

Week 8

Task 1 Using the diagrams given in the presentation calculate how much (%) is the effect of applying different modifications (changing the gas, adding an extra pane, using a low emissivity coating) on the U value with respect to a benchmark case of double layer with air and no coating ? (keep the gap thickness to be 13 mm)

Answer:

The U value of window:

$$U_{\text{window}} = (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}}) / A_{\text{window}}$$

Disregarding the thermal resistances of glass layers, the thermal resistance and U-factor of a double-pane window can be expressed as:

$$\frac{1}{U_{\text{double-pane(centerregion)}}} \cong \frac{1}{h_i} + \frac{1}{h_{\text{space}}} + \frac{1}{h_o}$$

$$h_{\text{space}} = h_{\text{rad,space}} + h_{\text{conv,space}}$$

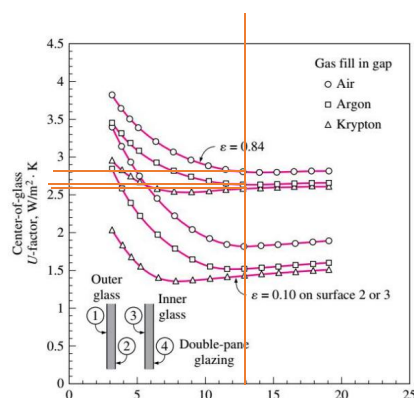
Note from the formula, the U value is related with h space, roughly half of the heat transfer through the air space of a double-pane window is by radiation and the other half is by conduction (or convection, if there is any air motion). Therefore, there are two ways to minimize h space and thus the rate of heat transfer through a double-pane window:

- 1) Minimize radiation heat transfer through the air space.
- 2) Minimize conduction heat transfer through air space.

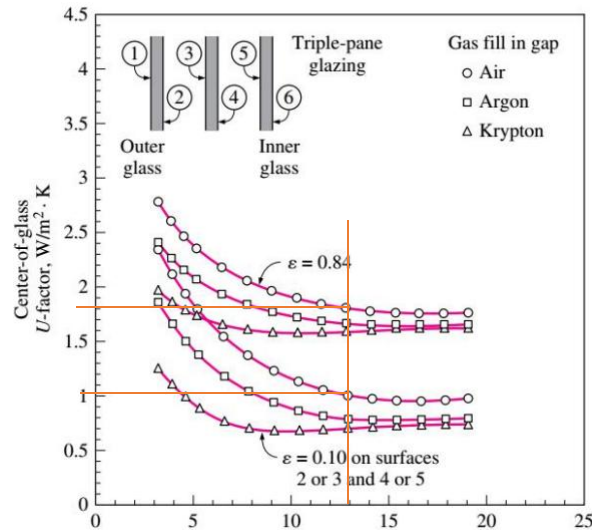
According to different modifications:

1. **changing the gas:** to use a less-conducting fluid such as argon or krypton to fill the gap between the glasses instead of air is a way of reducing conduction heat transfer through a double-pane window.

Note from the figure that when the thickness of gas is 13mm, if we change the air into less-conducting fluid like **Argon**, the value of U-FACTOR changes from **2.8W/m² •k** into **2.65W/m² •k**, which means the value decrease by **6.43%**. And if we change the air into less-conducting fluid like **Krypton**, the value of U-FACTOR changes from **2.8W/m² •k** into **2.6 W/m² •k**, which means the value of U_factor decrease by **7.14%**.



2. **adding an extra pane:** Note from the figure that when the thickness of air gap is 13mm and filled gas is air, the U-factor in triple-panes glazing is $1.8 \text{ W/m}^2 \cdot \text{K}$. So from the double-pane to triple-pane, the value of U changes from $2.8 \text{ W/m}^2 \cdot \text{K}$ into $1.8 \text{ W/m}^2 \cdot \text{K}$, which means the value decreases by 55.6%, about one-third.



3. **using a low emissivity coating:** By coating glass surface with low-emissivity material can reduce the emissivity of them. Recall that the effective emissivity of two parallel plates of emissivities ϵ_1 and ϵ_2 is given by:

$$\epsilon_{\text{effective}} = \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

The emissivity of an ordinary glass surface is 0.84.

$$\epsilon_{\text{effective}} = \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1} = \frac{1}{1/0.84 + 1/0.84 - 1} \approx 0.72$$

Therefore, the effective emissivity of two parallel glass surfaces facing each other is 0.72.

