integration DM scheme, the voice signal is sampled at a rate of 64 kHz. The maximum signal amplitude is 1 Volt.

1. Determine the minimum value of step size to avoid slope overload.
2. Determine granular noise power $N_0$, if the voice signal bandwidth is 3.5 kHz.
3. Assuming signal to be sinusoidal, calculate signal power $S_o$ and signal to noise ratio (SNR).
4. Assuming that the voice signal amplitude is uniformly distributed in the range, (– 1, 1), determine $S_o$ and SNR. [redacted]

**Soln. :**

**Given :** $f_s = 64$ kHz $\qquad A = 1$ Volt

**1. Minimum step size to avoid slope overload :**

$$A \leq \frac{\delta f_s}{2 \pi f_m},$$

$$\therefore \delta_{min} = \frac{2 \pi f_m A}{f_s}$$

$$= \frac{2 \pi \times 3.5 \times 10^3 \times 1}{64 \times 10^3}$$

$$\therefore \delta_{min} = 0.3436 \text{ Volt}$$ ...**Ans.**

**2. Granular noise power :**

$$N_q = \frac{\delta^2}{3} \times \frac{f_m}{f_s}$$

$$= \frac{(0.3436)^2}{3} \times \frac{3.5}{64}$$

$$\therefore N_q = 2.15 \times 10^{-3} \text{ W}$$ ...**Ans.**

**3. Signal power $S_o$ and $SNR_o$ :**

As the signal is sinusoidal, the normalized output signal power

$$S_o = [A/\sqrt{2}]^2 = A/2 = 1/2 \text{ Watt.}$$ ...**Ans.**

$$\therefore SNR_o = \frac{S_o}{N_q} = \frac{0.5}{2.15 \times 10^{-3}} = 232.3 \text{ or } 23.66 \text{ dB.}$$

**4. Signal power for uniformly distributed signal :**

The signal PDF for a uniformly distributed signal is as shown in Fig. P. 3.20.9.

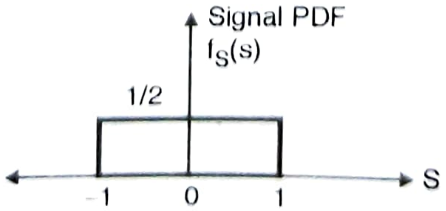



**Fig. P. 3.20.9**

$\therefore$ Mean square value of the signal, $E[S^2] = \int_{-1}^{1} S^2 \cdot f_s(S)\, dS$

$$= \frac{1}{2} \int_{-1}^{1} S^2\, dS$$

$$= \frac{1}{2} [S^3 / 3]_{-1}^{1} = 1/3$$

Assuming R = 1.

$$\text{Normalized signal power } S_o = \text{Mean square value}$$

$$= 1/3 \text{ W}$$ ...Ans.

$$\text{Signal to noise ratio} = \text{SNR} = \frac{1/3 \text{ W}}{2.15 \times 10^{-3} \text{ W}}$$

$$= 155.03 \text{ or } 21.9 \text{ dB}$$ ...Ans.

---

**Ex. 3.20.10 :** The information in an analog signal voltage waveform is to be transmitted over a PCM system with an accuracy of ± 0.1 % full scale accuracy. The analog voltage waveform has a bandwidth of 100 Hz and an amplitude range of – 10 to + 10 Volts.

(a) Determine the minimum sampling rate required.

(b) Determine the number of bits in each PCM word.

(c) Determine the minimum bit rate required in the PCM system.

(d) Determine the minimum absolute channel bandwidth required for the transmission of the PCM signal.

[illegible]

**Soln. :** It has been given that,

1. Accuracy of ± 0.1 % of full scale is expected.
2. W = 100 Hz and amplitude range is – 10 to + 10 V

**(a) Sampling rate $f_s$ :**

By sampling theorem the minimum sampling rate is

$$f_{s\,(\min)} = 2W$$

$$= 200 \text{ Hz}$$ ...Ans.

**(b) Number of bits per word (N) :**

As accuracy is expected to be ± 0.1 % of full scale, the maximum quantization error should be ± 0.1 % of full scale.

$$\therefore \quad \epsilon_{max} = \pm 0.1 \% \text{ of full scale}$$

$$= \pm 0.001 [10 - (-10)]$$

$$= \pm 0.001 \times 20$$

$$\therefore \quad \epsilon_{max} = \pm 0.02 \text{ Volts} \qquad ...(1)$$

We know that the maximum value of the quantization error is

$$\epsilon_{max} = \pm S/2 \qquad \ldots(2)$$

$$\therefore \quad \pm S/2 = \pm 0.02$$

$$\therefore \quad S = 0.04 \text{ Volt} \qquad \ldots(3)$$

$$\text{But } S = \frac{V_H - V_L}{Q}$$

$$\text{where} \quad V_H = 10 \text{ V}$$

$$\text{and} \quad V_L = -10 \text{ V}$$

$$\therefore \quad Q = \frac{10 + 10}{0.04}$$

$$= \frac{20}{0.04} = 500 \qquad \ldots(4)$$

$$\text{But } Q = 2^N$$

$$\therefore \quad N \log_{10} 2 = \log_{10} 500$$

$$\therefore \quad N = 8.96 \approx 9$$

...**Ans.**

**(c) System bit rate :**

[redacted]

$$\text{System bit rate (r)} = N f_s$$

$$= 9 \times 200$$

$$= 1800 \text{ bits/sec}$$

...**Ans.**

**(d) Transmission channel bandwidth ($B_T$) :**

$$B_T \geq \frac{1}{2} N f_s$$

$$\therefore \quad B_T \geq 900 \text{ Hz}$$

...**Ans.**

---

**Ex. 3.20.11 :** The information in an analog waveform with a maximum frequency $f_m$ = 3 kHz is to be transmitted over an M level PCM system, where the number of pulse levels is M = 16. The quantization distortion is specified not to exceed 1% of peak to peak analog signal.

