Газизов Булат

Дз-2.

9.16.

1. $\frac{(1+z^2)^2}{1-z^2}$

$1+z^2=0$ $\quad$ $1-z^2=0$

$z=\pm i$ $\quad$ $z=\pm 1$

$\frac{(z-i)^2(z+i)^2}{(z-1)(z+1)} \Rightarrow$ $z=i$ – нуль 2 порядка

$z=-i$ н 2 п.

$z=1$ – полюс I порядка

$z=-1$ – полюс I п.

2. $ctg\, z = \frac{\cos z}{\sin z}$

$\cos z = 0$

$z=\frac{\pi}{2}+\pi n$. $\quad n \in \pm 1, \pm 2$ – нуль I порядка

$\sin z = 0$

$z = 2\pi n$ $\quad n - 0, \pm 1, \pm 2$ – полюс I п.

3. $z\, tg^2 z = \frac{z \cdot \sin^2 z}{\cos^2 z}$

$tg\, z = 0$

$z \cdot \sin^2 z = 0$

$z=0$ – нуль III п.

$z = 2\pi n$ $\quad n \in Z$ нуль I п.

$z = \frac{\pi}{2} + 2\pi n$ $\quad n \in Z$ – полюс II п.

20.01.

1. $\sum_{n=-\infty}^{\infty} 2^{-|n|} z^n = \frac{1}{2^{|n|}} z^n$

$R = \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right|$ $R = \lim_{n\to\infty} \frac{1}{\sqrt[n]{|c_n|}}$

$r = \lim \left|\frac{c_{-n-1}}{c_{-n}}\right|$

$R = \lim_{n\to\infty} \left| \frac{\frac{1}{2^{|n|}}}{\frac{1}{2^{|n+1|}}} \right| = \left| \frac{1}{2^{|n|}} 2^{|n|} \cdot 2 \right| = 2.$

$\left(r = \lim \left|\frac{c_{n-1}}{c_{-n}}\right| = \frac{\frac{1}{3^{-n-1}+1}}{\frac{1}{3^{-n}+1}} = \frac{3^{-n}+1}{3^{-n-1}+1} = \frac{\frac{1}{3^n}+1}{3^{-(n+1)}+1} = \frac{\frac{1}{3^n}+1}{\frac{1}{3^n}\cdot\frac{1}{3}+1} = \right.$

$\left. = \frac{1}{e\frac{1}{3}+1} = \frac{1}{1} = 1. \right)$

$\mathcal{N}2$

Область сходимости: $1 < |z| < 2.$

$r = \lim \left|\frac{c_{-n-1}}{c_{-n}}\right| = \frac{\frac{1}{2^{1-n-1}}}{\frac{1}{2^{1-n|}}} = \frac{\frac{1}{2^{1-(n+1)}}}{2^{1-n|}} = \frac{1}{2}$

2. $\sum_{n=-\infty}^{\infty} \frac{z^n}{3^n+1} = \frac{1}{3^n+1} \cdot z^n$

$R = \lim_{n\to\infty} \left| \frac{\frac{1}{3^n+1}}{\frac{1}{3^{n+1}+1}} \right| = \frac{1}{3^n+1} \cdot (3^n \cdot 3 + 1) = \lim_{n\to\infty} \frac{3^n\left(3 + \frac{1}{3^n}\right)}{3^n\left(1+\frac{1}{3^n}\right)} = \frac{3}{1} = 3$

$r = \lim \left|\frac{c_{-n-1}}{c_{-n}}\right| = \frac{\frac{1}{3^{-n-1}+1}}{\frac{1}{3^{-n}+1}} = \frac{3^{-n}+1}{3^{-n-1}+1} = \frac{\frac{1}{3^n}+1}{\frac{1}{3^n}\cdot\frac{1}{3}+1} = \frac{1}{0\cdot\frac{1}{3}+1} = \frac{1}{1} = 1$

область сходимости: $1 < |z| < 3$

4. $\sum\limits_{n=-\infty}^{\infty} 2^{-n^2}(z+1)^n$

$R = \lim \left|\frac{c_n}{c_{n+1}}\right| = \frac{2^{-n^2}}{2^{-(n+1)^2}} = \frac{\frac{1}{2^{n^2}}}{\frac{1}{2^{(n+1)^2}}} = \frac{2^{(n+1)^2}}{2^{n^2}} =$

$= \frac{2^{n^2+2n+1}}{2^{n^2}} = 2^{2n}\cdot 2 = \infty$

$r = \lim \left|\frac{c_{-n-1}}{c_{-n}}\right| = \frac{2^{-(-n-1)^2}}{2^{-(-n)^2}} = \frac{2^{-(n^2+2n+1)}}{2^{-n^2}} = 2^{-2n}\cdot 2^{-1} = 0$

область сходимости: $0 < |z+1| < \infty$

20.09.

