Queen's University Belfast

ELE8088: Control & Estimation Theory

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Lab I - Assignment

Question 1 [Optimisation problem]. (i)This paper chooses the machine epsilon [1] to be a reasonable threshold, $\epsilon > 0$, below which a number is considered to be practically zero. That is, x is considered to be (practically) equal to zero if $\|x\| < \epsilon$. The Python code is: Question 1. The figure of $\mathrm{sp}(x^\star(\mu))$ against μ for $0 \leq \mu \leq \mu_{\max}$ is shown below as Figure 1:

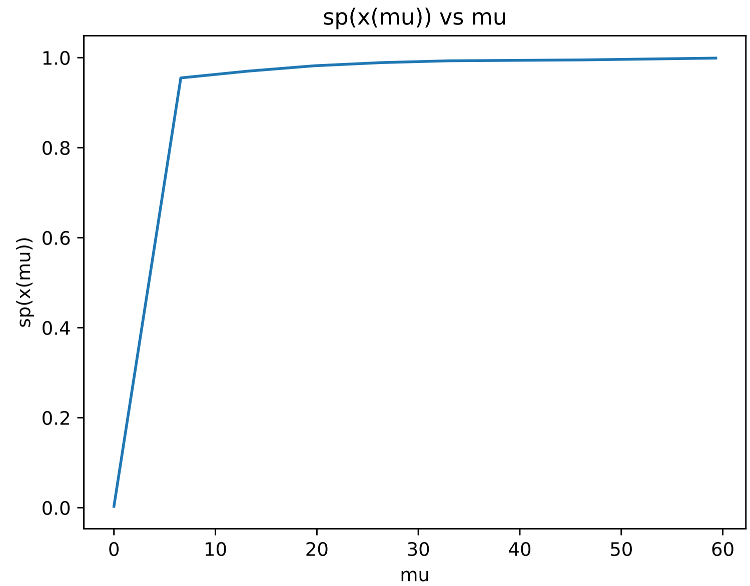


Figure 1. $\mathrm{sp}(x^\star(\mu))$ against μ for $0 \leq \mu \leq \mu_{\max}$

(ii)The figure of $\|Ax^\star(\mu) - b\|$ against μ for $0 \leq \mu \leq \mu_{\max}$ is shown below as Figure 2:

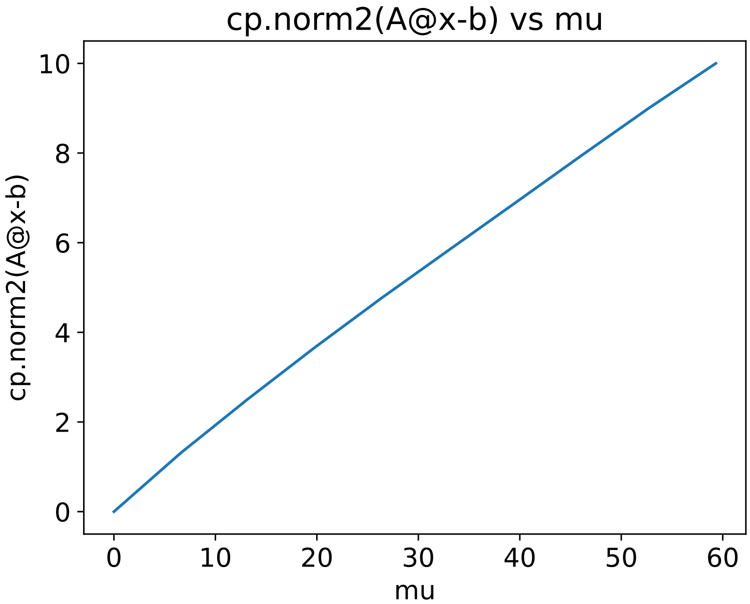


Figure 2. $\|Ax^\star(\mu) - b\|$ against μ for $0 \leq \mu \leq \mu_{\max}$

(iii) The Figure 1 shows that the the sparsness of $x^\star(\mu)$ increases rapidly with μ increasing in the beginning, and when μ more than a value which is approximately 8, the $\mathrm{sp}(x^\star(\mu))$ increases very slowly. And the Figure 2 shows that the $\|Ax^\star(\mu)-b\|$ and μ can be approximately regarded as a linear relationship.

