y)

Lista 11

1) a) $\int (1+x)\,dx = \int 1\,dx + \int x\,dx = x + \frac{x^2}{2} + C$

b) $\int (1-\sqrt{x})^2 dx = \int (1 - 2\sqrt{x} + x)\,dx = \int (1 - 2 \cdot x^{1/2} + x)\,dx =$

$= x - 2 \cdot \frac{x^{3/2}}{3/2} + \frac{x^2}{2} + C = x - \frac{4}{3} x^{3/2} + \frac{x^2}{2} + C$ //

c) $\int (2+x)^2 dx = \int (4 + 4x + x^2)\,dx = 4x + 4\frac{x^2}{2} + \frac{x^3}{3} + C =$

$= 4x + 2x^2 + \frac{x^3}{3} + C$

d) $\int x\sqrt{x}\,dx = \int (x \cdot x^{1/2})\,dx = \int x^{3/2}\,dx = \frac{x^{5/2}}{5/2} + C = \frac{2}{5} x^{5/2} + C$

e) $\int \frac{1-x^5}{1-x}\,dx = \int \frac{\cancel{-}(x^4+x^3+x^2+x+1)(x-1)}{\cancel{-}(-1+x)}\,dx = \int (x^4 + x^3 + x^2 + x + 1)\,dx =$

$= \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + C$ //

1	-1	0	0	0	0	1
	-1	-1	-1	-1	-1	0

$-x^4 - x^3 - x^2 - x - 1\,(x-1)$

$-(x^4 + x^3 + x^2 + x + 1)(x-1)$

f)

$$\frac{1}{\cos x} = \sec x \qquad (\tan x)' = \sec^2 x$$

$$\int \left(4\cos x - \frac{1}{\cos^2 x}\right) dx = \int 4\cos x \, dx - \int \sec^2 x \, dx =$$

$$= 4 \operatorname{sen} x - \tan x + c$$

g) $$\int \frac{3}{x^3} dx = \int 3x^{-3} dx = 3 \cdot \frac{x^{-2}}{-2} + c = \frac{-3}{2x^2} + C$$

h) $$\int \frac{1+x^2}{\sqrt{x}} dx = \int \frac{1}{\sqrt{x}} dx + \int \frac{x^2}{\sqrt{x}} dx =$$

$$= \int x^{-1/2} dx + \int x^{3/2} dx = \frac{x^{1/2}}{1/2} + \frac{x^{5/2}}{5/2} + c = 2\sqrt{x} + \frac{2}{5} x^{5/2} + C$$ //

i) $$\int \frac{4}{1+x} dx = 4 \int \frac{1}{1+x} dx = 4 \ln|1+x| + c$$

j) $$\int \frac{2x}{1+x^2} \underbrace{dx}_{du} = \int \frac{1}{u} du = \ln|u| = \ln|1+x^2| + c$$

$u = 1 + x^2$

$du = 2x \, dx$

k) $$\int \underbrace{\cos x \, dx}_{du} \cdot \operatorname{sen}^3 x = \int u^3 du = \frac{u^4}{4} = \frac{\operatorname{sen}^4 x}{4} + C$$ //

$u = \operatorname{sen} x$

$du = \cos x \, dx$

l) $\int \tan^5 x \sec^2 x \, dx = \int u^5 \, du = \frac{u^6}{6} = \frac{\tan^6 x}{6} + c$

$u = \tan x$

$du = \sec^2 x \, dx$

m) $\int \frac{e^x \operatorname{sen}(e^x)}{\cos(e^x)} \, dx = \int e^x . \tan(e^x) \, dx = \int \tan(u) \, du =$

$u = e^x$

$du = e^x \, dx$

$= \int \frac{\operatorname{sen} u}{\cos u} \, du = - \int \frac{1}{v} = \ln |v| =$

$= \ln |\cos u| = \ln |\cos e^x| + C$

$v = \cos u$

$dv = -\operatorname{sen} u \, du$

n) $\int \frac{x^2}{2+2x^3} \, dx = \int \frac{1}{6u} \, du = \frac{1}{6} \int \frac{1}{u} \, du = \frac{1}{6} \ln |u| =$

$u = 2 + 2x^3$

$= \frac{1}{6} \ln |2+2x^3| + C$

$du = 6x^2 \, dx$

$\frac{du}{6} = x^2 \, dx$

o) $\int \frac{2x + 1}{x^2 + x - 1} \, dx = \int \frac{1}{u} \, du = \ln |u| = \ln |x^2 + x - 1| + c$

$u = x^2 + x - 1$

$du = 2x + 1 \, dx$

p) $\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \int \sec^2 x \, dx - \int 1 \, dx =$

$= \tan x - x + c$

q) $\int -3\cos^5 x \, \operatorname{sen} x \, dx = 3 \int u^5 \, du = 3 \cdot \frac{u^6}{6} = \frac{\cos^6 x}{2} + C$ //

$u = \cos x$

$du = -\operatorname{sen} x \, dx$

r) $\int \frac{1+x}{1+x^2} dx = \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx = \arctan(x) +$
$\frac{1}{2} \ln|1+x^2| + C$ //

$* \int \frac{1}{1+x^2} dx = \arctan(x)$

$* \int \frac{x}{1+x^2} dx = \int \frac{1}{2u} du = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| =$

$= \frac{1}{2} \ln|1+x^2|$

$u = 1+x^2$

$du = 2x \, dx$

$\frac{du}{2} = x \, dx$

2)

a) $\int (1-3x)^5\,dx = \int \frac{u^5}{-3}\,du = -\frac{u^6}{3.6} = -\frac{(1-3x)^6}{18} + C$

