

# Logarithmic Differentiation

## Logarithmic Differentiation

The derivatives of positive functions given by formulas that involve products, quotients, and powers can often be found more quickly if we take the natural logarithm of both sides before differentiating. This enables us to use the laws of logarithms to simplify the formulas before differentiating. The process, called **logarithmic differentiation**, is illustrated in the next example.

**EXAMPLE 6** Find  $dy/dx$  if

$$y = \frac{(x^2 + 1)(x + 3)^{1/2}}{x - 1}, \quad x > 1.$$

**Solution** We take the natural logarithm of both sides and simplify the result with the algebraic properties of logarithms from Theorem 1 in Section 1.6:

$$\begin{aligned}\ln y &= \ln \frac{(x^2 + 1)(x + 3)^{1/2}}{x - 1} \\&= \ln ((x^2 + 1)(x + 3)^{1/2}) - \ln (x - 1) && \text{Rule 2} \\&= \ln (x^2 + 1) + \ln (x + 3)^{1/2} - \ln (x - 1) && \text{Rule 1} \\&= \ln (x^2 + 1) + \frac{1}{2} \ln (x + 3) - \ln (x - 1). && \text{Rule 4}\end{aligned}$$

We then take derivatives of both sides with respect to  $x$ , using Equation (2) on the left:

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2 + 1} \cdot 2x + \frac{1}{2} \cdot \frac{1}{x + 3} - \frac{1}{x - 1}.$$

Next we solve for  $dy/dx$ :

$$\frac{dy}{dx} = y \left( \frac{2x}{x^2 + 1} + \frac{1}{2x + 6} - \frac{1}{x - 1} \right).$$

Finally, we substitute for  $y$ :

$$\frac{dy}{dx} = \frac{(x^2 + 1)(x + 3)^{1/2}}{x - 1} \left( \frac{2x}{x^2 + 1} + \frac{1}{2x + 6} - \frac{1}{x - 1} \right).$$

■