Lista 8

1)

a) $y = \tan\left(\frac{x+1}{2}\right)$

$u = \frac{x+1}{2}$ $\qquad y = \tan(u)$

$\frac{du}{dx} = \frac{1}{2}$ → Regra 3 $\qquad \frac{dy}{du} = \sec^2 u$ → Regra 14

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Então:

$\frac{dy}{dx} = \sec^2 u \cdot \frac{1}{2} = \sec^2\left(\frac{x+1}{2}\right) \cdot \frac{1}{2} =$

$= \frac{\sec^2\left(\frac{x+1}{2}\right)}{2}$ //

b) $y = \sqrt{1 + 2\tan x}$

$u = 1 + 2\tan x$ $\qquad y = \sqrt{u} = u^{1/2}$

$\frac{du}{dx} = 2 \cdot \sec^2 x$ → 14 $\qquad \frac{dy}{du} = \frac{1}{2\sqrt{u}}$ → 2

$\frac{dy}{dx} = 2\sec^2 x \cdot \frac{1}{2\sqrt{u}} = \cancel{2}\sec^2 x \cdot \frac{1}{\cancel{2}\sqrt{1+2\tan x}} =$

$= \frac{\sec^2 x}{\sqrt{1 + 2\tan x}}$ //

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e) $y = \sqrt{\tan(x/2)}$

$u = \tan(x/2)$ → Regra (11) e (14)

$y = \sqrt{u} = u^{1/2}$

$\frac{du}{dy} = \sec^2(x/2) \cdot \frac{1}{2}$

$\frac{dy}{du} = \frac{1}{2\sqrt{u}}$

$\frac{dy}{dx} = \frac{\sec^2(x/2)}{2} \cdot \frac{1}{2\sqrt{u}} = \frac{\sec^2(x/2)}{4\sqrt{\tan(x/2)}}$ //

$\frac{1}{x} \cdot x^{-1} = -\frac{x}{?} = |-\frac{x}{?}|$

d) $y = sen(\sqrt{1+x^2})$

$u = \sqrt{1+x^2}$ → regra 10

$y = sen(u)$ → regra 12

$\frac{du}{dx} = \frac{1}{\cancel{2}} \cdot (1+x^2)^{-1/2} \cdot \cancel{2}x$

$\frac{dy}{du} = cos(u)$

$\frac{du}{dx} = \frac{x}{\sqrt{1+x^2}}$

$\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}} \cdot cos(u) = \frac{x}{\sqrt{1+x^2}} \cdot cos(\sqrt{1+x^2}) =$

$= \frac{x \, cos(\sqrt{1+x^2})}{\sqrt{1+x^2}}$ //

e) $y = \sqrt{1 + tan(x+1/x)}$

$u = 1 + tan(x + 1/x)$

$y = \sqrt{u}$

$\frac{du}{dx} = sec^2(x+1/x) \cdot (1 - 1/x^2)$

$\frac{dy}{du} = \frac{1}{2\sqrt{u}}$

$\frac{dy}{dx} = sec^2\left(\frac{x^2+1}{x}\right) \cdot \left(\frac{x^2-1}{x^2}\right) \cdot \frac{1}{2\sqrt{1+Tan\left(\frac{x^2+1}{x}\right)}} =$

$= \frac{sec^2\left(\frac{x^2-1}{x^2}\right) \cdot (x^2-1)}{2\sqrt{1+tan\left(\frac{x^2+1}{x}\right)}}$ //

$\sqrt{x} = x^{1/2}$

data / /

f) $y = \cos^2\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$

$u = \frac{1-\sqrt{x}}{1+\sqrt{x}}$

$y = \cos^2 u$

$\frac{dy}{du} = 2 \cdot \cos u \cdot (-\mathrm{sen}\, u)$

$= -2\cos u \cdot \mathrm{sen}\, u$

$= -\,\mathrm{sen}(2u)$ //

$\frac{du}{dx} = \frac{-\frac{1}{2\sqrt{x}} \cdot (1+\sqrt{x}) - (1-\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2} =$

$= -\frac{1}{2\sqrt{x}} \left[(1+\sqrt{x}) + (1-\sqrt{x})\right] \cdot \frac{1}{(1+\sqrt{x})^2} =$

$= \frac{-2}{2\sqrt{x}(1+\sqrt{x})^2} = \frac{-1}{\sqrt{x}(1+\sqrt{x})^2}$

$\frac{dy}{dx} = \frac{-1}{\sqrt{x}(1+\sqrt{x})^2} \cdot (-\,\mathrm{sen}(2u)) =$

$= \frac{\mathrm{sen}\left(\frac{2(1-\sqrt{x})}{1+\sqrt{x}}\right)}{\sqrt{x}(1+\sqrt{x})^2}$ //

$\to$ de fora p/ dentro

data / /

g) $f(x) = \cos(\text{sen}(\cos(1-x^2)))$

$f'(x) = -\text{sen}(\text{sen}(\cos(1-x^2))).\cos(\cos(1-x^2)).(-\text{sen}(1-x^2).$
$.(-2x)$

