Probability and Statistics (UCS410)

Experiment 7: Chi-square, t-distribution, F-distribution

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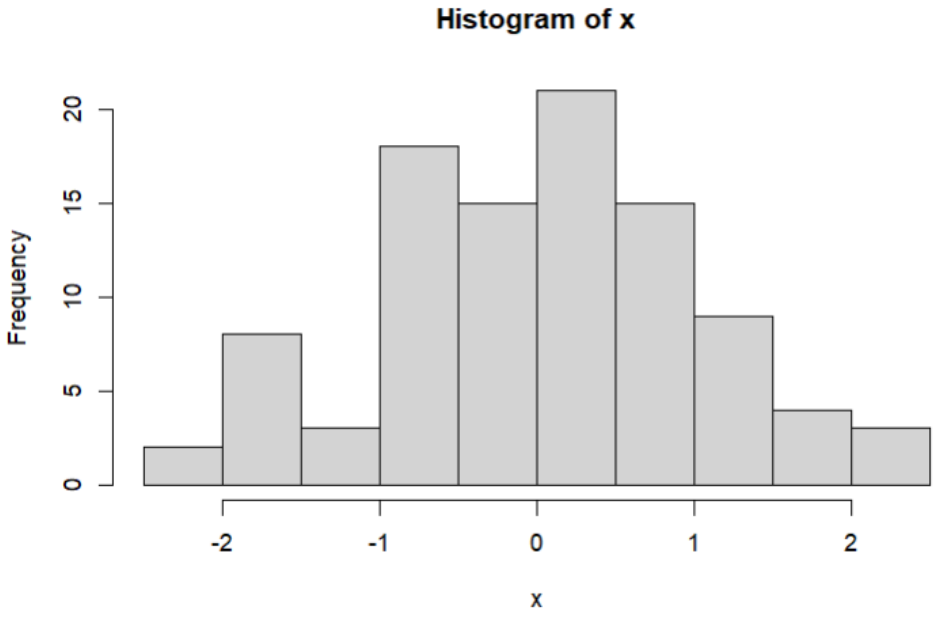
(1) Use the $rt(n, df)$ function in r to investigate the t-distribution for $n = 100$ and $df = n - 1$ and plot the histogram for the same.

```
#(1)
x = rt(n=100,df=n-1)
x
hist(x)
```

Output:

```
> #(1)
> x = rt(n=100,df=n-1)
Warning message:
In rt(n = 100, df = n - 1) : NAs produced
> x
  [1]         NaN -0.88467843 -0.84479725  1.33726118  0.14383657 -0.34891128 -0.15948434  0.43304197  1.29280689
 [10]  1.08974325  0.51408205 -1.60686342 -0.84815799  0.38291861 -0.17401391 -1.12488580  0.07534728 -0.33440563
 [19] -0.10132550 -0.26945476  1.03834697 -1.68866426  0.31119468 -1.56762952  1.11756140  2.08518084 -0.53954303
 [28]  1.16127769  2.29286106  0.79889889  1.55153255 -0.73502498 -0.39540494 -1.59179674  0.23810248 -1.76436204
 [37] -0.51881303 -0.68894062  0.20025897 -1.05754786 -0.87888343  0.83743585  0.52232154 -0.06533572  0.87260928
 [46]  1.61338936 -0.67653334  0.35005416  0.35737475  1.44808293         NaN  0.75962923  0.47979915  0.12582744
 [55]  1.24894557 -0.11951678  0.86600245 -1.65775637  0.35306076 -0.88213320 -1.58078929  0.53367020  0.06323351
 [64]  0.36715442 -1.73133068 -0.11266262 -0.91356781  0.75851614  0.37117999  0.21044151  1.70923250  0.03600394
 [73]  1.01236598 -0.94795295  0.75511077  0.57346071 -0.53051107  0.32426799  0.63292528 -0.03887271 -1.12331605
 [82]  0.41892142  1.69212546 -0.30837032 -0.80808569 -0.66158091 -0.39127932  0.68239001 -2.02074338  0.53443462
 [91] -0.57701247  0.05189164 -0.76762618  0.07891754  0.94218334 -0.85945619  2.44087453 -2.15468218 -0.19457176
[100] -0.40392021
> hist(x)
```