But when the glass surfaces are coated with a film that has an emissivity of 0.1,

$$\epsilon_{\text{effective}} = \frac{1}{1/\epsilon_{\text{coat1}} + 1/\epsilon_{\text{coat2}} - 1} = \frac{1}{1/0.1 + 1/0.1 - 1} \approx 0.05$$

the effective emissivity reduces to 0.05, which is **one-fourteenth of 0.72**. Using a low-emissivity coating, the value of U changes from $1.8 \text{ W/m}^2 \cdot \text{K}$ into $1.0 \text{ W/m}^2 \cdot \text{K}$, which means the value decreases by 44.4%.

we are assuming that there is no internal or external shading!

Table 10 Peak Irradiance, W/m²

Exposure		Latitude								
		20°	25°	30°	35°	40°	45°	50°	55°	60°
North	E_D	125	106	92	84	81	85	96	112	136
	E_d	128	115	103	93	84	76	69	62	55
	E_t	253	221	195	177	166	162	164	174	191
Northeast/Northwest	E_D	460	449	437	425	412	399	386	374	361
	E_d	177	169	162	156	151	147	143	140	137
	E_t	637	618	599	581	563	546	529	513	498
East/West	E_D	530	543	552	558	560	559 555	547	537	
	E_d	200	196	193	190	189	188 187	187	187	
	E_t	730	739	745	748	749	747	742	734	724
Southeast/Southwest	E_D	282	328	369	405	436	463	485	503	517
	E_d	204	203	203	204	205	207	210	212	215
	E_t	485	531	572	609	641	670	695	715	732
South	E_D	0	60	139	214	283	348	408	464	515
	E_d	166	193	196	200	204	209	214	219	225
	E_t	166	253	335	414	487	557	622	683	740
Horizontal	E_D	845	840	827	806	776	738	691	637	574
	E_d	170	170	170	170	170	170	170	170	170
	E_t	1015	1010	997	976	946	908	861	807	744

Exposure	Single Family	Detached	Multifamily
North	0.44		0.27
Northeast	0.21		0.43
East	0.31		0.56
Southeast	0.37		0.54
South	0.47		0.53
Southwest	0.58		0.61
West	0.56		0.65
Northwest	0.46		0.57
Horizontal	0.58		0.73

$$PXI_{\text{window}_{\text{east}}} = E_D + E_d = 559 + 188 = 747$$

$$SHGC = 0.54$$

$$IAC = 1$$

$$FFs = 0.31$$

$$CF_{\text{window}_{\text{east}} \text{irradiationPart}} = PXI \times SHGC \times IAC \times FFs = 747 * 0.54 * 1 * 0.31 = 125.1$$

$$CF_{\text{window}_{\text{east}}} = CF_{\text{window}_{\text{east}} \text{heatTransferPart}} + CF_{\text{window}_{\text{east}} \text{irradiationPart}} = 6.9 + 125.1 = 132 W / m^2$$

$$Q_{\text{window}_{\text{east}}} = CF_{\text{window}_{\text{east}}} \times A_{\text{window}_{\text{east}}} = 132 * 14.4 = 1900.8 W$$

NOW, I change the frame of the window from wooden one to ALUMINIUM :