$u = 1-3x$

$du = -3\,dx$

$\frac{du}{-3} = dx$

b) $\int \frac{x-3}{x^2-6x+4}\,dx = \int \frac{1}{2u}\,du = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| =$

$= \frac{1}{2}\ln|x^2-6x+4| + C$ //

$u = x^2-6x+4$

$du = (2x-6)\,dx$

$\frac{du}{2} = (x-3)\,dx$

c) $\int x\sqrt{x^2+1}\,dx = \int \frac{\sqrt{u}}{2}\,du = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} = \frac{1}{3}u^{3/2} =$

$= \frac{1}{3}(x^2+1)^{3/2} + C$ //

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

d) $\int \frac{e^{1/x}}{x^2}\,dx = -\int e^u\,du = -e^u = -e^{1/x} + C$ //

$u = \frac{1}{x}$

$du = -\frac{1}{x^2}\,dx$

$-du = \frac{1}{x^2}\,dx$

e) $\int e^{3x}\,dx = \int \frac{e^{u}}{3}\,du = \frac{1}{3}\,e^{u} = \frac{1}{3}\,e^{3x} + C$ //

$u = 3x$

$du = 3\,dx$

$\frac{du}{3} = dx$

$* \int a^{x}\,dx = \frac{a^{x}}{\ln(a)}$

$\int a^{x}\,dx = \int e^{\ln a^{x}} = \int e^{x\ln a} = \frac{e^{x\ln a}}{\ln a} = \frac{a^{x}}{\ln a}$ //

f) $\int 4^{2-3x}\,dx = \int \frac{4^{u}}{-3}\,du = -\frac{1}{3}\int 4^{u}\,du =$

$u = 2-3x$

$du = -3\,dx$

$-\frac{du}{3} = dx$

$= -\frac{1}{3}\cdot\frac{4^{u}}{\ln 4} = -\frac{1}{3}\cdot\frac{4^{2-3x}}{\ln 4} =$

$= -\frac{1}{3}\cdot\frac{4^{2-3x}}{\ln 2^{2}} = -\frac{1}{3}\,\frac{(2^{2})^{2-3x}}{2\ln 2} =$

$= -\frac{1}{3}\,\frac{2^{4-6x}}{2\ln 2} = -\frac{1}{3}\,\frac{2^{-6x+3}}{\ln 2} = -\frac{2^{-6x+3}}{3\ln 2} + C$ //

g) $\int \frac{x^{2}}{\sqrt{x^{3}+2}}\,dx = \int \frac{1}{3\sqrt{u}}\,du = \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du =$

$= \frac{1}{3}\int u^{-1/2}\,du = \frac{1}{3}\,\frac{u^{1/2}}{1/2} = \frac{2}{3}\sqrt{x^{3}+2} + C$ //

$u = x^{3}+2$

$du = 3x^{2}$

$\frac{du}{3} = x^{2}$

$\cancel{* \int \frac{1}{\sqrt{a^{2}-x^{2}}}\,dx = \arcsin\left(\frac{x}{a}\right)}$

h) $\int \frac{1}{\sqrt{1-4x^{2}}}\,dx = \int \frac{1}{\sqrt{1-(2x)^{2}}}\,dx = \int \frac{1}{\sqrt{1-u^{2}}}\,\frac{du}{2} = \frac{1}{2}\arcsin u =$

$= \frac{1}{2}\arcsin(2x) + C$ //

$u = 2x$

$du = 2\,dx$

i) $\int \frac{x^2}{1+x^6}\,dx = \int \frac{x^2}{1+(x^3)^2}\,dx = \int \frac{1}{1+u^2}\,\frac{du}{3} = \frac{1}{3}\arctan u =$

$u = x^3$

$du = 3x^2\,dx$

$= \frac{1}{3}\arctan(x^3) + C$ //

j)

$$\int \frac{dx}{9-4x^2} = \int \frac{dx}{3^2-(2x)^2} = \int \frac{3/2\, du}{3^2-(3u)^2} = \frac{3}{2}\cdot\frac{1}{9}\int \frac{du}{1-u^2} =$$

$$\left.\begin{aligned} 2x &= 3u \\ x &= \frac{3u}{2} \\ dx &= \frac{3}{2}\,du \end{aligned}\right\} \quad \left.\begin{aligned} u &= \operatorname{sen} t \\ du &= \cos t\, dt \end{aligned}\right\} = \frac{1}{6}\int \frac{\cos t\, dt}{1-\operatorname{sen}^2 t} = \frac{1}{6}\int \frac{\cos t\, dt}{\cos^2 t} =$$

$$= \frac{1}{6}\int \frac{1}{\cos t} = \frac{1}{6}\int \sec t\, dt =$$

$$= \frac{1}{6}\ln\left|\sec t + \tan t\right| + C = \frac{1}{6}\ln\left|\frac{1}{\cos t} + \frac{\operatorname{sen} t}{\cos t}\right| + C =$$

$$= \frac{1}{6}\ln\left|\frac{1+\operatorname{sen} t}{\cos t}\right| + C = \frac{1}{6}\ln\left|\frac{1+2x/3}{\sqrt{1-(2x/3)^2}}\right| + C //$$

$*$ $\cos^2 t = 1 - \operatorname{sen}^2 t$

$\cos t = \sqrt{1-\operatorname{sen}^2 t}$

(*) $$\int \sec t\, dt = \int \frac{\sec t \cdot (\sec t + \tan t)}{\sec t + \tan t}\, dt =$$

$$\left.\begin{aligned} u &= \sec t + \tan t \\ du &= (\sec t \cdot \tan t + \sec^2 t)\, dt \end{aligned}\right\} = \int \frac{\sec^2 t + \sec t \cdot \tan t}{\sec t + \tan t}\, dt =$$

$$= \int \frac{1}{u}\, du = \ln|u| = \ln\left|\sec t + \tan t\right| + C //$$

k) $\int \sqrt[3]{(x^2+x)^2}\,(2x+1)\,dx = \int \sqrt[3]{u^2}\,du = \int u^{2/3}\,du = \dfrac{u^{5/3}}{5/3} =$

