Lista 1

1)

a) $f(x) = 3 - 2x, \quad P = (-1, 5)$

$$m = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$ passa no ponto $(c, f(c))$

$c = -1$

$$m = \lim_{\Delta x \to 0} \frac{f(-1 + \Delta x) - f(-1)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{3 - 2(-1 + \Delta x) - 5}{\Delta x} = \lim_{\Delta x \to 0} \frac{3 + 2 - 2\Delta x - 5}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-2\Delta x}{\Delta x} = -2 \,//$$

$m = -2$

b) $g(x) = \frac{3}{2}x + 1, \quad P(-2, -1)$ $\qquad c = -2$

$$m = \lim_{\Delta x \to 0} \frac{f(-2 + \Delta x) - f(-2)}{\Delta x} = \lim_{\Delta x \to 0} \frac{3/2(-2 + \Delta x) + 1 + 2}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-3 + 3\Delta x/2 + 3}{\Delta x} = \lim_{\Delta x \to 0} \frac{3\cancel{\Delta x}}{2} \cdot \frac{1}{\cancel{\Delta x}} = \frac{3}{2}$$

$m = 3/2$

data / /

c) $g(x) = x^2 - 4$, $P = (1,-3)$ $e = 1$

$m = \lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(1+\Delta x)^2 - 4 + 3}{\Delta x} =$

$= \lim_{\Delta x \to 0} \frac{1 + 2\Delta x + \Delta x^2 - 4 + 3}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\Delta x + \Delta x^2}{\Delta x} =$

$= \lim_{\Delta x \to 0} 2 + \Delta x = 2$

$m = 2$

d) $h(x) = 5 - x^2$, $P = (2,1)$ $e = 2$

$m = \lim_{\Delta x \to 0} \frac{f(2+\Delta x) - f(2)}{\Delta x} = \lim_{\Delta x \to 0} \frac{5 - (2+\Delta x)^2 - 1}{\Delta x} =$

$= \lim_{\Delta x \to 0} \frac{5 - (4 + 4\Delta x + \Delta x^2) - 1}{\Delta x} = \lim_{\Delta x \to 0} \frac{5 - 4 - 4\Delta x - \Delta x^2 - 1}{\Delta x} =$

$= \lim_{\Delta x \to 0} \frac{-4\Delta x - \Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} -4 - \Delta x = -4$

e) $f(t) = 3t - t^2$, $P = (0,0)$ $e = 0$

$m = \lim_{\Delta t \to 0} \frac{f(0+\Delta t) - f(0)}{\Delta t} = \lim_{\Delta t \to 0} \frac{3\Delta t - \Delta t^2 - 0}{\Delta t} =$

$= \lim_{\Delta t \to 0} 3 - \Delta t = 3$

f) $h(t) = t^2 + 3, \quad P = (-2, 7)$ $\quad e = -2$

$$m = \lim_{\Delta t \to 0} \frac{f(-2+\Delta t) - f(-2)}{\Delta t} = \lim_{\Delta t \to 0} \frac{(-2+\Delta t)^2 + 3 - 7}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} \frac{4 - 4\Delta t + \Delta t^2 + 3 - 7}{\Delta t} = \lim_{\Delta t \to 0} \frac{-4\Delta t + \Delta t^2}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} -4 + \Delta t = -4 \,//$$

2)

a) $f(x) = 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{3 - 3}{\Delta x} = 0$$

Ñ precisa substituir o Δx.

b) $h(s) = 3 + \frac{2s}{3}$

$$h'(s) = \lim_{\Delta s \to 0} \frac{h(s+\Delta s) - h(s)}{\Delta s} =$$

$$= \lim_{\Delta s \to 0} \frac{3 + 2(s+\Delta s)/3 - (3 + 2s/3)}{\Delta s} =$$

$$= \lim_{\Delta s \to 0} \frac{3 + 2s/3 + 2\Delta s/3 - 3 - 2s/3}{\Delta s} =$$

$$= \lim_{\Delta s \to 0} \frac{2\Delta s/3}{\Delta s} = \frac{2}{3} \,//$$

data / /

c) $f(t) = 2t^2 + t - 1$

$$f'(t) = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} \frac{2(t+\Delta t)^2 + (t+\Delta t) - 1 - (2t^2 + t - 1)}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} \frac{\cancel{2t^2} + 4t\Delta t + 2\Delta t^2 + \cancel{t} + \Delta t \cancel{- 1} \cancel{- 2t^2} \cancel{- t} \cancel{+ 1}}{\Delta t} =$$

