

1- Intro to deep learning and population dynamics

Data Science and AI for Neuroscience Summer School 2022, Chen Institute at Caltech

Chethan Pandarinath, Ph.D.

(he/him)

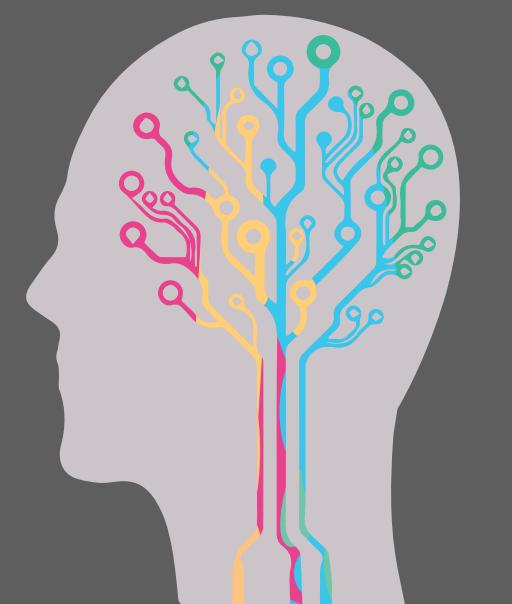
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Emory University & Georgia Tech

 @chethan



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EMORY
UNIVERSITY

Georgia
Tech



What you'll hear about in this lecture

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Neural network basics

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Deep autoencoders

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Intro to neural population dynamics

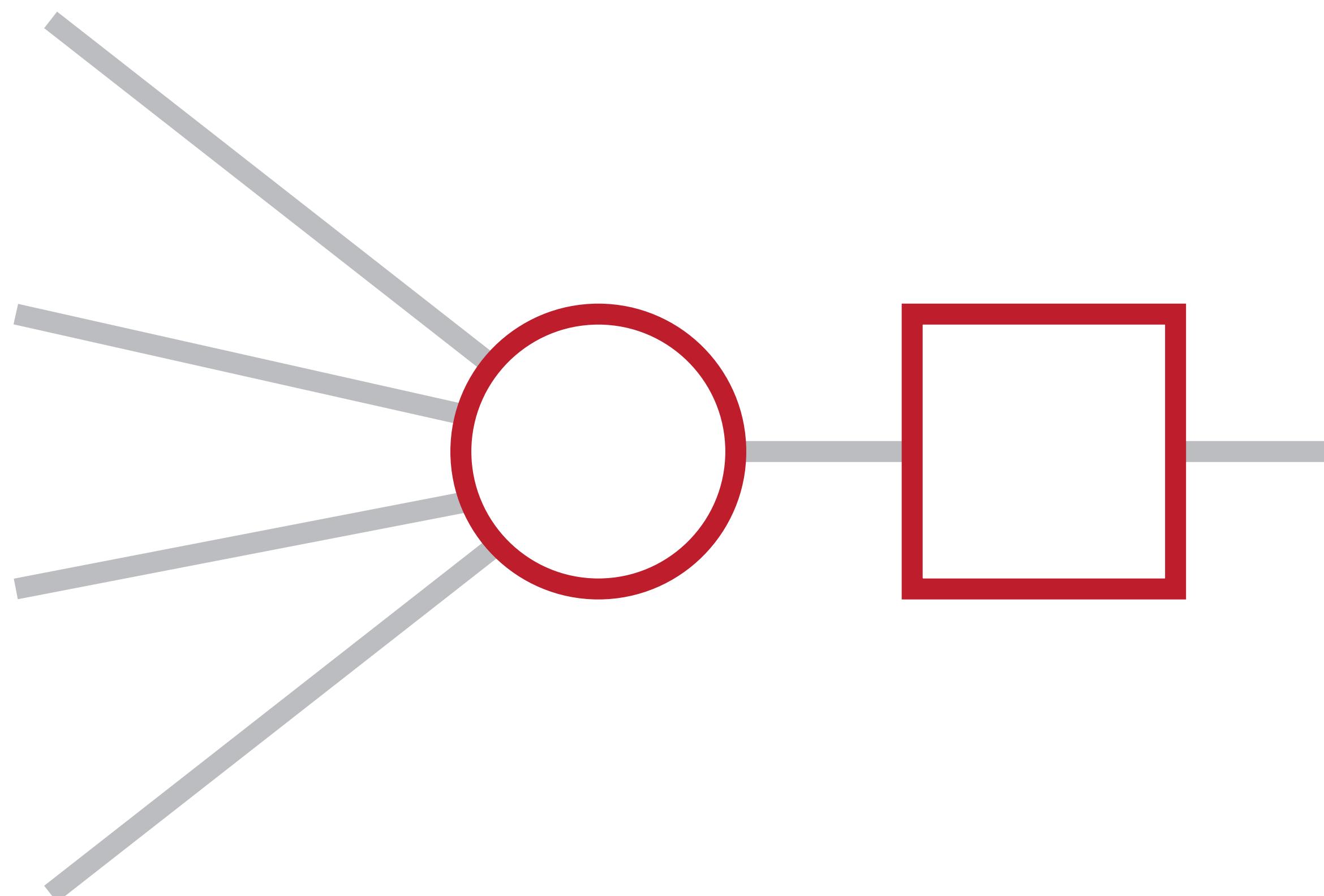
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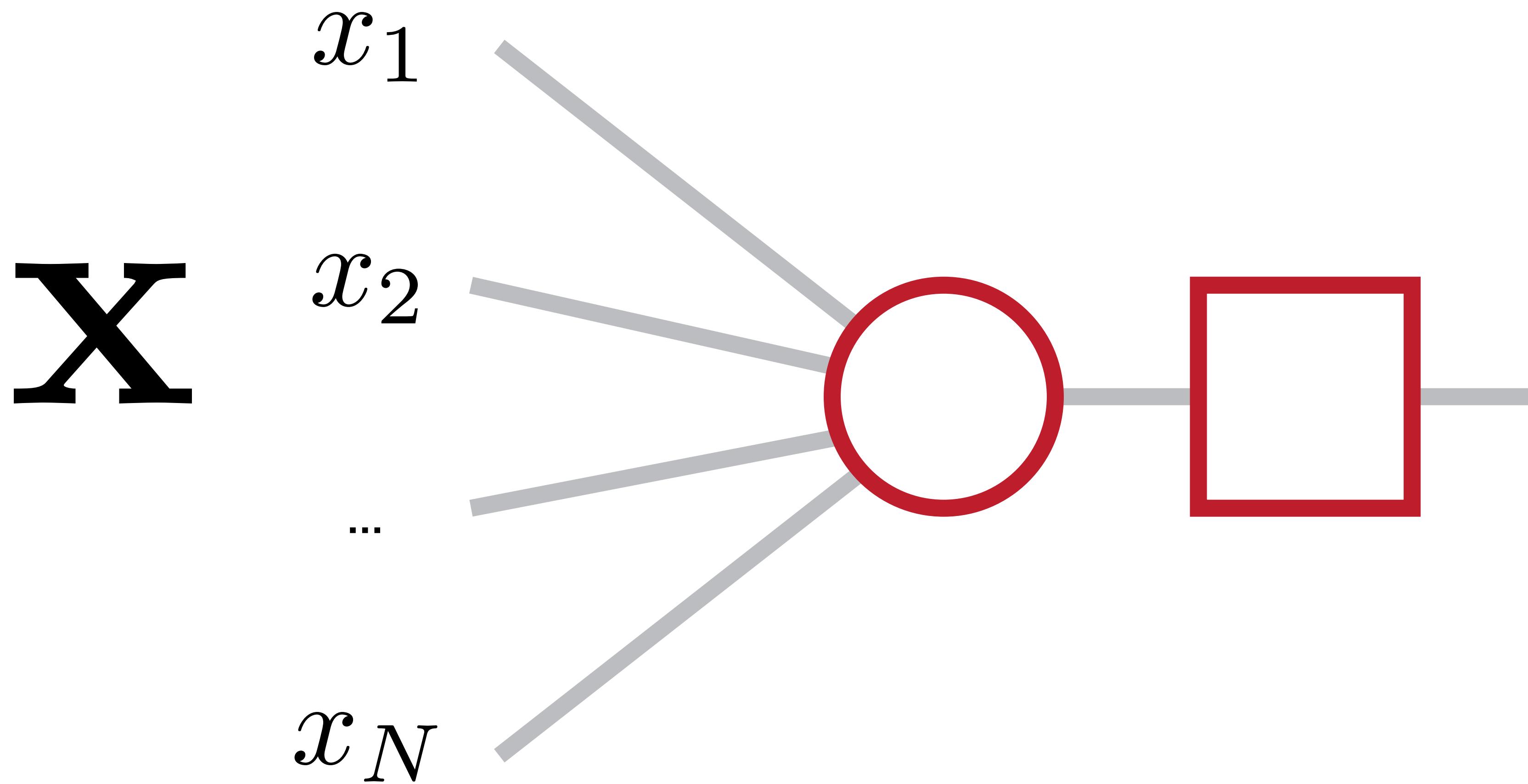
Intro to neural population dynamics

Artificial neurons

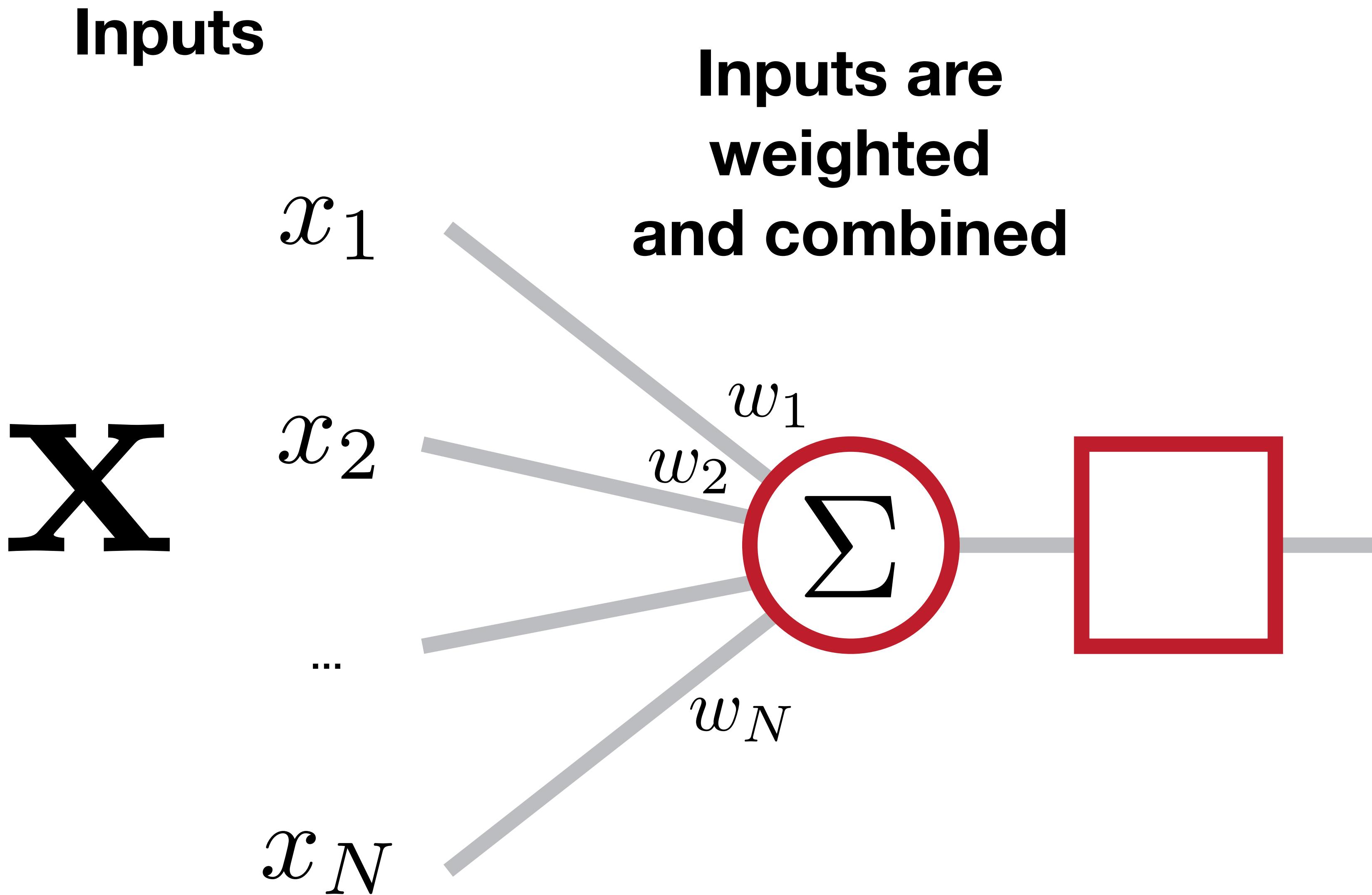


Artificial neurons

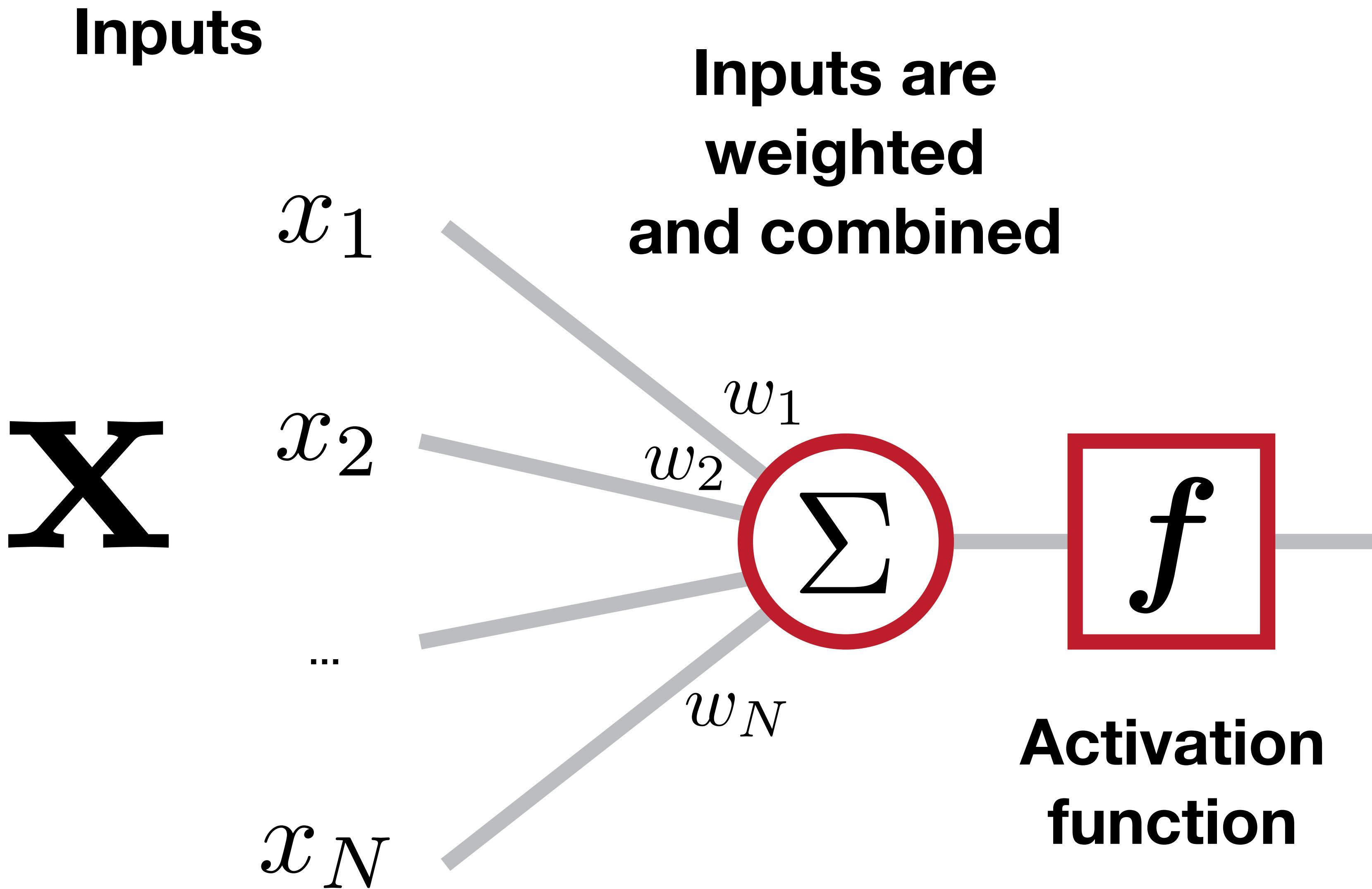
Inputs



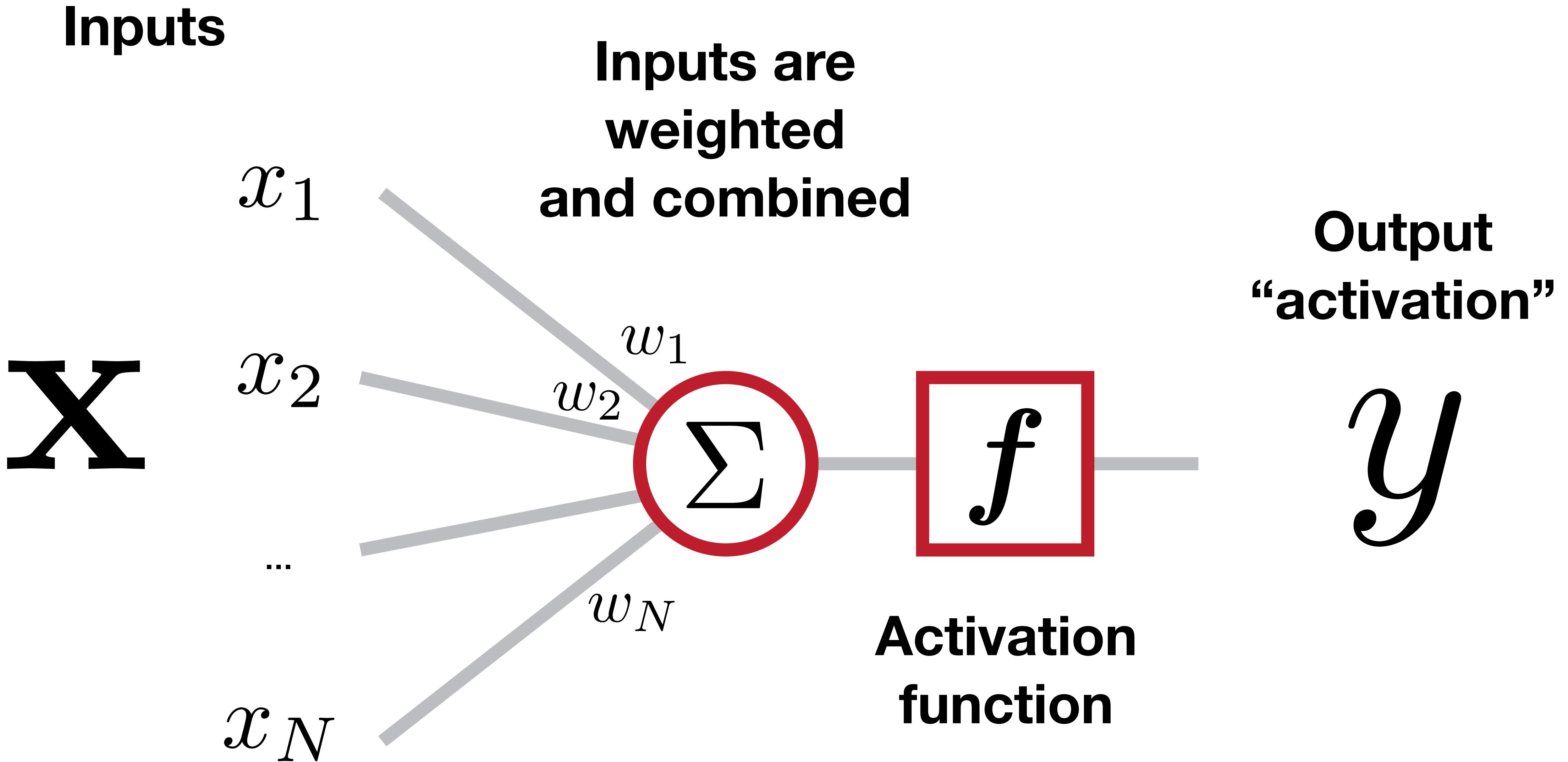
Artificial neurons



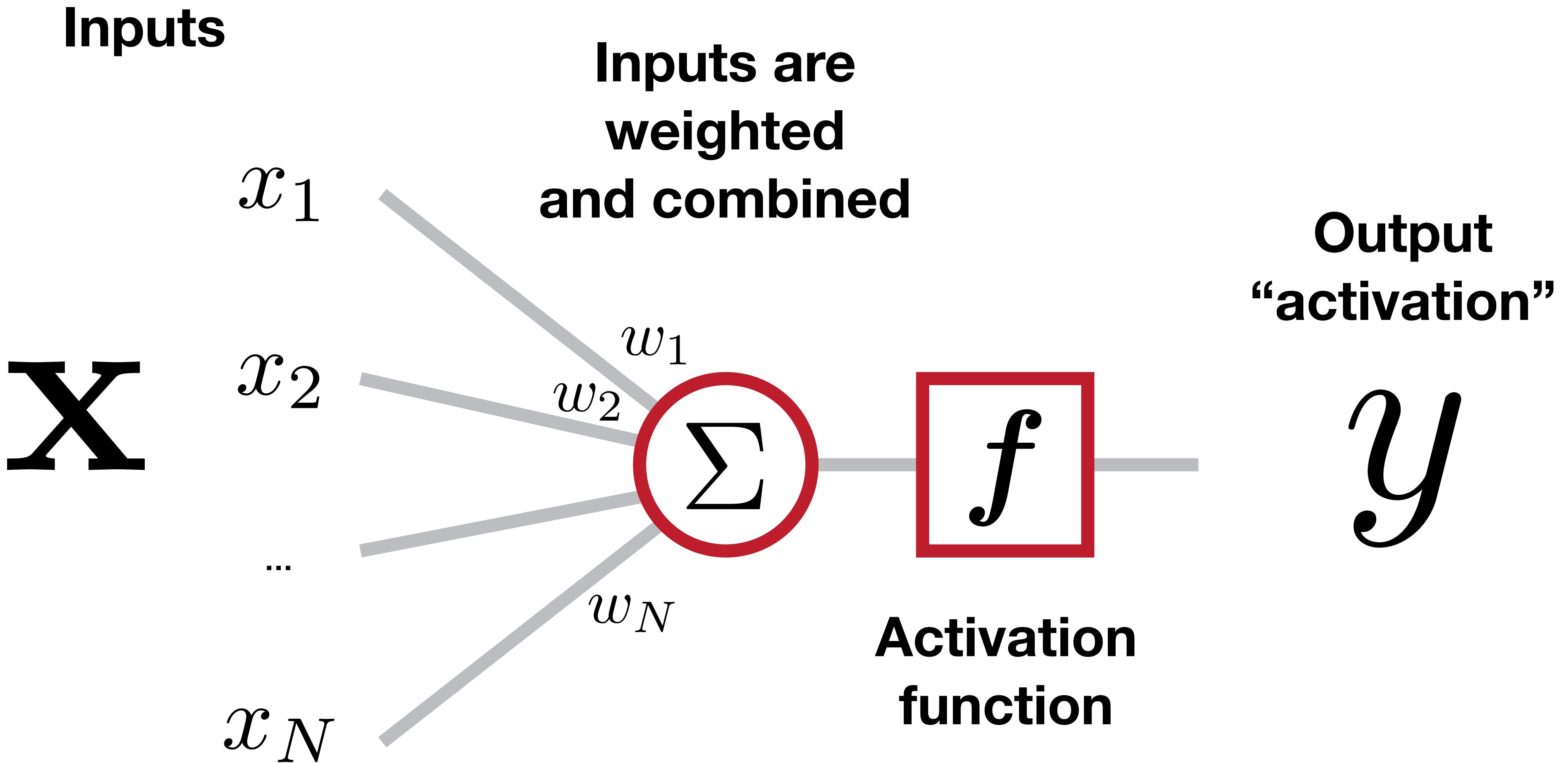
Artificial neurons



Artificial neurons



Artificial neurons

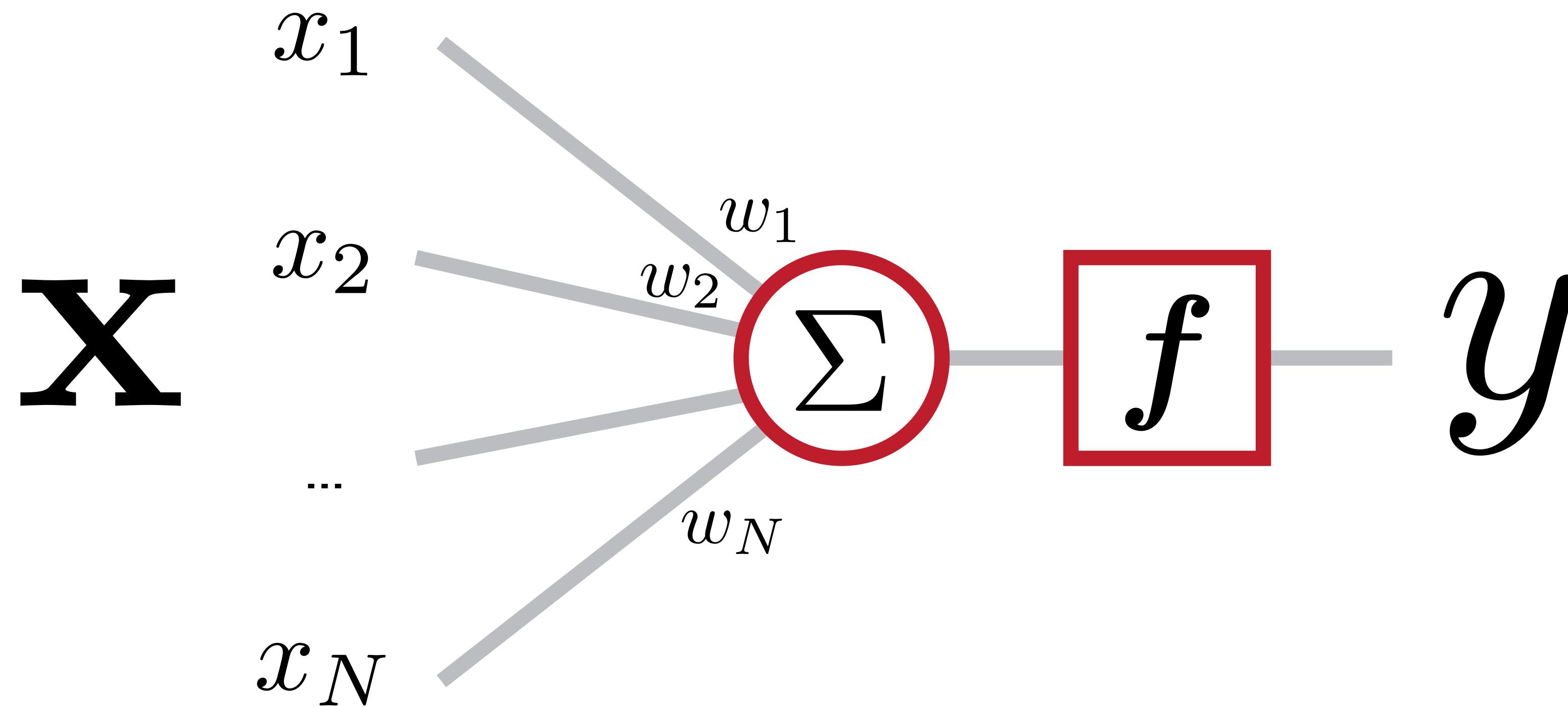


Warren S. McCulloch and Walter Pitts (1943)

"A logical calculus of the ideas immanent in nervous activity."

Bulletin of Mathematical Biophysics 5:115-133

Artificial neurons



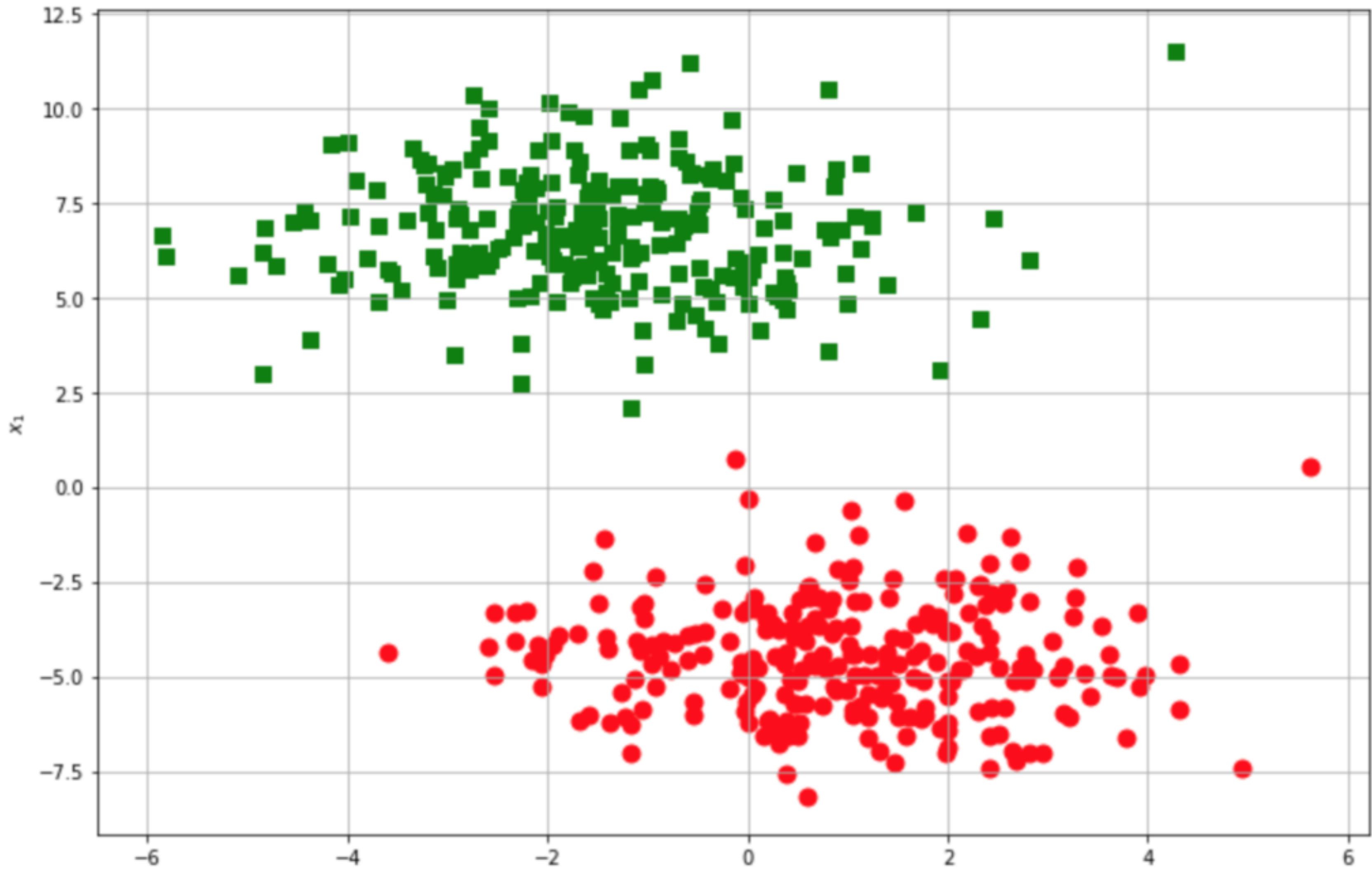
$$y = f(w_1 * x_1 + w_2 * x_2 + \dots + b)$$

Simple problem - 2 dimensional classification

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Green: class 0

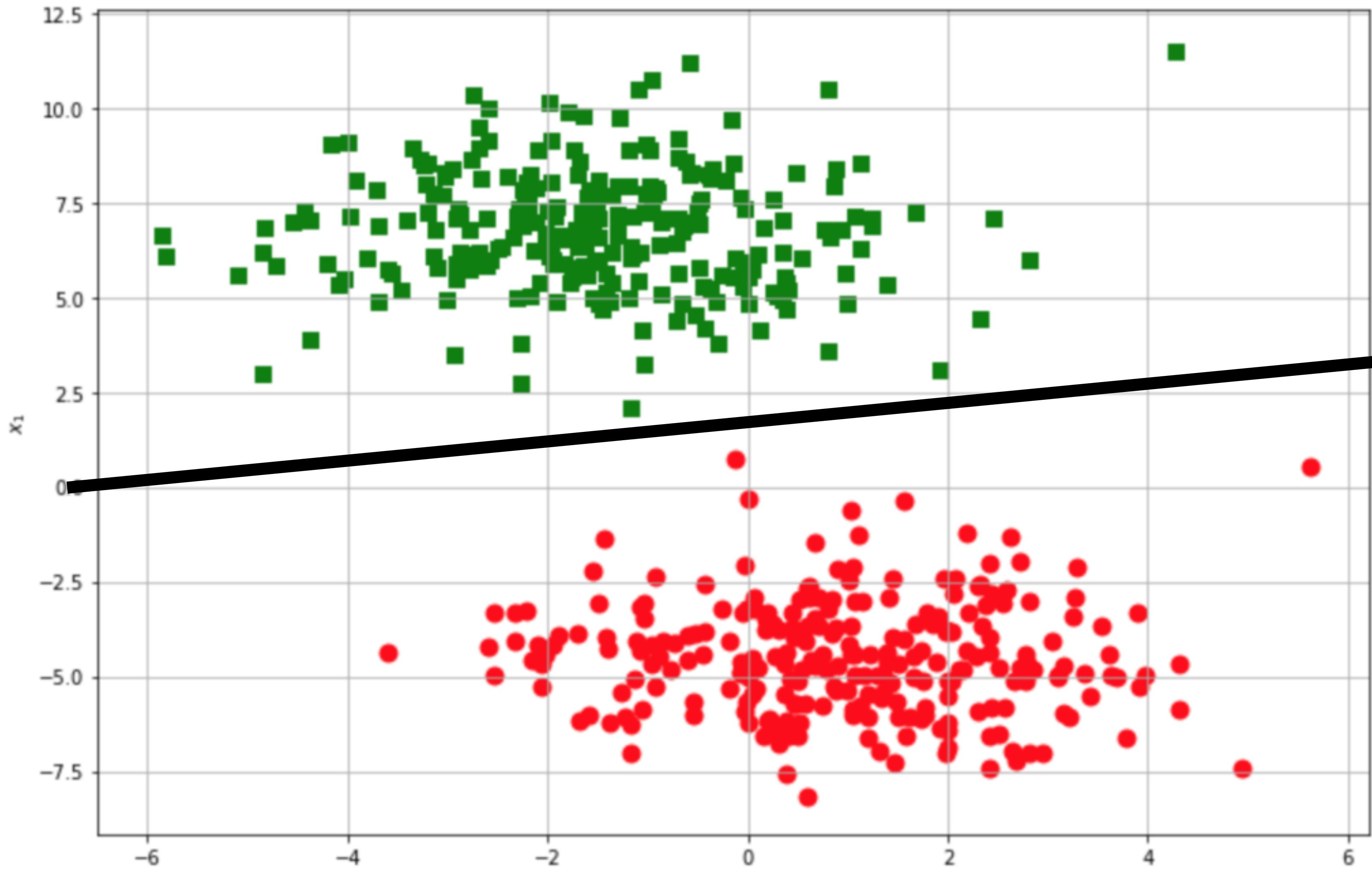
Red: class 1



Simple problem - 2 dimensional classification

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

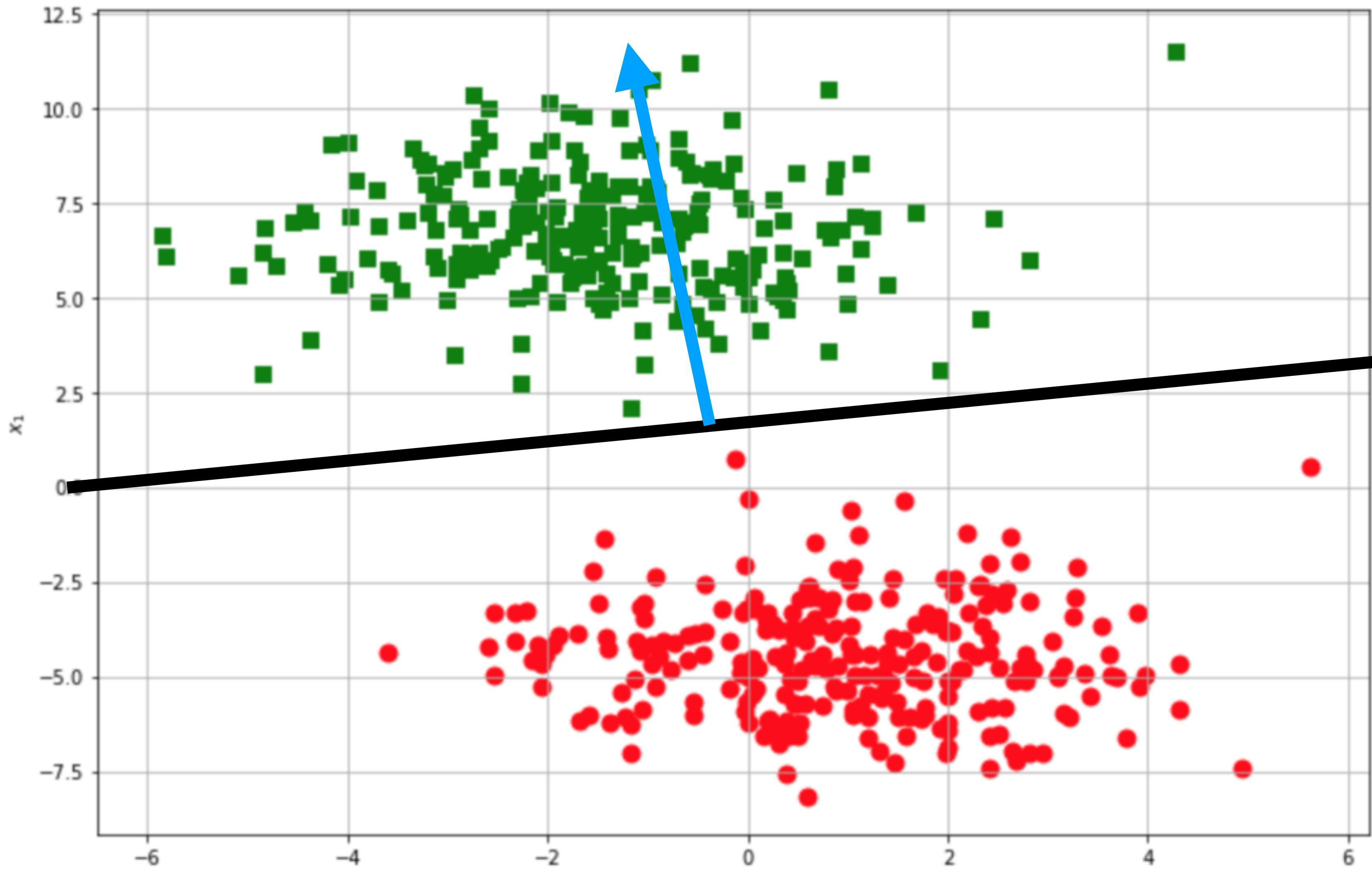
Green: class 0
Red: class 1



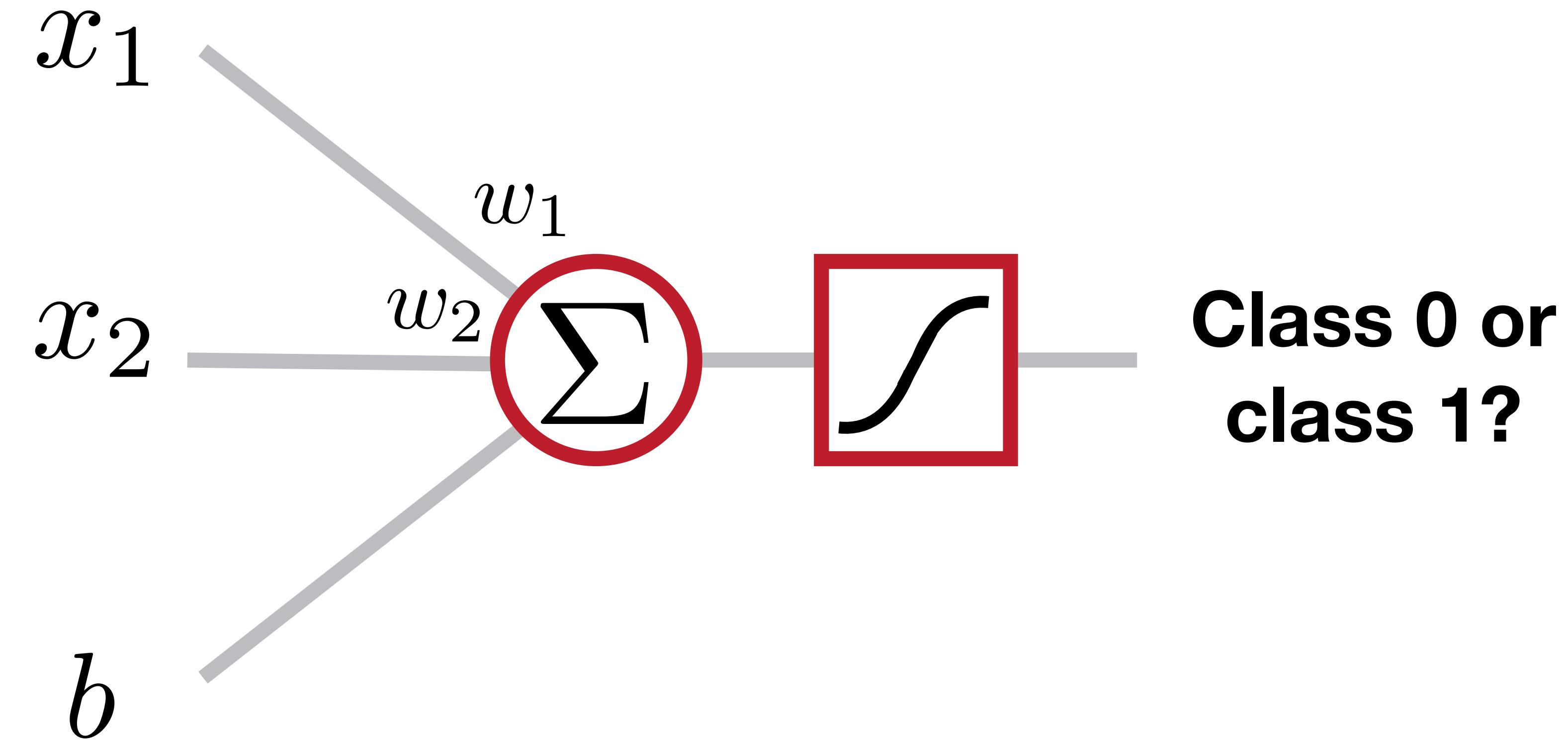
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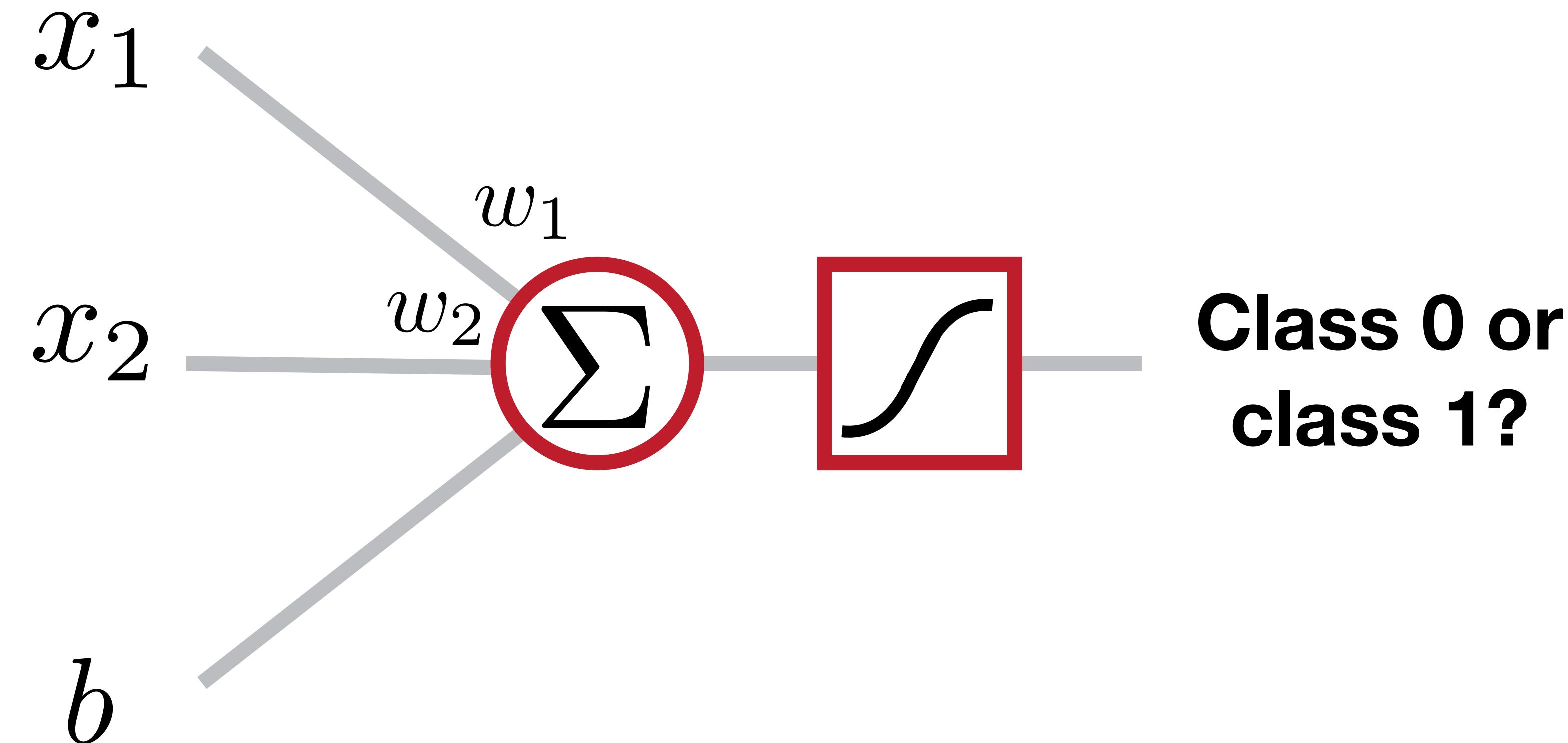
Green: class 0
Red: class 1



Simple classifier - Perceptron



Simple classifier - Perceptron

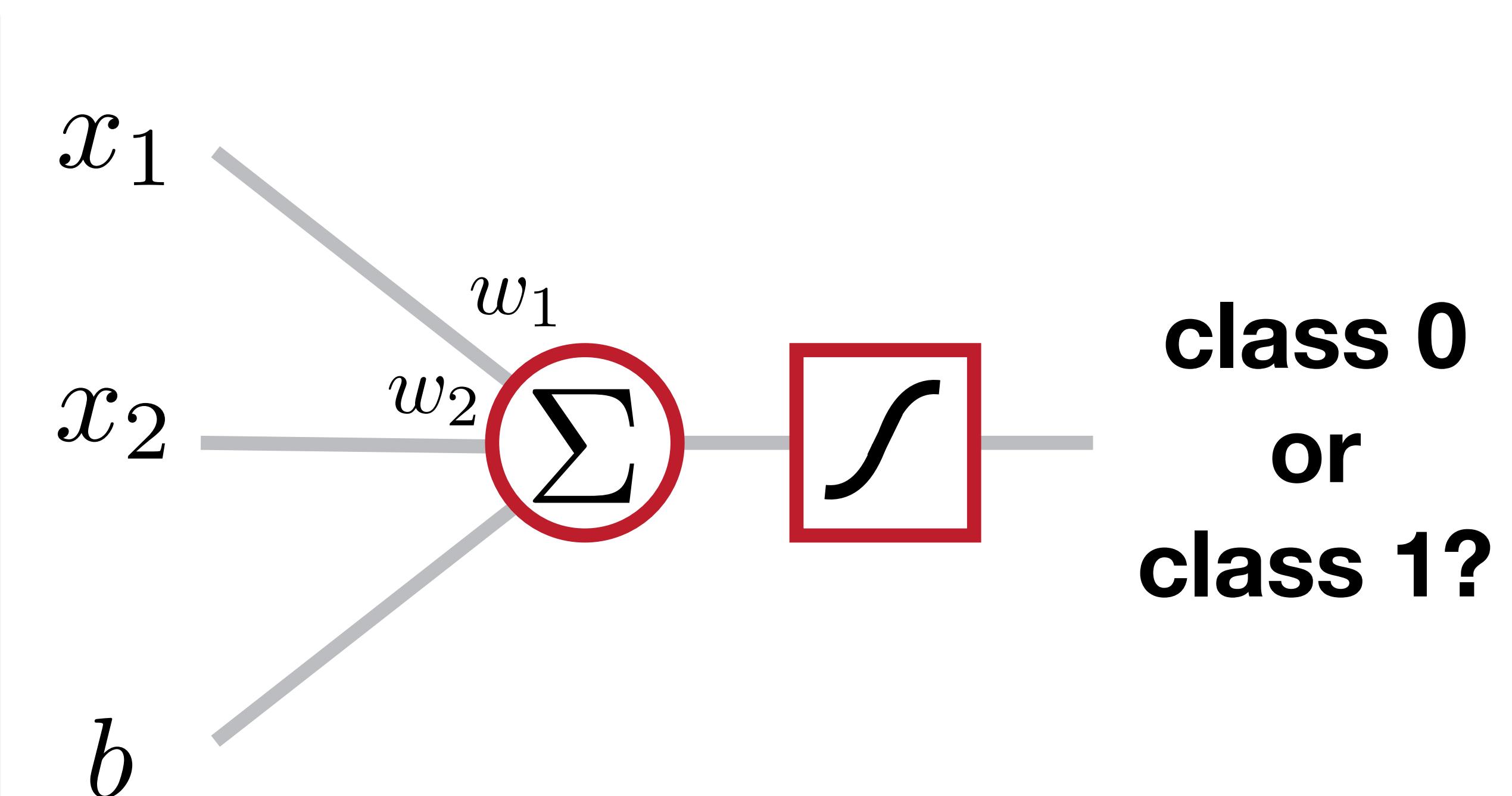
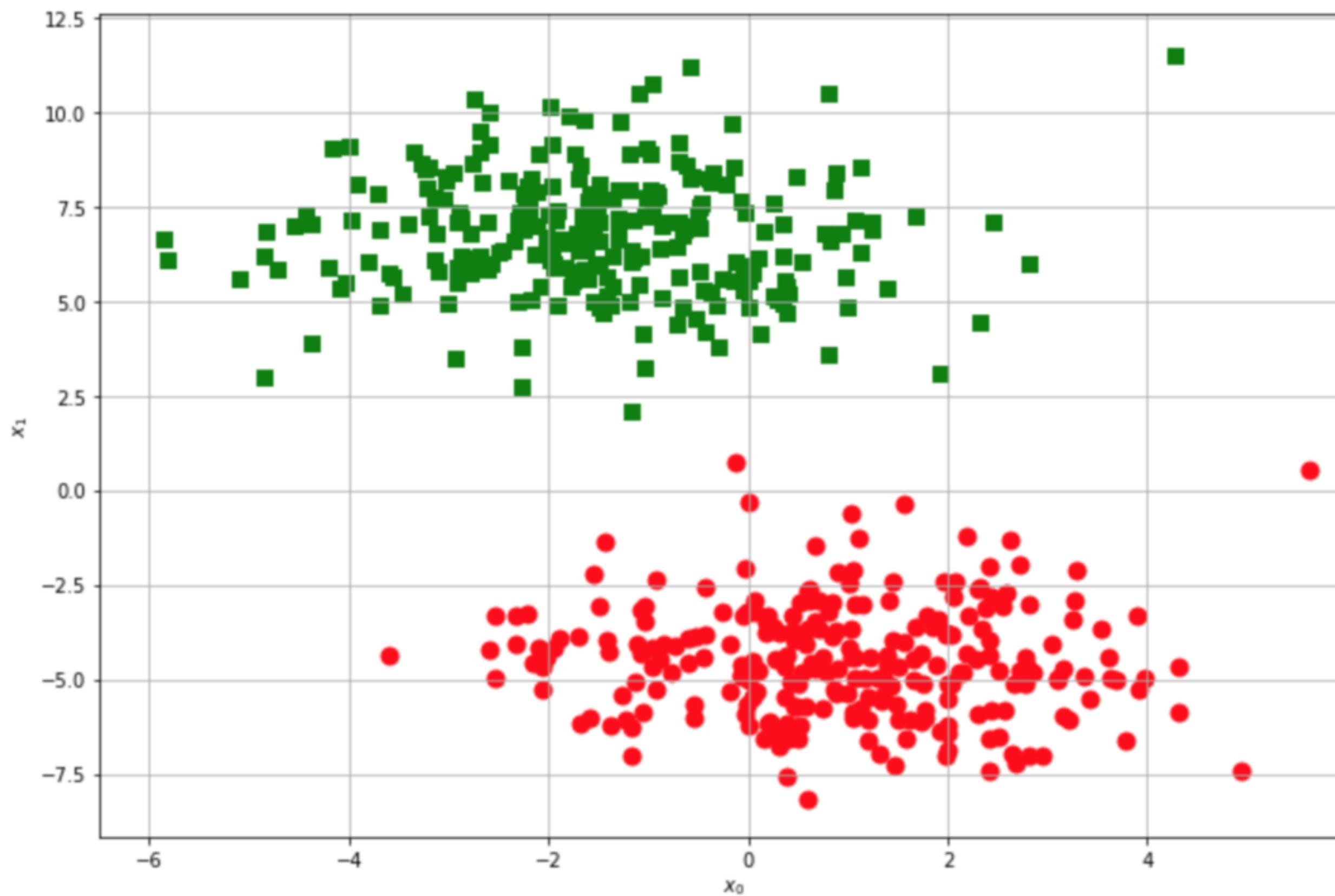


Frank Rosenblatt (1957)

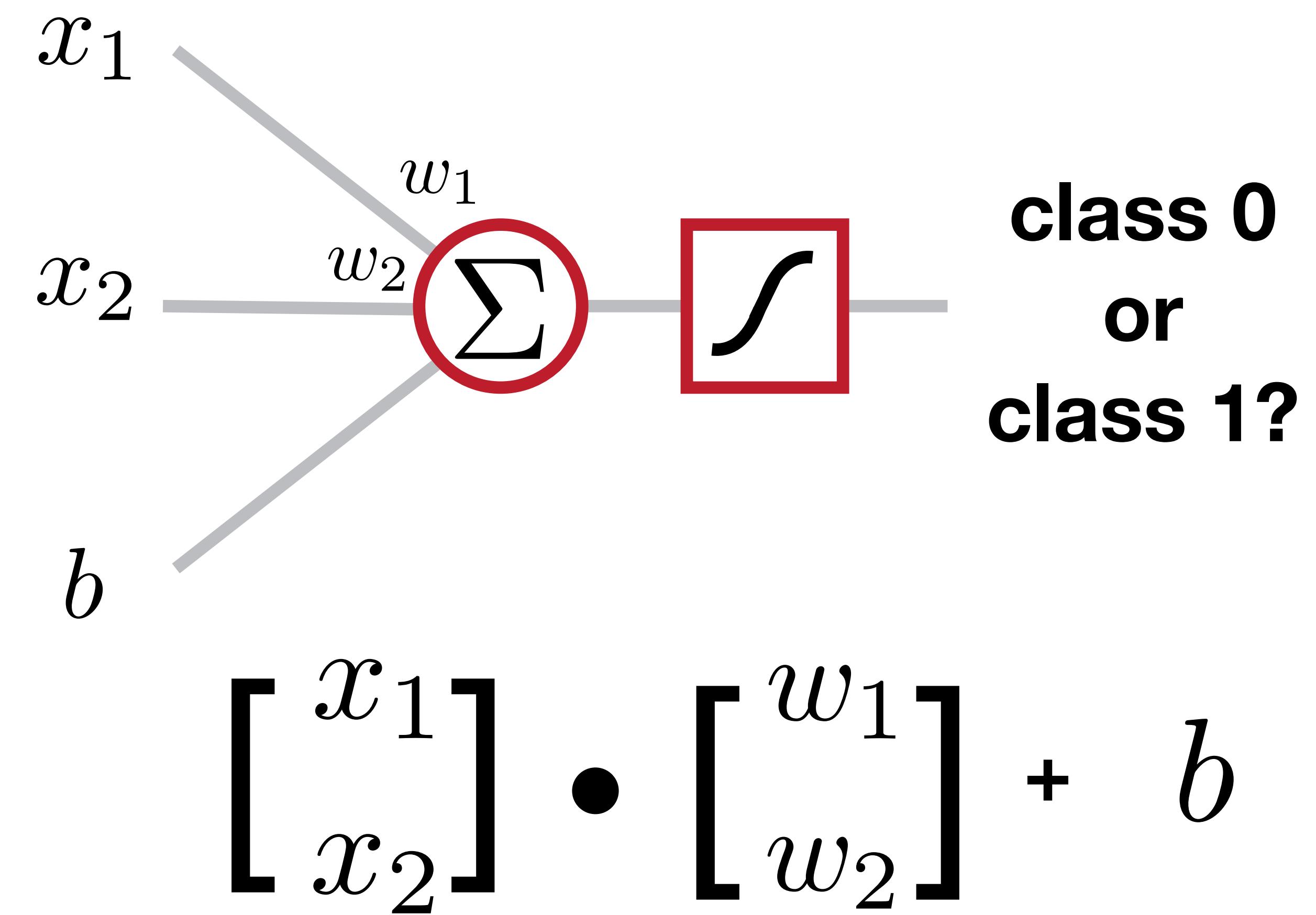
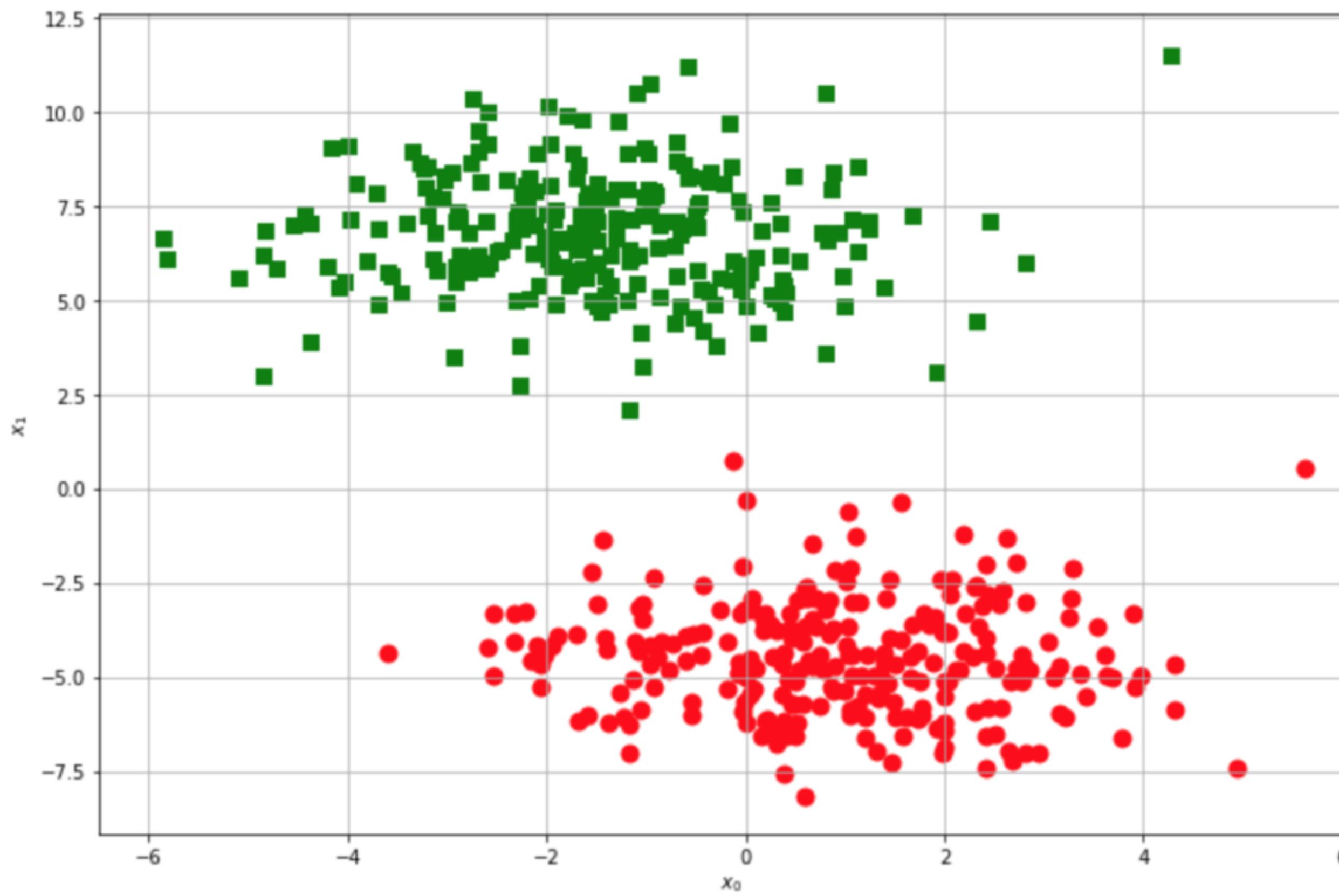
"The Perceptron — a perceiving and recognizing automaton."

Report 85-460-1, Cornell Aeronautical Laboratory.

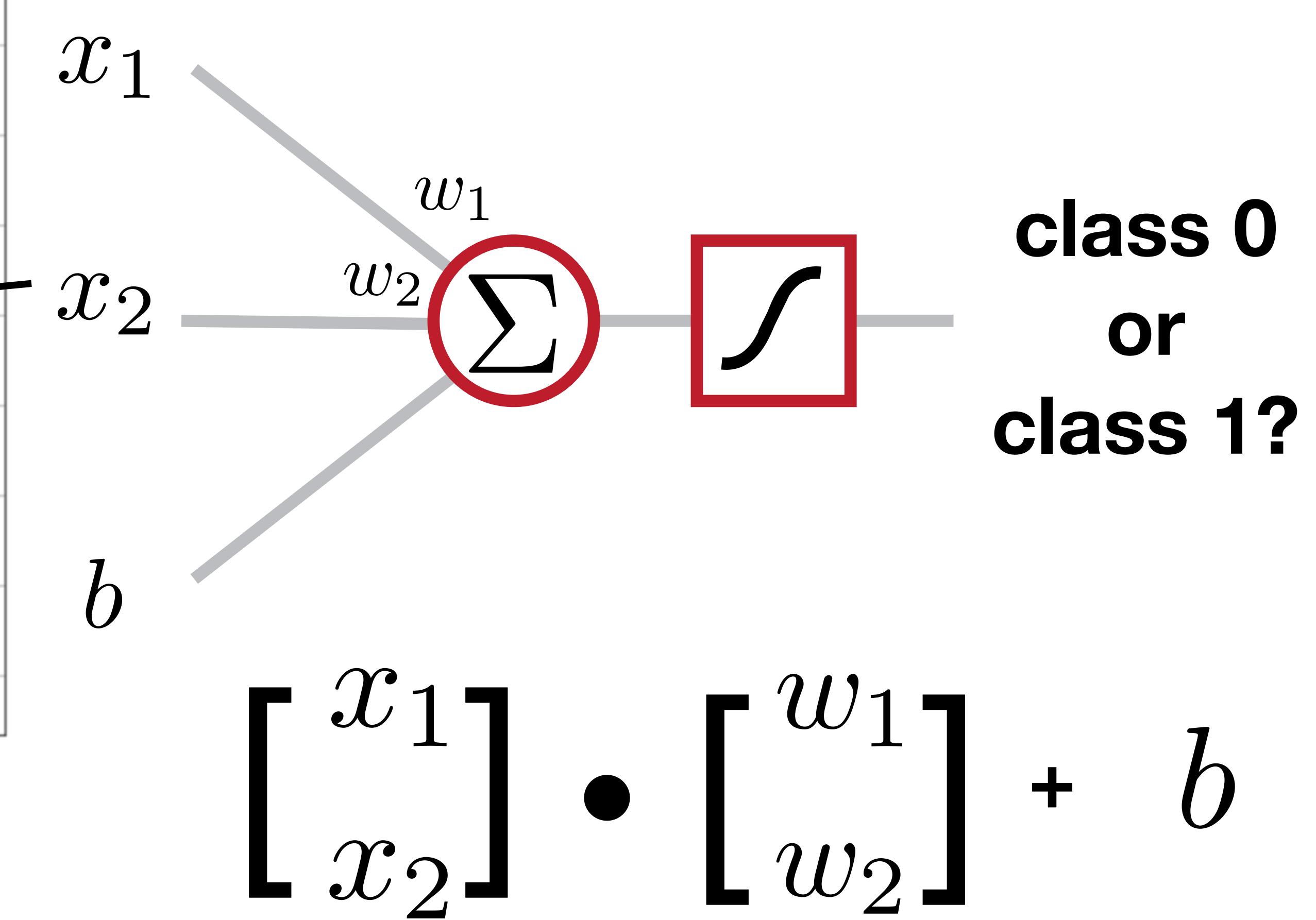
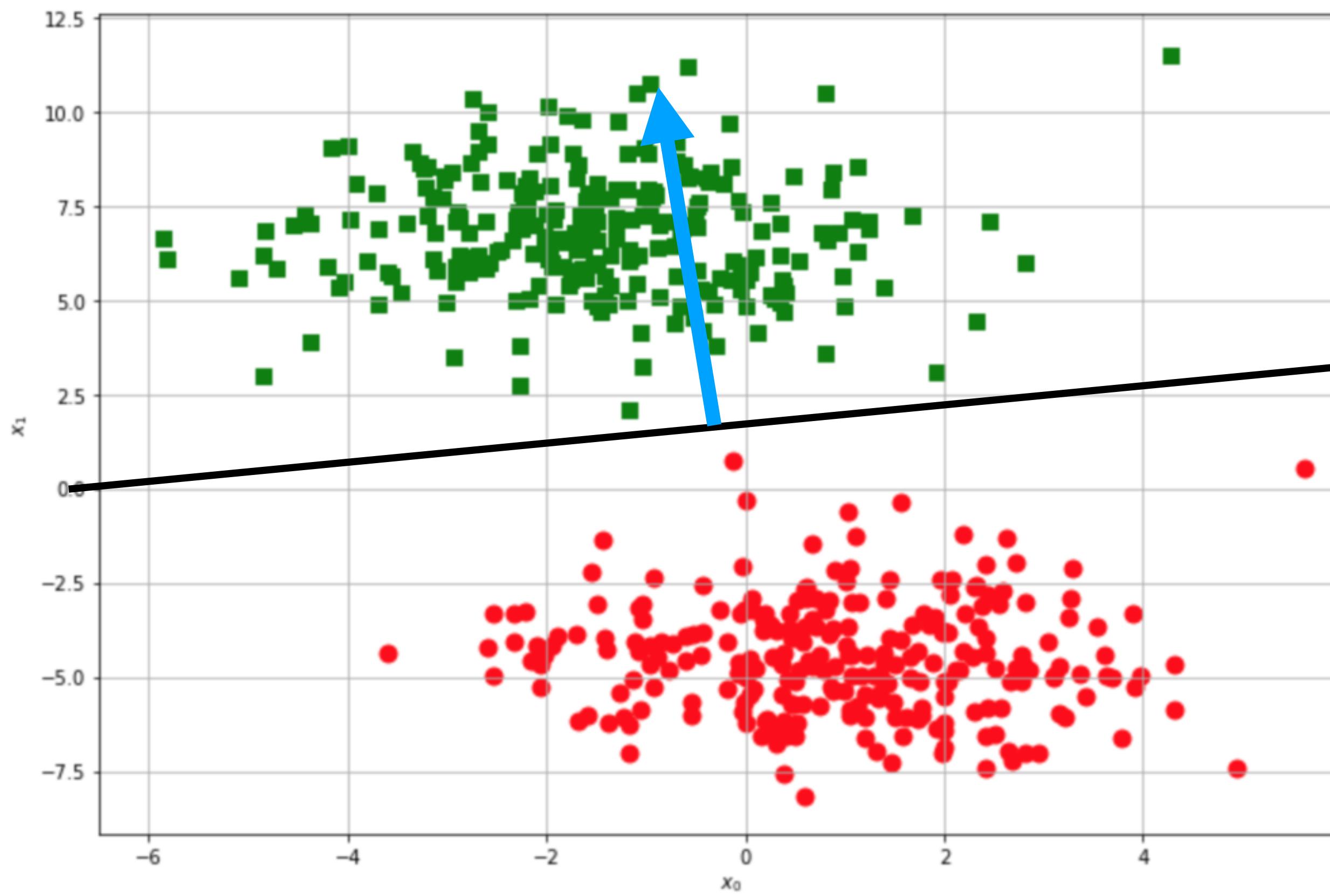
Classification with the perceptron