$u = x^2 + x$

$du = (2x+1)\,dx$

$= \dfrac{3}{5}\,(x^2+x)^{5/3} + c$ //

l) $\int (\ln x)^{-2}\,\dfrac{dx}{x} = \int \dfrac{1}{x \ln^2 x}\,dx = \int \dfrac{1}{u^2}\,du = \int u^{-2}\,du =$

$u = \ln x$

$du = \dfrac{1}{x}\,dx$

$= \dfrac{u^{-1}}{-1} = -\dfrac{1}{\ln x} + c$ //

m) $\int \dfrac{1}{x(\ln(2x))^3}\,dx = \int \dfrac{1}{x \cdot \ln^3(2x)}\,dx = \int \dfrac{1}{\frac{u}{\cancel{2}} \cdot \ln^3(u)}\,\dfrac{du}{2} =$

$u = 2x$

$du = 2\,dx$

$= \int \dfrac{1}{u \cdot \ln^3 u}\,du = \int \dfrac{1}{v^3}\,dv = \int v^{-3}\,dv = \dfrac{v^{-2}}{-2} =$

$v = \ln(u)$

$dv = \dfrac{1}{u}\,du$

$= \dfrac{\ln(2x)^{-2}}{-2} = -\dfrac{1}{\ln^2(2x)} + c$ //

n) $\int \cos^3(2x)\,\sin(2x)\,dx = \int u^3 \cdot -\frac{du}{2} = \frac{-1}{2}\int u^3\,du = \frac{-1}{2}\,\frac{u^4}{4} =$

$u = \cos(2x)$ $\qquad = \frac{-1}{2}\,\frac{\cos^4(2x)}{4} = -\frac{1}{8}\cos^4(2x) + C$ //

$du = -\sin(2x) \cdot 2\,dx$

o) $\int \frac{\ln(x+2)}{x+2}\,dx = \int \frac{\ln u}{u}\,du = \int v\,dv = \frac{v^2}{2} = \frac{\ln^2(x+2)}{2} + C$ //

$u = x+2$ $\quad v = \ln u$

$du = dx$ $\quad dv = \frac{1}{u}\,du$

p) $\int (\tan(3x)\sec(3x))^2\,dx = \int \tan^2 u \cdot \sec^2 u\,\frac{du}{3} = \frac{1}{3}\int v^2\,dv =$

$u = 3x$ $\quad v = \tan u$ $\quad = \frac{1}{3}\,\frac{v^3}{3} = \frac{1}{9}\tan^3(3x) + C$ //

$du = 3\,dx$ $\quad dv = \sec^2 u\,du$

q) $\int \frac{\ln(\ln x)}{x\ln x}\,dx = \int \frac{\ln u}{u}\,du = \int v\,dv = \frac{v^2}{2} = \frac{\ln^2(\ln x)}{2} + C$ //

$u = \ln x$ $\quad v = \ln u$

$du = \frac{1}{x}\,dx$ $\quad dv = \frac{1}{u}\,du$

r) $\int \frac{e^{\sqrt{x}} - 3}{\sqrt{x}}\,dx = \int (e^u - 3)\,2\,du = 2\left[\int e^u\,du - \int 3\,du\right] =$

$u = \sqrt{x}$ $\quad = 2e^u - 2 \cdot 3u = 2e^{\sqrt{x}} - 6\sqrt{x} + C$ //

$du = \frac{1}{2\sqrt{x}}\,dx$

s) $\int \frac{1}{4+x^2}\,dx = \int \frac{1}{4+4u^2}\,2du = \frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\arctan(u) =$

$x = 2u$ $\quad = \frac{1}{2}\arctan(x/2) + C$ //

$dx = 2\,du$

$\text{r)} \int \frac{1}{x^2+2x+5}\,dx =$

$* \; x^2+2x+5 = (x^2+2x+1)+5-1 = (x+1)^2+4$

$$\int \frac{1}{x^2+2x+5}\,dx = \int \frac{1}{(x+1)^2+4}\,dx = \int \frac{1}{u^2+4}\,du = \int \frac{1}{4v^2+4}\,2dv =$$

$u = x+1 \qquad u = 2v$

$du = dx \qquad du = 2dv$

$$= \frac{1}{2}\int \frac{1}{v^2+1}\,dv = \frac{1}{2}\arctan(v) =$$

$$= \frac{1}{2}\arctan\left(\frac{x+1}{2}\right) + c \;//$$

$\text{u)} \int \frac{5}{x^2-8x+25}\,dx =$

$* \; x^2-8x+25 = x^2-8x+16+25-16 = (x-4)^2+9$

$$5\int \frac{1}{(x-4)^2+9}\,dx = 5\int \frac{1}{u^2+9}\,du = 5\int \frac{1}{9v^2+9}\,3dv = \frac{15}{9}\int \frac{1}{v^2+1}\,dv =$$

$u = x-4 \qquad u = 3v$

$du = dx \qquad du = 3dv$

$$= \frac{5}{3}\arctan(v) = \frac{5}{3}\arctan\left(\frac{x-4}{3}\right) + C \;//$$

v) $\displaystyle\int \frac{1}{16-x^2}\,dx = \int \frac{1}{4^2-(4u)^2}\,4\,du = \frac{4}{16}\int \frac{1}{1-u^2}\,du =$ lo mesmo calculo da (δ)

