Export under risk and expectation dependence

Abstracting from self-protection and self-insurance effects of export production choices, exporting firms usually have access to a number of risk sharing markets that have an efficient risk management role. Two of the most striking results achieved from the existence of risk sharing markets are the separation theorem and the and full-hedging theorem. This note examines the optimal production for exports and hedging decisions of a risk-averse firm facing both hedgeable exchange rate risk and non-hedgeable (background) risk. While the separation property holds in this context, the full-hedging property does not. The correlation between the non-hedgeable income risk and the hedgeable foreign exchange rate risk is pivotal. We show that the concept of expectation dependence is useful in determining the optimal financial risk management.

Keywords Export, Background risk, Exchange Rate Risk, Expectation Dependence, Hedging

JEL Classification D81, D84, F11, F30, F31

1 Introduction

The purpose of risk management is to control the possible adverse economic consequences of output (or input) price uncertainty for national and international firms and investors. Several instruments are available to firms of developed economies to hedge interest rate, price and exchange rate risk. Hedging can reduce or even eliminate the firm's risks. Price con- tingent contracts are traded on well-organized futures markets. Risk management activities in general do not seek to increase profit per se but rather involve shifting income from more favourable states of nature to less favourable ones, thus increasing the expected utility of a risk-averse firm.

The literature on financial risk management argues that a passive match of an open risk position with a countervailing risk increases enterprise value (Adam et al., 2015; Boyer and Marin, 2015). In fact, two of the most striking theoretical results achieved from the existence of risk sharing markets are the separation theorem and the full-hedging theorem (see, for example, Battermann et al. (2000)). The separation theorem states that a risk-averse exporter, who faces foreign exchange rate uncertainty for the commodity it supplies, will produce a level of output that is independent of the probability distribution of the exchange rate risk and of the risk attitude. Optimal output depends exclusively on the forward foreign exchange rate, which is known with certainty when inputs are committed to production, and hence the production or investment activity is riskless. This separation result was first proved by Holthausen (1979), and Feder, Just and Schmitz (1980). Later this result has been extended to different types of national and international firms and investors.[1]

The full-hedging theorem states that if forward/futures markets are unbiased, a full-hedge is the optimal hedging policy. However, the empirical literature provides large evidence that companies do not pursue a congruent hedging strategy, but instead actively vary the size of hedging and tend to hedge less (Brown et al., 2006; Carter et al., 2006; Haushalter, 2000; Jin und Jorion, 2006; Tufano, 1996; Hoang and Ruckes, 2017).

In this note, we consider a risk-averse exporting firm that faces both hedgeable exchange rate risk and non-hedgeable income risk. Franke et al. (2011) offer several reasons for the existence of non-hedgeable income risk, i.e. additive and multiplicate background risks. This is particularly the case when informational asymmetries prevail or when assets are non-marketable. We show that the separation property remains valid in this context. This result applies to more general settings and includes uncertainty in other fi- nancial activities of the international firm which are not directly related to the production decision for the export market. However, we also show that the full-hedging property no longer holds, when the exporter faces the addi- tional income risk from other assets, which is correlated with the exchange rate risk. The correlation between the non-hedgeable income risk and the hedgeable exchange rate risk affects the firm's hedging decision, thereby in- validating the full-hedging property. To characterize the exporter's optimal use of financial instruments, we show that the concept of expectation de- pendence *a la* Wright (1987) is useful. Expectation dependence provides a general bivariate dependence structure. The results have direct implications

[1]See, for example, Kawai and Zilcha (1986), Broll et al. (2001), Wong (2003), Broll and Wong (2013) and others.

for risk management decision-making and for international trade.

The paper is organized as follows. First we delineate a model of an exporter's income risk facing exchange rate uncertainty and income risk from other assets, i.e. financial investment. We incorporate a tractable covariance structure into the underlying risks, i.e. the expectation dependence. Then the main results of this model are derived and discussed. The final section concludes.

2 The exporting firm under risks

Consider an exporting firm, hereafter referred to as the exporter, who produces a single commodity, Q, according to a domestic cost function, $C(Q)$, where $C'(Q) > 0$ and $C''(Q) > 0$. The world price in foreign currency, p, is given. The exporter has to make his production decision for export at date 0. The output, Q, is ready for sale at date 1 at the then prevailing foreign exchange rate, $\tilde{e}$, which is not known ex ante. Throughout the paper, we signify a random variable with a tilde (˜).