$f'(x) = -\text{sen}(\text{sen}(\cos(1-x^2))).\cos(\cos(1-x^2)). \, 2x\,\text{sen}(1-x^2)$

h) $f(x) = \tan^3(\text{sen}^2(4ax+b)) = [\tan(\text{sen}(4ax+b))^2]^3$

$f'(x) = 3.\tan^2(\text{sen}^2(4ax+b)).\sec^2.(\text{sen}^2(4ax+b)).$
$.2.\text{sen}(4ax+b).\cos(4ax+b).\,4a =$
$= 24a\,\tan^2(\text{sen}^2(4ax+b)).\sec^2(\text{sen}^2(4ax+b)).$
$.\text{sen}(4ax+b).\cos(4ax+b).$

i) $g(x) = \text{sen}(x^2 + \underbrace{\text{sen}(x^2 + \text{sen}(x^2))})$

$g'(x) = \cos(x^2 + \text{sen}(x^2 + \text{sen}(x^2))).(2x + \cos(x^2 + \text{sen}\,x^2).$
$.(2x + \cos(x^2).\,2x))$

( )

j) $f(x) = \text{sen}((x+1)^2(x+2))$

$f'(x) = \cos((x+1)^2(x+2)).[2(x+1).1.(x+2) + (x+1)^2.1]$

$= \cos((x+1)^2(x+2)).[(2x+2)(x+2) + (x+1)^2]$
$= \cos((x+1)^2(x+2)).[2x^2+4x+2x+4 + x^2+2x+1]$
$= \cos((x+1)^2(x+2))(3x^2+8x+5)$

2) $f(x) = \dfrac{x}{x^2-4}$, $[0, \infty) - 2$

Supondo $f(x_1) = f(x_2)$

$$\frac{x_1}{x_1^2-4} = \frac{x_2}{x_2^2-4}$$

$$x_1(x_2^2-4) = x_2(x_1^2-4)$$
$$x_1 x_2^2 - 4x_1 = x_2 x_1^2 - 4x_2$$
$$x_1 x_2^2 - x_2 x_1^2 = 4x_1 - 4x_2$$
$$x_1 x_2 (x_2 - x_1) = -4(x_2 - x_1)$$

Se $x_1 \neq x_2$ então $x_1 . x_2 = -4$
↳ não pode, pois $x_1, x_2 \geq 0$

Então $x_1 = x_2$ e f é injetiva

Se $x_2 \in \mathbb{R}$, $\exists x_1$ /
$f(x_1) = x_2$
$$\frac{x_1}{x_1^2-4} = x_2 \Rightarrow x_1 = x_1^2 x_2 - 4x_2$$
$$x_2 x_1^2 - x_1 - 4x_2 = 0$$
$$\Delta = 1 - 4 . x_2 . (-4x_2) =$$
$$= 1 + 16 x_2^2 > 0$$
↳ tem raiz real (tem solução)

Ou seja f é sobrejetiva

Como é injetiva e sobrejetiva é bijetiva e admite inversa

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$$(f^{-1})'[y] = \frac{1}{f'(f^{-1}(y))}$$

$$(f^{-1})'(3/5) = \frac{1}{f'(f^{-1}(3/5))} = \frac{1}{f'(3)} = \frac{1}{-13/25} = \frac{-25}{13}$$

$$\frac{x}{x^2-4} = \frac{3}{5}$$

$* f(3) = 3/5 \Rightarrow f^{-1}(3/5) = 3$

$$f'(x) = \frac{-x^2-4}{(x^2-4)^2}$$

$$3x^2 - 12 = 5x$$

$$3x^2 - 5x - 12 = 0$$

$$\Delta = 25 - 4.3.(-12) = 169$$

$$f'(3) = \frac{-13}{25}$$

$$x = \frac{5 \pm 13}{6}$$

$x_1 = 3$

$x_2 = \frac{-8}{6} = \frac{-4}{3}$

3)

* $\arcsin x = ?$

Seja $f(x) = \operatorname{sen} x$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(\operatorname{arcsen} x)} = \frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1-x^2}} \; //$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \; //$$

* $f'(x) = \cos(x)$

$f'(\arcsin x) = \cos(\arcsin x)$

* $\cos^2 x + \operatorname{sen}^2 x = 1$

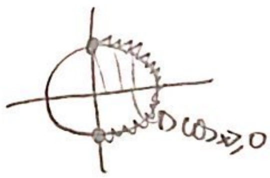
$\cos^2(\arcsin x) + \operatorname{sen}^2(\operatorname{arcsen} x) = 1$

$\cos^2(\arcsin x) = 1 - x^2$

$\cos(\arcsin x) = \sqrt{1-x^2}$

↳ positivo, pois

$-\pi/2 \leq \arcsin x \leq \pi/2$



$\cos(\arcsin x) \geq 0$

* $\arccos x = ?$

Seja $f(x) = \cos x$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(\arccos x)} = \frac{1}{-\operatorname{sen}(\arccos x)} = -\frac{1}{\sqrt{1-x^2}} \,//$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

* $f'(x) = -\operatorname{sen} x$

$f'(\arccos x) = -\operatorname{sen}(\arccos x)$

* $\cos^2 x + \operatorname{sen}^2(x) = 1$

$\cos^2(\arccos x) + \operatorname{sen}^2(\arccos x) = 1$

$x^2 + \operatorname{sen}^2(\arccos x) = 1$

$\operatorname{sen}^2(\arccos x) = 1 - x^2$

$\operatorname{sen}(\arccos x) = \sqrt{1-x^2}$

$\to$ positivo, pois

$0 \leq \arccos x \leq \pi$

$\operatorname{sen}(\arccos x) \geq 0 \,//$

arctan x ?