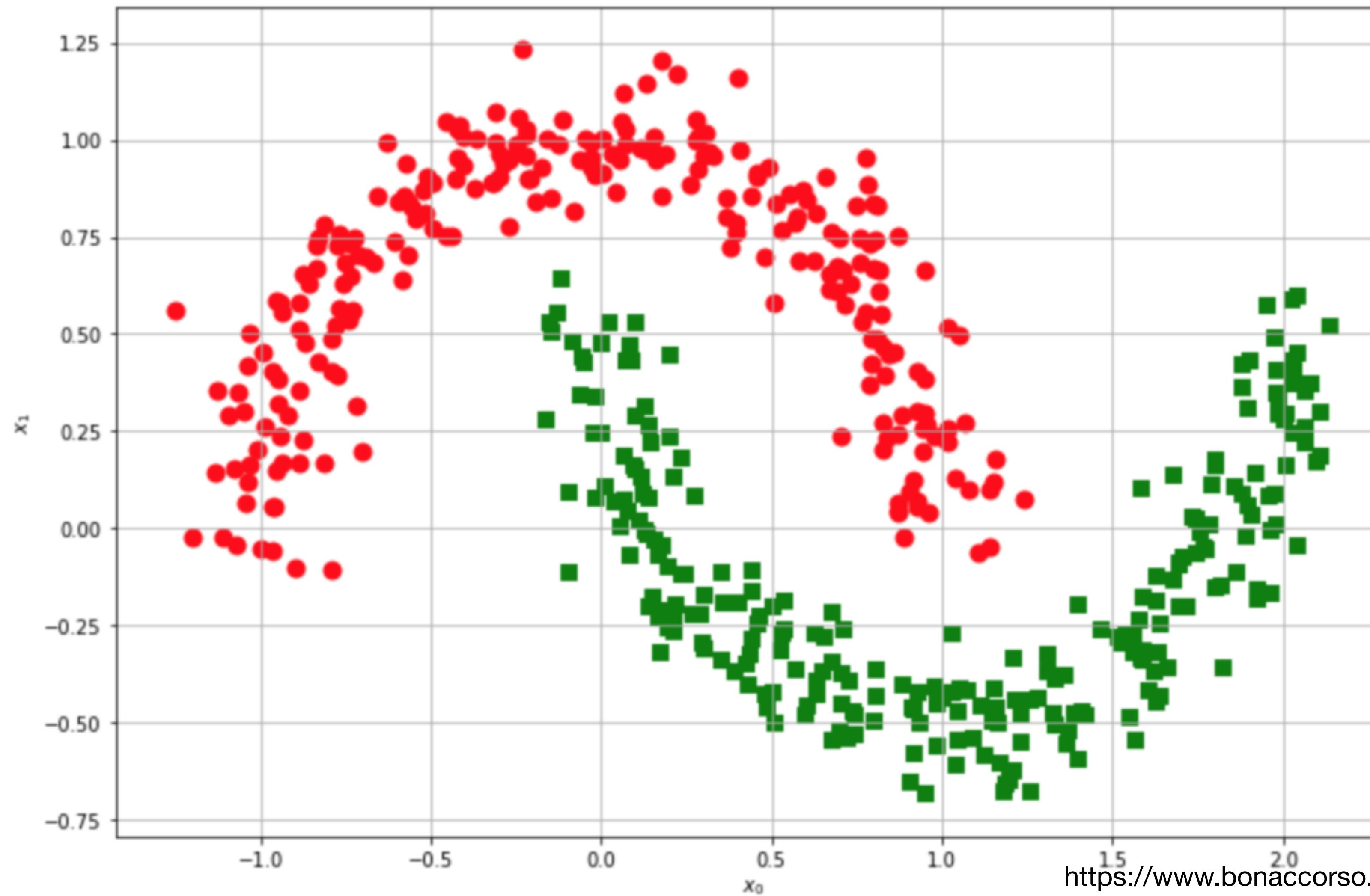
Classification with the perceptron



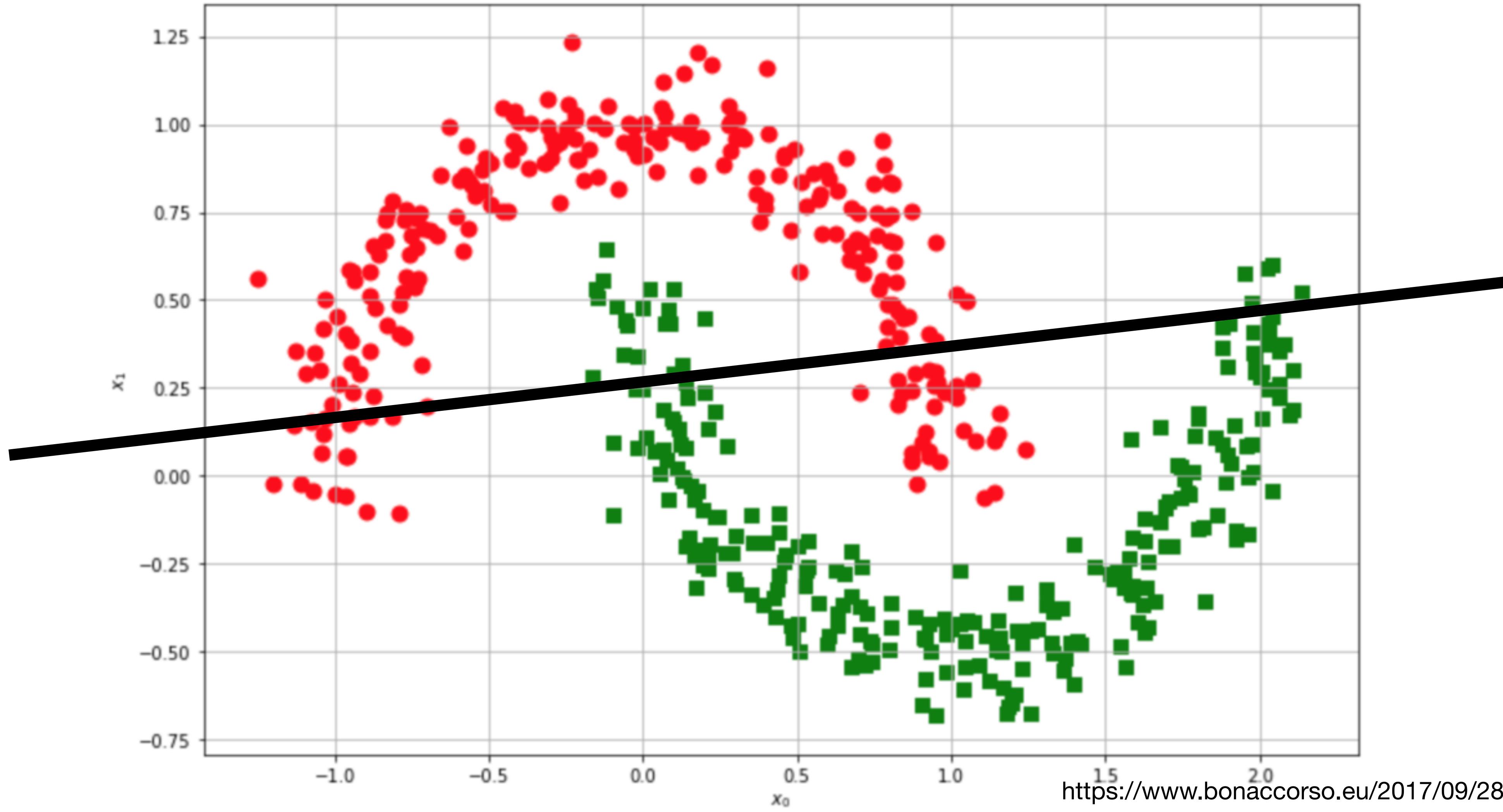
Classification with the perceptron



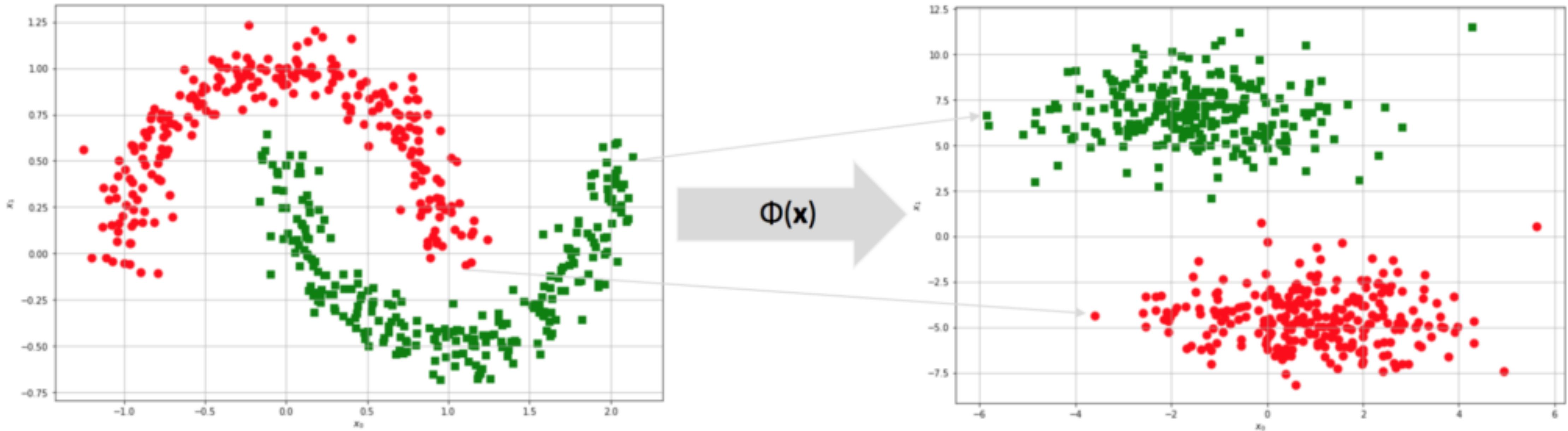
Harder problem



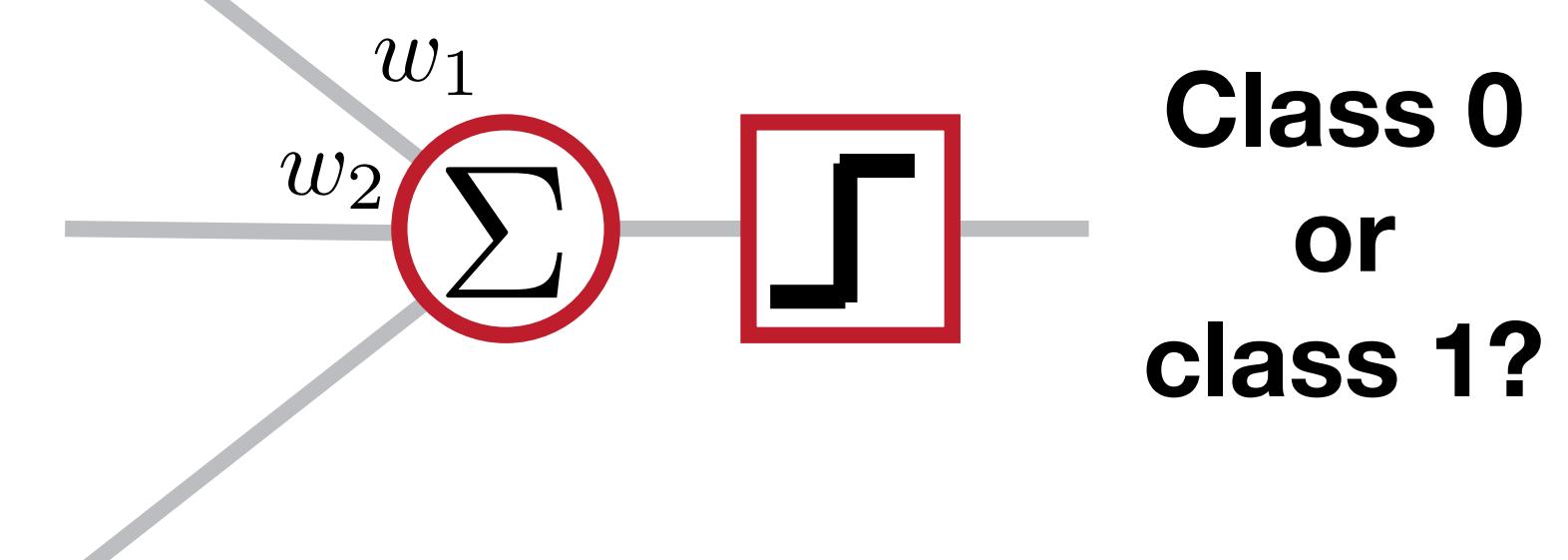
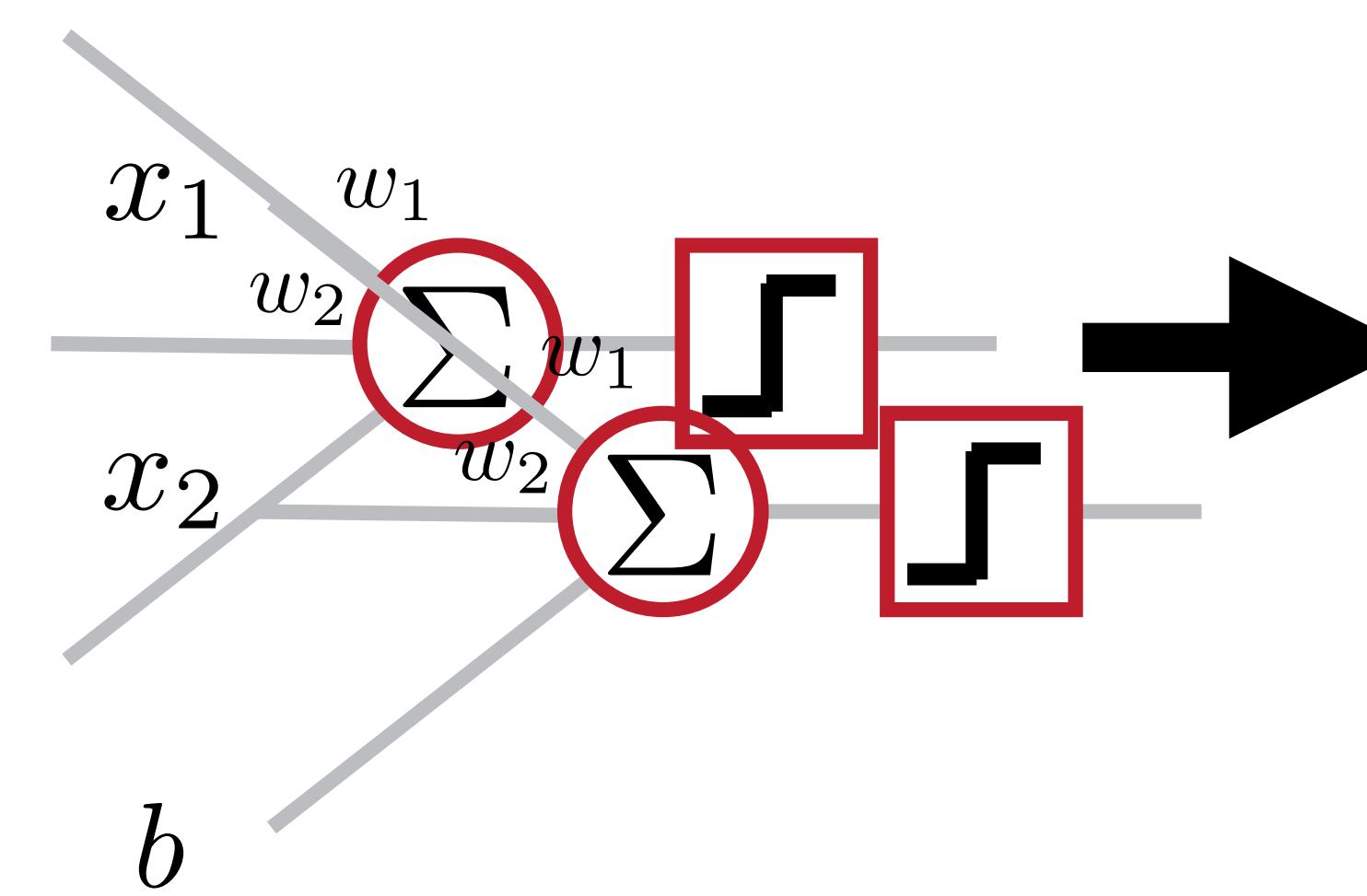
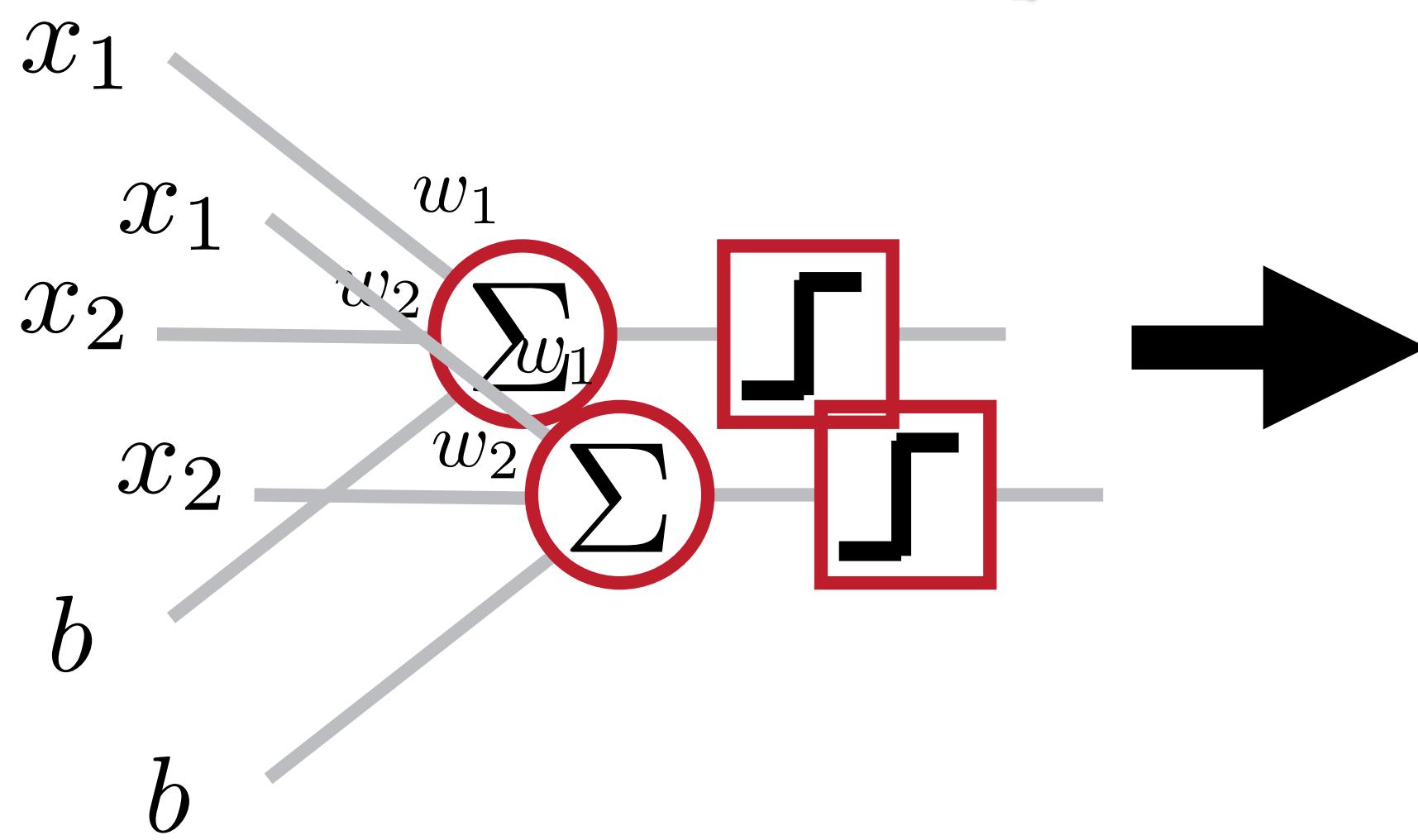
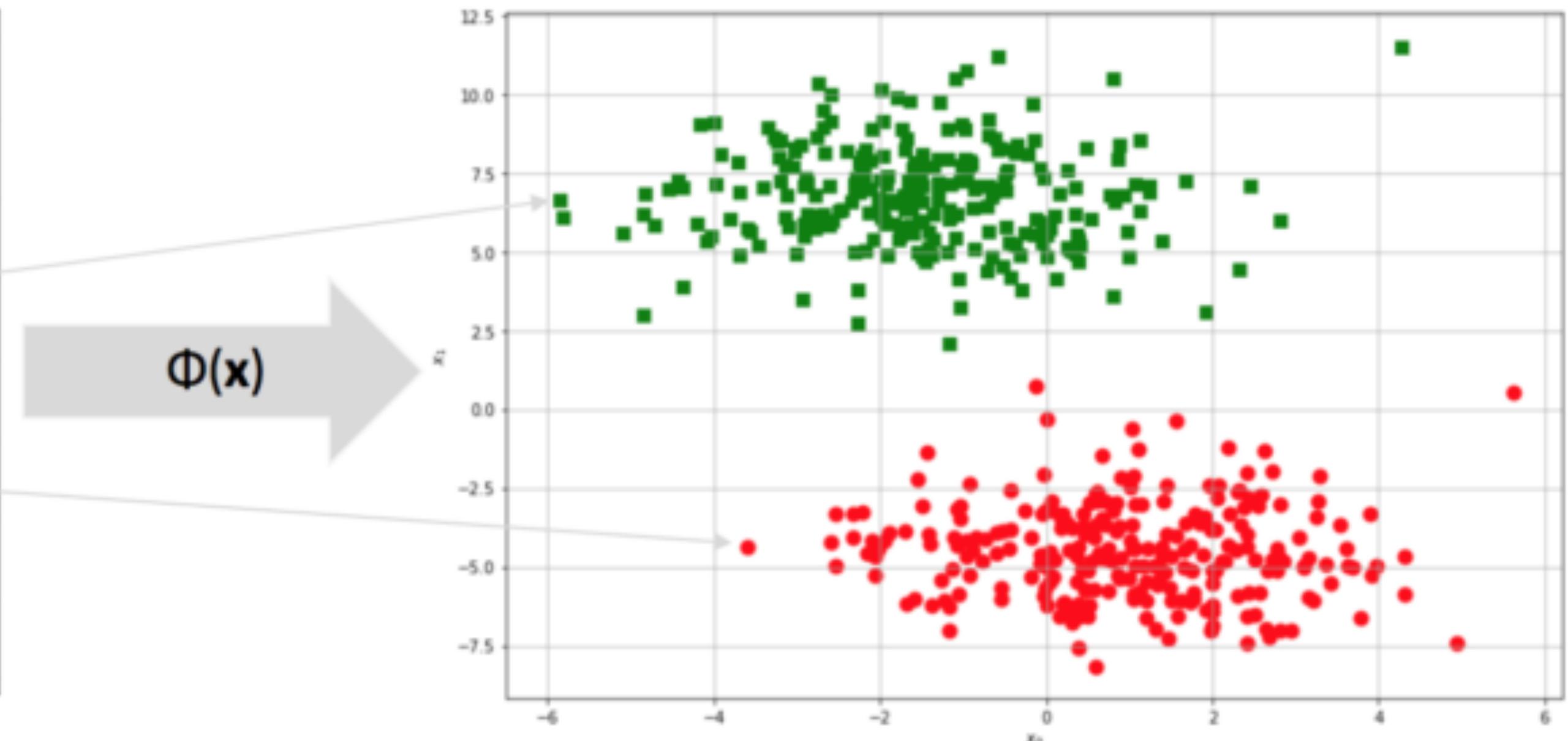
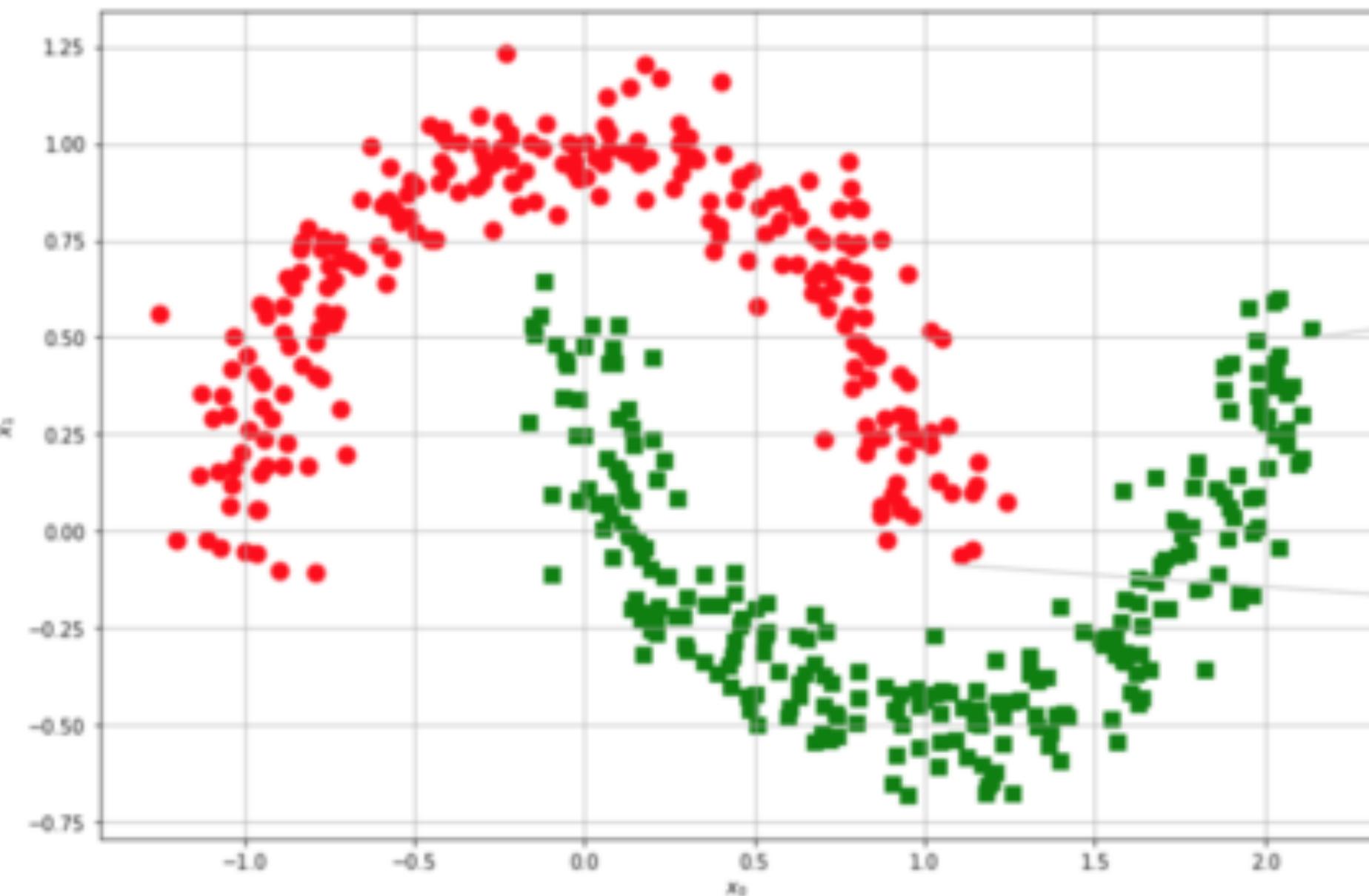
Harder problem



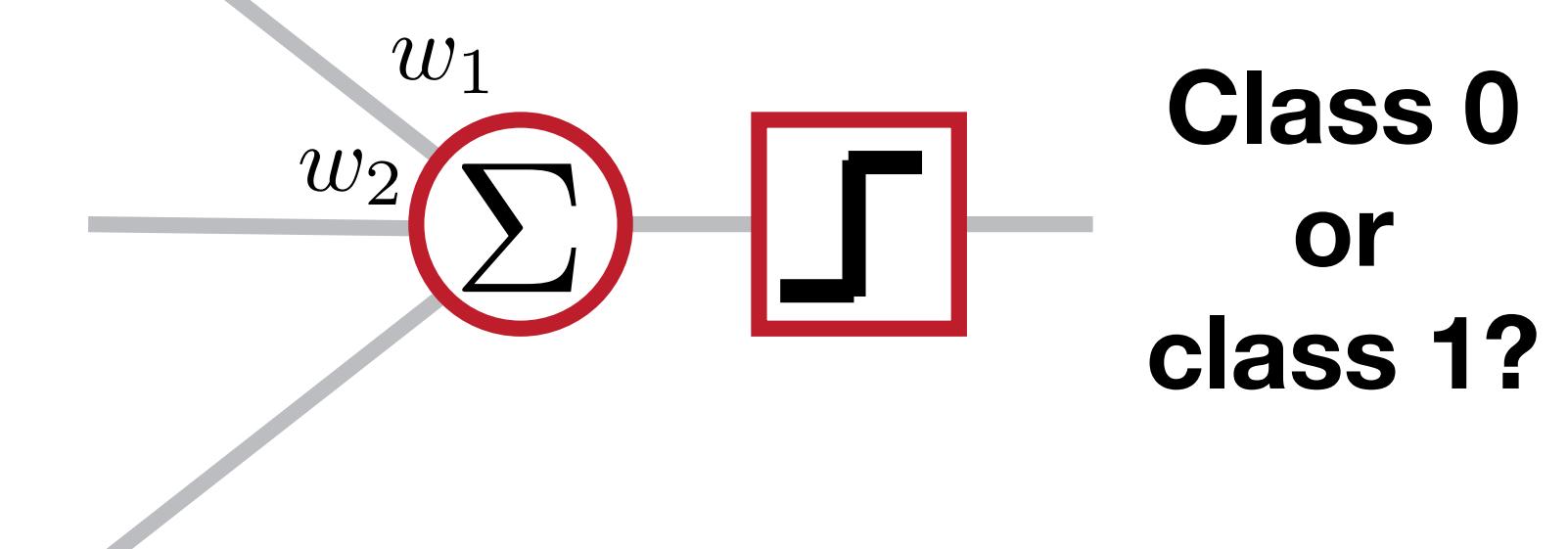
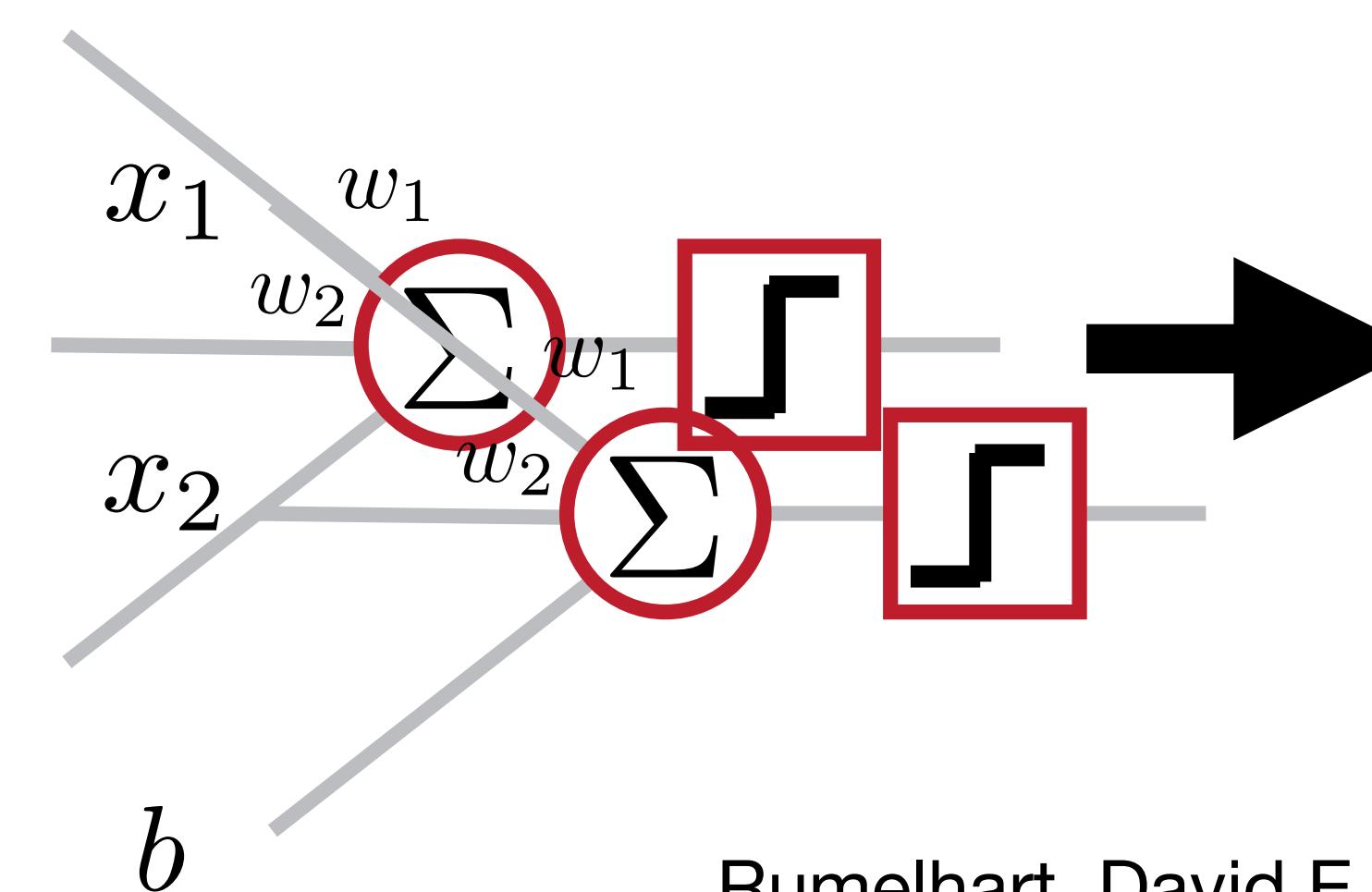
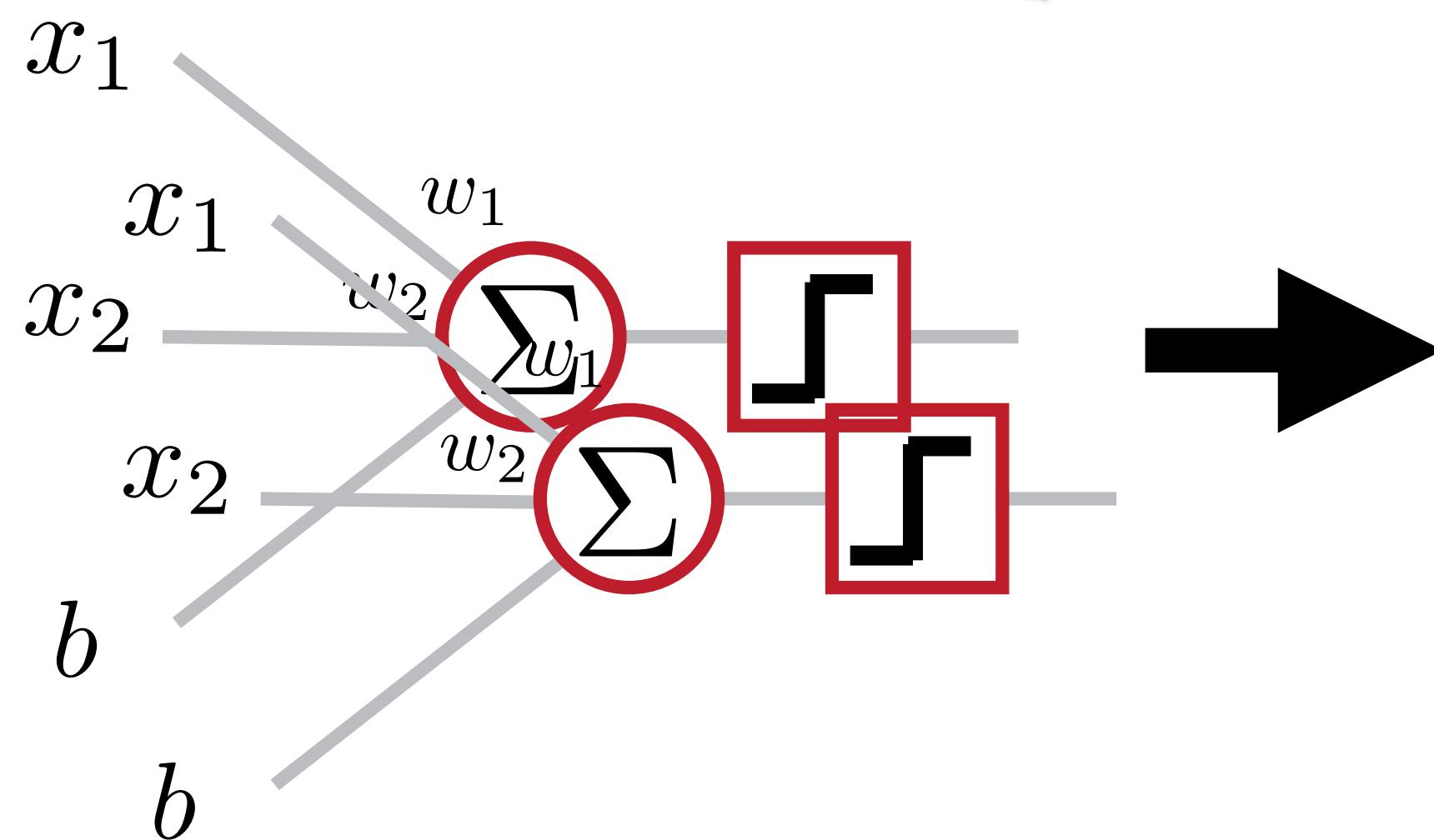
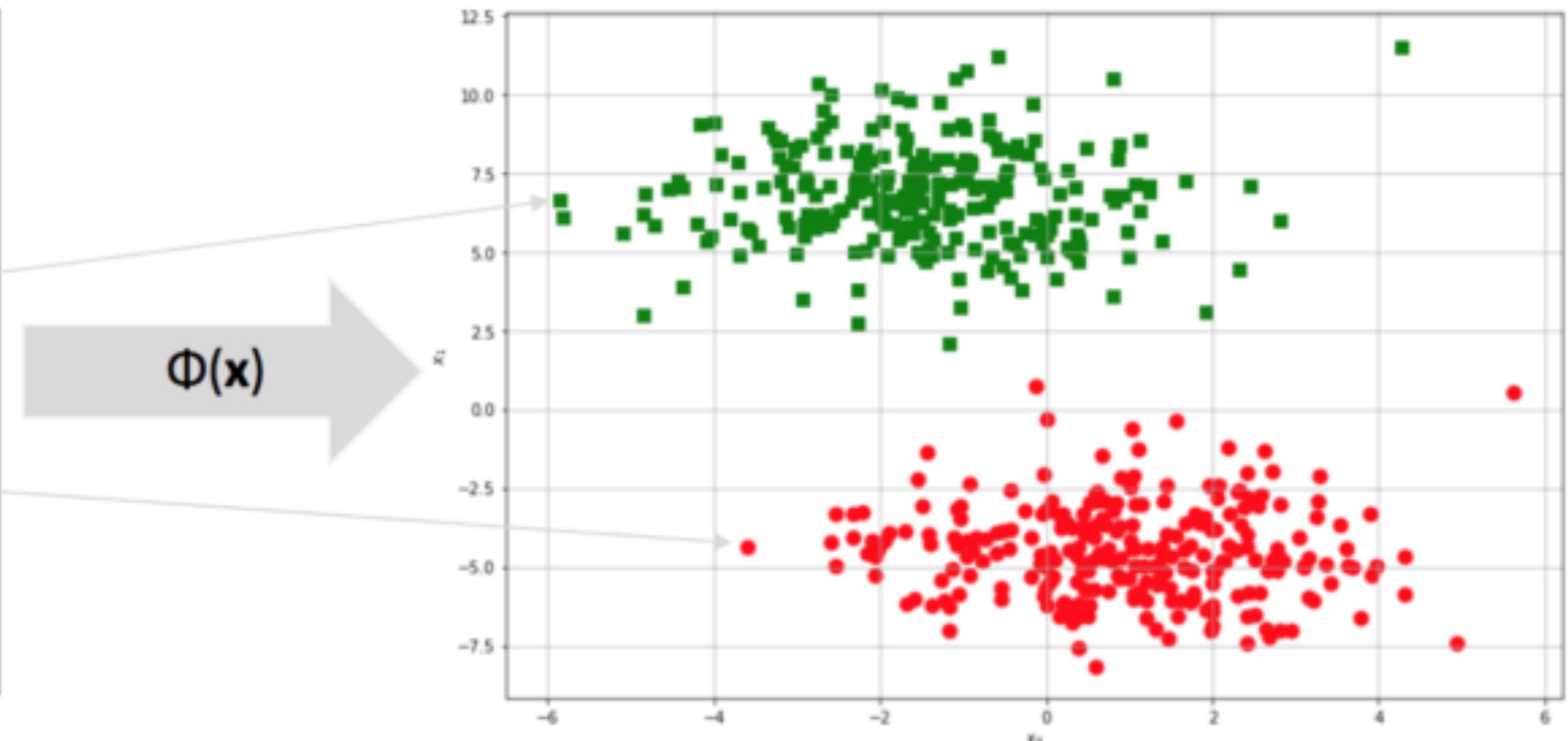
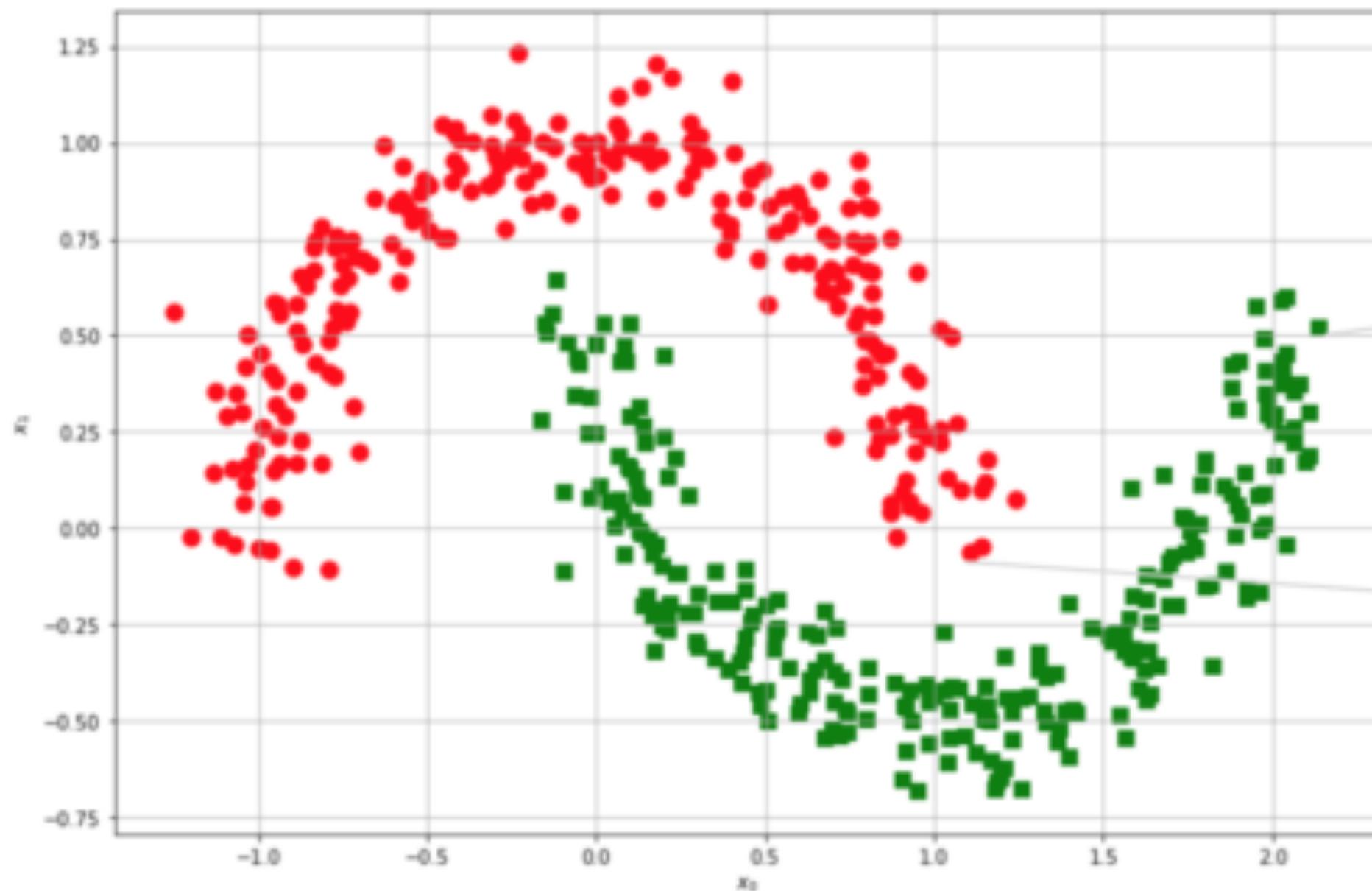
Strategy - multiple transformations



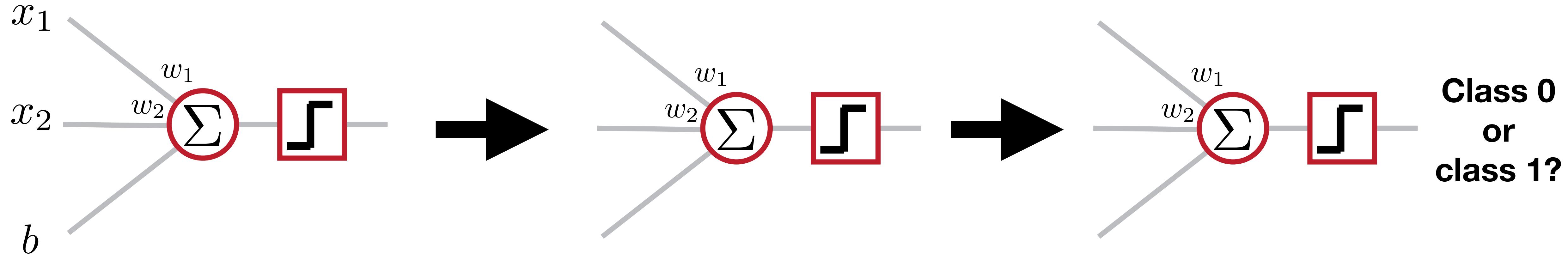
Multi-layer Perceptron (MLP)

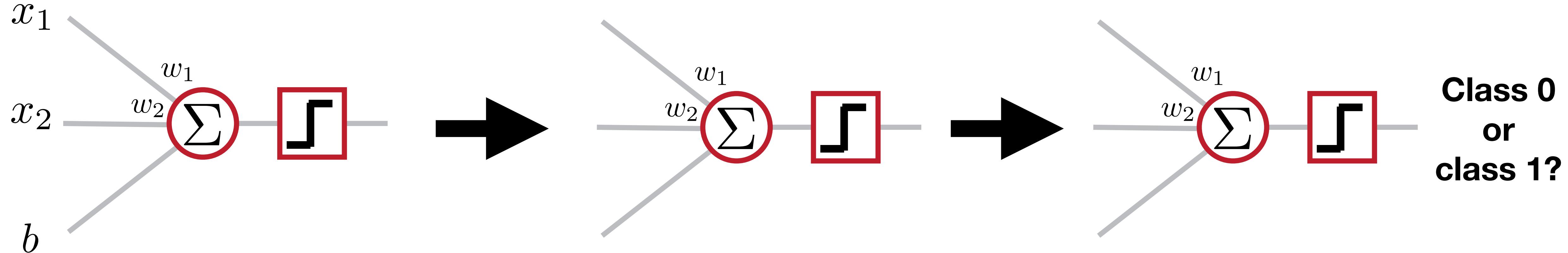


Multi-layer Perceptron (MLP)

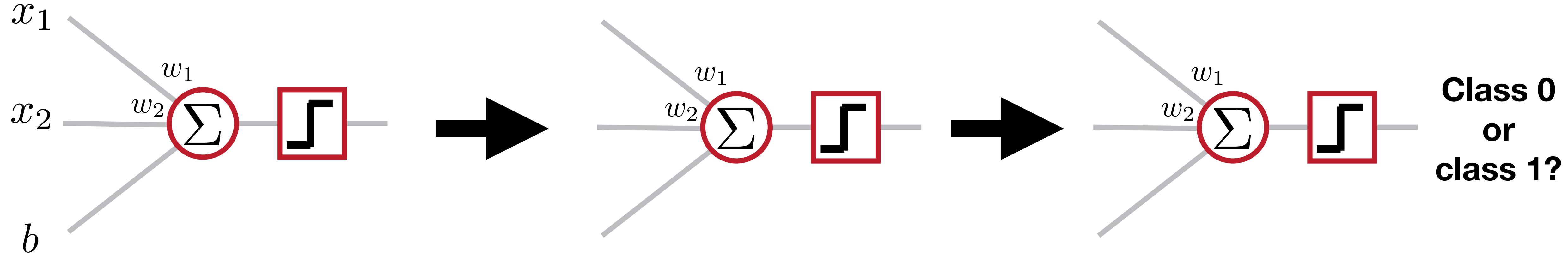


Rumelhart, David E., Geoffrey E. Hinton, and R. J. Williams. (1986)
“Learning Internal Representations by Error Propagation”. *Parallel distributed processing*



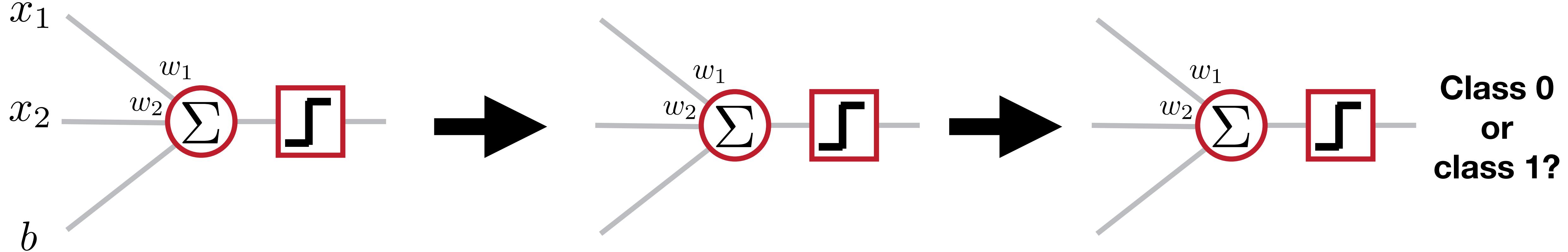


What are the parameters?



What are the parameters?

What is the objective (loss)?



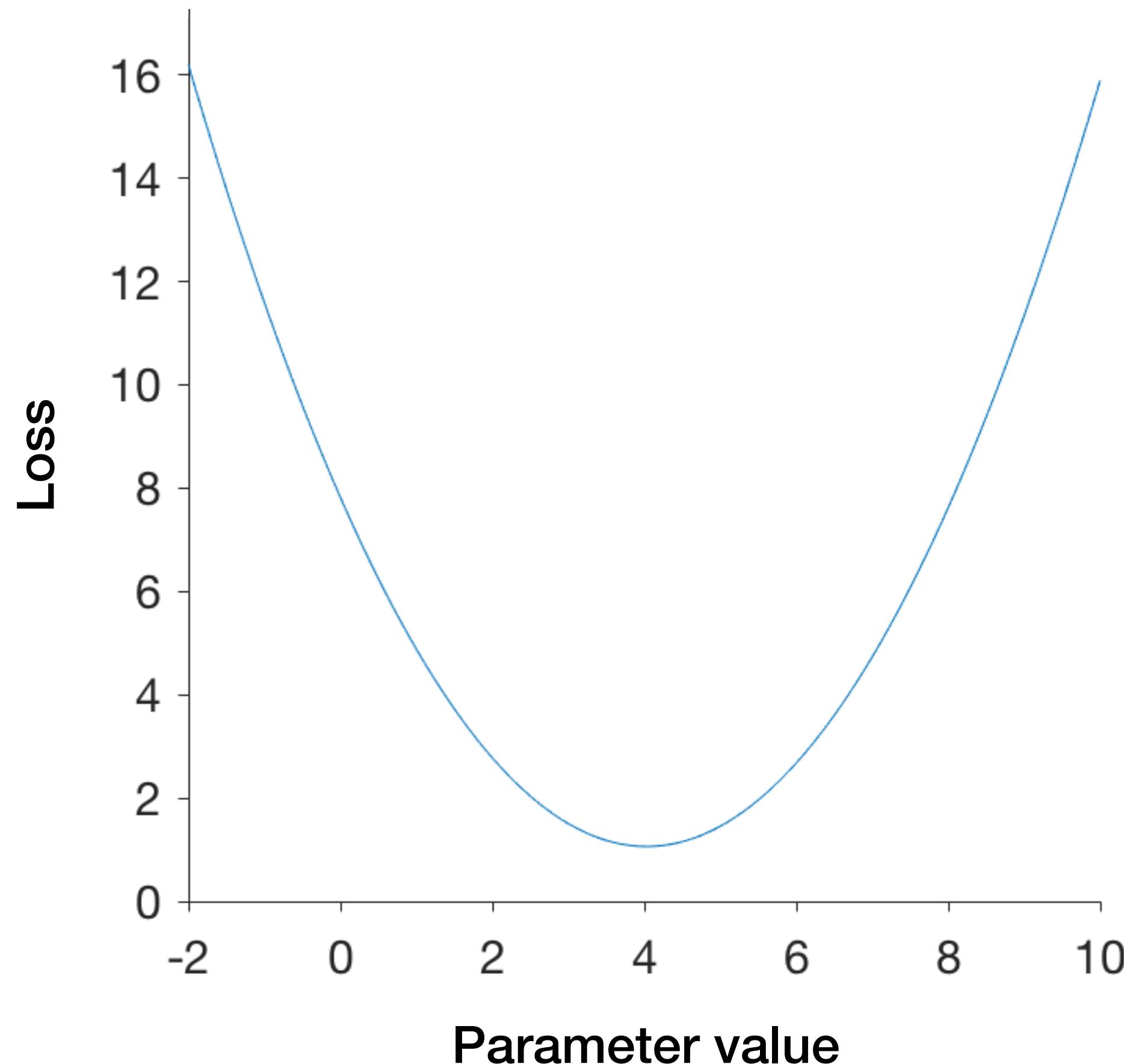
What are the parameters?

What is the objective (loss)?

How do we find the parameters?

Gradient descent for one parameter (a.k.a., “try something, then keep trying to make it better”)

**Loss landscape = loss for
different parameter values**

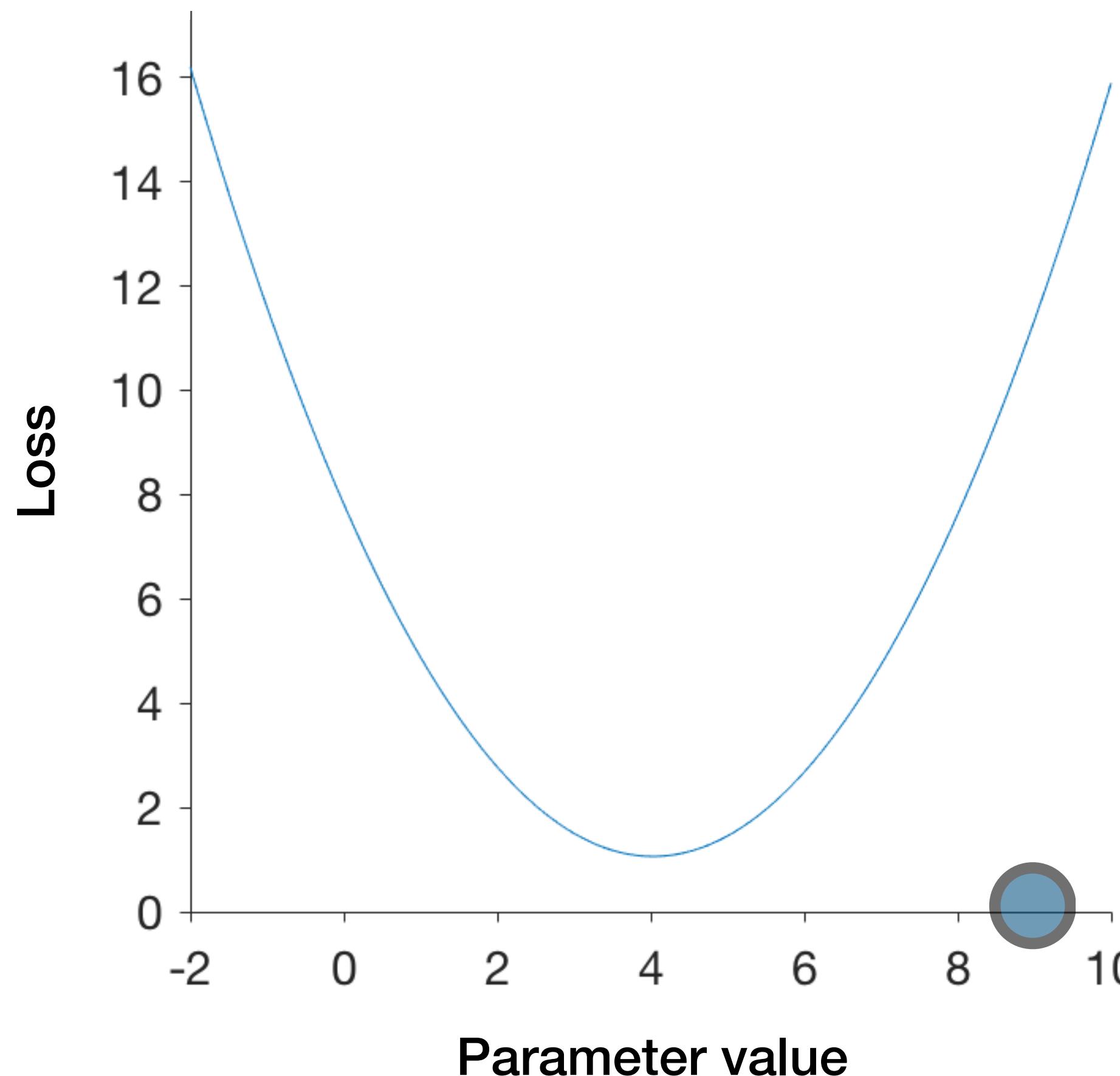


Gradient descent for one parameter

(a.k.a., “try something, then keep trying to make it better”)

**Loss landscape = loss for
different parameter values**

1. Guess a parameter value

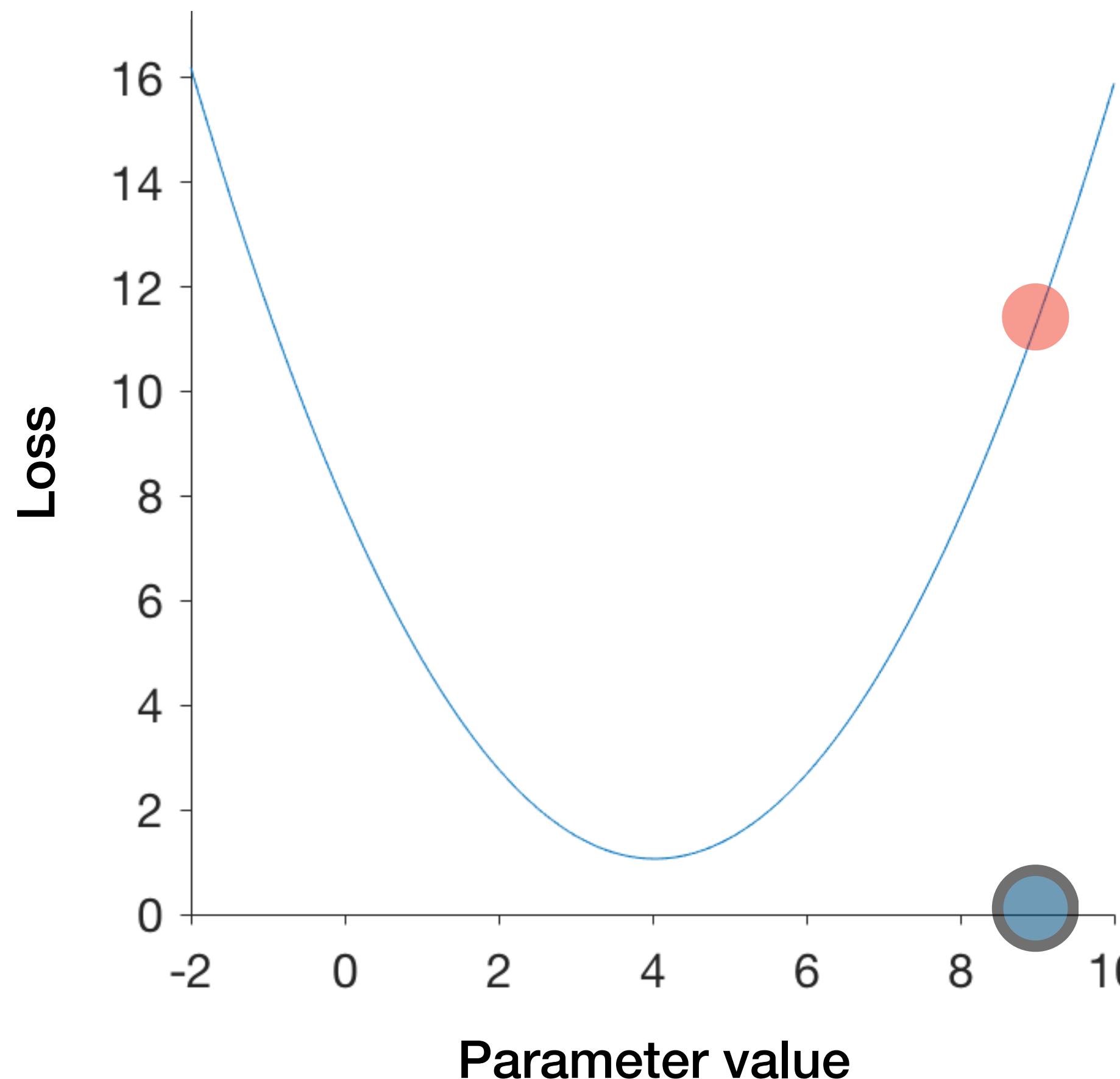


Gradient descent for one parameter

(a.k.a., “try something, then keep trying to make it better”)

**Loss landscape = loss for
different parameter values**

1. Guess a parameter value
2. Evaluate the loss

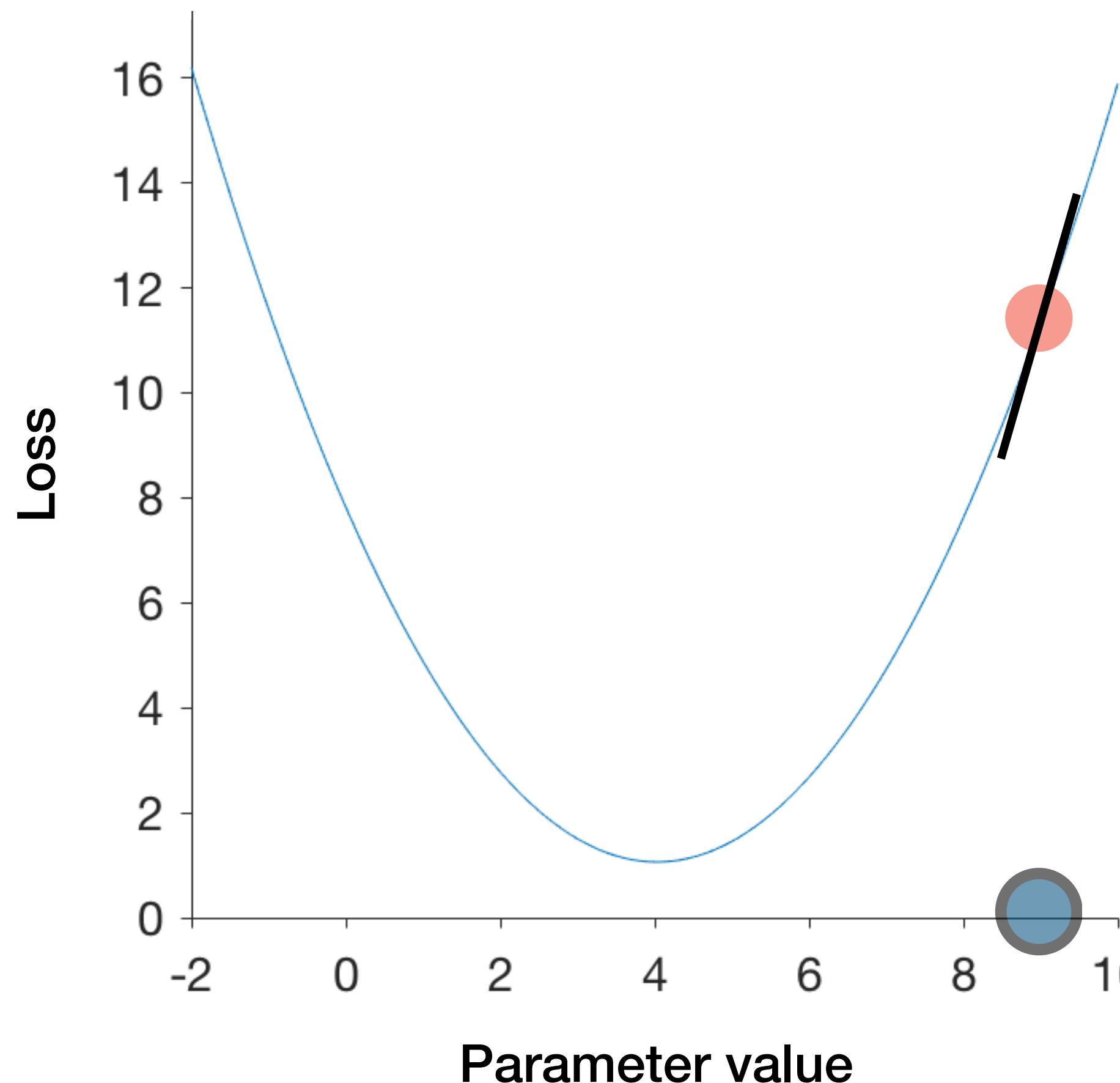


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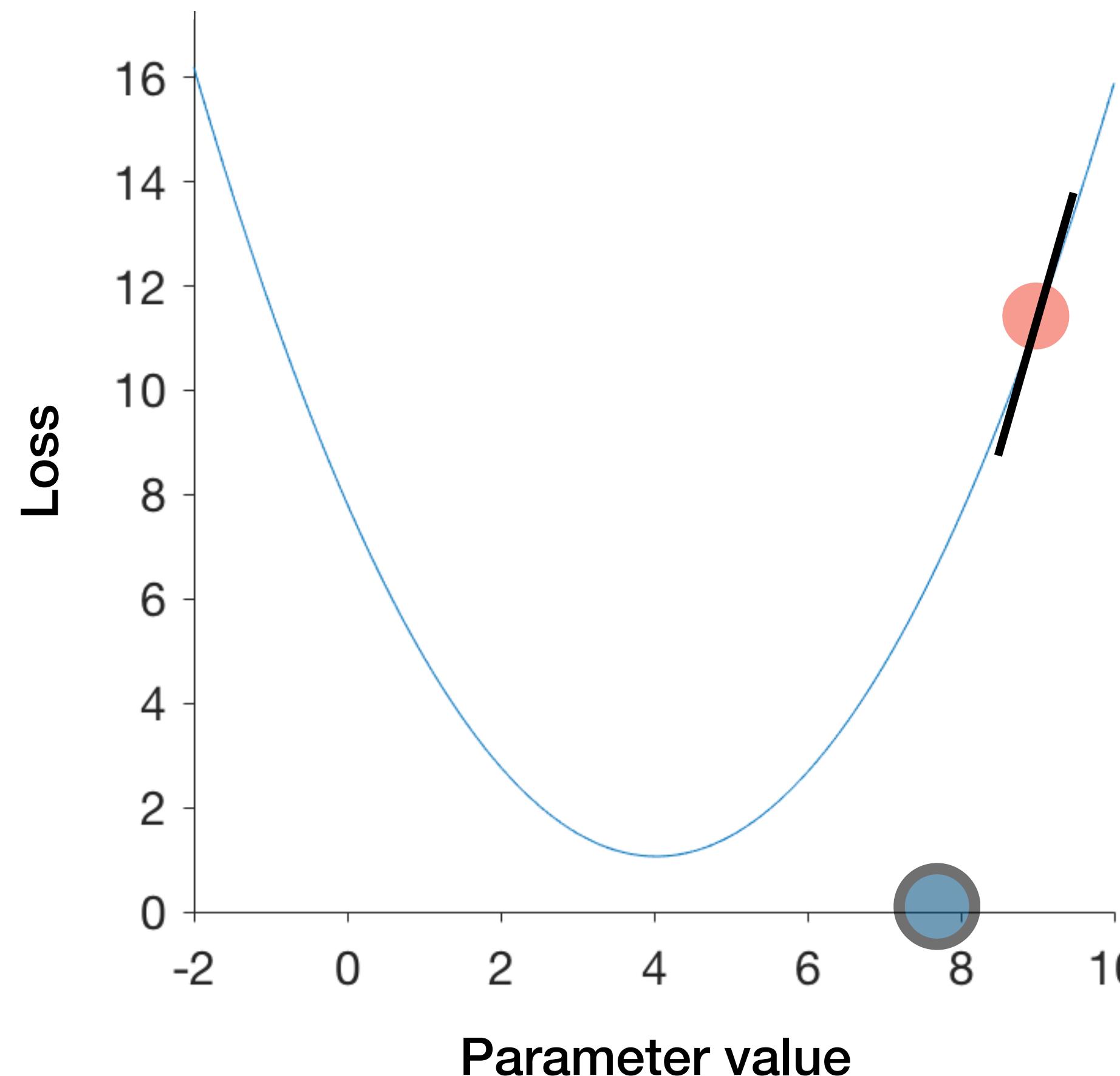
1. Guess a parameter value
2. Evaluate the loss
3. Calculate loss derivative w.r.t. parameter (gradient)



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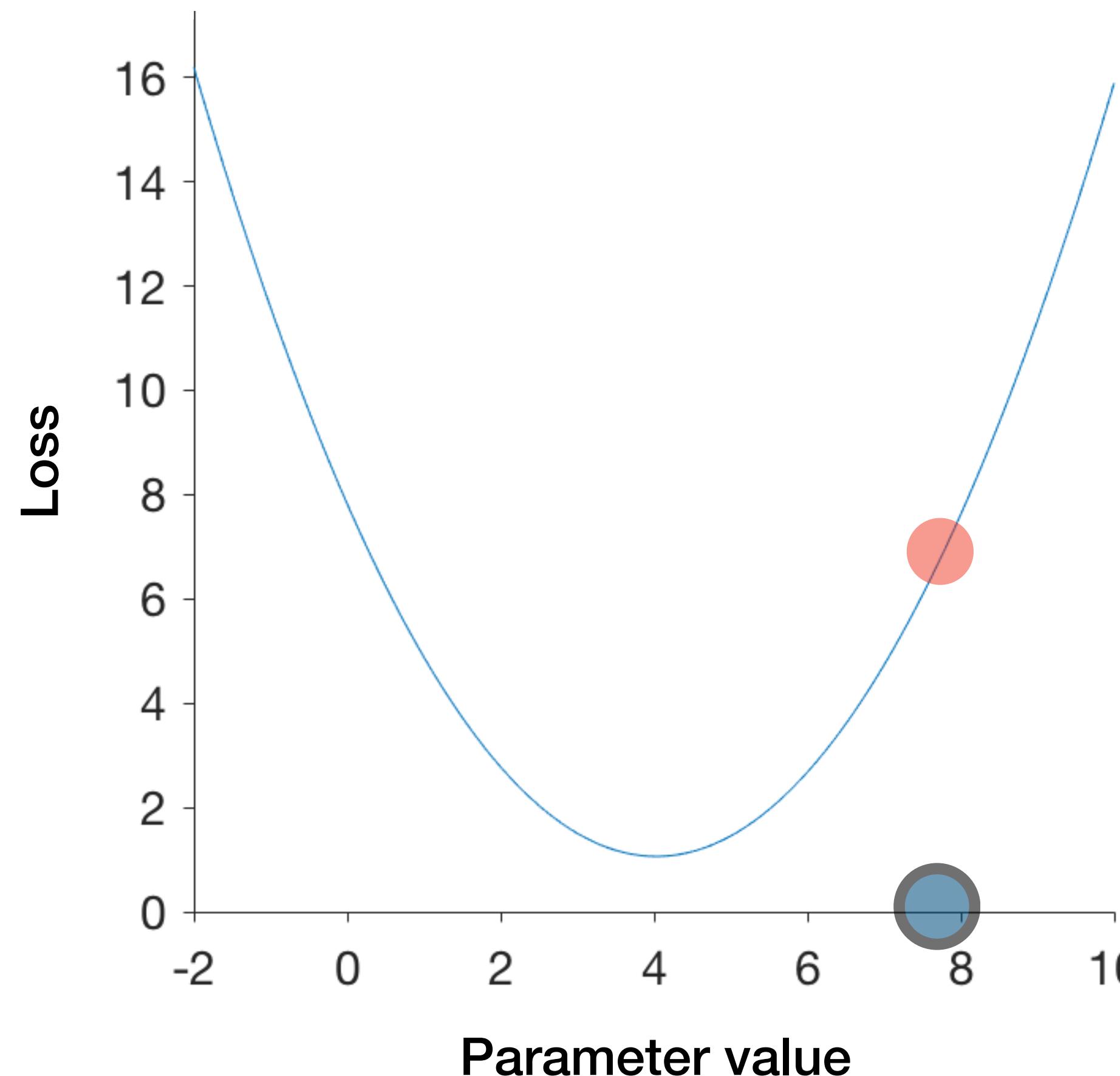


1. Guess a parameter value
2. Evaluate the loss
3. Calculate loss derivative w.r.t. parameter (gradient)
4. Step down the gradient

Gradient descent for one parameter

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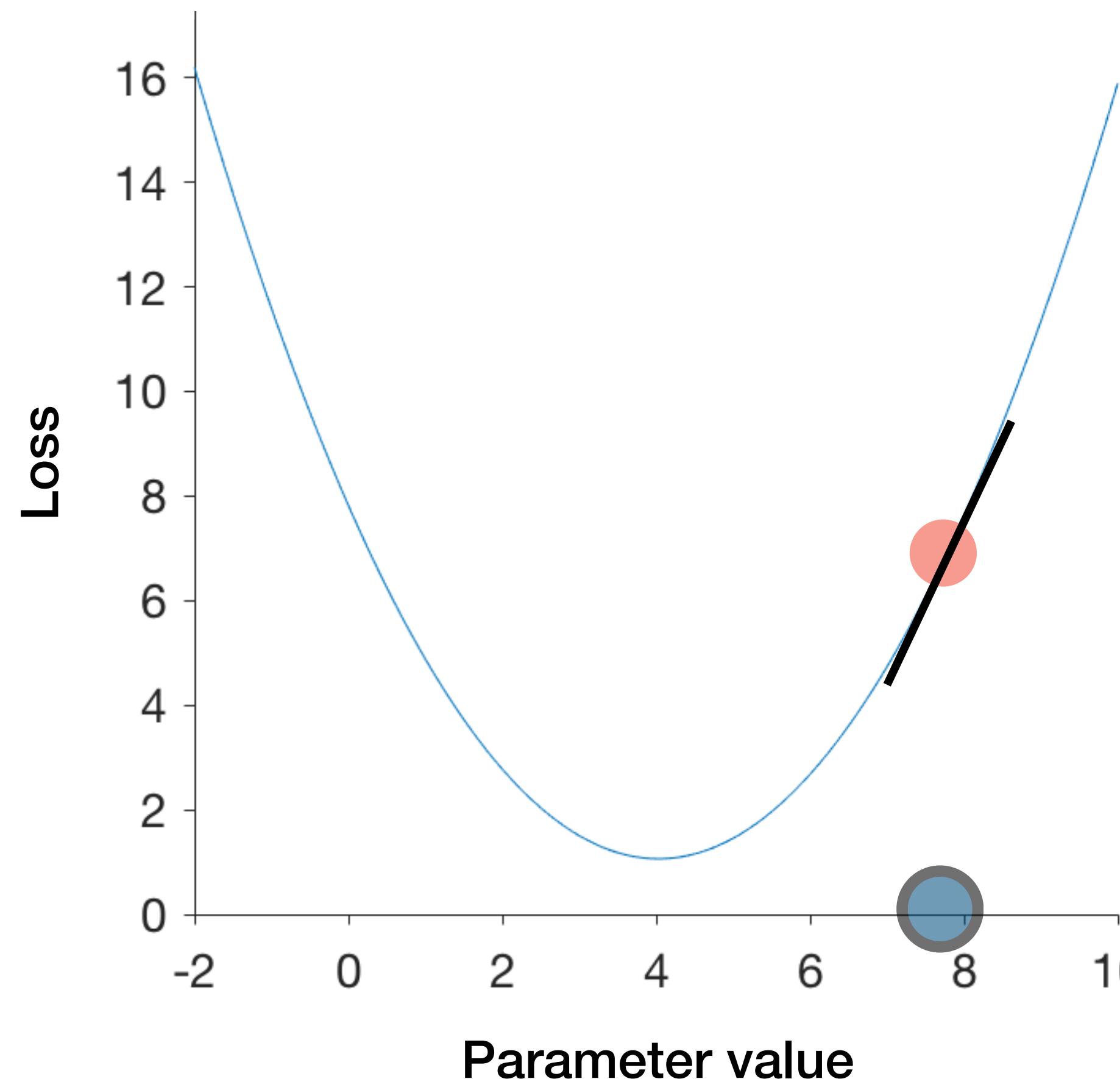


1. Guess a parameter value
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3. Calculate loss derivative w.r.t. parameter (gradient)
4. Step down the gradient
5. Repeat!

Gradient descent for one parameter

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Loss landscape = loss for different parameter values

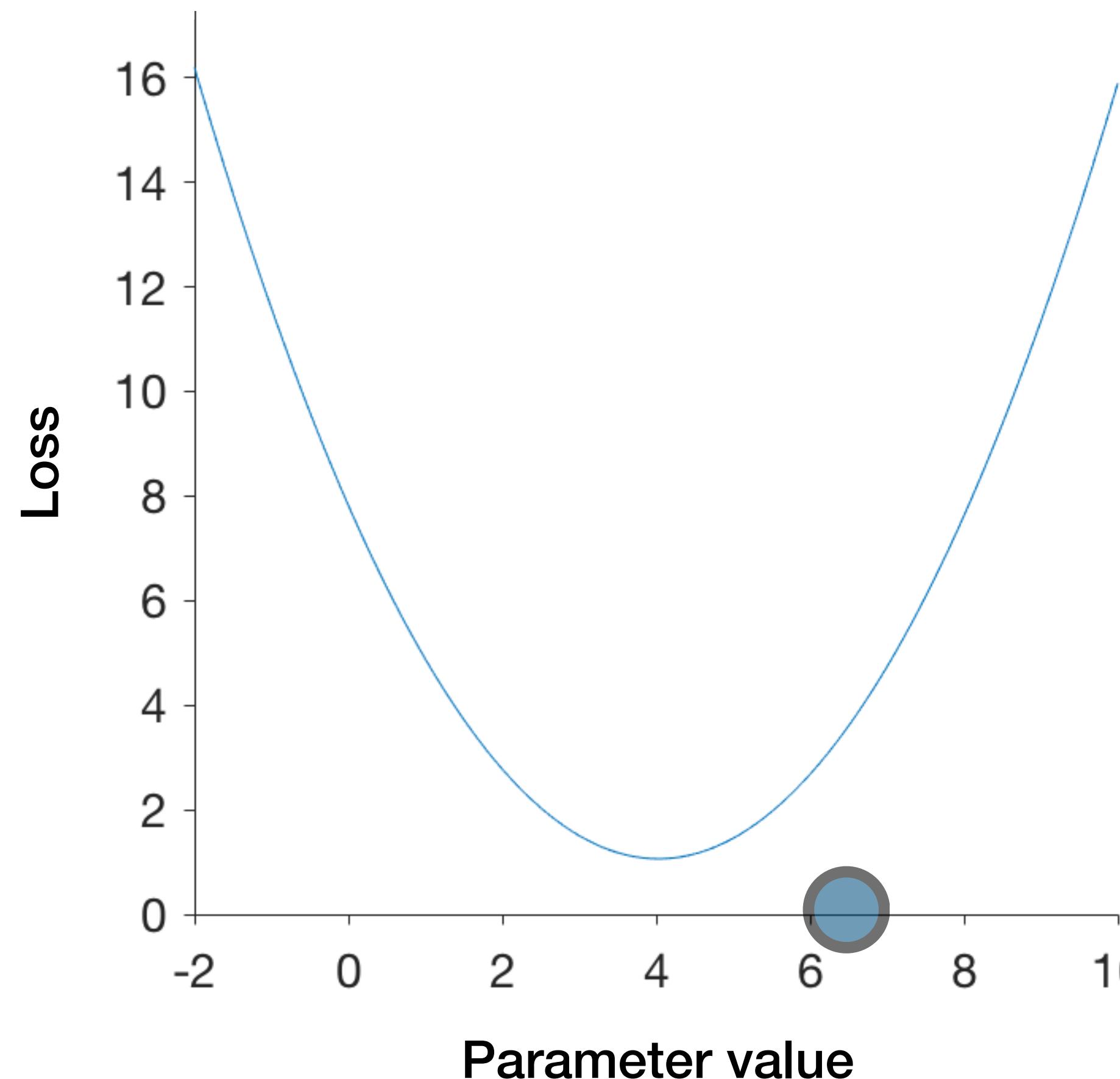


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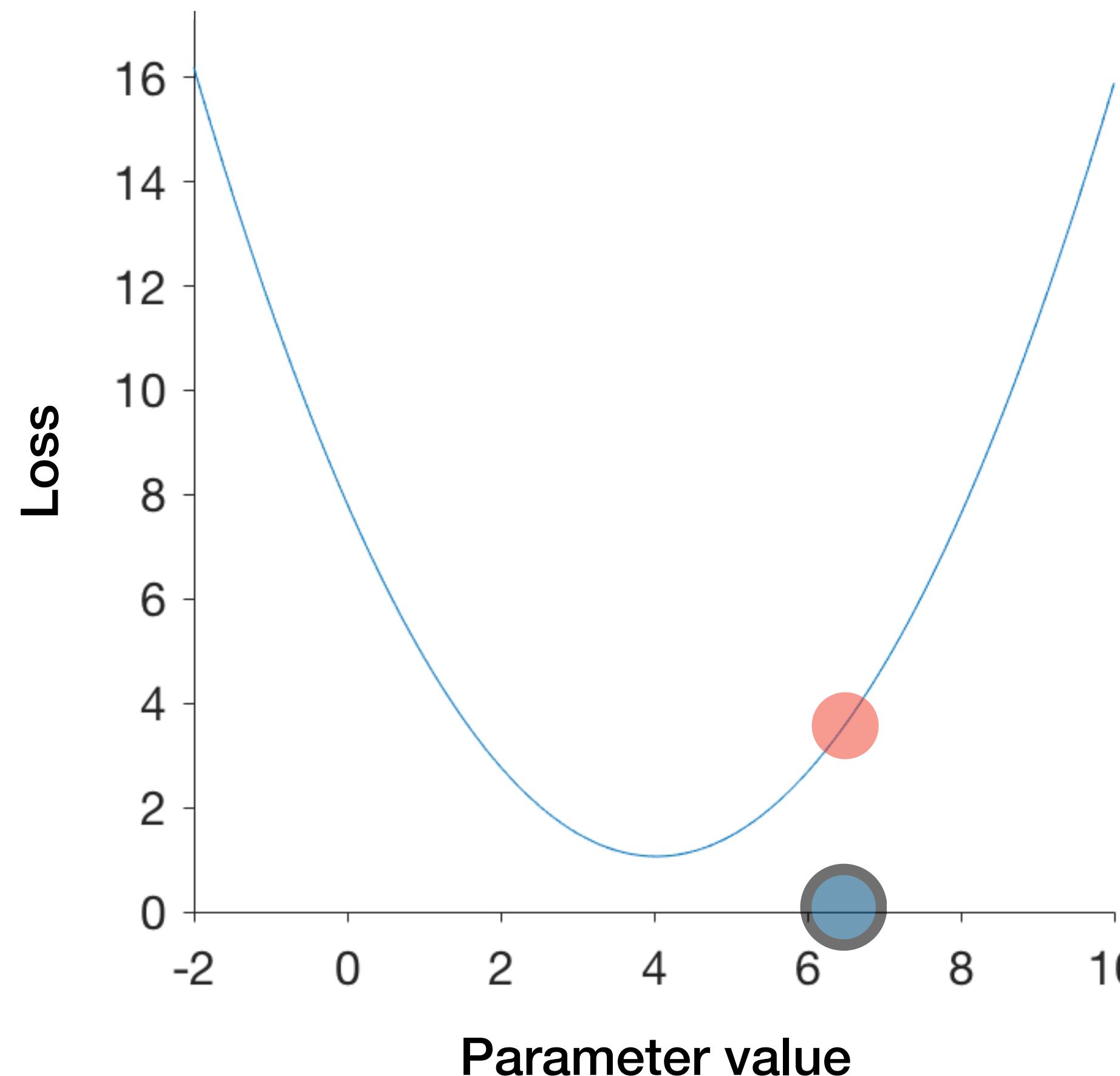


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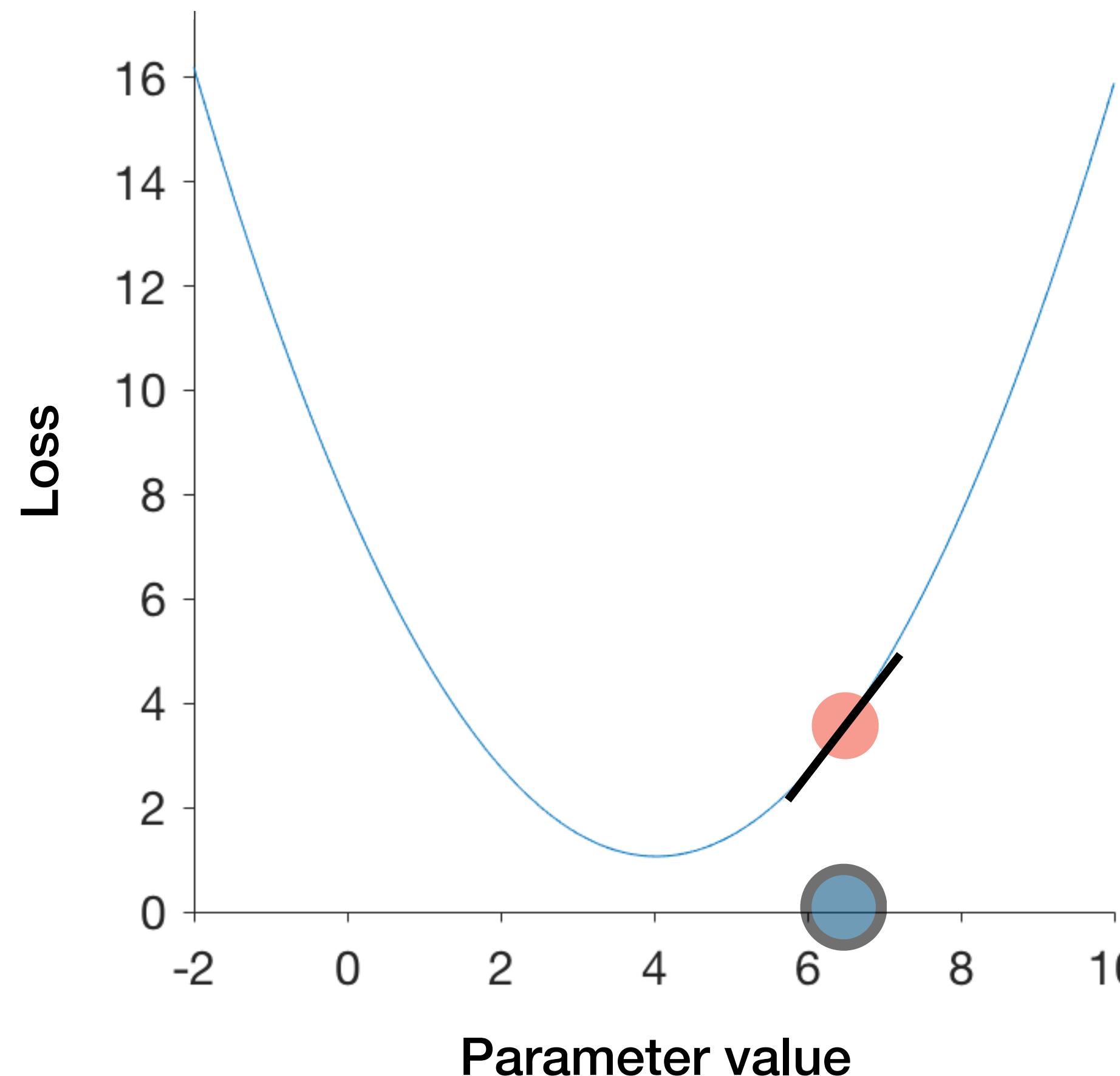


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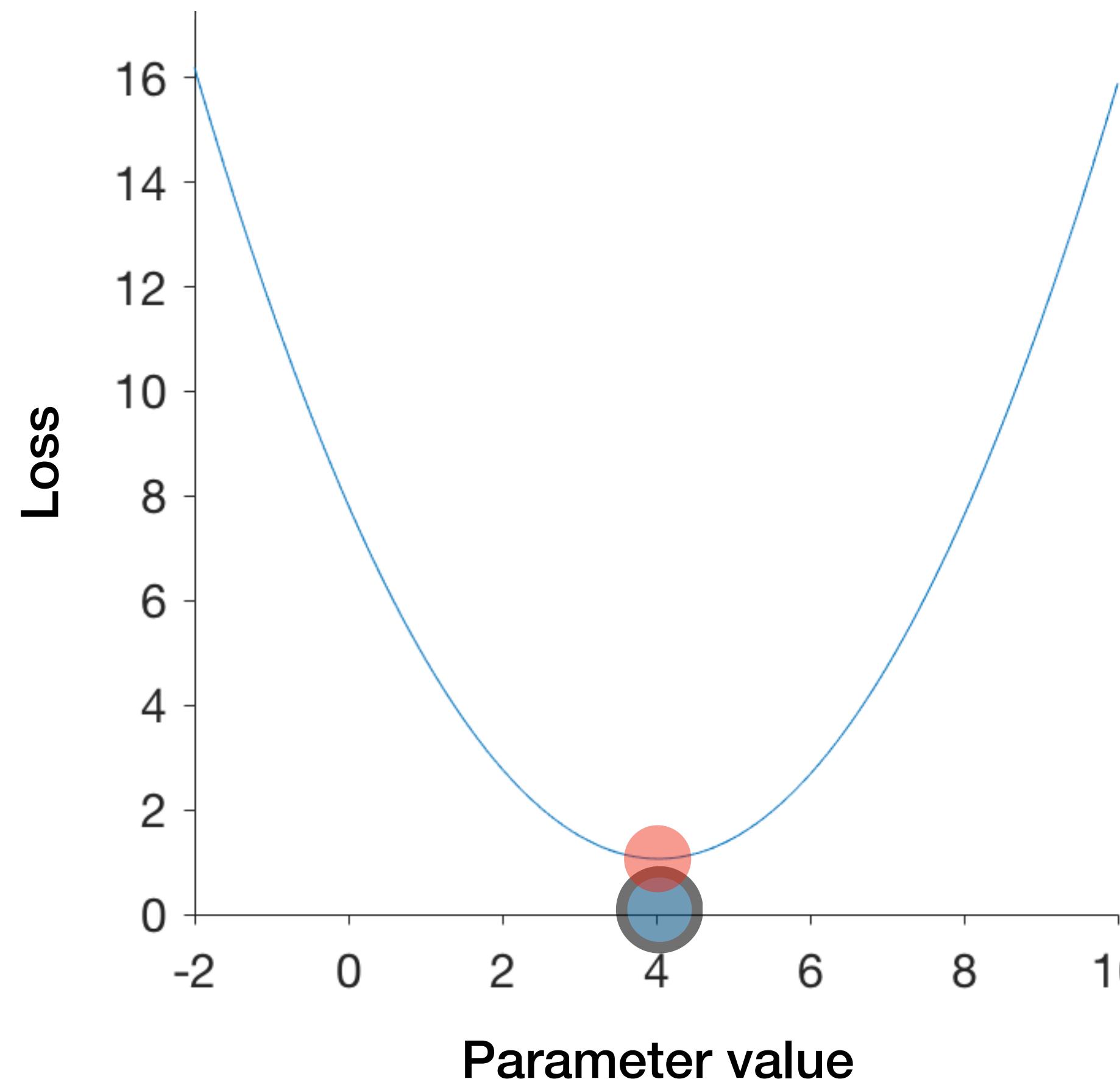


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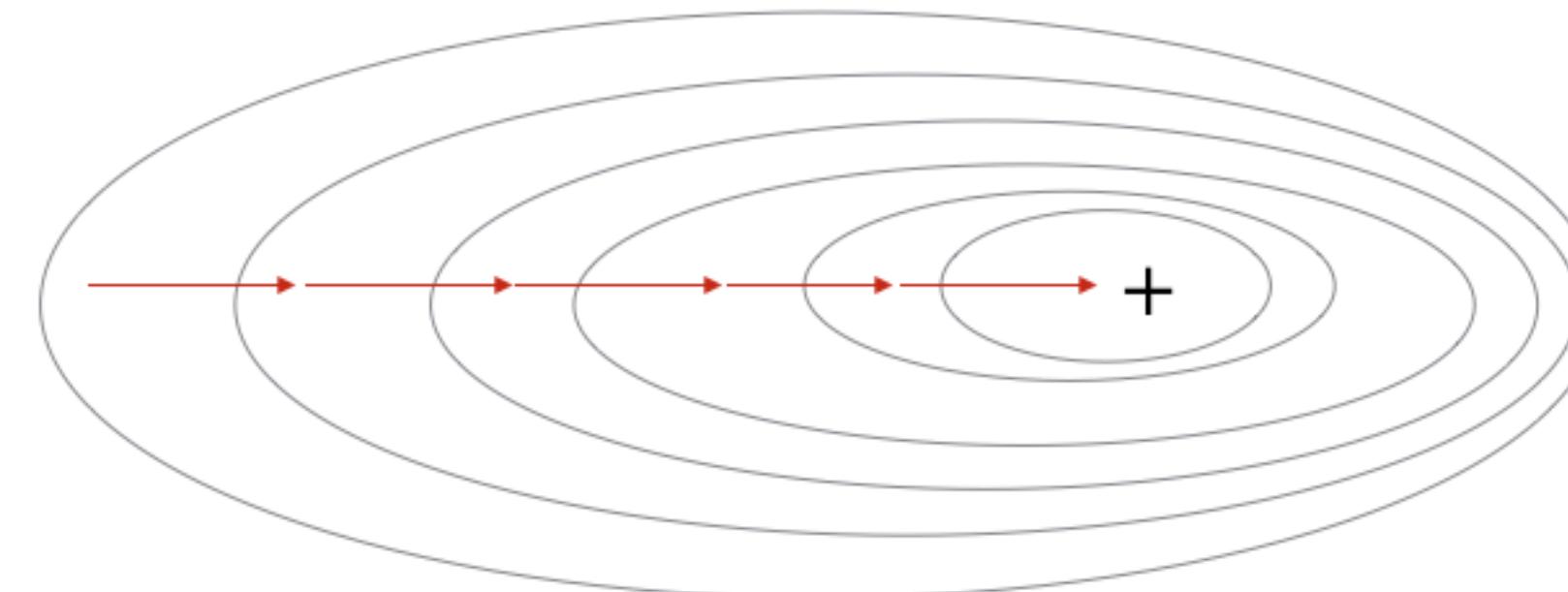
Loss landscape = loss for different parameter values



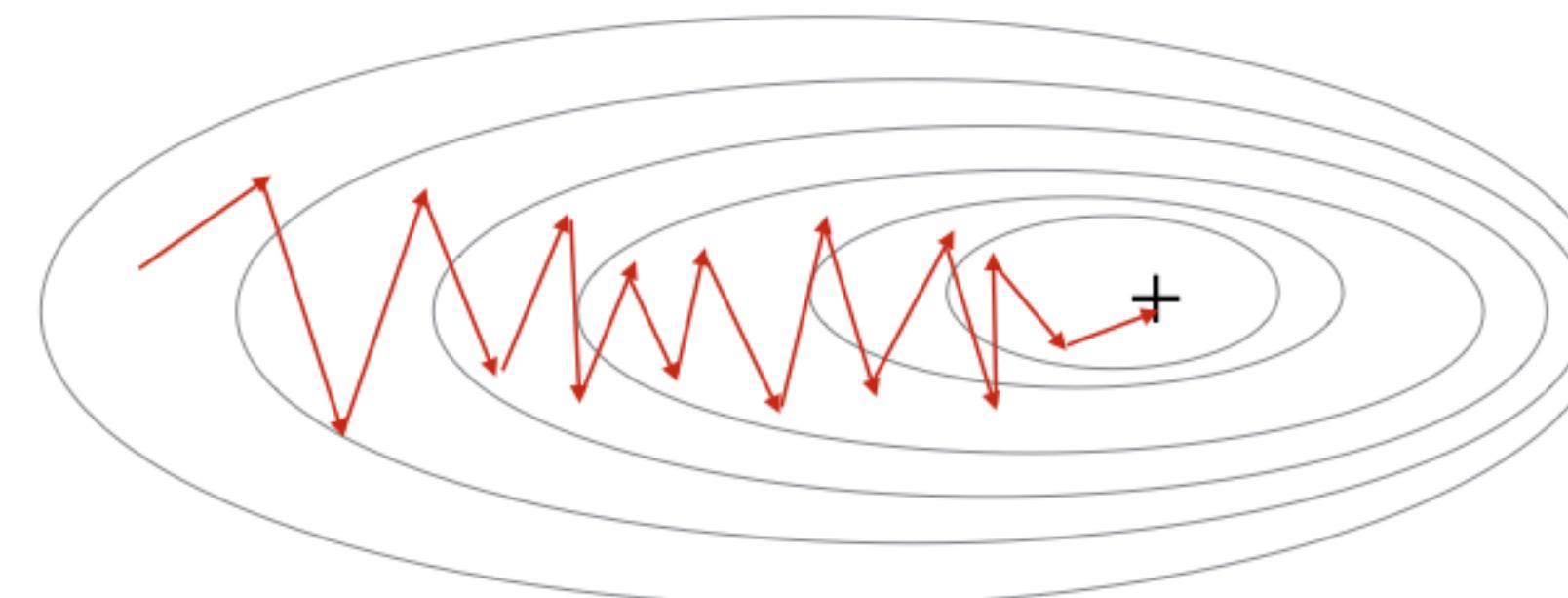
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4. Step down the gradient
5. Repeat!