$4u = x$ | $u = \operatorname{sen} t$
$4\,du = dx$ | $du = \cos t\,dt$

$\displaystyle = \frac{1}{4}\ln\left|\frac{1+\operatorname{sen} t}{\cos t}\right| + C =$

$\displaystyle = \frac{1}{4}\ln\left|\frac{1+x/4}{\sqrt{1-(x/4)^2}}\right| + C$ //

$\cos t = \sqrt{1-\operatorname{sen}^2 t} = \sqrt{1-(x/4)^2}$

w) $\displaystyle\int \frac{1}{x^2-9}\,dx = \int \frac{1}{(3u)^2-3^2}\,3\,du = \frac{1}{3}\int \frac{1}{u^2-1}\,du =$

$x = 3u$ $u = \sec t$
$dx = 3\,du$ $du = \sec t . \operatorname{tg} t\,dt$

$\displaystyle \frac{1}{3}\int \frac{\sec t . \operatorname{tg} t\,dt}{\sec^2 t - 1} =$

$\displaystyle = \frac{1}{3}\int \frac{\sec t . \operatorname{tg} t}{\operatorname{tg}^2 t}\,dt = \frac{1}{3}\int \frac{\sec t}{\operatorname{tg} t}\,dt = \frac{1}{3}\int \frac{1/\cos t}{\operatorname{sen} t/\cos t}\,dt =$

$\displaystyle = \frac{1}{3}\int \frac{1}{\operatorname{sen} t}\,dt = \frac{1}{3}\int \csc t\,dt = -\frac{1}{3}\ln|\csc t + \cot t| + C =$

$\displaystyle = -\frac{1}{3}\ln\left|\frac{1}{\operatorname{sen} t} + \frac{\cos t}{\operatorname{sen} t}\right| + C = -\frac{1}{3}\ln\left|\frac{1+\cos t}{\operatorname{sen} t}\right| + C =$

$\displaystyle = -\frac{1}{3}\ln\left|\frac{1+3/x}{\sqrt{1-(3/x)^2}}\right| + C$ //

$\displaystyle * \; u = \frac{1}{\cos t} \Rightarrow \cos t = \frac{1}{u} = \frac{1}{x/3} = \frac{3}{x}$

$\displaystyle * \int \csc t\,dt = \int \frac{\csc t(\csc t + \cot t)}{\csc t + \cot t}\,dt = \int \frac{\csc^2 t + \cot t \csc t}{\csc t + \cot t} =$

$u = \csc t + \cot t$
$du = -\csc t . \cot t - \csc^2 t$

$\displaystyle = -\int \frac{1}{u}\,du = -\ln|\csc t + \cot t| + C$

j) $\int \frac{2x-3}{x^2-6x+10}\,dx =$

$* x^2-6x+10 = x^2-6x+9+10-9 = (x-3)^2+1$

$\int \frac{2x-3}{(x-3)^2+1}\,dx = \int \frac{2(u+3)-3}{u^2+1}\,du = \int \frac{2u+3}{u^2+1}\,du =$

$u = x-3$

$du = dx$

$= \int \frac{2u}{u^2+1}\,du + \int \frac{3}{u^2+1}\,du = \ln|u^2+1| + 3\arctan(u) =$

$= \ln|(x-3)^2+1| + 3\arctan(x-3) =$

$= \ln|x^2-6x+10| + 3\arctan(x-3) + C$ //

$\int \frac{2u}{u^2+1}\,du = \int \frac{1}{v}\,dv =$

$= \ln|v| =$

$= \ln|u^2+1|$

$v = u^2+1$

$dv = 2u\,du$

$\int \frac{3}{u^2+1}\,du = 3\int \frac{1}{u^2+1}\,du =$

$= 3\arctan(u)$

g) $\displaystyle\int \frac{2x+1}{4x^2+12x+13}\,dx =$

$* \; 4x^2+12x+13 = 4\left(x^2+3x+13/4\right) = 4\left(x^2+3x+\frac{9}{4}+\frac{13}{4}-\frac{9}{4}\right) =$

$= 4\left[\left(x+3/2\right)^2+1\right]$

$$\frac{1}{4}\int \frac{2x+1}{(x+3/2)^2+1}\,dx = \frac{1}{4}\int \frac{2(u-3/2)+1}{u^2+1}\,dx = \frac{1}{4}\int \frac{2u-2}{u^2+1}\,du =$$

$u = x+3/2$

$du = dx$

$$= \frac{1}{4}\left[\int \frac{2u}{u^2+1}\,du - \int \frac{2}{u^2+1}\,du\right] =$$

↳ item g

$$= \frac{1}{4}\left[\ln|u^2+1| - 2\arctan u\right] =$$

$$= \frac{1}{4}\ln\left|(x+3/2)^2+1\right| - \frac{\arctan(x+3/2)}{2} + C$$ //

3) $\int u\,dv = uv - \int v\,du$

a) $\int e^{\sqrt{x}}\,dx = \int e^{t}.\,2.t\,dt = 2\int e^{t}.t\,dt =$

$t = \sqrt{x}$ $\qquad$ $u = t$ $\qquad$ $dv = e^{t}dt$

$dt = \frac{1}{2\sqrt{x}}dx$ $\qquad$ $du = dt$ $\qquad$ $v = e^{t}$

$dt.2\sqrt{x} = dx$

$\Rightarrow = 2\left(t.e^{t} - \int e^{t}dt\right) = 2\left(t.e^{t} - e^{t}\right) =$

$= 2\left(\sqrt{x}.e^{\sqrt{x}} - e^{\sqrt{x}}\right) + C$ //

b) $\int \frac{x}{e^{x}}dx = x.-e^{-x} - \int -e^{-x}dx = -xe^{-x} + \int e^{-x}dx =$