$f(x) = \tan x$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(\arctan x)} = \frac{1}{\sec^2(\arctan x)} =$$

$$= \frac{1}{x^2+1}$$

$$(\arctan x)' = \frac{1}{x^2+1} \quad //$$

$* \; f'(x) = \sec^2 x$

$f'(\arctan x) = \sec^2(\arctan x)$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2(\arctan x) + 1 = \sec^2(\arctan x)$

$x^2 + 1 = \sec^2(\arctan x)$

arccot x ?

$f(x) = \cot x$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(\text{arccot} x)} = \frac{1}{-\text{cosec}^2(\text{arccot} x)} =$$

$$= \frac{-1}{1+x^2} \quad // \qquad (\text{arcot } x)' = \frac{-1}{x^2+1} \quad //$$

$* \; f'(x) = -\text{cosec}^2 x$

$f'(\text{arccot} x) = -\text{cosec}^2(\text{arcot } x)$

$* \; 1 + \cot^2 x = \text{cosec}^2 x$

$1 + \cot^2(\text{arccot} x) = \text{cosec}^2(\text{arccot} x)$

$1 + x^2 = \text{cosec}^2(\text{arccot } x)$

arccosec x

Seja $f(x) = \operatorname{cosec} x$ → domínio $|x| \geq 1$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(\operatorname{arccosec} x)} = \frac{1}{-x \cdot \cot(\operatorname{arccosec} x)} =$$

$$= -\frac{1}{x \cdot \sqrt{x^2-1}}, \quad |x| > 1 \qquad (\operatorname{arccosec} x)' = -\frac{1}{x\sqrt{x^2-1}}, \ |x| > 1$$

$f'(x) = -\operatorname{cosec} x \cdot \cot x$

$f'(\operatorname{arccosec} x) = -\operatorname{cosec}(\operatorname{arccosec} x) \cdot \cot(\operatorname{arccosec} x)$

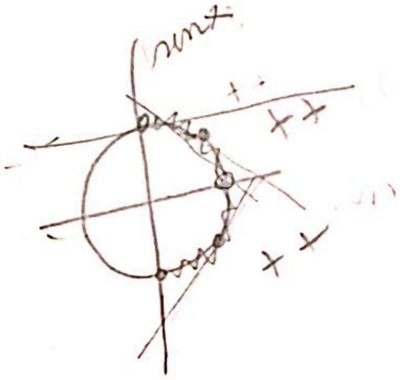
$f'(\operatorname{arccosec} x) = -x \cdot \cot(\operatorname{arccosec} x)$

$* \ 1 + \cot^2 x = \operatorname{cosec}^2 x$

$1 + \cot^2(\operatorname{arccosec} x) = \operatorname{cosec}^2(\operatorname{arccosec} x)$

$\cot^2(\operatorname{arccosec} x) = x^2 - 1$

$\cot(\operatorname{arccosec} x) = \sqrt{x^2-1}$



Lo positivo pois,

$\operatorname{arccosec} x : [-\pi/2, \pi/2] - \{0\}$

$\cot(\operatorname{arccosec} x) > 0$

arcsec x ?

$f(x) = \sec x$ → Domínio $(-\infty,-1] \cup [1,+\infty)$

$|x| \geq 1$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(\operatorname{arcsec} x)} = \frac{1}{x \tan(\operatorname{arcsec} x)} =$$

$$= \frac{1}{x \cdot \sqrt{x^2-1}}, \ |x| > 1 \qquad (\operatorname{arcsec} x)' = \frac{1}{x\sqrt{x^2-1}}, \ |x|>1$$

$*$ $f'(x) = \sec x \cdot \tan x$

$f'(\operatorname{arcsec} x) = \sec(\operatorname{arcsec} x) \cdot \tan(\operatorname{arcsec} x)$

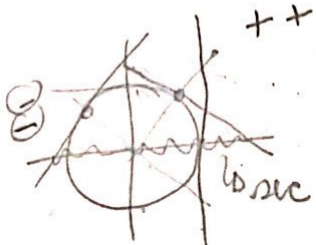
$f'(\operatorname{arcsec} x) = x \cdot \tan(\operatorname{arcsec} x)$

