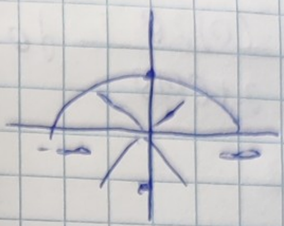
ДЗ – 3 Газизов Б.Р.

$$\int_{-\infty}^{+\infty} \frac{x^4}{1+x^6} \to \lim_{R\to\infty} \int_{-R}^{R} \frac{x^4}{1+x^6}$$



$$\oint = \int + \int_{C_R}$$

$$R\to\infty \quad f(z) = \frac{1}{z^2}$$

$$z = R\cdot e^{i\varphi} \quad dz = R e^{i\varphi} i\, d\varphi \quad d\varphi = \frac{dz}{zi}$$

$$\int_0^{\pi} \frac{dz}{zi z^2} = \frac{dz}{z^3 i} \to 0 \quad R\to\infty \Rightarrow \int_{C_R} \to 0$$

$$\oint = \int$$

$$z^6 = -1$$

$$e^{6i\varphi} = e^{i\pi + 2\pi k}$$

$$\varphi = \frac{\pi}{6} + \frac{\pi k}{3}$$

$$\varphi_1 = \frac{\pi}{6} \qquad \operatorname{res}_{\frac{\pi}{6}} f(z) = \frac{z^4}{6z^5} = \frac{1}{6}\cdot\frac{1}{z} = \frac{1}{6}\cdot\frac{1}{e^{i\frac{\pi}{6}}} = \frac{1}{6} e^{-i\frac{\pi}{6}}$$

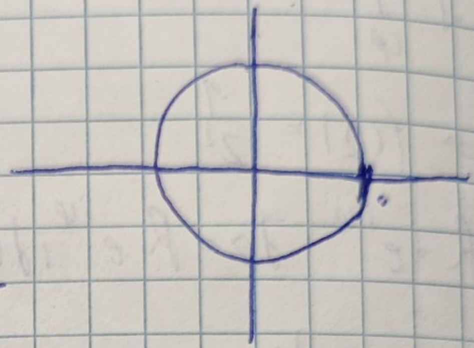
$$\varphi_2 = \frac{\pi}{2} \qquad \operatorname{res}_{\frac{\pi}{2}} f(z) = \frac{1}{6}\cdot\frac{1}{z} = \frac{1}{6} e^{-i\frac{\pi}{2}} = \frac{1}{6}(-i)$$

$$\varphi_3 = \frac{5\pi}{6} \qquad \operatorname{res}_{\frac{5\pi}{6}} f(z) = \frac{1}{6}\cdot\frac{1}{z} = \frac{1}{6} e^{-\frac{5\pi}{6}i}$$

$$\int = 2\pi i \sum \frac{1}{6}\left(e^{-\frac{\pi}{6}i} + e^{-\frac{5\pi}{6}i} + (-i)\right) = 2\pi i \cdot \frac{1}{6}\left(2i\left(-\frac{1}{2}\right)\cdot(-i)\right) =$$

$$= \frac{2\pi i}{6}\cdot -2i = \frac{2\pi}{3}$$

$$\int_0^{2\pi} \frac{\cos 2\theta}{2+\cos\theta}\, d\theta = \left(\int_0^{2\pi} \left(\frac{\cos^2\theta}{2+\cos\theta} - \frac{\sin^2\theta}{2+\cos\theta} \right) d\theta \right)$$



$\cos\theta + 2 = 0$ -

$Z = Re^{i\theta}$ $\quad dz = Re^{i\theta} i \, d\theta$ $\quad d\theta = \frac{dz}{Zi}$

$$f(z) = z^3 \cos\frac{1}{z-2}$$

$$\operatorname{res}_{z=\infty} f(z) = \lim_{z\to\infty}(-z\cdot f(z))$$

$\operatorname{res}_{z=\infty} f(z)$ - ?

$$\operatorname{res}_{z=\infty} f(z) = -C_{-1}$$

$$\cos\frac{1}{z-2} = 1 - \frac{(z-2)^2}{2!} + \frac{(z-2)^4}{4!} = \sum_0^\infty \frac{(-1)^n \cdot (z-2)^{2n}}{(2n)!}$$