$= -xe^{-x} - e^{-x} = \frac{-x}{e^{x}} - \frac{1}{e^{x}} + C$ //

$u = x$ $\qquad$ $dv = \frac{1}{e^{x}}$

$du = dx$ $\qquad$ $v = -e^{-x}$

c) $\int x.2^{-x}dx = \frac{-x2^{-x}}{\ln 2} - \int \frac{-2^{-x}}{\ln 2}dx = \frac{-x2^{-x}}{\ln 2} + \frac{1}{\ln 2}\int 2^{-x}dx =$

$u = x$ $\qquad$ $dv = 2^{-x}dx$ $\qquad$ $= \frac{-x2^{-x}}{\ln 2} + \frac{1}{\ln 2}.\left(\frac{-2^{-x}}{\ln 2}\right) =$

$du = dx$ $\qquad$ $v = -\frac{2^{-x}}{\ln 2}$

$\Rightarrow \int a^{x}dx = \frac{a^{x}}{\ln a}$

$\int (\tfrac{1}{2})^{x}dx = \frac{(1/2)^{x}}{\ln(1/2)} = -\frac{2^{-x}}{\ln 2}$ $\qquad$ $= -\frac{x.2^{-x}}{\ln 2} - \frac{2^{-x}}{\ln^{2}(2)} + C$ //

ℓi^{x}

$\frac{\left(\frac{1}{2}\right)^{x}}{\ln(1/2)} = $ $\qquad$ $\ln(1/2) = \frac{\ln 1}{\ln 2}$

$\ln 1 - \ln 2$

d) $\int x^2 3^x\,dx = \frac{x^2 \cdot 3^x}{\ln 3} - \int \frac{3^x\, 2x}{\ln 3}\,dx =$

$u = x^2 \qquad dv = 3^x\,dx \qquad = \frac{x^2\, 3^x}{\ln 3} - \frac{2}{\ln 3}\int 3^x \cdot x\,dx \overset{(*)}{=}$

$du = 2x\,dx \qquad v = \frac{3^x}{\ln 3}$

$= \frac{x^2\, 3^x}{\ln 3} - \frac{2}{\ln 3}\left(\frac{x\, 3^x}{\ln 3} - \frac{3^x}{\ln^2 3}\right) = \frac{x^2\, 3^x}{\ln 3} - \frac{2x3^x}{\ln^2 3} + \frac{2 \cdot 3^x}{\ln^3 3} + C$ //

(*) $\int 3^x \cdot x\,dx = \frac{x \cdot 3^x}{\ln 3} - \int \frac{3^x}{\ln 3}\,dx = \frac{x\,3^x}{\ln 3} - \frac{1}{\ln 3}\int 3^x\,dx =$

$x = u \qquad dv = 3^x\,dx \qquad = \frac{x\,3^x}{\ln 3} - \frac{3^x}{\ln^2 3} + C$ //

$dx = du \qquad v = \frac{3^x}{\ln 3}$

e) $\int \frac{x^2}{e^{3x}}\,dx =$

$u = x^2 \qquad dv = e^{-3x}\,dx \longrightarrow \int e^{-3x}\,dx = \frac{-1}{3}\int e^u = \frac{1}{3}e^u = -\frac{1}{3}e^{-3x}$ //

$du = 2x\,dx \qquad v = \frac{-1}{3}e^{-3x} \qquad u = -3x$

$du = -3\,dx$

$= -\frac{x^2 \cdot e^{-3x}}{3} - \int \frac{-1}{3}e^{-3x} \cdot 2x\,dx = -\frac{x^2 \cdot e^{-3x}}{3} + \frac{1}{3}\int e^{-3x} \cdot 2x\,dx \overset{(*)}{=}$

$= -\frac{x^2\, e^{-3x}}{3} + \frac{1}{3}\left(-\frac{2x\,e^{-3x}}{3} - \frac{2}{9}e^{-3x}\right) + C$ //

(*) $\int e^{-3x} \cdot 2x\,dx \quad = -\frac{2x \cdot e^{-3x}}{3} - \int \frac{-1}{3}e^{-3x} \cdot 2\,dx =$

$u = 2x \qquad dv = e^{-3x}\,dx \qquad = -\frac{2x \cdot e^{-3x}}{3} + \frac{2}{3}\int e^{-3x}\,dx =$

$du = 2\,dx \qquad v = -\frac{1}{3}e^{-3x}$

$= -\frac{2x\,e^{-3x}}{3} + \frac{2}{3} \cdot -\frac{1}{3}e^{-3x}$

$= -\frac{2x\,e^{-3x}}{3} - \frac{2}{9}e^{-3x} + C$ //

f) $\int \arctan x \, dx = x \arctan x - \int \frac{x}{1+x^2} dx \overset{(*)}{=}$

$u = \arctan x \qquad dv = dx$

$du = \frac{1}{1+x^2} dx \qquad v = x$

$= x \arctan x - \frac{1}{2} \ln|x^2+1| + C$ //

(*) $\int \frac{x}{1+x^2} dx = \int \frac{1}{2u} du = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| = \frac{1}{2} \ln|1+x^2| + C$

$u = 1 + x^2$
$du = 2x \, dx$

g) $\int \arcsen x \, dx = x \arcsen x - \int \frac{x}{\sqrt{1-x^2}} dx \overset{(*)}{=}$

$u = \arcsen x \qquad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}} dx \qquad v = x$

$= x \arcsin x + \sqrt{1-x^2} + C$ //

(*) $\int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{1}{-2\sqrt{u}} du = -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \frac{u^{1/2}}{1/2} =$

$= -\sqrt{u} = -\sqrt{1-x^2} + C$ //

$u = 1 - x^2$

$du = -2x \, dx$

$\frac{du}{-2} = x \, dx$

h) $\int 4x \ln(2x)\,dx = 4\int x \ln(x)\,dx = 4\left(\frac{x^2}{2}\ln 2x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx\right) =$