Question 2 [IHOCP and DARE]. (i)The Python code is: Question 2. P_t for $t \in \mathbb{N}_{[1,5]}$ is shown below as P_1, P_2, P_3, P_4 and P_5:

$$P_1 = \begin{bmatrix} 3.48006722 & 0.42287465 \\ 0.42287465 & 3.72528667 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 10.78513153 & 1.63773992 \\ 1.63773992 & 9.60377259 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 27.12835962 & 5.93967313 \\ 5.93967313 & 24.97062138 \end{bmatrix}, P_4 = \begin{bmatrix} 62.63517376 & 10.22520071 \\ 10.22520071 & 53.7344441 \end{bmatrix},$$

$$P_5 = \begin{bmatrix} 109.29093751 & 5.18670927 \\ 5.18670927 & 112.97575136 \end{bmatrix}.$$

(ii)Yes, if N is large enough, P_N is approximately equal to a matrix P that solves DARE. We can let $N = 50$, thus

$$P_N = \begin{bmatrix} 3052.97899665 & -4621.98828315 \\ -4621.98828315 & 7588.73677777 \end{bmatrix}$$

A matrix P that solves DARE:

$$P = \begin{bmatrix} 3052.97899664 & -4621.98828313 \\ -4621.98828313 & 7588.73677774 \end{bmatrix}$$

(iii)The figure of $P_{N,1,1}$, $P_{N,1,2}$ and $P_{N,2,2}$ against N is shown below as Figure 3:

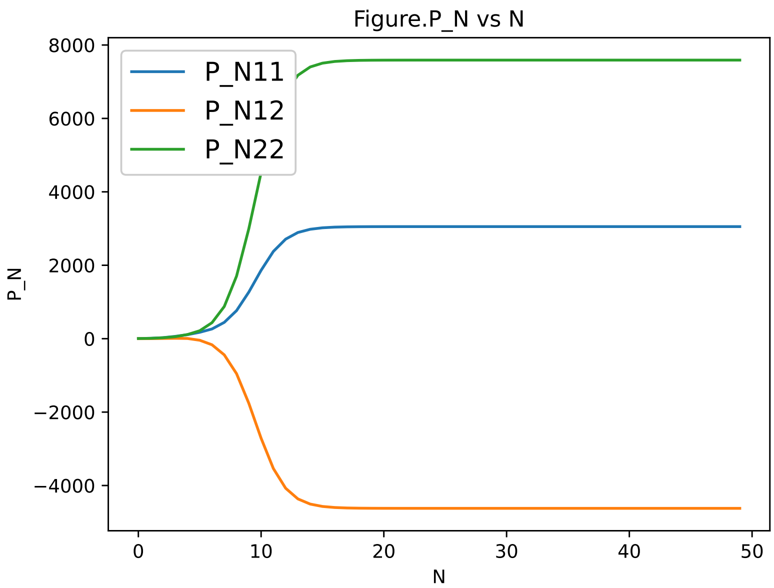


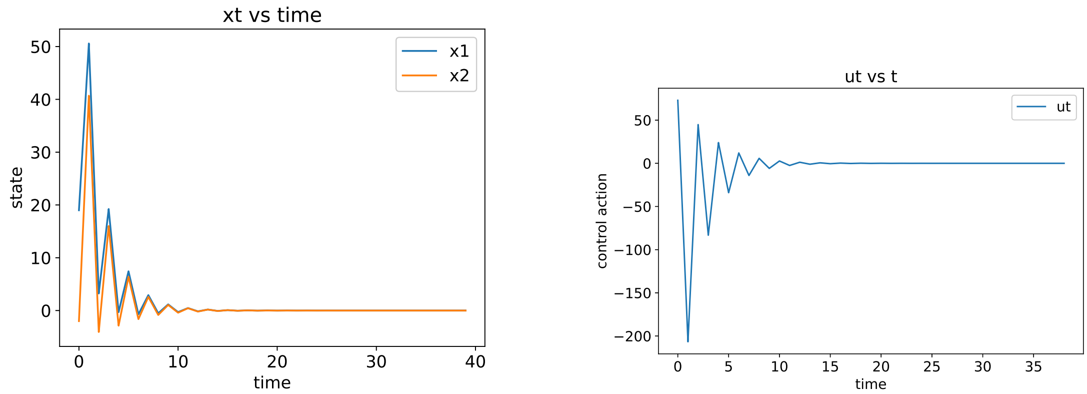
FIGURE 3. $P_{N,1,1}$, $P_{N,1,2}$ and $P_{N,2,2}$ against N

The Figure 3 shows that $P_{N,1,1}$, $P_{N,1,2}$ and $P_{N,2,2}$ converged to different values, and $P_{N,1,1}$ plus $P_{N,1,2}$ approximately equal to $P_{N,2,2}$.

