

Hyperbolic Functions

The hyperbolic functions are formed by taking combinations of the two exponential functions e^x and e^{-x} . The hyperbolic functions simplify many mathematical expressions and occur frequently in mathematical applications. In this section we give a brief introduction to these functions, their graphs, and their derivatives.

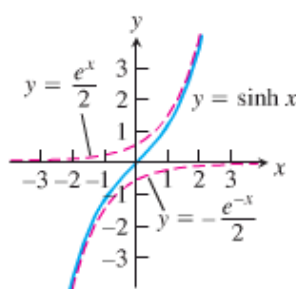
Definitions and Identities

The hyperbolic sine and hyperbolic cosine functions are defined by the equations

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad \text{and} \quad \cosh x = \frac{e^x + e^{-x}}{2}.$$

We pronounce $\sinh x$ as “cinch x ,” rhyming with “pinch x ,” and $\cosh x$ as “kosh x ,” rhyming with “gosh x .” From this basic pair, we define the hyperbolic tangent, cotangent, secant, and cosecant functions. The defining equations and graphs of these functions are shown in Table 7.3. We will see that the hyperbolic functions bear many similarities to the trigonometric functions after which they are named.

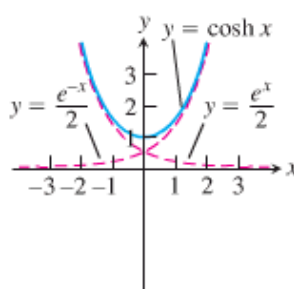
TABLE 7.3 The six basic hyperbolic functions



(a)

Hyperbolic sine:

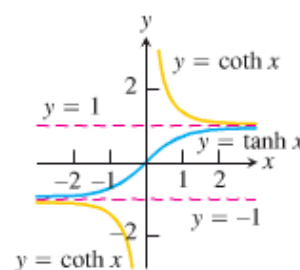
$$\sinh x = \frac{e^x - e^{-x}}{2}$$



(b)

Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2}$$



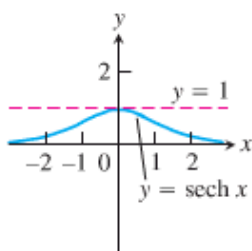
(c)

Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Hyperbolic cotangent:

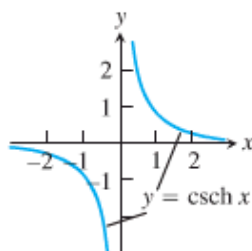
$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$



(d)

Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$



(e)

Hyperbolic cosecant:

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$