$u = \ln 2x \qquad dv = x\,dx$

$du = \frac{2}{2x} = \frac{1}{x}\,dx \qquad v = \frac{x^2}{2}$

$= 2x^2\ln(2x) - \frac{4}{2}\int x\,dx =$

$= 2x^2\ln(2x) - 2\frac{x^2}{2} =$

$= 2x^2\ln(2x) - x^2 + C$ //

i) $\int \sqrt{x}\ln x\,dx = \frac{2}{3}\ln x\, x^{3/2} - \int \frac{2}{3}x^{3/2}\frac{1}{x}\,dx =$

$u = \ln x \qquad dv = \sqrt{x}\,dx$

$du = \frac{1}{x}\,dx \qquad dv = x^{1/2}\,dx$

$v = \frac{2}{3}x^{3/2}$

$= \frac{2}{3}x^{3/2}\ln(x) - \frac{2}{3}\int x^{1/2}\,dx =$

$= \frac{2}{3}x^{3/2}\ln(x) - \frac{2}{3}\cdot\frac{2}{3}x^{3/2} =$

$= \frac{2}{3}x^{3/2}\ln(x) - \frac{4}{9}x^{3/2} + C$ //

j) $\int \cos^3 x\,dx = \int \cos^2 x\cdot\cos x\,dx = \operatorname{sen} x\cdot\cos^2 x - \int -2\cos x\,\operatorname{sen}^2 x\,dx = \ldots =$

$u = \cos^2 x \qquad dv = \cos x\,dx$

$du = 2\cdot\cos x\cdot(-\operatorname{sen} x)\,dx \qquad v = \operatorname{sen} x$

$du = -2\cos x\,\operatorname{sen} x\,dx$

$= \operatorname{sen} x\cos^2 x + 2\int \cos x\,\operatorname{sen}^2 x\,dx =$

$= \operatorname{sen} x\cos^2 x + \frac{2}{3}\operatorname{sen}^3 x + C$ //

(*) $\int \cos x\,.\,\operatorname{sen}^2 x\,dx = \int u^2\,du = \frac{u^3}{3} = \frac{\operatorname{sen}^3 x}{3} + C$

$u = \operatorname{sen} x$

$du = \cos x\,dx$

k) $\int \theta \cdot \cos(3\theta)\, d\theta = \int \frac{t}{3} \cdot \cos(t) \cdot \frac{dt}{3} = \int \frac{t \cdot \cos(t)}{9}\, dt =$

$t = 3\theta$
$dt = 3\, d\theta$

$u = t \qquad dv = \cos t\, dt$
$du = dt \qquad v = \operatorname{sen} t$

$= \frac{1}{9}\left(t \operatorname{sen} t - \int \operatorname{sen} t\, dt\right) =$

$= \frac{t}{9} \operatorname{sen} t + \frac{1}{9} \cos t =$

$= \frac{3\theta}{9} \operatorname{sen}(3\theta) + \frac{1}{9} \cos(3\theta) = \frac{\theta \operatorname{sen}(3\theta)}{3} + \frac{\cos 3\theta}{9} + C$ //

l) $\int x^5 \cdot \cos(x^3)\, dx = \int \frac{t \cdot \cos t}{3}\, dt = \frac{1}{3} \int t \cos t\, dt =$

$t = x^3$
$dt = 3x^2$

$dt.t = 3x^5$
$\frac{dt\, t}{3} = x^5$

$= \frac{1}{3}\left(t \operatorname{sen} t - \int \operatorname{sen} t\, dt\right) =$

$= \frac{1}{3} x^3 \operatorname{sen} x^3 - \frac{1}{3} \cos(x^3) + C$ //

m) $\int (t^2 + 5t) \cdot \cos(2t)\, dt = (t^2+5t) \cdot \frac{1}{2} \operatorname{sen} 2t - \int \frac{1}{2} (\operatorname{sen} 2t)(2t+5)\, dt =$

p. subs.

$u = t^2 + 5t \qquad dv = \cos 2t\, dt$
$du = 2t + 5\, dt \qquad v = \frac{1}{2} \operatorname{sen} 2t$

$= \frac{(t^2+5t) \operatorname{sen}(2t)}{2} + \frac{1}{4}(2t+5) \cos(2t) - \frac{1}{4} \operatorname{sen} 2t + C$ //

(*) $\int (\operatorname{sen} 2t)(2t+5)\, dt = (2t+5)\left(-\frac{\cos 2t}{2}\right) - \int -\frac{1}{\cancel{2}} \cos 2t \cdot \cancel{2} dt =$

p. subs

$u = 2t + 5 \qquad dv = \operatorname{sen} 2t$
$du = 2\, dt \qquad v = -\frac{1}{2} \cos 2t$

$= \frac{-(2t+5) \cos(2t)}{2} + \frac{1}{2} \operatorname{sen} 2t + C$ //

n) $\int \sec^3\theta \, d\theta =$

$\tan^2 x + 1 = \sec^2 x$
$\tan^2 x = \sec^2 x - 1$

$u = \sec\theta$ $\quad dv = \sec^2\theta \, d\theta$

$du = \sec\theta \tan\theta \, d\theta$ $\quad v = \tan\theta$

$\int \sec^3\theta \, d\theta = \int \sec^2\theta . \sec\theta \, d\theta = \sec\theta \tan\theta - \int \sec\theta . \tan^2\theta \, d\theta =$

$\int \sec^3\theta \, d\theta = \sec\theta \tan\theta - \int \sec^3\theta \, d\theta + \ln|\tan\theta + \sec\theta|$

