1)

a) $\int \frac{2x-1}{x^2-3x+2}\,dx = \int \frac{2x-1}{(x-1)(x-2)}\,dx = \int \frac{-1}{(x-1)}\,dx + \int \frac{3}{x-2}\,dx =$

$x^2 - 3x + 2 = 0$

$x_1 + x_2 = 3 \quad x_1 = 1$

$x_1 . x_2 = 2 \quad x_2 = 2$

$= -\ln|x-1| + 3\ln|x-2| + C$

própria ⇒ numerador menor que denominador

$$\frac{2x-1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)} = \frac{Ax-2A+Bx-B}{(x-1)(x-2)} = \frac{x(A+B)-2A-B}{(x-1)(x-2)}$$

$$\begin{cases} A + B = 2 \\ -2A - B = -1 \end{cases}$$

$-A = 1 \quad B = 3$

$A = -1$

1) f) $\int \frac{1}{x^2(x-2)^2}\,dx = \int \frac{1/4}{x}\,dx + \int \frac{1/4}{x^2}\,dx + \int \frac{-1/4}{x-2}\,dx + \int \frac{1/4}{(x-2)^2}$

$= \frac{1}{4}\ln|x| + \frac{1}{4}\frac{x^{-1}}{-1} - \frac{1}{4}\ln|x-2| + \frac{1}{4}\left(-\frac{1}{x-2}\right) + C =$

$(x-2)^2 = x^2-4x+4$

$= \frac{1}{4}\ln|x| - \frac{1}{4x} - \frac{1}{4}\ln|x-2| - \frac{1}{4x-8} + C$ //

$$\frac{1}{x^2(x-2)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-2)} + \frac{D}{(x-2)^2}$$

$$\frac{1}{x^2(x-2)^2} = \frac{A(x(x-2)^2) + B(x-2)^2 + C(x^2(x-2)) + Dx^2}{x^2(x-2)^2} =$$

$$\frac{1}{x^2(x-2)^2} = \frac{A(x^3-4x^2+4x) + B(x^2-4x+4) + C(x^3-2x^2) + Dx^2}{x^2(x-2)^2}$$

$$\begin{cases} A + C = 0 \Rightarrow C = -1/4 \\ -4A + B - 2C + D = 0 \Rightarrow -4\cdot\frac{1}{4} + \frac{1}{4} + \frac{2}{4} + D = 0 \Rightarrow D = 1 - \frac{3}{4} = \frac{1}{4} \\ 4A - 4B = 0 \Rightarrow 4A - 1 = 0 \Rightarrow 4A = 1 \Rightarrow A = 1/4 \\ 4B = 1 \Rightarrow B = 1/4 \end{cases}$$

$$\int \frac{1}{(x-2)^2}\,dx = \int \frac{1}{u^2}\,du = \int u^{-2}\,du = \frac{u^{-1}}{-1} =$$

$u = x-2$ $\quad = -\frac{1}{x-2} + C$

$du = dx$

1g) $\int \frac{1}{x^2-25}\,dx = \int \frac{1/10}{(x-5)}\,dx + \int \frac{-1/10}{(x+5)}\,dx =$

$= \frac{1}{10}\ln|x-5| - \frac{1}{10}\ln|x+5|$

$$\frac{1}{x^2-25} = \frac{1}{(x-5)(x+5)} = \frac{A}{x-5} + \frac{B}{x+5} = \frac{A(x+5)+B(x-5)}{(x-5)(x+5)}$$

$$\begin{cases} A+B=0 \\ 5A-5B=1 \end{cases} \sim \begin{cases} 5A+5B=0 \\ 5A-5B=1 \end{cases}$$

$10A = 1$

$A = 1/10 \quad B = -1/10$

l) $\int \frac{x^3 - 2x^2 + 3x - 8}{(x^2+4)^2}\,dx$

$$\frac{x^3 - 2x^2 + 3x - 8}{(x^2+4)^2} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2} = \frac{(Ax+B)(x^2+4) + Cx + D}{(x^2+4)^2}$$

$$x^3 - 2x^2 + 3x - 8 = Ax^3 + Ax4 + Bx^2 + B4 + Cx + D$$

$$\begin{cases} A = 1 \\ B = -2 \\ 4A + C = 3 \Rightarrow C = -1 \\ 4B + D = -8 \Rightarrow D = 0 \end{cases}$$

$$\int \frac{x-2}{x^2+4}\,dx + \int \frac{-x}{(x^2+4)^2}\,dx =$$

$$= \int \frac{x}{x^2+4}\,dx - 2\int \frac{1}{x^2+4}\,dx - \int \frac{x}{(x^2+4)^2}\,dx =$$