$\frac{1}{z(z-3)^2}$ $\qquad [a=1 \quad D: 1<|z-1|<2]$

$\frac{1}{z(z-3)^2} = \frac{A}{z} + \frac{B_1}{z-3} + \frac{B_2}{(z-3)^2} = \frac{1}{z(z-3)^2}$

$A(z-3)^2 + B_1 z(z-3) + B_2 z = 1$

$z=3: \quad 3B_2 = 1 \quad B_2 = \frac{1}{3}$

$z=0: \quad 9A = 1 \quad A = \frac{1}{9}$

$z=2: \quad A + -2B_1 + 2B_2 = 1$

$\frac{1}{9} - 2B_1 + \frac{2}{3} = 1$

$\frac{7}{9} - 1 = 2B_1$

$-\frac{2}{9} = 2B_1$

$B_1 = -\frac{1}{9}$

$\frac{1}{9z} - \frac{1}{9(z-3)} + \frac{1}{3(z-3)^2}$

$$\frac{1}{9z} - \frac{1}{9(z-1)} + \frac{1}{3(z-3)^2} \qquad 1 < |z-1| < 2$$

$$\bullet\ \frac{1}{9z} = \frac{1}{9(z-1)+1} = \frac{1}{9} \cdot \frac{1}{(z-1)+1} = \frac{1}{9} \frac{1}{(z-1)\left(1+\frac{1}{z-1}\right)} = \frac{1}{9}\frac{1}{z-1} \cdot \frac{1}{1+\frac{1}{z-1}} = \frac{1}{9}\frac{1}{z-1} \cdot$$

$$\cdot \sum (-1)^n \cdot \frac{1}{(z-1)^n} = \sum_{n=0}^{\infty} \frac{1}{9} \frac{(-1)^{n+1}}{(z-1)^{n+1}}$$

$$\bullet\ -\frac{1}{9(z-3)} = \frac{1}{9((z-1)-2)} = \frac{1}{9} \cdot \frac{1}{(z-1)-2} = \frac{1}{9} \cdot \frac{1}{-2\left(1-\frac{(z-1)}{2}\right)} = -\frac{1}{9} \cdot \frac{1}{-2} \sum \frac{(z-1)^n}{2^n} =$$

$$= \sum \frac{1}{9} \frac{(z-1)^n}{2^{n+1}}$$

$$\bullet\ \frac{1}{3(z-3)^2} = \frac{1}{3} \frac{1}{((z-1)-2)^2} = \frac{1}{3} \frac{1}{\left(-2\left(1-\frac{(z-1)}{2}\right)\right)^2} = \overbrace{\frac{1}{3} \frac{1}{-2^2}}^{a} \cdot \frac{1}{\left(1-\frac{(z-1)}{2}\right)^2}$$

$$\left\{\frac{z-1}{2} = u\right\} = \frac{1}{3} \frac{1}{(-2)^2} \quad \left\{\frac{1}{(1-u)^2} = \frac{d}{du}\left(\frac{1}{1-u}\right) = a \sum \frac{d}{du} \frac{1}{1-u} = a \sum u^n = a \sum \frac{d}{du} u^n =\right.$$

$$= a \sum n u^{n-1} = \sum n \frac{(z-1)^{n-1}}{2^{n-1}} \cdot \frac{1}{3} \cdot \frac{1}{4} = \sum \frac{1}{3} n \cdot \frac{(z-1)^{n-1}}{2^{n+1}}$$

20.16.

1. $z^3 \cdot e^{\frac{1}{z}} \qquad (a = 0 \quad 0 < |z| < \infty)$

$$z^3 \cdot \left(1 + \frac{1}{z\,1!} + \frac{1}{z^2\,2!} + \frac{1}{z^3\,3!} + \frac{1}{z^4\,4!} + \frac{1}{z^5\,5!} + \frac{1}{z^6\,6!}\right) \dots$$

$$= z^3 + z^2 \frac{1}{1!} + z \cdot \frac{1}{2!} + \frac{1}{3!} + \left(\underset{1}{\frac{1}{z\,4!}} + \underset{2}{\frac{1}{z^2\,5!}} \dots\right) =$$

$$= z^3 + z^2 + \frac{1}{2} z + \frac{1}{6} + \sum \frac{z^{-n}}{(n-3)!}$$

2. $z^2 \sin \pi \frac{z+1}{z}$ $\quad [a=0 \quad 0<|z|<\infty]$

$$z^2 \cdot \left(u - \frac{u^3}{3!} + \frac{u^5}{5!} \dots \sum \frac{(-1)^n u^{2n+1}}{(2n+1)!} \right).$$

2C.21

1. $\frac{z}{(z+2)^2} \quad (z_0 = -2)$

$$\frac{z}{(z+2)^2} = \frac{A}{z+2} + \frac{B}{(z+2)^2} = \frac{z}{(z+2)^2}$$

$A(z+2) + B = z$

$z = -2: \quad B = -2$

$z = 0 \quad 2A - 2 = 0$

$\quad 2A = 2 \quad A = 1$

$$\frac{1}{(z+2)} + -\frac{2}{(z+2)^2}$$

2. $$\frac{e^z + 1}{e^z - 1} = \frac{\left\{1 + \frac{z}{1!} + \frac{z^2}{2!} \dots\right\} + 1}{\left\{1 + \frac{z}{1!} + \frac{z^2}{2!} \dots\right\} - 1} = \frac{2 + \frac{z}{1!} + \frac{z^2}{2!} \dots}{\frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!}} =$$

$=$