Glazing Type	Glazing Layers	ID ^b	Property ^{c,d}	Center of Glazing	Frame									
					Operable					Fixed				
					Aluminum	Aluminum with Thermal Break	Reinforced Vinyl/Aluminum Clad Wood	Wood/Vinyl	Insulated Fiberglass/Vinyl	Aluminum	Aluminum with Thermal Break	Reinforced Vinyl/Aluminum Clad Wood	Wood/Vinyl	Insulated Fiberglass/Vinyl
Clear	1	1a	U	5.91	7.24	6.12	5.14	5.05	4.61	6.42	6.07	5.55	5.55	5.35
			SHGC	0.86	0.75	0.75	0.64	0.64	0.64	0.78	0.78	0.75	0.75	0.75
	2	5a	U	2.73	4.62	3.42	3.00	2.87	5.83	3.61	3.22	2.86	2.84	2.72
			SHGC	0.76	0.67	0.67	0.57	0.57	0.57	0.69	0.69	0.67	0.67	0.67
	3	29a	U	1.76	3.80	2.60	2.25	2.19	1.91	2.76	2.39	2.05	2.01	1.93
			SHGC	0.68	0.60	0.60	0.51	0.51	0.51	0.62	0.62	0.60	0.60	0.60
Low-e, low-solar	2	25a	U	1.70	3.83	2.68	2.33	2.21	1.89	2.75	2.36	2.03	2.01	1.90
			SHGC	0.41	0.37	0.37	0.31	0.31	0.31	0.38	0.38	0.36	0.36	0.36
	3	40c	U	1.02	3.22	2.07	1.76	1.71	1.45	2.13	1.76	1.44	1.40	1.33
			SHGC	0.27	0.25	0.25	0.21	0.21	0.21	0.25	0.25	0.24	0.24	0.24
Low-e, high-solar	2	17c	U	1.99	4.05	2.89	2.52	2.39	2.07	2.99	2.60	2.26	2.24	2.13
			SHGC	0.70	0.62	0.62	0.52	0.52	0.64	0.64	0.61	0.61	0.61	0.61
	3	32c	U	1.42	3.54	2.36	2.02	1.97	1.70	2.47	2.10	1.77	1.73	1.66
			SHGC	0.62	0.55	0.55	0.46	0.46	0.46	0.56	0.56	0.54	0.54	0.54
Heat-absorbing	1	1c	U	5.91	7.24	6.12	5.14	5.05	4.61	6.42	6.07	5.55	5.55	5.35
			SHGC	0.73	0.64	0.64	0.54	0.54	0.54	0.66	0.66	0.64	0.64	0.64
	2	5c	U	2.73	4.62	3.42	3.00	2.87	2.53	3.61	3.22	2.86	2.84	2.72
			SHGC	0.62	0.55	0.55	0.46	0.46	0.46	0.56	0.56	0.54	0.54	0.54
	3	29c	U	1.76	3.80	2.60	2.25	2.19	1.91	2.76	2.39	2.05	2.01	1.93
			SHGC	0.34	0.31	0.31	0.26	0.26	0.26	0.31	0.31	0.30	0.30	0.30
Reflective	1	1l	U	5.91	7.24	6.12	5.14	5.05	4.61	6.42	6.07	5.55	5.55	5.35
			SHGC	0.31	0.28	0.28	0.24	0.24	0.24	0.29	0.29	0.27	0.27	0.27
	2	5p	U	2.73	4.62	3.42	3.00	2.87	2.53	3.61	3.22	2.86	2.84	2.72
			SHGC	0.29	0.27	0.27	0.22	0.22	0.22	0.27	0.27	0.26	0.26	0.26
	3	29c	U	1.76	3.80	2.60	2.25	2.19	1.91	2.76	2.39	2.05	2.01	1.93
			SHGC	0.34	0.31	0.31	0.26	0.26	0.26	0.31	0.31	0.30	0.30	0.30

HEATING:

$$A_{\text{east}} = 14.4$$

$$U'_{\text{window}_{\text{east}}} = 3.61 W / m^2 \cdot K$$

$$HF'_{\text{window}_{\text{east}}} = U'_{\text{window}_{\text{east}}} \times \Delta T_{\text{heating}} = 3.61 \times 24.8 = 89.5 W / m^2 \cdot K$$

$$Q'_{\text{window}_{\text{east}}} = HF'_{\text{window}_{\text{east}}} \times A_{\text{window}_{\text{east}}} = 89.5 \times 14.4 = 1289.2 W$$

COOLING:

$$CF'_{\text{window}_{\text{east}} \text{heatTransferPart}} = U'_{\text{window}_{\text{east}}} (\Delta T_{\text{cooling}} - 0.46DR) = 3.61 * (7.9 - 0.46 * 11.9) = 8.7 W / m^2$$

$$PXI_{\text{window}_{\text{east}}} = E_D + E_d = 559 + 188 = 747$$

$$SHGC' = 0.56$$

$$IAC = 1$$

$$FFs = 0.31$$

$$CF'_{\text{window}_{\text{east}} \text{irradiationPart}} = PXI \times SHGC' \times IAC \times FFs = 747 * 0.56 * 1 * 0.31 = 129.6$$

$$CF'_{\text{window}_{\text{east}}} = CF'_{\text{window}_{\text{east}} \text{heatTransferPart}} + CF'_{\text{window}_{\text{east}} \text{irradiationPart}} = 8.7 + 129.6 = 138.3 W / m^2$$

$$Q'_{\text{window}_{\text{east}}} = CF'_{\text{window}_{\text{east}}} \times A_{\text{window}_{\text{east}}} = 138.3 * 14.4 = 1991.5 W$$

