Lista 8

⑥ $f(x) = e^{-\frac{1}{2}x}$ $\quad x \in [0, \frac{1}{4}]$ $\quad (f^{-1})''(e^{-\frac{1}{8}})$

$$(f^{-1})' = \frac{1}{f'(f^{-1}(x))} = [f'(f^{-1}(x))]^{-1}$$

$$(f^{-1}(x))'' = -1[f'(f^{-1}(x))]^{-2} f''(f^{-1}(x))(f^{-1}(x))'$$

$$(f^{-1}(x))'' = \frac{-1}{[f'(f^{-1}(x))]^{2}} \cdot f''(f^{-1}(x)) \cdot \frac{1}{[f'(f^{-1}(x))]^{1}} = \frac{f''(f^{-1}(x))}{[f'(f^{-1}(x))]^{3}}$$

$$f(x) = e^{-\frac{1}{2}x}$$

$$f'(x) = e^{-\frac{1}{2}x} \cdot \left(-\frac{1}{2}x\right)' = -\frac{1}{2} e^{-\frac{1}{2}x}$$

$$f''(x) = -\frac{1}{2} e^{-\frac{1}{2}x} \cdot \left(-\frac{1}{2}\right) = \frac{1}{4} e^{-\frac{1}{2}x}$$

$$e^{-\frac{1}{2}x} = e^{-\frac{1}{8}}$$

$$-\frac{1}{2}x = -\frac{1}{8}$$

$$x = \frac{-1 \cdot 2}{8 \cdot (-1)} = \frac{2}{8} = \frac{1}{4} \quad \mathrm{I}$$

SUBSTITUINDO I em $f'(x)$ e $f''(x)$

$$f'\left(\frac{1}{4}\right) = -\frac{1}{2} e^{-\frac{1}{2} \cdot \frac{1}{4}} = -\frac{1}{2} e^{-\frac{1}{8}}$$

$$f''\left(\frac{1}{4}\right) = \frac{1}{4} e^{-\frac{1}{2} \cdot \frac{1}{4}} = \frac{1}{4} e^{-\frac{1}{8}}$$

SUBSTITUINDO $f'(1/4)$ e $f''(1/4)$ em $(f^{-1}(x))''$, temos:

$$(f^{-1}(x))'' = \frac{-1/4\, e^{-1/8}}{\left[-1/2\, e^{-1/8}\right]^3} = \frac{-1\, e^{-1/8}}{4} \cdot \frac{8}{-1\, e^{-3/8}} = 2\, e^{+1/4}$$

$$= 2\sqrt[4]{e}$$

$$-\frac{1}{8} - \left(-\frac{3}{8}\right) = \frac{2}{8} = \frac{1}{4}$$

Lista 10

(11) $V = \frac{\pi r^2 \cdot h}{3}$

$\phi = 2r$

$h = \frac{3}{8} \cdot 2r = \frac{6}{8} r = \frac{3}{4} r$ m

$h = \frac{3}{4} r$

$r = \frac{4h}{3}$ //

(a) $h = \frac{3}{4} r$

$\frac{dh}{dt} = \frac{3}{4} \cdot \frac{dr}{dt}$ //

OBS: SUBSTITUIR o dr/dt do item (b)

$\frac{dh}{dt} = \frac{3}{4} \cdot \frac{15}{32 \cdot \pi} = \frac{45}{128\pi}$ m/min //

(b) $V = \frac{\pi r^2}{3} \cdot \frac{3}{4} r = \frac{\pi r^3}{4}$ //

$\frac{dV}{dt} = \frac{\pi}{4} \cdot 3r^2 \frac{dr}{dt} = \frac{\pi}{4} \cdot 3 \left(\frac{4}{3} 4\right)^2 \frac{dr}{dt}$

SUBSTITUINDO $\frac{dV}{dt} = 10$ e $h = 4$, Temos:

$10 = \frac{\pi}{4} \cdot 3 \left(\frac{4 \cdot 4}{3}\right)^2 \cdot \frac{dr}{dt}$

$40 = \cancel{3} \pi \cdot \frac{16^2}{\cancel{3} \cdot 3} \cdot \frac{dr}{dt}$

$$40 = \pi \cdot \frac{256}{3} \frac{dh}{dt}$$

$$\frac{3 \cdot 40}{\pi \cdot 256} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{120}{256\pi} = \frac{15}{32\pi} \text{ m/min} \times 100 = \frac{1500}{32\pi} \text{ cm/min}$$