Gradient Descent and Stochastic Gradient Descent

Gradient Descent

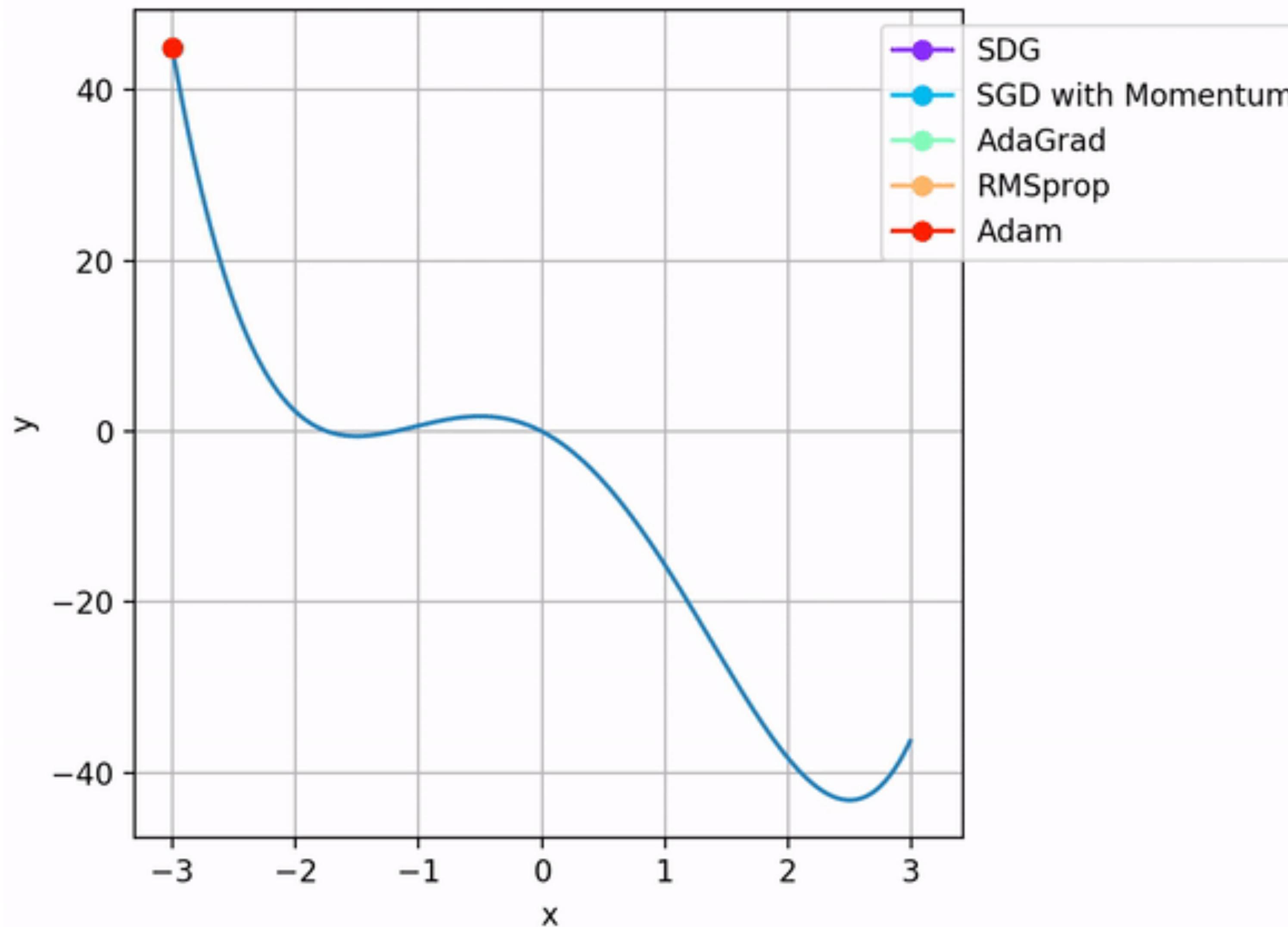


Stochastic Gradient Descent



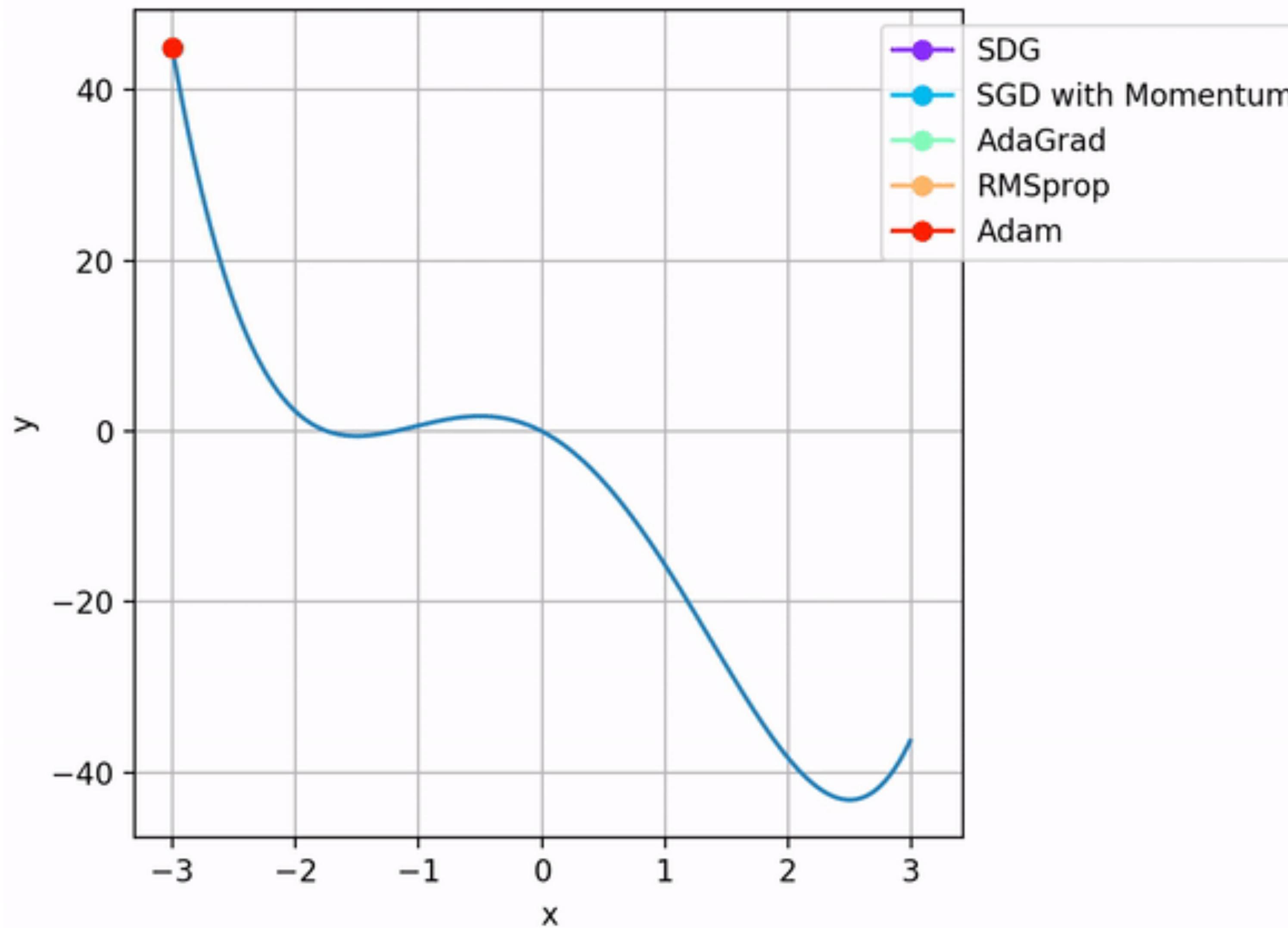
<https://towardsdatascience.com/complete-guide-to-adam-optimization-1e5f29532c3d>

Optimizer Comparison



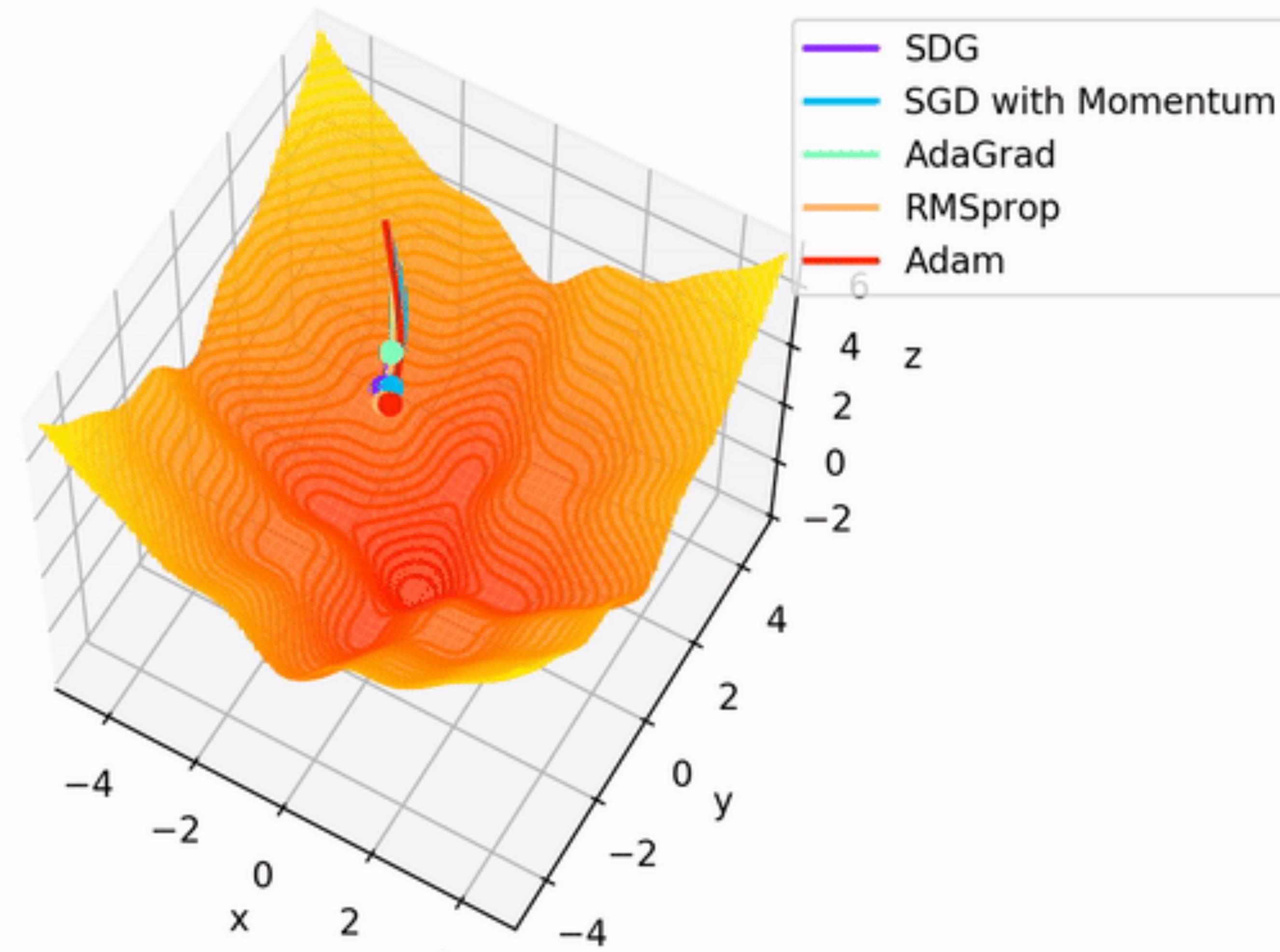
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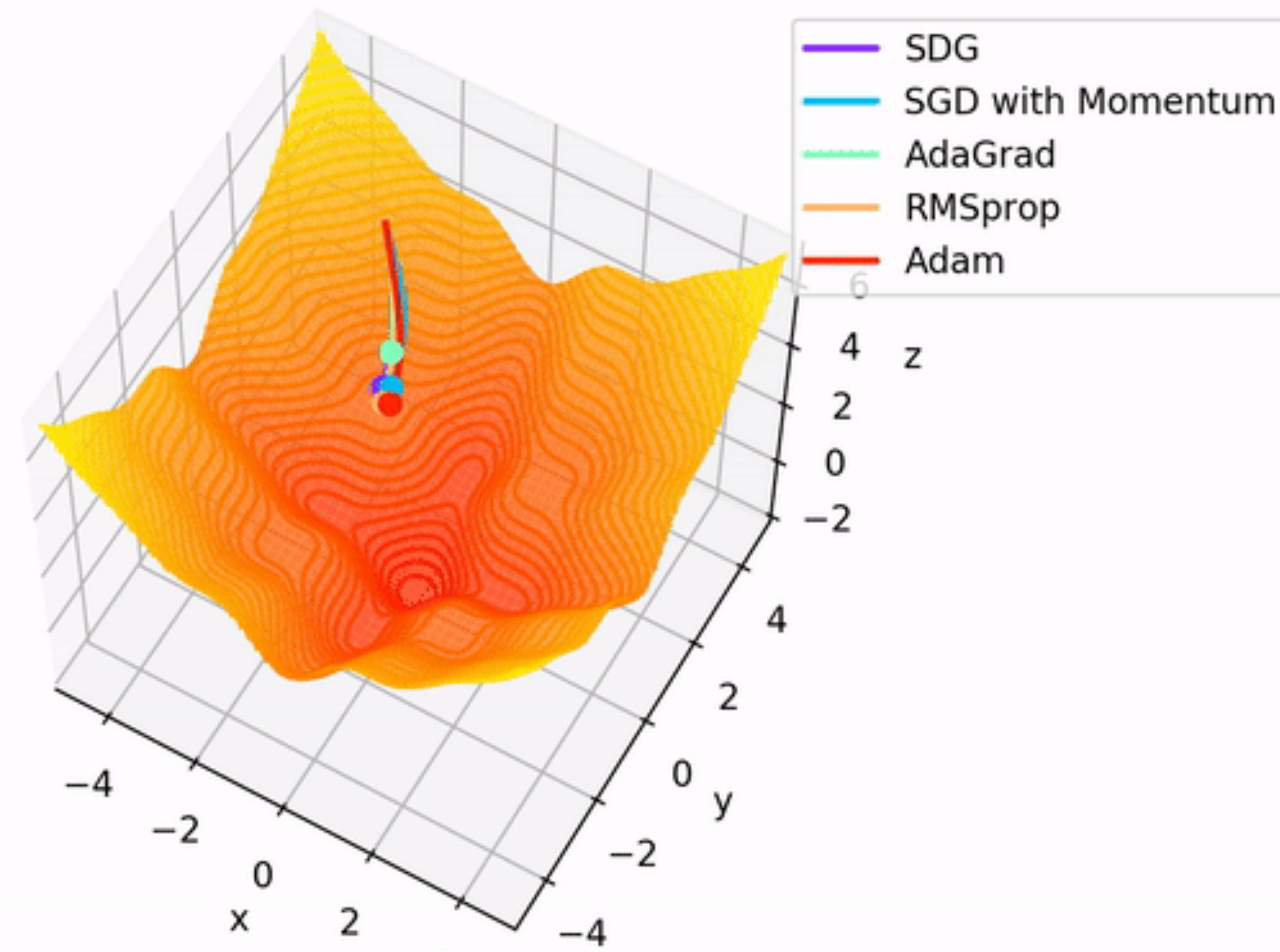
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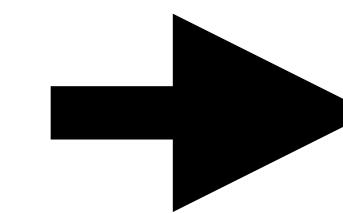
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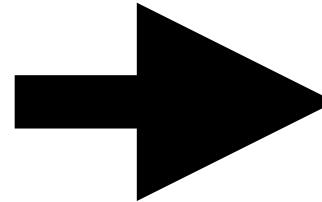


<https://towardsdatascience.com/complete-guide-to-adam-optimization-1e5f29532c3d>

High-D problems



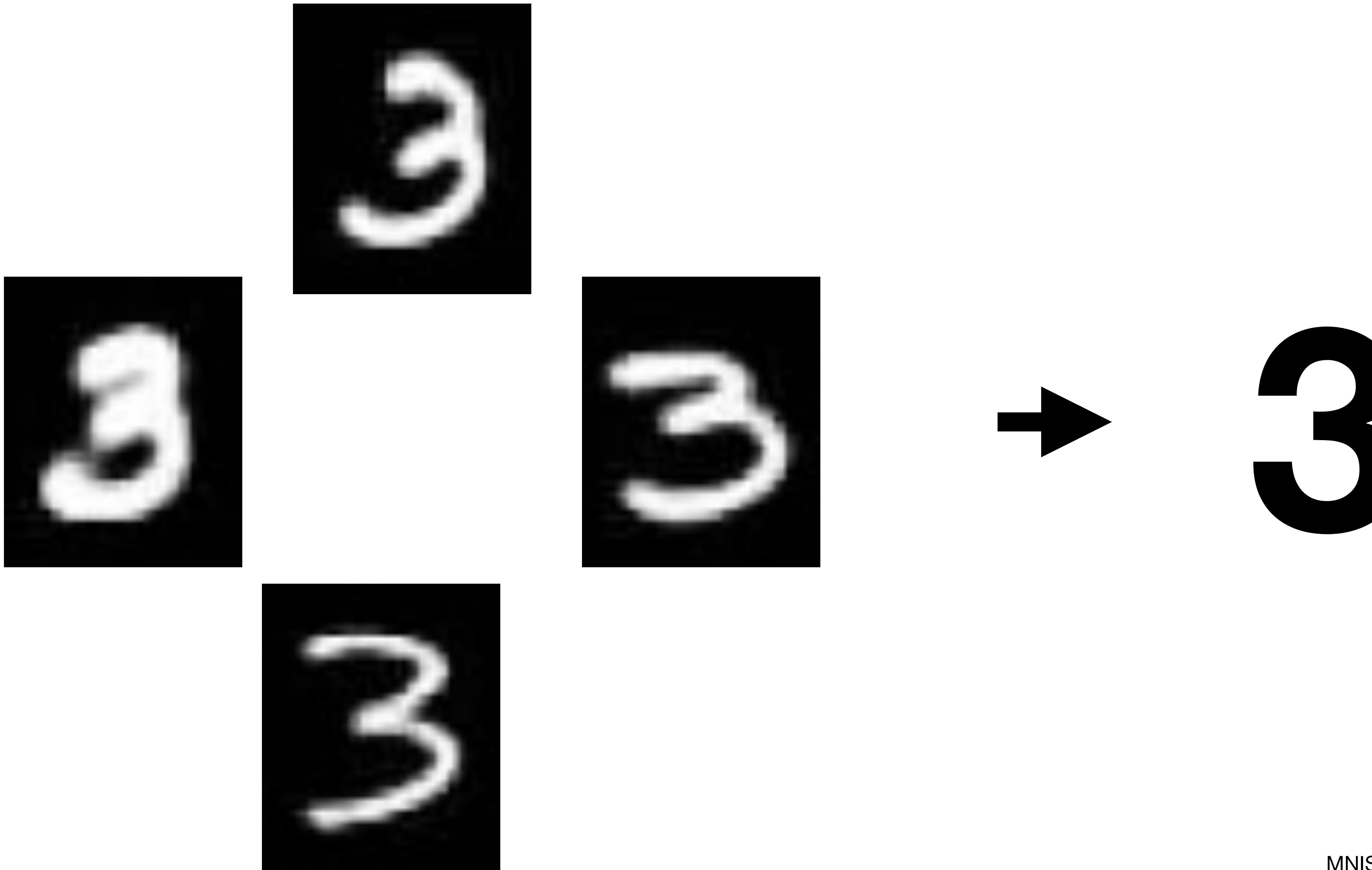
High-D problems



High-D problems



High-D problems



28 pixels

MNIST - 784 total pixels each ranges from 0 to 1

3Blue1Brown (YouTube)

But what is a Neural Network? | Deep learning, chapter 1

28 pixels

28 pixels

MNIST - 784 total pixels each ranges from 0 to 1



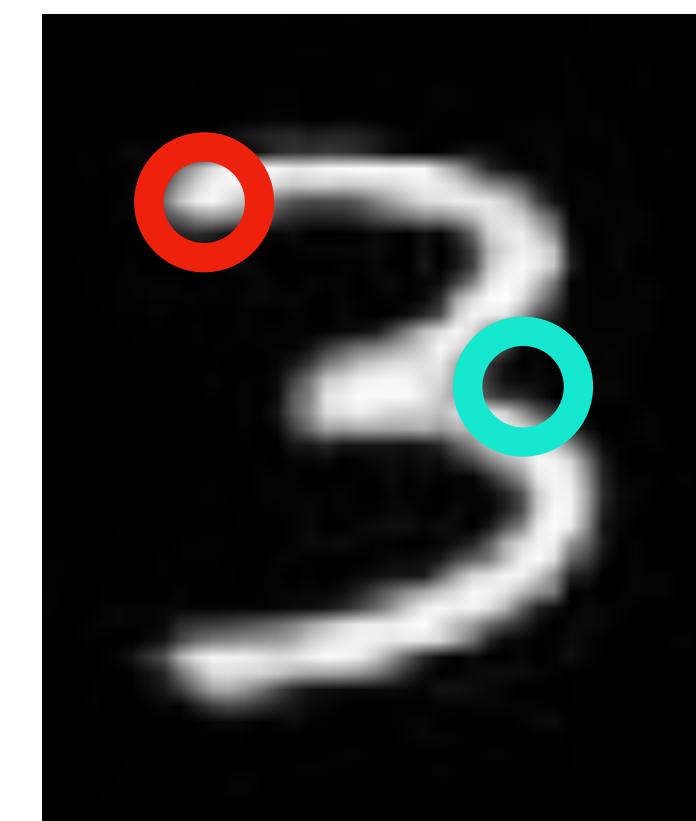
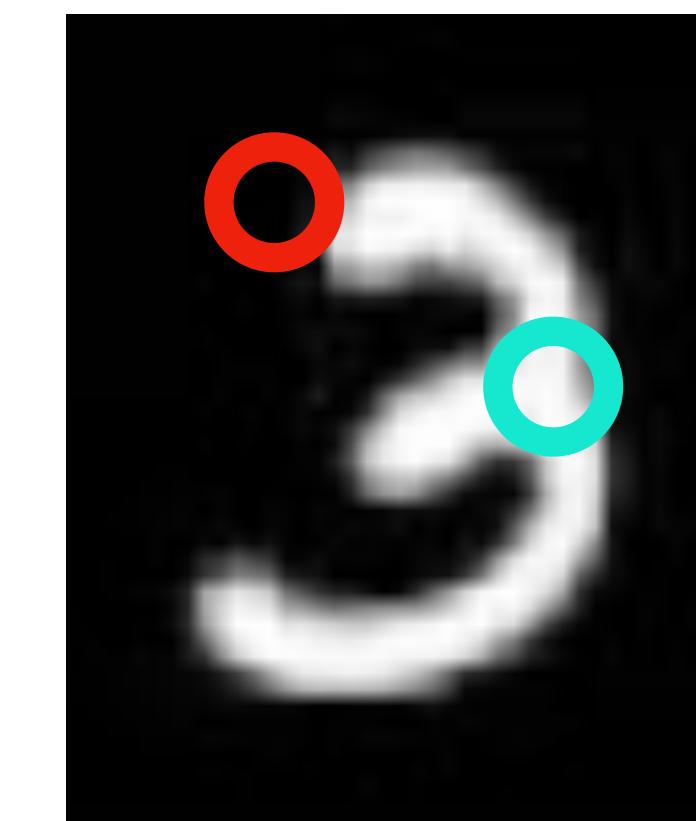
3Blue1Brown (YouTube)

But what is a Neural Network? | Deep learning, chapter 1

28 pixels

28 pixels

**MNIST - 784 total pixels
each ranges from 0 to 1**



Specific pixel values are very different between images

3Blue1Brown (YouTube)

But what is a Neural Network? | Deep learning, chapter 1

28 pixels

A thick black double-headed horizontal arrow, pointing left and right, centered above the section title.

**MNIST - 784 total pixels
each ranges from 0 to 1**



Specific pixel values are very different between images



3Blue1Brown (YouTube)

But what is a Neural Network? | Deep learning, chapter 1

THIS IS YOUR MACHINE LEARNING SYSTEM?

YUP! YOU POUR THE DATA INTO THIS BIG
PILE OF LINEAR ALGEBRA, THEN COLLECT
THE ANSWERS ON THE OTHER SIDE.

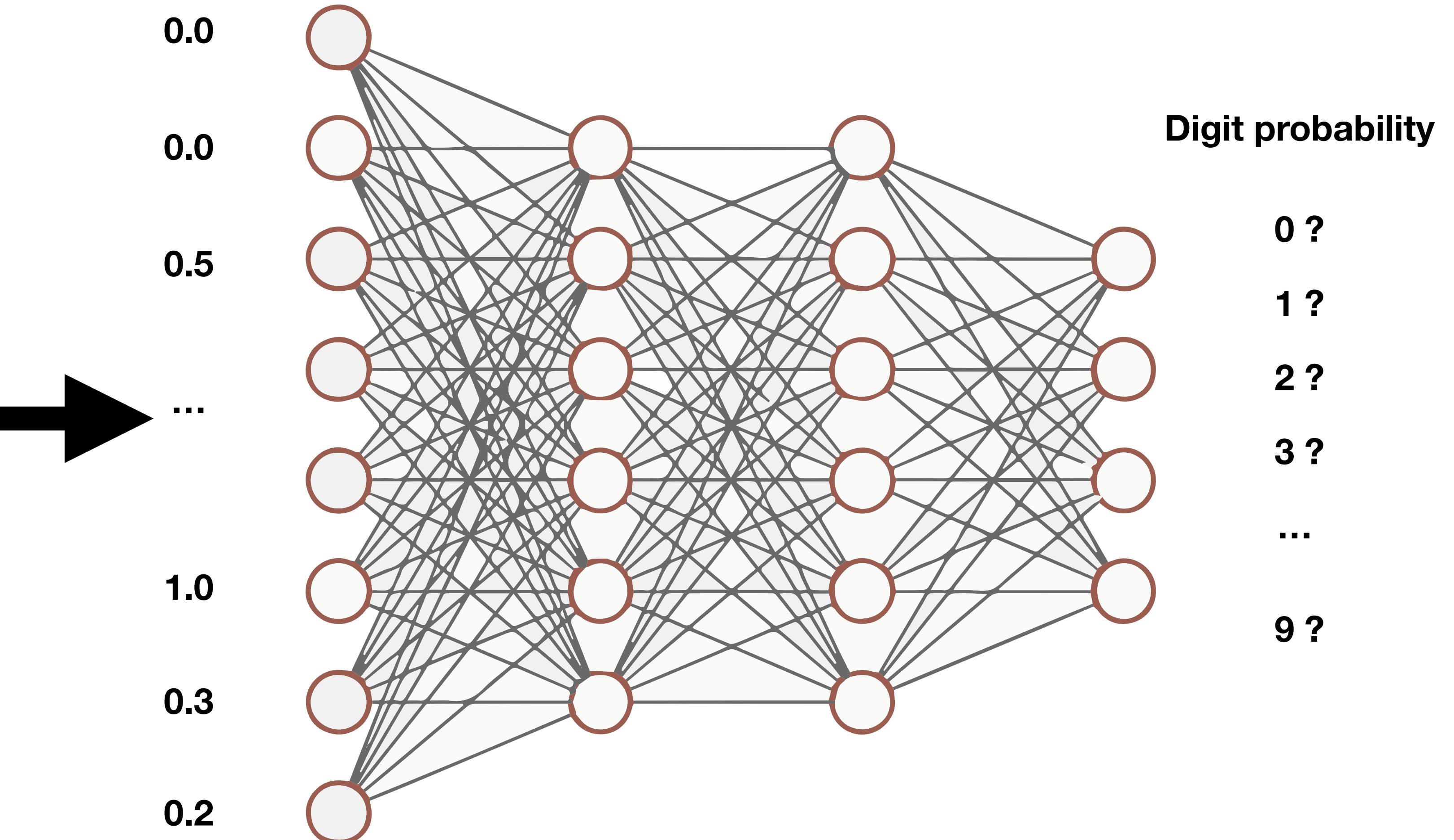
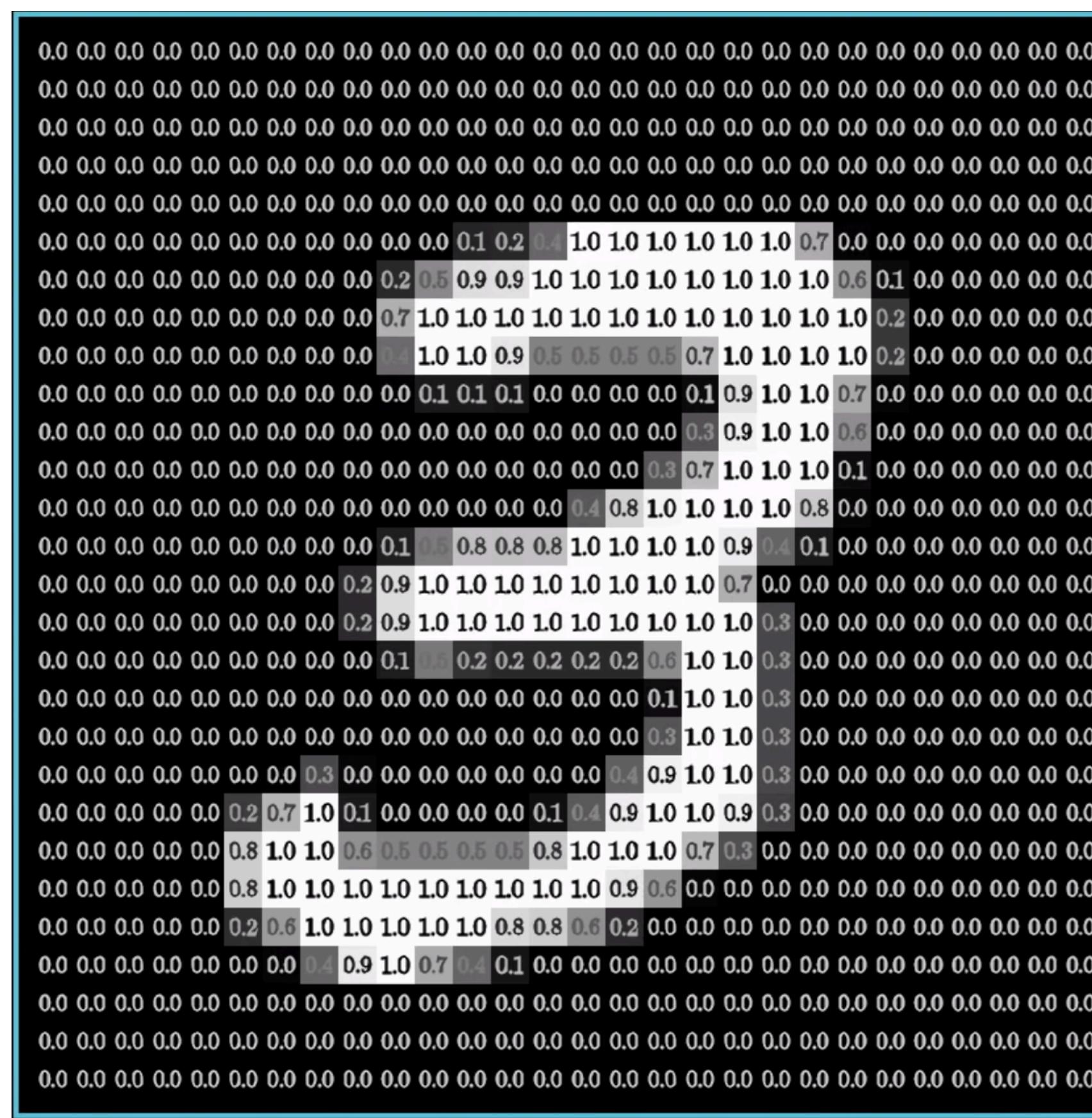
WHAT IF THE ANSWERS ARE WRONG?

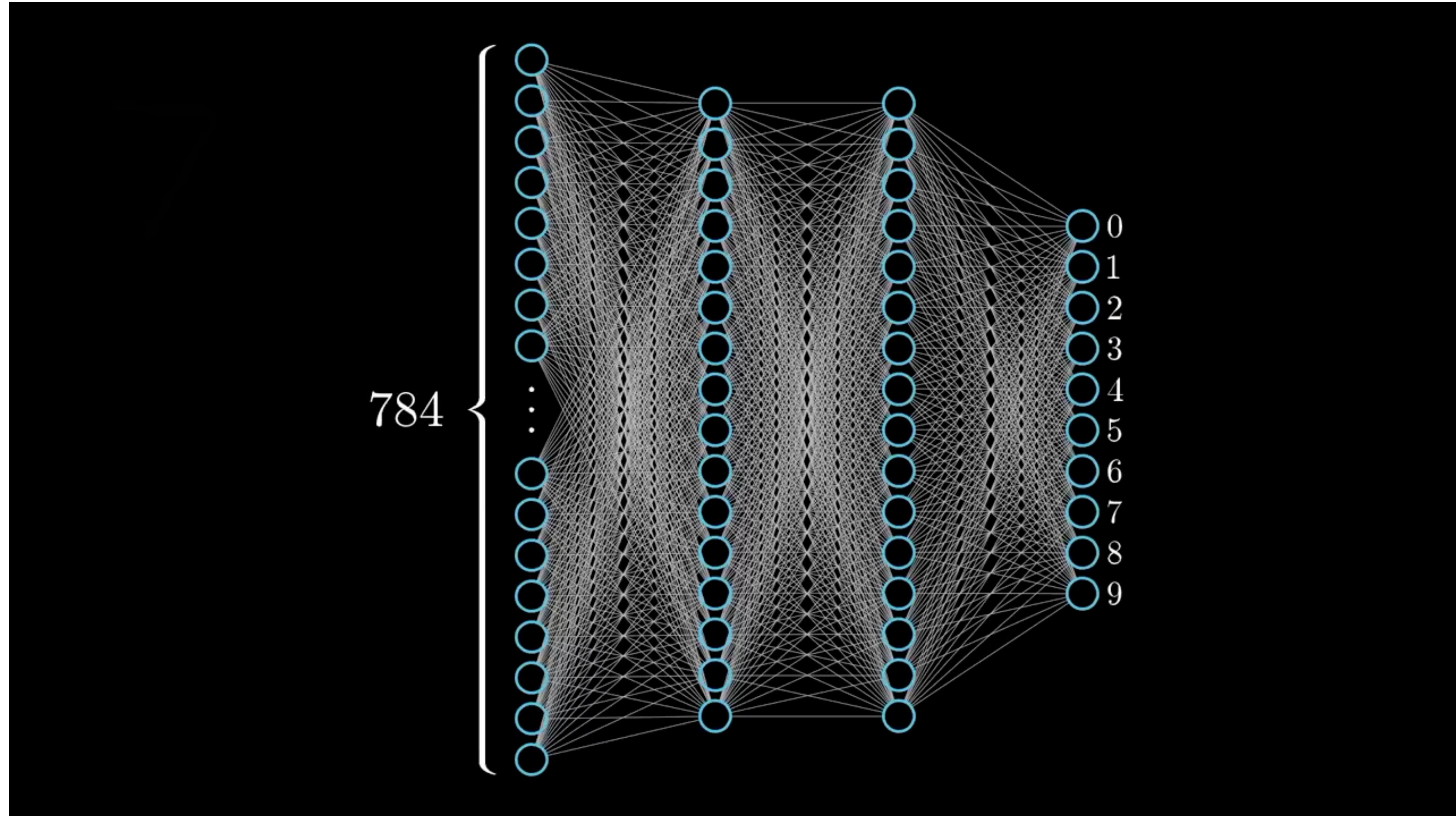
JUST STIR THE PILE UNTIL
THEY START LOOKING RIGHT.



MLP for digit classification

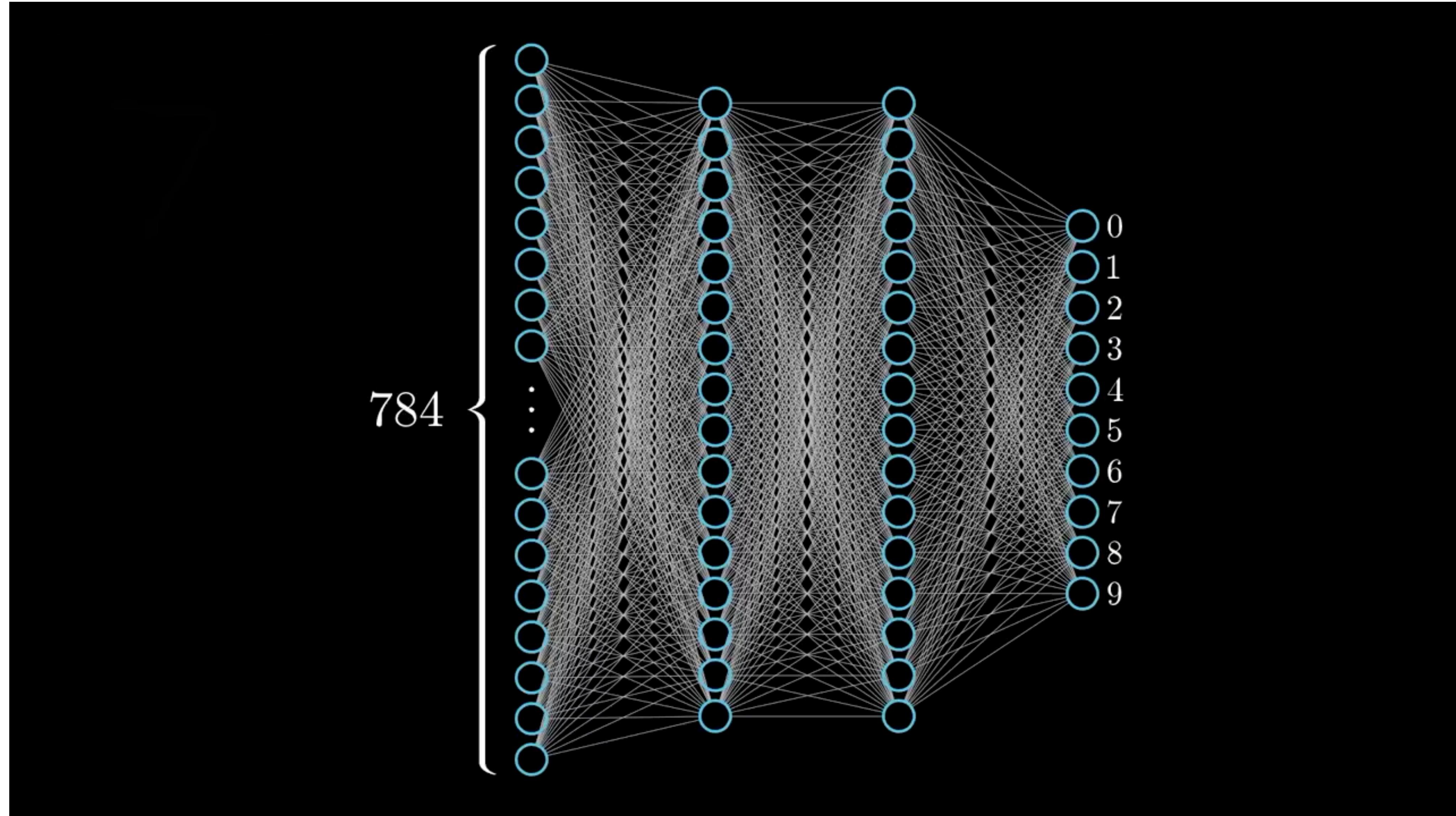
Pixel values





3Blue1Brown (YouTube)

But what is a Neural Network? | Deep learning, chapter 1



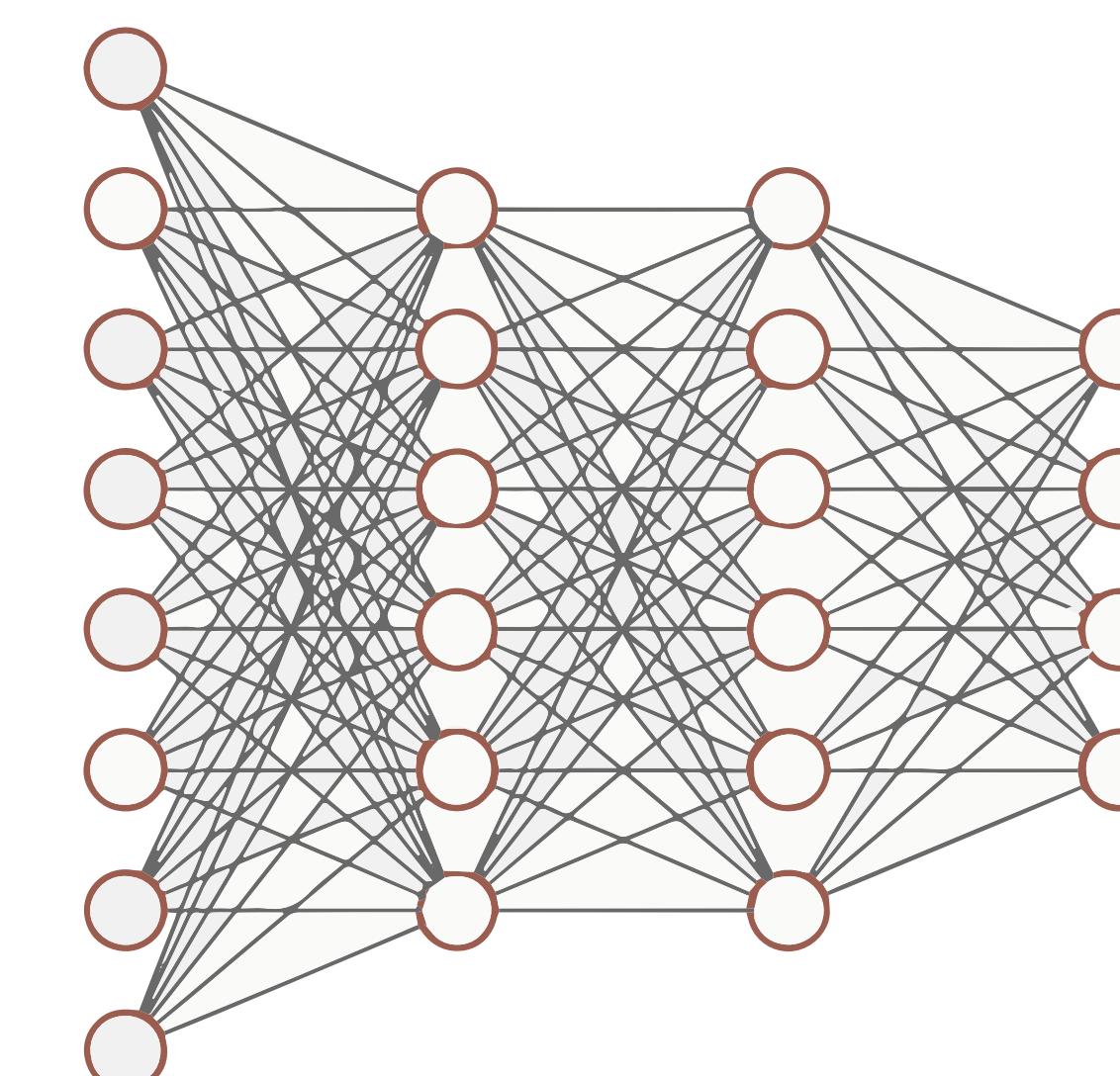
3Blue1Brown (YouTube)

But what is a Neural Network? | Deep learning, chapter 1

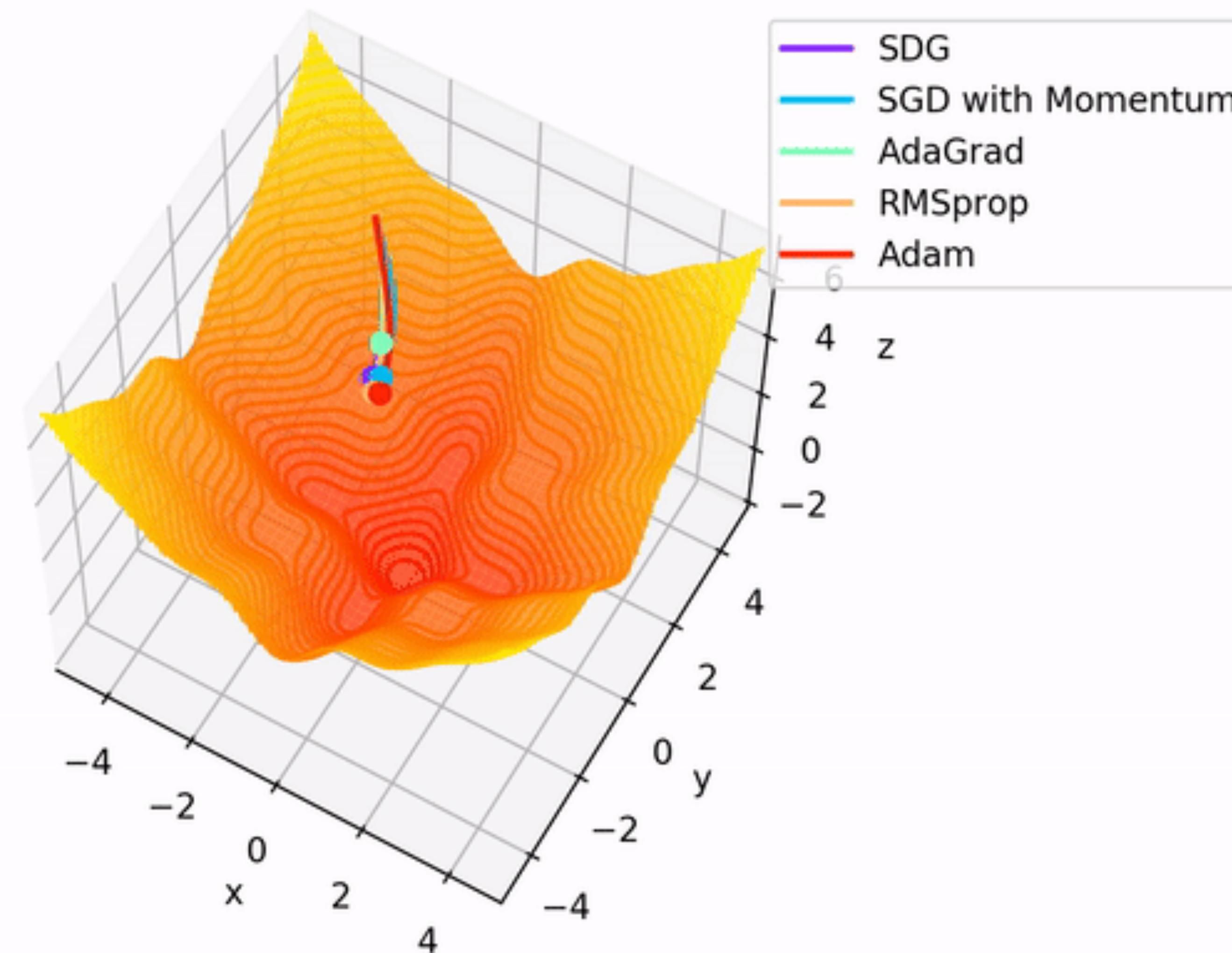
28 pixels



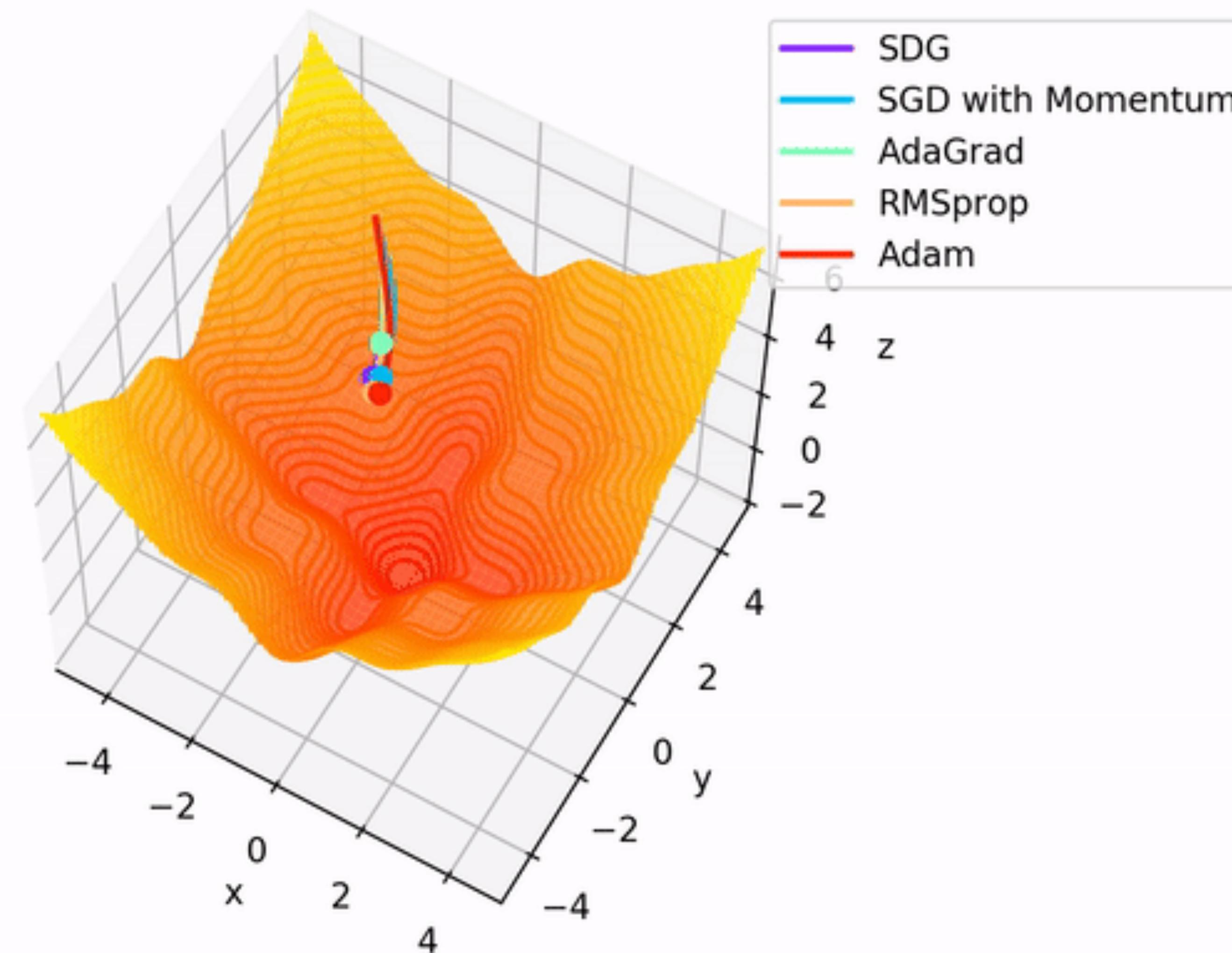
784-dimensional input



Optimizer Comparison



Optimizer Comparison



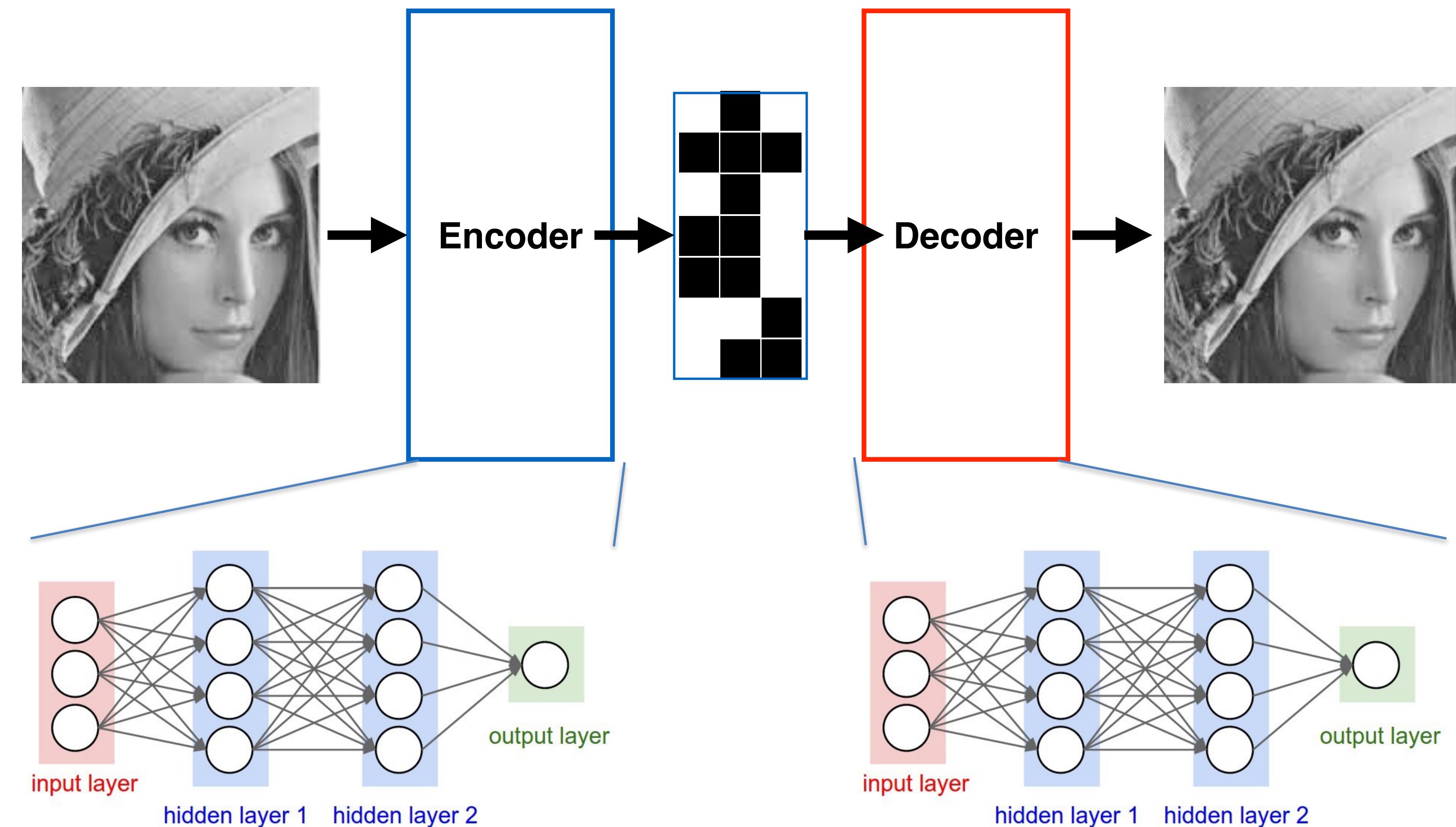
What you'll hear about in this lecture

Neural network basics

Deep autoencoders

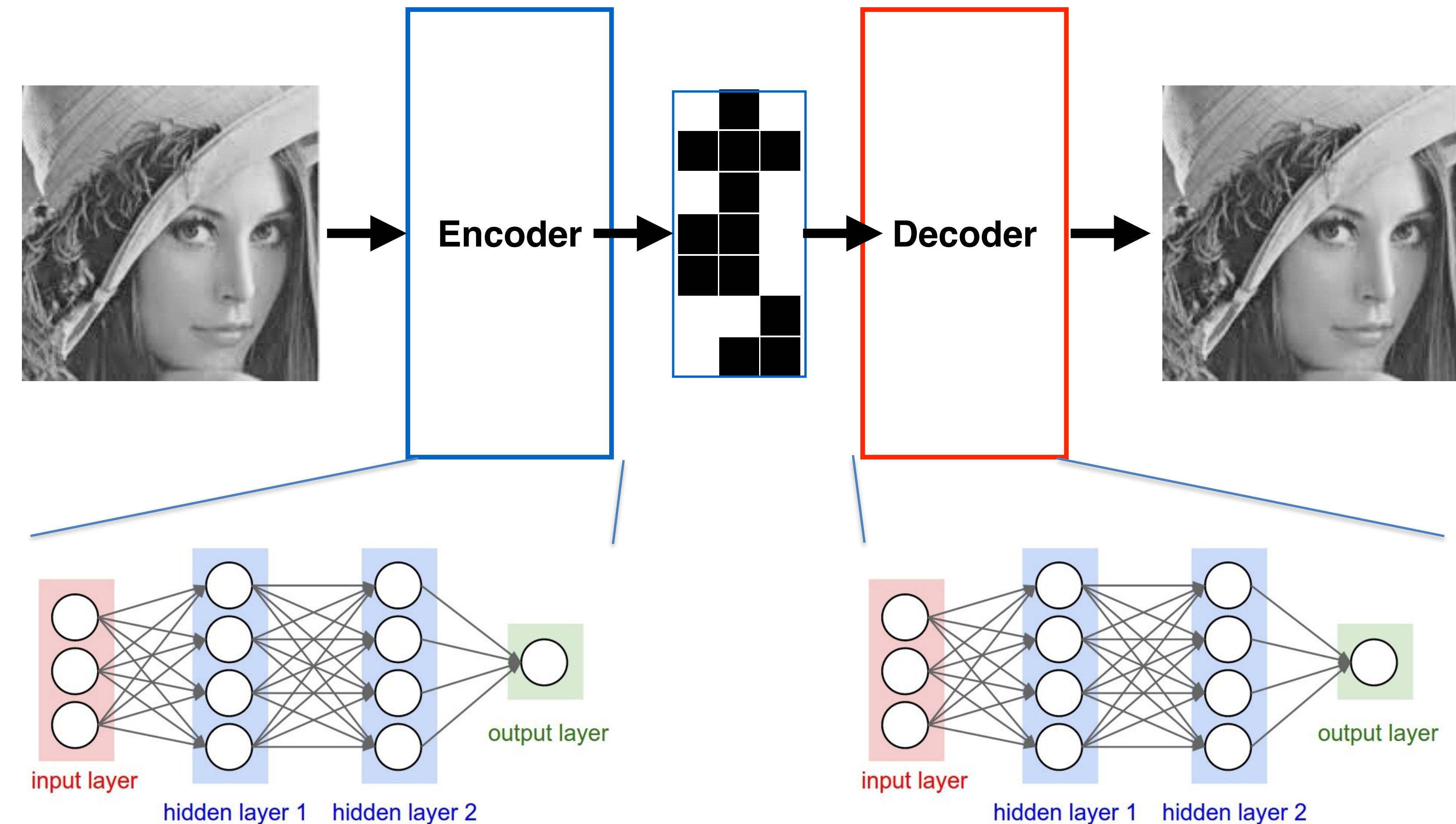
Intro to neural population dynamics

Latent (compressed) representation



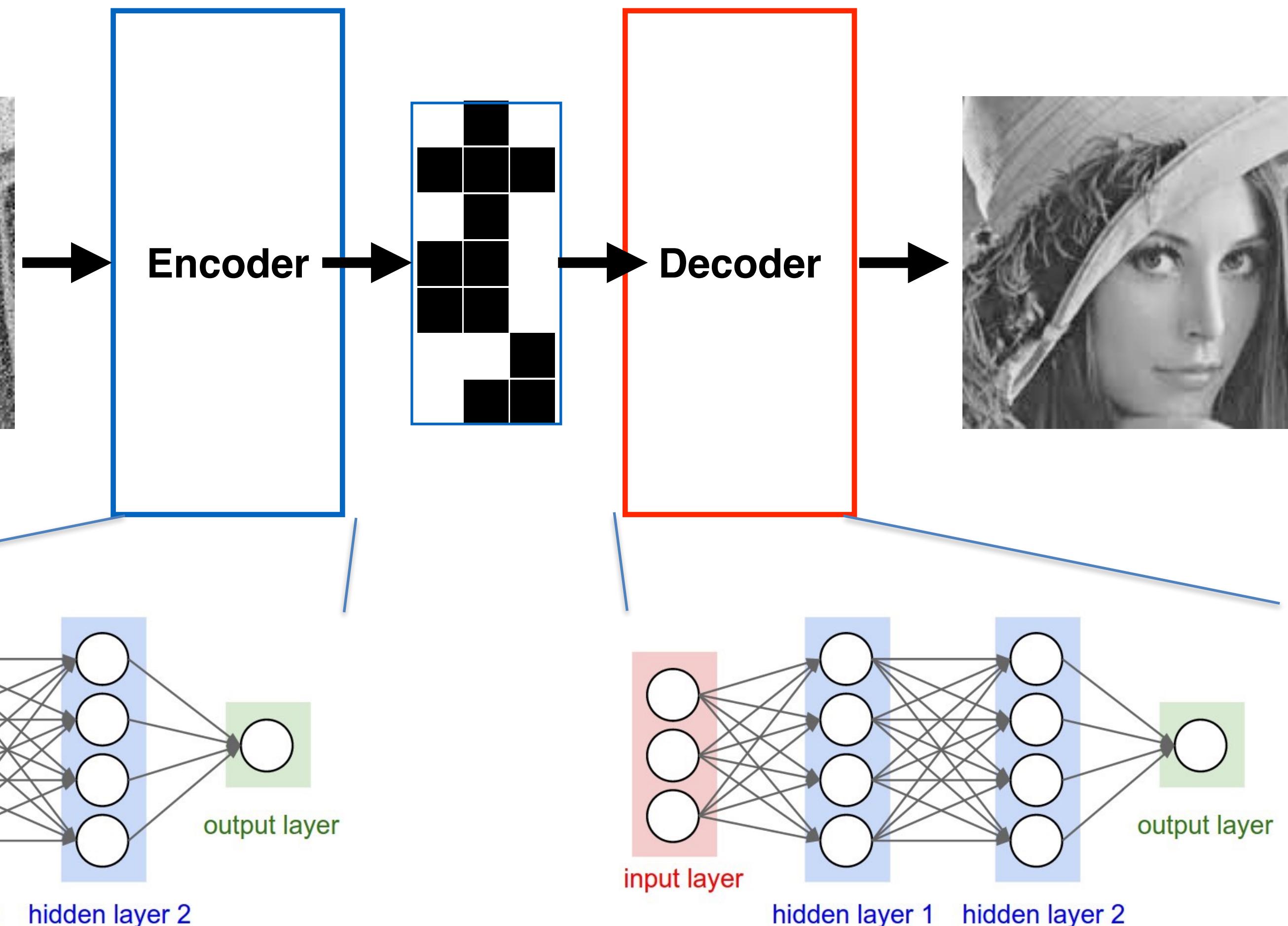
- Autoencoders consist of a pair of networks: an “encoder” and a “decoder”
- The encoder compresses the input down to a low-D representation. The decoder attempts to reconstruct the same input based on the low-D representation

Latent (compressed) representation



- The compressed representation is a bottleneck - the encoder is forced to throw out irrelevant information and extract the “true” structure underlying the data
- Completely unsupervised - there are no data labels

Latent (compressed) representation



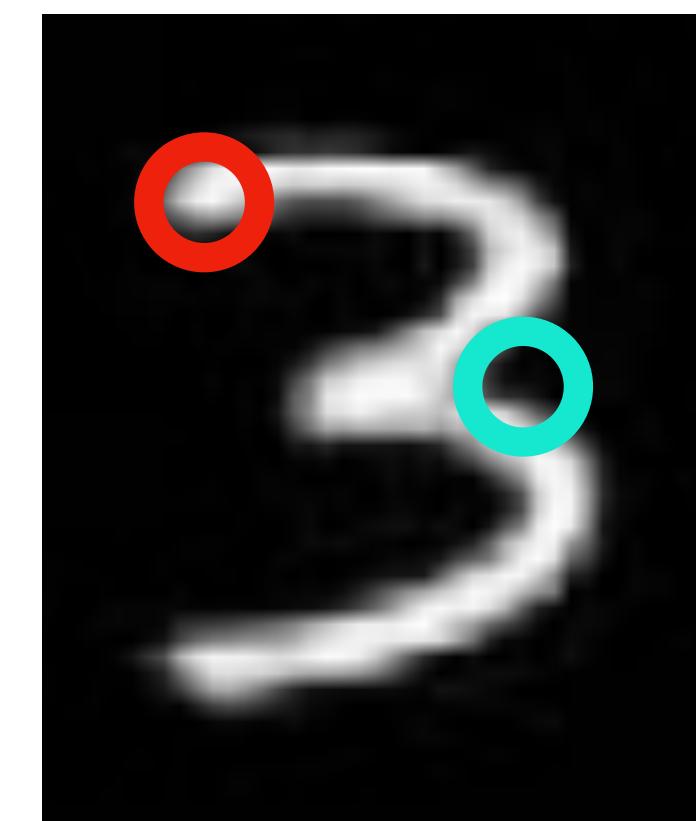
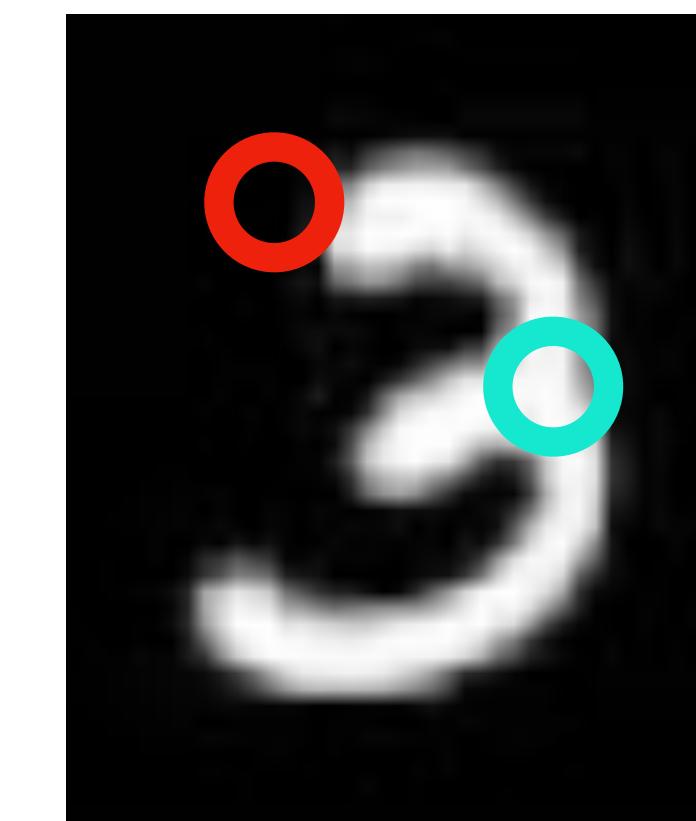
Applications

- An autoencoder can reject noise, that is, it filters out signals that don't correspond to the structure underlying the data
- The decoder must also learn to generate the data, i.e., it is a generative model

28 pixels

28 pixels

**MNIST - 784 total pixels
each ranges from 0 to 1**

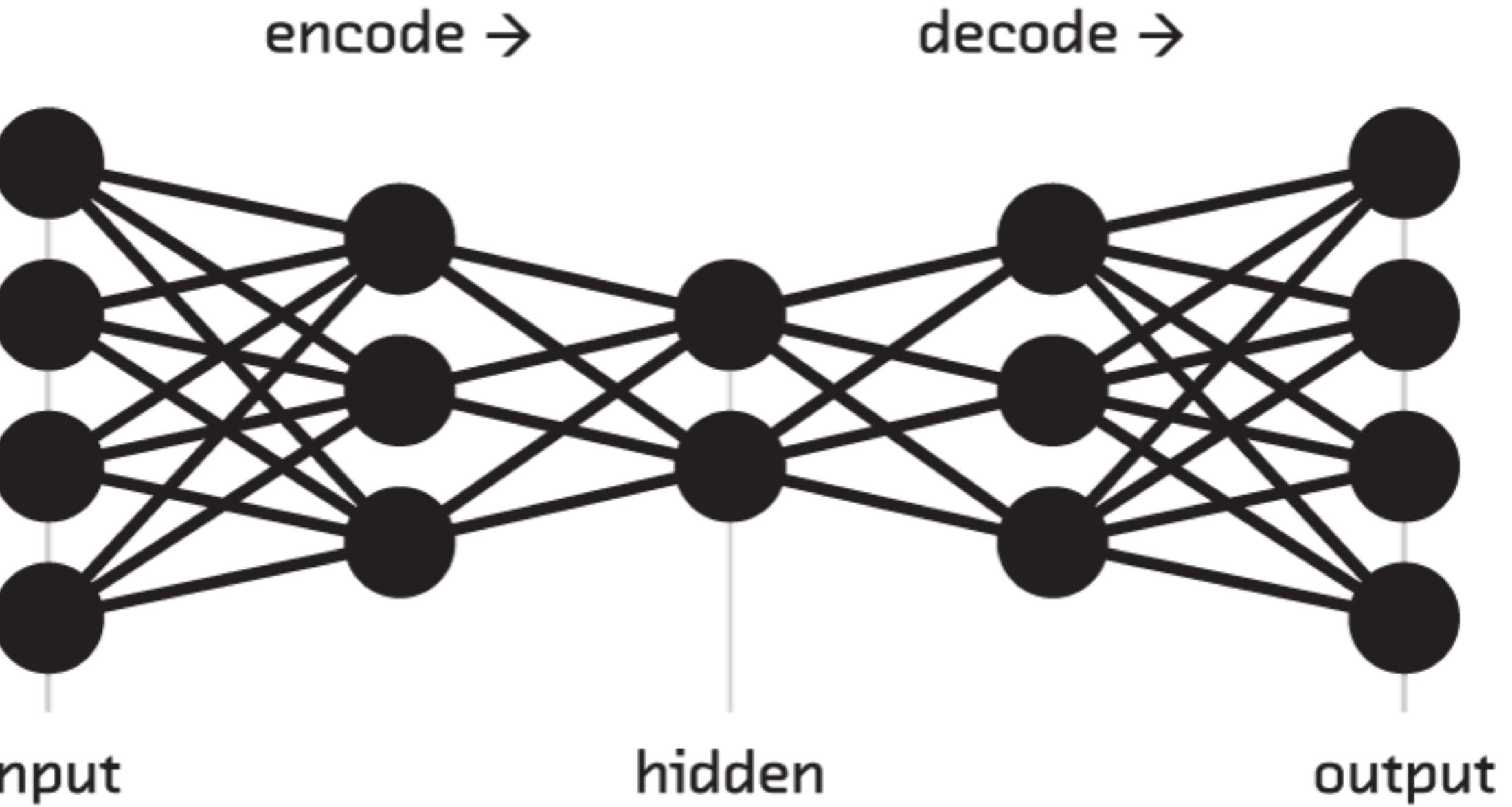


Specific pixel values are very different between images

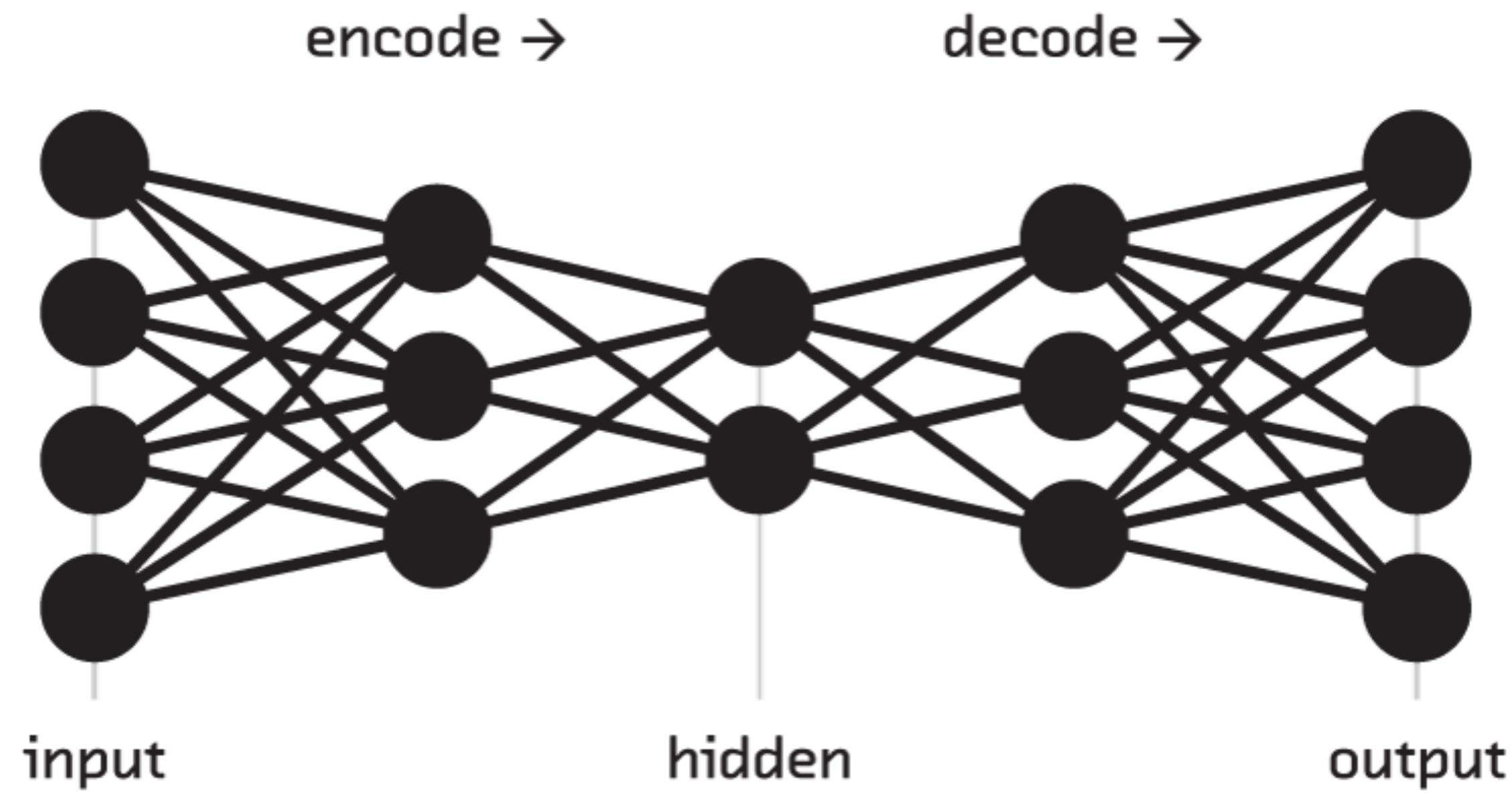


3Blue1Brown (YouTube)

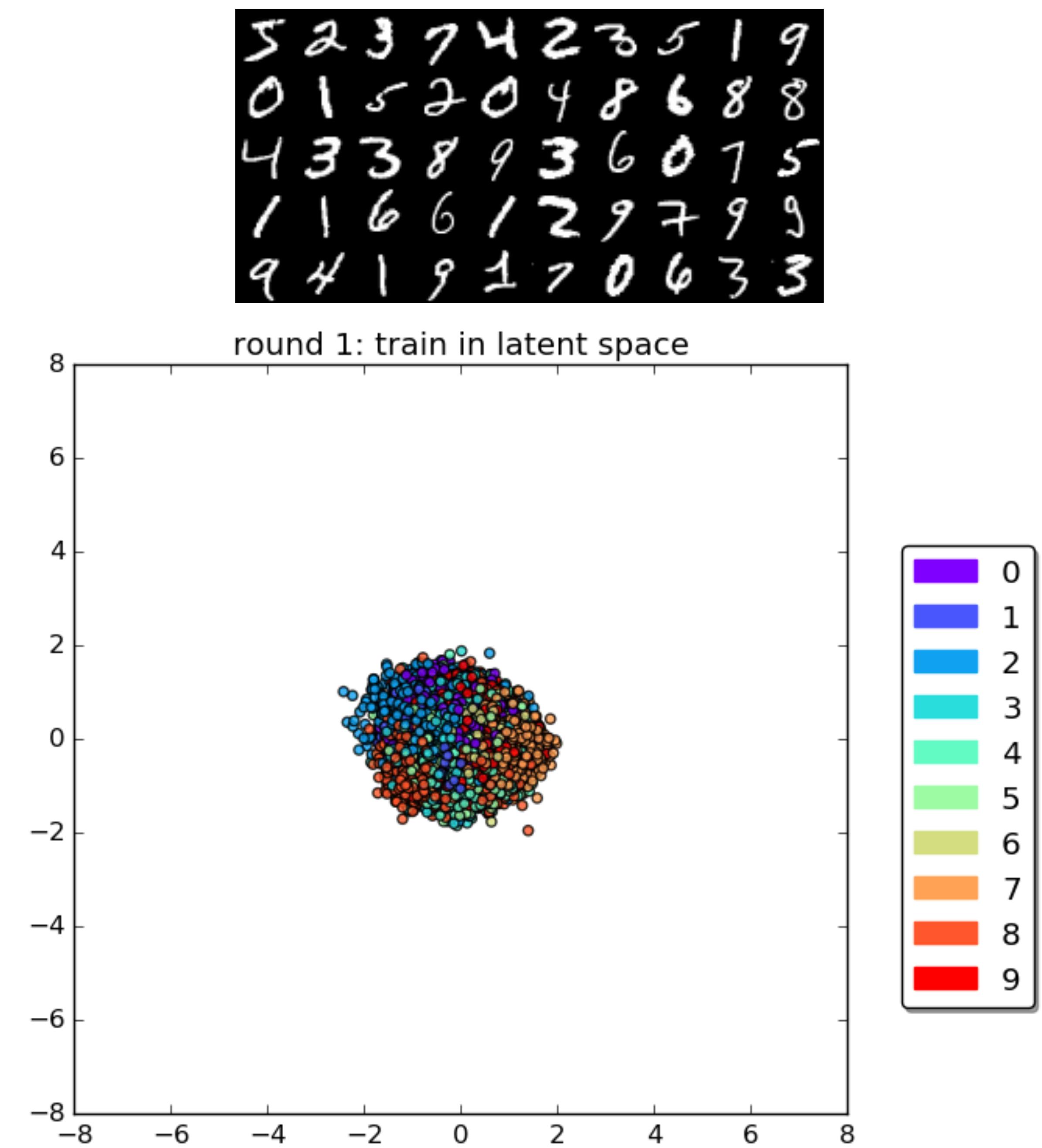
But what is a Neural Network? | Deep learning, chapter 1

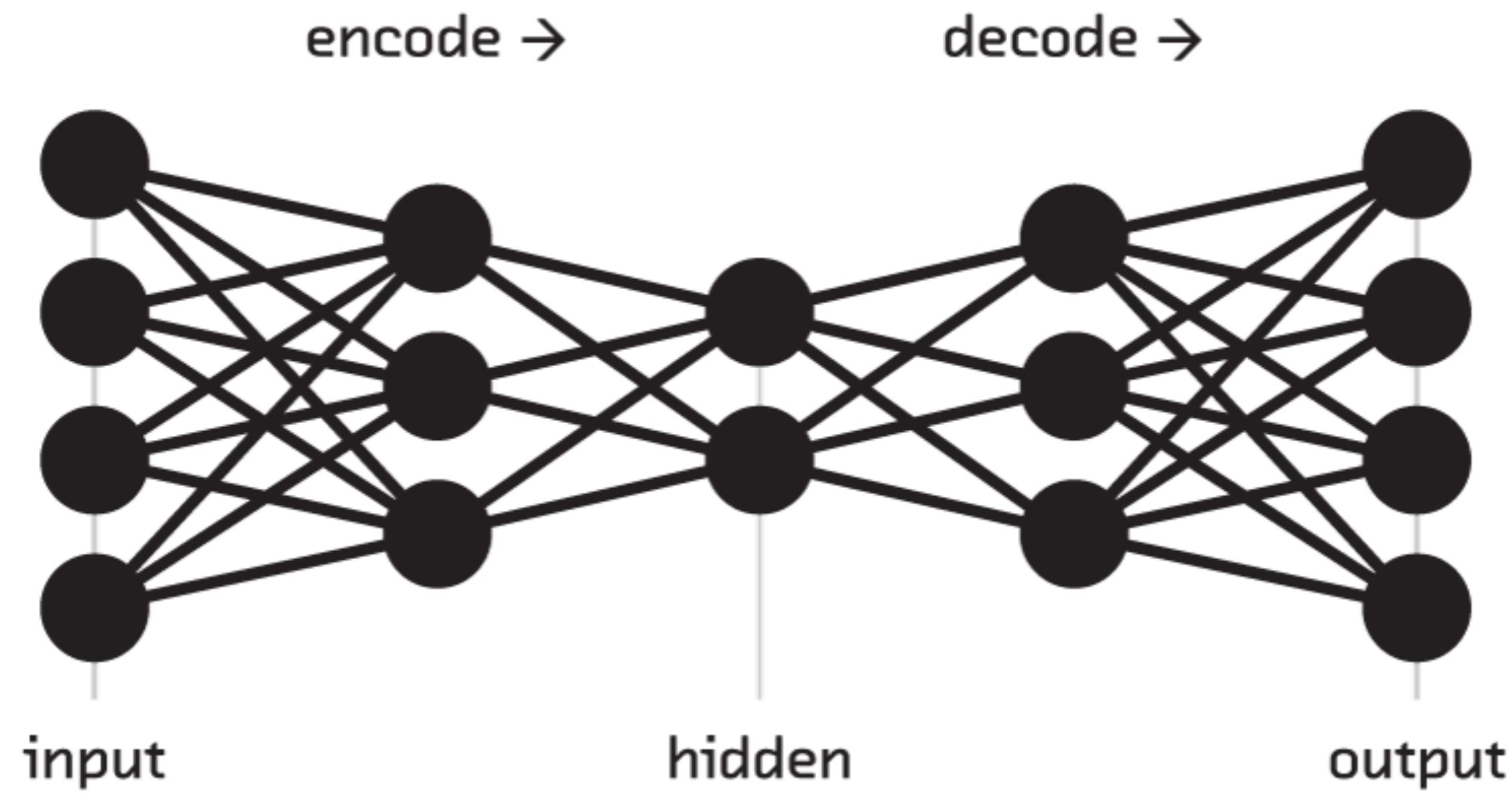


- Network weights are typically initialized completely randomly
 - Through the course of training, the latent representation begins to exhibit structure that matches the structure in the data

