

Antiderivatives

We have studied how to find the derivative of a function. However, many problems require that we recover a function from its known derivative (from its known rate of change). For instance, we may know the velocity function of an object falling from an initial height and need to know its height at any time. More generally, we want to find a function F from its derivative f . If such a function F exists, it is called an *antiderivative* of f . We will see in the next chapter that antiderivatives are the link connecting the two major elements of calculus: derivatives and definite integrals.

Finding Antiderivatives

DEFINITION A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I .

The process of recovering a function $F(x)$ from its derivative $f(x)$ is called *antidifferentiation*. We use capital letters such as F to represent an antiderivative of a function f , G to represent an antiderivative of g , and so forth.

EXAMPLE 1 Find an antiderivative for each of the following functions.

(a) $f(x) = 2x$

Solution We need to think backward here: What function do we know has a derivative equal to the given function?

(a) $F(x) = x^2$

Each answer can be checked by differentiating. The derivative of $F(x) = x^2$ is $2x$.



The function $F(x) = x^2$ is not the only function whose derivative is $2x$. The function $x^2 + 1$ has the same derivative. So does $x^2 + C$ for any constant C . Are there others?

Corollary 2 of the Mean Value Theorem in Section 4.2 gives the answer: Any two antiderivatives of a function differ by a constant. So the functions $x^2 + C$, where C is an **arbitrary constant**, form *all* the antiderivatives of $f(x) = 2x$. More generally, we have the following result.

THEOREM 8 If F is an antiderivative of f on an interval I , then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Thus the most general antiderivative of f on I is a *family* of functions $F(x) + C$ whose graphs are vertical translations of one another. We can select a particular antiderivative from this family by assigning a specific value to C . Here is an example showing how

EXAMPLE 3 Find the general antiderivative of each of the following functions.

(a) $f(x) = x^5$ (b) $g(x) = \frac{1}{\sqrt{x}}$

TABLE 4.2 Antiderivative formulas, k a nonzero constant

Function	General antiderivative
1. x^n	$\frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$

Solution In each case, we can use one of the formulas listed in Table 4.2.

(a) $F(x) = \frac{x^6}{6} + C$ Formula 1
with $n = 5$

(b) $g(x) = x^{-1/2}$, so

$G(x) = \frac{x^{1/2}}{1/2} + C = 2\sqrt{x} + C$ Formula 1
with $n = -1/2$

Other derivative rules also lead to corresponding antiderivative rules. We can add and subtract antiderivatives and multiply them by constants.

TABLE 4.3 Antiderivative linearity rules

	Function	General antiderivative
1. <i>Constant Multiple Rule:</i>	$kf(x)$	$kF(x) + C$, k a constant
2. <i>Negative Rule:</i>	$-f(x)$	$-F(x) + C$
3. <i>Sum or Difference Rule:</i>	$f(x) \pm g(x)$	$F(x) \pm G(x) + C$