$u = x^2+4$ $x = 2u$ $u = x^2+4$

$du = 2x\,dx$ $dx = 2du$ $du = 2x\,dx$

$$= \frac{1}{2}\int \frac{du}{u} - 2\int \frac{2\,du}{(2u)^2+4} - \frac{1}{2}\int \frac{du}{u^2} =$$

$$= \frac{1}{2}\ln|u| - \frac{2\cdot 2}{4}\int \frac{du}{u^2+1} - \frac{1}{2}\frac{u^{-1}}{-1} =$$

$$= \frac{1}{2}\ln|x^2+4| - \arctan(x/2) + \frac{1}{2x^2+8} + C$$

2)

$* \sqrt{a^2 - x^2}$ $\Rightarrow x = a \operatorname{sen} u$

a) $\displaystyle\int \frac{1}{x\sqrt{1-x^2}}\,dx = \int \frac{\cos u \, du}{\operatorname{sen} u \sqrt{1-\operatorname{sen}^2 u}} =$

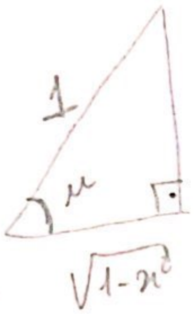
$a^2 = 1 \Rightarrow a = 1$

$x = 1 \operatorname{sen} u$

$dx = \cos u \, du$

$\displaystyle = \int \frac{\cos u \, du}{\operatorname{sen} u \, (1-\operatorname{sen}^2 u)^{1/2}} = \int \frac{\cos u \, du}{\operatorname{sen} u \cdot \cos u} =$

$\displaystyle = \int \frac{1}{\operatorname{sen} u}\, du = \int \csc u \, du =$



$= -\ln \left| \csc u + \cot u \right| + C =$

$\displaystyle = -\ln \left| \frac{1}{\operatorname{sen} u} + \frac{\cos u}{\operatorname{sen} u} \right| + C =$

$\Rightarrow \cos u = \sqrt{1-x^2}$

$\operatorname{sen} u = x$

$\displaystyle = -\ln \left| \frac{1}{x} + \frac{\sqrt{1-x^2}}{x} \right| + C$

$\displaystyle = -\ln \left| \frac{1 + \sqrt{1-x^2}}{x} \right| + C$ //

b) aula de ocon (02-06)

$* \sqrt{x^2 - a}$ $\Rightarrow x = a \sec x$
$dx = a \sec x . \tan x \, dx$

c) $\displaystyle\int \sqrt{x^2-1}\, dx = \int \sqrt{\sec^2 u - 1} \cdot \sec u \tan u \, du =$

$a^2 = 1 \Rightarrow a = 1$

$x = \sec u$

$dx = \sec u . \tan u \, du$

$\displaystyle = \int \sqrt{\tan^2 u} \cdot \sec u \tan u \, du =$

$\displaystyle = \int \tan^2 u \cdot \sec u \, du = \int (\sec^2 u - 1) \sec u \, du =$

$\displaystyle = \int \sec^3 u \, du - \int \sec u \, du =$

$\displaystyle = \int \sec u \tan^2 u \, du - \ln \left| \sec u + \tan u \right|$

$\displaystyle \int \tan^2 u \sec u \, du = \tan u \sec u - \int \sec u \tan^2 u \, du$

$\displaystyle \int \tan^2 u \sec u \, du = \frac{\tan u \sec u}{2} - \frac{\ln \left| \sec u + \tan u \right|}{2} + C$ //