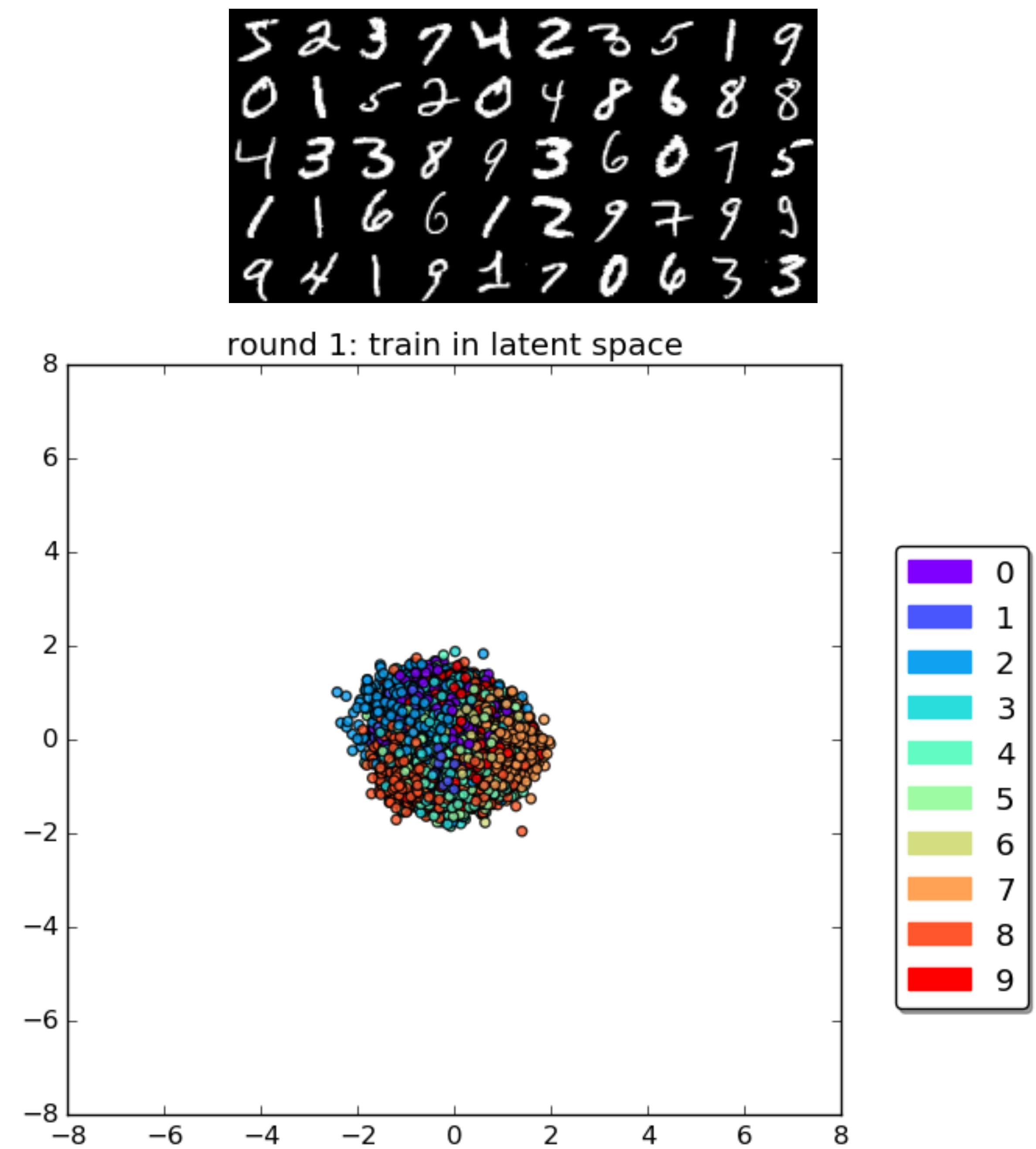


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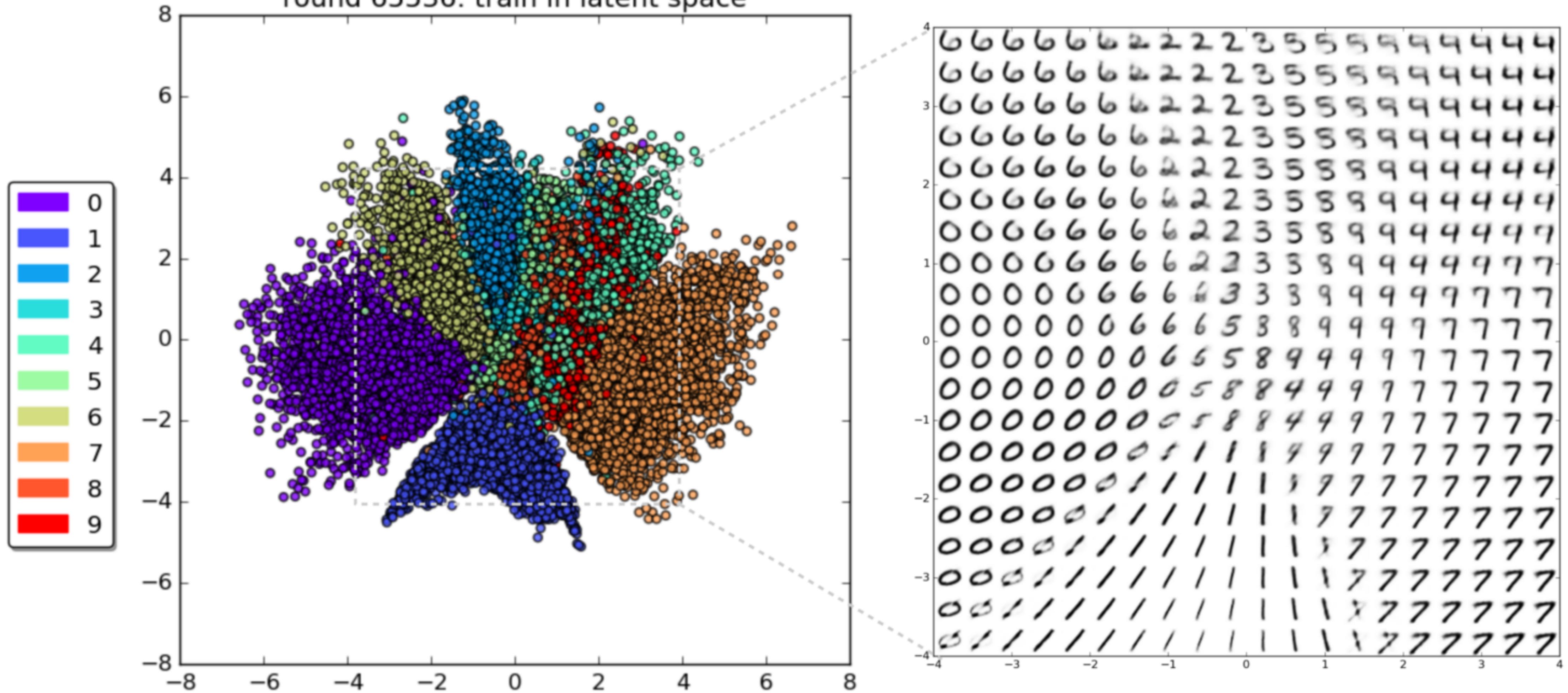




- Through the course of training, the latent representation begins to exhibit structure that matches the structure in the data



round 65536: train in latent space



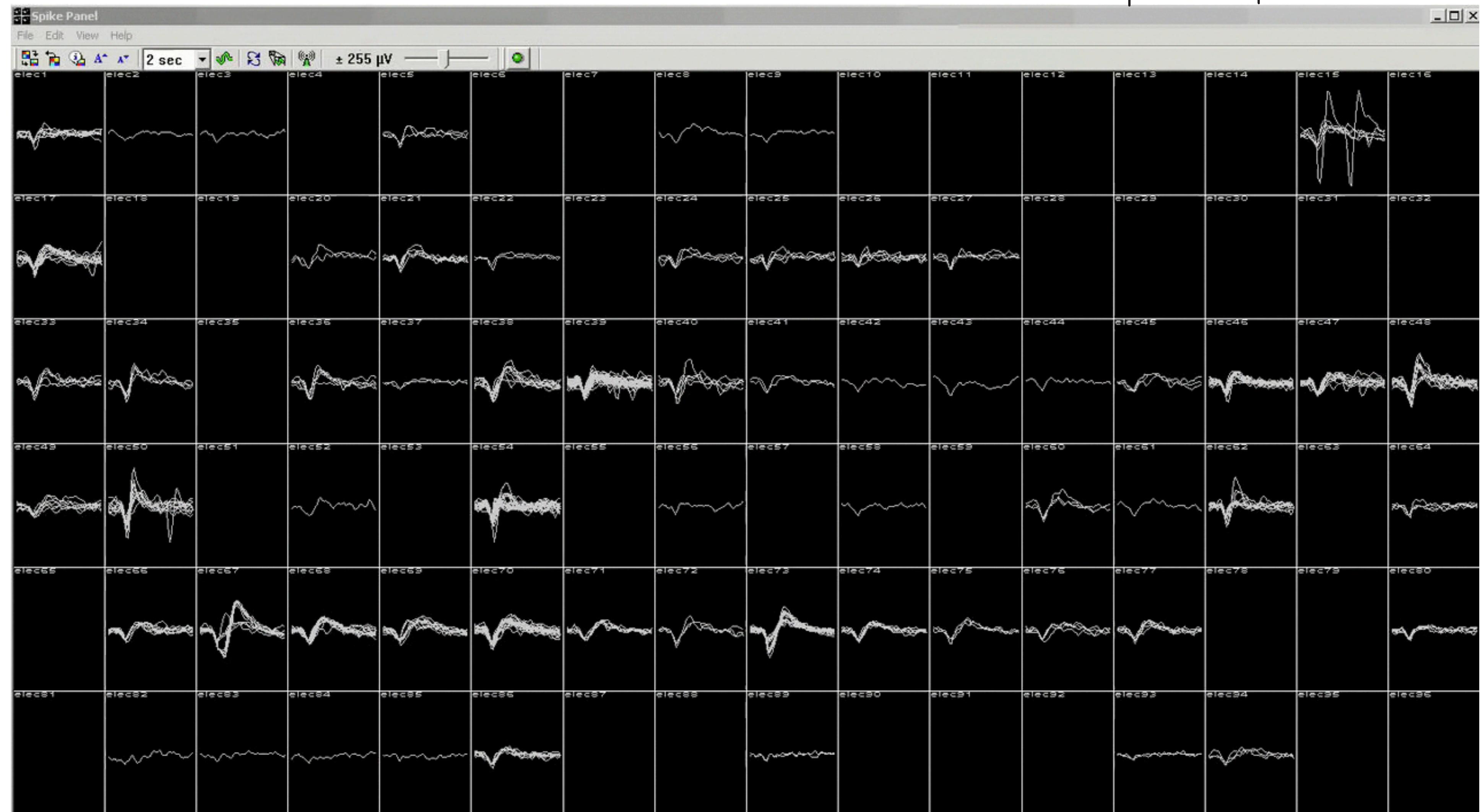
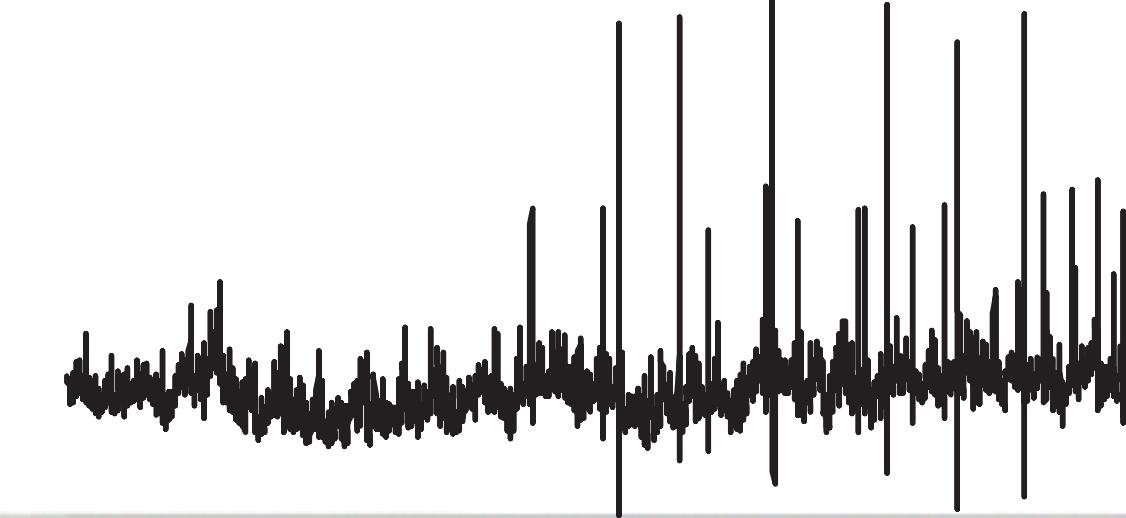
What you'll hear about in this lecture

Neural network basics

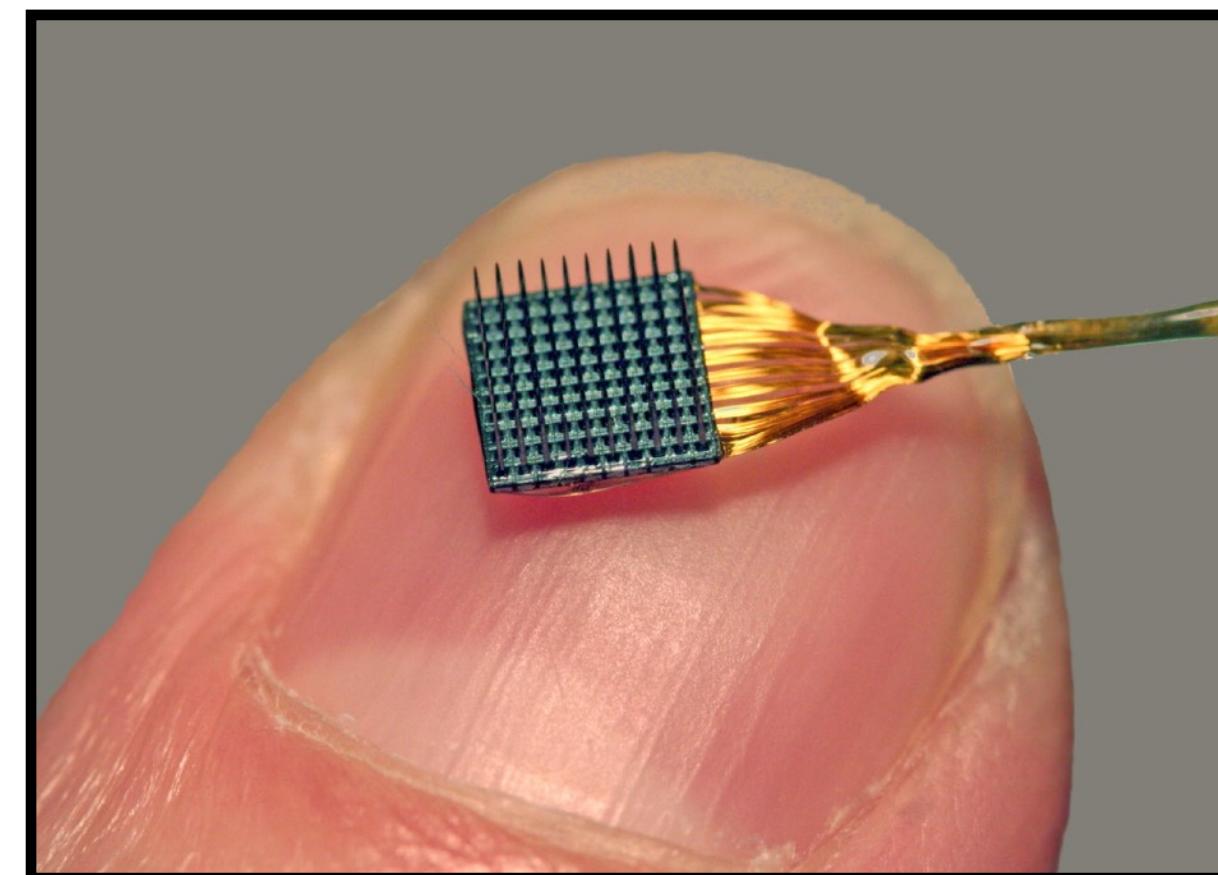
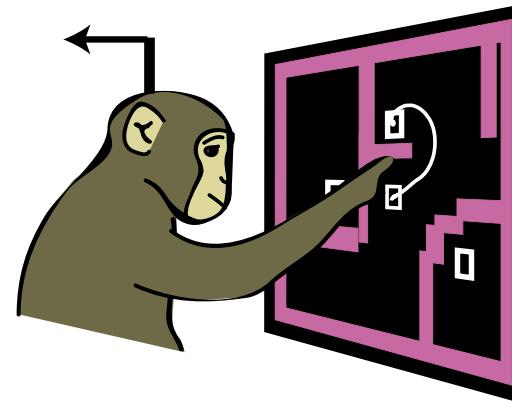
Deep autoencoders

Intro to neural population dynamics

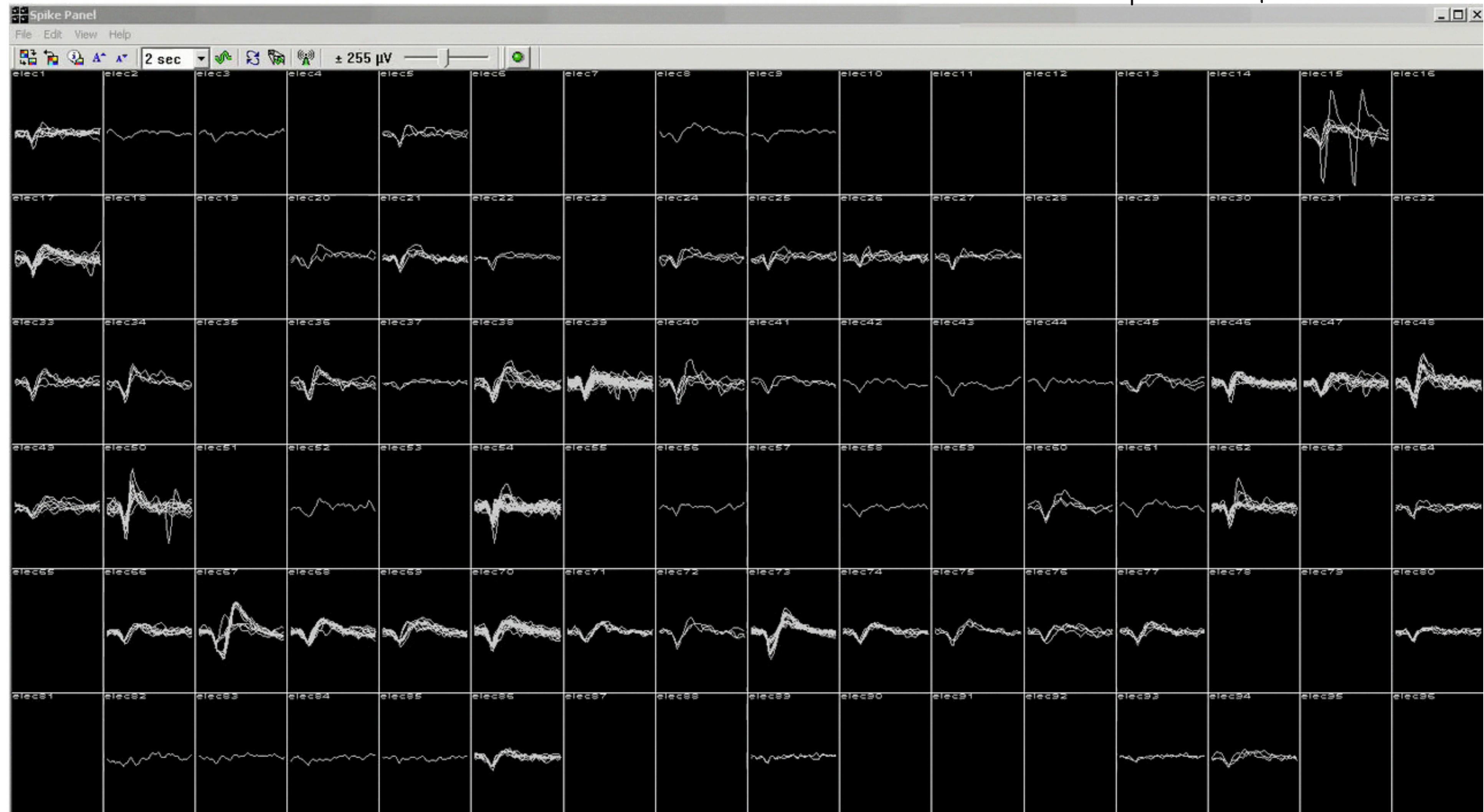
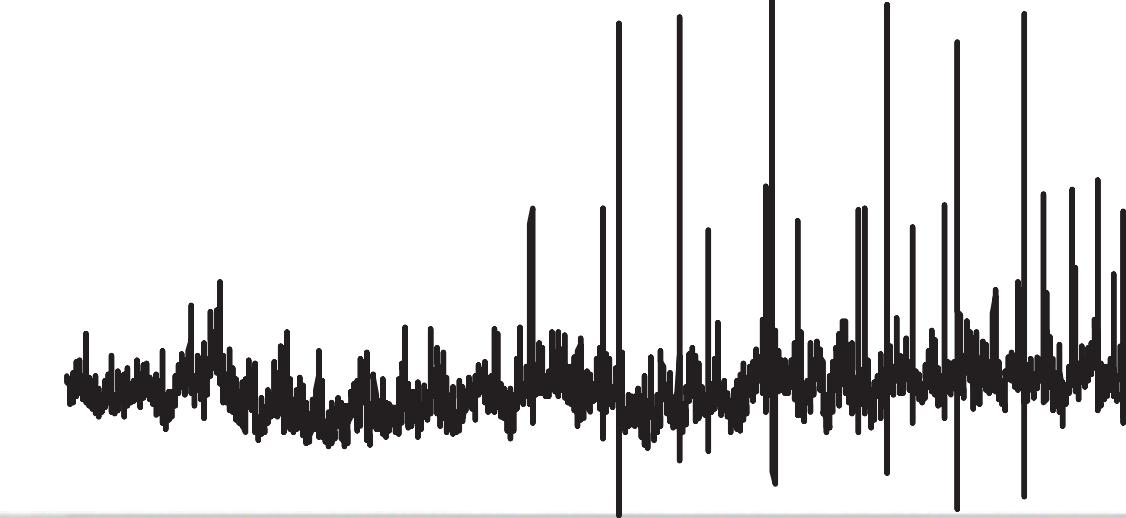
Intracortical electrophysiological recordings



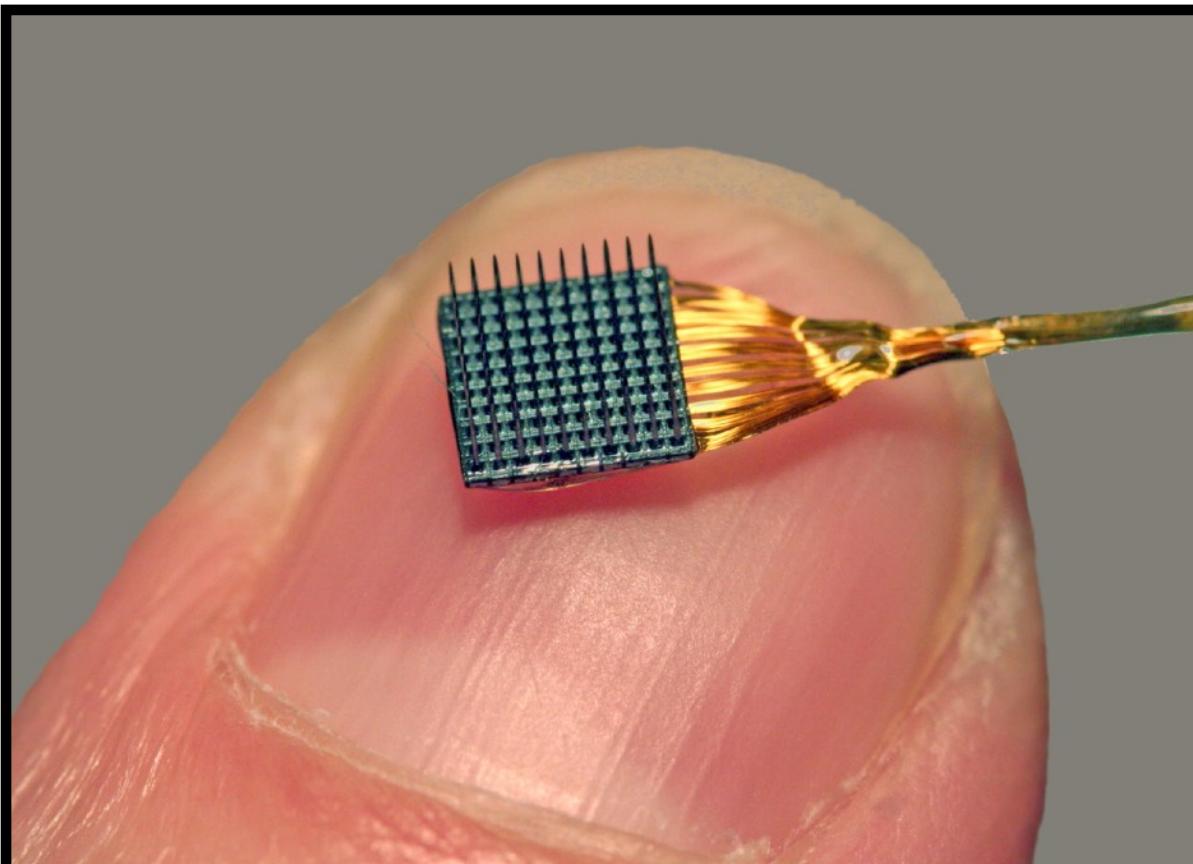
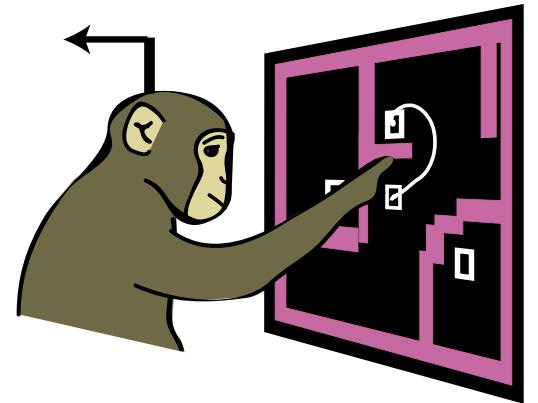
M1/
PMd



Intracortical electrophysiological recordings



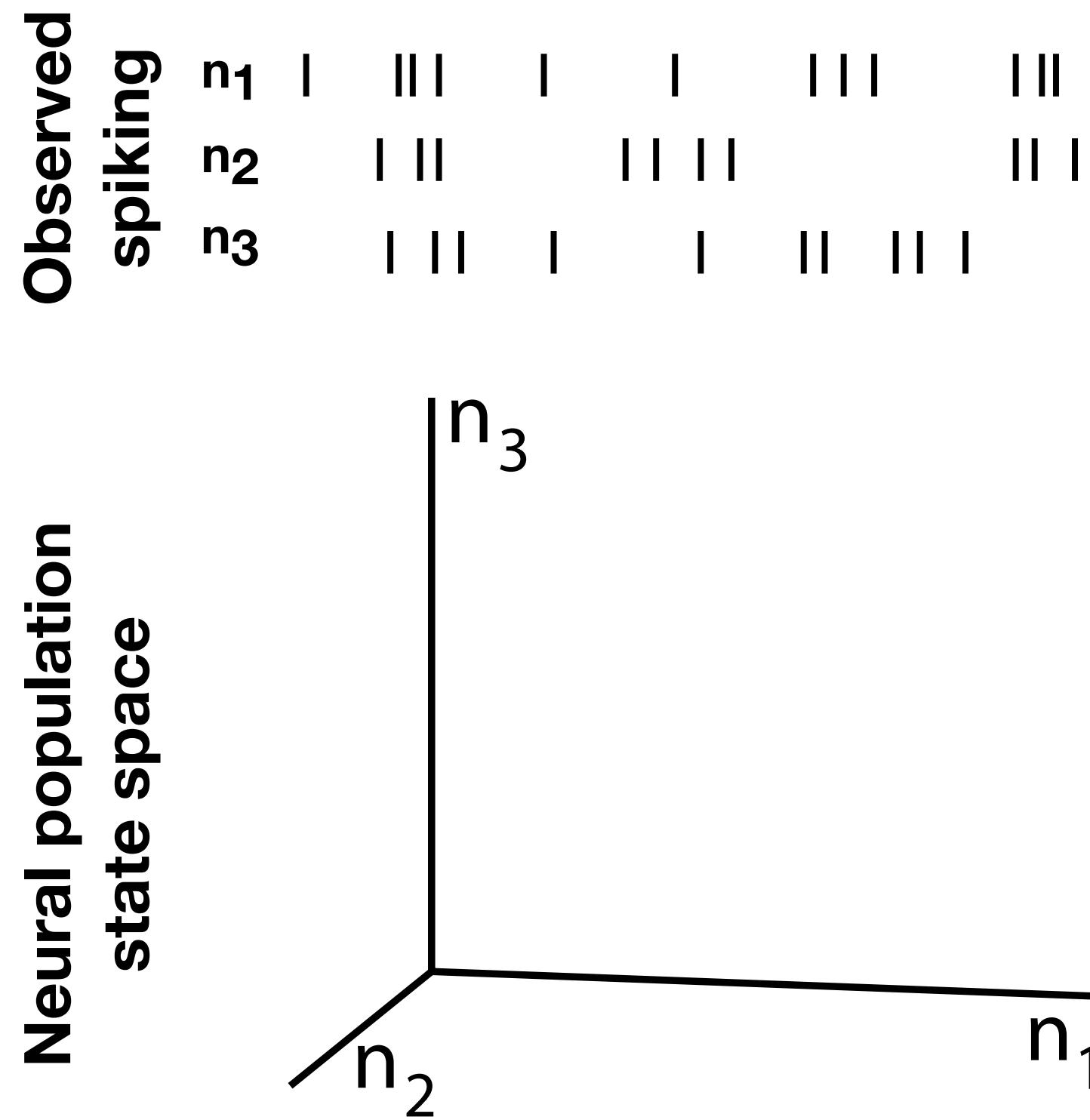
M1/
PMd



Neural population state spaces

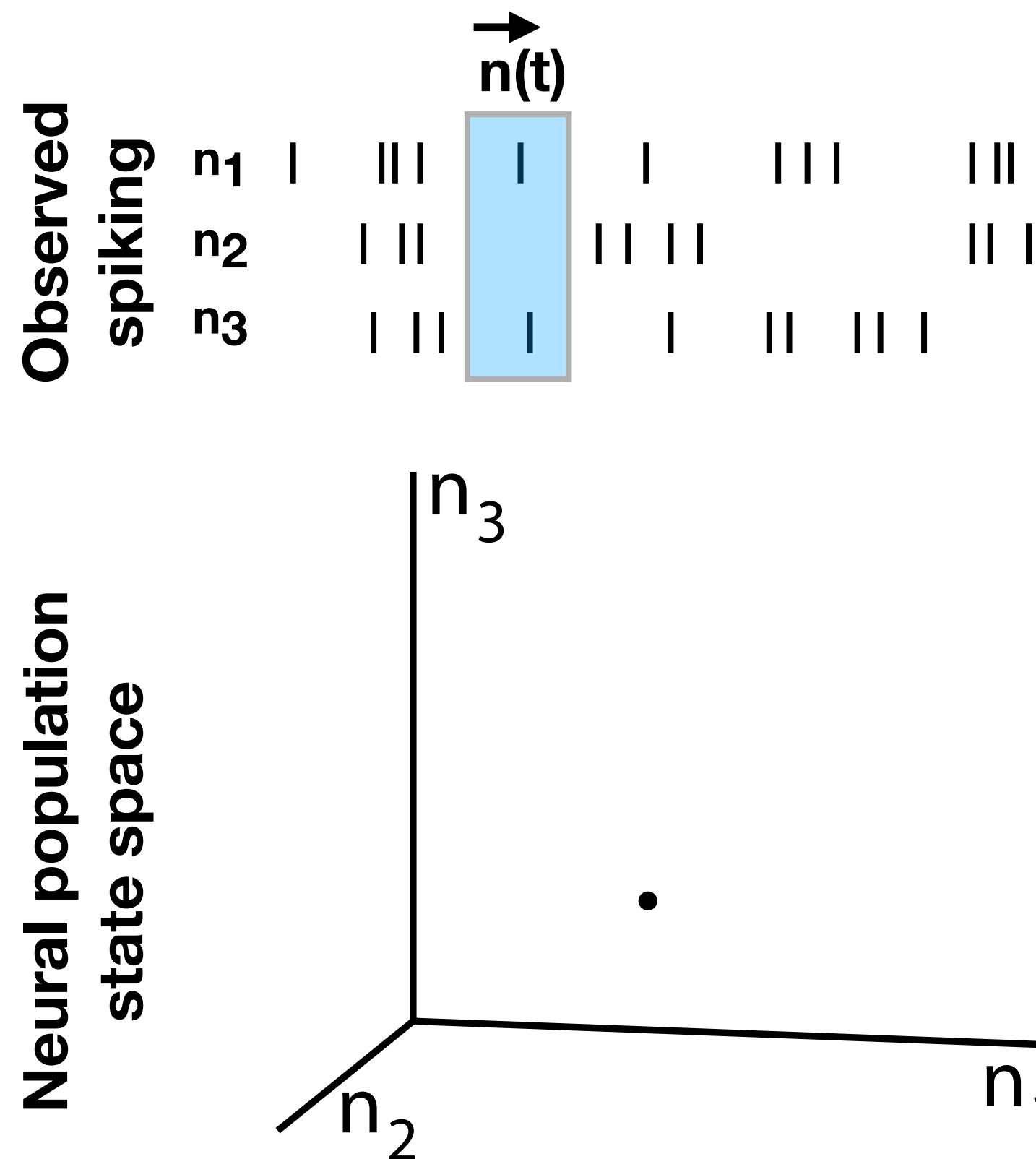
Observed spiking	n ₁							
n ₂								
n ₃								

Neural population state spaces



Adapted from Cunningham & Yu, *Nat Neuro* 2014
Dimensionality reduction for large-scale neural recordings

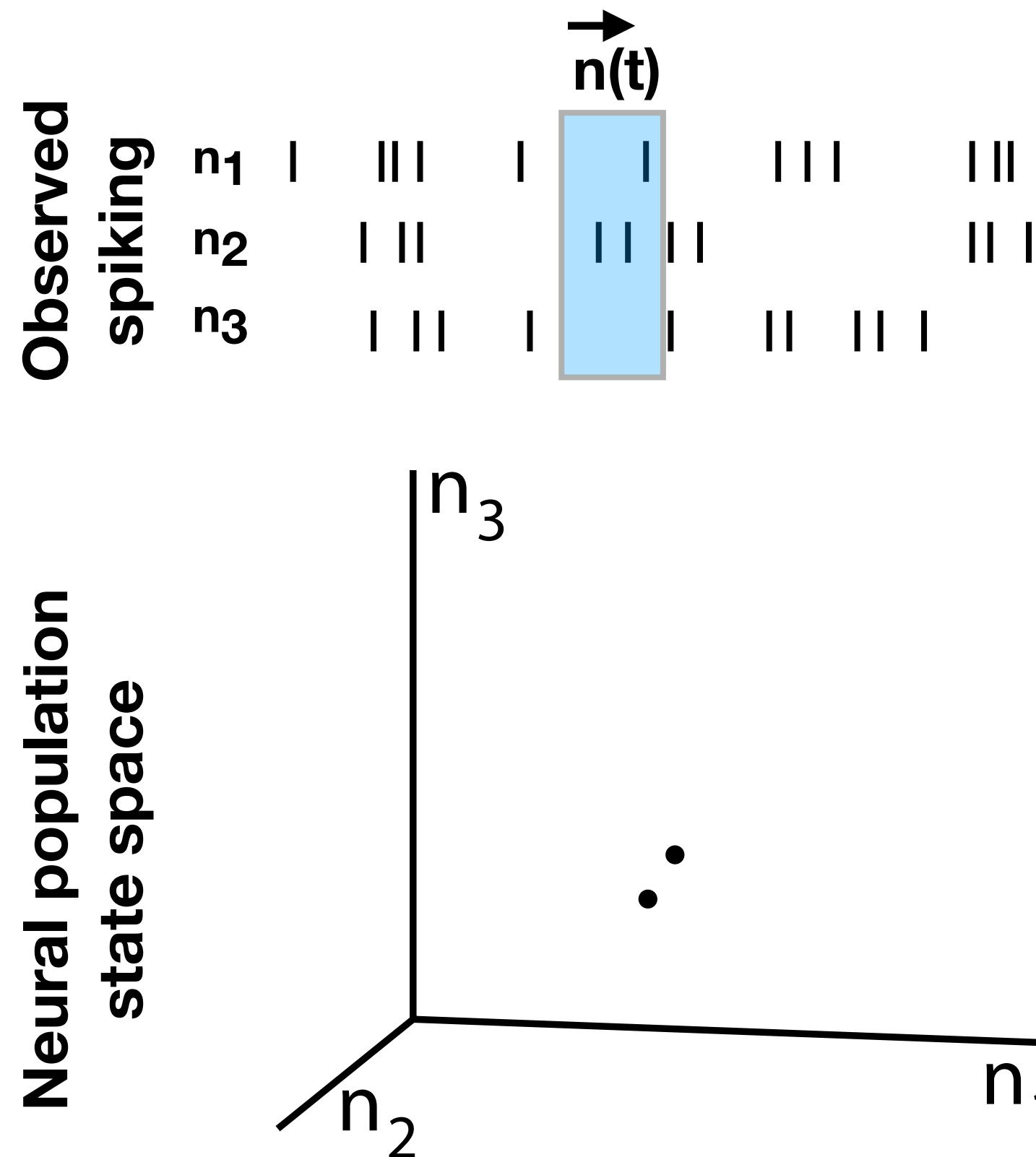
Neural population state spaces



- The current state of the system is captured by the firing rates of all of its neurons

Adapted from Cunningham & Yu, *Nat Neuro* 2014
Dimensionality reduction for large-scale neural recordings

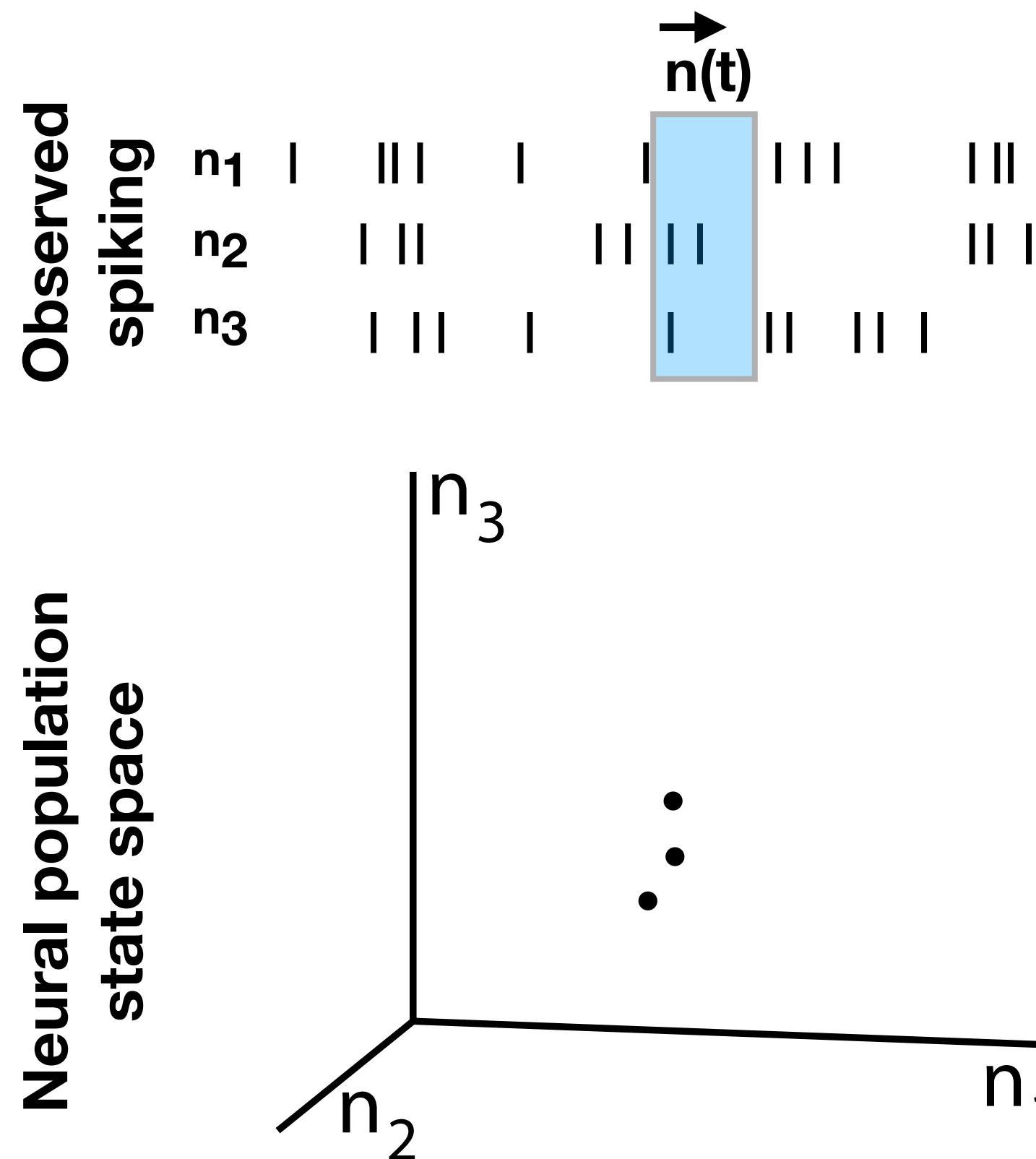
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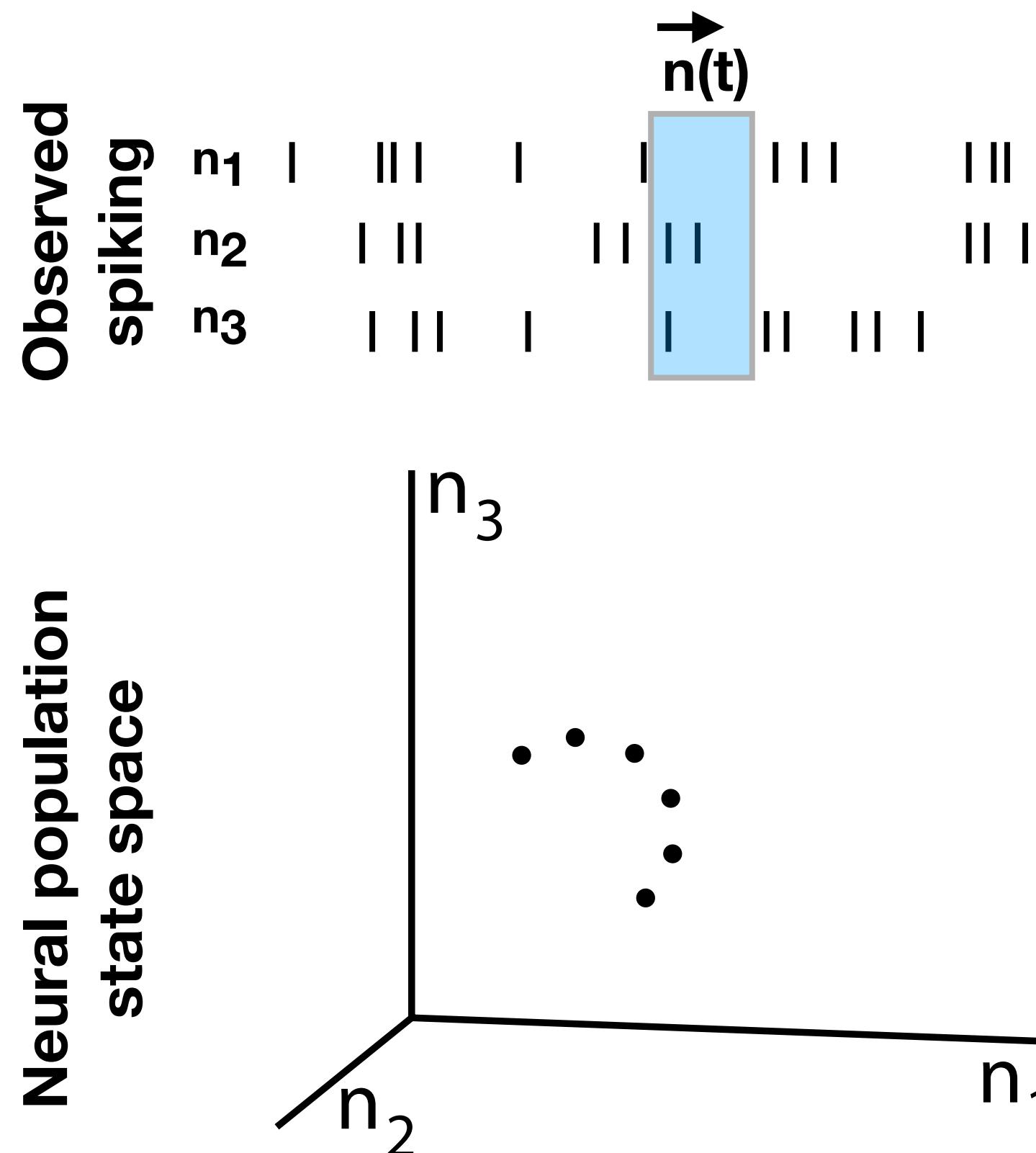
Neural population state spaces



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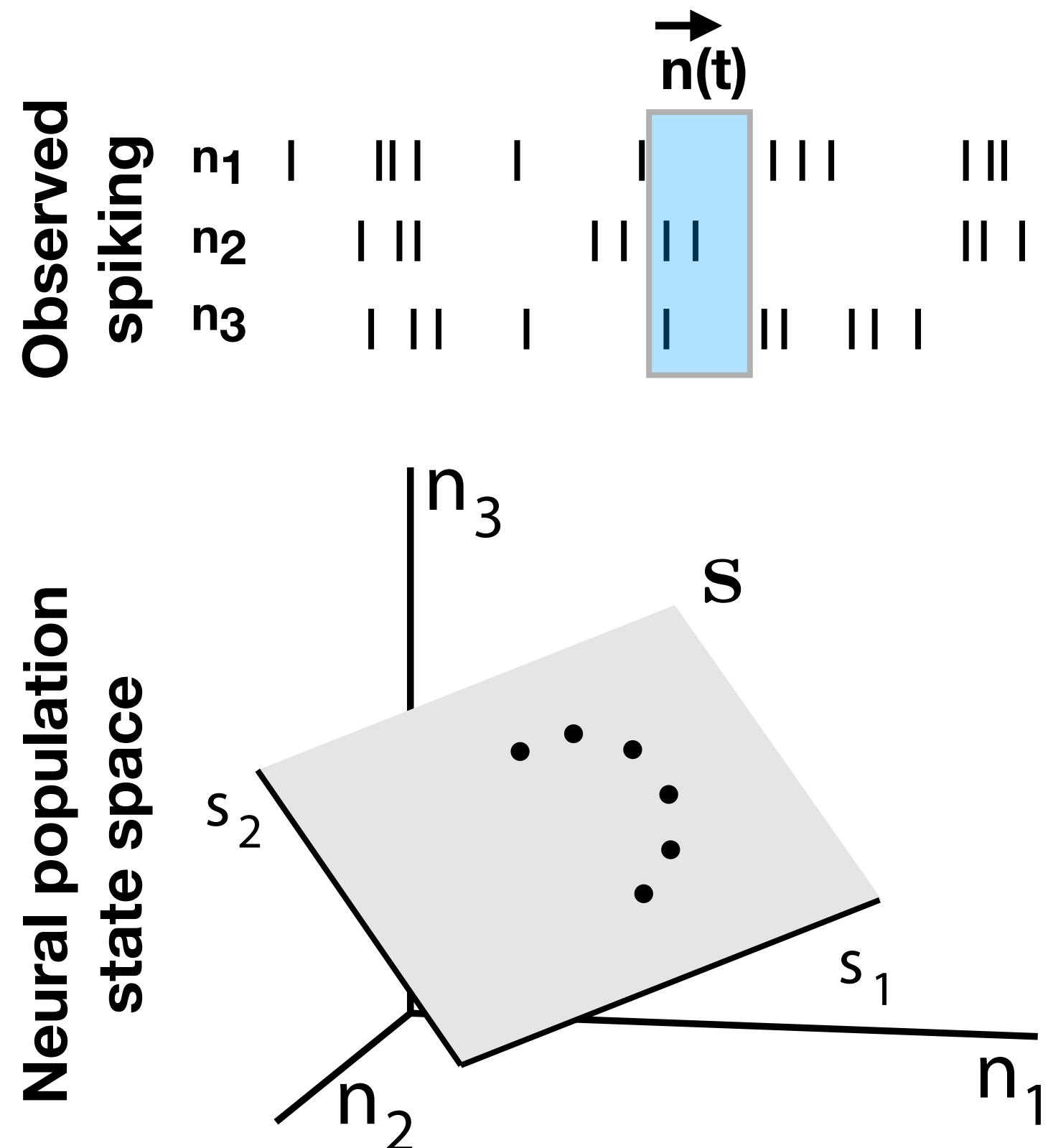
Neural population state spaces



- The current state of the system is captured by the firing rates of all of its neurons
- Activity does not uniformly fill the space - i.e., not all patterns of neural firing rates are equally likely

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Neural population state spaces

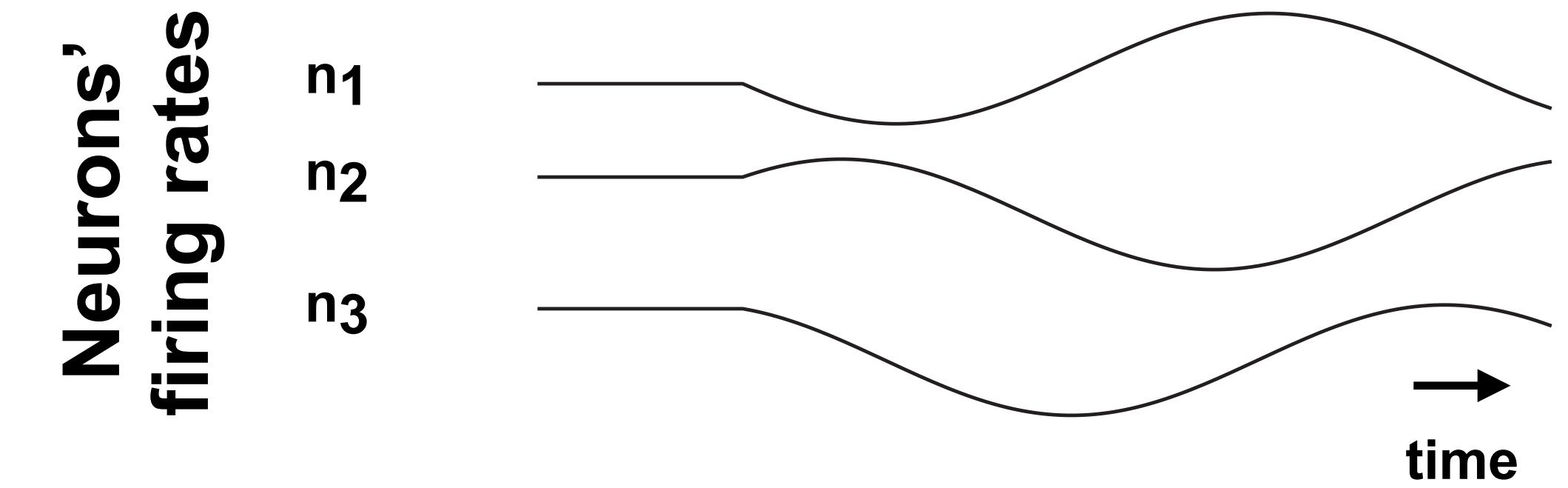


Adapted from Cunningham & Yu, *Nat Neuro* 2014
Dimensionality reduction for large-scale neural recordings

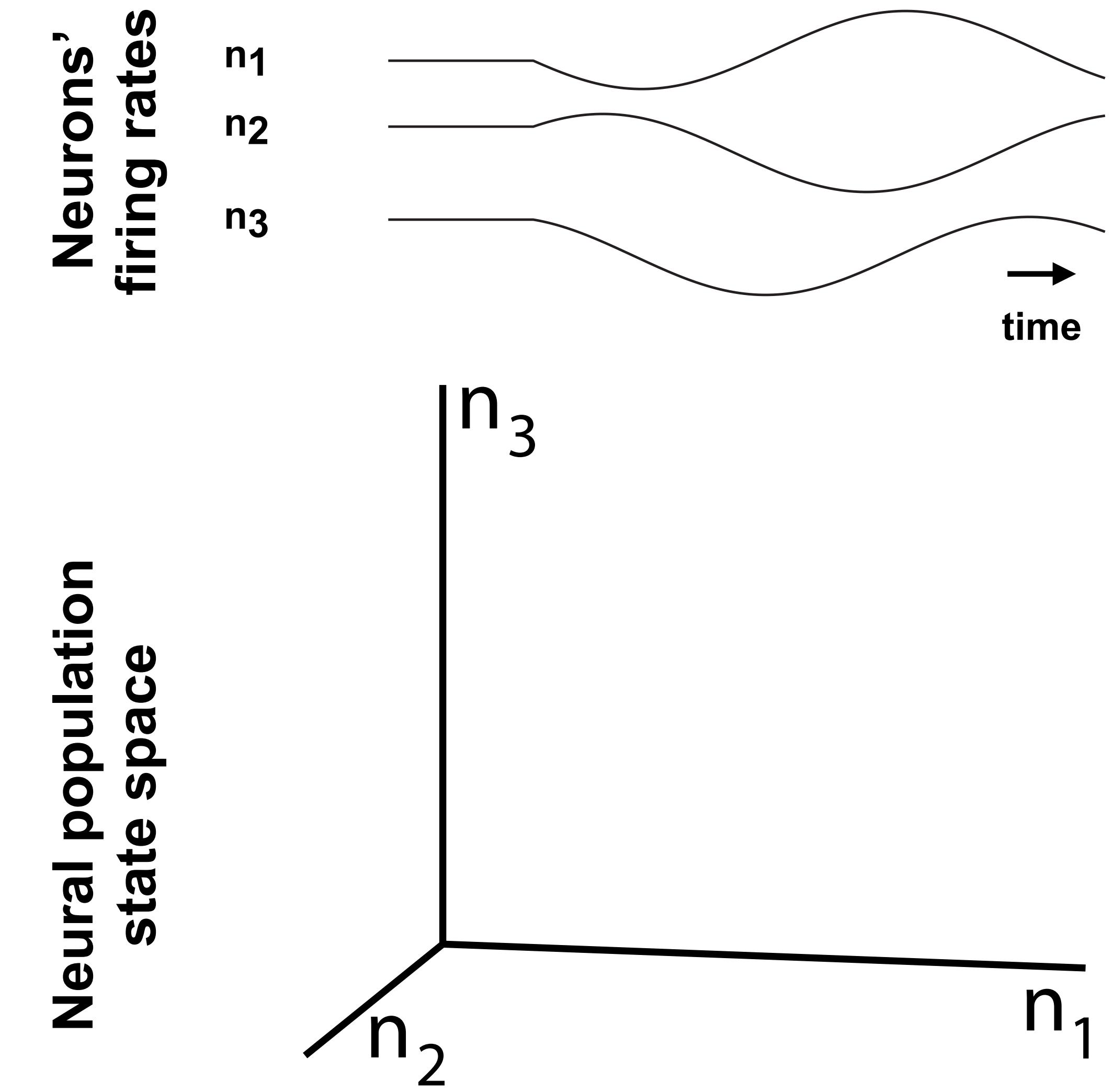
- The current state of the system is captured by the firing rates of all of its neurons
- Activity does not uniformly fill the space - i.e., not all patterns of neural firing rates are equally likely
- We can usually identify shared structure underlying the firing rates of the recorded neurons

Changes in neurons' firing rates are coordinated

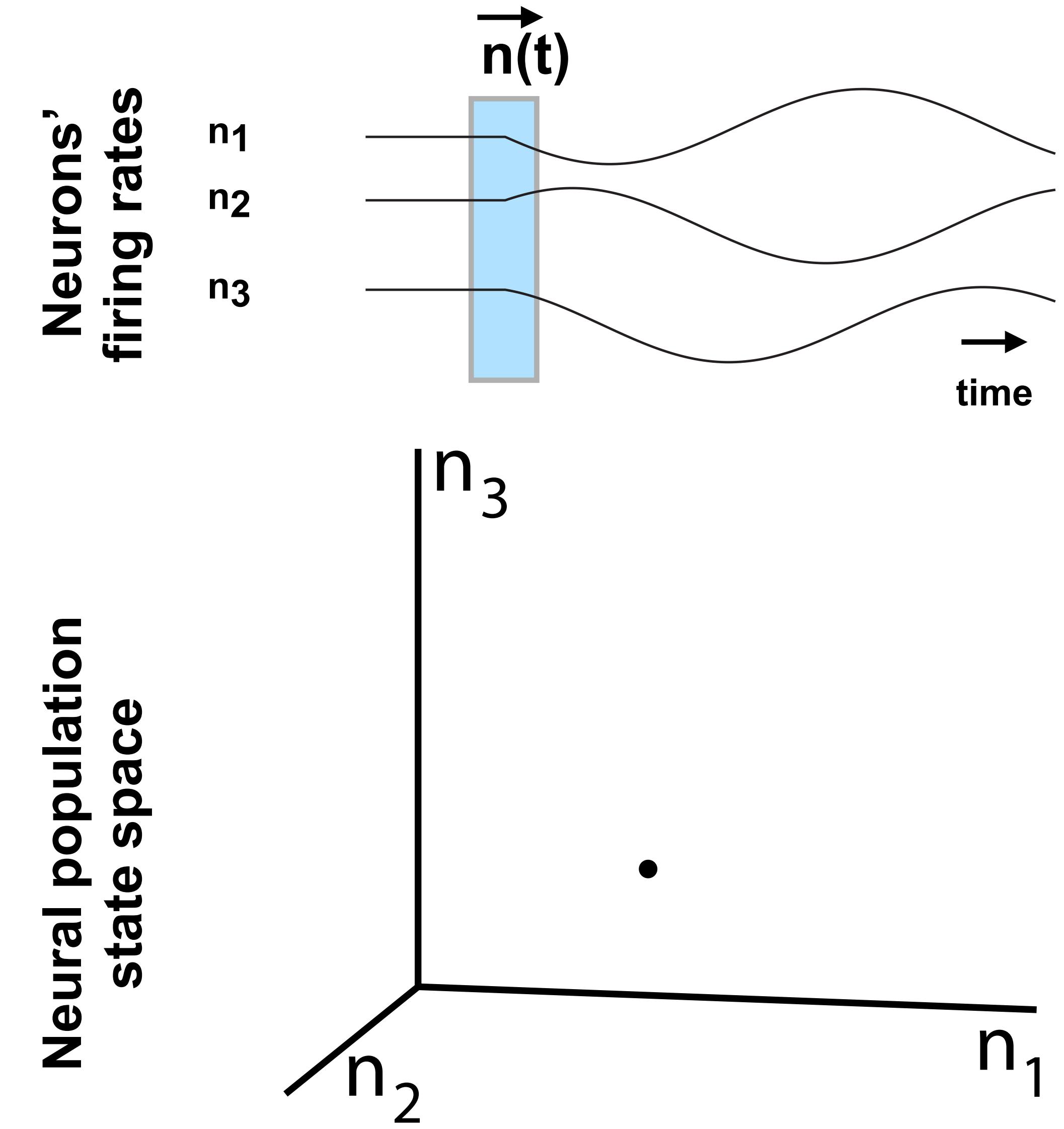
Changes in neurons' firing rates are coordinated



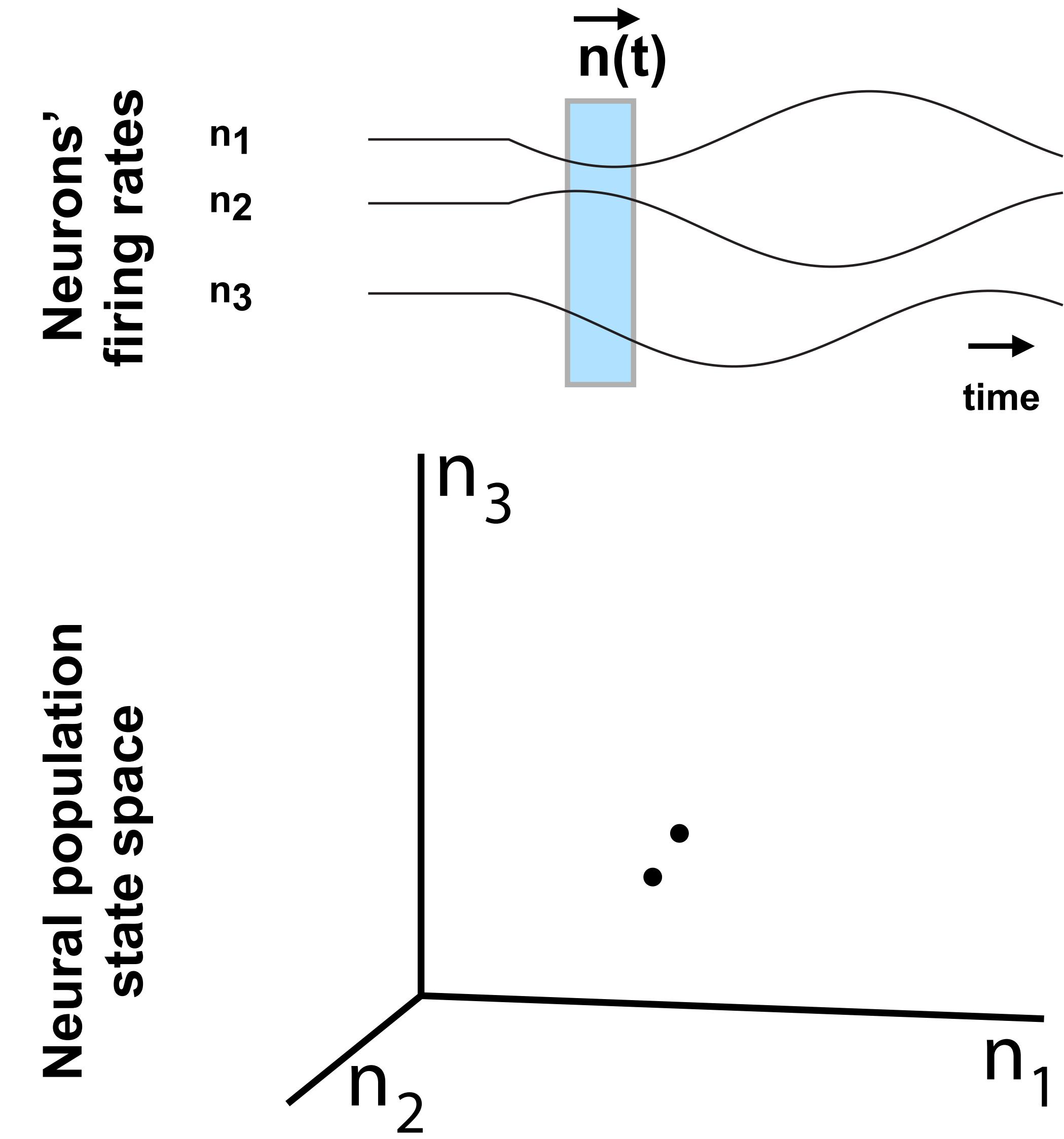
Changes in neurons' firing rates are coordinated



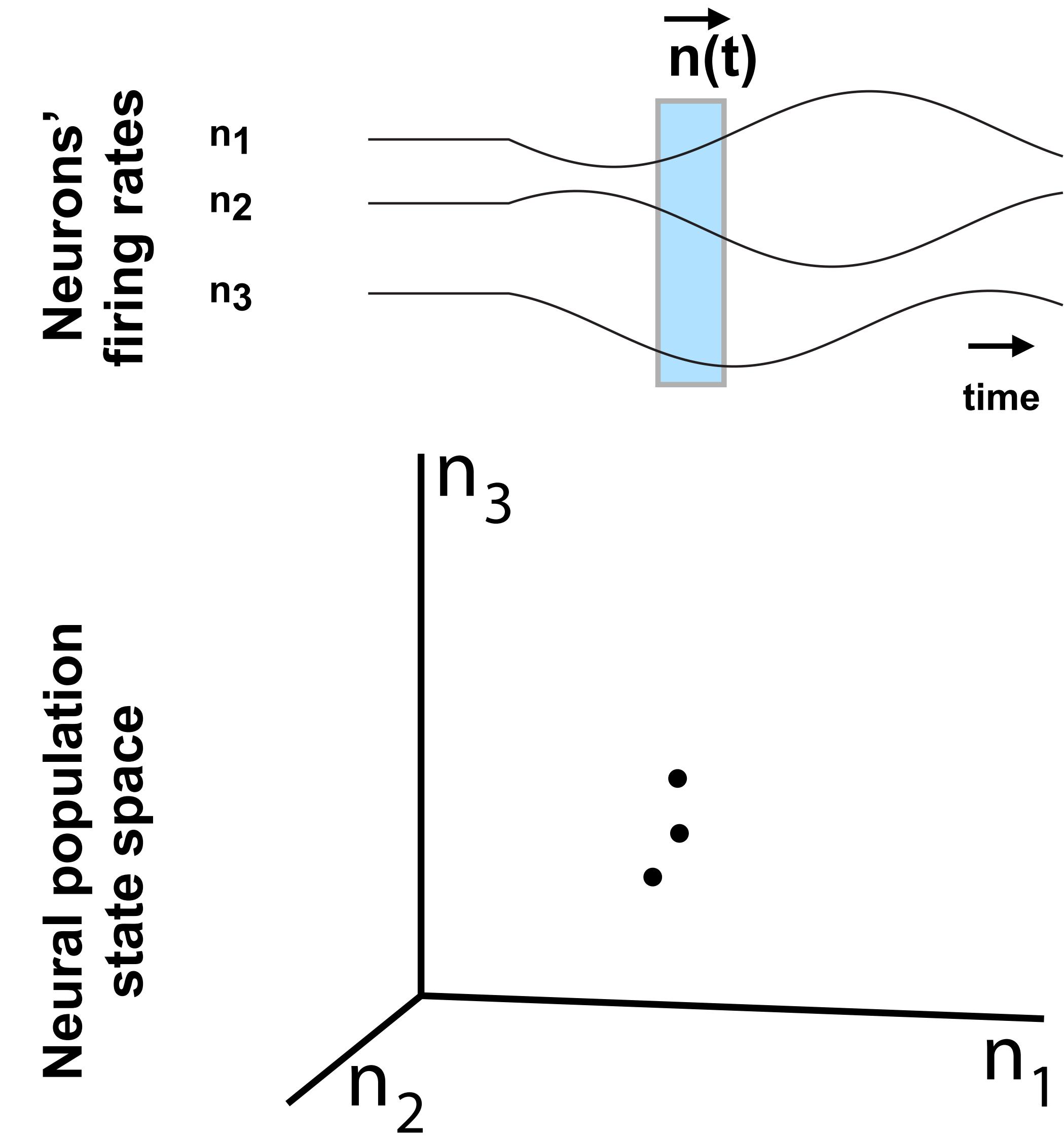
Changes in neurons' firing rates are coordinated



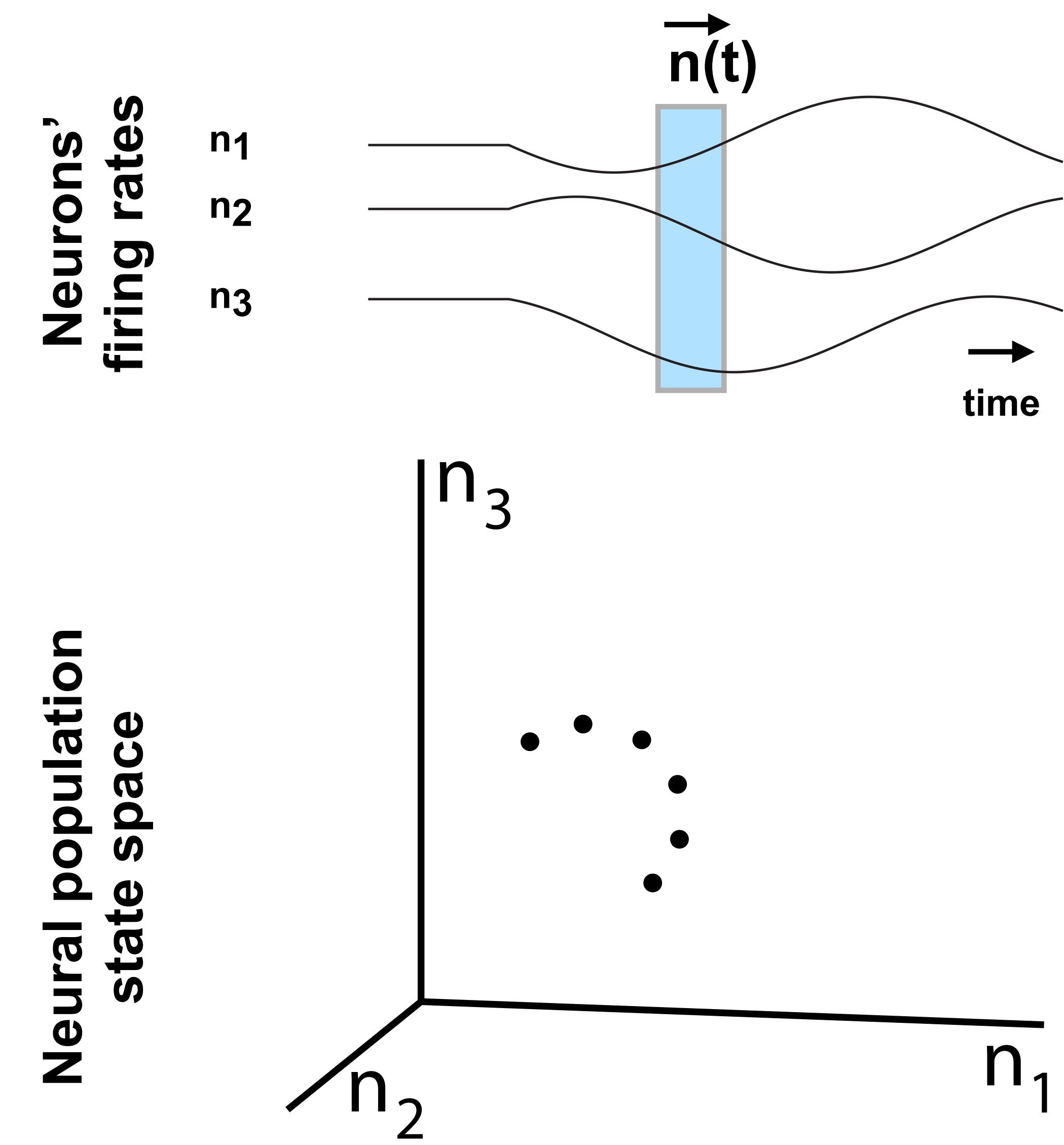
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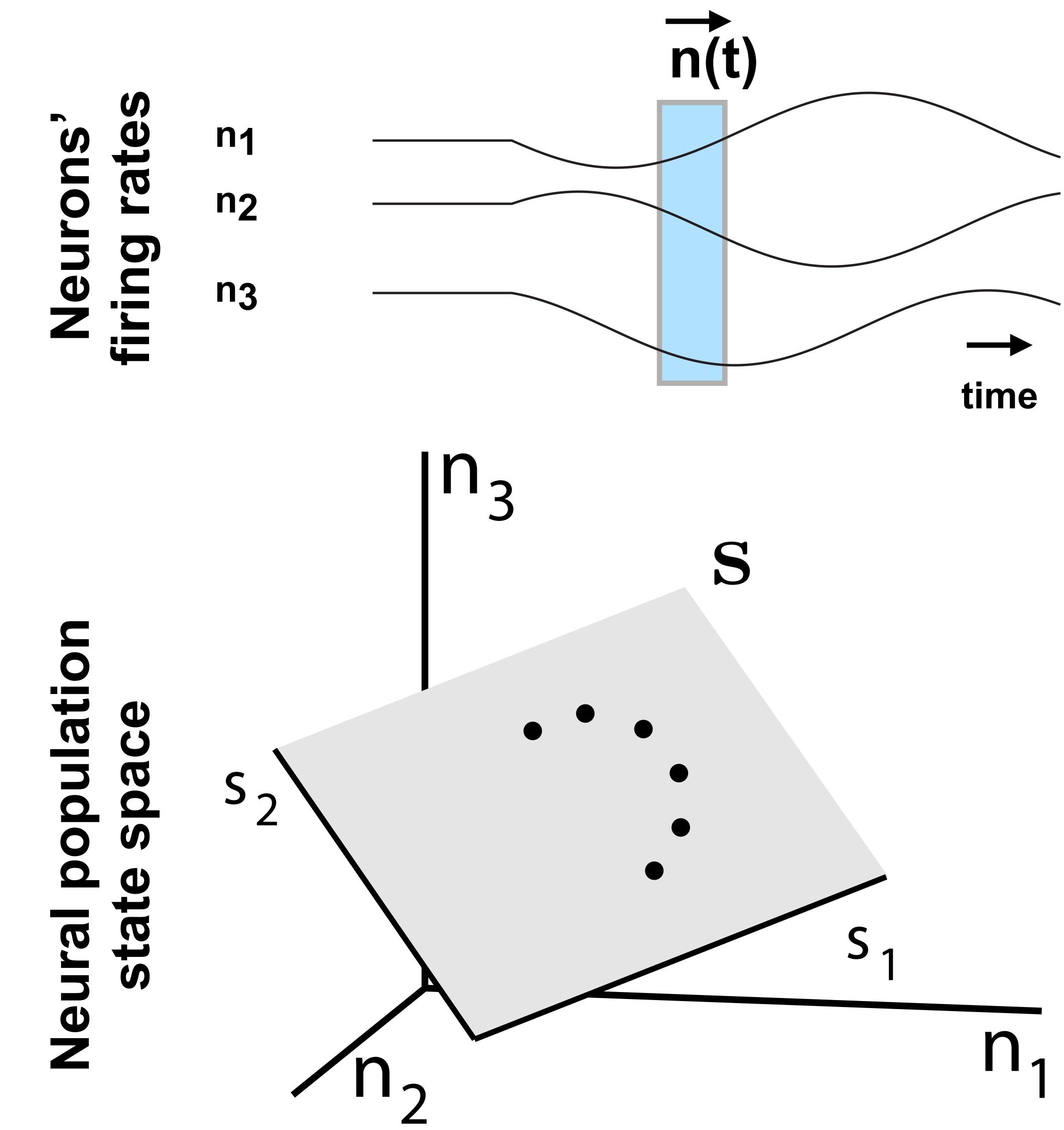
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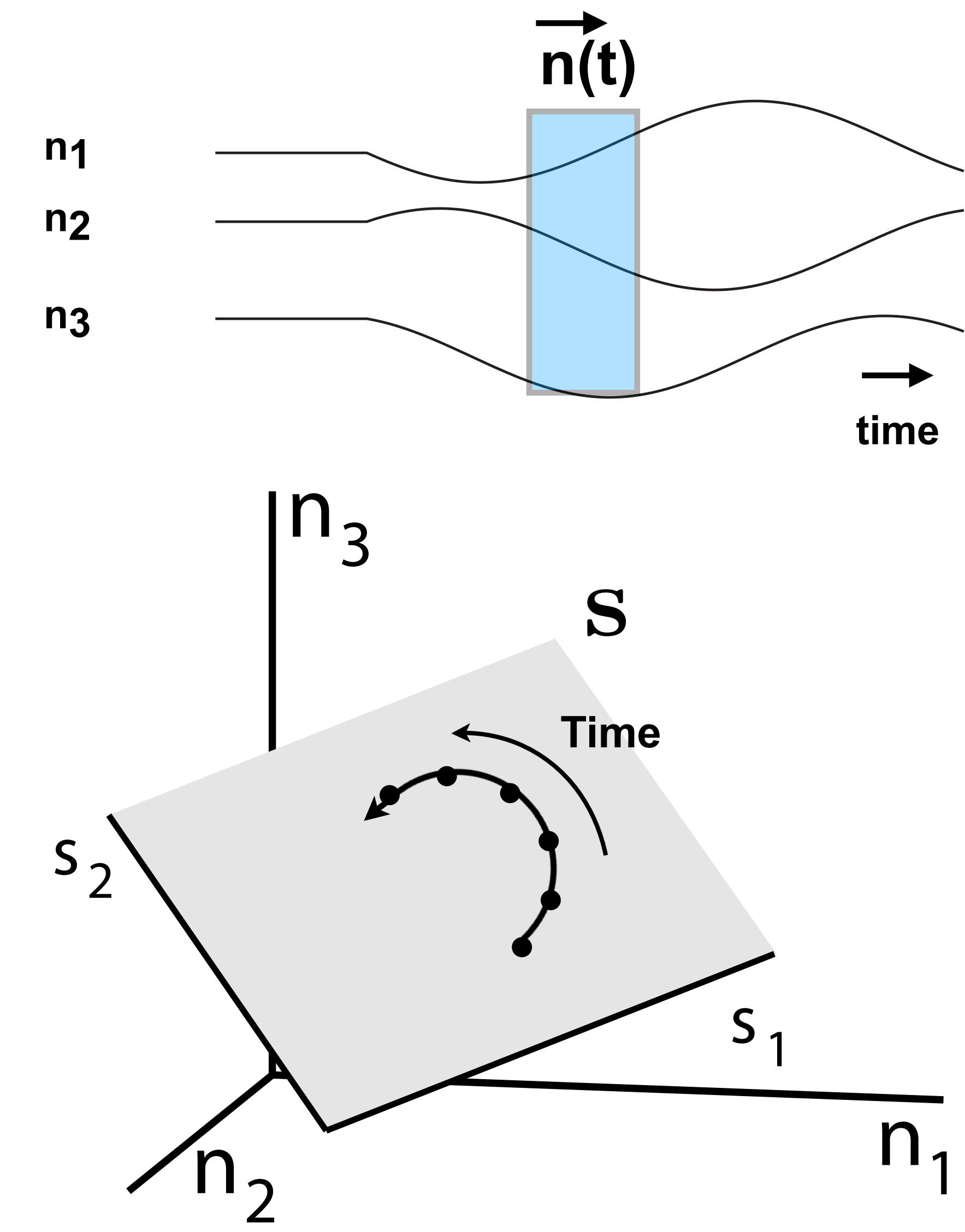


Changes in neurons' firing rates are coordinated



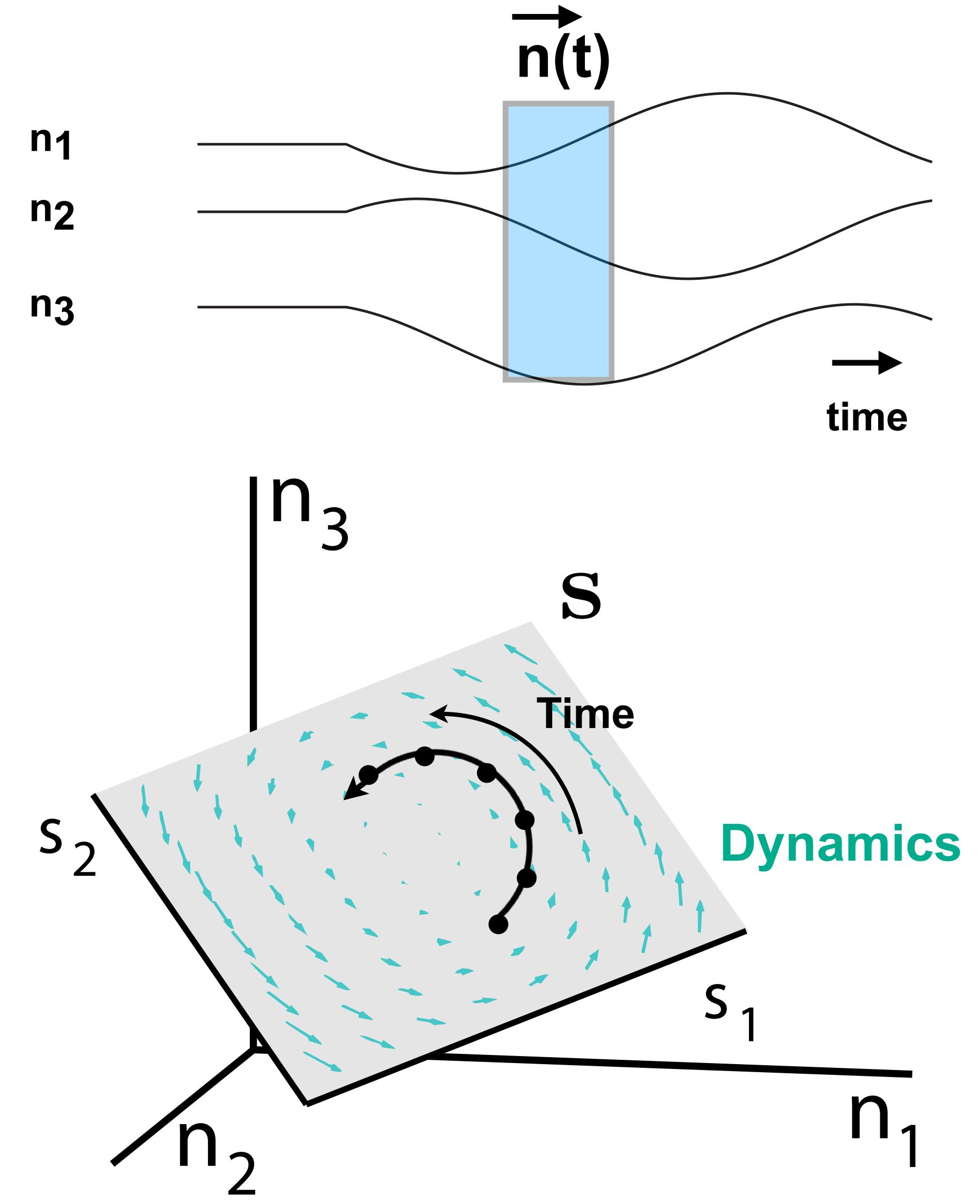
Neural population dynamics

Neurons'
firing rates



Neural population dynamics

Neurons' firing rates

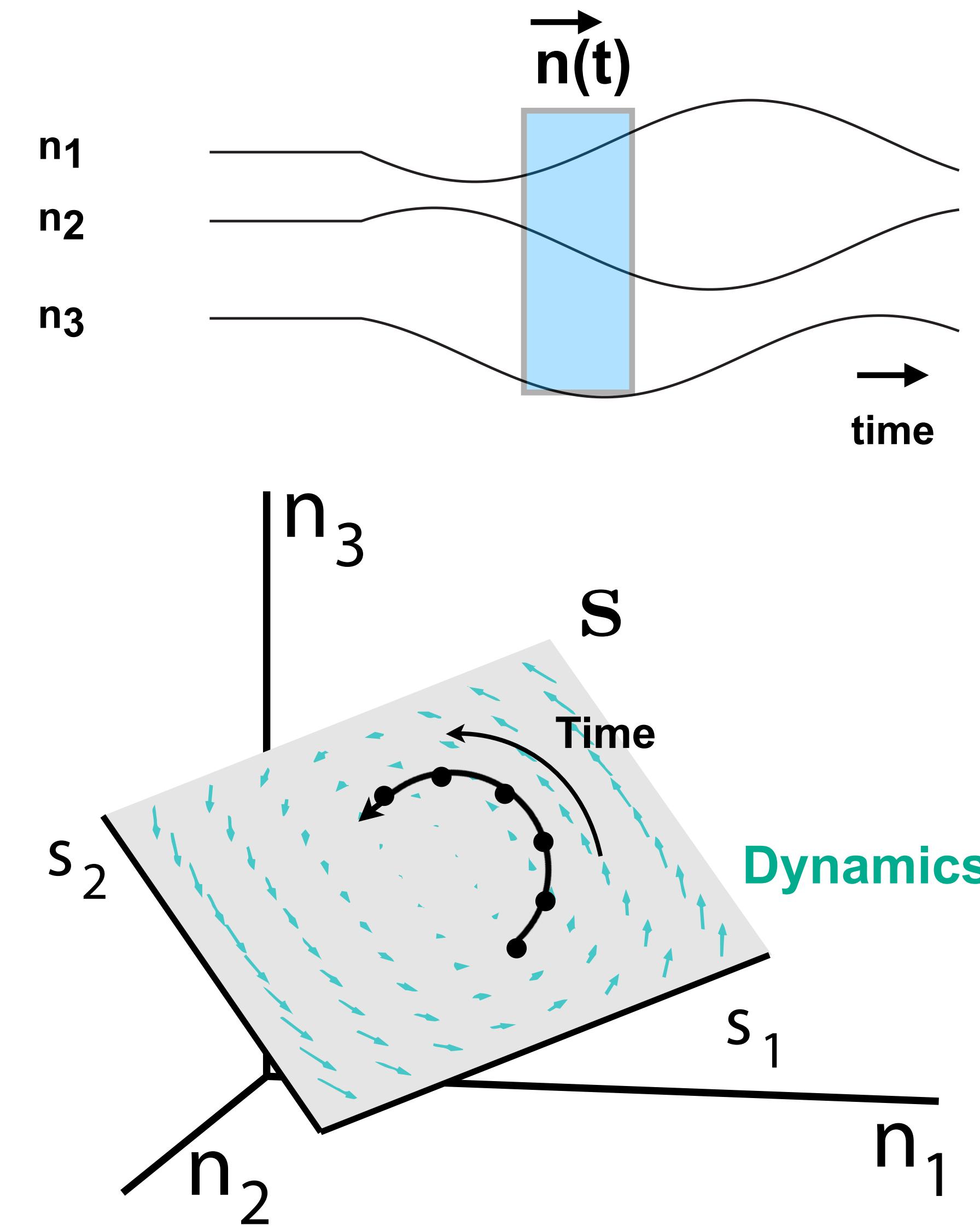


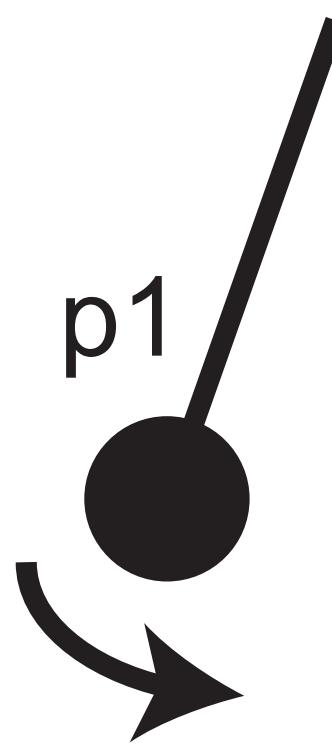
Neural population dynamics

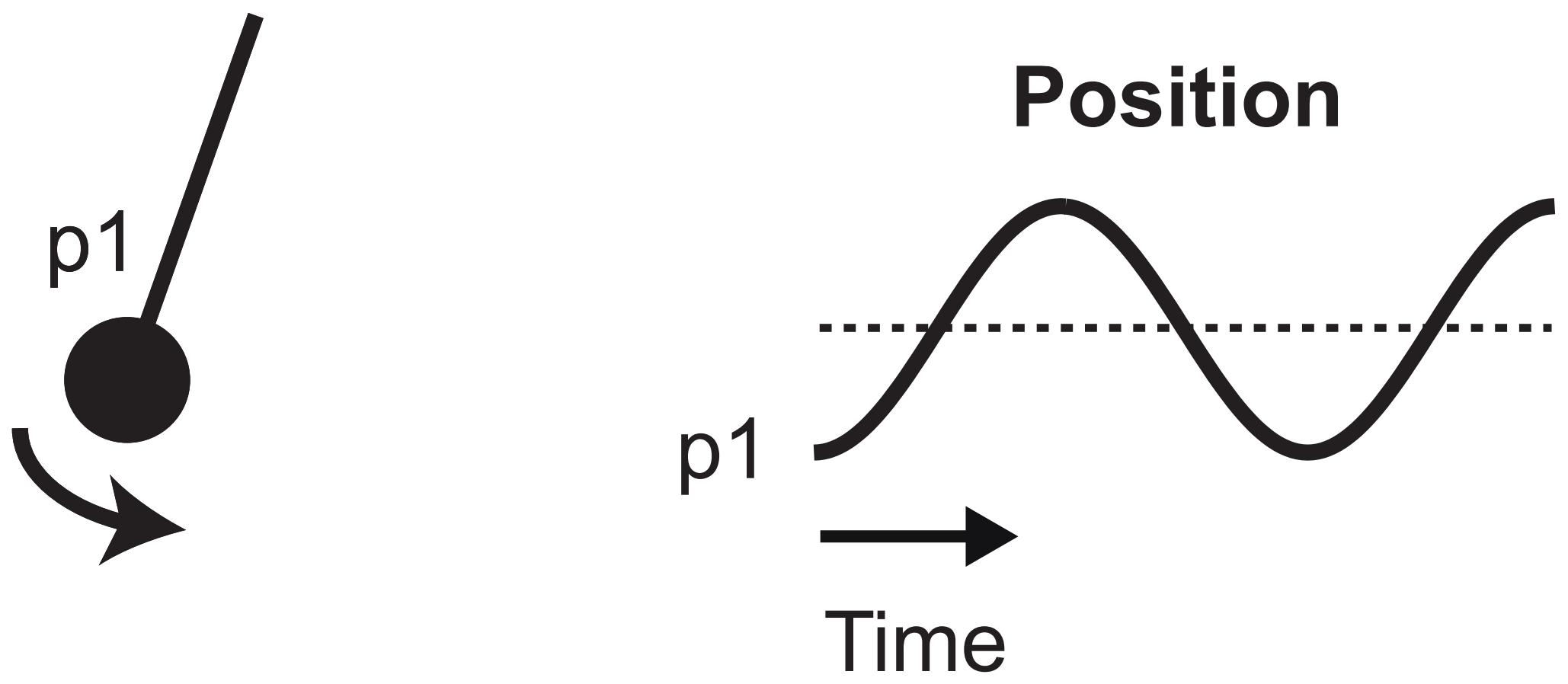
Predictable activity -
modeled by
autonomous
dynamics