The exporter is endowed with some financial assets that are worth $\tilde{I}$ at date 1. While this stochastic income from the financial assets is not insurable, the firm can sell (purchases if negative) H units of his foreign revenue at the predetermined forward exchange rate, e_f, at date 0 to hedge against his foreign exposure. As will be shown shortly, the exporter's hedging decision is affected in a systematic way depending on how $\tilde{I}$ and $\tilde{e}$ are correlated. Let $F(I, e)$ be the joint probability distribution of $\tilde{I}$ and $\tilde{e}$ over support $[\underline{I}, \bar{I}] \times [\underline{e}, \bar{e}]$, where $0 < \underline{I} < \bar{I}$ and $0 < \underline{e} < \bar{e}$. Likewise, let $F_I(I)$ and $F_e(e)$ be the marginal probability distribution of $\tilde{I}$ over support $[\underline{I}, \bar{I}]$, and that of $\tilde{e}$ over support $[\underline{e}, \bar{e}]$, respectively.

The exporter's random final wealth at date 1, $\tilde{W}$, is given by

$$\tilde{W} = \tilde{I} + \tilde{e}pQ - C(Q) + (e_f - \tilde{e})H. \qquad (1)$$

Production costs are implicitly compounded to the end of the period using the constant market interest rate. The exporter is risk averse and possesses a von Neumann-Morgenstern utility function, $U(W)$, defined over his wealth at date 1, W. Risk aversion implies that $U'(W) > 0$ and $U''(W) < 0$.

The exporter's ex-ante decision problem is to choose a level of output for export, Q, and an amount of forward sales, H, so as to maximize the expected utility of his random wealth at date 1:

$$\max_{Q\geq 0,\, H} \ \mathrm{E}[U(\tilde{W})], \tag{2}$$

where $\mathrm{E}(\cdot)$ is the expectation operator with respect to the joint probability distribution of $\tilde{I}$ and $\tilde{e}$, and $\tilde{W}$ is defined in Eq. (1).

The first-order conditions for program (2) are given by

$$\mathrm{E}\{U^{\mathrm{r}}(\tilde{W}^{*})[\tilde{e}p - C^{\mathrm{r}}(Q^{*})]\} = 0, \tag{3}$$

$$\mathrm{E}[U^{\mathrm{r}}(\tilde{W}^{*})(e_f - \tilde{e})] = 0, \tag{4}$$

where an asterisk ($*$) indicates an optimal level. The second-order conditions for program (2) are satisfied given risk aversion and the strict convexity of the cost function $C(Q)$.

3 Production decision for exports

This section examines the exporter's optimal production decision. Substituting Eq. (4) into Eq. (3) yields

$$\mathrm{E}\{U^{\mathrm{r}}(\tilde{W}^{*})[e_f p - C^{\mathrm{r}}(Q^{*})]\} = 0.$$

Since $U^{\mathrm{r}}(W) > 0$, we obtain

$$C^{\mathrm{r}}(Q^{*}) = e_f p, \tag{5}$$

thereby invoking the following proposition.

Proposition 1. Endowing the exporter with other sources of random income does not invalidate the separation property. Namely, the firm's production decision depends neither on his attitude towards risk nor on his belief about the joint probability distribution of the underlying uncertainty.

An immediate implication of our first result is that the firm's optimal production for international trade depends neither on the utility function nor on the joint CDF, $H(I, e)$. If the conditions underlying this separation property can be regarded as applicable, there are important economic implications. If one individual (the owner) were to delegate production decisions to another (the agent), the latter would not have had to know anything about the sub- jective preferences of the former. The separation property assures that the owner is made as well off as possible.

4 Exporter's hedging decision

This section examines the exporter's optimal hedging decision. It is evident from equation (4) that the exporter's optimal forward position, H^*, depends on both the utility and joint probability distribution functions. To determine whether this position is an under-hedge ($H^* < Q^*$), a full-hedge ($H^* = Q^*$), or an over-hedge ($H^* > Q^*$), we partially differentiate $\mathrm{E}[U(\tilde{W})]$ with respect to H and evaluate the resulting derivative at $Q = H = Q^*$ to yield

$$\frac{\partial \mathrm{E}[U'(\tilde{W})]}{\partial H}\bigg|_{Q=H=Q^*} = \mathrm{E}\{U'[\tilde{I} + e^f pQ^* - C(Q^*)](e^f - \tilde{e})\}. \tag{6}$$

It follows from Eq. (4) and the second-order conditions that $H^* < Q^*$ or $H^* > Q^*$ depending on whether the right-hand side of Eq. (6) is negative or positive, respectively.

To focus on the exporter's pure hedging motive, we assume hereafter that the forward exchange rate is unbiased in that e^f is set equal to the unconditional expected value of $\tilde{e}$. In this unbiased case, we can write Eq. (6) as[2]

$$\frac{\partial \mathrm{E}[U'(\tilde{W})]}{\partial H}\bigg|_{Q=H=Q^*} = -\mathrm{Cov}\{U'[\tilde{I} + e_f pQ^* - C(Q^*)], \tilde{e}\}. \tag{7}$$

where $\mathrm{Cov}(\cdot, \cdot)$ is the covariance operator with respect to the joint probability distribution of $\tilde{I}$ and $\tilde{e}$.