(2) Use the $rchisq(n, df)$ function in r to investigate the chi-square distribution with $n = 100$ and $df = 2, 10, 25$.

```
#(2)
x = rchisq(n=100,df=2)
x
x = rchisq(n=100,df=10)
x
x = rchisq(n=100,df=25)
x
```

Output:

```
> #(2)
> x = rchisq(n=100,df=2)
> x
  [1] 0.106165885 1.882849418 0.147872552 2.328270841 1.432636376 1.641834895 0.503127191 0.201730852 3.878903413
 [10] 2.537498570 3.507632730 3.196338862 1.834927000 3.463104232 3.802238841 0.427365625 1.825025603 0.868156221
 [19] 4.468084611 2.054057201 3.890005701 1.016618428 2.861813783 1.118483416 1.932291597 2.834248135 5.611577150
 [28] 3.219462001 1.198146013 1.229288237 0.543613856 0.247960072 1.943727730 7.190110090 0.335028369 6.328549053
 [37] 0.359184444 9.414786602 5.216274067 1.311690326 0.189256391 1.287720721 1.392410498 0.500957163 1.766091120
 [46] 0.311405105 0.882584986 3.918884500 0.284610813 0.245382986 2.915529477 0.002403691 0.932111040 0.421144977
 [55] 1.118412965 2.405620652 0.936471130 1.832623020 0.249078674 1.061837795 1.551489323 2.616378270 4.914556038
 [64] 4.444270531 0.452600977 1.250140473 1.397327519 0.502043711 0.911595859 0.969900637 0.080062542 2.714948112
 [73] 1.592133468 0.523359729 4.926280421 1.709387627 0.339356241 0.191489588 1.287844479 4.861074034 1.991061347
 [82] 0.574288591 1.615736280 1.417092270 0.270389325 0.528218379 0.849641461 1.367114955 0.690120583 0.094455818
 [91] 2.499554574 0.008724947 0.750035148 1.188569769 0.128153112 1.214062826 0.360087838 0.993208233 2.071710678
[100] 0.921174155
> x = rchisq(n=100,df=10)
> x
  [1]  8.907775 12.806975 10.774273  9.402924 16.743886 10.263010  7.227599  4.519767 12.602298  7.874086 10.420293
 [12]  9.110854  2.485444  6.234351 18.088850 10.263953 13.415233  9.969004  4.955717  6.320928  8.085852  5.293995
 [23]  4.144947  9.891102  5.552144  4.498565 12.500239 13.334018 17.800607  9.220176 11.534852  8.551305  7.333296
 [34] 10.124278  4.156235  5.785473 14.105456  7.139252  7.015440 10.845208  6.244189  7.739865  7.129666 12.863947
 [45] 12.962201 14.915603  1.635913  7.710264  4.437467 11.212289 26.946635 12.856883  4.880779  6.891079  9.869568
 [56] 13.811944  5.213436 18.215341  6.673661 11.070386 10.506336  5.628593  9.906277  9.109177 11.704816 10.163815
 [67]  3.270587  3.985603  7.220922  5.507172  7.347678  7.168388 15.032911 17.374231 15.150480 14.155009 14.030211
 [78] 10.269355 11.022561  6.333161  6.290615 21.106920 14.667813  9.287957  3.130596  8.867740 15.032434 15.040364
 [89]  7.859993 11.223466  4.640879  7.854259 16.718775 21.744477  3.813995  3.878632  7.852120  6.270131  7.681286
[100]  8.724575
> x = rchisq(n=100,df=25)
> x
  [1] 22.67155 26.58728 17.75044 31.23614 23.29508 21.01532 16.42759 30.71466 30.64281 28.27439 12.40468 21.66272
 [13] 12.50002 35.07397 24.41995 16.86407 35.92554 27.86095 20.82753 13.41164 25.29608 22.06885 11.93602 23.76201
 [25] 20.81730 28.97443 18.77827 26.77937 15.26819 19.00325 32.25472 19.04877 18.09255 28.59633 17.76720 31.53347
 [37] 40.67548 19.37350 30.22950 30.59823 15.69750 24.45892 24.29402 21.64494 40.37227 36.80992 21.45207 15.13358
 [49] 37.16978 18.60188 26.81138 26.13530 23.78470 23.74797 29.41393 17.23204 18.59963 19.79490 31.99483 31.93556
 [61] 25.17006 22.93391 25.62386 24.62787 24.20100 30.03161 19.37922 25.48948 27.92128 32.68406 19.33063 34.31264
 [73] 27.12872 26.84894 24.24013 23.14371 33.70851 27.65405 18.37443 30.92609 11.55105 25.60599 22.77706 27.53110
 [85] 24.25249 24.13926 26.10585 29.26695 15.69494 34.91190 35.05035 23.61479 24.86919 34.05768 43.27343 29.62954
 [97] 27.56651 33.79489 25.26320 25.96091
```

