Definition of Probability

Session 4

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Definition of Probability
(1)Mathematical(classical/a priori) definition

Definition. If a trial results in n exhaustive, mutually exclusive and equally likely cases and m of them are favourable to the happening of an event E, then the probability 'p' of happening of E is given by

$$p = P(E) = \frac{\textit{Favourable number of cases}}{\textit{Exhaustive number of cases}} = \frac{m}{n}$$

Since the number of cases favourable to the 'non-happening' of the event E are $(n - m)$, the probability 'q' that E will not happen is given by

$$q = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p \quad \Rightarrow \quad p + q = 1$$

Obviously p as well as q are non-negative and cannot exceed unity, *i.e*, $0 \le p \le 1$, $0 \le q \le 1$.

Remarks. 1. Probability 'p' of the happening of an event is also known as the probability of success and the probability 'q' of the non- happening of the event as the probability of failure.

2. If $P(E) = 1$, E is called a *certain event* and if $P(E) = 0$, E is called an *impossible* event.

3. **Limitations of Classical Definition.** This definition of Classical Probability breaks down in the following cases :

(i) If the various outcomes of the trial are not equally likely or equally probable. For example, the probability that a candidate will pass in a certain test is not 50% since the two possible outcomes, *viz.*, sucess and failure (excluding the possibility of a compartment) are not equally likely.

(ii) If the exhaustive number of cases in a trial is infinite.

Definition of Probability
(2)Statistical or Empirical Probability

Definition (Von Mises). *If a trial is repeated a number of times under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event happens to the number of trials, as the number of trials become indefinitely large, is called the probability of happening of the event. (It is assumed that the limit is finite and unique).*

Symbolically, if in n trials an event E happens m times, then the probability 'p' of the happening of E is given by

$$p = P(E) = \lim_{n \to \infty} \frac{m}{n}$$

Limitations of Statistical or Empirical Probability

- The experimental conditions may not remain essentially homogeneous and identical in a large number of repetitions of the experiment.
- The relative frequency $\frac{m}{n}$ may not attain a unique value no matter however large n may be.

(3)Probability Function (Axiomatic Definition of Probability)

- The modern theory of probability is based on the axiomatic approach introduced by Kolmogorov in 1930s.
- This probability definition includes both classical and empirical definition of probability and at the same time, is free from the drawbacks of first two definitions.
- This definition is based on certain axioms.
- Axiomatic probability : Given a sample space of a random experiment, the probability of occurrence of any event A is defined as the set function P(A) satisfying the following axioms.

Axioms

- 1) P(A) is defined, is real and non-negative. That is P(A)≥0 (Axiom of positiveness)
- 2) P(S) = $P(\Omega) = P(U) = 1$(Axiom of certainty)
- 3) If A_1, A_2,… A_n is a finite or infinite sequence of disjoint (mutually exclusive)events of S, then
- $P(A_1 U A_2 U \ldots U A_n) = P(A_1)+P(A_2)+\ldots+P(A_n)$

$$P\left(\bigcup_{i=1}^{n} A_n\right) = \sum_{i=1}^{n} P(A_i)$$

- $P(A_1 U A_2 U \ldots) = P(A_1)+P(A_2)+\ldots$
- (Axiom of additivity)

Example 1

- What is the chance that a leap year selected at random will contain 53 sundays ?

Solution. In a leap year (which consists of 366 days) there are 52 complete weeks and 2 days over. The following are the possible combinations for these two 'over' days:

(i) Sunday and Monday, *(ii)* Monday and Tuesday, *(iii)* Tuesday and Wednesday, *(iv)* Wednesday and Thursday, *(v)* Thursday and Friday, *(vi)* Friday and Saturday, and *(vii)* Saturday and Sunday.

In order that a leap year selected at random should contain 53 Sundays, one of the two 'over' days must be Sunday. Since out of the above 7 possibilities, 2 *viz.*, *(i)* and *(vii)*, are favourable to this event,

$$\therefore \qquad \text{Required probability} = \frac{2}{7}$$