We define the probability distribution function of $\tilde{I}$ conditional on the event that $\tilde{e} \leq e$ as

$$F(e|\tilde{I} \leq I) = \frac{F(I, e)}{F_I(I)}. \tag{8}$$

Let $\mathrm{E}(\tilde{e}|\tilde{I} \leq I)$ be the expected value of $\tilde{e}$ with respect to the conditional probability distribution function, $F(e|\tilde{I} \leq I)$. The following bivariate dependence structure, known as expectation dependence, is due to Wright (1987).

Definition 1. The random exchange rate, $\tilde{e}$, is said to be positively (negatively) expectation dependent on the random asset value, $\tilde{I}$, if

$$\mathrm{ED}(\tilde{e}|I) = \mathrm{E}(\tilde{e}) - \mathrm{E}(\tilde{e}|\tilde{I} \leq I) \geq (\leq)\ 0, \tag{9}$$

[2]For any two random variables, $\tilde{X}$ and $\tilde{Y}$, we have $\mathrm{Cov}(\tilde{X}, \tilde{Y}) = \mathrm{E}(\tilde{X}\tilde{Y}) - \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y})$.

for all $I \in [\underline{I}, \bar{I}]$, where the inequality is strict for some non-degenerate intervals.

Eq. (9) implies that the expected value of the random exchange rate, $\tilde{e}$ is revised upward (downward) whenever one discovers that the random asset value, $\tilde{I}$, is small, in the precise sense that one is given the truncation, $\tilde{I} \leq I$. To see further how Definition 1 defines dependence, we write Eq. (9) as

$$\begin{aligned} \mathrm{ED}(\tilde{e}|I) &= \int_{\underline{e}}^{\bar{e}} e \,\mathrm{d}F_e(e) - \int_{\underline{e}}^{\bar{e}} e \,\mathrm{d}F(e|\tilde{I} \leq I) \\ &= \int_{\underline{e}}^{\bar{e}} [F(e|\tilde{I} \leq I) - F_e(e)] \,\mathrm{d}e, \end{aligned} \tag{10}$$

where the second equality follows from integration by parts. It is evident from Eq. (10) that $\mathrm{ED}(\tilde{e}|I) \geq (\leq)\ 0$ if $F(e|\tilde{I} \leq I) \geq (\leq)\ F_e(e)$ for all $e \in [\underline{e}, \bar{e}]$, i.e., if $F_e(e)$ dominates (is dominated by) $F(e|\tilde{I} \leq I)$ in the sense of first-order stochastic dominance. Hence, positive (negative) expectation dependence is implied by the fact that small asset value increases (decreases) the riskiness of the random exchange rate in the sense of first-order stochastic dominance, which can be verified empirically using tests of stochastic dominance.

Cuadras (2002) proves that $\mathrm{Cov}[A(\tilde{I}), B(\tilde{e})]$ can be written in terms of the CDFs, $F_I(I)$, $F_e(e)$, and $F(I, e)$, as follows:

$$\mathrm{Cov}[A(\tilde{I}), B(\tilde{e})] = \int_{\underline{I}}^{\bar{I}} \int_{\underline{e}}^{\bar{e}} [F(I, e) - F_I(I)F_e(e)] \,\mathrm{d}A(I) \,\mathrm{d}B(e), \tag{11}$$

where $A(\cdot)$ and $B(\cdot)$ are functions of bounded variation. Using Eq. (11) with $A(\tilde{I}) = U'[\tilde{I} + e_f pQ^* - C(Q^*)]$ and $B(\tilde{e}) = \tilde{e}$ yields

$$\begin{aligned} &\mathrm{Cov}\{U'[\tilde{I} + e_f pQ^* - C(Q^*)], \tilde{e}\} \\ &= \int_{\underline{I}}^{\bar{I}} ED(\tilde{e}|I)\, U''[I + e_f pQ^* - C(Q^*)] F_I(I) \,\mathrm{d}I < (>)\ 0, \end{aligned} \tag{12}$$

where the equality follows from Eq. (11) and the inequality follows from the fact that $\tilde{e}$ is positively (negatively) expectation dependent on $\tilde{I}$. The following proposition follows immediately from Eqs. (7) and (12).

Proposition 2. The exporter optimally opts for an over-hedge (under-hedge), i.e., $H^* > (<)\ Q^*$, if the random exchange rate, $\tilde{e}$, is positively (negatively) expectation dependent on $\tilde{I}$.

5 Concluding remarks

In this paper, we have studied the optimal production and hedging decisions of a risk-averse exporting firm facing both hedgeable exchange rate risk and non-hedgeable income risk from financial assets. While the separation prop- erty is unaffected in the presence of the additional additive income risk, the full-hedging property is not. We show that the concept of expectation depen- dence is useful in determining the exporting firm's optimal futures position.

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