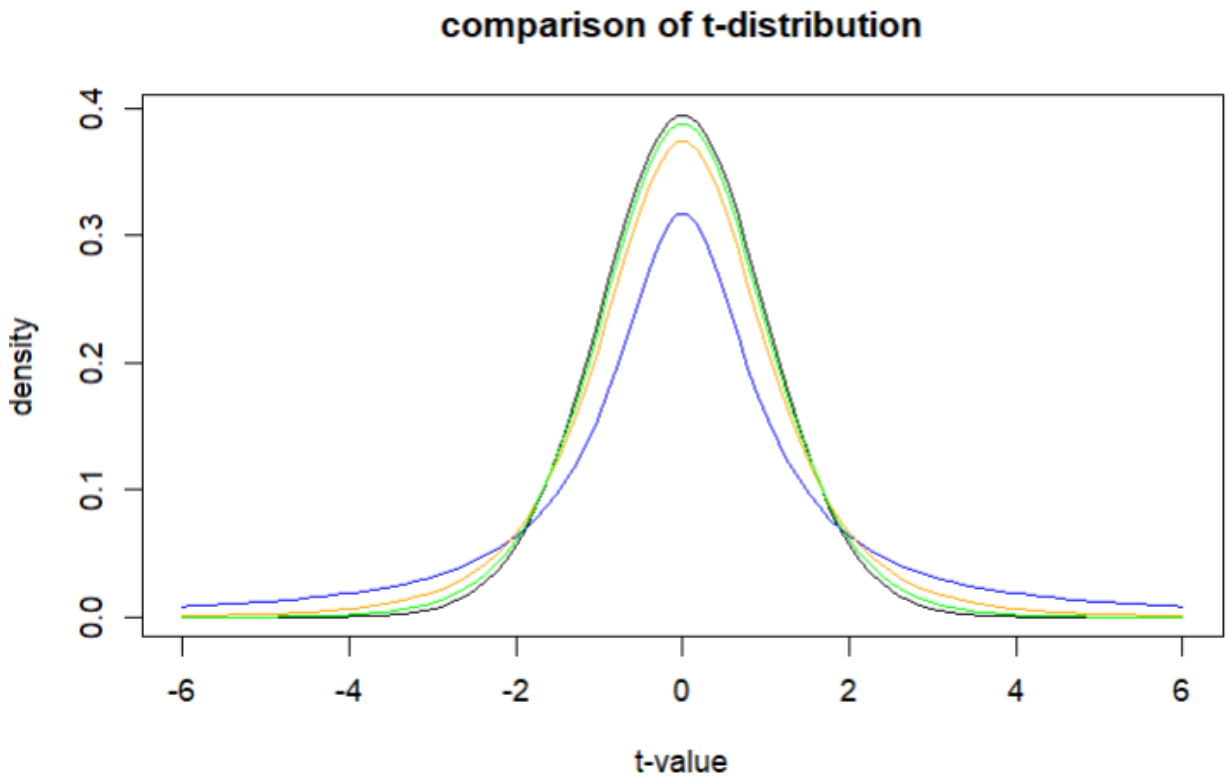
(3) Generate a vector of 100 values between -6 and 6. Use the $dt()$ function in r to find the values of a t-distribution given a random variable x and degrees of freedom 1,4,10,30. Using these values plot the density function for students t-distribution with degrees of freedom 30. Also shows a comparison of probability density functions having different degrees of freedom (1,4,10,30).

```r
#(3)
x=seq(-6,6,length=100)
x

df=c(1,4,10,30)
colour=c('blue','orange','green','black')

   #find the values of t-dist
dt(x,df[1])
dt(x,df[2])
dt(x,df[3])
dt(x,df[4])

plot(x,dt(x,df[4]),type='l',xlab='t-value',ylab='density',
     main='comparison of t-distribution',col=colour[4])
for(i in 1:3){
  lines(x,dt(x,df[i]),type='l',col=colour[i])
}
```