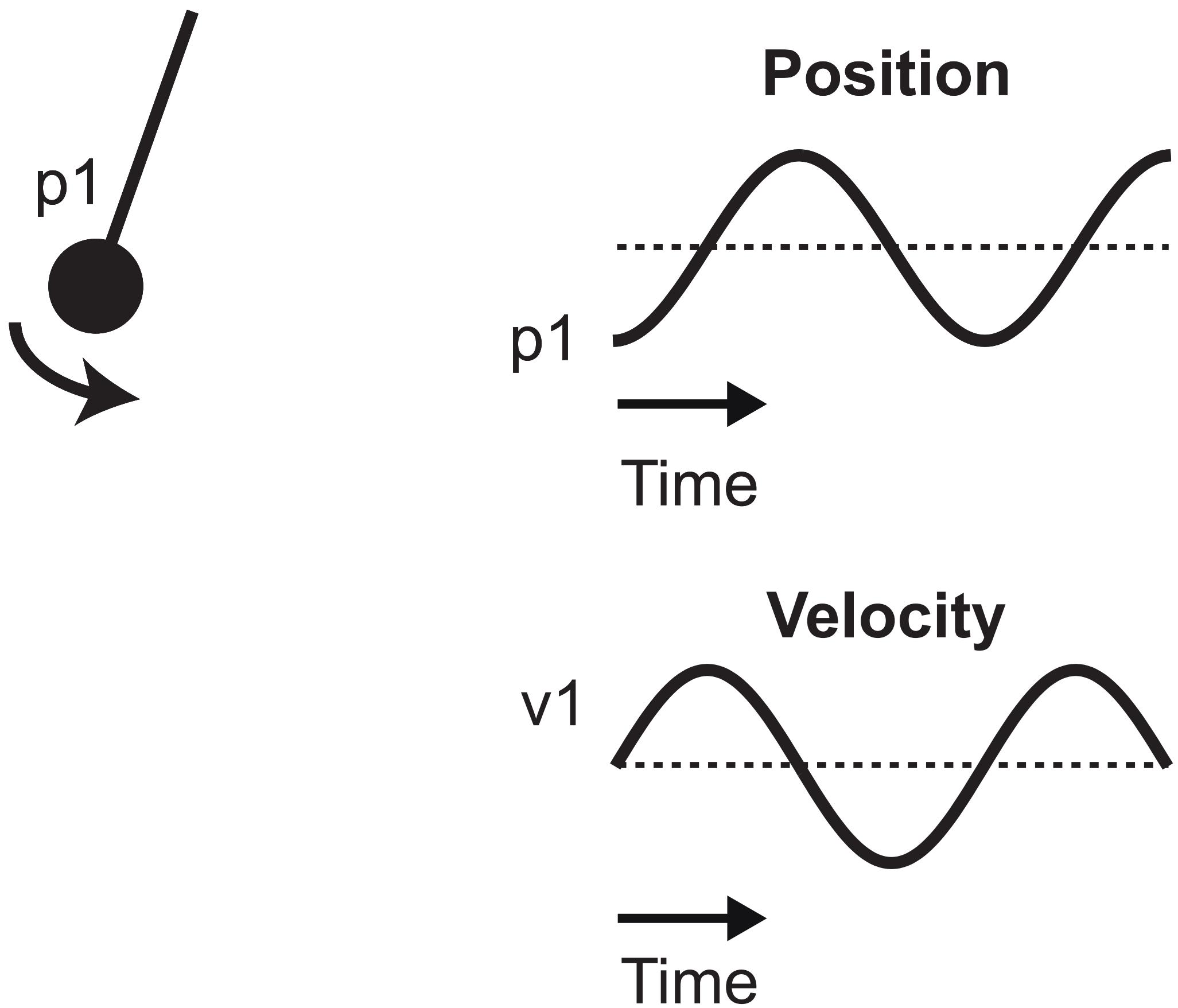
$$\frac{ds}{dt} = f(s)$$

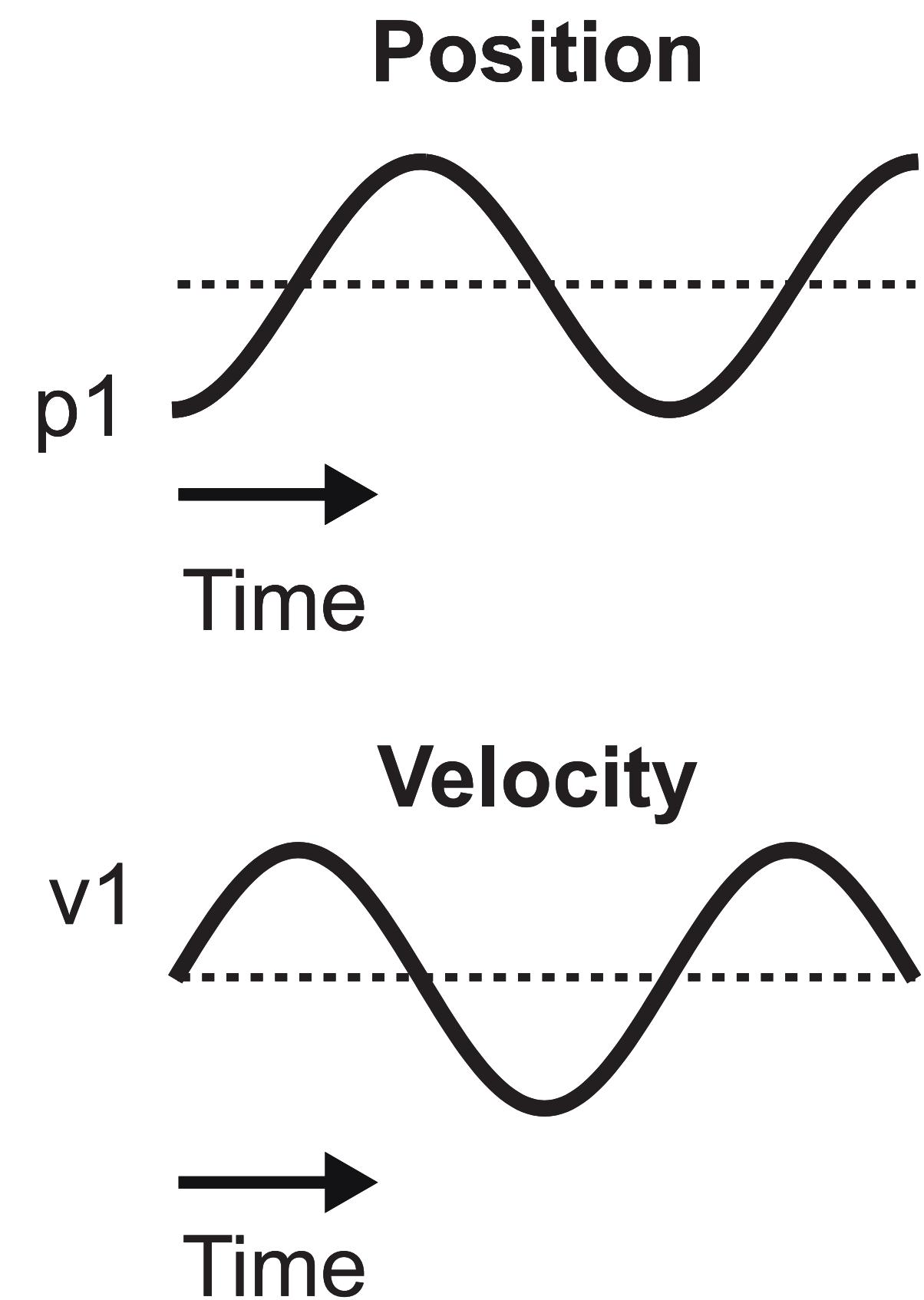
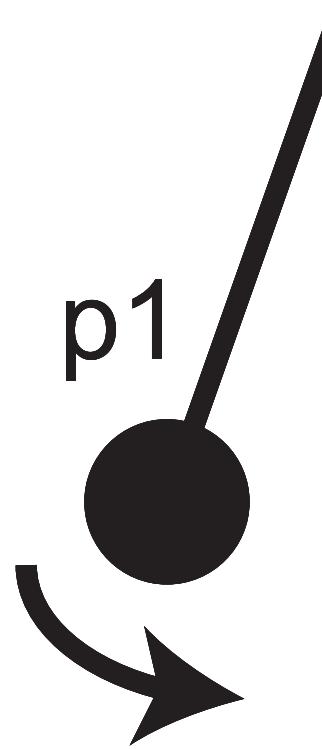
Neurons'
firing rates



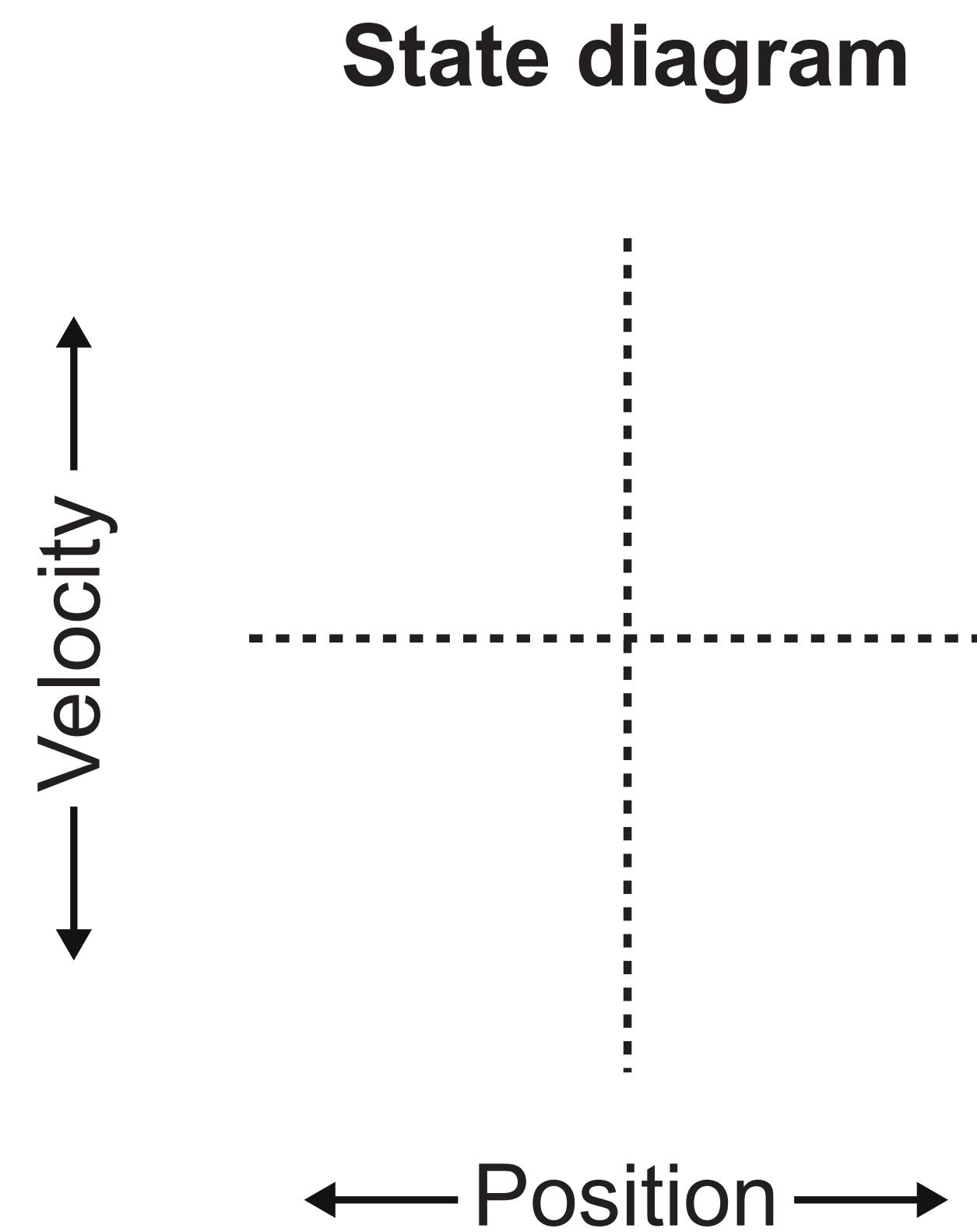


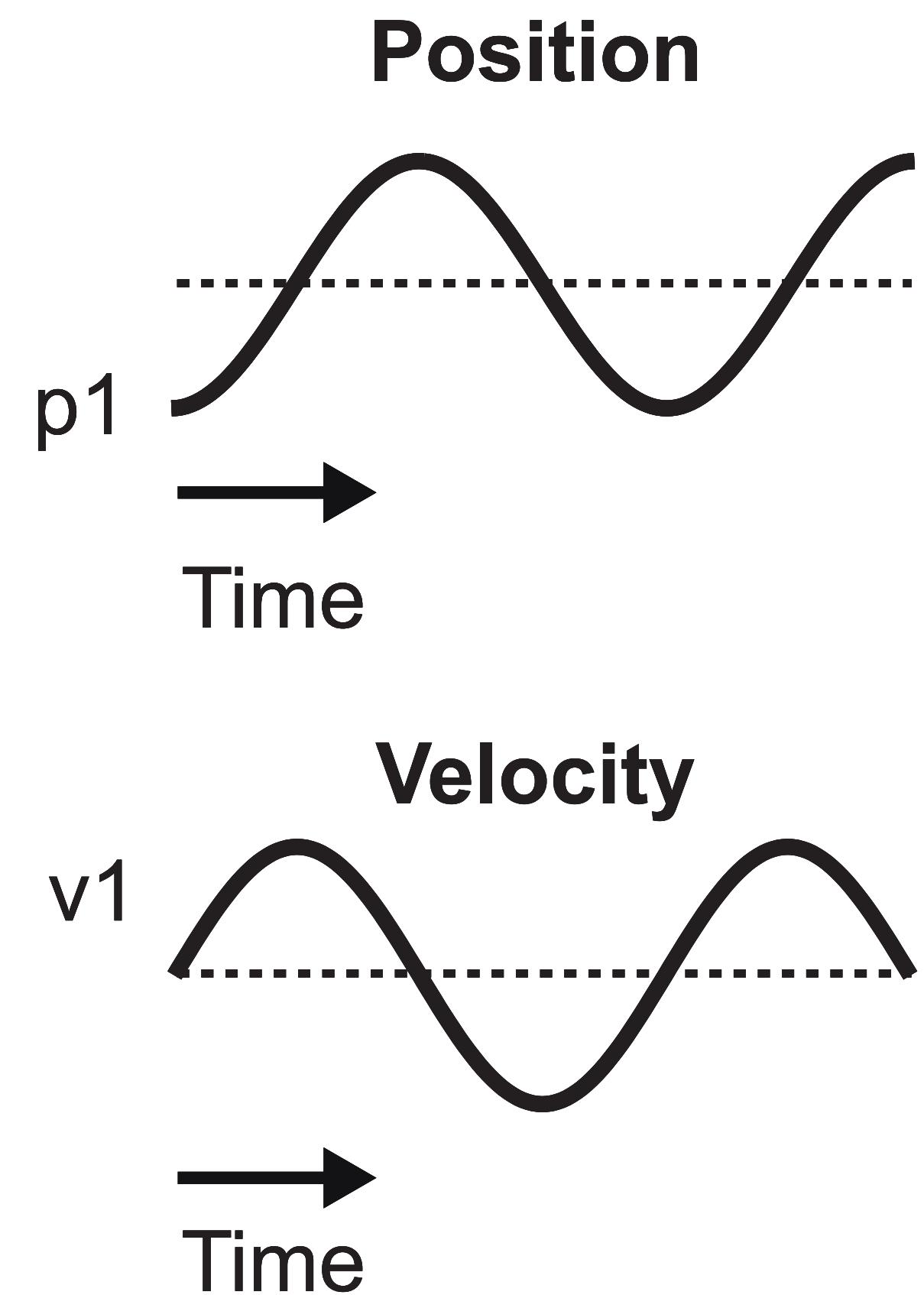
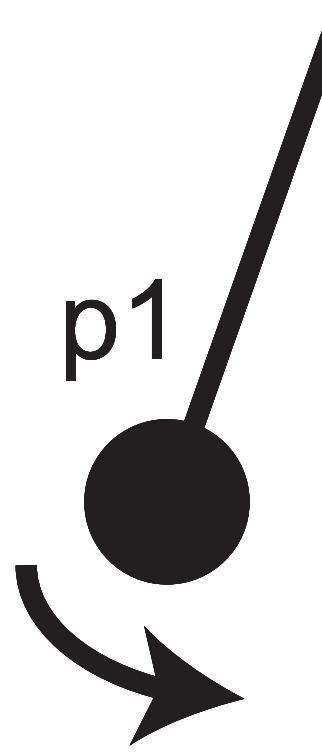




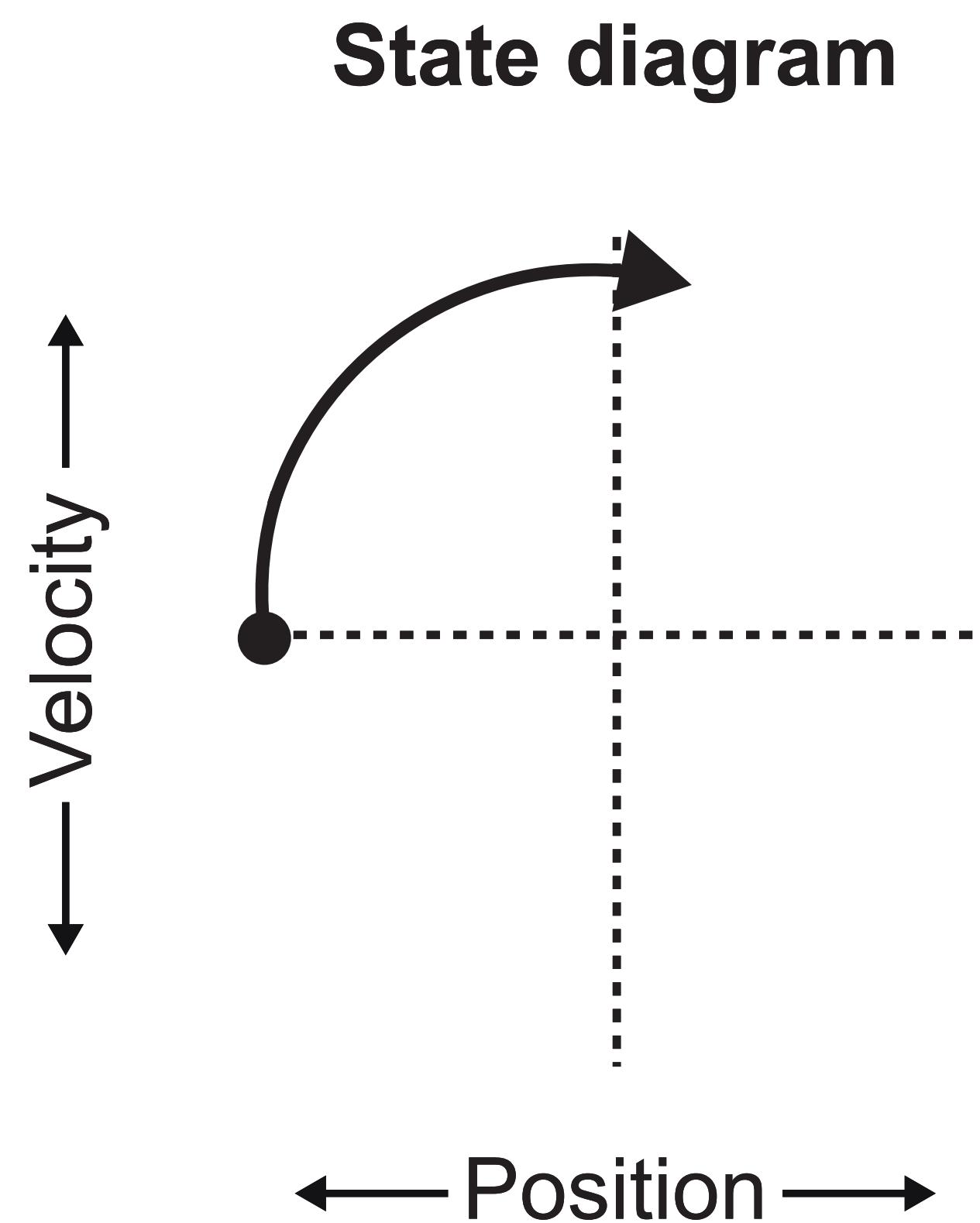


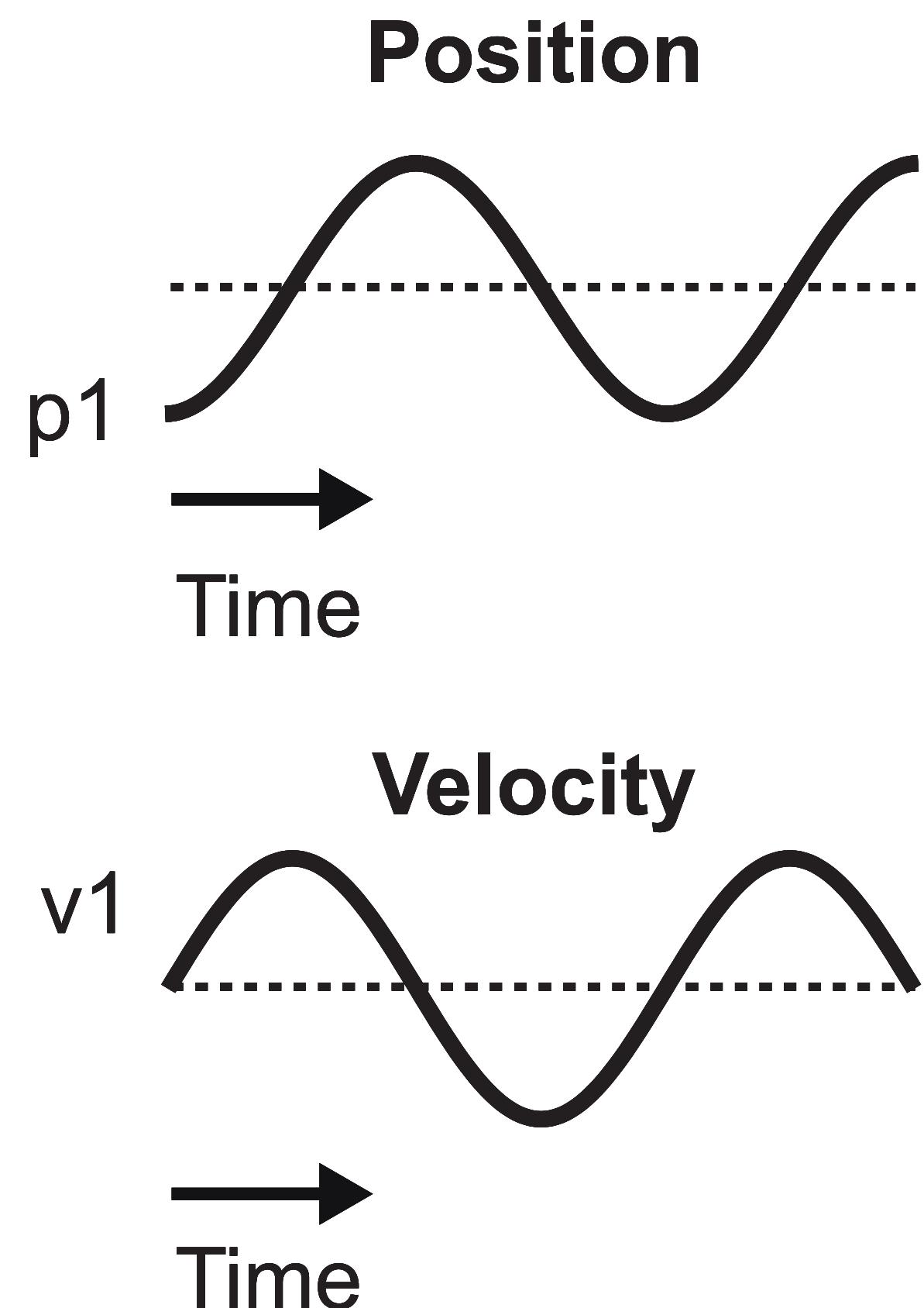
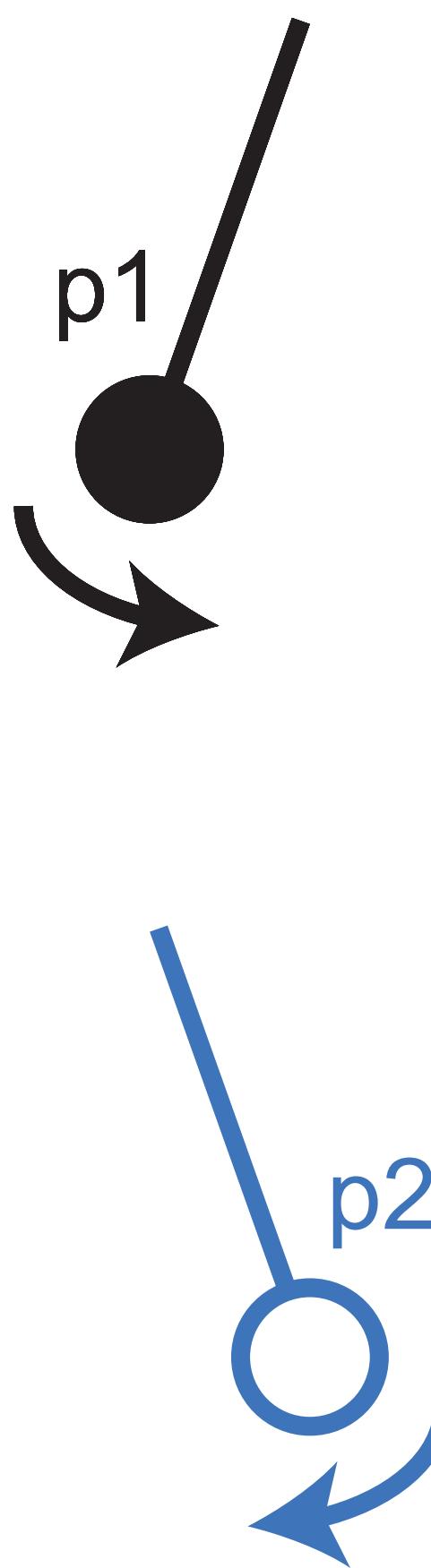
$$\mathbf{x} = \begin{bmatrix} p \\ v \end{bmatrix}$$



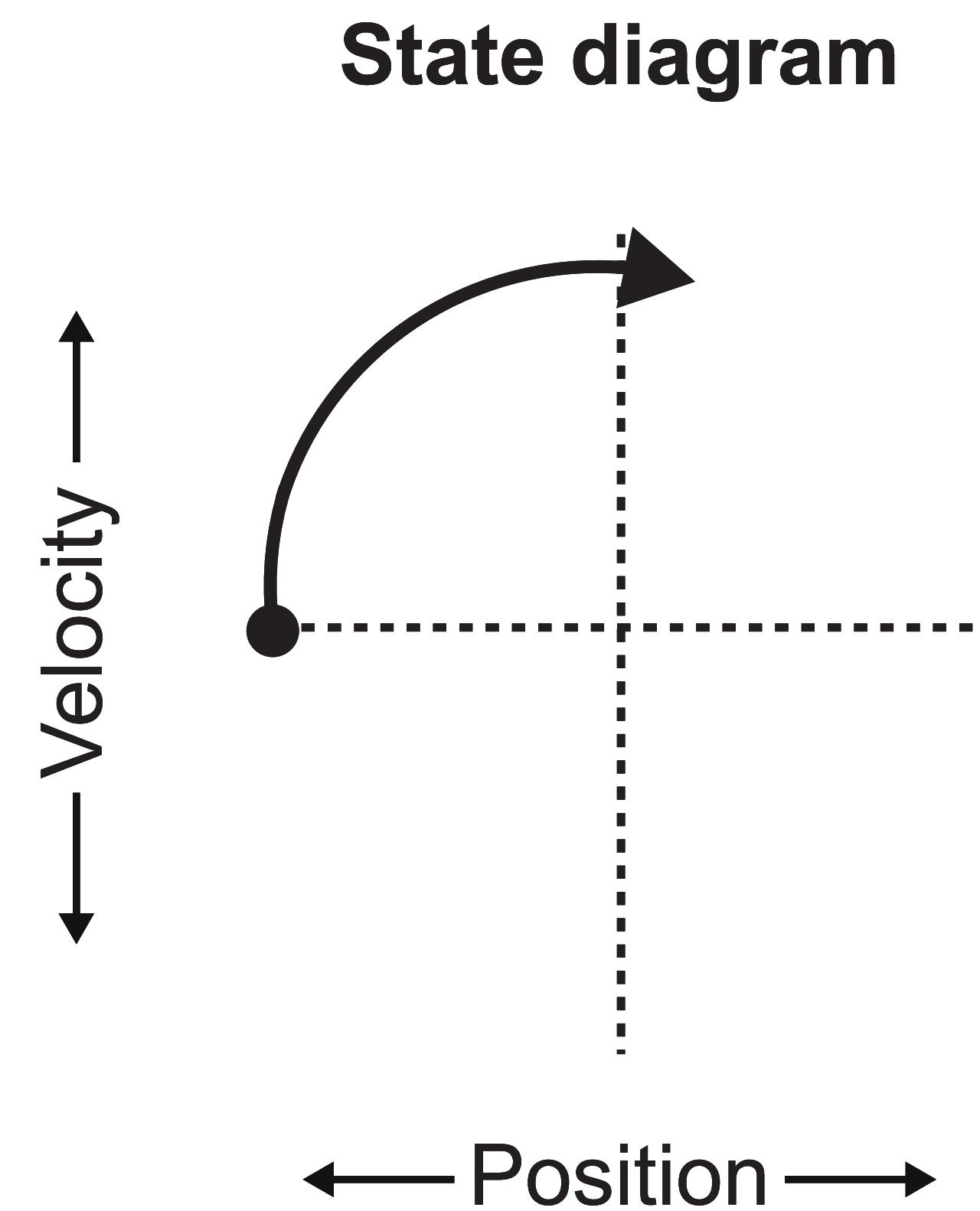


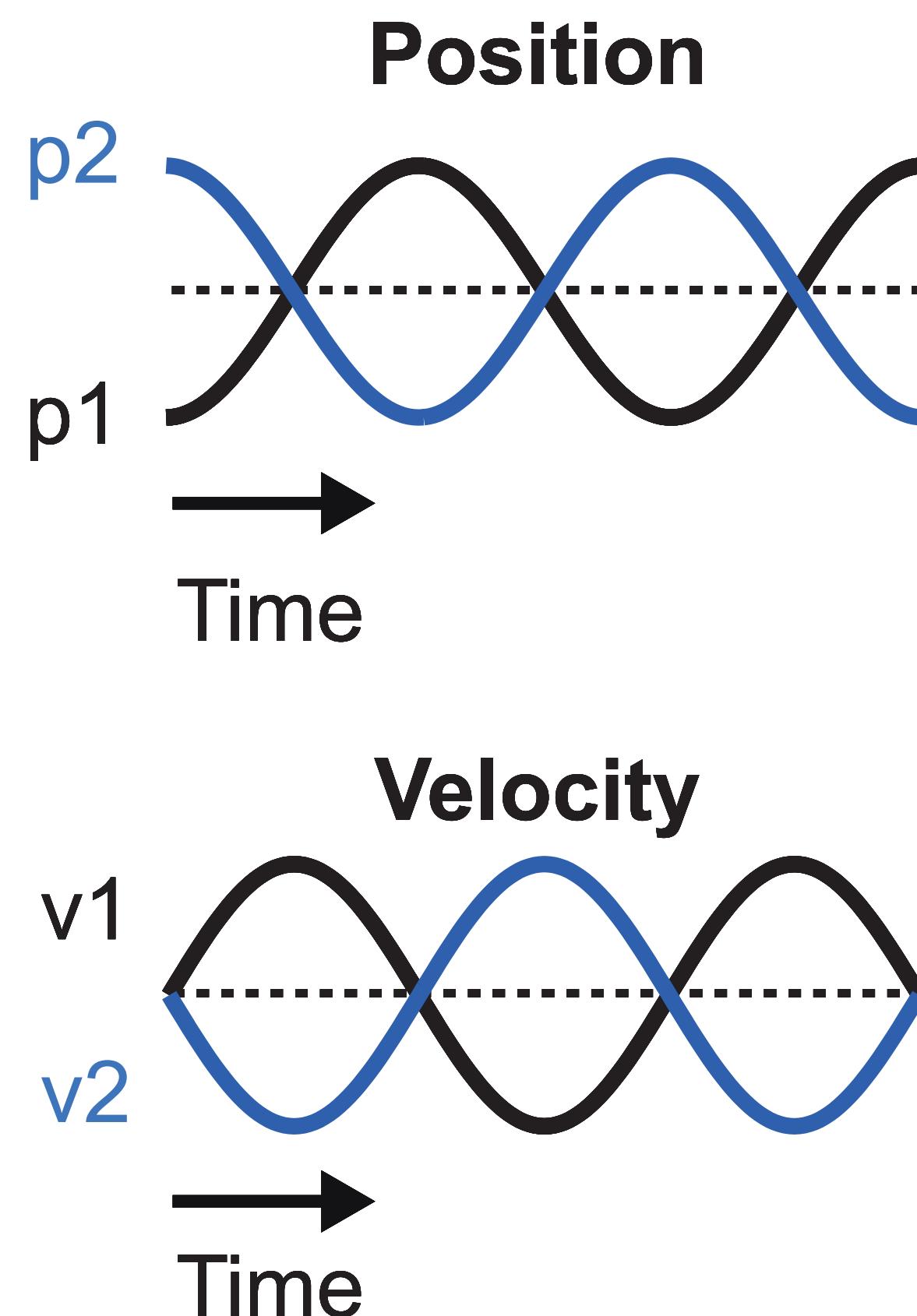
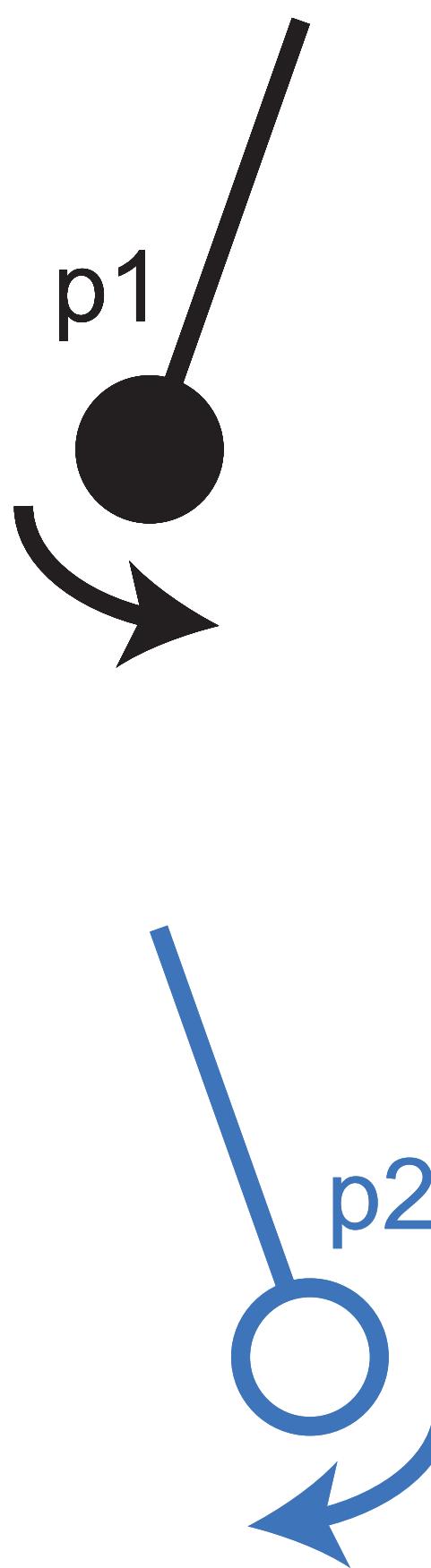
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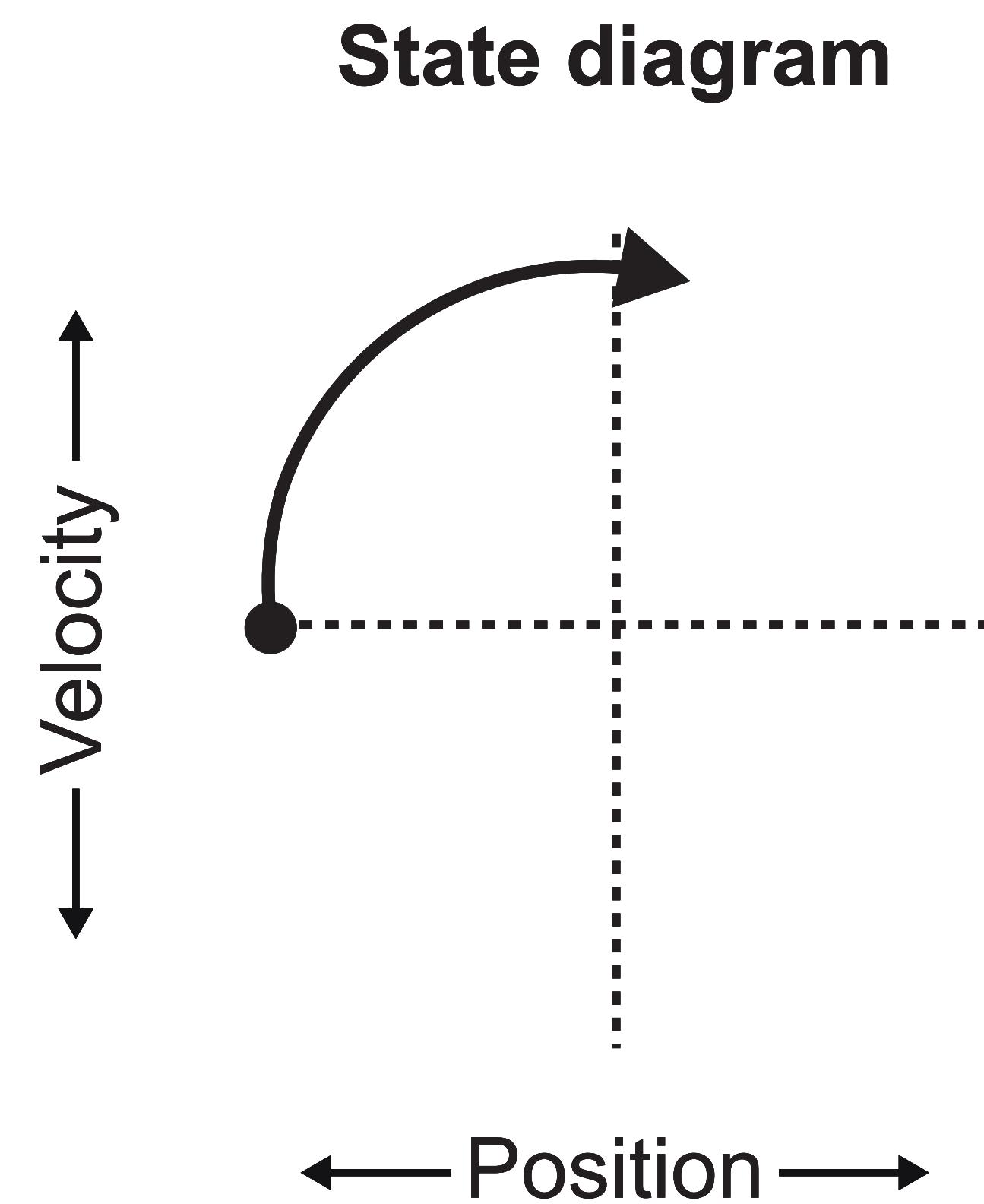


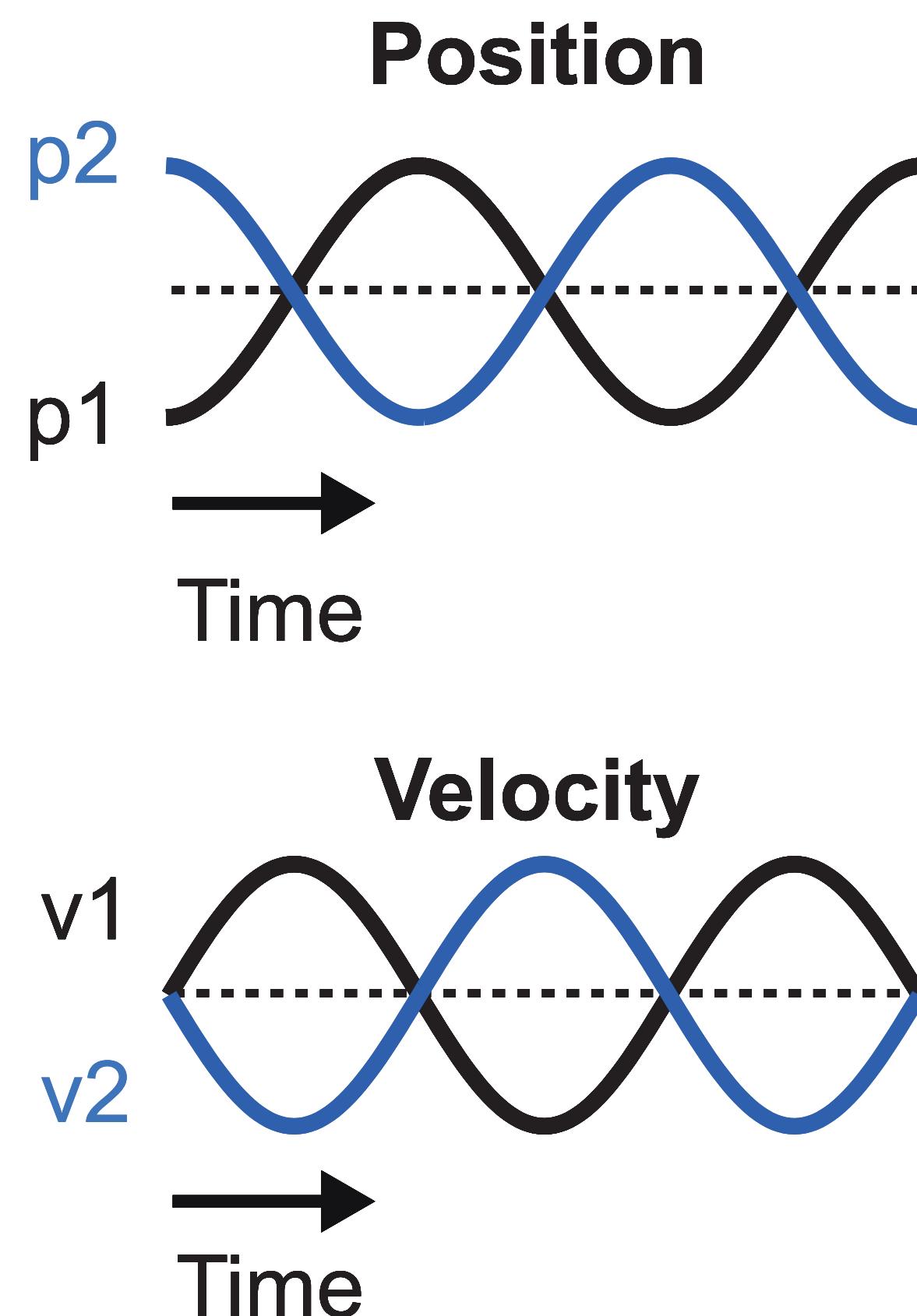
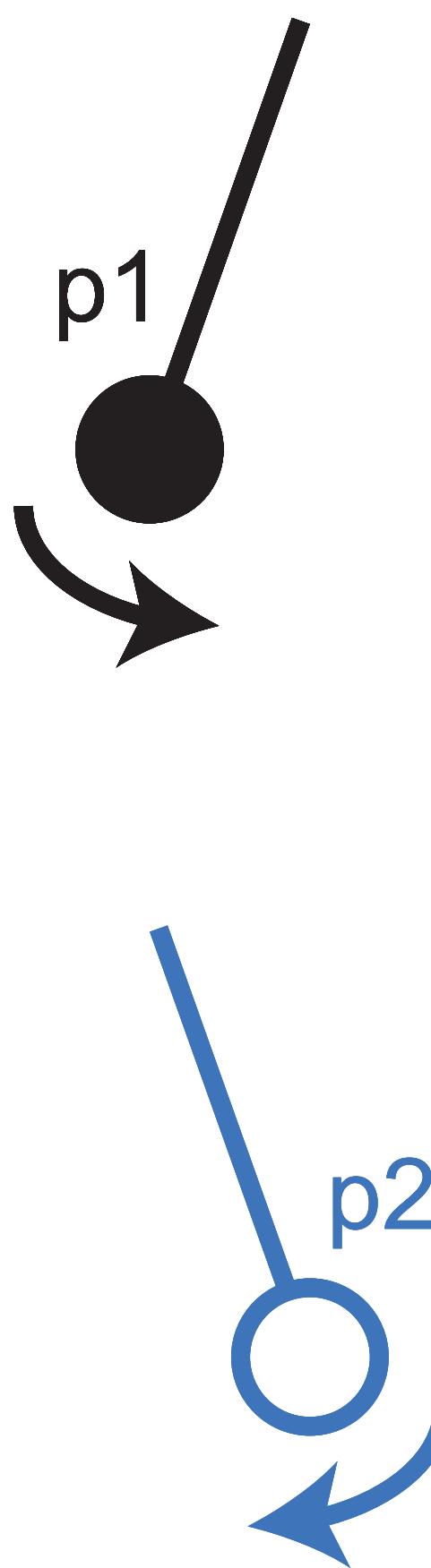
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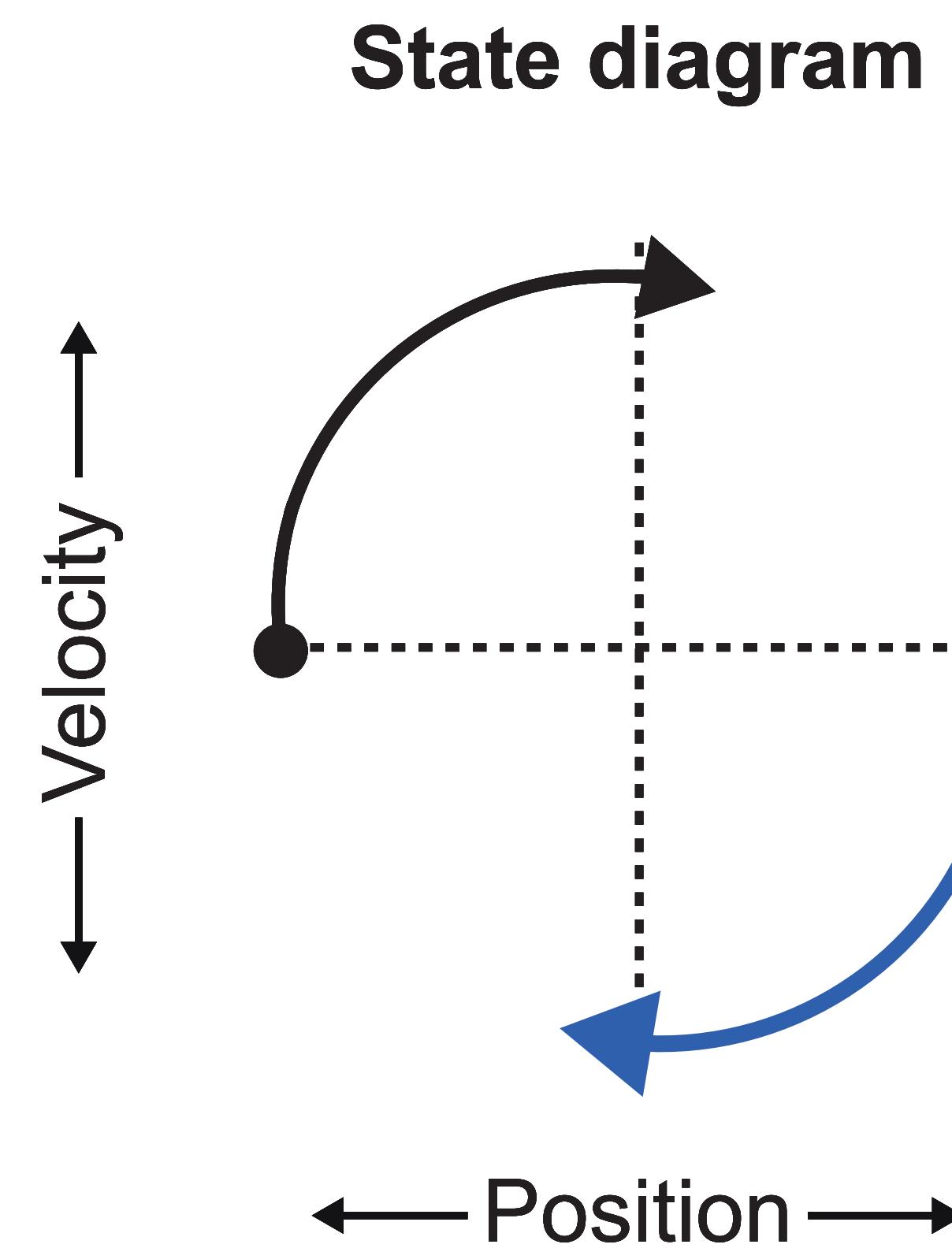


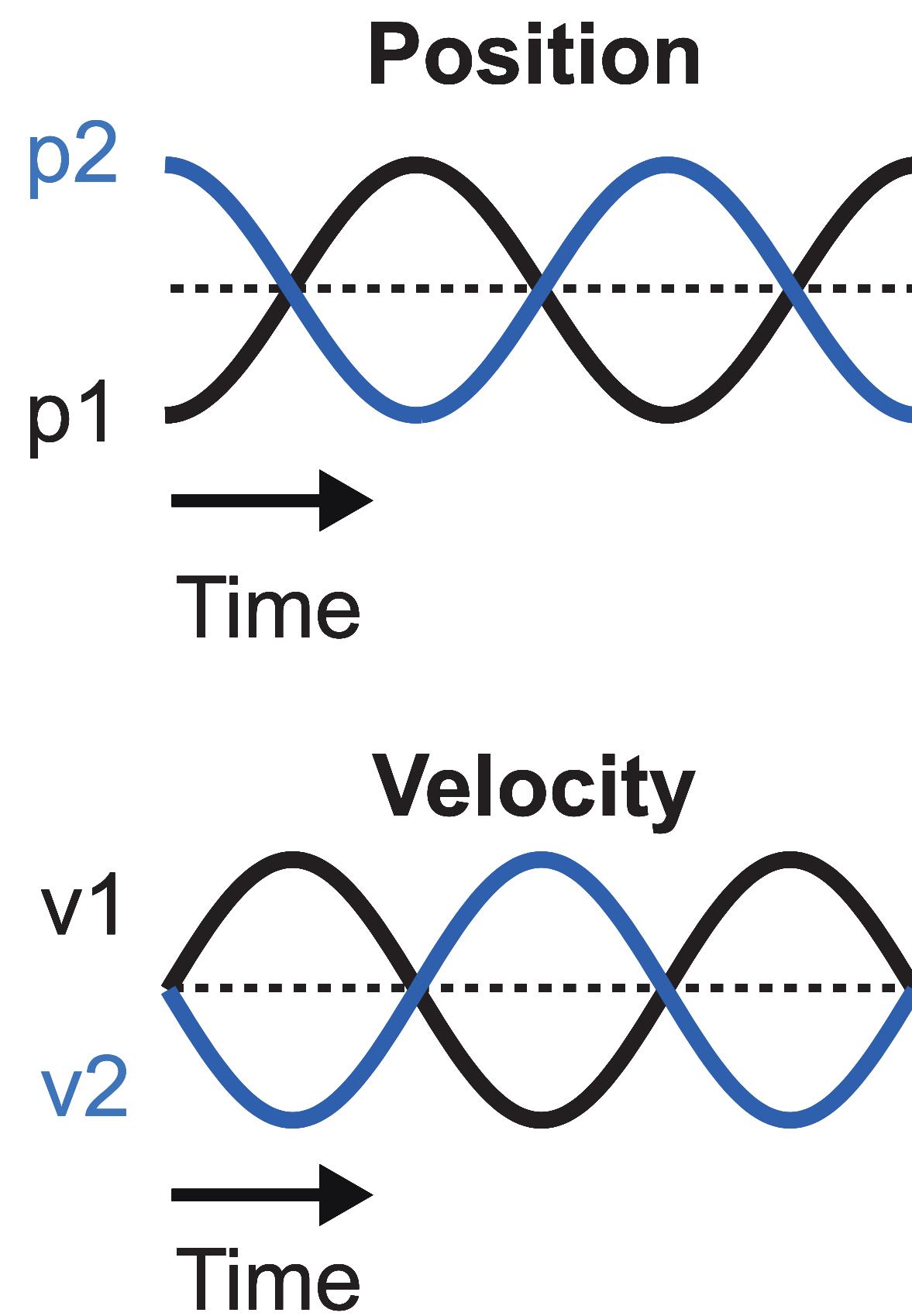
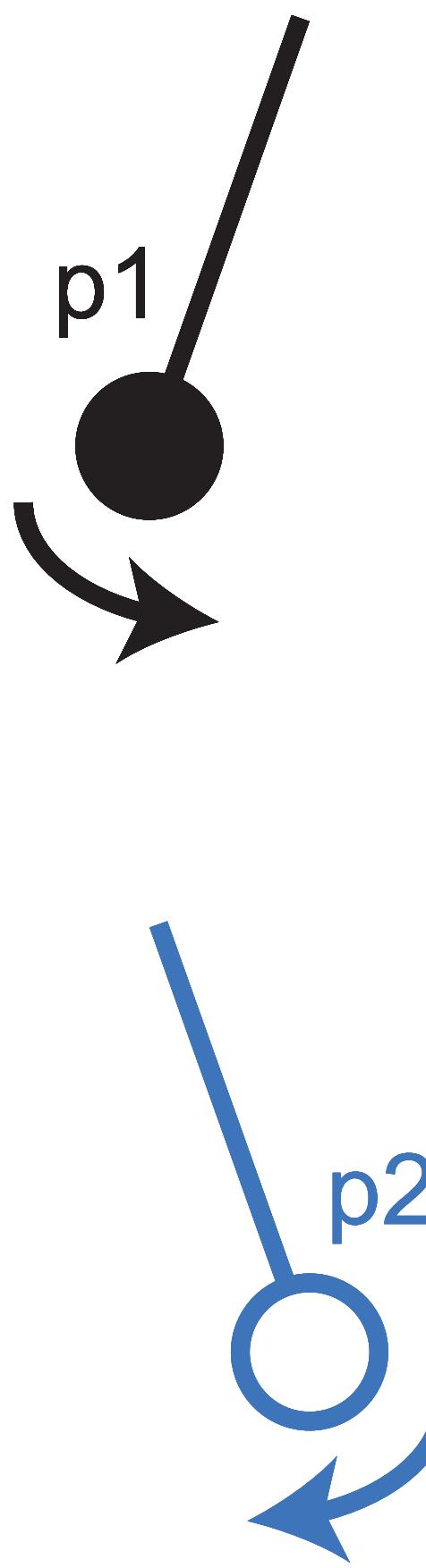
$$\mathbf{x} = \begin{bmatrix} p \\ v \end{bmatrix}$$





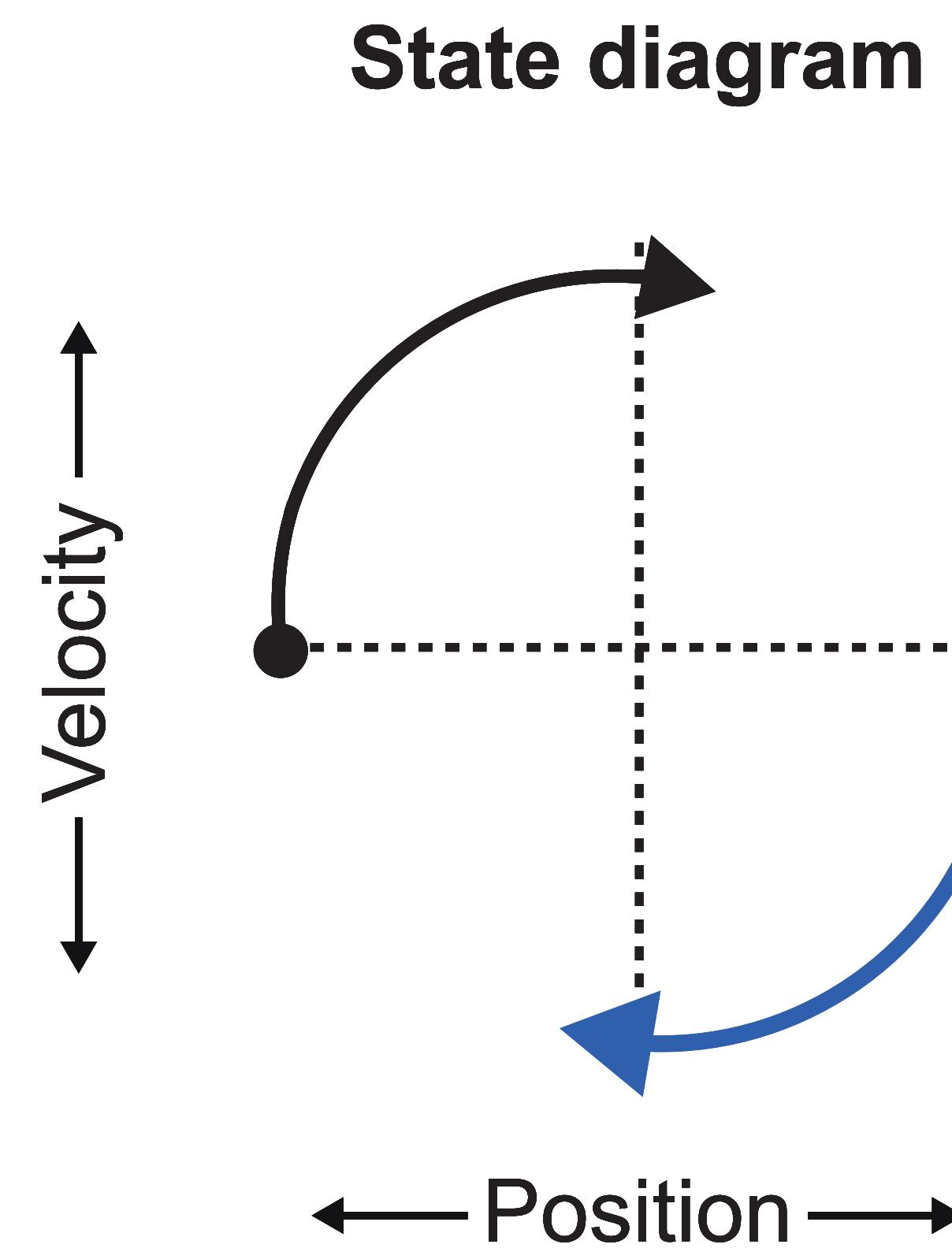
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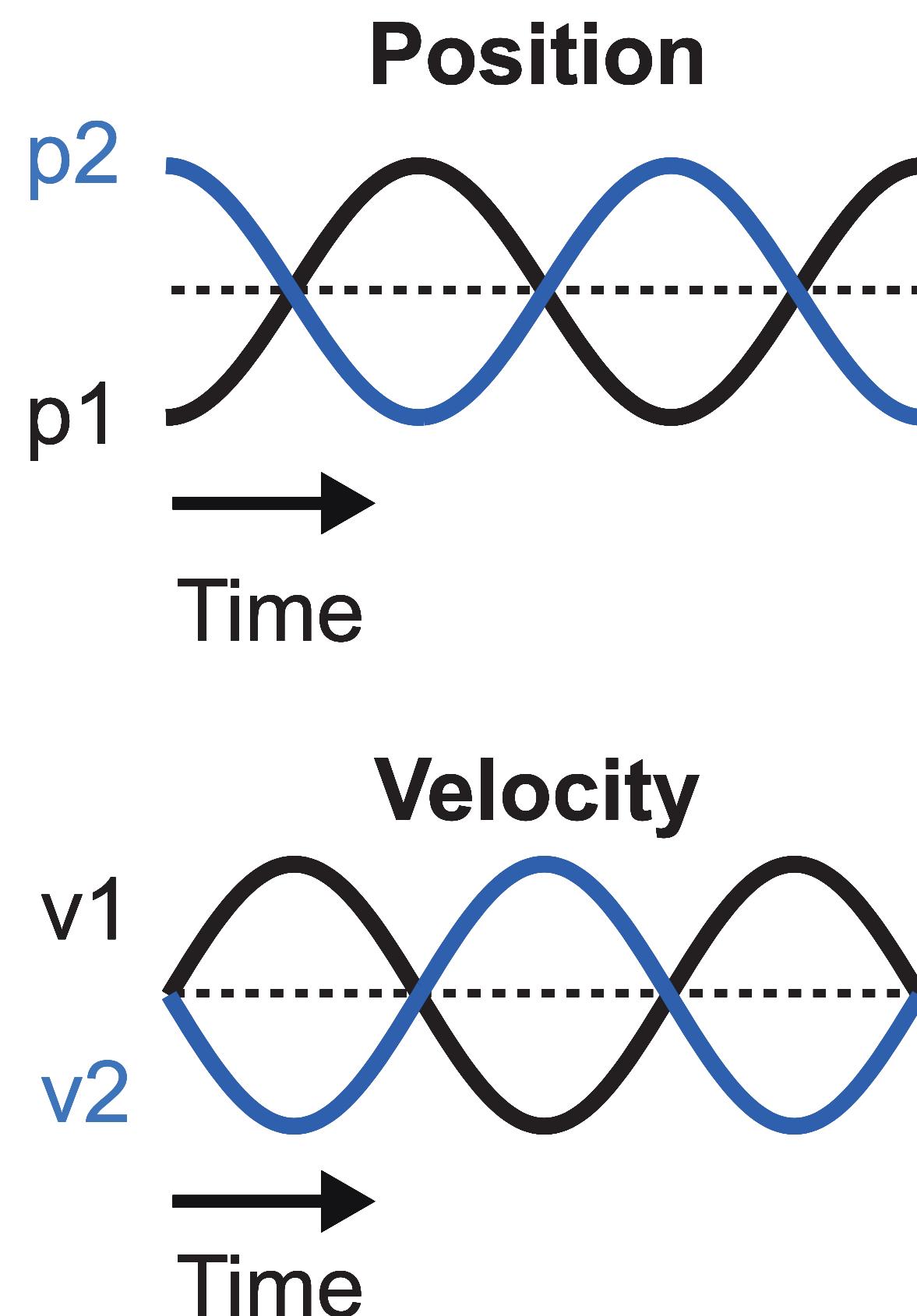
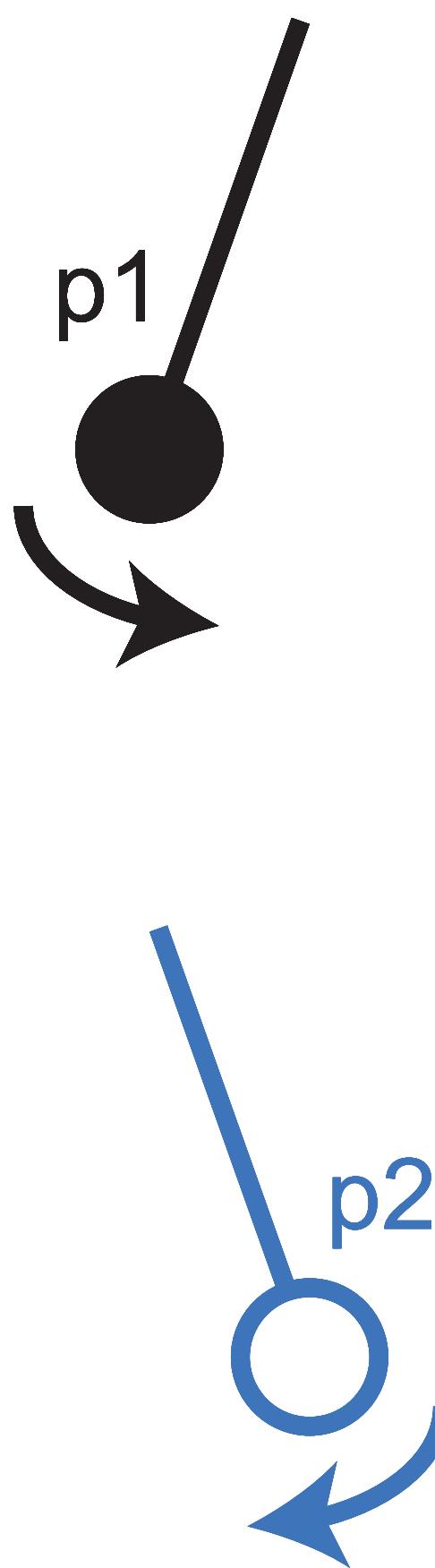




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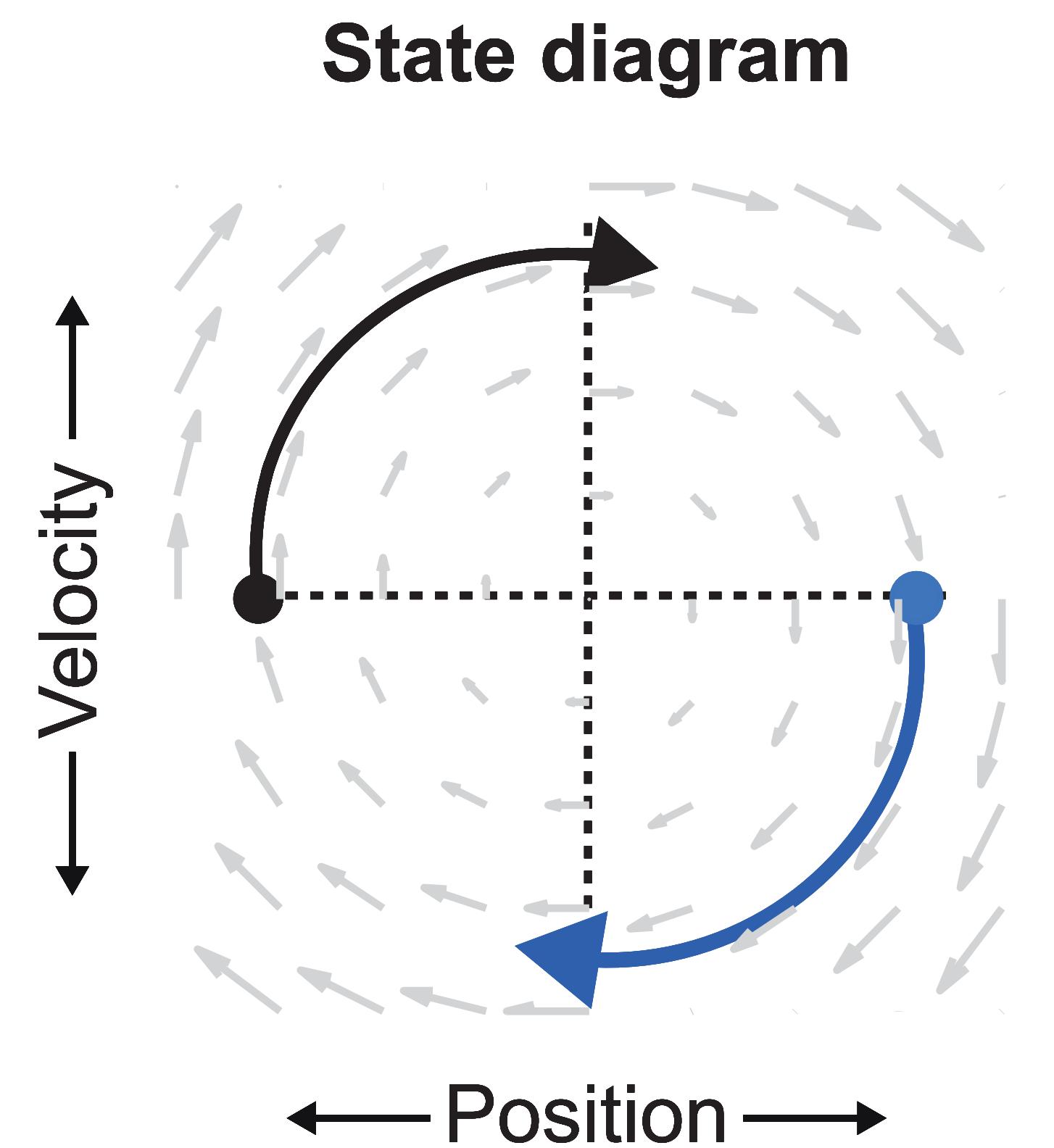
$$\dot{\mathbf{x}} = f(\mathbf{x})$$





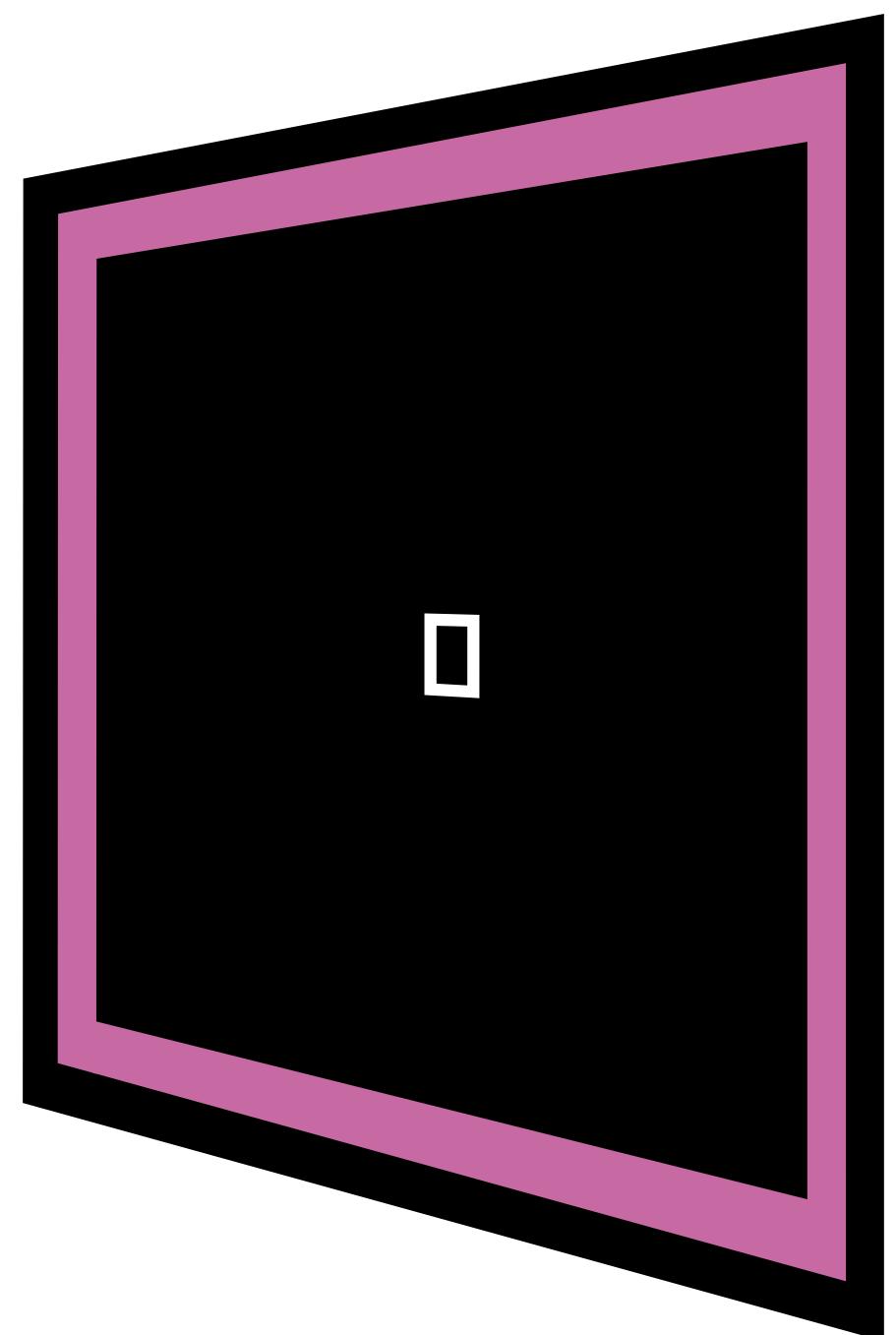
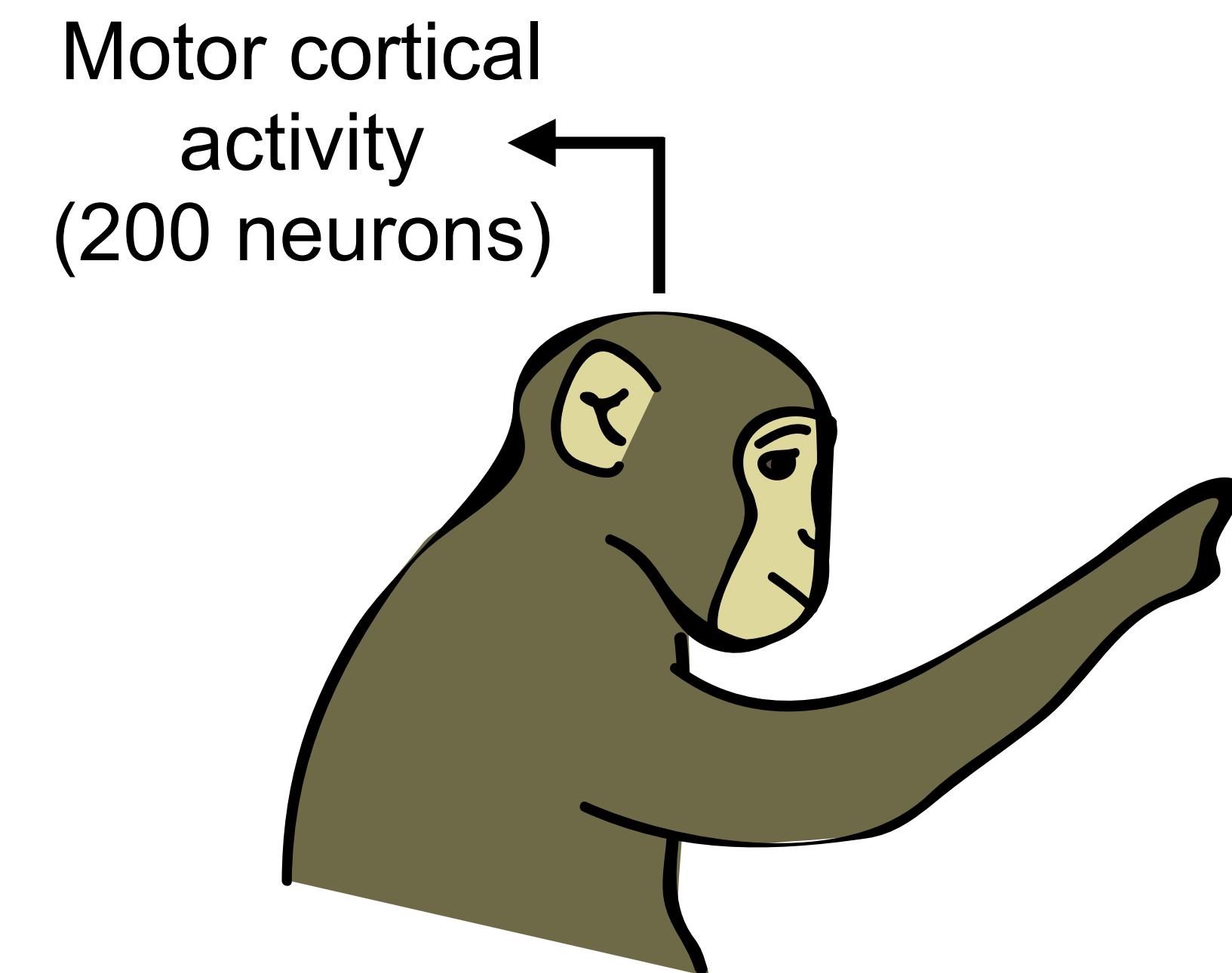
$$\mathbf{x} = \begin{bmatrix} p \\ v \end{bmatrix}$$

