LISTA 13 - INTEGRAL

① USE AS PROPRIEDADES PARA ENCONTRAR OS VALORES ENTRE OS QUAIS ESTÃO COMPREENDIDAS AS INTEGRAIS

ⓐ $\int_2^4 (x+5)\,dx$ $\quad \rightarrow 7 \leq x+5 \leq 9 \quad \forall x \in [2,4]$ então

$$7(4-2) \leq \int_2^4 (x+5)\,dx \leq 9(4-2)$$

$f(4) = 4+5 = 9$

$f(2) = 2+5 = 7$

$$14 \leq \int_2^4 x+5\,dx \leq 18$$

ⓑ $\int_0^4 \frac{dx}{1+x^2}$

$$\frac{1}{17}(4-0) \leq \int_0^4 \frac{dx}{1+x^2} \leq 1(4-0)$$

$f(0) = 1$

$f(4) = \frac{1}{17}$

$$\frac{4}{17} \leq \int_0^4 \frac{dx}{1+x^2} \leq 4$$

ⓒ $\int_0^{\pi/2} \operatorname{sen} x\,dx$ $\qquad 0 \leq \operatorname{sen} x \leq 1 \;\forall x \in [0, \pi/2]$

então $\quad 0\left(\frac{\pi}{2}-0\right) \leq \int_0^{\pi/2} \operatorname{sen} x\,dx \leq 1\left(\frac{\pi}{2}-0\right)$

$$0 \leq \int_0^{\pi/2} \operatorname{sen} x\,dx \leq \pi/2$$

(2) Se $\int_{-2}^{3} [f(x)+3]\,dx = 6$, Calcule $\int_{-2}^{3} f(x)\,dx$

$$\int_{-2}^{3} f(x)\,dx + \int_{-2}^{3} 3\,dx = 6$$

$$\int_{-2}^{3} f(x)\,dx = 6 - \int_{-2}^{3} 3\,dx = 6 - 3[3-(-2)] = 6 - 3\cdot 5 = -9$$ //

(3) Use a propriedade para determinar se cada desigualdade é Verdadeira ou falsa.

(a) $\int_{0}^{1} x\,dx \leq \int_{0}^{1} dx$

$$\left(\frac{x^2}{2}\right|_0^1 = \frac{1}{2} - 0 = \frac{1}{2} \qquad \left(x\right|_0^1 = 1 \Rightarrow \frac{1}{2} \leq 1$$ (V)

(b) $\int_{1}^{2} x^2\,dx < \int_{1}^{2} x\,dx$

$$\left(\frac{x^3}{3}\right|_1^2 < \left(\frac{x^2}{2}\right|_1^2$$

$$\frac{2^3}{3} - \frac{1^3}{3} < \frac{2^2}{2} - \frac{1^2}{2}$$

$$\frac{8}{3} - \frac{1}{3} < \frac{4}{2} - \frac{1}{2}$$

$$\frac{7}{3} < \frac{3}{2}$$

(F) pois $\frac{7}{3} > \frac{3}{2}$

c) $0 \leq \int_0^1 \frac{dx}{1+x^2}$

$0 \leq \operatorname{arctg} x \Big|_0^1 \Rightarrow 0 \leq \operatorname{arctg} 1 - \operatorname{arctg} 0$

$0 \leq \frac{\pi}{4}$ (V)

d) $\int_0^1 x^5\,dx \leq \int_0^1 x^6\,dx$

$\left(\frac{x^6}{6} \Big|_0^1 \leq \left(\frac{x^7}{7} \Big|_0^1\right.\right.$

$\frac{1}{6} \leq \frac{1}{7}$ (F) pois $1/6 \geq 1/7$

4) Em cada caso, calcule dy/dx

a) $y = \int_0^x (t^2+1)\,dt$

OBS: $F(x) = \int_0^{x(x)} f(t)\,dt$

$F'(x) = f(x(x)) \cdot x'(x)$

$\frac{dy}{dt} = (x^2+1) \cdot x' = x^2 + 1$ //

b) $y = \int_1^x (w^3 - 2w + 1)\,dw = (x^3 - 2x + 1) \cdot x' = x^3 - 2x + 1$ //

c) $y = \int_{-1}^x \frac{ds}{1+s^2}$ $\quad \frac{dy}{dx} = \frac{1}{1+x^2} \cdot x' = \frac{1}{1+x^2}$ //

d) $y = \int_0^x \frac{ds}{1+s} + \int_2^x \frac{ds}{1+s}$

$\frac{dy}{dx} = \frac{1}{1+x} + \frac{1}{1+x} = \frac{2}{1+x}$ //

(e) $y = \int_1^{3x} (5t^3+1)^7 \, dt$

$$\cancel{\frac{dy}{dx} = (5x^3+1)^7 \cdot (3x)' = 3(5x^3+1)^7}$$

$$\frac{dy}{dx} = (5(3x)^3+1)^7 \cdot (3x)' = 3(135x^3+1)^7$$

(f) $y = \int_1^{5x+1} \frac{dt}{9+t^2}$

$$\frac{dy}{dx} = \frac{1}{9(5x+1)^2}(5x+1)' = \frac{1 \cdot 5}{9(25x^2+10x+1)} = \frac{5}{225x^2+90x+9}$$

(g) $y = \int_1^{x-1} \sqrt{t^2-1} \, dt$

$$\frac{dy}{dx} = \sqrt{(x-1)^2-1}\,(x-1)' = \sqrt{(x^2-2x+1)-1} = \sqrt{x^2-2x}$$

(h) $y = \int_{x^2+1}^{2} \sqrt[3]{t-1} \, dt$

OBS: $F(x) = \int_{x_1}^{x_2} f(t)\,dt$

$F'(x) = f(x_2)x_2' - f(x_1) \cdot x_1'$

$$\frac{dy}{dx} = \sqrt[3]{2-1}\,(2)' - \sqrt[3]{x^2+1-1}\,(x^2+1)' = 0 - \sqrt[3]{x^2} \cdot 2x = -2x\sqrt[3]{x^2}$$

(i) $y = \int_x^{3x^2+2} \sqrt[4]{t^4+17} \, dt$

$$\frac{dy}{dx} = \sqrt[4]{(3x^2+2)^4+17}\,(3x^2+2)' - \sqrt[4]{x^4+17} \cdot x'$$

$$= 6x\sqrt[4]{(3x^2+2)^4+17} - \sqrt[4]{x^4+17}$$

(i) $y = \int_{x^3}^{x-x^2} \sqrt{t^3+1}\, dt$

$$\frac{dy}{dx} = \sqrt{(x-x^2)^3+1}\,(x-x^2)' - \sqrt{(x^3)^3+1}\,(x^3)'$$

$$= \sqrt{(x-x^2)^3+1}\,(1-2x) - 3x^2\sqrt{x^9+1} \;//$$

(5) Use o Teorema Fundamental do Cálculo para calcular cada integral.

