

1) Draw the Voronoi diagram for the case when the region centers are $x_1 = (0, 0)^T$, $x_2 = (0, 2)^T$, $x_3 = (3, 0)^T$, $x_4 = (4, 5)^T$.

2) Consider the discrete cosine transform (DCT), for a one-dimensional case with length ($N = 4$).

a) Write the DCT transform equation.

b) Plot the four basis functions.

c) Write the transform matrix G .

d) Is G orthonormal? (No proof is needed, but write the definition of orthonormal.)

3) Consider a transform code where the transform vectors (or basis functions) are given by (assume 4-dimensional case):

$$f_0(i) = \frac{1}{2}, \quad i = 0, 1, 2, \text{ or } 3$$

$$f_1(i) = \begin{cases} \frac{1}{2} & \text{if } i = 0 \text{ or } i = 1 \\ -\frac{1}{2} & \text{if } i = 2 \text{ or } i = 3 \end{cases}$$

$$f_2(i) = \begin{cases} \frac{1}{2} & \text{if } i = 0 \text{ or } i = 3 \\ -\frac{1}{2} & \text{if } i = 1 \text{ or } i = 2 \end{cases}$$

$$f_3(i) = \begin{cases} \frac{1}{2} & \text{if } i = 0 \text{ or } i = 2 \\ -\frac{1}{2} & \text{if } i = 1 \text{ or } i = 3 \end{cases}$$

a) Plot the four basis functions.

b) Write the transform matrix A .

c) Is A orthonormal? Prove.

d) Consider the vector $(1, -1, 1, 1)^T$. Find the transform coefficients.

e) Consider the two vectors: $(1, 0, -1, 0)^T$ and $(1.5, -0.5, -1.5, 0.5)^T$. Find the two basis functions that most efficiently encode these two vectors. If we encode the two vectors using the transform coefficients of the two basis functions you found, will the encoding be lossless or lossy?