Output:

```
> #(3)
> x=seq(-6,6,length=100)
> x
  [1] -6.00000000 -5.87878788 -5.75757576 -5.63636364 -5.51515152 -5.39393939 -5.27272727 -5.15151515 -5.03030303
 [10] -4.90909091 -4.78787879 -4.66666667 -4.54545455 -4.42424242 -4.30303030 -4.18181818 -4.06060606 -3.93939394
 [19] -3.81818182 -3.69696970 -3.57575758 -3.45454545 -3.33333333 -3.21212121 -3.09090909 -2.96969697 -2.84848485
 [28] -2.72727273 -2.60606061 -2.48484848 -2.36363636 -2.24242424 -2.12121212 -2.00000000 -1.87878788 -1.75757576
 [37] -1.63636364 -1.51515152 -1.39393939 -1.27272727 -1.15151515 -1.03030303 -0.90909091 -0.78787879 -0.66666667
 [46] -0.54545455 -0.42424242 -0.30303030 -0.18181818 -0.06060606  0.06060606  0.18181818  0.30303030  0.42424242
 [55]  0.54545455  0.66666667  0.78787879  0.90909091  1.03030303  1.15151515  1.27272727  1.39393939  1.51515152
 [64]  1.63636364  1.75757576  1.87878788  2.00000000  2.12121212  2.24242424  2.36363636  2.48484848  2.60606061
 [73]  2.72727273  2.84848485  2.96969697  3.09090909  3.21212121  3.33333333  3.45454545  3.57575758  3.69696970
 [82]  3.81818182  3.93939394  4.06060606  4.18181818  4.30303030  4.42424242  4.54545455  4.66666667  4.78787879
 [91]  4.90909091  5.03030303  5.15151515  5.27272727  5.39393939  5.51515152  5.63636364  5.75757576  5.87878788
[100]  6.00000000
> df=c(1,4,10,30)
> colour=c('blue','orange','green','black')
>    #find the values of t-dist
> dt(x,df[1])
  [1] 0.008602970 0.008951310 0.009321021 0.009713870 0.010131806 0.010576983 0.011051792 0.011558887 0.012101221
 [10] 0.012682086 0.013305165 0.013974580 0.014694962 0.015471523 0.016310143 0.017217477 0.018201075 0.019269524
 [19] 0.020432624 0.021701588 0.023089287 0.024610541 0.026282468 0.028124906 0.030160921 0.032417419 0.034925891
 [28] 0.037723307 0.040853208 0.044367012 0.048325591 0.052801137 0.057879356 0.063661977 0.070269505 0.077844030
 [37] 0.086551677 0.096583858 0.108155840 0.121499988 0.136849375 0.154405107 0.174278263 0.196396298 0.220368383
 [46] 0.245321632 0.269758339 0.291538659 0.308123970 0.317144983 0.317144983 0.308123970 0.291538659 0.269758339
 [55] 0.245321632 0.220368383 0.196396298 0.174278263 0.154405107 0.136849375 0.121499988 0.108155840 0.096583858
 [64] 0.086551677 0.077844030 0.070269505 0.063661977 0.057879356 0.052801137 0.048325591 0.044367012 0.040853208
 [73] 0.037723307 0.034925891 0.032417419 0.030160921 0.028124906 0.026282468 0.024610541 0.023089287 0.021701588
 [82] 0.020432624 0.019269524 0.018201075 0.017217477 0.016310143 0.015471523 0.014694962 0.013974580 0.013305165
 [91] 0.012682086 0.012101221 0.011558887 0.011051792 0.010576983 0.010131806 0.009713870 0.009321021 0.008951310
[100] 0.008602970
> dt(x,df[2])
  [1] 0.001185854 0.001299674 0.001426572 0.001568291 0.001726840 0.001904535 0.002104055 0.002328498 0.002581463
 [10] 0.002867130 0.003190370 0.003556866 0.003973266 0.004447354 0.004988268 0.005606751 0.006315456 0.007129303
 [19] 0.008065920 0.009146149 0.010394664 0.011840692 0.013518866 0.015470216 0.017743327 0.020395643 0.023494940
 [28] 0.027120922 0.031366892 0.036341391 0.042169621 0.048994381 0.056976082 0.066291261 0.077128754 0.089682498
 [37] 0.104139687 0.120662946 0.139365306 0.160277437 0.183307807 0.208198657 0.234483644 0.261456453 0.288162552
 [46] 0.313426933 0.335927310 0.354313737 0.367362749 0.374140500 0.374140500 0.367362749 0.354313737 0.335927310
 [55] 0.313426933 0.288162552 0.261456453 0.234483644 0.208198657 0.183307807 0.160277437 0.139365306 0.120662946