1. What is the maximum number of bits per sample that should be used in this PCM system ?
2. What is the minimum sampling rate and what is the resulting bit transmission rate ?

[redacted]

**Soln. :**

**Given :** $f_m$ = 3 kHz,

umber of quantization levels M = 16.

**1. Number of bits/sample (N) :**

We know that, number of quantization levels $M = 2^N$.

$$\therefore \quad 2^N = 16$$

$$\therefore \quad N = 4$$

...**Ans.**

**2. Minimum sampling rate :**

By sampling theorem : $f_{s\,(min)} = 2\, f_m$

$= 2 \times 3$ kHz

$\therefore \quad f_{s\,(min)} = 6$ kHz ...Ans.

**3. Bit transmission rate :**

$r = N f_s$

$= 4 \times 6$ kHz

$\therefore \quad r = 24$ kbits/sec. ...Ans.

$\therefore$ Number of quantization levels, $Q = 2^N = 2^7 = 128$ ...Ans.

---

**Ex. 3.20.12 :** A TV signal with a bandwidth of 4.2 MHz is transmitted using binary PCM. The number of quantization levels is 512. Calculate :

1. Code word length
2. Transmission bandwidth
3. Final bit rate
4. Output signal to quantization noise ratio.

**Soln. :**

**Given :** $f_m = 4.2$ MHz and $Q = 512$.

**1. Code word length (N) :**

$Q = 2^N$

$\therefore \quad N = \dfrac{\log 512}{\log 2}$

$\therefore \quad N = 9$ bits/word ...Ans.

**2. Transmission bandwidth :**

$B_T = \dfrac{1}{2} N f_s$

$= \dfrac{1}{2} N\,(2 f_m)$

$\therefore \quad B_T = 9 \times 4.2$ MHz

$= 37.8$ MHz ...Ans.

**3. Final bit rate (r) :**

$r = N f_s = 9 \times 2 \times f_m = 18 \times 4.2$ MHz

$\therefore \quad r = 75.6$ Mb/s ...Ans.

4. **Signal to quantization noise ratio :**

Since the TV signal is not a sinusoidal signal, let us use the general expression of signal to quantization noise ratio.

$$\left[\frac{S}{N_q}\right] = 4.8 + 6\,N\,\text{dB}$$

$$= 4.8 + (6 \times 9)$$

$$\therefore \quad \frac{S}{N_q} = 58.8\ \text{dB} \qquad \text{...Ans.}$$

This is the maximum signal to noise ratio that we are expected to get from this system.

---

**Ex. 3.20.13 :** For a typical quantizer in a PCM system, amplitude of input signal m (t) is confined to the range of ($-M_p$, $+M_p$). Assuming this range, divided in L zones, each of step size $\Delta$, derive the expression for quantization error in terms of $M_p$ and L. Sketch input/output characteristics of this quantizer. What are the advantages of non-uniform quantizing over uniform quantizing ? State the laws followed regarding non-uniform quantizing in practice.

May 2000, 10 Marks

**Soln. :**

**Given :**

1. Amplitude of input signal is confined to the range ($-M_p$, $+M_p$).
2. This range is divided into L zones, each of step size $\Delta$. We are asked to obtain the expression for quantization error $N_q$ in terms of $M_p$ and L. For this derivation, refer to section 3.2.5. There the expression for quantization noise power has been derived. Equation (3.2.3) states that,

Normalized quantized noise power,

$$N_q = \frac{S^2}{12} \qquad \text{...(1)}$$

where S = Step size.

Here step size is $\Delta$.

$$\therefore \quad N_q = \Delta^2/12 \qquad \text{...(2)}$$

$$\text{But step size } \Delta = \frac{\text{Peak to peak amplitude of signal m (t)}}{\text{Number of quantization levels}}$$

$$\therefore \quad \Delta = \frac{M_p - (-M_p)}{L}$$

$$= \frac{2\,M_p}{L} \qquad \text{...(3)}$$

Substituting Equation (3) into Equation (2) we get,

$$N_q = \frac{[2\,M_p/L]^2}{12} = \frac{4\,M_p^2}{12\,L}$$

$$\therefore \quad N_q = \frac{M_p^2}{3L} \qquad \text{...Ans.}$$

This is the expression for quantization noise.

This is a uniform quantizer and the input/output characteristic of the quantizer is as shown in Fig. P. 3.20.13.