$2\int \sec^3\theta \, d\theta = \sec\theta \tan\theta + \ln|\tan\theta + \sec\theta| + C$ //

$\int \sec^3\theta \, d\theta = \frac{\sec\theta \tan\theta}{2} + \frac{\ln|\tan\theta + \sec\theta|}{2}$

(*) $\int \sec\theta . \tan^2\theta \, d\theta = \int \sec\theta . (\sec^2\theta - 1) \, d\theta =$

$= \int (\sec^3\theta - \sec\theta) \, d\theta = \int \sec^3\theta \, d\theta - \int \sec\theta \, d\theta =$

$\hookrightarrow \int \sec x \, dx = \ln|\tan x + \sec x|$

$= \int \sec^3\theta \, d\theta - \ln|\tan\theta + \sec\theta|$ //

o) $\int e^x . \text{sen}(x) \, dx = -e^x \cos x - \int -\cos x . e^x \, dx =$

$= -e^x \cos x + \int \cos x . e^x \, dx =$

$u = e^x$ $\quad dv = \text{sen}\, x \, dx$

$du = e^x dx$ $\quad v = -\cos x$

$\int e^x . \text{sen}\, x \, dx = -e^x \cos x + \text{sen}\, x . e^x - \int \text{sen}\, x \, e^x dx$

$\int e^x \text{sen}\, x \, dx = -\frac{e^x \cos x}{2} + \frac{e^x \text{sen}\, x}{2} + C$

(*) $\int \cos x . e^x \, dx = \text{sen}\, x . e^x - \int \text{sen}\, x . e^x \, dx$

$u = e^x$ $\quad dv = \cos x \, dx$

$du = e^x dx$ $\quad v = \text{sen}\, x$

q) $\int x \cdot \operatorname{sen} x \cdot \cos x \, dx =$ $\quad \operatorname{sen}(2x) = 2 \operatorname{sen} x \cos x$

$= \int x \cdot \frac{\operatorname{sen}(2x)}{2} dx = \frac{1}{2} \int x \operatorname{sen}(2x) \, dx =$

$u = x \qquad dv = \operatorname{sen}(2x) \, dx$

$du = dx \qquad v = -\frac{1}{2}\cos(2x)$

$\xrightarrow{\text{subs}} \frac{1}{2}\left(-\frac{1}{2} x \cos 2x - \int -\frac{1}{2}\cos(2x)\, dx\right) =$

$= -\frac{1}{4} x \cos(2x) + \frac{1}{2}\frac{1}{2}\int \cos 2x \, dx =$

$= -\frac{1}{4} x \cos(2x) + \frac{1}{4} \cdot \frac{1}{2} \operatorname{sen} 2x =$

$= -\frac{1}{4} x \cos(2x) + \frac{1}{8} \operatorname{sen} 2x + C$ //

r) $\int x^2 \ln(x) \, dx = \frac{x^3}{3} \ln(x) - \int \frac{x^3}{3} \cdot \frac{1}{x} dx =$

$u = \ln(x) \qquad dv = x^2 \, dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^3}{3}$

$= \frac{x^3}{3} \ln(x) - \frac{1}{3} \int x^2 \, dx =$

$= \frac{x^3}{3} \ln(x) - \frac{1}{3} \frac{x^3}{3} =$

$= \frac{x^3}{3} \ln(x) - \frac{x^3}{9} + C$ //

$\frac{1}{\sqrt{x}} \cdot x^{1/2}$

s) $\int \frac{\ln(x)}{\sqrt{x}} dx = 2\sqrt{x} \ln(x) - \int \frac{2\sqrt{x}}{x} dx =$

$u = \ln(x) \qquad dv = \frac{1}{\sqrt{x}} dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^{1/2}}{1/2} = 2\sqrt{x}$

$= 2\sqrt{x} \ln(x) - 4\sqrt{x} + C$ //

$* \int \frac{2\sqrt{x}}{x} dx = \int 2 \frac{x^{1/2}}{x} dx = 2\int x^{-1/2} dx \quad 2 \cdot \frac{x^{1/2}}{1/2} = 4\sqrt{x} + C$

I) $\int e^{at} \cdot \cos(bt)\,dt = \frac{e^{at}}{b} \operatorname{sen}(bt) - \int \frac{e^{at} \cdot a}{b} \operatorname{sen}(bt)\,dt =$

$$\left.\begin{array}{ll} u = e^{at} & dv = \cos bt\,dt \\ du = e^{at} \cdot a\,dt & v = \frac{1}{b} \operatorname{sen}(bt) \end{array}\right\} =$$

$$\int e^{at} \cdot \cos(bt)\,dt = \frac{e^{at}}{b} \operatorname{sen}(bt) - \frac{a}{b} \int e^{at} \operatorname{sen}(bt)\,dt$$

$$\int e^{at} \cdot \cos(bt)\,dt = \frac{e^{at}}{b} \operatorname{sen}(bt) - \frac{a}{b}\left(-\frac{1}{b} e^{at} \cos(bt) + \frac{a}{b} \int e^{at} \cdot \cos(bt)\,dt\right)$$

$$\int e^{at} \cdot \cos(bt)\,dt = \frac{e^{at}}{b} \cdot \operatorname{sen}(bt) + \frac{a}{b^2} e^{at} \cos(bt) - \underbrace{\frac{a^2}{b^2} \int e^{at} \cos(bt)\,dt}$$

$$(*) \int e^{at} \operatorname{sen}(bt)\,dt = -\frac{1}{b} e^{at} \cos(bt) - \int -\frac{a}{b} \cdot e^{at} \cdot \cos(bt)\,dt =$$