● **WINDOW2: WEST (14.4 m², fixed, wooden frame)**

HEATING:

$$A_{\text{west}} = 14.4$$

$$U_{\text{window}_{\text{west}}} = 2.84 W / m^2 \cdot K$$

$$HF_{\text{window}_{\text{west}}} = U_{\text{window}_{\text{west}}} \times \Delta T_{\text{heating}} = 2.84 \times 24.8 = 70.4 W / m^2 \cdot K$$

$$Q_{\text{window}_{\text{west}}} = HF_{\text{window}_{\text{west}}} \times A_{\text{window}_{\text{west}}} = 70.4 \times 14.4 = 1014.2 W$$

COOLING:

$$CF_{\text{window}_{\text{west}} \text{heatTransferPart}} = U_{\text{window}_{\text{west}}} (\Delta T_{\text{cooling}} - 0.46DR) = 2.84 * (7.9 - 0.46 * 11.9) = 6.9 W / m^2$$

we are assuming that there is no internal or external shading!

$$PXI_{\text{window}_{\text{east}}} = E_D + E_d = 559 + 188 = 747$$

$$SHGC = 0.54$$

$$IAC = 1$$

$$FFs = 0.56$$

$$CF_{\text{window}_{\text{west}} \text{irradiationPart}} = PXI \times SHGC \times IAC \times FFs = 747 * 0.54 * 1 * 0.56 = 225.9 W / m^2$$

$$CF_{\text{window}_{\text{west}}} = CF_{\text{window}_{\text{west}} \text{heatTransferPart}} + CF_{\text{window}_{\text{west}} \text{irradiationPart}} = 6.9 + 225.9 = 232.8 W / m^2$$

$$Q_{\text{window}_{\text{west}}} = CF_{\text{window}_{\text{west}}} \times A_{\text{window}_{\text{west}}} = 232.8 * 14.4 = 3352.3 W$$

NOW, I change the frame of the window from wooden one to ALUMINIUM :

HEATING:

$$A_{\text{west}} = 14.4$$

$$U'_{\text{window}_{\text{west}}} = 3.61 W / m^2 \cdot K$$

$$HF'_{\text{window}_{\text{west}}} = U'_{\text{window}_{\text{west}}} \times \Delta T_{\text{heating}} = 3.61 \times 24.8 = 89.52 W / m^2$$

$$Q'_{\text{window}_{\text{west}}} = HF'_{\text{window}_{\text{west}}} \times A_{\text{window}_{\text{west}}} = 89.52 \times 14.4 = 1289.1 W$$

COOLING:

$$CF'_{\text{window}_{\text{west}} \text{heatTransferPart}} = U'_{\text{window}_{\text{west}}} (\Delta T_{\text{cooling}} - 0.46DR) = 3.61 * (7.9 - 0.46 * 11.9) = 8.7W / m^2$$

$$PXI_{\text{window}_{\text{east}}} = E_D + E_d = 559 + 188 = 747$$

$$SHGC' = 0.56$$

$$IAC = 1$$

$$FFs = 0.56$$

$$CF'_{\text{window}_{\text{west}} \text{irradiationPart}} = PXI \times SHGC' \times IAC \times FFs = 747 * 0.56 * 1 * 0.56 = 234.26W / m^2$$

$$CF'_{\text{window}_{\text{west}}} = CF'_{\text{window}_{\text{west}} \text{heatTransferPart}} + CF'_{\text{window}_{\text{west}} \text{irradiationPart}} = 8.7 + 234.26 = 242.96W / m^2$$

$$Q'_{\text{window}_{\text{west}}} = CF'_{\text{window}_{\text{west}}} \times A_{\text{window}_{\text{west}}} = 242.96 * 14.4 = 3498.6W$$

● **WINDOW3: SOUTH (3.6 m², fixed; 3.6 m² operable, wooden frame)**

HEATING:

$$A_{\text{south}_{\text{fixed}}} = A_{\text{south}_{\text{operable}}} = 3.6$$

$$U_{\text{window}_{\text{south}_{\text{fixed}}}} = 2.84W/m^2 \cdot K$$

$$U_{\text{window}_{\text{south}_{\text{operable}}}} = 2.87W/m^2 \cdot K$$

$$HF_{\text{window}_{\text{south}_{\text{fixed}}}} = U_{\text{window}_{\text{south}_{\text{fixed}}}} \times \Delta T_{\text{heating}} = 2.84 \times 24.8 = 70.4W/m^2$$

$$HF_{\text{window}_{\text{south}_{\text{operable}}}} = U_{\text{window}_{\text{south}_{\text{operable}}}} \times \Delta T_{\text{heating}} = 2.87 \times 24.8 = 71.17W/m^2$$

