

Homework DM Functions

March 10, 2016

1

Let $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c, d, e\}$. Define $g : X \rightarrow Y$ as follows:
 $g(1) = a, g(2) = a, g(3) = a, g(4) = d$

1. Draw an arrow diagram for g .
2. Let $A = \{2, 3\}$, $C = \{a\}$, $D = \{b, c\}$. Find $g(A)$, $g(X)$, $g^{-1}(C)$, $g^{-1}(D)$, $g^{-1}(Y)$

2

Let X and Y be any sets, $A \subset X$, $B \subset X$, $C \subset Y$, $D \subset Y$.
Is the following formula:

1. $F(A \cap B) \subseteq F(A) \cap F(B)$
2. $F(F^{-1}(C)) \subseteq C$

true for all functions F from X to Y ? Justify your answer.

3

Define $F : Z^+ \times Z^+ \rightarrow Z^+$ and $G : Z^+ \times Z^+ \rightarrow Z^+$ as follows:
For all $(n, m) \in Z^+ \times Z^+$

$$F(n, m) = 3^n 5^m \quad G(n, m) = 3^n 6^m$$

1. Prove or disprove that F and G are one-to-one functions.
2. Prove or disprove that F and G are onto functions.

4

Suppose $f : Z \rightarrow Y$ and $g : X \rightarrow Z$ are both one-to-one and onto. Prove that $(f \circ g)^{-1}$ exists and that $(f \circ g)^{-1} = f^{-1} \circ g^{-1}$.

5

Suppose $F : X \rightarrow Y$ is one-to-one.

1. Prove that $\forall A \subseteq X, \quad F^{-1}(F(A)) = A$
2. Prove that $\forall A_1 \subseteq X, \forall A_2 \subseteq X, \quad F(A_1 \cap A_2) = F(A_1) \cap F(A_2)$

6

A factorial $n!$, $n \in \mathbb{Z}$ can be decomposed into its canonical form:

$$n! = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3} \cdot \dots \cdot m^{a_i} \cdot \dots$$

where p_i, m are prime numbers and $m < n$. For example:

$$20! = 2^{18} \cdot 3^8 \cdot 5^4 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19$$

Your task is to write a script with a function $F(n, m) = \sum_{i=1}^{m^i < n} \left[\frac{n}{m^i} \right]$ that calculates a_i , which is a power of a prime factor m . It should work fast and with relatively big numbers (for example $n = 1000$, so you should count the a_i value in $1000!$).