$$\left.\begin{array}{ll} u = e^{at} & dv = \operatorname{sen}(bt)\,dt \\ du = a \cdot e^{at}\,dt & v = -\frac{1}{b} \cos(bt) \end{array}\right\} = -\frac{1}{b} e^{at} \cos(bt) + \frac{a}{b} \int e^{at} \cdot \cos(bt)\,dt \;//$$

$$\int e^{at} \cos(bt)\,dt \left(1 + \frac{a^2}{b^2}\right) = \frac{e^{at}}{b} \operatorname{sen}(bt) + \frac{a}{b^2} e^{at} \cos(bt) \quad (\times\, b^2)$$

$$1 + \frac{a^2}{b^2} \to \frac{b^2 + a^2}{b^2}$$

$$\int e^{at} \cos(bt)\,dt\,(b^2 + a^2) = b\,e^{at} \operatorname{sen}(bt) + a\,e^{at} \cos(bt)$$

$$\int e^{at} \cos(bt)\,dt = \frac{b e^{at} \cdot \operatorname{sen}(bt)}{a^2 + b^2} + \frac{a e^{at} \cdot \cos(bt)}{a^2 + b^2} + C \;//$$

u) $\int (x^2-2x+5)\, e^{-x}\, dx = -e^{-x}(x^2-2x+5) - \int -e^{-x}\cdot(2x-2)\, dx =$

$u = x^2-2x+5 \qquad dv = e^{-x}$ ∂ subs

$du = 2x-2\, dx \qquad v = -e^{-x}$

$= -e^{-x}(x^2-2x+5) - 2x e^{-x} =$

$= -e^{-x}\cdot x^2 + 2x e^{-x} - 5e^{-x} - 2x e^{-x} =$

$= -e^{-x}\cdot x^2 - 5e^{-x} + C$ //

(*) $\int e^{-x}(2x-2)\, dx = -(2x-2)\cdot e^{-x} - \int -e^{-x}\cdot 2\, dx =$

$u = 2x-2 \qquad dv = e^{-x}$

$du = 2\, dx \qquad v = -e^{-x}$

$= -2x\cdot e^{-x} + 2e^{-x} + 2\int e^{-x}\, dx =$

$= -2x e^{-x} + 2e^{-x} + 2\cdot(-e^{-x}) =$

$= -2x e^{-x} + C$

v) $\int \ln^2(x)\, dx = x\ln^2 x - \int x\cdot \frac{2\ln x}{x}\, dx = x\ln^2 x - 2\int \ln x\, dx =$

$u = \ln^2 x \qquad dv = dx$

$du = 2\cdot \ln x \cdot \frac{1}{x} \qquad v = x$

$du = \frac{2\ln x}{x}\, dx$

$= x\ln^2 x - 2(x\ln x - x) + C$ //

(*) $\int \ln x\, dx = x\ln x - \int \frac{x}{x}\, dx = x\ln x - x + C$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\, dx \qquad v = x$

w) $\int \ln(x\sqrt{1+x^2})\, dx = \int \left(\ln(x) + \frac{1}{2}\ln(x^2+1)\right) dx =$

$= \int \ln(x)\,dx + \frac{1}{2}\int \ln(x^2+1)\, dx =$

$= x\ln x - x + \frac{1}{2} x\ln(x^2+1) - \frac{1}{\cancel{2}}2x + \frac{1}{2}\,2\arctan(x) =$

$= x\ln x - 2x + \frac{x\ln(x^2+1)}{2} + \arctan(x) + C$ //

$* \int \ln(x)\, dx = x\ln x - x + C$ (item v)

$* \int \ln(x^2+1)\, dx = x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}\, dx =$

$= x\ln(x^2+1) - 2x + 2\arctan(x) + C$

$u = \ln(x^2+1)$ $\quad dv = dx$

$du = \frac{2x}{x^2+1}\, dx$ $\quad v = x$

reescrever como:

$* \int \frac{x^2}{x^2+1}\, dx = \int 1 - \frac{1}{x^2+1}\, dx = \int 1\, dx - \int \frac{1}{x^2+1}\, dx =$

$= x - \arctan(x) + C$ //

x) $\int sen(\ln(x))\,dx = x\,sen(\ln(x)) - \int \cos(\ln(x))\,dx =$

$u = sen(\ln(x))$ $\quad dv = dx$

$du = \cos(\ln(x)) \cdot \frac{1}{x}\,dx$ $\quad v = x$

$\int sen(\ln(x))\,dx = x\,sen(\ln(x)) - x\cos(\ln(x)) - \int sen(\ln(x))\,dx$

$\int sen(\ln(x))\,dx = \frac{x\,sen(\ln(x))}{2} - \frac{x\cos(\ln(x))}{2} + C$ //

$* \int \cos(\ln(x))\,dx = x\cos(\ln(x)) - \int -sen(\ln(x))\,dx =$

$u = \cos(\ln(x))$ $\quad dv = dx$ $\quad = x\cos(\ln(x)) + \int sen(\ln(x))\,dx$ //

$du = -\frac{sen(\ln(x))}{x}\,dx$ $\quad v = x$

3) $\int y^3 \cdot e^{-y^2}\, dy = \int \frac{e^t \cdot t\, dt}{2} = \frac{1}{2}\left(e^t \cdot t - \int e^t\, dt\right) =$

$t = -y^2$

$dt = -2y\, dy$

$t \cdot dt = 2y^3\, dy$

$\frac{t\, dt}{2} = y^3\, dy$

$u = t \qquad dv = e^t\, dt$

$du = dt \qquad v = e^t$

$= \frac{1}{2} e^t \cdot t - \frac{1}{2} e^t =$

$= \frac{1}{2} e^{-y^2} \cdot (-y^2) - \frac{1}{2} e^{-y^2} =$

$= -\frac{1}{2} e^{-y^2} y^2 - \frac{1}{2} e^{-y^2} + C$ //