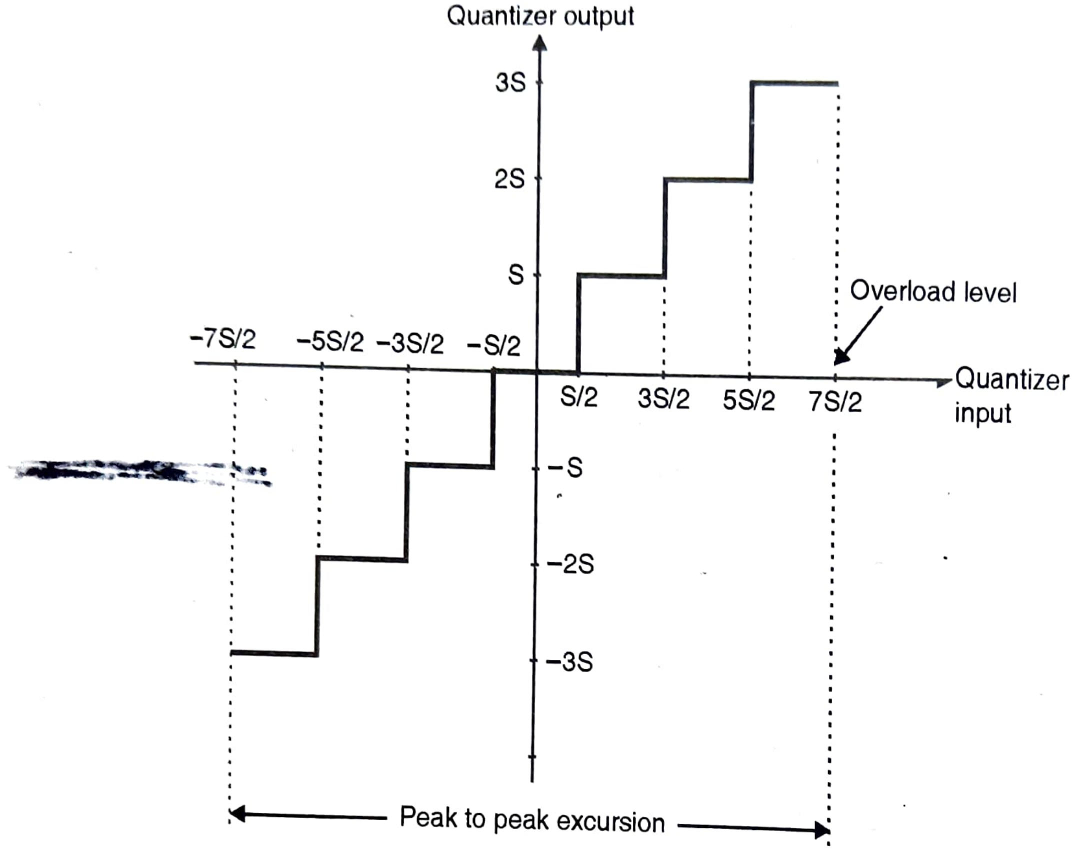



**Fig. P. 3.20.13 : Input/output characteristic of the quantizer**

Advantages of non-uniform quantizing over uniform quantizing are as follows :

1. Signal to quantization ratio almost remains constant as shown in Fig. P. 3.20.13(b) irrespective of the input signal strength.
2. Dynamic range increases. (Dynamic range is the range of input signal power in dB over which the output SNR is higher than about 30 dB).
3. Non-uniform quantizing is used for speech and music signals which have a high crest factor.

The laws followed for non-uniform quantization in practice are μ-law and A-law companding.

**Ex. 3.20.14 :** The output signal to noise ratio (SNR) of a 10 bit PCM was found to be 30 dB. The desired SNR is 42 dB. It was decided to increase the SNR to the desired value by increasing the number of quantization levels. Find fractional increase in the transmission bandwidth required for this SNR.

**Soln. :**

**Given :** SNR = 30 dB, N = 10

Desired value of SNR = 42 dB

1. **To find "N" for SNR = 42 dB**

With increase in N by 1 bit the value of SNR increases by 6 dB. Therefore to increase the value of SNR by 12 dB it is necessary to increase N by 2.

$$\therefore \quad N = 10 + 2 = 12 \qquad \text{...Ans.}$$

2. **Fractional increase in BW :**

$$\text{BW of PCM system} = \frac{1}{2} N f_s$$

$$\therefore \text{ BW with N} = 10 \text{ is given by, } BW_{10} = \frac{1}{2} \times 10\, f_s = 5\, f_s$$

$$\text{and BW with N} = 12 \text{ is given by, } BW_{12} = \frac{1}{2} \times 12\, f_s = 6\, f_s$$

$$\therefore \text{ Change in BW} = \Delta\, BW$$

$$= 6\, f_s - 5\, f_s = f_s$$

$$\therefore \text{ Fractional change in BW} = \frac{\Delta\, BW}{BW_{10}}$$

$$= \frac{f_s}{5\, f_s} \times 100\ \%$$

$$= 20\ \% \qquad \text{...Ans.}$$

---

**Ex. 3.20.15 :** A telephone signal with cut-off frequency of 4 kHz is digitised into 8 bit PCM, sampled at Nyquist rate. Calculate the baseband transmission bandwidth and quantization S/N ratio.

**May 2004, [illegible] Marks**

**Soln. :**

1. The Nyquist rate $= 2 \times 4 \text{ kHz} = 8 \text{ kHz}.$

2. Transmission bandwidth, $B_T = \frac{1}{2} \cdot N f_s$

$$= \frac{1}{2} \times 8 \times 8 \text{ kHz} = 32 \text{ kHz} \qquad \text{...Ans.}$$

3. $SNR_q = (1.8 + 6N)$ dB

$$= 1.8 + (6 \times 8) = 49.8 \text{ dB} \qquad \text{...Ans.}$$

---

**Ex. 3.20.16 :** In a single integration DM scheme the voice signal is sampled at a rate of 64 kHz. The maximum signal amplitude is 2 Volts. Voice signal bandwidth is 3.5 kHz. Determine the minimum value of step size to avoid slope overload and granular noise power.

**[illegible] 2004, 4 Marks**

**Soln. :**

**Given :** $f_s = 64$ kHz, $A_{max} = 2$ V, $f_m = 3.5$ kHz

1. **Minimum step size to avoid slope overload :**

We know that

$$A_{max} = \frac{\delta_{min}\, f_s}{2\,\pi\, f_m}$$

$$\therefore \quad \delta_{min} = \frac{2\,\pi\, f_m\, A_{max}}{f_s}$$

$$= \frac{2\,\pi \times 3.5 \times 10^3 \times 2}{64 \times 10^3}$$

$$\therefore \quad \delta_{min} = 0.6872 \text{ Volt}$$ ...Ans.

2. **Granular noise power :**

$$N_q = \frac{\delta^2}{3} \times \frac{f_m}{f_s} = \frac{(0.6872)^2}{3} \times \frac{3.5}{64}$$

$$\therefore \quad N_q = 8.6 \times 10^{-3} \text{ W}$$ ...Ans.

---

**Ex. 3.20.17 :** A television signal (video and audio) has a bandwidth of 4.5 MHz. This signal is sampled, quantized and binary coded to obtain a PCM signal.

1. Determine the sampling rate if the signal is to be sampled at a rate 20% above the Nyquist rate.
2. If the samples are quantized into 1024 levels, determine the number of binary pulses required to encode each sample.
3. Determine the binary pulse rate (bits per second) of binary coded signal and the minimum bandwidth required to transmit the signal.