$$Q_{\text{window}_{\text{south}_{\text{fixed}}}} = HF_{\text{window}_{\text{south}_{\text{fixed}}}} \times A_{\text{window}_{\text{south}_{\text{fixed}}}} = 70.4 \times 3.6 = 253.6W$$

$$Q_{\text{window}_{\text{south}_{\text{operable}}}} = HF_{\text{window}_{\text{south}_{\text{operable}}}} \times A_{\text{window}_{\text{south}_{\text{operable}}}} = 71.17 \times 3.6 = 256.2W$$

$$Q_{\text{window}_{\text{south}}} = Q_{\text{window}_{\text{south}_{\text{fixed}}}} + Q_{\text{window}_{\text{south}_{\text{operable}}}} = 509.8W$$

COOLING:

$$CF_{\text{window}_{\text{south}_{\text{fixed}} \text{heatTransferPart}}} = U_{\text{window}_{\text{south}_{\text{fixed}}}} (\Delta T_{\text{cooling}} - 0.46DR) = 2.84 * (7.9 - 0.46 * 11.9) = 6.9W / m^2$$

$$CF_{\text{window}_{\text{south}_{\text{operable}} \text{heatTransferPart}}} = U_{\text{window}_{\text{south}_{\text{operable}}}} (\Delta T_{\text{cooling}} - 0.46DR) = 2.87 * (7.9 - 0.46 * 11.9) = 6.96W / m^2$$

$$PXI_{\text{window}_{\text{south}}} = E_D + E_d = 348 + 209 = 557$$

$$SHGC_{\text{fixed}} = 0.54$$

$$SHGC_{\text{operable}} = 0.46$$

$$IAC = 1$$

$$FFs = 0.47$$

$$CF_{\text{window}_{\text{southfixed}} \text{irradiationPart}} = PXI \times SHGC_{\text{fixed}} \times IAC \times FFs = 557 * 0.54 * 1 * 0.47 = 120.4 W / m^2$$

$$CF_{\text{window}_{\text{southsouth}} \text{irradiationPart}} = PXI \times SHGC_{\text{operable}} \times IAC \times FFs = 557 * 0.46 * 1 * 0.47 = 141.4 W / m^2$$

$$CF_{\text{window}_{\text{southfixed}}} = CF_{\text{window}_{\text{southfixed}} \text{heatTransferPart}} + CF_{\text{window}_{\text{southfixed}} \text{irradiationPart}} = 6.9 + 120.4 = 127.3 W / m^2$$

$$CF_{\text{window}_{\text{southoperable}}} = CF_{\text{window}_{\text{southoperable}} \text{heatTransferPart}} + CF_{\text{window}_{\text{southoperable}} \text{irradiationPart}} = 6.96 + 141.4 = 148.36 W / m^2$$

$$Q_{\text{window}_{\text{southfixed}}} = CF_{\text{window}_{\text{southfixed}}} \times A_{\text{window}_{\text{southfixed}}} = 127.3 * 3.6 = 458.28 W$$

$$Q_{\text{window}_{\text{southoperable}}} = CF_{\text{window}_{\text{southoperable}}} \times A_{\text{window}_{\text{southoperable}}} = 148.3 * 3.6 = 533.88 W$$

$$Q_{\text{window}_{\text{south}}} = Q_{\text{window}_{\text{southoperable}}} + Q_{\text{window}_{\text{southfixed}}} = 533.88 + 458.28 = 992.16 W$$

NOW, I change the frame of the window from wooden one to ALUMINIUM :

HEATING:

$$A_{\text{south}_{\text{fixed}}} = A_{\text{south}_{\text{operable}}} = 3.6$$

$$U'_{\text{window}_{\text{southfixed}}} = 3.61 W / m^2 \cdot K$$

$$U'_{\text{window}_{\text{southoperable}}} = 4.62 W / m^2 \cdot K$$

$$HF'_{\text{window}_{\text{southfixed}}} = U'_{\text{window}_{\text{southfixed}}} \times \Delta T_{\text{heating}} = 3.61 \times 24.8 = 89.5 W / m^2$$

$$HF'_{\text{window}_{\text{southoperable}}} = U'_{\text{window}_{\text{southoperable}}} \times \Delta T_{\text{heating}} = 4.62 \times 24.8 = 114.58 W / m^2$$

$$Q'_{\text{window}_{\text{southfixed}}} = HF'_{\text{window}_{\text{southfixed}}} \times A_{\text{window}_{\text{southfixed}}} = 89.6 \times 3.6 = 322.56 W$$

$$Q'_{\text{window}_{\text{southoperable}}} = HF'_{\text{window}_{\text{southoperable}}} \times A_{\text{window}_{\text{southoperable}}} = 114.58 \times 3.6 = 411.488 W$$

$$Q'_{\text{window}_{\text{south}}} = Q'_{\text{window}_{\text{southfixed}}} + Q'_{\text{window}_{\text{southoperable}}} = 735.05 W$$