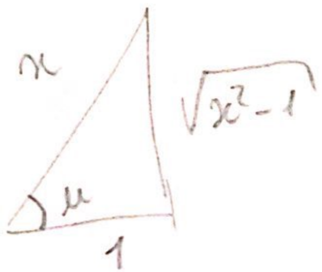
$\Rightarrow$

(*) $\int \sec^3 x \, dx = \tan x . \sec x - \int \sec x \tan^2 x \, dx$ //

$u = \sec x \qquad dv = \sec^2 x \, dx$

$du = \sec x . \tan x \, dx \qquad v = \tan x$

(*) $\int \sec x \, dx = \ln |\sec x + \tan x|$



$\Rightarrow \tan u = \sqrt{x^2 - 1}$

$\int \sqrt{x-1} \, dx = \frac{1}{2} x . \sqrt{x^2 - 1} - \frac{1}{2} \ln \left| x + \sqrt{x^2 - 1} \right| + C$ //

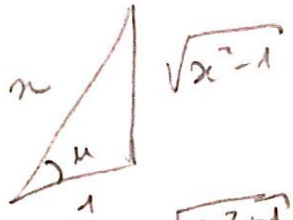
2 d) $\int \sqrt{(x^2-1)^3}\,dx = \int (\sec^2 u - 1)^{3/2} \cdot \sec u \tan u \, du =$

$a = 1$

$x = \sec u$ $\quad = \int \tan^3 u \cdot \sec u \cdot \tan u \, du =$

$dx = \sec u \cdot \tan u \, du$

$= \int \tan^4 u \, \sec u \, du = \int (\sec^2 u - 1)^2 \cdot \sec u \, du =$



$= \int (\sec^4 u - 2\sec^2 u + 1) \cdot \sec u \, du =$

$\sin u = \frac{\sqrt{x^2-1}}{x}$

$\cos u = \frac{1}{x}$

$= \int \sec^5 u - 2\sec^3 u + \sec u \, du =$

$= \frac{\tan x \sec^3 x}{4} + \frac{3 \tan x \sec x}{8} + \frac{3 \ln|\sec x + \tan x|}{8} - \tan x \sec x - \ln|\sec x + \tan x|$

$+ \ln|\sec x + \tan x| + C$

$= \frac{\tan x \sec^3 x}{4} - \frac{5 \tan x \sec x}{8} + \frac{3 \ln|\sec x + \tan x|}{8} + C =$

$= \frac{\sqrt{x^2+1} \cdot x^3}{4} - \frac{5 \cdot \sqrt{x^2+1} \cdot x}{8} + \frac{3 \ln|x + \sqrt{x^2+1}|}{8} + C$ //

$$* \int \sec^3 x \, dx = \tan x \sec x - \int \sec x \tan^2 x \, dx =$$

$u = \sec x \qquad dv = \sec^2 x \, dx$

$du = \sec x \tan x \, dx \qquad v = \tan x$

$$= \tan x \sec x - \int \sec x (\sec^2 x - 1) \, dx =$$

$$= \tan x \sec x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$\int \sec^3 x \, dx = \tan x \sec x - \int \sec^3 x \, dx + \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x \, dx = \frac{\tan x \sec x}{2} + \frac{\ln|\sec x + \tan x|}{2} + C$$

$$* \int \sec^5 x \, dx = \tan x \sec^3 x - \int 3\sec^3 x \cdot \tan^2 x \, dx =$$

$u = \sec^3 x$

$du = 3\sec^2 x \cdot \sec x \tan x \, dx$

$dv = \sec^2 x \, dx$

$v = \tan x$

$$= \tan x \sec^3 x - 3\int \sec^3 x \cdot (\sec^2 x - 1) \, dx =$$

$$= \tan x \sec^3 x - 3\int \sec^5 x \, dx + 3\int \sec^3 x \, dx$$

$$\int \sec^5 x \, dx = \tan x \sec^3 x - 3\int \sec^5 x \, dx + 3 \cdot \frac{\tan x \sec x}{2} + \frac{3\ln|\sec x + \tan x|}{2} + C$$

$$\int \sec^5 x \, dx = \frac{\tan x \sec^3 x}{4} + \frac{3\tan x \sec x}{8} + \frac{3\ln|\sec x + \tan x|}{8} + C$$

e) $\int \frac{\sqrt{x^2-1}}{x^3}\,dx = \int \frac{\overbrace{\sqrt{\sec^2 u - 1}}^{\tan u}}{\sec^3 u} \cdot \sec u . \tan u \, du =$

