Probability and Statistics (UCS410)

Experiment 6: Joint probability mass and density functions

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(1) The joint probability density of two random variables X and Y is

$$f(x,y) = \begin{cases} 2(2x+3y)/5; & 0 \leq x, y \leq 1 \\ 0; & elsewhere \end{cases}$$

Then write a R-code to

(i) check that it is a joint density function or not? (Use integral2())

(ii) find marginal distribution $g(x)$ at $x = 1$.

(iii) find the marginal distribution $h(y)$ at $y = 0$.

(iv) find the expected value of $g(x, y) = xy$.

```
#(1)
#(i)
f<-function(x,y) {2*(2*x + 3*y)/5}
install.packages("pracma")
library("pracma")
I<-integral2(f, xmin=0, xmax=1, ymin=0, ymax=1)
I

#(ii)
f1 <- function(y) {f(1,y)}
I<-integrate(f1,0,1)
I

#(iii)
f2 <- function(x) {(f(x,0))}
I<-integrate(f2,0,1)
I

#(iv)
f<-function(x,y) {x*y*2*(2*x + 3*y)/5}
I<-integral2(f, xmin=0, xmax=1, ymin=0, ymax=1)
I
```

Output:

```
> #(1)
> #(i)
> f<-function(x,y) {2*(2*x + 3*y)/5}
> library("pracma")
> I<-integral2(f, xmin=0, xmax=1, ymin=0, ymax=1)
> I
$Q
[1] 1

$error
[1] 6.938894e-17

> #(ii)
> f1 <- function(y) {f(1,y)}
> I<-integrate(f1,0,1)
> I
1.4 with absolute error < 1.6e-14
> #(iii)
> f2 <- function(x) {(f(x,0))}
> I<-integrate(f2,0,1)
> I
0.4 with absolute error < 4.4e-15
> #(iv)
> f<-function(x,y) {x*y*2*(2*x + 3*y)/5}
> I<-integral2(f, xmin=0, xmax=1, ymin=0, ymax=1)
> I
$Q
[1] 0.3333333

$error
[1] 5.89806e-17
```

(2) The joint probability mass function of two random variables X and Y is

$$f(x,y) = \{(x+y)/30; \;\; x = 0,1,2,3; \;\; y = 0,1,2\}$$

Then write a R-code to

(i) display the joint mass function in rectangular (matrix) form.

(ii) check that it is joint mass function or not? (use: Sum())

(iii) find the marginal distribution $g(x)$ for $x = 0,1,2,3$. (Use:apply())

(iv) find the marginal distribution $h(y)$ for $y = 0,1,2$. (Use:apply())

(v) find the conditional probability at $x = 0$ given $y = 1$.

(vi) find $E(x), E(y), E(xy), Var(x), Var(y), Cov(x,y)$ and its correlation coefficient.

```r
#(2)
#(i)
f<-function(x,y) {(x+y)/30}
m<-matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)) , nrow=4, ncol=3, byrow=TRUE)
print(m)

#(ii)
sum(m)

#(iii)
g_x = apply(m,1,sum)
g_x

#(iv)
h_y = apply(m,2,sum)
h_y

#(v)
t=m[1,2]/h_y[2]
t

#(vi)
x<-c(0:3)
Ex = sum(x*g_x)
Ex

y<-(0:2)
Ey = sum(y*h_y)
Ey

f<-function(x,y) {x*y*(x+y)/30}
m<-matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)) , nrow=4, ncol=3, byrow=TRUE)
Exy = sum(m)
Exy

E_x2 = sum((x^2)*g_x)
var_x = E_x2 - (Ex^2)
var_x

E_y2 = sum((y^2)*h_y)
var_y = E_y2 - (Ey^2)
var_y

cov_xy = Exy - Ex*Ey
cov_xy

corr_coeff = cov_xy/sqrt(var_x*var_y)
corr_coeff
```

Output:

```
> print(m)
           [,1]       [,2]       [,3]
[1,] 0.00000000 0.03333333 0.06666667
[2,] 0.03333333 0.06666667 0.10000000
[3,] 0.06666667 0.10000000 0.13333333
[4,] 0.10000000 0.13333333 0.16666667
> #(ii)
> sum(m)
[1] 1
> #(iii)
> g_x = apply(m,1,sum)
> g_x
[1] 0.1 0.2 0.3 0.4
> #(iv)
> h_y = apply(m,2,sum)
> h_y
[1] 0.2000000 0.3333333 0.4666667
> #(v)
> t=m[1,2]/h_y[2]
> t
[1] 0.1
> #(vi)
> x<-c(0:3)
> Ex = sum(x*g_x)
> Ex
[1] 2
> y<-(0:2)
> Ey = sum(y*h_y)
> Ey
[1] 1.266667
> f<-function(x,y) {x*y*(x+y)/30}
> m<-matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)) , nrow=4, ncol=3, byrow=TRUE)
> Exy = sum(m)
> Exy
[1] 2.4
> E_x2 = sum((x^2)*g_x)
> var_x = E_x2 - (Ex^2)
> var_x
[1] 1
> E_y2 = sum((y^2)*h_y)
> var_y = E_y2 - (Ey^2)
> var_y
[1] 0.5955556
> cov_xy = Exy - Ex*Ey
> cov_xy
[1] -0.1333333
> corr_coeff = cov_xy/sqrt(var_x*var_y)
> corr_coeff
[1] -0.1727737
```