(a) $\int_2^3 (3x+4)\,dx = \int_2^3 3x\,dx + \int_2^3 4\,dx$

$$= \left(\frac{3x^2}{2}\right|_2^3 + \left(4x\right|_2^3 = \frac{3.3^2}{2} - \frac{3.2^2}{2} + 4.3 - 4.2$$

$$= \frac{27}{2} - \frac{12}{2} + 4 = \frac{15}{2} + 4 = \frac{23}{2} \;//$$

(b) $\int_{-3}^{-1} 4 - 8x + 3x^2\,dx$

$$= \left(4x - \frac{8x^2}{2} + \frac{3x^3}{3}\right|_{-3}^{-1} = \left(4x - 4x^2 + x^3\right|_{-3}^{-1}$$

$$= 4(-1) - 4(-1)^2 + (-1)^3 - 4(-3) + 4(-3)^2 - (-3)^3$$

$$= -4 - 4 - 1 + 12 + 36 + 27 = -9 + 75 = 66 \;//$$

c) $\int_1^5 x^3 - 3x^2 + 1\,dx = \left(\frac{x^4}{4} - \frac{3x^3}{3} + x\right|_1^5$

$= \frac{5^4}{4} - 5^3 + 5 - \frac{1}{4} + 1 - 1 = \frac{625}{4} - 125 + 5 - \frac{1}{4} = 156 - 120 = 36$ //

d) $\int_1^3 (x-1)(x^2+x+1)\,dx$

$\int_1^3 x^3 + x^2 + x - x^2 - x - 1\,dx = \int_1^3 x^3 - 1\,dx = \left(\frac{x^4}{4} - x\right|_1^3$

$= \frac{3^4}{4} - 3 - \frac{1}{4} + 1 = \frac{81}{4} - \frac{1}{4} - 2 = \frac{80}{4} - 2 = 18$ //

e) $\int_0^1 (x^2+2)^2\,dx = \int_0^1 x^4 + 4x^2 + 4\,dx$

$= \left(\frac{x^5}{5} + \frac{4x^3}{3} + 4x\right|_0^1 = \frac{1}{5} + \frac{4}{3} + 4 - 0 = \frac{3+20}{15} + 4$

$= \frac{23}{15} + 4 = \frac{83}{15}$ //

f) $\int_1^5 \frac{x^4 - 16}{x^2+4}\,dx = \int_1^5 \frac{(x^2+4)(x^2-4)}{x^2+4}\,dx$

$= \int_1^5 x^2 - 4\,dx = \left(\frac{x^3}{3} - 4x\right|_1^5 = \frac{5^3}{3} - 4\cdot 5 - \frac{1}{3} + 4$

$= \frac{125}{3} - 20 - \frac{1}{3} + 4 = \frac{124}{3} - 16 = \frac{76}{3}$

(g) $\int_1^{32} \frac{1 + \sqrt[5]{t^2}}{\sqrt[3]{t}} \, dt$

$\int_1^{32} t^{-1/3} + t^{-1/15} \, dt$

$= \left(\frac{t^{2/3}}{2/3} + \frac{t^{14/15}}{14/15} \right|_1^{32}$

$= \frac{3}{2} t^{2/3} + \frac{15}{14} t^{14/15}$

$= \frac{3}{2} \sqrt[3]{32^2} + \frac{15}{14} \sqrt[15]{32^{14}} - \frac{3}{2} - \frac{15}{14}$

$= \frac{3}{2}\left(32^{2/3} - 1\right) + \frac{15}{14}\left(32^{14/15} - 1\right)$ //

(h) $\int_0^1 \frac{x^2}{(x^3+1)^5} \, dx = \int_0^1 \frac{x^2}{u^5} \frac{du}{3x^2}$

$u = x^3 + 1$

$du = 3x^2 \, dx$

$dx = \frac{du}{3x^2}$

$= \frac{1}{3} \int_0^1 u^{-5} \, du$

$= \frac{1}{3} \frac{u^{-4}}{-4} = \frac{-1}{12} \frac{1}{u^4} = \frac{-1}{12} \frac{1}{(x^3+1)^4}$

$= \left(\frac{-1}{12} \left(\frac{1}{x^3+1} \right)^4 \right|_0^1 = \frac{-1}{12} \frac{1}{(1^3+1)^4} - \frac{-1}{12} \frac{1}{(0^3+1)^4}$

$= -\frac{1}{12 \cdot 2^4} + \frac{1}{12} = \frac{-1}{192} + \frac{1}{12} = \frac{-1+16}{192} = +\frac{15}{192}$

$= \frac{5}{64}$ //

$3 - x^2 \geq 0$

$-x^2 \geq -3$

$x^2 \leq 3$

$x \leq \sqrt{3}$

$x \geq -\sqrt{3}$

(i) $\int_0^3 |3 - x^2| \, dx$

$= \int_0^{\sqrt{3}} 3 - x^2 + \int_{\sqrt{3}}^3 -(3 - x^2) \, dx$

$\left(3x - \frac{x^3}{3} \right|_0^{\sqrt{3}} + \left(-3x + \frac{x^3}{3} \right|_{\sqrt{3}}^3$

$3\sqrt{3} - \frac{\sqrt{3}^3}{3} \cancel{- 3.3 + \frac{3^3}{3}} + 3\sqrt{3} - \frac{\sqrt{3}^3}{3} = 6\sqrt{3} - \frac{2\sqrt{3}^3}{3}$

$6\sqrt{3} - \frac{2}{3}\sqrt{3}^3 = 6\sqrt{3} - \frac{2}{3} \cdot 3\sqrt{3} = 4\sqrt{3}$ //

(k) $\int_0^3 y\,|2-y|\,dy$

$\int_0^2 y(2-y)\,dy + \int_2^3 y(-2+y)\,dy$

$\int_0^2 2y - y^2\,dy + \int_2^3 -2y + y^2\,dy$

$\left(\frac{2y^2}{2} - \frac{y^3}{3}\right|_0^2 + \left(\frac{-2y^2}{2} + \frac{y^3}{3}\right|_2^3$

$= 2^2 - \frac{2^3}{3} + \left(-(3)^2 + \frac{3^3}{3} + 2^2 - \frac{2^3}{3}\right)$

$= 4 - \frac{8}{3} + \left(-9 + 9 + 4 - \frac{8}{3}\right) = 4 + 4 - \frac{8}{3} - \frac{8}{3} = 8 - \frac{16}{3} = \frac{8}{3}$ //

$= \cancel{4 + 4 + \frac{8}{3} + \frac{8}{3}}$

(j) $\int_{-1}^3 \sqrt[3]{2(|x| - x)}\,dx$

$\int_{-1}^0 \sqrt[3]{2(-x-x)}\,dx + \int_0^3 \sqrt[3]{2(x-x)}\,dx = \int_{-1}^0 \sqrt[3]{-4x}\,dx$

$\int_{-1}^0 (-4x)^{1/3}\,dx = (-4)^{1/3} \int_{-1}^0 x^{1/3}\,dx$

$= (-4)^{1/3} \left(\frac{x^{4/3}}{4/3}\right|_{-1}^0 = \left((-4)^{1/3} \cdot \frac{3}{4}\, x^{4/3}\right|_{-1}^0$

$= 0 - (-4)^{1/3} \cdot \frac{3}{4} \sqrt[3]{(-1)^4}$

$= +(4)^{1/3} \cdot \frac{3}{4} = \frac{3}{4^{2/3}}$ //

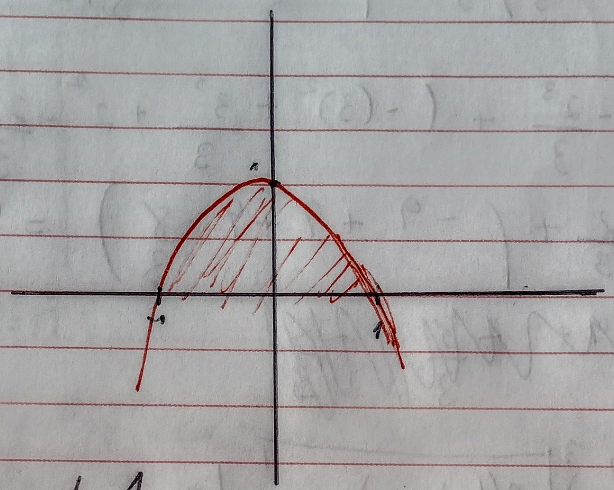
6. Em cada caso, calcule a área da região limitada pelo gráfico de cada função e as retas $x=a$, $x=b$ e $y=0$. Esboce o gráfico

a) $f(x) = 1-x^2 \quad a = -1 \quad b = 1$

$$\int_{-1}^{1} 1-x^2\,dx = \left(x - \frac{x^3}{3}\right|_{-1}^{1} = 1-\frac{1}{3} - (-1) + \frac{(-1)^3}{3}$$