$a = 1$

$x = \sec u$

$dx = \sec u \tan u \, du$

$$= \int \frac{\tan^2 u}{\sec^2 u}\, du \quad \int \left(\frac{\tan u}{\sec u}\right)^2 du =$$

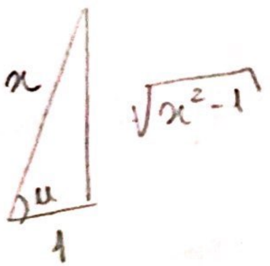
angulo duplo

$$= \int \left(\frac{\frac{\text{sen}\, u}{\cancel{\cos u}}}{1/\cos u} \right)^2 du = \int \text{sen}^2 u \, du = \int \frac{1 - \cos(2u)}{2}\, du =$$

$$= \frac{1}{2}\int 1\, du - \frac{1}{2}\int \cos(2u)\, du = \frac{1}{2}u - \frac{1}{2} \cdot \frac{1}{2}\,\text{sen}(2u) + C$$

$$= \frac{1}{2}\,\text{arcosec}(x) - \frac{1}{4}\,\frac{2\sqrt{x^2-1}}{x^2} + C =$$

$$= \frac{1}{2}\,\text{arcosec}(x) - \frac{\sqrt{x^2-1}}{2x^2} + C //$$



$\text{sen}\, u = \frac{\sqrt{x^2-1}}{x}$

$\cos u = \frac{1}{x}$

$\text{sen}(2u) = 2\,\text{sen}\, u \cos u$

$\text{sen}(2u) = 2\,\frac{\sqrt{x^2-1}}{x} \cdot \frac{1}{x}$

f) $\displaystyle\int \frac{1}{x\sqrt{x^2+1}}\,dx = \int \frac{\sec^2 u\,du}{\tan u \cdot \sqrt{\tan^2 u+1}} = \int \frac{\sec^2 u}{\tan u \cdot \sec u}\,du =$

$a^2 = 1 \Rightarrow a = 1$

$x = 1\tan u$ $\qquad \displaystyle = \int \frac{\sec u}{\tan u}\,du = \int \frac{1/\cos u}{\operatorname{sen} u/\cos u}\,du = \int \frac{1}{\operatorname{sen} u}\,du =$

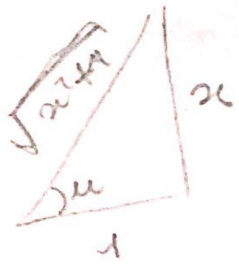
$dx = \sec^2 u\,du$

$= \displaystyle\int \csc u = -\ln\left|\csc u + \cot u\right| + C =$

$= \displaystyle -\ln\left|\frac{1}{\operatorname{sen} u} + \frac{\cos u}{\operatorname{sen} u}\right| + C =$

$= \displaystyle -\ln\left|\frac{\sqrt{x^2+1}}{x} + \frac{1}{x}\right| + C =$

$= \displaystyle -\ln\left|\frac{\sqrt{x^2+1}+1}{x}\right| + C$ //



Lo $\operatorname{sen} u = \dfrac{x}{\sqrt{x^2+1}}$

Lo $\cos u = \dfrac{1}{\sqrt{x^2+1}}$

g) $\displaystyle\int \frac{\sqrt{x^2+1}}{x^2}\,dx = \int \frac{\sqrt{\tan^2 u+1}}{\tan^2 u}\cdot \sec^2 u\,du =$

$a = 1$

$x = \tan u$

$dx = \sec^2 u\,du$

$\displaystyle = \int \frac{\sqrt{\sec^2 u}}{\tan^2 u}\sec^2 u\,du = \int \frac{\sec u \cdot \sec^2 u}{\tan^2 u}\,du =$

$\displaystyle = \int \frac{\sec^3 u}{\tan^2 u}\,du = \int \frac{1/\cos^3 u}{\frac{\operatorname{sen}^2 u}{\cos^2 u}}\,du = \int \frac{1}{\cos^3 u}\cdot\frac{\cos^2 u}{\operatorname{sen}^2 u}\,du =$