If above linear PCM system is converted to companded PCM, will the output bit rate changed ? Justify. [redacted]

**Soln. :**

**Given :**

$$W = 4.5 \text{ MHz}$$

1. $$\text{Sampling rate} = 1.2 \times \text{Nyquist rate} = 1.2 \times 2\text{ W}$$

$$= 1.2 \times 2 \times 4.5 \times 10^6 = 10.8 \text{ MHz}$$ ...Ans.

2. Given that number of quantization levels, Q = 1024

$$\text{But } Q = 2^N$$

$\therefore$ Number of binary pulse per word, $N = \log_2 Q$

$$\therefore \quad N = \log_2 1024$$

$$\therefore \quad N = 10$$ ...Ans.

3. $$\text{Binary pulse rate (bit rate)} = N\, f_s = 10 \times 10.8 \text{ MHz} = 108 \text{ Mbps}$$ ...Ans.

$$\text{Bandwidth} = \frac{1}{2} \text{ bit rate} = 54 \text{ MHz}$$ ...Ans.

**Ex. 3.20.18 :** A compact disc (CD) records audio signals digitally by using PCM. Assume the audio signal bandwidth to be 15 kHz.

1. What is Nyquist rate ?
2. If the Nyquist samples are quantized into L = 65,536 levels and then binary coded, determine the number of binary digits required to encode a sample.
3. Determine the number of binary digits per second (bit/s) required to encode the audio signal.
4. For practical reasons, the signals are sampled at a rate well above Nyquist rate at 44,100 samples per second. If L = 65,536, determine number of bits per second required to encode the signal and transmission bandwidth of encoded signal.

**Soln. :**

**Given :** $W = 15$ kHz

1. Nyquist rate $= 2\,W = 2 \times 15$ kHz $= 30$ kHz. **...Ans.**
2. Number of quantization levels $Q = 65{,}536$

$$\text{we know that } Q = 2^N$$

$$\therefore\ 2^N = 65{,}536$$

$$\therefore\ N = 16 \text{ bits.}$$ **...Ans.**

$\therefore$ Number of bits to encode each sample is $N = 16$.

3. Number of bits per second = Number of samples/sec. × Number of bits/sample

$$= 15 \times 10^3 \times 16$$

$$= 240 \text{ k bits/sec.}$$ **...Ans.**

4. Practical sampling rate $f_s = 44.1$ kHz

$$\therefore \text{ Number of bits per second } = 44.1 \times 16 \times 10^3 = 705.6\text{k bits/sec.}$$

$$\therefore \text{ Transmission bandwidth } B = \frac{\text{Bit rate}}{2} = \frac{705.6 \times 10^3}{2}$$

$$= 352.8 \text{ kHz.}$$ **...Ans.**

---

**Ex. 3.20.19 :** A Delta Modulator system operates at 3 times Nyquist rate for signal with 3.3 kHz bandwidth. The quantisation step is 250 mV. Determine the maximum amplitude of a 1 kHz input sinusoid for which the DM does not show slope overload.

**Soln. :**

**Given :** $W = 3.3$ kHz, $f_m = 1$ kHz, $\delta = 250$ mV, $f_s = 3 \times 2W = 19.8$ kHz.

1. Let the maximum amplitude of 1 kHz input sinusoid be "A".

The condition to avoid the slope overload is

$$A \leq \frac{\delta}{\omega_m T_s}$$

$$\therefore \text{ Maximum value of A} = \frac{\delta}{\omega_m T_s} = \frac{\delta f_s}{2 \pi f_m}$$

Substituting the values we get,

$$A_{max} = \frac{250 \times 10^{-3} \times 19.8 \times 10^3}{2 \pi \times 1 \times 10^3}$$

$$\therefore A_{max} = 0.787$$

---

**Ex. 3.20.20 :** An audio signal with highest frequency component 3300 Hz is pulse code modulated with a sampling rate of 8000 samples/sec. The required signal-to-quantisation noise ratio is 40 dB.

1. What is the minimum number of uniform quantising levels needed ?
2. What is the minimum number of bits per sample needed ?
3. Calculate the minimum number of bits per sample needed ?

**Soln. :**

**Given :** $f_s$ = 8000 samples/sec. , $f_m$ = 3300 Hz. $\frac{S}{N_q}$ = 40 dB

**1. To calculate Q and N :**

Assuming the input signal to be non sinusoidal we write

$$S/N = (4.8 + 6\,N)\text{ dB}$$

$$\therefore \quad 40 = 4.8 + 6N$$

$$\therefore \quad N = 5.86 \approx 6$$ ...Ans.

Thus the number of bits per word is 6.

$$Q = 2^N = 2^6 = 64$$ ...Ans.

Thus the number of quantization level is 64.

**2. The signaling rate (r) :**

$$r = N\,f_s = 6 \times 8 \times 10^3 = 48\text{ kbits/sec.}$$

**3. Transmission bandwidth :**

$$\text{Minimum BW,} \quad B_T = \frac{1}{2} r = 24\text{ kHz.}$$

**Ex. 3.21.1 :** Delta modulator (DM) gives output pulses + p (t) of – p (t). The output of DM is + p (t) when instantaneous sample is larger than previous sample value and is – p(t) when instantaneous sample is smaller than previous sample value (last sample). The p(t) has 2 microsecond duration and 628 mV amplitude and repeats every 10 microsecond. Plot the input and output of DM on graph paper one below other to same scale if the input to DM is 1 Volt sine wave of frequency 10 kHz for one cycle of input wave. What is the maximum frequency with 1 Volt amplitude that can be used in this system without slope overload distortion.

**Soln.**

**Part I : To plot the DM signal**

The D.M. Signal is as shown in Fig. P. 3.21.1.