$$= 2 - \frac{2}{3} = \frac{4}{3}$$

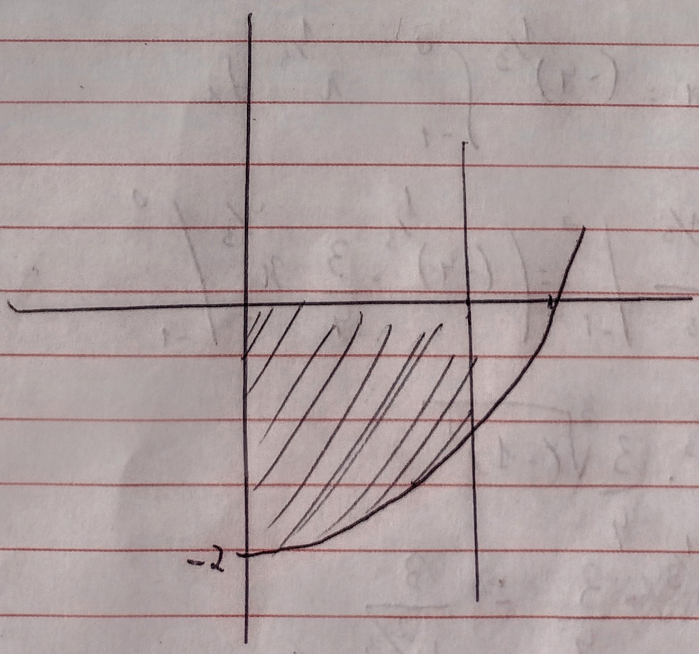
//



b) $g(x) = x^3 - 2 \quad a = 0 \quad b = 1$

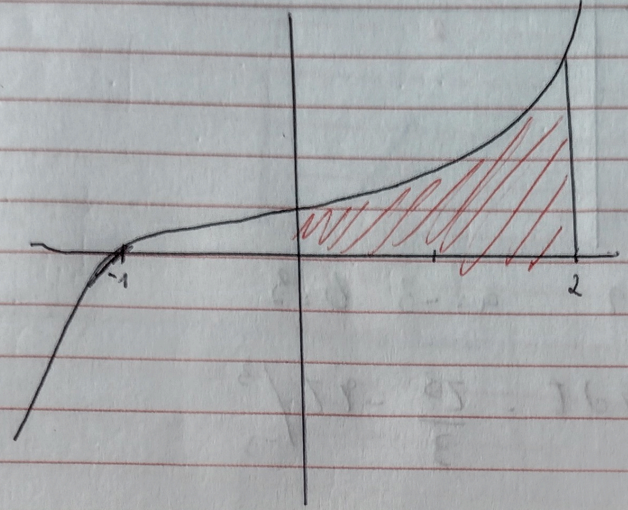
$$\int_{0}^{1} x^3-2\,dx = \left(\frac{x^4}{4} - 2x\right|_{0}^{1} = \frac{1}{4} - 2 = -\frac{7}{4}$$

//



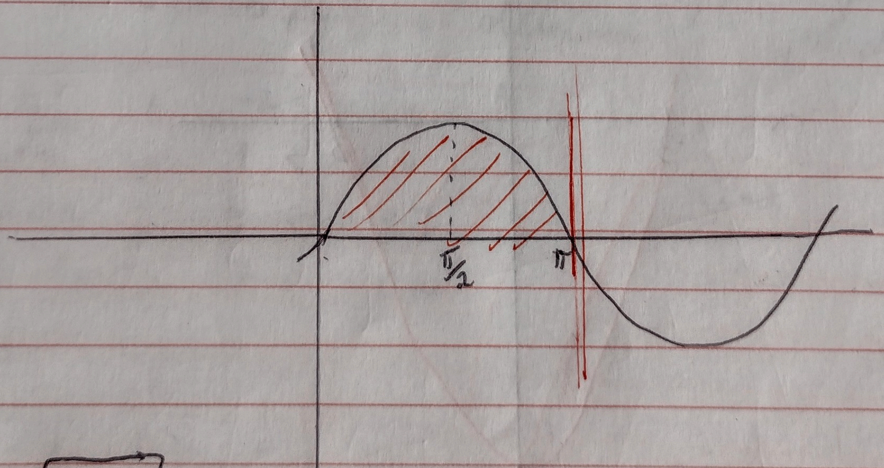
c) $h(x) = x^3 + 1 \qquad a = 0 \qquad b = 2$

$$\int_0^2 x^3 + 1\, dx = \frac{x^4}{4} + x \Big|_0^2 = \frac{2^4}{4} + 2 = 6$$



d) $T(x) = \operatorname{sen} x \qquad a = 0 \quad b = \pi$

$$\int_0^{\pi} \operatorname{sen} x\, dx = -\cos x \Big|_0^{\pi} = -\cos\pi - (-\cos 0) = -(-1) + 1 = 2$$



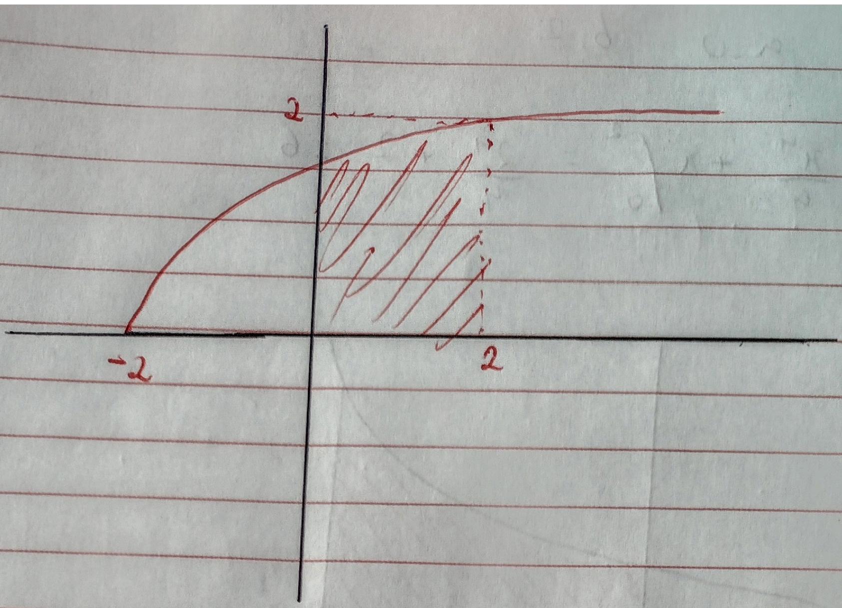
e) $l(u) = \sqrt{u+2} \quad a = 0 \quad b = 2$

$$\int_0^2 \sqrt{u+2}\, du = \int_0^2 x^{1/2}\, dx = \left(\frac{x^{3/2}}{3/2} \right|_0^2 = \left(\frac{(u+2)^{3/2}}{3/2} \right|_0^2$$