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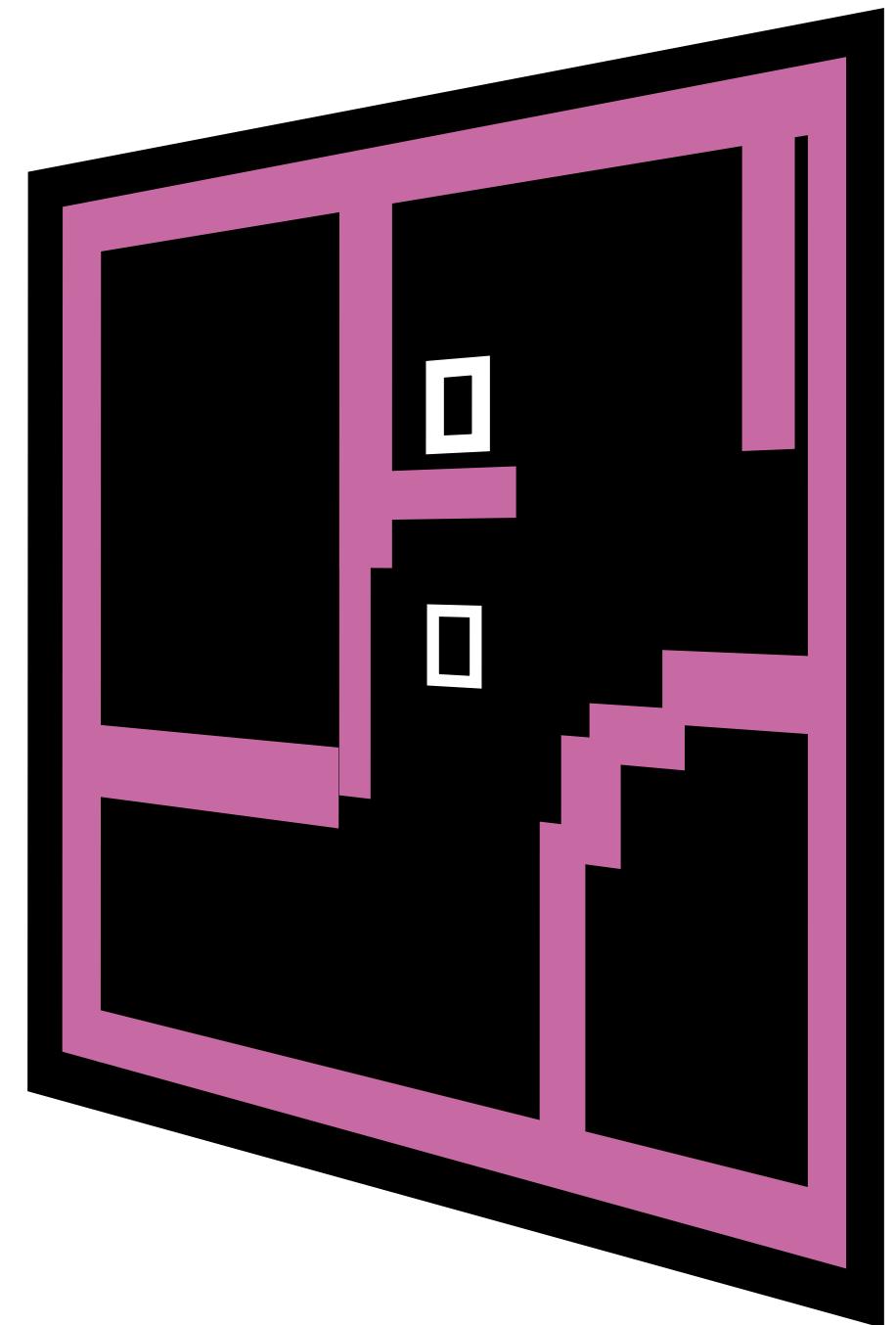
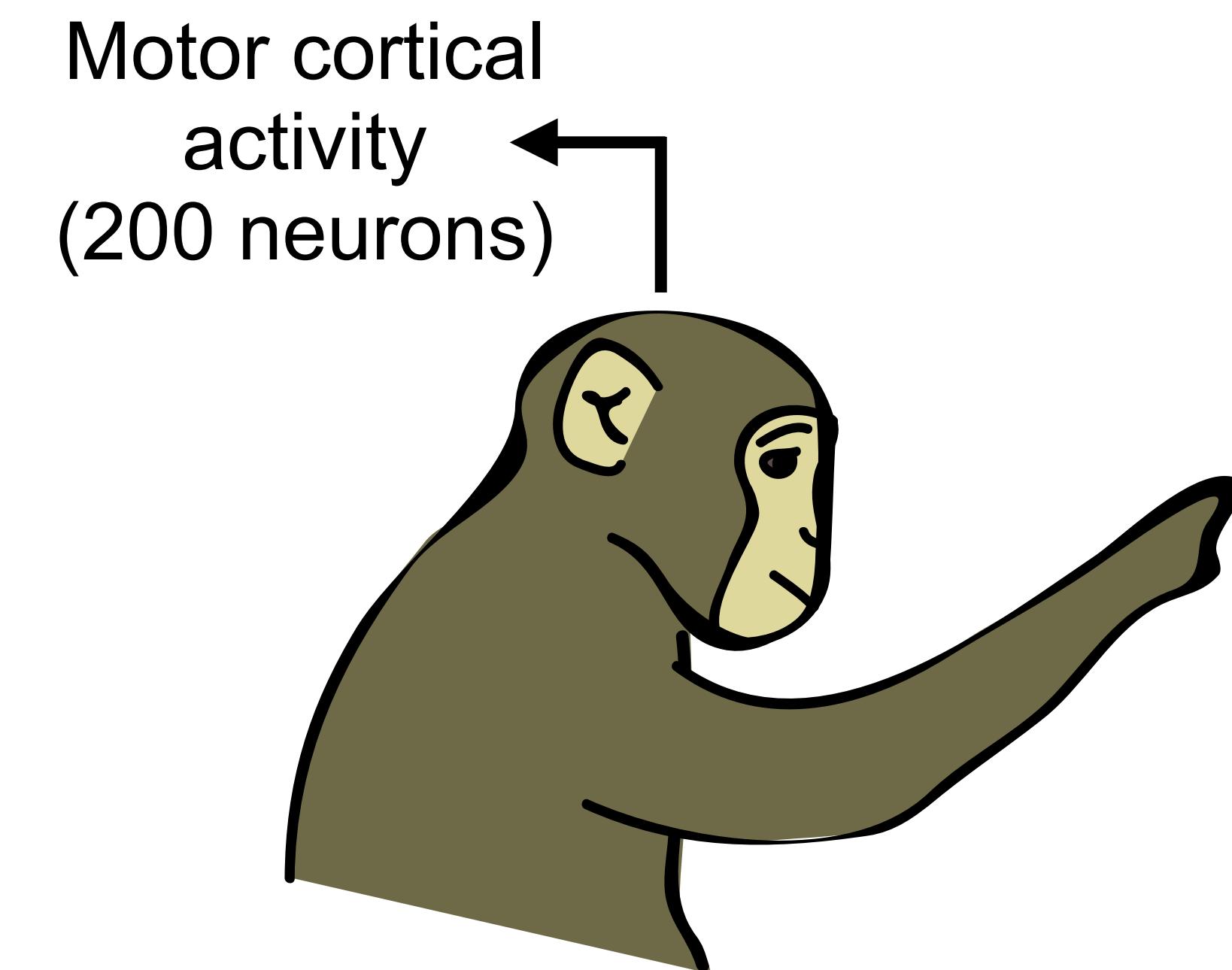
Predictable activity: delayed-reaching

- Motor cortex is set to an initial state during the preparatory phase
- Activity during movement execution is highly predictable based on initial state



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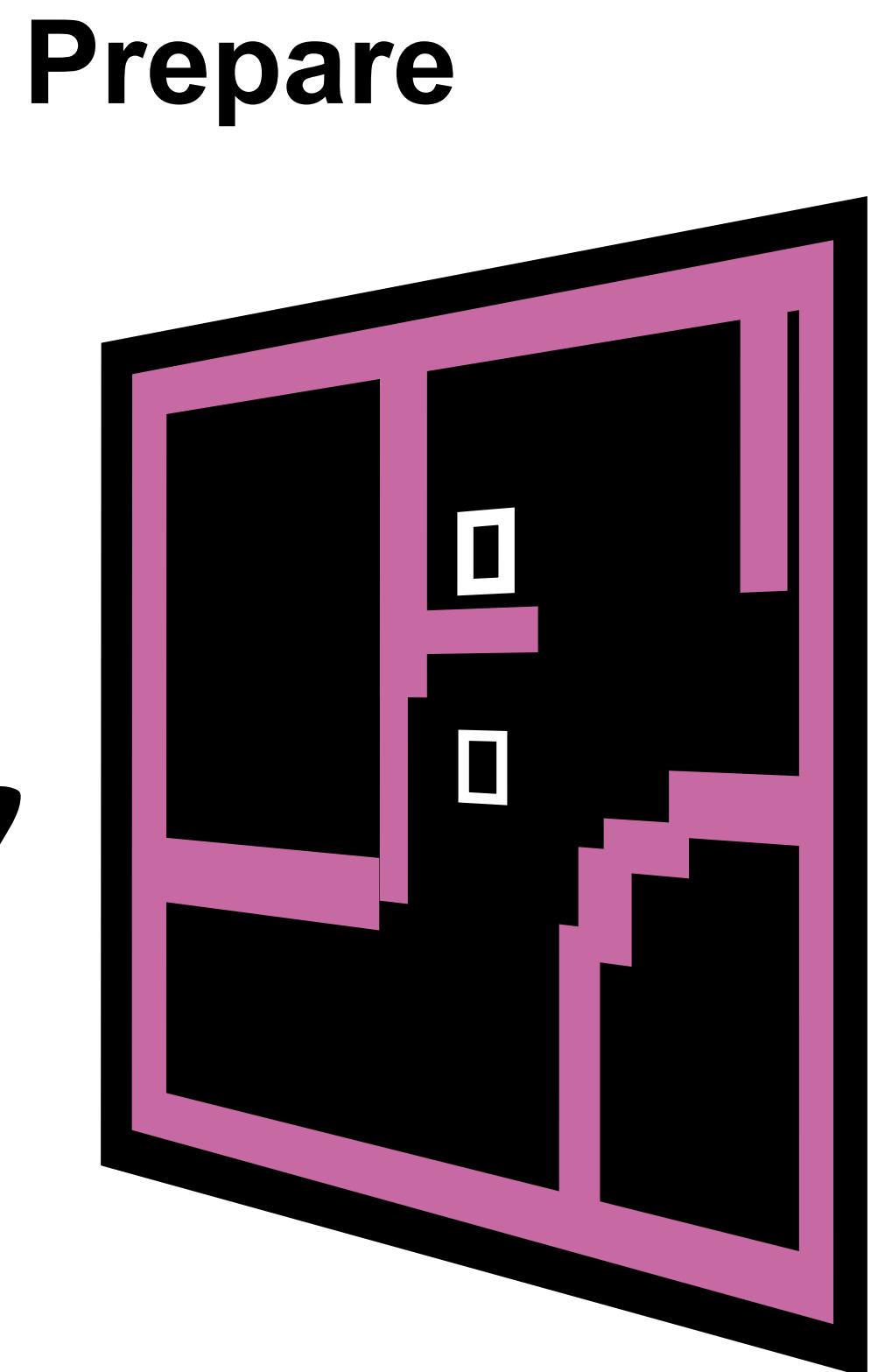
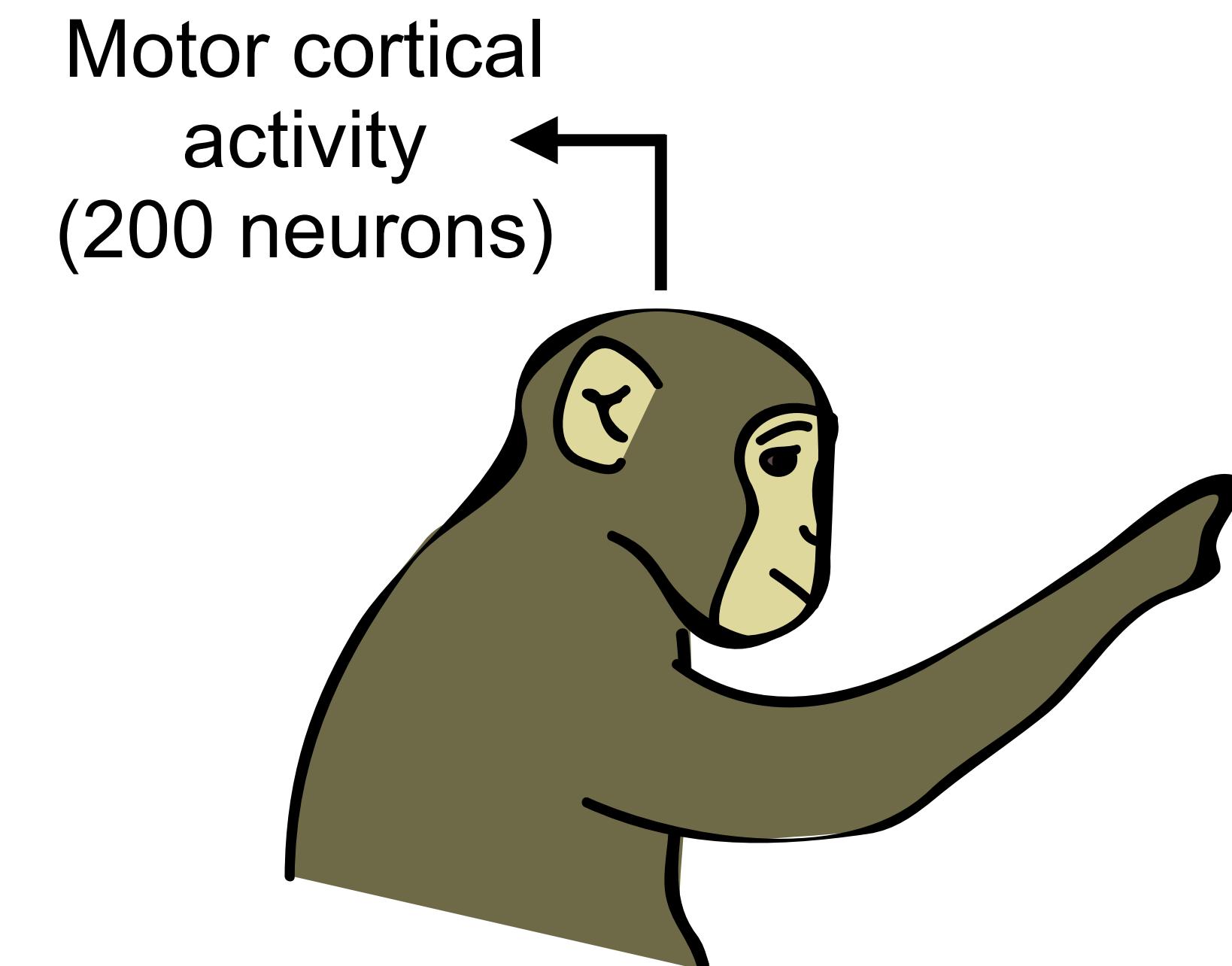
Condition 1

Churchland*, Cunningham* ... Shenoy, *Nature* 2012

Shenoy, Sahani, Churchland, *Ann Rev Neuro* 2013

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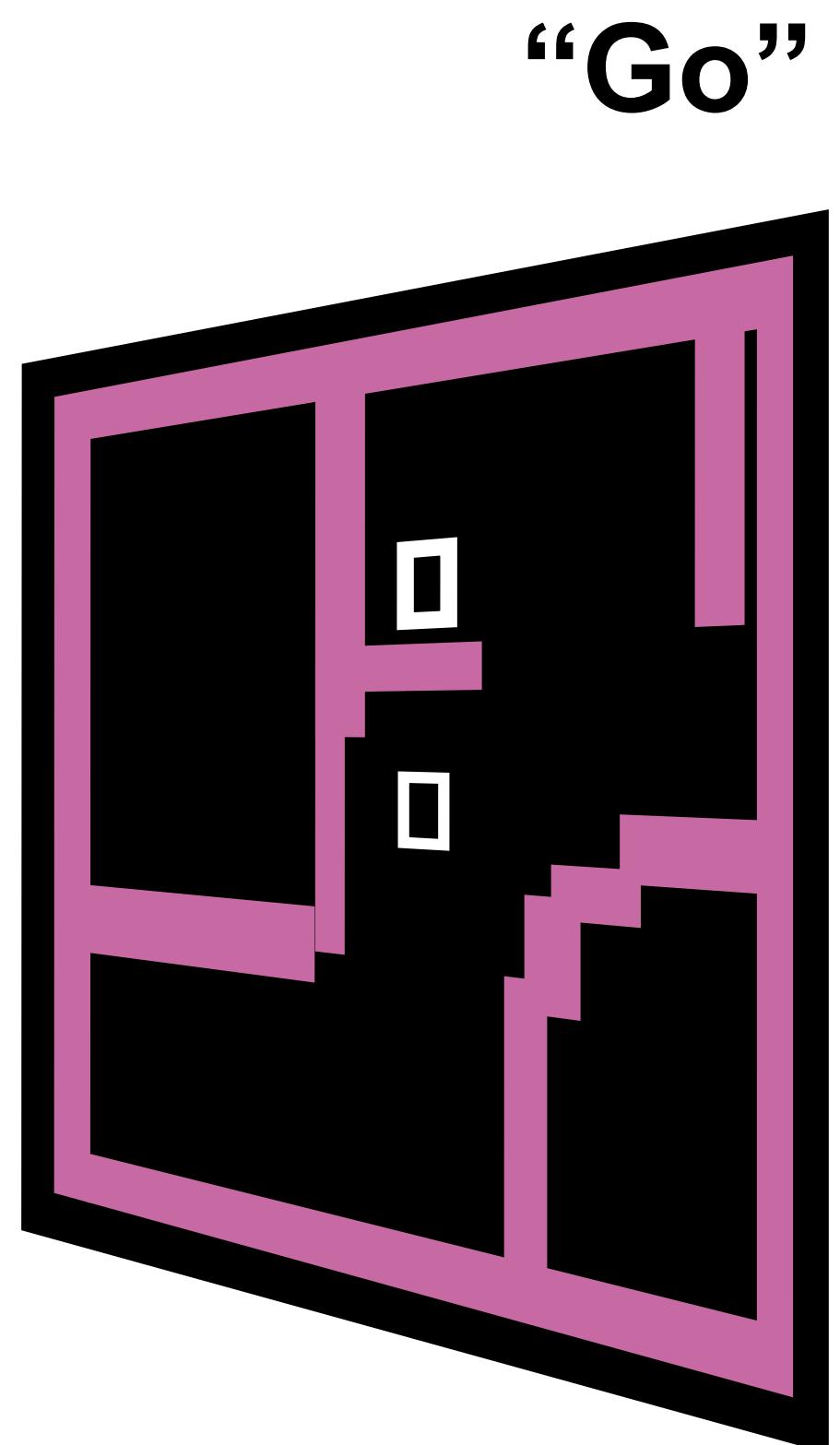
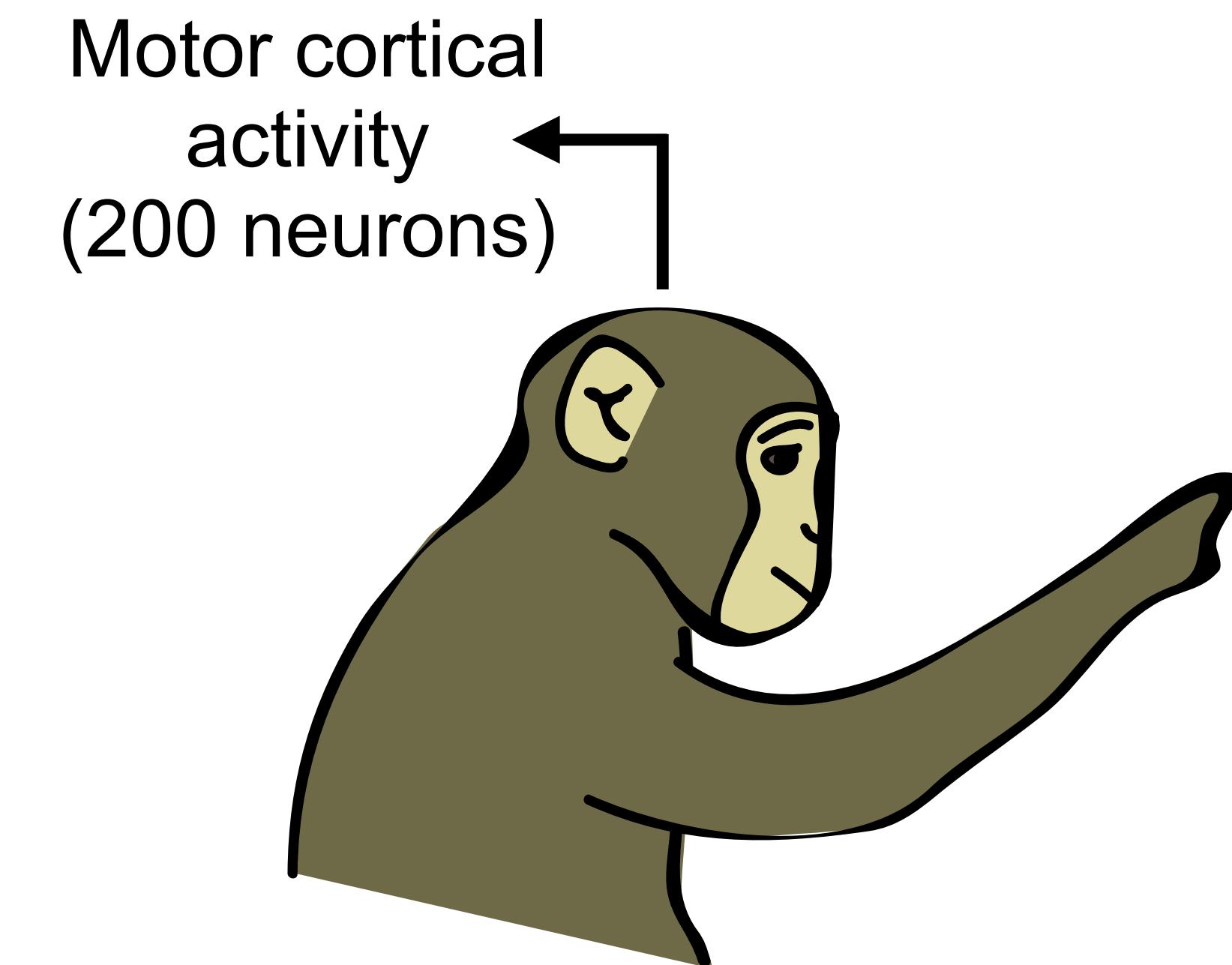


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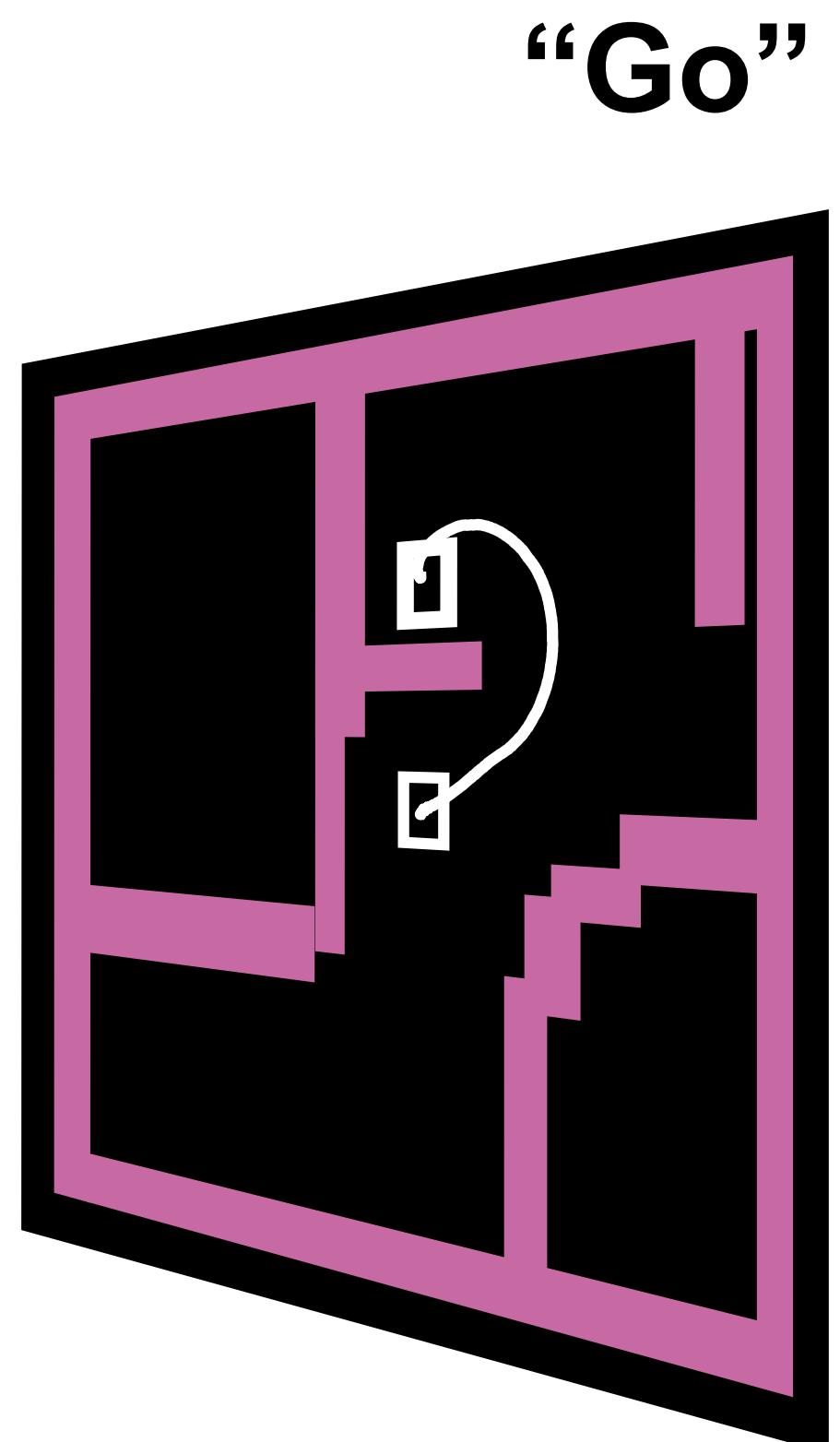
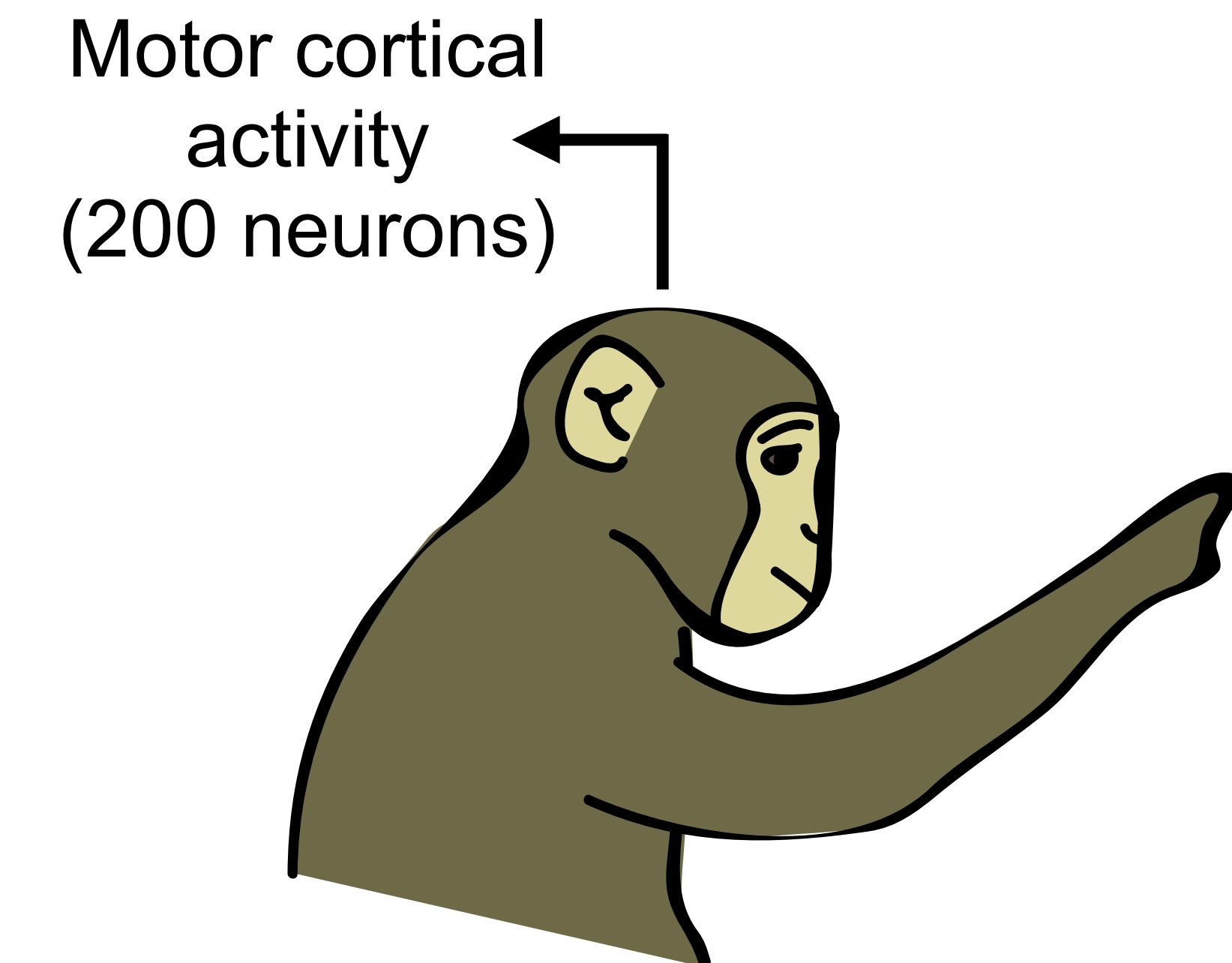


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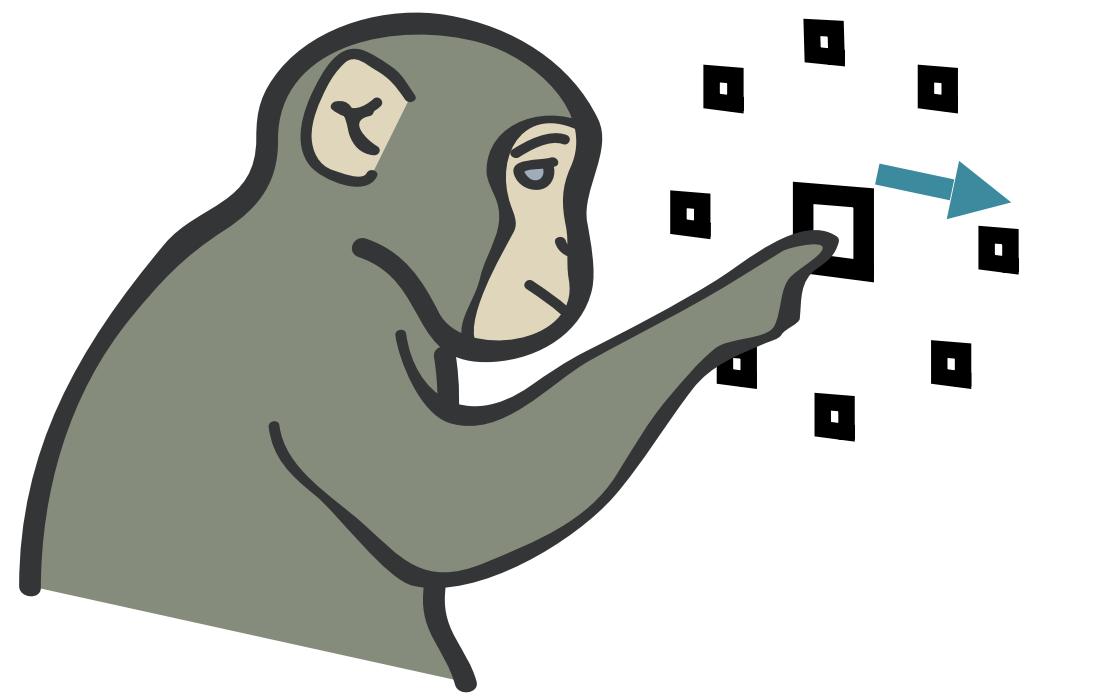
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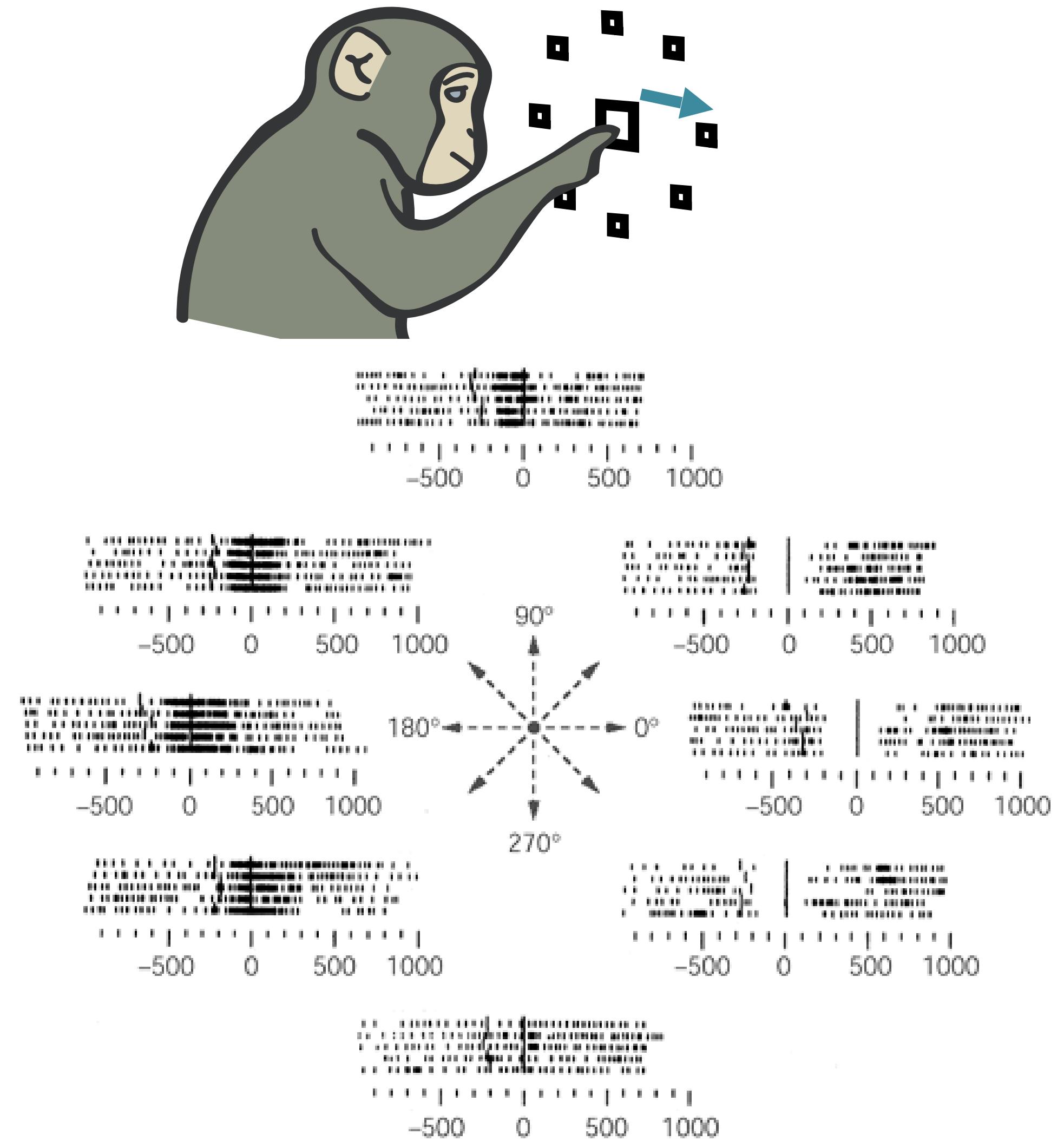


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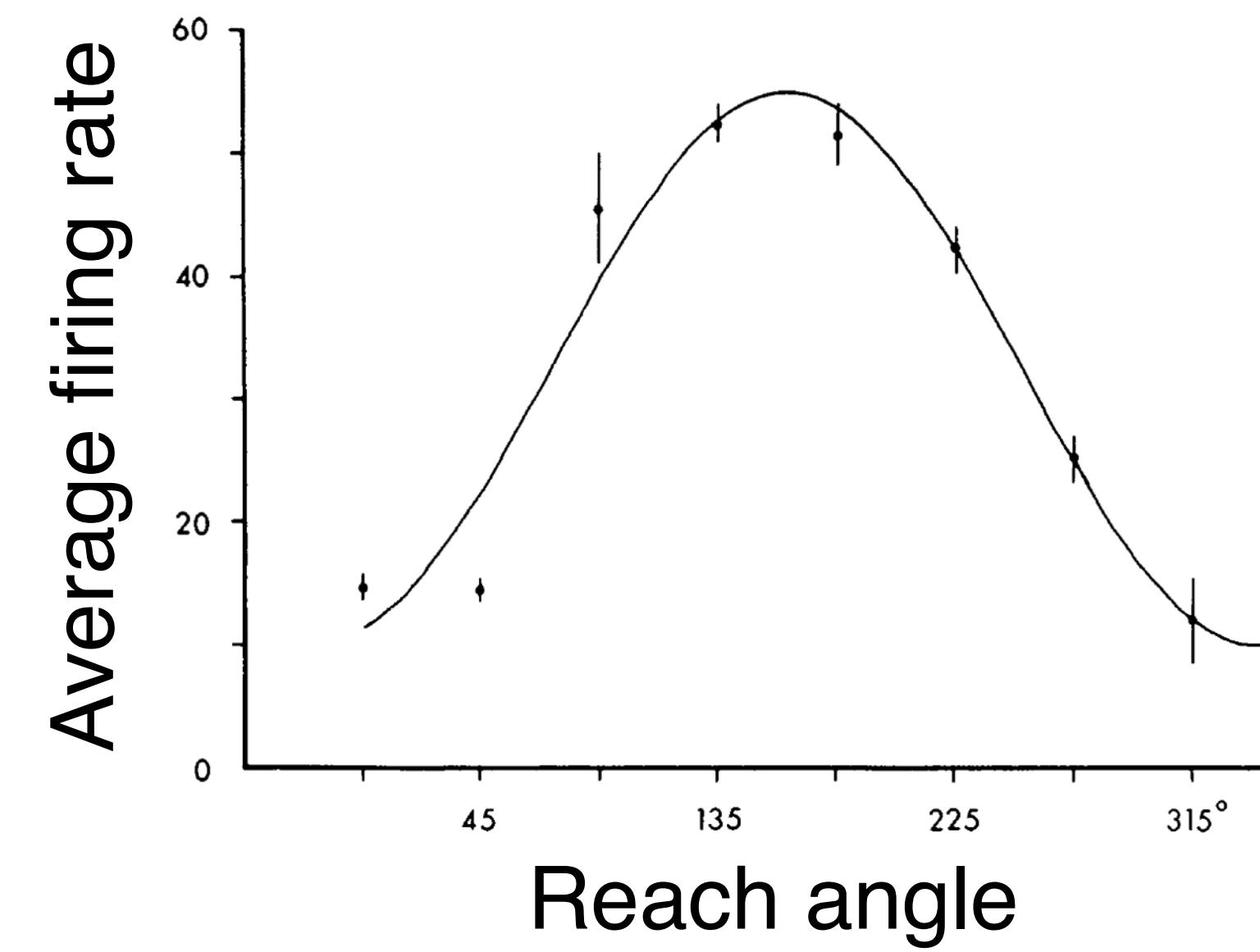
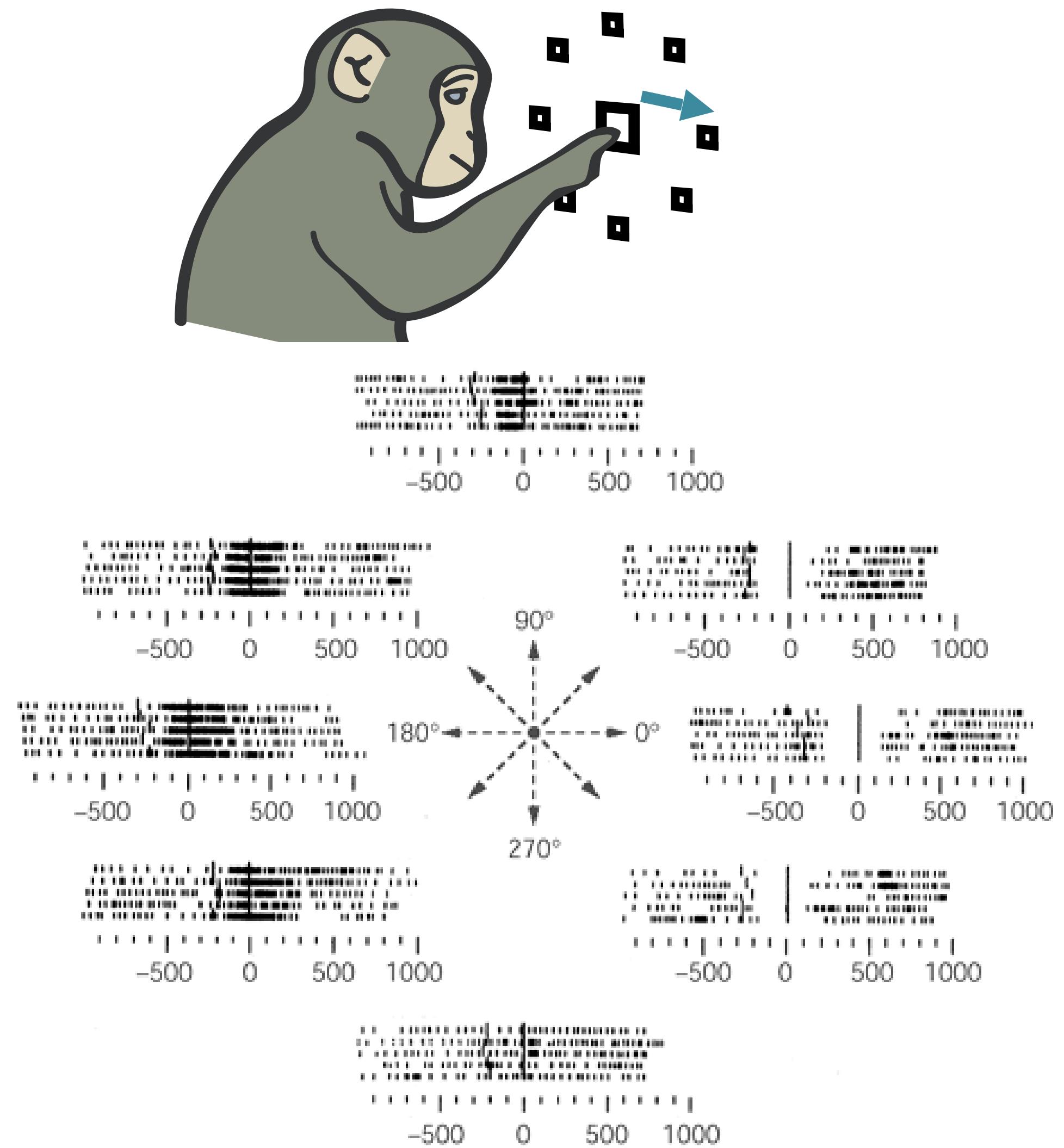
Shenoy, Sahani, Churchland, *Ann Rev Neuro* 2013



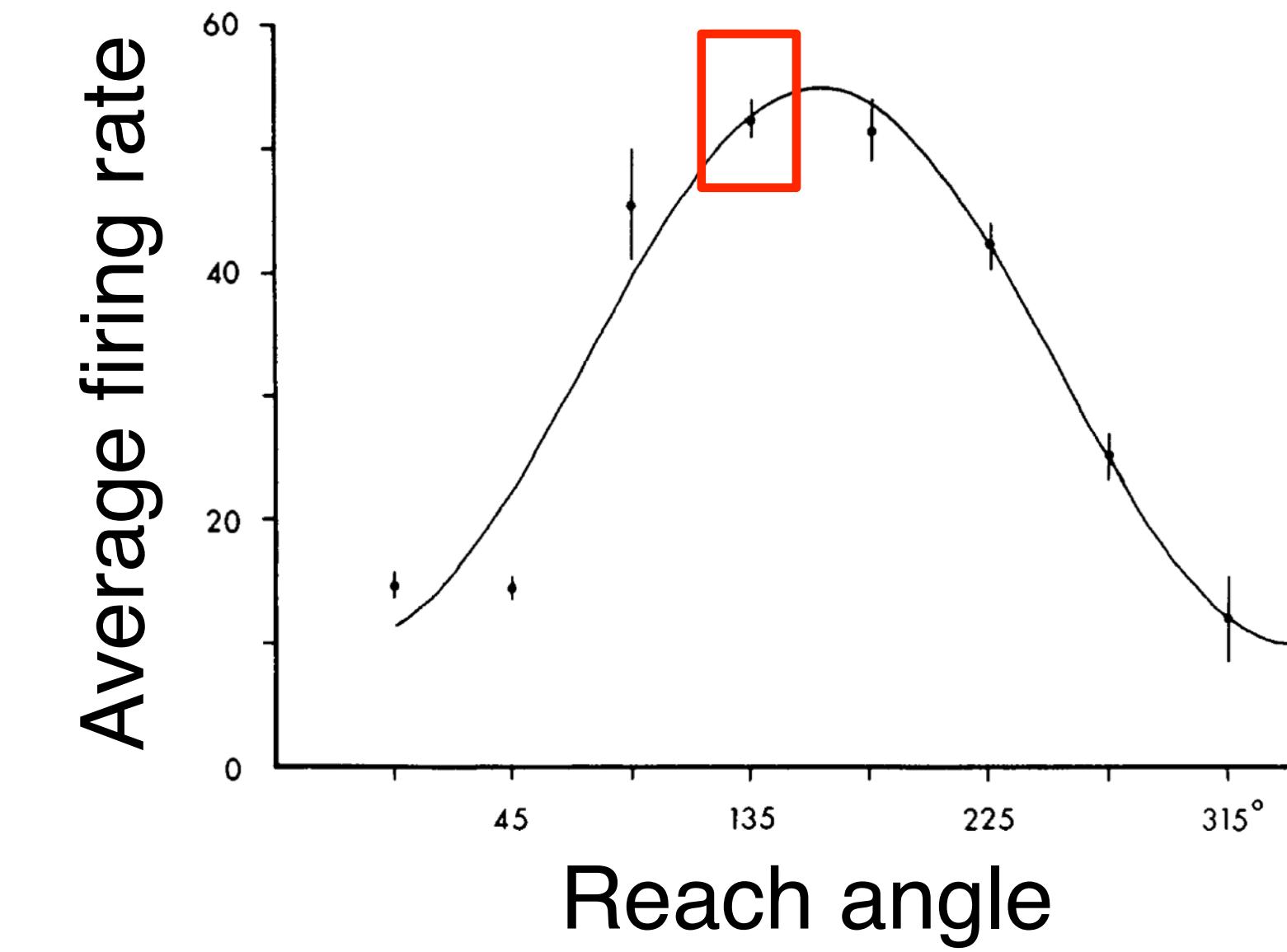
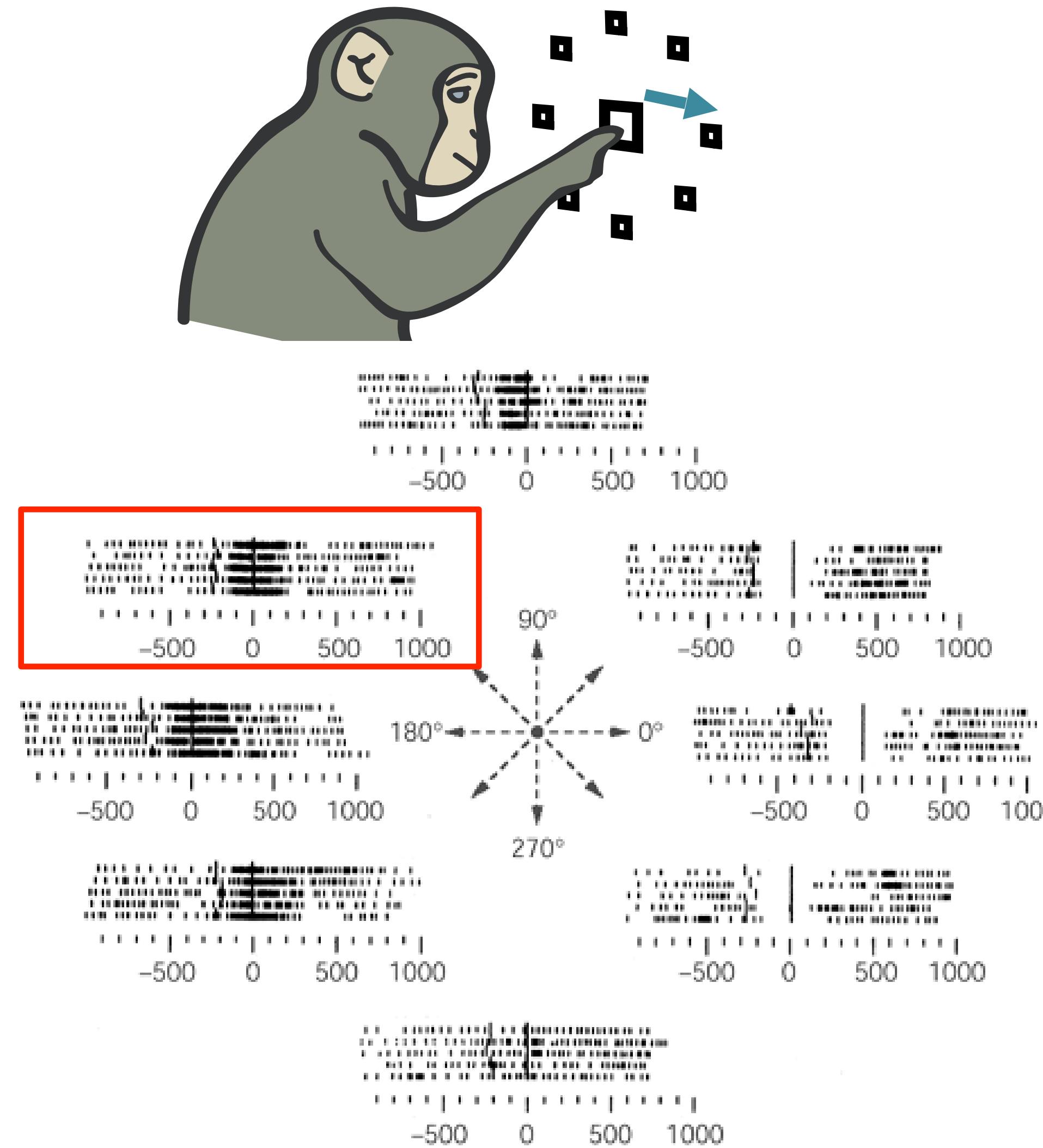
Georgopoulos et al. (1982)
Schwartz et al. (1988)



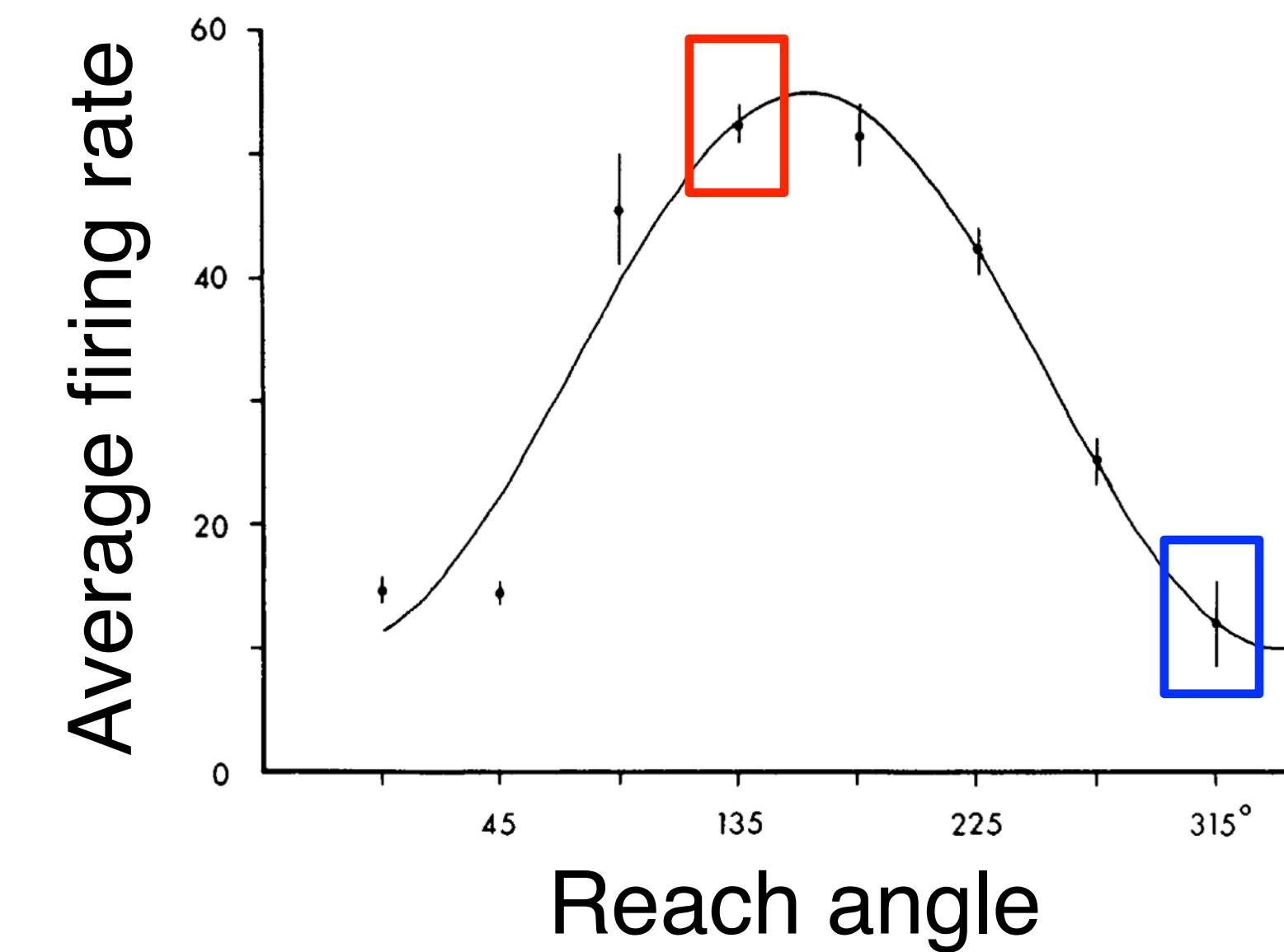
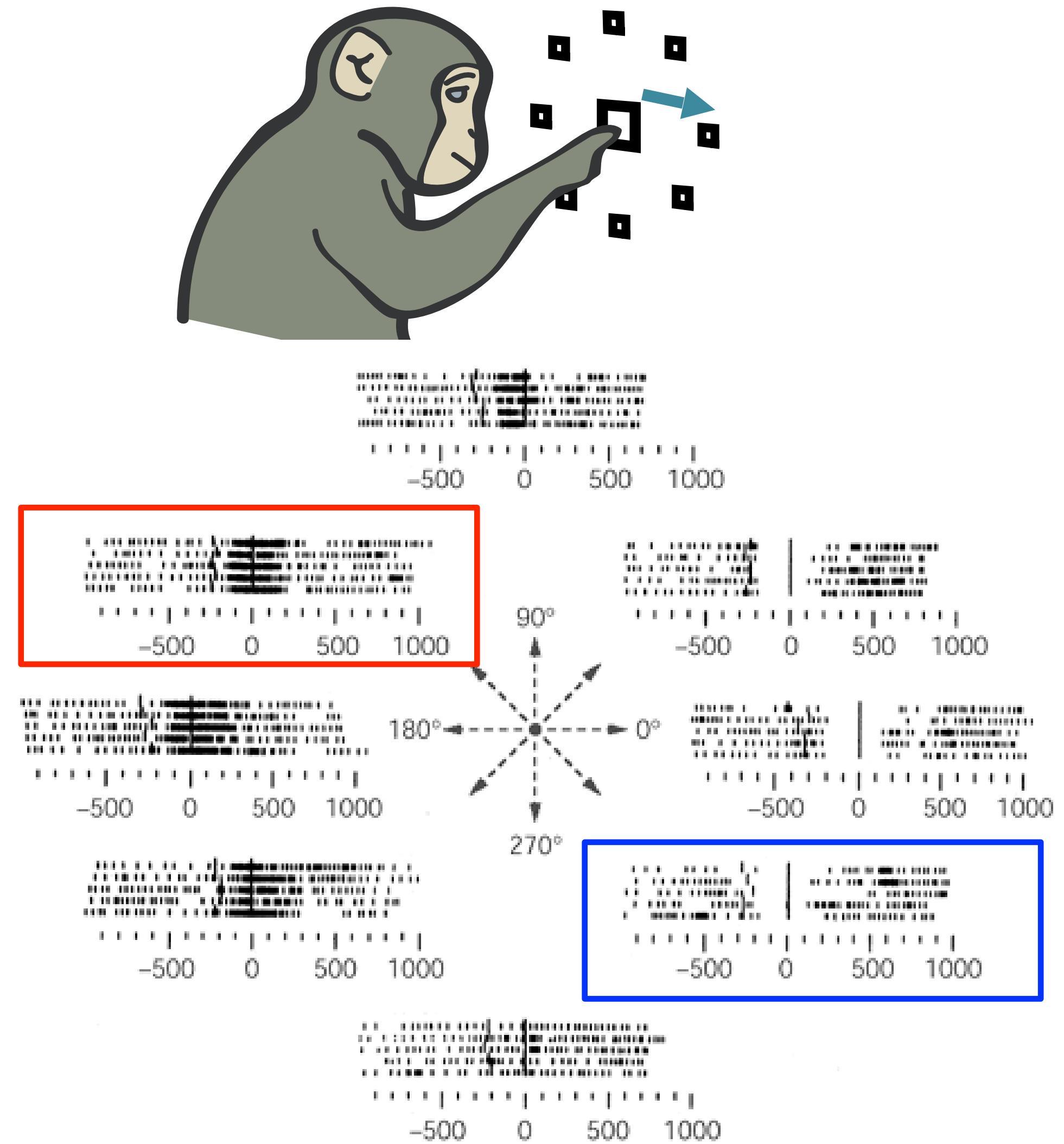
Georgopoulos et al. (1982)
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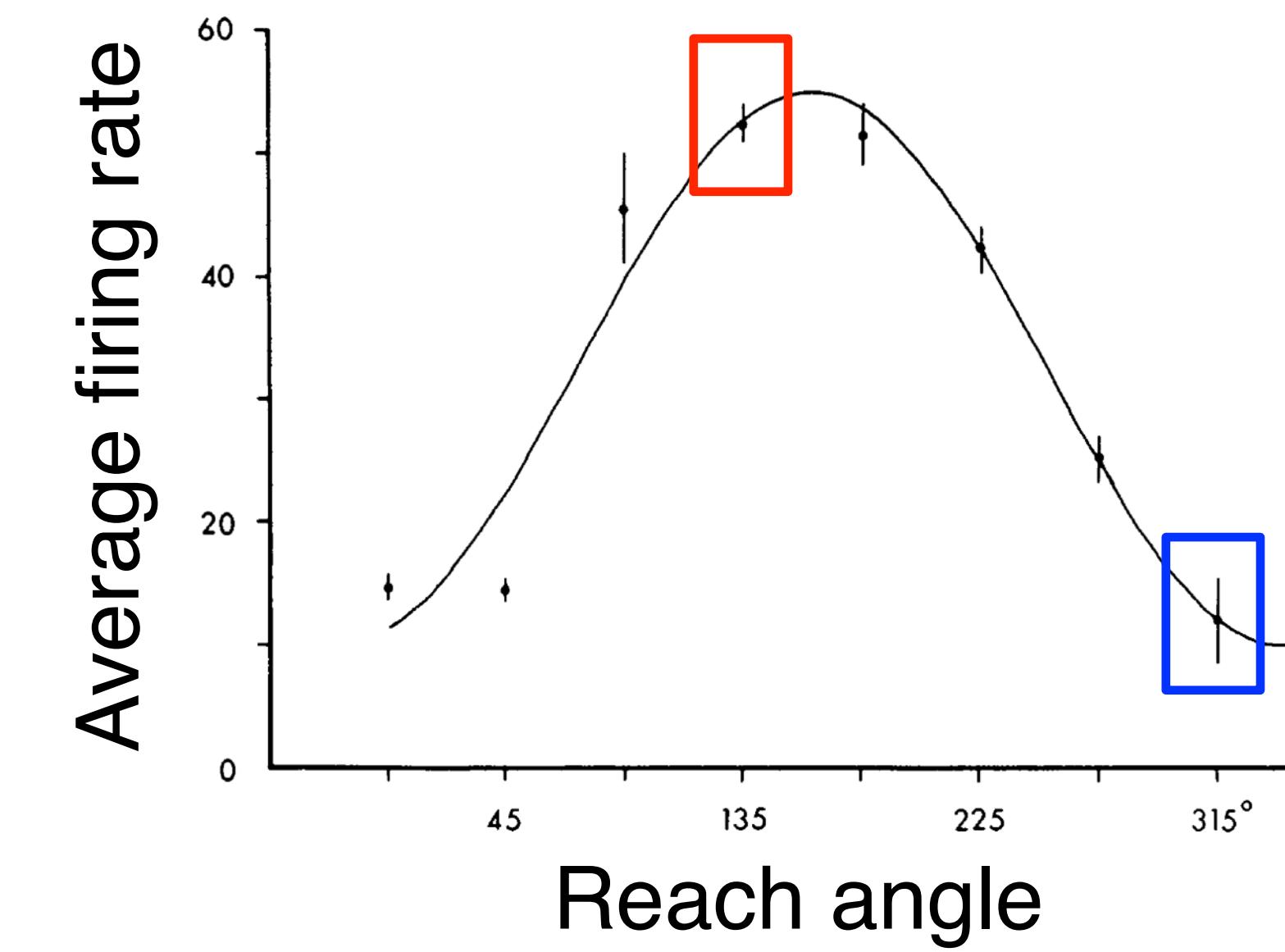
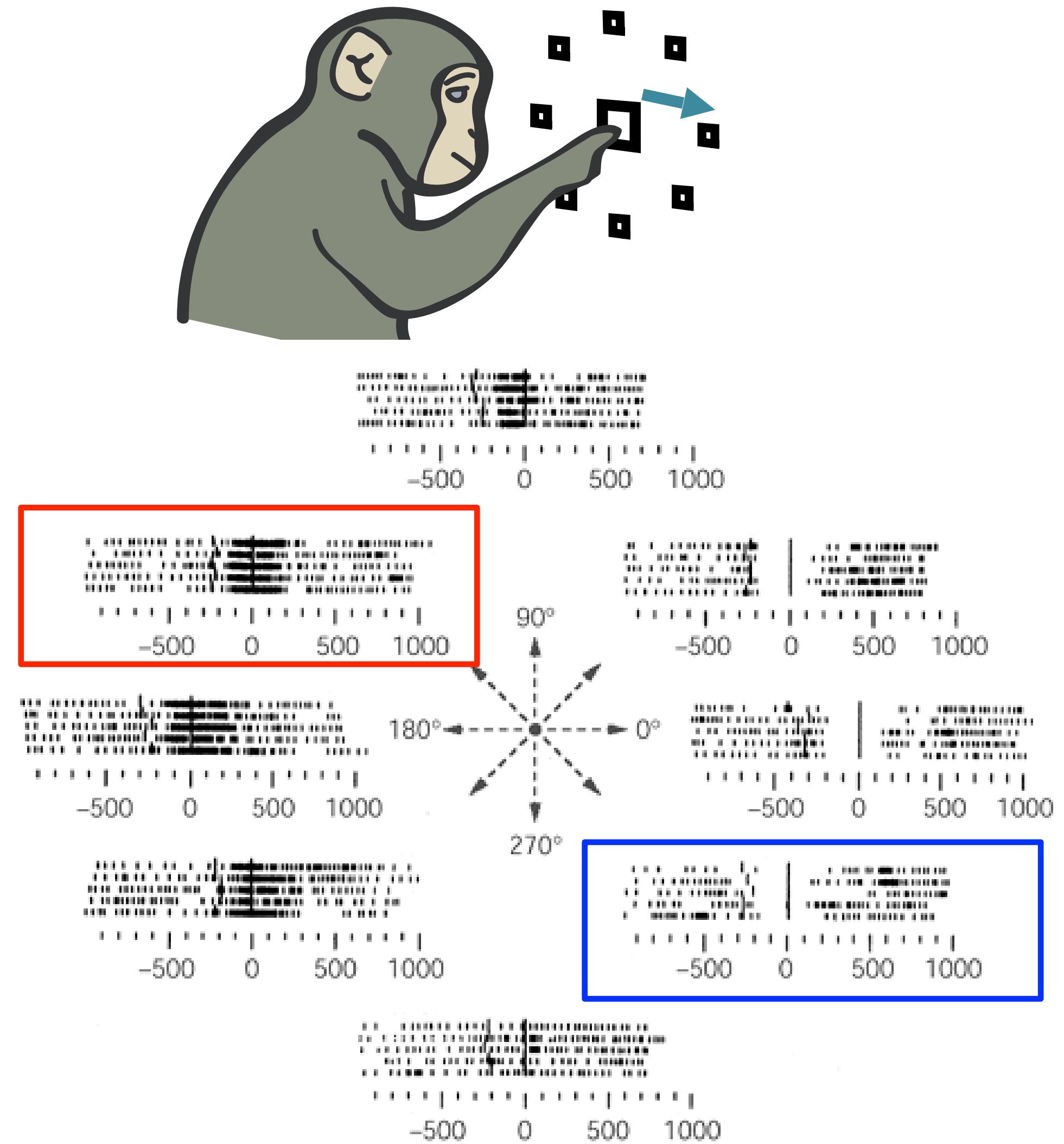
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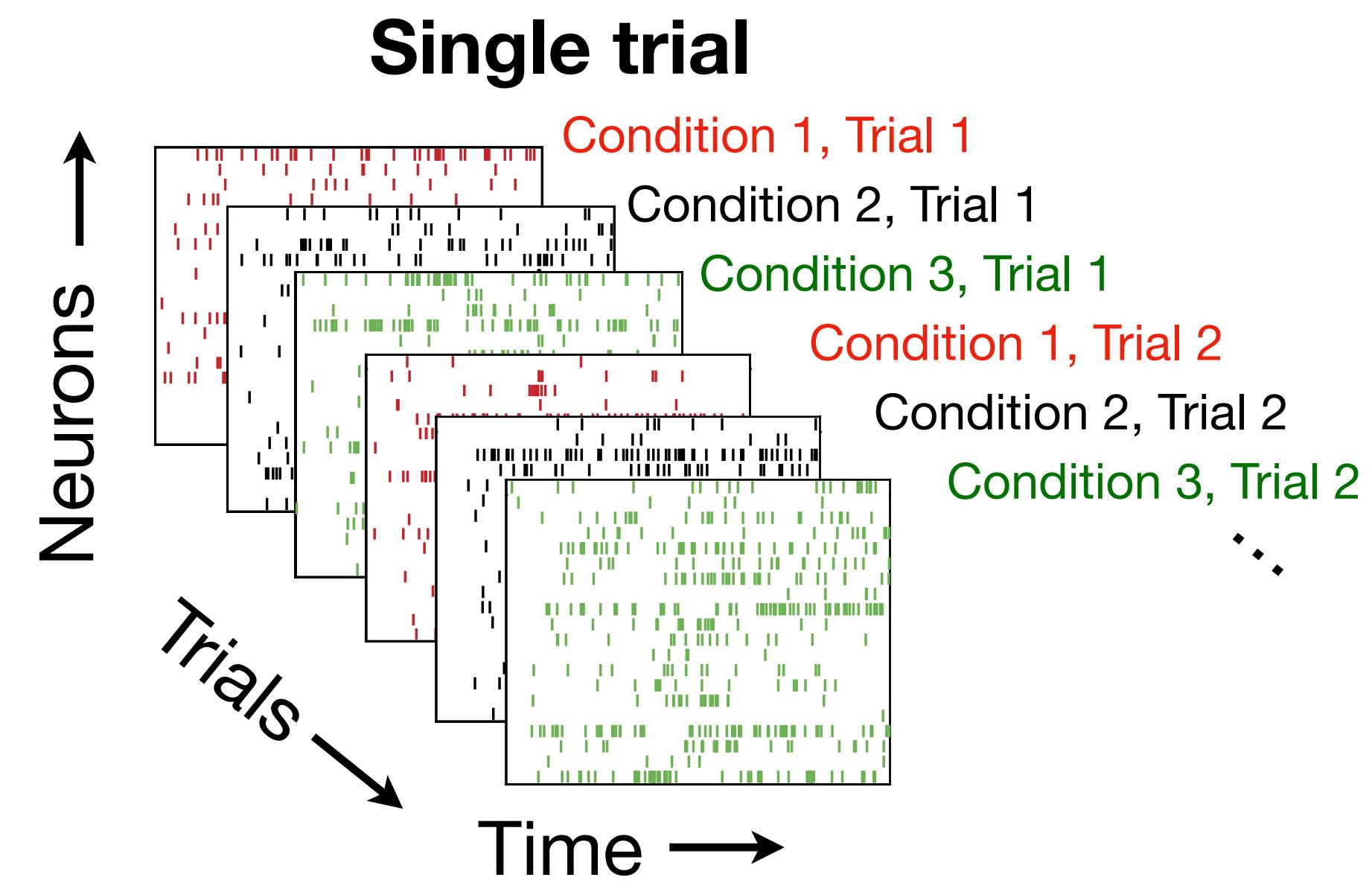


$$z = f(\mathbf{k})$$

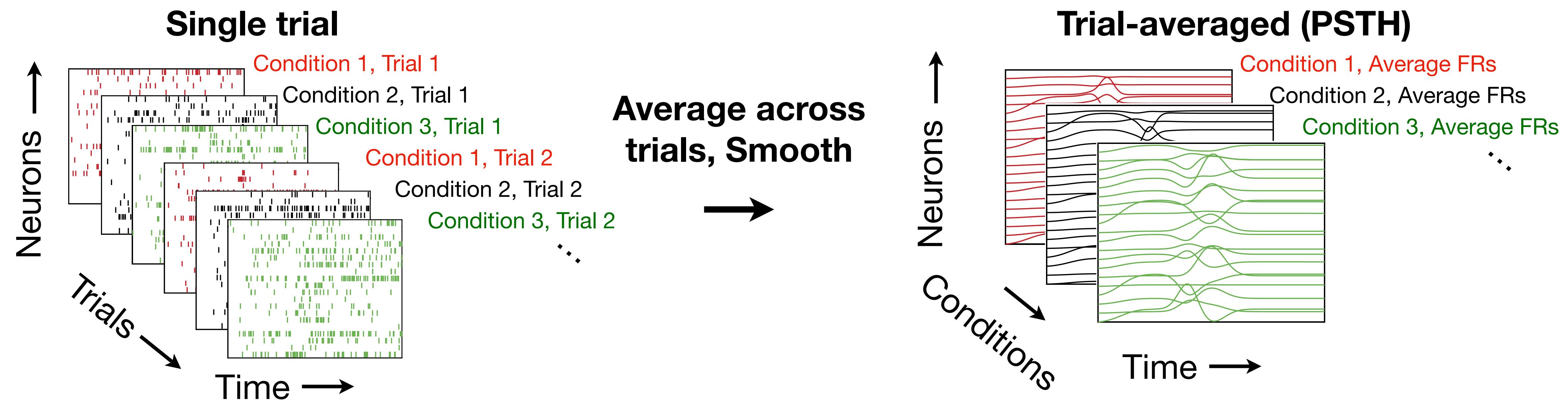
Neurons' firing rate Kinematics or kinetics

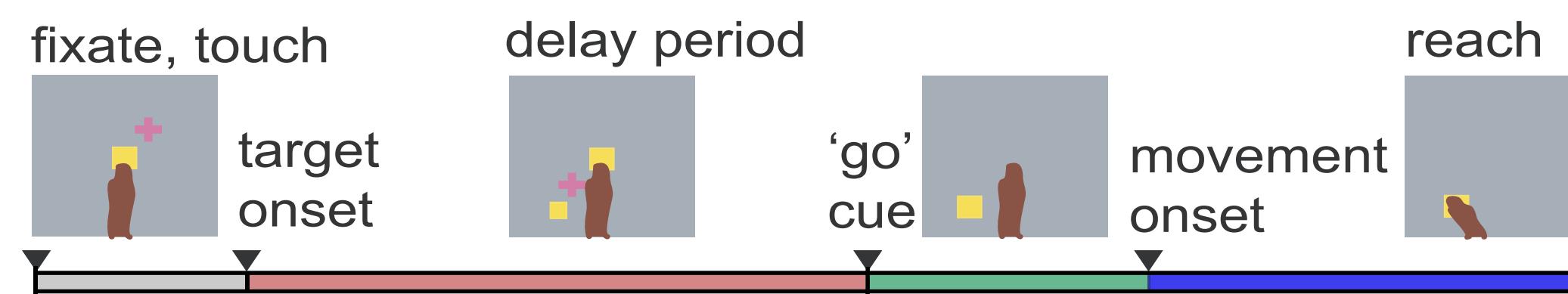
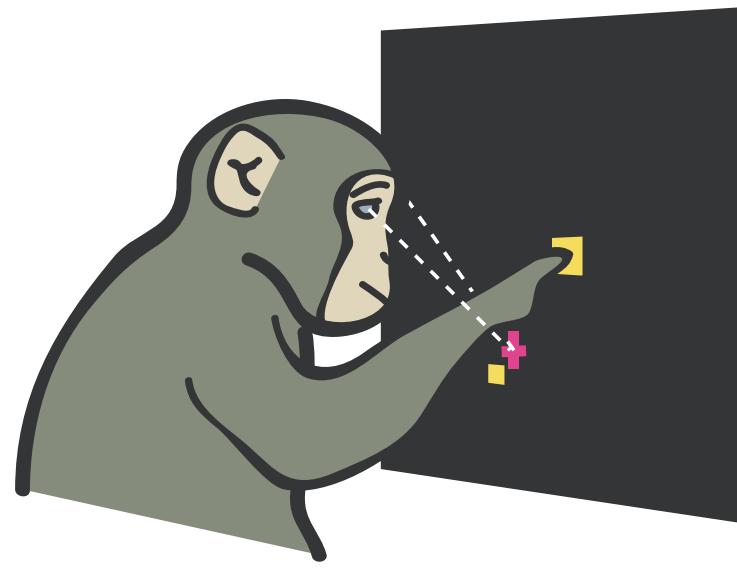
Georgopoulos et al. (1982)
Schwartz et al. (1988)

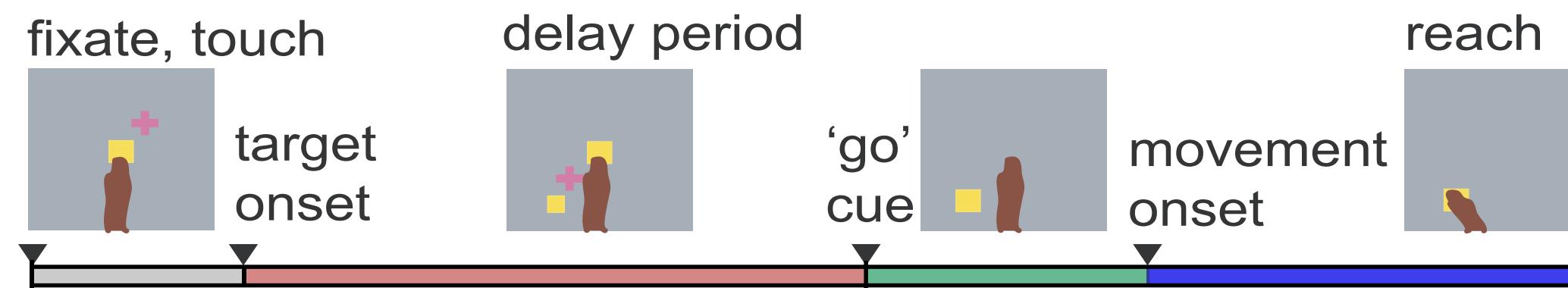
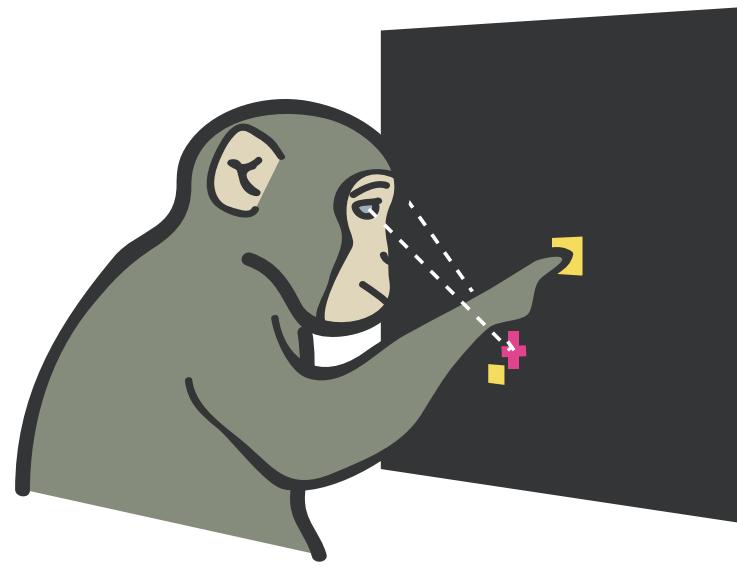
Visualizing the data



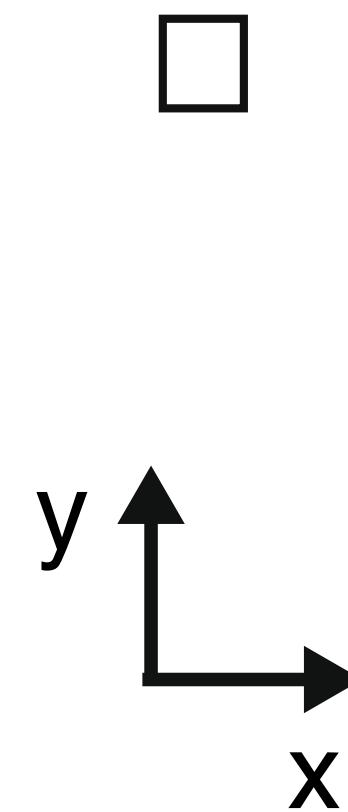
Visualizing the data







Reach trajectories



Average firing rate
(spikes / sec)

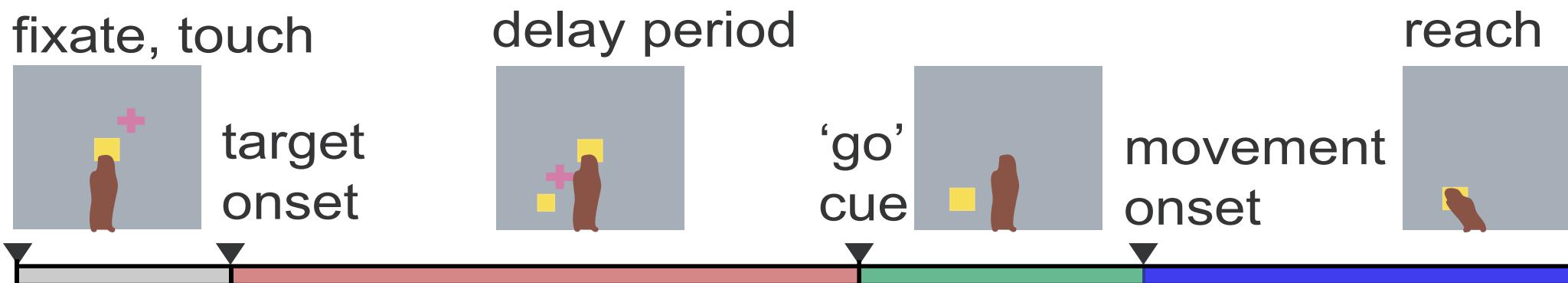
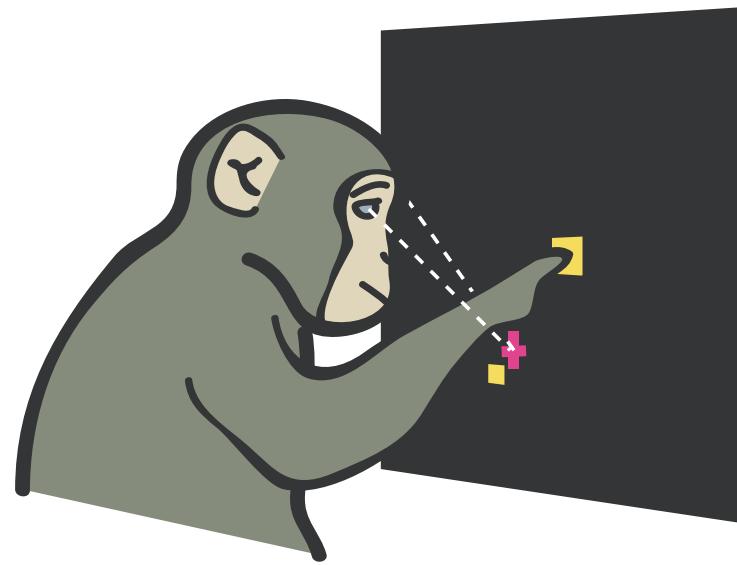
Target on



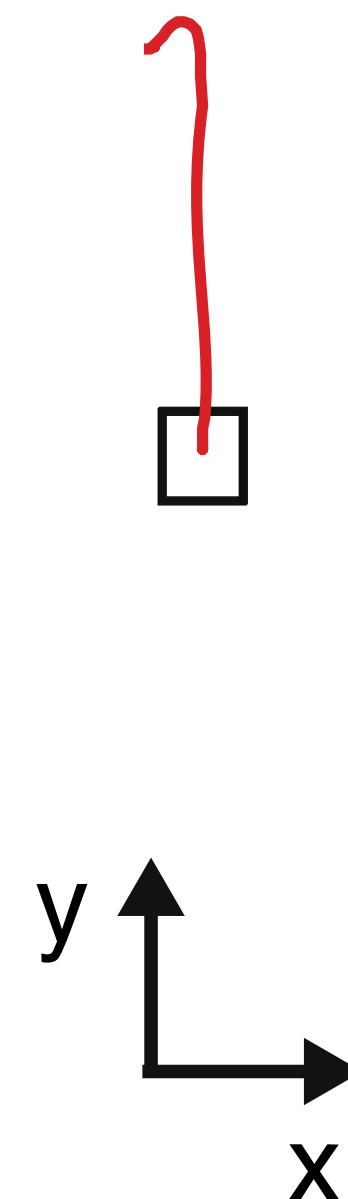
■ Go ■ Move

Single neuron firing rate (PSTH)

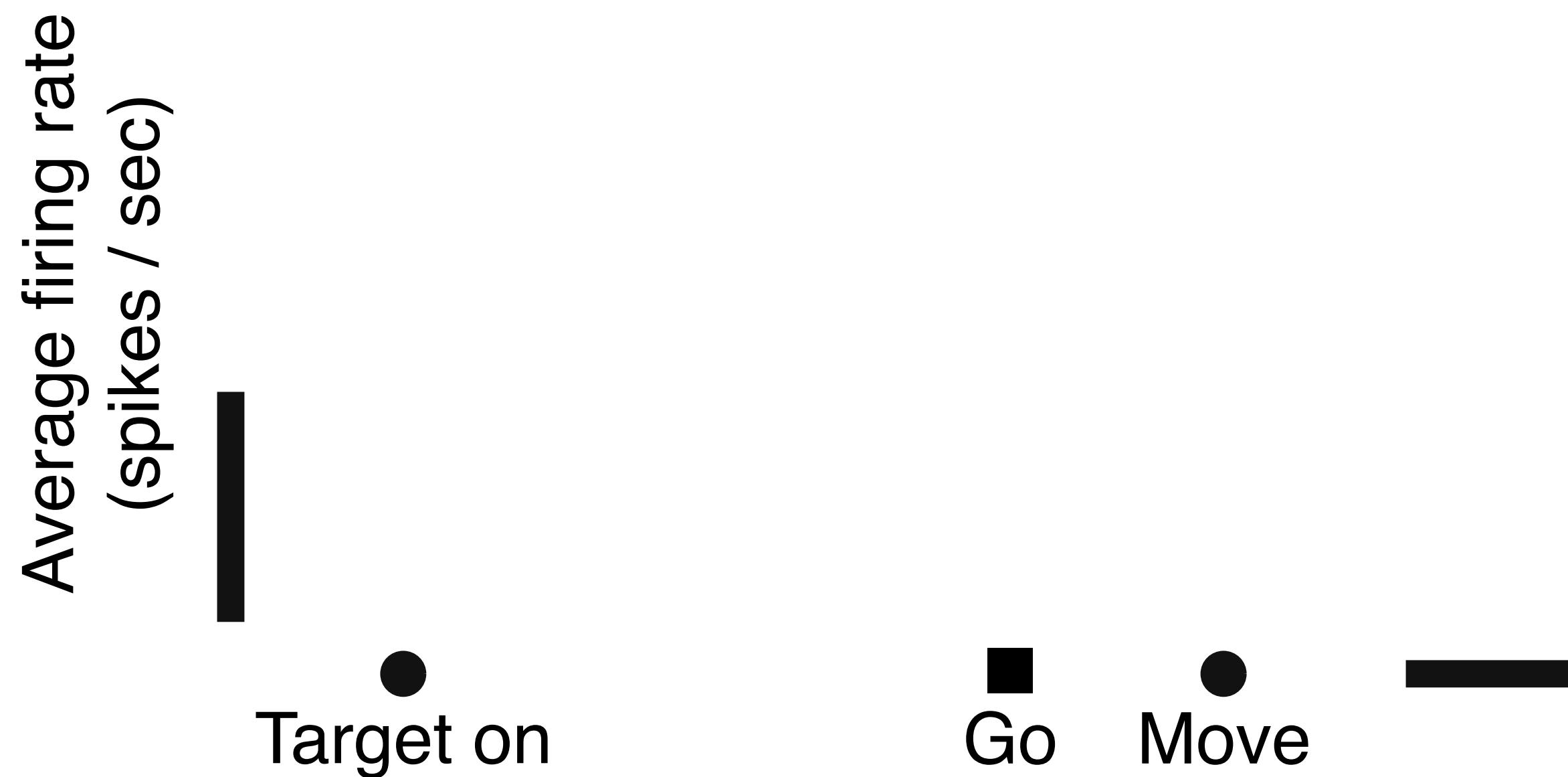
Cell 12
Monkey B



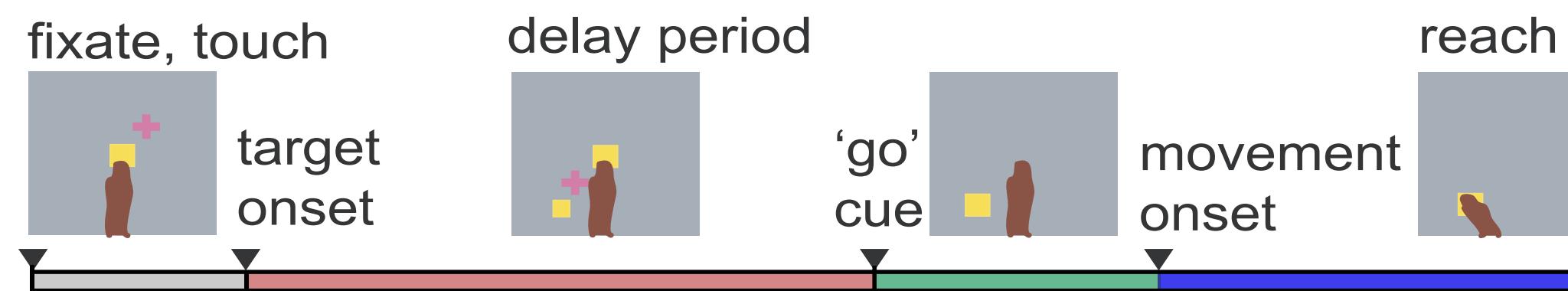
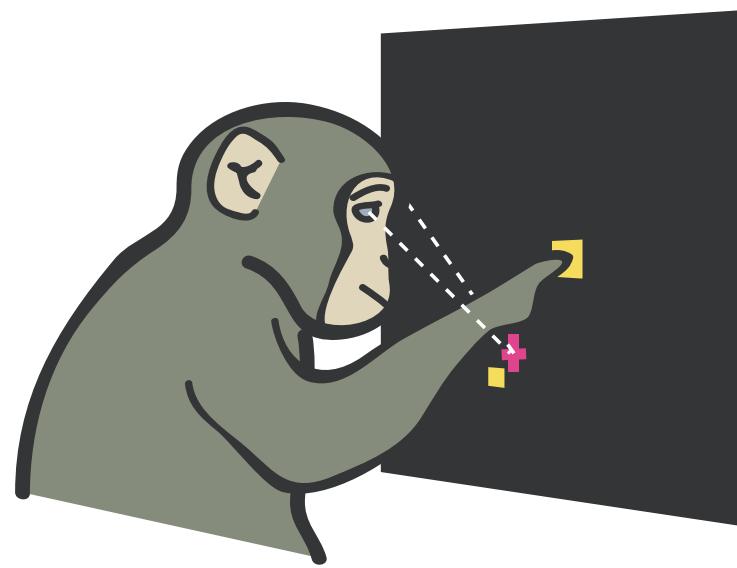
Reach trajectories



