Probability and Statistics (UCS410)

Experiment 4: Mathematical Expectation, Moments and Functions of Random Variables

Name: Ketan Singh Group: 3CO14 Roll no.: 102003349

---

1. The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given as

x	0	1	2	3	4
$p(x)$	0.41	0.37	0.16	0.05	0.01

Find the average number of imperfections per 10 meters of this fabric.

(Try functions **sum( )**, **weighted.mean( )**, c(a **%*%** b) to find expected value/mean.

```
#(1)
#using sum()
x = c(0,1,2,3,4)
y = c(0.41,0.37,0.16,0.05,0.01)

mean = sum(x*y)
mean

#using weighted.mean()
weighted.mean(x,y)

#using c(a %*% b)
mean(x%*%y)
```

Output:

```
> #(1)
> #using sum()
> x = c(0,1,2,3,4)
> y = c(0.41,0.37,0.16,0.05,0.01)
> mean = sum(x*y)
> mean
[1] 0.88
> #using weighted.mean()
> weighted.mean(x,y)
[1] 0.88
> #using c(a %*% b)
> mean(x%*%y)
[1] 0.88
```

2. The time T, in days, required for the completion of a contracted project is a random variable with probability density function $f(t) = 0.1\ e^{(-0.1t)}$ for $t > 0$ and 0 otherwise. Find the expected value of T.
Use function **integrate( )** to find the expected value of continuous random variable T.

```
#(2)
f<-function(t) {t*0.1*(exp(-0.1*t))}

EV = integrate(f,lower=0,upper=Inf)
EV
(EV$value)
```

Output:

```
> #(2)
> f<-function(t) {t*0.1*(exp(-0.1*t))}
> EV = integrate(f,lower=0,upper=Inf)
> EV
10 with absolute error < 6.7e-05
> (EV$value)
[1] 10
```

3. A bookstore purchases three copies of a book at \$6.00 each and sells them for \$12.00 each. Unsold copies are returned for \$2.00 each. Let X = {number of copies sold} and Y = {net revenue}. If the probability mass function of X is

x	0	1	2	3
$p(x)$	0.1	0.2	0.2	0.5

Find the expected value of Y.

```
#(3)
x = c(0,1,2,3)
probx = c(0.1,0.2,0.2,0.5)
expvalx = sum(x*probx)
expvaly=12*expvalx + 2*(3-expvalx) - 18
expvaly
```

Output:

```
> #(3)
> x = c(0,1,2,3)
> probx = c(0.1,0.2,0.2,0.5)
> expvalx = sum(x*probx)
> expvaly=12*expvalx + 2*(3-expvalx) - 18
> expvaly
[1] 9
```

4. Find the first and second moments about the origin of the random variable X with probability density function $f(x) = 0.5e^{-|x|}$, $1 < x < 10$ and 0 otherwise. Further use the results to find Mean and Variance.
(kth moment = $E(X^k)$, Mean = first moment and Variance = second moment – Mean^2.

```
#(4)
f<-function(x) {x*0.5*exp(-abs(x))}

first_moment = integrate(f,lower=1,upper=10)
first_moment
(first_moment$value)

f2<-function(x){x*x*0.5*exp(-abs(x))}

second_moment = integrate(f2,lower=1,upper=10)
second_moment
(second_moment$value)


mean = first_moment
mean
variance = second_moment$value - (first_moment$value)^2
variance
```

Output:

```
> #(4)
> f<-function(x) {x*0.5*exp(-abs(x))}
> first_moment = integrate(f,lower=1,upper=10)
> first_moment
0.3676297 with absolute error < 1.7e-14
> (first_moment$value)
[1] 0.3676297
> f2<-function(x){x*x*0.5*exp(-abs(x))}
> second_moment = integrate(f2,lower=1,upper=10)
> second_moment
0.9169292 with absolute error < 6e-13
> (second_moment$value)
[1] 0.9169292
> mean = first_moment
> mean
0.3676297 with absolute error < 1.7e-14
> variance = second_moment$value - (first_moment$value)^2
> variance
[1] 0.7817776
```

5. Let X be a geometric random variable with probability distribution

$$f(x) = \frac{3}{4}\left(\frac{1}{4}\right)^{x-1}, x = 1,2,3, \ldots$$

Write a function to find the probability distribution of the random variable Y = X^2 and find probability of Y for X = 3. Further, use it to find the expected value and variance of Y for X = 1,2,3,4,5.

```
#(5)
#prob of y=x^2 at x=3
f<-function(y) {(3/4)*(1/4)^(sqrt(y)-1)}
x = 3
y = x^2
proby = f(y)
proby

#prob distribution of y for x=1,2,3,4,5
x = c(1,2,3,4,5)
y = x^2
proby = f(y)
proby

#expected value of y (mean(y)) using sum() or weighted.mean()
expval = sum(y*proby)
expval

y=x^2
weighted.mean(y,proby)

#variance = E(y^2) - E(y)^2
z = y^2
Ey2 = sum(z*proby)
var = Ey2 - expval^2
var
```

Output:

```
> #(5)
> #prob of y=x^2 at x=3
> f<-function(y) {(3/4)*(1/4)^(sqrt(y)-1)}
> x = 3
> y = x^2
> proby = f(y)
> proby
[1] 0.046875
> #prob distribution of y for x=1,2,3,4,5
> x = c(1,2,3,4,5)
> y = x^2
> proby = f(y)
> proby
[1] 0.750000000 0.187500000 0.046875000 0.011718750 0.002929688
> #expected value of y (mean(y)) using sum() or weighted.mean()
> expval = sum(y*proby)
> expval
[1] 2.182617
> y=x^2
> weighted.mean(y,proby)
[1] 2.184751
> #variance = E(y^2) - E(y)^2
> z = y^2
> Ey2 = sum(z*proby)
> var = Ey2 - expval^2
> var
[1] 7.614112
```