ISE - 364/464: Introduction to Machine Learning
Homework Assignment 1

The goal of this assignment is to provide a series of problems that strengthen skills in critical thinking, linear algebra, calculus, and Python.

Grading: This assignment is due on Coursesite by E.O.D. 09/20/2024. All problems are worth the same number of points. If a problem has multiple parts, each of those parts will be worth equal amounts and will sum to the total number of points of the original problem (Example: If each problem is worth a single point, and problem 1 has 4 parts, each part will be worth 1/4th of a point). ISE - 364 students are only required to answer problems 1 through 4; however, you are allowed to answer the 5th graduate-level question (if done so correctly, you will receive extra credit in the amount that the 5th problem will be worth for the ISE - 464 students). ISE - 464 students are required to answer all 5 problems.

Submitting: Only electronic submissions on Coursesite are accepted.

1 Problems

1. **a)** What is the use of employing skepticism (in the form of Feynman's Method or Descartes' slate-clearing doubt)? Explain what the goal of such a procedure is and explain what it is not.

b) Suppose that one accepts the semantic content (the actual idea expressed by a statement, ignoring the language used to communicate it) of the axiomatic statement "I think; therefore, I am". List at least two other "axioms" that are implied by this simple statement alone (p.s., there are more than two that you could list).

c) Give an example of a justified, true belief that would not necessarily be considered to be knowledge.

d) Explain the difference between soundness and completeness and why they are important properties to have for a formal system.

e) Is Logicism a valid or invalid belief? Thoroughly explain your reasoning.
(Hint: Explain why you either agree or disagree with the proposition "Mathematics can ultimately be reduced to logic")

f) Explain what The Problem of Induction is. How does this connect to the Scientific Method?

2. Compute the gradient $\nabla f(x)$ and the Hessian $\nabla^2 f(x)$ of the following functions.
(Hint: Consult the reference material on taking derivatives with respect to vectors and matrices if need be. Page 8 of the matrix cookbook may also be useful)

a) $f(x) = \frac{1}{2}x^\top Hx$, where $x \in \mathbb{R}^n$ and $H \in \mathbb{R}^{n\times n}$. What if H is symmetric?

b) $f(x) = \|x\|_2$, where $x \in \mathbb{R}^n$ and $\|\cdot\|_2$ denotes the euclidean 2-norm.

3. Derive the partial derivative with respect to μ of the natural logarithm (ln) of the n-dimensional Gaussian PDF (probability density function) parameterized with the mean vector $\mu \in \mathbb{R}^n$ and covariance matrix $\Sigma \in \mathbb{R}^{n\times n}$. More specifically, given that $x \in \mathbb{R}^n$ is a random vector with a multivariate Gaussian PDF, written as $x \sim \mathcal{N}(x; \mu, \Sigma)$, derive the expression for $\frac{\partial}{\partial\mu} \ln\left[\mathcal{N}(x; \mu, \Sigma)\right]$.
(Hint 1: Use the expression that you obtained for the natural logarithm of the multivariate Gaussian PDF in problem 1 of the "Intermediate Calculus Section" of the Placement Exam as a starting point, then simply apply the rules of differentiation. Consult the reference material on taking derivatives with respect to vectors and matrices if need be. Page 8 of the matrix cookbook may also be useful)
(Hint 2: Use the fact that the inverse of an invertable symmetric matrix is also symmetric, i.e., $\left(\Sigma^{-1}\right)^\top = \Sigma^{-1}$. Notice that you should have proven this fact in problem 2 of the "Intermediate Linear Algebra Section" of the Placement Exam)

4. Finish and submit the following five coding review exercises (as Jupyter notebooks) that have been posted on CourseSite.

a) *PythonExercise.ipynb.*

b) *NumpyExercise.ipynb.*

c) *PandasExcercise.ipynb.*

d) *MatplotlibExercise.ipynb.*

e) *SeabornExercise.ipynb.*

5. **(ISE-464 Graduate Students)** Let $x \in \mathbb{R}^n$. Given that $\|\cdot\|_p$ denotes the p-norm, prove that $\|x\|_2 \le \|x\|_1$.
(Hint: I would suggest using a proof by induction over $\forall i \in \{1, 2, ..., n\}$)