Question 3 [Simulations of $\kappa^\star_\infty$-controlled system]. (i)The Python code is: Question 3. We can solve the discrete-time algebraic Riccati equation and determine an optimal stationary control law, $\kappa^\star_\infty = Kx$ (i.e., determine K). K is shown below:

$$K = \begin{bmatrix} 2.92322706 & -8.71230855 \end{bmatrix}$$

(ii) This paper chooses $x_0 = \begin{bmatrix}19 & -2\end{bmatrix}^\intercal$ as the initial state. The figure of the states against time is shown below as Figure 4(a) and the figure of the control actions against time is shown below as Figure 4(b):



(a) States against time (b) Control actions against time

FIGURE 4. States and control actions against time

(iii)The figure of $V^\star_\infty(x_t)$ against t is shown below as Figure 5:

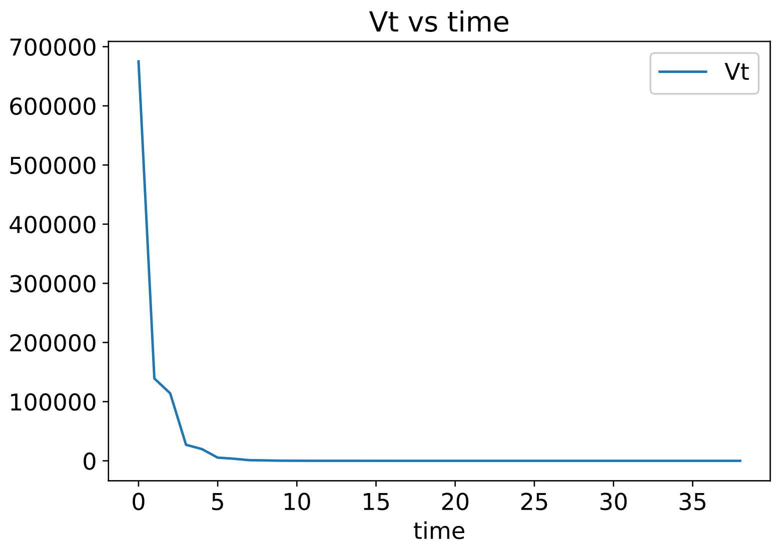
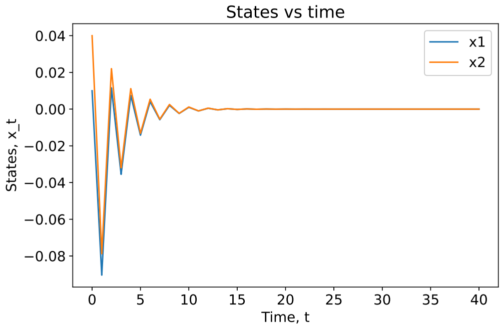
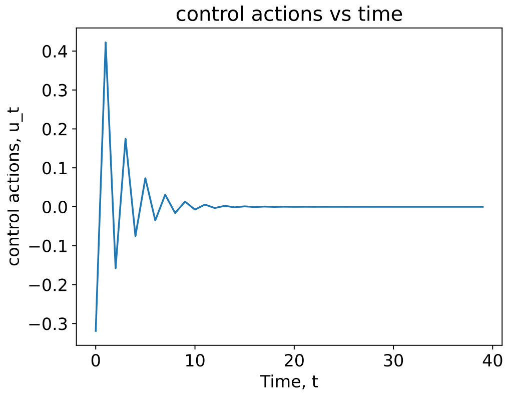