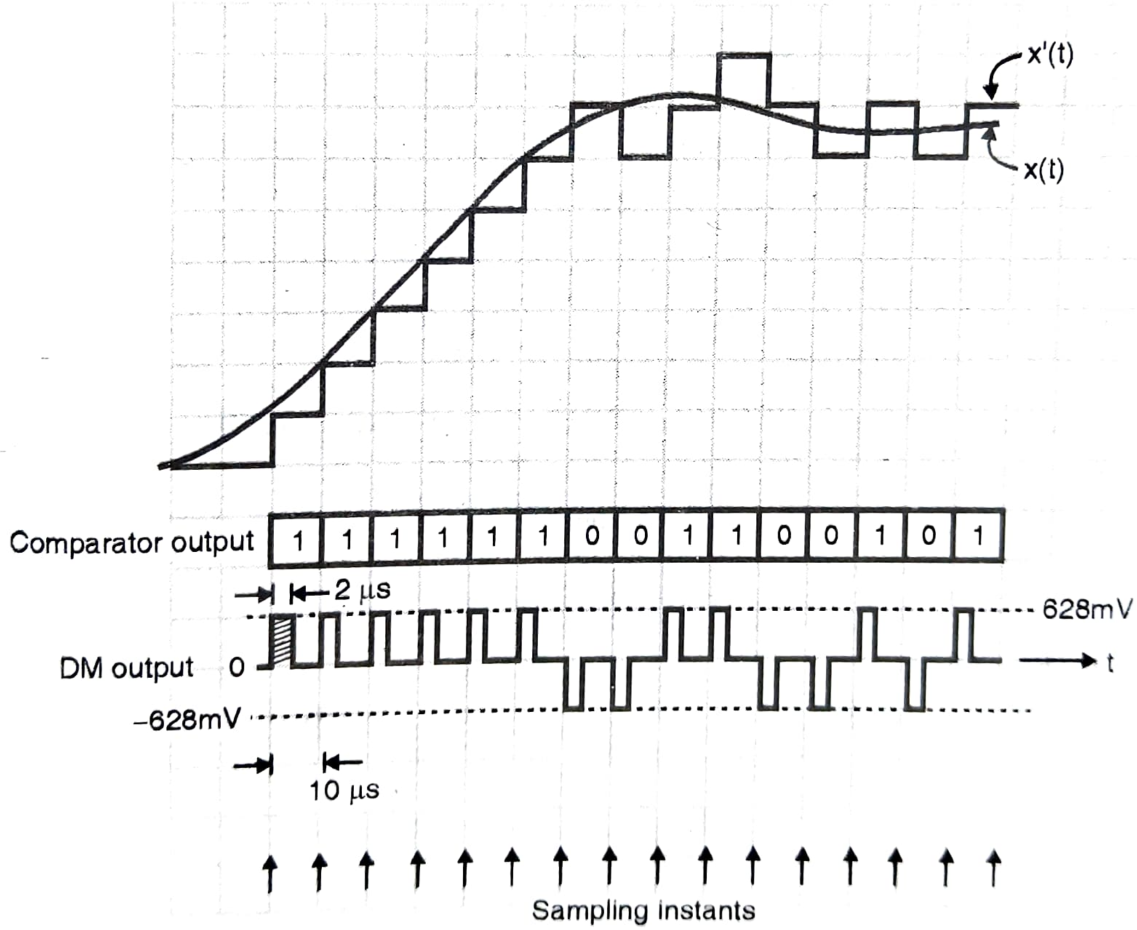



**Fig. P. 3.21.1 : DM signal**

**Part II : Maximum frequency :**

**Given :**

$A = 1V, \ f_m = 10 \text{ kHz},$

$f_s = 1/T_s = 1/10 \ \mu s = 100 \text{ kHz}, \ \delta = 265 \text{ mV}$

The condition for avoiding the slope overload is

$$A > \frac{\delta}{\omega_m T_s}$$

$$\therefore \quad 1 = \frac{256 \times 10^{-3}}{\omega_m \times 10 \times 10^{-6}}$$

$$\therefore \quad \omega_n = 25.6 \times 10^3 \text{ rad/sec.}$$

$$\therefore \quad f_{max} = 4074.36 \text{ Hz} \qquad \text{...Ans.}$$

This is the maximum input frequency without introducing any slope overload distortion.

---

**Ex. 3.21.2 :** Express μ law of companding. For $\mu = 255$ determine the maximum advantage over linear quantizer if the peak power to average power ratio is 9 and dynamic range of input signal is 30 dB and quantizer uses 256 levels.

**Soln. :**

For μ law compounding refer section 3.9.3.

The advantage over linear quantization can be shown by calculating the output signal to noise ratio and the companding gain of the μ law compander.

1. **Signal to noise ratio :**

$$SNR_o = \frac{3\,Q^2}{[\log_e (1+\mu)]^2}$$

Where Q = Number of quantization levels.

$$\therefore \quad SNR_o = \frac{3 \times (256)^2}{[\log_e (1+255)]^2}$$

$$= 6393.96 \text{ or } 38.05 \text{ dB}$$

2. **Companding gain :**

$$G_c = \frac{\mu}{\log_e (1+\mu)} = \frac{255}{\log_e (1+255)} = 45.98$$

$$G_c \text{ (dB)} = 20 \log_{10} (45.98) = 33.25 \text{ dB.}$$

---

**Ex. 3.21.3 :** An analog waveform with bandwidth 15 kHz is to be quantized with 200 levels and transmitted via binary PCM signal. Find rate of transmission and bandwidth required. If 10 such signals are to be multiplexed find the bandwidth requirement

**Soln. :**

**Given :** $f_m = 15$ kHz, $Q = 200$.

**To find :**

1. Rate of transmission
2. Bandwidth.

**Step 1 : Find $f_s$ :**

$$f_s = 2f_m = 2 \times 15 \times 10^3 = 30 \times 10^3$$

**Step 2 : Find N :**

[redacted]

$$Q = 2^N$$

$$\text{Number of bits/sample } N = \log_2 Q = \frac{\log_{10} 200}{\log_{10} 2}$$

$$\therefore \quad N = 7.6438 \approx 8$$

**Step 3 : Calculate rate of transmission :**

$$\text{Rate of transmission} = Nf_s$$

$$= 8 \times 30 \times 10^3 = 240 \text{ kbps}$$

**Step 4 : Transmission bandwidth :**

$$B = \frac{1}{2} Nf_s = \frac{1}{2} \times 240 = 120 \text{ kHz.}$$

---

**Ex. 3.21.4 :** A binary channel with bit rate 36 kbps is available for PCM voice transmission. Find appropriate values of number of quantization levels, number of bits per sample, and sampling frequency. Given that voice signal is band limited to 3.4 kHz.