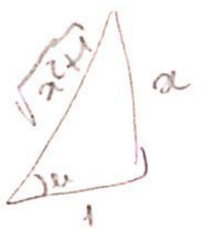
$\displaystyle = \int \frac{1}{\cos u\,\operatorname{sen}^2 u}\,du = \int \frac{1}{\cos u(1-\cos^2 u)}\,du = \int \frac{1}{\cos u - \cos^3 u}\,du =$

$\displaystyle = \int \frac{1}{\cos u}\,du - \int \frac{1}{\cos^3 u}\,du = \int \sec u\,du - \int \sec^3 u\,du =$

$\displaystyle = \ln|\sec u + \tan u| - \frac{\tan u \sec u}{2} - \frac{\ln|\sec u + \tan u|}{2} + C =$

$\displaystyle = \frac{\ln|\sec u + \tan u|}{2} - \frac{\tan u \sec u}{2} + C =$

$\displaystyle = \frac{\ln|\sqrt{x^2+1} + x|}{2} - \frac{x\cdot\sqrt{x^2+1}}{2} + C$ //



$\operatorname{sen} u = \dfrac{x}{\sqrt{x^2+1}}$

$\cos u = \dfrac{1}{\sqrt{x^2+1}}$

$\sec u = \sqrt{x^2+1}$

$\tan u = x$

h) $$\int \frac{x^3}{\sqrt{x^2-1}}\,dx = \int \frac{\sec^2 u}{\sqrt{\sec^2 u - 1}}\sec u \cdot \tan u \, du =$$

$a = 1$

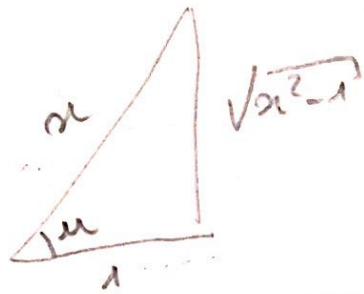
$x = \sec u$

$dx = \sec u \cdot \tan u \, du$

$$= \int \frac{\sec^3 u \cancel{\tan u}\, du}{\cancel{\tan u}} = \int \sec^3 u \, du =$$

$$= \frac{\tan u \sec u}{2} + \frac{\ln|\sec u + \tan u|}{2} + C =$$

$$= \frac{\sqrt{x^2-1}\cdot x}{2} + \frac{\ln|x + \sqrt{x^2-1}|}{2} + C$$



$\sin u = \frac{\sqrt{x^2-1}}{x}$

$\cos u = \frac{1}{x}$

$\sec u = x$

$\tan u = \sqrt{x^2-1}$

b) $\int \frac{x^2}{\sqrt{25-9x^2}}\,dx = \int \frac{25v^2/9}{\sqrt{25-9\cdot\frac{25v^2}{9}}}\cdot\frac{5}{3}\,dv =$

$x = \frac{5}{3}v$

$dx = \frac{5}{3}\,dv$

$= \frac{25}{9}\cdot\frac{1}{5}\cdot\frac{5}{3}\int\frac{v^2}{\sqrt{1-v^2}}\,dv =$

$= \frac{25}{27}\int\frac{v^2}{\sqrt{1-v^2}}\,dv = \frac{25}{27}\int\frac{sen^2 u}{\sqrt{1-sen^2 u}}\cos u\,du =$

$v = sen\,u$

$dv = \cos u\,du$

$= \frac{25}{27}\int\frac{sen^2 u\,\cancel{\cos u}\,du}{\cancel{\cos u}}\;\frac{25}{27}\int sen^2 u\,du =$

$= \frac{25}{27}\cdot\frac{1}{2}u - \frac{25}{27}\cdot\frac{1}{4}sen(2u) + C =$

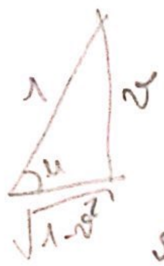
$= \frac{25}{54}\,arcsen(v) - \frac{25}{108}\cdot 2v + C =$

$= \frac{25}{54}\,arcsen(3x/5) - \frac{25}{54}\cdot\frac{3x}{5} + C$

$= \frac{25}{54}\,arcsen(3x/5) - \frac{5}{18}x + C$ //

$sen\,u = \frac{v}{\sqrt{1-v^2}}$

$\cos u = \sqrt{1-v^2}$



$sen(2u) = 2\cdot\frac{v}{\sqrt{1-v^2}}\cdot\sqrt{1-v^2}$

$sen(2u) = 2v$

* $\int sen^2 u\,du = \int\frac{1-\cos(2u)}{2}\,du = \frac{1}{2}\int 1^{du} - \frac{1}{2}\int\cos(2u)\,du =$