FIGURE 5. $V^\star_\infty(x_t)$ against t

Question 4 [MPC design and simulations]. (i)The Python code is: Question 4 i&ii. This paper chooses $x_0 = \begin{bmatrix}0.01 & 0.04\end{bmatrix}^\intercal$ as the initial state. The figure of states x_t against time for the MPC-controlled system is shown below as Figure 6(a) and the figure of control actions u_t against time for the MPC-controlled system is shown below as Figure 6(b):



(a) States against time



(b) Control actions against time

FIGURE 6. States and control actions against time

(ii) The figure of $V_N^\star(x)$ against time for the MPC-controlled system is shown below as Figure 7:

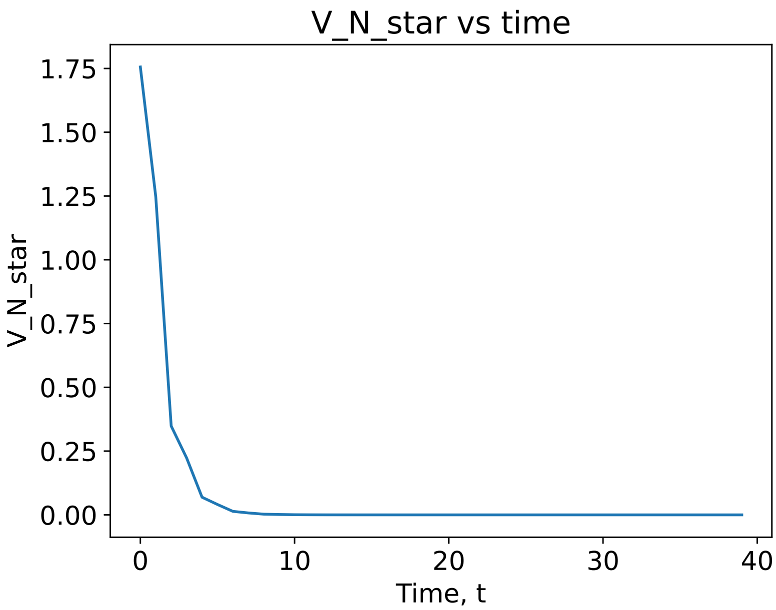


FIGURE 7. $V_N^\star(x_t)$ against t

(iii)The Python code is: Question 4 iii. The finite horizon optimal control problem:

$$\mathbb{P}_N(x) : \underset{x\in\mathbb{R}^n}{\text{Minimise}} \sum_{t=0}^{N-1} \tfrac{1}{2}\left(x_t^\intercal Q x_t + R u_t^2\right) + \tfrac{1}{2} x_N^\intercal P x_N, \tag{1a}$$

$$\text{subject to: } x_{t+1} = Ax_t + Bu_t, t \in \mathbb{N}_{[0,N-1]}, \tag{1b}$$

$$x^\intercal P x \leq \alpha, \tag{1c}$$

$$x_{\min} \leq x_t \leq x_{\max}, t \in \mathbb{N}_{[1,N]}, \tag{1d}$$

$$u_{\min} \leq u_t \leq u_{\max}, t \in \mathbb{N}_{[0,N-1]}, \tag{1e}$$

$$x_0 = x. \tag{1f}$$

We want to limit the magnitude of the successive changes of the state; in other words, we need to limit the velocity of the state. In particular, we want to impose the constraint:

$$\|x_{t+1} - x_t\|_\infty \leq 0.05$$

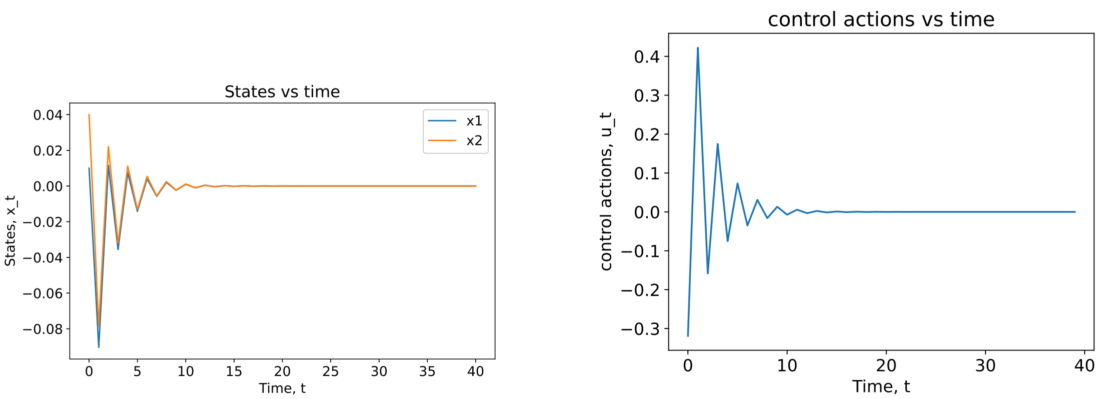
We can use the formula (Equation 1b): $x_{t+1} = Ax_t + Bu_t, t \in \mathbb{N}_{[0,N-1]}$, thus, we can get