② a) $y = \dfrac{1}{2x^2}$ $\Delta x = 0{,}001 \longrightarrow dx = 0{,}001$ $x = 1$

$\hookrightarrow y = \dfrac{1}{2} x^{-2}$

$\Delta y = f(x + \Delta x) - f(x)$

$dy = f'(x) \cdot dx$

$\Delta y = \dfrac{1}{2(x+\Delta x)^2} - \dfrac{1}{2x^2}$

SUBSTITUINDO Δx e x, TEMOS:

$$\Delta y = \frac{1}{2(1+0{,}001)^2} - \frac{1}{2 \cdot 1^2} = \frac{1}{2 \cdot 1{,}001^2} - \frac{1}{2} = 0{,}49900 - 0{,}5 = -0{,}000998$$

$dy = \dfrac{1}{2} \cdot (-2) \cdot x^{-3} \, dx$

$dy = \dfrac{-1}{x^3} \, dx$

SUBSTITUINDO x e dx, TEMOS:

$dy = \dfrac{-1}{1^3} \cdot 0{,}001 = -0{,}001$

b) 1) $\Delta y - dy$

$y = 2\sqrt{x} = 2 \cdot x^{1/2}$

$\Delta y = f(x+\Delta x) - f(x)$ $\qquad dy = f'(x) \cdot dx$

$\Delta y = 2\sqrt{x+\Delta x} - 2\sqrt{x}$ $\qquad dy = 2 \cdot \frac{1}{2} x^{-1/2} \cdot dx$

$= 2\left(\sqrt{x+\Delta x} - \sqrt{x}\right)$ // $\qquad dy = \frac{1}{\sqrt{x}} dx$

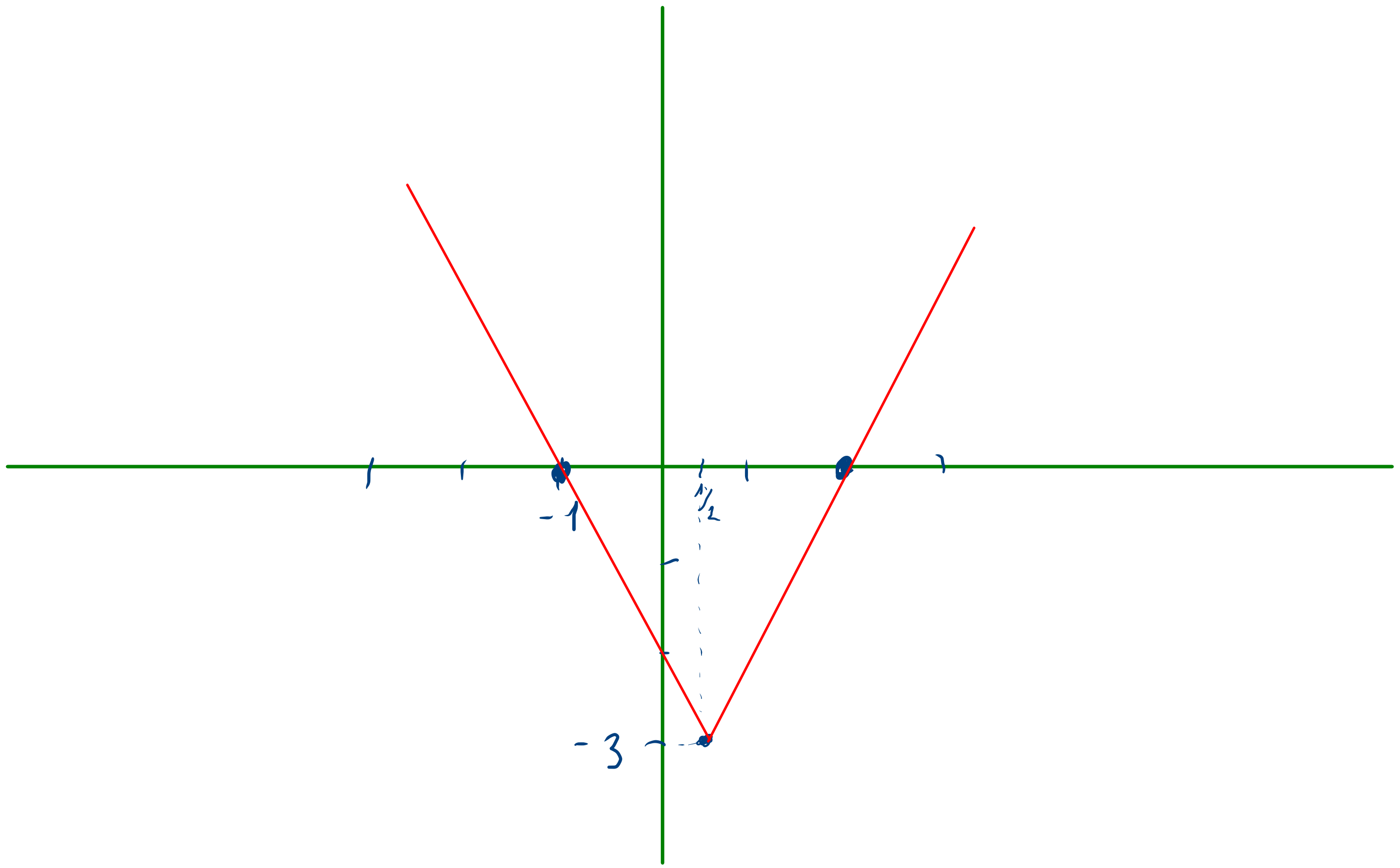
$\Delta y - dx = 2\left(\sqrt{x+\Delta x} - \sqrt{x}\right) - \frac{1}{\sqrt{x}} dx$ //