COOLING:

$$CF'_{\text{window}_{\text{southfixed}} \text{heatTransferPart}} = U'_{\text{window}_{\text{southfixed}}} (\Delta T_{\text{cooling}} - 0.46 DR) = 3.61 * (7.9 - 0.46 * 11.9) = 8.76 W / m^2$$

$$CF'_{\text{window}_{\text{southoperable}} \text{heatTransferPart}} = U'_{\text{window}_{\text{southoperable}}} (\Delta T_{\text{cooling}} - 0.46 DR) = 4.62 * (7.9 - 0.46 * 11.9) = 11.2 W / m^2$$

$$PXI_{\text{window}_{\text{south}}} = E_D + E_d = 348 + 209 = 557$$

$$SHGC'_{\text{fixed}} = 0.56$$

$$SHGC'_{\text{operable}} = 0.55$$

$$IAC = 1$$

$$FFs = 0.47$$

$$CF'_{\text{window}_{\text{southfixed}} \text{irradiationPart}} = PXI \times SHGC'_{\text{fixed}} \times IAC \times FFs = 557 * 0.56 * 1 * 0.47 = 146.6 W / m^2$$

$$CF'_{\text{window}_{\text{southsouth}} \text{irradiationPart}} = PXI \times SHGC'_{\text{operable}} \times IAC \times FFs = 557 * 0.55 * 1 * 0.47 = 143.98 W / m^2$$

$$CF'_{\text{window}_{\text{southfixed}}} = CF'_{\text{window}_{\text{southfixed}} \text{heatTransferPart}} + CF'_{\text{window}_{\text{southfixed}} \text{irradiationPart}} = 8.76 + 146.6 = 155.36 W / m^2$$

$$CF'_{\text{window}_{\text{southoperable}}} = CF'_{\text{window}_{\text{southoperable}} \text{heatTransferPart}} + CF'_{\text{window}_{\text{southoperable}} \text{irradiationPart}} = 11.2 + 143.98 = 155.18 W / m^2$$

$$Q'_{\text{window}_{\text{southfixed}}} = CF'_{\text{window}_{\text{southfixed}}} \times A_{\text{window}_{\text{southfixed}}} = 155.36 * 3.6 = 559.296 W$$

$$Q'_{\text{window}_{\text{southoperable}}} = CF'_{\text{window}_{\text{southoperable}}} \times A_{\text{window}_{\text{southoperable}}} = 155.18 * 3.6 = 558.648 W$$

$$Q'_{\text{window}_{\text{south}}} = Q'_{\text{window}_{\text{southoperable}}} + Q'_{\text{window}_{\text{southfixed}}} = 559.296 + 558.648 = 1117.944 W$$

SO,

HEATING:

$$Q_{TOTAL \text{ HEATING-WOODEN}} = Q_{\text{window}_{\text{east}}} + Q_{\text{window}_{\text{west}}} + Q_{\text{window}_{\text{south}}} = 1014.2 + 1014.2 + 509.8 = 2538.2 W$$

$$Q_{TOTAL \text{ HEATING-ALUMINUM}} = Q'_{\text{window}_{\text{east}}} + Q'_{\text{window}_{\text{west}}} + Q'_{\text{window}_{\text{south}}} = 1289.2 + 1289.1 + 735.05 = 3321.8 W$$

COOLING:

$$Q_{TOTAL \text{ COOLING-WOODEN}} = Q_{\text{window}_{\text{east}}} + Q_{\text{window}_{\text{west}}} + Q_{\text{window}_{\text{south}}} = 1900.8 + 3352.3 + 992.16 = 6245.26 W$$

$$Q_{TOTAL \text{ COOLING-ALUMINUM}} = Q'_{\text{window}_{\text{east}}} + Q'_{\text{window}_{\text{west}}} + Q'_{\text{window}_{\text{south}}} = 1991.5 + 3498.6 + 1117.944 = 6608.044 W$$