$= \frac{1}{2}u - \frac{1}{2}\cdot\frac{1}{2}sen(2u) + C = \frac{1}{2}u - \frac{1}{4}sen(2u)$

3)

a) $\int \operatorname{sen}^3(x)\cos^4(x)\,dx = \int \operatorname{sen}^2(x)\cdot\cos^4(x)\cdot\operatorname{sen}x\,dx = \int(1-\cos^2(x))\cdot\cos^4 x\cdot\operatorname{sen}x\,dx =$

$= \int \cos^4 x\cdot\operatorname{sen}x\,dx - \int \cos^6 x\,\operatorname{sen}x\,dx =$

$= -\int u^4\cdot du + \int u^6\,du = -\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C$

$u = \cos x$

$du = -\operatorname{sen}x$

b) $\int \operatorname{sen}^2(x)\cos^5(x)\,dx = \int \operatorname{sen}^2(x)\cdot\cos^4(x)\cos x\,dx =$

$= \int \operatorname{sen}^2 x\cdot(1-\operatorname{sen}^2 x)^2\cdot\cos x\,dx =$

$= \int \operatorname{sen}^2 x\,(1-2\operatorname{sen}^2 x+\operatorname{sen}^4 x)\cos x\,dx =$

$= \int \operatorname{sen}^2 x\cos x\,dx - 2\int \operatorname{sen}^4 x\cos x\,dx + \int \operatorname{sen}^6 x\cos x\,dx =$

$= \int u^2\,du - 2\int u^4\,du + \int u^6\,du =$

$= \frac{\operatorname{sen}^3 x}{3} - \frac{2}{5}\operatorname{sen}^5 x + \frac{\operatorname{sen}^7 x}{7} + C$

$u = \operatorname{sen}x$

$du = \cos x\,dx$

c) $\int \tan^3 x \sec^4 x \, dx = \int \tan^3 x \,(1+\tan^2 x) \cdot \sec^2 x \, dx =$

$= \int \tan^3 x \sec^2 x \, dx + \int \tan^5 x \sec^2 x \, dx =$

$= \int u^3 \, du + \int u^5 \, du = \dfrac{\tan^4 x}{4} + \dfrac{\tan^6 x}{6} + C$ //

$u = \tan x$

$du = \sec^2 x \, dx$

d) $\int \operatorname{sen}^4(x) \cos^2(x) \, dx = \int \left(\dfrac{1-\cos 2x}{2}\right)^2 \left(\dfrac{1+\cos 2x}{2}\right) dx =$

$= \int \left(\dfrac{1 - 2\cos 2x + \cos^2 2x}{4}\right)\left(\dfrac{1+\cos 2x}{2}\right) dx =$

$\int \operatorname{sen}^2(2x) \cos 2x \, dx = \int u^2 \, du/2 = u^3/6$

$u = \operatorname{sen} 2x$

$du = \cos(2x) \cdot 2 \, dx$

$= \dfrac{1}{8} \int (1 + \cos 2x - 2\cos 2x - 2\cos^2 2x + \cos^2 2x + \cos^3 2x) \, dx =$

$= \dfrac{1}{8} \int (1 - \cos 2x - \cos^2 2x + \cos^3 2x) \, dx =$

$= \dfrac{1}{8} \left[\int 1 \, dx - \int \cos 2x \, dx - \int \dfrac{1+\cos(4x)}{2} dx + \int (1-\operatorname{sen}^2 2x) \cos 2x \, dx \right]$

subs

$= \dfrac{1}{8} \left[x - \cancel{\dfrac{\operatorname{sen} 2x}{2}} - \dfrac{1}{2}x - \dfrac{1}{8} \operatorname{sen}(4x) + \cancel{\dfrac{\operatorname{sen} 2x}{2}} - \dfrac{\operatorname{sen}^3 2x}{6} \right]$