$*$ $\tan^2 x + 1 = \sec^2 x$

$\tan^2(\operatorname{arcsec} x) + 1 = \sec^2(\operatorname{arcsec} x)$

$\tan^2(\operatorname{arcsec} x) = x^2 - 1$

$\tan(\operatorname{arcsec} x) = \sqrt{x^2-1}$



↳ positivo pois,

arcsec x : $[0,\pi] - \frac{\pi}{2}$ → injetiva

$\tan(\operatorname{arcsec} x) > 0$

data / /

4)

a) $f'''(x)$ para $f(x) = 7x^3 - 6x^5$

$f'(x) = 21x^2 - 30x^4$
$f''(x) = 42x - 120x^3$
$f'''(x) = 42 - 360x^2$ //

b) $f''(x)$ para $f(x) = \frac{x}{1+x}$

$f'(x) = \frac{1.(1+x) - x.1}{(1+x)^2} = \frac{1+x-x}{(1+x)^2} = \frac{1}{(1+x)^2} = (1+x)^{-2}$

$f''(x) = -2.(1+x)^{-3}.1 = \frac{-2}{(1+x)^3}$

c) $\frac{d^2y}{dx^2}$ para $y = x^2 - \frac{1}{x^2} = x^2 - x^{-2}$

$\frac{dy}{dx} = 2x + 2x^{-3}$

$\frac{d^2y}{dx^2} = 2 - 6x^{-4} = 2 - \frac{6}{x^4}$ //

d) $\frac{d^4y}{dx^4}$ para $y = ax^4$

$\frac{dy}{dx} = 4ax^3$

$\frac{d^2y}{dx^2} = 12ax^2$

$\frac{d^3y}{dx^3} = 24ax$

$\frac{d^4y}{dx^4} = 24a$ //

e) $\frac{d^3y}{dx^3}$ para $y = (1+2x)^3$

$\frac{dy}{dx} = 3(1+2x)^2 \cdot 2 = 6(1+2x)^2$

$\frac{d^2y}{dx^2} = 12(1+2x) \cdot 2 = 24(1+2x) = 24 + 48x$

$\frac{d^3y}{dx^3} = 48$ //

f) $\frac{d^3y}{dx^3}$ para $y = (1+5x)^2$

$\frac{dy}{dx} = 2(1+5x) \cdot 5 = 10 + 50x$

$\frac{d^2y}{dx^2} = 50 \Rightarrow \frac{d^3y}{dx^3} = 0$

~~g) $\frac{d^2}{dx^2}(x^3 + d$~~

g) $\frac{d^2}{dx}\left(\frac{1-x}{1+x}\right)$

$\frac{d}{dx}\left(\frac{1-x}{1+x}\right) = \frac{-1(1+x)-(1-x)\,1}{(1+x)^2} = \frac{-1-x-1+x}{(1+x)^2} = \frac{-2}{(1+x)^2}$

$\frac{d^2}{dx}\left(\frac{1-x}{1+x}\right) = 4(1+x)^{-3}\,1 = \frac{4}{(1+x)^3}$

h) $\frac{d^2}{dx^2}\left(x^3 + \frac{1}{x^3}\right)$

$\frac{d}{dx}\left(x^3 + x^{-3}\right) = 3x - 3x^{-4}$

$\frac{d^2}{dx^2}\left(x^3 + x^{-3}\right) = 3 - 12x^{-5} = 3 - \frac{12}{x^5}$ //

i) $\frac{d^2}{dx^2}\left(\frac{ax+b}{cx+d}\right)$

$\frac{d}{dx}\left(\frac{ax+b}{cx+d}\right) = \frac{a(cx+d) - (ax+b).c}{(cx+d)^2} =$

$= \frac{acx + ad - acx - bc}{(cx+d)^2} = \frac{ad - bc}{(cx+d)^2} = (ad-bc)(cx+d)^{-2}$

$\frac{d^2}{dx^2}\left(\frac{ax+b}{cx+d}\right) = -2(ad-bc)(cx+d)^{-3}.c =$

$= -(2ad + 2bc) . \frac{c}{(cx+d)^3}$ //

data / /

j) $\frac{d}{dx}\left[x \cdot \frac{d}{dx}(1+x^2)\right]$

$\frac{d}{dx}(1+x^2) = 2x$

$\frac{d}{dx}\left[x \cdot 2x\right] = \frac{d}{dx}2x^2 = 4x$ //

$1 = (x+1)^{-1}$

k) $\frac{d}{dx}\left[x \frac{d^2}{dx^2}\left(\frac{1}{1+x}\right)\right] =$

$\frac{d}{dx}\left(\frac{1}{1+x}\right) = -(x+1)^{-2}$

$\frac{d^2}{dx^2}\left(\frac{1}{1+x}\right) = 2(x+1)^{-3}$

$\frac{d}{dx}\left[x \cdot \frac{2}{(x+1)^3}\right] = \frac{2 \cdot (x+1)^3 - 2x \cdot 3(x+1)^2 \cdot 1}{(x+1)^6} =$