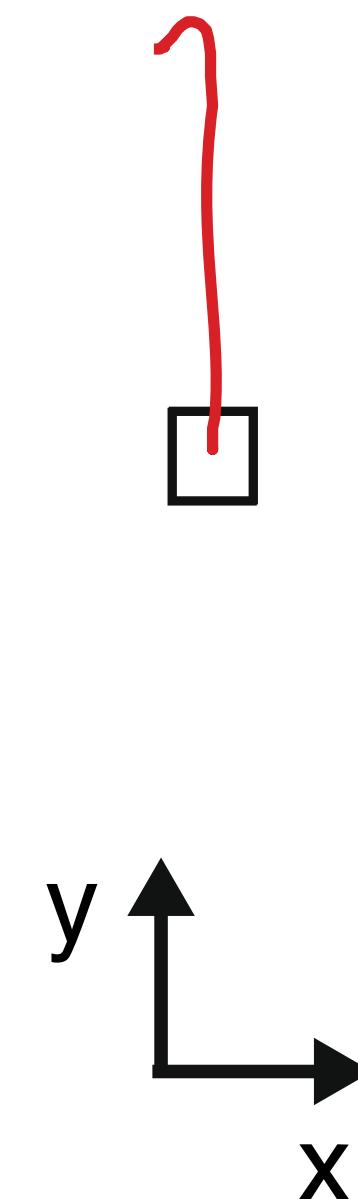
Single neuron firing rate (PSTH)



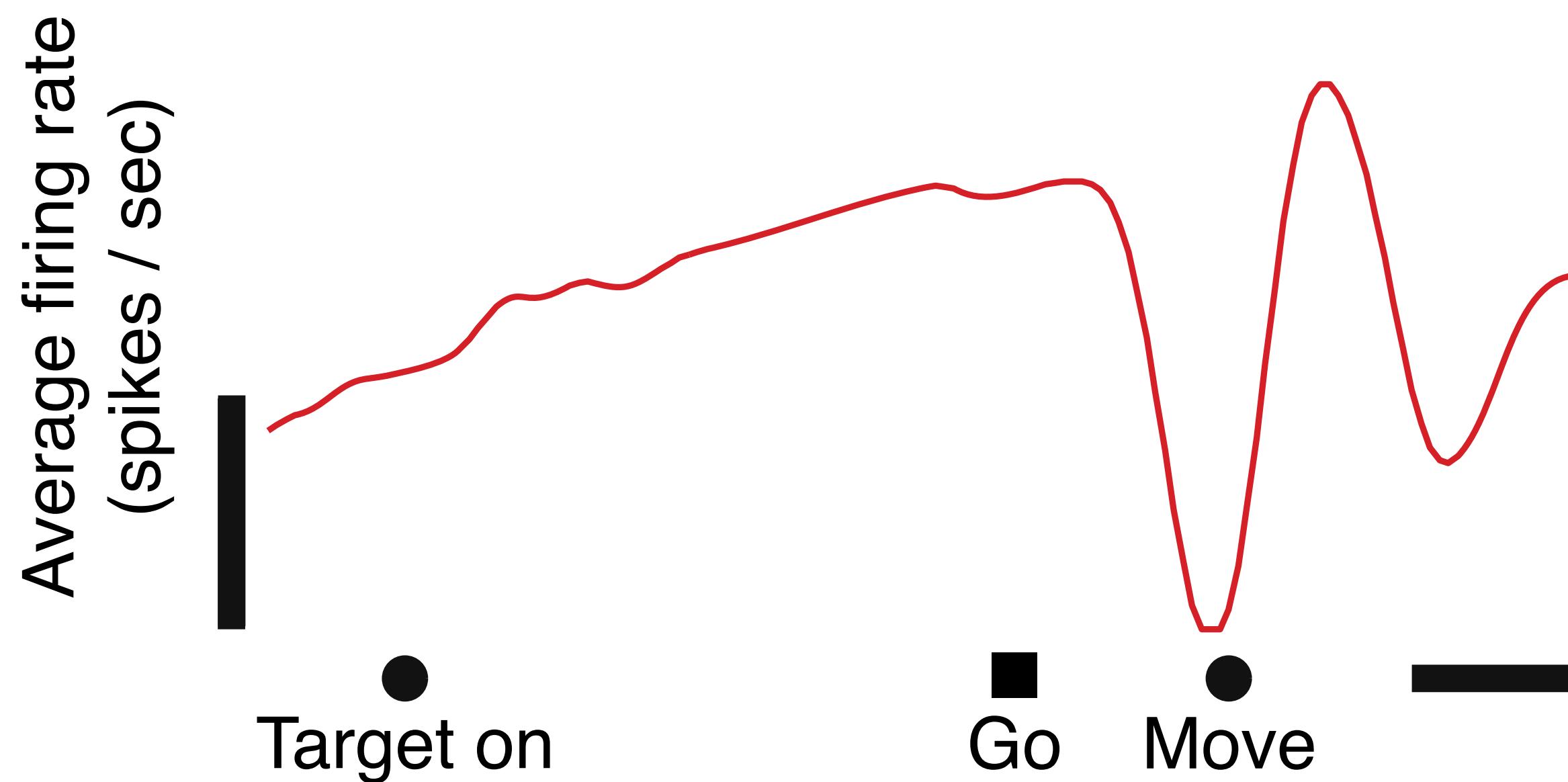
Cell 12
Monkey B

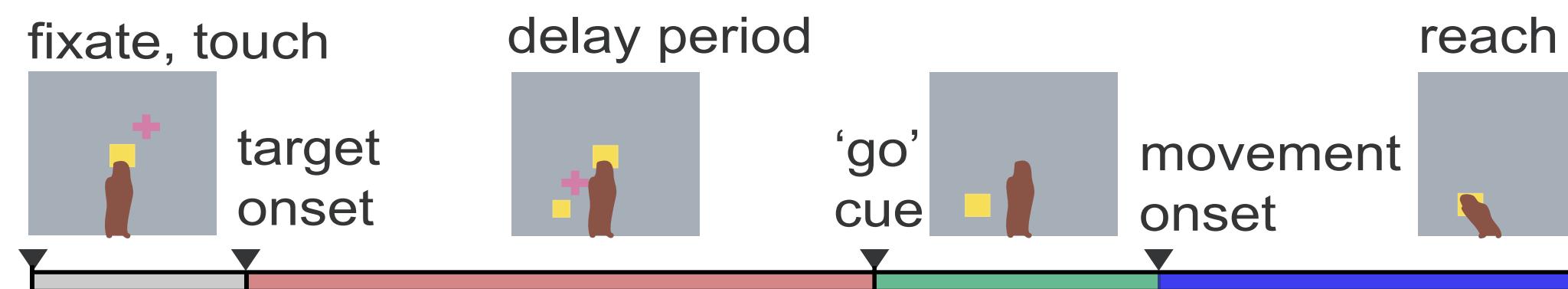
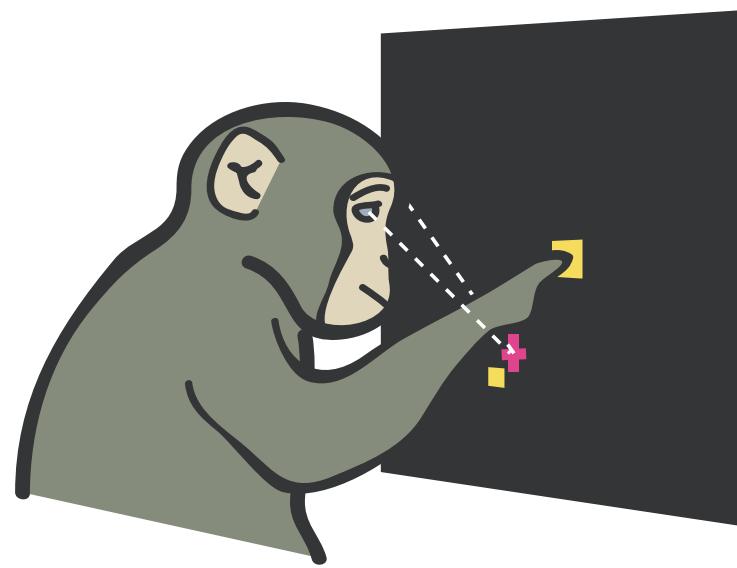


Reach trajectories

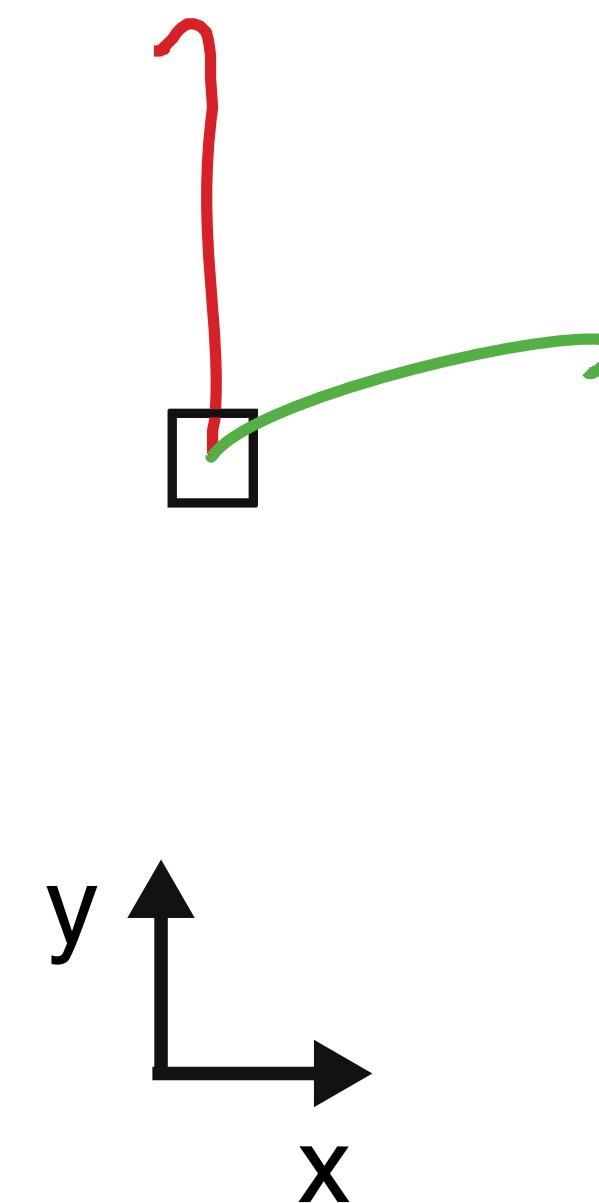


Single neuron firing rate (PSTH)

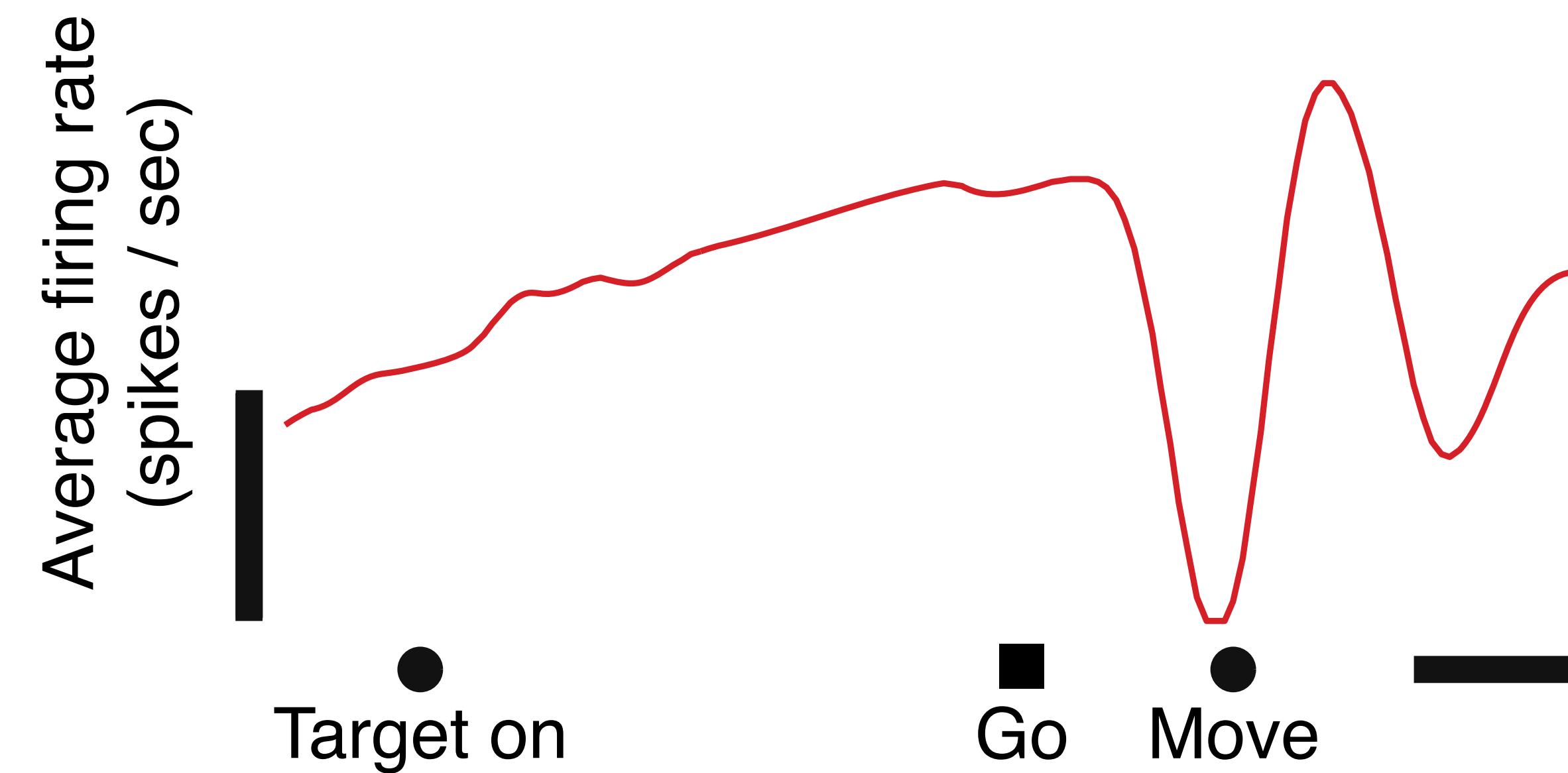


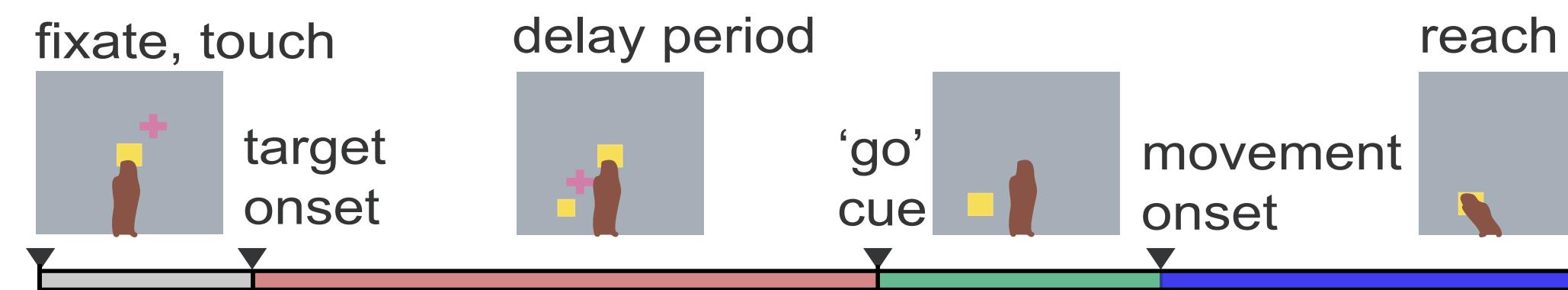
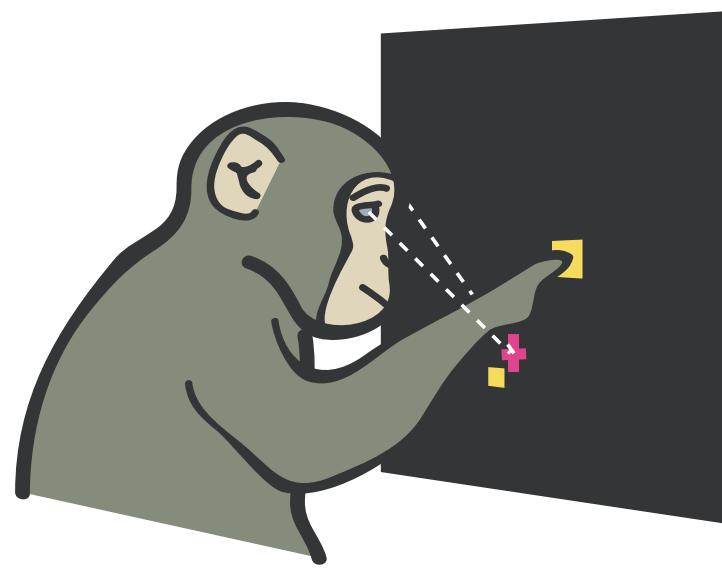


Reach trajectories

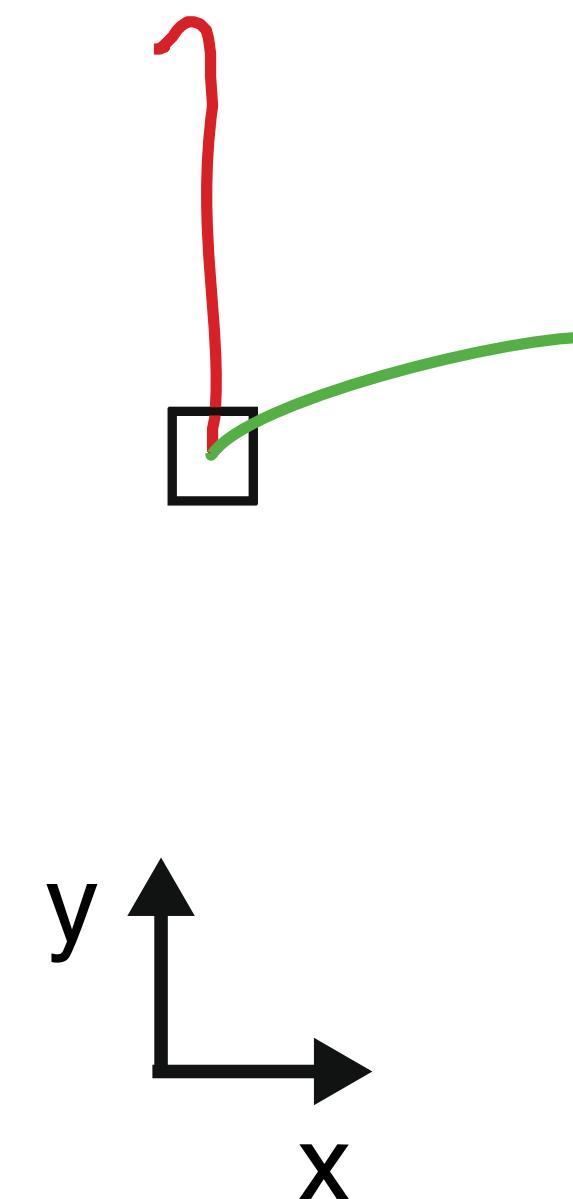


Single neuron firing rate (PSTH)



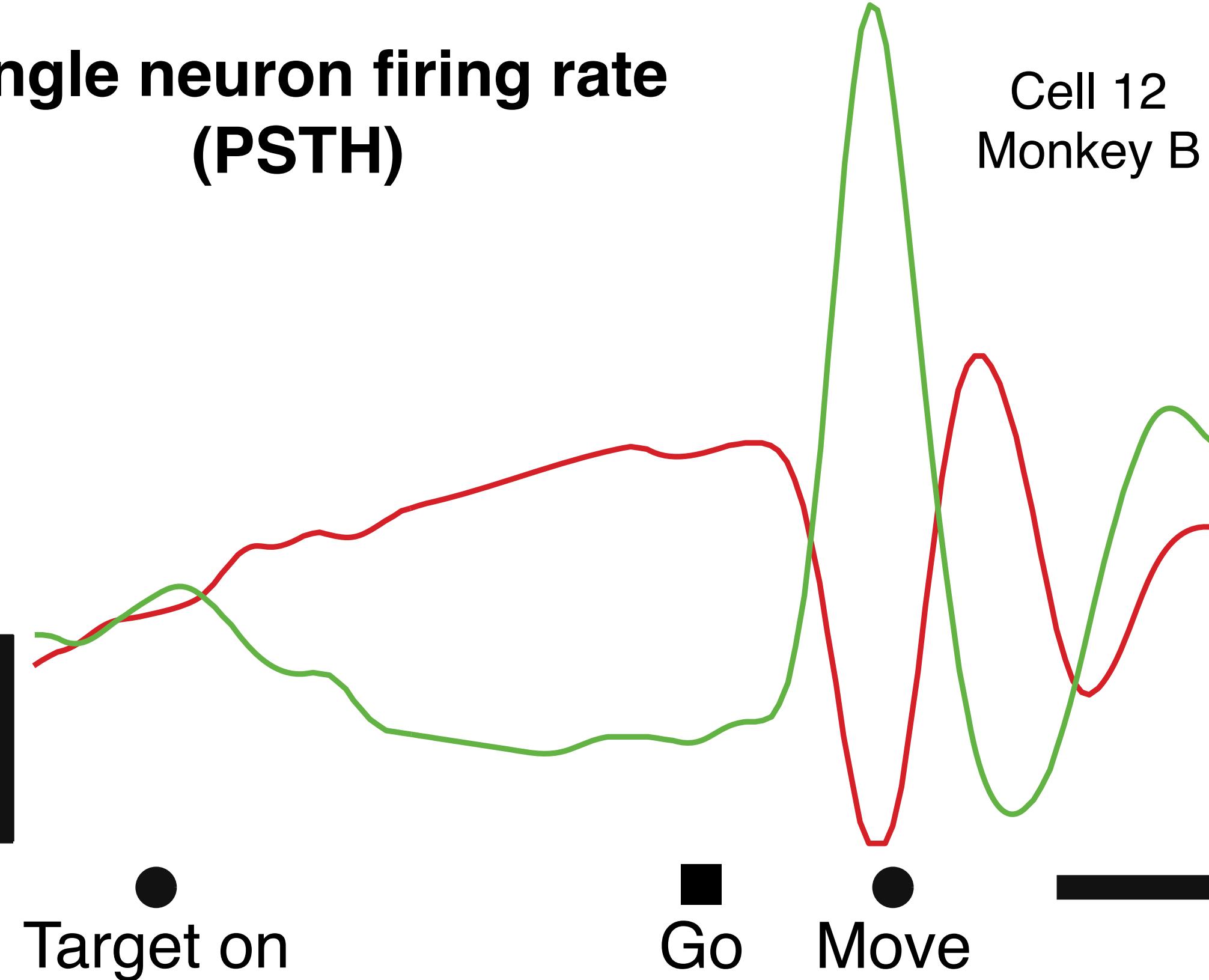


Reach trajectories

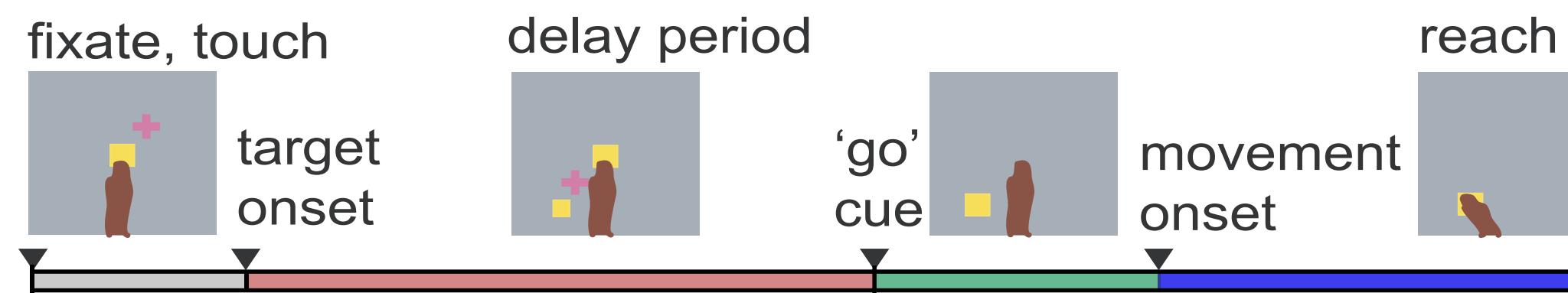
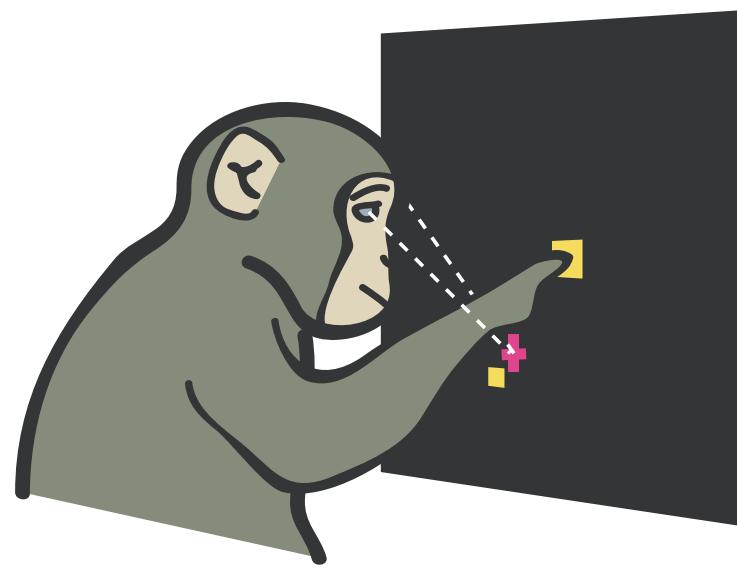


Single neuron firing rate (PSTH)

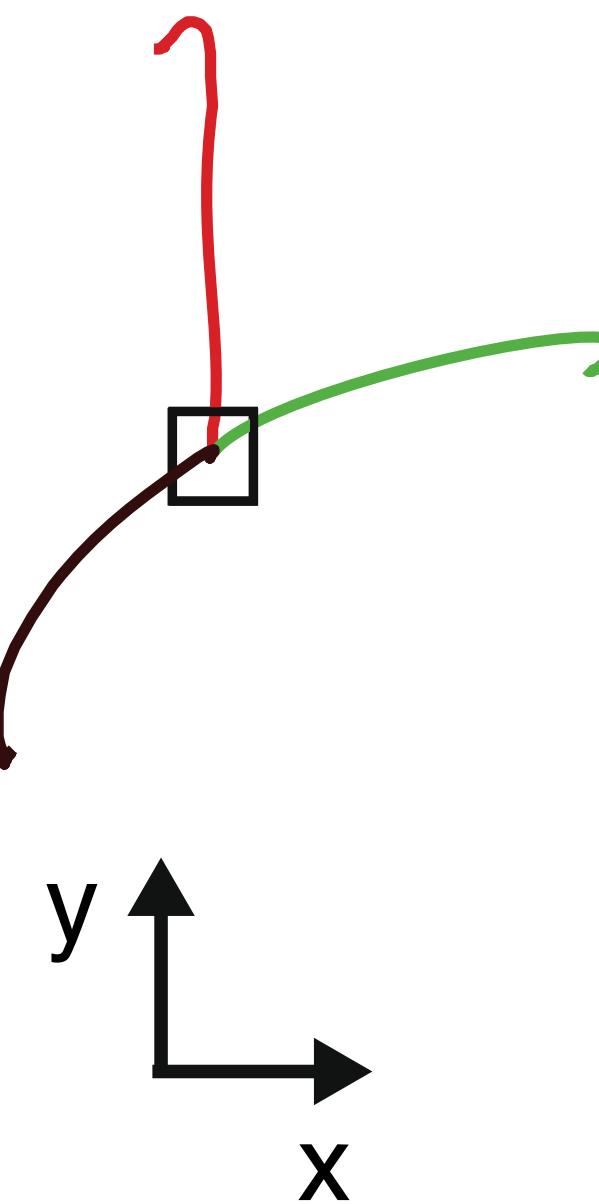
Average firing rate
(spikes / sec)



Cell 12
Monkey B

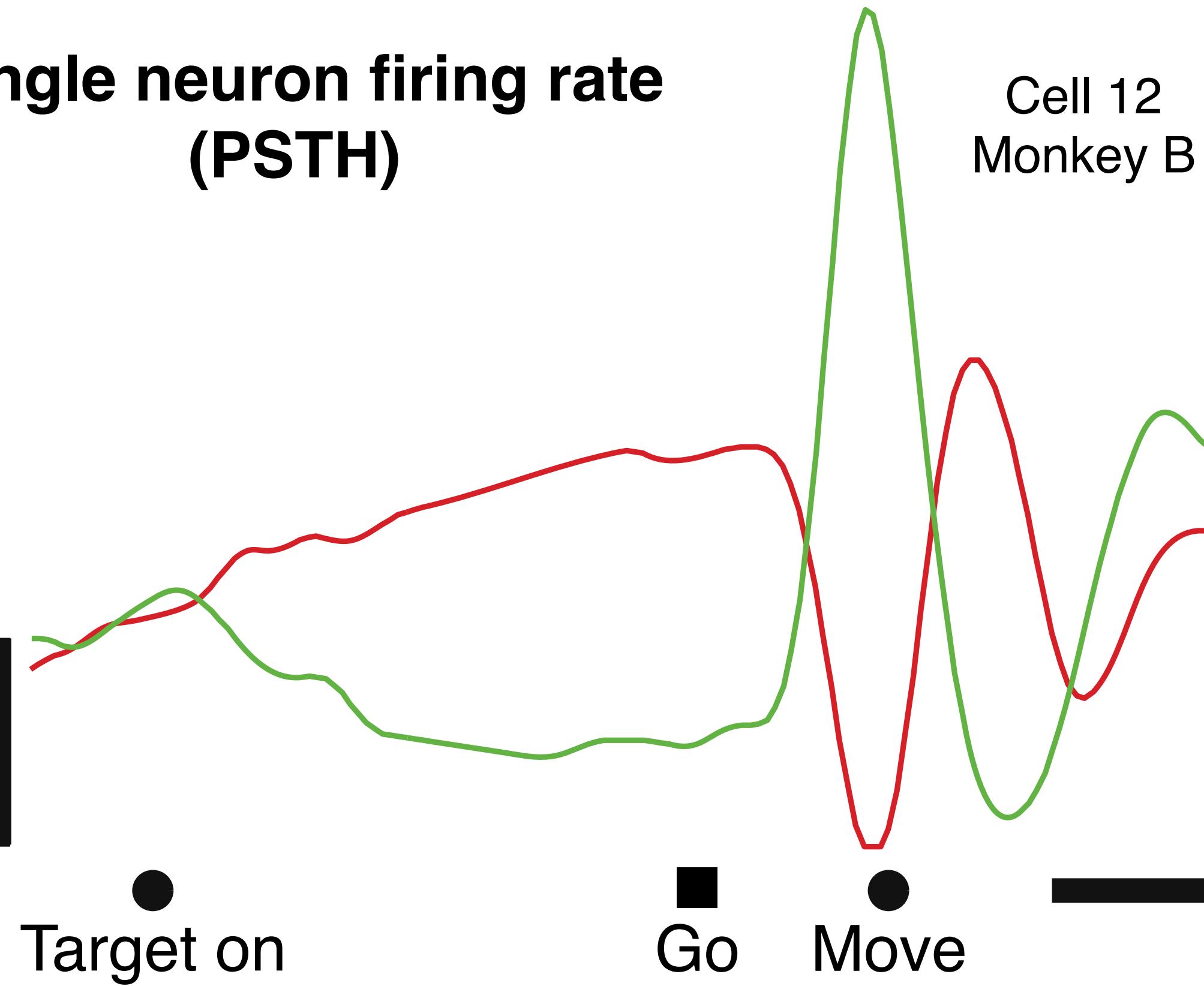


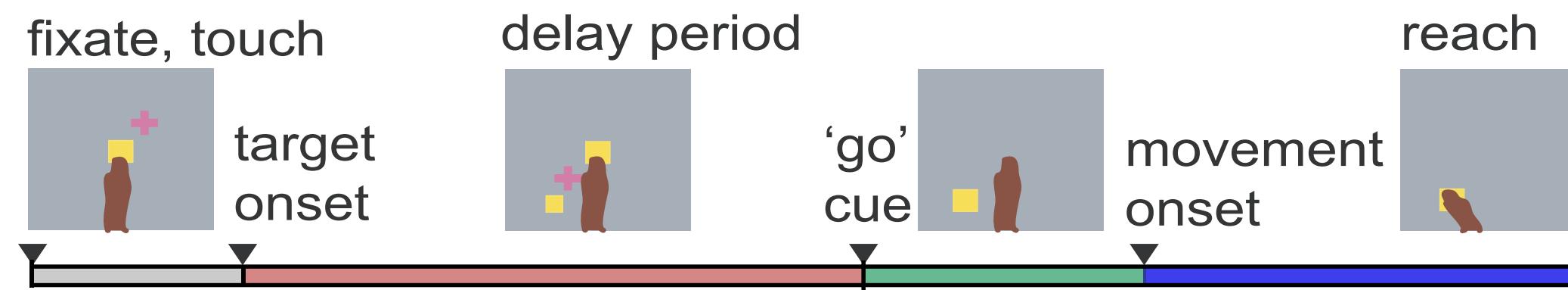
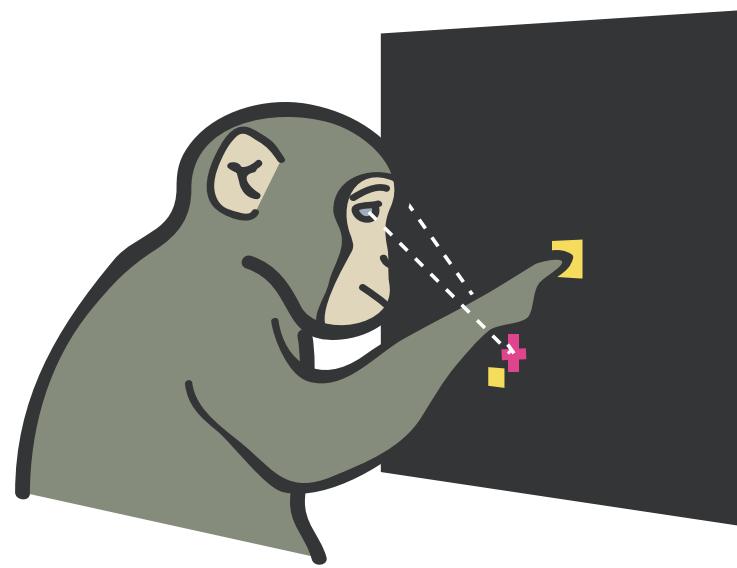
Reach trajectories



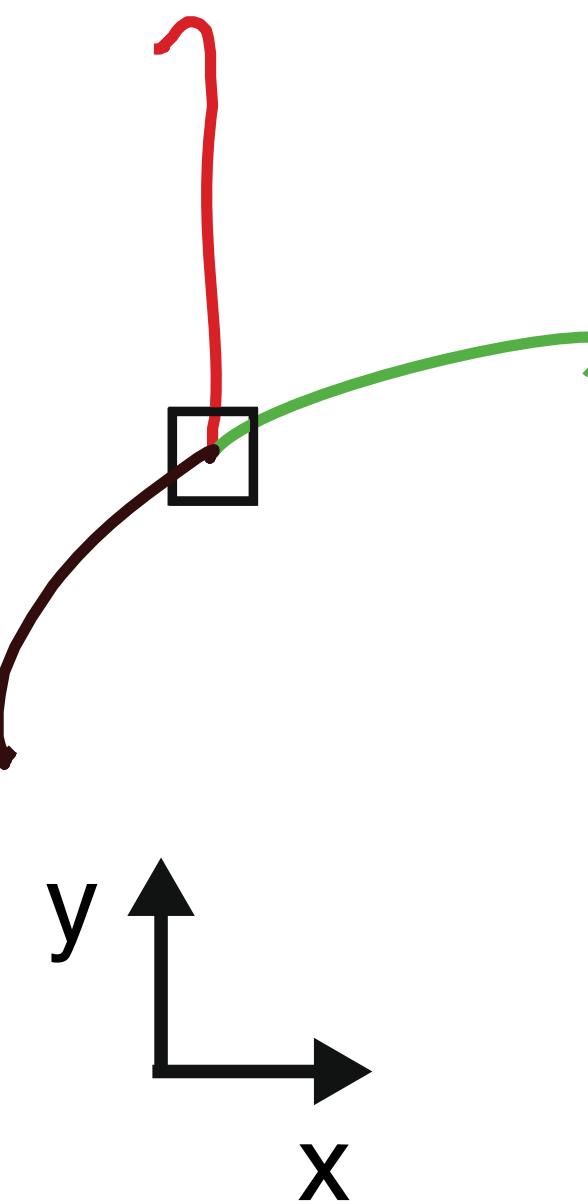
Single neuron firing rate (PSTH)

Average firing rate
(spikes / sec)



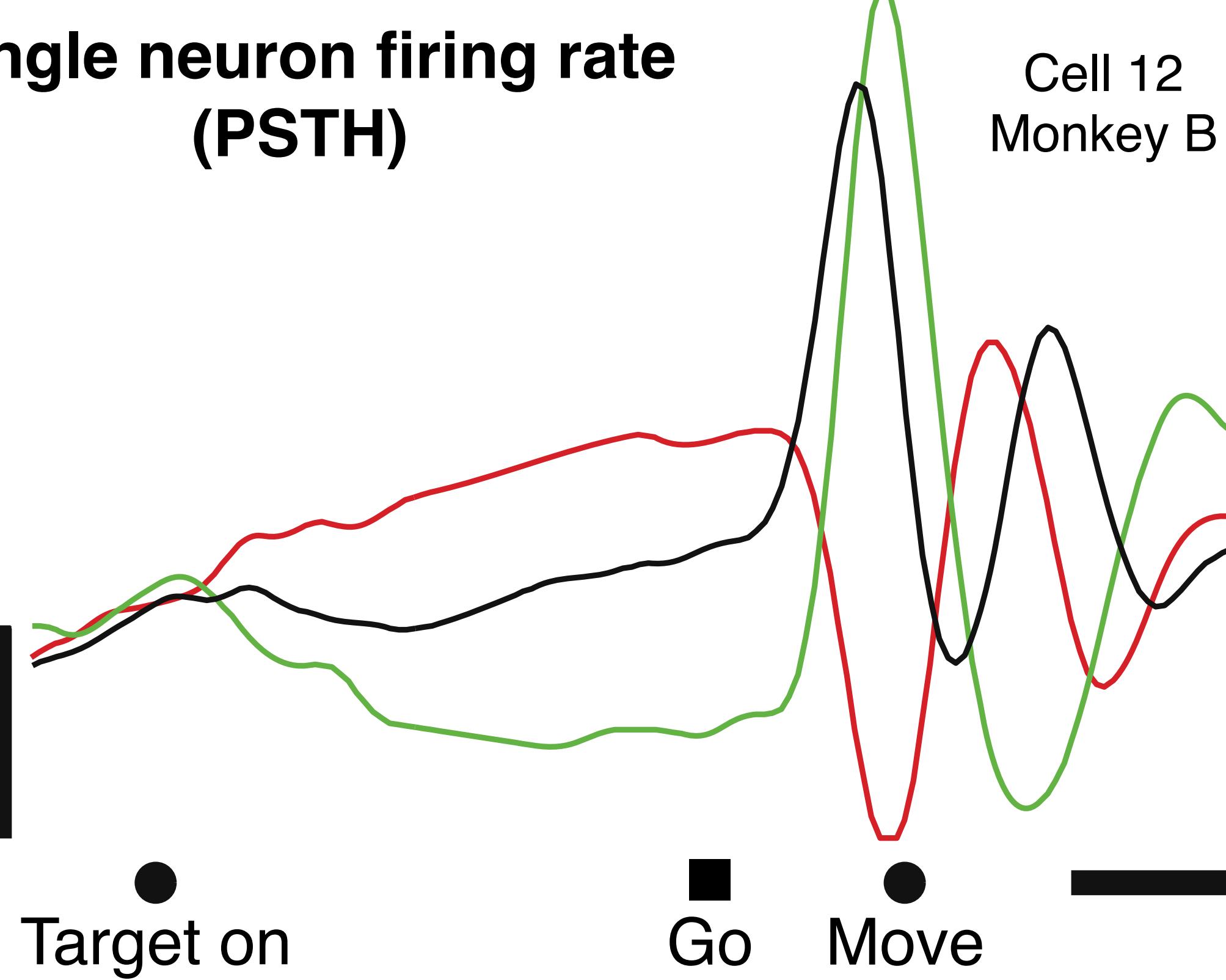


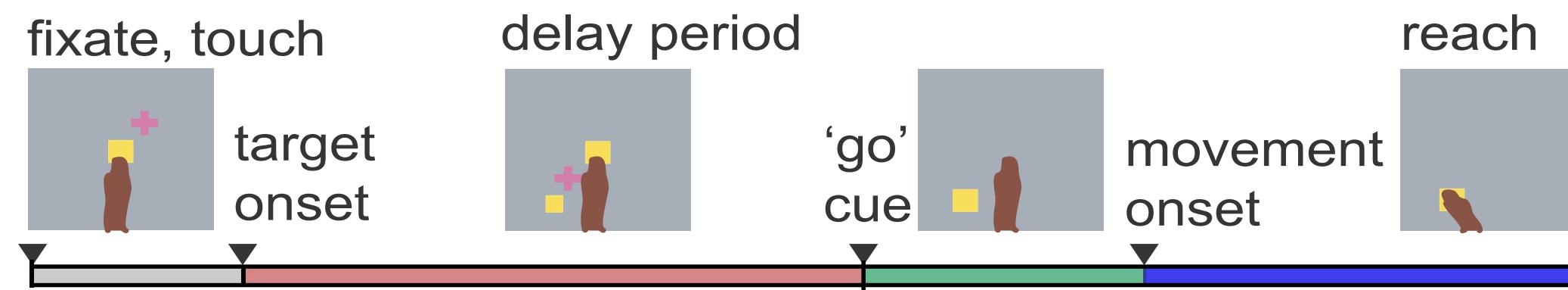
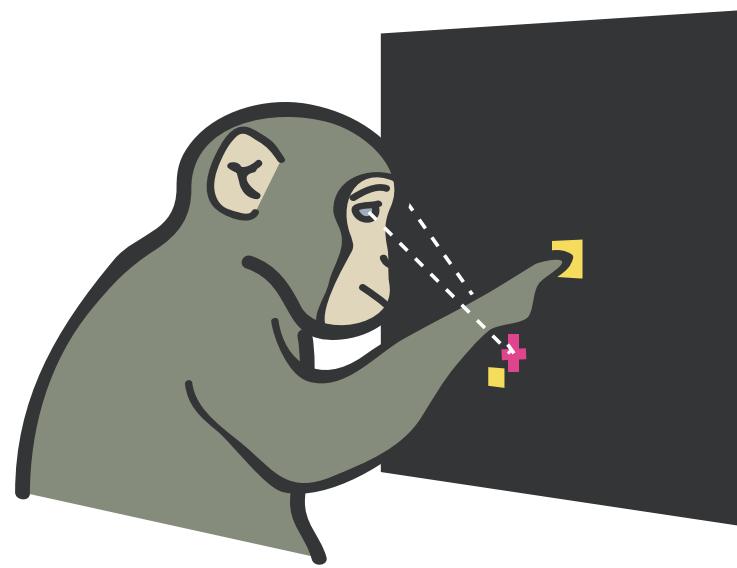
Reach trajectories



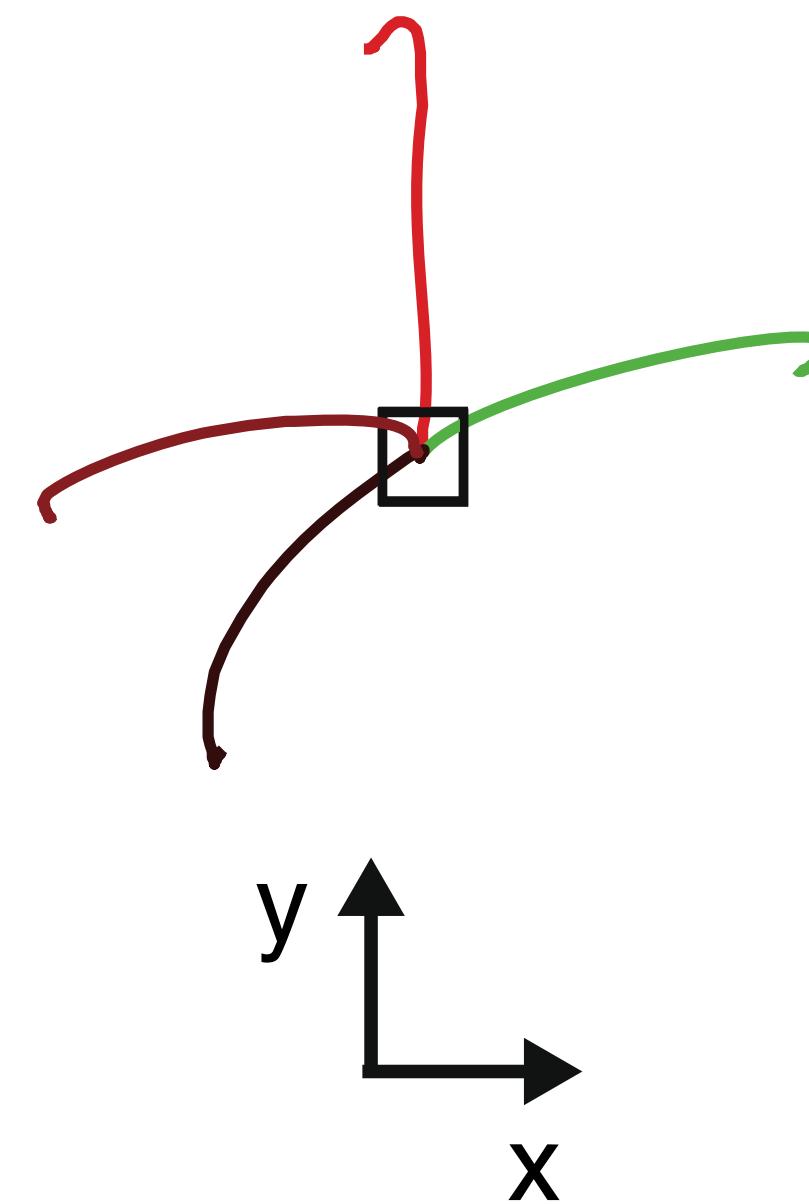
Single neuron firing rate (PSTH)

Average firing rate
(spikes / sec)

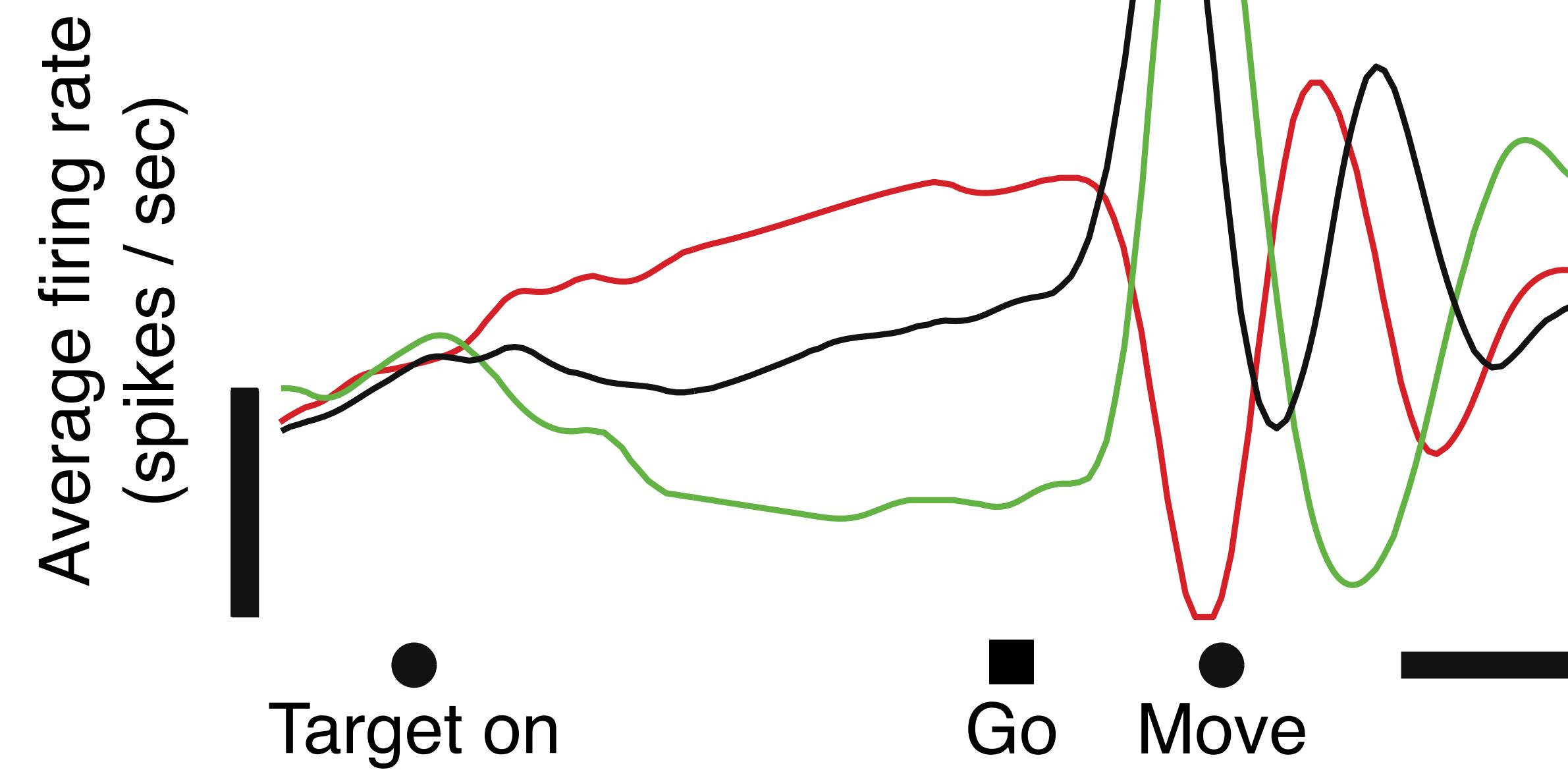


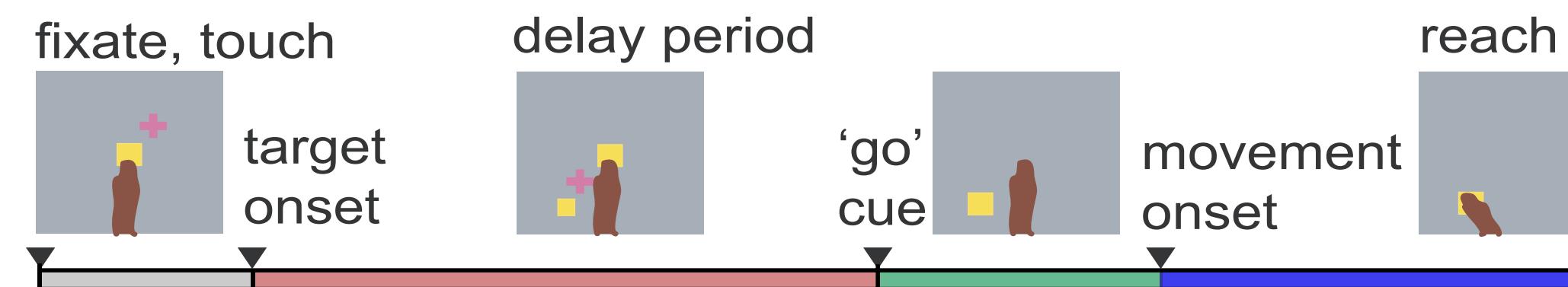
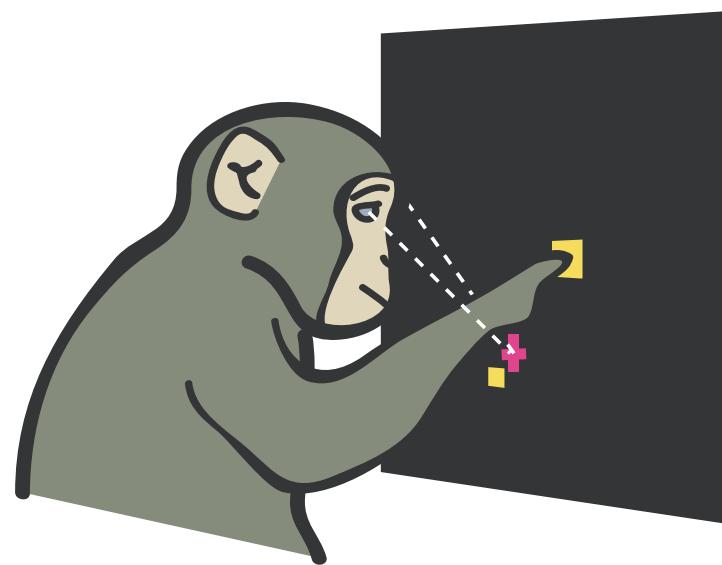


Reach trajectories

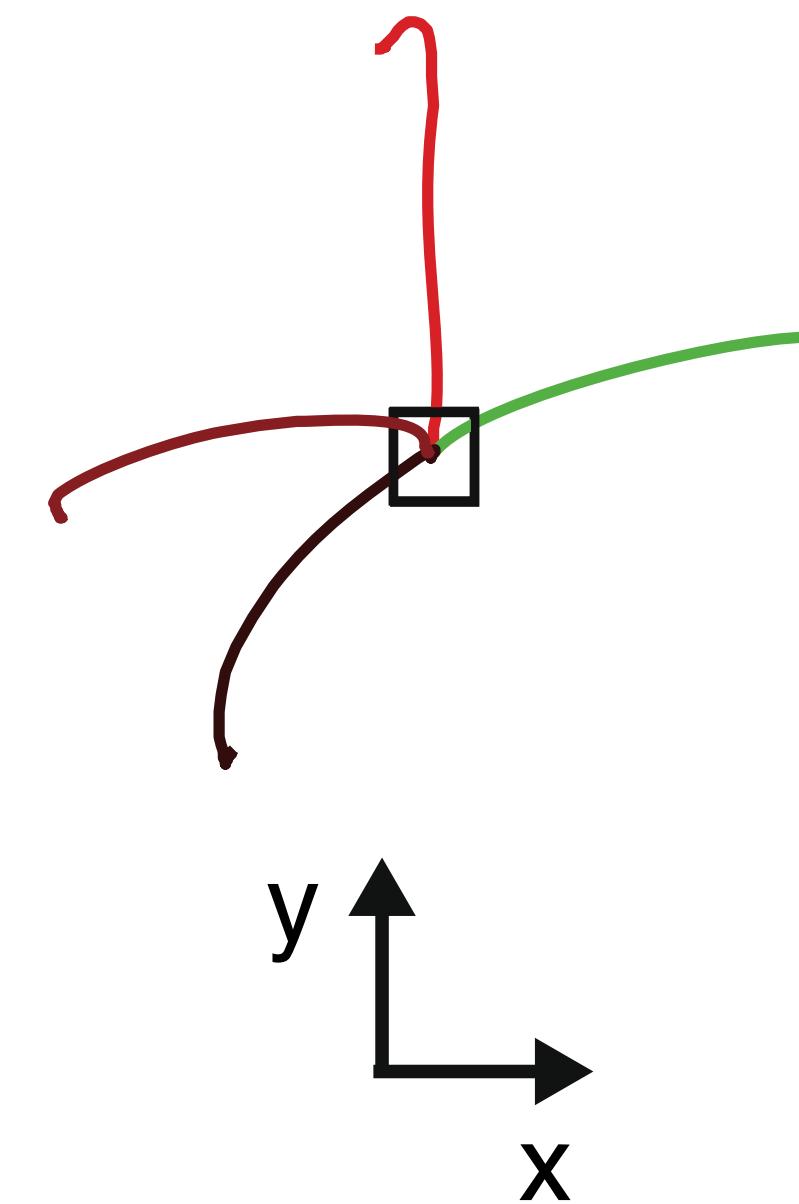


Single neuron firing rate (PSTH)



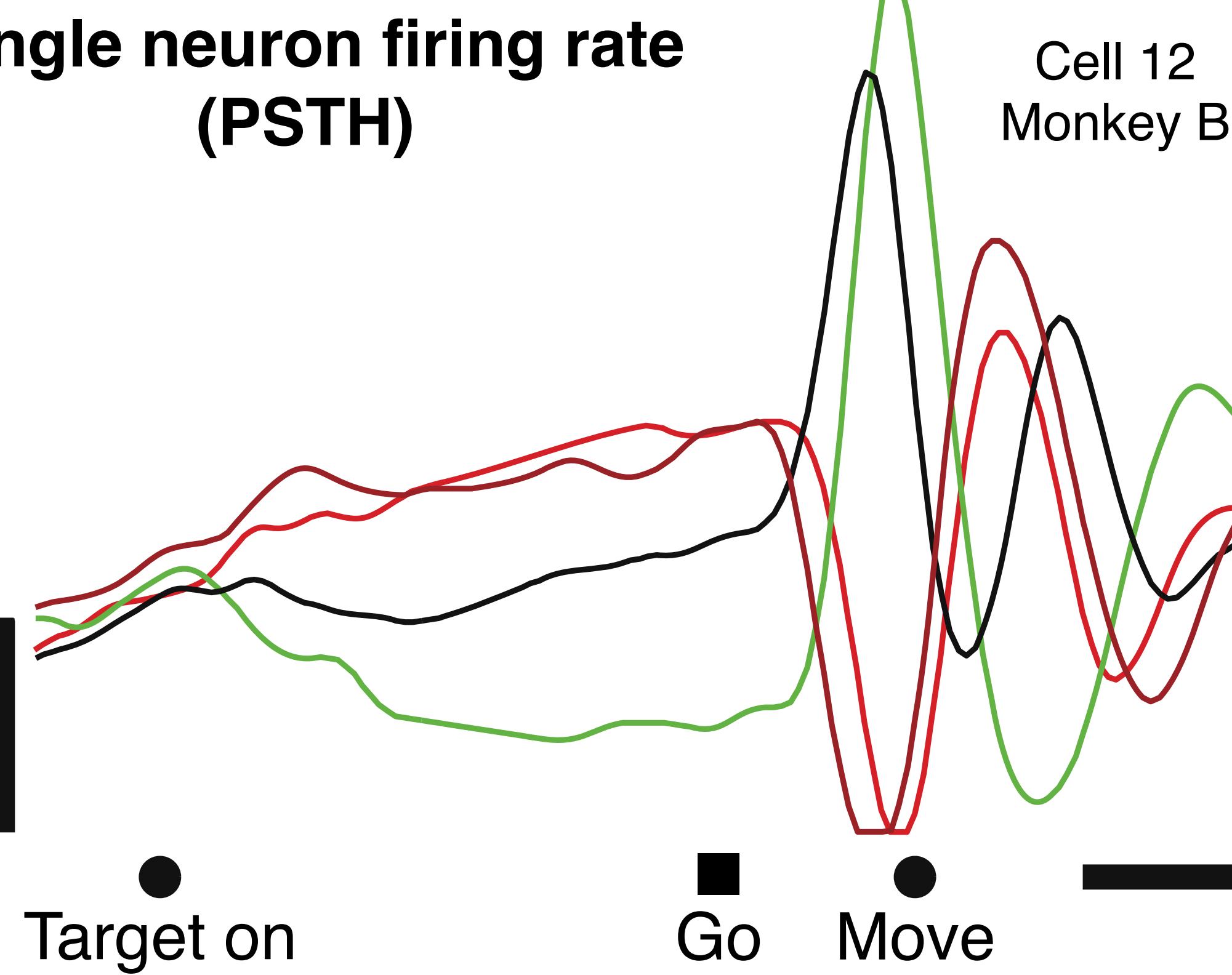


Reach trajectories

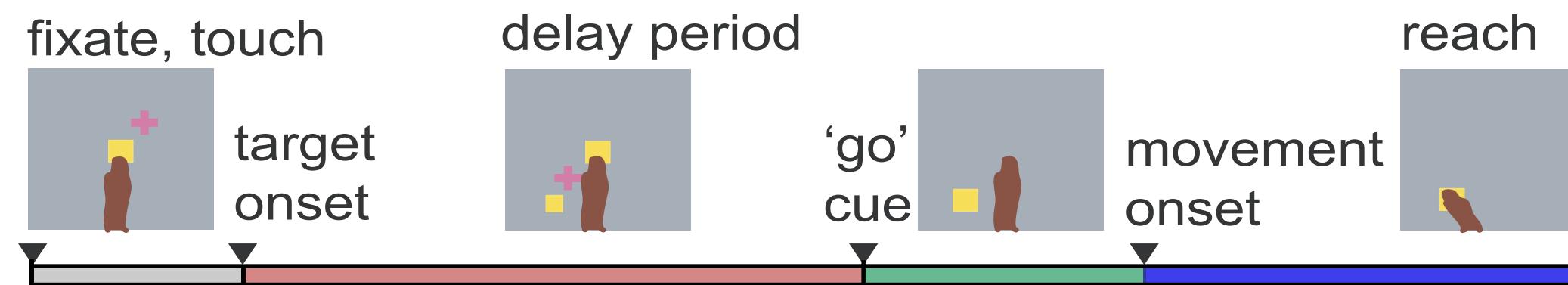
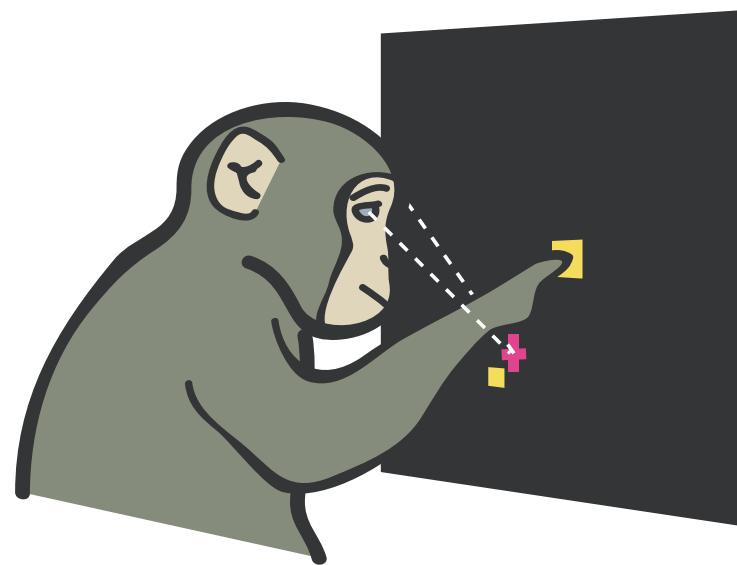


Single neuron firing rate (PSTH)

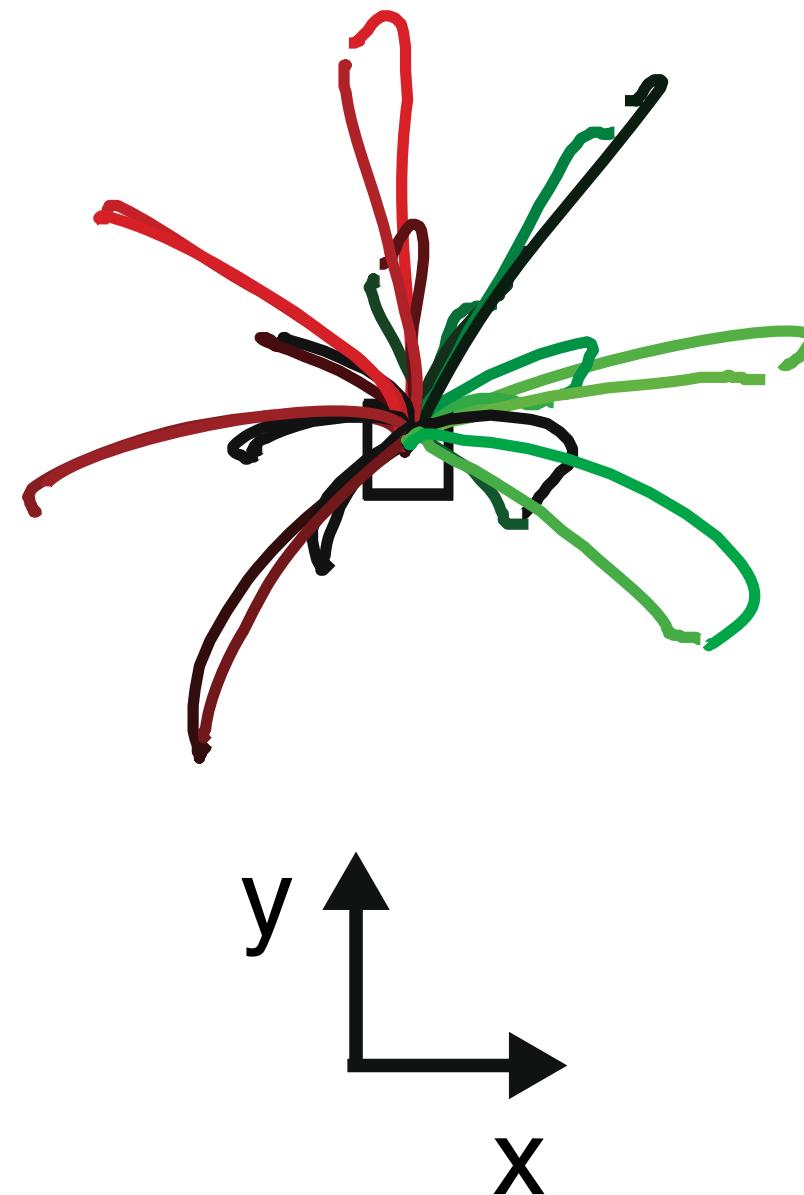
Average firing rate
(spikes / sec)



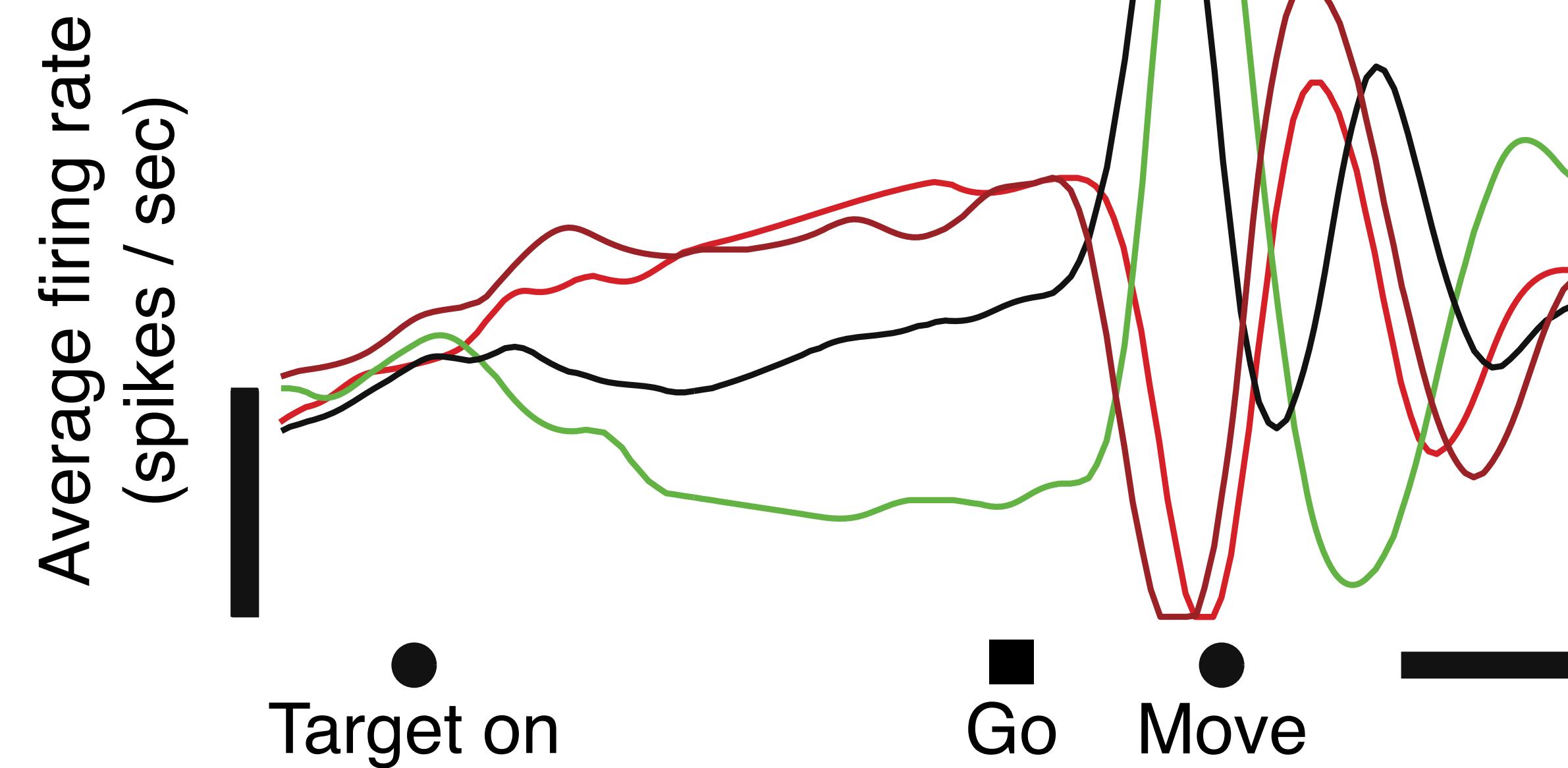
Cell 12
Monkey B

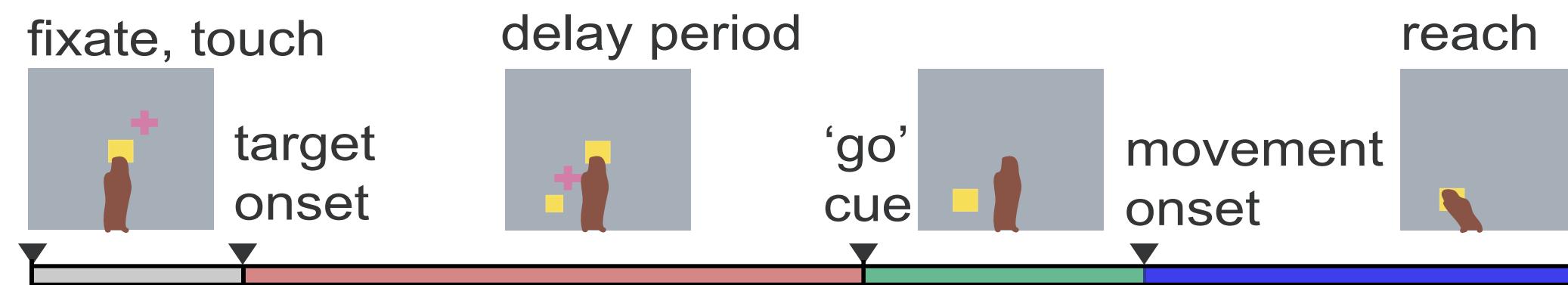
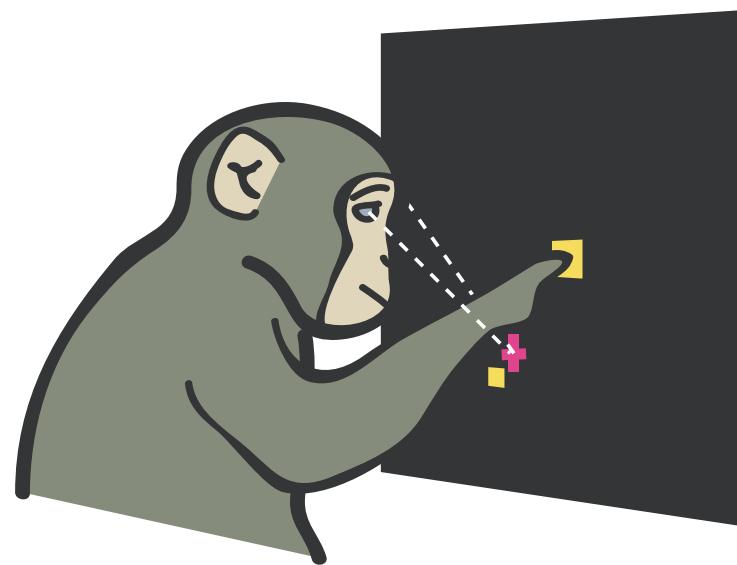


Reach trajectories

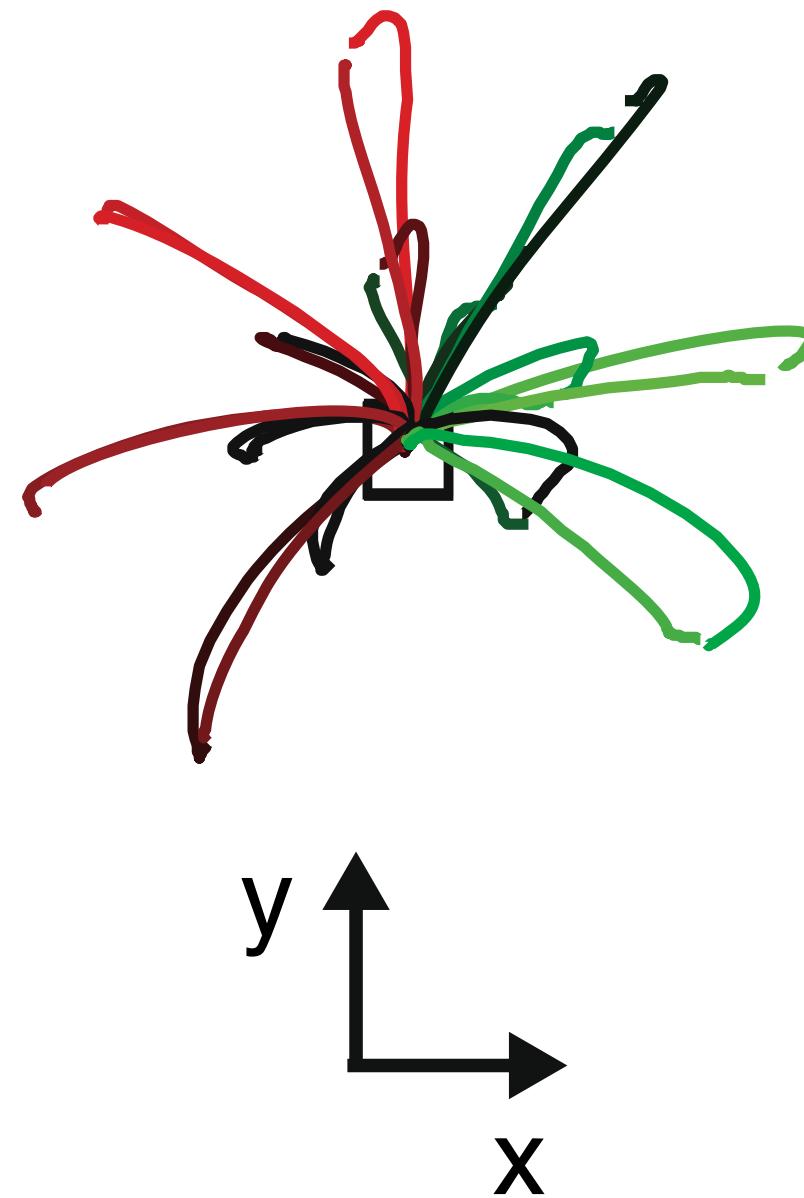


Single neuron firing rate (PSTH)

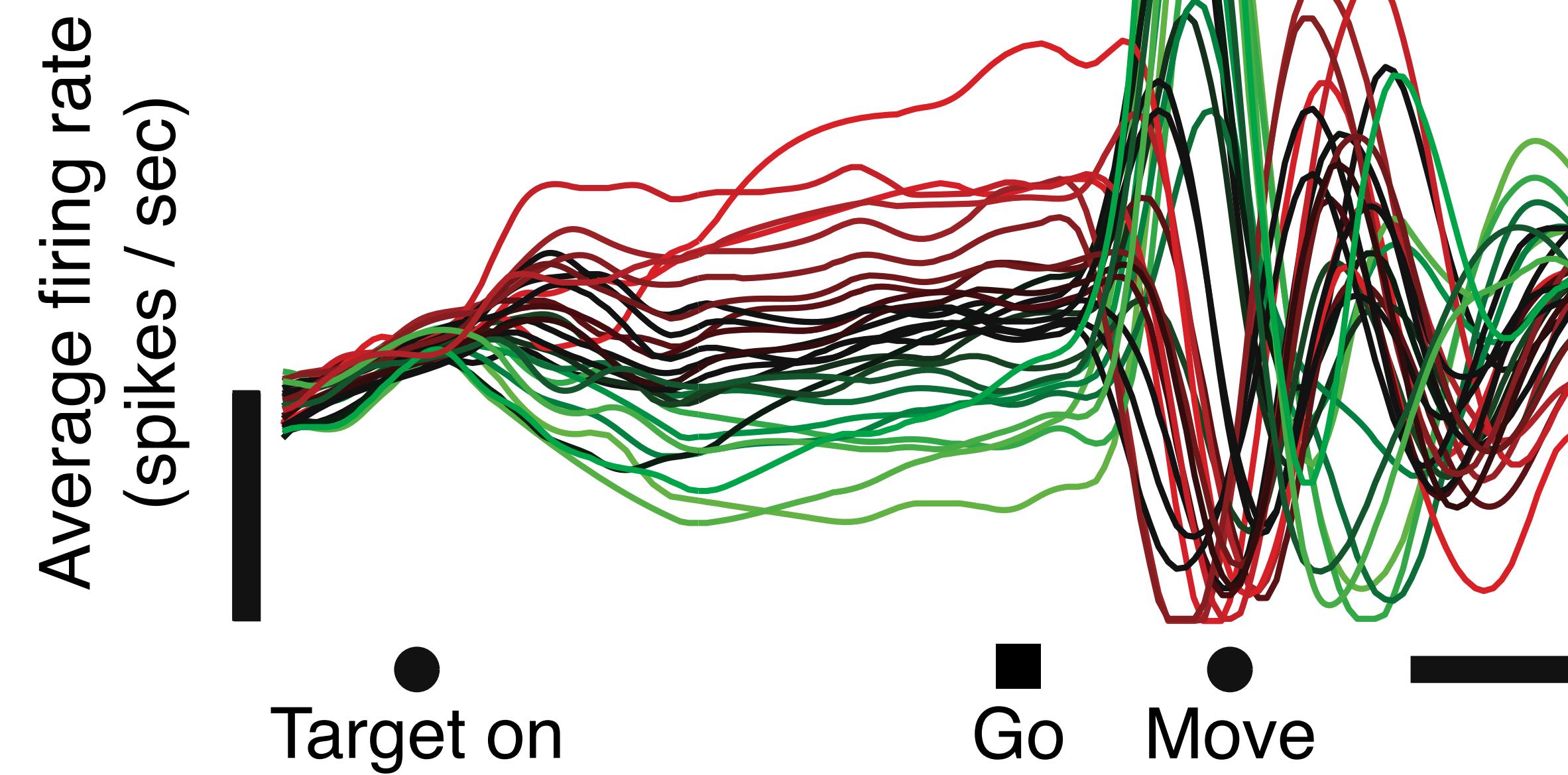


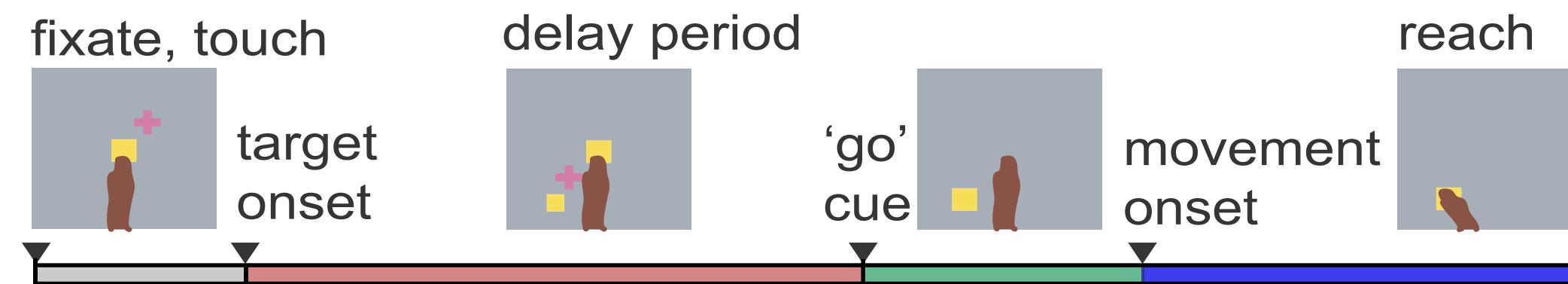
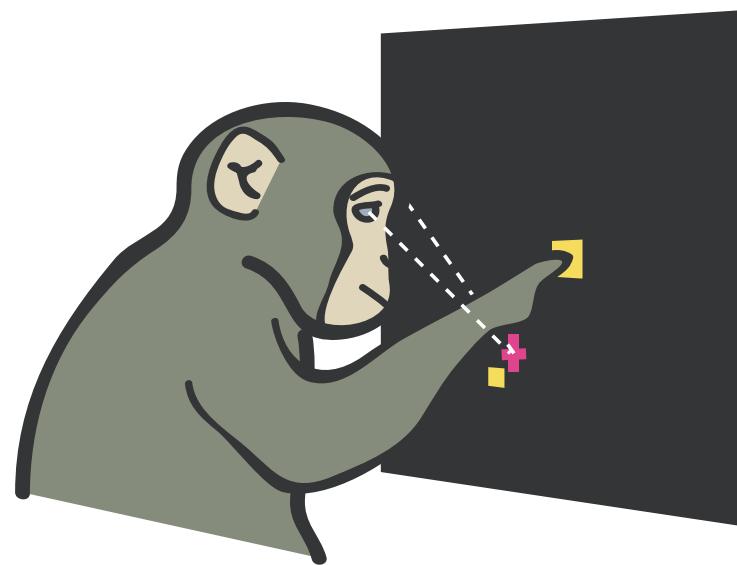