$= \dfrac{1}{8} \left[\dfrac{x}{2} - \dfrac{1}{8} \operatorname{sen} 4x - \dfrac{\operatorname{sen}^3 2x}{6} \right] + C$ //

e) $\int \tan^2 x \sec^2 x \, dx = \int u^2 \, du = \frac{u^3}{3} + C = \frac{\tan^3 x}{3} + C$ //

$u = \tan x$

$du = \sec^2 x \, dx$

f) $\int \operatorname{sen}^4 x \cdot \cos^3 x \, dx = \int \operatorname{sen}^4 x \, (1 - \operatorname{sen}^2 x) \cos x \, dx =$

$= \int \operatorname{sen}^4 x \cos x \, dx - \int \operatorname{sen}^6 x \cos x \, dx =$

$= \int u^4 \, du - \int u^6 \, du = \frac{\operatorname{sen}^5 x}{5} + \frac{\operatorname{sen}^7 x}{7} \, dx$

$u = \operatorname{sen} x$

$du = \cos x$

g) $\int \tan^3(x) \, dx = \int \tan x \, (\sec^2 x - 1) \, dx =$

$= \int \tan x \sec^2 x \, dx - \int \tan x \, dx =$

$= \int u \, du + \ln|\cos x| = \frac{\tan^2 x}{2} + \ln|\cos x| + C$ //

$u = \tan x$

$du = \sec^2 x \, dx$

$(*)$ $\int \tan x \, dx = \int \frac{\operatorname{sen} x}{\cos x} \, dx = - \int \frac{du}{t} =$

$t = \cos x$

$du = -\operatorname{sen} x$

$= - \ln|t| = - \ln|\cos x|$

4)

a) $\int \frac{dx}{x - 4\sqrt{x+5}} = \int \frac{2u}{u^2-5-4u}\, du = 2\int \frac{u}{u^2-5-4u}\, du =$

$u = \sqrt{x+5} \Rightarrow u^2 = x+5 \Rightarrow u^2 - 5 = x$ $= 2\int \frac{u}{(u-5)(u+1)}\, du =$

$du = \frac{1}{2\sqrt{x+5}}\, dx$

$dx = 2\underbrace{\sqrt{x+5}}_{u}\, du$

$= 2\int \frac{5/6}{u-5}\, du + 2\int \frac{1/6}{u+1}\, du =$

$= \frac{5}{3} \ln|u-5| + \frac{1}{3} \ln|u+1| + C =$

$= \frac{5}{3} \ln\left|\sqrt{x+5} - 5\right| + \frac{1}{3} \ln\left|\sqrt{x+5} + 1\right| + C$ //

$* \; u^2 - 4u - 5 = 0$

$\Delta = 16 - 4 \cdot 1 \cdot (-5) = 36$

$u = \frac{4 \pm 6}{2}$ $\Rightarrow u = 5$; $\Rightarrow u_2 = -1$

$\frac{u}{(u-5)(u+1)} = \frac{A}{(u-5)} + \frac{B}{(u+1)} = \frac{A(u+1) + B(u-5)}{(u-5)(u+1)}$

$\begin{cases} A + B = 1 \\ A - 5B = 0 \end{cases} \sim \begin{cases} -A - B = -1 \Rightarrow A = 1 - B \Rightarrow 1 - \frac{1}{6} = \frac{5}{6} \\ A - 5B = 0 \end{cases}$ //

$-6B = -1$

$B = 1/6$

b) $\int \frac{2x\,dx}{1+\sqrt{x+1}} = 2\int \frac{x}{1+\sqrt{x+1}}\,dx = 2\int \frac{u-1}{1+\sqrt{u}}\,du =$

$u = x+1$
$du = dx$

$= 2\int \sqrt{u} - 1\,du = 2\int \sqrt{u}\,du - 2\int 1\,du =$

$= 2 \cdot \frac{u^{3/2}}{3/2} - 2u = \frac{4}{9}(x+1)^{3/2} - 2(x+1) + C$ //

$\frac{u-1}{1+\sqrt{u}} \cdot \frac{\sqrt{u}-1}{\sqrt{u}-1} = \frac{(u-1)(\sqrt{u}-1)}{u-1} = \sqrt{u} - 1$

c) $\int x\sqrt{x+1}\,dx = \int (u-1)\sqrt{u}\,du = \int u\sqrt{u}\,du - \int \sqrt{u}\,du =$