$= \frac{\cancel{(x+1)^2}\left[2(x+1) - 6x\right]}{\cancel{(x+1)^2} \cdot (x+1)^4} = \frac{-4x+2}{(x+1)^4}$

m) $\frac{d^{100}}{dx^{100}}(x^9 - 20x^7 + x^5 + 1)$

$y' = 9x^8 - 140x^6 + 5x^4$

$y'' = 72x^7 - 840x^5 + 20x^3$

$\vdots$

$y^{100} = 0$

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m) $\frac{d^5}{dx^5}(x^5+c^5)$

$y' = 5x^4$

$y'' = 20x^3$

$y''' = 60x^2$

$y'''' = 120x$

$y^{IV} = 120$

o) $f'''(x)$ para $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{1} + 1$

$f'(x) = x^2 + x + 1$

$f''(x) = 2x + 1$

$f'''(x) = 2$

r) $\frac{d^4}{dx^4}[(1-x)^4]$

$y' = 4(1-x)^3\cdot(-1) = -4(1-x)^3$

$y'' = -12(1-x)^2\cdot(-1) = 12(1-x)^2$

$y''' = 24(1-x)^2\cdot(-1) = -24(1-x)^2$

$\frac{d}{dx^4}\left((1-x)^4\right) = -48(1-x)\cdot(-1) = 48-48x$

s) $\frac{d^n y}{dx^n}$ para $y=(1+x)^n$

$\frac{dy}{dx} = n(1+x)^{n-1}\cdot 1$

$\frac{d^2y}{dx^2} = n(n-1)(1+x)^{n-2}$

$\frac{d^3y}{dx^3} = n(n-1)(n-2)(1+x)^{n-3}$

$\vdots$

$\frac{d^n y}{dx^n} = n(n-1)(n-2)\cdots(n-(n-1))\cdot(1+x)^{n-n}$

$= n(n-1)(n-2)\cdots 1 = n!$

data / /

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4) $\frac{d^n y}{dx^n}$ para $y = \frac{1}{x+1} = (x+1)^{-1}$

$\frac{dy}{dx} = -1(x+1)^{-2}$

$\frac{d^2y}{dx^2} = (-1)(-2)(x+1)^{-3}$

$\frac{d^3y}{dx^3} = (-1)(-2)(-3).(x+1)^{-4}$

$\frac{d^4y}{dx^4} = (-1)(-2)(-3)(-4)(x+1)^{-5}$

$\frac{d^n y}{dx^n} = (-1)(-2)(-3)\ldots(-n)(x+1)^{-(n+1)}$

5) $\frac{d^n y}{dx^n}$ para $y = e^{ax}$ é $y^{(n)} = a^n e^{ax}$

$y^1 = e^{ax}.a.\underbrace{\ln e}_{1}$

$y^2 = a.e^{ax}.a = a^2 e^{ax}$,

$y^3 = a^2.e^{ax}.a = a^3 e^{ax}$

$y^{(n)} = a^n.e^{ax}$ //

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7)

a) $x^2+y^2-4x+10y-20=0$

$$\frac{d(x^2)}{dx}+\frac{d(y^2)}{dx}-\frac{d(4x)}{dx}+\frac{d(10y)}{dx}-\frac{d(20)}{dx}=\frac{d(0)}{dx}$$

$$2x+2y\frac{dy}{dx}-4+10\frac{dy}{dx}-0=0$$

$$2y\frac{dy}{dx}+10\frac{dy}{dx}=-2x+4$$

$$\frac{dy}{dx}\left(2y+10\right)=-2x+4$$

$$\frac{dy}{dx}=\frac{-2x+4}{2y+10}=\frac{-x+2}{y+5}\ //$$

b) $x^2+xy-3y^2-2x+6y=0$

$$2x+1.y+x.\frac{dy}{dx}-6y\frac{dy}{dx}-2+6\frac{dy}{dx}=0$$

$$\frac{dy}{dx}\left(x-6y+6\right)=-2x-y+2$$

$$\frac{dy}{dx}=\frac{-2x-y+2}{x-6y+6}\ //$$

c) $(x^2+y^2)^2=x^2y$

$$2.(x^2+y^2).\left(2x+2y\frac{dy}{dx}\right)=2xy+x^2 1.\frac{dy}{dx}$$

$$4x^3+4xy^2+4x^2y\frac{dy}{dx}+4y^3\frac{dy}{dx}=2xy+x^2\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(4x^2y+4y^3-x^2\right)=2xy-4x^3-4xy^2$$

$$\frac{dy}{dx}=\frac{2xy-4x^3-4xy^2}{4x^2y+4y^3-x^2}\ //$$

a) $y^2(1-x) = x^3$

$y^2 - y^2x = x^3$

$2y\frac{dy}{dx} - \left(2y\frac{dy}{dx}\cdot x + y^2\cdot 1\right) = 3x^2$

$2y\frac{dy}{dx} - 2y\frac{dy}{dx}\cdot x - y^2 = 3x^2$

$\frac{dy}{dx}\left(2y - 2yx\right) = 3x^2 + y^2$

$\frac{dy}{dx} = \frac{3x^2+y^2}{2y(1-x)}$ //

e) $(y-2)^2\cdot(x^2+y^2) = y^2$

$(y^2-4y+4)(x^2+y^2) = y^2$

$y^2x^2 + y^4 - 4x^2y - 4y^3 + (4x^2) + 4y^2 = y^2$

$y^2x^2 + y^4 - 4x^2y - 4y^3 + 4x^2 + 3y^2 = 0$

$2y\frac{dy}{dx}\cdot x^2 + y^2\cdot 2x + 4y^3\frac{dy}{dx} - 4\left(2xy + x^2\frac{dy}{dx}\right) - 12y^2\frac{dy}{dx} + 8x + 6y\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(2yx^2 + 4y^3 - 4x^2 - 12y^2 + 6y\right) = 2xy^2 - 8xy + 8x$