$$= \lim_{\Delta t \to 0} \frac{4t\Delta t + 2\Delta t^2 + \Delta t}{\Delta t} = \lim_{\Delta t \to 0} 4t + 2\Delta t + 1 = 4t + 1 //$$

d) $f(x) = \frac{1}{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \left(\frac{1}{x+\Delta x-1} - \frac{1}{x-1} \right) \cdot \frac{1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x} - \cancel{1} - \cancel{x} - \Delta x + \cancel{1}}{(x+\Delta x-1)(x-1)} \cdot \frac{1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}}{(x+\Delta x-1)(x-1)} \cdot \frac{1}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x-1)(x-1)} = \frac{1}{(x-1)^2} //$$

data / /

S T Q Q S S D

e) $g(x) = \frac{1}{x^2}$

$$g'(x) = \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \left(\frac{1}{(x+\Delta x)^2} - \frac{1}{x^2} \right) \cdot \frac{1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{x^2 - (x+\Delta x)^2}{(x+\Delta x)^2 \cdot x^2} \cdot \frac{1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{x^2 - x^2 - 2x\Delta x - \Delta x^2}{(x+\Delta x)^2 \cdot x^2} \cdot \frac{1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-2x\Delta x - \Delta x^2}{(x+\Delta x)^2 \cdot x^2} \cdot \frac{1}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{(x+\Delta x)^2 \cdot x^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

data / /

S T Q Q S S D

f) $h(s) = \frac{4}{\sqrt{s}}$

$$h'(s) = \lim_{\Delta s \to 0} \frac{h(s+\Delta s) - h(s)}{\Delta s} =$$

$$= \lim_{\Delta s \to 0} \left(\frac{4}{\sqrt{s+\Delta s}} - \frac{4}{\sqrt{s}} \right) \cdot \frac{1}{\Delta s} =$$

$$= \lim_{\Delta s \to 0} \left(\frac{4\sqrt{s} - 4\sqrt{s+\Delta s}}{\sqrt{s+\Delta s} \cdot \sqrt{s}} \right) \cdot \frac{1}{\Delta s} =$$

$$= \lim_{\Delta s \to 0} \left(\frac{4\sqrt{s} - 4\sqrt{s+\Delta s}}{\sqrt{s+\Delta s} \cdot \sqrt{s} \cdot \Delta s} \cdot \frac{4\sqrt{s} + 4\sqrt{s+\Delta s}}{4\sqrt{s} + 4\sqrt{s+\Delta s}} \right) =$$

$$= \lim_{\Delta s \to 0} \frac{16s - 16(s+\Delta s)}{\sqrt{s+\Delta s}\sqrt{s} \cdot \Delta s (4\sqrt{s} + 4\sqrt{s+\Delta s}} =$$

$$= \lim_{\Delta s \to 0} \frac{-16\Delta s}{\sqrt{s+\Delta s}\sqrt{s} \cdot \Delta s (4\sqrt{s} + 4\sqrt{s+\Delta s})} =$$

$$= \lim_{\Delta s \to 0} \frac{-16}{\sqrt{s+\Delta s}\sqrt{s} \cdot (4\sqrt{s} + 4\sqrt{s+\Delta s})} =$$

$$= \frac{-16}{\sqrt{s} \cdot \sqrt{s} (4\sqrt{s} + 4\sqrt{s})} = \frac{-16}{s(8\sqrt{s})} = \frac{-2}{s\sqrt{s}} \,//$$

3)

a) $f(x) = x^2 + 1$, $P = (2,5)$

$m = f'(x) = 2x \Rightarrow m = 2.2 = 4$

$y = mx + n$

$y = 4x + n \Rightarrow 5 = 4.2 + n$

$n = -3$

R: $y = 4x - 3$

b) $g(x) = x^2 + 2x + 1$, $P = (-3,4)$

$m = g'(x) = 2x + 2 \Rightarrow m = 2.(-3) + 2 = -4$

$y = mx + n$

$y = -4x + n \Rightarrow 4 = -4.(-3) + n$

$n = -8$

R: $y = -4x - 8$ //

c) $g(x) = x^3$, $P = (2,8)$

$m = g'(x) = 3x^2 \Rightarrow m = 3.2^2 = 12$

$y = 12x + n$ R: $y = 12x - 16$ //

$8 = 12.2 + n$

$n = 8 - 24 = -16$

data / /
S T Q Q S S D

d) $h(x) = \sqrt{x} = x^{1/2}$, $P = (1,1)$ $\quad m = h'(1)$

$m = h'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$ $\qquad m = \frac{1}{2\sqrt{1}} = \frac{1}{2}$