[redacted]

**Soln.:** [redacted]

**Given :** Bit rate = 36 kbps, $f_m = 3.4$ kHz

**To find :** 1. Q 2. N 3. $f_s$

1. Sampling frequency $f_s \geq 2f_m$

$$\therefore \quad f_s \geq 2 \times 3.4 \text{ kHz}$$

$$\therefore \quad f_s \geq 6.8 \text{ kHz}$$

For voice transmission the standard value of sampling frequency is $f_s = 8$ kHz.

2. Bit rate = $Nf_s$

$$\therefore \quad 36 \times 10^3 = N \times 8 \times 10^3$$

$$\therefore \quad N = 4.5 \approx 5$$

$$\therefore \quad Q = 2^N = 2^5 = 32.$$

---

**Ex. 3.21.5 :** A signal having band of 300 – 3000 Hz is sampled at 8000 samples/sec and is coded for PCM. Assume that the ratio of peak signal power to average quantization noise power required at the output is 30 dB.

1. What is minimum number of quantization level needed ? Derive the formula used.
2. Compute the bandwidth required for transmission. [redacted]

**Soln. :** For solution of the example refer Ex. 3.9.1 and for the derivation refer section 3.5.

---

**Ex. 3.21.6 :** A speech signal band limited to 3.4 kHz having maximum amplitude of 1 V is to be delta modulated at 20 kbps. What is appropriate step size for the same ? Derive the formula used.

[illegible]

**Soln. :**

**Given :** $f_m$ = 3.4 kHz, A = 1 V, Delta modulation, $f_s$ = 20 kHz.

**To find :** Step size $\delta$.

The amplitude required for avoiding the slope overload is given by,

$$A = \frac{\delta}{2\pi}\left[\frac{f_s}{f_m}\right]$$

$$\therefore \quad 1 = \frac{\delta}{2\pi}\left(\frac{20 \times 10^3}{3.4 \times 10^3}\right)$$

$$\therefore \quad \delta = 1.068 \text{ volt}$$ ...Ans.

For the derivation of the formula used, refer section 3.13.11.

---

**Ex. 3.21.7 :** A signal of bandwidth 3.5 kHz is sampled and quantized and coded by a PCM system. The coded signal is then transmitted over a transmission channel with transmission rate 50 kbps. Calculate the maximum signal to Noise ratio that can be obtained by the system. The input signal has peak to peak value of 4 V and rms value of 0.2 V.

May 2007, [illegible] Marks

**Soln. :**

**Given :** $f_m$ or W = 3.5 kHz, Bit rate = 50 kbps, $V_{PP}$ = 4 V.

**To find :** $SNR_{q\,(max)}$.

- From the values of peak to peak voltage and rms voltage we conclude that the input signal is nonsinusoidal.
- So,

$$SNR_q\,(dB)_{max} = 4.8 + 6\,N \qquad ...(1)$$

$$\text{or } SNR_{q\,(max)} = 3 \times 2^{2N} \qquad ...(2)$$

$$\text{i.e. } N = \text{Number of bits per word.}$$

**Step 1 : Calculate N :**

- Bit rate or transmission rate

$$r = N\,f_s$$

- For a voice signal $f_s$ = 8 kHz is a standard value.

$$\therefore \quad 50 \times 10^3 = N \times 8 \times 10^3$$

$$\therefore \quad N = \frac{50}{8} = 6.25 \approx 7 \text{ bits}$$

**Step 2 : Calculate $SNR_q$ :**

Substituting in Equation (1) we get.

$$SNR_q\,(dB)_{max} = 4.8 + (6 \times 7)$$

$$= 46.8\ dB \qquad \text{...Ans.}$$

---

**Ex. 3.21.8 :** A signal band-limited to 1 MHz is sampled at a rate 50% higher than the Nyquist rate and quantized into 256 levels using a $\mu$ law quantizer with $\mu = 255$.

1. Determine the signal to quantization noise ratio

2. The SNR found in part I was unsatisfactory. It must be increased by atleast 10 dB. Would you be able to obtain the desired SNR without increasing the transmission bandwidth. If it was found that a sampling rate 20% above the nyquist rate is adequate ? If so explain how. What is maximum SNR that can be realized in this way.

[redacted]

**Soln. :**

$f_m$ or $w$ = 1 MHz, $\qquad f_s = 1.5 \times 2w = 3MHz$,

$Q = 256$, $\qquad \mu = 255$

**1. Signal to noise ratio :**

$$SNR_o = \frac{3Q^2}{[\log_e(1+\mu)]^2} = \frac{3 \times (256)^2}{[\log_e(1+255)]^2} = 6393.96 \text{ or } 38\ dB$$

2. The new sampling rate is $1.2 \times 2w = 2.4$ MHz.
The signal to noise ratio is dependent on Q and $Q = 2^N$ where N is number of bits per word.