$u = x+1$
$du = dx$

$= \int u^{3/2}\,du - \int u^{1/2}\,du = \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C =$

$= \frac{2(x+1)^{5/2}}{5} - \frac{2}{3}(x+1)^{3/2} + C$

g) $\int \frac{(\cos x + 3)}{\operatorname{sen}^2(x)}\,dx = \int \frac{\cos x}{\operatorname{sen}^2(x)}\,dx + \int \frac{3}{\operatorname{sen}^2(x)}\,dx =$

$= -\csc x + 3 \cdot \int \frac{1}{\operatorname{sen}^2 x}\,dx = -\csc x + 3\int \csc^2 x\,dx =$

$-\csc x + 3(-\cot x) + C = -\csc x - 3\cot x + C$ //

* $\int \frac{\cos x}{\operatorname{sen}^2 x}\,dx = \int \frac{du}{u^2} = \int u^{-2}\,du = \frac{u^{-1}}{-1} + C =$

$= -\frac{1}{\operatorname{sen} x} = -\csc x + C$ //

$u = \operatorname{sen} x$
$du = \cos x\,dx$

h) $\int \frac{(e^{3x} + 3e^{x} - 2e^{-4x})\,dx}{e^{5x}} = \int \frac{1}{e^{2x}} + \frac{3}{e^{4x}} - \frac{2}{e^{9x}}\,dx =$

$= \int e^{-2x} + 3 \cdot e^{-4x} - 2e^{-9x}\,dx =$

subs

$= -\frac{1}{2}e^{-2x} - \frac{3}{4}e^{-4x} + \frac{2}{9}e^{-9x} + C$

f) $\int \frac{\sec^2 x + \tan x}{\sec^2 x}\,dx = \int 1 + \frac{\tan x}{\sec^2 x}\,dx =$

$= x + \frac{\operatorname{sen}^2 x}{2} + C$ //

* $\int \frac{\tan x}{\sec^2 x}\,dx = \int \frac{\operatorname{sen} x / \cos x}{1/\cos^2 x}\,dx = \int \operatorname{sen} x \cos x\,dx =$

$u = \operatorname{sen} x$

$du = \cos x\,dx$ $\quad = \int u\,du = \frac{u^2}{2} + C = \frac{\operatorname{sen}^2 x}{2} + C$

e) $\int (x-2) \cdot \operatorname{sen} x\,dx = -\cos x\,(x-2) - \int -\cos x\,dx =$

$u = x - 2 \quad dv = \operatorname{sen} x\,dx$

$du = dx \quad v = -\cos x$

$= -\cos x\,(x-2) + \operatorname{sen}(x) + C$ //

m) $\displaystyle\int \frac{x^{-1}}{(\ln x-3)(\ln x-2)}\,dx = \int \frac{1}{(\ln x-3)(\ln x-2)x}\,dx =$

$u = \ln x - 3 \Rightarrow u+1 = \ln x - 2$
$du = \frac{1}{x}\,dx$

$\displaystyle = \int \frac{1}{u(u+1)}\,du =$

$\displaystyle = \int \frac{1}{u}\,du + \int \frac{-1}{u+1}\,du = \ln|u| - \ln|u+1| + C =$

$= \ln|\ln(x)-3| - \ln|\ln(x)-2| + C$

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1} = \frac{(u+1)A + Bu}{u(u+1)}$$

$$\begin{cases} A + B = 0 \Rightarrow B = -1 \\ A = 1 \end{cases}$$

n) $\displaystyle\int x(2x+5)^{10}\,dx = \int \left(\frac{u-5}{2}\right) \cdot u^{10} \cdot \frac{du}{2} = \int \frac{(u-5)\cdot u^{10}}{4}\,du =$

$u = 2x+5 \Rightarrow x = u-5/2$
$du = 2\,dx$

$\displaystyle = \frac{1}{4}\int u^{11} - 5u^{10}\,du =$

$\displaystyle = \frac{1}{4}\cdot\frac{u^{12}}{12} - \frac{5}{4}\,\frac{u^{11}}{11} + C = \frac{(2x+5)^{12}}{48} - \frac{5(2x+5)^{11}}{44} + C$