 [64] 0.104139687 0.089682498 0.077128754 0.066291261 0.056976082 0.048994381 0.042169621 0.036341391 0.031366892
 [73] 0.027120922 0.023494940 0.020395643 0.017743327 0.015470216 0.013518866 0.011840692 0.010394664 0.009146149
 [82] 0.008065920 0.007129303 0.006315456 0.005606751 0.004988268 0.004447354 0.003973266 0.003556866 0.003190370
 [91] 0.002867130 0.002581463 0.002328498 0.002104055 0.001904535 0.001726840 0.001568291 0.001426572 0.001299674
[100] 0.001185854
```

```
> dt(x,df[3])
  [1] 8.808511e-05 1.049214e-04 1.252258e-04 1.497602e-04 1.794627e-04 2.154911e-04 2.592754e-04 3.125844e-04
  [9] 3.776092e-04 4.570665e-04 5.543283e-04 6.735831e-04 8.200373e-04 1.000165e-03 1.222017e-03 1.495608e-03
 [17] 1.833383e-03 2.250800e-03 2.767036e-03 3.405837e-03 4.196543e-03 5.175295e-03 6.386451e-03 7.884205e-03
 [25] 9.734397e-03 1.201647e-02 1.482550e-02 1.827413e-02 2.249422e-02 2.763790e-02 3.387746e-02 4.140377e-02
 [33] 5.042225e-02 6.114577e-02 7.378367e-02 8.852619e-02 1.055239e-01 1.248621e-01 1.465323e-01 1.704005e-01
 [41] 1.961789e-01 2.234026e-01 2.514189e-01 2.793936e-01 3.063382e-01 3.311623e-01 3.527460e-01 3.700297e-01
 [49] 3.821091e-01 3.883232e-01 3.883232e-01 3.821091e-01 3.700297e-01 3.527460e-01 3.311623e-01 3.063382e-01
 [57] 2.793936e-01 2.514189e-01 2.234026e-01 1.961789e-01 1.704005e-01 1.465323e-01 1.248621e-01 1.055239e-01
 [65] 8.852619e-02 7.378367e-02 6.114577e-02 5.042225e-02 4.140377e-02 3.387746e-02 2.763790e-02 2.249422e-02
 [73] 1.827413e-02 1.482550e-02 1.201647e-02 9.734397e-03 7.884205e-03 6.386451e-03 5.175295e-03 4.196543e-03
 [81] 3.405837e-03 2.767036e-03 2.250800e-03 1.833383e-03 1.495608e-03 1.222017e-03 1.000165e-03 8.200373e-04
 [89] 6.735831e-04 5.543283e-04 4.570665e-04 3.776092e-04 3.125844e-04 2.592754e-04 2.154911e-04 1.794627e-04
 [97] 1.497602e-04 1.252258e-04 1.049214e-04 8.808511e-05
> dt(x,df[4])
  [1] 1.948678e-06 2.742971e-06 3.862943e-06 5.442161e-06 7.668593e-06 1.080643e-05 1.522639e-05 2.144773e-05
  [9] 3.019610e-05 4.248311e-05 5.971486e-05 8.383942e-05 1.175458e-04 1.645301e-04 2.298498e-04 3.203887e-04
 [17] 4.454635e-04 6.176038e-04 8.535416e-04 1.175449e-03 1.612457e-03 2.202481e-03 2.994355e-03 4.050262e-03
 [25] 5.448382e-03 7.285618e-03 9.680204e-03 1.277386e-02 1.673306e-02 2.174888e-02 2.803476e-02 3.582149e-02
 [33] 4.534868e-02 5.685228e-02 7.054761e-02 8.660837e-02 1.051419e-01 1.261628e-01 1.495662e-01 1.751045e-01
 [41] 2.023705e-01 2.307906e-01 2.596315e-01 2.880217e-01 3.149896e-01 3.395167e-01 3.606011e-01 3.773274e-01
 [49] 3.889359e-01 3.948821e-01 3.948821e-01 3.889359e-01 3.773274e-01 3.606011e-01 3.395167e-01 3.149896e-01
 [57] 2.880217e-01 2.596315e-01 2.307906e-01 2.023705e-01 1.751045e-01 1.495662e-01 1.261628e-01 1.051419e-01
 [65] 8.660837e-02 7.054761e-02 5.685228e-02 4.534868e-02 3.582149e-02 2.803476e-02 2.174888e-02 1.673306e-02
 [73] 1.277386e-02 9.680204e-03 7.285618e-03 5.448382e-03 4.050262e-03 2.994355e-03 2.202481e-03 1.612457e-03
 [81] 1.175449e-03 8.535416e-04 6.176038e-04 4.454635e-04 3.203887e-04 2.298498e-04 1.645301e-04 1.175458e-04
 [89] 8.383942e-05 5.971486e-05 4.248311e-05 3.019610e-05 2.144773e-05 1.522639e-05 1.080643e-05 7.668593e-06
 [97] 5.442161e-06 3.862943e-06 2.742971e-06 1.948678e-06
> plot(x,dt(x,df[4]),type='l',xlab='t-value',ylab='density',
+      main='comparison of t-distribution',col=colour[4])
> for(i in 1:3){
+   lines(x,dt(x,df[i]),type='l',col=colour[i])
+ }
```