If signal to noise ratio is to be increased by 10 dB then we have to increase Q to a new value Q′ as calculated below :

$$SNR_o' = 38 + 10 = 48\ dB = 63095.73$$

$$\therefore\ 63095.73 = \frac{3\,(Q')^2}{[\log_e(256)]^2}$$

$$\therefore\ Q' = 804.12$$

New value of number of bits/word $= N' = \log_2 Q' = \log_2 804.12 = 9.65 \approx 10$

$$\therefore\ \text{Bit rate} = N' f_s = 10 \times 2.4 \times 10^6 = 24\ Mb/s$$

$$\therefore\ \text{Transmission B.W.} = \frac{1}{2} \times \text{bit rate} = 12\ Mb/sec$$

$$\text{Old transmission B.W.} = \frac{1}{2} N f_{s1} = \frac{1}{2} \times 8 \times 3\ MHz = 12\ Mb/sec$$

**Conclusion :**

If we increase N from 8 bits to 10 bits then it is possible to increase SNR by 10 dB with increasing the transmission bandwidth.

$$\text{Maximum } SNR_o = \frac{2 \times (2^{10})^2}{[\log_e (256)]^2} = 68202.3 = 48.34 \text{ dB}$$

---

**Ex. 3.21.9 :** A 1 kHz signal of voice channel is sampled at 4 kHz using 12-bit PCM and a DM system. If 25 cycles of voice signal are digitized find in each case :

1. Signaling rate
2. Bandwidth required
3. Number of bits required to be transmitted
4. Comment on results.

[illegible]

**Soln. :**

**Step 1 : Signaling rate :**

**1. For PCM :**

$N = 12$ and $f_s = 4$ kHz.

$$\therefore \text{ Signaling rate} = Nf_s = 12 \times 4 \times 10^3 = 48 \text{ kbps}$$ ...Ans.

**2. For DM :**

$$\text{Signaling rate} = f_s = 4 \text{ kbps}$$ ...Ans.

**Step 2 : Bandwidth :**

1. For PCM $\quad BW \geq \frac{1}{2} N f_s \quad \therefore \; BW \geq 24$ kHz ...Ans.

2. For DM $\quad BW \geq \frac{1}{2} \times f_s \quad \therefore \; BW \geq 2$ kHz ...Ans.

**Step 3 : Number of bits required to be transmitted :**

Number of cycles to be digitized = 25

$$1 \text{ cycle} \equiv 1/1 \text{ kHz} = 1 \text{ ms}$$

$$\therefore \; 25 \text{ cycles} \equiv 25 \text{ ms}$$

1. For PCM, bit rate = 48 kbps

$$\therefore \text{ Number of bits required to be transmitted} = 25 \times 10^{-3} \times 48 \times 10^3$$

$$= 1200 \text{ bits}$$ ...Ans.

2. For DM, bit rate = 2kbps

$$\therefore \text{ Number of bits required to be transmitted} = 25 \times 10^{-3} \times 2 \times 10^3$$

$$= 50 \text{ bits}$$ ...Ans.

**Step 4 : Comments :**

In order to send the same voice signal, the D. M. system needs less bandwidth and less number of bits as compared to the PCM system.

**Ex. 3.21.10 :** A signal m(t) bandlimited to 4 kHz is sampled at a rate 50% higher than Nyquist rate. The maximum acceptable error in the sample amplitude is 1% of peak amplitude. The quantized samples are binary coded. Find minimum bandwidth of a channel required to transmit the encoded binary signal. **Dec. 2010, [illegible] Marks**

**Soln. :**

**Given :** W = 4 Hz, Maximum error in the sampled amplitude = 1% of peak amplitude

**To find :** Minimum channel bandwidth $B_T$.

**1. Sampling rate :**

$$f_s = 1.5 \times \text{Nyquist rate} = 1.5 \times 2W$$

$$= 1.5 \times 2 \times 4 \text{ kHz} = 12 \text{ kHz} \qquad \ldots(1)$$

**2. Number of bits per word :**

As the acceptable error is 1% is 0.01 of the peak amplitude, the maximum quantization error should be ± 1% of the peak.

$$\therefore \in_{max} = \pm 0.01 \, V_H$$

$$\text{Assume } V_H = 10 \text{ V}$$

$$\therefore \in_{max} = \pm 0.01 \times 10 = \pm 0.1 \text{ Volts}$$

$$\text{But } \in_{max} = \pm S/2$$

$$\therefore \pm 0.1 = \pm S/2$$

$$\therefore S = 0.2 \text{ Volts}$$

$$\text{But } S = \frac{V_H - V_L}{Q}$$

$$\text{Let } V_L = -10 \text{ V}$$

$$\therefore S = \frac{10 - (-10)}{Q}$$

$$\therefore Q = \frac{20}{0.2} = 100$$

$$\text{But } Q = 2^N$$

$$N \log_{10} 2 = \log_{10} Q$$

$$\therefore 0.3010 \, N = \log_{10} 100 = 2$$

$$\therefore N = \frac{2}{0.3010} = 6.6445$$

Round off N to 7

$\therefore$ Number of bits per word i.e. $N = 7$ ...(2)

**3. System bit rate :**

$$\text{System bit rate } r = N f_s = 7 \times 12 \text{ kHz}$$

$$= 84 \text{ kbps} \qquad ...(3)$$

**4. Minimum channel bandwidth :**

$$B_{T\,(min)} = \frac{1}{2} \times r = \frac{1}{2} \times 84 \text{ kHz} = 42 \text{ kHz}$$ ...Ans.

---

**Ex. 3.21.11 :** A signal having bandwidth 3 kHz is to be encoded using :

1. 8 bit PCM system
2. DM system.

If 10 cycles of the signal are digitized, state how many bits will be digitized, output in each case if sampling frequency is 10 kHz. Also find bandwidth required in each case.

Dec. 2010, 3 Marks

**Soln. :**

**Step 1 : Signaling rate :**

$f_s = 10$

**1. For PCM :**

$N = 8$ bits and $f_m$ or $W = 3$ kHz, $f_s = 10$ kHz.

