

Example: Heat loss through a composite wall

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A 3 m high and 5 m wide wall consists of long 32 cm 22 cm cross section horizontal bricks ($k = 0.72 \text{ W/m} \cdot ^\circ\text{C}$) separated by 3 cm thick plaster layers ($k = 0.22 \text{ W/m} \cdot ^\circ\text{C}$). There are also 2 cm thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$) on the inner side of the wall. The indoor and the outdoor temperatures are 20°C and 10°C , and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_2 = 25 \text{ W/m}^2 \cdot ^\circ\text{C}$, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

Answer:

$$A = (0.015 + 0.22 + 0.015) \times 1 = 0.25 \text{ m}^2$$

$$R_{1, \text{conv.}} = \frac{1}{h_1 A_1} = \frac{1}{10 \times 0.25} = 0.4 \frac{\text{C}}{\text{W}}$$

$$R_f = \frac{L_f}{K_f A_1 - \text{dimen}} = \frac{0.03}{0.026 \times 0.25} = 4.615 \frac{\text{C}}{\text{W}}$$

$$R_{p1} = R_{p2} = \frac{L_p}{K_p \times A_p} = \frac{0.02}{0.22 \times 0.15 \times 1} = 96.97 \frac{\text{C}}{\text{W}} \quad (\text{p: 2cm thick plastic layers})$$

$$R_b = \frac{L_b}{K_b \times A_b} = \frac{0.32}{0.72 \times 0.22 \times 1} = 2.02 \frac{\text{C}}{\text{W}}$$

$$\frac{1}{R_{\text{total, parallel}}} = \frac{1}{96.97} + \frac{1}{2.02} + \frac{1}{96.97} = 0.516 \frac{\text{C}}{\text{W}} \quad (\text{Brick and two plastic layers})$$

$$R_{\text{total parallel}} = \frac{1}{0.516} = 1.94 \frac{\text{C}}{\text{W}}$$

$$R_{p1} = R_{p2} = \frac{L_p}{K_p \times A_p} = \frac{0.02}{0.22 \times 0.25 \times 1} = 0.363 \frac{\text{C}}{\text{W}} \quad (\text{p: 2cm thick plastic layers})$$

$$R_{2, \text{conv.}} = \frac{1}{h_2 \times A_1} = \frac{1}{40 \times 0.25} = 0.1 \frac{\text{C}}{\text{W}}$$

$$R_{\text{wall total}} = 0.4 + 4.615 + 0.363 + 1.94 + 0.363 + 0.1 = 7.781 \frac{\text{C}}{\text{W}}$$

The Rate of heat transfer loss: $\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{wall total}}} = \frac{20 - (-10)}{7.781} = 3.86 \text{ W}$

But $R_{\text{wall total}}$ when Thickness of brick = 16 mm

$$= 6.81 \frac{\text{C}}{\text{W}}$$

SO, the heat transfer rate is:

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{wall total}}} = \frac{20 - (-10)}{6.81} = 4.41 \text{ W}$$

So By comparing the two results 4.41w and 3.86w, we found that: double the thickness of a brick inside a composite wall increases slightly the thermal resistance of the wall, so the rate of heat transfer slightly decreases.