$\frac{dy}{dx} = \frac{xy^2 - 4xy + 4x}{yx^2 + 2y^3 - 2x^2 - 6y^2 + 3y}$ //

$$\frac{2}{3} \cdot \frac{1}{3} = \frac{-1}{3}$$

data / /

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f) $x^{2/3} + y^{2/3} = 1$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$\frac{2}{3}y^{-1/3}\frac{dy}{dx} = -\frac{2}{3}x^{-1/3}$$

$$y^{-1/3}\frac{dy}{dx} = -x^{-1/3}$$

$$\frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}}$$

$$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}} = -\left(\frac{y}{x}\right)^{1/3} = -\sqrt[3]{y/x} \ //$$

g) $x^{1/2} + y^{1/2} = 9$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$$

$$y^{-1/2}\frac{dy}{dx} = -x^{-1/2}$$

$$\frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}}$$

$$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/2}$$

$$\frac{dy}{dx} = -\sqrt{y/x}$$

h) $\sqrt{xy} = x - 2y$

$$(xy)^{1/2} = x - 2y$$

$$\frac{1}{2}(xy)^{-1/2} \cdot \left(1 \cdot y + x\frac{dy}{dx}\right) = 1 - 2\frac{dy}{dx}$$

$$\frac{y}{2\sqrt{xy}} + \frac{x}{2\sqrt{xy}}\frac{dy}{dx} = 1 - 2\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{x}{2\sqrt{xy}} + 2\right) = -\frac{y}{2\sqrt{xy}} + 1$$

$$\frac{dy}{dx}\left(\frac{x + 4\sqrt{xy}}{2\sqrt{xy}}\right) = \frac{-y + 2\sqrt{xy}}{2\sqrt{xy}}$$

$$\frac{dy}{dx} = \left(\frac{-y + 2\sqrt{xy}}{2\sqrt{xy}}\right) \frac{2\sqrt{xy}}{(x + 4\sqrt{xy})}$$

$$\frac{dy}{dx} = \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}} \ //$$

i) $\operatorname{sen} x + 2\cos 2y = 1$

$\cos x + 2.(-\operatorname{sen}(2y)). 2\frac{dy}{dx} = 0$

$-4\operatorname{sen}(2y)\frac{dy}{dx} = -\cos x$

$\frac{dy}{dx} = \frac{\cos x}{4\operatorname{sen}(2y)}$

j) $(\operatorname{sen}\pi x + \cos\pi y)^2 = 2$

$2.(\operatorname{sen}\pi x + \cos\pi y).\left(\cos(\pi x).\pi + (-\operatorname{sen}(\pi y).\pi\frac{dy}{dx}\right) = 0$

$(2\operatorname{sen}(\pi x) + 2\cos(\pi y)).(\pi\cos(\pi x) - \pi\operatorname{sen}(\pi y)\frac{dy}{dx}) = 0$

$(2\operatorname{sen}(\pi x).\pi.\cos(\pi x) - 2\operatorname{sen}(\pi x).\pi.\operatorname{sen}(\pi y)\frac{dy}{dx} +$
$+ 2\cos(\pi y).\pi.\cos(\pi x) - 2\cos(\pi y).\pi.\operatorname{sen}(\pi y)\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(-2\operatorname{sen}(\pi x).\pi.\operatorname{sen}(\pi y) - 2\cos(\pi y).\pi.\operatorname{sen}(\pi y)\right) =$

$= 2\operatorname{sen}(\pi x).\pi.\cos(\pi x) + 2\cos(\pi y).\pi.\cos(\pi x)$

$\frac{dy}{dx} = \frac{\cancel{\pi}(\overbrace{2\operatorname{sen}(\pi x).\cos(\pi x)}^{\operatorname{sen}(2\pi x)} + 2\cos(\pi y).\cos(\pi x))}{-\cancel{\pi}(2\operatorname{sen}(\pi x)\operatorname{sen}(\pi y) + \underbrace{2\cos(\pi y).\operatorname{sen}(\pi y)}_{\operatorname{sen}(2\pi y)})} =$