$y = \frac{1}{2}x + n$

$1 = \frac{1}{2} \cdot 1 + n$ $\qquad$ R: $y = \frac{1}{2}x + \frac{1}{2}$

$n = 1 - \frac{1}{2} = \frac{1}{2}$

e) $f(t) = t + \frac{4}{t}$, $P = (4,5)$

$m = f'(t) = 1 + \frac{0 \cdot t - 4 \cdot 1}{t^2} = 1 - \frac{4}{t^2} \Rightarrow m = 1 - \frac{4}{4^2} = \frac{12}{16} = \frac{3}{4}$

$y = \frac{3}{4}x + n$

$5 = \frac{3}{4} \cdot 4 + n$ $\qquad$ R: $y = \frac{3}{4}x + 2$

$5 = 3 + n$

$n = 2$

f) $h(t) = \frac{1}{t+1}$, $P = (0,1)$

$m = h'(t) = \frac{0 \cdot (t+1) - 1 \cdot 1}{(t+1)^2} = \frac{-1}{(t+1)^2} \Rightarrow m = \frac{-1}{(0+1)^2} = -1$

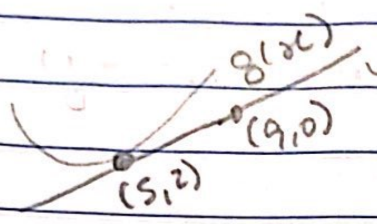
$y = -1x + n$

$1 = -1 \cdot 0 + n$ $\qquad$ $y = -x + 1$

$n = 1$

4) $y = g(x)$ no ponto $(5,2)$ passa por $(9,0)$

$g(5)$ e $g'(5)$



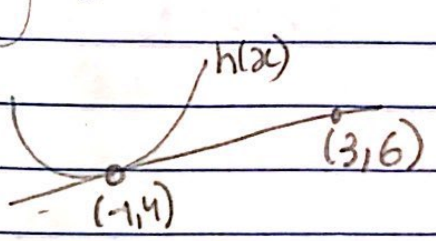
$y = mx + n$

$m = \frac{y_F - y_I}{x_F - x_I} = \frac{0-2}{9-5} = \frac{-1}{2}$

$g'(x) = m$ $\quad$ $g(5) = 2$ //

$g'(5) = -1/2$

5) $y = h(x)$ $\quad$ $(-1, 4)$ $\quad$ passa por $(3, 6)$



$y = mx + n$

$m = \frac{6-4}{3+1} = \frac{2}{4} = \frac{1}{2}$

$h'(x) = m$ $\quad$ $h(-1) = 4$

$h'(-1) = 1/2$

6)

a) $f(0) = 2$, $f'(x) = -3$, $-\infty < x < \infty$

$f(x) = -3x + 2$

b) $f(0) = 4$, $f'(0) = 0$; $f'(x) < 0$ para $x < 0$

$f'(x) > 0$ para $x > 0$

$f(x) = x^2 + 4 \Rightarrow f'(x) = 2x$

$f(x) = x^4 + 4 \Rightarrow f'(x) = 4x^3$

c) $f(0) = 0$, $f'(0) = 0$; $f'(x) > 0$ se $x \neq 0$

$f(x) = x^2$ X $\quad$ $f'(x) = 2x$ X

$f(x) = x^3 \Rightarrow f'(x) = \underbrace{3x^2}_{\Rightarrow\, > 0}$ ✓

data / /

S T Q Q S S D

$\frac{1}{3} - 1 = -\frac{2}{3}$

7)