(16) $f(x) = |2x-1| - 3$ $\quad f(-1) = f(2) = 0 \quad f' \neq 0$

$f(-1) = |2(-1)-1| - 3 = |-2-1| - 3 = |-3| - 3 = 3 - 3 = 0$

$f(2) = |2.2 - 1| - 3 = |4-1| - 3 = |3| - 3 = 3 - 3 = 0$

$\Big\}\ f(-1) = f(2) = 0$



$|2x-1| - 3 = -3$

$|2x-1| = 0$

$2x - 1 = 0 \qquad -2x + 1 = 0$

$x = 1/2 \qquad -2x = -1$

$x = \frac{1}{2}$

$(1/2, -3)$

$$f(x) = \begin{cases} 2x - 1 - 3, & x \geq 1/2 \\ -2x + 1 - 3, & x < 1/2 \end{cases} = \begin{cases} 2x - 4, & x \geq 1/2 \\ -2x - 2, & x < 1/2 \end{cases}$$

Temos que f é contínua em $[-1, 2]$, $f(-1) = f(2)$, mas ela não é derivável em $(-1, 2)$ pois $\nexists\, f'(\frac{1}{2})$ e daí não satisfaz plenamente as hipóteses e assim não contradiz o Teorema de Rolle.

$$f'_+\left(\tfrac{1}{2}\right) = 2 \qquad f'_-\left(\tfrac{1}{2}\right) = -2$$

$$f'_+\left(\tfrac{1}{2}\right) = \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^+} \frac{f(\frac{1}{2} + h) - (-3)}{h}$$

$$= \lim_{h \to 0^+} \frac{2(\frac{1}{2} + h) - 4 + 3}{h} = \lim_{h \to 0^+} \frac{1 + 2h - 1}{h} = \lim_{h \to 0^+} \frac{2\cancel{h}}{\cancel{h}} = 2$$

$$f'_-\left(\tfrac{1}{2}\right) = \lim_{h \to 0^-} \frac{-2(\frac{1}{2} + h) - 2 + 3}{h} = \lim_{h \to 0^-} \frac{-1 - 2h + 1}{h} = \lim_{h \to 0^-} \frac{-2h}{h} = -2$$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad f'_+\left(\tfrac{1}{2}\right) \neq f'_-\left(\tfrac{1}{2}\right)$$

$$\nexists\, f'\left(\tfrac{1}{2}\right)$$

17 b $f(x) = 3\sqrt{x} - 4x$, $a = 1$ $b = 4$

$\sqrt{x} \to x^{1/2}$

$x \geq 0$ $[1, 4]$

f é contínua em $[1, 4]$

f é derivável em $(1, 4)$

DERIVANDO $f(x)$, Temos:

$$f'(x) = 3 \cdot \frac{1}{2} \cdot x^{-1/2} - 4 = \frac{3}{2\sqrt{x}} - 4$$

$$f'(c) = \frac{3}{2\sqrt{c}} - 4 \quad \text{I}$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f'(c) = \frac{3\sqrt{4} - 4 \cdot 4 - (3\sqrt{1} - 4 \cdot 1)}{4 - 1}$$

$$f'(c) = \frac{3 \cdot 2 - 16 - 3 + 4}{3}$$

$$f'(c) = \frac{-9}{3} = -3 \quad \text{II}$$

IGUALANDO I e II, temos:

$$\frac{3}{2\sqrt{c}} - 4 = -3$$

$$\frac{3}{2\sqrt{c}} = -3 + 4$$

$$\frac{3}{2\sqrt{c}} = 1$$

$$2\sqrt{c} = 3$$

$$\sqrt{c} = 3/2$$

$$c = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$ //

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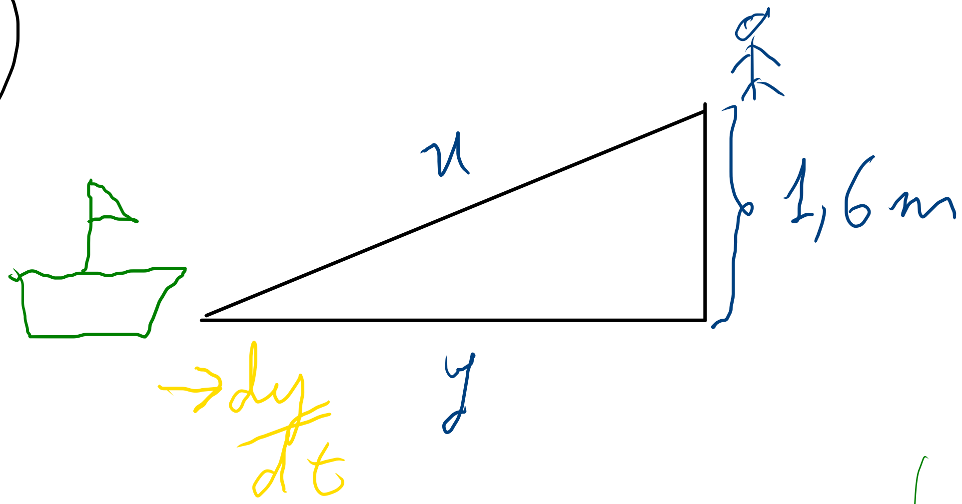

$$\frac{dx}{dt} = 5\,m/min$$

$$x^2 = 1{,}6^2 + y^2$$

$$2x\frac{dx}{dt} = 0 + 2y\frac{dy}{dt}$$

$$x^2 = y^2 + 1{,}6^2$$

$$x = 2$$

$$2^2 = y^2 + 1{,}6^2$$

$$4 = y^2 + 2{,}56$$

$$y^2 = 4 - 2{,}56 \Rightarrow y^2 = 1{,}44$$

$$y = \sqrt{1{,}44}$$

$$y = 1{,}2\,m$$

SUBSTITUINDO x, y e $\frac{dx}{dt}$, temos:

$$2 \cdot 2 \cdot 5 = 2 \cdot 1{,}2\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{20}{2{,}4} = \frac{10}{1{,}2} = \frac{100}{12}\,m/min$$