Reach trajectories

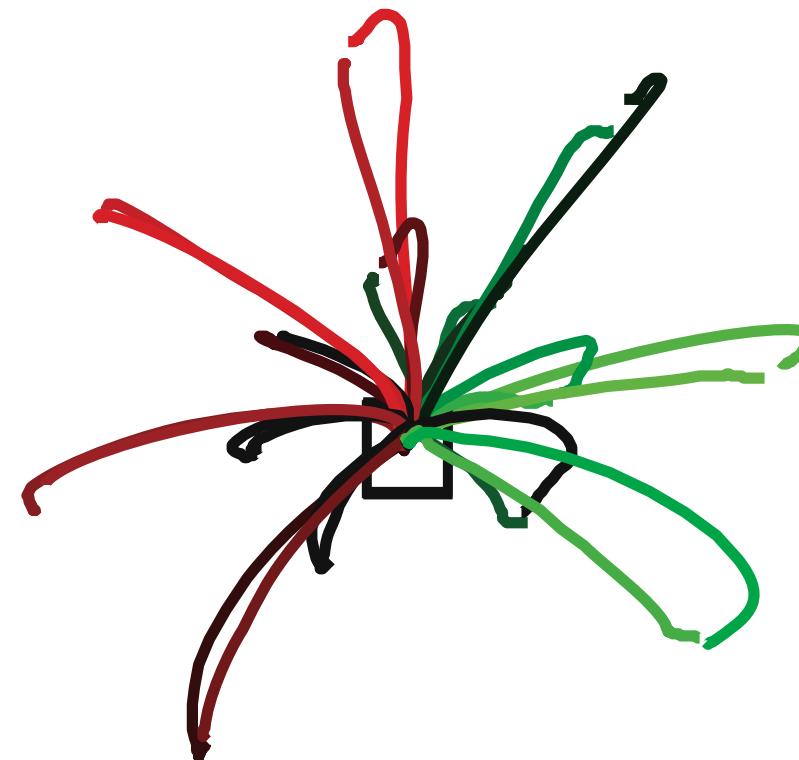


Single neuron firing rate (PSTH)

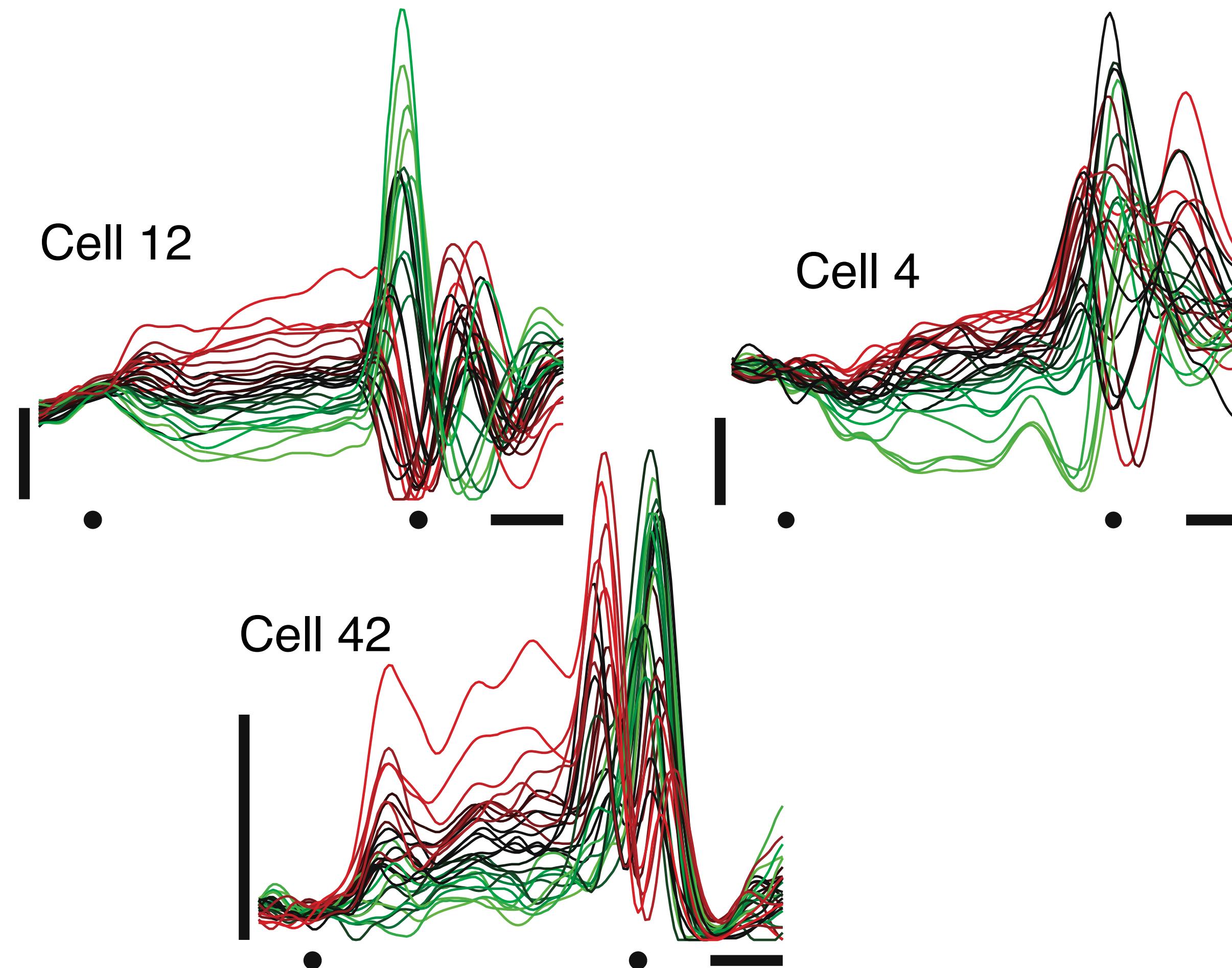




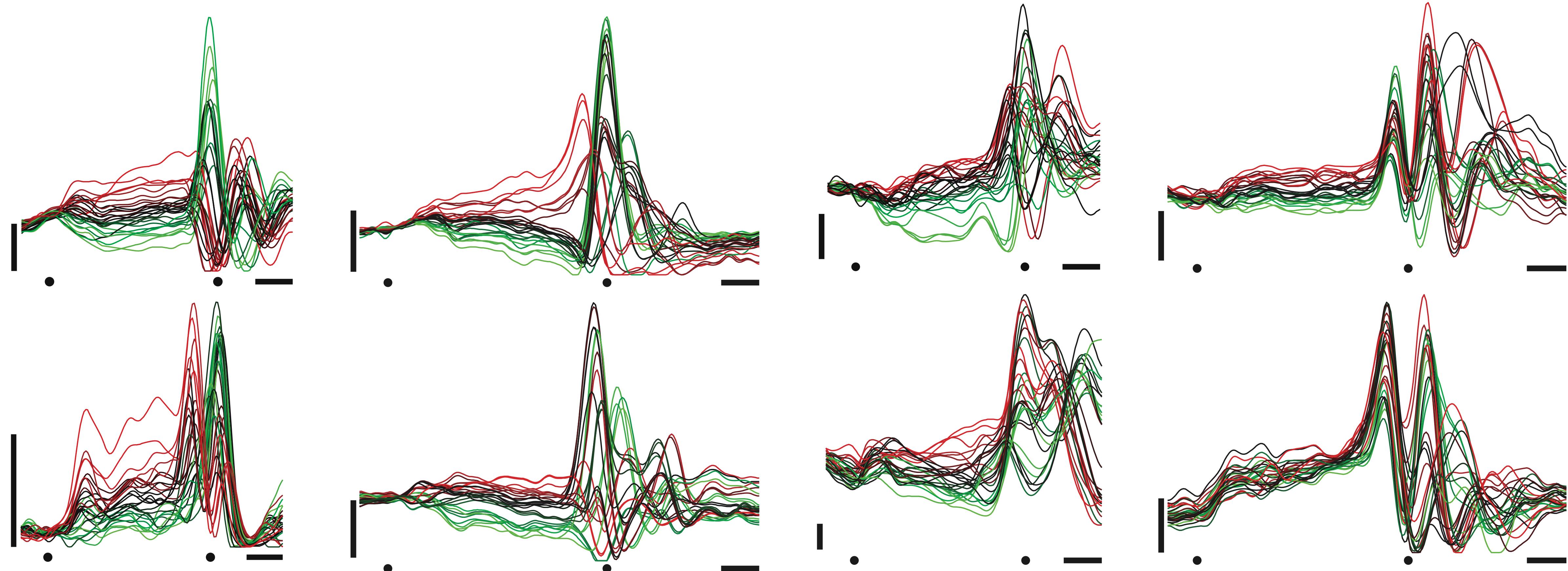
Reach
trajectories



Cell 12



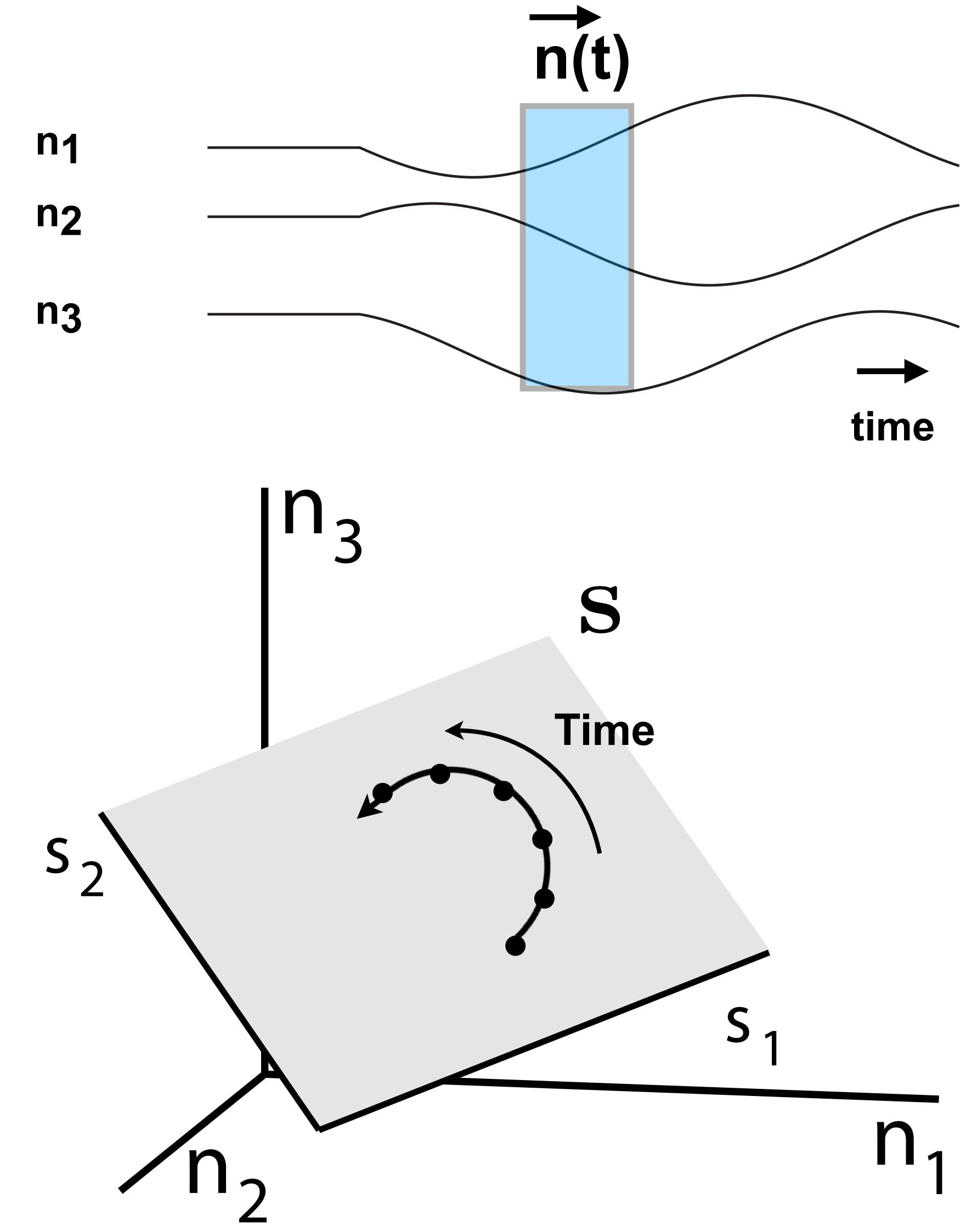
Single-neuron data is confusing



Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

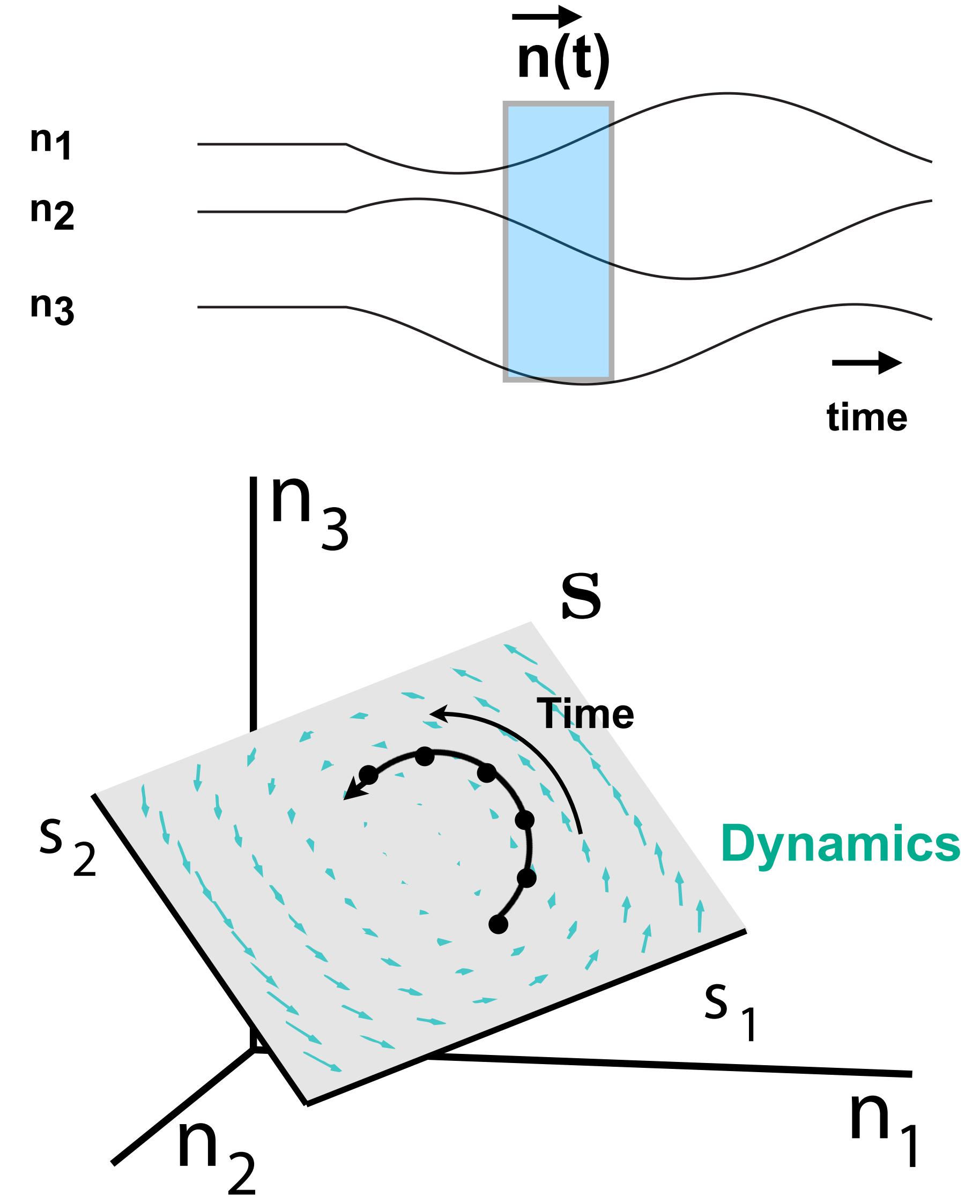
Uncovering neural population dynamics

Neurons' firing rates



Uncovering neural population dynamics

Neurons' firing rates

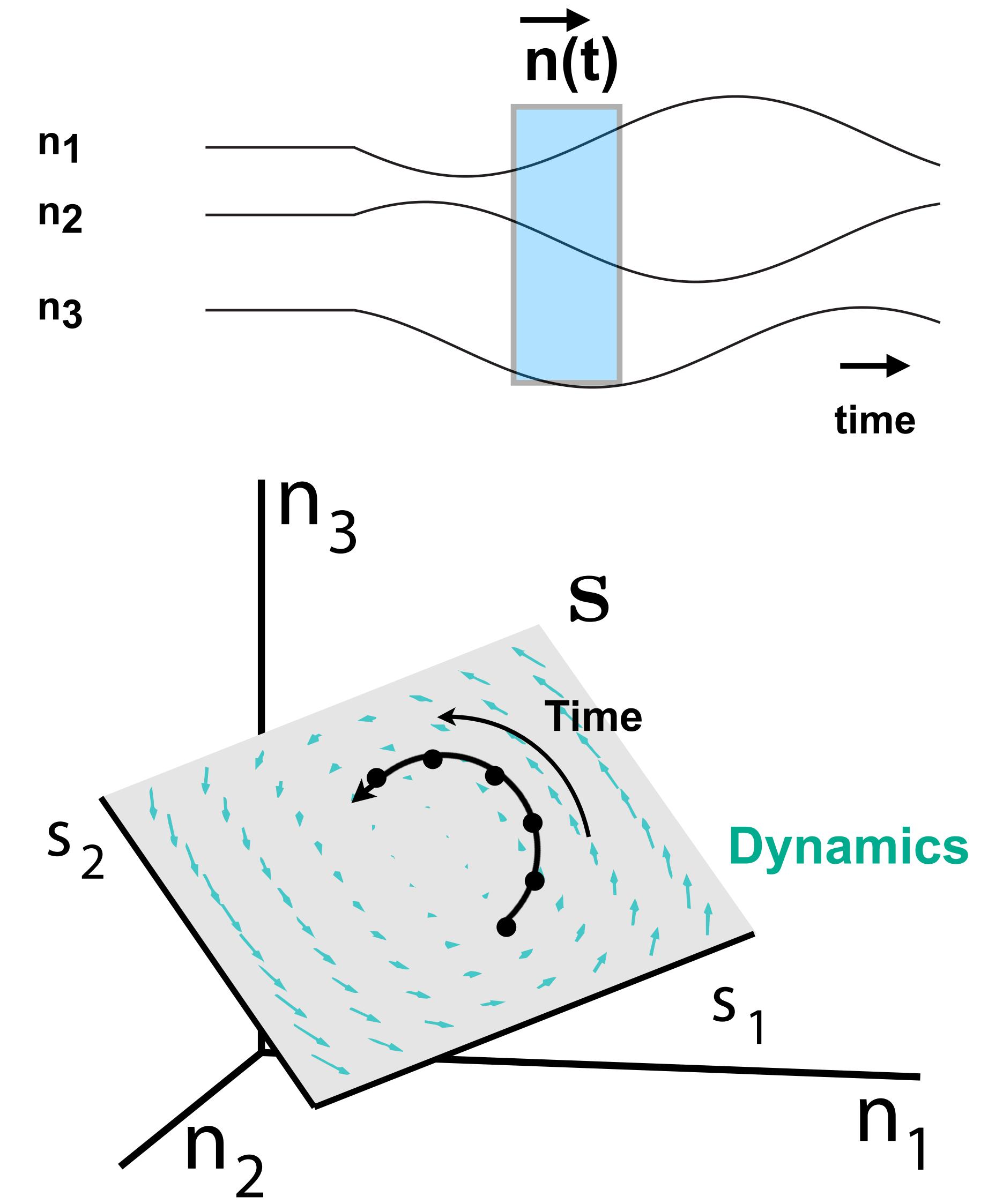


Uncovering neural population dynamics

Predictable activity -
modeled by
autonomous
dynamics

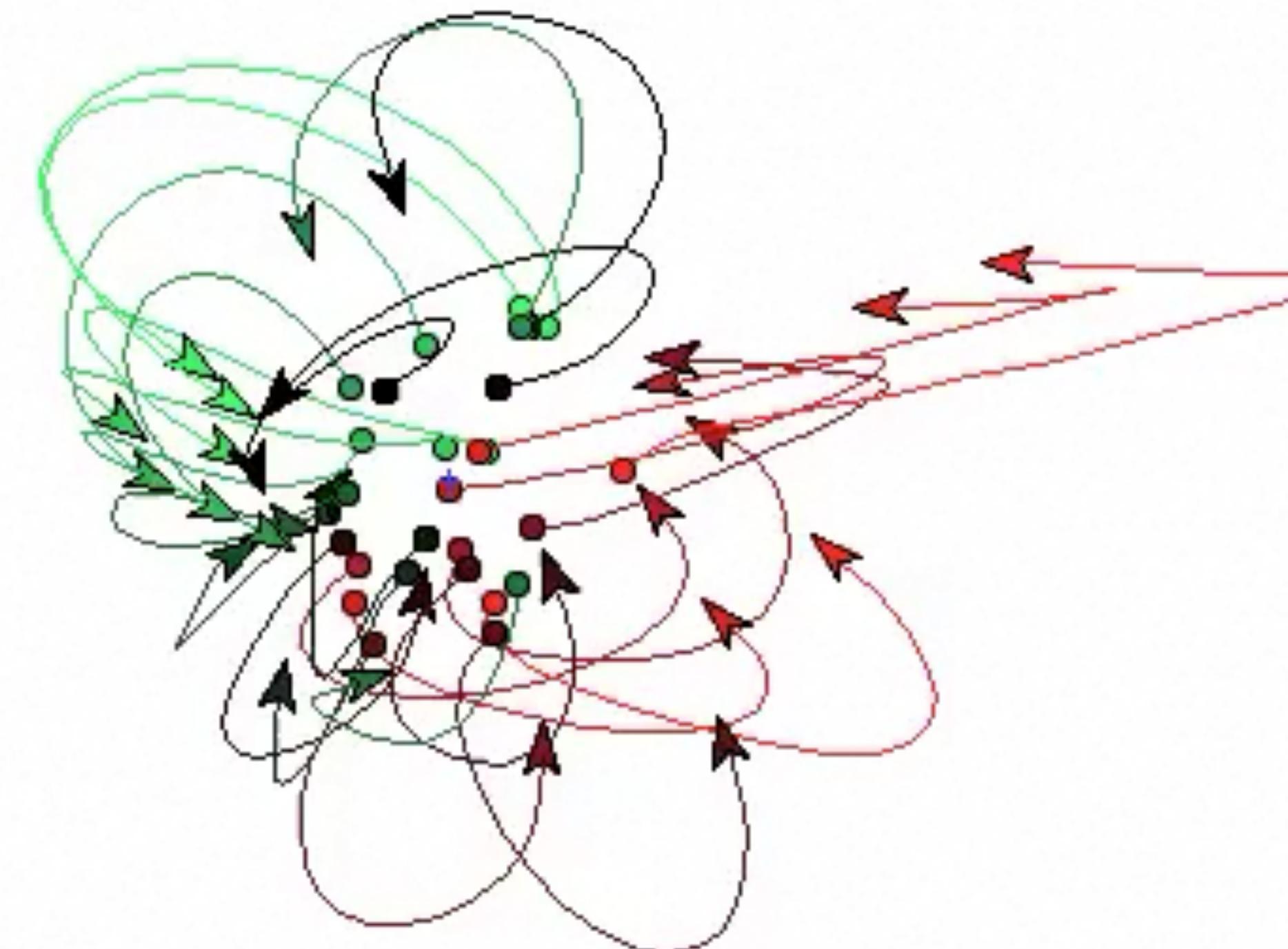
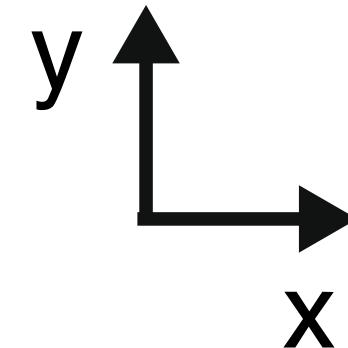
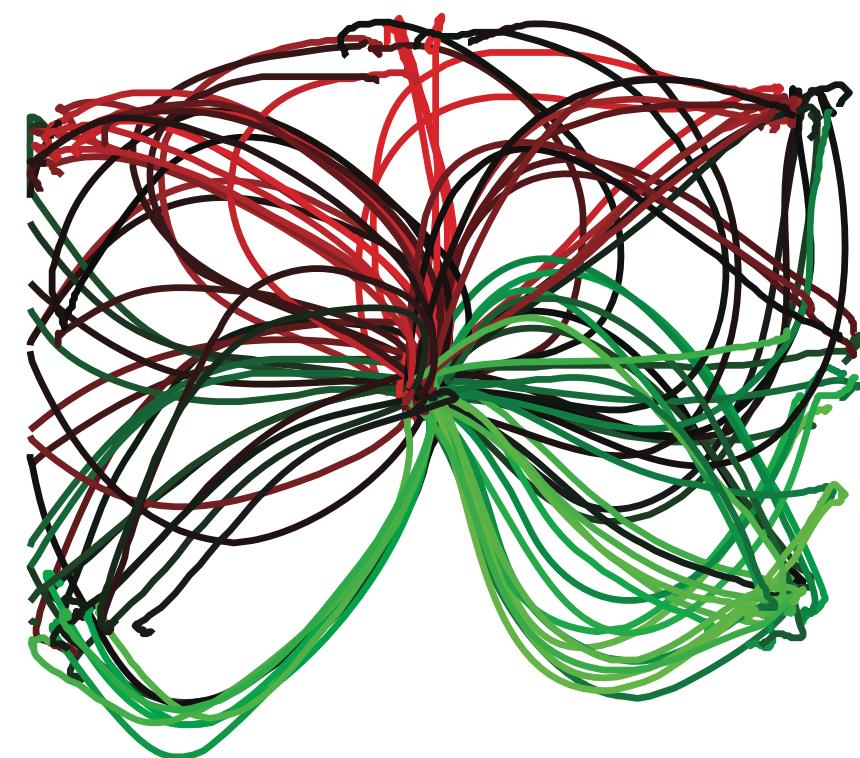
$$\frac{ds}{dt} = f(s)$$

Neurons'
firing rates



PCA / jPCA reveals underlying structure

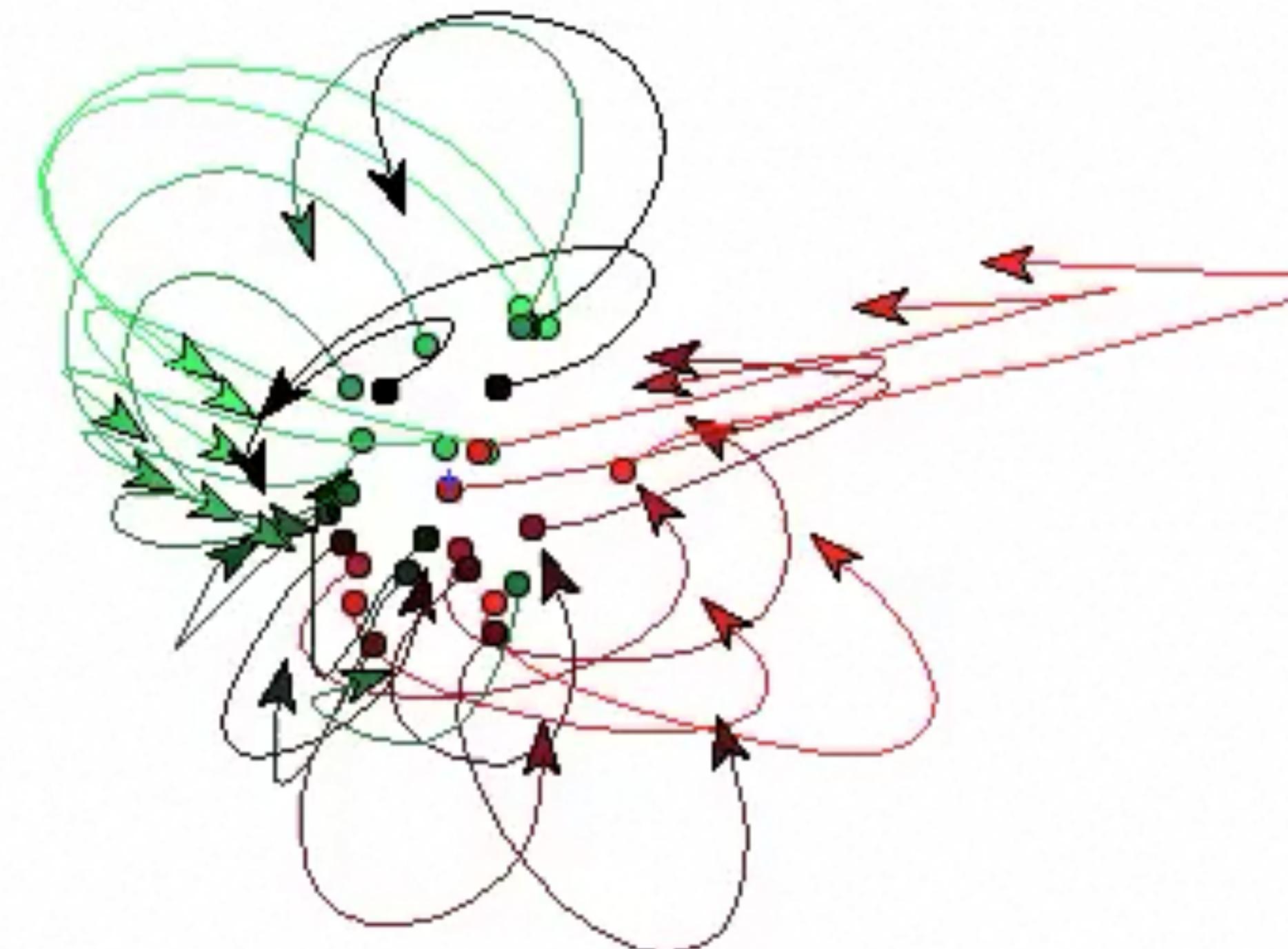
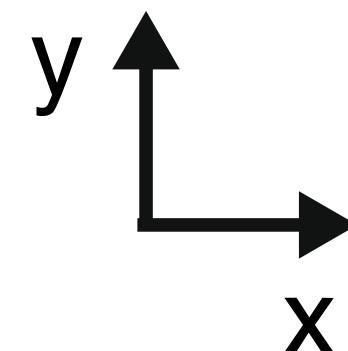
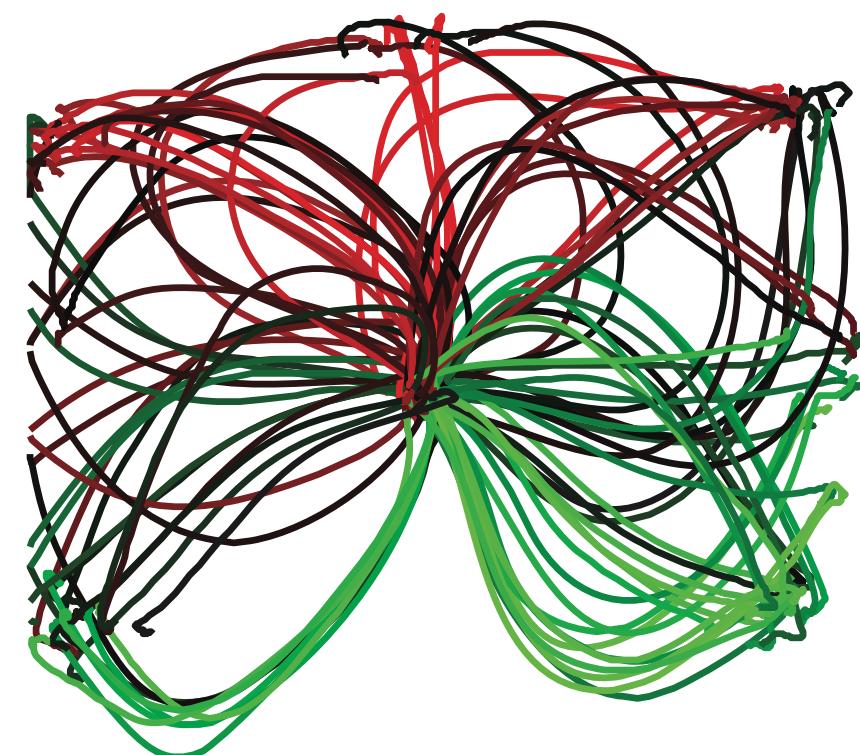
Reach
trajectories



Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

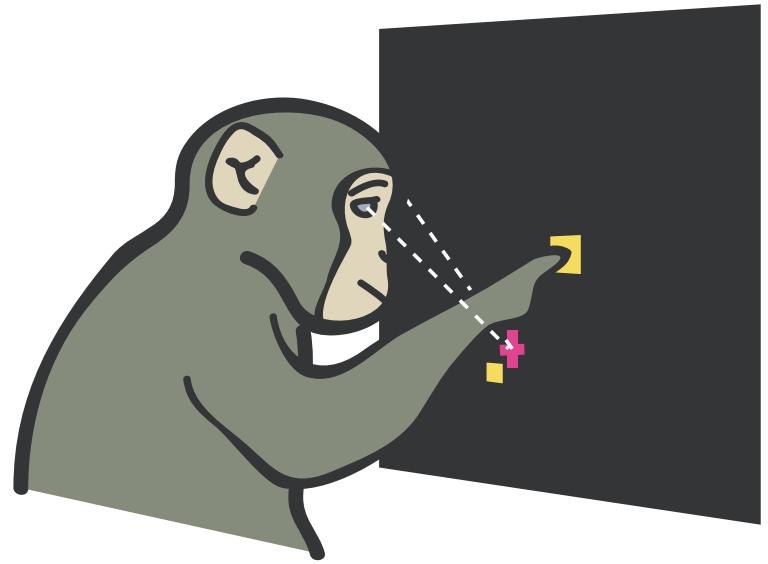
PCA / jPCA reveals underlying structure

Reach
trajectories

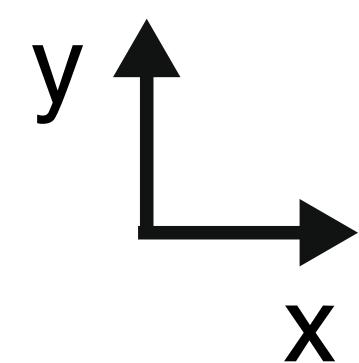
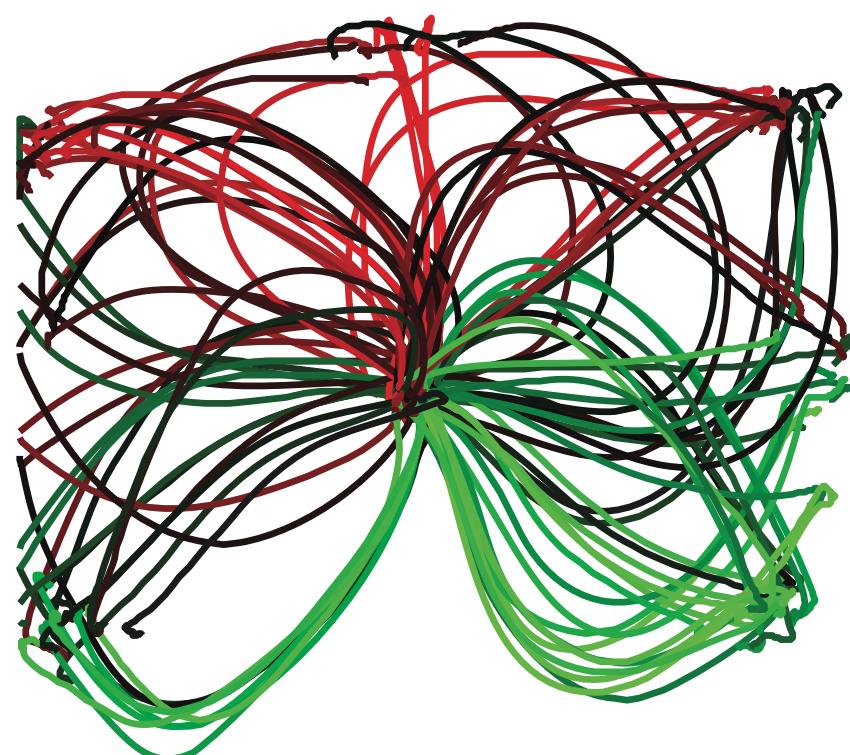


Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

Consistent rotational dynamics

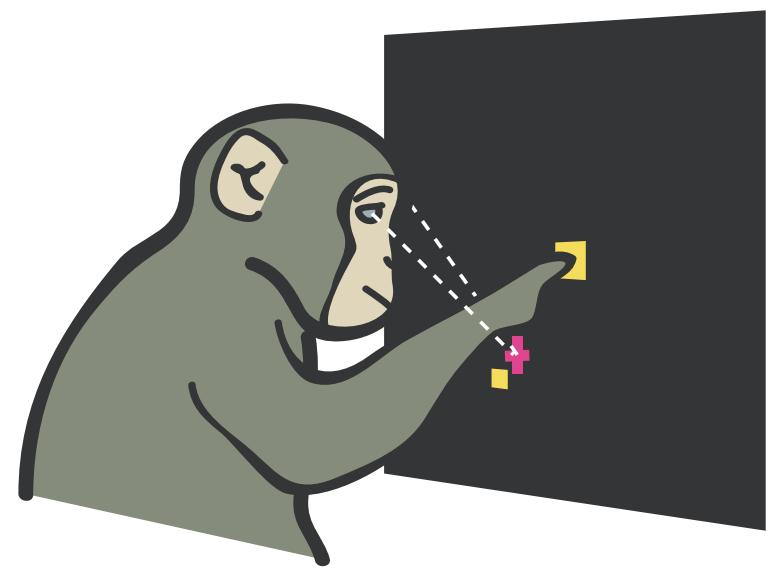


Reach
trajectories

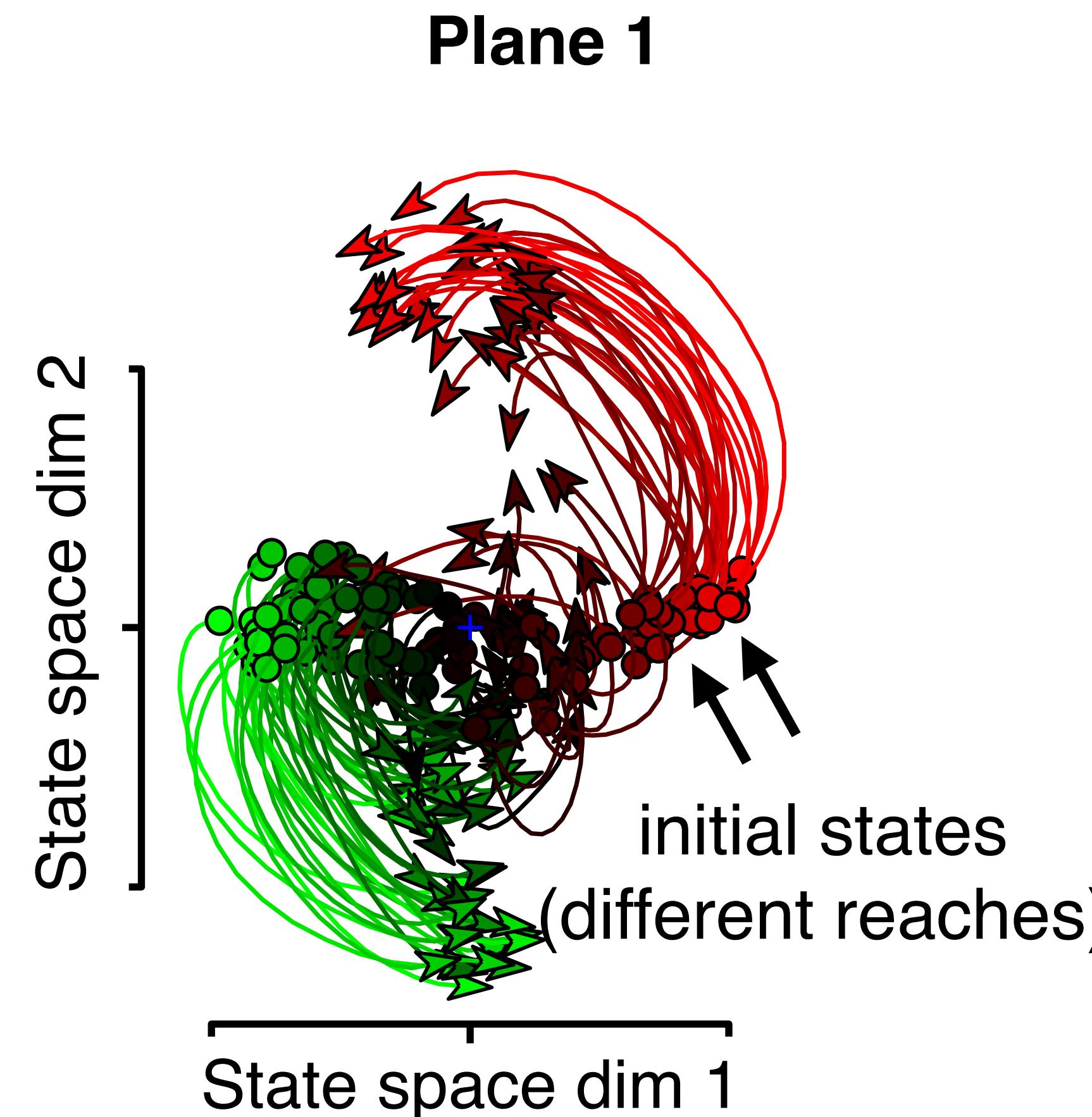
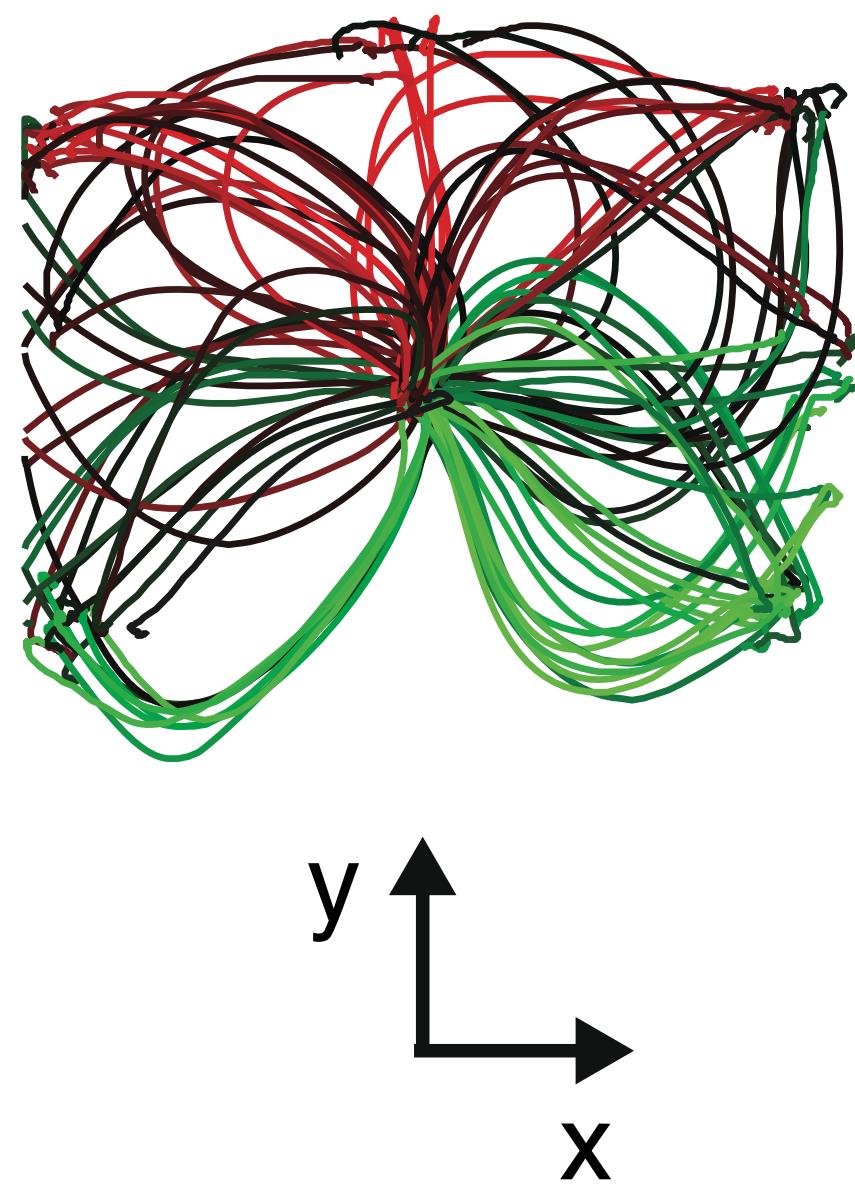


Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

Consistent rotational dynamics

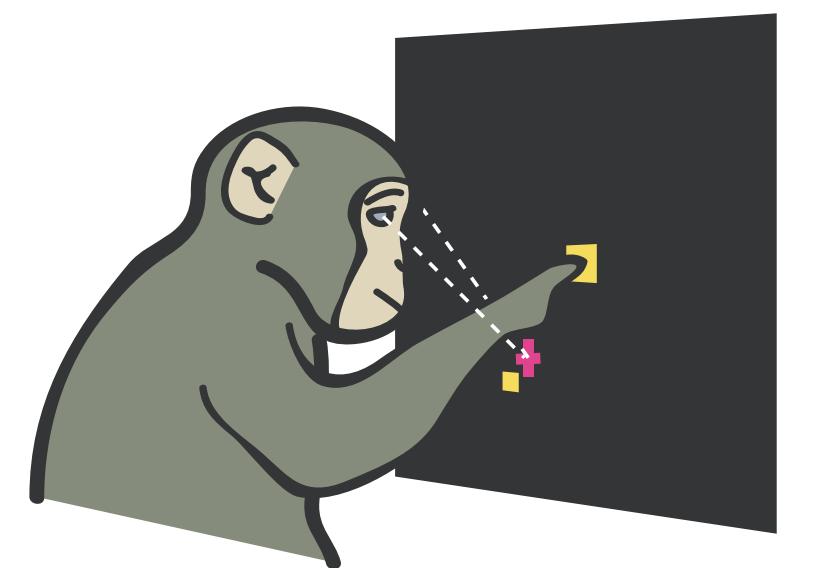


Reach
trajectories

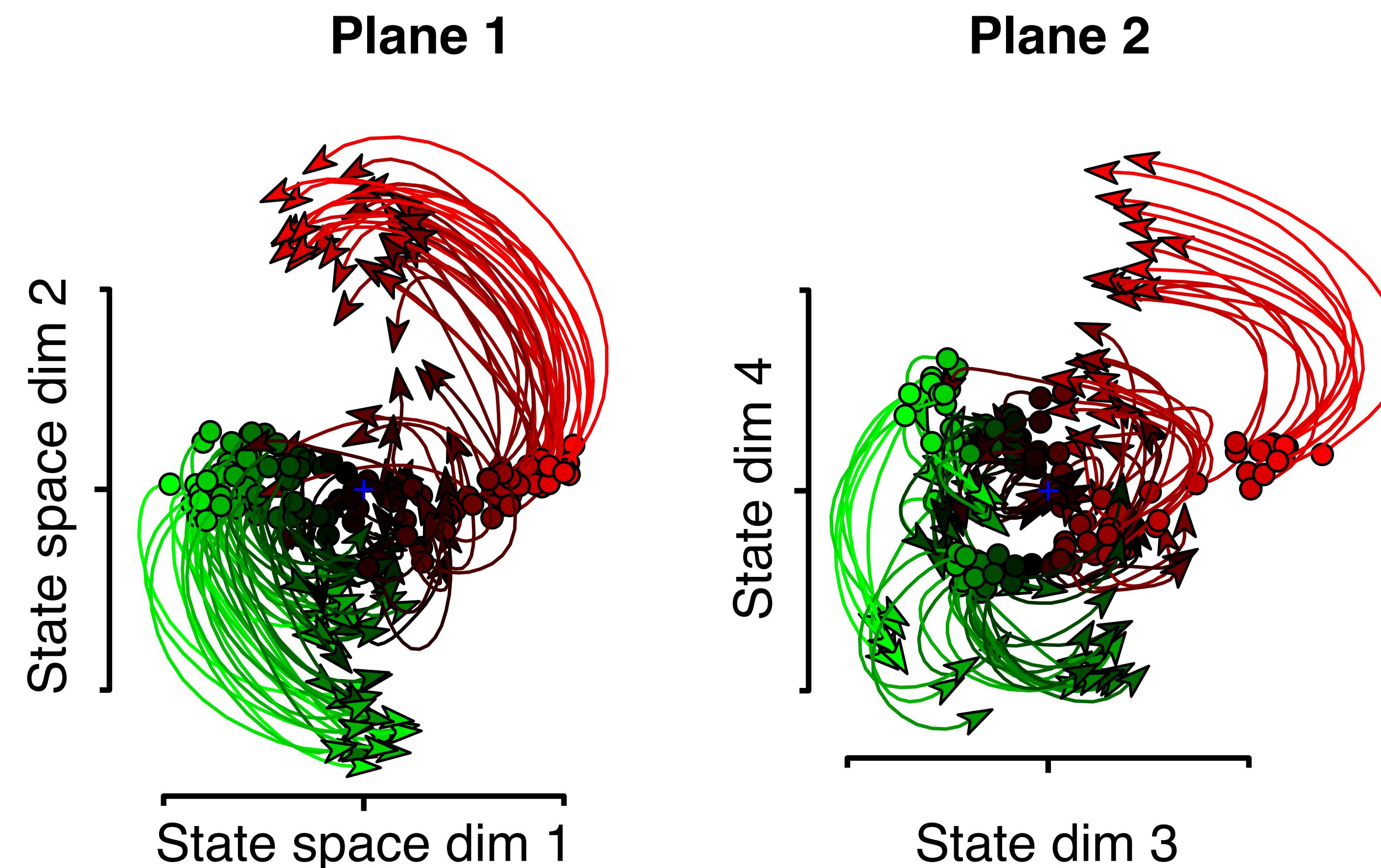
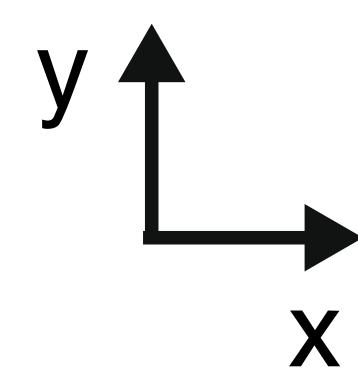


Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

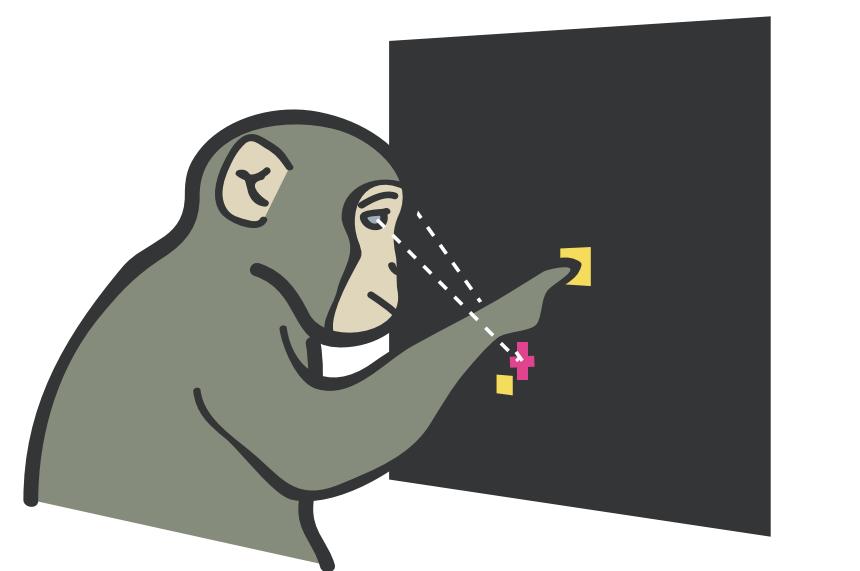
Consistent rotational dynamics



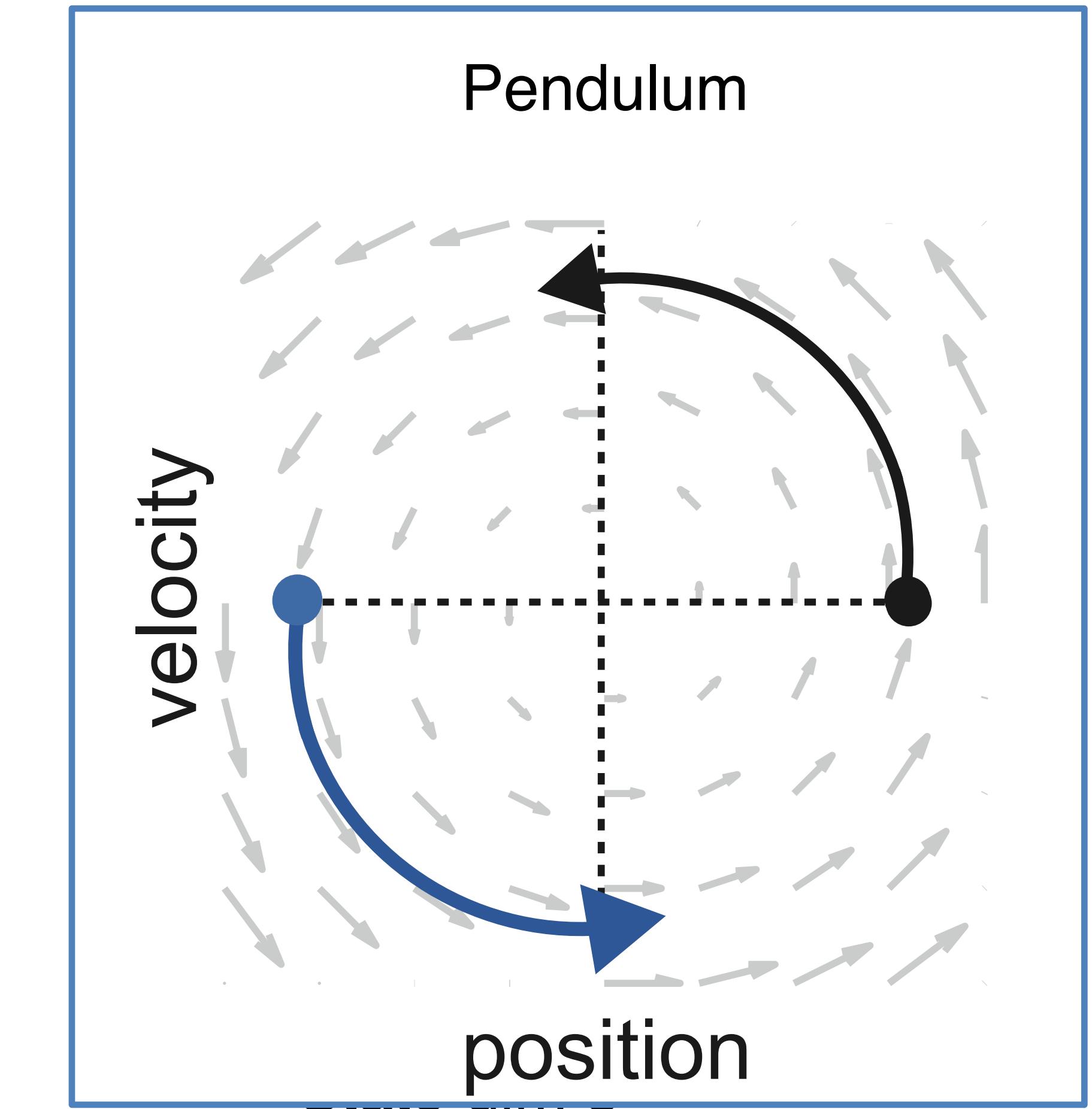
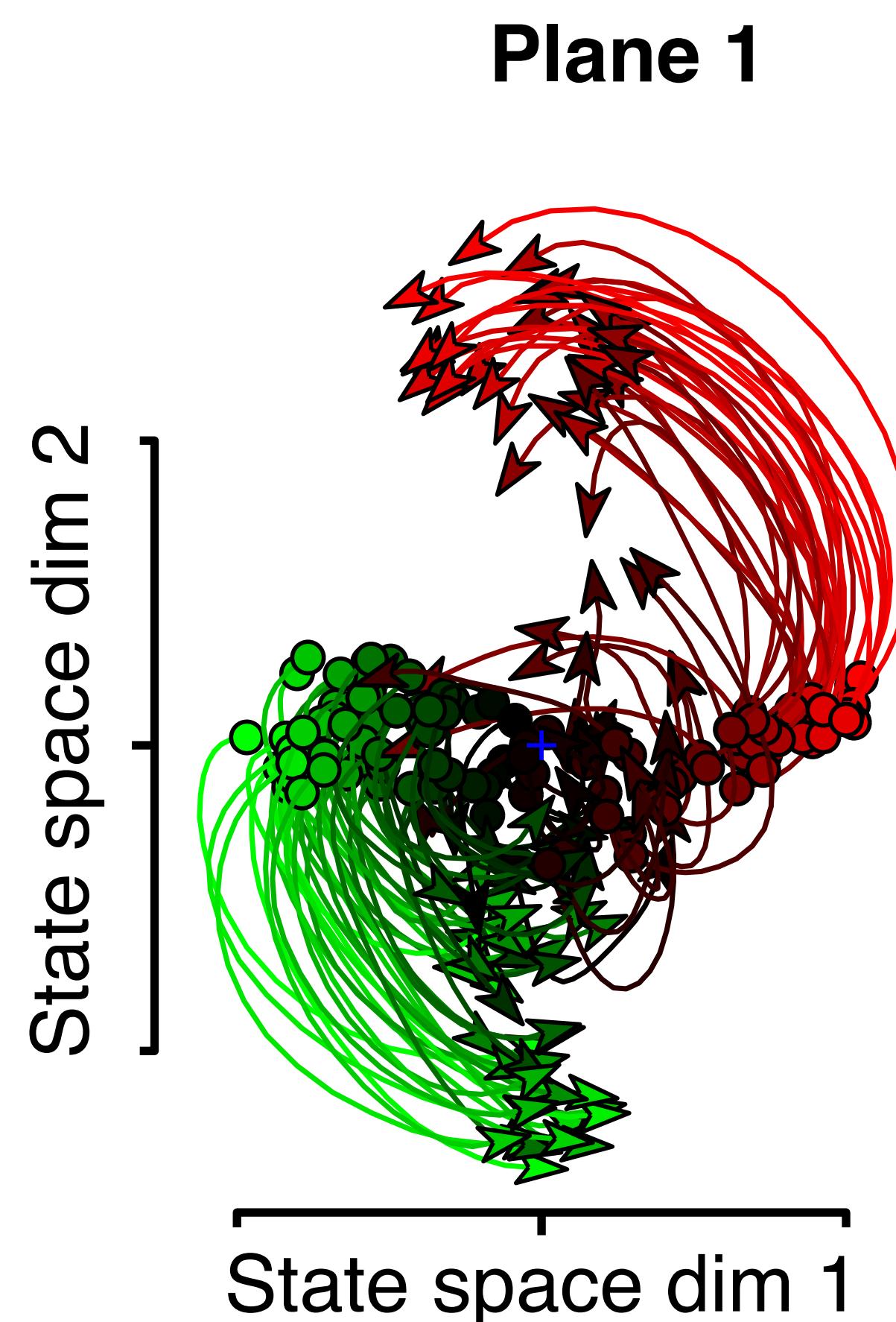
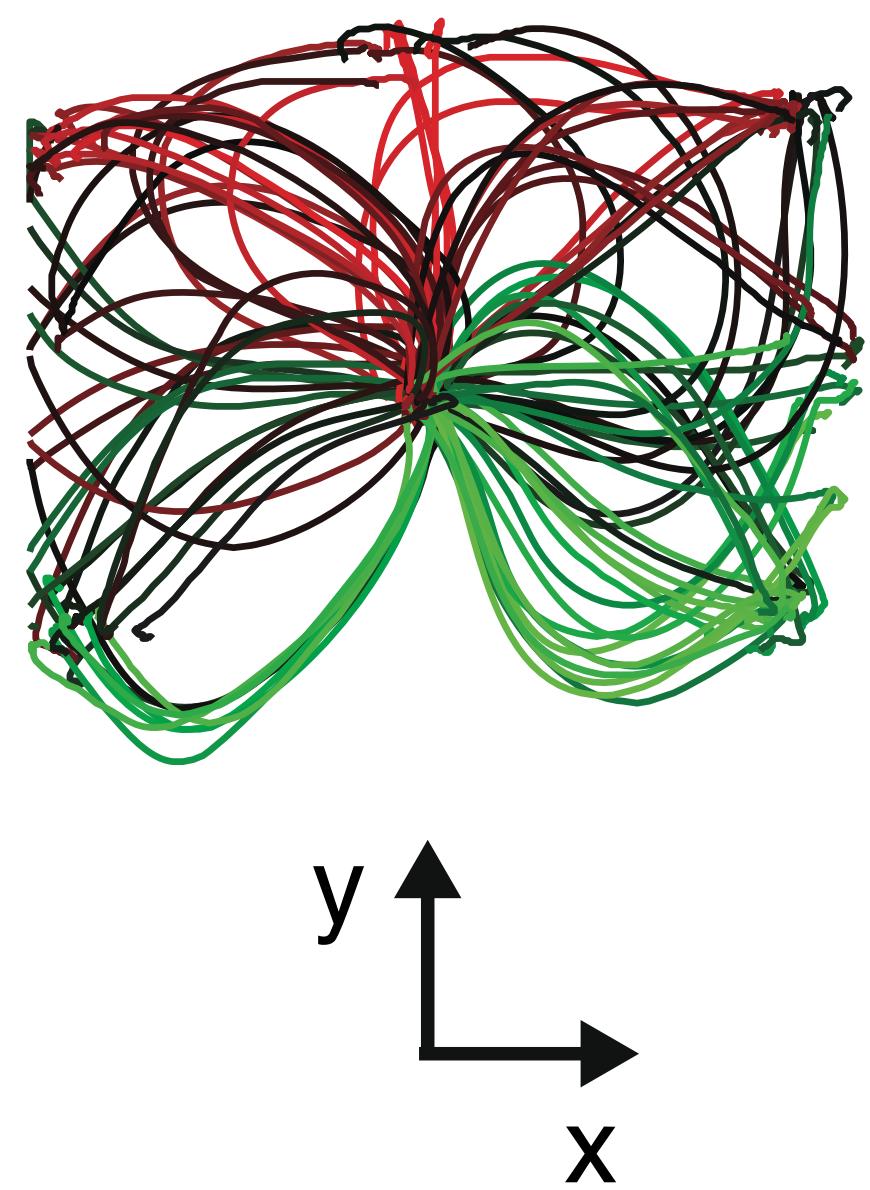
Reach
trajectories

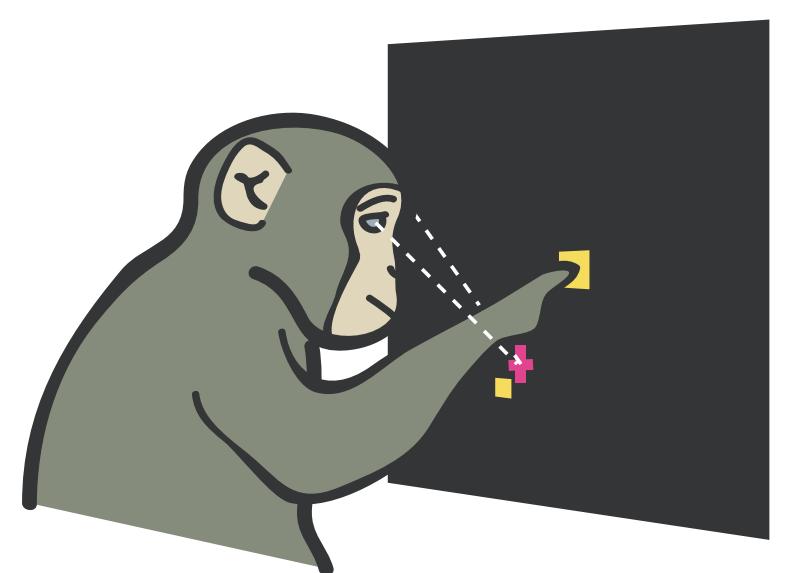


Consistent rotational dynamics

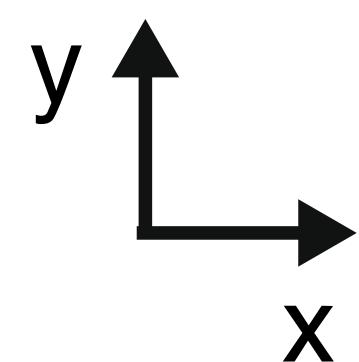
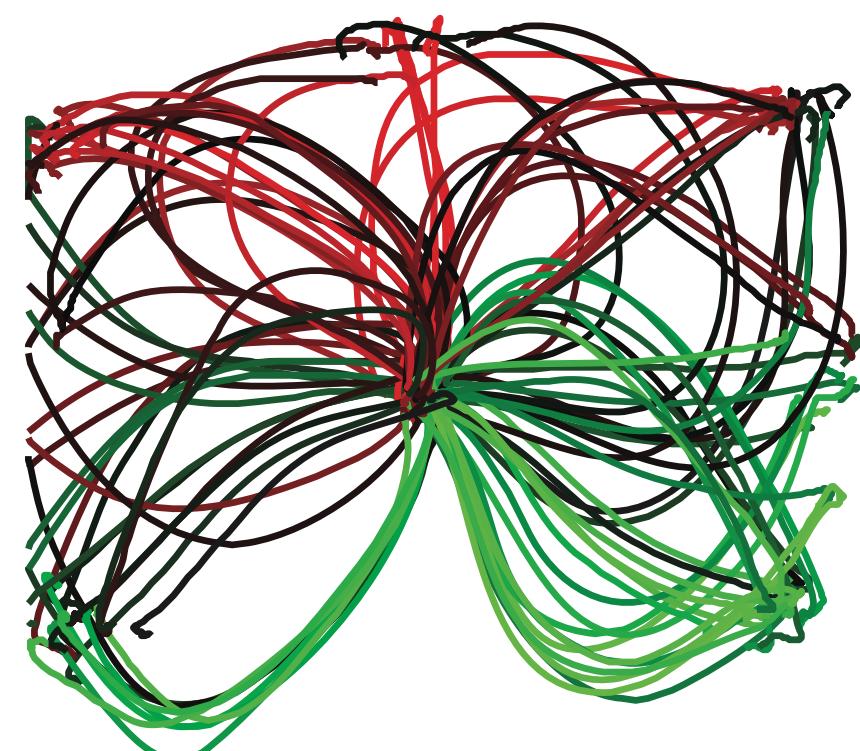


Reach
trajectories

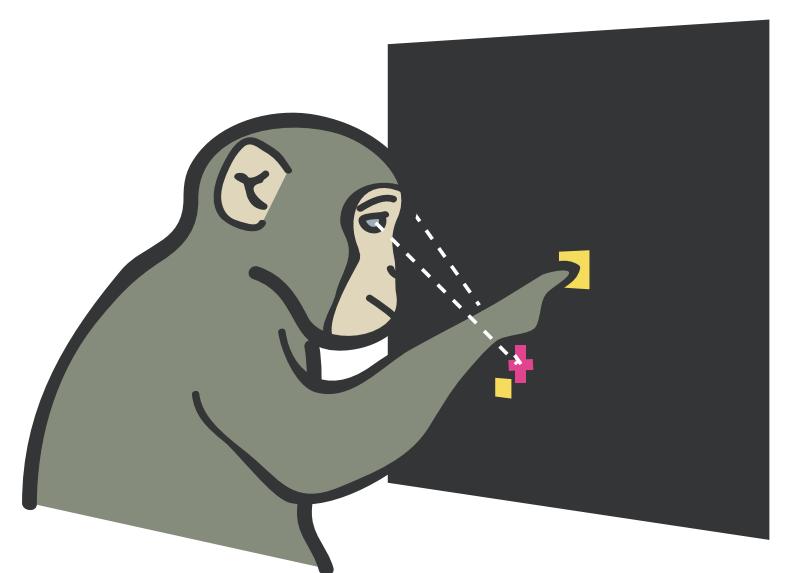




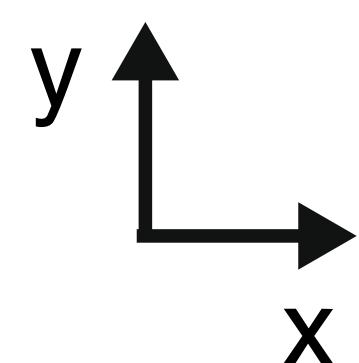
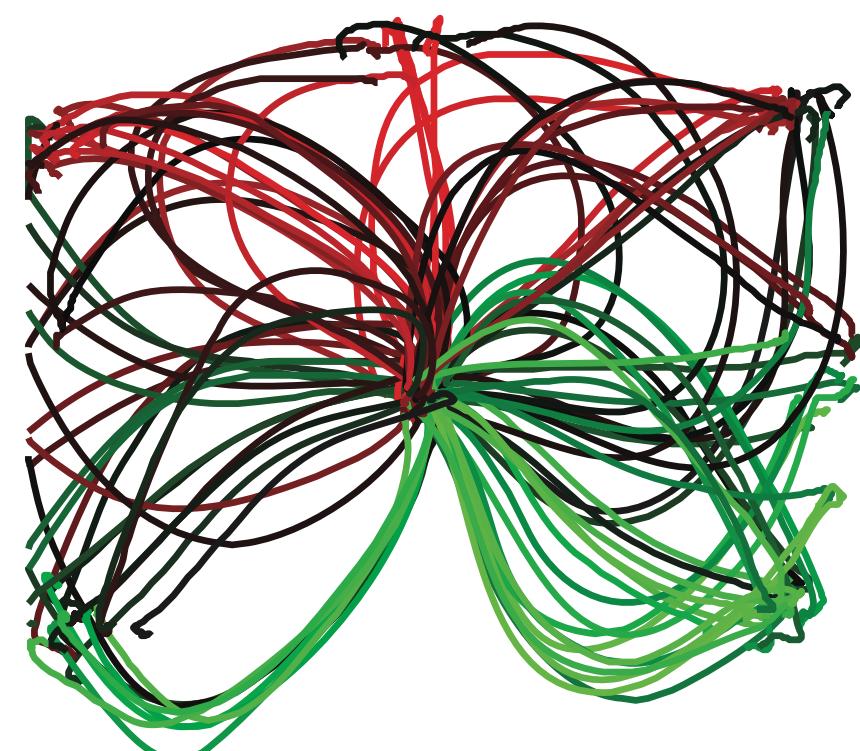
Reach
trajectories



Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

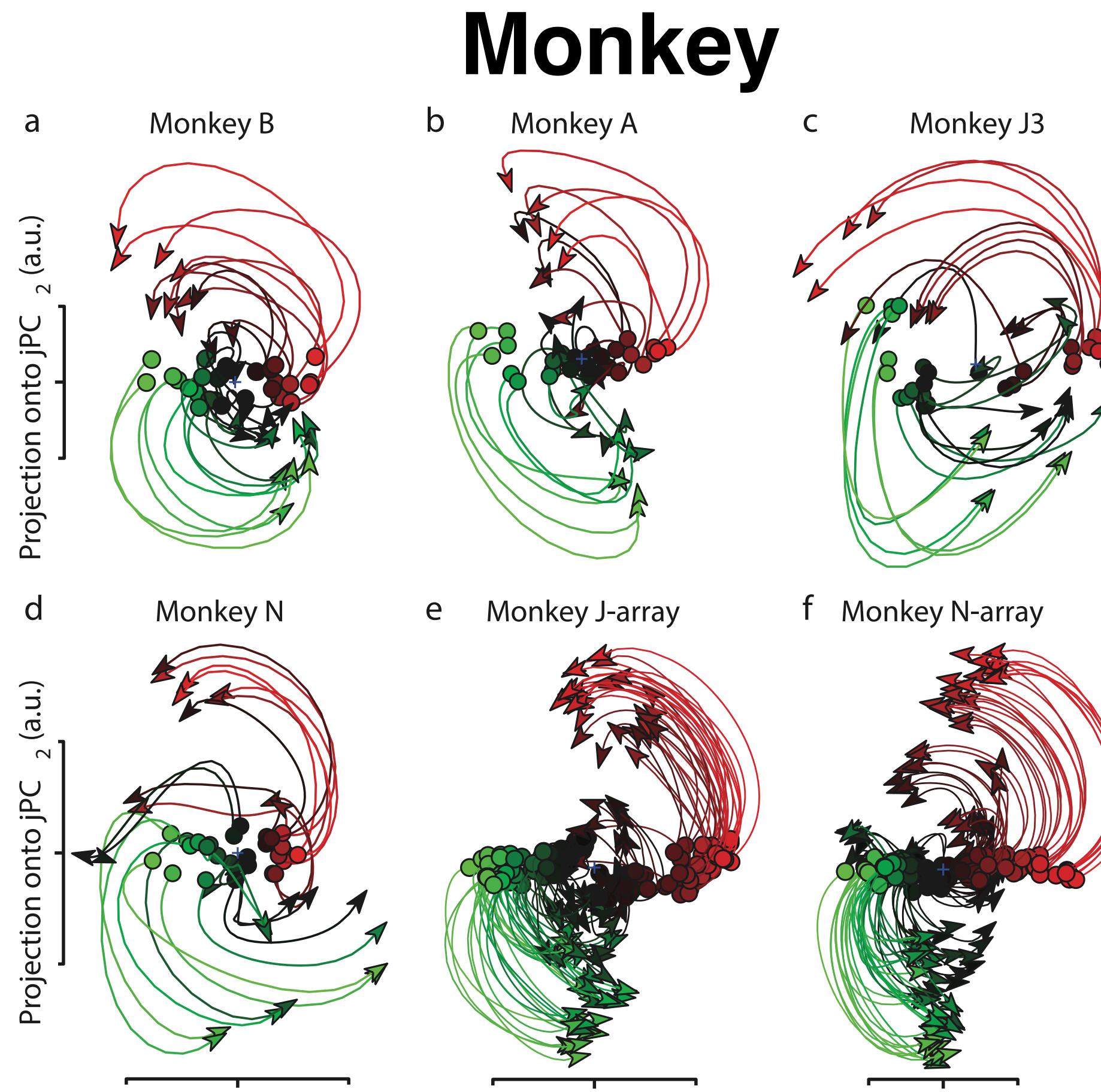


Reach
trajectories



Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

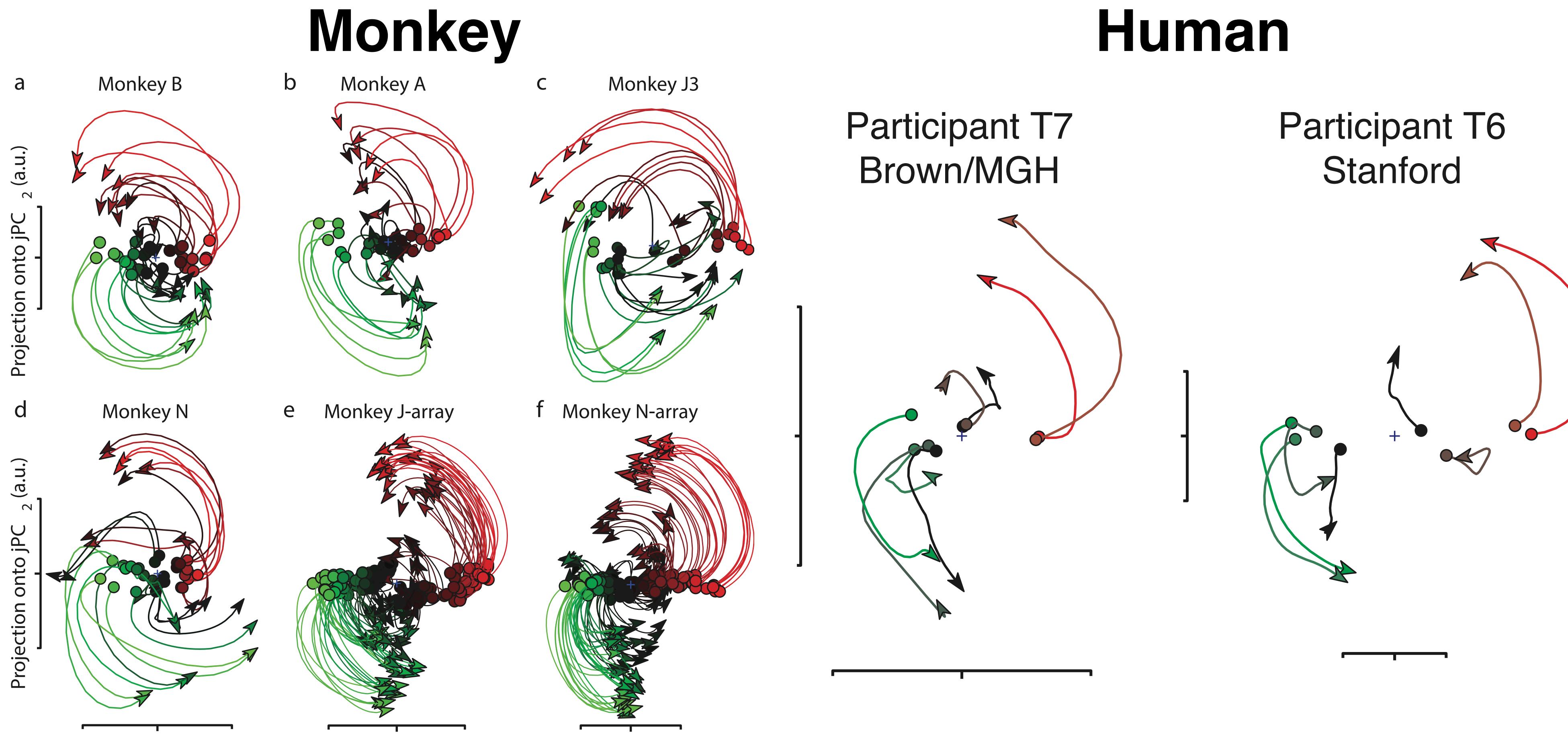
Rotational dynamics in monkeys & humans



Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

Pandarinath, ... Hochberg, Henderson*, Shenoy*, *eLife* (2015)

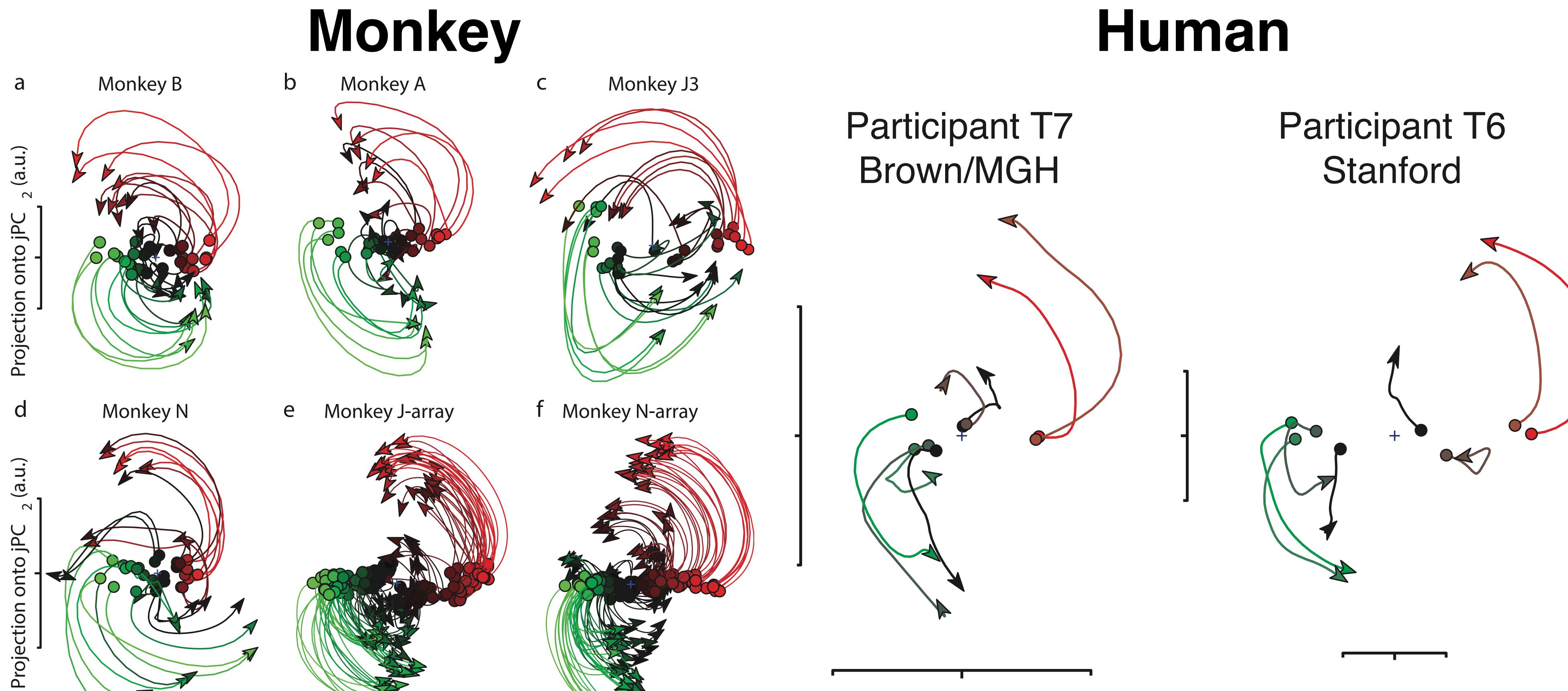
Rotational dynamics in monkeys & humans



Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

Pandarinath, ... Hochberg, Henderson*, Shenoy*, *eLife* (2015)

Rotational dynamics in monkeys & humans



Churchland*, Cunningham* ... Shenoy, *Nature* (2012)

Pandarinath, ... Hochberg, Henderson*, Shenoy*, *eLife* (2015)

What you'll hear about in this lecture

Neural network basics

Deep autoencoders

Intro to neural population dynamics