$$\|(A - I)x_t + Bu_t\|_\infty \leq 0.05, t \in \mathbb{N}_{[0,N-1]}$$

Then we can modify the problem $\mathbb{P}_N(x)$ (Equation (1)) and update the constraints of the problem to achieve this. The finite horizon optimal control problem after modifying:

$$\begin{aligned}
\mathbb{P}_N(x) : \underset{x \in \mathbb{R}^n}{\text{Minimise}} & \sum_{t=0}^{N-1} \tfrac{1}{2}\left(x_t^\intercal Q x_t + R u_t^2\right) + \tfrac{1}{2} x_N^\intercal P x_N, && \text{(2a)} \\
\text{subject to: } & x_{t+1} = Ax_t + Bu_t, t \in \mathbb{N}_{[0,N-1]}, && \text{(2b)} \\
& \|(A - I)x_t + Bu_t\|_\infty \leq 0.05, t \in \mathbb{N}_{[0,N-1]}, && \text{(2c)} \\
& x_{\min} \leq x_t \leq x_{\max}, t \in \mathbb{N}_{[1,N]}, && \text{(2d)} \\
& u_{\min} \leq u_t \leq u_{\max}, t \in \mathbb{N}_{[0,N-1]}, && \text{(2e)} \\
& x_0 = x. && \text{(2f)}
\end{aligned}$$

Then, we can simulate it by Python. This paper chooses $x_0 = \begin{bmatrix} 0.01 & 0.04 \end{bmatrix}^\intercal$ as the initial state. The figure of states x_t against time is shown below as Figure 8(a) and the figure of control actions u_t against time for the MPC-controlled system is shown below as Figure 8(b):



(a) States against time (b) Control actions against time

FIGURE 8. States and control actions against time

The figure of $V_\infty^\star(x_t)$ against t is shown below as Figure 9:

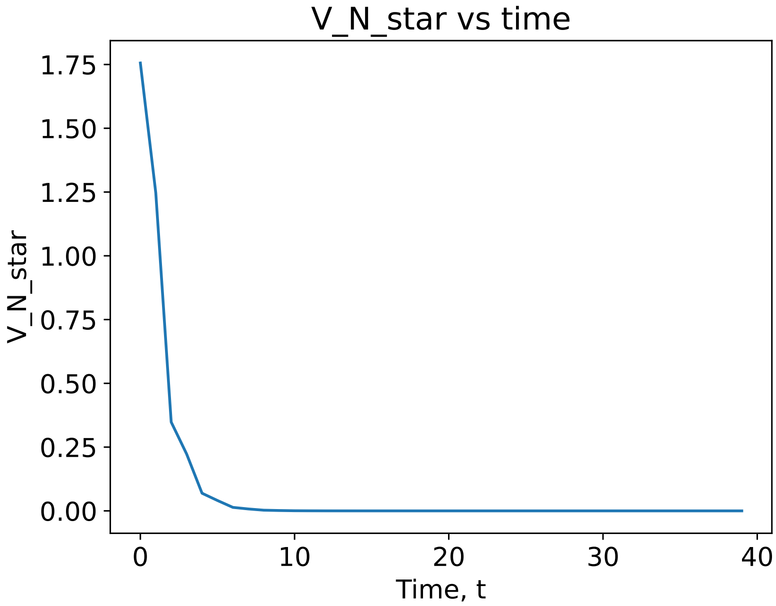


Figure 9. $V^{\star}_{\infty}(x_t)$ against t

References

[1] alim, "Realistic float value for "about zero"," https://stackoverflow.com/questions/28035240/realistic-float-value-for-about-zero, 2015.