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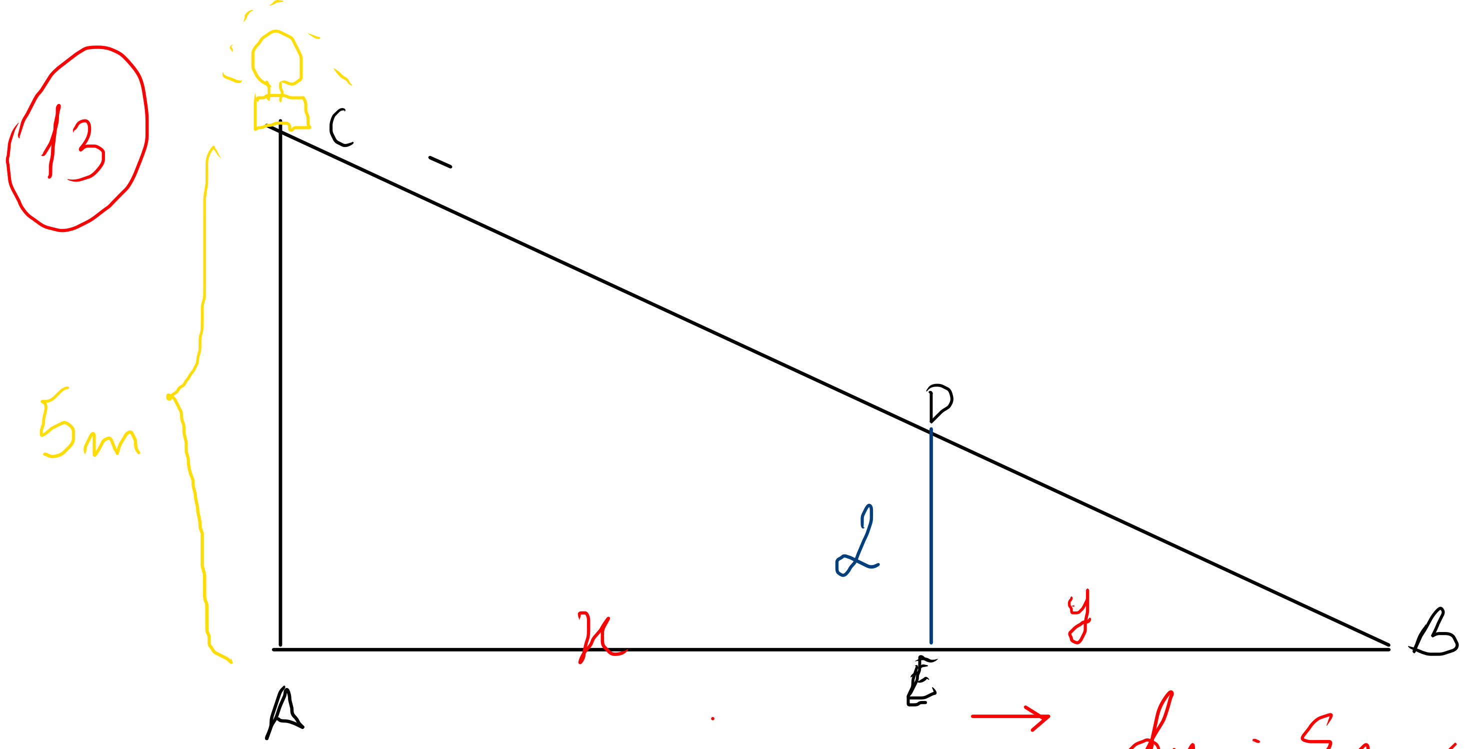


$$\frac{dx}{dt} = 5\,m/s$$

RELACIONANDO ΔABC com ΔBDE

$$\frac{5}{x+y} = \frac{2}{y}$$

$$5y = 2x + 2y$$

$$5y - 2y = 2x$$

$$3y = 2x \quad \text{(I)}$$

DERIVANDO (I) em relação a t:

$$3\frac{dy}{dt} = 2\frac{dx}{dt}$$

$$3\frac{dy}{dt} = 2 \cdot 5$$

$$\frac{dy}{dt} = \frac{10}{3}\,m/s$$

(14)

$$\frac{dx}{dt} + \frac{dy}{dt} = 5 + \frac{10}{3} = \frac{25}{3} \text{ m/s}$$

(19.)