$= -\frac{\operatorname{sen}(2\pi x) + 2\cos(\pi y).\cos\pi x}{\operatorname{sen}(2\pi y) + 2\operatorname{sen}(\pi x).\operatorname{sen}(\pi y)}$ //

data / /

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k) $\sin x = x(1+\tan y)$

$\sin x = x + x.\tan y$

$\cos x = 1 + 1.\tan y + x.\sec^2 y \frac{dy}{dx}$

$x \sec^2 y \frac{dy}{dx} = \cos x - 1 - \tan y$

$\frac{dy}{dx} = \frac{\cos x - 1 - \tan y}{x \sec^2 y}$

l) $\cot y = x - y$

$-\operatorname{cosec}^2 y . \frac{dy}{dx} = 1 - 1\frac{dy}{dx}$

$\frac{dy}{dx}(-\operatorname{cosec}^2 y + 1) = 1$

$\frac{dy}{dx} = \frac{1}{\operatorname{cosec}^2 y + 1}$ //

m) $y = \sin(xy)$

$\frac{dy}{dx} = \cos(xy).\left(1.y + x\frac{dy}{dx}\right)$

$\frac{dy}{dx} = \cos(xy).y + \cos(xy).x\frac{dy}{dx}$

$\frac{dy}{dx}\left(1 - \cos(xy).x\right) = \cos(xy).y$

$\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$

data / /

$\frac{1}{y} = y^{-1}$

n) $x = \sec 1/y$

$1 = \sec(1/y) . \tan(1/y) . -1\, y^{-2} \frac{dy}{dx}$

$1 = -\frac{\sec(1/y) . \tan(1/y)}{y^2} \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-y^2}{\sec(1/y) . \tan(1/y)}$

8)

a) $xy = 4$ $P = (-4,-1)$

$1y + x\frac{dy}{dx} = 0$ $P = (-4,-1)$

$\frac{dy}{dx} = -\frac{y}{x}$ $\left.\frac{dy}{dx}\right|_{\substack{x=-4\\y=-1}} = \frac{+1}{-4} = \frac{-1}{4}$

b) $x^2 - y^3 = 0$, $(1,1)$

$2x - 3y^2\frac{dy}{dx} = 0$ $(1,1)$

$\frac{dy}{dx} = \frac{2x}{3y^2}$ $\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=1}} \frac{2.1}{3.1^2} = \frac{2}{3}$ //

c) $y^2 = \dfrac{x^2-4}{x^2+4}$, $(0,1)$

$$2y\frac{dy}{dx} = \frac{2x(x^2+4)-(x^2-4)2x}{(x^2+4)^2}$$

$$\frac{dy}{dx} = \frac{2x^3+8x-2x^3+8x}{2y(x^2+4)^2}$$

$$\frac{dy}{dx} = \frac{16x}{2(x^2+4)^2y} = \frac{8x}{(x^2+4)^2y}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=0\\y=1}} = \frac{8\cdot 0}{(0+4)^2\cdot 1} = 0$$

d) $(x+y)^3 = x^3+y^3$, $(-1,1)$

$$3(x+y)^2\cdot\left(1+\frac{dy}{dx}\right) = 3x^2+3y^2\frac{dy}{dx}$$

$$3x^2+6xy+3y^2+3x^2\frac{dy}{dx}+6xy\frac{dy}{dx}+3y^2\frac{dy}{dx} = 3x^2+3y^2\frac{dy}{dx}$$

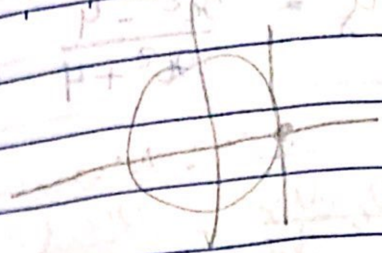
$$\frac{dy}{dx}(3x^2+6xy+\cancel{3y^2}-\cancel{3y^2}) = 3x^2-3x^2-6xy-3y^2$$

$$\frac{dy}{dx}(3x^2+6xy) = -6xy-3y^2$$

$$\frac{dy}{dx} = \frac{-6xy-3y^2}{3x^2+6xy} = \frac{-2xy-y^2}{x^2+2xy} = \frac{-y(2x+y)}{x(x+2y)}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=-1\\y=1}} = \frac{-1(2\cdot(-1)+1)}{-1(-1+2\cdot 1)} = \frac{-(-1)}{-(1)} = \frac{1}{-1} = -1 \,//$$

e) $\tan(x+y) = x$ $\quad (0,0)$



$\sec^2(x+y).(1+\frac{dy}{dx}) = 1$

$\sec^2(x+y) + \sec^2(x+y)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1-\sec^2(x+y)}{\sec^2(x+y)}$

$\left.\frac{dy}{dx}\right|_{\substack{x=0\\y=0}} = \frac{1-\sec^2(0)}{\sec^2(0)} = \frac{1-1}{1} = 0$

f) $x.\cos y = 1$, $(2, \pi/3)$

$1.\cos y + x.(-\operatorname{sen} y)\frac{dy}{dx} = 0$

$\cos y - x \operatorname{sen} y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{\cos y}{x \operatorname{sen} y}$

$60°$

$\left.\frac{dy}{dx}\right|_{\substack{x=2\\y=\pi/3}} = \frac{\cos \pi/3}{2 \operatorname{sen} \pi/3} = \frac{1/2}{2 . \sqrt{3}/2} = \frac{1/2}{\sqrt{3}} = \frac{1}{2\sqrt{3}}$ //