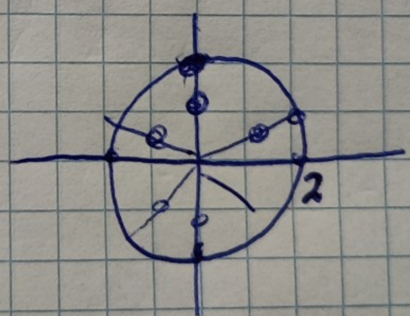
f

$z-2=u \qquad z=u+2$

$$(u+2)^3 \cdot \left(1 - \frac{u^2}{2!} + \frac{u^4}{4!} - \dots\right.$$

$$(u^3+6u^2+12u+8)\left(1-\frac{u^2}{2!}+\frac{u^4}{4!}\dots\right) \Rightarrow -C_{-1} = 0$$

$$\int_C \frac{z^5\,dz}{1+z^6}$$



$$\oint = 2\pi i \sum \operatorname{res} f(z) \qquad z = e^{i\varphi}$$

$$e^{6i\varphi} = e^{i\pi + 2\pi k}$$

$$\operatorname{res}\frac{h(z)}{g(z)} = \frac{h(z_0)}{g'(z_0)} = \frac{z^5}{6z^5} = \frac{1}{6}$$

$$\varphi = \frac{\pi}{6} + \frac{\pi k}{3}$$

$$\sum \operatorname{res} = \frac{1}{6}\cdot 6 = 1 \qquad \oint = 2\pi i \cdot 1 = 2\pi i$$

$f(z) = z^3 \cos \frac{1}{z-2}$

$\cos \frac{1}{z-2}$

$$\int_0^\infty \frac{x \sin \alpha x}{x^2+k^2}\,dx = \frac{1}{2i} \int \frac{x e^{ix\alpha}}{x^2+k^2}\,dx - \frac{1}{2i} \int \frac{x e^{-i\alpha x}}{x^2+k^2}$$

$$\frac{1}{2i} \int \frac{x e^{-i\alpha x}}{x^2+k^2}$$

$\underset{ik}{\operatorname{res}}$:

III $\qquad a>0$

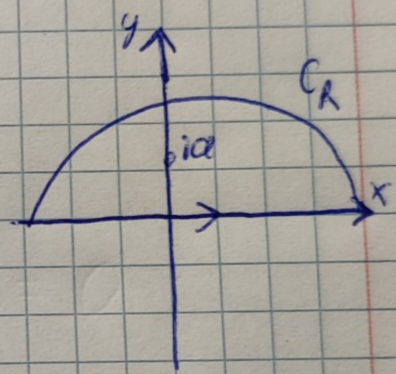
$$I = \int_0^\infty \frac{\cos x}{x^2+a^2}\,dx$$

$x \to -x$

$$I = \frac{1}{2}\int_0^\infty \frac{e^{iz}}{x^2+a^2}\,dx + \frac{1}{2}\int_0^\infty \frac{e^{-ix}}{x^2+a^2}\,dx$$

$$\int_0^{-\infty} \frac{e^{ix}}{x^2+a^2}(-dx) = \int_{-\infty}^0 \frac{e^{ix}}{x^2+a^2}$$

$$\Rightarrow I = \frac{1}{2}\int \frac{e^{ix}}{x^2+a^2}\,dx$$



$$f(z) = \frac{e^{iz}}{z^2+a^2}$$

$$\oint f(z)\,dz = \int_{-\infty}^{+\infty} + \cancel{\int_{C_R}} \to 0$$

$$\lambda = 1 \quad g(z) = \frac{1}{z^2+a^2} \xrightarrow[z\to\infty]{} 0$$

$$\operatorname{res} f(z) = \frac{e^{iz}}{(z+ia)(z-ia)}(z-ia)\Big|_{z=ia} = \frac{e^{iz}}{z+ia}\Big|_{ia} = \frac{e^{-a}}{2ia}$$

$$\Rightarrow 2\pi i \cdot \frac{1}{2} \cdot \frac{e^{-a}}{2ia}$$

III