$x = u + 2$

$dx = du$

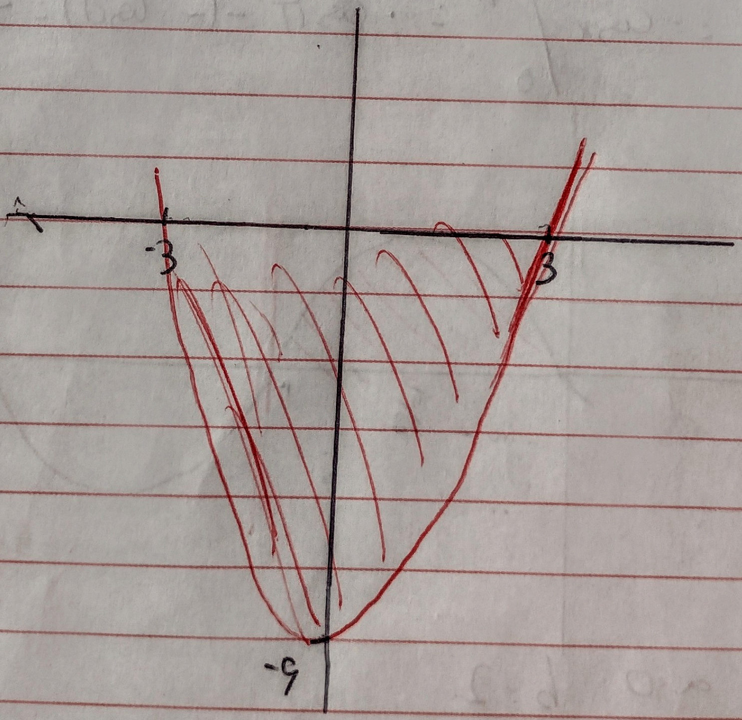
$$= (2+2)^{3/2} \cdot \frac{2}{3} - (0+2)^{3/2} \cdot \frac{2}{3} = \frac{\sqrt{4^3} \cdot 2}{3} - \frac{2}{3}\sqrt{2^3} = \frac{16}{3} - \frac{2\sqrt{8}}{3} \;//$$



(2) $f(t) = t^2 - 9 \qquad a = -3 \quad b = 3$

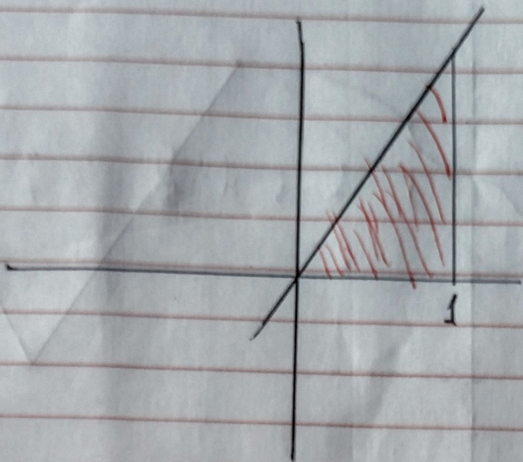
$$\int_{-3}^{3} t^2 - 9\,dt = \frac{t^3}{3} - 9t \Big|_{-3}^{3}$$

$$\frac{3^3}{3} - 9.3 - \frac{(-3)^3}{3} + 9(-3) = 9 - 27 + 9 - 27 = -36$$



(g) $m(x) = x^n \quad a=0 \quad b=1$

$$\int_0^1 x^n \, dx = \frac{x^{n+1}}{n+1}\Big|_0^1 = \frac{1^{n+1}}{n+1} - 0 = \frac{1^{n+1}}{n+1} = \frac{1}{n+1}$$ //



7- Em cada caso, Esboce o gráfico da função f, calcule a área da região limitada pelo gráfico de cada função e as retas $x=a$, $x=b$, $y=0$ e determine $\int_a^b f(x)\,dx$

(a)

$$f(x) = \begin{cases} x^3, & -2 \leq x \leq 1 \\ \sqrt{x}, & 1 < x \leq 4 \\ 10-2x, & 4 < x \leq 7 \\ 2x-18, & 7 < x \leq 12 \end{cases}$$

$a = -2 \quad b = 12$

$f(x) = x^3$

$f(-2) = (-2)^3 = -8$

$f(1) = 1^3 = 1$

$(-2, -8)$

$(1, 1)$

$\sqrt{x}$

$f(1) = 1$

$f(4) = 2$

$f(x) = 10-2x$

$f(4) = 10-2\cdot 4 = 2$

$f(7) = 10-2\cdot 7 = -4$

$10-2x=0$

$-2x=-10$

$x=-5$

$2x-18$

$f(7) = 2x-18$

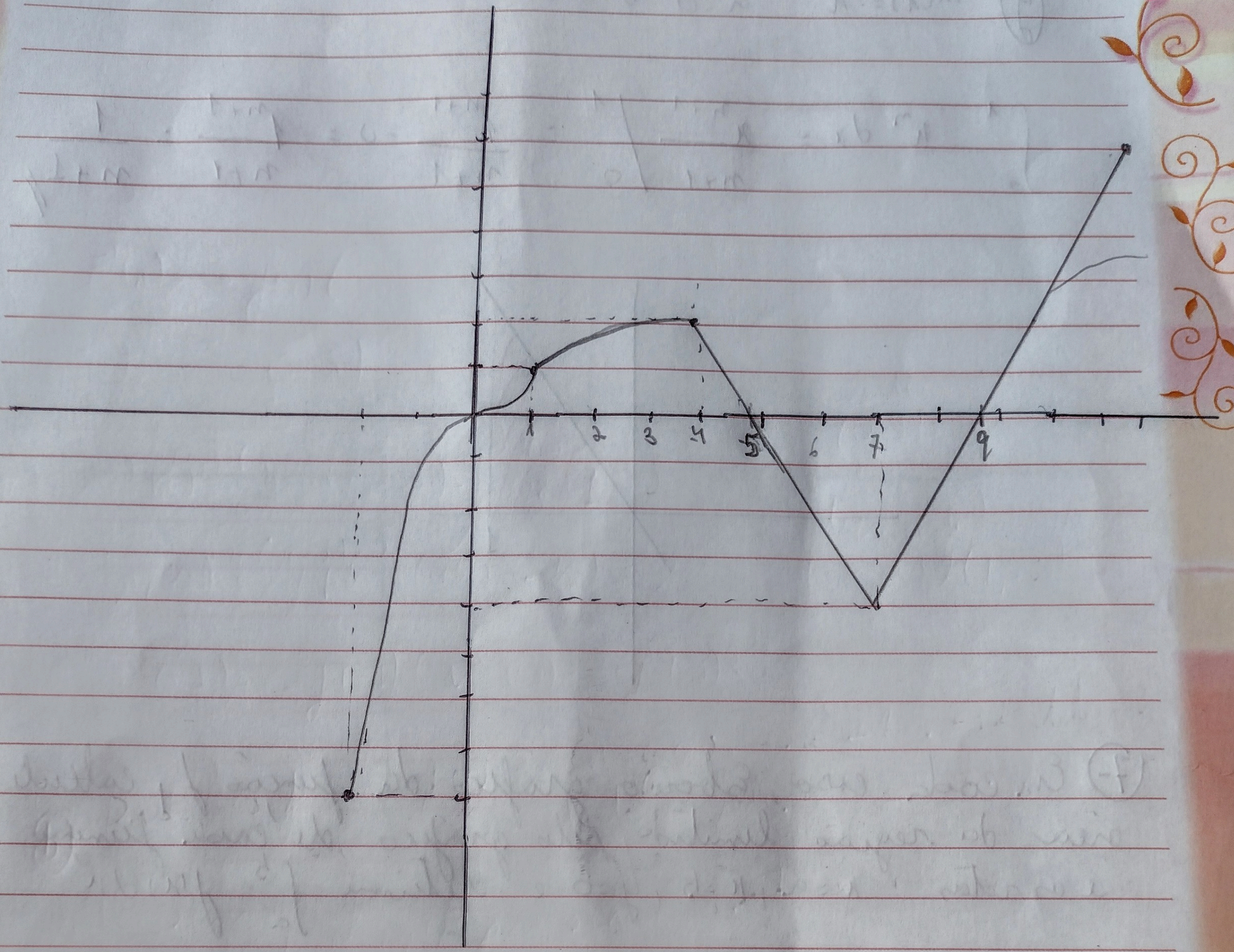
$= -4$

$f(12) = 2\cdot 12 - 18$

$= 6$

CALCULAR TAMBÉM:

$$\int_{-2}^{12} f(x)\,dx = \int_{-2}^{1} x^3\,dx + \int_{1}^{4} \sqrt{x}\,dx + \int_{4}^{7} (10-2x)\,dx + \int_{7}^{12} (2x-18)\,dx$$



$$\int_{-2}^{12} f(x)\,dx = -\int_{-2}^{0} x^3\,dx + \int_{0}^{1} x^3\,dx + \int_{1}^{4} \sqrt{x}\,dx + \int_{4}^{5} 10-2x\,dx - \int_{5}^{7} 10-2x\,dx$$

$$-\int_{7}^{9} 2x-18\,dx + \int_{9}^{12} 2x-18\,dx$$

$$= -\overbrace{\left(\frac{x^4}{4}\right|_{-2}^{0}}^{8} + \overbrace{\left(\frac{x^4}{4}\right|_{0}^{1}}^{1/4} + \overbrace{\left(x^{3/2}\cdot\frac{2}{3}\right|_{1}^{4}}^{16/3 - 2/3} + \overbrace{\left(10x - \frac{2x^2}{2}\right|_{4}^{5}}^{25-24} - \overbrace{\left(10x - \frac{2x^2}{2}\right|_{5}^{7}}^{21-25}$$

$$-\underbrace{\left(\frac{2x^2}{2} - 18x\right|_{7}^{9}}_{\substack{-(-81+77)\\ +4}} + \underbrace{\left(\frac{2x^2}{2} - 18x\right|_{9}^{12}}_{\substack{-72+81\\ 9}} = 8 + \frac{1}{4} + \frac{16}{3} - \frac{2}{3} + 1 - (-4) + 4 + 9$$

$$26 + \frac{14}{3} + \frac{1}{4} = \frac{92}{3} + \frac{1}{4} = \frac{368+3}{12} = \frac{371}{12}$$

x^2+6x-7

$f(-7)=0$

$f(-6)=-7$

$f(-6)=-36+24+5=-7$

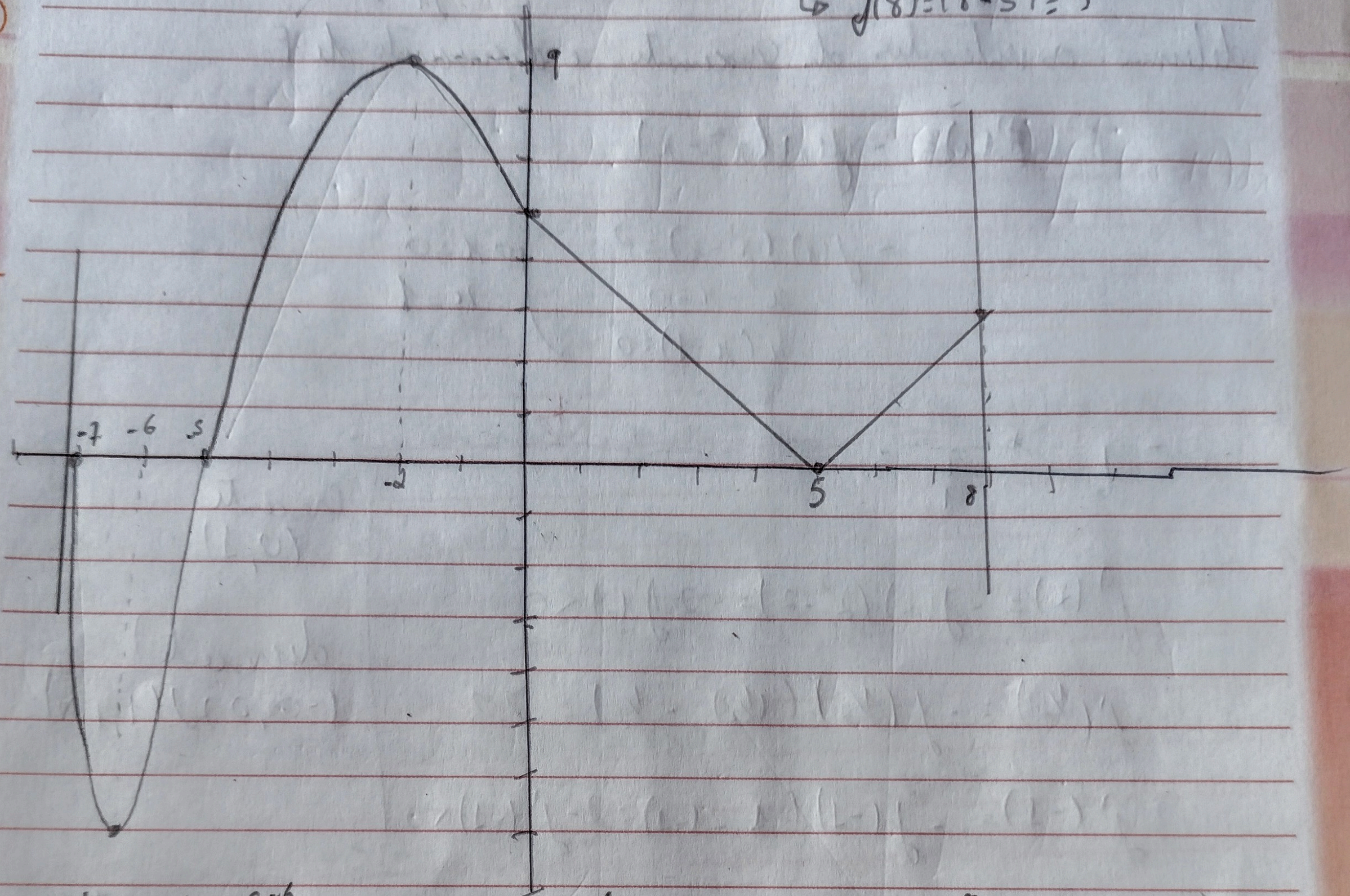
$f(0)=5$

$-x^2-4x+5=0$

$-25 \quad -25$

(b) $f(x)=\begin{cases} x^2+6x-7, & -7 \leq x \leq -6 \\ -x^2-4x+5, & -6 < x \leq 0 \\ |x-5|, & 0 < x \leq 8 \end{cases}$ $\to (-5,0);\ (-2,9)$ $\quad a=-7 \quad b=8$

$\to f(8)=|8-5|=3$



$$\int_{-7}^{8} f(x)\,dx = \int_{-7}^{-6} x^2+6x-7\,dx - \int_{-6}^{-5} -x^2-4x+5\,dx + \int_{-5}^{0} -x^2-4x+5\,dx +$$

$$+\int_{0}^{5} -x+5\,dx + \int_{5}^{8} x-5\,dx$$

$$= -\left(\frac{x^3}{3}+3x^2-7x\right|_{-7}^{-6} - \left(\frac{-x^3}{3}-2x+5x\right|_{-6}^{-5} + \left(-\frac{x^3}{3}-2x^2+5x\right|_{-5}^{0} +$$

$$+\left(-\frac{x^2}{2}+5x\right|_{0}^{5} + \left(\frac{x^2}{2}-5x\right|_{5}^{8} = \frac{172}{3} \Big\}\ \text{área}$$

CALCULAR TAMBÉM:

$$\int_{-7}^{8} f(x)\,dx = \int_{-7}^{-6} x^2+6x-7\,dx + \int_{-6}^{0} -x^2-4x+5\,dx + \int_{0}^{8} |x-5|\,dx$$

(8-) Resolver os seguintes problemas:

ⓐ $\forall x \in \mathbb{R}: f(x) > 0$ e f contínua, $F(x) = \int_x^0 f(t)(t^2 - t)\,dt$, determine os intervalos de crescimento e decrescimento de F