Uma função é dita diferenciável quando sua derivada existe em cada ponto do seu domínio

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9)

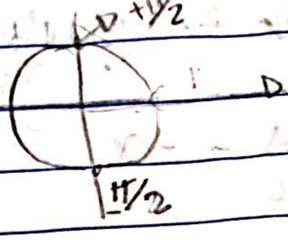
a) $\tan y = x$

$\sec^2 y \neq 0$

$\to D: (-\infty, -1] \cup [1, +\infty)$

$\sec^2 y \frac{dy}{dx} = 1$

$\to \frac{1}{\cos^2 y} \neq 0$



$\frac{dy}{dx} = \frac{1}{\sec^2 y}$

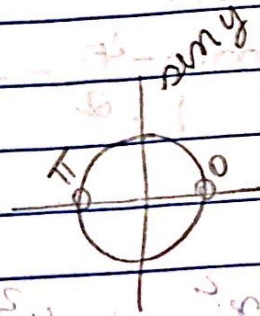
$\cos^2 y \neq 0$

$-\pi/2 < y < \pi/2$

b) $\cos y = x$

$-\sin y \cdot \frac{dy}{dx} = 1$

$\sin y \neq 0$



$\frac{dy}{dx} = \frac{-1}{\sin y}$

$0 < y < \pi$

data / /

10) $\frac{x^2}{2} + \frac{y^2}{8} = 1$ $(1,2)$

$x + \frac{y}{4}\frac{dy}{dx} = 0$

$\frac{y}{4}\frac{dy}{dx} = -x$

$\frac{dy}{dx} = \frac{-4x}{y} \Rightarrow$

$m = \frac{-4}{2} = -2$

$(y-y_0) = m(x-x_0)$

$(y-2) = -2(x-1)$

$y = -2x + 2 + 2$

$y = -2x + 4$ //

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (x_0, y_0)

$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$

$\frac{2y}{b^2}\frac{dy}{dx} = -\frac{2x}{a^2}$

$\frac{dy}{dx} = \frac{-2x}{a^2} \cdot \frac{b^2}{2y} = \frac{-xb^2}{ya^2}$

$m = \frac{-x_0 b^2}{y_0 a^2}$

$(y-y_0) = m(x-x_0)$

$(y-y_0) = \frac{-x_0 b^2}{y_0 a^2}(x-x_0)$

$\frac{y_0(y-y_0)}{b^2} = \frac{-x_0(x-x_0)}{a^2}$

$\frac{y_0 y - y_0^2}{b^2} + \frac{x_0 x - x_0^2}{a^2} = 0$

$\frac{y_0 y}{b^2} + \frac{x_0 x}{a^2} = \underbrace{\frac{y_0^2}{b^2} + \frac{x_0^2}{a^2}}_{1}$

$\frac{y_0 y}{b^2} + \frac{x_0 x}{a^2} = 1$ //

11)

a) $2x^2 - 3y^4 = 0$

(i) $4x - 12y^3 \frac{dy}{dx} = 0$ //

(ii) $4x \frac{dx}{dt} - 12y^3 \frac{dy}{dt} = 0$

b) $x^2 - 3xy^2 + y^3 = 10$

(i) $y = y(x)$

$2x - 3(1 \cdot y^2 + x \cdot 2y y') + 3y^2 \cdot y' = 0$

$2x - 3y^2 - 6xy y' + 3y^2 \cdot y' = 0$

(ii) $\left.\begin{matrix} y = y(t) \\ x = x(t) \end{matrix}\right\}$ $2x x'(t) - 3(x'(t) \cdot y^2 + x \cdot 2y y'(t)) + 3y^2 y'(t) =$

$2x x'(t) - 3x'(t) y^2 - 6xy y'(t) + 3y^2 y'(t) = 0$

data / /

c) $\cos\pi y - 3 \operatorname{sen}\pi x = 1$

(i) $-\operatorname{sen}(\pi y) \cdot \pi \frac{dy}{dx} - 3 \cdot \cos(\pi x) \cdot \pi = 0$

(ii)

$-\operatorname{sen}(\pi y) \cdot \pi \frac{dy}{dt} - 3 \cos(\pi x) \cdot \pi \frac{dx}{dt} = 0$

d) $4 \operatorname{sen} x \cdot \cos y = 1$

(i) $y = y(x)$

$4(\cos x \cdot \cos y + \operatorname{sen} x \cdot (-\operatorname{sen} y) \cdot y') = 0$

$4 \cos x \cos y - 4 \operatorname{sen} x \cdot \operatorname{sen} y \cdot y' = 0$

(ii) $y = y(t)$ e $x = x(t)$

$4(\cos x \cdot x'(t) \cdot \cos y + \operatorname{sen} x \cdot (-\operatorname{sen} y)\, y'(t)) = 0$

$4 \cos x \cdot x'(t) \cdot \cos y - 4 \operatorname{sen} x \cdot \operatorname{sen} y \cdot y'(t) = 0$