(4) Write a r-code

(i) To find the 95^{th} percentile of the F-distribution with $(10, 20)$ degrees of freedom.

(ii) To calculate the area under the curve for the interval $[0, 1.5]$ and the interval $[1.5, +\infty)$ of a F-curve with $v_1 = 10$ and $v_2 = 20$ (USE $pf()$).

(iii) To calculate the quantile for a given area (= probability) under the curve for a F-curve with $v_1 = 10$ and $v_2 = 20$ that corresponds to $q = 0.25, 0.5, 0.75$ and 0.999. (use the $qf()$)

(iv) To generate 1000 random values from the F-distribution with $v_1 = 10$ and $v_2 = 20$ (use $rf()$)and plot a histogram.

Output:

```
#(4)
#(i)
qf(1-0.05,10,20)

#(ii)
#for [0,1.5]
x=1.5
pf(x,df1=10,df2=20,lower.tail=TRUE)
#for [1.5,inf]
pf(x,df1=10,df2=20,lower.tail=FALSE)

#(iii)
q=c(0.25,0.5,0.75,0.999)
qf(q,df1=10,df2=20)

#(iv)
x = rf(n=1000,df1=10,df2=20)
hist(x)
```

```
> #(4)
> #(i)
> qf(1-0.05,10,20)
[1] 2.347878
> #(ii)
> #for [0,1.5]
> x=1.5
> pf(x,df1=10,df2=20,lower.tail=TRUE)
[1] 0.7890535
> #for [1.5,inf]
> pf(x,df1=10,df2=20,lower.tail=FALSE)
[1] 0.2109465
> #(iii)
> q=c(0.25,0.5,0.75,0.999)
> qf(q,df1=10,df2=20)
[1] 0.6563936 0.9662639 1.3994874 5.0752462
> #(iv)
> x = rf(n=1000,df1=10,df2=20)
> hist(x)
```