$F'(x) = -f(x)(x^2 - x)$

$-f(x)(x^2-x) = 0$
$x^2 - x = 0$
$x(x-1) = 0$

$\to x = 0$
$x = 1$

$-\quad 0 \quad + \quad 1 \quad -$

$f'(2) = -f(2)(2^2-2) = -2f'(2) < 0$

$f'(1/2) = -f(1/2)((1/2)^2 - 1/2) = > 0$

$f'(-1) = -f(-1)(1-(-1)) = 2 - f'(-1) < 0$

Crescente $(0, 1)$

decrescente) $(-\infty, 0) \cup (1, +\infty)$

ⓑ Dada as funções $f(x) = 2x^2$ e $g(x) = -ax^2 + 16a + 32$, determine $a > 0$ para que a área limitada pelos gráficos de f e g seja de 180 unidades geométricas.

iguala $f(x)$ e $g(x)$ para achar as retas a e b

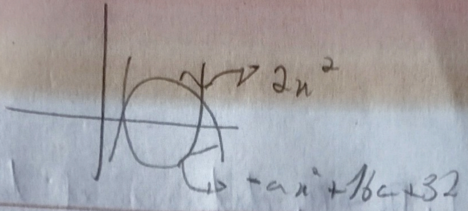
$2x^2 = -ax^2 + 16a + 32$

$(2+a)\,x^2 = 16a + 32$

$x^2 = \dfrac{16a+32}{2+a}$

$x = \pm\sqrt{\dfrac{16a+32}{2+a}}$

$x = \pm\sqrt{16\left(\dfrac{a+2}{a+2}\right)} = \pm 4$ //



$$\int_{-4}^{4} g(x) - f(x)\,dx = 180$$

$$\Rightarrow \int_{-4}^{4} (-ax^2 + 16a + 32 - 2x^2)\,dx = 180$$

$$\Rightarrow \int_{-4}^{4} -(a+2)\,x^2 + (16a+32)\,dx = 180$$

$$-(a+2)\frac{x^3}{3} + (16a+32)x \Big|_{-4}^{4} = 180$$

$$\Rightarrow -\frac{(a+2)\cdot 4^3}{3} + (16a+32)\cdot 4 + (a+2)\frac{(-4)^3}{3} - (16a+32)(-4) = 180$$

$$-\frac{64(a+2)}{3} - \frac{64(a+2)}{3} + 8(16a+32) = 180$$

$$\Rightarrow -\frac{128a - 256}{3} + 128a + 256 = 180$$

$$\Rightarrow -\frac{128a}{3} + 128a - \frac{256}{3} + 256 = 180$$

$$\Rightarrow \frac{256a}{3} + \frac{512}{3} = 180$$

$$\Rightarrow 256a = 3\cdot 180 - 512$$

$$\Rightarrow a = \frac{28}{256}$$

$$a = \frac{7}{64}\ u.a$$

//

(c) Seja $f: \mathbb{R} \to \mathbb{R}$ contínua tal que $\int_0^x f(t)\,dt = x^2 + \ln(x^3) + 5$. Calcule $f(4)$

$$\int_0^x f(t)\,dt = x^2 + \ln(x^3) + 5$$

$$f(x) = 2x + \frac{1}{x^3}(3x^2)$$

$$f(4) = 2\cdot 4 + \frac{1}{4^3}(3\cdot 4^2) = 8 + \frac{1\cdot 3\cdot 16}{64} = 8 + \frac{3}{4} = \frac{35}{4}$$

//

(1) Seja $g: \mathbb{R} \to \mathbb{R}$ diferenciável tal que $g(e) = 5$ e $\int_1^e g'(x) \ln x \, dx = 6$. Calcule $\int_1^e \frac{g(x)}{x} dx$

integral por partes

$$\int_1^e \frac{g(x)}{x} dx = g(x) \ln x \Big|_1^e - \int_1^e \ln x \, g'(x) dx$$

$u = g(x)$

$du = g'(x) dx$

$dv = \frac{1}{x} dx$

$v = \ln x$

$$= g(e) \ln e - g(1) . \overbrace{\ln 1}^{0} - 6$$

$$= 5.1 - 0 - 6 = 5 - 6 = -1$$

(2) Encontre o polinômio $p(x)$ de terceiro grau tal que $p(0) = p(-2) = 0$, $p(1) = 15$ e $\int_{-2}^0 p(x) dx = \frac{4}{3}$

$p(x) = ax^3 + bx^2 + cx + d$

$p(0) = p(-2) = 0$

$p(0) = d \Rightarrow d = 0$

$0 = a(-2)^3 + b(-2)^2 + c(-2)$

$-8a + 4b - 2c = 0$ //

$p(1) = 15$

$\hookrightarrow a + b + c = 15$ //

$$\int_{-2}^0 p(x) dx = \frac{4}{3}$$

$$\int_{-2}^0 ax^3 + bx^2 + cx \, dx = \frac{4}{3}$$

$$\frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} \Big|_{-2}^0$$

$$= -\frac{(-2)^4 a}{4} - \frac{(-2)^3 b}{3} - \frac{(-2)^2 c}{2}$$

$$= -4a + \frac{8b}{3} - 2c = \frac{4}{3}$$

$$\begin{cases} a + b + c = 15 \\ -8a + 4b - 2c = 0 \\ -4a + \frac{8}{3}b - 2c = \frac{4}{3} \end{cases}$$

$a = 3$

$b = 8$

$c = 4$

$p(x) = 3x^3 + 8x^2 + 4x$ //

8. (f) Encontre k para que $\int_{-2}^{6} f(x)\,dx = 1$ se

$$f(x) = \begin{cases} x, & -2 < x \leq 0 \\ \frac{1}{2} - kx, & 0 < x \leq 4 \\ x - 4, & 4 < x \leq 6 \end{cases}$$

$$\int_{-2}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx + \int_{4}^{6} f(x)\,dx = 1$$

$$\int_{-2}^{0} x\,dx + \int_{0}^{4} \frac{1}{2} - kx\,dx + \int_{4}^{6} x - 4\,dx = 1$$

$$\frac{x^2}{2}\Big|_{-2}^{0} + \left(\frac{1}{2}x - \frac{kx^2}{2}\Big|_{0}^{4}\right. + \left(\frac{x^2}{2} - 4x\Big|_{4}^{6}\right. = 1$$

$$-\frac{(-2)^2}{2} + \left(\frac{4}{2} - \frac{16}{2}k\right) + \left(\frac{36}{2} - 24 - \frac{16}{2} + 16\right) = 1$$

$$\cancel{-2} + \cancel{2} - 8k + \cancel{18} - \cancel{24} + \cancel{8} = 1$$

$$2 - 8k = 1$$

$$-8k = -1$$

$$k = \frac{1}{8}$$

9 ⓓ $6xy = x^4 + 3 \qquad 1 \leq x \leq 2$

$$f(x) = y = \frac{x^4+3}{6x} \qquad f'(x) = \frac{x^4-1}{2x^2}$$

$$l = \int_1^2 \sqrt{1 + [f'(x)]^2}\, dx = \int_1^2 \sqrt{1 + \left(\frac{x^4-1}{2x^2}\right)^2}\, dx$$

$$\int_1^2 \sqrt{\frac{4}{4} + \frac{(x^8 - 2x^4 + 1)}{4x^4}}\, dx$$

$$= \frac{1}{2}\int_1^2 \sqrt{4 + \frac{x^8 - 2x^4 + 1}{x^4}}\, dx$$

$$= \frac{1}{2}\int_1^2 \sqrt{\frac{4x^4 + x^8 - 2x^4 + 1}{x^4}}\, dx$$

$$= \frac{1}{2}\int_1^2 \sqrt{\frac{2x^4 + x^8 + 1}{x^4}}\, dx = \frac{1}{2}\int_1^2 \sqrt{\frac{(x^4+1)^2}{x^4}}\, dx$$

$$= \frac{1}{2}\int_1^2 \frac{x^4+1}{x^2}\, dx = \frac{1}{2}\int_1^2 x^2 + x^{-2}\, dx$$