$$f(x) = (x-a)(x-b)(x-c)$$

$$f(x) = (x^2 - bx - ax + ab)(x-c)$$

$$f(x) = x^3 - cx^2 - bx^2 + bcx - ax^2 + acx + abx - abc$$

$$f'(x) = 3x^2 - 2cx - 2bx + bc - 2ax + ac + ab$$

$$f''(x) = 6x - 2c - 2b - 2a$$

$$6x - 2c - 2b - 2a = 0$$

$6x = 2a + 2b + 2c$

$6x = 2(a + b + c)$

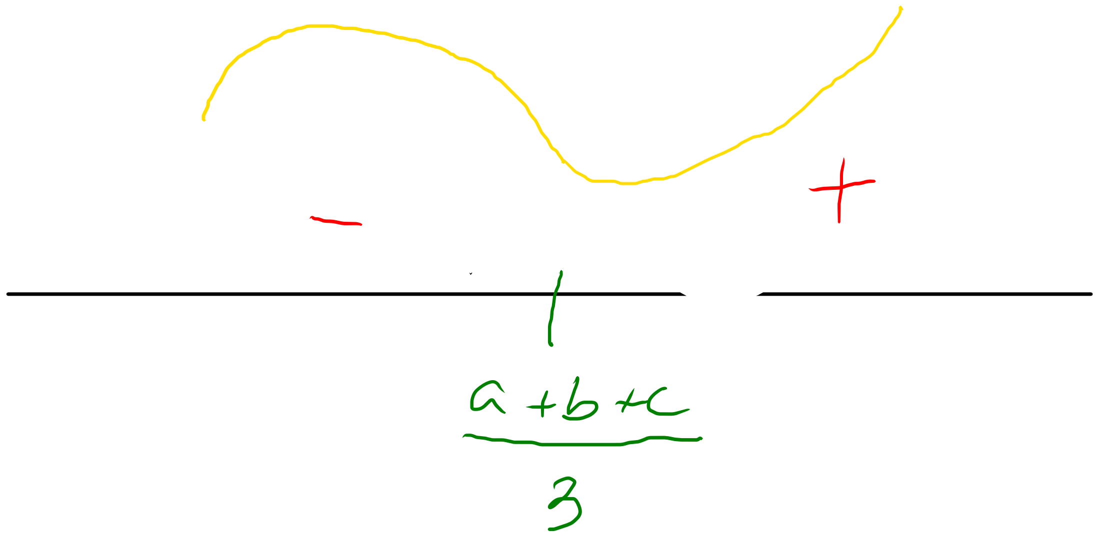
$x = \frac{2(a+b+c)}{6} = \frac{a+b+c}{3}$

$x - a$

$x = a$

PONTO DE INFLEXÃO

$d = \frac{a+b+c}{3}$ //



$f''(x) > 0$ $\cup$

$f''(x) < 0$ $\cap$

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$$f(x) = cx^2 + \boxed{\frac{1}{x^2}} \to x^{-2}$$

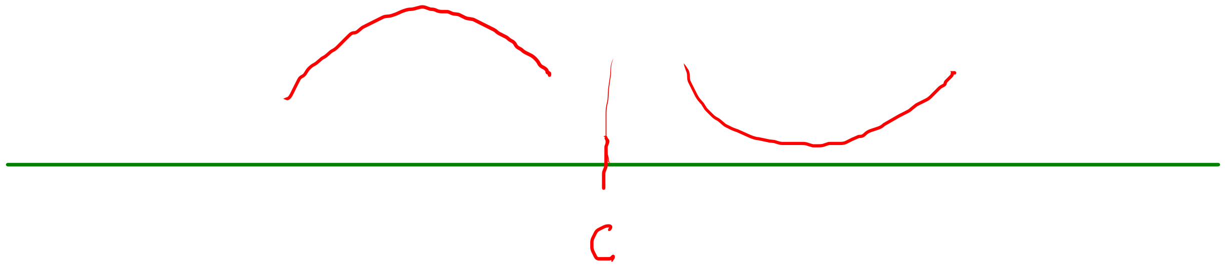
$$f'(x) = 2cx - \frac{2}{x^3}$$

$$f''(x) = 2c - 2(-3)\cdot x^{-4} = 2c + \frac{6}{x^4}$$

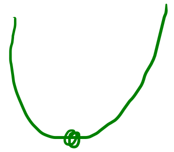
$$2c + \frac{6}{x^4} = 0 \Rightarrow \frac{6}{x^4} = -2c \Rightarrow x^4 = \frac{6}{-2c} \Rightarrow \overset{\text{positivo}}{x^4} = \underbrace{\frac{-3}{c}}_{c<0}$$

$$f''(1) = 2c + \frac{6}{1^4} = 2c + 6 \quad //$$

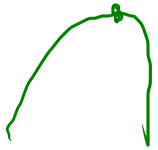
$c < 0$ $\qquad$ $f''(1) = 2c + 6$



$f''(c) > 0$



$f''(c) < 0$



$$2c + 6 > 0 \qquad 2c + 6 < 0$$
$$2c > -6 \qquad 2c < -6$$
$$c > -3 \qquad c < -3$$

PONTO DE INFLEXÃO EM $c = -3$ POIS É ONDE MUDA-SE A CONCAVIDADE