a) $x^5 - 3x^3 + 8 = y$

$y' = 5x^4 - 9x^2$

b) $y = \sqrt{x} + \sqrt[3]{x} + \frac{1}{x}$

$y = x^{1/2} + x^{1/3} + x^{-1}$

$y' = \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-2/3} - 1x^{-2}$

$y' = \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}} - \frac{1}{x^2}$

c) $f(x) = \frac{x^4}{\pi} + \frac{x^2}{e} + x$

$f'(x) = \frac{4x^3}{\pi} + \frac{2x}{e} + 1$ //

d) $g(x) = ex^3 - \frac{x}{\pi} + \pi^2$

$g'(x) = 3ex^2 - \frac{1}{\pi}$

e) $y = (1+4x^3)(1+2x^2)$

$y' = (1+4x^3)' \cdot (1+2x^2) + (1+4x^3) \cdot (1+2x^2)' \Rightarrow$

$y' = 12x^2(1+2x^2) + (1+4x^3)4x \Rightarrow$

$y' = 12x^2 + 24x^4 + 4x + 16x^4$

$y' = 40x^4 + 12x^2 + 4x$ //

data / /

S T Q Q S S D

f) $h(x) = x \cdot \ln x$

$h'(x) = x' \cdot \ln x + x \cdot \ln x'$

$h'(x) = 1 \cdot \ln x + \cancel{x} \frac{1}{\cancel{x}}$

$h'(x) = \ln x + 1$ //

g) $f(t) = (2t-1)(t^2-6t+3)$

$f'(t) = 2(t^2-6t+3) + (2t-1)(2t-6)$

$f'(t) = 2t^2-12t+6 + 4t^2-12t-2t+6$

$f'(t) = 6t^2-26t+12$

h) $y = x(3x+2)(2x+3)$

$y = (3x^2+2x)(2x+3)$

$y' = (6x+2)(2x+3) + (3x^2+2x) \cdot 2$

$y' = 12x^2+18x+4x+6+6x^2+4x$

$y' = 18x^2+26x+6$ //

i) $f(x) = \dfrac{2x^4}{4-x^2}$

$f'(x) = \dfrac{(2x^4)' \cdot (4-x^2) - (2x^4) \cdot (4-x^2)'}{(4-x^2)^2}$

$f'(x) = \dfrac{8x^3(4-x^2) - (2x^4)(-2x)}{(4-x^2)^2}$

$f'(x) = \dfrac{32x^3 - 8x^5 + 4x^5}{(4-x^2)^2}$

$f'(x) = \dfrac{-4x^5+32x^3}{(4-x^2)^2}$ //

data / /

S T Q Q S S D

f) $y = \dfrac{5-x}{5+x}$

$y' = \dfrac{-1(5+x) - (5-x).1}{(5+x)^2}$

$y' = \dfrac{-5-x-5+x}{(5+x)^2}$ $\rightarrow$ $y' = \dfrac{-10}{(5+x)^2}$ //

k) $y = \dfrac{x^3}{1+x^2}$

$y' = \dfrac{3x^2(1+x^2) - x^3(2x)}{(1+x^2)^2}$

$y' = \dfrac{3x^2+3x^4-2x^4}{(1+x^2)^2}$

$y' = \dfrac{x^4+3x^2}{(1+x^2)^2}$

$\rightarrow t^4-2t^3-6t^2-2t+1$

$\hookrightarrow$ -1 é raiz

-1	1	-2	-6	-2	1
	1	-3	-3	1	0

t^3-3t^2-3t+1

$\hookrightarrow$ -1 é raiz

-1	1	-3	-3	1
	1	-4	1	0

t^2-4t+1

l) $f(t) = \dfrac{t^3+1}{t^2-t-2}$

$f'(t) = \dfrac{3t^2(t^2-t-2)-(t^3+1)(2t-1)}{(t^2-t-2)^2}$

$f'(t) = \dfrac{3t^4-3t^3-6t^2-2t^4+t^3-2t+1}{((t+1)(t-2))^2}$

$f'(t) = \dfrac{t^4-2t^3-6t^2-2t+1}{((t+1)(t-2))^2}$

$f'(t) = \dfrac{\cancel{(t+1)^2}(t^2-4t+1)}{\cancel{(t+1)^2}(t-2)^2} = \dfrac{t^2-4t+1}{(t-2)^2}$

data / /

S T Q Q S S D

m) $y = x^{\sqrt{2}}$

$y' = \sqrt{2}\, x^{\sqrt{2}-1}$ //

n) $f(x) = (\sqrt{2})^x$

$y = a^x \Rightarrow y' = a^x . \ln a$

$y = a^{f(x)} \Rightarrow y' = a^x . f'(x) \ln a$

$f'(x) = (\sqrt{2})^x . \ln(\sqrt{2})$ //

o) $y = e^{3x}$

$y' = e^{3x} . (3x)' . \ln e$

$y' = e^{3x} . 3 . 1$

$y' = 3e^{3x}$ //