$$l = \frac{1}{2}\left(\frac{x^3}{3} - \frac{1}{x}\right|_1^2 = \frac{1}{2}\left(\frac{2^3}{3} - \frac{1}{2} - \frac{1}{3} + 1\right)$$

$$= \frac{1}{2}\left(\frac{7}{3} + \frac{1}{2}\right) = \frac{1}{2}\ \frac{14+3}{6} = \frac{17}{12}\ u.m$$

$$l = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

(7) Calcule o comprimento do arco:

a) $y = x$, $0 \leq x \leq 1$

$f(x) = x$

$f'(x) = 1 \qquad f'(x)^2 = 1 \qquad 1 + f'(x)^2 = 2$

$$\int_0^1 \sqrt{2}\, dx = \sqrt{2}\, x \Big|_0^1 = \sqrt{2}\ u.m$$

b) $y = \frac{1}{3}(x^2+2)^{3/2} \quad 0 \leq x \leq 2$

$f(x) = \frac{1}{3}(x^2+2)^{3/2}$

$f'(x) = \frac{1}{3} \cdot \frac{3}{2}(x^2+2)^{1/2} \cdot 2x = x\sqrt{x^2+2} \qquad f'(x)^2 = x^2(x^2+2)$

$$\int_0^2 \sqrt{1 + x^2(x^2+2)}\, dx$$

$$\int_0^2 \sqrt{1 + x^4 + 2x^2}\, dx$$

$$= \int_0^2 \sqrt{(x^2+1)^2}\, dx = \int_0^2 x^2 + 1\, dx = \frac{x^3}{3} + x \Big|_0^2$$

$$= \frac{2^3}{3} + 2 - 0 = \frac{8}{3} + 2 = \frac{14}{3}$$

c) $y = \frac{1}{8}x^4 + \frac{1}{4x^2} \qquad 1 \leq x \leq 2$

$f(x) = \frac{1}{8}x^4 + \frac{1}{4}x^{-2}$

$f'(x) = \frac{4}{8}x^3 + \frac{1}{4}(-2)x^{-3}$

$f'(x) = \frac{x^3}{2} - \frac{1}{2x^3}$

$[f'(x)]^2 = \left(\frac{x^3 - x^{-3}}{2}\right)^2$

$[f'(x)]^2 = \frac{1}{4}(x^3 - x^{-3})(x^3 - x^{-3})$

$= \frac{1}{4}(x^6 - 1 - 1 + x^{-6})$

$= \frac{1}{4}x^6 - \frac{1}{2} + \frac{1}{4x^6}$

$$1 + [f'(x)]^2 = 1 - \frac{1}{2} + \frac{x^6}{4} + \frac{1}{4x^6} = \frac{1}{2} + \frac{x^6}{4} + \frac{1}{4x^6}$$

$$= \frac{2 + x^6 + x^{-6}}{4}$$

$$\int_1^2 \sqrt{\frac{2 + x^6 + x^{-6}}{4}}\, dx$$

$$\frac{1}{2}\int_1^2 \sqrt{2 + x^6 + x^{-6}}\, dx = \frac{1}{2}\int_1^2 \sqrt{(x^3 - x^{-3})^2}\, dx$$

$$\frac{1}{2}\int_1^2 x^3 + x^{-3}\, dx = \frac{1}{2}\left[\frac{x^4}{4} + \frac{x^{-2}}{-2} \Big|_1^2 \right]$$

$$= \frac{1}{2}\left[\frac{2^4}{4} - \frac{1}{2 \cdot 2^2} - \frac{1}{4} + \frac{1}{2} \right] = \frac{1}{2}\left[4 - \frac{1}{8} - \frac{1}{4} + \frac{1}{2} \right]$$

$$\frac{1}{2}\left[4 - \frac{1 - 2 + 4}{8} \right] = \frac{1}{2}\left[\frac{32 + 1}{8} \right] = \frac{1}{2} \cdot \frac{33}{8} = \frac{33}{16} \;//$$

$16x^2+40x+26$

$16\left(x+\frac{5}{4}\right)^2+1$

② $y = 2x^2 + 5x$, embaixo do eixo Ox $\quad -5/2 \leq x \leq 0$

$f'(x) = 4x+5$

$f'(x)^2 = (4x+5)^2 = 16x^2+40x+25$

$\sec^2\theta = 1+Tg^2\theta$

$Tg\theta = 4x+5$

$x = \frac{Tg\theta - 5}{4}$

$dx = \frac{1}{4}\sec^2\theta\, d\theta$

$$l = \int_{-5/2}^{0} \sqrt{1+(4x+5)^2}\, dx$$

$$l = \int_{-5/2}^{0} \sqrt{1+Tg\theta^2}\, \frac{1}{4}\sec^2\theta\, d\theta = \frac{1}{4}\int_{-5/2}^{0} \sqrt{\sec^2\theta}\cdot \sec^2\theta\, d\theta$$

$$= \frac{1}{4}\int_{-5/2}^{0} \sec^3\theta\, d\theta.$$

$$^{*}\int \sec^2\theta \sec\theta\, d\theta = \sec\theta\cdot Tg\theta - \overbrace{\int Tg\theta\sec\theta\cdot Tg\theta\, d\theta}^{**}$$

$u = \sec\theta$

$du = \sec\theta\cdot Tg\theta\, d\theta$

$dv = \sec^2\theta$

$v = Tg\theta$

$$= \sec\theta\cdot Tg\theta - \int \sec^3\theta\, d\theta + \ln|\sec\theta + Tg\theta|$$

$$2\int \sec^3\theta\, d\theta = \sec\theta\cdot Tg\theta + \ln|\sec\theta + Tg\theta|$$

$$\int \sec^3\theta\, d\theta = \frac{1}{2}\left[\sec\theta\cdot Tg\theta + \ln|\sec\theta + Tg\theta|\right]$$

$$^{**}\int Tg^2\theta\sec\theta\, d\theta = \int \sec\theta(\sec^2\theta - 1)\, d\theta$$

$$= \int \sec^3\theta - \sec\theta\, d\theta = \int \sec^3\theta\, d\theta - \ln|\sec\theta + Tg\theta|$$

$$\Rightarrow \frac{1}{4}\left[\frac{1}{2}\left(\sqrt{1+(4x+5)^2}\cdot(4x+5) + \ln\left|\sqrt{1+(4x+5)^2} + (4x+5)\right|\right)\right]\Bigg|_{-5/2}^{0}$$

$$= \frac{1}{8}\left(\sqrt{26}\cdot 5 + \ln|\sqrt{26}+5| - \sqrt{26}(-5) + \ln|\sqrt{26}-5|\right)$$

$$l = \frac{1}{8}\left(10\sqrt{26} + \ln|\sqrt{26}+5| - \ln|\sqrt{26}-5|\right)$$

//

1) $F(x) = \int_0^x \sqrt{e^{2t} - 1}\, dt \quad 0 \leq x \leq 1$

$F'(x) = \sqrt{e^{2x} - 1} \qquad F'(x)^2 = e^{2x} - 1$

$l = \int_0^1 \sqrt{1 + e^{2x} - 1}\, dx = \int_0^1 \sqrt{e^{2x}}\, dx$

$= \int_0^1 (e^{2x})^{1/2}\, dx = \int_0^1 e^{x}\, dx$

$l = e^x \Big|_0^1 = e^1 - e^0 = e - 1$

$F(x) = \int_0^x f(t)\, dt \Rightarrow F'(x) = f(x) \cdot x'$

OBS: $\pi \int_a^b f(x)^2 \, dx$

(1a) Calcule o volume dos seguintes sólidos de revolução.