$N$

$$\therefore \text{ Signaling rate } = Nf_s = 8 \times 10 = 80 \text{ kbps}$$

40

**2. For DM :**

$80 \times 10^3$

$$\text{Singaling rate } = f_s = 10 \text{ kbps}$$

1s

5

**Step 2 : Number of bits to be digitized :**

3.33

$$\text{Number of cycles digitized} = 10$$

$$1 \text{ cycle} = 1/3 \text{ kHz} = 0.33 \text{ ms}$$

$$10 \text{ cycles} \equiv 3.3 \text{ ms}$$

1. For PCM, bit rate = 80 kbps.

$$\text{Number of bits to be digitized in 10 cycles} = 3.3 \times 10^{-3} \times 80 \times 10^3$$

$$= 264 \text{ bits.}$$ ...Ans.

2. For DM, bit rate = 10 kbps

$$\therefore \text{ Number of bits to be digitized} = 3.3 \times 10^{-3} \times 10 \times 10^3 = 33 \text{ bits}$$ ...Ans.

**Step 3 : Bandwidth :**

1. For PCM $\quad BW \geq \frac{1}{2} Nf_s \qquad \therefore \; BW \geq 40$ kHz. ...Ans.

2. For DM $\quad BW \geq \frac{1}{2} f_s \qquad \therefore \; BW \geq 5$ kHz. ...Ans.

**Ex. 3.21.12 :** In a 8 bit PCM scheme, the voice signal is sampled at a rate of 8 kHz. The maximum signal amplitude is 1V, voice signal bandwidth is 3.5 kHz. The quantization noise signal amplitude is uniformly distributed as shown in Fig. P. 3.21.12. Calculate the signal to noise ratio of the system. [redacted]

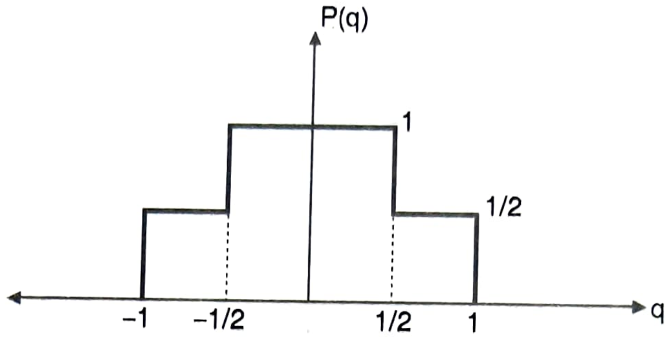



**Fig. P. 3.21.12**

**Soln. :**

**Given :** PCM system, $f_s = 8$ kHz, V = 1V, W or $f_m = 3.5$ kHz.

**To find :** Signal to noise ratio.

**Step 1 : Calculate the noise power :**

$$\text{Mean square value of noise} = \int_{-1}^{-1/2} \epsilon^2 \times \frac{1}{2} \times d\epsilon + \int_{-1/2}^{1.2} \epsilon^2 \times 1 \times d\epsilon + \int_{1/2}^{1} \epsilon^2 \times \frac{1}{2} \times d\epsilon$$

$$= \left[\frac{\epsilon^3}{3}\right]_{-1}^{-1/2} \times \frac{1}{2} + \left[\frac{\epsilon^3}{3}\right]_{-1/2}^{1/2} + \frac{1}{2}\left[\frac{\epsilon^3}{3}\right]_{1/2}^{1}$$

$$= \frac{1}{6}\left[-\frac{1}{8} + 1\right] + \frac{1}{3}\left[\frac{1}{8} - \left(-\frac{1}{8}\right)\right] + \frac{1}{6}\left[1 - \frac{1}{8}\right]$$

$$= 0.1458 + 0.08333 + 0.1458$$

$$\therefore \; V_n^2 = 0.375$$

$$\therefore \; \text{Normalized noise power } N_q = \frac{V_n^2}{1} = 0.375 \text{ W} \quad \ldots(1)$$

**Step 2 : Calculate the signal power :**

$$\text{Normalized signal power} = P = \frac{V_m^2}{1} = \frac{(1)^2}{2} = 0.5 \text{ W} \quad \ldots(2)$$

**Step 3 : Signal to noise ratio :**

$$(SNR)_q = \frac{P}{N_q} = \frac{0.5}{0.375} = 1.333$$ **...Ans.**

$$(SNR)_q = 10 \log_{10} (1.333) = 1.25 \text{ dB}$$ **...Ans.**

---

**Ex. 3.21.13 :** **The information in an analog waveform, whose maximum frequency $f_m = 4000$ Hz is to be transmitted using a 16-level PAM system.** The quantization distortion must not exceed **± 1% of the peak to peak analog signal.**

(a) What is the minimum number of bits per sample that should be used in the transmission system ?

(b) What is the minimum required sampling rate and bit rate of the system.

(c) 'What is the 16-ary PAM symbol transmission rate ? **Dec. 2011, 3 Marks**

**Soln. :**

**Given :** $W = 4$ kHz, $Q = 16$, $\epsilon_{max} = \pm 1$ % of peak to peak amplitude

Let peak amplitude of analog signal be 10 volts.

**(a) Number of bits/sample (N) :**

Number of quantization levels $Q = 16$

But $Q = 2^N$ where $N$ = Number of bits per sample

$\therefore \quad 16 = 2^N$

$\therefore \quad N = 4$ bits/sample ...**Ans.**

$T_s = 4T_b$

**(b) Minimum sampling rate and bit rate of the system :**

Minimum sampling rate $f_{s\,(min)} = 2\,W = 2 \times 4\text{ kHz} = 8\text{ kHz}$ ...**Ans.**

Bit rate of system $(r) = N \times f_{s\,(min)} = 4 \times 8$ kHz

$= 32$ kbits/sec ...**Ans.**

**(c) Symbol transmission rate :**

1 symbol $\equiv$ N bits

$$\therefore \text{ Symbol transmission rate } = \frac{\text{Bit rate (r)}}{N} = \frac{N\,f_s}{N}$$

$= f_s$ symbols/sec.

$= 8000$ symbols/sec