

Probability and Statistics (UCS410)

Experiment 3: Probability distributions

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(1) Roll 12 dice simultaneously, and let X denote the number of 6's that appear. Calculate the probability of getting 7, 8 or 9, 6's using R. (Try using the function `pbinom`; If we set $S = \{\text{get a 6 on one roll}\}$, $P(S) = 1/6$ and the rolls constitute Bernoulli trials; thus $X \sim \text{binom}(\text{size}=12, \text{prob}=1/6)$ and we are looking for $P(7 \leq X \leq 9)$.

```
#using dbinom
sum(dbinom(7:9,12,1/6))
#using pbinom
sum(pbinom(9,12,1/6) - pbinom(6,12,1/6))
```

Output:

```
> sum(dbinom(7:9,12,1/6))
[1] 0.001291758
> sum(pbinom(9,12,1/6) - pbinom(6,12,1/6))
[1] 0.001291758
```

(2) Assume that the test scores of a college entrance exam fits a normal distribution. Furthermore, the mean test score is 72, and the standard deviation is 15.2. What is the percentage of students scoring 84 or more in the exam?

```
pnorm(84,72,15.2,lower.tail=TRUE)
```

Output:

```
> pnorm(84,72,15.2,lower.tail=TRUE)
[1] 0.7850824
```

(3) On the average, five cars arrive at a particular car wash every hour. Let X count the number of cars that arrive from 10AM to 11AM, then $X \sim \text{Poisson}(\lambda = 5)$. What is probability that no car arrives during this time. Next, suppose the car wash above is in operation from 8AM to 6PM, and we let Y be the number of customers that appear in this period. Since this period covers a total of 10 hours, we get that $Y \sim \text{Poisson}(\lambda = 5 \times 10 = 50)$. What is the probability that there are between 48 and 50 customers, inclusive?

Output:

```
dpois(0,5)
sum(dpois(48:50,50))

> dpois(0,5)
[1] 0.006737947
> sum(dpois(48:50,50))
[1] 0.1678485
```

(4) Suppose in a certain shipment of 250 Pentium processors there are 17 defective processors. A quality control consultant randomly collects 5 processors for inspection to determine whether or not they are defective. Let X denote the number of defectives in the sample. Find the probability of exactly 3 defectives in the sample, that is, find $P(X = 3)$.

Output:

```
# with replacement
dhyper(3,17,233,5)
# without replacement
dbinom(3,5,17/250)
```

```
> # with replacement
> dhyper(3,17,233,5)
[1] 0.002351153
> # without replacement
> dbinom(3,5,17/250)
[1] 0.002731232
```

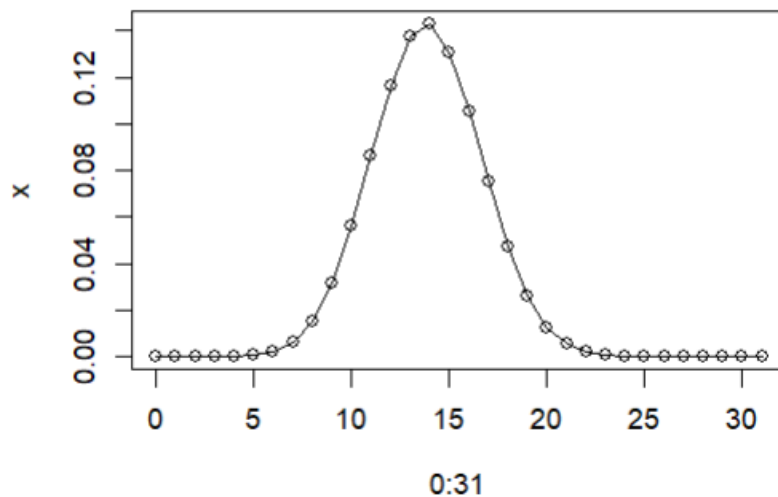
(5) A recent national study showed that approximately 44.7% of college students have used Wikipedia as a source in at least one of their term papers. Let X equal the number of students in a random sample of size $n = 31$ who have used Wikipedia as a source.

(a) How is X distributed?

```
#  $x \sim \text{binom}(\text{size}=31, \text{prob}=0.447)$ 
```

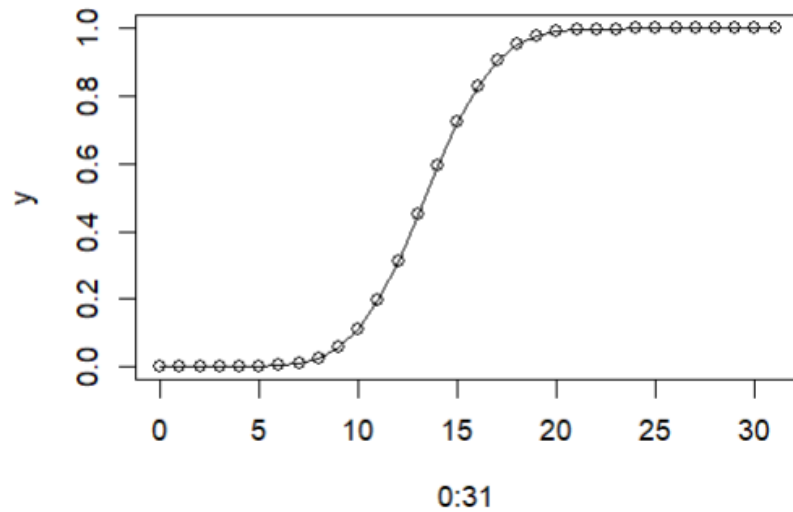
(b) Sketch the probability mass function.

```
x = dbinom(0:31,31,0.447)
plot(0:31,x,type='o')
```



(c) Sketch the cumulative distribution function.

```
y = pbinom(0:31,31,0.447)
plot(0:31,y,type='o')
```



(d) Find mean, variance and standard deviation of X.

```
x = dbinom(0:31,31,0.447)
mean(x)
var(x)
sd(x)

n=31          #size
p=0.447       #prob of success
mean = n*p    #mean
mean

q=1-0.447     #prob of failure
var = n*p*q
var

standard_dev = sqrt(var)
standard_dev
```

Output:

```
> x = dbinom(0:31,31,0.447)
> mean(x)
[1] 0.03125
> var(x)
[1] 0.002266289
> sd(x)
[1] 0.04760556
> n=31          #size
> p=0.447       #prob of success
> mean = n*p    #mean
> mean
[1] 13.857
> q=1-0.447     #prob of failure
> var = n*p*q
> var
[1] 7.662921
> standard_dev = sqrt(var)
> standard_dev
[1] 2.768198
```