(a) $y = \sqrt{x}$ $0 \leq x \leq 3$ em torno do eixo Ox

$$V = \pi \int_0^3 \sqrt{x}^2 \, dx = \pi \int_0^3 x \, dx = \pi \left(\frac{x^2}{2} \right|_0^3$$

$$\pi \left(\frac{3^2}{2} - 0 \right) = \frac{9\pi}{2} \text{ u.v}$$

(b) $y = x^2 + 1$ $-1 \leq x \leq 1$ em torno do eixo Ox

$$V = \pi \int_{-1}^1 (x^2+1)^2 \, dx = \pi \int_{-1}^1 x^4 + 2x^2 + 1 \, dx$$

$$= \pi \left(\frac{x^5}{5} + \frac{2x^3}{3} + x \right|_{-1}^1 = \pi \left[\left(\frac{1}{5} + \frac{2}{3} + 1 \right) - \left(\frac{-1}{5} - \frac{2}{3} - 1 \right) \right]$$

$$= \pi \left[\frac{2}{5} + \frac{4}{3} + 2 \right] = \pi \left[\frac{6 + 20 + 30}{15} \right] = \frac{56}{15} \pi \text{ u.v}$$

(c) $y = \frac{x^2}{4}$ $0 \leq x \leq 4$ em torno do eixo Ox

$$V = \pi \int_0^4 \left(\frac{x^2}{4} \right)^2 dx = \pi \int_0^4 \frac{x^4}{16} \, dx = \frac{\pi}{16} \int_0^4 x^4 \, dx$$

$$= \frac{\pi}{16} \left(\frac{x^5}{5} \right|_0^4 = \frac{\pi}{16} \cdot \frac{4^5}{5} = \frac{1024 \pi}{16 \cdot 5} = \frac{64\pi}{5} \text{ u.v}$$

(d) $x = y^3$ $0 \leq x \leq 8$ em torno de Oy

$$V = \pi \int_0^2 [y^3]^2 \, dy = \pi \int_0^2 y^6 \, dy \; \pi \left(\frac{y^7}{7} \right|_0^2$$

$$\pi \, \frac{2^7}{7} = \frac{128\pi}{7} \text{ u.v}$$

(e) $x = y$ $\quad y = 2x$ (curva) $\quad y = 4$ em torno de Oy

$$x = \frac{y}{2} \qquad x = 2x$$

$$V = \pi \int_0^4 [y]^2 - \left[\frac{y}{2}\right]^2 dy = \pi \int_0^4 y^2 - \frac{y^2}{4} \, dy$$

$$V = \pi \left(\frac{y^3}{3} - \frac{y^3}{3 \cdot 4} \right|_0^4 = \pi \left(\frac{4^3}{3} - \frac{4^3}{12} \right) - 0$$

$$\pi \left(\frac{64}{3} - \frac{64}{3 \cdot 4} \right) = \pi \left(\frac{64}{3} - \frac{16}{3} \right) = 16 \pi \; uV$$

(f) $y = 2 - x^2 + 4x$ $\quad 0 \leq x \leq 4$ em torno da reta $y = 2$

$$V = \pi \int [f(x) - 2]^2 dx$$

$$V = \pi \int_0^4 (2 - x^2 + 4x - 2)^2 dx = \pi \int_0^4 (-x^2 + 4x)^2 dx$$

$$V = \pi \int_0^4 (x(-x+4))^2 dx$$

$$V = \pi \int_0^4 x^2 (-x+4)^2 dx = \pi \int_0^4 x^2 (x^2 - 8x + 16) \, dx$$

$$V = \pi \int_0^4 x^4 - 8x^3 + 16x^2 dx = \pi \left(\frac{x^5}{5} - \frac{8x^4}{4} + \frac{16x^3}{3} \right|_0^4$$

$$V = \pi \left(\frac{4^5}{5} - 2 \cdot 4^4 + \frac{16}{3} \cdot 4^3 \right) = \pi \left(\frac{1024}{5} - 512 + \frac{1024}{3} \right)$$

$$= \pi \left(-\frac{1536}{5} + \frac{1024}{3} \right) = \pi \left(\frac{-4608 + 5120}{15} \right) = \frac{512}{15} \pi \; uV$$

g) $y = x^3$, $1 \leq y \leq 8$ em torno de Oy

$y=1$ | $x = \sqrt[3]{y}$

$y=8$

$$V = \pi \int_1^8 [y^{1/3}]^2 dy = \pi \int_1^8 y^{2/3} dy = \pi \left(\frac{y^{5/3}}{5/3} \right|_1^8$$

$$= \pi \cdot \frac{3}{5} y^{5/3} \Big|_1^8 = \frac{3\pi}{5}\left(\sqrt[3]{8^5} - 1\right) = \frac{3\pi}{5}(32-1)$$

$$V = \frac{93\pi}{5} \text{ u.v}$$

h) $y = \sqrt{x}$ $\rightarrow x = y^2$, $0 \leq y \leq 2$ em torno da reta $x=4$

$y=0 \Rightarrow x=0$

$y=2 \Rightarrow x=4$

$$V = \pi \int_0^4 [\sqrt{x} - 4]^2 dx$$

$$V = \pi \int_0^4 x - 8\sqrt{x} + 16 \, dx = \pi \left(\frac{x^2}{2} - \frac{8 x^{3/2}}{3/2} + 16x \right|_0^4$$

$$V = \pi \left[\frac{4^2}{2} - \frac{8 \cdot 2}{3} \sqrt{4^3} + 16 \cdot 4 \right] = \pi \left(8 - \frac{16}{3} \cdot 4\sqrt{4} + 64 \right)$$

$$= \pi \left(72 - \frac{64}{3}\sqrt{4} \right) \text{u.v}$$

i) $\begin{cases} xy = 4 \\ x + y = 5 \end{cases}$ em torno do eixo Oy

$x = \frac{4}{y}$ $\quad x = 5 - y$ | $y = \frac{4}{x}$ $\quad y = 5 - x$

$\frac{4}{x} = 5 - x$

$4 = 5x - x^2$

$-x^2 + 5x - 4 = 0$ $\rightarrow$ $x = 1$, $x = 4$

$x=1$, $y=4$

$x=4$, $y=1$

$$V = \pi \int_1^4 [5-y]^2 - \left[\frac{4}{y}\right]^2 dy$$

$$V = \pi \int_1^4 25 - 10y + y^2 - \frac{16}{y^2}\, dy$$

$$V = \pi \left(25y - \frac{10y^2}{2} + \frac{y^3}{3} + \frac{16}{y} \right|_1^4 = \pi \left(25y - 5y^2 + \frac{y^3}{3} + \frac{16}{y} \right|_1^4$$

$$= \pi \left(25 \cdot 4 - 5 \cdot 4^2 + \frac{4^3}{3} + \frac{16}{4} - 25 + 5 - \frac{1}{3} - 16 \right)$$

$$= \pi \left(100 - 80 + \frac{64}{3} + 4 - 20 - \frac{1}{3} - 16 \right) = \pi \left(-12 + \frac{63}{3} \right) = 9\pi \text{ u.V}$$

j) $\begin{cases} y^3 = x \\ y = x^2 \end{cases}$ em torno do eixo Oy → passa por cima

$y = \sqrt[3]{x}$ e $y = x^2$

$\sqrt[3]{x} = x^2$

$x = x^6$

$x^6 - x = 0$

$x(x^5 - 1) = 0$

$x = 0$ ou $x = 1$

$\to y = 0$ $\to y = 1$

$$V = \pi \int_0^1 [\sqrt{y}]^2 - [y^3]^2\, dy$$

$$V = \pi \int_0^1 y - y^6\, dy$$

$$V = \pi \left[\frac{y^2}{2} - \frac{y^7}{7} \right] \Big|_0^1$$

$$V = \pi \left(\frac{1}{2} - \frac{1}{7} \right) = \pi \left(\frac{7-2}{14} \right) = \frac{5\pi}{14} \text{ u.V}$$